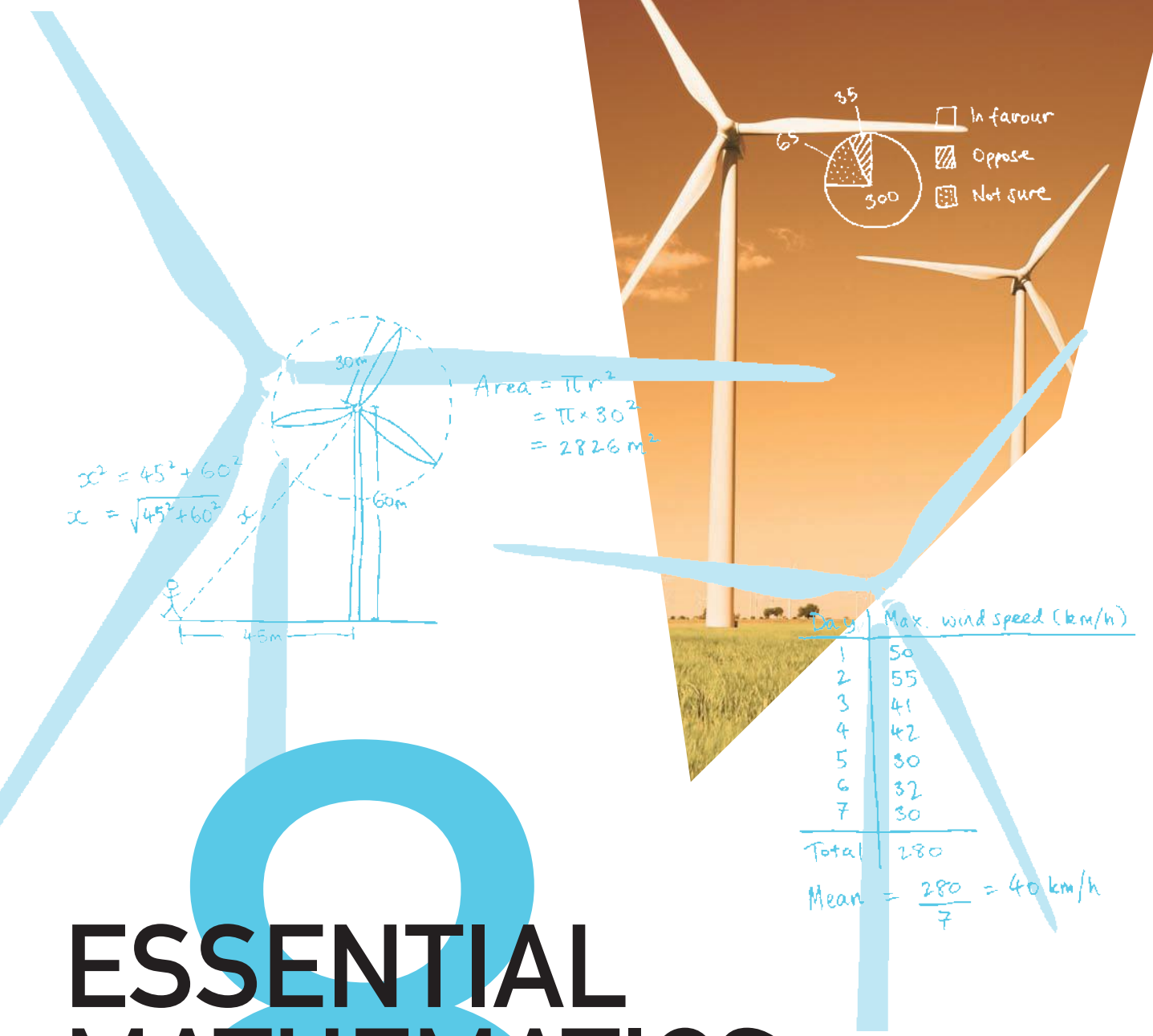


ESSENTIAL MATHEMATICS

FOR THE VICTORIAN CURRICULUM

8

DAVID GREENWOOD | BRYN HUMBERSTONE
JUSTIN ROBINSON | JENNY GOODMAN
JENNIFER VAUGHAN | STUART PALMER

CAMBRIDGE UNIVERSITY PRESS

Essential Mathematics for the Victorian Curriculum Year 8

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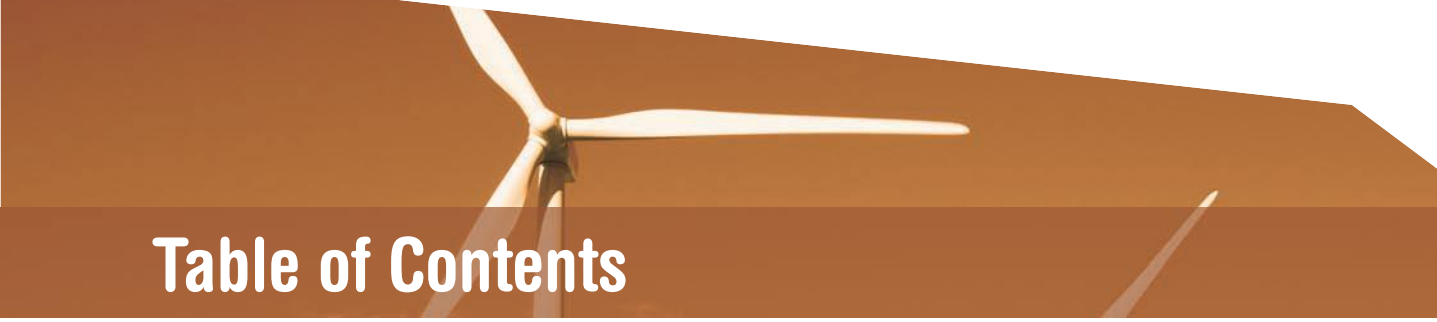


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About the Authors



David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has over 21 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.



Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, is currently teaching both junior and senior mathematics in Victoria and is the Head of Mathematics (Senior School) at Caulfield Grammar School Wheelers Hill. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.



Justin Robinson is the inaugural Director of the Institute of Positive Education, based at Geelong Grammar School, where he leads a team of staff dedicated to promoting the theory and practice of Positive Education. Prior to this, he spent 20 years teaching mathematics, covering all levels of secondary education and with significant experience teaching VCE, IB and A-Levels. Justin is passionate about challenging students within a safe learning environment and instils in his students a growth mindset. He has a keen interest in promoting student engagement through a wide variety of effective teaching methods and materials.



Jenny Goodman has worked for 20 years in comprehensive State and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.



Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.



Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and The University of Western Sydney.

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Introduction

Essential Mathematics for the Victorian Curriculum has been published to the new Victorian Curriculum, which schools in Victoria will implement from December 2016.

The Victorian Curriculum series includes additional content to cover new content descriptions and elaborations not found in the Australian Curriculum.

In particular, a new chapter on Algorithmic Thinking has been added to each book in the series. Instead of exercises and worked examples, this chapter contains a range of activities that show how algorithms and programming can be used as powerful tools for solving mathematical problems across all three curriculum content strands (Number and Algebra, Measurement and Geometry, Statistics and Probability). The activities utilise a range of readily-available technology, can be completed at any time during the year, and assume no prior knowledge of algorithmics.

Other features of the *Essential Mathematics for the Victorian Curriculum* series:

- New topics, worked examples and questions provide full coverage of the Victorian Curriculum.
- Victorian Curriculum content carried across from the Australian Curriculum has been further refined and improved.
- Revision and extension topics are marked as 'Consolidating' or 'Extending' to help customise the course to each classroom's needs.
- The working programs are subtly embedded in each exercise to differentiate three student pathways: Foundation, Standard and Advanced.
- A 'progress quiz' appears approximately two-thirds of the way into each chapter, allowing students to check and consolidate their learning – in time to address misunderstandings or weaknesses prior to completing the chapter.
- Pre-tests have been revised and are located in the Interactive Textbook.

Features of the all-new Interactive Textbook:

- Seamlessly blended with Cambridge HOTmaths, allowing enhanced learning opportunities in blended classrooms, revision of previous years' work, and access to *Scorcher* and the HOTmaths games library
- Every worked example in the book is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'
- Hundreds of interactive widgets and walkthroughs
- Built-in scientific calculator
- Searchable dictionary of mathematical terms and pop-up definitions in the text
- Automatically-marked quizzes and assessment tests, with saved scores
- Hundreds of printable worksheets (HOTsheets) suitable for homework or class group work.

Features of the Online Teaching Suite, also powered by Cambridge HOTmaths:

- Test generator, with ready-made tests
- Printable worked solutions for all questions
- Powerful learning management system with task-setting, progress-tracking and reporting functions.

The chart on the next pages shows how the components of this resource are integrated.

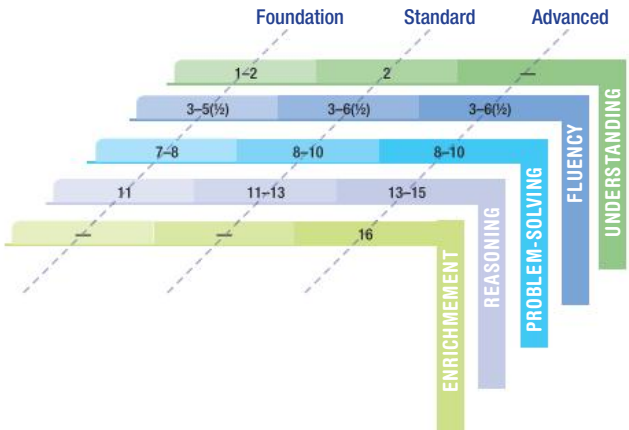
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Guide to the working programs

Essential Mathematics for the Victorian Curriculum contains working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Foundation, Standard and Advanced students.

Each exercise is structured in subsections that match the four Australian Curriculum proficiency strands (Understanding, Fluency, Problem-solving and Reasoning) as well as Enrichment (Challenge). The questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest shaded colour) is the Foundation pathway
- The middle column (medium shaded colour) is the Standard pathway
- The right column (darkest shaded colour) is the Advanced pathway.



Gradients within exercises and proficiency strands

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Understanding through to Reasoning and Enrichment – but also within each proficiency strand; the first few questions in Fluency, for example, are easier than the last few, and the last Problem-solving question is more challenging than the first Problem-solving question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Foundation pathway will likely need more practice at Understanding and Fluency, but should also attempt the easier Problem-solving and Reasoning questions. An Advanced student will likely be able to skip the Understanding questions, proceed through the Fluency questions (often half of each question), focus on the Problem-solving and Reasoning questions, and attempt the Enrichment question. A Standard student would do a mix of everything.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them.

If required, the chapter pre-tests (now found online) can also be used as a diagnostic tool. The following are recommended guidelines:

- A student who gets 40% or lower in the pre-test should complete the Foundation questions
- A student who gets between 40% and 85% in the pre-test should complete the Standard questions
- A student who gets 85% or higher in the pre-test should complete the Advanced questions.

For schools that have classes grouped according to ability, teachers may wish to set one of the Foundation, Standard or Advanced pathways as their default setting for their entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : do not complete any of the questions in this section.

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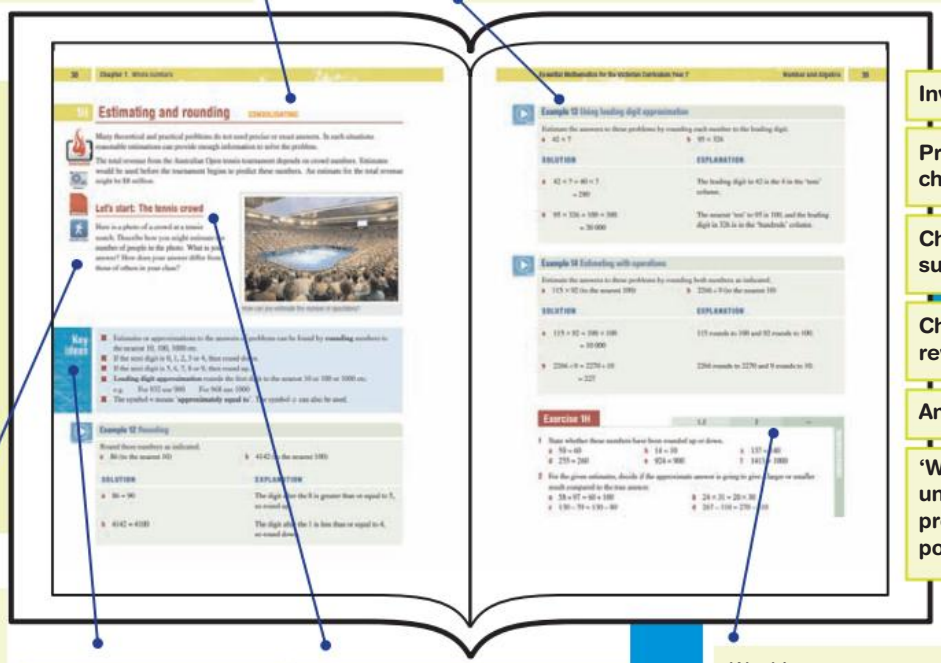
An overview of the *Essential Mathematics for the Victorian Curriculum* complete learning suite

For more detail, see the guide in the online Interactive Textbook

All non-core topics are marked Consolidating or Extending to assist with course planning

Examples with fully worked solutions and explanations show the thinking behind each step

PRINT TEXTBOOK



- Investigations
- Problems and challenges
- Chapter summaries
- Chapter reviews
- Answers
- 'Working with unfamiliar problems' poster

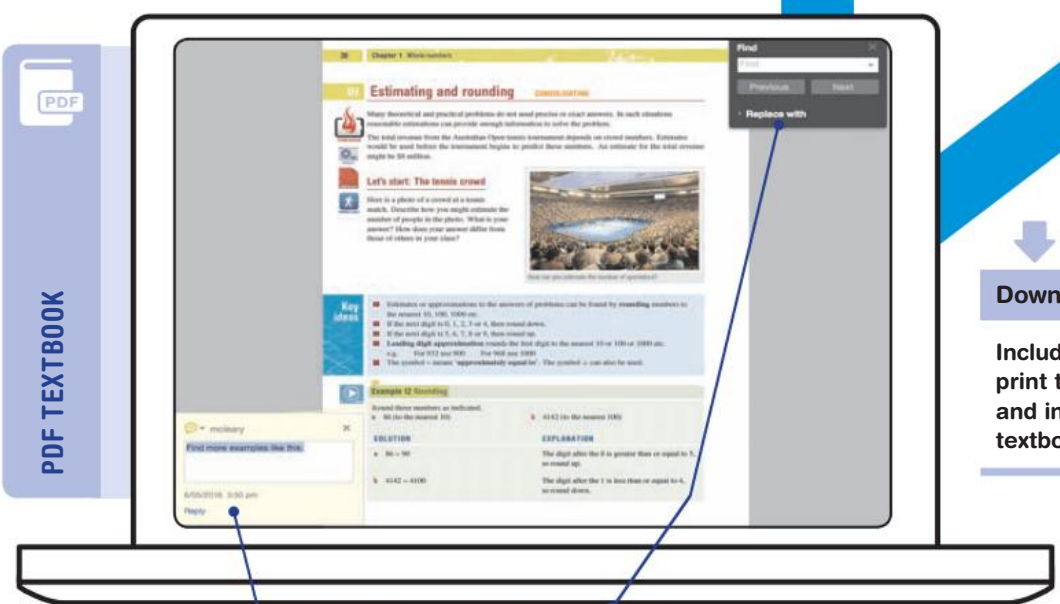
Each topic in the print book comes with interactive *HOTmaths* widgets, walkthroughs and HOTsheets in the interactive textbook

Key ideas summarise key knowledge and skills for each lesson

Let's start activities get students thinking critically and creatively about the topic

Working programs subtly embedded in each Proficiency Strand to provide three learning pathways through the book

PDF TEXTBOOK



Downloadable

Included with print textbook and interactive textbook

Note-taking

Search functions

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Easy navigation within sections without scrolling

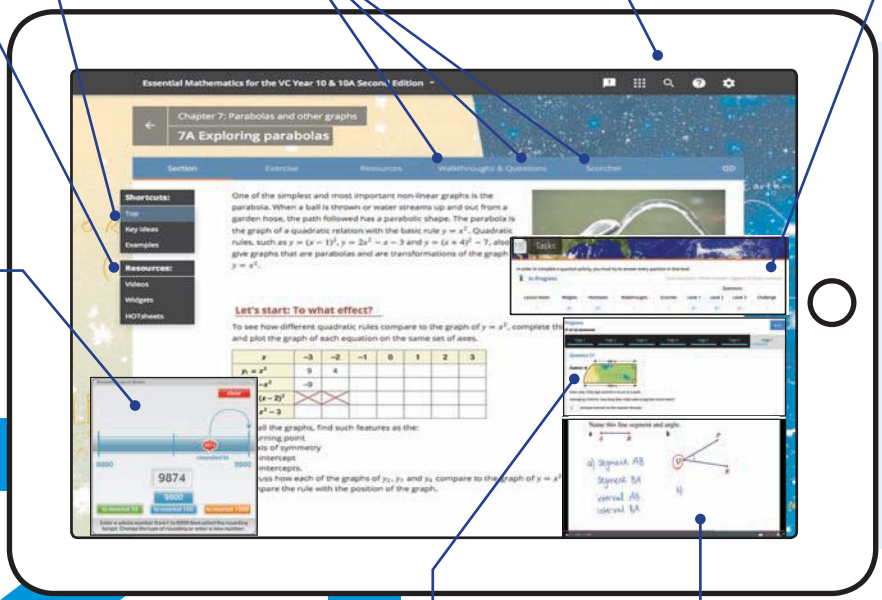
Walkthroughs, Scorcher and levelled question sets in every section

Interactive navigation and searching

Tasks sent by teacher

Access to interactive resources at any time without leaving the page

Hundreds of interactive widgets



INTERACTIVE TEXTBOOK POWERED BY *HOTmaths*

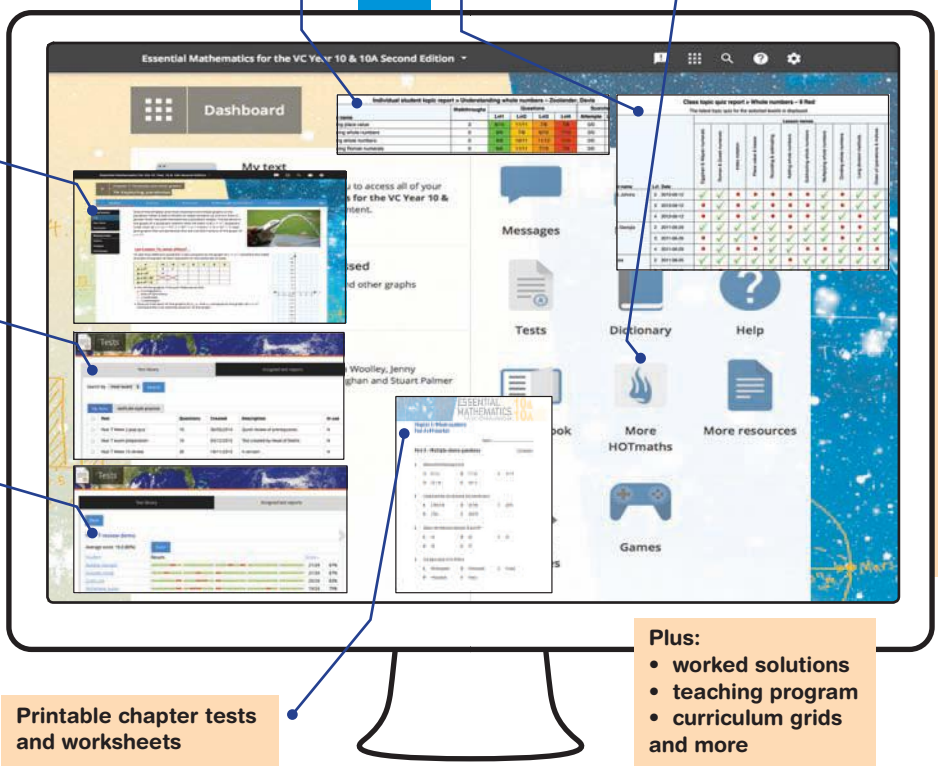
Online tests sent by teacher

Video demonstration for every worked example

Student reporting

Class reporting

Access to all *HOTmaths* Australian Curriculum courses



ONLINE TEACHING SUITE POWERED BY *HOTmaths*

Teacher's copy of interactive textbook

Test generator and ready-made tests

Student results

Printable chapter tests and worksheets

Plus:
- worked solutions
- teaching program
- curriculum grids

and more

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Working with unfamiliar problems: Part 1

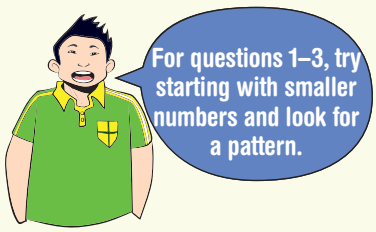
The questions on the next four pages are designed to provide practice in solving unfamiliar problems. Use the 'Working with unfamiliar problems' poster at the back of this book to help you if you get stuck.

In Part 1, apply the suggested strategy to solve these problems, which are in no particular order. Clearly communicate your solution and final answer.

1 Find the sum of the first 100 counting numbers.

2 If you start with 10 and then add half of it and then keep adding half of the previous number, what total do you achieve?

3 Find the total of $9 + 99 + 999 + 999 + 9999 + \ldots + 999999999$



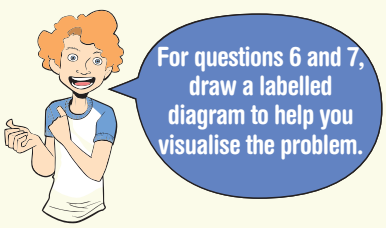
4 In the Melbourne Cup there are 24 horses that enter the race. If you select 3 horses that you think will win, then a Box Trifecta allows your selections to finish first, second and third in any order. How many mystery Box Trifectas are possible in the Melbourne Cup?

5 Arrange all the digits 1, 2, 3, 4 and 5 into the form $\square\square\square \times \square\square$ so that the three digit number multiplied by the two digit number gives the smallest possible answer.

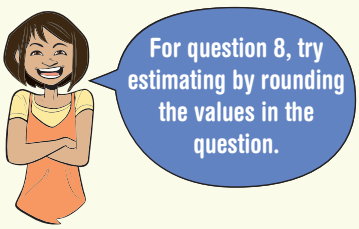


6 Arrange the first 16 counting numbers in a four by four magic square with a magic number of 34. What do the four numbers in the middle of the magic square add to?

7 A rectangle is divided up into 3 rows of 8 small squares (so 24 identical squares in total). If the rectangle has perimeter 55 cm, find the perimeter and area of each small square.

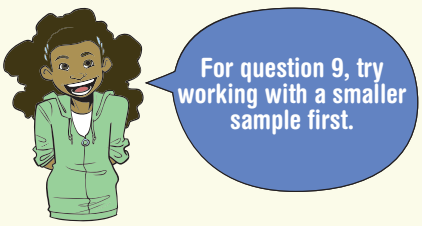


8 What is the best integer estimate for $6756.893 \div \frac{4}{3}$?



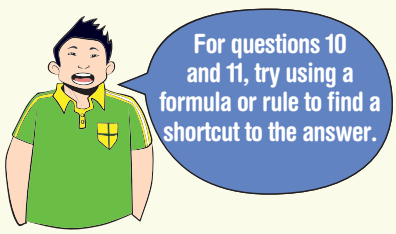
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9 How many people in your school have used the canteen in the last week?

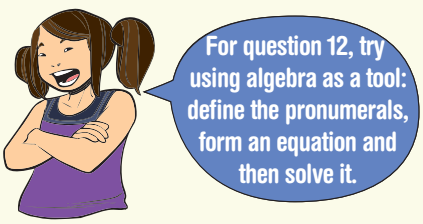


10 A square has a diagonal of 46 cm, what is the area of this square?

11 Find the size of the largest angle of a triangle if it is 10 degrees more than the second largest angle, which in turn is 10 degrees more than the smallest angle?

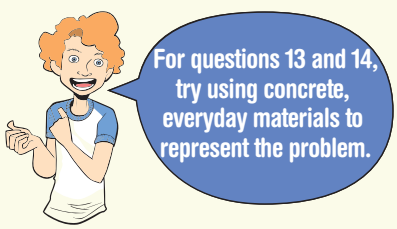


12 The sum of three consecutive even integers is 390. What is the product of the largest and smallest of the integers?

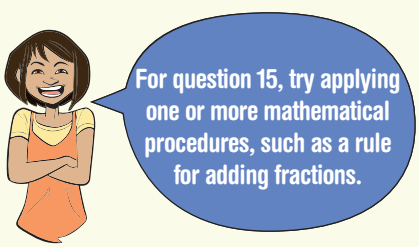


13 What are the dimensions of the rectangle with the largest area if the perimeter is 96 cm?

14 Each student in a round robin table tennis competition must play every other student once. How many (singles) matches are played if there are 12 students in the competition?



15 Ken can wash a car in 10 minutes. Sean takes 15 minutes to wash the same size car. How long should it take them to wash 10 same size cars, if they work as a team of two, washing each car together?



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Working with unfamiliar problems: Part 2

For the questions in Part 2, again use the 'Working with unfamiliar problems' poster at the back of this book, but this time choose your own strategy (or strategies) to solve each problem. Clearly communicate your solution and final answer.

1 The number of bacteria in a petri dish double every 30 seconds. If 5 bacteria are present at 1 p.m., how many will be present after 10 minutes?

2 If each letter represents a different digit (0–9), find the value of each letter so that the statement FOUR + FIVE = NINE is true.

3 A red and a blue four-sided die are each numbered $-4, -2, 0, 3$. If these dice are rolled and the uppermost results are multiplied together, what is the probability that the result is positive?

4 A four-digit number has the following properties: the second and fourth digits form a number that is a multiple of 11; the first two digits form a number divisible by 3 and less than 40; the first and third digits form the square of a whole number. Find all such four digit numbers that are divisible by:

a 4 **b** 5 **c** 6 **d** 9

5 In a school science competition there were 20 boys and 30 girls who entered. Prizes were awarded to 40% of the boys and 30% of the girls. Find the total percentage of entrants who received a prize.

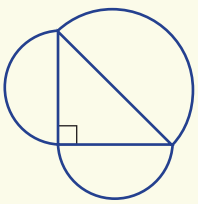


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6 Jake built a solid cube of side 9 cm with plastic 1 cm^3 linking blocks. Riley pulled Jake's cube apart and built a larger cube that looked solid from the outside but was hollow on the inside. What is the side length of the largest cube that Riley could build? How many 1 cm^3 blocks are left over?

7 In a list of five consecutive odd integers the middle integer is shown by the expression $2a - 3$. Find two simplified equivalent expressions for the sum of these five integers.

8 A right-angled triangle has a semi-circle drawn onto each side so each side is a diameter. The largest semi-circle has an arc length of 26.5π cm and another semi-circle has an area of 98π cm^2. Find the radius of the third semi-circle.



9 **a** Insert brackets into $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ to show how the sum of the first ten consecutive odd numbers is equal to the sum of the first four perfect cubes.

b Show how perfect squares can be formed by the addition of consecutive odd numbers.

c How many consecutive odd numbers add to give the sum of the first seven perfect cubes?

d What number when squared equals the sum of the first:

i 7 perfect cubes? **ii** 50 perfect cubes?

10 Four students stand alongside each other and start counting, each person giving the next number. This is how they start:

Amelia	Beau	Claire	Dominic
1 →	2 →	3 →	4
7 ←	6 ←	5	↙
	8 →	9 →	10
13 ←	12 ←	11	↙

a List out the next 6 numbers that Claire will say.

b List out the next 6 numbers that Amelia will say.

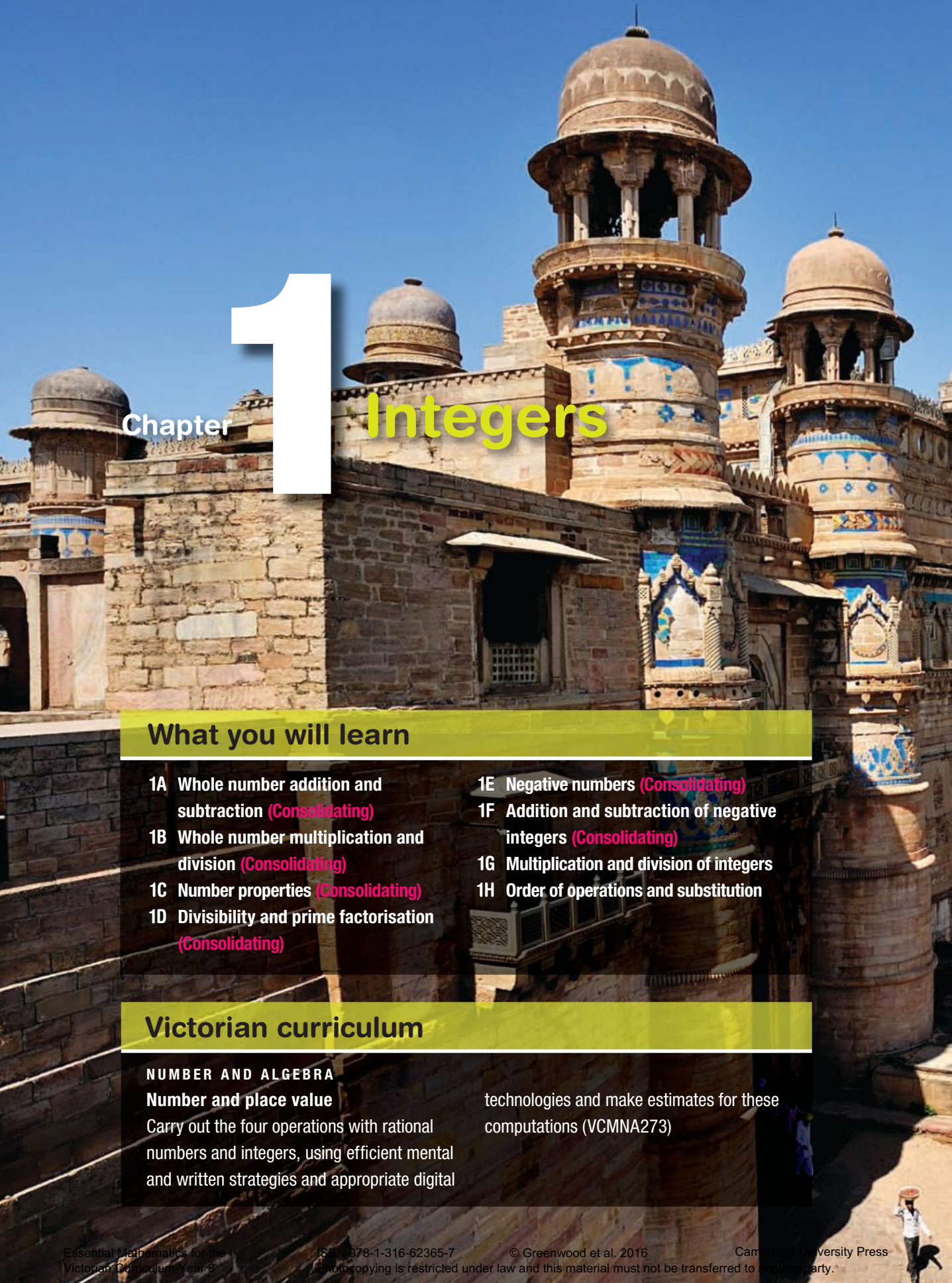
Complete parts **c**–**e** without listing out all the numbers.

c Who will say the number 256?

d Who will say the number 3002?

e Dominic says the number 1234. How many numbers has Dominic said before he says 1234?

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Chapter 1 Integers

What you will learn

1A Whole number addition and subtraction (Consolidating)
1B Whole number multiplication and division (Consolidating)
1C Number properties (Consolidating)
1D Divisibility and prime factorisation (Consolidating)
1E Negative numbers (Consolidating)
1F Addition and subtraction of negative integers (Consolidating)
1G Multiplication and division of integers
1H Order of operations and substitution

Victorian curriculum

NUMBER AND ALGEBRA

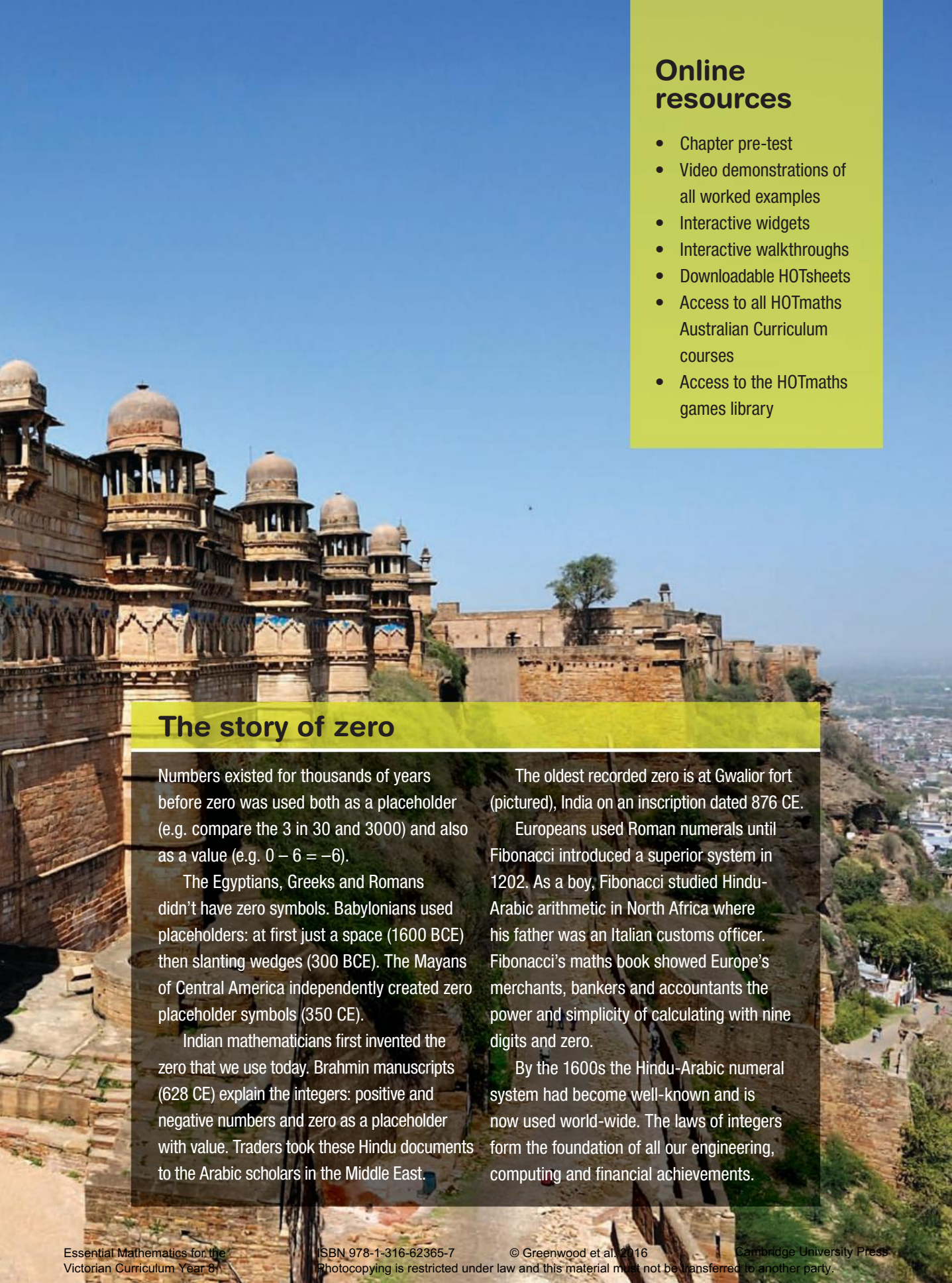
Number and place value

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies and make estimates for these computations (VCMNA273)

Essential Mathematics for the Victorian Curriculum Year 8 ISBN 978-1-316-62365-7
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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library



The story of zero

Numbers existed for thousands of years before zero was used both as a placeholder (e.g. compare the 3 in 30 and 3000) and also as a value (e.g. $0 - 6 = -6$).

The Egyptians, Greeks and Romans didn't have zero symbols. Babylonians used placeholders: at first just a space (1600 BCE) then slanting wedges (300 BCE). The Mayans of Central America independently created zero placeholder symbols (350 CE).

Indian mathematicians first invented the zero that we use today. Brahmin manuscripts (628 CE) explain the integers: positive and negative numbers and zero as a placeholder with value. Traders took these Hindu documents to the Arabic scholars in the Middle East.

The oldest recorded zero is at Gwalior fort (pictured), India on an inscription dated 876 CE.

Europeans used Roman numerals until Fibonacci introduced a superior system in 1202. As a boy, Fibonacci studied Hindu-Arabic arithmetic in North Africa where his father was an Italian customs officer. Fibonacci's maths book showed Europe's merchants, bankers and accountants the power and simplicity of calculating with nine digits and zero.

By the 1600s the Hindu-Arabic numeral system had become well-known and is now used world-wide. The laws of integers form the foundation of all our engineering, computing and financial achievements.

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1A Whole number addition and subtraction

CONSOLIDATING









The number system that we use today is called the Hindu–Arabic or decimal system and uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers 1, 2, 3, 4, … Two numbers can be added to find a sum or subtracted to find a difference. If 22 child tickets and 13 adult tickets were purchased for fairground rides, the sum of the number of tickets (35) is found using addition and the difference between the number of child and adult tickets (9) is found using subtraction.



A child or an adult ticket can be purchased for fairground rides.

Let's start: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 88 and their difference is 14.
- The sum of two numbers is 317 and their difference is 3.

Describe the meaning of the words 'sum' and 'difference' and discuss how you found the pair of numbers in each case.

Key ideas

- Two numbers can be added in any order. This is called the **commutative law** for addition. The commutative law does not hold for subtraction.
  $a + b = b + a$ For example: $7 + 11 = 11 + 7$
  $a - b \neq b - a$ For example: $5 - 2 \neq 2 - 5$
- Three or more numbers can be added in any order. This uses the **associative law** for addition. The associative law does not hold for subtraction.
  $(a + b) + c = a + (b + c)$ For example: $(2 + 5) + 4 = 2 + (5 + 4)$
  $(a - b) - c \neq a - (b - c)$ For example: $(9 - 5) - 2 \neq 9 - (5 - 2)$
- Addition and subtraction **algorithms** can be used for larger numbers.
  For example:

$$\begin{array}{r} {}^{1}4\,{}^{1}3\,9 \\ +\,1\,8\,2 \\ \hline 6\,2\,1 \end{array} \qquad \begin{array}{r} {}^{2}\not{3}\,{}^{1}1\,4 \\ -\,1\,4\,2 \\ \hline 1\,7\,2 \end{array}$$

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Key ideas

- Strategies for mental arithmetic include:
  - **Partitioning** For example: $247 + 121 = (200 + 100) + (40 + 20) + (7 + 1) = 368$
    $85 - 22 = (80 - 20) + (5 - 2) = 63$
  - **Compensating** For example: $134 + 29 = 134 + 30 - 1 = 163$
    $322 - 40 = 320 - 40 + 2 = 282$
  - **Doubling or halving** For example: $35 + 37 = 2 \times 35 + 2 = 72$
    $240 - 123 = 240 \div 2 - 3 = 117$
- Estimates for sums and differences can be made by firstly rounding each number to the nearest 10, 100, 1000 etc.

  For example: Rounding to the nearest 10
  $348 - 121 \approx 350 - 120$
  $= 230$
  Rounding to the nearest dollar
  $\$1.95 + \$3.10 \approx \$2 + \$3$
  $= \$5$

Example 1 Using mental arithmetic

Evaluate this difference and sum mentally.

a $347 - 39$ **b** $125 + 127$

SOLUTION

a $347 - 39 = 308$
b $125 + 127 = 252$

EXPLANATION

$347 - 39 = 347 - 40 + 1 = 307 + 1 = 308$
$125 + 127 = 2 \times 125 + 2 = 250 + 2 = 252$

Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

a
$$\begin{array}{r} 938 \\ +\ 217 \end{array}$$

b
$$\begin{array}{r} 141 \\ -\ 86 \end{array}$$

SOLUTION

a
$$\begin{array}{r} {}^{1}9\,{}^{1}3\,8 \\ +\ 2\,1\,7 \\ \hline 1\,1\,5\,5 \end{array}$$

b
$$\begin{array}{r} 1\,{}^{3}\!\not{4}\,{}^{1}\!\not{1} \\ -\ \ 8\,6 \\ \hline \ \ 5\,5 \end{array}$$

EXPLANATION

$8 + 7 = 15$ (carry the 1 to the tens column)
$1 + 3 + 1 = 5$
$9 + 2 = 11$
Borrow from the tens column then subtract 6 from 11. Then borrow from the hundreds column and then subtract 8 from 13.

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Exercise 1A

1–2(½) | 2(½) | —

UNDERSTANDING

1 Write the number that is:

a 26 plus 17
b 43 take away 9
c 134 minus 23
d 451 add 50
e the sum of 19 and 29
f the sum of 111 and 236
g the difference between 59 and 43
h the difference between 339 and 298
i 36 more than 8
j 142 more than 421
k 32 less than 49
l 120 less than 251

2 Write the digit missing from these sums and differences.

a
$$\begin{array}{r} 237 \\ +\ 4\square \\ \hline 279 \end{array}$$

b
$$\begin{array}{r} 49 \\ +38 \\ \hline 8\square \end{array}$$

c
$$\begin{array}{r} 493 \\ +214 \\ \hline 7\square 7 \end{array}$$

d
$$\begin{array}{r} 1\square 4 \\ +392 \\ \hline 556 \end{array}$$

e
$$\begin{array}{r} 38 \\ -19 \\ \hline 1\square \end{array}$$

f
$$\begin{array}{r} 128 \\ -\ 8\square \\ \hline 39 \end{array}$$

g
$$\begin{array}{r} 3\square 4 \\ -162 \\ \hline 142 \end{array}$$

h
$$\begin{array}{r} 251 \\ -1\square 4 \\ \hline 87 \end{array}$$

3–4(½) | 3–4(½), 5 | 3–5(½)

FLUENCY

Example 1

3 Evaluate these sums and differences mentally.

a 94 – 62
b 146 + 241
c 1494 – 351
d 36 + 19
e 138 + 25
f 251 – 35
g 99 – 20
h 441 – 50
i 350 + 351
j 115 + 114
k 80 – 41
l 320 – 159

Example 2

4 Use an algorithm to find these sums and differences.

a
$$\begin{array}{r} 128 \\ +\ 46 \\ \hline \end{array}$$

b
$$\begin{array}{r} 94 \\ +337 \\ \hline \end{array}$$

c
$$\begin{array}{r} 9014 \\ +\ 927 \\ +\ 421 \\ \hline \end{array}$$

d
$$\begin{array}{r} 814 \\ +1439 \\ +\ 326 \\ \hline \end{array}$$

e
$$\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$$

f
$$\begin{array}{r} 421 \\ -204 \\ \hline \end{array}$$

g
$$\begin{array}{r} 1726 \\ -1699 \\ \hline \end{array}$$

h
$$\begin{array}{r} 14072 \\ -\ 328 \\ \hline \end{array}$$

i
$$\begin{array}{r} 428 \\ +324 \\ +107 \\ +\ 29 \\ \hline \end{array}$$

j
$$\begin{array}{r} 1004 \\ +\ 2407 \\ +\ 9116 \\ +10494 \\ \hline \end{array}$$

k
$$\begin{array}{r} 3017 \\ -2942 \\ \hline \end{array}$$

l
$$\begin{array}{r} 10024 \\ -\ 936 \\ \hline \end{array}$$

5 Estimate these sums and differences by first rounding each given amount to the nearest dollar.

a $1.95 + $3.02
b $3.05 + $4.99
c $10.19 + $0.95
d $8.99 – $3.03
e $20.95 – $2.10
f $8.69 – $5.79

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1A

FLUENCY

6 By first rounding each given number as instructed, estimate the answer.

a 79 + 32 (nearest 10)
b 38 – 17 (nearest 10)
c 916 + 1401 (nearest 100)
d 2993 – 1210 (nearest 100)
e 3.01 – 0.95 (nearest 1)
f 39271 + 3648 (nearest 1000)

7–9 8–10 9–11

PROBLEM-SOLVING

7 A racing bike's odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. What was the total distance of the race?



Casey Stoner racing at the Malaysian Grand Prix.

8 The sum of two numbers is 39 and their difference is 5. What is the larger number?

9 Find the missing digits in these sums and differences.

a
$$\begin{array}{r} 2\ 3\ \square \\ +\ \square\ 9\ 4 \\ \hline 6\ \square\ 1 \end{array}$$

b
$$\begin{array}{r} \square\ 3\ \square \\ +\ \ \square\ 2 \\ \hline 2\ 1\ 9 \end{array}$$

c
$$\begin{array}{r} \square\ 3\ 7 \\ +\ 4\ 9\ \square \\ \hline 7\ \square\ 2 \end{array}$$

d
$$\begin{array}{r} \square\ 3 \\ -\ 2\ 9 \\ \hline 6\ \square \end{array}$$

e
$$\begin{array}{r} 3\ \square\ 2 \\ -\ \square\ 3\ \square \\ \hline 1\ 0\ 4 \end{array}$$

f
$$\begin{array}{r} 2\ \square\ \square\ 5 \\ -\ \ 6\ 8\ \square \\ \hline \square\ 3\ 1\ 8 \end{array}$$

10 Wally has two more marbles than Ashan and together they have 88 marbles. How many marbles does Ashan have?

11 Evaluate the following without the use of a calculator.

a 1 + 2 + 3 + 4 + … + 99 + 100 – 99 – 98 – … – 2 – 1
b 1 – 2 + 3 – 4 + 5 – 6 + … – 98 + 99

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1A

12 | 12, 13 | 12–14

REASONING

12 Explain why these number puzzles cannot be solved.

a
$$\begin{array}{r} \square\,2\,\square \\ +\,3\,\square\,6 \\ \hline 2\,3\,4\,9 \end{array}$$

b
$$\begin{array}{r} 3\,\square\,6 \\ -\,3\,2\,\square \\ \hline 8\,2 \end{array}$$

13 x, y and z represent any three numbers. Complete these statements.

a $x+y+z=__+x+y$ **b** $x-y+z=z-__+__=x+__-__$

14 How many different combinations of numbers make the following true? List the combinations and explain your reasoning.

a
$$\begin{array}{r} 1\,\square\,4 \\ +\,2\,\square\,\square \\ \hline 4\,2\,7 \end{array}$$

b
$$\begin{array}{r} 3\,\square\,\square \\ -\,1\,\square\,4 \\ \hline 1\,4\,3 \end{array}$$

Magic triangles

— | — | 15

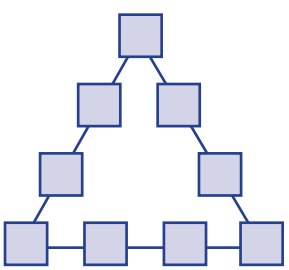
ENRICHMENT

15 The sides of a magic triangle all sum to the same total.

a Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

b Show how it is possible to arrange the same digits to a different total. How many different totals can you find?

c In how many different ways can you obtain each total? Switching the two middle numbers on each side does not count as a new combination.





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1B Whole number multiplication and division

CONSOLIDATING









It is useful to know how to multiply and divide numbers without the use of technology. Mental strategies can be used in some problems, and algorithms can be used for more difficult cases. Calculating the cost of 9 tickets at $51 each, for example, can be done mentally, but the short division algorithm might be useful when calculating the number of trips a dump truck with a capacity of 140 tonnes will need to shift 1000 tonnes of coal.



A typical large mining truck has a capacity of 140 tonnes.

Let's start: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people get if a packet of 32 cookies is shared equally between them
- The cost of paving 30 square metres of courtyard at a cost of $41 per square metre
- The number of sheets of paper in a shipment of 4000 boxes of 5 reams each (1 ream is 500 sheets)
- The number of hours I can afford a plumber at $75 per hour if I have a fixed budget of $1650

Make up your own situation that requires the use of multiplication and another for division.



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Key ideas

- Finding a **product** is the result of using multiplication. We say the product of 11 and 9 is 99.
- The **multiplication algorithm** can be used when products cannot be found mentally.
- Using division results in finding a **quotient** and a **remainder**.

$$\begin{array}{r} 217 \\ \times\ 26 \\ \hline 1302 \\ 4340 \\ \hline 5642 \end{array} \quad \begin{array}{l} \\ \\ \leftarrow\ 217 \times 6 \\ \leftarrow\ 217 \times 20 \\ \leftarrow\ 1302 + 4340 \end{array}$$

For example: $38 \div 11 = 3$ and 5 remainder

dividend, divisor, quotient

$$7\overline{)51^{2}2^{1}4}\quad 7\ 3\ 2$$

- The short division algorithm can be used when quotients cannot be found mentally.
- The **commutative law** holds for multiplication but not division.
  $7 \times 5 = 5 \times 7$ but $21 \div 3 \neq 3 \div 21$
- The **associative law** holds for multiplication but not division.
  $(5 \times 4) \times 2 = 5 \times (4 \times 2)$ but $(5 \div 4) \div 2 \neq 5 \div (4 \div 2)$
- Mental strategies for multiplication
  - Using the commutative and associative laws
    For example: $5 \times 17 \times 4 = 5 \times 4 \times 17 = 20 \times 17 = 340$
  - Using the **distributive law**
    For example: $4 \times 87 = (4 \times 80) + (4 \times 7) = 320 + 28 = 348$
    or $4 \times 87 = (4 \times 90) - (4 \times 3) = 360 - 12 = 348$
  - Doubling and halving
    For example: $4 \times 74 = 2 \times 148 = 296$
- Mental strategies for division
  - Halving both numbers
    For example: $132 \div 4 = 66 \div 2 = 33$
  - Using the distributive law
    For example: $96 \div 3 = (90 \div 3) + (6 \div 3) = 30 + 2 = 32$
    or $147 \div 3 = (150 \div 3) - (3 \div 3) = 50 - 1 = 49$

Example 3 Using mental strategies

Use a mental strategy to evaluate the following.

a 5×160 **b** 7×89 **c** $464 \div 8$

SOLUTION	EXPLANATION
a $5 \times 160 = 800$	Double one and halve the other so 5×160 becomes 10×80
b $7 \times 89 = 623$	Use the distributive law so 7×89 becomes $(7 \times 90) - (7 \times 1) = 630 - 7$
c $464 \div 8 = 58$	Halve both numbers repeatedly so $464 \div 8$ becomes $232 \div 4 = 116 \div 2$

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Example 4 Using multiplication and division algorithms

Use an algorithm to evaluate the following.

a 412×25

b $938 \div 13$

SOLUTION

a

$$\begin{array}{r} 412 \\ \times\ 25 \\ \hline 2060 \\ 8240 \\ \hline 10300 \end{array}$$

b

$$13\overline{)93^{2}8} \quad 7\ 2 \text{ Rem } 2$$

So $938 \div 13 = 72$ and 2 remainder.

EXPLANATION

$412 \times 5 = 2060$ and $412 \times 20 = 8240$

Add these two products to get the final answer.

$93 \div 13 = 7$ and 2 remainder

$28 \div 13 = 2$ and 2 remainder

Exercise 1B

1–3(½) 3(½) —

UNDERSTANDING

1 Find the results for the following.

a The product of 7 and 8

b The product of 13 and 100

c The remainder when 2 is divided into 19

d The remainder when 9 is divided into 104

e The quotient of 13 divided by 4

f The quotient of 120 divided by 59

2 Use your knowledge of the multiplication table to write down the answers to the following.

a 5×8	**b** 11×9	**c** 6×7	**d** 9×8
e 11×6	**f** 12×11	**g** 8×4	**h** 7×9
i $100 \div 10$	**j** $88 \div 8$	**k** $121 \div 11$	**l** $144 \div 12$
m $56 \div 7$	**n** $33 \div 3$	**o** $65 \div 5$	**p** $78 \div 6$

3 Decide if these simple equations are true or false.

a $4 \times 13 = 13 \times 4$

b $2 \times 7 \times 9 = 7 \times 9 \times 2$

c $6 \div 3 = 3 \div 6$

d $60 \div 20 = 30 \div 10$

e $14 \div 2 \div 7 = 7 \div 2 \div 14$

f $51 \times 7 = (50 \times 7) + (1 \times 7)$

g $79 \times 13 = (80 \times 13) - (1 \times 13)$

h $93 \div 3 = (90 \div 3) + (3 \div 3)$

i $133 \div 7 = (140 \div 7) - (7 \div 7)$

j $33 \times 4 = 66 \times 8$

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1B

4–7(½) | 4–7(½) | 4–7(½)

FLUENCY

Example 3

4 Use a mental strategy to evaluate the following.

a $5 \times 13 \times 2$ **b** $2 \times 26 \times 5$ **c** 4×35 **d** 17×4

e 17×1000 **f** 136×100 **g** 59×7 **h** 119×6

i 9×51 **j** 6×61 **k** 4×252 **l** 998×6

m $128 \div 8$ **n** $252 \div 4$ **o** $123 \div 3$ **p** $508 \div 4$

q $96 \div 6$ **r** $1016 \div 8$ **s** $5 \times 12 \times 7$ **t** $570 \div 5 \div 3$

Example 4a

5 Use a multiplication algorithm to evaluate the following.

a 67×9 **b** 129×4 **c** 294×13 **d** 1004×90

e 690×14 **f** 4090×101 **g** 246×139 **h** 1647×209

Example 4b

6 Use the short division algorithm to evaluate the following.

a $3\overline{)85}$ **b** $7\overline{)214}$ **c** $10\overline{)4167}$ **d** $11\overline{)143}$

e $15\overline{)207}$ **f** $19\overline{)3162}$ **g** $28\overline{)196}$ **h** $31\overline{)32690}$

7 Estimate answers to the following by firstly rounding each dollar/cent amount to the nearest dollar. For example, 3 packets at \$1.95 ≈ 3 × \$2 = \$6

a 5 packets at \$2.95 each **b** 7 packets at \$9.99 each

c 20 boxes at \$19.80 each **d** \$29.90 divided into 6 parts

e \$120.35 divided into 5 parts **f** \$999.80 divided into 20 parts

8–10 | 9–11 | 11, 12

PROBLEM-SOLVING

8 A university student earns \$550 for 22 hours work. What is the student's pay rate per hour?

9 Packets of biscuits are purchased by a supermarket in boxes of 18. The supermarket orders 220 boxes and sells 89 boxes in one day. How many packets of biscuits remain in the supermarket?

10 Riley buys a fridge, which he can pay for by the following options.

A 9 payments of \$183 **B** \$1559 up front

Which option is cheaper and by how much?

11 The shovel of a giant mechanical excavator can move 13 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

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1B

PROBLEM-SOLVING

12 Find the missing digits in these problems.

a
$$\begin{array}{r} 2\ \square \\ \times\ 2\ \ 1 \\ \hline 2\ \ 3 \\ 4\ \square\ \ 0 \\ \hline 4\ \ 8\ \ \square \end{array}$$

b
$$\begin{array}{r} 1\ \square\ 7 \\ \times\ \ \ 2\ \square \\ \hline \square\ \ 4\ \ 8 \\ 2\ \square\ \ 4\ \ 0 \\ \hline \square\ \square\ \ 8\ \ 8 \end{array}$$

c $7\overline{)6\ \square\ 4}$ $=\ 9\ 4$ with 6 remainder

d $17\overline{)\square\ \square\ 6}$ $=\ 1\ 9$ with 3 remainder

	13	13, 14	14, 15

REASONING

13 If a represents any number except 0, simplify the following.

a $a \div a$ **b** $a \div 1$ **c** $0 \div a$ **d** $25 \times a \div a$

14 A mental strategy for division involves separately dividing a pair of factors of the divisor. For example:

$114 \div 6 = 144 \div 2 \div 3$ (Note: 2 and 3 are factors of 6)

$= 57 \div 3$

$= 19$

Use this technique to evaluate the following.

a $204 \div 6$ **b** $144 \div 8$ **c** $261 \div 9$ **d** $306 \div 18$

15 Evaluate the following without using an algorithm.

a $(99 \times 17) + (1 \times 17)$ **b** $(82 \times 7) - (2 \times 7)$

c $(493 \times 12) + (507 \times 12)$ **d** $(326 \times 15) - (306 \times 15)$

Maximum ticket numbers	—	—	16

ENRICHMENT

16 **a** Gen spends exactly \$80 to buy child tickets at \$7 each and adult tickets at \$12 each. Find the maximum number of tickets that could be purchased.

b Alfred spends exactly \$141 to buy child tickets at \$9 each and adult tickets at \$15 each. Find the maximum number of tickets that could be purchased.

c Explain your method for solving the above two questions. Make up your own similar question and test it on a friend.



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1C Number properties CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

The properties of numbers are at the foundation of mathematical problem-solving. A class of 63 students, for example, can be divided into 7 equal groups of 9, but a class of 61 cannot be divided into smaller equal groups greater than 1. This is because 61 is a prime number with no other factors apart from 1 and itself; 63 is a multiple of 9 and the numbers 9 and 7 are factors of 63.



Into how many equal groups could these people be divided?

Let's start: How many in 60 seconds?

In 60 seconds, write down as many numbers as you can that fit each description.

- Multiples of 7
- Factors of 144
- Prime numbers

Compare your lists with the results of the class. For each part decide if there are any numbers less than 100 that you missed.

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Key ideas

- A **multiple** of a number is obtained by multiplying the number by the **counting numbers** 1, 2, 3, …
  For example: Multiples of 9 include 9, 18, 27, 36, 45, …
- The **lowest common multiple** (LCM) is the smallest multiple of two or more numbers that is common.
  For example: Multiples of 3 are 3, 6, 9, 12, (15), 18, …
  For example: Multiples of 5 are 5, 10, (15), 20, 25, …
  The LCM of 3 and 5 is therefore 15.
- A **factor** of a number has a remainder of zero when divided into the given number.
  For example, 11 is a factor of 77 since $77 \div 11 = 7$ with 0 remainder.
- The **highest common factor** (HCF) is the largest factor of two or more numbers that is common.
  For example: Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
  For example: Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.
  The HCF of 24 and 36 is therefore 12.
- **Prime numbers** have only two factors, the number itself and 1.
  - 2, 13 and 61 are examples of prime numbers.
  - 1 is not considered to be a prime number.
- **Composite numbers** have more than two factors.
  - 6, 20 and 57 are examples of composite numbers.
- The **square** of a number x is $x^2 = x \times x$.
  - We say x^2 as 'x squared' or 'the square of x' or 'x to the power 2'.
  - $3^2 = 9$ and $11^2 = 121$ ($3^2 = 3 \times 3$ and $11^2 = 11 \times 11$)
  - If x is a whole number then x^2 is called a perfect square. 4 and 121 are examples of perfect squares.
- The **square root** of a number is written with the symbol $\sqrt{\ }$.
  - $\sqrt{b} = a$ if $a^2 = b$ (and a is positive or zero), for example, $\sqrt{9} = 3$ since $3^2 = 9$.
    Note: $\sqrt{16}$ is a positive number only and is equal to 4 not ± 4.
- The **cube** of a number x is $x^3 = x \times x \times x$.
  - We say x^3 as 'x cubed or 'the cube of x' or 'x to the power 3'.
  - $2^3 = 2 \times 2 \times 2 = 8$ and $5^3 = 5 \times 5 \times 5 = 125$
- The **cube root** of a number is written with the symbol $\sqrt[3]{\ }$.
  - $\sqrt[3]{b} = a$ if $a^3 = b$, for example, $\sqrt[3]{8} = 2$ since $2^3 = 8$.

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Example 5 Finding the LCM and HCF

a Find the LCM of 6 and 8.

b Find the HCF of 36 and 48.

SOLUTION	EXPLANATION
a Multiples of 6 are: 6, 12, 18, 24, 30, … Multiples of 8 are: 8, 16, 24, 32, 40, … The LCM is 24.	First, list some multiples of 6 and 8. Continue the lists until there is at least one in common. Choose the smallest number that is common to both lists.
b Factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36 Factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 The HCF is 12.	First, list factors of 36 and 48. Choose the largest number that is common to both lists.

Example 6 Finding squares, cubes, square roots and cube roots

Evaluate the following.

a 6^2 **b** $\sqrt{81}$ **c** 2^3 **d** $\sqrt[3]{64}$

SOLUTION	EXPLANATION
a $6^2 = 6 \times 6$ $= 36$	Find the product of 6 with itself.
b $\sqrt{81} = 9$	$9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$ Note: $\sqrt{x}$ cannot be negative so $\sqrt{81} \neq -9$.
c $2^3 = 2 \times 2 \times 2$ $= 8$	In general $x^3 = x \times x \times x$.
d $\sqrt[3]{64} = 4$	$4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$

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Exercise 1C

1–3, 4(½) | 2, 4(½) | —

UNDERSTANDING

1 Circle or write down the number in each list which is not a multiple of the first number listed.

a 3, 6, 9, 12, 14, 18, 21
b 11, 22, 33, 45, 55, 66
c 21, 43, 63, 84, 105
d 13, 26, 40, 52, 65

2 Write down the missing factor from each list.

a Factors of 18: 1, 2, 3, 9, 18
b Factors of 24: 1, 2, 4, 6, 8, 12, 24

3 Classify these numbers as prime or composite.

a 7 **b** 12 **c** 29 **d** 69
e 105 **f** 117 **g** 221 **h** 1 046 734

4 Evaluate the following.

a $2 \times 2 \times 2$ **b** $3 \times 3 \times 3$ **c** $4 \times 4 \times 4$ **d** $5 \times 5 \times 5$
e $6 \times 6 \times 6$ **f** $7 \times 7 \times 7$ **g** $8 \times 8 \times 8$ **h** $9 \times 9 \times 9$

5–8(½) | 5–8(½), 9 | 5–9(½)

FLUENCY

Example 5a **5** Find the LCM of these pairs of numbers.

a 2, 3 **b** 5, 9 **c** 8, 12
d 4, 8 **e** 25, 50 **f** 4, 18
g 8, 60 **h** 12, 20

Example 5b **6** Find the HCF of these pairs of numbers.

a 6, 8 **b** 18, 9 **c** 16, 24
d 24, 30 **e** 7, 13 **f** 19, 31
g 72, 36 **h** 108, 64

Example 6a Example 6b **7** Evaluate these squares and square roots.

a 4^2 **b** 10^2 **c** 13^2
d 15^2 **e** 100^2 **f** 20^2
g $\sqrt{25}$ **h** $\sqrt{49}$ **i** $\sqrt{121}$
j $\sqrt{900}$ **k** $\sqrt{1600}$ **l** $\sqrt{256}$

Example 6c Example 6d **8** Evaluate these cubes and cube roots.

a 2^3 **b** 4^3 **c** 7^3
d 5^3 **e** 6^3 **f** 10^3
g $\sqrt[3]{27}$ **h** $\sqrt[3]{8}$ **i** $\sqrt[3]{125}$
j $\sqrt[3]{512}$ **k** $\sqrt[3]{729}$ **l** $\sqrt[3]{1\,000\,000}$

9 Find:

a the LCM of 8, 12 and 6
b the LCM of 7, 3 and 5
c the HCF of 20, 15 and 10
d the HCF of 32, 60 and 48

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1C

10, 11 | 11–13 | 12–14

PROBLEM-SOLVING

10 A teacher has 64 students to divide into small equal groups of greater than 2 with no remainder. In how many ways can this be done?

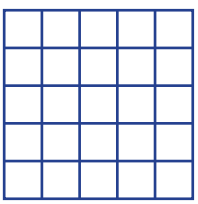
11 Three sets of traffic lights (A, B and C) all turn red at 9 a.m. exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?



12 How many prime numbers less than 100 are there?

13 **a** How many squares of any size are there on this grid?

b What do you notice about the number of squares of each size? Do you notice a pattern?



14 Cyclist A rides a lap of a circular course every 3 minutes. Cyclist B rides a lap of the same course every 5 minutes. If both cyclists start at the same place at the same time, how long will it take before they are both back together at the starting position?

15 | 15, 16 | 16–18

REASONING

15 Using the definitions (descriptions) in the **Key ideas**, explain why the number one (1) is not considered a prime or a composite number.

16 Explain why all prime numbers except the number 2 are odd.

17 Explain why all square numbers (1, 4, 9, 16, …) have an odd number of factors.

18 Decide if the following statements are always true. If they are not, give an example that shows that the statement is not always true. Assume that a and b are different numbers.

a The LCM of two numbers a and b is $a \times b$.

b The LCM of two prime numbers a and b is $a \times b$.

c The HCF of two prime numbers a and b is 1.

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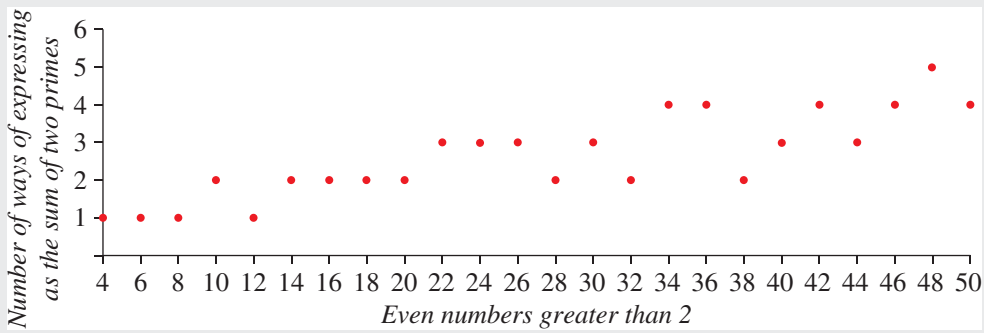
ENRICHMENT

19 In 1742, Goldbach wrote a letter to Euler suggesting that every even number greater than 5 is the sum of three primes. Euler replied saying that this was equivalent to saying that every even number greater than 2 is the sum of two primes. If the number 1 is not considered to be prime (the modern convention), the idea becomes *Every even number greater than 2 is the sum of two primes*. This is known today as Goldbach's conjecture.

a Show ways in which the following numbers can be written as a sum of two primes.

i 28 **ii** 62 **iii** 116

b Goldbach's conjecture does not discuss the odd numbers. Are there any odd numbers greater than 4 and less than 20 which cannot be written as a sum of two primes? If there are any, list them.



A graph illustrating Goldbach's conjecture up to and including 50, is obtained by plotting the number of ways of expressing even numbers greater than 2 as the sum of two primes.

20 Twin primes are pairs of prime numbers that differ by 2. It has been conjectured that there are infinitely many twin primes. List the pairs of twin primes less than 100.



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1D Divisibility and prime factorisation

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

The fundamental theorem of arithmetic says that every whole number greater than 1 can be written as a product of prime numbers, for example, $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$. For this reason it is often said that prime numbers are the building blocks of all other whole numbers.

Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.



Prime numbers can be thought of as the building blocks or foundations of all other whole numbers.

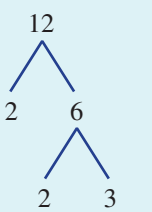
Let's start: Remembering divisibility tests

To test if a number is divisible by 2, we simply need to see if the number is even or odd. All even numbers are divisible by 2. Try to remember the divisibility tests for each of the following. As a class, can you describe all the tests for the following?

- Divisible by 3
- Divisible by 4
- Divisible by 5
- Divisible by 6
- Divisible by 8
- Divisible by 9
- Divisible by 10

Key ideas

- **Prime factorisation** involves writing a number as a product of prime numbers.
  For example: $12 = 2 \times 2 \times 3$
  - $2^2 \times 3$ is the **prime factor** form of 12.
  - The prime numbers are usually written in ascending order.
  - A prime factor tree can help to determine the prime factor form.



$\therefore 12 = 2^2 \times 3$

- The **lowest common multiple** (LCM) of two numbers in their prime factor form is the product of all the different primes raised to their highest power.
  For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$
  So the LCM of 12 and 30 is $2^2 \times 3 \times 5 = 60$.
- The **highest common factor** (HCF) of two numbers in their prime factor form is the product of all the common primes raised to their smallest power.
  For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$
  So the HCF of 12 and 30 is $2 \times 3 = 6$.

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Key ideas

Divisibility tests

A number is divisible by:

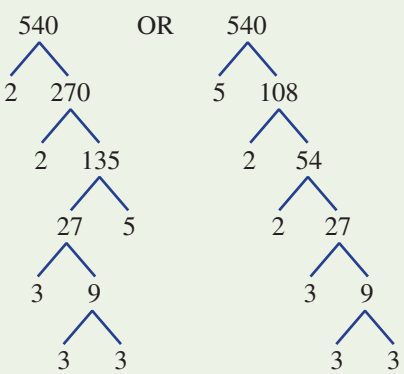
- 2 if it ends with the digit 0, 2, 4, 6 or 8, for example, 384 ends with a 4 and is an even number
- 3 if the sum of all the digits is divisible by 3
  For example, 162 where $1 + 6 + 2 = 9$, which is divisible by 3
- 4 if the number formed by the last two digits is divisible by 4
  For example, 148 where 48 is divisible by 4
- 5 if the last digit is a 0 or 5
  For example, 145 or 2090
- 6 if it is divisible by both 2 and 3
  For example, 456 where 6 is even and $4 + 5 + 6 = 15$, which is divisible by 3
- 8 if the number formed from the last 3 digits is divisibly by 8
  For example, 2112 where 112 is divisible by 8
- 9 if the sum of all the digits is divisible by 9
  For example, 3843 where $3 + 8 + 4 + 3 = 18$, which is divisible by 9
- 10 if the last digit is a 0
  For example, 4230

There is no simple test for 7.

Example 7 Finding prime factor form

Use a factor tree to write 540 as a product of prime factors.

SOLUTION



So $540 = 2^2 \times 3^3 \times 5$

EXPLANATION

First, divide 540 into the product of any two factors.
Since 540 is even it is easy to choose 2 as one of the factors but 3 or 5 could also be chosen.

Continue dividing numbers into prime factors until all the factors are prime numbers.

Write the factors in ascending order.

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Example 8 Testing for divisibility

Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

SOLUTION	EXPLANATION
Not divisible by 2 since 7 is odd.	The last digit needs to be even.
Divisible by 3 since $6 + 2 + 7 = 15$ and this is divisible by 3.	The sum of all the digits needs to be divisible by 3.
Not divisible by 4 as 27 is not divisible by 4.	The number formed from the last two digits needs to be divisible by 4.
Not divisible by 5 as the last digit is not a 0 or 5.	The last digit needs to be a 0 or 5.
Not divisible by 6 as it is not divisible by 2.	The number needs to be divisible by both 2 and 3.
Not divisible by 8 as the last 3 digits together are not divisible by 8.	The number formed from the last three digits needs to be divisible by 8.
Not divisible by 9 as $6 + 2 + 7 = 15$ is not divisible by 9.	The sum of all the digits needs to be divisible by 9.

Example 9 Finding the LCM and HCF

Find the LCM and HCF of 105 and 90, using prime factorisation.

SOLUTION	EXPLANATION
$105 = 3 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$	First, express each number in prime factor form. Note that 3 and 5 are common primes.
$\text{LCM} = 2 \times 3^2 \times 5 \times 7$ $= 630$	For the LCM include all the different primes, raising the common primes to their highest power.
$\text{HCF} = 3 \times 5$ $= 15$	For the HCF include only the common primes raised to the smallest power.

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Exercise 1D

1–4 | 4 | —

UNDERSTANDING

1 Write down all the factors of these numbers.

a 15 **b** 24 **c** 40 **d** 84

2 Write down the first 10 prime numbers. Note that 1 is not a prime number.

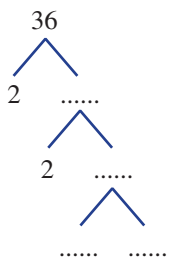
3 Evaluate the following.

a $2^2 \times 3$ **b** $2 \times 3^2 \times 5$ **c** $2^3 \times 5 \times 7$ **d** $3^3 \times 11$

Example 7

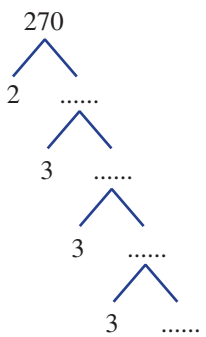
4 Copy and complete these factor trees to help write the prime factor form of the given numbers.

a



$\therefore 36 = 2^2 \times .....$

b



$\therefore 270 = 2 \times ..... \times .....$

5–7(½) | 5–7(½) | 5–7(½)

FLUENCY

5 Use a factor tree to find the prime factor form of these numbers.

a 20 **b** 28 **c** 40 **d** 90
e 280 **f** 196 **g** 360 **h** 660

6 How many different primes make up the prime factor form of these numbers?

a 30 **b** 63 **c** 180 **d** 2695

Example 8

7 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 51 **b** 126 **c** 248 **d** 387
e 315 **f** 517 **g** 894 **h** 3107

8, 9(½) | 9(½), 10, 11 | 9(½), 10, 11

PROBLEM-SOLVING

8 Find the highest common prime factor of each of these pairs of numbers.

a 10, 45 **b** 42, 72 **c** 24, 80 **d** 539, 525

Example 9

9 Find the LCM and the HCF of these pairs of numbers, using prime factorisation.

a 10, 12 **b** 14, 28 **c** 15, 24 **d** 12,15
e 20, 28 **f** 13, 30 **g** 42, 9 **h** 126, 105

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1D

PROBLEM-SOLVING

10 Aunt Elly's favourite nephew visits her every 30 days. The other nephew visits her every 42 days. If both nephews visit Aunt Elly on one particular day, how long will it be before they both visit her again on the same day?

11 Two armies face each other for battle. One army has 1220 soldiers and the other has 549 soldiers. Both armies are divided into smaller groups of equal size called platoons. Find the largest possible number of soldiers in a platoon if the platoon size is equal for the two armies.



12 | 12, 13 | 13, 14

REASONING

12 Decide if the following statements are true or false. If a statement is false, give an example to show this.

- **a** All numbers divisible by 9 are divisible by 3.
- **b** All numbers divisible by 3 are divisible by 9.
- **c** All numbers divisible by 8 are divisible by 4.
- **d** All numbers divisible by 4 are divisible by 8.

13 If a number is divisible by 2 and 3, then it must be divisible by 6. Use this idea to complete these sentences.

- **a** A number is divisible by 14 if it is divisible by ____ and ____.
- **b** A number is divisible by 22 if it is divisible by ____ and ____.
- **c** A number is divisible by 15 if it is divisible by ____ and ____.
- **d** A number is divisible by 77 if it is divisible by ____ and ____.

14 Powers higher than 3 can be used in prime factorisation.
e.g. $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
Write the prime factor form of each number.

a 162 **b** 96 **c** 5625 **d** 1792

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ENRICHMENT

15 There is a more complex test to see if a number is divisible by 11.

a Divide each of these numbers by 11.

i 22 **ii** 88 **iii** 121 **iv** 165

v 308 **vi** 429 **vii** 1034 **viii** 9020

b For the number 2035 (which is divisible by 11):

i Find the sum of the first and third digits.

ii Find the sum of the second and fourth digits.

iii Subtract your answer to part **ii** from your answer to part **i**. What do you notice?

c Repeat all the tasks in part **b** for the number 8173 (which is divisible by 11).

d Now find the sum of all the alternate digits for these numbers which are divisible by 11. Subtract the second sum from the first. What do you notice?

i 4092 **ii** 913

iii 2475 **iv** 77

e Can you now write down a divisibility test for dividing by 11? Test it on some numbers.



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1E Negative numbers

CONSOLIDATING











Although the Indian mathematician Brahmagupta set out the rules for the use of negative numbers in the 7th century, a British mathematician Maseres claimed in 1758 that negative numbers 'darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple'. Despite this view that negative numbers were unnatural and had little meaning, they have found their way into the practical world of science, engineering and commerce. We can use negative numbers to distinguish between left and right, up and down, financial profits and losses, warm and cold temperatures, and the clockwise and anticlockwise rotation of a wheel.





How are negative numbers associated with these images?

Let's start: A negative world

Describe how to use negative numbers to describe these situations.

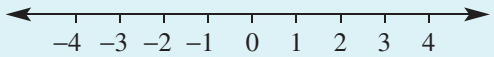
- 6°C below zero
- A loss of $4200
- 150 m below sea level
- A turn of 90° anticlockwise
- The solution to the equation $x + 5 = 3$

Can you describe another situation in which you might make use of negative numbers?

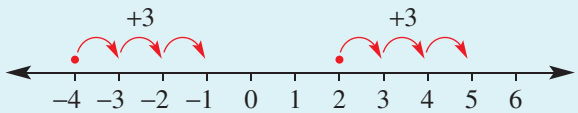
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Key ideas

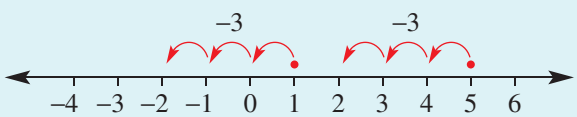
- Negative numbers are numbers less than zero.
- The integers are … , – 4, –3, –2, –1, 0, 1, 2, 3, 4, …
  These include positive integers (natural numbers), zero and negative integers. These are illustrated clearly on a number line.



- Adding or subtracting a positive integer can result in a positive or negative number.



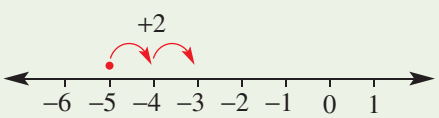
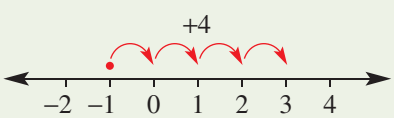
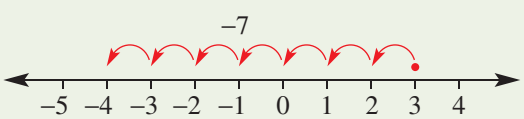
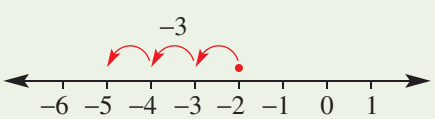
  - Adding a positive integer
    For example: $2 + 3 = 5$
    $-4 + 3 = -1$
  - Subtracting a positive integer
    For example: $1 - 3 = -2$
    $5 - 3 = 2$



Example 10 Adding and subtracting a positive integer

Evaluate the following.

a $-5 + 2$ **b** $-1 + 4$ **c** $3 - 7$ **d** $-2 - 3$

SOLUTION	EXPLANATION
a $-5 + 2 = -3$	
b $-1 + 4 = 3$	
c $3 - 7 = -4$	
d $-2 - 3 = -5$	

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Exercise 1E

1–3 | 3 | —

UNDERSTANDING

1 Write the symbol < (less than) or > (greater than) to make these statements true.

a 5 ____ −1 **b** −3 ____ 4 **c** −10 ____ 3 **d** −1 ____ −2

e −20 ____ −24 **f** −62 ____ −51 **g** 2 ____ −99 **h** −61 ____ 62

2 Write the missing numbers in these patterns.

a −3, −2, ____, 0, 1, ____, 3 **b** 1, 0, ____, −2, −3, ____, −5

c −10, −8, −6, ____, ____, 0, 2 **d** 20, 10, ____, ____, −20, −40

3 What is the final temperature?

a 10°C is reduced by 12°C **b** 32°C is reduced by 33°C

c −11°C is increased by 2°C **d** −4°C is increased by 7°C

4–7(½) | 4–8(½) | 4–8(½)

FLUENCY

Example 10a Example 10b

4 Evaluate the following.

a −1 + 2 **b** −3 + 7 **c** −10 + 11 **d** −4 + 12

e −20 + 35 **f** −100 + 202 **g** −7 + 2 **h** −15 + 8

i −26 + 19 **j** −38 + 24 **k** −173 + 79 **l** −308 + 296

Example 10c Example 10d

5 Evaluate the following.

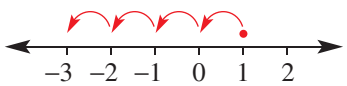
a 4 − 5 **b** 10 − 15 **c** 0 − 26 **d** 14 − 31

e 103 − 194 **f** 316 − 390 **g** −4 − 7 **h** −11 − 20

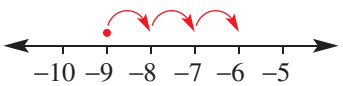
i −14 − 15 **j** −10 − 100 **k** −400 − 37 **l** −348 − 216

6 Write the sum (e.g. −3 + 4 = 1) or difference (e.g. 1 − 5 = −4) to match these number lines.

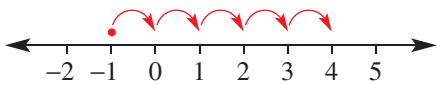
a



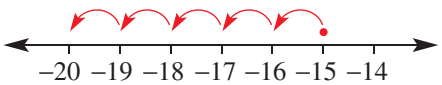
b



c



d



7 Write the missing number.

a −1 + ____ = 5 **b** ____ + 30 = 26

c ____ + 11 = −3 **d** −32 + ____ = −21

e 5 − ____ = −10 **f** ____ − 17 = −12

g ____ − 4 = −7 **h** −26 − ____ = −38

8 Work from left to right to evaluate the following.

a −3 + 4 − 8 + 6 **b** 0 − 10 + 19 − 1

c 26 − 38 + 14 − 9 **d** 9 − 18 + 61 − 53

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1E

	9, 10	10–12	10–12

PROBLEM-SOLVING

9 In a high-rise building there are 25 floors above ground floor (floor 1, floor 2, …) and 6 floors below ground floor. A lift starts at floor 3 and moves 5 floors down then 18 floors up, 4 more floors up, 26 floors down and finally 6 floors up. At which floor does the lift finish?



10 Insert a + and/or a − sign into these statements to make them true.

a 5____7 = −2 **b** 4____6____3 = 1

c −2____5____4 = −11

11 On Monday Milly borrows $35 from a friend. On Tuesday she pays her friend $40. On Friday she borrows $42 and pays back $30 that night. How much does Milly owe her friend in the end?

12 The temperature in Greenland on a sunny day rises 19°C from its minimum temperature to a maximum of −4°C. What was the minimum temperature on the day?



	13	13, 14	14–16

REASONING

13 If a and b are positive integers, decide if the following are *always* true.

a $a + b > 0$ **b** $a - b < 0$ **c** $b - a < 0$

d $-a - b < 0$ **e** $-a + b > 0$ **f** $-b + a < 0$

14 If a and b are positive integers and $a > b$, decide if the following are true or false.

a $b < a$ **b** $a - b < 0$ **c** $b - a < 0$

15 For what value of a is $a = -a$?

16 Find a method to evaluate the following without using a calculator or algorithm. Explain your method.

$$-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - \ldots - 997 + 998 - 999 + 1000$$

Simultaneous integers	—	—	17

ENRICHMENT

17 Find the pairs of integers (a, b) that satisfy both equations in each part.

a $a + b = 5$ and $a - b = -3$ **b** $a + b = -4$ and $a - b = -10$

c $a + 2b = -1$ and $a - 2b = -9$ **d** $a + b = -8$ and $a - 2b = -14$

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1F Addition and subtraction of negative integers

CONSOLIDATING

Interactive

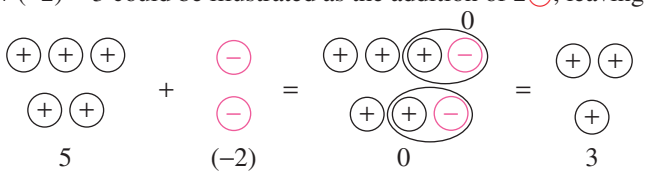
Widgets

HOTsheets

Walkthroughs

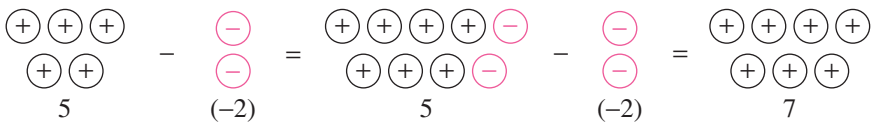
If ⊕ represents +1 and ⊖ represents −1 then ⊕⊖ added together has a value of zero.

Using these symbols 5 + (−2) = 3 could be illustrated as the addition of 2⊖, leaving a balance of 3.



So 5 + (−2) is the same a 5 − 2.

Also 5 − (−2) = 7 could be illustrated first as 7⊕ and 2⊖ together then subtracting the 2⊖.



So 5 − (−2) is the same a 5 + 2.

When adding or subtracting negative integers, we follow the rules set out by the above two illustrations.

Let's start: Circle arithmetic

Use ⊕ and ⊖ as shown in the introduction to illustrate and calculate the answers to these additions and subtractions.

- 3 + (−2)
- −2 + (−4)
- −5 + (−2)
- 3 − (−2)
- −3 − (−2)
- −1 − (−4)



Negative numbers can be used to describe the depth of divers below sea level. The deeper the dive, the larger the negative number.

Key ideas

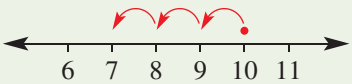
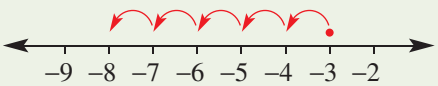
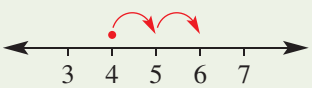
- Adding a negative number is the same as subtracting its opposite.
  For example:
  2 + (−3) = 2 − 3 = −1
  −4 + (−7) = −4 − 7 = −11
- Subtracting a negative number is the same as adding its opposite.
  For example:
  2 − (−5) = 2 + 5 = 7
  −6 − (−4) = −6 + 4 = −2

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Example 11 Adding and subtracting negative integers

Evaluate the following.

a $10 + (-3)$ **b** $-3 + (-5)$ **c** $4 - (-2)$ **d** $-11 - (-6)$

SOLUTION	EXPLANATION
a $10 + (-3) = 10 - 3$ $= 7$	Adding -3 is the same as subtracting 3. 
b $-3 + (-5) = -3 - 5$ $= -8$	Adding -5 is the same as subtracting 5. 
c $4 - (-2) = 4 + 2$ $= 6$	Subtracting -2 is the same as adding 2. 
d $-11 - (-6) = -11 + 6$ $= -5$	Subtracting -6 is the same as adding 6. 

Exercise 1F

1(½), 2, 3(½) | 3(½) | —

UNDERSTANDING

1 -3 and 3 are opposites. Write down the opposites of these numbers.

a -6 **b** 10 **c** 38 **d** -46
e -32 **f** 88 **g** 673 **h** -349

2 Write the words 'add' or 'subtract' to suit each sentence.

a To add a negative number _____ its opposite.
b To subtract a negative number _____ its opposite.

3 Decide if the following statements are true or false.

a $5 + (-2) = 5 + 2$ **b** $3 + (-4) = 3 - 4$ **c** $-6 + (-4) = -6 - 4$
d $-1 + (-3) = 1 - 3$ **e** $8 - (-3) = 8 + 3$ **f** $2 - (-3) = 2 - 3$
g $-3 - (-1) = 3 + 1$ **h** $-7 - (-5) = -7 + 5$ **i** $-6 - (-3) = 6 + 3$

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1F

FLUENCY

4–5(½) 4–6(½) 4–6(½)

Example 11a
Example 11b

4 Evaluate the following.

a $6 + (-2)$ **b** $4 + (-1)$ **c** $7 + (-12)$ **d** $20 + (-5)$
e $2 + (-4)$ **f** $26 + (-40)$ **g** $-3 + (-6)$ **h** $-16 + (-5)$
i $-18 + (-20)$ **j** $-36 + (-50)$ **k** $-83 + (-22)$ **l** $-120 + (-139)$

Example 11c
Example 11d

5 Evaluate the following.

a $2 - (-3)$ **b** $4 - (-4)$ **c** $15 - (-6)$ **d** $24 - (-14)$
e $59 - (-13)$ **f** $147 - (-320)$ **g** $-5 - (-3)$ **h** $-8 - (-10)$
i $-13 - (-16)$ **j** $-10 - (-42)$ **k** $-88 - (-31)$ **l** $-125 - (-201)$

6 Write down the missing number.

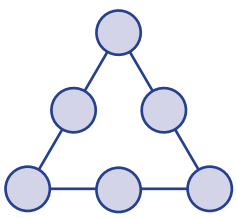
a $4 + ____ = 1$ **b** $6 + ____ = 0$ **c** $-2 + ____ = -1$
d $____ + (-8) = 2$ **e** $____ + (-5) = -3$ **f** $____ + (-3) = -17$
g $12 - ____ = 14$ **h** $8 - ____ = 12$ **i** $-1 - ____ = 29$
j $____ - (-7) = 2$ **k** $____ - (-2) = -4$ **l** $____ - (-436) = 501$

PROBLEM-SOLVING

7, 8 8–10 10–12

7 Place the integers from -3 to 2 in this magic triangle so that each side adds to the given number.

a -3 **b** 0



8 A magic square has each row, column and main diagonal adding to the same magic sum. Complete these magic squares.

a

		1
0	−2	−4

b

−12		
	−15	
	−11	−18

9 Find a pair of negative integers a and b that satisfy both equations in each part.

a $a + b = -8$ and $a - b = 2$
b $a + b = -24$ and $a - b = -6$

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1F

PROBLEM-SOLVING

10 A bank account has an initial balance of $150. Over a one-week period the following occurred.

- $180 was spent on shoes.
- $300 of debt was added to the account as a cash advance.
- $250 of debt was repaid.
- $110 of debt was added because of a bank fee.
- $150 of debt was removed with a cash deposit.



What was the balance of the account at the end of the week?

11 The sum of two integers is −5 and their difference is 11. What are the two numbers?

12 The sum of two integers is 11 and their difference is 19. What are the two numbers?

REASONING

	13	13, 14	14, 15

13 Describe the error made in the working shown.

a $5-(-2)=5-2$
$=3$

b $-2+(-3)=2-3$
$=-1$

14 If a and b are both negative integers and $a > b$, decide if the following are *always* less than zero.

a $a+b$ **b** $b+a$ **c** $a-b$ **d** $b-a$

15 If a is a negative number, decide if the following are equal to zero.

a $a+a$ **b** $a-a$ **c** $a+(-a)$ **d** $a-(-a)$

ENRICHMENT

Applying rules	—	—	16, 17

16 A rule linking two integers x and y is given by $y = 5 - x$.

a Complete this table.

x	−2	−1	0	1	2	3
y						

b Find a value for y if $x = -13$.

c Find a value for x if $y = 50$.

17 A rule linking two integers x and y is given by $x - y = -3$.

a Complete this table.

x	−3	−2	−1	0	1	2	3
y							

b Find a value for y if $x = 12$.

c Find a value for x if $y = -6$.

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Progress quiz

1A **1** Evaluate these sums and differences mentally.

a $86 - 53$ **b** $28 + 14$ **c** $213 + 145$ **d** $462 - 70$

1A **2** Use an algorithm to find these sums and differences.

a $\begin{array}{r} 58 \\ +265 \\ \hline \end{array}$ **b** $\begin{array}{r} 82 \\ -45 \\ \hline \end{array}$ **c** $\begin{array}{r} 378 \\ +26 \\ +139 \\ \hline \end{array}$ **d** $\begin{array}{r} 5024 \\ -2957 \\ \hline \end{array}$

1B **3** Use a mental strategy to evaluate the following.

a 5×140 **b** 6×49 **c** $128 \div 8$ **d** $1692 \div 4$

1B **4** Use an algorithm to evaluate the following.

a $\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$ **b** $\begin{array}{r} 307 \\ \times 219 \\ \hline \end{array}$ **c** $7\overline{)427}$ **d** $15\overline{)347}$

1C **5** **a** Find the LCM of 8 and 12. **b** Find the HCF of 24 and 30.

1C **6** Evaluate these squares and square roots.

a 6^2 **b** 30^2 **c** $\sqrt{64}$ **d** $\sqrt{2500}$

1C **7** Evaluate these cubes and cube roots.

a 2^3 **b** 100^3 **c** $\sqrt[3]{27}$ **d** $\sqrt[3]{125}$

1D **8** Use a factor tree to write 360 as a product of prime factors.

1D **9** Use divisibility tests to decide if the number 126 is divisible by 2, 3, 4, 5, 6, 8 or 9. State a reason for each answer.

1D **10** Find the HCF and LCM of these pairs of numbers, using prime factorisation.

a 42 and 18 **b** 105 and 90

1E **11** Evaluate the following.

a $-6 + 20$ **b** $-5 - 12$ **c** $-206 + 132$

d $-218 - 234$ **e** $-5 + 7 - 9 - 6$ **f** $12 - 46 + 27 - 63$

1F **12** Evaluate the following.

a $8 + (-14)$ **b** $-220 + (-146)$ **c** $17 - (-8)$ **d** $-12 - (-61)$

1D **13** Three schools are competing at a sports carnival. Each school has a different coloured sports uniform. The numbers of Year 8 students competing are: 162 with a green uniform, 108 with a red uniform and 144 with a blue uniform. All the Year 8 students are to be split up into equal sized teams.

a What is the largest possible team size so every Year 8 student is in a team of students all from their own school?

b How many of these Year 8 teams will be formed from each school?

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1G Multiplication and division of integers









As a repeated addition, $3 \times (-2)$ can be written as $(-2) + (-2) + (-2) = -6$. So $3 \times (-2) = -6$ and, since $a \times b = b \times a$ for all numbers a and b, then -2×3 is also equal to -6.

For division we can write the product $3 \times 2 = 6$ as a quotient $6 \div 2 = 3$. Similarly, if $3 \times (-2) = -6$ then $-6 \div (-2) = 3$. Also if $-2 \times 3 = -6$ then $-6 \div 3 = -2$.

These observations suggest that the quotient of two negative numbers results in a positive number and the product or quotient of two numbers of opposite sign is a negative number.

$6 \div (-2) = -3$ can also be rearranged to $-3 \times (-2) = 6$, which also suggests that the product of two negative numbers is a positive number.

Let's start: Logical patterns

Complete the patterns in these tables to discover the rules for the product of integers.

□	△	□ × △
3	2	6
2	2	
1	2	
0	2	
−1	2	
−2	2	
−3	2	

□	△	□ × △
3	−2	−6
2	−2	−4
1	−2	
0	−2	
−1	−2	
−2	−2	
−3	−2	

Use the table results to complete these sentences.

- $3 \times 2 = 6$ so $6 \div$ ____ $= 3$
- $-3 \times 2 =$ ____ so $-6 \div 2 =$ ____
- $3 \times (-2) =$ ____ so ____ $\div (-2) = 3$
- $-3 \times (-2) =$ ____ so $6 \div (-2) =$ ____

What do these observations tell us about multiplying and dividing positive and negative numbers?

Key ideas

- The product or quotient of two integers of the same sign is a positive integer.
  - Positive × Positive = Positive
  - Positive ÷ Positive = Positive
  - Negative × Negative = Positive
  - Negative ÷ Negative = Positive
- The product or quotient of two integers of opposite signs is a negative integer.
  - Positive × Negative = Negative
  - Positive ÷ Negative = Negative
  - Negative × Positive = Negative
  - Negative ÷ Positive = Negative

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Example 12 Finding products and quotients of integers

Evaluate the following.

a $3 \times (-7)$ **b** $-4 \times (-12)$ **c** $-63 \div 7$ **d** $-121 \div (-11)$

SOLUTION	EXPLANATION
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative.
b $-4 \times (-12) = 48$	-4 and -12 are both negative and so the product will be positive.
c $-63 \div 7 = -9$	The two numbers are of opposite sign so the answer will be negative.
d $-121 \div (-11) = 11$	-121 and -11 are both negative so the quotient will be positive.

Example 13 Combining multiplication and division

Work from left to right to evaluate $-2 \times 9 \div (-3) \times (-5)$

SOLUTION	EXPLANATION
$-2 \times 9 \div (-3) \times (-5) = -18 \div (-3) \times (-5)$	First, evaluate $-2 \times 9 = -18$
$= 6 \times (-5)$	$-18 \div (-3) = 6$
$= -30$	$6 \times (-5) = -30$

Exercise 1G

1–3 | 3 | —

UNDERSTANDING

1 Write the missing numbers in these tables. You should create a pattern in the third column.

a

□	△	□ × △
3	5	15
2	5	
1	5	
0	5	
−1	5	
−2	5	
−3	5	

b

□	△	□ × △
3	−5	−15
2	−5	−10
1	−5	
0	−5	
−1	−5	
−2	−5	
−3	−5	

2 Write the missing numbers in these sentences. Use the tables in Question **1** to help.

a $3 \times 5 =$ ____ so $15 \div 5 =$ ____ **b** $-3 \times 5 =$ ____ so $-15 \div 5 =$ ____

c $3 \times (-5) =$ ____ so $15 \div (-5) =$ ____ **d** $-3 \times (-5) =$ ____ so $15 \div (-5) =$ ____

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1G

UNDERSTANDING

3 Decide if these statements are true or false.

a Any integer multiplied by zero is equal to zero.

b The product of two positive integers is negative.

c The product of two positive integers is positive.

d The quotient of two integers of opposite sign is negative.

e The quotient of two integers of the same sign is negative.

FLUENCY

4–6(½) 4–6(½), 7 4–7(½)

Example 12a Example 12b

4 Evaluate the following.

a $4 \times (-5)$ **b** $6 \times (-9)$ **c** -4×10 **d** -11×9

e $-2 \times (-3)$ **f** $-5 \times (-21)$ **g** $-20 \times (-20)$ **h** $-100 \times (-3)$

i -4×38 **j** $41 \times (-3)$ **k** $-18 \times (-3)$ **l** $-51 \times (-15)$

Example 12c Example 12d

5 Evaluate the following.

a $-10 \div 2$ **b** $-38 \div 19$ **c** $-60 \div 15$ **d** $-120 \div 4$

e $32 \div (-16)$ **f** $52 \div (-2)$ **g** $180 \div (-4)$ **h** $900 \div (-25)$

i $-6 \div (-2)$ **j** $-30 \div (-10)$ **k** $-45 \div (-5)$ **l** $-300 \div (-50)$

6 Write the missing number.

a $____ \times 3 = -9$ **b** $____ \times (-7) = 35$ **c** $____ \times (-4) = -28$

d $-3 \times ____ = -18$ **e** $-19 \times ____ = 57$ **f** $____ \div (-9) = 8$

g $____ \div 6 = -42$ **h** $85 \div ____ = -17$ **i** $-150 \div ____ = 5$

Example 13

7 Evaluate the following by working from left to right.

a $-4 \times 2 \div (-8)$ **b** $30 \div (-15) \times (-7)$ **c** $48 \div (-3) \times (-10)$

d $-1 \times 58 \times (-2) \div (-4)$ **e** $-110 \div (-11) \times 12 \div (-1)$ **f** $-15 \times (-2) \div (-3) \times (-2)$

PROBLEM-SOLVING

8, 9 9–11 10–12

8 Insert $\times$ signs and/or $\div$ signs to make these equations true.

a $-2 ____ 3 ____ (-6) = 1$ **b** $10 ____ (-5) ____ (-2) = 25$

c $6 ____ (-6) ____ 20 = -20$ **d** $-14 ____ (-7) ____ (-2) = -1$

e $-32 ____ (-3) ____ (-2) = -48$ **f** $130 ____ (-4) ____ (-8) = 65$

9 The average of three numbers is -4. A new number is added to the list making the average -3. What is the new number?

10 The average of 10 numbers is -5. A new number is added to the list making the average -6. What is the new number?

11 The product of two numbers is -24 and their sum is -5. What are the two numbers?

12 The quotient of two numbers is -4 and their difference is 10. What are the two numbers?

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1G

13 | 13, 14 | 14–17

REASONING

13 Remember that $a^2 = a \times a$ and $a^3 = a \times a \times a$.

a Evaluate these expressions.

i $(-2)^2$ **ii** $(-3)^3$ **iii** $(-4)^3$ **iv** $(-5)^2$

b Will the square of a negative number always be positive? Explain why.

c Will the cube of a negative number always be negative? Explain why.

14 a and b are both positive integers with $a > b$. Decide if the following are true or false.

a $-a < b$ **b** $-a \times b > 0$ **c** $-a \div b < 0$

15 Consider the rule $y = -2x - 4$.

a Find the value of y if $x = -3$.

b Find the value of x if $y = -2$.

c Find the value of x that makes $y = 0$.

d Find the value of x that makes $y = -100$.

16 We know that $3^2 = 9$ and $(-3)^2 = 9$. Explain why $\sqrt{-9}$ is not a real number.

17 Is it possible to find the cube root of a negative number? Explain why and give some examples. $\sqrt[3]{-1} = ?$

What's my integer rule?

— | — | 18

ENRICHMENT

18 Find a rule linking x and y for these tables. Start your rules by making y the subject, e.g. $y = -2x + 1$.

a

x	y
−3	8
−2	5
−1	2
0	−1
1	−4
2	−7

b

x	y
−3	18
−2	11
−1	4
0	−3
1	−10
2	−17

c

x	y
−4	17
−2	5
0	1
2	5
4	17
6	37



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1H Order of operations and substitution









An expression such as $a + 2b$ can be evaluated if we know the values of a and b. The expression includes the operations addition (listed first) and multiplication ($2b$ is $2 \times b$); however, by convention, we know that multiplication is done before the addition. In this section we will deal with order of operations, using both positive and negative integers.

Let's start: Equal to 1

Maria makes up a difficult problem for which the answer is equal to 1 but forgets to bring the piece of paper she wrote it on to class. Maria remembers the numbers but not the extra sets of brackets that were involved.

$$-5 \times (-4) + 2 + (-40) \div 5 + 3 - 4 = 1$$

She remembers that there were 2 extra sets of brackets that should be inserted. Can you decide where they should go?

Key ideas

- The rules for order of operations are:
  - Deal with operations inside brackets first.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

For example:

$$\begin{aligned}(-2 + 3) \times 4 + 6 - 5 \div (-5) &= 1 \times 4 + 6 - 5 \div (-5) \\ &= 4 + 6 - (-1) \\ &= 10 + 1 = 11\end{aligned}$$

- Expressions can be evaluated by substituting numbers for the given pronumerals.
  For example: If $a = -2$ and $b = -3$ then $\begin{aligned}a + 5b &= -2 + 5 \times (-3) \\ &= -2 + (-15) \\ &= -17\end{aligned}$
  - Remember, for example, that $5b$ means $5 \times b$ and $\frac{a}{3}$ means $a \div 3$.



Example 14 Using order of operations

Evaluate the following.

a $5 - 6 \times (-2)$ **b** $-21 \div (5 - (-2))$

SOLUTION	EXPLANATION
a $\begin{aligned}5 - 6 \times (-2) &= 5 - (-12) \\ &= 17\end{aligned}$	Do the multiplication before the addition and remember that $5 - (-12) = 5 + 12$.
b $\begin{aligned}-21 \div (5 - (-2)) &= -21 \div 7 \\ &= -3\end{aligned}$	Deal with brackets first and remember that $5 - (-2) = 5 + 2$.

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Example 15 Substituting integers

Substitute the given integers to evaluate the expressions.

a $a - 3b$ with $a = -2$ and $b = -4$

b $(a + b) \div (-5)$ with $a = -7$ and $b = 2$

c $a^2 - b^3$ with $a = -2$ and $b = -3$

SOLUTION	EXPLANATION
a $a - 3b = -2 - 3 \times (-4)$ $= -2 - (-12)$ $= 10$	Substitute $a = -2$ and $b = -4$ and then evaluate, noting that $-2 - (-12) = -2 + 12$.
b $(a + b) \div (-5) = (-7 + 2) \div (-5)$ $= -5 \div (-5)$ $= 1$	Substitute $a = -7$ and $b = 2$ and then deal with the brackets before the division.
c $a^2 - b^3 = (-2)^2 - (-3)^3$ $= 4 - (-27)$ $= 4 + 27$ $= 31$	Use brackets when substituting into expressions with powers. $(-2)^2 = -2 \times (-2) = 4$ $(-3)^3 = -3 \times (-3) = -27$

Exercise 1H

1–3(½) | 2–3(½) | —

UNDERSTANDING

1 Decide if both sides of these simple statements are equal.

a $(2 + 3) - 1 = 2 + 3 - 1$

b $(3 + (-2)) - (-1) = 3 + (-2) - (-1)$

c $5 \times (2 + (-3)) = 5 \times 2 + (-3)$

d $-8 \times 2 - (-1) = -8 \times (2 - (-1))$

e $-10 \div 2 - 4 = -10 \div (2 - 4)$

f $-2 \times 3 + 8 \div (-2) = (-2 \times 3) + (8 \div (-2))$

2 Copy and complete the working for each problem.

a $-12 \div (6 + (-2)) = -12 \div$ ____
$=$ ____

b $(-8 + 2) \times (-3) =$ __ $\times (-3)$
$=$ ____

c $(-2 + (-1)) \div (15 \div (-5))$
$=$ ____ $\div (15 \div (-5))$
$=$ ____ $\div (-3)$
$=$ ____

d $6 \times (-1 - 5) \div 9 = 6 \times$ ____ $\div 9$
$=$ ____ $\div 9$
$=$ ____

3 Complete the working for these substitutions.

a $a + 2b$ $(a = -3, b = 4)$
$a + 2b = -3 + 2 \times 4$
$=$ ____ + ____
$=$ ____

b $6a - 2b$ $(a = 3, b = -4)$
$6a - 2b = 6 \times 3 - 2 \times (-4)$
$= 18 -$ ____
$=$ ____

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1H

UNDERSTANDING

c $3 \times (a - b)$ $(a = 5, b = -1)$

$3 \times (a - b) = 3 \times (5 - (-1))$

$= 3 \times$ ____

$=$ ____

d $-2 \times (a - 1) \div b$ $(a = 11, b = -5)$

$-2 \times (a - 1) \div b = -2 \times ($ ____ $- 1) \div$ ____

$= -2 \times$ ____ $\div$ ____

$=$ ____ $\div$ ____

$=$ ____

4–6(½), 8(½) | 4–9(½) | 5–10(½)

FLUENCY

Example 14a

4 Evaluate the following. Remember to use the normal order of operations.

a $-2 \times 3 \times 5$ **b** $-6 - 2 \times 3$ **c** $4 - 8 \times (-1)$

d $-3 \div (-1) + 7 \times (-2)$ **e** $6 \times (-2) - 10 \div (-5)$ **f** $4 + 8 \times (-2) \div (-16)$

g $20 - 10 \div (-5) \times 2$ **h** $0 \times (-3) + 2 \times (-30)$ **i** $35 - 10 \div (-2) + 0$

Example 14b

5 Use order of operations to evaluate the following.

a $3 \times (2 - 4)$ **b** $(7 - (-1)) \times 3$ **c** $(-8 + (-2)) \div (-5)$

d $40 \div (8 - (-2)) + 3$ **e** $0 \times (38 - (-4)) \times (-6)$ **f** $-6 \times (-1 + 3) \div (-4)$

g $((-2) + 1) \times (8 - (-3))$ **h** $(-6 - 4) \div (50 \div (-10))$ **i** $-2 \times (8 - 7 \times (-2))$

j $(3 - (-2) \times 2) \div 7$ **k** $-4 \times (2 - (-6) \div 6)$ **l** $-5 \div (1 - 3 \times 2)$

Example 15a

Example 15b

6 Evaluate these expressions using $a = -2$ and $b = 1$.

a $a + b$ **b** $a - b$ **c** $2a - b$

d $b - a$ **e** $a - 4b$ **f** $3b - 2a$

g $b \times (2 + a)$ **h** $(2b + a) - (b - 2a)$

7 Evaluate these expressions using $a = -3$ and $b = 5$.

a ab **b** ba **c** $a + b$ **d** $a - b$

e $b - a$ **f** $3a + 2b$ **g** $(a + b) \times (-2)$ **h** $(a + b) - (a - b)$

Example 15c

8 Evaluate these expressions using $a = -3$ and $b = 5$.

a $a + b^2$ **b** $a^2 - b$ **c** $b^2 - a$ **d** $b^3 + a$

e $a^3 - b$ **f** $a^2 - b^2$ **g** $b^3 - a^3$ **h** $(b - a^2)^2$

9 Evaluate these expressions using $a = -4$ and $b = -3$.

a $3a + b$ **b** $b - 2a$ **c** $4b - 7a$ **d** $-2a - 2b$

e $4 + a - 3b$ **f** $ab - 4a$ **g** $-2 \times (a - 2b) + 3$ **h** $ab - ba$

i $3a + 4b + ab$ **j** $a^2 - b$ **k** $a^2 - b^2$ **l** $b^3 - a^3$

10 Evaluate the following.

a $3 \times (-2)^2$ **b** $-2 \times (-2)^3$ **c** $-16 \div (-2)^3$

d $-4 + \sqrt{25}$ **e** $7 - \sqrt{16}$ **f** $-26 + \sqrt[3]{27}$

g $-4 + 2 \times \sqrt[3]{8}$ **h** $-8 \div \sqrt[3]{-64} + 1$ **i** $-3 \times (-2)^3 + 4$

j $(3 - (-4)^2) \times (-2)$ **k** $(\sqrt[3]{-27} + 3) \div (-1)$ **l** $\sqrt[3]{-8} \times (\sqrt[3]{1000} + 1)$

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1H

11	11, 12	11, 12

PROBLEM-SOLVING

11 The temperature in a mountain hut is 15°C at 9 p.m. on Monday night. It drops by 2°C per hour for 11 hours and then the next morning rises by 1°C per hour for the next 4 hours. What is the temperature at midday on Tuesday?



A mountain hut in Tasmania

12 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$

b $-10 \div 3 - (-2) = -2$

c $-8 \div (-1) + 5 = -2$

d $-1 - 4 \times 2 + (-3) = 5$

e $-4 + (-2) \div 10 + (-7) = -2$

f $20 + 2 - 8 \times (-3) = 38$

g $1 - (-7) \times 3 \times 2 = 44$

h $4 + (-5) \div 5 \times (-2) = -6$

13	13, 14	13–15

REASONING

13 If a, b and c are integers, decide whether or not the following equations are always true.

a $(a + b) + c = a + (b + c)$

b $(a - b) - c = a - (b - c)$

c $(a \times b) \times c = a \times (b \times c)$

d $(a \div b) \div c = a \div (b \div c)$

e $a - b = b - a$

f $-(a - b) = b - a$

14 We can write $(a + b) \div c$ without brackets in the form $\frac{a+b}{c}$. Evaluate these expressions if $a = -5$, $b = -3$ and $c = -2$.

a $\frac{a+b}{c}$ **b** $\frac{a-b}{c}$ **c** $\frac{2c-5a}{b}$ **d** $\frac{-c-2a}{b}$

15 We can use brackets within brackets for more complex expressions. The inside brackets are dealt with first. Evaluate these.

a $(-6 \times (-2 + 1) + 3) \times (-2)$ **b** $(2 - (3 - (-1))) \times (-2)$ **c** $-10 \div (2 \times (3 - (-2)))$

Tricky brackets

—	—	16–18

ENRICHMENT

16 Insert one or more sets of brackets to make these statements true.

a $1 - 3 \times (-4) \div (-13) = -1$

b $4 \div 3 + (-7) \times (-5) = 5$

c $6 - 7 \div (-7) + 6 = 1$

d $-1 - 5 + (-2) \times 1 - 4 = 8$

17 By inserting one extra set of brackets, how many different answers could be obtained from $-4 \times 3 - (-2) + 8$?

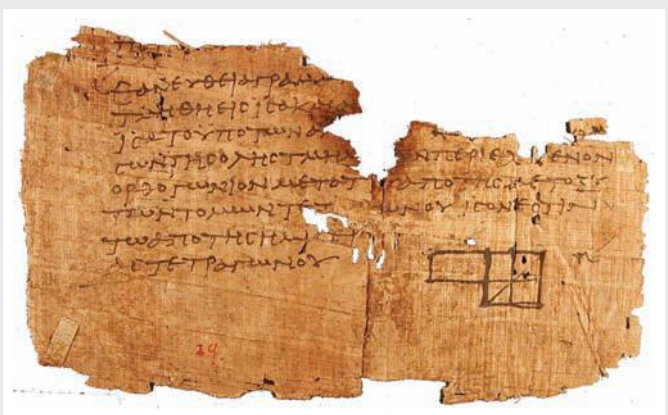
18 Make up your own statement like that in Questions **16** and then remove any brackets. Ask a friend to see if they can find where the brackets should go.

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Investigation

Euclidean division algorithm

The Euclidean division algorithm is a method for finding the highest common factor (also called the greatest common divisor) of two numbers. It is a method that can be performed by hand or programmed into a computer to quickly find the result. Euclid, the famous Greek mathematician, first published the algorithm in his well-known books titled *Elements* in about 300 BCE. The algorithm is used today in many mathematical situations. It is also an important part of today's public key encryption method that is used to code and decipher electronic information in the world of commerce.



This is a fragment of an Egyptian papyrus from a nearly 2000-year-old copy of Euclid's *Elements*, written in ancient Greek. The diagram shows that the text concerns the relationship of squares and rectangles derived from a straight line divided into unequal parts.

Source: Bill Casselman

In simple terms this is how the algorithm works.

- Let the two numbers be a and b where $a > b$.
- Let $c = a - b$.
- Let the new a and b be the smallest pair from the previous a, b and c. Make $a > b$.
- Repeat the above two steps until $a = b$. The HCF is the value of a (or b) at this point.
- If $a - b = 1$ then the HCF = 1.

The algorithm uses the fact that if two numbers a and b have a common divisor then $a - b$ will also have the same common divisor.

Examples

1 Find the HCF of 12 and 30.

Step	a	b	$a - b = c$
1	30	12	30 – 12 = 18
2	18	12	18 – 12 = 6
3	12	6	12 – 6 = 6
4	6	6	0

The HCF of 12 and 30 is therefore 6.

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2 Find the HCF of 7 and 15.

Step	*a*	*b*	*a* – *b* = *c*
1	15	7	15 – 7 = 8
2	8	7	8 – 7 = 1

The HCF of 7 and 15 is therefore 1.

Using the algorithm

3 Use the Euclidean division algorithm to find the HCF of these pairs of numbers. Set your steps out in a table similar to the example above.

a 12 and 8 **b** 13 and 29 **c** 42 and 24

d 184 and 136 **e** 522 and 666 **f** 91 and 137

Using a spreadsheet

4 a Set up a spreadsheet using the formulas shown. Leave the cells A1 and B1 empty, as this is where you will enter your two starting numbers.

	A	B	C	D
1			=A1-B1	
2	=MEDIAN(A1,B1,C1)	=MIN(A1,B1,C1)		

b Enter the two numbers 30 and 12 into cells A1 and B1. Put the larger number into A1. Now fill down each formula until the value in column C is 0. The HCF is therefore 6.

	A	B	C
1	30	12	18
2	18	12	6
3	12	6	6
4	6	6	0

c Enter the two numbers 15 and 7 into cells A1 and B1. Put the larger number into A1. Now fill down each formula until the value in column C is 1. The HCF is therefore 1.

	A	B	C
1	15	7	8
2	8	7	1

d Test your spreadsheet on the pairs of numbers that you worked out by hand in Question **3** above. Here they are again.

i 12 and 8 **ii** 13 and 29 **iii** 42 and 24

iv 184 and 136 **v** 522 and 666 **vi** 91 and 137

e Now choose a pair of large numbers and use your spreadsheet to find the HCF.

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Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 List the numbers less than 50 that are the product of two prime numbers.

2 **a** Two squares have side lengths 5 cm and 12 cm. Determine the side length of a single square with an area equal to the combined area of these two squares.

b Three cubes have side lengths 1 cm, 6 cm and 8 cm. Determine the side length of a single cube equal in volume to the combined volume of these three cubes.

3 What is the smallest number divisible by all the digits 2, 3, 4, 5, 6, 7, 8 and 9?

4 Evaluate the following expressions given $x = -2$ and $y = -5$.

a $y + y^2 + y^3$ **b** $10 - 2(y - x)$ **c** $60 + 3(x^3 - y^2)$

5 The brackets are missing from these statements. Insert brackets to make them true.

a $-5 \times 3 \div (-3) + 2 - 4 + (-3) = -6$ **b** $-100 \div 4 \times (-2) - 2 \times 3 - (-2) = 32$

6 $n!$(n factorial) $= n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$, so $5! = 5 \times 4 \times 3 \times 2 \times 1$. Evaluate these without the use of a calculator.

a $6! \div 5!$ **b** $1000! \div 999!$ **c** $15! \div 13!$ **d** $5! - 4!$

7 Find a rule linking y and x in each table. Make y the subject of each, e.g. $y = -2x + 3$.

a

x	y
−7	10
−6	9
−5	8
−4	7

b

x	y
−4	13
−3	6
−2	1
−1	−2

c

x	y
−5	−121
−3	−23
−1	3
1	5

d

x	y
−27	−7
−8	−5
−1	−3
1	1

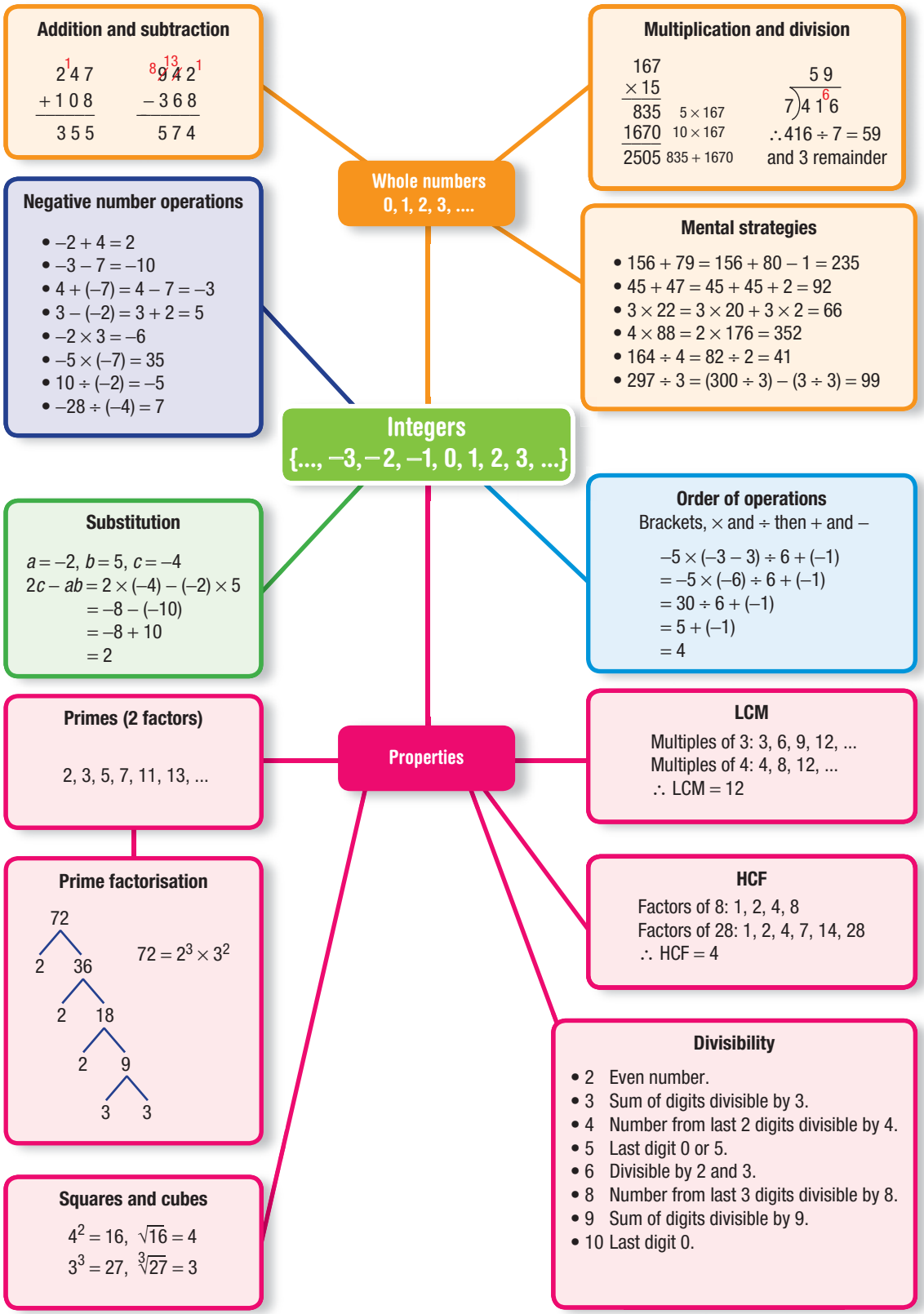
8 Determine the remainder when each of the following numbers is divided by 5.

a $4^{567} + 1$ **b** $4^{678} + 1$

9 Two different prime numbers a and b, are both less than 8. Determine which values of a and b give the largest HCF of $3a^2b$ and $2ab^2$ and state the value of the HCF.

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Chapter summary



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Multiple-choice questions

1A **1** 127 – 79 is the same as:

A 127 – 80 – 1 **B** 127 – 80 + 1 **C** 127 – 100 + 19
D 127 – 70 + 9 **E** 130 – 80 + 1

1A **2** The sum and difference of 291 and 147 are:

A 448 and 154 **B** 428 and 156 **C** 438 and 144
D 338 and 144 **E** 438 and 154

1A/B **3** Which of these four statements is/are true?

i $3 - 1 = 1 - 3$ **ii** $15 \div 5 = 5 \div 15$
iii $89 \times 3 = 90 \times 3 - 1 \times 3$ **iv** $171 + 50 = 170 + 50 - 1$

A **i** and **iii** **B** **ii** and **iv** **C** **i**, **ii** and **iii**
D **iv** only **E** **iii** only

1B **4** This division problem gives no remainder.

$$6\overline{)2\square 6}\quad (4\ 6)$$

The missing number is:

A 2 **B** 3 **C** 4 **D** 6 **E** 7

1C **5** The HCF and LCM (in that order) of 21 and 14 are:

A 7 and 14 **B** 14 and 21 **C** 42 and 7 **D** 7 and 28 **E** 7 and 42

1E **6** The temperatures of two countries on a particular day are –13°C and 37°C. The difference between the two temperatures is:

A 40°C **B** 36°C **C** 50°C **D** 46°C **E** 24°C



1F **7** The missing number in the statement $-4 - ____ = -1$ is:

A –3 **B** 3 **C** 5 **D** –6 **E** –5

1G **8** The missing number in the statement $____ \div (-7) = 8$ is:

A 42 **B** –42 **C** –6 **D** 56 **E** –56

1H **9** $-9 \times (-6 + (-2)) \div -12$ is equal to:

A 6 **B** –6 **C** –3 **D** –3 **E** –4

1G **10** Two negative numbers add to –5 and their product is 6. The two numbers are:

A –3, 2 **B** –4, –1 **C** –5, –1 **D** –3, –2 **E** –7, –2

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Chapter review

Short-answer questions

1A **1** Use a mental strategy to evaluate the following.

a $324 + 173$ **b** $592 - 180$ **c** $89 + 40$ **d** $135 - 68$

e $55 + 57$ **f** $280 - 141$ **g** $1001 + 998$ **h** $10\,000 - 4325$

1A **2** Use a mental strategy to find these sums and differences.

a $392 + 147$ **b** $1031 + 999$ **c** $147 - 86$ **d** $3970 - 896$

1B **3** Use a mental strategy for these products and quotients.

a $2 \times 17 \times 5$ **b** 3×99 **c** 8×42 **d** 141×3

e $164 \div 4$ **f** $357 \div 3$ **g** $618 \div 6$ **h** $1005 \div 5$

1B **4** Find these products and quotients using an algorithm.

a 139×12 **b** 507×42 **c** $3\overline{)843}$ **d** $7\overline{)854}$

1B **5** Find the remainder when 673 is divided by these numbers.

a 5 **b** 3 **c** 7 **d** 9

1C **6** Evaluate:

a $\sqrt{81}$ **b** $\sqrt{121}$ **c** 7^2 **d** 20^2

e $\sqrt[3]{27}$ **f** $\sqrt[3]{64}$ **g** 5^3 **h** 10^3

1C **7** **a** Find all the factors of 60.

b Find all the multiples of 7 between 110 and 150.

c Find all the prime numbers between 30 and 60.

d Find the LCM of 8 and 6.

e Find the HCF of 24 and 30.

1D **8** Write these numbers in prime factor form. You may wish to use a factor tree.

a 36 **b** 84 **c** 198

1D **9** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 84 **b** 155 **c** 124 **d** 621

1D **10** Write the numbers 20 and 38 in prime factor form and then use this to help find the following.

a LCM of 20 and 38 **b** HCF of 20 and 38

1E **11** Evaluate:

a $-6 + 9$ **b** $-24 + 19$ **c** $5 - 13$ **d** $-7 - 24$

e $-62 - 14$ **f** $-194 - 136$ **g** $-111 + 110$ **h** $-328 + 426$

1F **12** Evaluate:

a $5 + (-3)$ **b** $-2 + (-6)$ **c** $-29 + (-35)$ **d** $162 + (-201)$

e $10 - (-6)$ **f** $-20 - (-32)$ **g** $-39 - (-19)$ **h** $37 - (-55)$

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1F/G **13** a and b are both negative integers with $a > b$. Classify these as true or false.

a $b < a$ **b** $a + b > 0$ **c** $a \times b < 0$ **d** $a \div b > 0$

1G **14** Evaluate:

a -5×2 **b** $-11 \times (-8)$ **c** $9 \times (-7)$ **d** $-100 \times (-2)$

e $-10 \div (-5)$ **f** $48 \div (-16)$ **g** $-32 \div 8$ **h** $-81 \div (-27)$

1H **15** Evaluate using the order of operations.

a $2 + 3 \times (-2)$ **b** $-3 \div (11 + (-8))$

c $-2 \times 3 + 10 \div (-5)$ **d** $-20 \div 10 - 4 \times (-7)$

e $5 \times (-2 - (-3)) \times (-2)$ **f** $0 \times (-2 + 11 \times (-3)) + (-1)$

g $-19 \div (-18 - 1) \div (-1)$ **h** $15 \div (-2 + (-3)) + (-17)$

1H **16** Let $a = -2$, $b = 3$ and $c = -5$ and evaluate these expressions.

a $ab + c$ **b** $a^2 - b$ **c** $ac - b$ **d** abc

e $a^3 - bc$ **f** $c^3 - b$ **g** $bc \div b$ **h** $5b^3 - 2c$

Extended-response questions

1 A monthly bank account show deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

Details	P + W	Deposits	Balance
Opening balance	–	–	$250
Water bill	–$138	–	*a*
Cash withdrawal	–$320	–	*b*
Deposit	–	*c*	$115
Supermarket	*d*	–	–$160
Deposit	–	$400	*e*

a Find the values of a, b, c, d and e.

b If the water bill amount was $150, what would be the new value for letter e?

c What would the final deposit need to be if the value for e was $0? Assume the original water bill amount is $138 as in the table above.

2 Two teams compete at a club games night. Team A has 30 players while team B has 42 players.

a How many players are there in total?

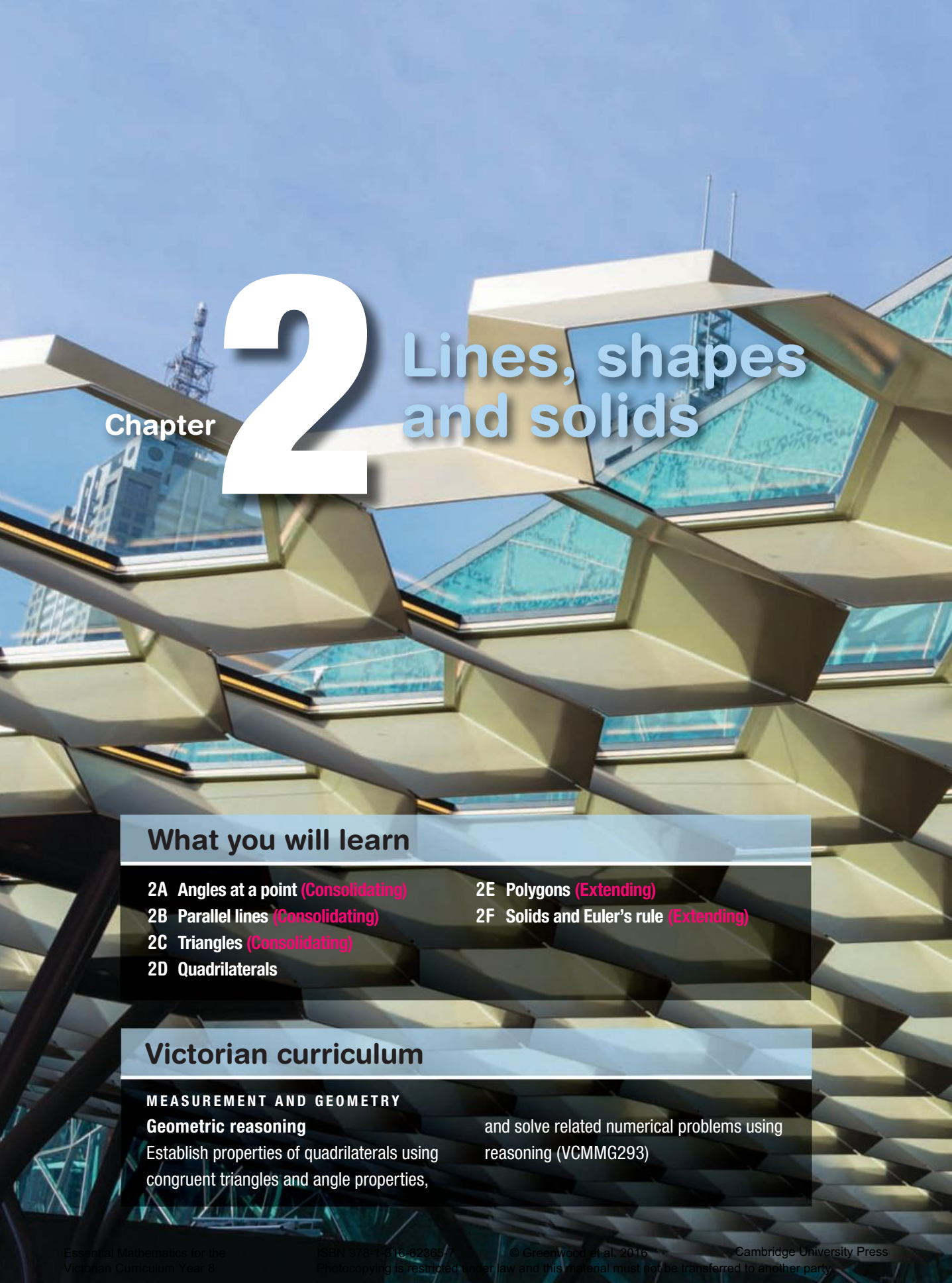
b Write both 30 and 42 in prime factor form.

c Find the LCM and HCF of the number of players representing the two teams.

d Teams are asked to divide into groups with equal numbers of players. What is the largest group size possible if team A and team B must have groups of the same size?

e In a game of 'scissors, paper, rock', each team forms a line in single file. Player 1 from team A plays against player 1 from team B, then the second pair play against each other, and so on. Once each game is complete, the players go to the back of their line. How many games are played before the first pair plays each other again?

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Chapter 2 Lines, shapes and solids

What you will learn

2A Angles at a point (Consolidating)
2B Parallel lines (Consolidating)
2C Triangles (Consolidating)
2D Quadrilaterals
2E Polygons (Extending)
2F Solids and Euler's rule (Extending)

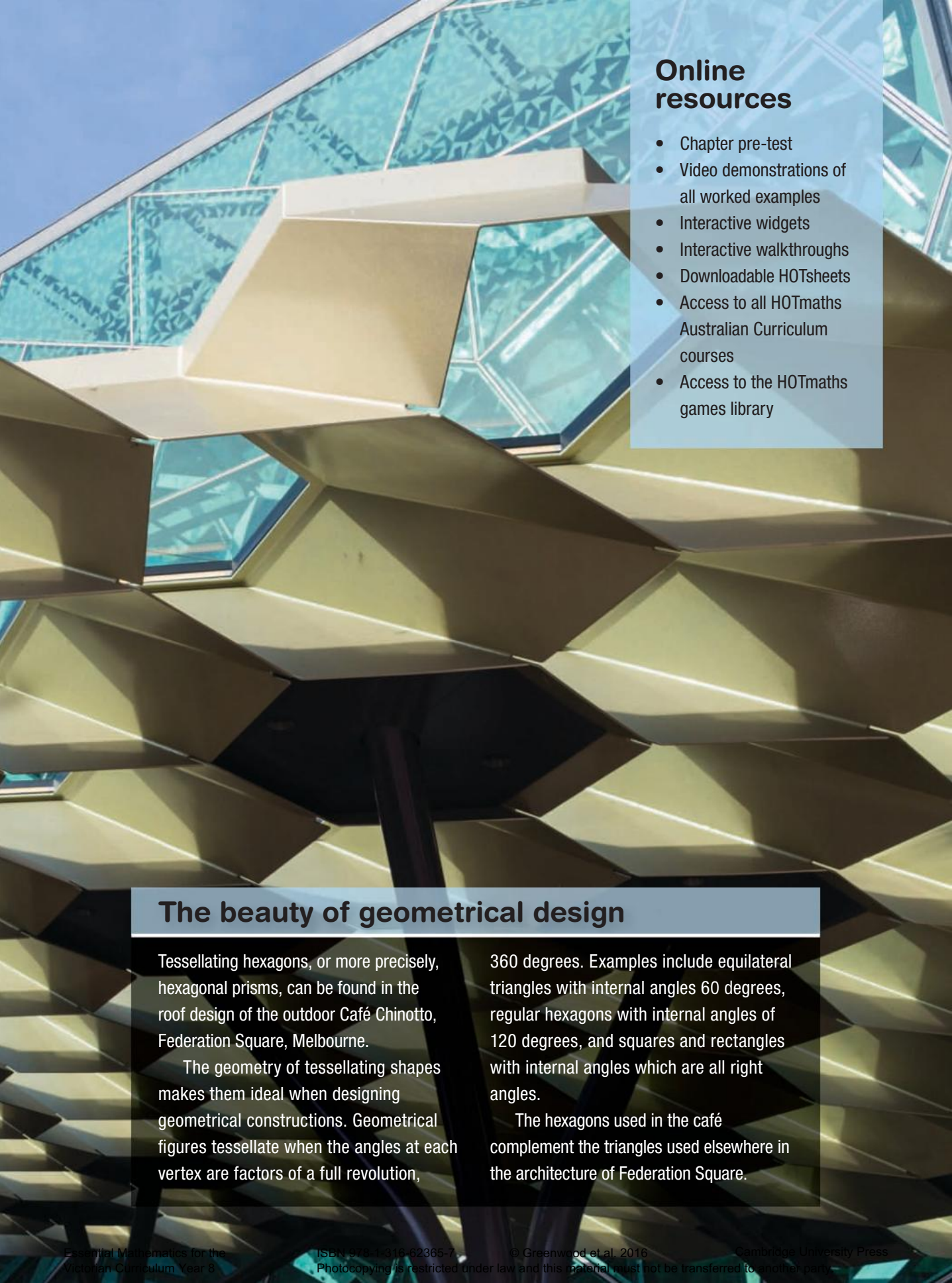
Victorian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (VCMMG293)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The beauty of geometrical design

Tessellating hexagons, or more precisely, hexagonal prisms, can be found in the roof design of the outdoor Café Chinotto, Federation Square, Melbourne.

The geometry of tessellating shapes makes them ideal when designing geometrical constructions. Geometrical figures tessellate when the angles at each vertex are factors of a full revolution, 360 degrees. Examples include equilateral triangles with internal angles 60 degrees, regular hexagons with internal angles of 120 degrees, and squares and rectangles with internal angles which are all right angles.

The hexagons used in the café complement the triangles used elsewhere in the architecture of Federation Square.

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2A Angles at a point

CONSOLIDATING









For more than 2000 years, geometry has been based on the work of Euclid, the Greek mathematician who lived in Egypt in about 300 BCE. Before this time, the ancient civilisations had demonstrated and documented an understanding of many aspects of geometry, but Euclid was able to produce a series of 13 books called *Elements*, which contained a staggering 465 propositions. This great work is written in a well-organised and structured form, carefully building on solid mathematical foundations. The most basic of these foundations, called axioms, are fundamental geometric principles from which all other geometry can be built. There are five axioms described by Euclid:

- Any two points can be joined by a straight line.
- Any finite straight line (segment) can be extended in a straight line.
- A circle can be drawn with any centre and any radius.
- All right angles are equal to each other.
- Given a line and a point not on the line, there is only one line through the given point and in the same plane that does not intersect the given line.

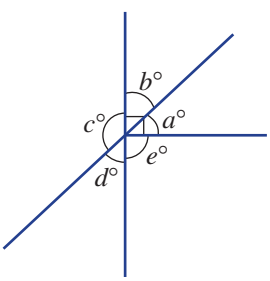


Euclid produced a series of 13 books called *Elements*.

These basic axioms are considered to be true without question and do not need to be proven. All other geometrical results can be derived from these axioms.

Let's start: Create a sentence or definition

The five pronumerals a, b, c, d and e represent the size of five angles in this diagram. Can you form a sentence using two or more of these pronumerals and one of the following words? Using simple language, what is the meaning of each of your sentences?

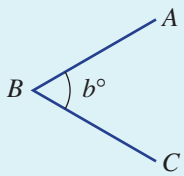


- Supplementary
- Adjacent
- Vertically opposite
- Revolution
- Complementary
- Right

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Key ideas

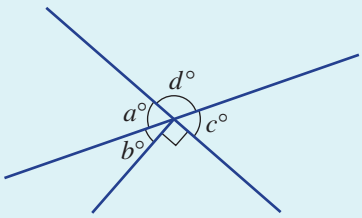
- The angle shown (to the right) could be named $\angle ABC$, $\angle CBA$, $\angle B$ or $A\hat{B}C$.
  - The size of the angle is $b°$.



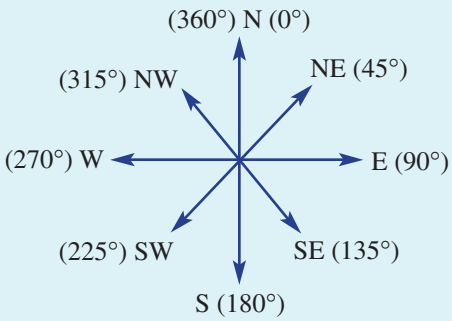
- Types of angles

Acute (0–90°)	**Right** (90°)	**Obtuse** (90–180°)
Straight (180°)	**Reflex** (180–360°)	**Revolution** (360°)

- Special pairs of angles at a point include:
  - **Complementary** angles (sum to 90°) $a + b = 90$
  - **Supplementary** angles (sum to 180°) $a + d = 180$
  - **Vertically opposite** angles (are equal) $a = c$



- Angles in a **revolution** sum to 360°.
- Two lines are **perpendicular** if they are at right angles (90°).
- Eight point compass bearing
  - Bearings are usually measured clockwise from north.



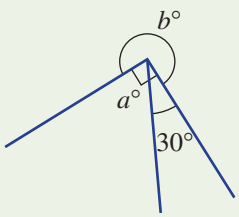


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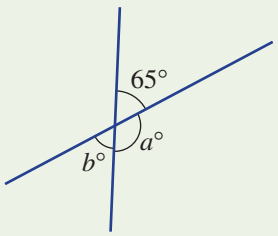
Example 1 Finding angles at a point

Determine the value of the pronumerals in these diagrams.

a



b



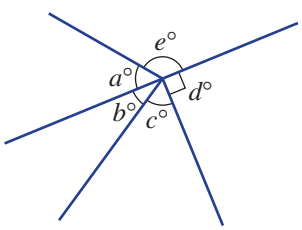
SOLUTION	EXPLANATION
a $a + 30 = 90$	$a°$ and 30° are the sizes of two angles which make a complementary pair, adding to 90°.
$a = 60$	
$b + 90 = 360$	Angles in a revolution add to 360°.
$b = 270$	
b $a + 65 = 180$	$a°$ and 65° are the sizes of two angles which make a supplementary pair, adding to 180°.
$a = 115$	
$b = 65$	The $b°$ and 65° angles are vertically opposite.

Exercise 2A

1–3 | 3 | —

UNDERSTANDING

1 Complete these sentences for this diagram.



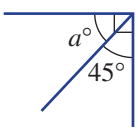
a $b°$ and $c°$ are _____ angles.

b $a°$ and $e°$ are _____ angles.

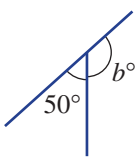
c $a°$, $b°$, $c°$, $d°$ and $e°$ form a _____.

2 State the value of the pronumeral (letter) in these diagrams.

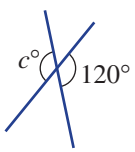
a



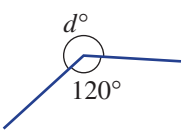
b



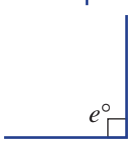
c



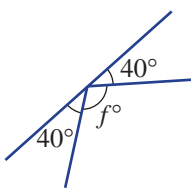
d



e



f



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2A

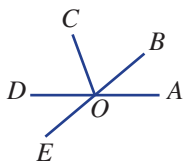
UNDERSTANDING

3 Estimate the size of these angles.

a $\angle AOB$

b $\angle AOC$

c Reflex $\angle AOE$



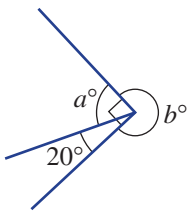
FLUENCY

4(½), 5, 6(½) 4(½), 5, 6–7(½) 4–7(½)

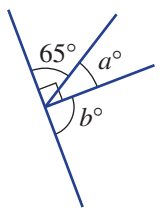
Example 1

4 Determine the value of the pronumerals in these diagrams.

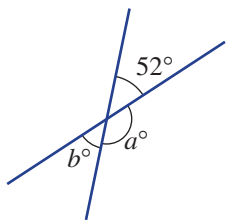
a



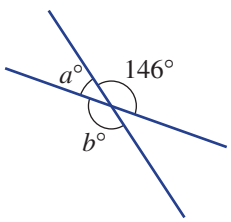
b



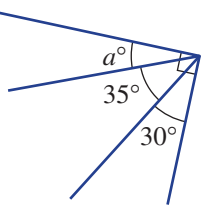
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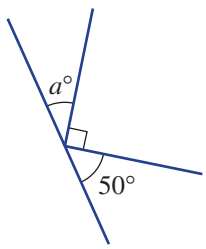
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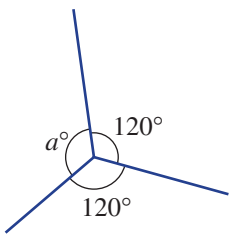
e



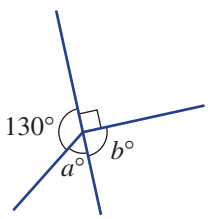
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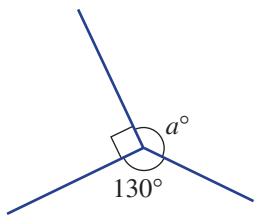
g



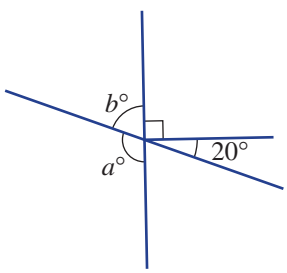
h



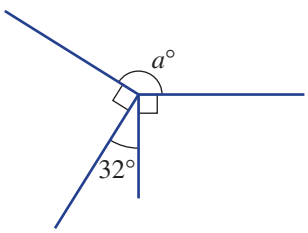
i



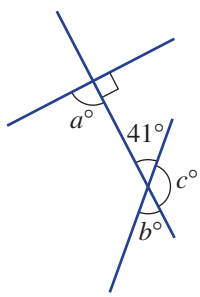
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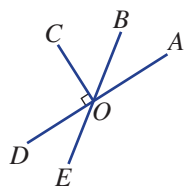
5 For the angles in this diagram, name an angle that is:

a vertically opposite to $\angle AOB$

b complementary to $\angle BOC$

c supplementary to $\angle AOE$

d supplementary to $\angle AOC$



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2A

FLUENCY

6 Give the compass bearing, in degrees, for these directions.

a West (W)	**b** East (E)	**c** North (N)	**d** South (S)
e NW	**f** SE	**g** SW	**h** NE

7 In which direction (e.g. north-east or NE) would you be walking if you were headed on these compass bearings?

a 180°	**b** 360°	**c** 270°	**d** 90°
e 45°	**f** 315°	**g** 225°	**h** 135°

8, 9(½) | 8, 9–10(½) | 9–10(½)

PROBLEM-SOLVING

8 A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

a everyone receives an equal share?

b Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have equal shares of the rest.)

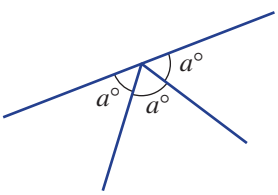
c Jack receives four times as much as everyone else?

d Jack receives ten times as much as everyone else?

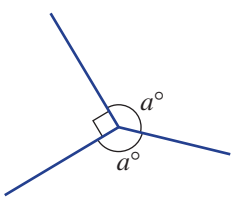


9 Find the value of a in these diagrams.

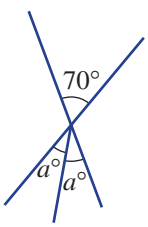
a



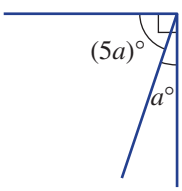
b



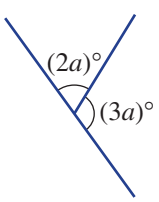
c



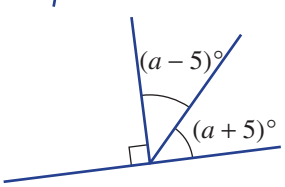
d



e



f



10 What is the angle between the hour hand and minute hand on a clock at these times?

a 2 : 30 p.m.	**b** 5 : 45 a.m.	**c** 1 : 40 a.m.	**d** 10 : 20 p.m.
e 2 : 35 a.m.	**f** 12 : 05 p.m.	**g** 4 : 48 p.m.	**h** 10 : 27 a.m.

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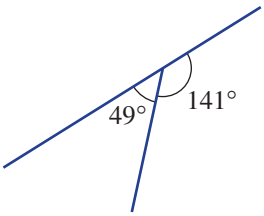
2A

11 | 11, 12 | 12, 13

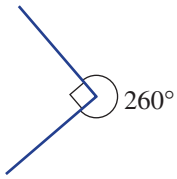
REASONING

11 Explain, with reasons, what is wrong with these diagrams.

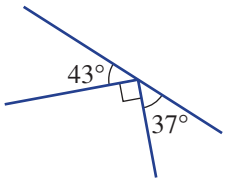
a



b

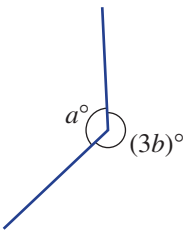


c

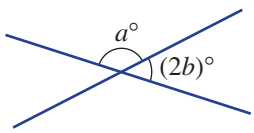


12 Write down an equation (e.g. $2a + b = 90$) for these diagrams.

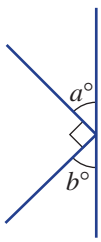
a



b



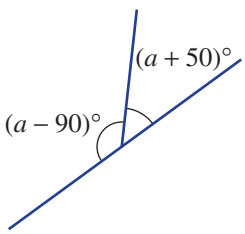
c



13 Consider this diagram (not drawn to scale).

a Calculate the value of a.

b Explain what is wrong with the way the diagram is drawn.

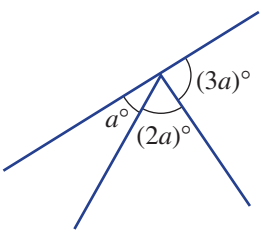


Geometry with equations — | — | 14

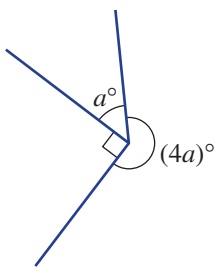
ENRICHMENT

14 Equations can be helpful in solving geometric problems in which more complex expressions are involved. Find the value of a in these diagrams.

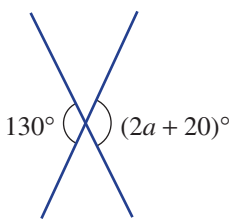
a



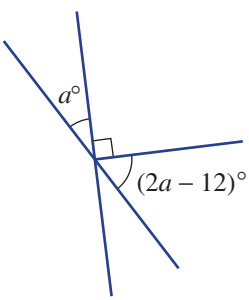
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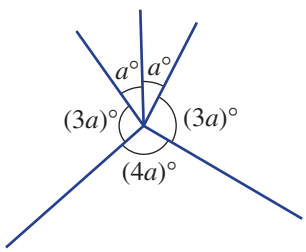
c



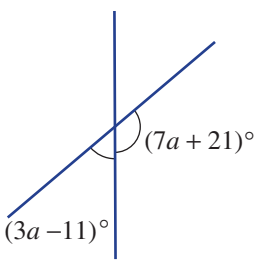
d



e



f



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2B Parallel lines CONSOLIDATING

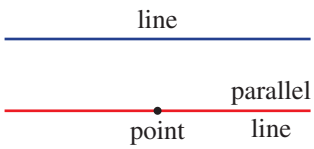
Interactive

Widgets

HOTsheets

Walkthroughs

Euclid's 5th axiom is: Given a line (shown in blue) and a point not on the line, there is only one line (shown in red) through the given point and in the same plane that does not intersect the given line.



In simple language, Euclid's 5th axiom says that parallel lines do not intersect.

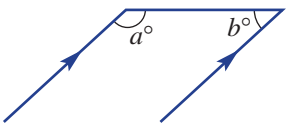


Parallel lines never intersect.

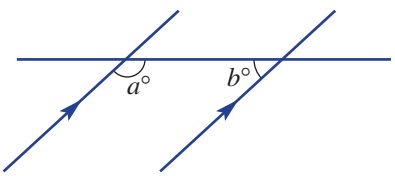
All sorts of shapes and solids both in the theoretical and practical worlds can be constructed using parallel lines. If two lines are parallel and are cut by a third line called a transversal, special pairs of angles are created.

Let's start: Hidden transversals

This diagram can often be found as part of a shape such as a parallelogram or another more complex diagram. To see the relationship between a and b more easily, you can extend the lines to form this second diagram. In this new diagram you can now see the pair of parallel lines and the relationships between all the angles.

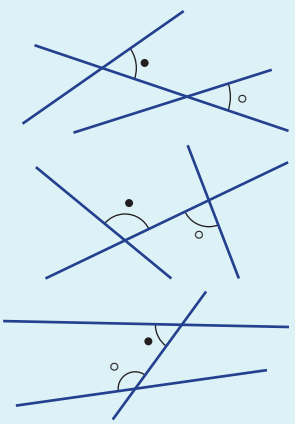


- Copy the new diagram.
- Label each of the eight angles formed with the pronumeral a or b, whichever is appropriate.
- What is the relationship between a and b? Can you explain why?



Key ideas

- A **transversal** is a line cutting at least two other lines.
- Pairs of angles formed by transversals can be:
  - **corresponding** (in corresponding positions)
  - **alternate** (on opposite sides of the transversal and inside the other two lines)
  - **co-interior** (on the same side of the transversal and inside the other two lines).

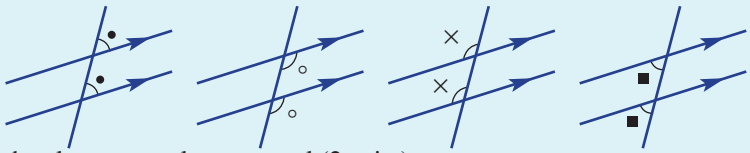


- Lines are parallel if they do not intersect.
  - Parallel lines are marked with the same number of arrows.

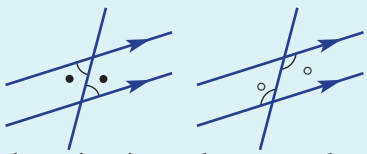
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Key ideas

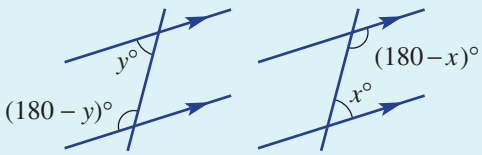
- If two parallel lines are cut by a transversal:
  - the corresponding angles are equal (4 pairs)



  - the alternate angles are equal (2 pairs)



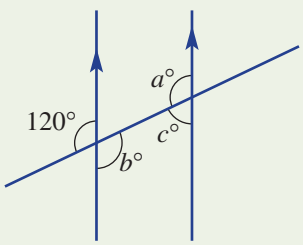
  - the co-interior angles are supplementary (sum to 180°) (2 pairs).



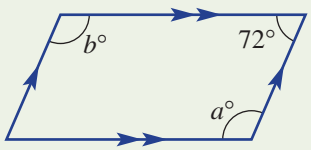
Example 2 Finding angles involving parallel lines

Find the value of the pronumerals in these diagrams, stating reasons.

a



b



SOLUTION	EXPLANATION
a $a = 120$ The angles of size $a°$ and 120° are corresponding and lines are parallel.	Corresponding angles on parallel lines are equal.
$b = 120$ The angles of size $a°$ and $b°$ are alternate and lines are parallel.	Alternatively, the angle of size $b°$ is vertically opposite to the angle marked 120°.
$c + 120 = 180$ $c = 60$	The angles of size $b°$ and $c°$ are co-interior and sum to 180°. Alternatively, look at the angles of size $a°$ and $c°$, which are supplementary.
b $a + 72 = 180$ $a = 108$	The pairs of angles are co-interior, which are supplementary if the lines are parallel.
$b + 72 = 180$ $b = 108$	
Co-interior angles in parallel lines are supplementary.	

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Exercise 2B

1, 2(½) | 2(½) | —

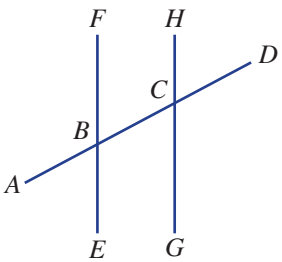
UNDERSTANDING

1 Two parallel lines are cut by a transversal. Write the missing word (*equal* or *supplementary*).

- **a** Corresponding angles are_____.
- **b** Co-interior angles are_____.
- **c** Alternate angles are_____.

2 Name the angle that is:

- **a** corresponding to $\angle ABF$
- **b** corresponding to $\angle BCG$
- **c** alternate to $\angle FBC$
- **d** alternate to $\angle CBE$
- **e** co-interior to $\angle HCB$
- **f** co-interior to $\angle EBC$
- **g** vertically opposite to $\angle ABE$
- **h** vertically opposite to $\angle HCB$





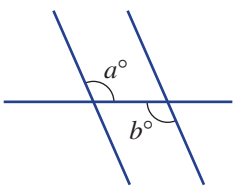
All sorts of shapes and solids can be constructed using parallel lines and transversals.

3–5(½) | 3–6(½) | 3–6(½)

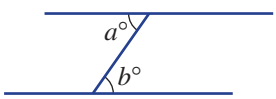
FLUENCY

3 State whether the following marked angles are corresponding, alternate or co-interior.

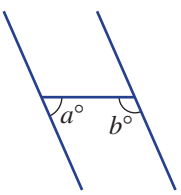
a



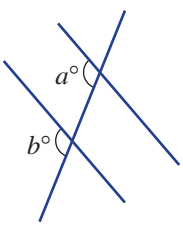
b



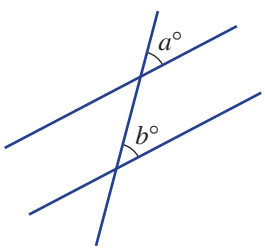
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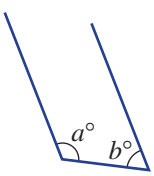
d



e



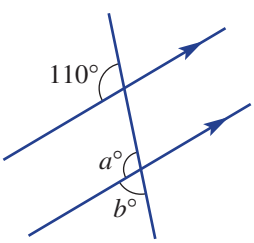
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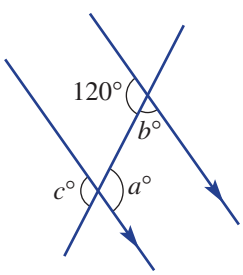
Example 2a

4 Find the value of the pronumerals in these diagrams, stating reasons.

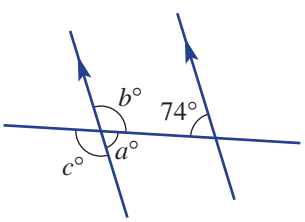
a



b

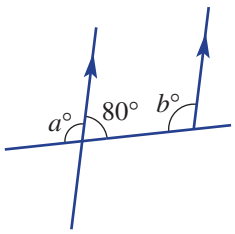


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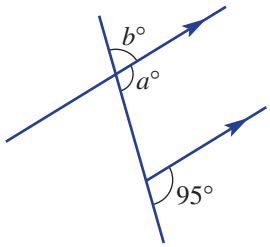


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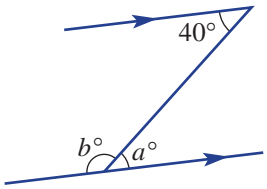
d



e



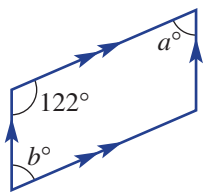
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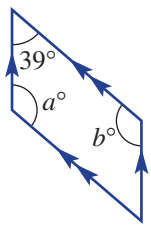
Example 2b

5 Find the value of the pronumerals in these diagrams, stating reasons.

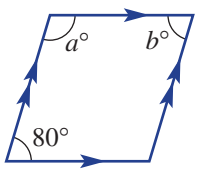
a



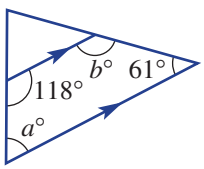
b



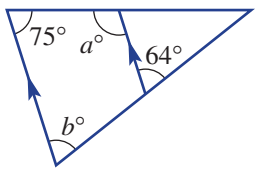
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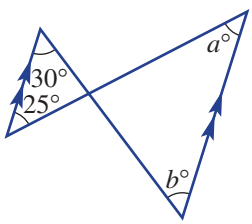
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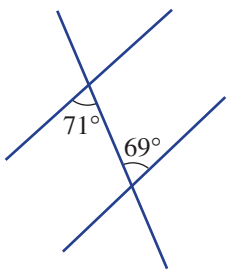


f

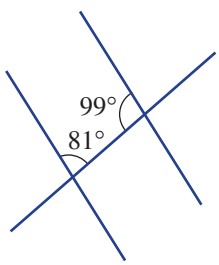


6 Decide if the following diagrams include a pair of parallel lines. Give a reason for each answer.

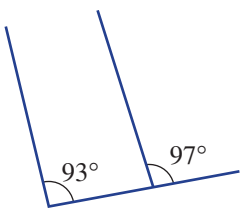
a



b



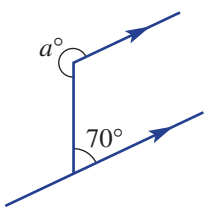
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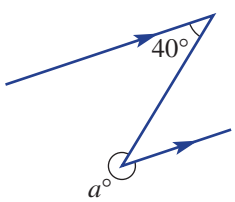
7(½) 7(½) 7

7 Find the value of a in these diagrams.

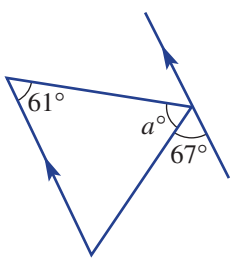
a



b



c

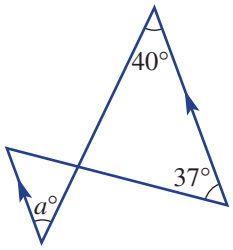


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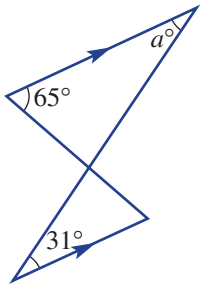
2B

PROBLEM-SOLVING

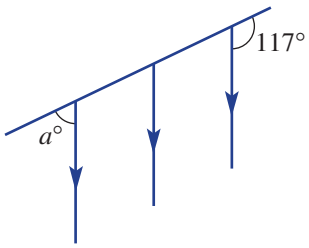
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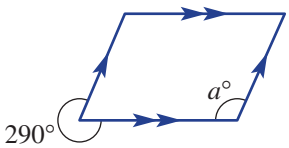
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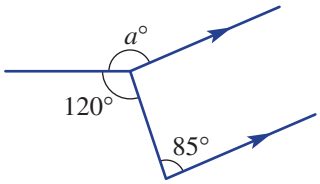
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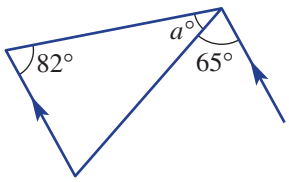
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h



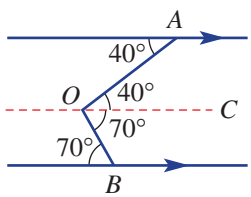
i



8(½) 8(½), 9 8(½), 9

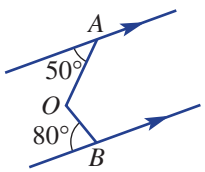
REASONING

8 Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in this diagram by first drawing the dashed line and finding $\angle AOC$ (40°) and $\angle COB$ (70°). So $\angle AOB = 40° + 70° = 110°$.

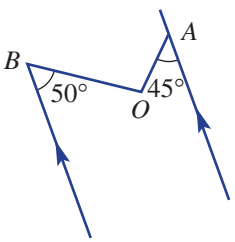


Apply a similar technique to find $\angle AOB$ in these diagrams.

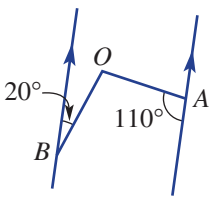
a



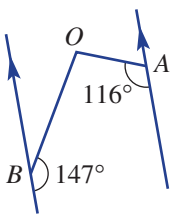
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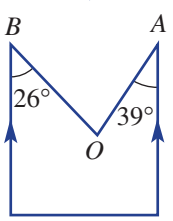
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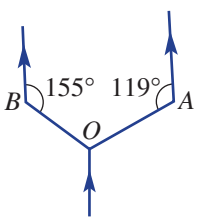
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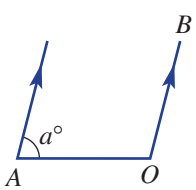


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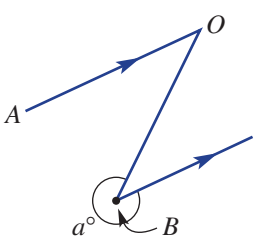


9 Write a rule for $\angle AOB$ using the given pronumerals, e.g. $\angle AOB = (90 - a°)$.

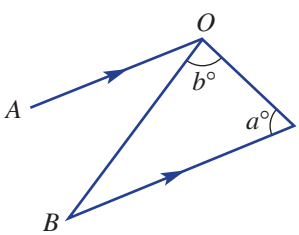
a



b



c



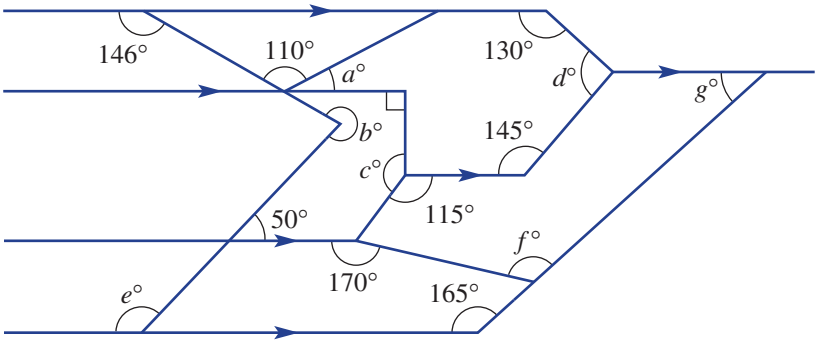
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Pipe networks — — 10

2B

ENRICHMENT

10 A plan for a natural gas plant includes many intersecting pipe lines some of which are parallel. Help the designers finish the plans by calculating the values of a–g.





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2C Triangles CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

A triangle is a shape with three straight sides. As a real life object, the triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

Knowing the properties of triangles can help to solve many geometrical problems and this knowledge can also be extended to explore other more complex shapes.

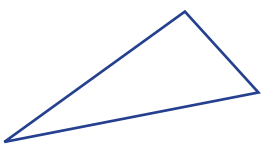


Triangular shapes are often used to striking effect in architecture, as shown by part of the National Gallery of Canada.

Let's start: Illustrating the angle sum

You can complete this task using a pencil and ruler or using computer dynamic geometry.

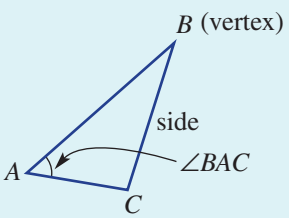
- Draw any triangle and measure each interior angle.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?



If dynamic geometry is used, drag one of the vertices to alter the interior angles. Now check to see if your conclusions remain the same.

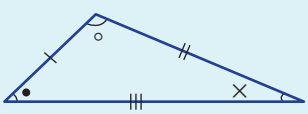
Key ideas

- A **triangle** has:
  - 3 sides
  - 3 vertices (vertices is the plural of vertex)
  - 3 interior angles.

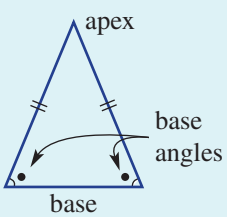


- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

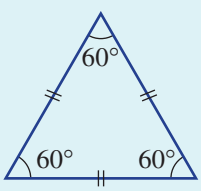
Scalene



Isosceles



Equilateral

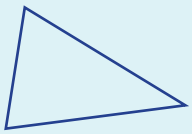


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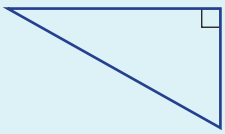
Key ideas

- Triangles classified by interior angles

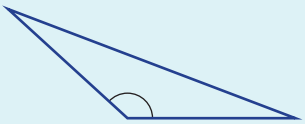
Acute
(All angles acute)



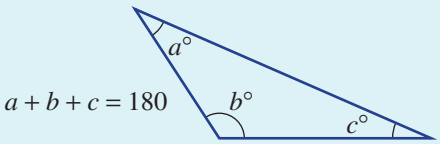
Right
(1 right angle)



Obtuse
(1 obtuse angle)

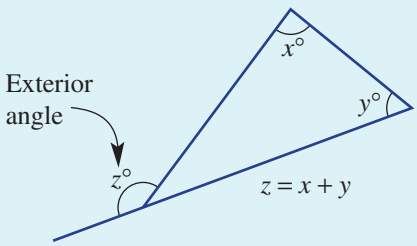


- The angle sum of a triangle is 180°.



$a + b + c = 180$

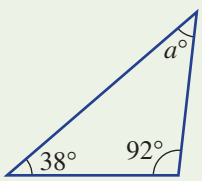
- The **exterior angle theorem:**
  The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



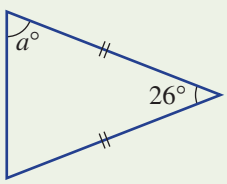
Example 3 Using the angle sum of a triangle

Find the value of a in these triangles.

a



b



SOLUTION

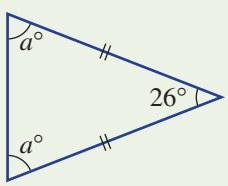
a $a + 38 + 92 = 180$
$a + 130 = 180$
$\therefore a = 50$

b $a + a + 26 = 180$
$2a + 26 = 180$
$2a = 154$
$\therefore a = 77$

EXPLANATION

The angle sum of the three interior angles of a triangle is 180°. Also $38 + 92 = 130$ and $180 - 130 = 50$.

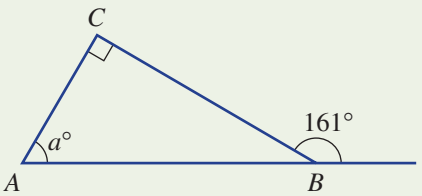
The two base angles in an isosceles triangle are equal.



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Example 4 Using the exterior angle theorem

Find the value of a in this diagram.



SOLUTION

$a + 90 = 161$

$\therefore a = 71$

or $\angle ABC = 180° - 161° = 19°$

so $a = 180 - (19 + 90)$

$= 71$

EXPLANATION

Use the exterior angle theorem for a triangle. The exterior angle (161°) is equal to the sum of the two opposite interior angles.

Alternatively find $\angle ABC$ (19°), then use the triangle angle sum to find the value of a.

Exercise 2C

1–3 | 2, 3 | —

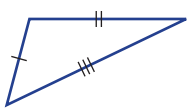
UNDERSTANDING

1 Give the common name of a triangle with these properties.

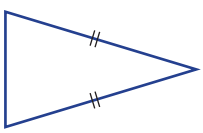
a One right angle
b 2 equal side lengths
c All angles acute
d All angles 60°
e One obtuse angle
f 3 equal side lengths
g 2 equal angles
h 3 different side lengths

2 Classify these triangles as scalene, isosceles or equilateral.

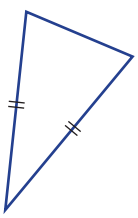
a



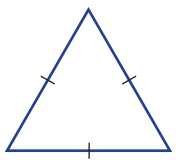
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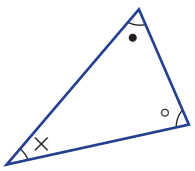
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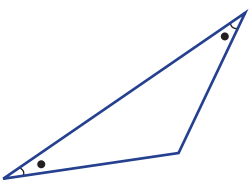
d



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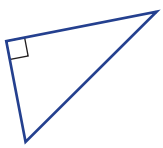


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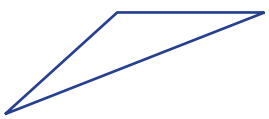


3 Classify these triangles as acute, right or obtuse.

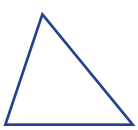
a



b



c

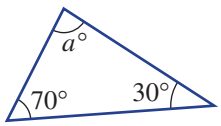


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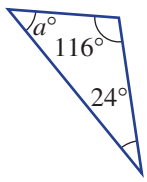
Example 3a

4 Use the angle sum of a triangle to help find the value of a in these triangles.

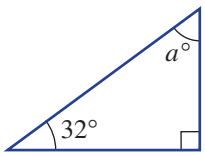
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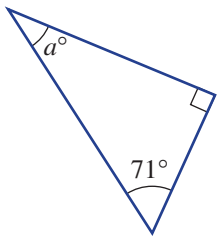
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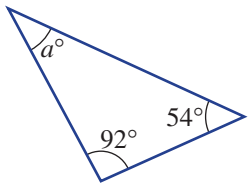
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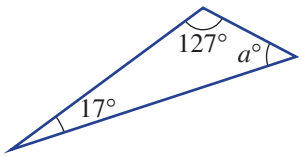
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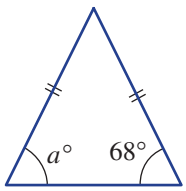


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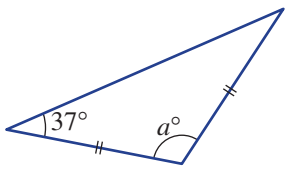


5 These triangles are isosceles. Find the value of a.

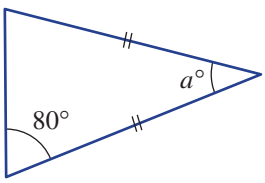
a



b



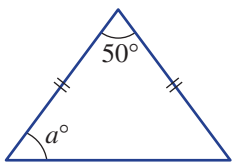
c



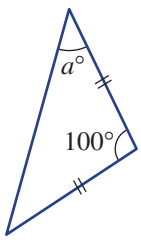
Example 3b

6 Find the value of a in these isosceles triangles.

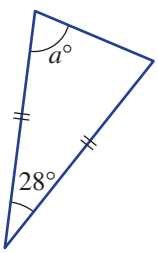
a



b



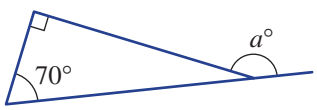
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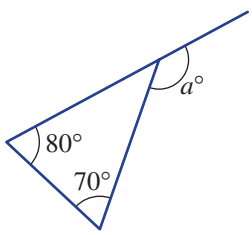
Example 4

7 Find the value of a in these diagrams.

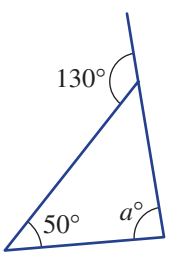
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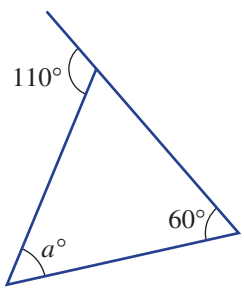
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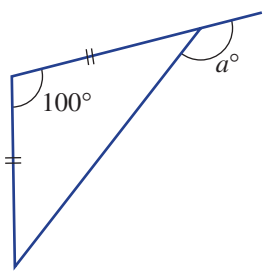
c



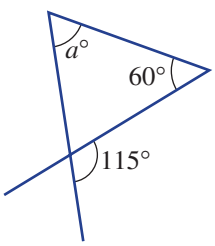
d



e



f



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2C

8, 9 | 8, 9 | 9, 10

PROBLEM-SOLVING

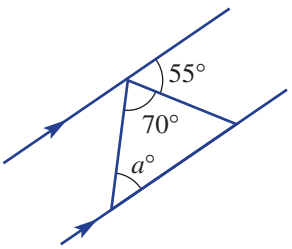
8 Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

a Right and scalene
b Obtuse and equilateral
c Right and isosceles
d Acute and isosceles
e Acute and equilateral
f Obtuse and isosceles

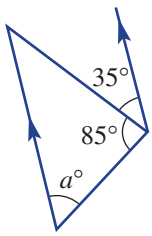


9 Use your knowledge of parallel lines and triangles to find the unknown angle a.

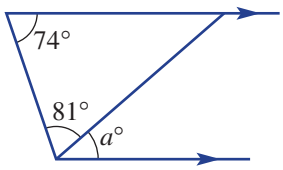
a



b

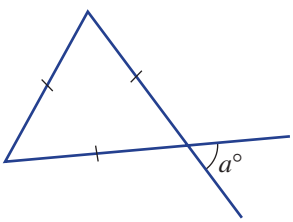


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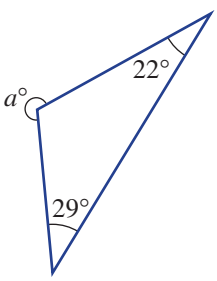


10 Find the value of a in these diagrams.

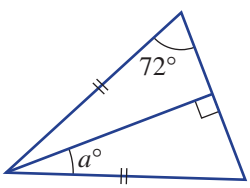
a



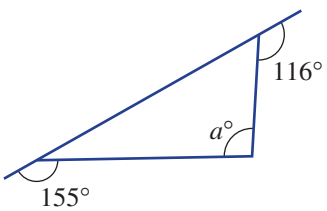
b



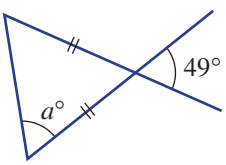
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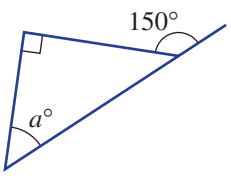
d



e



f

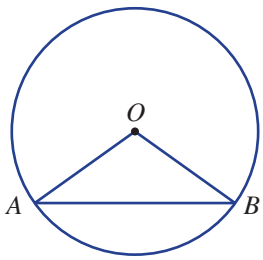


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11 11, 12 12–15 2C

REASONING

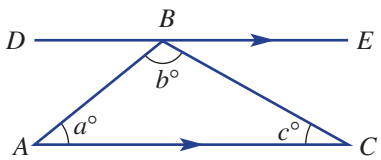
11 A triangle is constructed using a circle and two radius lengths.



- **a** What type of triangle is ΔAOB and why?
- **b** Name two angles that are equal.
- **c** Find $\angle ABO$ if $\angle BAO$ is 30°.
- **d** Find $\angle AOB$ if $\angle OAB$ is 36°.
- **e** Find $\angle ABO$ if $\angle AOB$ is 100°.

12 To prove that the angle sum of a triangle is 180°, work through these steps with the given diagram.

- **a** Using the pronumerals a, b or c, give the value of these angles and state a reason.
  - **i** $\angle ABD$ **ii** $\angle CBE$

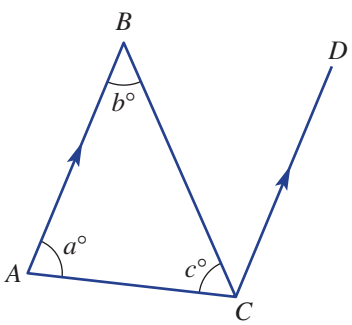


- **b** What is true about the three angles $\angle ABD$, $\angle ABC$ and $\angle CBE$ and why?
- **c** What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the angle sum of the triangle ABC?

13 Prove that a triangle cannot have two right angles.

14 Prove that an equilateral triangle must have 60° angles.

15 A different way of proving the angle sum of a triangle is to use this diagram.



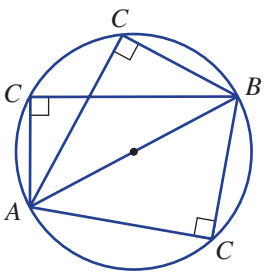
- **a** Give a reason why $\angle BCD = b°$.
- **b** What do you know about the two angles $\angle BAC$ and $\angle ACD$ and why?
- **c** What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the triangle ABC?

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2C Angle in a semicircle — — 16

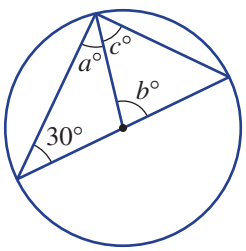
ENRICHMENT

16 The angle sum of a triangle can be used to prove other theorems, one of which relates to the angle in a semicircle. This theorem says that $\angle ACB$ in a semicircle is always 90° where AB is a diameter.

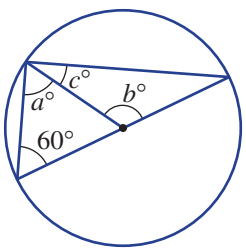


a Use your knowledge of isosceles triangles to find the value of a, b and c in this circle.

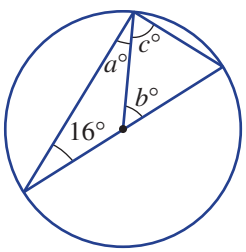
b What do you notice about the sum of the values of a and c?



c Repeat parts **a** and **b** above for this circle.



d Repeat parts **a** and **b** above for this circle.

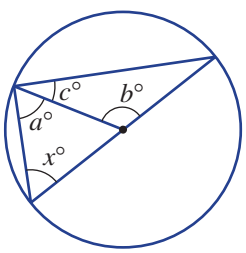


e What do you notice about the sum of the values of a and c for all the circles above?

f Prove this result generally by finding:

i a, b and c in terms of x

ii the value of $a + c$.



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2D Quadrilaterals

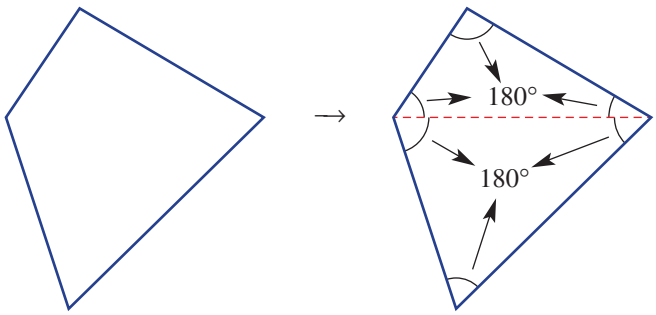
Interactive

Widgets

HOTsheets

Walkthroughs

Quadrilaterals are four-sided shapes with four interior angles. All quadrilaterals have the same angle sum, but other properties depend on such things as pairs of sides of equal length, parallel sides and lengths of diagonals. All quadrilaterals can be drawn as two triangles and, since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is $2 \times 180° = 360°$.



Let's start: Which quadrilateral?

Name all the different quadrilaterals you can think of that have the properties listed below. There may be more than one quadrilateral for each property listed. Draw each quadrilateral to illustrate the shape and its features.

- 4 equal length sides
- 2 pairs of parallel sides
- Equal length diagonals
- 1 pair of parallel sides
- 2 pairs of equal length sides
- 2 pairs of equal opposite angles

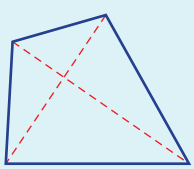


The Mathematical bridge in Cambridge University contains many quadrilaterals.

Key ideas

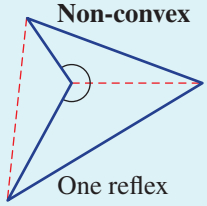
- **Quadrilaterals** can be convex or non-convex.
  - Convex quadrilaterals have all vertices pointing outwards.
    - The diagonals of a convex quadrilateral lie inside the Figure.
  - Non-convex (or concave) quadrilaterals have one vertex pointing inwards, and one reflex interior angle.
    - One diagonal will lie outside the figure.

Convex



All interior angles less than 180°

Non-convex

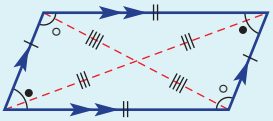


One reflex interior angle

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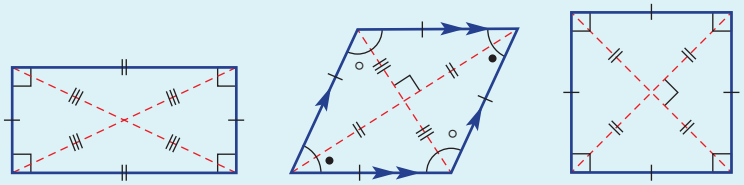
Key ideas

- Parallelograms are quadrilaterals with two pairs of parallel sides.
  - Other properties are illustrated in this diagram.



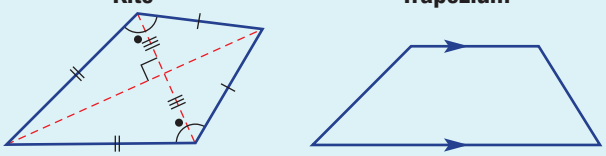
- Special parallelograms include:
  - Rectangle: Parallelogram with all angles 90°.
  - Rhombus: Parallelogram with all sides equal.
  - Square: Rhombus with all angles 90°.

Rectangle **Rhombus** **Square**

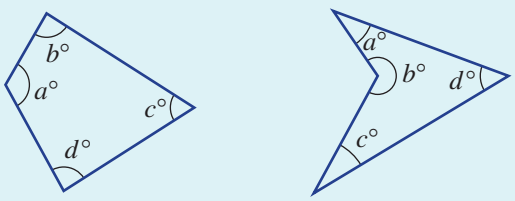


- Other special quadrilaterals include:
  - Kite: Quadrilateral with two adjacent pairs of equal sides.
  - Trapezium: Quadrilateral with at least one pair of parallel sides.

Kite **Trapezium**

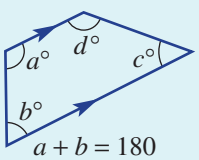


- The angle sum of any quadrilateral is 360°.



$$a + b + c + d = 360$$

- Quadrilaterals with parallel sides include two pairs of co-interior angles.



$c + d = 180$

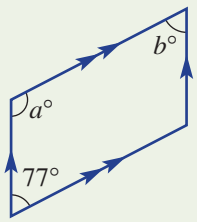
$a + b = 180$

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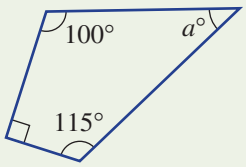
Example 5 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

a



b



SOLUTION

a $a + 77 = 180$

$\therefore a = 103$

$\therefore b = 77$

b $a + 100 + 90 + 115 = 360$

$a + 305 = 360$

$a = 360 - 305$

$\therefore a = 55$

EXPLANATION

Two angles inside parallel lines are co-interior and therefore sum to 180°.

Opposite angles in a parallelogram are equal.

The sum of angles in a quadrilateral is 360°.

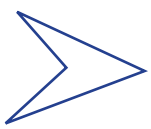
Exercise 2D

1, 2 | 2 | —

UNDERSTANDING

1 Decide if these quadrilaterals are convex or non-convex.

a



b



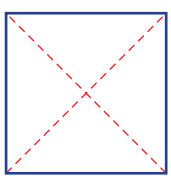
c



2 Refer to the diagrams in the **Key ideas** or accurately draw your own shapes, including the diagonals, to answer true or false to these statements.

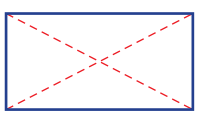
a Square

- **i** All sides are of equal length.
- **ii** Diagonals are not equal in length.
- **iii** All sides are parallel to each other.
- **iv** Diagonals intersect at right angles.



b Rectangle

- **i** All sides are always of equal length.
- **ii** All interior angles are 90°.
- **iii** Diagonals always intersect at right angles.
- **iv** There are two pairs of parallel sides.



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2D

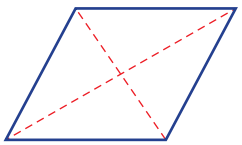
UNDERSTANDING

c Rhombus

i All interior angles are always equal.

ii All sides are of equal length.

iii Diagonals intersect at right angles.

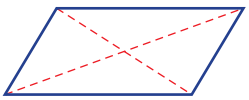


d Parallelogram

i There are two pairs of equal length and parallel sides.

ii Diagonals are always equal in length.

iii Diagonals always intersect at right angles.

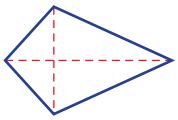


e Kite

i There are two pairs of sides of equal length.

ii There are always two pairs of parallel sides.

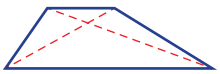
iii Diagonals intersect at right angles.



f Trapezium

i Diagonals are always equal in length.

ii There are always two pairs of parallel sides.



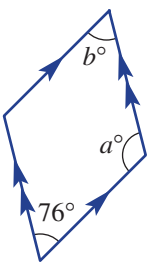
3, 4 | 3, 4(½), 5 | 3, 4(½), 5

FLUENCY

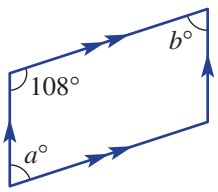
Example 5a

3 Find the value of the pronumerals in these quadrilaterals.

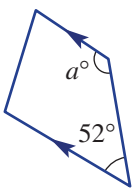
a



b



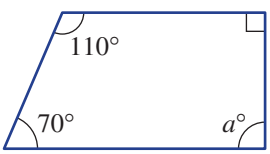
c



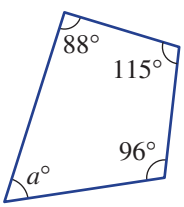
Example 5b

4 Use the quadrilateral angle sum to find the value of a in these quadrilaterals.

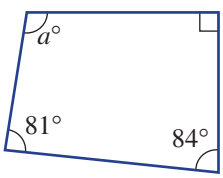
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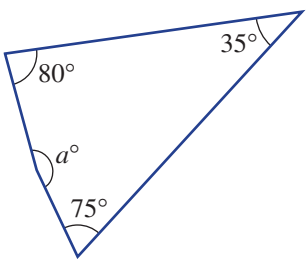
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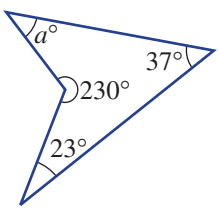
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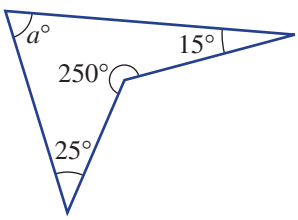
d



e



f

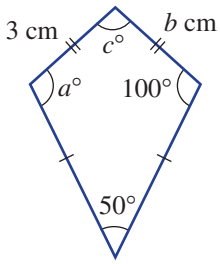


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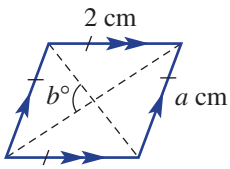
FLUENCY

5 By considering the properties of the given quadrilaterals, give the values of the pronumerals.

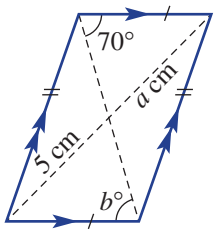
a



b



c

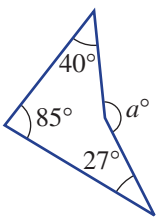


6 6 6(½), 7

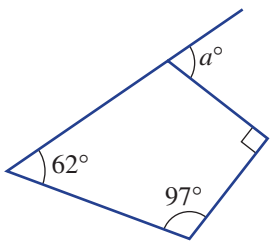
PROBLEM-SOLVING

6 Use your knowledge of geometry from the previous sections to find the values of a.

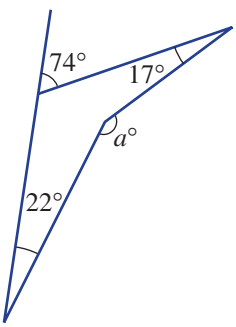
a



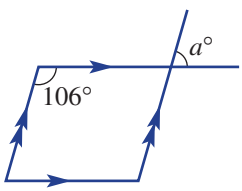
b



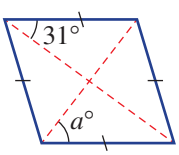
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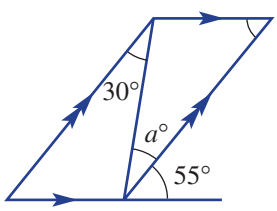
d



e

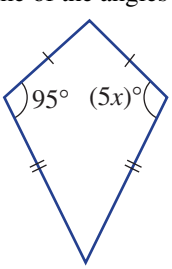


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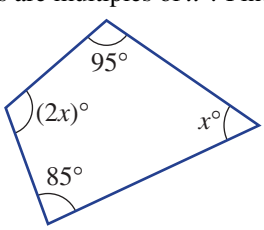


7 Some of the angles in these diagrams are multiples of $x°$. Find the value of x in each case.

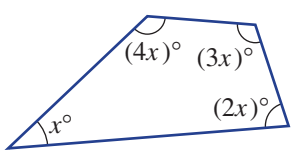
a



b



c



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2D

	8	8, 9	9, 10

REASONING

8 The word 'bisect' means to cut in half.

a Which quadrilaterals have diagonals that bisect each other?

b Which quadrilaterals have diagonals that bisect all their interior angles?

9 By considering the properties of special quadrilaterals, decide if the following are always true.

a A square is a type of rectangle.

b A rectangle is a type of square.

c A square is a type of rhombus.

d A rectangle is a type of parallelogram.

e A parallelogram is a type of square.

f A rhombus is a type of parallelogram.

10 Is it possible to draw a non-convex quadrilateral with two or more interior reflex angles? Explain and illustrate.

Quadrilateral Proofs

	—	—	11

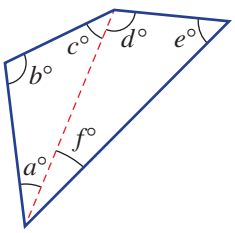
ENRICHMENT

11 Complete these proofs of two different angle properties of quadrilaterals.

a Angle sum $= a + b + c +$ _____ $+$ _____ $+$ _____

$=$ _____ $+$ _____(angle sum of a triangle)

$=$ _____

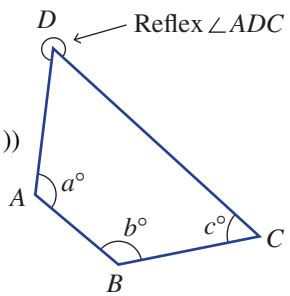


b $\angle ADC = 360° - ($ _____ $+$ _____ $+$ _____ $)$ (angle sum of a quadrilateral)

Reflex $\angle ADC = 360° - \angle ADC$

$= 360° - (360° - ($ _____ $+$ _____ $+$ _____ $))$

$=$ _____ $+$ _____ $+$ _____

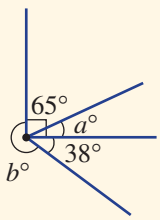


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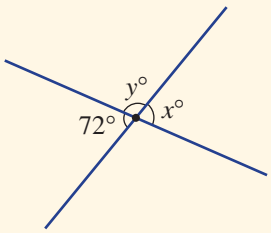
Progress quiz

2A/B **1** Determine the value of the pronumerals in these diagrams.

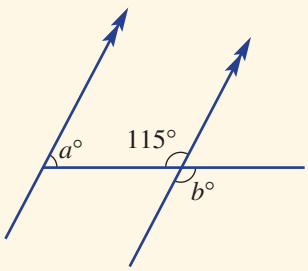
a



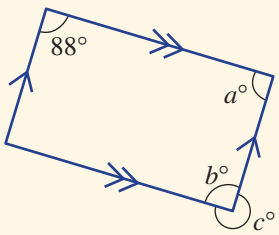
b



c



d

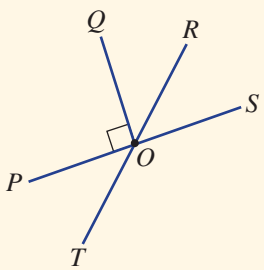


2A **2** For the angles in this diagram name the angle that is:

a vertically opposite to $\angle ROS$

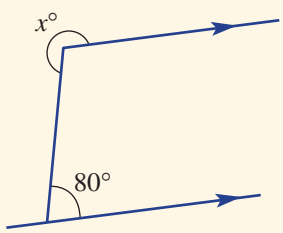
b complementary to $\angle QOR$

c supplementary to $\angle POT$

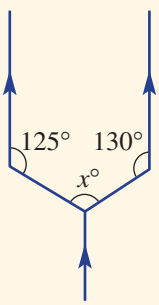


2B **3** Find the value of x in each of the following diagrams.

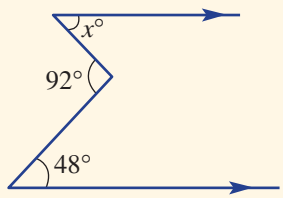
a



b

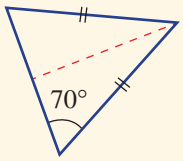


c



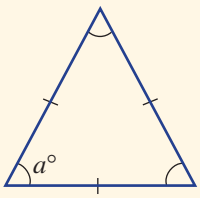
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4 Choose two words from: isosceles, scalene, equilateral, obtuse, acute and right, to describe this triangle.

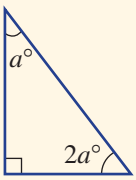


2C **5** Find the value of a in each of the following diagrams.

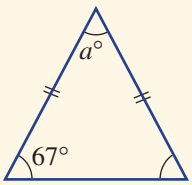
a



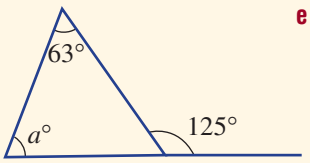
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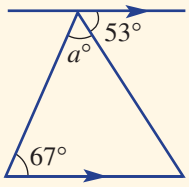
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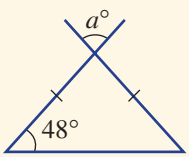
d



e

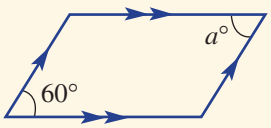


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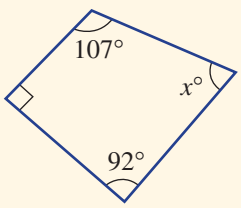


2D **6** Find the value of the pronumerals in these quadrilaterals.

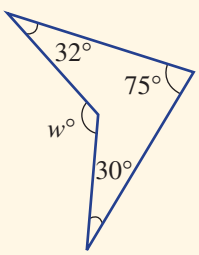
a



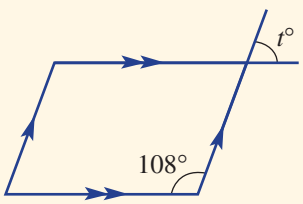
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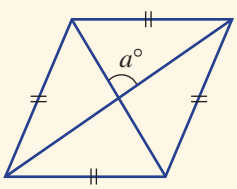
c



d



e



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2E Polygons EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

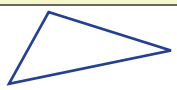
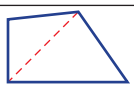
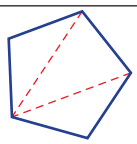
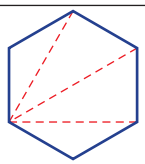
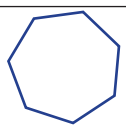
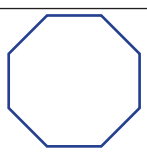
The word 'polygon' comes from the Greek words *poly*, meaning 'many', and *gonia*, meaning 'angles'. The number of interior angles equals the number of sides and the angle sum of each type of polygon depends on this number. Also, there exists a general rule for the angle sum of a polygon with n sides, which we will explore in this section.



The Pentagon is a famous government office building in Washington, USA.

Let's start: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is 180°, which was proved in an earlier section. Complete the table and try to write in the final row the general rule for the angle sum of a polygon.

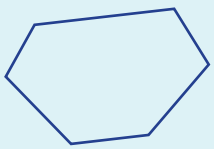
Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180° = 180°$
Quadrilateral 	4	2	____ × 180° = ____
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		(_____) × 180°

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Key ideas

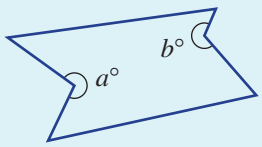
- **Polygons** are shapes with straight sides and can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex (or concave) polygons have at least one vertex pointing inwards and at least one reflex interior angle.

Convex



All interior angles less than 180°

Non-convex

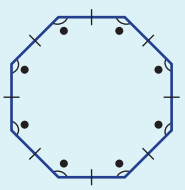


At least one reflex interior angle

- Polygons are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum S of a polygon with n sides is given by the rule: $S = (n - 2) \times 180°$.
- A **regular polygon** has sides of equal length and equal interior angles.



Regular octagon

Example 6 Finding the angle sum

Find the angle sum of a heptagon.

SOLUTION

$S = (n - 2) \times 180°$
$= (7 - 2) \times 180°$
$= 5 \times 180°$
$= 900°$

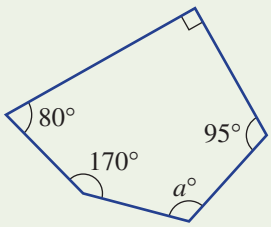
EXPLANATION

A heptagon has 7 sides so $n = 7$.
Simplify $(7 - 2)$ before multiplying by 180°.

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Example 7 Finding angles in polygons

Find the value of a in this pentagon.



SOLUTION	EXPLANATION
$S = (n-2) \times 180°$ $= (5-2) \times 180°$ $= 540°$	First calculate the angle sum of a pentagon using $n = 5$.
$a + 170 + 80 + 90 + 95 = 540$ $a + 435 = 540$ $a = 105$	Sum all the angles and set this equal to the angle sum of 540°. The difference between 540 and 435 is 105.

Example 8 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon.

SOLUTION	EXPLANATION
$S = (n-2) \times 180°$ $= (8-2) \times 180°$ $= 1080°$	First calculate the angle sum of a octagon using $n = 8$.
Angle size $= 1080° \div 8$ $= 135°$	All 8 angles are equal in size so divide the angle sum by 8.

Exercise 2E

1–4 | 4 | —

UNDERSTANDING

1 State the number of sides on these polygons.

a Hexagon **b** Quadrilateral **c** Decagon
d Heptagon **e** Pentagon **f** Dodecagon

2 Evaluate $(n-2) \times 180°$ if:

a $n = 6$ **b** $n = 10$ **c** $n = 22$

3 What is the common name given to these polygons?

a Regular quadrilateral **b** Regular triangle

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2E

UNDERSTANDING

4 Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

a Pentagon (540°) **b** Decagon (1440°) **c** Octagon (1080°)

5–7(½) | 5–6(½), 7 | 5–7(½)

FLUENCY

Example 6

5 Find the angle sum of these polygons.

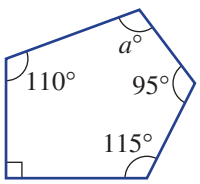
a Hexagon **b** Nonagon **c** Heptagon

d 15-sided polygon **e** 45-sided polygon **f** 102-sided polygon

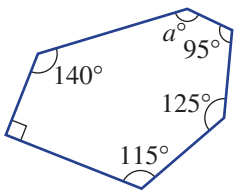
Example 7

6 Find the value of a in these polygons.

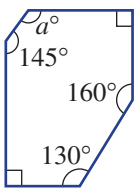
a



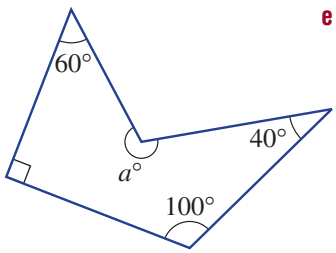
b



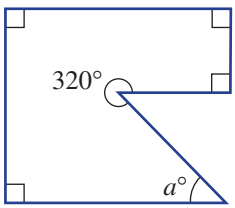
c



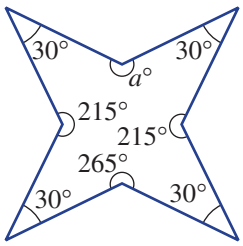
d



e



f



Example 8

7 Find the size of an interior angle of these regular polygons. Round the answer to one decimal place where necessary.

a Regular pentagon **b** Regular heptagon

c Regular undecagon **d** Regular 32-sided polygon

8 | 8, 9(½) | 8, 9(½)

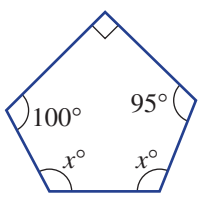
PROBLEM-SOLVING

8 Find the number of sides of a polygon with the given angle sums.

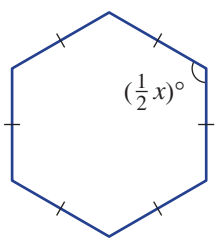
a 1260° **b** 2340° **c** 3420° **d** 29 700°

9 Find the value of x in these diagrams.

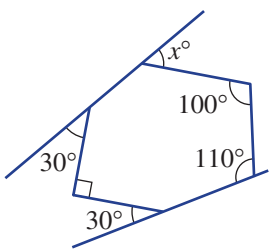
a



b



c

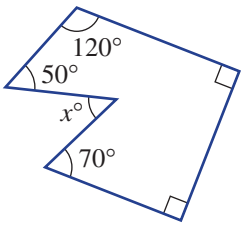


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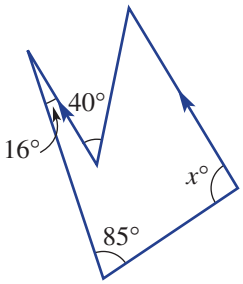
2E

PROBLEM-SOLVING

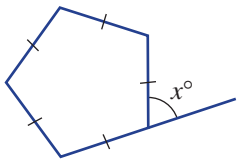
d



e



f



	10	10	10, 11

REASONING

10 Consider a regular polygon with a very large number of sides (n).

- **a** What shape does this polygon look like?
- **b** Is there a limit to the size of a polygon angle sum or does it increase to infinity as n increases?
- **c** What size does each interior angle approach as n increases?

11 Let S be the angle sum of a regular polygon with n sides.

- **a** Write a rule for the size of an interior angle in terms of S and n.
- **b** Write a rule for the size of an interior angle in terms of n only.
- **c** Use your rule to find the size of an interior angle of these polygons. Round to two decimal places where appropriate.
  - **i** Regular dodecagon
  - **ii** 82-sided regular polygon

Unknown challenges	—	—	12, 13

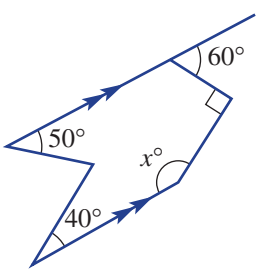
ENRICHMENT

12 Find the number of sides of a regular polygon if each interior angle is:

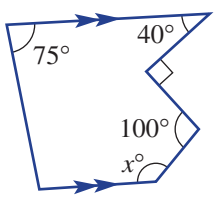
- **a** 120°
- **b** 162°
- **c** 147.272727…°

13 With the limited information provided, find the value of x in these diagrams.

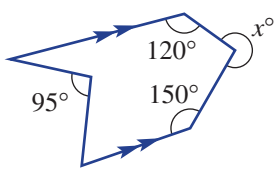
a



b



c



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2F Solids and Euler's rule EXTENDING



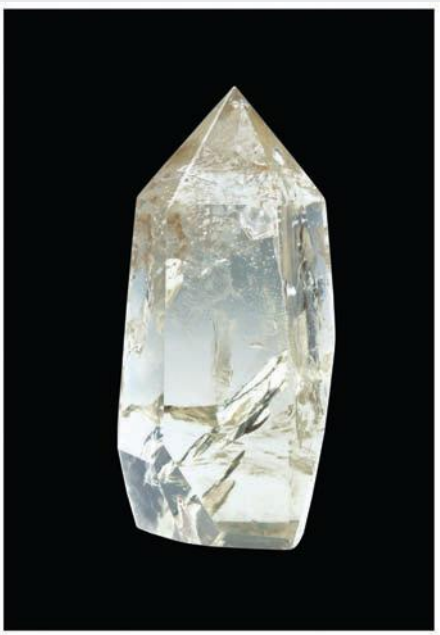






A solid occupies three-dimensional space and can take on all sorts of shapes. The outside surfaces could be flat or curved and the number of surfaces will vary depending on the properties of the solid. A solid with all flat surfaces is called a polyhedron, plural *polyhedra* or *polyhedrons*. The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.





The top of this Canary Wharf building in London (left) is a large, complex polyhedron. Polyhedra also occur in nature, particularly in rock or mineral crystals such as quartz (right).

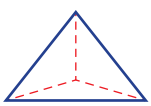
Let's start: Developing Euler's rule

Create a table with each polyhedron listed below in its own row in column 1 (see below). Include the name and a drawing of each polyhedron. Add columns to the table for faces (F), vertices (V), edges (E) and faces plus vertices added together ($F + V$).

Polyhedron	Drawing	Faces (F)	Vertices (V)	Edges (E)	$F + V$

Count the faces, vertices and edges for each polyhedron and record the numbers in the table.

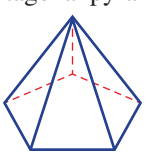
Tetrahedron



Hexahedron



Pentagonal pyramid



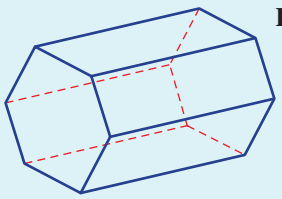
- What do you notice about the numbers in the columns for E and $F + V$?
- What does this suggest about the variables F, V and E? Can you write a rule?
- Add rows to the table, draw your own polyhedra and test your rule by finding the values for F, V and E.

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Key ideas

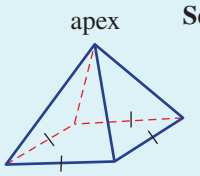
- A **polyhedron** (plural: polyhedra) is a closed solid with flat surfaces (faces), vertices and edges.
  - Polyhedra can be named by their number of faces.
    For example, tetrahedron (4 faces), pentahedron (5 faces) and hexahedron (6 faces).
- **Euler's rule** for polyhedra with F faces, V vertices and E edges is given by:
  $E = F + V - 2$
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name. The other faces are parallelograms. If these faces are rectangles, as shown, then the solid is called a **right prism**.

Hexagonal prism



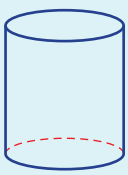
- **Pyramids** are polyhedra with a base face and all other triangular faces meeting at the same vertex point called the apex. They are named by the shape of the base.

Square pyramid

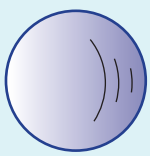


- Some solids have **curved** surfaces. Common examples include:

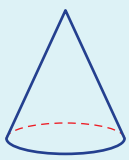
Cylinder



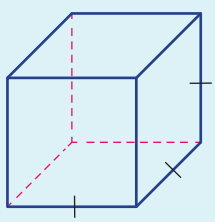
Sphere



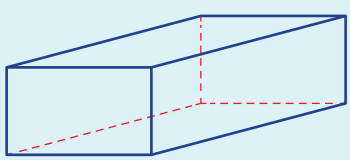
Cone



- A **cube** is a hexahedron with six square faces.



- A **cuboid** is a common name used for a rectangular prism.

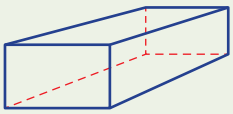


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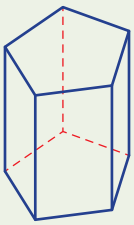
Example 9 Classifying solids

a Classify these solids by considering the number of faces, e.g. octahedron.

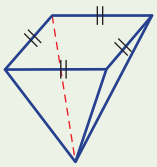
i



ii



iii



b Name these solids as a type of prism or pyramid, e.g. hexagonal prism or hexagonal pyramid.

SOLUTION		EXPLANATION
a i	Hexahedron	The solid has 6 faces.
ii	Heptahedron	The solid has 7 faces.
iii	Pentahedron	The solid has 5 faces.
b i	Rectangular prism	It has two rectangular ends with rectangular sides.
ii	Pentagonal prism	It has two pentagonal ends with rectangular sides.
iii	Square pyramid	It has a square base and four triangular faces meeting at an apex.

Example 10 Using Euler's rule

Use Euler's rule to find the number of faces on a polyhedron that has 10 edges and 6 vertices.

SOLUTION	EXPLANATION
$E = F + V - 2$	Write down Euler's rule and make the appropriate substitutions. Solve for F, which represents the number of faces.
$10 = F + 6 - 2$	
$10 = F + 4$	
$F = 6$	

Exercise 2F

1–4 | 4 | —

UNDERSTANDING

1 Write the missing word in these sentences.

a A polyhedron has faces, _____ and edges.

b A heptahedron has _____ faces.

c A prism has two _____ ends.

d A pentagonal prism has _____ faces.

e The base of a pyramid has 8 sides. The pyramid is called a _____ pyramid.

2 Find the value of the pronumeral in these equations.

a $E = 10 + 16 - 2$

b $12 = F + 7 - 2$

c $12 = 6 + V - 2$

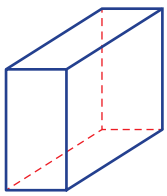
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2F

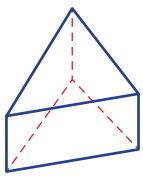
UNDERSTANDING

3 Count the number of faces, vertices and edges (in that order) on these polyhedra.

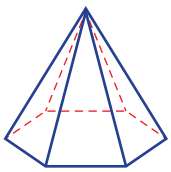
a



b



c



4 Which of these solids are polyhedra (i.e. have only flat surfaces)?

A Cube **B** Pyramid **C** Cone **D** Sphere
E Cylinder **F** Rectangular prism **G** Tetrahedron **H** Hexahedron

5–6(½), 7–10 5–6(½), 7–11 5–6(½), 7–11

FLUENCY

Example 9a

5 Name the polyhedron that has the given number of faces.

a 6 **b** 4 **c** 5 **d** 7
e 9 **f** 10 **g** 11 **h** 12

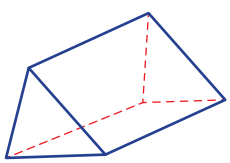
6 How many faces do you think these polyhedra have?

a Octahedron **b** Hexahedron **c** Tetrahedron **d** Pentahedron
e Heptahedron **f** Nonahedron **g** Decahedron **h** Undecahedron

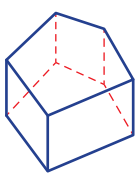
Example 9b

7 Name these prisms.

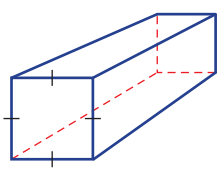
a



b



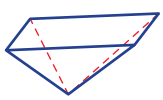
c



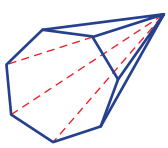
Example 9b

8 Name these pyramids.

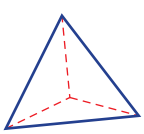
a



b



c



9 **a** Copy and complete this table.

Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	$F + V$
Cube				
Square pyramid				
Tetrahedron				
Octahedron				

b Compare the number of edges (E) with the value $F + V$ for each polyhedron. What do you notice?

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2F

Example 10

FLUENCY

10 Use Euler's rule to calculate the missing numbers in this table.

Faces (F)	Vertices (V)	Edges (E)
6	8	___
___	5	8
5	___	9
7	___	12
___	4	6
11	11	___

11 **a** A polyhedron has 16 faces and 12 vertices. How many edges does it have?

b A polyhedron has 18 edges and 9 vertices. How many faces does it have?

c A polyhedron has 34 faces and 60 edges. How many vertices does it have?

12 | 12, 13 | 13, 14

PROBLEM-SOLVING

12 Decide if the following statements are true or false. Make drawings to help.

a A tetrahedron is a pyramid.

b All solids with curved surfaces are cylinders.

c A cube and a rectangular prism are both hexahedrons.

d A hexahedron can be a pyramid.

e There are no solids with 0 vertices.

f There are no polyhedra with 3 surfaces.

g All pyramids will have an odd number of faces.

13 Decide if it is possible to cut the solid using a single straight cut, to form the new solid given in the brackets.

a Cube (rectangular prism)

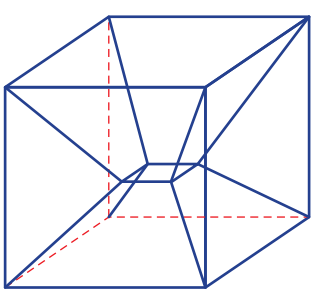
b Square based pyramid (tetrahedron)

c Cylinder (cone)

d Octahedron (pentahedron)

e Cube (heptahedron)

14 This solid is like a cube but is open at the top and bottom and there is a square hole in the middle forming a tunnel. Count the number of faces (F), vertices (V) and edges (E) then decide if Euler's rule is true for such solids.



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2F

15 15, 16 16–18

REASONING

15 **a** A cuboid is a common name for a solid with six rectangular faces. Name the solid in two other different ways.

b A pyramid has base with 10 sides. Name the solid in two ways.

16 Rearrange Euler's rule.

a Write V in terms of F and E. **b** Write F in terms of V and E.

17 Show that Euler's rule applies for these solids.

a Heptagonal pyramid **b** Octagonal prism **c** Octahedron

18 Decide if the following statements are true or false.

a For all pyramids, the number of faces is equal to the number of vertices.

b For all convex polyhedra, the sum $E + V + F$ is even.

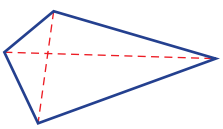
Convex solids

— — 19

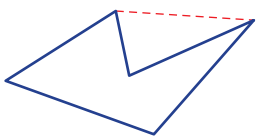
ENRICHMENT

19 Earlier you learned that a convex polygon will have all interior angles less than 180°. Notice also that all diagonals in a convex polygon are drawn *inside* the shape.

Convex polygon

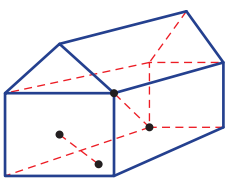


Non-convex polygon

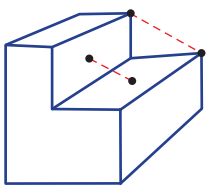


Solids can also be classified as convex or non-convex.

Convex solid



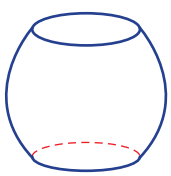
Non-convex solid



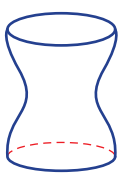
To test for a non-convex solid, join two vertices or two faces with a line segment that passes outside the solid.

a Decide if these solids are convex or non-convex.

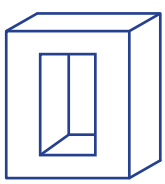
i



ii



iii



b Draw your own non-convex solids and check by connecting any two vertices or faces with a line segment outside the solid.

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Investigation

Constructions

Geometric construction involves a precise set of mathematical and geometric operations that do not involve any approximate measurements or other guess work. The basic tools for geometric construction are a pair of compasses, a straight edge and a pencil or drawing pen. Computer geometry or drawing packages can also be used, and include digital equivalents of these tools.

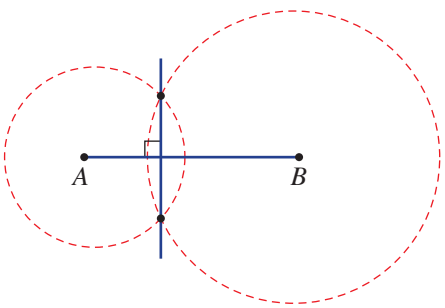




For the following constructions use only a pair of compasses, a straight edge and a pencil. Alternatively, use computer geometry software and adapt the activities where required.

