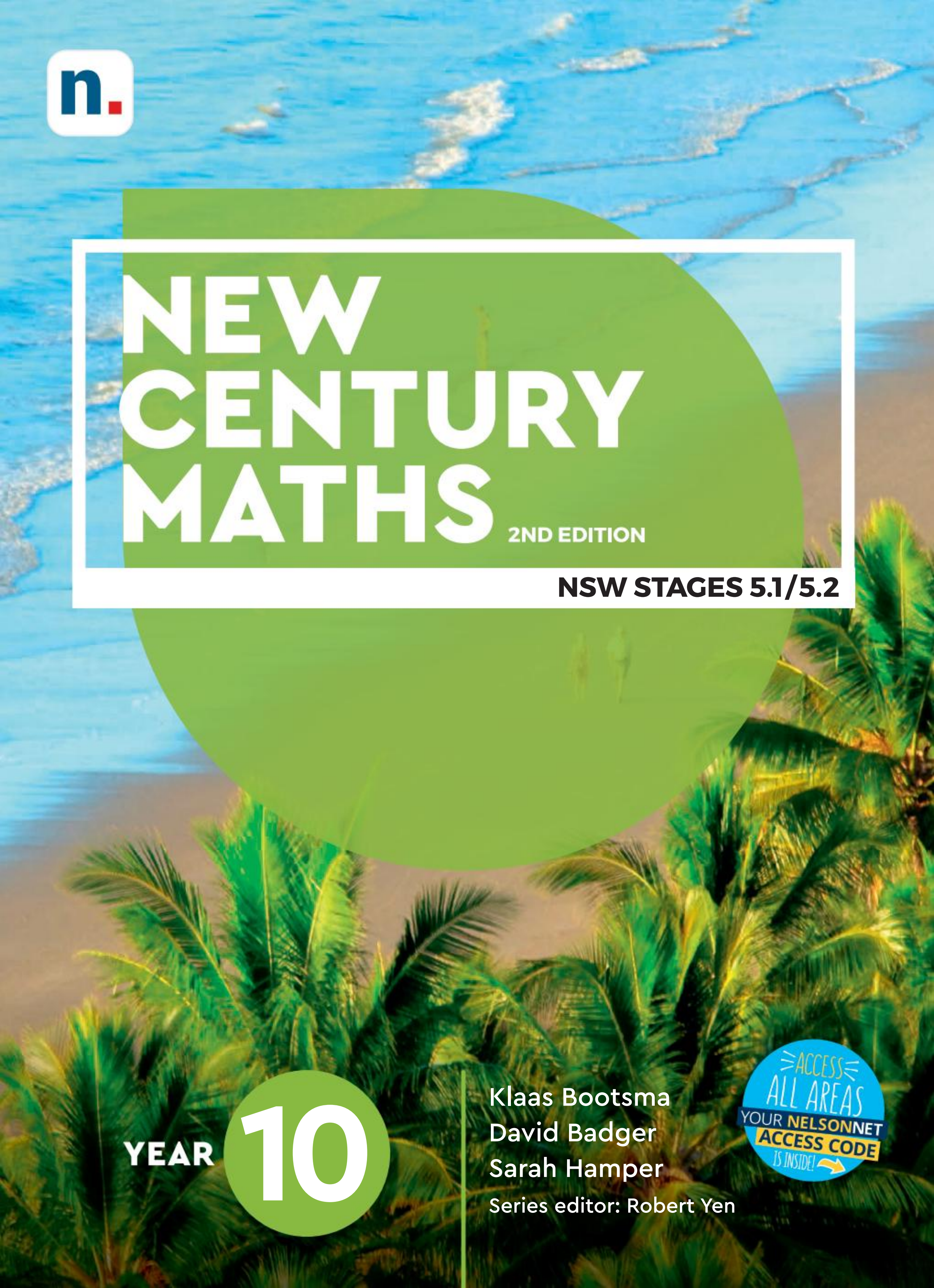

NEW CENTURY MATHS

2ND EDITION

NSW STAGES 5.1/5.2

YEAR 10

Klaas Bootsma
David Badger
Sarah Hamper
Series editor: Robert Yen

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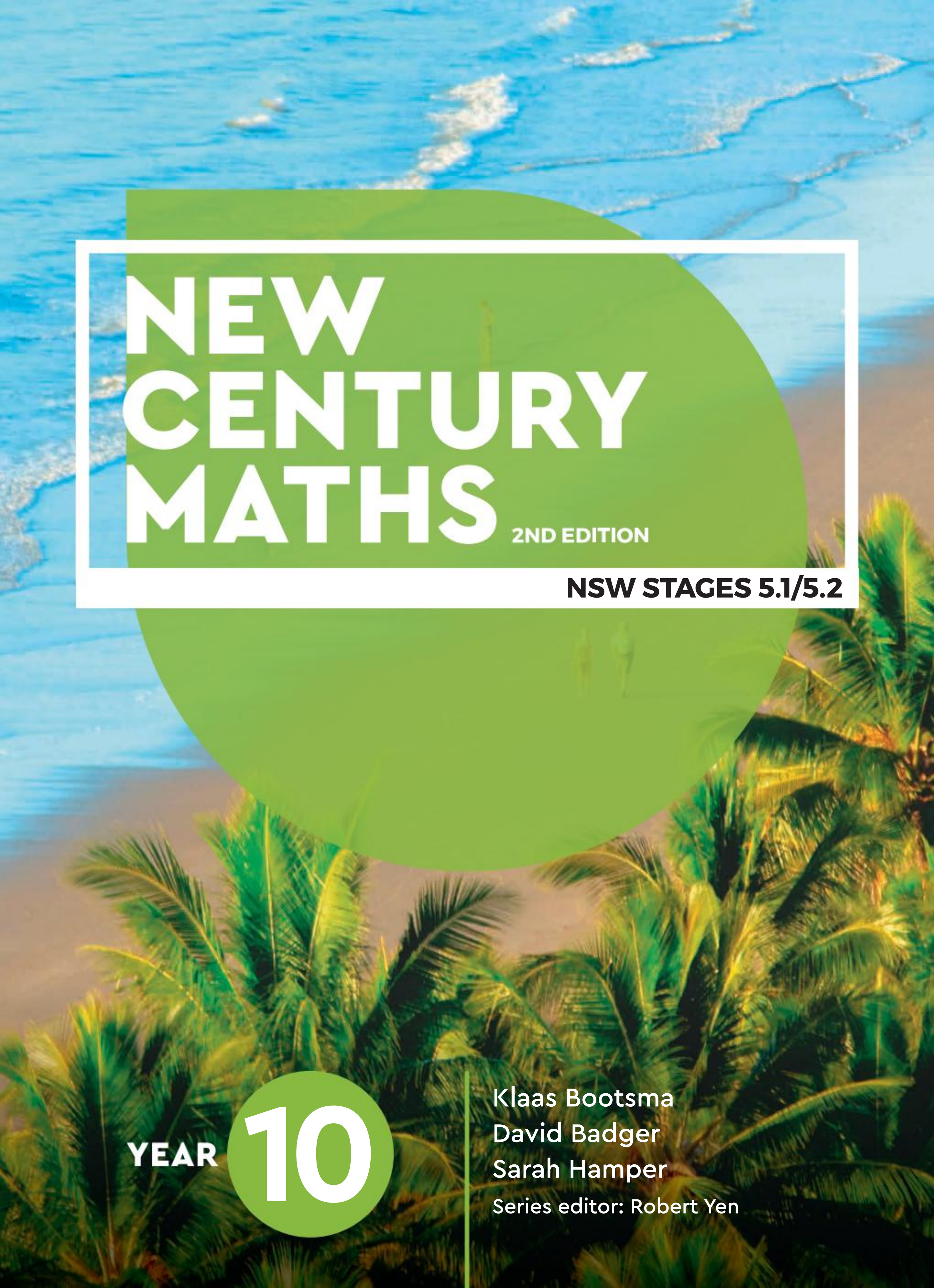

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NELSON
A Cengage Company

New Century Maths 10 Student Book
2nd Edition
Klaas Bootsma
David Badger
Sarah Hamper
ISBN 9780170453417

Publisher: Robert Yen
Project editor: Alan Stewart
Editor: Anna Pang
Original cover design by Chris Starr (MakeWork), Adapted by: Justin Lim
Text designer: Alba Design
Project designer: Justin Lim
Cover image: Alamy Stock Photo/Paul Dymond
Permissions researcher: Helen Mammides
Typeset by: KnowledgeWorks Global Ltd
Production controller: Karen Young

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ISBN 978 0 17 045341 7

Cengage Learning Australia
Level 7, 80 Dorcas Street
South Melbourne, Victoria Australia 3205

Cengage Learning New Zealand
Unit 4B Rosedale Office Park
331 Rosedale Road, Albany, North Shore 0632, NZ

For learning solutions, visit **cengage.com.au**

Printed in China by 1010 Printing International Limited.
1 2 3 4 5 6 7 25 24 23 22 21

PREFACE

Teachers, it's time to declutter and start teaching maths again. You asked and we listened.

New Century Maths 7–10 has been refreshed for the 2020s classroom, focusing on core skills to discourage 'teaching by syllabus dot points', featuring explicit grading of exercise questions, more 'flipped classroom' video tutorials, more applications and problem-solving questions, and worked solutions to every question.

In schools for over 25 years, *New Century Maths* is carefully mapped to the NSW syllabus and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

We publish 2 levels of mathematics books for Years 9 and 10 (Stage 5):

- *New Century Maths,* covering NSW Stages 5.1/5.2
- *New Century Maths Advanced,* covering NSW Stages 5.1/5.2/5.3

This book, *New Century Maths 10*, has been designed for Year 10 students progressing along Stages 5.1 or 5.2 of the continuum. It also contains **4 bonus Stage 5.3 chapters for extension**, especially for students heading towards the Mathematics Advanced course in Year 11 (see page vii).

The *NelsonNet* student and teacher websites contain worksheets, video tutorials, topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *New Century Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. So let's get started.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC marker and has worked on the HSC Advice Line. Klaas also co-wrote *New Century Maths 11–12 Mathematics Standard 2.*

David Badger teaches at Penrith Christian School. He was principal of Toongabbie Christian School, deputy principal at Mt Annan Christian College and head teacher of mathematics at Eagle Vale High School. He has been involved in HSC marking and has worked on the HSC Advice Line.

Sarah Hamper is head teacher of mathematics at Cheltenham Girls High School and has taught at Abbotsleigh, Meriden and Tara Anglican schools. She has an interest in gifted and talented (GAT) students and girls education. Sarah also co-wrote *New Century Maths 11–12 Mathematics Standard 2*.

Series editor **Robert Yen** taught at Hurlstone Agricultural High School. He co-wrote *New Century Maths 11–12 Mathematics Standard 2,* co-edited *Reflections,* the MANSW journal, and now works for Cengage as the mathematics publisher.

Contributing authors

Megan Boltze and **Robert Yen** wrote many of the *NelsonNet* worksheets and topic tests.

John Drake, **Katie Jackson**, **Joanne Magner**, **Scott Smith** and **Robert Yen** created the video tutorials.

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SYLLABUS GRIDS

NSW syllabus

Strand and substrand	*New Century Maths 9* chapter		*New Century Maths 10* chapter	
NUMBER AND ALGEBRA				
Financial mathematics (Stages 5.1, 5.2)	3	Working with numbers	1	Interest and depreciation
	8	Earning money		
Indices (Stages 5.1, 5.2)	5	Indices	4	Algebra
Linear relationships (Stages 5.1, 5.2)	7	Equations	2	Graphing lines
	11	Coordinate geometry and graphs		
Non-linear relationships (Stages 5.1, 5.2)	7	Equations	2	Graphing lines
	11	Coordinate geometry and graphs	7	Graphing curves
Ratios and rates (Stage 5.2)	3	Working with numbers	7	Graphing curves
	11	Coordinate geometry and graphs		
Algebraic techniques (Stage 5.2)	1	Algebra	4	Algebra
Equations (Stage 5.2)	7	Equations	6	Equations and inequalities
			9	Simultaneous equations
MEASUREMENT AND GEOMETRY				
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	10	Surface area and volume		
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Probability (Stages 5.1, 5.2)	12	Probability	10	Probability

Australian curriculum

Strand and substrand	New Century Maths 9 chapter		New Century Maths 10 chapter	
NUMBER AND ALGEBRA				
Real numbers	3	Working with numbers	7	Graphing curves
	5	Indices		
	11	Coordinate geometry and graphs		
Money and financial mathematics	3	Numeracy and calculation	1	Interest and depreciation
Patterns and algebra	1	Algebra	4	Algebra
	5	Indices		
	7	Equations		
Linear and non-linear relationships	7	Equations	2	Graphing lines
	11	Coordinate geometry and graphs	7	Graphing curves
			6	Equations and inequalities
			9	Simultaneous equations
MEASUREMENT AND GEOMETRY				
Using units of measurement	10	Surface area and volume	3	Surface area and volume
Geometric reasoning	6	Geometry	11	Geometry
	13	Congruent and similar figures		
Pythagoras and trigonometry	2	Pythagoras' theorem	8	Trigonometry
	4	Trigonometry		
STATISTICS AND PROBABILITY				
Chance	12	Probability	10	Probability
Data representation and interpretation	9	Analysing data	5	Comparing data

SYLLABUS GRIDS

Year 10 content descriptions

Australian Curriculum descriptions (© ACARA 2012).

Content description	New Century Maths 10 chapter	
NUMBER AND ALGEBRA		
Money and financial mathematics		
ACMNA299: Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies	1	Interest and depreciation
Patterns and algebra		
ACMNA230: Factorise algebraic expressions by taking out a common algebraic factor	4	Algebra
ACMNA231: Simplify algebraic products and quotients using index laws	4	Algebra
ACMNA232: Apply the four operations to simple algebraic fractions with numerical denominators	4	Algebra
ACMNA233: Expand binomial products and factorise monic quadratic expressions using a variety of strategies	4	Algebra
ACMNA234: Substitute values into formulas to determine an unknown	6	Equations and inequalities
Linear and non-linear relationships		
ACMNA235: Solve problems involving linear equations, including those derived from formulas	6	Equations and inequalities
ACMNA236: Solve linear inequalities and graph their solutions on a number line	6	Equations and inequalities
ACMNA237: Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology	9	Simultaneous equations
ACMNA238: Solve problems involving parallel and perpendicular lines	2	Graphing lines
ACMNA239: Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate	7	Graphing curves
ACMNA240: Solve linear equations involving simple algebraic fractions	6	Equations and inequalities
ACMNA241: Solve simple quadratic equations using a range of strategies	6	Equations and inequalities

9780170453417

Content description	New Century Maths 10 chapter	
MEASUREMENT AND GEOMETRY		
Using units of measurement		
ACMMG242: Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids	3	Surface area and volume
Geometric reasoning		
ACMMG243: Formulate proofs involving congruent triangles and angle properties	11	Geometry
ACMMG244: Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	11	Geometry
Pythagoras and trigonometry		
ACMMG245: Solve right-angled triangle problems including those involving direction and angles of elevation and depression	8	Trigonometry
STATISTICS AND PROBABILITY		
Chance		
ACMSP246: Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence.	10	Probability
ACMSP247: Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language	10	Probability
Data representation and interpretation		
ACMSP248: Determine quartiles and interquartile range	5	Comparing data
ACMSP249: Construct and interpret box plots and use them to compare data sets	5	Comparing data
ACMSP250: Compare shapes of box plots to corresponding histograms and dot plots	5	Comparing data
ACMSP251: Use scatter plots to investigate and comment on relationships between two numerical variables	5	Comparing data
ACMSP252: Investigate and describe bivariate numerical data where the independent variable is time	5	Comparing data
ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data	5	Comparing data

ABOUT THIS BOOK

Coverage of the syllabus

- *New Century Maths 10* covers both the NSW syllabus and the Australian curriculum, as shown by the table of contents and syllabus grids on the previous pages.
- This book contains Stages 5.1 and 5.2 content. Stage 5.2 content is marked by * and coded in orange, while NSW-only content that is not Australian curriculum is marked with #.
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiencies covered in each section.

Chapter outline

		Working mathematically				
7.01	Direct proportion*	U	F	PS	R	C
7.02	Inverse proportion*#	U	F	PS	R	C
7.03	Conversion graphs*	U	F	PS	R	C
7.04	The parabola $y = ax^2 + c$	U	F		R	C
7.05	The exponential curve	U	F		R	C
7.06	The circle	U	F		R	C
7.07	Identifying graphs*		F		R	C

*STAGE 5.2

U = UNDERSTANDING

Understanding is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.

F = FLUENCY

Fluency is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.

PS = PROBLEM SOLVING

Problem solving is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.

R = REASONING

Reasoning is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

C = COMMUNICATING

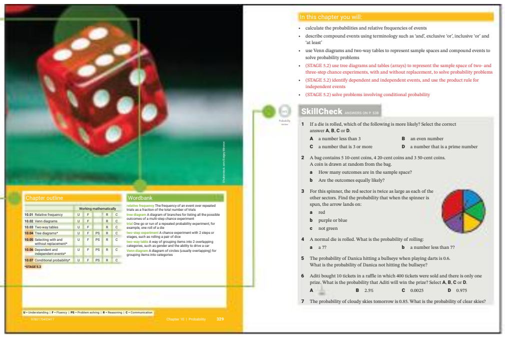
Communicating is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

- **Understanding** and **Fluency** can be found in every exercise and activity, while **Problem solving**, **Reasoning** and **Communicating** are found in the **Investigations**, **Technology**, **Mental skills**, **Language of Maths** and **Topic overview** activities, and explicitly labelled in every exercise. (see 'In each chapter' next page)

At the beginning of each chapter

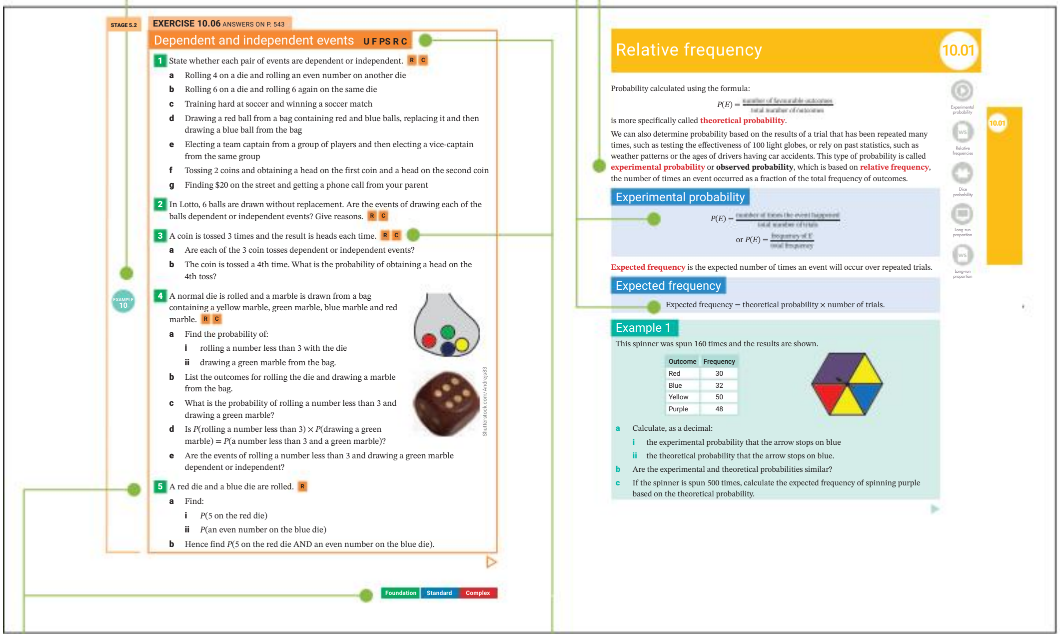
A **Chapter outline** and **Wordbank** chapter glossary (a full glossary at the back of the book).

SkillCheck reviews prerequisite skills and knowledge for the chapter.

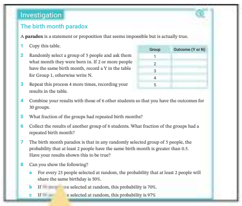


In each chapter

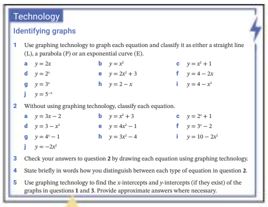
- Important facts and formulas are highlighted in a shaded box.
- Glossary terms are printed in **red**.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.



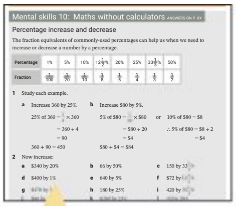
- The exercise questions are colour-coded for level of difficulty: **Foundation** **Standard** **Complex**
- Specific questions are also labelled **PS** for **Problem solving**, **R** for **Reasoning** and **C** for **Communicating**.



Investigations explore the syllabus in more detail, through group work, discovery and modelling activities.



Technology includes spreadsheets, online calculators, dynamic geometry software and the Internet.



Mental skills reinforce mental calculation strategies ('calculator-free maths')

Did you know? contains interesting facts and applications of the mathematics learned in the chapter.

At the end of each chapter

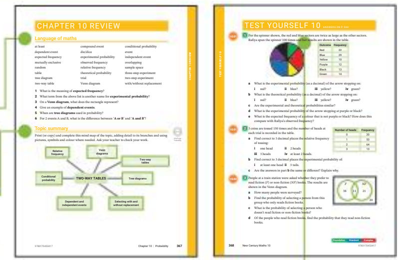
Power plus is an extension/challenge exercise.

Language of maths has a chapter word list and literacy questions.

Topic summary has a mind map activity with downloadable solutions.

Test Yourself contains chapter revision linked to the relevant exercise set.

Practice sets after every 3-4 chapters.

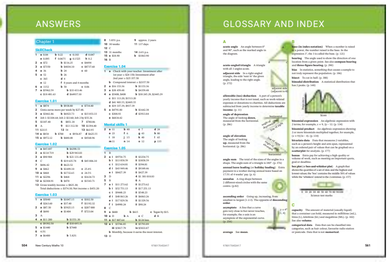


At the end of the book

General practice exercise

Answers (worked solutions are on the teacher website).

Glossary and index



NelsonNet student website

Visit the *NelsonNet* student website, **www.nelsonnet.com.au** to access 300+ worksheets, 100+ videos and more:

Worksheets and **puzzle sheets** (Worksheet, Puzzle sheet)	**Skillsheets** of examples and exercises (Skillsheet)

Video tutorials: worked examples explained by 'flipped classroom' teachers (Video tutorial)	**Technology worksheets** (Technology)	**Chapter quizzes:** interactive and self-marking (Chapter quiz)

NelsonNet teacher website

Teachers can access the *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, for:

- a **teaching program**
- **topic tests**
- **worked solutions** to every question
- **quiz generator and questionbank (Cognero)**
- **Chapter PDFs**

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

NelsonNetBook

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students

New Century Maths 7–10 workbooks

Each *New Century Maths* title has a companion workbook for students to write in, sold separately, containing 200 pages of worksheets, puzzle sheets, StartUp topic assignments and weekly homework assignments. Handy for homework, class assessment, practice, revision, relief classes or 'catch-up' lessons.

New Century Maths / Maths in Focus 7–12 series

MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study something in detail the parts of a situation

bisect: cut in half

calculate: *see* **evaluate**

classify, identify: state the type, category or feature of an item or situation

comment: express an observation or opinion about a result

complete: fill in detail to make a statement, diagram or table correct or finished

compare: show how 2 or more things are similar or different

construct: draw an accurate diagram

convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately.

evaluate, calculate: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression by, for example, expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

factorise: take out the highest common facor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand.**

give reasons: show the rules or thinking used when solving a problem. See also **justify.**

graph: display on a number line, number plane or statistical graph.

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: *see* **classify.**

interpret: find meaning in an answer or result

justify: give reasons or evidence to support your argument or conclusion. See also **give reasons.**

measure: use an instrument to find the size of something, for example, use a ruler to determine the length of a pen.

prove that: *see* **show that**

rationalise the denominator: simplify a fraction involving a surd by making its denominator rational (that is, not a surd).

recall: remember and state.

reduce (a fraction) to its lowest terms: see **simplify (a fraction)**.

round (a number): find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, \$12.9598 rounded to the nearest cent is \$12.96, 0.166 66 rounded to 3 decimal places is 0.167.

show that, prove: that (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

show working: show the steps you used to find the answer

simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

simplify (a fraction): reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$.

simplify (a ratio or rate): reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.

sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**

solve: find the value(s) of an unknown variable in an equation or inequality

state: *see* **write.**

substitute: replace a variable by a number and evaluate

verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write correct to: See **round (a number).**

write, state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

SYMBOLS AND ABBREVIATIONS

$=$	is equal to	$\sqrt{\ }$	square root, radical sign
$\neq$	is not equal to	$\sqrt[3]{\ }$	cube root
$\approx$	is approximately equal to	$P(E)$	the probability of event E occurring
$<$	is less than	$P(\bar{E})$	the probability of event E not occurring
$>$	is greater than	$P(B\|A)$	the probability of B given A
$\leq$	is less than or equal to	LHS	left-hand side
$\geq$	is greater than or equal to	RHS	right-hand side
( )	parentheses, round brackets	%	percentage
[ ]	(square) brackets	p.a.	per annum (per year)
{ }	braces	sin	sine ratio
$\pm$	plus or minus	cos	cosine ratio
-3	negative 3	tan	tangent ratio
π	pi = 3.14159	$\bar{x}$	the mean (average)
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152 ...	σ	the standard deviation
$^\circ$	degree	Σ	the sum of
42°17′54″	42 degrees, 17 minutes, 54 seconds	Q_1	1st quartile or lower quartile
$\angle A$	angle A	Q_2	median (2nd quartile)
$\triangle ABC$	triangle ABC	Q_3	3rd quartile or upper quartile
$\parallel$	is parallel to	IQR	interquartile range
$\perp$	is perpendicular to	α	alpha
$\equiv$	is congruent to	θ	theta
$\vert\vert\vert$	is similar to	ϕ	phi
$\therefore$	therefore	μ	micro-, mu
x^2	x squared, $x \times x$	m	gradient
x^3	x cubed, $x \times x \times x$		

NUMBER AND ALGEBRA

INTEREST AND DEPRECIATION

The value of an investment increases over time as a result of interest being added to it, whether it be simple or compound interest. On the other hand, the value of assets and items such as cars and office equipment decreases over time due to age and wear-and-tear. Compound interest and depreciation use formulas that involve repeated percentage increase and decrease respectively.

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Chapter outline

		Working mathematically				
1.01	Earning an income#	U	F	PS	R	
1.02	Income tax#	U	F	PS		C
1.03	Simple interest	U	F	PS	R	
1.04	Compound interest	U	F			
1.05	The compound interest formula*	U	F	PS	R	
1.06	Term payments#	U	F	PS	R	C
1.07	Depreciation*#	U	F	PS	R	C

***STAGE 5.2**

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as work-related expenses and donations to charities

compound interest Interest calculated on the principal invested as well as on any accumulated interest

depreciation The decrease in the value of items over time due to ageing

net pay Pay received after deductions from gross pay; 'take-home' pay

per annum (p.a.) Per year

principal The original amount of money invested or borrowed, for the purpose of calculating interest

repayment or **instalment** The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan

simple interest Interest calculated on the original principal invested only

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate weekly, fortnightly, monthly and yearly incomes
- calculate wages, salaries, overtime, commission, piecework and annual leave loading
- use tables to calculate income tax and PAYG tax
- solve problems involving simple interest
- solve problems involving compound interest by repeated percentage increase
- (STAGE 5.2) solve problems involving the compound interest formula $A = P(1 + r)^n$
- solve problems involving term payments
- (STAGE 5.2) solve problems involving depreciation

SkillCheck ANSWERS ON P. 504

Mental percentages

Percentage calculations

Percentage shortcuts

1 Convert each percentage to a decimal.

a 4% **b** 22% **c** 18.3% **d** 4.7%

e $9\frac{1}{2}\%$ **f** 6.75% **g** $15\frac{1}{4}\%$ **h** 20%

2 Find:

a 6% of $1200 **b** 2.5% of $4650 **c** 12% of $37 450

3 Increase:

a $7000 by 5% **b** $3955 by 2% **c** $8600 by 1.6%

4 How many months are there in:

a 3 years? **b** 2 years? **c** 5 years?

5 Copy and complete:

a One year = __________ weeks

b One year = __________ fortnights

c One year = __________ days

d 48 months = __________ years

e 84 days = _________ weeks

f 100 months = _________ years _________ months

6 If $P = mvt$, find:

a P when $m = 1600$, $v = 0.072$, $t = 10$

b m when $P = 120$, $v = 0.3$, $t = 8$

c v when $P = 18$, $m = 60$, $t = 5$

7 Evaluate, correct to the nearest cent:

a $\$5000 \times (1.045)^4$

b $\$28\,000 \times (1.03)^6$

c $\$15\,300 \times (1.065)^3$

d $\$32\,400 \times (1.072)^{10}$

Earning an income#

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1.01

Wages, salaries and overtime

A **wage** is calculated from the number of hours worked and is usually paid weekly. Wage earners can make more income by working extra hours (overtime).

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. Salary earners do not earn overtime pay but can receive benefits such as a computer, company car, expense account, shares in the company or paid medical expenses.

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Overtime

WS Earning money

1.01

Units of time for wages and salaries

1 year = 12 months

1 fortnight = 2 weeks

1 year = 52 weeks for wage earners

1 year = 52.18 weeks for salary earners

The 2 most common rates of **overtime** pay are:

- **time-and-a-half** = 1.5 × normal hourly rate
- **double time** = 2 × normal hourly rate

Example 1

Tyrone earns a salary of $70 400 p.a.
How much does he earn:

p.a. = per annum = per year

a each week? **b** each fortnight? **c** each month?

Solution

a Weekly income = $70 400 ÷ 52.18
= $1349.1759...
≈ $1349.18 — Rounded to the nearest cent.

b Fortnightly income = 2 × $1349.18 — 1 fortnight = 2 weeks
= $2698.36

c Monthly income = $70 400 ÷ 12 — 1 year = 12 months
= $5866.6666...
= $5866.67 — Rounded to the nearest cent.

Example 2

Noor earns \$22.65 per hour at normal rates. Last week, she worked 38 hours at normal rates, 6 hours at time-and-a-half and 3 hours at double time. Calculate Noor's total earnings for the week.

Solution

Normal pay = \$22.65 × 38
= \$860.70

Time-and-a-half pay = 6 × \$22.65 × 1.5 (6 hours)
= \$203.85

Double time pay = 3 × \$22.65 × 2 (3 hours)
= \$135.90

Total earnings = \$860.70 + \$203.85 + \$135.90
= \$1200.45

Commission, piecework and annual leave loading

Commission is earned by salespeople and agents, and is a percentage of the value of items sold or income made.

Piecework is earned according to the number of items made or tasks completed.

Annual leave loading or **holiday loading** is extra pay given during annual leave (holidays) and is 17.5% of 4 weeks' normal pay.

iStock.com/Allard1

Example 3

Georgia is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of \$750. How much will Georgia earn if she sells an apartment for \$590 000?

A **retainer** is a fixed amount paid regardless of how many items are sold.

Solution

Commission = 2.5% of \$590 000
= \$14 750

Total earnings = commission + retainer
= \$14 750 + \$750
= \$15 500

∴ Georgia earns \$15 500.

Example 4

Emad is a jewellery designer. He makes handmade jewellery and is paid at the following rates:

- $278 per necklace
- $72 per pair of earrings
- $105 per bracelet

This month, Emad made 23 necklaces, 7 pairs of earrings and 19 bracelets. How much did he earn?

Solution

Monthly earnings = 23 × $278 + 7 × $72 + 19 × $105
= $8893

Shutterstock.com/Anirut Thailand

Example 5

Kham's annual salary is $70 590. For his Christmas holidays, he received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. Calculate Kham's:

a normal weekly pay

b annual leave loading

c total pay for the Christmas holiday.

Solution

a Weekly pay = $70 590 ÷ 52.18
= $1352.8171…
≈ $1352.82

b Annual leave loading = 17.5% × $1352.82 × 4 — 17.5% of 4 weeks' pay
= $946.974
≈ $946.97

c Total holiday pay = (4 × $1352.82) + $946.97 — 4 weeks' pay + leave loading
= $6358.25

1.01

EXERCISE 1.01 ANSWERS ON P. 504

Earning an income **U F PS R**

Express all answers correct to the nearest cent where necessary.

1 Find the weekly wage for each person.

- **a** Mary earns $21.85 per hour and works for 40 hours.
- **b** Connor works 8 hours a day, Monday to Friday, and is paid $23.47 per hour.
- **c** Yoshe works on Monday and Tuesday from 8:30 a.m. until 4:00 p.m. and Thursday from midday until 9:00 p.m., and earns $30.60 per hour.

2 Greta earns $19.56 an hour and works for 31 hours each week. Chandler earns $21.44 per hour for his 27 hours of work. Who earns more per week and by how much?

EXAMPLE 1

3 Maggie earns a salary of $180 640 p.a. How much does she earn:

a each week? **b** each fortnight? **c** each month?

4 Rakitu considers 2 jobs, one locally with an annual salary of $57 640 p.a. and the other one in the city with a fortnightly pay of $2320. Calculate the weekly income for each job, determine which one pays more per week, and by how much. **R**

EXAMPLE 2

5 Anan works 38 hours at normal rates, 7 hours at time-and-a-half and 4 hours at double time. Calculate Anan's total earnings if he earns $19.60 per hour at normal rates.

6 Jacqui works 8.5 hours per day from Tuesday to Friday. She is paid $21.78 per hour. She also works on Saturday for 4.5 hours at a special rate of $24.59 per hour. How much did Jacqui earn for the week?

7 Idra works the following hours in a week at the clothing chain *Shop til U Drop*.

Day	Hours worked
Monday	9 a.m. – 5 p.m.
Tuesday	9 a.m. – 4 p.m.
Thursday	10 a.m. – 7:30 p.m.
Friday	10 a.m. – 5 p.m.
Saturday	10:30 a.m. – 5 p.m.

She is paid at the following rates.

Day	Rate of pay
Monday to Friday	$19.62 per hour
Saturday Thursday after 4:00 p.m.	$23.15 per hour

What is Idra's total income for the week? **PS** **R**

EXAMPLE 3

8 Fatimah is paid a commission of 2.5% on the value of goods she sells. She also receives a weekly retainer of $875. How much will Fatimah earn if she sells goods to the value of $41 600 in one week? Select the correct answer **A**, **B**, **C** or **D**.

A $1915 **B** $1061.88 **C** $2187.50 **D** $1018.13

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1.01

9 Nathan is a real estate agent whose commission is calculated on the value of the properties he sells:

- 3% paid on first $300 000
- 1.5% paid on next $250 000
- 0.75% paid on any value thereafter

How much commission did Nathan earn for selling a house for $625 000?

10 Briana designed an app, *KeyFinder*, that sells for $2.49. If she makes 70% profit on the sale price of each app sold, how much would she make from selling 800 units of this app?

EXAMPLE 4

11 Matt charges $60 for each lawn he mows and $45 for trimming hedges in each yard. In a week, he mows 24 lawns and trims 15 hedges. How much does he earn for the week?

Shutterstock.com/welcomia

12 *Clean* 2 *Swim* charges $86 to clean backyard pools. If this business earned $4644 in the first week of summer, how many pools were cleaned?

13 Jade makes homemade eco-friendly soaps, shampoos and cleaning products. A customer purchases 3 homemade soaps, 2 bottles of shampoo and 3 of the cleaning sprays. How much does Jade receive for these purchases?

Shutterstock.com/Volosina

Homemade soaps $5.60

Shutterstock.com/AlenKadr

Eco-friendly shampoo $12.70

Shutterstock.com/Lotus_studio

Natural cleaning spray $7.25

14 Calculate the annual leave loading for each person if it is 17.5% of 4 weeks' pay.

a Peter earns $1220 per week

b Jamilla earns $2000 per fortnight

c Samir earns $5944 per month

d Ellie earns $46 630 p.a.

EXAMPLE 5

15 For his annual holidays, Jake received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. If Jake's annual salary is $50 725, find his:

a normal weekly pay

b annual leave loading

c total pay for the 4-week holiday.

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Investigation

Workers' entitlements

The Australian Government sets the minimum standards for pay and conditions for all Australian workers. Different industries can have different needs from employees in terms of:

- normal and overtime hours worked, breaks allowed
- allowances
- dress codes, such as uniforms
- working conditions

1 Visit the Fair Work Ombudsman website **www.fairwork.gov.au** and select **Awards and agreements**.

2 Select 2 industries and identify any similarities and differences in the requirements of those industries.

3 Write a summary of your findings.

4 Give a report in class.

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Income tax#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

1.02

Not all of a person's income is taxed. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable deductions** (or tax deductions) and are not taxed. Examples of allowable deductions are tools of trade, uniforms, car/travel expenses, subscriptions to professional organisations and journals.

WS Income tax tables

WS Income tax and Medicare levy

Income tax

Income tax

Income tax is calculated on a person's **taxable income**, which is the gross income (total earnings) less all allowable deductions, rounded down to the nearest dollar.

Taxable income = gross income − allowable deductions

The more a person earns, the higher the rate of tax to be paid.

Tax rates for Australian residents	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $90 000	$3572 plus 32.5c for each $1 over $37 000
$90 001 – $180 000	$20 797 plus 37c for each $1 over $90 000
$180 001 and over	$54 097 plus 45c for each $1 over $180 000

Source: © Australian Taxation Office for the Commonwealth of Australia

Example 6

Sophia earned $62 348 last financial year and collected bank interest of $440.81. She had allowable deductions of $427.52 in work expenses and $110 in donations to charities.

a Calculate her taxable income.

b Use the tax table to calculate the income tax that Sophie must pay.

Solution

a Taxable income = $62348 + $440.81 – $427.52 – $110

= $62 251.29

≈ $62 251

Rounded down to the nearest dollar.

b According to the table, a taxable income of $62 251 is in the $37 001 − $90 000 tax bracket.

Income tax = $3572 + 0.325 × ($62 251 – $37 000)

= $11 778.575

≈ $11 778.58

'32.5c for each $1' means 32.5% or 0.325

PAYG tax and net pay

Income tax deducted from your pay by your employer every payday is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a tax refund.

Gross pay is the total amount a person earns or receives, but most workers have a variety of deductions taken from their pay before they receive it, including PAYG tax, superannuation contributions, union fees and health fund payments. The amount of income left after the deductions is called **net pay**.

Net pay

Net pay = gross pay − tax − other deductions

Example 7

Jayden earns a gross pay of \$2290.33 per fortnight. His deductions are for PAYG tax, \$44.10 for private health insurance and \$55.82 for superannuation.

Fortnightly earnings (\$)	PAYG tax withheld (\$)
2270–2275	460
2276–2281	462
2282–2287	464
2288–2293	466
2294–2299	468
2300–2305	470

a Use the PAYG tax table to find Jayden's PAYG tax per fortnight.

b Calculate Jayden's net pay.

c Calculate Jayden's total deductions as a percentage of his gross income (correct to one decimal place).

Solution

a In the table, \$2290.33 falls in the \$2288 – \$2293 range.

Fortnightly PAYG tax = \$466

b Net pay = \$2290.33 – (\$466 + \$44.10 + \$55.82)

= \$2290.33 – \$565.92

= \$1724.41

Net pay = gross pay − total deductions

c Total deductions = \$565.92

$$\text{Deductions percentage} = \frac{\$565.92}{\$2290.33} \times 100\%$$

$$= 24.7091\ldots\%$$

$$\approx 24.7\%$$

$$\frac{\text{Total deductions}}{\text{Gross pay}} \times 100\%$$

EXERCISE 1.02 ANSWERS ON P. 504

Income tax U F PS C

EXAMPLE 6

1.02

1 Shilpa earns $47 628 in a year and has allowable deductions of $1930.46.

a Calculate her taxable income, rounded down to the nearest dollar.

b Use the tax table on page 11 to calculate the income tax that Shilpa must pay.

2 Aiden is an environmental engineer who had a gross income of $118 742 this year and work-related expenses totalling $4022.80, which are tax-deductible. Calculate Aiden's:

a taxable income, rounded down to the nearest dollar

b income tax.

3 Ellie is a graphic designer who earns an annual salary of $90 541 and has collected $1029.45 in bank interest. She has allowable deductions of $379 for tools related to her work and $287 in donations to charity. Calculate: PS

a Ellie's taxable income

b the amount of tax payable.

4 Riley the builder had a gross income of $56 922 this year. He is entitled to these tax deductions: tools $1538, training courses $445 and outdoor protective clothing $506. How much should Riley pay in tax? Select the correct answer **A**, **B**, **C** or **D**. PS

A $13 046.65 **B** $10 855.58 **C** $9237.73 **D** $6884.27

5 Nicola is a nurse earning $87 996 per year. Her allowable deductions are the cost of non-slip footwear $225, the cost of laundering uniforms $1046, and union fees $297.60. How much should Nicola pay in tax? PS

6 Will owns a photography business and earned $196 000 last year. His allowable deductions were Internet costs for his website $968, photographic equipment $23 672, and travel to photo shoots $15 930. Calculate the amount that Will should pay in tax. PS

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7 Jackson earns a gross weekly income of $1075.26. His weekly deductions are $309.11 PAYG tax, $44.55 for private health insurance and $25.18 for superannuation. Calculate Jackson's net weekly pay. C

8 Isha earns a gross income of $788.20 per week. Her deductions are $132.44 tax and $32.24 for private health insurance. Calculate Isha's net income. C

Use the PAYG table from Example **7** on page 12 to answer questions **9** to **12**.

EXAMPLE 7

9 Every fortnight, Mr Singh earns $2278 and pays $22.80 in union fees and $94.10 in superannuation.

a Find how much PAYG tax he pays per fortnight.

b Calculate Mr Singh's fortnightly net pay.

c What percentage (correct to one decimal place) of his gross pay do the deductions make up?

10 Holly earns a gross pay of $2295 per fortnight. Her deductions are PAYG tax, $64.35 for superannuation and $30 for life insurance. Find Holly's:

a PAYG tax

b net pay

c total deductions as a percentage of her gross income (correct to one decimal place).

11 Stefan earns $1148 per week.

a If he is paid fortnightly, what is his fortnightly gross pay?

b Find the PAYG tax that is taken out of his gross pay.

c Stefan's deductions are $141.94 for his health fund and $51.33 for superannuation. Calculate Stefan's net fortnightly pay.

12 Agata earns a salary of $60 135 p.a. Each fortnight she has deductions of $256.20 for family health insurance and $35 for superannuation taken from her gross income.

a Calculate Agata's fortnightly gross income.

b How much PAYG tax does she pay per fortnight?

c Calculate Agata's fortnightly net income.

13 Copy and complete this pay slip. PS C

Employee: Ziad Chaker		Hourly pay rate: $19.65	
Hours worked		Deductions	
Normal	39	Tax: $205.72	Other: $168.38
Time-and-a-half	2	Gross weekly income	
Double time	0	Total deductions	
		Net weekly income	

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Technology

Online income tax calculators

The Australian Taxation Office (ATO) website **www.ato.gov.au** has online calculators for income tax and PAYG tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

1 Enter the taxable income $63 000 as '63000' (no spaces).

2 Select the current financial year.

3 Select 'Resident for full year' and click 'Next'.

4 The estimated tax payable will be shown on a new screen.

5 Repeat for at least 2 more taxable incomes.

6 Find the PAYG tax calculator and use it to find the PAYG tax payable and net pay for a gross pay of:

  a $1408 weekly

  b $2870 fortnightly

  c $5610 monthly

1.02

Simple interest 1.03

- When you invest money, you receive interest from your investment.
- When you borrow money, you pay interest on your loan.
- The original amount of money invested or borrowed is called the **principal**.
- This interest rate is a percentage of the principal, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or flat rate interest) is interest calculated simply on the original principal.

WS Simple interest

WS Simple interest table

Simple interest

The simple interest formula

$I = Prn$, where:

I is the simple interest

P is the principal

r is the interest rate per time period, expressed as a decimal

n is the number of time periods

Example 8

Find the simple interest on:

a $4000 at 3.5% p.a. for 6 years

b $13 500 at 5.5% p.a. for 7 months

c $75 640 at 0.42% per month for 2 years

Solution

a $P = \$4000$, $r = 3.5\% = 0.035$, $n = 6$ years

$I = Prn$
$= \$4000 \times 0.035 \times 6$
$= \$840$

$r = 0.035$ per year, $n = 6$ years, so the time period is **years**.

b $P = \$13\,500$, $r = 5.5\% = 0.055$, $n = \frac{7}{12}$ years

$I = Prn$
$= \$13\,500 \times 0.055 \times \frac{7}{12}$
$= \$433.125$
$\approx \$433.13$

$r = 0.055$ per year, $n = 7$ *months*, so we must change 7 months to years so that the time period is **years**.

rounded to the nearest cent

c $P = \$75\,640$, $r = 0.42\% = 0.0042$,
$n = 2 \times 12 = 24$ months

$I = Prn$
$= \$75\,640 \times 0.0042 \times 24$
$= \$7624.512$
$\approx \$7624.51$

$r = 0.0042$ per month, $n = 2$ *years*, so we must change 2 years to months so that the time period is **months**.

rounded to the nearest cent

Example 9

Petra invests $17 400 for 2 years at 3.75% p.a. flat rate interest. To what final value will her investment grow?

Solution

$P = \$17\,400$, $r = 3.75\% = 0.0375$, $n = 2$ years

n and r are in **years**.

$I = Prn$
$= \$17\,400 \times 0.0375 \times 2$
$= \$1305$

Value of investment $= \$17\,400 + \$1305$
$= \$18\,705$

Principal + interest

Example 10

After 4 years, an investment of $13 000 has earned $1092 in simple interest.

What is the annual interest rate?

Solution

$I = \$1092$, $P = \$13\,000$, $n = 4$ years

$$I = Prn$$

$$\$1092 = \$13\,000 \times r \times 4$$

$$\$1092 = \$52\,000r$$

$$r = \frac{\$1092}{\$52000}$$

$$= 0.021$$

$$= 2.1\%$$

$\therefore$ Annual interest rate = 2.1%

Example 11

For how many months will $10 000 need to be invested to earn $250 in simple interest at 3.25% p.a.?

Solution

$I = \$250$, $P = \$10\,000$, $r = 3.25\% = 0.0325$

$$I = Prn$$

$$\$250 = \$10000 \times 0.0325 \times n$$

$$\$250 = \$325n$$

$$n = \frac{\$250}{\$325}$$

$= 0.7692\ldots$ years — *n* is in years, so convert to months

$= 0.7692\ldots \times 12$ months

$= 9.230\ldots$ months

≈ 10 months — rounded *up* to the nearest month

EXERCISE 1.03 ANSWERS ON P. 504

Simple interest **U F PS R**

In this exercise, round all money answers to the nearest cent.

1 Calculate the simple interest earned on each investment. EXAMPLE 8

a $35 000 for 4 years at 3.6% p.a.

b $26 850 at 1.95% p.a. for 2 years

c $8200 invested for 5 months at 3% p.a.

d $6590 invested for 16 months at 0.25% per month

e $5250 invested for 250 days at 1.04% p.a.

f $18 400 invested for 3 years at 0.18% per month

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2 Calculate the flat rate interest charged on each loan.

a \$1250 for 2 years at 3.5% p.a.

b \$18 900 for $5\frac{1}{2}$ years at 5.7% p.a.

c \$1.15 million at 4.5% p.a. for 48 months

d \$12 000 for 10 months at 0.575% per month

e \$9750 for 2.5 years at 0.48% per month

f \$24 720 for 136 days at 7.85% p.a.

3 Harry owed \$783.26 on his credit card. The credit card company charged him one month's simple interest at 21% p.a. How much interest was he charged? Select the correct answer **A**, **B**, **C** or **D**.

A \$13.71 **B** \$16.45 **C** \$25.38 **D** \$37.30

EXAMPLE 9

4 Find the final value of each investment using simple interest.

a \$10 000 invested for 3 years at 4% p.a.

b \$1500 invested for 18 months at 0.19% per month

c \$8500 invested for 3.5 years at 0.25% per month

d \$9250 invested for 50 months at 3.75% p.a.

5 Liong borrowed \$6000 to go on an overseas holiday, at 12% p.a. flat rate interest for 2 years. Calculate:

a the total interest

b the total amount Liong must repay.

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6 The interest on a loan of $2500 over 4 years is $450. Calculate the flat rate of interest p.a. **R**

EXAMPLE 10

7 Katy took out a loan for $22 000 over 5 years. If her total loan repayments amounted to $28 400, calculate: **PS** **R**

a the interest charged

b the flat rate of interest p.a., correct to 2 decimal places.

8 After 5 years, the interest on a loan of $8000 amounts to $2340. Calculate the annual simple interest rate.

9 For how many years will $4200 need to be invested to earn $200 interest, if the interest rate is 2.5% p.a.? **R**

EXAMPLE 11

10 How many weeks will it take for $50 000 to earn $750 in interest if the rate is 2.6% p.a.? **R**

11 How many days will it take for $20 000 to earn $300 in interest if the rate is 4% p.a.? **R**

12 An online bank offered the following investment to its customers.

- A rate of 2.35% p.a. simple interest for the first 4 months only
- Then the principal and interest reinvested at 0.45% p.a. simple interest

What will Dariya's investment of $3480 be worth after 7 months?
Select **A**, **B**, **C** or **D**. **PS** **R**

A $31.21 **B** $374.10 **C** $3511.21 **D** $3854.10

13 For how many months will $20 000 need to be invested to amount to $22 000, if interest is paid at the rate of 0.33% per month? **PS** **R**

14 What is the flat rate of interest (as a percentage p.a., correct to one decimal place) when $1650 earns $85 in interest over 2 years? **R**

15 Toula used a credit card to buy a netbook computer for $799 and some extra accessories for $246. She pays off this debt in 30 days. The credit card charges 22% p.a. simple interest. **PS** **R**

a Calculate the simple interest charged.

b How much will Toula pay after 30 days?

16 Bhashine earned $185 interest each year for 4 years on an investment account. At the end of the 4 years, she closed her account and withdrew $6000 in total. What was the annual flat rate of interest paid into Bhashine's account?
Select **A**, **B**, **C** or **D**. **PS** **R**

A 3.1% **B** 3.5% **C** 12.3% **D** 14.1%

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1.04 Compound interest

Most investments earn **compound interest** rather than simple interest. With compound interest, the interest earned is **added** to the principal so that next time, the interest is calculated on a larger principal. This means that more interest is earned, because we are also earning interest on the interest we have already earned. The word **compound** means 'combined'.

