YEAR 12 ATAR COURSE REVISED EDITION

ACADEMIC TASK FORCE

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PHYSICS

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ROY SKINNER

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PHYSICS

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FIRST EDITION

ROY SKINNER

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First published 2015
Reprinted 2016, 2022

National Library of Australia ISBN: 978-1-74098-180-4

Printed in Western Australia on paper supporting responsible forestry.

FSC www.fsc.org MIX Paper | Supporting responsible forestry FSC® C113956

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Dr. Roy Skinner has a Master's Degree in Nuclear Physics and a PhD in Science Education. With over 40 years experience including exam marking he has taught in the UK, New Zealand and Australia. Dr Skinner has been a teacher and Head of Science at numerous schools and was awarded the West Australian de Laeter Medal for most outstanding science teacher.

Acknowledgements
- W.A. School Curriculum and Standards Authority – with permission to use extracts from the Physics syllabus and Physics Data Sheet.
- Images by CanStockPhoto and iStock

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How to Use this Study Guide

This book is meant to be used in conjunction with a class text book; the aim is to reinforce the most important principles and understandings. Contained are the essentials needed for students to teach themselves by absorbing the bare facts and concepts, then working through the many example questions and sets. This book is meant to be written in as a Work Book to produce progressive learning in a student-friendly manner.

An active learning style is promulgated through the use of simple pictures and diagrams within the text, making full understanding cognitively simpler.

Tests are supplied at the end of each topic and trial papers that are typical of those used in ATAR examinations.

One of the main thrusts in this new curriculum is the importance of experimental design and analysis. Specific thinking skills and a focus on practical work has become more prominent (e.g. control of variables, data analysis, graphing and evaluation skills). This book is unique in this aspect as it has a whole chapter devoted to Investigations (Experimental Physics), where the separate, discrete skills are each targeted, from "How to design experiments" to "Evaluation and improvement of techniques".

Dr Roy Skinner

Gravity and Motion

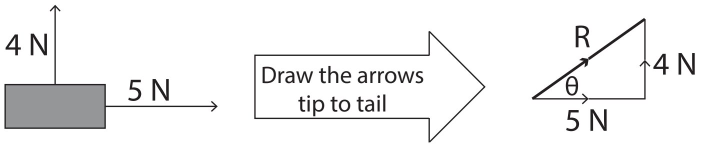
Chapter 1

1.1 VECTOR QUANTITIES

Adding vectors

Vectors are added by constructing a vector diagram or vector triangle to find the Resultant R.

e.g. a block of wood has forces of 5.0 N and 4.0 N acting on it at right angles.



$R^2 = 4.0^2 + 5.0^2$ So R = 6.40 N and $\tan\theta = 4/5$ so $\theta = 38.7^o$

Example 1

A crow can fly at 4.50 m s^{-1} in still air and heads due north with a wind blowing from the southeast at 12.0 m s^{-1}. Find the bird's resultant velocity with the wind blowing.

Solution 1

In this case the forces are not at right angles so the solution of this triangle requires the use of the cosine rule:

$R^2 = 4.5^2 + 12^2 - 2 \times 4.5 \times 12\cos 135$

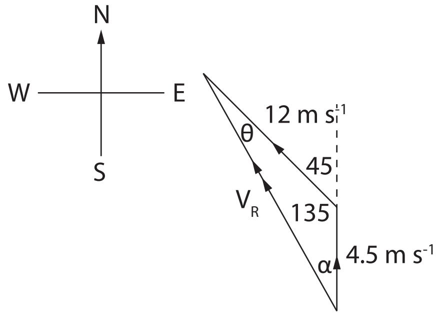
R = 15.5 m s^{-1}

To find θ the sine rule must be used: $\dfrac{15.5}{\sin 135} = \dfrac{4.5}{\sin\theta}$

$\sin\theta = \dfrac{4.5 \sin 135}{15.5} = 0.205$ so $\theta = 11.8^o$

Angle $\alpha = 180 - (11.8 + 135) = 33.2^o$

So R = 15.5 m s^{-1} at N 33.2^o W.

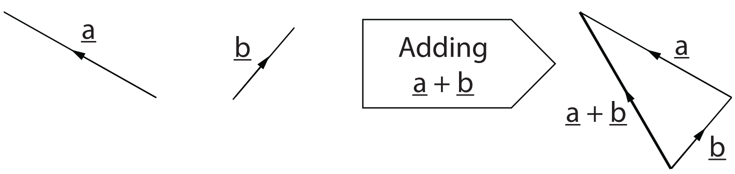


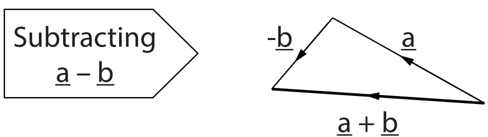
Subtracting vectors

Whenever a vector changes from one value to another we need to subtract vectors to find the change that has occurred. The value of the change = final vector – initial vector: $\Delta v = \underline{v} - \underline{u}.$

To subtract a vector we add the negative value to the initial vector (reverse the direction to find the negative value)

For example:





Example 2

A ball moving at 10.0 m s^{-1} strikes a wall at 45.0° and bounces off at 45.0° with a speed of 8.50 m s^{-1}. Find the ball's change in velocity.

Solution 2

The change in velocity (Δv) is given by: $\Delta v = \underline{v} - \underline{u}$

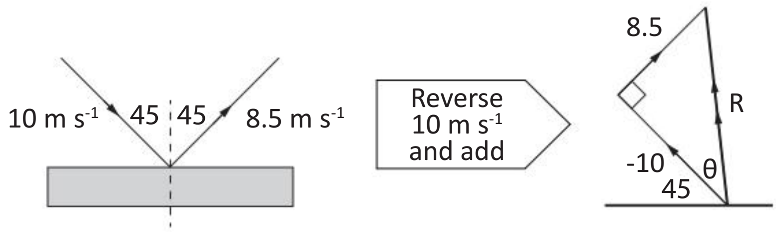
$\Delta v^2 = 10^2 + 8.5^2$

$\Delta v = 13.1$ m s^{-1}

$\tan\theta = \frac{8.5}{10}$ so $\theta = 40.4°$

Direction of $\Delta v = 45 + 40.4$

= 85.4° to wall.



Vector Components

The magnitude of a vector acting in one particular direction is called its Component in that direction. The horizontal and vertical components of a vector can be found by completing the other 2 sides of the vector triangle.

Example 3

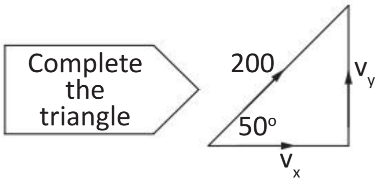
A bullet is fired at 50.0° to the horizontal and a speed of 200.0 m s^{-1}. How fast is it moving along the ground and how fast is the bullet rising in a vertical direction?

Solution 3

$\cos 50 = v_x/200$ So $v_x = 200 \cos 50 = 129$ m s^{-1}

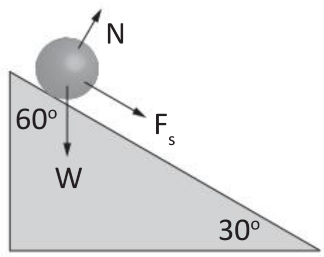
$\sin 50 = v_y/250$ So $v_y = 200 \sin 50 = 153$ m s^{-1}

The horizontal component of the rocket = 129 ms^{-1} and its vertical component = 153 m s^{-1}



Example 4

What is the net force on a 1.00 kg ball that is rolling down a slope of 30.0°?



Solution 4

Show all the forces acting (W and N):

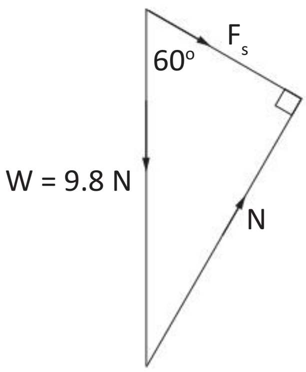
W is the weight of the ball

N is the normal reaction force from the surface

F_s is the force down the slope which is the resultant of W and N.

Construct the vector triangle with the forces shown in the correct directions $N + W = F_s$

$F_s = 9.8 \sin 30 = 4.9$ N

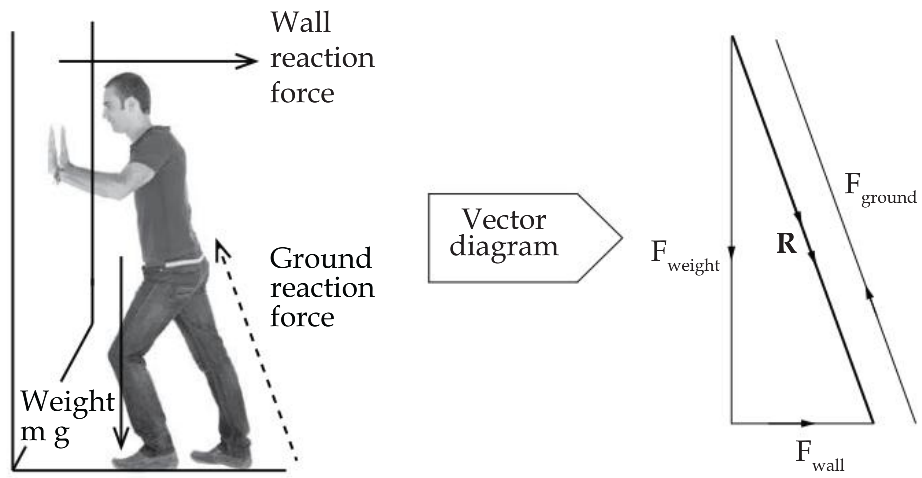


1.2 FREE-BODY DIAGRAMS

In the preceding diagram arrowed vectors have been drawn to show the direction of the forces acting (N and W) and another vector to show the direction of the net force or resultant (R).

These free-body diagrams are essential in solving problems involving vectors. In the above case R is the resultant force and so it indicates the direction the object will accelerate. In other cases where an object is at rest all the vectors must add up to zero. For example, a man leaning against a wall. As the man is in equilibrium any two of the vectors added to give a resultant (R) must be equal and opposite to the third vector, so the sum of all three is zero.

Below is a vector diagram showing the forces acting on a leaning man.

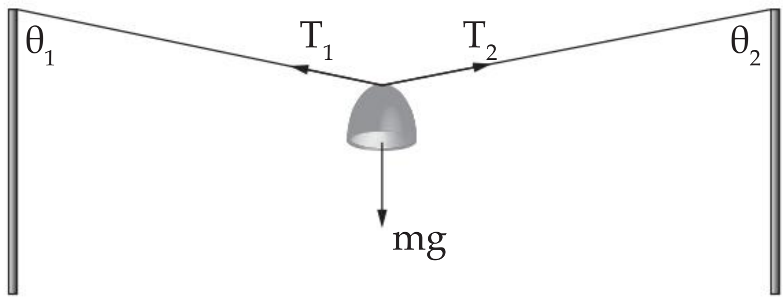


The resultant R of the forces F_{weight} and F_{wall} is equal and opposite to the force from the ground, F_{ground}.

The F_{ground} is also called the Equilibrant – that is the force that puts the other forces into Equilibrium.

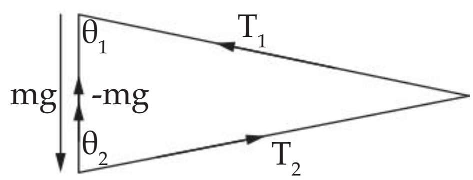
1.3 EQUILIBRIUM OF FORCES

If an object has no acceleration then all the forces acting on it must sum to zero in every direction i.e. $\Sigma F = 0$



For example: A street lamp is suspended on two wires.

The system is in equilibrium so the tensions in the wires must have upward vertical components that balance the weight of the lamp downwards.



Also, the left horizontal component of T_1 must equal the right horizontal component of T_2 for horizontal equilibrium to be stable.

So, vertically: $T_1\cos\theta_1 + T_2\cos\theta_2 = mg$

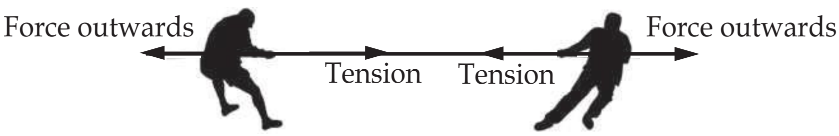
and horizontally: $T_1\sin\theta_1 = T_2\sin\theta_2$

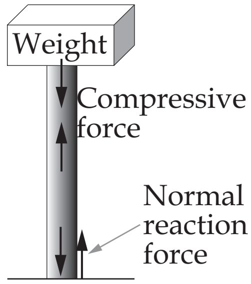
If we draw a clear vector triangle then the sum of T_1 and T_2 will give a resultant that will be equal and opposite to the weight of the lamp.

Therefore the net force on the system is mg + (-mg) = zero because no acceleration is occurring.

1.4 TENSILE AND COMPRESSIVE FORCES

When a rod/wire is being stretched it is under tension and when a rod/beam is being squeezed it is under compression.

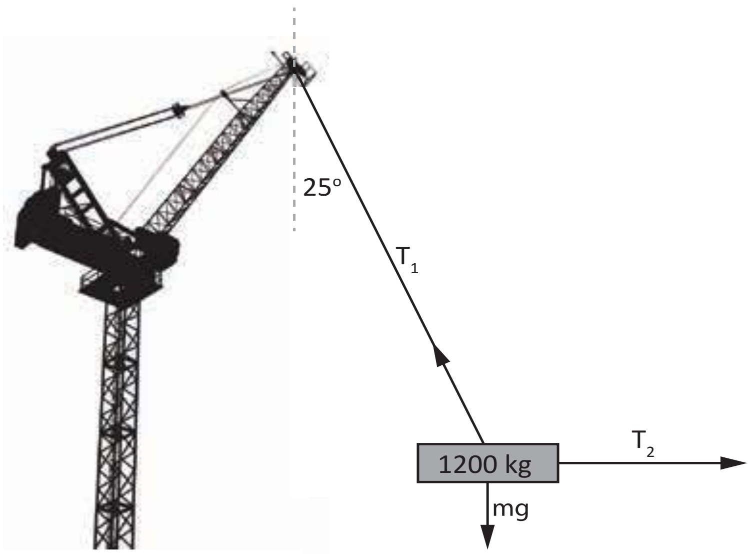




To maintain equilibrium at the ends, the tensile force must be equal and opposite to the applied force and the same is true for a compressive force. Hence, for a rod under tension, the direction of the tensile force at each end is inwards, whereas for a rod under compression the forces at each end are outwards.

Example 5

A crane has a rope at its end at 25.0° to the vertical holding a 1200.0 kg load. Another rope is also attached to the load and pulling it towards the deck of the ship making this rope horizontal. Calculate the tension in each rope.



Solution 5 (Components method)

Vertically: vertical component of T_1 = weight (mg)

So $T_1 \sin 65 = 11760 \rightarrow T_1 = 1.30 \times 10^4$ N

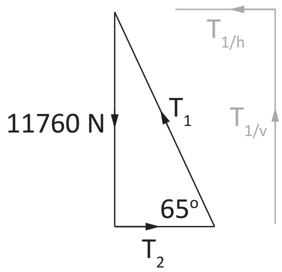
Horizontally: horizontal component of $T_1 = T_2$

So $T_2 = T_1 \cos 65 = 1.298 \times 10^4 \times \cos 65 = 5.48 \times 10^3$ N

(Vector triangle method)

Sin65 = 11760/T_1 So $T_1 = 1.30 \times 10^4$

Tan65 = 11760/ T_2 So $T_2 = 5.48 \times 10^3$ N

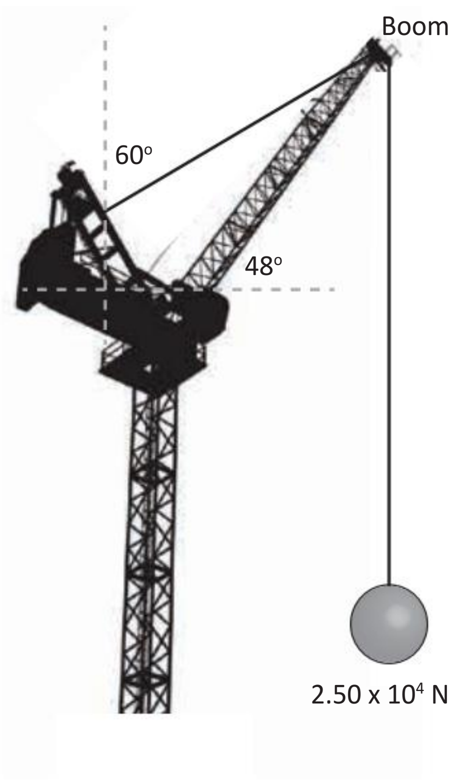


In general, the vector triangle method is a quicker method.

Example 6

A crane boom hangs at an angle of 48.0° to the ground and holds a load of 2.50×10^4 N.

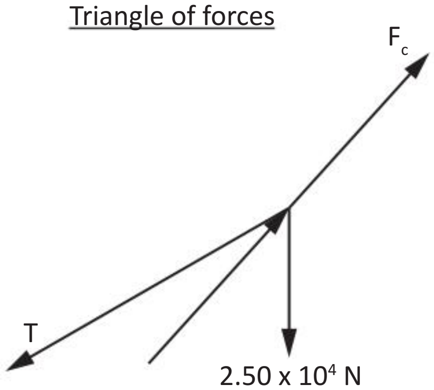
A cable is attached to the end of the boom at an angle of 60.0° to the vertical. Calculate the tension in the cable and the compressive force in the boom.

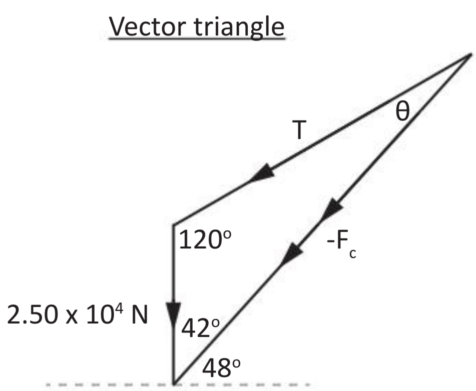


Solution 6 (Using a vector triangle method)

The compressive force in the boom (F_c) must act directly outwards along the boom.

The forces acting are shown in the force diagram.





$$\theta = 180 - (120 + 42) = 18^{\circ}$$

$$\text{Using the sine rule } \frac{25000}{\sin 18} = \frac{F_c}{\sin 120} = \frac{T}{\sin 42}$$

From this: F_c in the boom = 7.01×10^4 N. Tension in wire, T = 5.41×10^4 N

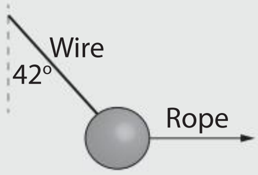
Note: The resultant of the weight plus tension acts downwards, so the compressive force in the boom (F_c) must be equal and opposite to this resultant so all the forces are in equilibrium and there is no movement of the system.

Set 1: Vectors

1 A truck loaded with rocks stops so that it has its wheels on two different sets of heavyweight weighing scales. The front scales read 11500 N and the rear scales read 28500 N.

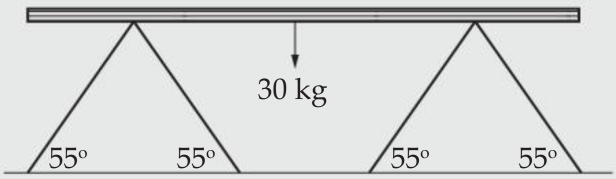
What is the mass of the truck?

2. A demolition ball has a mass of 500 kg and hangs on a steel wire but is pulled over to the right by a horizontal rope. If the wire is inclined at an angle of 42° to the vertical, calculate the tension in the wire and the force in the rope required to keep the ball in equilibrium.

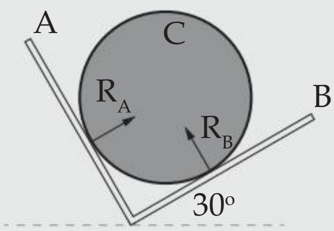


3. A painter's trestle table is constructed from a 30 kg flat sheet of wood supported on 4 legs at an angle of 55° to the ground.

Calculate the force in each supporting leg.

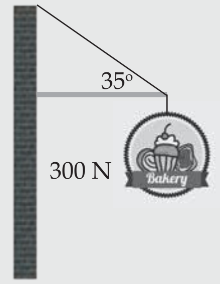


4. A cylindrical roller C is being held for machining in a 'V'-shaped block shown.The two faces of the 'V' are A and B, attached at 90° to each other and the right-hand face is inclined at 30° to the horizontal.

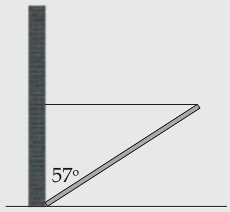


The normal reaction force from face A is called R_A and that from face B is called R_B. Calculate the ratio of the forces, R_A / R_B.

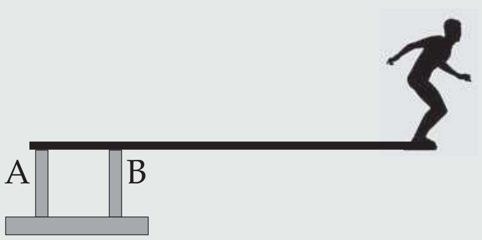
5. A heavy metal shop sign of mass 300 N hangs from a rod of negligible mass and is held horizontal by a cable attached to the wall. The cable fixes to the pole at an angle of 35° to the horizontal. If the structure is in equilibrium, calculate the tension in the cable and the force in the rod.



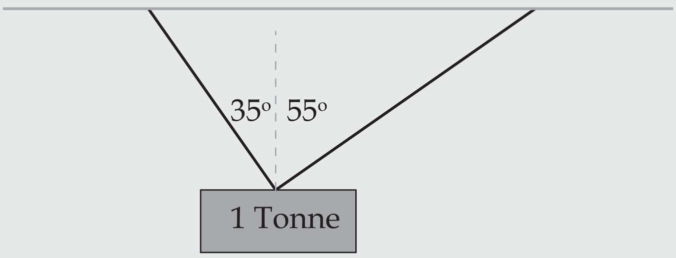
6. A uniform steel scaffolding pole of mass 110 kg has its bottom end between a wall and the floor. The pole is held at an angle of 57° to the vertical by a horizontal wire attached to the pole and a bolt in the wall. Calculate the tension in the wire and the force in the pole.



7. A uniform diving board has a mass of 30 kg and is supported at one end by two blocks A and B. A boy of mass 60 kg is standing at the other end ready to dive. If the force in block B is 3.10 kN upwards, what is the force in block A and its direction?

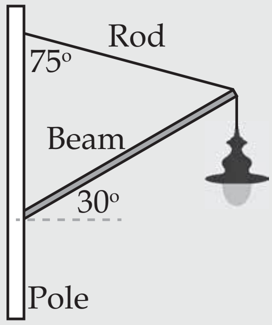


8.



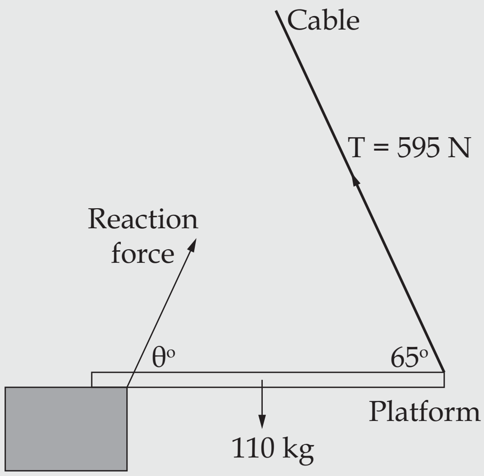
A 1.0 tonne load is supported from a roof by two steel wires. The wire on the left makes an angle with the vertical of 35° whilst the right-hand wire makes an angle of 55° with the vertical. Calculate the tensile forces in each wire.

9. A hanging lamp support at a railway station consists of a light wooden beam 1.8 m in length supported by a light rod securing it to an upright pole. The rod meets the pole at angle of 75° and the mass of the lamp being supported is 30 kg. Find the forces in the rod and beam and their directions.



10. A viewing point is constructed from a uniform wooden platform suspended over a cliff. The left end stands on a concrete base with the right hand end of the platform held up by a cable attached at 65° to the horizontal.

The mass of the platform is 110 kg and the tensile force in the wire supporting it is 595 N. The reaction force of the concrete on the platform is upwards making an angle of θ° to the ground, as shown.



Calculate the magnitude and direction of the reaction force at the left hand end of the platform.

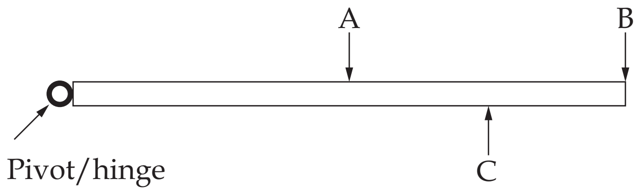
1.5 TORQUES

Objects that can rotate around a pivot have their own set of principles and equations which are different to objects that move in a straight line. Consider a door rotating about its hinges.

If one person pushes on the door at point B the door will open faster (accelerate) than if another person applied the same force at point A.

This is because person B is applying more leverage, or torque than person A.

Door - looking from above



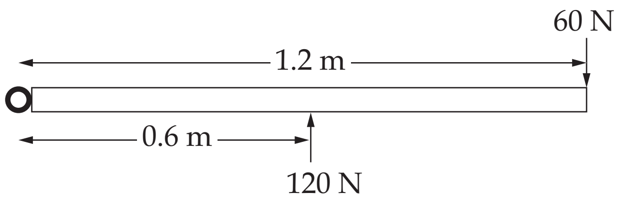
Torque is defined as the product of force x perpendicular distance (F.r) from the pivot, so, although A and B both apply the same forces, B exerts twice the torque, because the force is applied at twice the distance. The applied torque at A and B is called "Clockwise Torque" (CT) because it would make the door turn in a clockwise direction. To prevent the door from turning an opposite torque must be applied at, say, point C. A force upwards at C would give an anticlockwise torque (ACT).

The equivalent of Newton's First Law for rotating objects, then, is:

For equilibrium, the sum of all anticlockwise torques = the sum of all clockwise torques

$$\boxed{\Sigma ACT = \Sigma CT}$$

If a person is pushing at the end of a 1.2 m wide door with a force of 60 N, the value of the clockwise torque is



F.r = 60 x 1.2 = 72 newton metres (Nm).

For another person, pushing in the middle of the door, to stop the door from opening they will need to push with twice the force, or 120 N, to obtain the same torque

$72 = F.r = 120 \times 0.6$

The person pushing at the end has twice the leverage, and so needs only half the other force.

$\Sigma ACT = \Sigma CT$ applies if there is no rotation.

Newton's 1st Law must still apply if there is no translational acceleration (i.e. the door does not move up or down. Hence the net force on the door must be zero ($\Sigma F = 0$).

This means that there must be another force acting at the pivot which must be equal to 60 N and be acting downwards. Without this force at the pivot, the left-hand end of the door would move upwards.

Example 7

Little Johnny (mass 40.0 kg) sits on a see-saw 1.50 m from the fulcrum (pivot) and asks his uncle Jim (mass 100.0 kg) to sit on the other end so the see-saw just balances.

a) Where must Uncle Jim sit?

b) If Johnny's brother Jack (mass 30 kg) joins little Johnny and sits 2.0 m from the fulcrum, where must Uncle Jim now sit to balance both boys?

c) What is the reaction force at the fulcrum when all 3 people are sitting on the see-saw?

Solution 7

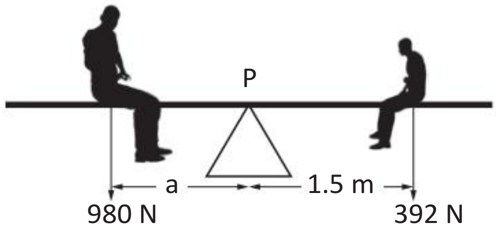
a) (Put all forces in on diagram)

Taking torques about the fulcrum P:

ΣACT = ΣCT

980 × a = 392 x 1.5

a = 0.600 m

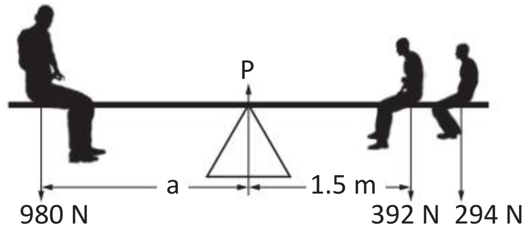


b) ΣACT = ΣCT

980 x a=

(392 x 1.5) + (294 x 2.0) = 1176

$a = \frac{1176}{980} = 1.20$ m



c) ΣFdown = ΣFup

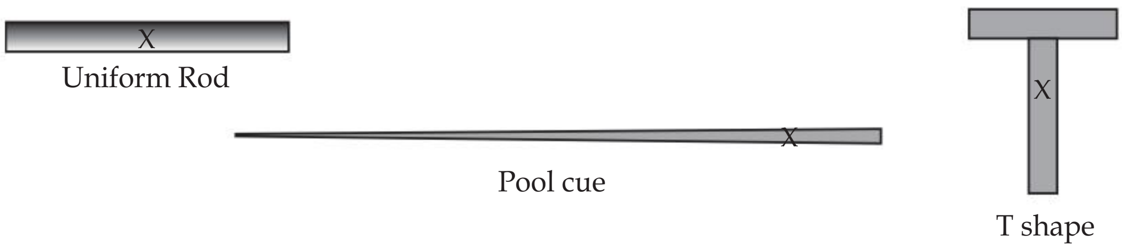
Σ Forces down = 980 + 392 + 294 = 1666 N

Therefore the force from the pivot must be 1.67×10^3 N upwards to 3 significant figures.

1.6 CENTRE OF MASS

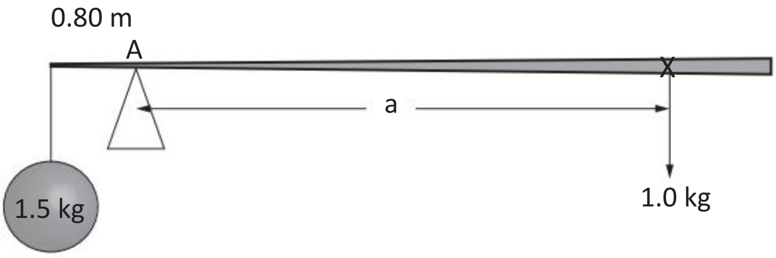
The centre of mass of any object is a point at which all the ACTs equal the CTs i.e. the exact balance point. For a symmetrical object the Centre of Mass (C of M) will be at its geometrical centre but for something like a pool cue the C of M (X) will be closer to one end.

(X shows the centre of mass of the object)



Example 8

A pool cue has a mass of 1000.0 or 1.00×10^3 grams. It is placed on a knife edge positioned 80.0 cm from its thin end with a mass of 1.50×10^3 g hanging from its tip.



If the cue is exactly balanced in this position, calculate the position of its centre of mass from the tip.

Solution 8

Taking torques about the fulcrum A

ΣACT = ΣCT

14.7 x 0.80 = 1.0 x 9.8 x a

a = 1.20 m from the pivot, or 2.00 m from the tip.

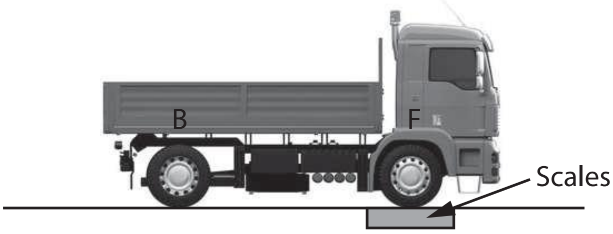
2-Pivot Problems

Some problems involve objects supported on two pivots and the method for calculating forces involves the same principles, except that torques are taken in turn about each pivot.

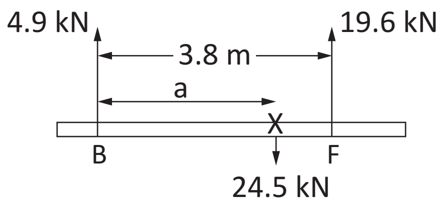
NB. If the system is in equilibrium then the sum of the torques about any chosen point must be zero but choosing a specific point makes the mathematics easier.

Example 9

A truck of mass 2.50 tonne is driven to the Council weighbridge where the force downwards on each set of wheels can be found. When the front wheels of the truck are on the scales the reading is 19.6 kN and with the back wheels on the scales a reading of 4.90 kN was obtained. If the distance between the axles of the truck is 3.80 m, where is the centre of mass of the truck?



Solution 9



Let the distance of the C of M from B be a.

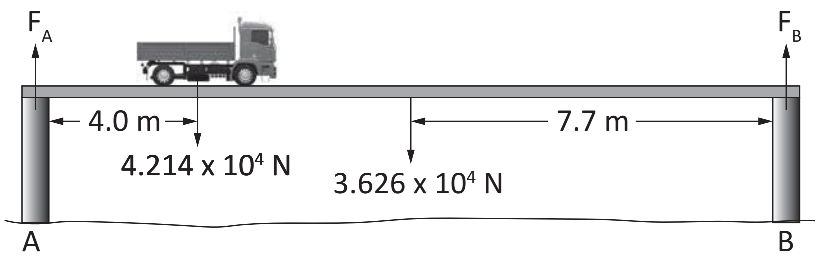
Taking torques about B: ΣACT = ΣCT

$3.8 \times 19.6 = 24.5 \times a$

so a = 3.04 m from the rear wheels.

Example 10

A bridge across a river consists of a uniform rectangular beam 15.4 m long of mass 3.70×10^3 kg held by two supporting poles, one at each end. If a truck of mass 4.30 tonne stopped at a distance of 4.00 m from one end of the bridge, what would be the force in each of the supporting poles?



Solution 10

Taking torques about end A:

ΣCT = ΣACT

$(4.0 \times 4.214 \times 10^4) + (7.7 \times 3.626 \times 10^4) = 15.4 \times F_B$

$F_B = 2.91 \times 10^4$ N

Now, taking torques about pole B:

ΣACT = ΣCT

$(7.7 \times 3.626 \times 10^4) + (11.4 \times 4.214 \times 10^4) = 15.4 \times F_A$

$F_A = 4.93 \times 10^4$ N

Check $\Sigma F = 0$: Net force down $= 4.214 \times 10^4 + 3.626 \times 10^4 = 7.84 \times 10^4$ N

Net force up $= F_A + F_B = 4.932 \times 10^4 + 2.908 \times 10^4 = 7.84 \times 10^4$ N (same as force down)

Set 2: Torques and Centre of mass

1. Before a footy team gets onto a double-decker bus the driver tells the men to fill up the lower deck first. The driver says that filling the top deck first could be dangerous when driving.

   The best scientific explanation for this would be:

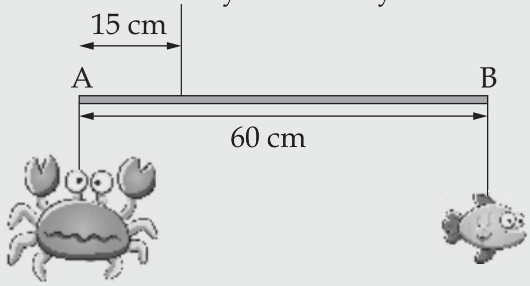
   A. Too much weight upstairs could cause the suspension springs to compress too much when going round corners

   B. Filling the top deck first could put an uneven distribution of weight on the axles, causing instability

   C. With a large weight upstairs the centre of gravity of the bus changes making toppling easier when rounding corners

   D. As the passengers climb the stairs this can cause an anticlockwise torque to be exerted on the axles, decreasing the lateral stability of the body.

2. A child's mobile hanging from the ceiling comprises two objects suspended from wires attached to a uniform horizontal rod 60 cm long.

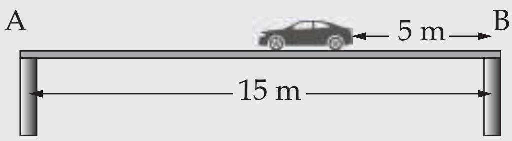
   

   At end B hangs a 200 g object whilst a heavier object hangs at end A.

   The rod has a mass of 100 g and the mobile balances at a point 15 cm from A.

   What is the mass of the object at end A?

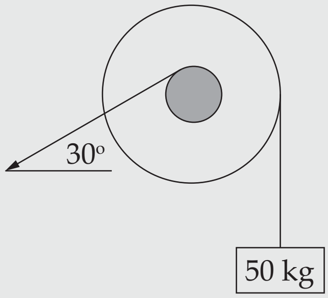
3. A uniform wooden bridge over a river is 15 m long and has a mass of 800 kg, supported at each end by wooden piles A and B. A car of mass 1100 kg stops 5.0 m from end B of the bridge.

   In this situation, calculate the reaction force on the bridge from pile A.

4. The wheel and axle machine shown is used to raise a 50 kg load from the floor by pulling on the left hand rope. The wheel has radius 12 cm and the axle has radius 5 cm.

   If a man pulls down on the left-hand rope at an angle of 30°, what is the force needed to raise the load?



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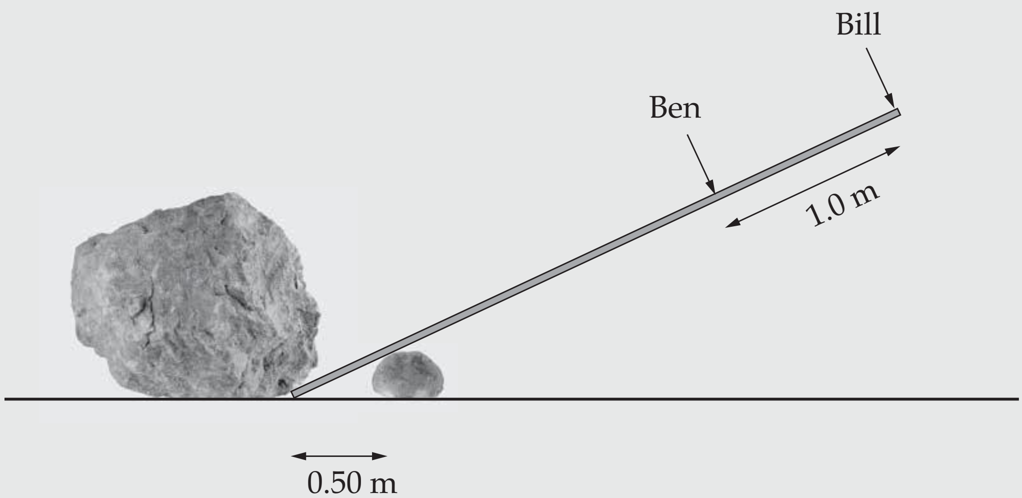

5. Bill and Ben want to move a 900 kg boulder in their garden. To do this they use a 3.5 m-long steel scaffolding pipe as a lever with a rock as a pivot. One end of the pipe is placed under the boulder whilst the rock is placed 0.50 m from that end. Bill pushes down at the other end of the pipe whilst Ben pushes down with an equal force at a point 1.0 m from Bill.

   What minimum force must each man exert to just lift the boulder?

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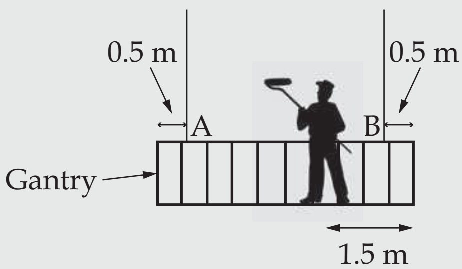
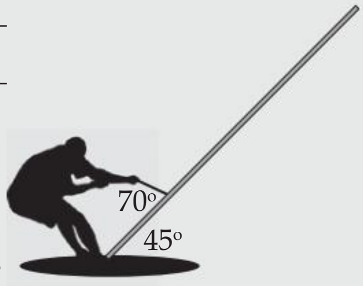


6. A painter stands on a gantry 5.0 m long and mass 80 kg. The gantry is supported by four ropes attached 0.5 m in from the ends and the painter (mass 95 kg) stands 1.5 m from end B.

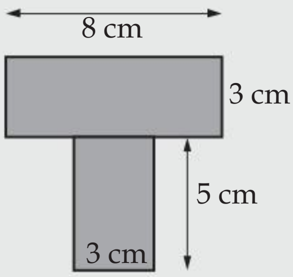
   What will the tension be in each of the ropes at end A and B

7. A windsurfer pulls on the mast rope to lift it up. The mast and sail have a mass of 12.0 kg, assumed to act half way along the mast which is 2.50 m long.



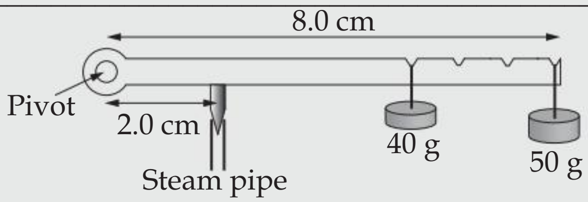
As he pulls, at one point the mast is at an angle of 45° to the horizontal and the rope is angled at 70° to the mast and attached 1.00 m from its base. What force must the windsurfer exert to hold the mast in equilibrium at this point?

8. A 'T'-shaped bracket is made from two 3.0 cm-wide bars welded together.



The lower bar is 5.0 cm long whilst the cross bar is 8.0 cm long.

Determine the position of the centre of mass of the whole bracket.



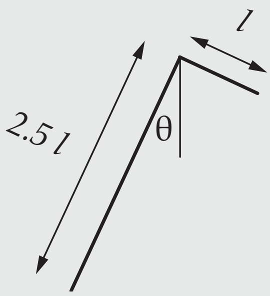
9. A steam pipe pressure limiter operates by the torque from weights acting on a tapered needle pushing into a steam pipe. The bar will lift when the force from the steam exceeds a critical value.

A 50 g weight sits permanently at a distance of 8.0 cm from the pivot but another, 40 g mass can be moved to different positions from the pivot.

Where must this 40 g mass be placed to balance an upward steam force of 2.85 N in the pipe?

10. *(Difficult!)*

A piece of wire is bent into a right-angled shape with sides in the ratio of 1:2.5 and then suspended from the ceiling by a string. The shape will come to an equilibrium position with the longer side at an angle of θ to the vertical.



Determine angle θ.

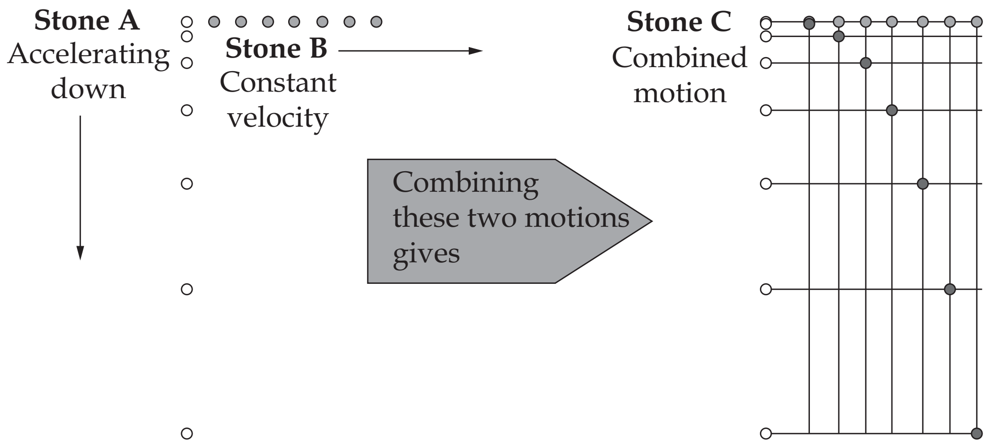
(Hint: the mass of a length of wire is proportional to its length, so we can write $m = kl$)

1.7 TWO DIMENSIONAL MOTION

Suppose a rock was thrown horizontally from the top of a cliff onto the beach below. Gravity would only have an effect on the vertical motion of the stone, causing it to accelerate downwards. The horizontal motion of the stone, however would not be affected by gravity, so its horizontal velocity would remain constant throughout its motion.

With 2-D motion we must always consider the vertical and horizontal motions as being totally separate. The only link between these two motions is time.

Below is a displacement plot of the motion of Stone A with time falling, directly downward under gravity and another Stone B which is moving horizontally with no downward force acting on it.



Stone C is projected horizontally from the top of a cliff and the Stone C graph shows the total x and y displacement added together: It accelerates downwards and moves horizontally at a constant rate at the same time.

The resultant path taken by the stone is parabolic, as can be seen from the following proof.

Horizontal motion is given by: $s_H = u_H t$ or $t = s_H/u_H$ (1)

Vertical motion is given by: $s_V = u_V t + \frac{1}{2}gt^2$ (2)

Substituting for t from (1): $s_V = u_V\left(\frac{s_H}{v_H}\right) + \frac{1}{2}\, g\left(\frac{s_H^2}{v_H}\right)$

Initially at t = 0 $u_V = 0$ $s_V = g\left(\frac{s_H}{v_H}\right)^2$

So vertical displacement depends on s_H squared.
This has the form of $y = kx^2$ which is a parabola.

Example 11

A boy throws a rock horizontally at 10.0 ms^{-1} from the top of a cliff 50.0 m high onto the beach below.

a) What is the time of flight?

b) Where does the stone land?

c) What is the stone's impact velocity?

Solution 11

Vertically

a) $u = 0$ m s^{-1} — $s = ut + \frac{1}{2}at^2$

$a = -9.8$ m s^{-2} — $-50 = 0 - 4.9t^2$ (N.B. 'downwards' is always negative.)

$t = ?$ — $t^2 = \frac{50}{4.9}$

$s = -50$ m — $t = 3.19$ s

b) Horizontally the stone is moving constantly at 10.0 m s^{-1} for a time of 3.19 s

So s = v x t = 10 x 3.19 = 31.9 m from the base of the cliff.

c) Vertically $v_V = u_V + gt = 0 – 9.8 \times 3.19 = 31.3$ m s^{-1}. Therefore $v_V = -31.3$ m s^{-1} (i.e. downwards).

Horizontally $v_H = 10$ m s^{-1} constantly

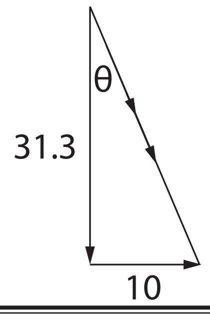
Adding these two vectors will give the resultant velocity:

Using Pythagoras' theorem:

$$v^2 = 31.3^2 + 10^2 = 1080 \text{ so } v = 32.9 \text{ m s}^{-1}$$

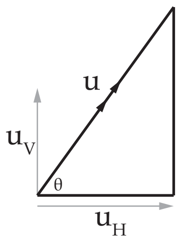
$$\tan\theta = \frac{10}{31.3} = 0.319,$$

$$\theta = 17.7^o \text{ to the vertical}$$



Projectiles

When an object is projected at an angle to the ground it will possess two different initial velocities in the horizontal and vertical directions – the horizontal and vertical components: u_H and u_V.



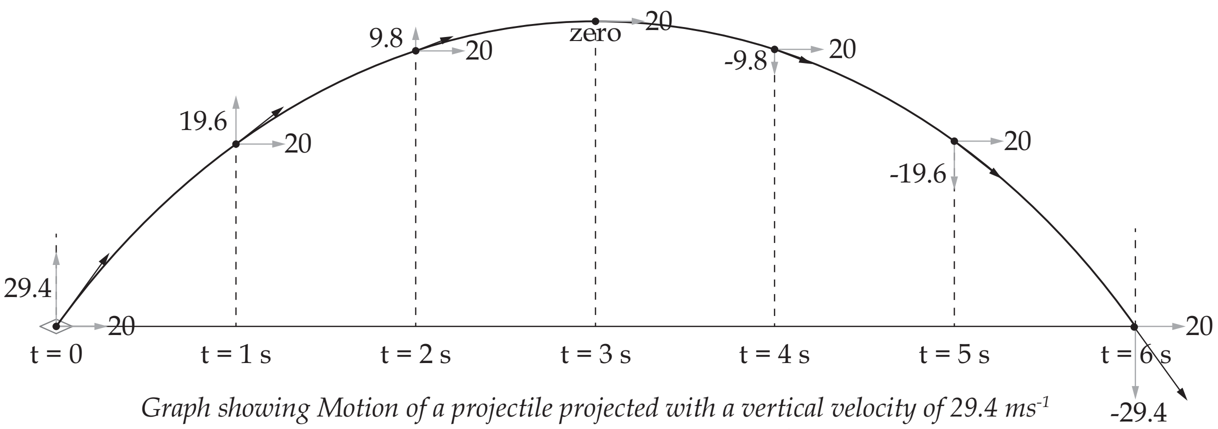
From the vector triangle
$u_V = u\sin\theta$ and
$u_H = u\cos\theta$

As in the last 2-D example u_H will again remain constant throughout the flight and u_V only will be affected by gravity.

In physics we always use a sign convention where the upward vector direction is taken as positive and a downward one as negative, so u_V is positive but the acceleration has a negative value:

a = -9.8 ms^{-2}. This means that u_V will become smaller with time and then become negative after the projectile reaches the top of its flight.

So the vertical flight vector changes but the horizontal one remains the same.



Graph showing Motion of a projectile projected with a vertical velocity of 29.4 ms^{-1} and a horizontal velocity of 20 m s^{-1}

Example 12

An arrow is fired into the air at a speed of 25.0 m s^{-1} at an angle of 60.0° to the horizontal.

a) How high will the arrow reach?

b) What is the arrow's speed after 3.00 s?

c) How far away will the arrow land?

Solution 12

Vertical velocity = 25sin60 = 21.65 m s^{-1}

Horizontal velocity = 25cos60 = 12.5 m s^{-1}

a) At top of flight $v_v = 0$, using $v^2 = u^2 + 2as$ $0 = 21.65^2 + 2(-9.8)s$

So s_{max} = 23.9 m

b) <u>Vertically</u>

u = 21.65 m s^{-1} v = u + at

v = ? v = 21.65 + (– 9.8) x 3.0

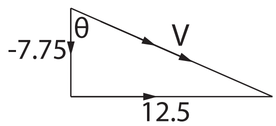
a = -9.8 m s^{-2} v = -7.75 (i.e. downwards)

t = 3.0

<u>Horizontally</u>

u = 12.5 m s^{-1}, constant

From the vector triangle $v^2 = 12.5^2 + (-7.75)^2$



v = 14.7 m s^{-1} $\tan\theta = \frac{12.5}{7.75}$ So = 58.2° to the vertical

c) To find the range firstly find the time of flight. In this example $s_{vertical}$ = 0 (starts on the ground and lands on the ground)

<u>Vertically</u>

u = 21.56 m s^{-1} a = -9.8 m s^{-2} s = 0 m t = ?

Using $s = ut + ½at^2$ $0 = 21.65t – 4.9t^2$ t = 4.42 s

<u>Horizontally</u>

u = 12.5 m s^{-1}, constant s = ut = 12.5 x 4.42

= 55.2 m

Example 13

WA cricketers can win a $10,000 prize if they hit a sponsor's sign which is 67.0 m away from the wicket and suspended on stands with its top edge 6.00 m up in the air. A batsman hits the ball with a velocity of 30.0 m s^{-1} at an angle of 65.0°.

Calculate whether the ball hits the sponsor's sign.

Solution 13

Vertical velocity = 30sin65 = 27.19 m s^{-1}
Horizontal velocity = 30cos65 = 12.68 m s^{-1}

<u>Horizontally</u>

Time to travel 67 m to the sign = $\frac{67}{12.68}$ = 5.28 s

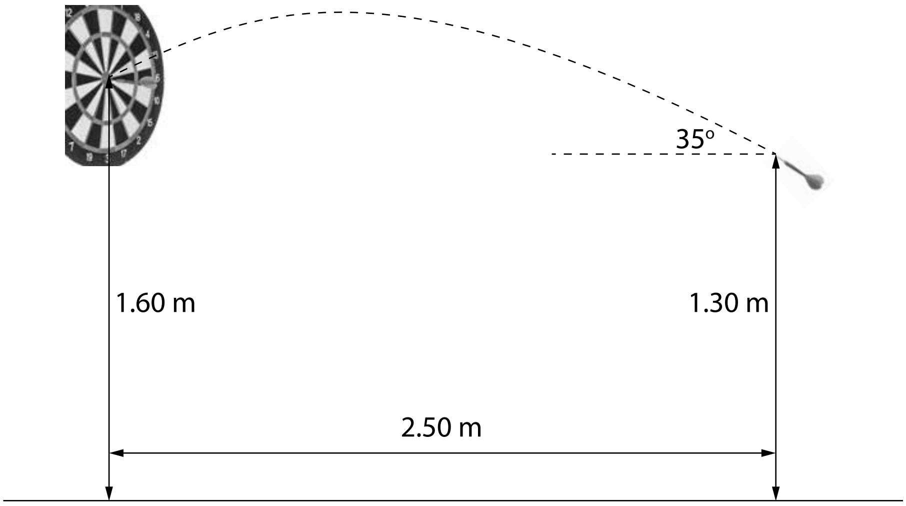
<u>Vertically</u>

u = 27.19 m s^{-1} Using $s = ut + ½at^2$
a = -9.8 m s^{-2} $s = 27.19 \times 5.28 + 0.5 \times (-9.8) \times (5.28)^2$
t = 5.28 s = + 6.96 m (+ means above the ground)
s = ?

The ball passes over the sign, missing it by 0.960 m

Example 14

A female physicist darts-player has perfected the skill of throwing her dart at an angle of exactly 35° above the horizontal. The dartboard is suspended on a wall 2.50 m away with its centre at a height of 1.60 m from the floor. If the woman throws the dart from a height of 1.30 m, how fast must she throw it if she is to score a bulls-eye?



Solution 14

Vertical velocity = usin35 Horizontal velocity = ucos35

<u>Horizontally</u>

Time of flight = $\frac{2.5}{u\cos 35} = \frac{3.05}{u}$ (this will be substituted into the vertical equation)

<u>Vertically</u>

u_v = usin35 = 0.5736u m s^{-1}

a = -9.8 m s^{-2}

t = $\frac{3.05}{u\ s}$

s_v = 1.6 – 1.3 = 0.3 m

<u>Vertically</u> using s = ut + ½at^2

$0.3 = 0.5736 \times 3.05 + \frac{1}{2}(-9.8)\left(\frac{3.05}{u}\right)^2$

$0.3 = 1.749 - \frac{45.58}{u^2}$ 45.58/u^2

$u^2 = \frac{-45.58}{-1.449}$

u = 5.61 m s^{-1}

Set 3: Projectiles

1. A stone is thrown horizontally at 15 m s^{-1} from the top of a cliff which is 30 m above the sea.

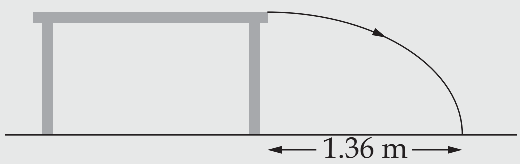
   a) How far out at sea does it land?

   b) How fast must the stone be thrown if it is to land on top of a floating piece of wood 25 m from the base of the cliff?

   c) How long will it take to strike the wood?

   d) With what velocity will it strike?

2. You are wiping the surface of the main dining table at home after a meal, when quite accidentally you strike a fork that then slides off the table at a horizontal speed of 3.70 m s^{-1}.



   If the fork lands on the ground, 1.36 m horizontally from the edge of the table, how high off the ground is the top of the table?

3. A TV news helicopter is ascending vertically at 4.00 m s^{-1} whilst travelling horizontally forwards at 16.0 m s^{-1}. When he is 110 m above the ground the news cameraman accidentally drops his camera out of the helicopter.

   a) How long does it take for the camera to reach the ground?

   b) What was the maximum height of the camera above the ground?

   c) How far forward does the camera travel before landing on the ground?

4. Big Bertha was the name of a huge cannon used by the Germans in the First World War to fire across the Channel to England. This gun was mounted on a train and could be moved to different positions. The shell's muzzle velocity was 750 km h^{-1} (Ignore air resistance).

a) If the gun were angled at 40° to the horizontal what would be the shell's velocity measured along the ground?

b) What would be the maximum height that the shell reached above the ground?

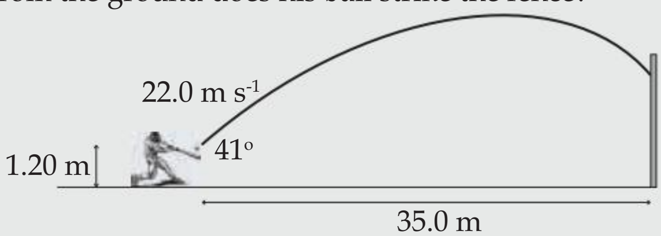
c) What is the range of the gun at 40°?

d) What would be the gun's maximum range?

e) What would be the velocity and angle of the shell 5.0 seconds after firing?

5. In training, a baseball batter hits a ball from a height of 1.20 m off the ground at 22.0 m s^{-1} at an angle of 41° towards a vertical metal fence that is 35.0 m away.

a) How far from the ground does his ball strike the fence?



b) At what angle does it strike the fence?

6. Jonathon, a world class archer, is shown preparing to shoot an arrow. At the instant of release, the 0.050 kg arrow has a velocity of 28.5 m s^{-1} at an angle of 30° to the ground. The arrow is 1.40 m from the ground when released.

(a) Calculate the vertical and horizontal components of the arrow's initial velocity.

(b) How far away from Jonathon does the arrow hit the ground?

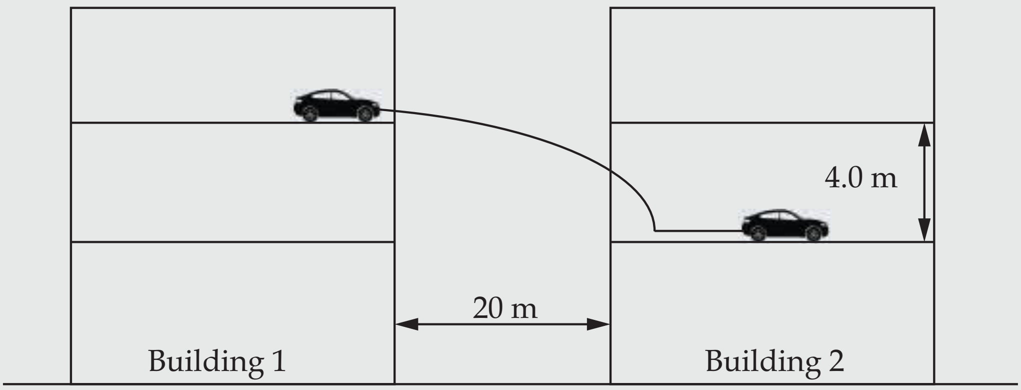
(c) (No calculations are required for the following):

Sketch the trajectory of the arrow on a graph. Label this (i) on the graph

Sketch on the graph the trajectory of the arrow showing how it would be changed if you allowed for air resistance. Label this as (ii) on the graph and explain why the trajectory changes in this way.

Sketch on the graph the path of the arrow if it is fired at an angle of 40° rather than 30° (neglecting air resistance). Label this as (iii) on the graph.

7. In the movie Car Escape, Taylor and Jones drove their sports car across a horizontal car park in building 1 and landed it in the car park of building 2, one floor lower. Building 2 is 20 metres from building 1, as shown in the diagram. The floor where the car lands is 4.0 m below the floor from which it started in building 1.

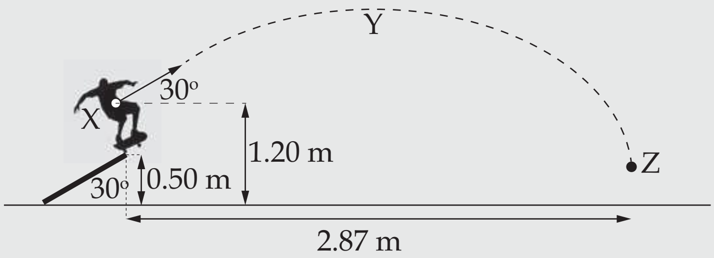


In the following questions, treat the car as a point particle and assume that air resistance is negligible.

a) Calculate the minimum speed, at which the car should leave building 1 in order to land in the car park of building 2. Show your working.

b) In order to be sure of landing in the car park of building 2, Taylor and Jones, in fact, left building 1 at a speed of 25 m s^{-1}. Calculate the magnitude of the velocity of the car just prior to landing in the car park of building 2.

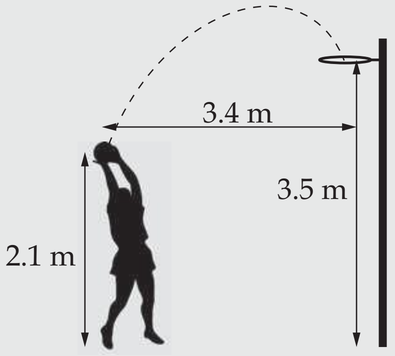
8. A skateboarder rides up a ramp as shown in the diagram below. At the instant the skateboard leaves the ramp, the centre of mass of the skateboard and rider is 1.20 m above the ground (as shown) and is initially moving with a speed of 5.00 m s^{-1}, at an angle of 30.0° above the horizontal. The parabola XYZ is the path of the centre of mass of the skateboard and rider. Assume that air resistance is negligible.



a) Calculate the height above the ground of the centre of mass at the highest point of the motion (Y).

b) When the skateboard touches the ground, the centre of mass has moved to point Z, as shown above. Calculate the time that the skateboarder is airborne.

9. A game of netball is being played in the gymnasium. A goalie shoots for a goal as described in the diagram. The ball takes 1.1 s to travel the trajectory shown.



(a) What is the horizontal component of the launch speed of the ball?

(b) At what angle to the horizontal was the ball launched?

(c) Throughout the flight the ball's horizontal component remains constant. What does this indicate about the atmospheric conditions that exist within the gym?

(d) The following weekend the same goalie is required to launch a ball in the same situation as before but this time she is playing on an outside court where there is a horizontal crosswind of 2.0 m s^{-1} blowing. (This is wind that blows at 90˚ to the direction of the throw). Calculate the adjustment to the horizontal component and the vertical component of the throw velocity that would need to be made, in order to successfully score a goal.

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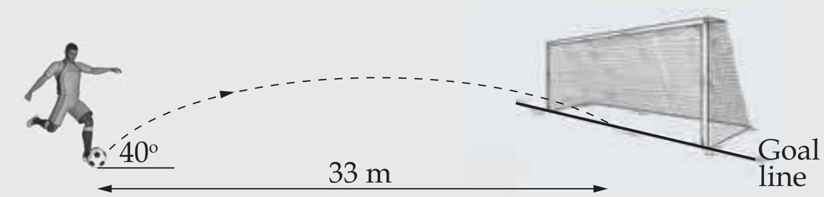
10. A soccer player kicks a ball from ground level at an angle of 40.0° to the horizontal in an attempt to score a goal. The goals are 33.0 m away. The ball lands on the goal line and bounces into the goal. (Assume the ball leaves the foot at ground level).



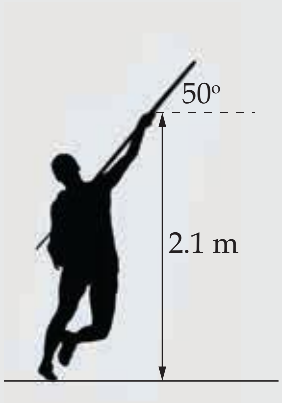
a) With what velocity does the ball leave the player's boot? (ignore air resistance)

b) What is the time of flight of the ball?

c) What is the highest point above the ground that the ball reaches?

d) At what horizontal distance from the kick-off point does the ball reach its maximum height?

11. Deanne, a world class javelin thrower, is shown executing a throw. At the instant of release, the 0.600 kg javelin has a velocity of 27.5 m s^{-1} at an angle of 50.0° to the ground. The javelin is 2.10 m from the ground when released.



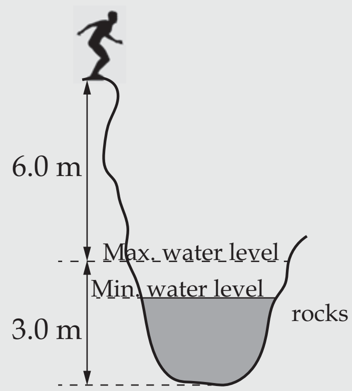
a) How long is it before the javelin reaches its maximum height?

b) What height above ground level does the javelin reach?

c) How far away from Deanne horizontally does the javelin hit the ground?

d) Sketch the trajectory of the javelin on a graph below. Label this (i) on the graph. Sketch on the graph the trajectory of the javelin showing how it would be changed if you allowed for air resistance. Label this (ii) on the graph. Explain why the trajectory changes in this way.

12. A diver wishes to make a difficult high dive from a cliff top into the water 6 metres below (see diagram). He always pushes off with zero vertical velocity. One problem is that the water is only deep enough to dive into when a large wave is directly below him. He must hit the water at the instant when the water level is at its maximum if he is to avoid injury.

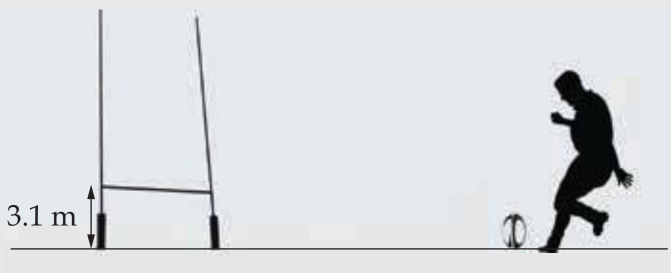


a) How long before the large wave arrives should he jump from the cliff top?

Another problem is that if he pushes off with too little horizontal speed he will land on the base of the cliff that he is standing on but if he pushes off with too much horizontal speed he will land on some rocks on the other side of the water. He needs to travel 2.5 metres in the horizontal direction during his dive to land safely in the water.

(b) What horizontal speed should he push off with?

(c) Calculate his speed and direction when he hits the water.



13. In a rugby football game, a place kicker kicks a football 36 m from the goalposts, and the ball must clear the crossbar which is 3.1 m from the ground, as shown in the diagram.

When kicked, the ball leaves the foot at 20 m s^{-1} at an angle of 53^{o} to the horizontal.

a) How long does it take the ball to travel the distance to the goalposts?

b) How far above or below the crossbar is the ball when it passes through the goal posts?

c) Show on a sketch the path of the football. Include the goalposts in your sketch. You must include the maximum height and overall range of the ball. Explain why you have drawn the path this way, showing any necessary working.

1.8 CIRCULAR MOTION

Consider a car moving around in a circle at constant speed from point A to point B.

At point A the inertia of the car should carry it tangentially in the direction of the arrow but at point B it is moving in a different direction i.e. the speed is the same but the velocity has changed by an amount Δv. Hence the car has accelerated. $\Delta v = v_2 + (-v_1)$ which can be found from a vector subtraction:

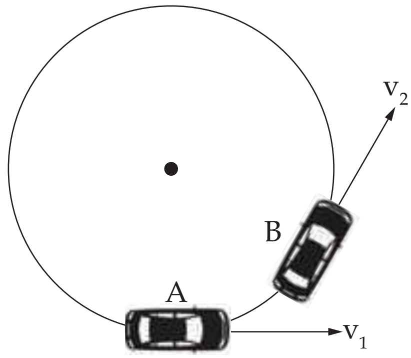


Diagram 1 Diagram 2

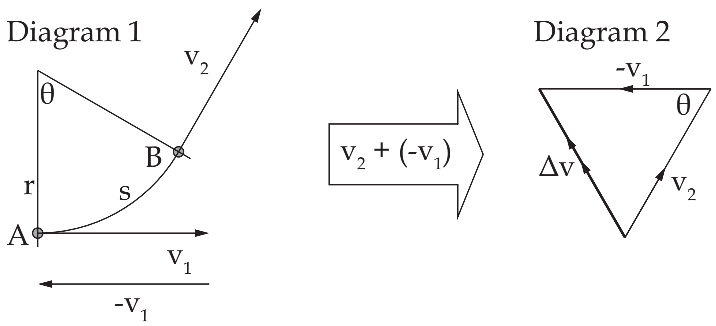


Diagram 1: For small angles $\tan\theta = \frac{s}{r}$

Diagram 2: $\tan\theta = \frac{\Delta v}{-v_1}$ (length of v_1 = length v_2)

$\frac{\Delta v}{-v_1} = \frac{s}{r}$ so $\Delta v = \frac{vs}{r}$

Acceleration $a = \frac{\Delta v}{\Delta t}$ (Δt = time to go from A to B)

$a_c = \frac{vs}{r\Delta t} = \frac{v}{r}\left(\frac{s}{\Delta t}\right)$ but $\frac{s}{\Delta t} = v$

So centripetal acceleration is $a_c = \frac{v^2}{r}$

The direction of a_c is the direction of the change in velocity vector (Δv) which is towards the centre of the circle.

For the acceleration to act towards the centre there must also be a force towards the centre because F = ma. This inwards force is called the centripetal force F_c.

In the case of the car, the friction of the tyres on the road provides this centripetal force. If the car were on ice then it could not be able to move in a circle when the steering wheel was turned because the inertia would cause the car to carry on going in a straight line — in the same direction as v_1.

Other systems where objects move in a circle must also have a force that pushes or attracts the objects towards the centre e.g.

System	Source of Centripetal force
Ball on a string	Tension force of the string
Moon in orbit	Gravitational force
Electron in orbit	Electrostatic force
Circling ice-skater	Reaction force of ice
Racing car turning on a banked track	Normal reaction force of track

Example 15

A car of mass 1.2×10^3 kg, moving at 72.0 km h^{-1} goes round a bend radius of exactly 100 m

a) What frictional force is exerted by the tyres?

b) If the maximum frictional force the tyres can provide is 7.20 kN, what is the minimum road radius that the car can traverse safely at 72.0 km h^{-1} without banking?

Solution 15

a) $72 \text{ km h}^{-1} = \frac{72}{3.6} = 20 \text{ m s}^{-1}$

$a_c = \frac{v^2}{r} = \frac{20^2}{100} = 4.0 \text{ m s}^{-2}$

Centripetal force from tyres $F_c = ma_c = 1200 \times 4 = 4.80$ kN

b) $F_{max} = 7.2 \times 10^3 = 1200a_c$

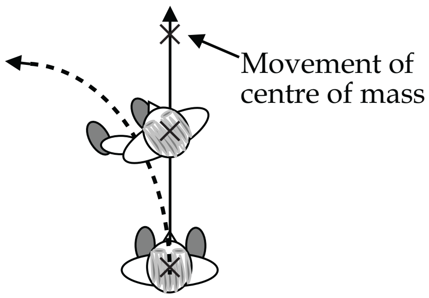
So $a_c = \frac{7200}{1200} = 6.0 \text{ m s}^{-2}$

$6.0 = \frac{v^2}{r} = \frac{20^2}{r}$ So $r = \frac{400}{6} = 66.7$ m

Normal Force

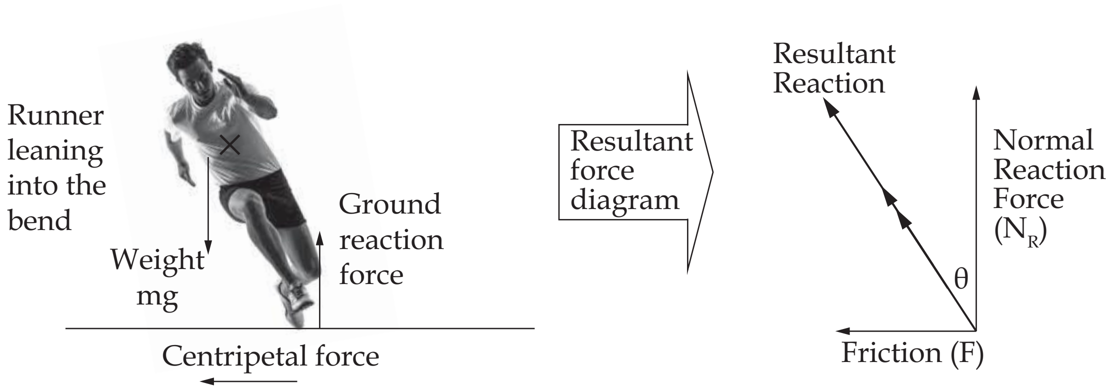
When sprinters run round a bend they have to lean in, otherwise their inertia will cause them to topple outwards. A runner's centre of mass will tend to carry on moving in a straight line because friction is the only force acting on him (at his feet). This causes his feet to move inwards.

Top view



By leaning towards the centre of the circle the runner would tend to topple inwards if it were not for the fact that the feet are also moving inwards. This produces a state of dynamic equilibrium.

Force Diagrams



NB. If the line of action of the resultant reaction force passes through the centre of mass of the runner then a dynamic equilibrium occurs and the runner will not topple. Zero toppling can only prevail if the angle of lean is correct.

To determine the angle: $\tan\theta = \frac{F}{N_R}$

Vertical equilibrium: $N_R = mg$

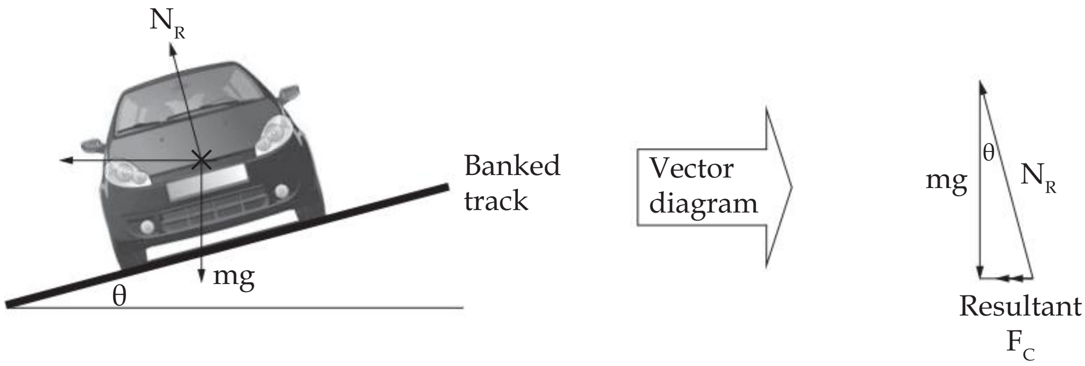
Horizontally: friction = centripetal force $F_c = \frac{mv^2}{r}$ and $\tan\theta = \frac{\frac{mv^2}{r}}{mg}$

Hence $\tan\theta = \frac{v^2}{rg}$

Curved bends in roads are often banked so that a component of the normal force from the road provides the inwards force. This means that the moving vehicles are less dependent on the friction from their tyres.

The equation for banking is the same as for the runner leaning at an angle: Tan $\theta = v^2/rg$

This gives the angle needed to go round the bend without the aid of friction i.e. the car would be able to go round a track made of ice if the banking was at an angle θ.