Perpendicular line

1 Construct:
- **a** a segment AB
- **b** a circle with centre A
- **c** a circle with centre B
- **d** a line joining the intersection points of the two circles

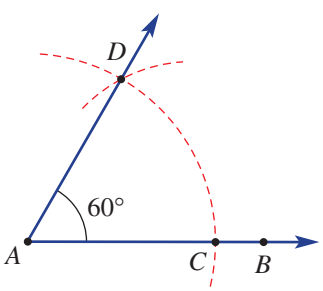


Perpendicular bisector

2 Repeat the construction for a perpendicular line, but ensure that the two circles have the same radius. If computer geometry is used, use the length of the segment AB for the radius of both circles.

A 60° angle

3 Construct:
- **a** a ray AB
- **b** an arc with centre A
- **c** the intersection point C
- **d** an arc with centre C and radius AC
- **e** a point D at the intersection of the two arcs
- **f** a ray AD



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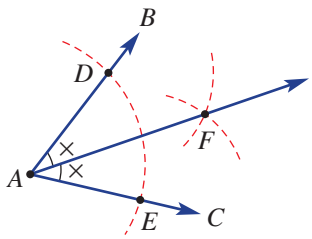
Equilateral triangle

4 Repeat the construction for a 60° angle then construct the segment CD.

Angle bisector

5 Construct:

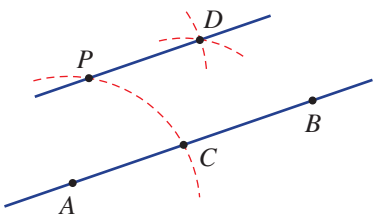
- **a** any angle $\angle BAC$
- **b** an arc with centre A
- **c** the two intersection points D and E
- **d** two arcs of equal radius with centres at D and E
- **e** the intersection point F
- **f** the ray AF



Parallel line through a point

6 Construct:

- **a** a line AB and point P
- **b** an arc with centre A and radius AP
- **c** the intersection point C
- **d** an arc with centre C and radius AP
- **e** an arc with centre P and radius AP
- **f** the intersection point D
- **g** the line PD



Rhombus

7 Repeat the construction for a parallel line through a point and construct the segments AP and CD.

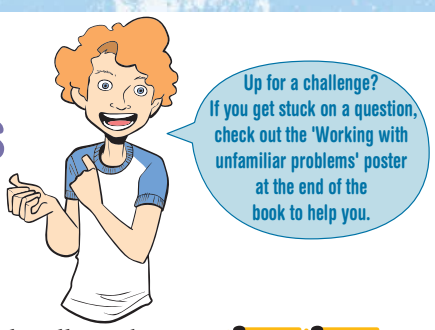
Construction challenges

8 For a further challenge try to construct these objects. No measurement is allowed.

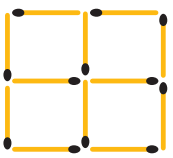
- **a** 45° angle
- **b** Square
- **c** Perpendicular line at the end of a segment
- **d** Parallelogram
- **e** Regular hexagon

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Problems and challenges



1 This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)



a Remove 2 matchsticks to form 2 squares.

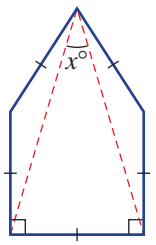
b Move 3 matchsticks to form 3 squares.

2 **a** Use 9 matchsticks to form 5 equilateral triangles.

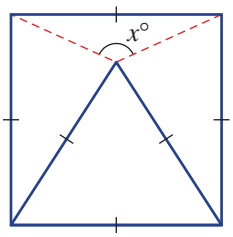
b Use 6 matchsticks to form 4 equilateral triangles.

3 Find the value of x in these diagrams.

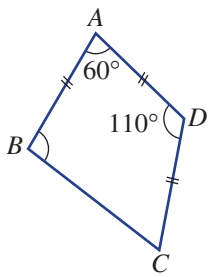
a



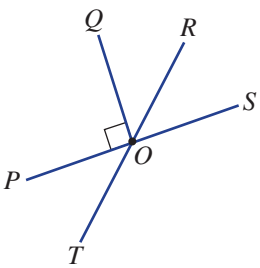
b



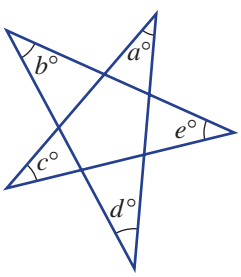
4 Find the size of $\angle ABC$ in this quadrilateral.



5 If $\angle ROS = 75°$ find the size of all other angles.



6 Find the value of $a + b + c + d + e$ in this star. Give reasons for your answer.



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Chapter summary

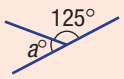
Angles at a point

Complementary angles



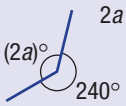
$a + 25 = 90$
$a = 65$

Supplementary angles



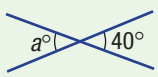
$a + 125 = 180$
$a = 55$

Revolution



$2a + 240 = 360$
$2a = 120$
$a = 60$

Vertically opposite angles

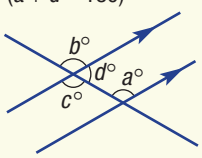


$a = 40$

Lines, shapes and solids

Parallel lines

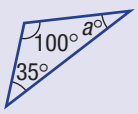
- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Co-interior angles are supplementary ($a + d = 180$)



Triangles

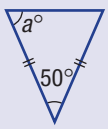
Angle sum = 180°

Scalene



$a + 135 = 180$
$a = 45$

Isosceles



$2a + 50 = 180$
$2a = 130$
$a = 65$

Polygons (Ext)

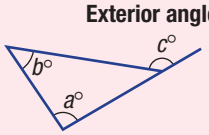


$S = (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$

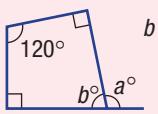
Regular pentagon

$a = \frac{540}{5} = 108$

Exterior angles



$c = a + b$



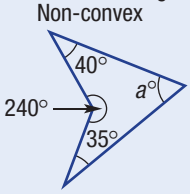
$b + 300 = 360$
$b = 60$
$a = 120$

Quadrilaterals

(Square, Rectangle, Rhombus, Parallelogram, Kite, Trapezium)

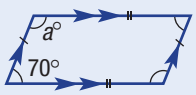
Angle sum = 360°

Non-convex



$a + 315 = 360$
$a = 45$

Parallelogram

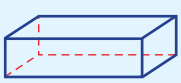


$a + 70 = 180$
$a = 110$

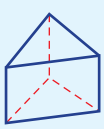
Polyhedra (Ext)

$E = F + V - 2$

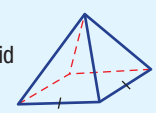
Hexahedron



Triangular prism



Square pyramid



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Cambridge University Press

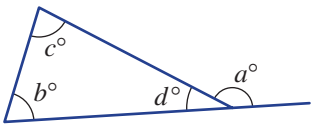
Chapter review

Multiple-choice questions

2A **1** What is the name given to two angles that sum to 90°?

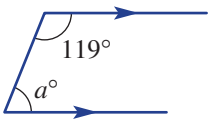
A Right **B** Supplementary
C Revolutionary **D** Complementary
E Vertically opposite

2C **2** The value of a in this diagram is equal to:



A $b + c$ **B** $c + d$ **C** $b + d$ **D** $180 - a$
E $d + 180$

2B **3** The value of a in this diagram is equal to:



A 45 **B** 122 **C** 241 **D** 119 **E** 61

2C **4** The size of an exterior angle on an equilateral triangle is:

A 60° **B** 120° **C** 180° **D** 100° **E** 45°

2D **5** The most general quadrilateral whose diagonals intersect at right angles and has 2 pairs of equal length sides is a:

A square **B** rhombus
C kite **D** parallelogram
E rectangle

2E Ext **6** The rule for the angle sum S of a polygon with n sides is:

A $S = n \times 180°$ **B** $S \times n = 180°$
C $S = (n - 1) \times 180°$ **D** $S = (n - 2) \times 180°$
E $S = (n + 2) \times 180°$

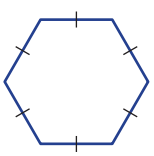
2E Ext **7** The name given to an eleven-sided polygon is:

A heptagon **B** elevenagon **C** decagon **D** dodecagon **E** undecagon

2F Ext **8** A polyhedron has 8 faces and 8 vertices. Its number of edges is:

A 14 **B** 16 **C** 18 **D** 15 **E** 10

2E Ext **9** The size of one interior angle of a regular hexagon is:



A 135° **B** 180° **C** 120° **D** 720° **E** 108°

2F Ext **10** How many edges does a rectangular prism have?

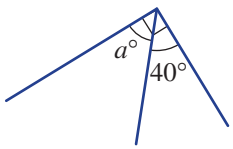
A 10 **B** 4 **C** 6 **D** 12 **E** 8

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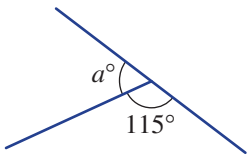
Short-answer questions

2A **1** Find the value of a in these diagrams.

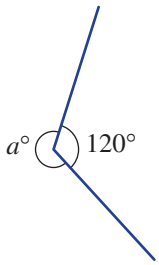
a



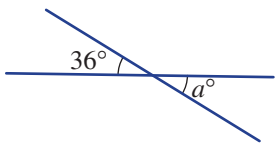
b



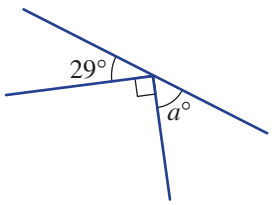
c



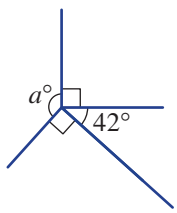
d



e

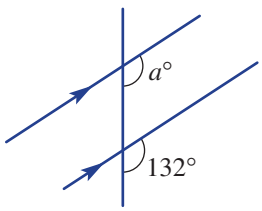


f

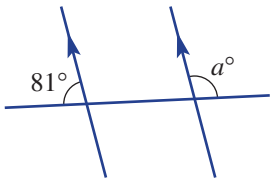


2B **2** These diagrams include parallel lines. Find the value of a.

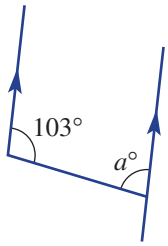
a



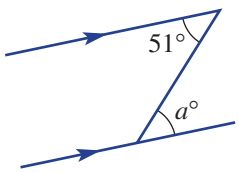
b



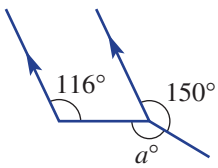
c



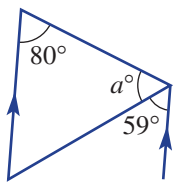
d



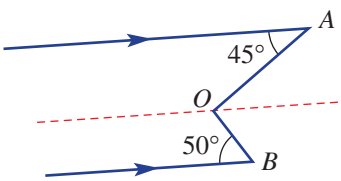
e



f

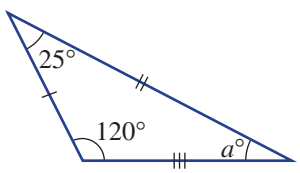


2B **3** Use the dashed construction line to help find the size of $\angle AOB$ in this diagram.

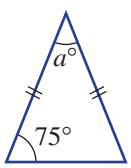


2C **4** Give a name for each triangle and find the value of a.

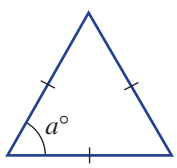
a



b



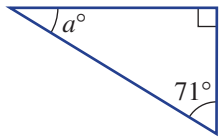
c



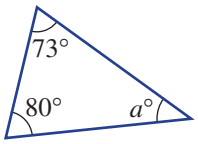
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Chapter review

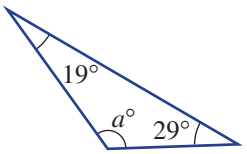
d



e

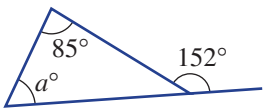


f

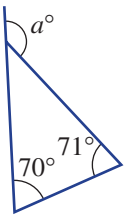


2C **5** These triangles include exterior angles. Find the value of a.

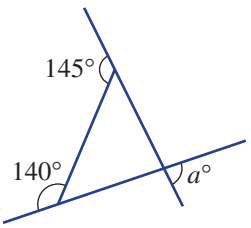
a



b

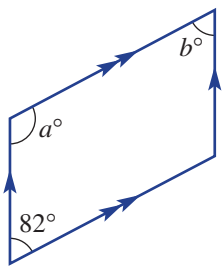


c

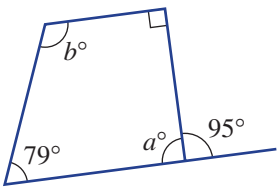


2D **6** Find the value of a and b in these quadrilaterals.

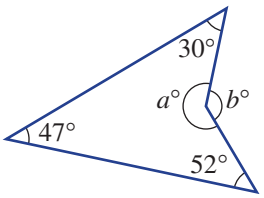
a



b



c



2E Ext **7** Find the angle sum of these polygons.

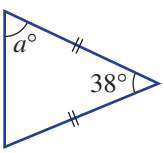
a Heptagon **b** Nonagon **c** 62-sided polygon

2E Ext **8** Find the size of an interior angle of these regular polygons.

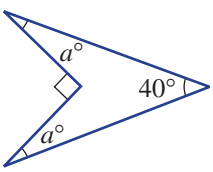
a Regular pentagon **b** Regular dodecagon

2E Ext **9** Find the value of a.

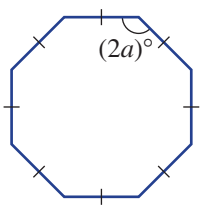
a



b



c

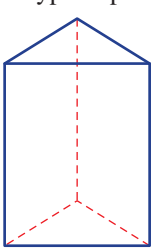


2F Ext **10** Name the polyhedron that has:

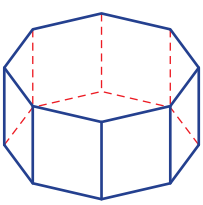
a 6 faces **b** 10 faces **c** 11 faces

2F Ext **11** What type of prism or pyramid are these solids?

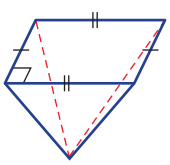
a



b



c

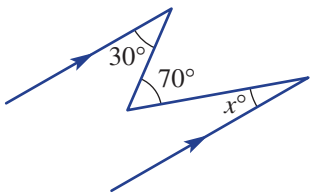


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2F Ext **12** Complete this table for polyhedra with number of faces F, vertices V and edges E.

F	V	E
5	5	
9		21
	10	15

2B **13** Find the value of x in this diagram.



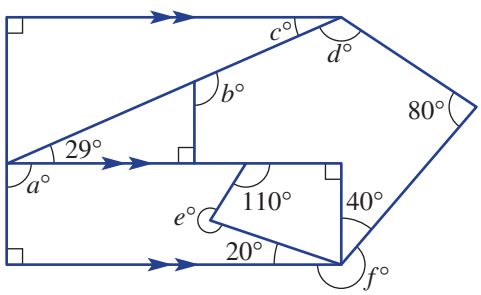
Extended-response questions

Ext **1** A regular polygon has 26 sides.

- **a** Find the angle sum.
- **b** Find the size of its interior angles correct to the nearest degree.
- **c** Find the size of its exterior angles correct to the nearest degree.
- **d** If there is one exterior angle showing for each vertex, find the sum of all the exterior angles.
- **e** The polygon is used to form the ends of a prism. For this prism find the number of:
  - **i** faces
  - **ii** vertices
  - **iii** edges

2 A modern house plan is shown here.

- **a** List the names of at least three different polygons that you see.
- **b** Find the values of the pronumerals a–f.





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Chapter 3

Fractions, decimals and percentages

What you will learn

- 3A Equivalent fractions (Consolidating)
- 3B Operations with fractions (Consolidating)
- 3C Operations with negative fractions
- 3D Understanding decimals (Consolidating)
- 3E Operations with decimals (Consolidating)
- 3F Terminating, recurring and rounding decimals
- 3G Converting fractions, decimals and percentages (Consolidating)
- 3H Finding a percentage and expressing as a percentage
- 3I Decreasing and increasing by a percentage
- 3J Calculating percentage change
- 3K Percentages and the unitary method (Extending)

Victorian curriculum

NUMBER AND ALGEBRA

Number and place value

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies and make estimates for these computations (VCMNA273)

Real numbers

Investigate terminating and recurring decimals (VCMNA274)

Solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital technologies (VCMNA276)

Money and financial mathematics

Solve problems involving profit and loss, with and without digital technologies (VCMNA278)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Calculating the value of gold jewellery

Before buying or selling gold jewellery it is useful to determine the value of the gold in it. Skill with fractions, percentages and decimals is needed for these calculations.

Gold is a soft and expensive metal so gold jewellery also contains other metals such as silver, copper or zinc. The proportion of gold present is measured on a scale of 0 to 24 carats. For example:

18-carat gold is $\frac{18}{24} \times 100 = 75\%$ pure gold.

8-carat gold is $\frac{8}{24} \times 100 = 33.3\%$ pure gold.

Suppose you wanted to make an offer on eBay for a gold necklace advertised as 'solid gold, 10k, 20g'. The word 'solid' does not mean it is pure gold. How much gold does this necklace contain and what is its value?

The gold purity is $\frac{10}{24} \times 100 = 41.7\%$.

The weight of gold is $\frac{41.7}{100} \times 20 = 8.34$ grams.

The price of gold is variable so we use the internet to check the current 'gold spot price' per gram. At a price of \$34.58/g, the gold in this eBay necklace is worth $8.34 \times 34.58 = \$288.40$.

Sometimes gold objects are rated by the number of grams of gold per 1000 g. Gold rated at 916.7 is 91.67% pure or 22 carat. A rating of 999.9 shows 99.99% purity or 24 carat and is called fine or pure gold.

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3A Equivalent fractions

CONSOLIDATING







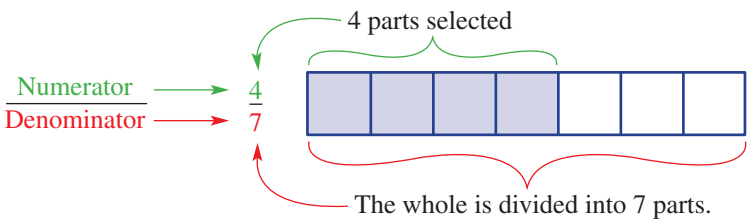


Fractions are extremely useful in practical situations whenever a proportion is required. Fractions are used by a chef measuring the ingredients for a cake, a builder measuring the ingredients for concrete and a musician using computer software to create music.

A fraction is formed when a whole number or amount is divided into equal parts. The bottom number is referred to as the **d**enominator (**d**own) and tells you how many parts the whole is divided up into. The top number is referred to as the n**u**merator (**u**p) and tells you how many of the parts you have selected.



Aerial view of farmland. The paddocks show the farmer's land divided into parts. Each paddock is a particular fraction of the farmer's land.



Equivalent fractions are fractions that represent equal portions of a whole amount and so are equal in value. The skill of generating equivalent fractions is needed whenever you add or subtract fractions with different denominators.

Let's start: Know your terminology

It is important to know and understand key terms associated with the study of fractions.

Working with a partner and using your previous knowledge of fractions, write a one-sentence definition or explanation for each of the following key terms:

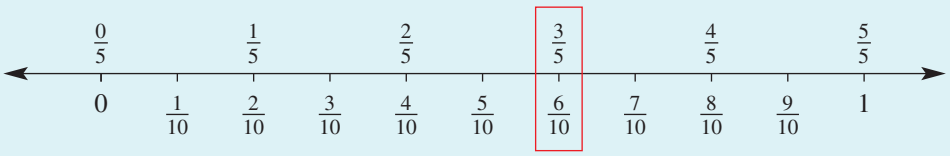
- Numerator
- Equivalent fraction
- Improper fraction
- Multiples
- Reciprocal
- Lowest common multiple
- Ascending
- Composite number
- Denominator
- Proper fraction
- Mixed numeral
- Factors
- Highest common factor
- Descending
- Lowest common denominator

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Key ideas

- **Equivalent fractions** are equal in value. They mark the same place on a number line.

  For example: $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions.



- Equivalent fractions are formed by multiplying or dividing a fraction by a number equal to 1, which can be written in the form $\frac{a}{a}$.

  For example: $\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$ $\therefore \frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.

- Equivalent fractions are therefore produced by multiplying the numerator and the denominator by the same whole number.

  For example: $\frac{2}{7} \overset{\times 5}{=} \frac{10}{35}$ $\therefore \frac{2}{7}$ and $\frac{10}{35}$ are equivalent fractions.

- Equivalent fractions are also produced by dividing the numerator and the denominator by the same common factor.

  For example: $\frac{6}{21} \overset{\div 3}{=} \frac{2}{7}$ $\therefore \frac{6}{21}$ and $\frac{2}{7}$ are equivalent fractions.

- The **simplest form** of a fraction is an equivalent fraction with the lowest possible whole numbers in the numerator and denominator. This is achieved by dividing the numerator and the denominator by their highest common factor (HCF). In the simplest form of a fraction, the HCF of the numerator and the denominator is 1.

  For example: $\frac{12}{18}$ The HCF of 12 and 18 is 6.

  $\frac{12}{18} \overset{\div 6}{=} \frac{2}{3}$ $\therefore \frac{12}{18}$ written in simplest form is $\frac{2}{3}$.

  This technique is also known as 'cancelling'.

  $\frac{12}{18} = \frac{2 \times \cancel{6}^{1}}{3 \times \cancel{6}^{1}} = \frac{2}{3}$

  The HCF is cancelled (divided) from the numerator and the denominator.

  $\frac{6}{6}$ 'cancels' to 1 because $6 \div 6 = 1$

- Two fractions are equivalent if they have the same numerator and denominator in simplest form.

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Example 1 Generating equivalent fractions

Rewrite the following fractions with a denominator of 40.

a $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{7}{4}$ **d** $\frac{36}{120}$

SOLUTION	EXPLANATION
a $\frac{3}{5} = \frac{24}{40}$	Denominator has been multiplied by 8. Numerator must be multiplied by 8.
b $\frac{1}{2} = \frac{20}{40}$	Multiply numerator and denominator by 20.
c $\frac{7}{4} = \frac{70}{40}$	Multiply numerator and denominator by 10.
d $\frac{36}{120} = \frac{12}{40}$	Divide numerator and denominator by 3.

Example 2 Converting to simplest form

Write the following fractions in simplest form.

a $\frac{8}{20}$ **b** $\frac{25}{15}$

SOLUTION	EXPLANATION
a $\frac{8}{20} = \frac{2 \times \cancel{4}^{1}}{5 \times \cancel{4}_{1}} = \frac{2}{5}$	The HCF of 8 and 20 is 4. Both the numerator and the denominator are divided by the HCF of 4.
b $\frac{25}{15} = \frac{5 \times \cancel{5}^{1}}{3 \times \cancel{5}_{1}} = \frac{5}{3}$	The HCF of 25 and 15 is 5. The 5 is 'cancelled' from the numerator and the denominator.

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Exercise 3A

1–3 | 3 | —

UNDERSTANDING

1 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\frac{3}{5} = \frac{\square}{10} = \frac{12}{\square} = \frac{120}{\square}$

b $\frac{4}{7} = \frac{8}{\square} = \frac{\square}{70} = \frac{80}{\square}$

c $\frac{3}{9} = \frac{\square}{3} = \frac{2}{\square} = \frac{14}{\square}$

d $\frac{18}{24} = \frac{6}{\square} = \frac{\square}{4} = \frac{15}{\square}$

2 Which of the following fractions are equivalent to $\frac{2}{3}$?

$\frac{2}{30}$ $\frac{4}{9}$ $\frac{4}{6}$ $\frac{20}{30}$ $\frac{1}{2}$ $\frac{4}{3}$ $\frac{10}{15}$ $\frac{20}{36}$

3 Are the following statements true or false?

a $\frac{1}{2}$ and $\frac{1}{4}$ are equivalent fractions.

b $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

c The fraction $\frac{8}{9}$ is written in its simplest form.

d $\frac{14}{21}$ can be simplified to $\frac{2}{3}$.

e $\frac{11}{99}$ and $\frac{1}{9}$ and $\frac{2}{18}$ are all equivalent fractions.

f $\frac{4}{5}$ can be simplified to $\frac{2}{5}$.

4–10(½) | 5–10(½) | 6–10(½)

FLUENCY

Example 1

4 Rewrite the following fractions with a denominator of 24.

a $\frac{1}{3}$ **b** $\frac{2}{8}$ **c** $\frac{1}{2}$ **d** $\frac{5}{12}$

e $\frac{5}{6}$ **f** $\frac{5}{1}$ **g** $\frac{3}{4}$ **h** $\frac{7}{8}$

5 Rewrite the following fractions with a denominator of 30.

a $\frac{1}{5}$ **b** $\frac{2}{6}$ **c** $\frac{5}{10}$ **d** $\frac{3}{1}$

e $\frac{2}{3}$ **f** $\frac{22}{60}$ **g** $\frac{5}{2}$ **h** $\frac{150}{300}$

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3A

FLUENCY

6 Find the missing value to make the equation true.

a $\frac{2}{5} = \frac{\square}{15}$ **b** $\frac{7}{9} = \frac{14}{\square}$ **c** $\frac{7}{14} = \frac{1}{\square}$ **d** $\frac{21}{30} = \frac{\square}{10}$

e $\frac{4}{3} = \frac{\square}{21}$ **f** $\frac{8}{5} = \frac{80}{\square}$ **g** $\frac{3}{12} = \frac{\square}{60}$ **h** $\frac{7}{11} = \frac{28}{\square}$

7 State the missing numerators for the following sets of equivalent fractions.

a $\frac{1}{2} = \frac{\square}{4} = \frac{\square}{6} = \frac{\square}{10} = \frac{\square}{20} = \frac{\square}{32} = \frac{\square}{50}$

b $\frac{3}{2} = \frac{\square}{4} = \frac{\square}{8} = \frac{\square}{20} = \frac{\square}{30} = \frac{\square}{100} = \frac{\square}{200}$

c $\frac{2}{5} = \frac{\square}{10} = \frac{\square}{15} = \frac{\square}{20} = \frac{\square}{35} = \frac{\square}{50} = \frac{\square}{75}$

8 State the missing denominators for the following sets of equivalent fractions.

a $\frac{1}{3} = \frac{2}{\square} = \frac{4}{\square} = \frac{8}{\square} = \frac{10}{\square} = \frac{25}{\square} = \frac{100}{\square}$

b $\frac{1}{5} = \frac{2}{\square} = \frac{3}{\square} = \frac{4}{\square} = \frac{5}{\square} = \frac{10}{\square} = \frac{12}{\square}$

c $\frac{5}{4} = \frac{10}{\square} = \frac{15}{\square} = \frac{35}{\square} = \frac{55}{\square} = \frac{100}{\square} = \frac{500}{\square}$

Example 2

9 Write the following fractions in simplest form.

a $\frac{3}{9}$ **b** $\frac{4}{8}$ **c** $\frac{10}{12}$ **d** $\frac{15}{18}$

e $\frac{11}{44}$ **f** $\frac{12}{20}$ **g** $\frac{16}{18}$ **h** $\frac{25}{35}$

i $\frac{15}{9}$ **j** $\frac{22}{20}$ **k** $\frac{120}{100}$ **l** $\frac{64}{48}$

10 Using your calculator, express the following fractions in simplest form.

a $\frac{23}{92}$ **b** $\frac{34}{85}$ **c** $\frac{375}{875}$ **d** $\frac{315}{567}$

e $\frac{143}{121}$ **f** $\frac{707}{404}$ **g** $\frac{1197}{969}$ **h** $\frac{2673}{1650}$

PROBLEM-SOLVING

11 | 11, 12 | 11–13

11 Three of the following eight fractions are not written in simplest form. Write down these three fractions and simplify them.

$\frac{17}{31}$ $\frac{14}{42}$ $\frac{5}{11}$ $\frac{51}{68}$ $\frac{23}{93}$ $\frac{15}{95}$ $\frac{1}{15}$ $\frac{13}{31}$

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3A

PROBLEM-SOLVING

12 Group the following 12 fractions into six pairs of equivalent fractions.

$\frac{5}{11}$ $\frac{3}{5}$ $\frac{7}{21}$ $\frac{8}{22}$ $\frac{2}{7}$ $\frac{20}{50}$ $\frac{6}{21}$ $\frac{9}{15}$ $\frac{15}{33}$ $\frac{1}{3}$ $\frac{16}{44}$ $\frac{6}{15}$

13 A 24-hour swim-a-thon was organised to raise funds for a local charity. The goal was to swim 1500 laps during the 24-hour event. After 18 hours, a total of 1000 laps had been swum.

a On the basis of time, what fraction, in simplest form, of the swim-a-thon had been completed?

b On the basis of laps, what fraction, in simplest form, of the swim-a-thon had been completed?

c Were the swimmers on target to achieve their goal? Explain your answer by using equivalent fractions.



14 14, 15 14–16

REASONING

14 **a** A particular fraction has a prime number in the numerator and a different prime number in the denominator. Using some examples, justify whether or not this fraction can be simplified.

b A fraction has a composite number in the numerator and a different composite number in the denominator. Using some examples, justify whether or not this fraction can be simplified.

15 **a** A pizza is cut into eight equal pieces. Can it be shared by four people in such a way that no two people receives an equivalent fraction of the pizza?

b A pizza is cut into 12 equal pieces. Can it be shared by four people in such a way that no-one receives the same amount of the pizza?

c What is the least number of pieces into which a pizza can be cut such that four people can share it and each receive a different amount?



16 Are there a finite or an infinite number of fractions that are equivalent to $\frac{1}{2}$? Justify your answer.

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ENRICHMENT

17 Algebraic fractions contain pronumerals (letters). $\frac{3a}{5}$ and $\frac{x}{y}$ are both examples of algebraic fractions.

a Find the value of the $\square$ to produce equivalent algebraic fractions.

i $\frac{2a}{3b} = \frac{4a}{\square}$ **ii** $\frac{x}{y} = \frac{\square}{5y}$ **iii** $\frac{3b}{20} = \frac{12b}{\square}$ **iv** $\frac{4de}{p} = \frac{\square}{3p}$

v $\frac{a}{b} = \frac{ac}{\square}$ **vi** $\frac{3k}{2t} = \frac{\square}{2tm}$ **vii** $\frac{4a}{5b} = \frac{\square}{20bc}$ **viii** $\frac{x}{y} = \frac{\square}{y^2}$

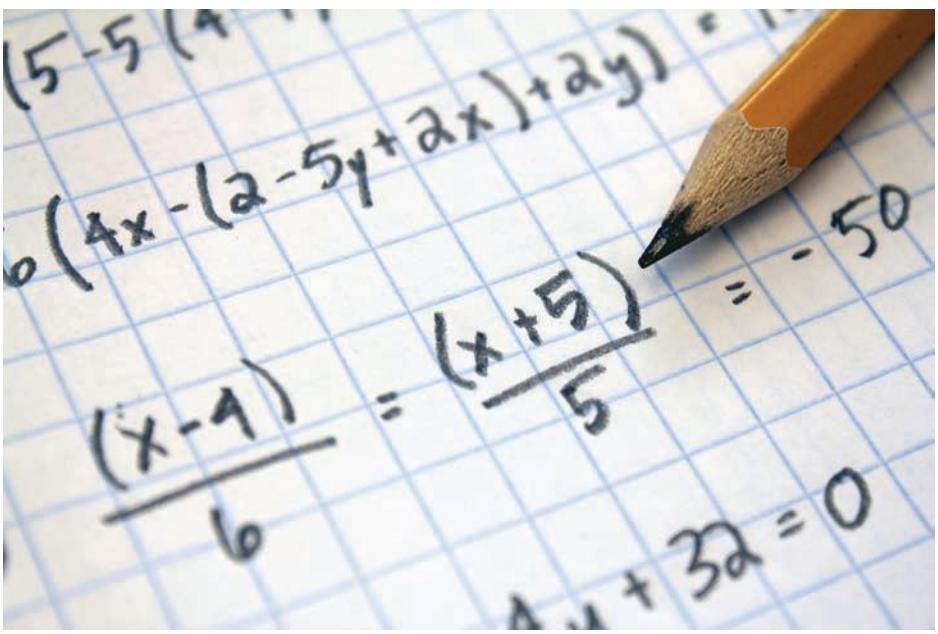
b Simplify the following algebraic fractions.

i $\frac{15b}{20}$ **ii** $\frac{4}{8y}$ **iii** $\frac{3x}{5x}$ **iv** $\frac{5y}{8xy}$

v $\frac{10p}{15qp}$ **vi** $\frac{120y}{12xy}$ **vii** $\frac{mnop}{mnpq}$ **viii** $\frac{15x}{5x^2}$

c Can an algebraic fraction be simplified to a non-algebraic fraction? Justify your answer by using examples.

d Can a non-algebraic fraction have an equivalent algebraic fraction? Justify your answer by using examples.



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3B Operations with fractions CONSOLIDATING

Interactive

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing fractions. Proper fractions, improper fractions and mixed numerals will be considered for each of the four mathematical operations.

Widgets

HOTsheets

Walkthroughs

Let's start: You write the question

Here are six different answers: $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 2, -\frac{1}{4}$.

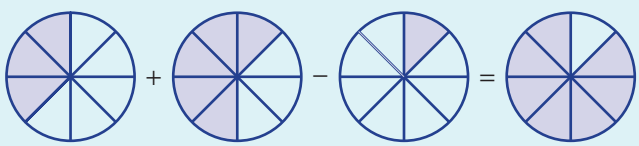
Your challenge is to write six different questions which will produce each of the above six answers. Each question must use only the two fractions $\frac{1}{2}$ and $\frac{1}{4}$ and one operation $(+, -, \times, \div)$.



Key ideas

Adding and subtracting fractions

- When we add or subtract fractions, we count how many we have of a certain 'type' of fraction. For example, we could count eighths: 3 *eighths* plus 5 *eighths* minus 1 *eighth* equals 7 *eighths*. When we count 'how many' *eighths*, the answer must be in *eighths*.



$\frac{3}{8} + \frac{5}{8} - \frac{1}{8} = \frac{7}{8}$

- Denominators *must* be the same before you can proceed with adding or subtracting fractions.
- If the denominators are different, use the lowest common multiple (LCM) of the denominators to find equivalent fractions.
  In this example we are counting sixths: 3 sixths plus 4 sixths equals 7 sixths.
- When the denominators are the same, simply add or subtract the numerators as required. The denominator remains the same.

① Equivalent fractions

② Add numerators

$$\frac{1}{2} + \frac{2}{3} \quad \leftarrow \text{Different denominators}$$
$$= \frac{3}{6} + \frac{4}{6} \quad \leftarrow \text{Same denominators}$$
$$= \frac{7}{6} \quad \leftarrow \text{Denominator stays the same}$$

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Key ideas

Common multiple and lowest common multiple (LCM)

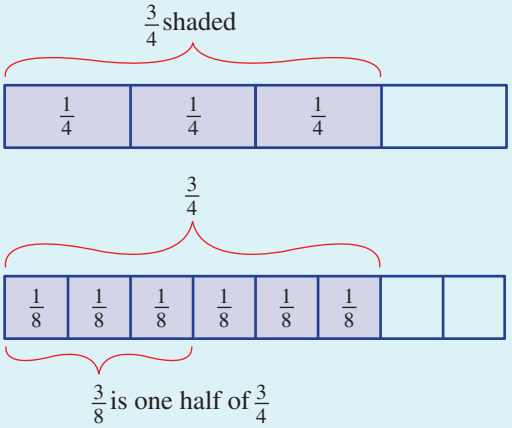
- A common multiple is found by multiplying whole numbers together.
- The LCM is found by multiplying the whole numbers together and then dividing by the HCF of the whole numbers.
  For example, consider the whole numbers 4 and 6.
  A common multiple is 24 (4×6). The LCM is 12 $\left(\frac{4 \times 6}{2}\right)$.
- The lowest common denominator (LCD) is the lowest common multiple (LCM) of the denominators.

Multiplying fractions

Example:

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{4}$$

$$= \frac{3}{8}$$



- Denominators *do not* need to be the same before you can proceed with fraction multiplication.
- Simply multiply the numerators together and multiply the denominators together.
- Mixed numerals must be converted to improper fractions before you can proceed.
- If possible, simplify or 'cancel' fractions before multiplying.

Example:

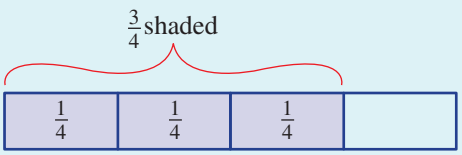
Multiply numerators together.

$$\frac{2}{3} \times \frac{5}{9} = \frac{2 \times 5}{3 \times 9} = \frac{10}{27}$$

Multiply denominators together.

Dividing fractions

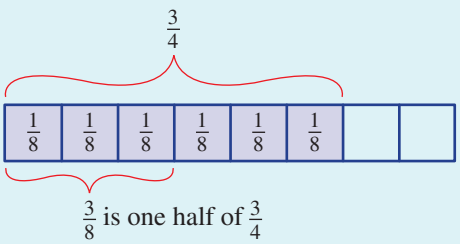
Example:



$\frac{3}{4} \div \frac{1}{4}$ is the same as 'how many quarters $\left(\frac{1}{4}\right)$ are in $\frac{3}{4}$?'

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1}$$

$$= 3$$



$\frac{3}{4} \div 2$ is the same as 'one half of $\frac{3}{4}$.'

$$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

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Key ideas

- Denominators *do not* need to be the same before you can proceed.
- Mixed numerals must be converted to improper fractions before you can proceed.
- To divide by a fraction, multiply by its reciprocal.
- The **reciprocal** of a fraction is found by swapping the numerator and the denominator. This is known as inverting the fraction.
  Proceed as for multiplying fractions.
  For example: $\frac{3}{8} \div \frac{5}{4} = \frac{3}{8} \times \frac{4}{5} = \frac{12}{40} = \frac{3}{10}$
  Instead of $\div, \times$ by the reciprocal of the fraction following the $\div$ sign.

■ **Checking your answer**

- Final answers should be written in simplest form.
- It is common to write answers involving improper fractions as mixed numerals.

Summary

■ **Adding and subtracting fractions**
When the denominators are the same, simply add or subtract the numerators. The denominator remains the same.

■ **Multiplying fractions**
Cancel where possible, then multiply the numerators together and multiply the denominators together.

■ **Dividing fractions**
To divide by a fraction, multiply by its reciprocal.

Example 3 Adding and subtracting fractions

Simplify:

a $\frac{3}{5} + \frac{4}{5}$

b $\frac{5}{3} - \frac{3}{4}$

SOLUTION

a $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$
$= 1\frac{2}{5}$

b $\frac{5}{3} - \frac{3}{4} = \frac{20}{12} - \frac{9}{12}$
$= \frac{11}{12}$

EXPLANATION

The denominators are the same, therefore count the number of fifths by simply adding the numerators.
The final answer is written as a mixed numeral.

LCM of 3 and 4 is 12.
Write equivalent fractions with a denominator of 12.
The denominators are the same, so subtract the numerators.

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Example 4 Adding and subtracting mixed numerals

Simplify:

a $3\frac{5}{8}+2\frac{3}{4}$ **b** $2\frac{1}{2}-1\frac{5}{6}$

SOLUTION | **EXPLANATION**

a $3\frac{5}{8}+2\frac{3}{4}=\frac{29}{8}+\frac{11}{4}$

$=\frac{29}{8}+\frac{22}{8}$

$=\frac{51}{8}=6\frac{3}{8}$

Convert mixed numerals to improper fractions.
The LCM of 8 and 4 is 8.
Write equivalent fractions with LCD.
Add numerators together, denominator remains the same. Convert the answer back to a mixed numeral.

Alternative method

$3\frac{5}{8}+2\frac{3}{4}=3+2+\frac{5}{8}+\frac{3}{4}$

$=5+\frac{5}{8}+\frac{6}{8}$

$=5+\frac{11}{8}=6\frac{3}{8}$

Add the whole number parts together.

The LCM of 8 and 4 is 8.
Write equivalent fractions with LCD.

Add fraction parts together and simplify the answer.

b $2\frac{1}{2}-1\frac{5}{6}=\frac{5}{2}-\frac{11}{6}$

$=\frac{15}{6}-\frac{11}{6}$

$=\frac{4}{6}=\frac{2}{3}$

Convert mixed numerals to improper fractions.
The LCD of 2 and 6 is 6.
Write equivalent fractions with LCD.
Subtract numerators and simplify the answer.

Example 5 Multiplying fractions

Simplify:

a $\frac{2}{5}\times\frac{3}{7}$ **b** $\frac{8}{5}\times\frac{7}{4}$ **c** $3\frac{1}{3}\times2\frac{2}{5}$

SOLUTION | **EXPLANATION**

a $\frac{2}{5}\times\frac{3}{7}=\frac{2\times3}{5\times7}$

$=\frac{6}{35}$

Multiply the numerators together.
Multiply the denominators together.
The answer is in simplest form.

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b $\frac{\cancel{8}^{2}}{5} \times \frac{7}{\cancel{4}_{1}} = \frac{2 \times 7}{5 \times 1}$

$= \frac{14}{5} = 2\frac{4}{5}$

Cancel first. Then multiply numerators together and denominators together.
Write the answer as an improper fraction or mixed numeral.

c $3\frac{1}{3} \times 2\frac{2}{5} = \frac{\cancel{10}^{2}}{\cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{\cancel{5}_{1}}$

$= \frac{2 \times 4}{1 \times 1}$

$= \frac{8}{1} = 8$

Convert to improper fractions first.
Simplify fractions by cancelling.
Multiply 'cancelled' numerators and 'cancelled' denominators together.
Write the answer in simplest form.

Example 6 Dividing fractions

Simplify:

a $\frac{2}{5} \div \frac{3}{7}$ **b** $\frac{5}{8} \div \frac{15}{16}$ **c** $2\frac{1}{4} \div 1\frac{1}{3}$

SOLUTION **EXPLANATION**

a $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}$

$= \frac{14}{15}$

Change ÷ sign to a × sign and invert the divisor (the second fraction).
Proceed as for multiplication.
Multiply numerators together and multiply denominators together.

b $\frac{5}{8} \div \frac{15}{16} = \frac{\cancel{5}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{\cancel{15}_{3}}$

$= \frac{1}{1} \times \frac{2}{3}$

$= \frac{2}{3}$

Change ÷ sign to a × sign and invert the divisor (the second fraction).
Proceed as for multiplication.

c $2\frac{1}{4} \div 1\frac{1}{3} = \frac{9}{4} \div \frac{4}{3}$

$= \frac{9}{4} \times \frac{3}{4}$

$= \frac{27}{16}$

$= 1\frac{11}{16}$

Convert mixed numerals to improper fractions.
Change ÷ sign to × sign and invert the divisor (the second fraction).
Proceed as for multiplication.

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Exercise 3B

1–6 | 4, 6 | —

UNDERSTANDING

1 **a** Which two operations require the denominators to be the same before proceeding?
b Which two operations do not require the denominators to be the same before proceeding?

2 **a** For which two operations must you first convert mixed numerals to improper fractions?
b For which two operations can you choose whether or not to convert mixed numerals to improper fractions before proceeding?

3 State the LCD for the following pairs of fractions.

a $\frac{1}{5}+\frac{3}{4}$ **b** $\frac{2}{9}+\frac{5}{3}$ **c** $\frac{11}{25}+\frac{7}{10}$ **d** $\frac{5}{12}+\frac{13}{8}$

4 Rewrite the following equations and fill in the empty boxes.

a $\frac{2}{3}+\frac{1}{4}$
$=\frac{8}{12}+\frac{\square}{12}$
$=\frac{11}{\square}$

b $\frac{7}{8}-\frac{9}{16}$
$=\frac{\square}{16}-\frac{9}{16}$
$=\frac{\square}{16}$

c $1\frac{4}{7}\times\frac{3}{5}$
$=\frac{\square}{7}\times\frac{3}{5}$
$=\frac{\square}{35}$

d $\frac{5}{7}\div\frac{2}{3}$
$=\frac{5}{7}\ \square\ \frac{3}{2}$
$=\frac{15}{\square}=\square\ \frac{\square}{14}$

5 State the reciprocal of the following fractions.

a $\frac{5}{8}$ **b** $\frac{3}{2}$ **c** $3\frac{1}{4}$ **d** $1\frac{1}{11}$

6 Write the 'first step' for the following division questions (no need to find the answer).

a $\frac{3}{13}\div\frac{9}{7}$ **b** $\frac{1}{2}\div\frac{1}{4}$ **c** $\frac{17}{10}\div\frac{3}{1}$ **d** $2\frac{1}{3}\div1\frac{3}{4}$

7–12(½) | 7–13(½) | 7–13(½)

FLUENCY

Example 3

7 Simplify:

a $\frac{1}{5}+\frac{2}{5}$ **b** $\frac{7}{9}-\frac{2}{9}$ **c** $\frac{5}{8}+\frac{7}{8}$ **d** $\frac{24}{7}-\frac{11}{7}$
e $\frac{3}{4}+\frac{2}{5}$ **f** $\frac{3}{10}+\frac{4}{5}$ **g** $\frac{5}{7}-\frac{2}{3}$ **h** $\frac{11}{18}-\frac{1}{6}$

Example 4

8 Simplify:

a $3\frac{1}{7}+1\frac{3}{7}$ **b** $7\frac{2}{5}+2\frac{1}{5}$ **c** $3\frac{5}{8}-1\frac{2}{8}$ **d** $8\frac{5}{11}-7\frac{3}{11}$
e $5\frac{1}{3}+4\frac{1}{6}$ **f** $17\frac{5}{7}+4\frac{1}{2}$ **g** $6\frac{1}{2}-2\frac{3}{4}$ **h** $4\frac{2}{5}-2\frac{5}{6}$

Example 5a
Example 5b

9 Simplify:

a $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{9}\times\frac{5}{7}$ **c** $\frac{7}{5}\times\frac{6}{5}$ **d** $\frac{5}{3}\times\frac{8}{9}$
e $\frac{4}{9}\times\frac{3}{8}$ **f** $\frac{12}{10}\times\frac{5}{16}$ **g** $\frac{12}{9}\times\frac{2}{5}$ **h** $\frac{24}{8}\times\frac{5}{3}$

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3B

FLUENCY

Example 5c

10 Simplify:

a $2\frac{3}{4} \times 1\frac{1}{3}$ **b** $3\frac{2}{7} \times \frac{1}{3}$ **c** $4\frac{1}{6} \times 3\frac{3}{5}$ **d** $10\frac{1}{2} \times 3\frac{1}{3}$

Example 6a

Example 6b

11 Simplify:

a $\frac{2}{9} \div \frac{3}{5}$ **b** $\frac{1}{3} \div \frac{2}{5}$ **c** $\frac{8}{7} \div \frac{11}{2}$ **d** $\frac{11}{3} \div \frac{5}{2}$

e $\frac{3}{4} \div \frac{6}{7}$ **f** $\frac{10}{15} \div \frac{1}{3}$ **g** $\frac{6}{5} \div \frac{9}{10}$ **h** $\frac{22}{35} \div \frac{11}{63}$

Example 6c

12 Simplify:

a $1\frac{4}{7} \div 1\frac{2}{3}$ **b** $3\frac{1}{5} \div 8\frac{1}{3}$ **c** $3\frac{1}{5} \div 2\frac{2}{7}$ **d** $6\frac{2}{4} \div 2\frac{1}{6}$

13 Simplify:

a $\frac{1}{2} \times \frac{3}{4} \div \frac{2}{5}$ **b** $\frac{3}{7} \div \frac{9}{2} \times \frac{14}{16}$ **c** $2\frac{1}{3} \div 1\frac{1}{4} \div 1\frac{3}{5}$ **d** $4\frac{1}{2} \times 3\frac{1}{3} \div 10$

14, 15 14–16 15–17

PROBLEM-SOLVING

14 Max and Tanya are painting two adjacent walls of equal area. Max has painted $\frac{3}{7}$ of his wall and Tanya has painted $\frac{2}{5}$ of her wall.

a What fraction of the two walls have Max and Tanya painted in total?

b What fraction of the two walls remains to be painted?

15 Eilish was required to write a 600 word Literature essay. After working away for one hour, the word count on her computer showed that she had typed 240 words. What fraction of the essay does Eilish still need to complete? Write your answer in simplest form.

16 Vernald ordered $12\frac{1}{4}$ kilograms of Granny Smith apples. Unfortunately $\frac{3}{7}$ of the apples were bruised and unusable. How many kilograms of good apples did Vernald have to make his apple pies?



17 For Mary's party, she asked her dad to buy 18 bottles of soft drink. Each bottle contained $1\frac{1}{4}$ litres. The glasses they had for the party could hold $\frac{1}{5}$ of a litre. How many glasses could be filled from the 18 bottles of soft drink?

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3B

18 | 18, 19 | 18–20

REASONING

18 Fill in the empty boxes to make the following fraction equations correct. More than one answer may be possible.

a $\frac{3}{\square} + \frac{2}{\square} = 1$

b $\frac{11}{7} - \frac{\square}{3} = \frac{5}{21}$

c $\frac{1}{\square} + \frac{\square}{3} = \frac{13}{15}$

d $5\frac{\square}{4} - 3\frac{\square}{2} = 1\frac{3}{4}$

19 Fill in the empty boxes to make the following fraction equations correct. More than one answer may be possible.

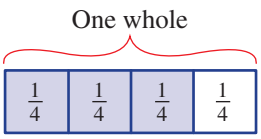
a $\frac{\square}{4} \times \frac{5}{6} = \frac{\square}{12}$

b $\frac{\square}{21} \div \frac{3}{7} = 2$

c $\frac{2}{\square} + \frac{\square}{5} = \frac{2}{5}$

d $1\frac{1}{4} \times \square\frac{\square}{\square} = 4\frac{1}{12}$

20 a What fraction is $\frac{1}{4}$ of $\frac{3}{4}$?



b How many 'lots' of $\frac{3}{4}$ are in 1 whole?

c Using multiplication find how many 'lots' of $\frac{3}{4}$ are in $3\frac{3}{4}$.

d Calculate $3\frac{3}{8} \div \frac{3}{4}$.

How small, how close, how large?

— | — | 21

ENRICHMENT

21 You have five different fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ and four different operations $+, -, \times, \div$ at your disposal. You must use each fraction and each operation once and only once. You may use as many brackets as you need. Here is your challenge:

a Produce an expression with the smallest possible positive answer.

b Produce an expression with an answer of 1 or as close to 1 as possible.

c Produce an expression with the largest possible answer.

An example of an expression using each of the five fractions and four operations is $\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right) \times \frac{1}{5} \div \frac{1}{6}$. This has an answer of $\frac{7}{10}$.

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3C Operations with negative fractions

Interactive

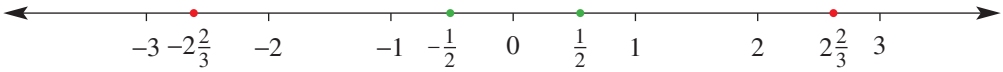
Widgets

HOTsheets

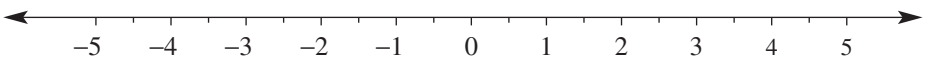
Walkthroughs

The English mathematician named John Wallis (1616–1703) invented a number line that displayed numbers extending in both the positive and negative directions.

So, just as we can have negative integers, we can also have negative fractions. In fact, each positive fraction has an opposite (negative) fraction. Two examples are highlighted on the number line below:



Let's start: Where do you end up?



You are given a starting point and a set of instructions to follow. You must determine where the finishing point is. The first set of instructions reviews the addition and subtraction of integers. The other two sets involve the addition and subtraction of positive and negative fractions.

- Starting point is 1. Add 3, subtract 5, add −2, subtract −4, subtract 3.
  Finishing point = ☐
- Starting point is 0. Subtract $\frac{3}{5}$, add $\frac{1}{5}$, add $-\frac{4}{5}$, subtract $\frac{2}{5}$, subtract $-\frac{3}{5}$.
  Finishing point = ☐
- Starting point is $\frac{1}{2}$. Subtract $\frac{3}{4}$, add $-\frac{1}{3}$, subtract $-\frac{1}{2}$, subtract $\frac{1}{12}$, add $\frac{1}{6}$.
  Finishing point = ☐

Key ideas

- The techniques for +, −, ×, ÷ positive fractions also apply to negative fractions.
- The arithmetic rules we observed for integers (Chapter 1) also apply to fractions.
- Subtracting a larger positive fraction from a smaller positive fraction will result in a negative fraction.
  For example: $\frac{1}{5}-\frac{2}{3}=\frac{3}{15}-\frac{10}{15}=-\frac{7}{15}$
- Adding a negative fraction is equivalent to subtracting its opposite.
  For example: $\frac{1}{2}+\left(-\frac{1}{3}\right)=\frac{1}{2}-\left(+\frac{1}{3}\right)=\frac{1}{2}-\frac{1}{3}$
- Subtracting a negative fraction is equivalent to adding its opposite.
  For example: $\frac{1}{2}-\left(-\frac{1}{3}\right)=\frac{1}{2}+\left(+\frac{1}{3}\right)=\frac{1}{2}+\frac{1}{3}$
- The product or quotient of two fractions of the same sign (positive or negative) is a positive fraction.
  - Product: $\frac{1}{3}\times\frac{2}{5}=\frac{2}{15}$ or $-\frac{1}{3}\times\left(-\frac{2}{5}\right)=\frac{2}{15}$
  - Quotient: $\frac{2}{15}\div\frac{1}{3}=\frac{2}{5}$ or $-\frac{2}{15}\div\left(-\frac{1}{3}\right)=\frac{2}{5}$

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Key ideas

- The product or quotient of two fractions of the opposite sign (positive and negative) is a negative fraction.
  - Product: $\frac{1}{2} \times \left(-\frac{1}{4}\right) = -\frac{1}{8}$ or $-\frac{1}{2} \times \frac{1}{4} = -\frac{1}{8}$
  - Quotient: $\frac{1}{8} \div \left(-\frac{1}{2}\right) = -\frac{1}{4}$ or $-\frac{1}{8} \div \frac{1}{2} = -\frac{1}{4}$



Example 7 Adding and subtracting negative fractions

Simplify:

a $\frac{2}{7} + \left(-\frac{5}{7}\right)$ **b** $\frac{2}{3} - \left(-\frac{4}{3}\right)$

c $\frac{1}{5} + \left(-\frac{1}{4}\right)$ **d** $-\frac{7}{3} - \left(-3\frac{2}{3}\right)$

SOLUTION	EXPLANATION
a $\frac{2}{7} + \left(-\frac{5}{7}\right) = \frac{2}{7} - \frac{5}{7}$	Adding $-\frac{5}{7}$ is equivalent to subtracting $\frac{5}{7}$.
$= -\frac{3}{7}$	
b $\frac{2}{3} - \left(-\frac{4}{3}\right) = \frac{2}{3} + \frac{4}{3}$	Subtracting $-\frac{4}{3}$ is equivalent to adding $\frac{4}{3}$.
$= \frac{6}{3} = 2$	
c $\frac{1}{5} + \left(-\frac{1}{4}\right) = \frac{1}{5} - \frac{1}{4}$	Adding $-\frac{1}{4}$ is equivalent to subtracting $\frac{1}{4}$.
$= \frac{4}{20} - \frac{5}{20}$	The LCM of 5 and 4 is 20. Write equivalent fractions with LCD of 20.
$= -\frac{1}{20}$	Subtract the numerators.
d $-\frac{7}{3} - \left(-3\frac{2}{3}\right) = -\frac{7}{3} + 3\frac{2}{3}$	Subtracting $-3\frac{2}{3}$ is equivalent to adding $3\frac{2}{3}$.
$= -\frac{7}{3} + \frac{11}{3}$	Convert mixed numeral to improper fraction. Denominators are the same, therefore add numerators $-7 + 11 = 4$.
$= \frac{4}{3} = 1\frac{1}{3}$	

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Example 8 Multiplying with negative fractions

Simplify:

a $\frac{2}{3} \times \left(-\frac{4}{5}\right)$ **b** $-\frac{6}{5} \times \left(-\frac{3}{4}\right)$

SOLUTION	EXPLANATION
a $\frac{2}{3} \times \left(-\frac{4}{5}\right) = -\frac{8}{15}$	The two fractions are of opposite sign so the answer is a negative.
b $-\frac{6}{5} \times \left(-\frac{3}{4}\right) = \frac{6}{5} \times \frac{3}{4}$	The two fractions are of the same sign, so the answer is a positive.
$= \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$	Cancel where possible, then multiply numerators and multiply denominators.

Example 9 Dividing with negative fractions

Simplify:

a $-\frac{2}{5} \div \left(-\frac{3}{4}\right)$ **b** $-1\frac{1}{3} \div 3$

SOLUTION	EXPLANATION
a $-\frac{2}{5} \div \left(-\frac{3}{4}\right) = -\frac{2}{5} \times \left(-\frac{4}{3}\right)$	The reciprocal of $\left(-\frac{3}{4}\right)$ is $\left(-\frac{4}{3}\right)$
$= \frac{2}{5} \times \frac{4}{3}$	The two fractions are of the same sign so the answer is a positive.
$= \frac{8}{15}$	The answer should be in simplest form.
b $-1\frac{1}{3} \div 3 = -\frac{4}{3} \times \frac{1}{3}$	The reciprocal of 3 is $\frac{1}{3}$.
$= -\frac{4}{9}$	The two numbers are of opposite sign, so the answer is a negative.

Exercise 3C

1–3 | 2, 3 | —

UNDERSTANDING

1 Draw a number line from -4 to 4 and mark on it the following negative and positive fractions.

a $-\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $1\frac{1}{2}$

d $-2\frac{1}{2}$ **e** $-3\frac{4}{5}$ **f** $-\frac{7}{3}$

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3C

UNDERSTANDING

2 Complete these sentences:

a Adding $\left(-\frac{1}{4}\right)$ is equivalent to subtracting ____.

b Adding $\frac{1}{3}$ is equivalent to subtracting ____.

c Subtracting $\left(-\frac{3}{5}\right)$ is equivalent to adding ____.

d Subtracting $\frac{2}{7}$ is equivalent to adding ____.

3 Do not evaluate the following expressions, just state whether the answer will be positive or negative.

a $-\frac{3}{5}\times\left(-\frac{1}{3}\right)$ **b** $-5\frac{1}{5}\times\frac{9}{11}$ **c** $\frac{5}{3}\div\left(-\frac{3}{5}\right)$ **d** $-2\frac{1}{7}\div\left(-8\frac{1}{3}\right)$

4–7(½) 4–7(½) 5–7(½)

FLUENCY

Example 7a Example 7b

4 Simplify:

a $-\frac{6}{7}+\frac{2}{7}$ **b** $-\frac{3}{5}+\frac{4}{5}$ **c** $-\frac{5}{9}-\frac{2}{9}$ **d** $-\frac{11}{3}-\frac{5}{3}$

e $\frac{1}{3}+\left(-\frac{2}{3}\right)$ **f** $\frac{1}{5}+\left(-\frac{3}{5}\right)$ **g** $\frac{1}{4}-\left(-\frac{5}{4}\right)$ **h** $\frac{3}{11}-\left(-\frac{4}{11}\right)$

Example 7c Example 7d

5 Simplify:

a $\frac{1}{4}+\left(-\frac{1}{3}\right)$ **b** $\frac{3}{7}+\left(-\frac{4}{5}\right)$ **c** $\frac{1}{2}-\left(-\frac{3}{5}\right)$ **d** $\frac{2}{9}-\left(-\frac{2}{3}\right)$

e $-\frac{3}{2}-\left(-\frac{5}{4}\right)$ **f** $-\frac{5}{8}-\left(-\frac{3}{4}\right)$ **g** $-\frac{7}{5}-\left(-1\frac{1}{4}\right)$ **h** $-\frac{8}{3}-\left(-2\frac{2}{5}\right)$

Example 8

6 Simplify:

a $\frac{3}{5}\times\left(-\frac{4}{7}\right)$ **b** $-\frac{2}{5}\times\frac{8}{11}$ **c** $-\frac{1}{3}\times\left(-\frac{4}{5}\right)$ **d** $-\frac{5}{9}\times\left(-\frac{3}{2}\right)$

e $-\frac{3}{9}\times\frac{4}{7}$ **f** $\frac{2}{6}\times\left(-\frac{3}{8}\right)$ **g** $-1\frac{1}{2}\times\left(-\frac{2}{7}\right)$ **h** $-\frac{3}{8}\times 3\frac{1}{5}$

Example 9

7 Simplify:

a $-\frac{5}{7}\div\frac{3}{4}$ **b** $-\frac{1}{4}\div\frac{5}{9}$ **c** $-\frac{2}{3}\div\left(-\frac{5}{4}\right)$ **d** $-\frac{4}{9}\div\left(-\frac{1}{3}\right)$

e $-\frac{4}{7}\div 2$ **f** $-\frac{3}{5}\div 4$ **g** $-1\frac{1}{2}\div(-2)$ **h** $-5\frac{1}{3}\div\left(-2\frac{2}{9}\right)$

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8, 9 | 9–11 | 9–11

3C

PROBLEM-SOLVING

8 Arrange these fractions from smallest to largest.

$\frac{3}{4}, -\frac{1}{2}, -\frac{5}{3}, -\frac{3}{4}, -1\frac{1}{2}, \frac{1}{16}, -\frac{1}{5}, 3\frac{1}{10}$

9 Toolapool has an average maximum temperature of $13\frac{1}{2}$°C and an average minimum temperature of $-3\frac{1}{4}$°C. Average temperature range is calculated by subtracting the average minimum temperature from the average maximum temperature. What is the average temperature range for Toolapool?



10 Xaio aims to get 8 hours sleep per week night. On Monday night he slept for $6\frac{1}{3}$ hours, on Tuesday night $7\frac{1}{2}$ hours, on Wednesday night $5\frac{3}{4}$ hours and on Thursday night $8\frac{1}{4}$ hours.

- **a** State the difference between the amount of sleep Xaio achieved each night and his goal of 8 hours. Give a negative answer if the amount of sleep is less than 8 hours.
- **b** After four nights, how much is Xaio ahead or behind in terms of his sleep goal?
- **c** If Xaio is to exactly meet his weekly goal, how much sleep must he get on Friday night?

11 Maria's mother wants to make 8 curtains that each require $2\frac{1}{5}$ metres of material in a standard width, but she only has $16\frac{1}{4}$ metres. She asks Maria to buy more material. How much more material must Maria buy?



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3C

12 | 12, 13 | 13, 14

REASONING

12 Place an inequality sign (< or >) between the following fraction pairs to make a true statement.

a $-\frac{1}{3}$ $-\frac{1}{2}$ **b** $-3\frac{1}{5}$ $-2\frac{3}{7}$ **c** $\frac{1}{4}$ $-\frac{1}{2}$ **d** $-\frac{3}{5}$ $\frac{1}{11}$

e $2\frac{1}{5}$ $-4\frac{3}{5}$ **f** 0 $-\frac{1}{100}$ **g** $\frac{4}{9}$ $\frac{5}{9}$ **h** $-\frac{4}{9}$ $-\frac{5}{9}$

13 Do not evaluate the following expressions, just state whether the answer will be positive or negative.

a $-\frac{2}{7}\times\left(-\frac{1}{7}\right)\times\left(-\frac{3}{11}\right)$ **b** $-4\frac{1}{5}\times\left(-\frac{9}{11}\right)^2$

c $-\frac{5}{6}\div\left(-\frac{2}{7}\right)\times\frac{1}{3}\times\left(-\frac{4}{9}\right)$ **d** $\left(-\frac{3}{7}\right)^3\div\left(-4\frac{1}{5}\right)^3$

14 If $a > 0$ and $b > 0$ and $a < b$, place an inequality sign between the following fractions pairs to make a true statement.

a $\frac{a}{b}$ $\frac{b}{a}$ **b** $\frac{a}{b}$ $-\frac{a}{b}$ **c** $-\frac{a}{b}$ $-\frac{b}{a}$ **d** $-\frac{b}{a}$ $-\frac{a}{b}$

Positive and negative averages | — | — | 15

ENRICHMENT

15 a Calculate the average (mean) of the following sets of numbers.

i $1\frac{1}{2}, \frac{3}{4}, \frac{5}{4}, 2\frac{1}{2}$

ii $-\frac{2}{3}, \frac{5}{6}, \frac{7}{6}, \frac{1}{3}, -1\frac{1}{3}$

iii $-2\frac{1}{5}, -\frac{3}{5}, 0, \frac{1}{5}, -1\frac{3}{5}$

iv $-7\frac{1}{3}, -2\frac{1}{2}, -5\frac{1}{6}, -3\frac{3}{10}$

b List a set of five different fractions that have an average of 0.

c List a set of five different fractions that have an average of $-\frac{3}{4}$.

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3D Understanding decimals

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

Decimals are another way of representing 'parts of a whole'. They are an extension of our base 10 number system. The term *decimal* is derived from the Latin word *decem* meaning 'ten'.

A decimal point is used to separate the whole number and the fraction part. Decimals have been studied extensively in Year 7. In this section we revisit the concepts of comparing decimals and converting between decimals and fractions.



Why do we use base 10?

Let's start: Order 10

The following 10 numbers all contain a whole number part and a fraction part. Some are decimals and some are mixed numerals.
Work with a partner. Your challenge is to place the 10 numbers in ascending order.

$4\frac{1}{11}$ $3\frac{5}{6}$ 3.3 $3\frac{72}{100}$ $3\frac{1}{3}$ 2.85 3.09 $2\frac{3}{4}$ $3\frac{2}{5}$ 3.9

Key ideas

- When dealing with decimal numbers, the place value table is extended to involve tenths, hundredths, thousandths, etc.
- The number 517.364 means:

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
5	1	7	.	3	6	4
5×100	1×10	7×1	.	$3 \times \frac{1}{10}$	$6 \times \frac{1}{100}$	$4 \times \frac{1}{1000}$
500	10	7	.	$\frac{3}{10}$	$\frac{6}{100}$	$\frac{4}{1000}$

- **Comparing and ordering decimals**
  To compare two decimal numbers with digits in the same place value columns, you must compare the left-most digits first. Continue comparing digits as you move from left to right until you find two digits that are different.
  For example, compare 362.581 and 362.549.

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
3	6	2	.	5	8	1
3	6	2	.	5	4	9

  Both numbers have identical digits in the hundreds, tens, units and tenths columns. Only when we get to the hundredths column is there a difference. The 8 is bigger than the 4 and therefore $362.581 > 362.549$.

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Key ideas

Converting decimals to fractions

- Count the number of decimal places used.
- This is the number of zeroes that you must place in the denominator.
- Simplify the fraction if required.

For example: $0.64 = \frac{64}{100} = \frac{16}{25}$

Converting fractions to decimals

- If the denominator is a power of 10, simply change the fraction directly to a decimal from your knowledge of its place value.

For example: $\frac{239}{1000} = 0.239$

- If the denominator is not a power of 10, try to find an equivalent fraction for which the denominator is a power of 10 and then convert to a decimal.

For example: $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$

- A method that will always work for converting fractions to decimals is to divide the numerator by the denominator. This can result in terminating and recurring decimals and is covered in Section 3F.

	A	B	C
1	Spreadsheet notation for fractions and division operations		
2			
3	Fraction notation for one quarter:	1/4	
4	Formula for the division operation 1 ÷ 4:	=1/4	
5	Result of division operation 1 ÷ 4:	0.25	
6			
7			

A fraction is equivalent to a division operation. In spreadsheets and some calculators, fractions and division operations are typed the same way, using a slash (/) between the numbers.

Example 10 Comparing decimals

Compare the following decimals and place the correct inequality sign between them.
57.89342 and 57.89631

SOLUTION	EXPLANATION
$57.89342 < 57.89631$	Digits are the same in the tens, units, tenths and hundredths columns. Digits are first different in the thousandths column. $\frac{3}{1000} < \frac{6}{1000}$

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Example 11 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.725 **b** 5.12

SOLUTION	EXPLANATION
a $\frac{725}{1000} = \frac{29}{40}$	Three decimal places, therefore three zeroes in denominator. 0.725 = 725 thousandths
b $5\frac{12}{100} = 5\frac{3}{25}$	Two decimal places, therefore two zeroes in denominator. 0.12 = 12 hundredths

Example 12 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{239}{100}$ **b** $\frac{9}{25}$

SOLUTION	EXPLANATION
a $\frac{239}{100} = 2\frac{39}{100} = 2.39$	Convert improper fraction to a mixed numeral. Denominator is a power of 10.
b $\frac{9}{25} = \frac{36}{100} = 0.36$	$\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100}$

Exercise 3D

1–4 | 4 | —

UNDERSTANDING

1 Which of the following is the mixed numeral equivalent of 8.17?

A $8\frac{1}{7}$ **B** $8\frac{17}{10}$ **C** $8\frac{1}{17}$ **D** $8\frac{17}{1000}$ **E** $8\frac{17}{100}$

2 Which of the following is the mixed numeral equivalent of 5.75?

A $5\frac{75}{10}$ **B** $5\frac{25}{50}$ **C** $5\frac{3}{4}$ **D** $5\frac{15}{20}$ **E** $5\frac{75}{1000}$

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3D

UNDERSTANDING

Example 10

3 Compare the following decimals and place the correct inequality sign (< or >) between them.

a 36.485 37.123 **b** 21.953 21.864

c 0.0372 0.0375 **d** 4.21753 4.21809

e 65.4112 64.8774 **f** 9.5281352 9.5281347

4 Complete the following statements to convert the fractions into decimals.

a $\frac{3}{5} = \frac{6}{\square} = 0.\square$ **b** $\frac{11}{20} = \frac{\square}{100} = 0.5\square$

5–7 | 5, 6–8(½) | 5, 6(½), 8(½), 9

FLUENCY

5 Arrange the following sets of decimals in descending order.

a 3.625, 3.256, 2.653, 3.229, 2.814, 3.6521

b 11.907, 11.891, 11.875, 11.908, 11.898, 11.799

c 0.043, 1.305, 0.802, 0.765, 0.039, 1.326

Example 11

6 Convert the following decimals to fractions in their simplest form.

a 0.31 **b** 0.537 **c** 0.815 **d** 0.96

e 5.35 **f** 8.22 **g** 26.8 **h** 8.512

i 0.052 **j** 6.125 **k** 317.06 **l** 0.424

Example 12a

7 Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $\frac{301}{1000}$ **c** $\frac{405}{100}$ **d** $\frac{76}{10}$

Example 12b

8 Convert the following fractions to decimals.

a $\frac{3}{25}$ **b** $\frac{7}{20}$ **c** $\frac{5}{2}$ **d** $\frac{7}{4}$

e $\frac{11}{40}$ **f** $\frac{3}{8}$ **g** $\frac{17}{25}$ **h** $\frac{29}{125}$

9 Convert the following mixed numerals to decimals and then place them in ascending order.

$2\frac{2}{5}$, $2\frac{1}{4}$, $2\frac{3}{8}$, $2\frac{7}{40}$, $2\frac{9}{50}$, $2\frac{3}{10}$

10, 11 | 11, 12 | 12, 13

PROBLEM-SOLVING

10 The distances from Nam's locker to his six different classrooms are listed below:

- Locker to room B5 (0.186 km)
- Locker to gym (0.316 km)
- Locker to room A1 (0.119 km)
- Locker to room C07 (0.198 km)
- Locker to room P9 (0.254 km)
- Locker to BW Theatre (0.257 km)

List Nam's six classrooms in order of distance of his locker from the closest classroom to the one furthest away.

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3D

PROBLEM-SOLVING

11 The prime minister's approval rating is 0.35, while the opposition leader's approval rating is $\frac{3}{8}$. Which leader is ahead in the popularity polls and by how much?



12 Lydia dug six different holes for planting six different types of fruit bushes and trees.
He measured the dimensions of the holes and found them to be:
Hole A depth 1.31 m, width 0.47 m
Hole B depth 1.15 m, width 0.39 m
Hole C depth 0.85 m, width 0.51 m
Hole D depth 0.79 m, width 0.48 m
Hole E depth 1.08 m, width 0.405 m
Hole F depth 1.13 m, width 0.4 m

a List the holes in increasing order of depth.

b List the holes in decreasing order of width.



13 | 13, 14 | 13, 14

REASONING

13 **a** Write a decimal that lies midway between 2.65 and 2.66.

b Write a fraction that lies midway between 0.89 and 0.90.

c Write a decimal that lies midway between 4.6153 and 4.6152.

d Write a fraction that lies midway between 2.555 and 2.554.

14 Complete the following magic square using a combination of fractions and decimals.

2.6		$1\frac{4}{5}$
	$\frac{6}{2}$	
4.2		

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3D Exchange rates — — 15

ENRICHMENT

15 The table below shows a set of exchange rates between the US dollar (US$), the Great Britain pound (£), the Canadian dollar (C$), the euro (€) and the Australian dollar (A$).

	USD (United States)	GBP (United Kingdom)	CAD (Canada)	EUR (European Union)	AUD (Australia)
USD	1	1.58781	0.914085	1.46499	0.866558
GBP	0.629795	1	0.575686	0.922649	0.545754
CAD	1.09399	1.73705	1	1.60269	0.948006
EUR	0.682594	1.08383	0.623949	1	0.591507
AUD	1.15399	1.83232	1.05484	1.69059	1

The following two examples are provided to help you to interpret the table.

- A$1 will buy US$0.866558.
- You will need A$1.15399 to buy US$1.

Study the table and answer the following questions.

a How many euros will A$100 buy?

b How many A$ would buy £100?

c Which country has the most similar currency rate to Australia?

d Would you prefer to have £35 or €35?

e C$1 has the same value as how many US cents?

f If the cost of living was the same in each country in terms of each country's own currency, list the five money denominations in descending order of value for money.

g A particular new car costs £30 000 in Great Britain and $70 000 in Australia. If it costs A$4500 to freight a car from Great Britain to Australia, which car is cheaper to buy? Justify your answer by using the exchange rates in the table.

h Research the current exchange rates and see how they compare to those listed in the table.



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3E Operations with decimals CONSOLIDATING

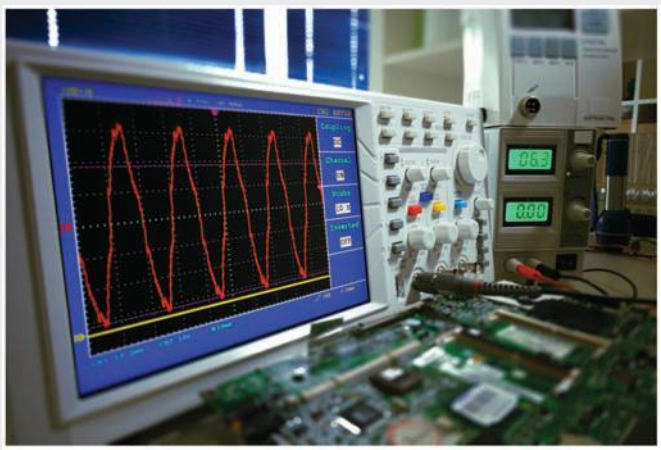
Interactive

Widgets

HOTsheets

Walkthroughs

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing decimals.



Precision electronic measuring instruments give a decimal read-out.

Let's start: Match the phrases

There are eight different sentence beginnings and eight different sentence endings below. Your task is to match each sentence beginning with its correct ending. When you have done this, write the eight correct sentences in your work book.

Sentence beginnings	Sentence endings
When adding or subtracting decimals	the decimal point appears to move two places to the right.
When multiplying decimals	the decimal point in the quotient goes directly above the decimal point in the dividend.
When multiplying decimals by 100	make sure you line up the decimal points.
When dividing decimals by decimals	the number of decimal places in the question must equal the number of decimal places in the answer.
When multiplying decimals	the decimal point appears to move two places to the left.
When dividing by 100	start by ignoring the decimal points.
When dividing decimals by a whole number	we start by changing the question so that the divisor is a whole number.

Key ideas

Adding and subtracting decimals

- Ensure digits are correctly aligned in similar place value columns.
- Ensure the decimal points are lined up directly under one another.

$37.56 + 5.231$

$$\begin{array}{r} 37.560 \\ +\ \underline{5.231} \end{array}$$ ✓

$$\begin{array}{r} 3\ 7.56 \\ \underline{5.2\,31} \end{array}$$ ✗

Multiplying and dividing decimals by powers of 10

- When multiplying, the decimal point appears to move to the *right* the same number of places as there are zeroes in the multiplier.

$13.753 \times 100 = 1375.3$

13.753

Multiply by 10 twice.

- When dividing, the decimal point appears to move to the *left* the same number of places as there are zeroes in the divisor.

$586.92 \div 10 = 58.692$

586.92

Divide by 10 once.

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Key ideas

Multiplying decimals

- Initially ignore the decimal points and carry out routine multiplication.
- The decimal place is correctly positioned in the answer according to the following rule: 'The number of decimal places in the answer must equal the total number of decimal places in the question.'

$5.73 \times 8.6 \longrightarrow \begin{array}{r} 573 \\ \times \quad 86 \\ \hline 49278 \end{array} \longrightarrow 5.73 \times 8.6 = 49.278$

(3 decimal places in question, 3 decimal places in answer)

Dividing decimals

The decimal point in the quotient goes directly above the decimal point in the **dividend**.

$56.34 \div 3$ Divisor $\longrightarrow 3\overline{)56.34}$ $\begin{array}{l} 18.78 \\ \leftarrow \text{Quotient (answer)} \\ \leftarrow \text{Dividend} \end{array}$

We avoid dividing decimals by other decimals. Instead we change the **divisor** into a whole number. Of course, whatever change we make to the divisor we must also make to the dividend, so it is equivalent to multiplying by 1 and the value of the question is not changed.

We avoid $\longrightarrow 27.354 \div 0.02$

Preferring to do $\longrightarrow 2735.4 \div 2$

$27.354 \div 0.02$

Example 13 Adding and subtracting decimals

Calculate:

a $23.07 + 103.659 + 9.9$ **b** $9.7 - 2.86$

SOLUTION

a
$$\begin{array}{r} 23.070 \\ 103.659 \\ 9.900 \\ \hline 136.629 \end{array}$$

b
$$\begin{array}{r} {}^{8}\not{9}.{}^{16}\not{7}\,{}^{1}\not{0} \\ -\ 2.\ 8\ 6 \\ \hline 6.\ 8\ 4 \end{array}$$

EXPLANATION

Make sure all decimal points and places are correctly aligned directly under one another. Fill in missing decimal places with zeroes. Carry out the addition of each column, working from right to left.

Align decimal points directly under one another and fill in missing decimal places with zeroes.
Carry out subtraction following the same procedure as for subtraction of whole numbers.

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Example 14 Multiplying and dividing by powers of 10

Calculate:

a $9.753 \div 100$ **b** $27.58 \times 10\,000$

SOLUTION	EXPLANATION
a $9.753 \div 100 = 0.09753$	Dividing by 100, therefore decimal point must move two places to the left. Additional zeroes are inserted as necessary. 09.753
b $27.58 \times 10\,000 = 275\,800$	Multiplying by 10 000, therefore decimal point must move four places to the right. Additional zeroes are inserted as necessary. 27.5800

Example 15 Multiplying decimals

Calculate:

a 2.57×3 **b** 4.13×9.6

SOLUTION	EXPLANATION
a $^{1}2^{2}5\,7 \times 3 = 7\,7\,1$ $2.57 \times 3 = 7.71$	Perform multiplication ignoring decimal point. There are two decimal places in the question, so two decimal places in the answer. Estimation is less than 10 ($\approx 3 \times 3 = 9$).
b $413 \times 96 = 2478 + 37170 = 39648$ $4.13 \times 9.6 = 39.648$	Ignore both decimal points. Perform routine multiplication. There is a total of three decimal places in the question, so there must be three decimal places in the answer. Estimation is about 40 ($\approx 4 \times 10 = 40$).

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Example 16 Dividing decimals

Calculate:

a $35.756 \div 4$ **b** $64.137 \div 0.03$

SOLUTION	EXPLANATION
a 8.939 $4\overline{)35.^{3}7^{1}5^{3}6}$ 8. 9 3 9	Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend.
b $64.137 \div 0.03$ $= 6413.7 \div 3$ $= 2137.9$ $3\overline{)64^{1}1^{2}3.^{2}7}$ 21 3 7. 9	Instead of dividing by 0.03, multiply both the divisor and the dividend by 100. Move each decimal point two places to the right. Carry out the division question $6413.7 \div 3$.

Exercise 3E

1–4 | 4 | —

UNDERSTANDING

1 Which of the following is correctly set up for the following addition problem?
$5.386 + 53.86 + 538.6$

A
$$\begin{array}{r} 5.386 \\ 53.86 \\ +\ 538.6 \\ \hline \end{array}$$

B
$$\begin{array}{r} 5.386 \\ 53.860 \\ +538.600 \\ \hline \end{array}$$

C
$$\begin{array}{r} 5.386 \\ 53.86 \\ +\ \ 538.6 \\ \hline \end{array}$$

D
$$\begin{array}{r} 538 + 53 + 5 \\ +\ 0.386 + 0.86 + 0.6 \\ \hline \end{array}$$

2 The correct answer to the problem $2.731 \div 1000$ is:
A 2731 **B** 27.31 **C** 2.731 **D** 0.02731 **E** 0.002731

3 If $56 \times 37 = 2072$, the correct answer to the problem 5.6×3.7 is:
A 207.2 **B** 2072 **C** 20.72 **D** 2.072 **E** 0.2072

4 Which of the following divisions would provide the same answer as the division question $62.5314 \div 0.03$?
A $625.314 \div 3$ **B** $6253.14 \div 3$ **C** $0.625314 \div 3$ **D** $625314 \div 3$

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3E

5(½), 8–11(½) | 5–11(½) | 7–12(½)

FLUENCY

Example 13

5 Calculate:

a 23.57 + 39.14 **b** 64.28 + 213.71 **c** 5.623 + 18.34 **d** 92.3 + 1.872

e 38.52 – 24.11 **f** 76.74 – 53.62 **g** 123.8 – 39.21 **h** 14.57 – 9.8

6 Calculate:

a 13.546 + 35.2 + 9.27 + 121.7 **b** 45.983 + 3.41 + 0.032 + 0.8942

c 923.8 + 92.38 + 9.238 + 0.238 **d** 4.572 + 0.0329 + 2.0035 + 11.7003

7 Calculate:

a 3.456 + 12.723 – 4.59 **b** 7.213 – 5.46 + 8.031

c 26.451 + 8.364 – 14.987 **d** 12.7 – 3.45 – 4.67

Example 14

8 Calculate:

a 36.5173 × 100 **b** 0.08155 × 1000 **c** 7.5 ÷ 10 **d** 3.812 ÷ 100

e 634.8 × 10 000 **f** 1.0615 ÷ 1000 **g** 0.003 × 10 000 **h** 0.452 ÷ 1000

Example 15

9 Calculate:

a 12.45 × 8 **b** 4.135 × 3 **c** 26.2 × 4.1 **d** 5.71 × 0.32

e 0.0023 × 8.1 **f** 300.4 × 2.2 **g** 7.123 × 12.5 **h** 81.4 × 3.59

Example 16a

10 Calculate:

a 24.54 ÷ 2 **b** 17.64 ÷ 3 **c** 0.0485 ÷ 5 **d** 347.55 ÷ 7

e 133.44 ÷ 12 **f** 4912.6 ÷ 11 **g** 2.58124 ÷ 8 **h** 17.31 ÷ 5

Example 16b

11 Calculate:

a 6.114 ÷ 0.03 **b** 0.152 ÷ 0.4 **c** 4023 ÷ 0.002 **d** 5.815 ÷ 0.5

e 0.02345 ÷ 0.07 **f** 16.428 ÷ 1.2 **g** 0.5045 ÷ 0.8 **h** 541.31 ÷ 0.4

12 Calculate:

a 13.7 + 2.59 **b** 35.23 – 19.71 **c** 15.4 × 4.3 **d** 9.815 ÷ 5

e 13.72 × 0.97 **f** 6.7 – 3.083 **g** 0.582 ÷ 0.006 **h** 7.9023 + 34.81

13, 14 | 14, 15 | 14–16

PROBLEM-SOLVING

13 The heights of Mrs Buchanan's five grandchildren are 1.34 m, 1.92 m, 0.7 m, 1.5 m, and 1.66 m. What is the combined height of Mrs Buchanan's grandchildren?

14 If the rental skis at Mt Buller were lined up end to end, they would reach from the summit of Mt Buller all the way down to the entry gate at Mirimbah. The average length of a downhill ski is 1.5 m and the distance from the Mt Buller summit to Mirimbah is 18.3 km. How many rental skis are there?

15 Joliet is a keen walker. She has a pedometer that shows she has walked 1 428 350 paces so far this year. Her average pace length is 0.84 metres. How many kilometres has Joliet walked so far this year? (Give your answer correct to the nearest kilometre.)

16 A steel pipe of length 7.234 m must be divided into four equal lengths. The saw blade is 2 mm thick. How long will each of the four lengths be?

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3E

17 | 17, 18 | 17, 18

REASONING

17 If $a = 0.12$, $b = 2.3$ and $c = 3.42$, find:

a $a + b + c$ **b** $c - (a + b)$ **c** $a \times b \times c$ **d** $c \div a - b$

18 If $a = 0.1$, $b = 2.1$ and $c = 3.1$, without evaluating, which of the following alternatives would provide the biggest answer?

A $a + b + c$ **B** $a \times b \times c$ **C** $b \div a + c$ **D** $c \div a \times b$

Target practice — | — | 19

ENRICHMENT

19 In each of the following problems, you must come up with a starting decimal number/s that will provide an answer within the target range provided when the nominated operation is performed.

For example: Find a decimal number that when multiplied by 53.24 will give an answer between 2.05 and 2.1.

You may like to use trial and error, or you may like to work out the question backwards.

Confirm these results on your calculator:

$0.03 \times 53.24 = 1.5972$ (answer outside target range – too small)

$0.04 \times 53.24 = 2.1296$ (answer outside target range – too large)

$0.039 \times 53.24 = 2.07636$ (answer within target range of 2.05 to 2.1)

Therefore a possible answer is 0.039.

Try the following target problems. (Aim to use as few decimal places as possible.)

Question	Starting number	Operation (instruction)	Target range
1	0.039	× 53.24	2.05–2.1
2		× 0.034	100–101
3		÷ 1.2374	75.7–75.8
4		× by itself (square)	0.32–0.33
5		÷ (–5.004)	9.9–9.99

Try the following target problems. (Each starting number must have at least two decimal places.)

Question	Two starting numbers	Operation (instruction)	Target range
6	0.05, 3.12	×	0.1–0.2
7		÷	4.1–4.2
8		×	99.95–100.05
9		+	0.001–0.002
10		–	45.27

You may like to make up some of your own target problems.

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3F Terminating, recurring and rounding decimals

Interactive

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HOTsheets

Walkthroughs

Not all fractions convert to the same type of decimal.
For example:

$$\frac{1}{2} = 1 \div 2 = 0.5$$

$$\frac{1}{3} = 1 \div 3 = 0.33333\ldots$$

$$\frac{1}{7} = 1 \div 7 = 0.142857\ldots$$

Decimals that stop (or terminate) are known as terminating decimals, whereas decimals that continue on forever with some form of pattern are known as repeating or recurring decimals.



The fraction of space occupied by each solar panel could be written as a terminating or recurring decimal.

Let's start: Decimal patterns

Carry out the following short divisions without using a calculator and observe the pattern of digits which occur.

For example: $\frac{1}{11} = 1 \div 11 = 0.0909090909\ldots$

$$11\overline{)1.\,{}^{1}0\,{}^{10}0\,{}^{1}0\,{}^{10}0\,{}^{1}0} \quad 0.\ 0\ 9\ 0\ 9\ldots$$

Try the following: $\frac{1}{3}, \frac{2}{7}, \frac{4}{9}, \frac{5}{11}, \frac{8}{13}, \frac{25}{99}$

Remember to keep adding zeroes to the dividend until you have a repetitive pattern.

Which fraction gives the longest repetitive pattern?

Key ideas

- A **terminating decimal** has a finite number of decimal places (i.e. it terminates).

  For example: $\frac{5}{8} = 5 \div 8 = 0.625$

  $$8\overline{)5.\,{}^{5}0\,{}^{2}0\,{}^{4}0} \quad 0.\ 6\ 2\ 5 \leftarrow \text{Terminating decimal}$$

- A **recurring decimal** (or **repeating decimal**) has an infinite number of decimal places with a finite sequence of digits that are repeated indefinitely.

  For example: $\frac{1}{3} = 1 \div 3 = 0.333\ldots$

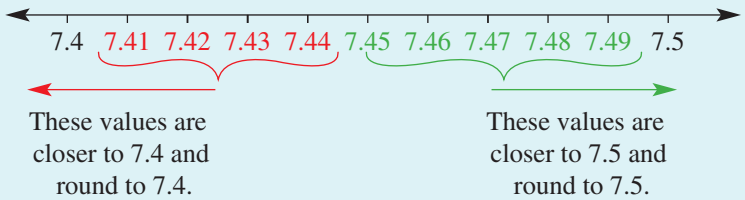
  $$3\overline{)1.\,{}^{1}0\,{}^{1}0\,{}^{1}0\,{}^{1}0} \quad 0.\ 3\ 3\ 3\ldots \leftarrow \text{Recurring decimal}$$

- A convention is to use dots placed above the digits to show the start and finish of a repeating cycle of digits.
  For example: $0.55555\ldots = 0.\dot{5}$ and $0.3412412412\ldots = 0.3\dot{4}1\dot{2}$
  Another convention is to use a horizontal line placed above the digits to show the repeating cycle of digits.
  For example: $0.55555\ldots = 0.\overline{5}$ and $0.3412412412\ldots = 0.3\overline{412}$

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Key ideas

- **Rounding decimals** involves approximating a decimal number to fewer decimal places.
  When rounding, the **critical digit** is the digit immediately after the rounding digit.
  - If the critical digit is less than five, the rounding digit is not changed.
  - If the critical digit is five or more, the rounding digit is increased by one.
    For example: 51.34721 rounded to two decimal places is 51.35.
    The critical digit is 7, hence the rounding digit is increased by 1.
  - An illustration of rounding to one decimal place:
    The decimals 7.41 to 7.44 are closer in value to 7.4 and will all round down to 7.4.
    The values 7.46 to 7.49 are closer in value to 7.5 and will all round up to 7.5.
    7.45 also rounds to 7.5.



Example 17 Writing terminating decimals

Convert the following fractions to decimals.

a $\frac{1}{4}$ **b** $\frac{7}{8}$

SOLUTION

a $\frac{1}{4} = 0.25$

b $\frac{7}{8} = 0.875$

EXPLANATION

$$4\overline{)1.^{1}0^{2}0}\ \ 0.\ 2\ 5$$

$$8\overline{)7.^{7}0^{6}0^{4}0}\ \ 0.\ 8\ 7\ 5$$

Example 18 Writing recurring decimals

Express the following fractions as recurring decimals.

a $\frac{2}{3}$ **b** $3\frac{5}{7}$

SOLUTION

a $\frac{2}{3} = 0.\dot{6}$

b $3\frac{5}{7} = 3.\dot{7}14285\dot{7}$ or $3.\overline{714285}$

EXPLANATION

$$3\overline{)2.^{2}0^{2}0^{2}0}\ \ 0.\ 6\ 6\ldots$$

$$8\overline{)5.^{5}0^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0}\ \ 0.\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ldots$$

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Example 19 Rounding decimals

Round each of the following to the specified number of decimal places.

a 15.35729 (3 decimal places) **b** 4.86195082 (4 decimal places)

SOLUTION	EXPLANATION
a 15.35729 = 15.357 (to 3 d.p.)	Critical digit is 2 which is less than 5. Rounding digit remains the same.
b 4.86195082 = 4.8620 (to 4 d.p.)	Critical digit is 5, therefore increase rounding digit by 1. There is a carry-on effect, as the rounding digit was a 9.

Example 20 Rounding recurring decimals

Write $\frac{3}{7}$ as a decimal correct to two decimal places.

SOLUTION

$\frac{3}{7} = 0.43$ (to 2 d.p.)

EXPLANATION

$$7\overline{)3.^{3}0^{2}0^{6}0^{4}0}\quad 0.\ 4\ 2\ 8$$

Exercise 3F

1–3 | 3 | —

UNDERSTANDING

1 State whether the following are terminating decimals (T) or recurring decimals (R).

a 5.47 **b** 3.1555… **c** $8.\dot{6}$ **d** 7.1834

e 0.333 **f** $0.\dot{5}3\dot{4}$ **g** 0.5615 **h** 0.32727…

2 Express the following recurring decimals using the convention of dots or a bar to indicate the start and finish of the repeating cycle.

a 0.33333… **b** 6.21212121… **c** 8.5764444…

d 2.135635635… **e** 11.2857328573… **f** 0.003523523…

3 Write down the 'critical' digit (the digit immediately after the rounding digit) for each of the following.

a 3.5724 (rounding to 3 decimal places) **b** 15.89154 (rounding to 1 decimal place)

c 0.004571 (rounding to 4 decimal places) **d** 5432.726 (rounding to 2 decimal places)

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3F

4, 5–6(½), 8, 9(½) | 4–9(½) | 4–6(½), 8–10(½)

FLUENCY

Example 17

4 Convert the following fractions to decimals.

a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{8}$ **d** $\frac{11}{20}$

Example 18

5 Express the following fractions as recurring decimals.

a $\frac{1}{3}$ **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{8}{11}$

e $\frac{3}{7}$ **f** $\frac{5}{13}$ **g** $3\frac{2}{15}$ **h** $4\frac{6}{7}$

Example 19

6 Round each of the following to the specified number of decimal places, which is the number in the bracket.

a 0.76581 (3) **b** 9.4582 (1) **c** 6.9701 (1) **d** 21.513426 (4)

e 0.9457 (2) **f** 17.26 (0) **g** 8.5974 (2) **h** 8.10552 (3)

7 Write each of the following decimals correct to two decimal places.

a 17.0071 **b** 5.1952 **c** 78.9963 **d** 0.0015

8 Round each of the following to the nearest whole number.

a 65.3197 **b** 8.581 **c** 29.631 **d** 4563.18

Example 20

9 Write each of the following fractions as decimals correct to two decimal places.

a $\frac{6}{7}$ **b** $\frac{2}{9}$ **c** $\frac{4}{11}$ **d** $\frac{5}{12}$

10 Estimate answers by firstly rounding each given number to one decimal place.

a $2.137 + 8.59 - 1.61$ **b** $15.03 - 6.991 + 3.842$ **c** 7.05×3

d 4×2.89 **e** $6.92 \div 3$ **f** $12.04 \div 3.99$

11, 12 | 12–14 | 12–14

PROBLEM-SOLVING

11 Round each of the following to the number of decimal places specified in the brackets.

a 7.699951 (4) **b** 4.95953 (1) **c** 0.0069996 (5)

12 Simone and Greer are two elite junior sprinters. At the Queensland State championships, Simone recorded her personal best 100 m time of 12.77 seconds, while Greer came a close second with a time of 12.83 seconds.

a If the electronic timing equipment could only display times to the nearest second, what would be the time difference between the two sprinters?

b If the electronic timing equipment could only record times to the nearest tenth of a second, what would be the time difference between the two sprinters?

c What was the time difference between the two girls, correct to two decimal places?

d If the electronic timing system could measure accurately to three decimal places, what would be the quickest time that Simone could have recorded?

e Assume that Simone and Greer ran at a consistent speed throughout the 100 m race. Predict the winning margin (correct to the nearest centimetre).

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3F

PROBLEM-SOLVING

13 When $\frac{3}{7}$ is expressed in decimal form, what is the digit in the 19th decimal place?

14 Express $\frac{1}{17}$ as a recurring decimal.

15 15, 16 15, 16

REASONING

15 Frieda stated that she knew an infinite non-recurring decimal. Andrew said that was impossible. He was confident that all decimals either terminated or repeated and that there was no such thing as an infinite non-recurring decimal. Who is correct?

16 Two students gave the following answers in a short test on rounding. Both students have one particular misunderstanding. Study their answers carefully and write a comment to help correct each student's misunderstanding.

Rounding question		Student A		Student B	
0.543	(2)	0.54	✓	0.50	✗
6.7215	(3)	6.721	✗	6.722	✓
5.493	(1)	5.5	✓	5.5	✓
8.2143	(3)	8.214	✓	8.210	✗
11.54582	(2)	11.54	✗	11.55	✓

Will it terminate or recur? — — 17

ENRICHMENT

17 Can you find a way of determining if a fraction will result in a terminating (T) decimal or a recurring (R) decimal?

a Predict the type of decimal answer for the following fractions, and then convert them to see if you were correct.

i $\frac{1}{8}$ **ii** $\frac{1}{12}$ **iii** $\frac{1}{14}$ **iv** $\frac{1}{15}$ **v** $\frac{1}{20}$ **vi** $\frac{1}{60}$

A key to recognising whether a fraction will result in a terminating or recurring decimal lies in the factors of the denominator.

b Write down the denominators from above in prime factor form.

c From your observations, write down a rule that assists the recognition of when a particular fraction will result in a terminating or recurring decimal.

d Without evaluating, state whether the following fractions will result in terminating or recurring decimals.

i $\frac{1}{16}$ **ii** $\frac{1}{9}$ **iii** $\frac{1}{42}$ **iv** $\frac{1}{50}$

v $\frac{1}{75}$ **vi** $\frac{1}{99}$ **vii** $\frac{1}{200}$ **viii** $\frac{1}{625}$

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Progress quiz

3A **1** State the missing numbers for the following sets of equivalent fractions.

a $\frac{1}{3} = \frac{\square}{6} = \frac{\square}{21} = \frac{\square}{45}$ **b** $\frac{5}{2} = \frac{20}{\square} = \frac{35}{\square} = \frac{125}{\square}$

3A **2** Write the following fractions in simplest form.

a $\frac{8}{24}$ **b** $\frac{12}{20}$ **c** $\frac{35}{45}$ **d** $\frac{120}{80}$

3B/C **3** Simplify:

a $\frac{2}{5} + \frac{1}{5}$ **b** $\frac{7}{10} - \frac{3}{5}$ **c** $\frac{2}{3} + \left(-\frac{5}{4}\right)$ **d** $-\frac{7}{15} - \left(-1\frac{2}{3}\right)$

3B/C **4** Simplify:

a $\frac{3}{5} \times \frac{2}{7}$ **b** $\frac{4}{15} \times \frac{20}{8}$ **c** $-1\frac{7}{8} \times 1\frac{1}{5}$ **d** $-4\frac{1}{5} \times \left(-1\frac{1}{9}\right)$

3B/C **5** Simplify:

a $\frac{3}{5} \div \frac{1}{4}$ **b** $\frac{6}{5} \div \left(-\frac{9}{10}\right)$ **c** $-1\frac{7}{8} \div \left(-1\frac{2}{3}\right)$ **d** $\frac{3}{4} \div \frac{9}{10} \times \frac{2}{5}$

3D **6** Compare the following decimals and place the correct inequality sign (< or >) between them.

a 0.2531 0.24876 **b** 17.3568 17.3572

3D **7** Convert the following decimals to fractions in their simplest form.

a 0.45 **b** 6.512

3D **8** Convert the following fractions to decimals.

a $\frac{28}{100}$ **b** $\frac{43}{1000}$ **c** $\frac{7}{4}$ **d** $\frac{9}{20}$

3E **9** Calculate:

a $17.537 + 26.8 + 4.01$ **b** $241.6 - 63.85$

3E **10** Calculate:

a 0.023×100 **b** $9.37 \div 1000$ **c** 5.23×7

d 3.16×5.8 **e** $36.52 \div 2$ **f** $26.460 \div 1.2$

3F **11** Express the following fractions as recurring decimals.

a $\frac{5}{9}$ **b** $\frac{8}{11}$ **c** $6\frac{2}{15}$

3F **12** Round each of the following to the specified number of decimal places.

a 23.6738 (2 decimal places) **b** 2.73968 (3 decimal places)

3F **13** Write each of the following fractions as decimals correct to two decimal places.

a $\frac{1}{3}$ **b** $\frac{5}{12}$

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3G Converting fractions, decimals and percentages

CONSOLIDATING

Interactive

Widgets

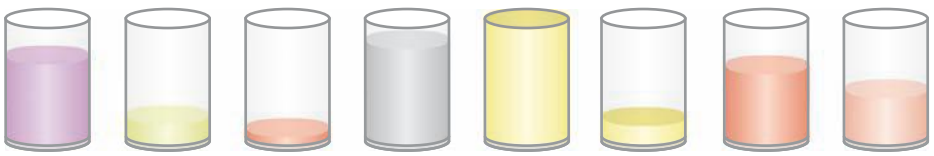
HOTsheets

Walkthroughs

Per cent is Latin for 'out of 100'. One dollar equals 100 **cents** and one **century** equals 100 years. We come across percentages in many everyday situations. Interest rates, discounts, test results and statistics are just some of the common ways we deal with percentages.

Percentages are closely related to fractions. A percentage is another way of writing a fraction with a denominator of 100. Therefore, 63% means that if something was broken into 100 parts you would have 63 of them (i.e. $63\% = \frac{63}{100}$).

Let's start: Estimating percentages



- Estimate the percentage of drink remaining in each of the glasses shown above.
- Discuss your estimations with a partner.
- Estimate what percentage of the rectangle below has been shaded in.



- Use a ruler to draw several 10 cm × 2 cm rectangles. Work out an amount you would like to shade in and using your ruler measure precisely the amount to shade. Shade in this amount.
- Ask your partner to guess the percentage you shaded.
- Have several attempts with your partner and see if your estimation skills improve.

Key ideas

- The symbol % means 'per cent'. It comes from the Latin words *per centum* which translates to 'out of 100'.
  For example: 23% means 23 out of 100 or $\frac{23}{100} = 0.23$.
- **To convert a percentage to a fraction**
  - Change the % sign to a denominator of 100.
  - Simplify the fraction if required.
  For example: $35\% = \frac{35}{100} = \frac{7}{20}$
- **To convert a percentage to a decimal**
  - Divide by 100. Therefore move the decimal point two places to the left.
  For example: $46\% = 46 \div 100 = 0.46$

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Key ideas

- **To convert a fraction to a percentage**
  - Multiply by 100.

    For example: $\frac{1}{8} \times 100 = \frac{1}{8} \times \frac{100}{1} = \frac{25}{2} = 12\frac{1}{2}$, so $\frac{1}{8} = 12\frac{1}{2}\%$
- **To convert a decimal to a percentage**
  - Multiply by 100. Therefore move the decimal point two places to the right.

    For example: $0.812 \times 100 = 81.2$, so $0.812 = 81.2\%$
  - Common percentages and their equivalent fractions are shown in the table below. It is helpful to know these.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.5	$0.\dot{3}$	0.25	0.2	0.125	$0.\dot{6}$	0.75
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$12\frac{1}{2}\%$	$66\frac{2}{3}\%$	75%

Example 21 Converting percentages to fractions

Convert the following percentages to fractions or mixed numerals in their simplest form.

a 160% **b** 12.5%

SOLUTION	EXPLANATION
a $160\% = \frac{160}{100}$	Change % sign to a denominator of 100.
$= \frac{8 \times 20}{5 \times 20}$	Cancel the HCF.
$= \frac{8}{5} = 1\frac{3}{5}$	Convert answer to a mixed numeral.
b $12.5\% = \frac{12.5}{100}$ or $= \frac{12.5}{100}$	Change % sign to a denominator of 100.
$= \frac{25}{200}$ $= \frac{125}{1000}$	Multiply numerator and denominator by 2 or by 10 to make whole numbers.
$= \frac{1}{8}$ $= \frac{1}{8}$	Simplify fraction by cancelling the HCF.

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Example 22 Converting percentages to decimals

Convert the following percentages to decimals.

a 723% **b** 13.45%

SOLUTION	EXPLANATION
a $723\% = 7.23$	$723 \div 100$ 723. Decimal point appears to move two places to the left.
b $13.45\% = 0.1345$	$13.45 \div 100$ 13.45

Example 23 Converting fractions to percentages

Convert the following fractions and mixed numerals into percentages.

a $\frac{3}{5}$ **b** $\frac{7}{40}$ **c** $2\frac{1}{4}$ **d** $\frac{2}{3}$

SOLUTION	EXPLANATION
a $\frac{3}{5} \times 100 = \frac{3}{1\cancel{5}} \times \frac{{}^{20}\cancel{100}}{1}$ $= 60 \therefore \frac{3}{5} = 60\%$	Multiply by 100. Simplify by cancelling the HCF.
b $\frac{7}{40} \times 100 = \frac{7}{2\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}$ $= \frac{35}{2} = 17\frac{1}{2} \therefore \frac{7}{40} = 17\frac{1}{2}\%$	Multiply by 100. Simplify by cancelling the HCF. Write the answer as a mixed numeral.
c $2\frac{1}{4} \times 100 = \frac{9}{1\cancel{4}} \times \frac{{}^{25}\cancel{100}}{1}$ $= 225 \therefore 2\frac{1}{4} = 225\%$	Convert mixed numeral to improper fraction. Cancel and simplify.
d $\frac{2}{3} \times 100 = \frac{2}{3} \times \frac{100}{1}$ $= \frac{200}{3} = 66\frac{2}{3} \therefore \frac{2}{3} = 66\frac{2}{3}\%$	Multiply by 100. Multiply numerators and denominators. Write answer as a mixed numeral.



This slice can be represented by $\frac{1}{4}$, 25% or 0.25 of the whole.

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Example 24 Converting decimals to percentages

Convert the following decimals to percentages.

a 0.458 **b** 17.5

SOLUTION	EXPLANATION
a $0.458 = 45.8\%$	0.458×100 0.458 The decimal point moves two places to the right.
b $17.5 = 1750\%$	17.5×100 17.50

Exercise 3G

1–4 | 3, 4 | —

UNDERSTANDING

1 The fraction equivalent of 27% is:

A $\frac{2}{7}$ **B** $\frac{27}{100}$ **C** 2700 **D** $\frac{13.5}{10}$

2 The decimal equivalent of 37% is:

A 0.037 **B** 0.37 **C** 3.7 **D** 37.00

3 The percentage equivalent of $\frac{47}{100}$ is:

A 0.47% **B** 4.7% **C** 47% **D** 470%

4 The percentage equivalent of 0.57 is:

A 57% **B** 5.7% **C** 570% **D** 0.57%

5–8, 11(½) | 5–8(½), 10–11(½) | 5–11(½)

FLUENCY

Example 21a **5** Convert the following percentages to fractions or mixed numerals in their simplest form.

a 39% **b** 11% **c** 20% **d** 75%
e 125% **f** 70% **g** 205% **h** 620%

Example 21b **6** Convert the following percentages to fractions in their simplest form.

a $37\frac{1}{2}\%$ **b** 15.5% **c** $33\frac{1}{3}\%$ **d** $66\frac{2}{3}\%$
e 2.25% **f** 4.5% **g** $10\frac{1}{5}\%$ **h** 87.5%

Example 22 **7** Convert the following percentages to decimals.

a 65% **b** 37% **c** 158% **d** 319%
e 6.35% **f** 0.12% **g** 4051% **h** 100.05%

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3G

FLUENCY

Example 23a Example 23b **8** Convert the following fractions to percentages.

a $\frac{2}{5}$ **b** $\frac{1}{4}$ **c** $\frac{11}{20}$ **d** $\frac{13}{50}$

e $\frac{9}{40}$ **f** $\frac{17}{25}$ **g** $\frac{150}{200}$ **h** $\frac{83}{200}$

Example 23c **9** Convert the following mixed numerals and improper fractions to percentages.

a $2\frac{3}{4}$ **b** $5\frac{1}{5}$ **c** $\frac{7}{4}$ **d** $\frac{9}{2}$

e $3\frac{12}{25}$ **f** $1\frac{47}{50}$ **g** $\frac{77}{10}$ **h** $\frac{183}{20}$

Example 23d **10** Convert the following fractions to percentages.

a $\frac{1}{3}$ **b** $\frac{1}{8}$ **c** $\frac{1}{12}$ **d** $\frac{1}{15}$

e $\frac{3}{8}$ **f** $\frac{2}{7}$ **g** $\frac{3}{16}$ **h** $\frac{27}{36}$

Example 24 **11** Convert the following decimals to percentages.

a 0.42 **b** 0.17 **c** 3.541 **d** 11.22

e 0.0035 **f** 0.0417 **g** 0.01 **h** 1.01

12, 13 13–15 14–16

PROBLEM-SOLVING

12 Which value is larger?

A 0.8 of a large pizza **B** 70% of a large pizza **C** $\frac{3}{4}$ of a large pizza

13 Complete the following conversion tables involving common fractions, decimals and percentages.

a

Fraction	Decimal	Percentage
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{4}{4}$		

b

Fraction	Decimal	Percentage
	$0.\dot{3}$	
	$0.\dot{6}$	
	$0.\dot{9}$	

c

Fraction	Decimal	Percentage
		20%
		40%
		60%
		80%
		100%

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3G

PROBLEM-SOLVING

14 Complete the following conversion tables.

a

Fraction	Decimal	Percentage
		15%
	0.24	
$\frac{3}{8}$		
$\frac{5}{40}$		
	0.7	
		62%

b

Fraction	Decimal	Percentage
$\frac{11}{5}$		
	0.003	
		6.5%
		119%
	4.2	
$\frac{5}{6}$		

15 A cake is cut into 8 equal pieces. What fraction, what percentage and what decimal does one slice of the cake represent?

16 The Sharks team has won 13 out of 17 games for the season to date. The team still has three games to play. What is the smallest and the largest percentage of games the Sharks could win for the season?



17 | 17, 18 | 17, 18

REASONING

17 **a** Explain why multiplying by 100% is the same as multiplying by 1.

b Explain why dividing by 100% is the same as dividing by 1.

18 Let A, B, C and D represent different digits.

a Convert BC% into a fraction.

b Convert CD.B% into a decimal.

c Convert A.BC into a percentage.

d Convert D.DBCC into a percentage.

e Convert $\frac{A}{D}$ into a percentage.

f Convert $B\frac{C}{A}$ into a percentage.

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Tangram percentages — — 19

ENRICHMENT

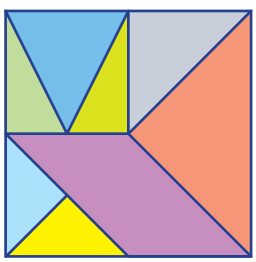
19 You are most likely familiar with the Ancient Chinese puzzle known as a tangram.
A tangram consists of 7 geometric shapes (tans) as shown.
The tangram puzzle is precisely constructed using vertices, midpoints and straight edges.



a Express each of the separate tan pieces as a percentage, a fraction and a decimal amount of the entire puzzle.

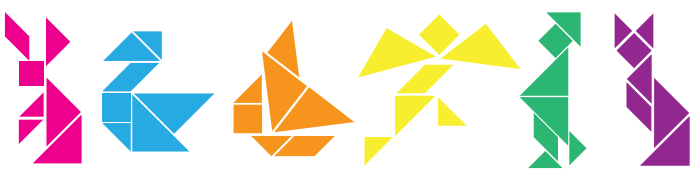
b Check your seven tans add up to a total of 100%.

c Starting with a square, design a new version of a 'modern' tangram puzzle. You must have at least six pieces in your puzzle. An example of a modern puzzle is shown.



d Express each of the separate pieces of your new puzzle as a percentage, a fraction and a decimal amount of the entire puzzle.

e Separate pieces of tangrams can be arranged to make more than 300 creative shapes and designs, some of which are shown. You may like to research tangrams and attempt to make some of the images.



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3H Finding a percentage and expressing as a percentage

Interactive

Widgets

HOTsheets

Walkthroughs

Showing values in percentages makes it easier for comparisons to be made. For example, Huen's report card could be written as marks out of each total or in percentages:

French test $\frac{14}{20}$	French test 70%
German test $\frac{54}{75}$	German test 72%

It is clear that Huen's German score was the stronger result. Comparison is easier when proportions or fractions are written as percentages (equivalent fractions with denominator of 100). Expressing one number as a percentage of another number is the technique covered in this section.

Another common application of percentages is to find a certain percentage of a given quantity. Throughout your life you will come across many examples in which you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, personal improvements, salary increases, commission rates and more.



Percentages are used to calculate retail discounts.

Let's start: What percentage has passed?

Answer the following questions.

- What percentage of your day has passed?
- What percentage of the current month has passed?
- What percentage of the current season has passed?
- What percentage of your school year has passed?
- What percentage of your school education has passed?
- If you live to an average age, what percentage of your life has passed?
- When you turned 5, what percentage of your life was 1 year?
- When you are 40, what percentage of your life will 1 year be?

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Key ideas

■ **To express one quantity as a percentage of another**

1 Write a fraction with the 'part amount' as the numerator and the 'whole amount' as the denominator.

2 Convert the fraction to a percentage by multiplying by 100.

For example: Express a test score of 14 out of 20 as a percentage.

14 is the 'part amount' that you want to express as a percentage out of 20, which is the 'whole amount'.

$$\frac{14}{20} \times 100 = \frac{14}{\cancel{20}_{1}} \times \frac{\cancel{100}^{5}}{1} = 70, \therefore \frac{14}{20} = 70\%$$

Alternatively $\frac{14}{20} \overset{\times 5}{=} \frac{70}{100} = 70\%$

■ **To find a certain percentage of a quantity**

1 Express the required percentage as a fraction.

2 Change the 'of' to a multiplication sign.

3 Express the number as a fraction.

4 Follow the rules for multiplication of fractions.

For example: Find 20% of 80.

$$20\% \text{ of } 80 = \frac{20}{100} \times \frac{80}{1} = \frac{\cancel{20}^{4}}{\cancel{100}_{5}} \times \frac{\cancel{80}^{4}}{1} = 16$$

Alternatively 20% of $80 = \frac{1}{5} \times 80$

$= 80 \div 5$

$= 16$

Example 25 Expressing one quantity as a percentage of another

Express each of the following as a percentage.

a 34 out of 40 **b** 13 out of 30

SOLUTION

a $\frac{34}{40} \times \frac{100}{1}\% = \frac{17}{20} \times \frac{100}{1}\%$

$= \frac{17}{1} \times \frac{5}{1}\%$

$= 85\%$

b $\frac{13}{30} \times \frac{100}{1}\% = \frac{13}{3} \times \frac{10}{1}\%$

$= \frac{130}{3}\%$

$= 43\frac{1}{3}\%$ or $43.\dot{3}\%$

EXPLANATION

Write as a fraction, with the first quantity as the numerator and second quantity as the denominator.
Multiply by 100.
Cancel and simplify.

Write quantities as a fraction and multiply by 100.
Cancel and simplify.
Express the percentage as a mixed numeral or a recurring decimal.

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Example 26 Converting units before expressing as a percentage

Express:

a 60 cents as a percentage of $5

b 2 km as a percentage of 800 m

SOLUTION

a $\frac{60}{500} \times \frac{100}{1}\% = \frac{60}{5}\%$

$= 12\%$

b $\frac{2000}{800} \times \frac{100}{1}\% = \frac{2000}{8}\%$

$= 250\%$

EXPLANATION

Convert $5 to 500 cents.
Write quantities as a fraction and multiply by 100.
Cancel and simplify.

Convert 2 km to 2000 m.
Write quantities as a fraction and multiply by 100.

Cancel and simplify.

Example 27 Finding a certain percentage of a quantity

Find:

a 25% of 128

b 155% of 60

SOLUTION

a $25\% \text{ of } 128 = \frac{25}{100} \times \frac{128}{1}$

$= \frac{1}{4} \times \frac{128}{1} = 32$

b $155\% \text{ of } 60 = \frac{155}{100} \times \frac{60}{1}$

$= \frac{155}{5} \times \frac{3}{1}$

$= \frac{31}{1} \times \frac{3}{1} = 93$

EXPLANATION

Write the percentage as a fraction over 100.
Cancel and simplify.

Write the percentage as a fraction over 100.
Cancel and simplify.

Exercise 3H

1–3 | 1–3 | —

UNDERSTANDING

1 The correct working line to express 42 as a percentage of 65 is:

A $\frac{42}{100} \times 65\%$ **B** $\frac{65}{42} \times 100\%$ **C** $\frac{100}{42} \times 65\%$ **D** $\frac{42}{65} \times 100\%$

2 The correct working line to find 42% of 65 is:

A $\frac{42}{100} \times 65$ **B** $\frac{65}{42} \times 100$ **C** $\frac{100}{42} \times 65$ **D** $\frac{42}{65} \times 100$

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3H

UNDERSTANDING

3 Copy and complete the following sentences.

a Finding 1% of a quantity is the same as dividing the quantity by _____.
b Finding 10% of a quantity is the same as dividing the quantity by _____.
c Finding 20% of a quantity is the same as dividing the quantity by _____.
d Finding 50% of a quantity is the same as dividing the quantity by _____.

4(½), 7(½), 9–10(½) | 4–11(½) | 4–11(½)

FLUENCY

Example 25

4 Express each of the following as a percentage.

a 20 out of 25 **b** 13 out of 20 **c** 39 out of 50 **d** 12 out of 40
e 26 out of 130 **f** 44 out of 55 **g** 24 out of 60 **h** 6 out of 16
i 11 out of 30 **j** 85 out of 120 **k** 17 out of 24 **l** 34 out of 36

5 Express the first quantity as a percentage of the second quantity, giving answers in fractional form where appropriate.

a 3, 10 **b** 9, 20 **c** 25, 80 **d** 15, 18
e 64, 40 **f** 82, 12 **g** 72, 54 **h** 200, 75

6 Express the first quantity as a percentage of the second quantity, giving answers in decimal form, correct to two decimal places where appropriate.

a 2, 24 **b** 10, 15 **c** 3, 7 **d** 18, 48
e 56, 35 **f** 15, 12 **g** 9, 8 **h** 70, 30

7 Express each quantity as a percentage of the total.

a 28 laps of a 50 lap race completed **b** Saved $450 towards a $600 guitar
c 172 people in a train carriage of 200 people **d** Level 7 completed of a 28 level video game
e 36 students absent out of 90 total **f** 14 km mark of a 42 km marathon

Example 26

8 Express:

a 40 cents as a percentage of $8 **b** 50 cents as a percentage of $2
c 3 mm as a percentage of 6 cm **d** 400 m as a percentage of 1.6 km
e 200 g as a percentage of 5 kg **f** 8 km as a percentage of 200 m
g 1.44 m as a percentage of 48 cm **h** $5.10 as a percentage of 85¢

Example 27a

9 Find:

a 50% of 36 **b** 20% of 45 **c** 25% of 68 **d** 32% of 50
e 5% of 60 **f** 2% of 150 **g** 14% of 40 **h** 70% of 250
i 15% of 880 **j** 45% of 88 **k** 80% of 56 **l** 92% of 40

Example 27b

10 Find:

a 130% of 10 **b** 200% of 40 **c** 400% of 25 **d** 155% of 140
e 125% of 54 **f** 320% of 16 **g** 105% of 35 **h** 118% of 60

11 Find:

a 20% of 90 minutes **b** 15% of $5 **c** 30% of 150 kg
d 5% of 1.25 litres **e** 40% of 2 weeks **f** 75% of 4.4 km

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3H

12, 13 | 13–15 | 14–16

PROBLEM-SOLVING

12 Find:

a $33\frac{1}{3}\%$ of 16 litres of orange juice

b $66\frac{2}{3}\%$ of 3000 marbles

c $12\frac{1}{2}\%$ of a \$64 pair of jeans

d 37.5% of 120 donuts



13 In a survey, 35% of respondents said they felt a penalty was too lenient and 20% felt it was too harsh. If there were 1200 respondents, how many felt the penalty was appropriate?

14 Four students completed four different tests. Their results were:
Maeheala: 33 out of 38 marks
Wasim: 16 out of 21 marks
Francesca: 70 out of 85 marks
Murray: 92 out of 100 marks
Rank the students in decreasing order of test percentage.

15 Jasper scored 22 of his team's 36 points in the basketball final. What percentage of the team's score did Jasper shoot? Express your answer as a fraction and as a recurring decimal.

16 Due to illness, Vanessa missed 15 days of the 48 school days in Term 1. What percentage of classes did Vanessa attend in Term 1?

17 | 17, 18 | 18–20

REASONING

17 Eric scored 66% on his most recent Mathematics test. He has studied hard and is determined to improve his score on the next topic test, which will be out of 32 marks. What is the least number of marks Eric can score to improve on his previous test score?

18 Calculate 40% of \$60 and 60% of \$40. What do you notice? Can you explain your findings?

19 Which of the following expressions would calculate $a\%$ of b?

A $\frac{100}{ab}$ **B** $\frac{ab}{100}$ **C** $\frac{a}{100b}$ **D** $\frac{100a}{b}$

20 Which of the following expressions would express x as a percentage of y?

A $\frac{100}{xy}$ **B** $\frac{xy}{100}$ **C** $\frac{x}{100y}$ **D** $\frac{100x}{y}$

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Two dimensional percentage increases — — 21

3H

ENRICHMENT



21 A rectangular photo has dimensions of 12 cm by 20 cm.

a What is the area of the photo in cm^2?

The dimensions of the photo are increased by 25%.

b What effect will increasing the dimensions of the photo by 25% have on its area?

c What are the new dimensions of the photo?

d What is the new area of the photo in cm^2?

e What is the increase in the area of the photo?

f What is the percentage increase in the area of the photo?

g What effect did a 25% increase in dimensions have on the area of the photo?

h Can you think of a quick way of calculating the percentage increase in the area of a rectangle for a given increase in each dimension?

i What would be the percentage increase in the area of a rectangle if the dimensions were:

i increased by 10%?

ii increased by 20%?

iii increased by $33\frac{1}{3}$%?

iv increased by 50%?

You might like to draw some rectangles of particular dimensions to assist your understanding of the increase in percentage area.

j What percentage increase in each dimension would you need to exactly double the area of the rectangle?

k You might like to explore the percentage increase in the volume of a three-dimensional shape when each of the dimensions is increased by a certain percentage.

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3I Decreasing and increasing by a percentage









Percentages are regularly used when dealing with money. Here are some examples.

- Decreasing an amount by a percentage (Discount)
  All items in the store were reduced by 30% for the three-day sale.
  The value of the car was depreciating at a rate of 18% per annum.
- Increasing an amount by a percentage (Mark-up)
  A retail shop marks up items by 25% of the wholesale price.
  The professional footballer's new contract was increased by 40%.



Column 1 of this stock market display of share prices shows the current share price, column 2 shows the opening price, column three shows the change (in dollars and cents), column 4 shows the percentage change. Minus signs indicate a decrease.

When dealing with questions involving money, you generally round your answers to the nearest cent. Do this by rounding correct to two decimal places. For example, $356.4781 rounds to $356.48 (suitable if paying by credit card).

As our smallest coin is the five cent coin, on many occasions it will be appropriate to round your answer to the nearest five cents. For example, $71.12 rounds to $71.10 (suitable if paying by cash).

Let's start: Original value ± % change = new value

The table below consists of three columns: the original value of an item, the percentage change that occurs and the new value of the item. However, the data in each of the columns have been mixed up. Your challenge is to rearrange the data in the three columns so that each row is correct.

This is an example of a correct row.

Original value	Percentage change	New value
$65.00	Increase by 10%	$71.50

Rearrange the values in each column in this table so that each row is correct.

Original value	Percentage change	New value
$102.00	Increase by 10%	$73.50
$80.00	Increase by 5%	$70.40
$58.00	Decreased by 2%	$76.50
$64.00	Decrease by 25%	$78.40
$70.00	Increase by 30%	$73.80
$82.00	Decrease by 10%	$75.40

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Key ideas

- Common words used to represent a *decrease* in price include **reduction**, **discount**, **sale**, **loss** or **depreciation**. In each case the new value equals the original value minus the decrease.
- Common words used to represent an *increase* in price include **mark-up** and **profit**. In each case the new value equals the original value plus the increase.
- Increasing or decreasing an item by a set percentage involves the technique of finding a percentage of a quantity (see Section 3H).
  For example: A 15% mark-up on $200 involves finding 15% of $200 and then adding this amount to $200.
- In retail terms:
  Selling price = retail price – discount
  Selling price = wholesale price + mark-up
- **GST** is a 10% Goods and Services Tax that is added to the cost price of many items. It is in effect a 10% mark-up. It is paid by the consumer and passed onto the government by the supplier.



Example 28 Finding new values

Find the new value when:

a $160 is increased by 40%

b $63 is decreased by 20%

SOLUTION

a $40\% \text{ of } \$160 = \frac{40}{100} \times \frac{160}{1} = \$64$

$\text{New price} = \$160 + \$64$

$= \$224$

b $20\% \text{ of } \$63 = \frac{20}{100} \times \frac{63}{1} = \$12.60$

$\text{New price} = \$63 - \$12.60$

$= \$50.40$

EXPLANATION

Calculate 40% of $160.
Cancel and simplify.
New price = original price + increase

Calculate 20% of $63.
Cancel and simplify.
New price = original price – decrease

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Example 29 Calculating discounts or mark-ups

a Find the cost of a $860 television that has been discounted by 25%.

b Find the cost of a $250 microwave oven that has been marked up by 12%.

SOLUTION	EXPLANATION
a Discount = 25% of $860	Calculate 25% discount.
$= \frac{25}{100} \times \frac{860}{1} = \$215$	Cancel and simplify.
Selling price = $860 − $215	Selling price = cost price − discount
= $645	
b Mark-up = 12% of $250	Calculate 12% of $250.
$= \frac{12}{100} \times \frac{250}{1} = \$30$	Cancel and simplify.
Selling price = $250 + $30	Selling price = cost price + mark-up
= $280	

Exercise 3I

1–4 | 4 | —

UNDERSTANDING

1 Calculate the new price when:
- **a** an item marked at $15 is discounted by $3
- **b** an item marked at $25.99 is marked up by $8
- **c** an item marked at $17 is reduced by $2.50
- **d** an item marked at $180 is increased by $45

2 Calculate the new price when:
- **a** an item marked at $80 is discounted by 50%
- **b** an item marked at $30 is marked up by 20%
- **c** an item marked at $45 is reduced by 10%
- **d** an item marked at $5 is increased by 200%

3 A toy store is having a sale in which everything is discounted by 10% of the recommended retail price (RRP). A remote-control car is on sale and has a RRP of $120.
- **a** Calculate the discount on the car (i.e. 10% of $120).
- **b** Calculate the selling price of the car (i.e. RRP − discount).

4 Shop A is advertising a watch for $350 with a discount of $80.

Shop B is advertising the same watch for $400 with a 30% discount.

Calculate the selling price of the watch in each shop.



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31

5–7(½) 5–8(½) 6–8(½)

FLUENCY

Example 28

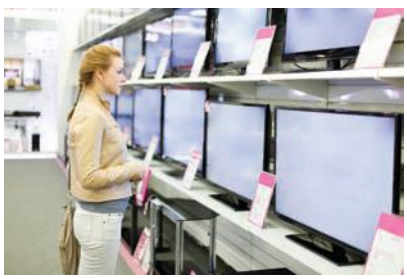
5 Find the new value when:

a $400 is increased by 10%
b $240 is increased by 15%
c $80 is decreased by 20%
d $42 000 is decreased by 2%
e $5000 is increased by 8%
f $60.60 is increased by 60%
g $15 is decreased by 10%
h $84 is decreased by 40%

Example 29

6 Find the cost of the following.

a a $600 television that has been discounted by 20%
b a $150 ripstick that has been reduced by 15%
c a $52 jumper that has depreciated by 25%
d a $80 framed Pink poster that has been marked up by 30%
e a $14 meal that has been increased by 10%
f a $420 stereo that has been marked up by 50%



7 Calculate the selling prices of the following items if they are to be reduced by 25%.

a $16 thongs
b $32 sunhat
c $50 sunglasses
d $85 bathers
e $130 boogie board
f $6.60 surfboard wax

8 Calculate the selling prices of the following items if they need to have 10% GST added to them. The prices listed do not include GST already.

a $35 t-shirt
b $75 backpack
c $42 massage
d $83 fishing rod
e $52.50 toaster
f $149.99 cricket bat

9–10 10–12 11–13

PROBLEM-SOLVING

9 Shop C and shop D purchase Extreme Game packages at a cost price of $60. Shop C has a mark-up of $20 for retailers and shop D has a mark-up of 25%. Calculate the selling price for the Extreme Game package at each shop.

10 A retail rug store marks up all items by 25% of the wholesale price. The wholesale price of a premier rug is $200 and for a luxury rug is $300. What is the customer price for the two different types of rugs?

11 A bookstore is offering a discount of 10%. Jim wants to buy a book with a RRP of $49.90. How much will it cost him if he pays by cash? How much will it cost him if he pays by credit card?

12 Shipshape stores were having a sale and reducing all items by 20%. Gerry purchased the following items, which still had their original price tags: jeans $75, long-sleeved shirt $94, T-shirt $38 and shoes $125. What was Gerry's total bill at the sale?

13 At the end of the winter season an outdoors store had a 20% discount on all items in the store. Two weeks later they were still heavily overstocked with ski gear and so they advertised a further 40% off already discounted items. Calculate the new selling price of a pair of ski goggles with a RRP of $175.00.

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3I

	14	14, 15	14, 15

REASONING

14 Georgie desperately wants to buy a new mountain bike that has a RRP of $350. She only has $220. Fortunately, her local bike store regularly has sales in which the discounts are multiples of 5%. What is the smallest discount at the bike store that will enable Georgie to purchase the bike?

15 Patrick wants to purchase a trail bike and has visited three different stores. For the same model he has received a different deal at each store.
Pete's Trail Bikes has the bike priced at $2400 and will not offer any discount.
Eastern Bikers has the bike priced at $2900 but will offer a 20% discount.
City Trail Bikes has the bike priced at $2750 but will offer a 15% discount.
What is the cheapest price for which Patrick can purchase his trail bike and which store is offering the best deal?

Commission	—	—	16

ENRICHMENT

16 Many sales representatives are paid by commission. This means their wage is calculated as a certain percentage of their sales. For example, Ben, a car salesman is paid 5% commission. In one week, Ben sold three cars for a total value of $42 000. His wage for this week was 5% of $42 000 = $2100. If in the next week Ben sells no cars, he would receive no wage.



a Calculate the following amounts.
- **i** 10% commission on sales of $850
- **ii** 3% commission on sales of $21 000
- **iii** 2.5% commission on sales of $11 000
- **iv** 0.05% commission on sales of $700 000

Generally sales representatives can be paid by commission only, by an hourly rate only, or by a combination of an hourly rate and a percentage commission. The last combination is common, as it provides workers with the security of a wage regardless of sales, but also the added incentive to boost wages through increased sales.
Solve the following three problems involving commission.

b Stuart sells NRL records. He earns $8.50 per hour and receives 5% commission on his sales. Stuart worked for five hours at the Brisbane Broncos vs Canberra Raiders match and sold 320 records at $4 each. How much did Stuart earn?

c Sam, Jack and Justin were all on different pay structures at work. Sam was paid an hourly rate of $18 per hour. Jack was paid an hourly rate of $15 per hour and 4% commission. Justin was paid by commission only at a rate of 35%.
Calculate their weekly wages if they each worked for 40 hours and each sold $2000 worth of goods.

d Clara earns an hourly rate of $20 per hour and 5% commission. Rose earns an hourly rate of $16 per hour and 10% commission. They each work a 40-hour week. In one particular week, Clara and Rose both sold the same value of goods and both received the same wage. How much did they sell and what was their wage?

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3J Calculating percentage change









If you sell something for more than you paid for it, you have made a profit. On the other hand, if you sell something for less than you paid for it, you have made a loss.

Retailers, business people and customers are all interested to know the percentage profit or the percentage loss that has occurred. In other words, they are interested to know about the percentage change. The percentage change provides valuable information over and above the simple value of the change.

For example:



Hat was $40
Discount $5
Now $35



Cap was $8
Discount $5
Now $3

The change for each situation is exactly the same, a $5 discount. However, the percentage change is very different.

Hat was $40. Discount 12.5%. Now $35.

Cap was $8. Discount 62.5%. Now $3.

For the sunhat, there is a 12.5% change, and for the cap there is a 62.5% change. In this case, the percentage change would be known as a **percentage discount**.

Let's start: Name the acronym

What is an acronym?

How many of the following nine business and finance-related acronyms can you name?

- RRP
- CBD
- GDP
- GST
- COD
- IOU
- ATM
- EFTPOS
- ASX

Can you think of any others?

How about the names of two of the big four Australian banks: NAB and ANZ?

How do these acronyms relate to percentages?

The following three acronyms are provided for fun and interest only. Do you know what they stand for?

- SCUBA diving
- LASER gun
- BASE jumping

Do you know what the acronym TLA stands for? (Three Letter Acronym)

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Key ideas

- **Profit** = selling price – cost price
- **Loss** = cost price – selling price
- Calculating a percentage change involves the technique of expressing one quantity as a percentage of another (see Section 3H).

$$\textbf{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$$

- A calculation **error** is the difference between an estimated or measured value and an exact, actual or given value.

$$\textbf{Percentage error} = \frac{\text{error}}{\text{actual value}} \times 100\%$$

Example 30 Calculating percentage change

Calculate the percentage change (profit/loss) when:

a \$25 becomes \$32 **b** \$60 becomes \$48

SOLUTION

a Profit = \$7

$$\%\ \text{Profit} = \frac{7}{25} \times \frac{100}{1}\%$$

$$= 28\%$$

b Loss = \$12

$$\%\ \text{Loss} = \frac{12}{60} \times \frac{100}{1}\%$$

$$= 20\%$$

EXPLANATION

Profit = \$32 – \$25

$$\text{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

Loss = \$60 – \$48

$$\text{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$$

Example 31 Solving worded problems

Ross buys a ticket to a concert for \$125, but is later unable to go. He sells it to his friend for \$75. Calculate the percentage loss Ross made.

SOLUTION

Loss = \$125 – \$75 = \$50

$$\%\ \text{Loss} = \frac{50}{125} \times \frac{100}{1}\%$$

$$= 40\%$$

Ross made a 40% loss on the concert ticket.

EXPLANATION

Loss = cost price – selling price

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

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Exercise 3J

1–4 | 3, 4 | —

UNDERSTANDING

1 Calculate the profit made in each of the following situations.

a Cost price = \$14, Sale price = \$21
b Cost price = \$75, Sale price = \$103
c Cost price = \$25.50, Sale price = \$28.95
d Cost price = \$499, Sale price = \$935

2 Calculate the loss made in each of the following situations.

a Cost price = \$22, Sale price = \$9
b Cost price = \$92, Sale price = \$47
c Cost price = \$71.10, Sale price = \$45.20
d Cost price = \$1121, Sale price = \$874

3 Which of the following is the correct formula for working out percentage change?

A $\%\ \text{change} = \dfrac{\text{change}}{\text{original value}}$

B $\%\ \text{change} = \dfrac{\text{original value}}{\text{change}} \times 100\%$

C $\%\ \text{change} = \text{change} \times 100\%$

D $\%\ \text{change} = \dfrac{\text{change}}{\text{original value}} \times 100\%$

4 The formula used for calculating percentage loss is $\%\ \text{loss} = \dfrac{\text{loss}}{\text{cost price}} \times 100\%$

Which of the following would therefore be the formula for calculating the percentage discount?

A $\%\ \text{discount} = \dfrac{\text{discount}}{\text{cost price}} \times 100\%$

B $\%\ \text{discount} = \dfrac{\text{cost price}}{\text{discount}} \times 100\%$

C $\%\ \text{discount} = \text{discount} \times 100\%$

D $\%\ \text{discount} = \dfrac{\text{discount}}{100\%} \times \text{cost price}$

5–7(½) | 5–8(½) | 6–8(½)

FLUENCY

Example 30

5 Find the percentage change (profit or loss) when:

a \$20 becomes \$36
b \$10 becomes \$13
c \$40 becomes \$30
d \$25 becomes \$21
e \$12 becomes \$20
f \$8 becomes \$11
g \$6 becomes \$4
h \$150 becomes \$117

6 Find the percentage change (increase or decrease) when:

a 15°C becomes 18°C
b 18°C becomes 15°C
c 4°C becomes 24°C
d 12°C becomes 30°C

7 Find the percentage change in population when:

a a town of 4000 becomes a town of 5000
b a city of 750 000 becomes a city of 900 000
c a country of 5 000 000 becomes a country of 12 000 000
d a region of 10 000 becomes 7500

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3J

FLUENCY

8 Calculate the percentage error in this situations. Use the formula:

$$\text{Percentage error} = \frac{\text{error}}{\text{actual value}} \times 100\%$$

- **a** Albert measures the length of a piece of timber at 24 cm which is actually 25 cm
- **b** Sally measures the weight of a bowl of flour at 150 g which is actually 160 g
- **c** Annette records a 100 m race time at 15.5 seconds against the actual time of 15.4 seconds. (Round to one decimal place.)
- **d** Kevin records a town's population as 5300 when the actual population is 5327. (Round to one decimal place.)

9, 10	9–12	11–13

PROBLEM-SOLVING

Example 31

9 Gari buys a ticket to a concert for $90, but is unable to go. She sells it to her friend for $72. Calculate the percentage loss Gari made.

10 Estelle purchased a piece of sporting memorabilia for $120. Twenty years later she sold it for $900. Calculate the percentage profit Estelle made.

11 Xavier purchased materials for $48 and made a dog kennel. He later sold the dog kennel for $84.



- **a** Calculate the profit Xavier made.
- **b** Calculate the percentage profit Xavier made.

12 Gemma purchased a $400 foal, which she later sold for $750.

- **a** Calculate the profit Gemma made.
- **b** Calculate the percentage profit Gemma made.

13 Lee-Sen purchased a $5000 car, which she later sold for $2800.

- **a** Calculate Lee-Sen's loss.
- **b** Calculate Lee-Sen's percentage loss.

14	14, 15	15, 16

REASONING

14 Explain why an increase of 10% followed by a decrease of 10% on the new price gives an answer that is less than the original amount.

15 Which of the following stores is offering a larger percentage discount?
Store A: Jeans originally selling for $60, now on sale for $51
Store B: Jeans originally selling for $84, now on sale for $73.50

16 The circulation of a student newspaper at Burrough High School was 60 copies in term 1, 120 copies in term 2, 200 copies in term 3 and 360 copies in term 4.

- **a** Calculate the percentage growth in circulation between:
  - **i** term 1 and term 2
  - **ii** term 2 and term 3
  - **iii** term 3 and term 4
- **b** In which term was there the greatest percentage growth in circulation?
- **c** What was the overall percentage growth in circulation over the course of the year?

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ENRICHMENT

17 Australia's population in early 2010 was 22 003 926 and the percentage population growth was 1.8% per year.

a Given this population size and percentage growth, how many more Australians would there be after one year?

b How many people would there be after:

i 2 years? **ii** 5 years? **iii** 10 years?



The Australian Bureau of Statistics carries out comprehensive population projections. One such projection assumes the following growth rates:

- One birth every 1 minute and 44 seconds
- One death every 3 minutes and 39 seconds
- A net gain of one international migrant every 1 minute and 53 seconds
- An overall total population increase of one person every 1 minute and 12 seconds

c Calculate the percentage growth per annum, using the 2010 population of 22 003 926 and the projected total population increase of one person every 1 minute and 12 seconds. Give your answer correct to one decimal place.

d Find out the current population of Australia and the population 10 years ago, and work out the percentage growth over the past decade.

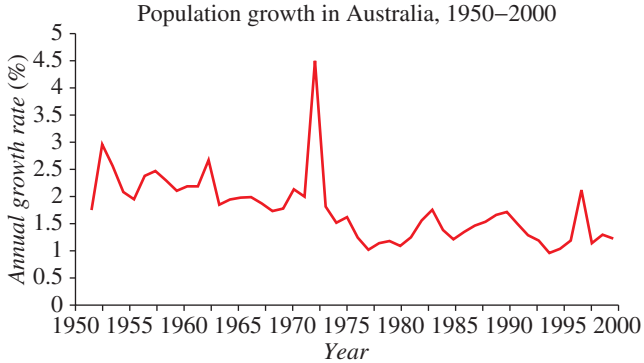
e How does Australia's percentage growth compare with that of other countries?

f Find out the current life expectancy of an Australian male and female. Find out their life expectancy 50 years ago. Calculate the percentage increase in life expectancy over the past 50 years.

g A key factor in population growth is the total fertility rate (TFR) (i.e. the number of babies per woman). For Australia the TFR in 2010 was at 1.74 expected births per woman. The peak of Australia's TFR over the past 100 years was 3.6 children per woman in 1961. What is the percentage decrease in TFR from 1961 to 2010?

h Carry out some of your own research on Australia's population.

Population growth in Australia, 1950–2000



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3K Percentages and the unitary method

EXTENDING









If you know a percentage of an amount, but do not actually know the amount, you can use a technique known as the unitary method to find the whole amount.

The unitary method involves finding the value of a unit and then using this value to calculate the value of a whole. In this section, the value of a unit will be the value of one per cent (1%).

Generally, the first step in all problems involving the unitary method is to divide the information given to find the value of a unit. The second step is to then multiply the value of a unit to find the value of the number of units required in the question.



Let's start: Using the unitary method

By first finding the value of '1 unit', answer the following questions using mental arithmetic only.

1. Four tickets to a concert cost $100. How much will 3 tickets cost?
2. Ten workers can dig 40 holes in an hour. How many holes can 7 workers dig in an hour?
3. Six small pizzas cost $54. How much would 10 small pizzas cost?
4. If 8 pairs of socks cost $64, how much would 11 pairs of socks cost?
5. Five passionfruit cost $2.00. How much will 9 passionfruit cost?
6. If a worker travels 55 km in 5 trips from home to the worksite, how far will they travel in 7 trips?

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to answer the question.
- When dealing with percentages, finding 'one unit' corresponds to finding one per cent (1%).
- Once the value of 1% of an amount is known, it can be multiplied to find the value of any desired percentage.



Example 32 Using the unitary method to find the full amount

If 8% of an amount of money is $48, what is the full amount of money?

SOLUTION

÷8 ↘ 8% of amount is $48
÷8 ↘ 1% of amount is $6
×100 ↘ 100% of amount is $600
×100 ↘
Full amount of money is $600.

EXPLANATION

Divide by 8 to find the value of 1%.
Multiply by 100 to find the value of 100%.

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Example 33 Using the unitary method to find a new percentage

If 11% of the food bill was $77, how much is 25% of the food bill?

SOLUTION

÷11 ↓ 11% of food bill is $77 → ÷11

∴ 1% of food bill is $7

×25 ↓ ∴ 25% of food bill is $175 → ×25

EXPLANATION

Divide by 11 to find the value of 1%.
Multiply by 25 to find the value of 25%.

Example 34 Using the unitary method to find the original price

A pair of shoes has been discounted by 20%. If the sale price was $120, what was the original price of the shoes?

SOLUTION

Only paying 80% of original price:

÷80 ↓ ∴ 80% of original price is $120 → ÷80

∴ 1% of original price is $1.50

×100 ↓ ∴ 100% of original price is $150 → ×100

The original price of the shoes was $150.

EXPLANATION

20% discount, so paying (100 – 20)%.

Divide by 80 to find the value of 1%.
Multiply by 100 to find the value of 100%.

Exercise 3K

1–3 | 2, 3 | —

UNDERSTANDING

1 If 10% of an amount of money is $75, how much is 1% of that amount?

- **A** $1
- **B** $7.50
- **C** $75
- **D** $750

2 If 10% of an amount of money is $22, how much is 100% of that amount?

- **A** $100
- **B** $2.20
- **C** $22
- **D** $220

3 Which alternative best describes the unitary method when dealing with percentages?

- **A** Work out the factor required to multiply the given percentage to make 100%.
- **B** Find the value of 1% and then multiply to find the value of percentage required.
- **C** Once you find the full amount, or 100% of the amount, this is known as 'the unit'.
- **D** Find what percentage equals $1 and then find the given percentage.

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3K

4, 5(½), 6–8 | 4, 5(½), 6, 7(½), 8, 9 | 4, 5(½), 6, 7(½), 8, 9

FLUENCY

Example 32

4 If 6% of an amount of money is $30, how much is the full amount of money?

5 Calculate the full amount of money for each of the following.

- **a** 3% of an amount of money is $27
- **b** 5% of an amount of money is $40
- **c** 12% of an amount of money is $132
- **d** 60% of an amount of money is $300
- **e** 8% of an amount of money is $44
- **f** 6% of an amount of money is $15



Example 33

6 If 4% of the total bill is $12, how much is 30% of the bill?

7 Calculate:

- **a** 20% of the bill, if 6% of the total bill is $36
- **b** 80% of the bill, if 15% of the total bill is $45
- **c** 3% of the bill, if 40% of the total bill is $200
- **d** 7% of the bill, if 25% of the total bill is $75

8 What is the total volume if 13% of the volume is 143 litres?

9 What is the total mass if 120% of the mass is 720 kg?

10, 11(½) | 11(½), 12 | 11(½), 12, 13

PROBLEM-SOLVING

Example 34

10 A necklace in a jewellery store has been discounted by 20%. If the sale price is $240, what was the original price of the necklace?

11 Find the original price of the following items.

- **a** A pair of jeans discounted by 40% has a sale price of $30.
- **b** A hockey stick discounted by 30% has a sale price of $105.
- **c** A second-hand computer discounted by 85% has a sale price of $90.
- **d** A second-hand textbook discounted by 80% has a sale price of $6.
- **e** A standard rose bush discounted by 15% has a sale price of $8.50.
- **f** A motorbike discounted by 25% has a sale price of $1500.

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3K

PROBLEM-SOLVING

12 Forty per cent of workers for a large construction company prefer to have an early lunch and 25% of workers prefer to work through lunch and leave an hour earlier at the end of the day. All other workers prefer a late lunch. If 70 workers prefer a late lunch, how many workers are employed by the construction company?



13 Daryl receives an amount of money from his grandparents for his birthday. He spends 70% of the money buying a new music CD. He then spends 50% of the remaining money buying more credit for his mobile phone. After these two purchases Daryl has $6 remaining. How much money did Daryl's grandparents give him for his birthday?

REASONING

	14	14, 15	15, 16

14 If 22% of an amount is $8540, which of the following would give the value of 1% of the amount?

A $8540 × 100 **B** $8540 ÷ 100 **C** $8540 × 22 **D** $8540 ÷ 22

15 If y% of an amount of money is $8, how much is the full amount of money?

16 If C% of an amount of money is $\$D$, how much is F% of the amount of money?

ENRICHMENT

GST (Goods and Services Tax)	—	—	17

17 Australia has a current GST rate of 10%. This means that for many items an extra 10% of the price is added on to the value of that item. Therefore the price including GST for these items is actually 110% of the original price.

For each of the following items, the cost including GST is given. Calculate the cost of each item before GST was added.

a A hair-dryer that sells for $77

b A complete Family Guy DVD set selling for $121

c A manicure costing $55

d A lawn mower selling for $495

e A airfare from Adelaide to Broome costing $715

f Carry out some research on the GST and then answer the following questions.

i In which year was the GST introduced in Australia?

ii Why was the GST introduced?

iii The GST is also known as a VAT and prior to the GST, Australia operated with a WST. What do VAT and WST stand for and what are the differences between GST, VAT & WST?

iv List several items that are GST exempt.

v List several organisations that are GST exempt.

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Investigation

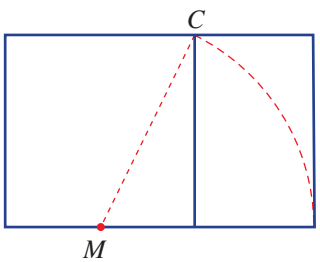
The Golden ratio involves a special number denoted by the Greek letter phi (Φ). It is often referred to as the golden section or divine proportion. There are remarkable observations of this ratio in nature and in a number of applications in mathematics, and many artists and architects have used this ratio in their paintings and designs. We will explore the Golden ratio further in this investigation.

Phi, the golden number

Constructing a golden rectangle

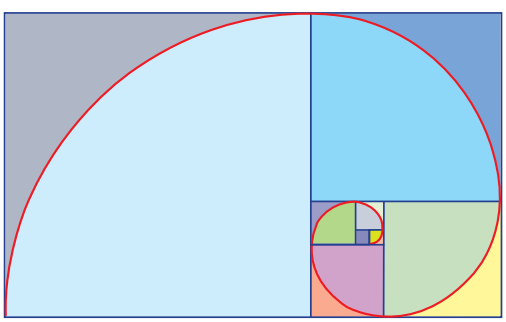
You will need a ruler, a pencil, a pair of compasses and a sheet of A4 paper.

1. Rule a square of side length about 15 cm on the left side of your paper.
2. Rule a line from the midpoint (M) of one side of the square to the opposite corner (C) as shown.
3. With your compass point on M and the compass pencil on C (radius MC), draw an arc from C as shown.
4. Extend the base of the square to meet this arc. This new length is the base length of a golden rectangle.
5. Complete the golden rectangle and erase the vertical line from C and the arc.
6. You now have two golden rectangles, one inside the other.



Constructing a golden spiral

7. In your smaller golden rectangle rule the largest square that can be drawn next to the first square.
8. Continue adding squares in each smaller golden rectangle until there are at least 7 adjacent squares arranged as shown below.
9. In each square mark the corners that are closest to the smallest square.
10. Using a compass with the point on these corners draw arcs equal to the side of each square.
11. Colour your golden spiral pattern.



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Calculating phi

12 Phi is the ratio of the length to the width (height) of a golden spiral or a golden rectangle. In the design you have drawn there are several golden rectangles.

a Measure the length and width of each golden rectangle and calculate the value of phi (length divided by width) for each.

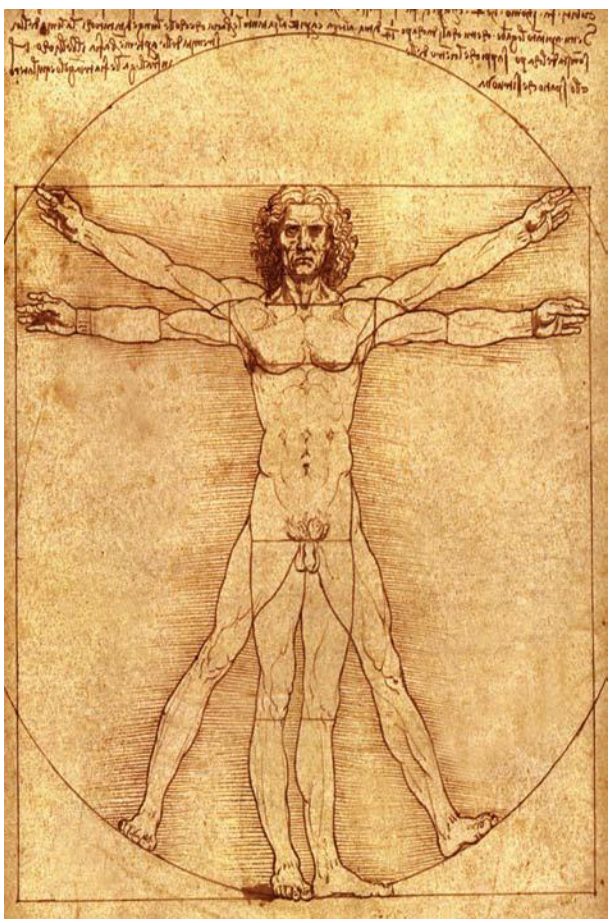
b Work out the average (mean) of all your calculations of phi. Compare your average with the actual value of $\frac{1+\sqrt{5}}{2} = 1.61803\ldots$

Golden rectangles in the human body

13 Investigate some of your own measurements and see how close they are to the golden ratio of phi: $1 \approx 1.6 : 1$.

Golden ratios in the human body include:

- total height: head to fingertips
- total height: navel (belly button) to floor
- height of head: width of head
- shoulder to fingertips: elbow to fingertips
- length of forearm: length of hand
- hip to floor: knee to floor
- length of longest bone of finger: length of middle bone of same finger
- eyes to the bottom of the chin: width of 'relaxed' smile
- width of two centre teeth: height of centre tooth



Research and class presentation

14 Research the internet to discover examples of golden rectangles in nature, human anatomy, architecture, art or graphic design. Present your findings to your class using a poster, report or technology.

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Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 The side lengths of a cube are all increased by 20%. As a percentage, by how much does its volume change?

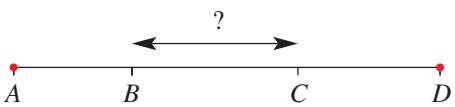
2 The cost of an item in a shop increased by 20% and was later decreased by 20% at a sale. Express the final price as a percentage of the original price.

3 Evaluate $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{999}{1000}$ without using a calculator.

4 Some prize money is shared between three students. Abby receives receives $\frac{1}{4}$ of the prize, Evie $\frac{1}{3}$ and Molly the remainder. If Molly was given \$20, what was the total value of the prize money?

5 Freshly squeezed orange juice contains 85% water. A food factory has 100 litres of fresh orange juice which is then concentrated by removing 80% of the water. What percentage of water is in the concentrated orange juice?

6 On line segment AD, AC is $\frac{2}{3}$ of AD and BD is $\frac{3}{4}$ of AD. What fraction of AD is BC?



7 Fifty people said they liked apples or bananas or both. Forty per cent of the people like apples and 70% of the people like bananas. What percentage of people like only apples?

8 A bank balance of \$100 has 10% of its value added to it at the end of every year, so that at the end of the second year the new balance is \$110 + \$11 = \$121.
How many full years will it take for the balance to be more than \$10 000?

9 From the time Lilli gets up until her bus leaves at 8:03 a.m., she uses one fifth of the time having a shower and getting dressed, then one quarter of the remaining time packing her lunch and school bag, one third of the remaining time having breakfast and then one half of the remaining time practising the piano which she finishes at exactly 7:45 a.m. At what time does Lilli get up?

10 Four numbers a, b, c, d are evenly spaced along a number line. Without using a calculator, find the values of b and c as fractions given that:

a $a = -1\frac{1}{3}, d = \frac{1}{6}$

b $a = 1\frac{7}{10}, d = 2\frac{1}{5}$

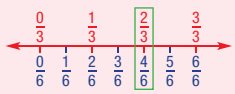
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Chapter summary

Fractions

Numerator
2 parts selected
$\frac{2}{3}$ =
3 parts in one whole
Denominator

Equivalent fractions represent equal proportions of one whole.



Simplest form: $\frac{2}{3} = \frac{4}{6} = \frac{20}{30} = \frac{10}{15} = \frac{2}{3}$ (×2, ×5, ÷2, ÷5)

Equivalent fractions have the same simplest form and are equal in value.

Mixed → Improper
$2\frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$

Improper → Mixed
$\frac{40}{15} = 2\frac{10}{15} = 2\frac{5\times2}{5\times3} = 2\frac{2}{3}$
OR
$\frac{40}{15} = \frac{5\times8}{5\times3} = \frac{8}{3} = 2\frac{2}{3}$

$-4 + 3 - \frac{1}{8} + \frac{5}{12}$
$= -1 - \frac{1\times3}{8\times3} + \frac{5\times2}{12\times2}$
$= -1 - \frac{3}{24} + \frac{10}{24}$
$= -1 + \frac{7}{24}$
$= -\frac{24}{24} + \frac{7}{24}$
$= -\frac{17}{24}$

OR

$-4\frac{1}{8} - (-3\frac{5}{12})$
$= -4\frac{1}{8} + 3\frac{5}{12}$
$= -\frac{33}{8}\times\frac{3}{3} + \frac{41}{12}\times\frac{2}{2}$
$= -\frac{99}{24} + \frac{82}{24}$
$= -\frac{17}{24}$

Adding and subtracting fractions

- Equivalent fractions
- 'Unit' is the denominator
- Add/subtract numerators
- Keep denominator the same

Lowest common denominator (LCD) is lowest common multiple (LCM)

$\frac{3}{35} + \frac{1}{40}$ HCF of 35 and 40 is 5
$= \frac{8}{8}\times\frac{3}{5\times7} + \frac{1}{5\times8}\times\frac{7}{7}$
$= \frac{24}{280} + \frac{7}{280}$
$= \frac{31}{280}$

Negative fractions

$\frac{5}{6} + (-\frac{1}{6})$ → Subtract opposite → $= \frac{5}{6} - \frac{1}{6}$

$\frac{5}{6} - (-\frac{1}{6})$ → Add opposite → $= \frac{5}{6} + \frac{1}{6}$

Dividing fractions

- Mixed numerals → Improper fractions
- Denominators do not need to be the same
- Multiply by reciprocal of divisor
- Cancel: Divide common factors in numerator and denominator

Negative fractions

$\frac{2}{3} \div (-\frac{1}{6})$
$= \frac{2}{3}\times(-\frac{1}{6})$
Different signs → Negative answer
$= -\frac{12}{3}$
$= -4$

$-\frac{5}{8} \div (-\frac{3}{8})$
$= -\frac{5}{8}\times(-\frac{8}{3})$
Same signs → Positive answer
$= \frac{5}{3}$
$= 1\frac{2}{3}$

$-\frac{7}{12} \div (-4\frac{2}{3})$
$= -\frac{7}{12} \div (-\frac{14}{3})$ Reciprocal of $-\frac{14}{3}$ is $-\frac{3}{14}$
$= -\frac{7}{12}\times-\frac{3}{14}$
$= \frac{1}{8}$

Multiplying fractions

- Mixed numerals → Improper fractions
- Denominators do not need to be the same
- Cancel: Divide common factors in numerator and denominator

Negative fractions

$\frac{2}{3}\times(-\frac{1}{4})$
Different signs → Negative answer
$= -\frac{2}{12}$
$= -\frac{1}{6}$

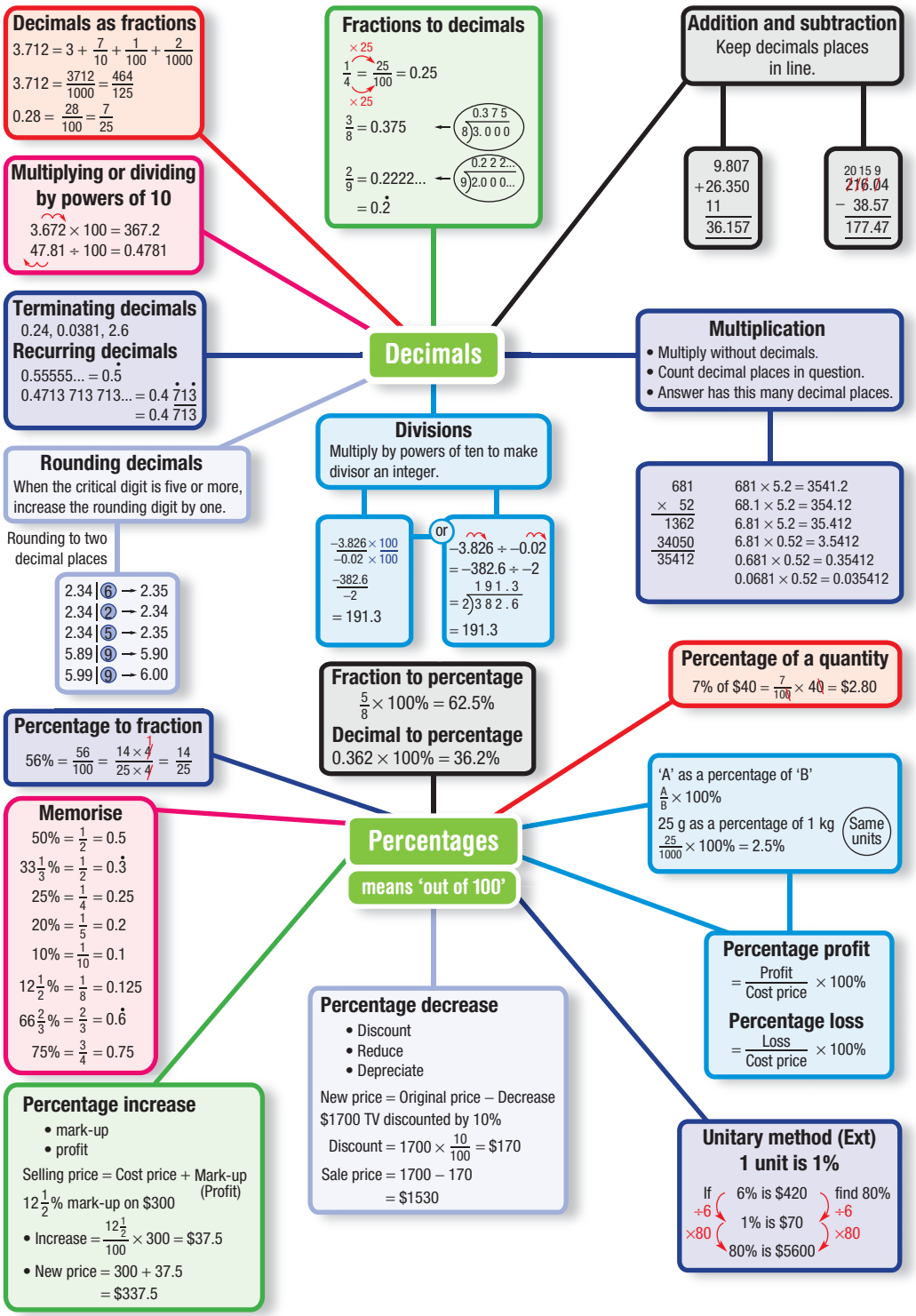
$-\frac{5}{8}\times(-\frac{2}{5})$
Same signs → Positive answer
$= \frac{10}{40}$
$= \frac{1}{4}$

$-\frac{5}{14}\times4\frac{1}{5}$
$= -\frac{5}{14}\times\frac{21}{5}$
$= -\frac{3}{2}$
$= -1\frac{1}{2}$

Note: $\frac{5}{5} = 5 \div 5 = 1$; $\frac{21}{14} = \frac{3}{2}$ (÷7)

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Chapter summary



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Multiple-choice questions

3A **1** 0.36 expressed as a fraction is:

A $\frac{36}{10}$ **B** $\frac{36}{100}$ **C** $\frac{3}{6}$ **D** $\frac{9}{20}$

3B **2** $\frac{2}{11}+\frac{5}{8}$ is equal to:

A $\frac{7}{19}$ **B** $\frac{10}{19}$ **C** $\frac{71}{88}$ **D** $\frac{31}{44}$

3E **3** When 21.63 is multiplied by 13.006, the number of decimal places in the answer is:

A 2 **B** 3 **C** 4 **D** 5

3A **4** $\frac{124}{36}$ is the same as:

A 88 **B** $\frac{34}{9}$ **C** $3\frac{4}{9}$ **D** 3.49

3B **5** When $5\frac{1}{3}$ is written as an improper fraction, its reciprocal is:

A $\frac{1}{53}$ **B** 53 **C** $\frac{16}{3}$ **D** $\frac{3}{16}$

3D **6** Which decimal has the largest value?

A 6.0061 **B** 6.06 **C** 6.016 **D** 6.0006

3E **7** $9.46 \times 100\,000$ is the same as:

A 94 600 000 **B** 946 000 **C** 94 605 **D** 0.0000946

3H **8** 75% of 84 is the same as:

A $\frac{84}{4} \times 3$ **B** $\frac{84}{3} \times 4$ **C** $84 \times 100 \div 75$ **D** $\frac{(0.75 \times 84)}{100}$

3G **9** 590% is the same as:

A 59.0 **B** 0.59 **C** 5.9 **D** 0.059

3I **10** \$790 increased by 15% gives:

A \$118.50 **B** \$908.50 **C** \$671.50 **D** \$805

Short-answer questions

3A **1** Copy and complete:

a $\frac{7}{20}=\frac{\square}{60}$ **b** $\frac{25}{40}=\frac{5}{\square}$ **c** $\frac{350}{210}=\frac{\square}{6}$

3A **2** Simplify:

a $\frac{25}{45}$ **b** $\frac{36}{12}$ **c** $\frac{102}{12}$

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Chapter review

3B **3** Evaluate each of the following.

a $\frac{5}{11}+\frac{2}{11}$ **b** $\frac{7}{8}-\frac{3}{4}$ **c** $3-1\frac{1}{4}$

d $3\frac{1}{4}-1\frac{2}{3}$ **e** $2\frac{2}{5}+3\frac{1}{4}$ **f** $1\frac{1}{2}+2\frac{2}{3}-\frac{3}{5}$

3B **4** Evaluate each of the following.

a $\frac{2}{3}\times 12$ **b** $\frac{3}{7}\times 1\frac{1}{12}$ **c** $2\frac{1}{3}\times 6$

d $3\div\frac{1}{2}$ **e** $\frac{2}{3}\div 12$ **f** $1\frac{1}{2}\div\frac{3}{4}$

3C **5** Evaluate each of the following.

a $\frac{1}{5}-\frac{2}{3}$ **b** $-\frac{3}{4}\times\frac{1}{5}$ **c** $\left(-\frac{3}{5}\right)^2$

d $\frac{3}{4}-\left(-\frac{1}{5}\right)$ **e** $\frac{5}{3}\div\left(-\frac{1}{3}\right)$ **f** $-6\frac{1}{4}+\left(-1\frac{1}{3}\right)$

3D **6** Insert >, < or = to make each of the statements below true.

a $\frac{11}{20}\square 0.55$ **b** $\frac{2}{3}\square 0.7$ **c** $0.763\square 0.7603$

3E **7** Evaluate:

a $12.31+2.34+15.73$ **b** $14.203-1.4$

c 569.74×100 **d** 25.14×2000

e 7.4×10^4 **f** $5-2.0963$

3E **8** Calculate:

a 2.67×4 **b** 2.67×0.04 **c** 1.2×12

d $1.02\div 4$ **e** $1.8\div 0.5$ **f** $9.856\div 0.05$

3F **9** Round these decimals to three decimal places.

a $0.\dot{6}$ **b** 3.57964 **c** 0.00549631

3G **10** Copy and complete this table of conversions.

0.1					0.75		
	$\frac{1}{100}$			$\frac{1}{4}$		$\frac{1}{3}$	$\frac{1}{8}$
		5%	50%				

3H **11** Express each of the following as a percentage.

a \$35 out of \$40 **b** 6 out of 24

c \$1.50 out of \$1 **d** 16 cm out of 4 m

e 15 g out of $\frac{1}{4}$ kg

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3H **12** Find:

a 30% of 80 **b** 15% of $70 **c** $12\frac{1}{2}$% of 84

3I **13** **a** Increase $560 by 10%.

b Decrease $4000 by 18%.

c Increase $980 by 5% and then decrease the result by 5%. Calculate the overall percentage loss.

3I **14** A new plasma television was valued at $3999. During the end-of-year sale it was discounted by 9%. What was the discount and the sale price?

3J **15** Jenni works at her local pizza takeaway restaurant and is paid $7.76 per hour. When she turns 16, her pay will increase to $10 an hour. What will be the percentage increase in her pay (to the nearest per cent)?

3J **16** Johan saved 15% of his weekly wage. He saved $5304 during the year. Calculate Johan's weekly wage.

3J **17** Mary measures the volume of a cup to be 240 mL when the actual volume is 250 mL. Find Mary's percentage error.

3K **18** If 5% of an amount equals 56, what is 100% of the amount?

Ext

3K **19** A shopping receipt quotes an A4 folder as costing $3.64 including 10% GST. What is the cost of the folder pre-GST, correct to the nearest cent?

Ext

Extended-response questions

1 The following table shows the value of A$1 (one Australian dollar) in foreign currency.

Indian rupee (INR)	42
Singapore dollar (SGD)	1.25
Thai baht (THB)	30
Hong Kong dollar (HKD)	7

Genevieve is planning an extended holiday to Asia, where she plans on visiting India, Singapore, Phuket and Hong Kong.

a She has decided to convert some Australian dollars to each of the above currencies before she flies out. How much of each currency will she receive if she converts A$500 to each currency?

b During the exchange she needs to pay a fee of 1.5% for each transaction. How much has she paid in fees (in A$)?

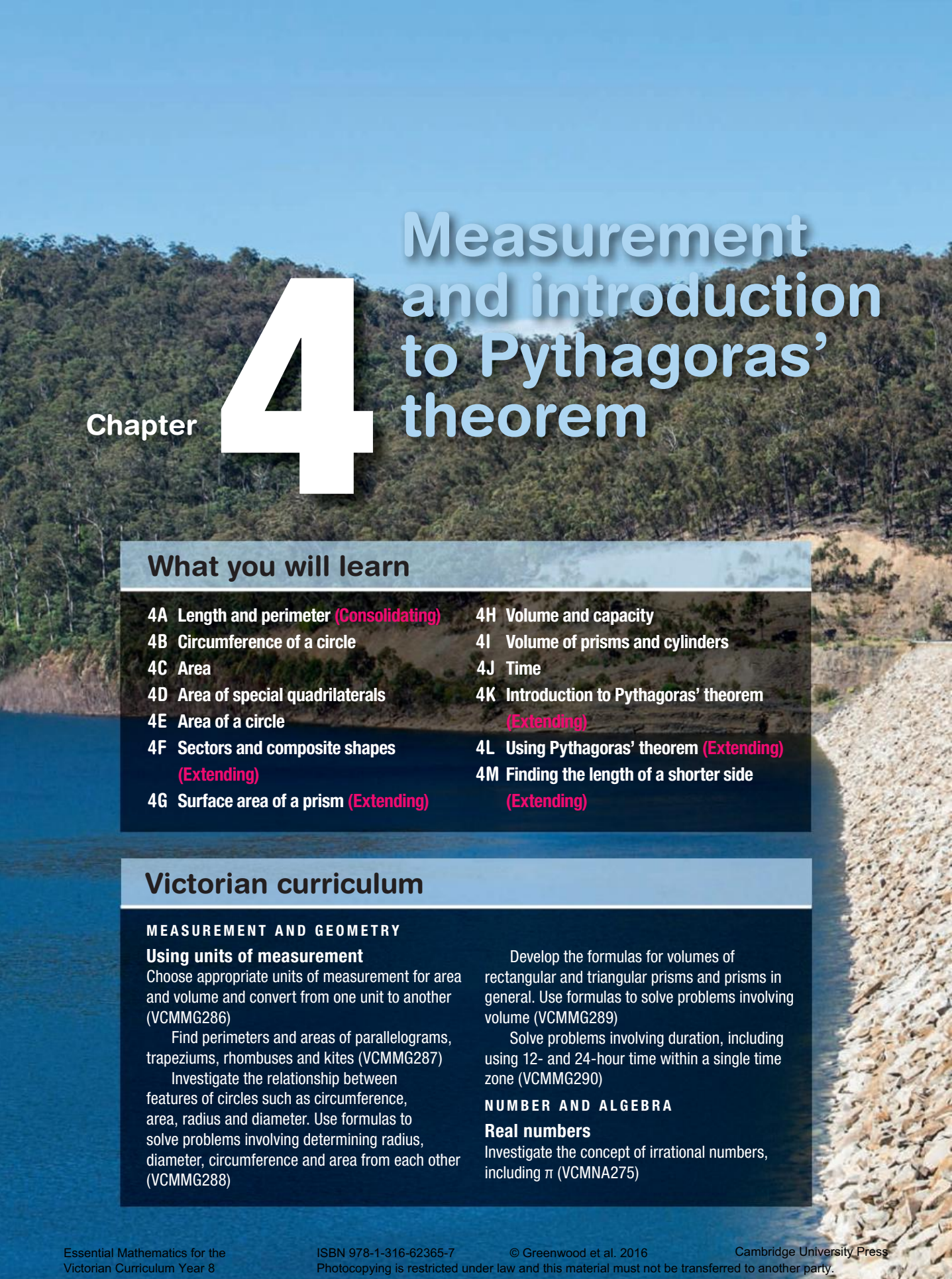
c **i** The day after she leaves, the exchange rate for Indian rupees is A$1 = 43.6 INR. How much more would she have received if she had waited until arriving in India to convert her Australian dollars? (Ignore transaction fees.)

ii Express this as a percentage (to one decimal place).

d **i** On her return to Australia, Genevieve has in her wallet 1000 INR, 70 SGD and 500 THB. Using the same exchange rate, how much is this in A$?

ii She also needs to pay the 1.5% transaction fee. How much does she receive in A$, and is it enough to buy a new perfume at the airport for $96?

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Chapter 4 Measurement and introduction to Pythagoras' theorem

What you will learn

4A Length and perimeter (Consolidating)
4B Circumference of a circle
4C Area
4D Area of special quadrilaterals
4E Area of a circle
4F Sectors and composite shapes (Extending)
4G Surface area of a prism (Extending)
4H Volume and capacity
4I Volume of prisms and cylinders
4J Time
4K Introduction to Pythagoras' theorem (Extending)
4L Using Pythagoras' theorem (Extending)
4M Finding the length of a shorter side (Extending)

Victorian curriculum

MEASUREMENT AND GEOMETRY

Using units of measurement

Choose appropriate units of measurement for area and volume and convert from one unit to another (VCMMG286)

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (VCMMG287)

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving determining radius, diameter, circumference and area from each other (VCMMG288)

Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (VCMMG289)

Solve problems involving duration, including using 12- and 24-hour time within a single time zone (VCMMG290)

NUMBER AND ALGEBRA

Real numbers

Investigate the concept of irrational numbers, including π (VCMNA275)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable hOTsheets
- Access to all hOTmaths Australian Curriculum courses
- Access to the hOTmaths games library

Water storage

Civilisations from ancient to modern times have all used various measurements in their day-to-day existence. From counting livestock, building pens, managing crop size to volumes of industrial production, measurement, in all its forms, has played a vital role in human life.

With increases in population and the challenges of climate change, water storage systems have become an important focus for governments around the world.

There are around 400 000 dams in Victoria, including the small dams on rural properties. The Thomson Reservoir in West Gippsland, Victoria, is the largest of Melbourne's reservoirs with a storage capacity of about 1 068 000 ML. This reservoir can supply 60% of Melbourne's total water needs. The dam wall has a height of 165 m and is 590 m long. The entire reservoir holds twice the volume of Sydney harbour. The daily volumes of water in all of Victoria's reservoirs are monitored constantly and the data is publicly available from Melbourne Water.

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4A Length and perimeter

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

For thousands of years, civilisations have found ways to measure length. The Egyptians, for example, used the cubit (length of an arm from the elbow to the tip of the middle finger), the Romans used the pace (5 feet) and the English developed their imperial system using inches, feet, yards and miles. The modern-day system used in Australia (and most other countries) is the metric system, which was developed in France in the 1790s and is based on the unit called the metre. We use units of length to describe the distance between two points, or the distance around the outside of a shape, called the perimeter.



The Romans would have measured the perimeter of the Colosseum in paces.

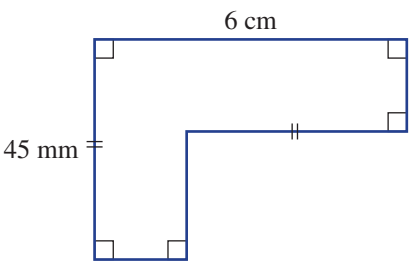
Let's start: Provide the perimeter

In this diagram some of the lengths are given. Three students were asked to find the perimeter.

- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is $9 + 12 = 21$ cm.
- Greta says that there is enough information and the answer is $90 + 12 = 102$ cm.

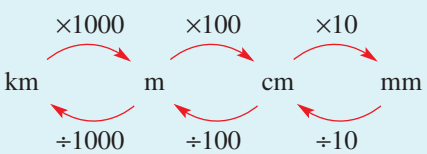
Who is correct?

Discuss how each person arrived at their answer.

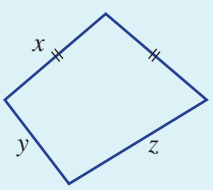


Key ideas

- The common metric units of length include the **kilometre** (km), **metre** (m), **centimetre** (cm) and **millimetre** (mm).



- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same type of markings (dashes) are of equal length.



$P = 2x + y + z$

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Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.

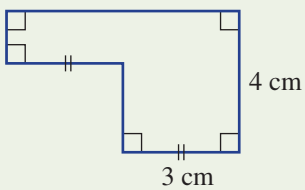
a 5.2 cm (mm)

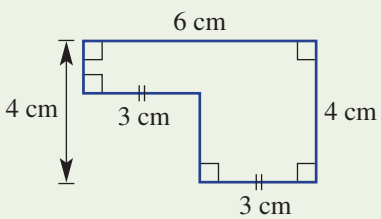
b 85 000 cm (km)

SOLUTION	EXPLANATION
a $5.2 \text{ cm} = 5.2 \times 10$ $= 52 \text{ mm}$	1 cm = 10 mm so multiply by 10. ×10 cm → mm
b $85\,000 \text{ cm} = 85\,000 \div 100 \div 1000$ $= 0.85 \text{ km}$	1 m = 100 cm and 1 km = 1000 m so divide by 100 and 1000.

Example 2 Finding perimeters

Find the perimeter of this shape.



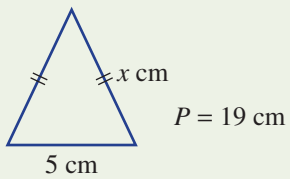
SOLUTION	EXPLANATION
$P = 2 \times (3 + 3) + 2 \times 4$ $= 12 + 8$ $= 20 \text{ cm}$	



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Example 3 Finding an unknown length

Find the unknown value x in this triangle if the perimeter is 19 cm.



$P = 19$ cm

SOLUTION

$2x + 5 = 19$

$2x = 14$

$x = 7$

EXPLANATION

$2x + 5$ makes up the perimeter.

$2x$ is the difference between 19 and 5.

If $2x = 14$ then $x = 7$ since $2 \times 7 = 14$.

Exercise 4A

1–4 | 4 | —

UNDERSTANDING

1 Write the missing number in these sentences.

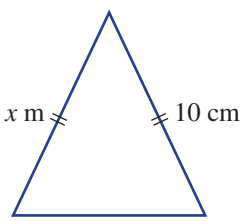
a There are ___ mm in 1 cm.
b There are ___ cm in 1 m.
c There are ___ m in 1 km.
d There are ___ cm in 1 km.
e There are ___mm in 1 m.
f There are ___ mm in 1 km.

2 Evaluate the following.

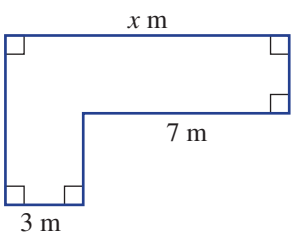
a 10×100
b 100×1000
c $10 \times 100 \times 1000$

3 State the value of x in these diagrams.

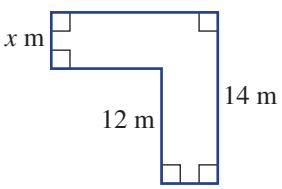
a



b



c



4 Find the perimeter of these quadrilaterals.

a Square with side length 3 m.
b Rectangle with side lengths 4 cm and 7 cm.
c Rhombus with side length 2.5 m.
d Parallelogram with side lengths 10 km and 12 km.
e Kite with side lengths 0.4 cm and 0.3 cm.
f Trapezium with side lengths 1.5 m, 1.1 m, 0.4 m and 0.6 m.

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5–6(½) 5–7(½) 5–7(½) 4A

FLUENCY

Example 1

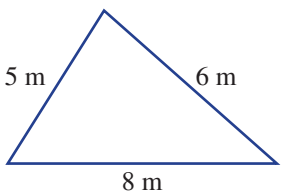
5 Convert these measurements to the units shown in the brackets.

a 3 cm (mm)	**b** 6.1 m (cm)	**c** 8.93 km (m)	**d** 3 m (mm)
e 0.0021 km (m)	**f** 320 mm (cm)	**g** 9620 m (km)	**h** 38 000 cm (km)
i 0.0043 m (mm)	**j** 0.0204 km (cm)	**k** 23 098 mm (m)	**l** 342 000 cm (km)
m 194 300 mm (m)	**n** 10 000 mm (km)	**o** 0.02403 m (mm)	**p** 994 000 mm (km)

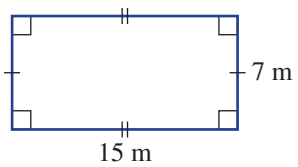
Example 2

6 Find the perimeter of these shapes.

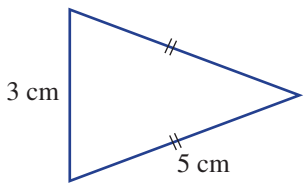
a



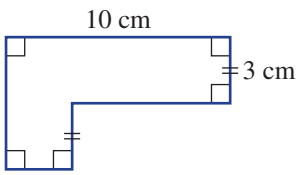
b



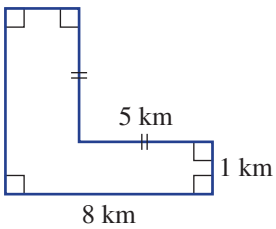
c



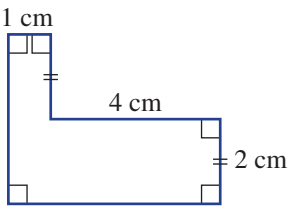
d



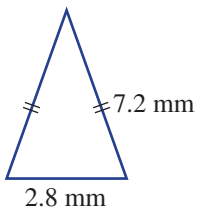
e



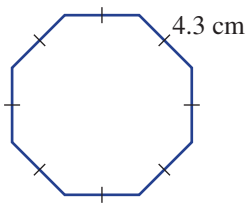
f



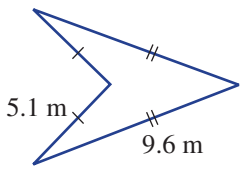
g



h



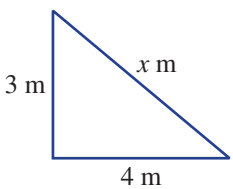
i



Example 3

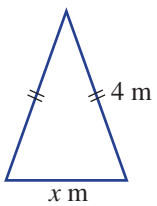
7 Find the unknown value x in these shapes with the given perimeter (P).

a



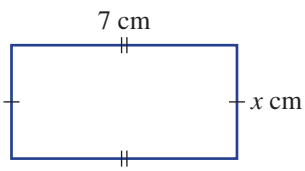
$P = 12$ m

b



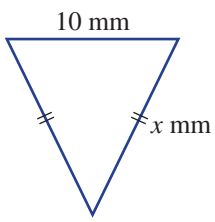
$P = 10$ m

c



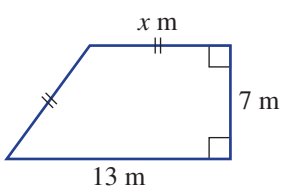
$P = 22$ cm

d



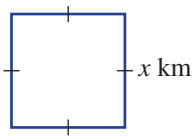
$P = 46$ mm

e



$P = 39$ m

f



$P = 26$ km

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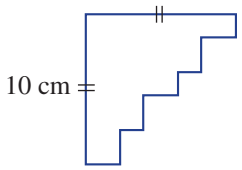
4A

8, 9 | 8–11 | 9–12

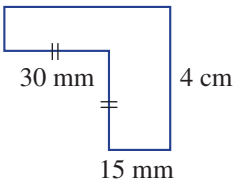
PROBLEM-SOLVING

8 Find the perimeter of these shapes. Give your answers in cm and assume that angles that look right-angled are 90°.

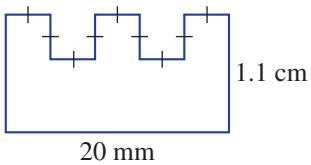
a



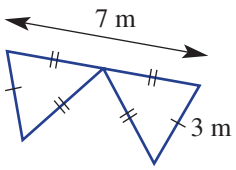
b



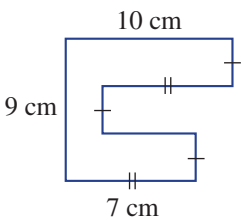
c



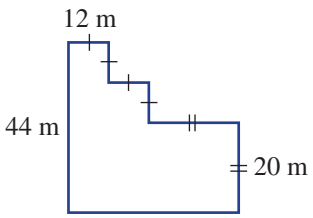
d



e

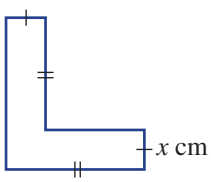


f



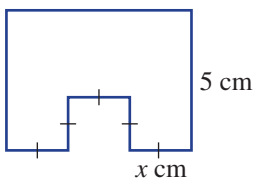
9 Find the unknown value x in these diagrams. All angles are 90°.

a



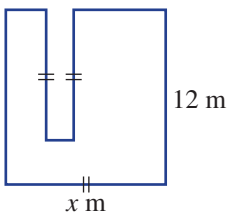
$P = 24$ cm

b



$P = 34$ cm

c



$P = 60$ m

10 Jennifer needs to fence her country house block to keep her dog in. The block is a rectangle with length 50 m and width 42 m. Fencing costs $13 per metre. What will be the total cost of fencing?



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4A

PROBLEM-SOLVING

11 Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Give your answer in minutes.

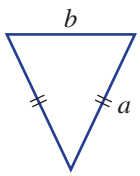
12 A rectangular picture of length 65 cm and width 35 cm is surrounded by a frame of width 5 cm. What is the perimeter of the framed picture?

13 13 13, 14

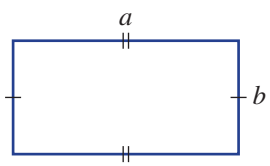
REASONING

13 Write down rules using the given letters for the perimeter of these shapes, e.g. $P = a + 2b$. Assume that angles that look right-angled are 90°.

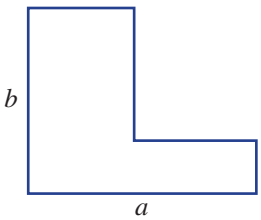
a



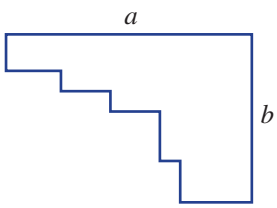
b



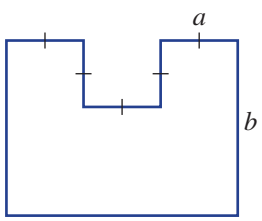
c



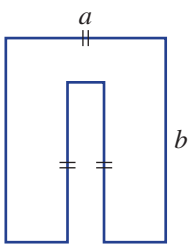
d



e

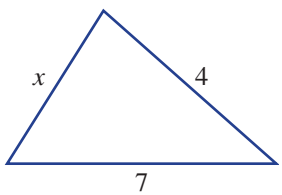


f

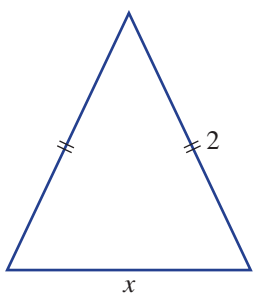


14 Write a rule for x in terms of its perimeter P, e.g. $x = P - 10$.

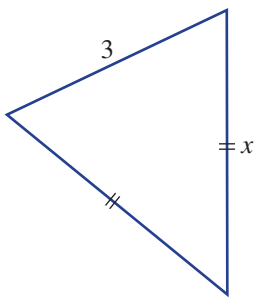
a



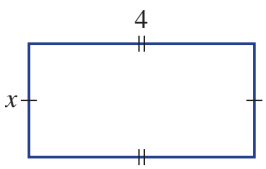
b



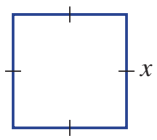
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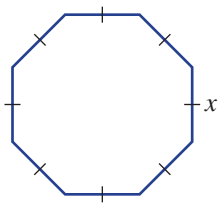
d



e



f



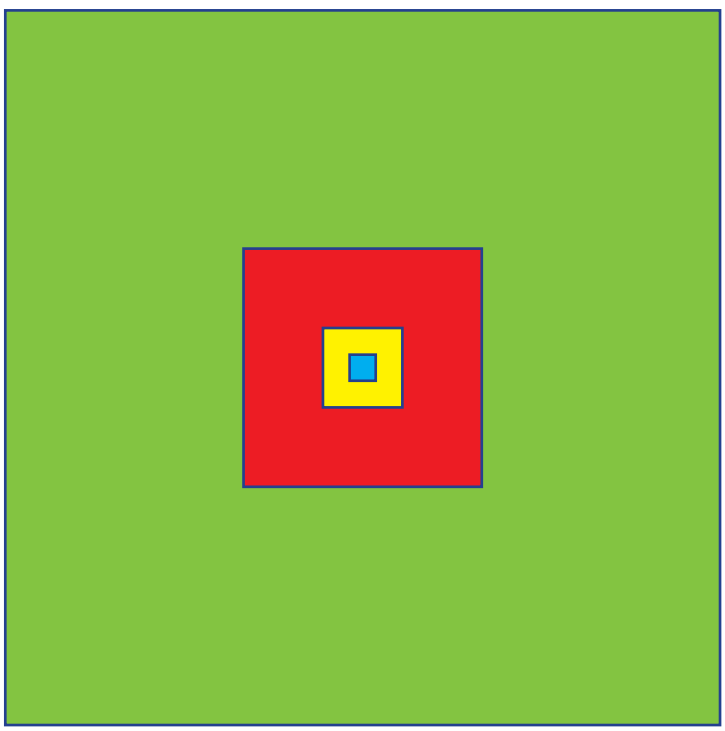
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ENRICHMENT

15 A square is drawn with a particular side length. A second square is drawn inside the square so that its side length is one-third that of the original square. Then a third square is drawn, with side length of one-third that of the second square and so on.

a What is the minimum number of squares that would need to be drawn in this pattern (including the starting square), if the innermost square has a perimeter of less than 1 hundredth the perimeter of the outermost square?

b Imagine now if the situation is reversed and each square's perimeter is 3 times larger than the next smallest square. What is the minimum number of squares that would be drawn in total if the perimeter of the outermost square is to be at least 1000 times the perimeter of the innermost square?



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4B Circumference of a circle

Interactive

Widgets

HOTsheets

Walkthroughs

Since the ancient times, people have known about a special number that links a circle's diameter to its circumference. We know this number as pi (π). π is a mathematical constant that appears in formulas relating to circles, but it is also important in many other areas of mathematics. The actual value of π has been studied and approximated by ancient and more modern civilisations over thousands of years. The Egyptians knew π was slightly more than 3 and approximated it to be $\frac{256}{81} \approx 3.16$. The Babylonians used $\frac{25}{8} = 3.125$ and the ancient Indians used $\frac{339}{108} \approx 3.139$.

It is believed that Archimedes of Syracus (287–212 BCE) was the first person to use a mathematical technique to evaluate π. He was able to prove that π was greater than $\frac{223}{71}$ and less than $\frac{22}{7}$. In 480 AD, the Chinese mathematician Zu Chongzhi showed that π was close to $\frac{355}{113} \approx 3.1415929$, which is accurate to seven decimal places.



Archimedes of Syracus (287–212 BCE)

Before the use of calculators, the fraction $\frac{22}{7}$ was commonly used as a good and simple approximation to π. Interestingly, mathematicians have been able to prove that π is an irrational number, which means that there is no fraction that can be found that is exactly equal to π. If the exact value of π was written down as a decimal, the decimal places would continue forever with no repeated pattern.

Let's start: Discovering π

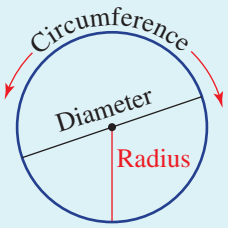
Diameter d (mm)	Circumference C (mm)	$C \div d$
4.46	14.01	
11.88	37.32	
40.99	128.76	
Add your own	Add your own	

Here are the diameters and circumferences for three circles correct to two decimal places. Use a calculator to work out the value of Circumference ÷ Diameter and put your results in the third column. Add your own circle measurements by measuring the diameter and circumference of circular objects such as a can.

- What do you notice about the numbers $C \div d$ in the third column?
- Why might the numbers in the third column vary slightly from one set of measurements to another?
- What rule can you write down which links C with d?

Key ideas

- Features of a circle
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre to the circle. Note $d = 2r$.



- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$
  - $r = \frac{C}{2\pi}$ or $r = \frac{C}{d}$
- **Pi** (π) ≈ 3.14159 (correct to five decimal places)
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet.
  - For a circle $\pi = \frac{C}{2r}$ or $\pi = \frac{C}{d}$

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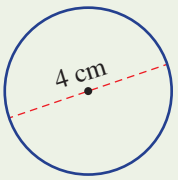
Example 4 Finding the circumference with a calculator

Find the circumference of these circles correct to two decimal places. Use a calculator for the value of π.

a



b

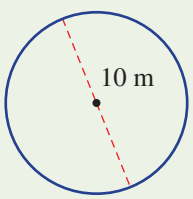


SOLUTION	EXPLANATION
a $C = 2\pi r$	
$= 2 \times \pi \times 3.5$	Since r is given, you can use $C = 2\pi r$.
$= 7\pi$	Alternatively use $C = \pi d$ with $d = 7$.
$= 21.99$ m (to 2 d.p.)	Round off as instructed.
b $C = \pi d$	
$= \pi \times 4$	Substitute into the rule $C = \pi d$ or use
$= 4\pi$	$C = 2\pi r$ with $r = 2$.
$= 12.57$ cm (to 2 d.p.)	Round off as instructed.

Example 5 Finding circumference without a calculator

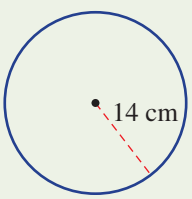
Calculate the circumference of these circles using the given approximation of π.

a



$\pi = 3.14$

b



$\pi = \frac{22}{7}$

SOLUTION	EXPLANATION
a $C = \pi d$	Use $\pi = 3.14$ and multiply mentally. Move the decimal point one place to the right.
$= 3.14 \times 10$	
$= 31.4$ m	Alternatively use $C = 2\pi r$ with $r = 5$.
b $C = 2\pi r$	Use $\pi = \frac{22}{7}$ and cancel the 14 with the 7 before calculating the final answer.
$= 2 \times \frac{22}{7} \times 14$	$2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2$
$= 88$ cm	

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Exercise 4B

1–4 | 4 | —

UNDERSTANDING

1 Evaluate the following using a calculator and round to two decimal places.

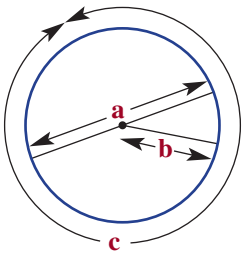
a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

2 Write down the value of π correct to:

a one decimal place **b** two decimal places

c three decimal places

3 Name the features of the circle as shown.



4 A circle has circumference (C) 81.7 m and diameter (d) 26.0 m correct to one decimal place. Calculate $C \div d$. What do you notice?

5(½), 6, 7 | 5(½), 6, 7, 8(½) | 5(½), 6, 7, 8(½)

FLUENCY

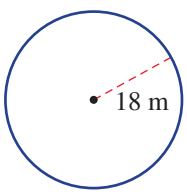
Example 4

5 Find the circumference of these circles correct to two decimal places. Use a calculator for the value of π.

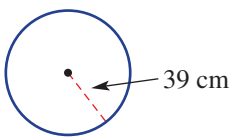
a



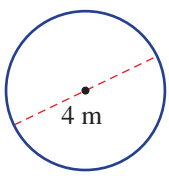
b



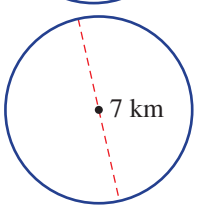
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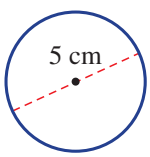
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e



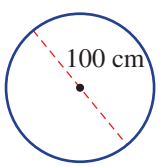
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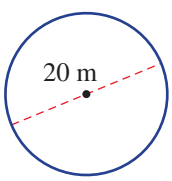
Example 5a

6 Calculate the circumference of these circles using $\pi = 3.14$.

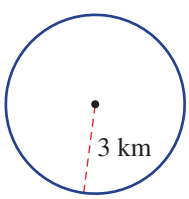
a



b



c



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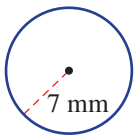
4B

FLUENCY

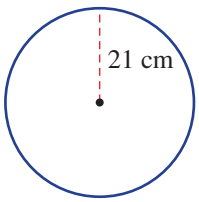
Example 5b

7 Calculate the circumference of these circles using $\pi = \frac{22}{7}$.

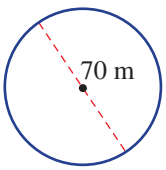
a



b



c



8 Using $r = \frac{C}{2\pi}$ or $d = \frac{C}{\pi}$ find the following correct to one decimal place.

a The diameter of a circle with circumference 20 cm
b The diameter of a circle with circumference 150 m
c The radius of a circle with circumference 43.8 mm
d The radius of a circle with circumference 2010 km

9–11 | 10–13 | 12–14

PROBLEM-SOLVING

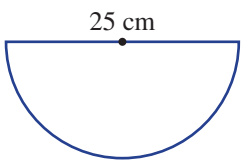
9 A water tank has a diameter of 3.5 m. Find its circumference correct to one decimal place.



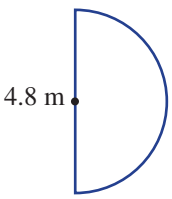
10 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

11 These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

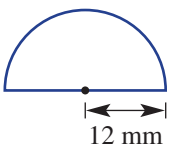
a



b

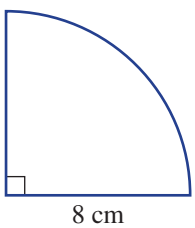


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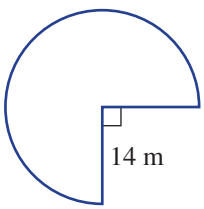


12 Calculate the perimeter of these diagrams correct to two decimal places.

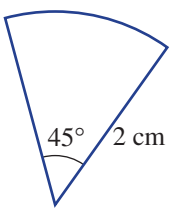
a



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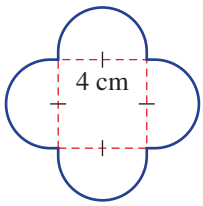
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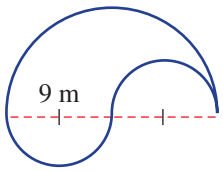
PROBLEM-SOLVING

13 Calculate the perimeter of these shapes correct to two decimal places.

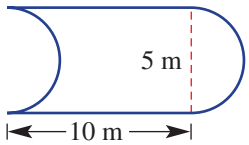
a



b



c



14 Here are some student's approximate circle measurements. Which students are likely to have incorrect measurements?

	r	C
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	13.8 m

15	15, 16	16–18

REASONING

15 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

16 It is more precise in mathematics to give 'exact' values for circle calculations in terms of π, e.g. $C = 2 \times \pi \times 3$ gives $C = 6\pi$. This gives the final exact answer and is not written as a rounded decimal. Find the exact answers for Question **5** in terms of π.

17 Find the exact answers for Question **13** above in terms of π.

18 We know that $C = 2\pi r$ or $C = \pi d$.

a Rearrange these rules to write a rule for:

i r in terms of C **ii** d in terms of C

b Use the rules you found in part **a** to find the following correct to two decimal places.

i The radius of a circle with circumference 14 m

ii The diameter of a circle with circumference 20 cm

Memorising π — — 19

ENRICHMENT

19 The box shows π correct to 100 decimal places. The World record for the most number of digits of π recited from memory is held by Akira Haraguchi, a retired Japanese engineer. He recited 111 700 digits on Pi day in 2015.

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory
100 000	World record

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4C Area

Interactive

Widgets

HOTsheets

Walkthroughs

Area is a measure of surface and is often referred to as the amount of space contained inside a two-dimensional space. Area is measured in square units and the common metric units are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2) and hectares (ha). The hectare is often used to describe area of land, since the square kilometre for such areas is considered to be too large a unit and the square metre too small. A school football oval might be about 1 hectare for example and a small forest might be about 100 hectares.

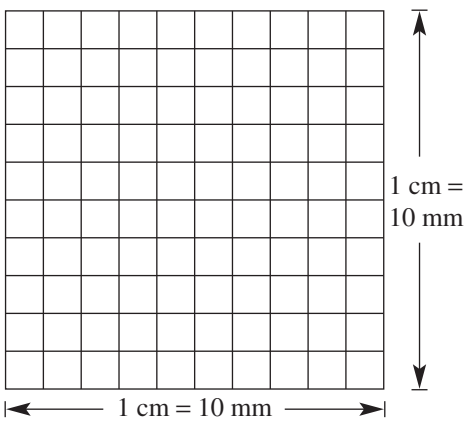


Large wheat farms in Australia can range from 1000 to 15 000 hectares.

Let's start: Squares of squares

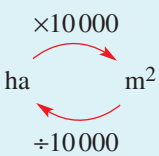
Consider this enlarged drawing of one square centimetre divided into square millimetres.

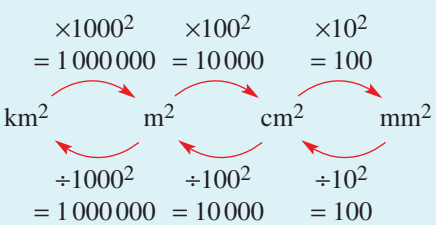
- How many square millimetres are there on one edge of the square centimetre?
- How many square millimetres are there in total in 1 square centimetre?
- What would you do to convert between mm^2 and cm^2 or cm^2 and mm^2 and why?
- Can you describe how you could calculate the number of square centimetres in one square metre and how many square metres in one square kilometre? What diagrams would you use to explain your answer?



Key ideas

- The common metric units for area include:
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)
  - hectares (ha)

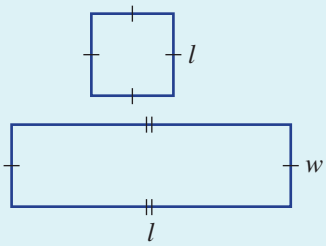




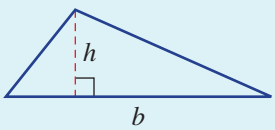
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Key ideas

- Area of squares, rectangles and triangles
  - Square $A = l \times l = l^2$

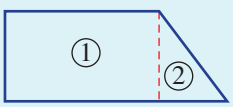


  - Rectangle $A = l \times w = lw$
  - Triangle $A = \frac{1}{2} \times b \times h = \frac{1}{2}bh$



The dashed line which gives the height is **perpendicular** (at right angles) to the base.

- Areas of **composite shapes** can be found by adding or subtracting the area of more basic shapes.

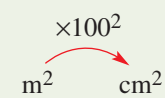


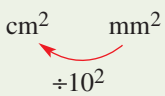
Example 6 Converting units of area

Convert these area measurements to the units shown in the brackets.

a $0.248\ m^2\ (cm^2)$ **b** $3100\ mm^2\ (cm^2)$

SOLUTION	EXPLANATION
a $0.248\ m^2 = 0.248 \times 10\,000$ $= 2480\ cm^2$	$1\ m^2 = 100^2\ cm^2$ $= 10\,000\ cm^2$ Multiply since you are changing to a smaller unit.
b $3100\ mm^2 = 3100 \div 100$ $= 31\ cm^2$	$1\ cm^2 = 10^2\ mm^2$ $= 100\ mm^2$ Divide since you are changing to a larger unit.



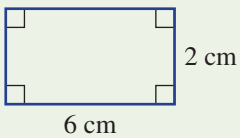


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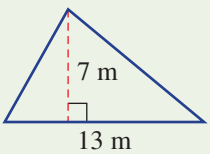
Example 7 Finding areas of rectangles and triangles

Find the area of these shapes.

a



b



SOLUTION

a $A = lw$

$= 6 \times 2$

$= 12 \text{ cm}^2$

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 7$

$= 45.5 \text{ m}^2$

EXPLANATION

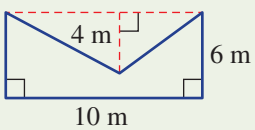
Write the formula for the area of a rectangle and substitute $l = 6$ and $w = 2$.

Remember that the height is measured using a line that is perpendicular to the base.

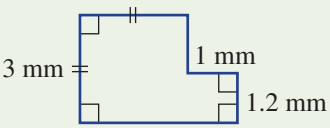
Example 8 Finding areas of composite shapes

Find the area of these composite shapes using addition or subtraction.

a



b



SOLUTION

a $A = lw - \frac{1}{2}bh$

$= 10 \times 6 - \frac{1}{2} \times 10 \times 4$

$= 60 - 20$

$= 40 \text{ m}^2$

b $A = l^2 + lw$

$= 3^2 + 1.2 \times 1$

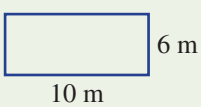
$= 9 + 1.2$

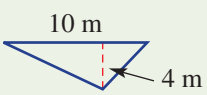
$= 10.2 \text{ mm}^2$

EXPLANATION

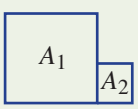
The calculation is done by subtracting the area of a triangle from the area of a rectangle.

Rectangle – triangle





The calculation is done by adding the area of a rectangle to the area of a square.



Area $= A_1 + A_2$

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Exercise 4C

1–3 | 2, 3 | —

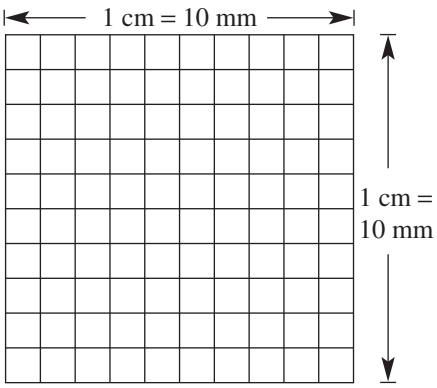
UNDERSTANDING

1 By considering the given diagrams answer the questions.

a **i** How many mm^2 in 1 cm^2?

ii How many mm^2 in 4 cm^2?

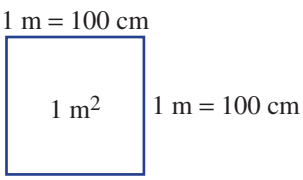
iii How many cm^2 in 300 mm^2?



b **i** How many cm^2 in 1 m^2?

ii How many cm^2 in 7 m^2?

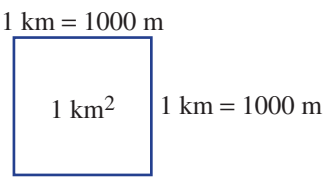
iii How many m^2 in 40 000 cm^2?



c **i** How many m^2 in 1 km^2?

ii How many m^2 in 5 km^2?

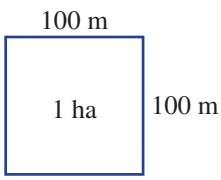
iii How many km^2 in 2 500 000 m^2?



d **i** How many m^2 in 1 ha?

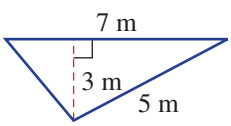
ii How many m^2 in 3 ha?

iii How many ha in 75 000 m^2?

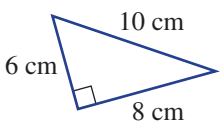


2 Which length measurements would be used for the *base* and the *height* (in that order) to find the area of these triangles?

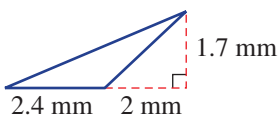
a



b



c



3 One hectare is how many square metres?

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4C

4–6(½) 4–6(½) 5–6(½)

FLUENCY

Example 6

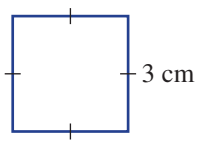
4 Convert these area measurements to the units shown in the brackets.

a 2 cm^2 (mm^2)
b 7 m^2 (cm^2)
c 0.5 km^2 (m^2)
d 3 ha (m^2)
e 0.34 cm^2 (mm^2)
f 700 cm^2 (m^2)
g 3090 mm^2 (cm^2)
h 0.004 km^2 (m^2)
i 2000 cm^2 (m^2)
j 450 000 m^2 (km^2)
k 4000 m^2 (ha)
l 3210 mm^2 (cm^2)
m 320 000 m^2 (ha)
n 0.0051 m^2 (cm^2)
o 0.043 cm^2 (mm^2)
p 4802 cm^2 (m^2)
q 19 040 m^2 (ha)
r 2933 m^2 (ha)
s 0.0049 ha (m^2)
t 0.77 ha (m^2)
u 2.4 ha (m^2)

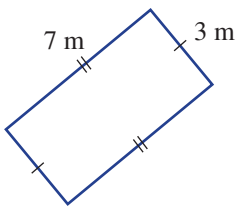
Example 7

5 Find the areas of these squares, rectangles and triangles.

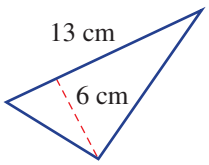
a



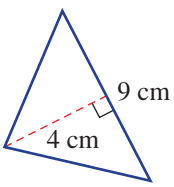
b



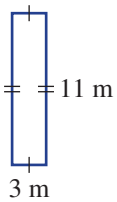
c



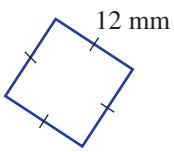
d



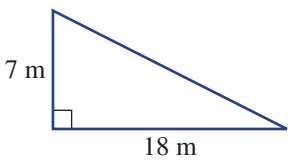
e



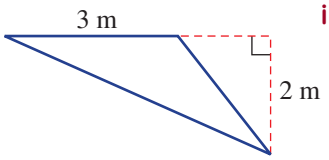
f



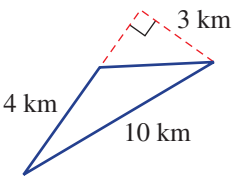
g



h



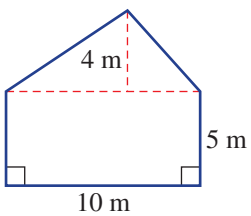
i



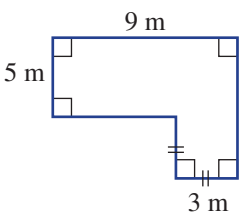
Example 8

6 Find the area of these composite shapes by using addition or subtraction.

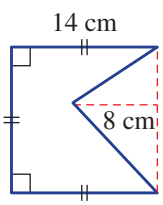
a



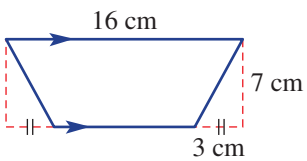
b



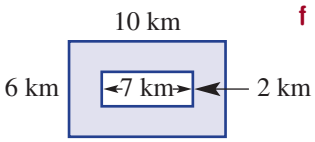
c



d

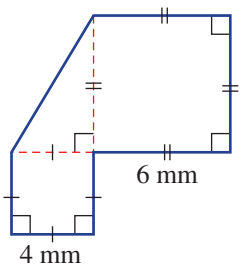


e



(Find the shaded area)

f



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4C

PROBLEM-SOLVING

7(½), 8 | 7–10(½) | 7, 8–10(½), 11, 12

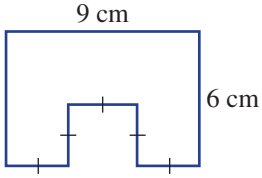
7 Use your knowledge of area units to convert these measurements to the units shown in the brackets.

a 0.2 m^2 (mm^2) **b** 0.000043 km^2 (cm^2) **c** 374 000 cm^2 (km^2)

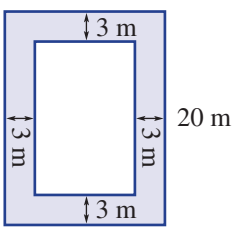
d 10 920 mm^2 (m^2) **e** 0.0000002 ha (cm^2) **f** 1 000 000 000 mm^2 (ha)

8 Find the area of these composite shapes. You may need to determine some side lengths first. Assume that angles that look right-angled are 90°.

a

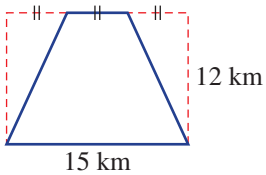


b



(Find the shaded area)

c



9 Find the side length of a square if its area is:

a 36 m^2 **b** 2.25 cm^2

10 **a** Find the area of a square if its perimeter is 20 m.

b Find the area of a square if its perimeter is 18 cm.

c Find the perimeter of a square if its area is 49 cm^2.

d Find the perimeter of a square if its area is 169 m^2.

11 A triangle has area 20 cm^2 and base 4 cm. Find its height.

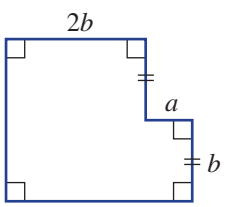
12 Paint costs $12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of 10 m^2. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

REASONING

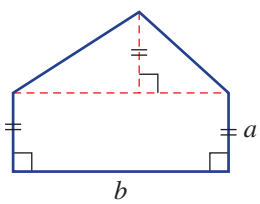
13 | 13, 14 | 13–15

13 Write down expressions for the area of these shapes in simplest form using the letters a and b (e.g. $A = 2ab + a^2$).

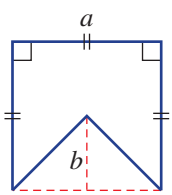
a



b



c



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REASONING

14 Assuming length and width can only be whole numbers, find:

- **a** how many distinct (different) rectangles have an area of 24 square units?
- **b** how many distinct squares have an area of 16 square units?

15 Write down rules for:

- **a** the width of a rectangle (w) with area A and length l
- **b** the side length of a square (l) with area A
- **c** the height of a triangle (h) with area A and base b

ENRICHMENT

The acre — — 16

16 Two of the more important imperial units of length and area that are still used today are the mile and the acre. Many of our country and city roads, farms and house blocks were divided up using these units.

Here are some conversions.

1 square mile = 640 acres
1 mile ≈ 1.609344 km
1 hectare = 10 000 m^2

- **a** Use the given conversions to find:
  - **i** the number of square kilometres in 1 square mile (round to two decimal places)
  - **ii** the number of square metres in 1 square mile (round to the nearest whole number)
  - **iii** the number of hectares in 1 square mile (round to the nearest whole number)
  - **iv** the number of square metres in 1 acre (round to the nearest whole number)
  - **v** the number of hectares in 1 acre (round to one decimal place)
  - **vi** the number of acres in 1 hectare (round to one decimal place)
- **b** A dairy farmer has 200 acres of land. How many hectares is this? (Round your answer to the nearest whole number.)



- **c** A house block is 2500 m^2. What fraction of an acre is this? (Give your answer as a percentage rounded to the nearest whole number.)

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4D Area of special quadrilaterals

Interactive

Widgets

HOTsheets

Walkthroughs

The formulas for the area of a rectangle and a triangle can be used to develop the area of other special quadrilaterals. These quadrilaterals include the parallelogram, the rhombus, the kite and the trapezium. Knowing the formulas for the area of these shapes can save a lot of time dividing shapes into rectangles and triangles.

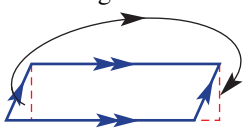


The area of each quadrilateral needs to be calculated to work out how many pavers are needed.

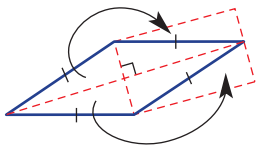
Let's start: Developing formulas

These diagrams contain clues as to how you might find the area of the shape using only what you know about rectangles and triangles. Can you explain what each diagram is trying to tell you?

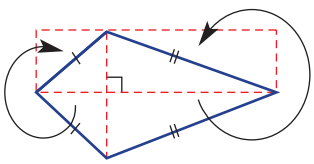
- Parallelogram



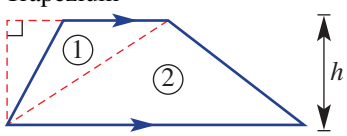
- Rhombus



- Kite



- Trapezium

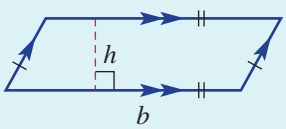


Key ideas

- Area of a **parallelogram**

  Area = base × perpendicular height

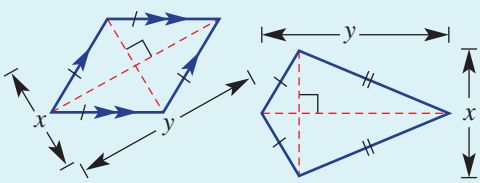
  or $A = bh$



- Area of a **rhombus** or **kite**

  $\text{Area} = \frac{1}{2} \times \text{diagonal } x \times \text{diagonal } y$

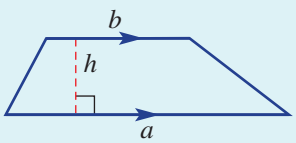
  or $A = \frac{1}{2}xy$



- Area of a **trapezium**

  $\text{Area} = \frac{1}{2} \times$ sum of parallel sides × perpendicular height

  or $A = \frac{1}{2}(a+b)h$

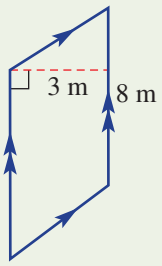


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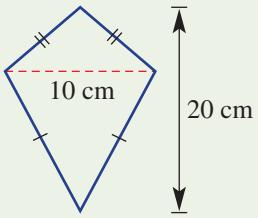
Example 9 Finding areas of special quadrilaterals

Find the area of these shapes.