Example 12

A principal of \$23 000 is invested at 4% p.a. interest, compounded yearly for 2 years.

a What is the total value of the investment after 2 years?

b What is the amount of compound interest earned?

Solution

a The interest for each year is calculated separately.

After the first year:

$I = \$23\,000 \times 0.04$

$= \$920$

Investment $= \$23\,000 + \$920$ — Principal + interest

$= \$23\,920$

After the second year:

$I = \$23\,920 \times 0.04$

$= \$956.80$

Investment $= \$23\,920 + \$956.80$ — New principal + interest

$= \$24\,876.80$

b Compound interest earned = final investment – principal

$= \$24\,876.80 - \$23\,000$

$= \$1876.80$

Notice that compound interest involves repeated percentage increase. In the above example, to calculate compound interest on a principal of \$23 000 over 2 years at 4% p.a., we are actually increasing \$23 000 by 4% twice. Adding 4% to the principal is the same as increasing the principal by 4%, which is the same as multiplying the principal by 104% or 1.04.

Investment after 1st year $= \$23\,000 \times 1.04 = \$23\,920$

Investment after 2nd year $= \$23\,920 \times 1.04 = \$24\,876.80$

We can even combine these 2 steps into one step by repeated percentage increases:

Investment after 2nd year $= \$23\,000 \times 1.04 \times 1.04 = \$24\,876.80$

Using repeated percentage increases can simplify our compound interest calculations.

Example 13

A principal of \$9000 is invested at 3.7% p.a. compounded yearly over 3 years. What is:

a the value of the investment after 3 years?

b the compound interest earned?

Solution

a Adding 3.7% interest to the principal is the same as multiplying the principal by 1.037.

$\therefore$ Investment after 3 years $= \$9000 \times 1.037 \times 1.037 \times 1.037$

$= \$9000 \times (1.037)^3$

$= \$10\,036.4188...$

$\approx \$10\,036.42$ rounded to the nearest cent.

b Compound interest earned = final investment – original principal

$= \$10\,036.42 - \$9000$

$= \$1036.42$

1.04

EXERCISE 1.04 ANSWERS ON P. 504

Compound interest U F

In this exercise, round all money answers to the nearest cent.

EXAMPLE 12

1 A principal of \$23 000 is invested at 5% p.a. interest, compounded yearly over 2 years.

a Copy and complete the following working to calculate the value of the investment after 2 years.

After the first year:

$I = \$23\,000 \times 0.05$ Investment = \$23 000 + \$_________

= \$_________ = \$_________

After the second year:

$I =$ \$_________ $\times 0.05$ Investment = \$_______ + _______

= \$_________ = \$_________

b Copy and complete the following working to calculate the amount of compound interest earned.

Compound interest earned = final investment – principal

= \$_________ – \$23 000

= \$_________

2 Finn invests \$15 000 at 2.5% p.a. compounded yearly over 3 years. Show all working (as in question **1**) to find:

a the value of the investment after 3 years

b the total amount of compound interest earned.

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3 Selina invests \$34 100 at 6.2% p.a. interest compounded yearly over 2 years. Calculate:

a the final value of the investment **b** the compound interest earned.

EXAMPLE 13

4 Use repeated percentage increases to calculate the final value of each investment compounded annually, then calculate the compound interest earned.

a \$5000 for 2 years at 4% p.a.
b \$27 800 for 3 years at 2.85% p.a.
c \$9600 for 3 years at 5% p.a.
d \$39 500 for 2 years at 3% p.a.
e \$18 400 for 4 years at 1.25% p.a.

5 For each investment, calculate the compound interest earned.

a \$30 400 at 5% p.a. interest for 3 years.
b \$19 150 at 4.2% p.a. interest for 2 years.
c \$8750 at 1.75% p.a. interest for 2 years.
d \$36 000 at 3.5% p.a. interest for 3 years.
e \$18 960 at 6.35% p.a. interest for 5 years.

Mental skills 1: Maths without calculators ANSWERS ON P. 504

Percentage of a quantity

Learn these commonly-used percentages and their fraction equivalents.

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$.
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

Now we will use them to find a percentage of a quantity.

1 Study each example.

a $20\% \times 25 = \frac{1}{5} \times 25$
$= 5$

b $50\% \times 120 = \frac{1}{2} \times 120$
$= 60$

c $12.5\% \times 32 = \frac{1}{8} \times 32$
$= 4$

d $75\% \times 56 = \frac{3}{4} \times 60$
$= \left(\frac{1}{4} \times 60\right) \times 3$
$= 15 \times 3$
$= 45$

e $33\frac{1}{3}\% \times 27 = \frac{1}{3} \times 27$
$= 9$

f $66\frac{2}{3}\% \times 60 = \frac{2}{3} \times 60$
$= \left(\frac{1}{3} \times 60\right) \times 2$
$= 20 \times 2$
$= 40$

2 Now simplify each expression.

a $25\% \times 44$ **b** $33\frac{1}{3}\% \times 120$ **c** $20\% \times 35$ **d** $66\frac{2}{3}\% \times 36$

e $10\% \times 230$ **f** $12\frac{1}{2}\% \times 48$ **g** $50\% \times 86$ **h** $20\% \times 400$

i $75\% \times 24$ **j** $33\frac{1}{3}\% \times 45$ **k** $25\% \times 160$ **l** $10\% \times 650$

m $12.5\% \times 88$ **n** $66\frac{2}{3}\% \times 21$ **o** $20\% \times 60$ **p** $75\% \times 180$

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The compound interest formula 1.05

There is a formula for calculating the final amount of an investment earning compound interest. Note the following pattern.

- Final amount of \$23 000 at 4% p.a. interest for 2 years = \$23 000 × $(1.04)^2$
- Final amount of \$9000 at 3.7% p.a. interest for 3 years = \$9000 × $(1.037)^3$
- Final amount of \$18 960 at 6.35% p.a. interest for 5 years = \$18 960 × $(1.0635)^5$

STAGE 5.2

Compound interest with annual rests

Compound interest with non-annual rests

Comparing interest rates

Simple and compound interest calculator

The compound interest formula

Simple and compound interest

Interesting facts

Compound interest formula

$A = P(1 + r)^n$, where:

A is the total (final) amount of the investment

P is the principal

r is the interest rate per compounding period, expressed as a decimal

n is the number of compounding periods

The compound interest is then calculated using this formula:

$$\text{Compound interest} = \text{total amount} - \text{principal}$$
$$I = A - P$$

Example 14

For each investment, calculate:

i the total amount of the investment

ii the compound interest earned if interest is compounded annually

a \$26 750 is invested at 4% p.a. for 3 years

b \$52 000 is invested at 3.8% p.a. for 5 years

Solution

a **i** $P = \$26\,750$, $r = 4\% = 0.04$, $n = 3$

$$\begin{aligned} A &= P(1 + r)^n \\ &= \$26\,750(1 + 0.04)^3 \\ &= \$26\,750(1.04)^3 \\ &= \$30\,090.112\ldots \\ &\approx \$30\,090.11 \end{aligned}$$

The total amount of the investment is \$30.090.11.

ii Compound interest = \$30 090.11 – \$26750 $\quad I = A - P$

= \$3340.11

STAGE 5.2

b **i** $P = \$52\ 000$, $r = 3.8\% = 0.038$, $n = 5$

$A = P(1 + r)^n$

$= \$52\ 000(1 + 0.038)^5$

$= \$52\ 000(1.038)^5$

$= \$62\ 659.9597...$

$\approx \$62\ 659.96$

ii Compound interest $= \$62\ 659.96 - \$52\ 000$

$= \$10\ 659.96$

Compound interest

Example 15

Calculate the compound interest when \$24 500 is invested at 6.3% p.a. for 5 years:

a compounded annually

b compounded monthly.

Solution

a $P = \$24\ 500$, $r = 0.063$, $n = 5$

$A = \$24\ 500\ (1 + 0.063)^5$

$= \$24\ 500\ (1.063)^5$

$= \$33\ 253.1205...$

$\approx \$33\ 253.12$

$I = \$33\ 253.12 - \$24\ 500$

$= \$8753.12$

b Because interest is compounded monthly, r and n must be expressed in months, not years.

$P = \$24\ 500$, $r = \frac{0.063}{12} = 0.00525$ per month, $n = 5 \times 12 = 60$ months

$A = \$24\ 500(1 + 0.005\ 25)^{60}$

$= \$24\ 500(1.005\ 25)^{60}$

$= \$33\ 543.701\ 98$

$\approx \$33\ 543.70$

$I = \$33\ 543.70 - \$24\ 500$

$= \$9043.70$

Note: More interest is earned when it is compounded monthly rather than yearly. Why do you think this is so?

EXERCISE 1.05 ANSWERS ON P. 504

The compound interest formula U F PS R

EXAMPLE 14

1.05

In this exercise, round all money answers to the nearest cent.

1 An amount of \$13 000 is invested at 5% p.a. interest, compounded annually over 2 years. Which expression represents the total value of the investment? Select the correct answer **A**, **B**, **C** or **D**.

A $13\,000 \times 0.05 \times 2$ **B** $13\,000\,(1 + 0.05)^2$

C $13\,000 \times (0.05)^2$ **D** $13\,000\,(1 - 0.05)^2$

2 For each investment, where interest is compounded yearly, calculate:

i the total amount of the investment, A **ii** the compound interest, I, earned.

a \$6500 invested at 7% p.a. for 6 years **b** \$10 000 invested at 8.5% p.a. for 4 years

c \$12 240 invested at 1.6% p.a. for 2 years **d** \$34 600 invested at 4.9% p.a. for 5 years

e \$8000 invested at 1.75% p.a. for 3 years

3 Calculate the amount of interest earned on an investment of \$6500 if it is invested at 2.5% p.a. compounded annually for 8 years. Select **A**, **B**, **C** or **D**.

A \$131.14 **B** \$832.81 **C** \$1300 **D** \$1419.62

4 Find the amount of interest earned on one million dollars invested at 14.9% p.a. compounded annually for 6 years.

5 Find the amount of interest charged on a loan of \$25 000 if it is borrowed over 10 years at 8% p.a. compounded annually. Select **A**, **B**, **C** or **D**.

A \$31 250 **B** \$28 973.12 **C** \$28 589.72 **D** \$20 000

6 Yasmin was given \$2000 when she turned 3 years old. Her parents invested it at a 2% p.a. compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Yasmin had in the account when she turned 16? Select **A**, **B**, **C** or **D**. **R**

A $2000\,(1 + 0.02)^{13}$ **B** $2000\,(1 + 0.2)^{13}$

C $2000\,(1 + 0.02)^{16}$ **D** $2000\,(1 + 0.2)^{16}$

EXAMPLE 15

7 For each investment, calculate:

i the total amount **ii** the interest earned.

a \$10 000 for 5 years at 2.4% p.a., compounded monthly

b \$35 500 for 10 years at 2% per half-year, compounded half-yearly

Half-yearly means 'twice a year' or 'every 6 months'

c \$8900 for 2 years at 3% p.a., compounded quarterly

Quarterly means '4 times per year' or 'every 3 months'

d \$42 000 for 5 years at 0.225% per month, compounded monthly

e \$16 500 for 3 years at 2.6% p.a., compounded half-yearly

f \$4900 for 1 year at 0.005% per day, compounded daily

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STAGE 5.2

8 Find the total value of an investment of $4300 over 5 years at 4.6% p.a. interest, compounded every 6 months. Select the correct answer **A**, **B**, **C** or **D**.

A $4817.78 **B** $5384.27 **C** $5397.90 **D** $8506.24

9 **a** Reese invested $6000 for 2 years at a flat rate of 5% p.a. Calculate the amount of interest earned.

b Tegan invested $6000 for 2 years at an interest rate of 5% p.a. compounded annually. Calculate the amount of interest earned.

c Whose investment earned more interest? How much more?

10 A principal of $5000 is invested at 3.5% p.a. for 3 years. Match each compounding period for this investment to its correct expression for the final value of the expression.

a compounded yearly

b compounded half-yearly

c compounded quarterly

d compounded monthly

A $5000\left(1+\frac{0.035}{2}\right)^{3\times 2}$

B $5000\left(1+\frac{0.035}{12}\right)^{3\times 12}$

C $5000\left(1+\frac{0.035}{4}\right)^{3\times 4}$

D $5000\,(1 + 0.035)^3$

11 Lisa is setting up a trust account for her new grandson Stefan. In 18 years time, she wants the investment to be worth $30 000, to help with the cost of university fees or the purchase of a car. Suppose the interest rate for the account is 2.94% p.a. compounding yearly. PS R

a How much should Lisa invest now to achieve the $30 000 target?

b If Lisa opened a trust account that earns 2.94% p.a. compounding monthly instead, how much less would she need to invest?

12 Zoe is 5 years old and about to start school. Her parents want to invest $25 000, for her high school education expenses, in an account that earns 3% p.a. over 7 years. R

a Calculate the total interest earned if interest is compounded:

i yearly **ii** half-yearly **iii** quarterly **iv** monthly

b Which compounding period should Zoe's parents choose? Why?

13 A principal of $10 000 is invested for 4 years, earning interest at the rate of 3% p.a., compounded monthly. Which expression represents the total value of the investment? Select **A**, **B**, **C** or **D**.

A $10\,000\left(1+\frac{3}{100}\right)^{4}$ **B** $10\,000\left(1+\frac{3}{100}\right)^{48}$

C $10\,000\times\left(1+\frac{3}{1200}\right)^{4}$ **D** $10\,000\left(1+\frac{3}{1200}\right)^{48}$

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STAGE 5.2

Spreadsheets

1.05

Technology

Comparing simple with compound interest

In this activity, you will compare the interest earned on an investment of $1000 for 10 years at 8% p.a. simple interest and 8% p.a. compound interest, compounded annually.

1 Create this spreadsheet. The principal (*P*) is entered in cell A1 and the annual interest rates (in decimal form) in cells B1 and C1.

	A	B	C
1	$ 1,000.00	0.08	0.08
2			
3	Years	Simple Interest	Compound Interest
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		

2 To calculate the simple interest in column B, in cell B4 enter the formula **=$A$1*$B$1*A4**. Now **Fill Down** from cell B4 to B13.

3 To calculate the compound interest in column C, in cell C4 enter the formula **=$A$1*(1+$C$1)^ A4-$A$1**. Now **Fill Down** from cell C4 to C13.

4 Highlight cells A3 to C13. **Insert 'Scatter with Smooth lines and markers'**.

5 When the interest rate is the same, which account pays better interest: simple or compound interest? (Type your answer in cell A15)

6 Now compare the interest earned on an investment of $1000 for 10 years at 9% p.a. simple interest and 7% p.a. compound interest, compounded annually. Change the interest rates in cells B1 (0.09) and C1 (0.07) respectively.

Answer the following questions in the spreadsheet cells indicated in brackets.

7 After how many years did the compound interest rate pay more than the simple interest rate? (A16)

8 How much extra interest did the compound interest rate pay at the end of the 10 years? (A17)

9 Change the interest rate in B1 to 10% (0.1) and C1 to 9% (0.09). How does the change in interest rate affect the amount of interest paid? Include calculations to justify your answer. (A18)

10 Change the interest rate in B1 to 12% (0.12) and C1 to 8.5% (0.085). After how many years did the amount of compound interest earned overtake the amount of simple interest earned? (A19)

11 What is the difference in the amount of compound interest earned for the 10-year period compared to the simple interest investment? Is it a significant amount? Justify your answer. (A20)

1.06 Term payments#

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Term paymentss

Many customers buy expensive household items **on terms**, which means 'paying off' the item by regular instalments over time, after paying a deposit. A **term payments** plan is also called **hire-purchase** because the purchaser actually hires the item until it is completely paid off. Special offers can include interest-free periods, but there may be other conditions such as establishment fees and extra charges if the regular **repayments** are not paid on time. Also, if the purchaser fails to keep up with the payments, higher interest may be charged or the item may be **repossessed** (taken back).

Example 16

Sonia purchases a new fridge and dishwasher package valued at \$4925. She pays a 10% deposit and repays the balance in monthly repayments over 3 years. Interest on the balance is charged at a flat rate of 12% p.a.

iStock.com/geremne

Find:

a the deposit paid

b the balance owing

c the interest charged on the balance

d the total to be repaid

e the amount of each monthly repayment

f the total price paid for the package.

Solution

a Deposit = 10% × \$4925
= \$492.50

b Balance owing = \$4925 – \$492.50 or 90% × \$4925
= \$4432.50

c Interest charged on the balance is flat or simple interest.

P = \$4432.50, $r = 0.12$, $n = 3$

$I = Prn$

= \$4432.50 × 0.12 × 3

= \$1595.70

d Total to be repaid = balance + interest
= \$4432.50 + \$1595.70
= \$6028.20

e Number of repayments = 3 × 12 months

= 36 months

Monthly repayment = $6028.20 ÷ 36

= $167.45

f Total price paid = deposit + total repayments or cash price + interest

= $492.50 + $6028.20 = $4925 + $1595.70

= $6520.70

1.06

Deferred payment plan

With a **deferred payment plan**, the customer does not make any repayments until a later date, such as after 6 months.

Deferred means 'delayed'.

Example 17

Ilhea and Robert purchase a $1600 home theatre system on a deferred payment plan over 2 years. They make no repayments for the first 3 months, then pay $105 per month.

Shutterstock.com/Viktorus

a Calculate the total repayments.

b How much interest did they pay?

c Ilhea and Robert were also charged the following fees for the plan.

- One-off establishment fee of $25
- Account service fee of $4.95 per month

i How much was paid in fees over the 2-year period?

ii What percentage (correct to one decimal place) of the purchase price was paid in fees?

Solution

a Total cost = $105 × 21   2 years − 3 months = 21 months

= $2205

b Interest = total cost – cash price

= $2205 – $1600

= $605

c **i** Total fees paid = $25 + $4.95 × 24

= $143.80

ii Percentage = $\frac{\$143.80}{\$1600} \times 100\%$

= 8.9875%

≈ 9.0%

EXERCISE 1.06 ANSWERS ON P. 505

ANSWERS ON P. 505

Term payments U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 16

1 Gaspard purchases a laundry package of washing machine and dryer valued at $1755. He pays a 10% deposit and repays the balance in 18 months. Interest on the balance is charged at a flat rate of 13.85% p.a. Find: PS R C

- **a** the deposit Gaspard paid
- **b** the balance owing
- **c** the interest charged on the balance
- **d** the total to be repaid
- **e** the amount of each monthly instalment
- **f** the total price paid for the package

2 Maree bought a backyard swimming pool for $25 500. She paid a 5% deposit and will repay the balance over 5 years, with flat-rate interest charged 8.6% p.a. Calculate: PS R C

- **a** the deposit
- **b** the balance owing
- **c** the interest charged
- **d** the total to be repaid
- **e** the monthly instalment
- **f** the total price paid.

3 Sarah wants to buy a new 3-piece lounge suite that she saw advertised at the lounge sale pictured, costing $6895. R

- **a** How much deposit will she have to pay?
- **b** What will be the flat-rate interest on the balance?
- **c** Find the size of each monthly repayment.

4 Peter pays a $1200 deposit on his holiday and then $185 per month for a year. The cash price of the holiday is $2700.

- **a** How much did Peter pay for his holiday?
- **b** Calculate the interest paid.
- **c** Calculate the balance owing after the deposit was paid.
- **d** Calculate, as a percentage correct to one decimal place, the flat interest rate charged p.a.

5 Mrs Mitchell buys a used car for her daughter by paying a 20% deposit and $275 per month for 4 years. If the car has a cash price of $10 400, find: R C

- **a** the deposit Mrs Mitchell paid
- **b** the balance owing after the deposit is paid
- **c** the total amount Mrs Mitchell repaid in instalments
- **d** the interest charged
- **e** the flat interest rate (p.a.), correct to one decimal place.

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6 Xuan wanted to buy a new smartphone costing $1044. R C

a How much deposit did she pay?

b What is the total amount repaid in repayments for the smartphone?

c Calculate the total interest paid for the year.

d What was the annual flat interest rate for this purchase, correct to 1 decimal place?

Alamy Stock Photo/STANCA SANDA

1.06

EXAMPLE 17

7 Diego bought a laptop priced at $800 on a deferred payment plan: nothing to pay for 3 months and then 9 monthly payments of $110. However, a monthly account fee of $6.95 was added to the plan. PS R

a Find the total cost of the laptop to Derek.

b How much in excess of the cash price was paid?

'in excess' means 'above' or 'more than'

c What was the flat rate of interest p.a. (correct to one decimal place) charged?

8 Tahlia bought a new outdoor setting and BBQ for her backyard that retails for $2899. She paid $300 deposit, no payments for 6 months and then fortnightly payments of $63 for 2 years. PS R

a Find the balance owing after Tahlia had paid the deposit.

b Calculate the total cost of the outdoor setting and BBQ.

c How much in excess of the cash price did Tahlia pay under this plan?

d What is the annual flat rate of interest charged, correct to one decimal place?

9 Sofia bought a home cinema system priced at $2100 on interest-free terms for one year with no repayments for the first 4 months. R

a If Sofia makes 8 equal monthly repayments, what is the amount of each payment?

b There is a service charge of $12.95 every month for this deferred payment plan. What percentage (correct to one decimal place) of the purchase price was paid in service charges?

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1.07 Depreciation#

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STAGE 5.2

Depreciation is the decrease in value of an item over time. When items lose value because of age or frequency of use, they are said to **depreciate**.

The compound interest formula can be adapted to find the depreciated value of an item. While compound interest involves repeated percentage increases, depreciation involves repeated percentage *decreases*, so its formula has a minus sign.

Depreciation formula

$A = P(1 - r)^n$, where:

- A is the final value of the item
- P is the original value of the item
- r is the rate of depreciation per period, expressed as a decimal
- n is the number of periods of depreciation

The amount of depreciation is then calculated using this formula:

Depreciation = original value − final value

Example 18

An accountant's computer system depreciates by 15% each year.

a If the computer system is currently valued at \$2600, what will its value be in 5 years?

b What is the depreciation over this time?

Solution

a $P = \$2600$, $r = 15\% = 0.15$, $n = 5$

$$\begin{aligned} A &= P(1 - r)^n \\ &= \$2600\,(1 - 0.15)^5 \\ &= \$2600\,(0.85)^5 \\ &= \$1153.6338\ldots \\ &\approx \$1153.63 \end{aligned}$$

The value of the computer system in 5 years will be \$1153.63.

b Depreciation = \$2600 − \$1153.63 (original value − final value)

= \$1446.37

STAGE 5.2

1.07

Example 19

An industrial oven in a restaurant originally costs $19 800, then depreciates at a rate of 12% p.a.

a Find the value of the oven after 6 years, correct to the nearest dollar.

b Express the depreciated value as a percentage of the cost price, correct to one decimal place.

Solution

a $P = \$19\,800$, $r = 0.12$, $n = 6$

$$\begin{aligned} A &= P(1 - r)^n \\ &= \$19\,800\,(1 - 0.12)^6 \\ &= \$19\,800\,(0.88)^6 \\ &= \$9195.2009\ldots \\ &\approx \$9195 \end{aligned}$$

b $$\begin{aligned} \text{Percentage of cost price} &= \frac{\$9195}{\$19800} \times 100\% \\ &= 46.4393\ldots\,\% \\ &\approx 46.4\% \end{aligned}$$

This means that after 6 years, the oven is worth approximately 46% of its original price (or has lost 54% of its original value).

Shutterstock.com/otnaydur

STAGE 5.2

EXERCISE 1.07 ANSWERS ON P. 505

Depreciation U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 18

1 Find the value of a photocopier after 5 years if its purchase price was $2850 and the annual depreciation rate is 20%.

2 **a** Find the value of a car after 7 years if it is purchased new for $49 990 and it depreciates at 12% p.a.

b Find the amount of depreciation over this time.

3 For each item shown in the table, calculate:

i its value after 4 years of depreciation

ii its value after 4 years as a percentage of its original value, correct to one decimal place.

	Item	Original value	Depreciation rate (p.a.)
a	Stove	$1100	12%
b	Fishing boat	$38 500	18%
c	Library	$8460	12%
d	Computer	$2500	20%
e	Furniture	$27 500	15.5%
f	Bike	$2900	22%
g	Electrical tools	$870	17.5%
h	Air conditioner	$2600	9%

EXAMPLE 19

4 A smartphone originally valued at $1729 depreciates at 37% p.a. R

a What percentage (to 2 decimal places where necessary) of the original value remains after:

i 1 year? **ii** 3 years? **iii** 6 years?

b Approximately how long would it take the smartphone to halve its original value?

5 A security system costs a company $12 500 to buy new. It depreciates at a yearly rate of 20%.

a Find the value of the system after:

i 1 year **ii** 2 years **iii** 5 years

b Find the value of the system after 5 years as a percentage of its original value. Answer correct to one decimal place.

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STAGE 5.2

1.07

6 Parveen pays $25 490 for a new car. The car will depreciate in value by an average of 11% p.a.

a Find correct to the nearest dollar the market value of the car in 3 years.

b Calculate the amount of depreciation in the car after 3 years.

7 Adam has spent $175 000 on equipment to set up his hairdressing salon. The equipment depreciates at 20% per year. R

a Find the value of the equipment after 4 years.

b Find the amount of depreciation in the equipment after 4 years.

c Find, by trial and error, how long it will take for the value to be over $50 000. Answer in years and months.

d Find the value of Adam's equipment after 9 years as a percentage of its original value, correct to one decimal place.

8 Kamal says that, at 10% p.a. depreciation, a car will lose half its value after 7 years.

Is he correct? Show all working to justify your answer. PS R C

9 Office equipment that is worth $12 000 when new, depreciates at 15% p.a. as shown in the table. PS R C

Year	Depreciated value
0	$12 000
1	$10 200
2	$8670
3	$7369.50
4	$6264.08
5	$5324.46
6	$4525.79
7	$3846.93
8	$3269.89
9	$2779.40
10	$2362.49
11	$2008.12
12	$1706.90
13	$1450.87
14	$1233.24

a How much did the office equipment lose in value in the first year?

b After how many years did the office equipment fall below half its original value?

c By how much did the office equipment depreciate between the 5th and 6th years?

d Will the value of the office equipment ever fall below $100?

e Will the value of the office equipment ever be zero?

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Power plus ANSWERS ON P. 505

1 How long, in years and days, will it take an investment of $3000 to earn $500 in simple interest at 4% p.a.?

2 What amount should Owen invest to earn $100 in simple interest if the investment will last for 9 months and the interest rate is 3% p.a.?

3 A principal of $10 000 is invested for 5 years at an interest rate of 5% p.a., with interest compounded weekly. Calculate the final value of the investment.

4 Meghan needs $80 000 in 4 years' time. What amount should she invest now at an interest rate of 6% p.a., with interest compounded annually, to reach her target?

5 A painting appreciates in value at a rate of 3% p.a. while a computer depreciates in value at a rate of 10% p.a. If I bought the painting for $1200 and the computer for $1500 new, what would be their combined value in 5 years time?

6 A bacteria colony is growing at a rate of 20% per hour. If there are 10 000 bacteria now, use the compound interest formula to calculate how many there will be after 1 day. (Give your answer correct to the nearest 10 000.)

7 a You invest $2000 in a bank account at an interest rate of 4% p.a. with interest compounded annually. How long will it take for your investment to double in value?

b If you invested $4000 instead of $2000 at the same interest rate, how long will it take to double in value?

c Does the size of the principal make any difference to the time taken for it to double?

CHAPTER 1 REVIEW

Qz
Money and finance

Language of maths

allowable deductions	annual leave loading	compound interest	deposit
depreciation	double time	flat rate	fortnightly
gross pay	income tax	interest	net pay
overtime	PAYG tax	per annum (p.a.)	principal
quarterly	repayment	salary	simple interest
taxable income	term payments	time-and-a-half	wage

1 When investing, why is compound interest better than simple interest?

2 What do the P and r stand for in the formulas $I = Prn$ and $A = P(1 + r)^n$?

3 What is another name for flat-rate interest?

4 What word means a decrease in the value of an item over time?

5 Why is gross pay higher than net pay?

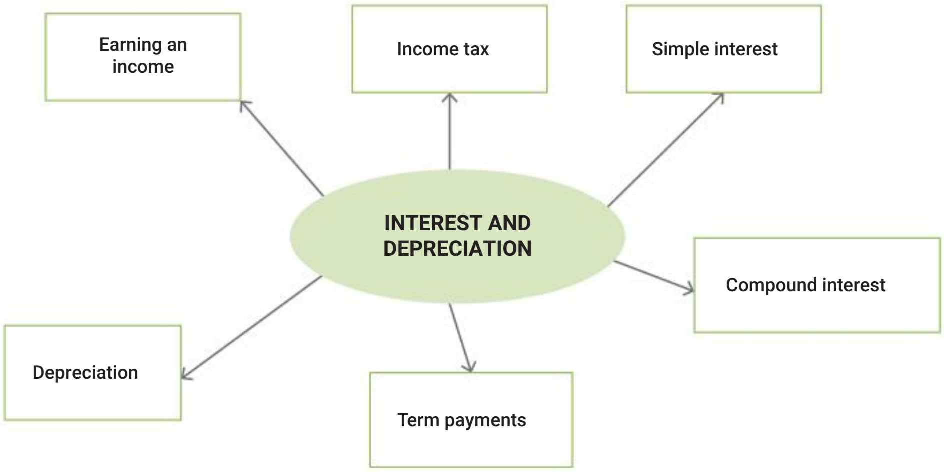
6 Use a dictionary to find at least two different meanings of principal.

Topic summary

WS
Mind map: Interest and depreciation

- Which parts of this chapter were revision of Year 9 knowledge and skills?
- Which parts of this chapter were new to you?
- Do you know how to use the simple interest and compound interest formulas?
- How is income tax calculated?
- How is the depreciation formula similar to the compound interest formula?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 1

ANSWERS ON P. 505

In this exercise, round all money answers to the nearest cent.

1.01 **1** Hayley is paid a commission of 2.5% on the value of the properties she sells. She also receives a weekly retainer of $1150. How much will Hayley earn if she sells a house for $475 830?

1.01 **2** Caleb earns a salary of $70 400 p.a. How much is he paid each week?

1.01 **3** A supermarket cashier is employed under the following award.

Normal rate: $21.45 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 40 hours

b 46 hours.

1.01 **4** For his Christmas holidays, Jacan received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Jacan's annual salary is $54 920, find:

a his normal weekly pay

b his leave loading

c his total pay for the 4-week holiday.

1.02 **5** Alia earns a salary of $68 650 p.a. Her allowable deductions are donations to charities of $540 and work-related expenses of $385.

a Calculate Alia's taxable income.

b Use the tax table on page 11 to calculate the income tax Alia should pay.

1.03 **6** Calculate the simple interest earned on each investment.

a $20 000 invested for 3 years at 4% p.a.

b $7850 invested at 2.5% p.a for 15 months

c $4500 invested for 6 months at 0.17% per month

d $25 200 invested for 100 days at 3.45% p.a.

1.04 **7** An amount of $5000 is invested at 2.35% p.a. interest, compounded annually over 3 years.

a What is the total value of the investment after 3 years?

b What is the amount of compound interest earned?

Foundation Standard Complex

9780170453417

8 Calculate the value of the investment when $34 200 is invested at 3% p.a. for 2 years, with interest compounded annually.

STAGE 5.2

1.05

9 Find the final value if $11 000 is invested for 4 years at 2.4% p.a., with interest compounded monthly.

1.05

10 Find the interest earned when $4895 is invested at 1.95% p.a. for 3 years, with interest compounded annually.

1.05

11 Calculate the interest earned when $46 230 is invested for 9 years at 2.8% p.a., with interest compounded half-yearly.

1.05

12 Yang purchases a furniture package valued at $4875. She pays a 10% deposit and repays the balance in 36 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find:

1.06

a the deposit Yang paid

b the balance owing

c the interest charged

d the total to be repaid

e the amount of each instalment

f the total price Yang paid for the package.

13 Caroline bought a new car for $24 990, which depreciates by 10% p.a.

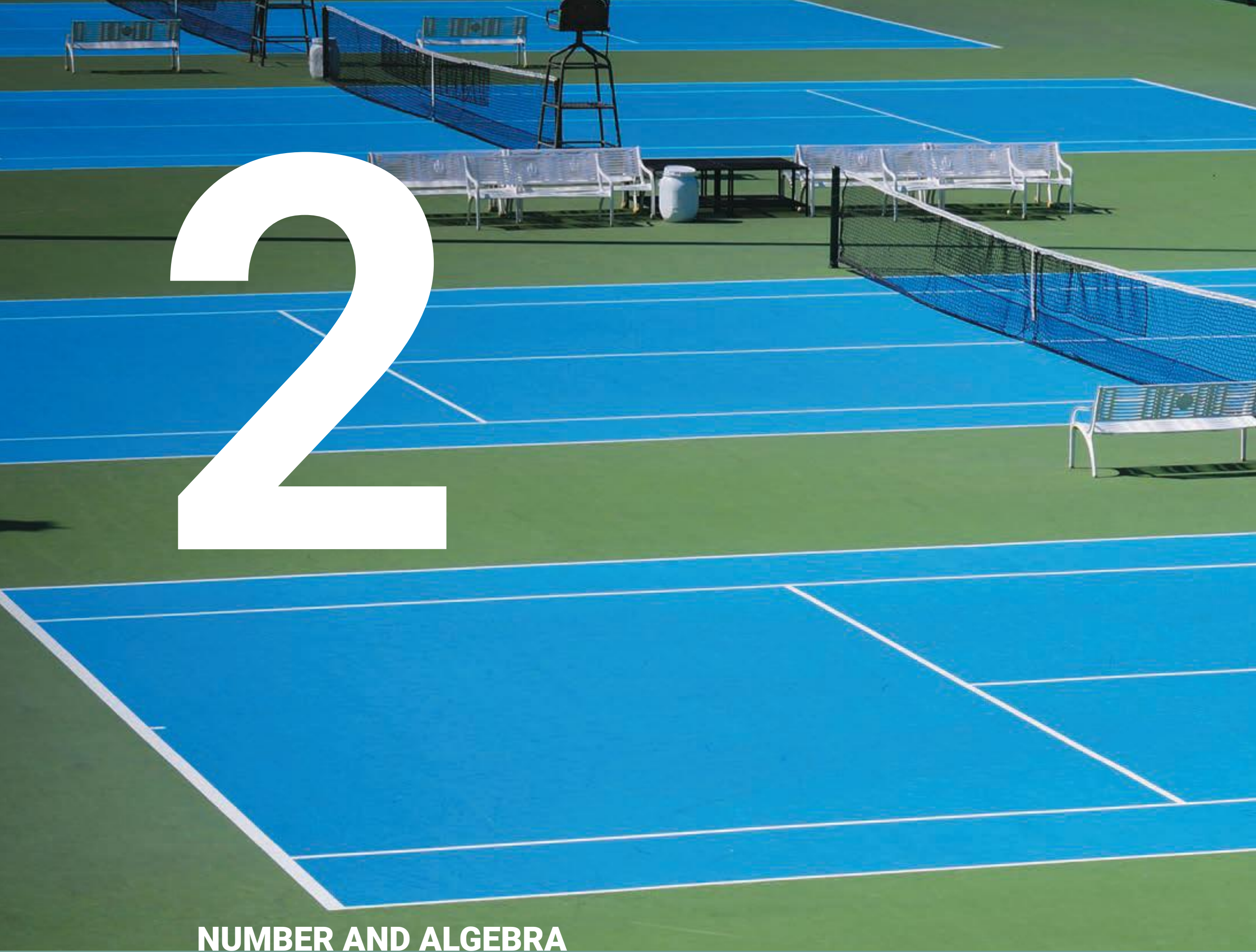
STAGE 5.2

1.07

a Find correct to the nearest dollar the depreciated value of the car after 5 years.

b What is the depreciation over this time?

c Express the depreciated value as a percentage of the original price (correct to one decimal place).



2

NUMBER AND ALGEBRA

GRAPHING LINES

Straight lines are an important part of our environment. We play sport on courts using parallel and perpendicular lines, and skyscrapers would not be standing without straight lines. We can also use straight lines to model different types of data and predict future outcomes.

9780170453417

Chapter outline

	Working mathematically				
2.01 Length, midpoint and gradient of an interval	U	F	PS	R	C
2.02 Parallel and perpendicular lines	U	F			
2.03 Graphing linear equations	U	F		R	C
2.04 The gradient–intercept equation $y = mx + c$*	U	F		R	
2.05 The general equation $ax + by + c = 0$*	U	F		R	C
2.06 Finding the equation of a line*	U	F		R	C
2.07 Equations of parallel and perpendicular lines*	U	F		R	C

***STAGE 5.2**

Wordbank

general form Any linear equation expressed as $ax + by + c = 0$, where a, b and c are integers and a is positive

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

gradient–intercept form Any linear equation expressed as $y = mx + c$, where m is the gradient and c is the y-intercept

linear equation An equation whose graph is a straight line

parallel lines Lines that point in the same direction and have the same gradient

perpendicular lines Lines that cross at right angles (90°) and have gradients whose product is −1

***x*-intercept** The x-value at which a graph cuts the x-axis

***y*-intercept** The y-value at which a graph cuts the y-axis

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- find the distance between 2 points on a number plane
- find the midpoint and gradient of an interval on a number plane
- find the properties of the gradients of parallel lines
- (STAGE 5.2) find the properties of the gradients of perpendicular lines
- graph a linear equation on a number plane
- test whether a point lies on a line
- (STAGE 5.2) use the gradient–intercept equation of a straight line $y = mx + c$
- (STAGE 5.2) use the general form of a linear equation $ax + by + c = 0$ and convert it to the gradient–intercept equation
- (STAGE 5.2) find the equation of a line from its graph
- (STAGE 5.2) find the equation of a line that is parallel or perpendicular to a given line

Pythagoras' theorem

SkillCheck ANSWERS ON P. 505

1 For this number plane, find:

a the midpoint of interval BC

b the midpoint of interval HE

c the length of interval GC

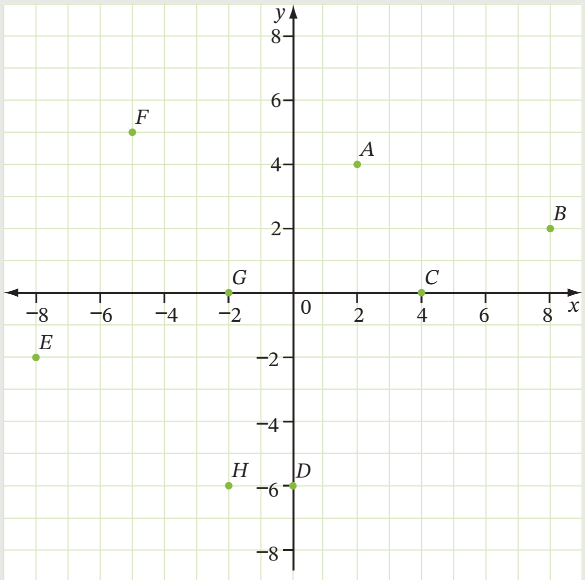
d the length of interval GH

e the lengths of AC and BC, correct to one decimal place

f the type of triangle $\triangle ABC$ is

g the gradient of GE

h the gradient of EH



2 For each linear equation, copy and complete the table of values and graph the equation.

a $y = x - 3$

x	0	1	2	3
y				

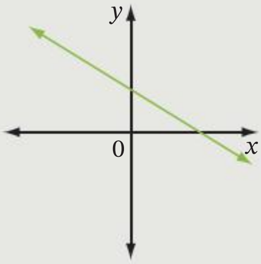
b $y = 3x + 2$

x	−2	−1	0	1
y				

c $y = 1 - 2x$

x	−1	0	1	2
y				

3 State whether each line's gradient is positive, negative or neither.

a


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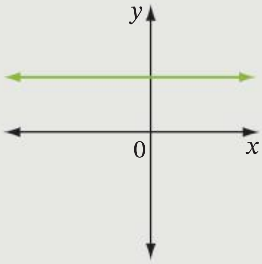
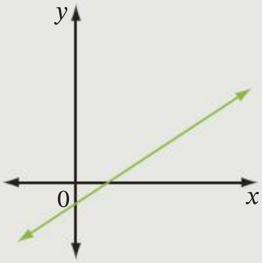
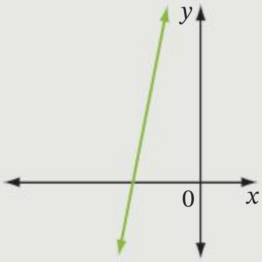
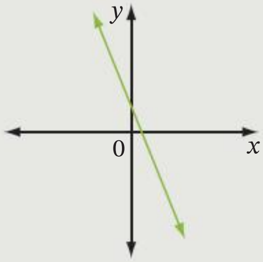
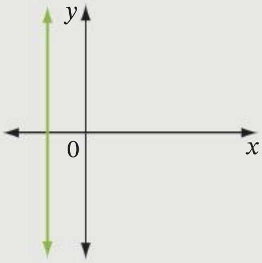


c


d


e


f


2

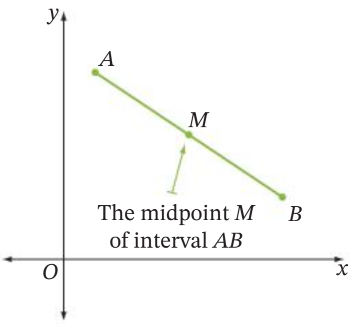
Length, midpoint and gradient of an interval

2.01

The **length** of an interval AB (or the **distance** between A and B) can be calculated using Pythagoras' theorem if we know the coordinates of A and B.

The **midpoint** of an interval AB is the point in the middle of AB or halfway between A and B.

- Its x-coordinate is the average of the x-coordinates of A and B.
- Its y-coordinate is the average of the y-coordinates of A and B.



WS Gradient, midpoint and distance

WS Intervals match-up

Midpoint and distance between 2 points

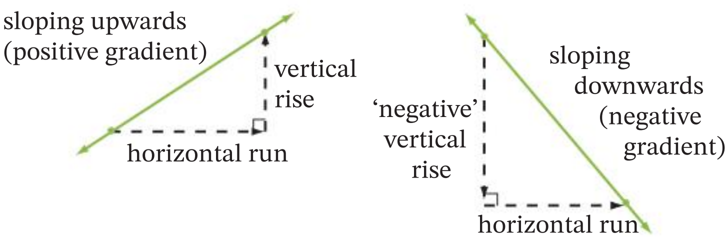
Midpoint and distance between 2 points

The gradient of a line

WS Gradient between two points

The **gradient** of an interval measures its steepness. It is given by the formula:

$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$



- A line **sloping upwards** has a **positive rise** and **a positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.

Example 1

For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place.

ii the midpoint of the interval

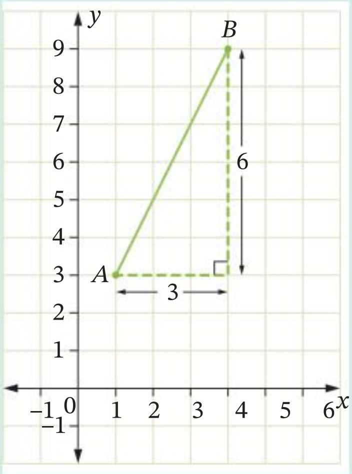
iii the gradient of the interval

a $A(1, 3)$ and $B(4, 9)$

b $P(-5, 8)$ and $Q(3, 6)$

Solution

a **i** Draw a right-angled triangle on the number plane, with AB as the hypotenuse.



Height $= 9 - 3 = 6$ — Difference between y-coordinates

Base $= 4 - 1 = 3$ — Difference between x-coordinates

$AB^2 = 6^2 + 3^2$ — by Pythagoras' theorem

$= 45$

$AB = \sqrt{45}$

$= 6.7082...$

≈ 6.7 units

ii For $A(1, 3)$ and $B(4, 9)$, the average of the x-coordinates is $\frac{1+4}{2} = 2\frac{1}{2}$.

The average of the y-coordinates is $\frac{3+9}{2} = 6$.

$\therefore$ The midpoint of AB is $\left(2\frac{1}{2}, 6\right)$.

From the diagram above, a midpoint at $\left(2\frac{1}{2}, 6\right)$ looks reasonable.

iii The rise is 6 units.

The run is 3 units.

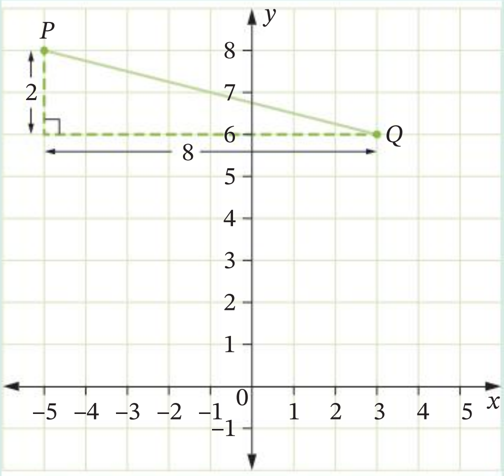
The gradient is positive, the line is sloping upwards.

$$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$$

b **i** Draw a right-angled triangle on the number plane with PQ as the hypotenuse.

The height of the triangle is 2 units.

The base of the triangle is 8 units.



$PQ^2 = 2^2 + 8^2$ by Pythagoras' theorem

$= 68$

$PQ = \sqrt{68}$

$= 8.2462...$

≈ 8.2 units

ii For $P(-5, 8)$ and $Q(3, 6)$, the average of the x-coordinates is $\frac{-5+3}{2} = -1$.

From the diagram above, a midpoint at $(-1, 7)$ looks reasonable.

The average of the y-coordinates is $\frac{8+6}{2} = 7$.

$\therefore$ The midpoint of PQ is $(-1, 7)$.

iii The rise is -2 units. Line slopes downwards.

The run is 8 units.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{8} = -\frac{1}{4}$$

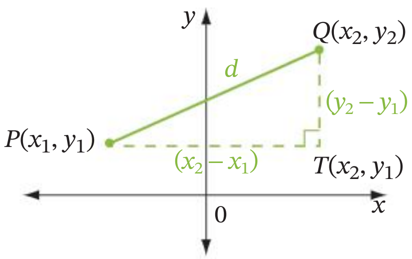
The gradient is negative, the line slopes downward.

2.01

Optional: The distance, midpoint and gradient formulas

The methods for finding the length, midpoint and gradient of an interval can each be summarised by a formula.

The **distance formula** is used to calculate the distance (d) between 2 points $P(x_1, y_1)$ and $Q(x_2, y_2)$, that is, the length of the interval PQ.



By Pythagoras' theorem:

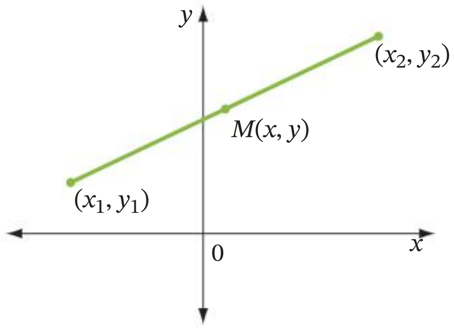
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **midpoint formula** gives the coordinates of the point M, the midpoint of the interval joining $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

We've used '≡' ('is identical to') rather than '=' because we are referring to the point (x, y), not a number.



The **gradient formula** gives the gradient of the interval or line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Vertical rise = difference in y-coordinates = $y_2 - y_1$

Horizontal run = difference in x-coordinates = $x_2 - x_1$

$$\text{Gradient, } m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance, midpoint and gradient formulas

Example 2

For the interval joining $P(-5, 8)$ and $Q(3, 6)$ from Example **1b**, use a formula to find:

a the length of the interval, correct to one decimal place

b the midpoint of the interval

c the gradient of the interval.

Solution

For $P(-5, 8)$ and $Q(3, 6)$: $x_1 = -5, y_1 = 8, x_2 = 3, y_2 = 6$.

(x_1, y_1) (x_2, y_2)

a
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-5))^2 + (6 - 8)^2} \\ &= \sqrt{68} \\ &= 8.2462\ldots \\ &\approx 8.2 \text{ units} \end{aligned}$$

Apply the distance formula.

b $M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ — Apply the midpoint formula.

$$\equiv \left(\frac{-5+3}{2}, \frac{8+6}{2}\right)$$

$$\equiv (-1, 7)$$

c $m = \frac{\text{difference in } y}{\text{difference in } x}$ — Apply the gradient formula.

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-8}{3-(-5)}$$

$$= \frac{-2}{8}$$

$$= -\frac{1}{4}$$

Example 3

a Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.

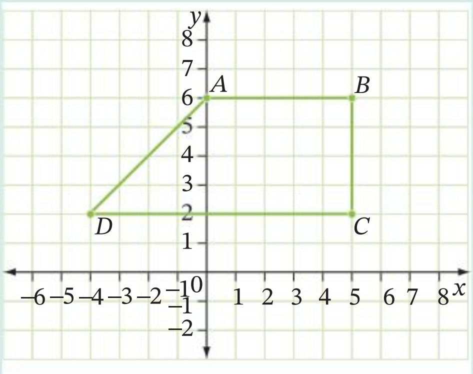
b What type of quadrilateral is $ABCD$?

c Find the exact length of AD.

d Hence find the perimeter of $ABCD$, correct to 2 decimal places.

Solution

a



Join the points in the correct order.

b Since $AB \parallel CD$, the quadrilateral is a trapezium.

c $AD^2 = 4^2 + 4^2$

$$= 32$$

$AD = \sqrt{32}$ units — In exact surd form.

d By counting grid squares, $AB = 5$, $BC = 4$, $CD = 9$.

Perimeter of $ABCD = 5 + 4 + 9 + \sqrt{32}$

$$= 23.656...$$

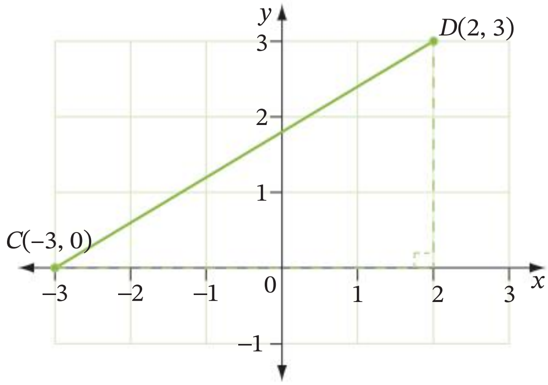
$$\approx 23.66 \text{ units}$$

EXERCISE 2.01 ANSWERS ON P.506

ANSWERS ON P.506

Length, midpoint and gradient of an interval U F PS R C

Questions **1**, **2** and **3** refer to this diagram of interval *CD*.