Example 16

A cyclist and her bike have a combined mass of 1.10×10^2 kg. She wants to negotiate a bend in a flat track where the radius is 50.0 m.

a) If the maximum frictional force from the tyres is 6.00×10^2 N, at what angle must she lean to get round the bend safely?

b) If the track were waterlogged, what angle of banking would allow the cyclist to go round the bend safely at a speed of 12.0 m s^{-1} without relying on the grip from her tyres?

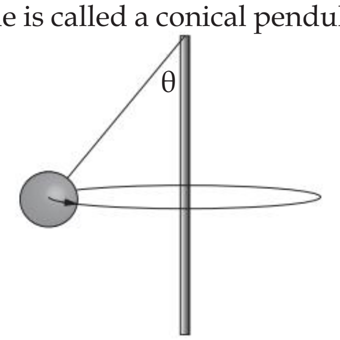
Solution 16

a) $\text{Tan}\theta = \dfrac{600}{(110 \times 9.8)} \Rightarrow \theta = 29.1^\circ$

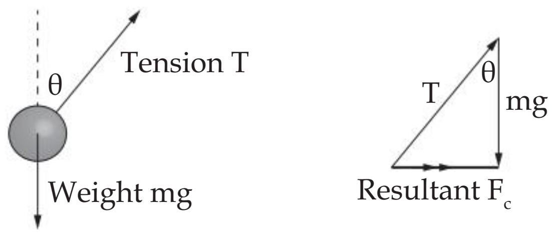
b) $\text{Tan}\theta = \dfrac{v^2}{rg} = \dfrac{12^2}{50 \times 9.8} \Rightarrow \theta = 16.4^\circ$

The Conical Pendulum

A ball hanging on a string and going round in a horizontal circle is called a conical pendulum. One example of where this can be seen is in athletics hammer-throwing events.



The two forces acting on the ball are shown below:



The sum of the 2 forces acting is equal to the centripetal force F_c – found from the vector triangle.

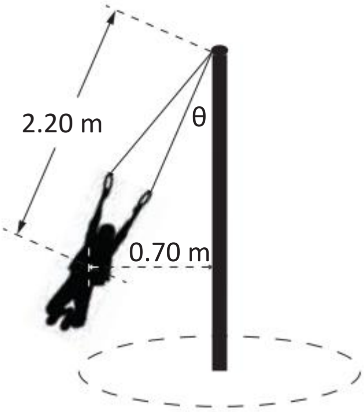
F_c is equal to the horizontal component of the string's tension, which provides the centripetal force needed to accelerate the ball inwards towards the centre of the circle.

$$F_c = T \sin \theta$$

There is no net vertical force so: $mg = T \cos \theta$

Example 17

A boy of mass 50.0 kg swings round a maypole in a circle of radius 0.700 m. Suspended on the two ropes, his centre of mass is 2.20 m from the top of the pole.



Find:

a) The tension in the string

b) The centripetal acceleration

c) The time for the boy to go round once.

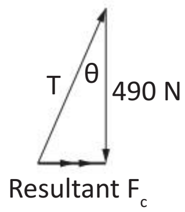
Solution 17

a) From the dimensions triangle, θ at the top is given by:

$\sin\theta = \frac{0.70}{2.20}$

so $\theta = 18.6°$

From the vector triangle:



Vertically: $T\cos 18.6 = 50 \times 9.8$

So T = 517 N (258 N each rope)

b) Centripetal force $F_c = T\sin\theta = 517\sin 18.6 = 165$ N

Centripetal acceleration $a_c = \frac{F_c}{m} = \frac{165}{50} = 3.30$ m s^{-2}

$a_c = \frac{v^2}{r}$ $\quad v^2 = a_c r = 3.30 \times 0.7 = 2.31$

So v = 1.52 m s^{-1}

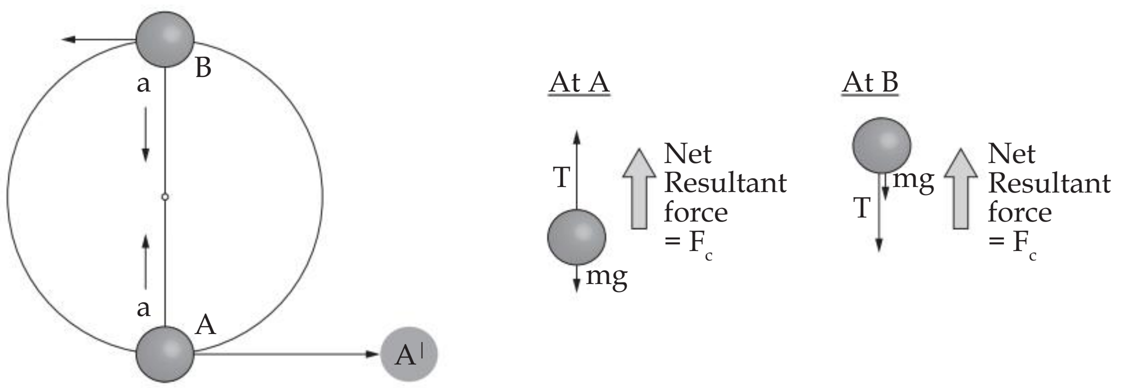
c) $v = \frac{2\pi r}{t}$ so $t = \frac{2\pi r}{v}$

$t = \frac{2\pi \times 0.7}{1.52} = 2.90$s

Vertical Circles

Consider the motion of a ball on a string whirling in a vertical circle and the forces acting in the diagram below:

At point A and point B the net force on the ball is making it accelerate towards the centre of the circle. At A the ball is accelerating upwards and at B its acceleration is downwards.



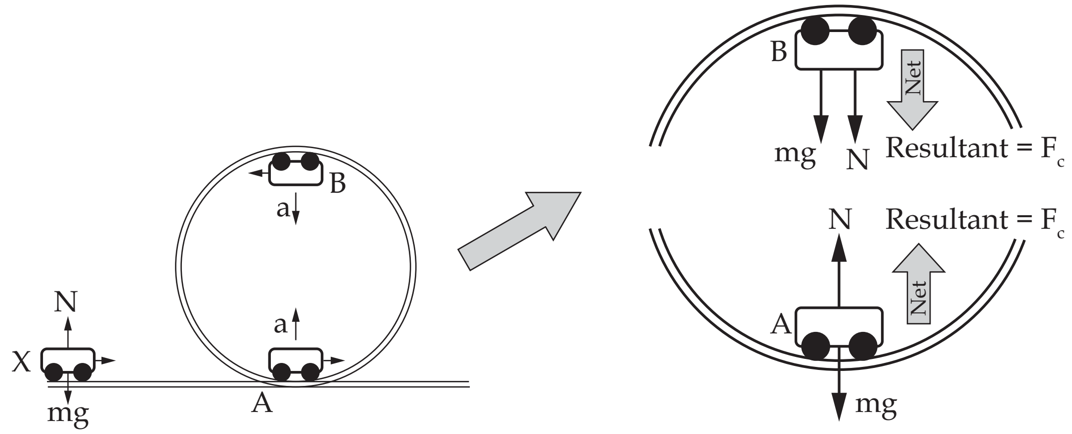
When the ball is at point A, with no net force on it the ball should move horizontally in a straight line to point A| due to its inertia. But the tension in the string pulls the ball back into the circular path i.e. towards the centre.

At A: $F_c = T - mg$ So $T = F_c + mg$ (tension > centripetal force)

At B: $F_c = T + mg$ So $T = F_c - mg$ (tension < centripetal force)

NB If Fc = mg then tension is zero and the string can go slack.

Roller Coaster



At X, before going into the loop, normal force = weight.

At A the truck is accelerating upwards and so:

F_c = N – mg (weight is a negative vector as upwards is taken as +)

$\Rightarrow$ N = F_c + mg

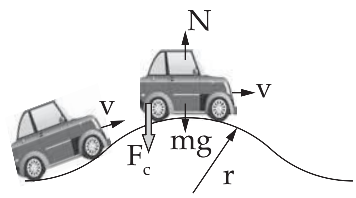
i.e. the normal reaction on passengers is greater than their usual weight and they will feel heavier. *(Note that our perception of weight comes from the force that the ground exerts on our feet i.e. Normal reaction).*

At B: F_c = N + mg and so N = F_c – mg

i.e. the normal reaction on the passenger is less than their usual weight and they will feel lighter but as long as N is greater than zero the seat will be pressing on the passenger and they will stay in their seat. When N = 0 there is no contact with the seat and they will be in freefall.

Speed Humps

When the car goes over the curved speed hump the normal reaction becomes less than the weight of the car i.e. there will be a net force downwards (F_c) causing it to move towards the centre of the circle.



F_c = mg – N

So: N = mg - F_c or N = mg – mv^2/r

For the car to stay on the road N must be greater than zero so mg > mv^2/r or g > v^2/r

e.g. for a 5 m radius speed hump the maximum velocity for the car to stay in contact with the road is given by $v = \sqrt{5 \text{x} 9.8}$
So v = 7.0 m s^{-1}

Example 18

A girl has a yo-yo of mass 1.00×10^2 g which she swings in a vertical circle of radius 80.0 cm.

a) If the yo-yo rotates 2 times per second, calculate the tension in the string at the top and bottom of the swing.

b) If the string has a breaking strain of 18.0 N, what is the maximum rate of rotation possible?

Solution 18

Time period = 0.5 s. $\quad v = \dfrac{2\pi \times 0.8}{0.5} = 10.1 \text{ m s}^{-1}$

a) $F_c = \dfrac{mv^2}{r} = \dfrac{0.1 \times 10.1^2}{0.8} = 12.6$ N.

<u>At top</u> Resultant F_c = T + mg so T = Fc – mg = 12.63 – 0.100 x 9.8 = 11.6 N

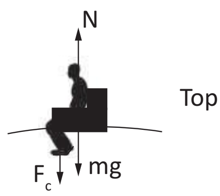
<u>At bottom</u> Resultant F_c = T - mg so T = Fc + mg T = 12.63 + 0.98 = 13.6 N

b) $F_c = \frac{mv^2}{r}$ so $18 = \frac{0.1 \times v^2}{0.8} = 12.6$ N.

So $v_{max} = 12.0$ m s^{-1}

$t = \frac{2\pi \times 0.8}{12} = 0.419$ s

Frequency = 1/time period = 1/0.419
= 2.39 times per second. (2.39 Hz)



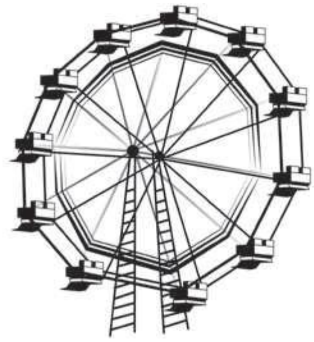
Example 19

The Big Wheel at a showground has a radius of 7.50 m and rotates once every 10.0 seconds.

What would the apparent weight be for a 65.0 kg boy when he is:

a) at the top of the rotation?

b) at the bottom of the rotation?



Solution 19

Boy's normal weight = 65 x 9.8 = 637 N

Velocity of wheel $v = \frac{2\pi \times 7.5}{10} = 4.71$ m s^{-1}

Centripetal acceleration: $a_c = \frac{4.71^2}{7.5} = 2.96$ m s^{-2}

At top: F_c = mg - N So N = 65 × 9.8 - 65 × 2.96 = 445 N (up)

At bottom: Fc = N – mg So N = 2.96 × 65 + 65 × 9.8 = 829 N (up)

Example 20

A car park company wants to construct a speed hump in the car park with a radius of 6.00 m. Calculate the maximum speed that a vehicle can travel over it and still stay in contact with the surface.

Solution 20

$a_c = \frac{v^2}{r}$ For N to be zero $9.8 \le a_c$

r = 6.0 m so $9.8 = \frac{v^2}{6.0}$

$v_{max} = \sqrt{6 \times 9.8}$

$V_{max} = 7.67$ m s^{-1} (27.6 km h^{-1})

Set 4: Circular Motion

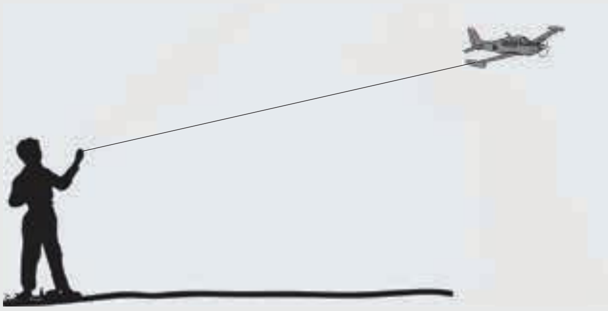
1. In an 800 metre race a girl, running at a constant speed goes into a bend of radius 60 m. The girl enters the bend at point A and exits at point B 31.4 s later.

If the bend has a radius of 60 m, what was the rate of change of the girl's velocity between entering and leaving the bend?

2. An aeroplane flying due north at 720 km h^{-1} suddenly changes its direction to north-west whilst maintaining constant speed. Which vector below correctly indicates the direction of the change in velocity of the aeroplane, if any?

A. No change B. C. D.

3. A boy has a 220 g model aeroplane with an engine which is attached to a fishing line so it will fly round in a horizontal circle. The plane is flown at a speed of 25 m s^{-1} on a line of length 45 m.



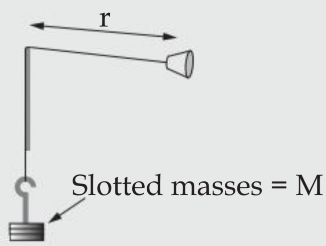
What is the tension in the line?

4. The maximum frictional force on a car's tyres on a dry road is 10 kN which reduces to 1.2 kN on the same road when it is icy. On the dry road this car can drive round a circular road of radius 50 m at a maximum speed of 20 m s^{-1} without sliding. ______________________________
   What is the maximum speed for the same car to travel around the same bend on an icy road?

   ____

   ____

5. In a class physics experiment, a rubber bung is whirled round in a circle on a fishing line. The slotted masses will remain in a stable position at a certain speed of rotation of the bung. If the radius (r) is held constant but the time of rotation (t) is varied, different values on M are required.



   In this case the relationship between t and M (where k is a constant) is given by:

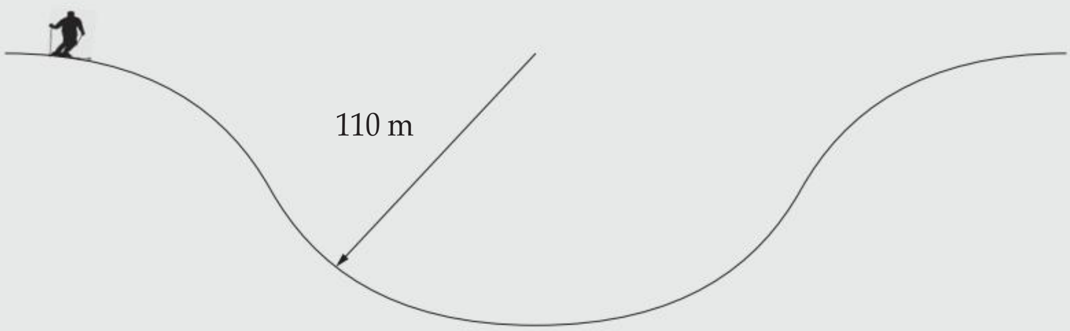
   A. $M^2 = kt$ B. $M = k/t$ C. $M = kt^2$ D. $M = k/t^2$

6. Using the same apparatus as in the previous question (Q. 5), the mass M was held constant whilst the radius of rotation (r) was changed. Different values of t were obtained for various radii.

   Which of the following plots would result in a straight-line graph?

   A. $\sqrt{t}$ versus r B. t versus r C. t^2 versus r D. $1/t^2$ versus r

7. A 70 kg skier goes down a hill on the ski fields into a small circular valley with a radius of 110 m.



   If the skier reaches the bottom of the valley at a speed of 30 m s^{-1}, what will be the reaction force upwards on her feet at the bottom?

   ____

   ____

   ____

   ____

   ____

   ____

8. A council decides to install a speed hump in a car park so cars travelling at speeds over 30 km h^{-1} cannot stay in contact with the road. What would need to be the radius of this speed hump?

9. A woman leaves her case on a roundabout in a playground at a distance of 2.0 m from the axis of rotation. Children spin the roundabout faster and faster. The case has a mass of 400 g and the maximum frictional force that can be exerted by the surface on the bag is 3.0 N.

At what rotational speed will the bag slide off?

10. On the roundabout mentioned in the previous question (Q. 9) a girl of mass 50 kg stands on the platform 2.8 m from the axis of rotation. When the roundabout is rotating once every 6.0 s the girl finds she must lean inwards at an angle to maintain her balance. The correct explanation for this is:

A. Her feet have a centrifugal force acting outwards on them which causes her body to pivot at an angle

B. The force on her upper body is greater than that on her feet and hence her head moves inwards

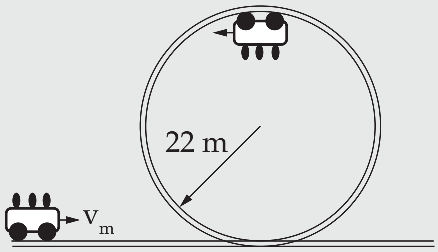
C. The frictional force on her feet and her normal reaction force combine to form a total reaction vector angled inwards

D. The torque exerted on her upper body must be counterbalanced by an equal torque about her centre of mass.

11. In the last question (Q. 10) at what angle must the girl lean (measured to the vertical) in order to maintain her equilibrium?

12. A roller-coaster car goes into a loop at a speed sufficient to prevent the passengers falling out at the top whilst they are upside down.

    If the radius of the loop is 22 m, what must be the minimum velocity (v_m) of the car as it enters the loop?



13. Because the Earth is rotating, the effective value of g is slightly different to that on a non-rotating Earth (call the rotating value g'). The correct explanation of this is:

    A. The Earth's spin tends to throw objects off its surface, which causes a tangential vector and makes g' larger than g

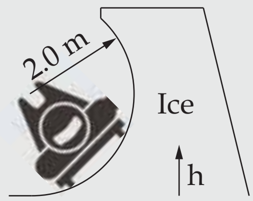
    B. As when standing in an accelerating lift, the gravitational force (g') seems less than g due to the centripetal acceleration of the surface supporting the object

    C. Due to rotation, the radius of the Earth is slightly different at the Equator, which causes the gravitational pull to be slightly more than normal

    D. The inertial mass of a person on a moving surface means their body will be "left behind" compared with their feet. This produces an effect where g' is slightly greater than g.

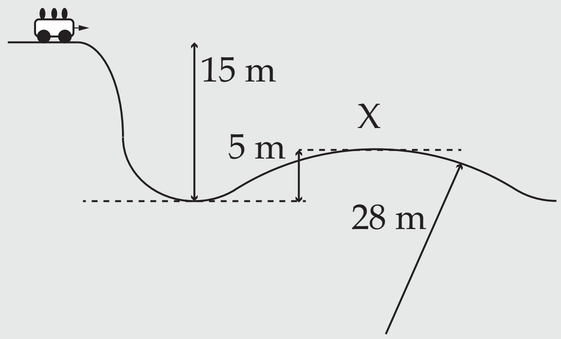
14. A toboggan cannot rely on the frictional force of ice to produce centripetal acceleration round bends. The bend on an Austrian toboggan run has a radius of 40 m. The ice at the bend has a curved surface of radius 2.0 m.

    If the toboggan goes round this bend on the run at a speed of 19.8 m s^{-1}, to what vertical height (h) must the toboggan rise up the ice surface above the ground?



15. On the Indianapolis 500 racetrack one bend has a radius of 350 m and a banking angle of 42.5° to the horizontal. If the maximum frictional force a car of mass 1200 kg can obtain from its tyres is 9 kN, what is the theoretical maximum speed this car can go round the bend without skidding?

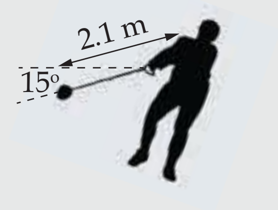
16.



A loaded roller-coaster car just moves towards the beginning of a 15 m downward slope leading to a curved, convex part of the track which has a radius of 28 m.

Point X is at the top of the convex track 5.0 m above the lowest part of the downward slope. When the car is at point X gravity appears to be less inside the car due to its motion. Neglecting frictional losses, what is the value of this apparent gravitational acceleration in the car at this point?

17. A hammer thrower swings his 4.0 kg hammer around in a circle so the wire attached to it makes an angle of 15° to the horizontal.



If the effective distance of the hammer to the thrower's shoulder is 2.1 m then what is the tension in the wire?

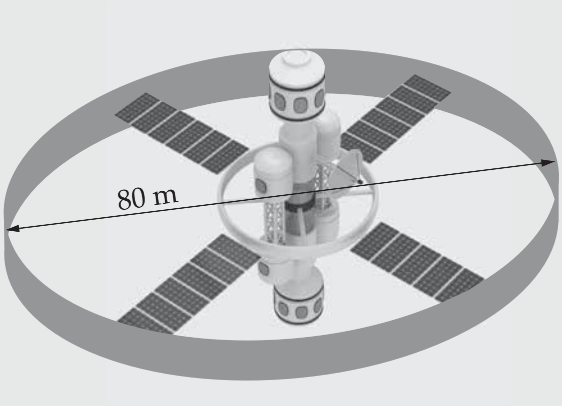
18. In the previous question (Q. 17), what would be the time taken for the hammer thrower to rotate one complete turn with the hammer?

19. A yo-yo has a mass of 120 g and swings at the end of a string 65.0 cm in length. A girl swings the yo-yo up in the air in a vertical circle with a speed of 3.50 m s^{-1}.

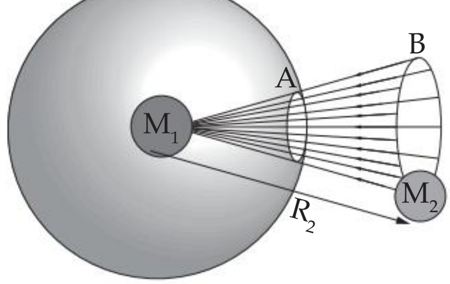
    What would be the tension in the yo-yo string when it is at the bottom of its circular swing?

20. A space station rotates in space so as to simulate gravity in the region of its outer ring which has a diameter of 80.0 m.

    At what rate (revolutions per minute) must the space station rotate to give a value for artificial gravity in the outer ring of 8.90 m s^{-2}?



1.9 GRAVITATION



Sir Isaac Newton was the first scientist to talk about the idea that any mass will attract any other mass with a force called Gravity. He imagined that gravity force exerted itself through field lines, which were indicated by the way one object moved towards the other object e.g. an apple falling to the earth in a straight line. All of these field lines converge towards the centre of the Earth and get spaced out more as they get further from the Earth.

Newton linked the spacing of the lines to the strength of the field so at point A on the diagram, the field strength is larger than at point B because they are concentrated into a smaller area. If B is twice as far away from the Earth's centre as point A then area B will be 4 x area A from similar triangles (radius of B is 2× radius A and area = πR_2^2). Hence the Inverse Square Law was formulated stating that gravitational force reduces in the ratio of the square of the distance from the centre of mass $F_g \alpha \frac{1}{R^2}$

The direction of any field line indicates the direction in which another object will move. In the case of electric or magnetic force, the lines can go inwards (attractive) or outwards from the object (repulsive). Gravitational fields can only be attractive.

The gravitational force will also depend on how large the mass of the Earth is (M_1) and how large the mass of the object is (M_2). The proportional constant throughout the Universe is given the symbol 'G'.

Newton's Universal gravitational formula is: $F_g = \frac{GM_1M_2}{R^2}$

The value of G has been found to be 6.67×10^{-11} N m^2 kg^{-2} and is the same throughout the Universe.

From this formula, Newton could "weigh the Earth" using the following logic:

At the Earth's surface, the radius is 6.38×10^6 m from the centre, a 1.0 kg mass has a weight of 9.8 newtons.

Using this equation: $9.8 = \frac{6.67 \times 10^{-11} M_e \times 1}{(6.38 \times 10^6)^2}$

Earth's mass, $M_e = 5.96 \times 10^{24}$ kg. (A more accurate value is 5.97×10^{24} kg.)

Gravitational force is extremely small: The attraction between two 1.0 kg masses one metre apart would be: $\frac{6.67 \times 10^{-11} \times 1 \times 1}{(1.0)^2} = 6.67 \times 10^{-11}$ N

A force of this size would not be measurable.

Example 21

A meteorite is moving at a height of 5.0×10^3 km above the earth's surface. What would the value of the gravitational field strength (g') be at this height?

Solution 21

Gravitational force on the meteorite is $M_2 g = \frac{GM_1M_2}{R^2}$ so $g = \frac{GM_1}{R^2}$

Distance from Earth's centre R = $6.38 \times 10^6 + 5.00 \times 10^6 = 1.138 \times 10^7$ m

So $g' = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(1.138 \times 10^7)^2} = 3.07$

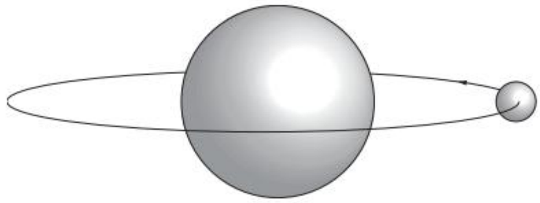
Gravitational field strength at this height = 3.07 N kg^{-1} (or m s^{-2})

Alternative (Proportion) Method

The force of gravity varies inversely with the square of the distance from the Earth, so if the gravitational field strength on the earth's surface is 9.80 N kg^{-1} then it will be proportionally less 5000 km away by a factor of $\left(\frac{6.38 \times 10^6}{1.138 \times 10^7}\right)^2$. Since $\frac{g^I}{g} = \left(\frac{R_E}{R^I}\right)^2$

So, at this height g will be $9.8 \times \left(\frac{6.38 \times 10^6}{1.138 \times 10^7}\right)^2 = 3.08\ N\ kg^{-1}$

1.10 SATELLITES



The force providing centripetal acceleration and pulling a satellite into a circular orbit equals the gravitational attraction, so we can equate the two forces: $\frac{M_2 v^2}{R} = \frac{GM_1 M_2}{R^2}$

This gives: $v^2 = \frac{GM_1}{R}$ from which we can obtain the velocity of a satellite.

Example 22

How fast must a satellite be moving if it is to stay orbiting 5.00 × 10^3 km above the Earth's surface?

Solution 22

$v^2 = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.38 \times 10^6 + 5000 \times 10^3}$

$v = 5.92 \times 10^3\ m\ s^{-1}$

Potential Energy

The potential energy of an object is given by the formula $E_p = mgR$, where R is the distance from the centre of the Earth.

Replacing g with the previous formula of $g = \frac{GM_1}{R^2}$ shows that $E_p = \frac{mGM_1 R}{R^2}$ (Force × distance) which means that E_p is proportional to 1/R if we take the centre of the Earth as zero potential. Scientists normally take a position at an infinite distance from the Earth as a zero position of E_p because it is the point where no attractive force would be felt. This means that all gravitational potential energies are counted as negative values.

So $E_p = -\frac{GmM_1}{R}$

Kinetic energy of a mass above the Earth would be given by $E_p = \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = \frac{GmM_1}{2R}$ so the total of E_p and E_k of a satellite is

$E_{Tot} = \frac{GmM_1}{2R} - \frac{GmM_1}{R} = -\frac{GmM_1}{2R}$

Because the total energy is negative, this means that the orbit of a satellite is stable.

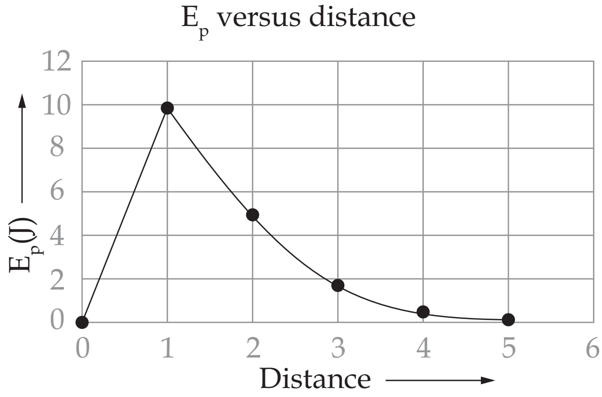
A golf ball down a hole has negative E_p and is also stable where it is because positive energy would be needed to lift it out of the hole.

Equipotential lines show the positions where the E_p of a satellite would be exactly the same. These would be circles (actually spheres) around the Earth with a fixed radius.

Anywhere on the Earth's surface (R = 6.38 × 10^6 m) one kilogram mass would have an equal potential energy of 9.8 J. The point above the Earth's surface where the E_p would be 4.9 J would be at double that radius (½ g) and at 3 times the Earth's radius, the E_p would be 1/3 of the E_p on the Earth's surface due to the inverse proportionality.

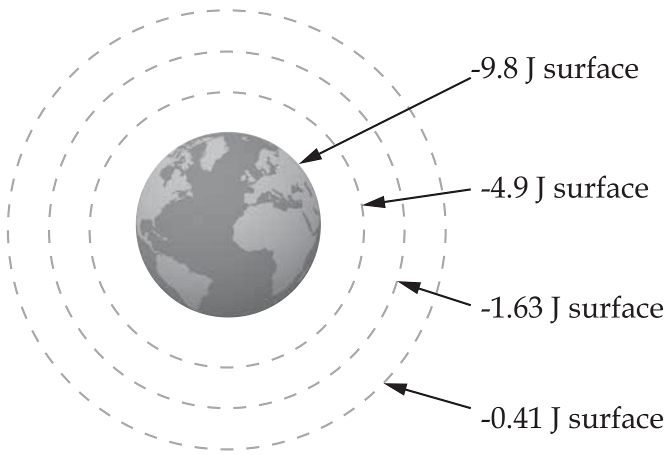
Below is a table of how the E_p changes with distance from the Earth's centre, measured in Earth radii.

Radius (No. of Earth's radii)	E_p (joule)
1	-9.80
2	-4.90
3	-1.63
4	-0.41
5	-0.08



Equipotential surfaces

Anywhere along the surface of the dotted lines the value of E_p is the same. To move from one point on the equipotential surface to another on the same surface would require zero energy input.



1.11 KEPLER'S LAW

Johannes Kepler derived an empirical law from knowing the radii of the orbits of planets (R) and their times to orbit the Sun (T). He found the complex relationship: $T^2 \alpha R^3$ for all planets, i.e. the period of revolution squared divided by the orbital radius cubed was a constant.

Why this empirical relationship should be true was a complete mystery until Newton proved it by using his gravitational force equations as shown below:

$v^2 = \frac{GM}{R}$. But $v = \frac{2\pi R}{T}$ so $\frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^2}{T^2}$

Rearranging gives: $GMT^2 = 4\pi^2 R^3$ which conforms to Kepler's Law and shows T^2 is directly proportional to R^3 or T^2/R^3 = constant.

Showing that the motion of the planets conformed exactly to a mathematical equation, like a machine, stunned nations and led to the term "The mechanical Universe"

1.12 GEOSTATIONARY SATELLITES

A satellite must orbit around the centre of mass of the Earth as it is attracted to this point. This rotation can occur in a polar orbit or an equatorial orbit. A polar orbiting satellite travels north to south and for photographic purposes can scan the whole Earth, whereas an equatorial satellite with a period of 24 hours, can remain above the same point on the Earth. This Geostationary orbit is a useful one for radio or TV transmissions from one country to another. Satellites are always launched as close to the equator as possible so that the tangential velocity of the ground at that point can be utilised (about 450 m s^{-1} at the equator). Launching to the east gives a 'slingshot' effect to the initial rocket and saves fuel as, before launching, the rocket is already moving at 450 m s^{-1}.

Example 23

Calculate the height above the Earth at which a satellite must remain to be geostationary.

Solution 23

Time period of a geostationary satellite = (24 × 3600) seconds = 8.64×10^4 s

Using Kepler's Law equation: $GMT^2 = 4\pi^2R^3$

$6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (8.64 \times 10^4)^2 = 4\pi^2R^3$ So R = 4.223×10^7 m from the centre of the Earth,

The height = $4.223 \times 10^7 - 6.38 \times 10^6 = 3.585 \times 10^7$ m above the Earth's surface (35,850 km).

1.13 USES OF SATELLITES

Since the first earth-orbiting satellite, Sputnik, was launched by USSR in 1957, satellites have had an enormous effect on our society. Sputnik could only receive and send weak analogue radio signals and was very primitive. Nowadays geostationary satellites can be used to provide continuous communication coverage for areas fairly close to the equator - but for countries such as Russia low orbiting satellites are used, as they are less expensive to launch and give a stronger signal. A single satellite cannot be used there as they only give coverage for about 10 minutes per day, so several are needed to give continuous reception of signals.

The first geostationary communications satellite was launched by USA in 1964 to transmit TV pictures of the Tokyo Olympics.

The first 3-axis stabilised geostationary satellite was launched in 1974, which used a gyroscope to maintain constant altitude and orientation for communication.

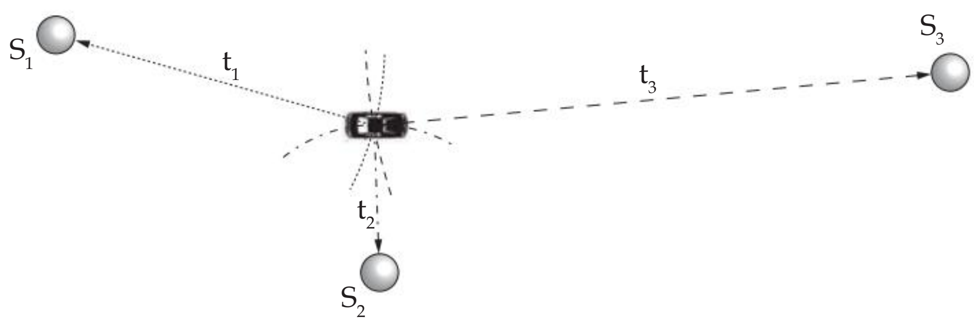
Today satellites are used for such things as: Astronomy (e.g. The Hubble Space Telescope); mobile telephone communication; navigation, tracking and GPS; weapons guidance; international TV, internet and radio transmission; atmospheric studies and weather prediction.

Some other uses are: Search and rescue missions; national security from terrorists; agricultural studies; disaster management; geology, hydrology and mining information; marine surveillance; mapping.

Satellites can receive information (TV mobile phone and Internet signals) transmitted on the gigahertz wavebands and retransmit them to a different location. In this way telephone calls and satellite TV transmissions can be sent and received using a parabolic microwave aerial.

GPS systems calculate the position of a transmitter by finding the time for a signal from an Earth position to travel to four satellites orbiting above the Earth, knowing the speed of light accurately. Each satellite must have a very accurate clock on board to make timing calculations accurate. A Master satellite with an on-board atomic clock sends signals to all the other satellites to correct the accuracy of their clocks by a few nanoseconds.

Triangulation between 3 satellites allows the calculation of the x, y and z co-ordinates of a transceiver (e.g GPS system in a car). The 4th satellite is used to calculate the height above sea level.



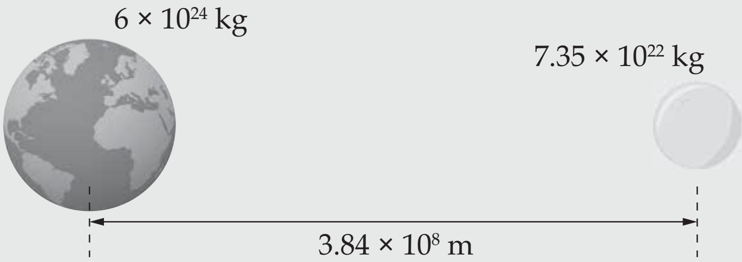
For example: If the time for the signal to be transmitted from satellite S1 and return is 360 μs then the car's distance from this satellite is $360 \times 10^{-6} \times 2.998 \times 10^8$ m = 108 km.

Set 5: Gravitation

1. A satellite of mass 50 kg is in orbit around the Earth 1270 km above its surface.

   What force does the satellite exert upon the Earth?

2. The Earth and the Moon both rotate about their common centre of mass.



   Using the data given, calculate the distance of the centre of mass of the Earth-Moon system from the Earth's surface?
   (NB The Earth and the Moon both rotate about this point).

3. A communications satellite is positioned 400 km above the Earth's surface. Calculate its orbital speed around the Earth?

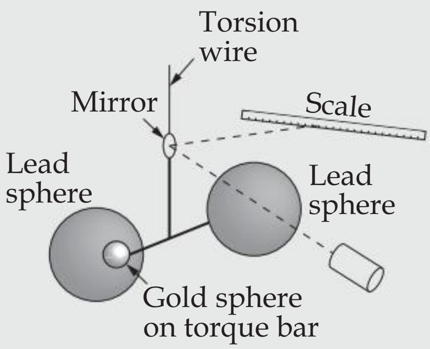
4. The record for high jump on the Earth is 2.45 m currently. If the same record-holder jumped on the surface of the moon, what height is he likely to achieve?

5. Neptune's mass is 17 times that of the Earth and it has a planetary radius of 2.27×10^4 km.

   Calculate a value for the gravitational field (g) on Neptune's surface.

6. A planet orbits around a neutron star at a radius of 5.2×10^9 km in a time period of 5.5×10^9 s. Calculate the mass of the neutron star.

7. A scientist called Boys devised a method for finding the universal gravitational constant (G) by measuring the attractive force between two gold spheres and two lead spheres. Two gold spheres of mass 500 g were suspended on the end of a torque bar attached to a mirror.



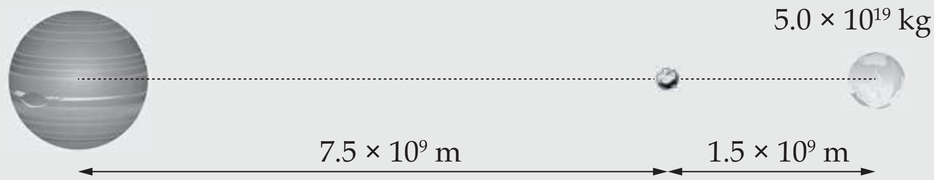
   When two 10 kg lead spheres were brought near to the gold spheres the force of attraction between the masses was determined from the angular deflection of the torque bar. A beam of light reflecting from the mirror onto a glass scale gave a value of the turning torque.

   In one set-up, lead spheres of radius 6.00 cm and gold spheres of radius 1.80 cm were used. Each lead sphere was positioned so its edge was exactly 1.0 mm away from the edge of a gold sphere. A force of attraction of 5.00×10^{-8} N was recorded between each lead and gold sphere. From these results, calculate a value for 'G' (in N m^2 kg^{-2}).

8. Referring to the previous question (Q. 7), the distance between the centres of masses of the gold spheres on the torque bar was 7.55 cm. The value for the torsional constant of the torsion wire was 9.2 μN m per degree (ie a torque of 9.2×10^{-6} Nm will turn the bar through an angle of 1^o).

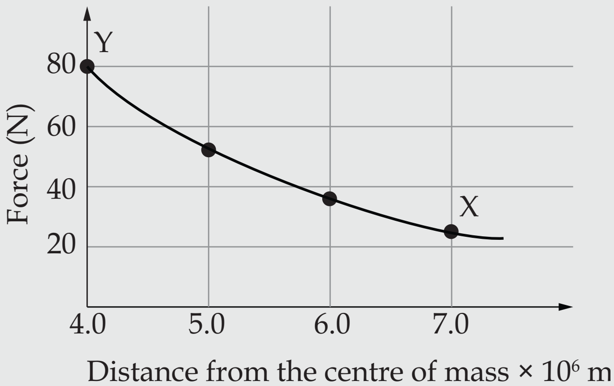
What angle would the mirror have turned through after the lead spheres had been placed in position 1 mm from the gold spheres?

9. A meteor of mass 500 kg is 7.5×10^9 m from the centre of a planet (X), 1.5×10^9 m from the centre of its moon (Y) and positioned along the planetary axis between the two planets. The planet has a mass of 5.0×10^{22} kg and the moon has a mass of 5.0×10^{19} kg.



What would be the resultant gravitational force on the meteor from the 2 bodies?

10. The graph shows the gravitational force acting on an 1100 kg asteroid as it approaches a planet. The force is measured in newtons and the distance is measured from the centre of mass of the planet as it moves from point X that is 7.0×10^6 m away to point Y, 4.0×10^6 m away. As the asteroid moves from X to Y the gravitational force on it increases, causing it to gain velocity.



Make an estimate of energy change from the graph and hence a value for the change in velocity of the asteroid.

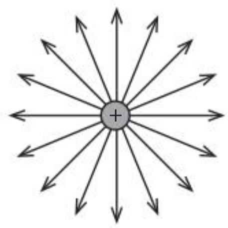
Chapter 2

Electromagnetism

2.1 ELECTRIC CHARGES

When any insulator, such as plastic, is rubbed with a cloth some electrons are transferred from the cloth to the rod. The energy to separate these electrons from their atoms comes from the work done in rubbing against friction. Uncharged objects have an equal number of + and – charges. So if extra electrons are added to an object it becomes negatively charged. A positive charge on an object actually arises, not through addition of + charge, but because electrons are removed because positive charges in a solid cannot move.

As with gravitational fields, a point charge has a radial field pattern and the strength of the field decreases with the distance from the charge.

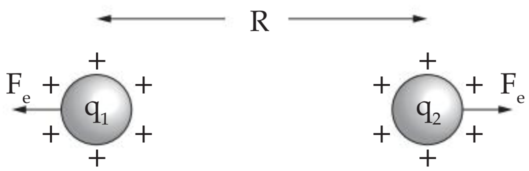


This is another example of the Inverse Square Law.

Unlike charges (+ and –) attract and like charges (– and – OR + and +) repel.

The attractive or repulsive force between two charges is given by Coulomb's Law, which makes use of Newton's idea of force fields conforming to an inverse square law:

Coulomb's Law for charges: $F_e = \dfrac{q_1 q_2}{4\pi\epsilon R^2}$



F_e = the electric force
q_1 and q_2 are the two charges
R is the distance between the charges
e is called the permittivity of the material between the charges (= 8.85 x 10^{-12} for air)

A more simplified equation is $F_e = k\dfrac{q_1 q_2}{R^2}$ where k = 9.0 x 10^9 Nm^2 kg^{-2} (to 2 sf).

Example 1

Find the force of attraction between an electron and a proton in a hydrogen atom where they are 53 pm apart (1 picometre = 1 x 10^{-12} m)

Solution 1

Charge on a proton and an electron = 1.6 x 10^{-19} C.

R = 53 x 10^{-12}

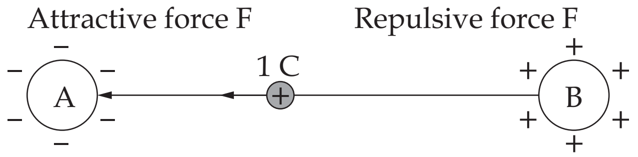
$F_E = k\dfrac{q_1 q_2}{R^2} = 9 \times 10^9 \times \dfrac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(53 \times 10^{-12})^2}$

F_E = 8.20 x 10^{-8} N.

Charge tends to gather at areas where the radius of a conductor is smallest (sharpest) and hence lightning will always strike the sharpest object around, such as a tree branch or a church steeple. In a storm, it is the large electric field between a cloud and the Earth that causes the air to become ionised and makes it able to conduct electricity.

2.2 ELECTRIC FIELDS

A negative charge will have a higher potential energy than a positive charge because work can be done when the electrons are repelled from an area of negative to positive charge. Heat will be generated as the charge moves through a medium. However, due to our "conventional" definition of current as the movement of positive charge, we define a conventional view of high potential as an area with excess + charge (imagining + charges moving). The potential difference between two points A and B is defined as the work done in transferring one coulomb of + charge from A to B.



Sphere B is at high potential as it has a + charge. To move 1 coulomb of + charge from A to B requires work to be done against the repulsive force F, so the potential difference (p.d.) between A and B will be the work done, or energy used, in moving this 1 C charge from A to B.

Potential difference is measured in volts and is obviously equal to the energy per coulomb for this transfer. This leads to the definition of voltage as:

$$V = \frac{W}{q}$$

W = energy used, or work done

q = charge

It follows that Work Done, W = Vq

Example 2

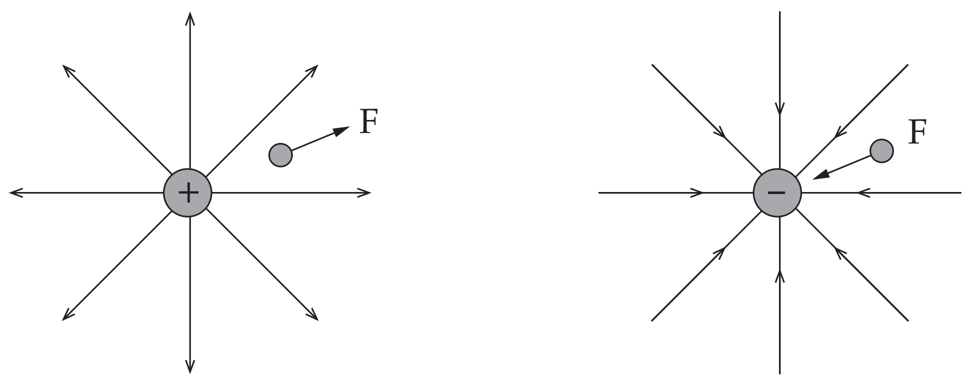
What energy is required to transfer 35 mC of charge between two charged plates having a potential difference of 15 volts?

Solution 2

$W = Vq$ so $W = 15 \times 35 \times 10^{-3} = 0.525$ J

A field is a region where a non-contact force is felt and its direction is shown by the direction of the force on a positive charge. Hence the field direction is from B to A in the diagram above. Obviously there must be an electric field existing between the two charges because of the force of repulsion or attraction experienced by the +1 C charge.

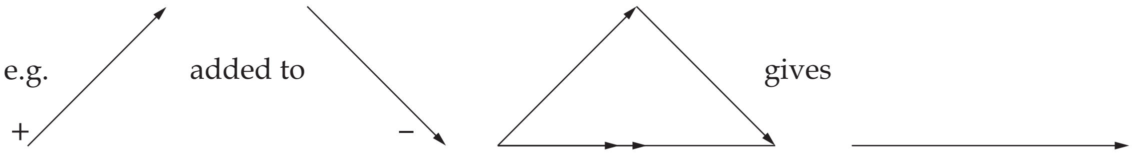
Considering single, charged spheres the field lines must radiate directly out from the sphere, showing the direction of motion of a free positive charge if placed at that point.



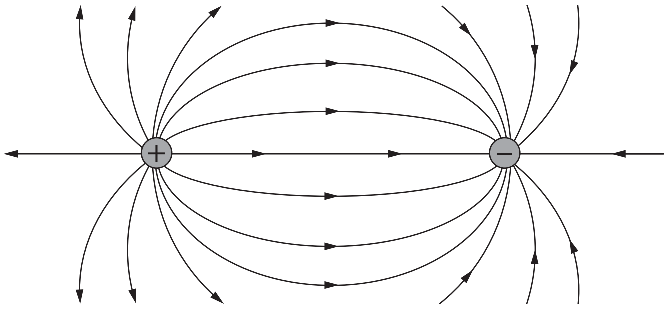
NB The closer the lines of force the stronger the electric field.

2.3 INTERACTION OF FIELDS

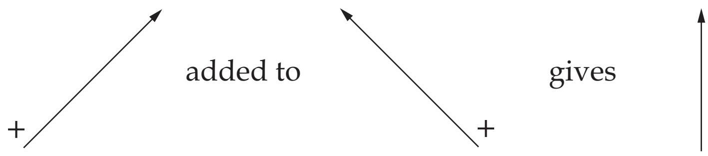
Field lines are really vectors that show the net force on a + charge placed at that point, so it two charges come close to each other the field line vectors will add to produce a net vector direction.

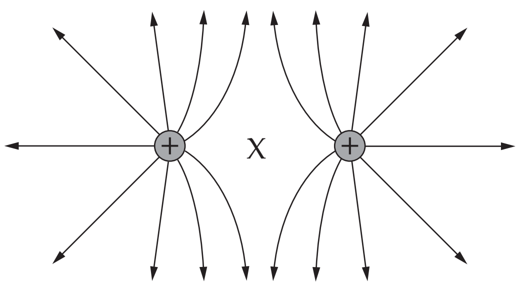


The diagram below shows how the field lines add up when a + charge is placed close to a – charge.



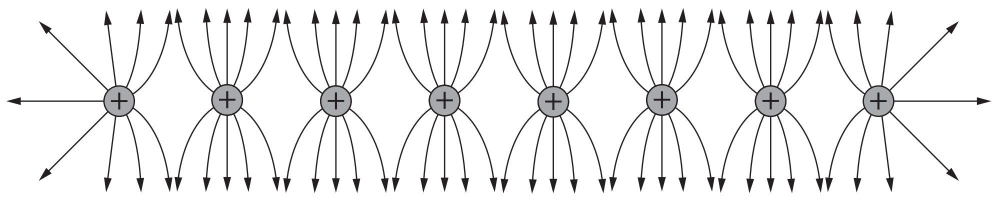
The resultant field shape of two positive charges interacting is shown below.



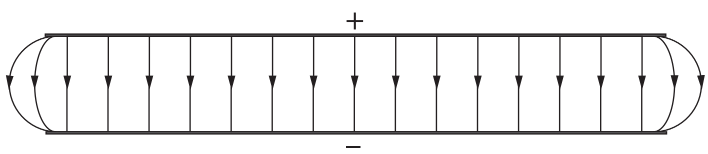


(At point X the field strength is zero)

Lots of + charges in a line will give a resultant field that is vertical (except at the ends)



Parallel plates with opposite charges give a field inbetween the plates which is uniform (except at the ends)



The formula for the electric field strength between the plates is given by:

$$E = \frac{V}{d}$$

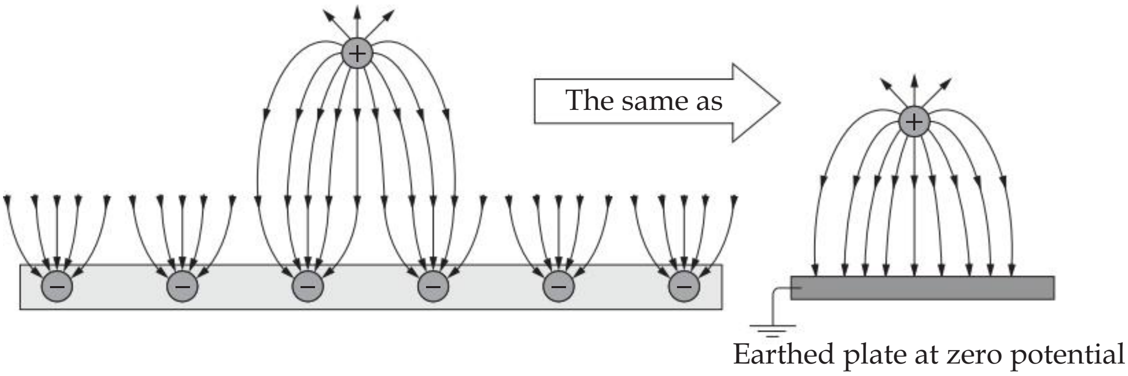
V = p.d. between the plates and d = distance between the plates.

As with other field strength relationships, the force on a charge in an electric field E is give by:

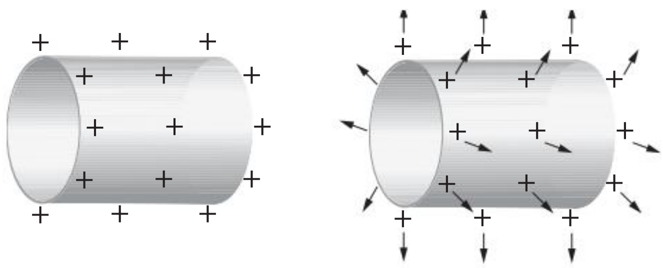
Electric force $F_e = Eq$

(Compare with $F_w = mg$, where g is the gravitational field strength)

A single charge brought near to a plate of negative charge will give the resultant field shown below:



2.4 FIELD INSIDE A CONDUCTOR



If a charge is placed on a hollow conductor, all the charges repel so as to get as far away as possible from each other and hence they must all reside on the outside of the cylinder; there can be no charge on the inside surface.

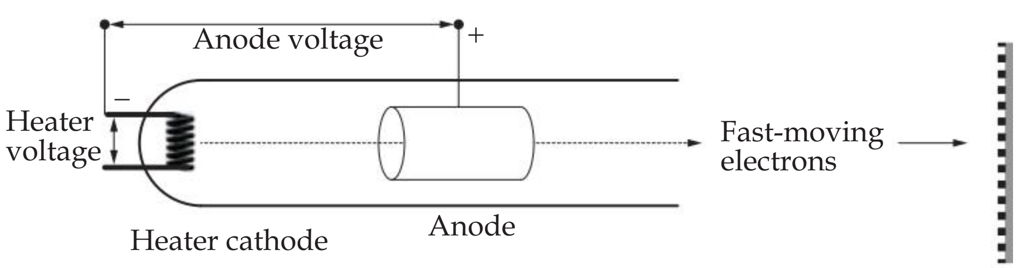
Since all field lines must come out from charges, it follows that there can be no field inside a hollow conductor and that the electric field E will spread outwards from the outer surface.

The fact that there was no field inside a conductor was first demonstrated by Michael Faraday who had a woman sit inside a wire cage that was charged to such a high voltage that sparks jumped from the outside wire netting. In spite of this she felt no shock or any effects of the large voltage from inside the cage because no field existed where she sat.

This knowledge is used in electronics to “shield” components from stray electromagnetic fields and waves. Electric guitars will pick up radio signals unless their components are shielded inside a “Faraday cage” i.e. an earthed metal box surrounding the circuits.

2.5 CHARGES IN AN ELECTRIC FIELD

An electron can be accelerated by allowing it to be attracted by a large positive potential. The electron is emitted by a hot cathode (negative) and moves through a vacuum towards an anode at a high positive potential.



Television tube

This principle is used in the older, glass-screen televisions and Cathode Ray Tubes. The electrons gain kinetic energy from acceleration in the electric field and by the time they reach the anode they are moving close to the speed of light. However, the anode is hollow, so the electrons can pass straight through it to collide with a phosphor on the TV screen. Here the electron's KE is converted into a spot of light which goes to make up the TV picture. The electrons can be deflected by current-carrying coils placed either side of the tube which can position the emerging electrons to any point on the screen to produce an image.

Example 3

An electron is accelerated by an electric field situated between two charged metal plates 2.50 cm apart with a potential difference of 3.50 kV between them.

a) Calculate the electric field strength

b) Find the acceleration of the electron

Solution 3

a) $E = V/d = 3500/0.025 = 1.40 \times 10^5$ V m^{-1}

b) $F = Eq = 1.4 \times 10^5 \times 1.6 \times 10^{-19} = 2.24 \times 10^{-14}$ N

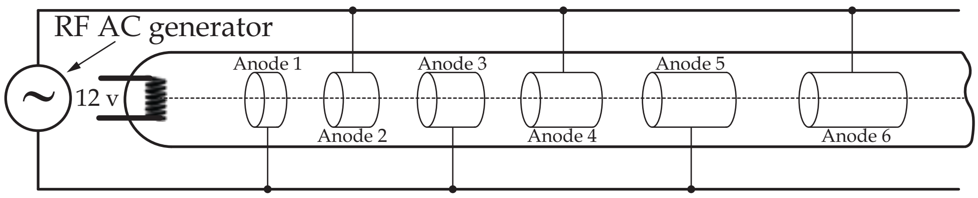
$a = F/m = 2.24 \times 10^{-14} / 9.11 \times 10^{-31} = 2.46 \times 10^{16}$ m s^{-2}

(Electron mass = 9.11×10^{-31} kg)

2.6 PARTICLE ACCELERATORS

The first kind of ion accelerator used a Van der Graaf generator to produce the high voltage necessary to accelerate particles like the electrons up to very high velocities. Voltages up to 10^6 volts from the Van der Graaf generator meant that particles gained 1 million electron volts of kinetic energy (1 MeV). (1eV = 1.6×10^{-19} J)

Using a linear accelerator (e.g. at Stanford University), larger energies than this became available (> 25 MeV) by progressive acceleration between adjacent anodes.



The Linear Accelerator

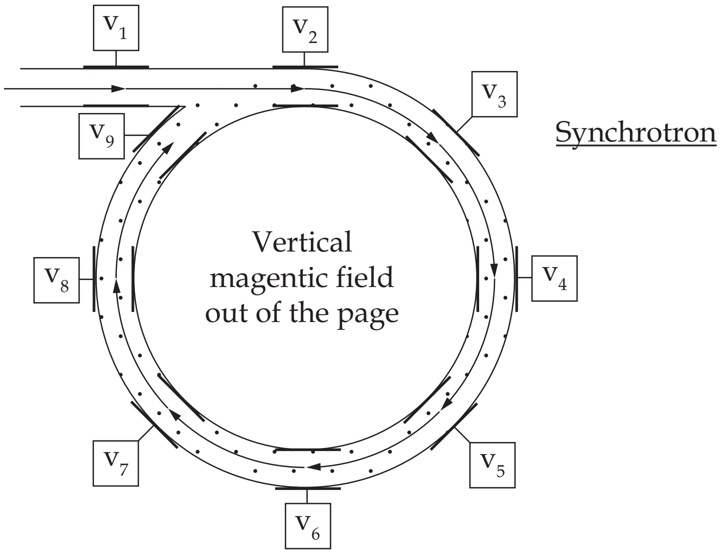
A very high frequency AC generator attached across the anodes alternates the voltage between the top and bottom rails so, when anode 1 (A1) is +1000 V, anode 2 (A2) is – 1000 V. The electron is then attracted to anode 1 but as it reaches it the voltages reverse so that A1 is now -1000 V and A2 is +1000V so the electron is repelled from A1 and attracted to A2 through a potential difference of 2000 V. A3 then becomes +1000 V and so this alternation between +1000 V and -1000 V at radio frequencies (108 Hz) will keep accelerating the electron up to the equivalent voltage of 25 million volts.

Cyclotrons

The first circular accelerator, called the Cyclotron, was built in the 1950s to create very fast moving particles that could collide with atoms and produce sub-atomic particles. The cyclotron works by accelerating charged particles between successive chambers at higher and higher potentials with magnetic fields at right angles which deflect the particles into a circular path. This has the advantage over the linear accelerator in that after one accelerating cycle, the charged particle can then go round many more times, increasing the velocity even further.

The charged particles are accelerated between anodes as in the Linac using RF frequency voltage applied alternately. However, the time for the particles to reach the next anode will be shorter each time because the velocity is increasing so the time the next anode in the sequence is turned on must get shorter and shorter i.e. frequency is gradually increased and synchronised.

With the Synchrotron, the cycle of acceleration could occur continuously many times to give the particles energies in the GeV range (10^9 eV)



Cyclotrons of the 1960s and 70s used superconducting magnets which were coils of wire cooled to a very low temperature so their resistance was virtually zero. This meant that extremely high currents in the coils could produce massive fields and result in more energetic particles without much heat generation.

The latest Large Hadron Collider can produce particles with energies of 7 TeV (1 Teraelectron-volt is 10^{12} eV). In a cyclotron called the Tevatron, during approximately 20 seconds, the magnetic field rises from 0.66 Tesla to 3.54 Tesla as the beam energy increases from 150 GeV to 800 GeV.

In this time, the beam makes 1,000,000 turns around the 6.28 km circumference circle.

With such high energies scientists are hoping to simulate the same energy conditions that existed when the Universe was first created in the Big Bang and produce more basic sub-atomic particles that haven't been discovered yet.

Set 6: Electric Charges

1. Explain why:

   a) It is dangerous for a person to shelter under a tree in stormy weather where there is lightning.

   b) Tall buildings often have spiked, earthed rods on the topmost point.

2. Some metalloids, like selenium, can hold a charge, but lose it if light is shone on to the metalloid. Explain how this phenomenon is used in a photocopier or laser printer.

3. A glass rod has 2×10^4 electrons on it in a static position.

   a) What charge is on the rod?

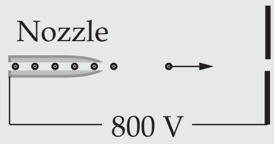
   A piece of charged floating dust comes within 3 mm of the rod.

   b) If the charge on the dust is $+1.6 \times 10^{-5}$ C, what is the force of attraction between the dust and the glass rod?

4. Two metal plates A and B are at potentials of -1.80 V and + 6.70V respectively. A small sphere with a charge of + 0.045 C is transferred from plate A to plate B.

   How much energy is used to make this transfer?

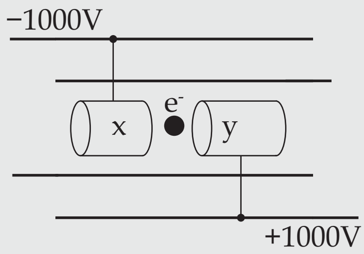
5. In an ink-jet printer the drops of ink acquire a charge of -5.50 x 10^{-6} C and become attracted to an anode plate which has a potential of +800 V above the nozzle potential. Mass of an oil drop is 2.00 × 10^{-12} kg



a) Calculate the work done in bringing the ink drop to the anode plate.

b) Calculate the velocity of the ink drop as it reaches the hole in the anode plate.

6. A linear accelerator maintains a potential difference of 2000 V between two adjacent anodes as an electron is accelerated.

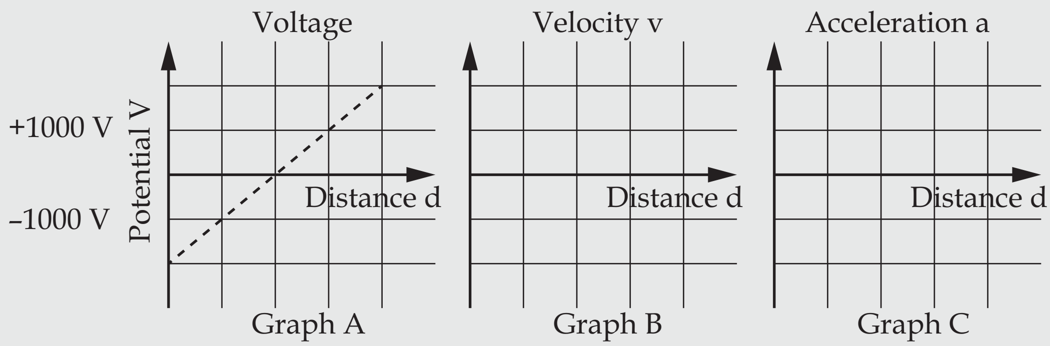


a) Explain how a machine that only generates a p.d. of 2000 V is able to give an electron a final energy of 800 MeV.

b) How many anodes would be needed to achieve this value of energy?

7. The graph A below shows how the potential varies as the electron passes from point x on one electrode to point y on the next electrode.

Draw on the graph scales provided, a graph B showing how the velocity of the particle would vary with distance, and graph C showing how the acceleration of the particle varies with distance.



8. An electron moving at 1.5×10^{7} m s^{-1} is injected horizontally at a mid-position between two horizontal metal plates 10.0 cm long which have a potential difference between them of 1000 V. The plates are separated by a distance of 5.00 cm.

   a) Calculate the electric field strength between the plates.

   b) What force is exerted on the electron by the field?

   c) Calculate the time it takes for the electron to pass between the length of the plates.

9. From the calculations in the previous question (Q8) find the electron's acceleration and from the velocity-time equations determine whether the electron is able to emerge at the other end of the plates.

10. In a Millikan experiment an oil drop with a radius of 0.015 mm is charged negatively and then injected between two horizontal plates which are spaced 3.50 cm apart. The voltage on the plates is adjusted until the oil drop "hovers".

   a) If the mass of the oil is 850 kg for 1 m^3 (its density), calculate the mass of the oil drop. ($m_{oil} = \left(\frac{4\pi r^3}{3}\right) \times \text{density}$)

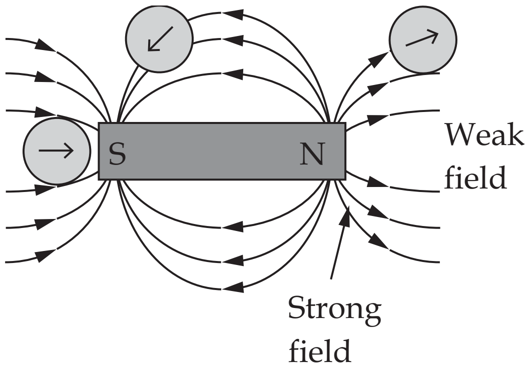
   b) If the electric force upwards exactly balances the weight of the oil drop, what is the value of the electric force?

   c) How many electrons were there on the oil drop?

2.7 MAGNETIC FIELDS

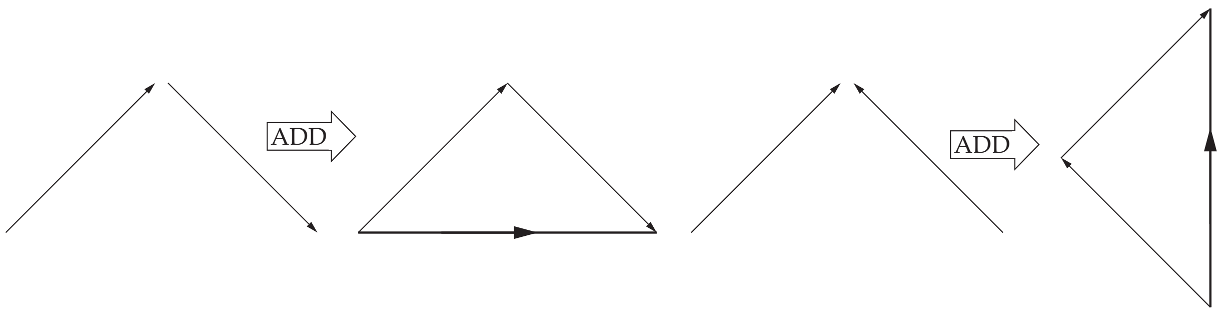
A magnetic field (or flux) is a region where a force will be felt on a magnet. Newton drew imaginary lines to illustrate magnetic fields with arrows showing the direction a free North pole would move (actually, you cannot get a free North pole, without a South pole). Newton also drew magnetic field lines closer together to illustrate a region where the field was stronger. A strong field is a region where there would be a greater force on another magnet and is a vector quantity.

The field lines around a single bar magnet can be visualised either by sprinkling iron filings near it or by plotting the direction in which a small compass points when placed around the magnet. The arrow of the compass is a small north magnetic pole – repelled by another north and attracted to a south.The number of lines coming from a pole is called the total **flux**, measured in webers The concentration of lines (i.e. lines per square metre) is called the **flux density**, measured in webers/m^2 or tesla.

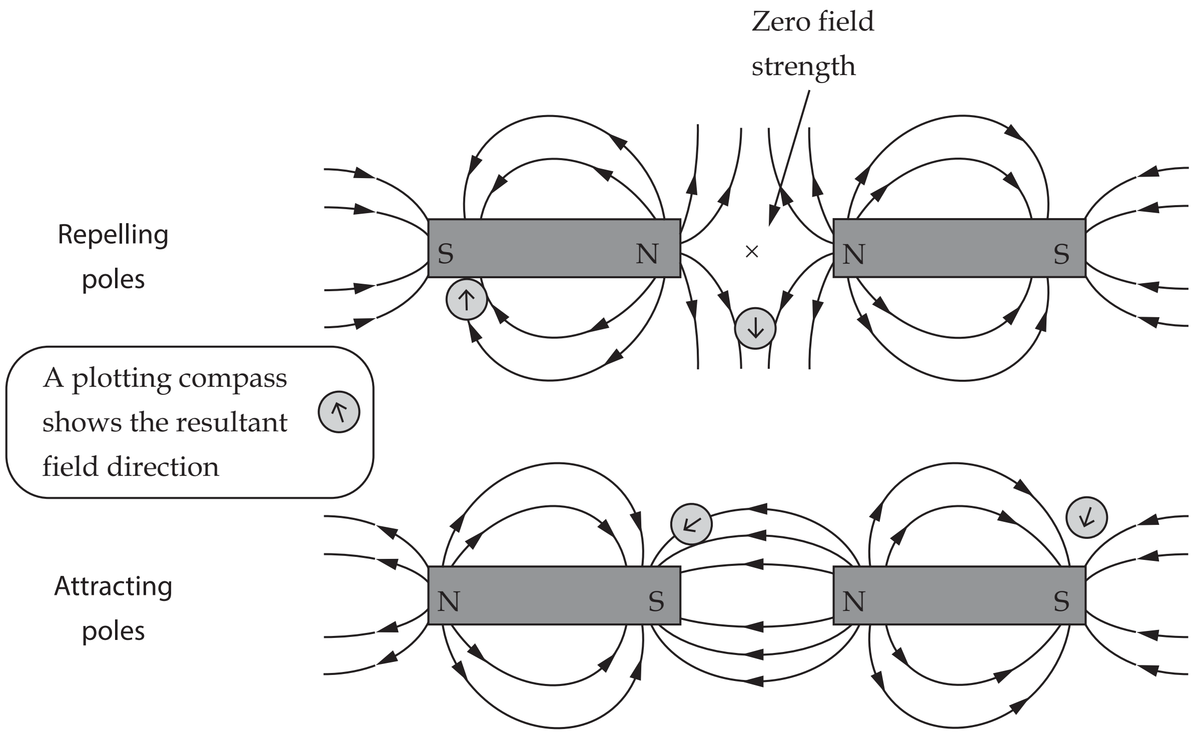


Interaction of fields

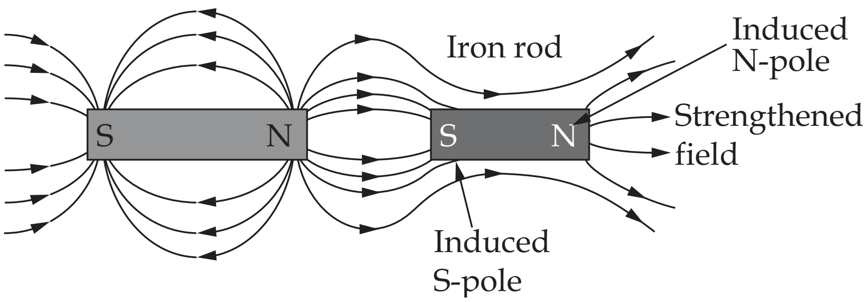
Magnetic field lines can interact and add, just like forces do. To find the resultant flux density and direction we must construct a vector diagram where vectors are added tip-to-tail. The resultant will run from start to finish. For example:



Applying this principle to two magnets placed together, we can map the resultant field pattern which is distorted due to interaction between the fields.



Ferromagnetic materials are ones that become strongly magnetic when placed in a magnetic field. Field lines become diverted towards the ferromagnetic substance thereby concentrating the field lines and making the flux density larger.

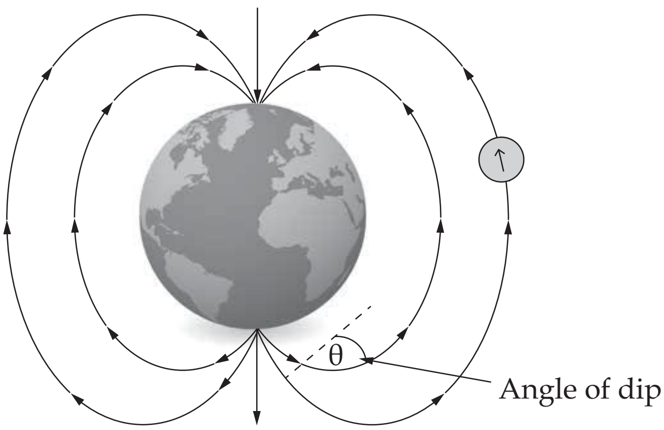


Ferromagnetic materials are: Iron, steel, nickel and cobalt. All other metals are very weakly magnetic.

2.8 THE EARTH'S MAGNETIC FIELD

The Earth has its own magnetic field produced by its north and south poles. It is thought that this magnetic field is produced as a result of its core being made of iron, floating inside a molten iron outer core. The rotation of the core acts like a giant dynamo and the resultant electric currents produce a magnetic effect.

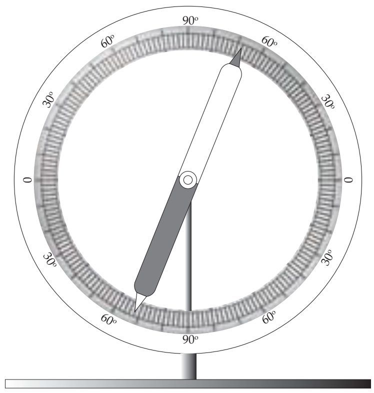
The Earth's magnetic field pattern is the same as that of a bar magnet. A compass needle arrow will point towards the north magnetic pole of the Earth, showing it is an opposite pole to the compass north pole.



The compass needle arrow end should be called the "North-seeking pole" because it points towards the north – a bit confusing, but remember that magnetic lines of force go into the north pole of the Earth.

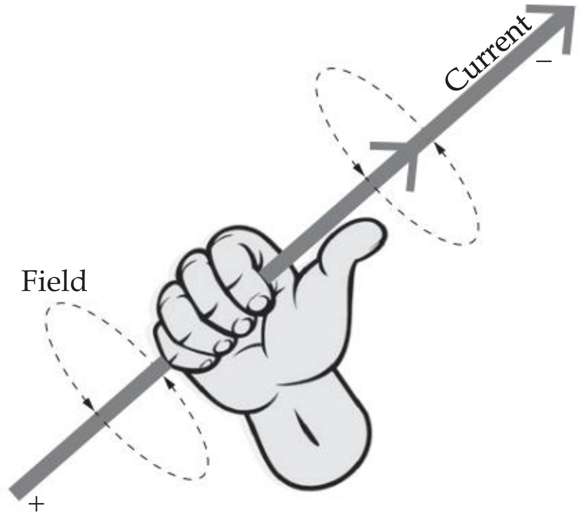
The angle that the Earth's field lines make with the horizontal at any point on the Earth is called the angle of dip (θ) which can be measured with a dip circle. In Perth the angle of dip is about 68^o.