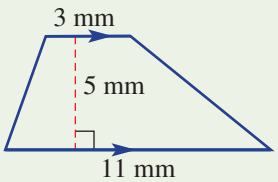
a



b



c



SOLUTION

a $A = bh$

$= 8 \times 3$

$= 24 \text{ m}^2$

b $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 10 \times 20$

$= 100 \text{ cm}^2$

c $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (11+3) \times 5$

$= \frac{1}{2} \times 14 \times 5$

$= 35 \text{ mm}^2$

EXPLANATION

The height is measured at right angles to the base.

Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus.

The two parallel sides are 11 mm and 3 mm in length. The perpendicular height is 5 mm.

Exercise 4D

1, 2 | 1 | —

UNDERSTANDING

1 Find the value of A using these formulas and given values.

a $A = bh$ $(b = 2, h = 3)$

b $A = \frac{1}{2}xy$ $(x = 5, y = 12)$

c $A = \frac{1}{2}(a+b)h$ $(a = 2, b = 7, h = 3)$

d $A = \frac{1}{2}(a+b)h$ $(a = 7, b = 4, h = 6)$

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4D

UNDERSTANDING

2 Complete these sentences.

a A perpendicular angle is _____ degrees.

b In a parallelogram, you find the area using a base and the _____.

c The two diagonals in a kite or a rhombus are _____.

d To find the area of a trapezium you multiply $\frac{1}{2}$ by the sum of the two _____ sides and then by the _____ height.

e The two special quadrilaterals that have the same area formula using diagonal lengths x and y are the _____ and the _____.

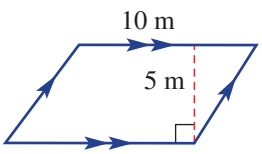
3(½), 4 3(½), 4 3(½), 4

FLUENCY

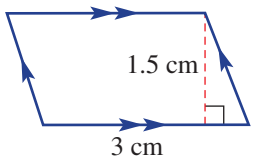
Example 9

3 Find the area of these special quadrilaterals. First state the name of the shape.

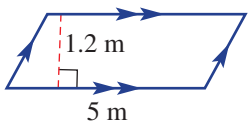
a



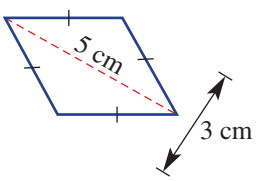
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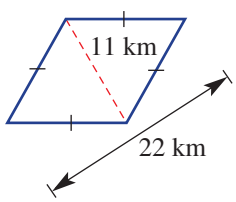
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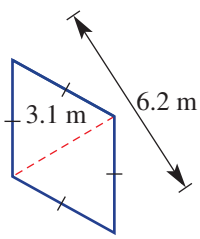
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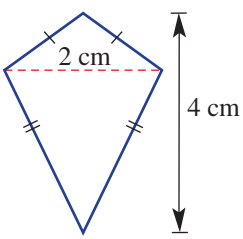
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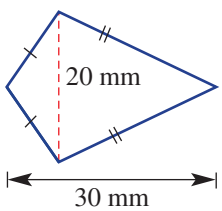
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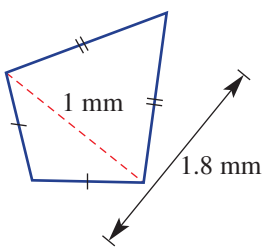
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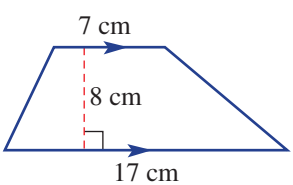
h



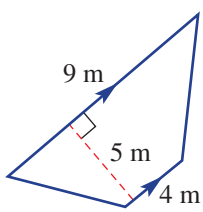
i



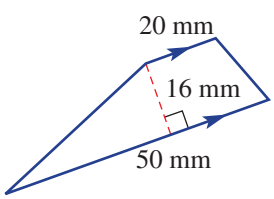
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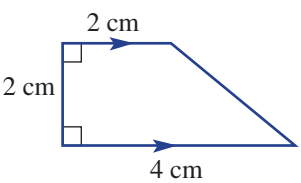


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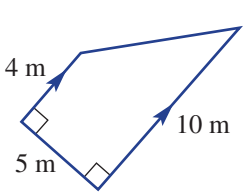


4 These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

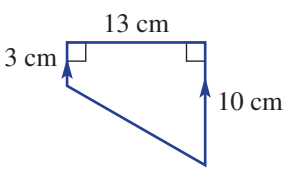
a



b



c



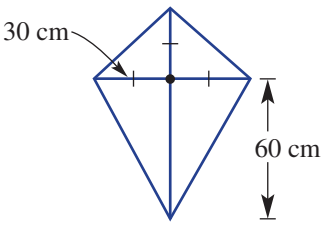
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4D

5, 6 | 5–7 | 6–8

PROBLEM-SOLVING

5 A flying kite is made from four centre rods all connected near the middle of the kite as shown. What area of plastic, in square metres, is needed to cover the kite?



6 A parallelogram has an area of 26 m^2 and its base length is 13 m. What is its perpendicular height? Draw a diagram to illustrate.

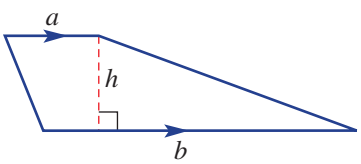
7 A landscape gardener charges $20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are 8 m and 14.5 m. What would be the cost of laying this lawn?

8 The parallel sides of a trapezium are 2 cm apart and one of the sides is 3 times the length of the other. If the area of the trapezium is 12 cm^2, what are the lengths of the parallel sides?

9 | 9, 10 | 10, 11

REASONING

9 Consider this shape.

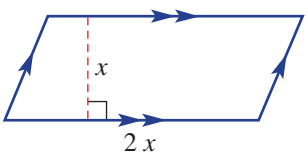


a What type of shape is it?

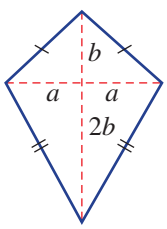
b Find its area if $a = 5$, $b = 8$, and $h = 3$.
All measurements are in cm.

10 Write an expression for the area of these shapes in simplest form (e.g. $A = 2a + 3ab$).

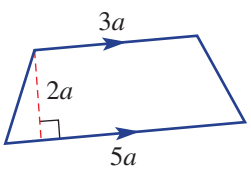
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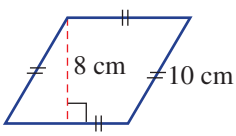
b



c



11 Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus?



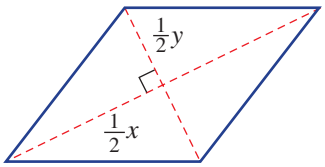
Explain why or why not.

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ENRICHMENT

12 Complete these proofs to give the formula for the area of a rhombus and a trapezium.

a Rhombus



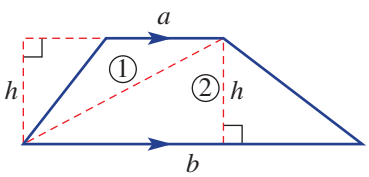
$A = 4$ triangle areas

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times _____ \times _____$

$= _____$

b Trapezium 1



$A = \text{Area (triangle 1)} + \text{Area (triangle 2)}$

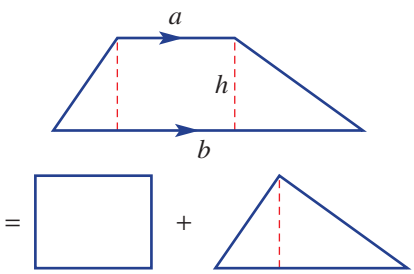
$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times _____ \times _____ + \frac{1}{2} \times _____ \times _____$

$= _____ + _____$

$= _____$

c Trapezium 2



$A = \text{Area (rectangle)} + \text{Area (triangle)}$

$= \text{length} \times \text{width} + \frac{1}{2} \times \text{base} \times \text{height}$

$= _____ \times _____ + \frac{1}{2} \times _____ \times _____$

$= _____ + _____ - _____$

$= _____ + _____$

$= _____$

13 Design an A4 poster for one of the proofs in Question **12** to be displayed in your class.



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4E Area of a circle

Interactive

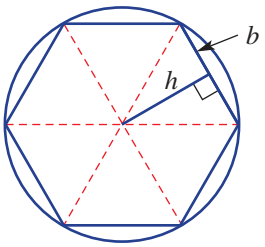
Widgets

HOTsheets

Walkthroughs

We know that the link between the perimeter of a circle and its radius has challenged civilisations for thousands of years. Similarly, people have studied the link between a circle's radius and its area.

Archimedes (287–212 BCE) attempted to calculate the exact area of a circle using a particular technique involving limits. If a circle is approximated by a regular hexagon, then the approximate area would be the sum of the areas of 6 triangles with base b and height h.

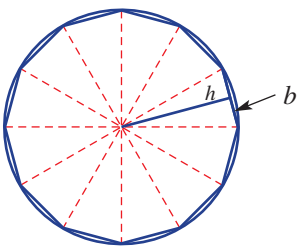


Hexagon ($n = 6$)

$A = 6 \times \frac{1}{2}bh$

So $A \approx 6 \times \frac{1}{2}bh$

If the number of sides (n) on the polygon increases, the approximation would improve. If n gets larger, the error in estimating the area of the circle gets smaller.



Dodecagon ($n = 12$)

$A = 12 \times \frac{1}{2}bh$

Proof

$$A = n \times \frac{1}{2}bh$$

$$= \frac{1}{2} \times nb \times h$$

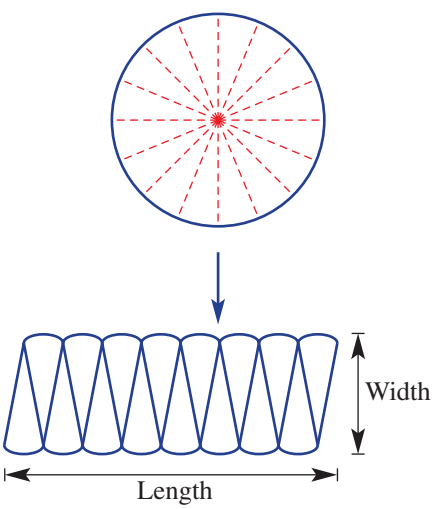
$$= \frac{1}{2} \times 2\pi r \times r \quad \text{(As } n \text{ becomes very large, } nb \text{ becomes } 2\pi r \text{ as } nb$$

$$= \pi r^2 \quad \text{is the perimeter of the polygon, and } h \text{ becomes } r.\text{)}$$

Let's start: Area as a rectangle

Imagine a circle cut into small sectors and arranged as shown.

Now try to imagine how the arrangement on the right would change if the number of sector divisions was not 16 (as shown) but a much higher number.



- What would the shape on the right look like if the number of sector divisions was a very high number? What would the length and width relate to in the original circle?
- Try to complete this proof.

A = length × width

$$= \frac{1}{2} \times _____ \times r$$

$$= _____$$

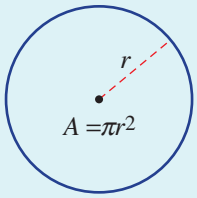
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Key ideas

- The ratio of the area of a circle to the square of its radius is equal to π.

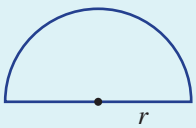
  $\frac{A}{r^2} = \pi$ so $A = \pi r^2$

  Rearranging $r = \sqrt{\frac{A}{\pi}}$



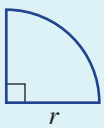
- A half circle is called a **semicircle**.

  $A = \frac{1}{2}\pi r^2$



- A quarter circle is called a **quadrant**.

  $A = \frac{1}{4}\pi r^2$



Example 10 Finding circle areas using a calculator

Use a calculator to find the area of this circle correct to two decimal places.



SOLUTION

$A = \pi r^2$

$= \pi \times 2^2$

$= 12.57 \text{ cm}^2$ (to 2 d.p.)

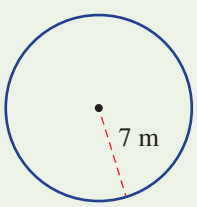
EXPLANATION

Use the π button on the calculator and enter $\pi \times 2^2$ or $\pi \times 4$.

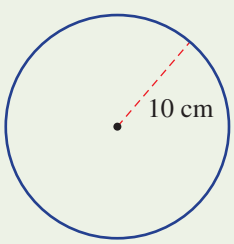
Example 11 Finding circle areas without technology

Find the area of these circles using the given approximate value of π.

a $\pi = \frac{22}{7}$



b $\pi = 3.14$



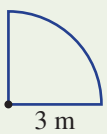
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SOLUTION	EXPLANATION
a $A = \pi r^2$ $= \frac{22}{7} \times 7^2$ $= 154 \text{ m}^2$	Always write the rule. Use $\pi = \frac{22}{7}$ and $r = 7$. $\frac{22}{7} \times 7 \times 7 = 22 \times 7$
b $A = \pi r^2$ $= 3.14 \times 10^2$ $= 314 \text{ cm}^2$	Use $\pi = 3.14$ and substitute $r = 10$. 3.14×10^2 is the same as 3.14×100

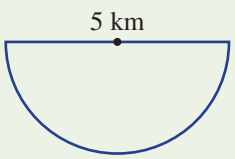
Example 12 Finding areas of semicircles and quadrants

Find the area of this quadrant and semicircle correct to two decimal places.

a



b



SOLUTION	EXPLANATION
a $A = \frac{1}{4} \times \pi r^2$ $= \frac{1}{4} \times \pi \times 3^2$ $= 7.07 \text{ m}^2$ (to 2 d.p.)	The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius.
b $r = \frac{5}{2} = 2.5$ $A = \frac{1}{2} \times \pi r^2$ $= \frac{1}{2} \times \pi \times 2.5^2$ $= 9.82 \text{ km}^2$ (to 2 d.p.)	The radius is half the diameter. The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius.

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Exercise 4E

1–3 | 3 | —

UNDERSTANDING

1 Evaluate without the use of a calculator.

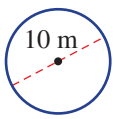
a 3.14×10 **b** 3.14×4 **c** $\frac{22}{7} \times 7$ **d** $\frac{22}{7} \times 7^2$

2 Use a calculator to evaluate these to two decimal places.

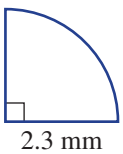
a $\pi \times 5^2$ **b** $\pi \times 13^2$ **c** $\pi \times 3.1^2$ **d** $\pi \times 9.8^2$

3 What radius length (r) would be used to help find the area of these shapes?

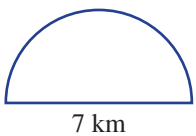
a



b



c



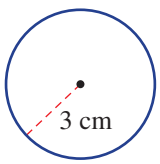
4–6(½) | 4–6(½), 7 | 4–6(½), 7

FLUENCY

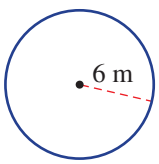
Example 10

4 Use a calculator to find the area of these circles correct to two decimal places.

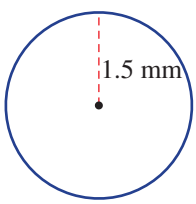
a



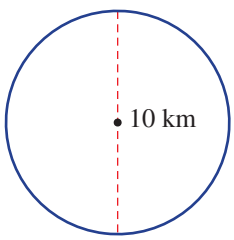
b



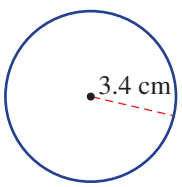
c



d



e



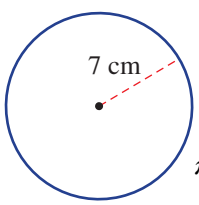
f



Example 11

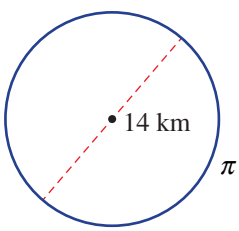
5 Find the area of these circles, using the given approximate value of π.

a



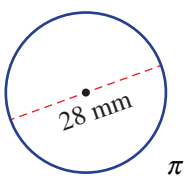
$\pi = \frac{22}{7}$

b



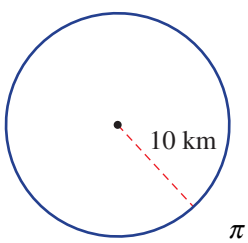
$\pi = \frac{22}{7}$

c



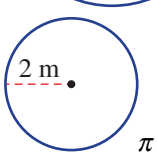
$\pi = \frac{22}{7}$

d



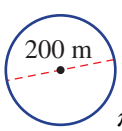
$\pi = 3.14$

e



$\pi = 3.14$

f



$\pi = 3.14$

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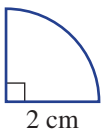
4E

Example 12

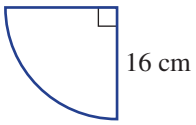
FLUENCY

6 Find the area of these quadrants and semicircles correct to two decimal places.

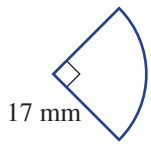
a



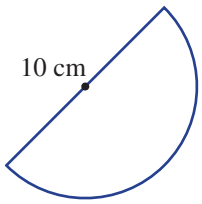
b



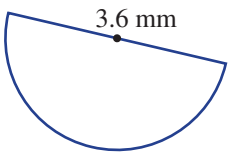
c



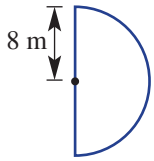
d



e



f



7 Using the formula $r = \sqrt{\frac{A}{\pi}}$ find the following correct to one decimal place.

a The radius of a circle with area 35 cm^2

b The diameter of a circle with area 7.9 m^2

8, 9 | 10–12 | 11, 13

PROBLEM-SOLVING

8 A pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2.

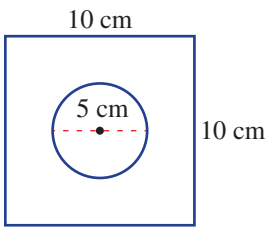
9 A tree trunk is cut to reveal a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 m^2 and, if so, by how much? Round your answer to the nearest whole number of cm^2.

10 A circular oil slick has a diameter of 1 km. The newspaper reported an area of more than 1 km^2. Is the newspaper correct?

11 Two circular plates have radii 12 cm and 13 cm. Find the difference in their area correct to two decimal places.

12 Which has the largest area, a circle of radius 5 m, a semicircle of radius 7 m or a quadrant of radius 9 m?

13 A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.



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4E

	14	14, 15	15, 16

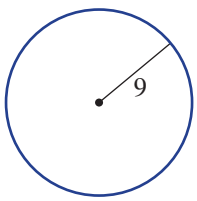
REASONING

14 A circle has radius 2 cm.

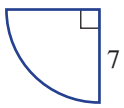
- **a** Find the area of the circle using $\pi = 3.14$.
- **b** Find the area if the radius is doubled to 4 cm.
- **c** What is the effect on the area if the radius is doubled?
- **d** What is the effect on the area if the radius is tripled?
- **e** What is the effect on the area if the radius is quadrupled?
- **f** What is the effect on the area if the radius is multiplied by n?

15 The area of a circle with radius 2 could be written exactly as $A = \pi \times 2^2 = 4\pi$. Write the exact area of these shapes.

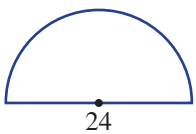
a



b



c



16 We know that the diameter d of a circle is twice the radius r, i.e. $d = 2r$ or $r = \frac{1}{2}d$.

- **a** Substitute $r = \frac{1}{2}d$ into the rule $A = \pi r^2$ to find a rule for the area of a circle in terms of d.
- **b** Use your rule from part **a** to check that the area of a circle with diameter 10 m is 25π m^2.

Reverse problems	—	—	17

ENRICHMENT

17 Reverse the rule $A = \pi r^2$ to find the radius in these problems.

- **a** If $A = 10$, use your calculator to show that $r \approx 1.78$.
- **b** Find the radius of circles with these areas. Round the answer to two decimal places.
  - **i** 17 m^2 **ii** 4.5 km^2 **iii** 320 mm^2
- **c** Can you write a rule for r in terms of A? Check that it works for the circles defined in part **b**.



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4F Sectors and composite shapes EXTENDING

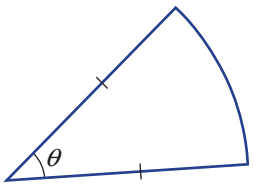








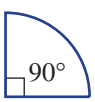
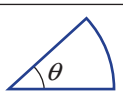
A slice of pizza or a portion of a round cake cut from the centre forms a shape called a sector. The area cleaned by a windscreen wiper could also be thought of as a difference of two sectors with the same angle but different radii. Clearly the area of a sector depends on its radius, but it also depends on the angle between the two straight edges.





Let's start: The sector area formula

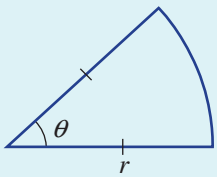
Complete this table to develop the rule for finding the area of a sector.

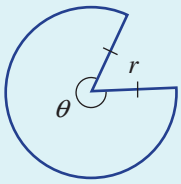
Angle	Fraction of area	Area rule	Diagram
180°	$\frac{180}{360} = \frac{1}{2}$	$A = \frac{1}{2} \times \pi r^2$	
90°	$\frac{90}{360} =$ _____	$A =$ _____ $\times \pi r^2$	
45°			
30°			
θ		$A =$ _____ $\times \pi r^2$	

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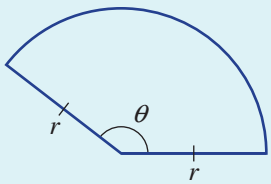
Key ideas

- A **sector** is formed by dividing a circle with two radii.

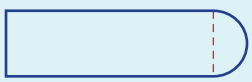




- A sector's area is determined by calculating a fraction of the area of a circle with the same radius.
  - Fraction is $\frac{\theta}{360}$
  - Sector area $= \frac{\theta}{360} \times \pi r^2$



- The area of a **composite shape** can be found by adding or subtracting the areas of more basic shapes.

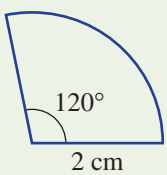


$A = lw + \frac{1}{2}\pi r^2$

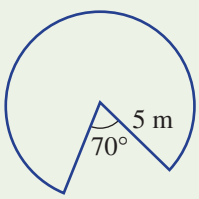
Example 13 Finding areas of sectors

Find the area of these sectors correct to two decimal places.

a



b

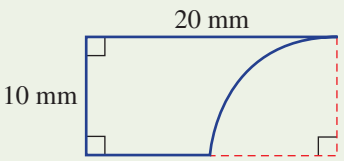


SOLUTION	EXPLANATION
a $A = \frac{\theta}{360} \times \pi r^2$	First write the rule for the area of a sector.
$= \frac{120}{360} \times \pi \times 2^2$	Substitute $\theta = 120$ and $r = 2$. Note that $\frac{120}{360}$ simplifies to $\frac{1}{3}$.
$= \frac{1}{3} \times \pi \times 4$	
$= 4.19 \text{ cm}^2$ (to 2 d.p.)	
b $\theta = 360 - 70 = 290$	First calculate the angle inside the sector and remember that a revolution is 360°. Then substitute $\theta = 290$ and $r = 5$.
$A = \frac{\theta}{360} \times \pi r^2$	
$= \frac{290}{360} \times \pi \times 5^2$	
$= 63.27 \text{ m}^2$ (to 2 d.p.)	

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Example 14 Finding areas of composite shapes

Find the area of this composite shape correct to the nearest whole number of mm^2.



SOLUTION

$$A = lw - \frac{1}{4}\pi r^2$$
$$= 20 \times 10 - \frac{1}{4} \times \pi \times 10^2$$
$$= 200 - 25\pi$$
$$= 121 \text{ mm}^2 \text{ (to nearest whole number)}$$

EXPLANATION

The area can be found by subtracting the area of a quadrant from the area of a rectangle.

Exercise 4F

1–3 | 3 | —

UNDERSTANDING

1 Simplify these fractions.

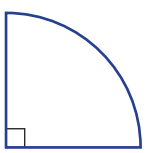
a $\frac{180}{360}$ **b** $\frac{90}{360}$ **c** $\frac{60}{360}$ **d** $\frac{45}{360}$

2 Evaluate the following using a calculator. Give your answer correct to two decimal places.

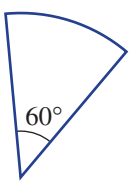
a $\frac{80}{360} \times \pi \times 2^2$ **b** $\frac{20}{360} \times \pi \times 7^2$ **c** $\frac{210}{360} \times \pi \times 2.3^2$

3 What fraction of a circle in simplest form is shown by these sectors?

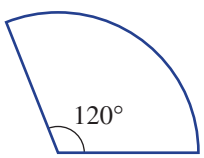
a



b



c



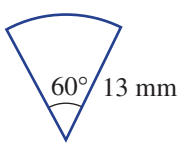
4(½), 6(½) | 4(½), 5, 6(½) | 4(½), 5, 6(½)

FLUENCY

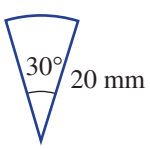
Example 13a

4 Find the area of these sectors correct to two decimal places.

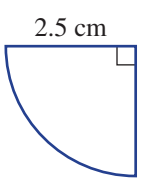
a



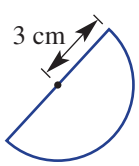
b



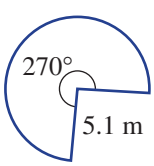
c



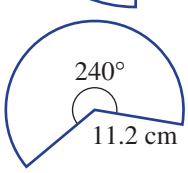
d



e



f

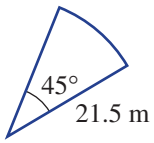


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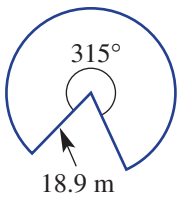
4F

FLUENCY

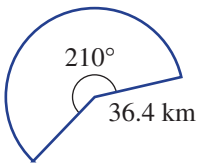
g



h



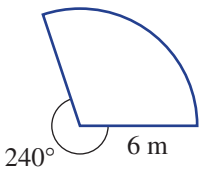
i



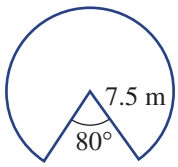
Example 13b

5 Find the area of these sectors correct to two decimal places.

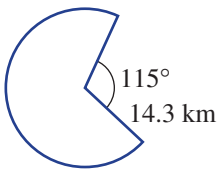
a



b



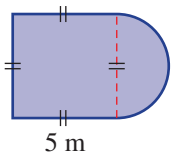
c



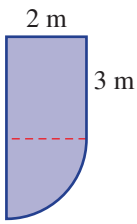
Example 14

6 Find the areas of these composite shapes using addition or subtraction. Round the answer to two decimal places.

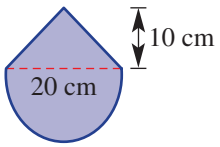
a



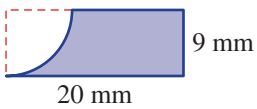
b



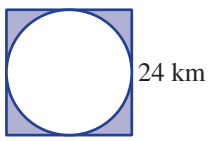
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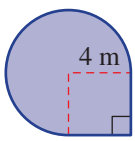
d



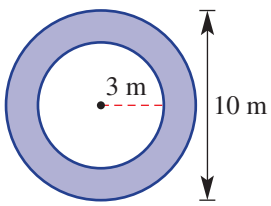
e



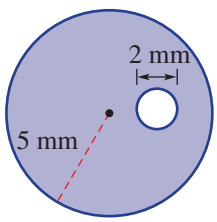
f



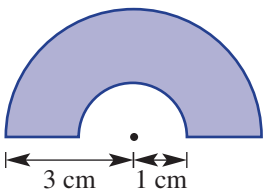
g



h



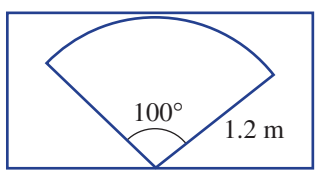
i



PROBLEM-SOLVING

7, 8 | 8, 9 | 8–10

7 A simple bus wiper blade wipes an area over 100° as shown. Find the area wiped by the blade correct to two decimal places.



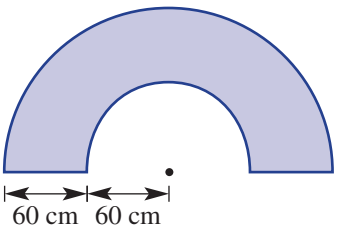
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PROBLEM-SOLVING

8 At Buy-by-the-sector Pizza they offer a sector of a 15 cm radius pizza with an angle of 45° or a sector of a 13 cm radius pizza with an angle of 60°. Which piece gives the bigger area and by how much? Round the answer to two decimal places.

9 An archway is made up of an inside and outside semicircle as shown. Find the area of the arch correct to the nearest whole cm^2.

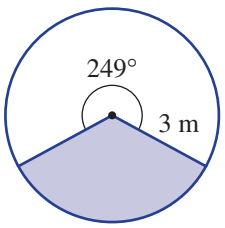


10 What percentage of the total area is occupied by the shaded region in these diagrams? Round the answer to one decimal place.

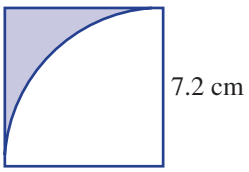
a



b



c

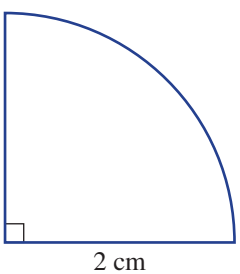


11(½) 11(½) 11(½), 12

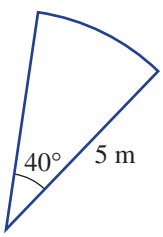
REASONING

11 An exact area measure in terms of π might look like $\pi \times 2^2 = 4\pi$. Find the exact area of these shapes in terms of π. Simplify your answer.

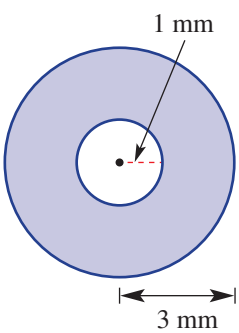
a



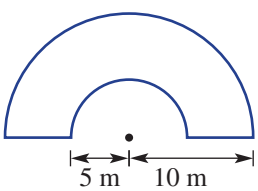
b



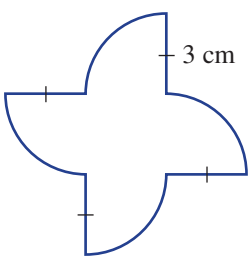
c Find the shaded area



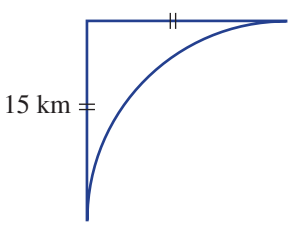
d



e



f

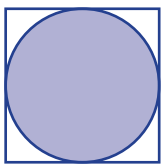


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REASONING

12 Consider the percentage of the area occupied by a circle inside a square and touching all sides as shown.



a If the radius of the circle is 4 cm, find the percentage of area occupied by the circle. Round the answer to one decimal place.

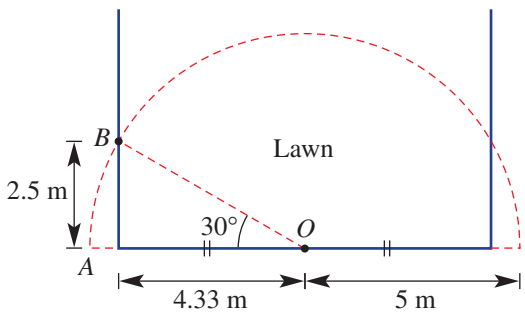
b Repeat part **a** for a radius of 10 cm. What do you notice?

c Can you prove that the percentage area is always the same for any radius r? Hint: Find the percentage area using the pronumeral r for the radius.

Sprinkler waste — — 13

ENRICHMENT

13 A rectangular lawn area has a 180° sprinkler positioned in the middle of one side as shown.



a Find the area of the sector OAB correct to two decimal places.

b Find the area watered by the sprinkler outside the lawn area correct to two decimal places.

c Find the percentage of water wasted, giving the answer correct to one decimal place.



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4G Surface area of a prism EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

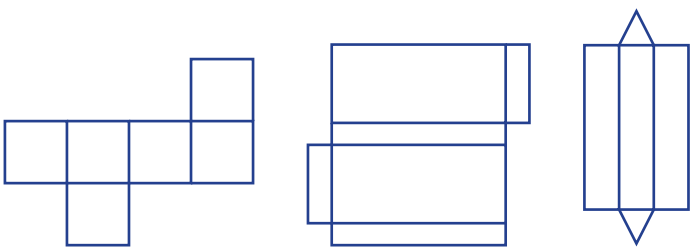
Many problems in three dimensions can be solved by looking at the problem or parts of the problem in two dimensions. Finding the surface area of a solid is a good example of this, as each face can usually be redrawn in two-dimensional space. The surface area of the walls of an unpainted house, for example, could be calculated by looking at each wall separately and adding to get a total surface area.



Let's start: Possible prisms

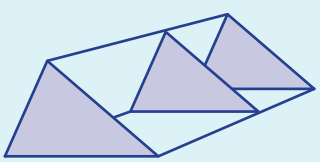
Here are three nets that fold to form three different prisms.

- Can you draw and name the prisms?
- Try drawing other nets of these prisms that are a different shape to the nets given here.

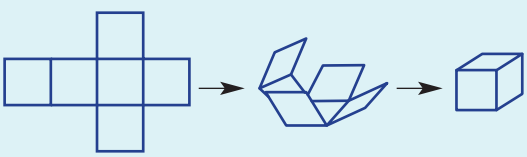


Key ideas

- A **prism** is a polyhedron with a constant (uniform) **cross-section**.
  - The cross-section is parallel to the two identical (congruent) ends.
  - The other sides are parallelograms (or rectangles for right prisms).



- A **net** is a two-dimensional representation of all the surfaces of a solid. It can be folded to form the solid.

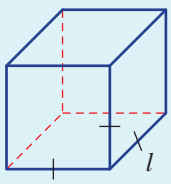


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Key ideas

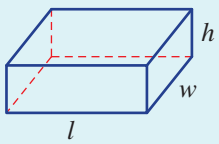
- The **total surface area** (TSA) of a prism is the sum of the areas of all its faces.

Cube



$TSA = 6l^2$

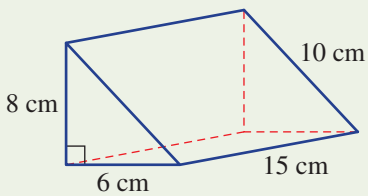
Rectangular prism



$TSA = 2lw + 2lh + 2wh$

Example 15 Finding surface areas

Find the surface area of this prism.



SOLUTION

Area of 2 triangular ends

$$A = 2 \times \frac{1}{2} \times bh$$
$$= 2 \times \frac{1}{2} \times 6 \times 8$$
$$= 48 \text{ cm}^2$$

Area of 3 rectangles

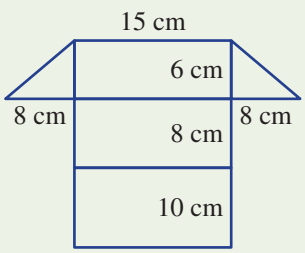
$$A = (6 \times 15) + (8 \times 15) + (10 \times 15)$$
$$= 360 \text{ cm}^2$$

Surface area

$$TSA = 48 + 360$$
$$= 408 \text{ cm}^2$$

EXPLANATION

One possible net is



Work out the area of each shape or group of shapes and find the sum of their areas to obtain the total surface area.



The surface area of one wedge piece of this chocolate can be estimated by a similar process to that shown in Example 15.

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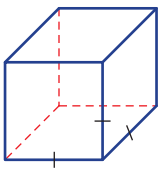
Exercise 4G

1–3 | 3 | —

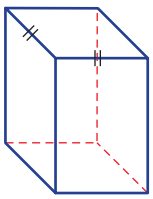
UNDERSTANDING

1 How many faces are there on these right prisms? Also name the types of shapes that make the different faces.

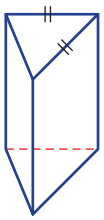
a



b

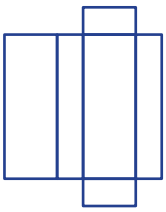


c

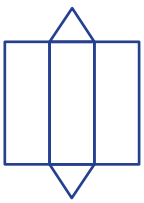


2 Match the net to its solid.

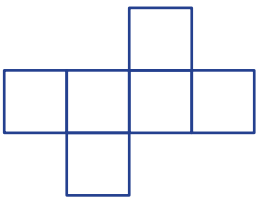
a



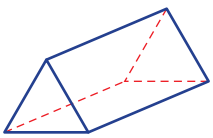
b



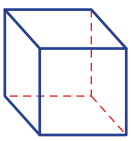
c



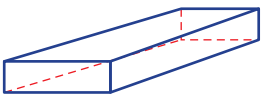
A



B



C



3 How many rectangular faces are on these solids?

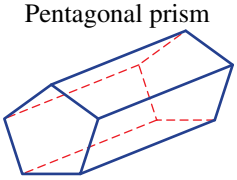
a Triangular prism

b Rectangular prism

c Hexagonal prism

d Pentagonal prism

Pentagonal prism



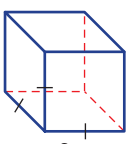
4(½) | 4(½), 5 | 4(½), 5

FLUENCY

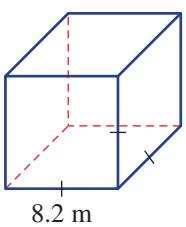
Example 15

4 Find the surface area of these right prisms.

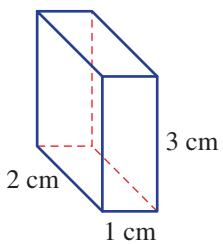
a



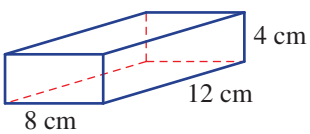
b



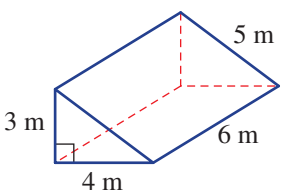
c



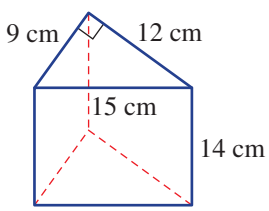
d



e



f

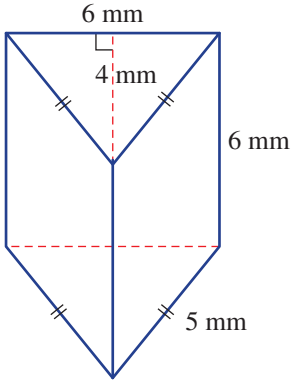


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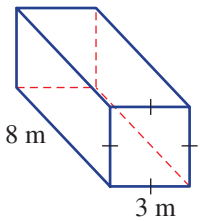
4G

FLUENCY

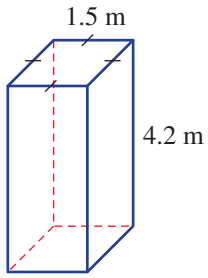
g



h



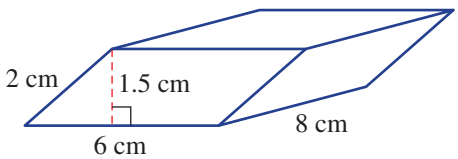
i



5 This prism has two end faces that are parallelograms.

a Use $A = bh$ to find the combined area of the two ends.

b Find the surface area of the prism.



PROBLEM-SOLVING

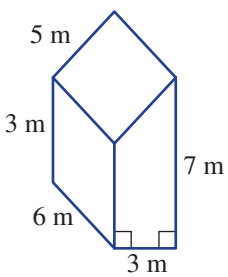
6, 7 6–8 7–9

6 An open box (with no lid) is in the shape of a cube and is painted on the outside including the base. What surface area is painted if the side length of the box is 20 cm?

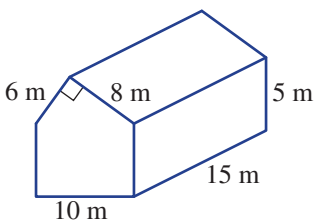
7 A book 20 cm long, 15 cm wide and 3 cm thick is wrapped in plastic. What area of plastic is needed to wrap 1000 books? Convert your answer to m^2.

8 Find the surface area of these solids.

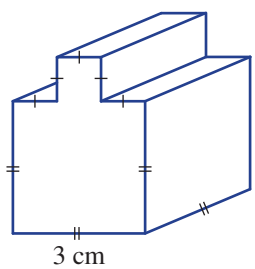
a



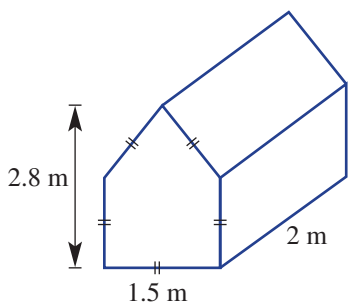
b



c



9 The floor, sides and roof of this tent are made from canvas at a cost of $5 per square metre. The tent's dimensions are shown in the diagram. What is the cost of the canvas for the tent?



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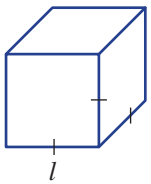
4G

10 | 10, 11 | 10, 11

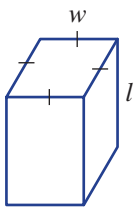
REASONING

10 Write down a rule for the surface area for these right prisms in simplest form.

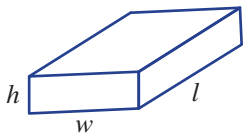
a



b



c



11 A cube of side length 1 cm has a surface area of 6 cm^2.

a What is the effect on the surface area of the cube if:

i its side length is doubled?

ii its side length is tripled?

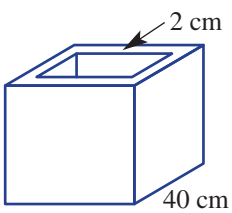
iii its side length is quadrupled?

b Do you notice a pattern from your answers to part **a**? What effect would multiplying the side length by a factor of n have on the surface area?

ENRICHMENT

The thick wooden box | — | — | 12

12 An open box (with no lid) in the shape of a cube is made of wood that is 2 cm thick. Its outside side length is 40 cm.



a Find its total surface area both inside and out.

b If the box was made with wood that is 1 cm thick, what would be the change in surface area?



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4H Volume and capacity

Interactive

Widgets

HOTsheets

Walkthroughs

Volume is a measure of the space occupied by a three-dimensional object. It is measured in cubic units. Common metric units for volume given in abbreviated form include mm^3, cm^3, m^3 and km^3. We also use mL, L, kL and ML to describe volumes of fluids or gas. The volume of space occupied by a room in a house for example might be calculated in cubic metres (m^3) or the capacity of fuel tanker might be measured in litres (L) or kilolitres (kL).



A fuel tanker can carry about 32 000 litres.

Let's start: Packing a shipping container

There are 250 crates of apples to be shipped from Australia to Japan. Each crate is 1 m long, 1 m wide and 1 m high. The shipping container used to hold the crates is 12 m long, 4 m wide and 5 m high.

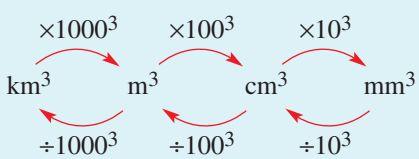


The fruit picker says that the 250 crates will 'fit in, no problems'. The forklift driver says that the 250 crates will 'just squeeze in'. The truck driver says that 'you will need more than one shipping container'.

- Explain how the crates might be packed into the container. How many will fit into one end?
- Who (the fruit picker, forklift driver or truck driver) is the most accurate? Explain your choice.
- What size shipping container and what dimensions would be required to take all 250 crates with no space left over? Is this possible or practical?

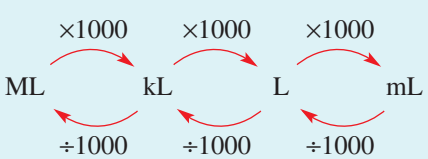
Key ideas

- **Volume** is measured in cubic units. Common metric units are:
  - cubic millimetres (mm^3)
  - cubic centimetres (cm^3)
  - cubic metres (m^3)
  - cubic kilometres (km^3)



- **Capacity** is the volume of fluid or gas that a container can hold.

  Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

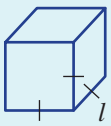


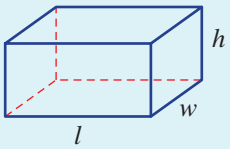
- Some common conversions are:
  - 1 mL = 1 cm^3
  - 1 L = 1000 mL
  - 1 kL = 1000 L = 1 m^3

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Key ideas

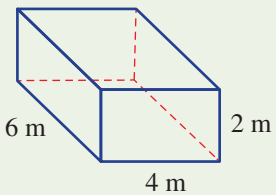
- Volume of a rectangular prism
  - Volume = length × width × height
    $V = lwh$
- Volume of a cube
  - $V = l^3$





Example 16 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION	EXPLANATION
$V = lwh$ $= 6 \times 4 \times 2$ $= 48 \text{ m}^3$	First write the rule and then substitute for the length, width and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc.

Example 17 Finding capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

SOLUTION	EXPLANATION
$V = lwh$ $= 20 \times 10 \times 15$ $= 3000 \text{ cm}^3$ $= 3000 \div 1000$ $= 3 \text{ L}$	First calculate the volume of the container in cm^3. Then convert to litres using $1 \text{ L} = 1000 \text{ cm}^3$.

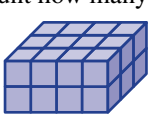
Exercise 4H

1–3 | 3 | —

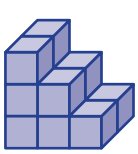
UNDERSTANDING

1 Count how many cubic units are shown in these cube stacks.

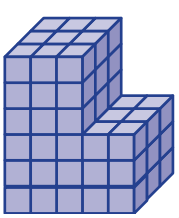
a



b



c



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4H

UNDERSTANDING

2 Find the missing number.

a $3 \times 4 \times 8 =$ _____ **b** _____ $\times 5 \times 20 = 600$

c $8 \times$ _____ $\times 12 = 192$ **d** $20 \times 2 \times$ _____ $= 200$

3 Write the missing number in the following unit conversions.

a 1 L = _____ mL **b** _____ kL = 1000 L **c** 1000 kL = _____ ML

d 1 mL = _____ cm^3 **e** 1000 cm^3 = _____ L **f** 1 m^3 = _____ L

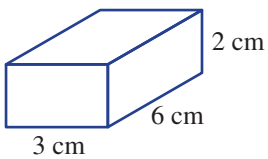
FLUENCY

4–6(½) 4–6(½) 4–6(½)

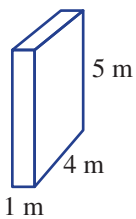
Example 16

4 Find the volume of these rectangular prisms.

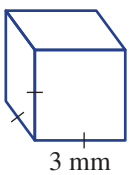
a



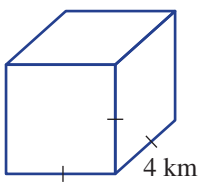
b



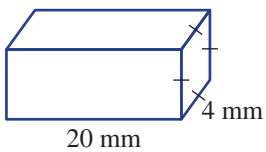
c



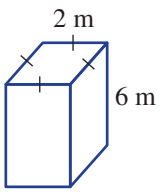
d



e



f



5 Convert the measurements to the units shown in the brackets.

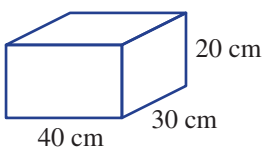
a 2 L (mL) **b** 5 kL (L) **c** 0.5 ML (kL) **d** 3000 mL (L)

e 4 mL (cm^3) **f** 50 cm^3 (mL) **g** 2500 cm^3 (L) **h** 5.1 L (cm^3)

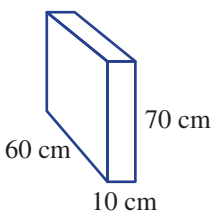
Example 17

6 Find the capacity of these containers, converting your answer to litres.

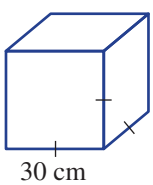
a



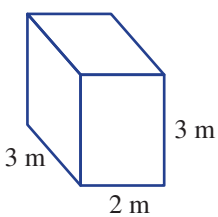
b



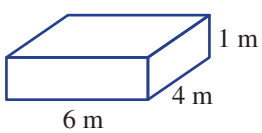
c



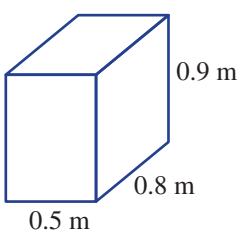
d



e



f



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4H

PROBLEM-SOLVING

7–9 | 8–10 | 9–11

7 A oil tanker has a capacity of 60 000 m^3.

a What is the ship's capacity in:

i litres?

ii kilolitres?

iii megalitres?

b If the tanker leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out?
Assume the ship started with full capacity.

8 Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

9 A city skyscraper is a rectangular prism 50 m long, 40 m wide and 250 m high.

a What is the total volume in m^3?

b What is the total volume in ML?

10 If 1 kg is the mass of 1 L of water, what is the mass of water in a full container that is a cube with side length 2 m?

11 Using whole numbers only, give all the possible dimensions of rectangular prisms with the following volume. Assume the units are all the same.

a 12 cubic units **b** 30 cubic units **c** 47 cubic units

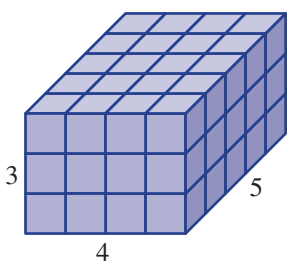
REASONING

12 | 12, 13 | 13, 14

12 Explain why a rectangular prism of volume 46 cm^3 cannot have all its side lengths (length, width and height) as whole numbers greater than 1. Assume all lengths are in centimetres.

13 How many cubic containers, with side lengths that are a whole number of centimetres, have a capacity of less than 1 litre?

14 Consider this rectangular prism.



a How many cubes are in the base layer?

b What is the area of the base?

c What do you notice about the two answers from above? How can this be explained?

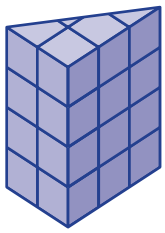
d If A represents the area of the base, explain why the rule $V = Ah$ can be used to find the volume of a rectangular prism.

e Could any side of a rectangular prism be considered to be the base when using the rule $V = Ah$? Explain.

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15 This question looks at using half of a rectangular prism to find the volume of a triangular prism.

a Consider this triangular prism.

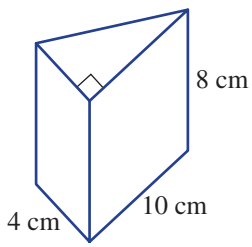


i Explain why this solid could be thought of as half a rectangular prism.

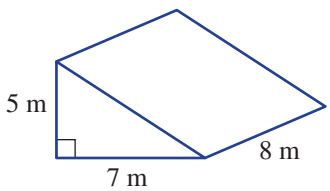
ii Find its volume.

b Using a similar idea, find the volume of these prisms.

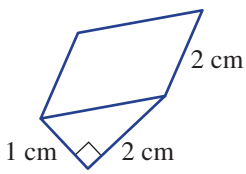
i



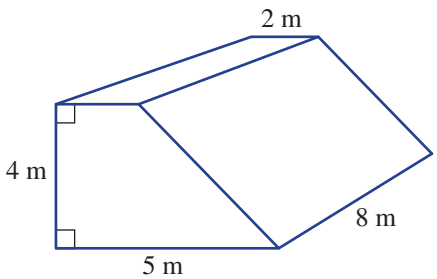
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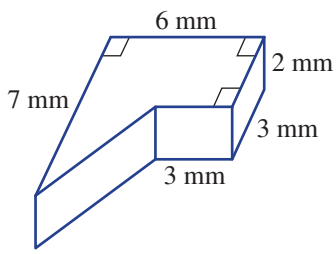
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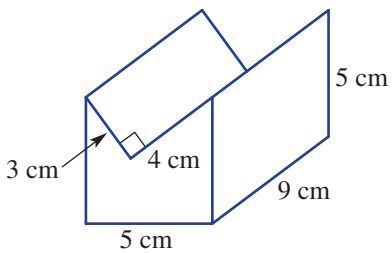
iv



v



vi





Models of architectural designs usually include different types of prisms.

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4I Volume of prisms and cylinders

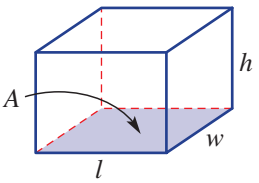
Interactive

Widgets

HOTsheets

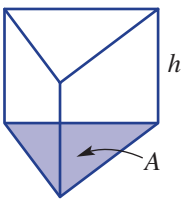
Walkthroughs

We know that for a rectangular prism its volume V is given by the rule $V = lwh$. Length × width (lw) gives the number of cubes on the base, but it also tells us the area of the base A. So $V = lwh$ could also be written as $V = Ah$.

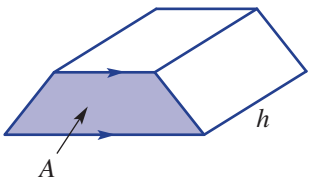


The rule $V = Ah$ can also be applied to prisms that have different shapes as their bases. One condition, however, is that the area of the base must represent the area of the cross-section of the solid. The height h is measured perpendicular to the cross-section. Note that a cylinder is *not* a prism as it does not have sides that are parallelograms; however, it can be treated like a prism when finding its volume because it has a constant cross-section, a circle.

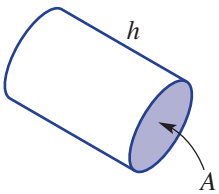
Here are some examples of two prisms and a cylinder with A and h marked.



Cross-section is a triangle



Cross-section is a trapezium



Cross-section is a circle

Let's start: Drawing prisms

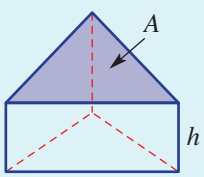
Try to draw prisms (or cylinders) that have the following shapes as their cross-sections.

- Circle
- Trapezium
- Parallelogram
- Triangle
- Pentagon

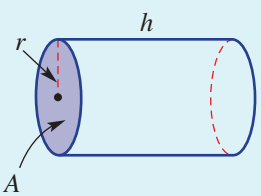
The cross-section of a prism should be the same size and shape along the entire length of the prism. Check this property on your drawings.

Key ideas

- A **prism** is a polyhedron with a constant (uniform) cross-section.
  - The sides joining the two congruent ends are parallelograms.
  - A right prism has rectangular sides joining the congruent ends.



- Volume of a prism = Area of cross-section × perpendicular height or $V = Ah$.
- Volume of a **cylinder** = $Ah = \pi r^2 \times h = \pi r^2 h$
  So $V = \pi r^2 h$

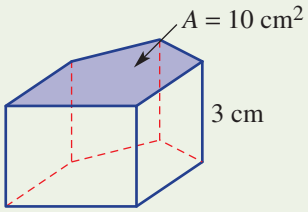


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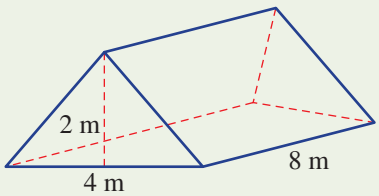
Example 18 Finding the volumes of prisms

Find the volumes of these prisms.

a



b



SOLUTION

a $V = Ah$
$= 10 \times 3$
$= 30 \text{ cm}^3$

b $V = Ah$
$= \left(\frac{1}{2} \times 4 \times 2\right) \times 8$
$= 32 \text{ m}^3$

EXPLANATION

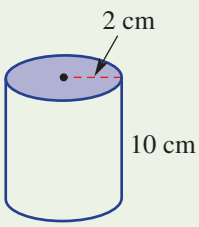
Write the rule and substitute the given values of A and h, where A is the area of the cross-section.

The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m.

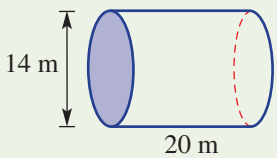
Example 19 Finding the volume of a cylinder

Find the volumes of these cylinders, rounding to two decimal places.

a



b



SOLUTION

a $V = \pi r^2 h$
$= \pi \times 2^2 \times 10$
$= 125.66 \text{ cm}^3$ (to 2 d.p.)

b $V = \pi r^2 h$
$= \pi \times 7^2 \times 20$
$= 3078.76 \text{ m}^3$ (to 2 d.p.)

EXPLANATION

Write the rule and then substitute the given values for π, r and h.
Round as required.

The diameter is 14 m so the radius is 7 m.
Round as required.

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Exercise 4I

1–3 | 2 | —

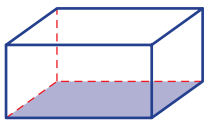
UNDERSTANDING

1 For these solids:

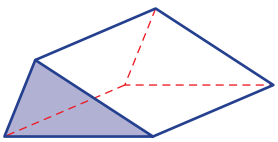
i state whether or not it looks like a prism

ii if it is a prism, state the shape of its cross-section.

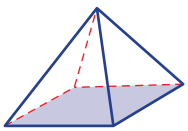
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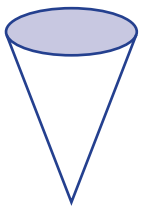
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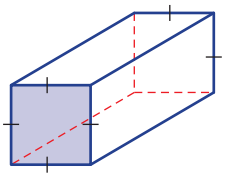
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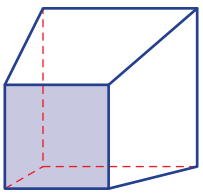
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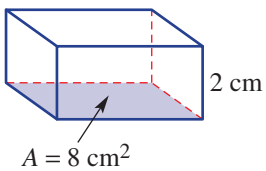


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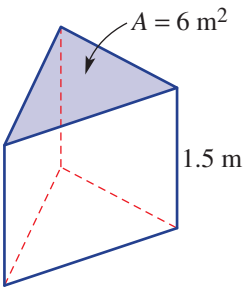


2 For these prisms and cylinder, state the value of A and the value of h that could be used in the rule $V = Ah$ to find the volume of the solid.

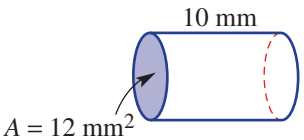
a



b



c



3 Evaluate the following.

a $2 \times 5 \times 4$ **b** $\frac{1}{2} \times 3 \times 4 \times 6$ **c** $\frac{1}{2}(2+3) \times 4$ **d** $3.14 \times 10^2 \times 20$

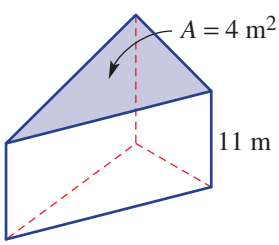
4–6(½) | 4–6(½) | 4–6(½)

FLUENCY

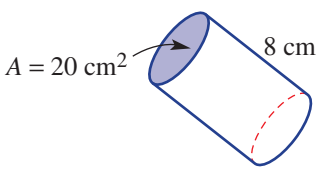
Example 18a

4 Find the volume of these solids using $V = Ah$.

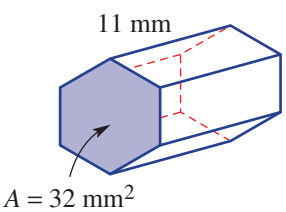
a



b



c



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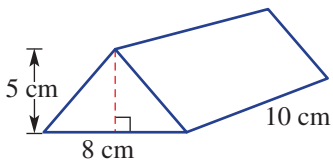
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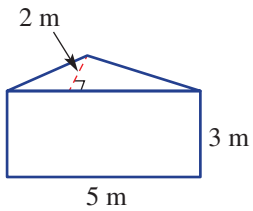
Example 18b

5 Find the volume of these prisms.

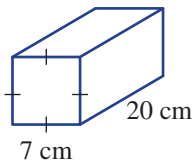
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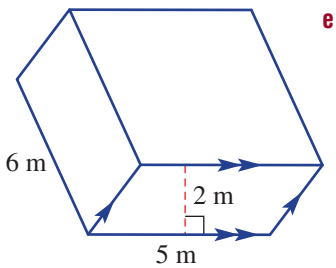
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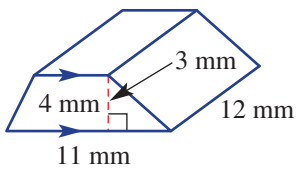
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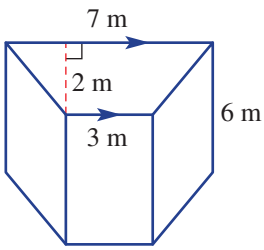
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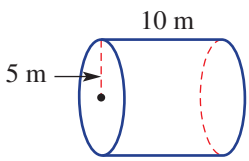
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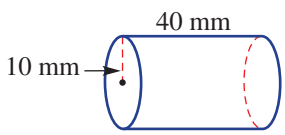
Example 19

6 Find the volume of these cylinders. Round the answer to two decimal places.

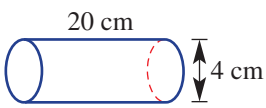
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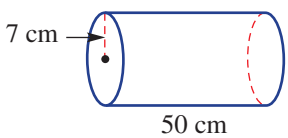
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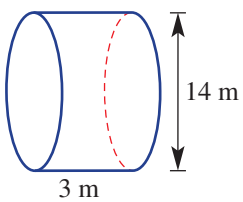
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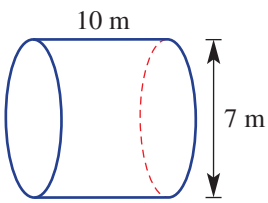
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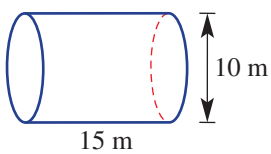
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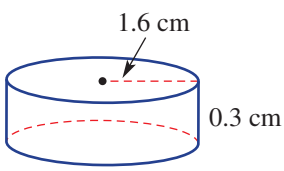
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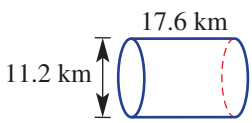
g



h



i



7, 8 8–10 9–11

PROBLEM-SOLVING

7 A cylindrical tank has a diameter of 3 m and height 2 m.

a Find its volume in m^3 correct to three decimal places.

b What is the capacity of the tank in litres?

8 Jack looks at buying either a rectangular water tank with dimensions 3 m by 1 m by 2 m or a cylindrical tank with radius 1 m and height 2 m.

a Which tank has the greater volume?

b What is the difference in the volume correct to the nearest litre?

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4I

PROBLEM-SOLVING

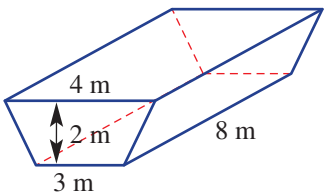
9 Susan pours water from a full 4 L container into a number of water bottles for a camp hike. Each water bottle is a cylinder with radius 4 cm and height 20 cm. How many bottles can be filled completely?

10 There are 80 liquorice cubes stacked in a cylindrical glass jar. The liquorice cubes have a side length of 2 cm and the glass jar has a radius of 5 cm and a height of 12 cm. How much air space remains in the jar of liquorice cubes? Give the answer correct to two decimal places.

11 A swimming pool is a prism with a cross-section that is a trapezium as shown. The pool is being filled at a rate of 1000 litres per hour.

a Find the capacity of the pool in litres.

b How long will it take to fill the pool?



REASONING

12 | 12, 13 | 13, 14

12 Using exact values (e.g. 10π cm^3) calculate the volume of cylinders with these dimensions.

a Radius 2 m and height 5 m

b Radius 10 cm and height 3 cm

c Diameter 8 mm and height 9 mm

d Diameter 7 m and height 20 m

13 A cylinder has a volume of 100 cm^3. Give three different combinations of radius and height measurements that give this volume. Give these lengths correct to two decimal places.

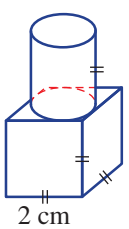
14 A cube has side length x metres and a cylinder has a radius also of x metres and height h. What is the rule linking x and h if the cube and the cylinder have the same volume?

ENRICHMENT

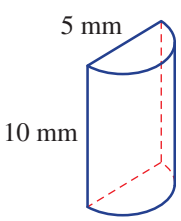
Complex composites — | — | 15

15 Use your knowledge of volumes of prisms and cylinders to find the volume of these composite solids. Round the answer to two decimal places where necessary.

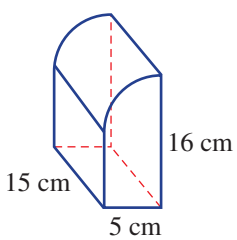
a



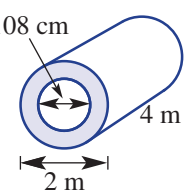
b



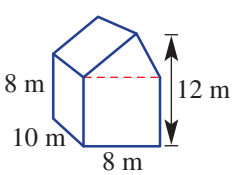
c



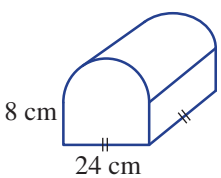
d



e



f



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Progress quiz

4A **1** Convert:

a 6.4 m into mm

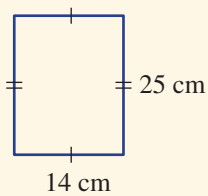
b 180 cm into m

c 97000 cm into km

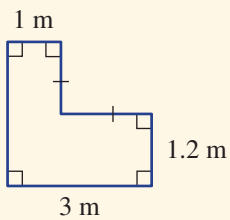
d $2\frac{1}{2}$ m into cm

4A **2** Find the perimeter of these shapes.

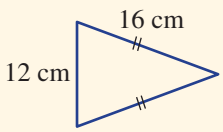
a



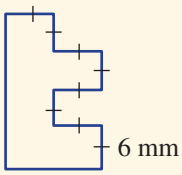
b



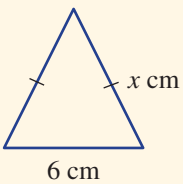
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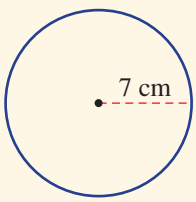


4A **3** If the perimeter of a triangle is 24 cm, find the value of x.

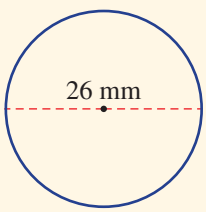


4B/E **4** Find the circumference and area of these circles, correct to two decimal places.

a



b



4C **5** Convert these area measurements to the units shown in brackets.

a 4.7 m^2 (cm^2)

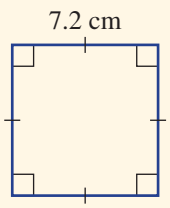
b 4100 mm^2 (cm^2)

c 5000 m^2 (ha)

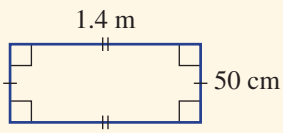
d 0.008 km^2 (m^2)

4C/D **6** Find the area of these shapes.

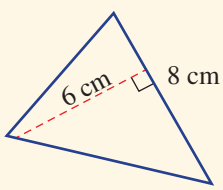
a



b

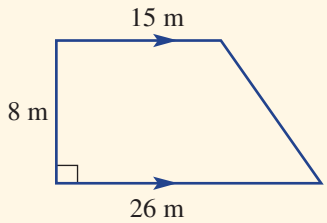


c

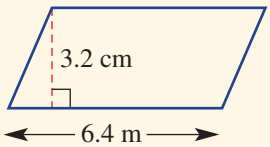


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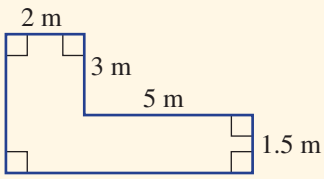
d



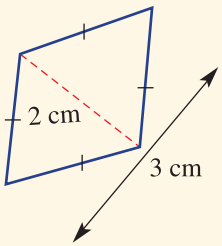
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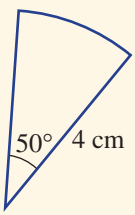
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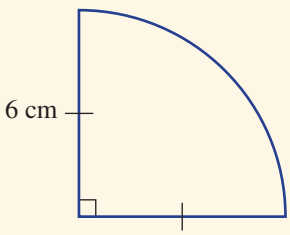
4F Ext

7 Find the area and perimeter of these sectors. Round to two decimal places.

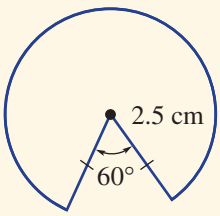
a



b



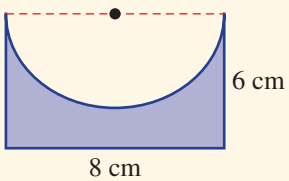
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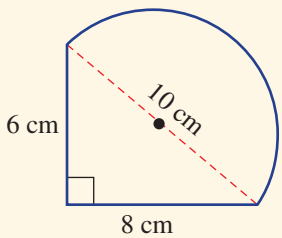
4F Ext

8 Find the area of these composite shapes correct to two decimal places.

a



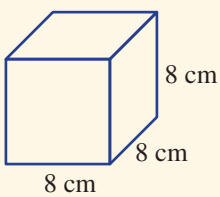
b



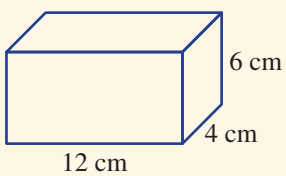
4G/H/I Ext

9 Find the surface area and volume of these prisms.

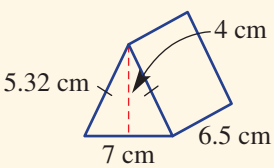
a



b



c

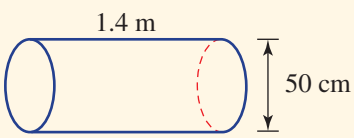


4I

10 For this cylinder, find:

a the volume in m^3 correct to four decimal places

b the capacity in litres to one decimal place.



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4J Time

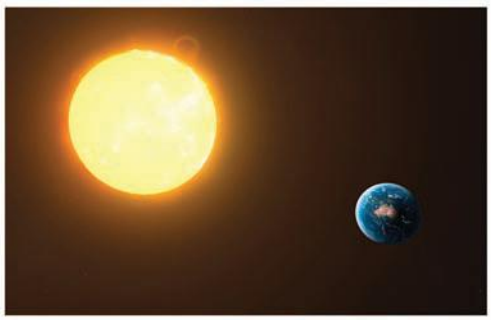
Interactive

Widgets

HOTsheets

Walkthroughs

Time in minutes and seconds is based on the number 60. Other units of time, including the day and year, are defined by the rate at which the Earth spins on its axis and the time that the Earth takes to orbit the Sun.



The Earth takes 1 year to orbit the Sun.

The origin of the units seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use a.m. (*ante meridiem*, which is Latin for 'before noon') and p.m. (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday). During the rule of Julius Caesar, the ancient Romans introduced the solar calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (in February) every 4 years.

Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- When is the next leap year?
- Why do we have a leap year?
- Which months have 31 days?
- Why are there different times in different countries or parts of a country?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

- The standard unit of time is the **second** (s).
- Units of time include:
  - 1 **minute** (min) = 60 seconds (s)
  - 1 **hour** (h) = 60 minutes (min)
  - 1 **day** = 24 hours (h)
  - 1 **week** = 7 days
  - 1 **year** = 12 months

day $\underset{\div 24}{\overset{\times 24}{\rightleftarrows}}$ hour $\underset{\div 60}{\overset{\times 60}{\rightleftarrows}}$ minute $\underset{\div 60}{\overset{\times 60}{\rightleftarrows}}$ second

- Units of time smaller than a second.
  - millisecond = 0.001 second (1000 milliseconds = 1 second)
  - microsecond = 0.000001 second (1 000 000 microseconds = 1 second)
  - nanosecond = 0.000000001 second (1 000 000 000 nanoseconds = 1 second)
- a.m. or p.m. is used to describe the 12 hours before and after noon (midday).
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 a.m.
  - 1530 is 3:30 p.m.

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Key ideas

- The Earth is divided into 24 time zones (one for each hour).
  - Twenty-four 15° lines of longitude divide the Earth into its time zones. Time zones also depend on a country's borders and its proximity to other countries. (See map on pages 231–232 for details.)
  - Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.

Example 20 Converting units of time

Convert these times to the units shown in brackets.

a 3 days (minutes)

b 30 months (years)

SOLUTION

a $3 \text{ days} = 3 \times 24 \text{ h}$
$= 3 \times 24 \times 60 \text{ min}$
$= 4320 \text{ min}$

b $30 \text{ months} = 30 \div 12 \text{ years}$
$= 2\frac{1}{2} \text{ years}$

EXPLANATION

1 day = 24 hours
1 hour = 60 minutes

There are 12 months in 1 year.

Example 21 Using 24-hour time

Write these times using the system given in brackets.

a 4:30 p.m. (24-hour time)

b 1945 hours (a.m./p.m.)

SOLUTION

a 4:30 p.m. = 1200 + 0430
= 1630 hours

b 1945 hours = 7:45 p.m.

EXPLANATION

Since the time is p.m., add 12 hours to 0430 hours.

Since the time is after 1200 hours, subtract 12 hours.

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Example 22 Using time zones

Coordinated Universal Time (UTC) and is based on the time in Greenwich, United Kingdom. Use the world time zone map (on pages 231–232) to answer the following.

a When it is 2 p.m. UTC, find the time in these places.

i France **ii** China

iii Queensland **iv** Alaska

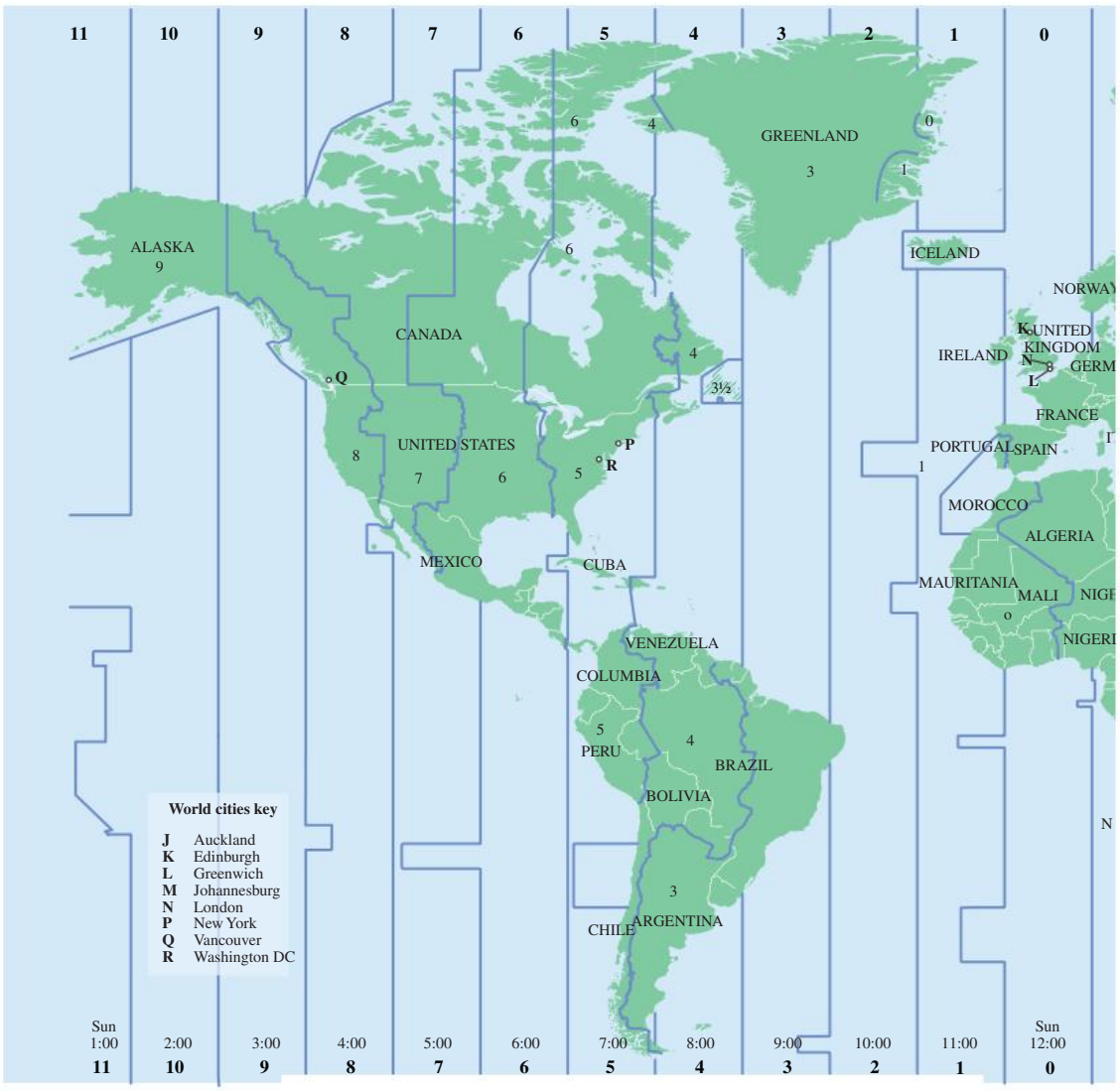
b When it is 9:35 a.m. in New South Wales, Australia, find the time in these places.

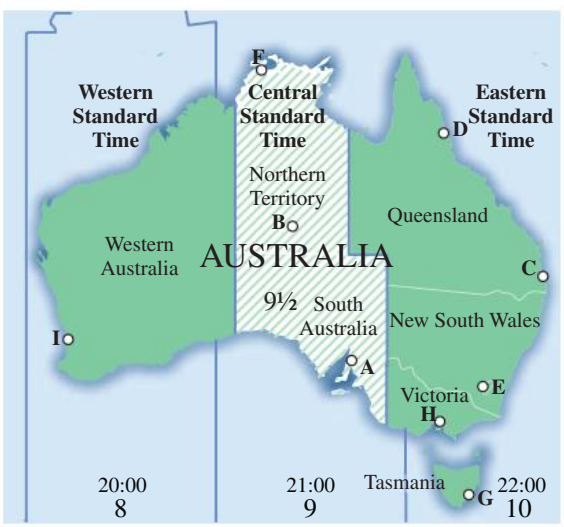
i Alice Springs **ii** Perth

iii London **iv** central Greenland

SOLUTION	EXPLANATION
a i 2 p.m. + 1 hour = 3 p.m.	Use the time zone map to see that France is to the east of Greenwich and is in a zone that is 1 hour ahead.
ii 2 p.m. + 8 hours = 10 p.m.	From the time zone map, China is 8 hours ahead of Greenwich.
iii 2 p.m. + 10 hours = 12 a.m.	Queensland uses Eastern Standard Time, which is 10 hours ahead of Greenwich.
iv 2 p.m. − 9 hours = 5 a.m.	Alaska is to the west of Greenwich, in a time zone that is 9 hours behind.
b i 9:35 a.m. $-\frac{1}{2}$ hour = 9:05 a.m.	Alice Springs uses Central Standard Time, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 9:35 a.m. − 2 hours = 7:35 a.m.	Perth uses Western Standard Time, which is 2 hours behind Eastern Standard Time.
iii 9:35 a.m. − 10 hours = 11:35 p.m. (the day before)	UTC (time in Greenwich, United Kingdom) is 10 hours behind EST.
iv 9:35 a.m. − 13 hours = 8:35 p.m. (the day before)	Central Greenland is 3 hours behind UTC in Greenwich, so is 13 hours behind EST.

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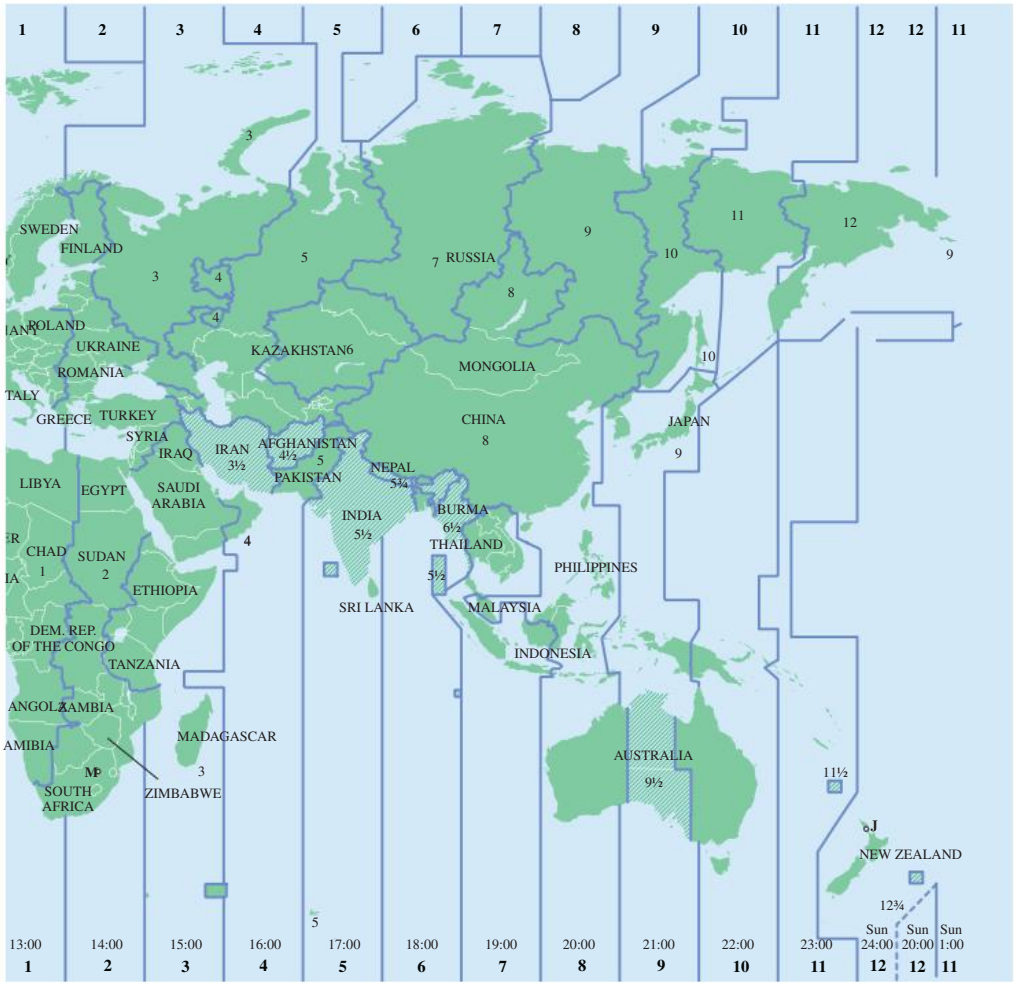


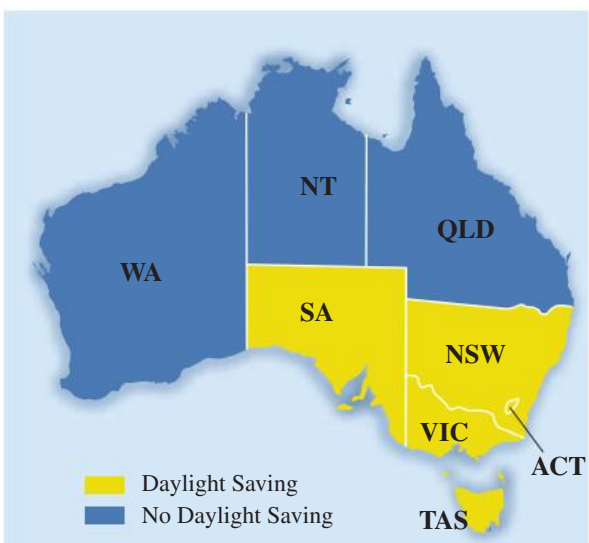


Australian cities key

A Adelaide
B Alice Springs
C Brisbane
D Cairns
E Canberra, ACT
F Darwin
G Hobart
H Melbourne
I Perth

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Exercise 4J

1–3 | 3 | —

UNDERSTANDING

1 From options **A** to **F**, match up the time units with the most appropriate description.

a single heartbeat
b 40 hours of work
c duration of a university lecture
d bank term deposit
e 200 m run
f flight from Australia to the UK

A 1 hour
B 1 minute
C 1 day
D 1 week
E 1 year
F 1 second

2 Find the number of:

a seconds in 2 minutes
b minutes in 180 seconds
c hours in 120 minutes
d minutes in 4 hours
e hours in 3 days
f days in 48 hours
g weeks in 35 days
h days in 40 weeks

3 What is the time difference between these times?

a 12 p.m. and 6:30 p.m. on the same day
b 12 a.m. and 10:45 a.m. on the same day
c 12 a.m. and 4:20 p.m. on the same day
d 11 a.m. and 3:30 p.m. on the same day

4–9(½) | 4–10(½) | 4–11(½)

FLUENCY

Example 20

4 Convert these times to the units shown in brackets.

a 3 h (min)
b 10.5 min (s)
c 240 s (min)
d 90 min (h)
e 6 days (h)
f 72 h (days)
g 1 week (h)
h 1 day (min)
i 14 400 s (h)
j 20 160 min (weeks)
k 2 weeks (min)
l 24 h (s)
m 5000 milliseconds (s)
n 2 500 000 microseconds (s)
o 7 000 000 000 nanoseconds (s)
p 0.4 s (milliseconds)
q 0.0000027 s (microseconds)
r 0.000000003 s (nanoseconds)

5 Write the time for these descriptions.

a 4 hours after 2:30 p.m.
b 10 hours before 7 p.m.
c $3\frac{1}{2}$ hours before 10 p.m.
d $7\frac{1}{2}$ hours after 9 a.m.
e $6\frac{1}{4}$ hours after 11:15 a.m.
f $1\frac{3}{4}$ hours before 1:25 p.m.

Example 21

6 Write these times using the system shown in brackets.

a 1:30 p.m. (24-hour)
b 8:15 p.m. (24-hour)
c 10:23 a.m. (24-hour)
d 11:59 p.m. (24-hour)
e 0630 hours (a.m./p.m.)
f 1300 hours (a.m./p.m.)
g 1429 hours (a.m./p.m.)
h 1938 hours (a.m./p.m.)
i 2351 hours (a.m./p.m.)

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4J

FLUENCY

7 Round these times to the nearest hour.

a 1:32 p.m. **b** 5:28 a.m. **c** 1219 hours **d** 1749 hours

8 What is the time difference between these time periods?

a 10:30 a.m. and 1.20 p.m. **b** 9:10 a.m. and 3:30 p.m. **c** 2:37 p.m. and 5:21 p.m.
d 10:42 p.m. and 7:32 a.m. **e** 1451 and 2310 hours **f** 1940 and 0629 hours

Example 22a

9 Use the time zone map on pages 231–232 to find the time in the following places, when it is 10 a.m. UTC.

a Spain **b** Turkey **c** Tasmania **d** Darwin
e Argentina **f** Peru **g** Alaska **h** Portugal

Example 22b

10 Use the time zone map on pages 231–232 to find the time in these places, when it is 3:30 p.m. in Victoria.

a United Kingdom **b** Libya **c** Sweden
d Perth **e** Japan **f** central Greenland
g Alice Springs **h** New Zealand

11 What is the time difference between these pairs of places?

a United Kingdom and Kazakhstan **b** South Australia and New Zealand
c Queensland and Egypt **d** Peru and Angola (in Africa)
e Mexico and Germany

PROBLEM-SOLVING

12–14 14–17 17–21

12 A scientist argues that dinosaurs died out 52 million years ago, whereas another says they died out 108 million years ago. What is the difference in their time estimates?

13 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

14 Adrian arrives at school at 8:09 a.m. and leaves at 3:37 p.m. How many hours and minutes is Adrian at school?

15 On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

16 A phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

a What is the cost of a 70-second call?
b What is the cost of a call that lasts 6 minutes and 20 seconds?

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PROBLEM-SOLVING

17 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day
b per hour
c per minute
d per second (in cents)

18 A 2 hour football match starts at 2:30 p.m. Eastern Standard Time (EST) in Newcastle, NSW. What time will it be in United Kingdom when the match finishes?

19 If the date is 29 March and it is 3 p.m. in Perth, what is the time and date in these places?

a Italy **b** Alaska **c** Chile

20 Monty departs on a 20 hour flight from Brisbane to London, United Kingdom, at 5 p.m. on 20 April. Give the time and date of his arrival in London.

21 Elsa departs on an 11 hour flight from Johannesburg, South Africa, to Perth at 6:30 a.m. on 25 October. Give the time and date of her arrival in Perth.

REASONING

22 | 22–23 | 23–25

22 When there are 365 days in a year, how many weeks are there in a year? Round your answer to two decimal places.

23 **a** To convert from hours to seconds, what single number do you multiply by?
b To convert from days to minutes, what single number do you multiply by?
c To convert from seconds to hours, what single number do you divide by?
d To convert from minutes to days, what single number do you divide by?

24 Assuming there are 365 days in a year and my birthday falls on a Wednesday this year, on what day will my birthday fall in 2 years' time?

25 **a** Explain why you gain time when you travel from Australia to Europe.
b Explain why you lose time when you travel from Germany to Australia.
c Explain what happens to the date when you fly from Australia to Canada across the International Date Line.

ENRICHMENT

Daylight saving | — | — | 26

26 Use the internet to investigate how daylight saving affects the time in some places. Write a brief report discussing the following points.

a **i** Name the States in Australia that use daylight saving.
ii Name five other countries that use daylight saving.
b Describe how daylight saving works, why it is used and what changes have to be made to our clocks.
c Describe how daylight saving in Australia affects the time difference between time zones. Use New South Wales and Greece as an example.

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4K Introduction to Pythagoras' theorem

EXTENDING

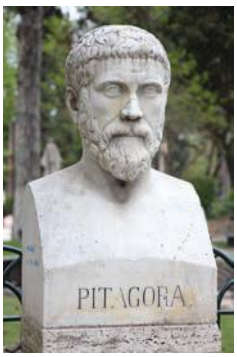
Interactive

Widgets

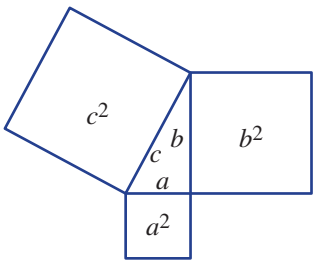
HOTsheets

Walkthroughs

Pythagoras was a philosopher in ancient Greece who lived in the 6th century BCE. He studied astronomy, mathematics, music and religion, but is most well known for the famous Pythagoras' theorem. Pythagoras was known to provide a proof for the theorem that bears his name, and methods to find Pythagorean triples, which are sets of three whole numbers that make up the sides of right-angled triangles.



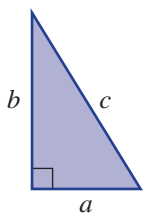
The ancient Babylonians, 1000 years before Pythagoras' time, and the Egyptians also knew that there was a relationship between the sides of a right-angled triangle. Pythagoras, however, was able to clearly explain and prove the theorem using mathematical symbols. The ancient theorem is still one of the most commonly used theorems today.

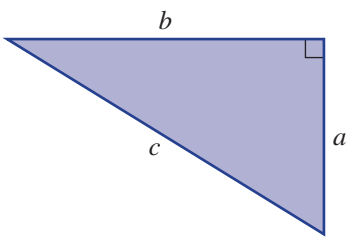


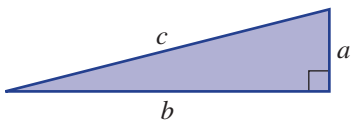
Pythagoras' theorem states that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. An illustration of the theorem includes squares drawn on the sides of the right-angled triangle. The area of the larger square (c^2) is equal to the sum of the two smaller squares ($a^2 + b^2$).

Let's start: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then complete the table.







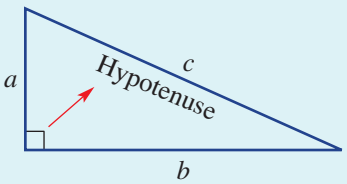
	a	b	c	a^2	b^2	c^2
Triangle 1						
Triangle 2						
Triangle 3						

- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.
- Research how you can cut the two smaller squares (with areas a^2 and b^2) to fit the pieces into the larger square (with area c^2).

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Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



- **Pythagoras' theorem**
  - The square of the length of the hypotenuse is the sum of the squares of the lengths of the other two shorter sides.
  - $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- A **Pythagorean triple** (or triad) is a set of three whole numbers which satisfy Pythagoras' theorem.

Example 23 Checking Pythagorean triples

Decide if the following are Pythagorean triples.

a 6, 8, 10 **b** 4, 5, 9

SOLUTION

a $a^2 + b^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100 \ (= 10^2)$
$\therefore$ 6, 8, 10 is a Pythagorean triple.

b $a^2 + b^2 = 4^2 + 5^2$
$= 16 + 25$
$= 41 \ (\neq 9^2)$
$\therefore$ 4, 5, 9 is not a Pythagorean triple.

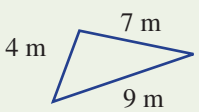
EXPLANATION

Let $a = 6$, $b = 8$ and $c = 10$ and check that $a^2 + b^2 = c^2$

$a^2 + b^2 = 41$ and
$9^2 = 81$ so
$a^2 + b^2 \neq c^2$

Example 24 Deciding if a triangle has a right angle

Decide if this triangle has a right angle.



SOLUTION

$a^2 + b^2 = 4^2 + 7^2$
$= 16 + 49$
$= 65 \ (\neq 9^2 = 81)$

EXPLANATION

Check to see if $a^2 + b^2 = c^2$. In this case $a^2 + b^2 = 65$ and $c^2 = 81$ so the triangle is not right angled.

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Exercise 4K

1–4 | 4 | —

UNDERSTANDING

1 Calculate these squares and sums of squares.

a 3^2 **b** 5^2 **c** 12^2 **d** 1.5^2

e $2^2 + 4^2$ **f** $3^2 + 7^2$ **g** $6^2 + 11^2$ **h** $12^2 + 15^2$

2 Decide if these equations are true or false.

a $2^2 + 3^2 = 4^2$ **b** $6^2 + 8^2 = 10^2$ **c** $7^2 + 24^2 = 25^2$

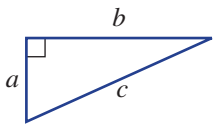
d $5^2 - 3^2 = 4^2$ **e** $6^2 - 3^2 = 2^2$ **f** $10^2 - 5^2 = 5^2$

3 Write the missing words in this sentence.

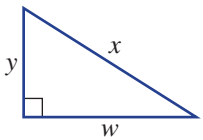
The _____ is the longest side of a right-angled _____.

4 Which letter represents the length of the hypotenuse in these triangles?

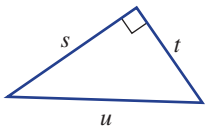
a



b



c



5(½), 7(½) | 5(½), 6, 7(½) | 5(½), 6, 7(½)

FLUENCY

Example 23

5 Decide if the following are Pythagorean triples.

a 3, 4, 6 **b** 4, 2, 5 **c** 3, 4, 5

d 9, 12, 15 **e** 5, 12, 13 **f** 2, 5, 6

g 9, 40, 41 **h** 10, 12, 20 **i** 4, 9, 12

6 Complete this table and answer the questions.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5				
6	8	10				
8	15	17				

a Which two columns give equal results?

b What would be the value of c^2 if:

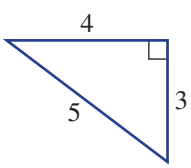
i $a^2 = 4$ and $b^2 = 9$? **ii** $a^2 = 7$ and $b^2 = 13$?

c What would be the value of $a^2 + b^2$ if:

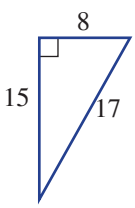
i $c^2 = 25$? **ii** $c^2 = 110$?

7 Check that $a^2 + b^2 = c^2$ for all these right-angled triangles.

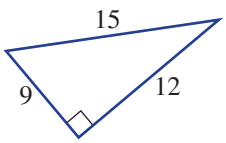
a



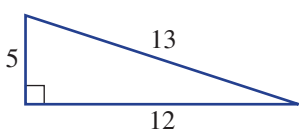
b



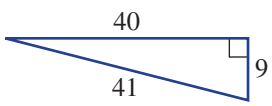
c



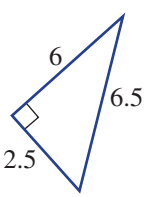
d



e



f



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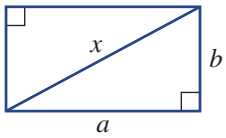
4K

8, 9 | 8, 9 | 9, 10

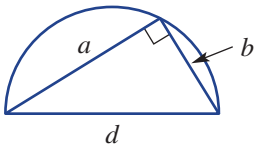
PROBLEM-SOLVING

8 Write down an equation using the pronumerals given these diagrams.

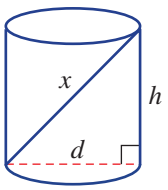
a



b



c



9 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

a Using $c = 40$, decide if $a^2 + b^2 = c^2$.

b Do you think the triangle formed by the mast and the cable is right angled? Give a reason.



10 (3, 4, 5) and (5, 12, 13) are Pythagorean triples since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$.

a Find 10 more Pythagorean triples using whole numbers all less than 100.

b Find the total number of Pythagorean triples with whole numbers all less than 100.

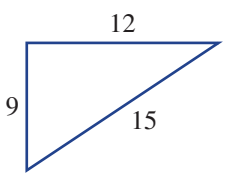
11 | 11, 12 | 11–13

REASONING

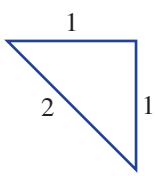
Example 24

11 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

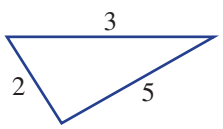
a



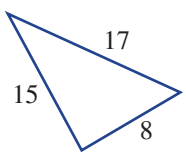
b



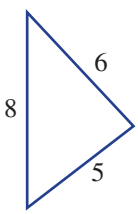
c



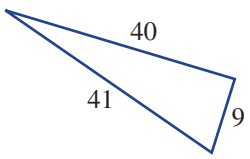
d



e



f



12 If $a^2 + b^2 = c^2$ is true, complete these statements.

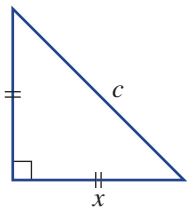
a $c^2 - b^2 =$ __ **b** $c^2 - a^2 =$ __ **c** $c =$ _____

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4K

REASONING

13 This triangle is isosceles. Write Pythagoras' theorem using the given pronumerals. Simplify if possible.



Pythagoras proof — — 14

ENRICHMENT

14 There are many ways to prove Pythagoras' theorem, both algebraically and geometrically.

a Here is an incomplete proof of the theorem that uses this illustrated geometric construction.

Area of inside square = c^2

Area of 4 outside triangles = $4 \times \frac{1}{2} \times$ base $\times$ height

= _____

Total area of outside square = (_____ + _____)^2

= $a^2 + 2ab + b^2$

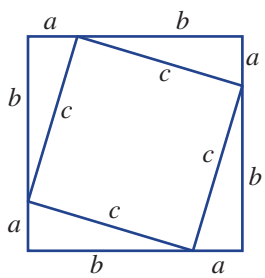
Area of inside square = Area (outside square) − Area of 4 triangles

= _____ − _____

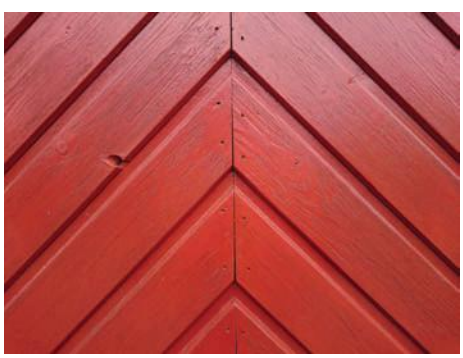
= _____

Comparing results from the first and last steps gives

c^2 = _____



b Use the internet to search for other proofs of Pythagoras' theorem. See if you can explain and illustrate them.



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4L Using Pythagoras' theorem EXTENDING

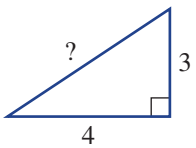
Interactive

Widgets

HOTsheets

Walkthroughs

From our understanding of algebra, we know that equations can be solved to find the value of an unknown. This is also the case for equations derived from Pythagoras' theorem, where, if two of the side lengths of a right-angled triangle are known, then the third can be found.



So if $c^2 = 3^2 + 4^2$ then $c^2 = 25$ and $c = 5$.

We also notice that if $c^2 = 25$ then $c = \sqrt{25} = 5$ (if $c > 0$).

This application of Pythagoras' theorem has a wide range of applications wherever right-angled triangles can be drawn.

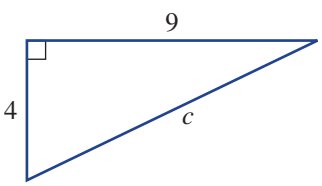
Note that a number using a $\sqrt{\ }$ sign may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers and neither can be written as a fraction. These types of numbers are called surds and are a special group of numbers (irrational numbers) that are often approximated using rounded decimals.



The exterior of the Australian Centre for the Moving Image is created from many right-angled triangles.

Let's start: Correct layout

Three students who are trying to find the value of c in this triangle using Pythagoras' theorem write their solutions on a board. There are only very minor differences between each solution and the answer is written rounded to two decimal places. Which student has all the steps written correctly? Give reasons why the other two solutions are not laid out correctly.

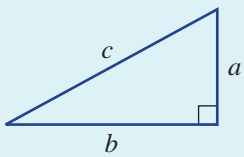


Student 1	Student 2	Student 3
$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$	$c = a^2 + b^2$
$= 4^2 + 9^2$	$= 4^2 + 9^2$	$= 4^2 + 9^2$
$= 97$	$= 97$	$= 97$
$= \sqrt{97}$	$\therefore c = \sqrt{97}$	$= \sqrt{97}$
$= 9.85$	$= 9.85$	$= 9.85$

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Key ideas

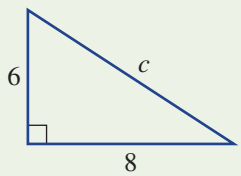
- **Surds** are numbers that have a $\sqrt{\ }$ sign when written in simplest form.
  - They are not a whole number and cannot be written as a fraction.
  - Written as a decimal, the decimal places would continue forever with no repeated pattern (just like the number pi). Surds are therefore classified as irrational numbers.
  - $\sqrt{2}, \sqrt{5}, 2\sqrt{3}$ and $\sqrt{90}$ are all examples of surds.
- Using Pythagoras' theorem.
  If $c^2 = a^2 + b^2$ then $c = \sqrt{a^2 + b^2}$.
- Note
  - $\sqrt{a^2 + b^2} \neq a + b$, for example, $\sqrt{3^2 + 4^2} \neq 3 + 4$
  - If $c^2 = k$ then $c = \sqrt{k}$ if $c \geq 0$.



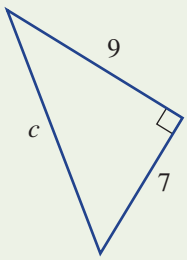
Example 25 Finding the length of the hypotenuse

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to two decimal places.

a



b

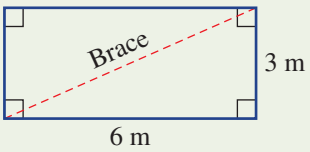


SOLUTION	EXPLANATION
a $c^2 = a^2 + b^2$	Write the equation for Pythagoras' theorem and substitute the values for the shorter sides.
$= 6^2 + 8^2$	
$= 100$	
$\therefore c = \sqrt{100}$	Find c by taking the square root.
$= 10$	
b $c^2 = a^2 + b^2$	First calculate the value of $7^2 + 9^2$.
$= 7^2 + 9^2$	
$= 130$	
$\therefore c = \sqrt{130}$	$\sqrt{130}$ is a surd, so round the answer as required.
$= 11.40$ (to 2 d.p.)	

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Example 26 Applying Pythagoras' theorem

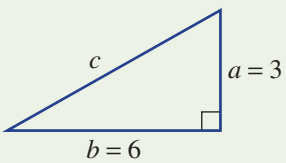
A rectangular wall is to be strengthened by a diagonal brace. The wall is 6 m wide and 3 m high. Find the length of brace required correct to the nearest cm.



SOLUTION

$c^2 = a^2 + b^2$
$= 3^2 + 6^2$
$= 45$
$\therefore c = \sqrt{45}$
$= 6.71$ m or 671 cm (to nearest cm)

EXPLANATION



Exercise 4L

1–3 | 3 | —

UNDERSTANDING

1 Decide if these numbers written with a $\sqrt{}$ sign simplify to a whole number. Answer Yes or No.

a $\sqrt{9}$ **b** $\sqrt{11}$ **c** $\sqrt{20}$ **d** $\sqrt{121}$

2 Round these surds correct to two decimal places using a calculator.

a $\sqrt{10}$ **b** $\sqrt{26}$ **c** $\sqrt{65}$ **d** $\sqrt{230}$

3 Copy and complete this working out.

a
$c^2 = a^2 + b^2$
$= 5^2 + 12^2$
$=$ ____
$\therefore c = \sqrt{\quad}$
$=$ ____

b
$c^2 =$ ____
$= 9^2 + 40^2$
$=$ ____
$\therefore c = \sqrt{\quad}$
$=$ ____

c
____ = ____
$= 9^2 + 12^2$
$=$ ____
$\therefore c = \sqrt{\quad}$
$=$ ____

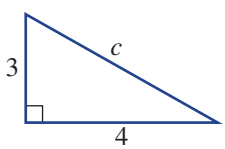
4, 5(½) | 4–5(½) | 4–5(½)

FLUENCY

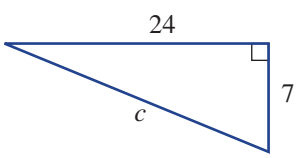
Example 25a

4 Find the length of the hypotenuse (c) of these right-angled triangles.

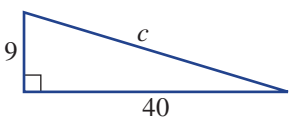
a



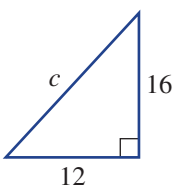
b



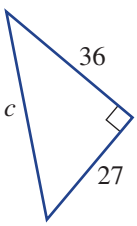
c



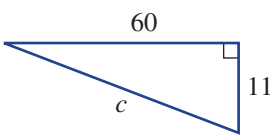
d



e



f



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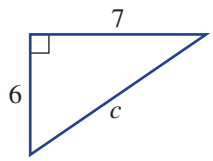
4L

FLUENCY

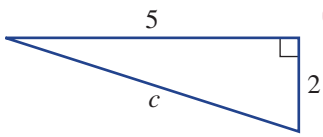
Example 25b

5 Find the length of the hypotenuse (c) of these right-angled triangles correct to two decimal places.

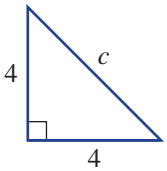
a



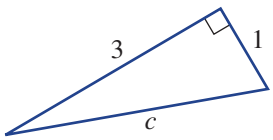
b



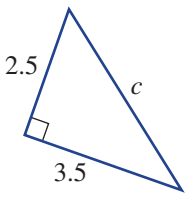
c



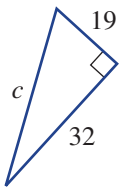
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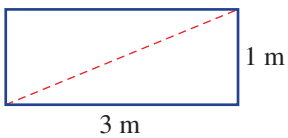


6, 7	7–9	7–9

PROBLEM-SOLVING

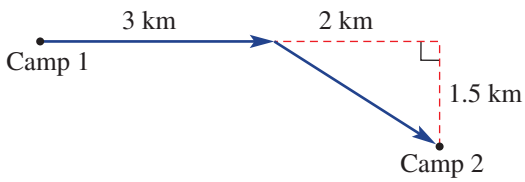
Example 26

6 A rectangular board is to be cut along one of its diagonals. The board is 1 m wide and 3 m high. What will be the length of the cut, correct to the nearest cm?

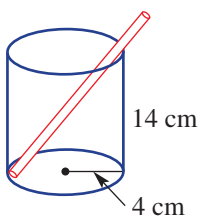


7 The size of a television screen is determined by its diagonal length. Find the size of a television screen that is 1.2 m wide and 70 cm high. Round the answer to the nearest cm.

8 Here is a diagram showing the path of a bushwalker from Camp 1 to Camp 2. Find the total distance rounded to one decimal place.



9 A 20 cm straw sits in a cylindrical glass as shown. What length of straw sticks above the top of the glass? Round the answer to two decimal places.





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4L

10 | 10 | 10, 11

REASONING

10 Explain the error in each set of working.

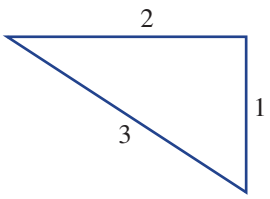
a
$c^2 = 2^2 + 3^2$
$\therefore c = 2 + 3$
$= 5$

b
$c^2 = 3^2 + 4^2$
$= 7^2$
$= 49$
$\therefore c = 7$

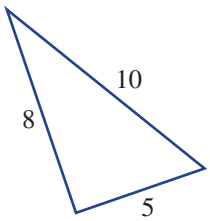
c
$c^2 = 2^2 + 5^2$
$= 4 + 25$
$= 29$
$= \sqrt{29}$

11 Prove that these are not right-angled triangles.

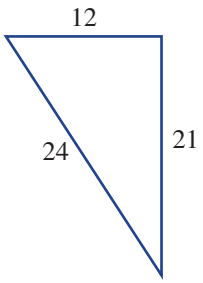
a



b



c



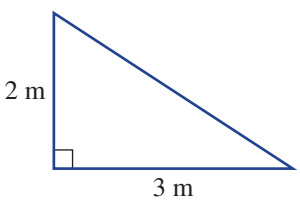
Perimeter and Pythagoras

— | — | 12

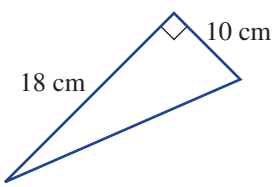
ENRICHMENT

12 Find the perimeter of these shapes correct to two decimal places.

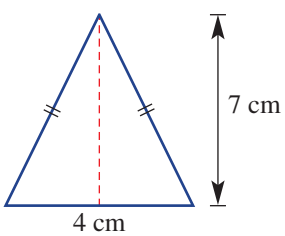
a



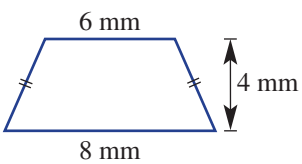
b



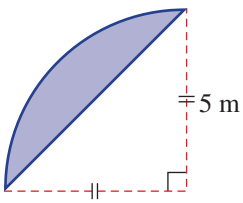
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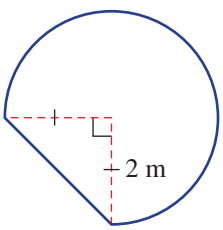
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e



f



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4M Finding the length of a shorter side EXTENDING

Interactive

Widgets

HOTsheets

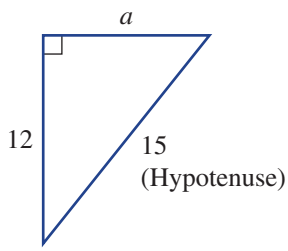
Walkthroughs

We know that if we are given the two shorter sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the hypotenuse. Generalising further, we can say that if given *any* two sides of a right-angled triangle we can use Pythagoras' theorem to find the length of the third side.

Let's start: What's the setting out?

The triangle shown has a hypotenuse length of 15 and one of the shorter sides is of length 12. Here is the setting out to find the length of the unknown side a.

Fill in the missing gaps and explain what is happening at each step.



$a^2 + b^2 = c^2$

$a^2 + ____^2 = ____^2$

$a^2 + ____ = ____$

$a^2 = ____$ (Subtract $____$ from both sides)

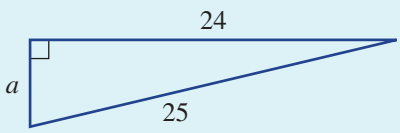
$\therefore a = \sqrt{____}$

$= ____$

Key ideas

- Pythagoras' theorem can be used to find the length of the shorter sides of a right-angled triangle if the length of the hypotenuse and another side are known.
- Use subtraction to make the unknown the subject of the equation.

For example:



$a^2 + b^2 = c^2$

$a^2 + 24^2 = 25^2$

$a^2 + 576 = 625$

$a^2 = 49$ (Subtract 576 from both sides.)

$\therefore a = \sqrt{49}$

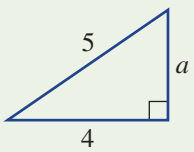
$= 7$



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Example 27 Finding the length of a shorter

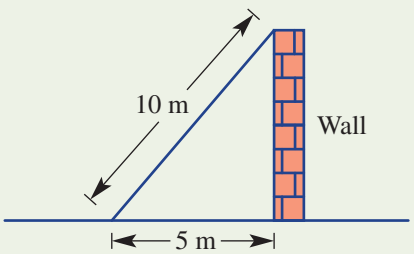
Find the value of a in this right-angled triangle.



SOLUTION	EXPLANATION
$a^2 + b^2 = c^2$	Write the equation using Pythagoras' theorem and substitute the known values.
$a^2 + 4^2 = 5^2$	
$a^2 + 16 = 25$	
$a^2 = 9$	Subtract 16 from both sides.
$\therefore a = \sqrt{9}$	
$= 3$	

Example 28 Applying Pythagoras to find a shorter side

A 10 m steel brace holds up a concrete wall. The bottom of the brace is 5 m from the base of the wall. Find the height of the concrete wall correct to two decimal places.



SOLUTION	EXPLANATION
Let a metres be the height of the wall.	Choose a letter (pronumeral) for the unknown height.
$a^2 + b^2 = c^2$	
$a^2 + 5^2 = 10^2$	Substitute into Pythagoras' theorem.
$a^2 + 25 = 100$	
$a^2 = 75$	Subtract 25 from both sides.
$\therefore a = \sqrt{75}$	$\sqrt{75}$ is the exact answer.
$= 8.66$ (to 2 d.p.)	Round as required.
The height of the wall is 8.66 metres.	Answer a worded problem using a full sentence.

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Exercise 4M

1(½), 2 | 2 | —

UNDERSTANDING

1 Find the value of a in these equations. (Assume a is a positive number.)

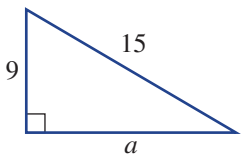
a $a^2 = 16$ **b** $a^2 = 49$ **c** $a^2 + 16 = 25$

d $a^2 + 9 = 25$ **e** $a^2 + 36 = 100$ **f** $a^2 + 441 = 841$

g $10 + a^2 = 19$ **h** $6 + a^2 = 31$ **i** $25 + a^2 = 650$

2 Copy and complete the missing steps.

a



$a^2 + b^2 = c^2$

$a^2 + 9^2 = ____$

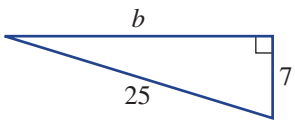
$a^2 + ____ = 225$

$a^2 = ____$

$\therefore a = \sqrt{____}$

$= ____$

b



$a^2 + b^2 = c^2$

$7^2 + b^2 = ____$

$____ + b^2 = ____$

$b^2 = 576$

$\therefore b = \sqrt{____}$

$= ____$

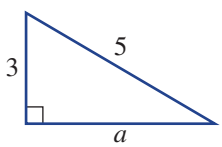
3, 4(½) | 3–4(½) | 3–4(½)

FLUENCY

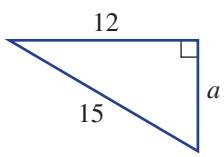
Example 27

3 Find the length of the unknown side in these right-angled triangles.

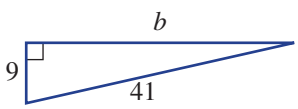
a



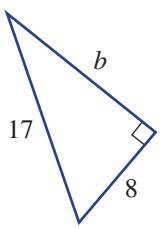
b



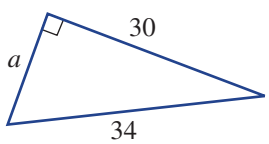
c



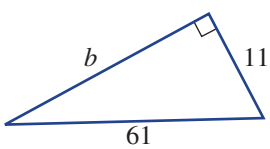
d



e



f



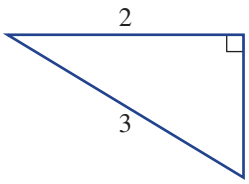
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4M

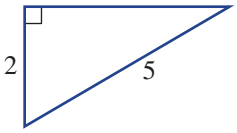
FLUENCY

4 Find the length of the unknown side in these right-angled triangles, giving the answer correct to two decimal places.

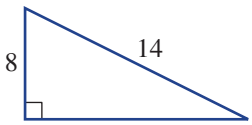
a



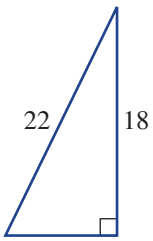
b



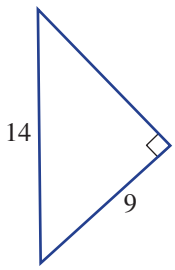
c



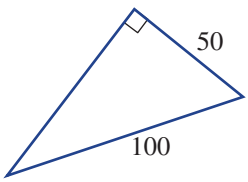
d



e



f

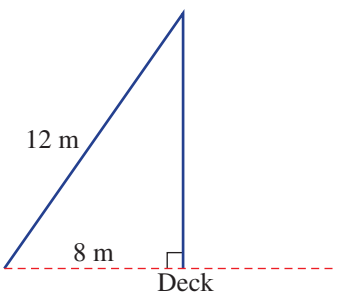


5, 6 | 5–7 | 6–8

PROBLEM-SOLVING

Example 28

5 A yacht's mast is supported by a 12 m cable attached to its top. On the deck of the yacht, the cable is 8 m from the base of the mast. How tall is the mast? Round the answer to two decimal places.



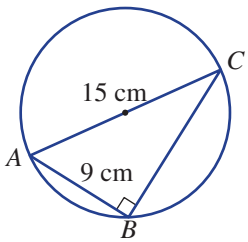


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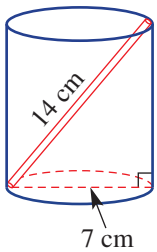
4M

PROBLEM-SOLVING

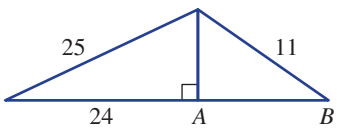
6 A circle's diameter AC is 15 cm and the chord AB is 9 cm. Angle ABC is 90°. Find the length of the chord BC.



7 A 14 cm drinking straw just fits into a can as shown. The diameter of the can is 7 cm. Find the height of the can correct to two decimal places.



8 Find the length AB is this diagram. Round to two decimal places.



REASONING

9 | 9, 10 | 10, 11

9 Describe what is wrong with the second line of working in each step.

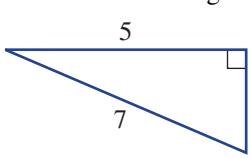
a $a^2 + 10 = 24$
$a^2 = 34$

b $a^2 = 25$
$= 5$

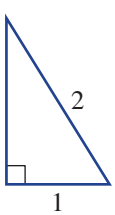
c $a^2 + 25 = 36$
$a + 5 = 6$

10 The number $\sqrt{11}$ is an example of a surd that is written as an exact value. Find the surd that describes the exact lengths of the unknown sides of these triangles.

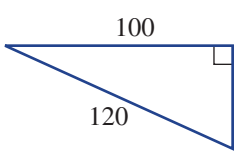
a



b



c



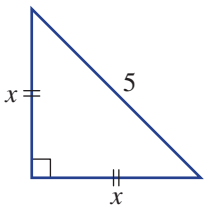
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4M

REASONING

11 Show how Pythagoras' theorem can be used to find the unknown length in these isosceles triangles. Complete the solution for part **a** and then try the others. Round to two decimal places.

a



$a^2 + b^2 = c^2$

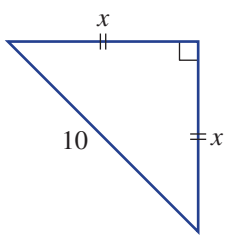
$x^2 + x^2 = 5^2$

$2x^2 = 25$

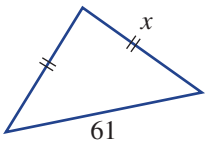
$x^2 =$ ____

$\therefore x = \sqrt{\quad}$

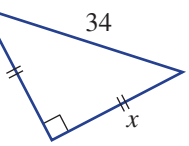
b



c



d



Pythagorean families — — 12

ENRICHMENT

12 (3, 4, 5) is called a Pythagorean triple because the numbers 3, 4 and 5 satisfy Pythagoras' theorem ($3^2 + 4^2 = 5^2$).

- **a** Explain why (6, 8, 10) is also a Pythagorean triple.
- **b** Explain why (6, 8, 10) is considered to be in the same family as (3, 4, 5).
- **c** List 3 other Pythagorean triples in the same family as (3, 4, 5) and (6, 8, 10).
- **d** Find another triple not in the same family as (3, 4, 5), but has all 3 numbers less than 20.
- **e** List 5 triples that are each the smallest triple of 5 different families.



Statue of Pythagoras, Samos, Greece.

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Investigation

GMT and travel

As discussed in Section 4J, the world is divided into 24 time zones, which are determined loosely by each 15° meridian of longitude. World time is based on the time at a place called Greenwich near London, United Kingdom. This time is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT). Places east of Greenwich are ahead in time and places west of Greenwich are behind.

In Australia, the Western Standard Time is 2 hours behind Eastern Standard Time and Central Standard Time is $\frac{1}{2}$ hour behind Eastern Standard Time. Use the world time zone map on pages 231–232 to answer these questions and to investigate how the time zones affect the time when we travel.

East and west

1 Name five countries that are:
- **a** ahead of GMT
- **b** behind GMT

Noon in Greenwich

2 When it is noon in Greenwich, what is the time in these places?
- **a** Sydney
- **b** Perth
- **c** Darwin
- **d** Washington, DC
- **e** Auckland
- **f** France
- **g** Johannesburg
- **h** Japan

2 p.m. EST

3 When it is 2 p.m. Eastern Standard Time (EST) on Wednesday, find the time and day in these places.
- **a** Perth
- **b** Adelaide
- **c** London
- **d** western Canada
- **e** China
- **f** United Kingdom
- **g** Alaska
- **h** South America

Adjusting your watch

4 Do you adjust your watch forwards or backwards when you are travelling to these places?
- **a** India
- **b** New Zealand

5 In what direction should you adjust your watch if you are flying over the Pacific Ocean?

Flight travel

6 You fly from Perth to Brisbane on a 4 hour flight that departed at noon. What is the time in Brisbane when you arrive?

7 You fly from Melbourne to Edinburgh on a 22 hour flight that departed at 6 a.m. What is the time in Edinburgh when you arrive?

8 You fly from Sydney to Los Angeles on a 13 hour flight that departed at 7:30 p.m. What is the time in Los Angeles when you arrive?

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9 Copy and complete the following table.

Departing	Arriving	Departure time	Flight time (hours)	Arrival time
Brisbane	Broome	7 a.m.	3.5	
Melbourne	London	1 p.m.	23	
Hobart	Adelaide		1.5	4 p.m.
London	Tokyo		12	11 p.m.
New York	Sydney		15	3 a.m.
Beijing	Vancouver	3:45 p.m.		7:15 p.m.

10 Investigate how daylight saving alters the time in some time zones and why. How does this affect flight travel? Give examples.

Pythagorean triples and spreadsheets

Pythagorean triples (or triads) can be grouped into families. The triple (3, 4, 5) is the base triple for the family of triples $(3k, 4k, 5k)$. Here are some triples in this same family.

k	1	2	3
triple	(3, 4, 5) (base triple)	(6, 8, 10)	(9, 12, 15)

1 Write down three more triples in the family $(3k, 4k, 5k)$.

2 Write down three triples in the family $(7k, 24k, 25k)$.

3 If $(3k, 4k, 5k)$ and $(7k, 24k, 25k)$ are two triple families, can you find three more families that have whole numbers less than 100.

4 Pythagoras discovered that if the smaller number in a base triple is a then the other two numbers in the triple are given by the rules:

$$\frac{1}{2}(a^2 + 1) \quad \text{and} \quad \frac{1}{2}(a^2 - 1)$$

Set up a spreadsheet to search for all the families of triples of whole numbers less than 200. Here is how a spreadsheet might be set up.

	A	B	C
1	Pythagorean triples		
2	a	b	c
3	1	=1/2*(A3^2-1)	=1/2*(A3^2+1)
4	2		
5	3		
6	4		

Fill down far enough so that c is a maximum of 200.

5 List all the base triples using whole numbers less than 200. How many are there?

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Problems and challenges

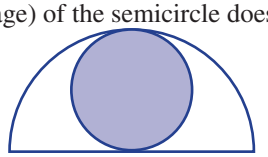


Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

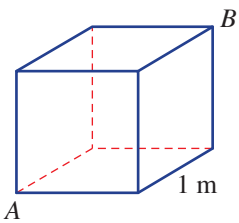
1 A cube has capacity 1 L. What are its dimensions in cm correct to one decimal place?

2 A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. Rocks with a volume of 3000 cm^3 are placed into the tank. Will the tank overflow?

3 What proportion (fraction or percentage) of the semicircle does the full circle occupy?

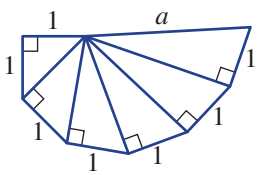


4 What is the distance AB in this cube? (Pythagoras' theorem is required.)

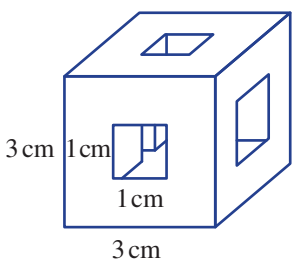


5 By what number do you multiply the radius of a circle to double its area?

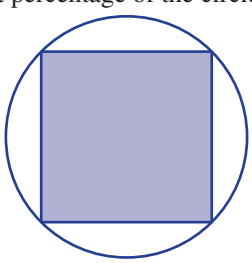
6 Find the exact value (as a surd) of a in this diagram. (Pythagoras' theorem is required.)



7 A cube of side length 3 cm has its core removed in all directions as shown. Find its total surface area both inside and out.



8 A square just fits inside a circle. What percentage of the circle is occupied by the square?



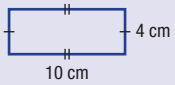
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Chapter summary

Measurement

Length

Perimeter



$P = 2 \times 10 + 2 \times 4$
$= 28$ cm

Units

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

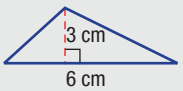
Circumference

$C = 2\pi r$ or πd
$= 2 \times \pi \times 3$
$= 6\pi$ m^2



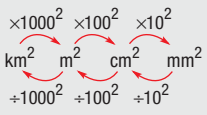
Area

Triangle



$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 6 \times 3$
$= 9$ cm^2

Units



1 ha = 10 000 m^2

Quadrilaterals

Square $A = l^2$
Rectangle $A = lw$
Parallelogram $A = bh$
Rhombus $A = \frac{1}{2}xy$
Kite $A = \frac{1}{2}xy$
Trapezium $A = \frac{1}{2}(a + b)h$

Circle

$A = \pi r^2$
$= \pi \times 7^2$
$= 49\pi$ cm^2



Sectors (Ext)

$A = \frac{\theta}{360} \times \pi r^2$
$= \frac{280}{360} \times \pi \times 2^2$
$= 9.77$ m^2



Surface area (Ext)

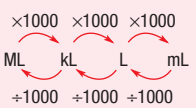


SA = 2×(1×2)+2×(1×3)+2×(2×3)
$= 4 + 6 + 12$
$= 22$ m^2

Volume

Units

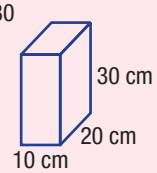
km^3, m^3, cm^3, mm^3



1 mL = 1 cm^3
1 m^3 = 1000 L

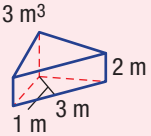
Rectangular prism

$V = lwh$
$= 10 \times 20 \times 30$
$= 6000$ cm^3
$= 6$ L

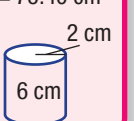


Prism and cylinders

$V = Ah$
$= \frac{1}{2} \times 3 \times 1 \times 2$
$= 3$ m^3



$V = \pi r^2 h$
$= \pi \times 2^2 \times 6$
$= 75.40$ cm^3

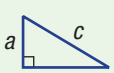


Time

1 min = 60 s
1 h = 60 min
0311 is 03:11 a.m.
2049 is 08:49 p.m.

Pythagoras' theorem (Ext)

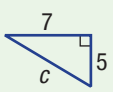
Theorem



$a^2 + b^2 = c^2$

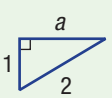
Finding *c*

$c^2 = 5^2 + 7^2$
$= 74$
$\therefore c = \sqrt{74}$



Finding a shorter side

$a^2 + 1^2 = 2^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \sqrt{3}$

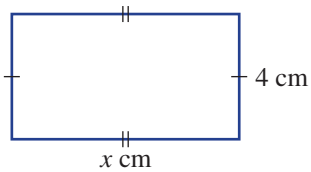


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Chapter review

Multiple-choice questions

4A **1** The perimeter of this rectangle is 20 cm. The unknown value x is:

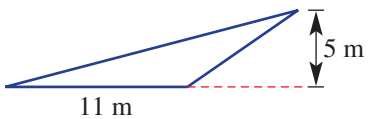


A 4 **B** 16 **C** 5
D 10 **E** 6

4B **2** A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:

A π, π^2 **B** $2\pi, \pi$ **C** $4\pi, 4\pi$
D 2, 1 **E** 4, 4

4C **3** The area of this triangle is:

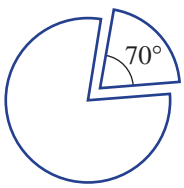


A 27.5 m^2 **B** 55 m **C** 55 m^2
D 110 m **E** 16 m^2

4E **4** Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

A 7850 m^2 **B** 314 m^2 **C** 31 400 m^2 **D** 78.5 m^2 **E** 628 m^2

4F Ext **5** A sector of a circle is removed as shown. The fraction of the circle remaining is:

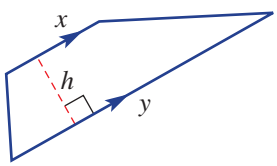


A 290 **B** $\frac{29}{36}$ **C** $\frac{7}{36}$
D $\frac{7}{180}$ **E** $\frac{3}{4}$

4G Ext **6** A cube has a surface area of 24 cm^2. The length of its sides is:

A 12 cm **B** 8 cm **C** 6 cm
D 4 cm **E** 2 cm

4D **7** The rule for the area of the trapezium shown is:



A $\frac{1}{2}xh$ **B** $\frac{1}{2}(x+y)$ **C** $\frac{1}{2}xy$
D πxy^2 **E** $\frac{1}{2}(x+y)h$

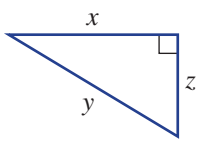
4H **8** The volume of a rectangular prism is 48 cm^3. If its width is 4 cm and height 3 cm, its length would be:

A 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

4I **9** A cylinder has radius 7 cm and height 10 cm. Using $\pi = \frac{22}{7}$, its volume would be:

A 1540 cm^2 **B** 440 cm^3 **C** 440 L **D** 1540 cm^3 **E** 220 cm^3

4K Ext **10** The rule for Pythagoras' theorem for this triangle would be:



A $a^2 - b^2 = c^2$ **B** $x^2 + y^2 = z^2$ **C** $z^2 + y^2 = x^2$
D $x^2 + z^2 = y^2$ **E** $y = \sqrt{x^2 - z^2}$

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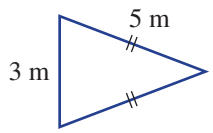
Chapter review

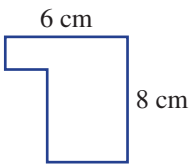
Short-answer questions

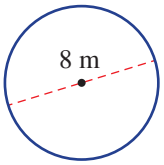
4A **1** Convert these measurements to the units given in the brackets.

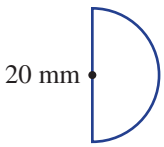
a 2 m (mm) **b** 50 000 cm (km) **c** 3 cm^2 (mm^2) **d** 4000 cm^2 (m^2)
e 0.01 km^2 (m^2) **f** 350 mm^2 (cm^2) **g** 400 cm^3 (L) **h** 0.2 m^3 (L)

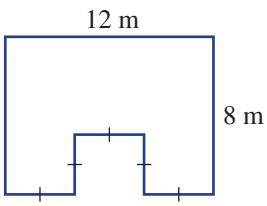
4A/B **2** Find the perimeter/circumference of these shapes. Round the answer to two decimal places where necessary.

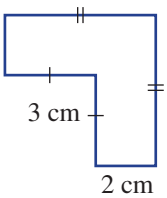
a


b


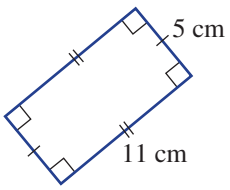
c


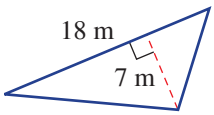
d


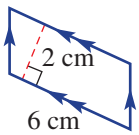
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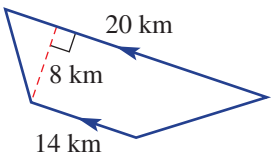
f


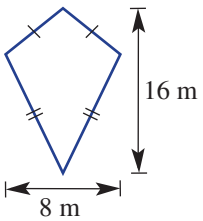
4C/D/E **3** Find the area of these shapes. Round the answer to two decimal places where necessary.

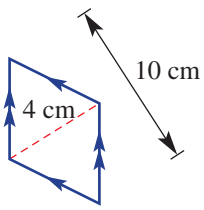
a


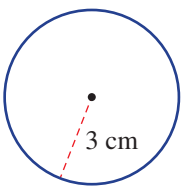
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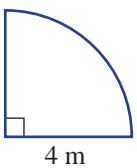
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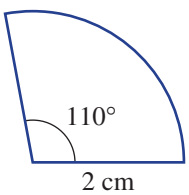
d


e


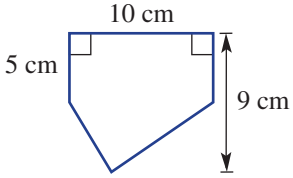
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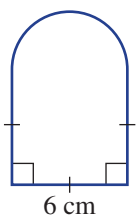
g


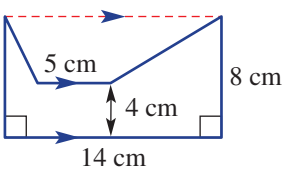
h


Ext **i**


4F Ext **4** Find the area of these composite shapes.

a


b


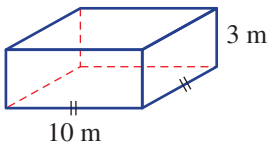
c


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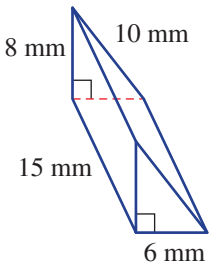
4G Ext

5 Find the surface area of each prism.

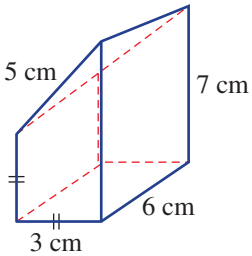
a



b



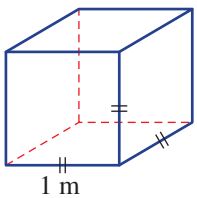
c



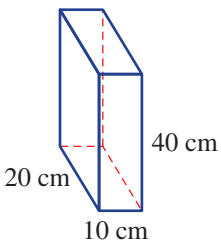
4H/I

6 Find the volume of each prism, giving your answer in litres. Remember 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

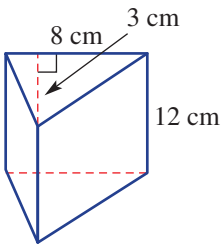
a



b



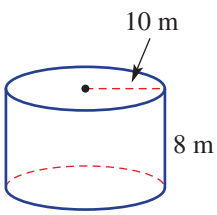
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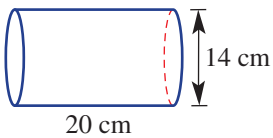
4I

7 Find the volume of these cylinders, rounding the answer to two decimal places.

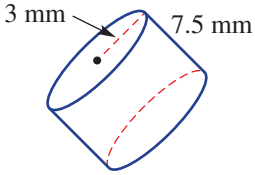
a



b



c



4J

8 An oven is heated from 23°C to 310°C in 18 minutes and 37 seconds. It then cools by 239°C in 1 hour, 20 minutes and 41 seconds.

a Give the temperature:

i increase **ii** decrease

b What is the total time taken to heat and cool the oven?

c How much longer does it take for the oven to cool down than heat up?

4J

9 **a** What is the time difference between 4:20 a.m. and 2:37 p.m.?

b Write 2145 hours in a.m./p.m. time.

c Write 11:31 p.m. in 24-hour time.

4J

10 When it is 4:30 p.m. in Western Australia, state the time in each of these places.

a New South Wales **b** Adelaide **c** United Kingdom

d China **e** Finland **f** South Korea

g Russia (eastern mainland) **h** New Zealand

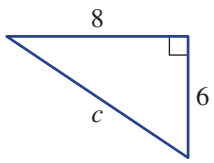
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Chapter review

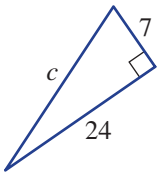
4L Ext

11 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round the answer to two decimal places in part **c**.

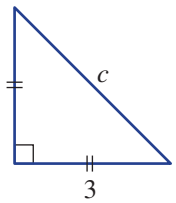
a



b



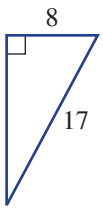
c



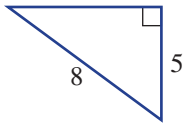
4M Ext

12 Use Pythagoras' theorem to find the unknown length in these right-angled triangles. Round the answer to two decimal places in parts **b** and **c**.

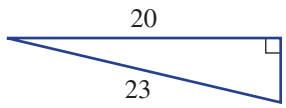
a



b

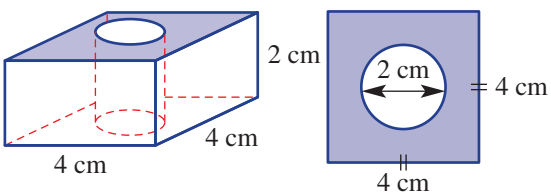


c



Extended-response questions

1 A company makes square nuts for bolts to use in building construction and steel structures. Each nut starts out as a solid steel square prism. A cylinder of diameter 2 cm is bored through its centre to make a hole. The nut and its top view are shown here.



The company is interested in how much paint is required to paint the nuts. The inside surface of the hole is not to be painted. Round all answers to two decimal places where necessary.

a Find the area of the top face of the nut.

b Find the total outside surface area of the nut.

c If the company makes 10 000 nuts, how many square metres of surface needs to be painted?

The company is also interested in the volume of steel used to make the nuts.

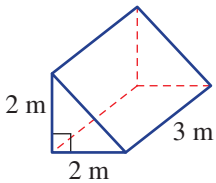
d Find the volume of steel removed from each nut to make the hole.

e Find the volume of steel in each nut.

f Assuming that the steel removed to make the hole can be melted and reused, how many nuts can be made from 1 L of steel?

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2 A simple triangular shelter has a base width of 2 m, a height of 2 m and a length of 3 m.



a Use Pythagoras' theorem to find the hypotenuse length of one of the ends of the tent. Round the answer to one decimal place.

b All the faces of the shelter including the floor are covered with canvas material. What area of canvas is needed to make the shelter? Round the answer to the nearest whole number of square metres.

c Every edge of the shelter is to be sealed with a special tape. What length of tape is required? Round to the nearest whole number of metres.

d The shelter tag says that is occupies 10 000 L of space. Show working to decide if this is true or false. What is the difference?



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Chapter 5 Algebra

What you will learn

5A The language of algebra (Consolidating)
5B Substitution and equivalence (Consolidating)
5C Adding and subtracting terms
5D Multiplying and dividing terms
5E Adding and subtracting algebraic fractions (Extending)
5F Multiplying and dividing algebraic fractions (Extending)
5G Expanding brackets
5H Factorising expressions
5I Applying algebra
5J Index laws: multiplying and dividing powers
5K Index laws: raising powers

Victorian curriculum

NUMBER AND ALGEBRA

Number and place value

Use index notation with numbers to establish the index laws with positive integral indices and the zero index (VCMNA272)

Patterns and algebra

Extend and apply the distributive law to the expansion of algebraic expressions (VCMNA279)

Factorise algebraic expressions by identifying numerical factors (VCMNA280)

Simplify algebraic expressions involving the four operations (VCMNA281)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Algebra and sport

Algebra formulas show the relation between real-life quantities and are a powerful tool for studying sport performance.

In 2014, Australian schoolboy Jack Hale ran 100 m in 10.42 seconds, setting an under-18s national record. Using the formula for speed $s = \frac{d}{t}$, we can calculate that Jack ran at an average speed of 9.597 m/s, or 34.55 km/h!

Algebra formulas are used to design computer models that simulate an athlete in action. By varying the input values, virtual experiments can discover how to improve an athlete's skills and their equipment for a better performance.

Sea kayaks, whitewater kayaks and competitive canoes each have a different hull shape for overcoming the water drag-force that acts on the underwater surface area. A short kayak is easier to manoeuvre in whitewater rapids but in calmer water longer kayaks are always faster. This is because longer kayaks have a faster maximum speed before the prow (i.e. the front) rises up as it tries to climb over its own bow wave, making sustained paddling impossible. Virtual models built with algebra can experiment with designs for a kayak's speed and stability.

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5A The language of algebra

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

A **pronumeral** is a letter that can represent one or more numbers. For instance, x could represent the number of goals a particular soccer player scored last year. Or p could represent the price (in dollars) of a book. The word **variable** is also used to describe a letter which represents an unknown value or quantity.



If a soccer player scored x goals last year they might hope to score $x + 1$ or more goals this year.

Let's start: Algebra sort

Consider the four expressions $x + 2$, $x \times 2$, $x - 2$ and $x \div 2$.

- If you know that x is 10, sort the four values from lowest to highest.
- Give an example of a value of x that would make $x \times 2$ less than $x + 2$.
- Try different values of x to see if you can:
  - make $x \div 2$ less than $x - 2$
  - make $x + 2$ less than $x - 2$
  - make $x \times 2$ less than $x \div 2$

Key ideas

- In algebra, letters can be used to represent one or more numbers. These letters are called **pronumerals** or **variables**.
- $a \times b$ is written ab and $a \div b$ is written $\frac{a}{b}$.
- $a \times a$ is written a^2.
- An **expression** is a combination of numbers and pronumerals combined with mathematical operations, for example, $3x + 2yz$ and $8 \div (3a - 2b) + 41$ are expressions.
- A **term** is a part of an expression with only pronumerals, numbers, multiplication and division, for example, $9a$, $10cd$ and $\frac{3x}{5}$ are all terms.
- A **coefficient** is the number in front of a pronumeral. If the term is being subtracted, the coefficient is a negative number, and if there is no number in front, the coefficient is 1.
  For the expression $3x + y - 7z$, the coefficient of x is 3, the coefficient of y is 1 and the coefficient of z is -7.
- A term that does not contain any variables is called a **constant term**.
- The **sum** of a and b is $a + b$.
- The **difference** of a and b is $a - b$.
- The **product** of a and b is $a \times b$.
- The **quotient** of a and b is $a \div b$.
- The **square** of a is a^2.

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Example 1 Using the language of algebra

a List the individual terms in the expression $4a + b - 12c + 5$.

b In the expression $4a + b - 12c + 5$ state the coefficients of a, b, c and d.

c What is the constant term in $4a + b - 12c + 5$?

d State the coefficient of b in the expression $3a + 4ab + 5b^2 + 7b$.

SOLUTION	EXPLANATION
a There are four terms: $4a, b, 12c$ and 5.	Each part of an expression is a term. Terms get added (or subtracted) to make an expression.
b The coefficient of a is 4. The coefficient of b is 1. The coefficient of c is -12. The coefficient of d is 0.	The coefficient is the number in front of a pronumeral. For b the coefficient is 1 because b is the same as $1 \times b$. For c the coefficient is -12 because this term is being subtracted. For d the coefficient is 0 because there are no terms with d.
c 5	A constant term is any term that does not contain a pronumeral.
d 7	Although there is a 4 in front of ab and a 5 in front of b^2, neither of these is a term containing just b, so they should be ignored.

Example 2 Creating expressions from a description

Write an expression for each of the following.

a The sum of 3 and k

b The product of m and 7

c 5 is added to one half of k

d The sum of a and b is doubled

SOLUTION	EXPLANATION
a $3 + k$	The word 'sum' means +.
b $m \times 7$ or $7m$	The word 'product' means $\times$.
c $\frac{1}{2}k + 5$ or $\frac{k}{2} + 5$	One half of k can be written $\frac{1}{2} \times k$ (because 'of' means $\times$), or $\frac{k}{2}$ because k is being divided by two.
d $(a + b) \times 2$ or $2(a + b)$	The values of a and b are being added and the result is multiplied by 2. Grouping symbols (the brackets) are required to multiply the whole result by two and not just the value of b.

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Exercise 5A

1–3 | 3 | —

UNDERSTANDING

Example 1a Example 1b

1 The expression $3a + 2b + 5c$ has three terms.

a List the terms.

b State the coefficient of:

i a **ii** b **iii** c

c Write another expression with three terms.

Example 1c Example 1d

2 The expression $5a + 7b + c - 3ab + 6$ has five terms.

a State the constant term.

b State the coefficient of:

i a **ii** b **iii** c

c Write another expression that has five terms.

Example 2a Example 2b

3 Match each of the following worded statements with the correct mathematical expression.

a The sum of x and 7	**A** $3 - x$
b 3 less than x	**B** $\frac{x}{3}$
c x is divided by 2	**C** $x - 3$
d x is tripled	**D** $3x$
e x is subtracted from 3	**E** $\frac{x}{2}$
f x is divided by 3	**F** $x + 7$

4–6(½) | 4–7(½) | 4–7(½)

FLUENCY

4 For each of the following expressions:

i state how many terms there are **ii** list the terms

a $7a + 2b + c$ **b** $19y - 52x + 32$

c $a + 2b$ **d** $7u - 3v + 2a + 123c$

e $10f + 2be$ **f** $9 - 2b + 4c + d + e$

g $5 - x^2y + 4abc - 2nk$ **h** $ab + 2bc + 3cd + 4de$

5 For each of the following expressions, state the coefficient of b.

a $3a + 2b + c$ **b** $3a + b + 2c$ **c** $4a + 9b + 2c + d$

d $3a - 2b + f$ **e** $b + 2a + 4$ **f** $2a + 5c$

g $7 - 54c + d$ **h** $5a - 6b + c$ **i** $4a - b + c + d$

j $2a + 4b^2 - 12b$ **k** $7a - b + c$ **l** $8a + c - 3b + d$

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5A

FLUENCY

Example 2c
Example 2d

6 Write an expression for each of the following.

a 7 more than y
b 3 less than x
c The sum of a and b
d The product of 4 and p
e Half of q is subtracted from 4
f One third of r is added to 10
g The sum of b and c multiplied by 2
h The sum of b and twice the value of c
i The product of a, b and c divided by 7
j A quarter of a added to half of b
k The quotient of x and $2y$
l The difference of a and half of b
m The product of k and itself
n The square of w

7 Describe each of the following expressions in words.

a $3 + x$
b $a + b$
c $4 \times b \times c$
d $2a + b$
e $(4 - b) \times 2$
f $4 - 2b$

PROBLEM-SOLVING

8, 9 | 9–11 | 10–12

8 Marcela buys 7 plants from the local nursery.

a If the cost is $\$x$ for each plant, write an expression for the total cost in dollars.
b If the cost of each plant is decreased by \$3 during a sale, write an expression for:
i the new cost per plant in dollars
ii the new total cost in dollars of the 7 plants



9 Francine earns $\$p$ per week for her job. She works for 48 weeks each year. Write an expression for the amount she earns:

a in a fortnight
b in one year
c in one year, if her wage is increased by \$20 per week after she has already worked 30 weeks in the year

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5A

PROBLEM-SOLVING

10 Jon likes to purchase DVDs of some TV shows. One show, *Numbers*, costs $\$a$ per season, and another show, *Proof by Induction*, costs $\$b$ per season. Write an expression for the cost of:

- **a** 4 seasons of *Numbers*
- **b** 7 seasons of *Proof by Induction*
- **c** 5 seasons of both shows
- **d** all 7 seasons of each show, if the total price is halved when purchased in a sale

11 A plumber charges a $70 call-out fee and then $90 per hour. Write an expression for the total cost of calling a plumber out for x hours.

12 A mobile phone call costs 20 cents connection fee and then 50 cents per minute.

- **a** Write an expression for the total cost (in cents) of a call lasting t minutes.
- **b** Write an expression for the total cost (in dollars) of a call lasting t minutes.
- **c** Write an expression for the total cost (in dollars) of a call lasting t hours.

REASONING

13 | 13, 14 | 13(½), 14, 15

13 If x is a positive number, classify the following statements as true or false.

- **a** x is always smaller than $2 \times x$.
- **b** x is always smaller than $x + 2$.
- **c** x is always smaller than x^2.
- **d** $1 - x$ is always less than $4 - x$.
- **e** $x - 3$ is always a positive number.
- **f** $x + x - 1$ is always a positive number.

14 If b is a negative number, classify the following statements as true or false. Give a brief reason.

- **a** $b - 4$ *must* be negative.
- **b** $b + 2$ *could* be negative.
- **c** $b \times 2$ *could* be positive.
- **d** $b + b$ *must* be negative.

15 What is the difference between $2a + 5$ and $2(a + 5)$? Give an English expression to describe each of them. Describe how the brackets change the meaning.

ENRICHMENT

Algebraic alphabet — | — | 16

16 An expression contains 26 terms, one for each letter of the alphabet. It starts $a + 4b + 9c + 16d + 25e + \ldots$

- **a** What is the coefficient of f?
- **b** What is the coefficient of z?
- **c** Which pronumeral has a coefficient of 400?
- **d** One term is removed and now the coefficient of k is zero. What was the term?
- **e** Another expression containing 26 terms starts $a + 2b + 4c + 8d + 16e + \ldots$ What is the sum of all the coefficients?

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5B Substitution and equivalence CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

One common thing to do with algebraic expressions is to replace the pronumerals with numbers. This is referred to as **substitution** or **evaluation**. In the expression $4 + x$ we can substitute $x = 3$ to get the result 7. Two expressions are said to be **equivalent** if they always give the same result when a number is substituted. For example, $4 + x$ and $x + 4$ are equivalent, because $4 + x$ and $x + 4$ will be equal numbers no matter what the value of x is substituted.

Let's start: AFL algebra

In Australian Rules football, the final team score is given by $6x + y$, where x is the number of goals and y is the number of behinds scored.

- State the score if $x = 3$ and $y = 4$.
- If the score is 29, what are the values of x and y? Try to list all the possibilities.
- If $y = 9$ and the score is a two-digit number, what are the possible values of x?



Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value.
  For example, if $a = 3$ and $b = 4$, then we can evaluate the expression $7a + 2b + 5$:
  $$\begin{aligned} 7a + 2b + 5 &= 7(3) + 2(4) + 5 \\ &= 21 + 8 + 5 \\ &= 34 \end{aligned}$$
- Two expressions are **equivalent** if they have equal values regardless of the number that is substituted for each pronumeral. The laws of arithmetic help to determine equivalence.
  - The **commutative** laws of arithmetic tell us that $a + b = b + a$ and $a \times b = b \times a$ for all values of a and b.
  - The **associative** laws of arithmetic tell us that $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$ for all values of a, b and c.

Example 3 Substituting values

Substitute $x = 3$ and $y = 6$ to evaluate the following expressions.

a $5x$ **b** $5x^2 + 2y + x$

SOLUTION	EXPLANATION
a $5x = 5(3)$ $= 15$	Remember that 5(3) is another way of writing 5×3.

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b $5x^2 + 2y + x = 5(3)^2 + 2(6) + (3)$
$= 5(9) + 12 + 3$
$= 45 + 12 + 3$
$= 60$

Replace all the pronumerals by their values and remember the order in which to evaluate (multiplication before addition).

Example 4 Deciding if expressions are equivalent

a Are $x - 3$ and $3 - x$ equivalent expressions?
b Are $a + b$ and $b + 2a - a$ equivalent expressions?

SOLUTION	EXPLANATION
a No.	The two expressions are equal if $x = 3$ (both equal zero). But if $x = 7$ then $x - 3 = 4$ and $3 - x = -4$. Because they are not equal for every single value of x, they are not equivalent.
b Yes.	Regardless of the values of a and b substituted, the two expressions are equal. It is not possible to check every single number but we can check a few to be reasonably sure they seem equivalent. For instance, if $a = 3$ and $b = 5$, then $a + b = 8$ and $b + 2a - a = 8$. If $a = 17$ and $b = -2$ then $a + b = 15$ and $b + 2a - a = 15$.

Exercise 5B

1–4 | 4 | —

UNDERSTANDING

1 What number is obtained when $x = 5$ is substituted into the expression $3 \times x$?

2 What is the result of evaluating $20 - b$ if b is equal to 12?

3 What is the value of $a + 2b$ if a and b both equal 10?

4 **a** State the value of $4 + 2x$ if $x = 5$.
b State the value of $40 - 2x$ if $x = 5$.
c Are $4 + 2x$ and $40 - 2x$ equivalent expressions?

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5B

FLUENCY

5–6(½), 9(½) | 5–10(½) | 6–10(½)

Example 3a

5 Substitute the following values of x into the expression $7x + 2$.

a 4 **b** 5 **c** 2 **d** 8
e 0 **f** –6 **g** –9 **h** –3

Example 3b

6 Substitute $a = 4$ and $b = -3$ into each of the following.

a $5a + 4$ **b** $3b$ **c** $a + b$
d $ab - 4 + b$ **e** $2 \times (3a + 2b)$ **f** $100 - (10a + 10b)$
g $\frac{12}{a} + \frac{6}{b}$ **h** $\frac{ab}{3} + b$ **i** $\frac{100}{a + b}$
j $a^2 + b$ **k** $5 \times (b + 6)^2$ **l** $a - 4b$

7 Evaluate the expression $2x - 3y$ when:

a $x = 10$ and $y = 4$ **b** $x = 1$ and $y = -10$ **c** $x = 0$ and $y = -2$
d $x = -10$ and $y = -6$ **e** $x = -7$ and $y = -9$ **f** $x = -2$ and $y = 9$

8 Evaluate the expression $4ab - 2b + 6c$ when:

a $a = 4$ and $b = 3$ and $c = 9$ **b** $a = -8$ and $b = -2$ and $c = 9$
c $a = -1$ and $b = -8$ and $c = -4$ **d** $a = 9$ and $b = -2$ and $c = 5$
e $a = -8$ and $b = -3$ and $c = 5$ **f** $a = -1$ and $b = -3$ and $c = 6$

Example 4

9 For the following state whether they are equivalent (E) or not (N).

a $x + y$ and $y + x$ **b** $3 \times x$ and $x + 2x$ **c** $4a + b$ and $4b + a$
d $7 - x$ and $4 - x + 3$ **e** $4(a + b)$ and $4a + b$ **f** $4 + 2x$ and $2 + 4x$
g $\frac{1}{2} \times a$ and $\frac{a}{2}$ **h** $3 + 6y$ and $3(2y + 1)$

10 For each of the following, two of the three expressions are equivalent. State the odd one out.

a $4x$, $3 + x$ and $3x + x$ **b** $2 - a$, $a - 2$ and $a + 1 - 3$
c $5t - 2t$, $2t + t$ and $4t - 2t$ **d** $8u - 3$, $3u - 8$ and $3u - 3 + 5u$

PROBLEM-SOLVING

11 | 11, 12 | 11, 13

11 Give three expressions that are equivalent to $2x + 4y + 5$.

12 Copy and complete the following table.

x	3		0.25		–2	
$4x + 2$	14	6				
$4 - 3x$	–5					–2
$2x - 4$				8		

13 Assume that a and b are two integers (positive, negative or zero).

a List the values they could have if you know that $ab = 10$.
b What values could they have if you know that $a + b = 10$?
c List the values they could have if you know that $ab = a + b$.

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5B

14 | 14, 15 | 16–18

REASONING

14 **a** Is it possible to substitute values of x and y so that $x + y$ and $x + 2y$ are equal? Try to describe all possible solutions.

b Does this imply that $x + y$ and $x + 2y$ are equivalent? Give reasons.

15 **a** Give an example to show that $a \div (b \times c)$ is not equivalent to $(a \div b) \times c$.

b Does this contradict the laws of associativity (see **Key ideas**)? Justify your answer.

c Is $a \div (b \div c)$ equivalent to $(a \div b) \div c$? Why or why not?

16 The expressions $4 - x$ and $x - 4$ are not equivalent because they are not always equal (for instance, when $x = 3$). They are sometimes equal (for instance, when $x = 4$).

a Give an example of two expressions that are only equal if $a = 5$.

b Give an example of two expressions that are only equal if $a = b$.

c Give an example of two expressions that are never equal, regardless of the value of the variables involved.

17 **a** By substituting a range of numbers for a and b, determine whether $(ab)^2$ is equivalent to a^2b^2.

b Is $(a + b)^2$ equivalent to $a^2 + b^2$?

c Is $\sqrt{ab}$ equivalent to $\sqrt{a}\sqrt{b}$?

d Is $\sqrt{a + b}$ equivalent to $\sqrt{a} + \sqrt{b}$?

e For pairs of expressions in **a** to **d** that are not equivalent, find an example of values for a and b that make the expressions equal.

18 Sometimes when two expressions are equivalent you can explain why they are equivalent. For example, $x + y$ is equivalent to $y + x$ 'because addition is commutative'. For each of the following pairs of expressions, try to describe why they are equivalent.

a $x \times y$ and $y \times x$

b $x + x$ and $2x$

c $y - y$ and 0

d $\frac{1}{2} \times x$ and $x \div 2$

Missing values

— | — | 19

ENRICHMENT

19 Find the missing values in the table below.

a	5	8			–20			
b	2		1					
a* + *b		10		10		7		–19
a* + 2*b				17	0		11	
a* – *b			1			13		
a* – 2*b							–29	20

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5C Adding and subtracting terms

Interactive

Widgets

HOTsheets

Walkthroughs

Recall from Year 7 that an expression such as $3x + 5x$ can be simplified to $8x$, but an expression such as $3x + 5y$ cannot be simplified. The reason is that $3x$ and $5x$ are **like terms** – they have exactly the same pronumerals. The terms $3x$ and $5y$ are not like terms. Also, $4ab$ and $7ba$ are like terms because ab and ba are equivalent, as multiplication is commutative. However, a^2b, ab, and ab^2 are all unlike terms, since a^2b means $a \times a \times b$, which is different from $a \times b$ and $a \times b \times b$.

Let's start: Like terms

- Put these terms into groups of like terms: $4a$ $5b$ $2ab$ $3ba$ $2a$ $7b^2$ $5a^2b$ $9aba$
- What is the sum of each group?
- Ephraim groups $5a^2b$ and $2ab$ as like terms, so he simplifies $5a^2b + 2ab$ to $7ab$. How could you demonstrate to him that $5a^2b + 2ab$ is not equivalent to $7ab$?

Key ideas

- **Like terms** contain exactly the same pronumerals with the same powers; the pronumerals do not need to be in the same order, for example, $4ab$ and $7ba$ are like terms.
- Like terms can be combined when they are added or subtracted to simplify an expression, for example, $3xy + 5xy = 8xy$.
- The sign in front of the term stays with the term even when it is moved.

$3x + 7y \boxed{-2x} + 3y \boxed{+x} - 4y = 3x - 2x + x + 7y + 3y - 4y$
$= 2x + 6y$

Example 5 Identifying like terms

a Are $5abc$ and $-8abc$ like terms?
b Are $12xy^2$ and $4y^2x$ like terms?
c Are $3ab^2$ and $7a^2b$ like terms?

SOLUTION	EXPLANATION
a Yes.	Both terms have exactly the same pronumerals: a, b and c.
b Yes.	When written out in full $12xy^2$ is $12 \times x \times y \times y$ and $4y^2x$ is $4 \times y \times y \times x$. Both terms include one x and two occurrences of y being multiplied, and the order of multiplication does not matter.
c No.	$3ab^2 = 3 \times a \times b \times b$ and $7a^2b = 7 \times a \times a \times b$. They are not like terms because the first includes only one a and the second includes two.

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Example 6 Simplifying by combining like terms

Simplify the following by combining like terms.

a $7t + 2t - 3t$ **b** $4x + 3y + 2x + 7y$ **c** $7ac + 3b - 2ca + 4b - 5b$

SOLUTION	EXPLANATION
a $7t + 2t - 3t = 6t$	These are like terms, so they can be combined: $7 + 2 - 3 = 6$.
b $4x + 3y + 2x + 7y$ $= 4x + 2x + 3y + 7y$ $= 6x + 10y$	Move the like terms next to each other. Combine the pairs of like terms.
c $7ac + 3b - 2ca + 4b - 5b$ $= 7ac - 2ca + 3b + 4b - 5b$ $= 5ac + 2b$	Move the like terms together. Recall that the subtraction sign stays in front of $2ca$ even when it is moved. $7 - 2 = 5$ and $3 + 4 - 5 = 2$

Exercise 5C

1, 2 | 1, 2 | 3

UNDERSTANDING

1 **a** If $x = 3$, evaluate $5x + 2x$.
b If $x = 3$, evaluate $7x$.
c $5x + 2x$ is equivalent to $7x$. True or false?

2 **a** If $x = 3$ and $y = 4$, evaluate $5x + 2y$.
b If $x = 3$ and $y = 4$, evaluate $7xy$.
c $5x + 2y$ is equivalent to $7xy$. True or false?

3 **a** Substitute $x = 4$ into the expression $10x - 5x + 2x$.
b Substitute $x = 4$ into:
i $3x$ **ii** $5x$ **iii** $7x$
c Which one of the expressions in part **b** is equivalent to $10x - 5x + 2x$?

4–7(½) | 4–8(½) | 5–8(½)

FLUENCY

Example 5a

4 Classify the following pairs as like terms (L) or not like terms (N).
a $3a$ and $5a$ **b** $7x$ and $-12x$ **c** $2y$ and $7y$ **d** $4a$ and $-3b$
e $7xy$ and $3y$ **f** $12ab$ and $4ba$ **g** $3cd$ and $-8c$ **h** $2x$ and $4xy$

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5C

FLUENCY

Example 5b
Example 5c

5 Classify the following pairs as like terms (L) or not like terms (N).

a $-3x^2y$ and $5x^2y$ **b** $12ab^2$ and $10b^2a$ **c** $2ab^2$ and $10ba^2$

d $7qrs$ and $-10rqs$ **e** $11q^2r$ and $10rq$ **f** $-15ab^2c$ and $-10cba^2$

Example 6a

6 Simplify the following by combining like terms.

a $3x + 2x$ **b** $7a + 12a$ **c** $15x - 6x$

d $4xy + 3xy$ **e** $16uv - 3uv$ **f** $10ab + 4ba$

g $11ab - 5ba + ab$ **h** $3k + 15k - 2k$ **i** $15k - 2k - 3k$

Example 6b
Example 6c

7 Simplify the following by combining like terms.

a $7f + 2f + 8 + 4$ **b** $10x + 3x + 5y + 3y$

c $2a + 5a + 13b - 2b$ **d** $10a + 5b + 3a + 4b$

e $10 + 5x + 2 + 7x$ **f** $10a + 3 + 4b - 2a - b$

g $10x + 31y - y + 4x$ **h** $11a + 4 - 2a + 12a$

i $7x^2y + 5x + 10yx^2$ **j** $12xy - 3yx + 5xy - yx$

k $-4x^2 + 3x^2$ **l** $-2a + 4b - 7ab + 4a$

m $10 + 7q - 3r + 2q - r$ **n** $11b - 3b^2 + 5b^2 - 2b$

8 For each expression choose an equivalent expression from the options listed.

a $7x + 2x$ **A** $10y + 3x$

b $12y + 3x - 2y$ **B** $9xy$

c $3x + 3y$ **C** $9x$

d $8y - 2x + 6y - x$ **D** $3y + 3x$

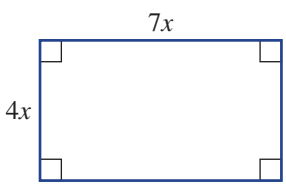
e $4xy + 5yx$ **E** $14y - 3x$

9, 10 10–12 12–14

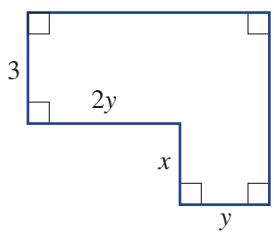
PROBLEM-SOLVING

9 Write expressions for the perimeters of the following shapes in simplest form.

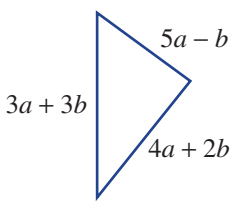
a



b



c



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5C

PROBLEM-SOLVING

10 Towels cost $\$c$ each at a shop.



a John buys 3 towels, Mary buys 6 towels and Naomi buys 4 towels. Write a fully simplified expression for the total amount spent on towels.

b On another occasion, Chris buys n towels, David buys twice as many as Chris and Edward buys 3 times as many as David. Write a simplified expression for the total amount they spent on towels.

11 State the missing numbers to make the following equivalences true.

a $10x + 6y - \square x + \square y = 3x + 8y$

b $5a - 7b + \square a + \square b = 11a$

c $\square c + \square d + \square = 4c + 2d + 1 + 3c + 7d + 4$

d $\square a^2b + \square b^2a + 2a^2b + b^2a = 7b^2a + 10a^2b$

12 Add the missing expressions to the puzzle to make all six equations true.

$5x$	+		=	$7x$
+		+		+
y	+		=	
=		=		=
	+		=	$7x + 3y$

13 In how many ways could the blanks below be filled if all the coefficients must be positive integers?

$\square a + \square b + \square a = 10a + 7b$

14 Simplify $a - 2a + 3a - 4a + 5a - 6a + \ldots + 99a - 100a$.

15 | 15, 16 | 16, 17

REASONING

15 Prove that $10x + 5y + 7x + 2y$ is equivalent to $20x - 3x + 10y - 3y$. Hint: Simplify both expressions.

16 **a** Make a substitution to prove that $4a + 3b$ is not equivalent to $7ab$.

b Is $4a + 3b$ ever equal to $7ab$? Try to find some values of a and b to make $4a + 3b = 7ab$ a true equation.

c Is $4a + 3a$ ever not equal to $7a$? Explain your answer.

17 **a** Decide whether $7x - 3x$ is equivalent to $7x + (-3x)$. Explain why or why not.

b Fill in the missing numbers to make the following equivalence true.

$14a + 3b + 2ab + \square a + \square b + \square ba = a$

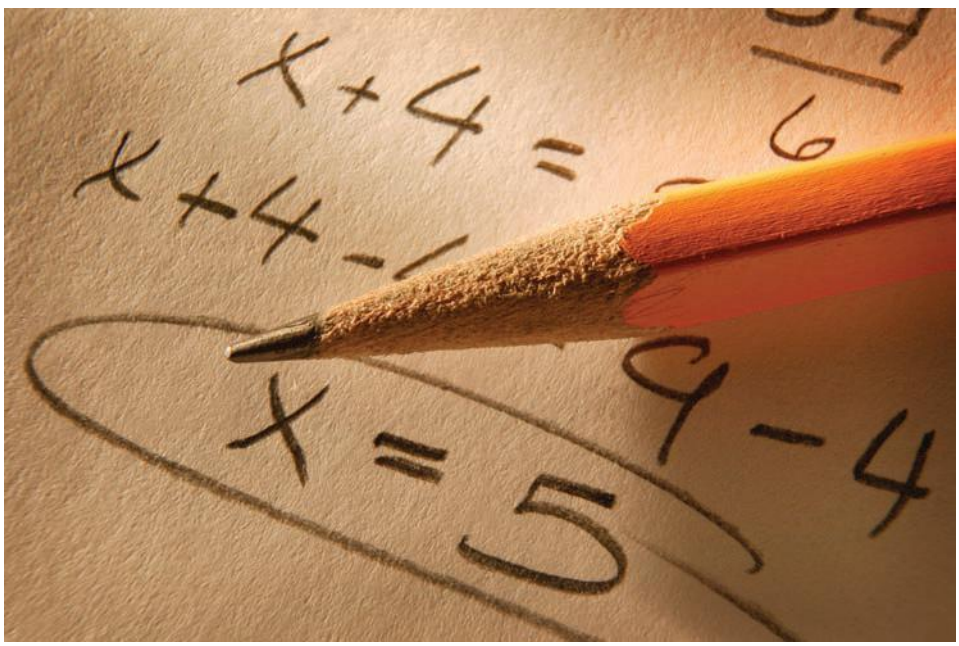
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ENRICHMENT

18 a Fill in the missing expressions to make all 8 equations true.

$5a$	+		+	$4a$	=	$10a + 3$
+		+		+		+	
$2b$	+	$4 + a$	+		=		
+		+		+		+	
	+		+	$-3a$	=	1
=		=		=		=	
	+	$3a + 8$	+		=	$10a + 2b + 8$

b Design your own 'missing values' puzzle like the one above. It should only have one possible solution.



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5D Multiplying and dividing terms









Recall that a term written as $4ab$ is shorthand for $4 \times a \times b$. Observing this helps to see how we can multiply terms.

$$\begin{aligned} 4ab \times 3c &= 4 \times a \times b \times 3 \times c \\ &= 4 \times 3 \times a \times b \times c \\ &= 12abc \end{aligned}$$

Division is written as a fraction so $\frac{12ab}{9ad}$ means $(12ab) \div (9ad)$. To simplify a division we look for common factors:

$$\frac{{}^{4}\cancel{12} \times \cancel{a} \times b}{{}^{3}\cancel{9} \times \cancel{a} \times d} = \frac{4b}{3d}$$

$a \div a = 1$ for any value of a except 0

So $\frac{a}{a}$ cancels to 1.

Let's start: Multiple ways

Multiplying $4a \times 6b \times c$ gives you $24abc$.

- In how many ways can positive integers fill the blanks in $\square a \times \square b \times \square c = 24abc$?
- In how many other ways can you multiply three terms to get $24abc$? For example, $12ab \times 2 \times c$. You should assume the coefficients are all integers.

Key ideas

- $12abc$ means $12 \times a \times b \times c$.
- When multiplying, the order is not important: $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$.
- x^2 means $x \times x$ and x^3 means $x \times x \times x$.
- When dividing, cancel any common factors.

For example: $\frac{{}^{3}\cancel{15}xy}{{}^{4}\cancel{20}yz} = \frac{3x}{4z}$

Example 7 Multiplying and dividing terms

a Simplify $7a \times 2bc \times 3d$.

b Simplify $3xy \times 5xz$.

c Simplify $\frac{10ab}{15bc}$.

d Simplify $\frac{18x^2y}{8xz}$.

SOLUTION

a $7a \times 2bc \times 3d$
$= 7 \times a \times 2 \times b \times c \times 3 \times d$
$= 7 \times 2 \times 3 \times a \times b \times c \times d$
$= 42abcd$

EXPLANATION

Write the expression with multiplication signs and bring the numbers to the front.

Simplify: $7 \times 2 \times 3 = 42$ and $a \times b \times c \times d = abcd$

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b $3xy \times 5xz$
$= 3 \times x \times y \times 5 \times x \times z$
$= 3 \times 5 \times x \times x \times y \times z$
$= 15x^2yz$

Write the expression with multiplication signs and bring the numbers to the front.

Simplify, remembering that $x \times x = x^2$.

c $\frac{10ab}{15bc} = \frac{{}^{2}\cancel{10} \times a \times \cancel{b}}{{}^{3}\cancel{15} \times \cancel{b} \times c}$
$= \frac{2a}{3c}$

Write the numerator and denominator in full, with multiplication signs. Cancel any common factors and remove the multiplication signs.

d $\frac{18x^2y}{8xz} = \frac{{}^{9}\cancel{18} \times \cancel{x} \times x \times y}{{}^{4}\cancel{8} \times \cancel{x} \times z}$
$= \frac{9xy}{4z}$

Write the numerator and denominator in full, remembering that x^2 is $x \times x$. Cancel any common factors and remove the multiplication signs.

Exercise 5D

1–4 | 4 | —

UNDERSTANDING

1 Which is the correct way to write $3 \times a \times b \times b$?
A $3ab$ **B** $3ab^2$ **C** ab^3 **D** $3a^2b$

2 Simplify these fractions by looking for common factors in the numerator and the denominator.
a $\frac{12}{20}$ **b** $\frac{5}{15}$ **c** $\frac{12}{8}$ **d** $\frac{15}{25}$

3 Which one of these is equivalent to $a \times b \times a \times b \times b$?
A $5ab$ **B** a^2b^3 **C** a^3b^2 **D** $(ab)^5$

4 Write these without multiplication signs.
a $3 \times x \times y$ **b** $5 \times a \times b \times c$ **c** $12 \times a \times b \times b$ **d** $4 \times a \times c \times c \times c$

5–7(½) | 5–7(½) | 6–7(½)

FLUENCY

Example 7a

5 Simplify the following.
a $7d \times 9$ **b** $5a \times 2b$ **c** $3 \times 12x$
d $4a \times 2b \times cd$ **e** $3a \times 10bc \times 2d$ **f** $4a \times 6de \times 2b$

Example 7b

6 Simplify the following.
a $8ab \times 3c$ **b** $a \times a$ **c** $3d \times d$
d $5d \times 2d \times e$ **e** $7x \times 2y \times x$ **f** $5xy \times 2x$
g $4xy \times 2xz$ **h** $4abc \times 2abd$ **i** $12x^2y \times 4x$
j $9ab \times 2a^2$ **k** $3x^2y \times 2x \times 4y$ **l** $-3xz \times (-2z)$
m $-5xy \times 2yz$ **n** $10ab^2 \times 7ba$ **o** $4xy^2 \times 4y$

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5D

Example 7c
Example 7d

FLUENCY

7 Simplify the following divisions by cancelling any common factors.

a $\frac{5a}{10a}$ **b** $\frac{7x}{14y}$ **c** $\frac{10xy}{12y}$ **d** $\frac{ab}{4b}$

e $\frac{7xyz}{21yz}$ **f** $\frac{2}{12x}$ **g** $\frac{-5x}{10yz^2}$ **h** $\frac{12y^2}{-18y}$

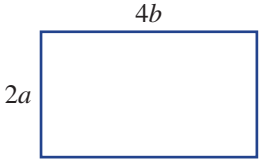
i $\frac{-4a^2}{8ab}$ **j** $\frac{21p}{-3q}$ **k** $\frac{-21p}{-3p}$ **l** $\frac{-15z}{-20z^2}$

8–9(½) | 8, 9 | 9(½), 10

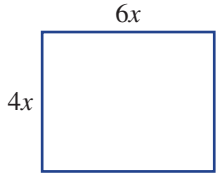
PROBLEM-SOLVING

8 Write a simplified expression for the area of the following shapes. Recall that rectangle area = width × length.

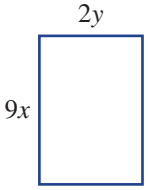
a



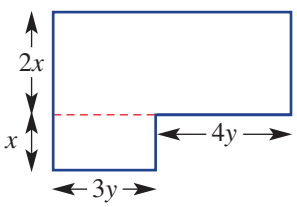
b



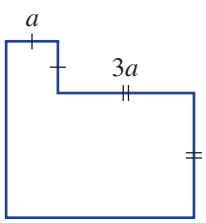
c



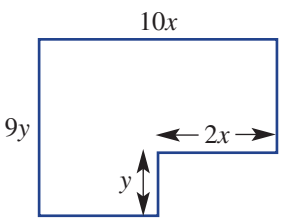
d



e



f



9 Fill in the missing terms to make the following equivalences true.

a $3x \times \square \times z = 6xyz$ **b** $4a \times \square = 12ab^2$ **c** $-2q \times \square \times 4s = 16qs$

d $\frac{\square}{4r} = 7s$ **e** $\frac{\square}{2ab} = 4b$ **f** $\frac{14xy}{\square} = -2y$

10 A box has a height of x cm. It is 3 times as wide as it is high, and 2 times as long as it is wide. Find an expression for the volume of the box, given that volume = length × width × height.



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5D

	11	11, 12	11–13

REASONING

11 A square has a width of x cm.

a State its area in terms of x.

b State its perimeter in terms of x.

c Prove that its area divided by its perimeter is equal to a quarter of its width.

12 Joanne claims that the following three expressions are equivalent: $\frac{2a}{5}, \frac{2}{5} \times a, \frac{2}{5a}$.

a Is she right? Try different values of a.

b Which two expressions are equivalent?

c There are two values of a that make all three expressions are equal. What are they?

13 Note that $\frac{14xy}{7x} = 2y$ and $7x \times 2y = 14xy$.

a You are told that $12x^2y^3 \times 3x^5$ is equivalent to $36x^7y^3$. What does $\frac{36x^7y^3}{12x^2y^3}$ simplify to?

b You are told that $-7a^5b \times -3b^2c^3$ is equivalent to $21a^5(bc)^3$. What does $\frac{21a^5(bc)^3}{-7a^5b}$ simplify to?

c Describe how you can find the missing value in this puzzle: term 1 $\times$ $\square$ = term 2.

Multiple operations	—	—	14

ENRICHMENT

14 Simplify the following expressions, remembering that you can combine like terms when adding or subtracting.

a $\frac{2ab \times 3bc \times 4cd}{4a \times 3bc \times 2d}$

b $\frac{12a^2b + 4a^2b}{4b + 2b}$

c $\frac{7x^2y - 5yx^2}{12xy}$

d $\frac{8a^2b + (4a \times 2ba)}{3ba - 2ba}$

e $\frac{10abc + 5cba + 5a \times bc}{4c \times 10ab}$

f $\frac{10x^2y - (4x \times 6xy)}{7xy^2}$

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5E Adding and subtracting algebraic fractions

EXTENDING









An algebraic fraction is a fraction that could include any algebraic expression in the numerator or the denominator.

$\frac{2}{3}$ $\frac{5}{7}$ $\frac{20}{3}$ — Fractions

$\frac{2x}{5}$ $\frac{3}{7a+4}$ $\frac{2x-46}{7a+9b}$ — Algebraic fractions

The rules for working with algebraic fractions are the same as the rules for normal fractions. For example, two fractions with the same denominator can be added or subtracted easily.

Normal fractions	Algebraic fractions
$\frac{2}{13}+\frac{7}{13}=\frac{9}{13}$	$\frac{5x}{13}+\frac{3x}{13}=\frac{8x}{13}$
$\frac{8}{11}-\frac{2}{11}=\frac{6}{11}$	$\frac{5x}{11}-\frac{2y}{11}=\frac{5x-2y}{11}$

If two fractions do not have the same denominator, they must be converted to have the lowest common denominator (LCD) before adding or subtracting.

Normal fractions	Algebraic fractions
$\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}$ $=\frac{13}{15}$	$\frac{2a}{3}+\frac{b}{5}=\frac{10a}{15}+\frac{3b}{15}$ $=\frac{10a+3b}{15}$

Let's start: Adding thirds and halves

Dallas and Casey attempt to simplify $\frac{x}{3}+\frac{x}{2}$. Dallas gets $\frac{x}{5}$ and Casey gets $\frac{5x}{6}$.

- Which of the two students has the correct answer? You could try substituting different numbers for x.
- How can you prove that the other student is incorrect?
- What do you think $\frac{x}{3}+\frac{x}{4}$ is equivalent to? Compare your answers with others in the class.

Key ideas

- An **algebraic fraction** is a fraction with an algebraic expression as the numerator or the denominator.
- The **lowest common denominator** (or LCD) of two algebraic fractions is the smallest multiple of the denominators.
- Adding and subtracting algebraic fractions requires that they both have the same denominator, for example, $\frac{2x}{5}+\frac{4y}{5}=\frac{2x+4y}{5}$.

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Example 8 Working with denominators

a Find the lowest common denominator of $\frac{3x}{10}$ and $\frac{2y}{15}$.

b Convert $\frac{2x}{7}$ to an equivalent algebraic fraction with the denominator 21.

SOLUTION	EXPLANATION
a 30	The multiples of 10 are 10, 20, 30, 40, 50, 60 etc. The multiples of 15 are 15, 30, 45, 60, 75, 90 etc. The smallest number in both lists is 30.
b $\frac{2x}{7} = \frac{3 \times 2x}{3 \times 7}$ $= \frac{6x}{21}$	Multiply the numerator and denominator by 3, so that the denominator is 21. Simplify the numerator: $3 \times 2x$ is $6x$.

Example 9 Adding and subtracting algebraic fractions

Simplify the following expressions.

a $\frac{3x}{11} + \frac{5x}{11}$ **b** $\frac{4a}{3} + \frac{2a}{5}$ **c** $\frac{6k}{5} - \frac{3k}{10}$ **d** $\frac{a}{6} - \frac{b}{9}$

SOLUTION	EXPLANATION
a $\frac{3x}{11} + \frac{5x}{11} = \frac{3x + 5x}{11}$ $= \frac{8x}{11}$	The two fractions have the same denominators, so the two numerators are added. $3x$ and $5x$ are like terms, so they are combined to $8x$.
b $\frac{4a}{3} + \frac{2a}{5} = \frac{5 \times 4a}{15} + \frac{3 \times 2a}{15}$ $= \frac{20a}{15} + \frac{6a}{15}$ $= \frac{26a}{15}$	The LCD = 15, so both fractions are converted to have 15 as the denominator. Simplify the numerators. Combine: $20a + 6a$ is $26a$.

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c $\frac{6k}{5} - \frac{3k}{10} = \frac{12k}{10} - \frac{3k}{10}$
$= \frac{12k - 3k}{10}$
$= \frac{9k}{10}$

LCD = 10, so convert the first fraction (multiplying numerator and denominator by 2).

Combine the numerators.

Simplify: $12k - 3k = 9k$.

d $\frac{a}{6} - \frac{b}{9} = \frac{3a}{18} - \frac{2b}{18}$
$= \frac{3a - 2b}{18}$

LCD = 18, so convert both fractions to have 18 as a denominator.

Combine the numerators. Note this cannot be further simplified since $3a$ and $2b$ are not like terms.

Exercise 5E

1–4 | 3–4(½) | —

UNDERSTANDING

1 Complete these sentences.

a For the fraction $\frac{2}{7}$ the numerator is 2 and the denominator is _____.

b For $\frac{4}{9}$ the numerator is _____ and the denominator is _____.

c The expression $\frac{12x}{5}$ is an example of an _____ fraction.

d The denominator of $\frac{5x+3}{7}$ is _____.

2 Find the lowest common denominator (LCD) of the following pairs of fractions.

a $\frac{1}{3}$ and $\frac{2}{5}$ **b** $\frac{1}{4}$ and $\frac{1}{5}$ **c** $\frac{3}{7}$ and $\frac{5}{6}$ **d** $\frac{2}{3}$ and $\frac{1}{6}$

3 Find the missing numerator to make the following equations true.

a $\frac{2}{3} = \frac{\square}{6}$ **b** $\frac{4}{7} = \frac{\square}{21}$ **c** $\frac{1}{3} = \frac{\square}{12}$ **d** $\frac{6}{11} = \frac{\square}{55}$

4 Evaluate the following, by first converting to a lowest common denominator.

a $\frac{1}{4} + \frac{1}{3}$ **b** $\frac{2}{7} + \frac{1}{5}$ **c** $\frac{1}{10} + \frac{1}{5}$ **d** $\frac{2}{5} - \frac{1}{4}$

5–8(½) | 5–9(½) | 6–9(½)

FLUENCY

Example 8a

5 Find the LCD of the following pairs of algebraic fractions.

a $\frac{x}{3}$ and $\frac{2y}{5}$ **b** $\frac{3x}{10}$ and $\frac{21y}{20}$ **c** $\frac{x}{4}$ and $\frac{y}{5}$ **d** $\frac{x}{12}$ and $\frac{y}{6}$

Example 8b

6 Copy and complete the following, to make each equation true.

a $\frac{x}{5} = \frac{\square}{10}$ **b** $\frac{2a}{7} = \frac{\square}{21}$ **c** $\frac{4z}{5} = \frac{\square}{20}$ **d** $\frac{3k}{10} = \frac{\square}{50}$

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5E

FLUENCY

Example 9a Example 9b

7 Simplify the following sums.

a $\frac{x}{4}+\frac{2x}{4}$ **b** $\frac{5a}{3}+\frac{2a}{3}$ **c** $\frac{2b}{5}+\frac{b}{5}$ **d** $\frac{4k}{3}+\frac{k}{3}$

e $\frac{a}{2}+\frac{a}{3}$ **f** $\frac{a}{4}+\frac{a}{5}$ **g** $\frac{p}{2}+\frac{p}{5}$ **h** $\frac{q}{4}+\frac{q}{2}$

i $\frac{2k}{5}+\frac{3k}{7}$ **j** $\frac{2m}{5}+\frac{2m}{3}$ **k** $\frac{7p}{6}+\frac{2p}{5}$ **l** $\frac{x}{4}+\frac{3x}{8}$

Example 9c

8 Simplify the following differences.

a $\frac{3y}{5}-\frac{y}{5}$ **b** $\frac{7p}{13}-\frac{2p}{13}$ **c** $\frac{10r}{7}-\frac{2r}{7}$ **d** $\frac{8q}{5}-\frac{2q}{5}$

e $\frac{p}{2}-\frac{p}{3}$ **f** $\frac{2t}{5}-\frac{t}{3}$ **g** $\frac{9u}{11}-\frac{u}{2}$ **h** $\frac{8y}{3}-\frac{5y}{6}$

i $\frac{r}{3}-\frac{r}{2}$ **j** $\frac{6u}{7}-\frac{7u}{6}$ **k** $\frac{9u}{1}-\frac{3u}{4}$ **l** $\frac{5p}{12}-\frac{7p}{11}$

Example 9d

9 Simplify the following expressions, giving your final answer as an algebraic fraction.

Hint: $4x$ is the same as $\frac{4x}{1}$.

a $4x+\frac{x}{3}$ **b** $3x+\frac{x}{2}$ **c** $\frac{a}{5}+2a$

d $\frac{8p}{3}-2p$ **e** $\frac{10u}{3}+\frac{3v}{10}$ **f** $\frac{7y}{10}-\frac{2x}{5}$

g $2t+\frac{7p}{2}$ **h** $\frac{x}{3}-y$ **i** $5-\frac{2x}{7}$

10 | 10, 11 | 11, 12

PROBLEM-SOLVING

10 Cedric earns an unknown amount $\$x$ every week. He spends $\frac{1}{3}$ of his income on rent and $\frac{1}{4}$ on groceries.

a Write an algebraic fraction for the amount of money he spends on rent.

b Write an algebraic fraction for the amount of money he spends on groceries.

c Write a simplified algebraic fraction for the total amount of money he spends on rent and groceries.

11 Egan fills the bathtub so it is a quarter full and then adds half a bucket of water. A full bathtub can contain T litres and a bucket contains B litres.



a Write the total amount of water in the bathtub as the sum of two algebraic fractions.

b Simplify the expression in part **a** to get a single algebraic fraction.

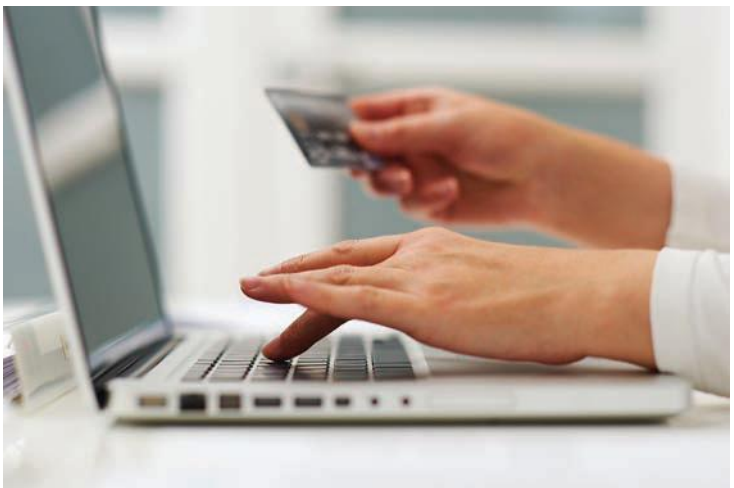
c If a full bathtub contains 1000 litres and the bucket contains 2 litres, how many litres of water are in the bathtub?

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5E

PROBLEM-SOLVING

12 Afshin's bank account is halved in value and then $20 is removed. If it initially had $\$A$ in it, write an algebraic fraction for the amount left.



13a, b | 13 | 13, 14

REASONING

13 a Illustrate that $\frac{x}{2}+\frac{x}{3}$ is equivalent to $\frac{5x}{6}$ by substituting at least three different values for x.

b Show that $\frac{x}{4}+\frac{x}{5}$ is not equivalent to $\frac{2x}{9}$.

c Is $\frac{x}{2}+\frac{x}{5}$ equivalent to $x-\frac{x}{3}$? Explain why or why not.

14 a Simplify the following differences.

i $\frac{x}{2}-\frac{x}{3}$ **ii** $\frac{x}{3}-\frac{x}{4}$ **iii** $\frac{x}{4}-\frac{x}{5}$ **iv** $\frac{x}{5}-\frac{x}{6}$

b What patterns did you notice in the above results?

c Write a difference of two algebraic fractions that simplifies to $\frac{x}{110}$.

Equivalent sums and differences | — | — | 15

ENRICHMENT

15 For each of the following expressions, find a single equivalent algebraic fraction.

a $\frac{z}{4}+\frac{z}{3}+\frac{z}{12}$ **b** $\frac{2x}{5}+\frac{x}{2}-\frac{x}{5}$ **c** $\frac{7u}{2}+\frac{3u}{4}-\frac{5u}{8}$ **d** $\frac{8k}{3}+\frac{k}{6}-\frac{5k}{12}$

e $\frac{p}{4}+\frac{p}{2}-3$ **f** $\frac{u}{3}+\frac{u}{4}+\frac{u}{5}$ **g** $\frac{5j}{12}-\frac{j}{3}+2$ **h** $\frac{7t}{5}-\frac{t}{3}+\frac{2r}{15}$

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5F Multiplying and dividing algebraic fractions EXTENDING









As with fractions, it is generally easier to multiply and divide algebraic fractions than it is to add or subtract them.

$$\frac{3}{5}\times\frac{2}{7}=\frac{6}{35}\leftarrow\begin{matrix}3\times2\\5\times7\end{matrix}$$

Fractions

$$\frac{4x}{7}\times\frac{2y}{11}=\frac{8xy}{77}$$

Algebraic fractions

Dividing is done by multiplying by the reciprocal of the second fraction.

$$\frac{4}{5}\div\frac{1}{3}=\frac{4}{5}\times\frac{3}{1}$$
$$=\frac{12}{5}$$

Fractions

$$\frac{2x}{5}\div\frac{3y}{7}=\frac{2x}{5}\times\frac{7}{3y}$$
$$=\frac{14x}{15y}$$

Algebraic fractions

Let's start: Always the same

One of these four expressions always gives the same answer, no matter what the value of x is.

$$\frac{x}{2}+\frac{x}{3}\qquad\frac{x}{2}-\frac{x}{3}\qquad\frac{x}{2}\times\frac{x}{3}\qquad\frac{x}{2}\div\frac{x}{3}$$

- Which of the four expressions always has the same value?
- Can you explain why this is the case?
- Try to find an expression involving two algebraic fractions that is equivalent to $\frac{3}{8}$.

Key ideas

- To multiply two algebraic fractions, multiply the **numerators** and the **denominators** separately. Then cancel any common factors in the numerator and the denominator. For example:

$$\frac{2x}{5}\times\frac{10y}{3}=\frac{\cancel{20}^{4}xy}{\cancel{15}^{3}}$$
$$=\frac{4xy}{3}$$

- The **reciprocal** of an algebraic fraction is formed by swapping the numerator and denominator.
  The reciprocal of $\frac{3b}{4}$ is $\frac{4}{3b}$.
- To divide algebraic fractions, take the reciprocal of the second fraction and then multiply. For example:

$$\frac{2a}{5}\div\frac{3b}{4}=\frac{2a}{5}\times\frac{4}{3b}$$
$$=\frac{8a}{15b}$$

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Example 10 Multiplying algebraic fractions

Simplify the following products.

a $\frac{2a}{5} \times \frac{3b}{7}$

b $\frac{4x}{15} \times \frac{3y}{2}$

SOLUTION	EXPLANATION
a $\frac{2a}{5} \times \frac{3b}{7} = \frac{6ab}{35}$	$2a \times 3b = 6ab$ and $5 \times 7 = 35$.
b Method 1: $\frac{4x}{15} \times \frac{3y}{2} = \frac{12xy}{30}$ $= \frac{2xy}{5}$	$4x \times 3y = 12xy$ $15 \times 2 = 30$ Divide by a common factor of 6 to simplify.
Method 2: $\frac{{}^{2}\cancel{4}x}{{}^{5}\cancel{15}} \times \frac{{}^{1}\cancel{3}y}{\cancel{2}^{1}} = \frac{2xy}{5}$	First divide by any common factors in the numerators and denominators: $4x$ and 2 have a common factor of 2. Also $3y$ and 15 have a common factor of 3.

Example 11 Dividing algebraic fractions

Simplify the following divisions.

a $\frac{3a}{8} \div \frac{b}{5}$

b $\frac{u}{4} \div \frac{15p}{2}$

SOLUTION	EXPLANATION
a $\frac{3a}{8} \div \frac{b}{5} = \frac{3a}{8} \times \frac{5}{b}$ $= \frac{15a}{8b}$	Take the reciprocal of $\frac{b}{5}$, which is $\frac{5}{b}$. Multiply as before: $3a \times 5 = 15a, 8 \times b = 8b$.
b $\frac{u}{4} \div \frac{15p}{2} = \frac{u}{4} \times \frac{2}{15p}$ $= \frac{2u}{60p}$ $= \frac{u}{30p}$	Take the reciprocal of $\frac{15p}{2}$, which is $\frac{2}{15p}$. Multiply as before: $u \times 2 = 2u$ and $4 \times 15p = 60p$. Cancel the common factor of 2.

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Exercise 5F

1, 2, 3(½) | 3(½) | —

UNDERSTANDING

1 Write the missing number.

a $\frac{2}{3} \times \frac{4}{5} = \frac{\square}{15}$ **b** $\frac{1}{2} \times \frac{3}{7} = \frac{\square}{14}$ **c** $\frac{4}{5} \times \frac{7}{11} = \frac{28}{\square}$ **d** $\frac{3}{4} \times \frac{5}{8} = \frac{15}{\square}$

2 Which one of the following shows the correct first step in calculating $\frac{2}{3} \div \frac{4}{5}$?

A $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} \times \frac{5}{4}$ **B** $\frac{2}{3} \div \frac{4}{5} = \frac{2}{4} \times \frac{3}{5}$ **C** $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$ **D** $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} \times \frac{4}{5}$

3 Calculate the following by hand, remembering to simplify your result.

a $\frac{2}{3} \times \frac{7}{10}$ **b** $\frac{1}{5} \times \frac{10}{11}$ **c** $\frac{1}{4} \times \frac{6}{11}$ **d** $\frac{5}{12} \times \frac{6}{10}$

e $\frac{2}{3} \div \frac{4}{5}$ **f** $\frac{1}{4} \div \frac{2}{3}$ **g** $\frac{4}{17} \div \frac{1}{3}$ **h** $\frac{2}{9} \div \frac{1}{2}$

4, 5–7(½) | 4–8(½) | 5–8(½)

FLUENCY

4 Simplify the following algebraic fractions.

a $\frac{2x}{6}$ **b** $\frac{5xy}{2y}$ **c** $\frac{7ab}{14b}$ **d** $\frac{2bc}{5bc}$

Example 10a

5 Simplify the following products.

a $\frac{x}{3} \times \frac{2}{5}$ **b** $\frac{1}{7} \times \frac{a}{9}$ **c** $\frac{2}{3} \times \frac{4a}{5}$

d $\frac{4c}{5} \times \frac{1}{5}$ **e** $\frac{4a}{3} \times \frac{2b}{5}$ **f** $\frac{3a}{2} \times \frac{7a}{5}$

Example 10b

6 Simplify the following products, remembering to cancel any common factors.

a $\frac{6x}{5} \times \frac{7y}{6}$ **b** $\frac{2b}{5} \times \frac{7d}{6}$ **c** $\frac{8a}{5} \times \frac{3b}{4c}$

d $\frac{9d}{2} \times \frac{4e}{7}$ **e** $\frac{3x}{2} \times \frac{1}{6x}$ **f** $\frac{4}{9k} \times \frac{3k}{2}$

Example 11

7 Simplify the following divisions, cancelling any common factors.

a $\frac{3a}{4} \div \frac{1}{5}$ **b** $\frac{2x}{5} \div \frac{3}{7}$ **c** $\frac{9a}{10} \div \frac{1}{4}$ **d** $\frac{2}{3} \div \frac{4x}{7}$

e $\frac{4}{5} \div \frac{2y}{3}$ **f** $\frac{1}{7} \div \frac{2}{x}$ **g** $\frac{4a}{7} \div \frac{2}{5}$ **h** $\frac{4b}{7} \div \frac{2c}{5}$

i $\frac{2x}{5} \div \frac{4y}{3}$ **j** $\frac{2y}{x} \div \frac{3}{y}$ **k** $\frac{5}{12x} \div \frac{7x}{2}$ **l** $\frac{4a}{5} \div \frac{2b}{7a}$

8 Simplify the following. (Recall that $3 = \frac{3}{1}$.)

a $\frac{4x}{5} \times 3$ **b** $\frac{4x}{5} \div 3$ **c** $2 \div \frac{x}{5}$ **d** $4 \times \frac{a}{3}$ **e** $5 \times \frac{7}{10x}$ **f** $1 \div \frac{x}{2}$

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5F

9 | 9, 10 | 10, 11

PROBLEM-SOLVING

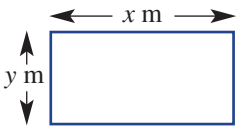
9 Helen's family goes to dinner with Tess' family. The bill comes to a total of $\$x$ and each family pays half.



a Write an algebraic fraction for the amount Helen's family pays.

b Helen says that she will pay for one third of her family's bill. Write an algebraic fraction for the amount she pays.

10 The rectangular field below has width x metres and length y metres.



a Write an expression for the area of the field.

b A smaller section is fenced off. It is $\frac{1}{2}$ the width and $\frac{3}{4}$ the length.

i Write an expression for the width of the smaller section.

ii Write an expression for the length of the smaller section.

iii Hence, write an expression for the area of the smaller section.

c To find the proportion of the field that is fenced off, you can divide the fenced area by the total area. Use this to find the proportion of the field that has been fenced.

11 Write an algebraic fraction for the result of the following operations.

a A number q is halved and then the result is tripled.

b A number x is multiplied by $\frac{2}{3}$ and the result is divided by $1\frac{1}{3}$.

c The fraction $\frac{a}{b}$ is multiplied by its reciprocal $\frac{b}{a}$.

d The number x is reduced by 25% and then halved.

12 | 12, 13 | 12, 14

REASONING

12 Recall that any value x can be thought of as the fraction $\frac{x}{1}$.

a Simplify $x \times \frac{1}{x}$.

b Simplify $x \div \frac{1}{x}$.

c Show that $x \div 3$ is equivalent to $\frac{1}{3} \times x$ by writing them both as algebraic fractions.

d Simplify $\frac{a}{b} \div c$.

e Simplify $a \div \frac{b}{c}$.

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5F

REASONING

13 **a** Simplify each of the following expressions.

i $\frac{x}{5}+\frac{x}{6}$ **ii** $\frac{x}{5}-\frac{x}{6}$ **iii** $\frac{x}{5}\times\frac{x}{6}$ **iv** $\frac{x}{5}\div\frac{x}{6}$

b Which one of the expressions above will always have the same value regardless of x?

14 Assume that a and b are any two whole numbers.

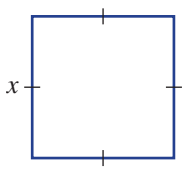
a Prove that $1\div\frac{a}{b}$ is the same as the reciprocal of the fraction $\frac{a}{b}$.

b Find the reciprocal of the reciprocal of $\frac{a}{b}$ by evaluating $1\div \quad 1\div\frac{a}{b}$.

ENRICHMENT

Irrational squares — — 15

15 Consider a square with side length x.



a Write an expression for the area of the square.

b The length of each side is now halved. Give an expression for the area of the new square.

c If each side of the original square is multiplied by $\frac{3}{5}$, show that the resulting area is less than half the original area.

d If each side of the original square is multiplied by 0.7, find an expression for the area of the square. Recall that $0.7=\frac{7}{10}$.

e Each side of the square is multiplied by some amount, which results in the square's area being halved. Find the amount by which they were multiplied correct to three decimal places.



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5G Expanding brackets

Interactive

Two expressions might look different when in fact they are equivalent. For example, $2(3 - 7b)$ is equivalent to $4b + 6(1 - 3b)$ even though they look different. One use for expanding brackets is that it allows us to easily convert between equivalent expressions.

Widgets

HOTsheets

Walkthroughs

Let's start: Room plans

An architect has prepared floor plans for a house but some numbers are missing. Four students have attempted to describe the total area of the plans shown.

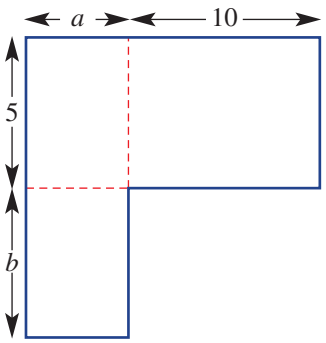
Alice says it is $5a + 50 + ab$.

Brendan says it is $5(a + 10) + ab$.

Charles says it is $a(5 + b) + 50$.

David says it is $(5 + b)(a + 10) - 10b$.

- Discuss which of the students is correct.
- How do you think each student worked out their answer?
- The architect later told them that $a = 4$ and $b = 2$. What value would each of the four students get for the area?



Key ideas

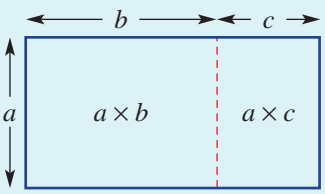
- To expand brackets, you can use the **distributive law**, which states that:

$a(b + c) = a \times b + a \times c$
$= ab + ac$

$a(b - c) = a \times b - a \times c$
$= ab - ac$

For example: $4(2x + 5) = 8x + 20$ and $3(5 - 2y) = 15 - 6y$

- The distributive law can be illustrated by considering rectangle areas.



Area $= a(b + c)$
Area $= ab + ac$

- The distributive law is used in arithmetic. For example:

$5 \times 31 = 5(30 + 1)$
$= 5(30) + 5(1)$
$= 150 + 5$
$= 155$

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Example 12 Expanding using the distributive law

Expand the following.

a $3(2x + 5)$ **b** $-8(7 + 2y)$ **c** $4x(2 - y)$

SOLUTION	EXPLANATION
a $3(2x + 5) = 3(2x) + 3(5)$	Distributive law: $3(2x + 5) = 3(2x) + 3(5)$
$= 6x + 15$	Simplify the result.
b $-8(7 + 2y) = -8(7) + (-8)(2y)$	Distributive law: $-8(7 + 2y) = -8(7) + (-8(2y))$
$= -56 + (-16y)$	Simplify the result.
$= -56 - 16y$	Adding $-16y$ is the same as subtracting positive $16y$.
c $4x(2 - y) = 4x(2) - 4x(y)$	Distributive law: $4x(2 - y) = 4x(2) - 4x(y)$
$= 8x - 4xy$	Simplify the result.

Example 13 Expanding and combining like terms

Expand the brackets in each expression and then combine like terms.

a $3(2b + 5) + 3b$ **b** $12xy + 7x(2 - y)$

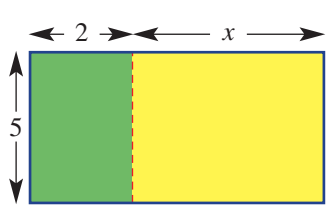
SOLUTION	EXPLANATION
a $3(2b + 5) + 3b = 3(2b) + 3(5) + 3b$	Use the distributive law.
$= 6b + 15 + 3b$	Simplify the result.
$= 9b + 15$	Combine the like terms.
b $12xy + 7x(2 - y)$	Use the distributive law.
$= 12xy + 7x(2) - 7x(y)$	Simplify the result.
$= 12xy + 14x - 7xy$	Combine the like terms.
$= 5xy + 14x$	

Exercise 5G

1–5 | 3, 5 | —

UNDERSTANDING

1 The area of this combined rectangle is $5(2 + x)$.



a What is the area of the green rectangle?

b What is the area of the yellow rectangle?

c Write an expression for the sum of these two areas.

d Hence complete the following:
The expanded form of $5(2 + x)$ is _____.

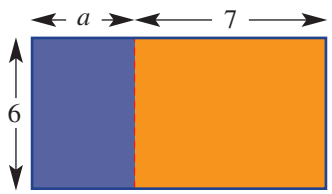
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5G

UNDERSTANDING

2 The expression $3(2 + 7x)$ is equivalent to $2 + 7x + 2 + 7x + 2 + 7x$. Simplify this expression by combining like terms.

3 Consider this combined rectangle, which has an area of $6(a + 7)$.



- **a** What is the area of the purple rectangle?
- **b** What is the area of the orange rectangle?
- **c** Write an expression for the sum of these two areas.
- **d** Write the expanded form for $6(a + 7)$.

4 Which of the following is the correct expansion of $4(3x + 7)$?

A $12x + 7$ **B** $4x + 28$ **C** $3x + 28$ **D** $12x + 28$

5 Complete the following expansions.

- **a** $2(b + 5) = 2b + \square$
- **b** $3(2x + 4y) = 6x + \square y$
- **c** $5(3a - 2) = 15a - \square$
- **d** $7(4x - 2y) = 28x - \square y$

6, 7–8(½) | 6–8(½), 10(½) | 6–10(½)

FLUENCY

Example 12

6 Use the distributive law to expand these expressions.

a $9(a + 7)$	**b** $2(2 + t)$	**c** $8(m - 10)$	**d** $3(8 - v)$
e $-5(9 + g)$	**f** $-7(5b + 4)$	**g** $-9(u + 9)$	**h** $-8(5 + h)$
i $5(6 - j)$	**j** $6(2 - m)$	**k** $3(10 - b)$	**l** $2(c - 8)$

7 Use the distributive law to expand the following.

a $8z(k - h)$	**b** $6j(k + a)$	**c** $4u(r - q)$	**d** $2p(c - v)$
e $m(10a + v)$	**f** $-2y(s + 5g)$	**g** $-3s(8q + g)$	**h** $-g(n + 4f)$
i $-8c(u + 10t)$	**j** $-j(t + 5s)$	**k** $u(2h - 9m)$	**l** $4m(5w - 10a)$

Example 13

8 Simplify the following by expanding and then collecting like terms.

a $7(9f + 10) + 2f$	**b** $8(2 + 5x) + 4x$
c $4(2a + 8) + 7a$	**d** $6(3v + 10) + 6v$
e $7(10a + 10) + 6a$	**f** $6(3q - 5) + 2q$
g $6(4m - 5) + 8m$	**h** $4(8 + 7m) - 6m$

9 The distributive law also allows expansion with more than two terms in the brackets, for instance $3(2x - 4y + 5) = 6x - 12y + 15$. Use this fact to simplify the following.

a $2(3x + 2y + 4z)$	**b** $7a(2 - 3b + 4y)$	**c** $2q(4z + 2a + 5)$
d $-3(2 + 4k + 2p)$	**e** $-5(1 + 5q - 2r)$	**f** $-7k(r + m + s)$

10 Simplify the following by expanding and then collecting like terms.

a $3(3 + 5d) + 4(10d + 7)$	**b** $10(4 + 8f) + 7(5f + 2)$	**c** $2(9 + 10j) + 4(3j + 3)$
d $2(9 + 6d) + 7(2 + 9d)$	**e** $6(10 - 6j) + 4(10j - 5)$	**f** $8(5 + 10g) + 3(4 - 4g)$

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11 12, 14 12–14

5G

PROBLEM-SOLVING

11 Write an expression for each of the following and then expand it.

- **a** A number t has 4 added to it and the result is multiplied by 3.
- **b** A number u has 3 subtracted from it and the result is doubled.
- **c** A number v is doubled, and then 5 is added. The result is tripled.
- **d** A number w is tripled, and then 2 is subtracted. The result is doubled.

12 Match each operation on the left with an equivalent one on the right. (Hint: First convert the descriptions into algebraic expressions.)

- **a** The number x is doubled and 6 is added.
- **b** The number x is reduced by 5 and the result is doubled.
- **c** The number x is added to double the value of x.
- **d** The number x is halved, then 3 is added and the result is doubled.
- **e** 2 is subtracted from one third of x and the result is tripled.

- **A** x is doubled and reduced by 10.
- **B** The number x is tripled.
- **C** x is decreased by 6.
- **D** x is increased by 3 and the result is doubled.
- **E** x is increased by 6.

13 The number of boys in a classroom is b and the number of girls is g. Each boy has 5 pencils and each girl has 3 pencils.

- **a** Write an expression for the total number of pencils in the class.
- **b** If the pencils cost \$2 each, write and expand an expression for the total cost of all the pencils in the room.
- **c** Each boy and girl also has one pencil case, costing \$4 each. Write a simplified and expanded expression for the total cost of all pencils and cases in the room.



- **d** If there are 10 boys and 8 girls in the room, what is the total cost for all the pencils and cases in the room?

14 **a** When expanded, $4(2a + 6b)$ gives $8a + 24b$. Find two other expressions that expand to give $8a + 24b$.

- **b** Give an expression that expands to $4x + 8y$.
- **c** Give an expression that expands to $12a - 8b$.
- **d** Give an expression that expands to $18ab + 12ac$.

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5G

15a, b | 15, 16 | 15, 17, 18

REASONING

15 The distributive law is often used in multiplication of whole numbers. For example, $17 \times 102 = 17 \times (100 + 2) = 17(100) + 17(2) = 1734$.

a Use the distributive law to find the value of 9×204. Start with $9 \times 204 = 9 \times (200 + 4)$.

b Use the distributive law to find the value of 204×9. Start with $204 \times 9 = 204 \times (10 - 1)$.

c Given that $a \times 11 = a \times (10 + 1) = 10a + a$, find the value of these multiplications.

i 14×11 **ii** 32×11 **iii** 57×11 **iv** 79×11

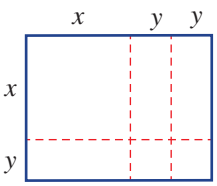
d It is known that $(x + 1)(x - 1)$ expands to $x^2 - 1$. For example, if $x = 7$ this tells you that $8 \times 6 = 49 - 1 = 48$. Use this fact to find the value of:

i 7×5 **ii** 21×19 **iii** 13×11 **iv** 201×199

e Using a calculator, or otherwise evaluate 15^2, 25^2 and 35^2. Describe how these relate to the fact that $(10n + 5)(10n + 5)$ is equivalent to $100n(n + 1) + 25$.

16 Prove that $4a(2 + b) + 2ab$ is equivalent to $a(6b + 4) + 4a$ by expanding both expressions.

17 Find an expanded expression for $(x + y)(x + 2y)$ by considering the diagram below. Ensure your answer is simplified by combining any like terms.



18 Prove that the following sequence of operations has the same effect as doubling a number.

1 Take a number, add 2.
2 Multiply by 6.
3 Subtract 6.
4 Multiply this result by $\frac{1}{3}$.
5 Subtract 2.

Expanding algebraic fractions

— | — | 19

ENRICHMENT

19 To simplify $\frac{x+5}{3} + \frac{x}{2}$ change both fractions to have a common denominator of 6, giving $\frac{2(x+5)}{6} + \frac{3x}{6}$. Then expand to finish off the simplification: $\frac{2x+10}{6} + \frac{3x}{6} = \frac{5x+10}{6}$.

Use this method to simplify the following sums.

a $\frac{x+1}{3} + \frac{x}{2}$ **b** $\frac{x+5}{5} + \frac{x}{3}$ **c** $\frac{3x}{8} + \frac{x-1}{4}$

d $\frac{x+2}{4} + \frac{x+1}{3}$ **e** $\frac{2x+1}{5} + \frac{3x+1}{10}$ **f** $\frac{2x-1}{7} + \frac{3x+2}{5}$

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Progress quiz

5A **1** Answer the following questions about the expression $3a - 9b - ab + c + 8$.

a How many terms are there?
b List the individual terms.
c State the coefficients of a, b, c and d.
d What is the constant term?
e State the coefficient of ab.

5A **2** Write an expression for each of the following.

a The sum of 5 and m.
b The product of k and 8.
c 7 less than p.
d 12 more than h.
e Double the sum of x and y.
f The quotient of a and b.
g The difference of half of k and one third of m.
h The product of a and c divided by 5.

5B **3** Substitute $x = 3$ and $y = -6$ to evaluate the following expressions.

a $4x + y$ **b** $3 \times (x + 2y)$ **c** $2x^2 + y^2$ **d** $\dfrac{36}{x - y}$

5B **4** For the following state whether they are equivalent (E) or not (N).

a $x + y$ and $y + x$ **b** $x - 5$ and $5 - x$ **c** $2(x + y)$ and $2x + y$ **d** $2 \times y$ and $y + y$

5C **5** Classify the following pairs as like terms (L) or not like terms (N).

a $5a$ and $-7a$ **b** $3xy$ and $5yx$ **c** $5p^2t$ and $8pt^2$ **d** $8abc$ and $-9bac$

5C **6** Simplify the following by combining like terms.

a $4h + 3h + 8 - 5$
b $12a + 7 - 8a + 1 + a$
c $8xy + 4x - 3yx - x$
d $-gk + 3g^2k + 12 - 8kg^2$

5D **7** Simplify the following.

a $3a \times 2b$ **b** $5d \times 2d$ **c** $5abc \times 3acd \times 2d$ **d** $4p^2q \times 3q$

e $\dfrac{12a}{48a}$ **f** $\dfrac{16x^2}{40x}$ **g** $\dfrac{8c}{24ac}$ **h** $\dfrac{-18m^2}{27mt}$

5E **8** Simplify the following expressions, giving your final answer as an algebraic fraction.

Ext

a $\dfrac{2m}{9} + \dfrac{5m}{9}$ **b** $\dfrac{4k}{3} + \dfrac{5k}{6}$ **c** $\dfrac{5a}{6} - \dfrac{3b}{8}$ **d** $5x - \dfrac{2x}{3}$

5F **9** Simplify the following.

Ext

a $\dfrac{3a}{5} \times \dfrac{2b}{7}$ **b** $\dfrac{8m}{9a} \times \dfrac{6a}{10}$ **c** $\dfrac{5}{2} \div \dfrac{15}{8y}$ **d** $\dfrac{2m}{5} \div \dfrac{4mp}{15}$

5G **10** Expand the brackets and simplify where possible.

a $3(x + 2)$ **b** $5(3a - 4b)$ **c** $4(x + 1) + 2x$ **d** $5(2b - 3) + 20$

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5H Factorising expressions

Interactive

Widgets

HOTsheets

Walkthroughs

Factorising is the opposite procedure to expanding. It allows us to simplify expressions and solve harder mathematical problems. Because $3(2x + 5)$ expands to $6x + 15$, this means that the factorised form of $6x + 15$ is $3(2x + 5)$. The aim in factorising is to write expressions as the product of two or more factors, just as with numbers we can factorise 30 and write $30 = 2 \times 3 \times 5$.

Let's start: Expanding gaps

- Try to fill in the gaps to make the following equivalence true: $\square(\square + \square) = 12x + 18xy$.
- In how many ways can this be done? Try to find as many ways as possible.
- If the aim is to make the term outside the brackets as large as possible, what is the best possible solution to the puzzle?

Key ideas

- The **highest common factor** (HCF) of a set of terms is the largest factor that divides into each term. For example:
  - HCF of $15x$ and $21y$ is 3.
  - HCF of $10a$ and $20c$ is 10.
  - HCF of $12x$ and $18xy$ is $6x$.
- To **factorise** an expression, first take the HCF of the terms outside the brackets and divide each term by it, leaving the result in brackets.
  For example:

  $10x + 15y$
  HCF $= 5$
  Result $5(2x + 3y)$
  HCF $10x \div 5$ $15y \div 5$
- To check your answer, expand the factorised form, for example, $5(2x + 3y) = 10x + 15y$ ✓

Example 14 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of:

a 20 and 35 **b** $18a$ and $24ab$ **c** $12x$ and $15x^2$

SOLUTION	EXPLANATION
a 5	5 is the largest number that divides into 20 and 35.
b $6a$	6 is the largest number that divides into 18 and 24, and a divides into both terms.
c $3x$	3 divides into both 12 and 15, and x divides into both terms.