EXAMPLES 1, 2

1 What is the length of interval *CD*? Select the correct answer **A**, **B**, **C** or **D**.

A 2 units **B** 5.8 units **C** 3.2 units **D** 8 units

2 What is the midpoint of *CD*? Select **A**, **B**, **C** or **D**.

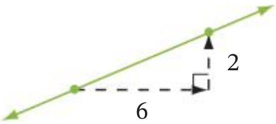
A (−1, 3) **B** (−5, 3) **C** (−0.5, 1.5) **D** (−2.5, 1.5)

3 What is the gradient of *CD*? Select **A**, **B**, **C** or **D**.

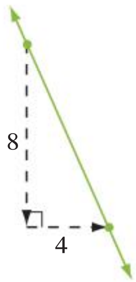
A $\frac{3}{5}$ **B** $-\frac{3}{5}$ **C** $-\frac{5}{3}$ **D** $\frac{5}{3}$

4 Calculate the gradient of each line.

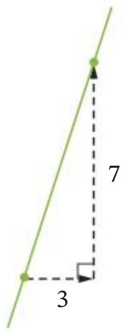
a



b



c



5 For the interval joining each pair of points given, find:

- **i** the length of the interval, correct to one decimal place
- **ii** the midpoint of the interval
- **iii** the gradient of the interval.

a *A*(5, 3) and *B*(7, 2) **b** *J*(−1, 0) and *K*(8, 6) **c** *M*(0, −3) and *N*(−5, 2)

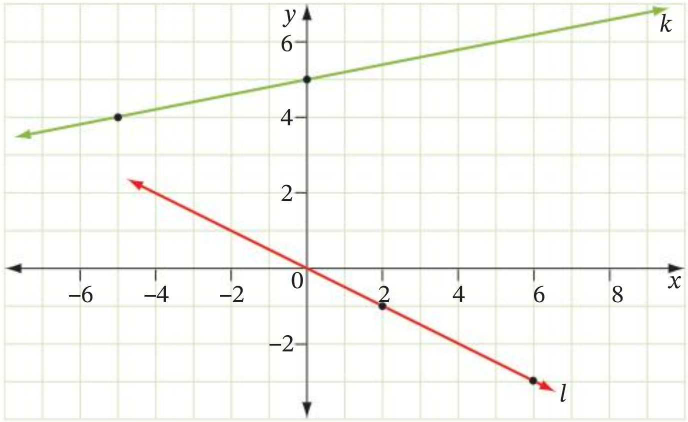
d *R*(−3, −6) and *S*(4, −9) **e** *A*(−7, 2) and *B*(−5, −8) **f** *U*(3, −2) and *V*(7, 2)

Foundation Standard Complex

6 Calculate, in exact (surd) form, the distance between each pair of points.

a (−8, −1) and (0, 4) **b** (12, −6) and (−1, −1) **c** (7, −2) and (−2, −3)

7 Find the gradient of the lines labelled *k* and *l*.



8 Which expression gives the *y*-coordinate of the midpoint of the interval joining points (3, 8) and (−1, 5)? Select **A**, **B**, **C** or **D**.

A $\frac{-1+5}{2}$ **B** $\frac{8+5}{2}$ **C** $\frac{8-5}{2}$ **D** $\frac{5-8}{2}$

9 The vertices of triangle *ABC* are *A*(−1, −1), *B*(1, 3) and *C*(3, 1). PS R C

EXAMPLE 3

a Draw △*ABC* on a number plane.

b Find the exact length of each side of the triangle.

c Are any sides of the triangle equal in length?

d What type of triangle is *ABC*?

e Find the perimeter of △*ABC*, correct to one decimal place.

10 The vertices of quadrilateral *KLMP* are *K*(1, 6), *L*(7, 2), *M*(3, −4) and *P*(−3, 0). PS R C

a Draw the quadrilateral on a number plane.

b What type of quadrilateral is *KLMP*?

c Find the gradients of sides *KL* and *PM*.

d Find the gradients of sides *KP* and *LM*.

e What do you notice about the gradients of the opposite sides of this quadrilateral? What does that mean about those sides?

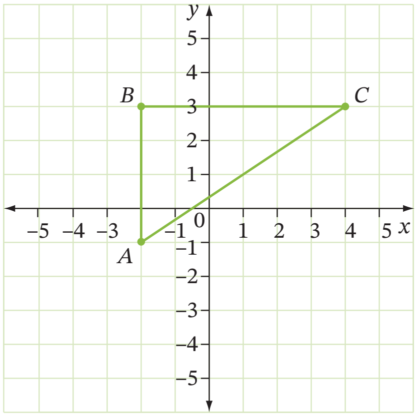
f Find the exact length of each side of *KLMP*.

g Find the perimeter of *KLMP*, correct to one decimal place.

h Find the area of *KLMP*.

Foundation Standard Complex

11 This diagram shows a right-angled triangle with vertices $A(-2, -1)$, $B(-2, 3)$ and $C(4, 3)$. R C



a Copy the diagram and find the coordinates of *P* and *Q*, the midpoints of *BA* and *BC* respectively. Mark *P* and *Q* on your diagram.

b Calculate, correct to one decimal place, the lengths of *PQ* and *AC*. What do you notice about your answers?

c Find the gradients of *PQ* and *AC*. What do you notice about your answers?

Investigation

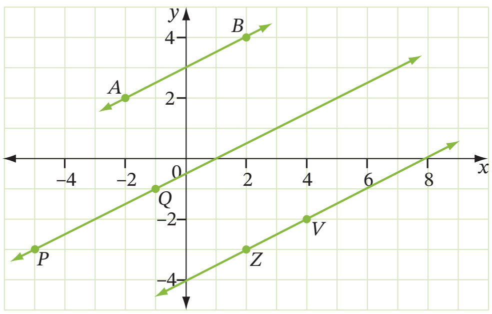
Parallel and perpendicular lines

1 These 3 lines are parallel. Calculate the gradient of:

a *AB*

b *PQ*

c *ZV*

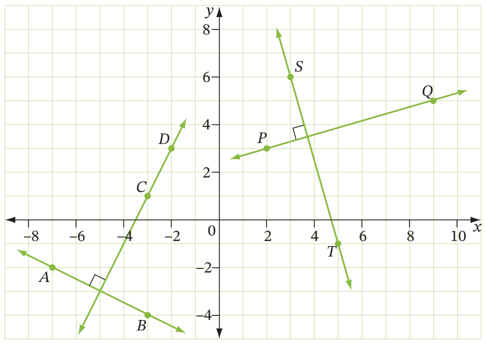


2 What can you conclude about the gradients of parallel lines?

3 This diagram shows 2 pairs of perpendicular lines. $AB \perp CD$ and $PQ \perp ST$. Calculate the gradient of:

a *AB* **b** *CD*

c *PQ* **d** *ST*



4 Is there a relationship between:

a the gradients of *AB* and *CD*? **b** the gradients of *PQ* and *ST*?

5 Calculate the product of:

a the gradients of *AB* and *CD* **b** the gradients of *PQ* and *ST*

6 What can you conclude about the gradients of perpendicular lines?

Parallel and perpendicular lines 2.02

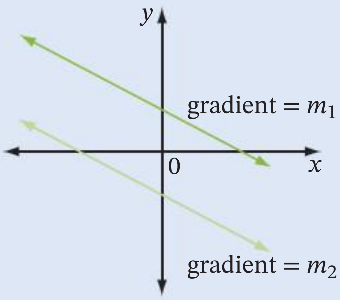
Gradients of parallel and perpendicular lines

2.02

Parallel lines

Parallel lines have the same gradient.

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.



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Perpendicular lines

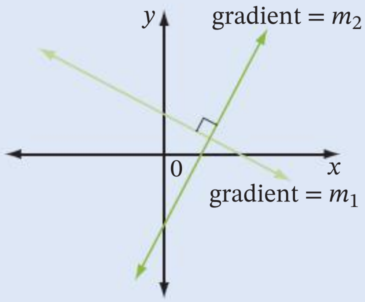
STAGE 5.2

Perpendicular lines have gradients with a product of −1.

If 2 lines with gradients m_1 and m_2 are perpendicular, then

$$m_1 \times m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}.$$

Note that m_2 is the **negative reciprocal** of m_1.



Example 4

State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{2}, m_2 = 2$ **b** $m_1 = 0.4, m_2 = \frac{2}{5}$ **c** $m_1 = 1\frac{3}{5}, m_2 = -\frac{5}{8}$

Solution

a $m_1 \neq m_2$ so the lines are not parallel.

$$\begin{aligned} m_1 \times m_2 &= \frac{1}{2} \times 2 \\ &= 1 \\ &\neq -1 \end{aligned}$$

so the lines are not perpendicular.

$\therefore$ The lines are neither parallel nor perpendicular.

b $m_2 = \frac{2}{5} = 0.4$

$m_1 = m_2$

$\therefore$ The lines are parallel.

c $m_1 = 1\frac{3}{5} = \frac{8}{5}$

$$\begin{aligned} m_1 \times m_2 &= \frac{8}{5} \times \left(-\frac{5}{8}\right) \\ &= -1 \end{aligned}$$

$\therefore$ The lines are perpendicular.

STAGE 5.2

Example 5

Find the gradient of a line that is perpendicular to a line with gradient:

a 2 **b** −3 **c** $\frac{3}{4}$ **d** −0.6

Solution

a $m_1 = 2$

$$\begin{aligned} m_2 &= \frac{-1}{m_1} \text{ for perpendicular lines} \\ &= \frac{-1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

The negative reciprocal of m_1.

The gradient is $-\frac{1}{2}$.

b $m_1 = -3$

$$\begin{aligned} m_2 &= \frac{-1}{m_1} \\ &= \frac{-1}{-3} \\ &= \frac{1}{3} \end{aligned}$$

The gradient is $\frac{1}{3}$.

c $m_1 = \frac{3}{4}$

$$\begin{aligned} m_2 &= \frac{-1}{m_1} \\ &= \frac{-1}{\left(\frac{3}{4}\right)} \\ &= -\frac{4}{3} \end{aligned}$$

The gradient is $-\frac{4}{3}$.

d $m_1 = -0.6 = -\frac{3}{5}$

$$m_2 = \frac{-1}{\left(-\frac{3}{5}\right)} = \frac{5}{3}$$

The gradient is $\frac{5}{3}$.

Example 6

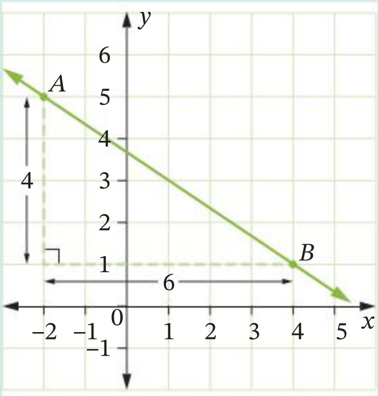
A line passes through the points $A(-2, 5)$ and $B(4, 1)$. What is the gradient of a line:

a parallel to AB? **b** perpendicular to AB?

Solution

Find the gradient of AB by calculating the rise and run.

Sometimes drawing a diagram helps you to understand the problem.



Rise = 1 − 5 = −4 — Difference between y-coordinates.

Run = 4 − (−2) = 6 — Difference between x-coordinates.

Gradient $AB = \frac{-4}{6} = -\frac{2}{3}$ — $\frac{\text{rise}}{\text{run}}$

a Any line parallel to AB will have the same gradient as AB.

$\therefore m = -\frac{2}{3}$

b The gradient of a line perpendicular to AB will be given by:

$m = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$

STAGE 5.2

EXERCISE 2.02 ANSWERS ON P.506

Parallel and perpendicular lines U F

EXAMPLE 4

1 State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{4}, m_2 = 4$ **b** $m_1 = 3, m_2 = -\frac{1}{3}$ **c** $m_1 = 0.5, m_2 = \frac{1}{2}$

d $m_1 = \frac{2}{7}, m_2 = \frac{7}{2}$ **e** $m_1 = \frac{3}{10}, m_2 = 0.3$ **f** $m_1 = 1\frac{1}{5}, m_2 = -\frac{6}{5}$

2 Find the gradient of a line that is parallel to a line with gradient:

a 4 **b** −2 **c** $\frac{1}{3}$ **d** −0.2

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STAGE 5.2

EXAMPLE 5

3 Find the gradient of a line that is perpendicular to a line with gradient:

a 1 **b** −6 **c** −1.5 **d** $\frac{5}{2}$

4 What is the gradient of a line that is perpendicular to a line with a gradient of 0.8? Select the correct answer **A**, **B**, **C** or **D**.

A 0.2 **B** −0.2 **C** 1.25 **D** −1.25

5 What is the gradient of a line that is parallel to a line that goes through $P(0, 3)$ and $Q(5, -2)$? Select **A**, **B**, **C** or **D**.

A 1 **B** −1 **C** $\frac{1}{5}$ **D** $-\frac{1}{5}$

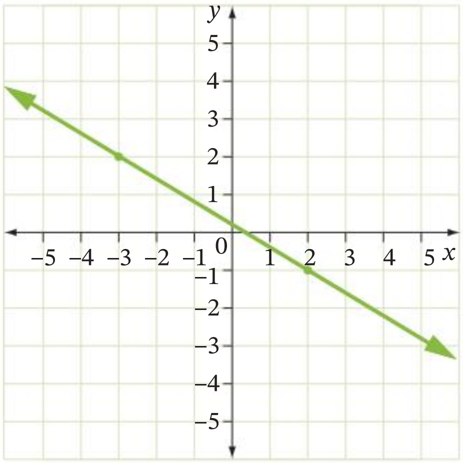
STAGE 5.2

EXAMPLE 6

6 What is the gradient of a line perpendicular to the line shown? Select **A**, **B**, **C** or **D**.

A $\frac{5}{3}$ **B** −5

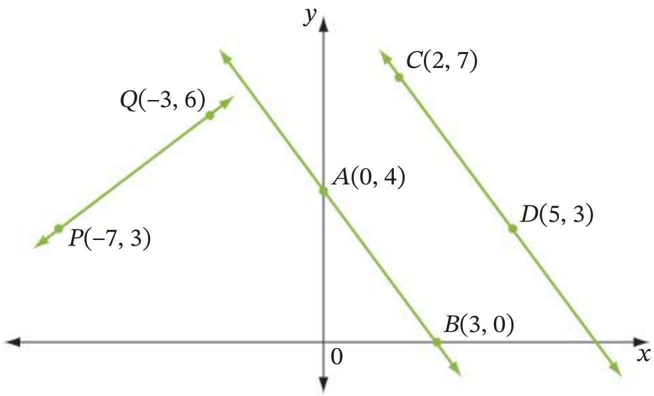
C $\frac{3}{5}$ **D** $\frac{1}{5}$



7 Calculate the gradient of each line shown and test whether:

a $AB \parallel CD$

b $PQ \perp CD$.



8 A line passes through the points $R(-5, 2)$ and $S(1, 4)$. What is the gradient of a line:

a parallel to RS?

b perpendicular to RS?

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Technology

Parallel and perpendicular lines

Use dynamic geometry software to find out if sets of linear equations represent parallel or perpendicular lines.

1 Parallel lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are parallel using $m_1 = m_2$.

a $5x - 3y = 0$ and $y = \frac{5x}{3}$

b $x + y + 4 = 0$ and $x + y - 6 = 0$

c $x - 2y = 0$ and $y = 0.5x$

d $y = 5x - 9$ and $5x - y - 1 = 0$

2 Perpendicular Lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are perpendicular using $m_1 \times m_2 = -1$.

a $y = 0.6x + 2$ and $y = \frac{5}{3}x$

b $x - 4y + 1 = 0$ and $y = -4x - 3$

c $3x - 2y = 0$ and $y = -\frac{2x}{3}$

d $y = 2x + 4$ and $x - 2y - 1 = 0$

2.02

Graphing linear equations 2.03

A relationship between 2 variables, x and y, whose graph is a straight line is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = 3x + 2$, is called a **linear equation**.

Graphing linear equations

A page of lines

A page of number planes

Graphing linear equations

Graphing linear equations

Example 7

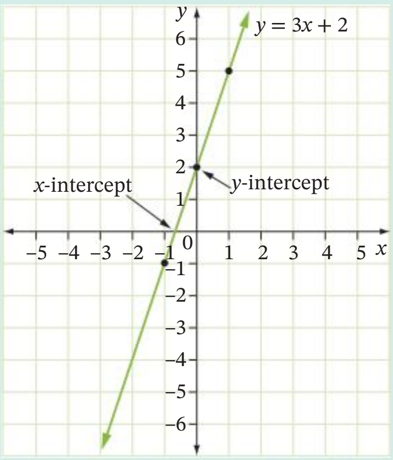
Graph $y = 3x + 2$ on a number plane and find its x- and y-intercepts.

Solution

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	2	5

Graph $(-1, -1)$, $(0, 2)$ and $(1, 5)$ on a number plane. Rule a straight line through the points, place arrows at each end, and label the line with its equation.



The x-intercept is the x-value where the line cuts the x-axis and can be found by substituting $y = 0$ into the equation $y = 3x + 2$:

$0 = 3x + 2$

$3x = -2$

$x = -\frac{2}{3}$ which agrees with what's shown on the graph

The x-intercept is $-\frac{2}{3}$.

The y-intercept is the y-value where the line cuts the y-axis and can be found by looking at the graph, or looking at the constant term in the equation $y = 3x + 2$, or substituting $x = 0$ into the equation $y = 3x + 2$ (which has already been done for the table of values).

The y-intercept is 2.

In the above example, every point on the line follows the linear equation $y = 3x + 2$. For example, (−1, 1), (0, 2) and (1, 5) lie on the line and follow the rule $y = 3x + 2$.

There are an infinite number of points that follow the rule. Arrows on both ends of the line indicate that it has infinite length.

Testing if a point lies on a line

Straight-line equations

Testing if a point lies on a line

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.

Example 8

Test whether each point lies on the line $x - 2y = 5$.

a (17, 6) **b** (8, −4)

Solution

- Separate the equation into its left-hand side (LHS) and right-hand side (RHS)
- Substitute the coordinates of the point into both sides
- If LHS = RHS, the point satisfies the equation and so lies on the line
- If LHS ≠ RHS, the point does not lie on the line.

a Substitute $x = 17$, $y = 6$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 17 - 2 \times 6$

$= 5$

RHS = 5

LHS = RHS, so (17, 6) lies on the line.

b Substitute $x = 8$, $y = -4$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 8 - 2 \times (-4)$

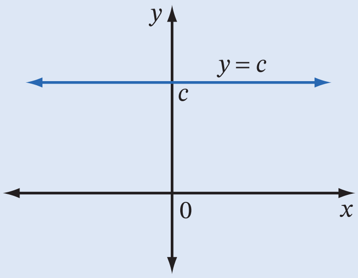
$= 16$

RHS = 5

LHS ≠ RHS, so (8, −4) does not lie on the line.

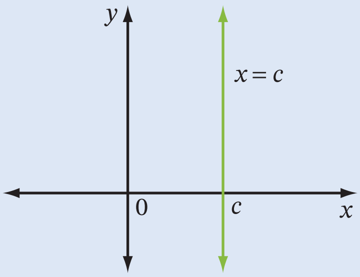
Horizontal and vertical lines

The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).



Note: The x-axis has equation $y = 0$.

The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).



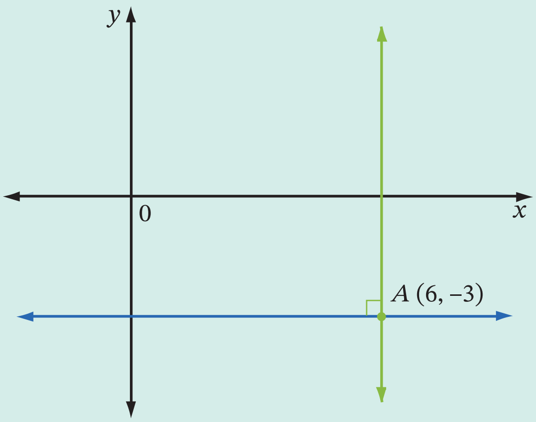
Note: The y-axis has equation $x = 0$.

Example 9

For the graph shown, find the equation of:

a the vertical line

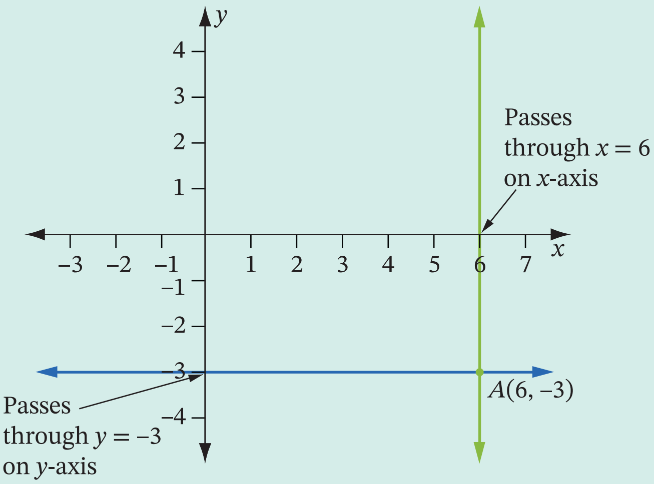
b the horizontal line



Solution

a The vertical line has an x-intercept of 6 and passes though $A(6, -3)$, so its equation is $x = 6$.

b The horizontal line has a y-intercept of -3 and passes though $A(6, -3)$, so its equation is $y = -3$.



Example 10

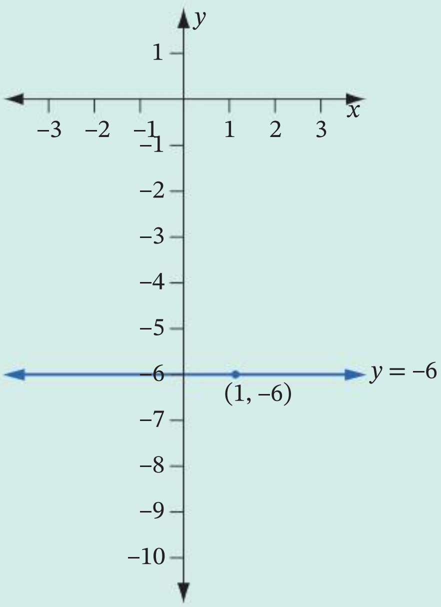
Find the equation of the line that is:

a parallel to the x-axis and passes through the point (1, −6)

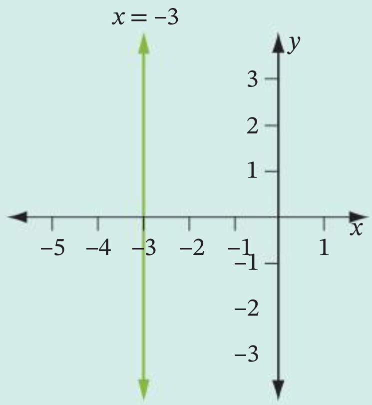
b always 3 units to the left of the y-axis.

Solution

a The equation of the line is $y = -6$.



b The equation of the line is $x = -3$.



EXERCISE 2.03 ANSWERS ON P.506

Graphing linear equations **U F R C**

EXAMPLE 7

1 Graph each linear equation on a number plane, and find:

i its x-intercept **ii** its y-intercept.

a $y = 3x - 1$ **b** $y = 2x + 5$ **c** $y = -x + 4$

d $y = -2x - 2$ **e** $y = 4x$ **f** $y = \frac{x}{2} + 3$

EXAMPLE 8

2 Test whether the point (3, −1) lies on each line.

a $y = 2x - 5$ **b** $x - y = 4$ **c** $y + 2x = 5$

d $y = x - 4$ **e** $x + y = 5$ **f** $3x + y + 8 = 0$

3 Which point lies on the line $y = 6x - 5$? Select the correct answer **A**, **B**, **C** or **D**.

A (−1, 11) **B** (3, −13) **C** (−2, −17) **D** (−5, 25)

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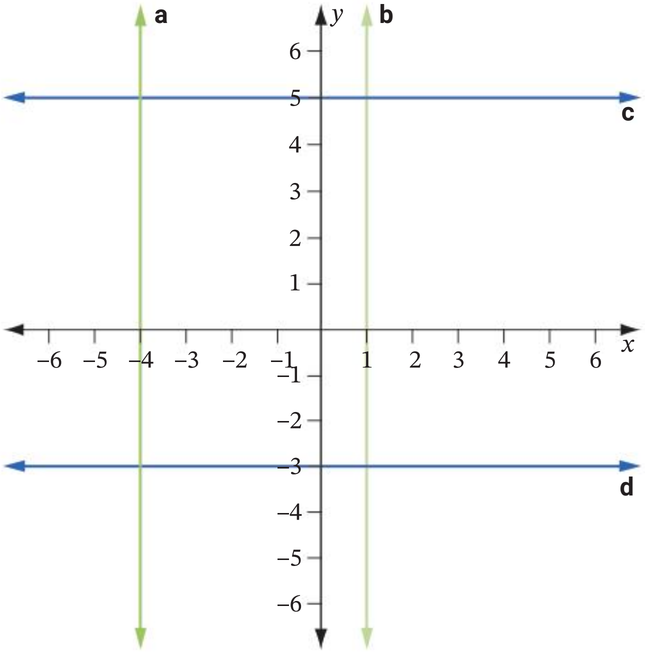
4 Find the equation of each line shown.

EXAMPLE 9



5 Graph each set of lines on a number plane.

a $x=2\frac{1}{2}, y=-3, y=1$

b $x=6, y=-2, x=-\frac{1}{2}$

6 Find the equation of the line that is: R C

EXAMPLE 10

a horizontal and passes through the y-axis at 2

b vertical with an x-intercept of 4

c parallel to the y-axis and passes through the point (−1, 4)

d parallel to the x-axis and passes through the point (0, −2)

e 3 units above the x-axis

f 1 unit to the left of the y-axis

g drawn through the points (−1, 6) and (2, 6)

h drawn through the points (−1, 8) and (−1, 2).

7 Which point lies on the line $4x + y = 1$? Select **A**, **B**, **C** or **D**.

A (−1, 5) **B** (−2, 7) **C** (6, 9) **D** $\left(-\frac{1}{2}, 1\right)$

8 Which line is horizontal and passes through the point (8, −2)? Select **A**, **B**, **C** or **D**.

A $y = 8$ **B** $x = 8$ **C** $y = -2$ **D** $x = -2$

9 **a** What is another name for the line $y = 0$? C

b What is another name for the line $x = 0$? C

2.03

Technology

Graphing $y = mx + c$

1 Use dynamic geometry software to graph the 4 lines $y = 3x + 2$, $y = 5x + 2$, $y = -2x + 2$ and $y = -0.1x + 2$. Make each line a different colour.

2 Find the gradient (slope) and y-intercept of each line and record your results in a table like this.

Equation	Gradient	*y*-intercept

3 What do you notice about your results?

4 Repeat the steps above for each set of equations.

a $y = -4x$ $\quad$ $y = -4x + 1$ $\quad$ $y = -4x - 10$

b $y = 2x + 3$ $\quad$ $y = -7x + 3$ $\quad$ $y = 0.2x + 3$

c $y = -x - 1$ $\quad$ $y = -x + 2$ $\quad$ $y = -x - 4$

5 For each set of lines drawn in question **4**, complete a table as shown in Step **2**.

6 What do you notice about each set of lines? Identify any key features of each set of graphs.

2.04 The gradient–intercept equation $y = mx + c$

STAGE 5.2

Graphing $y = mx + c$

Gradient and y-intercept of a line

The gradient–intercept form of a linear equation

The equation of a straight line is $y = mx + c$, where m is the **gradient** and c is the ***y*-intercept**.

For this reason, $y = mx + c$ is also called the **gradient–intercept form** of a linear equation.

Example 11

Find the gradient and y-intercept of the line with equation:

a $y = -4x + 9$ **b** $y = 10 - 6x$ **c** $y = \dfrac{5x+4}{2}$ **d** $3x + 2y - 6 = 0$

Solution

a $y = -4x + 9$ is in the form $y = mx + c$.

$\therefore$ Gradient $m = -4$ and y-intercept $c = 9$.

b $y = 10 - 6x$ can be rewritten as $y = -6x + 10$.

$\therefore$ Gradient $m = -6$ and y-intercept $c = 10$.

c $y = \dfrac{5x+4}{2}$ can be rewritten as $y = \dfrac{5x}{2} + \dfrac{4}{2} = \dfrac{5x}{2} + 2$.

$\therefore$ Gradient $m = \dfrac{5}{2}$ and y-intercept $c = 2$.

d $3x + 2y - 6 = 0$ can be rearranged in the form $y = mx + c$.

Make y the **subject** of the equation: 'solve' the equation for y.

$$3x + 2y - 6 = 0$$
$$3x + 2y - 6 - 3x = 0 - 3x$$
$$2y - 6 = -3x$$
$$2y - 6 + 6 = -3x + 6$$
$$2y = -3x + 6$$
$$\frac{2y}{2} = \frac{-3x+6}{2}$$
$$y = \frac{-3x}{2} + 3$$

$\therefore$ Gradient $m = -\dfrac{3}{2}$ and y-intercept $c = 3$.

Example 12

Write the equation of a line with a gradient of -4 and a y-intercept of -6.

Solution

$m = -4$, $c = -6$

$\therefore$ The equation of the line is $y = -4x - 6$.

Example 13

Graph each linear equation by finding the gradient and y-intercept first.

a $y = -2x + 5$ **b** $y = \dfrac{3}{4}x - 2$

STAGE 5.2

The gradient-intercept formula

Equation of a line

Equations in gradient form

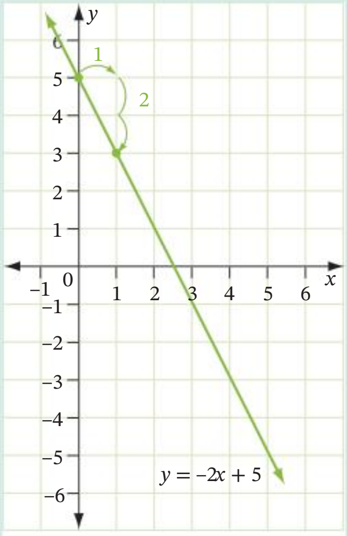
Drawing straight lines: $y = mx + c$

2.04

Solution

a $y = -2x + 5$ has a gradient of -2 and a y-intercept of 5.

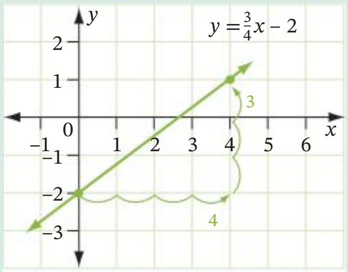
- Plot the y-intercept 5 on the y-axis.
- Make a gradient of $-2 = \frac{-2}{1}$ by moving **across** 1 unit (**run**) and **down** 2 units (**'negative' rise**) and marking the point at (1, 3).
- Rule a line through this point and the y-intercept.



Don't forget to label the line with its equation '$y = -2x + 5$'

b $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a y-intercept of -2.

- Plot the y-intercept -2 on the y-axis.
- Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at (4, 1).
- Rule a line through this point and the y-intercept.



Example 14

Test whether each line is parallel to $y = -2x + 3$.

a $y = 2x + 3$ **b** $y = -2x + 1$ **c** $y = -2x$ **d** $y = 5x + 3$

Solution

Parallel lines have the **same gradient**. The line $y = -2x + 3$ has the gradient $m = -2$.

a $y = 2x + 3$ has a different gradient, 2, so it is not parallel to $y = -2x + 3$.

b $y = -2x + 1$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

c $y = -2x$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

d $y = 5x + 3$ has a different gradient, 5, so it is not parallel to $y = -2x + 3$.

$\therefore$ The lines **b** $y = -2x + 1$ and **c** $y = -2x$ are parallel to $y = -2x + 3$.

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EXERCISE 2.04 ANSWERS ON P.507

The gradient–intercept equation $y = mx + c$ U F R

EXAMPLE 11

1 Find the gradient and y-intercept of each line.

a $y = 3x - 2$ **b** $y = -2x + 7$ **c** $y = x + 4$ **d** $y = 9 - x$

e $y = \frac{3x}{4} + 6$ **f** $y = x$ **g** $y = \frac{x}{2} - 11$ **h** $y = \frac{2x + 18}{3}$

i $y = \frac{-24 - x}{3}$ **j** $y = 2(x - 3)$ **k** $11 - 3x = y$ **l** $\frac{2x - 7}{2} = y$

EXAMPLE 12

2 Find the equation of a line with:

a a gradient of 2 and a y-intercept of 1 **b** a gradient of $\frac{3}{4}$ and a y-intercept of 2

c a gradient of -7 and a y-intercept of 5 **d** a gradient of $-\frac{2}{5}$ and a y-intercept of 3

e $m = -2, c = -3$ **f** $m = -3, c = \frac{1}{2}$

EXAMPLE 13

3 Graph each linear equation by finding the gradient and y-intercept first.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = 2x$ **d** $y = \frac{x}{2} - 1$

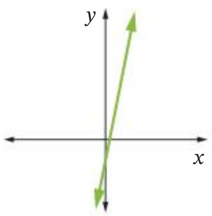
e $y = -2x + 3$ **f** $y = -\frac{3x}{4}$ **g** $y = \frac{-5x + 2}{2}$ **h** $y = \frac{3x - 20}{5}$

4 Write the equation of a line with a gradient of 2 that passes through the origin. R

5 Match each equation to its graph below. R

a $y = 4x + 1$ **b** $y = -4x + 1$ **c** $y = -4x - 1$ **d** $y = 4x - 1$

A



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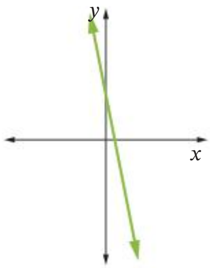
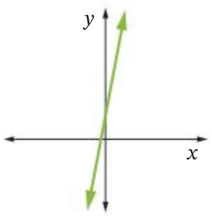
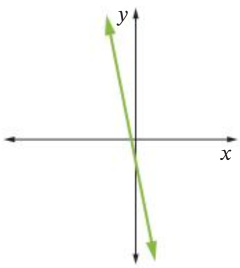


C



D



STAGE 5.2

EXAMPLE 14

6 For each given line, select the lines that are parallel. There may be more than one answer. R

a $y = x + 6$

A $y = 6x$ **B** $y = 6 - x$ **C** $y = x + 1$ **D** $y = 2x$

b $y = 3x + 10$

A $y = 10x + 3$ **B** $y = 3x - 1$ **C** $y = 1 - 3x$ **D** $y = 4 + 3x$

c $y = \frac{x}{2} + 5$

A $y = 2x - 1$ **B** $y = \frac{x+6}{2}$ **C** $y = 1 - \frac{x}{2}$ **D** $y = x + 2$

d $y = 6$

A $y = 2x + 6$ **B** $y = 6x$ **C** $y = -1$ **D** $y = 10$

e $y = 4x$

A $y = 4x - 2$ **B** $y = 4x + 3$ **C** $y = 4$ **D** $y = 1 - 4x$

f $x = 10$

A $y = 10$ **B** $y = 10x$ **C** $x = 2y$ **D** $x = -6$

7 For each set of linear equations, find a pair of equations whose graphs are parallel lines. R

a $y = 4x + 3$ $y = x + 2$ $y = 4x - 6$ $y = 2x$

b $y = 5x + 1$ $3x - y + 7 = 0$ $y = 3x - 2$ $y = -5x + 2$

Mental skills 2: Maths without calculators ANSWERS ON P. 508

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

- To find 10% or $\frac{1}{10}$ of a number, divide by 10
- To find 5% of a number, find 10% first, then halve it (since 5% is half of 10%).
- So to find 15% of a number, find 10% and 5% of the number separately, then add the answers together.

1 Study each example.

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80) = 8 + 4 = 12$

b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170) = \$17 + \$8.50 = \$25.50$

c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600) = 360 + 180 = 540$

d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28) = \$2.80 + \$1.40 = \$4.20$

2 Now find 15% of each amount.

a 120	**b** \$840	**c** 260	**d** \$202
e \$50	**f** 72	**g** \$180	**h** 400
i \$1600	**j** \$22	**k** 6000	**l** \$350

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To find $2\frac{1}{2}\%$ of a number, first find 5%, then halve it.

3 Study each example.

a $2\frac{1}{2}\% \times 600$

$10\% \times 600 = 60$

$5\% \times 600 = \frac{1}{2} \times 60 = 30$

$2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2}\% \times \$820$

$10\% \times \$820 = \$82$

$5\% \times \$820 = \frac{1}{2} \times 82 = \$41$

$2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

4 Now find $2\frac{1}{2}\%$ of each amount

a	400	**b**	6640	**c**	\$2000	**d**	\$880
e	1500	**f**	\$232	**g**	5400	**h**	\$904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

a $25\% \times 700$

$50\% \times 700 = \frac{1}{2} \times 700 = 350$

$25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$

$50\% \times \$86 = \frac{1}{2} \times \$86 = \$43$

$\therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

6 Now find 25% of each amount.

a	2000	**b**	\$80	**c**	18	**d**	\$25
e	\$324	**f**	\$140	**g**	66	**h**	298
i	\$780	**j**	\$1700	**k**	\$126	**l**	1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it. In other words, halve 3 times because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = \frac{1}{2} \times 400 = 200$

$25\% \times 400 = \frac{1}{2} \times 200 = 100$

$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$

$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$

$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

8 Now find $12\frac{1}{2}\%$ of each amount.

a	1280	**b**	\$12	**c**	60	**d**	\$260
e	\$540	**f**	\$250	**g**	304	**h**	1360

2.04

2.05 The general equation $ax + by + c = 0$

STAGE 5.2

WS Parallel and perpendicular lines

A linear equation written in **gradient–intercept form**, such as $y=-\frac{3}{4}x+2$, can also be written in **general form** ($3x + 4y - 8 = 0$). Note that, for the general form, all of the terms on the left-hand side of the equation are written with no fractions, and only 0 is on the right-hand side. Sometimes the general form is neater and more convenient.

The general form of a linear equation

The **general form of a linear equation** is written as $ax + by + c = 0$, where a, b and c are integers and a is positive.

The general equation $ax + by + c = 0$

Example 15

Write each linear equation in general form.

a $y = 6x + 2$ **b** $y=-\frac{2}{3}x+2$ **c** $y=2x-\frac{3}{5}$

Solution

a $y = 6x + 2$

$0 = 6x - y + 2$ Subtracting y from both sides.

$6x - y + 2 = 0$ Swapping sides so that 0 appears on the RHS.

b $y=-\frac{2}{3}x+2$

$3y=3\left(-\frac{2}{3}x+2\right)$ Multiplying both sides by 3 to remove the fraction.

$3y = -2x + 6$

$2x + 3y = 6$ Adding $2x$ to both sides.

$2x + 3y - 6 = 0$ Subtracting 6 from both sides.

c $y=2x-\frac{3}{5}$

$5y=5\left(2x-\frac{3}{5}\right)$ Multiplying both sides by 5.

$5y = 10x - 3$

$0 = 10x - 5y - 3$ Subtracting $5y$ from both sides.

$10x - 5y - 3 = 0$ Swapping sides so that 0 appears on the RHS.

STAGE 5.2

2.05

Example 16

Find the gradient and y-intercept of the line whose equation is $5x + 2y - 10 = 0$.

Solution

Rewrite $5x + 2y - 10$ in the form $y = mx + c$.

Make y the subject, solve for y.

$5x + 2y - 10 = 0$

$2y - 10 = -5x$ Subtracting $5x$ from both sides.

$2y = -5x + 10$ Adding 10 to both sides.

$y = \frac{-5x + 10}{2}$ Dividing both sides by 2.

$y = \frac{-5x}{2} + 5$

Aim to have y on its own on the LHS of the equation.

$\therefore$ Gradient: $m = -\frac{5}{2}$

y-intercept: $c = 5$

EXERCISE 2.05 ANSWERS ON P. 508

The general equation $ax + by + c = 0$ U F R C

EXAMPLE 15

1 Write each linear equation in general form. R C

- **a** $y = x + 2$ **b** $y = 3x - 1$ **c** $y = 8 + 5x$
- **d** $x + 2y = 3$ **e** $x - 2y = 6$ **f** $y = 8x + 2$
- **g** $y + 3 = 6x$ **h** $2y = x - 6$ **i** $y = \frac{3}{5}x + 2$

EXAMPLE 16

2 Find the gradient and y-intercept of the line with equation: R C

- **a** $2x + y = 6$ **b** $8x - 2y = 10$ **c** $3x - 2y + 4 = 0$
- **d** $y + 2x - 1 = 0$ **e** $2x + y + 5 = 0$ **f** $4x + 3y - 12 = 0$

3 Find the gradient, m, and the y-intercept, c, of the line with equation $x - 3y + 5 = 0$. Select the correct answer **A**, **B**, **C** or **D**.

A $m = -1, c = 5$ **B** $m = \frac{1}{3}, c = \frac{5}{3}$ **C** $m = 1, c = -5$ **D** $m = \frac{1}{3}, c = -\frac{5}{3}$

4 Which statement is FALSE about the line whose equation is $3x + y - 6 = 0$? Select **A**, **B**, **C** or **D**. R C

A Its gradient is -3. **B** Its y-intercept is -6.

C Its x-intercept is 2. **D** It is parallel to the line $y = -3x$.

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2.06 Finding the equation of a line

STAGE 5.2

Finding the equation of a line

Finding the equation of a line

A page of lines

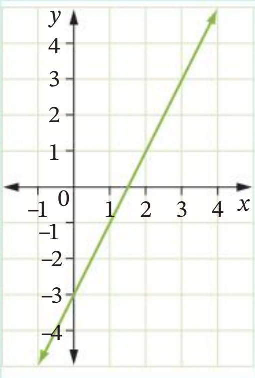
Straight-line equations

Finding the equation of a line

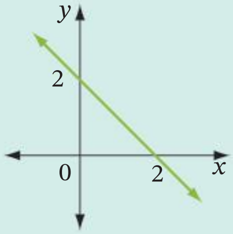
Example 17

Find the equation of each line.

a



b



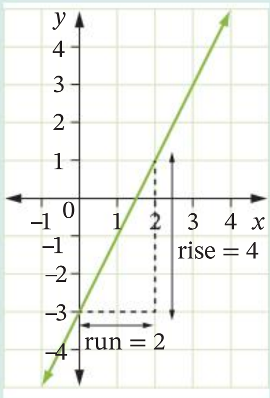
Solution

a Select 2 points on the line to find the gradient, say (0, −3) and (2, 1).

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ from the graph

y-intercept: $c = -3$

$\therefore$ The equation of the line is $y = 2x - 3$. $y = mx + c$



We can check that this equation is correct for any point on the line, say (3, 3).

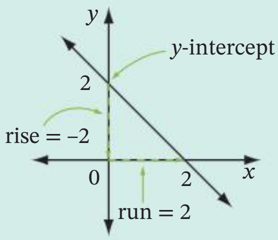
When $x = 3$, $y = 2 \times 3 - 3 = 3$.

b Find the gradient of the line passing through (0, 2) and (2, 0).

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$ from the graph

y-intercept: $c = 2$

$\therefore$ The equation of line is $y = -x + 2$. $y = mx + c$



EXERCISE 2.06 ANSWERS ON P.508

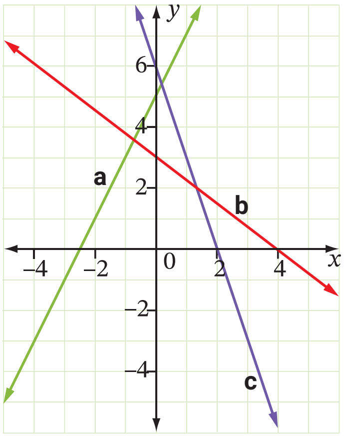
ANSWERS ON P.508

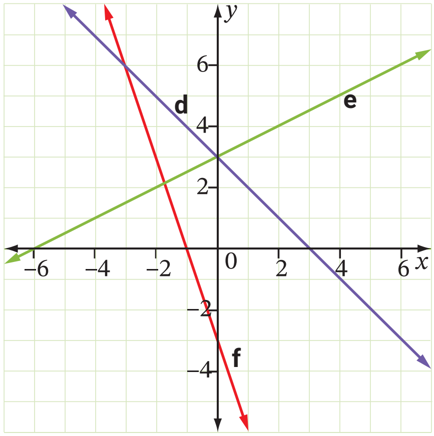
Finding the equation of a line U F R C

EXAMPLE 17

2.06

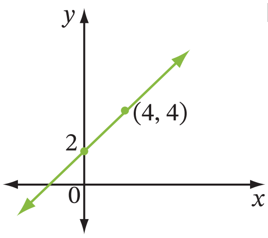
1 Find the equation of each line. R C





2 Find the equation of each line. R C

a



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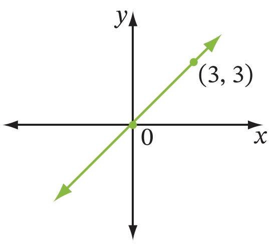

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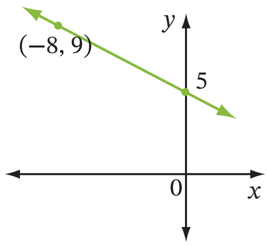

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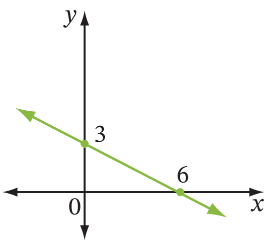

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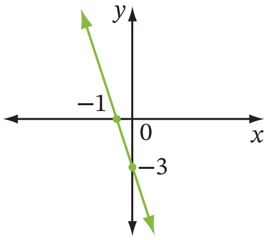

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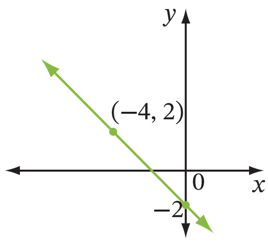

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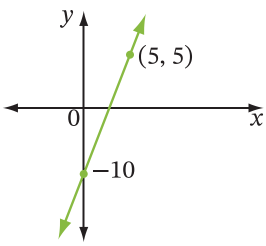

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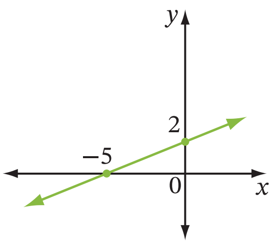

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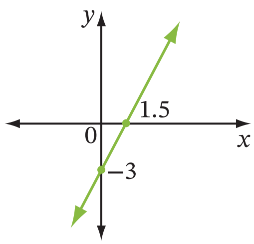

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Investigation

Sausage sizzle

A local football club is organising a sausage sizzle on Saturday to raise money to buy new equipment. It costs $50 to hire a gas bottle to run the barbecue and each sandwich costs $1 to make. They hope to sell 100 sandwiches.

Alamy Stock Photo/Boaz Rottem

1 Copy and complete this table to show the cost of making sausage sandwiches. Include the cost of hiring the gas bottle.

No. of sandwiches (x)	0	10	20	30	40	50	60	70	80	90	100
Cost ($$y$)	50	60									

2 Find the linear equation (formula) for y that represents the cost of making x sausage sandwiches.

3 Use an appropriate scale to construct a graph that shows the cost of making from $x = 0$ to 100 sandwiches. Label your axes and give your graph an appropriate title.

4 How much will it cost to make 35 sausage sandwiches?

5 How many sandwiches can be made for $132?

6 How much would it cost to make 120 sausage sandwiches?

7 **a** If the club sold all 100 sausage sandwiches for $5 each, how much money would they take?

b How much profit would the club make?

Equations of parallel and perpendicular lines

2.07

Shutterstock.com/ngoc tran

STAGE 5.2

Linear equations match-up

Equations of parallel lines

Gradients of parallel and perpendicular lines

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.

If 2 lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

Example 18

Find the equation of the line parallel to $y = 8 - 3x$ that passes through the point $(-1, 6)$.

Solution

For $y = 8 - 3x$ (or $y = -3x + 8$), the gradient is $m = -3$.

A line parallel to $y = 8 - 3x$, will also have $m = -3$.

The equation of this line is $y = mx + c = -3x + c$, where c is a constant.

To find the value of c, substitute the point $(-1, 6)$ into the equation:

$y = -3x + c$

$6 = -3 \times (-1) + c$ $\quad x = -1, y = 6$

$6 = 3 + c$

$c = 3$

$\therefore$ The equation is $y = -3x + 3$.

STAGE 5.2

Equations of perpendicular lines

Example 19

Find the equation of the line perpendicular to $3x - 4y + 6 = 0$ that passes through the point (5, 4).

Solution

To find the gradient of $3x - 4y + 6 = 0$, first convert it to the form $y = mx + c$:

$$3x - 4y + 6 = 0$$
$$3x + 6 = 4y$$
$$4y = 3x + 6$$
$$y = \frac{3x+6}{4}$$
$$y = \frac{3}{4}x + \frac{3}{2} \qquad y = mx + c$$

$\therefore$ Gradient $= \frac{3}{4}$

$\therefore$ Gradient of perpendicular line $= \frac{-1}{\left(\frac{3}{4}\right)}$ — The negative reciprocal of $\frac{3}{4}$.

$$= -\frac{4}{3}$$

$\therefore$ The equation of this line is $y = -\frac{4x}{3} + c$

To find the value of c, substitute the point (5, 4) into the equation.

$$4 = \left(-\frac{4}{3}\right) \times 5 + c \qquad x = 5, y = 4$$
$$= -\frac{20}{3} + c$$
$$4 + \frac{20}{3} = c$$
$$c = \frac{32}{3}$$

$\therefore$ The equation is $y = -\frac{4x}{3} + \frac{32}{3}$ or $y = \frac{-4x+32}{3}$ or, converting to the neater general form:

$$3y = -4x + 32$$
$$4x + 3y - 32 = 0$$

EXERCISE 2.07 ANSWERS ON P.508

Equations of parallel and perpendicular lines U F R C

EXAMPLE 18

1 Find the equation of the line that is parallel to: R C

a $y = 2x + 9$ and has a y-intercept of 4

b $y = 3x$ and has an x-intercept of -2

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STAGE 5.2

c $y = 5 - \frac{x}{2}$ and passes through (−1,6).

d $2x - y = 6$ and passes through (5, −2)

e $y = -5x - 8$ and passes through the midpoint of (3, −10) and (−5, −6)

f $2y = x - 3$ and passes through (6, −7)

2 Find the equation of a line that is perpendicular to: R C

EXAMPLE 19

2.07

a $y = \frac{x}{2}$ and has a y-intercept of −2

b $y = -5x$ and has an x-intercept of 1

c $y = 3x - 1$ and passes through the x-axis at 4

d $y = \frac{x-6}{3}$ and passes through (1, −6)

e $x + y - 6 = 0$ and passes through (−4, 2)

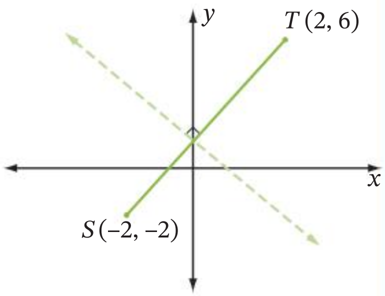
f $3x - y - 9 = 0$ and passes through (−10, −7)

3 a Find the gradient of interval ST in the diagram.

b Find the midpoint of ST.

c The dotted line is perpendicular to ST and passes through its midpoint. What is its gradient?