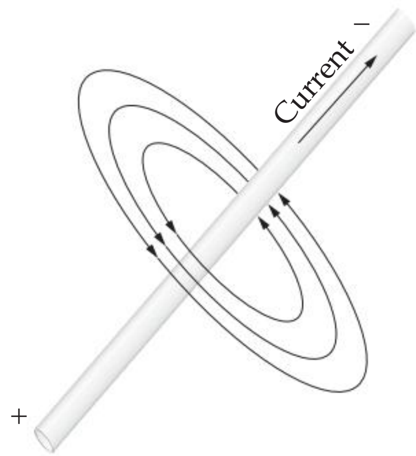
A magnetic dip needle



2.9 ELECTROMAGNETISM

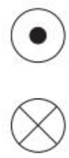
Oersted's experiments showed that whenever a charge moves or a current flows through a conductor, an associated magnetic field is produced. The field circles around the wire and reduces with distance away from the wire. If we point our thumb in the direction of conventional current flow (+ to -) then the fingers show the direction of the field (the way a N-pole would point)



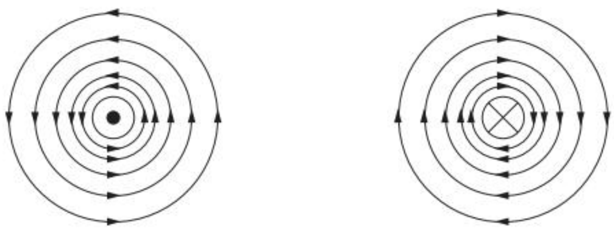


Looking from the top of a wire, if the conventional current is coming out of the page (towards us), it is represented by a circle with a dot in it.

A wire carrying current into the page is represented by a circle with a cross in it.



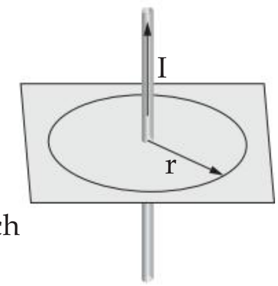
Hence the field lines around these wires will have these patterns:



Notice: field lines are closer near the wire.

Field around a wire formula

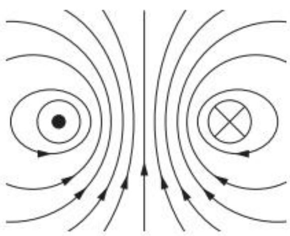
The greater the distance from the wire the smaller the field strength.



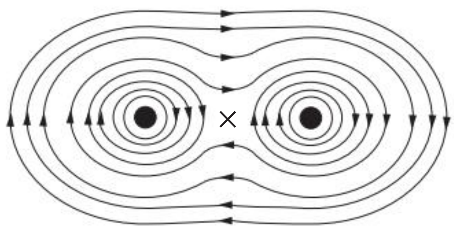
The formula for the field strength at a distance r from the wire is:

$$B = \frac{\mu_o I}{2\pi r}$$

Where I is the current and n_o is called the permeability of space which shows the magnetic effect of space and has a value of 1.26×10^{-6} T m A^{-1}.



One current in and one out of the page — fields add up



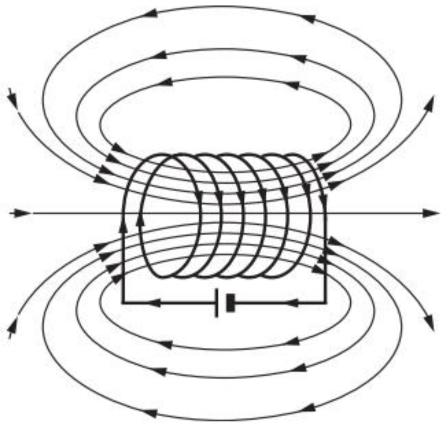
Both currents out of the page — fields subtract at the centre

The fields from two wires placed close to each other can interact to produce resultant, distorted field patterns, as shown above.

Coils

A loop of wire will have the same field pattern as in diagram below as the current will come up and out on one side and down and in on the other.

A coil (or solenoid) consists of lots of loops side by side. The fields from each loop add to give an overall flux pattern that is similar to that of a bar magnet.

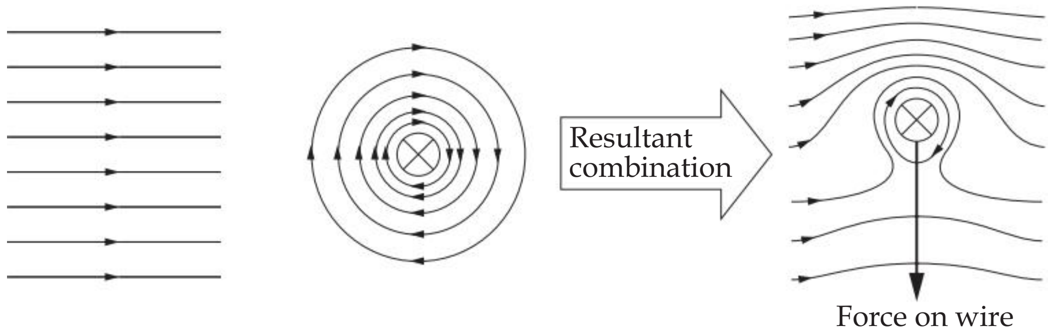


2.10 THE MOTOR EFFECT

Just as the magnetic fields from two permanent magnets can interact to produce motion (attraction/repulsion) so the magnetic field from a wire can interact with the field from a magnet to produce motion. The movement of a current-carrying wire in a magnetic field is due to the motor effect – the force used to turn an electric motor coil around.

The uniform (parallel) field from a magnet combines with the circular field around a current- carrying wire to produce a resultant (distorted) field

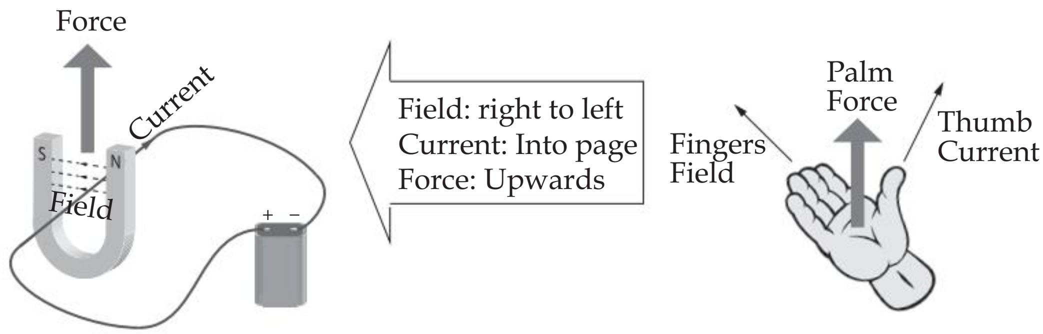
The uniform (parallel) field from a magnet combines with the circular field around a current-carrying wire to produce a resultant (distorted) field



The resultant field above the wire is now much stronger and the resultant field below the wire is weaker (or zero in one position). This causes a resultant force to act on the wire from the strong to weak field region.

A quick way to work out the direction of the force acting when a current flows through a wire situated in a magnetic field is to use the right-hand slap or right-hand palm rule.

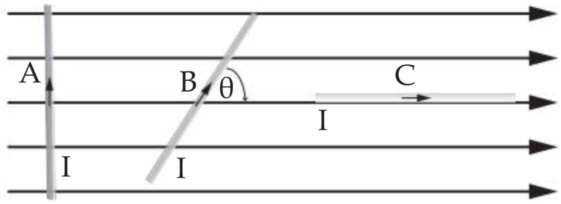
Right hand slap rule:
- Point fingers along field (N to S)
- Thumb indicates conventional current direction (+ to -)
- Palm (slap) shows resulting force



Force Formula

The size of the magnetic force F_M on a current carrying wire will depend on 4 factors:

- The strength of the magnetic field B
- The size of the current I
- The length of the wire in the field L
- The angle of the wire to the field θ



The formula for the magnetic force on a current-carrying wire in a field is therefore:

$F_M = BIL \sin\theta$

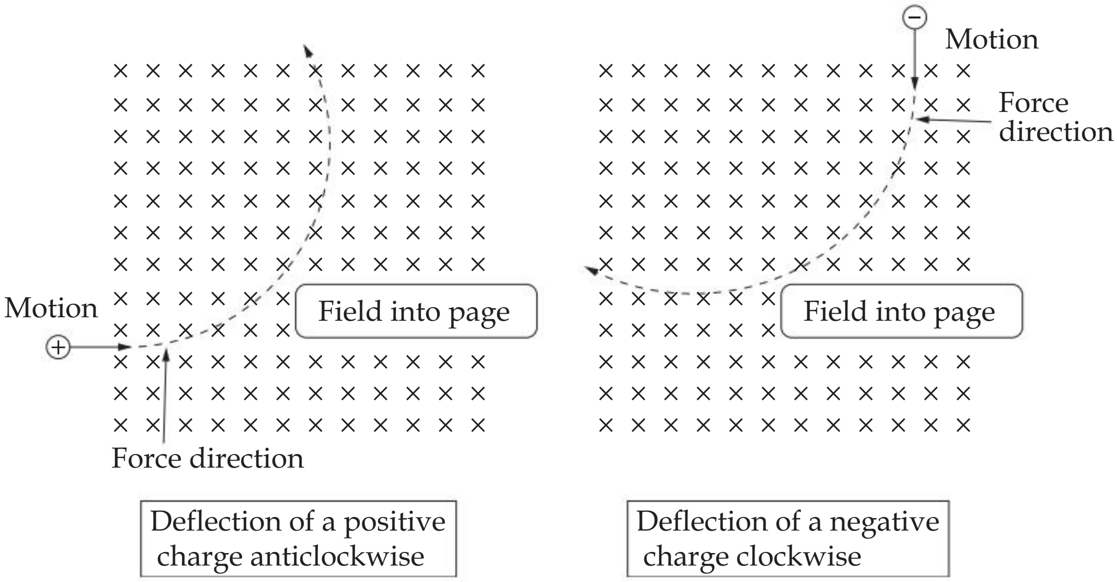
So in diagram above: Suppose the length of wire in the field is 8.0 cm, the field strength is 0.12 tesla, the current is 1.5 amps and the angle of the wire to the field is 30^o.

$F = BIL \sin\theta = 0.12 \times 1.5 \times 0.080 \times \sin30 = 7.20 \times 10^{-3}$ N (down into the page)

Note that when the angle between the wire and the field is zero (position C) then the force is zero as sinθ is zero.

2.11 THE FORCE ON A CHARGED PARTICLE

A charged particle that is moving is classified as a current. A positive charge moving in a magnetic field will have a force acting on it which is in the same direction as that on a wire carrying a conventional current. A negative charge moving will have a force acting opposite to a conventional current.



The effective current produced by a moving charge is equivalent to the charge passing a point per second (depending on charge and velocity) so the size of the force on a charged particle will depend on Flux density B, Charge q, and Velocity v.

$F_m = Bqv$

Also remember that for a particle moving in a circle, centripetal force $F_c = \frac{mv^2}{r}$

Equating these two forces: $\frac{mv^2}{r} = Bqv$ so $r = \frac{mv}{Bq}$

Example 4

The radius of curvature of a sub-atomic particle in a cloud chamber where the magnetic field strength is 235 mT is 5.20 cm. The particle is projected at a velocity of 5.86×10^5 m s^{-1} and carries two negative charges. Calculate the mass of the particle.

Solution 4

$$r = \frac{mv}{Bq} \text{ so } m = \frac{rBq}{v}$$

$$m = \frac{5.20 \times 10^{-2} \times 0.235 \times 2 \times 1.6 \times 10^{-19}}{5.86 \times 10^{5}}$$

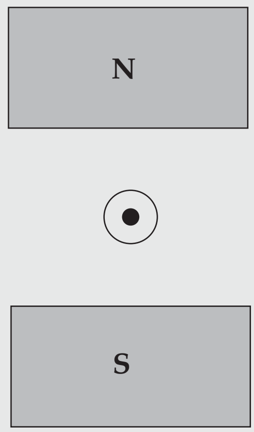
$m = 6.67 \times 10^{-27}$ kg.

1 atomic mass unit = 1.66×10^{-27} kg so this particle has a mass of 4 atomic mass units - possibly a Helium ion.

Set 7: Electromagnetism

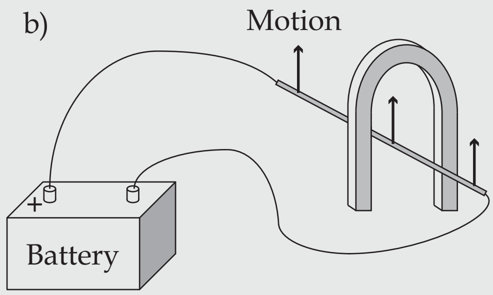
1. State the direction of the missing variable from each diagram below (force, field or conventional current direction).(You may need to use the words: " To the left, right, up, down, into or out of the page").

a)



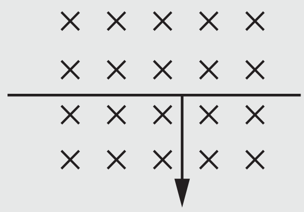
Motion is ______________________

b)



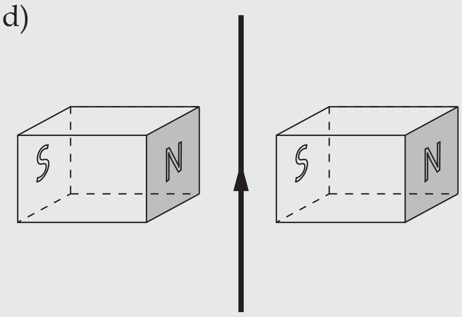
Field is ______________________

c)



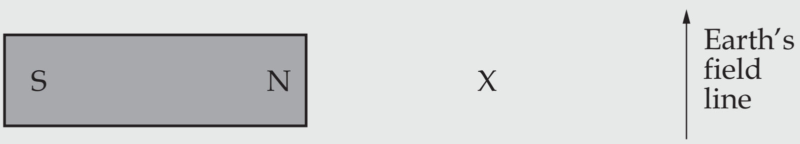
Current is ______________________

d)



Motion is ______________________

2.



(This diagram is the view of the apparatus looking down from above)

A letter X is drawn on a sheet of paper lying on a desk where the Earth's horizontal component of magnetic flux density is 2.1×10^{-5} T. A bar magnet is then placed to the west of point X so that it contributes a flux density of 8.4×10^{-5} T at point X. If a compass needle were placed at point X at what angle to the north would it reach equilibrium?

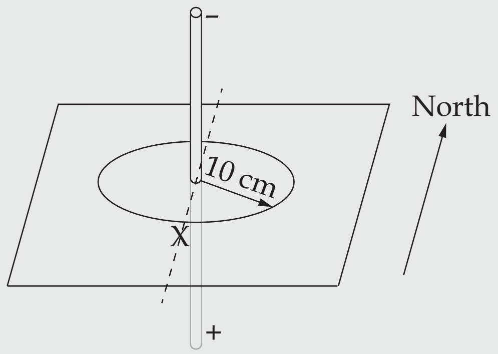
____

____

____

____

3.

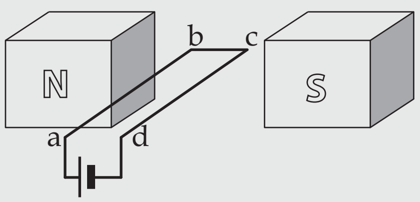


The magnetic flux density (B) at a distance r from a current-carrying wire is given by the formula:

I = current in wire $B = \frac{\mu_o I}{2\pi r}$ ($\mu_o = 1.26 \times 10^{-6}$ units)

If the current flowing is 5.0 amps, then what must be the value for B due to the wire at a point X due south of the wire, 10 cm from the wire?

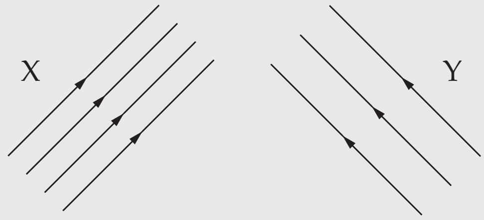
4. In the previous question (Q. 3) a compass needle is suspended on a cotton thread at point X so the Earth's field and the field due to the wire interact on the needle. The resultant field would cause the needle to set at an angle θ to the north. Calculate the value of θ (Assume the horizontal component due to the Earth's field is 2.10×10^{-5} T).



5.

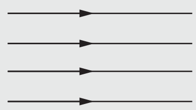
A rectangular coil abcd is placed between two magnetic poles as shown. In what direction is the resultant force on wire bc?

6.

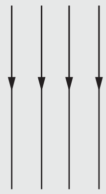


The magnetic fields from two different sources (X and Y) are shown above. If these two fields were overlapped, which diagram below would indicate the resultant field?

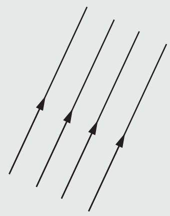
A.



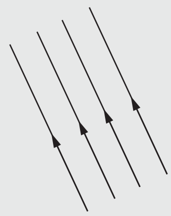
B.



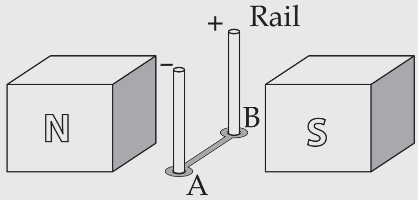
C.



D.



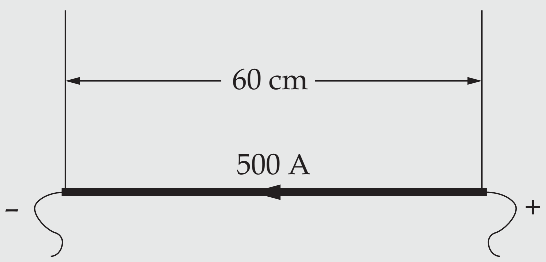
7.



Wire AB is connected by sliding ring contacts to vertical rails attached to a voltage supply. The rails and wire sit inside a uniform magnetic field of flux density 0.02 tesla. Wire AB has a diameter of 2.0 mm and is made of copper (density = 8.89×10^3 kg m^{-3})

What is the minimum current through AB that will allow it to become weightless and rise up the conductor rails? (Hint: what force is needed to lift the wire?)

8.



A copper wire is suspended from a wooden beam on light strings in an east-west orientation. It is in a region where the Earth's horizontal component of flux density is 2.0×10^{-5} T. If the copper wire has a mass of 2.5 grams per metre and has a 500 ampere current running through it from east to west, what would be the tension in each string?

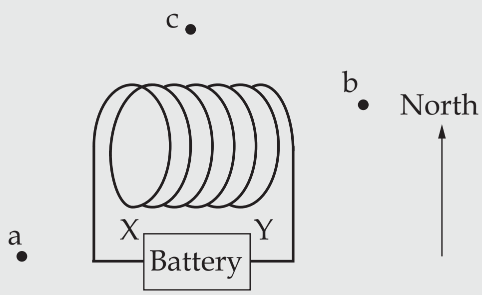
9. The Earth's magnetic field strength in a certain European country is 2.2×10^{-5} T with an angle of dip of 79^{o}. What is the magnitude and direction of the field strength

   a) measured along the ground,

   b) measured vertically.

   c) What other factors are likely to affect the value of field strength, apart from latitude?

10.



The coil shown is connected to the battery terminals X and Y. A compass needle that normally points north has the following orientations when placed in different positions:

Position	Orientation
a	Points north east
b	Points north east
c	Points west

Which of the following conclusions is not supported by the observations?

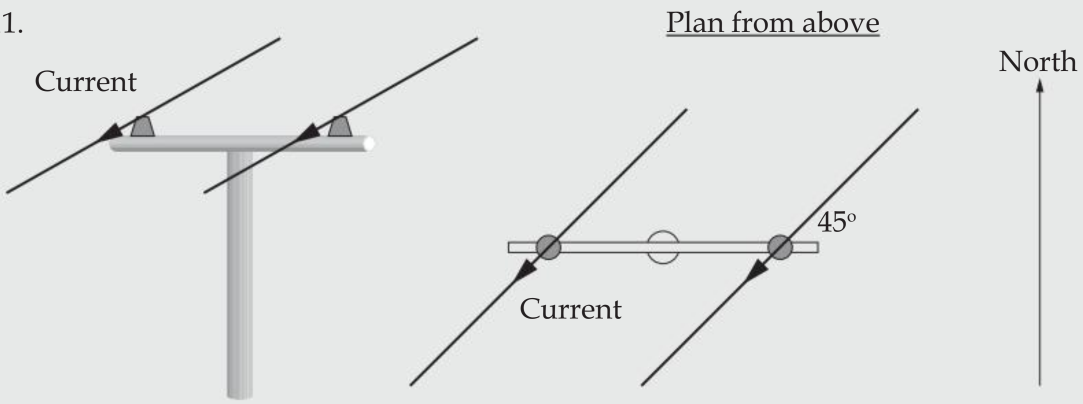
A. The battery is delivering a current

B. X is negative and Y is positive

C. The end of the coil near point b is a north pole

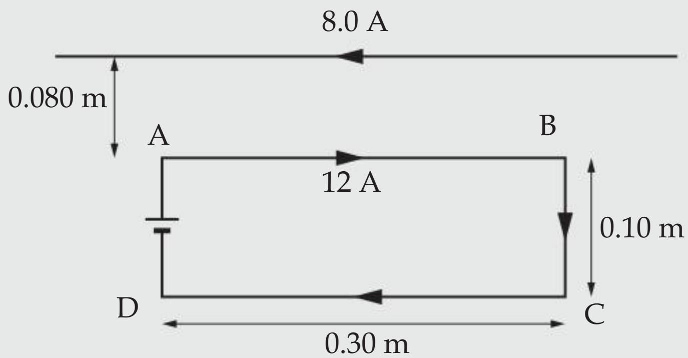
D. The coil's field and the Earth's field are adding at point a

11.



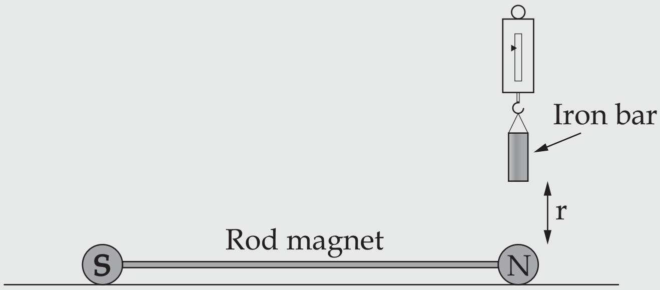
Power cables run on a pole to a farm in the direction N-E. At this latitude the horizontal component of the Earth's flux density is 2.1 x 10^{-5} T. If the cable carries a current of 94.3 amps down in a southwesterly direction, then what will be the magnetic force on each metre of cable?

12.



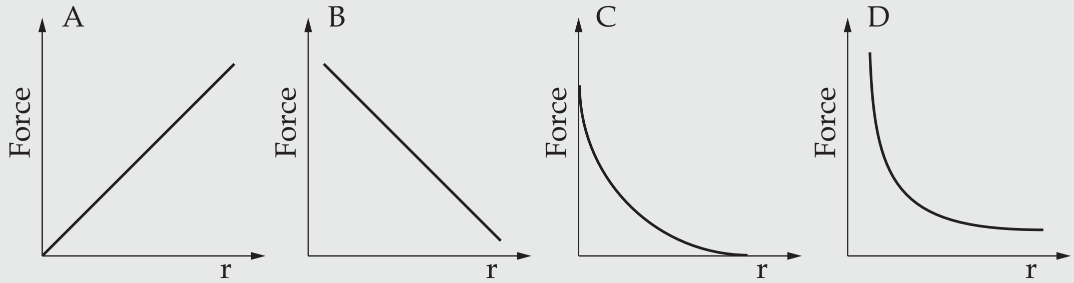
A rectangular loop of wire carrying a current of 12 A is placed 0.08 m from a straight wire carrying a current of 8 A in the opposite direction. Using the dimensions of the coil shown in the diagram calculate the net force on the coil using the formula given that flux density B at distance r from a current-carrying wire is given by: $B = \frac{\mu_o I}{2\pi r}$ (n_o = 1.26 x 10^{-6} units)

13.



In an investigation of magnetic field strength, a magnet is made in the form of a rod with poles exactly at its ends and widely separated. To test the field a spring balance is used to measure the force produced on a small bar of iron attached to it. Readings are taken of the spring balance and of the distance (r) of the bar from the magnetic pole.

Which graph correctly shows the graph obtained plotting force against distance r?



14.

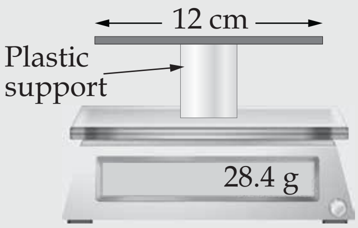


Diagram a

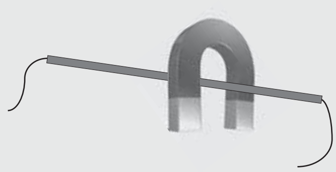


Diagram b

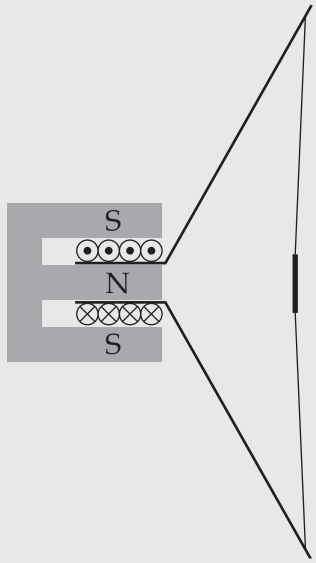
In an experiment to determine the flux density between the poles of a horseshoe magnet the apparatus above was used. A 12 cm long copper rod was attached to the pan of a digital balance by a plastic support. The balance read 28.4 g with nothing attached.

The horseshoe magnet was then fixed above the wire as in diagram b, so the wire was between the poles and a current of 16.8 A was sent through it from a power supply. The new reading on the balance was then 35.6 g. If the width of the field between the magnetic poles was 2.5 cm, then what is the calculated value for the magnetic flux density of the magnet between the poles?

15.

The diagram shows a cutaway view of a loudspeaker with the magnetic orientations and the direction of the current in the coil indicated. The following data apply to the speaker system:

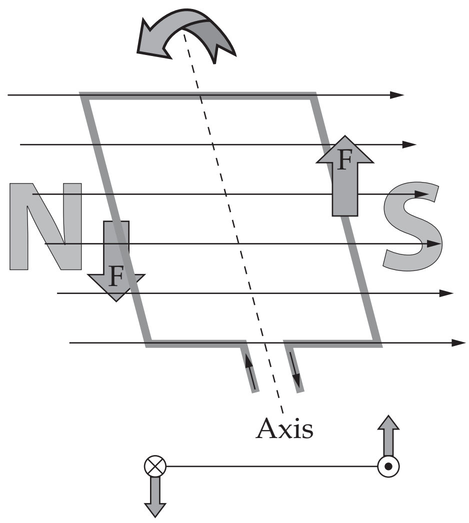
Coil diameter = 5.8 cm
Coil turns = 86
Coil and cone mass = 30 g
Magnetic flux density = 0.12 T
Current in the coil = 1.8 A



Use the data above to find the force on the coil and hence the acceleration of the cone when the current flows.

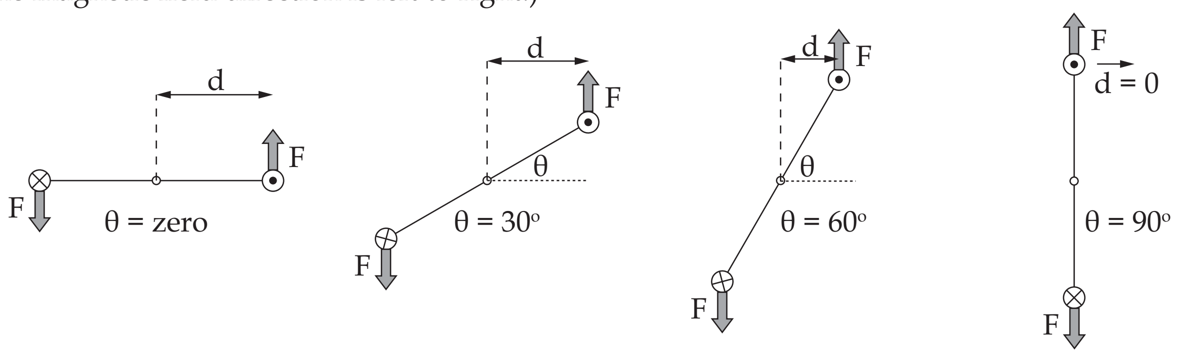
2.12 THE ELECTRIC MOTOR

By placing a current-carrying coil in a magnetic field it is possible to get it to rotate. This occurs because the current direction on one side of the coil is opposite to the current direction on the other side. One force (F) is upwards on the right and one force (F) downwards on the left. This produces two rotating torques on the coil (a 'couple')



The current/torque diagram shows how the currents in/out of the page produce upward/downward forces according to the right hand slap rule to make the coil rotate anticlockwise.

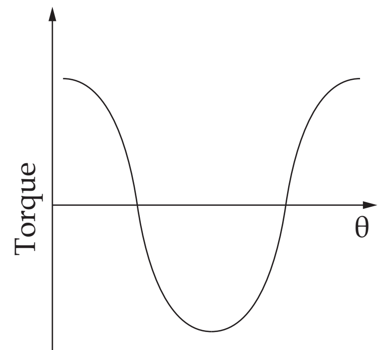
Consider what happens to the coil and its torque as the coil rotates anticlockwise. (Remember the magnetic field direction is left to right.)



The perpendicular distance (d) of the point of action of the force (F = BIL) from the pivot becomes less with angle so that the torque is a maximum at 0° and the torque is zero at 90°.

The graph of torque versus angle is shown here:

The negative part of the graph is a result of the torque reversing when the coil goes past 90° which would cause the coil to reverse to a clockwise direction.



Torque versus angle graph

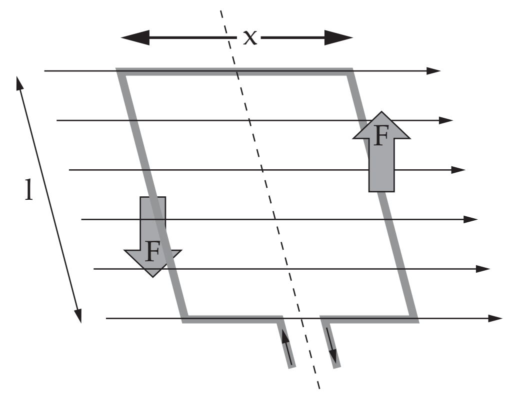
Torque formula

Imagine a coil of N turns, with side length L and width x.

The magnetic force on the sides is BILNcosθ but the side wires are always at 90° to the field, so force on left side is BILN downwards and force on right side is BILN upwards.

Torque = Fr where r is the horizontal distance of F from the pivot. r equals $\frac{x}{2}$ at zero degrees and changes with angle : $r = \frac{x}{2}\cos\theta$

The force upwards on the right and the force downwards on the left produces two torques, called a couple.

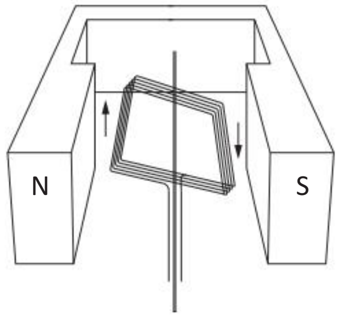


Hence total couple, $\tau = \frac{(BILx)}{(2\cos\theta)} \times 2 = BILN \times \cos\theta$

L_x = area of the coil so $\boldsymbol{\tau = BINA\cos\theta}$

Example 5

A simple DC electric motor has a wire coil made of four loops, connected to a battery supplying a current of 4.50 A. The coil is 15.0 cm long and 8.00 cm wide, sitting in a magnetic field strength between the poles is 6.00 x 10^{-4} T.



Calculate:

a) the force on the side of the coil at any time and

b) the maximum torque obtained from the motor.

Solution 5

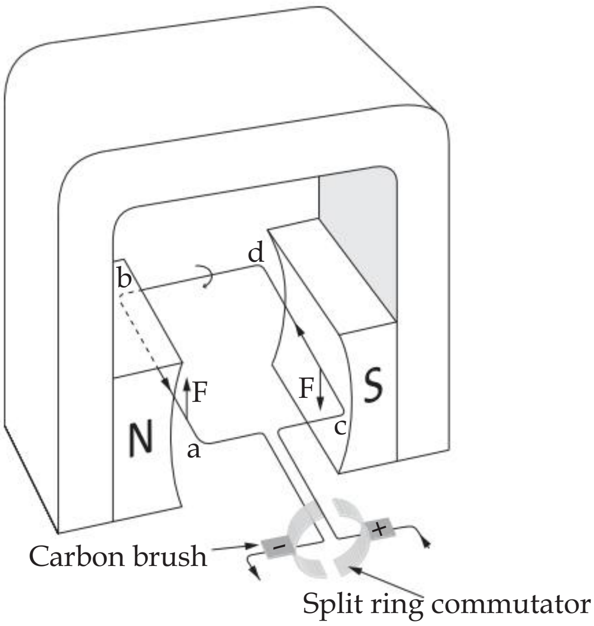
a) Force on side wires = BILN = 6.0 x 10^{-4} x 4.5 x 15 x 10^{-2} x 4 = 1.62 x 10^{-3} N

b) Couple acting = 2Fr = 2 x 1.62 x 10^{-3} x 4.0 x 10^{-2} = 1.30 x 10^{-4} N m

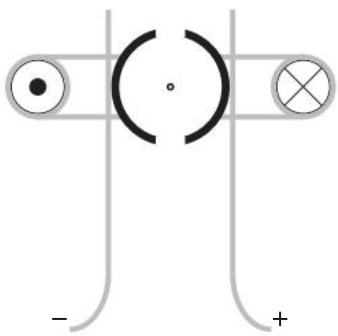
Commutator design

From the torque versus angle graph shown previously, it can be seen that if a permanent connection were made to the coils in the magnetic field then the coil would simply revolve until it was vertical and then remain upright.

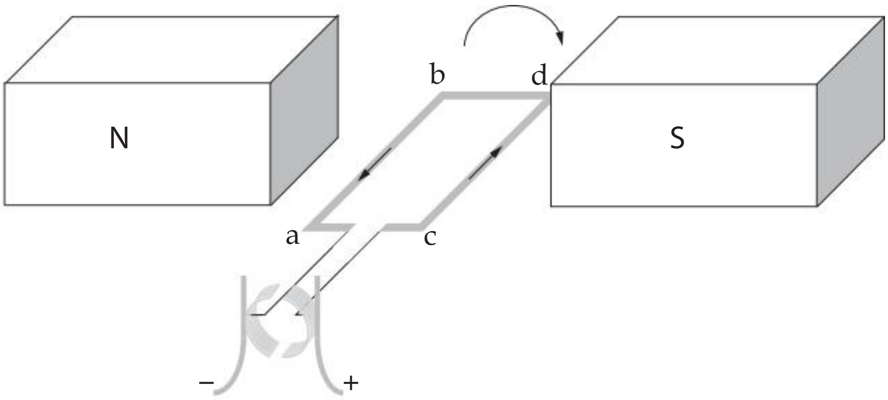
To make a motor continue to turn in the same direction a method is needed to reverse the current to the coil when it becomes vertical. Nikolai Tesla devised a way of doing this by connecting a split ring to make a commutator. A commutator allows current to flow into the coil through two carbon brushes that make a sliding connection with it.

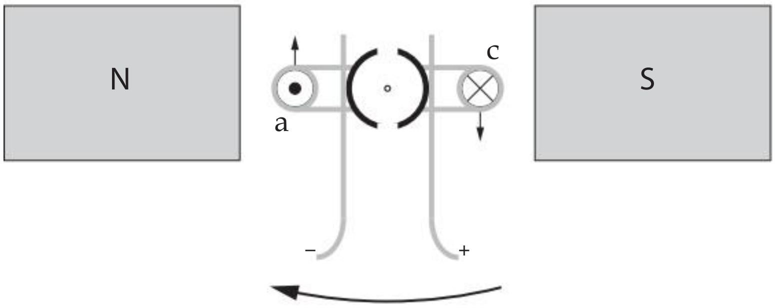


End-on view of commutator

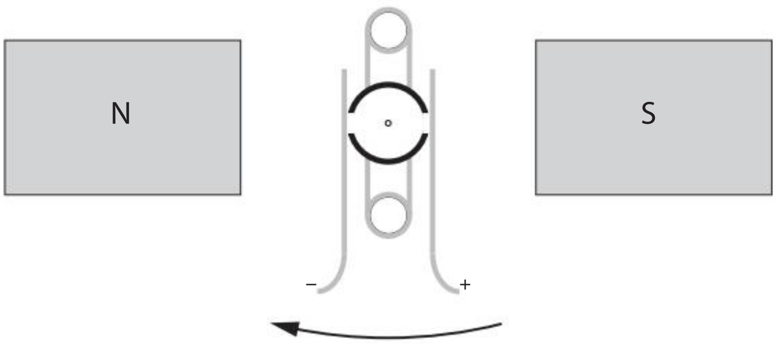


Phases of an electric motor

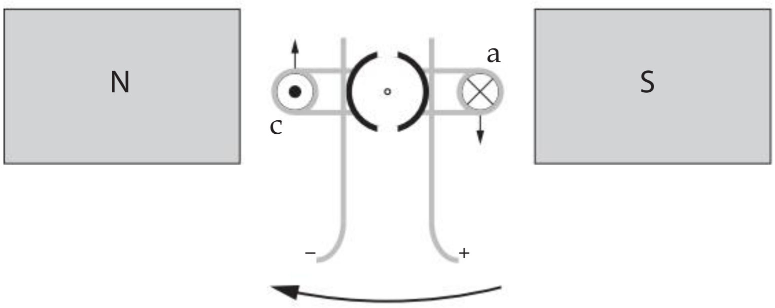




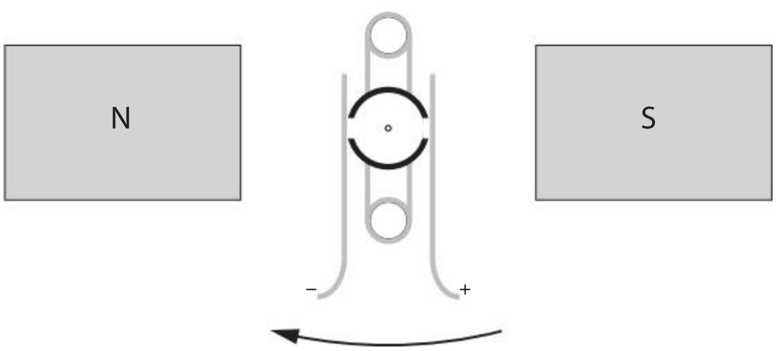
cd is touching the right hand + brush (force down) and comes out of the left side of the coil **ba** (force up) through the brushes.
The coil rotates **clockwise**



With the coil in the vertical position, the brushes are positioned on the gap in the commutator and no longer connected to the coil.
However, the inertia of the coil keeps it rotating clockwise



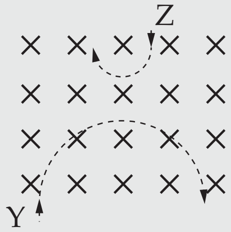
The coil has been carried over to its new position so that wire **ab** is now touching the + brush and current flows from a to **b** which is a reversal and now gives a force downwards on **ab**.



The coil is now in the vertical position again and the brushes are positioned on the gap in the commutator and no longer connected to the coil.
Inertia of the coil keeps it rotating clockwise

Set 8: Electromagnetism

1. Two ions Y and Z with the same velocity are fired into a magnetic field shown going into the page.



Which statement about Y and Z is definitely true?

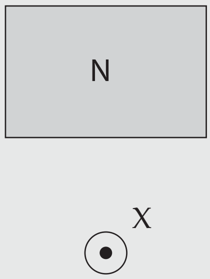
(A) Y and Z have unequal masses

(B) Y and Z have different charges

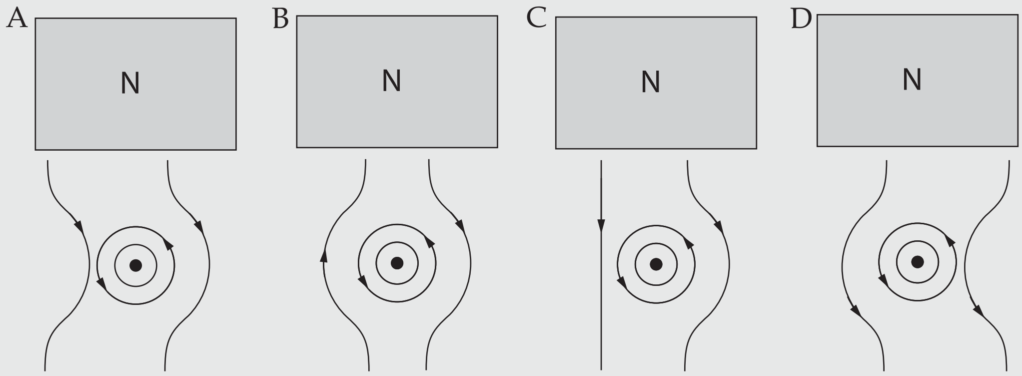
(C) Y and Z have different values for their charge per kilogram of mass

(D) Y and Z have different sized particles

2. When a current-carrying wire X is placed in a magnetic field the two fields interact to form a newly-shaped resultant field. A wire with current running out of the page is placed close to the North Pole of a fixed magnet.



Which diagram below correctly shows the resultant field below the magnet?



3. An alpha particle (mass = 6.64×10^{-27} kg) is fired into a field of flux density 2.75×10^{-3} T at a velocity of 3.4×10^{6} m s^{-1}.

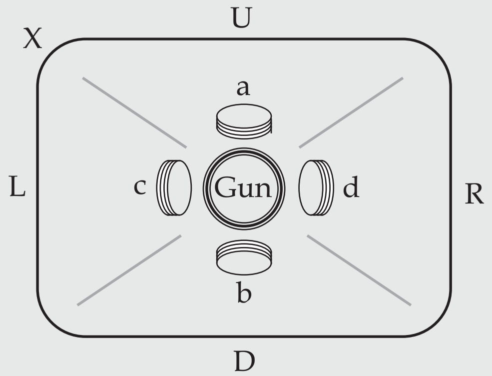
Calculate:

a) The force acting on an alpha particle moving in the field.

b) The acceleration of this particle.

c) The change in velocity of the α-particle after 1.0 nanosecond.

4. The diagram shows the rear view of a cathode ray tube with the electron gun closest to the viewer and electrons being fired into the page. Electrons from the gun normally strike the front screen at its exact centre but the 2 pairs of electromagnets a-b and c-d can deflect the beam up (U), down (D), left (L) or right (R).



At one particular instant coil d is a North Pole and coil c is a South Pole. The magnetic field from these magnets would cause the electron beam to move from the centre spot in which direction?

A. L B. R C. U D. D

5. Referring to the previous diagram (Q. 4), at one instant the electron beam is deflected to position X (left and up). For this to occur which of the following polarities must exist for the coils?

A. a - north, b-south, c- south, d- north

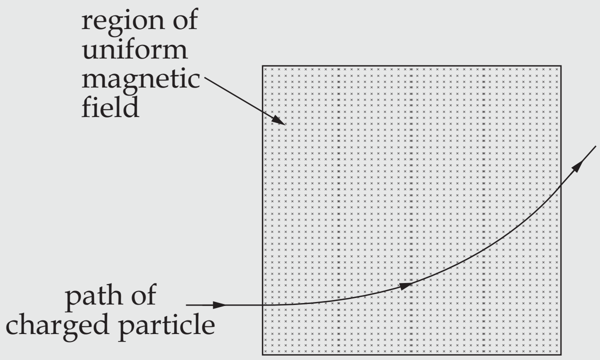
B. a - south, b-north, c-north, d- south

C. a - north, b-south, c- north, d-south

D. a - south, b-north, c-south, d- north

6. A charged particle passes through a region of uniform magnetic field of flux density 0.74 T, as shown.

The radius r of the path of the particle in the magnetic field is 23.0 cm.



a) If the particle is positively charged. State the direction of the magnetic field.

A. Left B. Right C. Into page D. Out of page

b)

(i) Use formulas you know to show that the charge to mass ratio is given by the expression $\frac{q}{m} = \frac{v}{Br}$

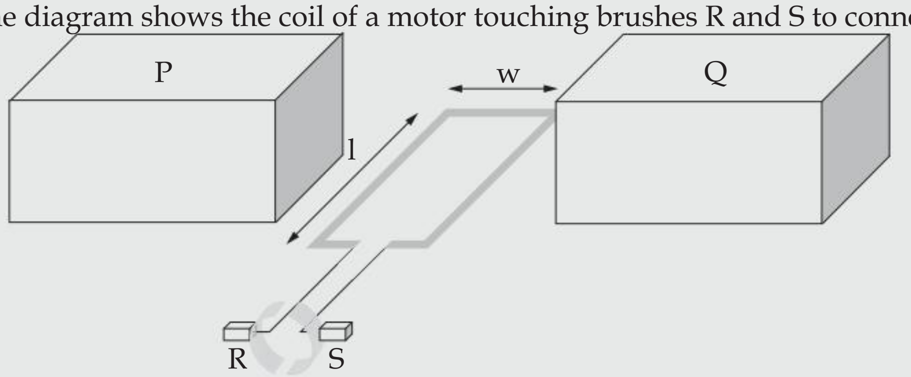
(q is the charge, m is the mass, v is the speed of the particle and B is the flux density of the field).

(ii) The speed of the particle 8.20×10^6 m s^{-1}. Calculate the charge to mass ratio of the particle (q/m).

(iii) It is thought that the particle may be the nucleus of a tritium atom (1 proton and 2 neutrons). Is this possible? Use calculations to justify your answer. (Proton mass = 1.007276 u, neutron mass = 1.008665 u, 1 u = 1.67×10^{-27} kg.]

7.

The diagram shows the coil of a motor touching brushes R and S to connect it to



a battery. Magnets P and Q supply a magnetic field. If the coil is meant to rotate in a clockwise direction, what are the polarities of the magnets P, Q and the brushes, R, S?

A. P – north, Q – south, R +, S -

B. P – south, Q – south, R - , S +

C. P – north, Q – south, R - , S +

D. P – south, Q – north, R + , S -

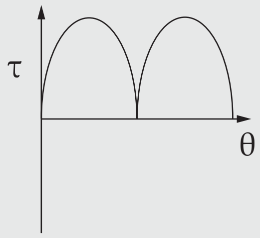
8. Referring to the last diagram (Q. 7), the coil has a length (l) of 35 mm and width (w) 22 mm, with 340 turns. The strength of magnetic flux between the magnets is 4.0×10^{-2} T.

   a) What is the maximum torque this motor can give when drawing a current of 750 mA from the power source?

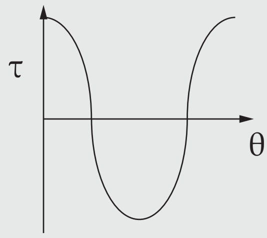
   b) What power does the motor use when connected to a 12 V battery?

9. The coil shown in Q. 7 is allowed to rotate through a complete circle under the influence of the current and magnetic field. Which graph of motor torque (x) versus angle (θ) below would be correct for one rotation of the coil?

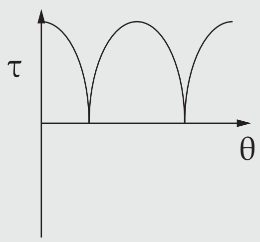
A.



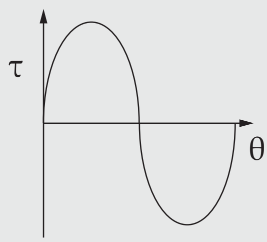
B.



C.



D.



10. At certain times of the year, the *Aurora Australis* (Southern Lights) can be seen at very southerly latitudes: the sky lights up with beautiful colouration.
    Some factors have been proposed as essential to the formation of the Aurora Australis:

    (i) Very fast moving positive particles are coming from the Sun

    (ii) There is a very low pressure of air at high altitudes

    (iii) Ions are deflected by the Earth's gravitational field

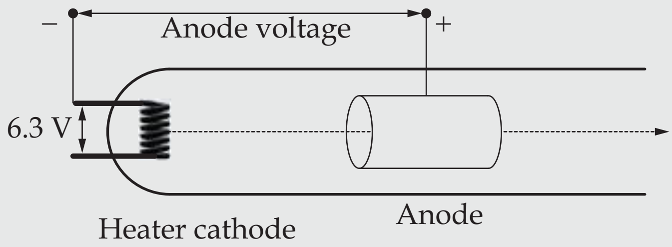
    (iv) Gas molecules are ionized

    (v) Centripetal force acts on the ions

Which of these factors are definite contributors to the Southern Lights phenomenon?

A. (i), (ii) (iv) B. (i), (iii), (v) C. (i), (iv), (v) D. (i), (ii), (v)

11. The electron gun in a TV set consists of a heater coil and a cylindrical anode, both sealed into an evacuated tube.



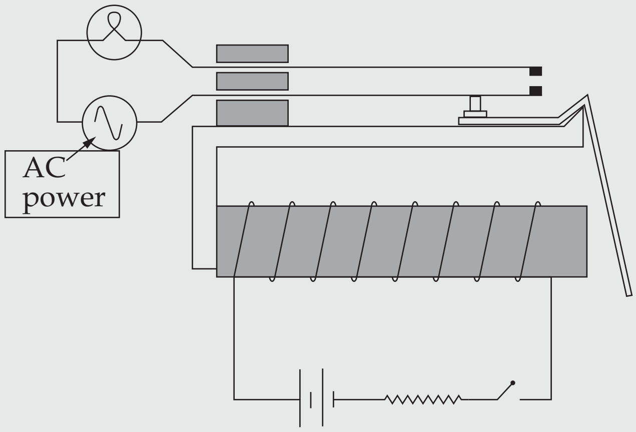
The heater voltage is 6.3 V and the anode voltage is 22 kV. Electrons have a mass of 9.1×10^{-31} kg and carry a charge of 1.6×10^{-19} C.

With what velocity would the electrons emerge after passing through the anode?

12. Referring to the diagram in the last question (Q.11), the emerging electrons are now accelerated by a different anode voltage to a velocity of 2.0×10^7 m s^{-1} and then pass between the coils of an electromagnet delivering a flux density of 2.1 millitesla. What is the radius of the circle in which the electron beam will be deflected?

13. A hairdryer motor contains a 250 turn rectangular coil of dimensions 120 mm long by 45 mm wide. The magnet's flux density is 0.45 T and the coil draws a current of 0.25 A when working normally. What torque does the coil produce?

14. A schematic diagram of a relay is shown. The relay uses a small DC battery to control a lamp which is connected to an AC power supply.



Explain how the relay manages to control the lamp in this way.

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15. In a cathode ray oscilloscope the electron beam current emerging from the cathode and striking the screen is measured as 20 nA. If the charge on an electron is 1.6×10^{-19} C what is the number of electrons striking the screen in one minute?

____

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____

____

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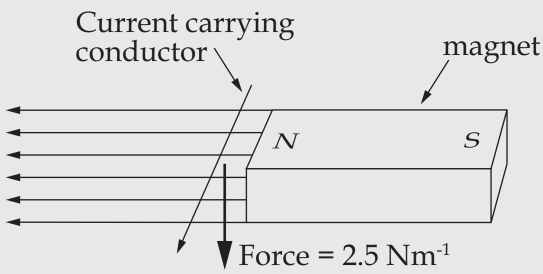
16. Refer to the diagram opposite. The force on the wire carrying a current of 40.0 A out of the page is 2.50 N m^{-1} vertically downwards. What is the flux density near the pole of the magnet?



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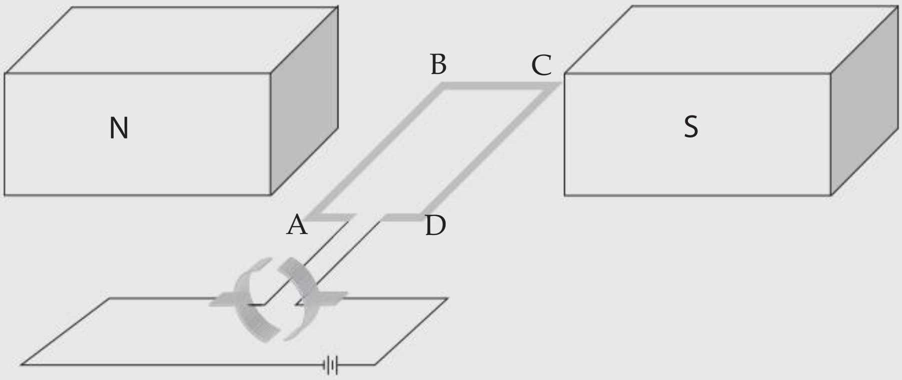
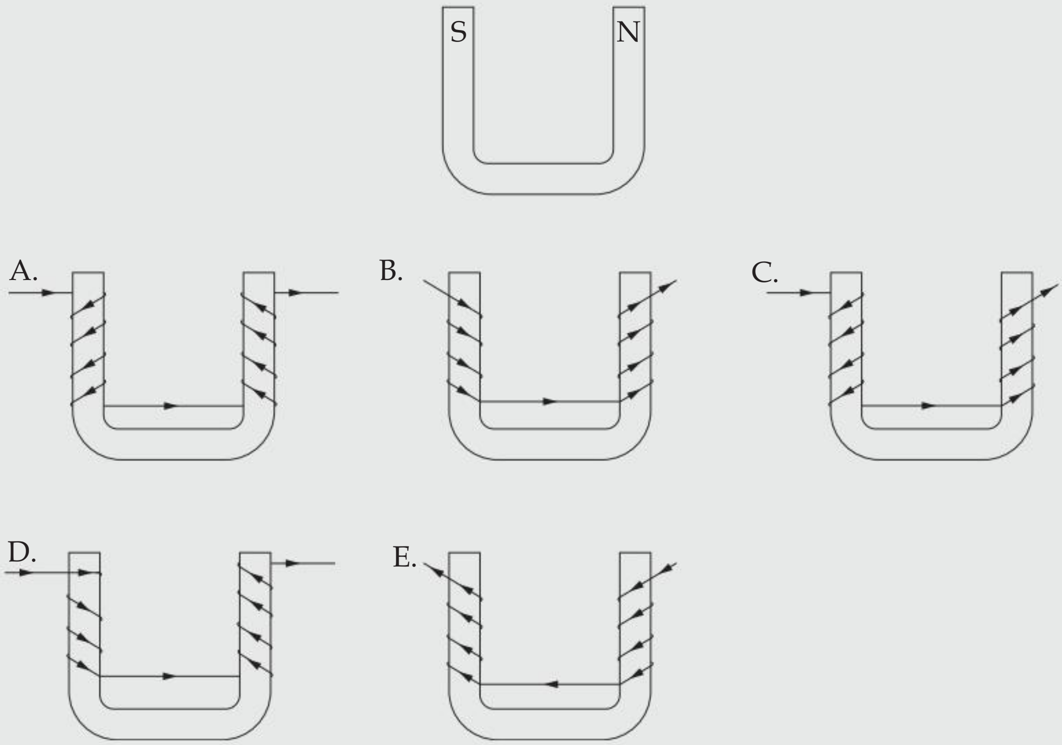
17. The diagram below shows a single rectangular loop of wire free to rotate in a uniform magnetic field of 0.06 T.

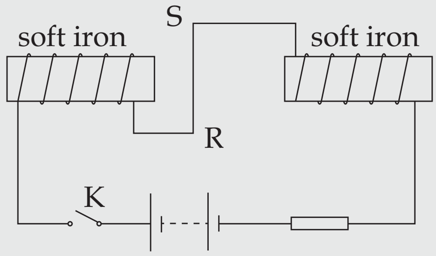


The loop is carrying a current of 2.50 A and is 8.0 cm long and 3.0 cm wide. What torque is being exerted on the loop when it has rotated through an angle of 30° from the position shown?

18. Which of the following winding arrangements will give the same polarity as the specimen magnet shown at the top?

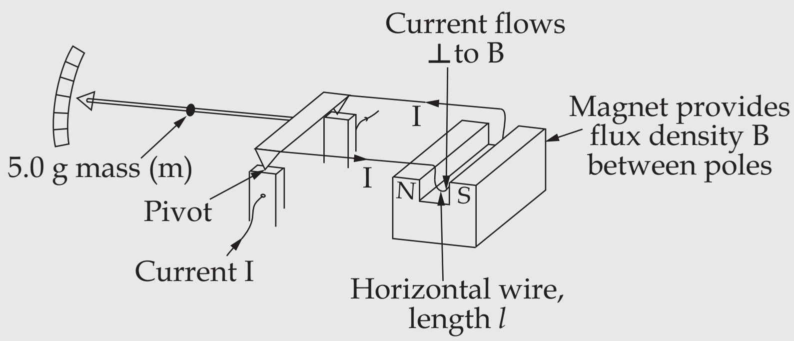


19. In the diagram below, RS is a wire that is free to move. In which direction will it move when the switch is closed?



20. An experiment is done to examine the force exerted on a current-carrying conductor when it is placed in a magnetic field. The set-up is called a "current balance".

The diagram below shows how the magnet, wire, pivot and pointer are arranged. Before any current (I) is passed through the conductor, the pointer was on the zero mark (horizontal). A current was then passed though the wire and the pointer brought back to zero by moving a 5.0 g mass to the position shown along the pointer.



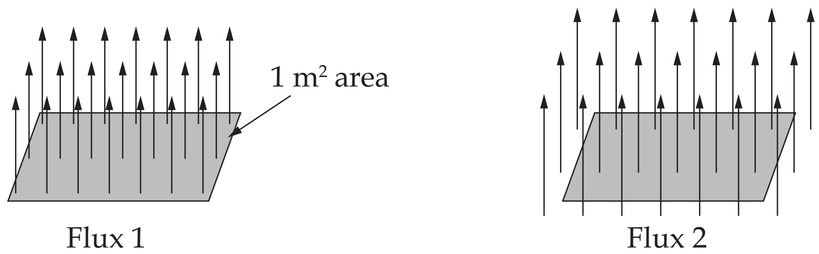
a) Which way would the pointer move (up or down) when the current (I) is passed through the conductor? Why?

b) With the current still being passed through the conductor, what adjustments are there that could be made to the current balance to bring the pointer back to the horizontal?

c) List the measurements you would need to make in order to calculate the magnetic force on the conductor.

2.13 ELECTROMAGNETIC INDUCTION

Flux: The total flux (symbol]) in a magnetic field equal the total number of lines of force present.

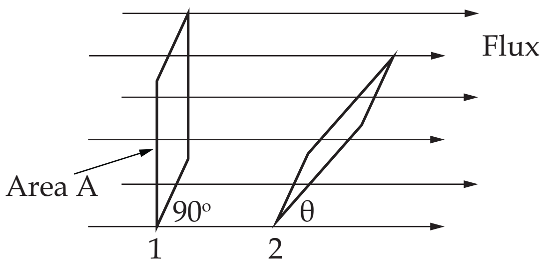


Both flux 1 and flux 2 contain 20 field lines but flux 1 is more concentrated (the field is stronger) because it contains more field lines per square metre. The field strength or flux density (symbol B) is defined as the flux per metre^2.

i.e. $B = \frac{\phi}{A}$

Flux (]) is measured in webers (Wb) and flux density (B) is measured in weber/m^2 or tesla (T).

Flux] = BA

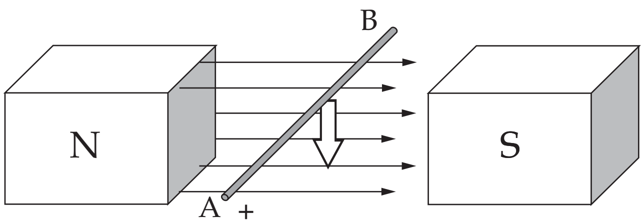


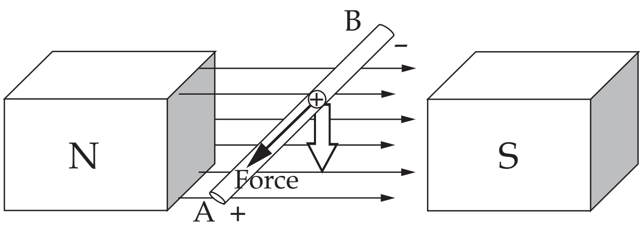
Coil 1 is at 90° to the flux lines so the flux through the coil = BA — a maximum value

The flux passing through the coil in position 2 is BAsin θ. When the angle to the field, sin 90 = 1.0, and so the number of lines passing through the coil will be a maximum value. When θ to the field is zero, (sin θ = 0) then the number of lines passing through the coil will be zero.

Induced Voltage

Faraday's experiments showed that when a wire was moved through a magnetic flux, a voltage or Electro Motive Force (emf) was produced between the ends of the wire.





If a copper wire AB was pushed down through the field as shown, then end A became positive (+) and end B became negative (–).

A potential difference (p.d.) was produced across the ends of the wire.

Faraday proposed a reason for the emf being produced based on the force on the positive charges in the wire due to the field (the electron was unknown then).

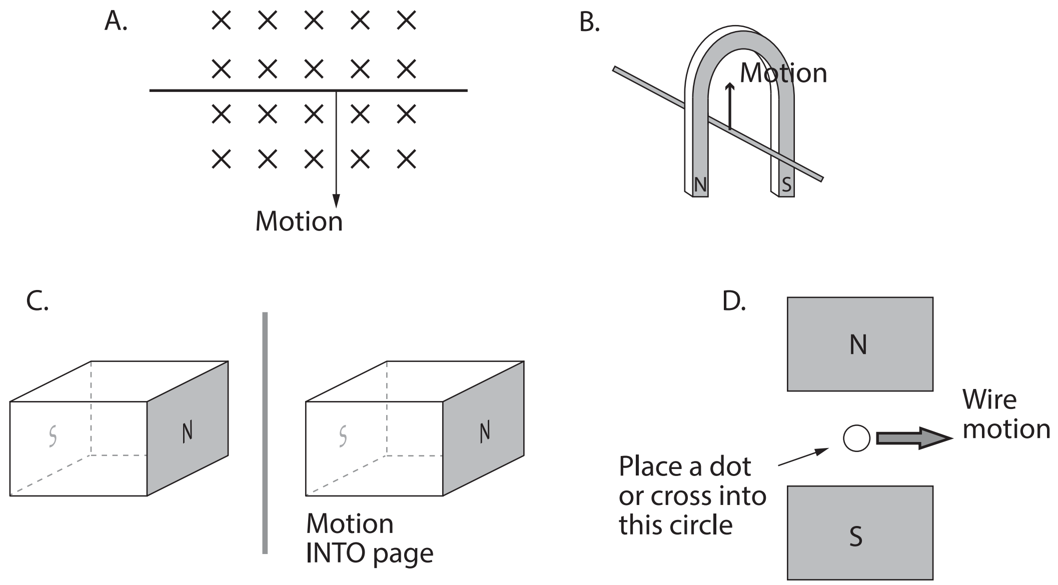
Consider a single + charge in the copper wire moving downwards. By the right-hand slap rule, there will be a force exerted on the charge which “slaps” it to end A of the wire. In actual fact positive charges cannot move – it is actually just the electrons that move in the opposite direction (towards end B).

If we use the right-hand slap rule in a different way we can predict the end of a wire that will become positive. For induction, we use

- Fingers for the direction of the field
- Thumb for the direction of the motion
- The slap direction tell us which way the positive charges will move (conventional current)

Example 6

Mark in which end of the wire will become positive from the motion shown in each of the diagrams below:



Solution 6

A. Right hand end of wire becomes positive.

B. Furthest end of wire becomes positive.

C. Bottom end of wire becomes positive.

D. Current is into the page (cross in circle).

2.14 FARADAY’S LAW

Faraday noticed that the size of the induced voltage depended on several things

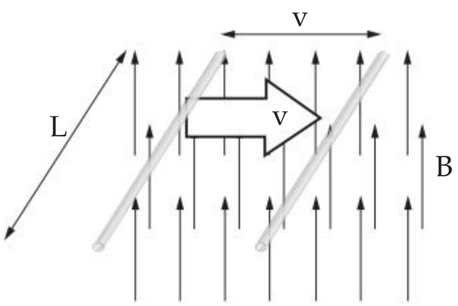
1. The strength of the field
2. The speed of motion of the wire
3. The length of the wire in the field

From these observations he made two generalised rules:

- A voltage is induced in any conductor when the magnetic flux around it changes
- The magnitude of the induced voltage depends on the rate at which the flux is changing.

Average induced Emf is given by $\epsilon = \frac{\Delta\varnothing}{\Delta t}$

Let us take the case of a wire moving through a magnetic field. The wire is "cutting through" magnetic flux, so the flux change occurring around the wire is what causes a potential difference (Emf) to be set up across the ends of the wire.



If a wire of length L moves through a field of strength B with a velocity v, then the area it covers in 1 second will be L v metres squared.

The flux cut in 1 second (φ) will therefore be given by: B x area = B L v

Flux cut per second, by Faraday's law equals the Emf

So the induced voltage from a moving wire is: $\varepsilon = B\,L\,v$

Example 7

A 12.0 cm-long wire is pushed through a magnetic field of strength 0.250 tesla with a velocity of 35.0 cms^{-1} What is the voltage induced across the ends of the wire?

Solution 7

$B = 0.25\ T$

$L = 0.12\ m$ $\varepsilon = B\,L\,v = 0.25 \times 0.12 \times 0.35 = 1.05 \times 10^{-2}\ V$

$v = 0.35\ m\ s^{-1}$

Example 8

A single square coil of wire with side 5.00 cm has a magnet close to it so that the flux density within the coil is 0.0550 T. The magnet is withdrawn from the coil to a distance so that the value of flux density in the coil is now 4.00×10^{-3} T. If the time to change the flux was 20.0 ms, what voltage is induced in the coil?

Solution 8

Initial B = 0.055 T

Final B = 0.004 T

Area = $(5 \times 10^{-2} \times 5 \times 10^{-2})$

$= 2.5 \times 10^{-3}\ m^2$

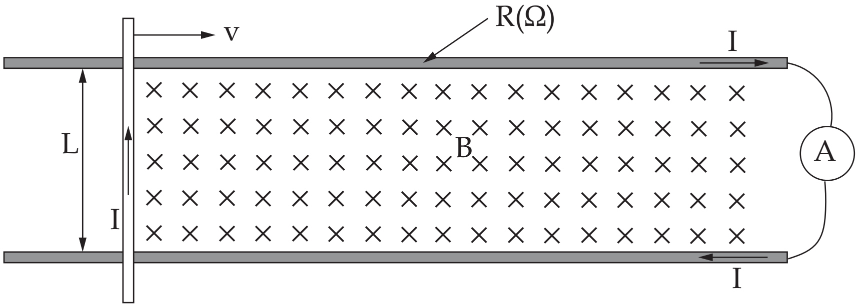
$\Delta t = 0.020\ s$

$$\epsilon = \frac{\Delta\varnothing}{\Delta t} = \frac{BA}{\Delta t}$$

$$\epsilon = \frac{(0.055 - 0.004) \times 2.5 \times 10^{-3}}{0.02} = 6.38\text{mV}$$

2.15 LENZ'S LAW

Lenz experimented with wires and coils connected into complete circuits so that current flowed. He realised that currents flowing in isolated circuits produced energy and that this energy had to come from another source.



If the wire shown is moved at velocity v through the field of strength B then the voltage induced is given by Emf = B L v (note that L is the **effective** length of the rod within the field).

The current flowing, (I), will be given by I = V/R (Ohm's Law).

The effect of the induced current flowing up the rod in the field is to generate a magnetic force on the rod according to the Right Hand Slap Rule i.e. a force to the left **opposing** the direction of motion. Because the rod is moving at a constant speed the force used to push the rod to the right must exactly balance the force opposing the movement of the rod (no net force if v is constant)

In this way, the work produced per second by the induced voltage always equals the work done in moving the rod against the opposing force.

Lenz's law then can be summarised as:

The induced voltage will always be produced in a direction that will oppose any change in field.

Lenz's Law is really a result of the Law of Conservation of Energy, which equates work done by the person moving the rod with the electrical energy output. For this to apply, the magnetic force must be in opposition to the motion, or – the magnetic effect produced by the induced current will always oppose the change that causes it.

Example 9

In the diagram above the distance between the rails is 12.0 cm, the flux density of the field is 0.150 T and the rod velocity is 65.0 cm s^{-1}.

a) If the electrical resistance of the rod and rails is 0.250 Ω then what current is produced?

b) What power is developed in the circuit?

c) What force needs to be exerted on the rod to move it?

d) What mechanical power is being used?

Solution 9

L = 0.12 m B = 0.15 T v = 0.65 m s^{-1}

a) Induced voltage Emf = B L v = 0.15 x 0.12 x 0.65 = 1.17 x 10^{-2} V

Induced current I = V/R = 1.17 x 10^{-2}/0.25 = 4.68 x 10^{-2} A

b) P = VI = 1.17 x 10^{-2} x 4.68 x 10^{-2} = 5.48 x 10^{-4} W

c) F = B I L = 0.15 x 4.68 x 10^{-2} x 0.12 = 8.42 x 10^{-4} N

d) Power used = Force x distance moved per second = 8.42 x 10^{-4} x 0.65

= 5.48 x 10^{-4} W (same as electrical power developed)

2.16 COILS

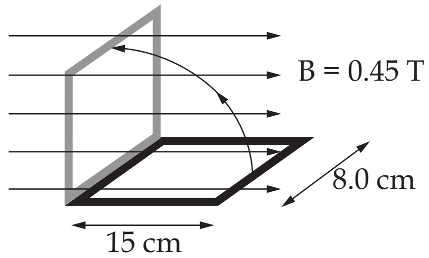
A coil really acts like lots of small wires (number = N) connected in series, so the induced voltages in each wire will be added together to produce an overall large voltage. To reflect the fact that Lenz's Law predicts the voltage to be in opposition to the change in flux, a negative sign is placed in front of Faraday's equation. Hence the final equation for induced emf in a coil when a flux occurs will be: $\epsilon = -N\frac{\Delta\varnothing}{\Delta t}$

Example 10

A coil of 250 turns with dimensions 15.0 cm x 8.00 cm is rotated through a quarter turn about a horizontal axis in a horizontal field of strength 0.450 T. If the rotation takes place in 0.120 s, what is the:

a) Average voltage induced

b) Maximum voltage induced



Solution 10

a) Area = 0.15 x 0.08 = 0.012 m^2

$\Delta\phi$ = BA = 0.45 x 0.012 = 5.4 x 10^{-3} Wb (flux through coil changes from zero to 5.4 x 10^{-3} Wb)

Average voltage is given by: $\epsilon = -N\frac{\Delta\varnothing}{\Delta t} = \frac{250 \times 5.4 \times 10^{-3}}{0.12}$ = 11.3V (3 sf)

b) Max voltage calculation considers the rotational motion of the moving 8.0 cm end of the wire so: E_{max} occurs when the coil is lying flat.

For ¼ turn v = ½ πr/t = ½ (π x 0.15)/0.12 = 1.963 m s^{-1} (Circumference of a circle = 2πr)

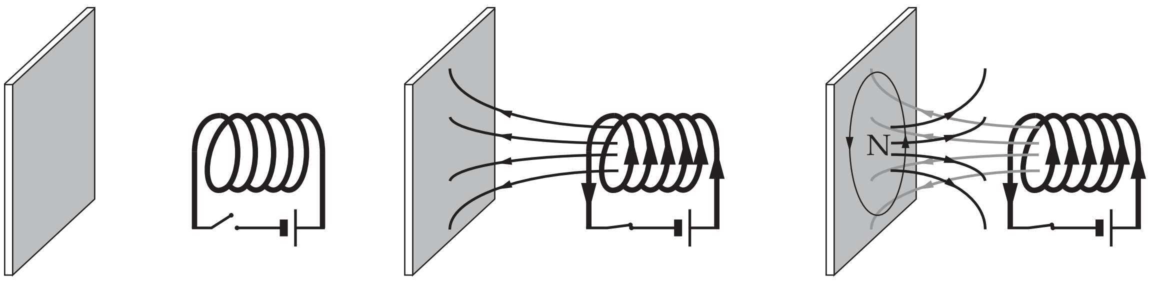
Effective length of wire cutting the flux = 0.08 x 250 = 20 m

E_{max} = B L v = 0.45 x 20 x 1.963 = 17.7 V

2.17 EDDY CURRENTS

Currents can be induced in any metal object that is in a position where there is a flux change occurring near it. These induced currents are called Eddy Currents.

Example 1



(i) (ii) (iii)

(i) A coil is shown on the right with a metal conductor facing it on the left.

(ii) When a current flows in the coil the magnetic field penetrates the metal plate and causes a flux change in the metal.

(iii) By Faraday's Law, the flux change will produce an induced current in the metal which must flow so as to oppose this flux change (Lenz's Law). Hence a north pole must be established in the metal by the current flowing in an anticlockwise or circular direction.

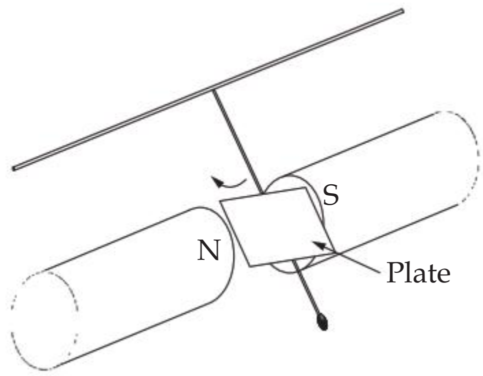
This flow of current in the conductor is like an eddy in a stream and hence is called an eddy current.

Exactly the same flow of current would occur in the case above if the north pole of a magnet were pushed towards the plate.

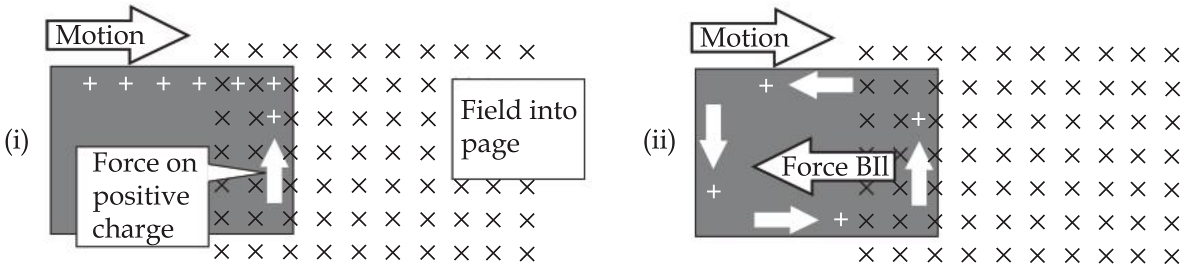
Waltenhofen's Pendulum

If a metal plate is moved in or out of a magnetic field, the flux change will cause a current to flow in the plate. This induced current will set up a field in opposition to the applied force.