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Example 15 Factorising expressions

Factorise the following expressions.

a $6x + 15$ **b** $12a + 18ab$ **c** $21x - 14y$

SOLUTION	EXPLANATION
a $6x + 15 = 3(2x + 5)$	HCF of $6x$ and 15 is 3. $6x \div 3 = 2x$ and $15 \div 3 = 5$
b $12a + 18ab = 6a(2 + 3b)$	HCF of $12a$ and $18ab$ is $6a$. $12a \div (6a) = 2$ and $18ab \div (6a) = 3b$
c $21x - 14y = 7(3x - 2y)$	HCF of $21x$ and $14y$ is 7. $21x \div 7 = 3x$ and $14y \div 7 = 2y$ The subtraction sign is included as in the original expression.

Exercise 5H

1–4 | 4(½), 5 | —

UNDERSTANDING

1 List the factors of:

a 20 **b** 12 **c** 15 **d** 27

2 The factors of 14 are 1, 2, 7 and 14. The factors of 26 are 1, 2, 13 and 26. What is the highest factor that these two numbers have in common?

3 Find the highest common factor of the following pairs of numbers.

a 12 and 18 **b** 15 and 25 **c** 40 and 60 **d** 24 and 10

4 Fill in the blanks to make these expansions correct.

a $3(4x + 1) = \square x + 3$ **b** $5(7 - 2x) = \square - 10x$

c $6(2 + 5y) = \square + \square y$ **d** $7(2a - 3b) = \square - \square$

e $3(2a + \square) = 6a + 21$ **f** $4(\square - 2y) = 12 - 8y$

g $7(\square + \square) = 14 + 7q$ **h** $\square(2x + 3y) = 8x + 12y$

5 Verify that $5x + 15$ and $5(x + 3)$ are equivalent by copying and completing the table below.

x	2	7	4	0	−3	−6
$5x + 15$						
$5(x + 3)$						

6–7(½) | 6–8(½) | 7–8(½)

FLUENCY

Example 14

6 Find the highest common factor (HCF) of the following pairs of terms.

a 15 and $10x$ **b** $20a$ and $12b$ **c** $27a$ and $9b$

d $7xy$ and $14x$ **e** $-2yz$ and $4xy$ **f** $11xy$ and $-33xy$

g $8qr$ and $-4r$ **h** $-3a$ and $6a^2$ **i** $14p$ and $25pq$

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5H

FLUENCY

Example 15a

7 Factorise the following by first finding the highest common factor. Check your answers by expanding them.

a $3x + 6$ **b** $8v + 40$ **c** $15x + 35$
d $10z + 25$ **e** $40 + 4w$ **f** $5j - 20$
g $9b - 15$ **h** $12 - 16f$ **i** $5d - 30$

Example 15b
Example 15c

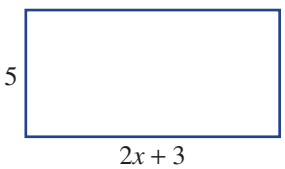
8 Factorise the following.

a $10cn + 12n$ **b** $24y + 8ry$ **c** $14jn + 10n$ **d** $24g + 20gj$
e $10h + 4z$ **f** $30u - 20n$ **g** $40y + 56ay$ **h** $12d + 9dz$
i $21hm - 9mx$ **j** $49u - 21bu$ **k** $28u - 42bu$ **l** $21p - 6c$

PROBLEM-SOLVING

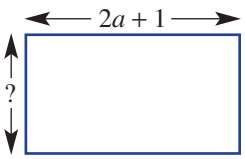
9 | 9, 10 | 9, 11

9 The rectangle shown has an area of $10x + 15$. Draw two different rectangles that would have an area $12x + 16$.

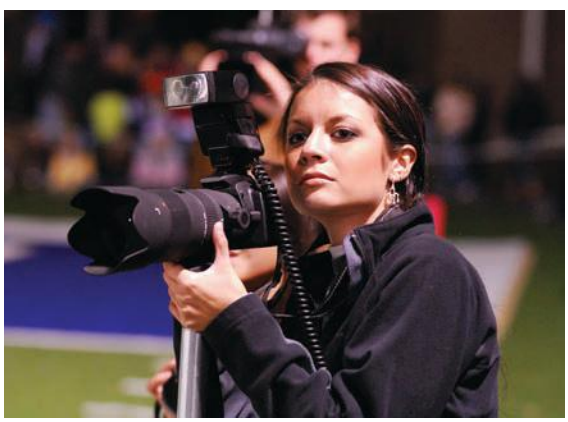


10 The area of the rectangle shown is $10a + 5$. One side's measurement is unknown.

a What is the value of the unknown measurement?
b Write an expression for the perimeter of the rectangle.



11 A group of students lines up for a photo. They are in 6 rows each with x students in each row. Another 18 students join the photo.



a Write an expression for the total number of students in the photo.
b Factorise the expression above.
c How many students would be in each of the 6 rows now? Write an expression.
d If the photographer wanted just 3 rows, how many students would be in each row? Write an expression.
e If the photographer wanted just 2 rows, how many students would be in each row? Write an expression.

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5H

	12	12, 13	12–14

REASONING

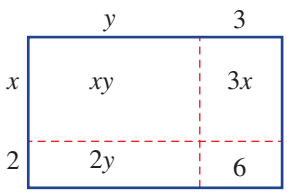
12 **a** Expand $2(x+1)+5(x+1)$ and simplify.

b Factorise your result.

c Make a prediction about the equivalence of $3(x+1)+25(x+1)$ if it is expanded, simplified and then factorised.

d Check your prediction by expanding and factorising $3(x+1)+25(x+1)$.

13 Consider the diagram shown to the right. What is the factorised form of $xy+3x+2y+6$?



14 In English, people often convert between 'factorised' and 'expanded' sentences. For instance, 'I like John and Mary' is equivalent in meaning to 'I like John and I like Mary'. The first form is factorised with the common factor that I like them. The second form is expanded.

a Expand the following sentences.

i I eat fruit and vegetables.

ii Rohan likes Maths and English.

iii Petra has a computer and a television.

iv Hayden and Anthony play tennis and chess.

b Factorise the following sentences.

i I like sewing and I like cooking.

ii Olivia likes ice cream and Mary likes ice cream.

iii Brodrick eats chocolate and Brodrick eats fruit.

iv Adrien likes chocolate and Adrien likes soft drinks, and Ben likes chocolate and Ben likes soft drinks.

Factorising fractions

	—	—	15

ENRICHMENT

15 Factorising can be used to simplify algebraic fractions. For example, $\frac{5x+10}{7x+14}$ can be simplified by first factorising the numerator and the denominator $\frac{5\cancel{(x+2)}}{7\cancel{(x+2)}}=\frac{5}{7}$. Factorise and then simplify the following fractions as much as possible.

a $\frac{2x+4}{5x+10}$ **b** $\frac{7x-7}{2x-2}$ **c** $\frac{3ac+5a}{a+2ab}$

d $\frac{4a+2b}{8c+10d}$ **e** $\frac{5q-15}{3q-9}$ **f** $\frac{7p+14pq}{9p+18pq}$

g $\frac{7a-21}{2a-6}$ **h** $\frac{12p}{8p+2pq}$ **i** $\frac{100-10x}{20-2x}$

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5I Applying algebra









The skills of algebra can be applied to many situations within other parts of mathematics as well as to other fields such as engineering, sciences and economics.



Algebra can be applied to both the engineering of the ride and the price of the tickets.

Let's start: Carnival conundrum

Alwin, Bryson and Calvin have each been offered special deals for the local carnival.

- Alwin can pay $50 to go on all the rides all day.
- Bryson can pay $20 to enter the carnival and then pay $2 per ride.
- Calvin can enter the carnival at no cost and then pay $5 per ride.

- Which of them has the best deal?
- In the end, each of them decides that they were happiest with the deal they had and would not have swapped. How many rides did they each go on? Compare your different answers.

Key ideas

- Different situations can be **modelled** with algebraic expressions.
- To apply a rule, the variables should first be clearly defined. Then known values are substituted for the variables.



Example 16 Writing expressions from descriptions

Write an expression for the following situations.

a The total cost of k bottles if each bottle cost $4

b The area of a rectangle if its width is 2 cm more than its length and its length is x cm

c The total cost of hiring a plumber for n hours if he charges $40 call-out fee and $70 per hour

SOLUTION	EXPLANATION
a $4 \times k = 4k$	Each bottle costs $4 so the total cost is $4 multiplied by the number of bottles purchased.
b $x \times (x + 2) = x(x + 2)$	Length = x so width = $x + 2$. The area is length × width.
c $40 + 70n$	$70 per hour means that the cost to hire the plumber would be $70 \times n$. Additionally $40 is added for the call-out fee, which is charged regardless of how long the plumber stays.

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Exercise 5I

1–3 | 3 | —

UNDERSTANDING

1 Evaluate the expression $3d + 5$ when:

a $d = 10$ **b** $d = 12$ **c** $d = 0$

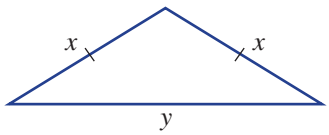
2 Find the value of $30 + 10x$ when:

a $x = 2$ **b** $x = -1$ **c** $x = -3$

3 Consider the isosceles triangle shown.

a Write an expression for the perimeter of the triangle.

b Find the perimeter when $x = 3$ and $y = 2$.



4–6 | 4–7 | 5–7

FLUENCY

Example 16a

4 Pens cost $3 each.

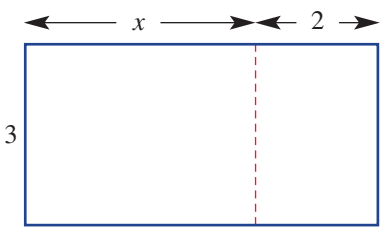
a Write an expression for the total cost of n pens.

b If $n = 12$, find the total cost.

Example 16b

5 **a** Write an expression for the total area of the shape shown.

b If $x = 9$, what is the area?



Example 16c

6 An electrician charges a call-out fee of $30 and $90 per hour. Which of the following represents the total cost for x hours?

A $x(30 + 90)$

B $30x + 90$

C $30 + 90x$

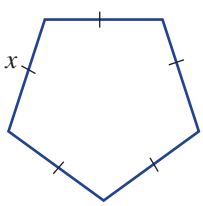
D $120x$



7 **a** Give an expression for the perimeter of this regular pentagon.

b If each side length were doubled, what would the perimeter be?

c If each side length were increased by 3, write a new expression for the perimeter.



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5I

8, 9 | 8–10 | 9–11

PROBLEM-SOLVING

8 An indoor soccer pitch costs $40 per hour to hire plus a $30 booking fee.

a Write an expression for the cost of hiring the pitch for x hours.

b Hence, find the cost of hiring the pitch for an 8-hour round-robin tournament.



9 A plumber says that the cost in dollars to hire her for x hours is $50 + 60x$.

a What is her call-out fee?

b How much does she charge per hour?

c If you had $200, what is the longest period you could hire the plumber?

10 A repairman says the cost in dollars to hire his services for x hours is $20(3 + 4x)$.

a How much would it cost to hire him for 1 hour?

b Expand the expression he has given you.

c Hence, state:

i his call-out fee

ii the amount he charges per hour.

11 Three deals are available at a fair.

Deal 1: Pay $10, rides cost $4/each.

Deal 2: Pay $20, rides cost $1/each.

Deal 3: Pay $30, all rides are free.

a Write an expression for the total cost of n rides using deal 1. (The total cost includes the entry fee of $10.)

b Write an expression for the total cost of n rides using deal 2.

c Write an expression for the total cost of n rides using deal 3.

d Which of the three deals is best for someone going on just two rides?

e Which of the three deals is best for someone going on 20 rides?

f Fill in the gaps:

i Deal 1 is best for people wanting up to _____rides.

ii Deal 2 is best for people wanting between _____ and _____ rides.

iii Deal 3 is best for people wanting more than _____ rides.

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5I

12 | 12, 13 | 13, 14

REASONING

12 In a particular city, taxis charge $4 to pick someone up (flagfall) and then $2 per minute of travel. Three drivers have different ways of calculating the total fare.

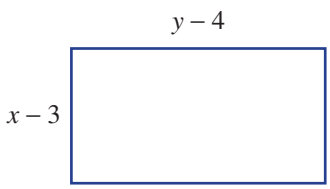
- Russell adds 2 to the number of minutes travelled and doubles the result.
- Jessie doubles the number of minutes travelled and then adds 4.
- Arash halves the number of minutes travelled, adds 1 and then quadruples the result.

a Write an expression for the total cost of travelling x minutes in:
i Russell's taxi **ii** Jessie's taxi **iii** Arash's taxi

b Prove that all three expressions are equivalent by expanding them.

c A fourth driver starts by multiplying the number of minutes travelled by 4 and then adding 8. What should he do to this result to calculate the correct fare?

13 Roberto draws a rectangle with unknown dimensions. He notes that the area is $(x-3)(y-4)$.



a If $x = 5$ and $y = 7$, what is the area?

b What is the value of $(x-3)(y-4)$ if $x = 1$ and $y = 1$?

c Roberto claims that this proves that if $x = 1$ and $y = 1$ then his rectangle has an area of 6. What is wrong with his claim? (Hint: Try to work out the rectangle's perimeter.)

14 Tamir notes that whenever he hires an electrician, they charge a call-out fee $\$F$ and an hourly rate of $\$H$ per hour.

a Write an expression for the cost of hiring an electrician for one hour.

b Write an expression for the cost of hiring an electrician for two hours.

c Write an expression for the cost of hiring an electrician for 30 minutes.

d How much does it cost to hire an electrician for t hours?

Ticket sales

— | — | 15

ENRICHMENT

15 At a carnival there are six different deals available to reward loyal customers.

Deal	Entry cost ($)	Cost per ride ($)
A	79	0
B	50	2
C	31	4
D	18	6
E	7	8
F	0	10



The queue consists of 100 customers. The first customer knows they will go on 1 ride, the second will go on 2 rides, and the pattern continues, with the 100th customer wanting to go on 100 rides. Assuming that each customer can work out their best deal, how many of each deal will be sold?

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5J Index laws: multiplying and dividing powers









Recall that x^2 means $x \times x$ and x^3 means $x \times x \times x$. **Index notation** provides a convenient way to describe repeated multiplication.

index or exponent

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

base

Notice that $3^5 \times 3^2 = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{3^5} \times \underbrace{3 \times 3}_{3^2}$ which means that $3^5 \times 3^2 = 3^7$.

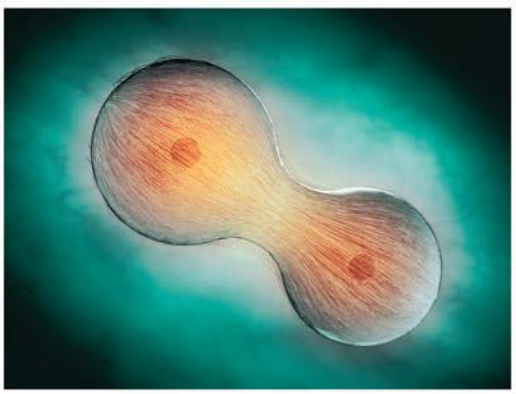
Similarly it can be shown that $2^6 \times 2^5 = 2^{11}$.

When dividing, note that:

$$\frac{5^{10}}{5^7} = \frac{5 \times 5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$$

$$= 5 \times 5 \times 5$$

So $5^{10} \div 5^7 = 5^3$. These observations are generalised in index laws 1 and 2.



Index notation has wide application, particularly in modelling growth and decay, in science, economics and computer applications.

Let's start: Comparing powers

- Arrange these numbers from smallest to largest.
  $2^3, 3^2, 2^5, 4^3, 3^4, 2^4, 4^2, 5^2, 1^{20}$
- Did you notice any patterns?
- If all the bases were negative, how would that change your arrangement from smallest to largest? For example, 2^3 becomes $(-2)^3$.

Key ideas

- Expressions involving repeated multiplication can be expressed using a **base** and an **index** (plural indices).

  index or exponent

  $a^n = \underbrace{a \times a \times \cdots \times a}_{n \text{ copies of } a}$

  base

  For example: $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
- An expression such as $4x^3$ can be written in **expanded form** as $4 \times x \times x \times x$.
- **Index law 1** is for multiplying powers with the same base: $a^m \times a^n = a^{m+n}$ (e.g. $a^4 \times a^2 = a^6$).
- **Index law 2** is for dividing powers with the same base: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ (e.g. $a^8 \div a^5 = a^3$).

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Example 17 Multiplying powers

Simplify the following using the index law for multiplication.

a $5^3 \times 5^7 \times 5^2$ **b** $x^3 \times x^4$ **c** $a^5 \times a \times a^3$ **d** $2x^4y^3 \times 5x^2y^8$

SOLUTION	EXPLANATION
a $5^3 \times 5^7 \times 5^2 = 5^{12}$	$3 + 7 + 2 = 12$ and the first index law applies (using $a = 5$).
b $x^3 \times x^4 = x^7$	Using the first index law, $3 + 4 = 7$, so $x^3 \times x^4 = x^7$.
c $a^5 \times a \times a^3 = a^5 \times a^1 \times a^3$ $= a^9$	Write a as a^1 so the index law can be used. $5 + 1 + 3 = 9$, so the final result is a^9.
d $2x^4y^3 \times 5x^2y^8$ $= 2 \times 5 \times x^4 \times x^2 \times y^3 \times y^8$ $= 10x^6y^{11}$	Bring all the numbers to the front of the expression and then bring the pronumerals together. $x^4 \times x^2 = x^6$ and $y^3 \times y^8 = y^{11}$ by the first index law, and $2 \times 5 = 10$.

Example 18 Dividing powers

Simplify the following using the index law for division.

a $\frac{10^8}{10^5}$ **b** $\frac{u^{20}}{u^5}$ **c** $\frac{10x^6}{4x^2}$ **d** $\frac{a^{10}b^6}{a^3b^2}$

SOLUTION	EXPLANATION
a $\frac{10^8}{10^5} = 10^3$	Using the second index law with $8 - 5 = 3$ and $a = 10$.
b $\frac{u^{20}}{u^5} = u^{15}$	Use the second index law, so $20 - 5 = 15$.
c $\frac{10x^6}{4x^2} = \frac{10}{4} \times \frac{x^6}{x^2}$ $= \frac{5}{2} \times x^4$ $= \frac{5x^4}{2}$	First separate the numbers into a separate fraction. Cancel the common factor of 2 and use the second index law. Combine the result as a single fraction.
d $\frac{a^{10}b^6}{a^3b^2} = a^7b^4$	The two letters are treated separately, with $10 - 3 = 7$ and $6 - 2 = 4$.

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Exercise 5J

1–4 | 4 | —

UNDERSTANDING

1 Fill in the gaps: In the expression 5^7 the base is _____ and the exponent is _____.

2 Which of the following expressions is the same as 3^5?

A 3×5 **B** $3 \times 3 \times 3 \times 3 \times 3$ **C** $5 \times 5 \times 5$ **D** $5 \times 5 \times 5 \times 5 \times 5$

3 **a** Calculate the value of:

i 2^2 **ii** 2^3 **iii** 2^5 **iv** 2^6

b Is $2^2 \times 2^3$ equal to 2^5 or 2^6?

4 **a** Write 5^3 in expanded form.

b Write 5^4 in expanded form.

c Write the result of multiplying $5^3 \times 5^4$ in expanded form.

d Which of the following is the same as $5^3 \times 5^4$?

A 5^{12} **B** 5^5 **C** 5^7 **D** 5^1

5, 6(½), 8 | 5, 6–7(½), 8, 9(½) | 5–9(½)

FLUENCY

Example 17a

5 Simplify the following, giving your answers in index form.

a $4^3 \times 4^5$ **b** $3^{10} \times 3^2$ **c** $2^{10} \times 2^5 \times 2^3$ **d** $7^2 \times 7 \times 7^3$

Example 17b, Example 17c

6 Simplify the following using the index law for multiplication.

a $m^3 \times m^4$ **b** $x^2 \times x^4$ **c** $q^{10} \times q^3$ **d** $r^7 \times r^2$

e $m^2 \times m^4 \times m^3$ **f** $a^2 \times a^4 \times a^3$ **g** $r^2 \times r^3 \times r^4$ **h** $z^{10} \times z^{12} \times z^{14}$

i $k \times k^3$ **j** $j^2 \times j$ **k** $m^4 \times m^3 \times m$ **l** $x^2 \times x \times x$

Example 17d

7 Simplify the following using the index law for multiplication.

a $4m^2 \times 5m^3$ **b** $2k^3 \times 5k^4$ **c** $7x^2 \times 4x^{12}$ **d** $4y^3 \times 7y^{10}$

e $m^2 \times n^3 \times m^4 \times n^7$ **f** $x^2y \times y^2$ **g** $3r^3s^2 \times s^5$ **h** $2y^{10}z^2 \times y^5z^3$

i $11x \times 10x^3$ **j** $3a \times 5a \times a^4$ **k** $2x^2y^2 \times 4x^3y^5$ **l** $7a^2b^3 \times 2a^3b$

m $-7x^2y^3 \times 2x^5y$ **n** $-4ab^2 \times a^4b$ **o** $2c^4d \times (-8c^2)$ **p** $7x \times 12x^3y^5$

Example 18a

8 Simplify the following, giving your answers in index form.

a $\frac{3^7}{3^2}$ **b** $\frac{10^{15}}{10^7}$ **c** $\frac{2^{10}}{2^5}$ **d** $\frac{5^{100}}{5^{98}}$

Example 18b, Example 18c, Example 18d

9 Simplify the following using the index law for division.

a $\frac{m^5}{m^2}$ **b** $\frac{z^5}{z^2}$ **c** $\frac{q^{10}}{q^3}$ **d** $\frac{r^{10}}{r}$

e $\frac{m^5n^7}{m^3n^2}$ **f** $\frac{a^{10}b^5}{a^5b^2}$ **g** $\frac{x^3y^{10}z^5}{x^2y^4z^3}$ **h** $\frac{x^4y^7z^3}{x^2y^4}$

i $\frac{4k^{10}}{k^7}$ **j** $\frac{10m^{20}}{5m^7}$ **k** $\frac{30x^{20}y^{12}}{18x^2y^5}$ **l** $\frac{a^3b}{2ab}$

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5J

10	10	10, 11

PROBLEM-SOLVING

10 John enters $2^{10000} \div 2^{9997}$ into his calculator and he gets the error message 'Number Overflow', because 2^{10000} is too large.

- **a** According to the second index law, what does $2^{10000} \div 2^{9997}$ equal? Give your final answer as a number.
- **b** Find the value of $(5^{2000} \times 5^{2004}) \div 5^{4000}$.
- **c** What is the value of $\dfrac{3^{700} \times 3^{300}}{3^{1000}}$?

11 Find values of a and b so that $a < b$ and $a^b = b^a$.

12	12, 13	14, 15

REASONING

12 A student tries to simplify $3^2 \times 3^4$ and gets the result 9^6.

- **a** Use a calculator to verify this is incorrect.
- **b** Write out $3^2 \times 3^4$ in expanded form, and explain why it is not the same as 9^6.
- **c** Explain the mistake they have made in attempting to apply the first index law.

13 Recall that $(-3)^2$ means $-3 \times (-3)$, so $(-3)^2 = 9$.

- **a** Evaluate:
  - **i** $(-2)^2$
  - **ii** $(-2)^3$
  - **iii** $(-2)^4$
  - **iv** $(-2)^5$
- **b** Complete the following generalisations.
  - **i** A negative number to an even power is _____.
  - **ii** A negative number to an odd power is _____.
- **c** Given that $2^{10} = 1024$, find the value of $(-2)^{10}$.

14 **a** Use the index law for division to write $\dfrac{5^3}{5^3}$ in index form.

- **b** Given that $5^3 = 125$, what is the numerical value of $\dfrac{5^3}{5^3}$?
- **c** According to this, what is the value of 5^0? Check whether this is also the result your calculator gives.
- **d** What is the value of 12^0?

15 **a** If $\dfrac{3^a}{3^b} = 9$, what does this tell you about the value of a and b?

- **b** Given that $\dfrac{2^a}{2^b} = 8$, find the value of $\dfrac{5^a}{5^b}$.

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5J Scientific notation for timescales — — 16

ENRICHMENT

16 Using indices we can express very large numbers and very small numbers easily. For example, 8 000 000 can be written as 8×10^6 (an 8 followed by six zeroes) and 0.0003 can be written as 3×10^{-4} (there are four zeroes and then a 3). This is called scientific notation.

a Express the following numbers in scientific notation.

i 50 000 **ii** 7 000 000 000 **iii** 0.005 **iv** 0.0000002

b The following timescales have been written in scientific notation. Rewrite them as regular numbers.

i 2×10^6 hours (the time it takes Pluto to orbit the Sun)

ii 4×10^7 days (the time it takes for light to travel from one side of the Milky Way to the other)

iii 3×10^{-3} seconds (the time it takes sound to travel one metre)

iv 3×10^{-9} seconds (the time it takes light to travel one metre).

c Sometimes scientific notation can be avoided by choosing a more appropriate unit of time (e.g. days instead of seconds). Rewrite the following timescales using the given units.

i 3×10^6 seconds (using days) **ii** 9×10^8 milliseconds (using hours)

iii 2×10^{-4} hours (using seconds) **iv** 5×10^{-8} days (using milliseconds)



The number of hydrogen atoms (the most abundant element) in the observable universe is estimated to be between 10^{79} and 10^{81}.

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5K Index laws: raising powers

Interactive

Widgets

HOTsheets

Walkthroughs

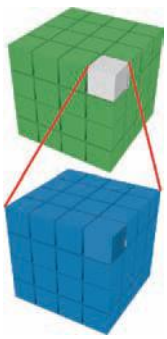
Consider what the expanded form of $(a^3)^4$ would be:

$$(a^3)^4 = a^3 \times a^3 \times a^3 \times a^3$$
$$= a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a$$
$$= a^{12}$$

Similarly:

$$(b^4)^2 = b^4 \times b^4$$
$$= b \times b \times b \times b \times b \times b \times b \times b$$
$$= b^8$$

This leads us to an index law: $(a^m)^n = a^{mn}$.



A pictorial representation of $(4^3)^3$. Each of the 4^3 green cubes in the top figure is made up of 4^3 tiny blue cubes shown magnified in the lower figure. How many blue cubes are there in total?

Let's start: How many factors?

The number 7 has two factors (1 and 7) and the number 7^2 has three factors (1, 7 and 49).

- Which of these has the most factors?

7^5 $\quad 7^2 \times 7^3$ $\quad (7^2)^3$ $\quad \dfrac{7^{10}}{7^6}$

- Which has more factors: 7^{10} or 10^7? Compare your answers with others in your class.

Key ideas

- $a^0 = 1$ for every value of a except 0. For example, $4^0 = 1$ and $(73xy)^0 = 1$.
- A power of a power can be simplified by multiplying indices: $(a^m)^n = a^{mn}$ (e.g. $(x^2)^5 = x^{10}$).
- Expressions involving powers can be expanded, so $(3x)^4 = 3^4x^4$ and $(2y)^{10} = 2^{10}y^{10}$.

Example 19 Working with zero powers

Simplify the following expressions using the index laws.

a $10^0 + 5^0$ b $(4x)^0 \times (8xy)^0$ c $4x^0 \times 8xy^0$

SOLUTION	EXPLANATION
a $10^0 + 5^0 = 1 + 1$ $= 2$	Recall $10^0 = 1$ and $5^0 = 1$ by the index law for zero powers.
b $(4x)^0 \times (8xy)^0 = 1 \times 1$ $= 1$	Any bracketed expression to the power 0 equals 1, so $(4x)^0 = 1$ and $(8xy)^0 = 1$.
c $4x^0 \times 8xy^0 = 4(1) \times 8x(1)$ $= 32x$	$4x^0$ means $4 \times x^0$, which is 4×1. Similarly $8xy^0$ means $8 \times x \times y^0 = 8 \times x \times 1$.

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Example 20 Simplifying powers of powers

Simplify the following expressions using the index laws.

a $(2^3)^5$ **b** $(5x^3)^2$ **c** $(u^2)^4 \times (7u^3)^2$

SOLUTION	EXPLANATION
a $(2^3)^5 = 2^{15}$	$3 \times 5 = 15$, so we can apply the index law easily.
b $(5x^3)^2 = 5^2(x^3)^2$ $= 5^2x^6$	Expand the brackets to square both terms within them. $3 \times 2 = 6$, so $x^3)^2 = x^6$
c $(u^2)^4 \times (7u^3)^2 = u^8 \times 7^2u^6$ $= 7^2u^{14}$	Apply the index law with $2 \times 4 = 8$. Apply the first index law: $u^8 \times u^6 = u^{14}$.

Exercise 5K

1–3 | 2, 3 | —

UNDERSTANDING

1 Which one of the following is the expanded form of 5^3?

A $5 \times 5 \times 5$ **B** 5×3 **C** $5 + 5 + 5$ **D** $5 \times 5 \times 3$

2 Which one of the following is equivalent to $(a^3)^2$?

A $a \times a \times a$ **B** $a^3 \times a^3$ **C** $a^2 \times a^2$ **D** $a \times 3 \times 2$

3 Which of the following is equivalent to $(3x)^2$?

A $3 \times x$ **B** $3 \times x \times x$ **C** $3 \times x \times 3 \times x$ **D** $3 \times 3 \times x$

4–6(½) | 4–7(½) | 4–7(½)

FLUENCY

Example 19

4 Simplify the following.

a 7^0 **b** $5^0 \times 3^0$ **c** $5b^0$

d $12x^0y^2z^0$ **e** $3x^2)^0$ **f** $13(m + 3n)^0$

g $2\ x^0y)^2$ **h** $4x^0(4x)^0$ **i** $3\ a^5y^2)^0\ a^2$

Example 20a, Example 20b

5 Simplify the following.

a $2^3)^4$ **b** $5^2)^8$ **c** $6^4)^9$

d $d^3)^3$ **e** $k^8)^3$ **f** $m^5)^{10}$

Example 20c

6 Simplify the following. Large numerical powers like 5^4 should be left in index form.

a $3x^5)^2$ **b** $2u^4)^3$ **c** $5x^5)^4$ **d** $12x^5)^3$

e $4x^4)^2$ **f** $7x^2)^2$ **g** $9x^7)^{10}$ **h** $10x^2)^5$

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5K

FLUENCY

Example 20c

7 Simplify the following using the index laws.

a $x^3)^2 \times x^5)^3$ **b** $y^2)^6 \times y^3)^2$ **c** $2k^4)^2 \times 5k^5)^3$

d $m^3)^6 \times 5m^2)^2$ **e** $4\ x^3)^2 \times 2\ x^4)^3$ **f** $5\ p^2)^6 \times 5p^2)^3$

g $\dfrac{(y^3)^4}{y^2}$ **h** $\dfrac{(p^7)^2}{(p^3)^2}$ **i** $\dfrac{(2p^5)^3}{2^2p^2}$

j $\dfrac{(3x^2)^{10}}{(x^3)^2}$ **k** $\dfrac{8h^{20}}{(h^3)^5}$ **l** $\dfrac{(q^2)^{10}}{(q^3)^6}$

8 8–10 9–11

PROBLEM-SOLVING

8 Find the missing value that would make the following simplifications correct.

a $(7^3)^{\square} = 7^{15}$ **b** $(x^{\square})^4 = x^{12}$

c $(x^2)^3 \times x^{\square} = x^{11}$ **d** $(x^4)^{\square} \times (x^3)^2 = x^{14}$

9 **a** Use the fact that $(x^2)^3 = x^6$ to simplify $((x^2)^3)^4$.

b Simplify $((x^3)^4)^5$.

c Put the following numbers into ascending order. (You do not need to calculate the actual values.)
$2^{100}, (2^7)^{10}, ((2^5)^6)^7, ((2^3)^4)^5$

10 **a** How many zeroes do the following numbers have?

i 10^2 **ii** 10^5 **iii** 10^6

b How many zeroes does the number $(10^5 \times 10^6 \times 10^7)^3$ have?

11 **a** Simplify $x^3 \times x^4$.

b Simplify $(x^3)^4$.

c Explain why $x^3 \times x^4$ is not equivalent to $(x^3)^4$.

d Find the two values of x that make $x^3 \times x^4$ and $(x^3)^4$ equal.

12 12, 13 13–15

REASONING

12 For this question you will be demonstrating why a^0 should equal 1 for any value of a other than zero.

a State the value of $\dfrac{5^2}{5^2}$.

b Use the index law for division to write $\dfrac{5^2}{5^2}$ as a power of 5.

c Use this method to prove that $3^0 = 1$.

d Use this method to prove that $100^0 = 1$.

e Explain why you cannot use this method to prove that $0^0 = 1$.

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5K

REASONING

13 Ramy is using his calculator and notices that $(2^3)^4 = (2^6)^2$.

a Explain why this is the case.

b Which of the following are also equal to $(2^3)^4$?

A $(2^4)^3$ **B** $(2^2)^6$ **C** $(4^2)^3$ **D** $(4^3)^2 \times (6^2)^2$

c Freddy claims that $(2^5)^6$ can be written in the form $(4^{\square})^{\square}$. Find one way to fill in the two missing values.

14 **a** According to the index laws, what number is equal to $(9^{0.5})^2$?

b What positive number makes the equation $x^2 = 9$ true?

c What should $9^{0.5}$ equal according to this? Check on a calculator.

d Use this observation to predict the value of $36^{0.5}$.

15 Alexios notices that $\dfrac{(a^3)^2 \times a^4}{(a^2)^5}$ is always equal to one, regardless of the value of a.

a Simplify the expression above.

b Give an example of two other expressions that will always equal 1 because of the index laws.

Combining index laws — — 16

ENRICHMENT

16 Simplify the following using the index laws.

a $\dfrac{(5x^2)^3 \times (5x^3)^4}{(5x^6)^3}$

b $\dfrac{(x^2)^4}{x^3} \times \dfrac{x^7}{(x^2)^2}$

c $\dfrac{(x^2y^3)^4 \times (x^3y^2)^5}{(xy)^7 \times (x^2y)^6}$

d $\dfrac{(a^2b^3c^4)^{10}}{a^{10}b^{20}c^{30}} \div \dfrac{a^3}{b^2}$

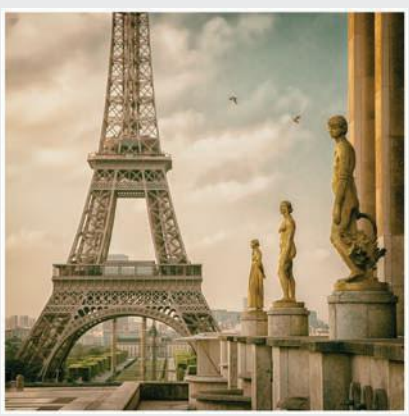
e $\dfrac{(x^{20}y^{10})^5}{(x^{10}y^{20})^2}$

f $\dfrac{(7^8)^9}{(7^{10})^7}$

g $\dfrac{(7^6)^5}{(7^5)^6}$

h $\dfrac{5^{11} \times 5^{13}}{(5^2)^{11}}$

i $\dfrac{100^{20}}{1000^{12}}$



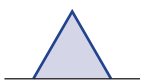
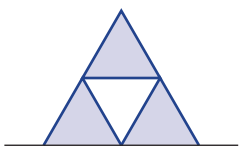
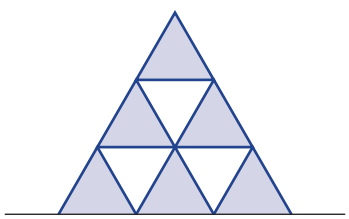
Paris – the Renaissance mathematician Nicolas Chuquet developed a form of index notation here in the 15th century, but our present index notation was not widely used before Rene Descartes' writing spread it in the 17th century.

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Investigation

Card pyramids

Using a pack of playing cards, build some pyramids on your desk like the ones illustrated below.

		
Pyramid 1 (One-triangle pyramid) (Two cards)	Pyramid 2 (Three-triangle pyramids) (Seven cards)	Pyramid 3 (Six-triangle pyramids) (Fifteen cards)

1 Copy and complete this table.

Number of triangle pyramids on base	1	2	3	5			
Total number of triangle pyramids	1	3				45	55
Total number of cards required	2		15		100		



2 Describe the number of pyramids in, and the number of cards required for, pyramid 20 (20 pyramids on the base). How did you get your answer?

3 If you had 10 decks of playing cards, what is the largest tower you could make? Describe how you obtained your answer.

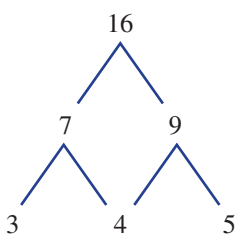
Number pyramids

Number pyramids with a base of three consecutive numbers

1 Can you explain how this number pyramid is constructed?

2 Draw a similar number pyramid starting with the number 4 on the left of the base.

3 Draw a similar number pyramid that has 44 as its top number. Remember the base of the pyramid must be consecutive numbers.



4 Can you draw a similar number pyramid that has 48 as its top number? Explain your answer.

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5 Draw several of these pyramids to investigate how the top number is related to the starting value.

a Set up a table showing starting values and top numbers.

b Can you work out an algebraic rule that calculates top numbers given the starting number?

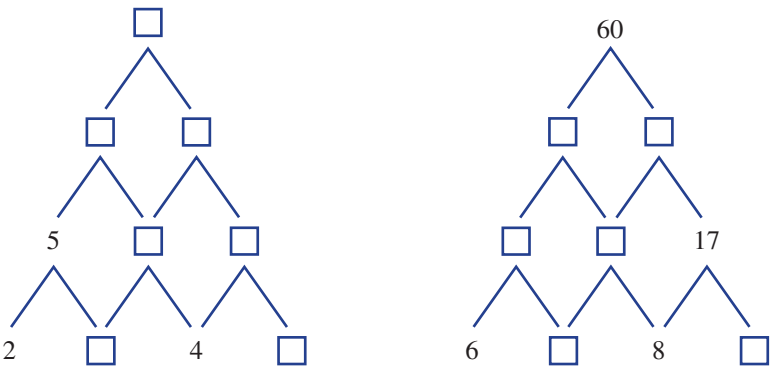
6 Draw a number pyramid that has a base row of $n, n+1$ and $n+2$. What is the algebraic expression for the top number? Check this formula using some other number pyramids.

7 What is the sum of all the numbers in a pyramid with base row $-10, -9, -8$?

8 Determine an expression for the sum of all the numbers in a pyramid starting with n on the base.

Number pyramids with four consecutive numbers on the base

9 Copy and complete the following number pyramids.



10 Investigate how the top number is related to the starting number. Can you show this relationship using algebra?

11 Write the sequence of all the possible top numbers less than 100.

12 What patterns can you see in this sequence of top numbers? Can you find some ways of showing these patterns using algebraic expressions? Let the bottom row start with n. (In the examples above $n = 6$ and $n = 2$.)

Number pyramids with many consecutive numbers on the base

13 Determine the algebraic rule for the value of the top number for a pyramid with a base of six consecutive numbers starting with n.

14 List out the algebraic expressions for the first number of each row for several different sized pyramids all starting with n. What patterns can you see occurring in these expressions for:

a the coefficients of n? **b** the constants?

15 What is the top number in a pyramid with a base of 40 consecutive numbers starting with 5?

16 Write an expression for the top number if the base had 1001 consecutive numbers starting with n.

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Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 Five consecutive even integers have $2m + 2$ as the middle integer. Find two simplified equivalent expressions for the sum of these five integers.

2 Re-arrange the order of the five expressions $4(a + 1), 6a - 5, 2 - a, a - 7, 6 - 2a$ so that the sum of the first three expressions and the sum of the last three expressions are both equal to $3(a + 1)$.

3 Write this list $16^{1000}, 8^{1334}, 4^{1999}, 2^{4001}$ in ascending order.

4 Finding the largest value

a If m can be any number, what is the largest value that $10 - m(m + 5)$ could have?

b If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?

c If a and b are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

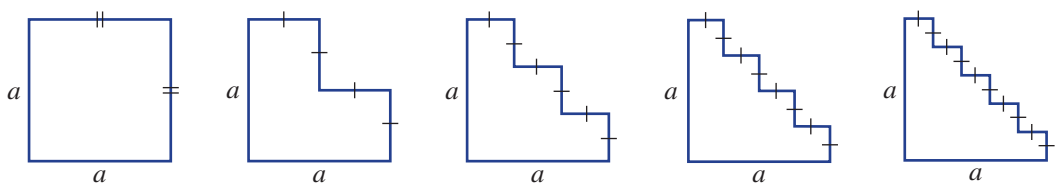
5 Simplify these algebraic expressions.

a $\frac{a}{5} + \frac{a+1}{6} - \frac{a}{2}$ **b** $\frac{x-1}{3} - \frac{2x-3}{7} + \frac{x}{6}$

6 The following three expressions all evaluate to numbers between 1 and 100, but most calculators cannot evaluate them. Find their values using the index laws.

a $\frac{2^{1001} \times 2^{2002}}{(2^{150})^{20}}$ **b** $\frac{5^{1000} \times 3^{1001}}{15^{999}}$ **c** $\frac{8^{50} \times 4^{100} \times 2^{200}}{(2^{250})^2 \times 2^{48}}$

7 Consider the following pattern.



The perimeter for the shape when $n = 1$ is given by the expression $4a$ and the area is a^2.

a Give expressions for the perimeter and area of the other shapes shown above and try to find a pattern.

b If $a = 6$ and $n = 1000$, state the perimeter and give the approximate area.

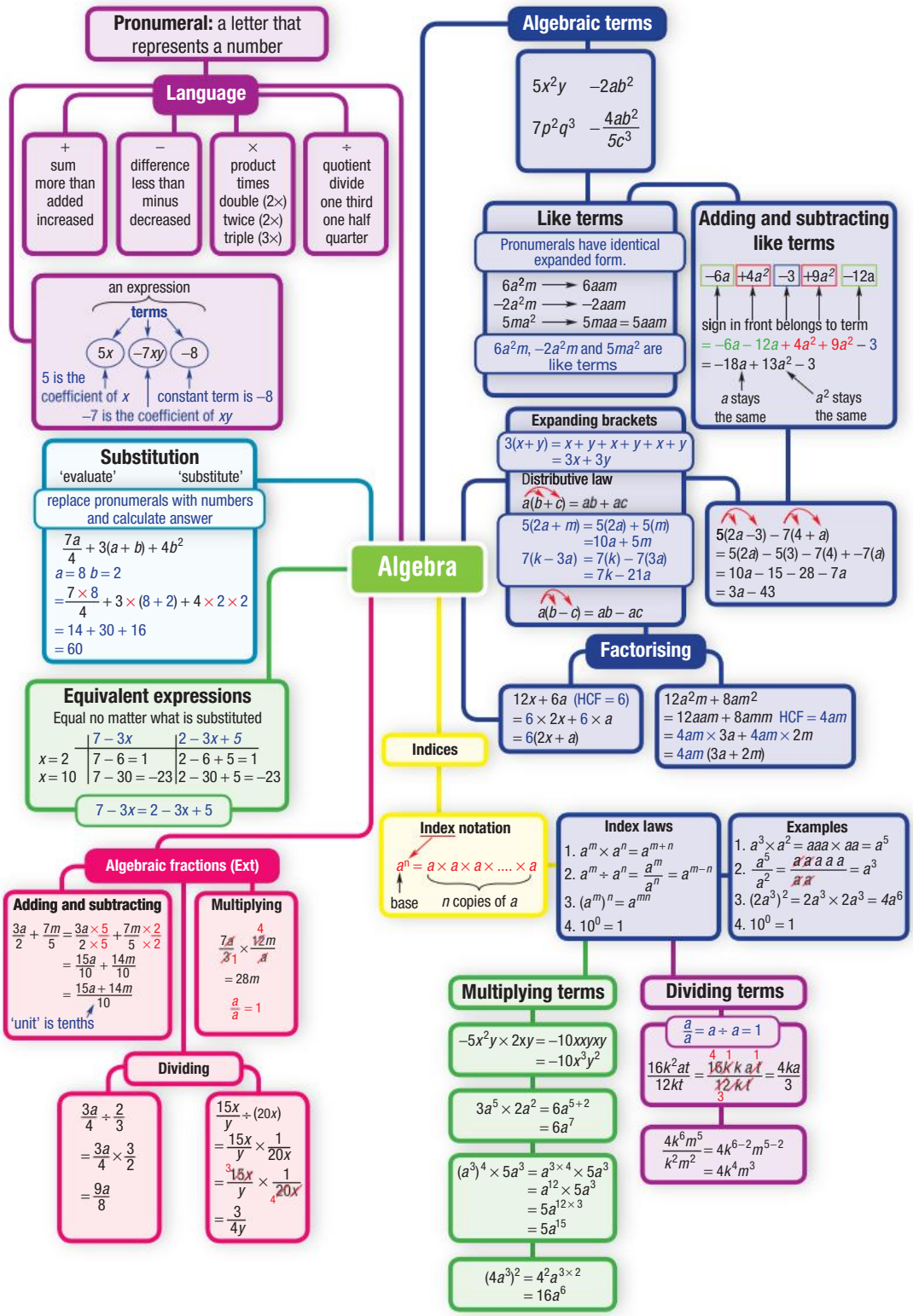
8 A cube has a side length of $2^x 3^y$ cm. Determine the volume and surface area of this cube, writing the answers in index form.

9 Determine the value of the pronumerals in each of the following equations.

a $5^x = 125$ **b** $3^a = 81$ **c** $2^b 3^c = 72$ **d** $25^x = 5$ **e** $8^k = 32$

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Chapter summary



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Multiple-choice questions

5A **1** Consider the expression $5a^2 - 3b + 8$. Which one of the following statements is true?

A The coefficient of a is 5. **B** It has 5 terms.
C The constant term is 8. **D** The coefficient of b is 3.
E The coefficient of a^2 is 10.

5A **2** Half the sum of double x and 3 can be written as:

A $\frac{1}{2} \times 2x + 3$ **B** $\frac{2x+6}{2}$ **C** $x + 6$ **D** $\frac{2x+3}{2}$ **E** $\frac{2(x-3)}{2}$

5C **3** The simplified form of $12x + 4y - 3x$ is:

A $15x + 4y$ **B** $9x + 4y$ **C** $16xy - 3x$ **D** $13xy$ **E** $12x + y$

5B **4** Find the value of $5 - 4a^2$ given $a = 2$.

A 3 **B** -3 **C** 21 **D** -11 **E** 13

5D **5** $3 \times x \times y$ is equivalent to:

A $3x + y$ **B** xy **C** $3 + x + y$ **D** $3x + 3y$ **E** $xy + 2xy$

5D **6** $\frac{12ab}{24a^2}$ can be simplified to:

A $2ab$ **B** $\frac{2a}{b}$ **C** $\frac{b}{2a}$ **D** $\frac{ab}{2}$ **E** $\frac{b}{2}$

5G **7** The expanded form of $2x(3 + 5y)$ is:

A $6x + 5y$ **B** $3x + 5y$ **C** $6x + 5xy$ **D** $6 + 10y$ **E** $6x + 10xy$

5D **8** Simplifying $3a \div (6b)$ gives:

A 2 **B** $\frac{a}{b}$ **C** $\frac{2a}{b}$ **D** $\frac{ab}{2}$ **E** $\frac{a}{2b}$

5J **9** $5^7 \times 5^4$ is equal to:

A 25^{11} **B** 5^{28} **C** 25^3 **D** 5^3 **E** 5^{11}

5H **10** The factorised form of $3a^2 - 6ab$ is:

A $3a^2(1 - 2b)$ **B** $3a(a - 2b)$ **C** $3a(a - b)$ **D** $6a(a - b)$ **E** $3(a^2 - 2ab)$

Short-answer questions

5A **1** State whether each of the following is true or false:

a The constant term in the expression $5x + 7$ is 5.
b $16xy$ and $5yx$ are like terms.
c The coefficient of d in the expression $6d^2 + 7d + 8abd + 3$ is 7.
d The highest common factor of $12abc$ and $16c$ is $2c$
e The coefficient of xy in the expression $3x + 2y$ is 0.

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Chapter review

5A **2** For the expression $6xy + 2x - 4y^2 + 3$, state:

a the coefficient of x **b** the constant term

c the number of terms **d** the coefficient of xy

5B **3** Substitute the following values of a to evaluate the expression $8 - a + 2a^2$.

a -1 **b** 2 **c** -3 **d** 0

5B **4** Substitute $x = 2$ and $y = -3$ into each of the following.

a $2y + 3$ **b** $3x + y$ **c** $xy + y$

d $4x - 2y$ **e** $\frac{5xy}{6} + 2$ **f** $\frac{-3x + 2y}{x + y}$

5B **5** For what value of x is $3 - x$ equal to $x - 3$?

5C **6** Simplify each of these expressions by collecting like terms.

a $7m + 9m$ **b** $3a + 5b - a$ **c** $x^2 - x + x^2 + 1$

d $5x + 3y + 2x + 4y$ **e** $7x - 4x^2 + 5x^2 + 2x$ **f** $-8m + 7m + 6n - 18n$

5D **7** Simplify these expressions.

a $9a \times 4b$ **b** $30 \times x \times y \div 2$ **c** $-8x \times 4y \div (-2)$

5D **8** Copy and complete the following equivalences.

a $3y \times \square = 15xy$ **b** $3ab \times \square = -12abc$

c $\frac{3x}{4} = \frac{\square}{20}$ **d** $9a^2b \div \square = 3a$

5E/F **9** Express each of the following in their simplest form.

Ext

a $\frac{5x}{12} - \frac{x}{6}$ **b** $\frac{2a}{5} + \frac{b}{15}$ **c** $\frac{6x}{5} \times \frac{15}{2x^2}$ **d** $\frac{4a}{7} \div \frac{8a}{21b}$

5G **10** Expand and simplify when necessary.

a $3(x - 4)$ **b** $-2(5 + x)$ **c** $k(3l - 4m)$ **d** $2(x - 3y) + 5x$

e $7 - 3(x - 2)$ **f** $10(1 - 2x)$ **g** $4(3x - 2) + 2(3x + 5)$

5H **11** Factorise fully.

a $2x + 6$ **b** $24 - 16g$ **c** $12x + 3xy$ **d** $7a^2 + 14ab$

5H **12** By factorising first, simplify the following fractions.

a $\frac{5a + 10}{5}$ **b** $\frac{12x - 24}{x - 2}$ **c** $\frac{16p}{64p + 48pq}$

5J **13** Find the missing values.

a $7^5 \times 7^2 = 7^{\square}$ **b** $5^4 \div 5 = 5^{\square}$ **c** $4^2 \times 4^{\square} = 4^8$ **d** $3^4 \times 3^{\square} = 3^5$

5J/K **14** Use the index laws to simplify each of the following expressions:

a $m^2 \times m^5$ **b** $3m^7 \times 4m$ **c** $\frac{m^5}{m^3}$

d $\frac{12a^6}{6a^2}$ **e** $(x^3)^4$ **f** $(2a^2)^3$

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5J/K **15** Simplify:

a $-10x^6y^3z^4 \div (5x^2yz^2)$ **b** $(y^5)^2$ **c** $7a^0$ **d** $(2x^3)^0 \times 2(x^3)^0$

e $(2y^3)^2 \times y^4$ **f** $(m^4)^3 \div (m^3)^2$ **g** $(2b)^3 \div (4b^2)$ **h** $\dfrac{(d^3e^3y^5)^2}{e^7} \times \dfrac{e}{(dy)^6}$

Extended-response questions

1 Two bus companies have different pricing structures.

Company A: \$120 call-out fee, plus \$80 per hour

Company B: \$80 call-out fee, plus \$100 per hour

a Write an expression for the total cost \$$A$ of travelling n hours with company A.

b Write an expression for the total cost \$$B$ of travelling for n hours with company B.

c Hence, state the cost of travelling for 3 hours with each company.

d For how long would you need to hire a bus to make company A the cheaper option?

e In the end, a school principal cannot decide which bus company to choose and hires 3 buses from company A and 2 buses from company B. Give an expanded expression to find the total cost for the school to hire the five buses for n hours.

f If the trip lasts for 5 hours, how much does it cost to hire the five buses for this period of time?



2 Consider the floor plan shown, labelled Plan A.

a Write an expanded expression for the floor's area in terms of x and y.

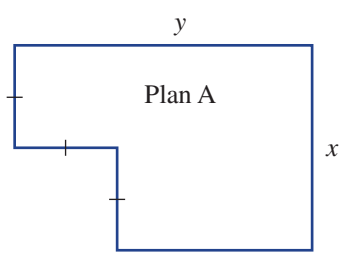
b Hence, find the floor's area if $x = 6$ metres and $y = 7$ metres.

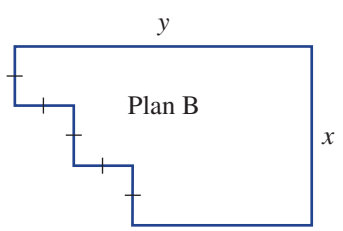
c Write an expression for the floor's perimeter in terms of x and y.

d Hence, find the floor's perimeter if $x = 6$ metres and $y = 7$ metres.

e Another floor plan (Plan B) is shown. Write an expression for the floor's area and an expression for its perimeter.

f Describe how the area and perimeter change when the floor plan goes from having two 'steps' to having three 'steps'.





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Semester review 1

Integers

Multiple-choice questions

1 $156 \div 4$ is the same as:

A $156 \div 2 \times 2$ **B** $156 \div 2 \div 2$ **C** $312 \div 2$ **D** $156 \times 2 \div 2$

2 $-24 + 6 \times (-3)$ is equal to:

A 6 **B** 42 **C** -42 **D** -6

3 What is the smallest number that can be added to 1923 to make the answer divisible by 9?

A 1 **B** 2 **C** 3 **D** 4

4 $(-15)^2$ equals:

A 225 **B** 30 **C** -30 **D** -225

5 Two numbers have a sum of -10 and a product of -56. The larger of the two numbers is:

A -4 **B** 4 **C** 14 **D** -14

Short-answer questions

1 Evaluate, without using a calculator:

a $4973 + 196$ **b** $1506 - 156$ **c** -96×3

d 139×5 **e** 14×99 **f** $14 \times 99 + 14 \times 101$

g 9^2 **h** 4^3 **i** $-9 - 7 - 3$

2 Evaluate:

a $10 - 6 \times 4$ **b** $15 \times 4 \div 2$ **c** $24 \div 2 \times 6$

d $-3 + (-10 - (-6))$ **e** $-81 \div (-3) \times 2$ **f** $73 - 72 - 7$

3 Find the HCF of:

a 24 and 42 **b** 35 and 42 **c** 100 and 60

d 15, 45 and 36

4 Write down the LCM of:

a 24 and 42 **b** 8 and 9 **c** 100 and 60

d $7^2 \times 5^2 \times 3^3$ and $2 \times 7^2 \times 5 \times 3^2$

5 If $a = -5$, $b = 4$ and $c = -2$, evaluate these expressions.

a $a + b + c$ **b** abc **c** $a^2 - c$

d $5(a - b + c)$ **e** a^2 **f** c^3

g $\dfrac{8a + \sqrt{b}}{c}$

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Extended-response question

The weather for a November day is given for different cities around the world.

	Minimum (°C)	Maximum (°C)
Amsterdam	3	12
Auckland	11	18
LA	8	14
Hong Kong	16	28
Moscow	6	8
Beijing	–3	0
New York	8	10
Paris	6	13
Tel Aviv	16	23
Wollongong	18	22

a Which city recorded the highest temperature on the day shown in the table?

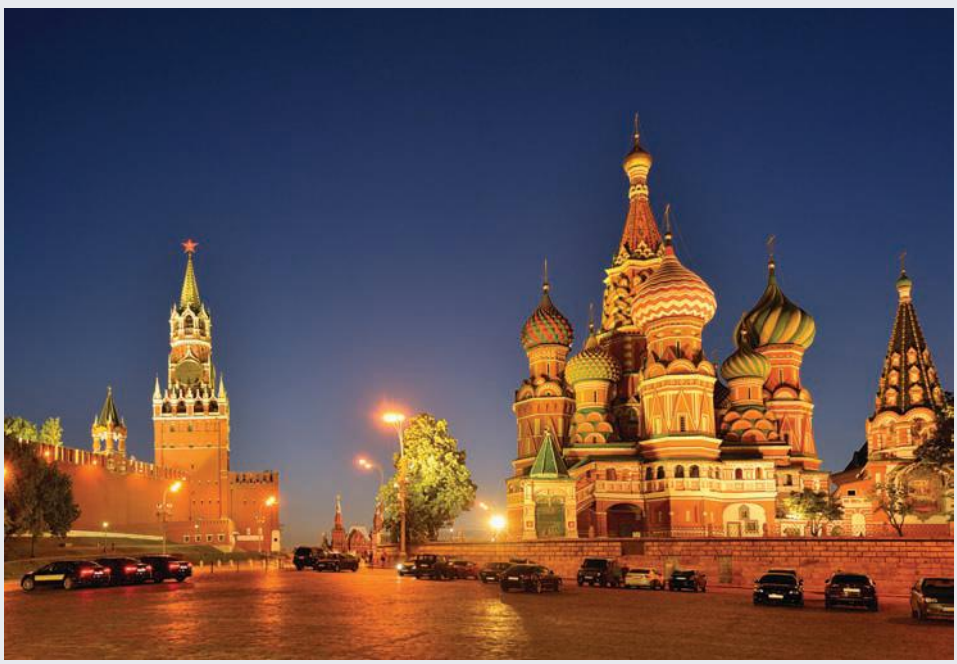
b Which two cities only had a two-degree variance in temperature?

c Which city had the largest variance in temperature on this November day?

d What was the mean (average) minimum temperature for the 10 cities listed in the table?

e What was the mean (average) maximum temperature?

f If Bangkok's temperature of 29 to 34 degrees were added to the table, what effect would this have on the means?



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Lines, shapes and solids

Multiple-choice questions

1 The supplementary angle to 80° is:

A 10° **B** 100° **C** 280° **D** 20°

2 In this diagram a equals:

A 150 **B** 220 **C** 70 **D** 80



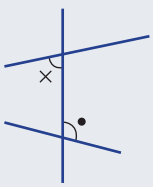
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3 The interior angle of a regular pentagon is:

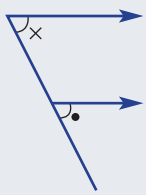
A 72° **B** 540° **C** 108° **D** 120°

4 Which diagram shows equal alternate angles?

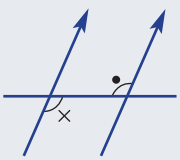
A



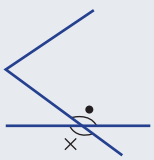
B



C



D



Ext

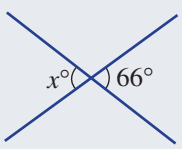
5 A polyhedron with 5 vertices, 8 edges and 5 faces could be described as a:

A triangular pyramid **B** square prism
C square pyramid **D** triangular prism

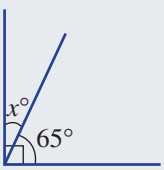
Short-answer questions

1 Find the value of x.

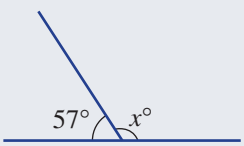
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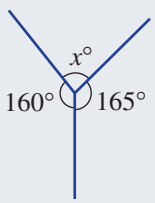
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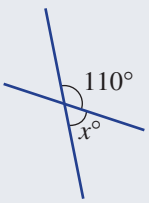
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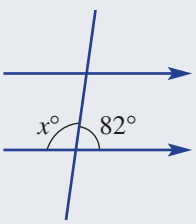
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e



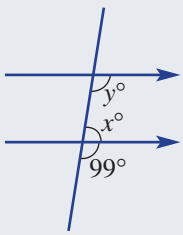
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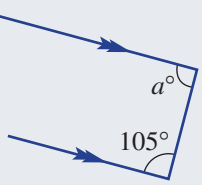
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2 Find the value of each pronumeral.

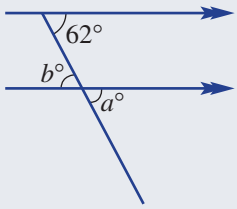
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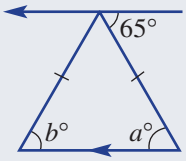
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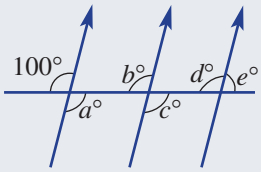
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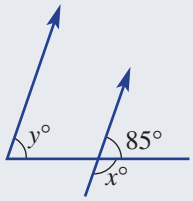
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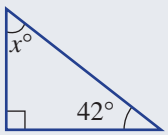


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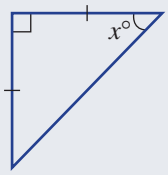


3 Find the value of the pronumeral in these triangles.

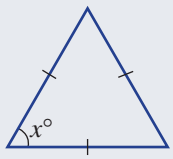
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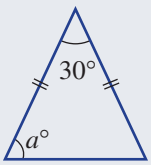
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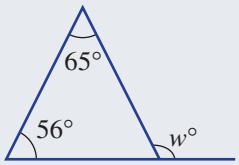
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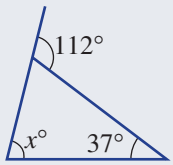
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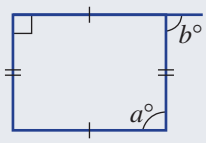


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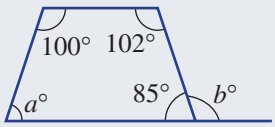


Ext **4** Find the value of a and b in these quadrilaterals.

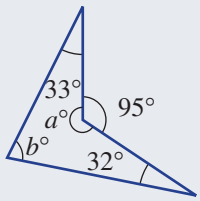
a



b



c

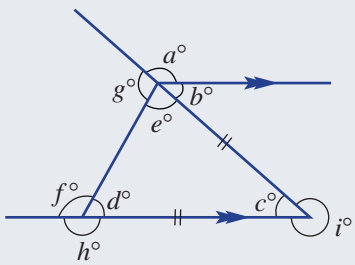


Ext **5** Find the interior angle of a regular hexagon.

Extended-response question

If $a = 115$, find the size of each angle marked. Give a reason for each answer. Write your answers in the order you found them.

Is the order the same for everybody in the class? Discuss any differences and the reasons associated with each.



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Fractions, decimals and percentages

Multiple-choice questions

1 $\frac{150}{350}$ simplifies to:

A $\frac{6}{14}$ **B** $\frac{3}{70}$ **C** $\frac{15}{35}$ **D** $\frac{3}{7}$

2 Sienna spends $\frac{3}{7}$ of \$280 her income on clothes and saves the rest. She saves:

A \$470 **B** \$120 **C** \$160 **D** \$2613

3 0.008×0.07 is equal to:

A 0.056 **B** 0.0056 **C** 0.00056 **D** 56

4 0.24 expressed as a fraction is:

A $\frac{1}{24}$ **B** $\frac{6}{25}$ **C** $\frac{12}{5}$ **D** $\frac{24}{10}$

5 If 5% of $x = y$, then 10% of $2x$ equals:

A $\frac{1}{2}y$ **B** $2y$ **C** $4y$ **D** $10y$

Short-answer questions

1 Copy and complete these equivalent fractions.

a $\frac{3}{5} = \frac{\square}{30}$ **b** $\frac{\square}{11} = \frac{5}{55}$ **c** $1\frac{4}{6} = \frac{\square}{3}$

2 Evaluate each of the following.

a $\frac{3}{4} - \frac{1}{2}$ **b** $\frac{4}{5} + \frac{3}{5}$ **c** $1\frac{1}{2} + 1\frac{3}{4}$

d $\frac{4}{7} - \frac{2}{3}$ **e** $\frac{4}{9} \times \frac{3}{4}$ **f** $1\frac{1}{2} \times \frac{3}{5}$

3 Write the reciprocal of:

a $\frac{2}{5}$ **b** 8 **c** $4\frac{1}{5}$

4 Evaluate:

a $2\frac{1}{2} \times 1\frac{4}{5}$ **b** $1\frac{1}{2} \div 2$ **c** $1\frac{1}{2} \times \frac{1}{4} \div \frac{3}{5}$

5 Calculate each of the following.

a $3.84 + 3.09$ **b** $10.85 - 3.27$ **c** $12.09 \div 3$

d $6.59 - 0.2 \times 0.4$ **e** 96.37×40 **f** $15.84 \div 0.02$

6 Evaluate:

a 5.3×103 **b** 9.6×105 **c** $61.4 \div 100$

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7 Copy and complete this table of decimals, fractions and percentages.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$				
Decimal								0.99	0.005
Percentage						80%	95%		

8 Find:

a 10% of 56 **b** 12% of 98 **c** 15% of 570 m

d 99% of \$2 **e** $12\frac{1}{2}$% of \$840 **f** 58% of 8500 g

9 **a** Increase \$560 by 25%.

b Decrease \$980 by 12%.

c Increase \$1 by 8% and then decrease the result by 8%.

10 A \$348 Charlie Brown dress sold for \$261. This represents a saving of x%. What is the value of x?

Extended-response question

A laptop decreases in value by 15% a year.

a Find the value of a \$2099 laptop at the end of:

i 1 year **ii** 2 years **iii** 3 years

b After how many years is the laptop worth less than \$800?

c Is the laptop ever going to have a value of zero dollars? Explain.

Measurement and introduction to Pythagoras' theorem

Multiple-choice questions

1 A cube has a volume of 8 cubic metres. The surface area of this cube is:

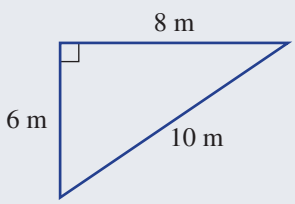
A 2 m^2 **B** 4 m^2

C 24 m^2 **D** 384 m^2

2 The area of this triangle is:

A 48 m^2 **B** 24 m^2

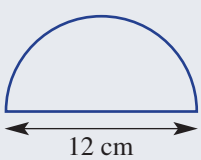
C 30 m^2 **D** 40 m^2



3 The perimeter of this semicircle is closest to:

A 38 cm **B** 30 cm

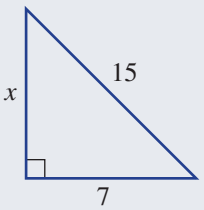
C 19 cm **D** 31 cm



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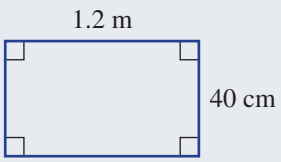
4 The value of x in this triangle is closest to:

A 176 **B** 13
C 274 **D** 17



5 The area of this rectangle is:

A 48 m^2 **B** 48 000 cm^2
C 480 cm^2 **D** 0.48 m^2



Short-answer questions

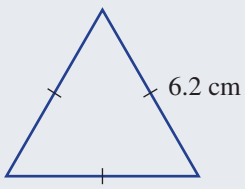
1 Complete these conversions.

a 5 m = _____cm **b** 1.8 m = _____cm **c** 9 m^2 = _____cm^2

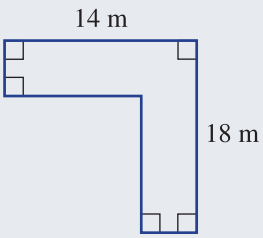
d 1800 mm = _____m **e** 4 L = _____cm^3 **f** $\frac{1}{100}$ km^2 = _____m^2

2 Find the perimeter of these shapes.

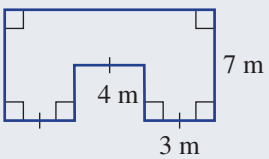
a



b



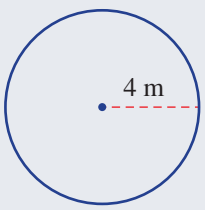
c



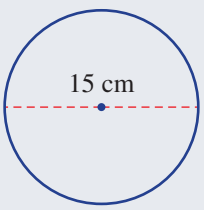
3 Find correct to two decimal places:

i the circumference **ii** the area

a



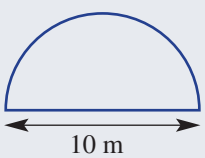
b



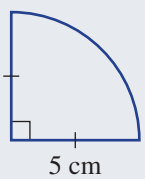
4 Find correct to two decimal places:

i the perimeter **ii** the area

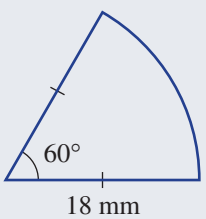
a



b



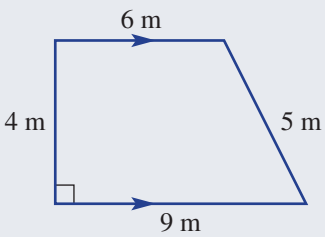
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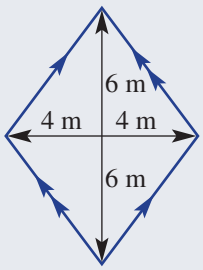
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5 Find the area of these shapes.

a

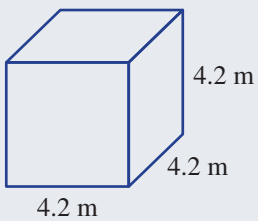


b

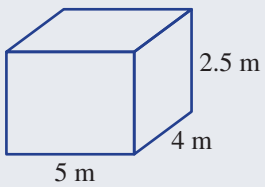


Ext **6** Find the surface area and volume of these solids.

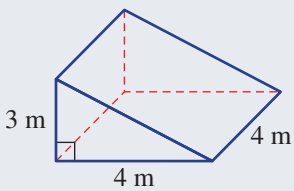
a



b

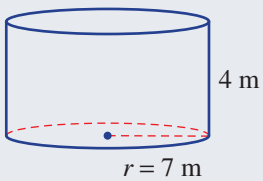


c

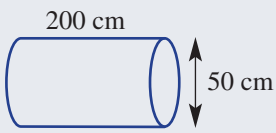


7 Find the volume of each cylinder correct to two decimal places.

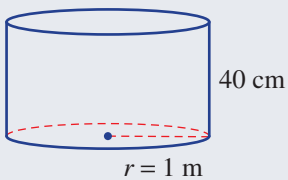
a



b

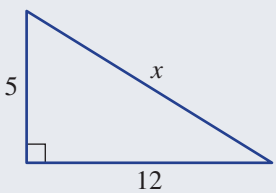


c

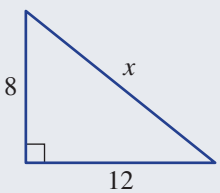


Ext **8** Find the value of x in these triangles. Round to two decimal places for part **b**.

a



b

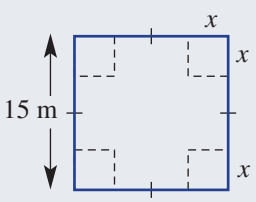


9 Write these times using 24-hour time.

a 3:30 p.m. **b** 7:35 a.m.

Extended-response question

A square sheet of metal 15 m by 15 m has equal squares of sides x m cut from each corner as shown. It is then folded to form an open tray.



a What is the length of the base of the tray? Write an expression.

b What is the height of the tray?

c Write an expression for the volume of the tray.

d If $x = 1$, find the volume of the tray.

e What value of x do you think produces the maximum volume?

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Algebra

Multiple-choice questions

1 $8^3 \times 8^4$ is the same as:

A 8^{-1} **B** 64^7 **C** 8^7 **D** 8^{12}

2 $4x + 5 + 3x$ is the same as:

A $7x + 5$ **B** $12x$ **C** $12 + x^2$ **D** $2x + 12$

3 $12m + 18$ factorises to:

A $2(6m - 9)$ **B** $-6(2m - 3)$ **C** $6(3 - 2m)$ **D** $6(2m + 3)$

4 $5a + 5 - 4a - 4 - a - 1$ equals:

A $a - 1$ **B** 0 **C** $2 - a$ **D** $a + 1$

5 Which answer is NOT equivalent to $(m \times n) \div (p \times q)$?

A $\frac{mn}{pq}$ **B** $m \times \frac{n}{pq}$ **C** $\frac{m}{p} \times \frac{n}{q}$ **D** $\frac{mnq}{p}$

Short-answer questions

1 Write an expression for:

a the sum of p and q **b** the product of p and 3

c half the square of m **d** the sum of x and y, divided by 2

2 If $a = 6$, $b = 4$ and $c = -1$, evaluate:

a $a + b + c$ **b** $ab - c$ **c** $a(b^2 - c)$

d $3a^2 + 2b$ **e** abc **f** $\frac{ab}{c}$

3 Simplify each algebraic expression.

a $4 \times 6k$ **b** $a + a + a$ **c** $a \times a \times a$ **d** $7p \div 14$

e $3ab + 2 + 4ab$ **f** $7x + 9 - 6x - 10$ **g** $18xy \div (9x)$ **h** $m + n - 3m + n$

Ext

4 Simplify:

a $\frac{5xy}{5}$ **b** $\frac{3x}{7} - \frac{2x}{7}$ **c** $\frac{w}{5} + \frac{w}{2}$ **d** $3a + \frac{a}{2}$

Ext

5 Simplify:

a $\frac{m}{5} \times \frac{5}{6}$ **b** $\frac{ab}{7} \div \frac{1}{7}$ **c** $\frac{m}{3} \times \frac{n}{2} \div \frac{mn}{4}$

6 Expand, and simplify where necessary.

a $6(2m - 3)$ **b** $10 + 2(m - 3)$ **c** $5(A + 2) + 4(A - 1)$

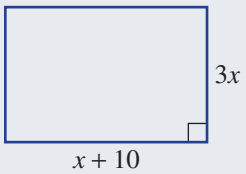
7 Factorise:

a $18a - 12$ **b** $6m^2 + 6m$ **c** $-8m^2 - 16mn$

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8 Write an expression for the rectangle's:

a perimeter **b** area



9 Simplify:

a $m^7 \times m^2$ **b** $8a^3 \times 4a$ **c** $12a^4b^6 \times (-4a^2b^3)$

d $a^{12} \div a^6$ **e** $a^7b^4 \div (a^3b^2)$ **f** $5a^6 \div (10a^6)$

10 Simplify:

a $(x^7)^2$ **b** $(2a^3)^4$ **c** $(-5a^4b^6)^2$

d x^0 **e** $(3x^2)^0$ **f** $-5(ab)^0c^2$

Extended-response question

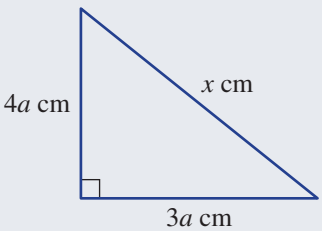
a Write an expression for the perimeter of this triangle.

b Write an expression for the area of this triangle.

c Use Pythagoras' theorem to find a relationship between x and a.

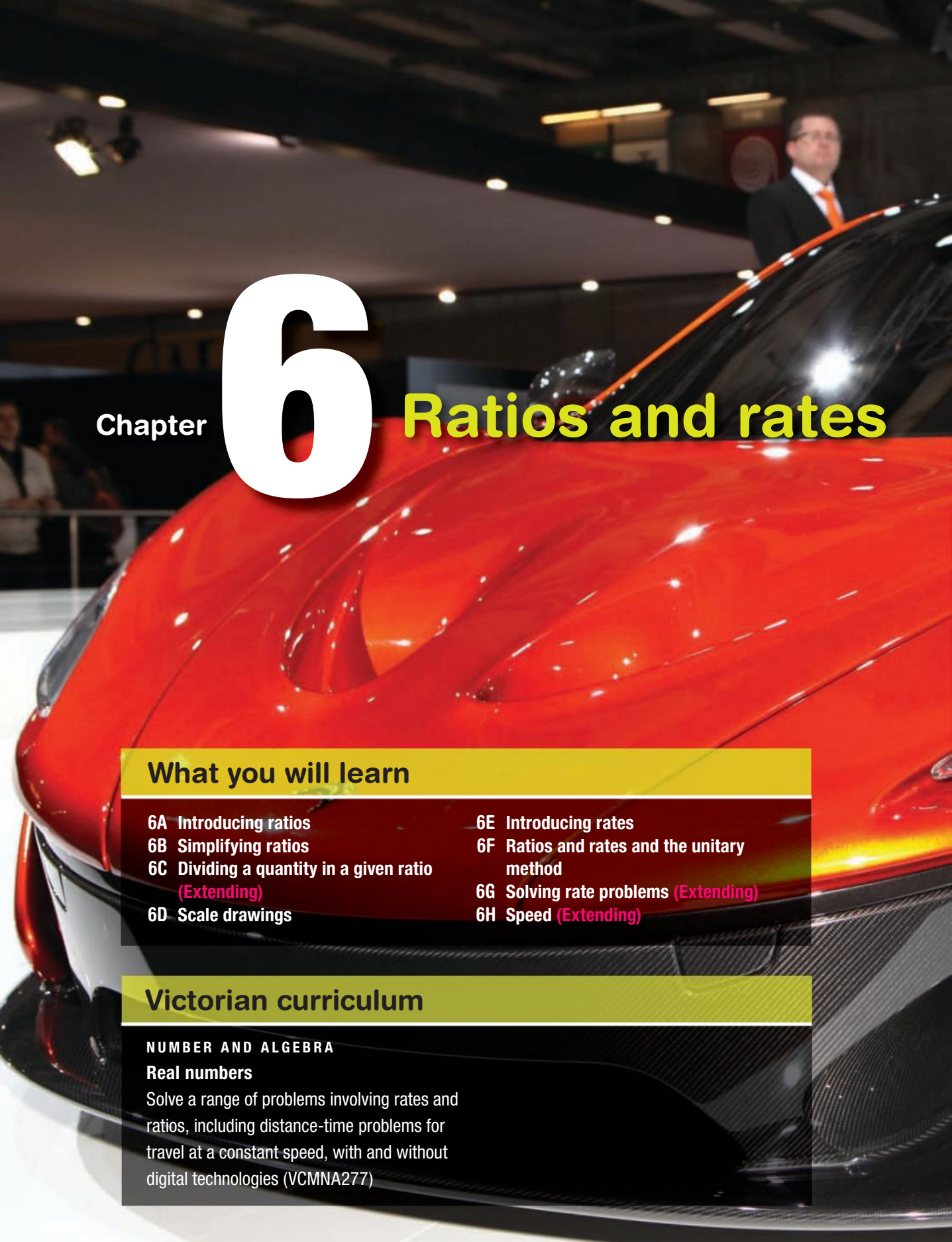
d Use your relationship to write an expression for the perimeter in terms of only a.

e If the perimeter equals 72 cm, what is the area of this triangle?





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Chapter 6 Ratios and rates

What you will learn

6A Introducing ratios
6B Simplifying ratios
6C Dividing a quantity in a given ratio (Extending)
6D Scale drawings
6E Introducing rates
6F Ratios and rates and the unitary method
6G Solving rate problems (Extending)
6H Speed (Extending)

Victorian curriculum

NUMBER AND ALGEBRA

Real numbers

Solve a range of problems involving rates and ratios, including distance-time problems for travel at a constant speed, with and without digital technologies (VCMNA277)

Essential Mathematics for the Victorian Curriculum Year 8
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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Gear ratios and Formula 1 racing

Gears are designed to transfer power from one moving part to another moving part. Gears are wheels with teeth. The motor rotates the 'drive' gear which in turn rotates the 'driven' gear. If the drive gear (e.g. with 30 teeth) makes two revolutions for every one revolution made by the driven gear (e.g. with 60 teeth), the gear ratio is 2:1. Riding a bicycle uphill with a gear ratio of 3:1 means that the pedals rotate 3 times in order to drive the rear wheel around once.

Cars cannot go faster than the gear ratios allow. In Formula 1 racing, such as the Australian Grand Prix, the race engineers use data from previous events to set the gear ratios. Each Formula 1 team must submit up to 30 pairs of gear ratios that they can choose from during the race season. Gear boxes must be removed whenever gear ratios are adjusted. The world famous McLaren P1 has 7 gears with a first gear ratio of 3.98:1 giving a maximum speed of 80 km/h in first gear. In seventh gear, the gear ratio is 0.69:1 and the top speed becomes 460 km/h.

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6A Introducing ratios









Ratios are regularly used in everyday life. They are used to show the relationship between two (or more) related quantities.

Here are five common uses of ratios:

- Ingredients – the ratio of different ingredients in a recipe (cooking, industrial)
- Maps – most maps include a scale which is written as a ratio
- Sporting success – showing a team's win to loss ratio or the ratio of kicking goals to points
- Comparing size – the ratio of length, area or volume of different shapes
- Legal requirements – minimum standards of supervision, staff to student ratios



The ratio of eggs to butter to sugar will affect the flavour and texture of a cake.

When dealing with ratios, the order in which the ratio is written is very important. For example, a team's win : loss ratio of 5 : 2 is very different to a team's win : loss ratio of 2 : 5.

Ratios compare quantities of the same type and given in the same unit. Therefore a ratio is not generally written with a unit of measurement.

Let's start: Estimating ratios

Estimate the following ratios.

- Ratio of boys to girls in your school
- Ratio of Year 8 students to Year 7 students in your school
- Ratio of your teacher's age to your age
- Ratio of hours you spend outside to hours you spend inside
- Ratio of hours you are awake to hours you are asleep
- Ratio of parents to children in an average Australian family
- Ratio of the length to the width of an A4 sheet of paper

Discuss your answers as a class.

Key ideas

- Ratios show the relationship between two (or more) related quantities.
  For example, a drink was made with the ratio of cordial to water of 1 : 3.
- The **colon :** is the mathematical symbol used to represent ratios.
  The written ratio of **a : b** is read as the ratio of '**a to b**' or '**a is to b**'.

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Key ideas

- The order in which the quantities are written in a ratio is important.
- Before ratios can be written and simplified, the quantities must be expressed in the same unit.
- If each value in a ratio is multiplied or divided by the same number an **equivalent ratio** is formed.
  For example, 1 : 3 and 2 : 6 and 10 : 30 are equivalent ratios.

Example 1 Writing ratios

A sample of mixed nuts contains 5 cashews, 12 peanuts and 2 macadamia nuts.
Write down:

a the ratio of cashews to peanuts to macadamias

b the ratio of cashews to the total number of nuts

c the ratio of peanuts to other nuts

SOLUTION	EXPLANATION
a 5 : 12 : 2	cashews : peanuts : macadamias
b 5 : 19	5 cashews, total nuts 19
c 12 : 7	12 peanuts, other nuts 7

Example 2 Producing equivalent ratios

Complete each pair of equivalent ratios.

a 4 : 9 = 16 : ☐ **b** 30 : 15 = ☐ : 5

SOLUTION	EXPLANATION
a $\times 4$ (4 : 9 → 16 : 36) $\times 4$	Both numbers are multiplied by 4.
b $\div 3$ (30 : 15 → 10 : 5) $\div 3$	Both numbers are divided by 3.

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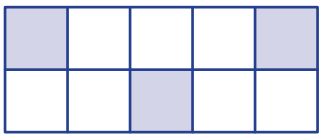
Exercise 6A

1–3 | 3 | —

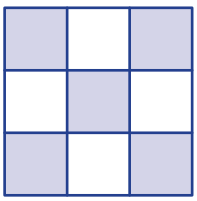
UNDERSTANDING

1 Write down the ratio of shaded parts to unshaded parts for each grid.

a



b

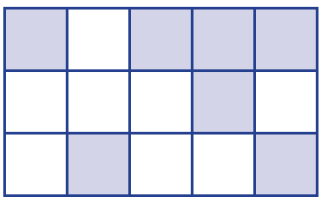


2 Write down the ratio of shaded parts to total parts for each grid.

a



b



3 Express each of the following pairs of quantities as a ratio.

a 9 goals to 4 goals

b 52 litres to 17 litres

c 7 potatoes to 12 carrots

d 3 blue marbles to 5 green marbles

4–6, 7–8(½) | 4–6, 7(½), 8–9 | 4–9(½)

FLUENCY

Example 1

4 A bag contains 5 green marbles, 7 blue marbles and 3 yellow marbles
Write down the ratio of:

a green marbles to yellow marbles

b blue marbles to total marbles

c yellow to blue to green marbles

d green to yellow to blue marbles



5 Over the past fortnight, it has rained on eight days and it has snowed on three days. Write down the ratio of:

a rainy days to snowy days

b snowy days to total days

c fine days to rainy and snowy days

d rainy days to non-rainy days

6 In a box of 40 flavoured icy poles there were 13 green, 9 lemonade, 11 raspberry and 7 orange icy poles. Write down the ratio of:

a green icy poles to orange icy poles

b raspberry icy poles to lemonade icy poles

c the four different flavours of icy poles

d green and orange icy poles to raspberry and lemonade icy poles

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6A

FLUENCY

Example 2

7 Complete each pair of equivalent ratios.

a 1 : 3 = 4 : ☐ **b** 1 : 7 = 2 : ☐ **c** 2 : 5 = ☐ : 10

d 3 : 7 = ☐ : 21 **e** 5 : 10 = 1 : ☐ **f** 12 : 16 = 3 : ☐

g 12 : 18 = ☐ : 3 **h** 20 : 50 = ☐ : 25 **i** 4 : 7 = 44 : ☐

j 4 : 17 = 12 : ☐ **k** 27 : 6 = ☐ : 2 **l** 121 : 66 = ☐ : 6

8 Complete each pair of equivalent ratios.

a 2 : 3 : 5 = 4 : ☐ : ☐ **b** 4 : 12 : 16 = ☐ : 6 : ☐

c 1 : 7 : 9 = ☐ : ☐ : 63 **d** 22 : 110 : 66 = 11 : ☐ : ☐

9 Write three equivalent ratios for each of the following ratios.

a 1 : 2 **b** 2 : 5 **c** 8 : 6 **d** 9 : 3

PROBLEM-SOLVING

10, 11 11–13 12–14

10 Sort the following ratios into three pairs of equivalent ratios.

2 : 5, 6 : 12, 7 : 4, 1 : 2, 4 : 10, 70 : 40

11 During a recent dry spell, it rained on only 3 days during the month of September.

a What was the ratio of wet days to total days for the month of September?

b Write equivalent ratios for a total of 10 days and 100 days.

12 On their way to work, Andrew passes 15 sets of traffic lights and Pauline passes 10 sets of traffic lights. One morning Andrew was stopped by 12 red traffic lights. How many green traffic lights would Pauline need to pass through to have the equivalent ratio of red to green traffic lights as Andrew?

13 Write the ratio of vowels to consonants for each of the following place names.

a Queensland **b** Canberra **c** Wagga Wagga **d** Australia

14 Name all Australian states that have a vowel to consonant ratio of 1 : 1.

REASONING

15 15, 16 16–18

15 Can 10 people be divided into two groups with a ratio of 1 : 2? Explain.

16 Bertrand wins one-third of his games of tennis this season. Write down his win to loss ratio.

17 **a** What is the ratio of black keys to white keys in one octave of the piano?

b What is the ratio of black keys to white keys for the entire 88 keys of the piano?

c Are these equivalent ratios?

18 Write the missing expression.

a $2 : x = 6 :$ ☐ **b** $2 : x = 5 :$ ☐ **c** $y : 2x =$ ☐ $: 8x$ **d** $12xy : 6y =$ ☐ $: 1$

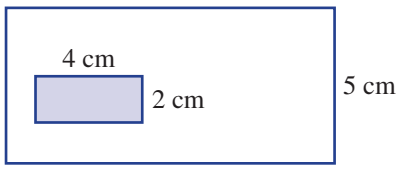
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6A Area ratios — — 19–21

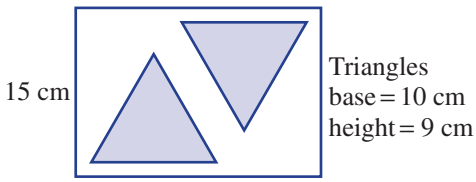
ENRICHMENT

19 Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

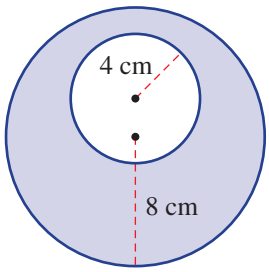
a



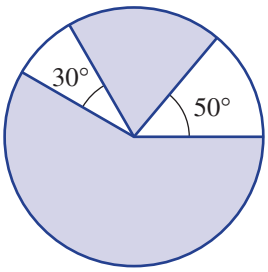
b



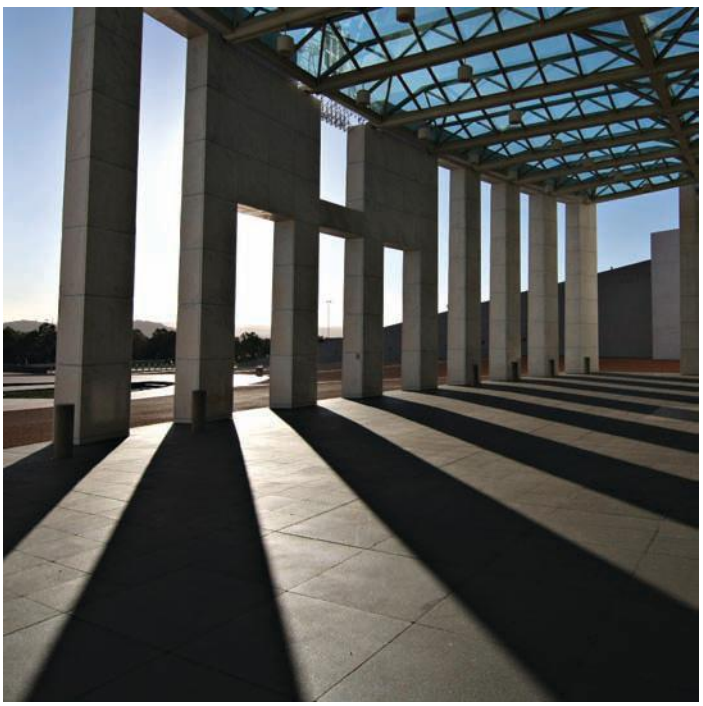
c



d



20 Estimate the ratio of the shaded floor to the unshaded floor in this photo.



21 Design your own diagram for which the ratio of shaded area to unshaded area is:

a 1 : 3 **b** 1 : 7

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6B Simplifying ratios

Interactive

Widgets

HOTsheets

Walkthroughs

In a similar way to fractions, ratios are simplified by dividing each term by a common factor. A ratio is said to be in its simplest form when it contains whole numbers only and the highest common factor (HCF) between the terms in the ratio is 1.

If a ratio contains fractions or decimals, it can be simplified by multiplying rather than dividing or cancelling.

Let's start: Class ratios

Look around your classroom and write down the following ratios:



a Ratio of girls to boys
b Ratio of teachers to students
c Ratio of wearing a watch to not wearing a watch
d Ratio of white socks to black socks
e Ratio of textbooks open to textbooks closed
f Ratio of not having a pencil case to having a pencil case
g Ratio of blonde hair to brown hair to black hair
h Ratio of blue eyes to brown eyes to other colour eyes

Design your own ratio question for your class or classroom.

Can any of your ratio answers be simplified?

Key ideas

- **Simplifying ratios**
  A ratio is simplified by dividing both numbers in the ratio by their highest common factor (HCF). For example, the ratio 15 : 25 can be simplified to 3 : 5.

$$15 : 25 \xrightarrow{\div 5} 3 : 5$$ ← Simplest form

- It is convention to express ratios in their simplest form.
- Ratios in simplest form use whole numbers only.
- If a ratio is expressed with fractions, it is simplified by converting the quantities to whole numbers. This is generally done by multiplying by the lowest common denominator (LCD).
- Before ratios are simplified the quantities must be expressed in the same unit.

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Example 3 Simplifying ratios

Simplify the following ratios.

a 7 : 21 **b** 450 : 200

SOLUTION

a $\div 7$ (7 : 21 → 1 : 3) $\div 7$

b $\div 50$ (450 : 200 → 9 : 4) $\div 50$

EXPLANATION

HCF of 7 and 21 is 7.
Divide both numbers by 7.

HCF of 450 and 200 is 50.
Divide both numbers by 50.

Example 4 Simplifying ratios involving fractions

Simplify the following ratios.

a $\frac{3}{5} : \frac{1}{2}$ **b** $2\frac{1}{3} : 1\frac{1}{4}$

SOLUTION

a $\times 10$ ($\frac{3}{5} : \frac{1}{2}$ → 6 : 5) $\times 10$

b $\times 12$ ($\frac{7}{3} : \frac{5}{4}$ → 28 : 15) $\times 12$

EXPLANATION

LCD of 5 and 2 is 10, so multiply by 10.
Alternatively, $\frac{3}{5} : \frac{1}{2} = \frac{6}{10} : \frac{5}{10} = 6 : 5$

Convert mixed numerals to improper fractions.
LCD of 3 and 4 is 12.
Multiply both numbers by 12.

Example 5 Changing quantities to the same units

First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 4 mm to 2 cm **b** 25 minutes to 2 hours

SOLUTION

a 4 mm to 2 cm = 4 mm to 20 mm
= 4 : 20
= 1 : 5

b 25 minutes to 2 hours
= 25 minutes to 120 minutes
= 25 : 120
= 5 : 24

EXPLANATION

2 cm = 20 mm
Once in same unit, write as a ratio.
Simplify ratio by dividing by the HCF of 4.

2 hours = 120 minutes
Once in same unit, write as a ratio.
Simplify ratio by dividing by the HCF of 5.

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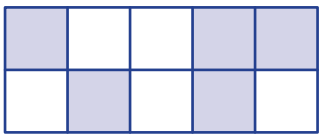
Exercise 6B

1–4 | 3, 4 | —

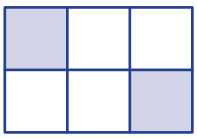
UNDERSTANDING

1 Write down the ratio of shaded parts to unshaded parts for each of the following in simplest form.

a



b



2 To express the ratio 4 : 16 in simplest form, you would:

A multiply both quantities by 2 **B** subtract 4 from both quantities
C divide both quantities by 2 **D** divide both quantities by 4

3 Decide which one of the following ratios is not written in simplest form.

A 1 : 5 **B** 3 : 9 **C** 2 : 5 **D** 11 : 17

4 Decide which one of the following ratios is written in simplest form.

A 2 : 28 **B** 15 : 75 **C** 14 : 45 **D** 13 : 39

5–7(½), 9(½) | 5–9(½) | 5–9(½)

FLUENCY

Example 3

5 Simplify the following ratios.

a 2 : 8 **b** 10 : 50 **c** 4 : 24 **d** 6 : 18
e 8 : 10 **f** 25 : 40 **g** 21 : 28 **h** 24 : 80
i 18 : 14 **j** 26 : 13 **k** 45 : 35 **l** 81 : 27
m 51 : 17 **n** 20 : 180 **o** 300 : 550 **p** 150 : 75
q 1200 : 100 **r** 70 : 420 **s** 200 : 125 **t** 90 : 75

6 Simplify the following ratios.

a 2 : 4 : 6 **b** 12 : 21 : 33 **c** 42 : 60 : 12 **d** 85 : 35 : 15
e 12 : 24 : 36 **f** 100 : 300 : 250 **g** 270 : 420 : 60 **h** 24 : 48 : 84

Example 4a

7 Simplify the following ratios.

a $\frac{1}{3} : \frac{1}{2}$ **b** $\frac{1}{4} : \frac{1}{5}$ **c** $\frac{2}{5} : \frac{3}{4}$ **d** $\frac{2}{7} : \frac{1}{5}$
e $\frac{3}{8} : \frac{1}{4}$ **f** $\frac{7}{10} : \frac{4}{5}$ **g** $\frac{11}{10} : \frac{2}{15}$ **h** $\frac{9}{8} : \frac{7}{12}$

Example 4b

8 Simplify the following ratios.

a $1\frac{1}{2} : \frac{3}{4}$ **b** $2\frac{1}{5} : \frac{3}{5}$ **c** $3\frac{1}{3} : 1\frac{2}{5}$ **d** $4\frac{1}{3} : 3\frac{3}{4}$

Example 5

9 First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 12 mm to 3 cm **b** 7 cm to 5 mm **c** 120 m to 1 km
d 60 mm to 2.1 m **e** 3 kg to 450 g **f** 200 g to 2.5 kg
g 2 tonnes to 440 kg **h** 1.25 L to 250 mL **i** 400 mL to 1 L
j 20 minutes to 2 hours **k** 3 hours to 15 minutes **l** 3 days to 8 hours
m 180 minutes to 2 days **n** 8 months to 3 years **o** 4 days to 4 weeks
p 8 weeks to 12 days **q** 50 cents to $4 **r** $7.50 to 25 cents

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6B

10, 11 | 11, 12 | 11–13

PROBLEM-SOLVING

10 Which of these ratios is the simplest form of the ratio $\frac{1}{2} : 2$?

A $2 : \frac{1}{2}$ **B** $1 : 4$ **C** $\frac{1}{4} : 1$ **D** $1 : 1$

11 Kwok was absent from school on 8 days in Term 1. There were 44 school days in Term 1. Write the following ratios in simplest form.

a Ratio of days absent to total days

b Ratio of days present to total days

c Ratio of days absent to days present

12 Over the past four weeks, the Paske family had eaten takeaway food for dinner on eight occasions. They had hamburgers twice, fish and chips three times and pizza three times. Every other night they had home-cooked dinners. Write the following ratios in simplest form.

a Ratio of hamburgers to fish and chips to pizza

b Ratio of fish and chips to pizza

c Ratio of takeaway dinners to home-cooked dinners

d Ratio of home-cooked dinners to total dinners

13 When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

a Write the ratio of the fruits in Lisa's fruit salad.

b Lisa wanted to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.

c Write these ratios in simplest form:

i bananas to strawberries **ii** strawberries to other fruits

14 | 14, 15 | 15–17

REASONING

14 Andrew incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Andrew's mistake and what is the correct simplified ratio?

15 Mariah has \$4 and Rogan has 50 cents. To write a ratio in simplest form, values must be written in the same unit.

a First convert both units to dollars, and then express the ratio of Mariah's money to Rogan's money in simplest form.

b As an alternative, convert both units to cents and express the ratio in simplest form. Do you arrive at the same simplified ratio?

16 a Write two quantities of time, in different units, which have a ratio of 2 : 5.

b Write two quantities of distance, in different units, which have a ratio of 4 : 3.

17 Simplify the following ratios.

a $2a : 4b$ **b** $15x : 3y$ **c** $a : a^2$

d $5f : 24f$ **e** $hk : 3k$ **f** $24xyz : 60yz$

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ENRICHMENT

18 Aspect ratio is the relationship between the width and height of the image as displayed on a screen.



A high-definition television screen (left) compared to the same image on an analogue television screen (right).

a An old analogue television has an aspect ratio of $1.\dot{3} : 1$. Write this ratio in simplest form.

b A high definition digital television has an aspect ratio of $1.\dot{7} : 1$. Write this ratio in simplest form.

c Although these two ratios are clearly not equivalent, there is a connection between the two. What is it?

d The digital television aspect ratio of $1.\dot{7} : 1$ was chosen as the best compromise to show widescreen movies on television. Many major films are presented in Panavision, which has an aspect ratio of 2.35 : 1. Write this ratio in simplest form.

e Investigate the history of aspect ratio in films and television.

f Research how formats of unequal ratios are converted through the techniques of cropping, zooming, letterboxing, pillarboxing or distorting.

g Investigate aspect ratios of other common media.

- **i** Find the aspect ratio for several different newspapers.
- **ii** Find the aspect ratio for several common sizes of photographs.
- **iii** Find the aspect ratio for a piece of A4 paper (a surd is involved!)



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6C Dividing a quantity in a given ratio EXTENDING









Ratios are closely connected to fractions. To help solve problems involving ratios, we can think of each quantity in terms of a 'number of parts'.

A drink made in the ratio of cordial to water of 1 : 4 means that there is '1 part' of cordial and '4 parts' of water. There is a total of '5 parts' in the drink. In terms of ratios, it does not matter whether the drink is a 250 mL glass, a 2 L bottle or a 50 L urn. Changing the size of the drink will not change the ratio of 'parts'; it will simply change the size of each part.

The fraction of cordial in the drink is $\frac{\text{number of cordial parts}}{\text{total number of parts}} = \frac{1}{5}$.

The fraction of water in the drink is $\frac{\text{number of water parts}}{\text{total number of parts}} = \frac{4}{5}$.

Thinking about ratios in terms of 'parts' helps us to divide quantities into particular ratios.

Let's start: Sharing $120

For each situation below you have $120 to share out in the given ratios.

With a partner, work out how much each person receives.

Situation 1: Two people, Jack and Pippa, ratio of 1 : 1

Situation 2: Two people, Andrew and Alex, ratio of 2 : 2

Situation 3: Two people, Jess and Teresa, ratio of 2 : 3

Situation 4: Two people, Kyle and Kara, ratio of 3 : 5

Situation 5: Three people, Matt, Leos and Djarrin, ratio of 1 : 1 : 1

Situation 6: Three people, Christine, Prue and Carolyn, ratio of 3 : 5 : 7

Discuss the strategy that you used to share the $120. Does your strategy work for every situation?

Key ideas

- Think of a ratio in terms of '**parts**'.
  A ratio of 2 : 3 has 2 parts of one quantity for every 3 parts of another quantity and a total of 5 parts.
- Using the **unitary method** to divide a quantity in a given ratio:
  - Find the total number of parts in the ratio.
  - Find the value of one part.
  - Find the value of the number of parts required in the ratio.

  For example: Share $20 in ratio of 2 : 3.
  Think of sharing $20 into 2 parts and 3 parts.
  Total number of parts = 2 + 3 = 5.
  Value of one part = $20 ÷ 5 = $4.
  Therefore 2 parts = $8, and 3 parts = $12.

$$\begin{array}{l} \div 5 \\ \times 2 \end{array} \begin{array}{c} \$20 = 5 \text{ parts} \\ \$4 = 1 \text{ part} \\ \$8 = 2 \text{ parts} \end{array} \begin{array}{l} \div 5 \\ \times 2 \end{array}$$

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Key ideas

- Using fractions to divide a quantity in a given ratio:
  - Fraction of the amount required $= \dfrac{\text{number in ratio}}{\text{total number of parts}}$
  - Calculate the fraction of the amount for each share of the ratio.

  For example, share \$20 in ratio of 2 : 3.

  Fractions of the amount required are $\frac{2}{5}$ and $\frac{3}{5}$.

  Therefore $\frac{2}{5}$ of \$20 = \$8 and $\frac{3}{5}$ of \$20 = \$12.
- To find a total quantity from a given ratio:
  - Use the concept of 'parts' and the unitary method to find the value of one part and therefore the value of the total parts can be calculated.

  Or
  - Use equivalent ratios to find the value of each quantity in the ratio and then add the numbers together to find the total.

Example 6 Dividing a quantity in a particular ratio

Divide 54 m in a ratio of 4 : 5.

SOLUTION

Unitary method

Total number of parts = 9

$\div 9$ (9 parts = 54 m) $\div 9$
$\times 4$ (1 part = 6 m) $\times 4$
(4 parts = 24 m)

$\times 5$ (1 part = 6 m) $\times 5$
(5 parts = 30 m)

54 m divided in a ratio of 4 : 5 is 24 m and 30 m.

Fractions method

$\frac{4}{9}$ of $54 = \frac{4}{9} \times \frac{54}{1} = 24$

$\frac{5}{9}$ of $54 = \frac{5}{9} \times \frac{54}{1} = 30$

54 m divided in a ratio of 4 : 5 is 24 m and 30 m.

EXPLANATION

Total number of parts = 4 + 5 = 9

Value of 1 part = 54 m ÷ 9 = 6 m

Check numbers add to total: 24 + 30 = 54

Fraction $= \dfrac{\text{number in ratio}}{\text{total number of parts}}$

Check numbers add to total:
24 + 30 = 54

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Example 7 Dividing a quantity in a ratio with three terms

Divide \$300 in the ratio of 2 : 1 : 3.

SOLUTION	EXPLANATION
Unitary method	
Total number of parts = 6	Total number of parts = 2 + 1 + 3 = 6
$\div 6$ (6 parts = \$300) $\div 6$ $\times 2$ (1 part = \$50) $\times 2$ 2 parts = \$100	$\times 3$ (1 part = \$50) $\times 3$ 3 parts = \$150
	Value of 1 part = \$300 ÷ 6 = \$50
\$300 divided in a ratio of 2 : 1 : 3 is	Check numbers add to total:
\$100, \$50 and \$150.	\$100 + \$50 + \$150 = \$300
Fractions method	
$\frac{2}{6}$ of $300 $= \frac{2}{6} \times \frac{300}{1} = 100$	Fraction = $\frac{\text{number in ratio}}{\text{total number of parts}}$
$\frac{1}{6}$ of $300 $= \frac{1}{6} \times \frac{300}{1} = 50$	
$\frac{3}{6}$ of $300 $= \frac{3}{6} \times \frac{300}{1} = 150$	
\$300 divided in a ratio of 2 : 1 : 3 is	Check numbers add to total:
\$100, \$50 and \$150.	\$100 + \$50 + \$150 = \$300



An example of dividing a quantity (a length of steel) in a particular ratio.

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Example 8 Finding a total quantity from a given ratio

The ratio of boys to girls at Birdsville College is 2 : 3. If there are 246 boys at the school, how many students attend Birdsville College?

SOLUTION	EXPLANATION
Unitary method	
$\div 2$ (2 parts = 246) $\div 2$ $\times 5$ (1 part = 123) $\times 5$ 5 parts = 615	Ratio of boys : girls is 2 : 3 Boys have '2 parts' = 246 Value of 1 part = $246 \div 2 = 123$ Total parts = $2 + 3 = 5$ parts 5 parts = $5 \times 123 = 615$
615 students attend Birdsville College.	
Equivalent ratios method	
boys : girls $\times 123$ (= 2 : 3 = 246 : 369) $\times 123$	Use equivalent ratios. Multiply each quantity by 123. Total number of students = 246 boys + 369 girls = 615
615 students attend Birdsville College.	

Exercise 6C

1–4 | 3, 4 | —

UNDERSTANDING

1 Find the total number of parts in the following ratios.

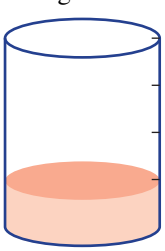
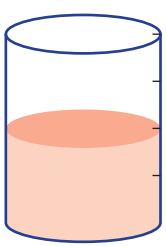
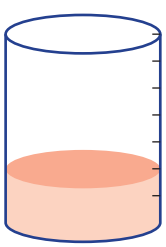
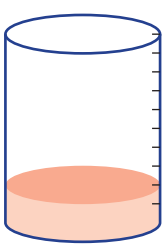
a 3 : 7 **b** 1 : 5 **c** 11 : 3 **d** 2 : 3 : 4

2 The ratio of girls to boys in a class is 3 : 5.

a What fraction of the class is girls?

b What fraction of the class is boys?

3 The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass. For each drink shown, what is the ratio of cordial to water?

a  **b**  **c**  **d** 

4 What fraction of each of the drinks above is cordial?

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6C

5(½), 6, 7(½) | 5–8(½) | 6–8(½)

FLUENCY

Example 6

5 Divide:

a $60 in the ratio of 2 : 3 **b** $110 in the ratio of 7 : 4 **c** $1000 in the ratio of 3 : 17

d 48 kg in the ratio of 1 : 5 **e** 14 kg in the ratio of 4 : 3 **f** 360 kg in the ratio of 5 : 7

g 72 m in the ratio of 1 : 2 **h** 40 m in the ratio of 3 : 5 **i** 155 m in the ratio of 4 : 1

6 Share $400 in the ratio:

a 1 : 3 **b** 2 : 3 **c** 3 : 5 **d** 9 : 11

Example 7

7 Divide:

a $200 in the ratio of 1 : 2 : 2 **b** $400 in the ratio of 1 : 3 : 4

c 12 kg in the ratio of 1 : 2 : 3 **d** 88 kg in the ratio of 2 : 1 : 5

e 320 kg in the ratio of 12 : 13 : 15 **f** $50 000 in the ratio of 1 : 2 : 3 : 4

8 Share 600 lollies in the ratio:

a 1 : 9 **b** 2 : 1 : 3 **c** 2 : 5 : 5 **d** 12 : 7 : 8 : 3

9, 10 | 10–13 | 12–15

PROBLEM-SOLVING

9 Evergreen Fertiliser is made up of the three vital nutrients nitrogen, potassium and phosphorus in a mass ratio of 4 : 5 : 3. How much of each nutrient is in a 1.5 kg bag?



10 The angles of a triangle are in the ratio of 2 : 3 : 4. Find the size of each angle.

11 Three friends, Cam, Molly and Seb, share a prize of $750 in a ratio of 3 : 4 : 8. How much more money does Seb receive than Cam?

12 Trudy and Bella made 80 biscuits. If Trudy made 3 biscuits in the time that Bella made 2 biscuits, how many biscuits did Trudy make?

Example 8

13 In Year 8, the ratio of boys to girls is 5 : 7. If there are 140 girls in Year 8, what is the total number of students in Year 8?

14 A light rye bread requires a ratio of wholemeal flour to plain flour of 4 : 3. A baker making a large quantity of loaves uses 126 cups of plain flour. What is the total amount of flour used by the baker?

15 A textbook contains three chapters and the ratio of pages in these chapters is 3 : 2 : 5. If there are 24 pages in the smallest chapter, how many pages are in the textbook?

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16 16, 17 16, 17 **6C**

REASONING

16 The ratio of the cost of a shirt to the cost of a jacket is 2 : 5. If the jacket cost $240 more than the shirt, find the cost of the shirt and the cost of the jacket.



17 In a class of 24 students the ratio of girls to boys is 1 : 2.

- **a** How many more girls are needed to make the ratio 1 : 1?
- **b** If 4 more girls and 4 more boys joined the class, what would be the new ratio?
- **c** On one day, the ratio was 3 : 7. How many boys and how many girls were absent?

A fair share

— — 18

ENRICHMENT

18 Three students Ramshid, Tony and Maria entered a group Geography competition.

- Tony spent 3 hours researching the topic.
- Maria spent $2\frac{1}{2}$ hours designing the poster.
- Ramshid spent 5 hours preparing the final presentation.

Their group won second prize in the competition and received a prize of $250. Ramshid, Tony and Maria decide to share the prize in a ratio of 3 : 2 : 1.

- **a** How much money did each student receive? Round the answer to nearest cent.
- **b** If the prize was divided up according to the time spent on the project, what would be the new ratio? Write the ratio in whole numbers.
- **c** How much money did each student receive with the new ratio? Round the answer to the nearest cent.
- **d** Although she spent the least time on the project, Maria would prefer to divide the prize according to time spent. How much more money did Maria receive with the new ratio?



- **e** Tony preferred that the original ratio remained. How much was Tony better off using the original ratio?
- **f** Which ratio would Ramshid prefer and why?
- **g** The group ended up going with a ratio based on time spent but then rounded amounts to the nearest $10. How much prize money did each student receive?

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6D Scale drawings

Interactive

Widgets

HOTsheets

Walkthroughs

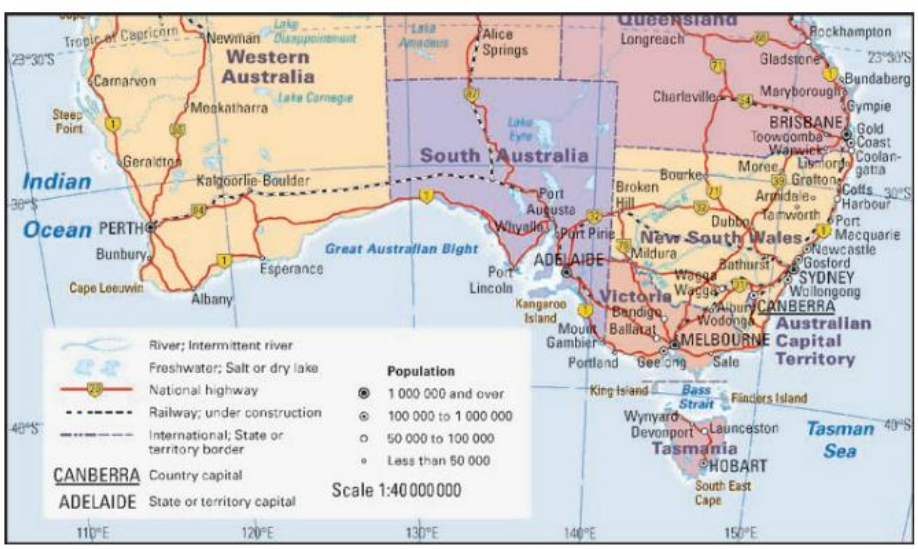
Scale drawings are a special application of ratios. The purpose of a scale drawing is to provide accurate information about an object which has dimensions that are either too large or too small to be shown on a page.

If a scale drawing has a ratio of 1 : 1, then the drawing would be exactly the same size as the actual (real-life) object. For example, it is not practical to have a map that is the same size as the actual distance you need to travel, so a much smaller map (a scaled drawing) is used. The map shows a scale to indicate the relationship of the map distance to the actual distance. The scale is displayed as a ratio.

Three common travel maps with their scales are shown below.







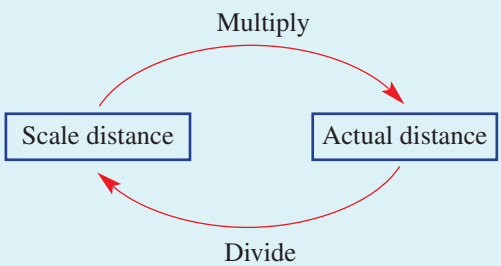
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Let's start: Scaling down and scaling up

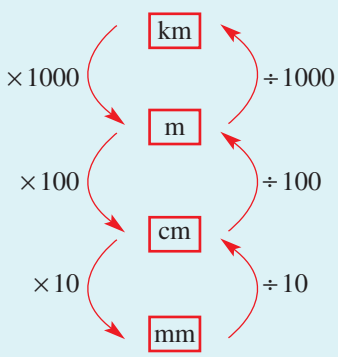
- Brainstorm specific examples where you have come across scale drawings. Think of examples where a scale drawing has been used to show very large objects or distances, and also think of examples where a scale drawing has been used to show very small objects.
- Share your list of examples with your partner.
- As a class, make a list on the board of specific examples of scale drawings.
- What are some examples of common scales that are used?

Key ideas

- A **scale drawing** has exactly the same shape as the original object, but a different size. All scale drawings should indicate the scale ratio.
- The scale on a drawing is written as a scale ratio:
  Drawing length : Actual length
  Drawing length represents the length on the diagram, map or model.
  Actual length represents the real length of the object.
- A **scale ratio** of 1 : 100 means the actual or real lengths are 100 times greater than the lengths measured on the diagram.
- A scale ratio of 20 : 1 means the scaled lengths on the model are 20 times greater than the actual or real lengths.
- It is helpful if scales begin with a 1. Then the second number in the ratio can be referred to as the **scale factor**. Scale ratios that do not start with a 1 can be converted using equivalent ratios.
- To convert a scaled distance to an actual distance you multiply by the scale factor.
- To convert an actual (real) distance to a scaled distance you divide by the scale factor.



- **Conversion of units**. It is important to remember how to correctly convert measurements of length when dealing with scales.
  1 km = 1000 m
  1 m = 100 cm
  1 cm = 10 mm



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Example 9 Converting scale distance to actual distance

A map has a scale of 1 : 20 000.
Find the actual distance for each scaled distance.

a 2 cm **b** 5 mm **c** 14.3 cm

SOLUTION	EXPLANATION
a Actual distance = 2 cm × 20 000	Scale factor = 20 000
= 40 000 cm	Multiply scaled distance by scale factor.
= 400 m (or 0.4 km)	Convert answer into sensible units.
b Actual distance = 5 mm × 20 000	Multiply scaled distance by scale factor.
= 100 000 mm	
= 100 m	Convert answer into sensible units.
c Actual distance = 14.3 cm × 20 000	Multiply scaled distance by scale factor.
= 286 000 cm	Shortcut: × 2, then × 10 000
= 2.86 km	Convert answer into sensible units.

Example 10 Converting actual distance to scaled distance

A model boat has a scale of 1 : 500.
Find the scaled length for each of these actual lengths.

a 50 m **b** 75 cm **c** 4550 mm

SOLUTION	EXPLANATION
a Scaled distance = 50 m ÷ 500	Scale factor = 500
= 5000 cm ÷ 500	Convert metres to centimetres.
= 10 cm (0.1 m)	Divide actual distance by scale factor. Convert answer into sensible units.
b Scaled distance = 75 cm ÷ 500	Convert centimetres to millimetres.
= 750 mm ÷ 500	Divide actual distance by scale factor.
= 1.5 mm	
c Scaled distance = 4550 mm ÷ 500	Divide actual distance by scale factor.
= 45.5 mm ÷ 5	Shortcut: ÷ 100, then ÷ 5 (or vice versa)
= 9.1 mm	

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Example 11 Determining the scale factor

State the scale factor in the following situations.

a A length of 4 mm on a scale drawing represents an actual distance of 50 cm.

b An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

SOLUTION	EXPLANATION
a Scale ratio = 4 mm : 50 cm	Write the ratio drawing length : actual length.
= 4 mm : 500 mm	Convert to 'like' units.
Scale ratio = 4 : 500	Write the scale ratio without units.
= 1 : 125	Divide both numbers by 4.
Scale factor = 125	Ratio is now in the form 1 : scale factor. The actual size is 125 times larger than the scaled drawing.
b Scale ratio = 3 cm : 0.1 mm	Write the ratio drawing length : actual length.
= 30 mm : 0.1 mm	Convert to 'like' units.
Scale ratio = 30 : 0.1	Write the scale ratio without units.
= 300 : 1	Multiply both numbers by 10.
$= 1 : \frac{1}{300}$	Divide both numbers by 300.
Scale factor = $\frac{1}{300}$	Ratio is now in the form 1 : scale factor. The actual size is 300 times smaller than the scaled drawing.

Exercise 6D

1–4 | 3, 4 | —

UNDERSTANDING

1 A map has a scale of 1 : 50 000. How many centimetres in real life does 1 cm on the map equal?

2 A map has a scale of 1 : 100. How many metres on the map does 300 m in real life equal?

3 Convert 10 000 cm to:

a mm **b** m **c** km

4 Convert 560 m to:

a km **b** cm **c** mm

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6D

5–6(½), 7 | 5–6(½), 7, 8(½) | 5–8(½)

FLUENCY

Example 9

5 Find the actual distance for each of the following scaled distances. Give your answer in an appropriate unit.

a	Scale 1 : 10 000	**i** 2 cm	**ii** 4 mm	**iii** 7.3 cm
b	Scale 1 : 20 000	**i** 80 cm	**ii** 18.5 mm	**iii** 1.25 m
c	Scale 1 : 400	**i** 16 mm	**ii** 72 cm	**iii** 0.03 m
d	Scale 1 : 300	**i** 5 mm	**ii** 8.2 cm	**iii** 7.1 m
e	Scale 1 : 2	**i** 44 m	**ii** 310 cm	**iii** 2.5 mm
f	Scale 1 : 5	**i** 4 m	**ii** 24 cm	**iii** 155 mm
g	Scale 1 : 0.5	**i** 12 cm	**ii** 3.2 mm	**iii** 400 m
h	Scale 100 : 1	**i** 3 cm	**ii** 11.5 km	**iii** 81.5 cm

Example 10

6 Find the scaled length for each of these actual lengths.

a	Scale 1 : 200	**i** 200 m	**ii** 4 km	**iii** 60 cm
b	Scale 1 : 500	**i** 10 000 m	**ii** 1 km	**iii** 75 cm
c	Scale 1 : 10 000	**i** 1350 m	**ii** 45 km	**iii** 736.5 m
d	Scale 1 : 20 000	**i** 12 km	**ii** 1800 m	**iii** 400 mm
e	Scale 1 : 250 000	**i** 5000 km	**ii** 750 000 m	**iii** 1250 m
f	Scale 1 : 10	**i** 23 m	**ii** 165 cm	**iii** 6.15 m
g	Scale 1 : 0.1	**i** 30 cm	**ii** 5 mm	**iii** 0.2 mm
h	Scale 200 : 1	**i** 1 mm	**ii** 7.5 cm	**iii** 8.2 mm

Example 11

7 Determine the scale ratio for each of the following.

a A length of 2 mm on a scale drawing represents an actual distance of 50 cm.
b A length of 4 cm on a scale drawing represents an actual distance of 2 km.
c A length of 1.2 cm on a scale drawing represents an actual distance of 0.6 km.
d A length of 5 cm on a scale drawing represents an actual distance of 900 m.
e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.
f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

8 Convert the two measurements provided in each scale into the same unit and then write the scale as a ratio of two numbers in simplest form.

a	2 cm : 200 m	**b**	5 mm : 500 cm
c	12 mm : 360 cm	**d**	4 mm : 600 m
e	4 cm : 5 m	**f**	1 cm : 2 km
g	28 mm : 2800 m	**h**	3 cm : 0.6 mm
i	1.1 m : 0.11 mm		

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9–11 10–13 11–14

6D

PROBLEM-SOLVING

9 A model city has a scale ratio of 1 : 1000.

a Find the actual height of a skyscraper that has a scaled height of 8 cm.

b Find the scaled length of a train platform that is 45 m long in real life.

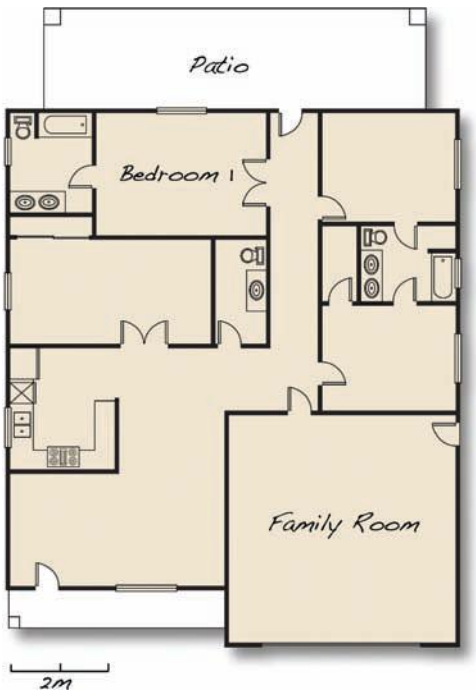
10 Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1 : 50 000. How far apart are the towns in real life?

11 Using the house plans shown below, state the actual dimensions of the following rooms.

a Bedroom 1

b Family room

c Patio



12 From the scaled drawing below, calculate the actual length and height of the truck, giving your answer to the nearest metre.



13 The photograph on the right shows Jackson enjoying a walk with his dog. Jackson is 1.8 m tall in real life.
Find a scale for the photograph and use this to estimate the height of Jackson's dog.



14 A scale length of $5\frac{1}{2}$ cm is equal to an actual distance of 44 km.

a How many kilometres does a scale length of 3 cm equal?

b How many kilometres does a scale length of 20 cm equal?

c How many centimetres does an actual distance of 100 km equal?

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6D

	15	15–17	17–19

REASONING

15 Group the six ratios below into pairs of equivalent ratios.

1 : 0.01 25 : 1 20 : 1 1 : 0.05 100 : 1 50 : 2

16 Which of the following scales would be most appropriate if you wanted to make a scale model of:

a a car **b** your school grounds **c** Mt Kosciuszko

A 1 : 10 000 **B** 1 : 1000 **C** 1 : 100 **D** 1 : 10

17 A map maker wants to make a map of a mountainous area that is 64 km^2 (side length of 8 km). She decides to use a scale of 1 : 1000. Describe the problem she is going to have.



18 A scientist says that the image of a 1 mm long insect is magnified by a scale of 1 : 1000 through his magnifying glass. Explain what might be wrong with the scientist's ratio.

19 Obtain a map of Australia.

a What is the scale of your map?

b Which two capital cities in Australia are the furthest apart? State the distance.

c Which two capital cities in Australia are closest together? State the distance.

d Which capital city is furthest away from Sydney? State the distance.

e Which capital city is closest to Adelaide? State the distance.

f Check the accuracy of your answers on the internet.

Scaled drawing

	—	—	20

ENRICHMENT

20 Design a scaled drawing of one of the following:

- Your classroom and the associated furniture
- Your bedroom and the associated furniture
- A room in your house (bathroom, family room, garage ...)

Your scaled drawing should fit comfortably onto an A4 page.

a Measure the dimensions of your chosen room and the dimensions of an A4 page, and determine the most appropriate scale for your diagram.

b Show size and location of doors and windows where appropriate.

c Produce a second scale diagram of your room, but show a different arrangement of the furniture.

Can you produce the scale diagram on a computer software package?

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6E Introducing rates







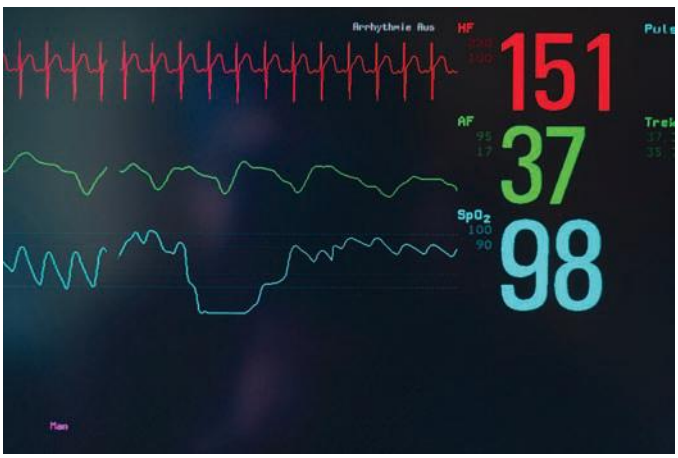


If you were to monitor what you said each day, you might well find that you speak about rates many times per day!

A **ratio** shows the relationship between the same type of quantities with the same units, but a **rate** shows the relationship between two different types of quantities with different units.

The following are all examples of rates:

- Cost of petrol was $1.45 per litre.
- Rump steak was on special for $18/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.



The top readout on this heart monitor shows heart rate in beats per minute.

Unlike ratios, a **rate** compares different types of quantities, and so units must be shown.

For example:

- The ratio of boys to girls in a school was 4 : 5.
- The average rate of growth of an adolescent boy is 12 cm/year.

Let's start: State the rate

For each of the following statements, write down a corresponding rate.

- The Lodges travelled 400 km in 5 hours.
- Gary was paid $98 for a 4-hour shift at work.
- Felicity spent $600 on a two-day shopping spree.
- Max had grown 9 cm in the last three months.
- Vuong paid $37 for half a cubic metre of crushed rock.
- Paul cycled a total distance of 350 km for the week.

What was the rate (in questions/minute) at which you answered these questions?

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Key ideas

- **Rates** compare quantities measured in different units.
- All rates must include units for each quantity.
- The two different units are separated by a slash '/', which is the mathematical symbol for 'per'.
  For example: 20 km/h = 20 km per hour = 20 km for each hour
- It is convention to write rates in their simplest form. This involves writing the rate for only one unit of the second quantity.
  For example: spent \$45 in 3 hours = \$45 in 3 hours ← Non simplified rate
  ÷3 ↓ ÷3
  = \$15 in 1 hour ← Simplified rate (\$15/h)
- The **average rate** is calculated by dividing the total change in one quantity by the total change in the second quantity.
  For example: reading a 400 page book in 4 days
  Average reading rate = 400 pages in 4 days
  ÷4 ↓ ÷4
  = 100 pages in 1 day
  Therefore average reading rate = 100 pages/day.

Example 12 Writing simplified rates

Express each of the following as a simplified rate.

a 12 students for two teachers

b \$28 for 4 kilograms

SOLUTION

a 6 students/teacher

b \$7/kg

EXPLANATION

12 students for 2 teachers
÷2 ↓ ÷2
6 students for 1 teacher

\$28 for 4 kg
÷4 ↓ ÷4
\$7 for 1 kg

Example 13 Finding average rates

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

SOLUTION

÷10 ↓ 65 cm in 10 years
6.5 cm in 1 year ÷10

Average rate of growth = 6.5 cm/year.

EXPLANATION

Growth = 185 – 120 = 65 cm

Divide both numbers by 10.

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Exercise 6E

1–3 | 3 | —

UNDERSTANDING

1 Which of the following are examples of rates?

A \$5.50 **B** 180 mL/min **C** \$60/h **D** $\frac{5}{23}$

E 4.2 runs/over **F** 0.6 g/L **G** 200 cm^2 **H** 84c/L

2 Match each rate in the first column with its most likely rate in the second column.

Employee's wage	90 people/day
Speed of a car	\$2100/m^2
Cost of building a new home	68 km/h
Population growth	64 beats/min
Resting heart rate	\$15/h

3 Write typical units for each of the following rates.

a Price of sausages **b** Petrol costs
c Typing speed **d** Goal conversion rate
e Energy nutrition information **f** Water usage in the shower
g Pain relief medication **h** Cricket team's run rate

4(½) | 4(½), 5 | 4–5(½)

FLUENCY

Example 12

4 Write each of the following as a simplified rate.

a 12 days in 4 years **b** 15 goals in 3 games
c \$180 in 6 hours **d** \$17.50 for 5 kilograms
e \$126 000 to purchase 9 acres **f** 36 000 cans in 8 hours
g 12 000 revolutions in 10 minutes **h** 80 mm rainfall in 5 days
i 60 minutes to run 15 kilometres **j** 15 kilometres run in 60 minutes

Example 13

5 Find the average rate of change for each situation.

a Relma drove 6000 kilometres in 20 days. **b** Holly saved \$420 over three years.
c A cricket team scored 78 runs in 12 overs. **d** Saskia grew 120 centimetres in 16 years.
e Russell gained 6 kilograms in 4 years. **f** The temperature dropped 5°C in 2 hours.

6–9 | 8–10 | 9–11

PROBLEM-SOLVING

6 A dripping tap filled a 9 litre bucket in 3 hours.

a What was the dripping rate of the tap in litres/hour?
b How long would it take the tap to fill a 21 litre bucket?

7 Martine grew at an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?

8 If 30 salad rolls were bought to feed 20 people at a picnic, and the total cost was \$120, find the following rates.

a Salad rolls/person **b** Cost/person **c** Cost/roll

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6E

PROBLEM-SOLVING

9 The number of hours of sunshine was recorded each day for one week in April. The results are listed.
Monday 6 hours, Tuesday 8 hours, Wednesday 3 hours, Thursday 5 hours
Friday 7 hours, Saturday 6 hours, Sunday 7 hours



a Find the average number of hours of sunshine:
- **i** per weekday
- **ii** per weekend day
- **iii** per week
- **iv** per day

b Given the above rates, how many hours of sunshine would you expect in April?

10 Harvey finished a 10 kilometre race in 37 minutes and 30 seconds. Jacques finished a 16 kilometre race in 53 minutes and 20 seconds. Calculate the running rate of each runner. Which runner had a faster running pace?

11 The Tungamah Football Club had 12 000 members. After five successful years and two premierships, they now have 18 000 members.

a What has been the average rate of membership growth per year for the past 5 years?

b If this membership growth rate continues, how many more years will it take for the club to have 32 000 members?

	12	12, 13	12, 13

REASONING

12 **a** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:
i km/L **ii** L/km

b How can you convert km/L to L/km?

13 The Teleconnect telecommunications company has a variable call charge rate for phone calls of up to 30 minutes. The charges are 50c/min for first 10 minutes, 75c/min for the second 10 minutes and $1/min for the third 10 minutes.

a Find the cost of phone calls of these given lengths.
i 8 minutes **ii** 13 minutes **iii** 24 minutes **iv** 30 minutes

b What is the average charge rate per minute for a 30 minute call?

Connectplus, a rival telecommunications company, charges a constant call rate of 60c/minute.

c If you normally made calls that were 15 minutes long, which company has the better deal for you?

d If you normally made calls that were 25 minutes long, which company has the better deal for you?

e What is the length of phone call for which both companies would charge the same amount?

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Target 155 — — 14

6E

ENRICHMENT

14 In Victoria, due to drought conditions, the state government in 2008 urged all residents to save water. The goal was set for each Victorian to use no more than 155 litres of water per day.



a How many people live in your household?

b According to the Victorian government, how many litres of water can your household use per day?

c Perform some experiments and calculate the following rates of water flow.

i Shower rate (L/min)

ii Washing machine (L/load)

iii Hose (L/min)

iv Toilet (L/flush, L/half flush)

v Running tap (L/min)

vi Drinking water (L/day)

vii Dishwasher (L/wash)

viii Water for food preparation (L/day)

d Estimate the average daily rate of water usage for your household.

e Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.

Before the initiative, Victorians were using an average of 164 litres/day/person. Twelve months after the initiative, Victorians were using 151 litres/day/person.

f How much water per year for the state of Victoria does this saving of 13 litres/day/person represent?

g What is the rate at which your family is charged for its water?



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Progress quiz

6A **1** An excursion to Sea World has 6 teachers and 144 students. Sixty-nine of the students are boys; the rest are girls. Write down, in simplest form, the ratio of:

a teachers to students **b** boys to girls

c girls to the total of people on the excursion

6A **2** Complete each pair of equivalent ratios.

a 2 : 5 = ____ : 15 **b** ____ : 3 = 12 : 9

c 24 : 8 = 12 : ____ **d** 7 : 8 : 3 = 21 : ____ : ____

6B **3** Simplify the following ratios.

a 24 : 36 **b** 24 : 36 : 40 **c** 3/4 : 9

d 0.9 : 12 **e** 40 cents to $1.20

6C Ext **4** Divide:

a $800 in the ratio of 5 : 3 **b** $7500 in the ratio of 8 : 3 : 1

c 1 km in the ratio of 2 : 3

6C Ext **5** Tony and Vi share their company's profits in the ratio of 15 : 7. Tony's share totals $1200.

a How much profit did the company make?

b What is Vi's share?

6D **6** A map uses a scale of 1 : 40 000. Find the actual distance, in metres, if the distance on the map measures 8 mm.

6D **7** A model car has a scale of 1 : 100. Find the scale length, in centimetres, of a Lexus that is 4.32 metres long.

6D **8** Determine the scale ratio for 4mm on a drawing representing an actual distance of 8 km.

6E **9** Express each of the following as a simplified rate.

a 180 students on 3 buses **b** $5.60 for 4 kg

c 186 km in $2\frac{1}{2}$ hours

6E **10** Find the average rate for each situation.

a Thelma drove 8000 km in 50 days **b** Callum saved $1250 in 6 months

c Ainslie grew 20 cm in $2\frac{1}{2}$ years

6E **11** Who earns the most, and by how much, if Kelly is paid $96 570 a year and Todd earns $7985 each month?

6E **12** Which is faster 70 km/h or 21 m/s ?

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6F Ratios and rates and the unitary method

Interactive

Widgets

HOTsheets

Walkthroughs

The concept of solving problems using the unitary method was introduced in Chapter 3. The unitary method involves finding the value of 'one unit'. This is done by dividing the amount by the given quantity. Once the value of 'one unit' is known, multiplying will find the value of the number of units required.

Let's start: Finding the value of '1 unit'?

For each of the following, find the value of 1 unit or 1 item.

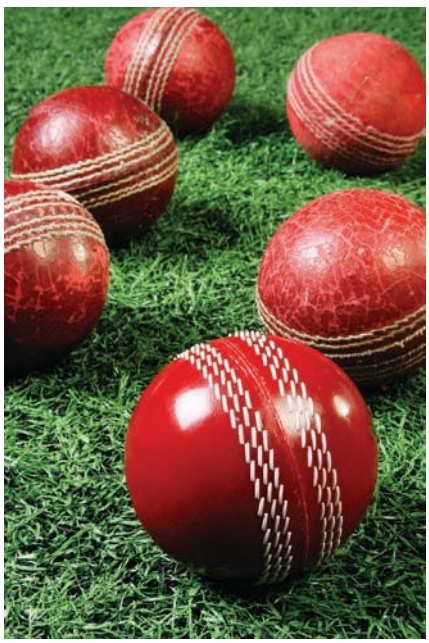
- 8 basketballs cost $200.
- 4 cricket bats cost $316.
- 5 kg of watermelon cost $7.50.

For each of the following, find the rate per 1 unit.

- Car travelled 140 km in 2 hours.
- 1000 L of water leaked out of the tank in 8 hours.
- $51 was the price paid for 3 kg of John Dory fish.

For each of the following, find the value of 1 'part'.

- Ratio of books to magazines read was 2 : 5. Milli had read 14 books.
- Ratio of pink to red flowers is 7 : 11. A total of 330 red flowers are in bloom.
- Ratio of girls to boys is 8 : 5. There are 40 girls in a group.



Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to solve the problem.
- When dealing with **ratios**, find the value of 1 'part' of the ratio.
  For example:
  Ratio of cars to motorbikes is 35 : 2 and there are 6 motorbikes.

$$\begin{array}{c} \div 2 \\ \text{2 parts} = \text{6 motorbikes} \\ \div 2 \\ \text{1 part} = \text{3 motorbikes} \end{array}$$

- When dealing with **rates**, find the value of the rate per 1 'unit'.
  For example: Pedro earned $64 for a 4 hour shift at work.
  Therefore, wage rate = $64 per 4 hours = $16 per hour = $16/h.
- Once the value of one 'part' or the rate per one 'unit' is known, the value of any number of parts or units can be found by multiplying.
- The technique of dividing and/or multiplying values in successive one-step calculations can be applied to the concept of converting rates from a set of given units to a different set of units.

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Example 14 Reviewing the unitary method

Andy travels 105 km in 7 identical car trips from home to school. How far would she travel in 11 such car trips?

SOLUTION

÷7 ×11

7 car trips = 105 km
1 car trip = 15 km
11 car trips = 165 km

÷7 ×11

Andy travels 165 km.

EXPLANATION

Find the value of 1 unit by dividing both quantities by 7.

Solve the problem by multiplying both quantities by 11.

Example 15 Solving ratio problems using the unitary method

The ratio of apples to oranges is 3 : 5. If there are 18 apples, how many oranges are there?

SOLUTION

÷3 ×5

3 parts = 18 apples
1 part = 6 pieces
5 parts = 30 oranges

÷3 ×5

There are 30 oranges.

EXPLANATION

Apples = 3 'parts', Oranges = 5 'parts'

Need to find the value of 1 'part'.

To find 5 'parts' multiply the value of 1 'part' by 5.

Example 16 Solving rate problems using the unitary method

A truck uses 4 L of petrol to travel 36 km. How far will it travel if it uses 70 L of petrol?

SOLUTION

Rate of petrol consumption

÷4 ×70

36 km for 4 L
9 km for 1 L
630 km for 70 L

÷4 ×70

Truck will travel 630 km on 70 L.

EXPLANATION

Find the petrol consumption rate of 1 unit by dividing both quantities by 4.

Solve the problem by multiplying both quantities by 70.



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Example 17 Converting units using the unitary method

Melissa works at the local supermarket and earns $57.60 for a 4 hour shift.
How much does she earn in c/min?

SOLUTION	EXPLANATION
Wage = $57.60 for 4 hours	Write down Melissa's wage rate.
÷4 ↘ $14.40 for 1 hour ↘ ÷4	Find the rate of $ per 1 hour.
÷60 ↘ 1440c for 60 minutes 24c for 1 minute ↘ ÷60	Convert $ to cents and hours to minutes.
Melissa earns 24c/min.	Divide rate by 60 to find rate of cents per minute.

Exercise 6F

1–4 | 4 | —

UNDERSTANDING

Example 14

1 If 8 kg of chicken fillets cost $72, how much would 3 kg of chicken fillets cost?

2 If one dozen tennis balls cost $9.60, how much would 22 tennis balls cost?

3 If three pairs of socks cost $12.99, how much would 10 pairs of socks cost?

4 If 500 g of mince meat costs $4.50, how much would 4 kg of mince meat cost?

5, 6, 8(½) | 5–7, 8(½) | 5–6(½), 7, 8(½)

FLUENCY

Example 15

5 Solve the following ratio problems.

a The required staff to student ratio for an excursion is 2 : 15. If 10 teachers attend the excursion, what is the maximum number of students who can attend?

b The ratio of commercials to actual show time for a particular TV channel is 2 : 3. How many minutes of actual show were there in 1 hour?

c A rectangle has length and width dimensions in a ratio of 3 : 1. If a particular rectangle has a length of 21 m, what is its width?

d Walter and William have a height ratio of 7 : 8. If William has a height of 152 cm, how tall is Walter?

Example 16

6 Solve the following rate problems.

a A tap is dripping at a rate of 200 mL every 5 minutes. How much water drips in 13 minutes?

b A professional footballer scores an average of 3 goals every 6 games. How many goals is he likely to score in a full season of 22 games?

c A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

d A computer processor can process 500 000 kilobytes of information in 4 seconds. How much information can it process in 15 seconds?

7 Leonie, Spencer and Mackenzie have just won a prize. They decide to share it in the ratio of 4 : 3 : 2. If Spencer receives $450, how much do Leonie and Mackenzie receive, and what was the total value of the prize?

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6F

Example 17

FLUENCY

8 Convert the following rates into the units given in the brackets.

a $15/h (c/min)
b $144/h (c/s)
c 3.5 L/min (L/h)
d 20 mL/min (L/h)
e 0.5 kg/month (kg/year)
f 120 g/day (kg/week)
g 60 g/c (kg/$)
h $38/m (c/mm)
i 108 km/h (m/s)
j 14 m/s (km/h)

9, 10	10–12	10–12

PROBLEM-SOLVING

9 The Mighty Oats breakfast cereal is sold in boxes of three different sizes: small (400 g) for $5.00, medium (600 g) for $7.20, large (750 g) for $8.25

a Find the value of each box in $/100 g.
b What is the cheapest way to buy a minimum of 4 kg of the cereal?

10 Gemma runs 70 metres in 8.4 seconds. If she maintains the same average speed, in what time will she run 100 metres?

11 Zana's hair grew 6 cm in 5 months.

a Find Zana's average rate of hair growth in cm/month and in m/year.
b How long would it take for Zana's hair to grow 30 cm?



12 Maria can paint 15 m^2 in 20 minutes.

a What is the rate at which Maria paints in m^2/h?
b What area can Maria paint in 20 hours?
c Maria must paint 1000 m^2 in 20 hours. Find the rate at which she will need to paint in m^2/min.

14a	13, 14	13–15

REASONING

13 If x donuts cost $\$y$:

a how much would 1 donut cost?
b how much would one dozen donuts cost?
c how much would z donuts cost?

14 **a** A triangle has side lengths in a ratio of 19 : 22 : 17. If the shortest side is 17 cm, find the lengths of the other two sides and the perimeter of the triangle.
b A triangle has side lengths in a ratio of 3 : 5 : 4. If the longest side is 35 cm, find the lengths of the other two sides and the perimeter of the triangle.

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6F

REASONING

15 In a faraway galaxy, a thriving alien colony uses the following units:
For money they have puks and paks: 1 puk (pu) = 1000 pak (pa)
For length they have doits and minidoits: 1 doit (D) = 80 minidoits (mD)
Polynaute rope is priced at 4 pu/D. Find the cost of the rope in terms of pa/mD.

Where will we meet? — — 16

ENRICHMENT

16 Phil lives in Perth and his friend Werner lives in Sydney. The distance, by road, between their two houses is 4200 km (rounded to the nearest 100 km).

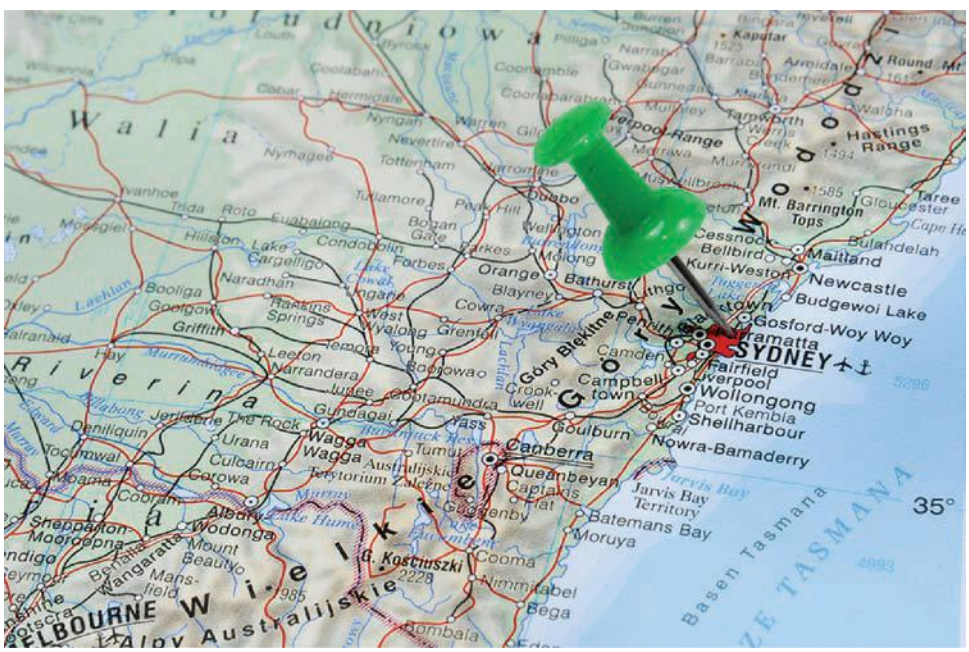
Phil decides to drive to Sydney and Werner decides to drive to Perth. They leave home at the same time and travel the same route, but in opposite directions.

Phil drives at a constant speed of 75 km/h and Werner drives at a constant speed of 105 km/h.

- **a** Will they meet on the road at a spot closer to Sydney or closer to Perth?
- **b** How long will it take Phil to travel to Sydney?
- **c** How long will it take Werner to travel to Perth?
- **d** State the location of each friend after they have been driving for 15 hours.
- **e** At what location (distance from Sydney and/or Perth) will they meet?

When they meet, Phil decides to change into Werner's car and they drive back to Werner's home at an average speed of 105 km/h.

- **f** How long did it take Phil to travel to Sydney?
- **g** Design a similar problem for two friends travelling at different constant speeds between two different capital cities in Australia.



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6G Solving rate problems EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

'One thing that is certain in life is that change is inevitable.'

We are constantly interested in rates of change and how things change over a period of time. We are also regularly faced with problems involving specific rates. Strong arithmetic skills and knowing whether to multiply or divide by the given rate allows many rate problems to be solved quickly and accurately.

Over the next few days, keep a record of any rates you observe or experience. Look out for the slash '/' sign and listen for the 'per' word.

Let's start: Estimate the rate

For each of the following statements, estimate a corresponding rate.



- Commercial rate: The number of commercials on TV per hour
- Typing rate: Your typing speed in words per minute
- Laughing rate: The number of times a teenager laughs per hour
- Growth rate: The average growth rate of a child from 0 to 15 years of age
- Running rate: Your running pace in metres per second
- Homework rate: The number of subjects with homework per night
- Clapping rate: The standard rate of audience clapping in claps per minute
- Thankyou rate: The number of opportunities to say thank you per day

Compare your rates. Which rate is the 'highest'? Which rate is the 'lowest'? Discuss your answers.

Key ideas

- When a rate is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
  For example: Patrick earns $20/hour. How much will he earn in 6 hours?

$$\times 6 \left(\begin{array}{c} \$20 \text{ for 1 hour} \\ \$120 \text{ for 6 hours} \end{array} \right) \times 6$$

  For example: Patrick earns $20/hour. How long will it take him to earn $70?

$$\times 3\tfrac{1}{2} \left(\begin{array}{c} \$20 \text{ for 1 hour} \\ \$70 \text{ for } 3\tfrac{1}{2} \text{ hours} \end{array} \right) \times 3\tfrac{1}{2}$$

- Carefully consider the units involved in each question and answer.

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Example 18 Solving rate problems

a Rachael can type at 74 words/minute. How many words can she type in 15 minutes?

b Leanne works in a donut van and sells on average 60 donuts every 15 minutes. How long is it likely to take her to sell 800 donuts?

SOLUTION

a

$\times 15$ (74 words in 1 minute → 1110 words in 15 minutes) $\times 15$

Rachael can type 1110 words in 15 minutes.

EXPLANATION

$$\begin{array}{r} 74 \\ \times\ 15 \\ \hline 370 \\ 740 \\ \hline 1110 \end{array}$$

b

$\div 15$ (60 donuts in 15 minutes → 4 donuts in 1 minute) $\div 15$

$\times 200$ (4 donuts in 1 minute → 800 donuts in 200 minutes) $\times 200$

Leanne is likely to take 3 hours and 20 minutes to sell 800 donuts.

Selling rate = 60 donuts/15 minutes

Divide both quantities by HCF of 15.

Multiply both quantities by 200.

Convert answer to hours and minutes.

Example 19 Solving combination rate problems

Three water hoses from three different taps are used to fill a large swimming pool. The first hose alone takes 200 hours to fill the pool. The second hose alone takes 120 hours to fill the pool and the third hose alone takes only 50 hours to fill the pool. How long would it take to fill the pool if all three hoses were used?

SOLUTION

In 600 hours:

hose 1 would fill 3 pools

hose 2 would fill 5 pools

hose 3 would fill 12 pools

Therefore in 600 hours the three hoses together would fill 20 pools.

$\div 20$ (600 hours for 20 pools → 30 hours for 1 pool) $\div 20$

It would take 30 hours to fill the pool if all three hoses were used.

EXPLANATION

LCM of 200, 120 and 50 is 600,

Hose 1 = 600 h ÷ 200 h/pool = 3 pools

Hose 2 = 600 h ÷ 120 h/pool = 5 pools

Hose 3 = 600 h ÷ 50 h/pool = 12 pools

Together = 3 + 5 + 12 = 20 pools filled.

Simplify rate by dividing by HCF.

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Exercise 6G

1, 2 | 2 | —

UNDERSTANDING

1 Fill in the gaps.

a
$\times 3$ (60 km in 1 hour) $\times 3$
(180 km in ___)

b
$\times 5$ ($25 in 1 hour) ___
($125 in ___)

c
___ (7 questions in 3 minutes) ___
(70 questions in ___)

d
___ (120 litres in 1 minute) ___
(___ in 6 minutes)

2 Fill in the gaps.

a
$\div 3$ ($36 for 3 hours) $\div 3$
(___ for 1 hour)
$\times 5$ (___ for 5 hours) ___

b
___ (150 rotations in 5 minutes) ___
___ (_____ in 1 minute) ___
___ (_____ in 7 minutes) ___

3–7 | 3–8 | 4–8

FLUENCY

Example 18a

3 A factory produces 40 plastic bottles/minute.

a How many bottles can the factory produce in 60 minutes?

b How many bottles can the factory produce in an 8 hour day of operation?



Example 18b

4 Mario is a professional home painter. When painting a new home he uses an average of 2.5 litres of paint per hour. How many litres of paint would Mario use in a week if he paints for 40 hours?

5 A truck travels 7 km per litre of fuel. How many litres are needed for the truck to travel 280 km?

6 Daniel practises his guitar for 40 minutes every day. How many days will it take him to log up 100 hours of practice?

7 A flywheel rotates at a rate of 1500 revolutions per minute.

a How many revolutions does the flywheel make in 15 minutes?

b How many revolutions does the flywheel make in 15 seconds?

c How long does it take for the flywheel to complete 15 000 revolutions?

d How long does it take for the flywheel to complete 150 revolutions?

8 Putra is an elite rower. When training, he has a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.

a How many times does Putra's heart beat during a 30 minute workout?

b How many times does Putra's heart beat during 30 minutes of 'rest'?

c If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

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6G

PROBLEM-SOLVING

9, 10 | 10–12 | 10–12



9 What is the cost of paving a driveway that is 18 m long and 4 m wide, if the paving costs $35 per square metre?

10 A saltwater swimming pool requires 2 kg of salt to be added for every 10 000 litres of water. Joan's swimming pool is 1.5 metres deep, 5 metres wide and 15 metres long. How much salt will she need to add to her pool?

11 The Bionic Woman gives Batman a 12 second start in a 2 kilometre race. If the Bionic Woman runs at 5 km/min, and Batman runs at 3 km/min, who will win the race and by how many seconds?

12 At a school camp there is enough food for 150 students for 5 days.

a How long would the food last if there were only 100 students?

b If the food ran out after only 4 days, how many students attended the camp?

REASONING

13 | 13–15 | 14–17

Example 19



13 Michelle can complete a landscaping job in 6 days and Danielle can complete the same job in 4 days. Working together, in how many days could they complete the job?

14 Three bricklayers Maric, Hugh and Ethan are cladding a new home. If Maric were to work alone, the job would take him 8 days to complete. If Hugh were to work alone, the job would take him 6 days to complete, and if Ethan were to work by himself, the job would take him 12 days to complete.

a If the three men work together, how long will it take them to complete the job?

b What fraction of the house will each bricklayer complete?

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6G

REASONING

15 Four cans of dog food will feed 3 dogs for 1 day.

- **a** How many cans are needed to feed 10 dogs for 6 days?
- **b** How many dogs can be fed for 9 days from 60 cans?
- **c** For how many days will 40 cans feed 2 dogs?

16 If it takes 4 workers, 4 hours to dig 4 holes, how long would it take 2 workers to dig 6 holes?

17 State the units required for the answer to each of the following rate calculations.

- **a** \$205/kg × 48 kg
- **b** 62 s × 12 m/s
- **c** 500 beats ÷ 65 beats/min (bpm)
- **d** 4000 revolutions ÷ 120 revs/min

Value for money — — 18

ENRICHMENT

18 Soft drink can be purchased from supermarkets in a variety of sizes.
Below are the costs for four different sizes of a certain brand of soft drink.

600 mL 'buddies'	1.25 L bottles	2 L bottles	10 × 375 mL cans
\$2.70 each	\$1.60 each	\$2.20 each	\$6.00 per pack

- **a** Find the economy rate (in \$/L) for each size of soft drink.
- **b** Find and compare the cost of 30 litres of soft drink purchased entirely in each of the four different sizes.
- **c** If you only have \$60 to spend on soft drink for a party, what is the difference between the greatest amount and the least amount of soft drink you could buy? Assume you have less than \$1.60 left.

Most supermarkets now include the economy rate of each item underneath the price tag to allow customers to compare value for money.

- **d** Carry out some research at your local supermarket on the economy rates of a particular food item with a range of available sizes (such as drinks, breakfast cereals, sugar, flour). Write a report on your findings.



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6H Speed EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

A rate that we come across almost every day is speed. **Speed** is the rate of distance travelled per unit of time.

On most occasions, speed is not constant and therefore we are interested in the average speed of an object. Average speed is calculated by dividing the distance travelled by the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Given the average speed formula, we can tell that all units of speed must have a unit of length in the numerator, followed by a unit of time in the denominator. Therefore 'mm/h' is a unit of speed and could be an appropriate unit for the speed of a snail!



This snail's average speed won't be high, but it can still be calculated.

The most common units of speed are m/s and km/h.

Let's start: Which is faster?

With a partner, determine the faster of the two listed alternatives.

a	Car A travelling at 10 m/s	Car B travelling at 40 km/h
b	Walker C travelling at 4 km/h	Walker D travelling at 100 m/min
c	Jogger E running at 1450 m/h	Jogger F running at 3 m/s
d	Plane G flying at 700 km/h	Plane H flying at 11 km/min

Key ideas

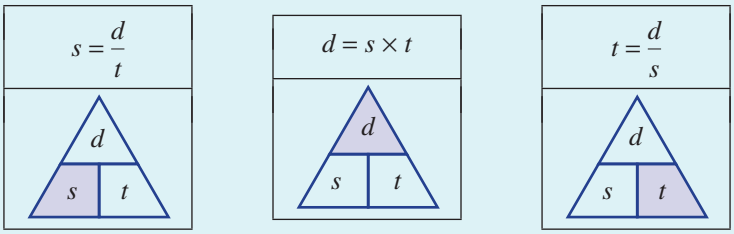
- **Speed** is a measure of how fast an object is travelling.
- If the speed of an object does not change over time, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.
- When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.
- Average speed is calculated by the formula:

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \quad \text{or} \quad s = \frac{d}{t}$$

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Key ideas

- Depending on the unknown value, the above formula can be rearranged to make d or t the subject. The three formulas involving s, d, and t are:



- Care must be taken with units for speed, and on occasions units will need to be converted. The most common units of speed are m/s and km/h.

Example 20 Finding average speed

Find the average speed in km/h of:

a a cyclist who travels 140 km in 5 hours

b a runner who travels 3 km in 15 minutes

SOLUTION

a $s = \frac{d}{t}$

$= \frac{140 \text{ km}}{5 \text{ h}}$

$= 28$ km/h

Alternative unitary method

$\div 5$ (140 km in 5 hours → 28 km in 1 hour) $\div 5$

Average speed = 28 km/h

b $s = \frac{d}{t}$

$= \frac{3 \text{ km}}{15 \text{ min}}$

$= 3 \div \frac{1}{4}$

$= 3 \times 4 = 12$ km/h

Alternative unitary method

$\times 4$ (3 km in 15 minutes → 12 km in 60 minutes) $\times 4$

Average speed = 12 km/h

EXPLANATION

The unknown value is speed.

Write the formula with s as the subject.

Distance travelled = 140 km

Time taken = 5 h

Speed unit is km/h.

Write down the rate provided in the question.

Divide both quantities by 5.

Distance travelled = 3 km

Time taken = 15 min or $\frac{1}{4}$ h

Dividing by $\frac{1}{4}$, is the same as multiplying by $\frac{4}{1}$.

Write down the rate provided in the question.

Multiply both quantities by 4.

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Example 21 Finding the distance travelled

Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

SOLUTION	EXPLANATION
$d = s \times t$	The unknown value is distance.
$= 95 \text{ km/h} \times 15 \text{ h}$	Write the formula with d as the subject.
$= 1425 \text{ km}$	Distance unit is km.
Alternative unitary method	
$\times 15$ (95 km in 1 hour → 1425 km in 15 hours) $\times 15$	Write the rate provided in the question.
Truck travels 1425 km in 15 hours.	Multiply both quantities by 15.

Example 22 Finding the time taken

Find the time taken for a hiker walking at 4 km/h to travel 15 km.

SOLUTION	EXPLANATION
$t = \frac{d}{s}$	The unknown value is time.
$= \frac{15 \text{ km}}{4 \text{ km/h}}$	Write the formula with t as the subject.
$= 3.75 \text{ h}$	The time unit is h.
$= 3 \text{ h } 45 \text{ min}$	Leave answer as a decimal or convert to hours and minutes.
	$0.75 \text{ h} = 0.75 \times 60 = 45 \text{ min}$
Alternative unitary method	
$\div 4$ (4 km in 1 hour → 1 km in $\frac{1}{4}$ hour) $\div 4$	Express the rate as provided in the question.
$\times 15$ (1 km in $\frac{1}{4}$ hour → 15 km in $\frac{15}{4}$ hours) $\times 15$	Divide both quantities by 4.
It takes 3 h 45 min to travel 15 km.	Multiply both quantities by 15.

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Example 23 Converting units of speed

a Convert 72 km/h to m/s. **b** Convert 8 m/s to km/h.

SOLUTION	EXPLANATION
a 72 km in 1 hour	Express rate in kilometres per hour.
÷60 ↓ 72 000 m in 60 minutes ↓ ÷60 1200 m in 1 minute	Convert km to m and hour to minutes. Divide both quantities by 60.
÷60 ↓ 1200 m in 60 seconds ↓ ÷60 20 m in 1 second	Convert 1 minute to 60 seconds. Divide both quantities by 60.
∴ 72 km/h = 20 m/s	Shortcut for converting km/h → m/s ÷ 3.6.
b 8 m in 1 second	Express rate in metres per second.
×60 ↓ 8 m in 1 second ↓ ×60 480 m in 1 minute	Multiply by 60 to find distance in 1 minute.
×60 ↓ 28 800 m in 1 hour ↓ ×60	Multiply by 60 to find distance in 1 hour.
∴ 28.8 km in 1 hour	Convert metres to kilometres.
	Shortcut: m/s × 3.6 → km/h. 8 m/s × 3.6 = 28.8km/h

Exercise 6H

1–4 | 3, 4 | —

UNDERSTANDING

1 Which of the following is not a unit of speed?

A m/s **B** km/h **C** cm/h **D** L/kg **E** m/min

2 If Average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$ then the Distance travelled must equal:

A Average speed × Time taken **B** $\frac{\text{Average speed}}{\text{Time taken}}$

C $\frac{\text{Time taken}}{\text{Average speed}}$

3 If Average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$ then Time taken must equal:

A Distance travelled × Average speed **B** $\frac{\text{Distance travelled}}{\text{Average speed}}$

C $\frac{\text{Average speed}}{\text{Distance travelled}}$

4 If an object travels 800 metres in 10 seconds, the average speed of the object is:

A 8000 m/s **B** 800 km/h **C** 80 km/h **D** 80 m/s

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5–9(½) 5–7(½), 8–10 5–10(½)

6H

FLUENCY

Example 20

5 Calculate the average speed of:



- **a** a sprinter running 200 m in 20 seconds
- **b** a skateboarder travelling 840 m in 120 seconds
- **c** a car travelling 180 km in 3 hours
- **d** a truck travelling 400 km in 8 hours
- **e** a train travelling 60 km in 30 minutes
- **f** a tram travelling 15 km in 20 minutes

Example 21

6 Calculate the distance travelled by:

- **a** a cyclist travelling at 12 m/s for 90 seconds
- **b** an ant travelling at 2.5 m/s for 3 minutes
- **c** a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h
- **d** a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h

Example 22

7 Calculate the time taken by:

- **a** a sports car to travel 1200 km at an average speed of 150 km/h
- **b** a bus to travel 14 km at an average speed of 28 km/h
- **c** a plane to fly 6900 km at a constant speed of 600 km/h
- **d** a ball moving through the air at a speed of 12 m/s to travel 84 m

Example 23a

8 Convert the following speeds to m/s.

a 36 km/h **b** 180 km/h **c** 660 m/min **d** 4 km/s

Example 23b

9 Convert the following speeds to km/h.

a 15 m/s **b** 2 m/s **c** 12 m/min **d** 1 km/s

10 Complete the following table.

	Speed *s*	Distance *d*	Time *t*
a		50 km	2 h
b	30 m/s	1200 m	
c	5 km/h		12 h
d		600 m	5 min
e	210 km/h		20 min
f	100 m/s	10 km	



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6H

11, 12 | 12–15 | 13–16

PROBLEM-SOLVING

11 A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 a.m. to 1:30 p.m. on the same day?

12 The wheels on Charlie's bike have a circumference of 1.5 m. When Charlie is riding fastest, the wheels rotate at a speed of five turns per second.



a What is the fastest speed Charlie can ride his bike, in km/h?

b How far would Charlie travel in 5 minutes at his fastest speed?

13 The back end of a 160-metre-long train disappears into a 700-metre-long tunnel. Twenty seconds later the front of the train emerges from the tunnel. Determine the speed of the train in m/s.

14 Anna rode her bike to school one morning, a distance of 15 km, at an average speed of 20 km/h. It was raining in the afternoon, so Anna decided to take the bus home. The bus trip home took 30 minutes. What was Anna's average speed for the return journey to and from school?

15 In Berlin 2009, Jamaican sprinter Usain Bolt set a new 100 m world record time of 9.58 seconds. Calculate Usain Bolt's average speed in m/s and km/h for this world record. (Round the answers correct to one decimal place.)

16 The Ghan train is an Australian icon. You can board The Ghan in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. (Round the answers correct to one decimal place.)



a If you board The Ghan in Adelaide on Sunday at 2:20 p.m. and arrive in Darwin on Tuesday at 5:30 p.m., what is the average speed of the train journey?

b There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine. Taking these breaks into account, what is the average speed of the train when it is moving?

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6H

	17	17, 18	18, 19

REASONING

17 Write two speeds in km/h that are between 40 m/s and 45 m/s.

18 Nina, Shanti and Belle run a 1000 m race at a constant speed. When Nina crossed the finish line first, she was 200 m ahead of Shanti and 400 m ahead of Belle. When Shanti crossed the finish line, how far ahead of Belle was she?

19 Julie and Jeanette enjoy finishing their 6 km morning run together. Julie runs at an average speed of 10 km/h and Jeanette runs at an average speed of 3 m/s. If Julie leaves at 8 a.m., at what time should Jeanette leave if they are to finish their run at the same time?

Speed research	—	—	20

ENRICHMENT

20 Carry out research to find answers to the following questions.



Light and sound

- **a** What is the speed of sound in m/s?
- **b** What is the speed of light in m/s?
- **c** How long would it take sound to travel 100 m?
- **d** How long would it take light to travel 100 km?
- **e** How many times quicker is the speed of light than the speed of sound?
- **f** What is a Mach number?



Spacecraft

- **g** What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.
- **h** What is the orbital speed of planet Earth? Give your answer in km/h and km/s.
- **i** What is the average speed of a space shuttle on a journey from Earth to the International Space Station?

Knots

Wind speed and boat speed are often given in terms of knots (kt).

- **j** What does a knot stand for?
- **k** What is the link between nautical miles and a system of locating positions on Earth?
- **l** How do you convert a speed in knots to a speed in km/h?



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Investigation

Fun run investigation

Three maths teachers, Mrs M, Mr P and Mr A, trained very hard to compete in a Brisbane Fun Run. The 10.44 km route passed through the botanical gardens, along the Brisbane River to New Farm Park, and back again. Their times and average stride lengths were:

- Mrs M: 41 minutes, 50 seconds: 8 strides per 9 m
- Mr P: 45 minutes, 21 seconds: 7 strides per 9 m
- Mr A: 47 minutes, 6 seconds: 7.5 strides per 9 m



Copy and complete the following table and determine which rates are the most useful representations of fitness. Give the answers rounded to one decimal place. Justify your conclusions.

Running rates	Mrs M	Mr P	Mr A	World record 10 km runners	Your family member or friend
Seconds per 100 m					
Seconds per km					
Metres per minute					
Km per hour					
Strides per 100 m					
Strides per minute					
Strides per hour					

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Fitness investigation

1 Using a stopwatch, measure your resting heart rate in beats per minute.

2 Run on the spot for one minute and then measure your working heart rate in beats per minute.

3 Using a stopwatch, time yourself for a 100 m run and also count your strides. At the end, measure your heart rate in beats per minute. Also calculate the following rates.
- Your running rate in m/s, m/min and km/h
- Your running rate in time per 100 m and time per km
- Your rate of strides per minute, strides per km and seconds per stride

4 Run 100 m four times without stopping and, using a stopwatch, record your cumulative time after each 100 m.
- Organise these results into a table.
- Draw a graph of the distance ran (vertical) vs time taken (horizontal).
- Calculate your running rate in m/min for each 100 m section.
- Calculate your overall running rate in m/min for the 400 m.
- Explain how and why your running rates changed over the 400 m.

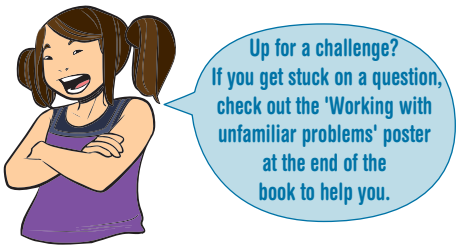
5 Try sprinting fast over a measured distance and record the time. Calculate your sprinting rate in each of the following units:
- minutes per 100 m
- metres per minute
- time per km
- km per hour

6 Research the running rate of the fastest schoolboy and schoolgirl in Australia. How do their sprinting rates compare to the running rates of Australian Olympian athletes?

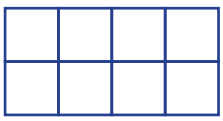


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Problems and challenges



1 This diagram is made up of 8 equal-sized squares.



How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

a 1 : 3? **b** 2 : 3? **c** 1 : 2?

2 Bottle A has 1 L of cordial drink with a cordial to water ratio of 3 : 7. Bottle B has 1 L of cordial drink with a cordial to water ratio of 1 : 4. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

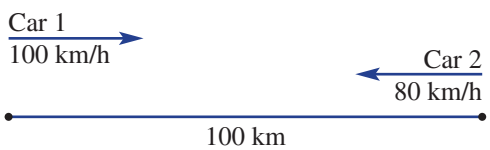
3 Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

4 **a** If 1 person takes 1 hour to dig 1 post hole, how long will it take 2 people to dig 2 post holes?

b If 3 people take 3 hours to make 3 wooden train sets, how many train sets can 6 people make in 6 hours?

5 At a market you can trade 2 cows for 3 goats or 3 goats for 8 sheep. How many sheep are 3 cows worth?

6 Two cars travel toward each other on a 100 km straight stretch of road. They leave at opposite ends of the road at the same time. The cars' speeds are 100 km/h and 80 km/h. How long does it take for the cars to pass each other?



7 A river is flowing downstream at a rate of 1 km/h. In still water Michael can swim at a rate of 2 km/h. Michael dives into the river and swims downstream then turns around and swims back to the starting point, taking 0.5 hours in total. How far did he swim?

8 A fitness fanatic walks at 4 km/h for time t_1, and then runs at 7 km/h for time t_2. He travels a total of 26 km. If he had run for the same time that he had walked (t_1) and walked for the same time that he had run (t_2), then he would have travelled 29 km. What was the total time spent walking and running?

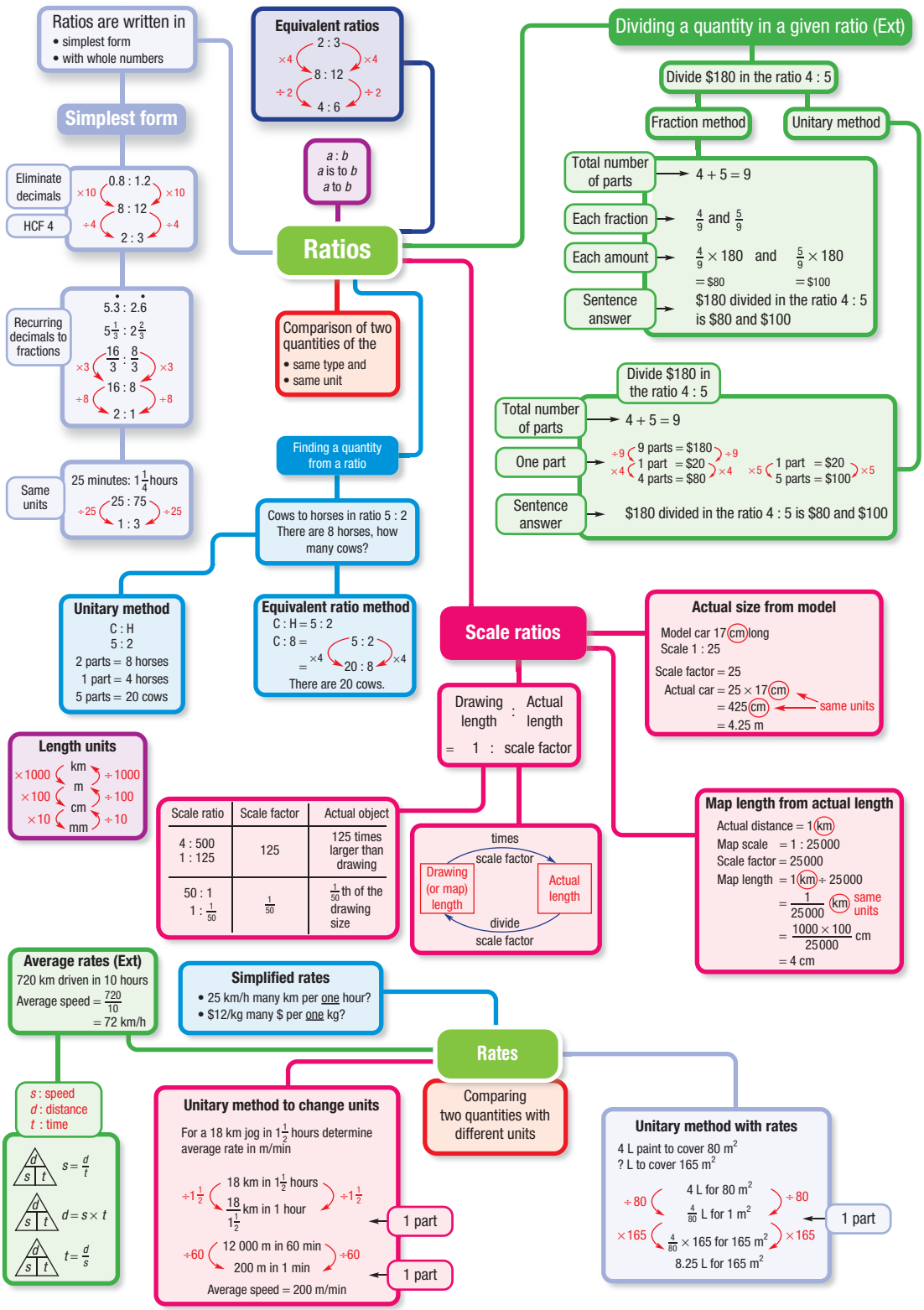
9 A top secret goo has been developed so that it doubles in volume every 60 seconds. At midnight a small amount is placed in a beaker and its growth observed. At 1 a.m. the beaker is full. At exactly what time did the goo fill only $\frac{1}{8}$ of the beaker?

10 A car averages 60 km/h for 20 km and then increases its speed averaging 80 km/h for the next 20 km. Calculate the car's average speed (to two decimal places) for the full 40 km trip.

11 Max travels at 50 km/h for 10 km. For the next 10 km Max wants to drive fast enough to make his average speed 100 km/h over the 20 km trip. Advise Max about reaching this average speed.

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Chapter summary



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Chapter review

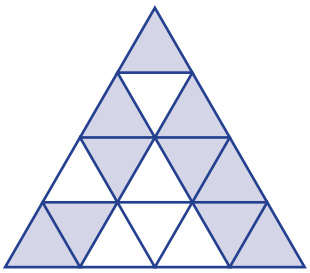
Multiple-choice questions

6A **1** A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:

A 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5

6A **2** Find the ratio of the shaded area to the unshaded area in this triangle.

A 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8



6B **3** The ratio 500 mm to $\frac{1}{5}$ m is the same as:

A 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 5 : 2

6B **4** The ratio $1\frac{1}{2} : \frac{3}{4}$ simplifies to:

A 2 : 1 **B** 1 : 2 **C** 3 : 4 **D** 4 : 3

6C Ext **5** $750 is divided in the ratio 1 : 3 : 2. The smallest share is:

A $250 **B** $125 **C** $375 **D** $750

6C Ext **6** The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 cm^2. What is the area of the smaller triangle?

A 12 cm^2 **B** 24 cm^2 **C** 30 cm^2 **D** 17 cm^2

6E **7** Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from $50 cash is:

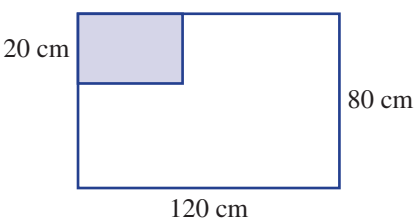
A $10 **B** $39.95 **C** $10.05 **D** $40

6F **8** 45 km/h is the same as:

A 0.25 m/s **B** 25 m/s **C** 12.5 m/s **D** 75 m/s

6G Ext **9** A flag is created by enlarging the shaded rectangle as shown. What is the length of the original rectangle?

A 20 cm
B 30 cm
C 40 cm
D 13 cm



6D **10** On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?

A 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000

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Short-answer questions

6A **1** In Lao's pencil case there are 6 coloured pencils, 2 black pens, 1 red pen, 5 textas, 3 lead pencils and a ruler. Find the ratio of:

- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** textas to all pencils

6B **2** True or false?

- **a** $1 : 4 = 3 : 6$
- **b** The ratio $2 : 3$ is the same as $3 : 2$.
- **c** The ratio $3 : 5$ is written in simplest form.
- **d** 40 cm : 1 m is written as $40 : 1$ in simplest form.
- **e** $1\frac{1}{4} : 2 = 5 : 8$

6B **3** Copy and complete.

- **a** $4 : 50 = 2 : \square$
- **b** $1.2 : 2 = \square : 20$
- **c** $\frac{2}{3} : 4 = 1 : \square$
- **d** $1 : \square : 5 = 5 : 15 : 25$

6B **4** Simplify the following ratios.

- **a** $10 : 40$
- **b** $36 : 24$
- **c** $75 : 100$
- **d** $8 : 64$
- **e** $27 : 9$
- **f** $5 : 25$
- **g** $6 : 4$
- **h** $52 : 26$
- **i** $6b : 9b$
- **j** $8a : 4$
- **k** $\frac{2}{7} : \frac{5}{7}$
- **l** $1\frac{1}{10} : \frac{2}{10}$
- **m** $2\frac{1}{2} : \frac{3}{4}$
- **n** $12 : 36 : 72$

6B **5** Simplify the following ratios by first changing to the same units.

- **a** 2 cm : 8 mm
- **b** 5 mm : 1.5 cm
- **c** 3 L : 7500 mL
- **d** 30 min : 1 h
- **e** 400 kg : 2 t
- **f** 6 h : 1 day
- **g** 120 m : 1 km
- **h** 45 min : $2\frac{1}{2}$ h

6C Ext **6** Divide:

- **a** \$80 in the ratio 7 : 9
- **b** 200 kg in the ratio 4 : 1
- **c** 40 m in the ratio 6 : 2
- **d** \$1445 in the ratio 4 : 7 : 6
- **e** \$1 in the ratio 3 : 1 : 1

6C Ext **7**

- **a** The ratio of the cost price of a TV to its retail price is 5 : 12. If its cost price is \$480, calculate its retail price.
- **b** The ratio of Sally's height to Ben's height is 12 : 17. If the difference in their heights is 60 cm, how tall is Sally?
- **c** Orange juice, pineapple juice and guava juice are mixed in the ratio 4 : 3 : 2. If 250 mL of guava juice is used, how many litres of drink does this make?

6E **8** Express each rate in simplest form.

- **a** 10 km in 2 hours
- **b** \$650 for 13 hours
- **c** 2800 km in 20 days

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Chapter review

6E **9** Copy and complete.

a 400 km on 32 litres = ____ km/L
= ____ L/100 km

b 5 grams in 2 min = ____ g/min
= ____ g/h

c \$1200 in $\frac{1}{2}$ day = ____ \$/day
= ____ \$/h

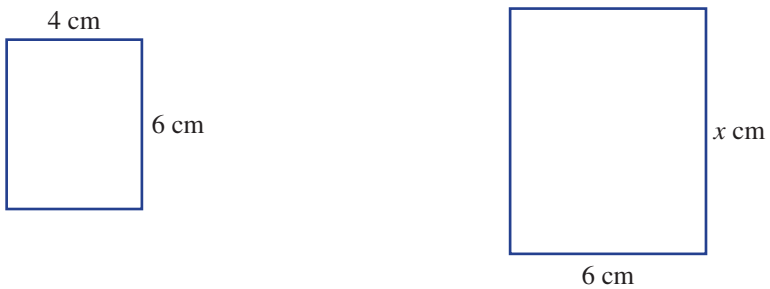
6D **10** For a scale of 1 : 1000, find the real length (in metres) if the scale length is given as:

a 0.0002 m **b** 2.7 cm **c** 140 mm

6D **11** Two cities are 50 km apart. How many millimetres apart are they on a map that has a scale of 1 : 100 000?

6D **12** Find the scale factor for the following pairs of similar figures, and find the value of x.

a



b



6F **13** Copy and complete.

a \$120/h = ____ c/min **b** 6 m/s = ____ km/h **c** 720 km/h = ____ m/s

6G Ext **14** **a** A truck uses 12 litres of petrol to travel 86 km. How far will it travel on 42 litres of petrol?

b Samira earns \$67.20 for a 12-hour shift. How much will she earn for a 7-hour shift?

c Tap 1 fills the pool in 12 hours, while tap 2 fills the same pool in 15 hours. How long does it take to fill this pool if both taps are used?

6H Ext **15** **a** Sandra drives to her mother's house. It takes 45 minutes. Calculate Sandra's average speed if her mother lives 48 km away.

b How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip?

c How far does Siri ride his bike if he rides at 4.5 km/h for 90 minutes?

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Extended-response questions

1 The Harrison family and the Nguyen family leave Wollongong at 8 a.m. on Saturday morning for a holiday in Melbourne. The Harrisons' 17-year-old son drives for the first 2 hours at 80 km/h. They then stop for a rest of $1\frac{1}{2}$ hours. Mr Harrison drives the rest of the way.

The Nguyen family drives straight to Melbourne with no stops. It takes them 6 hours and 15 minutes to drive the 627 km to Melbourne.

a At what time did the Nguyen family arrive in Melbourne?

b Calculate the average speed of the Nguyen's car. Round your answer to the nearest whole number.

c At what time did the Harrisons resume their journey after their rest stop?

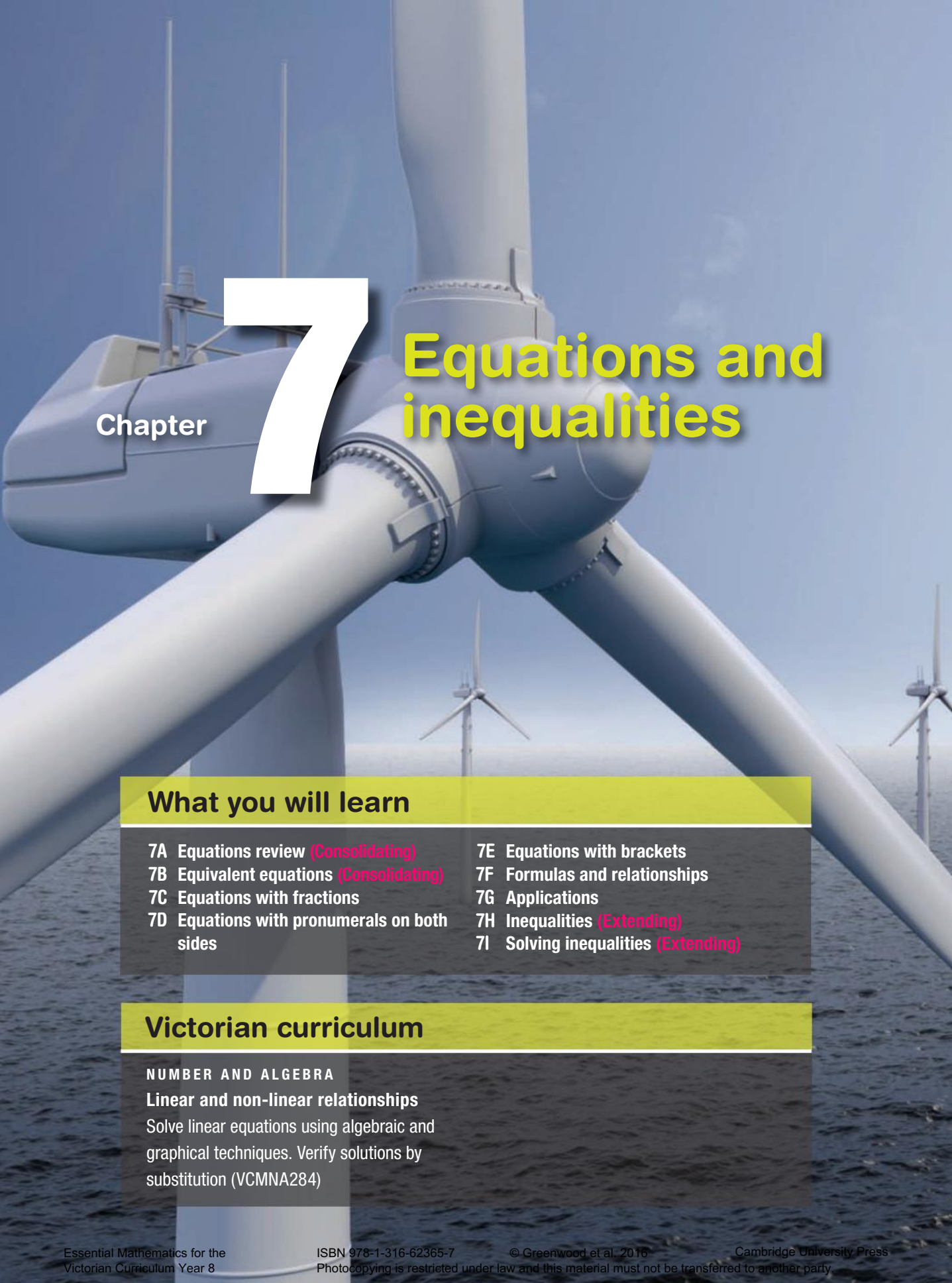
d How many kilometres did the Harrisons still have to cover after their rest break before arriving in Melbourne?

e If the Harrisons arrive in Melbourne at 4:30 p.m., for how long did Mr Harrison drive and at what speed?

f Calculate the cost of the petrol for each family's trip if petrol cost 125c/L and the Harrison car's consumption is 36 L/100 km, while the Nguyen's car uses 40 L/100 km.



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Chapter 7

Equations and inequalities

What you will learn

7A Equations review (Consolidating)
7B Equivalent equations (Consolidating)
7C Equations with fractions
7D Equations with pronumerals on both sides
7E Equations with brackets
7F Formulas and relationships
7G Applications
7H Inequalities (Extending)
7I Solving inequalities (Extending)

Victorian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (VCMNA284)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Maths and wind power

Powered freely by the wind, the giant blades of a wind turbine rotate at 30–60 rpm (revolutions per minute). A gear box drives the motor shaft at 1000–1800 rpm, generating electricity.

The maximum quantity of electrical power available from wind is proportional to the cube of the wind speed. If wind speed doubles, the available electrical power is $2^3 = 8$ times greater.

Many complex mathematical equations are used to design virtual models of wind turbines and to study the effects of weather conditions on them. The shape, thickness and total area of the blades must be designed with enough strength to survive huge wind forces as well as extracting as much wind energy as possible.

Mathematical calculations show that a wind turbine with three blades gives the most power per blade, and also that increasing the size of each turbine blade delivers a lot more power. Some wind farms that are built out to sea have giant blades of 80 m, similar in length to the wingspan of an Airbus!

Wind power is a low-cost, clean, renewable energy. Australian wind farms already produce more than 4% of our overall energy needs, helping to reduce global warming.

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7A Equations review CONSOLIDATING









An equation is a statement that two things are equal, such as:

$2 + 2 = 4$

$7 \times 5 = 30 + 5$

$4 + x = 10 + y$.

It consists of two expressions separated by the equals sign (=), and it is true if the left-hand side and right-hand side are equal. True equations include $7 + 10 = 20 - 3$ and $8 = 4 + 4$; examples of false equations are $2 + 2 = 7$ and $10 \times 5 = 13$.

If an equation has a pronumeral in it, such as $3 + x = 7$, then a **solution** to the equation is a value to substitute for the pronumeral to form a true equation. In this case a solution is $x = 4$ because $3 + 4 = 7$ is a true equation.

Let's start: Solving the equations

- Find a number that would make the equation $25 = b \times (10 - b)$ true.
- How can you prove that this value is a solution?
- Try to find a solution to the equation $11 \times b = 11 + b$.

Key ideas

- An **equation** is a mathematical statement that two expressions are equal, such as $4 + x = 32$. It could be true (e.g. $4 + 28 = 32$) or false (e.g. $4 + 29 = 32$).
- A false equation can be made into a true statement by using the $\neq$ sign. For instance, $4 + 29 \neq 32$ is a true statement.
- An equation has a left-hand side (LHS) and a right-hand side (RHS).
- A **solution** to an equation is a value that makes an equation true. The process of finding a solution is called **solving**. In an equation with a pronumeral, the pronumeral is also called an **unknown**.
- An equation could have no solutions or it could have one or more solutions.



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Example 1 Classifying equations as true or false

For each of the following equations, state whether they are true or false.

a $3 + 8 = 15 - 4$ **b** $7 \times 3 = 20 + 5$ **c** $x + 20 = 3 \times x$, if $x = 10$

SOLUTION	EXPLANATION
a True	Left-hand side (LHS) is $3 + 8$, which is 11. Right-hand side (RHS) is $15 - 4$, which is also 11. Since LHS equals RHS, the equation is true.
b False	LHS $= 7 \times 3 = 21$ RHS $= 20 + 5 = 25$ Since LHS and RHS are different, the equation is false.
c True	If $x = 10$ then LHS $= 10 + 20 = 30$. If $x = 10$ then RHS $= 3 \times 10 = 30$. LHS equals RHS, so the equation is true.

Example 2 Stating a solution to an equation

State a solution to each of the following equations.

a $4 + x = 25$ **b** $5y = 45$ **c** $26 = 3z + 5$

SOLUTION	EXPLANATION
a $x = 21$	We need to find a value of x that makes the equation true. Since $4 + 21 = 25$ is a true equation, $x = 21$ is a solution.
b $y = 9$	If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true.
c $z = 7$	If $z = 7$ then $3z + 5 = 3 \times 7 + 5$ $= 21 + 5$ $= 26$ Note: The fact that z is on the right-hand side of the equation does not change the procedure.

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Example 3 Writing equations from a description

Write equations for the following scenarios.

a The number k is doubled, then three is added and the result is 52.

b Akira works n hours, earning \$12 per hour. The total she earned was \$156.

SOLUTION	EXPLANATION
a $2k + 3 = 52$	The number k is doubled, giving $k \times 2$. This is the same as $2k$. Since 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description.
b $12n = 156$	If Akira works n hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$.

Exercise 7A

1–4 | 3, 4 | —

UNDERSTANDING

Example 1a
Example 1b

1 Classify these equations as true or false.

a $5 \times 3 = 15$ **b** $7 + 2 = 12 + 3$ **c** $5 + 3 = 16 \div 2$

d $8 - 6 = 6$ **e** $4 \times 3 = 12 \times 1$ **f** $2 = 8 - 3 - 3$

2 If the value of x is 3, what is the value of the following?

a $10 + x$ **b** $3x$ **c** $5 - x$ **d** $6 \div x$

3 State the value of the missing number to make the following equations true.

a $5 + \square = 12$ **b** $10 \times \square = 90$ **c** $\square - 3 = 12$ **d** $3 + 5 = \square$

4 Consider the equation $15 + 2x = x \times x$.

a If $x = 5$, find the value of $15 + 2x$.

b If $x = 5$, find the value of $x \times x$.

c Is $x = 5$ a solution to the equation $15 + 2x = x \times x$?

d Give an example of another equation with $x = 5$ as a solution.

5–6(½), 8(½) | 5–8(½) | 6–8(½)

FLUENCY

Example 1c

5 If $x = 2$, state whether the following equations are true or false.

a $7x = 8 + 3x$ **b** $10 - x = 4x$ **c** $3x = 5 - x$

d $x + 4 = 5x$ **e** $10x = 40 \div x$ **f** $12x + 2 = 15x$.

6 If $a = 3$, state whether the following equations are true or false.

a $7 + a = 10$ **b** $2a + 4 = 12$ **c** $8 - a = 5$

d $4a - 3 = 9$ **e** $7a + 2 = 8a$ **f** $a = 6 - a$

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7A

FLUENCY

7 Someone has attempted to solve the following equations. State whether the solution is correct (C) or incorrect (I).

- **a** $5 + 2x = 4x - 1$, proposed solution: $x = 3$
- **b** $4 + q = 3 + 2q$, proposed solution: $q = 10$
- **c** $13 - 2a = a + 1$, proposed solution: $a = 4$
- **d** $b \times (b + 3) = 4$, proposed solution, $b = -4$

Example 2

8 State a solution to each of the following equations.

- **a** $5 + x = 12$
- **b** $3 = x - 10$
- **c** $4v + 2 = 14$
- **d** $17 = p - 2$
- **e** $10x = 20$
- **f** $16 - x = x$
- **g** $4u + 1 = 29$
- **h** $7k = 77$
- **i** $3 + a = 2a$

9, 10(½) 9–10(½),11, 12 11–13

PROBLEM-SOLVING

Example 3

9 Write equations for each of the following problems. You do not need to solve the equations.

- **a** A number x is doubled and then 7 is added. The result is 10.
- **b** The sum of x and half of x is 12.
- **c** Aston's age is a. His father, who is 25 years older, is twice as old as Aston.
- **d** Fel's height is h cm and her brother Pat is 30 cm taller. Pat's height is 147 cm.
- **e** Coffee costs \$$c$ per cup and tea costs \$$t$. Four cups of coffee and three cups of tea cost a total of \$21.
- **f** Chairs cost \$$c$ each. To purchase 8 chairs and a \$2000 table costs a total of \$3600.

10 Find the value of the unknown number for each of the following.

- **a** A number is tripled to obtain the result 21.
- **b** Half of a number is 21.
- **c** Six less than a number is 7.
- **d** A number is doubled and the result is −16.
- **e** Three-quarters of a number is 30.
- **f** Six more than a number is −7.

11 Berkeley buys x kg of oranges at \$3.20 per kg. He spends a total of \$9.60.

- **a** Write an equation involving x to describe this situation.
- **b** State a solution to this equation.



12 Emily's age in 10 years' time will be triple her current age. She is currently E years old.

- **a** Write an equation involving E to describe this situation.
- **b** Find a solution to this equation.
- **c** How old is Emily now?
- **d** How many years will she have to wait until she is four times her current age?

13 Find two possible values of t that make the equation $t(10 - t) = 21$ true.

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7A

14 | 14, 15 | 14, 16

REASONING

14 **a** Why is $x = 3$ a solution to $x^2 = 9$?

b Why is $x = -3$ a solution to $x^2 = 9$?

c Find the two solutions to $x^2 = 64$ (Hint: one is negative).

d Explain why $x^2 = 0$ has only one solution, but $x^2 = 1$ has two.

e Explain why $x^2 = -9$ has no solutions.

15 **a** Explain why the equation $x + 3 = x$ has no solutions.

b Explain why the equation $x + 2 = 2 + x$ is true, regardless of the value of x.

c Show that the equation $x + 3 = 10$ is sometimes true and sometimes false.

d Classify the following equations as always true (A), sometimes true (S) or never true (N).

i $x + 2 = 10$ **ii** $5 - q = q$ **iii** $5 + y = y$

iv $10 + b = 10$ **v** $2 \times b = b + b$ **vi** $3 - c = 10$

vii $3 + 2z = 2z + 1$ **viii** $10p = p$ **ix** $2 + b + b = (b + 1) \times 2$

e Give a new example of another equation that is always true.

16 **a** The equation $p \times (p + 2) = 3$ has two solutions. State the two solutions.
Hint: One of them is negative.

b How many solutions are there for the equation $p + (p + 2) = 3$?

c Try to find an equation that has three solutions.

More than one unknown

— | — | 17, 18

ENRICHMENT

17 **a** There are six equations in the square below. Find the values of a, b, c, d and e to make all six equations true.

a	+	12	=	22
×		÷		−
2	×	b	=	c
=		=		=
d	÷	e	=	10

b Find the value of f that makes the equation $a \times b \times e = c \times d \times f$ true.

18 For each of the following pairs of equations, find values of c and d that make both equations true. More than one answer may be possible.

a $c + d = 10$ and $cd = 24$.

b $c - d = 8$ and $c + d = 14$

c $c \div d = 4$ and $c + d = 30$

d $cd = 0$ and $c - d = 7$

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7B Equivalent equations CONSOLIDATING









If we have an equation, we can obtain an **equivalent** equation by performing the same operation to both sides. For example, if we have $2x + 4 = 20$, we can add 3 to both sides to obtain $2x + 7 = 23$.

The new equation will be true for exactly the same values of x as the old equation. This observations helps us to solve equations algebraically. For example, $2x + 4 = 20$ is equivalent to $2x = 16$ (subtract 4 from both sides), and this is equivalent to $x = 8$ (divide both sides by 2). The bottom equation is only true if x has the value 8, so this means the solution to the equation $2x + 4 = 20$ is $x = 8$. We write this as:

$$\begin{array}{c} 2x + 4 = 20 \\ -4 \quad\quad -4 \\ 2x = 16 \\ \div 2 \quad\quad \div 2 \\ x = 8 \end{array}$$

Let's start: Attempted solutions

Below are three attempts at solving the equation $4x - 8 = 40$. Each has a problem.

Attempt 1

$$\begin{array}{c} 4x - 8 = 40 \\ +8 \quad\quad +8 \\ 4x = 48 \\ -4 \quad\quad -4 \\ x = 44 \end{array}$$

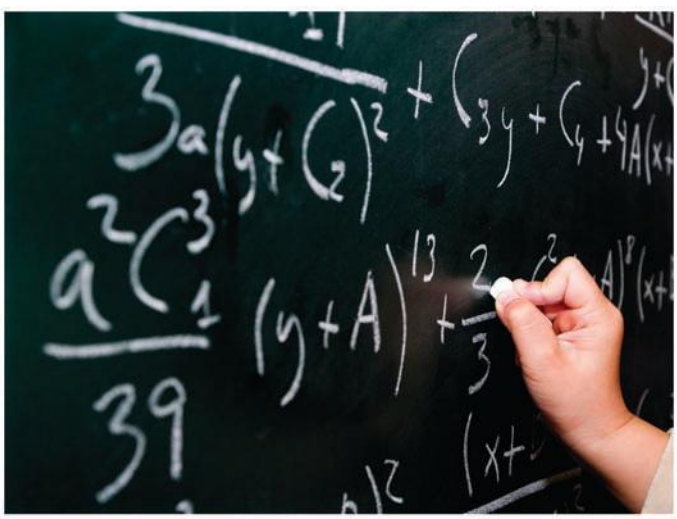
Attempt 2

$$\begin{array}{c} 4x - 8 = 40 \\ +8 \quad\quad -8 \\ 4x = 32 \\ \div 4 \quad\quad \div 4 \\ x = 8 \end{array}$$

Attempt 3

$$\begin{array}{c} 4x - 8 = 40 \\ \div 4 \quad\quad \div 4 \\ x - 8 = 10 \\ +8 \quad\quad +8 \\ x = 18 \end{array}$$

- Can you prove that these results are not the correct solutions to the equation above?
- For each one, find the mistake that was made.
- Can you solve $4x - 8 = 40$ algebraically?



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Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand sides of the equation.
- To solve an equation **algebraically**, repeatedly find an equivalent equation that is simpler.
  For example:

$$5x + 2 = 32 \quad (-2,\ -2)$$
$$5x = 30 \quad (\div 5,\ \div 5)$$
$$x = 6$$

- To check a solution, substitute the unknown's value in to both sides to see if the equation is true, e.g. LHS = 5(6) + 2 = 32 and RHS = 32.

Example 4 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

a $8y = 40$ [$\div 8$] **b** $10 + 2x = 36$ [-10] **c** $5a - 3 = 12$ [$+3$]

SOLUTION

a

$$8y = 40 \quad (\div 8,\ \div 8)$$
$$y = 5$$

b

$$10 + 2x = 36 \quad (-10,\ -10)$$
$$2x = 26$$

c

$$5a - 3 = 12 \quad (+3,\ +3)$$
$$5a = 15$$

EXPLANATION

Write the equation out and then divide both sides by 8.

$40 \div 8$ is 5 and $8y \div 8$ is y.

Write the equation out and then subtract 10 from both sides.

$36 - 10$ is 26

$10 + 2x - 10$ is $2x$

Write the equation out and then add 3 to both sides.

$12 + 3$ is 15

$5a - 3 + 3$ is $5a$

Example 5 Solving equations algebraically

Solve the following equations algebraically and check the solution by substituting.

a $x - 4 = 16$ **b** $2u + 7 = 17$ **c** $40 - 3x = 22$

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SOLUTION	EXPLANATION
a $x - 4 = 16$; $+4$ / $+4$; $x = 20$ So the solution is $x = 20$.	By adding 4 to both sides of the equation, we get an equivalent equation. Check: $20 - 4 = 16$ ✓
b $2u + 7 = 17$; -7 / -7; $2u = 10$; $\div 2$ / $\div 2$; $u = 5$ So the solution is $u = 5$.	To get rid of the +7, we subtract 7 from both sides. Finally we divide by 2 to reverse the $2u$. Remember that $2u$ means $2 \times u$ Check: $2(5) + 7 = 10 + 7 = 17$ ✓
c $40 - 3x = 22$; -40 / -40; $-3x = -18$; $\div -3$ / $\div (-3)$; $x = 6$ So the solution is $x = 6$.	We subtract 40 from both sides to get rid of the 40 at the start of the LHS. Since $-3 \times x = -18$, we divide by -3 to get the final solution. Check: $40 - 3(6) = 40 - 18 = 22$ ✓

Exercise 7B

1–4 | 2–4 | —

UNDERSTANDING

1 For each of the following equations add 4 to both sides to obtain an equivalent equation.

a $3x = 10$ **b** $7 + k = 14$ **c** $5 = 2x$

2 For each equation fill in the blank to get an equivalent equation.

a $5x = 10$; $+2$ / $+2$; $5x + 2 = __$

b $10 - 2x = 20$; $+5$ / $+5$; $15 - 2x = __$

c $3q + 4 = __$; -4 / -4; $3q = 12$

d $7z + 12 = 4z + 10$; -10 / -10; $7z + 2 = __$

3 Consider the equation $4x = 32$.

a Copy and complete the following working.

$4x = 32$; $\div 4$ / $\div 4$; $x = __$

b What is the solution to the equation $4x = 32$?

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7B

UNDERSTANDING

4 To solve the equation $10x + 5 = 45$, which of the following operations would you first apply to both sides?

A Divide by 5 **B** Subtract 5 **C** Divide by 10 **D** Subtract 45

5–6, 7–9(½) 5–10(½) 6–10(½)

FLUENCY

Example 4

5 For each equation, show the result of applying the given operation to both sides.

a $10 + 2x = 30$ [−10] **b** $4 + q = 12$ [−2] **c** $13 = 12 - q$ [+5]

d $4x = 8$ [× 3] **e** $7p = 2p + 4$ [+6] **f** $3q + 1 = 2q + 1$ [−1]

6 Copy and complete the following to solve the given equation algebraically.

a

$10x = 30$

÷ 10 ÷ 10

$x =$ __

b

$q + 5 = 2$

−5 −5

__ = __

c

$4x + 2 = 22$

−2 −2

$4x =$ __

÷ 4 ÷ 4

__ = __

d

$30 = 4p + 2$

−2 −2

__ = __

□ □

__ = __

e

$20 - 4x = 8$

−20 −20

$-4x = -12$

÷ −4 ÷ −4

$x =$ __

f

$p \div 3 + 6 = 8$

−6 −6

$p \div 3 = 2$

□ □

__ = __

Example 5a

7 Solve the following equations algebraically.

a $a + 5 = 8$ **b** $t \times 2 = 14$ **c** $7 = q - 2$

d $11 = k + 2$ **e** $19 = x + 9$ **f** $-30 = 3h$

g $-36 = 9l$ **h** $g \div 3 = -3$

Example 5b

8 Solve the following equations algebraically. Check your solutions using substitution.

a $5 + 9h = 32$ **b** $9u - 6 = 30$ **c** $13 = 5s - 2$

d $-18 = 6 - 3w$ **e** $-12 = 5x + 8$ **f** $-44 = 10w + 6$

g $8 = -8 + 8a$ **h** $4y - 8 = -40$

Example 5c

9 Solve the following equations algebraically and check your solutions.

a $20 - 4d = 8$ **b** $34 = 4 - 5j$ **c** $21 - 7a = 7$

d $6 = 12 - 3y$ **e** $13 - 8k = 45$ **f** $44 = 23 - 3n$

g $13 = -3b + 4$ **h** $-22 = 14 - 9b$

10 The following equations do not all have whole number solutions. Solve the following equations algebraically, giving each solution as a fraction.

a $2x + 3 = 10$ **b** $5 + 3q = 6$ **c** $12 = 10b + 7$

d $15 = 10 + 2x$ **e** $15 = 10 - 2x$ **f** $13 + 2p = -10$

g $22 = 9 + 5y$ **h** $12 - 2y = 15$

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7B

PROBLEM-SOLVING

11 11–13 12–14

11 For each of the following, write an equation and solve it algebraically.

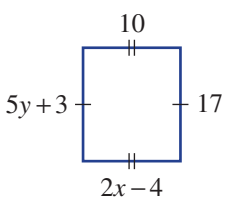
- **a** The sum of p and 8 is 15.
- **b** The product of q and -3 is 12.
- **c** 4 is subtracted from double the value of k and the result is 18.
- **d** When r is tripled and 4 is added the result is 34.
- **e** When x is subtracted from 10 the result is 6.
- **f** When triple y is subtracted from 10 the result is 16.

12 Solve the following equations algebraically. More than two steps are involved.

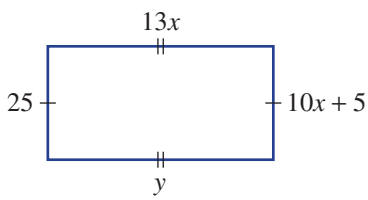
a $14 \times (4x + 2) = 140$ **b** $8 = (10x - 4) \div 2$ **c** $-12 = (3 - x) \times 4$

13 The following shapes are rectangles. By solving equations algebraically, find the value of the variables. Some of the answers will be fractions.

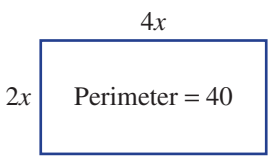
a



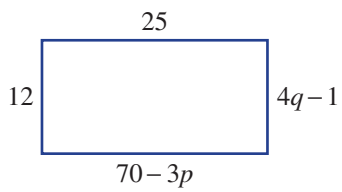
b



c



d

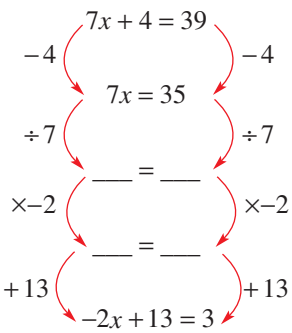


14 Sidney works for 10 hours at the normal rate of pay ($\$x$ per hour) and then the next three hours at double that rate. If he earns a total of \$194.88, write an equation and solve it to find his normal hourly rate.

REASONING

15 15, 16 16, 17

15 a Prove that $7x + 4 = 39$ and $-2x + 13 = 3$ are equivalent by filling in the missing steps.



b Prove that $10k + 4 = 24$ and $3k - 1 = 5$ are equivalent.

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7B

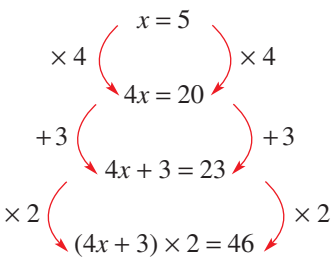
REASONING

16 **a** Prove that $4x + 3 = 11$ and $2x = 4$ are equivalent. Try to use just two steps to get from one equation to the other.

b Are the equations $5x + 2 = 17$ and $x = 5$ equivalent?

c Prove that $10 - 2x = 13$ and $14x + 7 = 20$ are not equivalent, no matter how many steps are used.

17 A student has taken the equation $x = 5$ and performed some operations to both sides.



a Solve $(4x + 3) \times 2 = 46$ algebraically.

b Describe how the steps you used in your solution compare with the steps the student used.

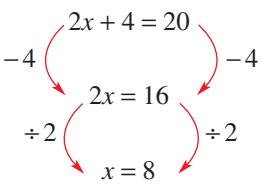
c Give an example of another equation that has $x = 5$ as its solution.

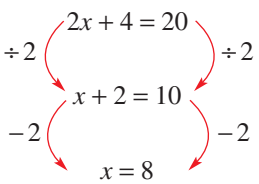
d Explain why there are infinitely many different equations with the solution $x = 5$.

ENRICHMENT

Dividing whole expressions — — 18

18 It is possible to solve $2x + 4 = 20$ by first dividing both sides by 2, as long as every term is divided by 2. So you could solve it in either of these fashions.





Note that $2x + 4$ divided by 2 is $x + 2$, not $x + 4$. Use this method of dividing first to solve the following equations and then check that you get the same answer as if you subtracted first.

a $2x + 6 = 12$ **b** $4x + 12 = 16$ **c** $10x + 30 = 50$

d $2x + 5 = 13$ **e** $5x + 4 = 19$ **f** $3 + 2x = 5$

g $7 = 2x + 4$ **h** $10 = 4x + 10$ **i** $12 = 8 + 4x$

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7C Equations with fractions

Interactive

Widgets

HOTsheets

Walkthroughs

Recall from algebra that a fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:

$$\frac{x}{3} = 10 \quad (\times 3 \text{ on both sides}) \quad x = 30$$

Let's start: Practising with fractions

- If x is a number greater than 1, evaluate these expressions and put them into ascending order (smallest to largest).

$$\frac{2x+1}{2} \qquad 2\left(\frac{x}{2}+1\right) \qquad \frac{2}{x+1} \qquad \frac{2+2x}{2} \qquad 2\left(x+\frac{1}{2}\right)$$

- Investigate how the order might change if x could be a number less than 1.

Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a fraction on one side, multiply both sides by the denominator. For example:

$$\frac{q}{4} = 12 \quad (\times 4 \text{ on both sides}) \quad q = 48$$

Example 6 Solving equations with fractions

Solve the following equations algebraically.

a $\frac{4x}{3} = 8$

b $\frac{4y+15}{9} = 3$

c $4 + \frac{5x}{2} = 29$

d $7 - \frac{2x}{3} = 5$

SOLUTION

a

$$\frac{4x}{3} = 8 \quad (\times 3 \text{ on both sides}) \quad 4x = 24 \quad (\div 4 \text{ on both sides}) \quad x = 6$$

EXPLANATION

Multiplying both sides by 3 removes the denominator of 3.

Both sides are divided by 4 to solve the equation.

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b

$$\frac{4y+15}{9}=3$$
$\times 9$ $\times 9$
$$4y+15=27$$
-15 -15
$$4y=12$$
$\div 4$ $\div 4$
$$y=3$$

Multiplying both sides by 9 removes the denominator of 9.

The equation $4y + 15 = 27$ is solved in the usual fashion (subtract 15, divide by 4).

c

$$4+\frac{5x}{2}=29$$
-4 -4
$$\frac{5x}{2}=25$$
$\times 2$ $\times 2$
$$5x=50$$
$\div 5$ $\div 5$
$$x=10$$

We must subtract 4 first because we do not have a fraction by itself on the left-hand side. Once there is a fraction by itself, multiply by the denominator (2).

d

$$7-\frac{2x}{3}=5$$
-7 -7
$$-\frac{2x}{3}=-2$$
$\times -1$ $\times -1$
$$\frac{2x}{3}=2$$
$\times 3$ $\times 3$
$$2x=6$$
$\div 2$ $\div 2$
$$x=3$$

Subtract 7 first to get a fraction. When both sides are negative, multiplying (or dividing) by -1 makes them both positive.

Exercise 7C

1–3 | 2, 3 | —

UNDERSTANDING

1 **a** If $x = 4$ find the value of $\frac{x}{2}+6$.

b If $x = 4$ find the value of $\frac{x+6}{2}$.

c Are $\frac{x}{2}+6$ and $\frac{x+6}{2}$ equivalent expressions?

2 Fill in the missing steps to solve these equations.

a

$$\frac{x}{3}=10$$
$\times 3$ $\times 3$
$$x=__$$

b

$$\frac{m}{5}=2$$
$\times 5$ $\times 5$
$$m=__$$

c

$$\square \quad 11=\frac{q}{2} \quad \square$$
$$__=q$$

d

$$\square \quad \frac{p}{10}=7 \quad \square$$
$$p=__$$

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7C

UNDERSTANDING

3 Match each of these equations with the correct first step to solve it.

a $\frac{x}{4} = 7$ **b** $\frac{x-4}{2} = 5$ **c** $\frac{x}{2} - 4 = 7$ **d** $\frac{x}{4} + 4 = 3$

A Multiply both sides by 2.
B Add 4 to both sides.
C Multiply both sides by 4.
D Subtract 4 from both sides.

4–5(½) 4–5(½) 4–5(½)

FLUENCY

Example 6a

4 Solve the following equations algebraically.

a $\frac{b}{5} = 4$ **b** $\frac{g}{10} = 2$ **c** $\frac{a}{5} = 3$ **d** $\frac{k}{6} = 3$

e $\frac{2l}{5} = 8$ **f** $\frac{7w}{10} = -7$ **g** $\frac{3s}{2} = -9$ **h** $\frac{5v}{4} = 15$

i $\frac{3m}{7} = 6$ **j** $\frac{2n}{7} = 4$ **k** $\frac{-7j}{5} = 7$ **l** $\frac{-6f}{5} = -24$

Example 6b
Example 6c
Example 6d

5 Solve the following equations algebraically. Check your solutions by substituting.

a $\frac{t-8}{2} = -10$ **b** $\frac{h+10}{3} = 4$ **c** $\frac{a+12}{5} = 2$ **d** $\frac{c-7}{2} = -5$

e $-1 = \frac{s-2}{8}$ **f** $\frac{5j+6}{8} = 2$ **g** $3 = \frac{7v}{12} + 10$ **h** $\frac{4n}{9} - 6 = -2$

i $\frac{7q+12}{5} = -6$ **j** $-4 = \frac{f-15}{3}$ **k** $15 = \frac{3-12l}{5}$ **l** $9 - \frac{4r}{7} = 5$

m $-6 = \frac{5x-8}{-7}$ **n** $\frac{5u-7}{-4} = -2$ **o** $\frac{5k+4}{-8} = -3$ **p** $20 = \frac{3+13b}{-7}$

q $\frac{7m}{12} - 12 = -5$ **r** $4 + \frac{-7y}{8} = -3$ **s** $4 = \frac{p-15}{-3}$ **t** $\frac{g-3}{5} = -1$

6 6(½), 7 6(½), 7, 8

PROBLEM-SOLVING

6 For the following puzzles, write an equation and solve it to find the unknown number.

a A number x is divided by 5 and the result is 7.
b Half of y is -12.
c A number p is doubled and then divided by 7. The result is 4.
d Four is added to x. This is halved to get a result of 10.
e x is halved and then 4 is added to get a result of 10.
f A number k is doubled and then 6 is added. This result is halved to obtain -10.

7 The average of two numbers can be found by adding them and then dividing the result by 2.

a If the average of x and 5 is 12, what is x? Solve the equation $\frac{x+5}{2} = 12$ to find out.
b The average of 7 and p is -3. Find p by writing and solving an equation.
c The average of a number and double that number is 18. What is that number?
d The average of $4x$ and 6 is 19. What is the average of $6x$ and 4? (Hint: Find x first.)

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7C

PROBLEM-SOLVING

8 A restaurant bill of $100 is to be paid. Blake puts in one-third of the amount in his wallet, leaving $60 to be paid by the other people at the table.

a Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.

b Solve the equation algebraically, and hence state how much money Blake has in his wallet.