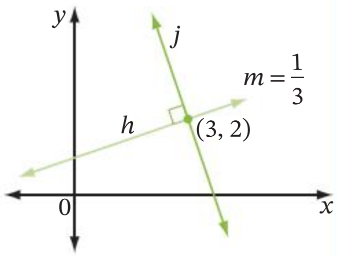
d Find the equation of the dotted line, in the form $y = mx + c$.



4 a Find the equation of line h in the diagram.

b Find the gradient of line j (which is perpendicular to line h).

c Find the equation of line j. R C



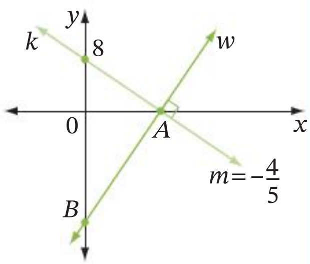
5 a Find the equation of line k.

b Find the coordinates of point A.

c Find the gradient of line w.

d Find the equation of line w.

e Find the coordinates of point B. R C



Did you know?

Constants

Expressions like '*k* is a constant' are often used in mathematics, but constants are also used in areas such as physics, chemistry, biology and astronomy. A constant may be:

- a numerical part of an algebraic expression. For example, in the expression $3x^2 + 5$, the 3 and 5 are constants and 5 is usually called the constant term.
- a quantity that has a fixed value for an expression or calculation. For example, in the equation of a line, $ax + by + c = 0$, the a, b and c are constants (while x and y are variables).
- a number or quantity that does not change in any circumstances. Examples are c (the speed of light) in the formula $E = mc^2$, and π.

Other constants that do not change include Faraday's constant, Planck's constant, Boltzmann's constant, Avogadro's number, 1 astronomical unit, and the gravitational constant.

1 **Find the symbol and value of each of the constants listed above.**

2 **Explain the meaning of the word 'constants' in this statement: 'There are only 2 constants in life—death and taxes'.**

Power plus ANSWERS ON P. 508

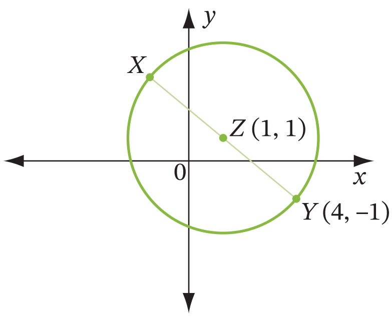
1 A line is drawn through the points $A(0, -2)$ and $B(3, 0)$. The x-coordinate of a point C on AB is 9. Find:

  a the gradient of AB **b** the equation of AB **c** the y-coordinate of C.

2 The point $(-1, 6)$ lies on the line $kx + 3y - 13 = 0$, where k is a constant number. Find k.

3 $Z(-1, 3)$ is the midpoint of the interval joining $A(-4, 7)$ and B. Find the coordinates of B.

4 The circle has XY as a diameter and centre Z. What are the coordinates of X?



5 Show that the points (4, 2), (10, –4) and (1, 5) are collinear.

CHAPTER 2 REVIEW

Language of maths

axes	distance	exact answer	general form
gradient	gradient–intercept form	horizontal	interval
length	linear	linear equation	midpoint
parallel	perpendicular	reciprocal	rise
run	surd	vertical	*x*-axis
x-intercept	*y*-axis	*y*-intercept	

Qz Graphing lines

1 What is the difference between the *y*-axis and the *y*-intercept?

2 When finding the length of an interval on a number plane, what is meant by an **exact** answer?

3 What measurement is the fraction given by the vertical rise of a line divided by the horizontal run?

4 What is the everyday meaning of the word **intercept**? Look it up in a dictionary.

5 What is the property of the gradients of perpendicular lines?

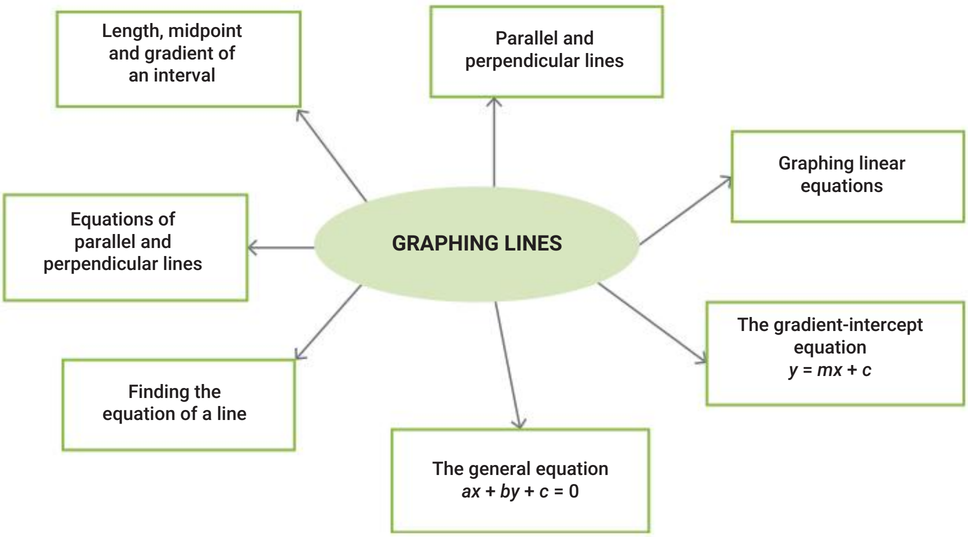
6 What form of the linear equation is $ax + by + c = 0$?

Topic summary

WS Mind map: Graphing lines

- How can you find the gradient of a line?
- What is $y = mx + c$?
- How can you test whether a pair of lines are parallel?
- What parts of this topic did you find difficult?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 2 ANSWERS ON P. 508

ANSWERS ON P. 508

2.01

1 An interval is formed by joining the points $K(5, 6)$ and $L(-7, 2)$.

a Find, correct to one decimal place, the length of interval KL.

b Find the midpoint of KL.

c Find the gradient of KL.

2.01

2 The vertices of a quadrilateral $HJKL$ are $H(-8, -5)$ $J(-1, -2)$ $K(2, 5)$ $L(-5, 2)$.

a Find the exact length of the sides of the quadrilateral.

b Find the gradient of each side of $HJKL$.

c Find the exact length of the diagonals HK and JL.

d What type of quadrilateral is $HJKL$?

2.02

3 A line passes through the points $V(8, -1)$ and $W(10, -2)$. What is the gradient of a line:

a parallel to VW? **b** perpendicular to VW?

2.03

4 Graph the linear equation $y = -5x - 1$ on a number plane.

2.03

5 Which point lies on the line of $3x + y = 2$? Select the correct answer **A**, **B**, **C** or **D**.

A $(1, 0)$ **B** $(2, 4)$ **C** $(-1, 5)$ **D** $(-1, -5)$

2.03

6 What is the equation of the line through $(-2, 3)$ and parallel to the x-axis? Select **A**, **B**, **C** or **D**.

A $x = -2$ **B** $x = 3$ **C** $y = -2$ **D** $y = 3$

STAGE 5.2

2.04

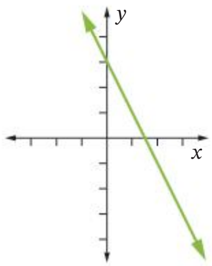
7 Write the gradient, m, and y-intercept, c, for each linear equation.

a $y = 2x - 10$ **b** $y = 4x + 3$ **c** $y = \frac{4 - 3x}{8}$

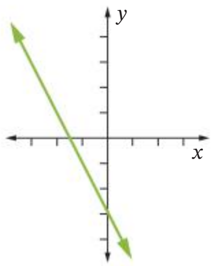
8 Match each equation to its graph below.

a $y = 2x - 3$ **b** $y = -2x - 3$ **c** $y = -2x + 3$ **d** $y = 2x + 3$

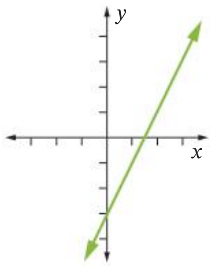
A



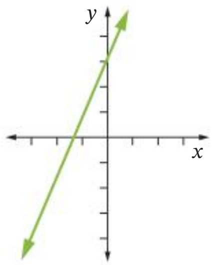
B



C



D



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STAGE 5.2

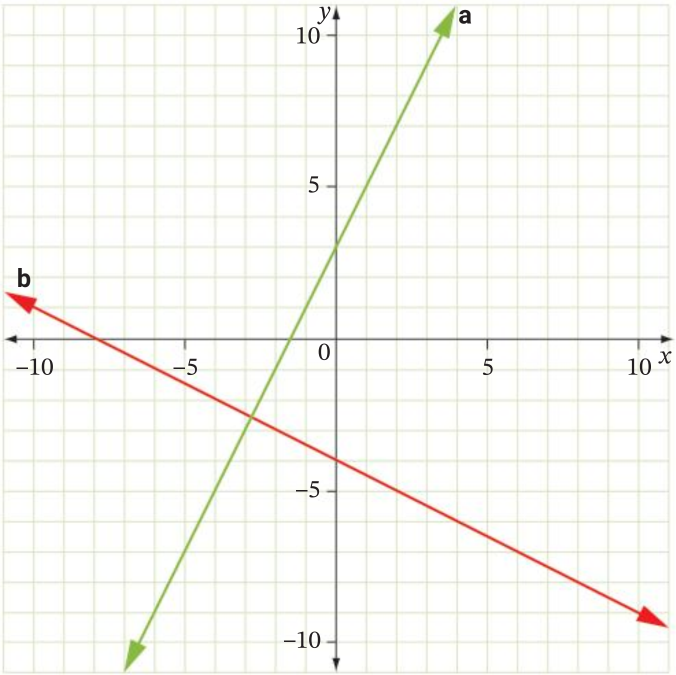
9 Convert each equation to general form $ax + by + c = 0$. 2.05

a $y = 3x + 5$ **b** $y = \frac{2x}{5} - 10$ **c** $x = 3y + 6$

10 Rewrite each equation in the form $y = mx + c$, then state the gradient, m, and the y-intercept, c. 2.05

a $x - y + 2 = 0$ **b** $2x - 8y + 8 = 0$ **c** $3x + y - 9 = 0$

11 Find the equation of each line and show that they are perpendicular. 2.06



12 Find the equation of a line that is: 2.07

a parallel to $y = 3x + 1$ and passes through the x-axis at 2

b perpendicular to $y = \frac{x}{2}$ and passes through the origin.

Foundation Standard Complex

3

MEASUREMENT AND GEOMETRY

SURFACE AREA AND VOLUME

Some theme parks have wave pools, which are big swimming pools that simulate the movement of the water at a beach. A large volume of water is quickly released into one end of the pool, which produces a large wave that moves from one end of the pool to the other. The excess water in the pool is recycled so that it can be used to produce more waves.

Chapter outline

		Working mathematically				
3.01	Areas of composite shapes	U	F	PS	R	
3.02	Surface area of a prism	U	F	PS	R	C
3.03	Surface area of a cylinder*	U	F	PS	R	
3.04	Surface areas of composite solids*	U	F	PS	R	
3.05	Volumes of prisms and cylinders	U	F		R	

*STAGE 5.2

Wordbank

capacity The amount of fluid (liquid or gas) in a container

composite shape A shape made up of 2 or more basic shapes

cross-section A 'slice' of a solid, taken across the solid rather than along it

curved surface area The area of the curved surface of a solid such as a cylinder or sphere. For example, the curved surface of a cylinder is a rectangle when flattened.

cylinder A can-shaped solid with identical cross-sections that are circles

prism A solid with identical cross-sections that are polygons

sector A region of a circle cut off by 2 radii, shaped like a piece of pizza

surface area The total area of all faces of a solid shape

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

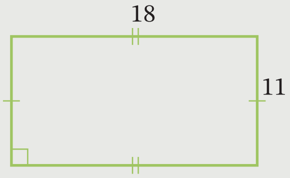
In this chapter you will:

- calculate the areas of triangles, quadrilaterals, circles, sectors and composite shapes
- calculate the surface areas of rectangular and triangular prisms
- (STAGE 5.2) calculate the surface areas of right prisms and cylinders
- calculate the volumes and capacities of right prisms and cylinders
- (STAGE 5.2) calculate the surface areas and volumes of composite solids

SkillCheck ANSWERS ON P. 509

1 Calculate the area of each shape. All measurements are in centimetres.

a



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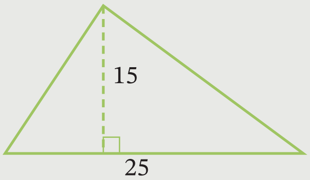

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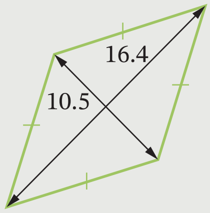

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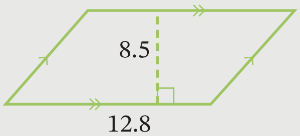

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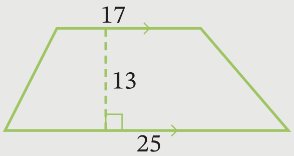

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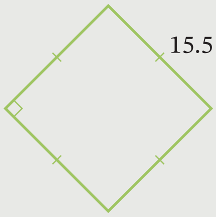

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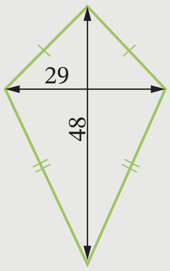

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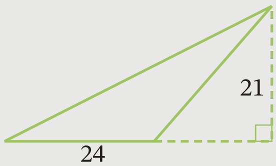

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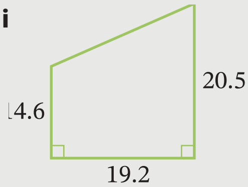
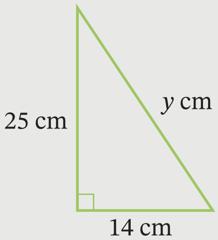


2 Use Pythagoras' theorem to find, correct to one decimal place, the value of each variable.

a



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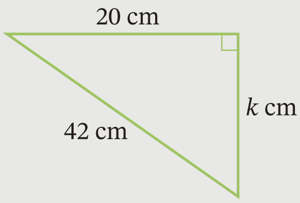

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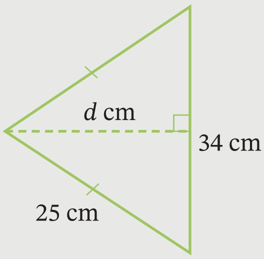
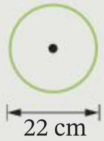


3 For each circle, find correct to one decimal place:

i its circumference   ii its area.

a


b


c


4 Calculate the volume of each solid. All measurements are in metres.

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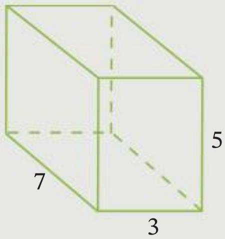

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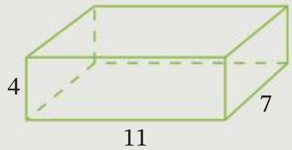

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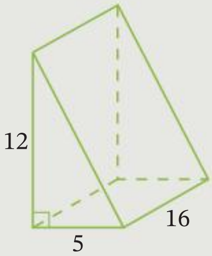

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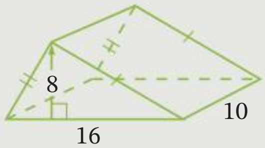

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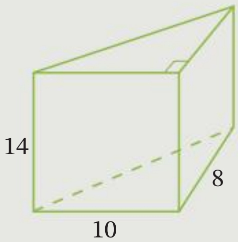


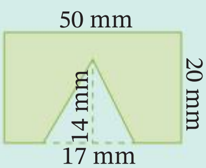
Solid shapes

What is volume?

Areas of composite shapes 3.01

Example 1

Find the area of each composite shape, correct to one decimal place where appropriate.

a


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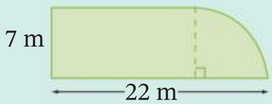

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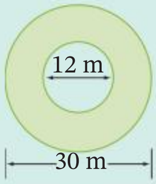


Area

A page of circular shapes

Solution

a Area $= 50 \times 20 - \frac{1}{2} \times 17 \times 14$ Area of rectangle – area of triangle

$= 881\ \text{mm}^2$

b The shape is made up of a rectangle and a quadrant.

Radius of quadrant $= 7$ m

Length of rectangle $= 22 - 7$

$= 15$ m

Area of shape = area of rectangle + quadrant

$= 15 \times 7 + \frac{1}{4} \times \pi \times 7^2$

$= 143.4845...$

$\approx 143.5\ \text{m}^2$

c This ring shape is an **annulus**, its area is enclosed by 2 circles with the same centre.

Radius of large circle $= \frac{1}{2} \times 30$ m

$= 15$ m

Radius of small circle $= \frac{1}{2} \times 12$ m

$= 6$ m

Area of annulus $= \pi \times 15^2 - \pi \times 6^2$ Large circle – small circle

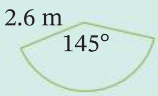
$= 593.7610....$

$= 593.8\ \text{m}^2$

Example 2

Calculate, correct to 2 decimal places, the area of each sector.

a



b

A **sector** is a fraction of a circle 'cut' along 2 radii, like a pizza slice.

Solution

a Area $= \frac{145}{360} \times \pi \times 2.6^2$ $\frac{145}{360} \times$ area of circle

$= 8.55385...$

$\approx 8.55\ \text{m}^2$

There are 360° in a circle, but a sector is a fraction of a circle

b Sector angle $= 360° - 115°$

$= 245°$

Area of sector $= \frac{245}{360} \times \pi \times 9^2$

$= 173.18029...$

$\approx 173.18\ \text{m}^2$

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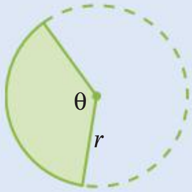
Area of a sector

$$\text{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$



3.01

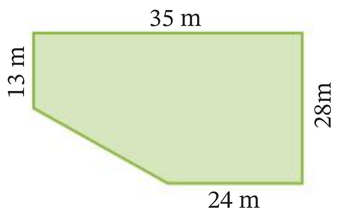
EXERCISE 3.01 ANSWERS ON P. 509

Area of composite shapes U F PS R

EXAMPLE 1

1 Find the area of each composite shape. PS R

a



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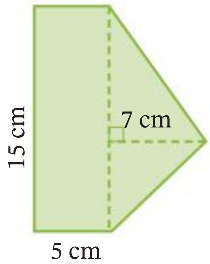

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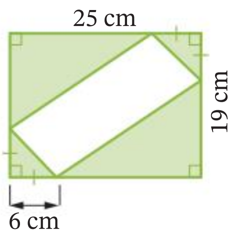

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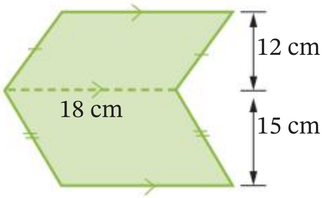

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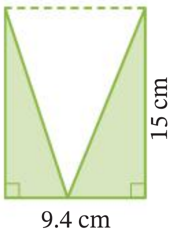

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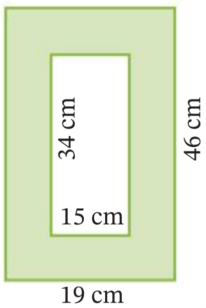

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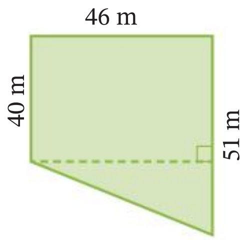

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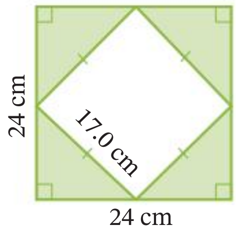

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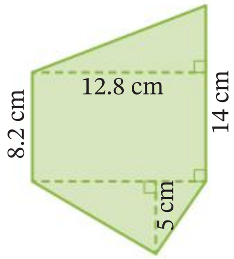

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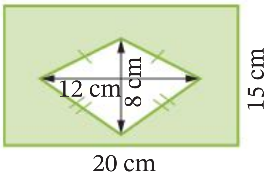

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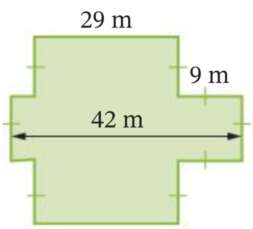

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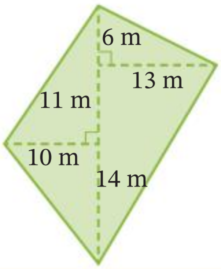

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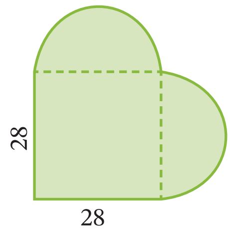
2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres. PS R

a



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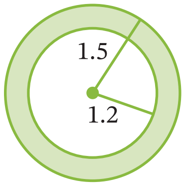

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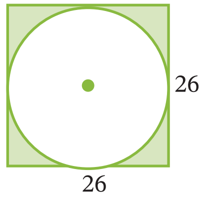

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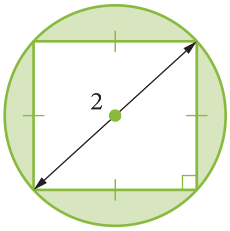

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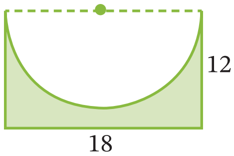

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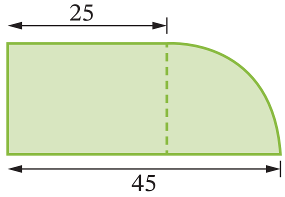

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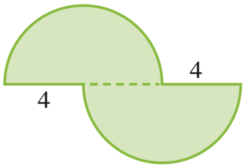

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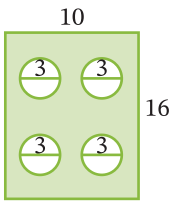

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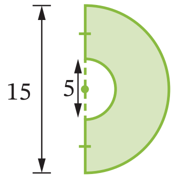

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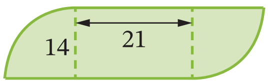

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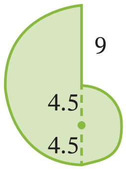

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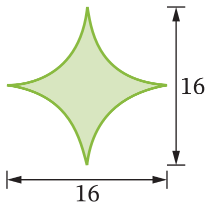
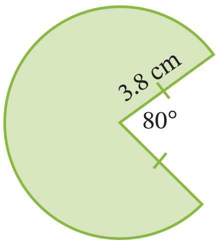


EXAMPLE 2

3 Find, correct to one decimal place, the area of each sector.

a



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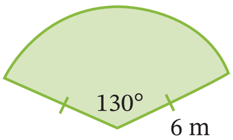

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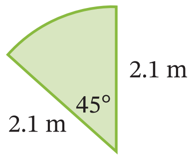

4 A bike tyre has a diameter of 715mm.

a How far will the bike travel in one revolution of the tyre? Give your answer in metres, correct to 2 decimal places.

b How many revolutions of the tyre are required to travel a distance of 5 km?

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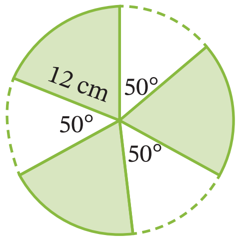
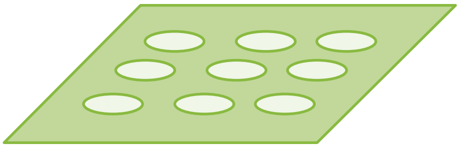
5 Calculate the area of the shaded region.
Select the correct answer **A**, **B**, **C** or **D**. R

A 362.7 cm² **B** 452.4 cm²

C 188.5 cm² **D** 263.9 cm²



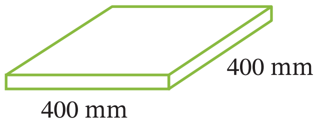
6 A rectangular metal plate with dimensions 2.5 m × 2.2 m has 9 holes of diameter 46 cm drilled in it.



a Find the total area of the holes that have been drilled. Give your answer in m², correct to 2 decimal places.

b What percentage of the metal plate remains?

7 A rectangular courtyard 15 m long and 8 m wide is to be covered with square pavers of side length 400 mm, costing \$79.29/m². PS R



a What is the area of one paver, in m²?

b How many pavers will be required to pave the courtyard?

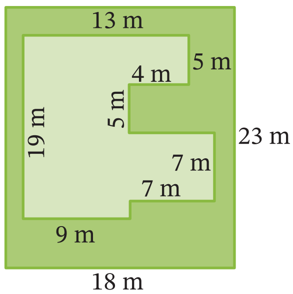
c Calculate the cost of paving the courtyard.

8 A circular sports ground of diameter 140 m has a rectangular soccer pitch measuring 110 m by 70 m inside it. The area outside the soccer pitch is to be painted in the team colour of red.

a Calculate the area that is to be painted red, correct to the nearest m².

b If the cost of paint is \$29.50 per 50 m², calculate the cost of painting this area.

9 The diagram shows the floor plan of a house on a block of land. PS R



a Calculate the area of the block.

b Calculate the area taken up by the house.

c What percentage of the area of the block is taken up by the house?

d The area not covered by the house and is to be turfed. Find the cost of turfing the yard at a cost of \$11.75/m².

3.01

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3.02 Surface area of a prism

WS Surface area

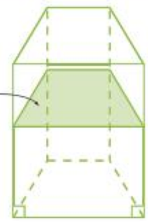
WS Nets of solids

Surface area of a prism

Surface area of prisms

A **cross-section** of a solid is a 'slice' of the solid cut *across* it, parallel to its end faces, rather than along it. A **prism** has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides).

The trapezoidal prism shown here has cross-sections that are trapeziums.



A right prism

Surface area

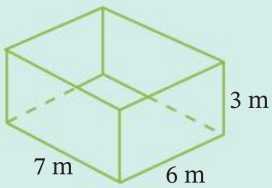
The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

It is often useful to draw the net of a solid when finding its surface area. A net may be used to form an open solid or a closed solid. A sealed cardboard box is an example of a **closed solid**. A cardboard box with the lid removed is an example of an **open solid**. For the surface area of an open solid, we only count the *external* surfaces, not the internal ones (so that we don't count each surface twice).

Example 3

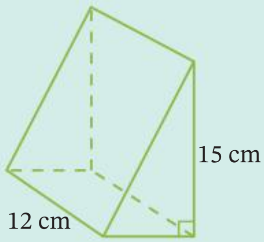
Find the surface area of each prism.

a



Open rectangular prism

b



Closed triangular prism

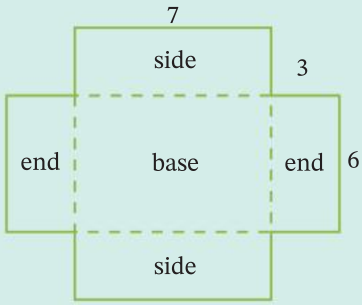
Solution

a This open prism has 5 faces.

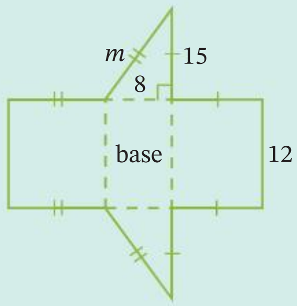
Surface area = 2 ends + 2 sides + base

$= 2 \times (3 \times 6) + 2 \times (3 \times 7) + (6 \times 7)$

$= 120\text{ m}^2$



b This closed prism has 5 faces: two identical triangles (front and back) and three different rectangles.



Using Pythagoras' theorem to find m, the hypotenuse of the triangle:

$m^2 = 8^2 + 15^2$

$= 289$

$m = \sqrt{289}$

$= 17$

Surface area = 2 triangles + 3 rectangles

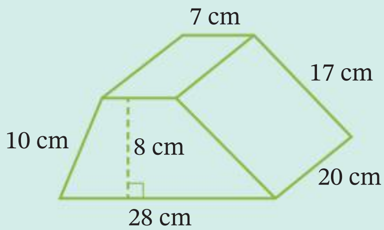
$$= 2 \times \left(\frac{1}{2} \times 8 \times 15\right) + (17 \times 12) + (8 \times 12) + (15 \times 12)$$

$= 600\text{ cm}^2$

Example 4

STAGE 5.2

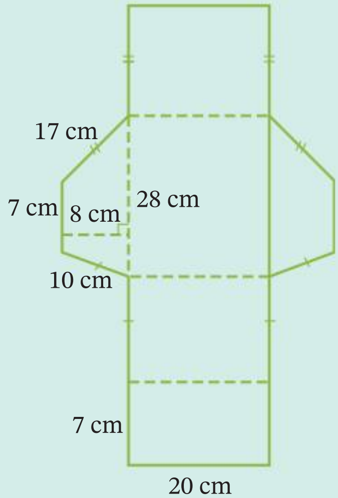
Calculate the surface area of this trapezoidal prism.



Solution

This trapezoidal prism has 6 faces:

2 identical trapeziums (front and back) and 4 different rectangles.



Area of each trapezium $= \frac{1}{2} \times (7 + 28) \times 8$

$= 140\text{ cm}^2$

Surface area $= (2 \times 140) + (20 \times 7) + (20 \times 10) + (20 \times 28) + (20 \times 17)$

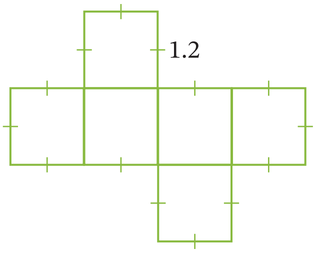
$= 1520\text{ cm}^2$

EXERCISE 3.02 ANSWERS ON P. 509

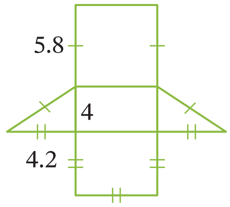
Surface area of a prism U F PS R C

1 Identify the prism that each net represents, then calculate the surface area of the prism. All lengths are in metres. R C

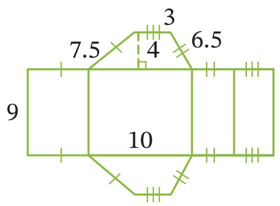
a



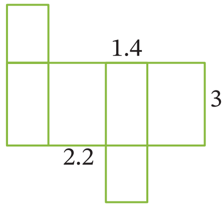
b



c



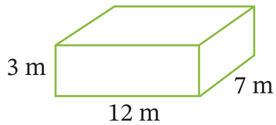
d



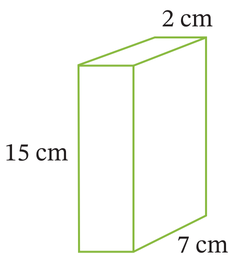
EXAMPLE 3

2 Find the surface area of each prism.

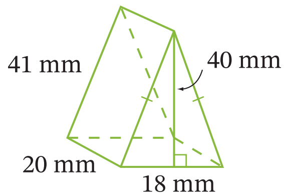
a



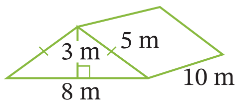
b



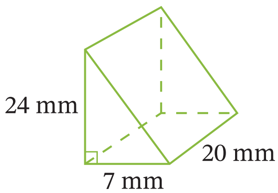
c



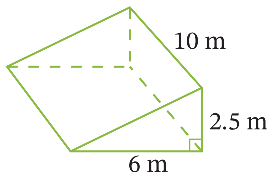
d



e



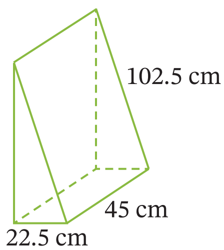
f



3 Calculate the surface area of this triangular prism. Select the correct answer **A**, **B**, **C** or **D**.

A 12 375 cm^2 **B** 11 250 cm^2

C 10 125 cm^2 **D** 12 431.25 cm^2



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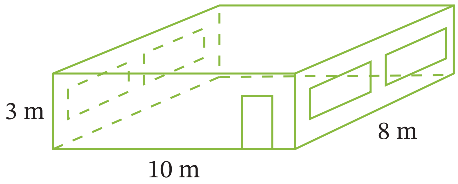
4 This classroom is being renovated. Find: PS

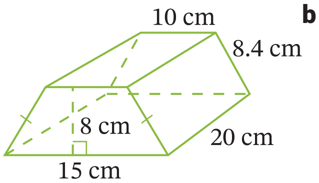


a the area of the floor to be carpeted and the total cost, at $55 per square metre.

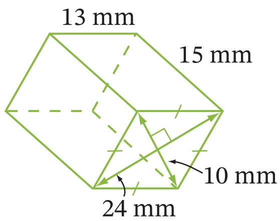
b the ceiling and wall area to be painted, if the room contains 4 windows, each 2.5 m by 1.5 m, and a doorway 2 m by 0.8 m.

5 Calculate the surface area of each prism.

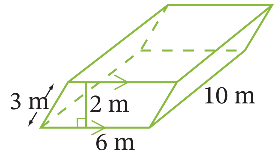
a



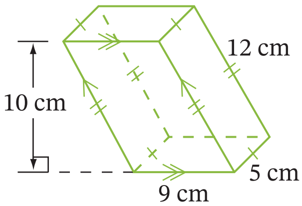
b



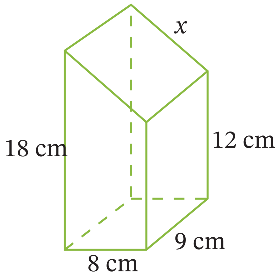
c



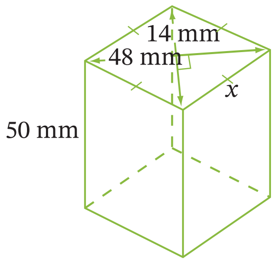
d



e



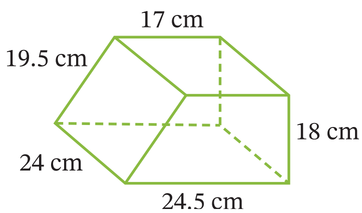
f



6 Calculate the surface area the trapezoidal prism. Select **A**, **B**, **C** or **D**.

A 10 584 cm² **B** 2643 cm²

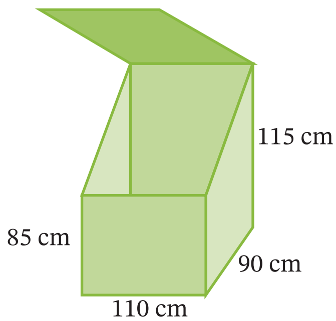
C 2082.75 cm² **D** 8964 cm²



7 The wooden toy box is in the shape of a trapezoidal prism.

a Calculate how much timber is required to make the toy box, correct to the nearest cm².

b If the price of the timber is $25/m², calculate the cost of making the box.



3.02

STAGE 5.2

EXAMPLE 4

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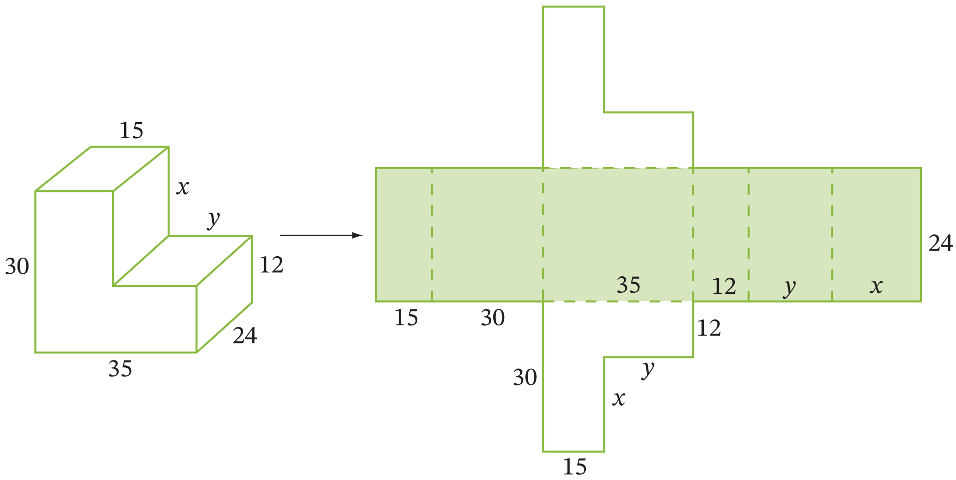
Investigation

A surface area shortcut

1 Consider this L-shaped prism and its net. We will find its surface area.



a Find x and y.

b This prism has 8 faces: 2 'L-shaped' ends and 6 rectangles. Instead of calculating the areas of the 6 rectangles separately, we can combine them into one long rectangle, as shaded in the net above. The length of the rectangle is the same as the perimeter of the L-shape. What is the length of this long rectangle?

c What is the area of this long rectangle?

d Copy and complete:

Length of shaded rectangle = p ______________ of the L-shape.

e Find the surface area of the prism by copying and completing the following:

Surface area = 2 'L-shaped' ends + shaded rectangle

$= 2 \times (15 \times 30 + 20 \times 12) +$ _________

$=$ ________

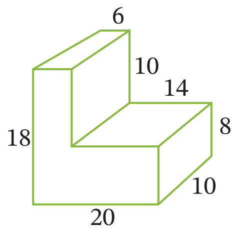
2 From question **1**, it can be seen that the surface area of any prism with end faces of area A and perpendicular height (distance between end faces) h can be calculated using the formula:

$$SA = 2A + Ph$$

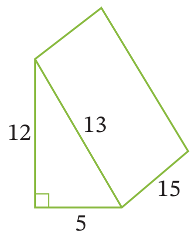
where P = perimeter of the end face.

Use this method to calculate the surface area of each prism. All measurements are in centimetres.

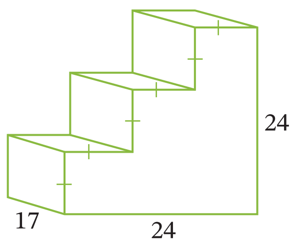
a



b



c



Surface area of a cylinder

3.03

STAGE 5.2

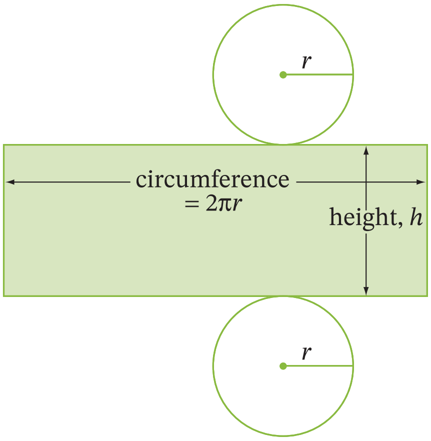
Surface area

Surface area of a cylinder

A closed cylinder has 3 faces made up of 2 circles (the circular ends) and a rectangle (the curved surface). The length of the rectangle is the **circumference** of the circular end, while the width of the rectangle is the height of the cylinder.

Surface area of a cylinder = area of 2 circles + area of rectangle

$$SA = 2 \times \pi r^2 + 2\pi r \times h$$
$$= 2\pi r^2 + 2\pi rh$$



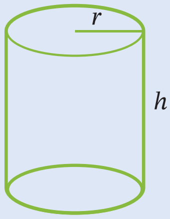
Surface area of a closed cylinder

$$SA = 2\pi r^2 + 2\pi rh$$

where r = radius of circular base

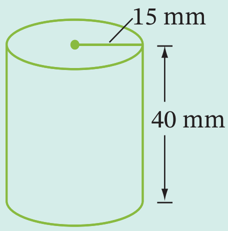
h = perpendicular height

The area of the 2 circular ends = $2\pi r^2$
and the area of the curved surface = $2\pi rh$.



Example 5

Find, correct to the nearest mm², the surface area of this cylinder.



Solution

Surface area = area of 2 ends + area of the curved surface $\quad r = 15, h = 40$

$$= 2\pi r^2 + 2\pi rh$$
$$= 2 \times \pi \times 15^2 + 2 \times \pi \times 15 \times 40$$
$$= 5183.627...$$
$$\approx 5184 \text{ mm}^2$$

STAGE 5.2

Example 6

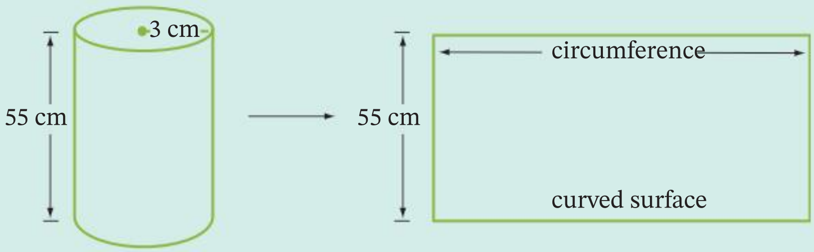
Find, correct to 2 significant figures, the surface area of:

a a cylindrical tube, open at both ends, with radius 3 cm and height 55 cm

b an open half-cylinder with radius 0.65 m and length 2.4 m.

Solution

a



Surface area = curved surface

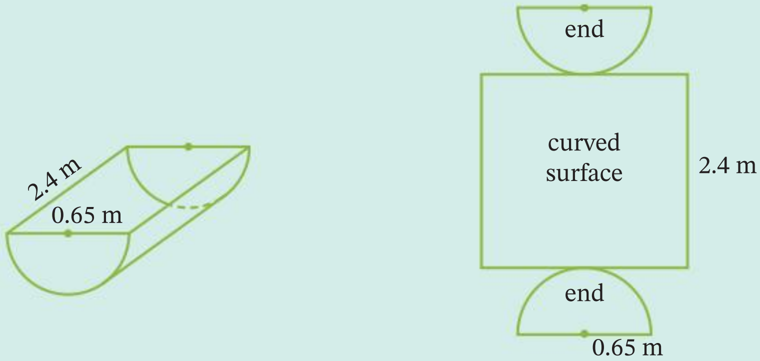
$= 2\pi rh$ $r = 3$ and $h = 55$

$= 2 \times \pi \times 3 \times 55$

$= 1036.725...$

$= 1000\ \text{cm}^2$

b



Surface area = 2 semicircle ends + $\frac{1}{2}$ × curved surface

$= 2 \times \left(\frac{1}{2} \times \pi \times 0.65^2\right) + \frac{1}{2} \times (2 \times \pi \times 0.65 \times 2.4)$

$= 6.2282...$

$\approx 6.2\ \text{m}^2$

EXERCISE 3.03 ANSWERS ON P. 509

Surface area of a cylinder U F PS R

STAGE 5.2

3.03

1 Calculate, correct to one decimal place, the surface area of a cylinder with: EXAMPLE 5

a radius 1.4 m, height 2.2 m **b** diameter 45 cm, height 65 cm

c diameter 9 cm, height 24 cm **d** radius 1.3 m, height 3.8 m

2 Find, correct to the nearest whole number, the curved surface area of a cylinder with:

a radius 1.5 m, height 3.75 m **b** diameter 27 cm, height 41 cm

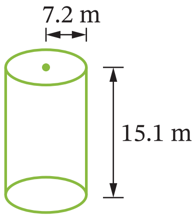
3 A container of potato chips is a cylinder with diameter 7 cm and height 23.2 cm. Calculate its surface area, correct to one decimal place.

4 Find the surface area of a cylinder, open at one end with diameter 12 mm and length 15 cm. Select the closest answer **A**, **B**, **C** or **D**. **R** EXAMPLE 6

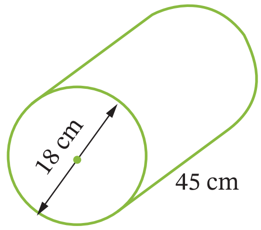
A 678.6 cm^2 **B** 1017 cm^2 **C** 6107.3 mm^2 **D** 5768.0 mm^2

5 Calculate, correct to the nearest whole number, the surface area of each solid. **R** EXAMPLE 6

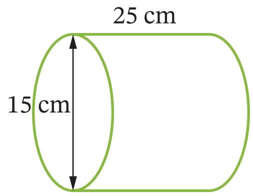
a closed cylinder



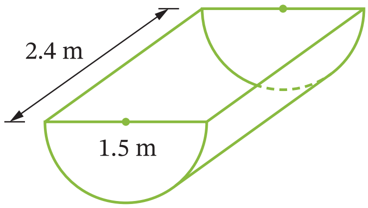
b closed cylinder



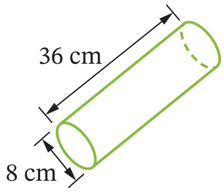
c cylinder with one open end



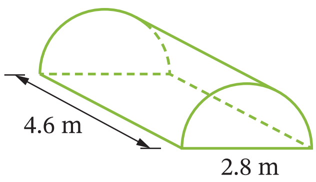
d half cylinder with open top



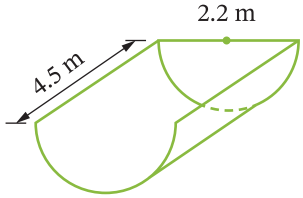
e cylinder open both ends



f closed half cylinder



g half cylinder with open top, one end open



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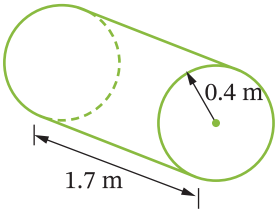
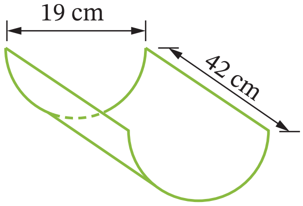
STAGE 5.2

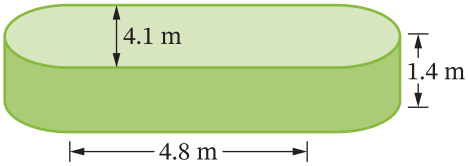
h closed cylinder



i half cylinder, open both ends



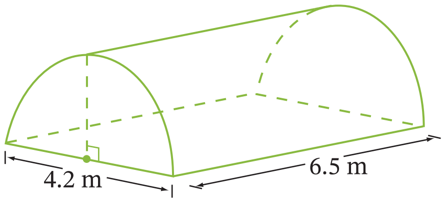
6 The inside of the swimming pool, including the floor, is to be repainted. Find: PS R



a the area to be repainted, correct to one decimal place

b the number of whole litres of paint needed if coverage is 9 m^2 per litre.

7 The diagram shows a tent to be made in the shape of half a cylinder. Find: PS R



a the area of the floor of the tent

b the surface area of the tent, excluding the floor.

c the total cost of materials for the tent if the material for the flooring costs \$18.50/m^2 and the canvas for the tent costs \$21.75/m^2.

Alamy Stock Photo/Robert Wyatt

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Surface area of composite solids 3.04

STAGE 5.2

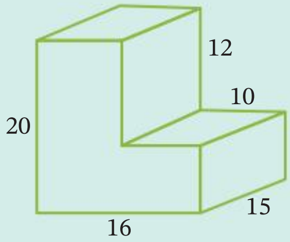
WS A page of prisms and cylinders

When calculating surface areas of composite solids, remember **not** to include the areas common to both solids.

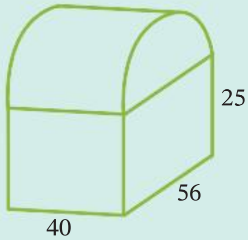
Example 7

Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres.

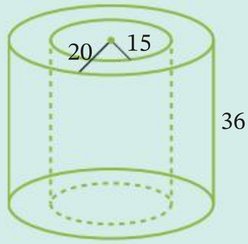
a



b



c

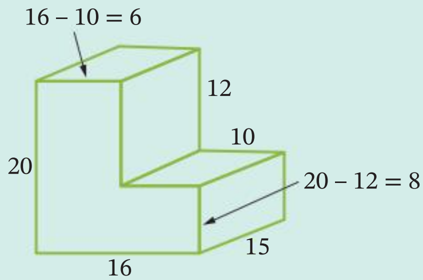


Solution

a This prism has 8 faces: 2 identical L-shapes (front and back) and 6 different rectangles.

Area of L-shape $= 16 \times 20 - 10 \times 12$

$= 200 \text{ cm}^2$



Surface area = Front and back L-faces + 1st top + 1st right + 2nd top + 2nd right + bottom + left

$= (2 \times 200) + (6 \times 15) + (12 \times 15) + (10 \times 15) + (8 \times 15) + (16 \times 15) + (20 \times 15)$

$= 1480 \text{ cm}^2$

Note that the 6 rectangles can also be thought of as one long rectangle of width 15 cm.

Length of long rectangle = perimeter of L

$= 6 + 12 + 10 + 8 + 16 + 20$

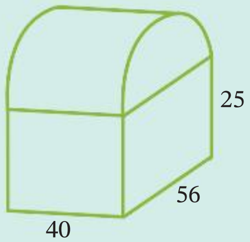
$= 72$

Surface area $= (2 \times 200) + (72 \times 15)$

$= 1480 \text{ cm}^2$

STAGE 5.2

b The solid is made up of a half-cylinder (3 faces) and a rectangular prism (5 faces).



Surface area of half-cylinder

$= 2$ semi-circular ends + curved surface area

$= 2 \times \frac{1}{2} \times \pi \times 28^2 + \frac{1}{2} \times 2 \times \pi \times 28 \times 40$

$= 5981.5924...\text{ cm}^2$

Radius of semi-circle $= \frac{1}{2} \times 56$

$= 28$

Do not round this partial answer, or the final answer will be inaccurate.

Surface area of rectangular prism = Front and back faces + 2 side faces + bottom face

$= (2 \times 40 \times 25) + (2 \times 56 \times 25) + (40 \times 56)$

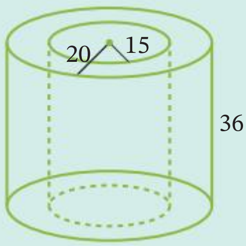
$= 7040\text{ cm}^2$

Total surface area $= 5981.5924... + 7040$

$= 13\,021.5924...$

$\approx 13\,021.6\text{ cm}^2$

c The hollow cylinder is made up of 2 annulus (ring) faces, an outside curved surface area and an inside curved surface area.