Waltenhofen made a metal plate in the form of a pendulum so that when the pendulum was released, instead of swinging freely, it stopped very quickly in the field due to the eddy currents.



Explanation



In diagram (i) the metal plate of the pendulum has just moved into the magnetic field. By the right-hand slap rule, positive charge will be pushed up to the top of the plate. This positive charge can move to the left out of the field to create an anticlockwise eddy current, shown in diagram (ii). The upward current on the right of the plate in the field causes a force (BIL), by the slap rule, which acts towards the left and slows the movement of the plate down so the pendulum stops quickly when the magnets are in place.

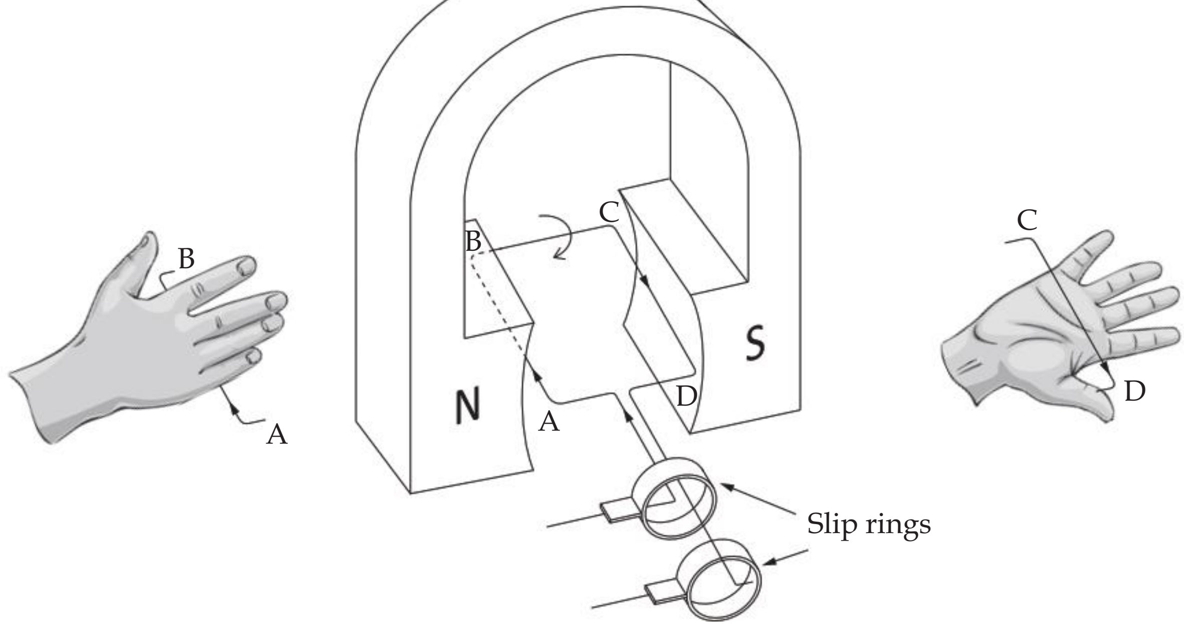
If slots are cut in the moving plate of the pendulum this prevents the eddy currents from flowing and so a pendulum that is slotted will oscillate normally and will not be slowed when in the field.

2.18 GENERATORS

Alternating current (AC) or direct (DC) current can be generated by rotating a coil in a magnetic field. The right-hand slap rule is used to find the direction of flow of (positive) current.

If the coil ABCD shown below is rotated clockwise then current is induced to flow from A to B and then from C to D.

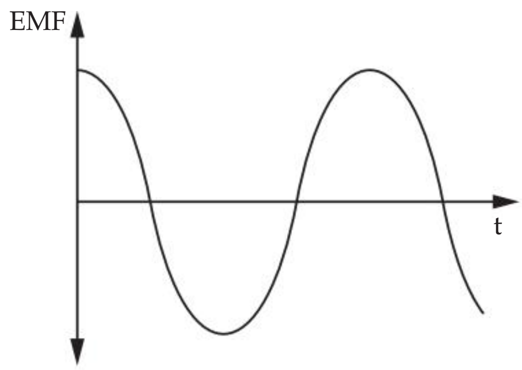
AC generator



The generator gives alternating current out to some external device connected to the slip-rings via the brushes, so current will go into ring 1 and out of ring 2 as it is shown (i.e. Ring 1 is – and ring 2 is +).

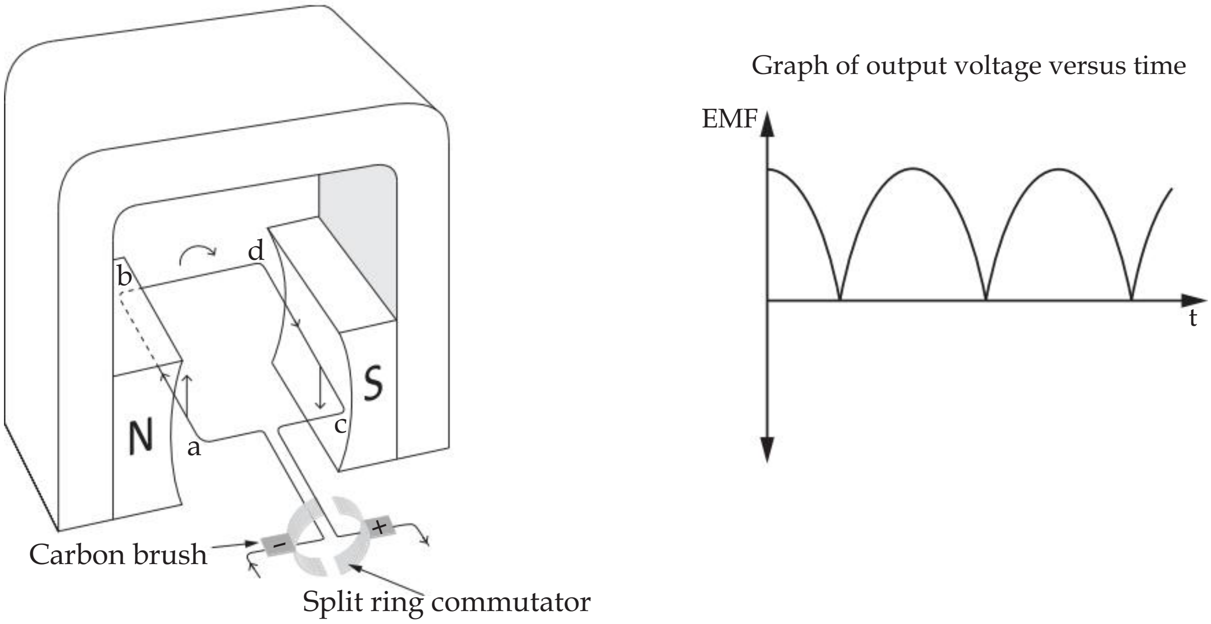
However, when side AB of the coil is going down on the right, the current will reverse to flow from B to A and out of ring 1 (Ring 1 is now +).

Graph of output voltage versus angle



With the coil in the position shown, the graph starts at a maximum emf value (i.e. is a cosine graph) because the wire AB is cutting the flux at right angles when the angle of the coil to the field is zero.

To produce direct current, split rings are attached to the coil so that, as the current begins to reverse, the connection is switched to the other brush. In this way one brush is always at a positive potential.



Generator Emf

A coil with an area A and number of turns N rotating f times per second in a field of flux density B will cut a flux of BA in a quarter of a rotation (see section 2.16). So in a complete rotation the flux cut will be 4 x BA and with a rotational frequency of f, the total flux cut per second will be 4BAf for each turn.

Hence the **average** emf generated is $\varepsilon = 4BANf$.

Example 11

A coil with 120 turns and with dimensions 12.0 cm x 8.50 cm rotates within a magnetic field of strength 0.150 T. If the coil rotates at a frequency of 1200 revolutions per minute, calculate the average emf generated.

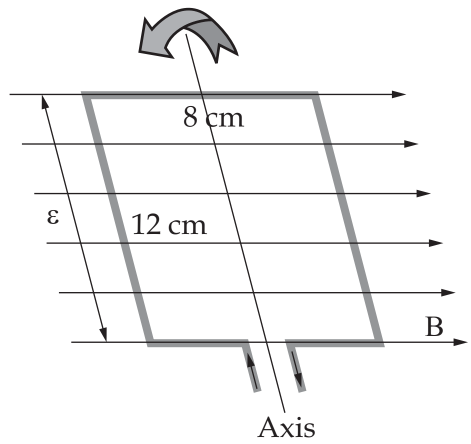
Solution 11

1200 rpm = 20 revolutions per second (f = 20 Hz).

Area = 0.12 x 0.085 = 0.0102 m^2

ε_{av} = 4BANf = 4 x 0.15 x 0.0102 x 120 x 20 = 14.7 V.

Peak emf



To obtain the maximum or Peak emf the formula BLv is used to find the voltage produced when one side of the coil is rotating in the field with a velocity v.

If the side wires of the coil are rotating f times per second then they will move at a velocity of 2πrf, so the voltage generated on one side will be BLv = BL × 2πrf × N turns.

But 2r × L = area A so $\varepsilon_{one\ side}$ = πBANf.

Each side of the coil generates this voltage (in series) so the total ε_{max} = 2πBANf.

For the generator in Example 11 the maximum voltage would be 2π × 0.15 × 0.0102 × 120 × 20

ε_{max} = 23.1 V

RMS values

With alternating current the average voltage over the whole cycle would be zero because it reverses but the power produced would not depend on which way the current flows. By calculating the Root Mean Square value of a sine wave we can obtain the value of the voltage or current which would give the equivalent power output to a constant direct current. To produce an RMS value we square the peak value, divide by 2 and then square root the answer. This turns out to be the same as taking the peak voltage and dividing by $\sqrt{2}$.

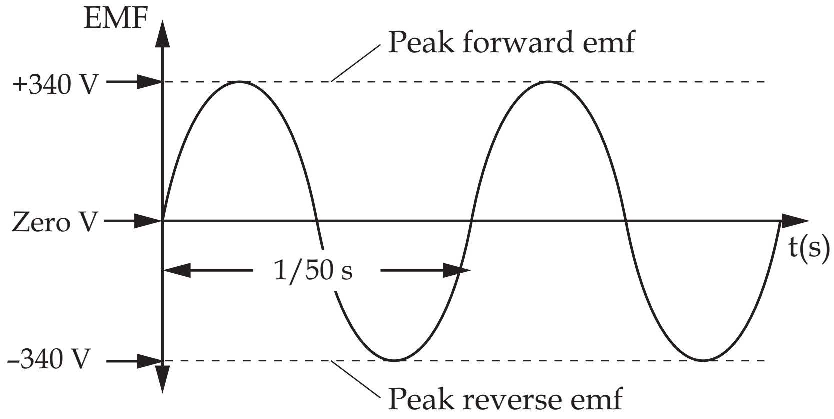
In the above example the RMS value of voltage would be $\frac{\epsilon_{peak}}{\sqrt{2}} = \frac{23.1}{1.414} = 16.3\text{V}$.

i.e. A constant DC source of 16.3 V would produce as much electrical power as an AC generator giving a peak voltage of 23.1 V.

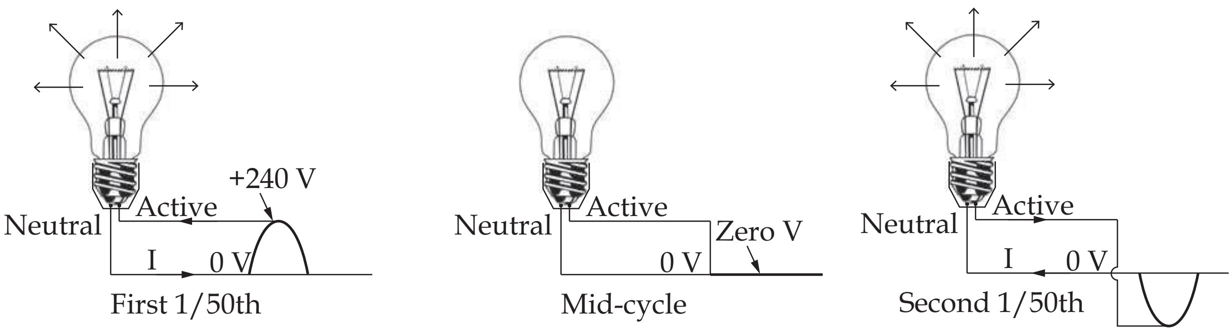
2.19 ALTERNATING AND DIRECT CURRENT.

The voltage considered so far has been from batteries, called direct current (DC) because the terminals give a voltage that is always in the same direction (one terminal remains + and the other is always -) In Australia all houses work on a 240 volt rms Alternating Current (AC) supply, running at 50 hertz. This means that all devices run from a supply voltage that changes 50 times per second from + 240 V rms to – 240 V rms and back again. The reason AC is supplied is because transformers in the system can be used to change the voltage easily to higher or lower values (e.g. a mobile telephone phone charger incorporates a transformer)

An electrical device working on a 2-pin plug has one pin at earth potential (zero volts which is called the neutral pin. The other (active) pin is connected to a voltage from the power station which changes from + 240 volts rms to – 240 volts rms and back again 50 times per second. This means that the current through an AC device will reverse every 0.01 seconds according to the sine wave supply.

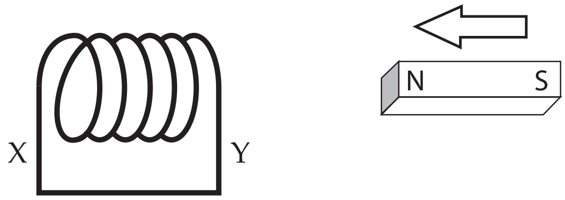


The current in a lamp or a heater is actually oscillating 50 times a second but light and heat energies are still given out regardless of current direction which is shown in the diagrams below.

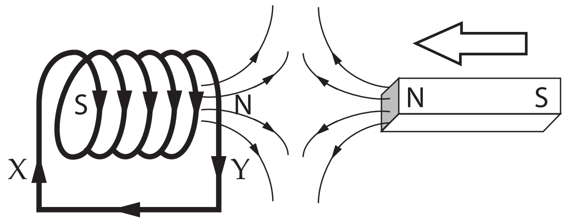


2.20 INDUCTION IN COILS

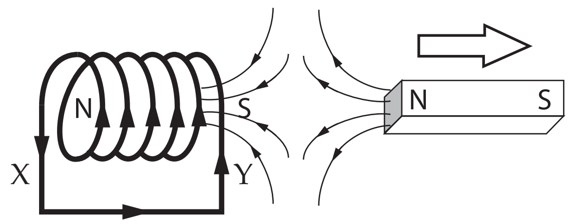
When a magnet is pushed towards a coil there will be a flux change occurring in the coil and so a voltage will be induced in the coil. If the coil connects to a complete circuit (i.e. there are no gaps in the wire) then a current will flow, otherwise a voltage will be present but no current.



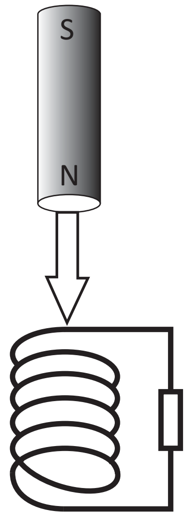
For energy to be conserved, the current must flow in the coil so that end Y produces a North pole and repel the magnet. By repelling, work is done by the magnet, which equals the energy produced in the coil. For end Y to become a N-pole current must flow from Y to X outside the coil. If an attempt is made to increase the flux in the coil then a flux is set up in opposition to the decrease of flux.



If the magnet is withdrawn from the coil then the flux in the coil will be decreasing. By Lenz's Law, a magnetic pole will be produced at end Y which will oppose this decrease in flux and will attract the magnet back. i.e. the current in the coil will reverse to make end Y a South pole.



Example 12



A magnet is dropped through the coil as shown in the diagram with a diameter of 5.00 cm and resistance 6.50 Ω.

a) Between time t = 0 and t = 0.0400 s the flux in the coil changes from a value of 5.00×10^{-3} Wb to 0.220 Wb. If the coil has 30 turns, calculate the current induced in the coil over this time.

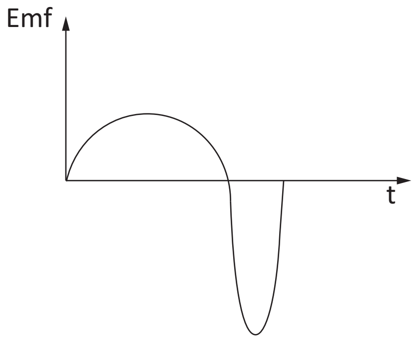
b) Sketch a graph of the voltage across the coil as the magnet approaches and passes right through the coil.

Solution 12

a) $\epsilon_{av} = \frac{-N\Delta\varnothing}{\Delta t} = \frac{30 \times (0.22 - 0.005)}{0.04} = 161.3\text{V}$

$I = \frac{V}{R}$ So $I = \frac{161.3}{6.5} = 24.8$ A

b) Notes on the shape of the voltage/time graph:



As the north pole of the magnet approaches the coil a positive voltage is induced but as it passes through the coil the north pole is receding and so the voltage will reverse to be negative. The magnet is also accelerating under gravity so its speed emerging from the coil will be larger than when entering the coil.

By Faraday's Law this means that:

(i) the time to emerge will be smaller than to enter

(ii) the voltage will be larger exiting than entering as the flux change is occurring in a shorter time.

Example 13

A generator consists of a coil of dimensions 325 mm long by 122 mm wide, containing 350 turns. The coil rotates at a frequency of 2400 revolutions per minute between the poles of a magnet with flux density 0.330 T. What average voltage does the coil generate?

Solution 13

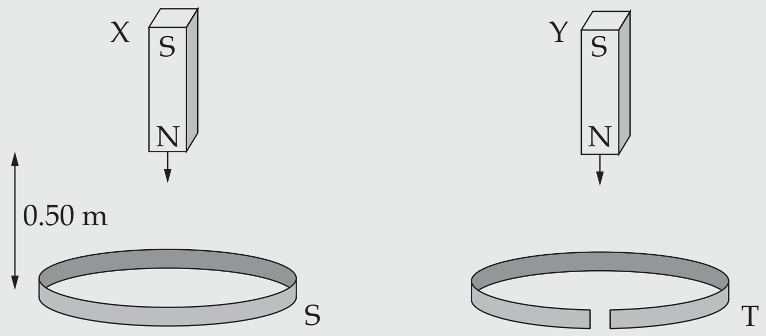
Flux through the coil = BA = (0.325 x 0.122) x 0.33 = 1.309×10^{-2} Wb

Frequency of revolution = 2400/60 = 40 Hz, so the time for 1 revolution = 1/40 s or 0.025 s

During 1 revolution there will be 4 flux changes (zero to max, max to zero, zero to –max, –max to zero), so time for each flux change = 0.025/4 = 6.25×10^{-3} s

$\epsilon_{av} = \frac{-N\Delta\varnothing}{\Delta t} = \frac{350 \times (1.309 \times 10^{-2} - 0)}{6.25 \times 10^{-3}} = 733\text{V}$

Set 9: Electromagnetic Induction



1. Two identical magnets X and Y are held 0.5 m above two metal rings S and T. Ring S is a complete circle whilst ring T has a small cut in it. The magnets are both released to fall at the same time.

   Which statement about the situation is true?

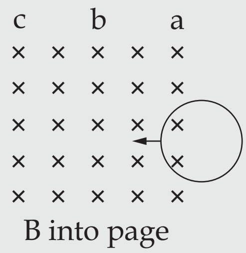
   A. Both magnets will fall at the same rate as they have the same weight and induce the same emf in the coils

   B. Both rings will have the same current induced in them whose resultant repulsive field will affect the rate of fall of both magnets equally

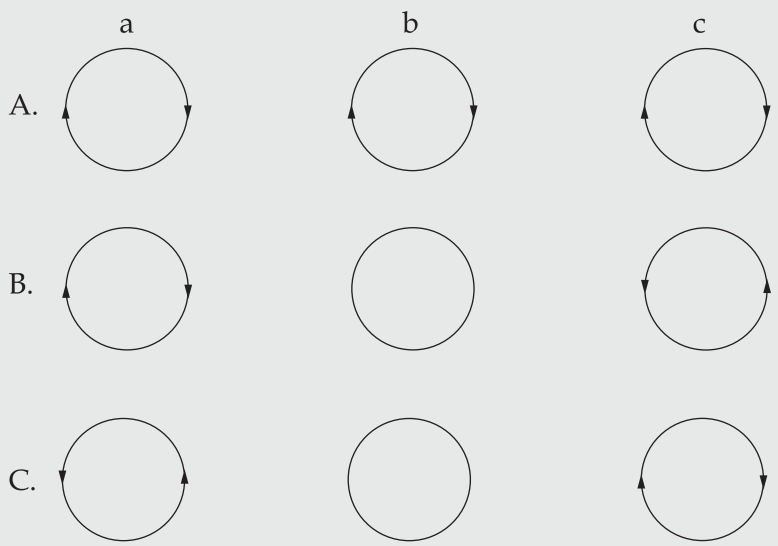
   C. The eddy currents in both rings will tend to reduce the effective weight of both falling magnets

   D. Magnet Y will fall faster than magnet X as it has no magnetic interaction force on it

2. This diagram shows a single loop of wire just entering a magnetic field that is directed into the page. Voltage is induced in the loop in 3 different positions: a, b and c.



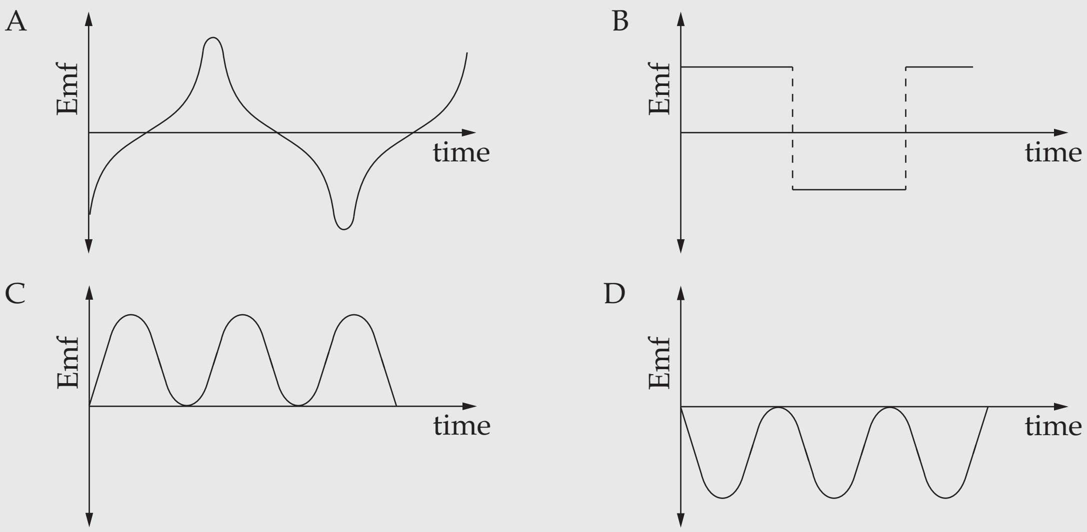
   The direction of the current flow in each of these positions is represented by which set of diagrams below?



3. A meteor of dimensions shown below enters the upper atmosphere of the Earth at a velocity of 25 km s^{-1} in an area where the vertical component of flux density is 35 μT.

What will be the emf generated between the sides of the meteor?

4. Cathy sits on a garden swing with an aluminium seat and swings back and forth so the seat cuts the Earth's magnetic field. Which of the following is a likely graph of the emf generated across the seat as a function of time, as Cathy swings to and fro? Explain your choice.



5. The Earth's magnetic field is about 50 μT, with an angle of dip at 68° in a certain gym. Estimate the maximum amount of magnetic flux that could be enclosed by a basketball ring.

6. A back-up generator for a theatre operates at a frequency of 50 Hz and generates an average voltage of 240 V AC. The area of the generator coil is 8.0×10^{-2} m^2 rotating within a field of flux density 0.30 T

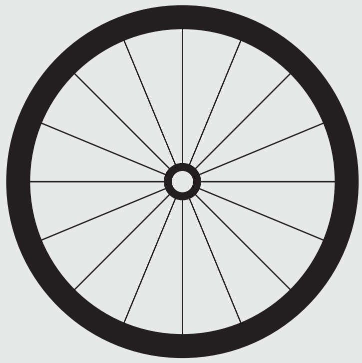
   a) Why must the coil constantly rotate to produce a voltage?

   b) How many turns does the coil have?

   c) What is the maximum voltage the generator produces?

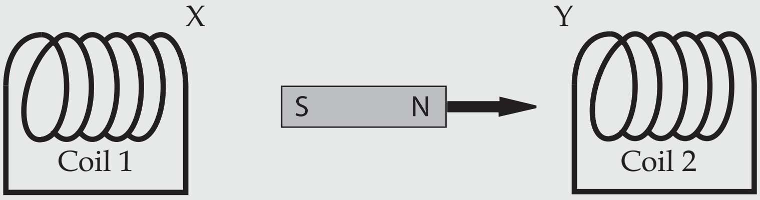
   d) Calculate the RMS value of the voltage.

7. A cyclist is riding at 36 km h^{-1} from east to west along a straight road on a bicycle with wheels of radius 32 cm. The cyclist realises that each spoke on the wheel is cutting the Earth's magnetic field and generating a voltage according to Faraday's Law.



   What is the approximate magnitude of the voltage induced between the axle and wheel rim at this speed?
   (Earth's horizontal field component on this road = 2.1×10^{-5} T, Earth's vertical field component on this road = 4.5×10^{-5} T)

8.



A magnet is placed between two coils (1 and 2) as shown. If the magnet is moved to the right a current is induced in both coils. By Lenz's Law, when the magnet is moved, the coils will become induced magnets. The polarities of the coil ends X and Y will be:

A. X-north, Y-south B. X-south, Y-north

C. X-north, Y-north D. X-south, Y-south

9. A helicopter has 4 blades each of length 5.20 m that rotate at a rate of 6300 revs per minute. If the Earth's magnetic field where the helicopter is operating has a total flux density of 3.80 μT at a dip angle of 66^{o}, what voltage is developed across the blades?

10. A man is sitting at night in his car in the dark with the engine running at 1100 revs per minute. He turned his headlights on and noticed that the engine speed immediately dropped to 950 revs per minute. A good explanation of why this happened would be:

A. The headlights need a greater voltage than before so the engine slows to compensate

B. More current is drawn now from the alternator so a greater torque is required to turn it round, which slows the engine

C. The back-emf from the battery causes a voltage drop along the wires which reduces the lamp brightness

D. Engine speed is indirectly proportional to the voltage drawn.

11. In a NASA experiment on electromagnetic induction several years ago a thin, stainless-steel wire was stretched from a space shuttle to a satellite, both orbiting the earth at different heights. The wire joining the two craft was 22.5 km long and the voltage obtained between the shuttle and satellite was 5.2 kV whilst the shuttle's velocity was 8.12 km s^{-1}.

What would these figures yield as a value for the Earth's flux density being cut by the wire?

12. The variation of resistance of a wire with temperature is given by the formula:

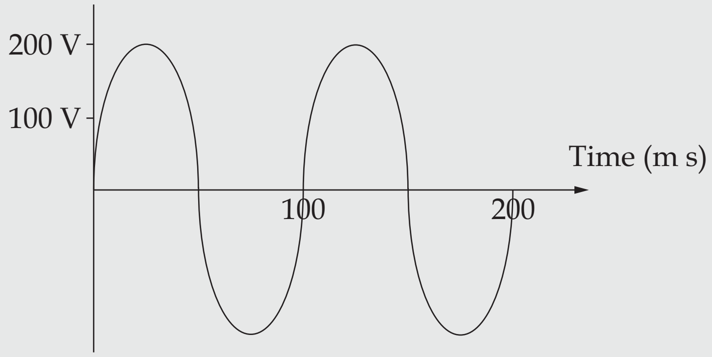
$$R = R_0 (1 + \alpha T)$$

R is the resistance at T degrees above the initial resistance value of R_0 and α is the temperature coefficient. α is defined as the fractional change in resistance per degree Celsius temperature rise.

The initial resistance of a nichrome electric heating wire at 20°C (R_0) is 24 Ω. The value of α for nichrome is 4.0×10^{-3} $^{o}C^{-1}$ and the normal operating temperature of the heater is 420° C which is connected to a 240 V supply.

What power is dissipated by the heater when it is at its normal operating temperature?

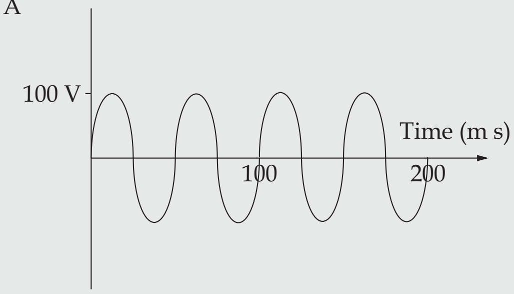
13. An Acme 1 generator is rotated by a small petrol engine at 600 rpm to produce an AC voltage displayed by the graph below.



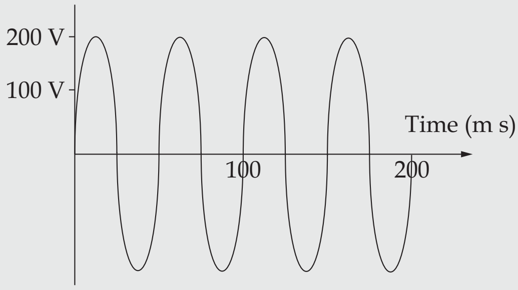
An Acme 2 generator has a magnetic field half as strong, it rotates at twice the speed and has a coil with half the number of turns as the Acme 1.

Which graph below best displays the voltage/time plot for the Acme 2 generator?

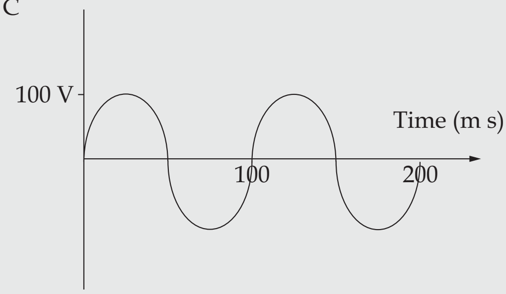
A



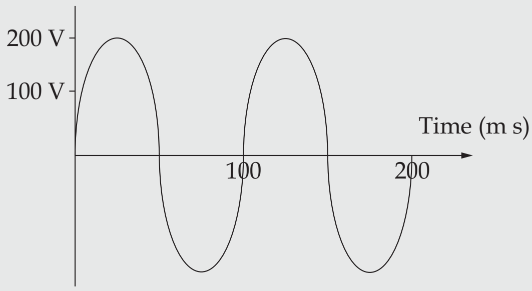
B



C



D



14. You have made a coil by winding a 1 metre length of copper wire around the cardboard tube in the centre of a toilet roll. You now place a magnet near the end of the coil. If the magnetic field near the end of the magnet is 0.12 T, estimate how quickly (i.e. in what time) you must pull the magnet away from the coil in order to generate an emf of about 1 V in the coil. State all your assumptions

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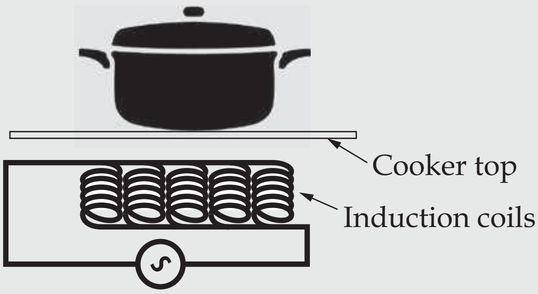
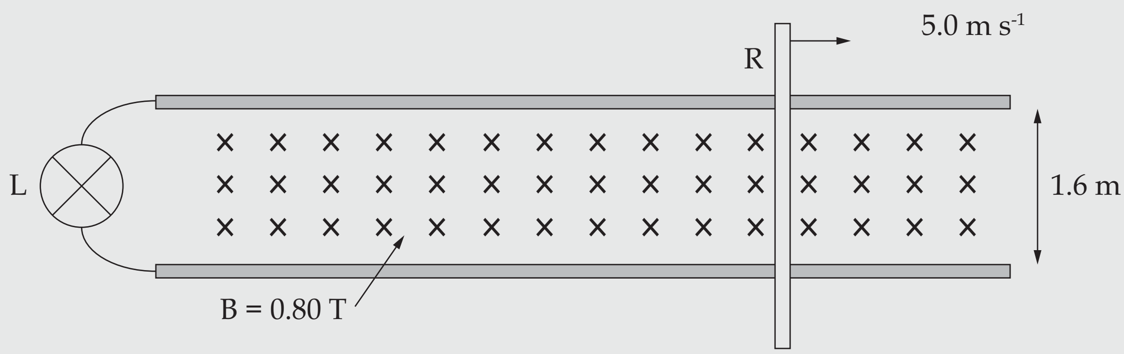
15. Induction cookers work by inducing large eddy currents in metal saucepans. This is done by switching a magnetic flux on and off very quickly, using a high frequency supply.



The induction coils shown above produce a magnetic field of flux density 0.045 T in the region above the cooker top which switches on and off at a rate of 10 kHz. The saucepan is made of aluminium with a resistance of 1.5 Ω and has a base diameter of 25 cm. Using these data calculate the heat supplied to the saucepan per second.

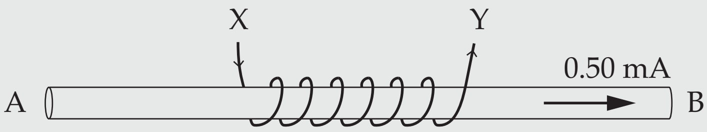
16.



In an experiment the apparatus is set up as shown, with two conducting rails containing a flux of density 0.8 T and joined at one end to a lamp L of resistance 96 Ω. A sliding copper rod R is moved to the right with a velocity of 5.0 m s^{-1}, generating a current which lights the lamp. When the current begins to flow a force is produced on the rod acting towards the left.

What is the size of this force?

17. A small coil, XY, is wound on a current carrying wire AB which carries a current



of 0.50 mA when a current is introduced into the coil XY, as shown:

(a) Will the current in AB:

Increase? Stay the same Decrease? (Circle the best answer)

(b) Briefly explain your answer.

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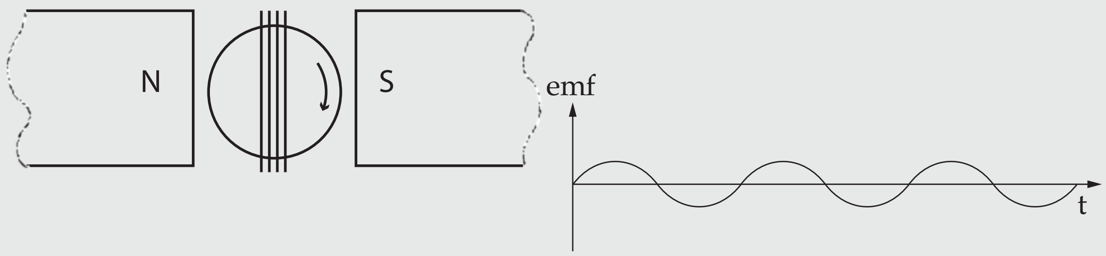
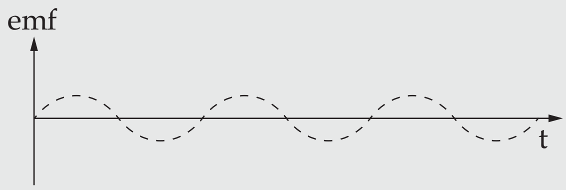
18. A coil is rotated in a magnetic field as shown. A graph of the voltage (emf) generated is also shown.

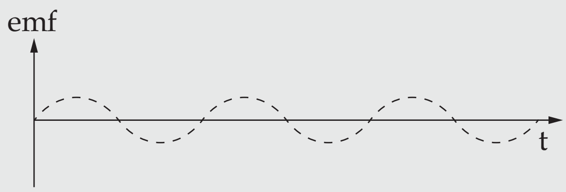


In each of the following cases, sketch (on the diagram) a graph of the output voltage when the changes indicated are made. Give reasons for your answers. The dotted line represents the voltage before the change has been applied.

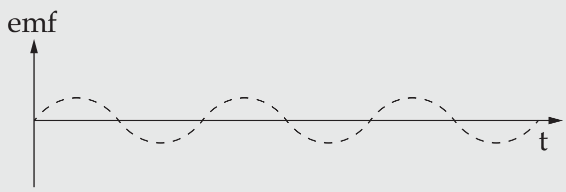
(i) The magnitude of the magnetic field is doubled.



(ii) The number of turns in the coil is doubled.



(iii) The rate of rotation of the coil is doubled.



19. A bicycle has an electric generator attached to the rear wheel to provide power for a headlamp. The barrel of the generator rubs against the bicycle wheel, which causes it to rotate. Inside the generator there are 400 turns of wire forming a coil with an average area of 140 mm^2. The coil rotates between the poles of a permanent magnet.

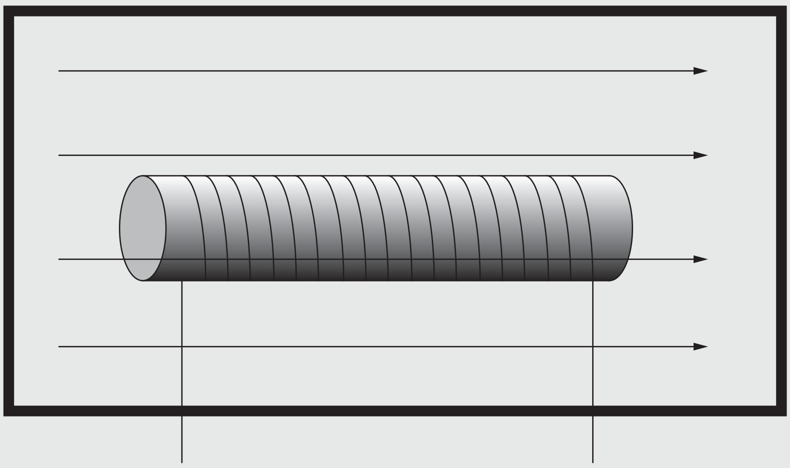
    When the bicycle is ridden at 1.50 m s^{-1}, the generator is rotating at 5100 revolutions per minute and generates an average voltage of 6.60 V AC across the lamp, whose resistance is 1.10 Ω.

    (i) What is the frequency of the AC voltage generated?

    (ii) What power is provided to the lamp?

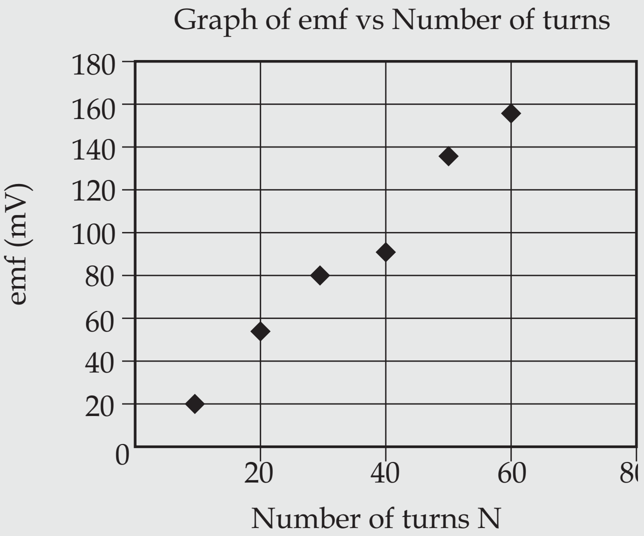
    (iii) What is the approximate magnetic field strength passing through the coil?

20. Henry investigated the relationship between the emf induced in a solenoid and the number of turns on it. First, he built six solenoids each made of the same insulated copper wire and consisting of circular loops 25 mm in radius



He then placed these solenoids, one at a time, into a device that contained a uniform magnetic field of intensity 0.420 T and arranged the solenoid so that the magnetic field was aligned with the long axis of the solenoid. The device had a switch that uniformly reduced the magnetic field intensity to zero over a standard time period t.

The graph Henry obtained from his experiments is shown below.



Draw a line of best fit for the points shown in the graph and determine its gradient.

Use the gradient to determine the standard time t over which Henry's device reduced its magnetic field to zero. Show your working.

2.21 TRANSFORMERS

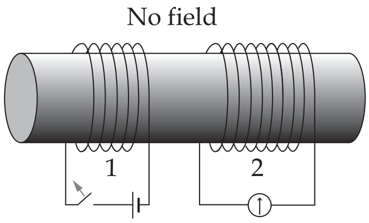


Diagram (i)

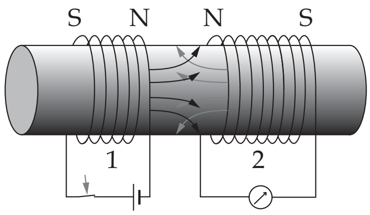


Diagram (ii) Positive voltage

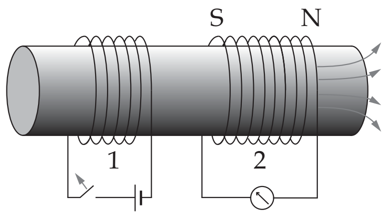


Diagram (iii) Negative voltage

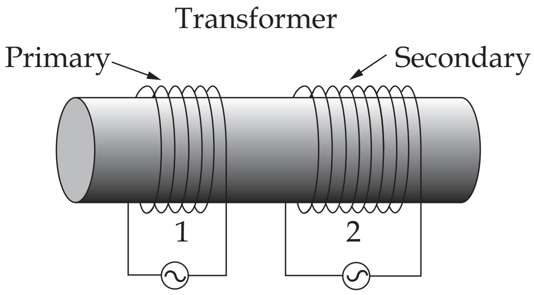
In a transformer a current change in one coil can induce a voltage in a second coil close to it.

In diagram (ii), when the battery is connected to coil 1, a flux is produced that links to coil 2. The north pole produced at the right hand end of coil 1 induces another north pole in coil 2. To oppose this change in flux, a current flows through the meter from left to right.

In diagram (iii) the current in coil 1 is turned off so the flux linked to coil 2 starts to collapse. This causes the flux produced by coil 2 to reverse in order to oppose the reduction in flux. This means that the current in coil 2 must reverse so it now flows from right to left.

Alternating Current

With alternating current, the voltage is constantly switching from positive to negative 50 times per second. This means that if an AC voltage is connected to the Primary coil of the transformer then the flux linked to the Secondary will constantly be changing and so an AC output from coil 2 will be produced which is out of phase with the input at coil 1.



Suppose the flux produced by the input voltage of the Primary is 0.5 Wb over a cycle – then the same flux change will occur in the Secondary (assuming no loss of flux).

However, if the Secondary has twice as many turns as the Primary then, according to Faraday's law, the emf induced will be twice as much. Hence we are able to obtain a larger voltage from the output of the Secondary than that which is applied to the Primary. A transformer set up like this is called a step-up transformer as the number of coils on the output side is greater than those on the input side and the voltage is stepped up.

Power output

Of course, we cannot obtain more power from a transformer than is put in because power input must equal the power output (V_1I_1 must equal $V_2\,I_2$), assuming no energy loss.

In coil 1, if the AC voltage input is 24 V at 0.50 amps and the number of turns is 100 and coil 2 has 2000 turns then, from the turns ratio (1:20), the output voltage will be 24 x 20 = 480 V i.e. 20 times more.

Transformer turns ratio formula:

$\frac{N_p}{N_s} = \frac{V_p}{V_s}$ (N.B. The ratio of the turns on the coils is the same as the ratio of the voltages.)

However, due to conservation of energy considerations, the maximum output current available will be **20 times less** (0.025 A).

Current formula: $V_p \times I_p = V_s \times I_s$

N.B. The current actually drawn from the output at any time will depend on the resistance connected to it (called the Load) e.g. If a 100,000 Ω resistor is connected to coil 2. The current will be 480/100,000 = 0.0048 A, but if a 1 Ω resistor is connected as a load then the current drawn can only be the maximum value of 0.025 A. Hence transformers limit the amount of current that can be drawn from a supply.

With a step-down transformer the number of turns on coil 2 is less than the number on coil 1 and hence a lower voltage is produced at the output but a higher current can be drawn.

Step-up transformers are used where a high voltage is required with small current (e.g. to power a TV tube) and a step-down transformer can give a low voltage with a high current (e.g. mobile 'phone charger or an arc welder)

Example 14

A 240 V battery charger transformer has a turns ratio of 24:1 and draws a current from the mains of 0.100 A. What voltage is produced at the output and what maximum current can be drawn?

Solution 14

This is a step-down transformer so: $\frac{N_p}{N_s} = \frac{V_p}{V_s}$ $\frac{24}{1} = \frac{240}{V_s}$

Output voltage will be 240/24 = 10 V

$V_p\ I_p = V_s\ I_s$ so $240 \times 0.10 = 10 \times I_s$

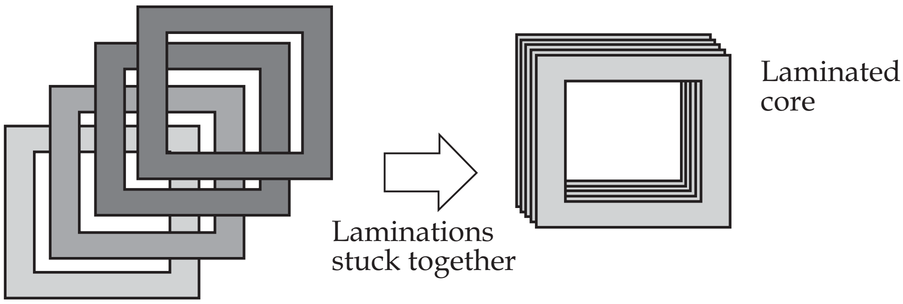
$I_s = 2.40$ A.

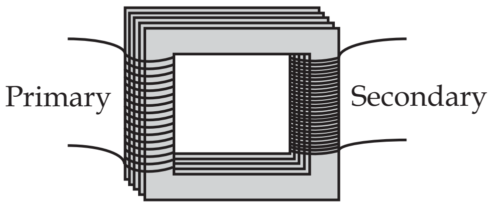
Commercial transformers.

The straight rod core shown in the previous diagrams suffers from a loss of flux through its ends which would lead to high power losses

Faraday's first transformer consisted of an output coil (Secondary) wound over the top of the input coil (Primary). The Primary and Secondary were wound onto a circular (or rectangular) iron core so the flux linkage between the two coils was 100%.

Modern transformers have a rectangular, laminated iron-alloy core with low magnetic energy loss. The insulated laminations prevent energy loss through eddy currents as these cannot flow from one lamination to another.





Other forms of energy loss in transformers are:

- Hysteresis loss – the energy needed to magnetise and then demagnetise the core
- Joule Heating – heat energy produced when a current runs through a wire.

Any energy losses within a transformer means that the power output would be smaller than the power input i.e. its efficiency is less than 100%. The coil connected to the power supply is called the Primary and the output coil is called the Secondary.

To minimise these energy losses efficient transformers have:

- A rectangular, laminated core design with efficient flux linkage from Primary to Secondary.
- Cores made from special alloys with low remanence (low retention of magnetic field) and low coercivity (a small amount of energy is required to magnetise the core)
- Thick, low resistance coil winding wires (usually copper)

Example 15

A step-up transformer is used to power a mains radio from a car battery. The 12 V DC supply is changed to alternating voltage by an electronic switch and then fed to a transformer. The step-up transformer used is 80% efficient and has a 240 V AC output

a) What is the transformer turns ratio?

b) If the output current is 200 mA, what is the power output of the radio?

c) What current is drawn from the battery to run the radio?

Solution 15

a) Ratio of turns = ratio of voltages = 12:240 or 1:20

b) P= VI = 240 x 0.2 = 48 W

c) P_{in} x 0.8 = P_{out} so P_{in} = 48/0.8 = 60 W

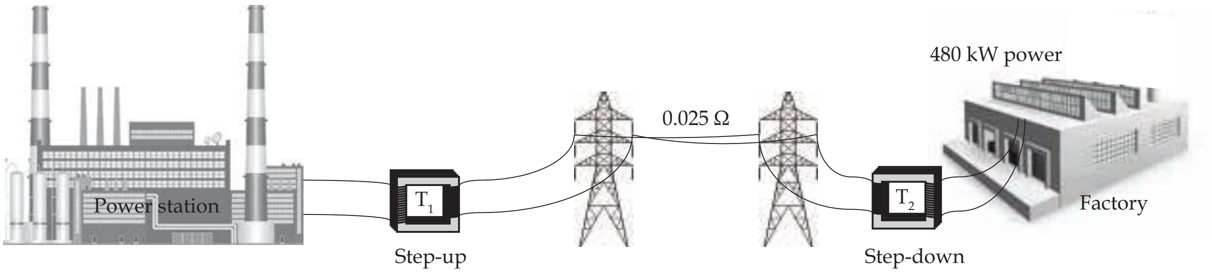
V_I x I_I = 60 so I_I = 60/12 = 5.0 A

2.22 AC POWER TRANSMISSION

Electrical power can be transmitted across the country at high or low voltages. Power stations generate alternating voltage so that transformers can be used to change the supply voltage to any convenient value (e.g. 5 volts for computers) without a great deal of power loss. However, when current runs through cables, energy is lost in the form of Joule heating (electrons colliding with metal atoms) – the greater the current, the greater will be the power loss, according to the equation:

$P_{LOSS} = I^2R$

In order to minimise the energy loss in hundreds of kilometres of transmission cables that make up the National Grid, power is transmitted at a high voltage and low current.



Suppose a power station needs to supply electricity to a factory requiring 480 kW of power at a supply voltage of 240 V AC through transmission cables with resistance 0.025 Ω.

Power could either be transmitted along the pylons at 240 V without the need for transformers or stepped up to 240,000 V for transmission and then stepped down again at the factory.

Let's look at these two alternative cases.

Case 1: 240 V supply

P = VI so for a total power of 480,000 W the current in the circuit will be 480,000/240 = 2000 A

The power loss in the cables would be $I^2R = (2000)^2 \times 0.025 = 100{,}000$ W = 100 kW

Hence, the power available for the factory will only be 480 – 100 = 380 kW, a 21% power loss.

The voltage drop across the cables will be IR = 2000 x 0.025 = 50 which leaves 240 – 50 = 190 V available for the factory. Case 1 is a very unsatisfactory situation!

Case 2: 240 kV supply

P = VI so for a total power of 480,000 W the current in the circuit at 240,000 V will be 480,000/240,000 = 2 A

The power loss in the cables will be $I^2R = (2)^2 \times 0.025 = 0.1$W

Hence, the power available for the factory will only be 480 – 0.0001 = 479.9999 kW, which represents a power loss of only 2.1×10^{-5} %.

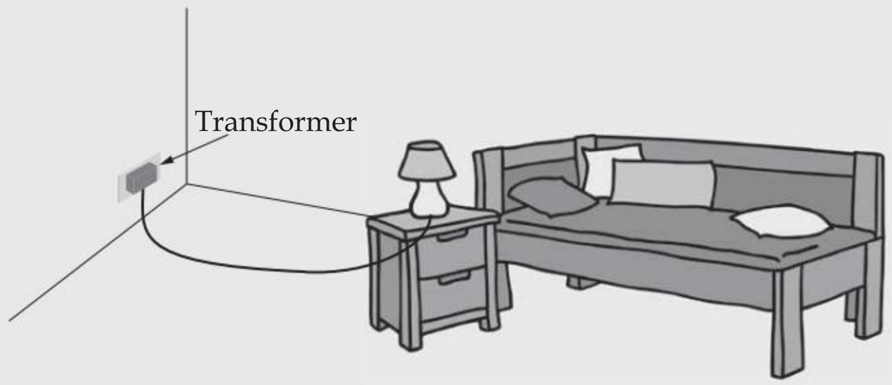
The voltage drop across the cables will be IR = 2 x 0.025 = 0.05 V, leaving 240 – 0.05 = 239.95 V available for the factory.

Clearly Case 2 is a much more efficient system!

The higher the transmission voltage, the lower the current and the lower the power that is lost in heating the cables - but there is a limit to the size of the transmission because higher voltages have associated problems with insulation. Insulators can break down and air can start conducting, especially when it is raining so a pylon transmission voltage of 330 kV is chosen as a compromise throughout Australia for transmitting power across the country.

Set 10: Power Transmission

1.



Jill has a small quartz-halogen bedside lamp that runs on 12 volts AC from a 12 V transformer. She has two choices for connection:

1. Plug the transformer into the wall socket 3 metres away and have a low voltage cord to the lamp running along the floor (as shown)
2. Have a 240 volt extension cord that is 3 metres long from the socket and plug the transformer into this behind the bedside cabinet.

Which statement about the two different choices of wiring is true?

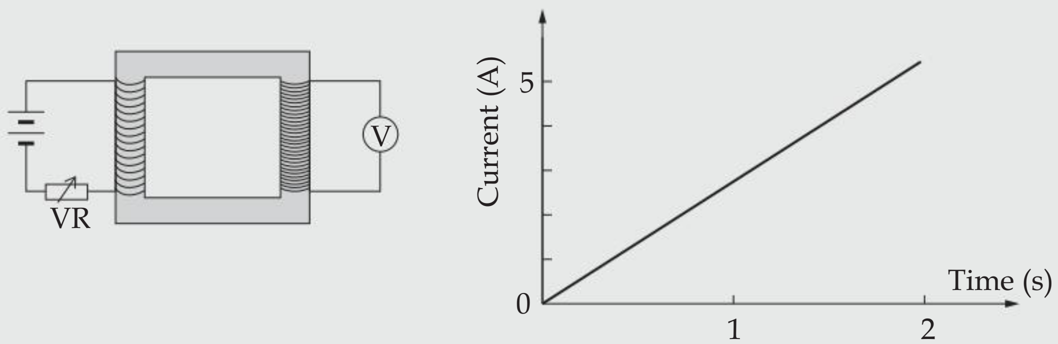
A. Choice 1 is better as there will only be a small current running out to the lamp which is safer

B. Choice 2 is better as the extension lead is thicker and can carry more current

C. Choice 1 is better as the power supplied to the lamp is reduced by the transformer, making it safer

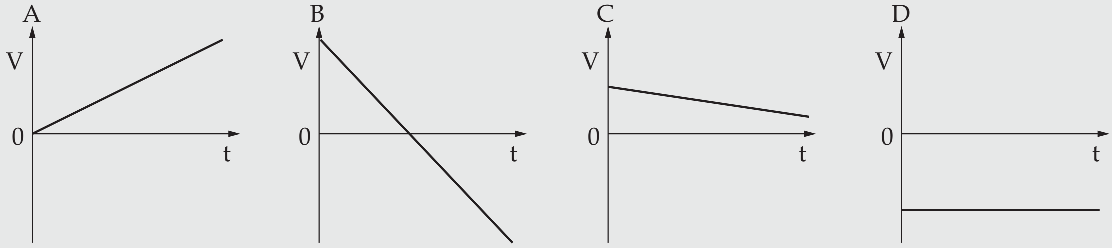
D. Choice 2 is better as there will be a higher voltage across the cord and less power loss

2.

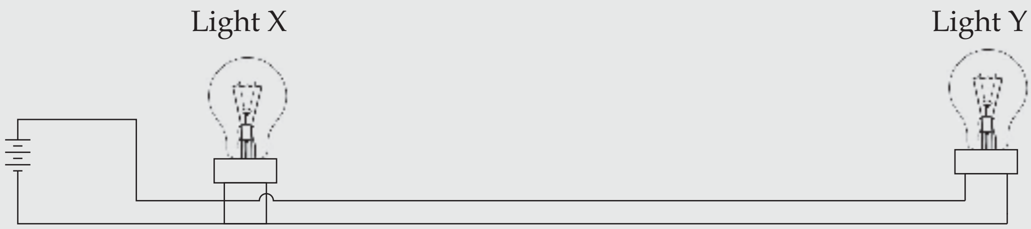


A DC supply was connected to the left hand coil shown above through a variable resistor connected in series. A voltmeter was then connected to the right hand coil which is linked through an iron core. The current in the left coil was then increased constantly with time as shown in the graph

Which graph below correctly describes the variation of Secondary coil voltage versus time?



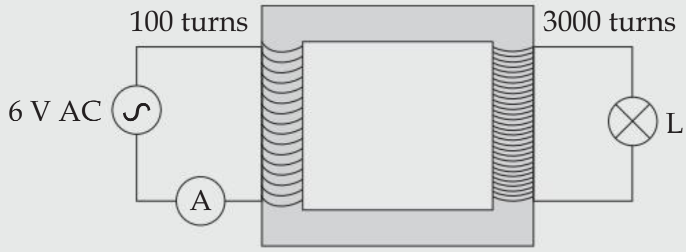
3. Two identical high power lights are connected to a battery in the manner shown in the diagram, the second being connected by an extremely long pair of wires, similar to an extension cord. Because of the length of the wires, their resistance is not negligible.



Explain any difference in the brightness of the lights.

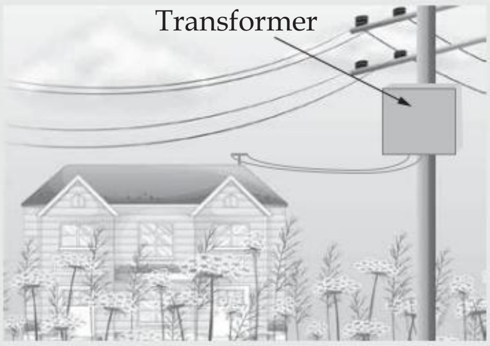
4. After replacing the lights with others of the same power but rated for the new voltage, would the answer to Question 3 change if you used a high voltage power source ? Explain.

5. A transformer has 100 turns in its primary coil and 3000 turns in its secondary. The primary coil is connected to a 6 V AC power source with an ammeter reading 0.9 amps.



What is the maximum current that the lamp L could theoretically draw?

6. A transformer supplies 240 volts AC to cables leading to a farmhouse that is 350 m from its position on a power pole. The cables to the farmhouse each have a resistance of 1.8×10^{-3} Ω per metre length. A cooker in the farmhouse of resistance 24 Ω and is turned on so that a current is drawn from the transformer.



What voltage would be delivered at the cooker socket?

7. The farmer in the house referred to in question 6 decides to build a work shed on his land close to the house, which contains a large saw-mill machine for cutting logs. This machine draws a current of 60 A whilst it is running. He connects the shed cables in parallel to the house circuit. The effect this machine will have on the voltage at the house and the correct reason is:

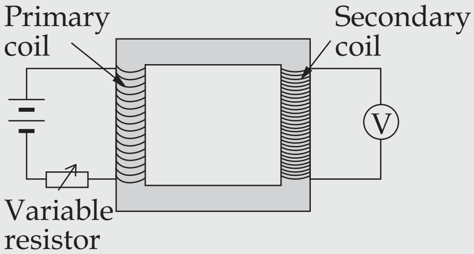
A. It will make no difference to the house voltage as the current drawn by the saw is on a different circuit.

B. There will be a large voltage drop in the supply wires as more current is drawn by the saw which will lower the house voltage.

C. The house voltage will remain constant with any current drawn by both circuits because total power cannot be created nor destroyed

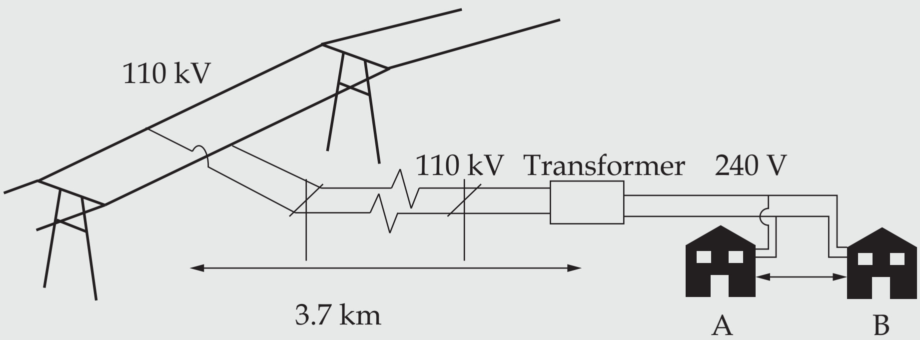
D. The voltage at the house will increase as the total power is now shared between two circuits, instead of just one.

8. The primary coil of a transformer is supplied with current from a battery that can be reduced to zero with a variable resistor. The maximum flux that links the two coils is 0.06 Wb. This flux passes completely through the Secondary coil which has 240 turns and has an attached voltmeter reading the Secondary emf.



Whilst the flux was reduced from maximum to zero at a constant rate the reading on the voltmeter was steady at 6.6 V. What was the time that was taken to reduce the flux to zero?

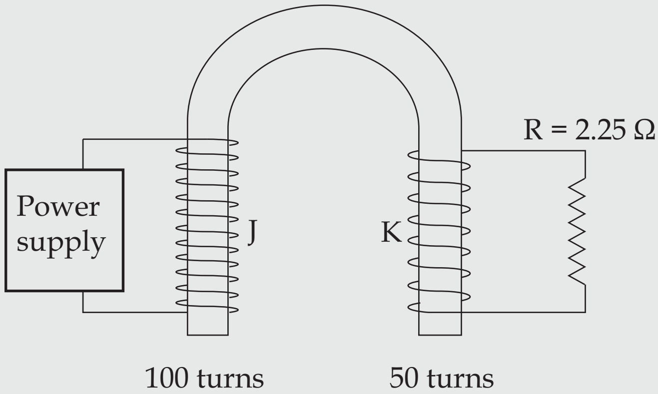
9. The buildings on a farm are situated 3.7 km from the 110 kV mains power lines. Two houses on the farm are to be connected to the electricity, as shown in the diagram. The wires from the transformer to the houses have a resistance of 2.250×10^{-3} Ω m^{-1}.



(a) Explain why the transformer, which drops the voltage from 110 kV to 240 V, is placed at the end of the 3.7 km line rather than at the beginning.

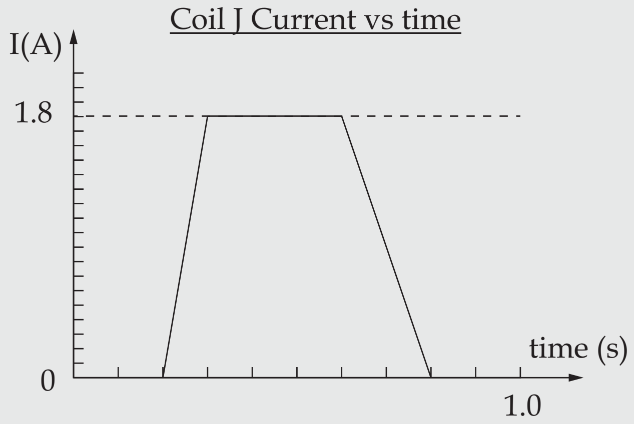
(b) The farmer turns on an electric kettle of resistance 28.8 Ω in Farmhouse A. The total distance from the transformer to the farmhouse along the wires is 70 m. What power is actually delivered to the kettle?

10. In carrying out an experiment on magnetic induction, a student wound two coils J and K on a soft iron core as shown in the diagram. A resistor R is connected to the Secondary coil K with a resistance of 2.25 Ω.



The soft iron core links the Primary to the Secondary coil with a cross-section area of 2.4×10^{-4} m^2 and the formula for the magnetic field strength in coil J is given by $B = 2.51 \times 10^{-3} \times I$ tesla

The student passes a current through coil J so that it varies as shown in the graph. Draw a sketch graph of the current in coil K (actual values of the current should be calculated).

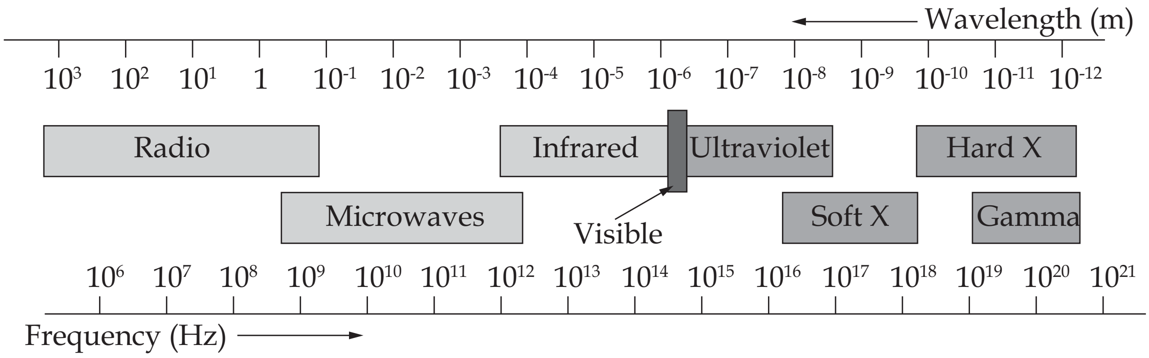


Chapter 3

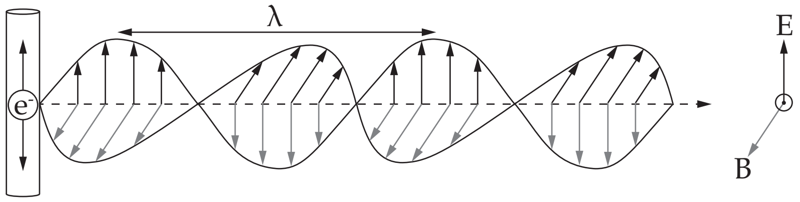
Waves and Quanta

3.1 ELECTROMAGNETIC WAVES

Electromagnetic waves exhibit the same phenomena as sound waves in that they can diffract, interfere, form standing waves etc. but they are generated in an entirely different way and they can travel much faster. They do not need a medium to propagate and so can travel through the vacuum of space faster than sound waves can in air. Examples of electromagnetic waves are: Light waves, radio waves, x-rays and microwaves. These all travel about 3×10^8 m s^{-1} but differ only in their wavelength. A spectrum of electromagnetic wavelengths is shown below.



All of these waves are caused by the same process whereby electrons in atoms are accelerated in some way. This causes a transverse electromagnetic wave to be emitted in all directions. If electrons are made to travel up and down a wire quite slowly a radio frequency wave is radiated outwards. If electrons fall from one shell in the atom to another visible light may be emitted and if electrons are fired at a metal so they decelerate very rapidly x-rays will given off. These waves are all the same kind of radiation but have different transmission properties in a medium.

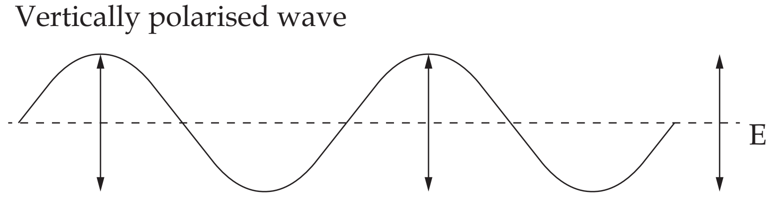


The diagram shows how an oscillating electron (e^-) produces a changing electric field (E) and this in turn generates an oscillating magnetic field (B) at right angles to it which goes out into space and is self-propagating: a changing E produces a changing B, which produces a changing E which produces a changing B, and so on. And so all electromagnetic waves contain no particle of any mass, but are simply moving, oscillating fields. When the wave strikes electrons in a material, it causes them to oscillate under the action of the fields. Hence we perceive light because the field from the incoming wave causes the electrons in our eyes' retinal cells (rods and cones) to oscillate and send a current down the optic nerve to the brain.

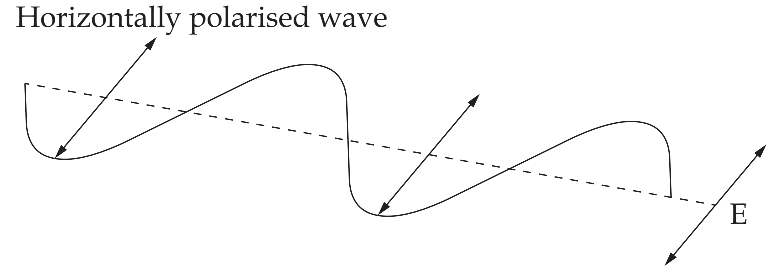
3.2 POLARISATION

This is a phenomenon that can only occur with electromagnetic waves because of their transverse nature. Light waves from most sources are not polarised i.e. the direction of vibration of the electric vector is random.

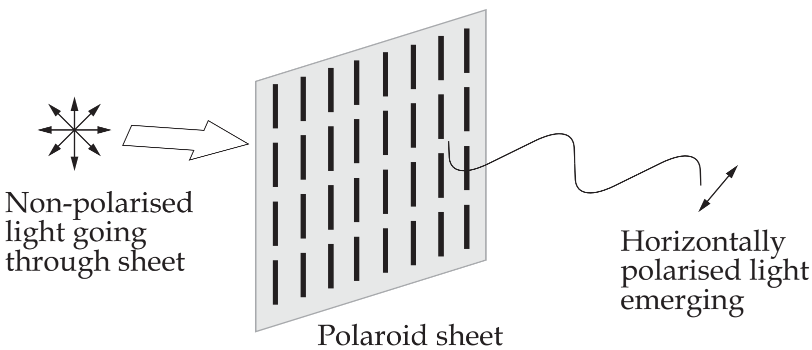
With vertically polarised light the vibration of the electric vector is up and down.



Horizontally polarised light has its electric vector vibrating side to side



Non-polarised light can be made to be polarised in a particular direction by passing it through a sheet of Polaroid plastic. As the wave passes through a Polaroid sheet only one direction of vibration will be allowed through. Polaroid is made of long chain molecules all orientated in the same direction and so all electric field vectors in that direction are absorbed. If the chains of molecules in the Polaroid are stretched vertically then only light waves that have their electric field in the horiontal direction will be allowed to pass through it. This is how Polaroid sunglasses reduce reflected glare from the sun and reflected light.



Example 1

Explain how Polaroid sunglasses reduce the glare from a reflective surface, such as a lake?

Solution 1

When unpolarised light strikes a reflective surface the vertically polarised electrical vectors of the light are absorbed by the surface, leaving only the horizontal electric vectors. The Polaroid lens material in the glasses have their long molecules arranged horizontally so the horizontally polarised vector is absorbed, reducing the glare.

3.3 REVOLUTIONS IN MODERN PHYSICS

At the beginning of the 19th Century new paradigms of atomic physics had emerged from Rutherford's experiments, such as Bohr's solar system model of the atom, where electrons were thought to rotate around a central nucleus.

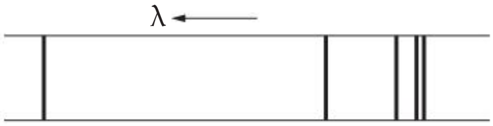
This new model produced some questions physicists could not answer from their current views of energy.

It was known that whenever a charge was accelerated it emitted energy in the form of an electromagnetic wave, just as radio waves are produced by accelerating the electrons in an aerial. However, physicists could not explain why electrons orbiting the nucleus in the Bohr Model and undergoing constant centripetal acceleration did not continuously radiate energy as electromagnetic waves. If they did it meant they would consequently spiral into the nucleus as their angular momentum and radius decreased.

The proposed electron orbital idea, then, would be non-existent – and yet the idea of orbiting electrons was essential in the Bohr model of the atom.

Another major problem at that time was that no-one could explain the pattern of the electromagnetic spectrum produced by glowing gases as they were excited when electricity was passed through them.

The sun produces a continuous spectrum, consisting of every colour of the rainbow gradually blending from one colour to the next.



Line spectrum of a gas

The spectrum of an excited gas, however, was completely different, comprising a set of single, coloured lines with black spaces in between.

It was known previously that the wavelengths of these lines bore a strict mathematical ratio – a quadratic function. The equation governing the wavelength of a line when the electrons in a gas were excited was of the form: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

n_1 and n_2 being integers and R is called the Rydberg Constant (= 1.097×10^7 m^{-1})

Example 2

Calculate the longest wavelength of light emitted in the Lyman Series where n_1 = 1 and n_2 = 2.

Solution 2

$\frac{1}{\lambda} = 1.097 \text{x} 10^7\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

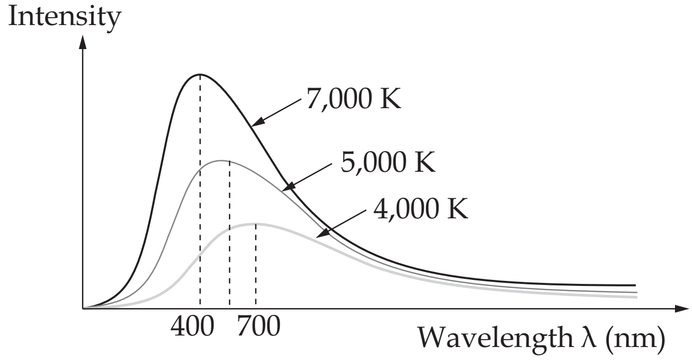
m = 122 nm

Another major dilemma for physicists at this time was also centred on their inability to explain the intensity/wavelength pattern for Black Body Radiation.

It was known that, as a black object (Black Body) is heated, the proportion of different electromagnetic wavelengths changed from long to shorter waves. i.e. cold objects radiate more IR and hot objects emit more blue and UV light. The true representation of a Black Body in reality can only be a cavity (hole) inside a metal ball which can emit and absorb all radiation.

The graphs below show the emission spectra for Black Bodies at different temperatures.

Blackbody radiation curves at several different temperatures.



Wien's Law, derived from these results, relates the wavelength at which the peak intensity λ_p, occurs. λp is inversely proportional to the kelvin temperature T.

Wien's Law: $\lambda_p = \frac{b}{T}$ or $\lambda_p \times T$ = constant

From the graph we can see, for example, that at 4000 kelvins 700 x 4000 = 2.8×10^6 and at 7000 kelvins 400 x 7000 also equals 2.8×10^6. This shows that Wien's Law ($\lambda_p \times T$ = constant) is valid.

b is called Wien's Constant and a more accurate value is given as 2.90×10^{-3} m K (note that the figures in the graph above are using nanometres as units).

Example 3

Calculate the peak intensity wavelength for a spotlight operating at a temperature of 1800 °C.