9	9, 10	9, 10(½)

REASONING

9 In solving $\frac{2x}{3} = 10$ we have first been multiplying by the denominator, but we could have written $2\left(\frac{x}{3}\right) = 10$ and divided both sides by 2.

a Solve $2\left(\frac{x}{3}\right) = 10$.

b Is the solution the same as the solution for $\frac{2x}{3} = 10$ if both sides are first multiplied by 3?

c Solve $\frac{147q}{13} = 1470$ by first:

i multiplying both sides by 13 **ii** dividing both sides by 147

d What is one advantage in dividing first rather than multiplying?

e Solve the following equations.

i $\frac{20p}{14} = 40$ **ii** $\frac{13q}{27} = -39$ **iii** $\frac{-4p}{77} = 4$ **iv** $\frac{123r}{17} = 246$

10 To solve an equation with a pronumeral on the denominator we can first multiply both sides by that pronumeral.

$$\frac{30}{x} = 10$$
$$\times x \quad \times x$$
$$30 = 10x$$
$$\div 10 \quad \div 10$$
$$3 = x$$

Use this method to solve the follows equations.

a $\frac{12}{x} = 2$ **b** $\frac{-15}{x} = -5$ **c** $\frac{1}{x} + 3 = 4$

d $4 + \frac{20}{x} = 14$ **e** $\frac{16}{x} + 1 = 3$ **f** $5 = \frac{-10}{x} + 3$

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Fractional solutions — — 11, 12

7C

ENRICHMENT

11 Solve the following equations. Note that the solutions should be given as fractions.

a $\frac{4x+3}{5} = 12$

b $\frac{8+3x}{5} = 6$

c $7 = \frac{x}{4} + \frac{1}{3}$

d $2 = \frac{10-3x}{4}$

12 Recall from Section 5E (Adding and subtracting algebraic fractions) that algebraic fractions can be combined by finding a common denominator, for example:

$$\frac{2x}{3} + \frac{5x}{4} = \frac{8x}{12} + \frac{15x}{12}$$

$$= \frac{23x}{12}$$

Use this simplification to solve the following equations.

a $\frac{2x}{3} + \frac{5x}{4} = 46$

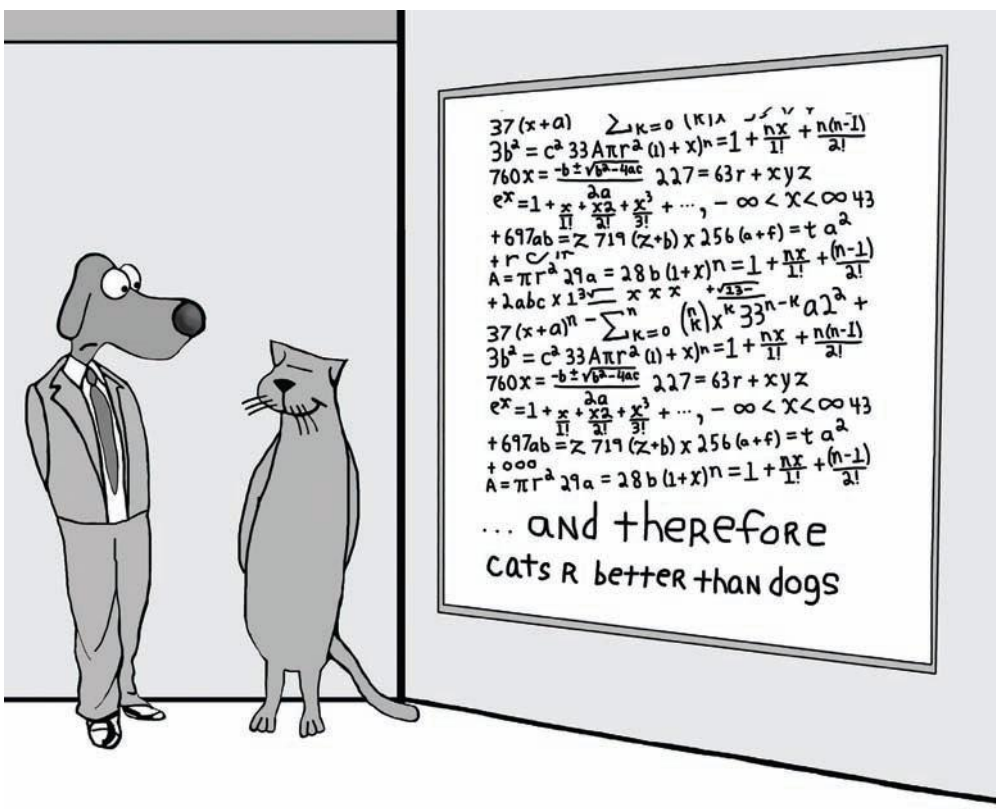
b $\frac{x}{5} + \frac{x}{6} = 22$

c $10 = \frac{x}{2} + \frac{x}{3}$

d $4 = \frac{x}{2} - \frac{x}{3}$

e $\frac{6x}{5} + \frac{2x}{3} = 28$

f $4 = \frac{3x}{7} - \frac{x}{3}$



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7D Equations with pronumerals on both sides

Interactive

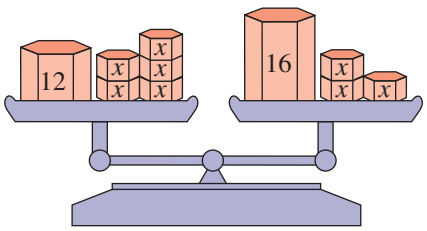
Widgets

HOTsheets

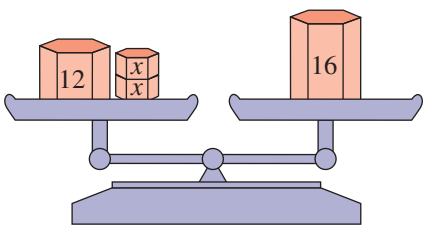
Walkthroughs

So far all the equations we have considered involved a pronumeral either on the left-hand side, for example, $2x + 3 = 11$ or on the right side, for example, $15 = 10 - 2x$. But how can you solve an equation with pronumerals on both sides, for example, $12 + 5x = 16 + 3x$? The idea is to look for an equivalent equation with pronumerals on just one side.

The equation $12 + 5x = 16 + 3x$ can be thought of as balancing scales.



Then $3x$ can be removed from both sides of this equation to get



The equation $12 + 2x = 16$ is straightforward to solve.

Let's start: Moving pronumerals

You are given the equation $11 + 5x = 7 + 3x$.

- Can you find an equivalent equation with x just on the left-hand side?
- Can you find an equivalent equation with x just on the right-hand side?
- Try to find an equivalent equation with $9x$ on the left-hand side.
- Do all of these equations have the same solution? Try to find it.



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Key ideas

- If both sides of an equation have the same term added or subtracted, the new equation will be equivalent to the original equation.
- If pronumerals are on both sides of an equation, add or subtract terms so that the pronumeral appears only one side.

For example:

$$10 + 5a = 13 + 2a \quad (-2a, -2a) \quad 10 + 3a = 13$$

$$4b + 12 = 89 - 3b \quad (+3b, +3b) \quad 7b + 12 = 89$$

Example 7 Solving equations with pronumerals on both sides

Solve the following equations and check your solutions using substitution.

a $7t + 4 = 5t + 10$ **b** $6x + 4 = 22 - 3x$ **c** $2u = 7u - 20$

SOLUTION

a

$$7t + 4 = 5t + 10 \quad (-5t, -5t)$$
$$2t + 4 = 10 \quad (-4, -4)$$
$$2t = 6 \quad (\div 2, \div 2)$$
$$t = 3$$

EXPLANATION

Pronumerals are on both sides of the equation, so subtract $5t$ from both sides.

Once $5t$ is subtracted, the usual procedure is applied for solving equations.

LHS $= 7(3) + 4$ RHS $= 5(3) + 10$
$= 25$ $= 25$ ✓

b

$$6x + 4 = 22 - 3x \quad (+3x, +3x)$$
$$9x + 4 = 22 \quad (-4, -4)$$
$$9x = 18 \quad (\div 9, \div 9)$$
$$x = 2$$

Pronumerals are on both sides. To get rid of $3x$, we add $3x$ to both sides of the equation. Alternatively, $6x$ could have been subtracted from both sides of the equation to get $4 = 22 - 9x$.

LHS $= 6(2) + 4$ RHS $= 22 - 3(2)$
$= 16$ $= 16$ ✓

c

$$2u = 7u - 20 \quad (-2u, -2u)$$
$$0 = 5u - 20 \quad (+20, +20)$$
$$20 = 5u \quad (\div 5, \div 5)$$
$$4 = u$$
$$\therefore u = 4$$

Choose to get rid of $2u$ by subtracting it.

Note that $2u - 2u$ is equal to 0, so the LHS of the new equation is 0.

LHS $= 2(4)$ RHS $= 7(4) - 20$
$= 8$ $= 8$ ✓

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Exercise 7D

1–3 | 2, 3 | —

UNDERSTANDING

1 If $x = 3$ are the following equations true or false?

a $5 + 2x = 4x - 1$ **b** $7x = 6x + 5$

c $2 + 8x = 12x$ **d** $9x - 7 = 3x + 11$

2 Fill in the blanks for these equivalent equations.

a $5x + 3 = 2x + 8$

$-2x$ ( ) $-2x$

$___ = 8$

b $9q + 5 = 12q + 21$

$-9q$ ( ) $-9q$

$___ = 3q + 21$

c $3p + 9 = 5 - 2p$

$+2p$ ( ) $+2p$

$___ = ___$

d $15k + 12 = 13 - 7k$

$+7k$ ( ) $+7k$

$___ = ___$

3 To solve the equation $12x + 2 = 8x + 16$, which one of the following first steps will ensure that x is only on one side of the equation?

A Subtract 2 **B** Subtract $8x$ **C** Add $12x$

D Subtract 16 **E** Add $20x$

4–5(½) | 4–6(½) | 5–7(½)

FLUENCY

Example 7a

4 Solve the following equations algebraically. Check your solution using substitution.

a $10f + 3 = 23 + 6f$ **b** $10y + 5 = 26 + 3y$ **c** $7s + 7 = 19 + 3s$

d $9j + 4 = 4j + 14$ **e** $2t + 8 = 8t + 20$ **f** $4 + 3n = 10n + 39$

g $4 + 8y = 10y + 14$ **h** $5 + 3t = 6t + 17$ **i** $7 + 5q = 19 + 9q$

Example 7b

5 Solve the following equations algebraically.

a $9 + 4t = 7t + 15$ **b** $2c - 2 = 4c - 6$ **c** $6t - 3 = 7t - 8$

d $7z - 1 = 8z - 4$ **e** $8t - 24 = 2t - 6$ **f** $2q - 5 = 3q - 3$

g $5x + 8 = 6x - 1$ **h** $8w - 15 = 6w + 3$ **i** $6j + 4 = 5j - 1$

Example 7c

6 Solve the following equations algebraically.

a $1 - 4a = 7 - 6a$ **b** $6 - 7g = 2 - 5g$ **c** $12 - 8n = 8 - 10n$

d $2 + 8u = 37 + 3u$ **e** $21 - 3h = 6 - 6h$ **f** $37 - 4j = 7 - 10j$

g $13 - 7c = 8c - 2$ **h** $10 + 4n = 4 - 2n$ **i** $10a + 32 = 2a$

j $10v + 14 = 8v$ **k** $18 + 8c = 2c$ **l** $2t + 7 = 22 - 3t$

m $6n - 47 = 9 - 8n$ **n** $3n = 15 + 8n$ **o** $38 - 10l = 10 + 4l$

7 Solve the following equation, giving your solutions as improper fractions where necessary.

a $3x + 5 = x + 6$ **b** $5k - 2 = 2k$ **c** $3 + m = 6 + 3m$

d $9j + 4 = 5j + 14$ **e** $3 - j = 4 + j$ **f** $2z + 3 = 4z - 8$

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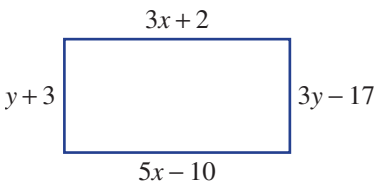
PROBLEM-SOLVING

8 Write an equation and solve it algebraically to find the unknown number in these problems.

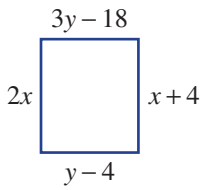
- **a** Doubling x and adding 3 is the same as tripling x and adding 1.
- **b** If z is increased by 9, this is the same as doubling the value of z.
- **c** The product of 7 and y is the same as the sum of y and 12.
- **d** When a number is increased by 10 this has the same effect as tripling the number and subtracting 6.

9 Find the value of x and y in the following rectangles.

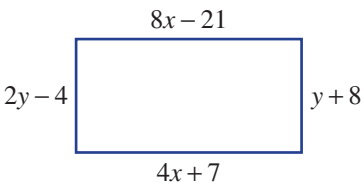
a



b



10 Find the area and the perimeter of this rectangle.



11 At a newsagency, Preeta bought 4 pens and a $1.50 newspaper, while her husband Levy bought 2 pens and a $4.90 magazine. To their surprise the cost was the same.

- **a** Write an equation to describe this, using p for the cost of a single pen in dollars.
- **b** Solve the equation to find the cost of pens.
- **c** If Fred has a $20 note, what is the maximum number of pens that he can purchase?



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7D

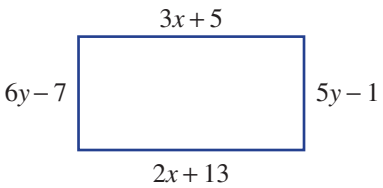
12 | 12, 13 | 13, 14

REASONING

12 To solve the equation $12 + 3x = 5x + 2$ you can first subtract $3x$ or subtract $5x$.

- **a** Solve the equation above by first subtracting $3x$.
- **b** Solve the equation above by first subtracting $5x$.
- **c** What is the difference between the two methods?

13 Prove that the rectangular shape shown to the right must be a square. (Hint: First find the values of x and y.)



14 **a** Try to solve the equation $4x + 3 = 10 + 4x$.

- **b** This tells you that the equation you are trying to solve has no solutions (because $10 = 3$ is never true). Prove that $2x + 3 = 7 + 2x$ has no solutions.
- **c** Give an example of another equation that has no solutions.

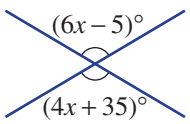
Geometric equations

— | — | 15

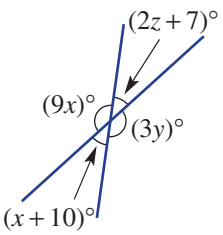
ENRICHMENT

15 Find the values of the unknown variables in the following geometric diagrams.

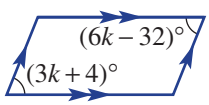
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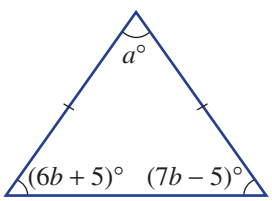
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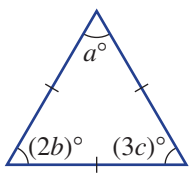
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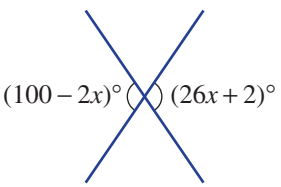
d



e



f



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7E Equations with brackets

Interactive

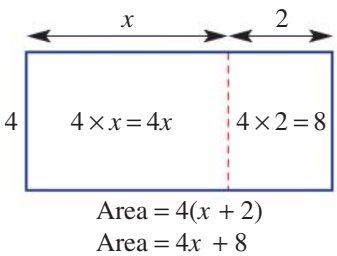
Widgets

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Walkthroughs

In Chapter 5 it was noted that expressions with brackets could be expanded by considering rectangle areas.

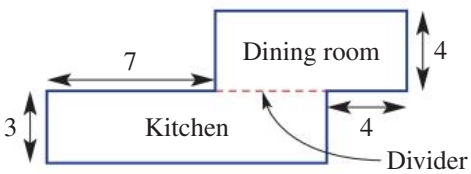
The diagram shows that $4(x+2)$ and $4x+8$ are equivalent. This becomes quite helpful when solving an equation like $4(x+2) = 5x+1$. We just solve $4x+8 = 5x+1$ using the techniques from the previous section.



Let's start: Architect's dilemma

In the following house plans, the kitchen and dining room are separated by a dividing door.

- If the width of the divider is x, what is the area of the kitchen? What is the area of the dining room?
- Try to find the width of the divider if the areas of the two rooms are equal.
- Is it easier to solve $3(7+x) = 4(x+4)$ or $21+3x = 4x+16$? Which of these did you solve when trying to find the width of the divider?



Key ideas

- To expand brackets, use the **distributive law**, which states that
  - $a(b+c) = ab+ac$. For example, $3(x+4) = 3x+12$.
  - $a(b-c) = ab-ac$. For example, $4(b-2) = 4b-8$.
- **Like terms** are terms that contain exactly the same pronumerals and can be collected to simplify expressions. For example, $5x+10+7x$ can be simplified to $12x+10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

Example 8 Solving equations with brackets

Solve the following equations by first expanding any brackets.

a $3(p+4) = 18$ **b** $-12(3q+5) = -132$

c $4(2x-5)+3x = 57$ **d** $2(3k+1) = 5(2k-6)$

SOLUTION

a

$$3(p+4) = 18$$
$$3p+12 = 18 \quad (-12)$$
$$3p = 6 \quad (\div 3)$$
$$p = 2$$

EXPLANATION

Use the distributive law to expand the brackets.

Solve the equation by performing the same operations to both sides.

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b

$$-12(3q+5) = -132$$
$$-36q + (-60) = -132$$
$$-36q - 60 = -132 \quad (+60,\ +60)$$
$$-36q = -72 \quad (\div -36,\ \div -36)$$
$$q = 2$$

Use the distributive law to expand the brackets.
Simplify $-36q + (-60)$ to $-36q - 60$.

Solve the equation by performing the same operations to both sides.

c

$$4(2x-5) + 3x = 57$$
$$8x - 20 + 3x = 57$$
$$11x - 20 = 57 \quad (+20,\ +20)$$
$$11x = 77 \quad (\div 11,\ \div 11)$$
$$x = 7$$

Use the distributive law to expand the brackets.
Combine the like terms: $8x + 3x = 11x$.

Solve the equation by performing the same operations to both sides.

d

$$2(3k+1) = 5(2k-6)$$
$$6k + 2 = 10k - 30 \quad (-6k,\ -6k)$$
$$2 = 4k - 30 \quad (+30,\ +30)$$
$$32 = 4k \quad (\div 4,\ \div 4)$$
$$8 = k$$
$$\therefore\ k = 8$$

Use the distributive law on both sides to expand the brackets.

Solve the equation by performing the same operations to both sides.

Exercise 7E

1–4 | 1, 4 | —

UNDERSTANDING

1 Fill in the missing numbers.

a $4(y+3) = 4y + \square$
b $7(2p-5) = \square p - 35$
c $2(4x+5) = \square x + \square$
d $10(5+3q) = \square + \square q$

2 Match each expression **a**–**d** with its expanded form **A**–**D**.

a $2(x+4)$
b $4(x+2)$
c $2(2x+1)$
d $2(x+2)$

A $4x+8$
B $2x+4$
C $2x+8$
D $4x+2$

3 Rolf is unsure whether $4(x+3)$ is equivalent to $4x+12$ or $4x+3$.

a Fill out the table below.

x	0	1	2
$4(x+3)$			
$4x+12$			
$4x+3$			

b What is the correct expansion of $4(x+3)$?

4 Simplify the following expressions by collecting like terms.

a $4x+3x$
b $7p+2p+3$
c $8x-2x+4$
d $2k+4+5k$
e $3x+6-2x$
f $7k+21-2k$

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7E

5, 6–7(½) | 5, 6–8(½) | 5–9(½)

FLUENCY

Example 8a

5 Solve the following equations by first expanding the brackets.

a $2(4u + 2) = 52$ **b** $3(3j - 4) = 15$ **c** $5(2p - 4) = 40$

d $15 = 5(2m - 5)$ **e** $2(5n + 5) = 60$ **f** $26 = 2(3a + 4)$

Example 8b

6 Solve the following equations involving negative numbers.

a $-6(4p + 4) = 24$ **b** $-2(4u - 5) = 34$ **c** $-2(3v - 4) = 38$

d $28 = -4(3r + 5)$ **e** $-3(2b - 2) = 48$ **f** $-6 = -3(2d - 4)$

Example 8c

7 Solve the following equations by expanding and combining like terms.

a $4(3y + 2) + 2y = 50$ **b** $5(2l - 5) + 3l = 1$ **c** $4(5 + 3w) + 5 = 49$

d $49 = 5(3c + 5) - 3c$ **e** $28 = 4(3d + 3) - 4d$ **f** $58 = 4(2w - 5) + 5w$

g $23 = 4(2p - 3) + 3$ **h** $44 = 5(3k + 2) + 2k$ **i** $49 = 3(2c - 5) + 4$

Example 8d

8 Solve the following equations by expanding brackets on both sides.

a $5(4x - 4) = 5(3x + 3)$ **b** $6(4 + 2r) = 3(5r + 3)$ **c** $5(5f - 2) = 5(3f + 4)$

d $4(4p - 3) = 2(4 + 3p)$ **e** $2(5h + 4) = 3(4 + 3h)$ **f** $4(4r - 5) = 2(5 + 5r)$

g $4(3r - 2) = 4(2r + 3)$ **h** $2(2p + 4) = 2(3p - 2)$ **i** $3(2a + 1) = 11(a - 2)$

9 Solve the following equations algebraically.

a $2(3 + 5r) + 6 = 4(2r + 5) + 6$ **b** $3(2l + 2) + 18 = 4(4l + 3) - 8$

c $2(3x - 5) + 16 = 3 + 5(2x - 5)$ **d** $3(4s + 3) - 3 = 3(3s + 5) + 15$

e $4(4y + 5) - 4 = 6(3y - 3) + 20$ **f** $3(4h + 5) + 2 = 14 + 3(5h - 2)$

10, 11 | 11, 12 | 11–13

PROBLEM-SOLVING

10 Desmond notes that in 4 years' time his age when doubled will give the number 50. Desmond's current age is d years old.

a Write an expression for Desmond's age in 4 years' time.

b Write an expression for double his age in 4 years' time.

c Write an equation to describe the situation described above.

d Solve the equation to find his current age.

11 Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then 3 more hours at an increased wage of $\$(w + 4)$.

a Write an expression for the total amount Rahda earns for the 8 hours.

b Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.



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7E

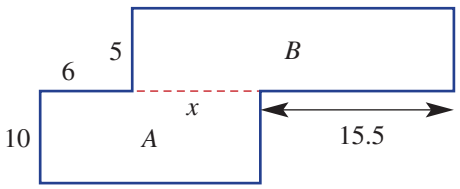
PROBLEM-SOLVING

12 Kate's age 5 years ago, when doubled, is equal to triple her age 10 years ago.

- **a** Write an equation to describe this, using k for Kate's current age.
- **b** Solve the equation to find Kate's current age.

13 Rectangles A and B have the same area.

- **a** What is the value of x?
- **b** State the perimeter of the shape shown at right.



REASONING

14	14,15	15,16

14 Abraham is asked how many people are in the room next door. He answers that if three more people walked in and then the room's population was doubled, this would have the same effect as quadrupling the population and then 11 people leaving. Explain why what Abraham said cannot be true.

15 Ajith claims that three times his age 5 years ago is the same as nine times how old he will be next year. Prove that what Ajith is saying cannot be true.

16 A common mistake when expanding is to write $2(n + 3)$ as $2n + 3$. These are not equivalent, since, for example, $2(5 + 3) = 16$ and $2 \times 5 + 3 = 13$.

- **a** Prove that they are never equal by trying to solve $2(n + 3) = 2n + 3$.
- **b** Prove that $4(2x + 3)$ is never equal to $8x + 3$ but it is sometimes equal to $4x + 12$.

ENRICHMENT

Challenging expansions	—	—	17

17 Solve the following equations. Note that your answers might not be integers.

- **a** $2(3x + 4) + 5(6x + 7) = 64x + 1$
- **b** $-5(3p + 2) + 5(2p + 3) = -31$
- **c** $-10(n + 1) + 20(2n + 13) = 7$
- **d** $4(2q + 1) - 5(3q + 1) = 11q - 1$
- **e** $x + 2(x + 1) + 3(x + 2) = 11x$
- **f** $m - 2(m + 1) - 3(m - 1) = 2(1 - 4m)$

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7F Formulas and relationships

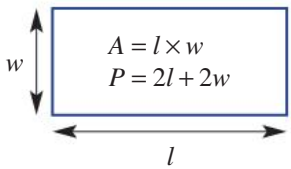
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Walkthroughs

Some equations involve two or more variables that are related. For example, you know from measurement that the area of a rectangle is related to its length and width, given by the formula $A = l \times w$ and its perimeter is given by $P = 2l + 2w$. Although these are often used as a definition for the area and a definition of perimeter, they are also examples of equations – two expressions written on either side of an equal sign.



Let's start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = l \times w$ and $P = 2l + 2w$.

- If $l = 10$ and $w = 7$, find the perimeter and the area.
- If $l = 2$ and $w = 8$, find the perimeter and the area.
- Notice that sometimes the area is bigger than the perimeter and sometimes the area is less than the perimeter. If $l = 10$, is it possible to make the area and the perimeter equal?
- If $l = 2$, can you make the area and the perimeter equal? Discuss.

Key ideas

- The **subject** of an equation is a pronumeral (or variable) that occurs by itself on the left-hand side.
  For example, V is the subject of $V = 3x + 2y$.
- A **formula** or **rule** is an equation containing two or more variables, one of which is the subject of the equation.
- To use a formula, substitute all known values and then solve the equation to find the unknown value.

Example 9 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2l + 2w$ to find:

a P when $l = 4$ and $w = 7$ **b** l when $P = 40$ and $w = 3$

SOLUTION	EXPLANATION
a $P = 2l + 2w$	Write the formula.
$P = 2(4) + 2(7)$	Substitute in the values for l and w.
$P = 22$	Simplify the result.
b $P = 2l + 2w$	Write the formula.
$40 = 2l + 2(3)$	Substitute in the values for P and w to obtain an equation.
$40 = 2l + 6$ -6 -6 $34 = 2l$ $\div 2$ $\div 2$ $17 = l$ $\therefore\ l = 17$	Solve the equation to obtain the value of l.

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Exercise 7F

1–3 | 1, 3 | —

UNDERSTANDING

1 **a** Substitute $x = 4$ into the expression $x + 7$.
b Substitute $a = 2$ into the expression $3a$.
c Substitute $p = 5$ into the expression $2p - 3$.
d Substitute $r = -4$ into the expression $7r$.

2 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?
A $10 = 6 + x$ **B** $10 = 3m + 2$
C $2 = 3m + 10$ **D** $P = 30 + 2$

3 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?
A $12 = 40 + Q$ **B** $L = 40 + 12$
C $12 = 410 + Q$ **D** $10 = 48 + Q$

4–7 | 4–9 | 5–9

FLUENCY

Example 9a

4 Consider the rule $A = 4p + 7$.
a Find A if $p = 3$.
b Find A if $p = 11$.
c Find A if $p = -2$.
d Find A if $p = \frac{13}{2}$.

Example 9b

5 Consider the rule $U = 8a + 4$.
a Find a if $U = 44$. Set up and solve an equation.
b Find a if $U = 92$. Set up and solve an equation.
c If $U = -12$, find the value of a.

6 Consider the relationship $y = 2x + 4$.
a Find y if $x = 3$.
b By solving an appropriate equation, find the value of x that makes $y = 16$.
c Find the value of x if $y = 0$.

7 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

8 Assume that x and y are related by the equation $4x + 3y = 24$.
a If $x = 3$, find y by solving an equation.
b If $x = 0$, find the value of y.
c If $y = 2$, find x by solving an equation.
d If $y = 0$, find the value of x.

9 Consider the formula $G = k(2a + p) + a$.
a If $k = 3$, $a = 7$ and $p = -2$, find the value of G.
b If $G = 78$, $k = 3$ and $p = 5$, find the value of a.

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7F

10 10, 11 11, 12

PROBLEM-SOLVING

10 The cost \$C to hire a taxi for a trip of length d km is $C = 3 + 2d$.

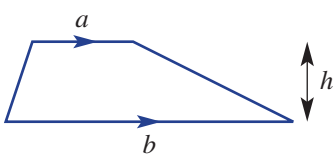


- **a** Find the cost of a 10 km trip (i.e. for $d = 10$).
- **b** A trip has a total cost of \$161.
  - **i** Set up an equation by substituting $C = 161$.
  - **ii** Solve the equation algebraically.
  - **iii** How far did the taxi travel? (Give your answer in km.)

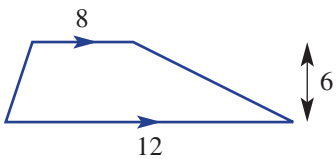
11 To convert temperature between Celsius and Fahrenheit, the rule is $F = 1.8C + 32$.

- **a** Find F if $C = 10$.
- **b** Find C if $F = 95$.
- **c** Vinod's body temperature is 100° Fahrenheit. What temperature is this in degrees Celsius? Answer correct to one decimal place.

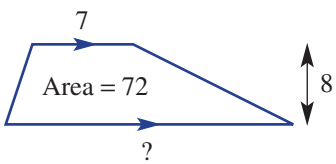
12 The formula for the area of a trapezium is $A = \frac{1}{2}(a + b)h$.



- **a** Find the area of the trapezium shown at right.



- **b** Find the value of h if $A = 20$, $a = 3$ and $b = 7$.
- **c** Find the missing value in the trapezium at right.



13 13, 14 15, 16

REASONING

13 Katy is a scientist who tries to work out the relationship between the volume of a gas, V mL, and its temperature T°C. She makes a few measurements.

V	10	20
T	10	15

- **a** What is a possible rule between V and T?
- **b** Use your rule to find the volume at a temperature of 27°C.
- **c** Prove that the rule $T = \frac{(V-10)^2}{20} + 10$ would also work for Katy's results.

14 Consider the rule $G = 120 - 4p$.

- **a** If p is between 7 and 11, what is the largest value of G?
- **b** If p and G are equal, what value do they have?

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7F

REASONING

15 Marie is a scientist who is trying to discover the relationship between the volume of a gas V, its temperature T and its transparency A. She makes a few measurements.

	Test 1	Test 2
V	10	20
A	2	5
T	15	12

Which one or more of the following rules are consistent with the experiment's results?

A $T = \frac{3V}{A}$ **B** $T = V + 2A$ **C** $T = 17 - A$

16 Temperatures in degrees Fahrenheit and Celsius are related by the rule $F = 1.8C + 32$.

a By substituting $F = x$ and $C = x$, find a value such that the temperature in Fahrenheit and the temperature in Celsius are equal.

b By substituting $F = 2x$ and $C = x$, find a temperature in Celsius that doubles to give the temperature in Fahrenheit.

c Prove that there are no Celsius temperatures that can be multiplied by 1.8 to give the temperature in Fahrenheit.

ENRICHMENT

Mobile phone plans — — 17

17 Two companies have mobile phone plans that factor in the number of minutes spent talking each month (t) and the total number of calls made (c).

Company A's cost in cents: $A = 20t + 15c + 300$
Company B's cost in cents: $B = 30t + 10c$

a In one month 12 calls were made, totalling 50 minutes on the phone. Find the cost in dollars that company A and company B would have charged.

b In another month, a company A user was charged \$15 (1500 cents) for making 20 calls. How long were these calls in total?

c In another month, a company B user talked for 60 minutes in total and was charged \$21. What was the average length of these calls?



d Briony notices one month that for her values of t and c, the two companies cost exactly the same. Find a possible value of t and c that would make this happen.

e Briony reveals that she made exactly 20 calls for the month in which the two companies' charges would be the same. How much time did she spend talking?

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Progress quiz

7A **1** For each of the following equations, state whether they are true (T) or false (F).

a $5 + 11 = 8 \times 2$ **b** $x + 9 = 18 - x$, if $x = 4$ **c** $a \times (a - 4) = 2a$, if $a = 6$

7A **2** State a solution to each of the following equations (no working required).

a $k + 5 = 21$ **b** $7 = c - 6$ **c** $4m = 32$ **d** $10 + t = 3t$

7A **3** Write equations for the following scenarios. You do not need to solve the equations.

a A number n is doubled and then 5 is added. The result is 17.

b Archie's age is a. Archie's mother, who is 26 years older than Archie, is triple Archie's age.

7B **4** Solve the following equations algebraically.

a $a + 8 = 15$ **b** $12 = 9 - k$ **c** $-42 = 6h$

d $5 + 3y = 29$ **e** $4u - 8 = 40$ **f** $52 = 8j - 4$

g $68 - 12d = 8$ **h** $59 = -13 - 9m$

7C **5** Solve the following equations algebraically.

a $\frac{5u}{2} = 100$ **b** $\frac{-3h}{7} = -6$ **c** $3 + \frac{4x}{3} = 15$ **d** $\frac{2w + 7}{3} = 5$

7D **6** Solve the following equations algebraically.

a $4n + 3 = 2n + 17$ **b** $9w - 7 = 4w - 17$ **c** $e + 8 = -28 - 3e$

7E **7** Solve the following equations by first expanding any brackets.

a $6(a + 2) = 42$ **b** $3(4w - 6) = 114$ **c** $5(2q - 1) - 3q = 30$ **d** $-8(2 - p) = 3(2p - 8)$

7F **8** **a** Apply the formula for a rectangle's perimeter $P = 2l + 2w$ to find the length l, when $P = 32$ cm and $w = 6$ cm.

b The rule for the area of a triangle is $A = \frac{bh}{2}$. By solving an appropriate equation, find the base length b, for a triangle of height $h = 3$ cm and area $A = 24$ cm^2.

7B/C **9** For each of the following, write an equation and solve it algebraically to find the unknown number.

a The product of q and -6 is 30.

b Two thirds of a number m gives a result of 12.

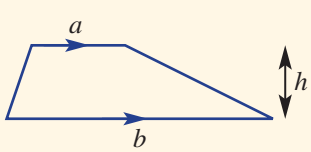
c A number k is tripled and then 4 is added. This result is halved to obtain -13.

d The average of $3x$ and 10 is 14.

7E **10** Maddie's age 8 years ago when multiplied by 5 is the same as triple Maddie's age in 2 years' time. Write and solve an equation to find Maddie's current age.

7F **11** The formula for the area of a trapezium can be written as $A = \frac{h}{2}(a + b)$.

Find the value of the height h, in a trapezium with $A = 162$ cm^2, $a = 12$ cm, $b = 15$ cm.



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7G Applications

Although knowing how to solve equations is useful, it is important to be able to recognise when real-world situations can be thought of as equations. This is the case whenever it is known that two values are equal. In this case, an equation can be constructed and solved. It is important to translate this solution into a meaningful answer within the real-world context.

Let's start: Sibling sum

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 42, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if x is used for John's age?
- How would the equation change if x is used for John's sister's age instead?



The difference in two people's ages can be expressed as an equation.

Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.
  1. Define pronumerals to stand for unknown numbers (e.g. let j = John's current age).
  2. Write an equation to describe the problem.
  3. Solve the equation algebraically if possible, or by inspection.
  4. Ensure you answer the original question, including the correct units (e.g. dollars, years, cm).
- Your final solution should be checked to see if it solves the problem correctly.

Example 10 Solving a problem using equations

The weight of 6 identical books is 1.2 kg. What is the weight of one book?

SOLUTION	EXPLANATION
Let b = weight of one book in kg. $6b = 1.2$	Define a pronumeral to stand for the unknown number. Write an equation to describe the situation.
$\div 6$ ($6b = 1.2$) $\div 6$ $b = 0.2$	Solve the equation.
The books weigh 0.2 kg each, or 200 g each.	Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number.

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Example 11 Solving a harder problem using equations

Purchasing 5 apples and a $2.40 mango costs the same as purchasing 7 apples and a mandarin that costs 60 cents. What is the cost of each apple?

SOLUTION	EXPLANATION
Let c = cost of one apple in dollars.	Define a pronumeral to stand for the unknown number.
$5c + 2.4 = 7c + 0.6$	Write an equation to describe the situation. Note that 60 cents must be converted to $0.6 to keep the units the same throughout the equation.
$5c + 2.4 = 7c + 0.6$ $-5c$ $-5c$ $2.4 = 2c + 0.6$ -0.6 -0.6 $1.8 = 2c$ $\div 2$ $\div 2$ $0.9 = c$	Solve the equation.
Apples cost 90 cents each.	Answer the original question. It is not enough to give a final answer as 0.9; this is not the cost of an apple, it is just a number.

Example 12 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

SOLUTION	EXPLANATION
Let l = Luke's age	Define a pronumeral for the unknown value. Once Luke's age is found, we can double it to find Jane's age.
$l + 2l = 60$	Write an equation to describe the situation. Note that Jane's age is $2l$ because she is twice as old as Luke.
$3l = 60$ $\div 3$ $\div 3$ $l = 20$	Solve the equation by first combining like terms.
Luke is 20 years old and Jane is 40 years old.	Answer the original question.

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Exercise 7G

1–3 | 2 | —

UNDERSTANDING

1 Match each of these worded descriptions with an appropriate expression.

a	The sum of x and 3	**A**	$2x$
b	The cost of 2 apples if they cost \$$x$ each	**B**	$x + 1$
c	The cost of x oranges if they cost \$1.50 each	**C**	$3x$
d	Triple the value of x	**D**	$x + 3$
e	One more than x	**E**	$1.5x$

2 For the following problems choose the equation to describe them.

a The sum of x and 5 is 11.

A $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$

b The cost of 4 pens is \$12. Each pen costs \$$p$.

A $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$

c Josh's age next year is 10. His current age is j.

A $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$

d The cost of n pencils is \$10. Each pencil costs \$2.

A $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10.$

3 Solve the following equations.

a $5p = 30$ **b** $5 + 2x = 23$ **c** $12k - 7 = 41$ **d** $10 = 3a + 1$

4–6 | 4–7 | 5–7

FLUENCY

Example 10

4 Jerry buys 4 cups of coffee for \$13.20.

a Choose a pronumeral to stand for the cost of one cup of coffee.

b Write an equation to describe the problem.

c Solve the equation algebraically.

d Hence state the cost of one cup of coffee.

5 A combination of 6 chairs and a table costs \$3000. The table alone costs \$1740.

a Define a pronumeral for the cost of one chair.

b Write an equation to describe the problem.

c Solve the equation algebraically.

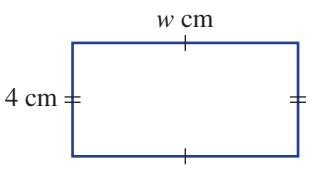
d Hence state the cost of one chair.

6 The perimeter of this rectangle is 72 cm.

a Write an equation to describe the problem, using w for the width.

b Solve the equation algebraically.

c Hence state the width of the rectangle.



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7G

FLUENCY

7 A plumber charges a $70 call-out fee and $52 per hour. The total cost of a particular visit was $252.



- **a** Define a variable to stand for the length of the visit in hours.
- **b** Write an equation to describe the problem.
- **c** Solve the equation algebraically.
- **d** State the length of the plumber's visit, giving your answer in minutes.

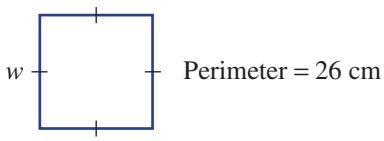
9	8–11	11–13

PROBLEM-SOLVING

Example 11

8 A number is tripled, then 2 is added. This gives the same result as if the number were quadrupled. Set up and solve an equation to find the original number.

9 A square has a perimeter of 26 cm.

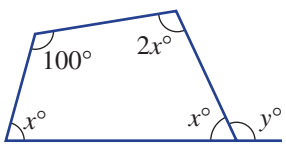


- **a** Solve an equation to find its width.
- **b** Hence state the area of the square.

Example 12

10 Alison and Flynn's combined age is 40. Given that Flynn is 4 years older than Alison, write an equation and use it to find Alison's age.

11 Recall that in a quadrilateral the sum of all angles is 360°. Find the values of x and y in the diagram shown.



12 The sum of three consecutive numbers is 357.

- **a** Use an equation to find the smallest of the three numbers.
- **b** What is the average of these three numbers?
- **c** If the sum of three consecutive numbers is 38 064, what is their average?

13 The width of a rectangular pool is 5 metres longer than the length. The perimeter of the pool is 58 metres.



- **a** Draw a diagram of this situation.
- **b** Use an equation to find the pool's length.
- **c** Hence state the area of the pool.

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7G

14 | 14 | 14, 15

REASONING

14 The average of two numbers can be found by adding them and dividing by 2.

- **a** If the average of x and 10 is 30, what is the value of x?
- **b** If the average of x and 10 is 2, what is the value of x?
- **c** If the average of x and 10 is some number R, create a formula for the value of x.

15 Sometimes you are given an equation to solve a puzzle, but the solution of the equation is not actually possible for the situation. Consider these five equations.

A $10x = 50$ **B** $8 + x = 10$ **C** $10 + x = 8$ **D** $10x = 8$ **E** $3x + 5 = x + 5$

- **a** You are told that the number of people in a room can be determined by solving an equation. Which of these equations could be used to give a reasonable answer?
- **b** If the length of an insect is given by the variable x cm, which of the equations could be solved to give a reasonable value of x?
- **c** Explain why equation **D** could not be used to find the number of people in a room but could be used to find the length of an insect.
- **d** Give an example of a puzzle that would make equation **C** reasonable.

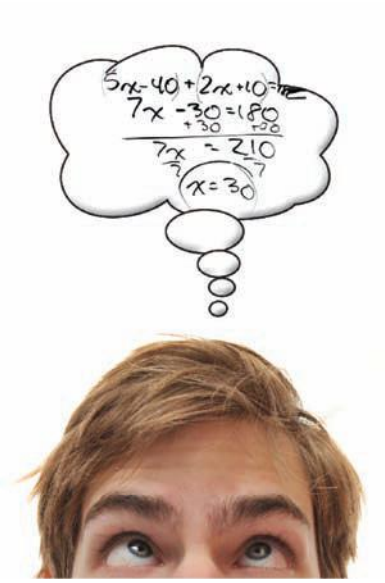
Unknown numbers

— | — | 16

ENRICHMENT

16 Find the unknown number using equations. The answers might not be whole numbers.

- **a** The average of a number and double the number is 25.5.
- **b** Adding 3 to twice a number is the same as subtracting 9 from half the number.
- **c** The average of a number and double the number gives the same result as adding one to the original number and then multiplying by one-third.
- **d** The product of 5 and a number is the same as the sum of four and twice the original number.
- **e** The average of 5 numbers is 7. When one more number is added the average becomes 10. What number was added?



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7H Inequalities EXTENDING



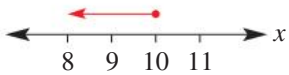






An **inequality** is like an equation but, instead of indicating that two expressions are equal, it indicates which of the two has a greater value. For example, $2 + 4 < 7$, $3 \times 5 \geq 15$ and $x \leq 10$ are all inequalities. The first two are true, and the last one could be true or false depending on the value of x. For instance, the numbers 9.8, 8.45, 7 and -120 all make this inequality true.

We could represent all the values of x that make $x \leq 10$ a true statement.



Let's start: Small sums

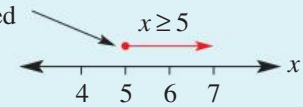
Two positive whole numbers are chosen: x and y. You are told that $x + y \leq 5$.

- How many possible pairs of numbers make this true? For example, $x = 2$ and $y = 1$ is one pair and it is different from $x = 1$ and $y = 2$.
- If $x + y \leq 10$, how many pairs are possible? Try to find a pattern rather than listing them all.
- If all you know about x and y is that $x + y > 10$, how many pairs of numbers could there be? Explain.

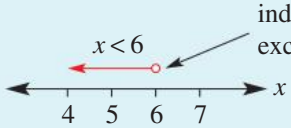
Key ideas

- An **inequality** is a statement of the form:
  - LHS > RHS (greater than). For example, $5 > 2$
  - LHS ≥ RHS (greater than or equal). For example, $7 \geq 7$ or $10 \geq 7$
  - LHS < RHS (less than). For example, $2 < 10$
  - LHS ≤ RHS (less than or equal). For example, $5 \leq 5$ or $2 \leq 5$
- Inequalities can be reversed: $3 < x$ and $x > 3$ are equivalent.
- Inequalities can be represented on a number line, using closed circles at the end points if the value is included, or open circles if it is excluded.

Closed circle indicates 5 is included



Open circle indicates 6 is excluded



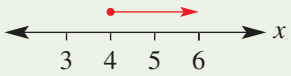
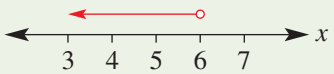
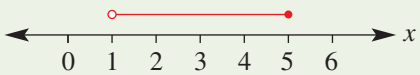
- A range can be represented as a segment on the number line using appropriate closed and open end points.

Example 13 Representing inequalities on a number line

Represent the following inequalities on a number line.

a $x \geq 4$ **b** $x < 6$ **c** $1 < x \leq 5$

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SOLUTION	EXPLANATION
a 	A circle is placed at 4 and then the arrow points to the right, towards all numbers greater than 4. The circle is filled (closed) because 4 is included in the set.
b 	A circle is placed at 6 and then the arrow points to the left, towards all numbers less than 6. The circle is hollow (open) because 6 is not included in the set.
c 	Circles are placed at 1 and 5, and a line goes between them to indicate that all numbers in between are included. The circle at 1 is open because the inequality is $<$ not $\leq$.

Example 14 Using inequalities to describe real-life situations

Describe the following situations as an inequality, using x to stand for the unknown quantity.

a Fred is shorter than 160 cm.

b John is at least as old as Maria, who is 10.

c Rose's test score is between 40 and 50 inclusive.

SOLUTION	EXPLANATION
a $x < 160$	Using x to stand for Fred's height in cm, x must be less than 160.
b $x \geq 10$	John is at least 10, so his age is greater than or equal to 10.
c $40 \leq x \leq 50$	x is between 40 and 50. The word 'inclusive' tells us that 40 and 50 are both included, so $\leq$ is used (rather than $<$ if the word 'exclusive' is used).

Exercise 7H

1–4 | 4 | —

UNDERSTANDING

1 Classify the following statements as true or false.

a $5 > 3$ **b** $7 < 5$ **c** $2 < 12$

d $13 < 13$ **e** $13 \leq 13$ **f** $4 \geq 2$

2 Match each of these inequalities with the appropriate description.

a $x > 5$ **b** $x < 5$ **c** $x \geq 3$ **d** $x \leq 3$

A The number x is less than 5.

B The number x is greater than or equal to 3.

C The number x is less than or equal to 3.

D The number x is greater than 5.

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7H

UNDERSTANDING

3 For each of the following, state whether they make the inequality $x > 4$ true or false.

a $x = 5$ **b** $x = -2$ **c** $x = 4$ **d** $x = 27$

4 If $x = 12$, classify the following inequalities as true or false.

a $x > 2$ **b** $x < 11$ **c** $x \geq 13$ **d** $x \leq 12$

5(½), 6 | 5(½), 6, 7(½) | 5(½), 6, 7(½)

FLUENCY

Example 13a, Example 13b

5 Represent the following inequalities on separate number lines.

a $x > 3$ **b** $x < 10$ **c** $x \geq 2$ **d** $x < 1$

e $x \geq 1$ **f** $x \geq 4$ **g** $x < -5$ **h** $x < -9$

i $x \leq 2$ **j** $x < -6$ **k** $x \geq -3$ **l** $x \leq 5$

m $10 > x$ **n** $2 < x$ **o** $5 \geq x$ **p** $-3 \leq x$

Example 13c

6 **a** List which of the following numbers make the inequality $2 \leq x < 7$ true.

8, 1, 3, 4, 6, 4.5, 5, 2.1, 7, 6.8, 2

b Represent the inequality $2 \leq x < 7$ on a number line.

7 Represent the following inequalities on separate number lines.

a $1 \leq x \leq 6$ **b** $4 \leq x < 11$ **c** $-2 < x \leq 6$ **d** $-8 \leq x \leq 3$

e $2 < x \leq 5$ **f** $-8 < x < -1$ **g** $7 < x \leq 8$ **h** $0 < x < 1$

8 | 8, 10 | 9, 10

PROBLEM-SOLVING

Example 14

8 For each of the following descriptions, choose an appropriate inequality from **A**–**H** below.

a John is more than 12 years old.

b Marika is shorter than 150 cm.

c Matthew is at least 5 years old but he is younger than 10.

d The temperature outside is between −12°C and 10°C inclusive.

A $x < 150$ **B** $x < 12$ **C** $x > 12$ **D** $x \leq 150$

E $10 \leq x \leq -12$ **F** $-12 \leq x \leq 10$ **G** $5 \leq x < 10$ **H** $5 < x \leq 10$

9 It is known that Tim's age is between 20 and 25 inclusive, and Nick's age is between 23 and 27 inclusive.



a If t = Tim's age and n = Nick's age, write two inequalities to represent these facts.

b Represent both inequalities on the same number line.

c Nick and Tim are twins. What is the possible range of their ages? Represent this on a number line.

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7H

PROBLEM-SOLVING

10 At a certain school the following grades are awarded for different scores.

Score	$x \geq 80$	$60 \leq x < 80$	$40 \leq x < 60$	$20 \leq x < 40$	$x < 20$
Grade	A	B	C	D	E

a Convert the following scores into grades.
i 15 **ii** 79 **iii** 80 **iv** 60 **v** 30

b Emma got a B on one test, but her sister Rebecca got an A with just 7 more marks. What is the possible range for Emma's score?

c Hugh's mark earned him a C. If he had scored half this mark, what grade would he have earned?

d Alfred and Reuben earned a D and a C respectively. If their scores were added together, what grade or grades could they earn?

e Michael earned a D and was told that if he doubled his mark he would have a B. What grade or grades could he earn if he got an extra 10 marks?

	11a, b	11	11, 12(½)

REASONING

11 Sometimes multiple inequalities can be combined to a simpler inequality.

a Explain why the combination $x \geq 5, x \geq 7$ is equivalent to the inequality $x \geq 7$.

b Simplify the following pairs of inequalities to a single inequality.
i $x > 5, x \geq 2$ **ii** $x < 7, x < 3$ **iii** $x \geq 1, x > 1$
iv $x \leq 10, x < 10$ **v** $x > 3, x < 10$ **vi** $x > 7, x \leq 10$

c Simplify the following pairs of inequalities to a single inequality.
i $3 < x < 5, 2 < x < 7$ **ii** $-2 \leq x < 4, -2 < x \leq 4$
iii $7 < x \leq 10, 2 \leq x < 8$ **iv** $5 \leq x < 10, 9 \leq x \leq 11$

12 Some inequalities, when combined, have no solutions; some have one solution and some have infinitely many solutions. Label each of the following pairs using 0, 1 or ∞ (infinity) to say how many solutions they have.

a $x \geq 5$ and $x \leq 5$ **b** $x > 3$ and $x < 10$
c $x \geq 3$ and $x < 4$ **d** $x > 3$ and $x < 2$
e $-2 < x < 10$ and $10 < x < 12$ **f** $-3 \leq x \leq 10$ and $10 \leq x \leq 12$
g $x > 2.5$ and $x \leq 3$ **h** $x \geq -5$ and $x \leq -7$

Working within boundaries	—	—	13, 14

ENRICHMENT

13 If it is known that $0 \leq x \leq 10$ and $0 \leq y \leq 10$, which of the following inequalities must be true? Justify your answers.

a $x + y \leq 30$ **b** $2x \leq 20$ **c** $10 \leq 2y \leq 20$
d $x \times y \leq 100$ **e** $0 \leq x - y \leq 10$ **f** $x + 5y \leq 100$

14 If it is known that $0 \leq a \leq 10, 0 \leq b \leq 10$ and $0 \leq c \leq 10$, what is the largest value that the following expressions could have?

a $a + b + c$ **b** $ab + c$ **c** $a(b + c)$ **d** $a \times b \times c$
e $a - b - c$ **f** $a - (b - c)$ **g** $3a + 4$ **h** $a - bc$

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7I Solving inequalities EXTENDING

Interactive

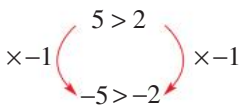
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HOTsheets

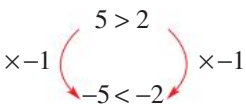
Walkthroughs

Sometimes a problem arises in which an inequality is more complicated than something such as $x > 5$ or $y \leq 40$. For instance, you could have the inequality $2x + 4 > 100$. To **solve** an inequality means to find all the values that make it true. For the inequality above, $x = 50$, $x = 90$ and $x = 10\,000$ are all part of the solution, but the solution is best described as $x > 48$, because any number greater than 48 will make this inequality true and any other number makes it false.

The rules for solving inequalities are very similar to those for equations: perform the same operation to both sides. The one exception occurs when multiplying or dividing by a negative number. We can do this, but we must flip the sign because of the following observation.



Incorrect method



Correct method

Let's start: Limousine costing

A limousine is hired for a wedding. The charge is a $50 hiring fee plus $200 per hour.

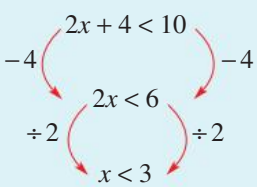
- If the total hire time was more than 3 hours, what can you say about the total cost?
- If the total cost is less than $850 but more than $450, what can you say about the total time the limousine was hired?

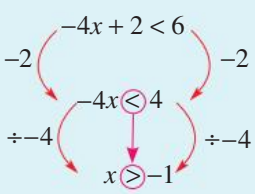


Key ideas

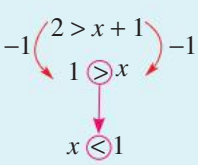
- Given an inequality, an **equivalent** inequality can be obtained by:
  - adding or subtracting an expression from both sides
  - multiplying or dividing both sides by any positive number
  - multiplying or dividing both sides by a negative number and reversing the inequality
  - switching sides and reversing the inequality.

  For example:





Sign is reversed



Sign is reversed

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Example 15 Solving inequalities

Solve the following inequalities.

a $5x + 2 < 47$ **b** $\frac{3 + 4x}{9} \geq 3$ **c** $15 - 2x > 1$

SOLUTION | **EXPLANATION**

a

$5x + 2 < 47$

-2 / -2

$5x < 45$

$\div 5$ / $\div 5$

$x < 9$

The inequality is solved in the same way as an equation is solved: 2 is subtracted from each side and then both sides are divided by 5. The sign does not change throughout.

b

$\frac{3 + 4x}{9} \geq 3$

$\times 9$ / $\times 9$

$3 + 4x \geq 27$

-3 / -3

$4x \geq 24$

$\div 4$ / $\div 4$

$x \geq 6$

The inequality is solved in the same way as an equation is solved. Both sides are multiplied by 9 first to eliminate 9 from the denominator.

c

$15 - 2x > 1$

-15 / -15

$-2x > -14$

$\div -2$ / $\div -2$

$x < 7$

15 is subtracted from each side.

Both sides are divided by -2. Because this is a negative number, the inequality is reversed from $>$ to $<$.

Exercise 7I

1–4 | 4 | —

UNDERSTANDING

1 If $x = 3$, classify the following inequalities as true or false.

a $x + 4 > 2$ **b** $5x \geq 10$ **c** $10 - x < 5$ **d** $5x + 1 < 16$

2 State whether the following choices of x make the inequality $2x + 4 \geq 10$ true or false.

a $x = 5$ **b** $x = 1$ **c** $x = -5$ **d** $x = 3$

3 **a** Copy and complete the following.

$2x < 8$

$\div 2$ / $\div 2$

$x <$ __

b What is the solution to the inequality $2x < 8$?

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7I

UNDERSTANDING

4 **a** Copy and complete the following.

$2x + 4 \geq 10$

-4 -4

$2x \geq __$

$\div 2$ $\div 2$

$x \geq __$

b What is the solution to the inequality $2x + 4 \geq 10$?

c If $x = 7.1328$, is $2x + 4 \geq 10$ true or false?

5–6(½) 5–7(½) 5–7(½), 8

FLUENCY

Example 15a

5 Solve the following inequalities.

a $x + 9 > 12$ **b** $4l + 9 \geq 21$ **c** $8g - 3 > 37$ **d** $2r - 8 \leq 6$

e $9k + 3 > 21$ **f** $8s - 8 < 32$ **g** $8a - 9 > 23$ **h** $2 + n \geq 7$

i $9 + 2d \geq 23$ **j** $8 + 6h < 38$ **k** $10 + 7r \leq 24$ **l** $6 + 5y < 26$

Example 15b

6 Solve the following inequalities involving fractions.

a $\frac{d-9}{2} > 10$ **b** $\frac{y+4}{2} \leq 7$ **c** $\frac{x-3}{4} > 2$ **d** $\frac{q+4}{2} \leq 11$

e $\frac{2x+4}{3} > 6$ **f** $\frac{7+3h}{2} < 5$ **g** $\frac{4+6p}{4} \geq 4$ **h** $\frac{8j+2}{7} < 6$

Example 15c

7 Solve the following inequalities involving negative numbers. Remember to reverse the inequality when multiplying or dividing by a negative number.

a $6 - 2x < 4$ **b** $24 - 6s \geq 12$ **c** $43 - 4n > 23$

d $34 - 2j < 14$ **e** $2 - 9v \leq 20$ **f** $2 - 7j \leq 37$

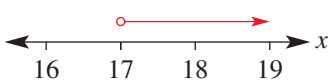
g $48 - 8c \geq 32$ **h** $42 - 8h \leq 42$ **i** $7 - 8s > 31$

j $6 - 8v > 22$ **k** $10 - 4v \geq 18$ **l** $4 - 5v < 29$

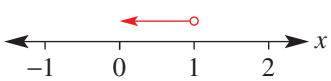
8 Match the following inequalities with their solutions depicted on a number line.

a $5x + 2 \geq 17$ **b** $\frac{x+1}{6} > 3$ **c** $9(x + 4) < 45$ **d** $5 - 2x < 3$

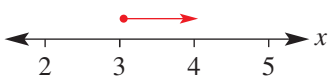
A



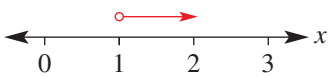
B



C



D



9, 10 9, 10 10, 11

PROBLEM-SOLVING

9 Kartik buys 4 cartons of milk and a $20 phone card. The total cost of his shopping was greater than $25.

a If c is the cost of a carton of milk, write an inequality to describe the situation above.

b Solve the inequality to find the possible values of c

c If the milk's cost is a multiple of 5 cents, what is the minimum price it could be?

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PROBLEM-SOLVING

10 In AFL football the score is given by $6g + b$ where g is the number of goals and b is the number of behinds. A team scored 4 behinds and their score was less than or equal to 36.

- **a** Write an inequality to describe this situation.
- **b** Solve the inequality.
- **c** Given that the number of goals must be a whole number, what is the maximum number of goals that they could have scored?



11 Recall that to convert degrees Celsius to Fahrenheit the rule is $F = 1.8C + 32$. Pippa informs you that the temperature is between 59° and 68° Fahrenheit inclusive.

- **a** Solve $1.8C + 32 \geq 59$.
- **b** Solve $1.8C + 32 \leq 68$.
- **c** Hence state the solution to $59 \leq 1.8C + 32 \leq 68$, giving your answer as a single inequality.
- **d** Pippa later realised that the temperatures she gave you should have been doubled – the range was actually 118° to 136° Fahrenheit. State the range of temperatures in Celsius, giving your answer as an inequality.

	12a, b	12, 14	13–15

REASONING

12 To say that a number x is positive is to say that $x > 0$.

- **a** If $10x - 40$ is positive, find all the possible values of x. That is, solve $10x - 40 > 0$.
- **b** Find all k values that make $2k - 6$ positive.
- **c** If $3a + 6$ is negative and $10 - 2a$ is positive, what are the possible values of a?
- **d** If $5a + 10$ is negative and $10a + 30$ is positive, what are the possible values of a?

13

- **a** Prove that if $5x - 2$ is positive then x is positive.
- **b** Prove that if $2x + 6$ is positive then $x + 5$ is positive.
- **c** Is it possible that $10 - x$ is positive and $10 - 2x$ is positive but $10 - 3x$ is negative? Explain.
- **d** Is it possible that $10 - x$ is positive and $10 - 3x$ is positive but $10 - 2x$ is negative? Explain.

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REASONING

14 A puzzle is given below with four clues.

Clue A: $3x > 12$
Clue B: $5 - x \leq 4$
Clue C: $4x + 2 \leq 42$
Clue D: $3x + 5 < 36$

a Two of the clues are unnecessary. State which two clues are not needed.
b Given that x is a whole number divisible by 4, what is the solution to the puzzle?

15 Multiplying or dividing by a negative number can be avoided by adding the variable to the other side of the equation. For example:

$-4x + 2 < 6$
$+4x$ $+4x$
$2 < 6 + 4x$
-6 -6
$-4 < 4x$
$\div 4$ $\div 4$
$-1 < x$

This can be rearranged to $x > -1$, which is the same as the answer obtained using the method shown in the **Key ideas**. Use this method to solve the following inequalities.

a $-5x + 20 < 10$ **b** $12 - 2a \geq 16$ **c** $10 - 5b > 25$ **d** $12 < -3c$

Pronumerals on both sides — — 16

ENRICHMENT

16 This method for solving inequalities allows both sides to have any expression subtracted from them. This allows us to solve inequalities with pronumerals on both sides. For example:

$12x + 5 \leq 10x + 11$
$-10x$ $-10x$
$2x + 5 \leq 11$

which can then be solved as usual. If we end up with a pronumeral on the right-hand side, such as $5 < x$, the solution is rewritten as $x > 5$.

Solve the following inequalities.

a $12x + 5 \leq 10x + 11$ **b** $7a + 3 > 6a$ **c** $5 - 2b \geq 3b - 35$
d $7c - 5 < 10c - 11$ **e** $14k > 200 + 4k$ **f** $9g + 40 < g$
g $4(2a + 1) > 7a + 12$ **h** $2(3k - 5) \leq 5k - 1$ **i** $2(3p + 1) > 4(p + 2) + 3$

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Investigation

Tiling a pool edge

The Sunny Swimming Pool Company constructs rectangular pools each 4 m wide with various lengths. There are non-slip square tiles, 50 cm by 50 cm, that can be used for the external edging around the pool perimeter where swimmers walk.



1 Draw a neat diagram illustrating the pool edge with one row of flat tiles bordering the perimeter of a rectangular pool 4 m wide and 5 m long.

2 Develop a table showing the dimensions of rectangular pools each of width 4 m and ranging in length from 5 m to 10 m. Add a column for the total number of tiles required for each pool when one row of flat tiles borders the outside edge of the pool.

3 Develop an algebraic rule for the total number of tiles, T, required for a bordering the perimeter of rectangular pools that are 4 m wide and x m long.

4 **a** Use your algebraic rule to form equations for each of the following total number of tiles when a single row of flat tiles is used for pool edging.

i 64 tiles **ii** 72 tiles **iii** 80 tiles **iv** 200 tiles

b By manually solving each equation, determine the lengths of the various pools that use each of the above numbers of tiles.

5 Develop an algebraic rule for the total number of tiles, T, required for two rows of flat tiles bordering rectangular pools that are 4 m wide and x m long.

6 **a** Use your algebraic rule to form equations for each of the following total numbers of tiles when two rows of flat tiles are used for pool edging.

i 96 tiles **ii** 120 tiles **iii** 136 tiles **iv** 248 tiles

b By manually solving each equation, determine the lengths of the pools that use these numbers of tiles.

7 Determine an algebraic rule for the total number of tiles, T, required for n rows of flat tiles bordering rectangular pools that are 4 m wide and x m in length.

8 Use this algebraic rule to form equations for each of the following pools, and then manually solve each equation to determine the length of each pool.

Pool	Width of pool 4 m	Length of pool x m	Number of layers n	Total number of tiles, T
A	4		3	228
B	4		4	288
C	4		5	500

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Hiring a tuk-tuk taxi

The cost of hiring a tuk-tuk taxi in Thailand is 50 baht hiring cost (flag fall) plus 15 baht every 2 km.

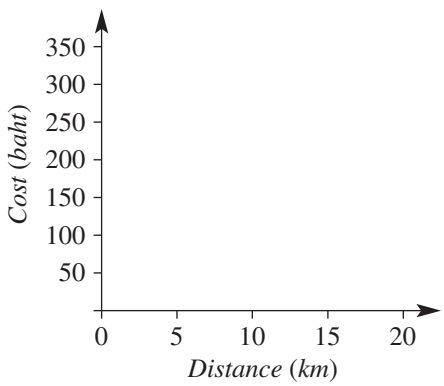


1 Using a table, show the costs of hiring a tuk-tuk for 2, 4, 6, 8, 10 and 20 km.

2 On graph paper, draw a graph of the hiring cost (vertical axis) versus the distance (horizontal axis) travelled. Use the grid at right as a guide.

3 On your graph rule lines to show the distance travelled for hiring costs of 125 baht and 170 baht.

4 Write an algebraic formula for the hiring cost C of an x km ride in a tuk-tuk taxi.



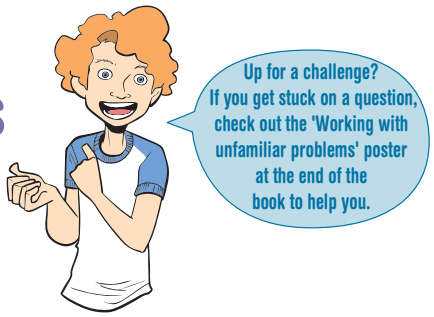
5 **a** Use your algebraic formula to form equations for each of the following trip costs:
i 320 baht **ii** 800 baht **iii** 1070 baht

b Use your equation-solving procedures to calculate the distance travelled for each trip.

6 A tourist was charged A$100 for the 116 km trip from Pattaya Beach to the Bangkok International airport. If 1 baht can be bought for A$0.03318, do you think that this was a fair price? Justify your answer with calculations.

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Problems and challenges



1 Find the unknown value in the following problems.

- **a** A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.
- **b** A number is doubled and then one third of the number is subtracted. The result is 5 larger than the original number.
- **c** In five years' time Alf will be twice as old as he was two years ago. How old is Alf now?
- **d** The price of a shirt is increased by 10% for GST and then decreased by 10% on a sale. The new price is $44. What was the original price?
- **e** One-third of a number is subtracted from 10 and then the result is tripled, giving the original number back again.
- **f** The sides of a quadrilateral are four consecutive integers. If the longest side is 26% of the perimeter, find the perimeter.

2 Consider the following 'proof' that $0 = 1$.

- **a** Which step caused the problem in this proof? (Hint: Consider the actual solution to the equation.)
- **b** Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

$$2x + 5 = 3x + 5$$
$$-5 \qquad\qquad -5$$
$$2x = 3x$$
$$\div x \qquad\qquad \div x$$
$$2 = 3$$
$$-2 \qquad\qquad -2$$
$$0 = 1$$

3 Find all the values of x that would make both these inequalities false.
$19 - 2x < 5$ and $20 + x > 4x + 2$

4 The following six expressions could appear on either side of an equation. Using just two of the expressions, create an equation that has no solution.

$2x$ $\quad 3x + 1$ $\quad 7x + 4$ $\quad 4(x + 7)$ $\quad 2 + 3(x + 1)$ $\quad 2(3 + x) - 1$

5 A certain pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

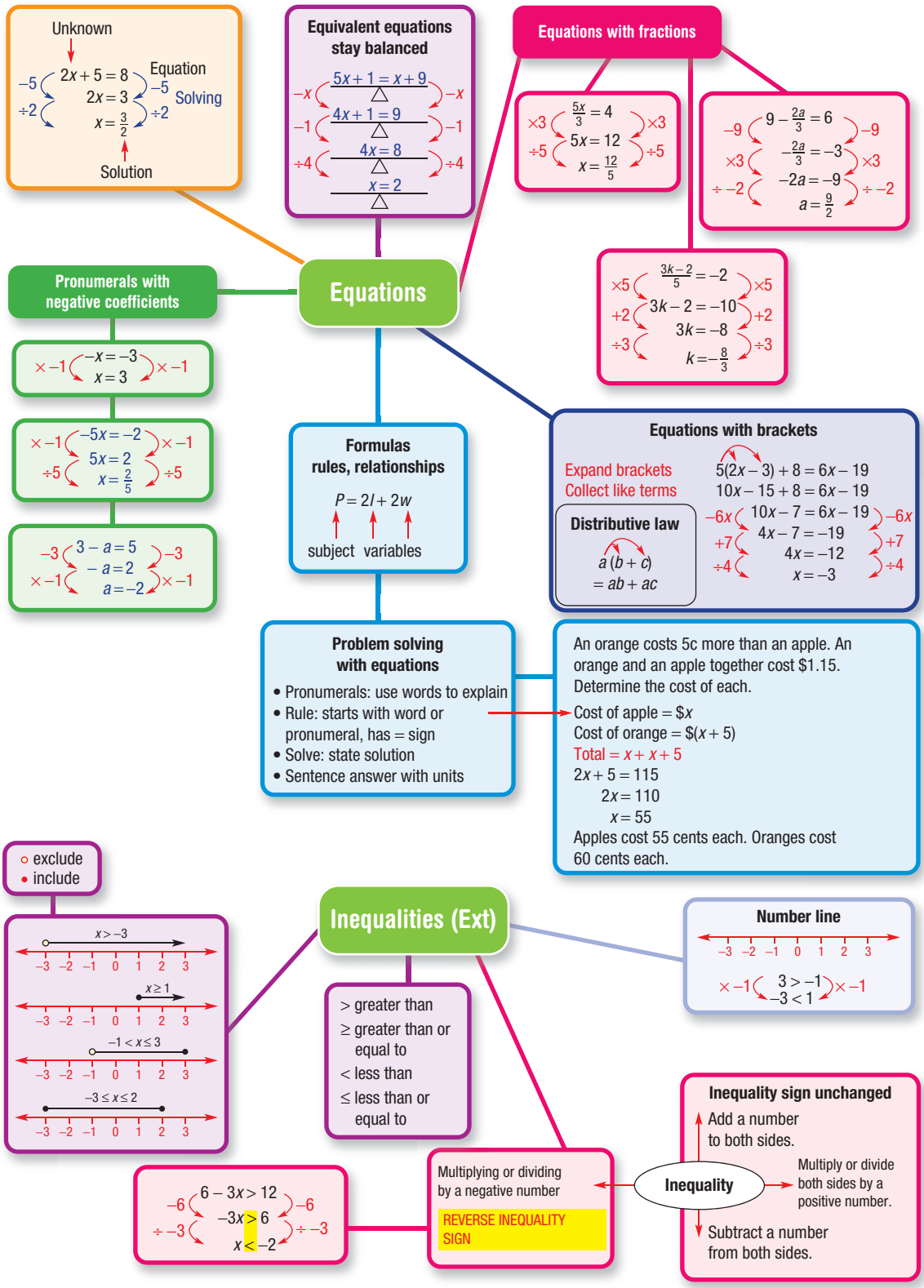
- **a** If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?
- **b** If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?
- **c** A group of four children who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much do they each weigh?

6 Each link in a chain has an outer length of 44 mm and is made of wire 4 mm thick.

- **a** Find the greatest length (in mm) of a chain with 5 links.
- **b** Determine an expression for the greatest length L (in mm) of a chain with n links.
- **c** Find the smallest number of links required to make a chain with length greater than 7 metres.

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Chapter summary



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Chapter review

Multiple-choice questions

7A **1** Which one of the following equations is true?

A $20 \div (2 \times 5) = 20 \div 10$ **B** $12 - 8 = 2$ **C** $5 \div 5 \times 5 = \frac{1}{5}$

D $15 + 5 \times 4 = 20 + 4$ **E** $10 - 2 \times 4 = 4 \times 2 - 5$

7A **2** Which one of the following equations does not have the solution $x = 9$?

A $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$

D $x + 9 = 0$ **E** $14 - x = 5$

7B **3** The solution to the equation $3a + 8 = 29$ is:

A $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

7G **4** 'Three less than half a number is the same as four more than the number' can be expressed as an equation by:

A $\frac{x}{2} - 3 = 4x$ **B** $\frac{(x-3)}{2} = x + 4$ **C** $\frac{x}{2} - 3 = x + 4$

D $\frac{x}{2} + 3 = x + 4$ **E** $\frac{x}{2} - 3 + 4$

7E **5** The solution to the equation $-3(m + 4) = 9$ is:

A $m = 7$ **B** $m = -7$ **C** $m = -1$ **D** $m = 1$ **E** $m = -3$

7D **6** If $12 + 2x = 4x - 6$ then x equals:

A 8 **B** 9 **C** 12 **D** 15 **E** 23

7H **7** If $\square < -4$, then $\square$ could have the value:

Ext **A** 0 **B** $-\frac{1}{4}$ **C** -3 **D** -5 **E** 3

7A **8** Which one of the following equations has the solution $n = 10$?

A $4 - n = 6$ **B** $2n + 4 = 3n + 5$ **C** $50 - 4n = 90$

D $2(n + 5) = 3(n + 1)$ **E** $70 - 6n = n$

7B **9** Malcolm solves an equation as follows:

$5 - 2x + 4 = 11$ line 1

$1 - 2x = 11$ line 2

$-2x = 10$ line 3

$x = -5$ line 4

Choose the correct statement.

A The only mistake was made in moving from line 1 to line 2.

B The only mistake was made in moving from line 2 to line 3.

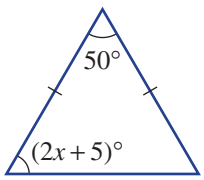
C The only mistake was made in moving from line 3 to line 4.

D A mistake was made in moving from line 1 to line 2 and from line 3 to line 4.

E No mistakes were made.

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7G **10** The value of x in this isosceles triangle is:



A 30 **B** 45 **C** 62.5
D 65 **E** 130

Short-answer questions

7A **1** Classify each of the following as true or false.

a If $3x = 6$ then $x = 3$.
b If $a = 21$ then $a + a - a = a$.
c $5 \times 4 = 10 + 10$

7B **2** Find the solutions to these equations.

a $4m = 16$ **b** $\frac{m}{3} = -4$ **c** $9 - a = 10$
d $10m = 2$ **e** $2m + 6 = 36$ **f** $a + a = 12$

7A **3** Write an equation to represent each of the following statements. You do not need to solve the equations.

a Double m plus 3 equals three times m
b The sum of n and four is multiplied by five; the answer is 20.
c The sum of two consecutive even numbers, the first being x, is 74.

7B **4** For each equation below, state the first operation you would apply to both sides.

a $15 + 2x = 45$ **b** $\frac{x}{2} - 5 = 6$ **c** $3a + 3 = 2a + 10$

7B **5** Solve the following equations.

a $a + 8 = 12$ **b** $6 - y = 15$ **c** $2x - 1 = -9$
d $5 + 3x = 17$ **e** $20 - 4x = 12$ **f** $8a - 8 = 0$

7C **6** Solve the following equations algebraically.

a $\frac{m}{3} = -2$ **b** $\frac{5x}{2} = 20$ **c** $\frac{-2y}{3} = 12$
d $\frac{k+3}{11} = -5$ **e** $\frac{8-2w}{3} = 4$ **f** $\frac{2a-8}{6} = 13$

7C **7** Solve these equations including fractions.

a $2 = \frac{x+2}{8}$ **b** $\frac{x}{6} + 2 = 3$ **c** $\frac{2x-10}{7} = 10$

7D **8** Solve these equations.

a $8a - 3 = 7a + 5$ **b** $2 - 3m = m$ **c** $3x + 5 = 7x - 11$
d $10 - 4a = a$ **e** $2x + 8 = x$ **f** $4x + 9 = 5x$

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Chapter review

7E **9** Solve the following equations by first expanding the brackets.

a $2(x+5)=16$ **b** $3(x+1)=-9$

c $18=-2(2x-1)$ **d** $3(2a+1)=27$

e $5(a+4)=3(a+2)$ **f** $2(3m+5)=16+3(m+2)$

g $8(3-a)+16=64$ **h** $2x+10=4(x-6)$

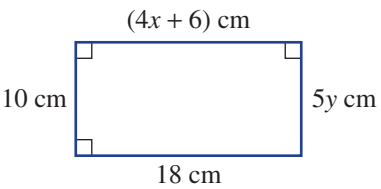
i $4(2x-1)-3(x-2)=10x-3$

7F **10 a** If $P=2(I+b)$, find I when $P=48$ and $b=3$.

b If $M=\dfrac{f}{f-d}$, find M when $f=12$ and $d=8$.

c If $F=2.5c+20$, find c when $F=30$.

d Find the value of x and y for this rectangle.



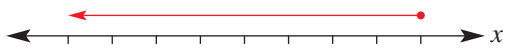
7G **11 a** The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

b Four times a number less 5 is the same as double the number plus 3. Write an equation to find the number.

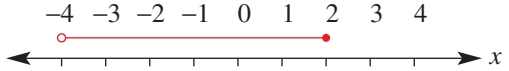
c The difference between a number and three times that number is 17. What is the number?

7H Ext **12** Write the inequality represented by each number line below.

a



b



7H Ext **13** Represent the following inequalities on separate number lines.

a $x \leq -1$ **b** $x > 2$ **c** $x < -1$

d $x \geq -2$ **e** $1 > x$ **f** $-3 \leq x$

g $x < 7$ **h** $-2 < x \leq 4$ **i** $-1 \leq x \leq 1$

7H Ext **14** Write an inequality to represent these situations.

a The profit of a company is at least $100 000.

b The cost of a new car cannot exceed $6700.

c To ride on the roller-coaster, a person's height must be between 1.54 m and 1.9 m inclusive.



7I Ext **15** Solve the following inequalities.

a $x+3>5$ **b** $x-2<6$ **c** $x-2<-6$

d $6x \geq 12$ **e** $4x<-8$ **f** $\dfrac{x}{4} \geq 2$

g $\dfrac{2x}{3}<-8$ **h** $\dfrac{2x+1}{3}>9$ **i** $\dfrac{6-5x}{4} \leq -1$

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Extended-response questions

1 To upload an advertisement to the www.searches.com.au website costs $20 and then 12 cents whenever someone clicks on it.

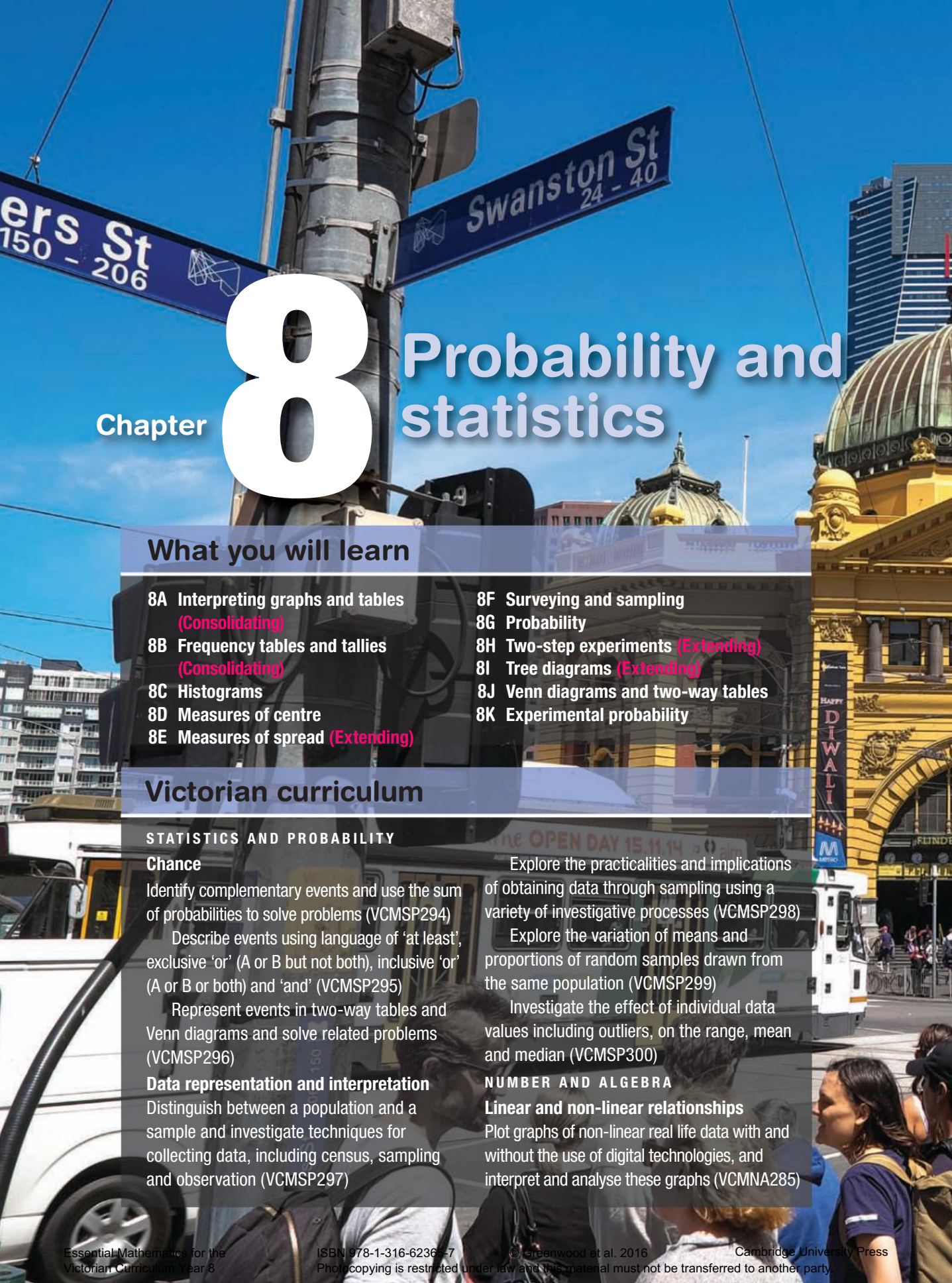
- **a** Write a formula relating the total cost ($\$S$) and the number of clicks (n) on the advertisement.
- **b** If the total cost is $23.60, write and solve an equation to find out how many times the advertisement has been clicked on.
- **c** To upload to the www.yousearch.com.au website costs $15 initially and then 20 cents for every click. Write a formula for the total cost $\$Y$ when the advertisement has been clicked n times.
- **d** If a person has at most $20 to spend, what is the maximum number of clicks they can afford on their advertisement at yousearch.com.au?
- **e** Set up and solve an equation to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to yousearch.com.au.

2 Mahni plans to spend the next 12 weeks saving some of her income. She will save $\$x$ a week for the first 6 weeks and then $\$(2x - 30)$ a week for the following 6 weeks.

- **a** Write an expression for the total amount saved over the 12 weeks.
- **b** If she managed to save $213 in the first six weeks, how much did she save:
  - **i** in the first week?
  - **ii** in the 7th week?
  - **iii** in total over the 12 weeks?
- **c** If Mahni wants to save a total of $270, write and solve an equation to find out how much she would have to save in the first week.
- **d** If Mahni wants to save the same amount in the first 6 weeks as in the last 6 weeks, how much would she save each week?
- **e** In the end Mahni decides that she does not mind exactly how much she saves but wants it to be between $360 and $450. State the range of x values that would achieve this goal, giving your answer as an inequality.



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Chapter 8

Probability and statistics

What you will learn

8A Interpreting graphs and tables (Consolidating)
8B Frequency tables and tallies (Consolidating)
8C Histograms
8D Measures of centre
8E Measures of spread (Extending)
8F Surveying and sampling
8G Probability
8H Two-step experiments (Extending)
8I Tree diagrams (Extending)
8J Venn diagrams and two-way tables
8K Experimental probability

Victorian curriculum

STATISTICS AND PROBABILITY

Chance

Identify complementary events and use the sum of probabilities to solve problems (VCMSP294)

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and' (VCMSP295)

Represent events in two-way tables and Venn diagrams and solve related problems (VCMSP296)

Data representation and interpretation

Distinguish between a population and a sample and investigate techniques for collecting data, including census, sampling and observation (VCMSP297)

Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (VCMSP298)

Explore the variation of means and proportions of random samples drawn from the same population (VCMSP299)

Investigate the effect of individual data values including outliers, on the range, mean and median (VCMSP300)

NUMBER AND ALGEBRA

Linear and non-linear relationships

Plot graphs of non-linear real life data with and without the use of digital technologies, and interpret and analyse these graphs (VCMNA285)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Pedestrian counting system

Governments and businesses have always been interested in the habits and needs of the population, from where they live and how much they earn to where they shop.

The City of Melbourne has installed a pedestrian counting system to measure the flow of people moving around the streets each day. The system is made up of sensors installed in key areas across the city. These sensors use multi-directional movement detectors to count the number of people who pass them. The data is stored in an onsite data logger before being transferred every 15 minutes to the central server. This in turn is uploaded to the web every hour.

The data collected helps to inform decisions around urban planning and management. It discovers where there is a need to improve mobility or safety for pedestrians and finds the impact of the pedestrian flow on business activity. In this way statistical records are used to improve urban design for both pedestrians and businesses in the city.

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8A Interpreting graphs and tables

CONSOLIDATING









Statistics give us a way to understand many parts of our world, from weather patterns to outcomes of horse races. By taking a large amount of data, people can understand complicated principles and even predict the future with high accuracy. Graphs and tables are the most common way to represent data that has been collected.



The statistics of a horse's past performance can help predict whether or not it will win a race.

Let's start: Representing results as graphs and tables

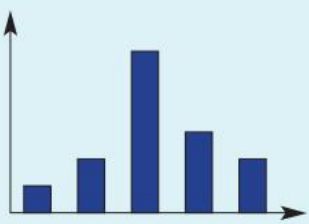
A number of people are surveyed about how many pets they have. They give the following results:

4, 0, 2, 2, 3, 1, 2, 5, 1, 4, 2, 1, 1, 0, 0, 2, 0, 3, 1, 2

- How could these results be put into a table to make them easier to understand?
- Try to represent the results in a graph. It could be a pie chart (sector graph), or a column graph or some other type of graph you have seen.
- Compare your graph with those of classmates and discuss why the graph is easier to understand than the original list of results.

Key ideas

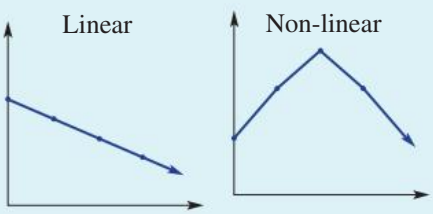
- Data can be **numerical** or **categorical**.
  - Numerical data can either be **discrete** (can only be particular numerical values) or **continuous** (can be any numerical value in a range).
  - Categorical data is non-numerical data that can be grouped into 'categories'.
- Data can be represented as a graph or a table.
- Common types of graphs include:



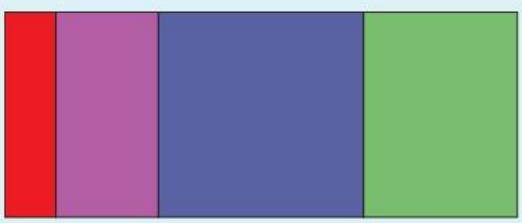
Column graphs



Pie charts (also called sector graphs)



Line graphs



Divided bar graphs

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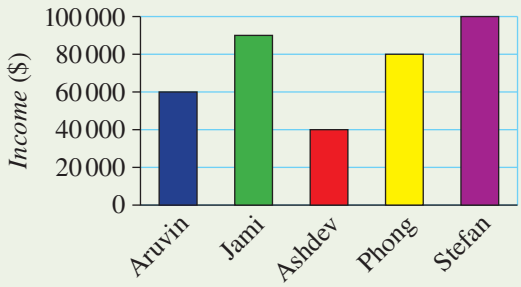
Example 1 Interpreting column graphs

This column graph represents the annual income of five different people.

a What is Aruvin's annual income?

b What is the difference between Jami's income and Ashdev's income?

c Who earns the most?



SOLUTION

EXPLANATION

a $60 000

Reading off the graph, Aruvin earns $60 000.

b $90 000 – 40 000 = $50 000

Jami earns $90 000 and Ashdev earns $40 000, so the difference is $50 000.

c Stefan earns the most.

With the highest column, Stefan earns the most ($100 000).

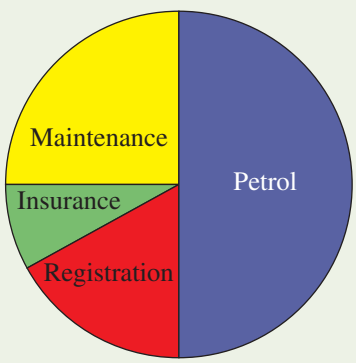
Example 2 Interpreting pie charts

A car owner graphs the amount of money spent per year on car-related expenses.

a What is the largest expense each year?

b What percentage of the car's expenses is devoted to maintenance?

c If the car owner spends $3000 per year on petrol, what is the total amount spent on the car each year?



SOLUTION

EXPLANATION

a Petrol

Since petrol occupies the largest area of the graph, it is the largest expense.

b 25%

Maintenance occupies $\frac{1}{4}$ of the chart's area, which equals 25%.

c

$\times 2$ (50% of expenses = $3000 → 100% of expenses = $6000) $\times 2$

The car owner spends $6000 each year.

Petrol occupies half the chart's area, which is 50%. This is doubled to find the total amount spent.

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Exercise 8A

1, 2 | 2 | —

UNDERSTANDING

1 The following table shows the population of some small towns over a 10-year period.

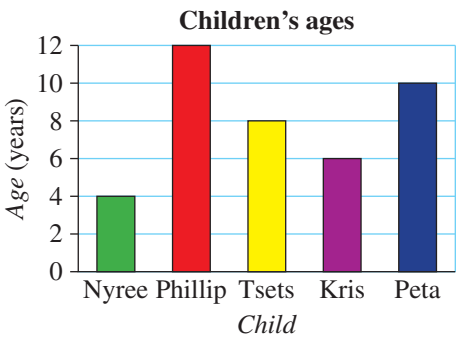
Year	Expton	Calcville	Statsland
2005	400	200	300
2010	320	240	310
2015	180	270	290

a What was the population of Expton in 2010?
b What was the population of Calcville in 2015?
c What was the population of Statsland in 2005?
d Which town's population decreased over time?
e Which town's population increased over time?

Example 1

2 The graph at right shows the age of five children.

a How old is Peta?
b How old is Kris?
c Who is the oldest of the five children?
d Who is the youngest of the five children?
e What is the difference in age between Tsets and Nyree?



3–6 | 3–7 | 4–7

FLUENCY

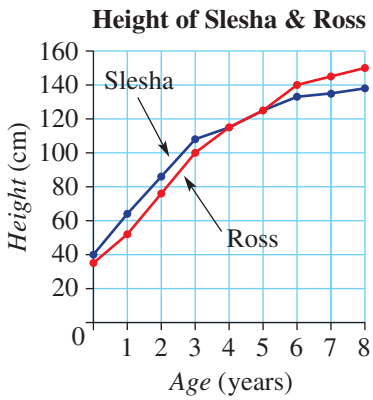
3 Six Year 8 classes are asked to vote for which sport they would like to do next in Physical Education. Their results are shown in the table.

Sport	8A	8B	8C	8D	8E	8F
Badminton	3	5	7	0	8	12
Water polo	9	9	8	14	11	9
Handball	12	10	11	11	7	3

a How many students in 8B want to do water polo?
b What is the most popular sport in 8C?
c How many students are in 8F?
d If the teachers decide all of Year 8 will do badminton, which class will this please the most?
e Which sport had the most votes in total?

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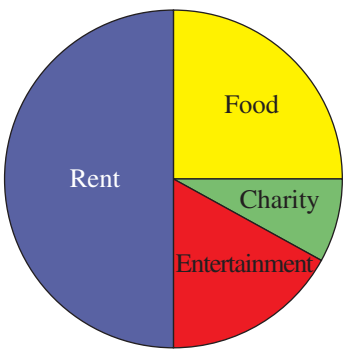
4 The line graph shows the height of Slesha and her twin brother Ross from the time they were born.



- **a** Which of the children was taller on their first birthday?
- **b** Which of the children was taller on their eighth birthday?
- **c** How old were the children when they were the same height?
- **d** Would you describe the general shape of the graphs as linear (straight line) or non-linear?

Example 2

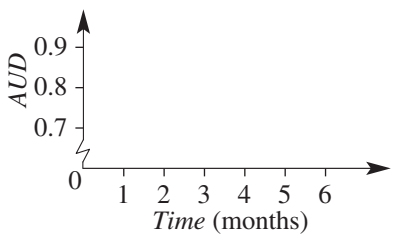
5 This sector graph shows one person's spending in a month.



- **a** What is the largest expense in that month?
- **b** What is the smallest expense in that month?
- **c** What percentage of the month's spending was on rent?
- **d** If the person spent a total of $600 on food in the month, what was their total spending?

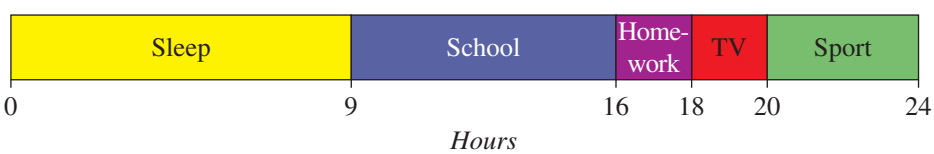
6 This table shows the value of the Australian dollar against the US dollar over a period of 6 months.

Time	0	1	2	3	4	5	6
AUD	0.90	0.87	0.85	0.81	0.78	0.76	0.72



- **a** Plot a graph of the value of the Australian dollar against the US dollar over the 6-month period. Use these axes to help you get started.
- **b** Would you describe the general shape of the graph as linear (straight line) or non-linear?
- **c** By how much did the Australian dollar decrease in:
  - **i** the first 3 months
  - **ii** the second month
- **d** Assuming this trend continued, what would the Australian dollar be worth after:
  - **i** 7 months
  - **ii** 9 months

7 A student has recorded how she spent her time on one day, in the divided bar graph shown below.



- **a** How much time did she spend doing homework on that day?
- **b** How much time was spent at school during that day?
- **c** What did she spend the most time doing?
- **d** What fraction of her day was spent playing sport?

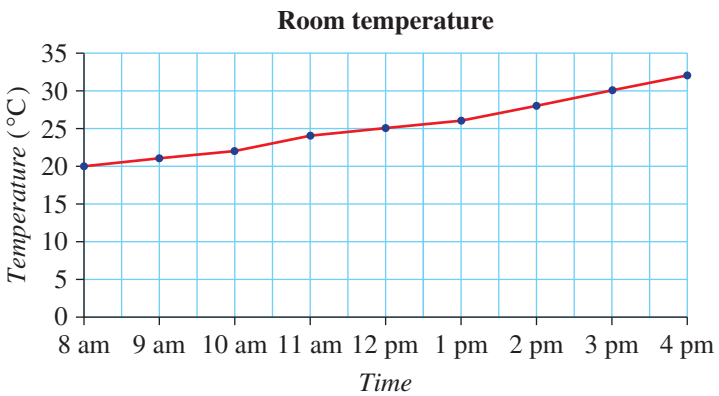
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8A

8 | 8, 9 | 8, 9

PROBLEM-SOLVING

8 The temperature in a classroom is graphed over an eight-hour period.



a What was the temperature at 8 a.m.?

b By how much did the temperature increase in the eight-hour period?

c Students complain that it is uncomfortably hot when the temperature is 25°C or greater. At what time does it become uncomfortably hot?

9 A diver jumps off a 10-metre diving board into a pool. The height of the top of her head above the water is given by the following table.

Time(s)	0	1	2	3	4	5	6	7
Diver height (m)	11.5	9.0	5.0	1.0	–1.5	–2.0	–0.2	0.2

a Plot this data on the given graph.

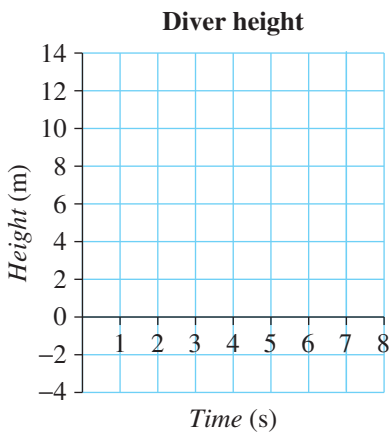
b Would you describe the general shape of the graph as linear (straight line) or non-linear.

c How tall is the diver?

d During which second does her head first enter the water?

e What is the deepest that her head reaches?

f Draw a graph that the diver might have if she were diving from a 3 metre spring board.



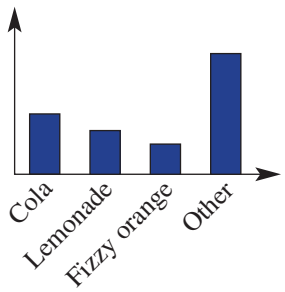


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10 10–12 11–13 8A

REASONING

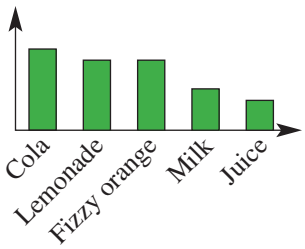
10 Three different surveys are conducted to establish whether soft drinks should be sold in the school canteen.



Survey 1: Favourite drink



Survey 2: Favourite type of drink



Survey 3: Sugar content per drink

a Which survey's graph would be the most likely to be used by someone who wished to show the financial benefit to the cafeteria of selling soft drinks?

b Which survey's graph would be the most likely to be used by someone who wanted to show there was not much desire for soft drink?

c Which survey's graph would be the most likely to be used by a person wanting to show how unhealthy soft drink is?

11 The population of three nearby towns is shown over a 10-year period.

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Town A	1414	1277	1204	1118	1026	1083	1171	1254	1317	1417
Town B	1062	1137	1188	1285	1371	1447	1502	1571	1665	1728
Town C	1042	1100	1174	1250	1312	1176	1075	992	895	783

a Describe how the population of each town has changed over time.

b A row is added to the bottom, containing the total of the three numbers above it. A column is added to the right-hand side, containing the total of all the numbers in each row. Give an example of a situation in which:

i the 'total' row at the bottom would be useful

ii the 'total' column at the right would be useful

12 Explain why you can use a sector graph for categorical data but you cannot use a line graph for categorical data.

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8A

REASONING

13 The 'water footprint' of different foods refers to the volume of fresh water that is used to produce the food. The water footprint of some foods is shown in the table below.

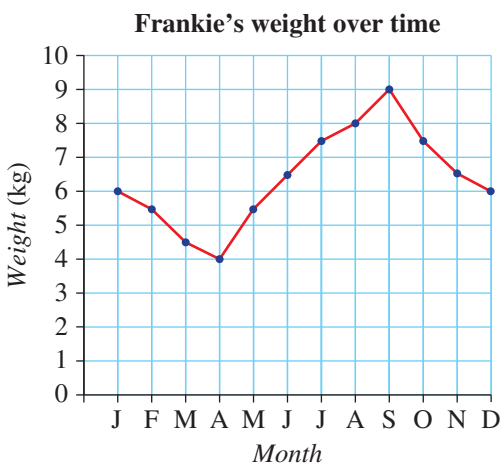
Food	bread	cheese	chicken	cucumber	lettuce	milk	potato	rice
Footprint (L/kg)	1608	3178	4325	353	237	1800	287	2497

a What type(s) of graph could be used for the data above? Justify your choice(s).
b Choose a suitable type of graph and depict the above numbers graphically.
c How is a food's water footprint related to how sustainable it is to produce?
d Estimate how many litres of water would be used for a chicken burger. Include your estimates of each item's weight.
e Another way to present the data is to say how many grams of each food is made from 1 kilolitre of water. Redraw the table above with a row for 'water efficiency' in g/kL.

ENRICHMENT

Weight over time — — 14

14 Frankie is a dog whose owner has graphed his weight over the year.





a If a normal weight for the dog is between 5 kg and 7 kg, fill out the following table for Frankie's weight.

	Underweight	Normal weight	Overweight	Total
Months				12

b Represent the results of the table as a pie chart.
c What is the advantage of a line graph over a pie chart?
d What is the advantage of a pie chart over a line graph?
e Draw a line graph showing another dog's weight over 12 months, given that the dog is underweight for 2 months, overweight for 3 months and the normal weight for 7 months.

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8B Frequency tables and tallies CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

Often the actual values in a set of data are not required – just knowing how many numbers fall into different ranges is often all the information that is needed. A **frequency table** allows us to do this by listing how common the different values are.

Frequency tables can be used for listing particular values or ranges of values.

Number of cars	Frequency
0	10
1	12
2	5
3	3

Age	Frequency
0–	7
5–	12
10–	10
15–	11

Let's start: Subject preferences

- Survey a group of peers to find their favourite school subject out of Maths, English, Science, Music and Sport.
- Represents your results in a table like the one below.

	Maths	English	Science	Music	Sport
Tally	𝍸	𝍸 |	𝍸 |||	||||	||
Frequency	5	6	8	4	2

- How would you expect the results to differ for different classes at your school, or for different schools?



Frequency tables can be used to show students' subject preferences.

- A **tally** is a tool used for counting as results are gathered. Numbers are written as vertical lines with every 5th number having a cross through a group of lines. For example, 4 is |||| and 7 is 𝍸 ||.
- **Frequency tables** show how common a certain value is in a frequency column. A tallying column is also often used as data is gathered.
- The items can be individual values or intervals of values.

Key ideas

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Example 3 Interpreting tallies

a The different car colours along a quiet road are noted. Convert the following tally into a frequency table.

White	Black	Blue	Red	Yellow
𝍸	𝍸 𝍸 |	𝍸 𝍸 𝍸 ||	𝍸 |	𝍸 ||||

b Hence state how many red cars were spotted.

SOLUTION

a

Colour	White	Black	Blue	Red	Yellow
Frequency	3	13	17	6	9

b 6 red cars were spotted.

EXPLANATION

Each tally is converted into a frequency. For example, black is two groups of 5 plus 3, giving $10 + 3 = 13$.

This can be read directly from the table.

Example 4 Constructing tables from data

Put the following data into a frequency table: 1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1.

SOLUTION

Number	1	2	3	4	5	6
Tally	𝍸 |	||	|	|||	|	|
Frequency	6	2	1	3	1	1

EXPLANATION

Construct the tally as you read through the list. Then go back and convert the tally to frequencies.

Exercise 8B

1, 2 | 2 | —

UNDERSTANDING

1 The table shows survey results for students' favourite colours. Classify the following as true or false.

Colour	Frequency
Red	5
Green	2
Orange	7
Blue	3

a Five people chose red as their favourite colour.
b Nine people chose orange as their favourite colour.
c Blue is the favourite colour of 3 people.
d More people chose green than orange as their favourite colour.

2 Fill in the blanks.

a The tally |||| represents the number _____.
b The tally 𝍸 || represents the number _____.
c The tally _____ represents the number 2.
d The tally _____ represents the number 11.

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Example 3

3 A basketball player's performance in one game is recorded in the following table.

	Passes	Shots at goal	Shots that go in	Steals
Tally	𝍸	𝍸 𝍸 ||	𝍸 |||	||
Frequency				

a Copy and complete the table, filling in the frequency row.
b How many shots did the player have at the goal?
c How many shots went in?
d How many shots did the player miss during the game?

Example 4

4 A student surveys her class to ask how many people are in their family.
The results are:
6, 3, 3, 2, 4, 5, 4, 5, 8, 5, 4, 8, 6, 7, 6, 5, 8, 4, 7, 6

a Construct a frequency table. Include a tally row and a frequency row.
b How many students have exactly 5 people in their family?
c How many students have at least 6 people in their family?

5 Braxton surveys a group of people to find out how much time they spend watching television each week. They give their answers rounded to the nearest hour.

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–168
Tally	𝍸	|||	𝍸 𝍸 ||	𝍸 𝍸 𝍸	𝍸 ||||	||||	||

a Draw a frequency table of his results, converting the tallies to numbers.
b How many people did he survey?
c How many people spend 15–19 hours per week watching television?
d How many people watch television for less than 5 hours per week?
e How many people watch television for less than 2 hours per day on average?

6 The heights of a group of 21 people are shown below, given to the nearest cm.

174 179 161 132 191 196 138 165 151 178 189
147 145 145 139 157 193 146 169 191 145

a Copy and complete the frequency table below.

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–189	190+
Tally							
Frequency							

b How many people are in the range 150–159 cm?
c How many people are 180 cm or taller?
d How many people are between 140 cm and 169 cm tall?

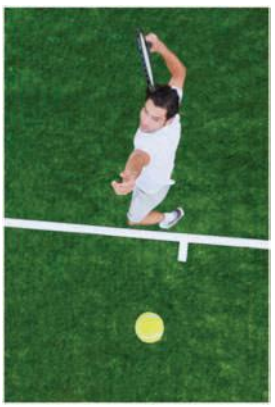
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8B

7, 8 | 7, 8 | 7–9

PROBLEM-SOLVING

7 A tennis player records the number of double faults they serve per match during one month.



Double faults	0	1	2	3	4	5
Frequency	4	2	1	0	2	1

a How many matches did they play in total during the month?
b How many times did they serve exactly 1 double fault?
c In how many matches did they serve no double faults?
d How many double faults did they serve in total during the month?

8 Match each of these data sets with the correct column (A, B, C or D) in the frequency table shown below.
a 1, 1, 2, 3, 3
b 1, 2, 2, 2, 3
c 1, 1, 1, 2, 3
d 1, 2, 3, 3, 3

Number	A	B	C	D
1	3	2	1	1
2	1	1	1	3
3	1	2	3	1

9 Five different classes are in the same building in different rooms. The ages of students in each room are recorded in the frequency table below.

Age	Room A	Room B	Room C	Room D	Room E
12	3	2	0	0	0
13	20	18	1	0	0
14	2	4	3	0	10
15	0	0	12	10	11
16	0	0	12	10	11
17	0	0	0	1	0

a How many students are in room C?
b How many students are in the building?
c How many 14-year-olds are in the building?
d What is the average (mean) age of students in room B? Answer to one decimal place.
e What is the average (mean) age of students in the building? Answer to one decimal place.

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REASONING

10 A number of students sat an exam for which they got a score out of 100. There were no part marks given. The results are presented in the frequency table below.

0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–100
0	0	3	1	2	5	8	12	10	2

a Redraw the table so that the intervals are of width 20 rather than 10 (i.e. so the first column is 0–19, the second is 20–39, and so on).

b Show how the table would look if it were drawn with the intervals 0–29, 30–59, 60–89, 90–100.

c Explain why it is not possible to fill out the frequency table below without additional information.

Score	0–24	25–49	50–74	75–100
Frequency				

d You are told that 2 students got less than 25, and 15 students got scores between 60 and 74. Use this information to fill out the frequency table above.

e What is the total of the frequency row in each of the tables you have drawn? Explain what this tells you.

11 **a** How could the stem-and-leaf plot below be represented as a frequency table using intervals 10–19, 20–29 and 30–39? (Remember that in a stem-and-leaf plot 1 | 2 represents the number 12, 3 | 5 represents 35 and so on.)

Stem	Leaf
1	2 5 8
2	1 7 9 9
3	5 5 7 7 8 9

b Give an example of a different stem-and-leaf plot that would give the same frequency table.

c Which contains more information: a stem-and-leaf plot or a frequency table?

d When would a frequency table be more appropriate than a stem-and-leaf plot?

12 An AFL player notes the number of points he scores in his 22 week season.

a Write out one possible list showing the scores he got for each of the 22 weeks. Note that the range 10– means at least 10 but less than 15.

b The list you wrote has a number from 10 to 14 written at least twice. Explain why this must be the case, regardless of what list you chose.

c Your list contains a number from 5 to 9 at least three times. Why must this be the case?

d Apart from the two numbers in parts **b** and **c**, is it possible to make a list that has no other repeated values?

e Is it possible to play 22 games and always score a different number of points?

Points	Frequency
0–	3
5–	11
10–	6
15–	1
20–	1

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13 Priscilla records the numbers of hours of homework she completes each evening from Monday to Thursday. Her results are shown in this frequency table.

Number of hours	Frequency
1	1
2	1
3	2

a One possibility is that she worked 3 hours on Monday, 2 hours on Tuesday, 3 hours on Wednesday and 1 hour on Thursday. Give an example of another way her time could have been allocated for the four nights.

b If she spent at least as much time doing homework each night as on the previous night, how long did she spend on Tuesday night?

c How many ways are there of filling in the table below to match her description?

Monday	Tuesday	Wednesday	Thursday
☐ hours	☐ hours	☐ hours	☐ hours

d If you know she spent two hours on Monday night, how many ways are there to fill in the table?

e If you know she spent three hours on Monday night, how many ways are there to fill in the table?

f Priscilla's brother Joey did homework on all five nights. On two nights he worked for 1 hour, on two nights he worked for 2 hours and on one night he worked for 3 hours. In how many ways could the table below be filled in to match his description?

Monday	Tuesday	Wednesday	Thursday	Friday
☐ hours	☐ hours	☐ hours	☐ hours	☐ hours



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8C Histograms









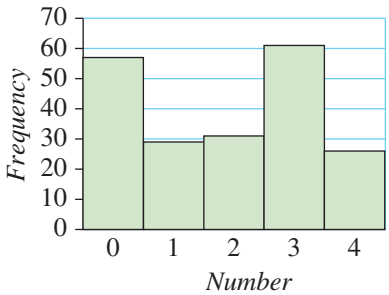
A **histogram** is a graphical representation of a frequency table so that patterns can be observed more easily.

For example, the data below is represented as a frequency table and as a histogram.

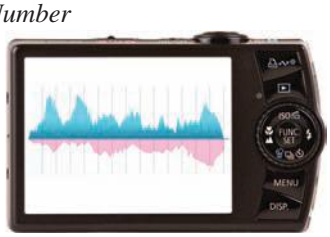
As a table

Number	Frequency
0	57
1	29
2	31
3	61
4	26

As a histogram



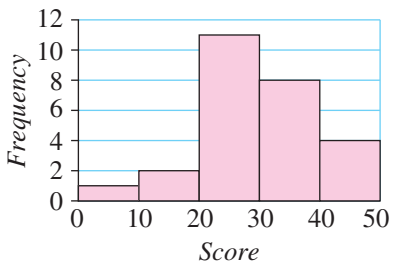
At a glance you can see from the histogram that 0 and 3 are about twice as common as the other values. This is harder to read straight from the table. Histograms are often used in digital cameras and photo editing software to show the brightness of a photo.



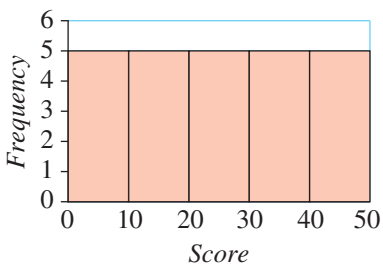
Let's start: Test analysis

The results for some end-of-year tests are shown for four different classes in four different histograms below.

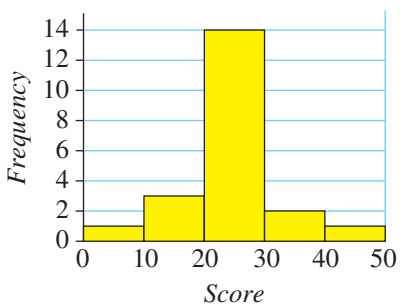
Class 1



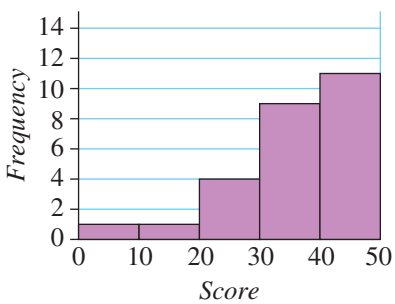
Class 2



Class 3



Class 4



- Describe each class on the basis of these graphs.
- Which class has the highest average score?
- Which class has the highest overall score?
- Which class would be the easiest to teach and which would be the hardest, do you think?

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Key ideas

- A **histogram** is a graphical representation of a frequency table. It can be used when the items are numerical.
- The vertical axis (y-axis) is used to represent the frequency of each item.
- Columns are placed next to one another with no gaps in between.
- A half-column-width space is sometimes placed between the vertical axis and the first column of the histogram if the first vertical bar does not start at zero.

Example 5 Constructing histograms from frequency tables

Represent the frequency tables below as histograms.

a

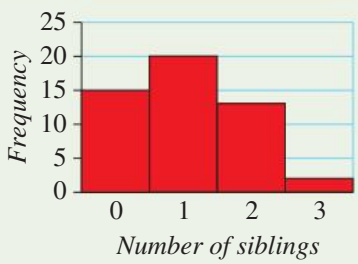
Number of siblings	Frequency
0	15
1	20
2	13
3	2

b

Number of words in story	Frequency
0–99	2
100–199	10
200–299	12
300–399	8
400–500	3

SOLUTION

a

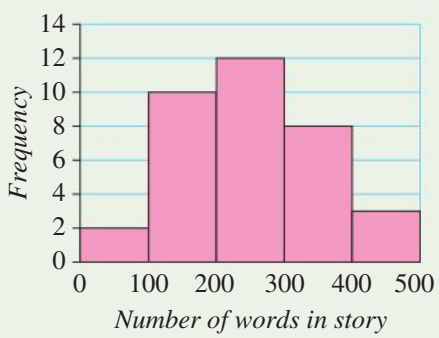


EXPLANATION

The scale 0–25 is chosen to fit the highest frequency (20).

Each different number of siblings in the frequency table is given a column in the graph.

b



The scale 0–14 is chosen to fit the highest frequency (12).

The different intervals (0–99 words, 100–199 words etc.) are displayed on the horizontal axis.

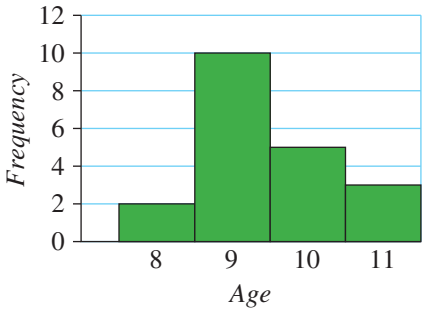
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Exercise 8C

1, 2 | 2 | —

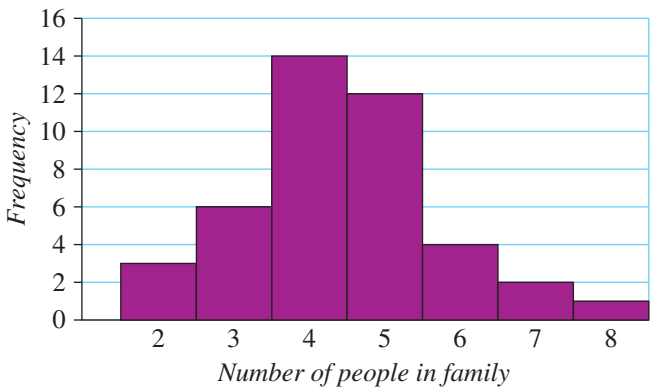
UNDERSTANDING

1 The histogram below shows the ages of people in an Art class.



- **a** How many 8-year-olds are in this class?
- **b** What is the most common age for students in this class?
- **c** What is the age of the oldest person in the class?

2 A survey is conducted of the number of people in different families. The results are shown.



- **a** What is the most likely number of people in a family, on the basis of this survey?
- **b** How many people responding to the survey said they had a family of 6?
- **c** What is the least likely number (from 2 to 8) of people in a family, on the basis of this survey?

3–5 | 3–4(½), 5 | 3–5(½)

FLUENCY

Example 5a

3 Represent the following frequency tables as histograms. Ensure that appropriate scales and labels are put on the axes.

a

Number	Frequency
0	5
1	3
2	5
3	2
4	4

b

Number	Frequency
0	3
1	9
2	3
3	10
4	7

c

Age	Frequency
12	15
13	10
14	25
15	20
16	28

d

Number of cars	Frequency
0	4
1	5
2	4
3	2

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8C

FLUENCY

4 For the following sets of data:

i create a frequency table **ii** hence draw a histogram

a 1, 2, 5, 5, 3, 4, 4, 4, 5, 5, 5, 1, 3, 4, 1

b 5, 1, 1, 2, 3, 2, 2, 3, 3, 4, 3, 3, 1, 1, 3

c 4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4

d 60, 52, 60, 59, 56, 57, 54, 53, 58, 56, 58, 60, 51, 52, 59, 59, 52, 60, 50, 52

Example 5b

5 Represent the following frequency tables as histograms.

a

Score	Frequency
0–19	1
20–39	4
40–59	10
60–79	12
80–100	5

b

Number	Frequency
0–5	5
6–10	12
11–15	14
16–20	11
21–25	5
26–30	8
31–35	2
36–40	1

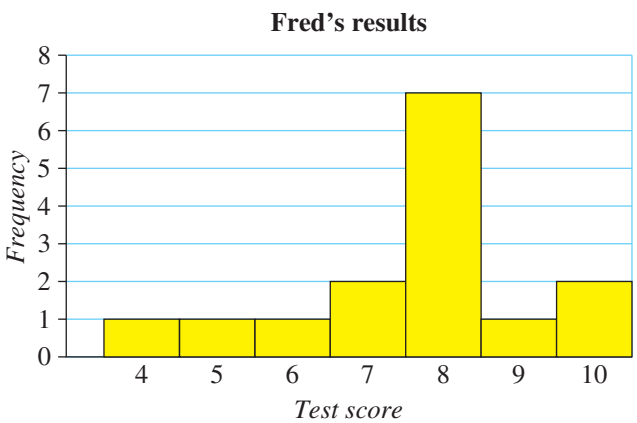
6, 7 6, 7 7, 8

PROBLEM-SOLVING

6 Edwin records the results for his spelling tests out of 10. They are 3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.

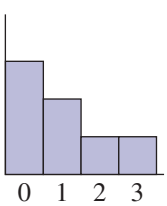
a Draw a histogram for his results.

b Is he a better or a worse speller generally than Fred, whose results are given by the histogram shown at right?

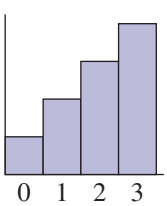


7 Some tennis players count the number of aces served in different matches. Match up the histograms with the descriptions.

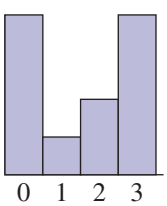
a



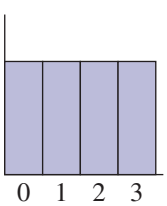
b



c



d



A Often serves aces.

B Generally serves 3 aces or 0 aces.

C Serves a different number of aces in each match.

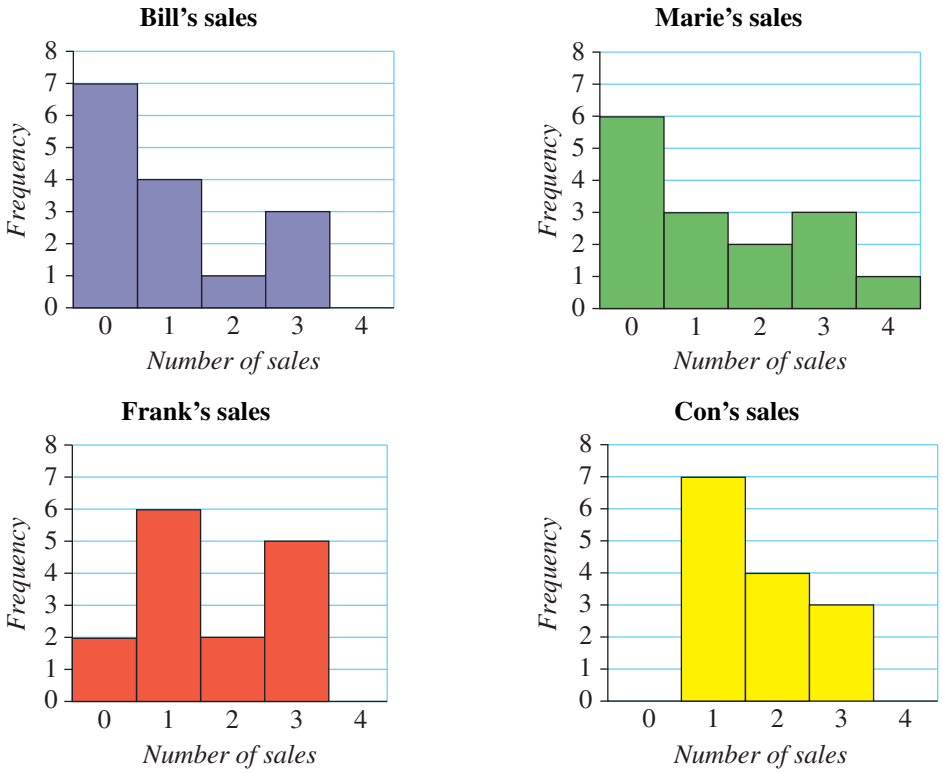
D Rarely serves aces.

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8C

PROBLEM-SOLVING

8 A car dealership records the number of sales each salesperson makes per day over three weeks.



a Which salesperson holds the record for the greatest number of sales in one day?

b Which salesperson made a sale every day?

c Over the whole period, which salesperson made the most sales in total?

d Over the whole period, which salesperson made the fewest sales in total?

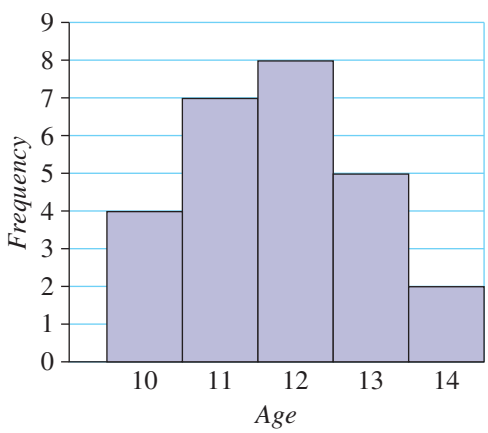
9 | 9, 10 | 9, 10

REASONING

9 This histogram shows the ages of a group of people in a room.

a Describe a histogram that shows the ages of this same group of people in 12 years' time.

b Describe a histogram that shows the ages of this same group of people 12 years ago. Include a rough sketch.



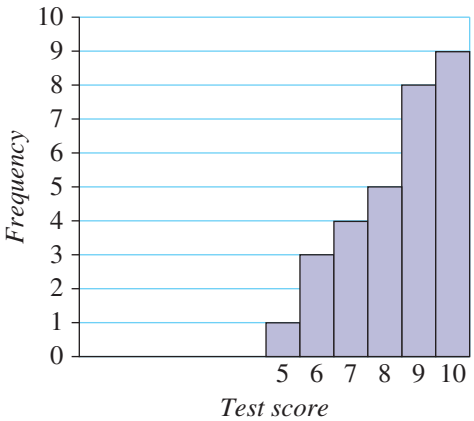
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8C

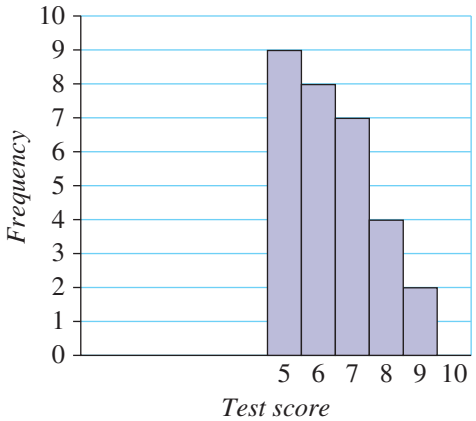
REASONING

10 Two students have each graphed a histogram that shows their results for a number of spelling tests. Each test is out of 10.

Manisha



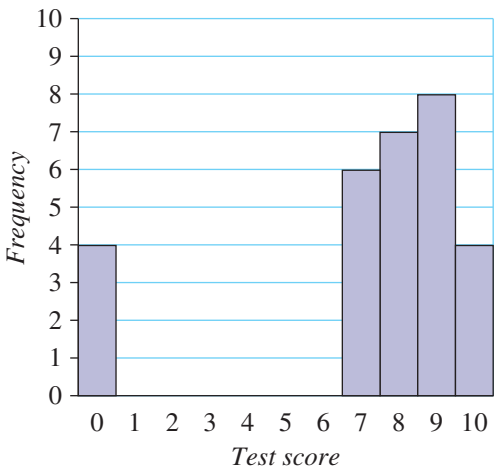
Ravi



a Does Manisha's graph show that her spelling is improving over the course of the year? Explain.

b Ravi's spelling has actually improved consistently over the course of the year. Give an example of a list of the scores he might have received for the 30 weeks so far.

c A third student has the following results. What is a likely explanation for the '0' results?





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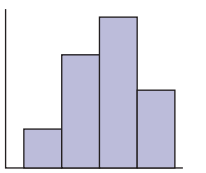
Heights, weights and ages mix-up — — 11

11 Three students survey different groups of people to find out their heights, weights and ages. Unfortunately they have mixed up all the graphs they obtained.

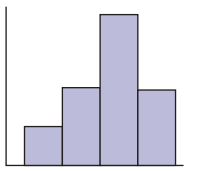
a Copy and complete the table below, stating which graph corresponds to which set of data.

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4		
Shopping centre			
Teachers common room			

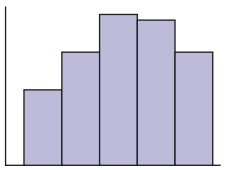
Graph 1



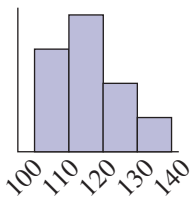
Graph 2



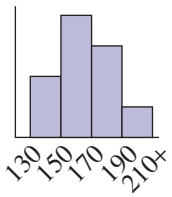
Graph 3



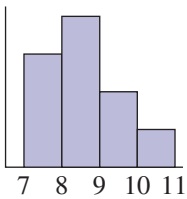
Graph 4



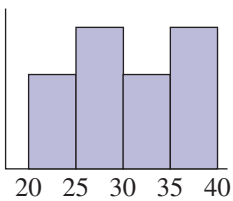
Graph 5



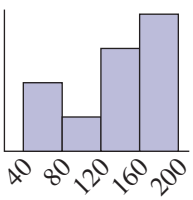
Graph 6



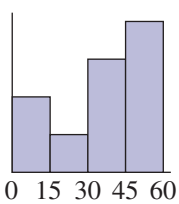
Graph 7



Graph 8



Graph 9



b Show with rough sketches how the height, weight and age graphs would look for:

i people in a retirement village

ii students at a secondary school

iii guests at a 30-year high school reunion

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8D Measures of centre









It is sometimes useful to summarise a large group of data as a single value. The concept of 'average' is familiar to most people, but more precise mathematical terms to use are 'mean', 'median' and 'mode'. The mean and the median are considered values that are approximately at the 'centre' of the data set, although this is not always the case.

Let's start: Tug-of-war competition

At a school there are 20 students in each class. They are to compete in a tug-of-war round-robin and it is known that a heavier team will always beat a lighter team. You are told some information about the classes.

8A: The mean weight is 53 kg.

8B: Half the class weighs more than 55 kg.

8C: Half the class weighs less than 52 kg.

8D: The mean weight is 56 kg.

8E: The most common weight is 60 kg.

8F: The mean weight is 54 kg.



Averages can let you predict the outcome of a tug-of-war contest.

- In a tug-of-war round-robin in which everyone in each class is involved, which results would you be able to determine from these facts alone? (For example, 8D would beat 8A.)
- Is it possible for 8A to beat 8E? Give an example of how this could happen.
- Is it possible for 8C to beat 8B? Explain.

Key ideas

- The **mean** (sometimes called the average) of a set of numbers is given by

  Mean = (sum of all the values) ÷ (total number of values)

  For example:

  $7 + 8 + 1 + 10 + 2 + 1 + 6 = 35$

  Mean $= 35 \div 7 = 5$

- The **median** is the middle value if the values are in order (ascending or descending). If there are two middle values then the average of them is taken, by adding them together and dividing by 2.

  For example:

  1 1 2 (6) 7 8 10

  Middle $\Rightarrow$ Median = 6

  For example:

  2 3 (5 9) 10 12

  $\frac{5+9}{2} \Rightarrow$ Median = 7

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Key ideas

- The **mode** is the most common value, i.e. the one with the highest frequency. There can be more than one mode.
  For example:
  (1) (1) 2 6 7 8 10
  Mode = 1
- An **outlier** is a data point that is significantly smaller or larger than the rest of the data.
  - The median and mode are generally unaffected by outliers whereas the mean can be affected significantly by an outlier.

Example 6 Finding the mean and the mode

For the set of numbers 10, 2, 15, 1, 15, 5, 11, 19, 4, 8 find:

a the mean **b** the mode

SOLUTION	EXPLANATION
a $10 + 2 + 15 + 1 + 15 + 5 + 11 + 19 + 4 + 8 = 90$ Mean $= 90 \div 10 = 9$	The numbers are added. The mean is found by dividing the total by the number of items (in this case 10).
b Mode = 15	The most common value is 15, so this is the mode.

Example 7 Finding the median

Find the median of:

a 16, 18, 1, 13, 14, 2, 11

b 7, 9, 12, 3, 15, 10, 19, 3, 19, 1

SOLUTION	EXPLANATION
a Sorted: 1, 2, 11, (13), 14, 16, 18 Median = 13	Sort the numbers. The middle value is 13.
b Sorted: 1, 3, 3, 7, (9, 10), 12, 15, 19, 19 Median $= \frac{9 + 10}{2} = 9.5$	Sort the numbers. There are two middle values (9 and 10) so we add them and divide by 2.

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Exercise 8D

1–4 | 4 | —

UNDERSTANDING

1 Fill in the blanks.

- **a** The most common value in a set of data is called the _____.
- **b** The sum of all values, divided by the number of values is called the _____.
- **c** The _____ can be calculated by finding the middle values of the numbers placed in ascending order.

2 Consider the set of numbers 1, 7, 1, 2, 4.

- **a** Find the sum of these numbers.
- **b** How many numbers are listed?
- **c** Hence find the mean.

3 Consider the values 5, 2, 1, 7, 9, 4, 6.

- **a** Sort these numbers from smallest to largest.
- **b** What is the middle value in your sorted list?
- **c** What is the median of this set?

4 Consider the set 1, 5, 7, 9, 10, 13.

- **a** State the two middle values.
- **b** Find the sum of the two middle values.
- **c** Divide your answer by 2 to find the median of the set.

5–7(½), 8 | 5–7(½), 8–10 | 5–7(½), 9, 10(½)

FLUENCY

Example 6

5 For each of the following sets find:

i the mean ii the mode

- **a** 5, 6, 3, 4, 4, 8
- **b** 2, 2, 1, 2, 1, 4, 2
- **c** 4, 3, 3, 10, 10, 2, 3
- **d** –10, – 4, 0, 0, –2, 0, –5
- **e** 3, 4, 5, –9, 6, –9
- **f** 3, –6, 7, – 4, –3, 3
- **g** 13, 15, 7, 7, 20, 9, 15, 15, 11, 17
- **h** 20, 12, 15, 11, 20, 3, 18, 2, 14, 16
- **i** 18, 12, 12, 14, 12, 3, 3, 16, 5, 16
- **j** 18, 5, 14, 5, 19, 12, 13, 5, 10, 3
- **k** –15, –6, –6, 16, 6, 13, 3, 2, 19, –8
- **l** –13, –6, –6, –13, –6, 10, –15, 6, 7, 2

Example 7a

6 For each of the following sets calculate the median.

- **a** 3, 5, 6, 8, 10
- **b** 3, 4, 4, 6, 7
- **c** 1, 2, 4, 8, 10, 13, 13
- **d** 2, 5, 5, 5, 8, 12, 14
- **e** 14, 15, 7, 1, 11, 2, 8, 7, 15
- **f** 4, 14, 5, 7, 12, 1, 12, 6, 11
- **g** 6, –10, 8, 1, 15, 8, 3, 1, 2
- **h** 5, –7, 12, 7, –3, 7, –3, 11, 12

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Example 7b

7 For each of the following sets calculate the median.

a 2, 2, 4, 6, 7, 9

b 1, 1, 2, 9, 9, 10

c 1, 3, 5, 7, 8, 10, 13, 14

d 0, 1, 9, 13, 1, 10, 7, 12, 9, 2

e 12, 17, 7, 10, 2, 17, –2, 15, 11, –8

f –2, –1, –3, 15, 13, 11, 14, 17, 1, 14

8 Bernie writes down how many hours he works each day for one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of hours	8	10	8	7	9

a What is the mean number of hours Bernie worked each day?

b What is the median number of hours Bernie worked each day?

c What is the mode number of hours Bernie worked each day?

d If the number of hours worked on Tuesday changed to 20 (becoming an outlier), which of your answers above would be affected?

9 Some people's ages are placed into a stem-and-leaf plot.

Stem	Leaf
1	8 9
2	0 3 5 7
3	1 2 2 7

a Write this set of data out as a list in ascending order. (Recall 1|8 means 18, 2|0 means 20 etc.)

b Find the median.

c Calculate the mean.

d State the mode.

10 State the mode (most common category) for the following frequency tables.

a Colours of cars are noted as they drive past

Colour	Red	Blue	Orange	White	Green	Black
Frequency	21	14	3	42	7	25

b Pizza preferences are noted within a group of teenagers

Hawaiian	Meat-lovers	Vegetarian	Cheese
5	7	4	2

c The favourite day of the week of a group of people

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Frequency	4	12	41	16	28

d The number of gymnasts in different states

New South Wales	Queensland	South Australia	Tasmania	Victoria	Western Australia
152	135	193	86	144	159

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8D

11, 12 | 12–14 | 13–14

PROBLEM-SOLVING

11 Jared measures the weights of 10 eggs at two shops. Both shops sell eggs at the same price.
Shop A (grams): 52.5, 49.6, 49.1, 47.8, 54.1, 53.7, 49.8, 45.7, 54.4, 53.3
Shop B (grams): 49.0, 48.1, 60.0, 55.4, 47.0, 53.9, 58.5, 46.5, 43.3, 42.2

a Find the mean weight of an egg in:
i shop A **ii** shop B

b Which shop has heavier eggs on average?

12 Federica is in a dancing competition and each week she is rated out of 10. Her results for one term are shown in the frequency table below.

Score	7	8	9	10
Frequency	3	0	3	4



a In how many weeks did she get 7 out of 10?

b What score did she receive the most often?

c What is her mean dancing score for the 10 weeks? (Hint: Write out the scores as a list.)

d What is her median dancing score for the 10 weeks?

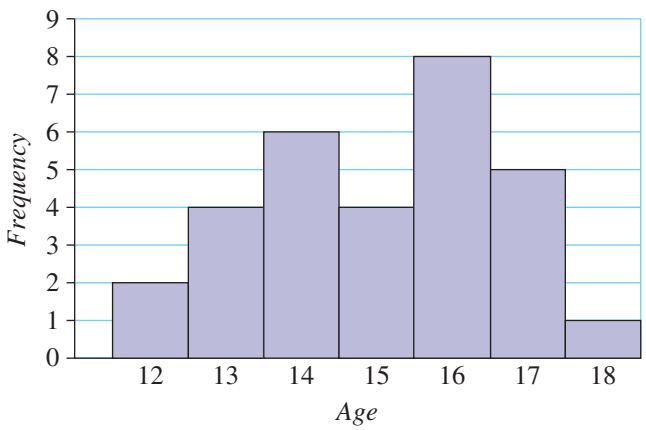
e Give an example of a single change that would reduce the mean score but keep the median the same.

13 The histogram at right shows the ages of all students in a school's chess club.

a What is the most common age?

b Calculate the mean age correct to two decimal places.

c Calculate the median age.



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8D

PROBLEM-SOLVING

14 In each case find the missing number.

- **a** The mean of the set 7, 12, ☐, 5 is equal to 10.
- **b** The mean of the set 4, 5, 6, ☐ is equal to 3.
- **c** The mode of the set 2, 6, ☐, 5 is equal to 5.
- **d** The median of the set 1, 9, 3, ☐, 100 is equal to 7.
- **e** The median of the set 15, 11, 2, ☐, 7, 23 is equal to 10.

15, 17 15–17 16–19

REASONING

15 Explain why you can calculate the mode for numerical or categorical data, but you can only calculate the mean and median for numerical data.

16 Consider the set 1, 5, 7, 8, 9.

- **a** Find the mean of this set.
- **b** What happens to the mean if every number is multiplied by 3?
- **c** What happens to the mean if every number has 4 added?
- **d** What happens to the mean if every number is squared?
- **e** What happens to the median if every number is squared?

17 The prices of all the houses in School Court are recorded: $520 000, $470 000, $630 000, $580 000, $790 000, $540 000, $710 000, $8.4 million, $660 000.

- **a** What is the mean house price in School Court, correct to the nearest dollar?
- **b** What is the median house price in School Court?
- **c** What effect does having a single $8.4 million mansion in School Court have on the mean?
- **d** What effect does having a single $8.4 million mansion in School Court have on the median?
- **e** Why might 'median house price' be a more informative measure than 'mean house price' when people are looking at living in a particular area?



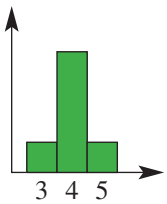
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8D

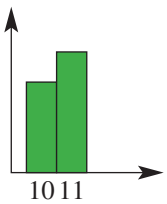
REASONING

18 For the following histograms drawn to scale, the frequencies have been omitted. Determine the median for each one.

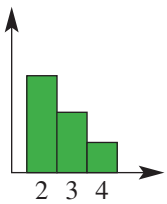
a



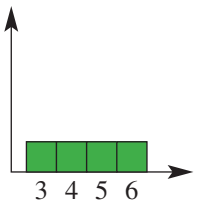
b



c



d



19 For the following descriptions of data sets, determine whether they are possible. If so, give an example; if not, explain why not.

- **a** This set has four items with a mean of 5, a median of 6 and a mode of 7.
- **b** This set has two items with a mean greater than the median.
- **c** This set has three elements and the mode is less than the median.
- **d** This set has four elements and the median is double the mean.

Report cards — — 20

ENRICHMENT

20 At a particular school the grades are given according to this guide.

Mark	80–100	60–79	40–59	20–39	0–19
Grade	A	B	C	D	E

In one semester Kathryn's grades were C, B, A, B, B, E, D, A. Her final mark is the mean of all the marks she obtained that semester.

- **a** Give a possible set of 8 marks that would give these grades.
- **b** Give the lowest possible set of 8 marks that would give these grades.
- **c** Give the highest possible set of 8 marks that would give these grades.
- **d** Is it possible for Kathryn to have a final mark of 80 on the basis of these grades?
- **e** What range of values could her final mark have?
- **f** What are the possible final grades that she could have?
- **g** Explain why the median must be a B, regardless of what the individual marks were.
- **h** Another student has just four results: A, B, A, A.
  - **i** What is the range of numbers for this student's final mark?
  - **ii** What are the possible grades that this student could have?

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8E Measures of spread EXTENDING

While a mean, median and a mode can give a summary of data, it is often helpful to know how spread out the data are. Two commonly used values are the **range** and the **interquartile range**. These are both examples of **measures of spread**.

Let's start: Golfing

Three golfers record their scores over 5 games. In golf, lower scores are better.

Alfred: 82, 87, 80, 112, 79
Brenton: 90, 89, 91, 92, 89
Charlie: 72, 103, 94, 83, 80

- Which golfer do you think is best? Explain.
- Which golfer is most consistent? Explain.
- Alfred's range of values (112 – 79 = 33) is larger than Charlie's (103 – 72 = 31). Discuss whether this means that Charlie is a more consistent golfer than Alfred.



Examining the range of results can help compare the skills of two golfers.

Key ideas

- The **range** of a set of data is given by:
  - Range = highest number – lowest number
- The **interquartile range** (or IQR) is found by the following procedure.
  - Sort the data into ascending order.
  - If there is an odd number of values, remove the middle one.
  - Split the data into two equal size groups.
  - The median of the lower half is called the **lower quartile**.
  - The median of the upper half is called the **upper quartile**.
  - IQR = upper quartile – lower quartile

For example: 1 5 7 9 10 11 | 13 17 19 20 23 28

lower half: 1 5 7 9 10 11 — Lower quartile = Median = 8

upper half: 13 17 19 20 23 28 — Upper quartile = Median = 19.5

$$IQR = 19.5 - 8 = 11.5$$

- The range and the interquartile range are **measures of spread**; they summarise the amount of spread in a set of numerical data. Outliers affect the range but not the IQR.

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Example 8 Finding the range

Find the range of the following sets of data.

a 1, 5, 2, 3, 8, 12, 4 **b** –6, –20, 7, 12, –24, 19

SOLUTION	EXPLANATION
a Range = 12 – 1 = 11	Maximum: 12, minimum: 1 Range = maximum – minimum
b Range = 19 – (–24) = 43	Maximum: 19, minimum: –24 Range = 19 – (–24) = 19 + 24

Example 9 Finding the IQR

Find the interquartile range of the following sets of data.

a 1, 15, 8, 2, 13, 10, 4, 14

b 2, 7, 11, 8, 3, 8, 10, 4, 9, 6, 8

SOLUTION	EXPLANATION
a Sorted: 1 2 4 8 10 13 14 15 1 2 4 8 10 13 14 15 Median $= \frac{2+4}{2} = 3$ Median $= \frac{13+14}{2} = 13.5$ IQR = 13.5 – 3 = 10.5	Values must be sorted from lowest to highest. Lower quartile = median of the lower half. Upper quartile = median of the upper half. IQR = upper quartile – lower quartile
b Sorted: 2 3 4 6 7 ~~8~~ 8 8 9 10 11 2 3 (4) 6 7 8 8 (9) 10 11 Lower quartile Upper quartile IQR = 9 – 4 = 5	Sort the data and remove the middle value (so there are two equal halves remaining). Lower quartile = median of the lower half. Upper quartile = median of the upper half. IQR = upper quartile – lower quartile

Exercise 8E

1–6 | 2, 3, 6 | 6

UNDERSTANDING

1 Consider the set of numbers 1, 3, 2, 8, 5, 6.

a State the largest number. **b** State the smallest number.

c Hence, state the range by finding the difference of these two values.

2 Consider the set of numbers 1, 2, 4, 5, 5, 6, 7, 9, 9, 10.

a State the median of 1, 2, 4, 5, 5. **b** State the median of 6, 7, 9, 9, 10.

c Hence state the interquartile range, by finding the difference of these two values.

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8E

UNDERSTANDING

3 Consider the set of numbers 1, 5, 6, 8, 10, 12, 14.

- **a** State the median of the lower half (1, 5, 6).
- **b** State the median of the upper half (10, 12, 14).
- **c** Hence state the interquartile range.

4 Complete these sentences.

- **a** The median of the lower half is called the _____ _____.
- **b** The difference between the highest and _____ values is the range.
- **c** In order to calculate the IQR you should first _____ the numbers from lowest to highest.
- **d** The upper quartile is the _____ of the upper half of the sorted data.
- **e** When calculating the IQR, you remove the middle term if the number of values is _____.
- **f** The range and IQR are measures of _____.

5 Calculate the median of the following sets.

- **a** 1, 3, 5, 6
- **b** 2, 7, 8, 12, 14, 15
- **c** 1, 4, 5, 6, 8, 10

6 Consider the set 8, 6, 10, 4, 3, 7, 5, 5.

- **a** Write this list in ascending order.
- **b** Calculate the lower quartile.
- **c** Calculate the upper quartile.
- **d** Calculate the IQR.

FLUENCY

7–9(½) 7–10(½) 7–10(½)

Example 8

7 Find the range of the following sets.

- **a** 5, 1, 7, 9, 10, 3, 10, 6
- **b** 9, 3, 9, 3, 10, 5, 0, 2
- **c** 4, 13, 16, 9, 1, 6, 5, 8, 11, 10
- **d** 16, 7, 17, 13, 3, 12, 6, 6, 3, 6
- **e** –7, 4, 12, –5, –18, –16, 7, 9
- **f** 16, –3, –5, –6, 18, –4, 3, –9
- **g** 3.5, 6.9, –9.8, –10.0, 6.2, 0.8
- **h** –4.6, 2.6, –6.1 2.6, 0.8, –5.4

Example 9a

8 Find the IQR of the following sets.

- **a** 1, 9, 11, 13, 28, 29
- **b** 7, 9, 13, 16, 20, 28
- **c** –17, –14, –12, –9, –8, 1, 9, 9, 12, 12
- **d** –20, –20, –10, –3, –1, 0, 3, 5, 14, 18
- **e** 0, 0, 4, 5, 7, 8, 8, 8, 11, 12, 17, 18
- **f** 1, 9, 9, 9, 12, 18, 19, 20
- **g** 0.28, 2.13, 3.64, 3.81, 5.29, 7.08
- **h** 1.16, 2.97, 3.84, 3.94, 4.73, 6.14

Example 9b

9 Find the IQR of the following sets.

- **a** 4, 4, 11, 16, 21, 27, 30
- **b** 10, 11, 13, 22, 27, 30, 30
- **c** –13, –13, –12, –3, 14, 15, 22, 23, 27
- **d** –19, –17, –6, 0, 3, 3, 18, 23, 26
- **e** 1, 1, 2, 9, 9, 12, 18, 18, 18
- **f** 2, 2, 8, 9, 12, 14, 15, 16, 19
- **g** 0.4, 0.9, 0.9, 1.9, 2.0, 3.9, 4.3, 4.4, 4.7
- **h** 0.5, 1.1, 1.2, 1.4, 1.5, 2.1, 2.2, 2.4, 4.8

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8E

FLUENCY

10 Find the IQR of the following sets. Remember to sort first.

a 0, 12, 14, 3, 4, 14
b 14, 0, 15, 18, 0, 3, 14, 7, 18, 12, 9, 5
c 6, 11, 3, 15, 18, 14, 13, 2, 16, 7, 7
d 6, 4, 6, 5, 14, 8, 10, 18, 16, 6
e 18, −15, 17, −15, −1, 2
f −12, −17, −12, 11, 15, −1
g −19, 8, 20, −10, 6, −16, 0, 14, 2, −2, 1
h −4, −9, 17, 7, −8, −4, −16, 4, 2, 5

11, 12 | 12–14 | 13–15

PROBLEM-SOLVING

11 Gary and Nathan compare the number of runs they score in cricket over a number of weeks.
Gary: 17, 19, 17, 8, 11, 20, 5, 13, 15, 15
Nathan: 39, 4, 26, 28, 23, 18, 37, 18, 16, 20

a Calculate Gary's range.
b Calculate Nathan's range.
c Who has the greater range?
d Which cricketer is more consistent, on the basis of their ranges only?

12 Winnie and Max note the amount of water they drink (in litres) every day for a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Winnie	2.4	1.3	3	2.2	2	4	1.2
Max	1.5	2.3	0.8	1.2	3	2.5	4.1

a Calculate Winnie's IQR.
b Calculate Max's IQR.
c Who has the larger IQR?

13 Over a 20-week period, Sara and Andy tally their spelling test results.

Score	0	1	2	3	4	5	6	7	8	9	10
Sara		II		I		𝍸	I	II	III	I	𝍸
Andy	I				I		IIII		𝍸	II	𝍸 II

a Find the range for
i Sara **ii** Andy.
b Find the IQR for
i Sara **ii** Andy.
c On the basis of the range only, which student is more consistent?
d On the basis of the IQR only, which student is more consistent?

14 Give an example of a set of numbers with mean = 10 and mean = 10 and:
a range = 2 **b** range = 20

15 Give an example of a set of numbers with range = 20 and:
a mean = 10 **b** mean = 1

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16 16, 17, 19 16, 18, 19 **8E**

REASONING

16 Consider the set of numbers 2, 3, 3, 4, 5, 5, 5, 7, 9, 10.

a Calculate the:

i range **ii** IQR

b If the number 10 changed to 100, calculate the new:

i range **ii** IQR

c Explain why the IQR is a better measure of spread if there are outliers in a data set.

17 Consider the set 1, 4, 5, 5, 6, 8, 11.

a Find the range.

b Find the IQR.

c Is it ever possible for the IQR of a set to be larger than the range? Explain.

d Is it possible for the IQR of a set to equal the range? Explain.

18 For a set of 3 numbers, what effect is there on the range if:

a each number is increased by 10?

b each number is doubled?

19 Why are the words 'upper quartile' and 'lower quartile' used? Think about what 'quart' might mean.

Changing the frequency

— — 20

ENRICHMENT

20 Consider the data below, given as a frequency table.

Number	1	2	3	4	5
Frequency	2	4	1	1	3

a What is the range?

b Calculate the IQR. (Hint: Write the data as a list first.)

c How would the range change if the frequency of each number were doubled?

d How would the IQR change if the frequency of each number were doubled?

e If the numbers themselves were doubled but the frequencies kept the same as in the table above, how would this change:

i the range? **ii** the IQR?

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8F Surveying and sampling









To find information about a large number of people it is generally not possible to ask everybody to complete a survey, so instead a sample of the population is chosen and surveyed. It is hoped that the information given by this smaller group is representative of the larger group of people. Choosing the right sample size and obtaining a representative sample are harder than many people realise.

Let's start: Average word length

To decide how hard the language is in a book, you could try to calculate the average length of the words in it. Because books are generally too large, instead you can choose a smaller sample. For this activity, you must decide or be assigned to the 'small sample', 'medium sample' or 'large sample' group. Then:

1 Pick a page from the book at random.



2 Find the average (mean) length of any English words on this page, choosing the first 10 words if you are in the 'small sample' group, the first 30 words for the 'medium sample' group and the first 50 words for the 'large sample' group.

Discuss as a class:

- Which of the groups would have the best estimate for the average word length in the book?
- What are the advantages and disadvantages of choosing a large sample?
- Does this sample help to determine the average length of words in the English language?
- How could the results of a whole class be combined to get the best possible estimate for average word length in the book?
- If all students are allowed to choose the page on which to count words, rather than choosing one at random, how could this bias the results?

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Key ideas

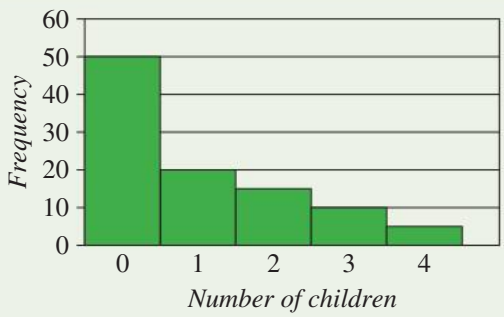
- A **population** is the set of all members of a group which is to be studied.
  For example: All the people in a town, looking at which local beach they prefer.
  All the kangaroos in a park, looking at the presence of any diseases.
- A **sample** is a subset (selected group) of a population.
  For example: 20 students selected from all Year 8 students in a school, looking at what their favourite football team is.
  1000 people selected and called via telephone regarding their preferred political party.
- A **survey** can be conducted to obtain information about a large group by using a smaller sample. A survey conducted on an entire population is called a **census**.
- The accuracy of the survey's conclusion can be affected by:
  - the **sample size** (number of participants or items considered)
  - whether the sample is **representative** of the larger group, or **biased**, which can result in a sample mean significantly different from the population mean
  - whether there were any **measurement errors**, which could lead to **outliers** – values that are noticeably different from the other values.
- Data represented as a histogram can be seen as **symmetric**, **skewed** or **bi-modal**.



- If a data distribution is symmetric, the mean and the median are approximately equal.

Example 10 Interpreting survey results

A survey is conducted asking 100 randomly selected adults how many children they have. The results are shown in this histogram.



a Assume that this sample is representative of the population.

i What proportion of the adult population has two or more children?

ii In a group of 9000 adults, how many would you expect to have 4 children?

b Is this distribution symmetric, skewed or bi-modal?

c Which of the following methods of conducting the survey could lead to bias?

Method 1 Asking people waiting outside a childcare centre

Method 2 Randomly selecting people at a night club

Method 3 Choosing 100 adults at random from the national census and noting how many children they claimed to have

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SOLUTION	EXPLANATION
a **i** $\frac{3}{10}$	$15 + 10 + 5 = 30$ adults have two or more children Proportion $= \frac{30}{100} = \frac{3}{10}$
ii 450	In the survey $\frac{5}{100} = \frac{1}{20}$ of the population have four children. $\frac{1}{20} \times 9000 = 450$
b Skewed	Many more people have 0 children, so the distribution is not symmetric.
c Methods 1 and 2 could both lead to bias.	If someone is waiting outside a childcare centre they are more likely to have at least one child. If someone is at a night club they are likely to be a younger adult, and so less likely to have a child.

Exercise 8F

1, 2 | 2 | —

UNDERSTANDING

1 Marieko wishes to know the average age of drivers in her city. She could survey 10 of her friends, or survey 1000 randomly selected drivers.

- **a** Which of these options would give a more accurate result?
- **b** Which would be easier for Marieko to perform?

Example 10a

2 Ajith looks at a random sample of penguins and notes that of the 50 he sees, 20 of them have spots on their bodies.

- **a** What proportion of the population has spots?
- **b** If there are 5000 penguins in a region, on the basis of this sample how many would have spots on their bodies?
- **c** If there are 500 penguins in a region, how many would you expect not to have spots on their bodies?

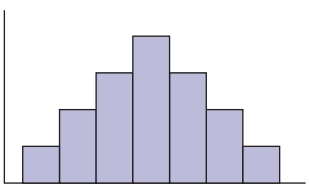
3–6 | 3–7 | 3, 4, 6–8

FLUENCY

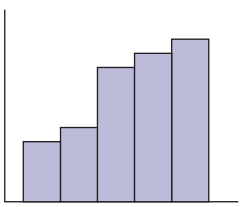
Example 10b

3 Use the guide in the **Key ideas** to classify the following as symmetric, skewed or bi-modal.

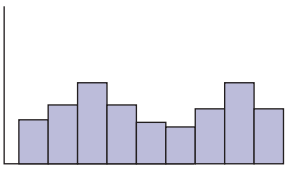
a



b

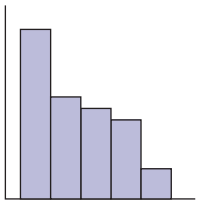


c

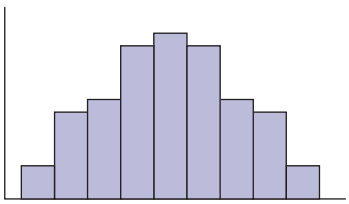


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d



e



4 Decide if the following is a *sample* or a *population*.

a A group of 10 chocolates is selected from a box of chocolates in a taste test.
b All the employees of a particular company are surveyed as to how they travel to work.
c Every electricity bill for the month of January in a state is analysed by the state government.
d 120 droplets of water are taken from a tank and analysed for their chemical content.

Example 10c

5 A survey is conducted asking people how many pets they own. You can assume it is a representative sample of the population.

Pets	0	1	2	3	4	Total
Responses	20	18	6	4	2	50

a Plot the results on a histogram.
b Is the distribution skewed, bi-modal or symmetric?
c Of a group of 1000 people, how many of them would you expect to have no pets?
d Of a group of 5000 people, how many of them would you expect to have 2 or more pets?
e Why would conducting this survey outside a veterinary clinic cause a bias in the results?

6 Fred attempts to find a relationship between people's ages and their incomes. He is considering some questions to put in his survey. For each question, decide whether it should be included in the survey, giving a brief explanation.

a What is your current age in years?
b Are you rich?
c Are you old?
d How much money do you have?
e What is your name?
f How much money did you earn in the past year?
g How much money did you receive today?



7 For each of the following survey questions, give an example of an unsuitable location and time to conduct the survey if you wish to avoid a bias.

a A survey to find the average number of children in a car.
b A survey to find how many people are happy with the current prime minister.
c A survey to find the proportion of Australians who are vegetarians.
d A survey to find the average cost of supermarket groceries.

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8F

FLUENCY

8 **a** Design a survey question to decide the mean number of siblings of a Year 8 student in your school.
b Conduct the survey among a random sample of students from your class.
c Present the results in a histogram.
d Comment on whether the distribution is symmetric, skewed or bi-modal.
e How might the mean vary based on the random sample you chose?

9 | 9, 10 | 9, 10

PROBLEM-SOLVING

9 In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency table.

Weight (g)	49	50	51	52	53	54	55	108
Frequency	2	5	10	30	42	27	11	1

a Which weight value is an outlier?
b How could the automatic weighing mechanism have caused this measurement error?
c Disregarding the 108 gram result, is this distribution skewed or symmetric?
d To find the spread of weights, the machine can calculate the range, or the IQR. Which would be a better value to use? Justify your answer.
e If measurement errors are not removed, would the mean or the median be a better guide to the 'central weight' of the bars?

10 A survey is being conducted to decide how many adults use Mathematics later in life.
a If someone wanted to make it seem that most adults do not use Mathematics, where and when could they conduct the survey?
b If someone wanted to make it seem that most adults use Mathematics a lot, where and when could they conduct the survey?
c How could the survey be conducted to provide less biased results?

11 | 11, 12 | 12, 13

REASONING

11 Robert wishes to find out how much time high school students spend on homework.
a Give some reasons why surveying just his Maths class might introduce bias.
b Why would surveying just the people on his soccer team introduce bias?
c He decides to choose 50 people from across the whole school. Who should he choose in order to minimise the bias? Justify your answer.
d Explain how the mean time spent on homework varies based on the sample chosen.

12 The height of Year 12 students at a particular school is graphed on a histogram. Explain why it is likely to form a bi-modal distribution at a co-educational school but not at an all girls school.

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8F

REASONING

13 Whenever a survey is conducted, even if the people being asked are randomly selected, bias can be introduced by the fact that some people will not answer the questions or return the surveys.

- **a** Assume 1000 surveys are mailed out to a random sample of people asking the question 'Do you think Australia should be a republic?' and 200 replies are received: 150 say 'yes' and 50 say 'no'.
  - **i** On the basis of the 200 people who returned the survey, what percentage are in favour of a republic?
  - **ii** If all 800 people who did not respond would have said 'yes', what percentage are in favour of a republic?
  - **iii** If all 800 people who did not respond would have said 'no', what percentage are in favour of a republic?
  - **iv** If the 1000 people receiving the survey are representative of the Australian population, what conclusion can be drawn about the popularity of a republic?
- **b** A survey is being conducted to decide how many people feel they are busy. Describe how bias is introduced by the people who agree to participate in the survey.
- **c** Give an example of another survey question that would cause a bias to be introduced simply on the basis of the people who participate.
- **d** Sometimes surveys ask the same question in different ways over a number of pages. Although this additional length makes it less likely that people will return the survey, why might the questioner wish to ask the same question in different ways?

Media bias — — 14

ENRICHMENT

14 Search a newspaper, magazine or website and find an example of a survey or poll that has been conducted.

- **a** Decide the following, with justifications:
  - **i** Can you tell if they chose a large enough sample for the survey?
  - **ii** Can you tell whether the sample chosen was representative or biased?
  - **iii** Would this newspaper, magazine or website have an incentive to choose or publish biased results?
- **b** Design a perfect survey to get the information that has been reported, and describe how you would choose the survey recipients.
- **c** Conduct the survey on a small random sample that is representative of the wider population, and compare your results with those reported.



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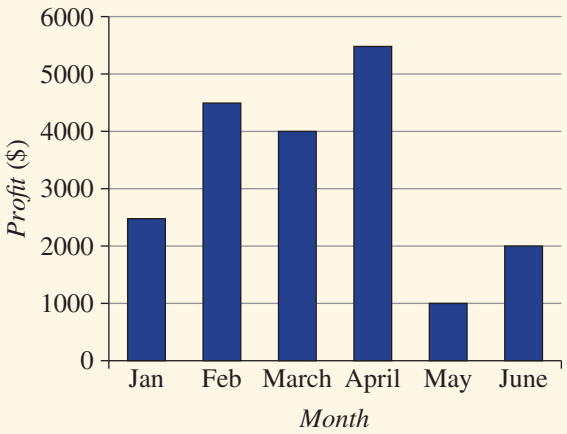
Progress quiz

8A **1** The column graph represents the monthly profit for a company in its first six months of operation.

a In which month did they have the greatest profit, and how much did they make?

b What is the difference in profit from February to March?

c What was the total profit for the six months shown?

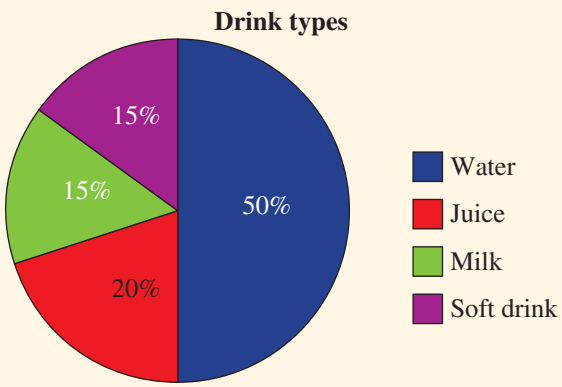


8A **2** A sector graph shows the favourite type of drink of a group of students.

a What is the angle of the sector indicating water?

b If 100 were surveyed, how many said water was their favourite drink?

c What two types of drinks had the same number of votes?



8A **3** The height, in centimetres, of a seedling is measured at the same time each day for a week and its height is shown in the line graph.

a On which day was the height:

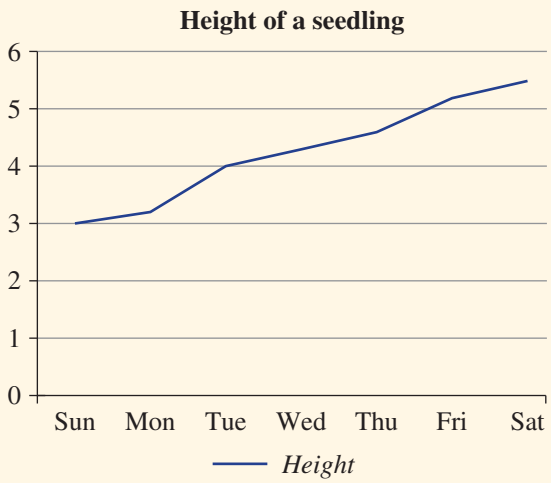
i 4 cm?

ii 4.6 cm?

iii 55 mm?

b What was the height of the seedling on day 1?

c How many millimetres did the seedling grow over the week?



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8B/C **4** **a** Put the following data into a frequency table.

2, 3, 2, 3, 4, 4, 4, 4, 5, 3, 4, 5, 3, 2, 5, 4, 3, 4, 4, 4, 5, 5, 2, 4, 3, 5, 5, 4, 5, 5

Number	2	3	4	5
Tally				
Frequency				

b Represent the frequency table as a histogram.

8D **5** For the data set 3, 7, 2, 8, 10:

a find the mean

b state the median

8D/E **6** For the data in Question **4**, find the:

a mean **b** median **c** mode Ext **d** range Ext **e** IQR

8F **7** When conducting a survey, it is important to avoid bias. In each of the following cases within a school, explain how the bias is introduced.

a Surveying people in a cafeteria to find out how many times per week students eat homemade food.

b Surveying Year 12 students to find out how much time an average high school student spends on homework.

c Surveying parents of Year 7 students to find out how many children the average adult has.



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8G Probability

Interactive

Widgets

HOTsheets

Walkthroughs

Most people would agree that being hit by lightning and getting rained upon are both possible when going outside, but that rain is more likely. **Probability** gives us a way to describe how much more likely one event is than another. A probability is a number between 0 and 1 where 0 means 'impossible' and 1 means 'certain'.

If the outcomes are equally likely, we find the probability of an event by counting the ways it can happen and dividing by the total number of outcomes.

Let's start: Estimating probabilities

Try to estimate the probability of the following events, giving a number between 0 and 1. Compare your answers with other students in the class and discuss any differences.

- Flipping a 'tail' on a 50-cent coin
- The next word you hear the prime minister say is 'good'
- Rolling three 6s in a row on a fair die
- Correctly guessing a number between 1 and 10
- Tomorrow being a rainy day
- A shuffled deck of cards having an ace on top

Are there some events for which there is more than one correct answer?



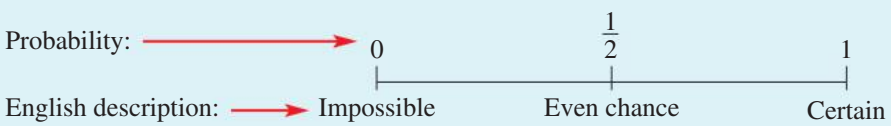
What is the probability of a rainy day tomorrow?

Key ideas

- An **experiment** is a situation involving chance which leads to a set of results.
- A **trial** is a process which can be repeated to produce results.
  Examples include: flipping a coin, rolling a die or spinning a spinner.
- An **outcome** is a possible result of the experiment, like rolling a 5 or a coin showing heads.
- An **event** is either a single outcome (e.g. rolling a 3) or a collection of outcomes (e.g. rolling a 3, 4 or 5).
- The **probability** of an event is a number between 0 and 1 that represents the chance that the event occurs. If all the outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of outcomes}}$$

- Probabilities are often written as fractions, but can also be written as decimals or percentages.



- The **sample space** is the set of possible outcomes of an experiment. For example, the sample space for the roll of a die is 1, 2, 3, 4, 5, 6.
- The **complement** of some event E is written E' (or not E). E' is the event that E does not occur. For example, the complement of 'rolling the number 3' is 'rolling a number other than 3'.
- For any event, either it or its complement will occur. That is, $\Pr(E) + \Pr(E') = 1$.

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Key ideas

- The following language is also commonly used in probability:
  - 'at least', for example, 'at least 3' means 3, 4, 5, …
  - 'at most', for example, 'at most 7' means …, 5, 6, 7
  - 'or', for example, 'rolling an even number or a 5' means rolling a 2, 4, 5, 6
  - 'and', for example, 'rolling an even number and a prime number' means rolling a 2

Example 11 Working with probabilities

The letters of the word PRINCE are written onto 6 equal-sized cards and one is chosen at random.

a State the sample space.

b Find Pr(the letter N is chosen).

c What is the sample space of the event V = choosing a vowel?

d Find Pr(V).

e State the sample space of the complement of choosing a vowel, written V'.

f Find Pr(V').

SOLUTION	EXPLANATION
a P, R, I, N, C, E	The sample space is all the possible outcomes when a single card is chosen. In this case each of the letters in the word.
b $\Pr(N) = \frac{1}{6}$	There are 6 equally likely cards and 1 of them has the letter N.
c I, E	The sample space V includes all the vowels in the word PRINCE.
d $\Pr(V) = \frac{2}{6}$ $= \frac{1}{3}$	There are 2 cards with vowels, so probability = $2 \div 6$.
e P, R, N, C	The complement of V is all the outcomes that are not in V, i.e. all the letters that are not vowels.
f $\Pr(V') = \frac{4}{6}$ $= \frac{2}{3}$	There are 4 cards that do not have vowels. Alternatively $\Pr(V') = 1 - \Pr(V)$ so $\Pr(V') = 1 - \frac{1}{3}$ $= \frac{2}{3}$

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Exercise 8G

1–3 | 3 | —

UNDERSTANDING

1 Match each experiment with the set of possible outcomes.

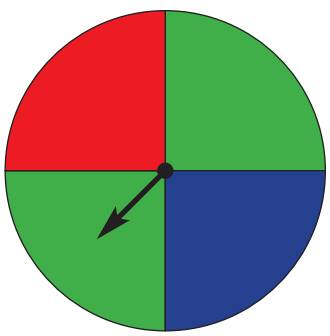
a Flipping a coin
b Choosing a number that is at least 2 and at most 5
c Choosing a letter of the word MATHS
d Rolling a die

A 1, 2, 3, 4, 5, 6 **B** Heads, Tails **C** 2, 3, 4, 5 **D** M, A, T, H, S

2 The following events are shown with their probabilities.
Event A: 0 Event B: 0.9 Event C: 1 Event D: 0.5

a Which of the four events is most likely to occur?
b Which of the four events is sure not to occur?
c Which is more likely – event B or event D?
d Which event is certain to occur?

3 The spinner is spun and could land with the pointer on any of the four sections. Answer true or false for the following.



a Red and blue are equally likely outcomes.
b Green is less likely to occur than blue.
c The probability of it landing yellow is 0.
d Red is less likely to occur than green.

4–7 | 4–8 | 4, 7, 9

FLUENCY

Example 11a
Example 11b
Example 11c
Example 11d

4 The letters of the word PIANO are written on 5 cards and then one card is drawn from a hat at random.

a List the sample space.
b Find Pr(the letter A is chosen).
c Find Pr(a vowel is chosen).
d Find Pr(a consonant is drawn).
e Find Pr(the letter chosen is not an N).

5 A group of 5 students walk into a classroom of a coeducational school and the number of boys in the group is counted.

a List the sample space.
b Find the probability of the following events.
  i An odd number of boys enters
  ii Exactly 2 boys enter
  iii All the students were boys
  iv At least 3 of the students were boys
c Using phrases like ‘at least’ and ‘at most’, give descriptions for the following events.
  i 0, 1, 2 **ii** 4, 5
  iii 2, 3, 4, 5 **iv** 0, 2, 4

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Example 11e
Example 11f

6 A fair die is rolled.

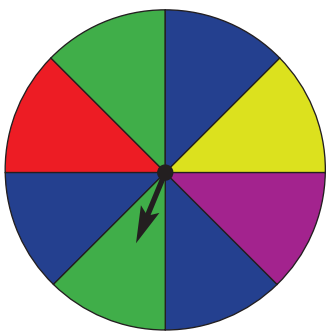


- **a** List the sample space.
- **b** Find Pr(5). That is, find the probability that a 5 is rolled.
- **c** Find Pr(even number).
- **d** State the sample space of the complement of 'rolling a 5'.
- **e** State the probability that a 5 is not rolled.
- **f** What is the probability of rolling a 14?

7 There are five red marbles, two green marbles and three black marbles. The 10 marbles are placed into a hat and one is picked out.

- **a** What is Pr(red)? That is, what is the probability that the picked marble is red?
- **b** Find Pr(green).
- **c** Find Pr(black).
- **d** Find Pr(a black or a red marble is drawn).
- **e** Find Pr(red′). That is, find the probability of not choosing a red marble.
- **f** Find Pr(black′).
- **g** Give an example of an event that has a probability of 0.

8 A spinner has the arrangement of colours as shown.



- **a** How many equally likely outcomes are there when this spinner is spun?
- **b** Find Pr(red).
- **c** State Pr(green).
- **d** Find Pr(blue).
- **e** List the sample space of the complement of the spinner landing on blue.
- **f** Find Pr(blue′).
- **g** Find Pr(red or green or blue).
- **h** What is an event that is equally likely to 'spinning red'?
- **i** Give an example of an event that has a probability of 0.

9 The numbers 1 to 10 are written on cards. A card is chosen at random.

- **a** List the sample space.
- **b** Find the probability of choosing a 5.
- **c** Find Pr(7 or 9).
- **d** Find Pr(a multiple of 3 is chosen).
- **e** Find Pr(at least 6).
- **f** Find Pr(a factor of 24).

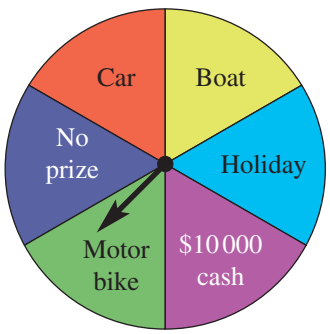
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8G

10, 11 | 11–13 | 11–14

PROBLEM-SOLVING

10 On a game show, a wheel is spun for a prize with the options as shown.



- **a** Joan wants to go on a $10 000 holiday so she is happy with the cash or the holiday. What is the probability she will get what she wants?
- **b** What is the probability of getting a prize that is not the cash?
- **c** What is Pr(car or motorbike)?
- **d** What is the probability of winning a prize?

11 Jamie has a collection of equally shaped and sized marbles in his pocket. Some are blue, some are green and some are white. It is known there is a 0.3 chance of a green marble being chosen, and a 0.75 chance of not choosing a blue marble.

- **a** What is the probability of not choosing a green marble?
- **b** What is the probability of choosing a blue marble?
- **c** Find the probability of choosing a white marble. Hint: the sum of the three colours' probabilities is 1.
- **d** What is the minimum number of marbles Jamie could have in his pocket?

12 A weighted die has the numbers 1 to 6, but the probability of each number occurring is unknown. Decide whether or not the following statements are guaranteed to be true for this die.

- **a** $\Pr(3) = \frac{1}{6}$
- **b** Pr(at least 4) = Pr(4) + Pr(5) + Pr(6)
- **c** Pr(at least 1) = 1
- **d** Pr(odd) = Pr(even)
- **e** Pr(odd) + Pr(even) = 1
- **f** Pr(at most 3) = 1 − Pr(at least 3)
- **g** Pr(at most 4) = 1 − Pr(at least 5)
- **h** Pr(at most 4) + Pr(at least 4) = 1 + Pr(4)

13 Six counters coloured red, purple or orange are placed in a pocket. You are told that $\Pr(\text{red or orange}) = \frac{1}{2}$ and $\Pr(\text{red or purple}) = \frac{2}{3}$.

- **a** How many counters of each colour are there?
- **b** State Pr(red).
- **c** Find Pr(purple).
- **d** Find Pr(orange′).

14 Draw a spinner that has Pr(red) = 0.2, Pr(blue) = 0.3 and Pr(green) = 0.5.

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8G

	15	15, 16a, b	15, 16

REASONING

15 In a large bucket there are 2 red balls and 8 blue balls.

- **a** State Pr(red).
- **b** One of each colour is added. What is the new Pr(red)?
- **c** The procedure of adding a red ball and a blue ball is repeated several times. How many balls are in the bucket when $\Pr(\text{red}) = \frac{1}{3}$?
- **d** Imagine the procedure is repeated many times. What value does Pr(red) eventually approach as more balls are added? It might be helpful to imagine 1000 balls of each colour are added and use decimals.

16 In a driving test, each student can pass or fail. That is, pass and fail are complements of each other. For Noni Pr(pass) = 0.4 and for Anthony Pr(pass) = 0.6.



- **a** Maria says she is twice as likely to pass as Noni. What is Maria's probability of passing?
- **b** Olivia says she is twice as likely to fail as Anthony. Find the following probabilities.
  - **i** Pr(Anthony fails)
  - **ii** Pr(Olivia fails)
  - **iii** Pr(Olivia passes)
- **c** If the probability of Noni passing is p, what is the probability of someone passing if they are twice as likely as Noni to pass? Write an expression involving p.
- **d** If the probability of Anthony passing is p, what is the probability of passing of someone who is twice as likely as Anthony to fail? Write an expression involving p.
- **e** Give an example to illustrate the difference between being twice as likely to fail and half as likely to pass.

Marble puzzle	—	—	17

ENRICHMENT

17 A bag initially contains 7 blue, 2 red and 6 green marbles. For each of the following, describe which marbles were removed from the bag.

- **a** 3 marbles are removed. Now $\Pr(\text{R}) = 0$, $\Pr(\text{B}) = \frac{1}{2}$.
- **b** 3 marbles are removed. Now $\Pr(\text{B}) = \Pr(\text{G})$ and $\Pr(\text{R}) = \frac{1}{6}$.
- **c** 5 marbles are removed. Now $\Pr(\text{G}) < \Pr(\text{R}) < \Pr(\text{B})$.
- **d** 12 marbles are removed. Now $\Pr(\text{B}) = \Pr(\text{G}) = \Pr(\text{R})$.
- **e** 1 marble is removed. Now $\Pr(\text{B}) = \frac{1}{2}$, $\Pr(\text{G}) = \frac{5}{14}$.
- **f** Some marbles are removed. Now $\Pr(\text{G}) \times 2 = \Pr(\text{R})$ and $\Pr(\text{R}) \times 3 = \Pr(\text{B})$.
- **g** Some marbles are removed. Now $\Pr(\text{R}) = \frac{1}{2}$ and $\Pr(\text{B}) = \Pr(\text{G}) = \frac{1}{4}$.

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8H Two-step experiments EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

Sometimes an experiment consists of two independent steps, such as when a coin is tossed and then a die is rolled. Or perhaps a card is pulled from a hat and then a spinner is spun. We can use tables to list the sample space.

Consider the following example in which a coin is flipped and then a die is rolled.

		Die					
		1	2	3	4	5	6
Coin	Heads	H1	H2	H3	H4	H5	H6
	Tails	T1	T2	T3	T4	T5	T6

There are 12 outcomes listed in the table. So the probability of getting a 'tail' combined with the number 5 is $\frac{1}{12}$.

Let's start: Monopoly mystery

In a board game, two dice are rolled and the player moves forward by their sum.

- What are the possible values that the sum could have?
- Are some values more likely than others? Discuss.
- How likely is it that the numbers showing on the two dice will add to 5?



Are you as likely to roll a 9 on two dice as any other number?

Key ideas

- If an experiment has two independent steps, the outcomes can be listed as a table.
- The probability is still given by:

$$\text{Pr(event)} = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}}$$

Example 12 Using a table for two-step experiments

A spinner with the numbers 1, 2, and 3 is spun, and then a card is chosen at random from the letters ATHS.

a Draw a table to list the sample space of this experiment.

b How many outcomes does the experiment have?

c Find the probability of the combination 2S.

d Find the probability of an odd number being spun and the letter H being chosen.

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SOLUTION	EXPLANATION
a	The sample space of the spinner (1, 2, 3) is put into the left column. The sample space of the cards (A, T, H, S) is put into the top row.
b There are 12 outcomes.	The table has $4 \times 3 = 12$ items in it.
c $\Pr(2S) = \frac{1}{12}$	All 12 outcomes are equally likely. Spinning 2 and choosing an S is one of the 12 outcomes.
d $\Pr(\text{odd, H}) = \frac{2}{12} = \frac{1}{6}$	Possible outcomes are 1H and 3H, so probability $= 2 \div 12$.

	A	T	H	S
1	1A	1T	1H	1S
2	2A	2T	2H	2S
3	3A	3T	3H	3S

Exercise 8H

1, 2 | 2 | —

UNDERSTANDING

1 A coin is flipped and then a spinner is spun. The possible outcomes are listed in the table below.

	1	2	3	4	5
H	H1	H2	H3	H4	H5
T	T1	T2	T3	T4	T5

a How many outcomes are possible?
b List the four outcomes in which an even number is displayed on the spinner.
c Hence, state the probability that an even number is displayed.
d List the outcomes for which tails is flipped and an odd number is on the spinner.
e What is Pr(T, odd number)?

2 Two coins are tossed and the four possible outcomes are shown below.

		20-cent coin	
		H	**T**
50-cent coin	**H**	HH	HT
	T	TH	TT

a What is the probability that the 50-cent coin will be heads and the 20-cent coin will be tails?
b For which outcomes are the two coins displaying the same face?
c What is the probability of the two coins displaying the same face?



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8H

3–5 3–5 4–6

FLUENCY

Example 12

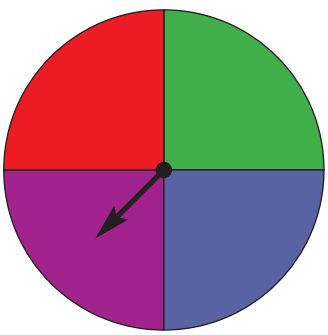
3 A coin is flipped and then a die is rolled.

a Draw a table to list the sample space of this experiment.
b How many possible outcomes are there?
c Find the probability of the pair H3.
d Find the probability of 'heads' on the coin with an odd number on the die.

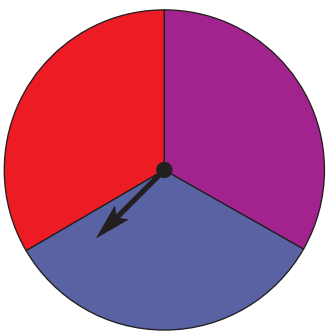
4 A letter is chosen from the word LINE and another is chosen from the word RIDE.

a Draw a table to list the sample space.
b How many possible outcomes are there?
c Find Pr(NR), i.e. the probability that N is chosen from LINE and R is chosen from RIDE.
d Find Pr(LD).
e Find the probability that two vowels are chosen.
f Find the probability that two consonants are chosen.
g Find the probability that the two letters chosen are the same.

5 The spinners shown below are each spun.



Spinner 1



Spinner 2

a Draw a table to list the sample space. Use R for red, P for purple and so on.
b Find the probability that spinner 1 will display red and spinner 2 will display blue.
c Find the probability that both spinners will display red.
d What is the probability that spinner 1 displays red and spinner 2 displays purple?
e What is the probability that one of the spinners displays red and the other displays blue?
f What is the probability that both spinners display the same colour?

6 A letter from the word EGG is chosen at random and then a letter from ROLL is chosen at random. The sample space is shown below.

	R	O	L	L
E	ER	EO	EL	EL
G	GR	GO	GL	GL
G	GR	GO	GL	GL

a Find Pr(ER).
b Find Pr(GO).
c Find Pr(both letters are vowels).
d Find Pr(both letters are consonants).

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8H

7, 8 | 7, 8 | 8, 9

PROBLEM-SOLVING

7 Two dice are rolled for a board game. The numbers showing are then added together to get a number between 2 and 12.

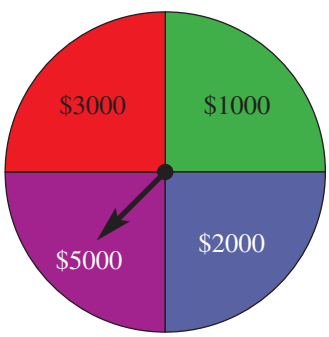
- **a** Draw a table to describe the sample space.
- **b** Find the probability that the two dice add to 5.
- **c** Find the probability that the two dice do not add to 5.
- **d** What is the most likely sum to occur?
- **e** What are the two least likely sums to occur between 2 and 12?

8 In Rosemary's left pocket she has two orange marbles and one white marble. In her right pocket she has a yellow marble, a white marble and 3 blue marbles. She chooses a marble at random from each pocket.

- **a** Draw a table to describe the sample space. Hint: The left-pocket outcomes are W, O, O.
- **b** Find the probability that she will choose an orange marble and a yellow marble.
- **c** What is the probability that she chooses a white marble and a yellow marble?
- **d** What is the probability that she chooses a white marble and an orange marble?
- **e** Find the probability that a white and a blue marble are selected.
- **f** What is the probability that the two marbles selected are the same colour?

9 In a game show a wheel is spun to determine the prize money and then a die is rolled. The prize money shown is multiplied by the number on the die to give the total winnings.

- **a** What is the probability that a contestant will win $6000?
- **b** What is the probability that they win more than $11 000?
- **c** What is the probability they will win $11 000 or less?



10 | 10, 11 | 10, 11

REASONING

10 Two separate experiments are conducted simultaneously. The first has 7 possible outcomes and the second has 9 outcomes. How many outcomes are there in the combined experiment?

11 In a deck of cards there are four suits ♡, ♦, ♣, ♠ and 13 cards in each suit (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K).

- **a** If a card is chosen at random, what is Pr(3♦)?
- **b** What is Pr(red king)?
- **c** If two cards are chosen at random from separate decks, what is the probability that they are both diamonds? (Hint: Do not draw a 52×52 table).

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8H

REASONING

d If two cards are chosen at random from separate decks, what is the probability that they are both red cards?

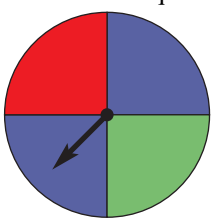
e What is the probability that 3♦ is chosen from both decks?

f Why is it important that the 2 cards are chosen from separate decks? How would your answers to parts **c–e** change if the two cards were drawn from the same deck?

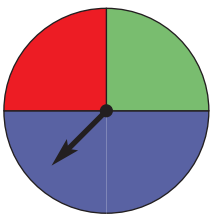
ENRICHMENT

Spinners with unequal areas — — 12

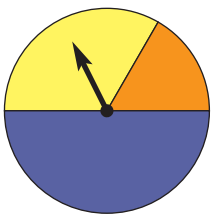
12 The two spinners below are identical in terms of their probabilities, even though the areas in Spinner 2 are unequal.



Spinner 1
Outcomes: R, G, B, B



Spinner 2
Outcomes: R, G, B



Spinner 3
Outcomes: B, Y, O

a Use the fact that Spinner 1 and Spinner 2 are equivalent to find the following probabilities for Spinner 2:

i Pr(red) **ii** Pr(blue)

b Spinner 2 is also equivalent to choosing a letter from the word RGBB. If Spinner 2 is spun twice, what is the probability of:

i two reds? **ii** two blues?
iii a red, then a green? **iv** a red and a green (in either order)?

c Spinner 3 has $\Pr(\text{orange}) = \frac{1}{6}$, $\Pr(\text{yellow}) = \frac{1}{3}$ and $\Pr(\text{blue}) = \frac{1}{2}$.

What 6 letters could be used to describe the 6 equally likely outcomes when Spinner 3 is spun?

d If Spinner 3 is spun twice, find the probability of obtaining:

i yellow twice **ii** the same colour twice
iii orange and then blue **iv** orange and blue (either order)
v at least one orange **vi** at least one blue

e Spinners 2 and 3 are both spun. Find the probability of obtaining:

i red then orange **ii** green then blue
iii orange and not blue **iv** both blue
v neither blue **vi** neither red

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8I Tree diagrams EXTENDING



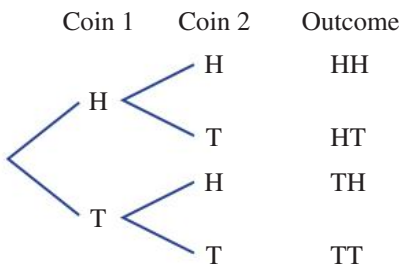






When two coins are flipped, we can draw a table to list the sample space. But if three coins are flipped, then we would need a three-dimensional table to list all outcomes. Imagine trying to find probabilities when five coins are flipped!

Another tool that mathematicians use for probability is the tree diagram. This tree diagram describes the four outcomes when two coins are flipped.



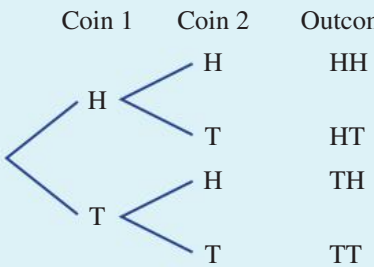
It is important to be able to read a tree diagram correctly. The first row (HH) represents the outcome where the first coin flipped was heads and the second coin flipped was heads. The third row (TH) represents the outcome where the first coin was tails and the second was heads.

Let's start: Coin puzzle

- If two coins are flipped, rank these outcomes from most likely to least likely.
  - Exactly two heads are flipped.
  - Exactly one head and exactly one tail are flipped.
  - At least one coin shows tails.
  - Three tails are shown.
- How might the order change if three coins are flipped? Compare your answers with other students.

Key ideas

- A **tree diagram** can be used to list the outcomes of experiments that involve two or more steps.



- At this stage, we will only consider tree diagrams for which each branch corresponds to an equally likely outcome.

Example 13 Using tree diagrams

Three fair coins are flipped.

a List the sample space using a tree diagram.

b How many possible outcomes are there?

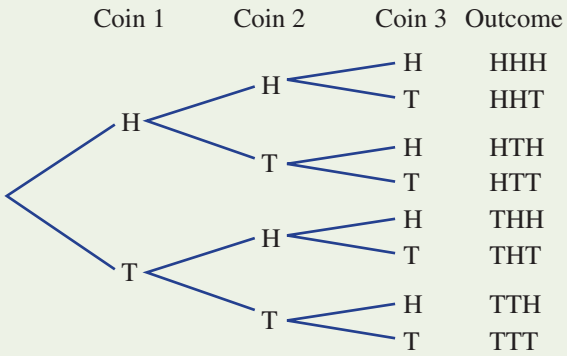
c Find the probability that the first coin is heads and the next two are tails.

d Find the probability that exactly two of the coins show heads.

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SOLUTION

a



EXPLANATION

Each coin has two outcomes: heads (H) and tails (T).
After the each coin is flipped, the next coin has two outcomes, so the tree branches out.

b There are 8 possible outcomes.

They are listed: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

c $\Pr(\text{HTT}) = \frac{1}{8}$

This is just one of the eight equally likely outcomes.

d Outcomes: HHT, HTH, THH

$\Pr(\text{exactly 2 heads}) = \frac{3}{8}$

List the outcomes with exactly two heads. There are 3 of them so the probability is $\frac{3}{8}$.

Exercise 8I

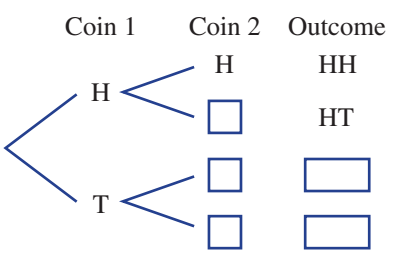
1, 2 | 2 | —

UNDERSTANDING

1 Two coins are flipped.

a Copy and complete the following tree diagram.

b How many equally likely outcomes are possible?



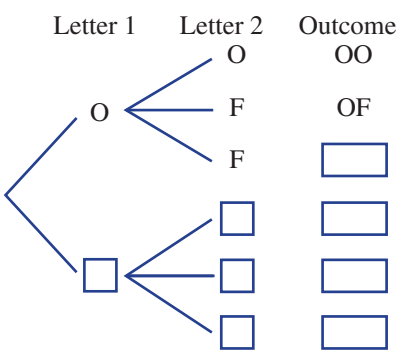
2 A letter from the word ON is chosen and then a letter from the word OFF is chosen.

a Copy and complete the tree diagram.

b Fill in the blanks: The sample space is OO, OF, ____, ____, ____, ____.

c How many equally likely outcomes are there in total?

d How many outcomes have two consonants?



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8I

3, 4 3–5 3, 5

FLUENCY

Example 13

3 A letter from the word CAT is chosen and then a letter from the word GO is chosen.

- **a** List the sample space using a tree diagram.
- **b** How many outcomes are possible?
- **c** Find Pr(C then G).
- **d** Find Pr(T then O).
- **e** Find Pr(2 consonants).

4 A spinner with numbers 1, 2 and 3 is spun twice.

- **a** Show the sample space in a tree diagram.
- **b** Find Pr(1 then 1).
- **c** Find Pr(1 then 2).
- **d** Find Pr(1 and 2 spun in either order).
- **e** Find Pr(both show the same number).
- **f** Find Pr(numbers add to 4).

5 A coin is tossed three times.

- **a** Draw a tree diagram to represent the sample space.
- **b** Find Pr(3 tails).
- **c** Find Pr(at least one head). Hint: this is the complement of 3 tails.
- **d** Find Pr(2 tails then 1 head).
- **e** Find Pr(2 tails and 1 head, in any order).
- **f** Which is more likely: getting exactly 3 tails or getting exactly 2 tails?
- **g** Find the probability of getting at least 2 tails.



6 6, 7 7, 8

PROBLEM-SOLVING

6 Two letters are chosen from the word CAR. Once a letter is chosen it cannot be chosen again.

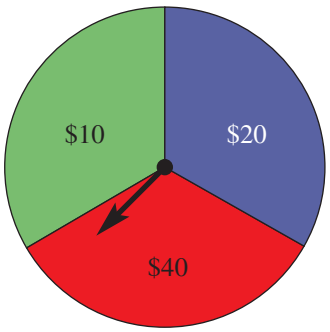
- **a** Draw a tree diagram of the six possible outcomes.
- **b** What is the probability that A and C will be chosen?
- **c** Find Pr(2 consonants).
- **d** Find Pr(2 vowels).
- **e** What is the probability that the letters chosen will be different?

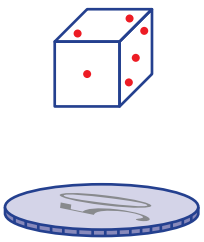
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8I

PROBLEM-SOLVING

7 In a game a prize wheel is spun, then a die is rolled and finally a coin is flipped. If the coin displays heads, you win the prize multiplied by the amount on the die. If the coin displays tails, you get nothing.





- **a** What amount do you win if you spin $20 then roll a 5 and then flip heads?
- **b** Draw a tree diagram showing the 36 possible outcomes.
- **c** What is the probability that you win $80?
- **d** Find Pr(win $100 or more).
- **e** Find Pr(receive less than $15). Include the possibility that you get nothing.

8 The letters of the word PIPE are placed on four cards. Two of the cards are chosen.

- **a** Draw a tree diagram showing all 12 outcomes.
- **b** Find Pr(2 vowels).
- **c** Find Pr(the same letter is on the 2 cards).
- **d** What is Pr(at least one letter is a P)?

9 | 10–12 | 11–13

REASONING

9 If 2 coins are tossed there are 4 outcomes. If 3 coins are tossed there are 8 outcomes. How many outcomes are there if 5 coins are tossed?

10 **a** If a coin is flipped 4 times, what is the probability that it will display heads four times?

- **b** If a coin is flipped 4 times, what is the probability that it will display H, T, T, H in that order?
- **c** If a coin is flipped 5 times, which is more likely: the result HHHHH or the result HTHHT?
- **d** If a coin is flipped 5 times, which is more likely: 5 heads or 3 heads?
- **e** Explain why your answers to parts **c** and **d** are different.

11 The words WORM and MORROW both have four different letters.

- **a** What is the difference between choosing 2 letters from WORM and choosing 2 letters from MORROW?
- **b** Give an example of an event that is impossible when choosing 2 letters from WORM but not impossible when choosing 2 from MORROW.

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8I

REASONING

12 If 3 coins are flipped, there are many possible events. For example, Pr(exactly 3 heads) $= \frac{1}{8}$ and Pr(first coin is tails) $= \frac{1}{2}$. Give an example of an event using 3 coins that has the following probabilities.

a $\frac{1}{4}$ **b** $\frac{5}{8}$ **c** $\frac{3}{4}$ **d** $\frac{7}{8}$ **e** 0 **f** 1

13 A word has 5 different letters that are written on cards.

a How many different outcomes will there be if 2 letters are chosen? The order matters, so EP and PE are different outcomes.

b How many different outcomes will there be if 5 letters are chosen? Again, the order does matter.

c Assume that 4 letters are chosen from the word MATHS. What is the probability that they are all consonants? Hint: Choosing 4 letters to consider is the same as choosing 1 letter to ignore.

d If 4 letters are chosen from the word MATHS, what is the probability that at least one is a vowel?

Lucky test — — 14

ENRICHMENT

14 In a test a student has to flip a coin 5 times. They get 1 point for every time they flip 'heads' and 0 when the flip 'tails'. Their final score is out of 5.

a Draw a tree diagram for this situation.

b What is Pr(5 out of 5)?

c What is Pr(0 out of 5)?

d To pass the test they must get at least 50%. What is Pr(pass the test)?

e What is Pr(2 out of 5)?

f If a student gets 2 out of 5 they may sit a re-test in which they flip 10 coins. What is the probability that a student sitting the re-test will pass? Try to list the outcomes systematically.

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8J Venn diagrams and two-way tables



When two events are being considered, Venn diagrams and two-way tables give another way to view the probabilities. They can be useful when survey results are being considered and converted into probabilities.



Let's start: Are English and Mathematics enemies?



Conduct a poll among students in the class, asking whether they like English and whether they like Maths. Use a tally like the one shown.



	Like Maths	Do not like Maths
Like English	𝍸 I	𝍸 IIII
Do not like English	𝍸 𝍸 I	𝍸 II

Use your survey results to debate these questions:

- Are the students who like English more or less likely to enjoy Maths?
- If you like Maths does that increase the probability that you will like English?
- Which is the more popular subject within your class?

Key ideas

- A **two-way table** lists the number of outcomes or people in different categories, with the final row and column being the total of the other entries in that row or column. For example:

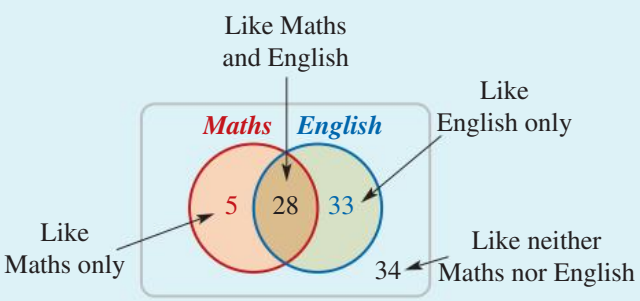
	Like Maths	Do not like Maths	Total
Like English	28	33	61
Do not like English	5	34	39
Total	33	67	100

- A two-way table can be used to find probabilities.
  For example:

$$\Pr(\text{like Maths}) = \frac{33}{100}$$

$$\Pr(\text{like Maths and not English}) \frac{5}{100} = \frac{1}{20}$$

- A **Venn diagram** is a pictorial representation of a two-way table without the total row and column. The two-way table above can be written as shown.



- The word 'or' can sometimes mean 'inclusive or' (A or B or both), and it can sometimes mean 'exclusive or' (A or B but not both).

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Example 14 Constructing Venn diagrams and two-way tables

A survey is conducted of 50 people, asking who likes coffee and who likes tea. It was found that 20 people liked both, 15 people liked coffee but not tea, and 10 people liked tea but not coffee.

a How many people liked neither tea nor coffee?

b Represent the survey findings in a Venn Diagram.

c How many people surveyed like tea?

d How many people like both coffee and tea?

e How many people like coffee or tea (or both)?

f Represent the survey findings in a two-way table.

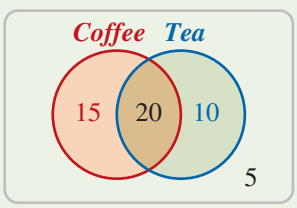
SOLUTION

EXPLANATION

a 5

$50 - 20 - 15 - 10 = 5$ people who do not like either.

b



The Venn diagram includes four numbers corresponding to the four possibilities.

For example, the number 15 means that 15 people like coffee but not tea.

c $20 + 10 = 30$

10 people like tea but not coffee, but 20 people like both. In total 30 people like tea.

d 20

20 out of 50 people like both coffee and tea.

e 45

$15 + 20 + 10 = 45$ people like either coffee or tea or both.

f

	Like coffee	Dislike coffee	Total
Like tea	20	10	30
Dislike tea	15	5	20
Total	35	15	50

The two-way table has the four numbers from the Venn diagram and also a 'total' column (e.g. $20 + 10 = 30$, $15 + 5 = 20$) and a 'total' row. Note that 50 in the bottom corner is both $30 + 20$ and $35 + 15$.



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Example 15 Using two-way tables to calculate probabilities

Consider the two-way table below showing the eating and sleeping preferences of different animals at the zoo.

	Eats meat	No meat	Total
Sleeps during day	20	12	32
Only sleeps at night	40	28	68
Total	60	40	100

a For a randomly selected animal find:
i Pr(sleeps only at night) **ii** Pr(eats meat or sleeps during day or both)

b If an animal is selected at random and it eats meat, what is the probability that it sleeps during the day?

c What is the probability that an animal that sleeps during the day does not eat meat?

SOLUTION | **EXPLANATION**

a i $\Pr(\text{sleeps only at night}) = \frac{68}{100}$

$= \frac{17}{25}$

The total of animals that sleep at night is 68.

So $\frac{68}{100} = \frac{17}{25}$

ii Pr(eats meat or sleeps during day or both)

$= \frac{72}{100}$

$= \frac{18}{25}$

$20 + 12 + 40 = 72$ animals eat meat or sleep during the day (or both). So $\frac{72}{100} = \frac{18}{25}$

b Pr(sleeps during day given that it eats meat)

$= \frac{20}{60}$

$= \frac{1}{3}$

Of the 60 animals that eat meat, 20 sleep during the day, so the probability is $\frac{20}{60} = \frac{1}{3}$

c Pr(does not eat meat given it sleeps during day)

$= \frac{12}{32}$

$= \frac{3}{8}$

Of the 32 animals that sleep during the day, 12 do not eat meat. The probability is $\frac{12}{32} = \frac{3}{8}$

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Exercise 8J

1, 2 | 2 | —

UNDERSTANDING

1 **a** Copy and complete the two-way table by writing in the missing totals.

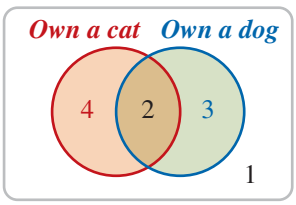
	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	
Total		35	75

b How many people like both apples and bananas?

c How many people dislike apples and dislike bananas?

d How many people were surveyed?

2 Consider the Venn diagram representing cat and dog ownership. State the missing number (1, 2, 3 or 4) to make the following statements true.



a The number of people who own a cat and a dog is _____.

b The number of people who own a cat but do not own a dog is _____.

c The number of people who own neither a cat nor a dog is _____.

d The number of people who own a dog but do not own a cat is _____.

3–6 | 3–7 | 4–7

FLUENCY

Example 14a
Example 14b
Example 14c
Example 14d
Example 14e

3 In a group of 30 students it is found that 10 play both cricket and soccer, 5 play only cricket and 7 play only soccer.

a How many people do not play cricket or soccer?

b Represent the survey findings in a Venn diagram.

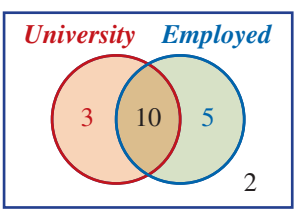
c How many of the people surveyed play cricket?

d How many of the people surveyed play either cricket or soccer or both? (This is inclusive 'or').

e How many of the people surveyed play either cricket or soccer but not both? (This is exclusive 'or').

Example 14f

4 Consider this Venn diagram showing the number of people who have a university degree and the number who are now employed.



a What is the total number of people in the survey who are employed?

b Copy and complete the two-way table shown below.

	Employed	Unemployed	Total
University degree			
No university degree			
Total			

c If the 10 in the centre of the Venn diagram changed to an 11, which cells in the two way table would change?

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8J

FLUENCY

Example 15a

5 The two-way table below shows the results of a poll conducted of a group of students who own mobile phones to see who pays their own bills.

	Boys	Girls	Total
Pay own bill	4	7	11
Do not pay own bill	8	7	15
Total	12	14	26

a How many people participated in this poll?
b How many boys were surveyed?
c How many of the people surveyed pay their own bill?
d Find the probability that a randomly selected person:
i is a boy and pays his own bill
ii is a girl and pays her own bill
iii is a girl
iv does not pay their own bill

Example 15a
Example 15b
Example 15c

6 The two-way table shows the results of a survey on car and home ownership at a local supermarket.

	Own car	Do not own car	Total
Own home	8	2	10
Do not own home	17	13	30
Total	25	15	40

a Represent the two-way table above as a Venn diagram.
b Find Pr(randomly selected person owns a car and a home).
c Find Pr(randomly selected person owns a car but not a home).
d What is the probability that a randomly selected person owns their own home?
e If a person from the group is selected at random and they own a car, what is the probability that they also own a home?
f If a person from the group is selected at random and they own a home, what is the probability that they also own a car?

7 The Venn diagram shows the number of people who like juice and/or soft drinks.



a What is the total number of people who like juice?
b What is the probability that a randomly selected person likes neither juice nor soft drink?
c What is the probability that a randomly selected person likes either juice or soft drink or both?
d What is the probability that a randomly selected person likes just one of the two drink types listed?
e What is the probability that a randomly selected person likes juice given that they like soft drink?

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8J

8a, 9 | 8–10 | 9–11

PROBLEM-SOLVING

8 Copy and complete the following two-way tables.

a

	B	Not B	Total
A	20		70
Not A			
Total		60	100

b

	B	Not B	Total
A		5	
Not A			7
Total	10		18

9 A car salesman notes that among his 40 cars, there are 15 automatic cars and 10 sports cars. Only two of the sports cars are automatic.

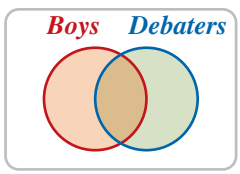
a Create a two-way table of this situation.

b What is the probability that a randomly selected car will be a sports car that is not automatic?

c What is the probability that a randomly selected car will be an automatic car that is not a sports car?

d If an automatic car is chosen at random, what is the probability that it is a sports car?

10 In Year 8 at a school there are 40 boys, half of whom are in the debating club. Of the 100 students in Year 8, 35 are in the debating club.



a Copy and complete the Venn Diagram shown to the right to describe the situation.

b What is the probability, as a percentage, that a randomly selected person in Year 8:

i is a boy in the debating club?

ii is a girl in the debating club?

iii is not a boy?

iv is not in the debating club?

c If a Year 8 debater is chosen at random, what is the probability they are a boy?

d If a girl from Year 8 is chosen, what is the probability she is in the debating club?

11 A page of text is analysed and, of the 150 words on it, 30 were nouns, 10 of which started with a vowel. Of the words that were not nouns, 85 of them did not start with vowels.

a If a word on the page is chosen at random, what is the probability that it is a noun?

b How many of the words on the page started with vowels?

c If a word on the page starts with a vowel, what is the probability that it is a noun?

d If a noun is chosen at random, what is the probability that it starts with a vowel?

12 | 12, 13 | 13, 14

REASONING

12 The word 'or' can mean inclusive 'or' and it can mean exclusive 'or'. Which has a higher probability: A inclusive-or B or A exclusive-or B? Justify your answer.

13 In a two-way table, there are 9 spaces to be filled with numbers.

a What is the minimum number of spaces that must be filled before the rest of the table can be determined? Explain your answer.

b If you are given a two-way table with 5 spaces filled, can you always determine the remaining spaces? Justify your answer.

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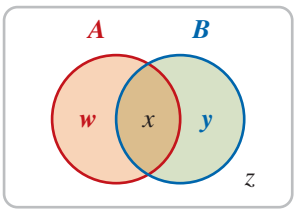
8J

REASONING

c Explain why the following two-way table must contain an error.

	B	B′	Total
A	20		
A′		29	
Total	62		81

14 In this Venn diagram, w, x, y and z are all unknown positive integers.



a Write an algebraic expression for Pr(both A and B) using any of the variables w, x, y and z.

b Write an algebraic expression for Pr(A) using any of the variables w, x, y and z.

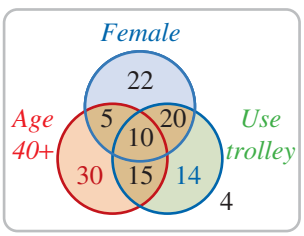
c Copy and complete this two-way table using algebraic expressions.

	B	B′	Total
A	x		
A′		z	
Total	$x + y$		

ENRICHMENT

Triple Venn diagrams — — 15

15 A group of supermarket shoppers is surveyed on their age, gender and whether they shop with a trolley or a basket. This Venn diagram summarises the results.



a How many shoppers were surveyed?

b How many of the shoppers were 40 or over?

c What is the probability that a randomly selected shopper:

i uses a trolley? **ii** is female?

iii is 40 or over? **iv** is male and uses a trolley?

v is female and under 40?

vi is under 40 and using a trolley?

d If a female shopper is chosen at random, what is the probability that she:

i uses a trolley? **ii** is 40 or over?

e If someone pushing a trolley is chosen at random, what is the probability that they:

i are male? **ii** are under 40?

f Describe what you know about the four people on the outside the three circles in the diagram.

g If all you know about someone is that they push a trolley, are they more likely to be male or female? Justify your answer.

h If a female is shopping, are they more likely to push a trolley or to use a basket?

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8K Experimental probability

Interactive

Sometimes the probability of an event is unknown or cannot be determined using the techniques learnt earlier. An experiment can be used to estimate an event's probability and this estimate is called an **experimental probability**.

Widgets

HOTsheets

Walkthroughs

Let's start: Dice roller

For this experiment, three dice are required per student or group.

- Roll the dice 20 times and count how many times the dice add to 10 or more.
- Each group should use this to estimate the probability that three dice will add to 10 or more when rolled.
- Combine the results from multiple groups to come up with a probability for the entire class. Discuss whether this should be more or less accurate than the individual estimates.



Comparing the results of repeated dice rolls lets you determine the experimental probability.

Key ideas

- The **experimental probability** of an event based on a particular experiment is defined as

$$\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$$

- The **expected number** of occurrences = probability × number of trials.
- Complex events can be simulated. A **simulation** is conducted using random devices such as coins, dice, spinners or random number generators.

Example 16 Working with experimental probability

A number of red, white and orange marbles are placed in a jar. Repeatedly, a marble is taken out, its colour is noted and the marble is replaced in the jar. The results are tallied in the table.

Red	White	Orange
𝍸 |	𝍸 𝍸 ||	𝍸 𝍸

a What is the experimental probability of a red marble being chosen next?

b What is the experimental probability of a red or a white marble being chosen?

c If the experiment is done 600 times, what is the expected number of times that an orange marble is selected?

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SOLUTION	EXPLANATION
a $\frac{8}{30} = \frac{4}{15}$	Experimental probability = $\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$
b $\frac{20}{30} = \frac{2}{3}$	Red or white marbles were selected 20 times out of the 30 trials.
c Expected number $= \frac{1}{3} \times 600 = 200$	$\Pr(\text{orange}) = \frac{1}{3}$ Expected number = probability × number of trials

Example 17 Designing and running a simulation

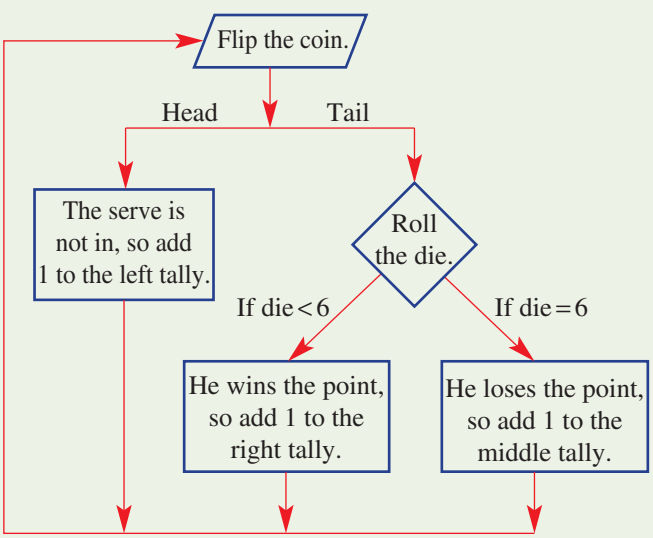
A tennis player has a 1 in 2 chance of getting his first serve in. If it goes in he has a 1 in 6 chance of losing the point. Design and run a simulation to estimate the probability that he gets his first serve in and wins the point.

SOLUTION

Create a tally for the results.

	Serve not in	Serve in but lose point	Serve in and win point
Tally			
Frequency			

Use one coin and one die.



Repeat the experiment as many times as desired.

EXPLANATION

The results will be stored in a tally.

Decide which random devices will be used for the simulation.

The simulation is described using the random devices.

The more times the simulation is repeated, the closer the experimental probability should be to the actual probability.

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Exercise 8K

1, 2 | 2 | —

UNDERSTANDING

1 A spinner is spun 10 times and the colour shown is recorded:
blue, blue, green, red, blue, green, blue, red, blue, blue

- **a** How many times was green shown?
- **b** What is the experimental probability of green being spun?
- **c** What is the experimental probability of blue being spun?

2 A fair die is rolled 100 times and the number 5 occurs 19 times.

- **a** What is the experimental probability of a 5 being rolled?
- **b** What is the actual probability of a 5 being rolled on a fair die?
- **c** For this experiment, which is greater: the experimental probability or the actual probability?

3–7 | 3–8 | 4–8

FLUENCY

Example 16

3 A spinner is spun 50 times and the results are shown in the frequency table below.

Red	Blue	White	Purple
30	5	2	13

- **a** What is the experimental probability of red?
- **b** What is the experimental probability of blue?
- **c** What is the experimental probability of red or purple?
- **d** If the spinner were spun 1000 times, what is the expected number of times that white would be spun?

4 A group of households are surveyed on how many cars they own. The results are shown.

0 cars	1 car	2 cars	3 cars	4 cars
𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	𝍸 |||	||

- **a** Write the tallied results as a frequency table.
- **b** How many households were surveyed?
- **c** What is the experimental probability that a randomly chosen household owns no cars?
- **d** What is the experimental probability that a randomly chosen household owns at least 2 cars?

5 A fair coin is tossed 100 times.

- **a** What is the expected number of times that it will land heads?
- **b** Is it possible that it will land heads 52 times?
- **c** Is it possible that it will land heads 100 times?

6 A die is painted so that 3 faces are blue, 2 faces are red and 1 face is green.

- **a** What is the probability that it will display red on one roll?
- **b** How many times would you expect it to display red on 600 rolls?
- **c** How many times would you expect it to display blue on 600 rolls?

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8K

FLUENCY

7 A spinner displays the numbers 1, 2, 3 and 4 on four sectors of different sizes. It is spun 20 times and the results are 1, 3, 1, 2, 2, 4, 1, 1, 3, 1, 2, 4, 4, 2, 4, 3, 1, 1, 3, 2.

a Give the experimental probability that the spinner will land on:
i 1 **ii** 2 **iii** 3 **iv** 4

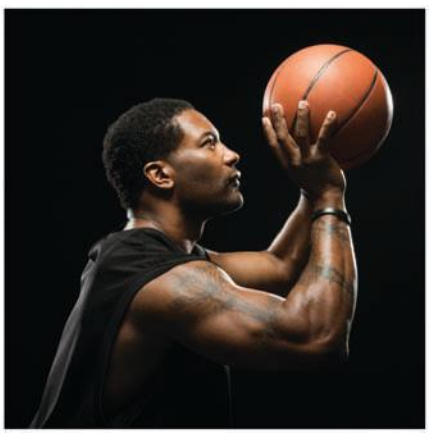
b On the basis of this experiment, what is the expected number of times in 1000 trials that the spinner will land on 3?

8 A basketball player has a 1 in 2 chance of getting a shot in from the free throw line. To simulate these use a coin: heads represents the shot going in, tails represents missing.

a Flip a coin 20 times and write down the results.

b Based on your experiment, what is the experimental probability that a shot will go in?

c Based on the actual probability of $\frac{1}{2}$, how many of 20 throws are expected to go in?



9 | 9, 10 | 10, 11

PROBLEM-SOLVING

Example 17

9 Ashwin sits a test consisting of 5 true or false questions. He plans to guess the answers. If he gets at least 3 questions correct he will pass the test.

a Design a simulation using 5 coins for this situation.

b Run your simulation 20 times and give the experimental probability that Ashwin passes the test.

10 A number of marbles are placed in a bag – some are red and some are green. A marble is selected from the bag and then replaced after its colour is noted. The results are shown in the table.

Red	Green
28	72

Based on the experiment, give the most likely answer to the following questions.

a If there are 10 marbles in the bag, how many are red?

b If there are 6 marbles in the bag, how many are red?

c If there are 50 marbles, how many are red?

d If there are 4 marbles, how many are green?

e If there are 14 green marbles in the bag, how many marbles are there in total?

f If there are 3 red marbles, how many green marbles are there?

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8K

PROBLEM-SOLVING

11 A baseball batter has up to 3 opportunities to hit a ball. Each time he has a 1 in 6 chance of hitting a home run, a 1 in 3 chance of hitting a small shot and a 1 in 2 chance of missing altogether. He uses a die to simulate each hitting opportunity.

Number	1	2 or 3	4 or 5 or 6
Outcome	Home run	Small shot	Miss

a He rolls the die up to three times. Match each of the simulation results **i–iv** with the correct outcome **A–D**.

i 5, 6, 1 **ii** 4, 3 **iii** 2 **iv** 6, 5, 6

A Hit small shot off first throw

B Miss twice then hit home run

C Miss 3 times

D Miss once, then small shot

b Conduct this simulation 50 times, keeping track of the result in a table like the one below.

Home runs	Small shots	3 Strikes (miss all three)

c Based on your simulation, estimate the probability that the batter hits a home run if he has up to three chances.

d Based on your simulation, how many times would the batter have 3 strikes if he has 500 attempts?

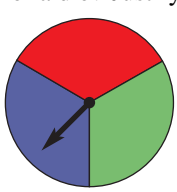
e The batter's results are recorded for 50 attempts: he hits 15 home runs, 29 small shots and 6 times he has three strikes. Estimate the number of home runs he would hit if he has 130 attempts.

12 12, 13 13, 14

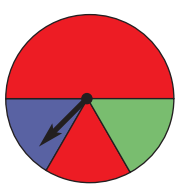
REASONING

12 It is possible to simulate a coin toss using a die by using the numbers 1–3 to stand for tails and 4–6 to stand for heads. Which of the following spinners could be simulated using a single roll of a die? Justify your answer.

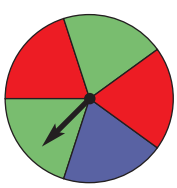
a



b



c



13 a An event has a (theoretical) probability of $\frac{1}{8}$. How could this be simulated using three coins?

b An event has a probability of $\frac{1}{12}$. How could this be simulated with a coin and a die?

c An event has a probability of $\frac{1}{9}$. How could this be simulated with two dice?

d How could an event with a probability of $\frac{1}{36}$ be simulated with two dice?

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REASONING

14 Four coins are tossed and the number of tails is noted.

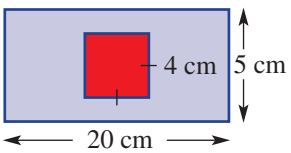
Number of tails	0	1	2	3	4
Frequency	1	3	5	2	0

a Based on this experiment, what is the experimental probability of obtaining 4 tails?
b Based on this experiment, what is the experimental probability of obtaining 3 heads?
c Use a tree diagram to give the actual probability of obtaining 4 tails.
d True or False? If the experimental probability is 0 then the theoretical probability is 0.
e True or False? If the theoretical probability is 0 then the experimental probability is 0.

Monte Carlo method — — 15

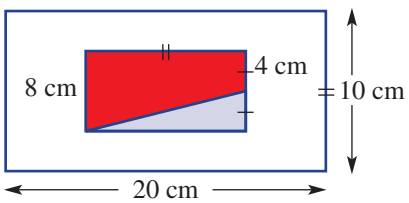
ENRICHMENT

15 Probability simulations can be used to find the area of an object by throwing darts randomly and seeing whether they land in the object.

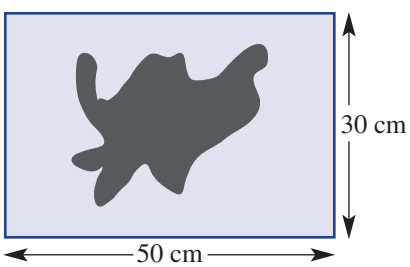


a Darts are thrown at the picture shown.
i Find the red area.
ii Find the total area of the picture.
iii A dart is thrown randomly at the picture. What is the probability that it hits the red part?
iv If 100 darts are thrown, how many would you expect to land in the blue area?

b 1000 darts are thrown at the picture on the right. How many of them would you expect them to hit:
i red? **ii** white? **iii** blue?



c The following map has had 1000 darts thrown at it. The darts landed in the black area 375 times. Based on this, estimate the area of the black shape.



d The map above has a scale of 1:1 000 000. Estimate the actual area of the island shown in the map.

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Investigation

The maths cup

The class race

1. The table below should be copied so that everyone in the race can see it and write on it.
2. Each student selects one horse as their 'own' (choose a winner!).
3. Each student rolls 2 dice and states the sum of the uppermost faces of the dice.
4. The total of each roll refers to the horse number. When its number is rolled, that horse moves another 100 m towards the finish line and a cross is placed next to its name.
5. The winning horse is the first to reach the finish!
6. Keep rolling the dice until first, second and third places are decided.