Surface area of annulus faces $= 2 \times (\pi \times 20^2 - \pi \times 15^2)$ 2 × area between 2 circles

$= 1099.5574...\text{ cm}^2$

Outside curved surface area $= 2 \times \pi \times 20 \times 36$

$= 4523.8934...\text{ cm}^2$

Inside curved surface area $= 2 \times \pi \times 15 \times 36$

$= 3392.9200...\text{ cm}^2$

Total surface area $= 1099.5574... + 4523.8934... + 3392.9200...$

$= 9016.3709...$

$= 9016.4\text{ cm}^2$

EXERCISE 3.04 ANSWERS ON P. 509

STAGE 5.2

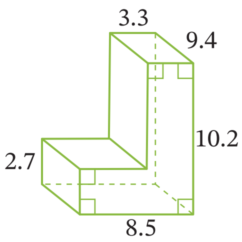
Surface areas of composite solids U F PS R

EXAMPLE 7

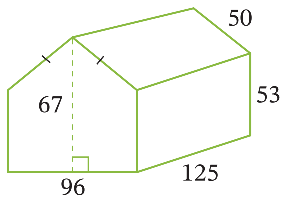
3.04

1 Find the surface area of each prism. All measurements are in centimetres. PS R

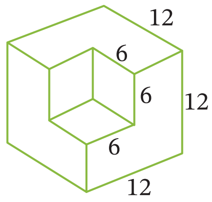
a



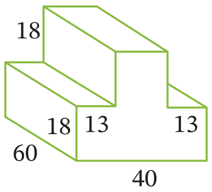
b



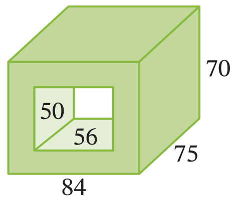
c



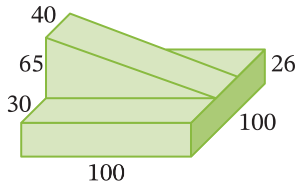
d



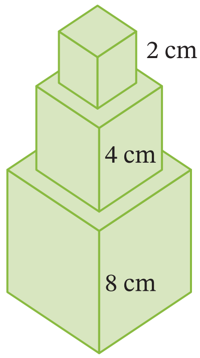
e



f



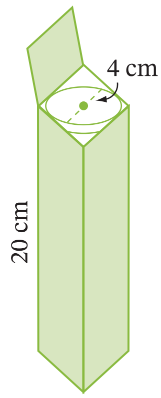
2 3 cubes of length 2 cm, 4 cm and 8 cm are glued on top of each other. Calculate the surface area of the new solid. PS R



3 Circular cracker biscuits of diameter 4 cm are packed in a cardboard box of length 20 cm.

a Calculate the surface area of the box.

b How much cardboard would be saved if the biscuits were packed into a cylindrical box?



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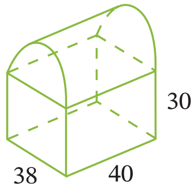
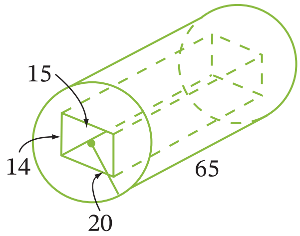
STAGE 5.2

4 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres. PS R

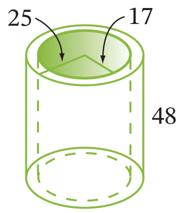
a



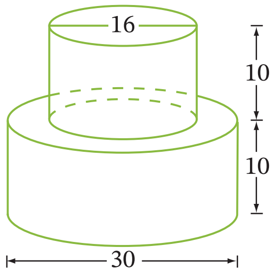
b



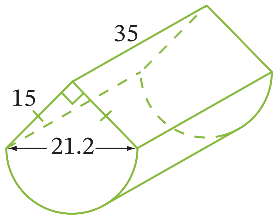
c



d



e



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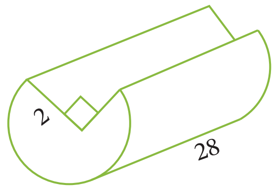
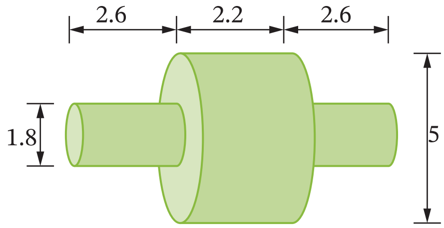


5 Calculate the surface area of the solid, correct to one decimal place. All measurements are in centimetres. R

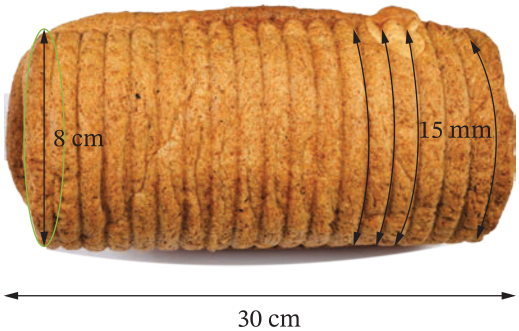


Select the correct answer **A**, **B**, **C** or **D**.

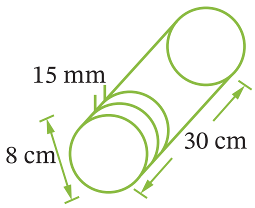
A 86.0 m^2 **B** 103.2 m^2

C 108.3 m^2 **D** 113.4 m^2

6 A cylindrical loaf of bread that is 30 cm long with a diameter of 8 cm is cut into slices 15 mm thick. R



Alamy Stock Photo/Clynt Garnham Food & Drink



a Calculate the surface area of the loaf of bread before it is sliced, correct to 2 decimal places.

b Find the number of slices in a loaf.

c Calculate the surface area of each slice, correct to the nearest cm^2.

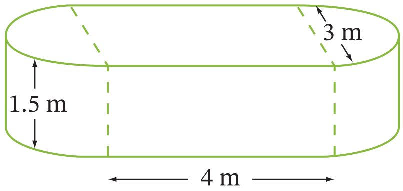
Foundation Standard Complex

7 A wedding cake with 3 tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total visible surface area, correct to the nearest cm². PS R

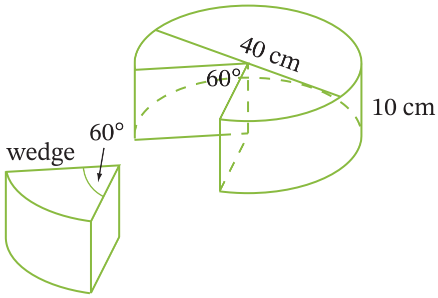
Shutterstock.com/Brian Barney

8 **a** Find, correct to 2 decimal places, the total external area of the wall of this above-ground swimming pool.

b Calculate the area of the water surface, correct to the nearest m². PS R



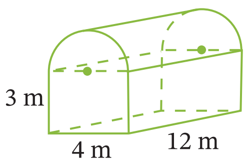
9 A wedge of cheese is cut from a cylindrical block of height 10 cm and diameter 40 cm. Find the total surface area of the wedge, correct to 2 decimal places. PS R



10 The curved roof of a greenhouse is to be covered in shade cloth.
PS R

a Calculate, correct to one decimal place, the area of shade cloth needed if there are no overlaps.

b Shade cloth is sold in 1.5 m wide rolls. How many linear metres of shade cloth are needed to cover the curved roof? Answer to the nearest 0.1 m.



STAGE 5.2

3.04

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Mental skills 3: Maths without calculators ANSWERS ON P. 509

Time differences

1 Study each example.

a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

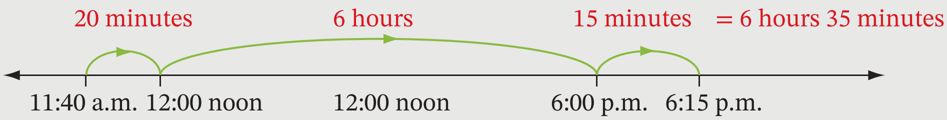
Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

5 hours + 20 min + 15 min = 6 hours 35 min

OR:



b What is the time difference between 20:30 and 01:20?

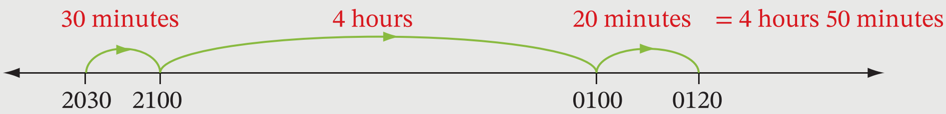
From 20:30 to 00:30 = 4 hours (24 – 20 = 4)

From 00:30 to 01:00 = 30 min

From 01:00 to 01:20 = 20 min

4 hours + 30 minutes + 20 minutes = 4 hours 50 minutes

OR:



2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.

b 6:20 pm. and 12:00 midnight

c 4:45 p.m. and 8:10 p.m.

d 2:35 a.m and 10:50 a.m.

e 1:05 p.m. and 12:30 a.m.

f 9:35 a.m. and 11:15 a.m.

g 04:25 and 09:35

h 14:40 and 20:25

i 7:55 a.m. and 3:50 p.m.

j 2:40 p.m. and 10:20 p.m.

Volumes of prisms and cylinders 3.05

The **volume** of a solid is the amount of space it takes up. Volume is measured in **cubic units**, for example, cubic metres (m^3) or cubic centimetres (cm^3).

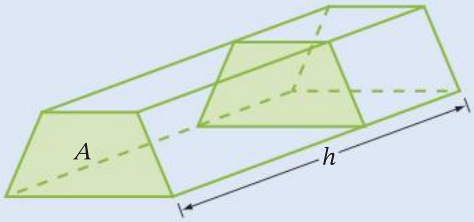
As a **prism** is made up of identical cross-sections, its volume can be calculated by multiplying the **area of its base** by its **perpendicular height** (the length or depth of the prism).

Volume of a prism

$V = Ah$

where A = area of base

h = perpendicular height



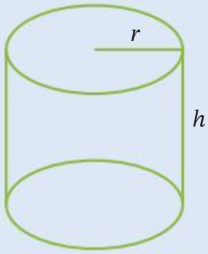
A **cylinder** is like a 'circular prism' because its cross-sections are identical circles. Because of this, we can also use $V = Ah$ to find the volume of a cylinder. But for a circle, $A = \pi r^2$, so:

Volume of a cylinder

$V = \pi r^2 h$

where r = radius of circular base

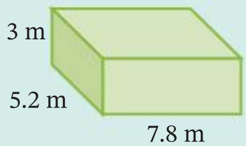
h = perpendicular height



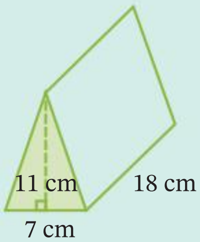
Example 8

Find the volume of each prism.

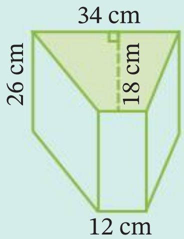
a



b



c



Solution

a $V = 7.8 \times 5.2 \times 3$

$= 121.68\ m^3$

For a rectangular prism,

Volume = length × width × height

$= lwh$

Volumes of prisms and cylinders

WS Volumes of solids

WS Volume and capacity

Formula matching game

Volumes of shapes

3.05

b $A = \frac{1}{2} \times 7 \times 11$ — Area of a triangle

$= 38.5 \text{ cm}^2$

$V = 38.5 \times 18$ — $V = Ah$, where height $h = 18$

$= 693 \text{ cm}^3$

c $A = \frac{1}{2} \times (12 + 34) \times 18$ — Area of a trapezium

$= 414 \text{ cm}^2$

$V = 414 \times 26$ — $V = Ah$, where height $h = 26$

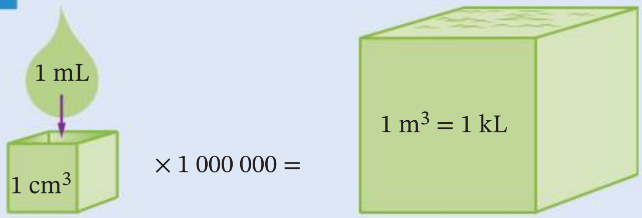
$= 10\,764 \text{ cm}^3$

The **capacity** of a container is the amount of fluid (liquid or gas) it holds, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

Volume and capacity

1 cm³ contains 1 mL

1 m³ contains 1000 L or 1 kL

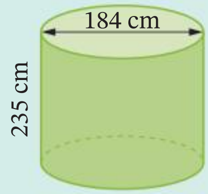


Example 9

For this cylinder, calculate:

a its volume, correct to the nearest cm³

b its capacity in kL, correct to 1 decimal place.



Solution

a Radius $= \frac{1}{2} \times 184$ — $\frac{1}{2}$ of diameter

$= 92 \text{ cm}$

$V = \pi \times 92^2 \times 235$ — $V = \pi r^2 h$

$= 6\,248\,753.452\ldots$

$\approx 6\,248\,753 \text{ cm}^3$

b Capacity $= 6\,248\,753 \text{ mL}$ — $1 \text{ cm}^3 = 1 \text{ mL}$

$= 6\,248\,753 \div 1000 \div 1000 \text{ kL}$

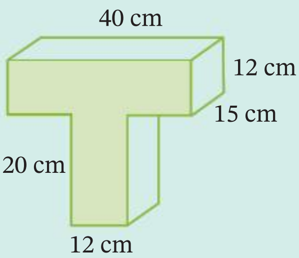
$= 6.248\,753 \text{ kL}$

$\approx 6.2 \text{ kL}$

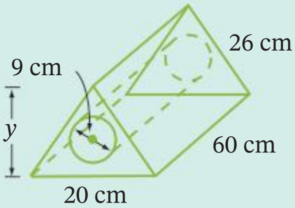
Example 10

Find, correct to the nearest whole number, the volume of each solid.

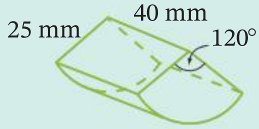
a



b



c



Solution

a $A = 40 \times 12 + 20 \times 12$ — Area of T cross-section

$= 720 \text{ cm}^2$

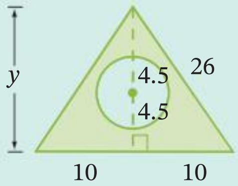
$V = Ah$

$= 720 \times 15$

$= 10\,800 \text{ cm}^3$

b Cross-section is a triangle minus a circle.

Use Pythagoras' theorem to find y.



$26^2 = y^2 + 10^2$

$y^2 = 26^2 - 10^2$

$= 576$

$y = \sqrt{576}$

$= 24 \text{ cm}$

Radius of circle $= \frac{1}{2} \times 9 = 4.5$

$A = \frac{1}{2} \times 20 \times 24 - \pi \times 4.5^2$ — Area of triangle – area of circle

$= 176.3827... \text{ cm}^2$ — Do not round this partial answer.

$V = Ah$

$= 176.3827... \times 60$

$= 10\,582.9649...$

$\approx 10\,583 \text{ cm}^3$

c $A = \frac{120}{360} \times \pi \times 25^2$ — Area of sector

$= 654.498... \text{ mm}^2$ — Do not round this partial answer.

$V = Ah$

$= 654.498... \times 40$

$= 26\,179.938...$

$\approx 26\,180 \text{ mm}^2$

3.05

EXERCISE 3.05 ANSWERS ON P. 509

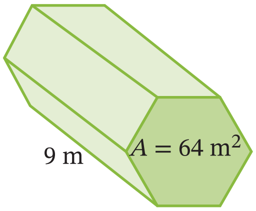
ANSWERS ON P. 509

Volumes of prisms and cylinders U F R

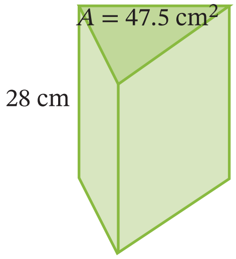
EXAMPLE 8

1 Find the volume of each solid, given the shaded area and height.

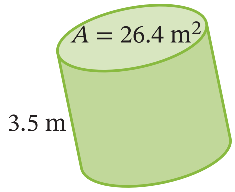
a



b

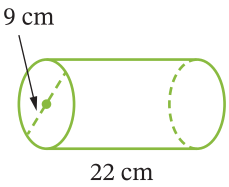


c

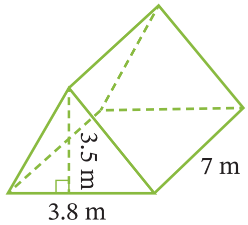


2 Calculate, correct to one decimal place, the volume of each solid.

a



b



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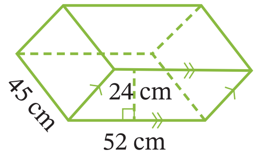

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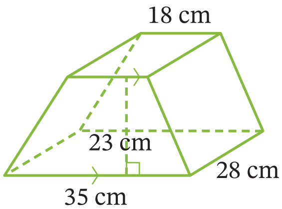

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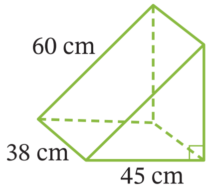

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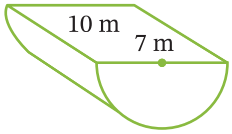
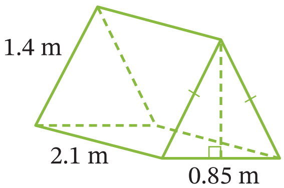


g



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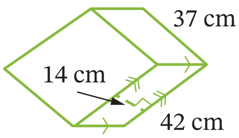
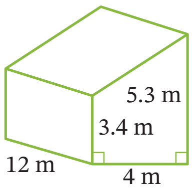


i



EXAMPLE 9

3 For each cylinder with the given measurements, calculate:

i its volume, correct to the nearest whole number

ii its capacity

a radius 7 m, height 10 m **b** diameter 35 cm, height 15 cm

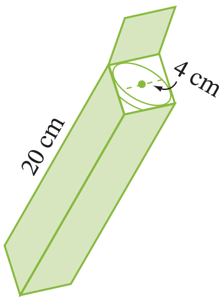
c diameter 6.2 m, height 7.5 m **d** radius 0.8 cm, height 2.35 cm

4 Rice crackers of diameter 4 cm are packed in a cardboard box of height 20 cm. Calculate, correct to one decimal place:

a the volume of the crackers in the box

b the volume of the box

c the percentage of the box that is empty space.



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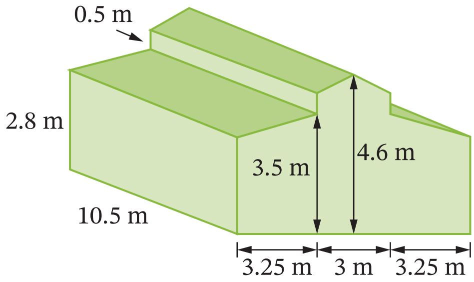
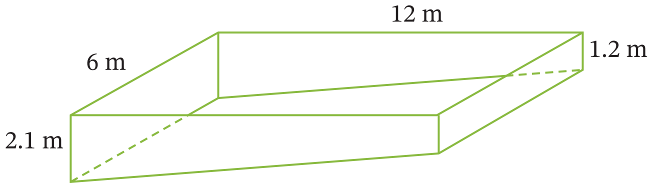
5 Calculate, correct to one decimal place, the volume of the shed. **R**



6 This swimming pool is 12 m long and 6 m wide. The depth of the pool ranges from 1.2 m to 2.1 m.



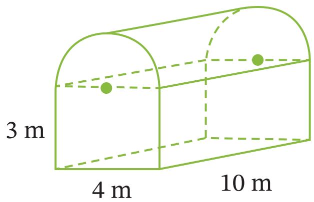
a Calculate the capacity of this pool in litres.

b If the pool is filled so that the water is 8 cm from the top of the pool, calculate the amount of water in the pool (to the nearest litre).

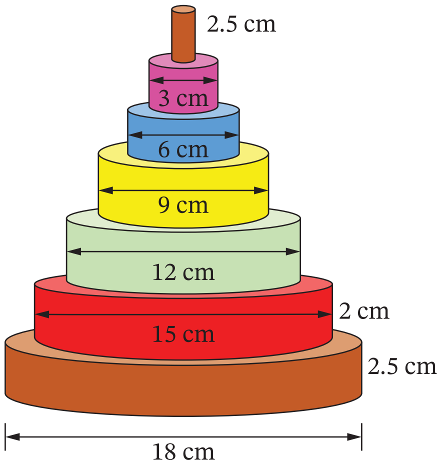
7 An Olympic sized swimming pool is 50 m long, 25 m wide and 2 m deep. What is the capacity of an Olympic pool in litres?

8 **a** Find, correct to 2 decimal places, the volume of this greenhouse.

b If this greenhouse costs 0.5c per m^3 per hour to heat, how much is this per day (correct to the nearest cent)?



9 A baby's toy involves placing 5 coloured cylinders on a wooden stick (diameter 1 cm) on a base (diameter 18 cm). The cylinders are of varying diameters with a height of 2 cm. Find the volume of the toy, including the base and peg, correct to one decimal place. **R**



3.05

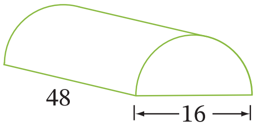
STAGE 5.2

Foundation Standard Complex

EXAMPLE 10

10 Find, correct to 2 decimal places where appropriate, the volume of each solid. All lengths shown are in centimetres. R

a



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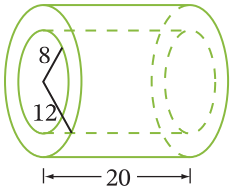

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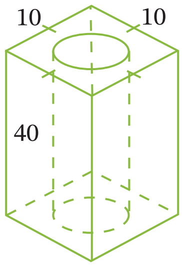
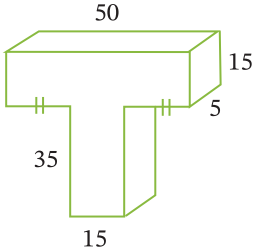


radius of circle = 4 cm

d



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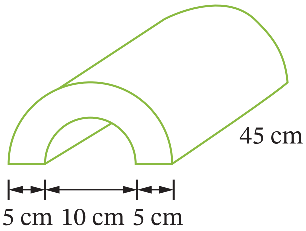

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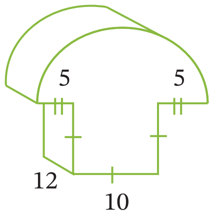

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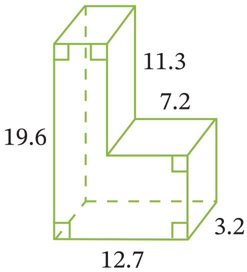

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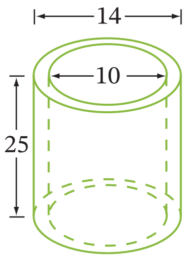

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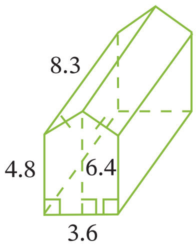

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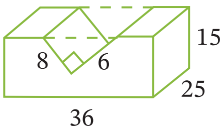

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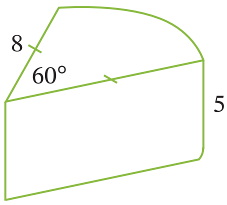

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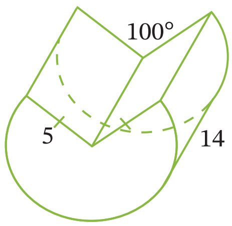

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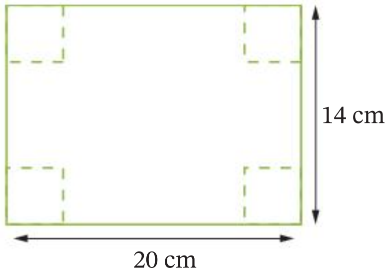
Technology

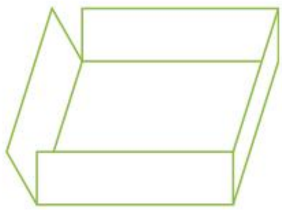
WS Biggest volume

3.05

Biggest volume

A rectangular sheet of metal measures 20 cm × 14 cm. Square corners are to be cut from it so that the remaining piece can be folded and welded to form an open tray.





What size must the cut-out squares be for the tray to have the largest possible volume? We will use a spreadsheet to solve this problem.

1 Create this spreadsheet.

	A	B	C	D	E
1	Side of square	Length	Width	Height	Volume
2	0	= 20–2*A2	= 14–2*A2	= A2	= B2*C2*D2
3					
4					
5					
6					
7					
8					

2 In cell A3, enter the formula **=A2+1**. Use **Fill Down** to copy corresponding formulas into cells A4 to A9.

3 Enter appropriate formulas for cells B3, C3, D3 and E3. Hint: Look at the formulas in row 2.

4 Use **Fill Down** to copy corresponding formulas into rows 4 to 9.

5 The length of the cut-out square cannot be more than 7 cm. Explain why this is so.

6 The spreadsheet suggests that a cut-out square length of 3 cm will give the biggest volume. Test values above and below 3 cm (correct to one decimal place) to see whether you can find a bigger volume.

7 What changes would we need to make to the spreadsheet if the starting dimensions were different?

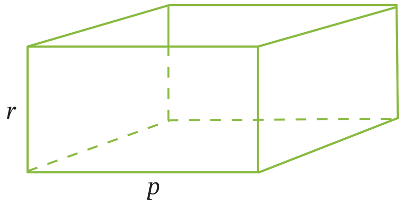
Power plus ANSWERS ON P. 509

1 The total surface area of a cube is 864 cm^2. Find its volume.

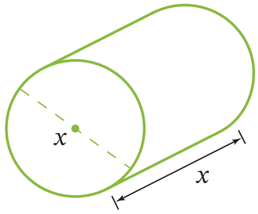
2 A cylinder has a volume of 3619.11 cm^3. Its height is 18 cm. Calculate to the nearest centimetre the radius of its base.

3 Find a formula for the surface area, *SA*, of:

a this open square prism of base length *p* and height *r*

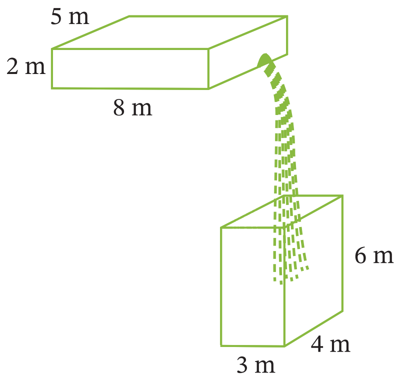


b this cylinder of diameter and height *x*

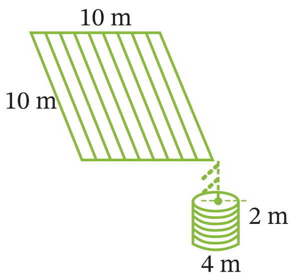


4 The surface area of the curved surface of a can is 27 143.4 mm^2. If its height is 120 mm, find the radius of the can.

5 Water flows from the top tank to the bottom tank at a constant rate. The level of water in the top tank falls at a rate of 15 cm/h. At what rate is the level of water rising in the bottom tank?



6 A 10 m flat square roof drains into a cylindrical rainwater tank with a diameter of 4 m. If 5 mm of rain falls on the roof, by how much (to the nearest millimetre) does the level of the water in the tank rise?



CHAPTER 3 REVIEW

Area and volume

Language of maths

annulus	area	base	capacity
circle	circumference	cross-section	cubic
curved surface	cylinder	diameter	external
kilolitre	litre	net	open
perpendicular height	prism	quadrant	radius
sector	solid	surface area	volume

1 Which word means a 'slice' of a prism or cylinder?

2 What is the formula for the curved surface area of a cylinder?

3 What is the formula $V = \pi r^2 h$ used for?

4 What is the difference between volume and capacity?

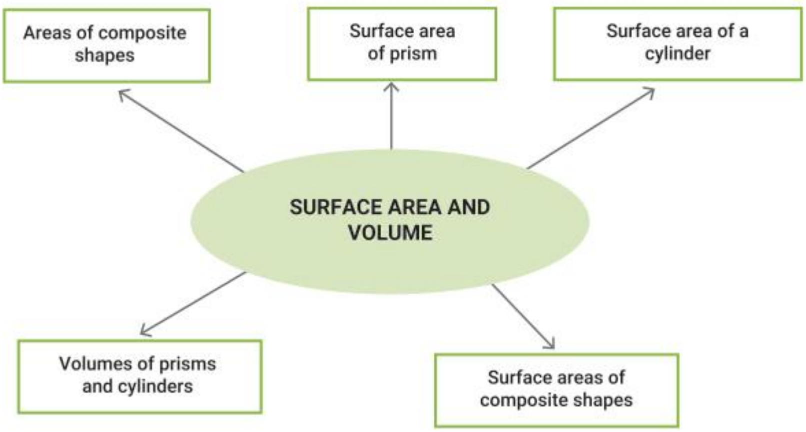
5 What is an annulus?

6 What type of measurement has units of cubic metres?

Topic summary

Mind map: Surface area and volume

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



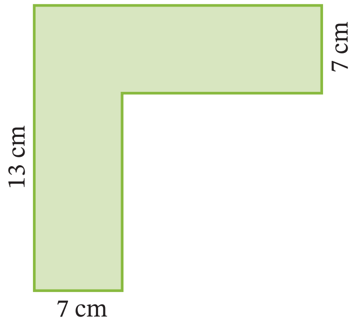
TEST YOURSELF 3

ANSWERS ON P. 510

3.01

1 Find the area of each shape. Give your answers correct to one decimal place where necessary.

a



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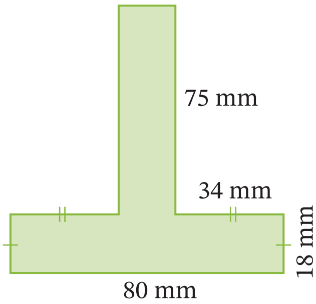

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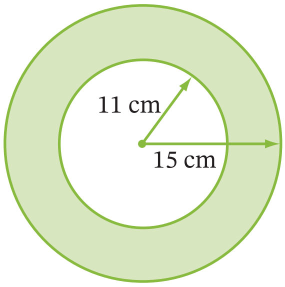

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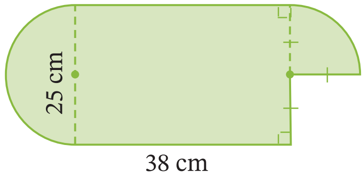

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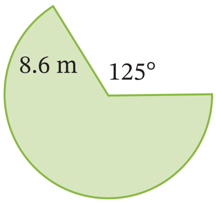

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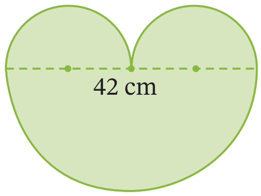

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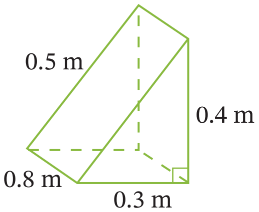
2 Find the surface area of each prism. 3.02

a



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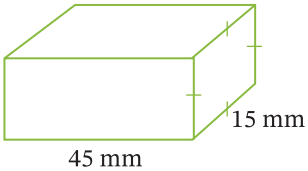

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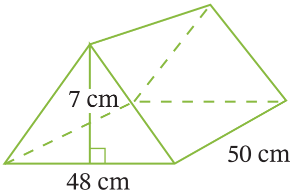

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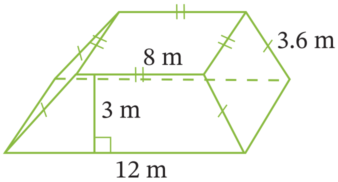

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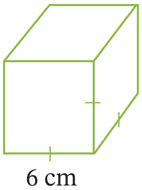

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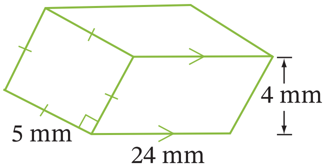
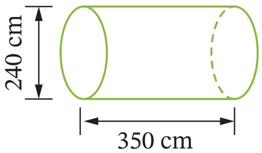


STAGE 5.2

3 Calculate, correct to one decimal place, the surface area of each solid. 3.03

a



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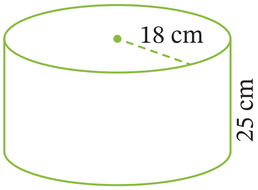

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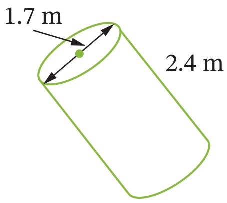
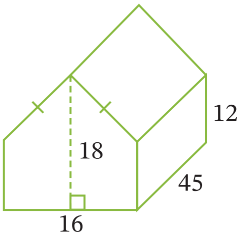


4 Calculate, correct to the nearest square centimetre, the surface area of each solid. All lengths shown are in centimetres. 3.04

a



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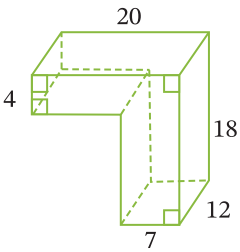

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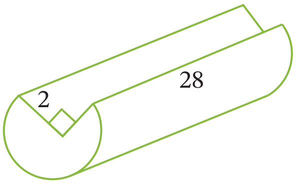

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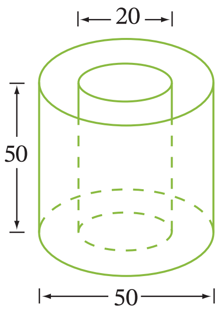

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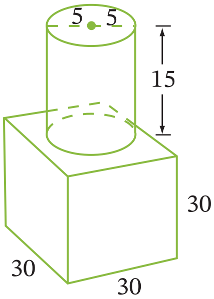
STAGE 5.2

e



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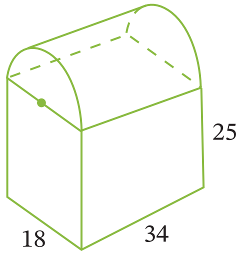
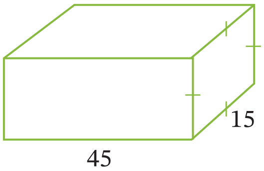


3.05

5 Calculate, correct to the nearest cubic metre, the volume of each solid. All lengths shown are in metres.

a



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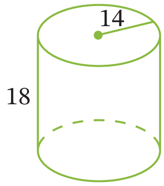

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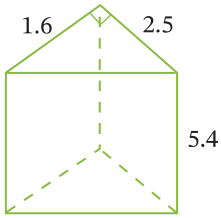

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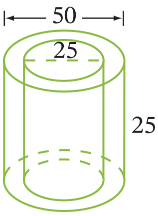

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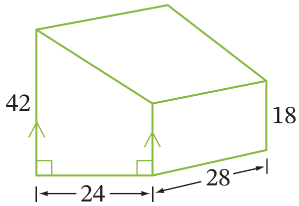

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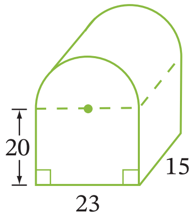

3.05

6 A rectangular fish tank measures 75 cm long by 55 cm wide by 35 cm deep. Find the capacity of the tank in litres if it is filled to 4 cm from the top.

3.05

7 A cylindrical rainwater tank has a radius of 2.8 m and a height of 2.4 m.

a Calculate, correct to 2 decimal places, the capacity of the tank in kilolitres.

b If the tank is 60% full, what is the height of the water in the tank? Answer correct to 2 decimal places.

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PRACTICE SET 1 ANSWERS ON P. 510

1 Cassie earns a salary of $114 750. How much is she paid each week? **1.01**

2 An interval is formed by joining the points $M(2, -3)$ and $N(-5, 9)$. **2.01**

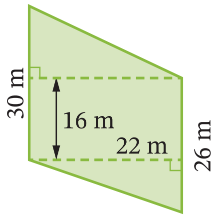
a Find the length of interval MN, correct to one decimal place.

b Find the midpoint of MN.

c Find the gradient of MN.

3 Find, correct to one decimal place, the area of each shape. **3.01**

a



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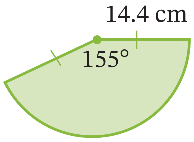

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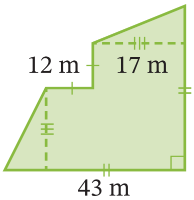

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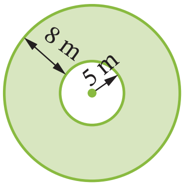

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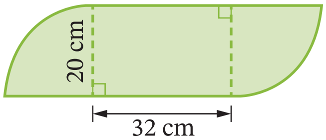

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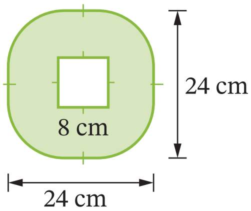


4 Naved is paid 4 weeks' normal pay plus 17.5% annual leave loading for his 4-week holiday. If Naved's salary is $78 580, find his: **1.01**

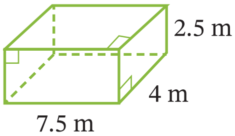
a normal weekly pay

b leave loading

c total pay for the 4-week holiday.

5 Find the surface area of each prism. **3.02**

a



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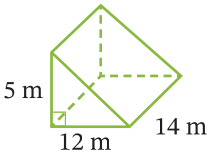

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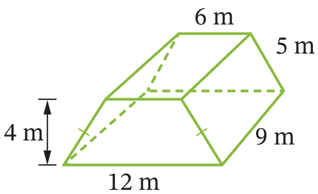

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1.01

6 A call centre operator is employed under the following award.

Normal rate: $28.75 per hour	
Normal rate	For 0 to 36 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 20 hours **b** 39 hours **c** 45 hours

STAGE 5.2

2.02

7 A line passes through the points $H(5, -3)$ and $K(8, -7)$. Calculate the gradient of the line:

a parallel to HK **b** perpendicular to HK

1.02

8 Nikita earns a weekly wage of $1485. She has annual deductions of $1756 for a health fund and $3560 for work expenses.

a Calculate Nikita's taxable income.

b Use the tax table on page 11 to calculate the income tax that Nikita should pay.

2.03

9 Graph the linear equations $y = 3x - 2$ and $y = -2x + 3$ on a number plane. Where do the lines intersect?

1.03

10 Calculate the simple interest earned on each investment.

a $25 000 invested for 4 years at 3% p.a.

b $500 invested at 1.5% p.a for 9 months

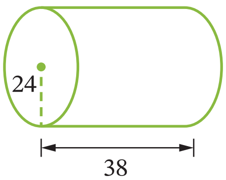
c $8500 for 2 years at 0.15% per month

STAGE 5.2

3.03

11 Calculate, correct to one decimal place, the external surface area of each solid. All lengths shown are in centimetres.

a



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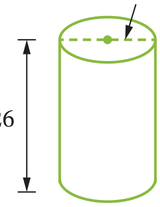

c

Cylinder, open at both ends

1.04

12 Joshua invests $12 000 at 4% p.a. interest, compounded annually for 2 years.

a What is the total value of his investment after 2 years?

b What is the amount of interest earned after 2 years?

2.03

13 Which of the following points lie on the line of $2x + y = 3$?
Select the correct answer **A**, **B**, **C** or **D**.

A (1, 0) **B** (2, −1) **C** (−1, −1) **D** (−1, −5)

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14 Find the equation of the line passing through (−3, 2) that is parallel to the *y*-axis. Select **A**, **B**, **C** or **D**. 2.03

A $x = 2$ **B** $x = -3$ **C** $y = 2$ **D** $y = -3$

15 Jovan purchases a home theatre system valued at $7680. He pays a 10% deposit and repays the balance in 48 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find: 1.06

a the deposit paid **b** the balance owing

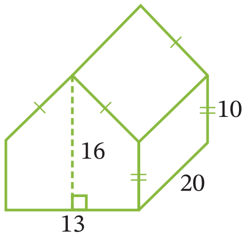
c the interest charged **d** the total amount to be repaid

e the amount of each instalment **f** the total price Jovan paid for the system.

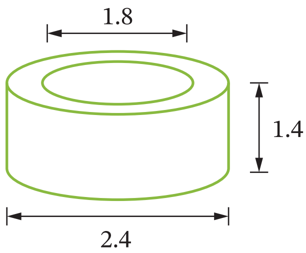
STAGE 5.2

16 Calculate, correct to nearest square metre, the surface area of each solid. All lengths shown are in metres. 3.04

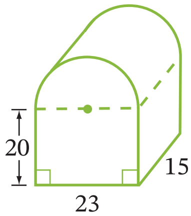
a



b



c



17 Calculate the value of an investment if $46 000 is invested at 4.2% p.a. for 3 years with interest compounded: 1.05

a annually **b** quarterly **c** monthly

18 Find the compound interest earned when $50 000 is invested for 10 years at 6.5% p.a., with interest compounded half-yearly. 1.05

19 Find the gradient, *m*, and *y*-intercept, *c*, for each linear equation. 2.04

a $y = -3x + 8$ **b** $y = 7 + x$ **c** $y = \frac{3-2x}{3}$

20 Convert each equation to general form $ax + by + c = 0$. 2.05

a $y = 2x - 3$ **b** $2x = y + 5$ **c** $y = \frac{3x}{4} + 6$

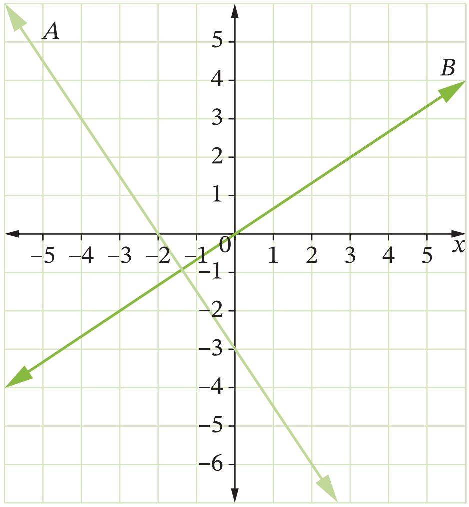
21 Rewrite each equation in the form $y = mx + c$. 2.05

a $5x - y + 3 = 0$ **b** $4x + y - 8 = 0$ **c** $2x - 6y + 8 = 0$

Foundation Standard Complex

2.06

22 Find the equation of each line.



2.07

23 Find the equation of a line that is:

a parallel to $y = 4x - 3$ and has an x-intercept of -8

b perpendicular to $y = -\frac{x}{5} + 3$ and passes through (0, 0).

1.07

24 Annieke purchases a new car for $39 990, which depreciates by 10% p.a.

a Find the depreciated value of the car after 4 years.

b What is the depreciation over this time?

c Express the depreciated value as a percentage of the cost price (correct to one decimal place).

2.05

25 Find the gradient, m, and the y-intercept, c, of the line with equation $2x + 5y - 3 = 0$. Select **A**, **B**, **C** or **D**.

A $m = -2, c = 3$

B $m = \frac{3}{5}, c = \frac{2}{5}$

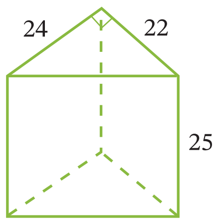
C $m = -\frac{2}{5}, c = \frac{3}{5}$

D $m = 2, c = -3$

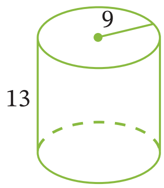
3.05

26 Calculate, correct to the nearest whole number, the volume of each solid. All lengths shown are in centimetres.

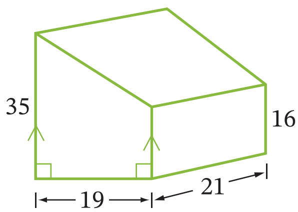
a



b



c



Foundation Standard Complex

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27 Find the equation of the line with: 2.04

a $m = -3, c = 7$ **b** $m = \frac{2}{3}, c = -5$ **c** $m = 0, c = 4$

28 Max's weekly wage is $1249.20. 1.01

a If he works 36 hours per week, find his hourly rate of pay.

b How much does Max earn per month?

29 A circular swimming pool has a radius of 3.5 m and a depth of 2 m. The pool is to be filled to within 30 cm from the top. 3.05

a Calculate the volume of water in the pool to the nearest m^3.

b If water costs $2.11 per kilolitre, find the cost of filling the pool.

30 What is the flat rate of interest per annum when $5600 earns $470 in interest over 3 years? Select **A**, **B**, **C** or **D**. 1.03

A 2.8% **B** 2.7% **C** 3.0% **D** 27.9%

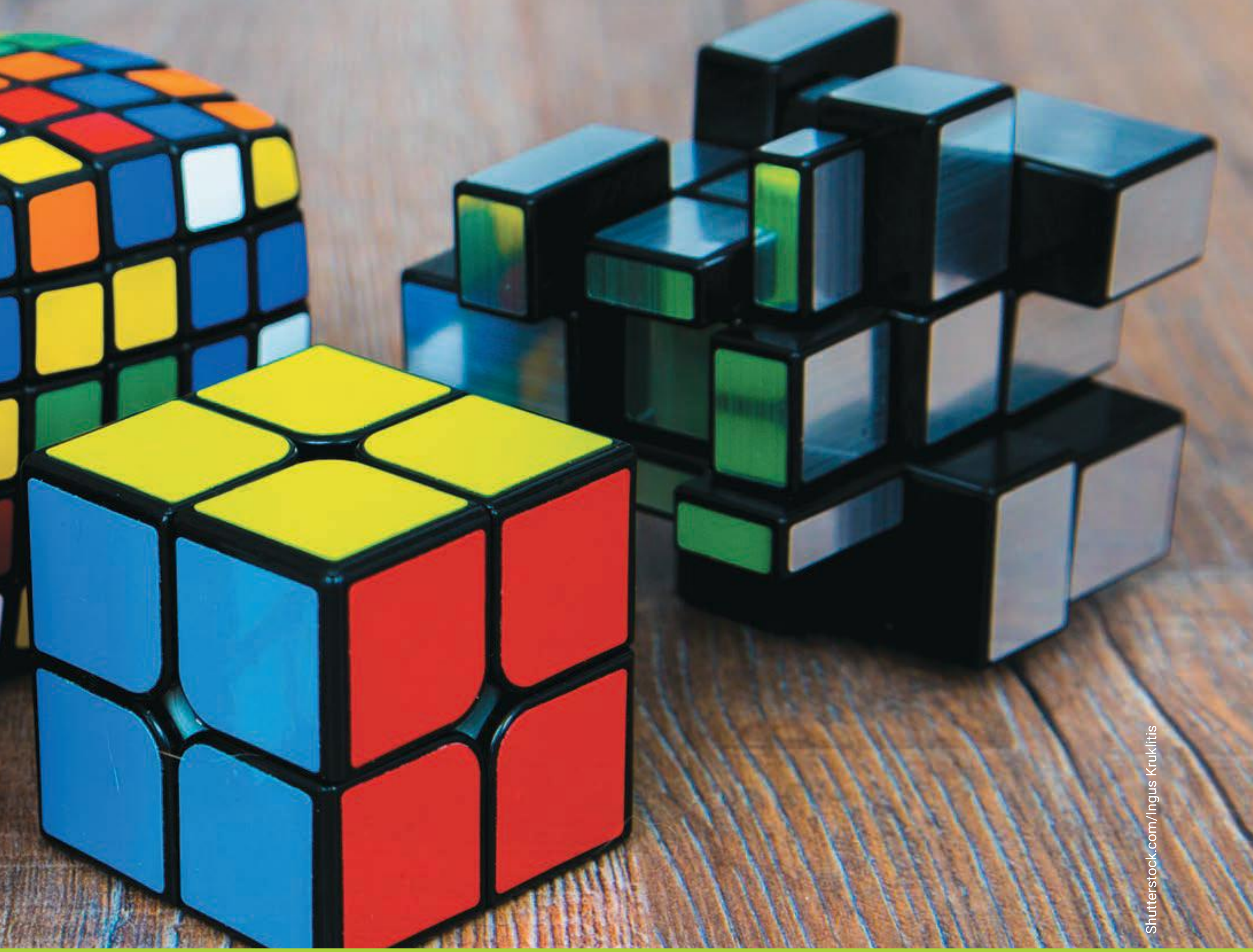
Foundation Standard Complex

4

NUMBER AND ALGEBRA

ALGEBRA

Rubik's Cube is a puzzle that was invented by Hungarian Professor of Architecture Erno Rubik in 1974. In Hungary, it was originally called the Magic Cube. The cube has $2^{10} \times 3^7 \times 8! \times 12! \approx$ 43 252 003 274 489 856 000 different possible arrangements (8! or '8 factorial' means $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$). However, the cube has been solved in as little as 20 moves in 6 seconds!



Chapter outline

		Working mathematically				
4.01	The index laws	U	F		R	C
4.02	Adding and subtracting algebraic fractions*	U	F			
4.03	Multiplying and dividing algebraic fractions*	U	F			
4.04	Expanding and factorising expressions	U	F			
4.05	Expanding binomial products*	U	F		R	C
4.06	Factorising quadratic expressions $x^2 + bx + c$*	U	F		R	

*STAGE 5.2

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product Binomials multiplied together, for example, $(x + 9)(3x - 4)$

expand To rewrite an expression such as $5(2k - 6)$ without grouping symbols; for example, $5(2k - 6)$ expands to $10k - 30$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor. Factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

index laws Rules for simplifying algebraic expressions involving powers of the same base; for example, $a^m \div a^n = a^{m-n}$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $2x^2 + 5x - 3$ or $x^2 + 2$

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- apply index laws to numerical expressions with integer indices
- (STAGE 5.2) apply index laws to algebraic expressions with integer indices
- interpret and use zero and negative indices
- (STAGE 5.2) add, subtract, multiply and divide simple algebraic fractions
- expand and factorise algebraic expressions
- (STAGE 5.2) expand and factorise algebraic expressions involving terms with indices
- (STAGE 5.2) expand binomial products and factorise quadratic expressions of the form $x^2 + bx + c$

SkillCheck ANSWERS ON P. 510

1 Simplify each expression.

a $g^4 \times g^5$ **b** $r^8 \div r^2$ **c** $(d^5)^3$

d $(-k)^2$ **e** $h \times h^9$ **f** $m^5 \div m$

g a^1 **h** a^0 **i** $3e^2 \times 2e^4$

j $18n^6 \div 6n^2$ **k** $(10w^3)^3$ **l** $25q^0$

m $(vw)^5$ **n** $\left(\frac{c}{p}\right)^3$ **o** y^{-1}

p k^{-2} **q** $\frac{35ad}{20a^2}$ **r** $4y^2 \times \frac{3}{8}y^3$

s $3g^{-2} \times 8g^4$ **t** $\left(\frac{2b}{3h}\right)^2$

2 Evaluate each expression.

a $\frac{a}{5}+\frac{3a}{4}$ **b** $\frac{7p}{2}-\frac{10p}{3}$ **c** $\frac{8}{w}\times\frac{5w}{24}$ **d** $\frac{x}{14}\div\frac{y}{2}$

3 Expand each expression.

a $6m(3m + 11)$ **b** $-5(3g - 8)$ **c** $3w(8y - 4w)$

4 Factorise each expression.

a $4x + 24$ **b** $20 - 15a$ **c** $q^2 + q$

d $18a^2 - 12a$ **e** $-2y - 30$ **f** $-18w + 24$

5 Find 2 numbers whose:

a product is 18 and sum is 9

b product is 8 and sum is −6

c product is −20 and sum is −1

d product is −16 and sum is 6

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The index laws

4.01

Index is another name for **power**. The plural of index is **indices** (pronounced 'in-de-sees'). The following rules are called **index laws**.