Solution 3

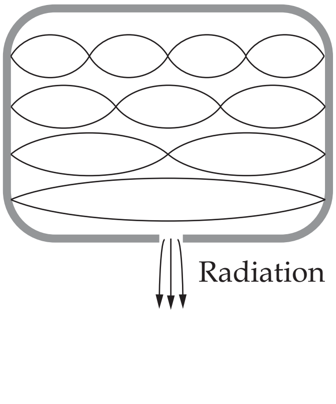
Temperature in kelvins = 1800 + 273 = 2073 K

$$\lambda_p = \frac{b}{T} \quad \frac{2.9 \times 10^{-3}}{2073} = 1.40 \times 10^{-6}\,\text{m}$$

The traditional explanation of the way emitted energy increases inside a Black Body cavity was that standing waves are set up which have the walls as boundaries. Existing ideas on energy in the early 20th century were that energy increased in a linear manner with temperature so, as the Black Body got hotter, more standing wave nodes and antinodes could be formed. Hence, as temperature rose the number of high energy (short wavelength) waves would increase.

Black Body cavity

Standing waves



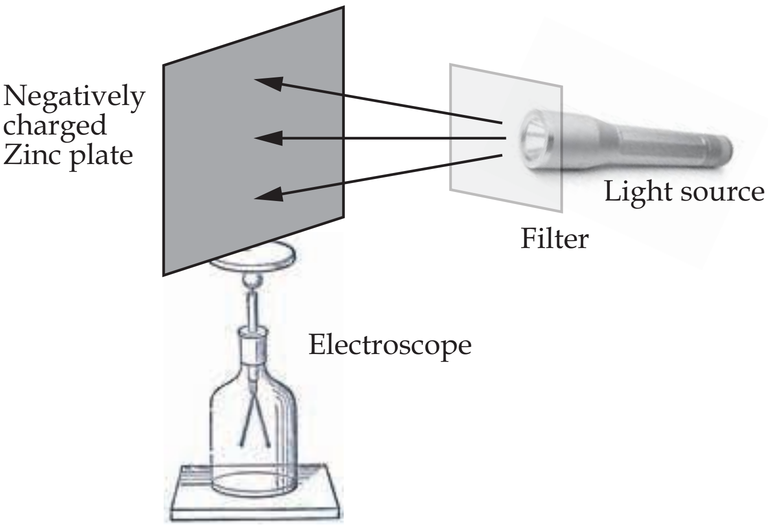
This Rayleigh-Jeans model predicted that the intensity of radiation would rise to infinity at shorter wavelengths which was completely contravened by the experimental results. This inability to explain the shape of the experimental radiation graphs was termed "The Ultraviolet Catastrophe", where the predicted energies massively refuted the experimental values.

The problem was in the fundamental assumption that, as the temperature rose the number of possible harmonics increased and that energy was independent of wavelength.

Hence at the beginning of the 20th Century there was a large chasm between existing conventional physics explanations and models, and the observed physical phenomena.

3.4 THE PHOTOELECTRIC EFFECT (MAX PLANCK)

Revelations to all of these major problematic areas in physics came as a result of Max Planck's seminal experiments with charged metal plates and the discovery of the Photoelectric Effect.



Planck was trying to see if he could discharge electrons from a zinc plate using the energy from light waves emitted by lamps of different wavelengths. Basically he was trying to "boil off" the electrons with the light energy available.

According to Classical Physics, this process should have been analogous to a candle heating up a beaker of water, where the water molecules gradually absorbed more and more energy up to boiling point.

In the Classical Model, when all the energy from the lamp had been "shared out" between the numerous electrons in the metal and they had all reached "boiling point", only then would they be able to leave the plate and the electroscope leaves would fall.

Planck's results initially stunned him and are summarised in the table below

λ	Dim light	Bright light
Red	No discharge	No discharge
Violet	No discharge	No discharge
UV 410 nm	No discharge	No discharge
UV 403 nm	Discharge	Discharge

From these data Planck concluded that only the 403 nm wavelength waves could provide enough energy to extract the electrons from the metal and that the intensity of the light seemed to play no part at all. There seemed to be a critical wavelength for discharge to occur because, no matter how long the light of 410 nm was shone onto the plate, no discharge occurred.

A calculation based on the amount of energy being shared between all the electrons predicted that "boiling" should occur after some thousands of seconds, whereas, in fact, the first electron was discharged after less than 1 nanosecond! (10^{-9} of a second).

What was obvious from the Planck experiments was that the shorter the wavelength the greater was the energy contained in the photon. This proportionality was expressed in Planck's equation: $E = hf$

E = energy of photon
f = frequency of wave
h is a constant that was determined later to have a value of 6.63×10^{-34} J s, called Planck's Constant.

Because $c = f\lambda$ this leads to another equation $E = \frac{hc}{\lambda}$

Example 4

Calculate the energy contained in a photon of ultraviolet light with a wavelength of 91.1 nm.

Solution 4

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{91.1 \times 10^{-9}} = 2.18 \times 10^{-18}\ \text{J}$$

There is also another revolutionary idea that came from the fact that the time of electron emission was so small that the energy from the light could not have been shared out between all the electrons, as would happen with a wave. The only possible interpretation of the fact that an electron was emitted so quickly was that all the energy from the light was given to one electron at a time and not shared between the large number of electrons (> 10^{20}). The energy in a wave would be spread over the whole wavefront but for all the energy to transfer to one electron meant that the energy must be channelled into a very small space. This means that the light energy must be arriving as a kind of particle. This "particle" of light is called a Photon.

The small size of h means that photon energies are extremely small (around 10^{-19} joules)

For example, to find the energy contained in a photon of light of wavelength 403 nanometres:

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{403 \times 10^{-9}} = 4.93 \times 10^{-19}\ \text{J}$$

Photon energies quoted in joules are quite unwieldy, so they are converted to electron-volts (eV) which gives much more convenient numbers. To convert from joules to eV we must divide by the energy contained in 1 eV, which is 1.60×10^{-19} J, so the energy above would be:

$$E = \frac{4.93 \times 10^{-19}}{1.60 \times 10^{-19}} = 3.08\ \text{eV}$$

Obviously a whole new theory of energy was required to explain this, which then evolved and was termed the Quantum Theory. With this model, all forms of energy are contained in very small packets, called quanta. With light, each photon had to give its energy to a single electron and energy can only be received or emitted as an integral number of photons.

The Classical Physics idea where energy existed as a continuum had to be discarded and replaced with the rather strange notion that, when dealing with extremely small objects, such as atoms, all energy is received and radiated in small packets which are indivisible. Each packet contains a quantum of energy equal to h times the frequency of the wave that is observed. This rule of energy emission and absorption would also apply to the macroscopic world, where large energies are involved but this "packets" idea had never been noticed because of the extremely small size of the packets (~ 10^{-19} of a joule).

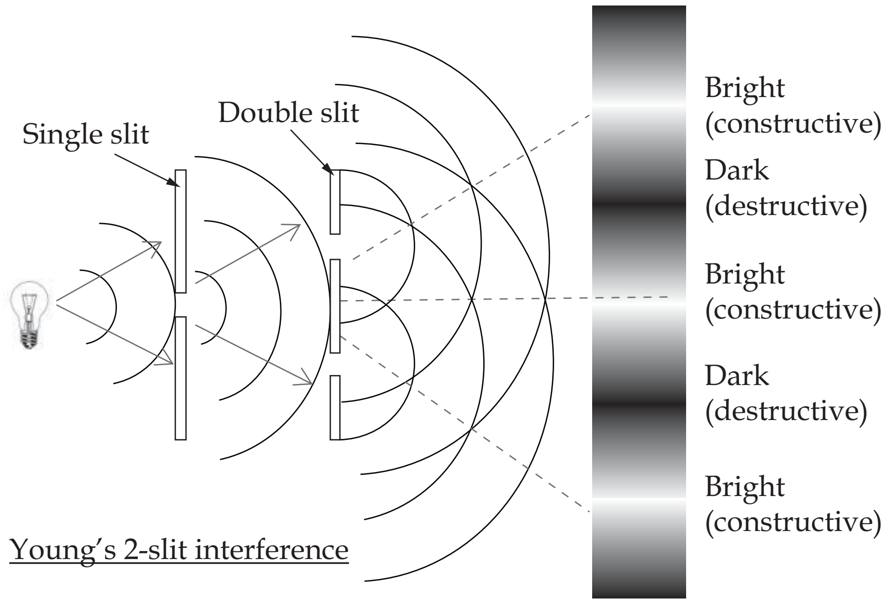
Of course, if we consider diffraction and interference phenomena, these can only be explained using a Wave Model - so there existed a dichotomy between the 2 models, where light could behave as a wave or a particle, depending on the circumstances.

Today physicists have to live with this idea of wave/particle duality, where the two totally different models of light as being waves and being particles (called Photons) are used separately in different situations (e.g. Diffraction and interference – Wave model; Spectra – Particle model.)

3.5 YOUNG'S 2 SLITS EXPERIMENT.

Young performed an experiment in 1801 where he managed to get 2 light sources to interfere with each other. This cannot be done with 2 separate lamps because we now know that light waves are emitted at random times from any source which would prevent superposition for more than a nanosecond or so. Young used a single lamp but passed its light through a single slit so the wave spread by diffraction and then caused the single wavefront to pass through another 2 slits. This effectively produced 2 sources of light which had to be coherent (i.e. in phase) and produce circular waves through diffraction again. The light source was also monochromatic (single colour, not white).

The set-up is shown below.



Young's 2-slit interference

The interference pattern is formed because the distance to a bright point is longer from 1 slit than the 2nd slit – there is a path difference (pd) between the two waves. Because one wave has travelled a longer distance it is "out of phase" with the other. If the pd equals a whole number of wavelengths then the two wave crests will overlap at that point (shown with dotted lines) and will be in phase (matching crests).

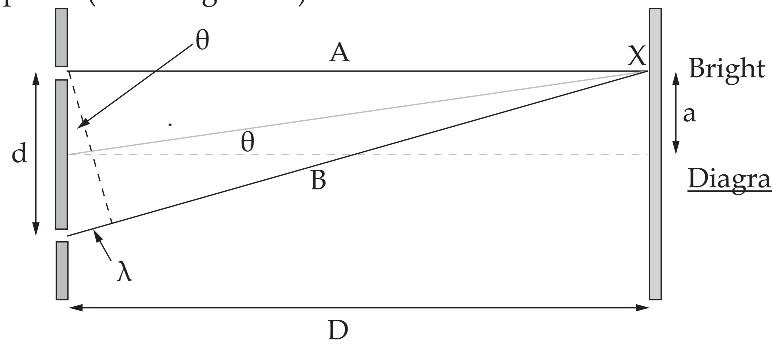


Diagram 1

In the diagram above wave A contains n wavelengths in reaching point X on the screen. If the length of wave B, in reaching X contains n + 1 wavelengths then the two waves will be in phase and point X will be bright due to constructive interference.

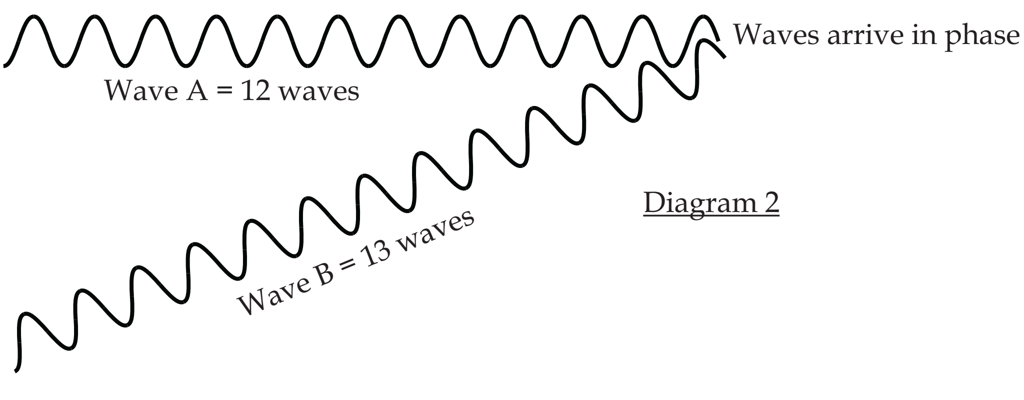


Diagram 2

The dark fringes occur because at that point wave B is an extra ½ wavelength longer than wave A (or any odd number of half wavelengths). This will cause the electric vectors to cancel due to destructive interference.

In the small triangle on the left of diagram 1: $\sin\theta = \frac{n\lambda}{d}$

And in the large triangle on the right $\sin\theta = \frac{a}{D}$ (true for small angles)

Hence the Young's double slit formula is

$\frac{n\lambda}{d} = \frac{a}{D}$ or $\lambda = \frac{ad}{nD}$

n is the number of bright fringes counted from the middle.

Young's interference fringes experiment demonstrates the wave nature of light.

Example 5

The screen in a Young's double slit experiment is placed 1.25 m from the slits and the angle where the 2nd bright fringe occurs is 0.230 degrees from the horizontal. If the distance between the 2 slits is 0.330 mm, calculate the wavelength of the light being used in this experiment.

Solution 5

$\sin\theta = \frac{n\lambda}{d}$

$n = 2$ so $\sin 0.230 = \frac{2\lambda}{0.330 \times 10^{-3}}$

$\lambda = \frac{\sin 0.23 \times 0.33 \times 10^{-3}}{2} = 6.62 \times 10^{-7}$ m

3.6 SOLUTIONS TO THE CLASSICAL PROBLEMS

The Young's 2 slit experiment can only be explained using a wave model of light but the Photoelectric Effect can only be explained by regarding light as a particle. This produced an enigma because both models could never be used concurrently.

The newly-determined notion that energy can only be absorbed or radiated in quantum packets served to resolve the enigmas created by the three problematic experimental observations mentioned previously:

- Stable electron orbitals
- Line spectra of gases
- The ultraviolet catastrophe.

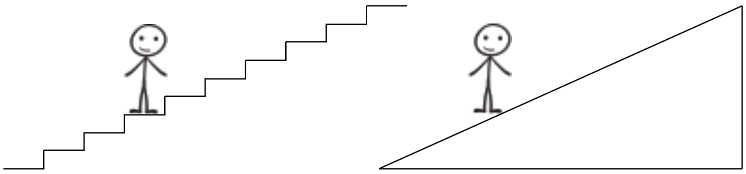
An electron rotating with kinetic energy around a nucleus would not be able to radiate electromagnetic waves as it accelerated unless it was able to lose an exact number of packets of energy. An excited electron can drop down to a lower energy state by emitting one or more photons but if it is in its lowest energy state then it cannot lose energy whilst accelerating. It is not possible to lose a part of a packet – it only ever loses one packet or none, which was in opposition to previous, Classical, ideas of energy.

The notion of energy levels in any atom comes about from the same "Packets of energy" idea, where a rotating electron can move up to the next Energy Level by absorbing a quantum of energy and can only fall down from one level to another when it loses energy. It is not possible to fall part-way down as it must always exist on one of the energy levels in the atom.

As it will be seen later, the difference between each energy level gets smaller ($1/x^2$ function) as the number of energy levels becomes greater. This leads to specific lines emitted by a hot gas as electrons fall from one prescribed level to another (see Chapter 4).

With Black Body radiation, the Classical Model maintained that each standing wave should have an equal chance of occurring and hence there would be an equal amount of energy in each harmonic. This lead to the prediction that there would be much more energy emitted in the UV region as there is more probability of having higher harmonics. However, Planck had established that the higher frequencies (shorter wavelengths) would contain more energy in their quanta as E = hf. As these UV packets are larger, there would be less of them, not more, and hence the energy graph comes down at the short wavelength end.

The question that arose from these results was whether **all** the energy in the Universe is quantised i.e. do the other forms of energy exist in packets or steps, rather than being continuous. This would mean, for instance, that a person standing on a hill could only take up fixed positions of potential energy, each step being one quantum level up from the next.



If energy were quantised the person could either stand on one step or the next step – but not half way between.

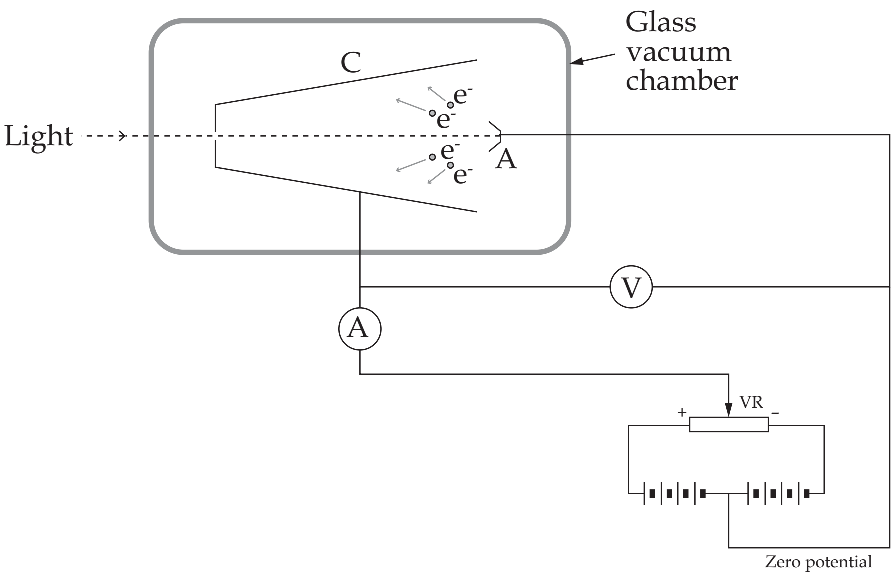
Einstein maintained that, yes, all forms of energy exist in packets but obviously we wouldn't notice these steps because they are so small (around 10^{-19} J)

With very small energy transitions, such as in atoms, the quantisation principle is much more noticeable because electron orbital levels have energies of around 10^{-19} J.

3.7 EINSTEIN'S PHOTOELECTRIC THEORY

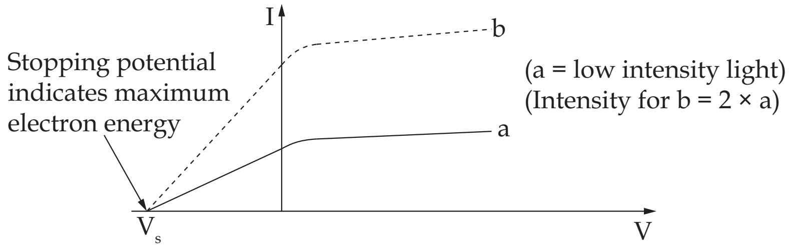
Here we will go back to the Planck experiment in more detail and outline Einstein's contribution to the interpretation and mathematical derivations for which he received the Nobel Prize.

Planck performed a more rigorous experiment than previously where he could measure the energies of the emitted photoelectrons. The apparatus is shown below.



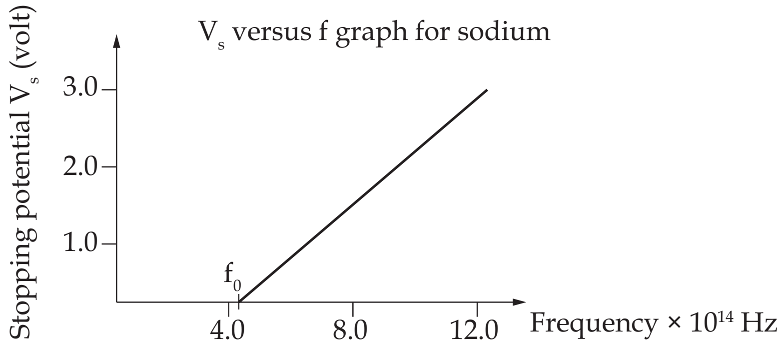
Light was shone through a glass window on to the Anode A to cause photoelectrons to be emitted as they gain kinetic energy from the photons of light. As these electrons strike the Cathode C a current would be recorded by the ammeter in the circuit reflecting the number of electrons emitted.

By moving the variable resistor (VR) to the left the cathode can be made more negative with respect to the anode, which is kept at zero potential. By giving the cathode a negative charge the photoelectrons could be repelled by the electric field and hence their kinetic energy could be determined by the formula $Vq = \frac{1}{2} mv^2$ (q = electron charge). The graph of current versus applied voltage for this experiment is shown overleaf.



As can be seen from the graph, if the intensity of light is increased more electrons are emitted (more current) but as the anode voltage becomes more negative a stopping potential V_s is reached. This is the voltage at which the photoelectrons have a maximum kinetic energy for the incident waves as the negative field is sufficient to suppress the emission of the fastest electrons.

According to Einstein's theory, an electron is ejected when the energy of the incident photon exceeds the energy binding the electron in the metal due to attractive forces inwards. This energy is akin to ionisation energy and is called the Work Function of the metal W_o. Each metal has a different value of W_o. If more energy than W_o is supplied by a photon of higher frequency, then the extra energy will go into ejecting the electron with a higher velocity (more E_K). This is expressed in Einstein's Photoelectric formula: $hf = E_{Kmax} + W_o$.



Another graph produced from the experiment plotted stopping potential Vs against frequency f.

This graph shows how, as the frequency of incident light increases the kinetic energy of the emitted electrons also increases in a linear manner. At 4.39 x 10^{14} Hz there was no electron current so this frequency (f_o) must represent the minimum energy needed to emit electrons i.e. the Work Function. $hf_o = W_o$

Hence the overall equation for the p/e effect is $hf = V_s q + W_o$

Or: $hf = V_s q + hf_o$

Giving: $V_s = \frac{hf}{q} - \frac{hf_o}{q}$ which would yield a straight line graph of V against f whose gradient is $\frac{h}{q}$ and whose intercept is $\frac{hf_o}{q}$.

Example 6

Using the gradient of the graph above, obtain a value for Planck's constant.

Solution 6

The gradient from the graph is $m = \frac{3.0}{(11.63 - 4.38) \times 10^{14}} = 4.14 \times 10^{-15}$

So $h = 4.14 \times 10^{-15} \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34}$ J s

Example 7

Use the graph above to obtain a value for the Work Function of sodium.

Solution 7

Taking an intercept value of 4.4 on the x-axis: $4.4 \times 10^{14} \times 6.63 \times 10^{-34} = 2.92 \times 10^{-19}$ J

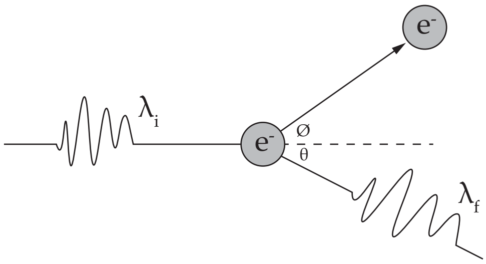
Hence W_o = 1.82 eV.

3.8 COMPTON'S EXPERIMENTS

A. H. Compton performed some important experiments where he directed x-rays at stationary electrons in a carbon matrix and observed the wavelengths of the incoming x-ray photons and the emitted photons. He found that the wavelength had changed to be longer after a collision had occurred.

Compton scattering

A photon with initial wavelength λ_i strikes an electron so that it recoils at an angle Ø. The emitted photon/wave is now emitted at an angle θ to the original direction and has a wavelength which is longer, λ_f.



Compton reasoned from the Law of Conservation of Momentum that if the electrons had gained momentum from their stationary positions then this must have been provided by the x-rays. Hence he deduced that electromagnetic waves must have momentum. As momentum is associated with particles and equal to mass x velocity, then this supported the idea of x-rays as photons, or packets of energy. Also, the energy carried by the incident x-ray must equal the sum of the energies of the emitted electron and the emitted x-ray photon.

Compton found that the change in wavelength (Δλ) between the incoming and outgoing waves depended on the angle of emission of the outgoing photon.

He derived an equation for this: $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_o c}(1 - \cos\theta)$

The constant $\frac{h}{m_o c}$ is called the Compton wavelength and gives the Δλ for a photon scattering at $\theta = 90^o$. The value for the Compton wavelength is 2.43×10^{-12} m, or 2.43 pm

Example 8

Calculate the wavelength shift expected when an x-ray strikes an electron, if waves are scattered at an angle of 45^o

Solution 8

$\Delta\lambda = 2.43 \times 10^{-12}(1 - 0.7071)$

$\Delta\lambda = 7.12 \times 10^{-13}$ m or 0.712 pm

3.9 DE BROGLIE MATTER WAVES

As physicists realised that waves could behave as particles in certain circumstances, De Broglie proposed that high speed particles might also behave as waves from the newly-discovered principle of wave/particle duality.

Sir Lawrence Bragg had previously won the Nobel Prize in 1915 for his investigation of X-ray crystallography. By shining x-rays onto the surface of crystals he could determine the atomic spacing from the diffraction pattern and hence the arrangement of atoms in the crystal. As the x-ray waves bounce off the vertical layers of atoms interference occurs between the waves due to path difference and this produces nodes and antinodes of intensity of the reflected light.

X-ray diffraction pattern produced by a single crystal of sodium chloride.

Imagine the surprise of the Scientific Community of the day when researchers Davisson and Germer, in 1928, produced similar results with fast-moving particles. Using a beam fast moving electrons incident on a nickel surface they produced a very similar pattern to the x-ray diffraction patterns of Bragg! The only way these bright and dark spots could arise from electrons fired onto a metal surface was if the fast-moving particles were behaving as waves to produce interference patterns - just as electromagnetic waves do.

De Broglie produced a formula linking the wavelength of a fast-moving particle with its momentum p. (p = mv)

De Broglie formula: $\lambda = \frac{h}{mv}\left(\lambda = \frac{h}{p}\right)$

λ is the wavelength associated with an electron of mass m moving with a velocity v.

This means that an electron moving with a speed of 1×10^7 m s^{-1} would have a momentum value of $9.11 \times 10^{-31} \times 1 \times 10^7 = 9.11 \times 10^{-24}$ kg m s^{-1}

The electron's wavelength $\lambda = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-24}} = 7.28 \times 10^{-11}$ m which is around the same value as an x-ray and comparable to the spacing of the atoms in a crystal which would allow it to produce interference patterns.

If moving objects exhibit wave-like properties, why do we not notice this in our everyday lives?

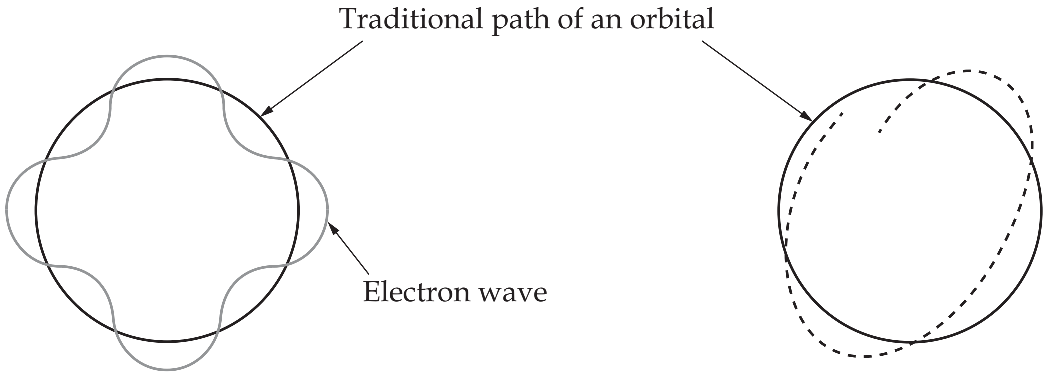
Consider a 20 gram bullet moving at 500 m s^{-1}. Its associated wavelength would be

$\lambda = \frac{6.63 \times 10^{-34}}{0.02 \times 500} = 6.63 \times 10^{-35}$ m, which could not possibly be observable.

So if particle momentum is high (e.g. bullet) the wave nature will be indiscernible, as there is no way to measure such small wavelengths. However, with fast electrons the momentum is tiny, which makes the associated wavelength comparable to that of an x-ray, from which observable interference phenomena can be seen. It is because of the extremely small value of h that the wave nature of matter could not reveal itself in Classical physics until the technology of measuring such small values could be made on the atomic scale.

3.10 ELECTRON ORBITALS

So far electrons in atoms have been regarded as orbiting in circles at very fast speeds (about 3 million orbits per second), but in view of the De Broglie principle of electrons having a wavelength, we can now regard the electron orbitals as standing electron waves set up around the nucleus. 'Resonance" occurs if a wave can superimpose upon itself around the orbital.

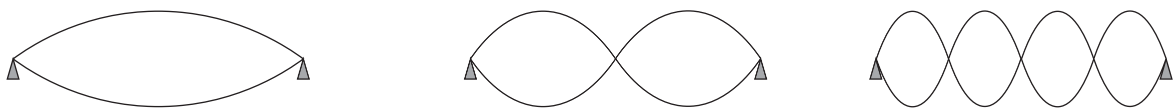


Whole number of wavelengths = Constructive interference – an allowed Energy level. Resonance.	Fractional number of wavelengths = Destructive interference – not an allowed Energy level

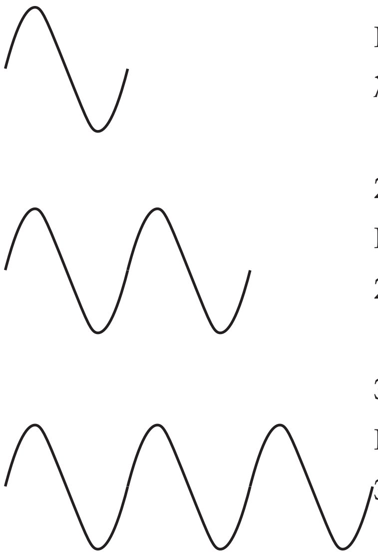
This idea of orbitals corresponding to resonance levels within an atom is similar to the allowed resonances in a vibrating string. Thus the total energy in an atom is quantised (exists in discrete steps) and not continuous, due to the allowed number of standing waves.

With electron wave resonances:

1st resonance level 2nd resonance level 3rd resonance level



1st resonance level.



Principal quantum number n = 1

$\lambda_1 = 2\pi r_1$

2nd resonance level.

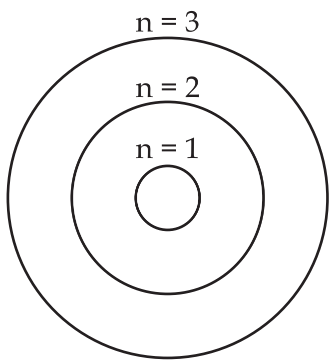
Principal quantum number n = 2

$2\lambda_2 = 2\pi r_2$

3rd resonance level.

Principal quantum number n = 3

$3\lambda_3 = 2\pi r_3$



Example 9

If the radius of a hydrogen atom (Bohr Radius) is 5.29 picometres (5.29×10^{-12} m), what is the value of the smallest wavelength of an electron that can produce constructive interference around the orbit?

Solution 9

Circumference of the hydrogen atom = $2\pi r = 2 \times \pi \times 5.29 \times 10^{-12}$ m = 3.32×10^{-11} m. For constructive interference, the circumference must equal the wavelength. So $\lambda = 3.32 \times 10^{-11}$ m.

3.11 HEISENBERG'S UNCERTAINTY PRINCIPLE

The principle of "Complementarity" means that the wave model and particle model are like two sides of a coin – you cannot observe both aspects at the same time. Similarly, when considering a particular effect (e.g. 2 slit interference), only one model (e.g. wave model) can be exclusively used: In revealing the wave character of matter, the particle character is suppressed.

Heisenberg considered how we make measurements and how the act of measuring a value actually affects the accuracy of that measurement. We could, for instance, measure the distance to the moon by shining a laser onto it and then measuring the time it took for the light to go out and come back. The impact of the laser photons used would actually cause the moon to recoil ever so slightly (Compton Effect) but, as the distance and masses of the objects involved are very large, this effect would be unobservable. Hence the position and momentum of the moon could be measured very accurately by this method. Not so with small objects however, as a photon fired at an electron in orbit would definitely affect its motion. Indeed, it might even knock the electron from its original position so we wouldn't know where it was in the first place! If we tried to reduce the impact of the photons by reducing their momentum, then, according to the De Broglie formula, the wavelength must be longer: $\lambda = \frac{h}{p}$.

If the wavelength was longer then more diffraction would occur, making the position of the electron more uncertain.

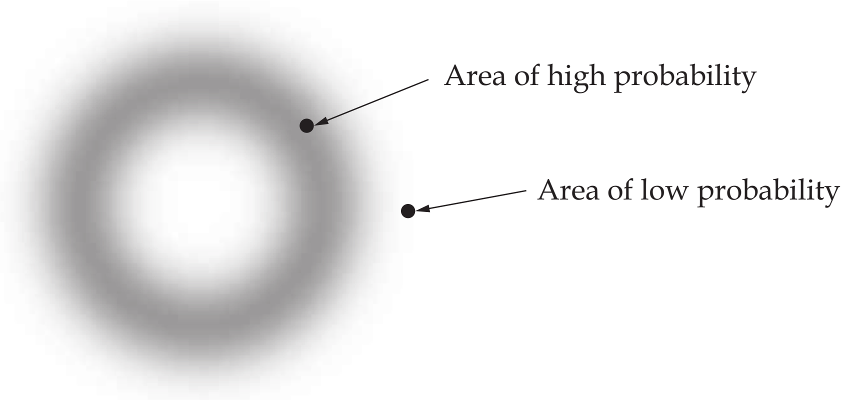
This notion of mutual exclusivity and complementarity led Heisenberg to the conclusion that the more accurately one variable can be measured, the less accurately another associated variable could be determined.

One formulation of Heisenberg's Uncertainty Principle is stated as: $\Delta x \Delta p \leq \frac{h}{4\pi}$

This equation shows that the accuracy with which the position of a particle is known (Δx) multiplied by the accuracy with which its momentum is known (Δp) will always be equal to a constant = $\frac{h}{4\pi}$.

In other words, if the momentum of, say, an electron is known very accurately then it would be impossible to know its position very accurately and vice versa.

This leads to a more statistical approach to descriptions of the atomic orbitals so that we can only determine the probability of finding an electron in a particular position.



If we were to specify a particular time to try to find where the electron was there would be a large uncertainty as to position. We could say the most probable position would be along the circular orbital but there would also be a range of possible places in which to find it decreasing as we go outwards from the orbital.

This view of uncertainty in the Universe led to another Heisenberg equation $\Delta E \Delta t \leq \frac{h}{4\pi}$ indicating that the more accurately the time is known for a particle, the less accurately its energy can be determined i.e. how far from the nucleus the electron is. The Uncertainty Principle also allows for a particle to come into existence for a very short time provided its mass/energy does not exceed a specific value, given by the above equation. So, in space, particles are popping into existence all the time from 'nothing' but only for short times of around 10^{-15} s (see "virtual photons" in the next section)

Example 10

Calculate the De Broglie wavelength of a proton in a Cosmic Ray shower that is moving at a speed of 3.40×10^7 m s

Solution 10

The momentum of the proton $m \times v = 1.67 \times 10^{-27} \times 3.40 \times 10^7 = 5.68 \times 10^{-20}$ kg m s^{-1}.

Using the De Broglie formula $\lambda = \frac{h}{p}$ so $\lambda = \frac{6.63 \times 10^{-34}}{5.68 \times 10^{-20}} = 1.17 \times 10^{-14}$ m.

Example 11

In the previous example, if we observed the track of that proton moving in a straight line, what would be the uncertainty in the position of the proton as it moved along the track?

Solution 11

The momentum of the proton is 5.68×10^{-20} kg m s^{-1} and according to Heisenberg's Uncertainty Principle:

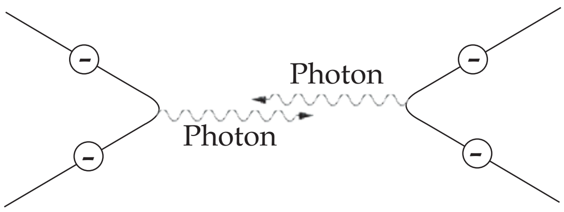
$\Delta x \Delta p \leq \frac{h}{4\pi}$ so the uncertainty in position $\Delta x = \frac{6.63 \times 10^{-34}}{4 \times \pi \times 5.68 \times 10^{-20}} = 9.29 \times 10^{-16}$ m

3.12 SUB-ATOMIC PARTICLES

The neutron was only discovered in the early 1930s, after which time it was believed that only three particles existed in an atom: the electron, proton and neutron. However, in the 1940s other particles began to be observed in cloud chamber pictures when nuclear collisions occurred. One of the great mysteries at this time was how protons, with their positive charge, could hold together in the nucleus against a massive electrostatic repulsive force. To account for this, another type of fundamental force was proposed, called the Strong Nuclear Force, which must over-ride this electrostatic force at very short distances inside the nucleus and cause attraction.

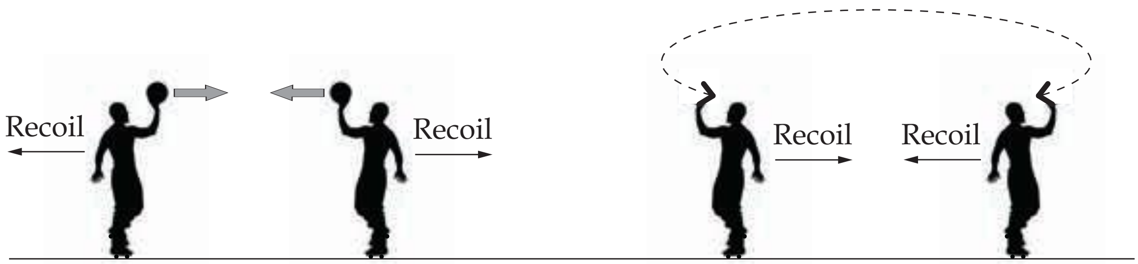
In the mid 1930's a Japanese physicist called Yukawa suggested that subatomic forces came about by the exchange of a particle or photon (remember, photons of light can be regarded as particles).

In Yukawa's model, as the electrons approach each other, both would emit virtual photons. This would not only cause the emitting electron to recoil, but would also cause the receiving electron to retreat in the opposite direction under the impact.



Virtual photons are not like the photons that travel through space, as they can only exist for an incredibly small time (which is allowed by Heisenberg's Uncertainty Principle). Their time of existence would be too small for them to be observed.

Hence, the force between two charges comes about by the exchange of photons. Imagine two people on skates throwing heavy balls to each other. Each person will recoil due to the momentum impact from throwing and the recoil from receiving the ball from the other person.



Repulsion Attraction

Attraction of particles (e.g. a + with a -) on this model occurs because of the recoil from the exchange of virtual photons, but in the opposite direction – rather like a boomerang being thrown backwards by the right-hand skater which then travels in an arc to be caught by the left-hand skater.

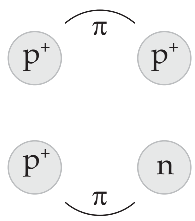
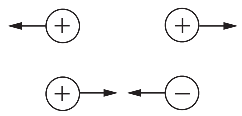
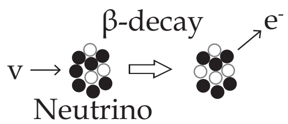
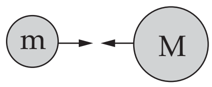
Thus was born the idea that all fundamental forces are brought about by the exchange of force-carrying particles called Bosons. Other than the electrostatic force, Yukawa suggested that the Strong Nuclear Force came about as a result of the exchange of another type of particle which he named the Meson. Particles that interact by the Strong Nuclear Force are jointly called Hadrons, which include Mesons and Baryons (protons and neutrons). Light particles are called Leptons and include Electrons, Mesons and Tau particles.

His calculations showed that this exchange particle would need to have a mass of about 250 times the mass of an electron, but, from the Uncertainty Principle, could only cause interactive attractions at a maximum distance apart of 10^{-15} m. The particle he predicted to exist was finally discovered in 1947 and named the pi meson (symbol π). Mesons were found to come in three types: π^-, π^o and π^+. Hence other fundamental particles, apart from the electron, proton and neutron were found to exist. Later on, even more different kinds of mesons were discovered, each having different masses and physical properties.

The notion that all fundamental forces in the Universe are brought about by the exchange of force-carrying particles later led scientists to search for the particles responsible for the Weak Nuclear Force which had been theorised to be involved in beta decay - where a neutron spontaneously decays into a proton and an electron. So-called W and Z bosons had been proposed to be responsible for the Weak Nuclear Force, but had never been observed in cloud chamber pictures due to the necessity of extremely high energies.

Finally, in 1983, a very high energy accelerator eventually revealed the existence of these particles and it has now been proposed that even the very weak gravitational force is mediated through particles called gravitons, however, so far, these have never been detected.

<u>Summary of forces and their relative strengths and ranges</u>

		Strength	Range	Boson
Strong Force (Holds nucleons together)		1	10^{-15} m	π (nucleus) Gluons (quarks)
Electromagnetic Force (Between charges)		1/137	∞	Photon
Weak Force		10^{-6}	10^{-18} m	W^+ W^- Z_o
Gravitational Force		~ 10^{-38}	∞	Graviton??

Antiparticles

The first antiparticle was discovered in 1932 – the positron. Positrons (β+) have the same mass as electrons, but have a positive charge instead of a negative one. Later, the antiparticles of the proton and the neutron were discovered and so the idea that for all normal particles a corresponding antiparticle will also exist.

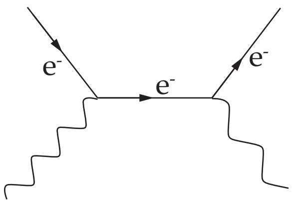
An antiproton is a "mirror image" of a proton, having the same mass but a negative charge however, an antineutron has a zero charge, like the normal neutron. An antiparticle doesn't just have an opposite charge, but is totally opposite in nature to its corresponding normal matter particle with virtually the same mass. If a normal particle comes into contact with its own antiparticle the two will annihilate each other and their total mass turned into energy in the form of gamma rays (γ):

$n + \overline{n} \rightarrow \gamma$, $p + \overline{p} \rightarrow \gamma$, $\beta^- + \beta^+ \rightarrow \gamma$

3.13 FEYNMAN DIAGRAMS.

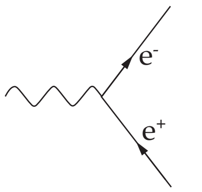
A very creative scientist, Richard Feynman developed a schematic system for showing particle interactions and decays. Incoming particles are shown on the left and outgoing particles are shown on the right i.e. time flows from left to right. Bosons are shown linking the incoming and outgoing particles. For instance, photons are shown as a wiggly line 

Examples of Feynman diagrams:



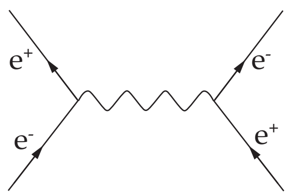
Compton Effect

Photon strikes a free electron which recoils and releases another photon with less energy than the first.



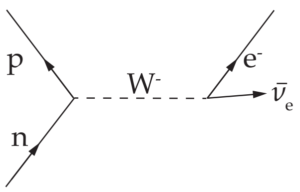
Pair production

The energy from a gamma ray can spontaneously convert to the mass of two particles (electron and a positron).



Electron-positron collision

(Note that antiparticles moving to the right are represented by an arrow pointing in the opposite direction). The two particles annihilate and produce a photon which, itself produces two antiparticles later in space.

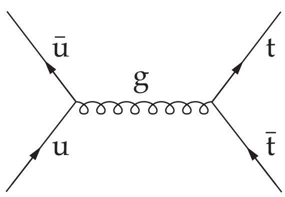


β-emission

A neutron changes into a proton with the emission of an electron and an antineutrino ($\bar{\nu}$).

Gluon interactions (see quark model later)

The gauge boson responsible for the forces between quarks, which exist inside hadrons, is called the Gluon. This causes attraction or repulsion in exactly the same way as the photon does with leptons (see the previous 'Skater" model) by boson exchange.



Here an Up quark interacts with an Anti-up quark to produce a Top and Anti-top quark through exchange of a gluon.

3.14 NEUTRINOS

In 1956, after carefully studying the mass and momentum of particles emitted during β-decay, physicists hypothesised that another type of particle might be involved in the reaction, due to slight apparent inequalities with both sides of the decay equation. Later, the very small particle emitted in addition to the proton and β particles was detected and was named the neutrino (symbol v). Neutrinos (and antineutrinos) had to have zero charge and an extremely small (if not zero) mass, with the ability to pass right through the Earth without interacting with it at all. This made the neutrino's initial detection extremely difficult and hence they were not discovered until the 1960s.

A neutron can decay in the nucleus: ${}^{1}_{0}n \rightarrow {}^{1}_{1}p^{+} + {}^{0}_{-1}e^{-} + \overline{\nu}$ (an electron and an antineutrino are emitted).

A proton can also decay in the nucleus: ${}^{1}_{1}p^{+} \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e^{+} + \nu$ (a positron and a neutrino are emitted).

The force holding a neutron or a proton together is called the Weak force which is mediated by another boson called the W boson.

Other types of neutrinos were thought to exist that were involved in the decay of other lighter subatomic particles. The μ neutrino (ν_{μ}) was later found to be emitted when a muon decays

(e.g. $\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$) and a τ neutrino($\nu_{\tau}$) is emitted in decays involving tauons.

Thus there are now known to be three different types of neutrinos. (ν_{e}, ν_{μ} and ν_{τ}), each with their own antiparticles ($\overline{\nu}_{e}$, $\overline{\nu}_{\mu}$ and $\overline{\nu}_{\tau}$). The electron, muon, tau and three types of neutrinos are collectively named Leptons (Light particles)

In summary, by the 1960's over a hundred sub-atomic particles had been found to exist. The lighter particles were considered to be truly elementary (e.g. electron and the neutrinos) but other, heavier, particles called baryons, seemed to possess some finer, internal composite structure. Because of the sheer number of these baryons, it was thought that they might be made up of smaller, truly elementary particles. Thus the Quark Theory was evolved by Gell-Mann and Zweig in 1963.

3.15 QUARKS

Gell-Mann and Zweig produced a much simpler model, where fewer basic components of matter were proposed, called quarks and which assumes that the basic unit of electric charge is not actually 1.6×10^{-19} coulomb but can be split into thirds of this value.

Quarks were deemed to have either $\frac{1}{3}$ or $\frac{2}{3}$ of an electronic charge which, when combined, could produce either a full negative or positive charge or a zero charge. It was later proposed that quarks had three attributes that give them different properties.

Quarks exist with different "flavours" i.e. different types.

Up or Down flavoured quarks (u = $+\frac{2}{3}$ charge, d = $-\frac{1}{3}$ charge.)

Charm or Strange flavoured quarks (c = $+\frac{2}{3}$ charge, s = $-\frac{1}{3}$ charge)

Top or Bottom flavoured quarks (t = $+\frac{2}{3}$ charge, b = $-\frac{1}{3}$ charge).

An example of how quarks can combine to form other particles is shown here:

Protons have 2 Up quarks and 1 Down quark (written as uud) with total charge = $+\frac{2}{3}$, $+\frac{2}{3}$ and $-\frac{1}{3}$, giving the proton a total of +1 electronic charge.

Neutrons are composed of 1 Up and 2 Down quarks (written as udd) with total charge = $+\frac{2}{3}$, $-\frac{1}{3}$ and $-\frac{1}{3}$, giving a total of zero electronic charge.

Antiquarks also exist to make such antiparticles as antiprotons and antineutrons. These are shown with a bar over the top e.g. anti-up is designated $\overline{u}$, so an antiproton would be composed of the quarks $\overline{uud}$ (2 anti ups and an antidown)

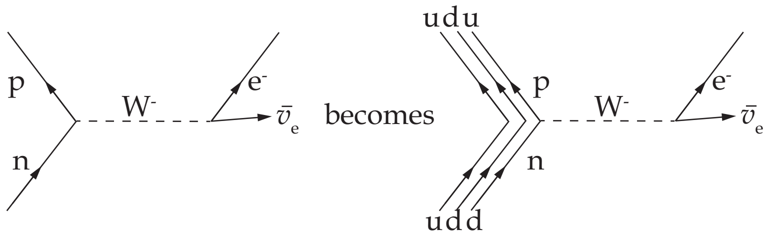
Other subatomic particles that are not baryons, such as the pion, contain only two quarks and are named Mesons,

e.g. Positive pion $\pi^+ = u\,\overline{d}$ ($+\frac{2}{3}, +\frac{1}{3} = 1+$ charge) and $\pi^- = \overline{u}\,d$ ($-\frac{2}{3}, -\frac{1}{3} = 1 -$ charge). Interestingly, a quark and its antiquark can co-exist within a meson e.g. π^o comprises u and $\overline{u}$ (zero charge).

The idea that meson exchange is responsible for the basic forces between particles has now been expanded to take into account the quark model. At present, it is believed that particles called gluons are exchanged by the quarks themselves to produce the forces holding them together and hence to hold the larger composite particles (baryons and mesons) together.

A modification of the Weak interaction would now indicate that one quark would change its flavour. The Feynman diagrams for this are shown below.

What actually happens here is that a Down quark becomes transformed into an Up quark –



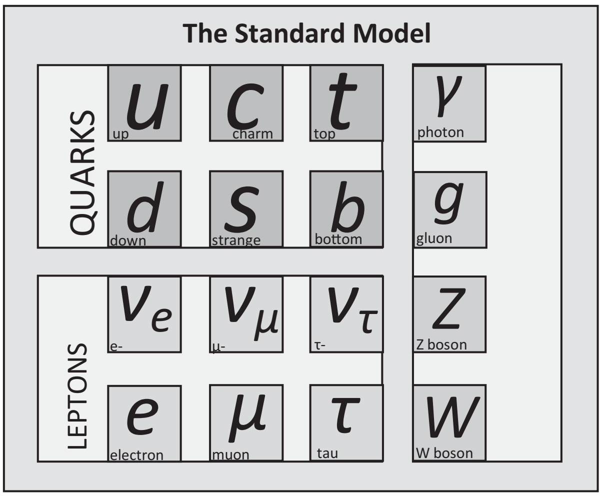
hence a neutron becomes a proton with the emission of an electron and an antineutrino.

3.16 THE STANDARD MODEL

In summary, in the current scientifically accepted model of matter there are 3 types of particles in all:

a. Those existing within the nucleus, called Hadrons, are all made up from quarks
b. Fundamental particles existing outside the nucleus, called Leptons (light particles)
c. Particles responsible for the 4 main forces existing in nature (electromagnetic, gravitational, strong nuclear and weak nuclear forces.)

A summary chart is shown here.



The combination of ideas laid out in this section, plus some later modifications, such as the 'coloured' and 'charmed' quarks, explain the nature and properties of the basic forces and particles of matter. This theory is termed The Standard Model.

Set 11: Waves and Quanta

1. What is the energy in electron-volts emitted by a radio aerial transmitting at a frequency of 94.5 MHz?

2. a) At what wavelength does the human body emit most of its energy? What assumption does your calculation involve?

   b) A star is emitting electromagnetic radiation at a wavelength of 1.00 nm at the peak of its spectrum. Calculate its surface temperature, assuming the star is a black-body radiator.

   c) What would be the peak wavelength of a black-body radiator that is at a temperature of 1727^{o}C.

3. a) What wavelength of light is required to just emit electrons from a metal which has a Work Function of 4.00 x 10^{-19} J?

b) If UV light of wavelength 300 nm is shone onto the metal, what is the maximum speed with which photoelectrons will be emitted?

4. In a photoelectric cell UV light of wavelength 300 nm is shone onto a metal with a Work Function of 4.00×10^{-19} J. Calculate the reverse voltage required to suppress all the electrons in the cell.

5. In a Young's 2 Slits experiment yellow light of wavelength 589 nm was used with a slits width of 0.15 mm.

   a) Calculate the spacing between the 1st and 2nd bright fringes projected on a screen which is 4.50 m from the slits.

   b) What would be the effect on the fringe spacing if light of a longer wavelength were used?

   c) What would be the effect on the fringe spacing if the slits width were decreased?

6. An x-ray photon of wavelength 2.50×10^{-11} m strikes an electron in a nitrogen atom. The electron is ejected and the x-ray photon is emitted at an angle of 60^{o} to its original direction. Calculate the wavelength of the emitted photon.

7. The velocity of an electron around a hydrogen nucleus is about 2×10^{6} m s^{-1}.

   a) If the mass of an electron is 9.11×10^{-31} kg, calculate the De Broglie wavelength of an orbiting electron.

   b) From the ideas of standing waves and resonance, show that the radius of the 1st energy level in a hydrogen atom is about 58 pm (5.8×10^{-11} m).

8. A motorist is recorded on a speed camera as doing over the 50 km h^{-1} speed limit. The photograph shows the car is doing 52.7 km h^{-1} and the camera is correct to ± 0.05 km h^{-1}. However, the motorist tells the policeman "According to Heisenberg, if you know my speed very accurately then you cannot know my position with any certainty. Hence my car may not have actually been in the 50 km h^{-1} zone at all!"

   a) Make a calculation using the uncertainty principle to find the error in knowing the position of the car, taking its mass as 1500 kg.

b) Compare the car situation to the uncertainty in position of an electron moving with a velocity of 3.00×10^{7} m s^{-1} with an uncertainty of ± 0.1%.

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9. a) Draw a Feynman diagram for β+-decay, where a proton converts to a neutron.

b) Show, on another Feynman diagram, the change in the quark structure of the nucleons.

10. For the sub-atomic particles shown in the table below state their overall charge and whether they are baryons, mesons or leptons.

Particle	Quark structure	Charge	Baryon/Meson/Lepton
Ω^-	s, s, s		
Λ_t^+	u, d, t		
K^+	$\bar{s}$, u		
Ξ^0	u, s, s		

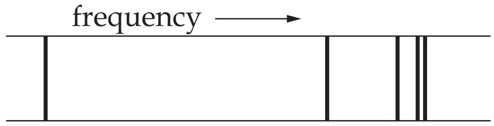
Chapter

4

Atomic physics and Relativity

4.1 ATOMIC SPECTRA

As mentioned in Chapter 3 the spectrum of a gas was found to consist of a series of coloured lines in a quadratic mathematical series to a limit which could only be explained if energies within atoms had quantised states.



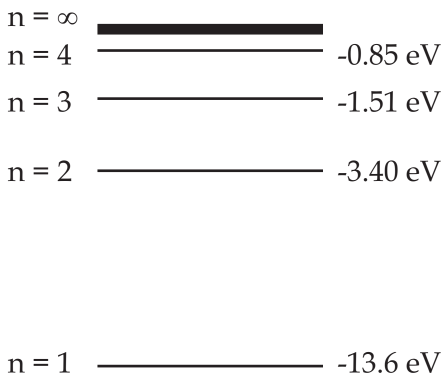
Line spectrum of a gas

The empirical Rydberg Equation (Chapter 3) showed the quadratic nature of the line spacing:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Each electron normally resides in its lowest energy state (Ground State) and absorbs energy by moving up, not bit-by-bit, but in "steps". If energy arrives in less than one complete quantum, then the electron will stay where it is as it cannot absorb that energy. Only if the energy arriving at the atom is exactly equal to the step size to the next energy level will it be absorbed. Surprisingly, even if the incoming energy is greater than the jump to the next level, it will not be absorbed. This is due to the "standing wave" view of energy levels, where an energy wave coming in must correspond exactly in frequency to the standing wavelength of the orbiting electron.

For hydrogen gas, it was calculated that the ground state energy was -13.6 eV. (All energy levels in atoms have negative values because they are stable Ep states below the zero level and potential energy must be given to the electron to lift it out of the atom).



Energy levels for hydrogen gas

We can see that the ionisation energy for hydrogen is 13.6 eV and that the spacing of the energy levels complies with a quadratic function:

$$E_n = \frac{13.6}{n^2}$$

Therefore:

$$E_1 = \frac{13.6}{1^2} = 13.6\text{eV} \qquad E_2 = \frac{13.6}{2^2} = 3.40\text{eV}$$

$$E_3 = \frac{13.6}{3^2} = 1.51\text{eV} \qquad E_4 = \frac{13.6}{4^2} = 0.85\text{eV}$$

When energy is absorbed by an atom it does so by promoting electrons to higher energy levels. For example, the electron in a hydrogen atom could absorb energy and move up from the -13.6 eV ground state to the first excited state of -3.40 eV. The electron is then in a metastable state at n = 2, as it can only remain there for a short while before it falls back down to the ground state. When it does fall back it emits energy in the form of an electromagnetic wave, or photon, with energy equal to the difference in energy levels ΔE, which is 13.6 – 3.40 = 10.2 eV. The wavelength of this wave would therefore be given by the equation:

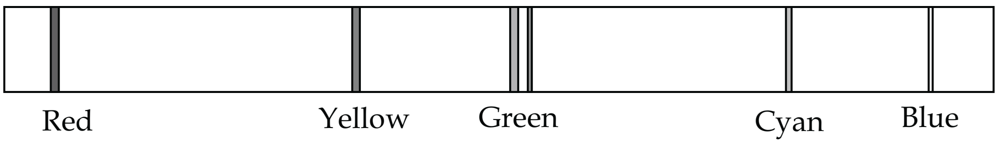
$$E = \frac{hc}{\lambda} \qquad 10.2 \times 1.60 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

$\lambda = 1.22 \times 10^{-7}$ m or 122 nm (UV)

Because of this energy level spacing, the emission spectrum lines must show the same pattern.

Each element has a unique pattern of lines which can be used to identify the element. If you have done the Flame Test in chemistry you will know that elements give off light of a particular colour when they are ionised in a high temperature flame of a Bunsen burner (e.g. copper glows green/blue, sodium – yellow, potassium – lilac, etc). The spectral light from a star can be used to tell what elements are in its atmosphere by passing it through a prism (or diffraction grating) to produce a spectrum.

Line emission spectrum of a gas



If the energy input to the atom is in the form of an electromagnetic wave (light striking the atom) then the wave energy must be exactly equal to the transition energy (10.2 eV) – rather like a resonance effect in sound.

However, if the energy input is mechanical (a particle striking the atom) then the transition energy can be extracted from the total energy available and then the particle will recoil with the initial energy minus the energy absorbed.

For example, if an electron of energy 12.5 eV strikes the hydrogen atom then 10.2 eV is transferred to the hydrogen electron, promoting it to the 1st excited state and the incoming electron recoils with a E_k equal to 12.5 – 10.2 = 2.3 eV.

Example 1

A photon of energy 12.75 eV is absorbed by a hydrogen atom (see diagram on previous page). Calculate the wavelengths of the spectral lines emitted by the atom.

Solution 1

12.75 eV is sufficient to promote the electron in the ground state into the 4th energy level. When it falls, it can come down to any of the energy levels below. Possible energies emitted are given by:

(a) n_4 to n_3, n_4 to n_2, n_4 to n_1.

Values: 1.51 – 0.85; 3.40 – 0.85; 13.6 – 0.85 giving lines of energies of 0.66 eV, 2.55 eV, 12.75 eV (1st series of lines)

(b) n_3 to n_2, n_3 to n_1.

3.40 – 1.51; 13.6 – 1.51 giving lines of energies of 1.89 eV, 12.09 eV (2nd series of lines)

(c) n_2 to n_1.

13.6 – 3.40 = 10.2 eV (3rd series of lines).

All 3 series of lines would appear together with wavelengths:

- 1st series lines: 1883 nm; 488 nm, 97.5 nm
- 2nd series lines: 658 nm, 103 nm
- 3rd series lines: 122 nm

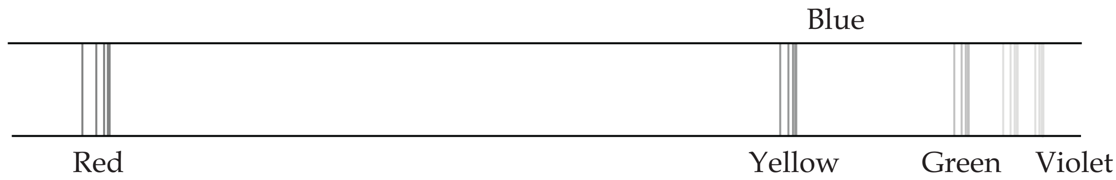
The only visible lines in this spectrum would be 488 nm (green), 658 (red).

1883 nm is in the infrared region and 97.5, 103, 122 nm are all in the ultraviolet region.

4.2 MOLECULAR SPECTRA

With a monatomic gas, the spectrum is the simple one outlined above, as each atom is widely separated from the next and will have no influence on it. However, with the spectra of large molecular gases there are other energy levels within the molecule that electrons can take up, due to the influence of neighbouring atoms present. This leads to a slightly different type of spectrum called a Band Spectrum.

e.g. spectrum of CO_2



Band spectrum

In solids the atoms are so closely packed that valence electrons tend to drift from one atom to another and so electron levels are 'degraded'. In other words, the step-like energy levels merge into a band of energies which is continuous.

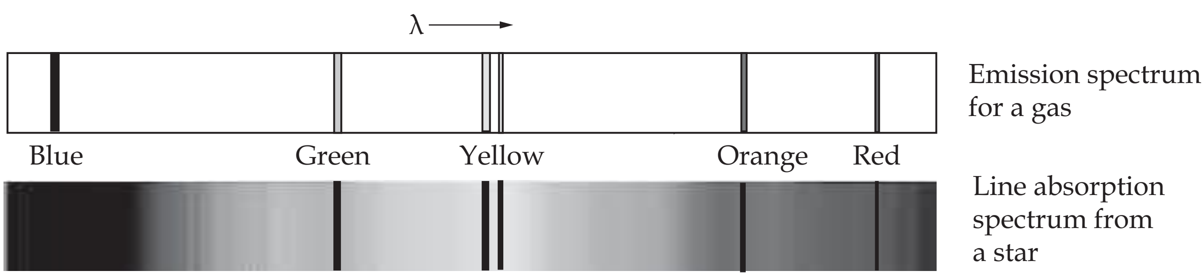
Continuous spectrum

This means that the electrons that are promoted into the band can fall to the ground state from anywhere within the band and will produce a continuous range of wavelengths. Glowing solid objects (e.g. light filaments and stars) always produce a continuous spectrum. Below is a table of the wavelengths in the spectrum and the colours perceived by the eye.

Colour	Wavelength (nm)	Frequency (Hz)
Infrared (not visible)	Above 750	$> 4.0 \times 10^{14}$
Red	750 – 630	4.0×10^{14}
Orange	630 – 600	5.0×10^{14}
Yellow	600 – 560	5.5×10^{14}
Green	560 – 490	6.0×10^{14}
Blue	490 – 420	6.5×10^{14}
Violet	420 - 400	7.5×10^{14}
Ultraviolet (invisible)	Below 400	$> 7.5 \times 10^{14}$

4.3 ABSORPTION SPECTRA

White light contains all possible wavelengths and energies lying continuously alongside each other. If white light passes through a gas then the wavelengths corresponding to the permissible electron transitions are absorbed. The absorbed energy is used to promote electrons into higher energy states.



Because these particular energies are absorbed from the spectrum black lines are left where these wavelengths are missing. The position of the black absorption lines will correspond to the emission lines of the elements in the gas absorbing the light.

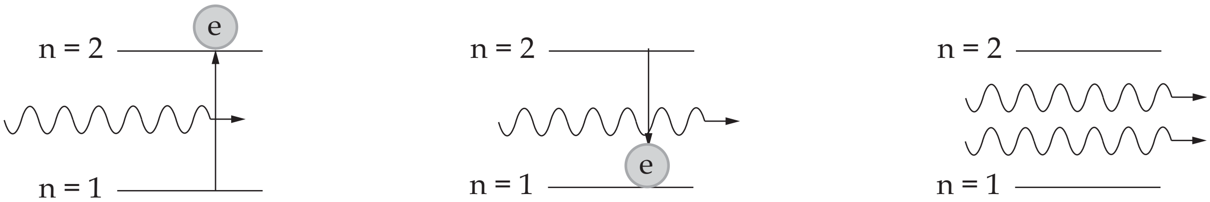
This absorption spectrum was first noticed in the sun and the lines are called Fraunhofer lines, after the discoverer. The absorption spectrum in the sun's light corresponds to the gas hydrogen, indicating that the sun is surrounded by hydrogen gas.

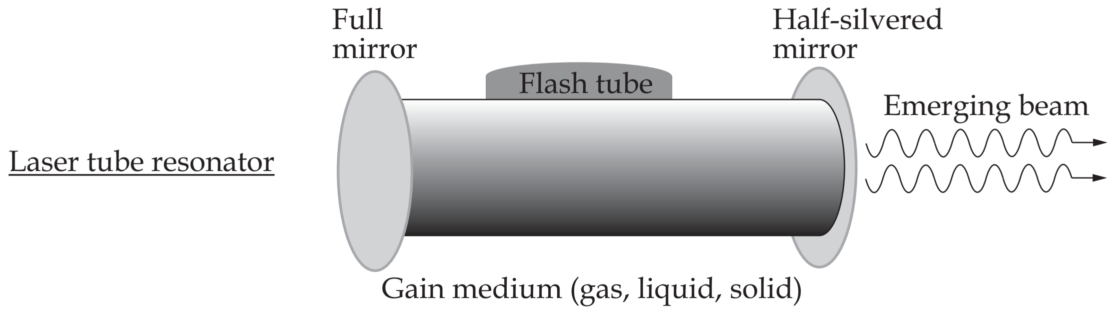
4.4 THE LASER

The word "Laser" stands for Light Amplification by the Stimulated Emission of Radiation and lasers today have many applications including uses for: lighting systems, surgical instruments, fibre optical telephony, CD reading, missile guidance and building wall levelling.

Lasers can utilise gases, liquids or solids, where in each case the electrons are stimulated to occupy a higher energy level for a relatively long time. A Helium/Neon gas laser emits light at 633 nm, a CO_2 laser emits IR radiation at 10.6 μm, giving out energies up to 100 W.

Maintaining electrons in higher levels is called "Population inversion" and is achieved through energy being absorbed from a flash tube or from an electric coil. This energy is called the Pump Energy for the laser to work – electrons are being "Pumped up".





When the flash gun fires, the electrons in the gas (e.g. He/Ne) are promoted to higher energy levels and then de-excite, emitting waves. These waves travel back and forth in the resonator, being reflected by the end mirrors.

Each time the wave moves through the gas, the electrons that are in the higher energy levels fall down and must travel in phase with the initiating wave, thereby producing more and more waves. The right-hand mirror is half-silvered so that a proportion of the wave energy can emerge as a beam, whilst the rest of the waves keep producing more coherent waves within the resonator.

Set 12: Spectra

1. Which of the following is NOT a fundamental principle of the Bohr Model of the atom?

   A. The energy of the orbiting electrons is quantised

   B. Centripetal force on electrons is provided by electrostatic attraction

   C. Spectral lines are a result of fixed energy levels

   D. Photons are emitted when electrons move into higher energy levels

2. The nucleus of a hydrogen atom has a radius of about 1.0×10^{-15} m with its electron at a distance of about 5.3×10^{-11} m from it. Taking this latter value as the radius of the hydrogen atom, make an estimate of the fraction of the hydrogen atom occupied by its nucleus.

3. A laser pen light emits a red beam of wavelength 620 nm. What is the energy content of each photon of this light?

4. An ultraviolet light-emitting diode (LED) gives out photons with energy of 5.0 eV. The rated power output of this LED is 10 mW. Using these figures, calculate the approximate number of photons emitted by the LED per second.

<u>This energy levels diagram for a hydrogen atom is referred to in questions 5 and 6 overleaf.</u>

n = 4 ———— -0.85 eV

n = 3 ———— -1.51 eV

n = 2 ———— -3.40 eV

n = 1 ———— -13.6 eV

5. Which of the following statements about this diagram is UNTRUE?
   A. The values shown in electron-volts follow a geometric progression of the form $E_n = E_1/n^2$
   B. Electron levels n =1 to n = 4 represent excited electron states
   C. At energy level n = ∞ there is no attractive force between the nucleus and the electron
   D. All electron level energies have negative values. This denotes stability of orbits

6. What energy would an electron promoted from the n = 1 level to the n = 4 level need to absorb?

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7. Match the spectrum type on the left with the correct conditions of formation:

a	Line emission spectrum	x	A star surrounded by a gaseous atmosphere
b	Band absorption spectrum	y	A heated metal
c	Continuous spectrum	z	Light from a flash of lightning

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8. A violet laser used for burning CD-ROMS emits 3.0×10^{18} photons per second at a wavelength of 420 nm. This laser is shone onto a point on the plastic of the CD for 1.0 millisecond. How much heat energy has been transferred to the plastic in this time?

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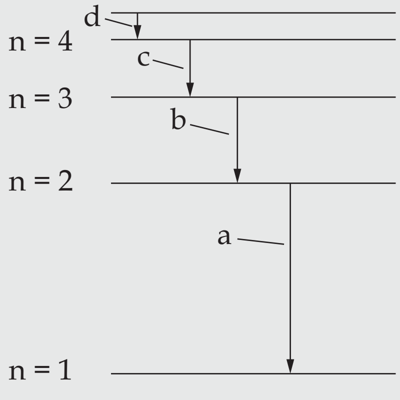
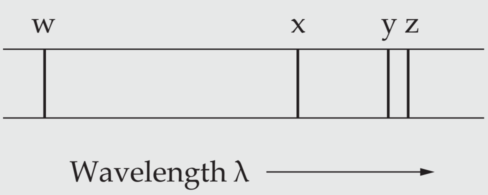




The electron transitions in the atom shown in the previous diagram give rise to the spectral lines shown on the right. The transitions a, b, c, d correspond to the wavelengths w, x, y, z in the following way:

A. a = z, b = y, c = x, d = w

B. a = y, b = w, c = x, d = z

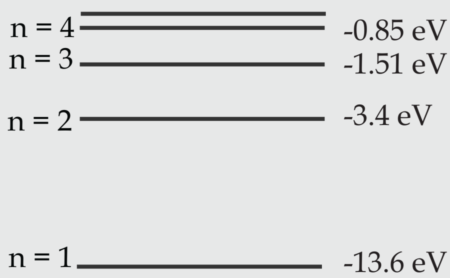
C. a = w, b = x, c = y, d = z

D. a = x, b = w, c = z, d = y

10. Explain why light from a 500 mW laser can cut through wood but the light from a 500 mW spotlight does not even burn a hole in paper.

11. Plants look green in the daylight but the chemical responsible for the photosynthesis (chlorophyll) is most sensitive to red light. Explain how this fact makes plants appear green.

12. A line in the Balmer spectral series for hydrogen has a wavelength of 487 nm. The electron transition responsible for the emission of this line is between which levels?



13. From the previous energy level diagram in Q. 12 what would be the energy, in joules, required to completely ionise a hydrogen atom?

14. Which of the photons in the following waves would contain the greatest amount of available energy?

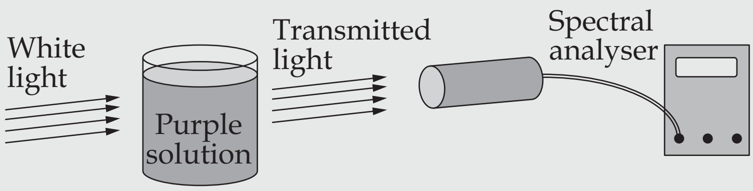
   A. TV wave of frequency 62 MHz

   B. Light of wavelength 589 nm

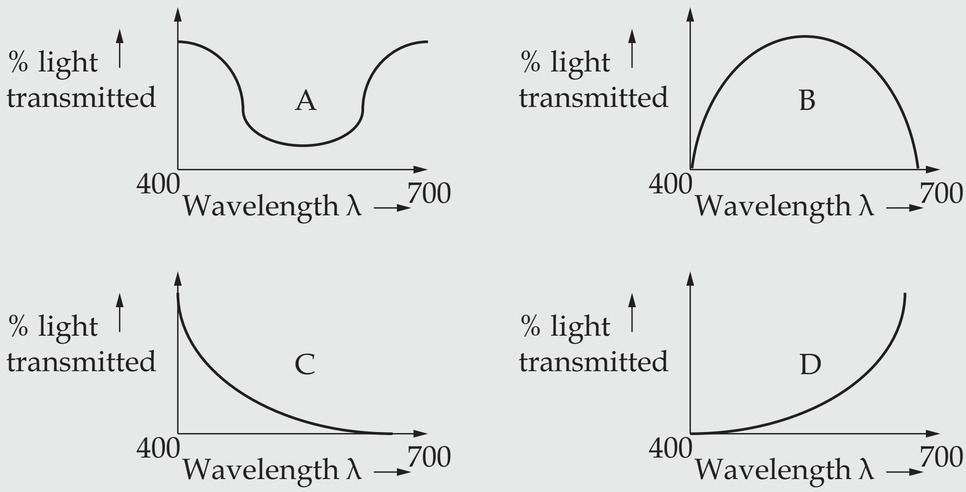
   C. A mobile phone carrier wave of frequency 22 GHz.

   D. A radar wave of wavelength 3.3 cm

15. White tungsten light from an overhead projector is shone through a strong, purple solution of potassium permanganate.



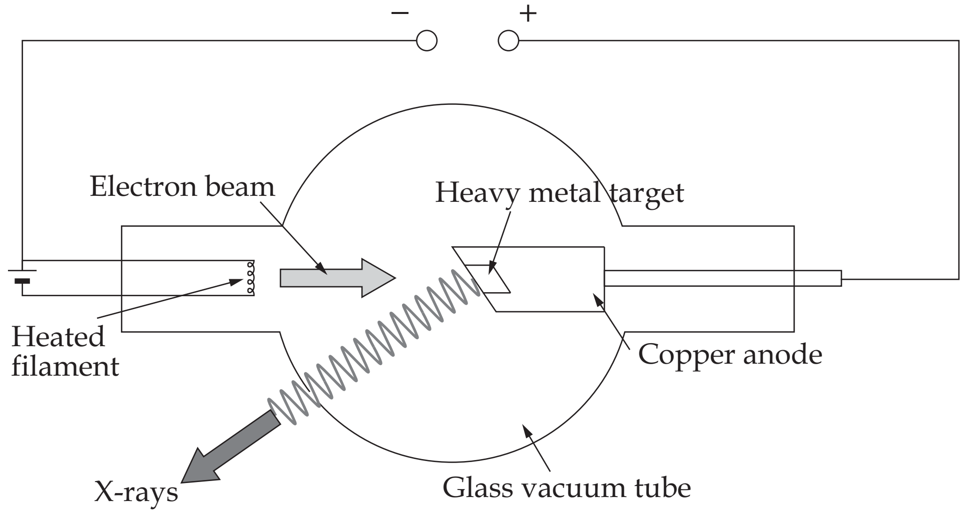
Which of the graphs below correctly represents the transmission spectrum of the light emerging from the purple solution?