	Horse	100 m	200 m	300 m	400 m	500 m	600 m	700 m	800 m	900 m	1000 m
1	SCRATCHED										
2	Greased Lightning										
3	Flying Eagle										
4	Quick Stix										
5	Break a Leg										
6	Slow and Steady										
7	The Donkey										
8	My Little Pink Pony										
9	Tooting Tortoise										
10	Ripper Racer										
11	Speedy Gonzales										
12	Phar Lap										

Small group races

1. Start a tally to record the number of wins for each horse.
2. Repeat the above activity several times individually, or in small groups. Maintain a tally of how many times each horse wins the race.
3. Collate the results into a frequency table and place this on a histogram.

Discussion questions

1. Which horse won most often? Would you expect that horse to always win? Why?
2. Do any of the horses have the same chance of winning? Do any of the horses have the same chance of gaining a place? Do any of the horses have the same chance of losing?
3. Are some horses luckier than others? In this game does luck affect which horse wins? How?
4. Write a report to explain to a jockey which horse would be expected to win, and which horses he needs to beat, as they are most likely to be close to the winner at the finish line. Mathematically justify your decision by demonstrating your understanding of probability.

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School food survey

A new food company GoodCheapFood is thinking of taking over the running of your school's cafeteria. Before they invest in your school, they would like to know the eating and drinking habits of high school students, in particular:

- how much money students spend in the cafeteria (per week)
- how many drinks they purchase (per week)
- how many days of the week students use the cafeteria
- how happy students are with the current cafeteria, on a scale from 1 to 5.

1 Design a survey for GoodCheapFood that could be used to get the data they would like. Think about how the questions should be phrased to ensure that you get accurate results that are free of bias.

2 Explain why you cannot conduct a census of the school population and describe at least three different ways you could choose random samples of 20 students. For each sample, comment on whether a bias would be introduced and give an explanation for why only selecting Year 7 girls might result in signifi cantly different results from only selecting Year 12 boys.

3 Conduct the survey on at least three different random samples of size 20 from within the school population and use a spreadsheet to tabulate the results. A separate sheet should be used for each of the samples and could be filled out as below.

	A	B	C	D
1	Money spent ($)	Drinks purchased	Days used	Happiness (1-5)
2	8.25	2	2	3
3	10	3	1	2
4				

4 GoodCheapFood is interested in the range of the amount of money spent in the cafeteria in a week. Use a formula like = MAX(A2 : A21) – MIN(A2 : A21) to calculate the range for each of your samples and give the results.

5 The company wishes to know the median number of drinks purchased. Explain why the median rather than the mean would be preferred by referring to any possible outliers and then use a formula like = MEDIAN(B2 : B21) to find the medians for each sample.

6 Find the mode number of days used in each of the samples and explain what this signifi es in plain English.

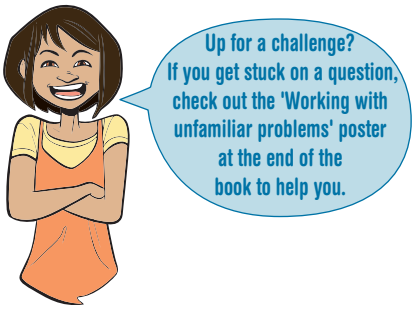
7 Find the mean happiness of students in each sample with a formula like = AVERAGE(D2 : D21).

8 Explain why your three or more different random samples drawn have different summary statistics (mean, median, mode and range) even though they are all drawn from the school population.

9 GoodCheapFood is grateful that you have gone to this much effort. Present your fi ndings as a brief summary, combining your samples to answer the four questions that they were interested in.

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Problems and challenges



1 Find a set of three numbers that have a range of 9, a mean of 10 and a median of 11.

2 The mean of a set of 20 numbers is 20. After the 21st number is added, the mean is now 21. What was the 21st number?

3 The median of a set of 10 numbers is 10. An even number is added, and the median is now 11 and the range is now 4. What number was added?

4 At the local sports academy, everybody plays netball or tennis. Given that half the tennis players also play netball and one-third of the netballers also play tennis, what is the probability that a randomly chosen person at the acadamy plays both?



5 For each of the following, find an English word that matches the description.

a $\Pr(\text{vowel}) = \frac{1}{2}$

b $\Pr(\text{F}) = \frac{2}{3}$

c $\Pr(\text{vowel}) = \frac{1}{4}$ and $\Pr(\text{D}) = \frac{1}{4}$

d $\Pr(\text{I}) = \frac{2}{11}$ and $\Pr(\text{consonant}) = \frac{7}{11}$

e $\Pr(\text{M}) = \frac{1}{7}$ and $\Pr(\text{T}) = \frac{1}{7}$ and $\Pr(\text{S}) = \frac{1}{7}$

f $\Pr(\text{vowel}) = 0$ and $\Pr(\text{T}) = \frac{1}{3}$

6 In a certain town, there are 22 women who can cook and 18 men who cannot cook. Given that half the town is male and 54% of the town can cook, how many men in the town can cook?

7 In the following game, the player flips a fair coin each turn to move a piece. If the coin shows 'heads' the piece goes right, and if it is 'tails' the coin goes left. What is the approximate probability that the player will win this game?

WIN				START					LOSE

8 If a person guesses all the answers on a 10-question true or false test, what is the probability that they will get them all right?

9 A bag contains 8 counters that are red, blue or yellow. A counter is selected from the bag, its colour noted and the counter replaced. If 100 counters were selected and 14 were red, 37 were blue and 49 were yellow, how many counters of each colour are likely to be in the bag?

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Chapter summary

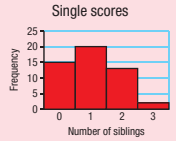
Representing Data

Graphs

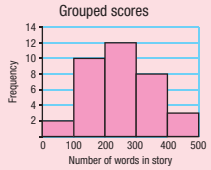
Column graph, Line graph, Sector graph, Divided bar graph

Histograms

Single scores



Grouped scores



Measures of centre

numerical order

0, 1, 4, (4, 5), 6, 9, 30

Median (middle) $= \frac{4+5}{2} = 4.5$

Mean (average) $= \frac{59}{8} = 7.4$

Mode (most frequent) = 4

Frequency tables

Single scores

Number of children (Score)	Tally	Frequency
0	|||	3
1	||||	5
2	|||| |||| |	11
3	||||	5
4	|||| |	6
		$n = 30$

Grouped scores

Age range	Frequency
0–4	3
5–9	7
10–14	5
	$n = 15$

Measures of spread (Ext)

Range = 28 – 1 = 27

lower half: 1 5 7 9 10 11 — Lower quartile = Median = 8

upper half: 13 17 19 20 23 28 — Upper quartile = Median = 19.5

IQR = 19.5 – 8 = 11.5 Interquartile range

Probability

Probabilities are written as • fractions • percentage • decimals

100% 1 Certain

Likely

0.5 50% $\frac{1}{2}$ Even chance (50–50)

Unlikely

0% 0 Impossible

Theoretical probability

Trial: Roll a fair die

Sample space (possible outcomes) { 1, 2, 3, 4, 5, 6 }

Pr(odd number) $= \frac{3}{6} = \frac{1}{2}$

Trial: Select a playing card and note its suit.

Sample space: spade, diamond, club, heart

Theoretical probabilities

Pr(black) $= \frac{26}{52} = \frac{1}{2}$

Pr(heart) $= \frac{13}{52} = \frac{1}{4}$

Pr(not spade) $= \frac{39}{52} = \frac{3}{4}$

Pr(either red or a spade) $= \frac{39}{52} = \frac{3}{4}$

Pr(red ace) $= \frac{2}{52} = \frac{1}{26}$

Experimental probability

Playing card selected and replaced 20 times, and its suit noted.

Outcome	Frequency	Experimental probability
Heart	4	$\frac{4}{20}$
Diamond	5	$\frac{5}{20}$
Club	4	$\frac{4}{20}$
Spade	7	$\frac{7}{20}$
	$n = 20$	

Expected number of outcomes

Outcome	Theoretical probability	Expected number in 20 trials
Heart	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Diamond	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Club	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Spade	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$

Sample space from experiments with two or more steps (Ext)

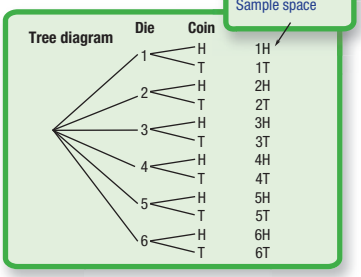
Table

Coin \ Die	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

(inside the table)

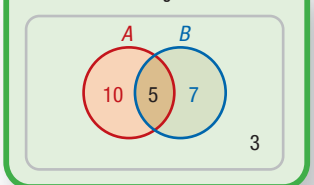
Possible outcomes
Sample space

Tree diagram



Two events

Venn diagrams



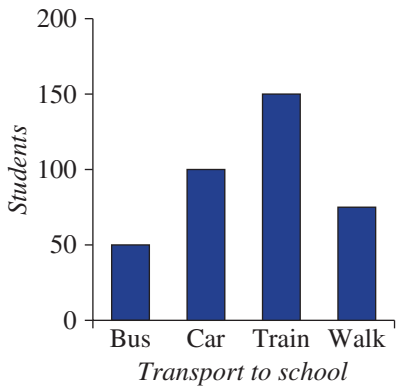
Two-way tables

	B	B′	Total
A	5	10	15
A′	7	3	10
Total	12	13	25

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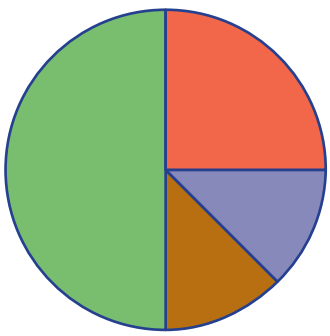
Multiple-choice questions

8A **1** Using the information in the column graph, how many students don't walk to school?



A 75 **B** 150 **C** 300
D 375 **E** 100

8A **2** The lollies in a bag are grouped by colour and the proportions shown in the sector graph. If there are equal numbers of blue and brown lollies, how many are blue, given that the bag contains 28 green ones?



A 112 **B** 7 **C** 56
D 14 **E** 28

8B **3** The tally below shows the number of goals scored by a soccer team over a season.

Goals	0	1	2	3	4
Tally	𝍸 ||	𝍸	|||	|||	||

Which one of the following statements is true?

A In the first game the team scored 5 goals.
B In four of the games the team scored two goals.
C In most of their games they did not score any goals.
D They had five games in which they scored one goal.
E The total number of goals scored for the season was 10.

8D **4** The median number of goals scored by the soccer team above is:

A 0 **B** 1 **C** 2 **D** 3 **E** 4

8D **5** Which is the best description of the mode in a set of test scores?

A The average of the scores
B The score in the middle
C The score with the highest frequency
D The difference between the highest and lowest score
E The lowest score

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Chapter review

8E Ext

6 For the set of data 1, 5, 10, 12, 14, 20 the interquartile range is:

A 1 **B** 19 **C** 4 **D** 11 **E** 9

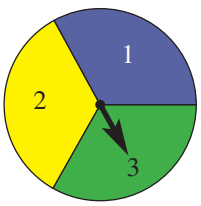
8G

7 The letters of the word STATISTICS are placed on 10 different cards and placed into a hat. If a card is drawn at random, the probability that it will show a vowel is:

A 0.2 **B** 0.3 **C** 0.4 **D** 0.5 **E** 0.7

8H Ext

8 A fair die is rolled and then the spinner to the right is spun. The probability that the die will display the same number as the spinner is:



A $\frac{1}{36}$ **B** $\frac{1}{18}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$ **E** 1

8I Ext

9 A coin is tossed three times. The probability of obtaining at least 2 tails is:

A $\frac{2}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{3}{8}$ **E** $\frac{1}{8}$

8K

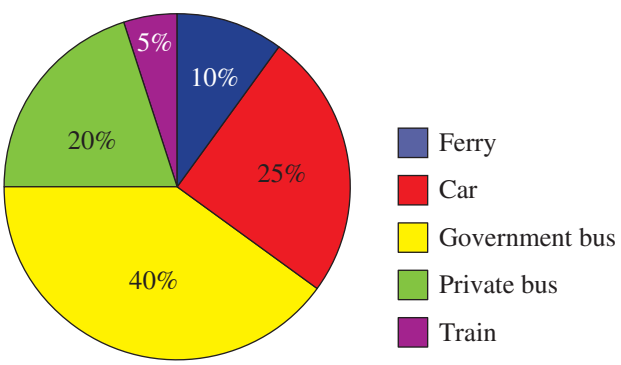
10 An experiment is conducted in which three dice are rolled and the sum of the faces is added. In 12 of the 100 trials, the sum of the faces was 11. Based on this, the experimental probability of having three faces add to 11 is:

A $\frac{11}{100}$ **B** $\frac{12}{111}$ **C** $\frac{3}{25}$ **D** 12 **E** $\frac{1}{2}$

Short-answer questions

8A

1 The sector graph shows the mode of transport office workers use to get to work every day.



a Which mode of transport is the most popular?

b Which mode of transport is the least popular?

c What angle of the 360° sector graph is represented by 'private buses'?

d If 20 000 workers were surveyed, how many people travelled to work each day by train?

e The year after this survey was taken, it was found that the number of people using government buses had decreased. Give a reason why this could have occurred.

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8B/C **2** Some students were asked how many hours of study they did before their half-yearly Maths exam. Their responses are represented in a tally.

0 hours	1 hour	2 hours	3 hours	4 hours
II	𝍸 I	III	III	𝍸 III

a How many students are in the class?
b Convert the tally above into a frequency table.
c Draw a histogram to represent the results of the survey.
d What proportion of the class did no study for the exam?
e Calculate the mean number of hours the students in the class spent studying for the exam, giving the answer correct to one decimal place.

8D/E **3** **a** Rewrite the following data in ascending order:

56 52 61 63 43 44 44 72 70 38 55
60 62 59 68 69 74 84 66 53 71 64

b What is the mode?
c What is the median for these scores?
Ext **d** Calculate the interquartile range.

8C/D **4** The ages of boys in an after-school athletics squad are shown in the table below.

a State the total number of boys in the squad.
b Display their ages in a histogram.
c Calculate the mean age of the squad, correct to two decimal places.
d What is the median age of the boys in the squad?

Age	Frequency
10	3
11	8
12	12
13	4
14	3

8B/F **5** A group of teenagers were weighed and their weights recorded to the nearest kilogram. The results are as follows:

56 64 72 81 84 51 69 69 63 57 59 68 72 73 72 80 78 61 61 70
57 53 54 65 61 80 73 52 64 66 66 56 50 64 60 51 59 69 70 85

a Find the highest and lowest weights.
b Create a grouped frequency distribution table using the groups 50–, 55–, 60– etc.
c Find:
  i the range
  ii the weight group with the most people
d Why is this sample not representative of the whole human population?

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Chapter review

8D/E **6** **a** Consider the data 5, 1, 7, 9, 1, 6, 4, 10, 12, 14, 6, 3. Find:

- **i** the mean
- **ii** the median
- Ext **iii** the lower quartile
- Ext **iv** the upper quartile
- Ext **v** the interquartile range (IQR)

b Repeat for the data 6, 2, 8, 10, 2, 7, 5, 11, 13, 15, 7, 4. What do you notice?

8F **7** In an attempt to find the average number of hours of homework that a Year 8 student completes, Samantha asks 10 of her friends in Year 8 how much homework they do.

a Explain two ways in which Samantha's sampling is inadequate to get the population average.

b If Samantha wished to convey to her parents that she did more than the average, how could she choose 10 people to bias the results in this way?

8G/H **8** An eight-sided die has the numbers 1, 2, 3, 4, 5, 6, 7, 8 on its faces.



a Find the probability that the number 4 is rolled.

b What is the probability that the number rolled is odd?

c What is the probability that the number rolled is both even and greater than 5?

d If P is the event that a prime number is rolled, state the sample space of P', the complement of P.

e If the die is rolled twice, what is the probability that the total of the two rolls is 20?

8G **9** The letters of the name MATHEMATICIAN are written on 13 cards. The letters are placed in a bag and one card is drawn at random.

a State the sample space.

b Find the probability of choosing the letter M.

c Find the probability of a vowel being drawn.

d What is the probability of a consonant being drawn?

e What is the probability that the letter chosen will be a letter in the word THEMATIC?

8H **10** A die is rolled and then a coin is flipped.

Ext

a Draw a table to list the sample space of this experiment.

b Find the probability that the die shows an even number and the coin shows tails.

c Find the probability of obtaining the pair (3, heads).

8I **11** A two-digit number is to be made from the digits 3, 4 and 5.

Ext

a Draw a tree diagram to show all outcomes if the digits can be used more than once.

b What is the probability of creating an even number?

c Find the probability that the number is divisible by 3.

d What is the probability that the sum of the two digits is greater than 8?

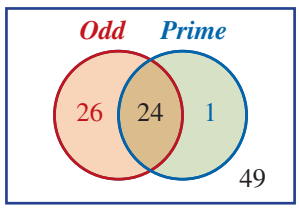
e Find the probability that the number starts with 3 or 5.

f If the numbers cannot be used more than once, what is the probability of creating an even number?

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8J

12 The Venn diagram below shows which numbers between 1 and 100 are odd and which are prime.
Consider the numbers 1–100.



- **a** How many are odd?
- **b** How many prime numbers are there?
- **c** What is the probability that a randomly selected number will be odd and prime?
- **d** What is the probability that a randomly selected number will be prime but not odd?
- **e** If an odd number is chosen, what is the probability that it is prime?
- **f** If a prime number is chosen, what is the probability that it is odd?

Extended-response questions

1 The two-way table below shows the results of a survey on car ownership and public transport usage. You can assume the sample is representative of the population.

	Uses public transport	Does not use public transport	Total
Own a car	20	80	
Do not own a car	65	35	
Total			

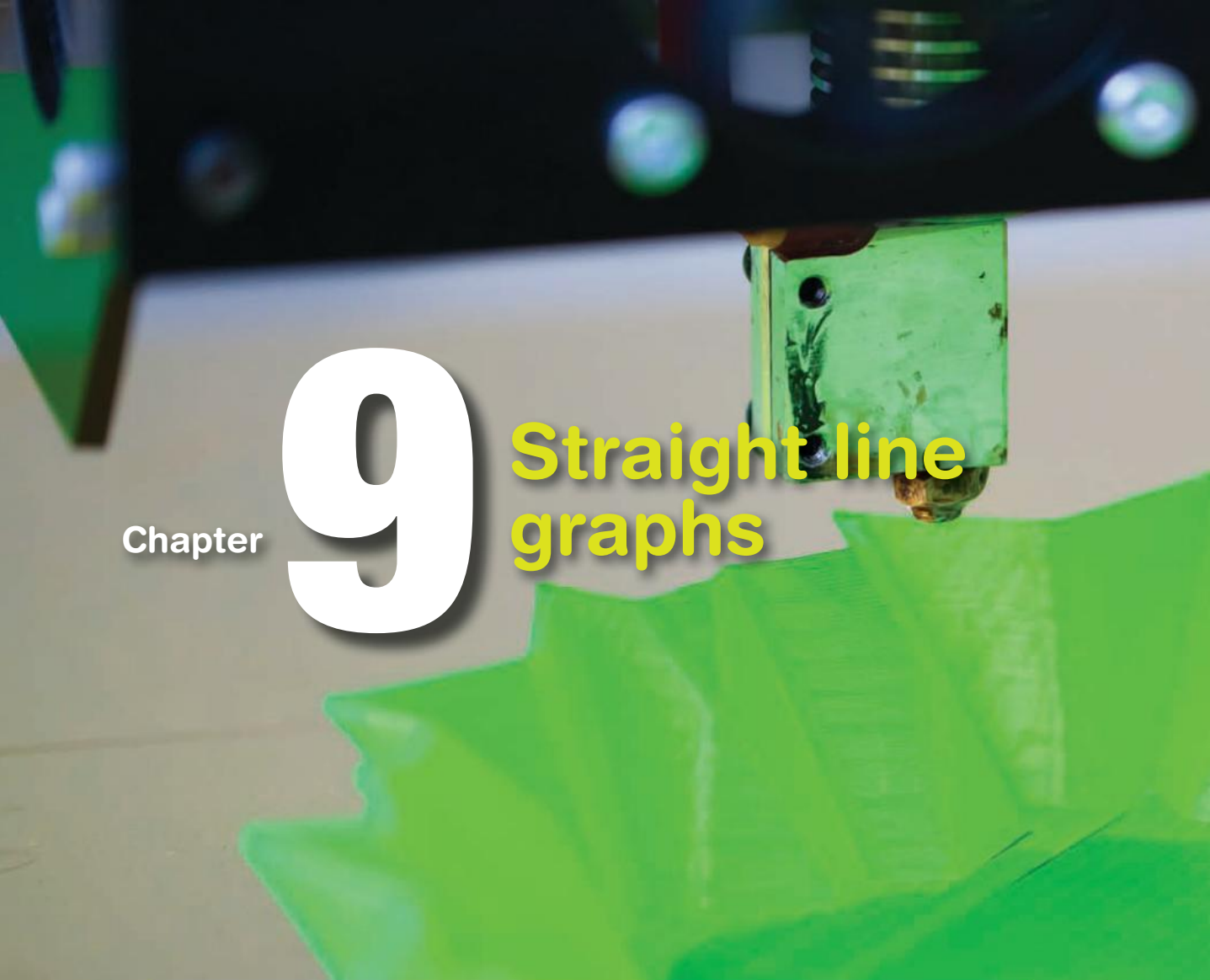
- **a** Copy and complete the table.
- **b** How many people were surveyed in total?
- **c** What is the probability that a randomly selected person will have a car?
- **d** What is the probability that a randomly selected person will use public transport and also own a car?
- **e** What is the probability that someone owns a car given that they use public transport?
- **f** If a car owner is selected, what is the probability that they will catch public transport?
- **g** In what ways could the survey produce biased results if it had been conducted:
  - **i** outside a train station?
  - **ii** in regional New South Wales?

2 A spinner is made using the numbers 1, 3, 5 and 10 in four sectors. The spinner is spun 80 times, and the results obtained are shown in the table.

Number on spinner	Frequency
1	30
3	18
5	11
10	21
	80

- **a** Display the data as a frequency histogram.
- **b** Which sector on the spinner occupies the largest area? Explain.
- **c** Two sectors of the spinner have the same area. Which two numbers do you think have equal areas, and why?
- **d** What is the experimental probability of obtaining a 1 on the next spin?
- **e** Draw an example of what you think the spinner might look like, in terms of the area covered by each of the four numbers.

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Chapter 9

Straight line graphs

What you will learn

9A The number plane (Consolidating)
9B Rules, tables and graphs
9C Finding the rule using tables
9D Gradient (Extending)
9E Gradient–intercept form (Extending)
9F The *x*-intercept (Extending)
9G Using graphs to solve linear equations
9H Applying linear graphs (Extending)
9I Non-linear graphs (Extending)

Victorian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Plot linear relationships on the Cartesian plane with and without the use of digital technologies (VCMNA283)

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (VCMNA284)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

3D printers and the number plane

Number plane geometry is the basis for 3-dimensional printing. Imagine a virtual number plane with positive x- and y-axes drawn on the print platform of a 3D printer. Computer software tells the printer head or nozzle how far across and how far up to move on this 2D number plane. The nozzle draws a line or curve forming a layer about 0.2 mm thick. Many different materials can be used in place of the traditional ink, such as plastic, glass, metal, wood, rubber or wax.

After one full layer, the print platform moves downward slightly and the computer directs the nozzle where to plot the next layer. Each layer is called a 'slice' and can be slightly different in shape from the previous slice. Thousands of slices slowly build a 3D object.

Objects made using 3D printing include: artificial body parts (e.g. hands, legs and ears); scale models of buildings; vehicle, plane or rocket engine parts including labels; jogging shoe samples testing colour and design features; smartphone cases; 3D maps; anatomy structures for a surgeon to study before a delicate operation (e.g. a patient's skull); replica fossils; models of engineering projects; and movie characters. There are endless possibilities for the practical applications of mathematics.

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9A The number plane

CONSOLIDATING









On a number plane, a pair of coordinates gives the exact position of a point. The number plane is also called the Cartesian plane after its inventor, Rene Descartes, who lived in France in the 17th century. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin and it provides a reference point for all other points on the plane.



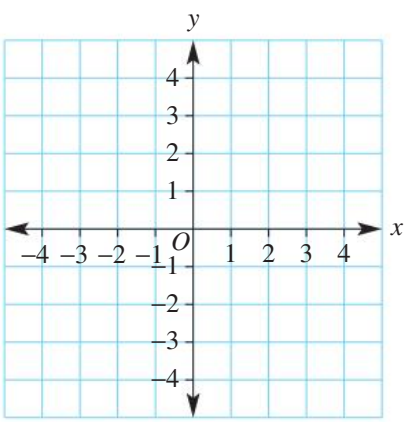
Coordinates on the number plane can be used to pinpoint locations just like you might locate points on a map.

Let's start: Make the shape

In groups or as a class, see if you can remember how to plot points on a number plane. Then decide what type of shape is formed by each set.

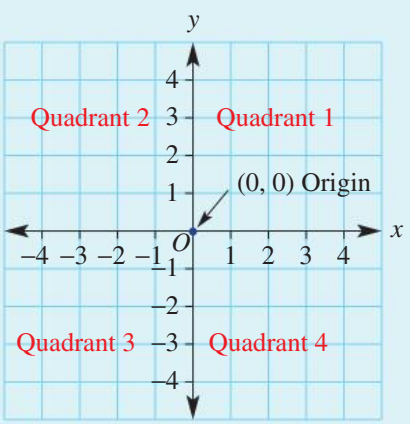
- $A(0, 0)$, $B(3, 1)$, $C(0, 4)$
- $A(-2, 3)$, $B(-2, -1)$, $C(-1, -1)$, $D(-1, 3)$
- $A(-3, -4)$, $B(2, -4)$, $C(0, -1)$, $D(-1, -1)$

Discuss the basic rules for plotting points on a number plane.



Key ideas

- A **number plane** (or **Cartesian plane**) includes a vertical y-axis and a horizontal x-axis intersecting at right angles.
  - There are 4 **quadrants** labelled as shown.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first followed by the y-coordinate.
- The point (0, 0) is called the **origin** (O).
- $(x, y) = \begin{pmatrix} \text{horizontal} \\ \text{units from} \\ \text{origin} \end{pmatrix}, \begin{pmatrix} \text{vertical} \\ \text{units from} \\ \text{origin} \end{pmatrix}$



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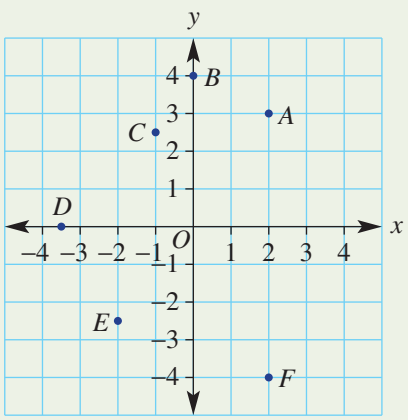
Example 1 Plotting points

Draw a number plane extending from −4 to 4 on both axes then plot and label these points.

a $A(2, 3)$ **b** $B(0, 4)$ **c** $C(-1, 2.5)$

d $D(-3.5, 0)$ **e** $E(-2, -2.5)$ **f** $F(2, -4)$

SOLUTION



EXPLANATION

The x-coordinate is listed first followed by the y-coordinate.

For each point start at the origin (0, 0) and move left or right or up and down to suit both x- and y-coordinates. For point $C(-1, 2.5)$, for example, move 1 to the left and 2.5 up.

Exercise 9A

1–3 | 3 | —

UNDERSTANDING

1 Complete these sentences.

a The coordinates of the origin are _____.

b The vertical axis is called the _____-axis.

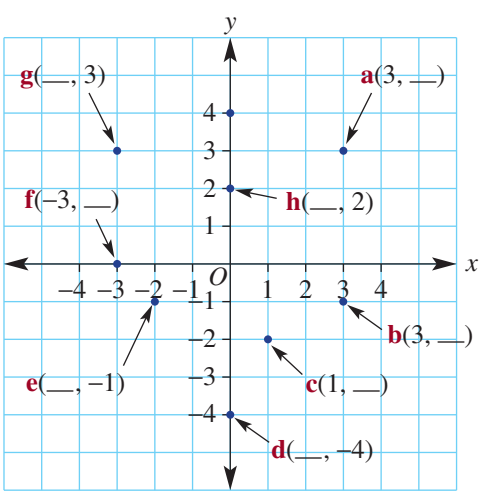
c The quadrant that has positive coordinates for both x and y is the _____ quadrant.

d The quadrant that has negative coordinates for both x and y is the _____ quadrant.

e The point $(-2, 3)$ has x-coordinate _____.

f The point $(1, -5)$ has y-coordinate _____.

2 Write the missing number for the coordinates of the points **a**–**h**.

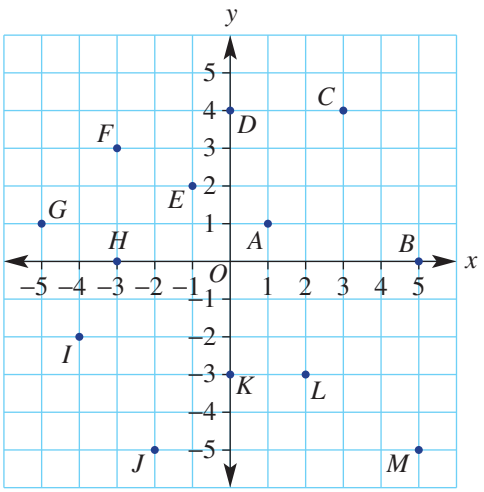


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9A

UNDERSTANDING

3 Write the coordinates of the points labelled A to M.





4(½) | 4(½), 5a | 4(½), 5b

FLUENCY

Example 1

4 Draw a number plane extending from −4 to 4 on both axes and then the plot and label these points.

a $A(4, 1)$	**b** $B(2, 3)$	**c** $C(0, 1)$	**d** $D(-1, 3)$
e $E(-3, 3)$	**f** $F(-2, 0)$	**g** $G(-3, -1)$	**h** $H(-1, -4)$
i $I(0, -2.5)$	**j** $J(0, 0)$	**k** $K(3.5, -1)$	**l** $L(1.5, -4)$
m $M(-3.5, -3.5)$	**n** $N(-3.5, 0.5)$	**o** $O(3.5, 3.5)$	**p** $P(2.5, -3.5)$

5 Using a scale extending from −5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

a $(-2, -2), (2, -2), (2, 2), (1, 3), (1, 4), \left(\frac{1}{2}, 4\right), \left(\frac{1}{2}, 3\frac{1}{2}\right), (0, 4), (-2, 2), (-2, -2)$

b $(2, 1), (0, 3), (-1, 3), (-3, 1), (-4, 1), (-5, 2), (-5, -2), (-4, -1), (-3, -1), (-1, -3), (0, -3), (2, -1), (1, 0), (2, 1)$

6–7(½) | 6–7(½) | 7–8(½)

PROBLEM-SOLVING

6 One point in each set is not 'in line' with the other points. Name the point in each case.

a $A(1, 2), B(2, 4), C(3, 4), D(4, 5), E(5, 6)$

b $A(-5, 3), B(-4, 1), C(-3, 0), D(-2, -3), E(-1, -5)$

c $A(-4, -3), B(-2, -2), C(0, -1), D(2, 0)$, E$(3, 1)$

d $A(6, -4), B(0, -1), C(4, -3), D(3, -2), E(-2, 0)$

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PROBLEM-SOLVING

7 Each set of points forms a basic shape. Describe the shape without drawing a graph if you can.

a $A(-2, 4)$, $B(-1, -1)$, $C(3, 0)$
b $A(-3, 1)$, $B(2, 1)$, $C(2, -6)$, $D(-3, -6)$
c $A(-4, 2)$, $B(3, 2)$, $C(4, 0)$, $D(-3, 0)$
d $A(-1, 0)$, $B(1, 3)$, $C(3, 0)$, $D(1, -9)$

8 The midpoint of a line segment (or interval) is the point that cuts the segment in half. Find the midpoint of the line segment joining these pairs of points.

a $(1, 3)$ and $(3, 5)$
b $(-4, 1)$ and $(-6, 3)$
c $(-2, -3)$ and $(0, -2)$
d $(3, -5)$ and $(6, -4)$

	9	9, 10	10, 11

REASONING

9 List all the points, using only integer values of x and y that lie on the line segment joining these pairs of points.

a $(1, -3)$ and $(1, 2)$
b $(-2, 0)$ and $(3, 0)$
c $(-3, 4)$ and $(2, -1)$
d $(-3, -6)$ and $(3, 12)$

10 If (a, b) is a point on a number plane, name the quadrant or quadrants that matches the given description.

a $a > 0$ and $b < 0$
b $a < 0$ and $b > 0$
c $a < 0$
d $b < 0$

11 A set of points has coordinates $(0, y)$ where y is any number. What does this set of points represent?

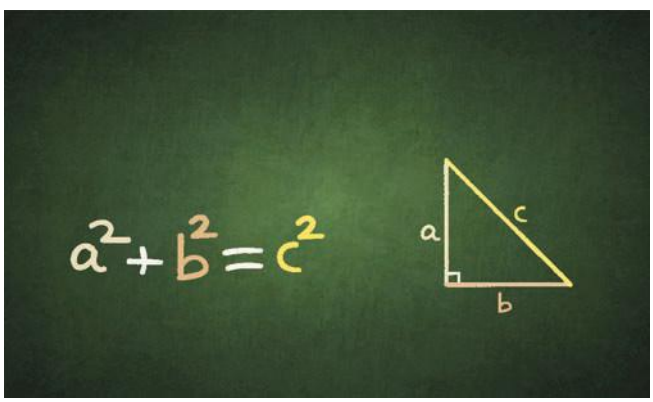
Distances between points	—	—	12, 13

ENRICHMENT

12 Find the distance between these pairs of points.

a $(0, 0)$ and $(0, 10)$
b $(0, 0)$ and $(-4, 0)$
c $(-2, 0)$ and $(5, 0)$
d $(0, -4)$ and $(0, 7)$
e $(-1, 2)$ and $(5, 2)$
f $(4, -3)$ and $(4, 1)$

13 When two points are not aligned vertically or horizontally, Pythagoras' theorem can be used to find the distance between them. Find the distance between these pairs of points.



a $(0, 0)$ and $(3, 4)$
b $(0, 0)$ and $(5, 12)$
c $(-3, -4)$ and $(4, 20)$
d $(1, 1)$ and $(4, -1)$
e $(-1, -2)$ and $(2, 7)$
f $(-3, 4)$ and $(3, -1)$

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9B Rules, tables and graphs

Interactive

Widgets

HOTsheets

Walkthroughs

From our earlier study of formulas we know that two (or more) variables that have a relationship can be linked by a rule. A rule with two variables can be represented on a graph to illustrate this relationship. The rule can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight line graphs are described as being **linear**.

For example, the rule linking degrees Celsius (°C) with degrees Fahrenheit (°F) is given by $°C = \frac{5}{9}(°F - 32)$ and gives a straight line graph.



The rule for converting Celsius to Fahrenheit gives a straight line graph.

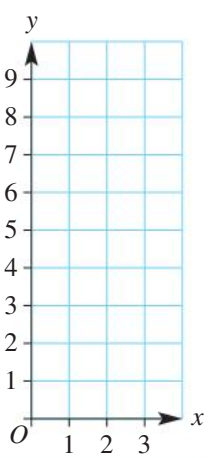
Let's start: They're not all straight

Not all rules give a straight line graph. Here are three rules which can be graphed to give lines or curves.

1 $y = \frac{6}{x}$ **2** $y = x^2$ **3** $y = 2x + 1$

- In groups, discuss which rule(s) might give a straight line graph and which might give curves.
- Use the rules to complete the given table of values.
- Discuss how the table of values can help you decide which rule(s) give a straight line.
- Plot the points to see if you are correct.

x	1	2	3
$y = \frac{6}{x}$			
$y = x^2$			
$y = 2x + 1$			



Key ideas

- A **rule** is an equation which describes the relationship between two or more variables.
- A **linear** relationship will result in a straight line graph.
- For two variables, a linear rule is often written with y as the subject. For example, $y = 2x - 3$ or $y = -x + 7$
- One way to graph a linear relationship using a rule is to follow these steps.
  - Construct a table of values finding a y-coordinate for each given x-coordinate by substituting each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

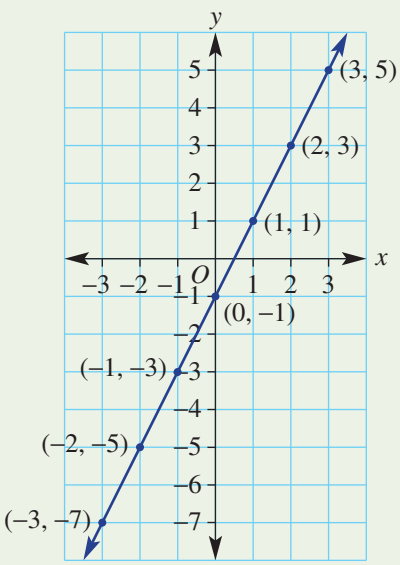
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Example 2 Plotting a graph from a rule

For the rule $y = 2x - 1$ construct a table and draw a graph.

SOLUTION

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



EXPLANATION

Substitute each x-coordinate in the table into the rule to find the y-coordinate.

Plot each point $(-3, -7)$, $(-2, -5)$ … and join them to form the straight line graph.

Example 3 Checking if a point lies on a line

Decide if the points $(1, 3)$ and $(-2, -4)$ lie on the graph of $y = 3x$.

SOLUTION

Substitute $(1, 3)$
$y = 3x$
$3 = 3 \times 1$ (True)
So $(1, 3)$ is on the line.

Substitute $(-2, -4)$
$y = 3x$
$-4 = 3 \times -2$ (False)
So $(-2, -4)$ is not on the line.

EXPLANATION

Substitute $(1, 3)$ into the rule for the line. The equation is true, so the point is on the line.

Substitute $(-2, -4)$ into the rule for the line. The equation is not true, so the point is not on the line.

Exercise 9B

1–3 | 3 | —

UNDERSTANDING

1 For the rule $y = 2x + 3$ find the y-coordinate for these x-coordinates.

a 1 **b** 2 **c** 0 **d** −1
e −5 **f** −7 **g** 11 **h** −12

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9B

UNDERSTANDING

2 Write the missing number in these tables for the given rules.

a $y = 2x$

x	0	1	2	3
y	0		4	6

b $y = x - 3$

x	−1	0	1	2
y	−4	−3		−1

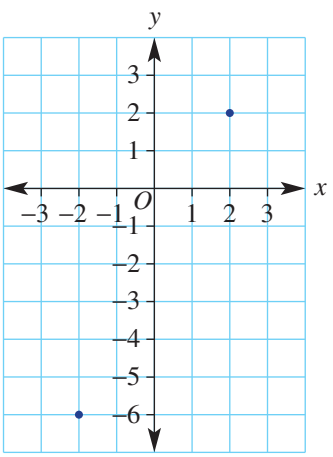
c $y = 5x + 2$

x	−3	−2	−1	0
y		−8	−3	2

3 Complete the graph to form a straight line from the given rule and table. Two points have been plotted for you.

$y = 2x - 2$

x	−2	−1	0	1	2
y	−6	−4	−2	0	2



4–5(½) | 4–5(½) | 4–5(½)

FLUENCY

Example 2

4 For each rule construct a table like the one shown here and draw a graph.

x	−3	−2	−1	0	1	2	3
y							

a $y = x + 1$ **b** $y = x - 2$ **c** $y = 2x - 3$ **d** $y = 2x + 1$

e $y = -2x + 3$ **f** $y = -3x - 1$ **g** $y = -x$ **h** $y = -x + 4$

Example 3

5 Decide if the given points lie on the graph with the given rule.

a Rule: $y = 2x$ Points: **i** (2, 4) and **ii** (3, 5)

b Rule: $y = 3x - 1$ Points: **i** (1, 1) and **ii** (2, 5)

c Rule: $y = 5x - 3$ Points: **i** (−1, 0) and **ii** (2, 12)

d Rule: $y = -2x + 4$ Points: **i** (1, 2) and **ii** (2, 0)

e Rule: $y = 3 - x$ Points: **i** (1, 2) and **ii** (4, 0)

f Rule: $y = 10 - 2x$ Points: **i** (3, 4) and **ii** (0, 10)

g Rule: $y = -1 - 2x$ Points: **i** (2, −3) and **ii** (−1, 1)

6 | 6–7(½) | 7, 8

PROBLEM-SOLVING

6 For x-coordinates from −3 to 3, construct a table and draw a graph for these rules. For parts **c** and **d** remember that subtracting a negative number is the same as adding its opposite, for example that $3 - (-2) = 3 + 2$.

a $y = \frac{1}{2}x + 1$ **b** $y = -\frac{1}{2}x - 2$ **c** $y = 3 - x$ **d** $y = 1 - 3x$

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PROBLEM-SOLVING

7 For the graphs of these rules, state the coordinates of the two points at which the line cuts the x- and y-axes.

a $y = x + 1$ **b** $y = 2 - x$ **c** $y = 2x + 4$

d $y = 10 - 5x$ **e** $y = 2x - 3$ **f** $y = 7 - 3x$

8 The rules for two lines are $y = x + 2$ and $y = 5 - 2x$. At what point do they intersect?

	9	9, 10	10, 11

REASONING

9 **a** What is the minimum number of points needed to draw a graph of a straight line?

b Draw the graph of these rules by plotting only two points. Use $x = 0$ and $x = 1$.

i $y = \frac{1}{2}x$ **ii** $y = 2x - 1$

10 **a** The graphs of $y = x$, $y = 3x$ and $y = -2x$ all pass through the origin (0, 0). Explain why.

b The graphs of $y = x - 1$, $y = 3x - 2$ and $y = 5 - 2x$ do not pass through the origin (0, 0). Explain why.

11 The y-coordinates of points on the graphs of the rules in Question **4** parts **a** to **d** increase as the x-coordinates increase. Also the y-coordinates of points on the graphs of the rules in Question **4** parts **e** to **h** decrease as the x-coordinates increase.

a What do the rules in Question **4** parts **a** to **d** have in common?

b What do the rules in Question **4** parts **e** to **h** have in common?

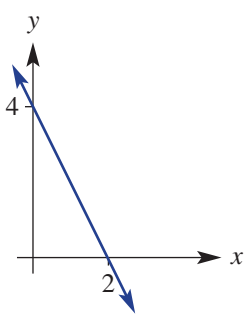
c What feature of a rule tells you that a graph increases or decreases as the x-coordinate increases?

Axes intercepts	—	—	12

ENRICHMENT

12 A sketch of a straight line graph can be achieved by finding only the x-and y-intercepts and labelling these on the graph. The y-intercept is the point where $x = 0$ and the x-intercept is the point where $y = 0$.

For example: $y = 4 - 2x$



y–intercept ($x = 0$)	x–intercept ($y = 0$)
$y = 4 - 2 \times 0$	$0 = 4 - 2x$
$= 4$	$-4 = -2x$
	$2 = x$

Sketch graphs of these rules using the method outlined above.

a $y = x + 4$ **b** $y = 2x - 4$ **c** $y = 5 - x$

d $y = -1 + 2x$ **e** $y = 7x - 14$ **f** $y = 5 - 3x$

g $y = 3 - 2x$ **h** $3y - 2x = 6$

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9C Finding the rule using tables

Interactive
Widgets
HOTsheets
Walkthroughs

A mathematical rule is an efficient way of describing a relationship between two variables. While a table and a graph are limited by the number of points they show, a rule can be used to find any value of y for any given x value quickly. Finding such a rule from a collection of points on a graph or table is an important step in the development and application of mathematics.



Businesses attempt to find mathematical rules from the data they collect and graph.

Let's start: What's my rule?

Each of the tables here describe a linear relationship between y and x.

x	y
0	4
1	5
2	6
3	7
4	8

x	y
−3	−5
−2	−3
−1	−1
0	1
1	3

x	y
−2	5
−1	4
0	3
1	2
2	1

- For each table write a rule making y the subject.
- Discuss your strategy for finding the three different rules. What patterns did you notice and how did these patterns help determine the rule?

Key ideas

- A **rule** is an equation that describes the relationship between the x-coordinate and y-coordinate in a table or graph.

coefficient of x

$$y = \square \times x + \square$$

constant

- Consider a linear rule of the form $y = \square \times x + \square$.
  - The **coefficient** of x will be the increase in y as x increases by 1. If there is a decrease in y, then the coefficient will be negative.

x	−2	−1	0	1	2
y	−1	1	3	5	7

+2 +2 +2 +2

$y = 2x + 3$

x	−2	−1	0	1	2
y	1	0	−1	−2	−2

−1 −1 −1 −1

$y = -x - 1$

  - The **constant** will be the value of y when $x = 0$.

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Key ideas

- If the value of y when $x = 0$ is not given in the table, substitute another pair of coordinates to find the value of the constant.

x	2	3	4	5
y	5	7	9	11

+2 +2 +2

$y = 2x + \square$

$5 = 2 \times 2 + \square$ substituting $(2, 5)$

So $\square = 1$

$\therefore\ y = 2x + 1$

Example 4 Finding rules from tables

Find the rule for these tables of values.

a

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

b

x	3	4	5	6	7
y	−5	−7	−9	−11	−13

SOLUTION

a Coefficient of x is 3.
Constant is −2.
$y = 3x - 2$

b Coefficient of x is −2.
$y = -2x + \square$
Substitute $(3, -5)$.
$-5 = -2 \times 3 + \square$
$-5 = -6 + \square$
So $\square = 1$.
$y = -2x + 1$

EXPLANATION

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

+3 +3 +3 +3

$y = 3x - 2$

x	3	4	5	6	7
y	−5	−7	−9	11	−13

−2 −2 −2 −2

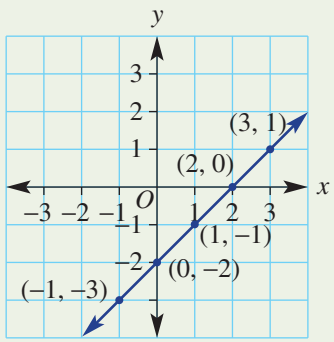
$y = -2x + \square$

To find the constant substitute a point and choose the constant so that the equation is true. This can be done mentally.

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Example 5 Finding a rule from a graph

Find the rule for this graph by first constructing a table of (x, y) values.



SOLUTION

x	−1	0	1	2	3
y	−3	−2	−1	0	1

Coefficient of x is 1.

When $x = 0$, $y = -2$

$y = x - 2$

EXPLANATION

Construct a table using the points given on the graph. Change in y is 1 for each increase by 1 in x.

x	−1	0	1	2	3
y	−3	−2	−1	0	1

+1 +1 +1 +1

$y = 1x + (-2)$ or $y = x - 2$

Exercise 9C

1–3 | 3 | —

UNDERSTANDING

1 By how much does y increase for each increase by 1 in x? If y is decreasing give a negative answer.

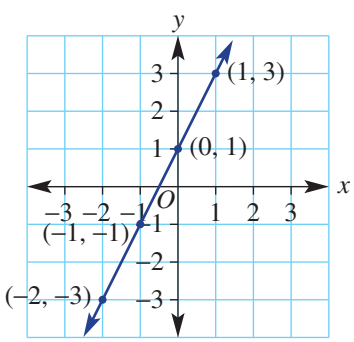
a

x	−2	−1	0	1	2
y	−1	1	3	5	7

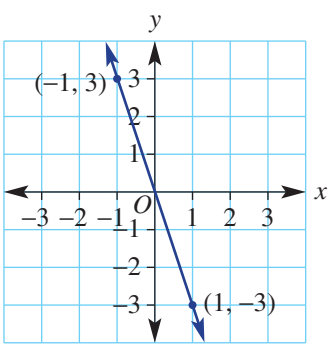
b

x	−3	−2	−1	0	1
y	4	3	2	1	0

c



d



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9C

UNDERSTANDING

2 For each of the tables and graphs in Question **1**, state the value of y when $x = 0$.

3 Find the missing number in these equations.

a $4 = 2 + \square$ **b** $3 = 5 + \square$ **c** $-1 = 2 + \square$

d $5 = 3 \times 2 + \square$ **e** $-2 = 2 \times 4 + \square$ **f** $-10 = 2 \times (-4) + \square$

g $-8 = -4 \times 2 + \square$ **h** $-12 = -5 \times 3 + \square$ **i** $16 = -4 \times (-3) + \square$

4–6(½) 4–6(½) 5–6(½)

FLUENCY

Example 4a

4 Find the rule for these tables of values.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

c

x	−3	−2	−1	0	1
y	4	3	2	1	0

d

x	−1	0	1	2	3
y	8	6	4	2	0

Example 4b

5 Find the rule for these tables of values.

a

x	1	2	3	4	5
y	5	9	13	17	21

b

x	−5	−4	−3	−2	−1
y	−13	−11	−9	−7	−5

c

x	5	6	7	8	9
y	−12	−14	−16	−18	−20

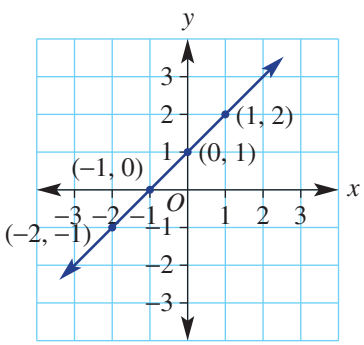
d

x	−6	−5	−4	−3	−2
y	10	9	8	7	6

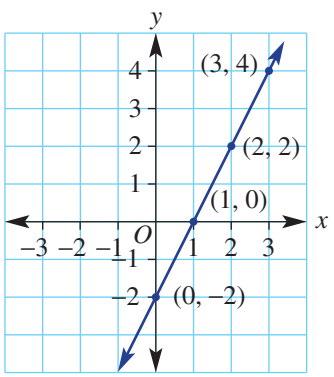
Example 5

6 Find the rule for these graphs by first constructing a table of (x, y) values.

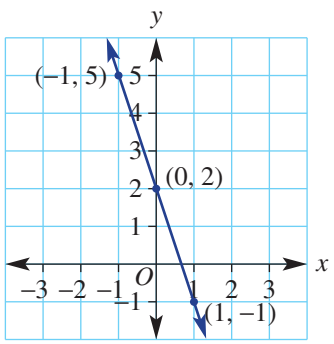
a



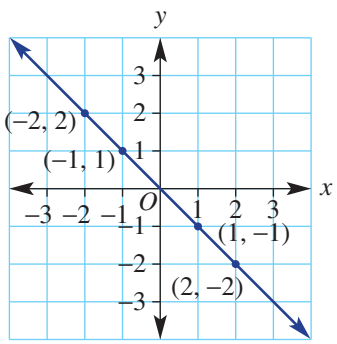
b



c



d



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9C

7, 8 | 7–9 | 8–10

PROBLEM-SOLVING

7 Find the rule for these set of points. Try to do it without drawing a graph or table.

a (1, 3), (2, 4), (3, 5), (4, 6)

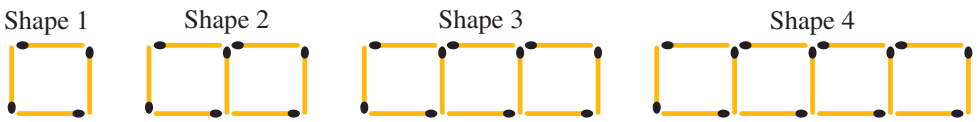
b (−3, −7), (−2, −6), (−1, −5), (0, − 4)

c (−1, −3), (0, −1), (1, 1), (2, 3)

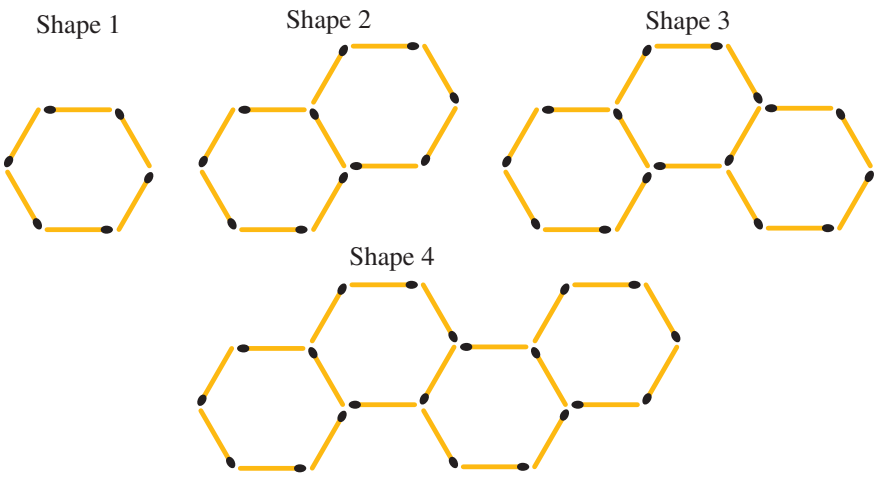
d (−2, 3), (−1, 2), (0, 1), (1, 0)

8 Write a rule for these matchstick patterns.

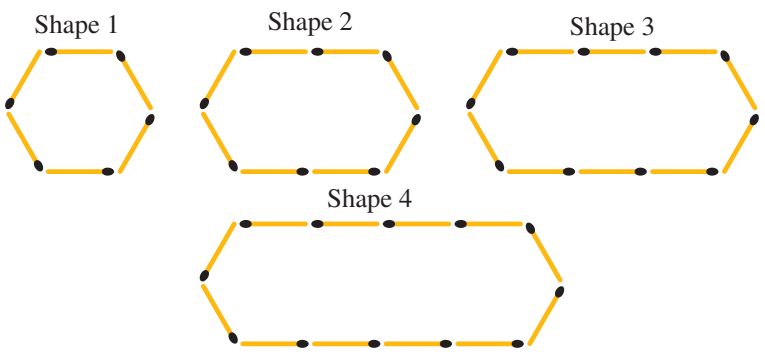
a x = number of squares y = number of matchsticks



b x = number of hexagons y = number of matchsticks



c x = number of matchsticks on top row y = number of matchsticks



9 A straight-line graph passes through the two points (0, −2) and (1, 6). What is the rule of the graph?

10 A straight-line graph passes through the two points (−2, 3) and (4, −3). What is the rule of the graph?

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REASONING

11 The rule $y = -2x + 3$ can be written as $y = 3 - 2x$. Write these rules in a similar form.

a $y = -2x + 5$ **b** $y = -3x + 7$ **c** $y = -x + 4$ **d** $y = -4x + 10$

12 A straight line has two points $(0, 2)$ and $(1, b)$.

a Write an expression for the coefficient of x in the rule linking y and x.

b Write the rule for the graph in terms of b.

13 A straight line has two points $(0, a)$ and $(1, b)$.

a Write an expression for the coefficient of x in the rule linking y and x.

b Write the rule for the graph in terms of a and b.

14 In Question **8a** you can observe that 3 extra matchsticks are needed for each new shape and 1 matchstick is needed to complete the first square (so the rule is $y = 3x + 1$). In a similar way, describe how many matchsticks are needed for the shapes in:

a Question **8b** **b** Question **8c**



ENRICHMENT

Skipping *x* values

— — 15

15 Consider this table of values.

x	−2	0	2	4
y	−4	−2	0	2

a The increase in y for each unit increase in x is not 2. Explain why.

b If the pattern is linear, state the increase in y for each increase by 1 in x.

c Write the rule for the relationship.

d Find the rule for these tables.

i

x	−4	−2	0	2	4
y	−5	−1	3	7	11

ii

x	−3	−1	1	3	5
y	−10	−4	2	8	14

iii

x	−6	−3	0	3	6
y	15	9	3	−3	−9

iv

x	−10	−8	−6	−4	−2
y	20	12	4	−4	−12

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9D Gradient EXTENDING

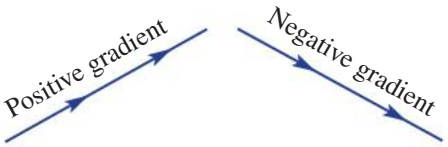
Interactive

Widgets

HOTsheets

Walkthroughs

The gradient of a line is a measure of how steep the line is. The steepness or slope of a line depends on how far it rises or falls over a given horizontal distance. This is why the gradient is calculated by dividing the vertical rise by the horizontal run between two points. Lines that rise (from left to right) have a positive gradient and lines that fall (from left to right) have a negative gradient.

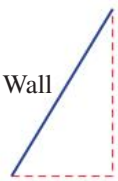




It's more difficult for a plane to take off at a steep gradient than a gradual one.

Let's start: Which is the steepest?

At a children's indoor climbing centre there are three types of sloping walls to climb. The blue wall rises 2 metres for each metre across. The red wall rises 3 metres for every 2 metres across and the yellow wall rises 7 metres for every 3 metres across.

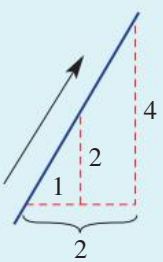


- Draw a diagram showing the slope of each wall.
- Label your diagrams with the information given above.
- Discuss which wall might be the steepest giving reasons.
- Discuss how it might be possible to accurately compare the slope of each wall.

Key ideas

- The **gradient** is a measure of **slope**.
  - It is the increase in y as x increases by 1.
  - It is the ratio of the change in y over the change in x.

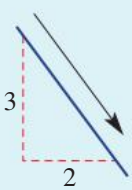
Positive gradient



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = \frac{2}{1} = 2$$

- **Gradient = $\frac{\text{rise}}{\text{run}}$**
- Rise = change in y
- Run = change in x

Negative gradient



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -\frac{3}{2}$$

- The run is always considered to be positive when moving from left to right on the Cartesian plane.

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Key ideas

- A gradient is negative if y decreases as x increases. The rise is considered to be negative.
- The gradient of a horizontal line is 0.
- The gradient of a vertical line is undefined.



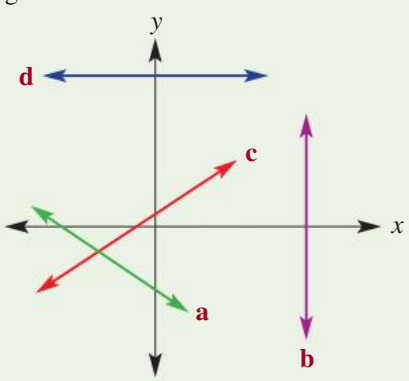
$$\text{Gradient} = \frac{0}{2} = 0$$



$$\text{Gradient} = \frac{2}{0} \text{ which is undefined}$$

Example 6 Defining a type of gradient

Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



SOLUTION	EXPLANATION
a Negative gradient	As x increases y decreases.
b Undefined gradient	The line is vertical.
c Positive gradient	y increases as x increases.
d Zero gradient	There is no increase or decrease in y as x increases.

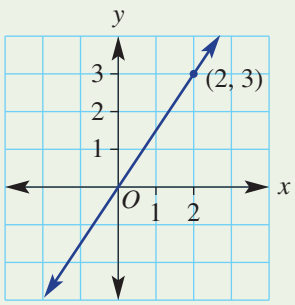


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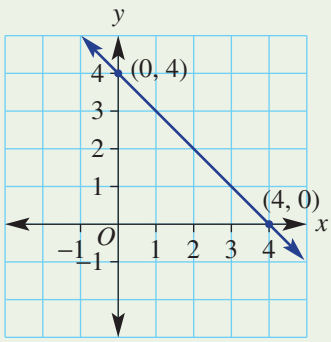
Example 7 Finding the gradient from a graph

Find the gradient of these lines.

a



b



SOLUTION

a Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{3}{2}$ or 1.5

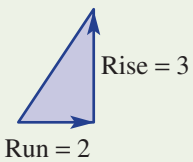
b Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{4}$

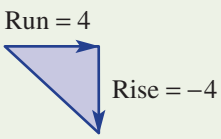
$= -1$

EXPLANATION

The rise is 3 for every 2 across to the right.



The y value falls 4 units while the x value increases by 4.



Exercise 9D

1–3(½) | 3(½) | —

UNDERSTANDING

1 Decide if these lines have a positive or negative gradient.

a



b



c



d



2 Simplify these fractions.

a $\frac{4}{2}$ **b** $\frac{12}{4}$ **c** $\frac{6}{4}$ **d** $\frac{14}{7}$

e $-\frac{6}{3}$ **f** $-\frac{20}{4}$ **g** $-\frac{16}{6}$ **h** $-\frac{15}{9}$

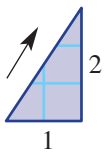
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9D

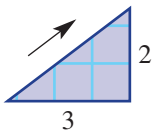
UNDERSTANDING

3 Write down the gradient, using $\frac{\text{rise}}{\text{run}}$ for each of these slopes.

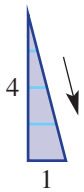
a



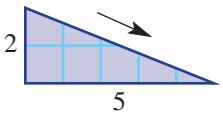
b



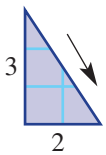
c



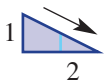
d



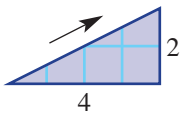
e



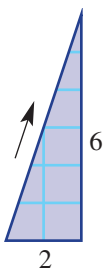
f



g



h

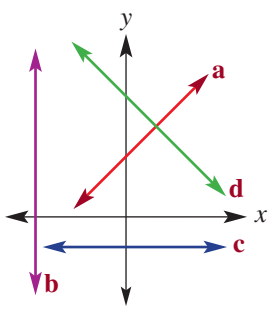


4, 5–6(½) 4, 5–6(½) 5–6(½)

FLUENCY

Example 6

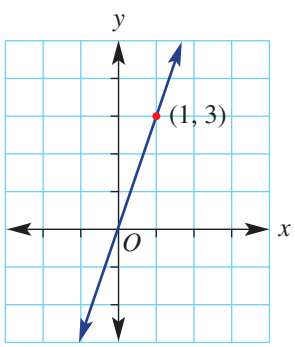
4 Decide if these lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



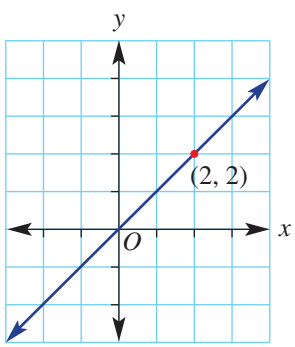
Example 7a

5 Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

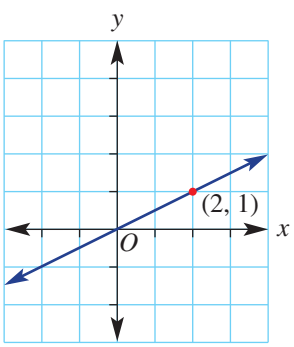
a



b



c

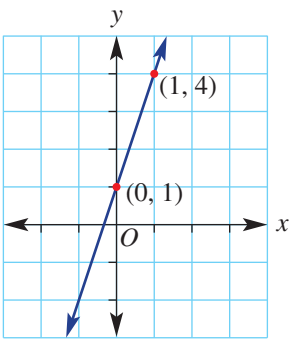


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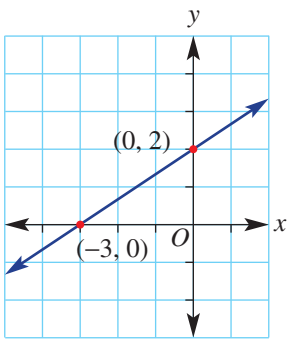
9D

FLUENCY

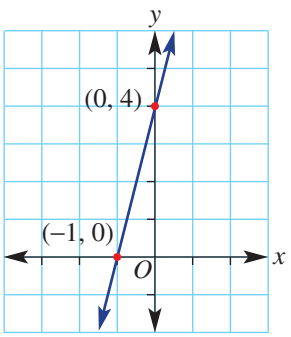
d



e



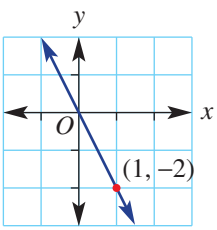
f



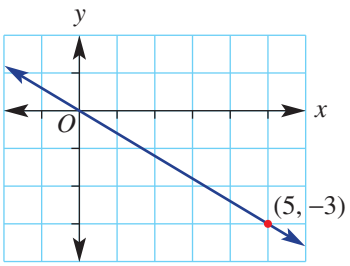
Example 7b

6 Find the gradient of these lines.

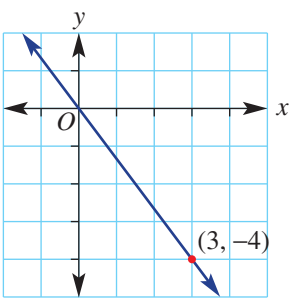
a



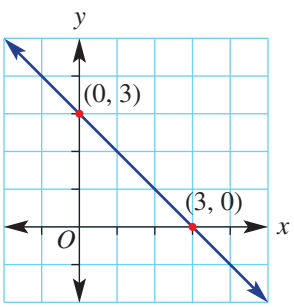
b



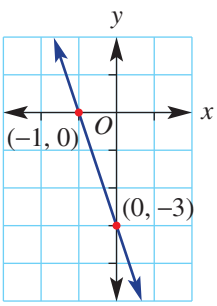
c



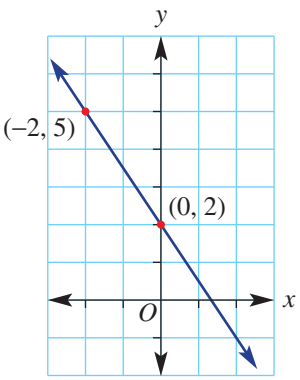
d



e



f



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9D

7, 8 8, 9 8–10

PROBLEM-SOLVING

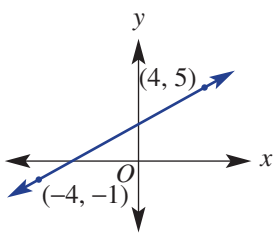
7 Abdullah climbs a rocky slope that rises 12 m for each 6 metres across. His friend Jonathan climbs a nearby grassy slope that rises 25 m for each 12 m across. Which slope is steeper?

8 A submarine falls 200 m for each 40 m across and a torpedo falls 420 m for each 80 m across in pursuit of the submarine. Which has the steeper gradient, the submarine or torpedo?

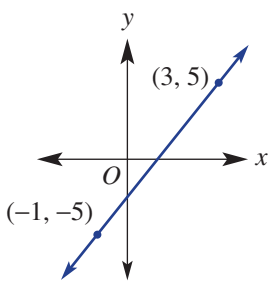


9 Find the gradient of these lines. You will need to first calculate the rise and run.

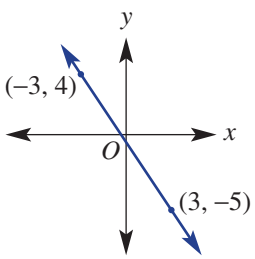
a



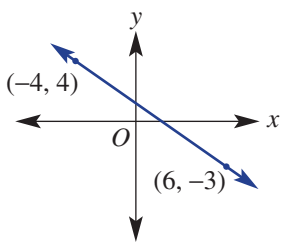
b



c



d



10 Find the gradient of the line joining these pairs of points.

a (0, 2) and (2, 7)
b (0, –1) and (3, 4)
c (–3, 7) and (0, –1)
d (–5, 6) and (1, 2)
e (–2, –5) and (1, 3)
f (–5, 2) and (5, –1)

11 11, 12 12, 13

REASONING

11 A line with gradient 3 joins (0, 0) with another point A.

a Write the coordinates of three different positions for A, using positive integers for both the x- and y-coordinates.

b Write the coordinates of three different positions for A using negative integers for both the x- and y-coordinates.

12 A line joins the point (0, 0) with the point (a, b) with a gradient of 2.

a If $a = 1$ find b.
b If $a = 5$ find b.
c Write an expression for b in terms of a.
d Write an expression for a in terms of b.

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9D

REASONING

13 A line joins the point (0, 0) with the point (a, b) with a gradient of $-\frac{1}{2}$.

a If $a = 1$ find b.

b If $a = 3$ find b.

c Write an expression for b in terms of a.

d Write an expression for a in terms of b.

Rise and run as differences — — 14

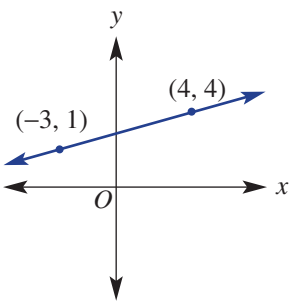
ENRICHMENT

14 The run and rise between two points can be calculated by finding the difference between the pairs of x- and pairs of y-coordinates.
For example:

$$\text{Rise} = 4 - 1 = 3$$

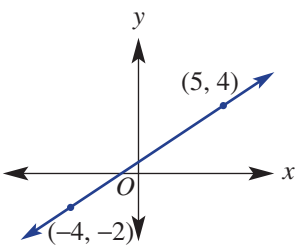
$$\text{Run} = 4 - (-3) = 7$$

$$\text{Gradient} = \frac{3}{7}$$

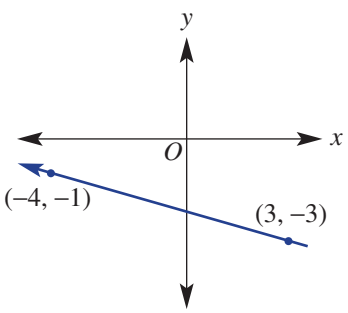


Use this method to find the gradient of the line joining the given points.

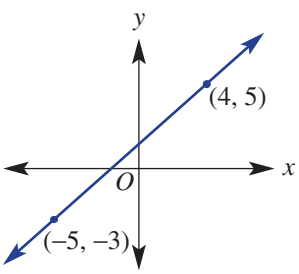
a



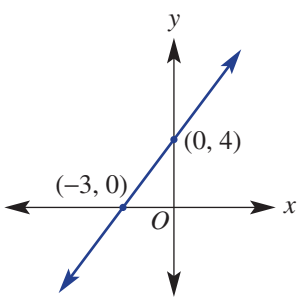
b



c



d



e (−3, 1) and (4, −2)

f (−2, −5) and (1, 7)

g (5, −2) and (−4, 6)

h $\left(-\frac{1}{2}, 2\right)$ and $\left(\frac{9}{2}, -3\right)$

i $\left(-4, \frac{2}{3}\right)$ and $\left(3, -\frac{4}{3}\right)$

j $\left(-\frac{3}{2}, \frac{2}{3}\right)$ and $\left(4, -\frac{2}{3}\right)$

k $\left(-\frac{7}{4}, 2\right)$ and $\left(\frac{1}{3}, \frac{1}{2}\right)$

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9E Gradient–intercept form EXTENDING

Interactive

Widgets

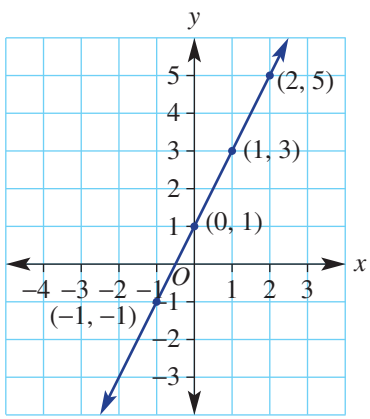
HOTsheets

Walkthroughs

From previous sections in this chapter, you may have noticed some connections between the numbers that make up a rule for a linear relationship and the numbers that are the gradient and y-coordinate with $x = 0$ (the y-intercept). This is no coincidence. Once the gradient and y-intercept of a graph are known, the rule can be written down without further analysis.

Let's start: What's the connection?

To explore the connection between the rule for a linear relationship and the numbers that are the gradient and the y-intercept, complete the missing details for the graph and table below.



x	−1	0	1	2
y	−1	1		

+2

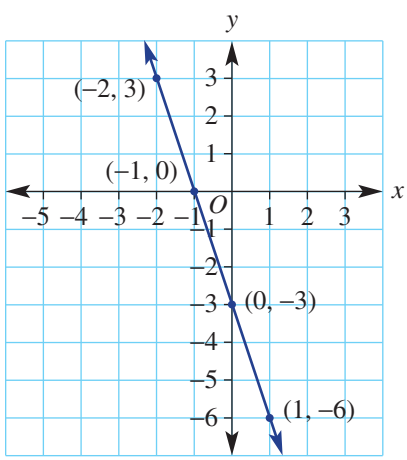
$y =$ ☐ $\times x +$ ☐

Gradient $= \frac{\text{rise}}{\text{run}}$

$=$ _____

y-intercept $=$ _____

- What do you notice about the numbers in the rule including the coefficient of x and the constant (below the table) and the numbers for the gradient and y-intercept?
- Complete the details for this new example below to see if your observations are the same.



x	−2	−1	0	1
y				

$y =$ ☐ $\times x +$ ☐

Gradient $= \frac{\text{rise}}{\text{run}}$

$=$ _____

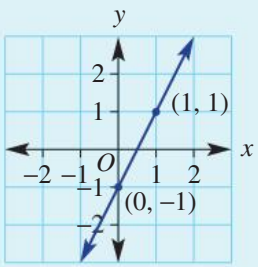
y-intercept $=$ _____

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Key ideas

- The rule for a straight line graph is given by $y = mx + c$ where:
  - m is the gradient
  - c is the **y-intercept**

  For example:



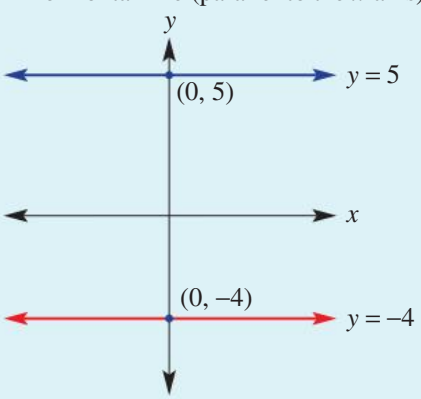
$m = \frac{2}{1} = 2$

$c = -1$

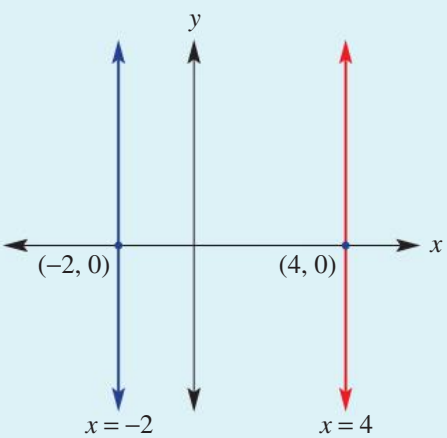
So $y = mx + c$ becomes

$y = 2x - 1$

- A horizontal line (parallel to the x-axis) has the rule $y = c$, since $m = 0$.



- A vertical line (parallel to the y-axis) has the rule $x = k$.



Example 8 Stating the gradient and y-intercept from the rule

Write down the gradient and y-intercept for the graphs of these rules.

a $y = 2x + 3$

b $y = \frac{1}{3}x - 4$

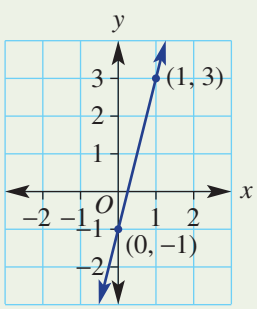
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SOLUTION	EXPLANATION
a $y = 2x + 3$ gradient $= 2$ y-intercept $= 3$	The coefficient of x is 2 and this number is the gradient. The y-intercept is the constant.
b $y = \frac{1}{3}x - 4$ gradient $= \frac{1}{3}$ y-intercept $= -4$	The gradient (m) is the coefficient of x. Remember that $y = \frac{1}{3}x - 4$ is the same as $y = \frac{1}{3}x + (-4)$ so the constant is -4.

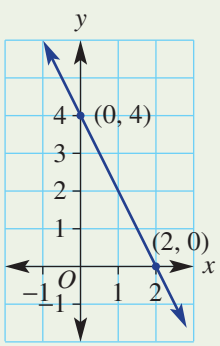
Example 9 Finding a rule from a graph

Find the rule for these graphs by first finding the values of m and the y-intercept.

a



b



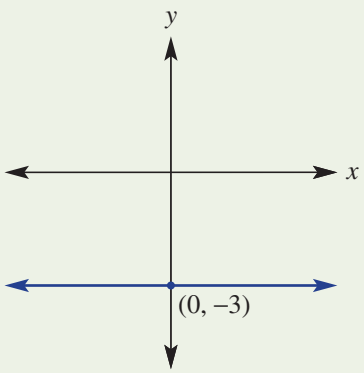
SOLUTION	EXPLANATION
a $m = \frac{\text{rise}}{\text{run}}$ $= \frac{4}{1}$ $= 4$ $c = -1$ $y = 4x - 1$	Between $(0, -1)$ and $(1, 3)$ the rise is 4 and the run is 1. The line cuts the y-axis at -1. Substitute the value of m and c into $y = mx + c$.
b $m = \frac{\text{rise}}{\text{run}}$ $= \frac{-4}{2}$ $= -2$ $c = 4$ $y = -2x + 4$	Between $(0, 4)$ and $(2, 0)$ y falls by 4 units as x increases by 2. The line cuts the y-axis at 4.

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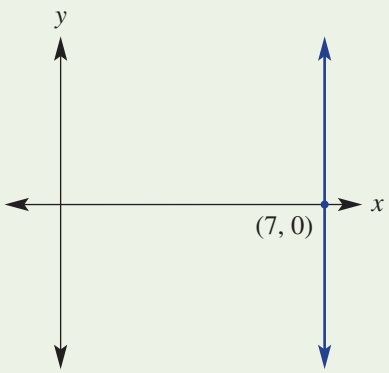
Example 10 Finding rules for horizontal and vertical lines

Write the rule for these horizontal and vertical lines.

a



b



SOLUTION	EXPLANATION
a $y = -3$	All points on the line have a y value of -3 and the gradient is 0.
b $x = 7$	Vertical lines take the form $x = k$. Every point on the line has an x value of 7.

Exercise 9E

1–2(½) | 2(½) | —

UNDERSTANDING

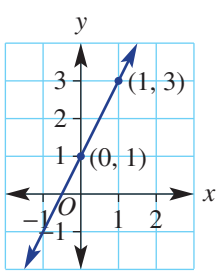
1 Substitute the given value of m and c into $y = mx + c$ to write a rule.

a $m = 2$ and $c = 3$ **b** $m = 4$ and $c = 2$ **c** $m = -3$ and $c = 1$

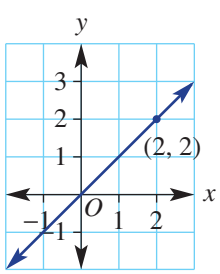
d $m = -5$ and $c = -3$ **e** $m = 2$ and $c = -5$ **f** $m = -10$ and $c = -11$

2 For these graphs write down the y-intercept and find the gradient using $\frac{\text{rise}}{\text{run}}$.

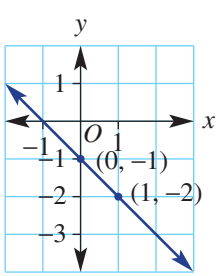
a



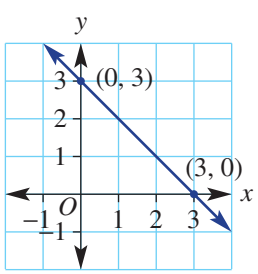
b



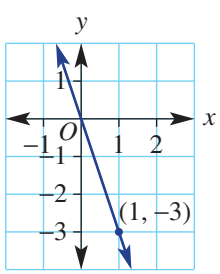
c



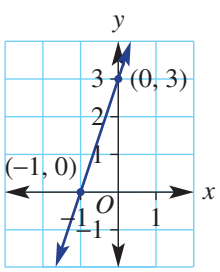
d



e



f



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3–6(½) 3–7(½) 3–7(½)

9E

FLUENCY

Example 8

3 Write down the gradient and y-intercept for the graphs of these rules.

a $y = 4x + 2$ **b** $y = 3x + 7$ **c** $y = \frac{1}{2}x + 1$ **d** $y = \frac{2}{3}x + \frac{1}{2}$

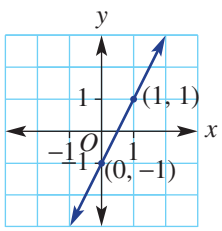
e $y = -2x + 3$ **f** $y = -4x + 4$ **g** $y = -7x - 1$ **h** $y = -3x - 7$

i $y = -x - 6$ **j** $y = -\frac{1}{2}x + 5$ **k** $y = -\frac{2}{3}x - \frac{1}{2}$ **l** $y = \frac{3}{7}x - \frac{4}{5}$

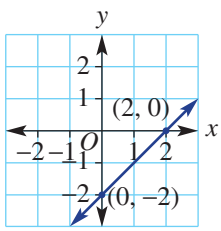
Example 9a

4 Find the rule for these graphs by first finding the values of m and c.

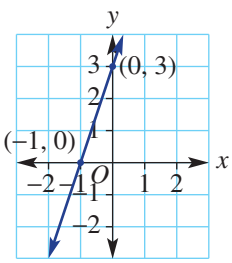
a



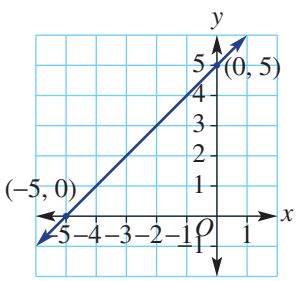
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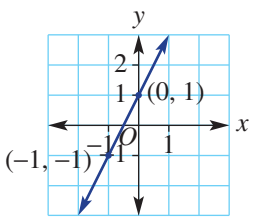
c



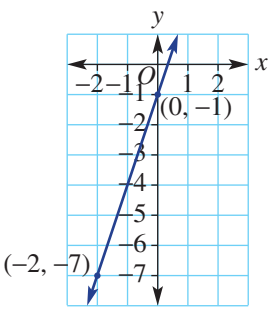
d



e



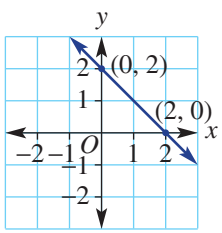
f



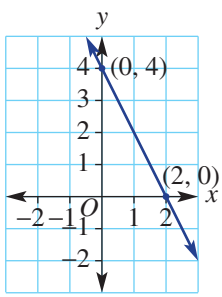
Example 9b

5 Find the rule for these graphs by first finding the values of m and c.

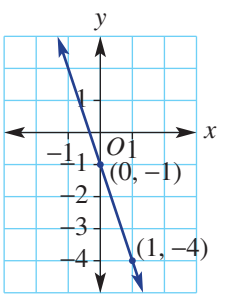
a



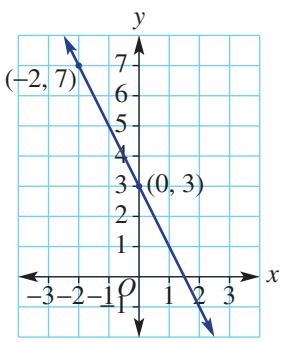
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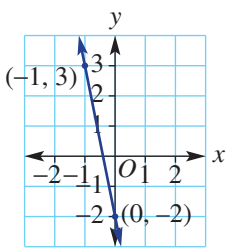
c



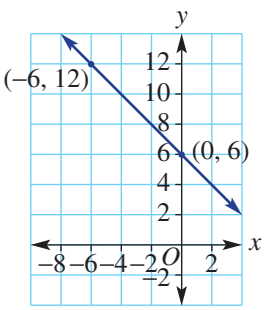
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e



f



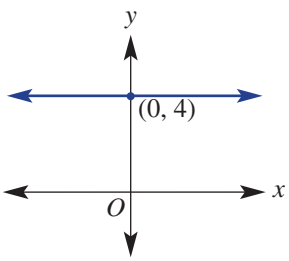
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9E

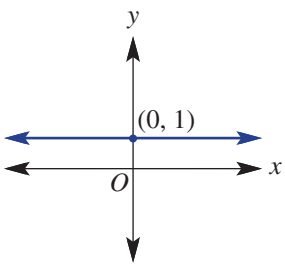
Example 10

6 Write the rule for these horizontal and vertical lines.

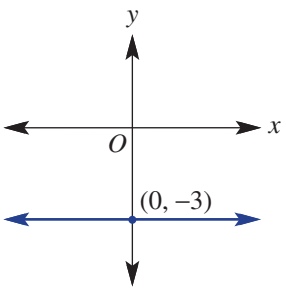
a



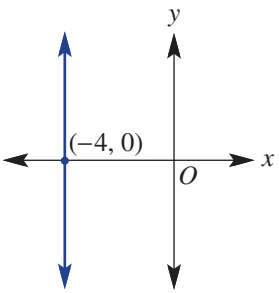
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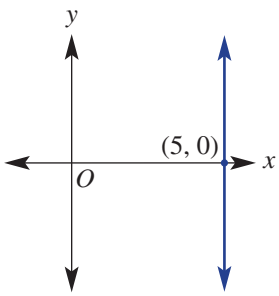
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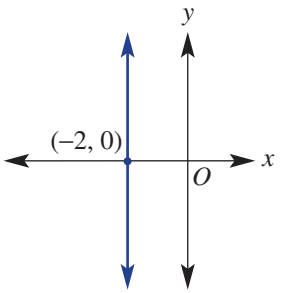
d



e



f



7 Sketch horizontal or vertical graphs for these rules.

a $y = 2$ **b** $y = -1$ **c** $y = -4$ **d** $y = 5$

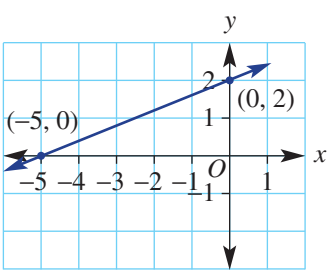
e $x = -3$ **f** $x = 4$ **g** $x = 1$ **h** $x = -1$

FLUENCY

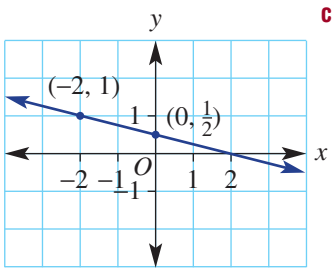
8, 9 8–10 9–11

8 These graphs have rules that involve fractions. Find m and c and write the rule.

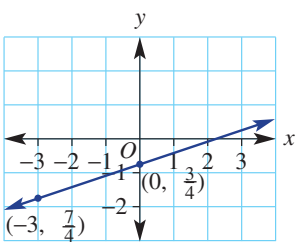
a



b



c



9 Find the rule for the graph of the lines connecting these pairs of points.

a (0, 0) and (2, 6) **b** (−1, 5) and (0, 0) **c** (−2, 5) and (0, 3) **d** (0, −4) and (3, 1)

10 A line passes through the given points. Note that the y-intercept is not given. Find m and c and write the linear rule. A graph may be helpful.

a (−1, 1) and (1, 5) **b** (−2, 6) and (2, 4) **c** (−2, 4) and (3, −1) **d** (−5, 0) and (2, 14)

11 Find the rectangular area enclosed by these sets of lines.

a $x = 4, x = 1, y = 2, y = 7$ **b** $x = 5, x = -3, y = 0, y = 5$

PROBLEM-SOLVING

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9E

12 12, 13 13, 14

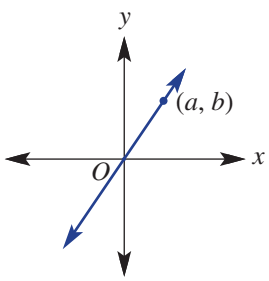
REASONING

12 **a** Explain why the rule for the x-axis is given by $y = 0$.

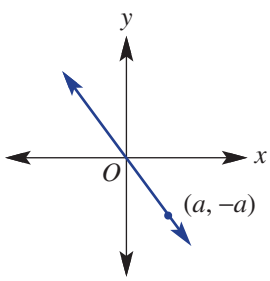
b Explain why the rule for the y-axis is given by $x = 0$.

13 Write the rule for these graphs. Your rule should include the pronumerals a and/or b.

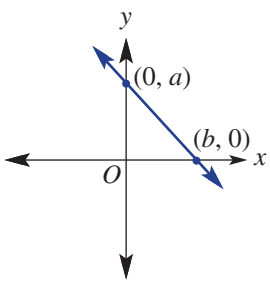
a



b



c



14 Some rules for straight lines may not be written in the form $y = mx + c$. The rule $2y + 4x = 6$, for example, can be rearranged to $2y = -4x + 6$ then to $y = -2x + 3$. So clearly $m = -2$ and $c = 3$. Use this idea to find m and c for these rules.

a $2y + 6x = 10$ **b** $3y - 6x = 9$ **c** $2y - 3x = 8$ **d** $x - 2y = -6$

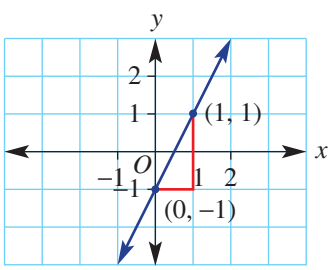
Sketching with m and c

— — 15

ENRICHMENT

15 The gradient and y-intercept can be used to sketch a graph without the need to plot more than two points.

For example, the graph of the rule $y = 2x - 1$ has $m = 2 \left(= \frac{2}{1}\right)$ and $c = -1$. By plotting the point $(0, -1)$ for the y-intercept and moving 1 to the right and 2 up for the gradient, a second point $(1, 1)$ can be found.



Use this idea to sketch the graphs of these rules.

a $y = 3x - 1$ **b** $y = 2x - 3$ **c** $y = -x + 2$ **d** $y = -3x - 1$

e $y = 4x$ **f** $y = -5x$ **g** $y = \frac{1}{2}x - 2$ **h** $y = -\frac{3}{2}x + 1$



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9F The *x*-intercept EXTENDING

Interactive

Widgets

HOTsheets

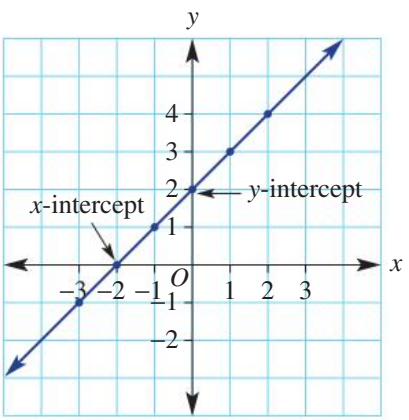
Walkthroughs

We know that the *y*-intercept marks the point where a line cuts the *y*-axis. This is also the value of *y* for the rule where $x = 0$. Similarly the *x*-intercept marks the point where $y = 0$. This can be viewed in a table of values or found using the algebraic method.

x-intercept where $y = 0$

x	−3	−2	−1	0	1	2
y	−1	0	1	2	3	4

y-intercept where $x = 0$



Let's start: Discover the method

These rules all give graphs that have *x*-intercepts at which $y = 0$.

A $y = 2x - 2$ **B** $y = x - 5$ **C** $y = 3x - 9$ **D** $y = 4x + 3$

- First try to guess the *x*-intercept by a trial and error (guess and check) method. Start by asking what value of *x* makes $y = 0$.
- Discuss why the rule for **D** is more difficult to work with than the others.
- Can you describe an algebraic method that will give the *x*-intercept for any rule? How would you show your working for such a method?

Key ideas

- The ***x*-intercept** is the point on a graph where $y = 0$.
- Find the *x*-intercept by substituting $y = 0$ into the rule. Solve using algebraic steps.

For example:

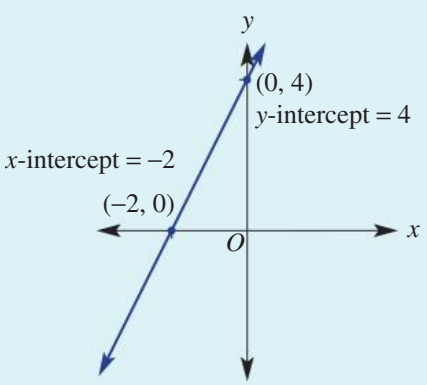
$y = 2x + 4$

$0 = 2x + 4$

$-4 = 2x$

$-2 = x$

$\therefore$ *x*-intercept is −2



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Example 11 Finding the *x*-intercept

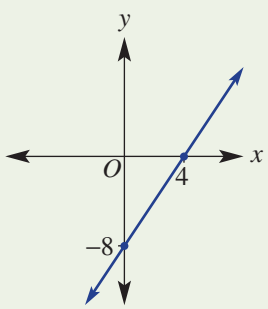
For the graphs of these rules, find the *x*-intercept.

a $y = 3x - 6$ **b** $y = -2x + 1$

SOLUTION	EXPLANATION
a $y = 3x - 6$	
$0 = 3x - 6$	Substitute $y = 0$ into the rule.
$6 = 3x$	Add 6 to both sides.
$2 = x$	Divide both sides by 3.
$\therefore x$-intercept is 2	
b $y = -2x + 1$	
$0 = -2x + 1$	Substitute $y = 0$ into the rule.
$-1 = -2x$	Subtract 1 from both sides.
$\frac{1}{2} = x$	Divide both sides by -2.
$\therefore x$-intercept is $\frac{1}{2}$	

Example 12 Sketching with intercepts

Find the *x*- and *y*-intercepts and then sketch the graph of the rule $y = 2x - 8$.

SOLUTION	EXPLANATION
$y = 2x - 8$	
$0 = 2x - 8$	Substitute $y = 0$ into the rule.
$8 = 2x$	Add 8 to both sides.
$4 = x$	Divide both sides by 2.
x-intercept is 4.	
y-intercept is -8.	The *y*-intercept is the value of c in $y = mx + c$. Alternatively substitute $x = 0$ to get $y = -8$.
	Sketch by showing the *x*- and *y*-intercepts. There is no need to show a grid.

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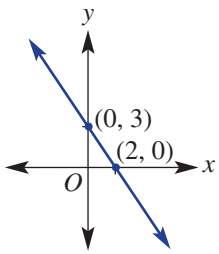
Exercise 9F

1–3(½) | 3(½) | —

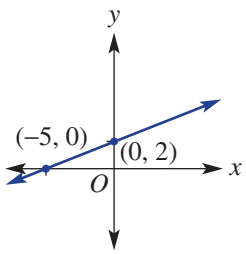
UNDERSTANDING

1 Look as these graphs and write down the x-intercept (the value of x where $y = 0$).

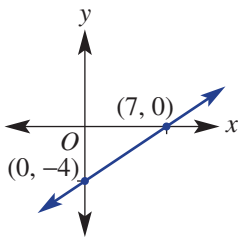
a



b



c



2 For each of these tables state the value of x that gives a y value of 0.

a

x	−2	−1	0	1	2
y	2	0	−2	−4	−6

b

x	1	2	3	4	5
y	3	2	1	0	−1

c

x	−4	−3	−2	−1	0
y	8	4	0	−4	−8

d

x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
y	−2	0	2	4	6

3 Solve each of these equations for x.

a $0 = x - 4$ **b** $0 = x + 2$ **c** $0 = x + 5$
d $0 = 2x + 10$ **e** $0 = 3x - 12$ **f** $0 = 4x + 28$
g $0 = 3x + 1$ **h** $0 = 5x + 2$ **i** $0 = -2x - 4$
j $0 = -5x - 20$ **k** $0 = -x + 2$ **l** $0 = -x + 45$

4–5(½) | 4–5(½) | 4–5(½)

FLUENCY

Example 11

4 For the graphs of these rules, find the x-intercept.

a $y = x - 1$ **b** $y = x - 6$ **c** $y = x + 2$
d $y = 2x - 8$ **e** $y = 4x - 12$ **f** $y = 3x + 6$
g $y = 2x + 20$ **h** $y = -2x + 4$ **i** $y = -4x + 8$
j $y = -2x - 4$ **k** $y = -x - 7$ **l** $y = -x + 11$

Example 12

5 Find the x- and y-intercepts and then sketch the graphs of these rules.

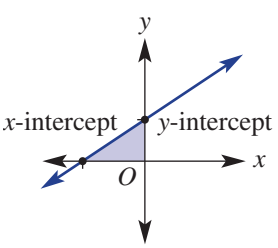
a $y = x + 1$ **b** $y = x - 4$ **c** $y = 2x - 10$
d $y = 3x + 9$ **e** $y = -2x - 4$ **f** $y = -4x + 8$
g $y = -x + 3$ **h** $y = -x - 5$ **i** $y = -3x - 15$

6, 7 | 6–8 | 7–9

PROBLEM-SOLVING

6 Find the area of the triangle enclosed by the x-axis, the y-axis and the given line. You will first need to find the x- and y-intercepts.

a $y = 2x + 4$ **b** $y = 3x - 3$
c $y = -x + 5$ **d** $y = -4x - 8$



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9F

PROBLEM-SOLVING

7 Write the rule for a graph that matches the given information.

- **a** y-intercept 4, x-intercept -4
- **b** y-intercept -1, x-intercept 2
- **c** x-intercept -2, gradient 3
- **d** x-intercept 5, gradient -5

8 The height of water (H cm) in a tub is given by $H = -2t + 20$ where t is the time in seconds.

- **a** Find the height of water initially (i.e. at $t = 0$).
- **b** How long will it take for the tub to empty?



9 The amount of credit (C cents) on a phone card is given by the rule $C = -t + 200$ where t is time in seconds.

- **a** How much credit is on the card initially ($t = 0$)?
- **b** For how long can you use the phone card before the money runs out?

REASONING

	10	10, 11	11, 12

10 Some lines have no x-intercept. What type of lines are they? Give two examples.

11 Decide if the x-intercept will be positive or negative for $y = mx + c$ under these conditions.

- **a** m is positive and c is positive, e.g. $y = 2x + 4$
- **b** m is positive and c is negative, e.g. $y = 2x - 4$
- **c** m is negative and c is negative, e.g. $y = -2x - 4$
- **d** m is negative and c is positive, e.g. $y = -2x + 4$

12 Write a rule for the x-intercept if $y = mx + c$. Your answer will include the pronumerals m and c.

ENRICHMENT

Using $ax + by = d$	—	—	13

13 The x-intercept can be found if the rule for the graph is given in any form. Substituting $y = 0$ starts the process whatever the form of the rule. Find the x-intercept for the graphs of these rules.

- **a** $x + y = 6$
- **b** $3x - 2y = 12$
- **c** $y - 2x = 4$
- **d** $2y - 3x = -9$
- **e** $y - 3x = 2$
- **f** $3y + 4x = 6$
- **g** $5x - 4y = -10$
- **h** $2x + 3y = 3$
- **i** $y - 3x = -1$

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Progress quiz

9A **1** For x-coordinates from -2 to 2, construct a table and draw a graph for the rule $y = 2x + 1$.

9B **2** Decide if the given points lie on the graph with the given rule.

a Rule: $y = 3x$ Points: **i** $(2, 6)$ **ii** $(4, 7)$

b Rule: $y = 4 - x$ Points: **i** $(5, 1)$ **ii** $(-2, 6)$

9C **3** Find the rule for these tables of values.

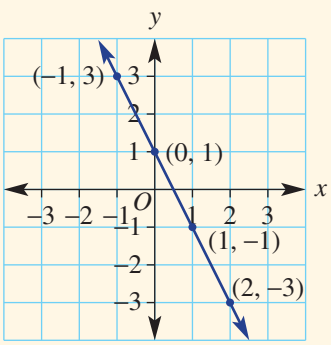
a

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

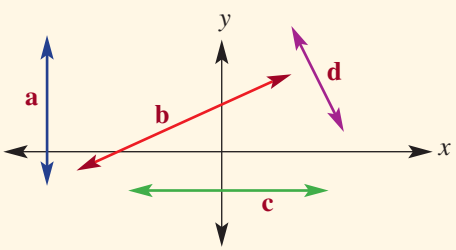
b

x	2	3	4	5	6
y	-2	-4	-6	-8	-10

9C **4** Find the rule for this graph by first constructing a table of (x, y) values.

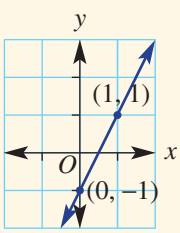


9D **Ext** **5** Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.

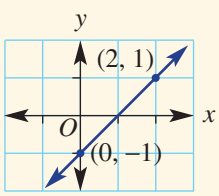


9D **Ext** **6** Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

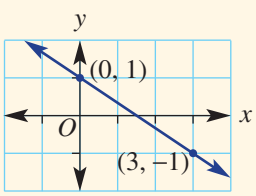
a



b



c



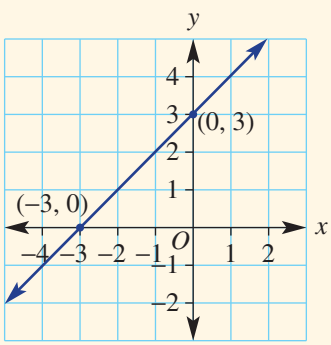
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9E Ext **7** Write down the gradient and y-intercept for the graphs of these rules.

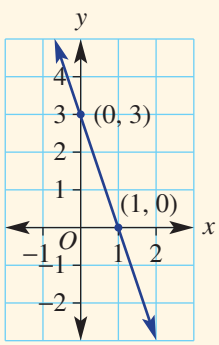
a $y = 3x + 2$ **b** $y = \frac{2}{3}x - 5$ **c** $y = -3x + 4$ **d** $y = -x - \frac{4}{5}$

9E Ext **8** Find the rule for these graphs by first finding the values of m and the y-intercept.

a

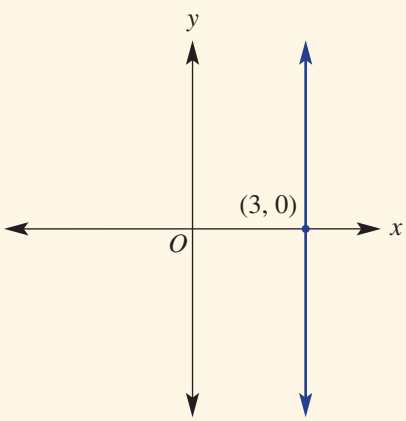


b

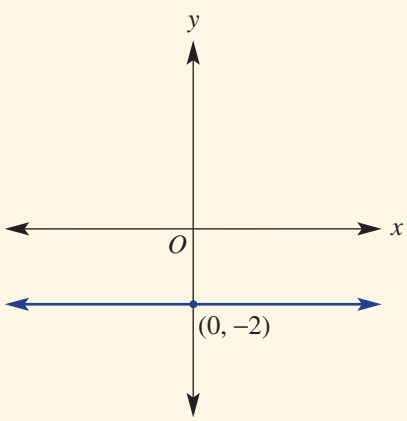


9E Ext **9** Write the rule for these horizontal and vertical lines.

a



b



9F Ext **10** For the graphs of these rules, find the x-intercept.

a $y = 2x - 6$ **b** $y = -3x - 12$

9F Ext **11** Find the x- and y-intercepts and then sketch the graph of the rule $y = 3x - 6$.

9E Ext **12** Find the rule for the graph of the line connecting this pair of points: $(0, -5)$ and $(2, -9)$.

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9G Using graphs to solve linear equations

Interactive

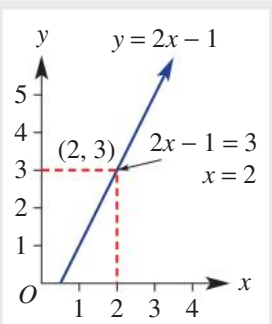
Widgets

HOTsheets

Walkthroughs

The rule for a straight line shows the connection between the x- and y-coordinate of each point on the line. We can substitute a given x-coordinate into the rule to calculate the y-coordinate. When we substitute a y-coordinate into the rule, it makes an equation that can be solved to give the x-coordinate. So, for every point on a straight line, the value of the x-coordinate is the solution to a particular equation.

The point of intersection of two straight lines is the shared point where the lines cross over each other. This is the only point with coordinates that satisfy both equations; that is, makes both equations true (LHS = RHS).



For example, the point (2, 3) on the line $y = 2x - 1$ shows us that when $2x - 1 = 3$ the solution is $x = 2$.

Let's start: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

- From the lists below, match each equation with a solution. Some equations have more than one solution.

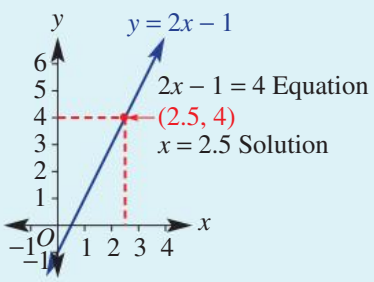
Equations	
$2x - 4 = 8$	$y = x + 4$
$3x + 2 = 11$	$y = 2x - 5$
$y = 10 - 3x$	$5x - 3 = 2$

Possible solutions			
$x = 1$	$(1, 5)$	$x = 2$	$(3, 1)$
$x = -1$	$x = 6$	$(2, -1)$	$(2, 6)$
$(-2, -9)$	$(-2, 16)$	$x = 3$	$(2, 4)$

- Which two equations share the same solution and what is this solution?
- List the equations that have only one solution. What is a common feature of these equations?
- List the equations that have more than one solution. What is a common feature of these equations?

Key ideas

- The x-coordinate of each point on the graph of a straight line is a solution to a particular linear equation.
  - A particular linear equation is formed by substituting a chosen y-coordinate into a linear relationship.
    For example, if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.
  - The solution to this equation is the x-coordinate of the point with the chosen y-coordinate.
    For example, the point (2.5, 4) shows that $x = 2.5$ is the solution to $2x - 1 = 4$.



- A point (x, y) is a solution to the equation for a line if its coordinates make the equation true.
  - An equation is true when LHS = RHS after the coordinates are substituted.
  - Every point on a straight line is a solution to the equation for that line.
  - Every point that is not on a straight line is a not a solution to the equation for that line.

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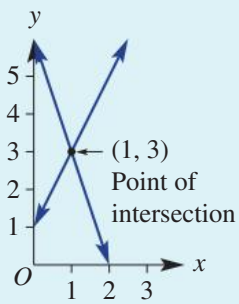
Key ideas

- The point of intersection of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that make both equations true.
    For example, (1, 3) is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.
    Substituting (1, 3)

$y = 6 - 3x$ $\quad$ $y = 2x + 1$

$3 = 6 - 3 \times 1$ $\quad$ $3 = 2 \times 1 + 1$

$3 = 3$ (True) $\quad$ $3 = 3$ (True)



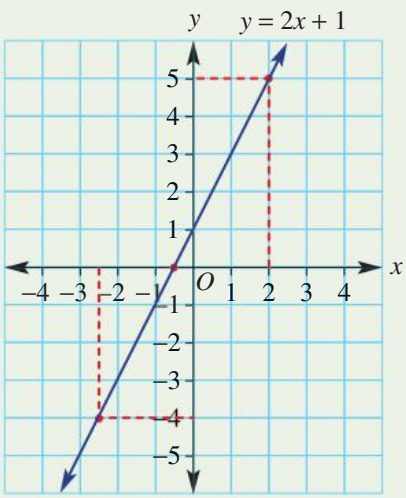
Example 13 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$ shown here to solve each of the following equations:

a $2x + 1 = 5$

b $2x + 1 = 0$

c $2x + 1 = -4$



SOLUTION	EXPLANATION
a $x = 2$	Locate the point on the line with y-coordinate 5. The x-coordinate of this point is 2 so $x = 2$ is the solution to $2x + 1 = 5$.
b $x = -0.5$	Locate the point on the line with y-coordinate 0. The x-coordinate of this point is -0.5 so $x = -0.5$ is the solution to $2x + 1 = 0$.
c $x = -2.5$	Locate the point on the line with y-coordinate -4. The x-coordinate of this point is 2.5 so $x = -2.5$ is the solution to $2x + 1 = -4$.

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Example 14 Using the point of intersection of two lines to solve an equation

Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

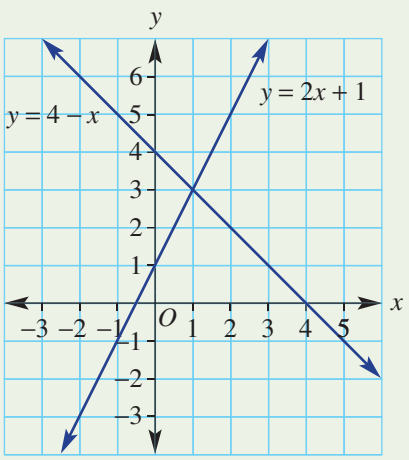
a Write two equations that each have $x = -2$ as a solution.

b Write four solutions (x, y) for the line with equation $y = 4 - x$.

c Write four solutions (x, y) for the line with equation $y = 2x + 1$.

d Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.

e Solve the equation $4 - x = 2x + 1$.



SOLUTION	EXPLANATION
a $4 - x = 6$ $2x + 1 = -3$	$(-2, 6)$ is on the line $y = 4 - x$ so $4 - x = 6$ has solution $x = -2$. $(-2, -3)$ is on the line $y = 2x + 1$ so $2x + 1 = -3$ has solution $x = -2$.
b $(-2, 6)$, $(-1, 5)$, $(1, 3)$, $(4, 0)$	Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line.
c $(-2, -3)$, $(0, 1)$, $(1, 3)$, $(2, 5)$	Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line.
d $(1, 3)$ $(1, 3)$ $y = 4 - x$ $y = 2x + 1$ $3 = 4 - 1$ $3 = 2 \times 1 + 1$ $3 = 3$ True $3 = 3$ True	The point of intersection (1, 3) is the solution that satisfies both equations. Substitute (1, 3) into each equation and show that it makes a true equation (LHS = RHS).
e $x = 1$	The solution to $4 - x = 2x + 1$ is the x-coordinate at the point of intersection. The value of both rules is equal for this x-coordinate.

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Exercise 9G

1–4 | 3, 4 | —

UNDERSTANDING

1 Use the given rule to complete this table and then plot and join the points to form a straight line. $y = 2x - 1$.

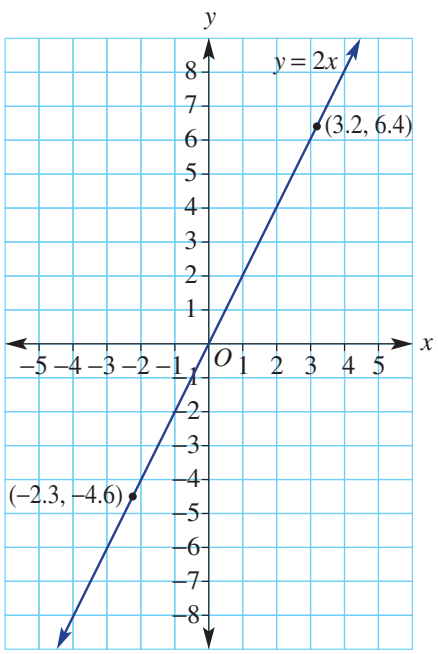
x	−2	−1	0	1	2
y					

2 Substitute each given y-coordinate into the rule $y = 2x - 3$, and then solve the equation algebraically to find the x-coordinate.

a $y = 7$ **b** $y = -5$

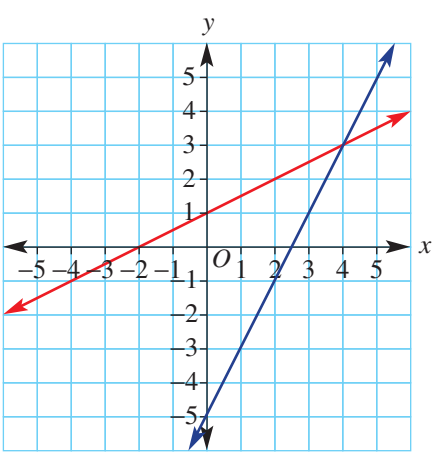
3 State the coordinates (x, y) of the point on this graph of $y = 2x$ where:

a $2x = 4$ (i.e. $y = 4$)
b $2x = 6.4$
c $2x = -4.6$
d $2x = 7$
e $2x = -14$
f $2x = 2000$
g $2x = 62.84$
h $2x = -48.602$
i $2x =$ *any number* (worded answer)

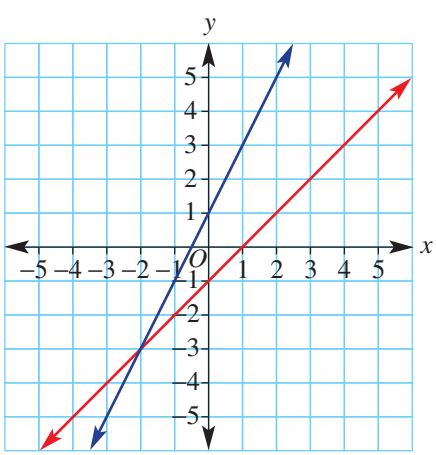


4 For each of these graphs write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

a



b



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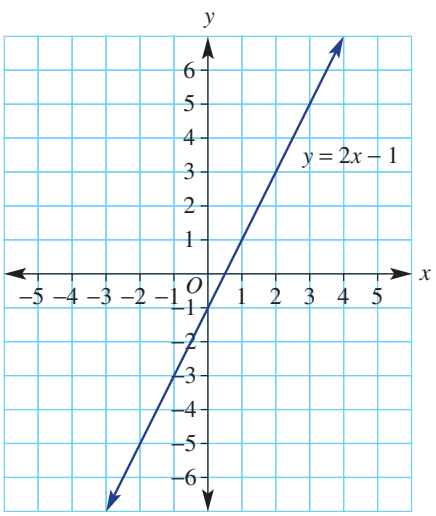
9G

Example 13

5–7(½), 8 | 5–7(½), 8 | 5–7(½), 9

FLUENCY

5 Use the graph of $y = 2x - 1$, shown here, to find the solution to each of these equations.



a $2x - 1 = 3$

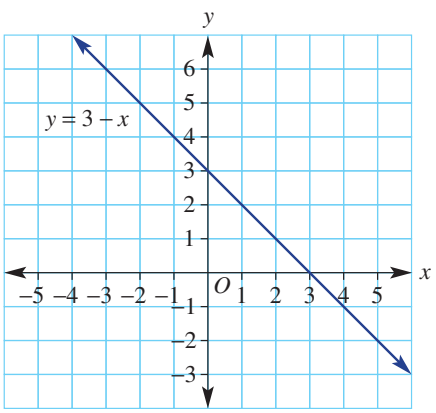
b $2x - 1 = 0$

c $2x - 1 = 5$

d $2x - 1 = -6$

e $2x - 1 = -4$

6 Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.



a $3 - x = 5.5$

b $3 - x = 0$

c $3 - x = 3.5$

d $3 - x = -1$

e $3 - x = -2$

7 Graph each pair of lines on the same set of axes and read off the point of intersection.

a $y = 2x - 1$

x	–2	–1	0	1	2	3
y						

$y = x + 1$

x	–2	–1	0	1	2	3
y						

b $y = -x$

x	–2	–1	0	1	2	3
y						

$y = x + 2$

x	–2	–1	0	1	2	3
y						

8 Use digital technology to sketch a graph of $y = 1.5x - 2.5$ for x and y values between -7 and 7. Use the graph to solve each of the following equations. Round answers to two decimal places.

a $1.5x - 2.5 = 3$ **b** $1.5x - 2.5 = -4.8$ **c** $1.5x - 2.5 = 5.446$

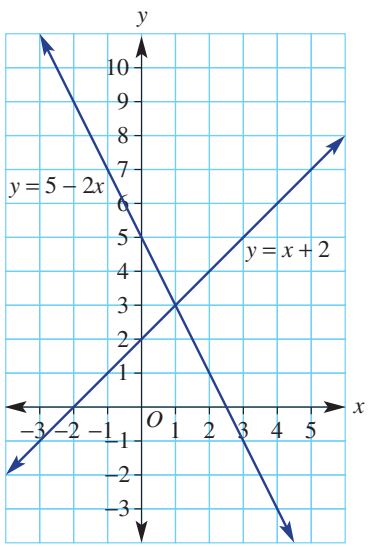
9 Use digital technology to sketch a graph of each pair of lines and find the coordinates of the points of intersection. Round answers to two decimal places.

a $y = 0.25x + 0.58$ and $y = 1.5x - 5.4$ **b** $y = 2 - 1.06x$ and $y = 1.2x + 5$

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Example 14

10 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

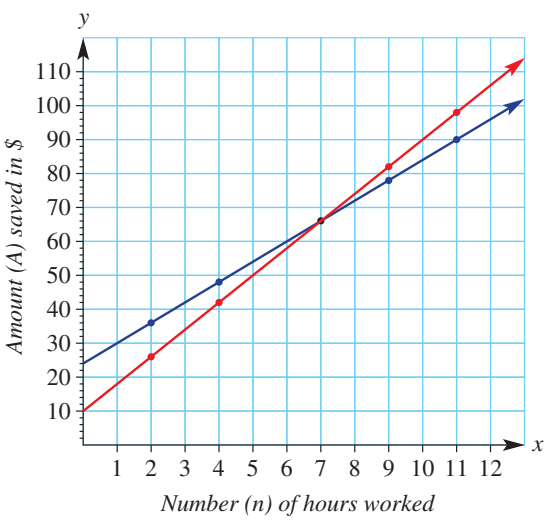


- **a** Write two equations that each have $x = -1$ as a solution.
- **b** Write four solutions (x, y) for the equation $y = 5 - 2x$.
- **c** Write four solutions (x, y) for the equation $y = x + 2$.
- **d** Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
- **e** Solve the equation $5 - 2x = x + 2$ from the graph.

11 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved \$24 and earns \$6 per hour mowing lawns.
- Ruby has saved \$10 and earns \$8 per hour babysitting.

This graph shows the total Amount (A) in dollars of their savings for the number (n) of hours worked.



- **a** Here are two rules for calculating the Amount (A) saved for working for n hours.
  $A = 10 + 8n$ and $A = 24 + 6n$
  Which rule applies to Ruby and which to Jayden? Explain why.
- **b** Use the appropriate line on the above graph to find the solution to the following equations.
  i $10 + 8n = 42$ **ii** $24 + 6n = 48$ **iii** $10 + 8n = 66$
  iv $24 + 6n = 66$ **v** $10 + 8n = 98$ **vi** $24 + 6n = 90$
- **c** From the graph write three solutions (n, A) that satisfy $A = 10 + 8n$.
- **d** From the graph write three solutions (n, A) that satisfy $A = 24 + 6n$.
- **e** Write the solution (n, A) that is true for both Ruby's and Jayden's equations and show that it satisfies both equations.
- **f** From the graph find the solution to the equation: $10 + 8n = 24 + 6n$ (i.e. find the value of n that makes Ruby's and Jayden's savings equal to each other).
- **g** Explain how many hours have been worked and what their savings are at the point of intersection of the two lines.

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9G

PROBLEM-SOLVING

12 Jessica and Max have a 10 second running race.

- Max runs at 6 m/second.
- Jessica is given a 10 m head-start and runs at 4 m/second.

a Copy and complete this table showing the distance run by each athlete.

Time (t) in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d) in metres	0										
Jessica's distance (d) in metres	10										

b Plot these points on a distance-time graph and join to form two straight lines labelling them 'Jessica' and 'Max'.

c Find the rule linking distance d and time t for Max.

d Using the rule for Max's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

e Find the rule linking distance d and time t for Jessica.

f Using the rule for Jessica's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

g Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

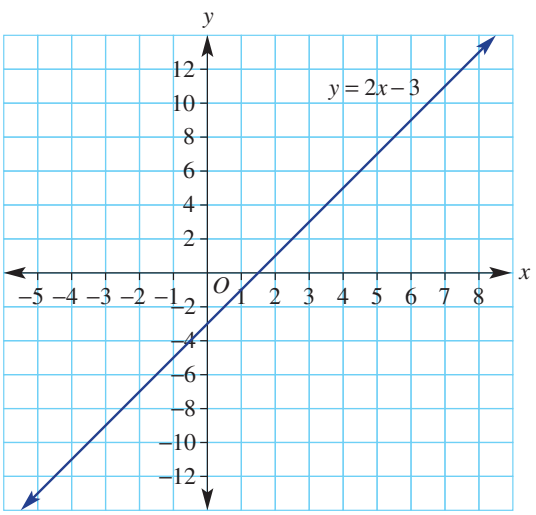
h Explain what is happening in the race at the point of intersection and for each athlete state the distance from the starting line and time taken.

13 Some equations can be re-arranged so that a given graph can be used to find a solution. For example $4x - 1 = 2(x - 3)$ can be solved using the graph of $y = 2x - 3$ with this re-arrangement:

$$\begin{aligned} & 4x - 1 = 2(x - 3) \\ -2x \swarrow\ & 4x - 1 = 2x - 6 \ \searrow -2x \\ -2 \swarrow\ & 2x - 1 = -6 \ \searrow -2 \\ & 2x - 3 = -8 \end{aligned}$$

$y = -8$ on the graph gives the solution $x = -2.5$

Re-arrange the following equations so the left side of each is $2x - 3$ and then use this graph of $y = 2x - 3$ to find each solution.



a $4x - 1 = 2(x - 5)$

b $5x + 7 = 3(x + 4)$

c $3 = 6 - 2(x - 3)$

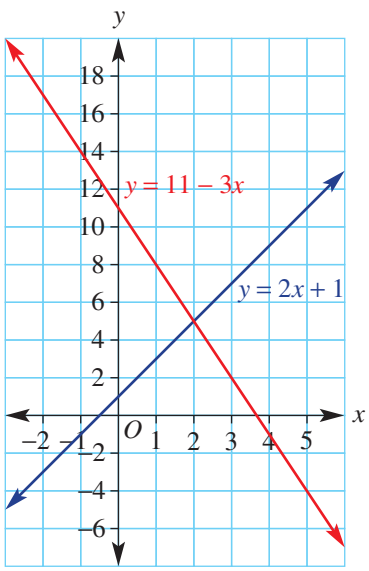
d $3 + 4x - 1 = x + 14 + x - 4$

e $3(x - 3) - 2x = 4x + 3 - 5x$

f $4(x - 1) + x = 5x - 7 - 2x$

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14 This graph shows two lines with equations $y = 11 - 3x$ and $y = 2x + 1$.



a Copy and complete the coordinates of each point that is a solution for the given linear equation.

i $y = 11 - 3x$

(−2, ?), (−1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

ii $y = 2x + 1$

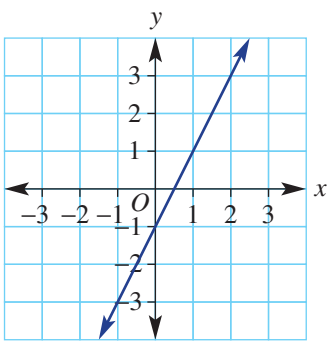
(−2, ?), (−1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

b State the coordinates of the point of intersection and show it is a solution to both equations.

c Explain why the point of intersection is the only solution that satisfies both equations.

15 Here is a table and graph for the line $y = 2x - 1$.

x	−1	−0.5	0	0.5	1	1.5	2
y	−3	−2	−1	0	1	2	3



a Luke says: *"Only seven equations can be solved from points on this line because the y values must be whole numbers."*

i What are three of the equations and their solutions that Luke could be thinking of?

ii Is Luke's statement correct? Explain your conclusion with some examples.

b Chloe says: *"There are sixty equations that can be solved from points on this line because y values can go to one decimal place."*

i What are three extra equations and their solutions that Chloe might have thought of?

ii Is Chloe's statement correct? Explain your conclusion with some examples.

c Jamie says: *"There are an infinite number of equations that can be solved from points on a straight line."*

i Write the equations that can be solved from these two points: (1.52, 2.04) and (1.53, 2.06).

ii Write the coordinates of two more points with x-coordinates between the first two points and write the equations that can be solved from these two points.

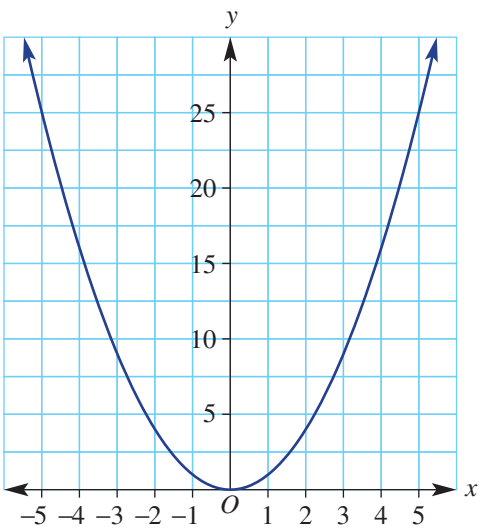
iii Is Jamie's statement correct? Explain your reasons.

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9G More than one solution — — 16

ENRICHMENT

16 a Use this graph of $y = x^2$ to solve the following equations.



- **i** $x^2 = 4$
- **ii** $x^2 = 9$
- **iii** $x^2 = 16$
- **iv** $x^2 = 25$

b Explain why there are two solutions to each of the equations in question **a** above.

c Use digital technology to graph $y = x^2$ and graphically solve the following equations, rounding answers to two decimal places.

- **i** $x^2 = 5$
- **ii** $x^2 = 6.8$
- **iii** $x^2 = 0.49$
- **iv** $x^2 = 12.75$
- **v** $x^2 = 18.795$

d Give one reason why the graph of $y = x^2$ does *not* give a solution to the equation $x^2 = -9$.

e List three more equations of the form x^2 = 'a number' that *cannot* be solved from the graph of $y = x^2$.

f List the categories of numbers that *will* give a solution to the equation: x^2 = 'a number'.

g Graph $y = x + 2$ and $y = x^2$ on the same screen and graphically solve $x^2 = x + 2$ by finding the x values of the points of intersection.

h Use digital technology to solve the following equations using graphical techniques. Round answers to two decimal places.

- **i** $x^2 = 3x + 16$
- **ii** $x^2 = 27 - 5x$
- **iii** $x^2 = 2x - 10$
- **iv** $x^2 = 6x - 9$



Parabolic solar reflector panels are based on the shape of the graph of $y = x^2$.

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9H Applying linear graphs EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

Rules and graphs can be used to help analyse many situations in which there is a relationship between two variables.

If the rate of change of one variable with respect to another is constant, then the relationship will be linear and a graph will give a straight line. For example, if a pile of dirt being emptied out of a pit increases at a rate of 12 tonnes per hour, then the graph of the mass of dirt over time would be a straight line. For every hour, the mass of dirt increases by 12 tonnes.



A constant rate of excavation creates a linear relationship and a straight line graph.

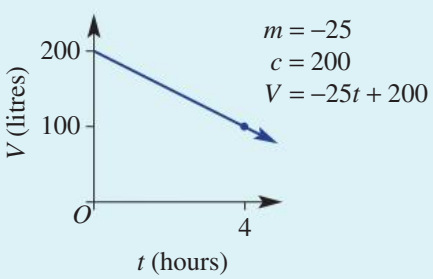
Let's start: Water storage

The volume of water in a tank starts at 1000 litres and with constant rainfall the volume of water increases by 2000 litres per hour for 5 hours.

- Describe the two related variables in this situation.
- Discuss whether or not the relationship between the two variables is **linear**.
- Use a table and a graph to illustrate the relationship.
- Find a rule that links the two variables and discuss how your rule might be used to find the volume of water in the tank at a given time.

Key ideas

- If the rate of change of one variable with respect to another is constant, then the relationship between the two variables is **linear**.
- When applying straight line graphs, choose letters to replace x and y to suit the variables. For example, V for volume and t for time.
- $y = mx + c$ can be used to help find the linear rule linking two variables.



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Example 15 Linking distance with time

A hiker walks at a constant rate of 4 kilometres per hour for 4 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 4.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking d with t.

d Use your rule to find the distance travelled for 2.5 hours of walking.

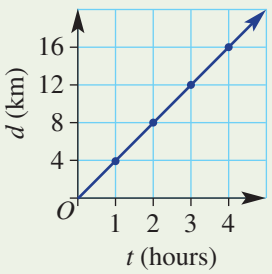
e Use your rule to find the time taken to travel 8 km.

SOLUTION

a

t	0	1	2	3	4
d	0	4	8	12	16

b



c $m = \frac{4}{1} = 4$

$c = 0$

$y = mx + c$

$d = 4t + 0$

So $d = 4t$

d $d = 4t$

$= 4 \times 2.5$

$= 10$

The distance is 10 km after 2.5 hours of walking.

e $d = 4t$

$8 = 4t$

$2 = t$

It takes 2 hours to travel 8 km.

EXPLANATION

d increases by 4 for every increase in t by 1.

Plot the points on a graph using a scale that matches the numbers in the table.

Using $y = mx + c$ find the gradient (m) and the y-intercept (c). Replace y with d and x with t.

Substitute $t = 2.5$ into your rule and find the value for d.

Substitute $d = 8$ into your rule then divide both sides by 4.

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Example 16 Applying graphs when the rate is negative

The initial volume of water in a dish in the sun is 300 mL. The water evaporates and the volume decreases by 50 mL per hour for 6 hours.

a Draw a table of values using t for time in hours and V for volume in millilitres.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking V with t.

d Use your rule to find the volume of water in the dish after 4.2 hours in the sun.

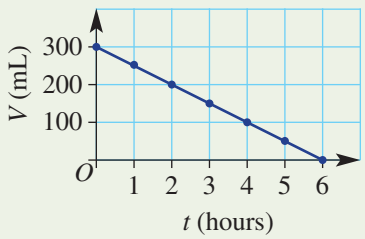
e Use your rule to find the time taken for the volume to reach 75 mL.

SOLUTION

a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

b



c $m = \dfrac{-50}{1} = -50$

$c = 300$

$y = mx + c$

$V = -50t + 300$

d $V = -50t + 300$

$= -50 \times 4.2 + 300$

$= 90$

The volume of water in the dish is 90 millilitres after 4.2 hours.

e $V = -50t + 300$

$75 = -50t + 300$

$-225 = -50t$

$4.5 = t$

It takes 4.5 hours for the volume to reach 75 mL.

EXPLANATION

The volume starts at 300 millilitres and decreases by 50 millilitres every hour.

Use numbers from 0 to 300 on the V-axis and 0 to 6 on the t-axis to accommodate all the numbers in the table.

The gradient m in $y = mx + c$ is given by $\dfrac{\text{rise}}{\text{run}} = \dfrac{-50}{1}$

c is the y-intercept = 300

Substitute $t = 4.2$ into your rule to find V.

Substitute $V = 75$ into your rule.
Subtract 300 from both sides.
Divide both sides by −50.

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Exercise 9H

1–4(½) | 4 | —

UNDERSTANDING

1 A rule linking distance d and time t is given by $d = 10t + 5$. Use this rule to find the value of d for the given values of t.

a $t = 1$ **b** $t = 4$ **c** $t = 0$ **d** $t = 12$

2 Write down the gradient (m) and the y-intercept (c) for these rules.

a $y = 3x + 2$ **b** $y = -2x - 1$ **c** $y = 4x + 2$
d $y = -10x - 3$ **e** $y = -20x + 100$ **f** $y = 5x - 2$

3 The height (in cm) of fluid in a flask increases at a rate of 30 cm every minute starting at 0 cm. Find the height of fluid in the flask at these times.

a 2 minutes **b** 5 minutes **c** 11 minutes

4 The volume of gas in a tank decreases from 30 L by 2 L every second. Find the volume of gas in the tank at these times.

a 1 second **b** 3 seconds **c** 10 seconds

5–7 | 5–8 | 6–8

FLUENCY

Example 15

5 A jogger runs at a constant rate of 6 kilometres per hour for 3 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 3.
b Draw a graph by plotting the points given in the table in part **a**.
c Write a rule linking d with t.
d Use your rule to find the distance travelled for 1.5 hours of jogging.
e Use your rule to find how long it takes to travel 12 km.

6 A paddle steamer moves up the Murray River at a constant rate of 5 kilometres per hour for 8 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t values between 0 and 8.
b Draw a graph by plotting the points given in the table in part **a**.
c Write a rule linking d with t.
d Use your rule to find the distance travelled after 4.5 hours.
e Use your rule to find how long it takes to travel 20 km.



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9H

FLUENCY

Example 16

7 The volume of water in a sink is 20 L. The plug is pulled out and the volume decreases by 4 L per second for 5 seconds.

- **a** Draw a table of values using t for time in seconds and V for volume in litres.
- **b** Draw a graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking V with t.
- **d** Use your rule to find the volume of water in the sink 2.2 seconds after the plug is pulled.
- **e** Use your rule to find how long it takes for the volume to fall to 8 L.

8 A weather balloon at a height of 500 m starts to descend at a rate of 125 m per minute for 4 minutes.

- **a** Draw a table of values using t for time in minutes and h for height in metres.
- **b** Draw a graph by plotting the points given in the table in part **a**.
- **c** Write a rule linking h with t.
- **d** Use your rule to find the height of the balloon after 1.8 minutes.
- **e** Use your rule to find how long it takes for the balloon to fall to a height of 125 m.

	9, 10	9, 10	10, 11

PROBLEM-SOLVING

9 A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.

- **a** Write a rule for the mass of gas M in terms of time t.
- **b** How long will it take for the gas bottle to empty?
- **c** How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

10 A cyclist races 50 km at an average speed of 15 km per hour.

- **a** Write a rule for the distance travelled d in terms of time t.
- **b** How long will it take the cyclist to travel 45 km?
- **c** How long will the cyclist take to complete the 50-km race? Give your answer in hours and minutes.

11 An oil well starts to leak and the area of an oil slick increases by 8 km^2 per day. How long will it take the slick to increase to 21 km^2? Give your answer in days and hours.



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9H

	12	12	12, 13

REASONING

12 The volume of water in a tank (in litres) is given by $V = 2000 - 300t$ where t is in hours.

- **a** What is the initial volume?
- **b** Is the volume of water in the tank increasing or decreasing? Explain your answer.
- **c** At what rate is the volume of water changing?

13 The cost of a phone call is 10 cents plus 0.5 cents per second.

- **a** Explain why the cost c cents of the phone call for t seconds is given by $c = 0.5t + 10$.
- **b** Explain why the cost C dollars of the phone call for t seconds is given by $C = 0.005t + 0.1$.

Danger zone	—	—	14

ENRICHMENT

14 Two small planes take off and land at the same airfield. One plane takes off from the runway and gains altitude at a rate of 15 metres per second. At the same time, the second plane flies near the runway and reduces its altitude from 100 metres at rate of 10 metres per second.

- **a** Draw a table of values using t between 0 and 10 seconds and h for height in metres of both planes.

t (s)	0	1	2	3	4	5	6	7	8	9	10
h_1 (m)											
h_2 (m)											

- **b** On the one set of axes draw a graph of the height of each plane during the 10-second period.
- **c** How long does it take for the second plane to touch the ground?
- **d** Write a rule for the height of each plane.
- **e** At what time are the planes at the same height?
- **f** At what time is the first plane at a height of 37.5 m?
- **g** At what time is the second plane at a height of 65 m?
- **h** At the same time, a third plane at an altitude of 150 m descends at a rate of 25 m per second. Will all three planes ever be at the same height at the same time? What are the heights of the three planes at the 4-second mark?



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9I Non-linear graphs EXTENDING

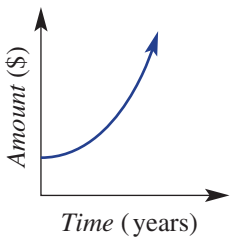
Interactive

Widgets

HOTsheets

Walkthroughs

Not all relationships between two variables are linear. The amount of money invested in a compound interest account, for example, will not increase at a constant rate. Over time, the account balance will increase more rapidly, meaning that the graph of the relationship between *Amount* and *Time* will be a curve and not a straight line.





Compound interest means your account balance increases at a more rapid pace over time.

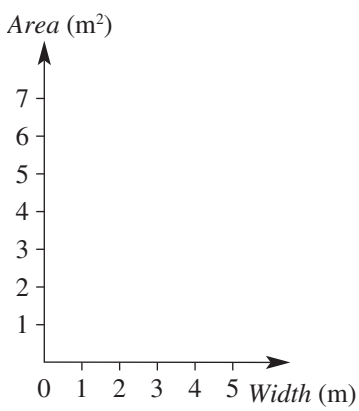
Let's start: The fixed perimeter play-pen

Imagine you have 10 metres of fencing material to make a rectangular play-pen.

- List some possible dimensions of your rectangle.
- What is the area of the play-pen for some of your listed dimensions?
- Complete this table showing all the positive integer dimensions of the play-pen.

Width (m)	1	2	3	4
Length (m)		3		
Area (m^2)		6		

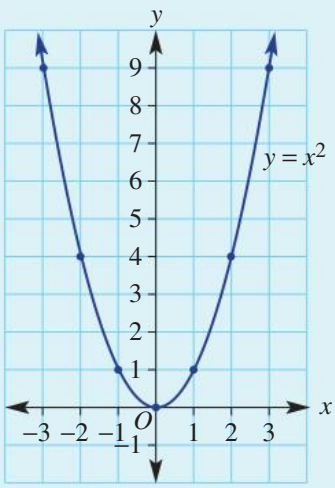
- Plot the *Area* against *Width* to form a graph.
- Discuss the shape of your graph.
- Discuss the situation and graphical points when the width is 1 m or 4 m
- What dimensions would deliver a maximum area and explain how your graph helps determine this?



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Key ideas

- To plot **non-linear** curves given their rule, follow these steps.
  - Construct a table of values using the rule.
  - Plot the points on a set of axes.
  - Join the plotted points to form a smooth curve.
- The graph of $y = x^2$ is an example of a non-linear graph called a **parabola**.

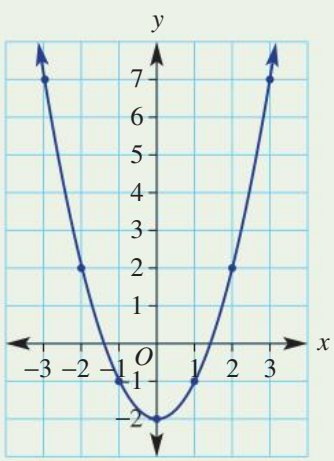


Example 17 Plotting a non-linear relationship

Plot points to draw the graph of $y = x^2 - 2$ using a table.

SOLUTION

x	−3	−2	−1	0	1	2	3
y	7	2	−1	−2	−1	2	7



EXPLANATION

Find the value of y by substituting each value of x into the rule.
Plot the points and join with a smooth curve. The curve is called a parabola.

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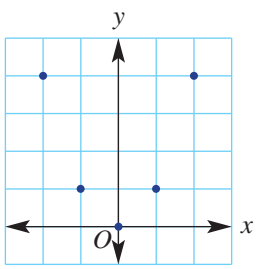
Exercise 9I

1–3 1–3 —

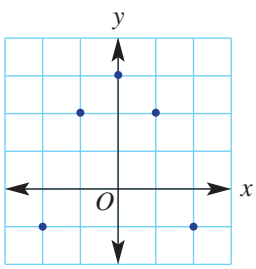
UNDERSTANDING

1 Copy these graphs then join the points to form smooth curves.

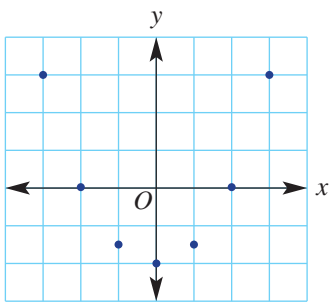
a



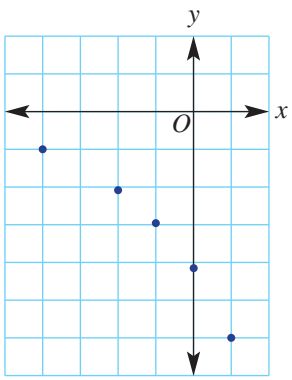
b



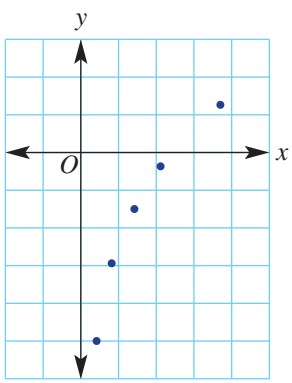
c



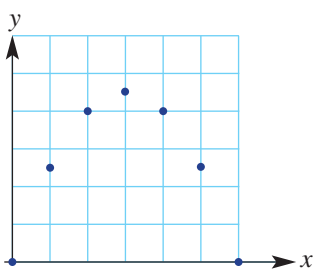
d



e



f



2 If $y = x^2 - 1$, find the value of y for these x values.

a $x = 0$ **b** $x = 3$

c $x = 2$ **d** $x = -4$

3 Decide if the following rules would give a non-linear graph.

a $y = x^2$ **b** $y = x$

c $y = 2x$ **d** $y = x^2 - 1$

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91

4, 5 | 4, 5 | 4(½), 5

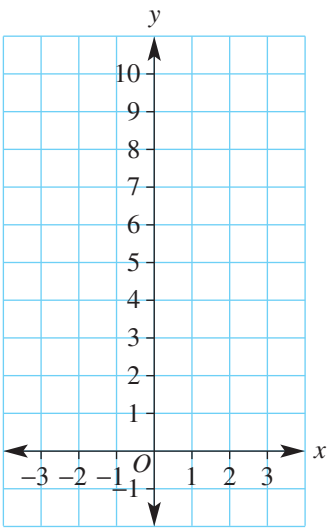
FLUENCY

Example 17

4 Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

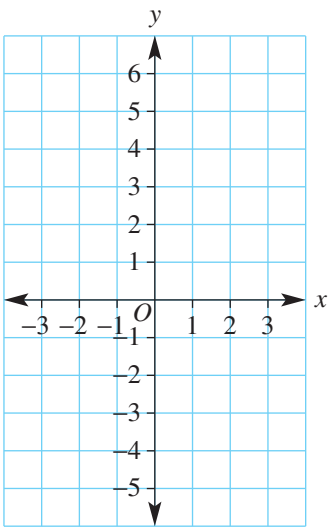
a $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9						



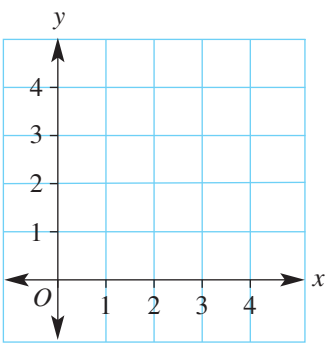
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5						



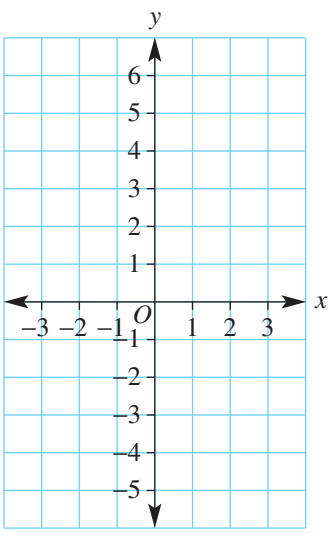
c $y = x(4 - x)$

x	0	1	2	3	4
y					



d $y = 5 - x^2$

x	−3	−2	−1	0	1	2	3
y		1					



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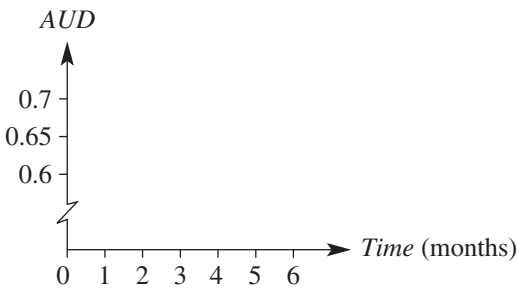
91

FLUENCY

5 The behaviour of the Australian dollar against the British pound over a 6-month period is summarised by the data in this table.

Time	0	1	2	3	4	5	6
AUD	0.69	0.64	0.61	0.6	0.61	0.64	0.69

a Plot the data on the given graph and join to form a smooth curve.



b Describe the shape of your graph.

c By how much has the Australian dollar:

i decreased in the first month?

ii increased in the fifth month?

d Estimate the value of the Australian dollar after 7 months.

6 6, 7 6–8

PROBLEM-SOLVING

6 James has 8 cm of string to form a rectangular space.

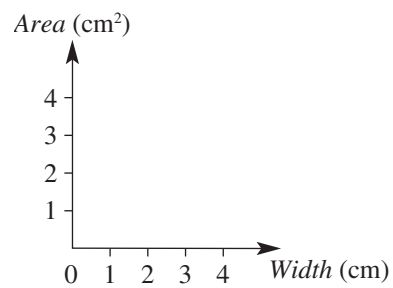
Width (cm)	0	1	2	3	4
Length (cm)			2		
Area (cm^2)			4		

a For the given width values, complete this table of values.

b Plot the *Area* against the *Width* to form a graph.

c Describe the shape of your graph.

d What rectangle dimensions appear to provide the maximum area?



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9I

PROBLEM-SOLVING

7 Lara has enough toy paint to cover 12 cm^2 of space. She intends to paint a rectangular area.

Width (cm)	1	2	3		6	
Length (cm)				3		
Perimeter (cm)						26

a For the given values, complete this table.

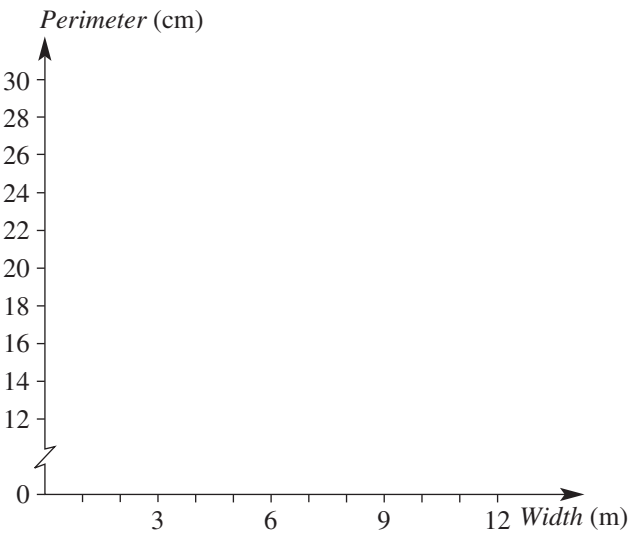
b Plot the *Perimeter* against *Width* to form a graph.

c Would you describe the curve to be linear or non-linear?

d Look at the point where there is a minimum perimeter.

i Estimate the width of the rectangle at this point.

ii Estimate the perimeter at this point.



8 What are the smallest and largest values of y on a graph of these rules if x ranges from -3 to 3?

a $y = x^2$ **b** $y = x^2 - 10$ **c** $y = 6 - x^2$

9	9, 10	9–11

REASONING

9 Explain why the graph of the rule $y = x^2$ is curved and not straight.

10 By mentally choosing and plotting points decide if the following rules would deliver linear or non-linear curves.

a $y = 5x$ **b** $y = 1 - x$ **c** $y = x^2 + 2$

d $y = \frac{1}{x}$ **e** $y = \frac{2}{x}$ **f** $y = x \times (x + 1)$

11 The sum of two numbers is 8. What is the maximum value of their product?

Families of parabolas	—	—	12

ENRICHMENT

12 For each family of parabolas, plot graphs by hand or use technology to draw each set on the same set of axes. Then describe the features of each family. Describe the effect on the graph when the number a changes.

a Family 1: $y = ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.

b Family 2: $y = -ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.

c Family 3: $y = x^2 + a$ Use $a = -3$, $a = -1$, $a = 0$, $a = 2$ and $a = 5$.

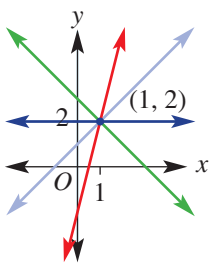
d Family 4: $y = (x - a)^2$ Use $a = -2$, $a = 0$, $a = 1$ and $a = 4$.

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Investigation

Families of straight lines

A set of lines are said to be in the same family if the lines have something in common. For example, four lines that pass through the point (1, 2).



The parallel family

1 Complete the table for these rules.

a $y_1 = 2x - 5$

b $y_2 = 2x - 2$

c $y_3 = 2x$

d $y_4 = 2x + 3$

x	–3	–2	–1	0	1	2	3
y_1							
y_2							
y_3							
y_4							

2 Plot the points given in your table to draw graphs of the four rules in part **1** on the one set of axes. Label each line with its rule.

3 What do you notice about the graphs of the four rules? Describe how the numbers in the rule relate to its graph.

4 How would the graphs for the rules $y = 2x + 10$ and $y = 2x - 7$ compare with the graphs you have drawn above? Explain.

The point family

1 Complete the table for these rules.

a $y_1 = x + 1$

b $y_2 = 2x + 1$

c $y_3 = 1$

d $y_4 = -x + 1$

e $y = -\frac{1}{2}x + 1$

x	–3	–2	–1	0	1	2	3
y_1							
y_2							
y_3							
y_4							
y_5							

2 Plot the points given in your table to draw graphs of the five rules in part **1** on one set of axes. Label each line with its rule.

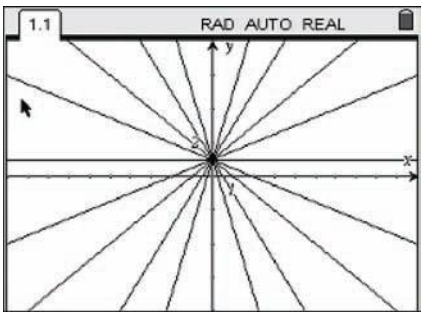
3 What do you notice about the graphs of the five rules? Describe how the numbers in the rule relate to its graph.

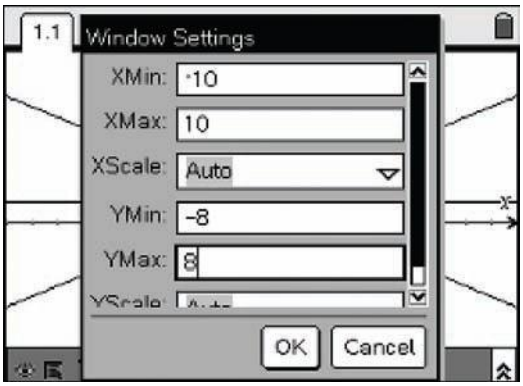
4 How would the graphs for the rules $y = 3x + 1$and $y = -\frac{1}{3}x + 1$ compare with the graphs you have drawn above? Explain.

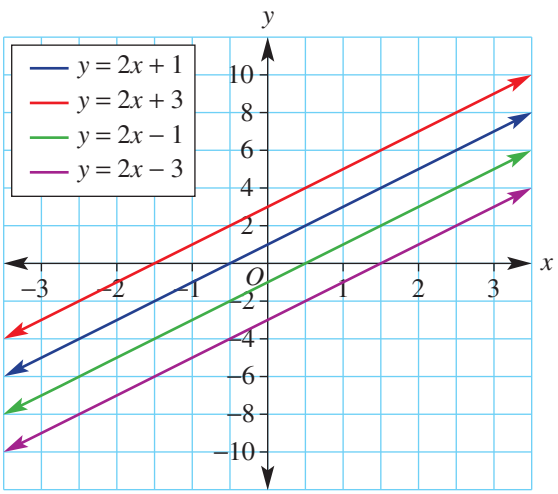
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Exploring families with technology

Graphics or CAS calculators and spreadsheets are useful tools to explore families of straight lines. Here are some screenshots showing the use of technology.





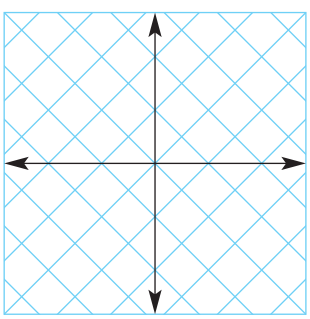


	A	B	C	D	E
1	x	y = 2x + 1	y = 2x + 3	y = 2x - 1	y = 2x - 3
2	-3	-5	-3	-7	-9
3	-2	-3	-1	-5	-7
4	-1	-1	1	-3	-5
5	0	1	3	-1	-3
6	1	3	5	1	-1
7	2	5	7	3	1
8	3	7	9	5	3

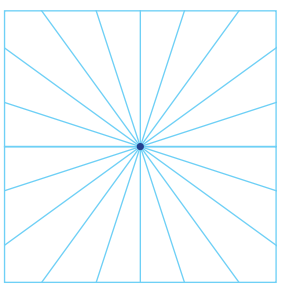
1 Choose one type of technology and sketch the graphs for the two families of straight lines shown in the previous two sections: the 'parallel family' and the 'point family'.

2 Use your chosen technology to help design a family of graphs that produces the patterns shown. Write down the rules used and explain your choices.

a



b



3 Make up your own design then use technology to produce it. Explain how your design is built and give the rules that make up the design.

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Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 A trekker hikes down a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker to catches up with the first?

2 Find the rules for the non-linear relations with these tables.

a

x	−2	−1	0	1	2
y	1	−2	−3	−2	1

b

x	−2	−1	0	1	2
y	6	9	10	9	6

c

x	0	1	4	9	16
y	1	2	3	4	5

d

x	−3	−2	−1	0	1
y	−30	−11	−4	−3	−2

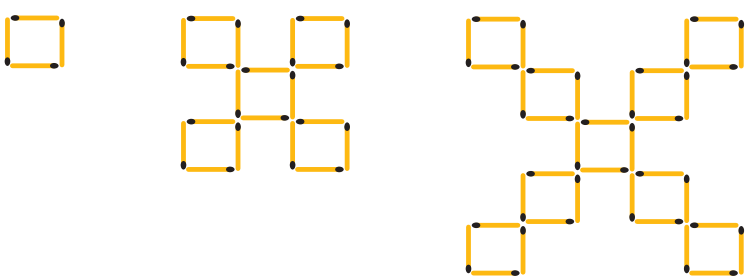
3 A line with a gradient of 3 intersects another line at right angles. Find the gradient of the other line.

4 Two cars travel toward each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

5 Find the y-intercept of a line joining the two points $(-1, 5)$ and $(2, 4)$.

6 Find the rule of a line that passes through the two points $(-3, -1)$ and $(5, 3)$.

7 Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



8 At a luxury car hire shop, a Ferrari costs \$300 plus \$40 per hour. A Porsche costs \$205 plus \$60 per hour. What hire time makes both cars the same cost? Give the answer in hours and minutes.



9 Find the area of the figure enclosed by the four lines: $x = 6, y = 4, y = -2$ and $y = x + 5$.

10 A rectangle $ABCD$ is rotated about A by 90° clockwise.

a In two different ways, explain why the diagonal AC is perpendicular to its image $A'C'$.

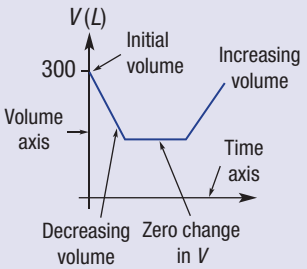
b If $AB = p$ and $BC = q$, find the simplified product of the gradients of AC and $A'C'$.

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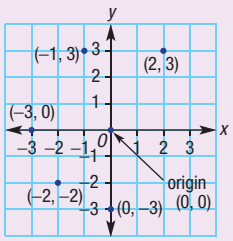
Chapter summary

Straight line graphs

Graphs



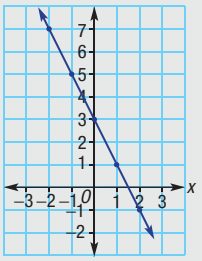
Number/Cartesian plane



Rules, tables and graphs

$y = -2x + 3$

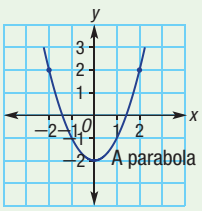
x	–2	–1	0	1	2
y	7	5	3	1	–1



Non-linear graphs (Ext)

$y = x^2 - 2$

x	–2	–1	0	1	2
y	2	–1	–2	–1	2



Finding the rule from tables

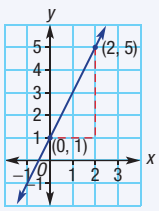
x	–2	–1	0	1	2
y	–8	–5	–2	1	4

+3 +3 +3 +3

$y = \square \times x + \square$
$= 3x + (-2)$
$= 3x - 2$

$y = mx + c$ (Ext)

Gradient y-intercept



$m = \frac{4}{2} = 2$
$c = 1$
$y = 2x + 1$

Applications (Ext)

- Distance increases by 20 km per hour.

t	0	1	2	3
d	0	20	40	60

$d = 20t$

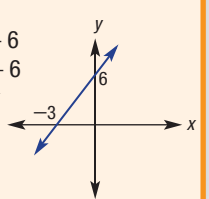
- Volume decreases from 1000 L by 200 L per minute.

t	0	1	2	3	4	5
V	1000	800	600	400	200	0

$V = -200t + 1000$

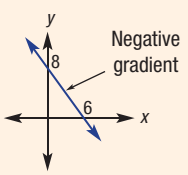
x-intercept (at $y = 0$) (Ext)

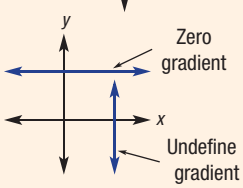
$y = 2x + 6$
$0 = 2x + 6$
$-6 = 2x$
$-3 = x$



Gradient (Ext)

$\frac{\text{rise}}{\text{run}} = -\frac{8}{6} = -\frac{4}{3}$

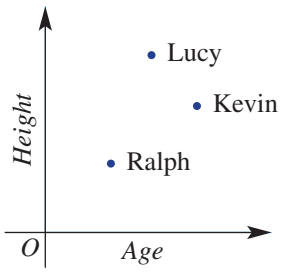




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Multiple-choice questions

9A **1** This graph shows the relationship between the height and age of 3 people. Who is the tallest person?



A Ralph
B Lucy
C Kevin
D Lucy and Ralph together
E Kevin and Lucy together

9A **2** The name of the point (0, 0) on a number (Cartesian) plane is:

A y-intercept **B** gradient **C** origin **D** axis **E** x-intercept

9A **3** Which point is not in line with the other points? $A(-2, 3), B(-1, 2), C(0, 0), D(1, 0), E(2, -1)$

A A **B** B **C** C **D** D **E** E

9B **4** Which of the points A(1, 2), $B(2, -1)$ or $C(3, -4)$ lie on the line $y = -x + 1$?

A C **B** A and C **C** A
D B **E** None

9E Ext **5** A rule gives this table of values.

x	−2	−1	0	1	2
y	3	2	1	0	−1

The y-intercept for the graph of the rule would be:

A 3 **B** 1 **C** 0
D −2 **E** −3

9D Ext **6** The gradient of a line joining the two points (0, 0) and (1, −6) is:

A 3 **B** 6 **C** 1
D −6 **E** −3

9E Ext **7** The gradient of a line is −1 and its y-intercept is −3. The rule for the line is:

A $y = -x - 3$ **B** $y = x - 3$ **C** $y = -x + 3$
D $y = x + 3$ **E** $-(x - 1)$

9E Ext **8** The rule for a horizontal line passing through (0, 6) is:

A $y = 6x + 1$ **B** $y = 6x$ **C** $y = x + 6$
D $y = x - 6$ **E** $y = 6$

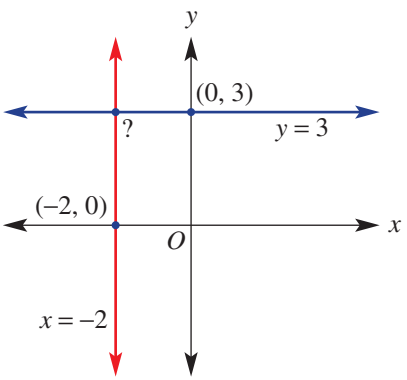
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Chapter review

9E Ext

9 The coordinates of the point of intersection of the graphs of $y = 3$ and $x = -2$ are:

A (0, 3) **B** (2, −3)
C (−2, −3) **D** (−2, 3)
E (2, 3)



9H Ext

10 The water level (h centimetres) in a dam starts at 300 cm and decreases by 5 cm every day for 10 days. The gradient of the graph of this relationship is:

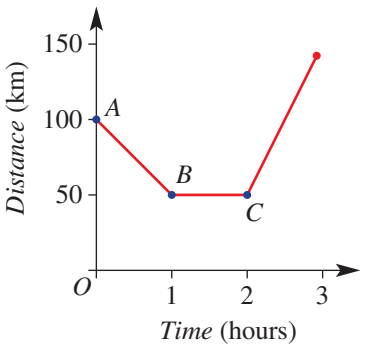
A 5 **B** 300 **C** −300
D −60 **E** −5

Short-answer questions

9A

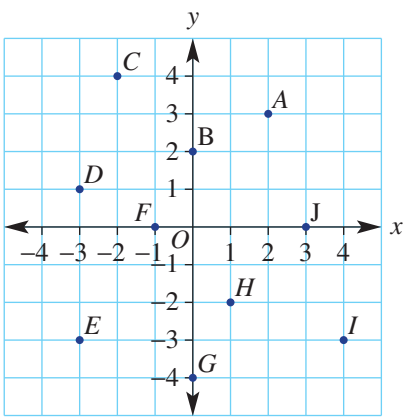
1 The graph shows how far a bird is from its nest.

a How far was the bird from its nest initially (at the start)?
b For how long did the bird rest?
c How far did the bird travel in:
i section A? **ii** section C?
d During which section did the bird fly the fastest?



9A

2 Write the coordinates of all the points $A - J$ in the graph below.



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9B **3** For each rule create a table using x values from -3 to 3 and plot to draw a straight line graph.

a $y = 2x$ **b** $y = 3x - 1$ **c** $y = 2x + 2$

d $y = -x + 1$ **e** $y = -2x + 3$ **f** $y = 3 - x$

9C **4** Write the rule for these tables of values.

a

x	−2	−1	0	1	2
y	−3	−1	1	3	5

b

x	−2	−1	0	1	2
y	−4	−1	2	5	8

c

x	3	4	5	6	7
y	6	7	8	9	10

d

x	−3	−2	−1	0	1
y	4	3	2	1	0

e

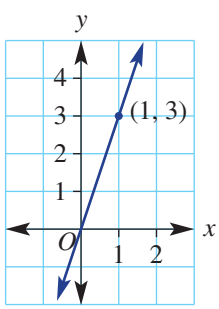
x	−1	0	1	2	3
y	3	−1	−5	−9	−13

f

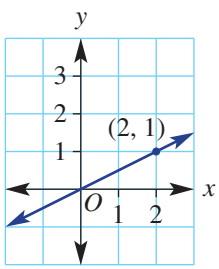
x	−6	−5	−4	−3	−2
y	8	7	6	5	4

9D Ext **5** Find the gradient of each of these lines.

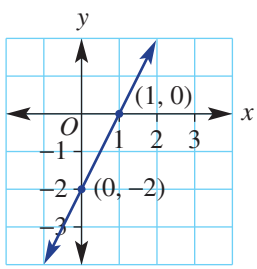
a



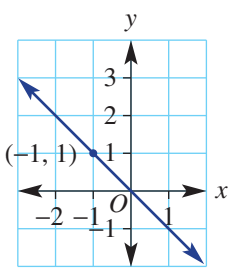
b



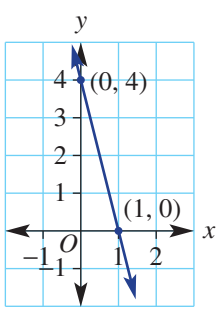
c



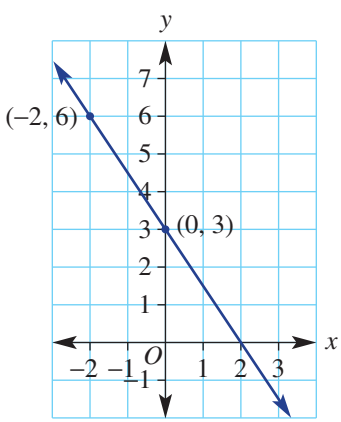
d



e



f



9D Ext **6** Find the gradient of the line joining these pairs of points.

a (0, 0) and (3, −12) **b** (−4, 2) and (0, 0)

c (1, 1) and (4, 4) **d** (−5, 3) and (1, −9)

9E Ext **7** Write the gradient (m) and y-intercept (c) for the graphs of these rules.

a $y = 5x + 2$ **b** $y = 2x - 4$ **c** $y = -3x + 7$ **d** $y = -x - \frac{1}{2}$

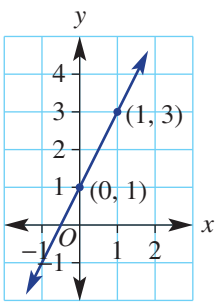
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Chapter review

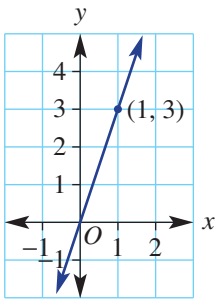
9E Ext

8 Write the rule for these graphs by first finding the values of m and c.

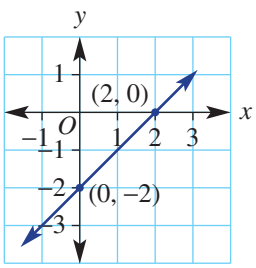
a



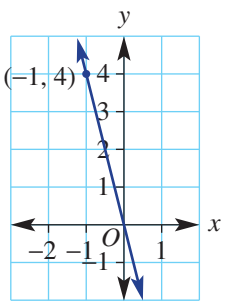
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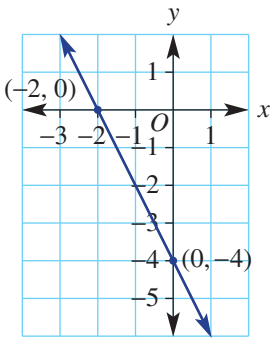
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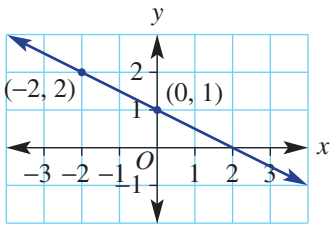
d



e



f



9E Ext

9 Find the rule for the graphs of the lines connecting these points by first finding the value of m and c.

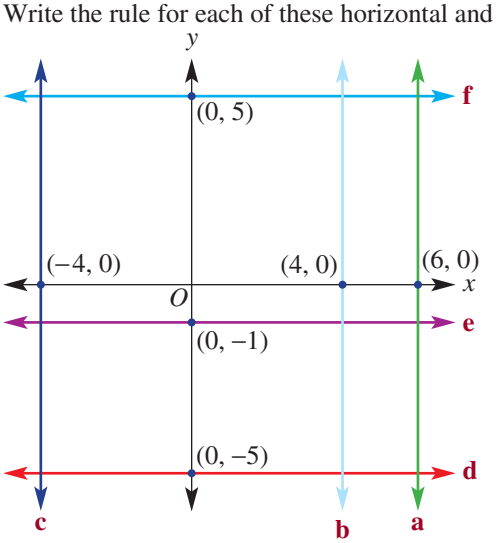
a (0, 0) and (1, 6)

b (−1, 4) and (0, 0)

c (−2, 3) and (0, 1)

d (0, −2) and (6, 1)

10 Write the rule for each of these horizontal and vertical lines.



9F Ext

11 Find the x-intercept for the graphs of the following rules.

a $y = 2x - 12$ **b** $y = 3x + 9$ **c** $y = -x - 4$ **d** $y = -4x + 8$

9F Ext

12 Find the x and y-intercepts for the graphs of these rules and then sketch a graph.

a $y = x + 3$ **b** $y = 2x - 10$ **c** $y = -4x + 8$

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9I

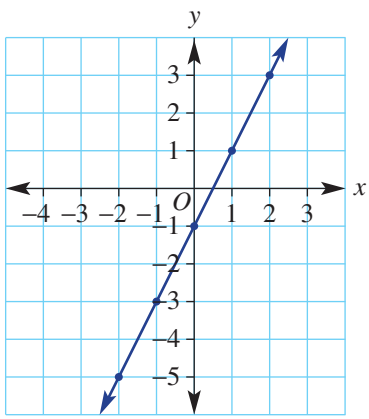
Ext

9G

13 Using a table with x values between -2 and 2 draw a smooth curve for the non-linear graphs of these rules.

a $y = x^2$ **b** $y = x^2 - 2$

14 Use this graph of $y = 2x - 1$ shown here to solve each of the following equations.



a $2x - 1 = 3$

b $2x - 1 = -5$

c $2x - 1 = 0$

Extended-response questions

1 A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.



a Construct a table of values using t for time in days and h for height in millimetres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c Find a rule linking h with t.

d Use your rule to find the height of the plant after 3.5 days.

e If the linear pattern continued, what would be the height of the plant after 10 days?

f How long will it be before the plant grows to 15 mm in height?

2 A speed boat at sea is initially 12 km from a distant buoy. The boat travels towards the buoy at a rate of 2 km per minute. The distance between the boat and the buoy will therefore decrease over time.



a Construct a table showing t for time in minutes and d for distance to the buoy in kilometres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c How long does it take the speed boat to reach the buoy?

d What is the gradient of the line drawn in part **b**?

e Find a rule linking d with t.

f Use your rule to find the distance from the buoy at the 2.5 minute mark.

g How long does it take for the distance to reduce to 3.5 km?

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Chapter 10

Transformation and congruence

What you will learn

10A Refl ction
10B Translation
10C Rotation
10D Congruent figures
10E Congruent triangles
10F Tessellations
10G Congruence and quadrilaterals
10H Similar figures (Extending)
10I Similar triangles (Extending)

Victorian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Define congruence of plane shapes using transformations and use transformations of congruent shapes to produce regular patterns in the plane including tessellations with and without the use of digital technology (VCMMG291)

Develop the conditions for congruence of triangles (VCMMG292)

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (VCMMG293)

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Online resources

- Chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Congruent triangles in architecture and design

Geometry has been the corner stone of building design from ancient times. Angles and symmetry are used to create strength and interest. Modern architects incorporate many geometrical ideas into new public buildings and private residences. Competitions are often held for architects to show their vision for public spaces, and also display their unique design perspectives.

In 1996 the Victorian government ran a competition which lead to the construction of Federation Square in Melbourne's CBD. The building project took 3 years and opened in October 2002. It covers 3.8 hectares above a section of Melbourne's railway system.

The modular system used in the design of the façade is made up of five congruent triangles, joined together to form a pinwheel grid. Five of these panels are then joined together to form a large, super panel which you can see in the structural frame of the outside of the buildings.

Federation Square displays the clever use of the geometry of shapes and different surfaces to create a space that, while different in each section, has an overall cohesion to it. This architecture has created an iconic tourist attraction and a usable space for the people of Melbourne.

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10A Reflection

Interactive

Widgets

HOTsheets

Walkthroughs

When an object is shifted from one position to another, rotated about a point, reflected over a line or enlarged by a scale factor, we say the object has been **transformed**. The names of these types of transformations are **reflection**, **translation**, **rotation** and **dilation**.

The first three of these transformations are called **isometric transformations** because the object's geometric properties are unchanged and the transformed object will be congruent to the original object. The word 'isometric' comes from the Greek words *isos* meaning 'equal' and *metron* meaning 'measure'. Dilation (or enlargement) results in a change in the size of an object to produce a 'similar' figure and this will be studied later in this chapter. The first listed transformation, reflection, can be thought of as the creation of an image over a mirror line.

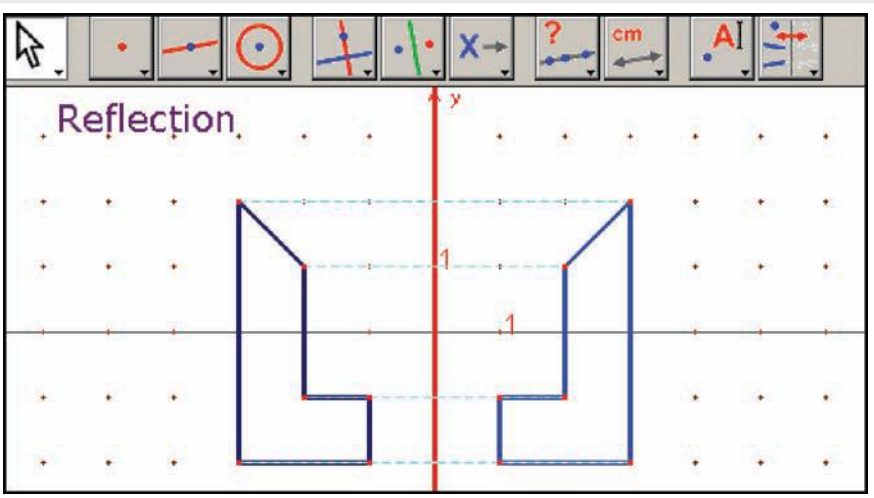


Reflection creates an image reversed as in a mirror or in water.

Let's start: Visualising the image

This activity could be done individually by hand on a page, in a group using a white board or using dynamic geometry projected onto a white board.

- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the Reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.

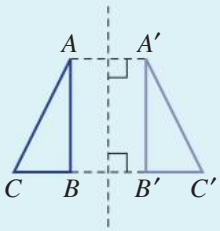


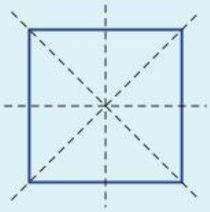
Dynamic geometry software provides a reflection tool.

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Key ideas

- **Reflection** is an **isometric transformation** in which the size of the object is unchanged.
- The **image** of a point A is denoted A'.
- Each point is reflected at right angles to the **mirror line**.
- The distance from a point A to the mirror line is equal to the distance from the image point A' to the mirror line.
- **Lines of symmetry** are mirror lines that result in an image being reflected onto itself.
  For example, a square has four lines of symmetry.

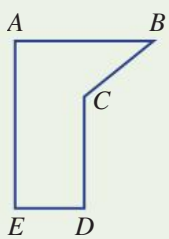




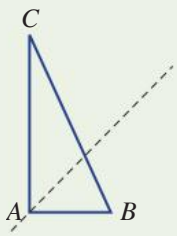
Example 1 Drawing reflected images

Copy the diagram and draw the reflected image over the given mirror line.

a

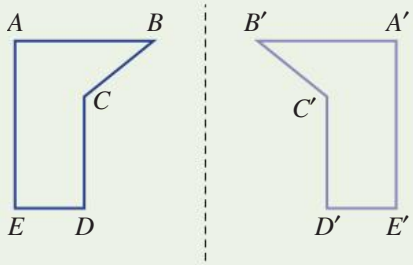


b

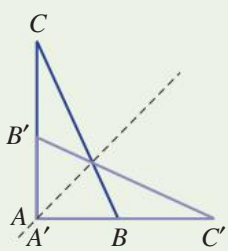


SOLUTION

a



b



EXPLANATION

Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.

Reflect points A, B and C at right angles to the mirror line to form A', B' and C'. Note that A' is in the same position as A as it is on the mirror line. Join the image points to form the image triangle.

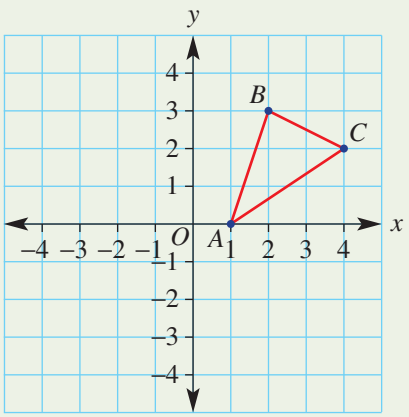
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Example 2 Using coordinates

State the coordinates of the vertices A', B' and C' after this triangle is reflected in the given axes.

a x-axis

b y-axis

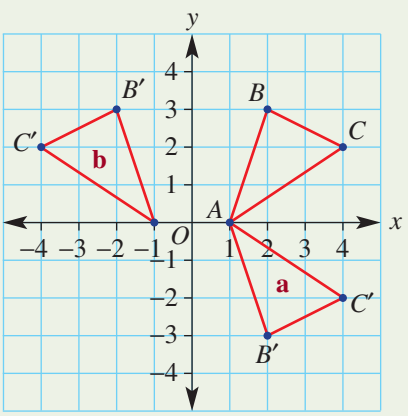


SOLUTION

a $A' = (1, 0)$
$B' = (2, -3)$
$C' = (4, -2)$

b $A' = (-1, 0)$
$B' = (-2, 3)$
$C' = (-4, 2)$

EXPLANATION



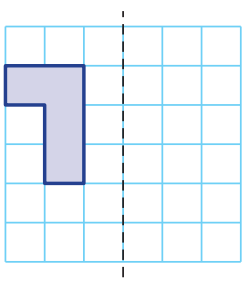
Exercise 10A

1, 2 | 2 | —

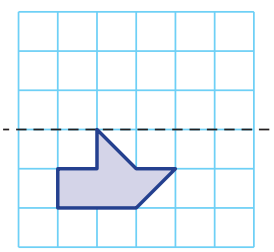
UNDERSTANDING

1 Use the grid to precisely reflect each shape in the given mirror line.

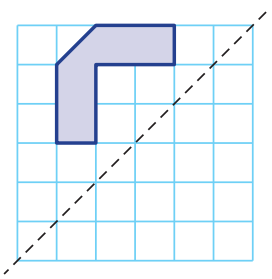
a



b



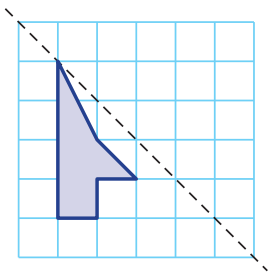
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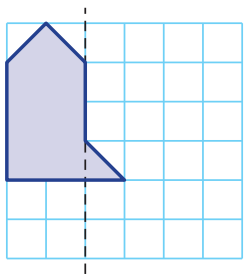


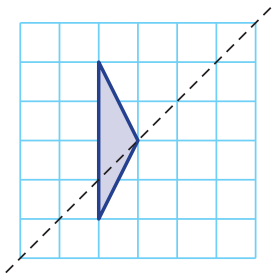
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10A

UNDERSTANDING

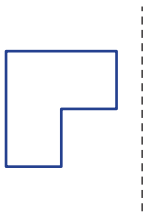
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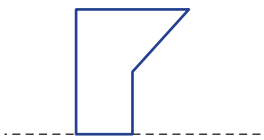
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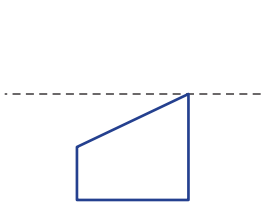
Example 1a

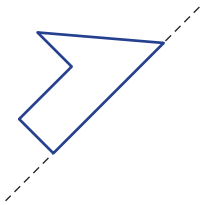
2 Copy the diagram and draw the reflected image over the given mirror line.

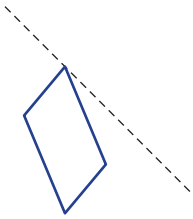
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c 

d 

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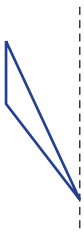
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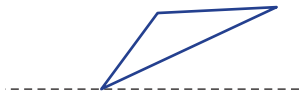
3–5 3, 4(½), 5, 6 3–4(½), 5, 6

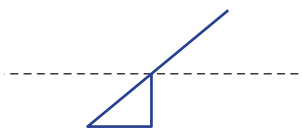
FLUENCY

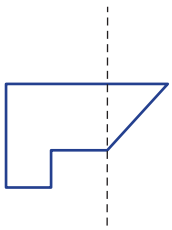
Example 1b

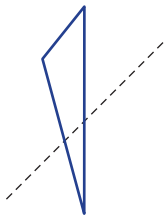
3 Copy the diagram and draw the reflected image over the given mirror line.

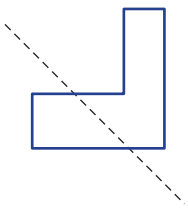
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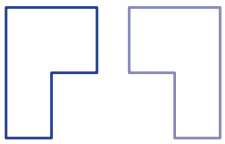
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10A

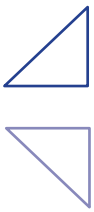
FLUENCY

4 Copy the diagram and accurately locate and draw the mirror line. Alternatively, pencil in the line on this page.

a



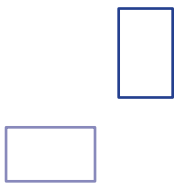
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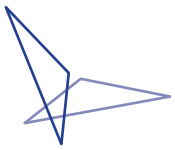
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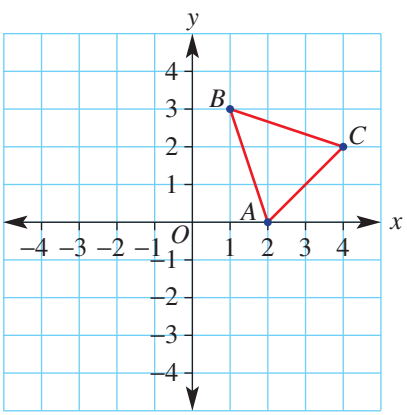
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Example 2

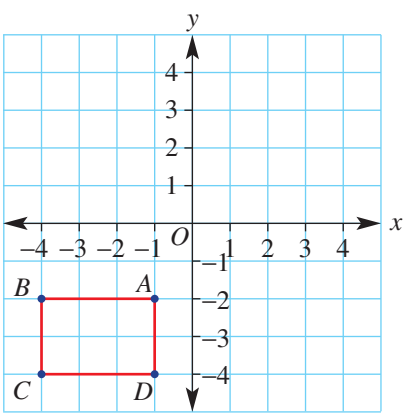
5 State the coordinates of the vertices A', B' and C' after the triangle (right) is reflected in the given axes.

a x-axis **b** y-axis



6 State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

a x-axis **b** y-axis



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10A

7	7, 8(½)	7, 8(½)

PROBLEM-SOLVING

7 How many lines of symmetry do these shapes have?

a Square

b Rectangle

c Rhombus

d Kite

e Trapezium

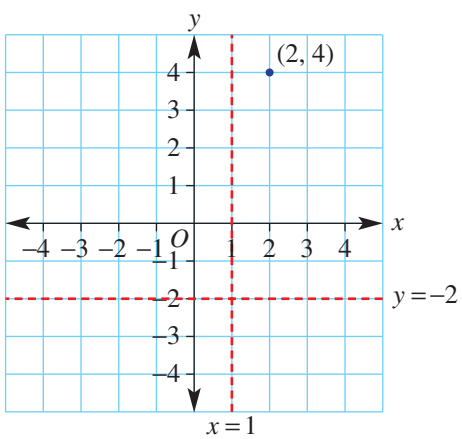
f Parallelogram

g Isosceles triangle

h Equilateral triangle

i Regular octagon

8 A point (2, 4) is reflected in the given horizontal or vertical line. State the coordinates of the image point. As an example, the graph of the mirror lines $x = 1$ and $y = -2$ are shown here.



The mirror lines are:

a $x = 1$ **b** $x = 3$ **c** $x = 0$

d $x = -1$ **e** $x = -4$ **f** $x = -20$

g $y = 3$ **h** $y = -2$ **i** $y = 0$

j $y = 1$ **k** $y = -5$ **l** $y = -37$

9, 11	9–11	11–13

REASONING

9 A shape with area 10 m^2 is reflected in a line. What is the area of the image shape? Give a reason for your answer.

10 How many lines of symmetry does a regular polygon with n sides have? Write an expression.

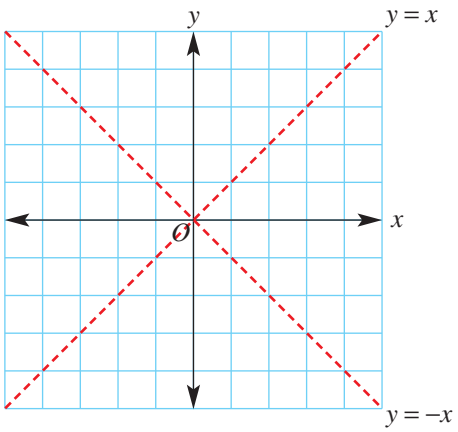
11 A point is reflected in the x-axis then in the y-axis and finally in the x-axis again. What single reflection could replace all three reflections?

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10A

REASONING

12 Two important lines of reflection on the coordinate plane are the line $y = x$ and the line $y = -x$ as shown.



a Draw the coordinate plane shown here. Draw a triangle with vertices $A(-1, 1)$, $B(-1, 3)$ and $C(0, 3)$. Then complete these reflections.

- **i** Reflect triangle ABC in the y-axis.
- **ii** Reflect triangle ABC in the x-axis.
- **iii** Reflect triangle ABC in the line $y = x$.
- **iv** Reflect triangle ABC in the line $y = -x$.

b Draw a coordinate plane and a rectangle with vertices $A(-2, 0)$, $B(-1, 0)$, $C(-1, -3)$ and $D(-2, -3)$. Then complete these reflections.

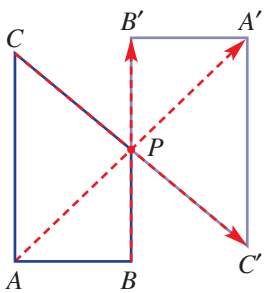
- **i** Reflect rectangle $ABCD$ in the y-axis.
- **ii** Reflect rectangle $ABCD$ in the x-axis.
- **iii** Reflect rectangle $ABCD$ in the line $y = x$.
- **iv** Reflect rectangle $ABCD$ in the line $y = -x$.

13 Points are reflected in a mirror line but do not change position. Describe the position of these points in relation to the mirror line.

Reflection through a point — — 14

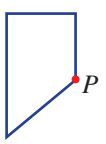
ENRICHMENT

14 Rather than completing a reflection in a line, it is possible to reflect an object through a point. An example of a reflection through point P is shown here. A goes to A', B goes to B' and C goes to C' all through P.

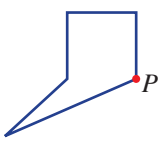


a Draw and reflect these shapes through the point P.

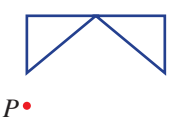
i



ii



iii

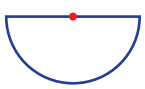


b Like line symmetry, shapes can have point symmetry if they can be reflected onto themselves through a point. Decide if these shapes have any point symmetry.

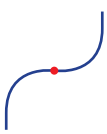
i



ii



iii



c How many special quadrilaterals can you name that have point symmetry?

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10B Translation

Interactive

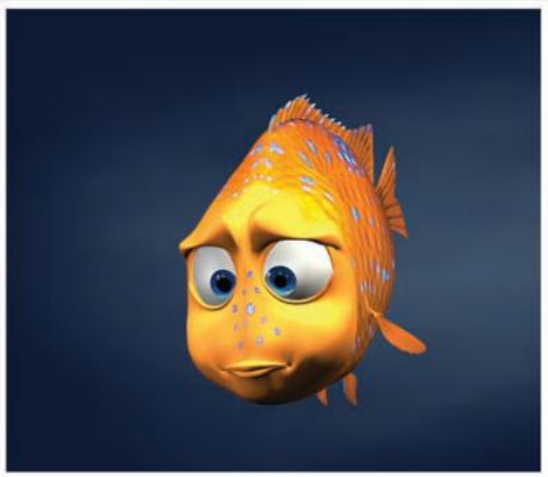
Widgets

HOTsheets

Walkthroughs

Translation is a shift of every point on an object in a given direction and by the same distance. The direction and distance is best described by the use of a translation vector. This vector describes the overall direction using a horizontal component (for shifts left and right) and a vertical component (for shifts up and down). Negative numbers are used for translations to the left and down.

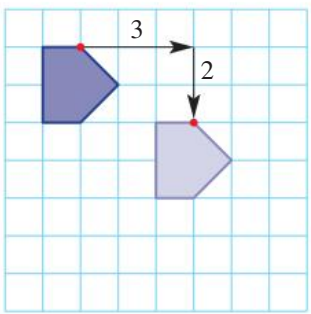
Designers of animated movies translate images in many of their scenes. Computer software is used and translation vectors help to define the specific movement of the objects and characters on the screen.



Animated characters move through a series of translations.

Let's start: Which is further?

Consider this shape on the grid. The shape is translated by the vector $(3, -2)$, which moves it 3 to the right and down 2.



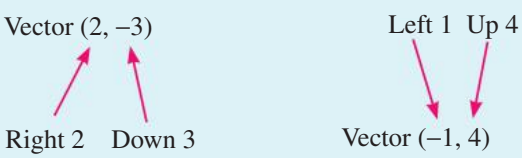
Now consider the shape being translated by these different vectors.

a $(3, -3)$ **b** $(-1, -4)$ **c** $(5, 0)$ **d** $(-2, 4)$

- By drawing and looking at the image from each translation, which vector do you think takes the shape furthest from its original position?
- Is there a way that you can calculate the exact distance? How?

Key ideas

- **Translation** is an isometric transformation that involves a shift by a given distance in a given direction.
- A **vector** (x, y) is used to describe the distance and direction of a translation.
  For example:

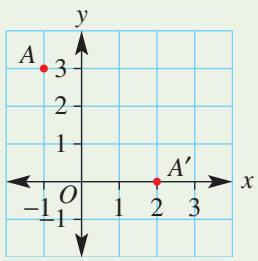


  - If x is positive you shift to the right.
  - If x is negative you shift to the left.
  - If y is positive you shift up.
  - If y is negative you shift down.
- The image of a point A is denoted A'.

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Example 3 Finding the translation vector

State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.



SOLUTION

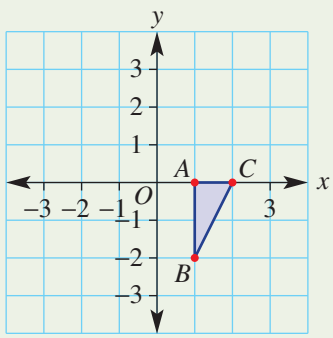
$(3, -3)$

EXPLANATION

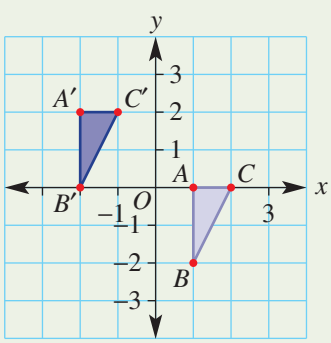
To shift A to A' move 3 units to the right and 3 units down.

Example 4 Drawing images using translation

Draw the image of the triangle ABC after a translation by the vector $(-3, 2)$.



SOLUTION



EXPLANATION

First translate each vertex A, B and C 3 spaces to the left, and then 2 spaces up.

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Exercise 10B

1–3 | 3 | —

UNDERSTANDING

1 Use the words *left*, *right*, *up* or *down* to complete these sentences.

a The vector (2, 4) means to move 2 units to the _____ and 4 units _____.

b The vector (−5, 6) means to move 5 units to the _____ and 6 units _____.

c The vector (3, −1) means to move 3 units to the _____ and 1 unit _____.

d The vector (−10, −12) means to move 10 units to the _____ and 12 units _____.

2 Write the vector (x, y) that describes these transformations.

a 5 units to the right and 2 units down

b 2 units to the left and 6 units down

c 7 units to the left and 4 units up

d 9 units to the right and 17 units up

3 Decide if these vectors describe vertical or horizontal translation.

a (2, 0) **b** (0, 7) **c** (0, −4) **d** (−6, 0)

4–5(½) | 4–6(½) | 4–6(½)

FLUENCY

Example 3

4 Write the vector that takes each point to its image. Use a grid to help you.

a $A(2, 3)$ to $A'(3, 2)$ **b** $B(1, 4)$ to $B'(4, 3)$ **c** $C(-2, 4)$ to $C'(0, 2)$

d $D(-3, 1)$ to $D'(-1, -3)$ **e** $E(-2, -4)$ to $E'(1, 3)$ **f** $F(1, -3)$ to $F'(-2, 2)$

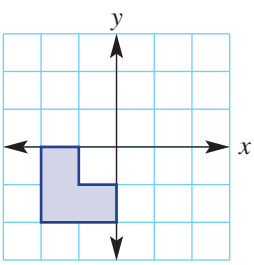
g $G(0, 3)$ to $G'(2, 0)$ **h** $H(-3, 5)$ to $H'(0, 0)$ **i** $I(5, 2)$ to $I'(-15, 10)$

j $J(-3, -4)$ to $J'(-12, -29)$

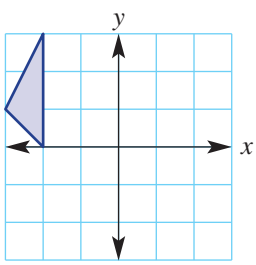
Example 4

5 Copy the diagrams and draw the image of the shapes translated by the given vectors.

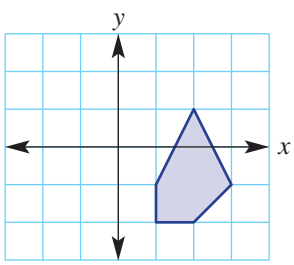
a Vector (2, 3)



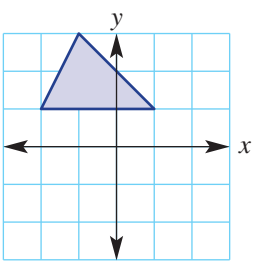
b Vector (4, −2)



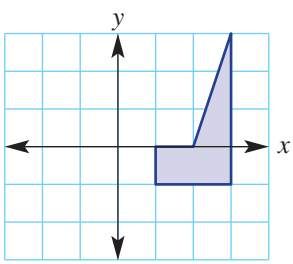
c Vector (−3, 1)



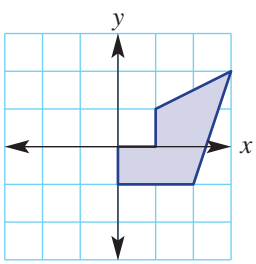
d Vector (0, −3)



e Vector (−4, −1)



f Vector (−3, 0)



6 Write the coordinates of the image of the point $A(13, -1)$ after a translation by the given vectors.

a (2, 3) **b** (8, 0) **c** (0, 7) **d** (−4, 3)

e (−2, −1) **f** (−10, 5) **g** (−2, −8) **h** (6, −9)

i (12, −3) **j** (−26, 14) **k** (−4, 18) **l** (−21, −38)

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10B

7, 8 | 7, 8 | 8, 9

PROBLEM-SOLVING

7 Which vector from each set takes an object the greatest distance from its original position? You may need to draw diagrams to help, but you should not need to calculate distances.

a $(-1, 3), (0, 3), (7, 0)$ **b** $(-1, -4), (4, 0), (0, 3)$

8 A car makes its way around city streets following these vectors:
$(2, 3), (-5, 1), (3, -3)$ and $(-2, -4)$

a What single vector would describe all these vectors combined?

b What vector takes the car back to the origin (0, 0), assuming it started at the origin?



9 A point undergoes the following multiple translations with these given vectors. State the value of x and y of the vector that would take the image back to its original position.

a $(3, 4), (-1, -2), (x, y)$ **b** $(2, 5), (-7, 2), (-1, -3), (x, y)$

c $(0, 4), (7, 0), (-4, -6), (x, y)$ **d** $(-4, 20), (12, 0), (-36, 40), (x, y)$

10 | 10 | 10, 11

REASONING

10 A reverse vector takes a point in the reverse direction by the same distance. Write the reverse vectors of these vectors.

a $(3, -2)$ **b** $(-5, 0)$ **c** (x, y) **d** $(-x, -y)$

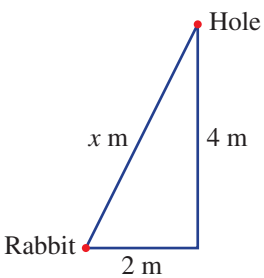
11 These translation vectors are performed on a shape in succession (one after the other). What is a single vector that would complete all transformations for each part in one go?

a $(2, 1), (-3, -4), (0, 3)$ **b** $(6, 4), (6, -2), (-11, 0)$ **c** $(a, b), (c, -a), (-a, a - c)$

How many options for the rabbit? | — | — | 12

ENRICHMENT

12 Hunters spot a rabbit on open ground and it has 1 second to find a hole before getting into big trouble with the hunter's gun. It can run a maximum of 5 metres in one second.



a Use Pythagoras' theorem to check that the distance x m in this diagram is less than 5 m.

b The rabbit runs a distance and direction described by the translation vector $(-4, 3)$. Is the rabbit in trouble?

c The rabbit's initial position is (0, 0) and there are rabbit holes at every point that has integers as its coordinates, for example, (2, 3) and $(-4, 1)$. How many rabbit holes can it choose from to avoid the hunters before its 1 second is up? Draw a diagram to help illustrate your working.



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10C Rotation









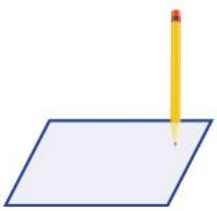
When the arm of a crane moves left, right, up or down, it undergoes a rotation about a fixed point. This type of movement is a transformation called a **rotation**. The pivot point on a crane represents the **centre of rotation** and all other points on the crane's arm move around this point by the same angle in a circular arc.



Every point on this crane's arm rotates around the pivot point when it moves.

Let's start: Parallelogram centre of rotation

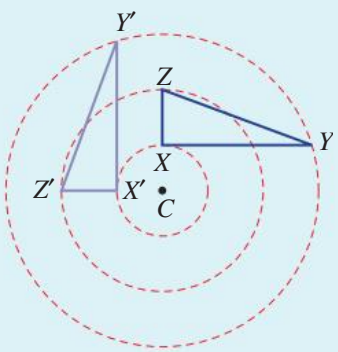
This activity will need a pencil, paper, ruler and scissors.



- Accurately draw a large parallelogram on a separate piece of paper and cut it out.
- Place the tip of a pencil at any point on the parallelogram and spin the shape around the pencil.
- At what position do you put the pencil tip to produce the largest circular arc?
- At what position do you put the pencil tip to produce the smallest circular arc?
- Can you rotate the shape by an angle of less than 360° so that it exactly covers the area of the shape in its original position? Where would you put the pencil to illustrate this?

Key ideas

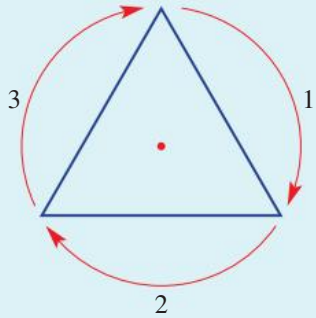
- **Rotation** is an isometric transformation about a centre point and by a given angle.
- An object can be rotated clockwise ↻ or anticlockwise ↺.
- Each point is rotated on a circular arc about the **centre of rotation** C.
  For example, this diagram shows a 90° anticlockwise rotation about the point C.



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Key ideas

- A shape has **rotational symmetry** if it can be rotated about a centre point to produce an exact copy covering the entire area of the original shape.
  - The number of times the shape can make an exact copy in a 360° rotation is called the **order of rotational symmetry**. If the order of rotational symmetry is 1, then it is said that the shape has no rotational symmetry.
    For example, this equilateral triangle has rotational symmetry of order 3.



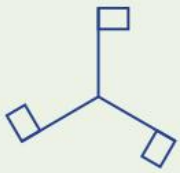
Example 5 Finding the order of rotational symmetry

Find the order of rotational symmetry for these shapes.

a



b

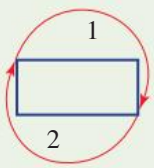


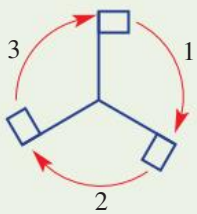
SOLUTION

a Order of rotational symmetry = 2

b Order of rotational symmetry = 3

EXPLANATION

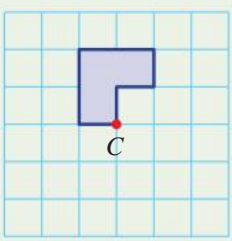




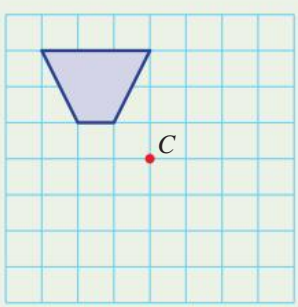
Example 6 Drawing a rotated image

Rotate these shapes about the point C by the given angle and direction.

a Clockwise by 90°



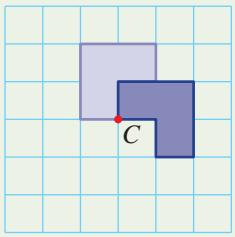
b 180°



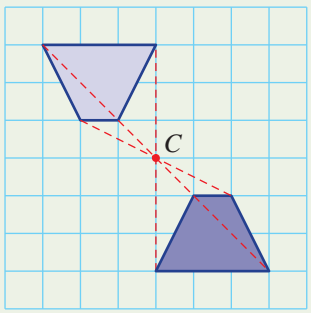
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SOLUTION

a



b



EXPLANATION

Take each vertex point and rotate about C by 90°, but it may be easier to visualise a rotation of some of the sides first. Horizontal sides will rotate to vertical sides in the image and vertical sides will rotate to horizontal sides in the image.

You can draw a dashed line from each vertex through C to a point at an equal distance on the opposite side.

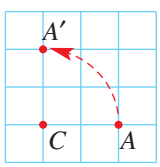
Exercise 10C

1, 2 | 2 | —

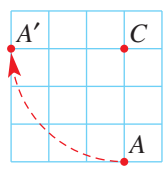
UNDERSTANDING

1 Point A has been rotated to its image point A'. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

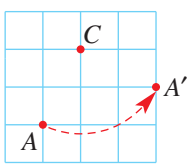
a



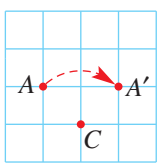
b



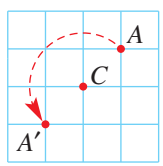
c



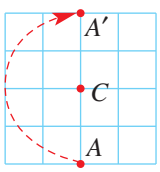
d



e



f



2 Complete these sentences.

a A rotation clockwise by 90° is the same as a rotation anticlockwise by _____.

b A rotation anticlockwise by 180° is the same as a rotation clockwise by _____.

c A rotation anticlockwise by _____ is the same as a rotation clockwise by 58°.

d A rotation clockwise by _____ is the same as a rotation anticlockwise by 296°.

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10C

3–5, 6(½) | 3, 4(½), 5, 6(½), 7 | 3, 4(½), 5, 6(½), 7

FLUENCY

Example 5

3 Find the order of rotational symmetry for these shapes.

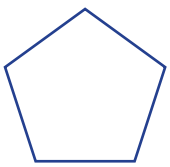
a



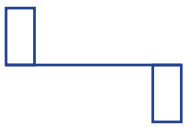
b



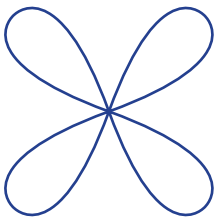
c



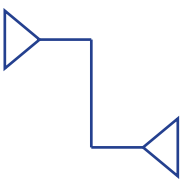
d



e



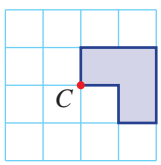
f



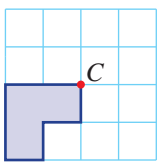
Example 6a

4 Rotate these shapes about the point C by the given angle and direction.

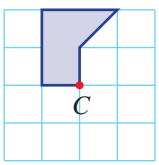
a Clockwise by 90°



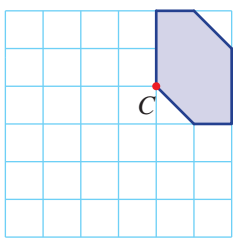
b Anticlockwise by 90°



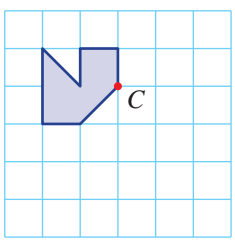
c Anticlockwise by 180°



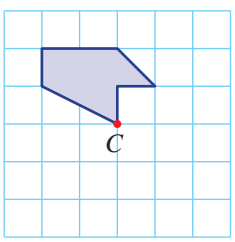
d Clockwise by 90°



e Anticlockwise by 180°



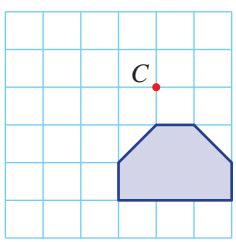
f Clockwise by 180°



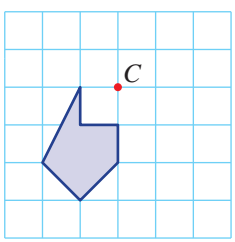
Example 6b

5 Rotate these shapes about the point C by the given angle and direction.

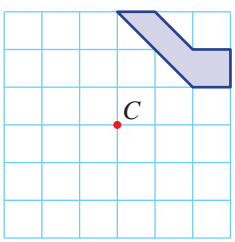
a Clockwise by 90°



b Anticlockwise by 90°



c Anticlockwise by 180°



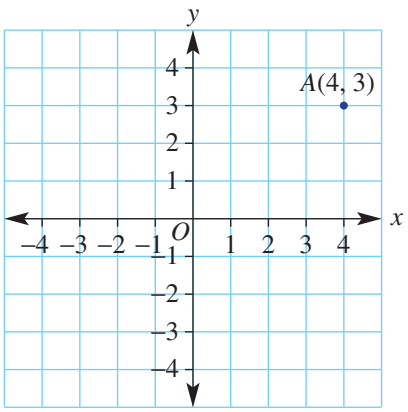
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10C

FLUENCY

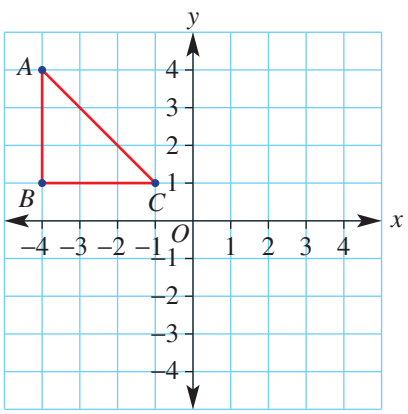
6 The point $A(4, 3)$ is rotated about the origin $C(0, 0)$ by the given angle and direction. Give the coordinates of A'.

a 180° clockwise
b 180° anticlockwise
c 90° clockwise
d 90° anticlockwise
e 270° clockwise
f 270° anticlockwise
g 360° clockwise



7 The triangle shown here is rotated about (0, 0) by the given angle and direction. Give the coordinates of the image points A', B' and C'.

a 180° clockwise
b 90° clockwise
c 90° anticlockwise

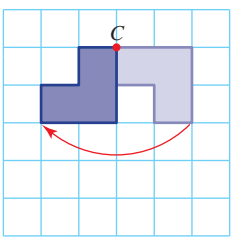


8, 9 8, 9 9, 10

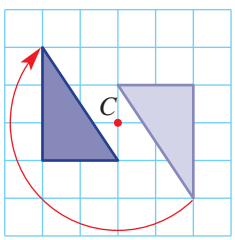
PROBLEM-SOLVING

8 By how many degrees have these shapes been rotated?

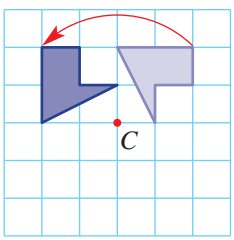
a



b



c



9 Which capital letters of the alphabet, as written below, have rotational symmetry of order 2 or more?

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

10 Draw an example of a shape that has these properties.

a Rotational symmetry of order 2 with no line symmetry
b Rotational symmetry of order 6 with 6 lines of symmetry
c Rotational symmetry of order 4 with no line symmetry

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10C

11 | 11, 12 | 12, 13

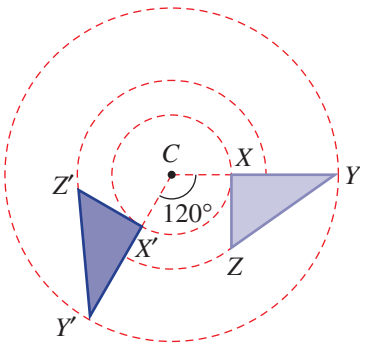
REASONING

11 What value of x makes these sentences true?

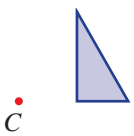
a Rotating x degrees clockwise has the same effect as rotating x degrees anticlockwise.

b Rotating x degrees clockwise has the same effect as rotating $3x$ degrees anticlockwise.

12 When working without a grid or without 90° angles, a protractor and compasses are needed to accurately draw images under rotation. This example shows a rotation of 120° about C.



a Copy this triangle with centre of rotation C onto a sheet of paper.

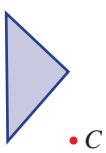


b Construct three circles with centre C and passing through the vertices of the triangle.

c Use a protractor to draw an image after these rotations.

i 120° anticlockwise **ii** 100° clockwise

13 Make a copy of this diagram and rotate the shape anticlockwise by 135° around point C. You will need to use compasses and a protractor as shown in Question **12**.

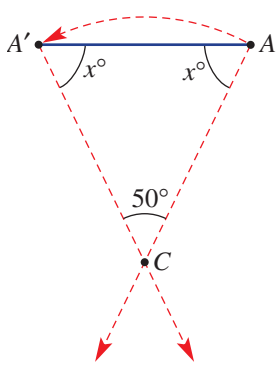


Finding the centre of rotation

— | — | 14

ENRICHMENT

14 Finding the centre of rotation if the angle is known involves the calculation of an angle inside an isosceles triangle. For the rotation shown, the angle of rotation is 50°. The steps are given:



1 Calculate $\angle CAA'$ and $\angle CA'A$.
($2x + 50 = 180$, so $x = 65$)

2 Draw the angles $\angle AA'C$ and $\angle A'AC$ at 65° using a protractor.

3 Locate the centre of rotation C at the point of intersection of AC and $A'C$.

a On a sheet of paper draw two points A and A' about 4 cm apart. Follow the steps above to locate the centre of rotation if the angle of rotation is 40°.

b Repeat part **a** using an angle of rotation of 100°.

c When a shape is rotated and the angle is unknown, there is a special method for accurately pinpointing the centre of rotation. Research this method and illustrate the procedure using an example.

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10D Congruent figures

Interactive

Widgets

HOTsheets

Walkthroughs

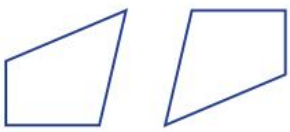
In mathematics, if two objects are identical we say they are congruent. If you ordered 10 copies of a poster from a printer, you would expect that the image on one poster would be congruent to the image on the next. Even if one poster was flipped over, shifted or rotated, you would still say the images on the posters were congruent.



Each brochure being printed is identical, so the images on them are congruent.

Let's start: Are they congruent?

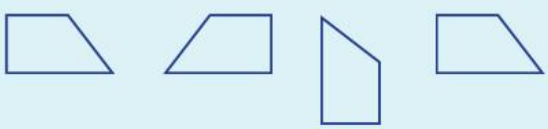
Here are two shapes. To be congruent they need to be exactly the same shape and size.



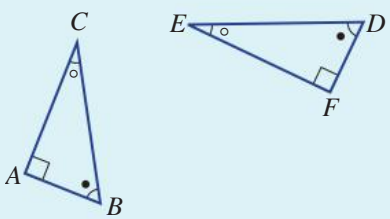
- Do you think they look congruent? Give reasons.
- What measurements could be taken to help establish whether or not they are congruent?
- Can you just measure lengths or do you need to measure angles as well? Discuss.

Key ideas

- A **figure** is a shape, diagram or illustration.
- Two figures are **congruent** if one of them can be transformed to the other using the transformations: reflection, translation and notation.
  - **Congruent figures** have the same size and shape.



- Corresponding (matching) parts of a figure have the same geometric properties.
  - Vertex C corresponds to vertex E.
  - Side AB corresponds to side FD.
  - $\angle B$ corresponds to $\angle D$.
- A congruent statement can be written using the symbol $\equiv$. For example, $\Delta ABC \equiv \Delta FDE$.
  - The symbol $\cong$ can also be used for congruence.
  - The symbol for triangle is Δ.
  - Vertices are usually listed in matching order.

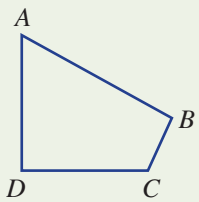


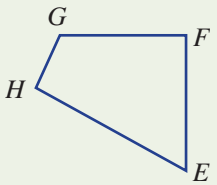
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Example 7 Naming corresponding pairs

These two quadrilaterals are congruent. Name the objects in quadrilateral *EFGH* that correspond to these objects in quadrilateral *ABCD*.

a Vertex *C* **b** Side *AB* **c** $\angle C$





SOLUTION	EXPLANATION
a Vertex *G*	*C* sits opposite *A* and $\angle A$ is the smallest angle. *G* sits opposite *E* and $\angle E$ is also the smallest angle.
b Side *EH*	Sides *AB* and *EH* are both the longest sides of their respective shapes. *A* corresponds to *E* and *B* corresponds to *H*.
c $\angle G$	$\angle C$ and $\angle G$ are both the largest angle in their corresponding quadrilateral.

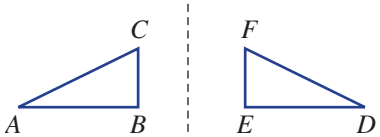
Exercise 10D

1–3 | 3 | —

UNDERSTANDING

1 In this diagram ΔABC has been reflected to give the image triangle ΔDEF.

a Is ΔDEF congruent to ΔABC, i.e. is $\Delta DEF \equiv \Delta ABC$?



b Name the vertex on ΔDEF which corresponds to:

i vertex *A* **ii** vertex *B* **iii** vertex *C*

c Name the side on ΔDEF which corresponds to:

i side *AB* **ii** side *BC* **iii** side *AC*

d Name the angle in ΔDEF which corresponds to:

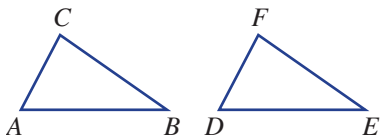
i $\angle B$ **ii** $\angle C$ **iii** $\angle A$

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10D

UNDERSTANDING

2 In this diagram ΔABC has been translated (shifted) to give the image triangle ΔDEF.



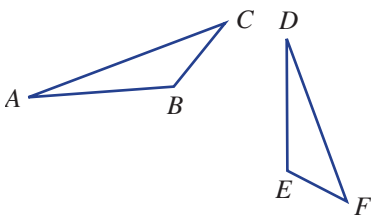
a Is ΔDEF congruent to ΔABC, i.e. is $\Delta DEF \equiv \Delta ABC$?

b Name the vertex on ΔDEF which corresponds to:
i vertex A **ii** vertex B **iii** vertex C

c Name the side on ΔDEF which corresponds to:
i side AB **ii** side BC **iii** side AC

d Name the angle in ΔDEF which corresponds to:
i $\angle B$ **ii** $\angle C$ **iii** $\angle A$

3 In this diagram ΔABC has been rotated to give the image triangle ΔDEF.



a Is ΔDEF congruent to ΔABC, i.e. is $\Delta DEF \equiv \Delta ABC$?

b Name the vertex on ΔDEF which corresponds to:
i vertex A **ii** vertex B **iii** vertex C

c Name the side on ΔDEF which corresponds to:
i side AB **ii** side BC **iii** side AC

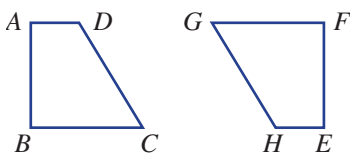
d Name the angle in ΔDEF which corresponds to:
i $\angle B$ **ii** $\angle C$ **iii** $\angle A$

4, 5 4, 6 5, 6

FLUENCY

Example 7

4 These two quadrilaterals are congruent. Name the object in quadrilateral $EFGH$ which corresponds to these objects in quadrilateral $ABCD$.