Index laws review | Index laws | Numbers and powers | Simplifying with the index laws | Negative indices | Index laws | Indices puzzle | Indices squaresaw

The index laws

When **multiplying terms with the same base**, add the powers

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, subtract the powers

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

When **raising a term with a power to another power**, multiply the powers

$$(a^m)^n = a^{m \times n}$$

When **raising a product of terms to a power**, raise each term to that power

$$(ab)^n = a^n b^n$$

When **raising a quotient of terms to a power**, raise each term to that power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Any number **raised to the power of zero** is equal to 1

$$a^0 = 1$$

A number **raised to a negative power** gives a fraction (with a numerator of 1)

$$a^{-n} = \frac{1}{a^n}$$

A number **raised to a power of −1** gives its reciprocal

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Example 1

Simplify each expression.

a $5p^2 q^3 \times 4p^3 q^4$ **b** $\frac{24e^6 n^{12}}{8en^4}$ **c** $(3t^5)^2$

d $(4d^4 q^2 r)^3$ **e** $\left(\frac{2c^2}{d}\right)^4$ **f** $31x^0 - (31x)^0$

Solution

a $5p^2 q^3 \times 4p^3 q^4 = 5 \times 4 \times p^2 \times p^3 \times q^3 \times q^4$

$= 20p^{2+3} q^{3+4}$

$= 20p^5 q^7$

b $\frac{24e^6 n^{12}}{8en^4} = \frac{{}^{3}\cancel{24}e^6 n^{12}}{{}_{1}\cancel{8}en^4}$

$= 3e^{6-1} n^{12-4}$

$= 3e^5 n^8$

c $(3t^5)^2 = 3t^5 \times 3t^5$
$= 9t^{5+5}$
$= 9t^{10}$

or $(3t^5)^2 = 3^2 \times t^{5\times 2}$
$= 9t^{10}$

d $(4d^4 q^2 r)^3 = 4^3 d^{4\times 3} q^{2\times 3} r^3$
$= 64d^{12} q^6 r^3$

e $\left(\frac{2c^2}{d}\right)^4 = \frac{(2c^2)^4}{d^4}$
$= \frac{2^4 c^{2\times 4}}{d^4}$
$= \frac{16c^8}{d^4}$

f $31x^0 - (31x)^0 = 31 \times 1 - 1$
$= 30$

STAGE 5.2

Example 2

Simplify each expression using a positive index.

a m^{-3} **b** $9x^{-2}$ **c** $(4q)^{-3}$ **d** $\left(\frac{2}{3}\right)^{-1}$

Solution

a $m^{-3} = \frac{1}{m^3}$

b $9x^{-2} = 9 \times x^{-2}$
$= 9 \times \frac{1}{x^2}$
$= \frac{9}{x^2}$

c $(4q)^{-3} = \frac{1}{(4q)^3}$
$= \frac{1}{64q^3}$

d $\left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)^1$
$= \frac{3}{2}$

Negative powers of quotients

Consider $\left(\frac{4}{5}\right)^{-2} = \frac{1}{\left(\frac{4}{5}\right)^2} = \frac{1}{\frac{16}{25}}$

$= 1 \div \frac{16}{25}$

$= 1 \times \frac{25}{16}$

$= \frac{25}{16}$

$= \frac{5^2}{4^2}$

$= \left(\frac{5}{4}\right)^2$

Negative powers of quotients

A number **raised to a power of $-n$** gives its reciprocal raised to the power of n.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

Example 3

STAGE 5.2

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-3}$ **b** $\left(2\frac{1}{2}\right)^{-2}$ **c** $\left(\frac{3a}{b^3}\right)^{-4}$

Solution

a $\left(\frac{4}{3}\right)^{-3}=\left(\frac{3}{4}\right)^{3}$

$=\frac{27}{64}$

b $\left(2\frac{1}{2}\right)^{-2}=\left(\frac{5}{2}\right)^{-2}$

$=\left(\frac{2}{5}\right)^{2}$

$=\frac{4}{25}$

c $\left(\frac{3a}{b^4}\right)^{-2}=\left(\frac{b^4}{3a}\right)^{2}$

$=\frac{b^8}{9a^2}$

4.01

EXERCISE 4.01 ANSWERS ON P. 510

The index laws U F R C

1 Simplify each expression. R C

EXAMPLE 1

a $5k^4\times 3k^7$ **b** $30y^{10}\div 3y^7$ **c** $12p^6\times\frac{3}{4}p^5$

d $(w^3)^5$ **e** $(2n^3)^5$ **f** $\frac{24h^5}{8h}$

g $3a^2d\times 4a^5d^4$ **h** $(3q^3)^2$ **i** $32w^5y^8\div 8w^3y^4$

j $(-2c^2)^7$ **k** $\frac{28a^2c^5d^7}{21c^4d^4}$ **l** $(5y^6)^3$

m $4h^3k^2\times 6hk^3w^5$ **n** $(2d^3g^2)^5$ **o** $\frac{8m^5p^{11}q}{12m^3p^7q}$

2 Simplify each expression. R C

a $(l^3m^5)^6$ **b** $\left(\frac{n}{2}\right)^3$ **c** $7x^0$ **d** $\left(\frac{w^2}{k^3}\right)^5$

e $\left(\frac{2}{3}\right)^0$ **f** $(-8ky^5)^2$ **g** $(16a)^0-16a^0$ **h** $\left(\frac{2b}{3d}\right)^4$

i $(5xy^2)^0$ **j** $(-5dy^2)^4$ **k** $\left(-\frac{3k^4}{10}\right)^3$ **l** $-9(a^2b^3)^0$

m $(2p^2q^3r^4)^4$ **n** $(4w^0)^3$ **o** $(-3g^6k)^2$ **p** $3a^0+(3ab)^0$

3 Evaluate each expression. R C

a 4^0 **b** $(-8)^0$ **c** 7×2^0 **d** $(5\times 3)^0$

e $(-2)^3$ **f** $(-3)^2$ **g** $(5^2)^2$ **h** 4^0+7^0

i $2^4\times 2^3$ **j** $(10^2)^0$ **k** $(2^3)^0-(2^0)^3$ **l** $4^5\div 4^2$

m $6^2\div 6^5$ **n** $9^5\div 9^5$ **o** $(8\times 3)^0-8\times 3^0$ **p** $5^2\div 5^0$

q $\left(\frac{2}{5}\right)^0+\left(\frac{5}{2}\right)^0$ **r** $3^{-2}\div 3^2$ **s** $\left(\frac{3}{4}\right)^0$ **t** $12^{-2}\times 12^2$

4 Express each term as a fraction. R C

a 5^{-2} **b** 2^{-5} **c** 20^{-1} **d** 10^{-3} **e** 3^{-4}

Foundation Standard Complex

5 Find the value of $(3x)^0 + 3 \times 2^0$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** 1 **C** 7 **D** 4

STAGE 5.2

6 Simplify $2c^{-3}$. Select **A**, **B**, **C** or **D**.

A $-6c$ **B** $\frac{1}{2c^3}$ **C** $\frac{2}{c^3}$ **D** $\frac{1}{8c^3}$

EXAMPLE 2

7 Simplify each expression using positive indices. R C

a 8^{-7} **b** 3^{-5} **c** y^{-1} **d** x^{-3}

e $(5b)^{-2}$ **f** $8h^{-3}$ **g** $(ab)^{-1}$ **h** $-pq^{-1}$

i $11w^{-3}$ **j** $(6x)^{-3}$ **k** $a^3 b^{-5}$ **l** mw^{-3}

m $8u^{-3} v^{-4}$ **n** $-2r^6 y^{-5}$ **o** $10e^{-1} f^3$ **p** $\frac{1}{2}g^4h^{-3}$

q $\frac{3}{4}d^7n^{-2}$ **r** $(-4c)^{-2}$ **s** $5x^2 y^{-1} w^{-2}$ **t** $2(mp)^{-1}$

8 Simplify each expression. R C

a $10r^{-6}$ **b** $\left(\frac{5}{2}\right)^{-1}$ **c** $\left(\frac{1}{3}\right)^{-1}$ **d** $\left(\frac{1}{x}\right)^{-1}$

e $\left(\frac{k}{3}\right)^{-1}$ **f** $5a^{-2}$ **g** $(6w)^{-2}$ **h** $\left(-\frac{m}{2}\right)^{-1}$

i $\left(\frac{5}{3g^3}\right)^{-1}$ **j** $\left(\frac{2r}{3h}\right)^{-1}$ **k** $m^3 n^2 p^{-2}$ **l** $\left(\frac{5b}{4a^2}\right)^{-1}$

9 Simplify each expression, writing your answer with positive indices. R C

a $(2x^5 y^2)^2 \times 5x^{-4}y^5$ **b** $(3m^8 n^{-2} \times m^{-2} n^3)^4$

c $\left(\frac{6k^3w^5}{9k^4w^3}\right)^2$ **d** $(5g^6 y^4)^2 \div 10g^8 y^{-4}$

e $\left(\frac{2a^4x^4}{3a^5x^2}\right)^{-3}$ **f** $-6a^2 d^3 \times (-2a^3d^4)^3$

g $3q^{-5} r^3 \div (6qr^2)^2$ **h** $(4h^3 k^2)^3 \times (hk^5)^2$

i $[24b^5q^6 \div (-2bq^2)^3]^2$

EXAMPLE 3

10 Simplify each expression. R C

a $\left(\frac{3}{4}\right)^{-2}$ **b** $\left(\frac{2}{3}\right)^{-3}$ **c** $\left(-\frac{1}{10}\right)^{-6}$ **d** $\left(\frac{5}{2}\right)^{-3}$

e $\left(-\frac{4}{3}\right)^{-5}$ **f** $\left(\frac{5}{4}\right)^{-4}$ **g** $\left(2\frac{1}{4}\right)^{-2}$ **h** $\left(1\frac{2}{5}\right)^{-3}$

i $\left(\frac{k}{3}\right)^{-2}$ **j** $\left(-\frac{3}{x}\right)^{-3}$ **k** $\left(\frac{a^2}{5}\right)^{-4}$ **l** $\left(\frac{4}{3g^3}\right)^{-2}$

Foundation Standard Complex

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4.02 Adding and subtracting algebraic fractions

Adding and subtracting fractions

To add or subtract fractions, convert them (if needed) so that they will have the same denominator, then simply add or subtract the numerators.

STAGE 5.2

Adding and subtracting algebraic fractions

Algebraic fractions

Algebraic fractions

Example 4

Simplify each expression.

a $\frac{a}{2}+\frac{a}{3}$ **b** $\frac{2x}{5}-\frac{x}{3}$ **c** $\frac{5}{6}+\frac{7b}{15}$

Solution

a $\frac{a}{2}+\frac{a}{3}=\frac{3\times a}{3\times 2}+\frac{2\times a}{2\times 3}$

Common denominator $= 2 \times 3 = 6$.

$=\frac{3a}{6}+\frac{2a}{6}$

$=\frac{5a}{6}$

b $\frac{2x}{5}-\frac{x}{3}=\frac{3\times 2x}{3\times 5}-\frac{5\times x}{5\times 3}$

$=\frac{6x}{15}-\frac{5x}{15}$

$=\frac{x}{15}$

c $\frac{5}{6}+\frac{7b}{15}=\frac{25}{30}+\frac{14b}{30}$

The lowest common denominator is 30.

$=\frac{25+14b}{30}$

EXERCISE 4.02 ANSWERS ON P. 511

Adding and subtracting algebraic fractions U F R C

EXAMPLE 4

1 Simplify each expression.

a $\frac{2k}{7}+\frac{3k}{7}$ **b** $\frac{8x}{11}-\frac{5x}{11}$ **c** $\frac{13m}{15}-\frac{7m}{15}$ **d** $\frac{9}{5g}+\frac{4}{5g}$

e $\frac{3w}{4}-\frac{2w}{5}$ **f** $\frac{a}{3}+\frac{3a}{7}$ **g** $\frac{4c}{5}+\frac{7c}{12}$ **h** $\frac{5u}{2}-\frac{u}{6}$

i $\frac{3}{4k}-\frac{1}{4k}$ **j** $\frac{9}{7x}+\frac{5}{7x}$ **k** $\frac{13}{5h}-\frac{3}{5h}$ **l** $\frac{5}{m}+\frac{7}{2m}$

m $\frac{5}{3a}-\frac{1}{a}$ **n** $\frac{7}{2h}+\frac{4}{3h}$ **o** $\frac{11}{8k}+\frac{4}{5k}$ **p** $\frac{16}{7p}-\frac{5}{3p}$

Foundation Standard Complex

STAGE 5.2

2 Simplify each expression.

a $\frac{m}{3}+\frac{n}{5}$ **b** $\frac{k}{7}-\frac{w}{6}$ **c** $\frac{x}{4}-\frac{p}{9}$ **d** $\frac{5}{8}+\frac{h}{3}$

e $\frac{n}{10}-\frac{4}{7}$ **f** $\frac{5y}{2}-\frac{2c}{5}$ **g** $\frac{3q}{11}+\frac{2d}{3}$ **h** $\frac{3a}{8}+\frac{5e}{6}$

i $\frac{2b}{9}+\frac{b}{5}$ **j** $\frac{9y}{7}-\frac{3y}{4}$ **k** $\frac{5c}{3}-\frac{2c}{11}$ **l** $\frac{8g}{5}+\frac{2g}{15}$

m $\frac{4h}{3}+\frac{h}{8}$ **n** $\frac{10x}{7}+\frac{4x}{5}$ **o** $\frac{12}{5}-\frac{8k}{3}$ **p** $\frac{7m}{12}-\frac{2m}{15}$

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4.03 Multiplying and dividing algebraic fractions

STAGE 5.2

WS Algebraic fractions

Algebraic fractions puzzle

Multiplying and dividing algebraic fractions

- To multiply fractions, cancel any common **factors**, then multiply the numerators and denominators separately.
- To divide by a fraction $\frac{a}{b}$, multiply by its **reciprocal** $\frac{b}{a}$.

Foundation Standard Complex

Example 5

Simplify each expression.

a $\frac{3x}{4} \times \frac{2x}{9}$ **b** $\frac{4}{k} \times \frac{3k}{16}$ **c** $\frac{2}{v} \div \frac{3}{w}$ **d** $\frac{xy}{5} \div \frac{3x}{25}$

Solution

a $\frac{3x}{4} \times \frac{2x}{9} = \frac{3x \times 2x}{4 \times 9}$
$= \frac{6x^2}{36}$
$= \frac{x^2}{6}$

or $\frac{3x}{4} \times \frac{2x}{9} = \frac{{}^1\cancel{3}x}{{}_2\cancel{4}} \times \frac{{}^1\cancel{2}x}{{}_3\cancel{9}}$
$= \frac{x^2}{6}$

b $\frac{4}{k} \times \frac{3k}{16} = \frac{{}^1\cancel{4}}{\cancel{k}} \times \frac{3\cancel{k}}{\cancel{16}_4}$
$= \frac{3}{4}$

c $\frac{2}{v} \div \frac{3}{w} = \frac{2}{v} \times \frac{w}{3}$
$= \frac{2w}{3v}$

d $\frac{xy}{5} \div \frac{3x}{25} = \frac{\cancel{x}y}{\cancel{5}_1} \times \frac{\cancel{25}^5}{3\cancel{x}}$
$= \frac{5y}{3}$

STAGE 5.2

Algebraic fractions

Upside-down fractions

4.03

EXERCISE 4.03 ANSWERS ON P. 511

Multiplying and dividing algebraic fractions U F

EXAMPLE 5

1 Simplify each product.

a $\frac{m}{5} \times \frac{2}{7}$ **b** $\frac{d}{3} \times \frac{h}{4}$ **c** $\frac{8}{y} \times \frac{3}{q}$

d $\frac{x}{11} \times \frac{5}{y}$ **e** $\frac{8}{k} \times \frac{v}{4}$ **f** $\frac{5}{3h} \times \frac{8}{15}$

g $\frac{12}{v} \times \frac{2}{9}$ **h** $\frac{6}{5b} \times \frac{10b}{18}$ **i** $\frac{4}{5y} \times \frac{7y}{12}$

j $\frac{14}{15p} \times \frac{3hp}{7}$ **k** $\frac{20a}{3k} \times \frac{9a}{5k}$ **l** $\frac{5}{12b} \times \frac{8ab}{15}$

2 Simplify the product $\frac{6ad}{7m} \times \frac{3y}{4d} \times \frac{14gm}{27ay}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{2ak}{7y}$ **B** $\frac{g}{3}$ **C** $\frac{14gm}{9}$ **D** $\frac{7m}{4}$

3 Simplify the quotient $\frac{5xy}{3k} \div \frac{20}{9ky}$. Select **A**, **B**, **C** or **D**.

A $\frac{3y}{4x}$ **B** $\frac{4x}{15y^2}$ **C** $\frac{45k}{60y}$ **D** $\frac{3xy^2}{4}$

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STAGE 5.2

4 Simplify each quotient.

a $\frac{h}{5} \div \frac{4}{9}$ **b** $\frac{x}{4} \div \frac{3p}{7}$ **c** $\frac{k}{6} \div \frac{a}{11}$

d $\frac{w}{y} \div \frac{4}{y}$ **e** $\frac{3c}{4} \div \frac{5c}{8}$ **f** $\frac{2m}{9} \div \frac{4am}{3}$

g $\frac{10e}{3} \div \frac{4}{e}$ **h** $\frac{8}{3xy} \div \frac{6x}{5y}$ **i** $\frac{8my}{27} \div \frac{4m}{9}$

j $\frac{8k}{15gh} \div \frac{12hk}{10g}$ **k** $\frac{6d^2}{25} \div \frac{3d}{10n}$ **l** $\frac{8y}{5} \div \frac{32y^2}{3g}$

5 Simplify each expression.

a $\frac{wy}{5} \div \frac{3y}{20}$ **b** $\frac{d}{3h} \div \frac{k}{6} \div \frac{2d}{hk}$ **c** $\frac{24pq}{7} \div 8p$

d $\frac{x}{4y} \times \frac{12m}{5g} \times \frac{10y}{9x}$ **e** $\frac{2w}{p} \times \frac{3h}{7} \div \frac{4h}{5p}$ **f** $10pg \div \frac{6p}{5}$

g $\frac{4k}{3m} \div \frac{3d}{m} \div \frac{5k}{2m}$ **h** $\frac{3d}{4} \div \frac{d}{a} \times \frac{4d}{15}$ **i** $\frac{8}{3p} \div \frac{4}{a} \times \frac{p}{6a}$

j $15g \div \frac{3g}{4x}$ **k** $\frac{5}{2y} \times \frac{4}{3q} \div \frac{10}{3y}$ **l** $\frac{12}{b} \div \frac{4}{ab}$

6 Simplify the expression $\frac{4km}{5} \div \frac{18d}{7} \div \frac{m}{30}$. Select **A**, **B**, **C** or **D**.

A $\frac{175}{12km}$ **B** $\frac{7km^2}{1350d}$ **C** $\frac{28k}{3d}$ **D** $\frac{4km}{5d}$

4.04 Expanding and factorising expressions

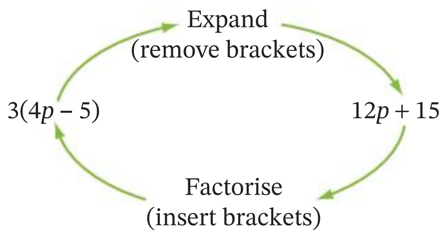
Algebra using diagrams

HCF by factor trees

Expanding and **factorising** are inverse operations.

When $3(4p - 5)$ is **expanded**, the answer is $12p - 15$.

When $12p - 15$ is **factorised**, the answer is $3(4p - 5)$.



Expanding an expression

Multiply each term inside the brackets by the term outside.

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Factorising using diagrams

Expanding and factorising

Algebra 4

Factorising puzzle

The distributive law

4.04

Example 6

Expand each expression.

a $4d(5d + 3)$

b $-5(3p - 8)$

Solution

a $4d(5d + 3) = 4d \times 5d + 4d \times 3$ — Multiplying out the brackets

$= 20d^2 + 12d$

b $-5(3p - 8) = -5 \times 3p - (-5) \times 8$ — Multiplying out the brackets

$= -15p - (-40)$

$= -15p + 40$

STAGE 5.2

Example 7

Expand and simplify by collecting **like terms**.

a $-3r^2(4r + 2) - 5r^3$

b $9(m - 3) + m(m - 10)$

Solution

a $-3r^2(4r + 2) - 5r^3 = -3r^2 \times 4r + (-3r^2) \times 2 - 5r^3$ — Expanding

$= -12r^3 + (-6r^2) - 5r^3$

$= -17r^3 - 6r^2$ — Collecting like terms to simplify.

b $9(m - 3) + m(m - 10) = 9 \times m - 9 \times 3 + m \times m - m \times 10$

$= 9m - 27 + m^2 - 10m$

$= m^2 + 9m - 10m - 27$ — It's neater to place m^2 at the front.

$= m^2 - m - 27$ — Collecting like terms to simplify.

Factorising an expression

- Find the **highest common factor (HCF)** of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets
- $ab + ac = a(b + c)$ $\qquad ab - ac = a(b - c)$
- To check that the factorised answer is correct, expand it.

Factorising expressions

Example 8

Factorise each expression.

a $18xy^2 - 24xy$ **b** $3m(5 + 2d) + 7(5 + 2d)$ **c** $-5k^2 + 15k$

Solution

a The HCF of $18xy^2$ and $24xy$ is $6xy$.

$\therefore 18xy^2 - 24xy = 6xy \times 3y - 6xy \times 4$ Rewrite the expression using the HCF $6xy$.

$= 6xy(3y - 4)$ Write the HCF at the front of the brackets.

b The HCF of $3m(5 + 2d) + 7(5 + 2d)$ is $(5 + 2d)$.

$\therefore 3m(5 + 2d) + 7(5 + 2d) = (5 + 2d) \times 3m + (5 + 2d) \times 7$

$= (5 + 2d)(3m + 7)$

c When factorising expressions that begin with a negative term, we use the 'negative' HCF.

The highest 'negative' common factor of $-5k^2$ and $15k$ is $-5k$.

$\therefore -5k^2 + 15k = (-5k) \times k + (-5k) \times (-3)$ $(-5k) \times (-3) = +15k$

$= (-5k)[k + (-3)]$

$= -5k(k - 3)$

STAGE 5.2

Example 9

Factorise each expression.

a $8a^3 + 4a^2$ **b** $20h^3k + 25h^4k - 10h^2k$

Solution

a The HCF of $8a^3$ and $4a^2$ is $4a^2$.

$\therefore 8a^3 + 4a^2 = 4a^2 \times 2a + 4a^2 \times 1$ Rewrite the expression using the HCF $4a^2$.

$= 4a^2(2a + 1)$

b The HCF is $5h^2k$.

$\therefore 20h^3k + 25h^4k - 10h^2k = 5h^2k \times 4h + 5h^2k \times 5h^2 - 5h^2k \times 2.$

$= 5h^2k(4h + 5h^2 - 2)$

EXERCISE 4.04 ANSWERS ON P. 511

Expanding and factorising expressions U F

EXAMPLE 6

1 Expand each expression.

a $5(d + 11)$	**b** $-3(r + 10)$	**c** $7(x - 9y)$	**d** $-4(a - 5w)$
e $-(2 - p^2)$	**f** $-10e(2e^2 + 3)$	**g** $6y(1 + 7y)$	**h** $4xy(3xy - 1)$
i $8rq(2q - r)$	**j** $3ab(4b - 7a)$	**k** $-6h^2(1 - 3h)$	**l** $-5x(5x^2 + 4y)$
m $-(3 + 8a)$	**n** $-2m^2(3m - 4n)$	**o** $5g(3 + 7g^2)$	**p** $-1(5e - 12)$

Foundation Standard Complex

2 Expand $-2y(5 + 7y)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3y - 5y^2$ **B** $-10y + 14y^2$ **C** $-10y - 14y^2$ **D** $-10y - 5y^2$

3 Use the substitution $x = 2$ to test whether each equation is correct or incorrect.

a $4(x + 10) = 4x + 40$ **b** $5(x - 1) = 5x - 6$ **c** $x(3 - x) = 3x - x^2$

STAGE 5.2

EXAMPLE 7

4.04

4 Expand and simplify by collecting like terms.

a $4k(3k - 5) - 9k^2$
b $5h - 7h(4 - h)$
c $9w^3 - 3w(5 + 2w^2)$
d $24x^3 - 5x^2(2x^2 - 5x)$
e $8 - 3(2 - 7d)$
f $4n(3 - 5n) - 6n^2$
g $4y(y - 4) + 5(2y + 1)$
h $5(1 - 2a) - a(3 + 4a)$
i $7(3 + 6w) - (5 - 8w)$
j $4y^2(5y + 5) + 4(2 - 7y^2)$
k $-v(2v + 7) + 6(v - 1)$
l $2(4 - 3a) - a(3 - a)$
m $2c(5c - 1) - 4(7c - 5)$
n $3m(m + 5m^2) - m^2(1 - 3m)$
o $4x(2y + 5) - 6y(10 - 3x)$

EXAMPLE 8

5 Factorise each expression.

a $15y - 20$
b $21 + 35w$
c $2p + p^2$
d $30y - 20y^2$
e $36d^2 + 24d$
f $28k^2 - 21k$
g $8(c - 5) - c(c - 5)$
h $m(3 + 2m) + 7(3 + 2m)$
i $-q^2 - 36q$
j $-8x + 12x^2$
k $b(3b + 5) - 2(3b + 5)$
l $-12cd^2 + 8cd$
m $-hn^2 + h^2n$
n $-15g^2 - 18g$
o $48q^2 - 54q$

6 Factorise $-16pw + 10xw$. Select **A**, **B**, **C** or **D**.

A $-2w(8p - 10x)$
B $-2w(8p - 5x)$
C $-8w(2p - x)$
D $-4w(5p + 2x)$

STAGE 5.2

EXAMPLE 9

7 Factorise each expression.

a $8m^2y^2 - 12my$
b $36ab^2c + 27bc$
c $24m^2n - 108mn^2$
d $20dg^2 - 35ag$
e $40wy^3 + 24w^2y^2$
f $75g^3h^2 - 125gh$
g $-4p^3 - 8p^2 + p$
h $6mn^2 + 3mn + 48m^2n$
i $32p^3g + 8pg^2 - 8pg$
j $18a^5 - 12a^2 + 15a^4$
k $28m^3h^2 - 21mh^2$
l $15kwp - 24wp^2 - 9kw$

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Did you know?

CAPTCHA

For security purposes on a website, have you ever been asked to enter letters, words or numbers that have been displayed in a wavy, difficult-to-read format like this?

This process is called **CAPTCHA**, which stands for **Completely Automated Public Turing Test to Tell Computers and Humans Apart**, invented in 1999. It is designed to ensure that the person accessing the website is a human and not another computer that could be hacking into the site, sending spam or viruses. CAPTCHA uses optical character recognition (OCR), which is a technology that can convert images of text into editable text.

How does CAPTCHA prove that you are a human and not a computer?

What is a Turing test, named after English computer scientist and mathematician Alan Turing?

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Mental skills 4: Maths without calculators ANSWERS ON P. 511

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$

$= (60 + 20 - 80) + (130 - 30)$

$= 0 + 100$

$= 100$ (Actual answer = 92)

b $78 \times 7 \approx 80 \times 7$

$= 560$ (Actual answer = 546)

c $510 \div 24 \approx 500 \div 20$

$= 50 \div 2$

$= 25$ (Actual answer = 21.25)

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$

b $55 + 34 - 22 - 46 + 136$

c $684 + 903$

d $35 + 81 + 110 + 22 + 7$

e $517 - 96$

f $210 - 38 - 71 + 151 - 49$

g $766 - 353$

h 367×2

i 83×81

j 984×16

k $828 \div 3$

l $507 \div 7$

3 Study each example involving decimals.

a $20.91 - 11.3 + 2.5 \approx 21 - 11 + 3$

$= 13$ (Exact answer = 12.11)

b $4.78 \times 19.2 \approx 5 \times 20$

$= 100$ (Exact answer = 91.776)

c $\frac{37.6+9.3}{41.2-12.7} \approx \frac{38+9}{40-13}$

$= \frac{47}{27}$

$= \frac{50}{30}$

$= 1.6$ (Exact answer = 1.6456...)

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$

b $14.807 + 6.6 - 7.22$

c 18.47×9.61

d 4.27×97.6

e $\frac{11.07+18.4}{12.2}$

f $\frac{38.18}{17.2-9.6}$

g $\frac{18.46\times 4.9}{39.72-15.2}$

h $62.13 \div 10.7$

i $(4.89)^2$

4.05 Expanding binomial products

STAGE 5.2

Expanding binomial products 1

Expanding binomial products 2

WS Algebra 2

Expanding brackets

Trinominoes

Expanding binomials

Expanding binomials

WS Area diagrams

Binomial means '2 terms'. $(m + 8)$ and $(m - 3)$ are **binomial expressions** because they each have exactly 2 terms. $(m + 8)(m - 3)$ is called a **binomial product** because it is a product of 2 binomial expressions.

Example 10

Expand each binomial product.

a $(m + 8)(m - 3)$

b $(4y - 5)(3y - 2)$

Solution

a $(m + 8)(m - 3) = m(m - 3) + 8(m - 3)$ — Each term in $(m + 8)$ is multiplied by $(m - 3)$

$= m^2 - 3m + 8m - 24$ — Expanding

$= m^2 + 5m - 24$ — Simplifying

One way of remembering which pairs of terms to multiply together in a binomial product is called the **FOIL method**, as shown.

F O I L

$(m + 8)(m - 3) = m^2 - 3m + 8m - 24$

$= m^2 + 5m - 24$

- **F** means multiply the **f**irst terms: $m \times m = m^2$
- **O** means multiply the **o**utside terms: $m \times (-3) = -3m$
- **I** means multiply the **i**nside terms: $8 \times m = 8k$
- **L** means multiply the **l**ast terms: $8 \times (-3) = -24$

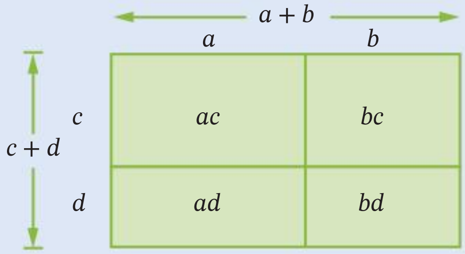
b $(4y - 5)(3y - 2) = 12y^2 - 8y - 15y + 10$ — Using FOIL

$= 12y^2 - 23y + 10$ — Simplifying

Expanding a binomial product

Multiply each term in the first binomial by each term in the second binomial.

$$(a + b)(c + d) = ac + ad + bc + bd$$



STAGE 5.2

WS Binomial products

Trinominoes

4.05

EXAMPLE 10

EXERCISE 4.05 ANSWERS ON P. 511

Expanding binomial products **U F R C**

1 Expand each binomial product.

a $(m + 4)(m + 3)$	**b** $(w + 5)(w + 5)$	**c** $(y + 12)(y - 12)$
d $(h + 7)(h - 9)$	**e** $(a - 5)(a + 3)$	**f** $(x - 11)(x - 4)$
g $(p + 3)(p + 8)$	**h** $(c - 7)(c - 12)$	**i** $(g - 1)(g - 2)$
j $(u - 8)(u + 7)$	**k** $(m + 4)(m - 10)$	**l** $(q - 11)(q + 6)$
m $(d - 5)(8 + d)$	**n** $(10 - e)(e - 7)$	**o** $(9 - h)(5 - h)$

2 Expand $(w - 8)^2$. Select the correct answer **A**, **B**, **C** or **D**.

A $w^2 - 64$ **B** $w^2 + 64w$ **C** $w^2 + 16w - 64$ **D** $w^2 - 16w + 64$

3 Expand $(12 - a)^2$. Select **A**, **B**, **C** or **D**.

A $144 - a^2$ **B** $144 - 24a + a^2$

C $a^2 + 24a + 144$ **D** $a^2 + 144$

4 Expand each binomial product.

a $(3y + 1)(y + 7)$	**b** $(4k + 9)(3k + 2)$	**c** $(3m - 5)(m + 3)$
d $(5p + 3)(2p - 5)$	**e** $(2w - 3)(w - 8)$	**f** $(7x + 4)(2x + 3)$
g $(3b - 4)(3b - 4)$	**h** $(5a + 6)(5a + 6)$	**i** $(2q - 7)(5q + 7)$
j $(6 + 5p)(p - 3)$	**k** $(4d - 11)(1 + 3d)$	**l** $(2r - 5)(9 - 4r)$
m $(7y + 3)(3 - 2y)$	**n** $(8h - 3)(8h + 3)$	**o** $(9 - 7w)(7w - 9)$
p $(4d - 1)(4d + 1)$	**q** $(f - 1)(1 + 3f)$	**r** $(6u - 5)(5 - 6u)$

5 Expand $(5 - 3k)(5 + 3k)$. Select **A**, **B**, **C** or **D**.

A $25 - 30k - 9k^2$ **B** $25 - 9k^2$

C $9k^2 - 30k - 25$ **D** $9k^2 - 25$

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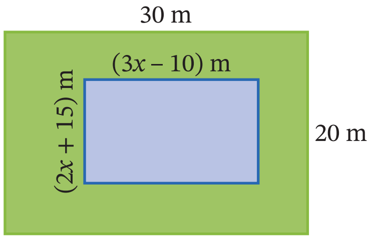
STAGE 5.2

6 This diagram shows a house $(3x - 10)$ m long and $(2x + 15)$ wide on a block of land with dimensions 30 m × 20 m. R C

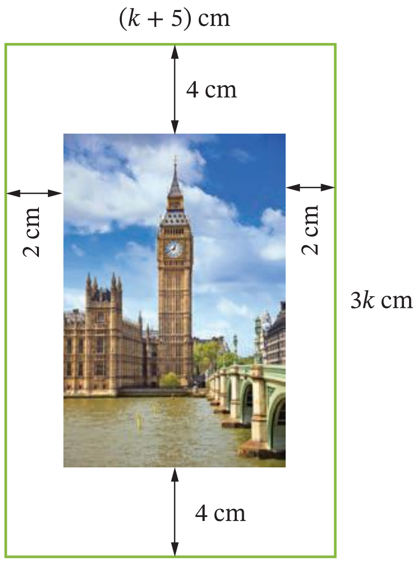


a Write down a binomial expression for the area of the house in square metres.

b Expand and simplify your expression for the area.

c The green area of the block of land not covered by the house is to be turfed. Write a simplified expression for this area in square metres.

7 A photograph frame is $(k + 5)$ cm wide and $3k$ cm long. The gap between the photograph and frame is 4 cm at the top and bottom and 2 cm on each side. R C



Shutterstock.com/S.Borisov

a What is the area of the photograph frame?

b Write down expressions for the length and width of the photograph.

c Write down a binomial expression for the area of the photograph.

d Expand and simplify your expression for the area of the photograph.

e Find an expression for the area of the frame not taken up by the photograph.

8 A family room in a house is to be extended. The room is a metres long and b metres wide. The length is to be increased by 3 metres and the width by 1 metre. R C

a Write down expressions for the new length and width in metres.

b Write down a binomial expression for the new area of the room in square metres.

c Expand and simplify your expression for the area.

d By how much has the area of the room increased in square metres?

9 Prove that: R C

a $(a - b)^2 = (b - a)^2$

b $(a + b)(a - b) = a^2 - b^2$

c $(a - b)^2 = a^2 - 2ab + b^2$

Foundation Standard Complex

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STAGE 5.2

Investigation

Factorising quadratic expressions

1 a Show that $(x + 3)(x + 5) = x^2 + 8x + 15$.

b The quadratic expression $x^2 + 8x + 15$ has 3 terms. The coefficient of x (the number in front of the x) is 8. How are the 3 and 5 in $(x + 3)(x + 5)$ related to the 8?

c The constant term in $x^2 + 8x + 15$ is 15, the number with no x at the end. How are the 3 and 5 related to the 15?

2 a Expand $(x + 9)(x + 2)$.

b What is the coefficient of x? How are 9 and 2 related to it?

c What is the constant term? How are 9 and 2 related to it?

3 a Expand $(x + 8)(x - 3)$.

b What is the coefficient of x? How are 8 and −3 related to it?

c What is the negative constant term? How are 8 and −3 related to it?

4 a Expand $(x - 4)(x - 1)$.

b What is the negative coefficient of x? How are −4 and −1 related to it?

c What is the positive constant term? How are −4 and −1 related to it?

5 In the expansion of any binomial product, how are the coefficient of x and the constant term related to the numbers in the binomials?

6 Copy and complete:

a $(x + \_\_\_)(x + \_\_\_) = x^2 + 5x + 4$

b $(x + \_\_\_)(x + \_\_\_) = x^2 + 8x + 15$

c $(x + \_\_\_)(x + \_\_\_) = x^2 + 7x + 12$

d $(x + \_\_\_)(x - \_\_\_) = x^2 - 4x - 32$

e $(x + \_\_\_)(x - \_\_\_) = x^2 + 2x - 3$

f $(x - \_\_\_)(x - \_\_\_) = x^2 - 9x + 20$

4.05

4.06 Factorising quadratic expressions $x^2 + bx + c$

STAGE 5.2

Factorominoes

A **quadratic expression** is an algebraic expression in which the highest power of the variable is 2, such as $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$.

A quadratic expression such as $x^2 - 5x + 7$ is called a **trinomial** because it has 3 terms.

The expansion of $(x + 2)(x + 4)$ is $x^2 + 6x + 8$, a quadratic trinomial.

$\therefore$ The factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

STAGE 5.2

Trinominoes

WS Simplifying algebraic fractions

Factorising quadratic trinomials

In the factorisation of a quadratic trinomial such as $x^2 + 6x + 8$:

- each factor must have an x term to give x^2 — $x^2 + 6x + 8 = (x + 2)(x + 4)$
- $2 + 4 = 6$, which is the **coefficient** of x, the number in front of the x — $x^2 + 6x + 8 = (x + 2)(x + 4)$
- $2 \times 4 = 8$, which is the **constant term** with no x — $x^2 + 6x + 8 = (x + 2)(x + 4)$

Factorising quadratic expressions 1

Example 11

Factorise each quadratic trinomial.

a $a^2 + 7a + 12$ **b** $x^2 + 9x + 8$

Solution

a Find the 2 numbers that have a sum of 7 and a product of 12.

It is best to test numbers that have a **product of 12** and then check if their sums equal 7.

Pair of numbers	Product	Sum
6, 2	6 × 2 = 12	6 + 2 = 8
3, 4	3 × 4 = 12	3 + 4 = 7

The correct numbers are 3 and 4.

$\therefore a^2 + 7a + 12 = (a + 3)(a + 4)$

b Find 2 numbers with a sum of 9 and a product of 8.

Test numbers that have a **product of 8** and check if their sums equal 9.

Pair of numbers	Product	Sum
2, 4	2 × 4 = 8	4 + 2 = 6
8, 1	8 × 1 = 8	1 + 8 = 9

The correct numbers are 8 and 1.

$\therefore x^2 + 9x + 8 = (x + 8)(x + 1)$

Factorising quadratic expressions of the form $x^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of c
- Use these 2 numbers to write a binomial product of the form $(x \_\_\_)(x \_\_\_)$

Example 12

Factorise each quadratic expression.

a $x^2 + x - 6$ **b** $a^2 - 2a - 15$ **c** $y^2 - 6y + 8$

Solution

a $x^2 + x - 6$

Find 2 numbers that have a product of –6 and a sum of 1.

Since the product is negative, one of the numbers must be negative.

They are +3 and −2.

$\therefore x^2 + x - 6 = (x + 3)(x - 2)$

b $a^2 - 2a - 15$

Product = −15, sum = −2.

Since the product is negative, one of the numbers must be negative.

They are −5 and +3.

$\therefore a^2 - 2a - 15 = (a - 5)(a + 3)$

c $y^2 - 6y + 8$

Product = 8, sum = −6.

Since the sum is negative, one of the numbers must be negative.

Since the product is positive, *both* of the numbers must be negative.

They are −2 and −4.

$\therefore y^2 - 6y + 8 = (y - 2)(y - 4)$

STAGE 5.2

Factorising quadratic expressions 1

4.06

EXERCISE 4.06 ANSWERS ON P. 512

Factorising quadratic expressions $x^2 + bx + c$ U F R

1 Find 2 numbers whose: R

EXAMPLE 11

a product is −15 and sum is 2

b product is 40 and sum is −14

c product is 42 and sum is 13

d product is −6 and sum is −1

e product is 45 and sum is 14

f product is −24 and sum is 2

g product is 54 and sum is 15

h product is −16 and sum is 0

i product is −10 and sum is 3

j product is 35 and sum is −12

2 Factorise each quadratic expression. R

EXAMPLE 12

a $y^2 + 8y + 12$ **b** $m^2 + 15m + 56$ **c** $g^2 + 9g + 14$

d $w^2 + 13x + 36$ **e** $p^2 + 11p + 24$ **f** $a^2 + 13a + 42$

g $e^2 + 12e + 27$ **h** $n^2 + 6n + 9$ **i** $c^2 + 10c + 21$

3 Factorise each quadratic trinomial. R

a $x^2 - 9x + 20$ **b** $h^2 - 13h + 30$ **c** $p^2 - 11p + 24$

d $e^2 - 11e + 30$ **e** $w^2 - 17w + 72$ **f** $k^2 - 10k + 9$

g $m^2 - 16m + 64$ **h** $u^2 - 5u + 6$ **i** $d^2 - 12c + 35$

STAGE 5.2

4 Factorise each quadratic expression. **R**

a $q^2 - 8q - 20$ **b** $h^2 - 5h - 36$ **c** $y^2 + 7y - 44$

d $x^2 - 2x - 63$ **e** $u^2 + 9u - 10$ **f** $e^2 + 7e - 30$

g $a^2 - a - 110$ **h** $y^2 + 6y - 27$ **i** $m^2 - 6m - 7$

j $c^2 + 7c - 18$ **k** $k^2 + 3k - 54$ **l** $r^2 - 9r - 22$

m $p^2 - 4p - 32$ **n** $u^2 + 12u - 45$ **o** $b^2 - 6b - 16$

5 Factorise $a^2 - 11a - 42$. Select the correct answer **A**, **B**, **C** or **D**.

A $(a - 7)(a + 6)$ **B** $(a + 7)(a - 8)$ **C** $(a - 21)(a + 2)$ **D** $(a + 3)(a - 14)$

6 Factorise each quadratic trinomial. **R**

a $h^2 - 2h + 1$ **b** $x^2 + 15x + 50$ **c** $r^2 + 20r + 96$

d $a^2 - 3a - 28$ **e** $u^2 - 7u - 60$ **f** $y^2 - 18y + 81$

g $v^2 - v - 56$ **h** $w^2 - 11w - 60$ **i** $g^2 + 3g - 18$

j $p^2 + 14p + 48$ **k** $e^2 + 7e - 8$ **l** $x^2 - 19x + 84$

Power plus ANSWERS ON P. 512

1 Simplify each expression using index notation.

a $13^7 \times (13^5 \div 13^{11})$ **b** $(8^{-4})^5 \div (8^5)^{-4}$ **c** $\left(\frac{(7^2)^3}{7^5 \times 7^2}\right)^{-1}$

d $\left(\frac{1}{10}\right)^{-10} \times \left(\frac{1}{10}\right)^{10}$ **e** $\left(-2\frac{1}{2}\right)^3$ **f** $\left(\frac{1}{2}\right)^{-10} - \left(\frac{1}{4}\right)^{-5}$

2 Simplify each expression.

a $\frac{1}{x} + \frac{1}{y} - \frac{1}{w}$ **b** $\frac{1}{x} \times \frac{1}{y} \div \frac{1}{w}$ **c** $\frac{1}{x} \times \left(\frac{1}{y} + \frac{1}{w}\right)$

3 Expand each expression.

a $(a - b)(a + b - c)$ **b** $(x - y + 1)^2$ **c** $t^{-1}(3t^2 + 4t - 1)$

4 Factorise each expression.

a $x^2 - 133x + 1000$ **b** $y^2 + 14y - 1800$

c $b^2 + 82b + 1681$ **d** $n^2 - 2500$

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CHAPTER 4 REVIEW

Qz
Numbers and powers

Language of maths

algebraic fraction	base	binomial	brackets
coefficient	constant term	denominator	expand
expression	factorise	highest common factor	index law
indices	numerator	product	power
quadratic expression	reciprocal	term	variable

1 Dividing by a fraction is the same as multiplying by its what?

2 Any number raised to the power of 0 is equal to what?

3 What is the difference between expand and factorise?

4 What power is associated with the reciprocal of a term or number?

5 In the quadratic expression $2x^2 - 3x + 6$, what is:

a the constant term? **b** the coefficient of x?

6 Copy and complete:

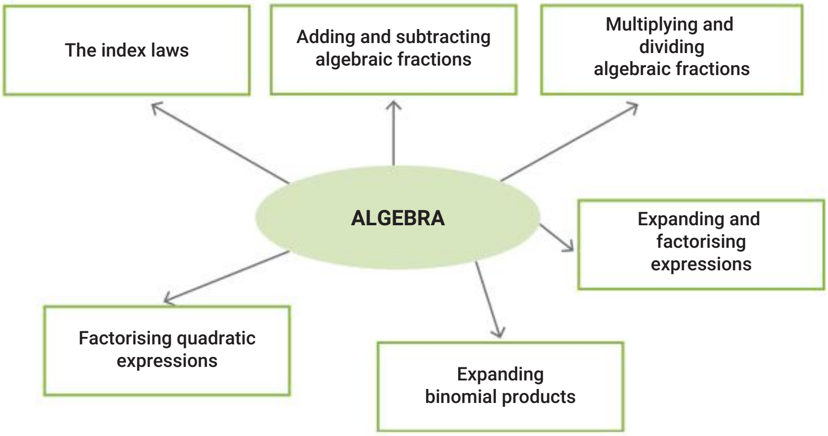
To factorise quadratic expressions of the form $x^2 + bx + c$, first find 2 numbers that have a _________ of b and a _________ of c.

Topic summary

WS
Mind map: Algebra

- What was this topic about? What was the main theme?
- What content was new and what was revision for you?
- Write 3 index laws in both words and symbols.
- Write 10 questions (with solutions) that could be used in a test for this chapter.
  Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 4

ANSWERS ON P. 512

4.01

1 Simplify each expression.

a $3v^4 w^2 \times 2vw^5$

b $\frac{24t^8h^8}{3th^2}$

c $(5xy^2)^2$

d $\left(\frac{2p}{3}\right)^0$

STAGE 5.2

e $(4k)^{-1}$

f $\left(\frac{5y}{2}\right)^3$

g $8^0 - 2^0$

h $3^{-1} + 4^0$

i $16a^3g^5 \div 18ag^4$

4.01

2 Simplify each expression.

a $(4m)^{-2}$

b $4m^{-2}$

c $\left(\frac{5b^8y^6}{b^2y^3}\right)^4$

d $(4t^4 u^5)^3 \times 8t^2 u$

e $45c^6 d^8 \div (-3cd^2)^2$

f $\left(\frac{45ab^4}{54a^2b^3}\right)^{-1}$

4.02

3 Simplify each expression.

a $\frac{t}{4} - \frac{2t}{5}$

b $\frac{5g}{3} + \frac{3g}{2}$

c $\frac{5x}{16} - \frac{x}{4}$

d $\frac{3b}{7} + \frac{2}{7}$

e $\frac{5w}{8} - \frac{3}{2}$

f $\frac{7}{12} - \frac{y}{5}$

g $\frac{5m}{7} + \frac{3n}{4}$

h $\frac{5p}{6} - \frac{p}{8}$

4.03

4 Simplify each expression.

a $\frac{3}{m} \times \frac{p}{4}$

b $\frac{1}{q} \times \frac{3}{q}$

c $\frac{2d}{3} \times \frac{15y}{14d}$

d $\frac{m}{3} \div \frac{2m}{9}$

e $\frac{5}{2a} \div \frac{4a}{7}$

f $\frac{12md}{5} \div \frac{3m}{10d}$

g $\frac{4xy}{5q} \div \frac{3x}{2y} \times \frac{9q}{4y}$

h $\frac{8ab}{3c} \div \frac{5}{2c} \div \frac{4b}{15c}$

4.04

5 Expand each expression.

a $9(m - 8)$

b $b(10a + b)$

c $-3y(4x^2 - 5y)$

d $8tp(7p - 5t)$

e $-(3n - 10)$

f $-5h^2(3h + 7)$

g $4y(5y - 7h)$

h $-wx(3x - 7w)$

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STAGE 5.2

6 Expand and simplify each expression. 4.04

a $5g - 3g(6 - 7g)$

b $4fg(g - 6f) - 6f^2g$

c $12(9 - n) - 5(2n + 3)$

d $x^2(6x + x^2) + 2x(3x^3 + x^2)$

e $3(7 - 2y) - 5y(7 - 2y)$

7 Factorise each expression. 4.04

a $8t - 72$

b $b^2 + 36b$

c $-3m - 33$

d $36wr^2 + 28w^2r$

e $-24p + 18q$

f $2(5x - 1) - 3x(5x - 1)$

8 Factorise each expression. 4.04

a $15xy^2 - 30x^3y^3$

b $6pt^2 + 12p^2t - 48p^3$

c $32r^2s^4 + 12r^4s^3$

d $50x^4y^3 - 75x^3y^4$

e $-8p^3q^3 + 48p^3q^6$

f $n(n^2 + 6) - (n^2 + 6)$

9 Expand each binomial product. 4.05

a $(b + 3)(b + 10)$

b $(d + 8)(d - 7)$

c $(t - 6)(9 - t)$

d $(5x + 7)(4x - 3)$

e $(7y - 5)(7y + 5)$

f $(3p - 8)(7p - 2)$

g $(3m + 7)(3m + 7)$

h $(4 - x)(3x + 2)$

i $(6 - 5d)(3 - 2d)$

10 Factorise each quadratic expression. 4.06

a $y^2 + 10y + 25$

b $x^2 - 21x + 20$

c $n^2 + 8n - 33$

d $a^2 - 11a + 28$

e $m^2 - 5m - 84$

f $p^2 + 3p - 54$

TEST YOURSELF 4

Foundation Standard Complex

5

STATISTICS AND PROBABILITY

COMPARING DATA

Is climate change affecting the amount of rainfall in different areas? What are the tourist numbers in different parts of Australia, and how much do they spend? How do we ensure accurate medical testing data to monitor the potential for pandemics such as COVID-19? To answer these questions, sets of data need to be collected and then compared by looking at the shape of their displays or by analysing their measures of centre and spread.