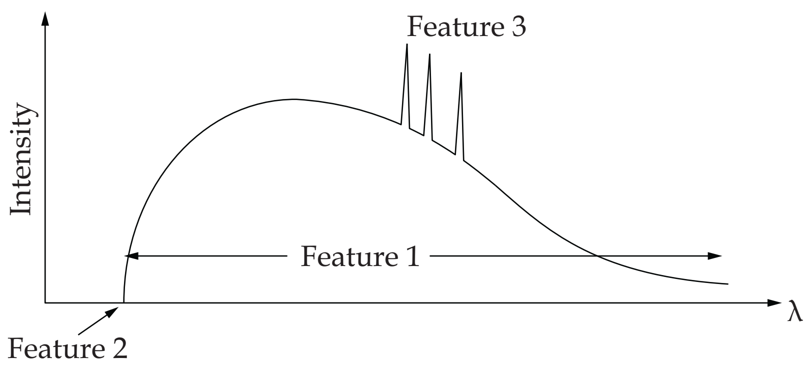
4.5 X-RAY SPECTRUM

In 1895 Wilhelm Rontgen found that when electrons were accelerated through a high potential difference and then collided with a heavy metal target, rays were given off which caused objects in his room to fluoresce. Later-on these mysterious X-rays were identified as very short wavelength electromagnetic waves that could penetrate solid objects.

A modern X-ray tube (Coolidge Tube) comprises an evacuated glass tube containing a metal target anode at one end and a heated filament cathode at the other.



When a voltage of about 30 kilovolts is applied fast electrons fly from the filament and strike the target (tungsten, copper or any heavy metal) and a large amount of heat is generated from the loss of E_k by the electrons. X-rays are emitted from the side of the tube.



X-ray intensity graph

A spectrum of X-rays from the tube shows 3 distinct features:

1. A continuous (whale-shaped) spectrum of wavelengths
2. A short wavelength cut-off point,
3. Spiked peaks superimposed over the continuous spectrum

Explanations of the special features of the X-ray spectrum

The electrons are accelerated as they leave the negative filament to cross the tube until they strike the target at a very high velocity (around half the speed of light) where they lose their kinetic energy in slowing down as they hit the metal. As these electrons approach the metal atoms they are repelled by the electron shells of the metal in a random way but as they decelerate they emit electromagnetic radiation of varying wavelengths. This area of radiation wavelengths seen in Feature 1 of the X-ray intensity graph is called Bremsstrahlung or 'braking radiation' – slowing down emission.

It is possible that, instead of the electron weaving its way through the metal electron shells and gradually slowing, it could lose all its energy in one lump by colliding head-on with a metal atom. This would result in the maximum amount of energy absorption from the electron and therefore the shortest emitted wavelength - shown as Feature 2 on the X-ray intensity graph.

This cut-off wavelength is given by the formula: Work done is $Vq = \frac{hc}{\lambda}$ as the energy of the electron is Vq after being accelerated through a voltage V.

Example 2

Calculate a value for the shortest wavelength emitted from an x-ray tube if the voltage across the tube is 30.0 kV.

Solution 2

$$Vq = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{Vq} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 10^{4} \times 1.6 \times 10^{-19}} = 4.14 \times 10^{-11}\,\text{m}.$$

The spikes shown as Feature 3 of the X-ray intensity graph constitute a line spectrum superimposed on the continuous spectrum. These lines are produced by the innermost orbiting electrons of the target metal being promoted to higher energy levels and then falling to emit X-rays of specific wavelengths. It requires a great deal of energy to promote the inner electrons in a heavy metal – in the region of tens of thousands of electron volts – because the repulsive force from the shells above the inner electrons.

4.6 X-RAY PROPERTIES

X-rays are extremely energetic and penetrating and so can cause ionization of atoms. Exposure in humans can cause cancers or can kill existing cancers, depending on exposure times. X-rays are absorbed by more dense materials, such as bone, and this property can be used to generate pictures that show the internal structures of biological systems (broken bones) or packages at airports.

Some galaxies are generating such a large amount of energy that they emit significant amounts of X-rays.

X-ray crystallography uses the reflection and interference of waves from crystal planes to determine their structure. The structure of DNA was determined from X-ray crystallography interference photographs.

Set 13: Accelerated Charges

1. An electron in an X-ray machine slows down at the surface of a tungsten target so that its kinetic energy changes from a value of 8.95×10^{-15} J to 2.65×10^{-15} J. What would be the wavelength of electromagnetic radiation given out during this E_k change?

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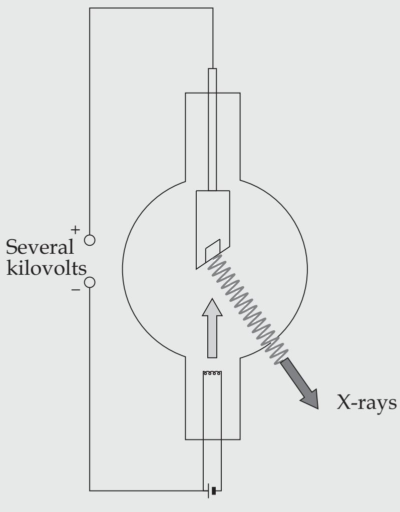
2. Which of the following statements about an X-ray tube is incorrect?



X-ray tube

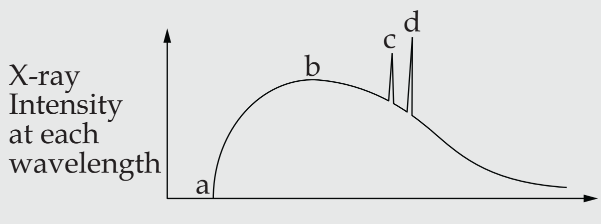
A. The kinetic energy gained by the electrons is transformed into heat in the target and into emitted photon energy

B. The cathode filament is heated so that electrons can be emitted more easily

C. Some of the electrons in the target are raised to higher energy levels, thus giving a line spectrum

D. Several thousand volts potential difference is needed across the tube to provide enough current for x-rays to be generated

3.



The graph above shows the distribution of energies for each wavelength of an X-ray tube running at 20 kV potential. Which of the following changes would occur if the metal that the target was made out of were changed to another type, still using the same anode voltage?

A. Points a and b would remain in the same position but c and d would change

B. Points b and c would remain in the same position but a and d would change

C. Points a and d would remain in the same position but b and c would change

D. Points a, b and c would remain in the same position and d only would change

4. (Refer to the graph shown in question 3).If the anode potential was 20 kV, what would the cut-off wavelength be at point a on the graph?

5. What would be the velocity of electrons that had a kinetic energy of 65 keV?

6. X-rays are used to produce photographic images of welded joints in steel vessels to locate fine cracks.

   a) Explain how these images are produced

   b) Why are x-rays so useful for this purpose?

7. Some of the stars we see in the sky are emitting large bursts of x-rays. Give a proposal for how you think x-rays can be produced in stars like this. How does the Earth's atmosphere help to protect us against these harmful rays?

8. X-rays can cause cancers in the human body and can also help to cure cancer. Explain this apparent contradiction.

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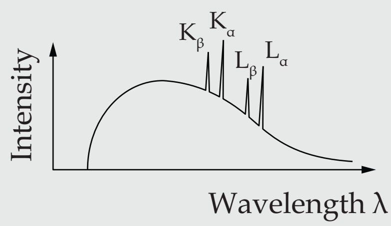
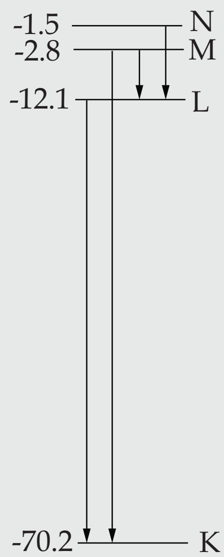
9. Graph showing how the intensity of X-rays varies with wavelength for a tungsten target



The diagram opposite shows some of the inner energy levels within an atom of a metal. The energies involved are given in keV.



(i) Label each arrowed transition in the diagram opposite to show which line in the spectrum it generates (eg K_α etc)

(ii) What is the wavelength of the electron transition from -2.80 keV to -70.2 keV?

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10. The anode of an x-ray tube, such as the Coolidge Tube, gets so hot that it has to be cooled by water. Explain why the anode gets so hot.

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4.7 STELLAR SPECTRA

By observing the emission spectra of stars and matching wavelengths against patterns from known elements we can deduce the elements that are present in the star. By studying the absorption spectrum of a star the make-up of gases in its atmosphere can also be determined, as the light emitted will be absorbed by these surrounding gases. Many astronomical objects are not only observable in visible light, but also emit radiation at radio, infrared and UV wavelengths.

Besides observing energetic objects such as pulsars and quasars, radio telescopes are able to "image" most astronomical objects, such as galaxies, nebulae, and even radio emissions from planets.

Infrared astronomy gives scientists the ability to measure the temperatures of planetary bodies, stars, and the dust in interplanetary space using Wien's Law (See Chapter 3). There are also many molecules that absorb infrared radiation strongly- hence the study of the composition of astrophysical bodies is often best done with infrared telescopes.

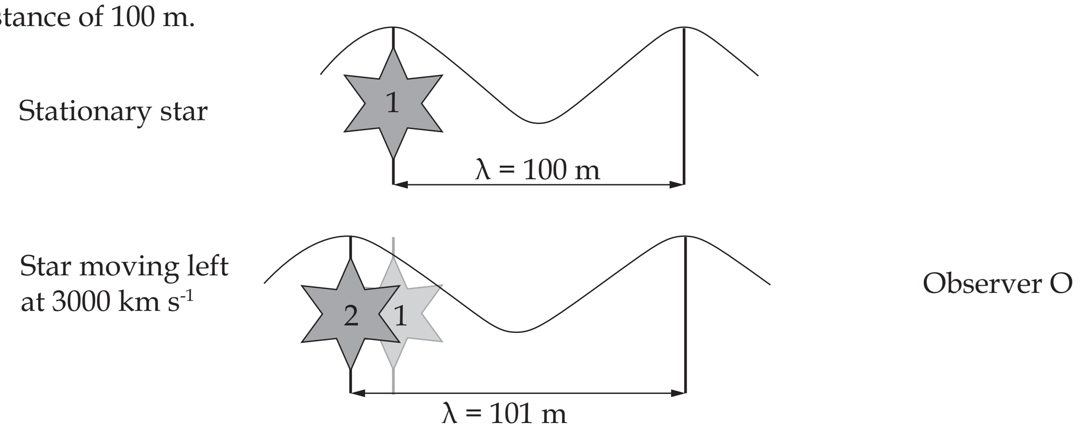
4.8 DOPPLER SHIFT

Doppler Shift is an effect whereby the wavelength of light is changed due to the motion of the star. If the star is moving away from us the wavelength of a particular line in its hydrogen spectrum, say, will be lengthened. By the time a wave is emitted, the star has moved away and so the wavelength appears longer to an observer. This apparent lengthening of wavelengths is called Red Shift because all waves move closer to the red end of the spectrum as a star moves away.

Suppose the light from a stationary star emits a wave with a frequency 3 x 10^6 Hz – the wavelength will be 100 m long $\left(\lambda = \frac{3 \times 10^8}{3 \times 10^6} = 100\text{m}\right)$.

The period between each wave crest will be $T = \frac{1}{3 \times 10^6} = 3.3\dot{3} \times 10^{-7}$ s

With a stationary star (1) in one time period T the wave crest will have moved to the right by a distance of 100 m.



With a moving star, it emits one wave crest at position 1 which will travel 100 m to the right in one period.

In that time, the star itself will have moved to position 2 (1.00 m to the left) by the time the next wave crest is emitted. Hence to an observer (O) the wave will arrive with an apparent wavelength of 101 m which is longer than the wavelength the star is actually emitting. This apparent lengthening of an electromagnetic wave (or apparent reduction in frequency) from a receding star is known as "Red Shift".

By the same logic, if a star is moving towards us then the wavelength will be compressed towards the shorter wavelength section (apparent frequency increase) of the spectrum and is known as "Blue Shift".

This shift in the frequency of an object due to its movement relative to an observer is called Doppler Shift and also occurs commonly with sound waves where, for instance a racing car passes an observer who notices the engine note drop in frequency as the car passes (eeaoow!)

Doppler equations for frequency shift are shown below.

Source receding: $f' = \dfrac{v}{v + v_s} f$

Source approaching: $f' = \dfrac{v}{v - v_s} f$

v = velocity of the wave

v_s = velocity of the source

f = original frequency emitted

f' = apparent frequency observed by the observer.

Example 3

A car is moving towards a woman at a speed of 60.0 km h^{-1} and is emitting a sound wave of frequency 256 Hz. If the speed of sound is 346 m s^{-1}, what frequency of sound does the observer perceive from the car?

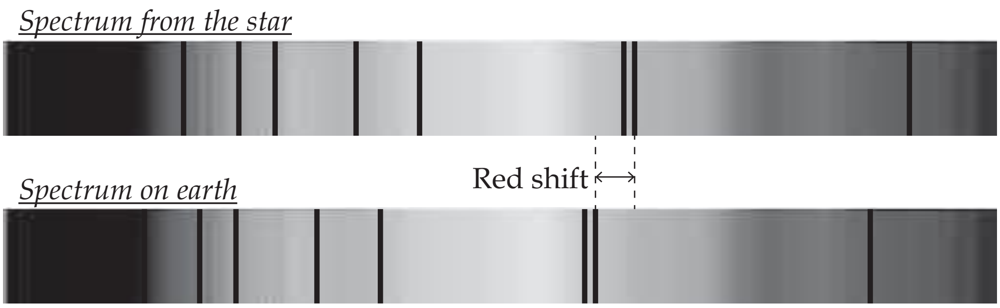
Solution 3

Using the Doppler equation $f' = \dfrac{v}{v - v_s} f$

$f' = \dfrac{346}{346 - 16.67} \times 256 = 269$ Hz.

The amount by which a known wavelength in a spectrum is shifted will also indicate how fast a star is moving away from us on Earth. But how do we know what the original wavelength was when the wave was emitted by the star?

For this we use the fact that all stars are composed of hydrogen whose spectral wavelengths we know from experiments on Earth (13.6 eV, 3.40 eV, etc., as mentioned before in section 4.1). The black lines in the Sun's absorption spectrum exactly match the emission lines of hydrogen because it is not moving towards or away from us, but with stars the hydrogen absorption lines are shifted because the star is moving.



Example 4

The blue line in the hydrogen spectrum has a wavelength of 486 nm on Earth but when observing a distant star, this same line appears to have a wavelength of 502 nm.

a. Is the star coming towards Earth or moving away?

b. Calculate the speed with which the star is moving.

Solution 4

a. As the wavelength has been stretched (Red Shifted), the star is receding.

b. $f' = \dfrac{c}{\lambda'}$

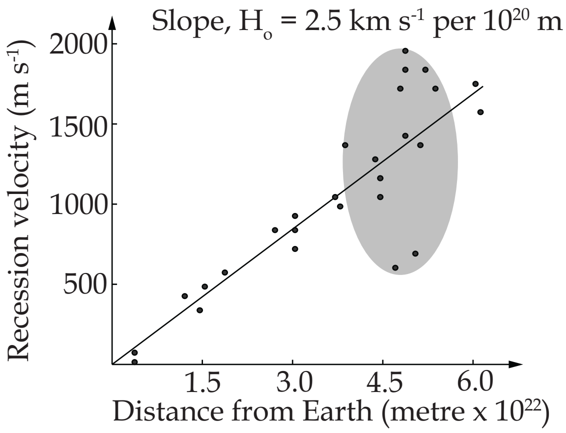
So $\dfrac{c}{\lambda'} = \dfrac{v}{v + v_s} \times \dfrac{c}{\lambda}$ $\qquad \dfrac{1}{502} = \dfrac{3 \times 10^8}{3 \times 10^8 + v_s} \times \dfrac{1}{486}$

$3 \times 10^8 + v_s = 3 \times 10^8 \times \dfrac{502}{486}$ $\qquad v_s = 9.88 \times 10^6$ m s^{-1}

From observing the red shift of all stars in the universe it was found that all observable stars (except those in the Andromeda Nebula) were receding from Earth and this gave rise to the Big Bang Theory by George Lemaitre. This theory postulates that the Universe began as a big agglomeration of hydrogen gas which collapsed inwards by gravitational pull and then exploded outwards about 14 billion years ago.

In any explosion, the particles further from the centre of the blast would move outwards at a faster rate, so Edward Hubble set out to investigate the relationship between the distance of a star and its velocity relative to Earth. His investigations and calculations resulted in what is now known as Hubble's Law.

From the graph Hubble produced, we can see quite a good line of best fit through the points taken for each star observed. How was the distance to each star found, one might ask?



For this, Hubble made an assumption about the brightness of stars becoming less with distance so he used measured values of brightness to estimate distances.

Hubble's Law can thus be expressed as: $\mathbf{v = H_0 d}$

v = velocity of the star (measured by its red-shift),
d = distance of the star from Earth,
H_0 is a constant, called Hubble's constant.

H_0 is actually the slope of this graph and has been determined as 20 km s^{-1} per mega light year. A light year is the distance travelled by light in one year (9.47 x 10^{15} m).

Another common measurement in astronomy is the Parsec (pc), which involves angular measure to express large distances. 1 pc is about 3.1 x 10^{16} m. Some stars are millions of parsecs away from Earth. H_0 can also be expressed as 68 km s^{-1} per mega parsec.

For example, using Hubble's Law, a line in the spectrum of a gas had a wavelength of 500 nm on a stationary Earth - but the same line for the same gas on a moving star has shifted to 501 nm due to its motion, this would equate to a calculated distance of 28 megalight years away from Earth.

4.9 THE BIG BANG

Using the red shift method, it was found that all stars but a few observed in the sky are moving away from Earth, which is evidence for the Big Bang Theory. If we extrapolate backwards to find the time that all the particles in the universe would have been at zero distance from each other, we get an answer of about 14 x 10^9 years. This is the predicted age of the Universe or the time that the Big Bang occurred.

Below is the predicted timescale of events from when the Universe first came into being:

Time from beginning	Occurrences	Temperature/Kelvin
10^{-10} s	Particle "soup" of quarks	10^{15} K
1 s	Neutrons and protons	10^{10} K
2 min	Helium nuclei	10^9 K
300,000 y	Microwaves fill the Universe	6000 K
½ million y	Temperature fall, IR emission	750 K
10^6 y	Atoms form	100 K
10^9 y	Stars _ Galaxies form	18 K
14 x 10^9 y	Present	2.7 K

The microwave background radiation, discovered in 1965 is another piece of evidence for the Big Bang theory – the remnants of the heat generated in the beginning is still there. Empty space is not at absolute zero as originally thought but at an average temperature of 2.7 kelvin.

This temperature was calculated from the microwave wavelengths coming in from every direction in the Universe.

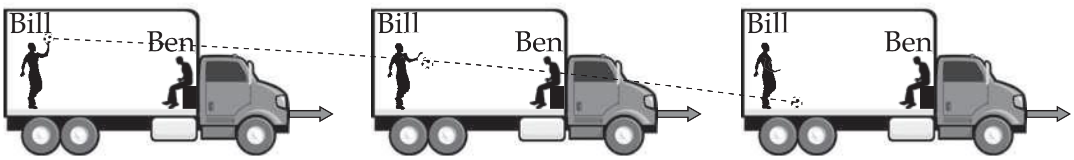
Until recently the bodies in the Universe were predicted to start slowing down and shrinking — to collapse once more into a single mass, due to the immense gravitational attraction towards its centre. However, accurate observations of speeds have recently shown that its rate of expansion is actually increasing. The only explanation for this is that the stars are being pushed outwards by another unknown force. It has now been proposed that there exists a mysterious "dark matter" also present in the Universe that we cannot see, but exerts a large gravitational pull on all the stars and planets. As matter and energy are now regarded as equivalent ($E = mc^2$) scientists talk about dark matter and dark energy pervading the Universe, but invisible due to its incredibly small density (7×10^{-27} kg per cubic metre of space). There seems to be no interaction with Dark Matter except by means of the gravitational force.

Calculations show that the whole Universe is composed of 27% dark matter, 68% dark energy and only 5% normal matter that we can see and measure. So far, we really do not know much about dark matter and dark energy.

4.10 EINSTEIN AND RELATIVITY

Einstein's ideas were quite revolutionary at the time as he discarded the notion of simultaneity — what happens on Earth at a particular time would be seen to happen at a different time by an observer situated a large distance away. For example, a star that you can see shining in the sky at Earth time may not actually even exist anymore in that particular area of space.

On the Earth we have a different Frame of reference to that of the star and so there are different realities, depending on where an observer is situated. If the star is 1000 light years away then the star could have exploded 500 years ago and we would not know it for another 500 years because the light from it would take 1000 years to get here. Einstein explained that there is no absolute frame of reference and that everything is relative in different frames of reference — time, distance and mass.



The diagram above illustrates how an occurrence can be observed in two entirely different ways because of differing Frames of Reference.

Bill and Ben are travelling in an open-sided truck at a constant speed of 5 m s^{-1} to the right. Bill holds his football up for Ben to see and then lets go and it lands 1 second later.

In the frame of reference of the truck, Ben sees the ball as falling straight down under gravity to land at Bill's feet.

Another observer standing on the pavement sees the same ball travelling forwards in a parabolic curve to land a distance of 5 m to the right inside the truck.

Because Bill and the ball both have a velocity of 5 m s^{-1} to the right when Bill lets the ball fall it is actually thrown forwards to land 5 m away but as the truck also moves 5 m, the ball lands directly under Bill's hands.

Did the ball fall straight down or did it move forwards in a parabola? The answer is — it depends on the frame of reference of the observer!

4.11 MASS TO ENERGY

One of the most famous equations in physics comes from Einstein's Special Theory of Relativity (SR): $\mathbf{E = mc^2}$

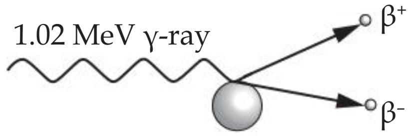
This equation shows the amount of energy that could be obtained from a mass of matter if it were to be completely converted. We have many examples of where this principle can be seen in practise.

For example, mass decreases when protons and neutrons are bonded together to form a nucleus, as in a fusion reaction. We find that the sum of the masses of the component nucleons is greater than the mass of the nucleus formed. This mass conversion to binding energy is the process by which we obtain energy output in nuclear reactors, atomic bombs and the Sun.

The opposite process can also occur i.e. energy can be converted into solid matter:

Using Einstein's equation of $E = mc^2$ the mass of an electron can be expressed in energy units, so the mass on an electron (9.11×10^{-31} kg) equates to 8.20×10^{-14} J or 0.512 MeV.

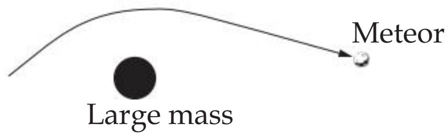
If a gamma ray with an energy equivalent to two electrons (1.02 MeV) passes close to a nucleus of an atom it can spontaneously produce two electrons — matter produced from energy. One electron will be a normal negative type and the other has to be its antiparticle called a positron. The other particle produced must be + so as to conserve charge.



This process occurs billions of times a day in the Earth's upper atmosphere.

4.12 SPACE-TIME

Einstein included Time as an addition to the three other dimensions and talked about Space-time as a single entity to be considered. He also proposed that space-time was distorted by large masses, such as the Sun or a black hole.



We would observe that a meteor passing close to the Sun would have its path bent by the gravitational pull of the Sun's mass. However, Einstein explained it a different way by saying that the meteor moved in a straight line through space-time but the space-time itself was bent by the Sun's mass. This is a bit like an aeroplane flying in a straight line north but because the air is moving west (the wind) the aircraft moves northwest, viewed from a frame of reference on the Earth's surface.

This 'curvature of space' idea leads us to the conclusion that space-time must be curved around the total mass of the Universe, so that, if we travelled in what we would consider to be a straight line for long enough, we would come back to where we started! If you compare the curvature of space with the curvature of the Earth we can see similarities: If you were to travel in a straight line any direction on the Earth for about 40,000 km you would come back to exactly where you started from.

4.13 POSTULATES OF THE SPECIAL THEORY OF RELATIVITY

A postulate is a claim or assumption made without proof. In Einstein's Special Theory (SR) the first postulate made was that the laws of physics would be the same, regardless of any particular frame of reference e.g. in a car crash the Law of Conservation of Energy would apply to the crash to an observer in the car or by the side of the road.

The second postulate of the SR was not so self-evident and caused a lot of dissent in the scientific community at the time Einstein revealed it in the early 1900s.This postulate was that the speed of light remained constant to an observer, regardless of his frame of reference.

Our experience of slow-moving objects dictates that if we were in a car moving east at 30 km h^{-1} and a car was passing us moving west at 50 km h^{-1} then we would observe the other car appearing to move towards us at a relative velocity of 80 km h^{-1} i.e. the sum of the two velocities. However, according to the 2nd postulate of the SR this mathematical principle would not apply at large velocities because the speed of light, nor the relative speed of light, could exceed 3×10^8 m s^{-1}.

This means that if one space ship is moving to the right at 3×10^8 m s^{-1} and encounters another space ship moving left at 3×10^8 m s^{-1} then the relative velocity of the two spacecraft will have a value of no greater than 3×10^8 m s^{-1}.

These figures seem counter-intuitive to us and are a result of two errors in our system of understanding, or "common sense":

(i) Our equations for relative velocity are only correct for the slow-moving objects that we normally have experience with,

(ii) Something was wrong with traditional basic ideas of space and time.

Maybe this idea of constancy of the velocity of light waves would seem logical if we considered a speedboat travelling on a river where the front of the boat emits waves on the water. The waves would actually travel at a set speed dependant on the medium (water), regardless of the speed of the boat. In fact, we can see that boats can actually travel faster than the water waves on the water being emitted when it leaves a bow wave.

4.14 TIME DILATION

For the 2nd Postulate to work the time elapsed for the space ship to travel a distance d must be perceived as different for a stationary observer compared with the time perception on a moving space ship. This is called time dilation.

Einstein derived an equation for this perceived difference in time in what is called a Relativistic Time Equation i.e. for very fast moving objects:

$$\frac{t_m}{t_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

t_s = time perceived by a stationary observer

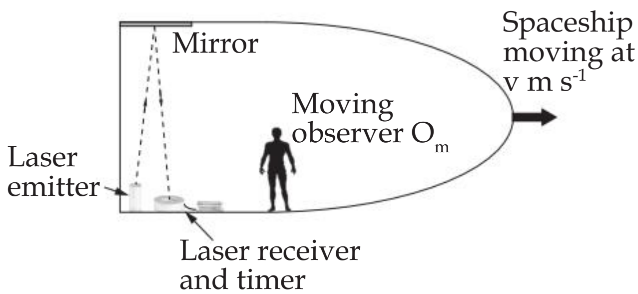
t_m = time perceived by a moving observer

v = velocity of moving observer

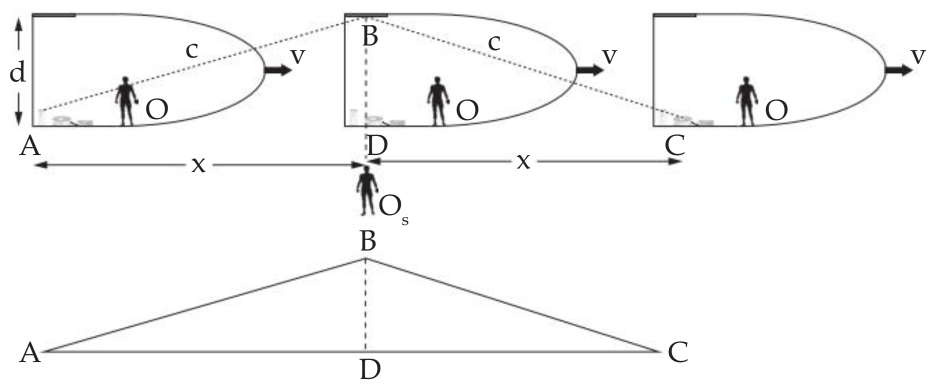
c = velocity of light in a vacuum

The derivation of Relativistic Time Equation formula is shown below out of interest.

Suppose that an astronaut is travelling on a spacecraft at a velocity v to the right. On board is a laser on the floor that shines onto a mirror on the ceiling of the spacecraft where it is reflected back again. A receiver on the floor is linked to a timer that can measure the time of flight of the laser light from floor to ceiling and back again.



A stationary observer on Earth O_s who can see into the spacecraft notes the time when he sees the light leave the laser and when he sees it enter the receiver. O_s records the time taken for this to occur as t_s (time for stationary observer), whilst, on the spacecraft the astronaut records this time on his timer as t_m (time for moving observer O_m).



In the spacecraft, the time for the laser to travel to the mirror and back is given by:

Inside the spacecraft, the time for the light to move from A to C is: $t_m = \frac{2BD}{c}$

A stationary observer on Earth sees the light leave the laser at point A, hit the mirror at point B and then reach the receiver at point C in time t_s. The spaceship will have travelled a distance AC in this time and the laser light will have travelled a diagonal distance equal to: AB + BC.

The time for the light to be emitted and then received would be:

$$t_s = \frac{AB + BC}{c}$$

But $BD^2 = AB^2 - AD^2$ or $BC^2 - DC^2$

Ratio of observed times $\frac{t_m}{t_s} = \frac{2BD}{AB + BC} = \frac{2\sqrt{AB^2 - AD^2}}{2AB}$ (AB = BC)

$$\frac{t_m}{t_s} = \sqrt{\frac{AB^2 - AD^2}{AB^2}} = \sqrt{1 - \left(\frac{AD^2}{AB^2}\right)}$$

AD is the distance that the rocket has travelled and AB is the distance that the light has travelled.

Hence: $\frac{t_m}{t_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$, which is Einstein's time dilation formula.

Suppose an observer on Earth is able to see the clock on a spaceship that is going past him at 0.25 times the velocity of light i.e. v = 0.25c.

His perception of one second tick measured on the clock of the moving space ship would be:

$$t_s = \frac{1.0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} = \frac{1.0}{\sqrt{1 - 0.25^2}} = 1.03\text{s.}$$

This would mean that the moving clock would appear to be slower in the rocket by about 3%, which would be hardly noticeable even at this high speed of 7.5×10^7 m s^{-1}.

The denominator of the equation above will always be less than 1 as v is always less than c, so t_s must always be greater than t_m.

In the 1960s an experiment was done to see if this time dilation could actually be noticed on Earth. A jumbo jet was flown around the Earth 4 times with an atomic clock in it which was then checked against another atomic clock that had been kept stationary. A time difference of 4 nanoseconds was recorded, which is very small, but significant, and exactly as calculated from the equation (see this experiment on YouTube).

As the speed of the space craft is increased, the effect due to time dilation becomes greater. At 0.99 x speed of light, 1 second on the moving observer's clock would seem to occur in a time of about 7 seconds measured by a stationary observer.

Einstein recognised a problem that would be caused by time dilation and expressed it in what was called the Twin Paradox: One twin leaves Earth on a space ship travelling at a speed close to the speed of light. His clock runs slower compared with the clock of the other stationary twin. When he returns to Earth he has aged by 1 year according to his measurement of time whilst his twin who stayed on Earth could have aged by 30 years!

On the surface of it, this does not seem logical, as the twin left behind could just as well be viewed by the other twin as moving backwards at close to the speed of light. Einstein resolved this dichotomy by proposing that the twin left behind was defined as being in an Inertial Frame of Reference (i.e. had not accelerated), whereas the twin who had left Earth had undergone acceleration and had therefore experienced the time dilation. The reality of biological effects of this kind of time travel have yet to be tried out.

Example 5

Suppose an astronaut travelled to the star Proxima Centauri 4.5 light years away at a speed of 0.999 times the speed of light.

a) How long would it take him to reach the star and return to Earth, measured on a clock on the Earth?

b) When the astronaut returned to Earth how long would the astronaut state that he had been gone?

Solution 5

a) If it takes light 9 years to get to the star and back at a speed of 3×10^8 m s^{-1} then at 0.999 c it would take time $t = \frac{9}{0.999} = 9.009$ years.

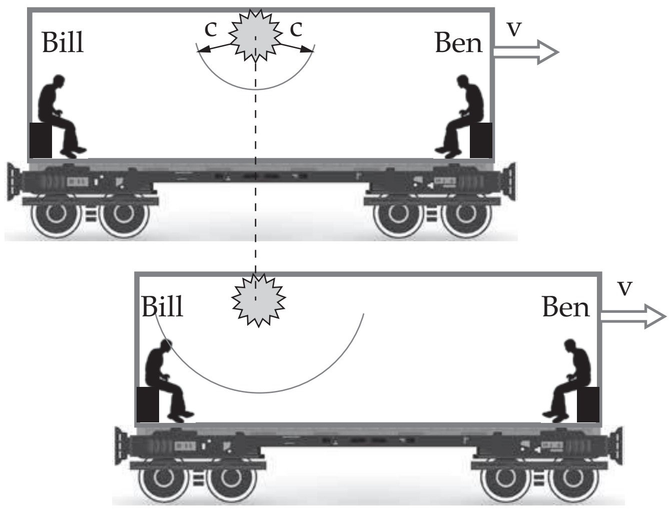
b) $t_m = t_s \sqrt{1 - \left(\frac{v^2}{c^2}\right)} = 9.009\sqrt{1 - (0.999)^2}$

$t_m = 0.403$ years – on the spaceship clock the trip only took 4 months and 3 weeks!

Synchronicity

Bill and Ben are sitting on a train moving with a velocity v when a mobile phone flash goes off in the centre of the carriage.

From the frame of reference of a stationary observer outside the carriage, Ben is moving away from the flash light, so it will take him longer to see the flash go off but Bill will see the flash as occurring sooner because he is moving towards the light source. To the outside observer the light reaching the two men will not be synchronous. If the light takes a time t to reach Bill then Ben would have moved a distance vt away from the light source.



The time for the light to reach Ben will be t + vt – a longer time. However, to the two men in the carriage, the light will be seen to reach both of them at the same time i.e. the event will appear synchronous in the moving frame of reference!

4.15 RELATIVISTIC ADDITION OF VELOCITIES

Suppose a passenger was in a train that was moving to the right with a velocity of 20 m s^{-1} and she herself walked along the corridor towards the front of the train with a velocity of 5 m s^{-1}.

From a Newtonian mechanics perspective, the passenger's velocity relative to the ground would be 20 + 5 = 25 m s^{-1} but how would we calculate this relative velocity if very high velocities (relativistic velocities) were involved?

A formula can be derived for this scenario from Einstein's equations: $u = \dfrac{u' + v}{1 + \dfrac{u'v}{c^2}}$ where u is the velocity of the man relative to a stationary frame of reference.

If the velocity of the train is small compared with the velocity of light, it can be seen that the equation reduces to $u = u' + v$.

If however, the scenario is that a rocket moving at 0.9 c fires a projectile forwards at 0.5 c then the velocity of the projectile would not be 1.4 c, but its velocity would be given by the above formula i.e. $u = \dfrac{0.5c + 0.9c}{1 + \dfrac{0.5c \times 0.9c}{c^2}} = 0.966c$.

Relative velocities cannot exceed the velocity of light!

4.16 DISTANCES

In the previous Example 5, if we took the time measured in the spaceship as being the correct time and we had travelled at a speed of 0.999c for 0.403 years, we would think that the distance we had travelled was 0.999 x 0.403 = 0.40 light years instead of 9 LY, so in the moving astronauts' frame of reference, we could say that the distance has been dilated.

Hence, we can also use a Length Dilation formula:

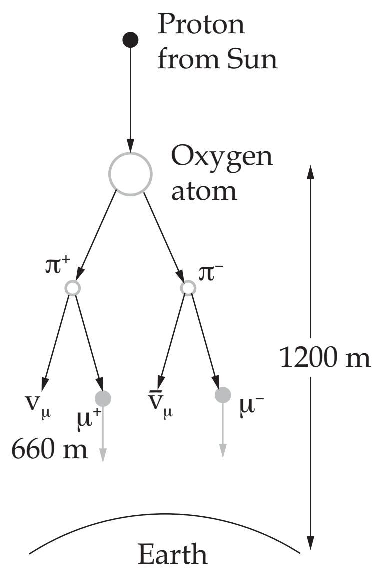
$$\frac{l_m}{l_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Using the values in the example: $l_m = 9\sqrt{1 - (0.999)^2} = 0.40$ LY

From the same SR time dilation equation a result is obtained whereby lengths also appear to shrink with velocity, so a spacecraft with a stationary length of 10 metres moving at a velocity of 0.25 c would appear to be about 3% shorter to an observer who is not moving.

The distance dilation equation predicts that a spaceship moving at the speed of light would appear to have no length at all! Looking at the implications of this apparent shrinkage of distance at high velocities we come to understand why the velocity of light can never be exceeded. It follows from the equation above that if v = c then the value of t_m approaches zero. In other words, if you were on a spaceship travelling at the speed of light any distance at all would shrink to zero and so it would take you zero time to move from a point to anywhere else in the Universe! You can't get there any quicker than that - so you can never exceed the speed of light!

4.17 EVIDENCE FOR TIME DILATION



Muons (μ) are produced when protons from the Sun collide with atoms of oxygen in the upper layer of our atmosphere. They are produced from the decay of pions and have a half-life of 2.2 μs, as measured in laboratories on Earth i.e. in 2.2 x 10^{-6} seconds half of them will have decayed. As they are moving at about 0.994 times the velocity of light (2.982 x 10^8 m s^{-1}), it would take a time of about 40 μs to reach the Earth.

As their half-life is 2.2 μs, this would mean the muons reaching the Earth would have gone through 40/2.2 = 18 half-lives. Therefore the fraction of muons reaching the Earth would be $(½)^{18}$ or about 3 millionths of the original number being produced. By this calculation, only a very tiny fraction could possibly ever reach the Earth's surface.

However, the 40 μs a stationary observer would record for the muon to reach the Earth would equate to a much smaller time experienced by the particle itself:

$t_m = 40x\sqrt{1 - 0.994^2} = 4.38$ μs.

Theoretically then, muons would be able to reach the Earth in a time of 4.37 μs, as measured in its own frame of reference, which is only about 2 half-lives – so actually about a quarter of the muons would remain, rather than the 3 millionths previously calculated.

So do muons in realty manage to reach the Earth? Yes, muons can actually be detected at the surface of the Earth, showing that time dilation proposal is actually occurring every day!

Masses

Einstein established that mass and energy are inter-convertible, so for an object moving through space, its total energy content would be its mass energy plus its kinetic energy.

$E_{Tot} = E_{Mass} + E_K$

If a force were applied to this mass then work would be done in accelerating it and its kinetic energy would be increased – correct? Yes, according to Classical Newtonian physics but we now know that kinetic energy can be converted to mass (e.g. pair production). As the velocity of an object increases, according to Einstein, the velocity of light cannot be exceeded so there will be a point where the work done on the object could not be used to increase its velocity and would go into increasing the mass, rather than the kinetic energy. Hence energy would be converted to mass, according to the equation $E = mc^2$. The mass of the object would become larger as measured by a stationary observer (relativity again!).

Here is another reason why the velocity of an object can never exceed the velocity of light – because the object's mass would become infinite and it would take an infinite amount of force to accelerate it further.

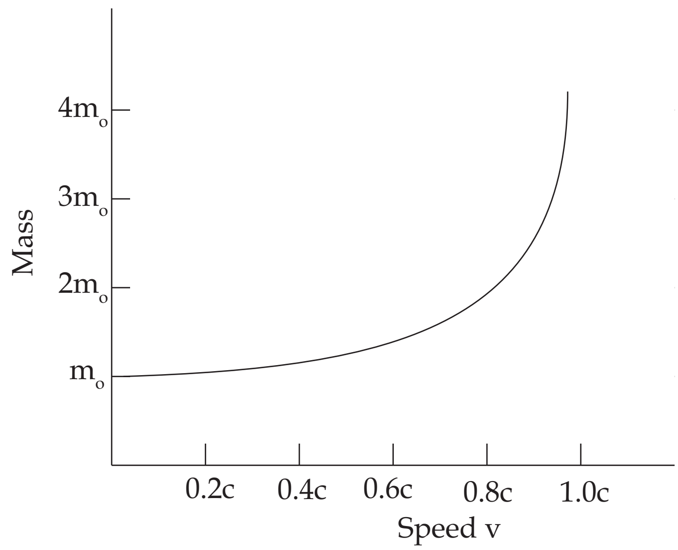
This apparent gain in mass is what limits the maximum speed of particles in a cyclotron because as they get faster their mass increases and they would not be able to bent into the same radius as when they were moving slower and therefore had less mass. A much larger magnetic field would have to be available and there is a technological limit on this.

Einstein's equation for mass dilation is:

$$m_m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

m_o is the Rest Mass of the object i.e. its mass measured in a stationary frame of reference.

The moving mass m_m will appear to get larger as v increases, as shown by the graph below.



Example 6

An electron is accelerated in a Linac to a speed which is 0.97 times the speed of light. Calculate its apparent proportional gain in mass.

Solution 6

$$m_m = \frac{m_o}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = \frac{m_o}{\sqrt{1-(0.97)^2}} = \frac{m_o}{0.243} = 4.1\ m_o$$

The mass of the electron will be measured as being over 4 times larger that its rest mass from the radius it moves in.

4.18 INTERSTELLAR DISTANCES

Taking the speed of light as 3 x 10^8 m s^{-1}, we can calculate the time it would take for light to reach us from a star if we know how far away the star is.

e.g. the Sun is about 150,000,000 km away, so the light emitted would reach us in a time of $\frac{1.5x10^{11}}{3x10^8}$ seconds which equals about 8.3 minutes, viewed from our Earth frame of reference, but zero time for the photons of light themselves . With fast-moving particles, like protons, emitted from the Sun, they would take longer than 8.3 minutes in our time but a shorter time in the proton's frame of reference, called the Relativistic Time. If v = 0.8c then the time taken for protons to reach Earth if you were on one of them would appear to be given by:

$$t_m = 8.3x\sqrt{(1-0.8^2)} = 5 \text{ minutes}$$

Maybe it will be possible after all for people to reach the nearest star outside the Solar System which is Proxima Centauri. This star is 4.2 light years away from Earth, which means a rocket would have to travel for 4.2 years at the speed of light to reach it. Converting to metres, this gives 4.2 x 365 x 24 x 3600 x 3 x 10^8 m, which is about 4 x 10^{16} m.

If a spacecraft could ever reach a speed of 0.99 c then, theoretically, the time to get to Proxima Centauri at a speed close to that of light would be $\frac{4 \text{ x } 10^{16}}{0.99 \text{ x } 3 \text{ x } 10^8}$ or 4.27 years.

This would be the time it took, as perceived from Earth. In the timeframe of the astronauts on board the spaceship the time taken t_m would be given by: $4.2 = \frac{t_m}{\sqrt{1-(0.99^2)}}$.

Which gives the time taken in the astronauts' frame of reference as about 0.6 years. You could say that, on a 4.27 year flight, the astronauts would only have aged by 0.6 years.

Quite often, with fast moving objects, like satellites and meteors, this small difference in time can make a big difference to results e.g. with GPS, where distances to objects are needed to the nearest metre, correction has to be made to the times because of the relativistic velocity of the Earth and the satellites.

4.19 BLACK HOLES

To escape from the gravitational pull of the Earth, a rocket would need to travel at about 11.3 km s^{-1} but from larger planets this escape velocity would have to be much higher.

On any star, which is basically a fusing mass of hydrogen, there is a dynamic equilibrium between the gravitational pull of the star and the radiation outwards which is escaping and preventing gravitational collapse. As a star burns out the radiated energy decreases and this equilibrium is disturbed so the star's mass starts to collapse into a smaller size. If the mass is big enough the gravitational pull is so large that all its matter collapses into what is called a Singularity (virtually zero size) — so it would not be able to be seen but its gravitational pull is still operating. Even light photons are pulled back, so the singularity appears black.

There will be a point at a distance from its centre where the escape velocity of any object would have to equal the velocity of light. The circumference of a sphere at this point is called the Event Horizon, which has a radius called the Schwarzschild radius $R = \frac{2GM}{c^2}$.

Example 7

What would the Schwarzschild radius be for a star which is 30 times the mass of the Sun?

Solution 7

$$R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 30 \times 1.99 \times 10^{30}}{(3 \times 10^8)^2} = 88.5\text{m.}$$

Any object closer than 88.5 m from its centre would not be able to escape. Also, anything that comes near a black hole would be drawn in and gobbled up by it! Light itself would be pulled back by the immense gravitational field and would not be able to escape inside the Event Horizon. Hence it would appear black!

Set 14: Astrophysics

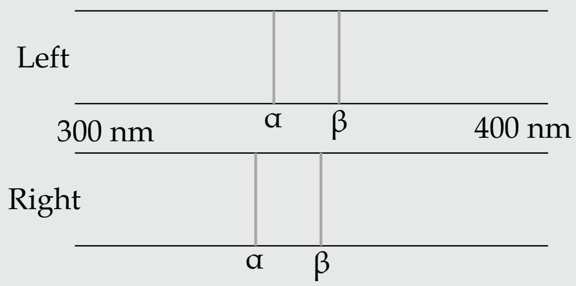
1. A star in a distant galaxy has an absorption line in its hydrogen spectrum blue shifted by 5%. How fast is the star moving?

2. Telescopes can operate so they are sensitive to different wavelengths of electromagnetic radiation, apart from just visible light e.g. IR, x-ray or radio telescopes.

   a) What advantages can be gained from using wavelengths that are not in the visible region?

   b) Why is it that very distant stars can only be observed with radio telescopes, such as the Square Kilometre Array being built in WA?

3. Shown below are two absorption spectra from the same distant galaxy NGC 1227. The absorption lines are in the spectrum of calcium contained in the atmosphere of that galaxy. One spectrum came from viewing the left-hand side of the galaxy and one from the right - and it was noticed that the lines did not match although they were from the same element because NGC 1227 was rotating edge-on to Earth.

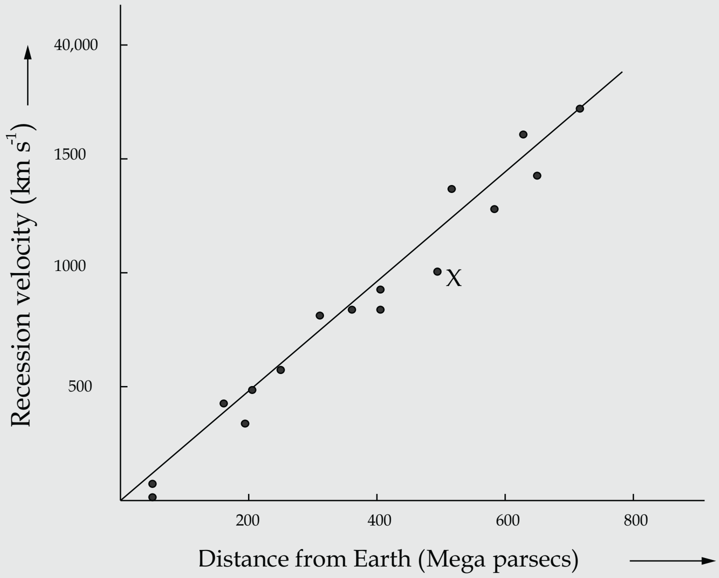


a) Which way was the galaxy rotating i.e. left or right side coming towards us?

On Earth the calcium α line has a wavelength of 339 nm but the wavelengths measured for the left and right sides of NGC 1227 respectively as it rotates are 341 nm and 337 nm.

b) Use these measured values to estimate the speed with which the edge of the galaxy is rotating.

4.



One of the first graphs produced linking the recessional velocity of stars with their distance from Earth is shown above. The values were fairly inaccurate for the time.

a) For the star shown as X, calculate the value of its red shift using estimates from the graph.

b) From the slope of the graph estimate Hubble's constant and hence the age of the Universe (1 Mega parsec = 3.09×10^{22} m).

5. The rest masses of an electron and a positron are both 9.109×10^{-31} kg. In Positron Emission Tomography a positron in the body meets an electron and the two annihilate to produce two photons of equal energy which are emitted at 180^{o} to each other.

   a) Calculate the energy of each of these photons in MeV.

   b) What would the wavelength of each emitted photon be?

6. When the electron in a hydrogen atom absorbs energy it can promoted from the n = 1 level to the n = 2 level. Here it would produce an absorption line in a continuous spectrum.

   a) Calculate the wavelength of this absorption line.

The redshift of galaxy 6C can be expressed as a parameter z where $z = \frac{\lambda_{0berved}}{\lambda_{earth}} - 1$

b) If the value of z for this galaxy is 4.15 calculate the wavelength of the hydrogen absorption line in part a) that would be observed in the spectrum of galaxy 6C.

7. A beam of pions is produced by bombarding a target with high energy protons. The pions are produced at the target moving with a velocity of 0.99 c. Pions will decay into muons with a half-life of 17.7 nanoseconds. Theoretically, half the pions would have decayed in a distance of 5.26 m from the source in this time.

a) Allowing for time dilation, calculate the actual distance from the source where half of the pions will have decayed.

b) If the muon is ejected forwards from the pion at a speed of 0.5 c relative to the pion, will the speed of the muon relative to the laboratory be 1.49 c? Explain.

8. The formula $u = \dfrac{u^| + v}{1 + \dfrac{u^| v}{c^2}}$ can be used to calculate the relative velocities of two objects moving in opposite directions.

A rocket moving through space at 2.5×10^8 m s^{-1} approaches a meteor which is travelling towards it at 1.8×10^8 m s^{-1}. Calculate the relative velocity of the rocket towards the meteor.

9. A spaceship is a distance of 1.2 light years from an Earth-like planet and moves towards it using Hyperdrive at a velocity of 0.5 c. Theoretically, the time to reach the planet should be 2.4 years but this does not allow for relativistic motion.

a) Calculate the actual time that the spaceship would take to reach the planet in its own frame of reference.

b) What would the relativistic distance be to the planet, as measured by the astronauts in the spaceship?

10. The spaceship in the previous question has a rest mass of 12,000 kg. An alien on the Earth-like planet can measure the mass of the approaching spaceship using a gravimeter but the measurement does not correspond to 12,000 kg due to relativistic motion.

What reading of mass would the alien observe?

Chapter

5

Experimental Physics

5.1 INVESTIGATIONS

In the new ATAR courses, quite a large proportion of the marks will be allocated to experimental investigations undertaken by students. Some of these investigations will be of the traditional type, where the teacher outlines the task and the method for conducting the experiment. Other investigations will be more open-ended in that the student is expected to organise and control the methods of the experiment and may even choose the topic to be investigated. Open-ended tasks tend to develop more independence and higher-level thinking processes, because the student is acting more like a research scientist and having to "think on their feet".

The mental abilities these kinds of investigations develop make people who have studied physics very valuable in the workplace. For instance, some physics graduates are actually employed analysing trends in the stock-market due to their ability to evaluate and identify trends in data. This chapter should give lots of ideas for topics for independent research projects in physics.

5.2 GOOD INVESTIGATIONS

Open investigations will be graded on performance on the following aspects:

- Ability to generate ideas (scanning for possible investigations or methods to use, problem-solving, creative thinking)
- Depth of planning (thinking ahead, modifying procedures and organising equipment)
- Ability to think independently and solve problems (coming up with alternative ideas after reflection upon results
- Efficiency in following procedures (using methods that minimise errors, using equipment efficiently and with precision)
- Estimation of errors (evaluating the precision of each measurement taken to produce an overall assessment of uncertainty in the result)
- Depth of interpretation of results (analysing trends, generating possible explanations)
- Effectiveness in evaluating outcomes (using critical thinking and self-evaluation)
- Quality of communication (using good verbal, written and graphical skills)

A good investigator will be aware of all these points throughout any investigation and be rewarded with good marks by the teacher.

5.3 REPORTING INVESTIGATIONS

A typical layout and order of subtitles for a scientific investigation report is demonstrated below:

- **Aim** – this tells us precisely what the investigation sets out to achieve
- **Hypothesis** – this outlines a proposal of what the investigation is expected to discover based on previous experience.
- **Theory** – this is where the facts and mathematical equations that relate to the experiment undertaken are laid out.
- **Method/Procedure** – for this section the processes undertaken in the experiment are written in the correct order. Explanations are given for the methods used and how the variables are controlled to make it a Fair Test.

- **Results** – the tables of readings and graphs are presented here.
- **Conclusions** – this is where the experimenter tries to make sense of the results and relates them to the hypothesis (do the results support it or not?)
- **Evaluation** – in this section a critique is given as to the validity of the findings, the methods used and the accuracy of the findings.
- **Discussion** – this is an overview of the whole project and its usefulness (how can the findings be applied in the "real world"?)
- **Appendices** – this section is really reference for all the more complex and numerous data that are not included in the "results" section. E.g. all the readings shown here but only the averages shown in the Results section. Any other relevant reference material (a web-site, scientific articles, etc.) should be included in an appendix.
- **Acknowledgements** – here any people who have given help in the project are recognised and named.

5.4 VARIABLE CONTROL

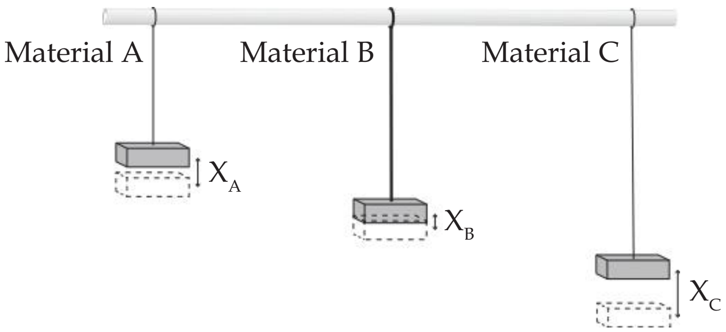
Let's use an example here of where several different types of fishing lines are being investigated to see which has the greatest breaking strain. Suppose we choose to investigate what effect a particular variable has on another, for instance:

How does the material of a fishing line affect its stretch?

For a good scientific investigation we would expect the student to consider the variables in the system and to control all of them except for the Independent and the Dependent variables.

The Independent Variable is the one that we want to change to see its effect (type of fishing line material) and the Dependent Variable is the one that we measure to see how it changes (stretch of fishing line). Consider the experimental set-up shown below to see which type of fishing line material stretches the most:

For this, the student tied a length of each different type of fishing line onto a beam and loaded it with a house brick and then measured how much each line stretched.



A competent student would realise that, although material C stretches the most, this is not a **fair test** i.e. variables have not been controlled scientifically: Line C is longer than the rest and Line B is thicker. It is also possible that all the bricks do not have the same mass.

There are several **Moderator Variables** that need to be controlled for each line material: Length of line; Cross-sectional area of line; Force on line. In actual fact with a bit of smart thinking it is not necessary to make all the line lengths, areas and masses the same if we use some mathematical logic:

Finding a value for the fractional stretch (X/L) takes into account the varying lengths and finding the force per unit area (F/A) allows for the fact that the cross sectional area of each line is different.

Now if we compare the fractional stretch with the force per unit area for each line the only variable that is being compared is the material each line is made of.

This is an example of the Scientific Method and can be seen in action in the determination of Young's Modulus, or Stiffness of materials.

5.5 PRELIMINARY TESTING

Before making an hypothesis or committing yourself to a particular experimental design, it is important, to “get a feel” for the variables and equipment involved. For instance, you could not make an hypothesis about which electric kettle was best unless you knew something about the power of the different kettles being tested, how much they cost, what materials they are made from, etc. This will involve some previous research before you start.

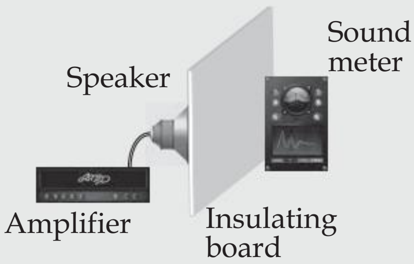
For instance, you might plan to put 100 grams of water into each kettle to heat for 5 minutes and measure the temperature rise, but then a preliminary experiment beforehand might show that the water in all the kettles had boiled away after only 2 minutes – and so all the results would be the same and no conclusion drawn.

Preliminary experiments allow you to get the values of variables roughly right for a full investigation, which saves time later.

If the equipment, volumes, times etc, can be chosen to give the largest variation produced in the results for the dependent variable then this will lead to the greatest validity. A valid experiment is one where we can be most sure about how the independent variable is affecting the dependent variable.

Set 15: Preliminary Testing

1. Some students are investigating the sound insulation properties of some fibreboard that they had in the garage. For this, they set up an amplifier which gave a constant sound level when connected to a loudspeaker with a diameter of 10 cm, as shown below.



The sound level on the other side of the board was recorded with a decibel sound meter. The students had available various sizes of square fibre boards of different thicknesses but were unsure what sized board to use in the main experiment so they set up a preliminary experiment with the aim of seeing the best sized board to use. Their results are shown below:

Board thickness (mm)	Board Size (cm x cm)	Sound Level (dB)
0	10 x 10	75
1		52
2		49
5		40
0	20 x 20	75
1		48
2		42
5		37
0	40 x 40	75
1		40
2		32
5		19

a) Which board size should be chosen for the main experiment from these preliminary tests that will yield the best results? (Look for the greatest variance.)

____

____

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b) Does there seem to be a definite relationship between the thickness of the boards and the resultant sound levels recorded? Explain.

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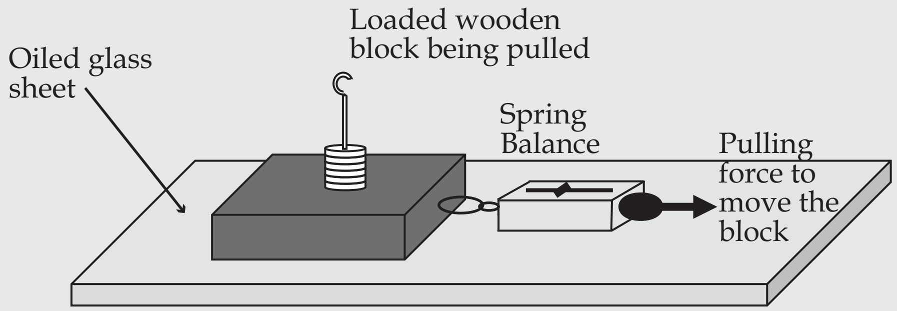
c) From the wave theory of sound, explain why the size of the board might affect the results.

d) Would you expect a lower frequency sound to give the same results as a high frequency sound in this experiment ? Explain, using your knowledge of wave theory.

2. Tamara's physics investigation on Forces involved her measuring the frictional force of a metal plate as it was pulled over a glass slab. She experimented with 3 different types of oil placed between the surface of the block and the glass to see which oil was the best lubricant.



Tamara set up a preliminary experiment using the equipment shown above, so she could get an idea of what weight would be best to use on top of the wooden block. The pulling forces needed to move the block for the different oils and different weights are shown below (all in newtons):

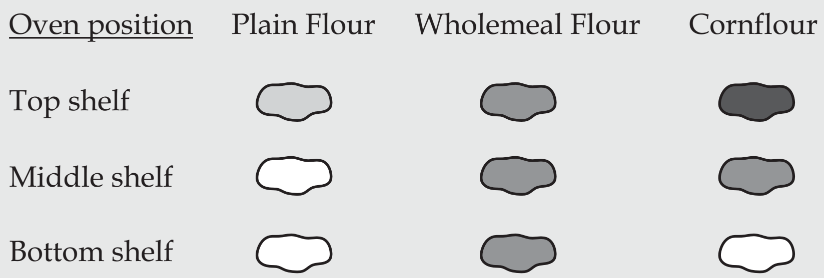
LOAD	OIL 1	OIL 2	OIL 3
0 grams	2.5 N	2.4 N	2.6 N
100 grams	4.2 N	4.3 N	4.5 N
200 grams	5.6 N	5.8 N	5.9 N
300 grams	7.1 N	7.8 N	8.3 N

a) From these results what weight would be best to use when she does the full investigation? Why?

b) Which of these oils seems to be the 'best' from the preliminary tests. Explain your choice.

3. David is investigating the difference in heating power at different positions in an oven. For this, he wants to use the "amount of browning" of some balls of dough to indicate the highest oven temperatures but he is not sure what type of flour would be best to show this. He sets up a preliminary experiment using 3 different types of flour dough balls placed in 3 different positions in the oven.

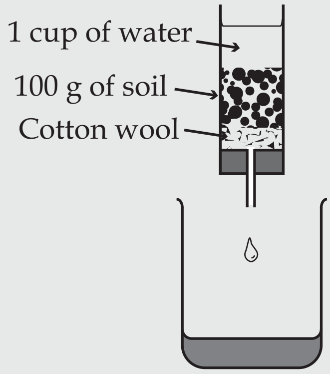
Below are the browning results from David's preliminary tests:



a) Comment on the useful information this preliminary test has given David about flour type and oven position for planning his full investigation.

b) What preliminary conclusions can be drawn about the temperatures at different positions in the oven?

4. Harry had heard that a lot of water just runs through the soil in W.A. and so was going to set up an investigation of the water retention of different soils around Perth. For his preliminary experiment Harry set up a tube-ful of each soil and poured a measured amount of water into the tube.



What Harry wanted to estimate from this preliminary experiment was the time needed for drainage of the tube that would give a good variation between the soils. The volumes of water passing through the tube for each soil in different times are shown in Harry's results table below.

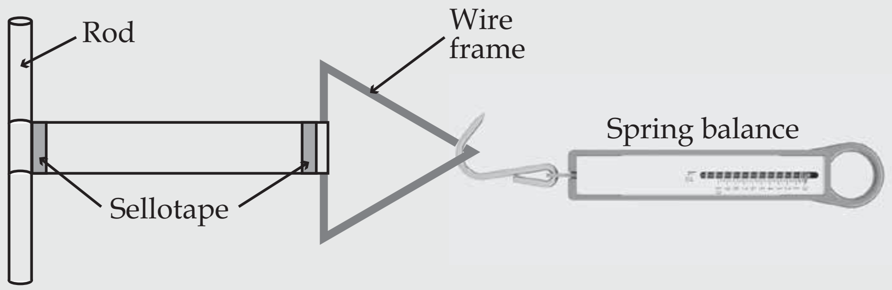
Collection time	SOIL 1	SOIL 2	SOIL 3
5 minutes	4 mL	74 mL	8 mL
10 minutes	39 mL	75 mL	11 mL
20 minutes	60 mL	75 mL	12 mL
60 minutes	75 mL	75 mL	12 mL

a) What collection time should Harry use to judge the degree of drainage for each soil?

b) What are the preliminary results for the relative water retention abilities for each soil? (Rank the soils from best retention to least).

5. For a physics investigation, John and James wanted to experiment with different types of paper to see which would be best for wrapping heavy items in a hardware shop. The boys only had 2 types of spring balances to work with – one that read up to 5 newtons and the other to 10 newtons. For their preliminary experiments they needed to find the best thickness of paper to use in the accurate experiment so they each set up their own test rig to measure breaking forces. John's test rig is shown:

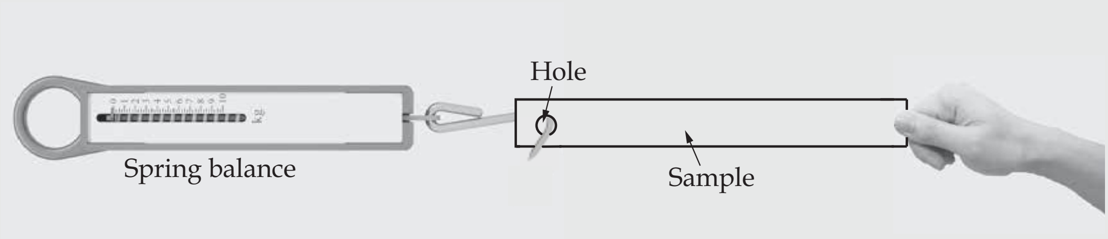
John's test results



Width	5 cm			2 cm			0.5 cm		
Paper	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3
Type A	10	4	8	8	6	2	1	1	1
Type B	7	3	9	7	4	7	1	1	1
Type C	N	N	N	10	5	10	1	1	1

(The letter 'N' in the table means the paper did not break)

James used a different test rig he designed, shown below.



James' results are shown in this table:

Width	5 cm			2 cm			0.5 cm		
Paper	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3
Type A	10	9	10	6	6	5	1	1	1
Type B	N	N	N	7	6	7	1	1	1
Type C	N	N	N	8	6	7	1	1	1

Consider the two different experimental designs and the results produced to answer the following questions:

a) Which seems to be the best experimental design out of the two, and why?

b) Comment on the variance shown in the results of John and James' investigations (variance should be low if it is a good experimental design)

c) From these preliminary results, what thickness of paper should be used in the accurate investigation to make the best comparison between the different papers? Explain your choice.

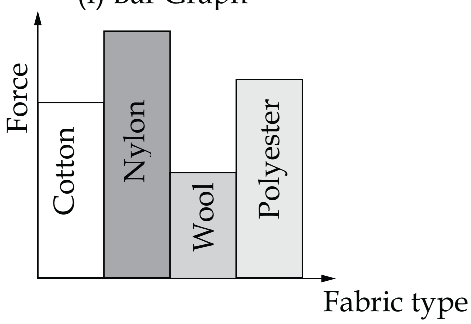
d) Which paper seems to be the strongest from the preliminary tests? Explain.

5.6 CONTROLLING VARIABLES

It has been mentioned before that the factor that is deliberately changed is called the Independent Variable and the factor that is then measured to see how it was affected is called the Dependent Variable. To see what effect the independent variable has on the dependent variable all other factors must be controlled e.g. if we wanted to see which fabric stretches the most we must control the size of each fabric so each has the same length and width. We then might hang weights on each fabric to then measure the stretch.

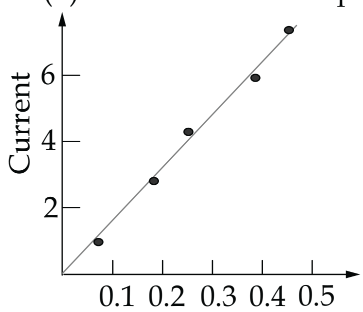
The independent variable is the one we change (e.g. the fabric type) and the dependent variable is the one we measure (the amount of stretch). In this case the Independent Variable is a "discrete" variable, where one value has no link to the next, whereas the "amount of stretch" is a "continuous" variable, where the values increase in increments. Because of this, unknown stretch values can be predicted from interpolation. In general, investigations involving discrete variables make use of Bar Graphs to display results, whereas a "line of best fit graph" would be appropriate for results where the variables are varied in a continuous fashion.

(i) Bar Graph



Breaking force for different materials

(ii) Line of Best Fit Graph



Voltage versus current for a wire

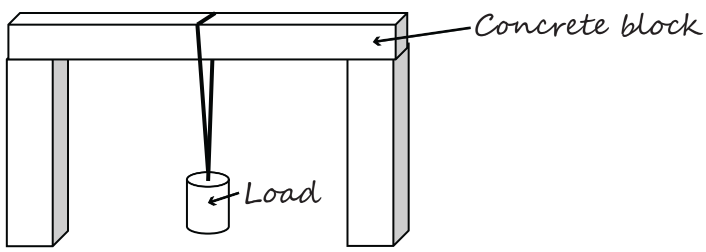
Only from a continuous graph, such as graph (ii) could we predict an unknown current from a given voltage.

5.7 EXAMPLE OF AN INVESTIGATION REPORT BY A STUDENT

Problem: Does the addition of fibreglass to concrete make it stronger?

This is an open-ended experiment as there is no set way of doing it in the book. My operational definition of 'strong' will be the greatest weight (in newtons) a concrete block can hold without breaking.

The independent variable here is "amount of fibreglass added" and a control will be used where an identical concrete block will be tested which has no fibreglass content. The dependent variable will be the amount of load placed on the block to make it break.



Possible set-up

Other variables present are:

Proportions of sand, cement dust and water for the mix
Setting time and temperature
Dimensions of the block
Spacing of the supports
Positioning of the load

Hypothesis

I know that some concrete used for building has other materials added to make it stronger (e.g. steel) so I hypothesise that the more fibreglass that is added the stronger the concrete will be. The amount of fibreglass in the mix is the independent variable as it is the one we vary and the average amount of weight needed to break the blocks is the dependent variable. Using creative thinking I have come up with 3 possible designs which I will criticise:

Possible experimental design	**Criticisms**
Drop a weight onto the block and measure the height needed to break it.	It fractures slightly every time and would give varying results
Push up onto it with a hydraulic jack until it breaks.	Good control of variables but how will I know how much force has been applied?
Load up block as above and measure the maximum load withstood.	How do I judge the exact point of fracture?

A Fair Test set up for my chosen design would be:

- Use a common mould to make all the samples of concrete
- Use a constant mix of sand, cement and water for all blocks (research what a normal mix is)
- Experimental blocks would contain a standard amount of fibreglass
- When set, each block is placed on the supports in the same position and a small load is added in the middle
- Weight is continually added until the block breaks and the load falls (my operational definition of "broken"). Weight added to break the block is the dependent variable.
- To increase the reliability of results 5 repetitions of loading for the experimental and control blocks are carried out.

Results Table of loads (newtons) added until blocks broke

TRIAL	CONTROL (N)	EXPERIMENTAL (N)
1	5.6	6.3
2	6.1	7.0
3	5.9	7.5
4	6.0	5.6
5	5.8	8.0

Conclusions

The addition of fibreglass seems to have increased the strength of the concrete as the average load taken by the blocks with no fibreglass was 5.88 N but with the fibreglass in the blocks could support an average load of 6.88 N – a 17% increase. However, there is a large variance in the readings for the Experimental blocks (range: 5.6 – 8.0) which indicates far less reliability in the results and hence more difficulty in being able to predict the breaking load of fibreglass-reinforced concrete blocks.

My method of testing the beam strength seems to have validity as it reflects the way concrete would be loaded when used in buildings.

Comments on the Student's Experiment

The method of variable control seems quite good in this case but the degree of variation in the values of breaking force (variance) seems quite high as it ranges from the lowest value of 5.6 N to a high of 8.0 N. The data show a variance of about 30%. A large degree of variance in experimental values indicates a serious fault with the experimental design. The conclusions about the fibreglass reinforced block being 17% stronger cannot be certain as the breaking force value of 6.88 N could be up to 15% larger or smaller, which puts its lowest value at about 5.85 N.

Also, as far as significant figures go, this means that the breaking force values should not be quoted with any more accuracy that about 1 or 2 significant figures. Points about significant figures are discussed later in this book (see 5.14).

With such a high variance in this case a competent experimenter would try to modify the method for making and breaking the blocks and check the data again with the aim of getting the results within a much smaller margin of variation.

Instead of judging the strength of the two concrete mixes simply by comparing the average values a much more rigorous statistical analysis would be advantageous. Statistical methods are commonly in use in subjects such as economics or biology and rarely discussed in physics, but for investigations the use of t-tests is very useful and easy to apply using the Excel program (also see later section 5.15). In this particular case a t-test between the two sets of data indicates that there is a significant difference between the maximum breaking strain values for the two different concrete compositions.

Set 16: Variable Control

1. A student wanted to find out which electric motor was the most powerful.
   a) Write a simple plan for how this could be achieved in the laboratory or at home and say what measurements would be taken.
   b) Say what you would take as the Independent variable and Dependent variable.
   c) Say what other variables are important and how these would be controlled.

2. Two students were arguing about whose skateboard was best.
   a) Derive a valid test for the two skateboards, defining exactly what is meant by "Best" in this case.
   b) Give the title of the Dependent Variable and a possible way of judging this accurately (you may need a diagram).
   c) List the other important variables in the Fair Test and say how each could be controlled.

3. Which brand of tissue absorbs most water?
   a) Give an hypothesis based on what you know about tissues available in the supermarket
   b) Name the Dependent and Independent variables in this investigation.
   c) How could you set up a Fair Test situation for this investigation, which allowed for the fact that all the tissues used are of different sizes?

4. Does the ear hear some frequencies of sound better than others? Using equipment available in the laboratory sound levels and frequencies can be controlled and measured.
   a) Explain how you could set up an experiment to investigate the question outlined above and how the accuracy of results could be maximised.
   b) What will you define as your Independent Variable and how will you control this?
   c) What will you measure as the Dependent Variable and how will you control all the other variables?

5. How does the amount of weight that can be lifted by your bicep muscle depend on the position of the weight along your arm?

   a) Explain how you could change the problem from one involving human judgement to a simulated situation, using wooden rods, weights and spring balances.

   b) Define what will be the Independent and Dependent Variables and how these will be measured.

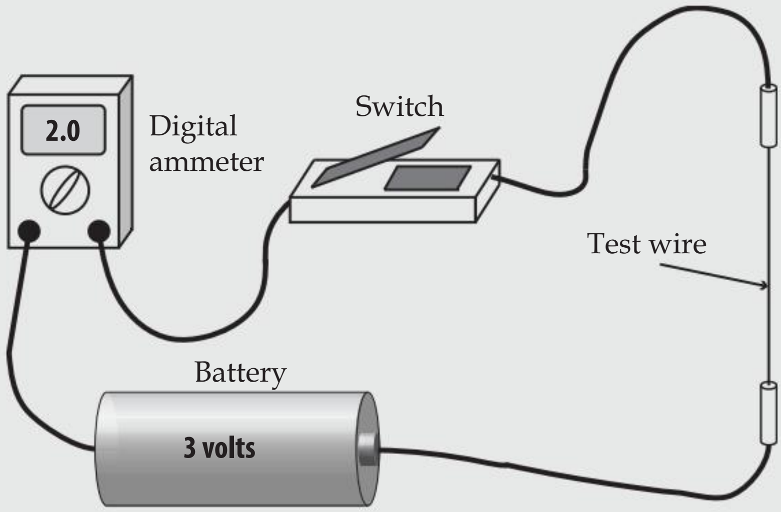
   c) What other factors must be controlled and how would you do this?

5.8 ANALYSING DATA AND DRAWING CONCLUSIONS

After performing an accurate science investigation good investigators will look at the results very carefully and use their critical thinking skills to "make sense of them" i.e. discover what the pattern or equation it is that links the Dependent Variable to the Independent Variable. If an obvious pattern is seen that can be used to accurately predict a result then it can be expressed as a mathematical equation (called a 'Law') and in a graph all points will lie on a line e.g. a graph of the number of cartons of milk against how much the milk weighs. If there is some unpredictability (or variance) then the points on the graph will lie around the trend line but will not follow an exact pattern e.g. the weights of people versus their heights.

Set 17: Analysing Data

1. Tim has decided to do a physics project on electricity and has set up a circuit with a 3 volt battery, some iron wire and a digital ammeter.



Tim wanted to find out how the current through a wire depends on its diameter, length and composition. In his investigation he clamped an iron wire with two different diameters and two different lengths in the position shown on the diagram. Tim repeated the whole experiment again with another wire called Constantan. His results table is shown below.