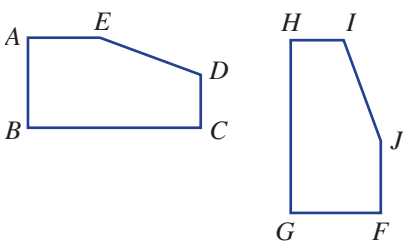


a **i** Vertex A **ii** Vertex D

b **i** Side AD **ii** Side CD

c **i** $\angle C$ **ii** $\angle A$

5 These two pentagons are congruent. Name the object in pentagon $FGHIJ$ which corresponds to these objects in pentagon $ABCDE$.



a **i** Vertex A **ii** Vertex D

b **i** Side AE **ii** Side CD

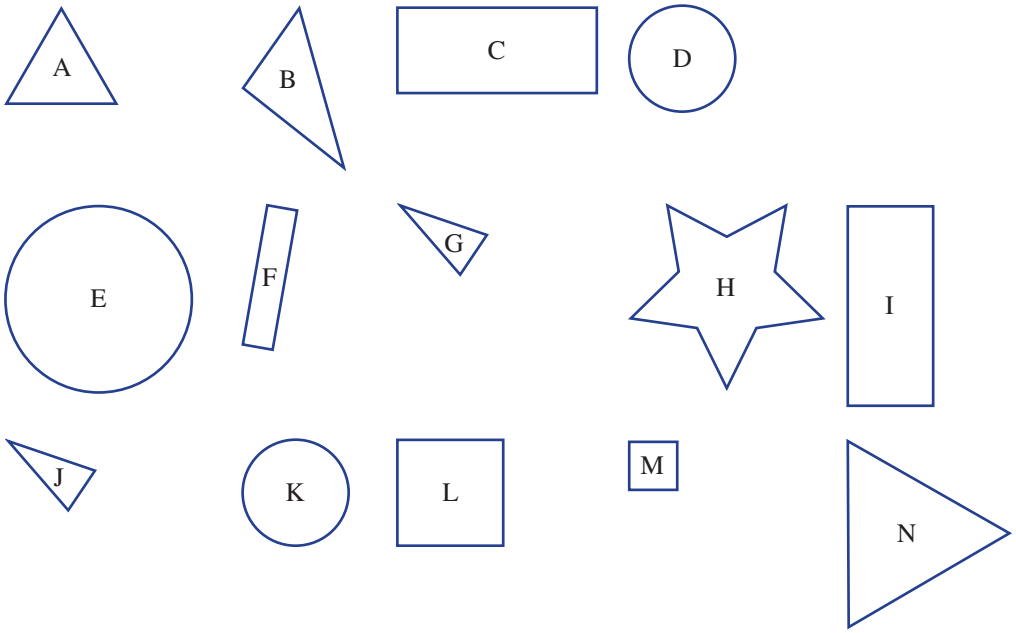
c **i** $\angle C$ **ii** $\angle E$

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10D

FLUENCY

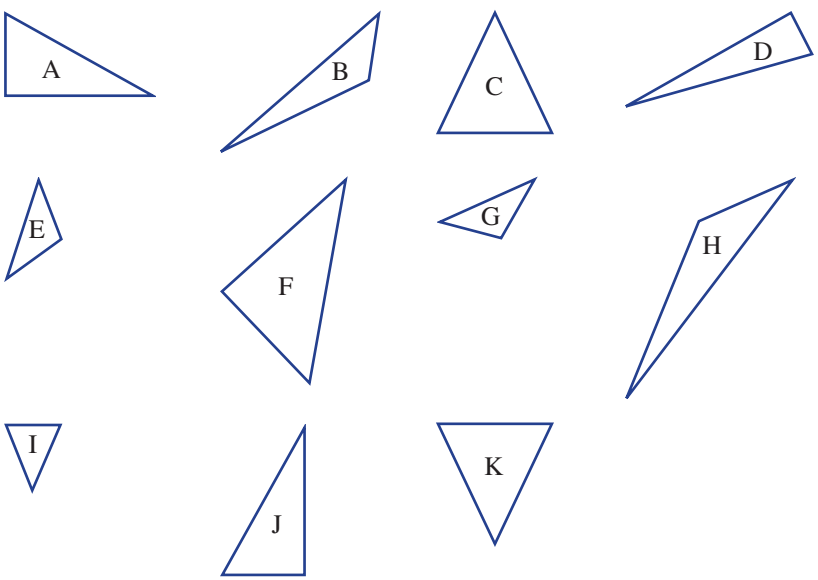
6 From all the shapes shown here, find three pairs that look congruent.



7, 8 | 7, 8 | 8, 9

PROBLEM-SOLVING

7 List the pairs of the triangles below that look congruent.



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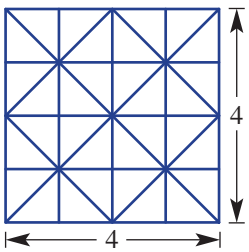
10D

PROBLEM-SOLVING

8 How many congruent triangles are there in this diagram with:

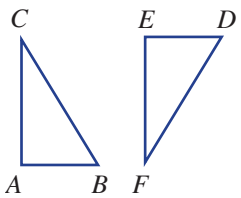
a area $\frac{1}{2}$? **b** area 1? **c** area 2?

d area 4? **e** area 8?

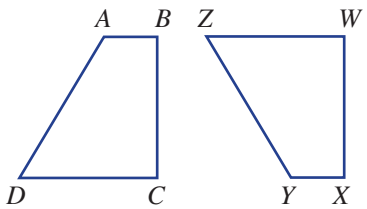


9 Write the pairs of corresponding vertices for these congruent shapes, e.g. $(A, E), (B, D)$.

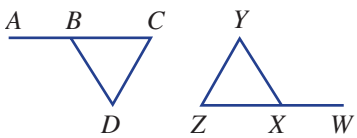
a



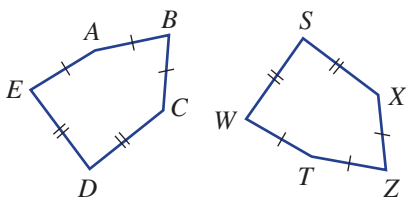
b



c



d



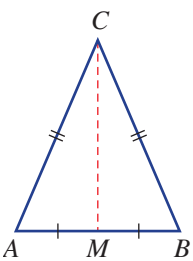
10 | 10, 11 | 11, 12

REASONING

10 An isosceles triangle is cut as shown, using the midpoint of AB.

a Name the two triangles formed.

b Will the two triangles be congruent? Give reasons.



11 A kite $ABCD$ has diagonals AC and BD.

a The kite is cut using the diagonal BD.

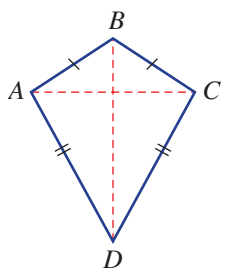
i Name the two triangles formed.

ii Will the two triangles be congruent? Give reasons.

b The kite is cut using the diagonal AC.

i Name the two triangles formed.

ii Will the two triangles be congruent? Give reasons.



12 If a parallelogram is cut by either diagonal will the two triangles be congruent?

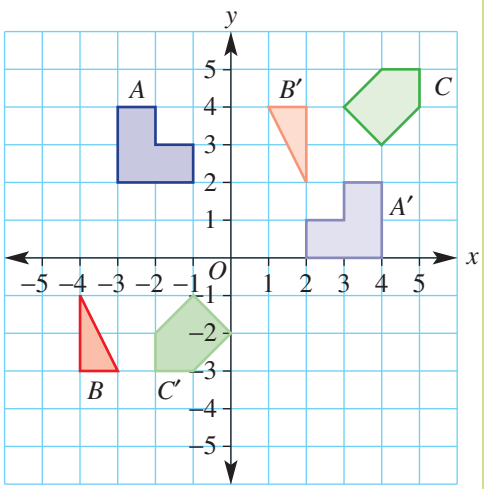
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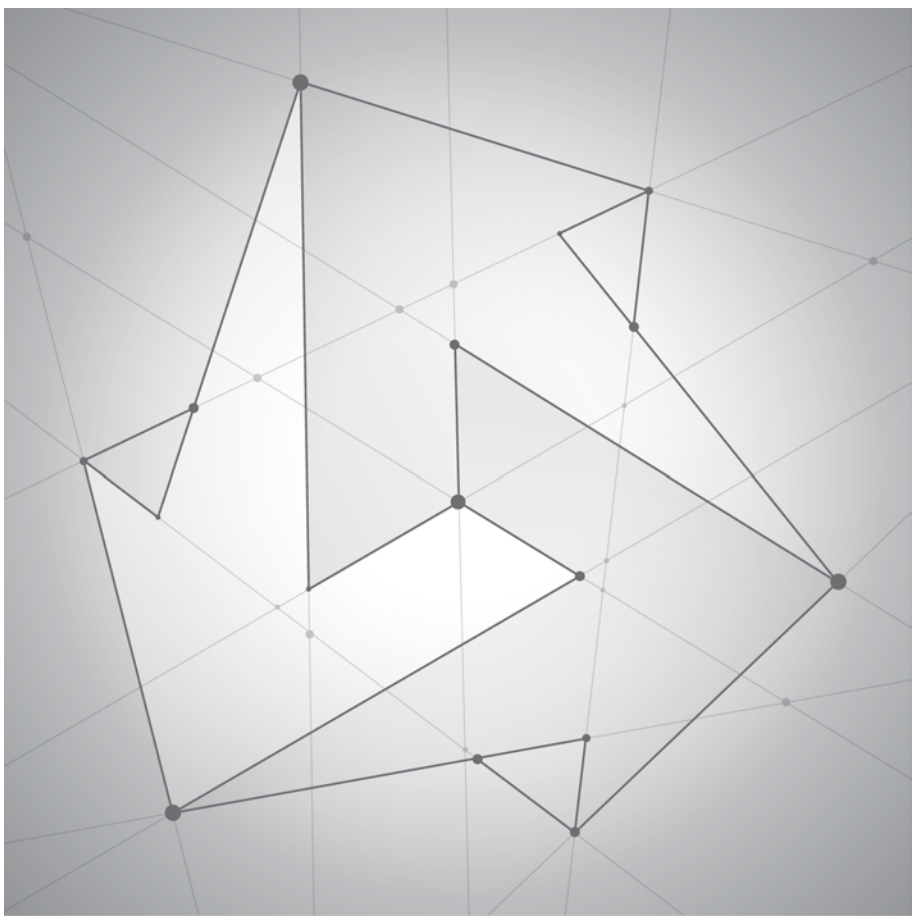
10D Combining isometric transformations — — 13

ENRICHMENT

13 Describe the combination of transformations (reflections, translations and/or rotations) that map each shape to its image under the given conditions. The reflections that are allowed include only those in the x- and y-axes and rotations will use (0, 0) as its centre.

a A to A' with a reflection and then a translation
b A to A' with a rotation and then a translation
c B to B' with a rotation and then a translation
d B to B' with two reflections and then a translation
e C to C' with two reflections and then a translation
f C to C' with a rotation and then a translation





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10E Congruent triangles

Interactive

Widgets

HOTsheets

Walkthroughs

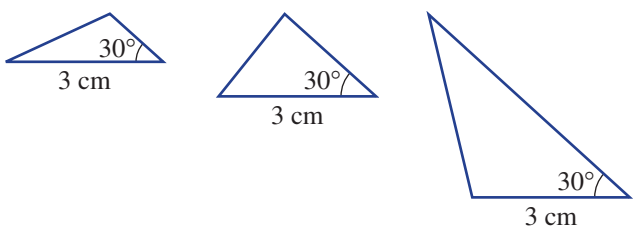
Imagine the sorts of design and engineering problems we would face if we could not guarantee that two objects such as window panes or roof truss frames were not the same size or shape. Further it might not be possible or practical to measure every length and angle to test for congruence. In the case of triangles, it is possible to consider only a number of pairs of sides or angles to test whether or not they are congruent.



The ceiling of the British Museum is made up of thousands triangles that give the illusion of being congruent (they are actually all slightly different shapes).

Let's start: How much information is enough?

Given one corresponding angle (say 30°) and one corresponding equal side length (say 3 cm), it is clearly not enough information to say two triangles are congruent. This is because more than one triangle can be drawn with the given information, that is, you cannot draw a unique triangle with this given information.



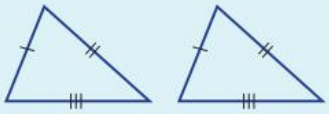
Decide if the following information is enough to determine if two triangles are congruent. If you can draw two non-identical triangles then there is not enough information. If you can only draw one unique triangle, then you have the conditions for congruence.

- ΔABC with $AC = 4$ cm and $\angle C = 40°$
- ΔABC with $AB = 5$ cm and $AC = 4$ cm
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $\angle = 45°$
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm
- ΔABC with $AB = 4$ cm, $\angle A = 40°$ and $\angle B = 60°$

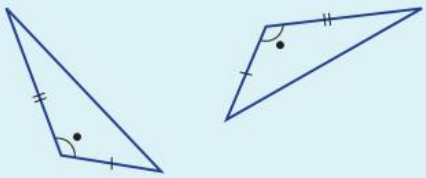
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Key ideas

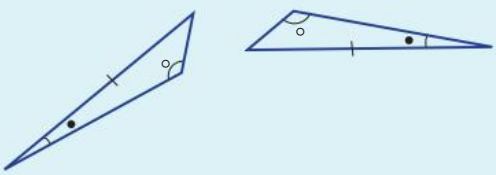
- Two triangles are congruent if one of these four sets of conditions is satisfied.
  - **SSS**
    3 equal corresponding sides



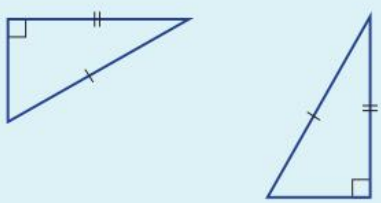
  - **SAS**
    2 equal corresponding sides and 1 equal corresponding angle between them.
    This angle is called the **included angle**.



  - **AAS**
    2 equal corresponding angles and 1 equal corresponding side. Any order is accepted AAS, ASA, SAA.



  - **RHS**
    2 right-angled triangles with equal hypotenuse lengths and 1 other pair of equal corresponding sides.

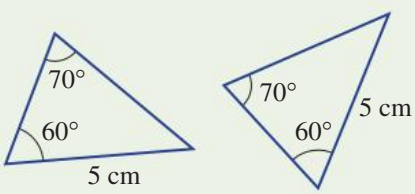


- If triangle ABC is congruent to triangle DEF, we write $\Delta ABC \equiv \Delta DEF$.
  - This is called a congruence statement.
- A **unique triangle** is one where any triangles matching the description are congruent to each other.

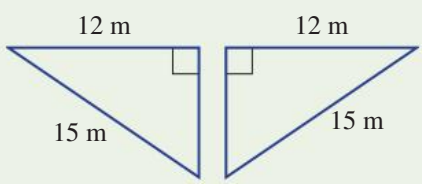
Example 8 Deciding on a test for congruence

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

a



b



SOLUTION

a AAS

EXPLANATION

There are two equal angles and 1 pair of equal corresponding sides. The side that is 5 cm is adjacent to the 60° angle on both triangles.

b RHS

There is a pair of right angles with equal hypotenuse lengths. A second pair of corresponding sides are also of equal length.

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Example 9 Constructing unique triangles

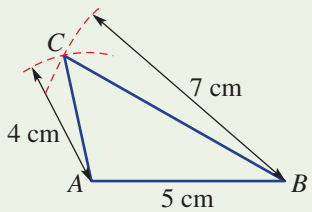
Use a ruler and a pair of compasses to construct these triangles. Decide if the triangle is unique and give a reason.

a $\triangle ABC$ with $AB = 5$ cm, $BC = 7$ cm and $AC = 4$ cm

b $\triangle DEF$ with $\angle D = 70°$, $\angle E = 50°$ and $EF = 4$ cm

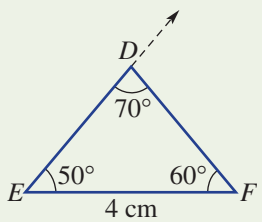
SOLUTION

a



$\angle ABC$ is unique (SSS)

b



$\angle DEF$ is unique (AAS)

EXPLANATION

Draw AB with length 5 cm. Construct two arcs with radii 4 cm and 7 cm centred at A and B respectively. Place point C at the intersection point of two arcs.

The triangle is unique since three sides are given (SSS).

Draw EF and the ray ED so that $\angle E = 50°$ using a protractor. Calculate $\angle F$ ($180° - 70° - 50° = 60°$). Measure $\angle F$ and draw in FD.

The triangle is unique since two angles and a side are given (AAS).

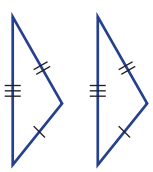
Exercise 10E

1, 2 | 2 | —

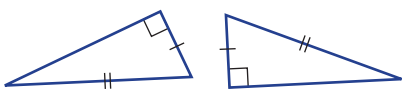
UNDERSTANDING

1 Pick the congruence test (SSS, SAS, AAS or RHS) that matches each pair of congruent triangles.

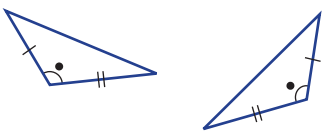
a



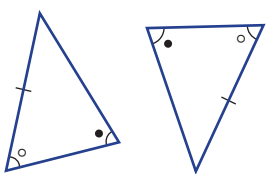
b



c



d



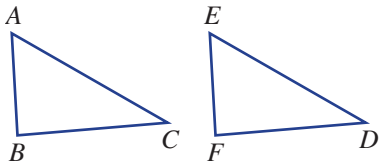
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10E

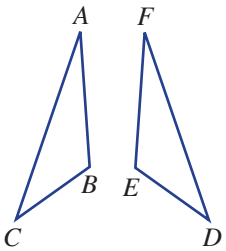
UNDERSTANDING

2 Write a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) for these pairs of congruent triangles. Try to match vertices by writing them in corresponding positions.

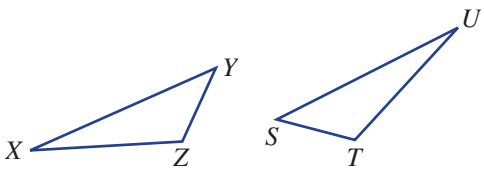
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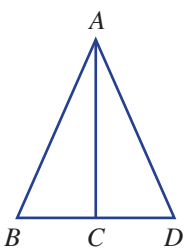
b



c



d



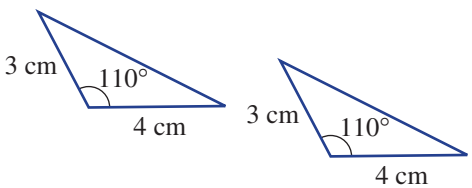
3, 4(½), 5 | 3, 4(½), 5 | 3–5(½), 6

FLUENCY

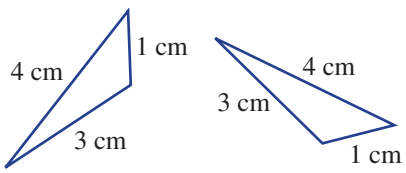
Example 8

3 Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

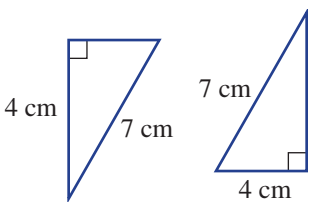
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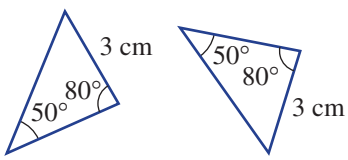
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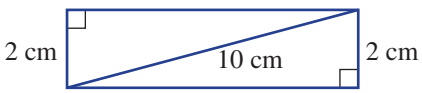
c



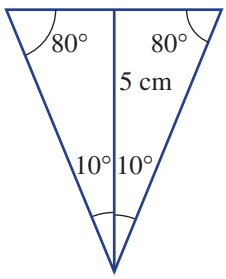
d



e



f



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Example 9

4 Use a ruler and a pair of compasses to construct these triangles. Which of the tests (SSS, SAS, AAS or RHS) show that the triangle is unique?

a ΔABC with $\angle A = 60°$, $AB = 5$ cm and $\angle B = 40°$

b ΔDEF with $DE = 5$ cm, $EF = 6$ cm and $DF = 7$ cm

c ΔSTU with $\angle S = 90°$, $ST = 4$ cm and $TU = 3$ cm

d ΔXYZ with $XY = 6$ cm, $\angle Y = 40°$ and $YZ = 4$ cm

e ΔABC with $AB = 4$ cm, $BC = 6$ cm and $AC = 3$ cm

f ΔSTU with $ST = 4$ cm, $\angle S = 65°$ and $\angle T = 45°$

g ΔPQR with $PQ = 5$ cm, $\angle P = 60°$ and $PR = 4$ cm

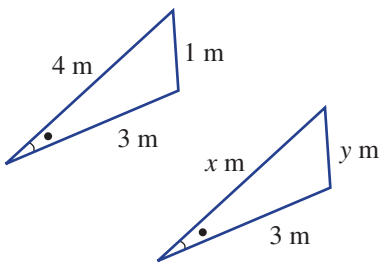
h ΔDEF with $\angle D = 40°$, $\angle E = 60°$ and $EF = 5$ cm

i ΔABC with $\angle B = 55°$, $BC = 6$ cm and $\angle A = 35°$

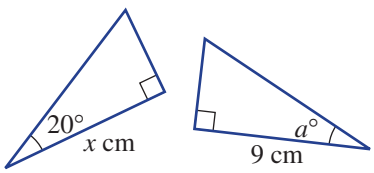
j ΔABC with $\angle B = 90°$, $BC = 5$ cm and $AC = 8$ cm

5 These pairs of triangles are congruent. Find the values of the pronumerals.

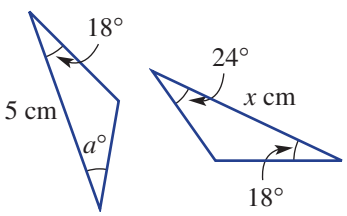
a



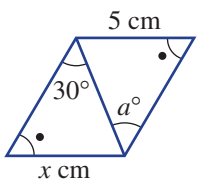
b



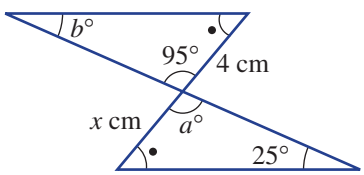
c



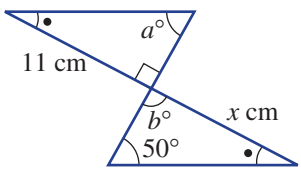
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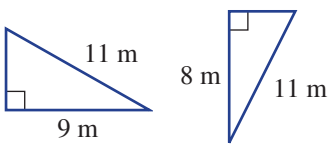


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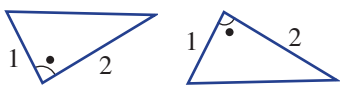


6 Decide if these pairs of triangles are congruent. If they are, give a reason.

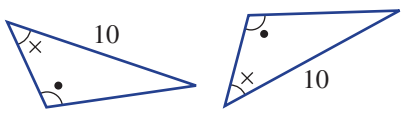
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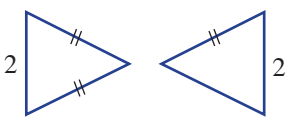
b



c



d



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10E

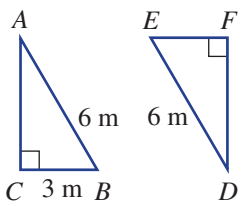
7 | 7–9 | 8–10

PROBLEM-SOLVING

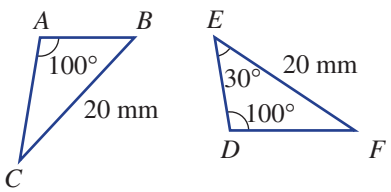
7 Decide which piece of information from the given list needs to be added to the diagram if the two triangles are to be congruent.

$\angle B = 30°, \angle C = 20°, EF = 3$ m, $FD = 3$ m, $AB = 6$ cm, $AC = 6$ cm, $\angle C = 20°, \angle A = 20°$

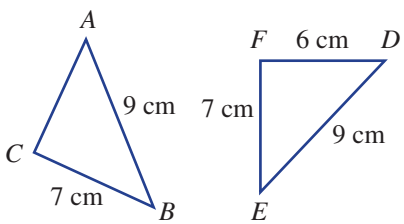
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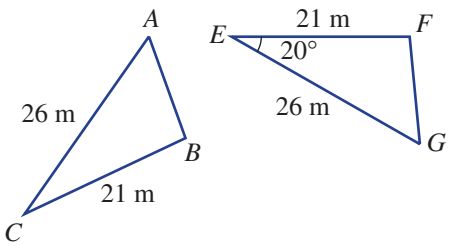
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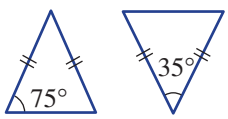


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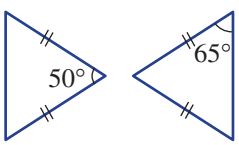


8 Decide if each pair of triangles is congruent. You may first need to use the angle sum of a triangle to help calculate some of the angles.

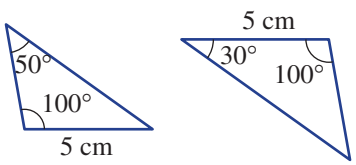
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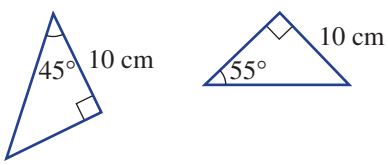
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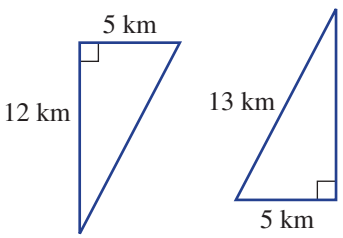
c



d



9 Two adjoining triangular areas of land have the dimensions shown. Use Pythagoras' theorem to help decide if the two areas of land are congruent.





10 **a** Explain why SSA is not a sufficient test to prove that two triangles are congruent. Draw diagrams to show your reasoning.

b Explain why AAA is not a sufficient test to prove that two triangles are congruent. Draw diagrams to show your reasoning.

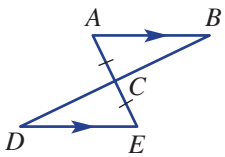
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11a 11a, b 11

10E

REASONING

11 Here is a suggested proof showing that the two triangles in this diagram are congruent.



In ΔABC and ΔEDC

$\angle CAB = \angle CED$ (alternate angles in parallel lines)

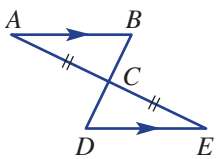
$\angle ACB = \angle ECD$ (vertically opposite angles)

$AC = EC$ (given equal and corresponding sides)

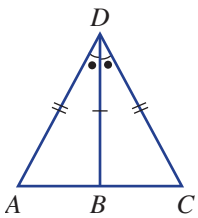
$\Delta ABC \equiv \Delta EDC$ (AAS)

Write a proof (similar to the above) for these pairs of congruent triangles. Any of the four tests (SSS, SAS, AAS or RHS) may be used.

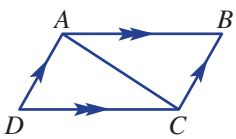
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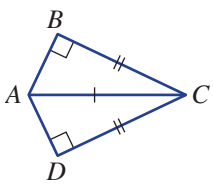
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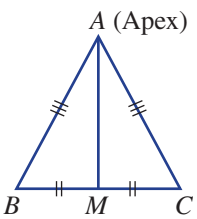
d



Proof challenge — — 12

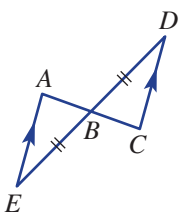
ENRICHMENT

12 Write a logical set of reasons (proof) as to why the following are true. Refer to Question **11** for an example of how to set out a simple proof.

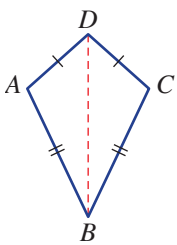


a The segment joining the apex of an isosceles triangle to the midpoint M of the base BC is at right angles to the base, i.e. prove $\angle AMB = \angle AMC = 90°$.

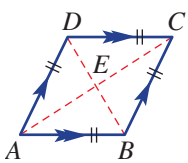
b The segment $AC = 2AB$ in this diagram.



c A kite has one pair of equal opposite angles.



d The diagonals of a rhombus intersect at right angles.



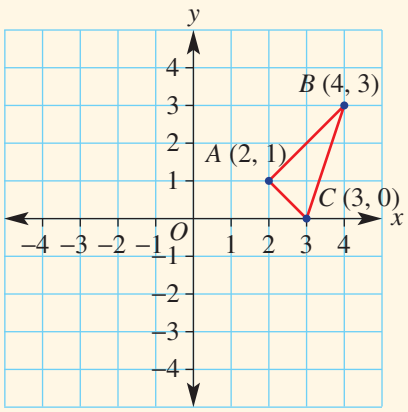
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Progress quiz

10A **1** Consider the triangle ABC.

a Copy the diagram and draw the reflected image using the y-axis as the mirror line.

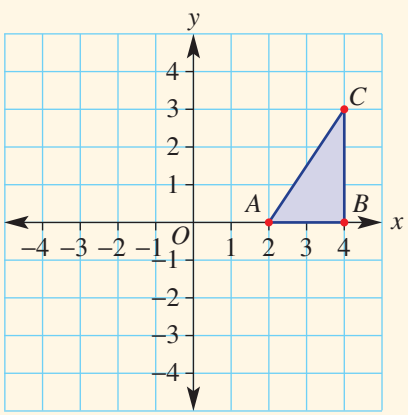
b State the coordinates of the vertices $A'B'C'$ after the triangle is reflected in the x-axis.



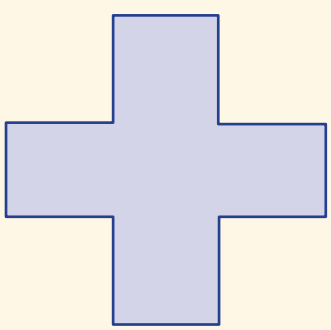
10B **2** State the translation vector that moves the point.

a $A(2, 1)$ to $A'(5, 6)$ **b** $B(-1, 0)$ to $B'(3, -2)$

10B **3** Draw the image of the triangle ABC after a translation by the vector $(-4, 1)$.

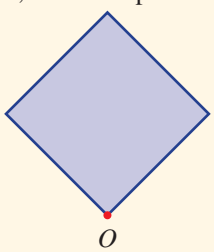


10C **4** State the order of rotational symmetry for diagram below.



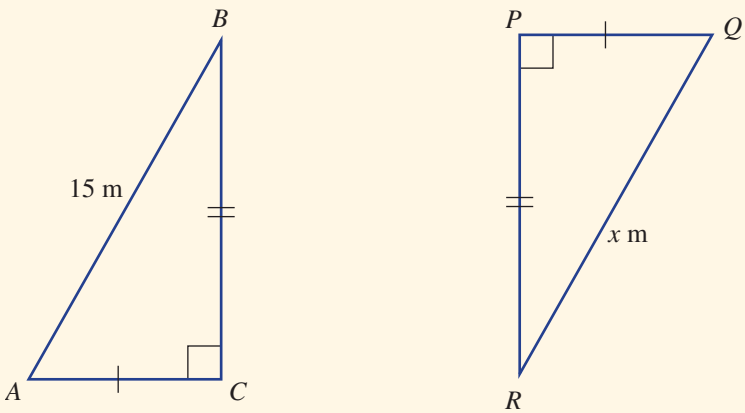
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10C **5** Rotate the following shape clockwise 90°, about the point O.



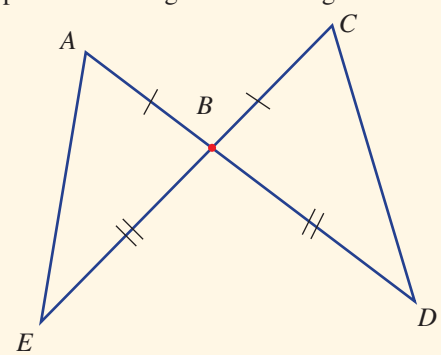
10C **6** Refer to the triangle in Question **3**. Give the coordinates of A', B' and C' after the original triangle ABC has been rotated about the origin 90°, anticlockwise.

10D/E **7** Consider the triangles ABC and PQR.



a Which congruency test would you choose for this pair of triangles?
b Write a congruency statement.
c Which side of triangle PQR corresponds to the side AB?
d Which angle in triangle ABC corresponds to angle RPQ?

10E **8** Write a congruency proof to prove that triangle ABE is congruent to triangle CBD.



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10F Tessellations









Architects, builders and interior designers have great interest in arranging basic congruent shapes to create interesting patterns within a new home. These patterns are often formed using tiles or pavers and can be found on bathroom walls, interior floors or exterior courtyards.



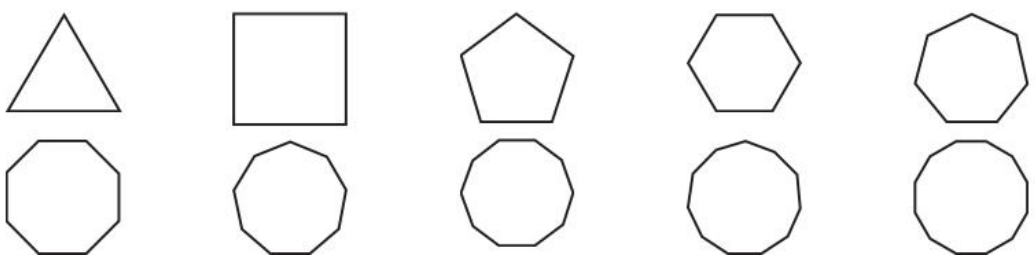
A common range of paving patterns that show how a rectangle can be arranged without leaving any gaps or holes. Which is your preferred pattern if you were wanting to pave an outdoor entertaining area?

The words *tessellate* and *tessellation* originate from the Latin noun, *tessera*, referring to a small tile used in the construction of a mosaic. Tessellated tile designs are commonly used throughout history in the fields of Art and Design and continue to be extensively employed today. It is most likely, that various tessellations exist within your home and your school.

Let's start: To tessellate or not to tessellate?

In Section 2E you were introduced to the concept of *regular polygons* as shapes with sides of equal length and equal interior angles. Can you remember the name given to the first ten polygons?

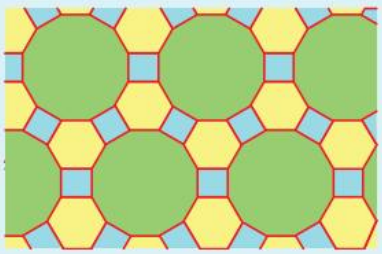
Johann Kepler, back in 1619, was the first mathematician to prove that there are only three polygons that will tessellate by themselves. Working with a partner, can you determine which polygons these are?



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Key ideas

- A **tessellation** is a pattern made up of shapes that fit together without any gaps and without any overlaps.
- Isometric transformations, such as **reflections, translations** and **rotations**, are used with appropriate shapes to produce tessellated patterns.
- **Regular tessellations** are formed by arranging multiple copies of one regular polygon. There are only three regular polygons that tessellate by themselves: triangle, square and hexagon.
- **Semi-regular tessellations** are formed by arranging multiple copies of two or more regular polygons. There are eight distinct semi-regular tessellations.
- Regular and semi-regular tessellations can be **named** by counting the number of sides each regular polygon has at any of the identical vertices. For example: the following semi-regular tessellation consists of squares, hexagons and dodecagons. It can be named as a 4.6.4.12 tessellation.



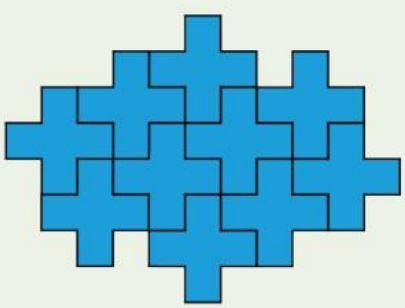
- Other tessellated patterns can be formed by any combination of shapes (regular, irregular, composite). Curved shapes and images can also be used to form tessellated patterns, like the one shown.



Example 10 Tessellating plus signs

Using the following 'plus sign' shape  draw ten identical plus signs to show that this shape will tessellate.

SOLUTION



EXPLANATION

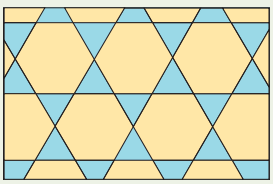
Translate each identical plus sign to make sure it fits without leaving any gaps and without any overlaps.

Note that the final shape does not need to be a neat rectangle to be classified as a tessellation. The only requirement is that the pattern can continue to grow without leaving any holes.

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Example 11 Naming tessellations

By considering any vertex, name the following semi-regular tessellation.

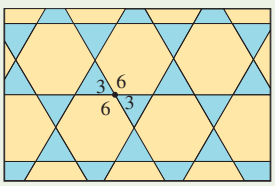


SOLUTION

(3.6.3.6)

EXPLANATION

Select any vertex and as you go around the vertex count the number of sides each polygon has.



Exercise 10F

When asked to construct tessellations you can do these by hand using grid or dot paper, alternatively you may prefer to use geometry software.

1–4 | 4 | —

UNDERSTANDING

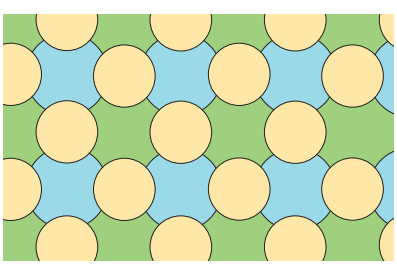
1 Which of the following phrases best describes what a *tessellation* is?

- **A** A group of shapes joined together
- **B** A group of shapes all stacked on top of one another to form a totem pole
- **C** A group of shapes arranged together without any overlaps or any gaps
- **D** A group of shapes positioned in such a way to form an attractive pattern

2 Which of the following words best matches the mathematical term *congruence*?

A Parallel **B** Similar **C** Related **D** Identical

3 Provide a reason why the following pattern cannot be called a tessellation.

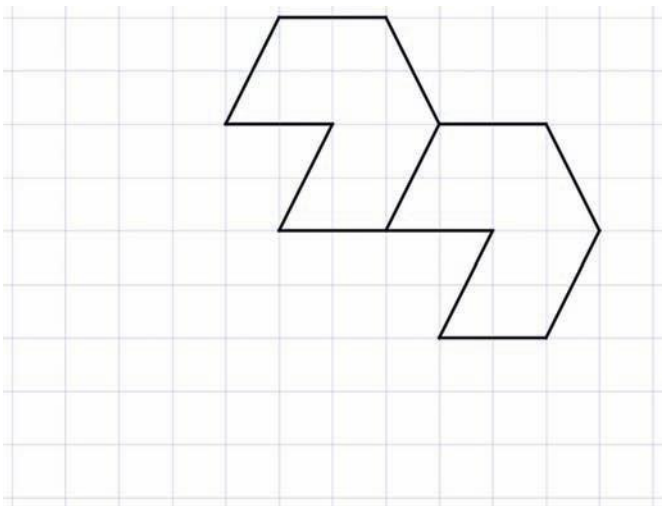


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10F

UNDERSTANDING

4 The following image shows the start of a tessellation, with two identical shapes joined together. Continue the tessellation by adding on another six identical shapes.



5–9 5–10 6–10

FLUENCY

Example 10

5 Using the following trapezium shape  draw ten identical shapes to show that a trapezium will tessellate. You can translate, rotate or reflect this shape to form your tessellation.

6 Draw regular tessellations using only:

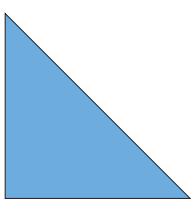
a triangles **b** squares **c** hexagons

Example 11

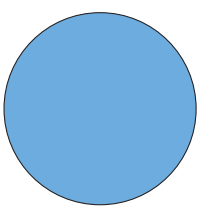
7 By looking at vertices, label each of the tessellations drawn in Question 6.

8 Which of the following shapes tessellate by themselves? Reflections, rotations and translations of the original shape can be used.

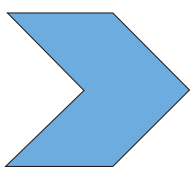
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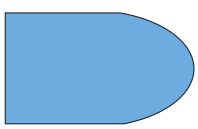
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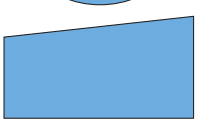
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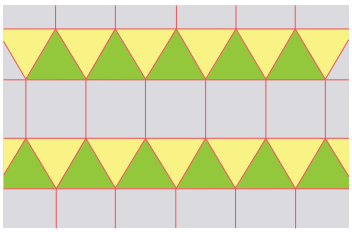
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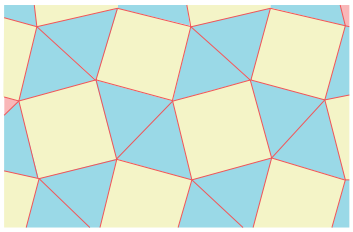
FLUENCY

9 By considering any vertex, name the following Semi-Regular tessellations.

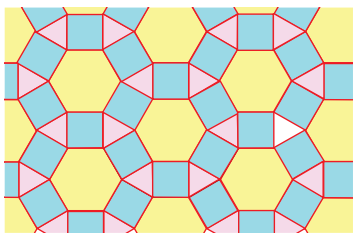
a



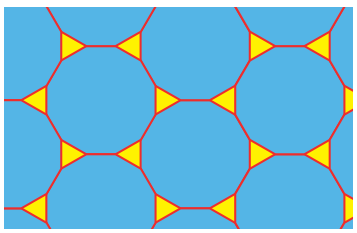
b



c



d

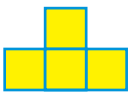


10 Design a tessellation using the following. Use rotations, reflections and translations if needed.

a only the following shape



b only the following shape



c any combination of the above two shapes

11 | 11, 12 | 12, 13

PROBLEM-SOLVING

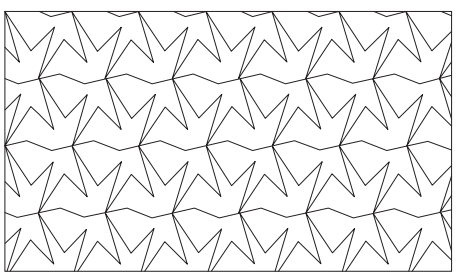
11 A landscaper is required to pave an outside entertaining area with the dimensions 10 m × 12 m. She must use rectangular pavers measuring 25 cm × 50 cm.

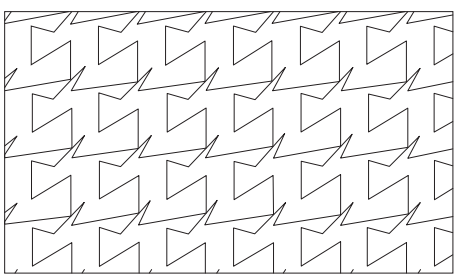
a How many pavers will be required to complete the job?

b Show the start of two possible tessellation patterns that could be used for the entertaining area.

12 Produce a tessellation using only regular octagons and squares.

13 **a** Shade in one unit shape within each of the following tessellations.





b Taking inspiration from the designs above, create your own irregular tessellation.

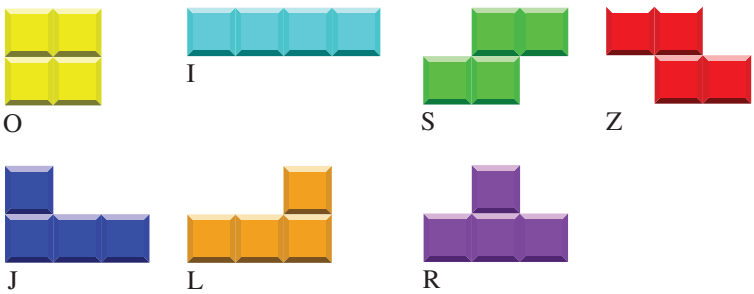
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14 14–15 14–16

10F

REASONING

14 The object of the game *Tetris* is to produce rows with no gaps, or in other words to produce a tessellation with the tiles as they appear. Using 1 cm grid paper, draw a large rectangle of width = 10 cm and height = 20 cm.
The following image shows the seven different *Tetris* pieces, with each small cube representing a 1 cm × 1 cm square.



a How many *Tetris* pieces will be needed to completely fill the 10 cm × 20 cm rectangle?

b Using at least three of each piece, design a tessellated pattern to fill the 10 cm × 20 cm rectangle.

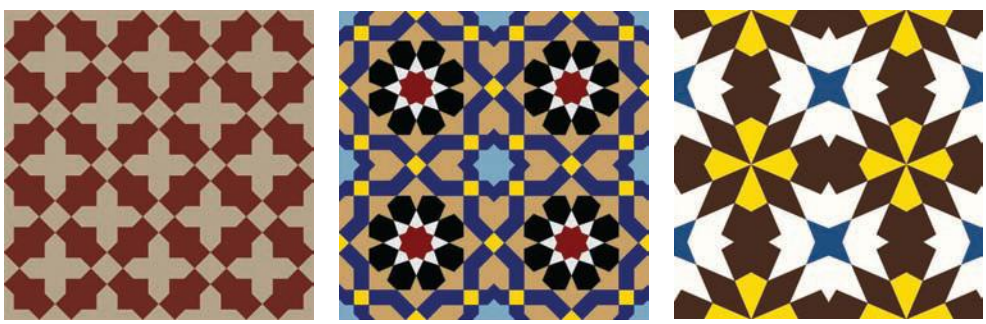
15 Explain why a circle can or cannot be used within a tessellation.

16 Using your knowledge of the interior angle of regular polygons, the angle size of a revolution and the vertex naming technique of tessellations, justify why there are only three regular polygons which tessellate by themselves.

Ancient, modern or cutting edge tessellations — — 17–19

ENRICHMENT

17 Ancient tessellations
During the Middle Ages the Moorish people, particularly of Spain, were well known for their distinctive and elaborate tile designs. Several images are shown below.



a Carry out research on Moorish tile designs and print two of your favourite tessellations.

b Using grid paper, design your own intricate 10 × 10 tile, consisting of a range of simple coloured shapes which tessellate and completely cover the tile.

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10F

ENRICHMENT

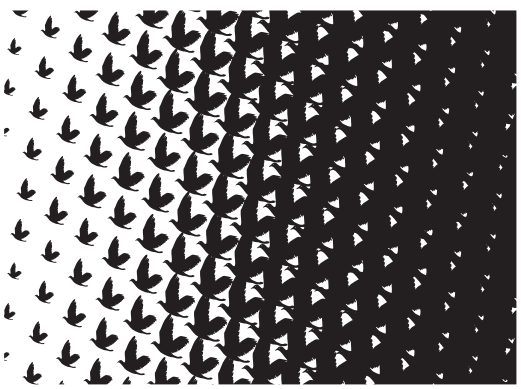
c Either by hand, or using appropriate geometry software, repeatedly draw your intricate tile to show how it tessellates and see how effective it looks as a design that could go in a modern home.



The Alcazar, a royal palace built for the Moorish Muslim kings in Seville, Spain is regarded as one of the most intricate and beautiful palaces in the world. It is the oldest royal palace still in use in Europe today and is a very popular tourist site for visitors to Spain. *The Alcazar* showcases many amazing mosaic and tessellating tiles.

18 Modern tessellations

The Dutch artist M.C. Escher (1898–1972) is famous for making irregular tessellations involving repeated images which gradually change form. An example of Escher-like tessellation art is shown.



a Carry out research on M.C. Escher and print two of your favourite Escher designs.

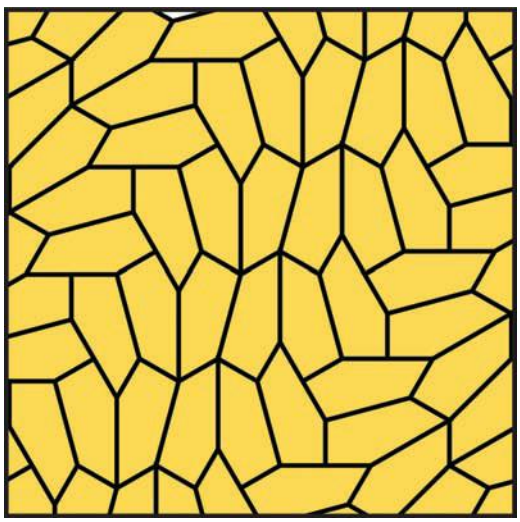
b Either by hand, or using appropriate geometry software, design your own irregular tessellation consisting of the one repeated image.

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19 Cutting-edge tessellations

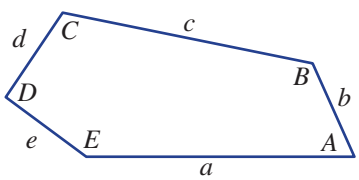
In 2015, Dr. Casey Mann, Associate Professor of Mathematics at the University of Washington, and his colleagues discovered a new irregular pentagon which tessellates. Reportedly, it is only the fifteenth such pentagon ever found and is the first new tessellating pentagon to be found in thirty years.

The image shows a tessellation involving only the new irregular pentagon.



a Carry out research on irregular pentagons which can tessellate by themselves.

b Either by hand, or using appropriate geometry software, replicate the newly identified pentagon using the information provided below.



$A = 60°$ $a = 1$

$B = 135°$ $b = \frac{1}{2}$

$C = 105°$ $c = \frac{1}{\sqrt{2}(\sqrt{3}-1)}$

$D = 90°$ $d = \frac{1}{2}$

$E = 150°$ $e = \frac{1}{2}$

c Create a tessellation using only the newly identified pentagon.

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10G Congruence and quadrilaterals



The properties of special quadrilaterals, including the parallelogram, rhombus, rectangle, square, trapezium and kite, can be examined more closely using congruence. By drawing the diagonals and using the tests for the congruence of triangles, we can prove many of the properties of these special quadrilaterals.

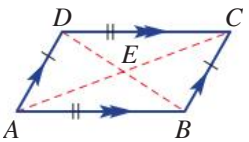






Let's start: Do the diagonals bisect each other?

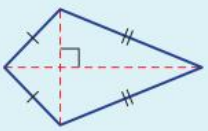
Here is a parallelogram with two pairs of parallel sides. Let's assume also that opposite sides are equal (a basic property of a parallelogram which we prove later in this exercise).



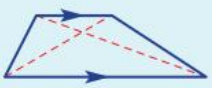
- First locate ΔABE and ΔCDE.
- What can be said about the angles $\angle BAE$ and $\angle DCE$?
- What can be said about the angles $\angle ABE$ and $\angle CDE$?
- What can be said about the sides AB and DC?
- Now what can be said about $\angle ABE$ and $\angle CDE$? Discuss the reasons.
- What does this tell us about where the diagonals intersect? Do they **bisect** each other and why? (To bisect means to cut in half.)

Key ideas

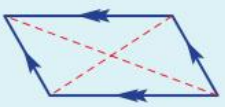
This is a summary of the properties of the special quadrilaterals.



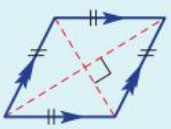
- Kite: A quadrilateral with two pairs of adjacent sides equal
  - Two pairs of adjacent sides of a kite are equal
  - One diagonal of a kite bisects the other diagonal
  - One diagonal of a kite bisects the opposite angles
  - The diagonals of a kite are perpendicular



- Trapezium: A quadrilateral with at least one pair of parallel sides
  - At least one pair of sides of a trapezium are parallel



- Parallelogram: A quadrilateral with both pairs of opposite sides parallel
  - The opposite sides of a parallelogram are parallel
  - The opposite sides of a parallelogram are equal
  - The opposite angles of a parallelogram are equal
  - The diagonals of a parallelogram bisect each other

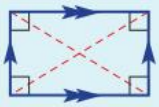


- Rhombus: A parallelogram with all sides equal in length
  - The opposite sides of a rhombus are parallel
  - All sides of a rhombus are equal
  - The opposite angles of a rhombus are equal
  - The diagonals of a rhombus bisect the angles
  - The diagonals of a rhombus bisect each other
  - The diagonals of a rhombus are perpendicular

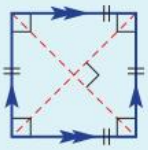
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Key ideas

- Rectangle: A parallelogram with a right angle
  - The opposite sides of a rectangle are parallel
  - The opposite sides of a rectangle are equal
  - All angles at the vertices of a rectangle are 90°
  - The diagonals of a rectangle are equal
  - The diagonals of a rectangle bisect each other

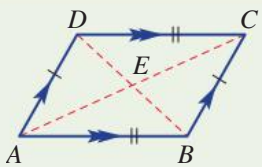


- Square: A rectangle with two adjacent sides equal
  - Opposite sides of a square are parallel
  - All sides of a square are equal
  - All angles at the vertices of a square are 90°
  - The diagonals of a square bisect the vertex angles
  - The diagonals of a square bisect each other
  - The diagonals of a square are perpendicular



Example 12 Proving that diagonals of a parallelogram bisect each other

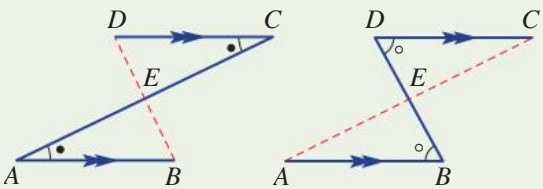
Prove that the diagonals of a parallelogram bisect each other and give reasons.



SOLUTION

$\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$AB = CD$ (given equal side lengths)
$\therefore \Delta ABE = \Delta CDE$ (AAS)
$\therefore BE = DE$ and $AE = CE$
$\therefore$ Diagonals bisect each other.

EXPLANATION



The two triangles are congruent using the AAS test. Since ΔABE and ΔCDE are congruent the corresponding sides are equal.

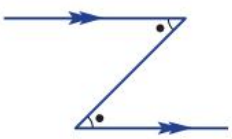
Exercise 10G

1–5 | 1–4, 5(½) | —

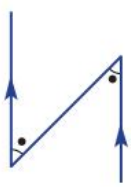
UNDERSTANDING

1 Give the reason why the two marked angles are equal.

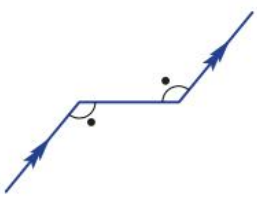
a



b



c



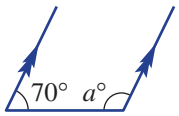
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10G

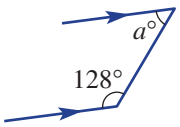
UNDERSTANDING

2 Give the reason why the two marked angles add to 180° and then state the value of a.

a



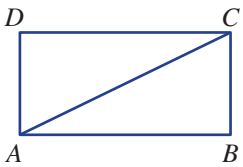
b



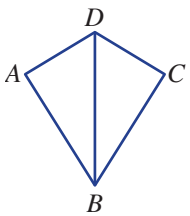
3 SSS is one test for congruence of triangles. Write down the other three.

4 Name the side (e.g. AB) that is common to both triangles in each diagram.

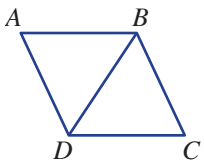
a



b

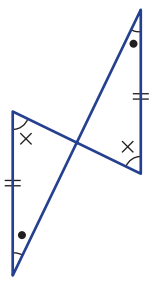


c

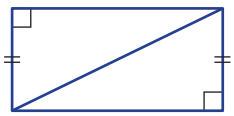


5 Which of the four tests for congruence of triangles would be used to prove that the triangles in each pair are congruent?

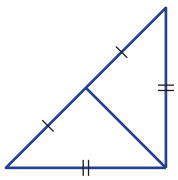
a



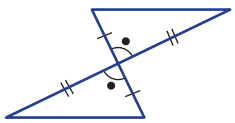
b



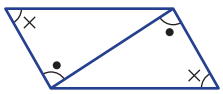
c



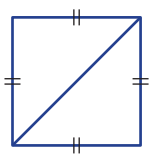
d



e



f

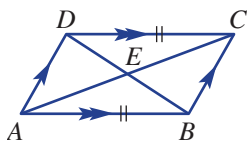


FLUENCY

6, 7 | 6–8 | 6–8

Example 12

6 Prove by giving reasons that the diagonals in a parallelogram bisect each other. You may assume here that opposite sides are equal so use $AB = CD$. Complete the proof by following these steps.



Step 1. List the pairs of equal angles in ΔABE and ΔCDE giving reasons why they are equal.

Step 2. List the pairs of equal sides in ΔABE and ΔCDE giving reasons why they are equal.

Step 3. Write $\Delta ABE \equiv \Delta CDE$ and give the reason SSS, SAS, AAS or RHS.

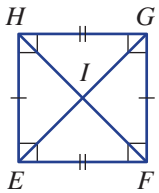
Step 4. State that $BE = DE$ and $AE = CE$ and give a reason.

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10G

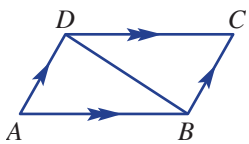
FLUENCY

7 Prove (as in Example 12) that the diagonals in a square *EFGH* bisect each other.



8 A parallelogram *ABCD* has two pairs of parallel sides.

- **a** What can be said about $\angle ABD$ and $\angle CDB$ and give a reason?
- **b** What can be said about $\angle BDA$ and $\angle DBC$ and give a reason?
- **c** Which side is common to both ΔABD and ΔCDB?
- **d** Which congruence test would be used to show that $\Delta ABD \equiv \Delta CDB$?
- **e** If $\Delta ABD \equiv \Delta CDB$, what can be said about the opposite sides of a parallelogram?

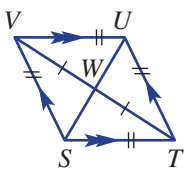


9 9, 10 9–11

PROBLEM-SOLVING

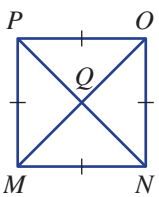
9 For this rhombus assume that $VW = WT$.

- **a** Give reasons why $\Delta VWU \equiv \Delta TWU$.
- **b** Give reasons why $\angle VWU = \angle TWU = 90°$.



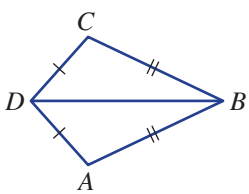
10 For this square assume that $MQ = QO$ and $NQ = PQ$.

- **a** Give reasons why $\Delta MNQ \equiv \Delta ONQ$.
- **b** Give reasons why $\angle MQN = \angle OQN = 90°$.
- **c** Give reasons why $\angle QMN = 45°$.



11 Use the information in this kite to prove these results.

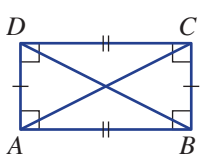
- **a** $\Delta ABD \equiv \Delta CBD$
- **b** $\angle DAB = \angle DCB$
- **c** $\angle ADB = \angle CDB$



12 12, 13 13–15

REASONING

12 Use Pythagoras' theorem to prove that the diagonals in a rectangle are equal in length. You may assume that opposite sides are equal.

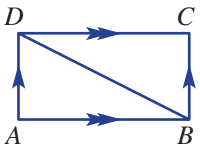


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10G

REASONING

13 Use the steps outlined in Question **6** to show that opposite sides of a rectangle $ABCD$ are equal. Give all reasons.

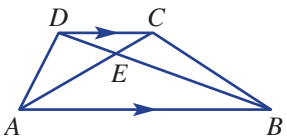


14 A trapezium $ABCD$ has one pair of parallel sides.

a Which angle is equal to $\angle BAE$?

b Which angle is equal to $\angle ABE$?

c Explain why ΔABE is not congruent to ΔCDE.

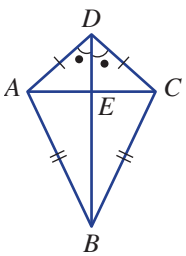


15 Use the information given for this kite to give reasons for the following properties. You may assume that $\angle ADE = \angle CDE$ as marked.

a $\angle DAE = \angle DCE$

b $\Delta AED \equiv \Delta CED$

c $\angle AED = \angle CED = 90°$

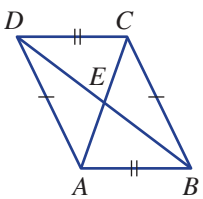


ENRICHMENT

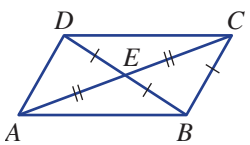
Prove the converse — — 16

16 Prove the following results, which are the converse (reverse) of some of the proofs completed earlier in this exercise.

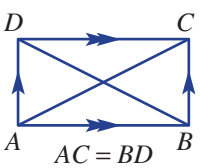
a If the opposite sides of a quadrilateral are equal then the quadrilateral is a parallelogram, i.e. show that the quadrilateral has two pairs of parallel sides.



b If the diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram.



c If the diagonals of a parallelogram are equal then the parallelogram is a rectangle.



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10H Similar figures

EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

When you look at an object through a telescope or a pair of binoculars, you expect the image to be much larger than the one you would see with the naked eye. Both images would be the same in every way except for their size. Such images in mathematics are called similar figures.

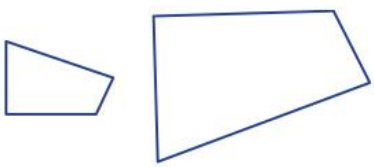
Images that have resulted in a reduction in size (rather than an enlargement in size) are also considered to be similar figures.



An item through a telescope is 'similar' to how it appears to the naked eye.

Let's start: Are they similar?

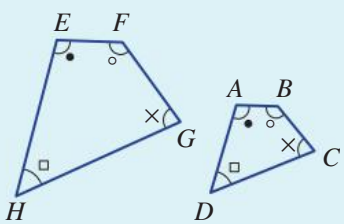
Here are two quadrilaterals.



- Do they look similar in shape? Why?
- How could you use a protractor to help decide if they are similar in shape? What would you measure and check? Try it.
- How could you use a ruler to help decide if they are similar in shape? What would you measure and check? Try it.
- Describe the geometrical properties that similar shapes have. Are these types of properties present in all pairs of shapes that are 'similar'?

Key ideas

- **Similar figures** have the same shape but can be of different size.
- All corresponding angles are equal.
- All corresponding sides are in the same ratio.
- The ratio of sides is often written as a fraction, shown here. It is often called the **scale factor**. Unless specified, the scale factor will be written as a number bigger than 1.

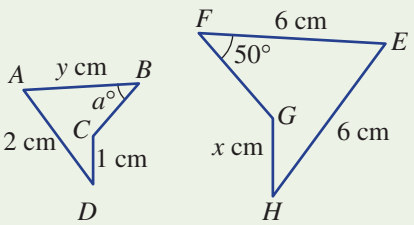


$$\text{Scale factor} = \frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}$$

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Example 13 Identifying corresponding features

For the following pair of similar figures complete these tasks.



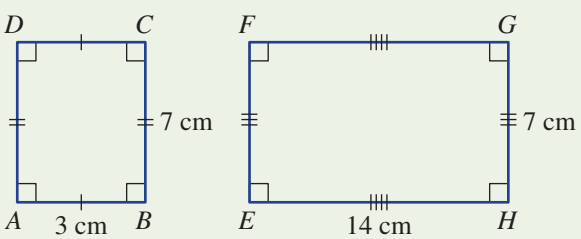
a List the pairs of corresponding sides.
b List the pairs of corresponding angles.
c Find the scale factor.
d Find the values of the pronumerals.

SOLUTION	EXPLANATION
a $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$	Pair up each vertex, noticing that G corresponds with C, H corresponds with D and so on.
b $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$	$\angle G$ and $\angle C$ are clearly the largest angles in their respective shapes. Match the other angles in reference to these angles.
c $\frac{HE}{DA} = \frac{6}{2} = 3$	HE and DA are corresponding sides both with given measurements. Divide the larger by the smaller.
d $a = 50$	$\angle B$ and $\angle F$ are equal corresponding angles.
$x = 3 \times 1 = 3$ cm	The scale factor $\frac{HE}{DA} = 3$ so $\frac{GH}{CD}$ should also equal 3.
$y = 6 \div 3 = 2$ cm	Alternatively, say that GH is 3 times the length CD. Similarly, EF should be 3 times the length AB (y cm).

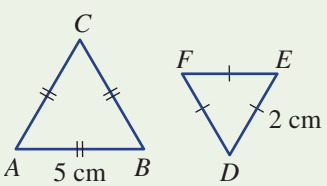
Example 14 Deciding if shapes are similar

Decide if these shapes are similar by considering corresponding angles and the ratio of sides.

a



b



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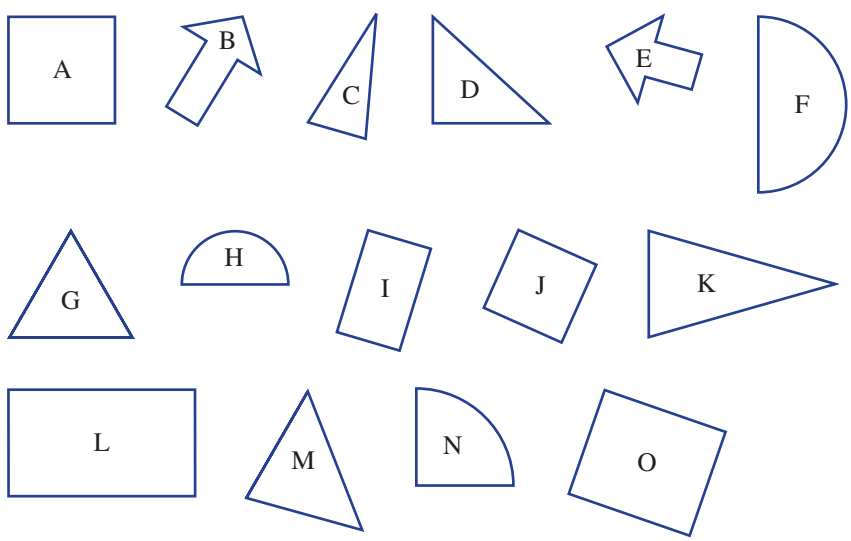
SOLUTION	EXPLANATION
a All corresponding angles are equal.	All angles are 90° and so corresponding angles are equal.
$\frac{EH}{BC} = \frac{14}{7} = 2$	Match pairs of sides to find scale factors.
$\frac{GH}{AB} = \frac{7}{3} = 2.\dot{3}$	The scale factor needs to be equal if the shapes are to be similar.
Scale factors are not equal. Shapes are not similar.	
b All angles are 60°	All angles in a equilateral triangle are 60°.
$\frac{AB}{DE} = \frac{5}{2} = 2.5$	All scale factors are equal so the triangles are similar.
$\frac{BC}{EF} = \frac{5}{2} = 2.5$	
$\frac{CA}{FD} = \frac{5}{2} = 2.5$	
Triangles are similar.	

Exercise 10H

1, 2 | 2 | —

UNDERSTANDING

1 List any pairs of shapes that look similar (i.e. same shape but different size).

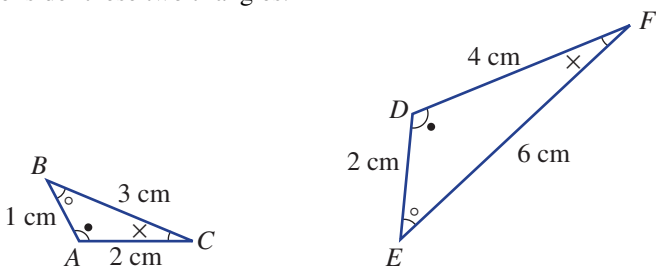


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10H

UNDERSTANDING

2 Consider these two triangles.



a Name the angle on the larger triangle which corresponds to:

i $\angle A$ **ii** $\angle B$ **iii** $\angle C$

b Name the side on the smaller triangle which corresponds to:

i DE **ii** EF **iii** FD

c Find these scale factors.

i $\frac{DE}{AB}$ **ii** $\frac{EF}{BC}$ **iii** $\frac{FD}{CA}$

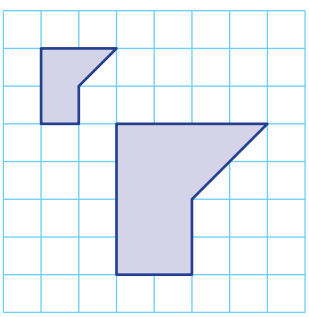
d Would you say that the two triangles are similar? Why or why not?

3, 4, 5(½) 3, 4, 5(½) 4, 5(½)

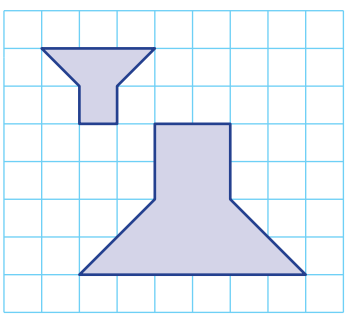
FLUENCY

3 Decide if the pairs of shapes on these grids are similar. If so, state the scale factor.

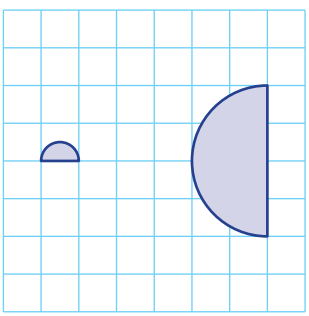
a



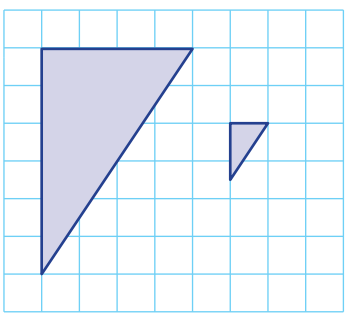
b



c



d



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Example 13

4 For the following pairs of similar figures, complete these tasks.

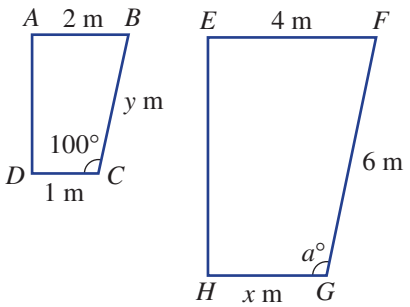
i List the pairs of corresponding sides.

ii List the pairs of corresponding angles.

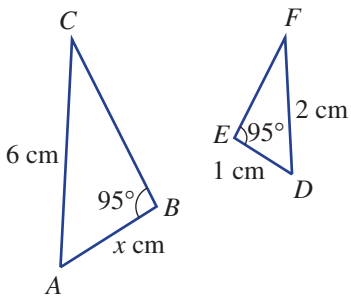
iii Find the scale factor.

iv Find the values of the pronumerals.

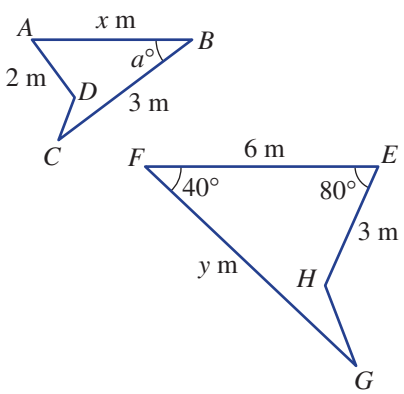
a



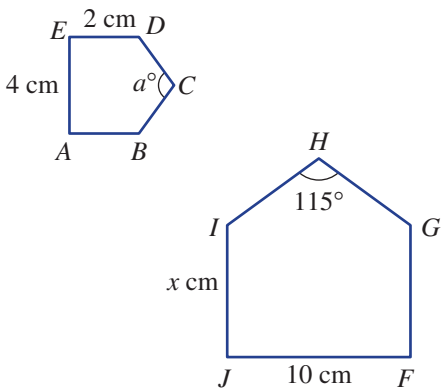
b



c



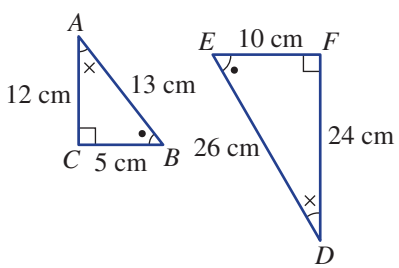
d



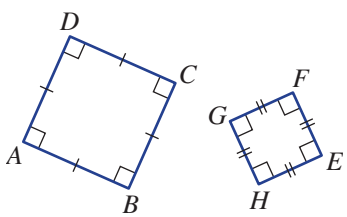
Example 14

5 Decide if these shapes are similar by considering corresponding angles and the ratios of sides.

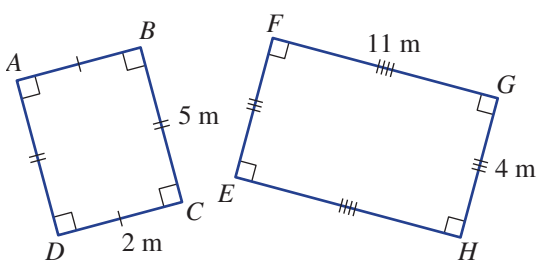
a



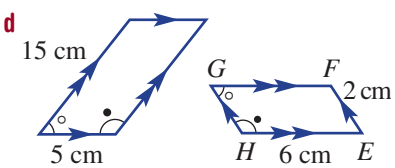
b



c



d



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10H

PROBLEM-SOLVING

6, 7 | 7–9 | 7–9

6 Two rectangular picture frames are similar in shape. A corresponding pair of sides are of length 50 cm and 75 cm. The other side length on the smaller frame is 70 cm. Find the perimeter of the larger frame.



7 Two similar triangles have a scale factor of $\frac{7}{3}$. If the larger triangle has a side length of 35 cm, find the length of the corresponding side on the smaller triangle.

8 One circle has a perimeter of 10π cm and another has an area of 16π cm^2. Find the scale factor of the diameters of the two circles.

9 A square photo of area 100 cm^2 is enlarged to an area of 900 cm^2. Find the scale factor of the side lengths of the two photos.

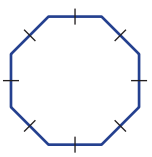
REASONING

10 | 10, 11 | 10, 12

10 Are the following statements true or false? Give reasons for your answers.

- **a** All squares are similar.
- **b** All rectangles are similar.
- **c** All equilateral triangles are similar.
- **d** All isosceles triangles are similar.
- **e** All rhombuses are similar.
- **f** All parallelograms are similar.
- **g** All kites are similar.
- **h** All trapeziums are similar.
- **i** All circles are similar.

11 **a** If a regular polygon such as this regular octagon is enlarged, do the interior angles change?



b Are all polygons with the same number of sides similar? Give reasons.

12 If a square is similar to another by a scale factor of 4, what is the ratio of the area of small square to the area of large square?

ENRICHMENT

Square designs | — | — | 13

13 A square design consists of a series of squares as shown. An inner square is formed by joining the midpoints of the sides of the next largest square.



- **a** Use Pythagoras' theorem to find the side length of:
  - **i** the second largest square
  - **ii** the third largest square
- **b** Find the scale factor of a pair of consecutive squares, e.g. first and second or second and third.
- **c** If the side length of the outside square was x cm, show that the scale factor of consecutive squares is equal to the result found in part **b**.
- **d** What is the scale factor of a pair of alternate squares, e.g. first and third or second and fourth?

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10I Similar triangles EXTENDING

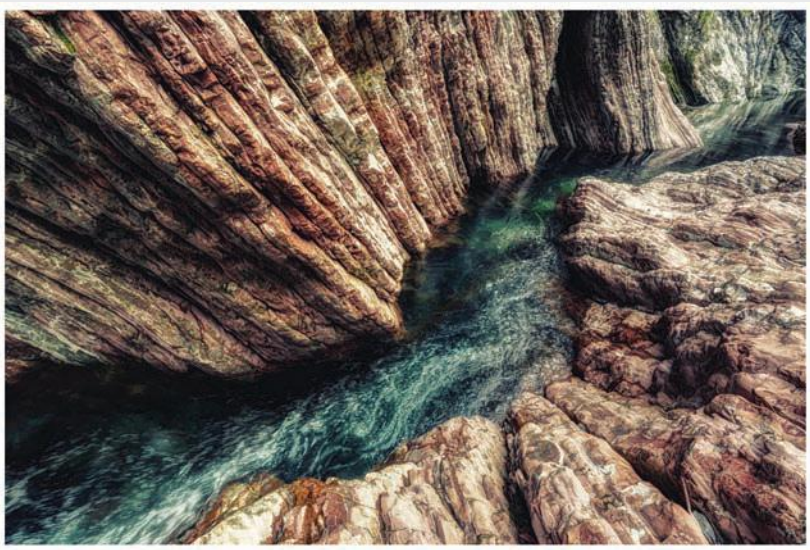
Interactive

Widgets

HOTsheets

Walkthroughs

Finding the approximate height of a tree or the width of a gorge using only simple equipment is possible without actually measuring the distance directly. Similar triangles can be used to calculate distances without the need to climb the tree or cross the gorge. It is important, however, to ensure that if the mathematics of similar triangles is going to be used, then the two triangles are in fact similar. We learned earlier that there were four tests that help to determine if two triangles are congruent. Similarly, there are four tests that help establish whether or not triangles are similar. Not all side lengths or angles are required to prove that two triangles are similar.



Similar triangles could be used to find the width of a gorge without the need to cross it.

Let's start: How much information is enough?

Given a certain amount of information, it may be possible to draw two triangles that are guaranteed to be similar in shape.

Decide if the information given is enough to guarantee that the two triangles will always be similar. If you can draw two triangles (ΔABC and ΔDEF) that are not similar, then there is not enough information provided.

- $\angle A = 30°$ and $\angle D = 30°$
- $\angle A = 30°, \angle B = 80°$ and $\angle D = 30°, \angle E = 80°$
- $AB = 3$ cm, $BC = 4$ cm and $DE = 6$ cm, $EF = 8$ cm
- $AB = 3$ cm, $BC = 4$ cm, $AC = 5$ cm and $DE = 6$ cm, $EF = 8$ cm, $DF = 10$ cm
- $AB = 3$ cm, $\angle A = 30°$ and $DE = 3$ cm, $\angle D = 30°$
- $AB = 3$ cm, $AC = 5$ cm, $\angle A = 30°$ and $DE = 6$ cm, $DF = 10$ cm, $\angle D = 30°$

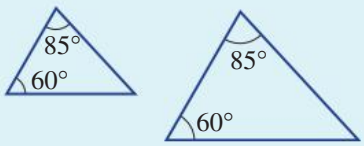
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Key ideas

- Two triangles are similar if:

AAA

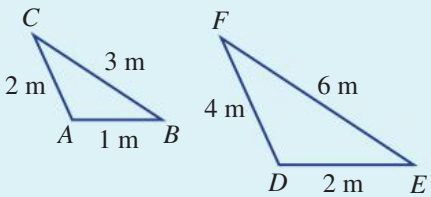
- Three pairs of corresponding angles are equal. If two pairs are known, then the angle sum of a triangle can be used to show that the third pair of angles are also equal.



Missing angle $= 180° - (60° + 85°)$
$= 35°$

SSS

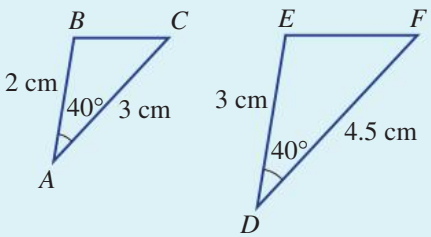
- All pairs of corresponding sides are in the same ratio.



$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = 2$

SAS

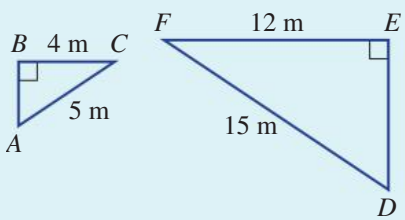
- Two pairs of corresponding sides are in the same ratio and the included angles are equal.



$\frac{DE}{AB} = \frac{DF}{AC} = 1.5$

RHS

- A pair of right angles, the pair of hypotenuse lengths and another pair of corresponding sides are in the same ratio.



$\frac{DF}{AC} = \frac{EF}{BC} = 3$

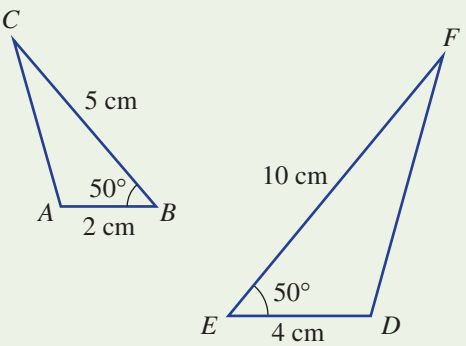
- If two triangles ΔABC and ΔDEF are similar, we write $\Delta ABC \sim \Delta DEF$ or $\Delta ABC \,|||\, \Delta DEF$.
  - This is called a similarity statement.

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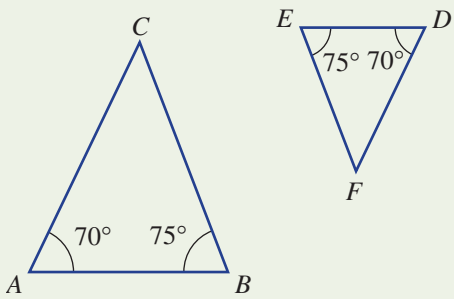
Example 15 Deciding if two triangles are similar

Decide, with reasons, why these pairs of triangles are similar.

a



b



SOLUTION

a $\frac{ED}{AB} = \frac{4}{2} = 2$

$\angle B = \angle E = 50°$

$\frac{EF}{BC} = \frac{10}{5} = 2$

$\therefore \Delta ABC$ is similar to ΔDEF

Using SAS

b $\angle A = \angle D$

$\angle B = \angle E$

$\therefore \Delta ABC$ is similar to ΔDEF

Using AAA

EXPLANATION

Work out the ratio of the two pairs of corresponding sides to see if they are equal. Note that the angle given is the included angle between the two given pairs of corresponding sides.

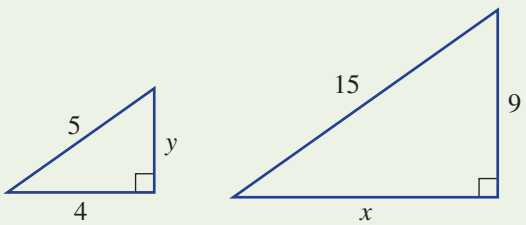
Two pairs of equal angles are given. This implies that the third pair of angles are also equal and that the triangles are similar.



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Example 16 Finding a missing length

Given that these pairs of triangles are similar, find the value of the pronumerals.



SOLUTION	EXPLANATION
Scale factor $= \frac{15}{5} = 3$	First find the scale factor.
$x = 4 \times 3 = 12$ $y = 9 \div 3 = 3$	Multiply or divide by the scale factor to find the values of the pronumerals.

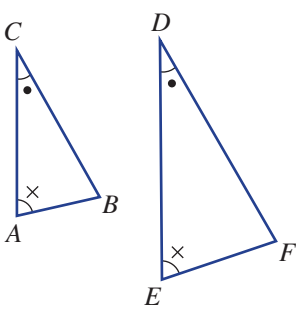
Exercise 10I

1, 2 | 2 | —

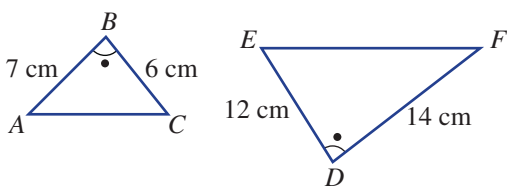
UNDERSTANDING

1 Write a similarity statement for these pairs of similar triangles, e.g. $\Delta ABC \sim \Delta FED$ or $\Delta ABC \,|||\, \Delta DEF$. Be careful with the order of letters and make sure each letter matches the corresponding vertex.

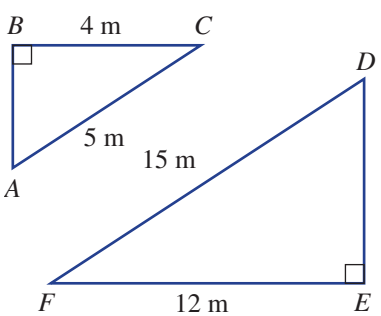
a



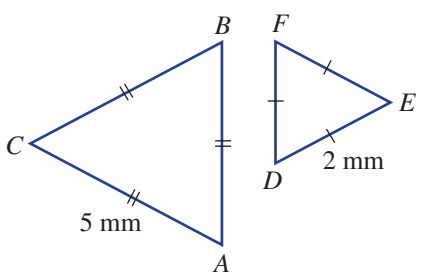
b



c



d



2 For the pairs of triangles in Question **1**, which of the four similar triangle tests (AAA, SSS, SAS or RHS) would be used to show their similarity? See the Key ideas for a description of each test. There is one pair for each test.

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3, 4 3, 4 3, 4

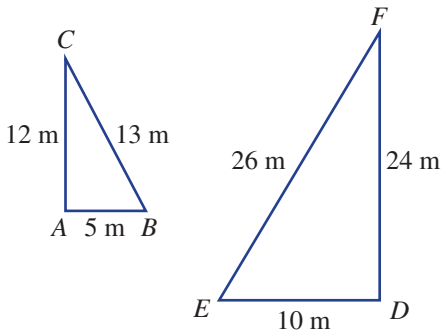
101

FLUENCY

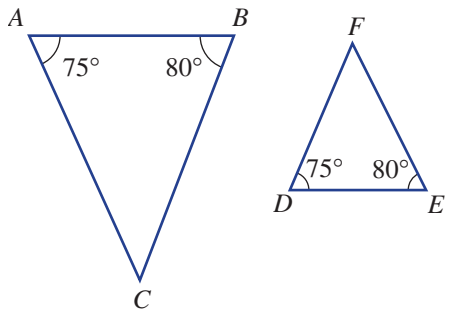
Example 15

3 Decide, with reasons, why these pairs of triangles are similar. State which of the four tests AAA, SSS, SAS or RHS applies.

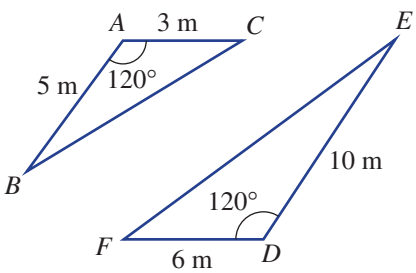
a



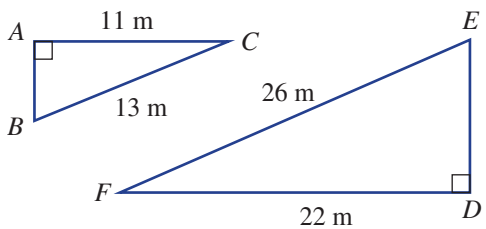
b



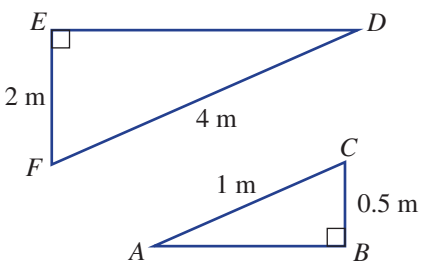
c



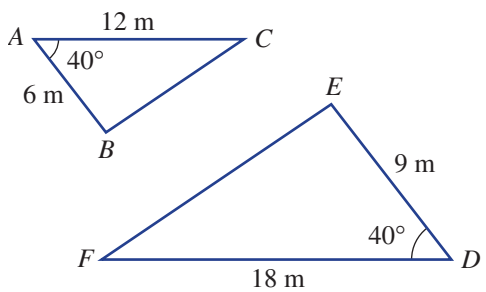
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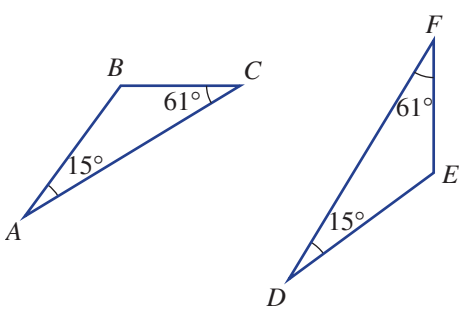
e



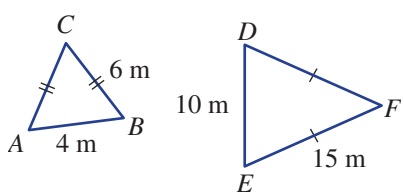
f



g



h



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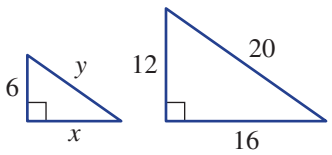
10I

Example 16

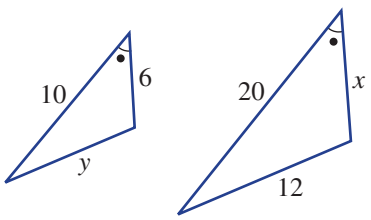
FLUENCY

4 Given that these pairs of triangles are similar, find the value of the pronumerals.

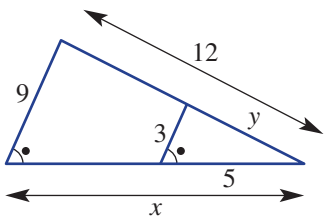
a



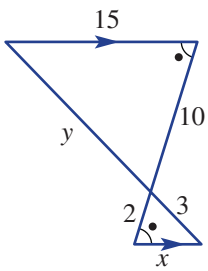
b



c



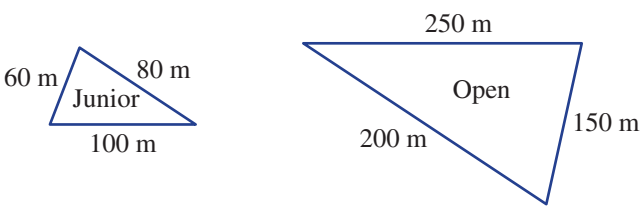
d



5–7 5–7 6–8

PROBLEM-SOLVING

5 Two triangular tracks are to be used for the junior and open divisions for a school event.



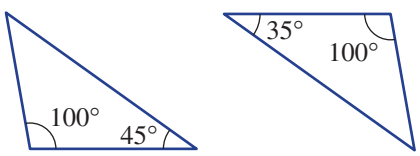
a Work out the scale factor for each of the three corresponding pairs of sides.

b Are the two tracks similar in shape? Give a reason.

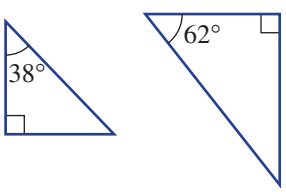
c How many times would a student need to run around the junior track to cover the same distance as running one lap of the senior track?

6 By using the angle sum of a triangle, decide if the given pairs of triangles are similar.

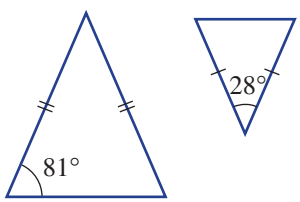
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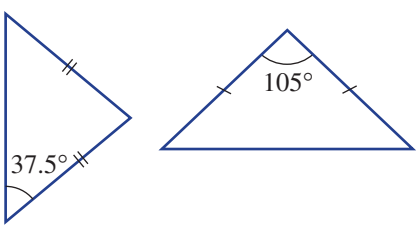
b



c

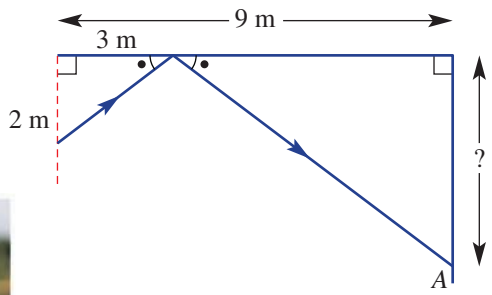


d



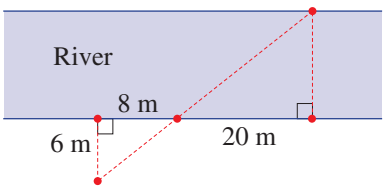
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7 In a game of minigolf, a ball bounces off a wall as shown. The ball proceeds and hits another wall at point A. How far down the right side wall does the ball hit?





8 Trees on a river bank are to be used as markers to form two triangular shapes. Each tree is marked by a red dot as shown in the diagram.



a Are the two triangles similar? Explain why.

b Calculate the scale factor.

c Calculate how far it is across the river.



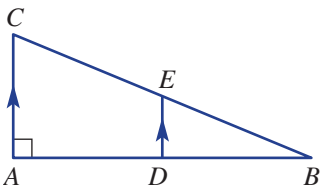
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REASONING

9 Explain why only two pairs of angles are required to prove that two triangles are similar.

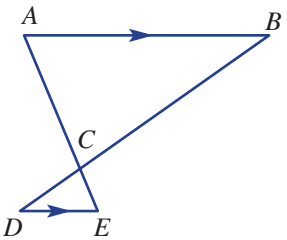
10 Give reasons why the pairs of triangles in these diagrams are similar. If an angle or side is common to both triangles, use the word 'common' in your reasoning.

a



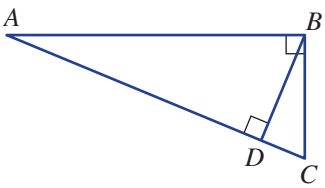
ΔABC and ΔDBE

b



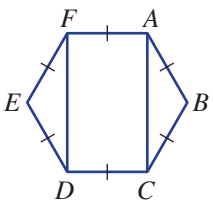
ΔABC and ΔEDC

c



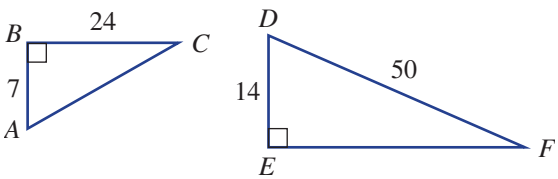
ΔABC and ΔADB

d



ΔABC and ΔFED

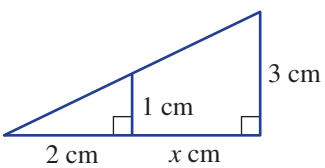
11 Show how Pythagoras' theorem can be used to prove that these two triangles are similar.



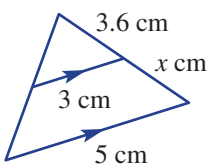
ENRICHMENT

12 The pairs of triangles here show a length x cm, which is not the length of a full side of the triangle. Find the value of x in each case.

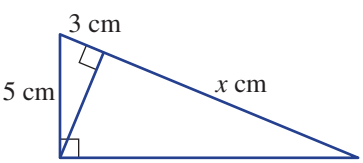
a



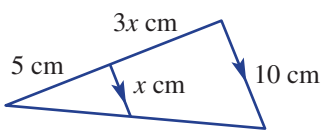
b



c



d



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Investigation

Applying similar triangles

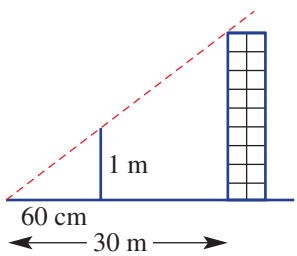
Similar triangles can be applied to situations where distances cannot be measured directly.

Height of a vertical object in the sun



1 Here is an illustration (not to scale) of a tall building with a shadow of 30 m. A 1-metre-long stick is placed vertically near the end of the shadow so that the top of the stick is just touched by the sun's rays. The distance from the base of the stick to the end of the shadow is 60 cm.

- **a** Give reasons why this illustration contains two similar triangles.
- **b** Show how the height of the building can be estimated using the scale factor for the pair of similar triangles.



2 Use the technique outlined in part **1** to help estimate the height of a tall object in the school grounds, at home or in a town or city. Show your workings and include diagrams.

Width of a gorge or river



3 Trees located on both sides of this gorge are chosen to create similar triangles, The distances between some of the trees on one side of the gorge are also recorded.

- **a** Explain how the positions of the trees must be chosen to create a pair of similar triangles. (See Exercise 10I Question **8** for suggestions.)
- **b** Show how the similar triangles can be used to estimate the distance across the gorge. Show your diagrams and workings.

4 Use the technique outlined in part **3** to help estimate the distance across an open space such as a gorge, river or area in the school ground or at home. Show your workings and include diagrams.

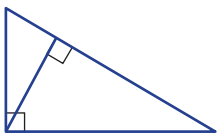
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Problems and challenges



Up for a challenge?
If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 How many similar triangles are there in this figure?
Give reasons.



2 Consider the capital letters of the alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

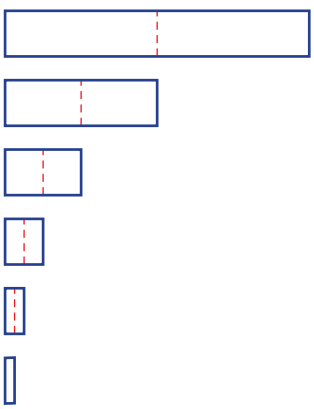
Which of the letters have:

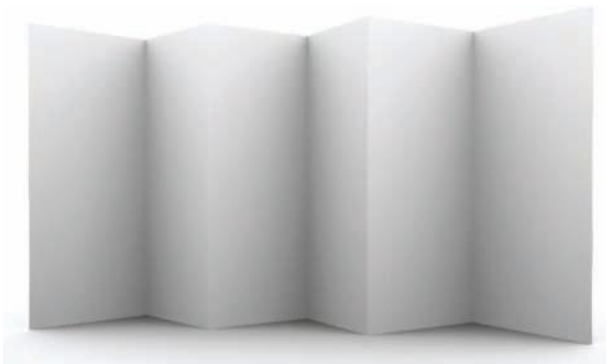
a horizontal line symmetry only?

b only vertical line symmetry?

c both vertical and horizontal line symmetry?

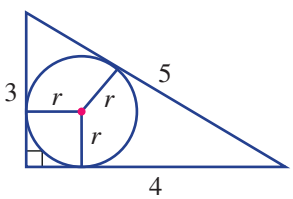
3 A strip of paper is folded 5 times in one direction only. How many creases will there be in the original strip when it is folded out?



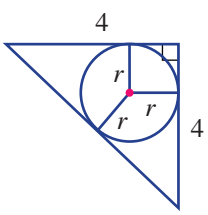


4 Use congruent triangles to find the radius r in these diagrams.

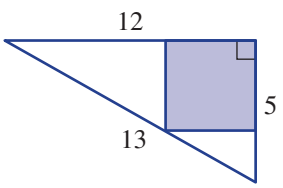
a



b



5 What is the side length of the largest square that can be cut out of this triangle? Use similar triangles.



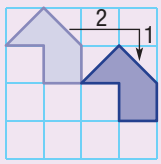
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Chapter summary

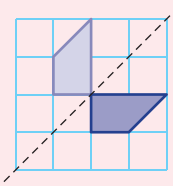
Transformation and congruence

Translation

Vector (2, –1)

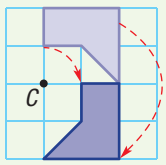


Reflection

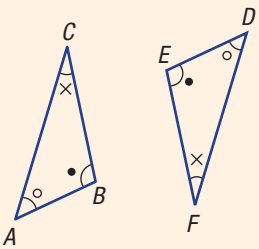


Rotation

Clockwise by 90°



Similar triangles (Ext)



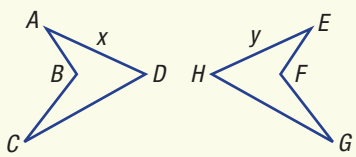
$\Delta ABC \text{ III } \Delta DEF$
or $\Delta ABC \sim \Delta DEF$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Tests: AAA, SSS, SAS or RHS (see Key ideas)

Congruent figures

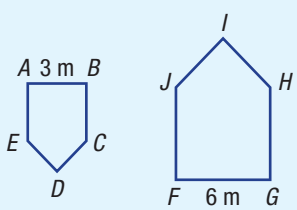


$ABCD \equiv EFGH$ $\quad x = y$

$AB = EF$

$\angle C = \angle G$

Similar figures (Ext)



Ratio $= \frac{FG}{AB} = \frac{6}{3} = 2$

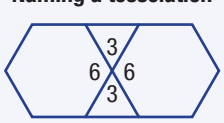
$\angle A = \angle F$ etc.

$GH = AE \times 2$

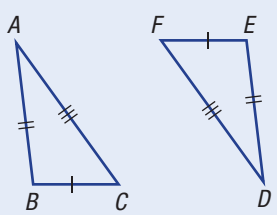
Tesselation

A tesselation is a pattern with no gaps made from shapes. Reflections, translations and rotations can be used.

Naming a tesselation



Congruent triangles



$\Delta ABC \equiv \Delta DEF$

$AB = DE, BC = EF, AC = DF$

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Tests: SSS, SAS, AAS, RHS

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Chapter review

Multiple-choice questions

10A **1** The number of lines of symmetry in a regular pentagon is:

A 10 **B** 5 **C** 2 **D** 1 **E** 0

10B **2** Which vector describes a translation of 5 units to the left and 3 units up?

A $(3, -5)$ **B** $(-3, 5)$ **C** $(5, -3)$ **D** $(-5, 3)$ **E** $(-5, -3)$

10B **3** The point $A(-3, 4)$ is translated to the point A' by the vector $(6, -4)$. The coordinates of point A' are:

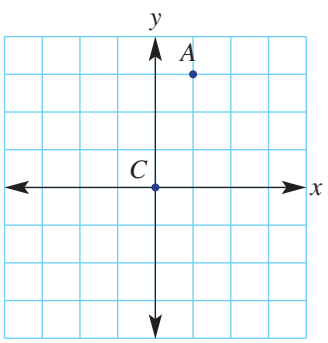
A $(3, 8)$ **B** $(-9, 8)$ **C** $(3, 0)$ **D** $(0, 3)$ **E** $(-9, 0)$

10C **4** An anticlockwise rotation of 125° about a point is the same as a clockwise rotation about the same point of:

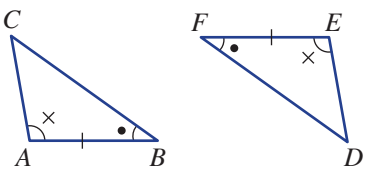
A 235° **B** 65° **C** 55° **D** 135° **E** 245°

10C **5** Point $A(1, 3)$ is rotated clockwise about C by 90° to A'. The coordinates of A' are:

A $(3, 0)$ **B** $(3, -1)$ **C** $(-3, 1)$
D $(-1, 3)$ **E** $(3, 1)$



Questions **6**, **7**, and **8** relate to this pair of congruent triangles.



10D/E **6** The angle on ΔDEF that corresponds to $\angle A$ is:

A $\angle C$ **B** $\angle B$ **C** $\angle F$ **D** $\angle D$ **E** $\angle E$

10D/E **7** If $AC = 5$ cm then ED is equal to:

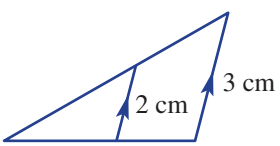
A 5 cm **B** 10 cm **C** 2.5 cm **D** 15 cm **E** 1 cm

10D/E **8** A congruent statement with ordered vertices for the triangles is:

A $\Delta ABC \equiv \Delta FED$ **B** $\Delta ABC \equiv \Delta EDF$
C $\Delta ABC \equiv \Delta DFE$ **D** $\Delta ABC \equiv \Delta DEF$
E $\Delta ABC \equiv \Delta EFD$

10I Ext **9** Which of the four tests (see Section 10H) would be chosen to show that these two triangles are similar?

A AAS **B** RHS **C** SAS
D AAA **E** SSS



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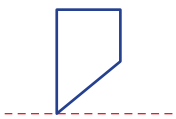
10H Ext **10** Two similar figures have a scale factor of 2.5 and the larger figure has a side length of 15 m. The length of the corresponding side on the smaller figure is:

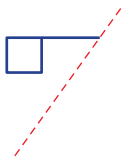
A 5 m **B** 10 m **C** 37.5 m **D** 9 m **E** 6 m

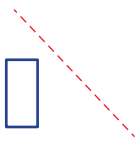
Short-answer questions

10A **1** Copy these shapes and draw the reflected image over the mirror line.

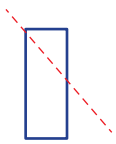
a 

b 

c 

d 

e 

f 

10A **2** The triangle $A(1, 2), B(3, 4), C(0, 2)$ is reflected in the given axis. State the coordinates of the image points A', B' and C'.

a x-axis **b** y-axis

10A **3** How many lines of symmetry do these shapes have?

a Square **b** Isosceles triangle **c** Rectangle

d Kite **e** Regular hexagon **f** Parallelogram

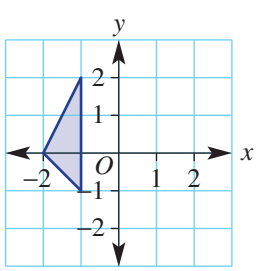
10B **4** Write the vectors that translate each point A to its image A'.

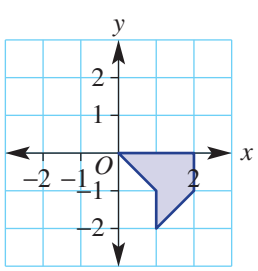
a $A(2, 5)$ to $A'(3, 9)$ **b** $A(-1, 4)$ to $A'(2, -2)$

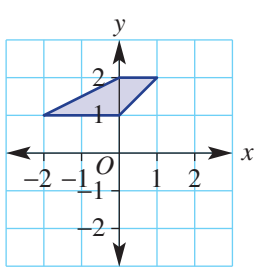
c $A(0, 7)$ to $A'(-3, 0)$ **d** $A(-4, -6)$ to $A'(0, 0)$

10B **5** Copy these shapes and draw the translated image using the given translation vector.

a $(3, -1)$ **b** $(-2, 2)$ **c** $(0, -3)$







10C **6** What is the order of rotational symmetry of these shapes?

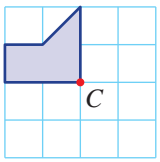
a Equilateral triangle **b** Parallelogram **c** Isosceles triangle

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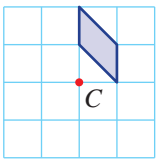
Chapter review

10C **7** Rotate these shapes about the point C by the given angle.

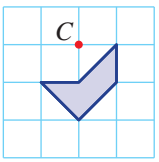
a Clockwise 90°



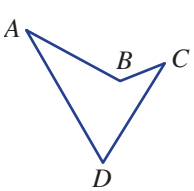
b Clockwise 180°

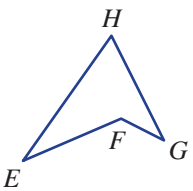


c Anticlockwise 270°



10D **8** For these congruent quadrilaterals, name the object in quadrilateral $EFGH$ that corresponds to the given object in quadrilateral $ABCD$.





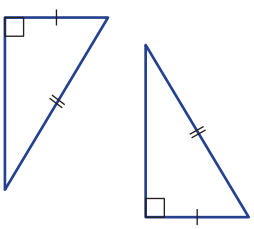
a **i** Vertex B **ii** Vertex C

b **i** Side AD **ii** Side BC

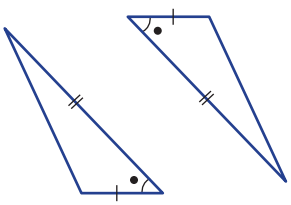
c **i** $\angle C$ **ii** $\angle A$

10E **9** Which of the tests SSS, SAS, AAS or RHS would you choose to explain the congruence of these pairs of triangles?

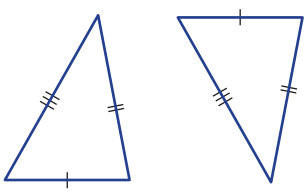
a



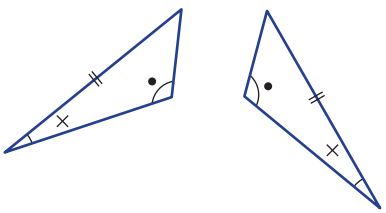
b



c

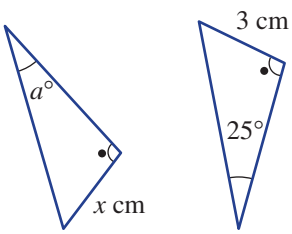


d

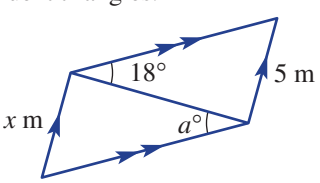


10E **10** Find the values of the pronumerals for these congruent triangles.

a



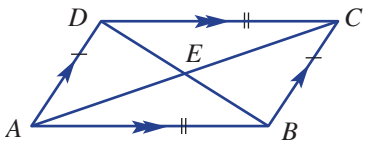
b



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10F **11** Name the three regular polygons which tessellate.

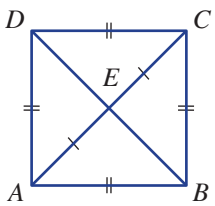
10G **12** This quadrilateral is a parallelogram with two pairs of parallel sides. You can assume that $AB = DC$ as shown.



a Prove that $\Delta ABE \equiv \Delta CDE$.

b Explain why BD and AC bisect each other.

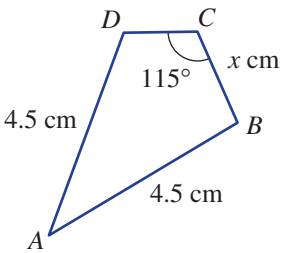
10G **13** This quadrilateral is a square with $AE = EC$.

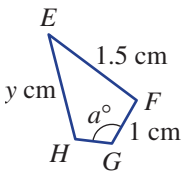


a Prove that $\Delta ABE \equiv \Delta CBE$.

b Explain why the diagonals intersect at right angles.

10H Ext **14** For these similar shapes complete the following tasks.





a List the pairs of corresponding sides.

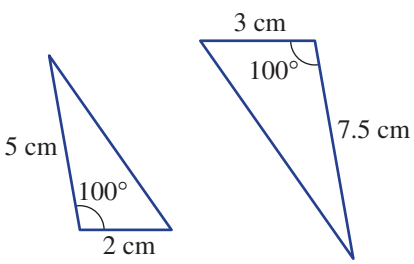
b List the pairs of corresponding angles.

c Find the scale factor.

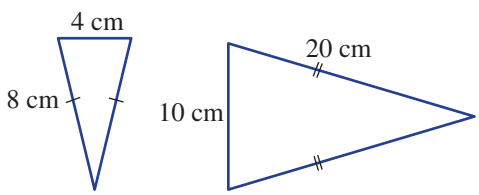
d Find the value of the pronumerals.

10I Ext **15** Decide, with reasons, if these pairs of triangles are similar. State which of tests AAA, SSS, SAS or RHS is used.

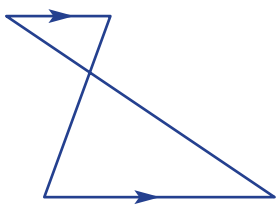
a



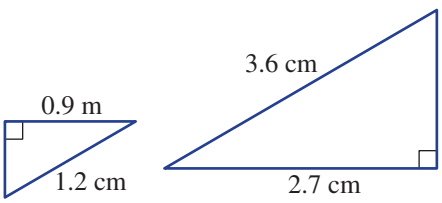
b



c



d



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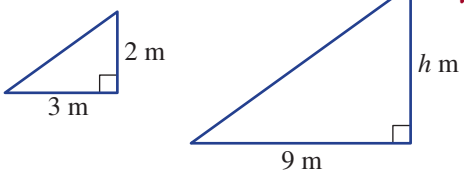
Chapter review

10I

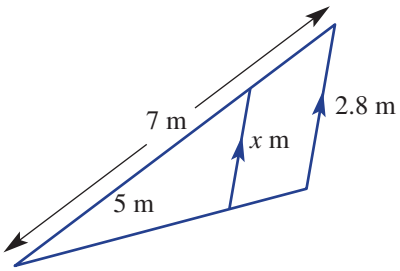
Ext

16 Given that the triangles are similar, find the values of the pronumerals.

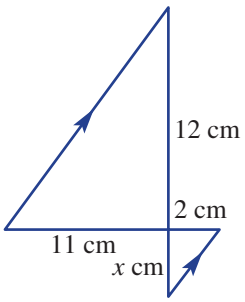
a



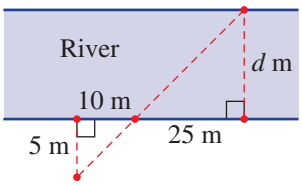
b



c

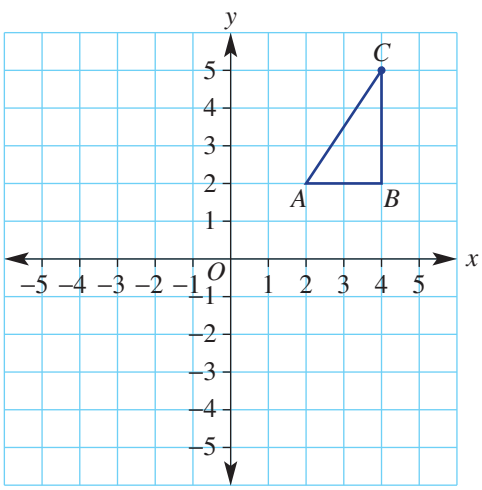


d



Extended-response questions

1 The shape on this set of axes is to be transformed by a succession of transformations. The image of the first transformation is used to start the next transformation. For each set of transformations, write down the coordinates of the vertices A', B' and C' of the final image.



Parts **a**, **b** and **c** are to be treated as separate questions.

a Set 1

- **i** Reflection in the x-axis.
- **ii** Translation by the vector $(-2, 1)$.
- **iii** Rotation about $(0, 0)$ by 180°.

b Set 2

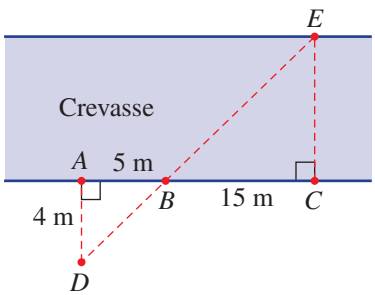
- **i** Rotation about $(0, 0)$ clockwise by 90°.
- **ii** Reflection in the y-axis.
- **iii** Translation by the vector $(5, 3)$.

c Set 3

- **i** Reflection in the line $y = -x$.
- **ii** Translation by the vector $(4, 2)$.
- **iii** Rotation about $(1, 0)$ by 90° clockwise.

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2 Three explorers come to a deep ice crevasse and wonder if they can jump across it. They notice a mound of snow on the other side of the crevasse (E) and decided to build 4 other mounds (A, B, C and D) on their side as shown.



a Why do you think the explorers built mounds B and D to be in line with the mound on the other side of the crevasse?

b What reasons are there to explain why the two triangles are similar?

c What is the scale factor?

d Can you help the explorers find the distance across the crevasse? What is the distance?



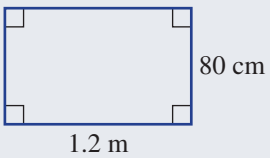
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Semester review 2

Ratios and rates

Multiple-choice questions

1 The ratio of the length to the width of this rectangle is:



A 12 : 80 **B** 3 : 20 **C** 3 : 2
D 20 : 3

2 Simplify the ratio 500 cm to $\frac{3}{4}$ km.

A 2 : 3 **B** 1 : 150 **C** 3 : 2 **D** 150 : 1

Ext

3 $18 is divided in the ratio 2 : 3. The larger part is:

A $3.60 **B** $7.20 **C** $10.80 **D** $12

4 Calvin spent $3 on his mobile phone card for every $4 he spent on his email account. Calvin spent $420 on his phone last year. How much did he spend on his email account for the same year?

A $140 **B** $315 **C** $560 **D** $240

Ext

5 A boat sailed 30 kilometres in 90 minutes. What was the average speed of the boat?

A 15 km/h **B** 45 km/h **C** 3 km/h **D** 20 km/h

Short-answer questions

1 Simplify these ratios.

a 24 to 36 **b** 15 : 30 : 45 **c** 0.6 m to 70 cm

d 15 cents to $2 **e** $\frac{3}{4}$ to 2 **f** 60 cm to 2 m

Ext

2 **a** Divide 960 cm in the ratio of 3 : 2. **b** Divide $4000 in the ratio of 3 : 5.
c Divide $8 in the ratio of 2 : 5 : 3.

3 A 20-metre length of wire is used to fence a rectangular field with dimensions in the ratio 3 : 1. Calculate the area of the field.

4 A business has a ratio of profit to costs of 5 : 8. If the costs were $12 400, how much profit was made?

5 Complete these rates.

a 5 g/min = _____ g/h **b** $240 in 8 hours = $_____/h

c 450 km in $4\frac{1}{2}$ h = _____ km/h

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6 A shop sells $1\frac{1}{2}$ kg bags of apples for $3.40. Find the cost of a one kilogram at this rate.

Ext 7 A car travels the 1035 km from southern Sydney to Melbourne in 11.5 hours. Calculate its average speed.

Extended-response question

A small car uses 30 litres of petrol to travel 495 km.

a At this rate, what is the maximum distance a small car can travel on 45 litres of petrol?

b What is the average distance travelled per litre?

c Find the number of litres used to travel 100 km, correct to one decimal place.

d Petrol costs 117.9 cents/litre. Find the cost of petrol for the 495 km trip.

Ext e A larger car uses 42 litres of petrol to travel 378 km. The smaller car holds 36 litres of petrol while the larger car holds 68 litres. How much further can the larger car travel on a full tank of petrol?

Equations and inequalities

Multiple-choice questions

1 The sum of a number and three is doubled. The result is 12. This can be written as:

A $x + 3 \times 2 = 12$ B $2x + 6 = 12$ C $2x + 3 = 12$ D $x + 3 = 24$

2 The solution to the equation $2m - 4 = 48$ is:

A $m = 8$ B $m = 22$ C $m = 20$ D $m = 26$

3 The solution to the equation $-5(m - 4) = 30$ is:

A $m = -10$ B $m = -2$ C $m = 2$ D $m = \frac{-34}{5}$

Ext 4 If $\Delta < -5$, then Δ can have the value:

A 0 B $-\frac{1}{5}$ C -6 D -4

Ext 5 Which of the following inequalities is represented by the number line below?



A $x \leq -1$ B $x \geq -1$ C $x > -1$ D $x < -1$

Short-answer questions

1 Solve each of these equations.

a $3w = 27$ b $12 = \frac{m}{6}$ c $4 - x = 3$

d $4a + 2 = 10$ e $2w + 6 = 0$ f $\frac{x}{5} - 1 = 6$

2 Solve each of these equations.

a $6 = 4 - 4m$ b $3a + 4 = 7a + 8$ c $3(x + 5) = 15$

d $\frac{x}{5} + \frac{x}{3} = 1$ e $2(5 - a) = 3(2a - 1)$ f $\frac{a + 7}{2a} = 3$

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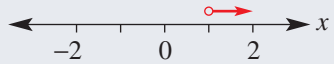
3 Double a number less three is the same as 9. What is the number?

4 A father is currently six times as old as his son. In 10 years' time his son will be 20 years younger than his dad. How old is the son now?

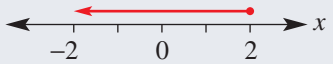
Ext

5 Write the inequality shown by each graph below.

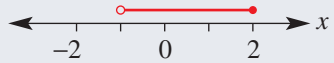
a



b



c



Ext

6 Solve each of these equations.

a $2x > -16$ **b** $3x + 8 \leq 17$ **c** $\frac{x}{5} - 6 \leq 0$

Extended-response question

EM Publishing has fixed costs of $1500 and production costs of $5 per book. Each book has a retail price of $17.

a Write an equation for the cost (C) of producing n books.

b Write an equation for the revenue (R) for selling n books.

c A company 'breaks even' when the cost of production equals the revenue from sales. How many books must the company sell to break even?

d Write an equation for the profit (P) of selling n books.

e Calculate the profit if 200 books are sold.

f What is the profit if 100 books are sold? Explain the significance of this answer.

Probability and statistics

Multiple-choice questions

1 For the set of numbers 3, 2, 1, 3, 5, 1, 3, 9, 3, 5 the mode is:

A 3 **B** 3.5 **C** 8 **D** 35

2 Consider the data 8, 9, 10, 10, 16, 19, 20, 20. Which of the following statements is true?

A Median = 13 **B** Mean = 13 **C** Mode = 13 **D** Range = 13

3 In a box there are 75 blue marbles and 25 pink marbles of the same size. If 12 marbles are drawn from the box, one at a time, at random and replaced each time, the most likely number of pink marbles is:

A 6 **B** 3 **C** 12 **D** 9

Ext

4 A coin and a six-sided die are tossed together. The number of elements in the sample space is:

A 2 **B** 6 **C** 12 **D** 8

5 A die is rolled 60 times. The number 4 appears exactly 24 times. The experimental probability of obtaining the number 4 is:

A 0 **B** $\frac{2}{5}$ **C** $\frac{2}{3}$ **D** $\frac{1}{6}$

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Short-answer questions

1 Find **i** the mean, **ii** the median and Ext **iii** the range of these data sets.

a 10, 15, 11, 14, 14, 16, 18, 12 **b** 1, 8, 7, 29, 36, 57

c 1.5, 6, 17.2, 16.4, 8.5, 10.4

2 For the data set 1, 1, 3, 5, 8, 8, 9, 10, 12, 17, 24, 30, find the:

a median score **b** lower quartile Ext **c** upper quartile Ext

d interquartile range Ext

3 The mean mark for a Chemistry quiz for a class of 20 students was 16 out of 20. On the same quiz, another class of 15 students had a mean of 18 out of 20. What is the combined mean of all the students? Give your answer correct to one decimal place.

4 Calculate the mean and median for:

a 1, 2, 5, 10, 10, 13, 13, 18, 19, 24, 25, 28, 28, 30, 35

b

Score	Frequency
10	15
11	29
12	11
13	5

5 Draw a histogram for the frequency table in Question **4b**.

Ext 6 A six-sided die and a coin are tossed together. Write down all the outcomes using a table.

7 A bag contains 16 balls of equal size and shape. Of these balls 7 are yellow, 1 is blue and the rest are black. If one ball is chosen from the bag at random, find the probability that it is:

a yellow **b** blue **c** not blue

d black **e** pink

8 The ages of 50 people at a party are shown in the table below.

Ages	0–9	10–19	20–29	30–39	40–49	50–59	60+
Frequency	3	7	1	28	6	2	3

If one person is chosen at random to a prepare a speech, find the probability that the person is aged:

a 0–9 **b** 30 or older **c** in their twenties

d not in their fifties

Extended-response question

Two groups of students have their pulse rates recorded as beats per minute. The results are listed here:

Group A: 65, 70, 82, 81, 67, 74, ☐, 81, 88, 84, 72, 65, 66, 81, 72, 68, 86, 86

Group B: 83, 88, 78, 60, 81, 89, 91, 76, 78, 72, 86, 80, 64, 77, 62, 74, 87, 78

a How many students are in group B?

b If the median pulse rate for group A is 76, what number belongs in the ☐?

c What is the median pulse rate for group B?

Ext **d** Which group has the largest interquartile range?

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Straight line graphs

Multiple-choice questions

1 The point $(2, -1)$ lies on the line with equation:

A $y = 3x - 1$ **B** $y = 3 - x$ **C** $x + y = 3$ **D** $y = 1 - x$

2 The coordinates of the point 3 units directly above the origin is:

A $(0, 0)$ **B** $(0, 3)$ **C** $(0, -3)$ **D** $(3, 0)$

3 The rule for the table of values shown is:

x	0	2	4
y	4	8	12

A $y = 2x$ **B** $y = 2x + 2$ **C** $y = 2(x + 2)$
D $y = x + 4$

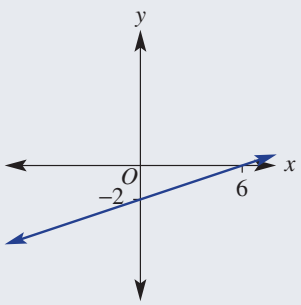
Ext

4 The gradient of the line through $A(4, 7)$ and $B(8, -1)$ is:

A $-\frac{1}{2}$ **B** 2 **C** $\frac{1}{2}$ **D** -2

Ext

5 The equation of the line shown is:



A $y = 6x - 2$
B $y = 3x - 2$
C $y = x - 2$
D $3y = x - 6$

Short-answer questions

1 In which quadrant does each point lie?

a $(5, 1)$ **b** $(-3, 4)$ **c** $(-5, -1)$ **d** $(8, -3)$

2 **a** Complete these tables of values.

i $y = 2x + 1$

x	0	1	2	3
y				

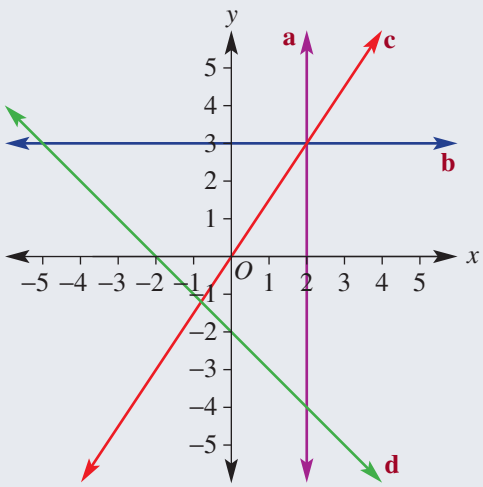
ii $y = 4 - x$

x	0	1	2	3
y				

b Sketch each line on the same number plane and state the point of intersection of the two lines.

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3 Give the equation of each line shown on this grid.



Ext **4** Consider the Cartesian plane shown in Question **3**.

a Which line(s) has/have:

i a positive gradient? **ii** a negative gradient?

iii a zero gradient? **iv** a gradient of 1.5?

b Which lines intersect at the point $(2, -4)$?

Ext **5** Sketch the curves $y = x^2$ and $y = 2x$ on the same number plane, by first completing a table of values with x from -2 to 3. Write down the point of intersection of the two graphs in the first quadrant.

Extended-response question

The cost ($\$C$) of running a coffee shop is given by the rule $C = 400 + 5n$, where n is the number of customers on any given day. On average, a customer spends \$13.

a Write a rule for the coffee shop's expected daily revenue (income from customers).

b What is the fixed cost for the coffee shop on any given day? What could this be?

c Show the graphs of the rules for the cost and the revenue on the same set of axes by first completing two tables of values for $n = 0, 5, 10, 15, \ldots, 60$.

d What is the 'break even' point for the coffee shop?

e If they are particularly busy on a Saturday, and serve 100 people, calculate the shop's profit.

Transformation and congruence

Multiple-choice questions

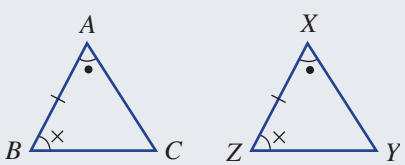
1 The number of lines of symmetry in a square is/are:

A 0 **B** 2 **C** 4 **D** 6

2 The side AC corresponds to:

A XZ **B** XY

C ZY **D** BC

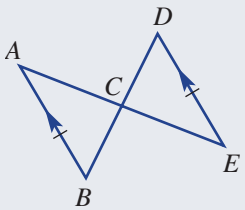


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3 A congruence statement for these triangles is:

A $\Delta ABC \equiv \Delta CED$ **B** $\Delta ABC \equiv \Delta CDE$

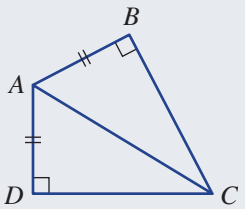
C $\Delta ABC \equiv \Delta ECD$ **D** $\Delta ABC \equiv \Delta EDC$



4 Which test is used to show triangle ABC congruent to triangle ADC?

A SSS **B** SAS

C AAS **D** RHS



5 Which of the following codes is not enough to prove congruence for triangles?

A SSS **B** AAS **C** AAA **D** SAS

Short-answer questions

1 How many lines of symmetry does each of these shapes have?

a Scalene triangle **b** Rhombus **c** Rectangle **d** Semicircle

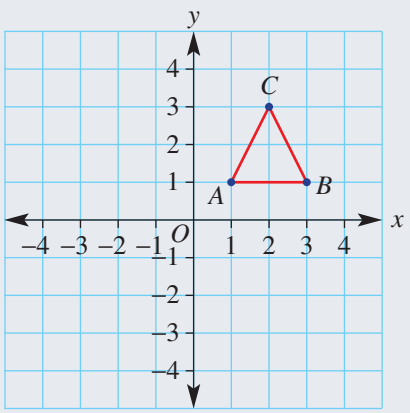
2 Write the vectors that translate each point P to its image P'.

a $P(1, 1)$ to $P'(3, 3)$ **b** $P(-1, 4)$ to $P'(-2, 2)$

3 Triangle ABC is on a Cartesian plane as shown. List the coordinates of the image points A', B' and C' after:

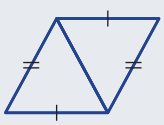
a a rotation 90° clockwise about (0, 0)

b a rotation 180° about (0, 0)

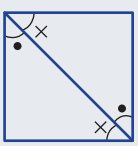


4 Write the congruency test that would be used to prove that the following pairs of triangles are congruent.

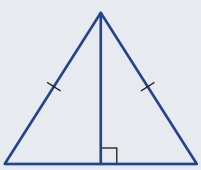
a



b

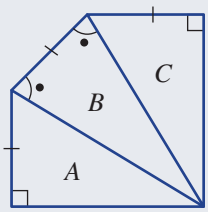


c



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5 Which two triangles are congruent?

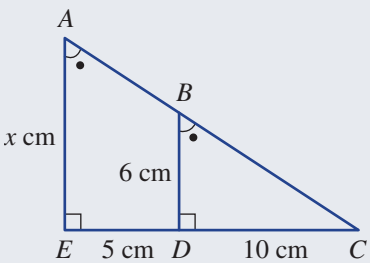


Ext

6 **a** Which two shapes in this diagram are similar and why?

b Which vertex in ΔAEC corresponds to vertex C in ΔBDC?

c Find the value of x.



Extended-response question

Ext

A circle of radius 3 cm is enlarged so that the ratio of old radius to new radius is 2 : 5.

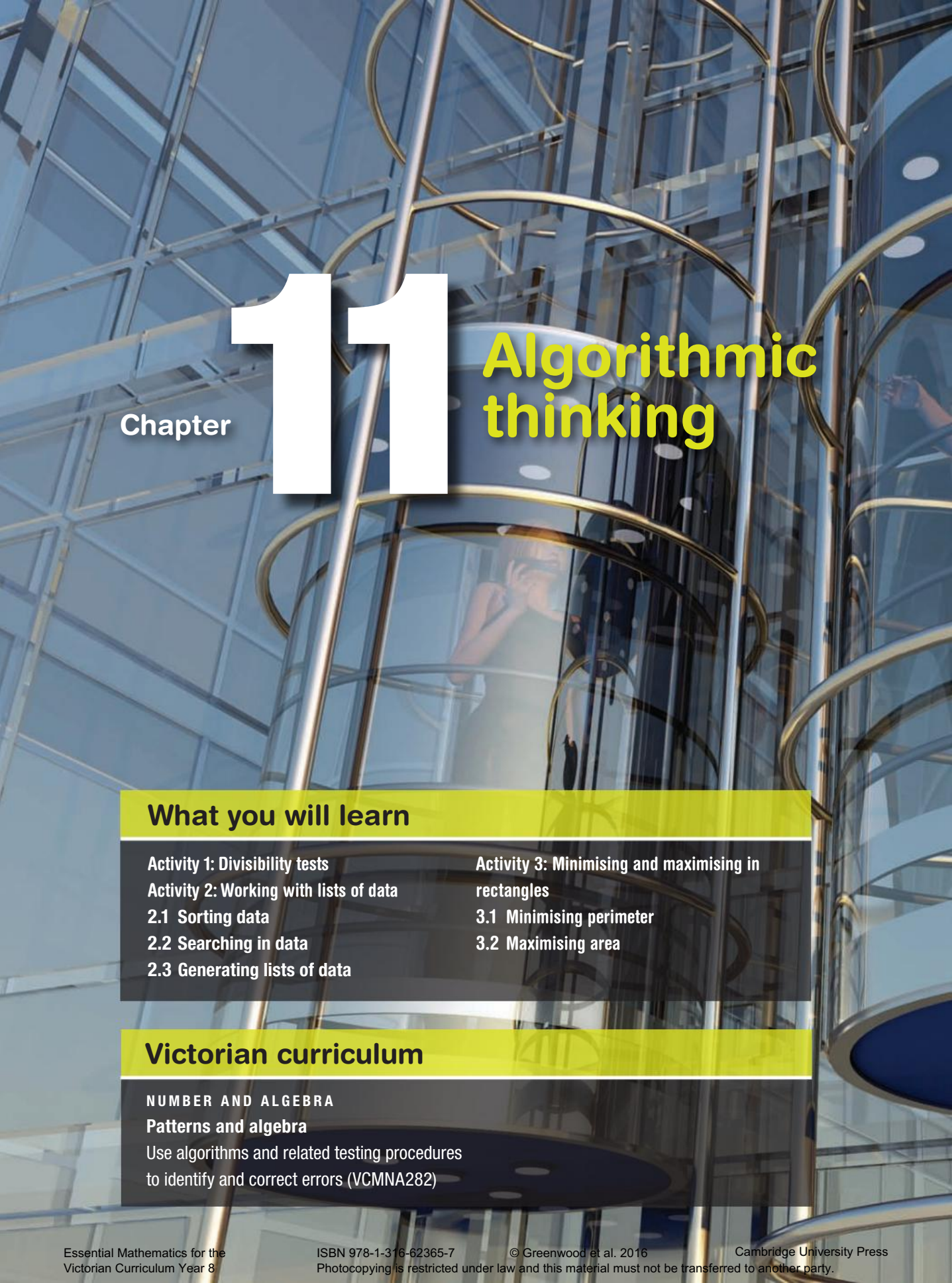
a What is the diameter of the enlarged image?

b What is the ratio of the circumference of the original circle to its image?

c What is the ratio of their areas?

d If the ratio of a circle's area to its image becomes 9 : 100, what is the radius of the image?

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Chapter 11

Algorithmic thinking

What you will learn

Activity 1: Divisibility tests
Activity 2: Working with lists of data
2.1 Sorting data
2.2 Searching in data
2.3 Generating lists of data
Activity 3: Minimising and maximising in rectangles
3.1 Minimising perimeter
3.2 Maximising area

Victorian curriculum

NUMBER AND ALGEBRA
Patterns and algebra
Use algorithms and related testing procedures to identify and correct errors (VCMNA282)

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Algorithms and elevators

The electronic controller for a building lift runs an algorithm which delivers people to their destination based on which buttons are pressed both inside the lift car and in lift waiting areas. The algorithm must accept the requests (inputs: which buttons are pressed) and deliver a result (output: the action of the lift car) so that passengers travel to where they want to go in an efficient manner. A basic algorithm for a lift is as follows:

- Continue travelling in the same direction if at least one request exists in that direction
- Remain stationary until new requests are made in either direction.

For multi-elevator systems, more complex algorithms are used. Requests are analysed and grouped so that requests for nearby building levels are serviced by one lift car, while other cars are left to service other levels. If lift waiting areas have two buttons (up or down) then this delivers more input to the algorithm. The algorithm can assess the direction of the request as well as the floor that the request was made. Further programming enhancements might cater for when a lift, stopping at multiple floors in one direction, is likely to be full. This may be the case early in the morning when many people head to work at a similar time.

An efficient lift algorithm will reduce waiting time for passengers and minimise wear and tear on the mechanics of the lift.

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Introduction

An **algorithm** is a procedure involving a number of steps that eventually leads to the answer of a problem. There should be clearly defined inputs and outputs.

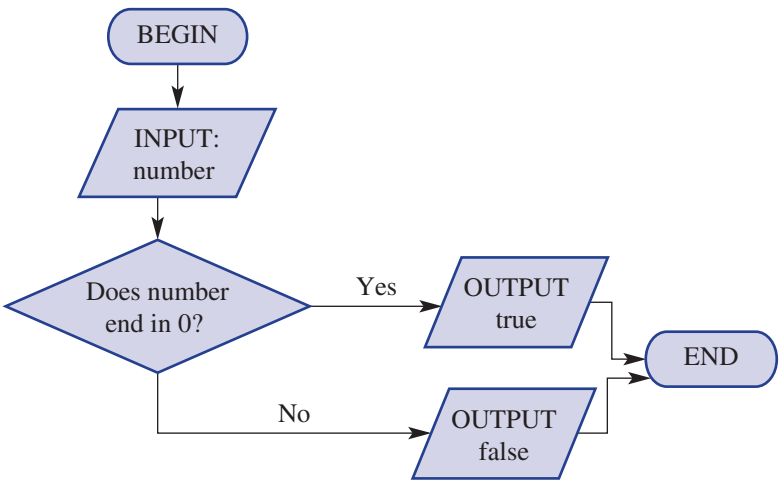
For example:

- The highest common factor (HCF) algorithm has two inputs like 36 and 54, and gives the output 18.
- The fraction division algorithm has two inputs like $\frac{2}{5}$ and $\frac{3}{4}$ and gives the output $\frac{8}{15}$.

Activity 1: Divisibility tests

Number and Algebra

a The flow chart below takes a number and tells you whether it is divisible by 10 or not.



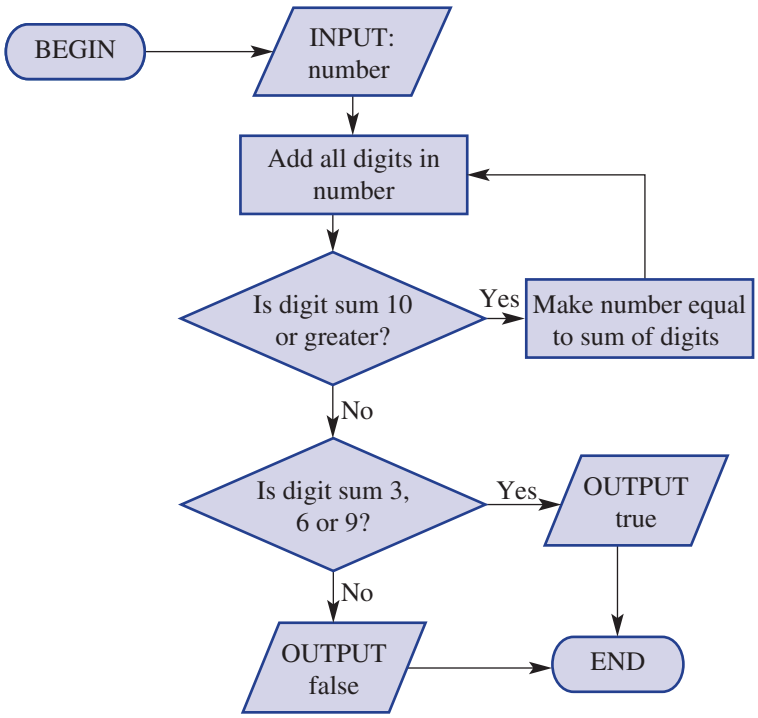
i What changes are required to the original flow chart to test if a number is divisible by 5?

ii What changes are required to the original flow chart to test if a number is divisible by 2?

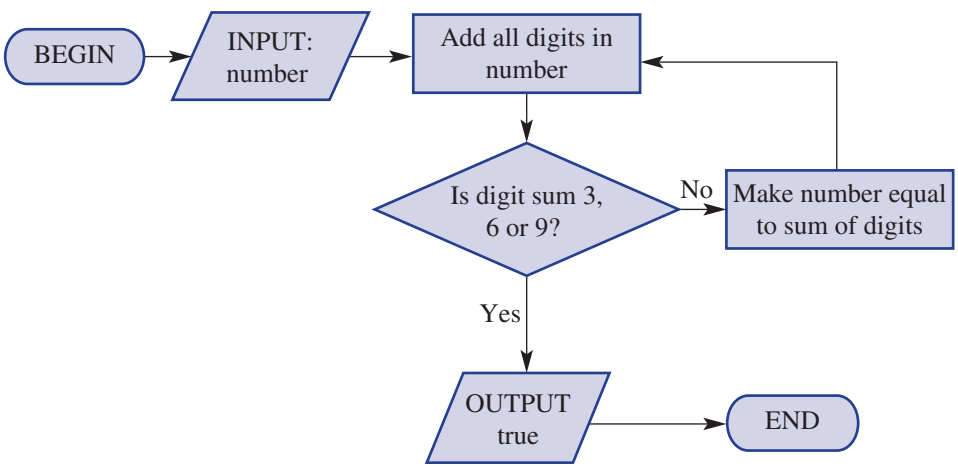
iii Jonty has attempted to test if a number is divisible by 4, so he has changed the conditional step to the following test 'Does number end in 0, 4 or 8?' Explain what is wrong with this, giving examples of numbers that it incorrectly classifies as divisible by 4 and numbers that it incorrectly classifies as not divisible by 4.

b i Testing for divisibility by 3 involves looping. Describe what happens if the input number is 831 in the flow chart below.

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ii Teale has simplified the flow chart to remove the test for numbers 10 or greater. This algorithm works on some numbers (for instance, 195) but not on other numbers (for instance, 196). Explain clearly what is wrong with it and why it does not always work.



iii Draw a flow chart for the algorithm that tests to see if a number is divisible by 9. Recall that divisibility by 9 involves adding up the digits to see if the sum is divisible by 9. Remember to make sure that your flow chart will not loop around forever.

c You can assume now you have algorithms to test if a number is divisible by the numbers 2, 3, 5, 9 and 10.

i Describe how you could combine these algorithms to test if a number is divisible by 6.

ii Describe how you could combine these algorithms to test if a number is divisible by 45.

iii Describe how you could combine these algorithms to test if a number is divisible by 9 but not by 18.

iv What other numbers can you test for divisibility by combining these algorithms?

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Activity 2: Working with lists of data
Probability and Statistics

2.1 Sorting data

In order to find the median or interquartile range of a list of numbers we first need to sort the list in ascending order.

Sample input: [12, 1, 3, 8, 2, 7]

Sample output: [1, 2, 3, 7, 8, 12]

One algorithm, called 'Selection Sort' is as follows:

Step 1. Make a new (empty) output list.
Step 2. Find the smallest number in the input list.
Step 3. Remove it from the input list and put it at the end of the output list.
Step 4. Repeat steps 2 and 3 until the input list is empty.

In a high-level programming language, it might look something like the following.

Line	Command	Explanation
1	`output ← []`	Set the output list to be empty
2	`loop:`	Perform lines 3 to 6 forever (until you hit `break`)
3	`  current ← min(input)`	Find the minimum value in the input list
4	`  input.remove(current)`	Remove one copy of current minimum from input
5	`  output.append(current)`	Put current minimum at the end of the output list
6	`  if input = []: break`	If we reach an empty input list, we leave this loop
7	`display output`	At the end we need to display the output

A computer can run this program line by line. The execution is shown below on the input list [7, 1, 5] and takes 15 steps.

Line	Command	Result
1	`output ← []`	`output` is set to the be the empty list
2	`loop:`	Get ready to repeat lines 3 to 6
3	`current ← min(input)`	`current` is now 1
4	`input.remove(current)`	`input` is now [7, 5]
5	`output.append(current)`	`output` is now [1]
6	`if input = []: break`	No effect, because `input` is not empty.
3	`current ← min(input)`	`current` is now 5
4	`input.remove(current)`	`input` is now [7]
5	`output.append(current)`	`output` is now [1, 5]
6	`if input = []: break`	No effect, because `input` is not empty.
3	`current ← min(input)`	`current` is now 7
4	`input.remove(current)`	`input` is now [ ]
5	`output.append(current)`	`output` is now [1, 5, 7]
6	`if input = []: break`	Exit the loop, because `input` is empty.
7	`display output`	Displays the list `output` which is [1, 5, 7]

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a Show the step-by-step result of running the program on the input list [5, 8, 2]. Your results should be shown in a table like the one below.

Line	Result
1	`output` is [ ]
2	Begin loop
3	`current` is 2
4	`input` is [5, 8]
5	`output` is [2]
6	No action
3	`current` is 5
…	…

b Estimate how many execution steps would be required on an input list with 100 numbers in it.

c A 'bug' in a program or algorithm is a step that means the algorithm no longer gives the desired results. For each of the following bugs, explain the effect of it by first demonstrating what the algorithm will do with a sample input list like [5, 8, 2].

i Describe the effect of changing the algorithm so that it puts the current minimum at the *start* of the output list rather than at the end.

ii Describe the effect of changing the algorithm so that it does not remove the current minimum from the input list.

iii Describe the effect of changing the variable `input` in line 6 to `output`.

2.2 Searching in data

In a set of data, it is sometimes necessary to search through for a particular value.

a Consider the following algorithm to search for the number 100 in the input list.

Step 1. Look at the first item in the list.
Step 2. If it is 100 then output the value 'Found'.
Step 3. Look at the next item in the list.
Step 4. Repeat steps 2 and 3 until you get to the end of the list or until you find 100.
Step 5. Output 'Not Found' if you have not found the number 100.

i If the input list is [3, 9, 100, 2, 17, 52], describe what this algorithm does.

ii If the input list is [2, 6, 1, 7, 3, 5], describe what this algorithm does.

iii What will be the output if the input list is [100, 3, 50, 100, 75]?

b With a large set of data, it is usually helpful to have the data sorted first in ascending order. For instance, a dictionary or a large phone list is sorted alphabetically to allow people to find values quickly. Page numbers in a book are also sorted, which can help you find a page easily. Describe how you could quickly find the page 381 in a book that was 1000 pages long.

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c Here is an algorithm to find page 381 in a book.

Line	Command	Explanation
1	`lower ← 0`	The page must be greater than 0
2	`upper ← 1000`	The page is at most 1000
3	`loop:`	Performs lines 4 to 7 indefinitely (until you hit `break`)
4	`  page = round((lower + upper) / 2)`	Find the middle page in the range
5	`  if page = 381:  break`	If we find the page, leave the loop
6	`  if page > 381:  upper = page`	If we are too high then this becomes the new upper bound
7	`  if page < 381:  lower = page`	If we are too low then this becomes the new lower bound
8	`display contents of page`	Outside the loop, display the contents of the page we found

i Construct a table like the one below to show how the bounds change each loop.

Loop number	lower	upper
0	0	1000
1	0	500
2	250	500
…	…	…

ii Describe how this algorithm performs compared to an algorithm that looks at each page from 1 to 381. You should include a comparison of roughly how many steps each takes.

d Emily has written an algorithm to search for page 210 in a 400 page book.

Line	Command
1	`lower ← 0`
2	`upper ← 400`
3	`loop:`
4	`  item = round((lower + upper) / 2)`
5	`  if page = 210:  break`
6	`  if page < 210:  upper = page`
7	`  if page > 210:  lower = page`
8	`display contents of page`

i Construct a table like the one below to show how the bounds change each loop.

Loop number	lower	upper
0	0	400
…	…	…

ii Explain what the error is in Emily's algorithm and how to fix it.

2.3 Generating lists of data (using simulation)

When investigating the probability of an event occurring, it is often helpful to use a simulation. For example, if two dice are rolled, we might want to know the probability of obtaining a sum of 10. An algorithm to do this is:

Step 1. Roll two dice 500 times.

Step 2. For each pair of dice, find the sum of the faces.

Step 3. Count how many times the number 10 occurs.

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Step 4. Divide this by 500 to estimate the probability.

This algorithm could be implemented in programming code like the following.

Line	Command
1	`trials ← 0`
2	`successes ← 0`
3	`loop:`
4	`  trials ← trials + 1`
5	`  die1 ← randint(1,6)`
6	`  die2 ← randint(1,6)`
7	`  if die1 + die2 = 10:  successes ← successes + 1`
8	`  if trials = 500:  break`
9	`display successes / trials`

An alternative way to explore the experimental probability is with a spreadsheet.

a Set up a table in a spreadsheet to randomly generate 500 outcomes for tossing two dice like the one below.

	A	B	C	D	E
1	Rolling two dice				
2					
3	Die 1	Die 2	Sum	Possible Sum	Count
4	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A4+B4	2	=COUNTIF(C$4:C$503,D4)
5	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B5	3	=COUNTIF(C$4:C$503,D5)
6	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B6	4	=COUNTIF(C$4:C$503,D6)
7	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B7	5	=COUNTIF(C$4:C$503,D7)
8	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B8	6	=COUNTIF(C$4:C$503,D8)
9	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B9	7	=COUNTIF(C$4:C$503,D9)
10	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B10	8	=COUNTIF(C$4:C$503,D10)
11	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B11	9	=COUNTIF(C$4:C$503,D11)
12	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B12	10	=COUNTIF(C$4:C$503,D12)
13	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B13	11	=COUNTIF(C$4:C$503,D13)
14	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B14	12	=COUNTIF(C$4:C$503,D14)
15	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B15		

b Using spreadsheet software, plot the number of times each sum from 2 to 12 is achieved as a histogram or 2D column graph. Comment on the shape of the graph that is produced.

c Comment on the number of times different sums occurred. For instance, which sums are most likely and which sums are least likely? Why do you think this is?

d Add a column in to calculate the experimental probability by dividing the count by the number of trials. To count the number of trials you could use a formula like =COUNT(A:A)

e Describe an algorithm that you could use to find the theoretical probability of two dice adding up to 10.

f (Extension) Modify your spreadsheet to conduct a large simulation (1000 or more rows) of rolling three dice and noting the sums; it should show the experimental probability of each sum from 3 to 18. Use the spreadsheet software to generate a frequency column graph of these experimental probabilities and comment on what you notice about these probabilities.

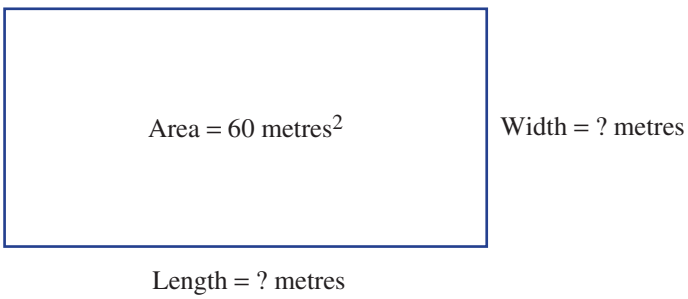
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Activity 3: Minimising and maximising in rectangles

Measurement and Geometry

3.1 Minimising perimeter

If a rectangle has an area of 60 metres^2, what is the smallest perimeter it can have?



One algorithm that could be used to solve this problem is:

Step 1. Guess a value for the width.
Step 2. Calculate the length by dividing 60 by the width.
Step 3. Calculate the perimeter of this rectangle ($2 \times$ width $+ 2 \times$ length).
Step 4. If this perimeter is smaller than the best answer found so far (or if this is the first time completing Step 4), make this the new 'best answer'.
Step 5. Reduce or increase the value for width by a small amount and repeat. When the perimeter cannot get smaller when you change the value of width in either direction, stop looping and give this as the minimum perimeter.

a Copy and complete the following table that reflects a few iterations of this algorithm.

Width	Length	Perimeter
20	3	46
10	6	
2		
5		
7.5		

Because a spreadsheet includes dynamic and interactive calculation abilities, it is a good tool to implement this algorithm.

b Fill out the following in a spreadsheet, using the initial guess of the width of 20. Change the width value to check your answer(s) in **a**.

	A	B	C
1	Width	Length	Perimeter
2	20	=60/A2	=A2*2+B2*2

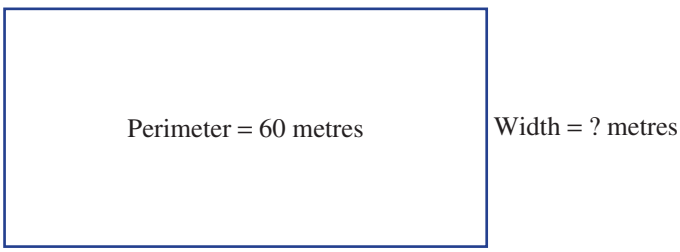
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c Using trial and error, find the minimum perimeter. Draw a picture of this rectangle roughly to scale. (Remember you are allowed to have a width and length that are decimals; keep going until you get the minimum possible perimeter.)

d *Extension.* Design another algorithm that takes as input a value for the area (like 60) and outputs the minimum perimeter without performing any loops/trial and error. Check that your algorithm gives the same result as **c**. above when the input is 60, and explain why it works in general.

3.2 Maximising area

If a rectangle has a perimeter of 60 metres, what is the maximum area it can have?



a **i** If the width is 20 metres, what is the length of the rectangle? Remember that the perimeter includes two lots of the length and two lots of the width.

ii Describe an algorithm for calculating the length of the rectangle once its width is known.

iii Describe an algorithm for calculating the area of the rectangle once its width is known.

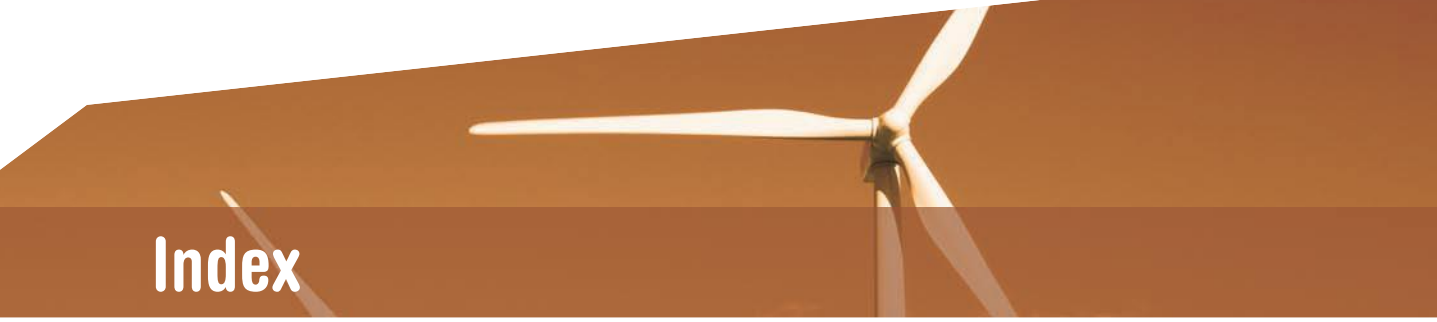
b Set up a spreadsheet as follows. What is the formula that needs to go in E2 to calculate the rectangle's area?

	A	B	C	D	E
1	Width	Double Width	Double Length	Length	Area
2	20	=A2*2	=60-B2	=C2/2	????

c Using trial and error, find the maximum value of the area of a rectangle with perimeter 60 metres. Draw a diagram of the rectangle that is formed.

d *Extension.* Design another algorithm that takes as input a value for the perimeter (like 60) and outputs the maximum area without performing any loops/trial and error. Check that your algorithm gives the same result as **c** above, and explain why it works in general.

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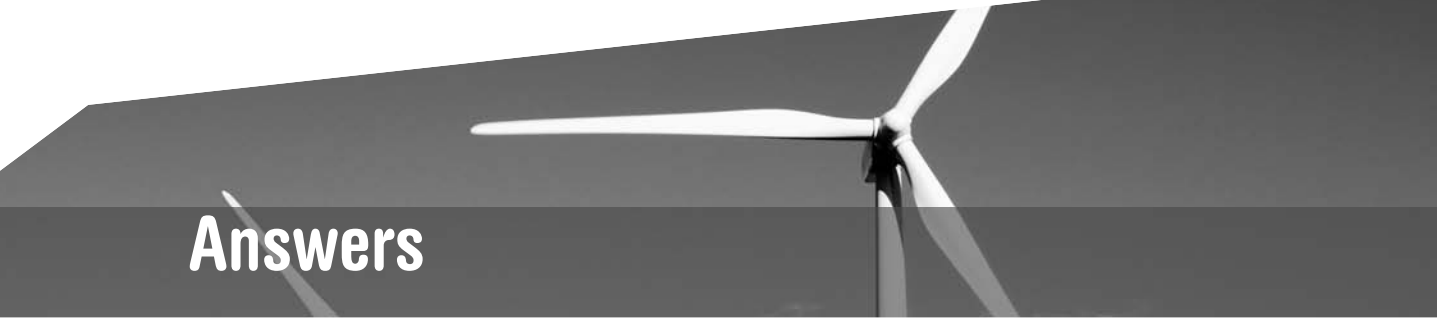
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Answers

Working with unfamiliar problems: Part 1

1 5050
2 almost 20
3 1 111 111 101
4 12 144
5 $245 \times 13 = 3185$
6

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

7 10 cm, 6.25 cm^2
8 5070
9 Answers vary
10 1058 cm^2
11 700
12 16 896
13 A square of side 24 cm
14 66
15 2h 5 min

Working with unfamiliar problems: Part 2

1 5 242 880
2 F = 2, O = 9, U = 7, R = 0, I = 1, V = 8, N = 5, E = 4
i.e. 2970 + 2184 = 5154
3 $\frac{5}{16}$
4 **a** 1868 **b** 1565 **c** 2454, 3666 **d** 2151, 3969
5 34%
6 side 16 cm, 8 blocks left over
7 $10a - 15, 5(2a - 3)$
8 22.5 cm
9 **a** $(1) + (3 + 5) + (7 + 9 + 11) + (13 + 15 + 17 + 19)$
$= 1^3 + 2^3 + 3^3 + 4^3$
b $1 = 1^2$
$1 + 3 = 2^2$
$1 + 3 + 5 = 3^2$
$1 + 3 + 5 + 7 = 4^2$
$1 + 3 + 5 + 7 + 9 = 5^2$
c 28
d **i** 28 **ii** 1275
10 **a** 15, 17, 21, 23, 27, 29 **b** 19, 25, 31, 37, 43, 49
c Dominic **d** Beau **e** 206 numbers

Chapter 1

Exercise 1A

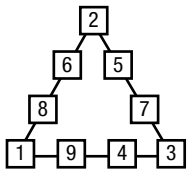
1 **a** 43 **b** 34 **c** 111 **d** 501
e 48 **f** 347 **g** 16 **h** 41
i 44 **j** 563 **k** 17 **l** 131
2 **a** 2 **b** 7 **c** 0 **d** 6
e 9 **f** 9 **g** 0 **h** 6
3 **a** 32 **b** 387 **c** 1143 **d** 55
e 163 **f** 216 **g** 79 **h** 391
i 701 **j** 229 **k** 39 **l** 161
4 **a** 174 **b** 431 **c** 10 362 **d** 2579
e 58 **f** 217 **g** 27 **h** 13 744
i 888 **j** 23 021 **k** 75 **l** 9088
5 **a** \$5 **b** \$8 **c** \$11 **d** \$6
e \$19 **f** \$3
6 **a** 110 **b** 20 **c** 2300 **d** 1800
e 2 **f** 43 000
7 678 km
8 22
9 Answers given from top row down and from left to right.
a 7, 3, 3 **b** 1, 7, 8 **c** 2, 5, 3 **d** 9, 4
e 4, 2, 8 **f** 0, 0, 7, 1
10 43 marbles
11 **a** 100 **b** 50
12 **a** The sum of two 3-digit numbers cannot be bigger than 1998.
b Subtracting 32_ from 3_6 will give a maximum of 76 (396 – 320).
13 **a** $x + y + z = z + x + y$
b $x - y + z = z - y + x = x + z - y$

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14 a The third number is always 3 and others are (9, 3), (8, 4), (7, 5), (6, 6), (5, 7), (4, 8), (3, 9), giving 7 combinations. A 1 has to be carried from the middle column.

b The second number is always 7 and others are (0, 6), (1, 7), (2, 8), (3, 9), giving 4 combinations. A 1 has to be used from the left column.

15 a



b 5 totals, 17, 19, 20, 21 and 23

c 17 in 2 ways, 19 in 4 ways, 20 in 6 ways, 21 in 4 ways and 23 in 2 ways

Exercise 1B

1 a 56 **b** 1300 **c** 1
d 5 **e** 3 **f** 2

2 a 40 **b** 99 **c** 42 **d** 72
e 66 **f** 132 **g** 32 **h** 63
i 10 **j** 11 **k** 11 **l** 12
m 8 **n** 11 **o** 13 **p** 13

3 a true **b** true **c** false **d** true
e false **f** true **g** true **h** true
i true **j** false

4 a 130 **b** 260 **c** 140 **d** 68
e 17 000 **f** 13 600 **g** 413 **h** 714
i 459 **j** 366 **k** 1008 **l** 5988
m 16 **n** 63 **o** 41 **p** 127
q 16 **r** 127 **s** 420 **t** 38

5 a 603 **b** 516 **c** 3822 **d** 90 360
e 9660 **f** 413 090 **g** 34 194 **h** 344 223

6 a 28 rem 1 **b** 30 rem 4 **c** 416 rem 7 **d** 13 rem 0
e 13 rem 12 **f** 166 rem 8 **g** 7 rem 0 **h** 1054 rem 16

7 a $15 **b** $70 **c** $400 **d** $5
e $24 **f** $50

8 $25

9 2358 packets

10 option B by $88

11 58 loads

12 Numbers are given from top down and left to right.
a 3, 6, 3 **b** 3, 4, 5, 7, 3, 2
c 6 **d** 3, 2

13 a 1 **b** a **c** 0 **d** 25

14 a 34 **b** 18 **c** 29 **d** 17

15 a 1700 **b** 560 **c** 12 000 **d** 300

16 a 10 (8 child and 2 adult)

b 15 (14 child and 1 adult)

c Take the maximum number of child tickets that leaves a multiple of the adult price remaining.

Exercise 1C

1 a 14 **b** 45 **c** 43 **d** 40

2 a 6 **b** 3

3 a prime **b** composite **c** prime
d composite **e** composite **f** composite
g composite **h** composite

4 a 8 **b** 27 **c** 64 **d** 125
e 216 **f** 343 **g** 512 **h** 729

5 a 6 **b** 45 **c** 24 **d** 8
e 50 **f** 36 **g** 120 **h** 60

6 a 2 **b** 9 **c** 8 **d** 6
e 1 **f** 1 **g** 36 **h** 4

7 a 16 **b** 100 **c** 169 **d** 225
e 10 000 **f** 400 **g** 5 **h** 7
i 11 **j** 30 **k** 40 **l** 16

8 a 8 **b** 64 **c** 343 **d** 125
e 216 **f** 1000 **g** 3 **h** 2
i 5 **j** 8 **k** 9 **l** 100

9 a 24 **b** 105 **c** 5 **d** 4

10 4 ways

11 30 minutes

12 25

13 a 55 **b** They are square numbers.

14 15 minutes

15 The number one (1) does not have two or more factors.

16 All even numbers greater than 2 have 2 as a factor as well as 1 and itself, and therefore have more than 2 factors.

17 All pairs of factors form groups of 2 except for the repeated factor with a square number, e.g. 9 has 1, 3 and 9 where 3 is the repeated factor.

18 a False, LCM of 4 and 8 is 8 not 32. **b** true
c true

19 a **i** $28 = 23 + 5$ **ii** $62 = 43 + 19$
iii $116 = 97 + 19$
b 11 and 17

20 (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)

Exercise 1D

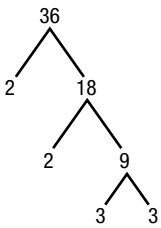
1 a 1, 3, 5, 15 **b** 1, 2, 3, 4, 6, 8, 12, 24
c 1, 2, 4, 5, 8, 10, 20, 40
d 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

3 a 12 **b** 90 **c** 280 **d** 297

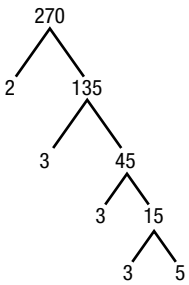
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4 a



$\therefore 36 = 2^2 \times 3^2$

b



$\therefore 270 = 2 \times 3^3 \times 5$

5 **a** $2^2 \times 5$ **b** $2^2 \times 7$ **c** $2^3 \times 5$ **d** $2 \times 3^2 \times 5$
e $2^3 \times 5 \times 7$ **f** $2^2 \times 7^2$ **g** $2^3 \times 3^2 \times 5$
h $2^2 \times 3 \times 5 \times 11$

6 **a** 3 : 2, 3, 5 **b** 2 : 3, 7 **c** 3 : 2, 3, 5 **d** 3 : 5, 7, 11

7 **a** divisible by 3 **b** divisible by 2, 3, 6, 9
c divisible by 2, 4, 8 **d** divisible by 3, 9
e divide by 3, 5, 9 **f** none
g 2, 3, 6 **h** none

8 **a** 5 **b** 3 **c** 2 **d** 7

9 **a** 60, 2 **b** 28, 14 **c** 120, 3 **d** 60, 3
e 140, 4 **f** 390, 1 **g** 126, 3 **h** 630, 21

10 210 days

11 61 soldiers

12 **a** true **b** false, 12 **c** true **d** false, 12

13 **a** 2 and 7 **b** 2 and 11 **c** 3 and 5 **d** 7 and 11

14 **a** 2×3^4 **b** $2^5 \times 3$ **c** $3^2 \times 5^4$ **d** $2^8 \times 7$

15 **a** **i** 2 **ii** 8 **iii** 11 **iv** 15
v 28 **vi** 39 **vii** 94 **viii** 820
b **i** 5 **ii** 5 **iii** result is 0
c result is 11
d **i** 11 **ii** 11 **iii** 0 **iv** 0
e The difference between the sum of the alternating digits is 0 or a multiple of 11.

Exercise 1E

1 **a** > **b** < **c** < **d** >
e > **f** < **g** > **h** <

2 **a** –1, 2 **b** –1, – 4 **c** – 4, –2 **d** 0, –10

3 **a** –2°C **b** –1°C **c** –9°C **d** 3°C

4 **a** 1 **b** 4 **c** 1 **d** 8
e 15 **f** 102 **g** –5 **h** –7
i –7 **j** –14 **k** –94 **l** –12

5 **a** –1 **b** –5 **c** –26 **d** –17
e –91 **f** –74 **g** –11 **h** –31
i –29 **j** –110 **k** –437 **l** –564

6 **a** 1 – 4 = –3 **b** –9 + 3 = –6
c –1 + 5 = 4 **d** –15 – 5 = –20

7 **a** 6 **b** – 4 **c** –14 **d** 11
e 15 **f** 5 **g** –3 **h** 12

8 **a** –1 **b** 8 **c** –7 **d** –1

9 ground floor

10 **a** – **b** –, + **c** –, –

11 $7

12 –23°C

13 **a** true **b** not always true **c** not always true
d true **e** not always true **f** not always true

14 **a** true **b** false **c** true

15 0

16 500, pair to give 500 pairs each with a total of 1.

17 **a** $a = 1, b = 4$ **b** $a = -7, b = 3$
c $a = -5, b = 2$ **d** $a = -10, b = 2$

Exercise 1F

1 **a** 6 **b** –10 **c** –38 **d** 46
e 32 **f** –88 **g** –673 **h** 349

2 **a** subtract **b** add

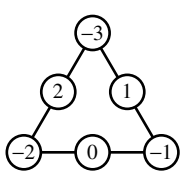
3 **a** false **b** true **c** true **d** false
e true **f** false **g** false **h** true
i false

4 **a** 4 **b** 3 **c** –5 **d** 15
e –2 **f** –14 **g** –9 **h** –21
i –38 **j** –86 **k** –105 **l** –259

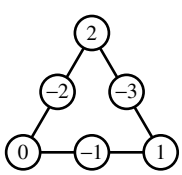
5 **a** 5 **b** 8 **c** 21 **d** 38
e 72 **f** 467 **g** –2 **h** 2
i 3 **j** 32 **k** –57 **l** 76

6 **a** –3 **b** –6 **c** 1 **d** 10
e 2 **f** –14 **g** –2 **h** –4
i –30 **j** –5 **k** –6 **l** 65

7 **a**



b



8 **a**

–1	–6	1
0	–2	–4
–5	2	–3

b

–12	–19	–14
–17	–15	–13
–16	–11	–18

9 **a** $a = -3, b = -5$ **b** $a = -15, b = -9$

10 –$40

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11 3 and –8

12 15 and – 4

13 **a** Should be 5 + 2. **b** Left off negative on the 2.

14 **a** true **b** true **c** not always true
d true

15 **a** no **b** yes **c** yes **d** no

16 **a**

x	–2	–1	0	1	2	3
y	7	6	5	4	3	2

b 18 **c** – 45

17 **a**

x	–3	–2	–1	0	1	2	3
y	0	1	2	3	4	5	6

b 15 **c** –9

Progress quiz

1 **a** 33 **b** 42 **c** 358 **d** 392

2 **a** 323 **b** 37 **c** 543 **d** 2067

3 **a** 700 **b** 294 **c** 16 **d** 423

4 **a** 222 **b** 67 233 **c** 61 **d** 23 remainder 2

5 **a** 24 **b** 6

6 **a** 36 **b** 900 **c** 8 **d** 50

7 **a** 8 **b** 1 000 000 **c** 3 **d** 5

8 $2^3 \times 3^2 \times 5$

9 Divisible by: 2 (last digit 6 is even); 3 (1 + 2 + 6 = 9 which is divisible by 3); 6 (divisible by both 2 and 3); 9 (1 + 2 + 6 = 9 which is divisible by 9)
Not divisible by: 4 (26 not divisible by 4); 5 (last digit not 0 or 5); 8 (last 3 digits not divisible by 8)

10 **a** HCF = 6; LCM = 126
b HCF = 15; LCM = 630

11 **a** 14 **b** –17 **c** –74 **d** –452
e –13 **f** –70

12 **a** –6 **b** –366 **c** 25 **d** 49

13 **a** Each team has 18 students
b 9 teams with green uniform; 6 teams with red uniform; 8 teams with blue uniform

Exercise 1G

1 **a**

□	△	□ × △
3	5	15
2	5	10
1	5	5
0	5	0
–1	5	–5
–2	5	–10
–3	5	–15

b

□	△	□ × △
3	–5	–15
2	–5	–10
1	–5	–5
0	–5	0
–1	–5	5
–2	–5	10
–3	–5	15

2 **a** 15, 3 **b** –15, –3 **c** –15, –3 **d** 15, –3

3 **a** true **b** false **c** true **d** true
e false

4 **a** –20 **b** –54 **c** –40 **d** –99
e 6 **f** 105 **g** 400 **h** 300
i –152 **j** –123 **k** 54 **l** 765

5 **a** –5 **b** –2 **c** –4 **d** –30
e –2 **f** –26 **g** –45 **h** –36
i 3 **j** 3 **k** 9 **l** 6

6 **a** –3 **b** –5 **c** 7 **d** 6
e –3 **f** –72 **g** –252 **h** –5
i –30

7 **a** 1 **b** 14 **c** 160 **d** –29
e –120 **f** 20

8 **a** ×, ÷ **b** ×, ÷ **c** ÷, × **d** ÷, ÷
e ×, ÷ **f** ×, ÷

9 0

10 –16

11 –8 and 3

12 8 and –2 or –8 and 2

13 **a** **i** 4 **ii** –27 **iii** –64 **iv** 25
b Yes, it will be a product of 2 numbers of the same sign.
c Yes, the product of 3 negative numbers will be negative.

14 **a** true **b** false **c** true

15 **a** 2 **b** –1 **c** –2 **d** 48

16 If $\sqrt{-9}$ was to exist, then you could find a value of a for which $a^2 = -9$. This is not possible using real numbers.

17 Yes, a cube of a negative number gives a negative number.
$(-3)^3 = -27$ so $\sqrt[3]{-27} = -3$

18 **a** $y = -3x - 1$ **b** $y = -7x - 3$
c $y = x^2 + 1$

Exercise 1H

1 **a** true **b** true **c** false
d false **e** false **f** true

2 Missing numbers are:
a 4, –3 **b** –6, 18 **c** –3, –3, 1 **d** –6, –36, – 4

3 Missing numbers are:
a –3, 8, 5 **b** –8, 26 **c** 6, 18
d 11, –5, 10, –5, –20, –5, 4

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4 **a** –30 **b** –12 **c** 12 **d** –11
e –10 **f** 5 **g** 24 **h** –60
i 40
5 **a** –6 **b** 24 **c** 2 **d** 7 **e** 0
f 3 **g** –11 **h** 2 **i** –44 **j** 1
k –12 **l** 1
6 **a** –1 **b** –3 **c** –5 **d** 3
e –6 **f** 7 **g** 0 **h** –5
7 **a** –15 **b** –15 **c** 2 **d** –8
e 8 **f** 1 **g** –4 **h** 10
8 **a** 22 **b** 4 **c** 28 **d** 122
e –32 **f** –16 **g** 152 **h** 16
9 **a** –15 **b** 5 **c** 16 **d** 14
e 9 **f** 28 **g** –1 **h** 0
i –12 **j** 19 **k** 7 **l** 37
10 **a** 12 **b** 16 **c** 2 **d** 1
e 3 **f** –23 **g** –8 **h** 3
i 28 **j** 26 **k** 0 **l** –22
11 –3°C
12 **a** $(-2 + 1) \times 3 = -3$ **b** $-10 \div (3 - (-2)) = -2$
c $-8 \div (-1 + 5) = -2$ **d** $(-1 - 4) \times (2 + (-3)) = 5$
e $(-4 + -2) \div (10 + (-7)) = -2$
f $20 + ((2 - 8) \times (-3)) = 38$
g $(1 - (-7) \times 3) \times 2 = 44$ **h** $(4 + -5 \div 5) \times (-2) = -6$
13 **a** true **b** not always true **c** true
d not always true **e** not always true **f** true
14 **a** 4 **b** 1 **c** –7 **d** –4
15 **a** –18 **b** 4 **c** –1
16 **a** $(1 - 3 \times (-4)) \div (-13) = -1$
b $4 \div (3 + (-7)) \times (-5) = 5$
c $6 - (7 \div (-7) + 6) = 1$
or $(6 - 7) \div ((-7) + 6) = 1$
d $-1 - (5 + (-2)) \times (1 - 4) = 8$
17 There are 5 answers.
18 Answers will vary.

Problems and challenges

1 4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49
2 **a** 13 cm **b** 9 cm
3 2520
4 **a** –105 **b** 16 **c** –39
5 **a** $-5 \times (3 \div (-3) + 2) - (4 + (-3)) = -6$
b $-100 \div (4 \times (-2) - 2) \times 3 - (-2) = 32$
6 **a** 6 **b** 1000 **c** 210 **d** 96
7 **a** $y = 3 - x$ **b** $y = x^2 - 3$
c $y = x^3 + 4$ **d** $y = y = 2\sqrt[3]{x} - 1$
8 **a** 0 **b** 2
9 $a = 7, b = 3$; HCF = 63

Multiple-choice questions

1 B **2** C **3** E **4** E **5** E
6 C **7** A **8** E **9** B **10** D

Short-answer questions

1 **a** 497 **b** 412 **c** 129 **d** 67
e 112 **f** 139 **g** 1999 **h** 5675
2 **a** 539 **b** 2030 **c** 61 **d** 3074
3 **a** 170 **b** 297 **c** 336 **d** 423
e 41 **f** 119 **g** 103 **h** 201
4 **a** 1668 **b** 21 294 **c** 281 **d** 122
5 **a** 3 **b** 1 **c** 1 **d** 7
6 **a** 9 **b** 11 **c** 49 **d** 400
e 3 **f** 4 **g** 125 **h** 1000
7 **a** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
b 112, 119, 126, 133, 140, 147
c 31, 37, 41, 43, 47, 53, 59
d 24 **e** 6
8 **a** $2^2 \times 3^2$ **b** $2^2 \times 3 \times 7$ **c** $2 \times 3^2 \times 11$
9 **a** divisible by 2, 3, 4, 6 **b** divisible by 5
c divisible by 2, 4 **d** divisible by 3, 9
10 **a** 380 **b** 2
11 **a** 3 **b** –5 **c** –8 **d** –31
e –76 **f** –330 **g** –1 **h** 98
12 **a** 2 **b** –8 **c** –64 **d** –39
e 16 **f** 12 **g** –20 **h** 92
13 **a** true **b** false **c** false **d** true
14 **a** –10 **b** 88 **c** –63 **d** 200
e 2 **f** –3 **g** –4 **h** 3
15 **a** –4 **b** –1 **c** –8 **d** 26
e –10 **f** –1 **g** –1 **h** –20
16 **a** –11 **b** 1 **c** 7 **d** 30
e 7 **f** –128 **g** –5 **h** 145

Extended-response questions

1 **a** $a = \$112, b = -\$208, c = \$323, d = -\$275, e = \$240$
b \$228 **c** \$160
2 **a** 72 **b** $30 = 2 \times 3 \times 5, 42 = 2 \times 3 \times 7$
c LCM = 210, HCF = 6
d 6 **e** 210

Chapter 2

Exercise 2A

1 **a** complementary **b** supplementary
c revolution
2 **a** 45 **b** 130 **c** 120 **d** 240 **e** 90 **f** 100
3 **a** 40° **b** 110° **c** 220°
4 **a** $a = 70, b = 270$ **b** $a = 25, b = 90$
c $a = 128, b = 52$ **d** $a = 34, b = 146$
e $a = 25$ **f** $a = 40$
g $a = 120$ **h** $a = 50, b = 90$
i $a = 140$ **j** $a = 110, b = 70$
k $a = 148$ **l** $a = 90, b = 41, c = 139$

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5 **a** $\angle DOE$ **b** $\angle AOB$ **c** $\angle DOE$ *or* $\angle AOB$
d $\angle COD$

6 **a** 270° **b** 90° **c** 0° (or 360°)
d 180° **e** 315° **f** 135°
g 225° **h** 45°

7 **a** S **b** N **c** W **d** E
e NE **f** NW **g** SW **h** SE

8 **a** 40° **b** 72° **c** 120° **d** 200°

9 **a** 60 **b** 135 **c** 35 **d** 15
e 36 **f** 45

10 **a** 105° **b** 97.5° **c** 170° **d** 170°
e 132.5° **f** 27.5° **g** 144° **h** 151.5°

11 **a** Supplementary angles should add to 180°.
b Angles in a revolution should add to 360°.
c Angles on straight line should add to 180°.

12 **a** $a + 3b = 360$
b $a + 2b = 180$
c $a + b = 90$

13 **a** $a = 110$ **b** $(a + 50)°$ should be the larger looking angle

14 **a** 30 **b** 54 **c** 55 **d** 34
e 30 **f** 17

Exercise 2B

1 **a** equal **b** supplementary **c** equal

2 **a** $\angle BCH$ **b** $\angle ABE$ **c** $\angle GCB$ **d** $\angle BCH$
e $\angle FBC$ **f** $\angle GCB$ **g** $\angle FBC$ **h** $\angle DCG$

3 **a** alternate **b** alternate
c co-interior **d** corresponding
e corresponding **f** co-interior

4 All reasons assume that lines are parallel.
a $a = 110$ (corresponding to 110°), $b = 70$ (supplementary to $a°$)
b $a = 120$ (alternate to 120°), $b = 60$ (co-interior to $a°$), $c = 120$ (corresponding to 120°)
c $a = 74$ (alternate to 74°), $b = 106$ (co-interior to 74°), $c = 106$ (supplementary to $a°$)
d $a = 100$ (supplementary to 80°), $b = 100$ (co-interior to 80°)
e $a = 95$ (corresponding to 95°), $b = 85$ (supplementary to $a°$)
f $a = 40$ (alternate to 40°), $b = 140$ (co-interior to 40°)

5 **a** $a = 58$, $b = 58$ (both co-interior to 122°)
b $a = 141$, $b = 141$ (both co-interior to 39°)
c $a = 100$ (co-interior to 80°), $b = 80$ (co-interior to $a°$)
d $a = 62$ (co-interior to 118°), $b = 119$ (co-interior to 61°)
e $a = 105$ (co-interior to 75°), $b = 64$ (corresponding to 64°)
f $a = 25$ (alternate to 25°), $b = 30$ (alternate to 30°)

6 **a** No, the alternate angles are not equal.
b Yes, the co-interior angles are supplementary.
c No, the corresponding angles are not equal.

7 **a** 250 **b** 320 **c** 52 **d** 40
e 31 **f** 63 **g** 110 **h** 145
i 33

8 **a** 130° **b** 95° **c** 90° **d** 97°
e 65° **f** 86°

9 **a** $\angle AOB = (180 - a)°$ **b** $\angle AOB = (360 - a)°$
c $\angle AOB = (180 - a - b)°$

10 $a = 36$, $b = 276$, $c = 155$, $d = 85$, $e = 130$, $f = 155$, $g = 15$

Exercise 2C

1 **a** right-angled triangle **b** isosceles triangle
c acute-angled triangle **d** equilateral triangle
e obtuse-angled triangle **f** equilateral triangle
g isosceles triangle **h** scalene triangle

2 **a** scalene **b** isosceles **c** isosceles **d** equilateral
e scalene **f** isosceles

3 **a** right **b** obtuse **c** acute

4 **a** 80 **b** 40 **c** 58 **d** 19
e 34 **f** 36

5 **a** 68 **b** 106 **c** 20

6 **a** 65 **b** 40 **c** 76

7 **a** 160 **b** 150 **c** 80 **d** 50
e 140 **f** 55

8 **a** yes **b** no **c** yes **d** yes
e yes **f** yes

9 **a** 55 **b** 60 **c** 25

10 **a** 60 **b** 231 **c** 18 **d** 91
e 65.5 **f** 60

11 **a** Isosceles, the two radii are of equal length.
b $\angle OAB$, $\angle OBA$ **c** 30° **d** 108°
e 40°

12 **a** **i** a, alternate angles in parallel lines
ii c, alternate angles in parallel lines
b They add to 180°, they are on a straight line.
c $a + b + c = 180$, angles in a triangle add to 180°.

13 Hint: Let $a°$ be the third angle.

14 Hint: Let $a°$ be the size of each angle.

15 **a** alternate to $\angle ABC$ in parallel lines
b supplementary, co-interior angles in parallel lines
c $a + b + c = 180$, angles in a triangle add to 180°.

16 **a** $a = 30$, $b = 60$, $c = 60$
b $a + c = 90$
c $a = 60$, $b = 120$, $c = 30$, $a + c = 90$
d $a = 16$, $b = 32$, $c = 74$, $a + c = 90$
e $a + c = 90$
f **i** $a = x$, $b = 2x$, $c = 90 - x$ **ii** 90

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Exercise 2D

1 **a** non-convex **b** non-convex **c** convex

2 **a** **i** true **ii** false **iii** false **iv** true
b **i** false **ii** true **iii** false **iv** true
c **i** false **ii** true **iii** true
d **i** true **ii** false **iii** false
e **i** true **ii** false **iii** true
f **i** false **ii** false

3 **a** $a = 104, b = 76$ **b** $a = 72, b = 72$
c $a = 128$

4 **a** 90 **b** 61 **c** 105 **d** 170 **e** 70 **f** 70

5 **a** $a = 100, b = 3, c = 110$
b $a = 2, b = 90$
c $a = 5, b = 70$

6 **a** 152 **b** 69 **c** 145 **d** 74 **e** 59 **f** 30

7 **a** 19 **b** 60 **c** 36

8 **a** square, rectangle, rhombus and parallelogram
b square, rectangle

9 **a** true **b** false **c** true **d** true
e false **f** true

10 No, the two reflex angles would sum to more than 360° on their own.

11 **a** angle sum $= a° + b° + c° + d° + e° + f° = 180° + 180°$
(angle sum of a triangle) $= 360°$
b $\angle ADC = 360° - (a + b + c)°$ (angle sum of a quadrilateral) reflex
$\angle ADC = 360° - \angle ADC$
$= 360° - (360° - (a + b + c))°$
$= (a + b + c)°$

Progress quiz

1 **a** $a = 25$ $b = 232$
b $x = 72$ $y = 108$
c $a = 65$ $b = 115$
d $a = 92$ $b = 88$ $c = 272$

2 **a** $\angle POT$ **b** $\angle ROS$ **c** $\angle TOS$ or $\angle POR$

3 **a** 260 **b** 105 **c** 44

4 isosceles, acute

5 **a** 60 **b** 30 **c** 46
d 62 **e** 60 **f** 84

6 **a** 60 **b** 71 **c** 137
d 72 **e** 90

Exercise 2E

1 **a** 6 **b** 4 **c** 10 **d** 7
e 5 **f** 12

2 **a** 720° **b** 1440° **c** 3600°

3 **a** square **b** equilateral triangle

4 **a** 108° **b** 144° **c** 135°

5 **a** 720° **b** 1260° **c** 900° **d** 2340°
e 7740° **f** 18 000°

6 **a** 130 **b** 155 **c** 105 **d** 250
e 40 **f** 265

7 **a** 108° **b** 128.6° **c** 147.3° **d** 168.8°

8 **a** 9 **b** 15 **c** 21 **d** 167

9 **a** 127.5 **b** 240 **c** 60 **d** 60
e 79 **f** 72

10 **a** circle **b** Increases to infinity. **c** 180°

11 **a** $\frac{S}{n}$ **b** $\frac{180(n-2)}{n}$
c **i** 150° **ii** 175.61°

12 **a** 6 **b** 20 **c** 11

13 **a** 150 **b** 130 **c** 270

Exercise 2F

1 **a** vertices **b** seven **c** congruent
d seven **e** octagonal

2 **a** 24 **b** 7 **c** 8

3 **a** 6, 8, 12 **b** 5, 6, 9 **c** 7, 7, 12

4 A, cube; B, pyramid; F, rectangular prism; G, tetrahedron; H, hexahedron

5 **a** hexahedron **b** tetrahedron
c pentahedron **d** heptahedron
e nonahedron **f** decahedron
g undecahedron **h** dodecahedron

6 **a** 8 **b** 6 **c** 4 **d** 5
e 7 **f** 9 **g** 10 **h** 11

7 **a** triangular prism **b** pentagonal prism
c square prism

8 **a** rectangular pyramid **b** heptagonal pyramid
c triangular pyramid

9 **a**

Solid	Faces (F)	Vertices (V)	Edges (E)	$F + V$
Cube	6	8	12	14
Square pyramid	5	5	8	10
Tetrahedron	4	4	6	8
Octahedron	8	6	12	14

b $F + V$ is 2 more than E.

10

Faces (F)	Vertices (V)	Edges (E)
6	8	12
5	5	8
5	6	9
7	7	12
4	4	6
11	11	20

11 **a** 26 **b** 11 **c** 28

12 **a** true **b** false **c** true **d** true
e false (sphere) **f** true **g** false

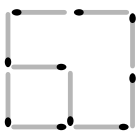
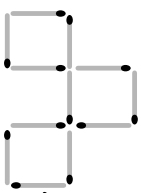
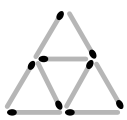
13 **a** yes **b** yes **c** no **d** yes
e yes

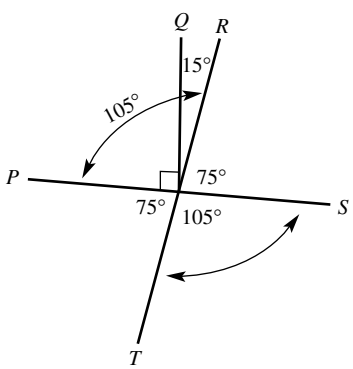
14 $F = 12$, $V = 12$ and $E = 24$ so Euler's rule does not apply.

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15 **a** hexahedron, rectangular prism
b undecahedron, decagonal pyramid
16 **a** $V = E - F + 2$ **b** $F = E - V + 2$
17 **a** $V = 8, E = 14, F = 8$ and $V + F - 2 = E$
b $V = 16, E = 24, F = 10$ and $V + F - 2 = E$
c $V = 6, E = 12, F = 8$ and $V + F - 2 = E$
18 **a** true **b** true
19 **a** **i** convex **ii** non-convex **iii** non-convex
b Answers will vary.

Problems and challenges

1 **a**

b

2 **a**

b

3 **a** 30 **b** 150
4 125°
5


6 180. Find each angle in the interior pentagon in terms of a, b, c, d and/or e. Then solve the sum equal to 540° for a pentagon.

Multiple-choice questions

1 D **2** A **3** E **4** B **5** C
6 D **7** E **8** A **9** C **10** D

Short-answer questions

1 **a** 50 **b** 65 **c** 240 **d** 36
e 61 **f** 138
2 **a** 132 **b** 99 **c** 77 **d** 51
e 146 **f** 41
3 95°
4 **a** scalene, 35 **b** isosceles, 30
c equilateral, 60 **d** right angle, 19
e acute, 27 **f** obtuse, 132
5 **a** 67 **b** 141 **c** 105
6 **a** $a = 98, b = 82$ **b** $a = 85, b = 106$
c $a = 231, b = 129$
7 **a** 900° **b** 1260° **c** 10 800°
8 **a** 108° **b** 150°
9 **a** 71 **b** 25 **c** 67.5
10 **a** hexahedron **b** decahedron **c** undecahedron
11 **a** triangular prism **b** octagonal prism
c rectangular pyramid
12

F	V	E
5	5	8
9	14	21
7	10	15

13 40

Extended-response questions

1 **a** 4320° **b** 166° **c** 14° **d** 360°
e **i** 28 **ii** 52 **iii** 78
2 **a** triangle, quadrilateral, pentagon, hexagon
b $a = 90, b = 119, c = 29, d = 121, e = 270, f = 230$

Chapter 3

Exercise 3A

1 **a** 6, 20, 200 **b** 14, 40, 140
c 1, 6, 42 **d** 8, 3, 20
2 $\frac{4}{6}, \frac{20}{30}, \frac{10}{15}$
3 **a** false **b** true **c** true
d true **e** true **f** false
4 **a** $\frac{8}{24}$ **b** $\frac{6}{24}$ **c** $\frac{12}{24}$ **d** $\frac{10}{24}$
e $\frac{20}{24}$ **f** $\frac{120}{24}$ **g** $\frac{18}{24}$ **h** $\frac{21}{24}$
5 **a** $\frac{6}{30}$ **b** $\frac{10}{30}$ **c** $\frac{15}{30}$ **d** $\frac{90}{30}$
e $\frac{20}{30}$ **f** $\frac{11}{30}$ **g** $\frac{75}{30}$ **h** $\frac{15}{30}$
6 **a** 6 **b** 18 **c** 2 **d** 7
e 28 **f** 50 **g** 15 **h** 44
7 **a** 2, 3, 5, 10, 16, 25 **b** 6, 12, 30, 45, 150, 300
c 4, 6, 8, 14, 20, 30
8 **a** 6, 12, 24, 30, 75, 300 **b** 10, 15, 20, 25, 50, 60
c 8, 12, 28, 44, 80, 400
9 **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{5}{6}$ **d** $\frac{5}{6}$
e $\frac{1}{4}$ **f** $\frac{3}{5}$ **g** $\frac{8}{9}$ **h** $\frac{5}{7}$
i $\frac{5}{3}$ **j** $\frac{11}{10}$ **k** $\frac{6}{5}$ **l** $\frac{4}{3}$

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10 **a** $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{3}{7}$ **d** $\frac{5}{9}$
e $\frac{13}{11}$ **f** $\frac{7}{4}$ **g** $\frac{21}{17}$ **h** $\frac{81}{50}$

11 $\frac{14}{42}=\frac{1}{3}$, $\frac{51}{68}=\frac{3}{4}$, $\frac{15}{95}=\frac{3}{19}$

12 $\frac{5}{11}=\frac{15}{33}$, $\frac{3}{5}=\frac{9}{15}$, $\frac{7}{21}=\frac{1}{3}$, $\frac{8}{22}=\frac{16}{44}$, $\frac{2}{7}=\frac{6}{21}$, $\frac{20}{50}=\frac{6}{15}$

13 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$

c No, $\frac{3}{4}=\frac{9}{12}$ of time complete. However, only $\frac{2}{3}=\frac{8}{12}$ of laps completed.

14 **a** Cannot be simplified, e.g. $\frac{3}{5}$, $\frac{7}{11}$… The HCF of the numerator and denominator is 1.

b Possibly. e.g. $\frac{15}{16}$, both are composite numbers, but HCF is 1 and therefore the fraction cannot be simplified.

However, in $\frac{15}{18}$, both numbers are composite, HCF is 3 and therefore the fraction can be simplified to $\frac{5}{6}$.

15 **a** no **b** yes **c** 10

16 infinite provided the denominator is twice the numerator then the fraction will be equivalent to $\frac{1}{2}$.

17 **a** **i** $6b$ **ii** $5x$ **iii** 80 **iv** $12de$
v bc **vi** $3km$ **vii** $16ac$ **viii** xy

b **i** $\frac{3b}{4}$ **ii** $\frac{1}{2y}$ **iii** $\frac{3}{5}$ **iv** $\frac{5}{8x}$
v $\frac{2}{3q}$ **vi** $\frac{10}{x}$ **vii** $\frac{o}{q}$ **viii** $\frac{3}{x}$

c Yes, $\frac{5x}{15x}=\frac{1}{3}$.

d Yes, $\frac{1}{3}=\frac{a}{3a}$.

Exercise 3B

1 **a** +, − **b** ×, ÷

2 **a** ×, ÷ **b** +, −

3 **a** 20 **b** 9 **c** 50 **d** 24

4 **a** 3, 12 **b** 14, 5 **c** 11, 33 **d** ×, 14, 1, 1

5 **a** $\frac{8}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{13}$ **d** $\frac{11}{12}$

6 **a** $\frac{3}{13}\times\frac{7}{9}$ **b** $\frac{1}{2}\times\frac{4}{1}$ **c** $\frac{17}{10}\times\frac{1}{3}$ **d** $\frac{7}{3}\times\frac{4}{7}$

7 **a** $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $1\frac{1}{2}$ **d** $1\frac{6}{7}$
e $1\frac{3}{20}$ **f** $1\frac{1}{10}$ **g** $\frac{1}{21}$ **h** $\frac{4}{9}$

8 **a** $4\frac{4}{7}$ **b** $9\frac{3}{5}$ **c** $2\frac{3}{8}$ **d** $1\frac{2}{11}$
e $9\frac{1}{2}$ **f** $22\frac{3}{14}$ **g** $3\frac{3}{4}$ **h** $1\frac{17}{30}$

9 **a** $\frac{3}{20}$ **b** $\frac{10}{63}$ **c** $1\frac{17}{25}$ **d** $1\frac{13}{27}$
e $\frac{1}{6}$ **f** $\frac{3}{8}$ **g** $\frac{8}{15}$ **h** 5

10 **a** $3\frac{2}{3}$ **b** $1\frac{2}{21}$ **c** 15 **d** 35

11 **a** $\frac{10}{27}$ **b** $\frac{5}{6}$ **c** $\frac{16}{77}$ **d** $1\frac{7}{15}$
e $\frac{7}{8}$ **f** 2 **g** $1\frac{1}{3}$ **h** $3\frac{3}{5}$

12 **a** $\frac{33}{35}$ **b** $\frac{48}{125}$ **c** $1\frac{2}{5}$ **d** 3

13 **a** $\frac{15}{16}$ **b** $\frac{1}{12}$ **c** $1\frac{1}{6}$ **d** $1\frac{1}{2}$

14 **a** $\frac{29}{70}$ **b** $\frac{41}{70}$

15 $\frac{3}{5}$

16 7 kg

17 112 glasses

18 Answers may vary.
a 5, 5 **b** 4 **c** 5, 2 **d** 1, 1

19 Answers may vary.
a 2, 5 **b** 18 **c** 10,1 **d** $3\frac{4}{15}$

20 **a** $\frac{3}{16}$ **b** $\frac{4}{3}$ **c** 5 **d** $4\frac{1}{2}$

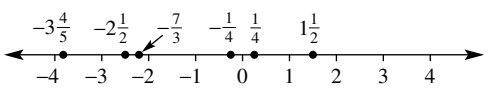
21 **a** $\left(\frac{1}{6}+\frac{1}{5}-\frac{1}{3}\right)\times\left(\frac{1}{4}\right)\div\left(\frac{1}{2}\right)=\frac{1}{60}$

b $\left(\frac{1}{2}\div\frac{1}{6}-\frac{1}{5}+\frac{1}{4}\right)\times\frac{1}{3}=1\frac{1}{60}$

c $\frac{1}{4}\div\left(\frac{1}{5}-\frac{1}{6}\right)\times\left(\frac{1}{3}+\frac{1}{2}\right)=6\frac{1}{4}$

Exercise 3C

1



2 **a** $\frac{1}{4}$ **b** $\frac{-1}{3}$ **c** $\frac{3}{5}$ **d** $\frac{-2}{7}$

3 **a** positive **b** negative **c** negative **d** positive

4 **a** $-\frac{4}{7}$ **b** $\frac{1}{5}$ **c** $-\frac{7}{9}$ **d** $-5\frac{1}{3}$
e $-\frac{1}{3}$ **f** $-\frac{2}{5}$ **g** $\frac{3}{2}$ **h** $\frac{7}{11}$

5 **a** $-\frac{1}{12}$ **b** $-\frac{13}{35}$ **c** $1\frac{1}{10}$ **d** $\frac{8}{9}$
e $-\frac{1}{4}$ **f** $\frac{1}{8}$ **g** $-\frac{3}{20}$ **h** $-\frac{4}{15}$

6 **a** $-\frac{12}{35}$ **b** $-\frac{16}{55}$ **c** $\frac{4}{15}$ **d** $\frac{5}{6}$
e $-\frac{4}{21}$ **f** $-\frac{1}{8}$ **g** $\frac{3}{7}$ **h** $-1\frac{1}{5}$

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7 **a** $-\frac{20}{21}$ **b** $-\frac{9}{20}$ **c** $\frac{8}{15}$ **d** $1\frac{1}{3}$
e $-\frac{2}{7}$ **f** $-\frac{3}{20}$ **g** $\frac{3}{4}$ **h** $2\frac{2}{5}$

8 $-\frac{5}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{5}, \frac{1}{16}, \frac{3}{4}, 3\frac{1}{10}$

9 $16\frac{3}{4}$°C

10 **a** Mon $= -1\frac{2}{3}$, Tue $= -\frac{1}{2}$, Wed $= -2\frac{1}{4}$, Thur $= \frac{1}{4}$
b $-4\frac{1}{6}$ **c** $12\frac{1}{6}$ hours

11 $1\frac{7}{20}$ metres

12 **a** > **b** < **c** > **d** <
e > **f** > **g** < **h** >

13 **a** negative **b** negative **c** negative **d** positive

14 **a** < **b** > **c** > **d** <

15 **a** **i** $1\frac{1}{2}$ **ii** $\frac{1}{15}$ **iii** $\frac{-21}{25}$ **iv** $-4\frac{23}{40}$
b Answers may vary: $-\frac{5}{8}, -\frac{3}{8}, -\frac{2}{8}, -\frac{1}{8}, \frac{11}{8}$
c Answers may vary: $-\frac{7}{4}, -\frac{5}{4}, -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}$

Exercise 3D

1 E

2 C

3 **a** < **b** > **c** <
d < **e** > **f** >

4 **a** 10, 6 **b** 55, 5

5 **a** 3.6521, 3.625, 3.256, 3.229, 2.814, 2.653
b 11.908, 11.907, 11.898, 11.891, 11.875, 11.799
c 1.326, 1.305, 0.802, 0.765, 0.043, 0.039

6 **a** $\frac{31}{100}$ **b** $\frac{537}{1000}$ **c** $\frac{163}{200}$ **d** $\frac{24}{25}$
e $5\frac{7}{20}$ **f** $8\frac{11}{50}$ **g** $26\frac{4}{5}$ **h** $8\frac{64}{125}$
i $\frac{13}{250}$ **j** $6\frac{1}{8}$ **k** $317\frac{3}{50}$ **l** $\frac{53}{125}$

7 **a** 0.17 **b** 0.301 **c** 4.05 **d** 7.6

8 **a** 0.12 **b** 0.35 **c** 2.5 **d** 1.75
e 0.275 **f** 0.375 **g** 0.68 **h** 0.232

9 2.175, 2.18, 2.25, 2.3, 2.375, 2.4

10 A1, B5, C07, P9, BW Theatre, gym

11 Opposition leader is ahead by 0.025.

12 **a** D, C, E, F, B, A **b** C, D, A, E, F, B

13 **a** 2.655 **b** $\frac{179}{200}$ **c** 4.61 525 **d** $2\frac{1109}{2000}$

14

2.6	4.6	$1\frac{4}{5}$
2.2	$\frac{6}{2}$	3.8
4.2	1.4	$3\frac{2}{5}$

15 **a** 59.15 **b** A$183.23 **c** Canada **d** £35
e 91.4c **f** AUD, CAD, USD, EUR, GBP
g Cheaper to buy car in GB and freight to Australia.
h Answers will vary.

Exercise 3E

1 B **2** E **3** C **4** B

5 **a** 62.71 **b** 277.99 **c** 23.963 **d** 94.172
e 14.41 **f** 23.12 **g** 84.59 **h** 4.77

6 **a** 179.716 **b** 50.3192 **c** 1025.656 **d** 18.3087

7 **a** 11.589 **b** 9.784 **c** 19.828 **d** 4.58

8 **a** 3651.73 **b** 81.55 **c** 0.75 **d** 0.03812
e 6 348 000 **f** 0.0010615 **g** 30 **h** 0.000452

9 **a** 99.6 **b** 12.405 **c** 107.42 **d** 1.8272
e 0.01863 **f** 660.88 **g** 89.0375 **h** 292.226

10 **a** 12.27 **b** 5.88 **c** 0.0097 **d** 49.65
e 11.12 **f** 446.6 **g** 0.322655 **h** 3.462

11 **a** 203.8 **b** 0.38 **c** 2 011 500 **d** 11.63
e 0.335 **f** 13.69 **g** 0.630625 **h** 1353.275

12 **a** 16.29 **b** 15.52 **c** 66.22 **d** 1.963
e 13.3084 **f** 3.617 **g** 97 **h** 42.7123

13 7.12 m

14 12 200 skis

15 1200 km

16 1807 mm, 1.807 m

17 **a** 5.84 **b** 1 **c** 0.94392 **d** 26.2

18 D 65.1

19 Answers may vary.

Exercise 3F

1 **a** T **b** R **c** R **d** T
e T **f** R **g** T **h** R

2 **a** $0.\dot{3}$ **b** $6.\dot{2}\dot{1}$ or $6.\overline{21}$
c $8.576\dot{4}$ **d** $2.135\dot{6}$ or $2.1\overline{356}$
e $11.\dot{2}8573\dot{}$ or $11.\overline{28573}$ **f** $0.00\dot{3}5\dot{2}$ or $0.00\overline{352}$

3 **a** 4 **b** 9 **c** 7 **d** 6

4 **a** 0.6 **b** 0.75 **c** 0.125 **d** 0.55

5 **a** $0.\dot{3}$ **b** $0.\dot{5}$ **c** $0.8\dot{3}$ **d** $0.\overline{72}$
e $0.\overline{428571}$ **f** $0.\overline{384615}$ **g** $3.1\dot{3}$ **h** $4.\overline{857142}$

6 **a** 0.766 **b** 9.5 **c** 7.0 **d** 21.5134
e 0.95 **f** 17 **g** 8.60 **h** 8.106

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7 **a** 17.01 **b** 5.20 **c** 79.00 **d** 0.00
8 **a** 65 **b** 9 **c** 30 **d** 4563
9 **a** 0.86 **b** 0.22 **c** 0.36 **d** 0.42
10 **a** 9.1 **b** 11.8 **c** 21.3 **d** 11.6
e 2.3 **f** 3
11 **a** 7.7000 **b** 5.0 **c** 0.00700
12 **a** 0 seconds **b** 0 seconds **c** 0.06 seconds
d 12.765 **e** 47 cm
13 4
14 $0.\overline{0588235294117647}$
15 Frieda is correct. Infinite, non-recurring decimals do exist. Examples include pi ($\pi = 3.1415926535\ldots$) and surds such as $\sqrt{3} = 1.73205080\ldots$
16 Student A: When dealing with a critical digit of 5 they incorrectly round down rather than round up. Student B: When rounding down, student B incorrectly replaces final digit with 0.
17 **a** **i** T **ii** R **iii** R **iv** R **v** T **vi** R
b $8 = 2^3, 12 = 2^2 \times 3, 14 = 2 \times 7, 15 = 3 \times 5,$ $20 = 2^2 \times 5, 60 = 2^2 \times 3 \times 5$
c Only denominators which have factors that are powers of 2 and/or 5 terminate.
d **i** T **ii** R **iii** R **iv** T **v** R **vi** R **vii** T **viii** T

Progress quiz

1 **a** $\frac{1}{3} = \frac{2}{6} = \frac{7}{21} = \frac{15}{45}$
b $\frac{5}{2} = \frac{20}{8} = \frac{35}{14} = \frac{125}{50}$
2 **a** $\frac{1}{3}$ **b** $\frac{3}{5}$ **c** $\frac{7}{9}$ **d** $\frac{3}{2}$
3 **a** $\frac{3}{5}$ **b** $\frac{1}{10}$ **c** $-\frac{7}{12}$ **d** $1\frac{1}{5}$
4 **a** $\frac{6}{35}$ **b** $\frac{2}{3}$ **c** $-2\frac{1}{4}$ **d** $4\frac{2}{3}$
5 **a** $2\frac{2}{5}$ **b** $-1\frac{1}{3}$ **c** $1\frac{1}{8}$ **d** $\frac{1}{3}$
6 **a** $0.2531 > 0.24876$ **b** $17.3568 < 17.3572$
7 **a** $\frac{9}{20}$ **b** $6\frac{64}{125}$
8 **a** 0.28 **b** 0.043 **c** 1.75 **d** 0.45
9 **a** 48.347 **b** 177.75
10 **a** 2.3 **b** 0.00937 **c** 36.61
d 18.328 **e** 18.26 **f** 22.05
11 **a** $0.\dot{5}$ **b** $0.\dot{7}\dot{2}$ **c** $6.1\dot{3}$
12 **a** 23.67 **b** 2.740
13 **a** 0.33 **b** 0.42

Exercise 3G

1 B **2** B **3** C **4** A
5 **a** $\frac{39}{100}$ **b** $\frac{11}{100}$ **c** $\frac{1}{5}$ **d** $\frac{3}{4}$
e $1\frac{1}{4}$ **f** $\frac{7}{10}$ **g** $2\frac{1}{20}$ **h** $6\frac{1}{5}$
6 **a** $\frac{3}{8}$ **b** $\frac{31}{200}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$
e $\frac{9}{400}$ **f** $\frac{9}{200}$ **g** $\frac{51}{500}$ **h** $\frac{7}{8}$
7 **a** 0.65 **b** 0.37 **c** 1.58 **d** 3.19
e 0.0635 **f** 0.0012 **g** 40.51 **h** 1.0005
8 **a** 40% **b** 25% **c** 55% **d** 26%
e 22.5% **f** 68% **g** 75% **h** 41.5%
9 **a** 275% **b** 520% **c** 175% **d** 450%
e 348% **f** 194% **g** 770% **h** 915%
10 **a** $33\frac{1}{3}\%$ **b** $12\frac{1}{2}\%$ **c** $8\frac{1}{3}\%$ **d** $6\frac{2}{3}\%$
e 37.5% **f** $28\frac{4}{7}\%$ **g** 18.75% **h** 75%
11 **a** 42% **b** 17% **c** 354.1% **d** 1122%
e 0.35% **f** 4.17% **g** 1% **h** 101%
12 A
13 **a**

Fraction	Decimal	%
$\frac{1}{4}$	0.25	25%
$\frac{2}{4}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{4}{4}$	1	100%

b

Fraction	Decimal	%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$
$\frac{3}{3}$	$0.\dot{9}$	100%

c

Fraction	Decimal	%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{5}{5}$	1	100%

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14 a

Fraction	Decimal	%
$\frac{3}{20}$	0.15	15%
$\frac{6}{25}$	0.24	24%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{40}$	0.125	12.5%
$\frac{7}{10}$	0.7	70%
$\frac{31}{50}$	0.62	62%

b

Fraction	Decimal	%
$\frac{11}{5}$	2.2	220%
$\frac{3}{1000}$	0.003	0.3%
$\frac{13}{200}$	0.065	6.5%
$1\frac{19}{100}$	1.19	119%
$4\frac{1}{5}$	4.2	420%
$\frac{5}{6}$	$0.8\dot{3}$	$83\frac{1}{3}\%$

15 $\frac{1}{8}$, 12.5%, 0.125

16 65%, 80%

17 a $\times\ 100\% = \times\ \frac{100}{100} = \times\ 1$ **b** $\div\ 100\% = \div\ \frac{100}{100} = \div\ 1$

18 a $\frac{BC}{100}$ **b** 0.CDB **c** ABC% **d** DDB.CC%

e $\frac{100A}{D}\%$ **f** $\frac{100(BA + C)}{A}\%$

19 a $\frac{1}{4} = 25\% = 0.25$

$\frac{1}{4} = 25\% = 0.25$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 12\frac{1}{2}\% = 0.125$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 12\frac{1}{2}\% = 0.125$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 12\frac{1}{2}\% = 0.125$

$\frac{1}{2} \times \frac{1}{8} = \frac{1}{16} = 6\frac{1}{4}\% = 0.0625$

$\frac{1}{2} \times \frac{1}{8} = \frac{1}{16} = 6\frac{1}{4}\% = 0.0625$

b–e Answers will vary.

Exercise 3H

1 D **2** A

3 a 100 **b** 10 **c** 5 **d** 2

4 a 80% **b** 65% **c** 78% **d** 30%

e 20% **f** 80% **g** 40% **h** 37.5%

i $36\frac{2}{3}\%$ **j** $70\frac{5}{6}\%$ **k** $70\frac{5}{6}\%$ **l** $94\frac{4}{9}\%$

5 a 30% **b** 45% **c** $31\frac{1}{4}\%$ **d** $83\frac{1}{3}\%$

e 160% **f** $683\frac{1}{3}\%$ **g** $133\frac{1}{3}\%$ **h** $266\frac{2}{3}\%$

6 a 8.33% **b** 66.67% **c** 42.86% **d** 37.5%

e 160% **f** 125% **g** 112.5% **h** 233.33%

7 a 56% **b** 75% **c** 86% **d** 25%

e 40% **f** $33\frac{1}{3}\%$

8 a 5% **b** 25% **c** 5% **d** 25%

e 4% **f** 4000% **g** 300% **h** 600%

9 a 18 **b** 9 **c** 17 **d** 16

e 3 **f** 3 **g** 5.6 **h** 175

i 132 **j** 39.6 **k** 44.8 **l** 36.8

10 a 13 **b** 80 **c** 100 **d** 217

e 67.5 **f** 51.2 **g** 36.75 **h** 70.8

11 a 18 minutes **b** \$0.75 **c** 45 kg

d 62.5 mL **e** 5.6 days **f** 3.3 km

12 a $5\frac{1}{3}$ L **b** 2000 marbles

c \$8 **d** 45 donuts

13 540

14 Murray, Maeheala, Francesca, Wasim

15 $61\frac{1}{9}\%$, $61.\dot{1}\%$

16 68.75%

17 22

18 \$24, \$24, They are the same. $\frac{40}{100} \times 60 = \frac{60}{100} \times 40$ as multiplication is commutative.

19 B

20 D

21 a 240 cm^2

b Area will increase by more than 25%

c 15 cm by 25 cm

d 375 cm^2

e 135 cm^2

f 56.25%

g 56.25% increase in area

h Multiply the percentage increase in each dimension.
$1.25 \times 1.25 = 1.5625 = 56.25\%$ increase

i **i** 21% **ii** 44% **iii** $77\frac{7}{9}\%$ **iv** 125%

j 41.42% increase ($\sqrt{2}$)

k Answers will vary.

Exercise 3I

1 a \$12 **b** \$33.99 **c** \$14.50 **d** \$225

2 a \$40 **b** \$36 **c** \$40.50 **d** \$15

3 a \$12 **b** \$108

4 shop A: \$270, shop B: \$280

5 a \$440 **b** \$276 **c** \$64 **d** \$41 160

e \$5400 **f** \$96.96 **g** \$13.50 **h** \$50.40

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6 **a** $480 **b** $127.50 **c** $39 **d** $104
e $15.40 **f** $630
7 **a** $12 **b** $24 **c** $37.50 **d** $63.75
e $97.50 **f** $4.95
8 **a** $38.50 **b** $82.50 **c** $46.20 **d** $91.30
e $57.75 **f** $164.99
9 shop C: $80, shop D: $75
10 premier rug: $250, luxury rug: $375
11 cash $44.90: credit card: $44.91
12 $265.60
13 $84
14 40%
15 Eastern Bikers, $2320
16 **a** **i** $85 **ii** $630 **iii** $275 **iv** $350
b $106.50
c Sam: $720, Jack: $680, Justin: $700
d Sold: $3200, Wage: $960

Exercise 3J

1 **a** $7 **b** $28 **c** $3.45 **d** $436
2 **a** $13 **b** $45 **c** $25.90 **d** $247
3 D
4 A
5 **a** 80% profit **b** 30% profit **c** 25% loss
d 16% loss **e** $66\frac{2}{3}$% profit **f** 37.5% profit
g $33\frac{1}{3}$% loss **h** 22% loss
6 **a** 20% increase **b** $16\frac{2}{3}$% decrease
c 500% increase **d** 150% increase
7 **a** 25% increase **b** 20% increase
c 140% increase **d** 25% decrease
8 **a** 4% **b** 6.25% **c** 0.6% **d** 0.5%
9 20% loss
10 650% profit
11 **a** $36 **b** 75% profit
12 **a** $350 **b** 87.5% profit
13 **a** $2200 **b** 44% loss
14 90% of (110% of x) is less than 100% of x.
15 store A
16 **a** **i** 100% **ii** $66\frac{2}{3}$% **iii** 80%
b term 2
c 500% growth
17 **a** 396 071
b **i** 22 803 197 **ii** 24 056 867
iii 26 301 345
c 2.0%
d–h Answers will vary.

Exercise 3K

1 B **2** D **3** B **4** $500
5 **a** $900 **b** $800 **c** $1100 **d** $500
e $550 **f** $250
6 $90
7 **a** $120 **b** $240 **c** $15 **d** $21
8 1100 litres **9** 600 kg **10** $300
11 **a** $50 **b** $150 **c** $600 **d** $30
e $10 **f** $2000
12 200 **13** $40 **14** D **15** $\frac{800}{y}$
16 $\frac{D}{C} \times F$ *or* $\frac{FD}{C}$
17 **a** $70 **b** $110 **c** $50
d $450 **e** $650 **f** Answers will vary.

Problems and challenges

1 72.8% **2** 96% **3** $\frac{1}{1000}$ **4** $48
5 53% **6** $\frac{5}{12}$ **7** 30% **8** 49 years
9 6:33 a.m.
10 **a** $b = -\frac{5}{6},\ c = -\frac{1}{3}$
b $b = 1\frac{13}{15},\ c = 2\frac{1}{30}$

Multiple-choice questions

1 B **2** C **3** D **4** C **5** D
6 B **7** B **8** A **9** C **10** B

Short-answer questions

1 **a** 21 **b** 8 **c** 10
2 **a** $\frac{5}{9}$ **b** 3 **c** $\frac{17}{2}$
3 **a** $\frac{7}{11}$ **b** $\frac{1}{8}$ **c** $1\frac{3}{4}$ **d** $1\frac{7}{12}$
e $5\frac{13}{20}$ **f** $3\frac{17}{30}$
4 **a** 8 **b** $\frac{13}{28}$ **c** 14 **d** 6
e $\frac{1}{18}$ **f** 2
5 **a** $-\frac{7}{15}$ **b** $-\frac{3}{20}$ **c** $\frac{9}{25}$ **d** $\frac{19}{20}$
e –5 **f** $-7\frac{7}{12}$
6 **a** = **b** < **c** >
7 **a** 30.38 **b** 12.803 **c** 56 974 **d** 50 280
e 74 000 **f** 2.9037
8 **a** 10.68 **b** 0.1068 **c** 14.4 **d** 0.255
e 3.6 **f** 197.12
9 **a** 0.667 **b** 3.580 **c** 0.005

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10

0.1	0.01	0.05	0.5	0.25	0.75	$0.\dot{3}$	0.125
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{8}$
10%	1%	5%	50%	25%	75%	$33\frac{1}{3}\%$	12.5%

11 **a** 87.5% **b** 25% **c** 150%
d 4% **e** 6%
12 **a** 24 **b** \$10.50 **c** 10.5
13 **a** \$616 **b** \$3280
c \$977.55, overall percentage loss is 0.25%
14 \$359.91, \$3639.09
15 29%
16 \$680
17 4%
18 1120
19 \$3.31

Extended-response questions

1 **a** 21 000 INR, 625 SGD, 15 000 THB, 3500 HKD
b \$30
c **i** 800 **ii** 3.8%
d **i** \$96.48 **ii** \$95.03, not enough to buy perfume.

Chapter 4

Exercise 4A

1 **a** 10 **b** 100 **c** 1000 **d** 100 000
e 1000 **f** 1 000 000
2 **a** 1000 **b** 100 000 **c** 1 000 000
3 **a** 10 **b** 10 **c** 2
4 **a** 12 m **b** 22 cm **c** 10 m **d** 44 km
e 1.4 cm **f** 3.6 m
5 **a** 30 mm **b** 610 cm **c** 8930 m **d** 3000 mm
e 2.1 m **f** 32 cm **g** 9.62 km **h** 0.38 km
i 4.3 mm **j** 2040 cm **k** 23.098 m **l** 3.42 km
m 194.3 m **n** 0.01 km **o** 24.03 mm **p** 0.994 km
6 **a** 19 m **b** 44 m **c** 13 cm **d** 32 cm
e 28 km **f** 18 cm **g** 17.2 mm **h** 34.4 cm
i 29.4 m
7 **a** 5 **b** 2 **c** 4 **d** 18
e 9.5 **f** 6.5
8 **a** 40 cm **b** 17 cm **c** 7.8 cm **d** 2000 cm
e 46 cm **f** 17 600 cm
9 **a** 2 **b** 3 **c** 9
10 \$2392 **11** 8 min **12** 240 cm
13 **a** $P = 2a + b$ **b** $P = 2a + 2b$
c $P = 2a + 2b$ **d** $P = 2a + 2b$
e $P = 8a + 2b$ **f** $P = 4a + 2b$
14 **a** $x = P - 11$ **b** $x = P - 4$
c $x = \frac{P-3}{2}$ **d** $x = \frac{P-8}{2}$
e $x = \frac{P}{4}$ **f** $x = \frac{P}{8}$
15 **a** 6 squares **b** 8 squares

Exercise 4B

1 **a** 15.71 **b** 40.84 **c** 18.85 **d** 232.48
2 **a** 3.1 **b** 3.14 **c** 3.142
3 **a** diameter **b** radius **c** circumference
4 Answer is close to pi.
5 **a** 12.57 mm **b** 113.10 m **c** 245.04 cm **d** 12.57 m
e 21.99 km **f** 15.71 cm
6 **a** 314 cm **b** 62.8 m **c** 18.84 km
7 **a** 44 mm **b** 132 cm **c** 220 m
8 **a** 6.4 cm **b** 47.7 m **c** 7.0 mm **d** 319.9 km
9 11.0 m
10 12 566 m
11 **a** 64.27 cm **b** 12.34 m **c** 61.70 mm
12 **a** 28.57 cm **b** 93.97 m **c** 5.57 cm
13 **a** 25.13 cm **b** 56.55 m **c** 35.71 m
14 Svenya and Andre
15 $d = 2r$, so $2\pi r$ is the same as πd.
16 **a** 4π **b** 36π **c** 78π **d** 4π
e 7π **f** 5π
17 **a** 8π **b** 18π **c** $5\pi + 20$
18 **a** **i** $r = \frac{C}{2\pi}$ **ii** $d = \frac{C}{\pi}$
b **i** 2.23 m **ii** 6.37 cm
19 Answers will vary.