Chapter outline

		Working mathematically				
5.01	The shape of a frequency distribution	U	F		R	C
5.02	Quartiles and interquartile range*	U	F			
5.03	Box plots*	U	F		R	C
5.04	Parallel box plots*	U	F		R	C
5.05	Comparing data sets*	U	F		R	C
5.06	Scatterplots*	U	F		R	C
5.07	Bivariate data involving time*	U	F		R	C
5.08	Statistics in the media	U	F		R	C

* STAGE 5.2

Wordbank

bivariate data Data that measures 2 variables, represented by an ordered pair of values that can be graphed on a scatterplot

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the box contains the middle 50% of values while the lines or 'whiskers' extend to the 2 extremes

five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value

interquartile range (IQR) The difference between the upper quartile and lower quartiles, IQR = $Q_3 - Q_1$, representing the middle 50% of values

quartile The values Q_1, Q_2, Q_3 that divide a set of data into quarters (4 equal parts)

scatterplot A graph consisting of dots on a number plane that represent bivariate data

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- describe the shape of a statistical distribution in terms of its symmetry and skewness, using terms such as 'skewed', 'symmetric' and 'bimodal'
- (STAGE 5.2) calculate quartiles and the interquartile range of a set of data
- (STAGE 5.2) calculate the five-number summary of a set of data and use it to construct a box plot
- (STAGE 5.2) construct histograms, dot plots, back-to-back stem-and-leaf plots and parallel box plots to compare 2 or more sets of data
- (STAGE 5.2) compare shapes of box plots, histograms and dot plots
- (STAGE 5.2) graph bivariate data on scatterplots to investigate patterns and relationships between independent and dependent variables, including strength and direction
- (STAGE 5.2) investigate bivariate data involving time
- evaluate statistics presented in the media and identify biases in sampling, sources, graphs, interpretation, claims and conclusions

SkillCheck ANSWERS ON P. 513

Statistical measures

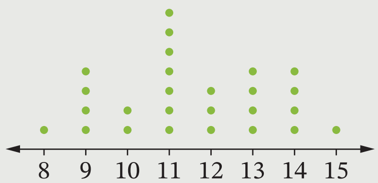
1 For each set of data, find:

i the range **ii** the mean (correct to one decimal place)
iii the median **iv** the mode

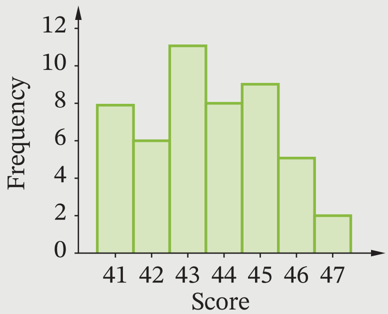
a 15 13 18 14 15 18 23 14 20 16 15

b 8°C 3°C –5°C 2°C –4°C 7°C 3°C 0°C

c



d



e

Stem	Leaf
1	0 3 6
2	1 4 4 7 8
3	2 3 4 5 5 7 9
4	0 5 7 8
5	2 6 8

f

Score	Frequency
0	2
1	5
2	8
3	4
4	3
5	1

2 A cricketer scored these numbers of runs in 10 innings.

34 21 78 30 26 19 41 36 16 32

a Find:

i the median **ii** the mean **iii** the range.

b Which score is the outlier?

c **i** Calculate the median, mean and range if the outlier is not included in the scores.

ii What effect does the outlier have on the mean, median and range?

Alamy Stock Photo/Action Foto Sport

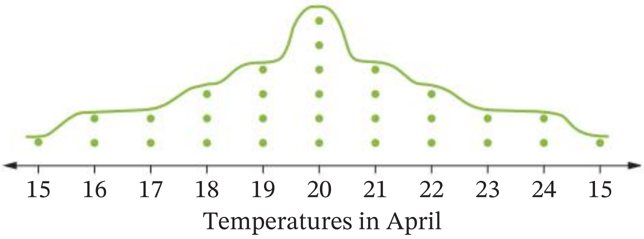
The shape of a frequency distribution 5.01

A **statistical distribution** shows how a **data** set is arranged, especially when graphed. When looking at **histograms, dot plots** and **stem-and-leaf plots**, an overall pattern can be seen from the shape of the display.

The shape of a statistical distribution shows how the data is spread. This can be seen by drawing a curve around the graph or display.

A distribution is **symmetrical** if the data is evenly spread or balanced about the centre.

Stem	Leaf
3	0 2 4
4	1 8 9 9
5	2 4 5 6 6 7 8 8
6	0 3 4 5 5 6 7 8 9 9
7	2 4 4 4 5 5 5 5
8	2 8 8 8
9	3 5 7



Statistical match-up

The shape of a frequency distribution

Skewness

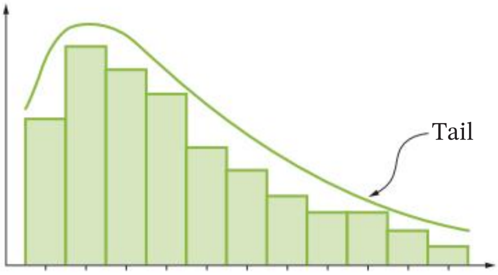
Skewness

Stem-and-leaf plots

WS
Analysing data

Analysing dot plots

A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other 'end' has a 'tail'.



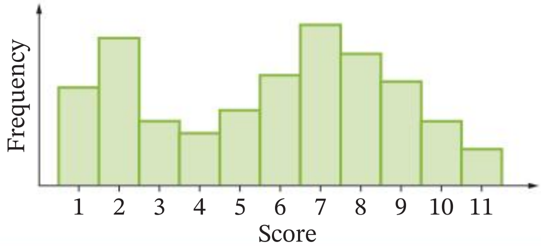
A distribution is **positively skewed** if its tail points to the right (higher values).

Stem	Leaf
0	3 5
1	0 6
2	5 7 8
3	0 3 8 9
4	1 1 2 3 4 8
5	0 0 1 1 2 2 5 5
6	3 5 7 5 6 6 7 7 9
7	0 2 2 4 5

A distribution is **negatively skewed** if its tail points to the left (lower values).

A distribution is **bimodal** if it has 2 peaks. The higher peak is the mode, while the other peak indicates another value that has a high frequency.

For example, this frequency histogram has 2 peaks at 2 and 7 so it is bimodal. The mode, however, is 7.

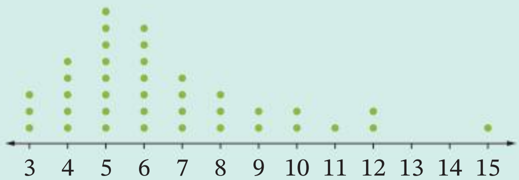


Example 1

For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters

a



b

Stem	Leaf
10	4 5
11	3 4 4 9
12	1 2 2 6 8
13	0 1 5 5 7 9 9 9
14	4 5 6 8 8
15	0 0 1 1
16	0 2

Solution

a **i** The shape is positively skewed (tail points towards the higher values).

ii 15 is an outlier and clustering occurs at 5 and 6.

b **i** The shape is symmetrical (the data is balanced about the stem of 13).

ii There are no outliers, but clustering occurs in the 130s.

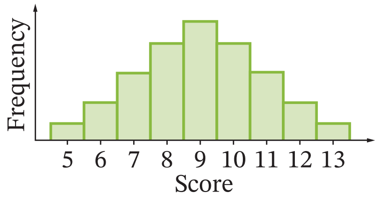
EXERCISE 5.01 ANSWERS ON P. 513

The shape of a distribution **U F R C**

1 For each statistical distribution: **R** **C**

i describe the shape **ii** identify any outliers and clusters.

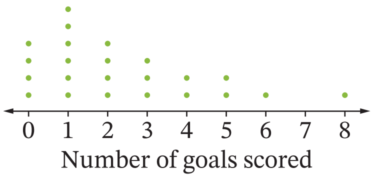
a



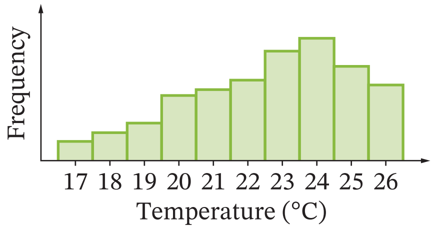
b

Stem	Leaf
2	4 5 6 9
3	1 2 3 3 4 5 7 8
4	0 4 4 6 8 9
5	4 5 5 8
6	0 0 2 3 5 6 7 8 9 9
7	3 5 7 8 8 9 9
8	1 1 3 5 6
9	0 3 5 6

c



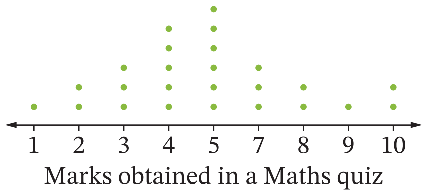
d



e

Stem	Leaf
12	0 2 4 9
13	2 4 6 7 8 8 8
14	3 3 4 4 5 5 8 9 9 9
15	0 1 1 5 7 8 9 9
16	1 1 5 6 7
17	2 4 5 8
18	0 3 9
19	5 8
20	6 8

f



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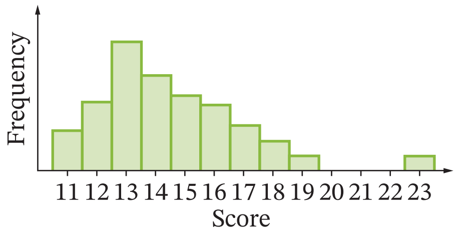

h

Stem	Leaf
5	3 4 4 6 7 8 9
6	0 0 5 9 9
7	2 4 5 6
8	5 7 8
9	3 3 6 7 8
10	2 4 6 8 8 8 8
11	
12	
13	6

EXAMPLE 1

5.01

2 These are the final round scores for players in a golf tournament. R C

66	70	67	72	75	72	70	74	75	72
74	72	73	71	71	69	70	71	71	74
72	69	75	73	69	75	73	69	69	67
74	72	72	73	71	73	77	68	72	72

a Arrange the data into a frequency table and construct a frequency histogram.

b Are there any outliers?

c Describe the shape of the distribution.

d Give a possible reason for the shape of the distribution.

e Where does clustering occur?

f Find the mode, the mean and the median and show their position in the histogram.

3 The stem-and-leaf plot shows the number of hours that students spend on their computers during the week. R C

Stem	Leaf
0	1 1 1 1 1 2 2 2 2 3 3 3 5 6 6 7 7 7 7 9 9
1	0 1 1 1 2 4 4 5 6 8 8 9
2	0 5 5 5 8 8
3	0 0 0 1 5
4	0 0

a How many students were surveyed?

b Where does the clustering occur?

c Are there any outliers?

d Describe the shape of the distribution.

e Give a possible reason for the shape of the distribution.

f Find the mean, median and mode.

4 The following values are the heights (in cm) of 30 Year 8 students. R C

162	155	153	162	182	173	165	165	142	167
164	168	150	155	143	153	123	163	170	169
153	162	161	170	160	162	172	151	160	171

a Arrange the data into an ordered stem-and-leaf plot.

b Describe the shape of the distribution.

c Are there any outliers?

d Where does clustering occur?

e Find the mode, median and mean.

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5 The daily maximum temperatures (correct to one decimal place) in Brisbane in November are shown in the stem-and-leaf plot. R C

Stem	Leaf
24	1
25	8
26	0 1 2 7 7
27	0 9 9
28	1 1 2 4 4 5 6 7 8 8
29	0 1 3 3 3
30	6 8 9
31	5
32	8

a Describe the shape of the distribution.

b Are there any outliers?

c What is the mode?

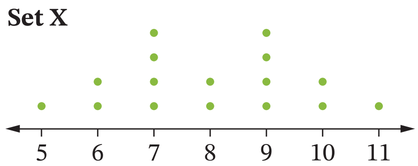
d Find the mean, correct to one decimal place.

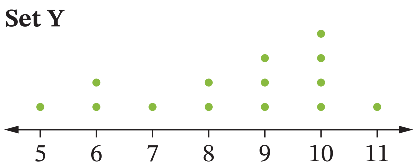
e What is the median?

f Find the range.

g Is the range a good indicator of the spread of the temperatures? Give reasons.

6 Which statement is true about the sets of data below?
Select the correct answer **A**, **B**, **C** or **D**.





A X is symmetrical and Y is positively skewed.

B X is bimodal and Y is negatively skewed.

C X has 2 peaks and the median of Y is 8.

D The median of X and Y is 8.

5.01

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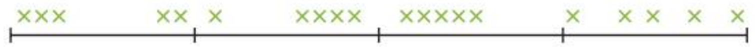
5.02 Quartiles and interquartile range

Quartiles

The **median**, being the middle value, divides a set of data into 2 equal parts (halves).

Quartiles are the values Q_1, Q_2 and Q_3 that divide the set of data into 4 equal parts (quarters).

Data values (in order)



The **1st quartile** Q_1, also called the **lower quartile** Q_L, is the value that divides the lower 25% of values. $\frac{1}{4}$ of the values lie below Q_1.

The **2nd quartile** Q_2 is the value that divides the lower 50% of values, so it is also the **median**. $\frac{1}{2}$ of the values lie below Q_2.

The **3rd quartile** Q_3, also called the **upper quartile** Q_U, is the value that divides the lower 75% of values from the upper 25% of values. $\frac{3}{4}$ of the values lie below Q_3, $\frac{1}{4}$ of the values lie above it.

Finding the quartiles of a data set

- Sort the values in order, find the median and call it Q_2
- Find the median of the bottom half of the values and call it Q_1 (or Q_L)
- Find the median of the top half of values and call it Q_3 (or Q_U).

Example 2

Find the quartiles for each set of data.

a	9	3	8	7	6	8	4	6	2	10	9		
b	15	18	7	16	23	9	15	20	16	14	13	11	19
c	65	84	75	82	97	70	68	76	93	48			
	79	54	80	79	82	96	63	85	72	70			

Solution

a Arranging the 11 values in ascending order, we have:

2 3 (4) 6 6 (7) 8 8 (9) 9 10

Lower quartile $Q_1 = 4$ | Median $Q_2 = 7$ | Upper quartile $Q_3 = 9$

Q_2 (median) = 7

Q_1 (lower quartile) = 4 — The middle of the 5 values below 7

Q_3 (upper quartile) = 9 — The middle of the 5 values above 7

b Arranging the 13 values in ascending order, we have:

7 9 11 13 14 15 (15) 16 16 18 19 20 23

$Q_1 = \frac{11+13}{2} = 12$ $Q_2 = 15$ $Q_3 = \frac{18+19}{2} = 18.5$

c Arranging the 20 values in ascending order, we have:

48 54 63 65 68 70 70 72 75 76 79 79 80 82 82 84 85 93 96 97

$Q_1 = \frac{68+70}{2} = 69$ $Q_2\,(\text{median}) = \frac{76+79}{2} = 77.5$ $Q_3 = \frac{82+84}{2} = 83$

5.02

The interquartile range

The **range** is a **measure of spread** because it gives an indication of how widely the values are spread in a set of data. Another measure of spread is the **interquartile range**.

The **interquartile range** is the difference between the upper and lower quartiles and so it is the range of the middle 50% of the data.

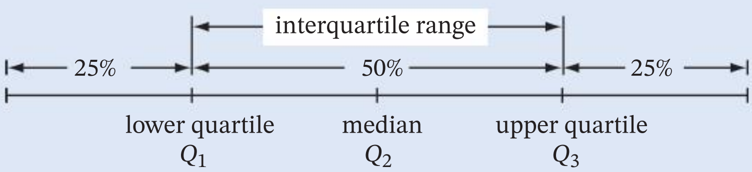
Interquartile range 1

Interquartile range 2

Interquartile range

Interquartile range (IQR) = upper quartile – lower quartile

$= Q_3 - Q_1$



The interquartile range takes into account the middle 50% of values and ignores very low or very high values (outliers), so sometimes it is better to use than the range as a measure of spread.

Example 3

The number of runs scored by the Sydney Sixers Twenty20 cricket team per match during one season were:

76 143 127 176 142 116 137 104 161 174 149 154 180 137 175

a Find the range.

b Find the interquartile range.

c Which is the better measure of spread of the points scored by the Sixers – the range or interquartile range?

Solution

First arrange the scores in order:

76 104 116 (127) 137 137 142 (143) 149 154 161 (174) 175 176 180

Lower quartile $Q_1 = 127$

Median $Q_2 = 143$

Upper quartile $Q_3 = 174$

a Range = 180 – 76
= 104

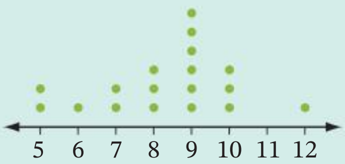
b Interquartile range = $Q_3 - Q_1$
= 174 – 127
= 47

c The interquartile range is the better measure of spread as the outlier of 76 is excluded. The score of 76 has affected the range, making it very big.

Example 4

Find the interquartile range of each data set.

a



b

Stem	Leaf
1	2 7
2	0 3 4 4 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

Solution

a There are 18 values, so the median is 'between' the 9th and 10th values, which are both 9s (see diagram).

Median, $Q_2 = \frac{9+9}{2} = 9$

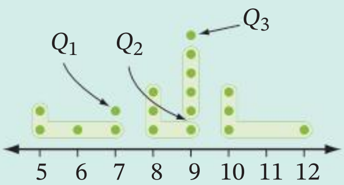
Q_1 is the median of the bottom half of 9 values (the 5th value).

$Q_1 = 7$.

Q_3 is the median of the top half of values (the 14th value).

$Q_3 = 9$.

$\therefore$ IQR = $Q_3 - Q_1$
= 9 – 7
= 2



b There are 21 values, so the median is the 11th value (see diagram).

$Q_2 = 34$

Lower quartile, $Q_1 = \dfrac{24+24}{2} = 24$

Upper quartile, $Q_3 = \dfrac{40+41}{2} = 40.5$

$\therefore \text{IQR} = 40.5 - 24$

$= 16.5$

Stem	Leaf
1	2 7
2	0 3 4 (4) 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

Q_1 between 5th-6th values

Q_2 11th value

Q_3 between 16th-17th values

5.02

EXERCISE 5.02 ANSWERS ON P. 513

ANSWERS ON P. 513

Quartiles and interquartile range **U F**

1 Find the quartiles for each set of data. EXAMPLE 2

a	3	7	9	5	5	6	2	8	9	7		
b	15	19	18	12	20	34	28	18	28	20	23	25
c	34	45	32	38	29	40	37	33	35			
	30	34	35	38	37	38	31	30	34			

2 Calculate the range and the interquartile range of each data set in question **1**. EXAMPLE 3

3 Calculate the interquartile range for each set of data below.

a	5	6	6	7	8	9	9	10	14	14	15	16	
b	2	0	3	5	2	1	0	6	4	3	8	4	2

4 The monthly rainfall figures in mm for Ulladulla one year were:

31 174 288 89 15 123 26 5 8 275 38 58

For this data, find:

a the range

b the interquartile range

Dreamstime.com/Merrillie Redden

Ulladulla Harbour

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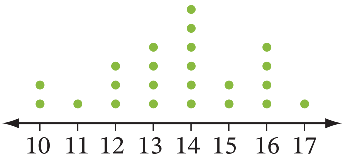
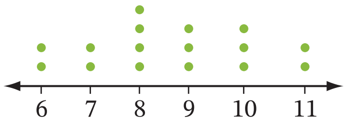
EXAMPLE 4

5 Find the interquartile range for each set of data.

a



b



c

Stem	Leaf
3	2 7
4	0 3 3 5
5	2 4 5 6 7 8 8
6	3 4 7
7	2

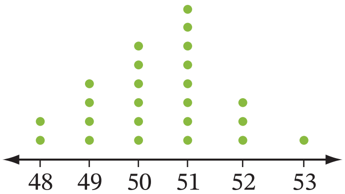
d

Stem	Leaf
1	3 5 8 9
2	0 1 3 3 4 5 6
3	5 8 9 9
4	1 3
5	4

e

Stem	Leaf
10	3 5 5 6 6
11	0 1 2
12	3 4 6 7 8
13	4 7
14	1

f



6 The pulse rates for a group of students are as follows.

82 81 72 58 79 77 62 66 92 78 80 67 91 75 72 68

a Find the range.

b Find the interquartile range.

c **i** List the values that lie between the lower and upper quartiles.

ii What percentage of values lie between Q_1 and Q_3?

d What percentage of values lie above the lower quartile?

7 The number of points per game scored by a basketball team during one season were:

55 35 49 53 51 55 42 48 63 43 48 48 62

a Find:

i the range

ii the interquartile range

b Which is the better measure of spread?

c List the values that lie between Q_1 and Q_3. What percentage of the values is this?

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5.02

Did you know?

How did statistics begin?

Alamy Stock Photo/Adam Ján Figel'

In prehistoric times, when the number of people and animals was recorded in pictures and symbols on the walls of caves, a simple form of statistics was being used.

Before 3000 BCE, ancient Babylonians used clay tablets to record crop yields and trade data, and around 2650 BCE the Egyptians 'surveyed' the population and wealth of their country before building the pyramids. Forms of statistics were also used in the Bible in the 'Book of Numbers' and the 'First Book of Chronicles'. Numerical records existed in China before 2000 BCE, and the Greeks (to help collect taxes) held a census in 594 BCE. The Roman Empire was the first government to collect information about the population. In 1086 a census was conducted in England. The information obtained in this census was recorded in the Domesday Book.

Use your library or the Internet to find out more about the Domesday Book. Write a one-page report suitable for a classroom presentation.

Box plots 5.03

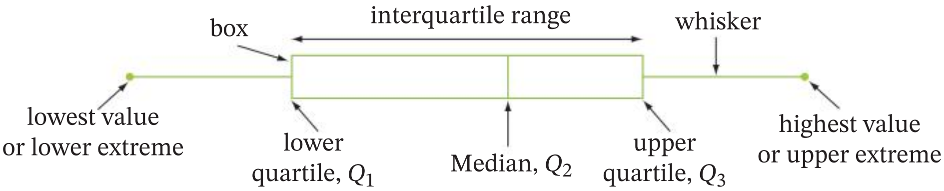
A **box plot** (or **box-and-whisker plot**) displays the quartiles of a set of data and the lowest and highest values (lower and upper extremes).

Box-and-whisker plots

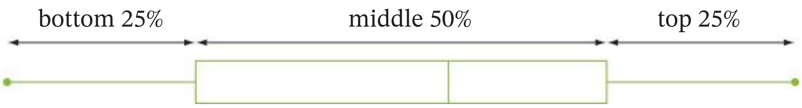
Statistics

Five-number summaries

Mode, median and mean



The 'box' represents the middle 50% of values and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of values.



Five-number summary

A box plot gives a **five-number summary** of a data set:

- the lower extreme (or lowest value)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the upper extreme (or highest value)

Example 5

The number of hours per week that Nick worked at the Big Chicken over summer were:

5	5	4	8	10	3	12
7	7	3	8	8	15	

a Find a five-number summary for this data.

b Represent this data on a box plot.

Alamy Stock Photo/Jeffrey Issac Greenberg 2+

Solution

a First arrange the values in order.

3 3 4 5 5 7 (7) 8 8 8 10 12 15

Q_1 (between 4 and 5), Median Q_2 (7), Q_3 (between 8 and 10)

Lowest value = 3

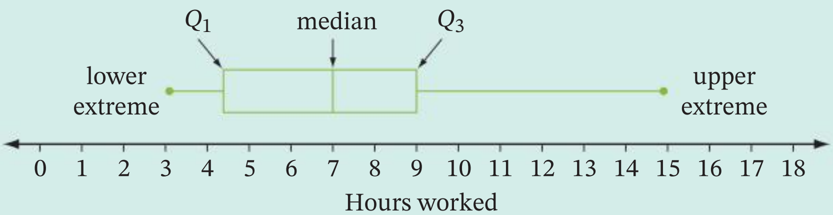
$Q_1 = \frac{4+5}{2} = 4.5$

Median = 7

$Q_3 = \frac{8+10}{2} = 9$

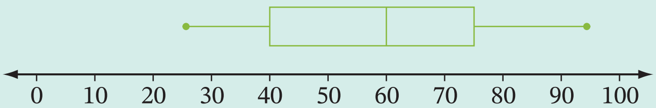
Highest value = 15

b



Example 6

The box plot represents the results of 80 students in a Science test.



a Find the range of the test results.

b Find the median test value.

c What is the interquartile range?

d How many students had a test mark between:

i 25 and 75? **ii** 40 and 60?

e What percentage of students scored more than 75?

Solution

a Range = highest value − lowest value
= 95 − 25
= 70

b Median = 60

c Interquartile range = $Q_3 - Q_1$
= 75 − 40
= 35

d **i** 25 is the lowest value and 75 is Q_3,
so 75% × 80 = 60 students had a mark between 25 and 75.

ii 40 is Q_1 and 60 is the median,
so 25% × 80 = 20 students had a mark between 40 and 60.

e 75 is Q_3, so 25% × 80 = 20 students scored more than 75.

EXERCISE 5.03 ANSWERS ON P. 513

Box plots U F R C

1 The number of orders taken per hour at Bernoulli's Pizza on a weekend were:

3	5	1	2	4	6	8	10	7	6
12	15	10	3	5	18	5	8	9	10

EXAMPLE 5

a Find the five-number summary for this data.

b Represent this data on a box plot.

2 The daily amount of snow (in cm) that fell at Thredbo during one ski season was:

2	5	5	2	5	7	1	2	2	2	2	2	12
20	12	5	40	50	10	40	13	30	5	35	2	6

a On how many days did it snow?

b Find a five-number summary for this data.

c Represent this data on a box plot.

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3 The monthly rainfall figures for Penrith one year were:

98 266 149 94 15 65 19 5 24 34 67 28

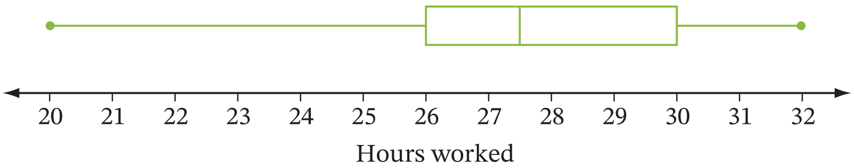
a Find the range.

b Find the five-number summary.

c Represent the data on a box plot.

EXAMPLE 6

4 This box plot represents the number of hours worked in one week by the staff at a supermarket.



a What is the median number of hours worked?

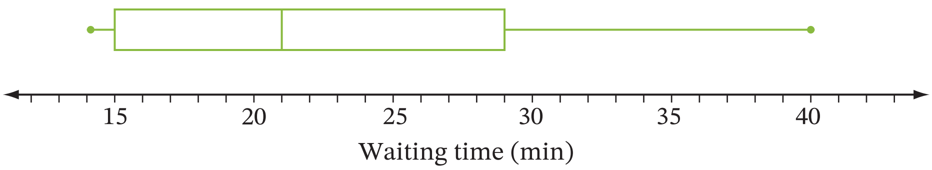
b What is the lower quartile?

c What is the upper quartile?

d Find the interquartile range.

e Estimate the percentage of staff who worked between 26 and 30 hours.

5 The ages of 16 people waiting at a bus stop are displayed in the box plot below.



a What is the range?

b What is the median age?

c Find the interquartile range.

d What percentage of people were aged from:

i 21 to 29? **ii** 15 to 40?

Getty Images/Mark Kolbe

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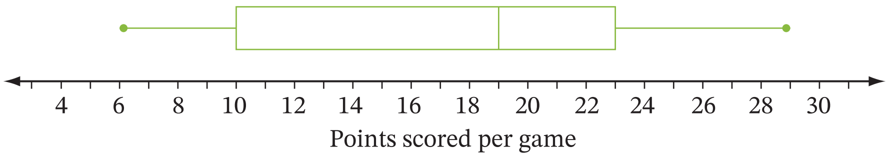
6 The box plot shows the number of points per game scored by Ben in 28 basketball games during the season.



a What is the five-number summary for the box plot?

b Find the interquartile range.

c In how many games did Ben score:

i more than 19 points? **ii** between 19 and 23 points?

iii less than 10 points? **iv** at least 10 points?

7 For each set of data, find the five-number summary and draw a box plot.

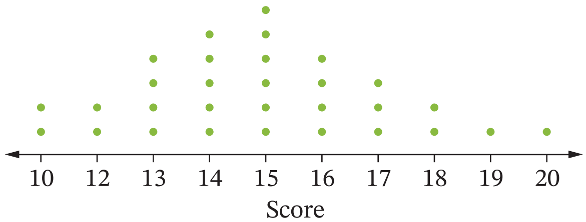
a

Stem	Leaf
2	0 2 3 5
3	3 7
4	4 6 7 8 8 9 9
5	0 1 1 5 6
6	0 3 3 8 8
7	2 5 6
8	5 5 7 8

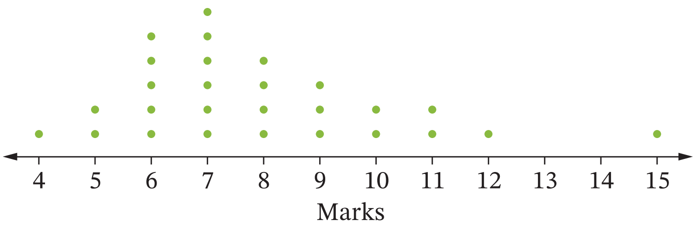
b

Stem	Leaf
3	0 7
4	2 6 6
5	1 2 5 9
6	0 4 7 7 9
7	2 3 5 6 8
8	3 4
9	5

c



8 The results of a general knowledge quiz (out of 15) taken by Year 10 students are displayed in the dot plot. R C



a Find the five-number summary for the dot plot and then draw a box plot.

b Describe the shape of the dot plot and compare it to the shape of the box plot.

c What is the outlier?

d Find the five-number summary for the data in the dot plot without the outlier and draw a box plot.

e Compare the 2 box plots. How are they:

i similar? **ii** different?

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Technology

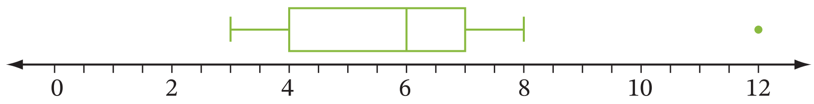
Box plots

Use graphing software or a spreadsheet to draw box plots.

1 Enter this set of data: 3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 12

2 Choose appropriate settings for the scale on your box plot.

3 Your box plot should look similar to the one below.



4 Write down the five-number summary for this data set.

5 Now enter the marks of an English exam completed by 2 classes 10A and 10B and create a box plot for each class:

10A: 21, 81, 33, 58, 67, 76, 64, 74, 56, 60, 54, 74, 49, 83, 66

10B: 77, 63, 63, 35, 51, 42, 54, 55, 71, 43, 41, 41, 40, 76, 72

6 Complete a five-number summary for each class.

7 What is the IQR for each class?

8 Which class had the highest mark?

9 Which class had the lowest mark?

10 Which class performed better? Give reasons for your answer, including explanations using the five-number summaries you found in step 7.

5.04 Parallel box plots

Double boxplots

Analysing data

Parallel box plots

Parallel box-and-whisker plots can be used to compare 2 or more sets of data. They are drawn on the same scale, but above each other.

Example 7

Two sprinters run the following times (in seconds) over 100 metres.

Sam:	10.9	10.5	11.0	9.9	10.7	10.5	10.0	11.2	11.5	10.3
Jesse:	11.0	11.4	10.1	9.8	10.8	11.4	10.7	10.3	11.1	11.6

a Find the five-number summary for each sprinter.

b Draw parallel box plots to display the data for both sprinters.

c Find the interquartile range for each sprinter.

d Find the range for each sprinter.

e Which sprinter is more consistent? Justify your answer.

Parallel box plots

5.04

Solution

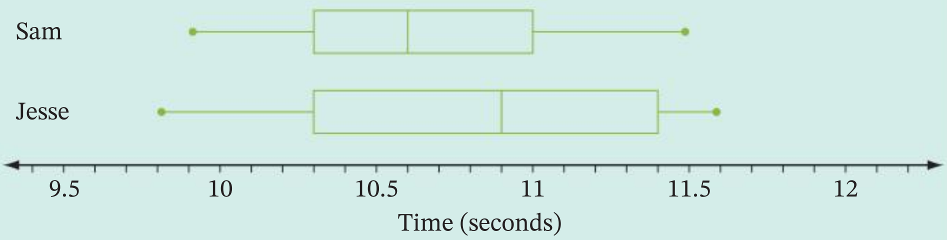
a Sam: 9.9 10.0 (10.3) 10.5 10.5 10.7 10.9 (11.0) 11.2 11.5

Lowest score = 9.9; Q_1 = 10.3; $Q_2 = \dfrac{10.5+10.7}{2} = 10.6$; Q_3 = 11.0; highest score = 11.5

Jesse: 9.8 10.1 (10.3) 10.7 10.8 11.0 11.1 (11.4) 11.4 11.6

Lowest score = 9.8; Q_1 = 10.3; $Q_2 = \dfrac{10.8+11.0}{2} = 10.9$; Q_3 = 11.4; highest score = 11.6

b



c Interquartile range for Sam = 11.0 − 10.3 = 0.7

Interquartile range for Jesse = 11.4 − 10.3 = 1.1

d Range for Sam = 11.5 − 9.9 = 1.6

Range for Jesse = 11.6 − 9.8 = 1.8

e Sam is the more consistent sprinter, since both the range and interquartile range of his times are lower than those of Jesse.

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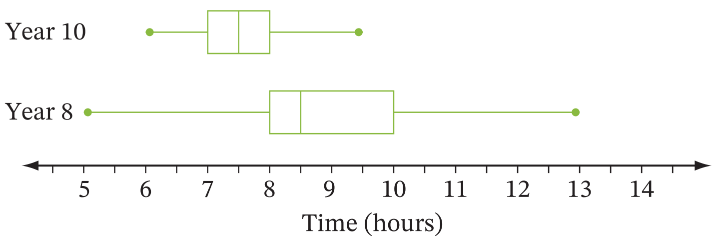
EXERCISE 5.04 ANSWERS ON P. 514

Parallel box plots U F R C

1 The parallel box plots show the number of hours of sleep that Year 8 and Year 10 students usually have on a school night.



a For each Year group, find:

i the range **ii** the median **iii** the interquartile range

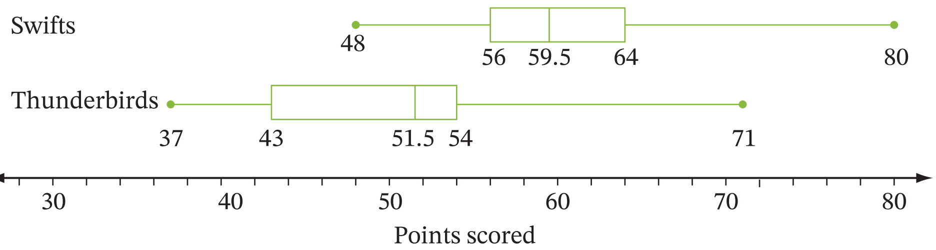
b What percentage of students usually had at most 8 hours of sleep on a school night in:

i Year 8? **ii** Year 10?

c 40 students in both Year 8 and Year 10 were surveyed. How many students usually had at least 10 hours of sleep in:

i Year 8? **ii** Year 10?

2 The number of points scored per match by the Adelaide Thunderbirds and the NSW Swifts netball teams during a season are shown in the parallel box plot. R C



a Find the range of points scored by:

i the NSW Swifts

ii the Adelaide Thunderbirds

b What is the median number of points scored for each team?

c Find the interquartile range for each team.

d Which team is more consistent?

e Which team performed better? Give reasons.

Getty Images/Matt King

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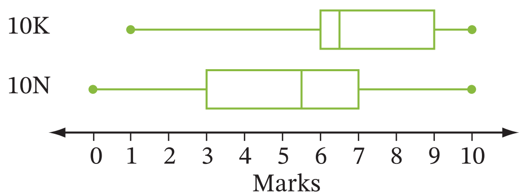
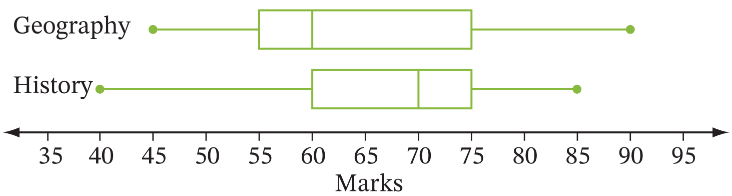
3 The box plots show the test results of students from 2 different classes. R C



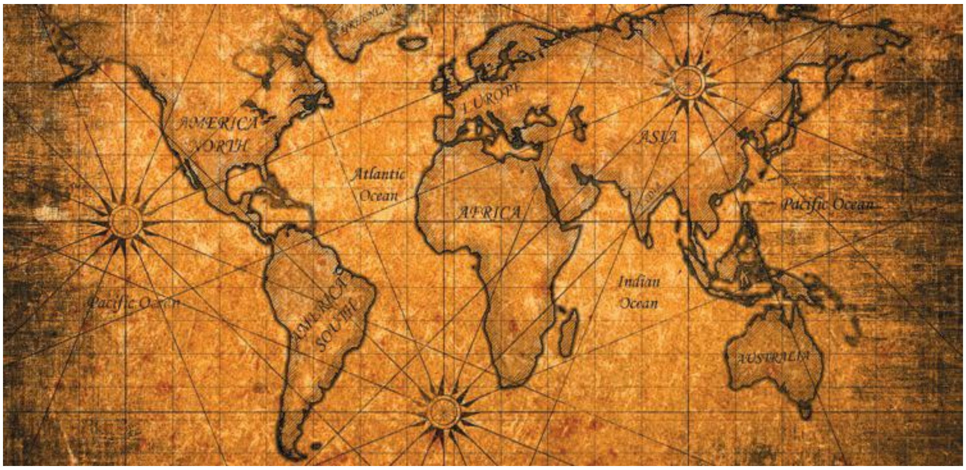
- **a** Find the range of marks for each class.
- **b** Find the median mark for each class.
- **c** Find the interquartile range for each class.
- **d** Which class is more consistent?
- **e** Find the percentage of students who scored 6 or more in 10K.

4 In a Year 10 class of 28 students, the marks for History and Geography tests were displayed using a double box plot. R



Which statement below could be true? Select the correct answer **A**, **B**, **C** or **D**.

- **A** In Geography, more students scored between 60 and 75 than between 55 and 60.
- **B** 14 students scored the same or more in History than the median mark in Geography.
- **C** More students scored 60 or more in History than they did in Geography.
- **D** The interquartile range for Geography is 5 less than the interquartile range for History.



Shutterstock.com/Dmitry Rukhlenko

5.04

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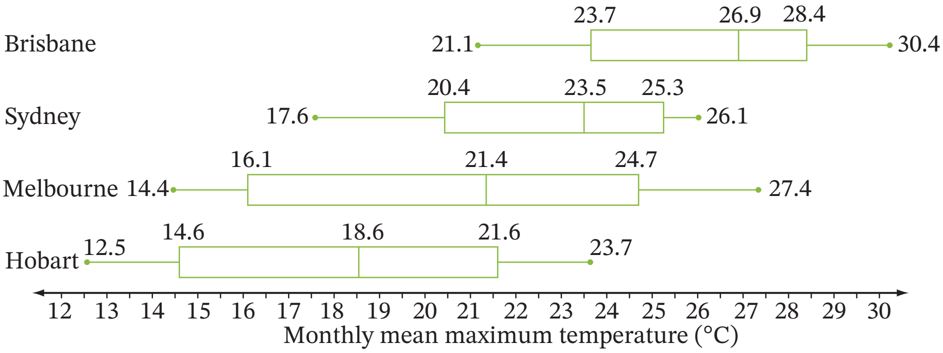
5 The monthly mean maximum temperatures for 4 state capitals are shown in the box plots. R C



a Find the median, range and interquartile range for each city.

b Which capital city had the most spread in temperature?

c Which capital city had the highest mean monthly temperatures? Justify your answer.

d Which city is warmer – Sydney or Melbourne? Give reasons.

e Which city was more consistent – Sydney or Melbourne? Give reasons.

EXAMPLE 7

6 The number of text messages received by a group of students in one hour are as follows. R C

Male:	2	0	3	0	1	2	5	6	2	1	3	2	3	7	4
Female:	4	5	6	3	7	5	8	7	4	2	4	5	10	4	3

a Find the five-number summary for each sex.

b Draw parallel box-and-whisker plots to display the data.

c Find the interquartile range for each sex.

d Find the range for each sex.

e Compare the number of text messages that males and females receive. Are there any significant differences between the spread of the 2 sets of data?

7 Students in a PE class had their heights measured (in cm). R C

Male:	174	167	164	175	189	145	165	166	165	167	171	169
Female:	163	155	171	162	165	183	172	175	166	163	150	186

a Find the five-number summary for each group and draw a parallel box plot to display the data.

b Find the range and interquartile range for each group.

c How does the spread of heights of male students compare with the spread of heights of female students?

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8 Students at a university were asked whether their frequency of exercise was high or low and then had their pulse taken. R C

Low:	90	78	80	84	70	66	92	80	80	77	64	88
High:	96	71	68	56	64	60	50	76	78	49	68	74

a Find a five-number summary for each group and then draw parallel box plots to show the information.

b Find the range and interquartile range for each group.

c Compare the spread between the 2 groups. Are there significant differences between them?

d Which group had the lower pulse rates?

9 The average maximum monthly temperatures (in °C) for Hobart and Darwin are shown. R C

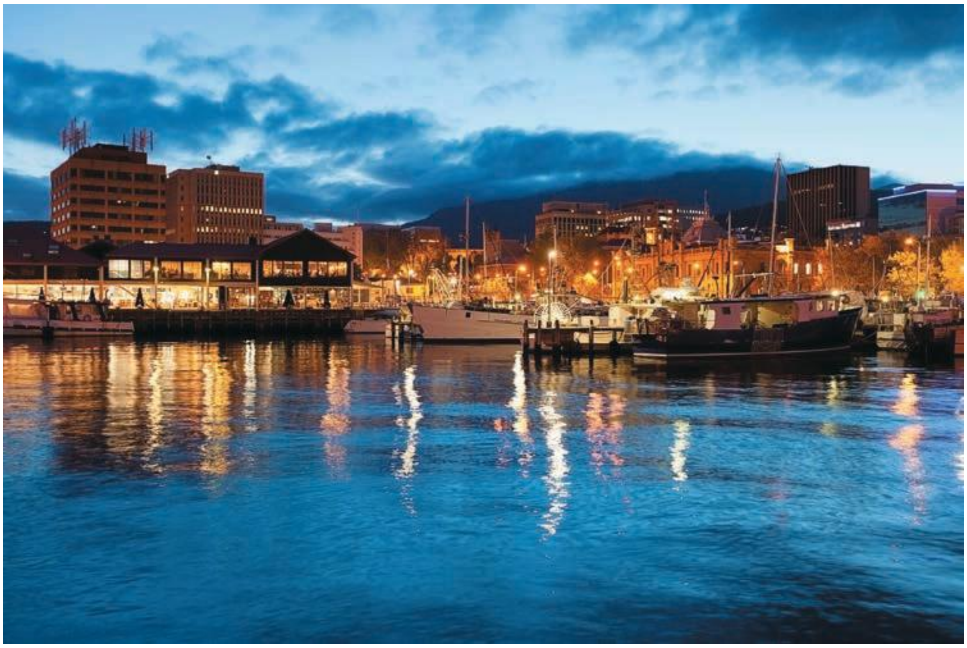
Hobart:	22.7	22.3	20.9	18.2	15.3	13.0	12.5	13.5	15.4	17.4	19.1	20.8
Darwin:	31.8	31.5	32.0	32.7	32.1	30.7	30.6	31.4	32.6	33.3	33.3	32.7

Source: © Bureau of Meteorology

a Find the five-number summary for each city and draw a parallel box plot.

b Find the range and interquartile range for each city.

c Which city had more consistent average maximum monthly temperatures? Give reasons.



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Hobart harbour

5.04

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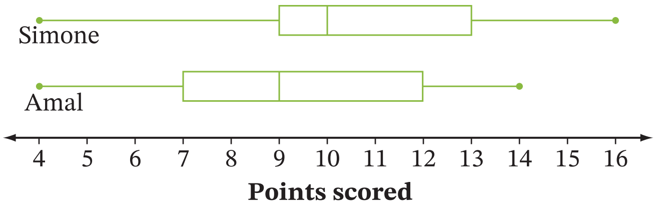
10 These box plots show the numbers of points scored by 2 basketball players during the season. R C



a Which player has the most points scored for a single game?

b What is the range of the points scored by each player?

c By just looking at the range, which player would seem to be more consistent? Justify your answer.

d Find the median score of each player.

e Find the interquartile range for each player.

f Which player is more consistent?

g Estimate the percentage of games in which Simone scored 9 or 10 points.

h Estimate the percentage of games in which Amal scored more than 12 points.

Mental skills 5: Maths without calculators ANSWERS ON P. 515

Multiplying by 9, 11, 99 and 101

We can use expansion when we multiply by a number near 10 or near 100.

1 Study each example.

a $25 \times 11 = 25 \times (10 + 1)$
$= 25 \times 10 + 25 \times 1$
$= 250 + 25$
$= 275$

b $14 \times 9 = 14 \times (10 - 1)$
$= 14 \times 10 - 14 \times 1$
$= 140 - 14$
$= 126$

c $32 \times 12 = 32 \times (10 + 2)$
$= 32 \times 10 + 32 \times 2$
$= 320 + 64$
$= 384$

d $7 \times 99 = 7 \times (100 - 1)$
$= 7 \times 100 - 7 \times 1$
$= 700 - 7$
$= 693$

e $27 \times 101 = 27 \times (100 + 1)$
$= 27 \times 100 + 27 \times 1$
$= 2700 + 27$
$= 2727$

f $18 \times 8 = 18 \times (10 - 2)$
$= 18 \times 10 - 18 \times 2$
$= 180 - 36$
$= 144$

2 Now evaluate each product.

a 16×11	**b** 33×11	**c** 29×9	**d** 45×9
e 62×11	**f** 7×101	**g** 18×101	**h** 36×99
i 19×8	**j** 45×12	**k** 21×102	**l** 6×98
m 32×9	**m** 7×99	**o** 39×101	**p** 71×12

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Comparing data sets

5.05

Example 8

The back-to-back stem-and-leaf plot shows the results in Year 10 Maths and Science tests.

Maths		Science
5 2	3	6 8
8 6 3 0	4	4 6
8 7 7 4 1	5	1 5 9
8 8 7 6 6 3 2 0	6	0 2 8 9
6 5 4 2 1 1	7	2 3 4 4 5 8 8
6 4 3	8	0 0 2 4 5 6 7 8 9
6 0	9	0 4 4

a Find the mean mark (correct to one decimal place) for each subject.

b Find the median for each subject.

c Find the range and interquartile range for each subject.

d For each subject:

i describe the shape **ii** identify any outliers and clusters.

e In which subject have the students performed better? Justify your answer.

Solution

a Mean for Maths $= \frac{1919}{30}$

$= 63.9666\ldots$

≈ 64.0

Mean for Science $= \frac{2151}{30} = 71.7$

b Median for Maths $= \frac{66+66}{2} = 66$ Average of the 15th and 16th values.

Median for Science $= \frac{74+75}{2} = 74.5$

c Range for Maths = 96 – 32 = 64

Interquartile range = 74 – 54 = 20 $Q_1 = 54, Q_3 = 74$

Range for Science = 94 – 36 = 58

Interquartile range = 85 – 60 = 25 $Q_1 = 60, Q_3 = 85$

d **i** The results for Maths are symmetrical, while the results for Science are negatively skewed.

ii No outliers. There is some clustering for the Maths results in the 60s and 70s, and in Science the clustering occurs in the 70s and 80s.

e The students have performed better in Science as the mean and median for it are greater than the mean and median for Maths. This can also be seen by the shape of the data on the stem-and-leaf plot.

STAGE 5.2

Comparing data sets

Back-to-back stem-and-leaf plots

WS Comparing city temperatures

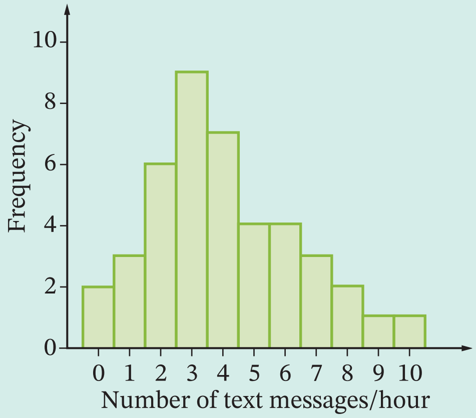
WS Comparing word lengths

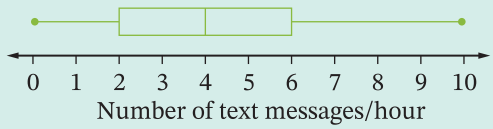
WS Investigating young drivers

STAGE 5.2

Example 9

The number of text messages received per hour by a group of teenagers are displayed in the frequency histogram and the box plot below.





a How many teenagers received more than 6 text messages per hour?

b Find:

i the mode **ii** the median

iii the range **iv** the interquartile range.

c The shape of the distribution is positively skewed. How is this shown by:

i the frequency histogram? **ii** the box plot?

d According to the box plot, what percentage of teenagers received 2 or more text messages?

e What information is better seen on:

i the frequency histogram? **ii** the box plot?

Solution

a Number of teenagers receiving more than 6 text messages

$= 3 + 2 + 1 + 1$ Using the frequency histogram.

$= 7$

b **i** Mode = 3 Using the frequency histogram.

ii Median = 4 Using the box plot.

iii Range = 10 – 0 Using the frequency histogram or box plot.

= 10

iv Interquartile range = 6 – 2 Using the box plot.