Iron Wire			Constantan Wire		
Diameter (mm)	Length (cm)	Current (amp)	Diameter (mm)	Length (cm)	Current (amp)
0.105	9.8	0.51	0.105	9.8	0.26
0.201	9.8	1.99	0.201	9.8	0.99
0.201	5	4.13	0.201	5	2.10
0.105	5	1.09	0.105	5	0.53

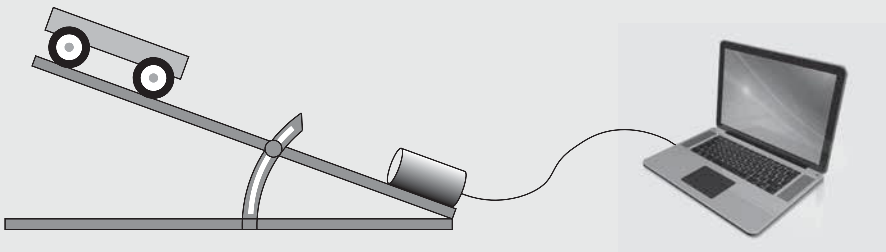
a) What conclusions could Tim reach about how the length of a wire affects the current flowing?

b) How does the diameter of the wire affect the current flowing? Is this a simple relationship?

c) Tim's friend Sarah thinks that the current may be directly related to the area of the wire, rather than the diameter. Do some calculations to see whether this hypothesis is supported.

d) What conclusions can be reached about the electrical properties of Constantan?

2. Samantha is investigating how the acceleration of a trolley on a slope depends on the angle of the slope.



For this, she adjusted the angle of a wooden slope, placed a trolley on it and let it run down. The acceleration of the trolley was measured using a motion sensor attached to a lap-top computer running a special program.

Samantha's results are shown in this table:

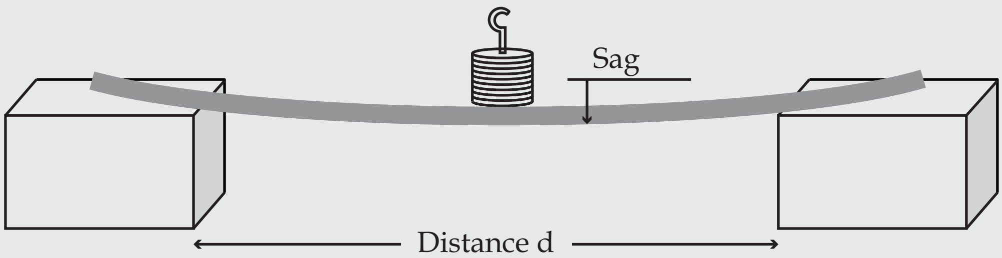
Slope angle (degrees)	Acceleration (m s^{-2})
10	1.7
20	3.3
30	4.9
40	6.3
50	7.5

a) Does the value of acceleration appear to be linked to the angle by a linear relationship? Explain.

b) Samantha's friend, Amy, suggests that the acceleration depends on the sine of the angle of slope. Perform calculations to see if Amy's idea is correct.

c) Write an equation in the form of y = mx + c linking the Dependent and Independent variables for this experiment.

3. Hiro always wanted to be an engineer like his mum and was particularly interested in bridge design. For his physics project, he made up a model wooden bridge held at both ends on wooden blocks as supports.



He put 100 gram weights on the bridge to simulate cars and measured how much the bridge sagged in the middle, keeping the distance between the supports (d) constant. This gave him an idea of the strength a real bridge would have.

Hiro's table of results is shown in Table 1.

Table 1

LOAD (grams)	SAG (mm)
0	0
100	4
200	9
300	12
400	15
500	20

a) Use table 1 to plot a graph of Sag (y) against Load (x).

b) What kind of graph is produced by this plot? What pattern is shown by the results i.e. how is the sag related to the load?

Draw graph here

In another investigation, Hiro kept the load the same, moved the wooden blocks further apart and measured the sag again. The results of this investigation are shown in Table 2

Table 2

Distance d (cm)	20	30	45	50	35	25	40	55
SAG s (mm)	0.8	2.7	9.1	12.5	4.2	1.6	6.4	16.6

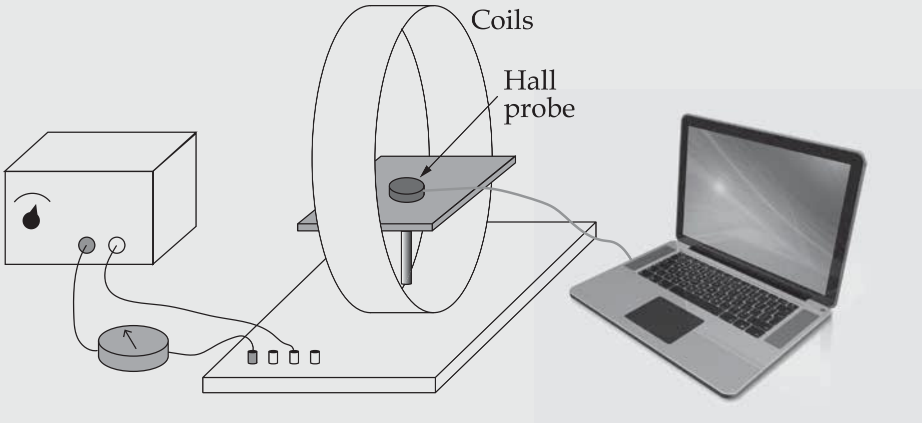
c) Use this to plot a graph of Sag (y) against Distance d (x).

d) What pattern is shown by the results i.e. how is the sag related to the distance apart of the supports?

e) What other variables might affect the sag for these model bridges.

4. The year 12 physics students at Middleton School were performing an investigation in class on the magnetic field inside a coil. The teacher supplied a vertical coil of wire with a horizontal platform in the middle, a power pack and a sensor called a Hall Probe which could measure the magnetic field strength (B) inside the coil.

A diagram of their set-up is shown below:



The magnetic field (B) was measured for:

(i) different numbers of turns on the coil and

(ii) different values of current.

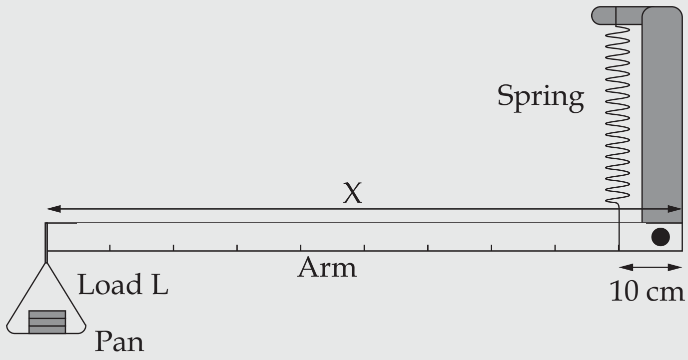
The results of the experiments are shown below.

	Field Strength B (T)		
Current →	I = 0.5 A	I = 1.0 A	I = 1.5 A
Number of turns N ↓			
50	25	40	73
100	48	79	112
200	108	140	197

a) How does the field strength appear to be connected to the number of turns?

b) How does the field strength appear to be connected to the current in the coils?

5. Arun and Sam are interested in the way the lever system in the human arm operates. They set up the apparatus shown below with a strong spring acting as a bicep muscle and a horizontal, pivoted metre rule as the forearm.



Arun found that, with the spring attached 10 cm from the pivot, a 200 g load could bring the arm down to a horizontal position when placed on the pan (100 cm from the pivot). The pan was then shifted to a different distance from the pivot and an appropriate load added until the arm came down to a horizontal position again. This process was repeated several times with different loads and different values of x to produce a horizontal arm.

In a second experiment the whole procedure was repeated, but with the spring attached to a point 15 cm from the pivot instead of 10 cm.

The results of this experiment are shown in Table 1.

Spring @ 10cm	Spring @ 15cm	Table 1
Load L_1 (grams)	**Load L_2 (grams)**	**x (cm)**
250	380	100
310	470	80
410	625	60
620	940	40

a) Name the Independent and Dependent Variables

b) How does the value of the load in each case depend on the distance the pan is placed from the pivot?

c) What kind of mathematical relationship exists between L and x?

d) When the attachment point of the spring is changed from 10 cm to 15 cm what effect does it have? Does the relationship from part c) still apply?

5.9 EVALUATING EXPERIMENTS

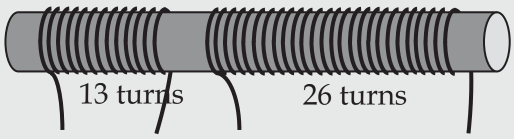
Evaluation is the highest order of thinking levels as it requires analytical, critical and creative thinking modes to make criticisms and judgements of experimental designs and to improve them. To do this, students must understand the good techniques involved with investigations and think effectively about how well the variables have been controlled. Good physics students should also be able to evaluate experiments in other subject areas, such as chemistry or biology, where the Scientific Method applies equally well. This is why many physicists are used in business circles to analyse data; not necessarily in the area of science (e.g. Stock Market share trends).

Set 18: Evaluating Investigations

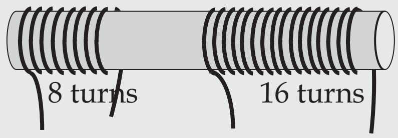
1. Two students were studying electromagnetism and transformers and wanted to investigate the effect of different cores on the voltage output from the Secondary coil. They wound several coils around different core materials with different shapes and used them in their home-made transformers. They then applied a 2.5 volt AC voltage across the terminals of the Primary coil of each one. In the experiment they measured the output voltage across the secondary coils to compare the effectiveness of different core materials. Four different cores were available from school.

   Below are sketches of the students' transformers and details of their set- ups with results shown in Table 1.

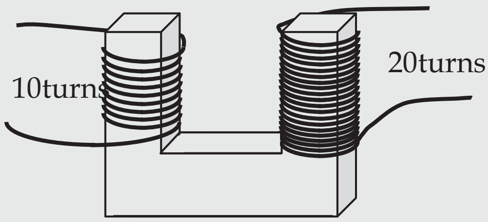
No. 1. 12 cm wooden core



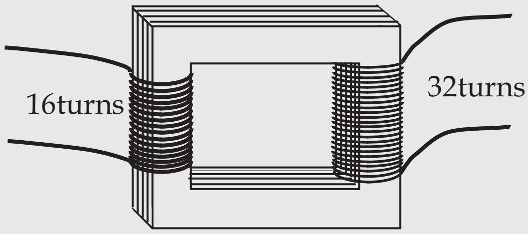
No. 2. 10 cm iron core



No. 3. 8 cm x 8 cm Alnico core



No. 4. 10 cm x 8 cm laminated iron core



Students' Results

Output voltages of each transformer (V_O) for an input voltage of 2.5 V ac.

Table 1

Coil Number	V_O (volts)
1	1.2
2	3.5
3	4.4
4	4.9

Students' Conclusions

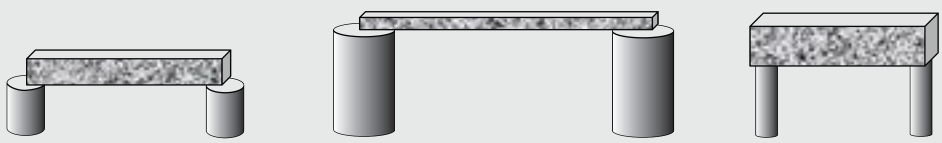
We found the best transformer material to be iron because Coil 4 gave the largest output voltage. Our results are a bit uncertain because we didn't manage to wind the same number of turns on each transformer and this could have affected our results. We also only had time to try the coils with a single voltage input of 2.5 V and we think that this might have been too low to be sure that No. 4 transformer was best at its normal 240 V operation. For safety reasons we had to use a low voltage.

a) Comment on the experimental design used by the students – was it a Fair Test?

b) The students criticised their own experimental methods in two areas: (i) the number of coils were different and (ii) the input voltage was very low. Give your opinion of how these two factors may have affected the conclusions about the transformer materials.

c) Coils No. 3 and 4 seemed to be the best performing but one of the students thought that Alnico could actually have been a better core material than iron. Give an explanation as to why this could be correct.

2. The diagram below shows a testing method used to find the strengths of 3 different types of concrete mixes. Concrete is made of sand, cement dust,



aggregate (small pebbles) and water.

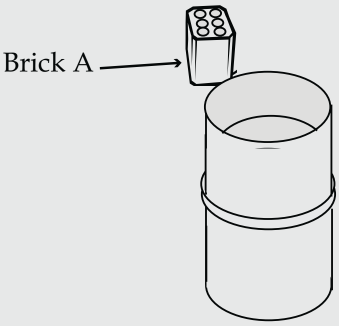
The method used for testing the strength of each block was to hit each one with a sledge hammer and to note how many hammer blows it took to break.

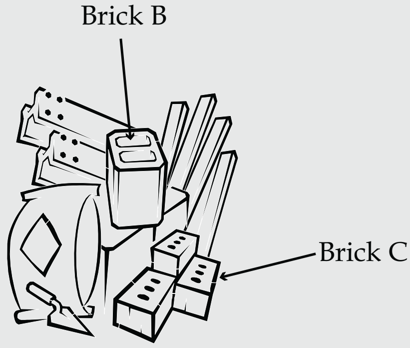
a) Evaluate this method as a fair test for the strength of concrete and write your criticisms. Identify and list the most important variables that would need to be controlled for a good test of different concrete mixes.

b) Make a sketch of a test rig that could find the breaking force of a concrete block in a more scientific way.

c) The Rule of Many says that single reading should never be relied upon. Explain why repeating the experiment several times would lead to a more accurate result in this investigation.

3. Jarrad's dad is a builder and so he wanted to investigate the water absorption of different types of bricks for his physics project. In his experiments he half-filled a 200 litre oil drum with water and marked the water level on the side.





Jared then put one type of brick into the water for 5 minutes and then marked the new water level (it went down as the brick absorbed water). In his maths lesson, he found out that the volume of water absorbed would be $\pi \times r^2 \times h$, where h is the drop in water level and r is the radius of the drum.

Jarrad's results were:

Water absorbed by brick A = 620 mL

Water absorbed by brick B = 1870 mL

Water absorbed by brick C = 1590 mL

He conclude from these results that brick B was made of the most absorbent type of material.

a) Discuss Jarrad's method of investigation, mentioning good and bad points.

b) Are there any points that could be improved with his method of measuring the amount of water absorption?

c) How could Jared have allowed for the fact that the bricks were of different sizes, (no cutting allowed).

4. Plastic Bag Investigations

The following two investigations were performed by two different students. You need to evaluate the quality of their experimental designs and conclusions

A. Jeffrey's Investigation.

Planning

We are going to see how strong 3 different types of plastic bags are by putting weights on them. We will keep the length of each bag being tested the same. We predict that the green bag will be stronger because it is coloured green like a strong shopping bag.

Diagram

We used lengths of pipe for weights but made sure they didn't fall on anyone's feet.

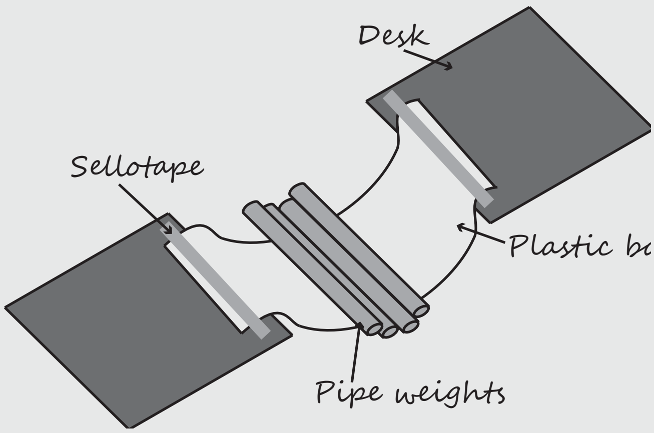
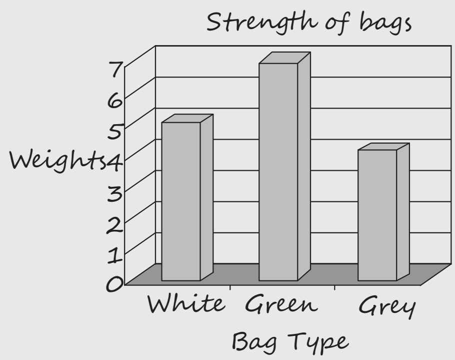


Table of weights needed to break bag

White bag	5 weights
Green bag	7 weights
Grey bag	4 weights



Conclusions

The green bag material was the strongest and the grey bag was the weakest. The prediction was correct which proves the colour made it the strongest plastic.

B) Karen's Investigation

Planning

I believe there are 4 major independent variables affecting the strength of a strip of plastic bag: length, width, thickness and type of plastic. If we want to see how the plastic type affects the strength we must keep all the other variables the same. My hypothesis is that the green plastic will be stronger because it feels the thickest.

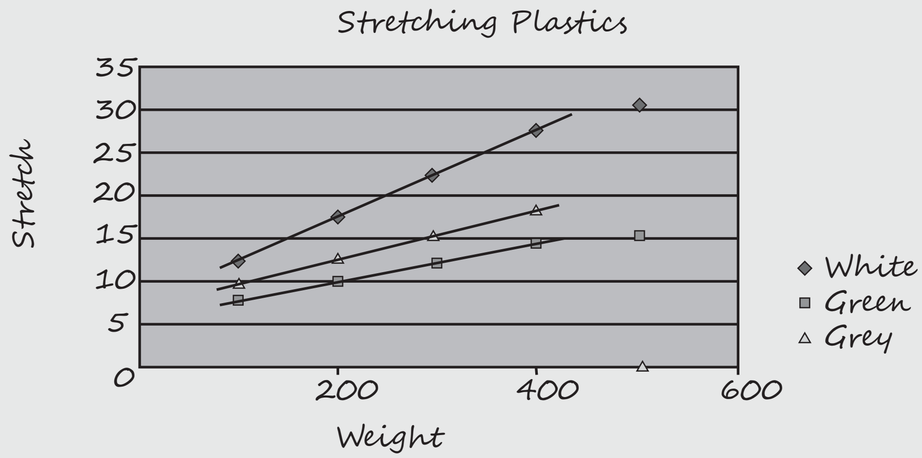
Method

1. Cut strips to be 30 cm long and 1 cm wide and measure the thickness to see if they are the same. We did this by putting 20 pieces on top of each other and measuring the thickness (then divide by 20)
2. Diagram
3. We did a preliminary experiment to find what size spring balance was needed. We found a 25 newton balance was good.
4. Pulled the balance and read the spring balance maximum value as the plastic just breaks.
5. We repeated each experiment 3 times with each plastic to get the averages
6. We drew up a table of values
7. (Extra experiment) Each plastic sample was pulled again with the force gradually increasing and we measured how the length changed for each one.
8. We plotted 2 graphs to show the data.

Results

Force to break each plastic strip (grams)

	1	2	3	Average
White bag	550	500	525	525
Green bag	720	780	930	810
Grey bag	420	460	440	440



Extra experiment (Results of stretch in cm)

Weight (grams)	100	200	300	400	500
White	12.5	17.2	22.2	28.1	31.0
Green	7.5	10.1	12.4	14.3	15.5
Grey	9.8	13	16	18.7	Broke

Conclusions

The green bag seems to be the strongest as it can hold about 810 g before breaking (We suspect the last reading of 930 g was an error). The grey bag was weakest.

From the extra experiment the white bag is the stretchiest as the slope of this graph is the steepest (see graph of the results table).

Explanation:

I think the molecules of the green plastic are longer and this causes the forces between them to be greater but I would have expected the green plastic to stretch less as it took more force to break it. There is not much difference between the green and grey plastic fort stretch. For wrapping food tightly (meat) the white plastic would be best as it would stretch tightly round the food without any slackness.

Give an evaluation of the two students' experiments i.e. compare the good and bad points in the following areas:

a) Evaluate the two experimental designs.

b) Evaluate the data presented by each student.

c) Evaluate the data processing used by each student (averages, graphs, etc).

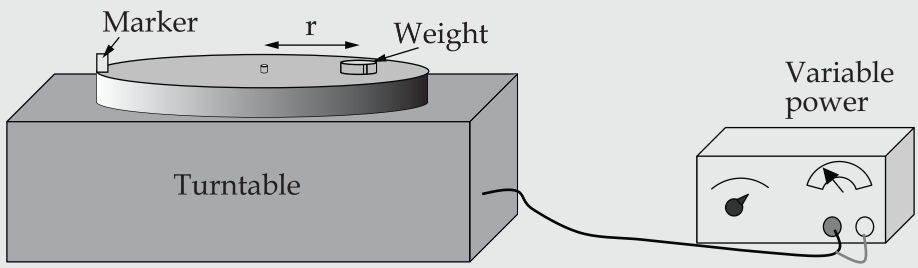
d) Evaluate the conclusions reached by each student.

5. Centripetal Forces

Ben and Alison had just completed the Motion section of the Year 12 syllabus and were interested in investigating centripetal motion. They discussed how they could use one of their father's old record turntables for this and decided it would do very well if they could change its speed of rotation. Alison found a power supply whose voltage (V) could be varied continuously to drive the turntable and thereby adjust its rotational velocity.

They placed a 100 g weight 20 cm from the centre of the turntable and increased the speed until the weight fell off. They argued that when the centripetal force became greater than frictional force on the weight it would begin to slide.

Their apparatus set-up is shown below.



Ben and Alison adjusted the turntable rotation rate and when the weight fell off, they then found how many rotations it performed in one minute by counting the number of times a marker on the turntable went past in a 60 second interval. They then changed the distance of the weight from the centre and repeated the experiment.

The students' results table is shown below:

Voltage (V)	No. per minute	Distance r (cm)
8.5	95	20
13.9	150	10
21.6	190	5

a) Using your knowledge of centripetal equations try to explain why, as the distance of the weight from the centre decreased the number of revolutions per minute increased.

b) In measuring 'r' the students measured from the centre pin of the turntable to the centre of the mass. If the mass was quite wide (5 cm) what error could this produce? Explain.

c) Ben insisted on cleaning the surface of the turntable before the experiments. Why is this necessary?

d) Does Alison's idea of using a Sellotape marker on the turntable seem an effective way of measuring the rotations per minute?

e) Is it important for the students to take account of the voltage of the power pack in this experiment?

5.10 PROCESSING DATA

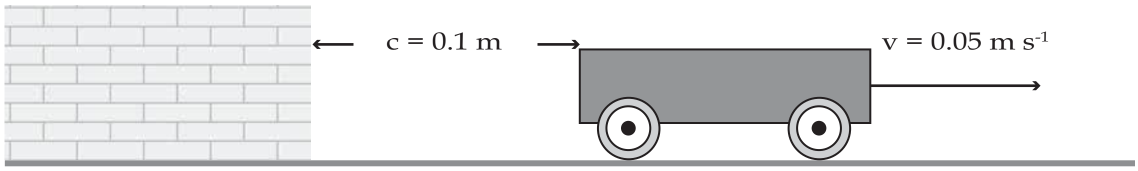
It is far easier to draw a line of best fit in physics than to estimate where a curve of best fit might be drawn through a set of data points.

Many relationships in physics are linear e.g.

- The number of pages in a book and its weight (N α W)
- The stretch of a spring versus the load on it (s α L)
- The temperature rise of an object and the amount of heat supplied (ΔT α H)

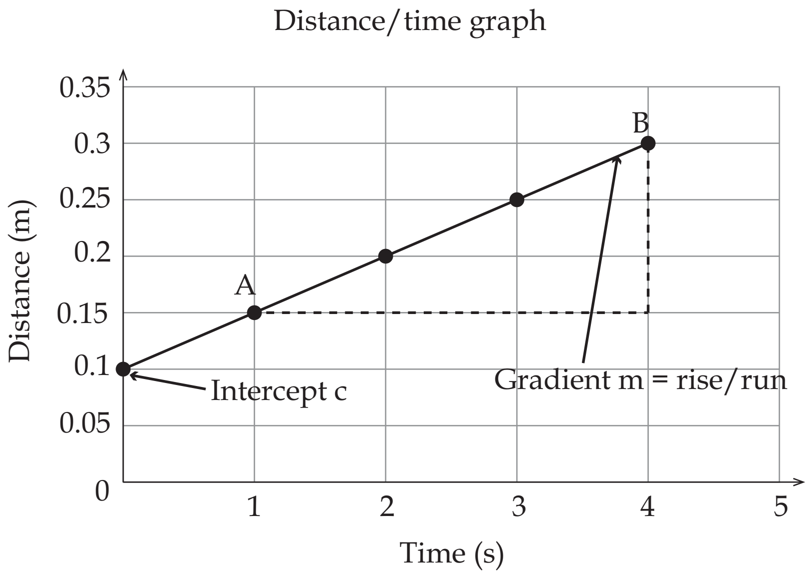
For all of these above, the mathematical relationship would be in the form of y = mx + c, a straight line.

In the example below a trolley starts 0.1 m from a wall and increases this distance by 0.05 m every second



The distance of the trolley from the wall will vary as shown below:

Distance (m)	Time (s)
0.10	0
0.15	1
0.20	2
0.25	3
0.30	4



y = mx + c

For the graph y = distance travelled and x = time. m will equal the gradient of the line (speed) and c will be the distance of the object from the assigned zero position.

To calculate the gradient of a straight line graph we use the formula: gradient $= \frac{\text{rise}}{\text{run}}$

For accuracy, the triangle chosen to calculate gradient must be large:

(NB. two adjacent points on the graph will be inaccurate!)

Suitable points would be point A and B shown above. So the gradient of the line would be:

$$\text{gradient} = \frac{y_B - y_A}{x_B - x_A} = \frac{0.3 - 0.15}{4 - 1} = 0.05 \text{ m s}^{-1}$$

The data used in the example above are similar to those seen in maths books, as they have been generated from an equation, Distance = 0.05t + 0.10

However, when data are measured, as with physics experiments, rather than found from an equation, the points will not lie directly on the line due to inaccuracies in making the measurements (ruler or stopwatch).

However, lots of relationships in the Universe do not conform to a linear relationship e.g :

- The height of a person versus their weight
- The brightness of a lamp versus the current flowing
- The force on an iron bar versus its distance from a magnet.

Graph Straightening

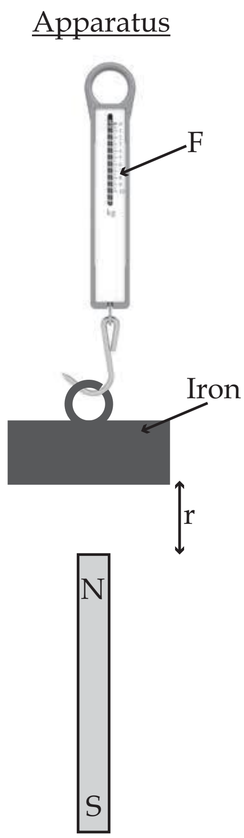
This last magnetic example involves the Inverse Square Law where force (F) should be inversely proportional to the distance from the magnet (r) according to the relationship:

$$F_m = \frac{k}{r^2}$$

i.e. if the distance away increases by a factor of 2 then the force reduced by a factor of 2^2 (4 times less)

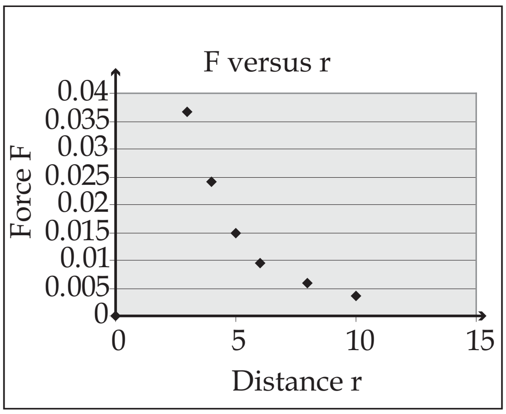
Here are some results obtained from an investigation of magnetic force at different positions, shown in Table 1.

r (cm)	F (N)
3	0.0367
4	0.0242
5	0.015
6	0.0096
8	0.0059
10	0.0035



If we plotted a graph of F versus r we would obtain a set of points that fit to an inverse square graph, where it would be difficult to draw a curve of best fit.

Straight lines of best fit are much easier to draw.



However, we can change this graph to a straight line graph (graph straightening) by plotting a different set of variables.

Written in the form $F_m = k\left(\frac{1}{r^2}\right)$ we can see that by plotting $\left(\frac{1}{r^2}\right)$ as a single, separate variable the equation is now in the form of $Y = mX$, where $Y = F$ and $X = \left(\frac{1}{r^2}\right)$

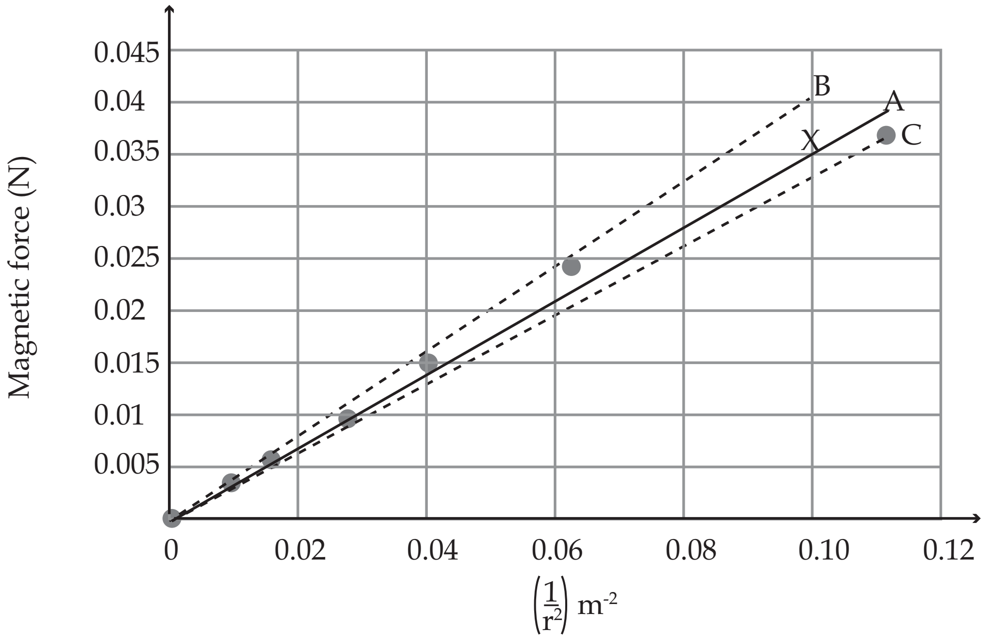
This will give a straight line of best fit.

A new table (Table 2) needs to be drawn up with a column at the end containing values of $\left(\frac{1}{r^2}\right)$

This is shown below with the resulting graph of F versus $\left(\frac{1}{r^2}\right)$

r	1/r2	F
3	0.1111	0.0367
4	0.0625	0.0242
5	0.0400	0.0150
6	0.0277	0.0096
8	0.0156	0.0059
10	0.0100	0.0035

Graph of F versus $\left(\frac{1}{r^2}\right)$



5.11 UNCERTAINTIES FROM GRAPHS

When the line of best fit (shown as A) is drawn through the points what we are doing is using statistics to estimate where the exact data line should be if there were no error. This is most probably correct statistically, but may not be the true line. One method of estimating the possible error or uncertainty in the data is to draw in two alternative lines – one showing the maximum slope and the other, the minimum slope.

From the graph (point X) we could conclude that the slope A is $\frac{0.035}{0.1} = 0.35$

Slope B = $\frac{0.04}{0.1} = 0.4$ and slope C is $\frac{0.035}{0.11} = 0.318$

We can calculate the percentage uncertainty in the slope (k) thus:

Maximum value (B) = 0.4, which gives % uncertainty from the slope of A as:

$\left(\frac{0.4 - 0.35}{0.35}\right) \times 100 = \pm 14\%$

Minimum value (C) = 0.318 which gives % uncertainty as

$\left(\frac{0.35 - 0.318}{0.35}\right) \times 100 = \pm 9\%$

So we could conclude that the value of the slope is 0.3 with an uncertainty of +14% and -9% i.e. the best value for k should be quoted as 0.35, but most probably lies between values of 0.32 and 0.4.

5.12 UNCERTAINTIES FROM SCALES

Every measuring instrument, no matter how expensive, will yield some uncertainty in its measurements. This uncertainty (or systematic error) can be gauged from the least division on its scale of values.

For example: if a ruler measurement is 8.4 cm (least division 0.1 cm) this implies that this value could have been corrected up from a smaller value of 8.35 or down from a larger value of 8.44 cm. An estimate of the measurement, with error, in this case would be 8.4 ± 0.05 cm i.e. error is half the least division. Here this constitutes an uncertainty in measurement of (0.05/8.4) x 100 = 0.6%.

If the measured length had been smaller (e.g. 4.2 cm) then the % error would be larger and more significant (1.2% if the length is 4.2 cm).

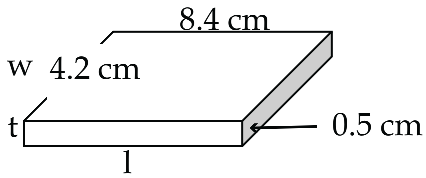
Very small distances, therefore, incur a larger error if measured with a ruler and so a more accurate measuring instrument would be required (e.g. micrometer).

To estimate the uncertainty in a final experimental result where a formula is used wherever numbers are being multiplied the percentage errors should be added.

e.g. The volume of a wooden block.

To find the volume we calculate l x w x t

8.4 x 4.2 x 0.5 = 17.64 cm^2



Suppose the error in l = 0.6%, error in w = 1.2%, error in t = 10%. Total % error = 11.8%.

11.8% of 17.64 = 2 cm^3 absolute error in the volume.

5.13 SIGNIFICANT FIGURES

Giving the above result as 17.64 ± 2 does not make sense because the number of significant figures quoted in the value 17.64 (4 s.f.) implies a rounding uncertainty in the 5th significant figure (1 in 10,000), whereas a value of ± 2 indicates that the 2nd significant figure in the answer (the '7' digit) could be as far out as ±2 in its true value.

The correct, scientific, way of quoting the final answer should be (18 ± 2) cm^3 which means it could vary anywhere in value from 16 to 20 cm^3.

NB: If a factor in an equation is squared then the % error must be doubled. E.g. if the radius of a circle has an uncertainty in measurement of 3%, then the uncertainty in the area($A = \pi r^2$) will be 6%.

Set 19: Uncertainties

1. A teacher takes a group of students out on the oval to measure the velocity of sound. The teacher measure out 100 m and stands at that point with a starting gun while 5 students have their stop-watches ready. The teacher fires the gun and when the students see the smoke each start their stop-watches and when they hear the sound of the explosion the button is pressed to stop it. The readings from the 5 students' watches are:

   0.33 s; 0.29 s; 0.31 s; 0.30 s and 0.27 s.

   a) Estimate the uncertainty in the time taken for the sound to reach the students by finding the variation from the average.

   b) Use the value from part a) to calculate the speed of sound on the oval.

   c) If the measurement of the distance was estimated to be within 0.7 m, make an estimate of the % error in the answer for the velocity of sound.

   d) What should the quoted value of the speed of sound be for this experiment, to the correct number of significant figures?

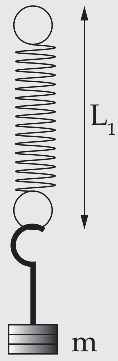
   e) One of the students suggests that more accurate stopwatches and better tape measures should be used to increase the accuracy of the answer. What do you think is the main source of error in this experiment and how could this be reduced most easily?

2. The spring constant k, of a spring is defined by the equation $k = F/\Delta L$.

   F is the load on the spring = mg

   ΔL is the change in length of the spring under the load.

   The readings taken in an experiment to find k for a spring are given below, with the degree of accuracy of the instruments used:

   Mass m = 150 g ± 0.1 g

   Initial length of spring, L_1 = 8.6 ± 0.1 cm

   Stretched length of spring, L_2 = 13.9 ± 0.1 cm

   a) Calculate a value for the spring constant k of the spring.

   b) Calculate a value for the % uncertainty of k found in part a).

   c) Quote a final value for k from this experiment with its absolute uncertainty to the correct degree of accuracy.

3. The formula for the density of a substance is given by D = m/V, where m is the mass and V its volume.

   A rectangular block of granite has measurements of 7.8 cm x 10.6 cm x 0.3 cm (measured with a millimetre rule) and a mass of 62 g (measured on the kitchen scales to the nearest gram).

a) Calculate the density of the granite

b) Calculate the % uncertainty in this value of density

c) Quote the density of granite and its absolute uncertainty to the correct number of s.f.

4. The time for a ball-bearing to fall from the bench top to the floor is given by the formula:

   $s = ut + \frac{1}{2} gt^2$ where s is the height of the bench (1.23 m ± 1 cm), t = time of fall, g is the acceleration due to gravity and u = zero, in this case.

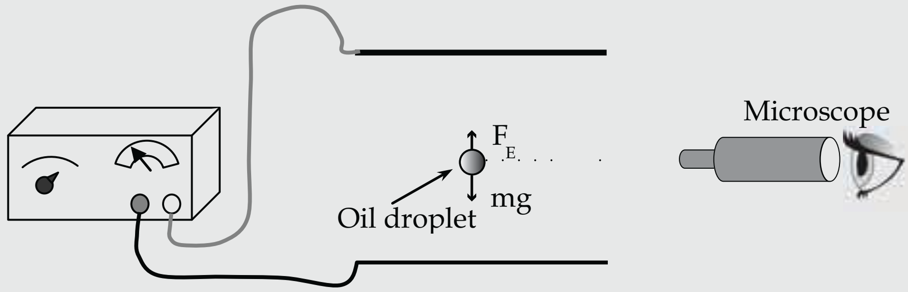
   The time for the fall was measured with an electronic timer correct to ± 0.01 s with the following trial times: 0.55 s; 0.50 s; 0.45 s; 0.55 s; 0.60 s.

   a) Calculate a value for 'g' from these data

   b) Estimate the % uncertainty in the answer.

   c) Quote a final answer for 'g' and its absolute uncertainty to the correct number of s.f.

5. In the original experiment to determine the charge on an electron, Millikan produced a tiny spherical oil droplet and gave it an unknown negative charge by ionising the air around it with X-rays. He then arranged for this droplet to be projected into a space between two charged metal plates so that is was attracted upwards towards the top positive plate in the electric field present.



The scientist adjusted the potential difference between the plates so that the attraction upwards of oil droplet exactly balanced its weight downwards and so the droplet was motionless. The motion of the sphere could be observed through a microscope. The measurements made then allowed the charge on the droplet to be ascertained.

<u>Relevant data for this experiment.</u>
Voltage (V) between the plates: 2100 V (± 100)
Spacing (d) of plates: 4.2 cm (± 0.1)
Radius (r) of oil droplet: 0.25 mm (± 0.05)
Density (ρ) of oil: 708 kg m^{-3} (± 2)
Relevant formulae for this experiment.
Electric force on a charge $F_E = Eq$
Electric field strength $E = V/d$
Mass of oil m = density x volume
Volume of a sphere: $\frac{4}{3}\pi r^3$

a) Use the equations and data given above to equate the weight of the oil droplet down with the electric force on it upwards to determine the charge on the droplet.

b) Calculate the percentage error in the value of charge calculated (Note that the error in r^3 will equal 3 times the error in r).

c) Write a final answer for the value of q and its absolute uncertainty, giving these to the correct number of significant figures.

5.14 GRAPH PLOTTING

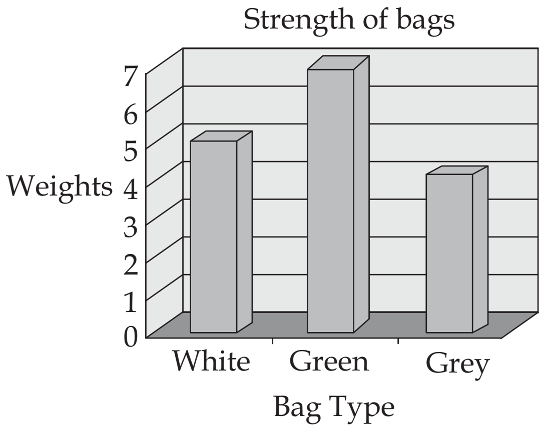
It is fairly easy to plot graphs on graphics calculators from a set of tables giving values for the Independent and Dependent Variables. However using the Excel program on your computer is much better as it is more visual, easier to manipulate and the graphs produced can easily be cut and pasted into an experimental write-up.

Graph Types

a) Bar Graphs

Bar graphs are rarely used in physics as they merely display "discrete variables" that are not connected or not continuously variable

e.g. Type of plastic and its strength – the different types of plastic have no relation to each other and so are just stand-alone named variables.

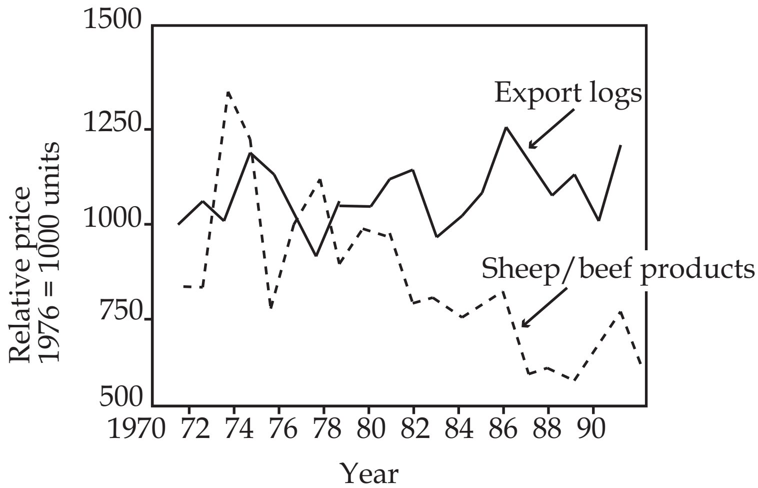


b) Point-to point graphs

These graphs are rarely used in physics as they have no equation linking one point to the next, although they may show a general trend overall.

e.g. Weather patterns or economics data.

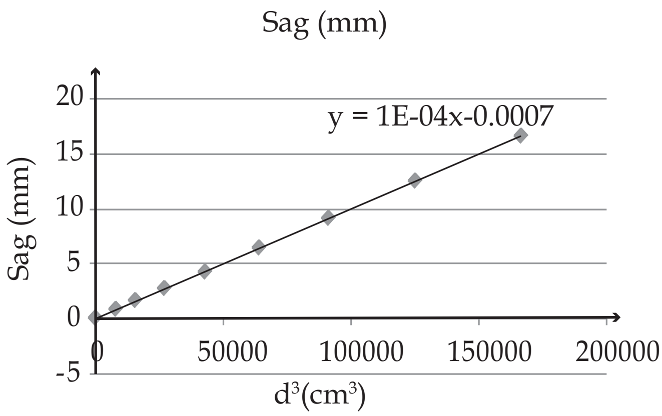
The 'export logs' graph shows a general trend upwards but the line between each point cannot really be trusted!



c) Line of best fit

In physics, we are really involved with finding the overall equation that governs the system but experimental data, as we have seen, has some uncertainty.

Hence by drawing an estimated line through the points we are assuming that this line follows valid equation describing the system.



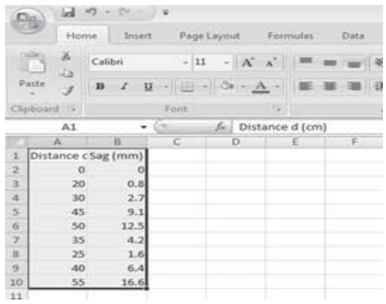
If we have as many points on one side of the line as the other then this is a way of averaging out the errors.

The more points there are the more certain we can be that the line is the correct one.

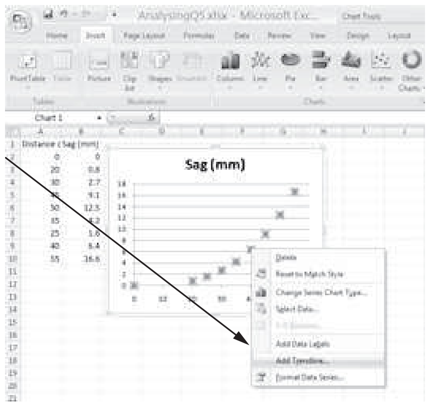
Using the Excel Program

If you have a set of data that you want to plot on Excel and draw a line of best fit through, then follow the steps below.

1. Open a new Excel spread sheet and type in your independent (x) values in column A and dependent values in column B.



2. Highlight both columns A and B by clicking and dragging, including the column headings ("Distance" and "Sag"), then click on the "Insert" tab at the top. Click on the box for XY Scatter graphs. The graph of the data in the table will be automatically plotted.
3. Right click onto one of the points of the graph and an option box will appear. This has an option of "add trend-line". If you click on this it will draw the line of best fit for you but the prompts ask for the kind of graph you want:



4. The options for types of graphs are:

   Exponential
   Linear
   Polynomial order? □
   Power
   Moving average
   (The different Orders of polynomial allow you to try a quadratic, cubic, quartic curves to see which fits the points best – the order {2, 3, 4 etc} is typed into the box).

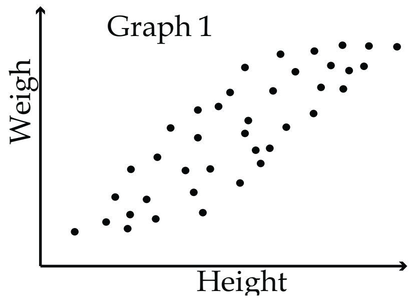
If the graph seems to be linear then clicking on the 'linear' box will draw in a straight line of best fit but if it is obviously curved like the one shown above then by choosing 'polynomial' the graph will be drawn according to what power polynomial is required.

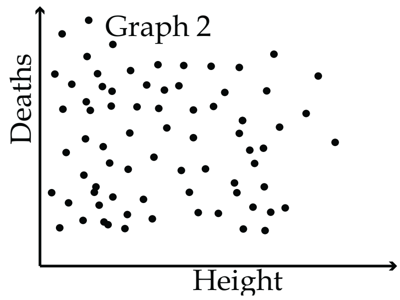
By trying each order in turn and then seeing which graph fits the points best, the equation of the line can be found by clicking in the box at the bottom of this window where it prompts "Display equation on graph".

5.15 TESTING DATA

If we suspect that there is a linear relationship between two variables there is a simple statistical test available that tells us the likelihood that the variables are linked, called the "T-test". This test is mostly used in areas such as biology or economics, where many exceptions to rules abound but there still may be a trend.

For instance, taller people are generally heavier but there are skinny tall people and plump short people which individually go against the trend. The graph of weight versus height would be like graph 1 below. Although there is a large variance, a trend-line could be drawn which may not be linear.





The link between number of deaths on the road and people's height may be totally random, in which case a graph like graph 2 might be found. If we subject the data to a T-test on Excel then it will tell us the likelihood that the link between the two variables can be found by pure chance, depending of the method of sampling used – with larger samples we can be sure that a trend-line is real. If the probability of finding a trend by pure chance is less than 5% then it is said to be very likely that the trend exists. The percentage that a T-test gives is called the confidence level. If you found a confidence level of less than 1% in a T-test then we still cannot say that the link is proven, but the evidence would be quite overwhelming.

5.16 T-TESTS

Distance (m)	Time (s)
0.15	0
0.22	1
0.23	2
0.31	3
0.34	4

Looking at the data from a trolley similar to the one in section 5.10, we can test to see how sure we are that distance is linked to time.

Pasting these data into an Excel spread sheet and clicking into a spare cell below then selecting from the Statistical Menu 'T-test' we will need to supply four variables:

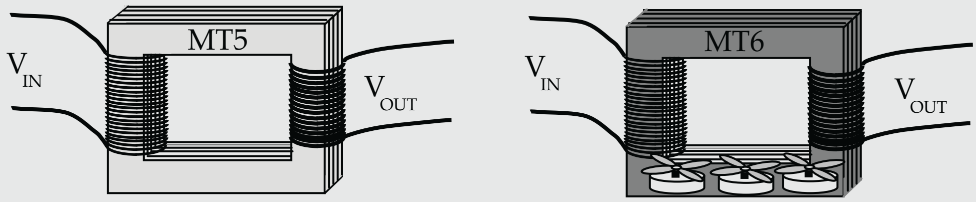
- The x-variable cells (A2:A6)
- The y-variable cells (B2:B6)
- Number of tails (1)
- Test type (1)

The result is a value of a 0.03 confidence level ($p \leq 3\%$) i.e. there is a 3% chance that the straight line graph could be just a chance event. The conclusion here would be that we are quite confident that distance is linked to time. Of course, the confidence limit would have to be an absolute zero for us to say a relationship has been "proven" – and this can never happen with real life data.

Set 20: Drawing Conclusions

1. Transformers

A Perth engineering company has developed a new type of air-cooled transformer called an MT6 which is supposed to have a better performance than the MT5 transformer it replaces.



Side-by-side tests have been done on both types of transformers to see how they performed at high and low input voltages. The tables below show the output voltages (VOUT) for the different input voltages (VIN) that were applied at the Primary coil.

MT5

V_{IN} (volt)	V_{OUT} (volt)
2.7	0.2
5.6	0.39
12.8	1.1
20.6	1.7
32.6	2.4
55.4	4.3
180	12
240	14.3

MT6

V_{IN} (volt)	V_{OUT} (volt)
240	25
150	15.6
110	11.2
97.6	10.3
86	8.9
43.7	4.7
20.9	2.2
14.2	1.5

a) Name the Independent and Dependent Variables in this experiment.

b) Plot graphs of VOUT against VIN for both transformers on the same graph.

c) Determine from the slopes of the graphs, a value for the Turns Ratio for the MT5 and the MT6. Are these values the same?

d) Looking at the two graphs, what feature is different for the MT6 compared with the MT5? What does the difference between the two graphs tell us is happening at higher voltages?

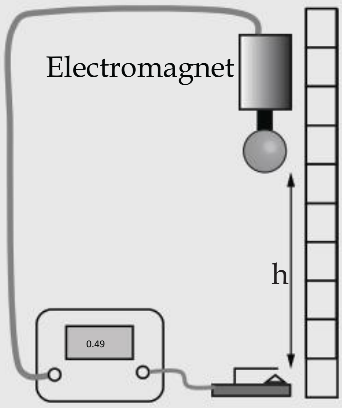
e) For higher voltages, the heat produced is likely to affect the performance of a transformer. What factor in the design of the MT6 can its superior performance can be attributed to?

f) Do you think the fact that the two transformers having different turns ratios affects our ability to compare their performance in a Fair Test? Discuss.

2. Free Fall

Nita and Ishani have set up an experiment to see how different sized balls fall under gravity by timing their fall over a known distance. For this they had a special electromagnet that holds a steel ball and lets it fall at the same time as an electronic clock starts.

When the ball hits a switch at the bottom the clock is stopped and the time of fall recorded on the timer. The apparatus is shown.



The two girls dropped ball 1 which was solid and obtained results for height (h) and t (time) for different heights then repeated the experiment with ball 2 which was much larger and was hollow. The results are displayed below.

Height h (cm)	Time t (s)	
0	0	
20	0.20	
50	0.32	
80	0.40	
120	0.49	
170	0.59	

Ball 1

Height h (cm)	Time t (s)	
0	0	
20	0.20	
50	0.33	
80	0.42	
120	0.52	
170	0.64	

Ball 2

From the physics books the link between distance fallen and time is $h = \frac{1}{2} gt^2$

Fill in the right hand column in each of the tables with a variable that will allow a straight line to be drawn between the variables h and t^2.

a) From the formula given, what variable should be plotted against h to obtain a straight line (assuming g is a constant)?

b) Use Excel to plot separate scatter graphs for the motion of Ball 1 and Ball 2 and write your opinion on whether they are both linear or whether there is an apparent difference between the two.

c) From the graph of Ball 1, write the equation of the line and calculate a value for 'g' the acceleration of the ball.

d) Try to explain why the two graphs might be a different shape.

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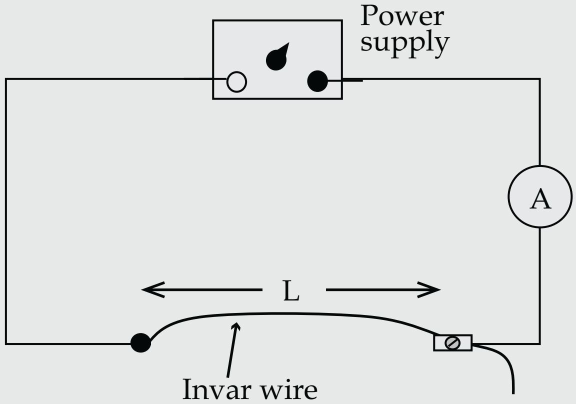
3. Wire Resistance

Catherine was investigating a resistance wire called Invar that her teacher had told her did not change its resistance with temperature. For her investigation she attached the wire into the circuit shown where she could alter the length of the wire and measure the current in the circuit.



Catherine's teacher gave her the formula linking the current with the length of wire: $I = \frac{V}{aL}$

I = current
V = voltage
α = resistance per metre of wire
L = length of wire

Catherine's results are shown in the table below:

Length L (cm)	Current I (mA)	
6.0	5.45	
6.9	4.81	
7.5	4.39	
8.3	3.98	
8.7	3.78	
9.5	3.50	

a) From the formula given what variable should be plotted against I to obtain a straight line (assuming R is constant)?

b) Insert values into the end column above and plot the graph on Excel that should give a straight line (make sure I is on the y-axis).

c) Write a conclusion from these data about whether the resistance of the wire changes with temperature or not, giving evidence.

d) Check the probability that your graph is a straight line by doing a T-test with the two variables. Write the confidence level as a percentage.

e) If the voltage of the power supply was 2.0 volts calculate a value for α from the gradient of your graph.

4. Stretching springs

Anna and Sarah were investigating how much springs stretch when weights are added to them. Anna placed an initial load of 50 g onto spring 1 and measured its length, as the load was increased. Sarah did the same experiment with spring 2.

The girls' results are shown below.

	Spring 1	Spring 2
Mass M (g)	Length L (cm)	Length L (cm)
50	8.4	12.8
100	10.5	15.3
150	12.5	17.7
200	14.7	20.3
250	16.1	22.8
300	18.8	25.2

a) Use Excel to plot a line of best fit with mass as the independent variable (x-axis) and length of spring as dependent variable (y-axis). Plot graphs for both springs on Excel and find the equation of both lines.

b) Write down the gradient for each graph:

c) One of the girls thinks she has an erroneous reading. Which reading do you think this could be? Perform a 1-tailed T-test on the data to find the probability that each graph is in fact a straight line. Write down the confidence levels (p) as %.

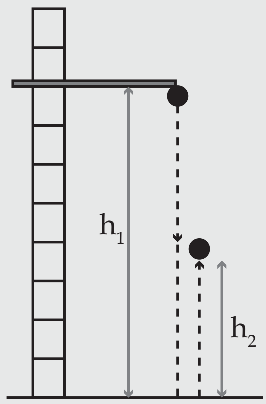
d) Is there evidence that the two springs are not made of the same material? Explain.

e) To be sure of your answer to part d), what other factors need to be controlled?

5. Squash ball bounce

Arian plays squash a lot and knows that before a game it is customary to 'warm up' the ball by hitting it hard against the wall to make it bounce properly.

He set up an investigation for his physics project where he placed a squash ball into a bath of water of known temperature and measured the bounce height of the ball after it had been dropped from a height of 80 cm above the bench. Arian's results are shown in Table 1.



80 cm drop height (h_1) Table 1

Temp	Trial 1	Trial 2	Trial 3	
T °C	h_1	h_1	h_1	Av h_1
20	21	27	25	
32	30	28	38	
44	40	47	36	
49	45	46	49	
58	54	58	59	
5	12	14	16	

Arian then repeated the whole experiment again, this time dropping the ball from a different height to investigate if the variable 'drop height' made any difference to the results.

50 cm drop height (h_2) Table 2

Temp	Trial 1	Trial 2	Trial 3	
T oC	h_2	h_2	h_2	Av h_2
21	16	19	22	
33	26	26	32	
42	32	37	33	
50	43	44	41	
62	57	52	53	
3	5	12	10	

a) Fill in the end columns to find the average values of h_1 and h_2 in each table.

b) Plot a graph from table 1 with temperature along the x-axis and bounce height on the y-axis.

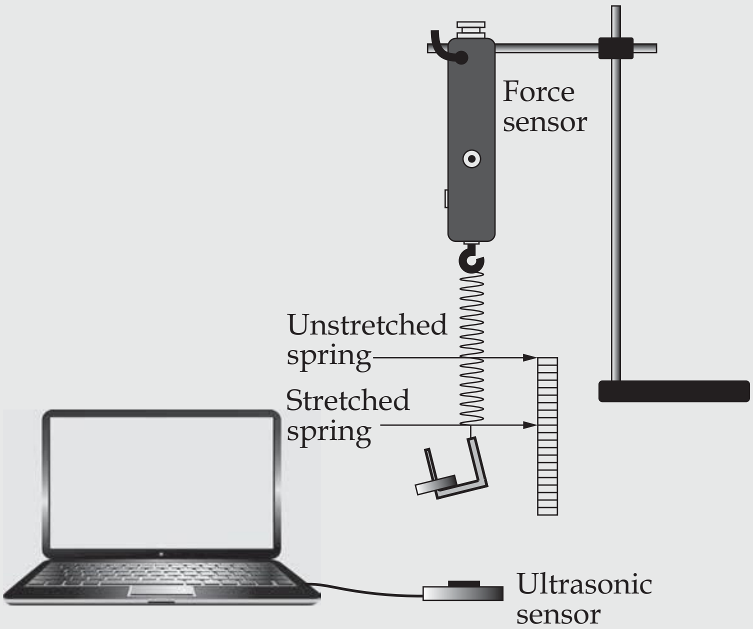
c) Try different equation lines (linear, power, exponential, etc) to see for which equation the line fits best. Write the equation for the line of best fit:

d) Plot another graph from table 2 and find the equation for the line of best fit using a 50 cm drop height.

e) State how the 'drop height' affects the 'bounce height'.

6 Oscillating spring

An experiment was set up in a university laboratory to investigate the factors affecting the time period of a mass bouncing up and down on a spring. The apparatus is shown here including a metal spring attached to force sensor with a mass loaded onto the spring. The time period of the oscillations were found from an ultrasonic sensor directed upwards which detected he movement of the mass and displayed its displacement using a computer program.



Readings of the added mass on the spring (m) and the time period of oscillation are shown in table 1.

Table 1.

Time t (s)		Mass m (kg)
0.314		0.1
0.385		0.15
0.446		0.25
0.566		0.325
0.666		0.45

The students performing the investigation have found a textbook that gives the formula linking the variables as:

T = time period
m = added mass
k = spring constant
Proposed formula is $T = 2\pi\sqrt{\frac{m}{k}}$

The students realise that they can use these data to find the spring constant of the spring by plotting a graph.

a) Explain why the students plotted t^2 against m instead of t against m.

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b) Fill in appropriate values in the middle column that will allow you to plot the correct graph and plot the graph using a graphics calculator or the Excel program.

c) Calculate a value for the gradient of the graph, in the correct units, showing how you obtained this value. Using the gradient obtained, calculate a value for the spring constant (k) of the spring. Show all working.

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One of the students suspects that one of the readings of time had an error in it. Which reading is this likely to be and estimate what the correct reading should be?

d) In physics, why do we manipulate data so we always have a straight line, and what are the advantages of drawing a line of best fit through the points, rather than joining one point to the next?

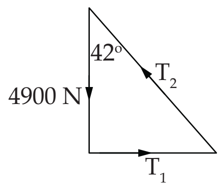
Solutions

Chapter 1 Solutions

Set 1 Vectors

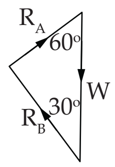
1. *Weight = sum of all the forces down*
   = 11,500 + 28,500 = 4.00 × 10^4 N
   Mass = 4.00 × 10^4/9.8 = 4.08 × 10^3 kg

2. *T_1 = 4900tan42 = 4.41 × 10^3 N*
   T_2 = 4900/cos 42 = 6.59 × 10^3 N,



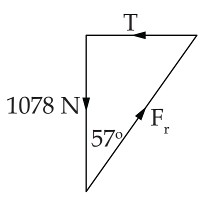
3. *Each end support holds a load of 15 × 9.8*
   = 147 N
   Upward vertical component of both legs = 2Fsin55 = 147
   F = 89.7 N

4. *From vector triangle*
   tan30 = R_A/ R_B = 0.577.



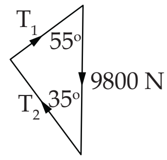
5. *From forces equilibrium:*
   vertical component of T = Tsin35 = 300 N
   T = 523 N
   Force in rod = horizontal component of T
   = 523cos35 = 428 N

6. *Weight of rod = 110 × 9.8 = 1078 N*
   From vector triangle T = 1078tan57 = 1.66 × 10^3 N
   F_r = 1078/cos57 = 1.98 × 10^3 N

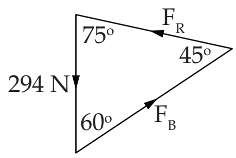


7. *$\Sigma F_{up} = \Sigma F_d$*
   so 3.1 × 10^3 + F_A = 294 + 588
   F_A = 2.22 × 10^3 N down

8. *From vector triangle*
   T_1 = 9800sin35 = 5.62 × 10^3 N
   T_2 = 9800cos35 = 8.03 × 10^3 N,



9. *From vector triangle – sine rule:*
   $\left(\frac{294}{sin45}\right) = \left(\frac{F_R}{sin60}\right) = \left(\frac{F_B}{sin75}\right)$
   F_R = 360 N *F_B = 402 N*



10. *From forces equilibrium:*
    R_v + 595sin65 (up) = 110 × 9.8 (down)
    R_v = 539 N, R_H = 595cos65 = 251 N
    $R_{Tot}{}^2$ = 539^2 + 251^2 so R_{Tot} 595 N
    Tanθ = 539/251 θ = 65°

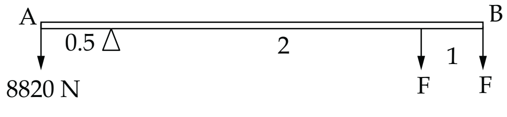
Set 2 Torques and Centre of mass

1. *C*
2. *Taking torques @ suspension wire:*
   15F = (15 × 100) + (45 × 200)
   (all in cm and grams)
   F = 700 g

3. *Taking torques @ B:*
   (5 × 10780) + (7.5 × 7840) = 15 F_A
   so F_A = 7.51 kN

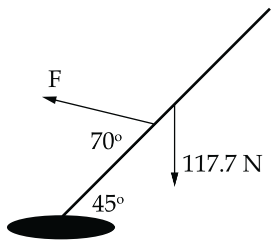
4. *Taking torques @ axle:*
   F × 5 = (50 × 9.8) × 12
   (note: angle is irrelevant).
   F = 1.18 kN

5. *Taking torques @ pivot:*
   0.5 × 8820 = 2F + 3F
   F = 882 N

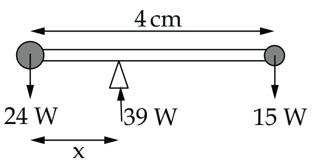


6. *Torques @ A:*
   $(2 \times 784) + (3 \times 931) = 4F_B$
   $F_B = 1.09 \times 10^3$ N
   ∴ *Each rope at end B has a tension = 545 N*
   Torques @ B:
   $(2 \times 784) + (1 \times 931) = 4F_A$
   $F_A = 6.25 \times 10^2$ N
   ∴ *Each rope at end A has a tension = 312 N*

7. *Taking torques about the base.*
   $117.7 \times 1.25\cos45 = 1\sin70 \times F$
   $F = 111$ N.



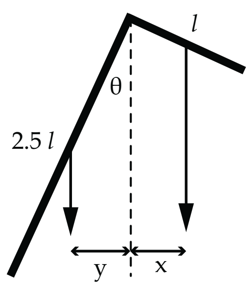
8. *Top rectangle mass = 24 units and bottom = 15 units at their centres.*
   Diagram reduces to:



   Torques @ LH end: $39Wx = 15W \times 4$
   $x = 1.538$ *cm from c of g of the top rectangle*
   Distance from top $= 1.538 + 1.5 = 3.04$ *cm*

9. *Torques @ pivot:*
   $(2 \times 2.85) = (8 \times 0.05 \times 9.8) + (L \times 0.04 \times 9.8)$ *[Lengths in cm]*
   L = from the pivot = 4.54 cm.

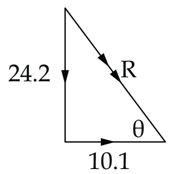
10 *Torque from the weight of part l of wire = Torque from the weight of part 2.5 l .*
   From triangles: $y = 1.25\sin\theta$ *and* $x = 0.5\cos\theta$, *so* $\sin\theta/\cos\theta = 0.5/3.125$
   Mass of rods is proportional to lengths



   Taking torques:
   $1.25l \times y = 0.5l \times x$
   $1.25 \times (1.25\sin\theta) = 0.5 \times (0.5\cos\theta)$
   i.e. $\tan\theta = 0.16$ *so* $\theta = 9.09^o$

Set 3 Projectiles

1. a) *Vertically:* $s = ut + \frac{1}{2}at^2$
      $-30 = 0 - 4.9t^2$
      $t^2 = 6.122$ s $t = 2.474$ s
      Horizontally: $s = ut$
      $= 15 \times 2.474 = 37.1$ m
   b) *The stone will still take the same time to fall*
      $s = ut$ $25 = u \times 2.474$
      $v = 10.1$ m s^{-1}
   c) *Vertically:* $s = ut + \frac{1}{2}at^2$
      $-30 = 0 - 4.9t^2$
      $t = 2.474$ s
   d) *Vertically:* $v = u + at$
      $v = 0 - 9.8 \times 2.474 = 24.24$ m s^{-1}
   Horizontally $v = 10.1$ m s^{-1}



   $R^2 = 24.2^2 + 10.1^2$ $R = 26.3$ m s^{-1}
   $\tan\theta = 24.24/10.1$ $\theta = 67.4$
   So the stone strikes with velocity 26.3 m s^{-1} *at an angle of* 67.4^o

2. *Horizontally:*
   Time of flight $= 1.36/3.70 = 0.3676$ s
   Vertically: $s = ut + \frac{1}{2}at^2$
   $s = 0 - 4.9 \times 0.3676^2 = -0.662$ m
   So table top is 0.662 m off the ground.

3. a) *Vertically:*

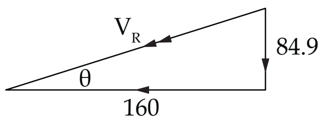
$u = 4$	$s = ut + \frac{1}{2}at2$
$t = ?$	$-110 = 4t - 4.9t^2$
$a = -9.8$	*Solver:* $t = 5.16$ s
$s = -110$	

   b) *Vertically:*

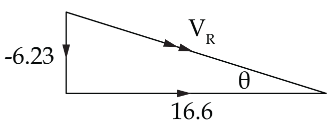
$u = 4$	$v^2 = u^2 + 2as$
$v = 0$	$0 = 4^2 - 19.6\ s$
$a = -9.8$	$s = 0.816$ m
$s = ?$	*Total height*
	$= 110 + 0.816$
	$= 110.816$ m

   c) *Horizontally:* $s = ut = 16 \times 5.16$
      $= 82.6$ m

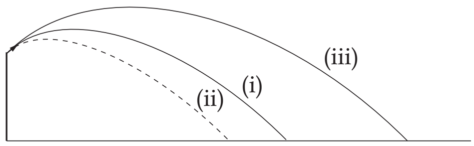
4. a) $v = 750/3.6 = 208.3$ m s-1
$u_H = 208.3\cos40 = 160\ m\ s^{-1}$
b) *Vertically:*
$u_v = 208.3\sin40 = 133.9\ m\ s^{-1}$
$v^2 = u^2 + 2as$
$0 = 133.9^2 - 19.6s$
$s = 915\ m$
c) *Vertically:*
$u = 133.9$
$a = -9.8$
$t = ?$ $\quad s = ut + \frac{1}{2}at^2$
$s = 0$ $\quad 0 = 133.9t - 4.9t^2$
$t = 133.9/4.9 = 27.32\ s$
Horizontally:
$s = ut = 160 \times 27.32 = 4.37 \times 10^3\ m$
d) *Max range when the angle is* 45^o
so $u_v = u_H = 208.3\sin45 = 147.3\ m\ s^{-1}$
Vertically: $\quad s = ut + \frac{1}{2}at^2$
$0 = 147.3t - 4.9t^2$
$t = 30.05\ s$
Horizontally :
$s = ut = 147.3 \times 30.05 = 4.43\ km$
e) *Vertically:* $\quad v = u + at$
$v = 133.9 - 9.8 \times 5 = 84.9\ m\ s^{-1}$
Horizontally: $v = 160$
$V_R^2 = 160^2 + 84.9^2 \quad V_R = 181\ m\ s^{-1}$
$\theta = 28.0^o$



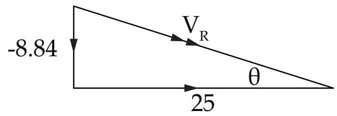
5. a) $u_v = 22\sin41 = 14.43\ ms^{-1}$
$u_H = 22\cos41 = 16.60\ m\ s^{-1}$
Horizontally: $\quad 35 = 16.6t$
so $t = 2.108\ s$
Vertically: $s = ut + \frac{1}{2}at^2$
$s_v = 14.43 \times 2.108 - 4.9(2.108)^2$
$= 8.64\ m.\ \therefore\ 8.64 + 1.20 = 9.84\ m$
above ground.
b) *Vertically:* $v = u + at$
$= 14.43 - 9.8 \times 2.108 = -6.228\ m\ s^{-1}$
$\tan\theta = 6.228/16.6$ *so* $\theta = 20.6^o$



6. a) $u_v = 28.5\sin30 = 14.25\ m\ s^{-1}$
$u_H = 28.5\cos30 = 24.68\ m\ s^{-1}$
b) *Vertically:*
$s = ut + \frac{1}{2}at^2$
$-1.4 = 14.25t - 4.9t^2$
Solver: $t = 3.00\ s$
Horizontally:
$s = 24.68 \times 3 = 74.0\ m$
c)



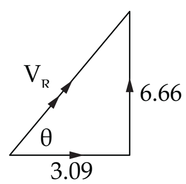
7. a) *Vertically:* $\quad s = ut + \frac{1}{2}at^2$
$-4 = 0 - 4.9t^2$
$t = 0.9035\ s$
Horizontally: $s = ut$
so $20 = u \times 0.9035 = 22.1\ m\ s^{-1}$
b)



Vertically: $v = u + at$
$= 0 - 9.8 \times 0.9035 = -8.854\ m\ s^{-1}$
$V_R^2 = 25^2 + 8.84^2$
so $V_R = 26.5\ m\ s^{-1}$ *at* 19.5^o

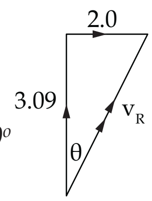
8. a) *Vertically:* $u_v = 5\sin30 = 2.5\ m\ s^{-1}$
$u_H = 5\cos30 = 4.33\ m\ s^{-1}$
$v^2 = u^2 + 2as$
$0 = 2.5^2 - 19.6s$
$s = 0.319\ m$ *above X*
so $s_m = 1.519\ m$ *above the ground.*
b) *Height of z is* $1.2 - 0.5 = 0.70\ m$ *so height fallen* $= 0.50\ m$
$s = ut + \frac{1}{2}at^2$
$-0.5 = 2.5t - 4.9t^2$
$t = 0.664\ s$

9. a) *Horizontally:* $s = 3.4 \quad t = 1.1$
so $u_H = 3.4/1.1 = 3.09\ m\ s^{-1}$
b) *Vertical height difference*
$= 3.5 - 2.1 = 1.4\ m$
Vertically: $\quad s = u_v t + \frac{1}{2}at^2$
so $1.4 = u_v \times 1.1 - 4.9(1.1)^2$
$7.329 = u_v \times 1.1$
so $u_v = 6.66\ m\ s^{-1}\ u = 7.34\ m\ s^{-1}$
$\tan\theta = 6.66/3.09$
$\theta = 65.1^o$



c) *If* u_H *is constant then air resistance is negligible (no wind)*

d) *Assuming the same angle of launch:*
Ball must follow the same horizontal and vertical path with same velocities.
Horizontally:
$v_R^2 = 2^2 + 3.09^2$
So $v = 3.68 \text{ m s}^{-1}$
$\tan\theta = 2/3.09 \quad \theta = 32.9^o$



NB the vertical velocity is not affected by the wind so it will remain the same.

10. a) *Horizontally:* $u_H = u\cos40 = 33/t$

so $t = \dfrac{33}{u\cos40}$

Vertically: $s = u_v t + \frac{1}{2}at^2$

$$0 = u\sin40\left(\frac{33}{u\cos40}\right) - 4.9\left(\frac{33}{u\cos40}\right)^2$$

$$0 = 33\tan40 - \frac{9093}{u^2}$$

$u = 18.1 \text{ m s}^{-1}$

b) $t = 33/18.1\cos40 = 2.38$ s

c) $v^2 = u^2 + 2as$
so $0 = (18.1\sin40)^2 - 19.6s$
$s = 6.91$ m

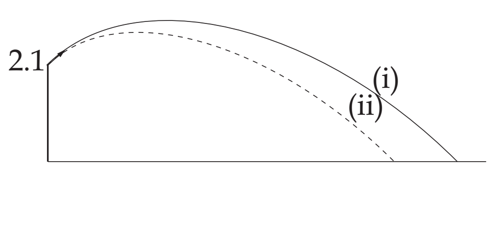
d) *Time when at max height*
$= 2.38/2 = 1.19$ s
$s = 18.1\cos40 \times 1.19 = 16.5$ m
(half-way, as expected)

11. a) *Vertically:* $u = 27.5\sin50 = 21.07 \text{ m s}^{-1}$
$v = u + at$, *at top* $0 = 21.07 - 9.8t$
$t = 2.15$ s

b) $v^2 = u^2 + 2as$
$0 = 21.07^2 - 19.6\, s$
$s = 22.65$ m *above launch height.*
Displacement from the ground
$= 22.65 + 2.10 = 24.8$ m

c) *Vertically: To find the time of flight when* $s = -2.10$ *(ground)*
$u = 21.07$
$a = -9.8$
Using $s = ut + \frac{1}{2}at^2$
$s = -2.10$
$-2.10 = 21.07t - 4.9t^2$
$t = ?$ *Solving for t:*
$t = 4.40$ s
Horizontally: $s = u_H t$
$s = 27.5\cos50 \times 4.403 = 77.8$ m

d)

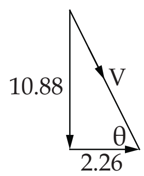


For graph ii) air resistance reduces the vertical and horizontal KE and so the javelin has less range and a lower maximum height.