Exercise 4C

1 **a** **i** 100 **ii** 400 **iii** 3
b **i** 10 000 **ii** 70 000 **iii** 4
c **i** 1 000 000 **ii** 5 000 000 **iii** 2.5
d **i** 10 000 **ii** 30 000 **iii** 7.5
2 **a** 7 m, 3 m **b** 8 cm, 6 cm (or other way around)
c 2.4 mm, 1.7 mm
3 10 000
4 **a** 200 mm^2 **b** 70 000 cm^2 **c** 500 000 m^2
d 30 000 m^2 **e** 34 mm^2 **f** 0.07 m^2
g 30.9 cm^2 **h** 4000 m^2 **i** 0.2 m^2
j 0.45 km^2 **k** 0.4 ha **l** 32.1 cm^2
m 32 ha **n** 51 cm^2 **o** 4.3 mm^2
p 0.4802 m^2 **q** 1.904 ha **r** 0.2933 ha
s 49 m^2 **t** 7700 m^2 **u** 24 000 m^2
5 **a** 9 cm^2 **b** 21 m^2 **c** 39 cm^2 **d** 18 cm^2 **e** 33 m^2
f 144 mm^2 **g** 63 m^2 **h** 3 m^2 **i** 6 km^2

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6 **a** 70 m^2 **b** 54 m^2 **c** 140 cm^2 **d** 91 cm^2
e 46 km^2 **f** 64 mm^2
7 **a** 200 000 mm^2 **b** 430 000 cm^2 **c** 0.0000374 km^2
d 0.01092 m^2 **e** 20 cm^2 **f** 0.1 ha
8 **a** 45 cm^2 **b** 168 m^2 **c** 120 km^2
9 **a** 6 m **b** 1.5 cm
10 **a** 25 m^2 **b** 20.25 cm^2 **c** 28 cm **d** 52 m
11 10 cm
12 \$48
13 **a** $A = 4b^2 + ab$ or $A = b(4b + a)$
b $A = 1.5ab$ or $A = \frac{3ab}{2}$
c $A = a^2 - \frac{ab}{2}$ or $A = a\left(a - \frac{b}{2}\right)$
14 **a** 4 **b** 1
15 **a** $w = \frac{A}{l}$ **b** $l = \sqrt{A}$ **c** $h = \frac{2A}{b}$
16 **a** **i** 2.59 km^2 **ii** 2 589 988 m^2
iii 259 ha **iv** 4047 m^2
v 0.4 ha **vi** 2.5 acres
b 81 ha
c 62%

Exercise 4D

1 **a** 6 **b** 30 **c** 13.5 **d** 33
2 **a** 90° **b** height **c** perpendicular
d parallel, perpendicular **e** rhombus, kite
3 **a** parallelogram, 50 m^2 **b** parallelogram, 4.5 cm^2
c parallelogram, 6 m^2 **d** rhombus, 7.5 cm^2
e rhombus, 121 km^2 **f** rhombus, 9.61 m^2
g kite, 4 cm^2 **h** kite, 300 mm^2
i kite, 0.9 mm^2 **j** trapezium, 96 cm^2
k trapezium, 32.5 m^2 **l** trapezium, 560 mm^2
4 **a** 6 cm^2 **b** 35 m^2 **c** 84.5 cm^2
5 0.27 m^2
6 2 m
7 \$1160
8 3 cm and 9 cm
9 **a** trapezium **b** 19.5 cm^2
10 **a** $A = 2x^2$ **b** $A = 3ab$ **c** $A = 8a^2$
11 No, use formula for parallelogram $A = bh$, as we already know these lengths.
12 **a** $A = 4$ triangle areas

$$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$$

$$= 4 \times \frac{1}{2} \times \frac{1}{2}x \times \frac{1}{2}y$$

$$= \frac{1}{2}xy$$

b A = Area (triange 1) + Area (triange 2)

$$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$$

$$= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$$

$$= \frac{1}{2}ah + \frac{1}{2}bh$$

$$= \frac{1}{2}(a + b)h$$

c A = Area (rectangle) + Area (triangle)

$$= \text{length} \times \text{width} + \frac{1}{2} \times \text{base} \times \text{height}$$

$$= a \times h + \frac{1}{2} \times (b - a) \times h$$

$$= ah + \frac{1}{2}bh - \frac{1}{2}ah$$

$$= \frac{1}{2}ah + \frac{1}{2}bh$$

$$= \frac{1}{2}(a + b)h$$

13 Answers will vary.

Exercise 4E

1 **a** 31.4 **b** 12.56 **c** 22 **d** 154
2 **a** 78.54 **b** 530.93 **c** 30.19 **d** 301.72
3 **a** 5 m **b** 2.3 mm **c** 3.5 km
4 **a** 28.27 cm^2 **b** 113.10 m^2 **c** 7.07 mm^2 **d** 78.54 km^2
e 36.32 cm^2 **f** 9.08 m^2
5 **a** 154 cm^2 **b** 154 km^2 **c** 616 mm^2 **d** 314 km^2
e 12.56 m^2 **f** 31 400 m^2
6 **a** 3.14 cm^2 **b** 201.06 cm^2 **c** 226.98 mm^2
d 39.27 cm^2 **e** 5.09 mm^2 **f** 100.53 m^2
7 **a** 3.3 cm **b** 3.2 m
8 707 cm^2
9 yes, by 1310 cm^2
10 no ($A = 0.79$ km^2)
11 78.54 cm^2
12 circle of radius 5 m
13 80 cm^2
14 **a** 12.56 cm^2 **b** 50.24 cm^2
c quadrupled ($\times 4$) **d** multiplied by 9
e multiplied by 16 **f** multiplied by n^2
15 **a** 81π **b** $\frac{49\pi}{4}$ **c** 72π
16 **a** $A = \frac{\pi d^2}{4}$ **b** true
17 **a** true
b **i** 2.33 m **ii** 1.20 km **iii** 10.09 mm
c $r = \sqrt{\frac{A}{\pi}}$

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Exercise 4F

1 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$ **d** $\frac{1}{8}$

2 **a** 2.79 **b** 8.55 **c** 9.69

3 **a** $\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$

4 **a** 88.49 mm^2 **b** 104.72 mm^2
c 4.91 cm^2 **d** 14.14 cm^2
e 61.28 m^2 **f** 262.72 cm^2
g 181.53 m^2 **h** 981.93 m^2
i 2428.12 km^2

5 **a** 37.70 m^2 **b** 137.44 m^2 **c** 437.21 km^2

6 **a** 34.82 m^2 **b** 9.14 m^2
c 257.08 cm^2 **d** 116.38 mm^2
e 123.61 km^2 **f** 53.70 m^2
g 50.27 m^2 **h** 75.40 mm^2
i 12.57 cm^2

7 1.26 m^2

8 13 cm radius pizza by 0.13 cm^2

9 16 965 cm^2

10 **a** 78.5% **b** 30.8% **c** 21.5%

11 **a** π cm^2 **b** $\frac{25\pi}{9}$ m^2 **c** 8π mm^2 **d** $\frac{75\pi}{2}$ m^2
e $9\pi + 9$ cm^2 **f** $225 - \frac{225\pi}{4}$ km^2

12 **a** 78.5%
b 78.5%, Same answers as for part **a**.
c Percentage area $= \frac{\pi r^2}{4} \div r^2 \times 100 = 25\pi \approx 78.5\%$

13 **a** 6.54 m^2 **b** 2.26 m^2 **c** 5.8%

Exercise 4G

1 **a** 6, squares **b** 6 squares and rectangles
c 6, isosceles triangles and rectangles

2 **a** C **b** A **c** B

3 **a** 3 **b** 6 **c** 6 **d** 5

4 **a** 24 cm^2 **b** 403.44 m^2 **c** 22 cm^2
d 352 cm^2 **e** 84 m^2 **f** 612 cm^2
g 120 mm^2 **h** 114 m^2 **i** 29.7 m^2

5 **a** 18 cm^2 **b** 146 cm^2

6 2000 cm^2 **7** 81 m^2

8 **a** 138 m^2 **b** 658 m^2 **c** 62 cm^2

9 $107.25

10 **a** $SA = 6l^2$ **b** $SA = 2w^2 + 4lw$
c $SA = 2wl + 2wh + 2lh$

11 **a** **i** quadrupled **ii** multiplied by 9
iii multiplied by 16
b multiplied by n^2

12 **a** 15 072 cm^2 **b** 456 cm^2

Exercise 4H

1 **a** 24 **b** 12 **c** 72

2 **a** 96 **b** 6 **c** 2 **d** 5

3 **a** 1000 **b** 1 **c** 1 **d** 1
e 1 **f** 1000

4 **a** 36 cm^3 **b** 20 m^3 **c** 27 mm^3 **d** 64 km^3
e 320 mm^3 **f** 24 m^3

5 **a** 2000 mL **b** 5000 L **c** 500 kL **d** 3 L
e 4 cm^3 **f** 50 mL **g** 2.5 L **h** 5100 cm^3

6 **a** 24 L **b** 42 L **c** 27 L **d** 18 000 L
e 24 000 L **f** 360 L

7 **a** **i** 60 000 000 L **ii** 60 000 kL **iii** 60 ML
b 200 days

8 80 minutes

9 **a** 500 000 m^3 **b** 500 ML

10 8000 kg

11 **a** (1, 1, 12) or (1, 2, 6) or (1, 3, 4) or (2, 2, 3)
b (1, 1, 30) or (1, 2, 15) or (1, 3, 10) or (1, 5, 6) or (2, 3, 5)
c (1, 1, 47)

12 Factors greater than 1 include only 2, 23 and 46, but 23 is prime so you cannot find 3 whole numbers that multiply to 46.

13 9

14 **a** 20 **b** 20
c Equal, the number of cubes on the base layer is the same as the number of squares on the base.
d If the number of cubes on the base layer is the same as the number of squares on the base, then Ah gives h layers of A cubes, giving the total.
e Yes, a rectangular prism could use 3 different bases.

15 **a** **i** By joining two of the same prisms together a rectangular prism could be formed.
ii 12 $units^3$
b **i** 160 cm^3 **ii** 140 m^3 **iii** 2 cm^3 **iv** 112 m^3
v 48 mm^3 **vi** 171 cm^3

Exercise 4I

1 **a** **i** prism **ii** rectangle
b **i** prism **ii** triangle
c **i** not a prism (pyramid)
d **i** not a prism (cone)
e **i** prism **ii** square
f **i** not a prism (truncated pyramid)

2 **a** 8, 2 **b** 6, 1.5 **c** 12, 10

3 **a** 40 **b** 36 **c** 10 **d** 6280

4 **a** 44 m^3 **b** 160 cm^3 **c** 352 mm^3

5 **a** 200 cm^3 **b** 15 m^3 **c** 980 cm^3 **d** 60 m^3
e 270 mm^3 **f** 60 m^3

6 **a** 785.40 m^3 **b** 12 566.37 mm^3 **c** 251.33 cm^3
d 7696.90 cm^3 **e** 461.81 m^3 **f** 384.85 m^3
g 1178.10 m^3 **h** 2.41 cm^3 **i** 1733.96 km^3

7 **a** 14.137 m^3 **b** 14 137 L

8 **a** cylindrical **b** 283 L

9 3 (almost 4 but not quite)

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10 302.48 cm^3

11 **a** 56 000 L **b** 56 hours

12 **a** 20π m^3 **b** 300π cm^3
c 144π mm^3 **d** 245π m^3

13 Answers may vary, an example is $r = 5$ cm and $h = 1.27$ cm.

14 $x = \pi h$

15 **a** 14.28 cm^3 **b** 98.17 mm^3
c 1119.52 cm^3 **d** 8.90 m^3
e 800 m^3 **f** 10 036.67 cm^3

Progress quiz

1 **a** 6400 mm **b** 1.8 m **c** 0.97 km **d** 250 cm

2 **a** 78 cm **b** 12.4 m **c** 44 cm **d** 84 mm

3 $x = 9$

4 **a** C = 43.98 cm A = 153.94 cm^2
b C = 81.68 mm A = 530.93 mm^2

5 **a** 47 000 cm^2 **b** 41 cm^2
c 0.5 ha **d** 8000 m^2

6 **a** 51.84 cm^2 **b** 0.7 m^2 **c** 24 cm^2 **d** 164 m^2
e 20.48 m^2 **f** 16.5 m^2 **g** 3 cm^2

7 **a** P = 11.49 cm A = 6.98 cm^2
b P = 21.42 cm A = 28.27 cm^2
c P = 18.09 cm A = 16.36 cm^2

8 **a** 22.87 cm^2 **b** 63.27 cm^2

9 **a** SA = 384 cm^2 V = 512 cm^3
b SA = 288 cm^2 V = 288 cm^3
c SA = 142.66 cm^2 V = 91 cm^3

10 **a** V = 0.2749 m^3 **b** capacity = 274.9 L

Exercise 4J

1 **a** F **b** D **c** A **d** E
e B **f** C

2 **a** 120 s **b** 3 min **c** 2 h **d** 240 min
e 72 h **f** 2 days **g** 5 weeks **h** 280 days

3 **a** 6 h 30 min **b** 10 h 45 min
c 16 h 20 min **d** 4 h, 30 min

4 **a** 180 min **b** 630 s **c** 4 min **d** 1.5 h
e 144 h **f** 3 days **g** 168 h **h** 1440 min
i 4 h **j** 2 weeks **k** 20 160 min
l 86 400 s **m** 5 s **n** 2.5 s **o** 7 s
p 400 milliseconds **q** 2.7 microseconds
r 3 nanoseconds

5 **a** 6:30 p.m. **b** 9 a.m. **c** 6:30 p.m. **d** 4:30 p.m.
e 5:30 p.m. **f** 11:40 a.m.

6 **a** 1330 h **b** 2015 h **c** 1023 h **d** 2359 h
e 6:30 a.m. **f** 1 p.m. **g** 2:29 p.m. **h** 7:38 p.m.
i 11:51 p.m.

7 **a** 2 p.m. **b** 5 a.m. **c** 1200 hours **d** 1800 hours

8 **a** 2 h 50 min **b** 6 h 20 min **c** 2 h 44 min
d 8 h 50 min **e** 8 h 19 min **f** 10 h 49 min

9 **a** 11 a.m. **b** 12 p.m. **c** 8 p.m. **d** 7:30 p.m.
e 7 a.m. **f** 5 a.m. **g** 1 a.m. **h** 10 a.m.

10 **a** 5:30 a.m. **b** 7:30 a.m. **c** 6:30 a.m. **d** 1:30 p.m.
e 2:30 p.m. **f** 2:30 a.m. **g** 3 p.m. **h** 5:30 p.m.

11 **a** 5 h **b** 2.5 h **c** 8 h **d** 6 h
e 7 h

12 56 million years **13** 17 min 28 s

14 7 h 28 min **15** 23 h 15 min

16 **a** 33c **b** 143c or $1.43

17 **a** $900 **b** $90 **c** $1.50 **d** 2.5c

18 6:30 a.m.

19 **a** 8 a.m. 29 March **b** 10 p.m. 28 March
c 3 a.m. 29 March

20 3 a.m. 21 April

21 11:30 p.m. 25 October

22 52.14 weeks

23 **a** 3600 **b** 1440 **c** 3600 **d** 1440

24 Friday

25 **a** You have to turn your clock back.
b You have to turn your clock forward.
c You adjust the date back one day.

26 Students' reports will vary.

Exercise 4K

1 **a** 9 **b** 25 **c** 144 **d** 2.25
e 20 **f** 58 **g** 157 **h** 369

2 **a** false **b** true **c** true **d** true
e false **f** false

3 hypotenuse, triangle

4 **a** c **b** x **c** u

5 **a** No **b** No **c** Yes **d** Yes
e Yes **f** No **g** Yes **h** No
i No

6

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
6	8	10	36	64	100	100
8	15	17	64	225	289	289

a $a^2 + b^2$ and c^2
b **i** 13 **ii** 20
c **i** 25 **ii** 110

7 **a** $3^2 + 4^2 = 5^2$ **b** $8^2 + 15^2 = 17^2$
c $9^2 + 12^2 = 15^2$ **d** $5^2 + 12^2 = 13^2$
e $9^2 + 40^2 = 41^2$ **f** $2.5^2 + 6^2 = 6.5^2$

8 **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$
c $d^2 + h^2 = x^2$

9 **a** no
b No, $a^2 + b^2 = c^2$ must be true for a right-angled triangle.

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10 a Answers will vary. See answer to part **b** for the list of possible answers.

b {(6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)}, {(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)}, {(7, 24, 25), (14, 48, 50), (21, 72, 75)}, {(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68), (40, 75, 85)}, {(9, 40, 41), (18, 80, 82)}, {(11, 60, 61)}, {(20, 21, 29), (40, 42, 58), (60, 63, 87)}. {(12, 35, 37), (24, 70, 74)}, {(28, 45, 53)}. {(33, 56, 65)}, {(16, 63, 65)}, {(48, 55, 73)}, {(13, 84, 85)}, {(36, 77, 85)}, {(39, 80, 89)}, {(65, 72, 97)}

11 a yes **b** no **c** no **d** yes
e no **f** yes

12 a a^2 **b** b^2 **c** $\sqrt{a^2 + b^2}$

13 $2x^2 = c^2$

14 a Area of inside square $= c^2$

Area of 4 outside triangles $= 4 \times \frac{1}{2} \times \text{base} \times \text{height} = 2ab$

Total area of outside square $= (a + b)^2 = a^2 + 2ab + b^2$

Area of inside square = Area (outside square) − Area of 4 triangles

$= a^2 + 2ab + b^2 - 2ab$

$= a^2 + b^2$

Comparing results from the first and last steps gives $c^2 = a^2 + b^2$.

b Answers will vary.

Exercise 4L

1 a yes **b** no **c** no **d** yes

2 a 3.16 **b** 5.10 **c** 8.06 **d** 15.17

3 a $c^2 = a^2 + b^2$
$= 5^2 + 12^2$
$= 169$
$\therefore c = \sqrt{169}$
$= 13$

b $c^2 = a^2 + b^2$
$= 9^2 + 40^2$
$= 1681$
$\therefore c = \sqrt{1681}$
$= 41$

c $c^2 = a^2 + b^2$
$= 9^2 + 12^2$
$= 225$
$\therefore c = \sqrt{225}$
$= 15$

4 a 5 **b** 25 **c** 41 **d** 20
e 45 **f** 61

5 a 9.22 **b** 5.39 **c** 5.66 **d** 3.16
e 4.30 **f** 37.22

6 3.16 m or 316 cm **7** 139 cm
8 5.5 km **9** 3.88 cm

10 a 2nd line is incorrect, cannot take the square root of each term.

b 2nd line is incorrect, cannot add $3^2 + 4^2$ to get 7^2.

c Last line should say $\therefore c = \sqrt{29}$.

11 a $1^2 + 2^2 \neq 3^2$ **b** $5^2 + 8^2 \neq 10^2$
c $12^2 + 21^2 \neq 24^2$

12 a 8.61 m **b** 48.59 cm **c** 18.56 cm **d** 22.25 mm
e 14.93 m **f** 12.25 m

Exercise 4M

1 a 4 **b** 7 **c** 3 **d** 4 **e** 8
f 20 **g** 3 **h** 5 **i** 25

2 a $a = 12$ **b** $b = 24$

3 a 4 **b** 9 **c** 40 **d** 15
e 16 **f** 60

4 a 2.24 **b** 4.58 **c** 11.49 **d** 12.65
e 10.72 **f** 86.60

5 8.94 m **6** 12 cm **7** 12.12 cm **8** 8.49

9 a Should subtract not add 10.

b Should say $a = 5$.

c Can't take the square root of each term.

10 a $\sqrt{24}$ **b** $\sqrt{3}$ **c** $\sqrt{4400}$

11 a 3.54 **b** 7.07 **c** 43.13 **d** 24.04

12 a $6^2 + 8^2 = 10^2$ **b** It is a multiple of (3, 4, 5).
c (9, 12, 15), (12, 16, 20), (15, 20, 25) **d** (8, 15, 17)
e (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), (9, 40, 41), etc.

Problems and challenges

1 10 cm each side **2** Yes, 1 L will overflow.
3 $\frac{1}{2}$ **4** $\sqrt{3} \approx 1.73$ m
5 $\sqrt{2}$ **6** $\sqrt{7}$
7 72 cm^2 **8** 63.66%

Multiple-choice questions

1 E **2** B **3** A **4** C **5** B
6 E **7** E **8** B **9** D **10** D

Short-answer questions

1 a 2000 mm **b** 0.5 km **c** 300 mm^2 **d** 0.4 m^2
e 10000 m^2 **f** 3.5 cm^2 **g** 0.4 L **h** 200 L

2 a 13 m **b** 28 cm **c** 25.13 m **d** 51.42 mm
e 48 m **f** 20 cm

3 a 55 cm^2 **b** 63 m^2 **c** 12 cm^2 **d** 136 km^2
e 64 m^2 **f** 20 cm^2 **g** 28.27 cm^2 **h** 12.57 m^2
i 3.84 cm^2

4 a 70 cm^2 **b** 50.14 cm^2 **c** 74 cm^2

5 a 320 m^2 **b** 408 mm^2 **c** 138 cm^2

6 a 1000 L **b** 8 L **c** 0.144 L

7 a 2513.27 m^3 **b** 3078.76 cm^3 **c** 212.06 mm^3

8 a i 287°C **ii** 239°C

b 1 h 39 min 18 s

c 1 h 2 min 4 s

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9 **a** 10 h 17 min **b** 9:45 p.m. **c** 2331 hours
10 **a** 6:30 p.m. **b** 6 p.m. **c** 8:30 a.m.
d 4:30 p.m. **e** 10:30 a.m. **f** 5:30 p.m.
g 8:30 p.m. **h** 8:30 p.m.
11 **a** 10 **b** 25 **c** 4.24
12 **a** 15 **b** 6.24 **c** 11.36

Extended-response questions

1 **a** 12.86 cm^2 **b** 57.72 cm^2 **c** 57.72 m^2
d 2.79 cm^3 **e** 25.72 cm^3
f 38 with some remainder
2 **a** 2.8 m **b** 24 m^2 **c** 23 m **d** No, 4000 L short

Chapter 5

Exercise 5A

1 **a** $3a, 2b, 5c$
b **i** $a = 3$ **ii** $b = 2$ **iii** $c = 5$
c $2x + 5y + 8z$. Other answers are possible.
2 **a** 6
b **i** $a = 5$ **ii** $b = 7$ **iii** $c = 1$
c $x + 2y + 3z + 4w + 91k$. Other answers are possible.
3 **a** F **b** C **c** E
d D **e** A **f** B
4 **a** **i** 3 **ii** $7a, 2b, c$
b **i** 3 **ii** $19y, 52x, 32$
c **i** 2 **ii** $a, 2b$
d **i** 4 **ii** $7u, 3v, 2a, 123c$
e **i** 2 **ii** $10f, 2be$
f **i** 5 **ii** $9, 2b, 4c, d, e$
g **i** 4 **ii** $5, x^2y, 4abc, 2nk$
h **i** 4 **ii** $ab, 2bc, 3cd, 4de$
5 **a** 2 **b** 1 **c** 9 **d** –2
e 1 **f** 0 **g** 0 **h** –6
i –1 **j** –12 **k** –1 **l** –3
6 **a** $y + 7$ **b** $x - 3$ **c** $a + b$ **d** $4p$
e $4 - \frac{q}{2}$ **f** $10 + \frac{r}{3}$ **g** $2(b + c)$ **h** $b + 2c$
i $\frac{abc}{7}$ **j** $\frac{a}{4} + \frac{b}{2}$ **k** $\frac{x}{2y}$ **l** $a - \frac{b}{2}$
m k^2 **n** w^2
7 **a** the sum of 3 and x **b** the sum of a and b
c the product of 4, b and c **d** double a is added to b
e b is subtracted from 4 and the result is doubled.
f b is doubled and the result is subtracted from 4.
8 **a** $7x$
b **i** $x - 3$ **ii** $7(x - 3)$
9 **a** $2p$ **b** $48p$ **c** $30p + 18(p + 20)$
10 **a** $4a$ **b** $7b$ **c** $5a + 5b$ **d** $\frac{7a + 7b}{2}$
11 $70 + 90x$
12 **a** $20 + 50t$ **b** $0.2 + \frac{t}{2}$ **c** $0.2 + 30t$
13 **a** true **b** true **c** false **d** true
e false **f** false
14 **a** true **b** true **c** false **d** true
15 In $2a + 5$, the number is doubled then 5 is added. In $2(a + 5)$, the number is increased by 5 and this result is doubled.
16 **a** 36 **b** 676 **c** t **d** $121k$
e 67 108 863

Exercise 5B

1 15 **2** 8 **3** 30
4 **a** 14 **b** 30 **c** no
5 **a** 30 **b** 37 **c** 16 **d** 58
e 2 **f** –40 **g** –61 **h** –19
6 **a** 24 **b** –9 **c** 1 **d** –19
e 12 **f** 90 **g** 1 **h** –7
i 100 **j** 13 **k** 45 **l** 16
7 **a** 8 **b** 32 **c** 6 **d** –2
e 13 **f** –31
8 **a** 96 **b** 122 **c** 24 **d** –38
e 132 **f** 54
9 **a** E **b** E **c** N **d** E
e N **f** N **g** E **h** E
10 **a** $3 + x$ **b** $2 - a$ **c** $4t - 2t$ **d** $3u - 8$
11 $4y + 2x + 5, 5 + 2x + 4y, 2(x + 2y) + 5$. Other answers are possible.
12

x	3	1	0.25	6	–2	2
$4x + 2$	14	6	3	26	–6	10
$4 - 3x$	–5	1	3.25	–14	10	–2
$2x - 4$	2	–2	–3.5	8	–8	0

13 **a** (a, b) : (1, 10), (2, 5), (5, 2), (10, 1), (–1, –10), (–2, –5), (–5, –2), (–10, –1)
b infinitely many solutions, e.g. $a = -42, b = 52$
c $a = 0, b = 0$ or $a = 2, b = 2$
14 **a** Yes, only when $y = 0$.
b No, need to be equal for all values of x and y.
15 **a** $24 \div (2 \times 3) = 4$ but $(24 \div 2) \times 3 = 36$ (other answers are possible)
b No, as there is a division rather than two multiplications
c No. For example, $24 \div (6 \div 2) = 8$ but $(24 \div 6) \div 2 = 4$.
16 **a** $5 - a$ and $a - 5$. Other answers are possible.
b $17(a - b)$ and $38(b - a)$. Other answers are possible.
c x and $x + 1$

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17 a They are equivalent

b No. For example, $(2+3)^2 = 25$ but $2^2 + 3^2 = 13$.

c Yes

d No. For example, $\sqrt{9+16} = 5$ but $\sqrt{9} + \sqrt{16} = 7$

e For part b. if $a = 0$ or $b = 0$ they are equal. For part d. if $a = 0$ or $b = 0$ they are equal.

18 a Multiplication is commutative (order unimportant).

b Adding a number to itself is double the number.

c A number subtracted from itself always results in zero.

d Dividing by 2 and multiplying by $\frac{1}{2}$ have the same effect.

19

a	5	8	2	3	–20	10	–9	–6
b	2	2	1	7	10	–3	10	–13
$a+b$	7	10	3	10	–10	7	1	–19
$a+2b$	9	12	4	17	0	4	11	–32
$a-b$	3	6	1	–4	–30	13	–19	7
$a-2b$	1	4	0	–11	–40	16	–29	20

Exercise 5C

1 a 21 **b** 21 **c** true

2 a 23 **b** 84 **c** false

3 a 28

b i 12 ii 20 iii 28

c $7x$

4 a L **b** L **c** L **d** N

e N **f** L **g** N **h** N

5 a L **b** L **c** N

d L **e** N **f** N

6 a $5x$ **b** $19a$ **c** $9x$ **d** $7xy$

e $13uv$ **f** $14ab$ **g** $7ab$ **h** $16k$

i $10k$

7 a $9f + 12$ **b** $13x + 8y$ **c** $7a + 11b$

d $13a + 9b$ **e** $12 + 12x$ **f** $8a + 3b + 3$

g $14x + 30y$ **h** $21a + 4$ **i** $17x^2y + 5x$

j $13xy$ **k** $-x^2$ **l** $2a + 4b - 7ab$

m $10 + 9q - 4r$ **n** $9b + 2b^2$

8 a C **b** A **c** D **d** E **e** B

9 a $22x$ **b** $6y + 6 + 2x$ **c** $12a + 4b$

10 a $13c$ **b** $\$9nc$

11 a 7, 2 **b** 6, 7 **c** 7, 9, 5 **d** 8, 6

12 From left to right, top to bottom: $2x$, $2y$, $3y$, $5x + y$, $2x + 2y$

13 9 ways

14 $-50a$

15 Both are equivalent to $17x + 7y$.

16 a If $a = 1, b = 2$: $4a + 3b = 10, 7ab = 14$. Other answers are possible.

b Yes. For example, if $a = 0$ and $b = 0$.

c No. They are equivalent.

17 a Yes, both are equivalent to $4x$.

b –13, –3, –2

18 a From top left to bottom right: $a + 3$, $-a$, $2b + 4$, $2a$, $a + 1$, $7a + 2b$, 0

b Answers will vary.

Exercise 5D

1 B

2 a $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$

3 B

4 a $3xy$ **b** $5abc$ **c** $12ab^2$ **d** $4ac^3$

5 a $63d$ **b** $10ab$ **c** $36x$ **d** $8abcd$

e $60abcd$ **f** $48abde$

6 a $24abc$ **b** a^2 **c** $3d^2$ **d** $10d^2e$

e $14x^2y$ **f** $10x^2y$ **g** $8x^2yz$ **h** $8a^2b^2cd$

i $48x^3y$ **j** $18a^3b$ **k** $24x^3y^2$ **l** $6xz^2$

m $-10xy^2z$ **n** $70a^2b^3$ **o** $16xy^3$

7 a $\frac{1}{2}$ **b** $\frac{x}{2y}$ **c** $\frac{5x}{6}$ **d** $\frac{a}{4}$

e $\frac{x}{3}$ **f** $\frac{1}{6x}$ **g** $\frac{-x}{2yz^2}$ **h** $\frac{-2y}{3}$

i $\frac{-a}{2b}$ **j** $\frac{-7p}{q}$ **k** 7 **l** $\frac{3}{4z}$

8 a $8ab$ **b** $24x^2$ **c** $18xy$ **d** $17xy$

e $13a^2$ **f** $88xy$

9 a $2y$ **b** $3b^2$ **c** -2 **d** $28rs$

e $8ab^2$ **f** $-7x$

10 $18x^3$

11 a x^2 **b** $4x$ **c** $\frac{x^2}{4x} = \frac{x}{4}$ = one quarter of width

12 a no **b** $\frac{2a}{5}$ and $\frac{2}{5} \times a$ **c** $a = 1, a = -1$

13 a $3x^5$ **b** $-3b^2c^3$ **c** simplify $\frac{\text{(term 2)}}{\text{(term 1)}}$

14 a bc **b** $\frac{8a^2}{3}$ **c** $\frac{x}{6}$ **d** $16a$

e $\frac{1}{2}$ **f** $\frac{-2x}{y}$

Exercise 5E

1 a 7 **b** 4, 9 **c** algebraic **d** 7

2 a 15 **b** 20 **c** 42 **d** 6

3 a 4 **b** 12 **c** 4 **d** 30

4 a $\frac{7}{12}$ **b** $\frac{17}{35}$ **c** $\frac{3}{10}$ **d** $\frac{3}{20}$

5 a 15 **b** 20 **c** 20 **d** 12

6 a $2x$ **b** $6a$ **c** $16z$ **d** $15k$

7 a $\frac{3x}{4}$ **b** $\frac{7a}{3}$ **c** $\frac{3b}{5}$ **d** $\frac{5k}{3}$

e $\frac{5a}{6}$ **f** $\frac{9a}{20}$ **g** $\frac{7p}{10}$ **h** $\frac{3q}{4}$

i $\frac{29k}{35}$ **j** $\frac{16m}{15}$ **k** $\frac{47p}{30}$ **l** $\frac{5x}{8}$

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8 **a** $\frac{2y}{5}$ **b** $\frac{5p}{13}$ **c** $\frac{8r}{7}$ **d** $\frac{6q}{5}$
e $\frac{p}{6}$ **f** $\frac{t}{15}$ **g** $\frac{7u}{22}$ **h** $\frac{11y}{6}$
i $\frac{-r}{6}$ **j** $\frac{-13u}{42}$ **k** $\frac{33u}{4}$ **l** $\frac{-29p}{132}$

9 **a** $\frac{13x}{3}$ **b** $\frac{7x}{2}$ **c** $\frac{11a}{5}$
d $\frac{2p}{3}$ **e** $\frac{100u+9v}{30}$ **f** $\frac{7y-4x}{10}$
g $\frac{4t+7p}{2}$ **h** $\frac{x-3y}{3}$ **i** $\frac{35-2x}{7}$

10 **a** $\frac{x}{3}$ **b** $\frac{x}{4}$ **c** $\frac{7x}{12}$

11 **a** $\frac{T}{4}+\frac{B}{2}$ **b** $\frac{T+2B}{4}$ **c** 251 litres

12 $\frac{A-40}{2}$

13 **a** For example, if $x = 12$, $\frac{x}{2}+\frac{x}{3} = 10$ and $\frac{5x}{6} = 10$.
b For example, if $x = 1$, $\frac{1}{4}+\frac{1}{5} \neq \frac{2}{9}$.
c No. If $x = 1$ they are different.

14 **a** **i** $\frac{x}{6}$ **ii** $\frac{x}{12}$ **iii** $\frac{x}{20}$ **iv** $\frac{x}{30}$
b Denominator is product of initial denominators, numerator is always x.
c $\frac{x}{10}-\frac{x}{11}$

15 **a** $\frac{2z}{3}$ **b** $\frac{7x}{10}$ **c** $\frac{29u}{8}$ **d** $\frac{29k}{12}$
e $\frac{3p-12}{4}$ **f** $\frac{47u}{60}$ **g** $\frac{24+j}{12}$ **h** $\frac{16t+2r}{15}$

Exercise 5F

1 **a** 8 **b** 3 **c** 55 **d** 32

2 C

3 **a** $\frac{7}{15}$ **b** $\frac{2}{11}$ **c** $\frac{3}{22}$ **d** $\frac{1}{4}$
e $\frac{5}{6}$ **f** $\frac{3}{8}$ **g** $\frac{12}{17}$ **h** $\frac{4}{9}$

4 **a** $\frac{x}{3}$ **b** $\frac{5x}{2}$ **c** $\frac{a}{2}$ **d** $\frac{2}{5}$

5 **a** $\frac{2x}{15}$ **b** $\frac{a}{63}$ **c** $\frac{8a}{15}$ **d** $\frac{4c}{25}$
e $\frac{8ab}{15}$ **f** $\frac{21a^2}{10}$

6 **a** $\frac{7xy}{5}$ **b** $\frac{7bd}{15}$ **c** $\frac{6ab}{5c}$ **d** $\frac{18de}{7}$
e $\frac{1}{4}$ **f** $\frac{2}{3}$

7 **a** $\frac{15a}{4}$ **b** $\frac{14x}{15}$ **c** $\frac{18a}{5}$ **d** $\frac{7}{6x}$
e $\frac{6}{5y}$ **f** $\frac{x}{14}$ **g** $\frac{10a}{7}$ **h** $\frac{10b}{7c}$
i $\frac{3x}{10y}$ **j** $\frac{2y^2}{3x}$ **k** $\frac{5}{42x^2}$ **l** $\frac{14a^2}{5b}$

8 **a** $\frac{12x}{5}$ **b** $\frac{4x}{15}$ **c** $\frac{10}{x}$ **d** $\frac{4a}{3}$
e $\frac{7}{2x}$ **f** $\frac{2}{x}$

9 **a** $\frac{x}{2}$ **b** $\frac{x}{6}$

10 **a** xy
b **i** $\frac{x}{2}$ **ii** $\frac{3y}{4}$ **iii** $\frac{3xy}{8}$
c $\frac{3}{8}$

11 **a** $\frac{3q}{2}$ **b** $\frac{x}{2}$ **c** 1 **d** $\frac{3x}{8}$

12 **a** 1 **b** x^2 **c** Both are $\frac{x}{3}$.
d $\frac{a}{bc}$ **e** $\frac{ac}{b}$

13 **a** **i** $\frac{11x}{30}$ **ii** $\frac{x}{30}$ **iii** $\frac{x^2}{30}$ **iv** $\frac{6}{5}$
b $\frac{x}{5} \div \frac{x}{6}$

14 **a** $1 \div \frac{a}{b} = \frac{1}{1} \times \frac{b}{a}$ = reciprocal of $\frac{a}{b}$ **b** $\frac{a}{b}$

15 **a** x^2 **b** $\frac{x^2}{4}$ **c** $\frac{9x^2}{25}$. Proportion is $\frac{9}{25} < \frac{1}{2}$.
d $\frac{49x^2}{100}$ or $0.49x^2$ **e** 0.707 (Exact answer is $\frac{1}{\sqrt{2}}$.)

Exercise 5G

1 **a** 10 **b** $5x$ **c** $10+5x$ **d** $10+5x$

2 $6+21x$

3 **a** $6a$ **b** 42 **c** $6a+42$ **d** $6a+42$

4 D

5 **a** 10 **b** 12 **c** 10 **d** 14

6 **a** $9a+63$ **b** $4+2t$
c $8m-80$ **d** $24-3v$
e $-45-5g$ **f** $-35b-28$
g $-9u-81$ **h** $-40-8h$
i $30-5j$ **j** $12-6m$
k $30-3b$ **l** $2c-16$

7 **a** $8zk-8zh$ **b** $6jk+6ja$
c $4ur-4uq$ **d** $2pc-2pv$
e $10ma+mv$ **f** $-2ys-10yg$
g $-24sq-3sg$ **h** $-gn-4fg$
i $-8cu-80ct$ **j** $-jt-5js$
k $2hu-9mu$ **l** $20mw-40am$

8 **a** $65f+70$ **b** $44x+16$ **c** $15a+32$ **d** $24v+60$
e $76a+70$ **f** $20q-30$ **g** $32m-30$ **h** $22m+32$

9 **a** $6x+4y+8z$ **b** $14a-21ab+28ay$
c $8qz+4aq+10q$ **d** $-6-12k-6p$
e $-5-25q+10r$ **f** $-7kr-7km-7ks$

10 **a** $55d+37$ **b** $115f+54$ **c** $32j+30$
d $75d+32$ **e** $4j+40$ **f** $68g+52$

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11 **a** $3(t+4) = 3t+12$ **b** $2(u-3) = 2u-6$
c $3(2v+5) = 6v+15$ **d** $2(3w-2) = 6w-4$
12 **a** D **b** A **c** B **d** E **e** C
13 **a** $5b+3g$ **b** $2(5b+3g) = 10b+6g$ **c** $14b+10g$
d \$220
14 **a** $2(4a+12b)$, $8(a+3b)$. Other answers are possible.
b $2(2x+4y)$. Other answers are possible.
c $4(3a-2b)$. Other answers are possible.
d $3a(6b+4c)$. Other answers are possible.
15 **a** 1836
b 1836
c **i** 154 **ii** 352 **iii** 627 **iv** 869
d **i** 35 **ii** 399 **iii** 143 **iv** 39 999
e $(D5)^2 = ?25$ where ? is $D \times (D+1)$
16 Both simplify to $8a+6ab$.
17 $x^2+3xy+2y^2$
18 $\frac{1}{3}(6(x+2)-6)-2$ simplifies to $2x$.
19 **a** $\frac{5x+2}{6}$ **b** $\frac{8x+15}{15}$ **c** $\frac{5x-2}{8}$ **d** $\frac{7x+10}{12}$
e $\frac{7x+3}{10}$ **f** $\frac{31x+9}{35}$

Progress quiz

1 **a** 5 **b** $3a, 9b, ab, c, 8$ **c** 3, –9, 1, 0
d 8 **e** –1
2 **a** $5+m$ **b** $8k$ **c** $p-7$
d $h+12$ **e** $2\times(x+y)$ **f** $\frac{a}{b}$
g $\frac{k}{2}-\frac{m}{3}$ **h** $\frac{ac}{5}$
3 **a** 6 **b** –27 **c** 54 **d** 4
4 **a** E **b** N **c** N **d** E
5 **a** L **b** L **c** N **d** L
6 **a** $7h+3$ **b** $5a+8$ **c** $5xy+3x$
d $-gk-5g^2k+12$
7 **a** $6ab$ **b** $10d^2$ **c** $30a^2bc^2d^2$ **d** $12p^2q^2$
e $\frac{1}{4}$ **f** $\frac{2x}{5}$ **g** $\frac{1}{3a}$ **h** $-\frac{2m}{3t}$
8 **a** $\frac{7m}{9}$ **b** $\frac{13k}{6}$ **c** $\frac{20a-9b}{24}$ **d** $\frac{13x}{3}$
9 **a** $\frac{6ab}{35}$ **b** $\frac{8m}{15}$ **c** $\frac{4y}{3}$ **d** $\frac{3}{2p}$
10 **a** $3x+6$ **b** $15a-20b$
c $6x+4$ **d** $10b+5$

Exercise 5H

1 **a** 1, 2, 4, 5, 10, 20 **b** 1, 2, 3, 4, 6, 12
c 1, 3, 5, 15 **d** 1, 3, 9, 27
2 2
3 **a** 6 **b** 5 **c** 20 **d** 2
4 **a** 12 **b** 35 **c** 12, 30 **d** $14a, 21b$
e 7 **f** 3 **g** $2, q$ **h** 4
5

$5x+15$	25	50	35	15	0	–15
$5(x+3)$	25	50	35	15	0	–15

6 **a** 5 **b** 4 **c** 9 **d** $7x$ **e** $2y$
f $11xy$ **g** $4r$ **h** $3a$ **i** p
7 **a** $3(x+2)$ **b** $8(v+5)$ **c** $5(3x+7)$
d $5(2z+5)$ **e** $4(10+w)$ **f** $5(j-4)$
g $3(3b-5)$ **h** $4(3-4f)$ **i** $5(d-6)$
8 **a** $2n(5c+6)$ **b** $8y(3+r)$
c $2n(7j+5)$ **d** $4g(6+5j)$
e $2(5h+2z)$ **f** $10(3u-2n)$
g $8y(5+7a)$ **h** $3d(4+3z)$
i $3m(7h-3x)$ **j** $7u(7-3b)$
k $14u(2-3b)$ **l** $3(7p-2c)$
9 For example: length = 2, width = $6x+8$. Other answers are possible.
10 **a** 5 **b** $4a+12$
11 **a** $6x+18$ **b** $6(x+3)$ **c** $x+3$ **d** $2x+6$
e $3x+9$
12 **a** $7x+7$ **b** $7(x+1)$ **c** Student prediction.
d $28(x+1)$
13 $(x+2)(y+3)$
14 **a** **i** I eat fruit and I eat vegetables.
ii Rohan likes Maths and Rohan likes English.
iii Petra has a computer and Petra has a television.
iv Hayden plays tennis and Hayden plays chess and Anthony plays tennis and Anthony plays chess.
b **i** I like sewing and cooking.
ii Olivia and Mary like ice cream.
iii Brodrick eats chocolate and fruit.
iv Adrien and Ben like chocolate and soft drinks.
15 **a** $\frac{2}{5}$ **b** $\frac{7}{2}$ **c** $\frac{3c+5}{1+2b}$ **d** $\frac{2a+b}{4c+5d}$ **e** $\frac{5}{3}$
f $\frac{7}{9}$ **g** $\frac{7}{2}$ **h** $\frac{6}{4+q}$ **i** 5

Exercise 5I

1 **a** 35 **b** 41 **c** 5
2 **a** 50 **b** 20 **c** 0
3 **a** $2x+y$ **b** 8
4 **a** $3n$ **b** \$36
5 **a** $3(x+2)$ or $3x+6$ **b** 33 units^2
6 C
7 **a** $5x$ **b** $10x$ **c** $5(x+3)$ or $5x+15$
8 **a** $30+40x$ **b** \$350
9 **a** \$50 **b** \$60 **c** 2 hours 30 minutes
10 **a** \$140 **b** $60+80x$
c **i** \$60 **ii** \$80

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11 **a** $10 + 4n$ **b** $20 + n$ **c** 30
d deal 1 **e** deal 3
f **i** 3 **ii** 4, 10 **iii** 10

12 **a** **i** $2(x + 2)$ **ii** $2x + 4$ **iii** $4\left(\frac{x}{2} + 1\right)$
b All expand to $2x + 4$.
c Divide by 2.

13 **a** 6 **b** 6
c Both dimensions are negative, so the shape cannot exist.

14 **a** $F + H$ **b** $F + 2H$ **c** $F + \frac{H}{2}$ **d** $F + tH$

15 3 buy deal F, 2 buy E, 1 buys D, 3 buy C, 5 buy B and 86 buy A.

Exercise 5J

1 5, 7

2 B

3 **a** **i** 4 **ii** 8 **iii** 32 **iv** 64
b 2^5

4 **a** $5 \times 5 \times 5$
b $5 \times 5 \times 5 \times 5$
c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
d C

5 **a** 4^7 **b** 3^{12} **c** 2^{18} **d** 7^6

6 **a** m^7 **b** x^6 **c** q^{13} **d** r^9
e m^9 **f** a^9 **g** r^9 **h** z^{36}
i k^4 **j** j^3 **k** m^8 **l** x^4

7 **a** $20m^5$ **b** $10k^7$ **c** $28x^{14}$ **d** $28y^{13}$
e m^6n^{10} **f** x^2y^3 **g** $3r^3s^7$ **h** $2y^{15}z^5$
i $110x^4$ **j** $15a^6$ **k** $8x^5y^7$ **l** $14a^5b^4$
m $-14x^7y^4$ **n** $-4a^5b^3$ **o** $-16c^6d$ **p** $84x^4y^5$

8 **a** 3^5 **b** 10^8 **c** 2^5 **d** 5^2

9 **a** m^3 **b** z^3 **c** q^7 **d** r^9
e m^2n^5 **f** a^5b^3 **g** xy^6z^2 **h** $x^2y^3z^3$
i $4k^3$ **j** $2m^{13}$ **k** $\frac{5x^{18}y^7}{3}$ **l** $\frac{a^2}{2}$

10 **a** $2^3 = 8$ **b** $5^4 = 625$ **c** 1

11 $a = 2, b = 4$ is the only solution.

12 **a** They are not equal.
b $(3 \times 3) \times (3 \times 3 \times 3 \times 3) \neq 9^6$
c The student mistakenly multiplied the bases.

13 **a** **i** 4 **ii** -8 **iii** 16 **iv** -32
b **i** positive **ii** negative
c 1024

14 **a** 5^0 **b** 1 **c** 1 **d** 1

15 **a** that $a - b = 2$ **b** 125

16 **a** **i** 5×10^4 **ii** 7×10^9
iii 5×10^{-3} **iv** 2×10^{-7}
b **i** 2000 000 hours **ii** 40 000 000 days
iii 0.003 seconds **iv** 0.000000003 seconds
c **i** 34.7 days **ii** 250 hours
iii 0.72 seconds **iv** 4.32 milliseconds

Exercise 5K

1 A **2** B **3** C

4 **a** 1 **b** 1 **c** 5 **d** $12y^2$ **e** 1
f 13 **g** $2y^2$ **h** 4 **i** $3a^2$

5 **a** 2^{12} **b** 5^{16} **c** 6^{36} **d** d^9
e k^{24} **f** m^{50}

6 **a** $9x^{10}$ **b** $8u^{12}$ **c** 5^4x^{20} **d** 12^3x^{15}
e $16x^8$ **f** $49x^4$ **g** $9^{10}x^{70}$ **h** 10^5x^{10}

7 **a** x^{21} **b** y^{18} **c** $500k^{23}$ **d** $25m^{22}$
e $8x^{18}$ **f** 5^4p^{18} **g** y^{10} **h** p^8
i $2p^{13}$ **j** $3^{10}x^{14}$ **k** $8h^5$ **l** q^2

8 **a** 5 **b** 3 **c** 5 **d** 2

9 **a** x^{24} **b** x^{60} **c** $((2^3)^4)^5$, $(2^7)^{10}$, 2^{100}, $((2^5)^6)^7$

10 **a** **i** 2 **ii** 5 **iii** 6
b 54

11 **a** x^7 **b** x^{12}
c For example, if $x = 2$ they give different values (128 vs 4096)
d $x = 0, x = 1$

12 **a** 1 **b** $5^{2-2} = 5^0$ **c** $3^0 = 3^{2-2} = \frac{3^2}{3^2} = \frac{9}{9} = 1$
d $100^0 = 100^{2-2} = \frac{100^2}{100^2} = \frac{10\,000}{10\,000} = 1$
e $\frac{0^2}{0^2}$ cannot be calculated (dividing by zero).

13 **a** Both equal 2^{12}. **b** A $(2^4)^3$, B $(2^2)^6$, C $(4^2)^3$
c 3, 5. Other answers are possible.

14 **a** 9 **b** 3 **c** 3 **d** 6

15 **a** 1 **b** Many answers are possible.

16 **a** 5^4 **b** x^8 **c** x^4y^9 **d** $a^7b^{12}c^{10}$
e $x^{80}y^{10}$ **f** $7^2 = 49$ **g** 1 **h** 25
i $10^4 = 10\,000$

Problems and challenges

1 $10m + 10 = 10(m + 1)$

2 Any list with $6 - 2a$ central; $2 - a$, $6a - 5$ together; $a - 7$, $4(a + 1)$ together. e.g. $2 - a$, $6a - 5$, $6 - 2a$, $a - 7$, $4(a + 1)$

3 $4^{1999}, 16^{1000}, 2^{4001}, 8^{1334}$

4 **a** $\frac{65}{4}$ **b** $\frac{225}{4}$ **c** 0

5 **a** $\frac{5 - 4a}{30}$ **b** $\frac{9x + 4}{42}$

6 **a** 8 **b** 45 **c** 4

7 **a** All perimeters $= 4a$
Areas: a^2, $\frac{3}{4}a^2$, $\frac{6}{9}a^2$, $\frac{10}{16}a^2$, $\frac{15}{25}a^2$
b $P = 24, A = \frac{9009}{500}$ or approximately 18.

8 $V = 2^{3x}\,3^{3y}\text{cm}^3$, $TSA = 2^{2x+1}3^{2y+1}\text{ cm}^2$

9 **a** $x = 3$ **b** $a = 4$ **c** $b = 3, c = 2$
d $x = \frac{1}{2}$ **e** $k = \frac{5}{3}$

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Multiple-choice questions

1 C 2 D 3 B 4 D 5 E
6 C 7 E 8 E 9 E 10B

Short-answer questions

1 a false b true c true d false
e true
2 a 2 b 3 c 4 d 6
3 a 11 b 14 c 29 d 8
4 a –3 b 3 c –9 d 14
e –3 f 12
5 3
6 a $16m$ b $2a + 5b$ c $2x^2 - x + 1$
d $7x + 7y$ e $9x + x^2$ f $-m - 12n$
7 a $36ab$ b $15xy$ c $16xy$
8 a $5x$ b $-4c$ c $15x$ d $3ab$
9 a $\frac{x}{4}$ b $\frac{6a+b}{15}$ c $\frac{9}{x}$ d $\frac{3b}{2}$
10 a $3x - 12$ b $-10 - 2x$ c $3kl - 4km$
d $7x - 6y$ e $13 - 3x$ f $10 - 20x$
g $18x + 2$
11 a $2(x + 3)$ b $8(3 - 2g)$
c $3x(4 + y)$ d $7a(a + 2b)$
12 a $a + 2$ b 12 c $\frac{1}{4+3q}$
13 a 7 b 3 c 6 d 1
14 a m^7 b $12m^8$ c m^2 d $2a^4$
e x^{12} f $8a^6$
15 a $-2x^4y^2z^2$ b y^{10} c 7 d 2
e $4y^{10}$ f m^6 g $2b$ h y^4

Extended-response questions

1 a $120 + 80n$
b $80 + 100n$
c A costs \$360, B costs \$380.
d any more than two hours
e $520 + 440n$
f \$2720
2 a $xy - \frac{x^2}{4}$ b 33 m^2 c $2x + 2y$ d 26 m
e Area $= xy - \frac{x^2}{3}$, Perimeter $= 2x + 2y$
f Area is reduced by $\frac{x^2}{12}$ and perimeter remains the same.

Semester review 1

Integers

Multiple-choice questions

1 B 2 C 3 C 4 A 5 B

Short-answer questions

1 a 5169 b 1350 c –288 d 695
e 1386 f 2800 g 81 h 64
i –19
2 a –14 b 30 c 72 d –7
e 54 f –6
3 a 6 b 7 c 20 d 3
4 a 168 b 72 c 300 d 66150
5 a –3 b 40 c 27 d –55
e 25 f –8 g 19

Extended-response question

a Hong Kong b Moscow, New York
c Hong Kong d 8.9°C e 14.8°C
f Min = 10.7°C, Max = 16.5°C

Lines, shapes and solids

Multiple-choice questions

1 B 2 D 3 C 4 C 5 C

Short-answer questions

1 a 66 b 25 c 123 d 35
e 70 f 98
2 a $x = 81, y = 99$ b $a = 75$ c $a = 62, b = 62$
d $a = 65,\ b = 65$ e $a = b = c = d = 100,\ e = 80$
f $x = 95,\ y = 85$
3 a 48 b 45 c 60 d 75
e 121 f 75
4 a $a = b = 90$
b $a = 73,\ b = 95$
c $a = 265,\ b = 30$
5 120°

Extended-response question

$b = 65$ (supplementary to a)
$c = 65$ (alternate to b)
$d = e = 57.5$ (isosceles triangle)
$f = 122.5$ (supplementary to d)
$g = 122.5$ (revolution angle 360)
$h = 180$ (straight angle)
$i = 295$ (revolution)

Fractions, decimals and percentages

Multiple-choice questions

1 D 2 C 3 C 4 B 5 C

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Short-answer questions

1 a 18 b 1 c 5

2 a $\frac{1}{4}$ b $\frac{7}{5}$ c $3\frac{1}{4}$ d $-\frac{2}{21}$
e $\frac{1}{3}$ f $\frac{9}{10}$

3 a $\frac{5}{2}$ b $\frac{1}{8}$ c $\frac{5}{21}$

4 a $\frac{9}{2}$ b $\frac{3}{4}$ c $\frac{5}{8}$

5 a 6.93 b 7.58 c 4.03 d 6.51
e 3854.8 f 792

6 a 545.9 b 1008 c 0.614

7

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{19}{20}$	$\frac{99}{100}$	$\frac{1}{200}$
Decimal	0.25	0.5	0.2	$0.\dot{3}$	$0.\dot{6}$	0.8	0.95	0.99	0.005
Percentage	25%	50%	20%	$33.\dot{3}\%$	$66.\dot{6}\%$	80%	95%	99%	0.5%

8 a 5.6 b 11.76 c 85.5 m d $1.98
e $105 f 4930 g

9 a $700 b $862.4 c $0.9936

10 25

Extended-response question

a i $1784.15 ii $1516.53 iii $1289.05

b 6 years

c No. There will always be 85% of the previous value.

Measurement and introductions to Pythagoras' theorem

Multiple-choice questions

1 C 2 B 3 D 4 B 5 D

Short-answer questions

1 a 500 cm b 180 cm c 90000 cm^2
d 1.8 m e 4000 cm^3 f 10000 m^2

2 a 18.6 cm b 64 m c 40 m

3 a i 25.13 m ii 50.27 m^2
b i 47.12 cm ii 176.71 cm^2

4 a i 25.71 m ii 39.27 m^2
b i 17.85 cm ii 19.63 cm^2
c i 54.85 mm ii 169.65 mm^2

5 a 30 m^2 b 48 m^2

6 a $SA = 105.84$ m^2, $V = 74.088$ m^3.
b $SA = 85$ m^2, $V = 50$ m^3
c $SA = 60$ m^2, $V = 24$ m^3

7 a 615.75 m^3 b 392 699.08 cm^3
c 1.26 m^3 or 1 256 637.06 cm^3

8 a 13 b 14.42

9 a 1530 b 0735

Extended-response question

a $15 - 2x$ b x c $x(15 - 2x)^2$
d 169 m^3 e $x = 2.5$

Algebra

Multiple-choice questions

1 C 2 A 3 D 4 B 5 D

Short-answer questions

1 a $p + q$ b 3p c $\frac{m^2}{2}$ d $\frac{x+y}{2}$

2 a 9 b 25 c 102 d 116
e −24 f −24

3 a $24k$ b $3a$ c a^3 d $\frac{p}{2}$
e $7ab + 2$ f $x - 1$ g $2y$ h $2n - 2m$

4 a xy b $\frac{x}{7}$ c $\frac{7w}{10}$ d $\frac{7a}{2}$

5 a $\frac{m}{6}$ b ab c $\frac{2}{3}$

6 a $12m - 18$ b $4 + 2m$ c $9A + 6$

7 a $6(3a - 2)$ b $6m(m + 1)$ c $-8m(m + 2n)$

8 a $8x + 20$ b $3x(x + 10)$

9 a m^9 b $32a^4$ c $-48a^6b^9$ d a^6
e a^4b^2 f $\frac{1}{2}$

10 a x^{14} b $16a^{12}$ c $25a^8b^{12}$ d 1
e 1 f $-5c^2$

Extended-response question

a $(7a + x)$ cm b $6a^2$ cm^2 c $x = 5a$
d $12a$ cm e 216 cm^2

Chapter 6

Exercise 6A

1 a 3 : 7 b 5 : 4

2 a 1 : 3 b 7 : 15

3 a 9 : 4 b 52 : 17 c 7 : 12 d 3 : 5

4 a 5 : 3 b 7 : 15 c 3 : 7 : 5 d 5 : 3 : 7

5 a 8 : 3 b 3 : 14 c 3 : 11 d 8 : 6

6 a 13 : 7 b 11 : 9 c 13 : 9 : 11 : 7
d 20 : 20 or 1 : 1

7 a 12 b 14 c 4 d 9
e 2 f 4 g 2 h 10
i 77 j 51 k 9 l 11

8 a 4 : 6 : 10 b 2 : 6 : 8 c 7 : 49 : 63 d 11 : 55 : 33

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9 Answers may vary.
a 2 : 4, 3 : 6, 5 : 10 **b** 4 : 10, 20 : 50, 200 : 500
c 4 : 3, 16 : 12, 40 : 30 **d** 3 : 1, 6 : 2, 18 : 6
10 2 : 5 and 4 : 10, 6 : 12 and 1 : 2, 7 : 4 and 70 : 40
11 **a** 3 : 30 **b** 1 : 10, 10 : 100
12 2
13 **a** 2 : 3 **b** 3 : 5 **c** 2 : 3 **d** 5 : 4
14 Victoria, Tasmania, South Australia
15 No, three is not a factor of 10.
16 1 : 2
17 **a** 5 : 8 **b** 9 : 13 **c** no
18 **a** $3x$ **b** $2.5x$ **c** $4y$ **d** $2x$
19 **a** 4 : 21 **b** 3 : 7 **c** 3 : 1 **d** 7 : 2
20 1 : 1
21 Answers will vary.

Exercise 6B

1 **a** 1 : 1 **b** 1 : 2
2 D **3** B **4** C
5 **a** 1 : 4 **b** 1 : 5 **c** 1 : 6 **d** 1 : 3
e 4 : 5 **f** 5 : 8 **g** 3 : 4 **h** 3 : 10
i 9 : 7 **j** 2 : 1 **k** 9 : 7 **l** 3 : 1
m 3 : 1 **n** 1 : 9 **o** 6 : 11 **p** 2 : 1
q 12 : 1 **r** 1 : 6 **s** 8 : 5 **t** 6 : 5
6 **a** 1 : 2 : 3 **b** 4 : 7 : 11 **c** 7 : 10 : 2 **d** 17 : 7 : 3
e 1 : 2 : 3 **f** 2 : 6 : 5 **g** 9 : 14 : 2 **h** 2 : 4 : 7
7 **a** 2 : 3 **b** 5 : 4 **c** 8 : 15 **d** 10 : 7
e 3 : 2 **f** 7 : 8 **g** 33 : 4 **h** 27 : 14
8 **a** 2 : 1 **b** 11 : 3 **c** 50 : 21 **d** 52 : 45
9 **a** 2 : 5 **b** 14 : 1 **c** 3 : 25 **d** 1 : 35
e 20 : 3 **f** 2 : 25 **g** 50 : 11 **h** 5 : 1
i 2 : 5 **j** 1 : 6 **k** 12 : 1 **l** 9 : 1
m 1 : 16 **n** 2 : 9 **o** 1 : 7 **p** 14 : 3
q 1 : 8 **r** 30 : 1
10 B
11 **a** 2 : 11 **b** 9 : 11 **c** 2 : 9
12 **a** 2 : 3 : 3 **b** 1 : 1 **c** 2 : 5 **d** 5 : 7
13 **a** 5 : 5 : 2 : 4 : 3 : 1 : 20
b 20 : 20 : 8 : 16 : 12 : 4 : 80
c **i** 1 : 4 **ii** 1 : 1
14 Andrew did not convert a.m.ounts to the same units. Correct ratio is 40 : 1.
15 **a** 8 : 1 **b** 8 : 1, yes
16 Answers may vary.
a 24 minutes to 1 hour **b** 2 kilometres to 1500 metres
17 **a** $a : 2b$ **b** $5x : y$ **c** $1 : a$ **d** 5 : 24
e $h : 3$ **f** $2x : 5$
18 **a** 4 : 3 **b** 16 : 9 **c** squared. $16 : 9 = 4^2 : 3^2$
d 47 : 20 **e–g** Answers will vary.

Exercise 6C

1 **a** 10 **b** 6 **c** 14 **d** 9
2 **a** $\frac{3}{8}$ **b** $\frac{5}{8}$
3 **a** 1 : 3 **b** 1 : 1 **c** 2 : 5 **d** 1 : 4
4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{2}{7}$ **d** $\frac{1}{5}$
5 **a** \$24 and \$36 **b** \$70 and \$40
c \$150 and \$850 **d** 8 kg and 40 kg
e 8 kg and 6 kg **f** 150 kg and 210 kg
g 24 m and 48 m **h** 15 m and 25 m
i 124 m and 31 m
6 **a** \$100 and \$300 **b** \$160 and \$240
c \$150 and \$250 **d** \$180 and \$220
7 **a** \$40, \$80, \$80 **b** \$50, \$150, \$200
c 2 kg, 4 kg, 6 kg **d** 22 kg, 11 kg, 55 kg
e 96 kg, 104 kg, 120 kg
f \$5000, \$10 000, \$15 000, \$20 000
8 **a** 60, 540 **b** 200, 100, 300 **c** 100, 250, 250
d 240, 140, 160, 60
9 nitrogen: 500 g, potassium: 625 g, phosphorus: 375 g
10 40°, 60°, 80°
11 \$250 **12** 48 **13** 240 **14** 294
15 120 pages
16 shirt \$160, jacket \$400
17 **a** 8 **b** 3 : 5
c 2 boys and 2 girls were absent or 5 boys and 9 girls.
18 **a** Ramshid: \$125, Tony: \$83.33, Maria: \$41.67
b 10 : 6 : 5
c Ramshid: \$119.05, Tony: \$71.43, Maria: \$59.52
d \$17.85 **e** \$11.90
f Original ratio, as he receives \$5.95 more.
g Ramshid: \$120, Tony: \$70, Maria: \$60

Exercise 6D

1 50 000 cm
2 3 m
3 **a** 100 000 mm **b** 100 m **c** 0.1 km
4 **a** 0.56 km **b** 56 000 cm **c** 560 000 mm
5 Numbers and units may vary.
a **i** 200 m **ii** 40 m **iii** 730 m
b **i** 16 km **ii** 370 m **iii** 25 km
c **i** 6.4 m **ii** 288 m **iii** 12 m
d **i** 150 cm **ii** 24.6 m **iii** 2.13 km
e **i** 88 m **ii** 620 cm **iii** 5 mm
f **i** 20 m **ii** 120 cm **iii** 77.5 cm
g **i** 6 cm **ii** 1.6 mm **iii** 200 m
h **i** 0.3 mm **ii** 115 m **iii** 8.15 mm
6 **a** **i** 1 m **ii** 20 m **iii** 3 mm
b **i** 20 m **ii** 2 m **iii** 1.5 mm

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c i 13.5 cm ii 4.5 m iii 7.365 cm
d i 60 cm ii 9 cm iii 0.02 mm
e i 20 m ii 3 m iii 5 mm
f i 2.3 m ii 16.5 cm iii 61.5 cm
g i 3 m ii 5 cm iii 2 mm
h i 20 cm ii 15 m iii 1.64 m

7 **a** 1 : 250 **b** 1 : 50 000 **c** 1 : 50 000 **d** 1 : 18 000
e 7 : 1 or 1 : $\frac{1}{7}$ **f** 600 : 1 or 1 : $\frac{1}{600}$

8 **a** 1 : 10 000 **b** 1 : 1000 **c** 1 : 300
d 1 : 150 000 **e** 1 : 125 **f** 1 : 200 000
g 1 : 100 000 **h** 50 : 1 **i** 10 000 : 1

9 **a** 80 m **b** 4.5 cm

10 8.5 km

11 **a** 3.5 m $\times$ 2.5 m **b** 4.6 m $\times$ 4.6 m
c 7.8 m $\times$ 2.1 m

12 length 11 m, height 4 m

13 55 cm

14 **a** 24 km **b** 160 km **c** 12.5 cm

15 1 : 0.01 and 100 : 1, 25 : 1 and 50 : 2, 20 : 1 and 1 : 0.05

16 **a** car: D, 1 : 10 **b** school grounds: B, 1 : 1000
c Mt Kosciuszko: A, 1 : 10 000

17 With chosen scale, the map will be 8 m wide by 8 m high, which is too big to be practical.

18 The ratio provided is the wrong way around. It should be 1000 : 1.

19 Answers will vary.

20 Answers will vary.

Exercise 6E

1 B, C, E, F, H

2 employee's wage: \$15/h
speed of a car: 68 km/h
cost of building new home: \$2100/m^2
population growth: 90 people/day
resting heart rate: 64 beats/min

3 **a** \$/kg **b** \$/L **c** words per minute
d goals/shots on goal **e** kJ/serve or kJ/100 g
f L/min **g** mL/kg or mg/tablet **h** runs/over

4 **a** 3 days/year **b** 5 goals/game **c** \$30/h
d \$3.50/kg **e** \$14 000/acre **f** 4500 cans/hour
g 1200 revs/min **h** 16 mm rainfall/day
i 4 min/km **j** 0.25 km/min or 250 m/min

5 **a** 300 km/day **b** \$140/year
c 6.5 runs/over **d** 7.5 cm/year
e 1.5 kg/year **f** dropped 2.5°C/h

6 **a** 3 L/h **b** 7 hours

7 158 cm

8 **a** 1.5 rolls/person **b** \$6/person **c** \$4/roll

9 **a** i 5.8 hours/day ii 6.5 hours/day
iii 42 hours/week iv 6 hours/day
b 180 hours

10 Harvey: 3.75 min/km, Jacques: 3.33 min/km; Jacques

11 **a** 1200 members/year **b** 12 years

12 **a** i 9 km/L ii $\frac{1}{9}$ L/km
b Find the reciprocal.

13 **a** i \$4 ii \$7.25 iii \$16.50 iv \$22.50
b 75c/minute **c** Teleconnect **d** Connectplus
e 16 $\frac{2}{3}$ min or 16 min and 40 seconds

14 Answers will vary.

Progress quiz

1 **a** 1 : 24 **b** 23 : 25 **c** 1 : 2

2 **a** 6 **b** 4 **c** 4 **d** 24 : 9

3 **a** 2 : 3 **b** 6 : 9 : 10 **c** 1 : 12
d 3 : 40 **e** 1 : 3

4 **a** \$500, \$300 **b** \$5000, \$1875, \$625
c 400 m, 600 m

5 **a** \$1760 **b** \$560

6 320 m **7** 4.32 cm **8** 1 : 2 000 000

9 **a** 60 students/bus **b** \$1.40/kg **c** 74.4 km/h

10 **a** 160 km/day **b** \$2500/year **c** 8 cm/year

11 Kelly earns the most by \$750
Kelly earns \$96570
Todd earns \$95820

12 21 m/s is faster
(21 m/s = 75.6 km/h)

Exercise 6F

1 \$27 **2** \$17.60 **3** \$43.30 **4** \$36

5 **a** 75 **b** 36 **c** 7 m **d** 133 cm

6 **a** 520 mL **b** 11 goals **c** 350 mm
d 1 875 000 kilobytes

7 Leonie \$600, Mackenzie \$300, total \$1350

8 **a** 25c/min **b** 4c/s **c** 210 L/h **d** 1.2 L/h
e 6 kg/year **f** 0.84 kg/week **g** 6 kg/\$
h 3.8c/mm **i** 30 m/s **j** 50.4 km/h

9 **a** small: \$1.25/100 g, medium; \$1.20/100 g, large: \$1.10/100 g
b 4 large, 1 medium, 1 small, \$45.20

10 12 s

11 **a** 1.2 cm/month, 0.144 m/year
b 25 months, 2 years and 1 month

12 **a** 45 m^2/h **b** 900 m^2 **c** $\frac{5}{6}$ m^2/min

13 **a** $\frac{\$y}{x}$ **b** $\frac{\$12y}{x}$ **c** $\frac{\$zy}{x}$

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14 **a** 19 cm, 22 cm, Perimeter = 58 cm
b 21 cm, 28 cm, Perimeter = 84 cm
15 50 pa/mD
16 **a** Perth **b** 56 hours **c** 40 hours
d Phil is 1125 km from Perth, Werner is 1575 km from Sydney.
e 2450 km from Sydney, 1750 km from Perth
f 46 hours and 40 minutes
g Answers will vary.

Exercise 6G

1 **a** 3 hours **b** 5 hours, $\times$ 5
c $\times$ 10, 30 minutes, $\times$ 10 **d** $\times$ 6, 720 litres, $\times$ 6
2 **a** \$12, \$60, $\times$ 5
b $\div$5, 30 rotations, $\div$5, $\times$ 7, 210 rotations, $\times$ 7
3 **a** 2400 **b** 19 200
4 100 litres **5** 40 litres **6** 150 days
7 **a** 22 500 **b** 375 **c** 10 minutes
d 6 seconds
8 **a** 3750 beats **b** 1380 beats **c** 80 minutes
9 \$2520 **10** 22.5 kg
11 Bionic woman wins by 4 seconds.
12 **a** $7\frac{1}{2}$ days **b** $187\frac{1}{2}$ students!
13 2.4 days
14 **a** $2\frac{2}{3}$ days **b** Matric: $\frac{1}{3}$, Hugh: $\frac{4}{9}$, Ethan: $\frac{2}{9}$
15 **a** 80 cans **b** 5 dogs **c** 15 days
16 12 hours
17 **a** \$ **b** m **c** min **d** min
18 **a** buddies: \$4.50/L, 1.25 L bottles: \$1.28/L, 2 L bottles: \$1.10/L, cans: \$1.60/L
b buddies: \$135, 1.25 L bottles: \$38.40, 2 L bottles: \$33, cans: \$48
c greatest amount = 54 L, least amount = 13.2 L
Difference = 40.8 L
d Answers will vary.

Exercise 6H

1 D **2** A **3** B **4** D
5 **a** 10 m/s **b** 7 m/s **c** 60 km/h **d** 50 km/h
e 120 km/h or 2 km/min **f** 45 km/h or 0.75 km/min
6 **a** 1080 m **b** 450 m **c** 36 km **d** 50 km
7 **a** 8 hours **b** $\frac{1}{2}$ hour or 30 minutes **c** 11.5 hours
d 7 seconds
8 **a** 10 m/s **b** 50 m/s **c** 11 m/s **d** 4000 m/s
9 **a** 54 km/h **b** 7.2 km/h **c** 0.72 km/h **d** 3600 km/h
10 **a** 25 km/h **b** 40 s **c** 60 km **d** 120 m/min
e 70 km **f** 100 s
11 2025 km
12 **a** 27 km/h **b** $2\frac{1}{4}$ km
13 27 m/s
14 24 km/h
15 10.4 m/s, 37.6 km/h
16 **a** 58.2 km/h **b** 69.4 km/h
17 Answers may vary; e.g. 150 km/h, 160 km/h
18 250 m
19 8 : 02 : 40; 2 minutes and 40 seconds after 8 a.m.
20 **a** 343 m/s **b** 299 792 458 m/s **c** 0.29 s
d 0.0003 s **e** 874 030
f how many times the speed of sound
(mach 1 = speed of sound)
g 40 000 km/h or 11.11 km/s
h 107 218 km/h, 29.78 km/s **i** 29.78 km/s
j–l Answers vary

Problems and challenges

1 **a** 2 **b** $3\frac{1}{5}$ **c** $2\frac{2}{3}$
2 1 : 3 **3** 9 km/h
4 **a** 1 hour **b** 12 sets
5 12 sheep **6** $\frac{5}{9}$ hour **7** 0.75 km
8 5 hours **9** 12.57 a.m. **10** 68.57 km/h
11 A 20 km trip at 100 km/h takes $\frac{1}{5}$ hour which Max has been used to drive the first 10 km. The second 10 km cannot be travelled in zero time; hence it is impossible for Max to achieve a 100 km/h average speed.

Multiple-choice questions

1 A **2** C **3** D **4** A **5** B
6 B **7** C **8** C **9** B **10** D

Short-answer questions

1 **a** 1 : 2 **b** 2 : 1 **c** 5 : 9
2 **a** false **b** false **c** true **d** false
e true
3 **a** 25 **b** 12 **c** 6 **d** 3
4 **a** 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8
e 3 : 1 **f** 1 : 5 **g** 3 : 2 **h** 2 : 1
i 2 : 3 **j** $2a$: 1 **k** 2 : 5 **l** 11 : 2
m 10 : 3 **n** 1 : 3 : 6
5 **a** 5 : 2 **b** 1 : 3 **c** 2 : 5 **d** 1 : 2
e 1 : 5 **f** 1 : 4 **g** 3 : 25 **h** 3 : 10
6 **a** \$35 : \$45 **b** 160 kg : 40 kg **c** 30 m : 10 m
d \$340 : \$595 : \$510 **e** 60c : 20c : 20c
7 **a** \$1152 **b** 144 cm **c** 1.125 L
8 **a** 5 km/h **b** \$50/h **c** 140 km/day
9 **a** 12.5 km/L, 8 L/100 km **b** 2.5 g/min, 150 g/h
c \$2400/day, \$100/h

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10 **a** 0.2 m **b** 27 m **c** 140 m
11 500 mm
12 **a** 1 : 1.5, $x = 9$ cm **b** 1:3, $x = 12$ cm
13 **a** 200c/min **b** 21.6 km/h **c** 200 m/s
14 **a** 301 km **b** \$39.20 **c** $6\frac{2}{3}$ hours
15 **a** 64 km/h **b** 108 min **c** 6.75 km

Extended-response questions

1 **a** 2:15 p.m. **b** 100 km/h **c** 11:30 a.m. **d** 467 km
e 5 hours at 93.4 km/h
f The Harrison's petrol cost \$282.15, the Nguyen's petrol cost \$313.50.

Chapter 7

Exercise 7A

1 **a** true **b** false **c** true **d** false
e true **f** true
2 **a** 13 **b** 9 **c** 2 **d** 2
3 **a** 7 **b** 9 **c** 15 **d** 8
4 **a** 25 **b** 25 **c** yes
d $2 + x = 7$ (other answers possible)
5 **a** true **b** true **c** false **d** false
e true **f** false
6 **a** true **b** false **c** true **d** true
e false **f** true
7 **a** C **b** I **c** C **d** C
8 **a** $x = 7$ **b** $x = 13$ **c** $v = 3$ **d** $p = 19$
e $x = 2$ **f** $x = 8$ **g** $u = 7$ **h** $k = 11$
i $a = 3$
9 **a** $2x + 7 = 10$ **b** $x + \frac{x}{2} = 12$
c $25 + a = 2a$ **d** $h + 30 = 147$
e $4c + 3t = 21$ **f** $8c + 2000 = 3600$
10 **a** 7 **b** 42 **c** 13 **d** –8
e 40 **f** –13
11 **a** $3.2x = 9.6$ **b** $x = 3$
12 **a** $E + 10 = 3E$ **b** $E = 5$ **c** 5 years old
d 15
13 $t = 3, t = 7$
14 **a** $3 \times 3 = 9$
b $-3 \times (-3) = 9$
c $x = 8, x = -8$
d $x = 0$ is the only solution and $-0 = 0$, but $x = 1$ and $x = -1$ are distinct solutions.
e Positive numbers square to give positives but so do negatives. (neg × neg = pos)
15 **a** No number can equal 3 more than itself.
b Addition is commutative.
c If $x = 7$ it is true, if $x = 6$ it is false.
d **i** S **ii** S **iii** N **iv** S
v A **vi** S **vii** N **viii** S
ix A
e $7 + x = x + 7$. Other answers are possible.
16 **a** $p = 1, p = -3$
b one
c Many answers possible, e.g. $(p - 2) \times (p - 3) \times (p - 5) = 0$ has 3 solutions.
17 **a** $a = 10, b = 6, c = 12, d = 20, e = 2$
b $f = \frac{1}{2}$
18 **a** $c = 4, d = 6$ (or $c = 6, d = 4$)
b $c = 11, d = 3$
c $c = 24, d = 6$
d $c = 7, d = 0$ (or $c = 0, d = -7$)

Exercise 7B

1 **a** $3x + 4 = 14$ **b** $11 + k = 18$
c $9 = 2x + 4$
2 **a** 12 **b** 25 **c** 16 **d** $4z$
3 **a** 8 **b** $x = 8$
4 B
5 **a** $2x = 20$ **b** $2 + q = 10$
c $18 = 17 - q$ **d** $12x = 24$
e $7p + 6 = 2p + 10$ **f** $3q = 2q$
6 **a** $x = 3$ **b** $q = -3$ **c** $x = 5$
d $p = 7$(missing operation: ÷ 4) **e** $x = 3$
f $p = 6$(missing operation: × 3)
7 **a** $a = 3$ **b** $t = 7$ **c** $q = 9$ **d** $k = 9$
e $x = 10$ **f** $h = -10$ **g** $l = -4$ **h** $g = -9$
8 **a** $h = 3$ **b** $u = 4$ **c** $s = 3$ **d** $w = 8$
e $x = -4$ **f** $w = -5$ **g** $a = 2$ **h** $y = -8$
9 **a** $d = 3$ **b** $j = -6$ **c** $a = 2$ **d** $y = 2$
e $k = -4$ **f** $n = -7$ **g** $b = -3$ **h** $b = 4$
10 **a** $x = \frac{7}{2}$ **b** $q = \frac{1}{3}$ **c** $b = \frac{1}{2}$
d $x = \frac{5}{2}$ **e** $x = -\frac{5}{2}$ **f** $p = -\frac{23}{2}$
g $y = \frac{13}{5}$ **h** $y = -\frac{3}{2}$
11 **a** $p + 8 = 15, p = 7$ **b** $q \times -3 = 12, q = -4$
c $2k - 4 = 18, k = 11$ **d** $3r + 4 = 34, r = 10$
e $10 - x = 6, x = 4$ **f** $10 - 3y = 16, y = -2$
12 **a** $x = 2$ **b** $x = 2$ **c** $x = 6$
13 **a** $x = 7, y = \frac{14}{5}$ **b** $x = 2, y = 26$
c $x = \frac{10}{3}$ **d** $p = 15,\ q = \frac{13}{4}$
14 $10x + 6x = 194.88$, hourly rate = \$12.18

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15 a
$7x + 4 = 39$ (−4) → $7x = 35$ (÷7) → $x = 5$
(×−2) $-2x = -10$ (+13) → $-2x + 13 = 3$

b
$10k + 4 = 24$ (−4) → $10k = 20$ (÷10) → $k = 2$
(×3) $3k = 6$ (−1) → $3k - 1 = 5$

16 a
$4x + 3 = 11$ (−3) → $4x = 8$ (÷2) → $2x = 4$

b no

c The two equations have different solutions and so cannot be equivalent.

17 a $x = 5$

b Opposite operations from bottom to top.

c For example, $7 - 3x = -8$.

d You can start with $x = 5$ and perform any operation to get a new equivalent equation (e.g. multiply by 2, multiply by 3, ….)

18 a $x = 3$ **b** $x = 1$ **c** $x = 2$ **d** $x = 4$
e $x = 3$ **f** $x = 1$ **g** $x = \frac{3}{2}$ **h** $x = 0$
i $x = 1$

Exercise 7C

1 a 8 **b** 5 **c** no
2 a 30 **b** 10 **c** × 2, 22 **d** × 10, 70
3 a C **b** A **c** B **d** D
4 a $b = 20$ **b** $g = 20$ **c** $a = 15$ **d** $k = 18$
e $l = 20$ **f** $w = -10$ **g** $s = -6$ **h** $v = 12$
i $m = 14$ **j** $n = 14$ **k** $j = -5$ **l** $f = 20$
5 a $t = -12$ **b** $h = 2$ **c** $a = -2$ **d** $c = -3$
e $s = -6$ **f** $j = 2$ **g** $v = -12$ **h** $n = 9$
i $q = -6$ **j** $f = 3$ **k** $l = -6$ **l** $r = 7$
m $x = 10$ **n** $u = 3$ **o** $k = 4$ **p** $b = -11$
q $m = 12$ **r** $y = 8$ **s** $p = 3$ **t** $g = -2$
6 a $x = 35$ **b** $y = -24$ **c** $p = 14$ **d** $x = 16$
e $x = 12$ **f** $k = -13$
7 a 19 **b** −13 **c** 12 **d** 26
8 a $100 - \frac{b}{3} = 60$ **b** \$120
9 a $x = 15$
b Yes
c **i** $q = 130$ **ii** $q = 130$
d Keeps numbers smaller, so can be done without calculator.
e **i** $p = 28$ **ii** $q = -81$ **iii** $p = -77$ **iv** $r = 34$
10 a $x = 6$ **b** $x = 3$ **c** $x = 1$ **d** $x = 2$
e $x = 8$ **f** $x = -5$
11 a $x = \frac{57}{4}$ **b** $x = \frac{22}{3}$ **c** $x = \frac{80}{3}$ **d** $x = \frac{2}{3}$
12 a $x = 24$ **b** $x = 60$ **c** $x = 12$ **d** $x = 24$
e $x = 15$ **f** $x = 42$

Exercise 7D

1 a true **b** false **c** false **d** true
2 a $3x + 3$ **b** 5 **c** $5p + 9 = 5$ **d** $22k + 12 = 13$
3 B
4 a $f = 5$ **b** $y = 3$ **c** $s = 3$ **d** $j = 2$
e $t = -2$ **f** $n = -5$ **g** $y = -5$ **h** $t = -4$
i $q = -3$
5 a $t = -2$ **b** $c = 2$ **c** $t = 5$ **d** $z = 3$
e $t = 3$ **f** $q = -2$ **g** $x = 9$ **h** $w = 9$
i $j = -5$
6 a $a = 3$ **b** $g = 2$ **c** $n = -2$ **d** $u = 7$
e $h = -5$ **f** $j = -5$ **g** $c = 1$ **h** $n = -1$
i $a = -4$ **j** $v = -7$ **k** $c = -3$ **l** $t = 3$
m $n = 4$ **n** $n = -3$ **o** $l = 2$
7 a $x = \frac{1}{2}$ **b** $k = \frac{2}{3}$ **c** $m = \frac{-3}{2}$ **d** $j = \frac{5}{2}$
e $j = \frac{-1}{2}$ **f** $z = \frac{11}{2}$
8 a $2x + 3 = 3x + 1$ so $x = 2$ **b** $z + 9 = 2z$ so $z = 9$
c $7y = y + 12$ so $y = 2$ **d** $n + 10 = 3n - 6$ so $n = 8$
9 a $x = 6$ and $y = 10$ **b** $x = 4$ and $y = 7$
10 Area = 700 units^2, Perimeter = 110 units
11 a $4p + 1.5 = 2p + 4.9$ **b** \$1.70 **c** 11
12 a $x = 5$ **b** $x = 5$
c Variable appears on RHS if subtract 3x.
13 $x = 8, y = 6,$ so length = width = 29.
14 a No solutions.
b Subtract $2x$, then $3 = 7$ (impossible).
c $5x + 23 = 5x + 10$. Other answers are possible.
15 a $x = 20$ **b** $x = 17, y = 51, z = 10$ **c** $k = 12$
d $b = 10, a = 50$ **e** $a = 60, b = 30, c = 20$
f $x = 3.5$

Exercise 7E

1 a 12 **b** 14 **c** 8, 10 **d** 50, 30
2 a C **b** A **c** D **d** B
3 a

x	0	1	2
$4(x + 3)$	12	16	20
$4x + 12$	12	16	20
$4x + 3$	3	7	11

b $4x + 12$
4 a $7x$ **b** $9p + 3$ **c** $6x + 4$ **d** $7k + 4$
e $x + 6$ **f** $5k + 21$

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5 **a** $u = 6$ **b** $j = 3$ **c** $p = 6$ **d** $m = 4$
e $n = 5$ **f** $a = 3$
6 **a** $p = -2$ **b** $u = -3$ **c** $v = -5$ **d** $r = -4$
e $b = -7$ **f** $d = 3$
7 **a** $y = 3$ **b** $l = 2$ **c** $w = 2$ **d** $c = 2$
e $d = 2$ **f** $w = 6$ **g** $p = 4$ **h** $k = 2$
i $c = 10$
8 **a** $x = 7$ **b** $r = 5$ **c** $f = 3$ **d** $p = 2$
e $h = 4$ **f** $r = 5$ **g** $r = 5$ **h** $p = 6$
i $a = 5$
9 **a** $r = 7$ **b** $l = 2$ **c** $x = 7$ **d** $s = 8$
e $y = 7$ **f** $h = 3$
10 **a** $d + 4$ **b** $2(d + 4)$ **c** $2(d + 4) = 50$
d 21
11 **a** $5w + 3(w + 4)$ **b** \$11.50
12 **a** $2(k - 5) = 3(k - 10)$ **b** 20
13 **a** 3.5 **b** 80
14 The equation is $2(x+3) = 4x-11$, which would mean 8.5 people next door.
15 Equation $3(x - 5) = 9(x + 1)$ has solution $x = -4$.
He cannot be -4 years old.
16 **a** $2n + 6 = 2n + 3$ implies $6 = 3$.
b $8x + 12 = 8x + 3$ implies $12 = 3$, but if $x = 0$ then $4(2x + 3) = 4x + 12$ is true.
17 **a** $x = \frac{3}{2}$ **b** $p = \frac{36}{5}$ **c** $n = -\frac{81}{10}$ **d** $q = 0$
e $x = \frac{8}{5}$ **f** $m = \frac{1}{4}$

Exercise 7F

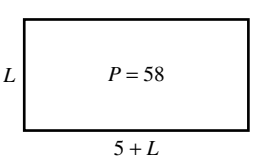
1 **a** 11 **b** 6 **c** 7 **d** -28
2 B
3 A
4 **a** $A = 19$ **b** $A = 51$ **c** $A = -1$ **d** $A = 33$
5 **a** $a = 5$ **b** $a = 11$ **c** $a = -2$
6 **a** $y = 10$ **b** $x = 6$ **c** $x = -2$
7 $m = 5.5$
8 **a** $y = 4$ **b** $y = 8$ **c** $x = 4.5$ **d** $x = 6$
9 **a** $G = 43$ **b** $a = 9$
10 **a** \$23
b **i** $161 = 3 + 2d$ **ii** $d = 79$ **iii** 79 km
11 **a** $F = 50$ **b** $C = 35$ **c** 37.8°C
12 **a** $A = 60$ **b** $h = 4$ **c** 11
13 **a** $T = \frac{V}{2} + 5$. Other answers are possible.
b 44 mL if using rule above. Other answers are possible.
c $\frac{(10-10)^2}{20} + 10 = 10$ and $\frac{(20-10)^2}{20} + 10 = 15$
14 **a** 92 **b** 24
15 A and C
16 **a** $-40°C = -40°F$ **b** $160°C = 320°F$
c $1.8x = 1.8x + 32$ implies $0 = 32$
17 **a** A charges \$14.80, B charges \$16.20.
b 45 minutes
c 2 minutes per call
d $t = 50$ and $c = 40$. Other answers are possible.
e 40 minutes

Progress quiz

1 **a** T **b** F **c** T
2 **a** $k = 16$ **b** $c = 13$ **c** $m = 8$ **d** $t = 5$
3 **a** $2n + 5 = 17$ **b** $a + 26 = 3a$
4 **a** $a = 7$ **b** $k = -3$ **c** $h = -7$ **d** $y = 8$
e $u = 12$ **f** $j = 7$ **g** $d = 5$ **h** $m = -8$
5 **a** $u = 40$ **b** $h = 14$ **c** $x = 9$ **d** $w = 4$
6 **a** $n = 7$ **b** $w = -2$ **c** $e = -9$
7 **a** $a = 5$ **b** $w = 11$ **c** $q = 5$ **d** $p = -4$
8 **a** $l = 10$ cm **b** $b = 16$ cm
9 **a** $-6q = 30$; $q = -5$
b $\frac{2m}{3} = 12$; $m = 18$
c $\frac{3k+4}{2} = -13$; $k = -10$
d $\frac{3x+10}{2} = 14$; $x = 6$
10 $5(m - 8) = 3(m + 2)$; $m = 23$
11 $h = 12$ cm

Exercise 7G

1 **a** D **b** A **c** E **d** C **e** B
2 **a** B **b** C **c** A **d** D
3 **a** $p = 6$ **b** $x = 9$ **c** $k = 4$ **d** $a = 3$
4 **a** Let c = cost of one cup. **b** $4c = 13.2$ **c** $c = 3.3$
d \$3.30
5 **a** Let c = cost of one chair. **b** $6c + 1740 = 3000$
c $c = 210$ **d** \$210
6 **a** $2(4 + w) = 72$ or $8 + 2w = 72$ **b** $w = 32$
c 32 cm
7 **a** Let t = time spent (hours). **b** $70 + 52t = 252$
c $t = 3.5$ **d** 210 minutes
8 2
9 **a** $4w = 26$, $w = 6.5$ **b** 42.25 cm^2
10 Alison is 18.
11 $x = 65$, $y = 115$
12 **a** 118 **b** 119 **c** 12 688
13 **a**



b 12 m **c** 204 m^2
14 **a** 50 **b** -6 **c** $x = 2R - 10$
15 **a** A, B and E
b A, B and D

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c Impossible to have 0.8 people in a room but can have 8 mm insect.

d The temperature this morning increased by 10°C and it is now 8°C. Other answers are possible.

16 a 17 **b** -8 **c** $\frac{2}{7}$ **d** $\frac{4}{3}$ **e** 25

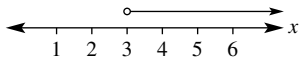
Exercise 7H

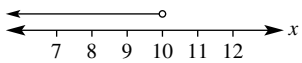
1 a true **b** false **c** true **d** false
e true **f** true

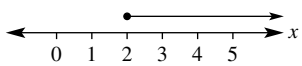
2 a D **b** A **c** B **d** C

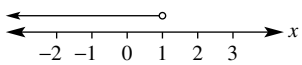
3 a true **b** false **c** false **d** true

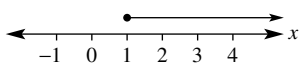
4 a true **b** false **c** false **d** true

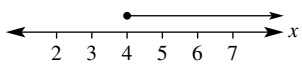
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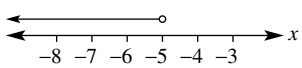
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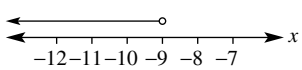
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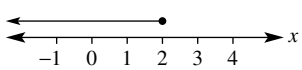
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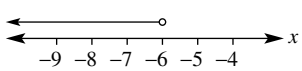
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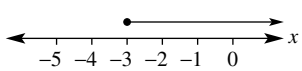
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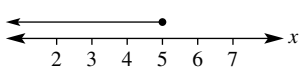
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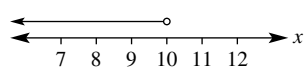
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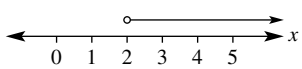
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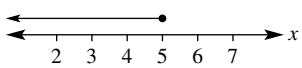
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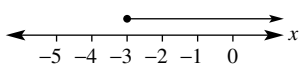
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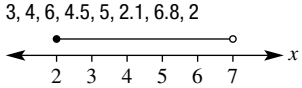
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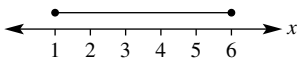
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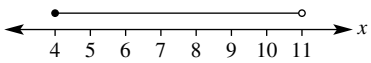
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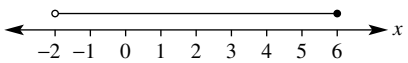
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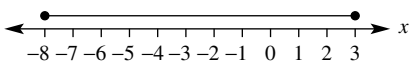
6 a 3, 4, 6, 4.5, 5, 2.1, 6.8, 2

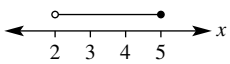
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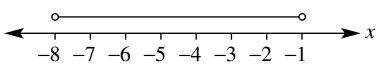
7 a


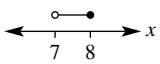
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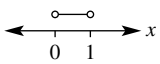
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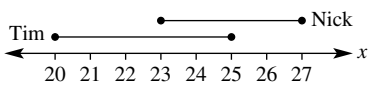
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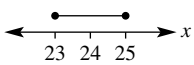
h


8 a C **b** A **c** G **d** F

9 a $20 \leq t \leq 25$ and $23 \leq n \leq 27$

b


c They are between 23 and 25 years old.



10 a **i** E **ii** B **iii** A **iv** B **v** D

b $73 \leq x < 80$

c D

d B or A

e C only

11 a If x is larger than 7 it must automatically be larger than 5.

b **i** $x > 5$ **ii** $x < 3$ **iii** $x > 1$ **iv** $x < 10$
v $3 < x < 10$ **vi** $7 < x \leq 10$

c **i** $3 < x < 5$ **ii** $-2 < x < 4$ **iii** $7 < x < 8$
iv $9 \leq x < 10$

12 a 1 **b** ∞ **c** ∞ **d** 0 **e** 0
f 1 **g** ∞ **h** 0

13 a Must be true. **b** Must be true.
c Not necessarily (e.g. $y = 0$). **d** Must be true.
e Not necessarily (e.g. $x = 3$ and $y = 5$). **f** Must be true.

14 a 30 **b** 110 **c** 200 **d** 1000
e 10 **f** 20 **g** 34 **h** 10

Exercise 7I

1 a true **b** true **c** false **d** false

2 a true **b** false **c** false **d** true

3 a 4 **b** $x < 4$

4 a 6, 3 **b** $x \geq 3$ **c** true

5 a $x > 3$ **b** $l \geq 3$ **c** $g > 5$ **d** $r \leq 7$
e $k > 2$ **f** $s < 5$ **g** $a > 4$ **h** $n \geq 5$
i $d \geq 7$ **j** $h < 5$ **k** $r \leq 2$ **l** $y < 4$

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6 **a** $d > 29$ **b** $y \leq 10$ **c** $x > 11$ **d** $q \leq 18$
e $x > 7$ **f** $h < 1$ **g** $p \geq 2$ **h** $j < 5$
7 **a** $x > 1$ **b** $s \leq 2$ **c** $n < 5$ **d** $j > 10$
e $v \geq -2$ **f** $j \geq -5$ **g** $c \leq 2$ **h** $h \geq 0$
i $s < -3$ **j** $v < -2$ **k** $v \leq -2$ **l** $v > -5$
8 **a** C **b** A **c** B **d** D
9 **a** $4c + 20 > 25$ **b** $c > 1.25$ **c** \$1.30
10 **a** $6g + 4 \leq 36$ **b** $g \leq \frac{16}{3}$ **c** 5 goals
11 **a** $C \geq 15$ **b** $C \leq 20$ **c** $15 \leq C \leq 20$
d $47.\dot{7} \leq C \leq 57.\dot{7}$
12 **a** $x > 4$ **b** $k > 3$ **c** $a < -2$ **d** $-3 < a < -2$
13 **a** If $5x - 2 > 0$ then $x > \frac{2}{5}$, so x is positive.
b If $2x + 6 > 0$ then $x > -3$, so $x + 5 > 2$ is positive.
c Yes, for example if $x = 4$.
d no
14 **a** clues B and D **b** $x = 8$.
15 **a** $x > 2$ **b** $a \leq -2$ **c** $b < -3$ **d** $c < -4$
16 **a** $x \leq 3$ **b** $a > -3$ **c** $b \leq 8$ **d** $c > 2$
e $k > 20$ **f** $g < -5$ **g** $a > 8$ **h** $k \leq 9$
i $p > 4.5$

Problems and challenges

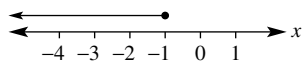
1 **a** 13 **b** 7.5 **c** 9 years **d** \$44.44
e 15 **f** 150 units
2 **a** 2nd step or 3rd line (can't divide by 0)
b
$$28 \begin{pmatrix} 0 = 1 \\ 0 = 28 \end{pmatrix} \times 28$$
$$22 \begin{pmatrix} \\ 22 = 50 \end{pmatrix} + 22$$
3 $6 \leq x \leq 7$
4 $2x = 2(3 + x) - 1$ or $3x + 1 = 2 + 3(x + 1)$
5 **a** 65 kg, 62 kg, 55 kg
b 70 kg, 60 kg, 48 kg
c 35 kg, 42 kg, 45 kg, 48 kg
6 **a** 188 mm
b $L = 8 + 36n$
c 195 links

Multiple-choice questions

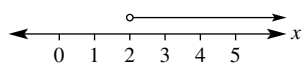
1 A **2** D **3** C **4** C **5** B
6 B **7** D **8** E **9** A **10** A

Short-answer questions

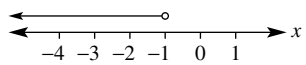
1 **a** false **b** true **c** true
2 **a** $m = 4$ **b** $m = -12$ **c** $a = -1$
d $m = \frac{1}{5}$ **e** $m = 15$ **f** $a = 6$
3 **a** $2m + 3 = 3m$ **b** $5(n + 4) = 20$
c $x + x + 2 = 74$
4 **a** subtract 15 **b** add 5 **c** subtract $2a$
5 **a** $a = 4$ **b** $y = -9$ **c** $x = -4$ **d** $x = 4$
e $x = 2$ **f** $a = 1$
6 **a** $m = -6$ **b** $x = 8$ **c** $y = -18$ **d** $k = -58$
e $w = -2$ **f** $a = 43$
7 **a** $x = 14$ **b** $x = 6$ **c** $x = 40$
8 **a** $a = 8$ **b** $m = \frac{1}{2}$ **c** $x = 4$ **d** $a = 2$
e $x = -8$ **f** $x = 9$
9 **a** $x = 3$ **b** $x = -4$ **c** $x = -4$ **d** $a = 4$
e $a = -7$ **f** $m = 4$ **g** $a = -3$ **h** $x = 17$
i $x = 1$
10 **a** $I = 21$ **b** $M = 3$ **c** $c = 4$ **d** $x = 3, y = 2$
11 **a** 12 **b** 4 **c** 8.5
12 **a** $x \leq 4$ **b** $1 < x \leq 7$
13 **a**



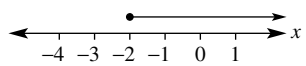
b



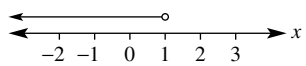
c



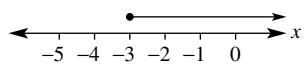
d



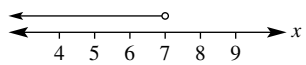
e



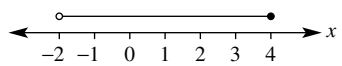
f



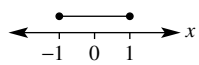
g



h



i



14 **a** $x \geq 100\,000$ **b** $x \leq 6700$ **c** $1.54 \leq x \leq 1.9$
15 **a** $x > 2$ **b** $x < 8$ **c** $x < -4$ **d** $x \geq 2$
e $x < -2$ **f** $x \geq 8$ **g** $x < -12$ **h** $x > 13$
i $x \geq 2$

Extended-response questions

1 **a** $S = 20 + 0.12n$ **b** 30 times **c** $Y = 15 + 0.2n$
d 25
e $20 + 0.12n = 15 + 0.2n$, $n = 62.5$, so 63 is the minimum number.
2 **a** $18x - 180$
b **i** \$35.50 **ii** \$41 **iii** \$459
c \$25
d \$30
e $30 \leq x \leq 35$

Chapter 8

Exercise 8A

1 **a** 320 **b** 270 **c** 300 **d** Expton **e** Calcville
2 **a** 10 **b** 6 **c** Phillip **d** Nyree **e** 4 years

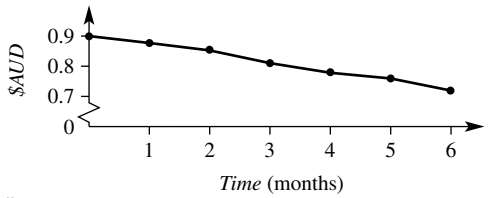
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3 **a** 9 **b** handball **c** 24 **d** 8F **e** water polo

4 **a** Slesha **b** Ross **c** 4 years old **d** non-linear

5 **a** rent **b** charity **c** 50% **d** $2400

6 **a**



b linear

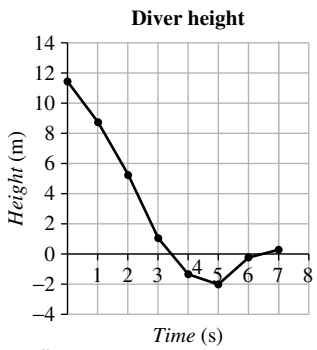
c **i** $ 0.09 **ii** $ 0.02

d **i** $ 0.69 **ii** $ 0.63

7 **a** 2 hours **b** 7 hours **c** sleeping **d** $\frac{1}{6}$

8 **a** 20°C **b** 12°C **c** midday

9 **a**



b non-linear **c** 1.5 metres

d the fourth second **e** 2 metres below surface

f Answers will vary.

10 **a** Survey 2 **b** Survey 1 **c** Survey 3

11 **a** Town A population decreased then increased. Town B steady increase, Town C population increased then decreased.

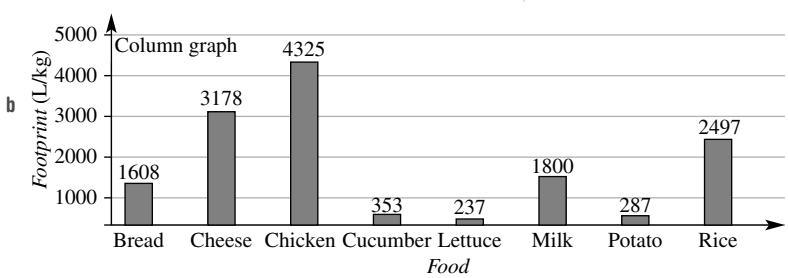
b **i** To find the total combined population in the 3 towns.

ii To work out the average population per year (total ÷ 10). Other answers are possible.

12 Need numbers for a meaningful axis but not for labels of each sector.

13 **a** Column graph – categorical data. (Sector graph is inappropriate as not measuring proportions of a whole.)

b



c As water becomes scarcer it is more difficult to produce these foods.

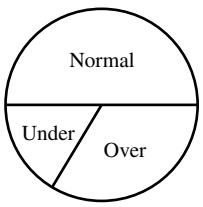
d Answers will vary.

e

	Bread	Cheese	Chicken	Cucumber	Lettuce	Milk	Potato	Rice
Efficien y (g/kL)	622	315	231	2833	4219	556	3484	400

14 **a** 2 months underweight, 6 months normal weight, 4 months overweight

b



c Can see how weight changes over time.

d Can see how much of the year the dog was underweight, overweight and normal weight.

e Answers will vary.

Exercise 8B

1 **a** true **b** false **c** true **d** false

2 **a** 4 **b** 7 **c** || **d** 𝍸 𝍸 |

3 **a**

	Passes	Shots at goal	Shots that go in	Steals
Frequency	3	12	8	2

b 12 **c** 8 **d** 4

4 **a**

People in family	2	3	4	5	6	7	8
Tally	|	||	𝍸	𝍸 |	𝍸 ||	||	𝍸 ||
Frequency	1	2	4	4	4	2	3

b 4 **c** 9

5

a

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–166
Tally	5	3	12	15	9	4	2

b 50 **c** 9 **d** 8 **e** 35

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6 a

Height (cm)	Tally	Frequency
130–139	III	3
140–149	IIII	5
150–159	II	2
160–169	III	3
170–179	III	3
180–189	I	1
190+	IIII	4

b 2 **c** 5 **d** 10

7 a 10 **b** 2 **c** 4 **d** 17

8 a B **b** D **c** A **d** C

9 a 28 **b** 130 **c** 19 **d** 13.1 years old **e** 14.4

10 a

Score	0–19	20–39	40–59	60–79	80–100
Frequency	0	4	7	20	12

b

Score	0–29	30–59	60–89	90–100
Frequency	3	8	30	2

c It is unknown how many of the 3 people in the 20s got less than 25 and how many got more.

d

Score	0–24	25–49	50–74	75–100
Frequency	2	4	20	17

e 43

11 a

Range	10–19	20–29	30–39
Frequency	3	4	6

b Many possible answers.

c a stem-and-leaf plot

d when individual numbers are not required but an overview is more important

12 a Many possible answers.

b There are 5 possible values and it happened 6 times, so one value is repeated.

c Even if each score was achieved twice that would only account for 10 weeks (not 11).

d yes

e yes

13 a Monday 3, Tuesday 2, Wednesday 1, Thursday 3

b 2 hours **c** 12 ways **d** 3 ways **e** 6 ways

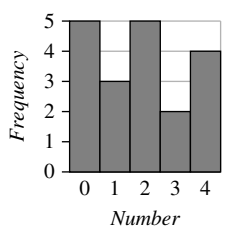
f 30 ways

Exercise 8C

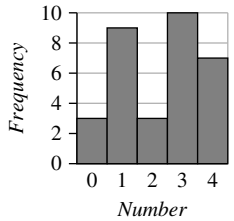
1 a 2 **b** 9 **c** 11 years old

2 a 4 **b** 4 **c** 8

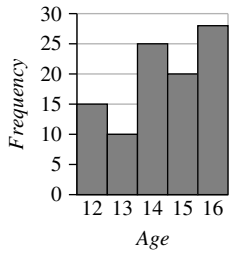
3 a



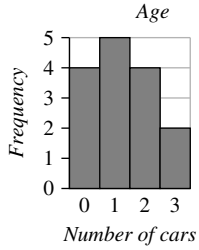
b



c



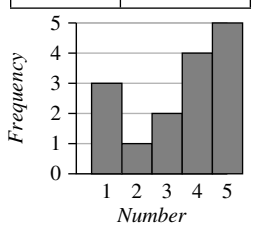
d



4 a i

Number	Frequency
1	3
2	1
3	2
4	4
5	5

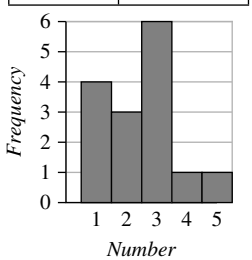
ii



b i

Number	Frequency
1	4
2	3
3	6
4	1
5	1

ii

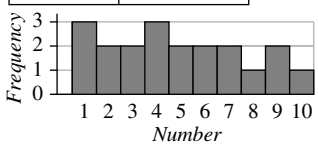


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c i

Number	Frequency
1	3
2	2
3	2
4	3
5	2
6	2
7	2
8	1
9	2
10	1

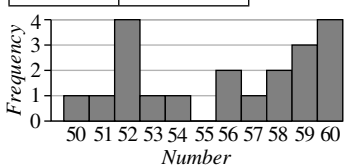
ii



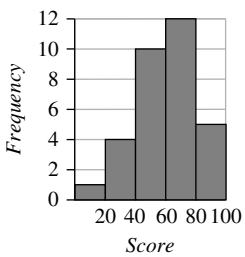
d i

Number	Frequency
50	1
51	1
52	4
53	1
54	1
55	0
56	2
57	1
58	2
59	3
60	4

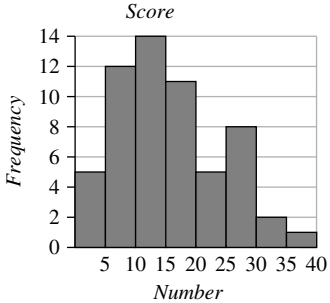
ii



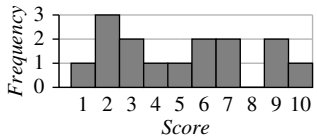
5 a



b



6 a



b Edwin is worse than Fred as most of Fred's scores are 8 or higher.

7 a D **b** A **c** B **d** C

8 a Marie **b** Con **c** Frank **d** Bill

9 a It would look identical but the x-axes labels would start at 22 and go to 26.

b It would look just like the right half (12–14, but labelled 0–2).

10 a No, just that she is more likely to get higher marks than lower marks

b 9 weeks of 5, then 8 weeks of 6, then 7 weeks of 7, then 4 weeks of 8, then 2 weeks of 9 out of 10

c They were absent from the test, or having a very bad day.

11 a

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4	Graph 7	Graph 6
Shopping centre	Graph 8	Graph 2	Graph 9
Teachers common room	Graph 5	Graph 3	Graph 1

b Answers will vary.

Exercise 8D

1 a mode **b** mean **c** median

2 a 15 **b** 5 **c** 3

3 a 1, 2, 4, 5, 6, 7, 9 **b** 5 **c** 5

4 a 7 and 9 **b** 16 **c** 8

5 a i 5 **ii** 4 **b i** 2 **ii** 2
c i 5 **ii** 3 **d i** –3 **ii** 0
e i 0 **ii** –9 **f i** 0 **ii** 3
g i 12.9 **ii** 15 **h i** 13.1 **ii** 20
i i 11.1 **ii** 12 **j i** 10.4 **ii** 5
k i 2.4 **ii** –6 **l i** –3.4 **ii** –6

6 a 6 **b** 4 **c** 8 **d** 5
e 8 **f** 7 **g** 3 **h** 7

7 a 5 **b** 5.5 **c** 7.5 **d** 8
e 10.5 **f** 12

8 a 8.4 **b** 8 **c** 8
d Only the mean would change (increase)

9 a 18, 19, 20, 23, 25, 27, 31, 32, 32, 37 **b** 26
c 26.4 **d** 32

10 a white **b** meat-lovers **c** wednesday
d South Australia

11 a i 51 grams **ii** 50.39 grams
b shop A

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12 **a** 3 **b** 10 **c** 8.8 **d** 9
e If one of the 7 scores become a 1. (others possible)
13 **a** 16 years old **b** 15.03 years old
c 15
14 **a** 16 **b** –3 **c** 5 **d** 7 **e** 9
15 Most frequent value makes sense for numbers or categories, but the mean requires adding (numbers) and the median requires ordering (numbers).
16 **a** 6 **b** It is multiplied by 3 (18).
c It is four higher (10). **d** It is now 44 (not 6 squared).
e It is squared.
17 **a** \$1 477 778 **b** \$630 000
c A strong effect — it makes the mean significantly higher.
d No effect — it is not factored in the median.
e Median is not easily distorted by a few very large values.
18 **a** 4 **b** 11 **c** 2 **d** 4.5
19 **a** Possible: 1, 5, 7, 7
b Impossible: median = mean for set with two items
c Impossible: must be x, x, y and then mode = median
d Possible: –5, 3, 5, 5
20 **a** Many answers possible.
b 40, 60, 80, 60, 60, 0, 20, 80
c 59, 79, 100, 79, 79, 19, 39, 100
d No
e between 50 and 69.25
f C or B
g When sorted from worst to best, she got E, D, C, B, B, B, A, A and the average of the two middle marks must be a B if they were both Bs.
h **i** 75 to 94.75 **ii** B or A

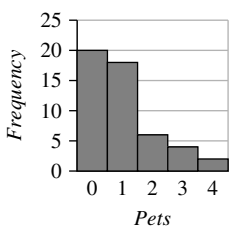
Exercise 8E

1 **a** 8 **b** 1 **c** 7
2 **a** 4 **b** 9 **c** 5
3 **a** 5 **b** 12 **c** 7
4 **a** lower quartile **b** lowest **c** sort (or order)
d median **e** odd **f** centre
5 **a** 4 **b** 10 **c** 5.5
6 **a** 3, 4, 5, 5, 6, 7, 8, 10 **b** 4.5 **c** 7.5 **d** 3
7 **a** 9 **b** 10 **c** 15 **d** 14
e 30 **f** 27 **g** 16.9 **h** 8.7
8 **a** 19 **b** 11 **c** 21 **d** 15 **e** 7
f 9.5 **g** 3.16 **h** 1.76
9 **a** 23 **b** 19 **c** 35 **d** 32
e 16.5 **f** 10.5 **g** 3.45 **h** 1.15
10 **a** 11 **b** 10.5 **c** 9 **d** 8
e 32 **f** 23 **g** 18 **h** 13
11 **a** 15 **b** 35 **c** Nathan **d** Gary
12 **a** 1.7 **b** 1.8 **c** Max
13 **a** **i** 9 **ii** 10
b **i** 4.5 **ii** 4.0
c Sara
d Andy
14 **a** 9, 10, 11. Other answers are possible.
b 0, 10, 20. Other answers are possible.
15 **a** 0, 20. Other answers are possible.
b –9, 11. Other answers are possible.
16 **a** **i** 8 **ii** 4
b **i** 98 **ii** 4
c A single outlier does not affect the IQR but the range is greatly affected.
17 **a** 10
b 4
c No, the range is the largest difference between two numbers.
d Yes, for instance, for a set like 2, 2, 2, 6, 6, 6.
18 **a** no effect **b** range is doubled
19 The lower quartile is the number above the bottom quarter of values and the upper quartile is the number below the top quarter of values.
20 **a** 4
b 3
c It would stay the same.
d It would stay the same.
e **i** It would double. **ii** It would double.

Exercise 8F

Some answers may vary.

1 **a** surveying 1000 randomly selected people
b surveying 10 friends
2 **a** $\frac{2}{5}$ **b** 2000 **c** 300
3 **a** symmetric **b** skewed **c** bi-modal **d** skewed
e symmetric
4 **a** sample **b** population **c** population **d** sample
5 **a** **b** skewed



c 400 **d** 1200
e More likely that people will have pets if near a vet clinic.
6 **a** Yes, it is required information.
b No, it is too vague or subjective.
c No, it is too vague or subjective.
d No, it addresses wealth but not income.
e No, it is irrelevant.
f Yes, it can be used to decide income.
g No, if it is not a pay day then results will be distorted.

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7 **a** at midday on a Thursday on a major road
b outside a political party office
c in a butcher's shop
d at 11 p.m., when people will buy just a few items

8 **a** 'How many siblings do you have?'
b–d Results will vary.
e Different samples from among the same population can yield different means.

9 **a** 108 g
b if two bars were on scales together
c skewed (slightly)
d IQR as it is not affected by outlier
e median

10 **a** at a professional dance studio in the afternoon
b at a university physics department
c Choose a large random sample.

11 **a** Only one year level. Possibly streamed class, so similar work ethic.
b Only males would be surveyed, also same age.
c A range of students in age, gender and results.
d A more studious sample would give a higher mean.

12 There will be two distinct taller sections, corresponding to the most common heights for boys and girls. At a single-sex school the distribution is likely to be symmetric.

13 **a** **i** 75% **ii** 95% **iii** 15%
iv Between 15% and 95% of the population are in favour.
b Busy people are less likely to participate and will not be counted.
c 'Do you enjoy participating in surveys?'
d Helps correct for measurement error, caused by people answering either randomly or mistakenly.

14 Answers vary

Progress quiz

1 **a** April, $5500 **b** $500 **c** $19500

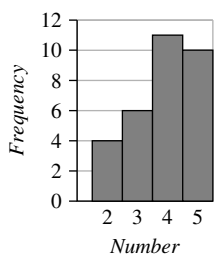
2 **a** 180° **b** 50 **c** milk and soft drink

3 **a** **i** Tuesday **ii** Thursday **iii** Saturday
b 3 cm
c 25 mm

4 **a**

Number	2	3	4	5
Tally	IIII	𝍸 I	𝍸 𝍸 I	𝍸 𝍸
Frequency	4	6	11	8

b



5 **a** 6 **b** 7

6 **a** 3.87 (2 dec pl) **b** 4 **c** 4 **d** 3
e 5 – 3 = 2

7 **a** Bias towards students who buy their lunch - less likely to eat homemade food.
b Bias towards students who spend longer on homework if only Year 12s considered.
c Bias towards adults with at least one child, excluding all childless adults.

Exercise 8G

1 **a** B **b** C **c** D **d** A

2 **a** Event C **b** Event A **c** Event B **d** Event C

3 **a** true **b** false **c** true **d** true

4 **a** P, I, A, N, O **b** $\frac{1}{5}$ **c** $\frac{3}{5}$
d $\frac{2}{5}$ **e** $\frac{4}{5}$

5 **a** 1, 2, 3, 4, 5
b **i** 1, 3, 5 **ii** 2 **iii** 5 **iv** 3, 4, 5
c **i** at most 2 boys **ii** at least 4 boys
iii at least 3 boys **iv** even number of boys

6 **a** 1, 2, 3, 4, 5, 6 **b** $\frac{1}{6}$ **c** $\frac{1}{2}$
d 1, 2, 3, 4, 6 **e** $\frac{5}{6}$ **f** 0

7 **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{3}{10}$ **d** $\frac{4}{5}$
e $\frac{1}{2}$ **f** $\frac{7}{10}$ **g** choosing a purple marble

8 **a** 8 **b** $\frac{1}{8}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$
e green, green, red, yellow, purple **f** $\frac{5}{8}$
g $\frac{3}{4}$ **h** spinning purple (or spinning yellow)
i spinning orange

9 **a** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **b** $\frac{1}{10}$ **c** $\frac{1}{5}$
d $\frac{3}{10}$ **e** $\frac{1}{2}$ **f** $\frac{3}{5}$

10 **a** $\frac{1}{3}$ **b** $\frac{5}{6}$ **c** $\frac{1}{3}$ **d** $\frac{5}{6}$

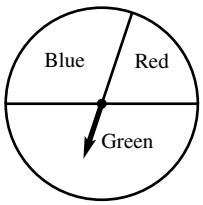
11 **a** 0.7 **b** 0.25 **c** 0.45 **d** 20

12 **a** no **b** yes
c yes **d** no
e yes **f** no
g no **h** yes

13 **a** 1 red, 2 orange, 3 purple
b $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$

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14



15 a $\frac{1}{5}$ b $\frac{1}{4}$ c 18

d It approaches $\frac{1}{2}$ or 0.5.

16 a 0.8

b i 0.4 ii 0.8 iii 0.2

c 2p

d $1 - 2(1 - p)$ or $2p - 1$

e If Pr(pass) = 0.6 then twice as likely to fail gives Pr(other person pass) = 0.2, which is not the same as half of 0.6.

17 a 2 red, 1 blue b 2 blue, 1 green

c 5 green d 6 blue, 1 red, 5 green e 1 green

f 1 blue, 5 green g 6 blue, 5 green

Exercise 8H

1 a 10 b H2, H4, T2, T4 c $\frac{2}{5}$

d T1, T3, T5 e $\frac{3}{10}$

2 a $\frac{1}{4}$ b HH, TT c $\frac{1}{2}$

3

a

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

b 12 c $\frac{1}{12}$ d $\frac{1}{4}$

4 a

	R	I	D	E
L	LR	LI	LD	LE
I	IR	II	ID	IE
N	NR	NI	ND	NE
E	ER	EI	ED	EE

b 16 c $\frac{1}{16}$ d $\frac{1}{16}$ e $\frac{1}{4}$

f $\frac{1}{4}$ g $\frac{1}{8}$

5 a

	R	P	B
R	RR	RP	RB
P	PR	PP	PB
G	GR	GP	GB
B	BR	BP	BB

b $\frac{1}{12}$ c $\frac{1}{12}$ d $\frac{1}{12}$ e $\frac{1}{6}$ f $\frac{1}{4}$

6 a $\frac{1}{12}$ b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{1}{2}$

7 a

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{9}$ c $\frac{8}{9}$ d 7 e 2 and 12

8 a

	Y	W	B	B	B
W	WY	WW	WB	WB	WB
O	OY	OW	OB	OB	OB
O	OY	OW	OB	OB	OB

b $\frac{2}{15}$ c $\frac{1}{15}$ d $\frac{2}{15}$ e $\frac{1}{5}$ f $\frac{1}{15}$

9 a $\frac{1}{8}$ b $\frac{1}{3}$ c $\frac{2}{3}$

10 63

11 a $\frac{1}{52}$ b $\frac{1}{26}$ c $\frac{1}{16}$ d $\frac{1}{4}$ e $\frac{1}{2704}$

f The second card would then depend on the first card (e.g. if a red card was selected, it would be a little less likely that the next card would be red).

12 a i $\frac{1}{4}$ ii $\frac{1}{2}$

b i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{1}{16}$ iv $\frac{1}{8}$

c OYYBBB (or some rearrangement of those letters)

d i $\frac{1}{9}$ ii $\frac{7}{18}$ iii $\frac{1}{12}$ iv $\frac{1}{6}$

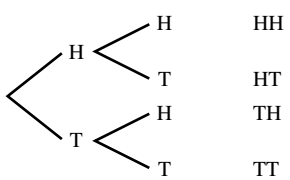
v $\frac{11}{36}$ vi $\frac{3}{4}$

e i $\frac{1}{24}$ ii $\frac{1}{8}$ iii $\frac{1}{12}$ iv $\frac{1}{4}$

v $\frac{1}{4}$ vi $\frac{3}{4}$

Exercise 8I

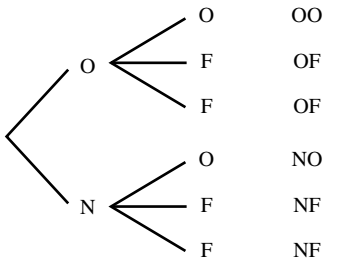
1 a



b 4

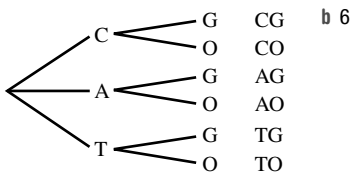
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2 a



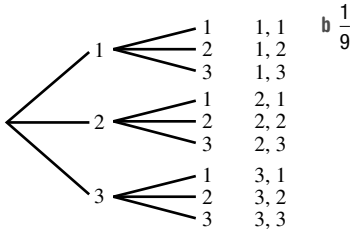
b OF, NO, NF, NF **c** 6 **d** 2

3 a



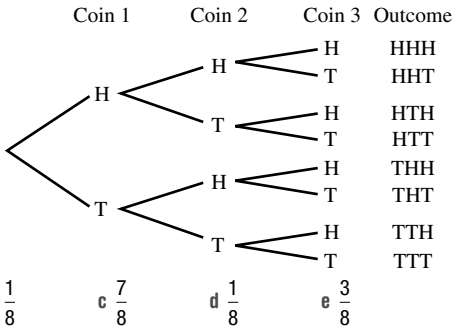
b 6 **c** $\frac{1}{6}$ **d** $\frac{1}{6}$ **e** $\frac{1}{3}$

4 a



b $\frac{1}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$ **e** $\frac{1}{3}$ **f** $\frac{1}{3}$

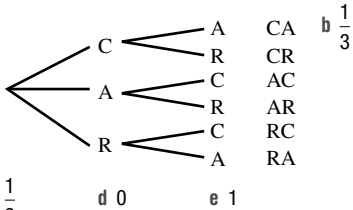
5 a



b $\frac{1}{8}$ **c** $\frac{7}{8}$ **d** $\frac{1}{8}$ **e** $\frac{3}{8}$

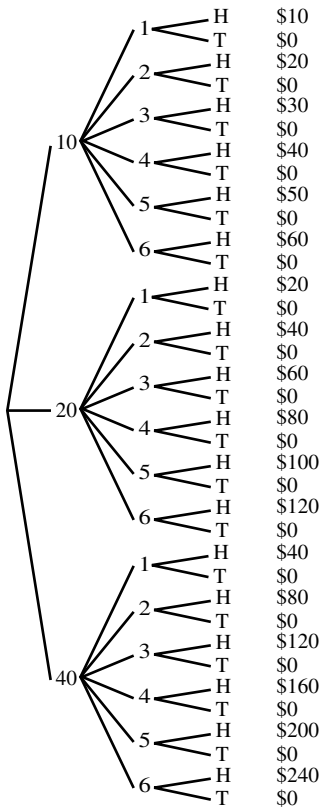
f exactly 2 **g** $\frac{1}{2}$

6 a



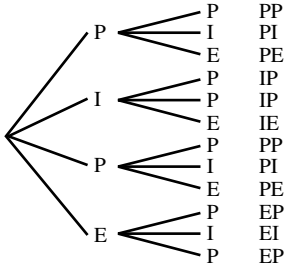
b $\frac{1}{3}$ **c** $\frac{1}{3}$ **d** 0 **e** 1

7 a \$100 **b**



c $\frac{1}{18}$ **d** $\frac{1}{6}$ **e** $\frac{19}{36}$

8 a



b $\frac{1}{6}$ **c** $\frac{1}{6}$ **d** $\frac{5}{6}$

9 32

10 a $\frac{1}{16}$ **b** $\frac{1}{16}$ **c** They are equally likely.

d 3 heads

e 3 heads could be HTHHT or HTTHH or THTHH etc. but heads must be HHHHH $\left(\frac{1}{32}\right)$.

11 a There are 12 outcomes when choosing from WORM and 30 when choosing from MORROW.

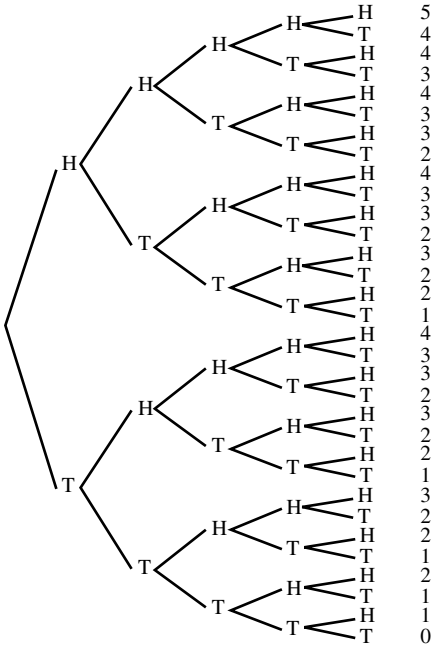
b You can't select two letters that are the same.

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12 **a** The first two coins are tails.
b There are not exactly two tails.
c The first and third coin are not both heads.
d at least one tail
e four heads
f at least one head or at least one tail

13 **a** 20 **b** 120 **c** $\frac{1}{5}$ **d** $\frac{4}{5}$

14 **a**



b $\frac{1}{32}$ **c** $\frac{1}{32}$ **d** $\frac{1}{2}$ **e** $\frac{5}{16}$

f $\frac{319}{512}$ (approximately 0.62)

Exercise 8J

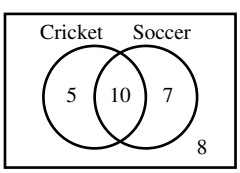
1 **a**

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	30
Total	40	35	75

b 30 **c** 20 **d** 75

2 **a** 2 **b** 4 **c** 1 **d** 3

3 **a** 8 **b**



c 15 **d** 22 **e** 12

4 **a** 15

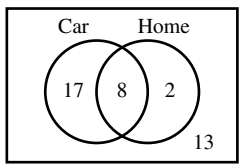
b

	Employed	Unemployed	Total
University degree	10	3	13
No university degree	5	2	7
Total	15	5	20

c The 10, 13, 15 and 20 would all increase by 1.

5 **a** 26
b 12
c 11
d **i** $\frac{2}{13}$ **ii** $\frac{7}{26}$ **iii** $\frac{7}{13}$ **iv** $\frac{15}{26}$

6 **a**



b $\frac{1}{5}$
c $\frac{17}{40}$ **d** $\frac{1}{4}$ **e** $\frac{8}{25}$ **f** $\frac{4}{5}$

7 **a** 12 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$ **d** $\frac{4}{5}$ **e** $\frac{1}{8}$

8 **a**

	B	Not B	Total
A	20	50	70
Not A	20	10	30
Total	40	60	100

b

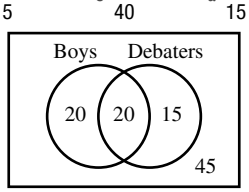
	B	Not B	Total
A	6	5	11
Not A	4	3	7
Total	10	8	18

9 **a**

	Sports	Not Sports	Total
Automatic	2	13	15
Not Automatic	8	17	25
Total	10	30	40

b $\frac{1}{5}$ **c** $\frac{13}{40}$ **d** $\frac{2}{15}$

10 **a**



b **i** 20% **ii** 15% **iii** 60% **iv** 65%
c $\frac{4}{7}$ **d** $\frac{1}{4}$

11 **a** $\frac{1}{5}$ **b** 45 **c** $\frac{2}{9}$ **d** $\frac{1}{3}$

12 A inclusive or B is more likely since it also includes all the values in the middle of a Venn Diagram.

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13 **a** 4

b No, not if the 5 spots are in the last row/column.

c Filling it requires a negative number, which is impossible.

14 **a** $\frac{x}{w+x+y+z}$ **b** $\frac{w+x}{w+x+y+z}$

c

	B	Not B	Total
A	x	w	$x+w$
Not A	y	z	$y+z$
Total	$x+y$	$w+z$	$w+x+y+z$

15 **a** 120

b 60

c **i** $\frac{59}{120}$ **ii** $\frac{19}{40}$ **iii** $\frac{1}{2}$ **iv** $\frac{29}{120}$

v $\frac{7}{20}$ **vi** $\frac{17}{60}$

d **i** $\frac{10}{19}$ **ii** $\frac{5}{19}$

e **i** $\frac{29}{59}$ **ii** $\frac{34}{59}$

f They are male, under 40 and not using a trolley.

g More likely to be female $\left(\frac{30}{59} > \frac{29}{59}\right)$.

h More likely to use trolley $\left(\frac{30}{57} > \frac{27}{57}\right)$.

Exercise 8K

1 **a** 2 **b** 0.2 **c** 0.6

2 **a** 0.19 **b** $\frac{1}{6}$ **c** experimental

3 **a** 0.6 **b** 0.1 **c** 0.86 **d** 40

4 **a**

No. of cars	0	1	2	3	4
Frequency	12	37	41	8	2

b 100

c 0.12 **d** 0.51

5 **a** 50 **b** yes **c** yes (but very unlikely)

6 **a** $\frac{1}{3}$ **b** 200 **c** 300

7 **a** **i** 0.35 **ii** 0.25 **iii** 0.2 **iv** 0.2

b 200

8 **a** Answers will vary. **b** Answers will vary.

c 10

9 **a** Flip 5 coins. Heads = correct answer, tails = incorrect answer. Count number of heads for score out of 5.

b Answers will vary.

10 **a** 3 **b** 2 **c** 14 **d** 3

e 19 **f** 8

11 **a** **i** B **ii** D **iii** A **iv** C

b Answers will vary.

c Answers will vary.

d Answers will vary, but should be approximately 60.

e 39

12 **a** Could be (Red: 1, 2, Blue: 3, 4, Green: 5, 6)

b Could be (Red: 1, 2, 3, 4 Blue: 5, Green: 6)

c Could not be; probability of $\frac{1}{5}$ cannot be achieved with single die roll.

13 **a** If all three coins show heads, then count the event as happening.

b If tails is flipped and the number 5 is rolled.

c If two 'small' numbers are rolled (counting small as 1 or 2).

d If the sum of the dice is 12.

14 **a** 0 **b** $\frac{3}{11}$ **c** $\frac{1}{16}$ **d** false

e true

15 **a** **i** 16 cm^2 **ii** 100 cm^2 **iii** $\frac{4}{25}$ **iv** 84

b **i** 300 **ii** 600 **iii** 100

c approximately 562.5 cm^2

d 56 250 km^2

Problems and challenges

1 5, 11, 14 **2** 41 **3** 12 **4** 0.25

5 **a** MOON **b** OFF **c** DING **d** PROBABILITY

e STUMBLE **f** TRY

6 32 **7** $\frac{5}{9}$ **8** 0.000977

9 1 red, 3 blue, 4 yellow

Multiple-choice questions

1 C **2** B **3** D **4** B **5** C

6 E **7** B **8** C **9** C **10** C

Short-answer questions

1 **a** government bus **b** train **c** 72°

d 1000

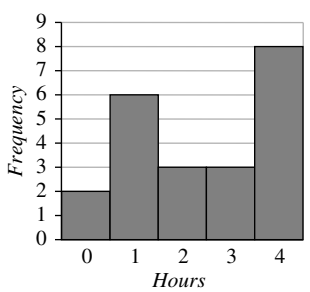
e Example: Prices went up for government buses.

2 **a** 22

b

Hours	0	1	2	3	4
Frequency	2	6	3	3	8

c



d $\frac{1}{11}$

e 2.4

3 **a** 38, 43, 44, 44, 52, 53, 55, 56, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 74, 84

b 44

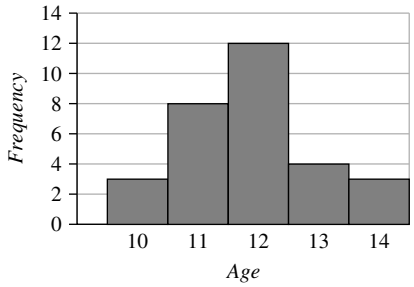
c 61.5

d 16

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4 **a** 30

b



c 11.87 years **d** 12

5 **a** lowest: 50 kg, highest 85 kg

b

Weight	Frequency
50–	6
55–	6
60–	8
65–	7
70–	7
75–	1
80–85	5

c **i** 35 kg **ii** 60–64 kg

d Only teenagers were chosen, not including children or adults.

6 **a** **i** 6.5 **ii** 6 **iii** 3.5 **iv** 9.5 **v** 6

b Mean and median have increased by 1, but IQR is the same because all numbers have just increased by 1.

7 **a** Not enough people, and her friends might work harder (or less hard) than other students.

b She could choose 10 people who worked less hard than her.

8 **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** 1, 4, 6, 8 **e** 0

9 **a** M, A, T, H, E, M, A, T, I, C, I, A, N **b** $\frac{2}{13}$

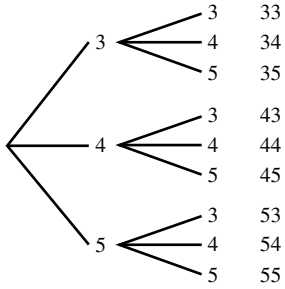
c $\frac{6}{13}$ **d** $\frac{7}{13}$ **e** $\frac{12}{13}$

10 **a**

	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

b $\frac{1}{4}$ **c** $\frac{1}{12}$

11 **a**



b $\frac{1}{3}$ **c** $\frac{1}{3}$ **d** $\frac{1}{3}$ **e** $\frac{2}{3}$ **f** $\frac{1}{3}$

12 **a** 50 **b** 25 **c** 24% **d** 1% **e** $\frac{12}{25}$ **f** $\frac{24}{25}$

Extended-response questions

1 **a**

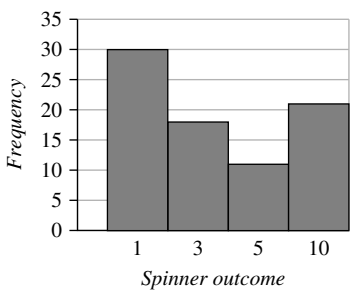
	Uses public transport	Does not use public transport	Total
Own a car	20	80	100
Do not own a car	65	35	100
Total	85	115	200

b 200 **c** $\frac{1}{2}$ **d** $\frac{1}{10}$ **e** $\frac{4}{17}$ **f** $\frac{1}{5}$

g **i** More public transport users expected.

ii People less likely to use public transport in regional area.

2 **a**

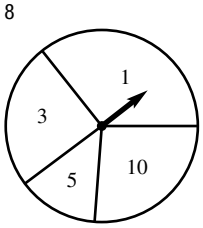


b 1, it has the most occurrences.

c 3 and 10, as they have the closest outcomes.

d $\frac{3}{8}$

e



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Chapter 9

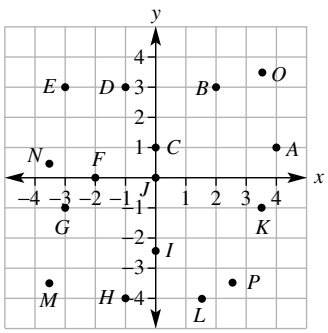
Exercise 9A

1 **a** (0, 0) **b** y **c** 1st quadrant
d 3rd quadrant **e** −2 **f** −5

2 **a** 3 **b** −1 **c** −2 **d** 0
e −2 **f** 0 **g** −3 **h** 0

3 $A(1, 1)$, $B(5, 0)$, $C(3, 4)$, $D(0, 4)$, $E(-1, 2)$, $F(-3, 3)$, $G(-5, 1)$, $H(-3, 0)$, $I(-4, -2)$, $J(-2, -5)$, $K(0, -3)$, $L(2, -3)$, $M(5, -5)$

4



5 **a** house **b** fish

6 **a** B **b** C **c** E **d** D

7 **a** triangle **b** rectangle **c** parallelogram
d kite

8 **a** (2, 4) **b** (−5, 2) **c** (−1, −2.5) **d** (4.5, −4.5)

9 **a** (1, −2), (1, −1), (1, 0), (1, 1)
b (−1, 0), (0, 0), (1, 0), (2, 0) **c** (−2, 3), (−1, 2), (0, 1), (1,0)
d (−2, −3), (−1, 0), (0, 3), (1, 6), (2, 9)

10 **a** quadrant 4 **b** quadrant 2
c quadrants 2 and 3 **d** quadrants 3 and 4

11 a line on the y-axis

12 **a** 10 **b** 4 **c** 7 **d** 11 **e** 6
f 4

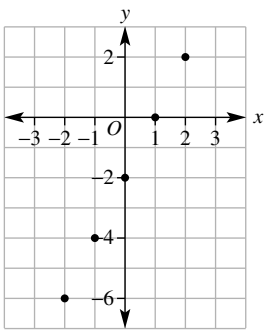
13 **a** 5 **b** 13 **c** 25 **d** $\sqrt{13}$
e $\sqrt{90}$ **f** $\sqrt{61}$

Exercise 9B

1 **a** 5 **b** 7 **c** 3 **d** 1
e −7 **f** −11 **g** 25 **h** −21

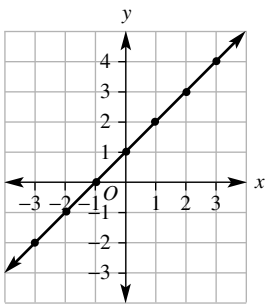
2 **a** 2 **b** −2 **c** −13

3



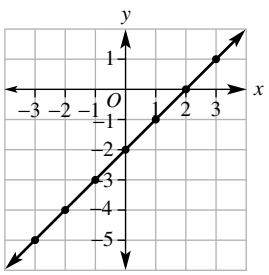
4 **a**

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4



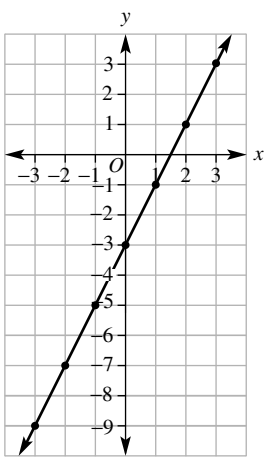
b

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1



c

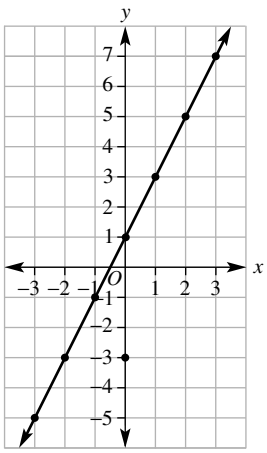
x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3



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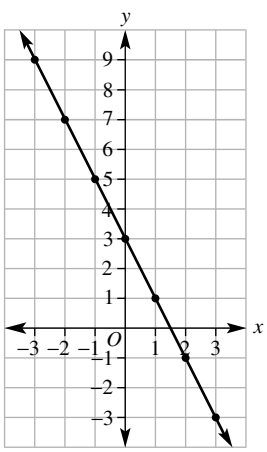
d

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7



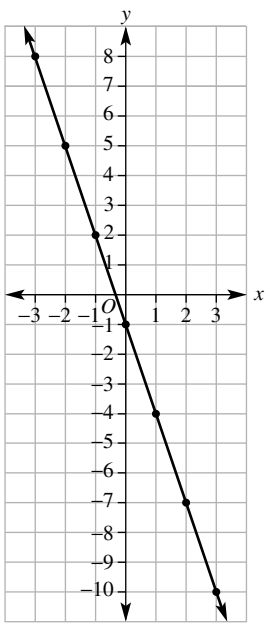
e

x	−3	−2	-1	0	1	2	3
y	9	7	5	3	1	−1	−3



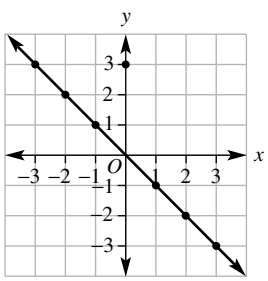
f

x	−3	−2	−1	0	1	2	3
y	8	5	2	−1	−4	−7	−10



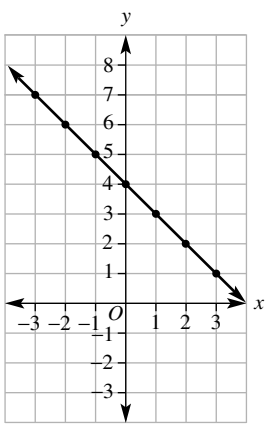
g

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



h

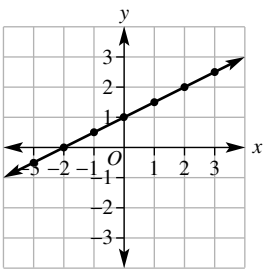
x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1



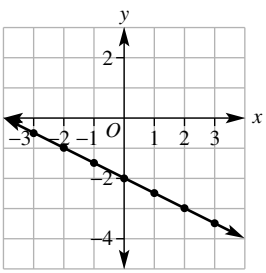
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5 **a** **i** yes **ii** no
b **i** no **ii** yes
c **i** no **ii** no
d **i** yes **ii** yes
e **i** yes **ii** no
f **i** yes **ii** yes
g **i** no **ii** yes

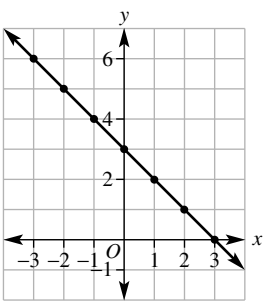
6 **a**



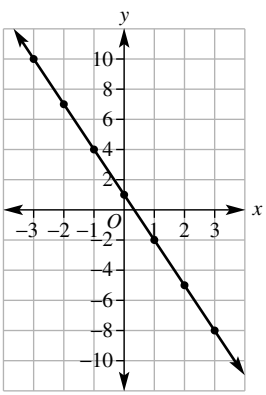
b



c



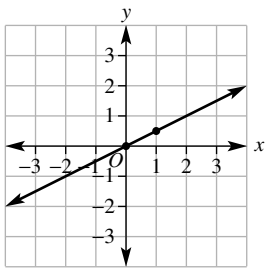
d



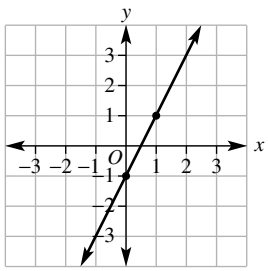
7 **a** (−1, 0), (0, 1) **b** (2, 0), (0, 2) **c** (−2, 0), (0, 4)
d (2, 0), (0, 10) **e** (1.5, 0), (0, −3)
f $(\frac{7}{3}, 0)$, (0, 7)

8 (1, 3)

9 **a** 2
b **i**



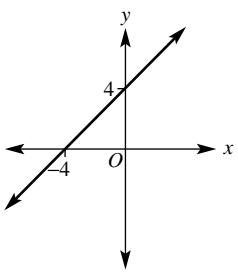
ii



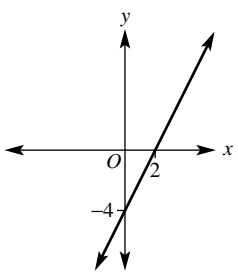
10 **a** When $x = 0$, $y = 0$ for all rules.
b All the rules have a constant so when $x = 0$ $y \neq 0$.

11 **a** The coefficient of x is positive.
b The coefficient of x is negative.
c the coefficient of x

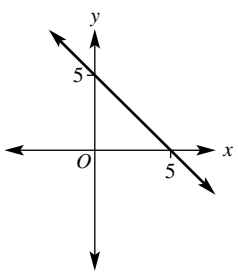
12 **a**



b

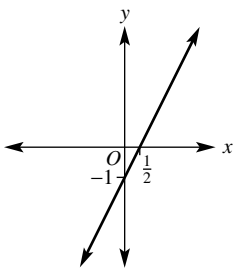


c

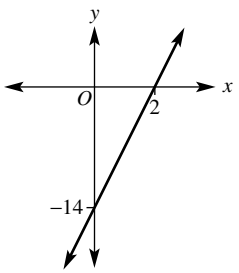


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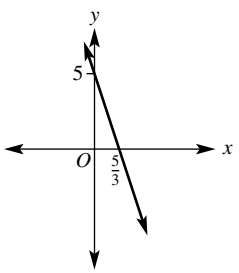
d



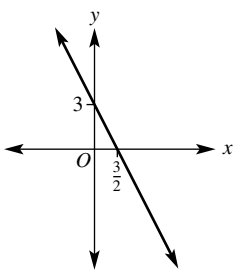
e



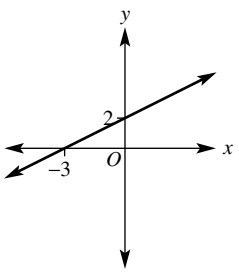
f



g



h



Exercise 9C

1 **a** 2 **b** −1 **c** 2 **d** −3

2 **a** 3 **b** 1 **c** 1 **d** 0

3 **a** 2 **b** −2 **c** −3 **d** −1
e −10 **f** −2 **g** 0 **h** 3 **i** 4

4 **a** $y = 2x + 4$ **b** $y = 3x - 1$
c $y = -x + 1$ **d** $y = -2x + 6$

5 **a** $y = 4x + 1$ **b** $y = 2x - 3$
c $y = -2x - 2$ **d** $y = -x + 4$

6 **a** $y = x + 1$ **b** $y = 2x - 2$
c $y = -3x + 2$ **d** $y = -x$

7 **a** $y = x + 2$ **b** $y = x - 4$
c $y = 2x - 1$ **d** $y = -x + 1$

8 **a** $y = 3x + 1$ **b** $y = 5x + 1$
c $y = 2x + 4$

9 $y = 8x - 2$

10 $y = -x + 1$

11 **a** $y = 5 - 2x$ **b** $y = 7 - 3x$
c $y = 4 - x$ **d** $y = 10 - 4x$

12 **a** $b - 2$ **b** $y = (b - 2)x + 2$

13 **a** $b - a$ **b** $y = (b - a)x + a$

14 **a** 5 extra matchsticks are needed for each new shape and 1 matchstick is needed for the first hexagon, so the rule is $y = 5x + 1$.
b 2 extra matchsticks are needed for each new shape and 4 matchsticks are needed for the sides, so the rule is $y = 2x + 4$.

15 **a** x is not increasing by 1.
b 1
c $y = x - 2$
d **i** $y = 2x + 3$ **ii** $y = 3x - 1$
iii $y = -2x + 3$ **iv** $y = -4x - 20$

Exercise 9D

1 **a** positive **b** negative **c** negative **d** positive

2 **a** 2 **b** 3 **c** $\frac{3}{2}$ **d** 2
e −2 **f** −5 **g** $-\frac{8}{3}$ **h** $-\frac{5}{3}$

3 **a** 2 **b** $\frac{2}{3}$ **c** −4 **d** $\frac{-2}{5}$
e $\frac{-3}{2}$ **f** $\frac{-1}{2}$ **g** $\frac{1}{2}$ **h** 3

4 **a** positive **b** undefined **c** zero **d** negative

5 **a** 3 **b** 1 **c** $\frac{1}{2}$ **d** 3
e $\frac{2}{3}$ **f** 4

6 **a** −2 **b** $\frac{-3}{5}$ **c** $\frac{-4}{3}$ **d** −1
e −3 **f** $\frac{-3}{2}$

7 grassy slope

8 torpedo

9 **a** $\frac{3}{4}$ **b** $\frac{5}{2}$ **c** $\frac{-3}{2}$ **d** $\frac{-7}{10}$

10 **a** $\frac{5}{2}$ **b** $\frac{5}{3}$ **c** $\frac{-8}{3}$ **d** $\frac{-2}{3}$
e $\frac{8}{3}$ **f** $\frac{-3}{10}$

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11 Answers may vary. Examples:

a (1, 3), (2, 6), (3, 9) **b** (–1, –3), (–2, –6), (–3, –9)

12 **a** 2 **b** 10 **c** $b = 2a$ **d** $a = \frac{b}{2}$

13 **a** $-\frac{1}{2}$ **b** $-\frac{3}{2}$ **c** $b = -\frac{a}{2}$ **d** $a = -2b$

14 **a** $\frac{2}{3}$ **b** $\frac{-2}{7}$ **c** $\frac{8}{9}$ **d** $\frac{4}{3}$

e $-\frac{3}{7}$ **f** 4 **g** $-\frac{8}{9}$ **h** –1

i $-\frac{2}{7}$ **j** $-\frac{8}{33}$ **k** $-\frac{18}{25}$

Exercise 9E

1 **a** $y = 2x + 3$ **b** $y = 4x + 2$ **c** $y = -3x + 1$

d $y = -5x - 3$ **e** $y = 2x - 5$ **f** $y = -10x - 11$

2 **a** $c = 1, m = 2$ **b** $c = 0, m = 1$

c $c = -1, m = -1$ **d** $c = 3, m = -1$

e $c = 0, m = -3$ **f** $c = 3, m = 3$

3 **a** $m = 4, c = 2$ **b** $m = 3, c = 7$

c $m = \frac{1}{2}, c = 1$ **d** $m = \frac{2}{3}, c = \frac{1}{2}$

e $m = -2, c = 3$ **f** $m = -4, c = 4$

g $m = -7, c = -1$ **h** $m = -3, c = -7$

i $m = -1, c = -6$ **j** $m = -\frac{1}{2}, c = 5$

k $m = -\frac{2}{3}, c = -\frac{1}{2}$ **l** $m = \frac{3}{7}, c = -\frac{4}{5}$

4 **a** $y = 2x - 1$ **b** $y = x - 2$ **c** $y = 3x + 3$

d $y = x + 5$ **e** $y = 2x + 1$ **f** $y = 3x - 1$

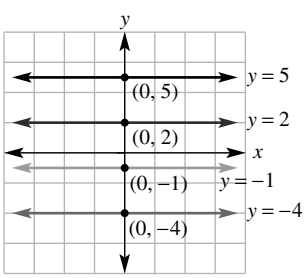
5 **a** $y = -x + 2$ **b** $y = -2x + 4$ **c** $y = -3x - 1$

d $y = -2x + 3$ **e** $y = -5x - 2$ **f** $y = -x + 6$

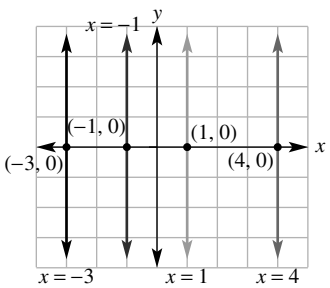
6 **a** $y = 4$ **b** $y = 1$ **c** $y = -3$

d $x = -4$ **e** $x = 5$ **f** $x = -2$

7 **a, b, c, d**



e, f, g, h



8 **a** $y = \frac{2}{5}x + 2$ **b** $y = -\frac{1}{4}x + \frac{1}{2}$ **c** $y = \frac{1}{3}x - \frac{3}{4}$

9 **a** $y = 3x$ **b** $y = -5x$ **c** $y = -x + 3$

d $y = \frac{5}{3}x - 4$

10 **a** $y = 2x + 3$ **b** $y = -\frac{1}{2}x + 5$ **c** $y = -x + 2$

d $y = 2x + 10$

11 **a** 15 square units **b** 40 square units

12 **a** It is a horizontal line passing through (0, 0).

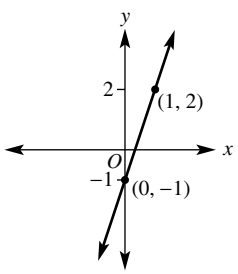
b It is a vertical line passing through (0, 0).

13 **a** $y = \frac{b}{a}x$ **b** $y = -x$ **c** $y = -\frac{a}{b}x + a$

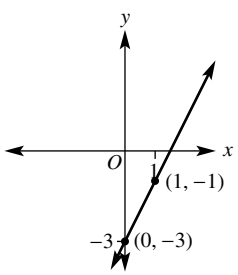
14 **a** $m = -3, c = 5$ **b** $m = 2, c = 3$

c $m = \frac{3}{2}, c = 4$ **d** $m = \frac{1}{2}, c = 3$

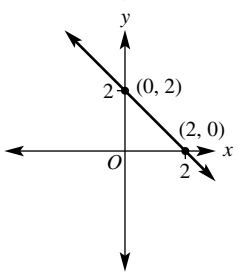
15 **a**



b

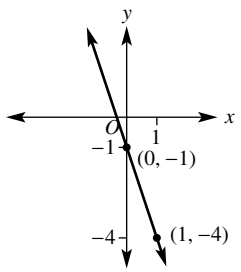


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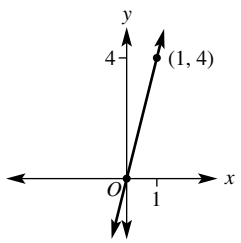


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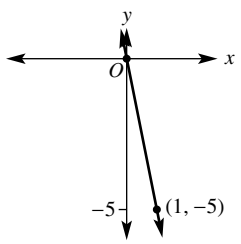
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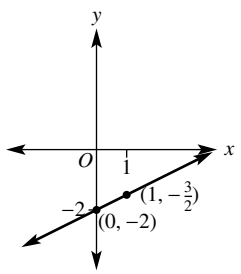
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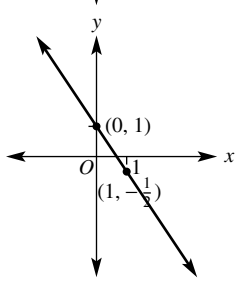
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g



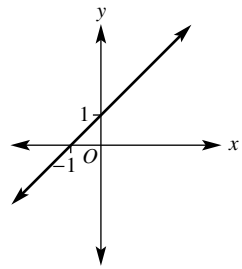
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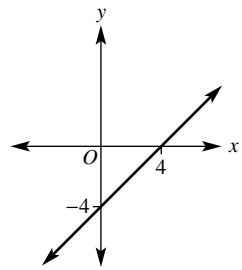
Exercise 9F

1	**a** 2	**b** −5	**c** 7	
2	**a** −1	**b** 4	**c** −2	**d** 0
3	**a** 4	**b** −2	**c** −5	**d** −5
	e 4	**f** −7	**g** $-\frac{1}{3}$	**h** $-\frac{2}{5}$
	i −2	**j** −4	**k** 2	**l** 45
4	**a** 1	**b** 6	**c** −2	**d** 4
	e 3	**f** −2	**g** −10	**h** 2
	i 2	**j** −2	**k** −7	**l** 11

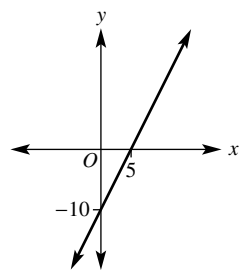
5 a



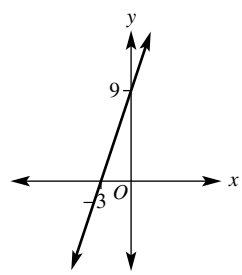
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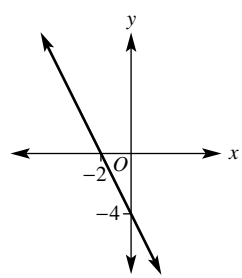
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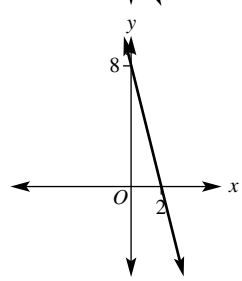
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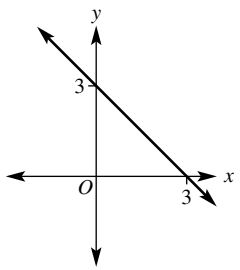


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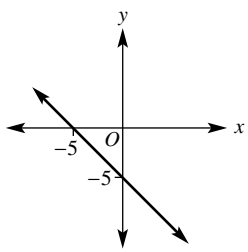


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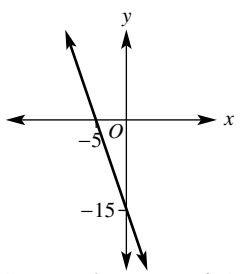
g



h



i



6 **a** 4 square units **b** 1.5 square units
c 12.5 square units **d** 8 square units

7 **a** $y = x + 4$ **b** $y = \frac{1}{2}x - 1$ **c** $y = 3x + 6$ **d** $y = -5x + 25$

8 **a** 20 cm **b** 10 seconds

9 **a** 200 cents **b** 200 seconds

10 horizontal lines; e.g. $y = 2, y = -5$

11 **a** negative **b** positive **c** negative **d** positive

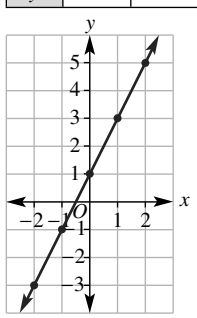
12 $x = -\frac{c}{m}$

13 **a** 6 **b** 4 **c** −2 **d** 3
e $-\frac{2}{3}$ **f** $\frac{3}{2}$ **g** −2 **h** $\frac{3}{2}$ **i** $\frac{1}{3}$

Progress quiz

1

x	−2	−1	0	1	2
y	−3	−1	1	3	5



2 **a** **i** yes **ii** no
b **i** no **ii** yes

3 **a** $y = 3x - 1$ **b** $y = -2x + 2$

4 $y = -2x + 1$

x	−1	0	1	2
y	3	1	−1	−3

5 **a** undefined **b** positive **c** zero **d** negative

6 **a** 2 **b** 1 **c** $\frac{-2}{3}$

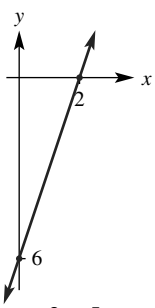
7 **a** $m = 3, c = 2$ **b** $m = \frac{2}{3}, c = -5$
c $m = -3, c = 4$ **d** $m = -1, c = -\frac{4}{5}$

8 **a** $y = x + 3$ **b** $y = -3x + 3$

9 **a** $x = 3$ **b** $y = -2$

10 **a** 3 **b** −4

11



12 $y = -2x - 5$

Exercise 9G

1

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

2 **a** $x = 5$ **b** $x = -1$

3 **a** (2, 4) **b** (3.2, 6, 4) **c** (−2.3, −4, 6)
d (3.5, 7) **e** (−7, −14) **f** (1000, 2000)
g (31.42, 62.84) **h** (−24.301, −48.602)
i $\left(\frac{\textit{any number}}{2}, \textit{any number}\right)$

4 **a** (4, 3) **b** (−2, −3)

5 **a** $x = 2$ **b** $x = 0.5$ **c** $x = 3$ **d** $x = -2.5$
e $x = -1.5$

6 **a** $x = -2.5$ **b** $x = 3$ **c** $x = -0.5$ **d** $x = 4$
e $x = 5$

7 **a** (2, 3) **b** (−1, 1)

8 **a** $x = 3.67$ **b** $x = -1.53$ **c** $x = 5.30$

9 **a** (4.78, 1.78) **b** (−1.33, 3.41)

10 **a** $x + 2 = 1$ and $5 - 2x = 7$
b Any point that lies on the line is correct,
e.g. (−2, 9) (0, 5) (1, 3) (2, 1)

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c Any point that lies on the line is correct,
e.g. (–2, 0) (0, 2) (1, 3) (3, 5)

d (1, 3)
$y = x + 2$ $y = 5 - 2x$
$3 = 1 + 2$ $3 = 5 - 2 \times 1$
$3 = 3$ True $3 = 3$ True

e $x = 1$

11 a $A = 10 + 8n$ applies to Ruby as she has \$10 to start with and adds to her savings by \$8 times the number (n) of hours worked. $A = 24 + 6n$ applies to Jayden as he has \$24 to start with and increases his savings by \$6 times the number (n) of hours worked.

b **i** $n = 4$ **ii** $n = 4$ **iii** $n = 7$ **iv** $n = 7$
v $n = 11$ **vi** $n = 11$

c Many solutions e.g. (2, 26) (4, 42) (7, 66) (9, 82) (11, 98)

d Many solutions e.g. (2, 36) (4, 48) (7, 66) (9, 78) (11, 90)

e

(7, 66)	(7, 66)
$A = 10 + 8n$	$A = 24 + 6n$
$66 = 10 + 8 \times 7$	$66 = 24 + 6 \times 7$
$66 = 10 + 56$	$66 = 24 + 42$
$66 = 66$ True	$66 = 66$ True

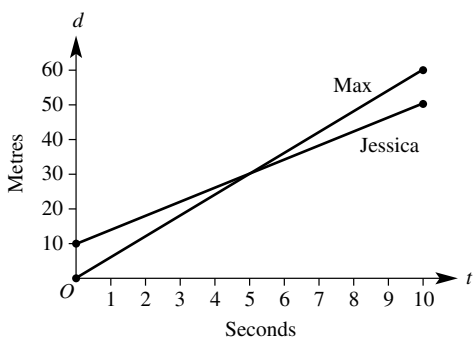
f $n = 7$

g Ruby and Jayden have both worked 7 hours and both have \$66 saved.

12 a

Time in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance in metres	0	6	12	18	24	30	36	42	48	54	60
Jessica's distance in metres	10	14	18	22	26	30	34	38	42	46	50

b



c $d = 6t$

d **i** $6t = 18$ **ii** $6t = 30$ **iii** $6t = 48$

e $d = 10 + 4t$

f **i** $10 + 4t = 22$ **ii** $10 + 4t = 30$
iii $10 + 4t = 42$

g

(5, 30)	(5, 30)
$d = 6t$	$d = 10 + 4t$
$30 = 6 \times 5$	$30 = 10 + 4 \times 5$
$30 = 30$ True	$30 = 30$ True

h Max catches up to Jessica. They are both 30 m from the starting line and have each run for 5 seconds.

13 a $2x - 3 = -12, x = -4.5$

b $2x - 3 = 2, x = 2.5$

c $2x - 3 = 6, x = 4.5$

d $2x - 3 = 5, x = 4$

e $2x - 3 = 9, x = 6$

f $2x - 3 = -6, x = -1.5$

14 a **i** (–2, 17) (–1, 14) (0, 11) (1, 8) (2, 5) (3, 2) (4, –1)(5, –4)

ii (–2, –3), (–1, –1) (0, 1) (1, 3) (2, 5) (3, 7) (4, 9)(5, 11)

b (2, 5) $y = 11 - 3x$ $y = 2x + 1$
$5 = 11 - 3 \times 2$ $5 = 2 \times 2 + 1$
$5 = 11 - 6$ $5 = 4 + 1$
$5 = 5$ True $5 = 5$ True

c It is the only shared point.

15 a **i** Any answer with y a whole number is correct:
$2x - 1 = -3, 2x - 1 = -1, 2x - 1 = 3$

ii No. Many possible correct examples,
e.g. $2x - 1 = 2.5, 2x - 1 = -1.75, 2x - 1 = 2.8$

b **i** Any answers with y-value to one decimal place are correct, e.g. $2x - 1 = -2.6, 2x - 1 = -1.8$, $2x - 1 = 0.7$

ii No. Many possible correct answers,
e.g. $2x - 1 = 0.42, 2x - 1 = -1.68, 2x - 1 = 2.88$

c **i** $2x - 1 = 2.04, 2x - 1 = 2.06$

ii Many possible correct answers, e.g. (1.521, 2.042) (1.529, 2.058); $2x - 1 = 2.042, 2x - 1 = 2.058$

iii Yes, for every two points on a line another point can be found in the between them so there are an infinite number of points on a line. Also an infinite number of equations can be solved from the points on a straight line if the graph has a suitable scale (digitally possible).

16 a **i** $x = 2, x = -2$ **ii** $x = 3, x = -3$
iii $x = 4, x = -4$ **iv** $x = 5, x = -5$

b For each y-coordinate there are two different points so two different solutions.

c **i** $x = 2.24, x = -2.24$ **ii** $x = 2.61, x = -2.61$
iii $x = 0.7, x = -0.7$ **iv** $x = 3.57, x = -3.57$
v $x = 4.34, x = -4.34$

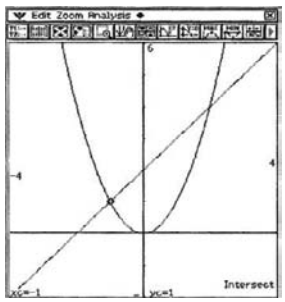
d The graph of $y = x^2$ does not include a point where $y = -9$.

e Many correct answers all with x^2 equal to a negative number, e.g. $x^2 = -5, x^2 = -10, x^2 = -20$

f Positive numbers or zero.

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g $x = -1, x = 2$



h **i** $x = -2.77, x = 5.77$ **ii** $x = -8.27, x = 3.27$
iii no solution **iv** $x = 3$

Exercise 9H

1 **a** 15 **b** 45 **c** 5 **d** 125

2 **a** $m = 3, c = 2$ **b** $m = -2, c = -1$
c $m = 4, c = 2$ **d** $m = -10, c = -3$
e $m = -20, c = 100$ **f** $m = 5, c = -2$

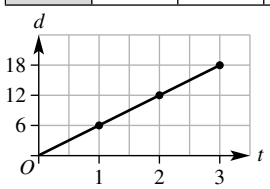
3 **a** 60 cm **b** 150 cm **c** 330 cm

4 **a** 28 L **b** 24 L **c** 10 L

5 **a**

t	0	1	2	3
d	0	6	12	18

b

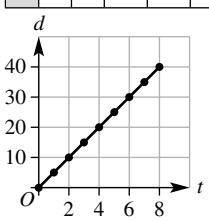


c $d = 6t$ **d** 9 km **e** 2 hours

6 **a**

t	0	1	2	3	4	5	6	7	8
d	0	5	10	15	20	25	30	35	40

b

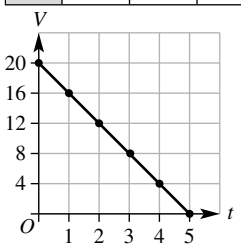


c $d = 5t$ **d** 22.5 km **e** 4 hours

7 **a**

t	0	1	2	3	4	5
v	20	16	12	8	4	0

b

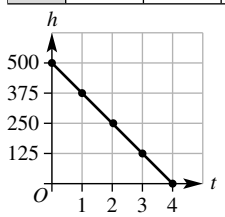


c $V = -4t + 20$ **d** 11.2 L **e** 3 seconds

8 **a**

t	0	1	2	3	4
h	500	375	250	125	0

b



c $h = -125t + 500$ **d** 275 m **e** 3 minutes

9 **a** $M = -0.5t + 3.5$ **b** 7 hours **c** 4.5 hours

10 **a** $d = 15t$
b 3 hours
c 3 hours 20 minutes

11 2 days 15 hours

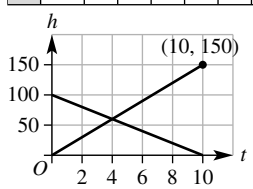
12 **a** 2000 L
b Decreasing; it has a negative gradient.
c 300 L/h

13 **a** Using cents, $m = \frac{1}{2}$ and $c = 10$.
b Using dollars, $m = 0.005$ and $c = 0.1$.

14 **a**

t	0	1	2	3	4	5	6	7	8	9	10
h_1	0	15	30	45	60	75	90	105	120	135	150
h_2	100	90	80	70	60	50	40	30	20	10	0

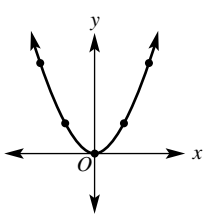
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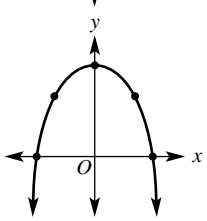
c 10 seconds **d** $h = 15t, h = -10t + 100$
e at 4 seconds **f** at 2.5 seconds
g at 3.5 seconds **h** no, 60 m, 60 m and 50 m

Exercise 9I

1 **a**

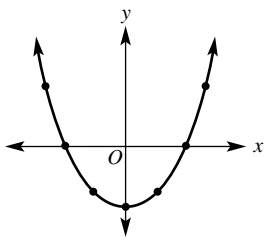


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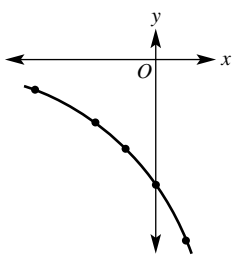


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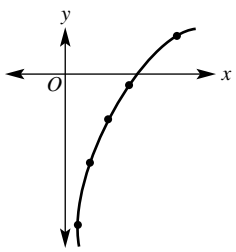
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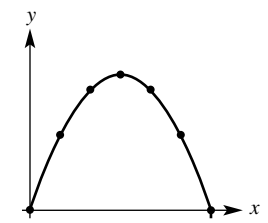
d



e



f

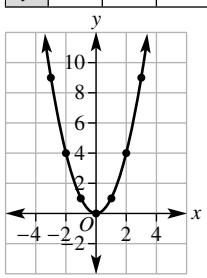


2 **a** –1 **b** 8 **c** 3 **d** 15

3 **a** Yes **b** No **c** No **d** Yes

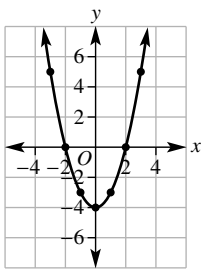
4 **a** $y = x^2$

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



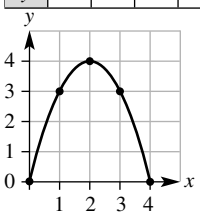
b $y = x^2 - 4$

x	–3	–2	–1	0	1	2	3
y	5	0	–3	–4	–3	0	5



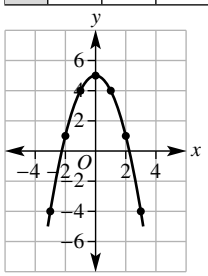
c $y = x(4 - x)$

x	0	1	2	3	4
y	0	3	4	3	0

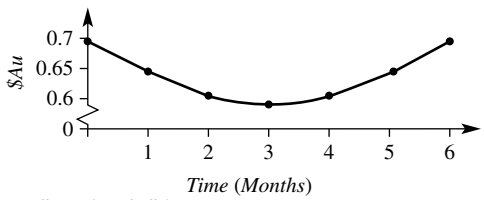


d $y = 5 - x^2$

x	–3	–2	–1	0	1	2	3
y	–4	1	4	5	4	1	–4



5 **a**



b non-linear (parabolic)

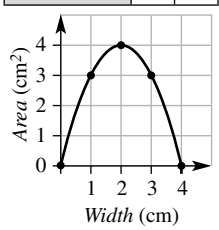
c **i** \$0.05 **ii** \$0.03

d ≈ 0.76

6 **a**

Width (cm)	0	1	2	3	4
Length (cm)	4	3	2	1	0
Area (cm^2)	0	3	4	3	0

b



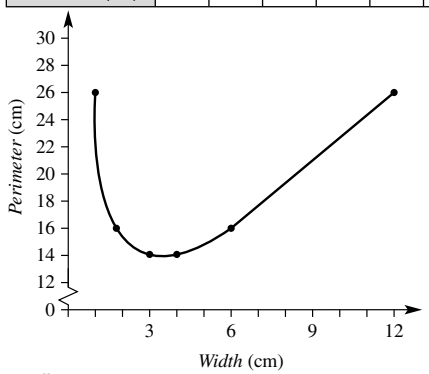
c non-linear (parabolic) **d** 2 cm by 2 cm

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7 a

Width (cm)	1	2	3	4	6	12
Length (cm)	12	6	4	3	2	1
Perimeter (cm)	26	16	14	14	16	26

b



c non-linear

d **i** ≈ 3.5 cm **ii** ≈ 13.9 cm

8 a 0 and 9 **b** -10 and -1 **c** -3 and 6

9 For each unit change in x there are variable changes in y.

10 a linear **b** linear **c** non-linear **d** non-linear **e** non-linear **f** non-linear

11 16

12 a Upright parabolas, as a increases the graphs become narrower.

b Inverted (upside down) parabolas, as a increases the graphs become narrower.

c Parabolas, as a increases the graphs shift up.

d Parabolas, as a increases the graphs shift right.

Problems and challenges

1 3 hours

2 a $y = x^2 - 3$ **b** $y = 10 - x^2$ **c** $y = \sqrt{x} + 1$ **d** $y = x^3 - 3$

3 $-\frac{1}{3}$ **4** 40 min **5** $\frac{14}{3}$

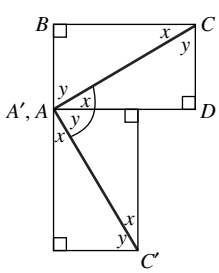
6 $y = \frac{1}{2}x + \frac{1}{2}$

7 1588

8 4 hours 45 minutes

9 60 units^2

10 a Diagonal AC has been rotated about A by 90^0 clockwise so the angle between AC and AC' is 90^0.



$x + y + 90^0 = 180^0$

$x + y = 90^0$

b $\frac{p}{q} \times \frac{-q}{p} = -1$

Multiple-choice questions

1 B **2** C **3** C **4** D **5** B
6 D **7** A **8** E **9** D **10** E

Short-answer questions

1 a 100 km

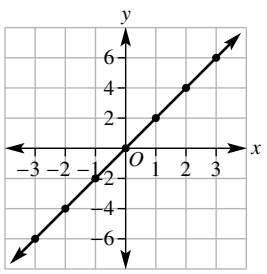
b 1 hour

c **i** 50 km **ii** 100 km

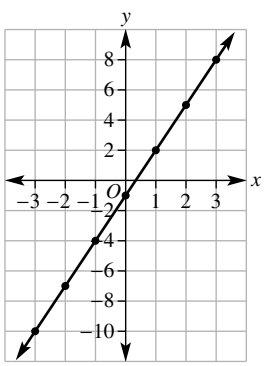
d section C

2 $A(2, 3)$, $B(0, 2)$, $C(-2, 4)$, $D(-3, 1)$, $E(-3, -3)$, $F(-1, 0)$, $G(0, -4)$, $H(1, -2)$, $I(4, -3)$, $J(3, 0)$

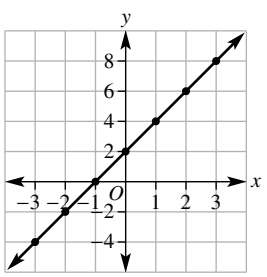
3 a



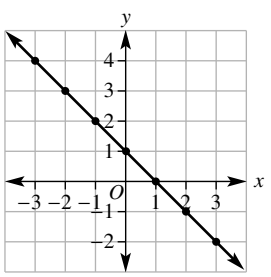
b



c

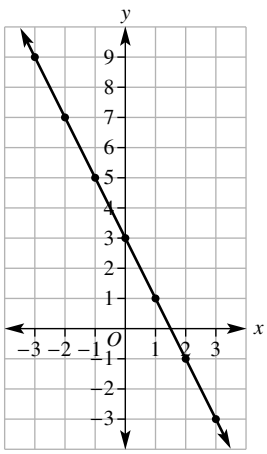


d

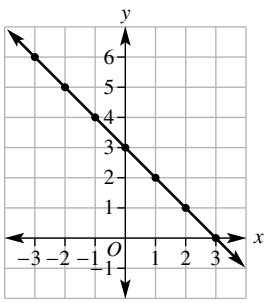


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e



f



4 a $y = 2x + 1$ b $y = 3x + 2$ c $y = x + 3$
d $y = -x + 1$ e $y = -4x - 1$ f $y = -x + 2$

5 a 3 b $\frac{1}{2}$ c 2 d –1
e –4 f $-\frac{3}{2}$

6 a –4 b $-\frac{1}{2}$ c 1 d –2

7 a $m = 5, c = 2$ b $m = 2, c = -4$
c $m = -3, c = 7$ d $m = -1, c = -\frac{1}{2}$

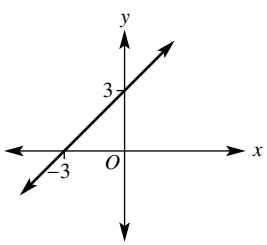
8 a $y = 2x + 1$ b $y = 3x$ c $y = x - 2$ d $y = -4x$
e $y = -2x - 4$ f $y = -\frac{1}{2}x + 1$

9 a $y = 6x$ b $y = -4x$ c $y = -x + 1$ d $y = \frac{1}{2}x - 2$

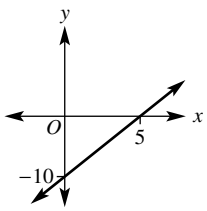
10 a $x = 6$ b $x = 4$ c $x = -4$ d $y = -5$
e $y = -1$ f $y = 5$

11 a 6 b –3 c –4 d 2

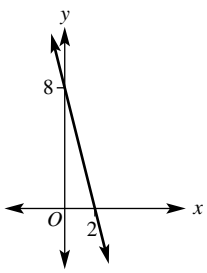
12 a



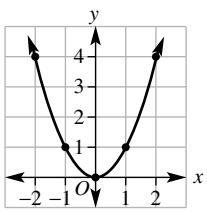
b



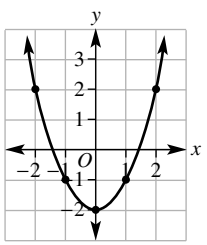
c



13 a



b



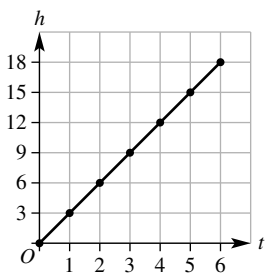
14 a 2 b –2 c $\frac{1}{2}$

Extended-response questions

1 a

t	0	1	2	3	4	5	6
h	0	3	6	9	12	15	18

b



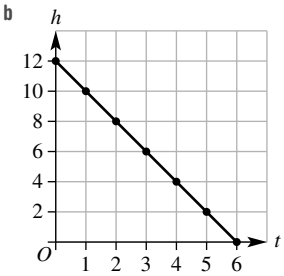
c $h = 3t$ d 10.5 mm e 30 mm f 5 days

2 a

t	0	1	2	3	4	5	6
h	12	10	8	6	4	2	0

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b



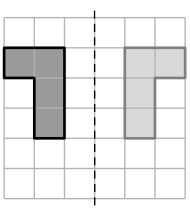
c 6 minutes d −2 e $h = -2t + 12$

f 7 km g 4 minutes 15 seconds

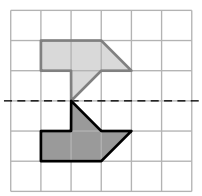
Chapter 10

Exercise 10A

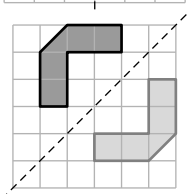
1 a



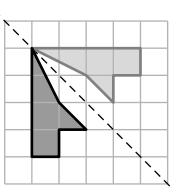
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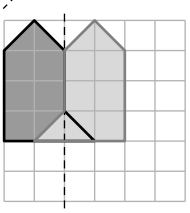
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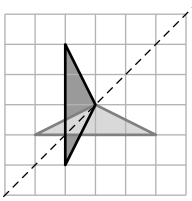
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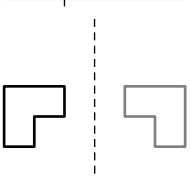
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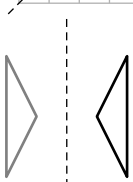
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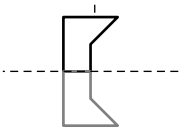
2 a



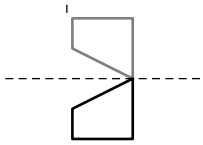
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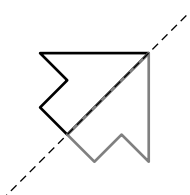
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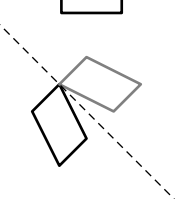
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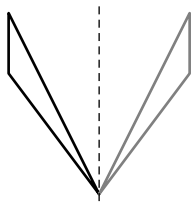
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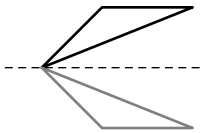
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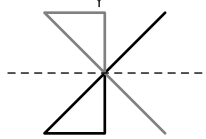
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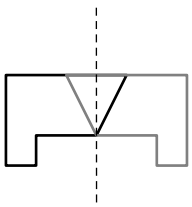
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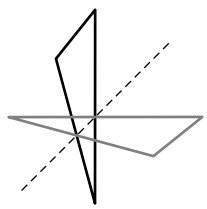
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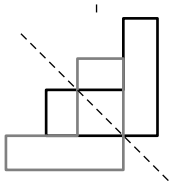
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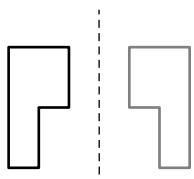
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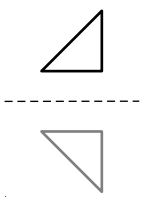
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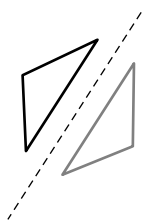
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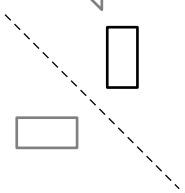
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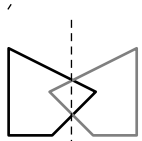
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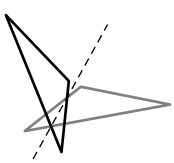
d



e



f



5 a $A'(2, 0)$, $B'(1, -3)$, $C'(4, -2)$

b $A'(-2, 0)$, $B'(-1, 3)$, $C'(-4, 2)$

6 a $A'(-1, 2)$, $B'(-4, 2)$, $C'(-4, 4)$, $D'(-1, 4)$

b $A'(1, -2)$, $B'(4, -2)$, $C'(4, -4)$, $D'(1, -4)$

7 a 4 b 2 c 2 d 1 e 0
f 0 g 1 h 3 i 8

8 a (0, 4) b (4, 4) c (−2, 4) d (−4, 4)
e (−10, 4) f (−42, 4) g (2, 2) h (2, −8)
i (2, −4) j (2, −2) k (2, −14) l (2, −78)

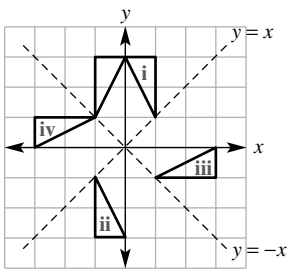
9 10 m^2, the area is unchanged after reflection.

10 n

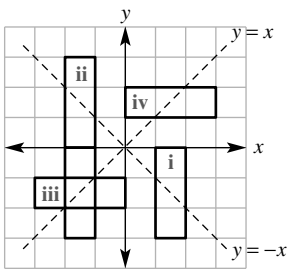
11 Reflection in the y-axis.

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12 a



b

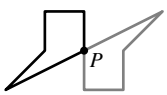


13 They are on the mirror line.

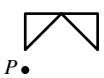
14 a i



ii



iii



b i yes ii no iii yes

c square, rectangle, rhombus, parallelogram

Exercise 10B

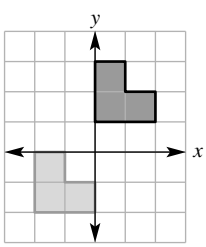
1 **a** right, up **b** left, up **c** right, down **d** left, left, down

2 **a** (5, –2) **b** (–2, –6) **c** (–7, 4) **d** (9, 17)

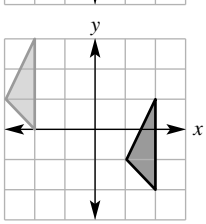
3 **a** horizontal **b** vertical **c** vertical **d** horizontal

4 **a** (1, –1) **b** (3, –1) **c** (2, –2) **d** (2, –4)
e (3, 7) **f** (–3, 5) **g** (2, –3) **h** (3, –5)
i (–20, 8) **j** (–9, –25)

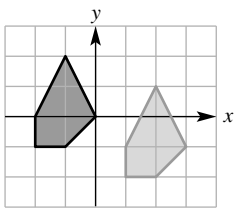
5 **a**



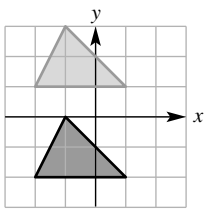
b



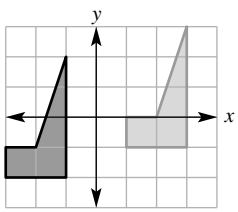
c



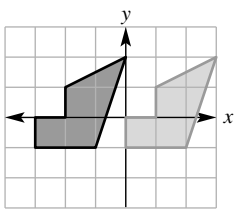
d



e



f



6 **a** (15, 2) **b** (21, –1) **c** (13, 6) **d** (9, 2)
e (11, –2) **f** (3, 4) **g** (11, –9) **h** (19, –10)
i (25, –4) **j** (–13, 13) **k** (9, 17) **l** (–8, –39)

7 **a** (7, 0) **b** (–1, –4)

8 **a** 2.3 km (or 2300 m) **b** (–2, –3)

9 **a** $x = -2, y = -2$ **b** $x = 6, y = -4$
c $x = -3, y = 2$ **d** $x = 28, y = -60$

10 **a** (–3, 2) **b** (5, 0) **c** $(-x, -y)$ **d** (x, y)

11 **a** (–1, 0) **b** (1, 2) **c** $(c, b - c)$

12 **a** $x \approx 4.47$ **b** just OK **c** 80 holes

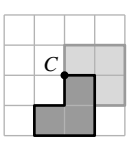
Exercise 10C

1 **a** anticlockwise, 90° **b** clockwise, 90°
c anticlockwise, 90° **d** clockwise, 90°
e anticlockwise, 180° **f** clockwise, 180°

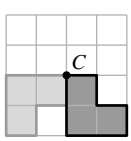
2 **a** 270° **b** 180° **c** 302° **d** 64°

3 **a** 4 **b** 2 **c** 5 **d** 2
e 4 **f** 2

4 **a**

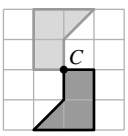


b

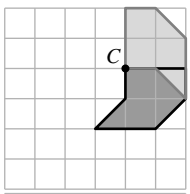


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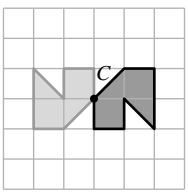
c



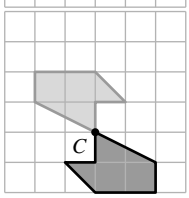
d



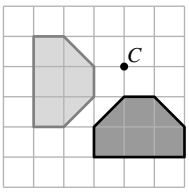
e



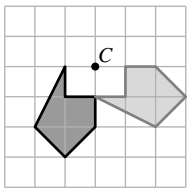
f



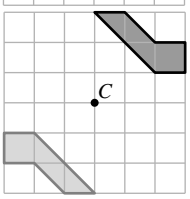
5 a



b



c



6 **a** (–4, –3) **b** (–4, –3) **c** (3, –4) **d** (–3, 4) **e** (–3, 4) **f** (3, –4) **g** (4, 3)

7 **a** $A'(4, -4), B'(4, -1), C'(1, -1)$

b $A'(4, 4), B'(1, 4), C'(1, 1)$

c $A'(-4, -4), B'(-1, -4), C'(-1, -1)$

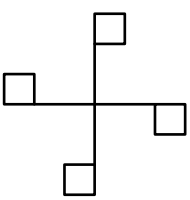
8 **a** 90° **b** 180° **c** 90°

9 H, I, N, O, S, X and Z

10 Answers may vary. Examples are:

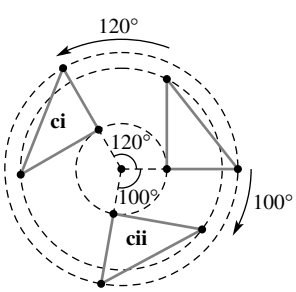
a parallelogram **b** regular hexagon

c

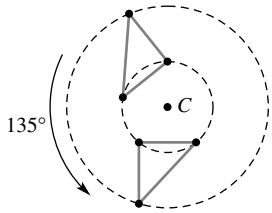


11 **a** 180° **b** 90°

12



13



14 **a** use $x = 70°$ **b** use $x = 40°$

c Answers will vary.

Exercise 10D

1 **a** yes

b **i** D **ii** E **iii** F

c **i** DE **ii** EF **iii** DF

d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$

2 **a** yes

b **i** D **ii** E **iii** F

c **i** DE **ii** EF **iii** DF

d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$

3 **a** yes

b **i** D **ii** E **iii** F

c **i** DE **ii** EF **iii** DF

d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$

4 **a** **i** E **ii** H

b **i** EH **ii** GH

c **i** $\angle G$ **ii** $\angle E$

5 **a** **i** F **ii** I

b **i** FJ **ii** HI

c **i** $\angle H$ **ii** $\angle J$

6 (J, G), (D, K), (C, I)

7 (A, J), (C, K), (E, G)

8 **a** 32 **b** 24 **c** 20 **d** 8 **e** 4

9 **a** $(A, E), (B, D), (C, F)$ **b** $(A, Y), (B, X), (C, W), (D, Z)$

c $(A, W), (B, X), (C, Z), (D, Y)$

d $(A, T), (B, Z), (C, X), (D, S), (E, W)$

10 **a** $\Delta AMC, \Delta BMC$

b Yes, all corresponding sides and angles will be equal.

11 **a** **i** $\Delta ABD, \Delta CBD$

ii Yes, all corresponding sides and angles will be equal.

b **i** $\Delta ABC, \Delta ACD$

ii No, sides and angles will not be equal.

12 yes

13 **a** reflection in the y-axis then translation by the vector (1, –2)

b rotation anticlockwise by 90° then translation by the vector (6, 3)

c rotation by 180° then translation by the vector (–2, 1)

d reflection in the x-axis, reflection in the y-axis and translation by the vector (–2, 1)

e reflection in the x-axis, reflection in the y-axis and translation by the vector (3, 2)

f rotation by 180° then translation by the vector (3, 2)

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Exercise 10E

1 **a** SSS **b** RHS **c** SAS **d** AAS

2 **a** $\Delta ABC \equiv \Delta EFD$ **b** $\Delta ABC \equiv \Delta FED$
c $\Delta XYZ \equiv \Delta UST$ **d** $\Delta ABC \equiv \Delta ADC$

3 **a** SAS **b** SSS **c** RHS **d** AAS
e RHS **f** AAS

4 **a** unique (AAS) **b** unique (SSS)
c unique (RHS) **d** unique (SAS)
e unique (SSS) **f** unique (AAS)
g unique (SAS) **h** unique (AAS)
i unique (AAS) **j** unique (RHS)

5 **a** $x = 4, y = 1$ **b** $x = 9, a = 20$
c $x = 5, a = 24$ **d** $x = 5, a = 30$
e $x = 4, a = 95, b = 25$ **f** $x = 11, a = 50, b = 90$

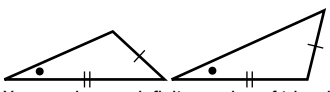
6 **a** no **b** yes, SAS **c** yes, AAS **d** no

7 **a** $EF = 3$ m **b** $\angle B = 30°$ **c** $AC = 6$ cm **d** $\angle C = 20°$

8 **a** no **b** yes **c** yes **d** no

9 Yes, show SSS using Pythagoras' theorem.

10 **a** You can draw two different triangles with SSA.



b You can draw an infinite number of triangles with the same shape but of different size.

11 **a** $\angle CAB = \angle CED$ (equal alternate angles)
$\angle ACB = \angle ECD$ (vertically opposite angles)
$AC = EC$ (given equal and corresponding sides)
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)

b $BD = BD$ (given and common equal side)
$\angle ADB = \angle CDB$ (given and equal angles)
$AD = CD$ (given equal sides)
$\therefore \Delta ADB \equiv \Delta CDB$ (SAS)

c $\angle ACB = \angle CAD$ (equal alternate angles)
$\angle CAB = \angle ACD$ (equal alternate angles)
$AC = AC$ (given and common equal side)
$\therefore \Delta ABC \equiv \Delta CDA$ (AAS)

d $\angle ABC = \angle ADC$ (given 90° angles)
$AC = AC$ (given and common equal side)
$BC = DC$ (given equal sides)
$\therefore \Delta ABC \equiv \Delta ADC$ (RHS)

12 **a** $AB = AC$ (given equal sides)
$BM = CM$ (given equal sides)
$AM = AM$ (given and common equal side)
$\therefore \Delta ABM \equiv \Delta ACM$ (SSS)
$\therefore \angle AMB = \angle AMC$
As $\angle AMB + \angle AMC = 180°$ then
$\angle AMB = \angle AMC = 90°$.

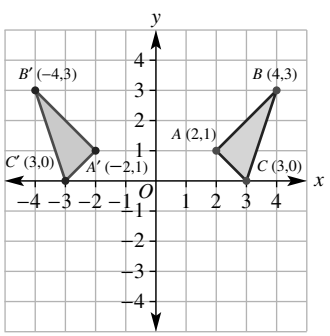
b $\angle AEB = \angle CDB$ (equal alternate angles)
$\angle EAB = \angle DCB$ (equal alternate angles)
$EB = BD$ (given equal sides)
$\therefore \Delta AEB \equiv \Delta CDB$ (AAS)
$\therefore AB = BC$ and $AC = 2AB$

c $AD = DC$ (given equal sides)
$AB = CB$ (given equal sides)
BD is a common side
$\therefore \Delta ABD \equiv \Delta CBD$ (SSS)
$\therefore \Delta DAB = \Delta DCB$

d $\Delta ACD \equiv \Delta ACB$ (SSS)
so $\angle DCA = \angle BCA$
Now $\Delta DCE \equiv \Delta BCE$ (AAS)
with $\angle CDE = \angle CBE$ (isosceles triangle)
So $\angle DEC = \angle BEC$
Since $\angle DEC$ and $\angle BEC$ are supplementary (sum to 180°)
So $\angle DEC = \angle BEC = 90°$
So diagonals intersect at right angles.

Progress quiz

1 **a**

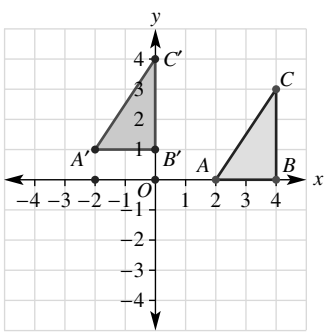


b A′(2, –1) B′(4, –3) C′(3, 0)

2 **a** (3, 5)

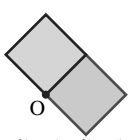
b (4, –2)

3



4 4

5



6 $A'(0, 2)$ $B'(0, 4)$ $C'(-3, 4)$

7 **a** SAS

b $\Delta ACB \equiv \Delta QPR (SAS)$

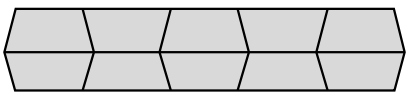
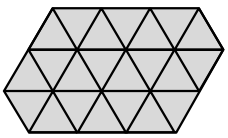
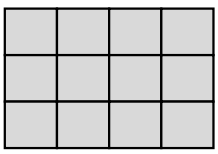
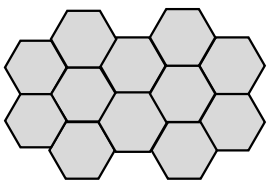
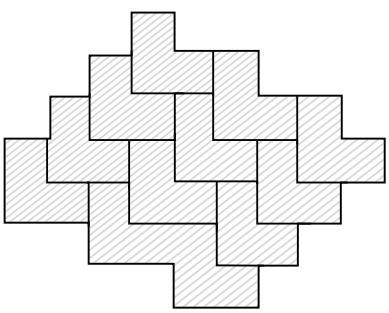
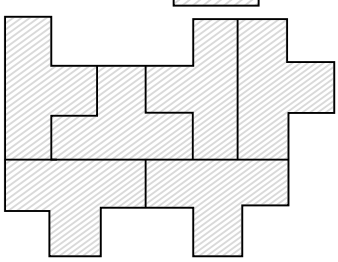
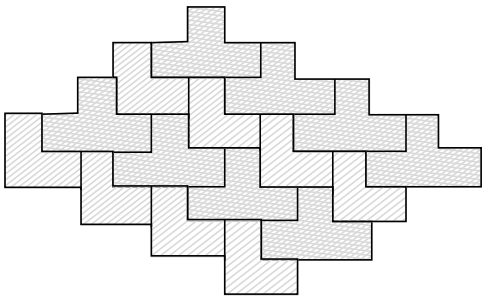
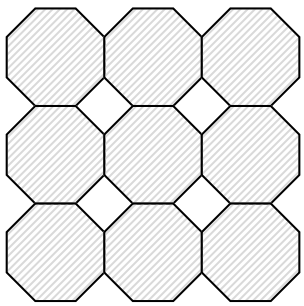
c QR

d angle BCA

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8 $\angle ABE = \angle CBD$ (vertically opposite)
$AB = CB$ (given)
$EB = DB$ (given)
$\therefore \Delta ABE \equiv \Delta CBD$ (SAS)

Exercise 10F

1 C
2 D
3 Overlaps exist
4 Answers will vary
5 
6 a 
b 
c 
7 a 3.3.3.3.3.3 b 4.4.4.4 c 6.6.6
8 a yes b no c yes d no
e yes
9 a 3.3.3.4.4 b 3.3.4.3.4 c 3.4.6.4 d 3.12.12
10 a 
b 
c 
11 a 960 b Answers will vary
12 
13 Answers will vary
14 a 50
b Answers will vary
15 Circles cannot be arranged together without any gaps unless overlaps are used. With different size circles and no overlaps the gaps can be made to be very small, but there will always be gaps
16 The size of a revolution angle is 360°. For a regular polygon to tessellate the interior angle of the polygon must be a factor of 360. An equilateral triangle has an interior angle of 30°, a square has an interior angle of 90° and a hexagon has an interior angle of 60°. These are the only polygons which have an interior angle which is a factor of 360° and therefore these are the only three regular polygons which will tessellate.
17 Answers will vary
18 Answers will vary
19 Answers will vary

Exercise 10G

1 a alternate angles in parallel lines
b alternate angles in parallel lines
c alternate angles in parallel lines
2 a co-interior angles in parallel lines, $a = 110$
b co-interior angles in parallel lines, $a = 52$
3 SAS, AAS and RHS
4 a AC b BD c DB
5 a AAS b RHS c SSS d SAS
e AAS f SSS

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6 $\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$\angle BAE = \angle DEC$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\Delta ABE \equiv \Delta CDE$ (AAS)
$BE = DE$ and $AE = CE$ because corresponding sides on congruent triangles are equal.

7 $\angle EFI = \angle GHI$ (alternate angles in parallel lines)
$\angle FEI = \angle HGI$ (alternate angles in parallel lines)
$EF = GH$ (given)
$\Delta EFI \equiv \Delta GHI$ (AAS)
$EI = GI$ and $FI = HI$ because corresponding sides on congruent triangles are equal.

8 **a** equal (alternate angles in parallel lines)
b equal (alternate angles in parallel lines) **c** BD
d AAS **e** They must be equal.

9 **a** $VU = TU$, $VW = TW$, UW is common.
So $\Delta VWU \equiv \Delta TWU$ by SSS.
b $\angle VWU = \angle TWU$ and since they add to 180° they must be equal and 90°.

10 **a** SSS (3 equal sides)
b They are equal and add to 180° so each must be 90°.
c Since ΔQMN is isosceles and $\angle MQN$ is 90° then $\angle QMN = 45°$.

11 **a** $AB = CB, AD = CD$ and BD is common.
So $\Delta ABD \equiv \Delta CBD$ by SSS.
b $\Delta ABD \equiv \Delta CBD$ so $\angle DAB = \angle DCB$
c $\Delta ABD \equiv \Delta CBD$ so $\angle ADB = \angle CDB$

12 Let $AD = BC = a$ and $AB = CD = b$.
Then show that both BD and AC are equal $\sqrt{a^2 + b^2}$.

13 $\angle ABD = \angle CDB$ (alternate angles in parallel lines)
$\angle ADB = \angle CBD$ (alternate angles in parallel lines)
BD is common
So $\Delta ABD \equiv \Delta CDB$
So $AB = CD$ and $AD = BC$

14 **a** $\angle DCE$ **b** $\angle CDE$
c There are no pairs of equal sides.

15 **a** ΔACD is isosceles.
b $AD = CD, \angle DAE = \angle DCE$ and $\angle ADE = \angle CDE$ (AAS)
c $\angle AED = \angle CED$ and sum to 180° so they are both 90°.

16 **a** First show that $\Delta ABD \equiv \Delta CDE$ by SSS.
So $\angle ABD = \angle CDE$ and $\angle ADB = \angle CBD$ and since these are alternate angles the opposite sides must be parallel.
b First show that $\Delta ABE \equiv \Delta CDE$ by SAS.
So $\angle ABE = \angle CDE$ and $\angle BAE = \angle DCE$ and since these are alternate angles the opposite sides must be parallel.
c First prove that $\Delta ABD \equiv \Delta BAC$ by SSS.
Now since $\angle DAB = \angle CBA$ and they are also co-interior angles in parallel lines then they must be 90°.

Exercise 10H

1 (A, J), (C, K), (F, H), (I, L)

2 **a** **i** $\angle D$ **ii** $\angle E$ **iii** $\angle F$
b **i** AB **ii** BC **iii** CA
c **i** 2 **ii** 2 **iii** 2
d Yes, all side ratios are equal and all interior angles are equal.

3 **a** yes, 2 **b** yes, 2 **c** yes, 4 **d** yes, 4

4 **a** **i** $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
ii $(\angle A, \angle E), (\angle B, \angle F), (\angle C, G), (\angle D, \angle H)$
iii 2 **iv** $a = 100,\ x = 2,\ y = 3$
b **i** $(AB, DE), (BC, EF), (CA, FD)$
ii $(\angle A, \angle D), (\angle B, \angle E), (\angle C, \angle F)$ **iii** 3
iv $x = 3$
c **i** $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
ii $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$
iii 1.5 **iv** $a = 40, x = 4, y = 4.5$
d **i** $(AB, FG), (BC, GH), (CD, HI), (DE, IJ), (EA, JF)$
ii $(\angle A, \angle F), (\angle B, \angle G), (\angle C, \angle H), (\angle D, \angle I), (\angle E, \angle J)$ **iii** 2.5
iv $a = 115,\ x = 5$

5 **a** Yes, all ratios are 2. **b** Yes, squares of different sizes.
c No, ratios are not equal.
d Yes, ratios are both 2.5 with equal angles.

6 360 cm **7** 15 cm **8** 1.25 **9** 3

10 **a** True, angles and side ratios will be equal.
b False, side ratios may be different.
c True, angles and side ratios will be equal.
d False, side ratios may be different.
e False, angles and side ratios may be different.
f False, side ratios and angles may be different.
g False, side ratios and angles may be different.
h False, side ratios and angles may be different.
i True, shape is always the same.

11 **a** no
b No, they can have different shapes.

12 16

13 **a** **i** $\sqrt{8}$ **ii** 2
b $\frac{2}{\sqrt{2}} = \sqrt{2}$ (small to big)
c Using Pythagoras' theorem, the side length of the 2nd square is $\sqrt{\left(\frac{x}{2}^2\right) + \left(\frac{x}{2}^2\right)} = \sqrt{\frac{x^2}{2}} = \frac{x}{\sqrt{2}}$.
So the scale factor is $x \div \frac{x}{\sqrt{2}} = x \times \frac{\sqrt{2}}{x} = \sqrt{2}$.
d 2 (small to big)

Exercise 10I

1 **a** $\Delta ABC \,|||\, \Delta EFD$ or use ~ instead of III
b $\Delta ABC \,|||\, \Delta FDE$
c $\Delta ABC \,|||\, \Delta DEF$
d $\Delta ABC \,|||\, \Delta DEF$ (order does not matter)

2 **a** AAA **b** SAS **c** RHS **d** SSS

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3 a SSS

$\frac{DE}{AB} = \frac{10}{5} = 2$

$\frac{DF}{AC} = \frac{24}{12} = 2$

$\frac{EF}{BC} = \frac{26}{13} = 2$

$\therefore \Delta ABC \sim \Delta DEF$

b AAA

$\angle A = \angle D$

$\angle B = \angle E$

$\therefore \Delta ABC \sim \Delta DEF$

c SAS

$\frac{DE}{AB} = \frac{10}{5} = 2$

$\angle D = \angle A$

$\frac{DF}{AC} = \frac{6}{3} = 2$

$\therefore \Delta ABC \sim \Delta DEF$

d RHS

$\angle D = \angle A = 90°$

$\frac{EF}{BC} = \frac{26}{13} = 2$

$\frac{DF}{AC} = \frac{22}{11} = 2$

$\therefore \Delta ABC \sim \Delta DEF$

e RHS

$\angle E = \angle B = 90°$

$\frac{DF}{AC} = \frac{4}{1} = 4$

$\frac{EF}{BC} = \frac{2}{0.5} = 4$

$\therefore \Delta ABC \sim \Delta DEF$

f SAS

$\frac{DE}{AB} = \frac{9}{6} = 1.5$

$\angle D = \angle A$

$\frac{DF}{AC} = \frac{18}{12} = 1.5$

$\therefore \Delta ABC \sim \Delta DEF$

g AAA

$\angle A = \angle D$

$\angle C = \angle F$

$\therefore \Delta ABC \sim \Delta DEF$

h SSS

$\frac{DE}{AB} = \frac{10}{4} = 2.5$

$\frac{DF}{AC} = \frac{15}{6} = 2.5$

$\frac{EF}{BC} = \frac{15}{6} = 2.5$

$\therefore \Delta ABC \sim \Delta DEF$

4 a $x = 8, y = 10$ **b** $x = 12, y = 6$
c $x = 15, y = 4$ **d** $x = 3, y = 15$

5 a 2.5 **b** yes (SSS) **c** 2.5

6 a yes **b** no **c** no **d** yes

7 4 m

8 a yes (AAA) **b** 2.5 **c** 15 m

9 If two angles are known then the third is automatically known using the angle sum of a triangle.

10 a AAA

$\angle A = \angle D$ (corresponding angles in parallel lines)

$\angle B = \angle B$ (Common)

b AAA

$\angle ACB = \angle ECD$ (vertically opposite)

$\angle E = \angle A$ (alternate angles in parallel lines)

c AAA

$\angle A = \angle A$ (common)

$\angle ABC = \angle ADB$ (given)

d SAS

$EF = BA$ (equal sides)

$\angle E = \angle B$ (equal interior angles in regular polygons)

$DE = CB$ (equal sides)

11 Using Pythagoras' theorem $AC = 25$.

$\frac{DF}{AC} = \frac{50}{25} = 2$

$\frac{ED}{AB} = \frac{14}{7} = 2$

$\therefore \Delta ABC \sim \Delta DEF$ (RHS)

12 a 4 **b** 2.4 **c** $\frac{16}{3}$ **d** $\frac{10}{3}$

Problems and challenges

1 3, Reason is AAA for each pair with a right angle and a common angle.

2 a B C D E K

b A M T U V W Y

c H I O X

3 31

4 a $(3 - r) + (4 - r) = 5$, so $r = 1$ **b** $r = 4 - 2\sqrt{2}$

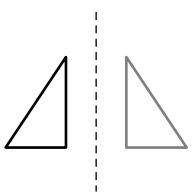
5 $\frac{60}{17}$

Multiple-choice questions

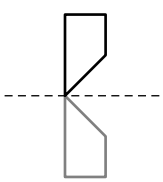
1 B **2** D **3** C **4** A **5** B
6 E **7** A **8** E **9** D **10** E

Short-answer questions

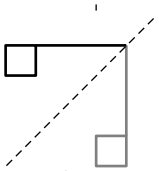
1 a



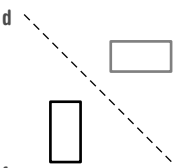
b



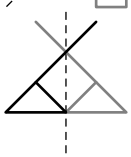
c



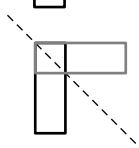
d



e



f



2 a $A'(1, -2), B'(3, -4), C'(0, -2)$

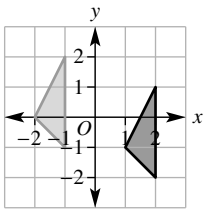
b $A'(-1, 2), B'(-3, 4), C'(0, 2)$

3 a 4 **b** 1 **c** 2 **d** 1 **e** 6
f 0

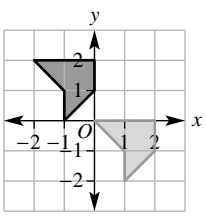
4 a (1, 4) **b** (3, –6) **c** (–3, –7) **d** (4, 6)

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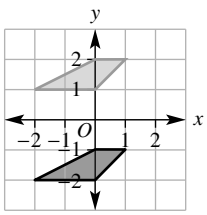
5 a



b

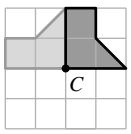


c

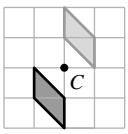


6 a 3 **b** 2 **c** no rotational symmetry

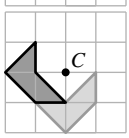
7 a



b



c



8 a i F **ii** G
b i EH **ii** FG
c i $\angle G$ **ii** $\angle E$

9 a RHS **b** SAS **c** SSS **d** AAS

10 a $x = 3, a = 25$ **b** $x = 5, a = 18$

11 triangle, square, hexagon.

12 a $\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
b $AE = CE$ and $BE = ED$ so AC and BD bisect each other.

13 a $AE = CE, AB = CB$ and BE is common
$\therefore \Delta ABE \equiv \Delta CBE$ (SSS)
b $\angle AEB = \angle CEB$ because $\Delta ABE \equiv \Delta CBE$ and since $\angle AEB + \angle CEB = 180°$
$\angle AEB = \angle CEB = 90°$ so AC and BD bisect at right angles.

14 a $(AB, EF), (BC, FG), (CD, GH), (DA, HE)$
b $(\angle A, \angle E), (\angle B, \angle F), (\angle C, \angle G), (\angle D, \angle H)$ **c** 3
d $y = 1.5, a = 115, x = 3$

15 a Yes, SAS **b** Yes, SSS **c** Yes, AAA **d** Yes, RHS

16 a $h = 6$ **b** $x = 2$ **c** $x = \frac{24}{11}$ **d** $d = 12.5$

Extended-response questions

1 a $A'(0, 1), B'(-2, 1), C'(-2, 4)$
b $A'(3, 1), B'(3, -1), C'(0, -1)$
c $A'(1, -1), B'(-1, -1), C'(-1, 2)$

2 a to form two similar triangles.
b AAA ($\angle DAB = \angle ECB$ and $\angle ABD = \angle CBE$)
c 3
d 12 m

Semester review 2

Ratios and rates

Multiple-choice questions

1 C **2** B **3** C **4** C **5** D

Short-answer questions

1 a 2 : 3 **b** 1 : 2 : 3 **c** 6 : 7 **d** 3 : 40
e 3 : 8 **f** 3 : 10
2 a 576 cm, 384 cm
b \$1500, \$2500
c \$1.60, \$4, \$2.40
3 18.75 m^2
4 \$7750
5 a 300 g/h **b** \$30/h **c** 100 km/h
6 \$2.27
7 90 km/h

Extended-response question

a 742.5 km **b** 16.5 km **c** 6.1 L **d** \$35.37
e 18 km

Equations and inequalities

Multiple-choice questions

1 B **2** D **3** B **4** C **5** B

Short-answer questions

1 a $w = 9$ **b** $m = 72$ **c** $x = 1$ **d** $a = 2$
e $w = -3$ **f** $x = 35$
2 a $m = -\frac{1}{2}$ **b** $a = -1$ **c** $x = 0$ **d** $x = \frac{15}{8}$
e $a = \frac{13}{8}$ **f** $a = \frac{7}{5}$
3 6
4 4 years
5 a $x > 1$ **b** $x \leq 2$ **c** $-1 < x \leq 2$
6 a $x > -8$ **b** $x \leq 3$ **c** $x \leq 30$

Extended-response question

a $C = 5n + 1500$ **b** $R = 17n$ **c** 125
d $P = 12n - 1500$ **e** \$900 **f** −\$300 (A loss)

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Probability and statistics

Multiple-choice questions

1 A **2** A **3** B **4** C **5** B

Short-answer questions

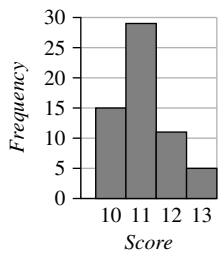
1 **a** i 13.75 ii 14 iii 8
b i 23 ii 18.5 iii 56
c i 10 ii 9.45 iii 15.7

2 **a** 8.5 **b** 4 **c** 14.5 **d** 10.5

3 16.9

4 **a** 17.4, 18 **b** 11.1, 11

5



6

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

7 **a** $\frac{7}{16}$ **b** $\frac{1}{16}$ **c** $\frac{15}{16}$ **d** $\frac{1}{2}$ **e** 0

8 **a** $\frac{3}{50}$ **b** $\frac{39}{50}$ **c** $\frac{1}{50}$ **d** $\frac{24}{25}$

Extended-response question

a 18 **b** 78 **c** 78 **d** Group A

Straight line graphs

Multiple-choice questions

1 D **2** B **3** C **4** D **5** D

Short-answer questions

1 **a** 1st **b** 2nd **c** 3rd **d** 4th

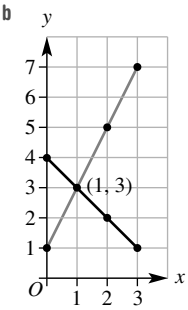
2 **a** i

x	0	1	2	3
y	1	3	5	7

ii

x	0	1	2	3
y	4	3	2	1

b

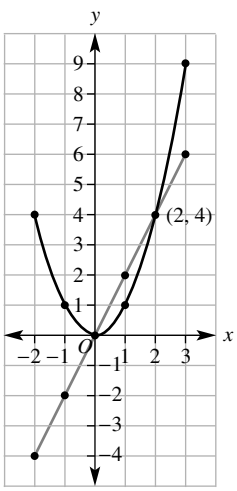


3 **a** $x = 2$
b $y = 3$
c $y = 1.5x$
d $y = -x - 2$

4 **a** i $y = 1.5x$ ii $y = -x - 2$
iii $y = 3$ iv $y = 1.5x$
b $y = -x - 2$ and $x = 2$

5

x	–2	–1	0	1	2	3
$y = x^2$	4	1	0	1	4	9
$y = 2x$	–4	–2	0	2	4	6

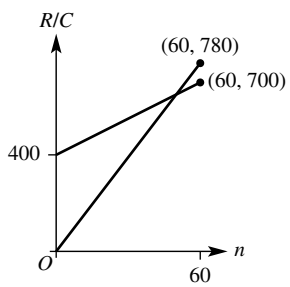


Extended-response question

a $R = 13n$

b \$400 (Wages)

c



d (50, 650)

e \$400

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Transformation and congruence

Multiple-choice questions

1 C **2** B **3** D **4** D **5** C

Short-answer questions

1 **a** 0 **b** 2 **c** 2 **d** 1

2 **a** (2, 2)

b (–1, –2)

3 **a** $A'(1, -1), B'(1, -3), C'(3, -2)$

b $A'(-1, -1), B'(-3, -1), C'(-2, -3)$

4 **a** SSS **b** AAS **c** RHS

5 A, C

6 **a** $\Delta BCD,\ \Delta ACE$ (AAA)

b Vertex C

c $x = 9$

Extended-response question

a 15 cm **b** 2 : 5 **c** 4 : 25 **d** 10 cm

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