12. a) *Vertically:* $s = ut + \frac{1}{2}at^2$
$6 = 4.9t^2$
$t = 1.11$ s
He needs to dive 1.11 s before the wave arrives to enter at the maximum wave height.

b) *She would need a horizontal velocity of* $v_H = 2.5/1.11$
$v_H = 2.26 \text{ m s}^{-1}$

c) *Vertically:* $v_v = u + at$
$= 0 - 9.8 \times 1.11$
$= -10.88 \text{ m s}^{-1}$

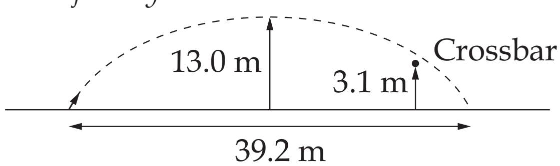


Horizontally: $v_H = 2.26 \text{ m s}^{-1}$
From the vector triangle:
$v^2 = 10.88^2 + 2.26^2$
$v = 11.1 \text{ m s}^{-1}$
$\tan\theta = 10.88/2.26$
so $\theta = 78.3^o$

13. a) *Horizontally:* $v_H = 20\cos53 = 12.04 \text{ m s}^{-1}$
Time to travel 36 m $= 36/12.04 = 2.99$ s

b) *To find height 36 m away:*
Vertically: $u = 20\sin53 = 15.97 \text{ m s}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
Using $s = ut + \frac{1}{2}at^2$
$t = 2.99$ s
$s = 15.97 \times 2.99 - 4.9(2.99)^2$
$s = 3.95$ m *above the ground so the ball clears the bar by 0.85 m*
Vertical time of flight: $s = ut + \frac{1}{2}at^2$
$0 = 15.97t - 4.9t^2$
$t = 15.97/4.9 = 3.26$ s
Range $= 12.04 \times 3.26 = 39.2$ m
Max vertical height when $v = 0$
$v^2 = u^2 + 2as$
so $0 = 15.97^2 - 19.6s$
$s = 13.0$ m

Trajectory:



Set 4 Circular Motion

1. $speed = \dfrac{\pi \times r}{t} = \dfrac{60\pi}{31.4} = 6.00\ m\ s^{-1}$

   At A $v_1 = 6\ m\ s^{-1}$ E and at B $v_2 = 6\ m\ s^{-1}$ W
   So $\Delta v = 6 - (-6) = 12\ m\ s^{-1}$ W
   $\dfrac{\Delta v}{\Delta t} = \dfrac{12}{31.4} = 0.382\ m\ s^{-1}$ W

2. Answer is D

3. Assuming horizontal motion (cannot happen!)
   $T = \dfrac{mv^2}{r} = \dfrac{0.22 \times 25^2}{45} = 3.06\ N$

4. Mass of car is:
   $m = \dfrac{Fr}{v^2} = \dfrac{10000 \times 50}{20^2} = 1250\ kg$
   $v^2 = \dfrac{Fr}{m} = \dfrac{1200 \times 50}{1250} = 48 \quad v = 6.93\ m\ s^{-1}$

5. Centripetal force
   $F_C = Mg = \dfrac{mv^2}{r}$ (m = mass of bung)
   But $v^2 = \dfrac{4\pi R^2}{t^2}$
   so $Mg = \dfrac{m \times 4\pi R^2}{t^2 R}$

   so $M = \dfrac{k}{t^2}$
   Answer is D

6. From the last question $t^2 = kR$ so the answer is C.

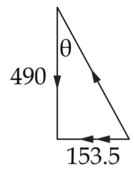
7. $F_R = \dfrac{mv^2}{r} + mg$
   $= \dfrac{70 \times 30^2}{110} + 70 \times 9.8$
   $= 1.26 \times 10^3\ N$

8. $30\ km\ h^{-1} = 8.33\ m\ s^{-1}$
   F_R will become zero when $a_c = 9.8$
   $9.8 = \dfrac{8.33^2}{r} \quad r = 7.08\ m$

9. Frictional force $F_F \geq F_c$
   $F_F = \dfrac{mv^2}{r}$
   $3 = \dfrac{0.4 \times v^2}{2} \quad v = 3.87\ m\ s^{-1}$

10. Answer is C

11. $v = (2\pi \times 2.8)/6 = 2.932\ m\ s^{-1}$
    $F_C = \dfrac{50 \times 2.932^2}{2.8} = 153.5\ N$

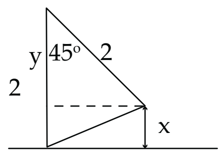


    $\tan \theta = 153.5/490 \qquad \theta = 17.4^o$

12. At top $a_c \geq 9.8$
    $9.8 = v^2/22 = 14.68\ m\ s^{-1}$
    $KE_{BOTTOM} = KE_{TOP} + PE_{TOP}$:
    $\tfrac{1}{2} \cancel{m} v_b^2 = \tfrac{1}{2} \cancel{m} 14.68^2 + \cancel{m} \times 9.8 \times 44$
    (m cancels)
    $v_b = 32.8\ m\ s^{-1}$

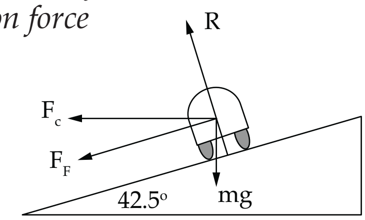
13. Answer is B (The Earth's surface under foot is accelerating downwards)

14. $F_C = \dfrac{m \times 19.8^2}{40} = 9.8m$
    Normal reaction must be at an angle where its vertical component = 9.8m and horizontal component (Fc) = 9.8m so the angle must be 45°.



    $y = 2\cos 45 = 1.41\ m$
    $x = 2 - 1.41 = 0.586\ m$

15. From vector triangle: $\tan 42.5 = F_c/mg$
    Horizontal F_c obtained from normal reaction force



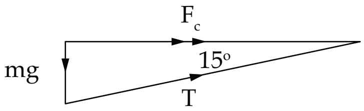
    $F_c = \tan 42.5 \times 1200 \times 9.8 = 10{,}776\ N$
    F_c from track friction = $9000 \cos 42.5 = 6{,}635\ N$
    Total $F_c = 10776 + 6635 = 17{,}411\ N$
    $17411 = \dfrac{m \times v^2}{r} = \dfrac{1200 \times v^2}{350}$
    $v = 71.26\ m\ s^{-1}$ ($256\ km\ h^{-1}$)

16. $PE_{TOP} = PE_X + KE_X$
$9.8 \times \cancel{m} \times 15 = 9.8 \times \cancel{m} \times 5 + ½\,\cancel{m} \times v^2$
(m cancels)
so $v = 14.0\ m\ s^{-1}$
$Fc = mg - N \rightarrow N = mg - Fc$
Let apparent g be $g^{|}$: $\cancel{m}g^{|} = \cancel{m}g - \cancel{m}v^2/r$
So $g^{|} = 9.8 - 14^2/28 = 2.80\ m\ s^{-2}$

17. $T = \dfrac{4 \times 9.8}{sin15} = 151\ N$

18. $Tan15 = \dfrac{mg}{F_C}$

$F_C = \dfrac{4 \times 9.8}{tan15} = 146.3\ N$



Horizontal radius $r = 2.1cos15 = 2.03\ m$

$F_C = \dfrac{mv^2}{r}$ $v^2 = \dfrac{146.3 \times 2.03}{4} = 74.25$

$v = 8.617\ m\ s^{-1}$ $8.617 = \dfrac{2\pi r}{t}$

So $t = \dfrac{2\pi \times 2.03}{8.617} = 1.48\ s$

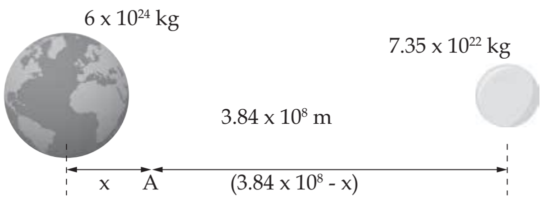
19. At the bottom: $T = mg + F_c$
$= 0.12 \times 9.8 + \dfrac{0.12 \times 3.5^2}{0.65}$
$= 3.44\ N$

20. Reaction force inwards:
$F_c = 8.9\cancel{m} = \cancel{m}v^2/40$
$v^2 = 8.9 \times 40 = 356$
$v = 18.87ms^{-1}$
$v = \dfrac{2\pi r}{t}$
$18.87 = \dfrac{2\pi \times 40}{t}$
so $t = 13.32\ s$
Rate per minute = $60/T = 4.51$ per min.

Set 5 Gravitation

1. $F = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 50}{(6.38 \times 10^6 + 1270 \times 10^3)^2}$
$= 340\ N$

2. Let distance of C of M be x metres from Earth



Taking torques about point A:
$(6 \times 10^{24})\ x = (3.84 \times 10^8 - x) \times 7.35 \times 10^{22}$
$(6 \times 10^{24})\ x = 2.82 \times 10^{31} - 7.35 \times 10^{22}x$
$6.073 \times 10^{24}\ x = 2.82 \times 10^{31}$
$x = 4.65 \times 10^6$ m from centre of Earth
(lies within the Earth's radius)

3. $r = 6.38 \times 10^6 + 4 \times 10^5 = 6.78 \times 10^6\ m$
$\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$
$v^2 = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.78 \times 10^6}$
$v = 7.66 \times 10^3\ m\ s^{-1}$

4. To find g on the Moon:
$g = \dfrac{GM}{r^2}$
$= \dfrac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{(1.74 \times 10^6)^2}$
$= 1.62\ m\ s^{-2}$

<u>On Earth</u>
$v^2 = u^2 + 2as$ so $0 = u^2 - 19.6 \times 2.45$
$u = 6.93\ m\ s^{-1}$ to reach this height
<u>On Moon</u>
$g = 1.62\ m\ s^{-2}$ so $v^2 = u^2 + 2as$
$0 = 6.93^2 - 3.24\ s$
$s = 14.8\ m$

5. $g = \dfrac{GMm}{r^2}$
$= \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 17}{(2.27 \times 10^7)^2}$
$= 13.2\ m\ s^{-2}$

6. From Kepler's Law:
$\dfrac{r^3}{T^2} = \dfrac{GM}{4\pi^2}$
So $\dfrac{(5.2 \times 10^{12})^3}{(5.5 \times 10^9)^2} = \dfrac{6.67 \times 10^{-11} M}{4\pi^2}$
$M = 2.75 \times 10^{30}\ kg$

7. Distance between the centres of masses of the spheres = (6 + 0.1 + 1.8) cm = 7.9 cm
$G = \dfrac{Fr^2}{m_1 m_2}$
$= \dfrac{5 \times 10^{-8}(7.9 \times 10^{-2})^2}{10 \times 0.5}$
$= 6.24 \times 10^{-11} Nm^2kg^{-2}$

8. Torque $T = 2Fr$ (radius = 7.55/2)
$T = 2 \times 3.775 \times 10^{-2} \times 5 \times 10^{-8}$
$= 3.775 \times 10^{-9}\ N\ m$
Angle turned = $3.775 \times 10^{-9} / 9.2 \times 10^{-6}$
$= 4.10 \times 10^{-4}$ degrees.

9. *Force from X* $= \dfrac{GMm}{r^2}$

$= \dfrac{6.67 \times 10^{-11} \times 5 \times 10^{22} \times 500}{(7.5 \times 10^{9})^2}$

$= 2.96 \times 10^{-5}$ *N left*

Force from Y

$= \dfrac{6.67 \times 10^{-11} \times 5 \times 10^{19} \times 500}{(1.5 \times 10^{9})^2}$

$= 7.41 \times 10^{-7}$ *N right*

Total force $= 2.96 \times 10^{-5} - 7.41 \times 10^{-7}$
$= 2.89 \times 10^{-5}$ *N left*

10. *Gain in KE = loss of PE*
= area under graph
= 6.8 rectangles, approx.
Each rectangle area $= 20 \times 1 \times 10^6$
$= 2 \times 10^7$ *so energy change =*
$2 \times 10^7 \times 6.8 = 1.4 \times 10^8$ *J*
$1.4 \times 10^8 = \tfrac{1}{2} \times 1100 \times v^2$
so $v = 500$ *m* s^{-1} *(approx)*

Chapter 2 Solutions

Set 6 Electric Charges

1. a) *During a storm a very large electric field is produced between the cloud and the Earth which can cause the air to conduct. The field strength will be greatest around pointed objects, such as tree branches which means that the tree is highly likely to be struck by lightning. If a person is sheltering under a tree a large current could run through them and electrocute them.*
   b) *Tall, pointed buildings are likely to become struck by lightning due to the large field produced around their sharp top. This could destroy part of the building. A lightning conductor forms a preferential site for the lightning to strike as it has a sharp spike and is a good conductor so the current will run down the copper connecting strip to Earth, leaving the building unharmed.*

2. *Selenium is a semiconductor which means it has a moderate attraction for electrons but there is enough energy in light rays for the electron to be released. The image of a reflected page is shone onto a selenium plate which is charged and where the light strikes the electrons will be discharged. The Selenium plate is then dipped into plastic dust (toner) which is attracted by electrostatic force to the charged areas. This plate is pressed onto a sheet of paper and heated which melts the dust onto the paper where the dark areas on the page were. Light areas would have no toner on them and appear white.*

3. a) *Charge* $= 2 \times 10^4 \times 1.6 \times 10^{-19}$
$= 3.2 \times 10^{-15}$ *C.*
   b) $F_E = k\dfrac{q_1 q_2}{R^2}$

$= 9 \times 10^9 \times \dfrac{3.2 \times 10^{-15} \times 15 \times 1.6 \times 10^{-5}}{(3 \times 10^{-3})^2}$

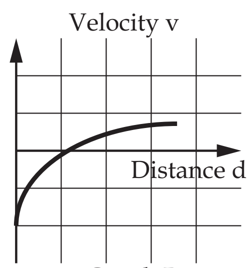
$= 5.12 \times 10^{-5}$ *N.*

4. *PD = 6.7 + 1.8 = 8.5 V*
$W = Vq = 8.5 \times 0.045 = 0.383$ *J.*

5. a) $W = Vq = 800 \times -5.5 \times 10^{-6} = -4.4 \times 10^{-3}$ *J*
(Work is done by the charge).
   b) $E_K = \tfrac{1}{2} mv^2$
so $v = \sqrt{\left(\dfrac{2 \times 4.4 \times 10^{-3}}{2.0 \times 10^{-12}}\right)}$
$= 6.63 \times 10^4$ *m* s^{-1}.

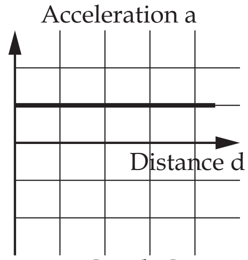
6. a) *One of the anodes is always going to be 2000 V above the other, so it is rather like a set of steps where each step lifts you up by 2000 but with many such steps the total energy gain can be in the millions of volts.*
   b) *With 400,000 steps the potential energy change in total will be* 800×10^6 *V.*

7. a)



Graph B

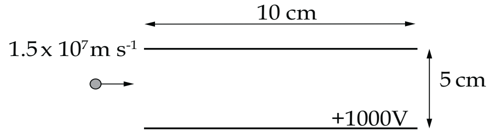
$v = \sqrt{\left(\dfrac{2Vq}{m}\right)}$
– a square root function.



Graph C

F = Eq and E is constant so a is constant.

8. a) $E = V/d = 1000/0.05 = 20{,}000\ V/m$



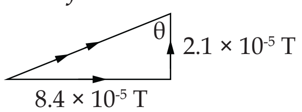
b) $F = Eq = 20{,}000 \times 1.6 \times 10^{-19}$
$= 3.2 \times 10^{-15}$ *N downwards.*
c) $t = d/v = 0.1 / 1.5 \times 10^{7}$
$= 6.67 \times 10^{-9}$ *s.*

9. $a = F/m = 3.2 \times 10^{-15} / 9.11 \times 10^{-31}$
$= 3.51 \times 10^{15}\ m\ s^{-2}$.
$s_{down} = ut + ½at^2$
$= 0 + ½ (3.51 \times 10^{15}) \times (6.67 \times 10^{-9})^2$
$= 7.8$ cm.
This is below the distance to the lower plate (2.5 cm) so the electron cannot emerge

10. a) $m_{oil} = \dfrac{4\pi\left(0.015 \times 10^{-3}\right)^3}{3} \times 850$
$= 1.20 \times 10^{-11}$ kg.
b) *If the forces balance exactly then the electric force = weight of droplet*
$F_E = 1.20 \times 10^{-11} \times 9.8$
$= 1.18 \times 10^{-10}$ N.
c) $F = Eq$
$$E = \frac{513}{0.035} = 14657\ Vm^{-1}$$
$$q = \frac{1.18 \times 10^{-10}}{14657} = 8.05 \times 10^{-15}$$
No of electrons N
$$N = \frac{8.05 \times 10^{-15}}{1.6 \times 10^{-19}}$$
$= 5.03 \times 10^{4}$ *electrons*

Set 7 Electromagnetism

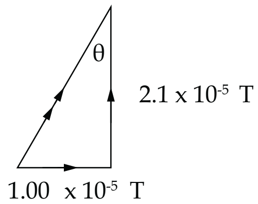
1. *a) Right, b) left to right, c) right to left, d) into page.*
2. *Vector diagram:*
Compass needle will point in the direction of the resultant field



$Tan\ \theta = (8.4 \text{ x } 10^{-5})/ (2.1 \times 10^{-5})$,
$\theta = 76.0^o$ *from the north.*

3. $$B = \frac{1.26 \times 10^{-6} \times 5}{2\pi \times 0.1} = 1.00 \times 10^{-5}\ T$$

4. *The field from the wire is to the right.*
Vector diagram:



$Tan\ \theta = (2.1 \times 10^{-5})/ (1.0 \times 10^{-5})$, $\theta = 64.5^o$ *from the north.*

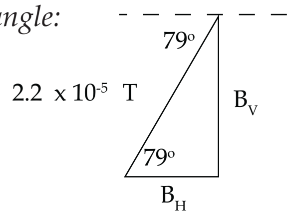
5. *No force on wire BC as it is parallel to the field. The current runs from A to B, AB will move down, so the coil will rotate anticlockwise.*

6. *Adding vector fields X (strong) and Y (weak) together gives field C.*

7. *Force up* $= BIL = 0.02 \times I \times L$
Force down (weight)
= *volume × density ×* g
$= \pi \times (1 \times 10^{-3})^2 \times L \times 8.89 \times 10^3 \times 9.8$
$= 0.274L$
Equating the force up and down:
$0.02I\cancel{L} = 0.274\cancel{L}$ so $I = 3.08 \times 10^{-5}/0.02$
$= 13.7A$

8. *Weight of wire (down)*
= *length × mass per metre ×* g
$= 0.6 \times 0.0025 \times 9.8 = 1.47 \times 10^{-2}$ N
Magnetic force (down) = BIL
$= 2.0 \times 10^{-5} \times 500 \times 0.6 = 6.0 \times 10^{-3}$ N
Total force down $= (1.47 + 0.6) \times 10^{-2}$
$= 2.07 \times 10^{-2}$ N
so tension $= 1.04 \times 10^{-2}$ *N per string.*

9. *Vector triangle:*



a) $B_H = 2.2 \times 10^{-5} \cos 79 = 4.20 \times 10^{-6}\ T$
b) $B_V = 2.2 \times 10^{-5} \sin 79 = 2.160 \times 10^{-6}\ T$
c) *Hidden metallic deposits under the ground.*

10. *Field must be going into the left end and out of the right end of the coil, so X is South and Y is North.*
By coil-fingers rule current must go from X to Y through the coil so assertion B is incorrect.

11. $F = BIL\sin\theta = 2.1 \times 10^{-5} \times 94.3 \times 1 \times \sin 45$
$= 1.40 \times 10^{-3}$ *N downwards.*

12. *Value of B at AB from top wire is:*
$$B_{AB} = \frac{2 \times 10^{-7} \times 8}{0.08} = 2.0 \times 10^{-5}\ T$$
Value of B at CD from top wire is:
$$B_{CD} = \frac{2 \times 10^{-7} \times 8}{0.18} = 8.89 \times 10^{-6}\ T$$
Repulsive force on AB
$= 2.0 \times 10^{-5} \times 12 \times 0.3 = 7.2 \times 10^{-5}\ N$
Attractive force on CD
$= 8.89 \times 10^{-6} \times 12 \times 0.3 = 3.2 \times 10^{-5}\ N$
Net force $= (7.2 \times 10^{-5} - 3.2 \times 10^{-5})$
$= 4.0 \times 10^{-5}$ *N repulsive.*

13. *Graph D is correct. The force follows an Inverse Square law* $(1/x^2)$

14. *Force on the wire* $= mg$
$= (35.6 \times 10^{-3} - 28.4 \times 10^{-3}) \times 9.8$
$= 7.056 \times 10^{-2}$
$F = BIL = B \times 16.8 \times 2.5 \times 10^{-2}$
$= 7.056 \times 10^{-2}$ so $B = 0.168\ T$

15. *Total length of wire in the coil* $= 2\pi r$
$= 2 \times \pi \times 2.9 \times 10^{-2} \times 86 = 15.67\ m$
Force on coil $= BIL$
$= 0.12 \times 1.8 \times 15.67 = 3.384\ N$
Acceleration $a = F/m$
$= 3.384/0.03 = 113\ m\ s^{-2}$

Set 8 Electromagnetism

1. *Correct answer is C. Electromagnetic force is directly proportional to charge and indirectly proportional to mass, so mass or charge could be varying. Charge divided by mass must be different.*

2. *Correct answer is D. Only D has the correct addition of the component fields.*

3. a) $F = Bqv$
$= 2.75 \times 10^{-3} \times (2 \times 1.6 \times 10^{-19}) \times 3.4 \times 10^{6}$
$= 3.00 \times 10^{-15}\ N$
b) $a = F/m = 3.00 \times 10^{-15}/ 6.64 \times 10^{-27}$
$= 4.51 \times 10^{11}\ m\ s^{-1}$
c) $\Delta v = a \times \Delta t = 451\ m\ s^{-1}$

4. *Correct answer is D. Field is left to right and conventional current is out of the page so deflection is down (right hand slap rule).*

5. *Correct answer is B. For an electron to go up and left, field from cd needs to be left to right and field ba needs to be upwards.*

6. a) *Field must be into the page*
b)
(i) *Centripetal force is given by* $F_c = \frac{mv^2}{r}$
Magnetic force on a charged particle is given by $F_m = Bqv$
Equating forces: $\frac{mv^2}{r} = Bqv$ so $\frac{q}{m} = \frac{v}{rB}$
(ii) *Charge to mass ratio is:*
$$\frac{q}{m} = \frac{v}{rB} = \frac{8.20 \times 10^{6}}{0.23 \times 0.74} = 4.82 \times 10^{7}\ C\ kg^{-1}$$
iii) *q/m for tritium (1 charge) is*
$$\frac{1.6 \times 10^{-19}}{(1.007276 + 2 \times 1.008665) \times 1.67 \times 10^{-27}}$$
$= 3.17 \times 10^{7}\ C\ kg^{-1}$
which is not the same q/m ratio

7. *For the left wire of the coil to go upwards P must be north and S must be + (answer C) Or: Q must be north and R must be + (answer D)*

8. a) $\tau_{max} = BANI$
$= 4 \times 10^{-2} \times (0.035 \times 0.022) \times 340 \times 0.75$
$= 7.85 \times 10^{-3}\ N\ m$
b) *Power* $= VI = 12 \times 0.75 = 9.00\ W$

9. *Correct answer is C for a DC motor starting at zero degrees with a split-ring commutator.*

10. *Correct answer is C. Protons from the Sun (solar wind) cross the Earth's magnetic field and spiral round. As they collide with the oxygen they ionise the atoms which produces light.*

11. *Energy to move a charge through a p.d. is given by* $W = Vq$
$= 22{,}000 \times 1.6 \times 10^{-19} = 3.52 \times 10^{-15}\ J$
This work is converted to KE of the electron so $Vq = \frac{1}{2} mv^2$
So $v^2 = \frac{2Vq}{m}$
$$= \frac{2 \times 22{,}000 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 7.736 \times 10^{15}$$
$v = 8.80 \times 10^{7}\ m\ s^{-1}$

12. *Equating centripetal and magnetic forces:*
$\frac{mv^2}{r} = Bqv$ so $r = \frac{mv}{qB}$
$$= \frac{9.1 \times 10^{-31} \times 2.0 \times 10^{7}}{1.6 \times 10^{-19} \times 2.1 \times 10^{-3}} = 0.0542\ m$$

13. $\tau = BANI$
$= 0.45 \times (0.12 \times 0.045) \times 250 \times 0.25$
$= 0.152\ N\ m$

14. *When the current is turned on this energises the electromagnet at the bottom. The electromagnet attracts the iron armature on the right. When the armature is attracted left it acts as a lever and pushes the electrical contacts together. When the contacts close this completes the upper AC circuit and the lamp will come on.*

15. *$I = q/t$ so the time between each electron striking is $t = q/I = 1.6 \times 10^{-19}/20 \times 10^{-6}$ $= 8.0 \times 10^{-15}$ s*
Hence, number of electrons striking per second $= 1/8.0 \times 10^{-15} = 1.25 \times 10^{14}$ s^{-1}
In 1 minute the number striking will be $60 \times 1.25 \times 10^{14} = 7.5 \times 10^{15}$

16. *$F = BIL$ so $2.50 = B \times 40 \times 1$ (for 1 metre of wire). Hence $B = 0.0625$ T*

17. *$\tau = BANI\cos\theta$*
$= 0.06 \times (0.08 \times 0.03) \times 1 \times 2.5\cos 30$
$= 3.12 \times 10^{-4}$ N m

18. *Correct answer is C.*
Poles on magnets (L to R) are:
A. S-S; B. N-N; C. S-N; D. N-S; E. S-S

19. *The wire will move out of the page. Left coil polarity is south and right is north so flux is right to left. Current is up from R to S.*

20. *a) By the RH palm rule the wire on the right in the field will move down so the pointer will move up.*
b) The 5 g mass could be moved further to the left to increase the anticlockwise torque.
c) Need to measure
(i) mass added,
(ii) distance of mass from pivot,
(iii) length of wire in the field
(iv) distance of wire from the pivot,
(v) current.

Set 9 Electromagnetic Induction

1. *D is correct. An emf is set up in both coils due to the changing flux in them as the magnet falls. However, current will only flow in coil S because coil T has no complete circuit. By Lenz's Law a current will only be induced in coil S to form a magnetic field in opposition to the falling magnet which slows it down.*
2. *C is the correct answer. As the coil moves left the flux inside the loop increases so the induced current in the coil will produce a field to oppose the field into the page. A field out of the page will be produced if the current runs anticlockwise. When the loop is totally inside the field the flux is constant so the induced current will be zero. Coming out from the other side of the flux the field in the loop is tending to decrease, so the induced current will produce a field which maintains the field into the page (clockwise).*

3. *$E = Blv = 35 \times 10^{-6} \times 45 \times 10^{-3} \times 25 \times 10^{3}$*
$= 3.94 \times 10^{-2}$ V
(Note: it is the 45 mm width that is cutting the field lines and is at the top of the meteor)

4. *A is the correct graph. When the swing is out from the centre the seat is cutting lines of force at a smaller angle so the voltage will be smaller. Maximum voltage will be induced in the centre (90° to the field) and the voltage will reverse as it moves upwards from there. Graph B shows a sudden change in flux, which does not occur.*

5. *Vertical component of field*
$= 50 \times 10^{-6} \times \sin 68 = 4.6 \times 10^{-5}$ T
Estimate the diameter of a basketball hoop to be 40 cm so r = 0.20 m
Flux $\varphi = BA = 4.6 \times 10^{-5} \times \pi \times 0.2^2$
$= 5.8 \times 10^{-6}$ Wb

6. *a) No induced voltage will be formed unless there is a change in flux in the coil. Only if the coil rotates will the wires be cutting lines of flux.*
b) A 50 Hz frequency of rotation means there are 4 × 50 flux changes per second so the time for 1 flux change to occur = 0.005 s.
Flux through the coil $\varphi = BA$
$= 0.3 \times 8.0 \times 10^{-2}$ Wb
$$emf = -\frac{N.\Delta\varphi}{\Delta t} \text{ so } 240 = \frac{N \times 0.024}{0.005}$$
N = 50
c) $emf_{MAX} = 2\pi BAnf$
$= 2\pi \times 0.3 \times 8 \times 10^{-2} \times 50 \times 50 = 377$ V
d) $E_{RMS} = \frac{E_{peak}}{\sqrt{2}} = 267$ V

7. *Velocity of bike = 36/3.6 = 10 m s^{-1}*
Circumference of wheel $= 2\pi r$
$= 2 \times \pi \times 0.32 = 2.01$ m
No. of revolutions of wheel per second,
n = 10/2.01 = 4.974 Hz
Flux through wheel $= B \times \pi r^2$
$= 2.1 \times 10^{-5} \times \pi(0.32)^2 = 6.756 \times 10^{-6}$ Wb
Flux change through wheel per second $= n\varphi$
$= 4.974 \times 6.756 \times 10^{-6} = 3.36 \times 10^{-5}$ Wb s^{-1}
Induced voltage = 33.6 μV

8. *Answer is C. By Lenz's Law:*
Y will become N to oppose the N-pole moving in.
X will become N to attract back the S-pole moving away.

9. *The vertical component of the Earth's field will be cut.*
This is $3.8 \times 10^{-6} \sin 66 = 3.47 \times 10^{-6}$ *T*
6300/min = 6300/60 = 105 Hz
Flux cut by each blade = BA
$= 3.47 \times 10^{-6} \times \pi \times 5.2^2 = 2.95 \times 10^{-4}$ *Wb*
emf $= n\,\varphi = 105 \times 2.95 \times 10^{-4} = 3.10 \times 10^{-2}$ *V*

10. *Answer is B. With no load on the dynamo, no work is being done by it. With the lights turned on a current is drawn from the coils, creating a reverse force (BIL) on the coil which will slow the engine.*

11. *emf = BLv, so*
$5200 = B \times 22.5 \times 10^3 \times 8.12 \times 10^3$
So $B = 2.85 \times 10^{-5}$ *T*

12. *Temp rise = 400 so*
$R = 24(1 + 400 \times 4 \times 10^{-3}) = 62.4\ \Omega$
$$P = \frac{V^2}{R} = \frac{240^2}{62.4} = 923\ W$$

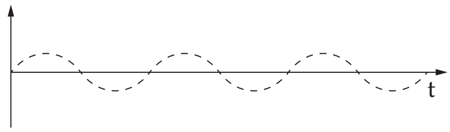
13. *The answer is A If B is halved, E is halved, if frequency is doubled emf is doubled, if number of turns is halved emf is halved so new emf will be ½ × 2 × ½ as large as the old emf i.e. half as much.*
So the new voltage will be half the old one but the frequency is doubled (twice the number of waves)

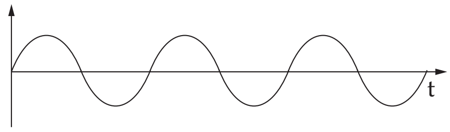
14. *Assume the diameter of a toilet roll to be 4 cm.*
Circumference $= 2\pi \times 2 \times 10^{-2} = 0.1257$ *m*
Hence, number of turns on coil = 1/0.1257 = about 8 turns.
Area of coil $= \pi(2 \times 10^{-2})^2 = 1.257 \times 10^{-3}\ m^2$
Flux through coil = BA
$= 0.12 \times 1.257 \times 10^{-3} = 1.51 \times 10^{-4}$ *Wb*
Induced voltage is given by:
$$emf = \frac{-Nd\varphi}{dt} = \frac{-8 \times 1.51 \times 10^{-4}}{dt} = 1\ volt$$
Time $(\Delta t) = 1.2 \times 10^{-3}$ *s*

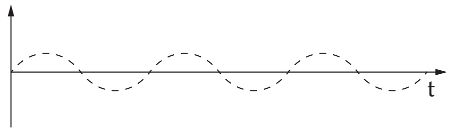
15. *Flux through the base of the saucepan = BA*
$= 0.045 \times \pi \times (12.5 \times 10^{-2})^2 = 2.21 \times 10^{-3}$ *Wb*
emf = rate of flux change
$= 2.21 \times 10^{-3} \times 10 \times 10^3 = 22.1$ *V*
Current flowing in the saucepan base = V/R
= 22.1/1.5 = 14.7 A
Power $= I^2R = (14.7)^2 \times 1.5 = 325$ *W.*

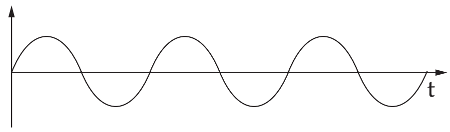
16. *emf* $= Blv = 0.8 \times 1.6 \times 5 = 6.4$ *V*
I = V/R = 6.4/96 = 0.0666 A
$F = BIL = 0.8 \times 0.0666 \times 1.6 = 8.53 \times 10^{-2}$ *N*

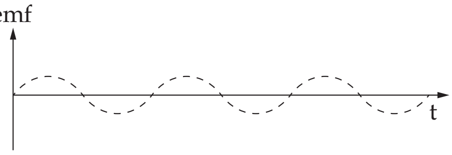
17. a) *Answer is "Stay the same"*
b) *Although there is a flux change, this change is in the same direction as the current and so will have no effect on the electrons by the RH palm rule.*

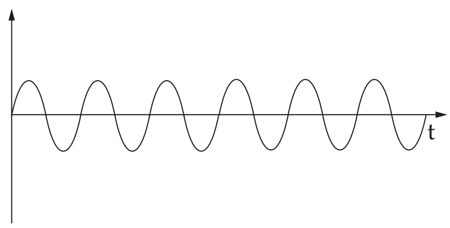
18. (i)
emf


Double Field
emf


(ii)
emf


Double Coils
emf


(iii)
emf


Double Rate of Rotation
emf


19. *(i) AC frequency = 5100/60 = 85 Hz*
(ii) $P = V^2/R = 6.60^2/1.1 = 39.6$ *W*
(iii) Area $= 1.40 \times 10^{-4}\ m^2$
$T = 1/85 = 1.176 \times 10^{-2}$ *s*
$t_{1/4} = 2.94 \times 10^{-3}$ *s*
$$6.6 = \frac{-Nd\varphi}{dt} = \frac{-400 \times B \times 1.40 \times 10^{-6}}{2.94 \times 10^{-3}}$$
So B = 34.7 T

20. *$\varphi = BA = 0.42 \times \pi \times (25 \times 10^{-3})^2$*
$= 8.247 \times 10^{-4}$ Wb
(Neglect point 4 on graph)
slope = $160 \times 10^{-3}/60$
$= 2.67 \times 10^{-3}$ Volts per turn
= E/N in the equation:
$$E = \frac{-Nd\varphi}{dt} \text{ so } \frac{E}{N} = \frac{d\varphi}{dt}$$
$$\text{or } 2.67 \times 10^{-3} = \frac{8.247 \times 10^{-4}}{t}$$
t = 0.309 s

Set 10 Power Transmission

1. *Answer D is correct*
With A a low voltage in the cord means a higher current must flow for the same current. This will lead to higher energy losses as power loss in the cord is proportional to current squared.

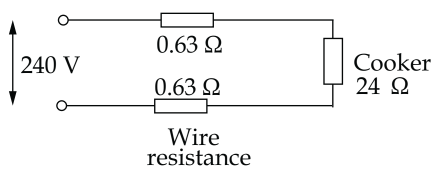
2. *Answer D is correct.*
The upper graph shows that the flux change in the coil is constant and positive. The induced emf depends on minus the rate of change of the flux which is constant. So emf will be a constant negative value.

3. *Light X will be brighter because is has less wire connected from it to the power supply – the wire will have some resistance and there will be a voltage drop in the wires so light Y will have less voltage applied to it.*

4. *With a higher voltage there is a lower current and less power loss in the connecting wires to light Y and so more power will be available for the light, but still slightly dimmer.*

5. *The secondary voltage will be 30 times the primary voltage (turns ratio = 1:30) but the current available will be 30 times smaller. 0.9/30 = 30 mA maximum.*

6. *Total wire resistance*
$= 2 \times 350 \times 1.8 \times 10^{-3} = 0.63 \times 2 = 1.26\ \Omega$
Theoretical circuit is shown:



Cooker is in series with the cable resistance.
Circuit current = V/R = 240/25.26 = 9.50 A
Voltage drop across the cooker, V = IR
$= 9.50 \times 24 = 228$ V

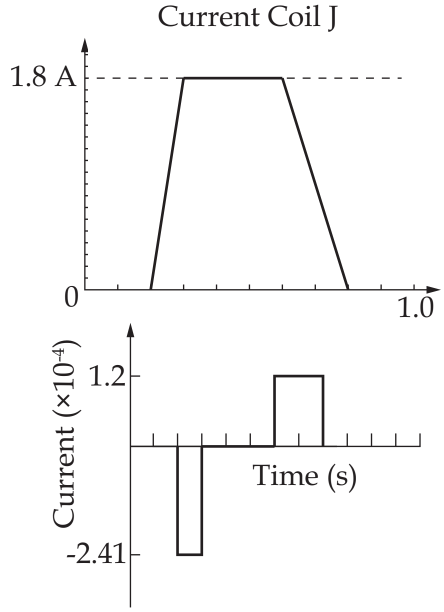
7. *Answer B is correct. More current will be drawn through the connecting cables with the saw working and so there will be more voltage drop in the wires. This means that the voltage supplied to the house must become less.*

8. $$E = \frac{-Nd\varphi}{dt} \text{ so } 6.6 = \frac{240 \times 0.06}{t}$$
t = 2.18 s

9. a) *By supplying a higher voltage over the 3.7 km less current is drawn for the same power delivery and so less power is lost ($P_{loss} = I^2R$)*
b) *Wire resistance for 140 m of wire*
$= 140 \times 2.25 \times 10^{-3} = 0.315\ \Omega$
$R_T = 28.8 + 0.315\ \Omega$
Current in wires = V/R
= 240/29.115 = 8.243 A
Voltage across kettle = IR
$= 8.243 \times 28.8 = 237.4$ V
Power delivered = 8.243×237.4
= 1.96 kW

10. *For a current of 1.8 A (graph) φ is BA*
$= 2.51 \times 10^{-3} \times 1.8 \times 2.4 \times 10^{-4}$
$= 1.084 \times 10^{-6}$ Wb
The flux rises to 1.084 μWb in 0.1 s then stays constant for 0.3 s then falls to zero in 0.2 s
$$E = \frac{-Nd\varphi}{dt}$$
$$E_1 = \frac{-50 \times 1.084 \times 10^{-6}}{0.1} = -5.42 \times 10^{-4} \text{ V}$$
$$E_2 = \frac{50 \times 1.084 \times 10^{-6}}{0.2} = 2.71 \times 10^{-4} \text{ V}$$
Current 1 will be V/R = $5.42 \times 10^{-4}/2.25$
$= -2.41 \times 10^{-4}$ A
Current 2 = $2.71 \times 10^{4}/2.25$
$= 1.20 \times 10^{-4}$ A

Current Coil J



Chapter 3 Solutions

Set 11 Waves and Quanta

1. $E = hf = 6.63 \times 10^{-34} \times 94.5 \times 10^{6} = 6.26 \times 10^{-26}\ J$

$$E = \frac{6.26 \times 10^{-26}}{1.6 \times 10^{-19}} eV = 3.92 \times 10^{-7}\ eV$$

2. a) *Assume the human body radiates as a Black Body with temperature = (37 +273) K*

$$\lambda_p = \frac{2.9 \times 10^{-3}}{310} = 9.35 \times 10^{-6} m \text{ (Infrared)}$$

b) *Wien's Law:* $\lambda p\ T = 2.9 \times 10^{-3}$ *so*

$$T = \frac{2.9 \times 10^{-3}}{1 \times 10^{-9}} = 2.9 \times 10^{6} K$$

c) $1727^oC = 2000\ K$

$$\lambda_p = \frac{2.9 \times 10^{-3}}{2000} = 1.45 \times 10^{-6} m$$

3. a) *Einstein Equation:* $hf = E_{Kmax} + W_o$ *so when* E_K *is just zero,* $hf = W_o$

$$f = \frac{4 \times 10^{-19}}{6.63 \times 10^{-34}} = 6.03 \times 10^{14} Hz$$

$$\lambda = \frac{3 \times 10^{8}}{6.03 \times 10^{14}} = 497nm$$

b) *Frequency* $f = \frac{3 \times 10^{8}}{300 \times 10^{-9}} = 1.00 \times 10^{15}$

$E_{Kmax} = hf - W_o$ *so*

$E_{Kmax} = 6.63 \times 10^{-34} \times 1 \times 10^{15} - 4.00 \times 10^{-19}$

$= 6.63 \times 10^{-19} - 4.00 \times 10^{-19} = 2.23 \times 10^{-19}\ J$

$E_{Kmax} = ½\ mv^2$ *so* $v = \sqrt{\frac{2E_K}{m}}$

$$v = \sqrt{\frac{2 \times 2.23 \times 10^{-19}}{9.11 \times 10^{-31}}} = 7.00 \times 10^{5} m\ s^{-1}$$

4. *Frequency* $f = \frac{3 \times 10^{8}}{300 \times 10^{-9}} = 1.00 \times 10^{15}$

Electrons will be suppressed if the energy from the reverse voltage is Vq

So $Vq = hf – Wo$ *which gives* $V = \frac{hf}{q} - \frac{W_o}{q}$

$$V = \frac{6.63 \times 10^{-34} \times 1 \times 10^{15}}{1.6 \times 10^{-19}} - \frac{4 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$= 1.64\ V.$

5. a) *Formula:* $\frac{n\lambda}{d} = \frac{a}{D}$

for the 1st fringe n = 1 so

$$a = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 4.5}{0.15 \times 10^{-3}} = 0.0177m$$

b) *From the formula, fringe spacing (a) is directly proportional to wavelength therefore if* λ *is larger the fringe spacing would also be larger.*

c) *From the formula fringe spacing (a) is inversely proportional to slit width, therefore if d gets smaller then the spacing becomes larger.*

6. $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_o c}(1 - cos\theta)$

$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}(1 - cos60)$$

$\Delta\lambda = 1.21 \times 10^{-12}\ m.$

New wavelength

$= 2.50 \times 10^{-11} – 0.12 \times 10^{-11} = 2.38 \times 10^{-11}\ m.$

7. a) *De Broglie wavelength equation is*

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 2 \times 10^{6}}$$

$= 3.54 \times 10^{-10}\ m$

b) *For constructive interference the circumference of the orbital must equal one wavelength for the lowest energy level, so* $3.54 \times 10^{-10} = 2\pi r$

$$r = \frac{3.54 \times 10^{-10}}{2\pi} = 5.79 \times 10^{-11} m.$$

8. a) *An error of* $\pm$ *0.05 km* h^{-1} *equates to 0.0139 m* s^{-1}*.*

Therefore the error in momentum of the car is $1500 \times 0.0139 = 20.8\ kg\ m\ s^{-1}$*.*

Using $\Delta p \Delta x \le \frac{h}{4\pi} \Delta x \le \frac{h}{4\pi \times \Delta p}$

$$= \frac{6.63 \times 10^{-34}}{4 \times \pi \times 20.8} = 2.53 \times 10^{-36} m$$

An incredibly small uncertainty in position!

b) *0.1% of* $3 \times 10^{7} = 3 \times 10^{4}\ m\ s^{-1}$ *so uncertainty in p* $= 9.11 \times 10^{-31} \times 3 \times 10^{4}$*.*

$\Delta p = 2.73 \times 10^{-26}\ kg\ m\ s^{-1}$

$$\Delta p \Delta x \le \frac{h}{4\pi} \Delta x \le \frac{h}{4\pi \times \Delta p}$$

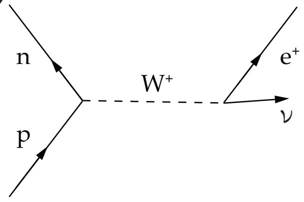
$$= \frac{6.63 \times 10^{-34}}{4 \times \pi \times 2.7 \times 10^{-26}} = 1.95nm$$

As the diameter of an electron is about 10^{-16} *m this figure shows an uncertainty in its position of about a billion times its diameter!*

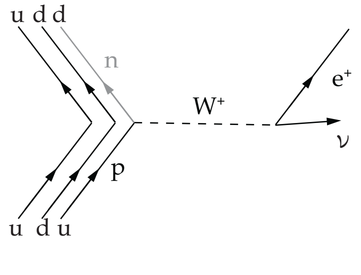
$${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + \nu$$

9. *The* β^+*-decay is shown as:*

a)



b)



10.

Particle	Quark structure	Charge	Baryon /Meson/ Lepton
Ω^-	s, s, s	-1	Baryon
Λ_t^+	u, d, t	+1	Baryon
K^+	$\bar{s}$, u	+1	Meson
Ξ^0	u, s, s	0	Baryon

Chapter 4 Solutions

Set 12 Spectra

1. D

2. $V = 4\pi r^3/3$ so $V_1/V_2 = r_1^3/r_2^3$
$= (10^{-15})^3/(5.3 \times 10^{-11})^3 = 6.72 \times 10^{-15}$

3. $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{620 \times 10^{-9}}$
$= 3.21 \times 10^{-19}$ J

4. E of each photon $= 5 \times 1.6 \times 10^{-19}$
$= 8.0 \times 10^{-19}$ J
$P = E/t = Ef$ So $f = P/E$
$= 0.01/8.0 \times 10^{-19} = 1.25 \times 10^{16}\ s^{-1}$

5. B

6. $\Delta E = 13.6 - 0.85 = 12.75 \times 1.6 \times 10^{-19}$
$= 2.04 \times 10^{-18}$ J

7. a –z, b- x, c – y

8. $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{420 \times 10^{-9}}$
$= 4.74 \times 10^{-19}$ J
Total energy $= 4.74 \times 10^{-19} \times 3 \times 10^{18}$
= 1.42 joules per second (= watts)
$E = Pt = 1.42 \times 10^{-3}$ J

9. C

10. A laser produces coherent waves all in phase which add up to give very high energy.

11. Red light is absorbed from the RGB spectrum leaving G and B to be reflected. This appears bluey-green

12. $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{487 \times 10^{-9}}$
$= 4.08 \times 10^{-19}$ J
$\Delta E = -2.55$ eV so transition is
$n = 4$ to $n = 2$

13. $E = 13.6 \times 1.6 \times 10^{-19} = 2.176 \times 10^{-18}$ J

14. B

15. A

Set 13 Accelerated Charges

1. $\Delta E = 8.95 \times 10^{-15} - 2.65 \times 10^{-15}$
$= 6.3 \times 10^{-15}$ J
$E = \frac{hc}{\lambda} \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6.30 \times 10^{-15}}$
$= 3.16 \times 10^{-11}$ m

2. D (The current is not important but the voltage is a measure of energy and is needed to accelerate the electrons).

3. A (The spikes are characteristic of the target metal so if the metal is changed the wavelength of the spikes would be different)

4. $E_{max} = 20{,}000 \times 1.6 \times 10^{-19} = 3.20 \times 10^{-15}$ J
$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3.2 \times 10^{-15}}$
$= 6.22 \times 10^{-11}$ m

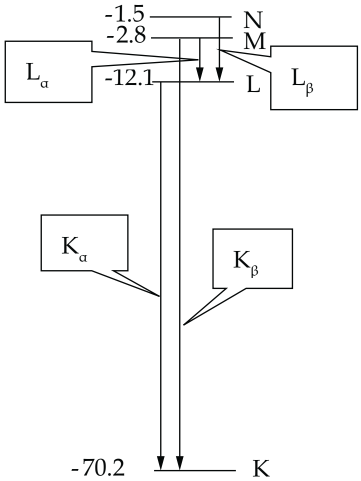
5. 65 keV gives $65 \times 10^3 \times 1.6 \times 10^{-19}$ J
$= 1.04 \times 10^{-14}$ J $= ½ \times 9.1 \times 10^{-31} v^2$
so $v = 1.51 \times 10^8\ m\ s^{-1}$.

6. a) X-rays will be absorbed by heavy metals, such as steel but pass through the crack to strike the photographic film. Hence white areas will show the position of the cracks.
b) Because X-rays have such a short wavelength ($\approx 10^{-11}$ m) very little diffraction will occur through the crack and hence a sharp image is produced.

7. For stars to produce X-rays there must be a source of electrons entering a medium where they slow down very rapidly – perhaps striking a very dense material.
The Earth's atmosphere will absorb some incoming X-rays so electrons in the oxygen atoms will move to higher energy levels, thereby absorbing X-ray energy. When the electrons return to the ground state they will emit electromagnetic waves of a much longer wavelength.

8. X-rays are very ionising so if they strike one of the atoms comprising a cell it can knock electrons out of it and hence cause the cell to function abnormally (cancerous cell).
In the same way, if a cancerous cell is located, X-rays can be targeted onto it to kill the cell and stop its abnormal growth.

9. (i) *Labels are shown (shortest wavelength corresponds to the largest energy transition.*



(ii) $\Delta E = 70.2\ keV - 2.8\ keV$

$= 67.4 \times 10^3 \times 1.6 \times 10^{-19}\ J$

$\Delta E = 1.078 \times 10^{-14}\ J$

$$E = \frac{hc}{\lambda} \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.078 \times 10^{-14}}$$

$$\lambda = 1.84 \times 10^{-11}\ m$$

10. *Electrons hit the target, moving close to the speed of light so they have a very large kinetic energy. As they are stopped within a few millimetres much of this KE is converted into heat which causes a large rise in temperature of the target metal.*

Set 14 Astrophysics

1. $$v_{star} = \frac{\Delta\lambda}{\lambda}c$$

$$\therefore v_{star} = 0.05 \times c$$

2. a) *Different wavelengths of radiation are emitted because of different energy changes so, although a reaction may not be seen with visible light other frequencies can display the source of energy. A cold star or interstellar gas may be invisible to us but is still emitting IR rays if it is above Absolute Zero and so can be perceived with an IR telescope. X-ray stars have high energy emission which can show more detail than mere visible light.*

b) *When a star is very distant – at the edge of the Universe – its light has been red-shifted so much that it is in the Radio wave frequency. Hence, very distant and old stars can only be perceived and recorded with these frequencies. Such distant stars would have been formed just after the Big Bang.*

3. a) *The right-hand side has a blue shift and so is moving faster than the left side, so the right must be coming towards us.*

b) *There is a 2 nm shift in 339 nm, so the velocity is given by:*

$$v = \frac{2}{339} \times 3 \times 10^8$$

$$v = 1.77 \times 10^6\ m\ s^{-1}$$

4. a) *From the graph the recessional velocity of star X is 1000 km s^{-1}.*

Fractional Red shift = 0.999994.

$$Z = \frac{1,000}{300,000} = 3.33 \times 10^{-3}$$

b) *Taking the point on the graph (620, 1500), gradient = 1500/620 = 2.42 km s^{-1} per MPc.*

Gradient = 7.83 × 10^{20}.

1/gradient = age of Universe

= 1.3 × 10^{19} seconds. = 4.1 × 10^{11} y.

5. a) *2 electrons: $E = mc^2$*

$= 2 \times 9.109 \times 10^{-31} \times (3 \times 10^8)^2$

$= 1.639 \times 10^{-13}\ J$

$$E = \frac{1.639 \times 10^{-13}}{1.6 \times 10^{-13}} = 1.02\ MeV$$

$\therefore$ *each photon has E = 0.51 MeV*

b) $$\frac{1}{2} \times 1.639 \times 10^{-13} = \frac{hc}{\lambda} \text{ so}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.5 \times 1.639 \times 10^{-13}}$$

$= 2.43 \times 10^{-12}\ m.$

6. a) $E_2 - E_1 = 13.6 - 3.4 = 10.2\ eV$

$10.2 \times 1.6 \times 10^{-19} = 1.63 \times 10^{-18}\ J$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.63 \times 10^{-18}}$$

$= 1.22 \times 10^{-7} m.$

b) $$4.15 = \frac{\lambda_{observed}}{1.22 \times 10^{-7}} - 1$$

$\lambda_{oberved} = 5.15 \times 1.22 \times 10^{-7}$

$= 6.28 \times 10^{-7}\ m.$

7. a) *Length dilation: the distance of 5.26 m will seem shorter by the relativistic equation:*

$$\frac{l_m}{l_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)} \text{ so}$$

$$l_m = \frac{5.26}{\sqrt{1 - 0.99^2}} = 37.3m.$$

b) *Addition of relativistic velocities:*

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Using velocities in multiples of c:

$$\frac{0.99 + 0.5}{1 + \frac{0.99 \times 0.5}{1^2}}$$

$= 0.997\ c.$

8. $$\frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{2.5 \times 10^8 + 1.8 \times 10^8}{1 + \frac{2.5 \times 10^8 \times 1.8 \times 10^8}{(3 \times 10^8)^2}}$$

$$= \frac{4.3 \times 10^8}{1 + 0.5}$$

$= 2.86 \times 10^8\ m\ s^{-1}.$

9. a) $\frac{t_m}{t_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$ *so*

$t_m = 2.4\sqrt{1 - 0.5^2} = 2.08y.$

It will actually take the astronauts only 2.08 years to reach the planet.

b) *From a length contraction point of view, the distance of 1.2 Ly will appear shorter from the formula:*

$\frac{l_m}{l_s} = \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$

$\therefore l_m = 1.2\sqrt{1 - 0.5^2} = 1.04Ly$

In the astronauts' frame of reference, the distance to the planet appears to be only 1.04 Ly away.

At a speed of 0.5 c, this will take the spaceship 1.04/0.5 = 2.08 years.

10. *Mass dilation:*

$$m_m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$= \frac{12000}{\sqrt{1 - (0.5)^2}} = 13,900kg$$

Some of its kinetic energy has been converted to extra mass.

Chapter 5 Solutions

Set 15 Preliminary Testing

1. a) *The 40 × 40 board is best for the main experiment because it gives the largest variation of results.*

   b) *In the case of all boards there is a definite reduction in sound level the thicker the board.*

   c) *The wave theory of sound shows that there will be less diffraction around the larger board. The reason there is less variation in sound levels for the smaller boards is probably because a lot of the sound energy is being diffracted around the boards and hence will reach the microphone.*

   d) *We would expect there to be more diffraction of the lower frequency sounds.*

2. a) *300 grams seems to be the best as it gives the greatest variation of frictional force between each oil.*

   b) *Oil 1 seems to be the best lubricant as it gives the least resistance to motion.*

3. a) *Cornflour would be the best to use in the main experiment as it gives a greater variation of colour at different temperatures.*

   b) *The top shelf seems to be the hottest position and the bottom shelf is the coolest.*

4. a) *20 minutes is best as it shows the greatest variation and allows a better comparison.*

   b) *Soil 3 retains water better than soil 1 and the least retentive is Soil 2.*

5. a) *James' experiment is best as his method of attachment ensures an even distribution of force on the paper. The hole used in John's experiment provides a weak point for breaking and would give large variance.*

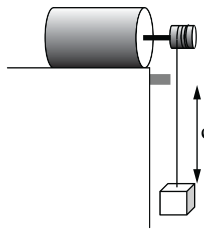
   b) *The variance in John's results (spread of values in the same conditions) is much larger and indicates a lack of variable control and more randomness.*

   c) *2 cm width seems the best as it shows clearer differences between the types of paper.*

   d) *Paper C seems stronger but is very close to Paper B*

Set 16 Variable Control

1. a) *A weight could be attached to each motor on a shaft and the time for each motor to lift the weight through the same vertical distance noted.*



   b) *Independent variable is the "motor type" and the dependent variable is the time taken for the lift.*

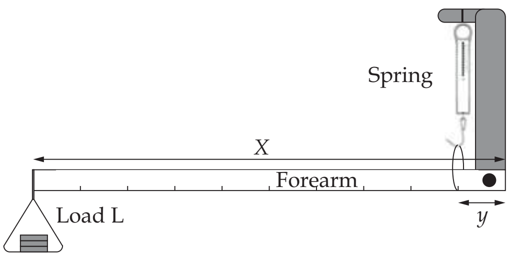
   c) *Other variables to be controlled : mass lifted, distance lifted, voltage on motor.*

2. a) *Best is defined as fastest.*
   b) *Dependent variable is the time taken to travel the same distance down a hill.*
   c) *Variables: Hill Slope (use the same hill), Weight of Board (add weights until each board weighs the same), Distance Travelled (measure out the same distance down the hill).*

3. a) *We would expect that the best tissue will be the one that costs the most.*
   b) *Independent Variable is "Tissue type" and Dependent Variable is "Mass of water absorbed".*
   c) *(i) weigh tissue, (ii) immerse in water and allow to drain, (iii) reweigh tissue, (iv) find the gain in mass of tissue (= water absorbed), (v) divide the % gain in mass by the area of tissue.*

4. a) *Apparatus: signal generator (where frequency and loudness can be adjusted) connected to a loudspeaker.*
   Procedure: A subject sits at a distance from the speaker and a certain frequency is played. The loudness is adjusted until the subject confirms that the note cannot be heard any more. This loudness is noted and the experiment is repeated at different frequencies.
   Accuracy can be increased by repeating the experiment with several different subjects and averaging the results.
   b) *Independent Variable is the frequency of the source – controlled by the dial on the generator.*
   c) *Dependent variable is the loudness at which the sound is just perceived by the subject. This is indicated by the meter on the signal generator.*
   Other variables are: Distance from the speaker to the subject (measure same distance each time), echoes from the room (use same room each time), subjectivity of the subject (repeat results with several different people).

5. a) *Suggested Apparatus:*



   b) *Independent Variable is the position of the load – distance X*
   Dependent Variable is the spring balance reading.
   c) *Controls: The angle of the forearm (must be kept horizontal)*
   The distance of attachment of the spring to the forearm (distance y same)
   The weight of the forearm (use the same apparatus)

Set 17 Analysing Data

1. a) *Comparing results 1 and 4, doubling the length will give half the current (inverse relationship).*
   b) *Comparing results 1 and 2, doubling the diameter will increase the current by a factor of 4.*
   c) *Area = πr^2. If $I = kA$ then k should be the same if length is the same ($k = I/A$).*
   Result 1: k = 64.2 and
   Result 2: k = 62.7 (very close).
   d) *For the same length or diameter the current through the constantan wire is about half of that for the iron wire. Hence the resistivity of Constantan must be about double that of iron.*

2. a) ***a** and θ are linked but they are not directly proportional to each other. At small angles doubling the angle from 10 – 20° produces a doubling of **a** but for larger angles there is no obvious linear relationship e.g. 20° to 40°, **a** increases by a factor of 1.85 and not 2.*
   b)

Sine θ	*a*	*a/sine θ*
0.174	1.7	9.77
0.342	3.4	9.94
0.500	4.9	9.80
0.642	6.3	9.81
0.766	7.5	9.79

   *Sine θ seems to be directly proportional to **a** because the ratio of **a** divided by Sine θ is virtually the same in each case.*
   c) *The average value of **a** is 9.82 so the equation of the line is $a = 9.82 \sin\theta$.*

3. a) *Sag is directly proportional to load (straight line graph)*
   b) *Graph is linear with a slope of 24.5 g cm^{-1}.*
   c) *It is not a linear graph – maybe a quadratic or cubic.*
   d) *A plot of d^3 versus Sag (s) is linear: $d^3 = ks$*

e) *Other variables: Bridge material; Width; Thickness; Anchoring method for ends; Position of weights.*

4. a) *The field strength B seems to be directly related to the number of turns (if turns doubled then B doubles).*
   b) *The field strength B seems to be directly related to the current.*
   However, as the current increases there is less linearity in the relationship as shown by the values of B divided by N in the table below. At low currents B/N is about 0.5 but when I = 1.5 the ratio of B/N changes.

No. of turns	***I = 0.5***	***I = 1.0***	***I = 1.5***
50	*0.50*	*0.80*	*1.46*
100	*0.48*	*0.79*	*1.12*
200	*0.54*	*0.70*	*0.98*

5. a) *Independent variable is the position of the pan (x)*
   Dependent Variable is the load (L)
   b) *As the load increases the value of x decreases. There seems to be an Inverse relationship between L and x.*
   c) *If inverse, then L = k/x, where k is a constant.*
   If L is proportional to 1/x then L times x should be a constant. From the table (below) we can see that this seems to apply.

Spring at 10 cm		
L	*1/x*	*Lx*
250	*0.010*	*2500*
310	*0.0125*	*2480*
410	*0.0167*	*2460*
620	*0.025*	*2480*

   d) *When x is increased to 15 cm a greater load is required to balance the torque of the load on the arm. However, the same equation seems to apply: L is proportional to 1/x and L times x is a constant, but the constant is larger.*

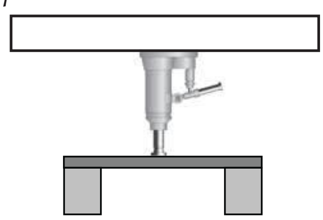
Spring at 15 cm		
L	*1/x*	*Lx*
380	*0.010*	*37000*
470	*0.0125*	*37600*
625	*0.0167*	*37500*

Set 18 Evaluating Investigations

1. a) *Criticism: If the students are trying to compare different core materials, as stated (wood, iron, Alnico, etc) then the core shape should be the same for each material as this could affect the output voltage (flux can get lost if the core is open, rather than closed).*
   b) *It should not matter if the actual number of turns is different for each transformer but what has been controlled is the turns ratio of each. At 1 : 2 this is actually the same for all the cores. Theoretically the size of the input voltage V_I should not affect the ratio of input to output voltages but there could be that with higher input voltages this ratio changes.*
   c) *The efficiencies of the different designs can be seen from the ratio of output to input voltages. With perfect efficiency this value should be 2.*
   Coil number 4 would appear to be the best but this may be due to its closed design, rather than core material. Hence, it is possible that Alnico is a more efficient core material but it would need the same construction as Coil 4 to make a Fair Test comparison.

Coil Number	***V_0 (volts)***	***Ratio V_0/V_I***
1	*1.2*	*0.40*
2	*3.5*	*1.40*
3	*4.4*	*1.76*
4	*4.8*	*1.92*

2. a) *The variables not controlled are:*
   Size of concrete block; width and spacing of supports.
   Hitting with a hammer is not controllable as invisible cracks can have formed – not a continuous variable.
   Important varaibles: Size and shape of concrete, Spacing of supports, Hammer force.
   b) *See diagram below. The car jack pushes upwards on a rigid beam and can exert a variable force downwards which could be measured from scales placed under the 2 supports.*



c) *Perform each test 5 times with the same variables controlled and average out the results.*

3. a) *The size of the bricks was not controlled but this could be overcome by finding the % absorption by mass for each brick type. The drum is too large for accuracy.*
   b) *With a narrower tank the drop in water level would be greater and more accurately measured.*
   c) *Comparing the water absorbed per kg of mass would be a Fair Test.*

4. a) *In both cases it is difficult to judge "Broken" as the bags will all stretch a lot first. Karen has controlled the width of the strips and the force is continuously variable and so will give more accurate results.*
   b) *Karen has repeated readings and averaged them but Jeffrey's results have a large average error of ± "1 pipe-weight". Karen's results also have a much lower variance.*
   c) *Bar graphs are OK to use with Independent Variables Like "Colour" but Karen's 2nd graph shows more detail on how the bag stretches.*
   d) *Karen's hypothesis on the strength due to molecular forces is more scientific than Jeffrey's one merely due to colour. Karen has also suggested a reasonable use for the weakest plastic with a large stretch. Overall, Karen's investigation is of a much higher quality.*

5. a) *The frictional force (F) on the weight is providing the centripetal force (F_c) for it to stay on. As the number of revs per minute increases so the speed of the turntable increases and F_c would become greater. However, as F_c depends on 1/r, if the weight is moved closer to the centre this increases the value of F_c at the position of the weight and so it cannot stay on the turntable.*
   b) *The 5 cm width of the weight would mean that the centripetal force at its inner edge would be smaller than that at its outer edge so the error in radius would be large. This would affect the results.*
   c) *The frictional force of the turntable on the weight would depend on how clean it was. By cleaning it each time the friction value should remain constant throughout the experiment.*
   d) *The marker would make it easier to count the revolutions as it hits the finger. However, if the rotation rate became too high it would be difficult to count accurately.*
   e) *It would not be necessary to record the voltage of the power pack because the only relevant factor is the rotation rate of the turntable. V does not figure in any calculations.*

Set 19 Uncertainties

1. a) *Average time = 0.30 s. Range of values is ± 0.03 s so error in timing = ± 0.03 s.*
   b) *Velocity = s/t = 100/0.30 = 333 m s^{-1}*
   c) *Δt = 0.03/0.3 = 0.1 (10%)*
   Δs = 0.7/100 = 0.005 (0.7%) Total error = 10.7%
   Error in v = 10.7% of 333 = 36 m s^{-1}
   d) *Correct quoted value is (330 ± 40) m s^{-1}*
   e) *Greatest %error comes from the timing with stopwatches (10%). However, the variation in times is not so much due to accuracy in timing as the human error in the judging of when to start and stop the watches. More accurate watches would not decrease this error.*
2. a) *k = F/x = (0.15 × 9.8)/0.053 = 27.74 N m^{-1}*
   b) *ΔL_o = 0.1/8.6 = 1.2%.*
   ΔL_1 = 0.1/13.9 = 0.7%
   Δm = 0.01/150 = 0.007%
   Total uncertainty = 1.9% which gives an absolute uncertainty of 0.53.
   c) *Correct quoted value is (27.7 ± 0.5) N m^{-1}*

3. a) *Volume = 7.8 × 10.6 × 0.3 = 24.8 cm^3*
   D = m/V = 62/24.8 = 2.50 g cm^{-3}
   b) *% errors are: 1.3%, 1%, 3.3% in lengths and 1.6% in mass. Total = 7.2%*
   c) *D = (2.5 ± 0.2) g cm^{-3}*

4. a) *Average t = 0.53 s. g = $2h/t^2$ = 8.76 m s^{-2}*
   b) *% h = 0.8%. %t is calculated from the spread about the mean = 0.6 – 0.53 = 0.07.*
   Error in t is 0.07/0.53 gives 26.4%. Error in t^2 = 53%. Total = 54%
   c) *(9 ± 5) m s^{-2}*

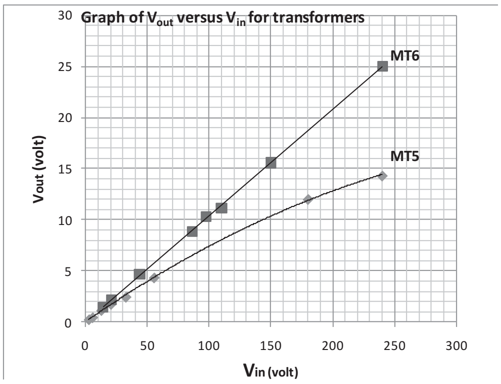
5. a) *E = V/d = 2100/0.042 = 5.0 × 10^4 V m^{-1}*
   Volume of oil = 4/3 × π × (0.25 × 10^{-3})^3 = 6.54 × 10^{-11} m^3
   Mass of oil drop = 6.54 × 10^{-11} × 708 = 4.63 × 10^{-8} kg
   Weight = 4.63 × 10^{-8} × 9.8 = 4.54 × 10^{-7} N
   W = Eq so 4.54 × 10^{-7} = 5 × 10^4q so q = 9.1 × 10^{-12} coulomb.

b) $\Delta V = 100/2100 = 4.8\%$,
$\Delta d = 0.1/4.2 = 2.4\%$,
$\Delta r = 0.01/0.25 = 4\%$ so
$\Delta r3 = 12\%$,
$\Delta D = 2/708 = 0.3\%$
Total % error = 19.5%

c) Absolute error = 19.5% of 9.1×10^{-12}
$= 1.8 \times 10^{-12}$
Correct quoted value is:
$(9 \pm 2) \times 10^{-12}$ coulomb.

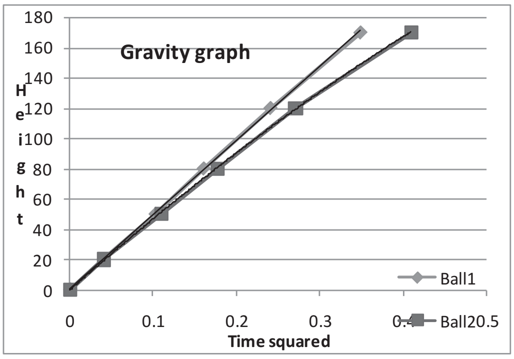
Set 20 Drawing Conclusions

1. a) The Independent Variable is Input voltage and the Dependent Variable (measured) is the Output Voltage.
   b) Graphs on Excel are shown:



   c) The MT6 slope is constant at 0.103 or 9.7 : 1 whereas the MT5 slope is 0.082 (12 : 1) at the start, decreasing with voltage.
   d) The MT6 shows a linear trend throughout the voltage range but the MT5 transformer graph curves over at higher voltages, showing that the ratio of V_o to V_i is decreasing.
   e) The fan seems to have a large effect on the characteristics of the MT6. Cooling the transformer with a fan seems to prevent the output from dropping off at higher voltages.
   f) Although the Turns Ratios of the transformers are different, the comparison between their output characteristics should still be valid as the important parameter is whether the graph of V_o and V_i is a straight one. The range of voltages is the same for both and so a comparison of efficiencies is valid.

2. Free Fall
   a) h should be plotted against t^2 for a straight line relationship (see graph)



   b) The graph of h versus t^2 for ball 1 is a good straight line but for ball 2 the graph is a slight curve. c)
   Equation is: h = 491t2 so g/2 = 491 and g equals 982 cm s-2
   d) Ball 2 being bigger and light (hollow) will be more affected by air resistance so it loses speed quicker, reducing the slope of the graph and the apparent value of g.

3. Wire Resistance
   a) A graph of I versus 1/L should be plotted
   b) Values of 1/L are 0.167; 0.145; 0.133; 0.120; 0.115; 0.105.
   c) I versus 1/L graph is a straight line but R is proportional to 1/I so R is directly proportional to L. Since the slope of the line does not change (not a curve) R must stay at a fixed value for larger I values (higher temperatures)
   d) $p < 6 \times 10^{-6}\%$
   e) Gradient = 32.2 so $\alpha = 62\ \Omega\ m^{-1}$

4. Stretching springs
   a) Spring 1 equation: L = 0.040 m + 6.4.
   Spring 2 equation: L = 0.049 m + 10.3
   b) Gradient 1 = 0.040. Gradient 2 = 0.049.
   c) The 5th reading on spring 1 seems wrong.
   Spring 1: P< 0.35%.
   Spring 2: P< 0.39%.
   d) The gradients are different. Spring 2 stretches more then spring 1 for a given load.
   e) Spring length and diameter must be controlled.

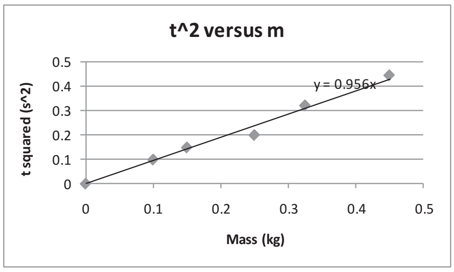
5. <u>Squash balls</u>
   a) *Average h_2 values in table 1 are 24.3; 32; 41; 46.7; 57; 14.*
   Average h_2 values in table 2 are: 19; 28; 34; 42.7; 54; 9.
   b) *The graph seems to show a slight curve (quadratic?)*
   c) *Equation from table 1 is*
   $h_2 = 0.005T^2 + 0.433T + 12.24$
   d) *Equation from table 2 is*
   $h_2 = 0.005T^2 + 0.428T + 7.735$
   e) *The larger the drop height the higher the bounce height, but the variables do not seem to be directly proportional. Comparing the bounce heights at the same temperatures for a 50 cm and 80 cm drop height we find that the ratio $h_2/h_1 = 0.84$ for 50 cm but $h_2/h_1 = 0.58$ for 80 cm.*

6. <u>Oscillating Mass</u>
   a) *From the formula supplied T^2 will be directly proportional to m so if T^2 is plotted against m it will result in a straight line graph, which is more desirable than a curved line.*

   b)

m	T^2
0.1	0.098596
0.15	0.148225
0.25	0.198916
0.325	0.320356
0.45	0.443556
0	0

   c)



Gradient = 0.443/0.45 = 0.98 s^2 kg^{-1} (from calculator gradient = 0.956).

$$\text{Gradient} = \frac{4\pi^2}{k} \text{ so } k = \frac{4\pi^2}{0.98}$$

$= 40.0$ N m^{-1}

3rd reading (0.25 kg) point is off the graph so is in error.
Using the calculated k value:
$T^2 = 4\pi^2 \times 0.25/40 = 0.247$ *so T should be* $\approx \sqrt{0.25} = 0.5$ s

d) *It is far easier to draw a straight line through a series of points than a curve. Drawing a line through the middle of a series of points averages out the errors. By drawing a line of best fit any outlier points can be ignored*

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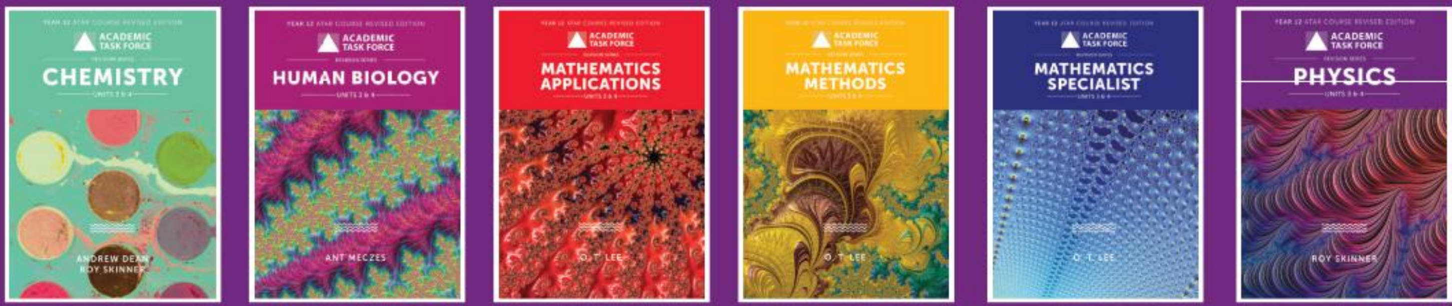

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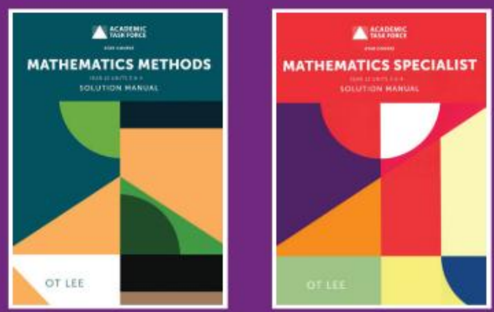

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