= 4

c **i** The tail of the frequency histogram leans towards the higher values.

ii The length of the box plot to the right of the median (Q_2) is greater than its length to the left of the median.

d $Q_1 = 2$, so 75% of teenagers received 2 or more text messages per hour.

e **i** The mode and information regarding the number of text messages received by teenagers can be determined from the frequency histogram.

ii The median, quartiles and interquartile range are easily determined from the box plot.

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EXERCISE 5.05 ANSWERS ON P. 515

Comparing data sets **U F R C**

EXAMPLE 8

1 The back-to-back stem-and-leaf plot shows the amount of cash (in dollars) carried by a sample of Year 11 students at Nelson Senior High. R C

Boys		Girls
5 5 3	0	5 5 6 8 9
8 5 5 2 0	1	0 2 2 5 5 8 8 9
9 6 5 5 5 0 0	2	0 5 6 8 8 8
8 5 5 4 3 2 0 0	3	0 1 4 5 6
5 4 4 2 2 0	4	0 0 5 6
6 6 5 4 3	5	0 3 5
4 2 2	6	5 5 8
5	7	0 4

a Find the mean amount of cash (correct to the nearest cent) carried by each group.

b Find the median amount of cash carried by each group.

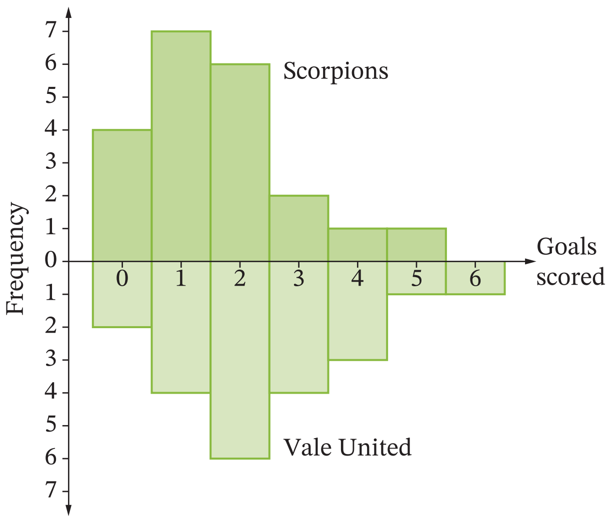
c Find the range and interquartile range for each group.

d For each group:

i describe the shape **ii** identify any outliers and clusters.

e Who generally carries more cash – boys or girls? Justify your answer.

2 The back-to-back histogram shows the number of goals scored by 2 football teams during a season. R C



a How many games were played by each team?

b How many goals were scored by

i the Scorpions **ii** Vale United?

c Find the mean number of goals scored by each team.

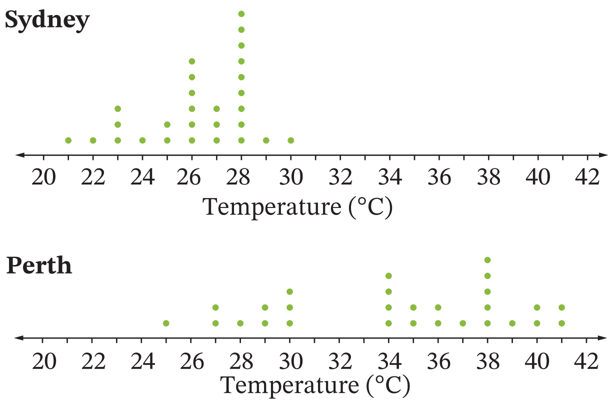
d What is the range for each team?

e Describe the shape of each team's results.

f Which team performed better? Give reasons.

STAGE 5.2

3 The daily maximum temperatures for Sydney and Perth in February are shown. R C



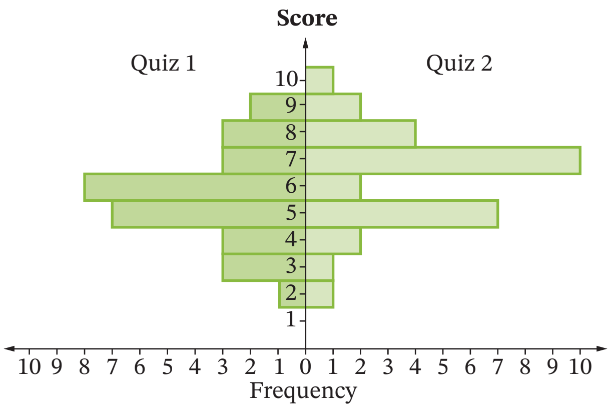
a Find the mean (to 1 decimal place), median and modal temperatures for each city.

b Find the range and interquartile range of temperatures for each city.

c Describe the distribution shape of the temperatures for each city and identify any outliers or clusters.

d Compare the temperatures in Sydney and Perth. Comment on **measures of central tendency** (the mean, median and mode), and measures of spread (range and interquartile range).

4 The results for 2 quizzes taken by a Year 10 History class are shown. R C



a How many students are in the Year 10 History class?

b Find the mean and mode for each quiz.

c Find the median for each quiz.

d For each quiz, find the:

i range **ii** interquartile range.

e Describe the distribution for each quiz, identifying any clusters and outliers.

f Are there significant differences between the results of the 2 quizzes? Justify your answer.

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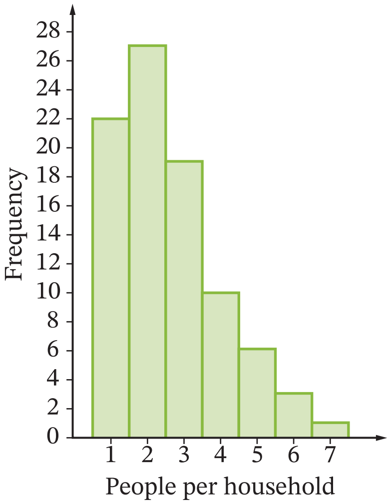
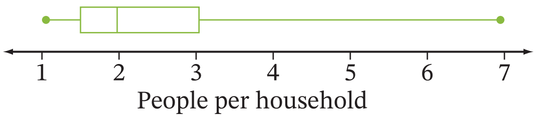
STAGE 5.2

EXAMPLE 9

5.05

5 A survey to determine the number of people per household was conducted in several shopping centres. The results are shown in the frequency histogram and box plot. R C





a How many households had 3 or more people?

b Find the:

i mode **ii** median

iii range **iv** interquartile range.

c Describe the shape of the distribution.

d According to the box plot, what percentage of households had 2 or more people?

e Clustering occurs at 1 to 3 people per household. How is this shown on the:

i frequency histogram? **ii** box plot?

f What information is better seen on the:

i frequency histogram? **ii** box plot?

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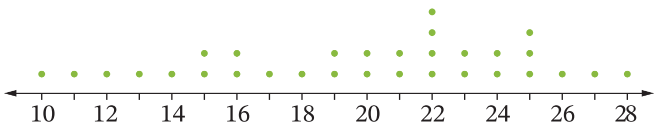
STAGE 5.2

6 The dot plot and box plot show the number of hours that Year 10 students spent watching TV during one week. R C



a How many students watched TV for:

i fewer than 15 hours per week? **ii** more than 20 hours per week?

b Find the:

i mode **ii** range **iii** interquartile range

c What is the shape of the distribution? How is this shown by the:

i dot plot? **ii** box plot?

d Which display of data, the dot plot or box plot, is better for finding the:

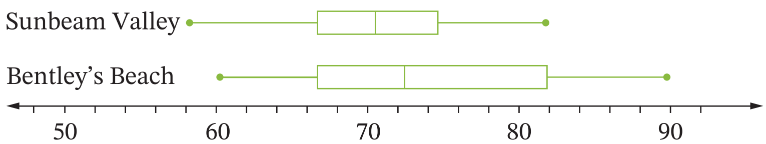
i mode? **ii** median?

iii number of students who watched TV for 25 hours?

iv interquartile range?

7 The speeds of cars were monitored along a main road in 2 different places. The results are shown in the back-to-back stem-and-leaf and parallel box plots. R C

Sunbeam Valley		Bentley's Beach
8	5	
9 8 8 7 4 3 3 3 2 0	6	0 0 1 2 3 5 5 7 8 9
9 9 6 5 5 4 4 3 3 2 2 1 1 0 0 0	7	0 0 2 2 3 3 5 5 5 6 6
2 0 0	8	0 2 3 4 5 5 5 8
	9	0



a Find the range, median and interquartile range for each place.

b What is the shape of the distribution for each place?

c Are there any clusters or outliers in either place?

d According to the box plot, what percentage of drivers in Bentley's Beach drive faster than all drivers in Sunbeam Valley?

e In which place do drivers generally drive faster? Give a possible reason for your answer.

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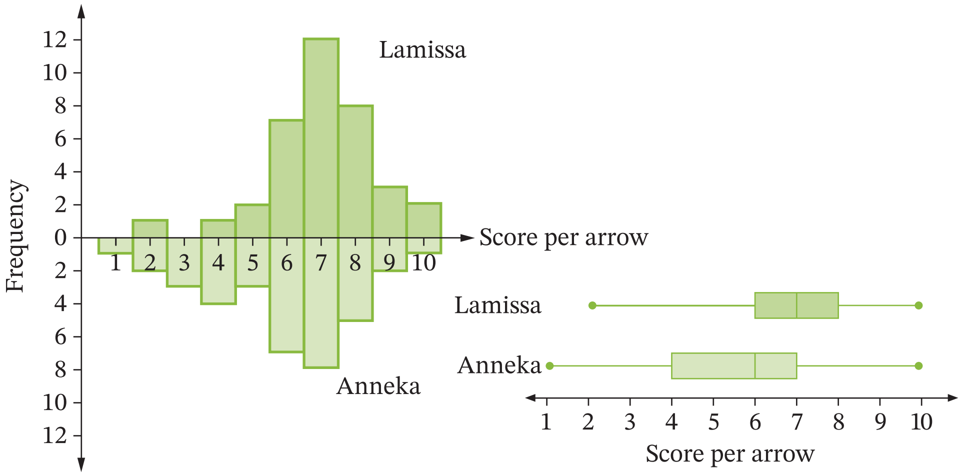
STAGE 5.2

5.05

8 Lamissa and Anneka shoot arrows at a target 50 m away during an archery contest. They scored 10 for a bullseye down to 1 for the outer ring. Their results are displayed in the back-to-back histogram and the parallel box plots. R C



a How many arrows did Lamissa and Anneka shoot?

b Find the mode and median score per arrow for each contestant.

c Find the range and interquartile range for each contestant.

d Describe the shape of the distribution for each contestant.

e According to the box plots, on what percentage of arrows shot was a score of 6 or less achieved by:

i Lamissa? **ii** Anneka?

f Who was the better archer during this contest? Justify your answer by referring to the measures of central tendency and spread.

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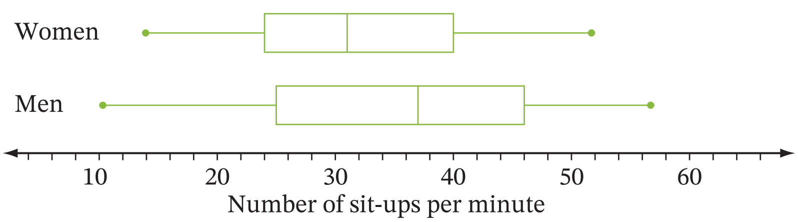
STAGE 5.2

9 The number of sit-ups per minute completed by men and women at the Full On Fitness Centre are displayed in the back-to-back histogram and parallel box plots. **R** **C**

Women		Men
8 7 5 4	1	0 6 7 9 9
9 9 9 8 8 7 4 4 3 3 1 0	2	0 2 3 4 4 5 5 7 7 8
7 6 5 5 5 4 3 2 1 0 0	3	0 2 4 5 6 6 7 7 8 8 8 8 9
7 5 4 3 2 0 0	4	1 3 4 6 6 6 6 6 7 7 9
2 1 0	5	0 1 3 4 7 7



a Why would a dot plot be an inappropriate way to display the data shown above?

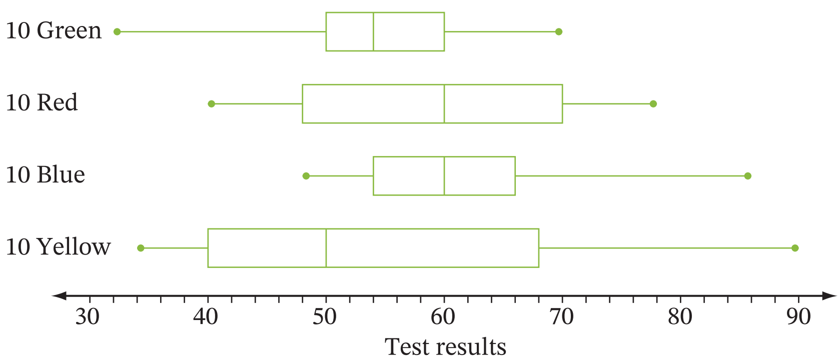
b What is the median number of sit-ups per minute completed by each group?

c Find the range and interquartile range for each group.

d Describe the shape of the distributions for each group.

e Which group has more spread in the number of sit-ups completed per minute? Give reasons for your answer.

10 The results of a Maths test given to 4 Year 10 classes are shown. **R** **C**



a What is the range of test results for:

i 10 Yellow? **ii** 10 Blue?

b For which class are the test results:

i positively skewed? **ii** negatively skewed? **iii** symmetrical?

c Which class had:

i the lowest interquartile range? **ii** the highest test score?

iii the highest median?

d Which class had the best test results overall? Give reasons.

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Scatterplots

5.06

STAGE 5.2

WS Scatterplots

Scatterplots matching game

Bivariate data is data that measures 2 variables, such as a person's height and arm span (distance between outstretched arms). Bivariate data is represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis.

A **scatterplot** is a graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern that may be linear or non-linear. If there is a pattern, then a relationship may exist between the 2 variables.

Example 10

The heights and arm spans of a group of students are shown in the table.

Height, *H* cm	162	182	153	145	172	163	150	142	183	145	192	171
Arm span, *S* cm	158	185	145	143	174	165	151	141	181	158	191	178

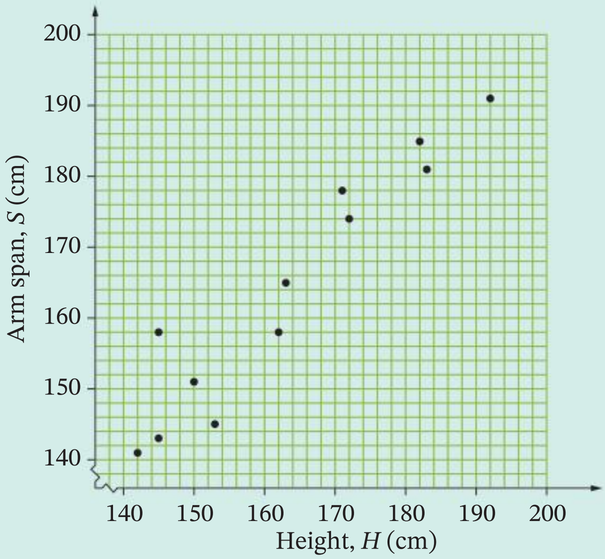
a Plot the data on a scatterplot.

b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and arm spans.

Solution

a



b The points form a linear pattern.

c As the heights of students increase, their arm spans tend to increase.

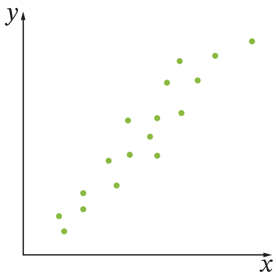
Strength and direction of linear relationships

The linear pattern will indicate the **strength** and **direction** of the relationship between the 2 variables. The strength of a relationship between 2 variables can be described as:

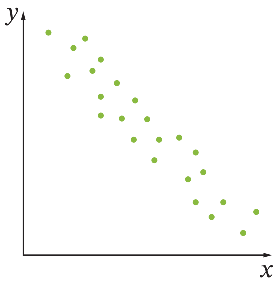
- **strong** if the points are close together
- **weak** if the points are more spread out
- **perfect** if all points lie on a straight line

The direction of a relationship can be described as **positive** or **negative**:

x and y have a **positive relationship** if y increases as x increases.

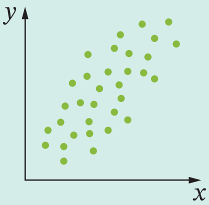


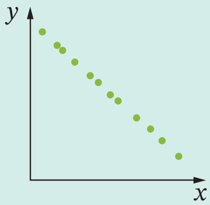
x and y have a **negative relationship** if y decreases as x increases.

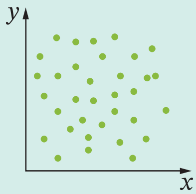


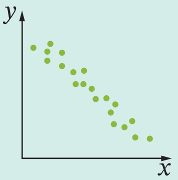
Example 11

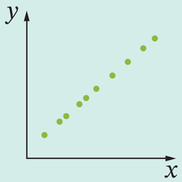
Describe the strength and direction of the relationship shown in each scatterplot.

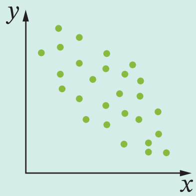
a


b


c


d


e


f


Solution

a Weak positive relationship — The points can be seen to form a line, but they are spread out.

b Perfect negative relationship — The points lie on a decreasing straight line.

c No relationship — The points are very spread out with no pattern.

d Strong negative relationship — The points can be seen to form a decreasing line and they are close together.

e Perfect positive relationship — The points lie on an increasing straight line.

f Weak negative relationship — The points can be seen to form a decreasing line, but they are spread out.

Dependent and independent variables

If a variable y depends on the value of the variable x, y is called the **dependent variable** and x is called the **independent variable**. For example, stride length (the length of a person's walking step or pace) depends on the person's height, so stride length is the dependent variable and height is the independent variable. When graphing, the dependent variable is shown on the vertical (y-) axis while the independent variable is shown on the horizontal (x-) axis.

EXERCISE 5.06 ANSWERS ON P. 516

Scatterplots U F R C

1 The heights and handspans of a group of students are shown in the table. C EXAMPLE 10

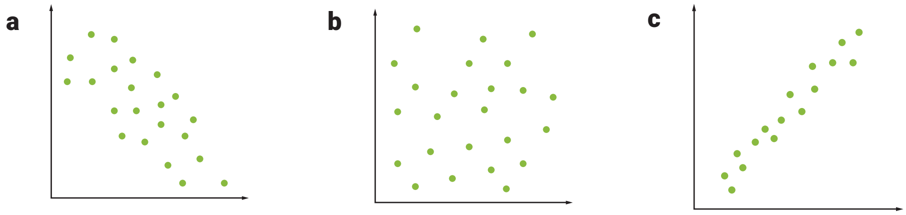
Height, H cm	168	175	175	156	160	173	171	180	185	175	182	180
Handspan, S cm	20.0	21.1	17.6	16.5	17.5	19.0	20.8	22.5	25.0	23.0	20.2	21.1

a Plot the data on a scatterplot.

b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and their hand spans.

2 Describe the strength and direction of the relationship shown in each scatterplot. C EXAMPLE 11



3 Describe the strength and direction between height, H and handspan, S in question **1**. C

4 The height and stride length measurements of some students are shown in the table. R C

Height, H cm	174	160	158	180	169	172	171	171	148	190	166	173
Stride length, L cm	72.2	64.0	66.4	74.7	70	71.5	70.9	71.2	61.4	78.9	68.0	71.9

a Explain why stride length is the dependent variable.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the relationship between the students' heights and stride lengths.

e Describe the strength and direction of the relationship.

f Predict the stride length of a student who is 175 cm tall.

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STAGE 5.2

5 The table lists the points scored for and against each NRL team one season. R C

Points scored for, *F*	Ponints scored against, *A*
568	369
579	361
559	438
497	403
597	445
545	536
445	441
481	447
405	438
506	551
449	477
448	488
462	626
497	609
409	575
431	674

a Graph this data on a scatterplot.

b Is the pattern of the points linear?

c Describe the strength and direction of the relationship between points scored for and points scored against.

6 Year 10 students were surveyed on the number of hours in a week they spent doing homework and the number of hours they spent on the computer. R C

Homework, *H*	2	15	12	5	4	2	4	15	14	5	2	5	20	4	2	11
Computer, *C*	25	30	18	35	6	30	20	22	6	40	8	3	20	30	5	8

a Plot the points on a scatterplot.

b Describe the strength and direction of the relationship between hours spent doing homework and hours spent on the computer.

7 A survey was conducted to see whether there was a relationship between height and the age of students in a high school. R C

Age, *A* (years)	14	16	15	13	11	14	17	15	12	11	14	16	13	18
Height, *H* (cm)	162	174	182	162	132	173	187	160	154	145	165	171	151	181

a Graph the points on a scatterplot.

b Which variable could be considered as the dependent variable? Give reasons.

c Describe the strength and direction of the relationship between the age and height of students.

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STAGE 5.2

5.06

Technology

Scatterplot patterns

Investigate one of the following pairs of bivariate data for a group of students or people. You will need instruments (measuring tapes and/or trundle wheels) and stopwatches to help you collect your data.

- Height vs arm span
- Reaction time vs hours of sleep
- Stride length vs 50 m sprint time

1 Enter your data into a spreadsheet. Graph it using Scatter with Smooth Lines and Markers.
2 Analyse your graph. What type of linear relationship does it show? Positive or negative? Strong or weak?
3 Write a brief summary describing the relationship between the 2 variables.

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Bivariate data involving time 5.07

STAGE 5.2

Bivariate data involving time, or time series data, is two-variable data that has time as the independent variable. Examples of time series data are population changes over time, weekly share prices, daily rainfall and patients' heart rates.

Example 12

This table shows the average household size between 1961 and 2016, according to the Census.

Year	People
1961	3.6
1966	3.5
1971	3.3
1976	3.1
1981	3.0
1986	2.9

Year	People
1991	2.8
1996	2.6
2001	2.6
2006	2.6
2011	2.6
2016	2.6

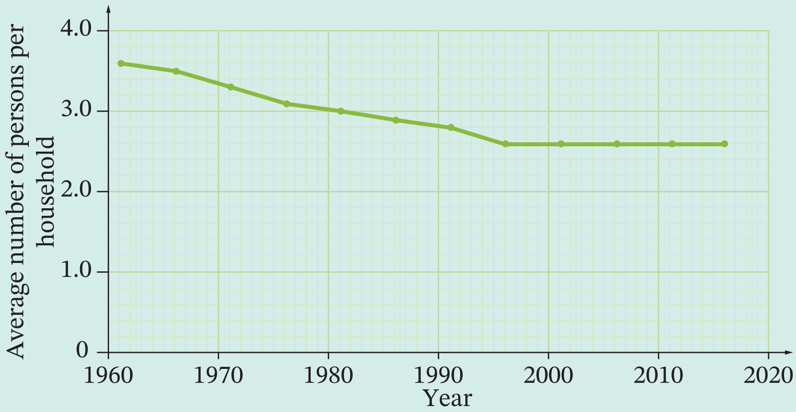
Source: CC © Copyright 2016, Commonwealth of Australia aifs.gov.au

a Graph the data on a scatterplot and join the points.
b Use your graph to describe the change in average household size from 1961 to 2016.
c Based on your time series graph, estimate the household size for 2021.

STAGE 5.2

Solution

a



Year is the independent variable.

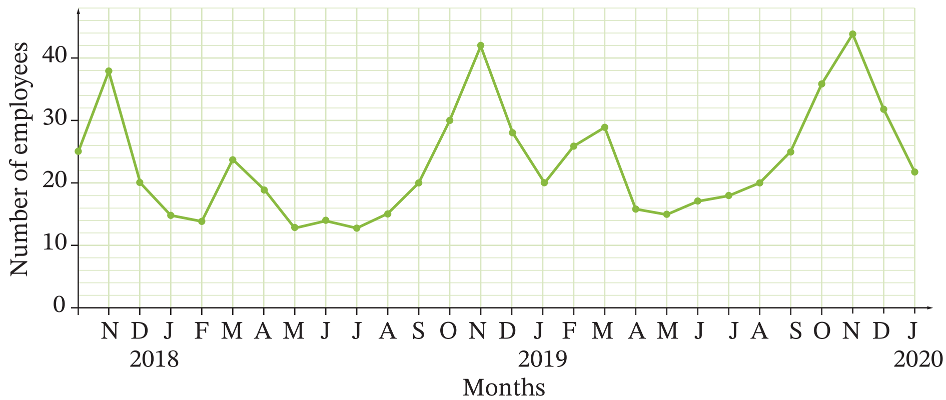
b The average household size decreased from 3.6 in 1961 to 2.6 in 1996 and since then there has been no change.

c 2.4 to 2.6 people per household.

EXERCISE 5.07 ANSWERS ON P. 517

Bivariate data involving time UFRC

1 The number of people employed per month at SupaSave supermarket from November 2017 to February 2020 is displayed on this time series graph. R C



a How many people were employed by the supermarket in:

i November 2017? **ii** December 2018? **iii** June 2019?

b In which month of the year were the most people employed by the supermarket? Suggest a reason why.

c In which month of the year were the least number of people employed? Suggest a reason why.

d Describe how the number of people employed by the supermarket changes from November 2017 to February 2020.

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STAGE 5.2

EXAMPLE 12

5.07

2 The population figures for Australia from 1960 to 2020 are given in the table. R C

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	2020
Population (millions)	10.28	11.39	12.51	13.89	14.70	15.76	17.07	18.07	19.15	20.39	22.3	23.8	25.4

a Graph the data on a scatterplot and join the points.

b Between which years was the greatest population increase?

c Use your graph to describe the change in Australia's population from 1960 to 2020.

d Based on your time series graph, estimate the population for Australia in:

i 2030 **ii** 2055.

3 This table shows the fatalities on NSW roads from 1950 to 2020. R C

Year	1950	1960	1970	1980	1990	2000	2010	2020
Fatalities	634	978	1309	1303	797	603	405	352

a Draw a time series graph for this data.

b Describe the change in road fatalities from 1950 to 2020.

c Give possible reasons for the reduction in road fatalities from a high of 1309 in 1970 to a low of 352 in 2020.

4 The annual mean maximum temperatures for Sydney from 1995 to 2019 are shown. R C

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003
Temp (°C)	21.8	22.1	22.4	22.7	22.1	22.7	23.1	23.1	22.7

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Temp (°C)	23.4	23.4	23.1	22.7	22.1	22.1	22.6	22.6	22.7

Year	2013	2014	2015	2016	2017	2018	2019
Temp (°C)	23.7	23.5	23.2	23.9	23.7	23.4	24.0

Source: Bureau of Meteorology

a Draw a time series graph for temperatures from:

i 1995 to 2005 **ii** 2006 to 2019.

b Has there been much change in Sydney's temperature from:

i 1995 to 2005? **ii** 2006 to 2019?

Justify your answer.

c Are there differences in temperature between the periods 1995–2005 and 2006–2019? Give reasons.

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STAGE 5.2

5 The table below shows the annual emissions of carbon (measured in Megatonnes) from 2008 to 2018. R C

Year	2008	2009	2010	2011	2012	2013
Annual emissions (Mt)	554	542.8	577.2	562.5	548.5	533.8

Year	2014	2015	2016	2017	2018
Annual emissions (Mt)	533.7	528.5	529.8	531.6	533

a Draw a time series graph for this data.

b Describe the change in carbon emissions from 2008 to 2018.

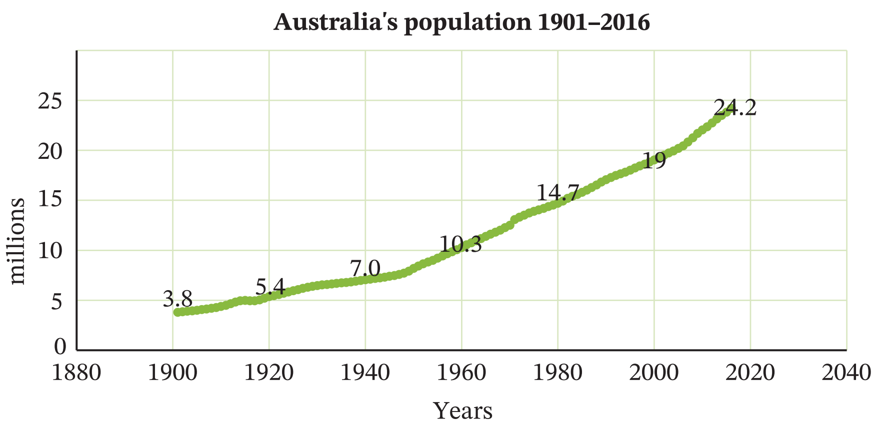
c What happens to the carbon emissions after 2011?

d Give a possible reason for your answer to part **c**.

e What is your estimate of carbon emissions for:

i 2025? **ii** 2035?

6 This graph shows Australia's population from 1901 to 2016. R C



Source: Australian Historical Population Statistics (3105.0.65.001); Australian Demographic Statistics (3101.0).

a What was Australia's population in 1901?

b By how much had Australia's population increased between 1901 and 2016?

c What was the average annual rate of increase in population between 2000 and 2016?

d If this trend continues, what is the expected population in 2030?

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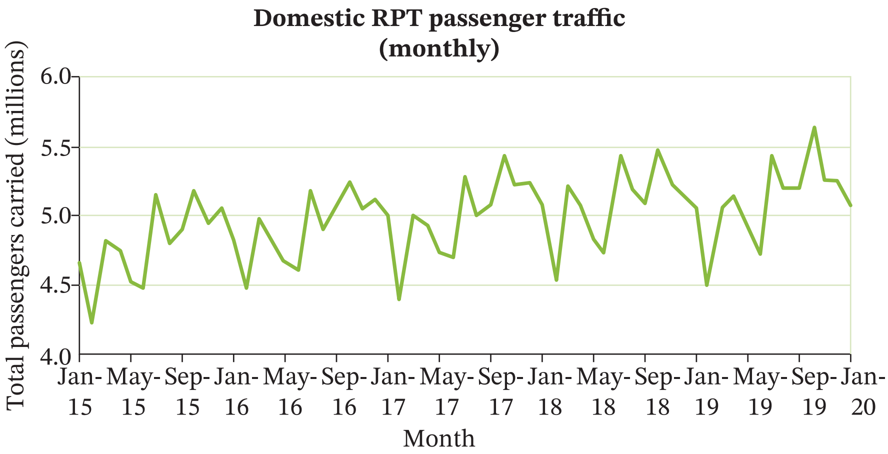
7 This time series graph shows the monthly amount of passenger traffic on Australian domestic commercial airlines. R C



Source: https://www.bitre.gov.au/statistics/aviation/domestic

a Describe the trend in domestic passenger traffic from 2015 to January 2020.

b What was the approximate amount of passenger traffic per month in:

i January 2015? **ii** January 2017?

iii January 2018 ? **iv** January 2020?

c What was the percentage increase in domestic passenger movements from January 2015 to January 2020?

STAGE 5.2

5.07

Investigation

Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas – from population, employment, weekly earnings, weight and obesity in adults, to health of children in Australia.

Visit the ABS website www.abs.gov.au to answer the following questions.

1 **a** What is the current population of Australia?

b What is the predicted population for:

i 2020? **ii** 2030? **iii** 2040?

c What is Australia's rate of population increase?

2 Go to 2016 Census Data by Location, and then to Data and analysis.

a What was the population in NSW and its increase from 2011?

b Which state had the:

i largest increase in population?

ii smallest increase in population?

5.08 Statistics in the media

We live in a world of 24-hour news, whether it is from newspapers, TV or the Internet, that often quote results from surveys. When survey data is used in the media we need to consider:

- where the news comes from and what **samples** the statistics are based on
- who supplied the information
- the number of samples and the sample size used
- the way in which the collected data has been presented

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STAGE 5.2

Example 13

What concerns could be raised about the following claim?

'The Daily Sun newspaper reports that it has an average issue readership of 1.39 million and that its Travel section has a readership of 1.46 million.'

Solution

The newspaper is reporting about its own readership, so it may be biased. It also states that its Travel section has a higher readership than its issue readership.

Example 14

The weights (in kg) of a large group of 18–20-year-olds attending university are:

57	58	62	84	64	74	57	55	56	90
68	63	49	66	63	65	60	60	46	70
85	60	70	41	73	75	67	63	70	85
51	49	75	77	87	54	60	75	58	68
55	65	66	57	85	75	56	60	62	75
74	58	51	62	50	55	71	57	58	100
72	58	103	64	52	55	80	96	45	87
81	80	48	54	65	54	59	50	78	60
74	70	64	59	72	78	104	63	102	95

a How many students were in the group?

b Randomly select 4 groups of 10 and for each sample calculate:

i the mean **ii** the median **iii** the interquartile range.

c Use your results to estimate the mean, median and interquartile range of the **population** from your 4 samples.

d Compare your estimates to the mean, median and interquartile range of the population.

STAGE 5.2

5.08

Solution

a There were 90 students in the group.

b Randomly select 4 samples of 10 from the 'population'.

Sample 1:	90	63	75	48	74	85	51	96	60	78
Sample 2:	62	75	103	64	65	54	55	54	60	75
Sample 3:	68	70	57	52	78	74	60	63	58	87
Sample 4:	72	54	52	80	45	87	49	77	54	58

The statistics for each group are:

Sample 1:	$\bar{x} = 72$	median = 74.5	interquartile range = 25
Sample 2:	$\bar{x} = 66.7$	median = 63	interquartile range = 20
Sample 3:	$\bar{x} = 66.7$	median = 65.5	interquartile range = 16
Sample 4:	$\bar{x} = 62.8$	median = 56	interquartile range = 25

c Taking averages, population statistics estimates are:

Mean $= \bar{x} = \dfrac{72 + 66.7 + 66.7 + 62.8}{4} = 67.1$ (correct to 1 decimal place)

Median $= \dfrac{74.5 + 63 + 65.5 + 56}{4} = 64.8$ (correct to 1 hbox*x* decimal place)

Interquartile range $= \dfrac{25 + 20 + 16 + 25}{4} = 21.5$

d The statistics for the population are:
Mean, $\bar{x} = 66.9$ (correct to 1 decimal place)
Median = 64
Interquartile range = 18
The estimates for the mean, median and interquartile range compare very favourably with the population statistics.

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EXERCISE 5.08 ANSWERS ON P. 518

Statistics in the media U F R C

EXAMPLE 13

1 A TV network surveys 300 people in shopping centres between 9 a.m. and 11 a.m. to get feedback on its new game show.

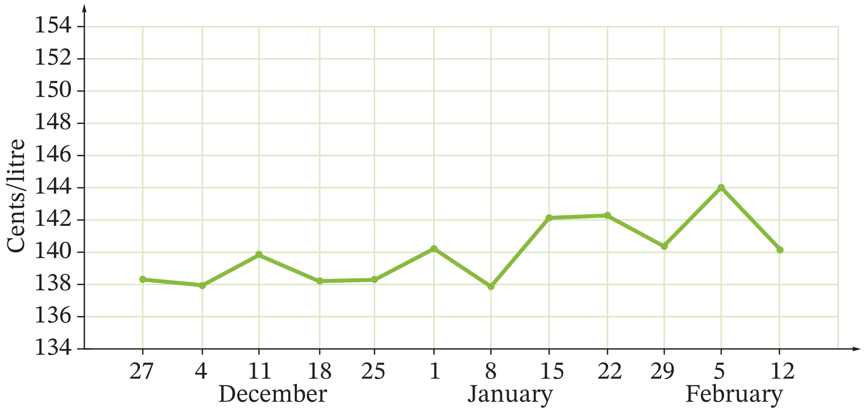
a How is this survey biased?

b Suggest a better method for getting feedback about its game show.

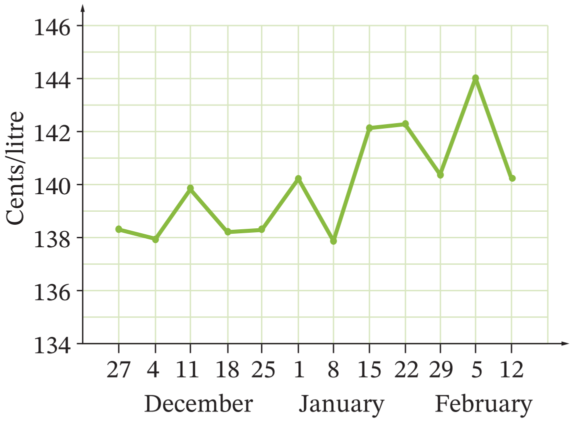
2 A report about hot water systems recommended a heat pump system. The report stated that people in Queensland who had the heat pump hot water system saved 30% on their electricity bills. The company is using this information to advertise their product in NSW and Victoria. How might this information be unsuitable for people in NSW and Victoria? Give reasons.

3 Two reports on petrol prices over a 12-week period were written by 2 different companies. Each company used line graphs to show the price of petrol for the same period. R C

Petrol pricing: Company A



Petrol pricing: Company B



a What message is being suggested about petrol prices in the line graph presented by:

i Company A? **ii** Company B?

b How could both graphs be improved to give a true picture of changing petrol prices?

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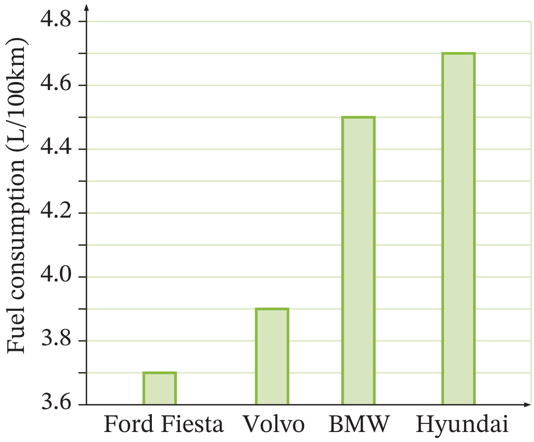
4 A report on the diesel fuel consumption of different cars was published in a motoring magazine, featuring this misleading graph. R C



a What is the graph suggesting about the fuel consumption of the different cars?

b What is the difference in fuel consumption between the:

i Ford Fiesta and the Volvo?

ii Ford Fiesta and the Hyundai?

iii BMW and the Hyundai?

c How should the graph be redrawn so that it is not biased towards the Ford Fiesta and the Volvo?

5 A company sells a new app. After 3 months, they conduct a survey and customers are asked to rate the product as Excellent, Good or Satisfactory. Is the survey biased? Justify your answer. R C

6 A market research company working for a car manufacturer needs to determine the most popular car colours. R C

a Give an example of a biased question for this survey.

b What other information should the market research company use, apart from the survey, to determine the most popular colour car?

5.08

STAGE 5.2

EXAMPLE 14

7 **a** Randomly select 4 samples of 10 weights from the population shown in Example **14**, and for each sample calculate the: R C

i mean **ii** median **iii** interquartile range.

b Use your results to estimate the mean, median and interquartile range of the population from your 4 samples.

c How do the statistics of your samples compare to the mean, median and interquartile range of the population?

d How do the estimated statistics compare to the population statistics?

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8 **a** Repeat the process of question **7** by taking 2 samples of size:

i 5 **ii** 15 **iii** 20

b Do the sample statistics become more accurate and move closer to the population statistics as sample size increases? R C

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Power plus ANSWERS ON P. 518

5.08

1 The strength and direction of the relationship between 2 variables can be measured by the correlation coefficient (r).

a Between which 2 values does the correlation coefficient lie?

b What is the strength and direction of the relationship if the correlation coefficient is zero?

c Write a possible value for the correlation coefficient for each relationship described.

i perfect positive   ii weak negative   iii strong negative

2 Two variables may have a strong relationship, but this does not mean a change in one variable causes a change in the other. Which of the following pairs of variables have a *causal* relationship?

a height and weight of people

b the time it takes to walk to school and the distance from home to school

c the number of children per household and the number of mobile phones per household

d the age of people and their reaction time

e the price of petrol and the amount of petrol sold

f the interest rate of loans and the number of new home loans

3 The following values are the test results on a History exam for a class of 20 students.

13	14	16	12	14	16	18	13	15	10
9	15	13	14	13	10	8	14	16	14

a Find the mean, median and mode.

b Find the range and interquartile range.

c An error was made in recording the values and 4 marks need to be added to each value. What effect will this have on the statistics calculated in parts **a** and **b**?

Qz
Statistics

CHAPTER 5 REVIEW

Language of maths

bias	bivariate data	box plot	cluster
dependent variable	five-number summary	independent variable	interquartile range
mean	measure of central tendency (or location)	measure of spread	median
mode	negatively skewed	outlier	parallel box plots
positively skewed	quartile	range	scatterplot
skewed	strong	symmetrical	weak

1 What is represented by the 'whiskers' on a **box plot**?

2 What are the **measures of central tendency** and the **measures of spread**?

3 What are the five things found in a **five-number summary**?

4 Describe a statistical distribution that is **positively-skewed**.

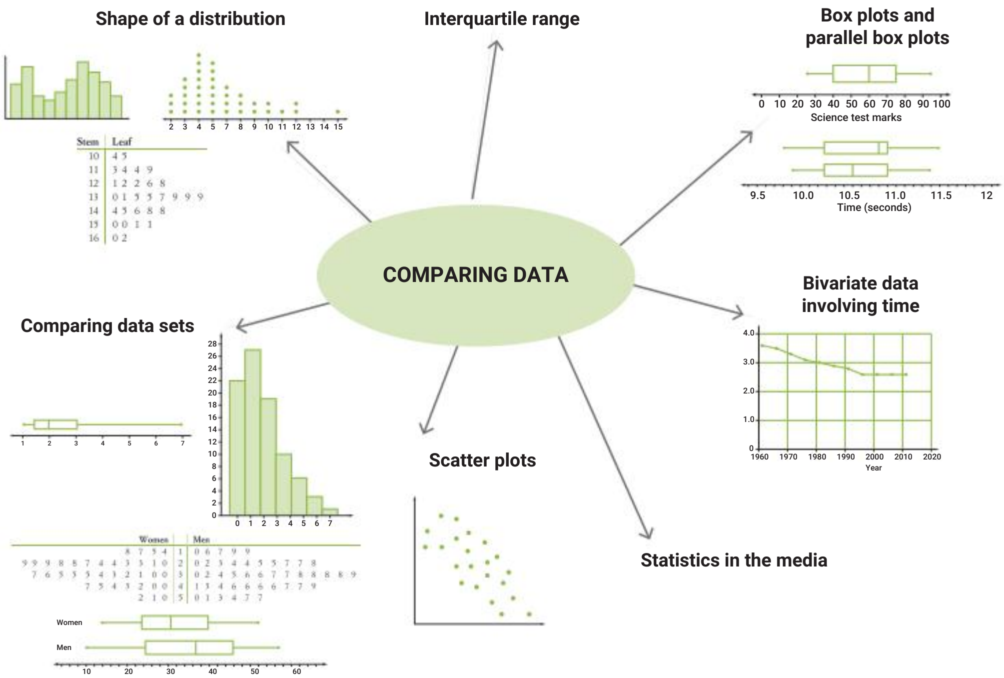
5 What type of graph is used to represent **bivariate data**?

6 Give 2 examples of how statistics can be misleading.

Topic summary

WS
Mind map: Comparing data

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

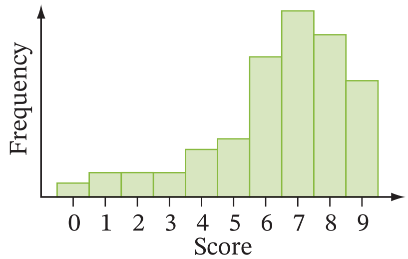


TEST YOURSELF 5 ANSWERS ON P. 518

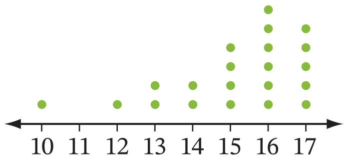
1 For each statistical distribution: 5.01

i describe the shape **ii** identify any outliers and clusters.

a



b



c

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

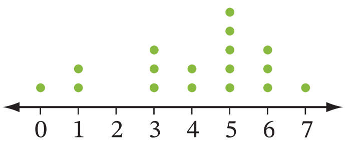
STAGE 5.2

2 Find the interquartile range of each set of data. 5.02

a 5 8 8 10 12 13 14 15 18

b 24 15 23 28 20 20 18 30 21 18

c



d

Stem	Leaf
3	0 1 2
4	3 5 8 8 9 9 9
5	4 5 6 6 8
6	0 1 3 7
7	2

e

Score	Frequency
10	3
11	8
12	15
13	18
14	10
15	5

3 The number of goals scored by the Under-18s Vale soccer team are: 5.03

2	0	0	4	2	1	1	2	3
2	3	7	4	3	1	0	4	2

a Find the range and interquartile range of the scores.

b Find the five-number summary for the data.

c Draw a box plot for the data.

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STAGE 5.2

5.04

4 The pulse rates of students were taken before and after exercising. The results were:

Before exercise:	78	80	66	70	56	64	68
	65	50	76	80	70	70	59
After exercise:	141	140	89	95	110	126	84
	82	90	88	146	98	96	92

a Find the five-number summary for the pulse rates before and after exercise.

b Construct parallel box plots for the 2 sets of data.

c Find the range and interquartile range of the pulse rates:

i before exercising

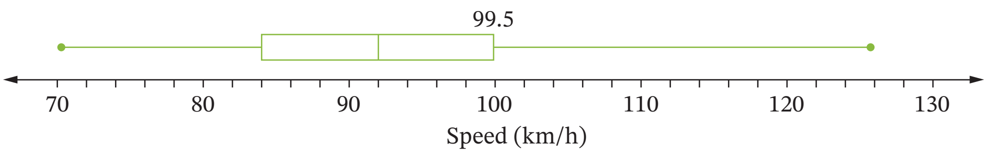
ii after exercising.

d Compare the 2 sets of pulse rates. Are there significant differences between them? Justify your answer.

5.05

5 The speeds of cars (in km/h) were monitored between 1:00 and 1:30 p.m. on a main road. The results are shown in the stem-and-leaf plot and box plot.

Stem	Leaf
7	0 3 7 9
8	0 2 2 3 5 6 8 8 9
9	0 0 1 3 5 5 7 8 9 9 9
10	0 0 4 4 6
11	0 1
12	6



a Which display best indicates:

i skewness?

ii clustering and outliers?

b Why would a dot plot be unsuitable for displaying the data?

c Find the:

i median

ii interquartile range.

d What percentage of cars were travelling at a speed of at least 92 km/h?

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STAGE 5.2

TEST YOURSELF 5

6 11 boxes containing 60 oranges each were placed in cold storage for different periods. After storage, the number of good oranges in each box was counted. 5.06

Weeks in storage (*W*)	2	5	12	8	14	6	5	9	10	3	11
Number of good oranges, (*N*)	58	50	33	40	28	50	52	38	35	55	33

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the relationship between the number of weeks in storage and the number of good oranges.

e Describe the strength and direction of the relationship between the variables *W* and *N*.

7 The mean maximum temperatures in Perth for January from 2010 to 2020 are shown. 5.07

Year (*t*)	2010	2011	2012	2013	2014	2015
Temperature (*T*, °C)	33.4	32.5	33.5	31.7	32.3	33.0
Year (*t*)	2016	2017	2018	2019	2020	
Temperature (*T*, °C)	32.0	31.0	30.8	30.5	30.5	

Source: Bureau of Meteorology

a What is the independent variable?

b Graph the data on a scatterplot and join the points.

c Has any change occurred to the temperatures in Perth for the month of January over the 11-year period? Give reasons for your answer.

8 An advertisement in a magazine states that a product is 75% fat-free. 5.08

a What impression is the advertisement trying to make about the product?

b What doesn't the advertisement say about the product?

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6

NUMBER AND ALGEBRA

EQUATIONS AND INEQUALITIES

In 1962, when astronaut John Glenn was preparing to be the first American to orbit the Earth, he called upon mathematician Katherine Johnson (1918–2020) to check by hand the complex calculations being performed on NASA's new electronic computers. Johnson worked for NASA for 35 years and her story was told in the 2016 book and film *Hidden Figures*. During her career, she solved equations that guided the paths of spacecrafts and helped the US send the first astronauts to the moon in 1969. Building on the ancient achievements of the Egyptians and Babylonians, Johnson opened the door to space travel and future space tourism.

Chapter outline

		Working mathematically				
6.01	Equations	U	F			
6.02	Equations with algebraic fractions*	U	F			
6.03	Quadratic equations $x^2 + bx + c = 0$*	U	F		R	C
6.04	Equation problems	U	F	PS	R	C
6.05	Equations and formulas*	U	F		R	C
6.06	Graphing inequalities on a number line*	U	F			C
6.07	Solving inequalities*	U	F		R	C

*STAGE 5.2

Wordbank

> 'is greater than'

< 'is less than'

equation A mathematical statement that 2 quantities are equal, involving algebraic expressions and an equals sign (=)

formula A rule written as an algebraic equation, using variables

inequality A mathematical statement that 2 quantities are not equal, involving algebraic expressions and an inequality sign (>, ≥, <, or ≤)

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$.

solution The answer to an equation, inequality or problem; the correct value(s) of the variable that makes an equation or inequality true

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve linear equations and problems involving equations
- (STAGE 5.2) solve linear equations involving simple algebraic fractions
- (STAGE 5.2) solve simple quadratic equations of the form $ax^2 = c$ and $x^2 + bx + c = 0$
- (STAGE 5.2) use formulas to solve problems
- (STAGE 5.2) graph inequalities on a number line
- (STAGE 5.2) solve linear inequalities

SkillCheck ANSWERS ON P. 519

1 Solve each equation.

a $8y = 16$ **b** $10x = 120$ **c** $\frac{m}{5} = 2$

d $w + 6 = 10$ **e** $m - 3 = 12$ **f** $n + 6 = -4$

2 Expand each expression.

a $5(x + 10)$ **b** $4(y - 1)$ **c** $2(5 - 3y)$

3 Solve each equation.

a $2x + 3 = 23$ **b** $3x - 5 = 19$ **c** $4a + 5 = 2a - 19$

d $\frac{3x+2}{5} = 4$ **e** $4(2 - x) = -24$

6.01 Equations

Equations with unknowns on both sides

Equations with variables on both sides

Equations with brackets

Equations

Example 1

Solve each equation.

a $3m - 6 = 12$ **b** $5 - 2a = 3a$

c $9x + 10 = 7x - 6$ **d** $5(p + 6) = 3p + 5$

Solution

a $3m - 6 = 12$

$3m - 6 + 6 = 12 + 6$ Adding 6 to both sides.

$3m = 18$

$\frac{3m}{3} = \frac{18}{3}$ Dividing both sides by 3.

$m = 6$

Check:

LHS $= 3 \times 6 - 6 = 12$

RHS $= 12$

LHS = RHS

b $5 - 2a = 3a$

$5 - 2a + 2a = 3a + 2a$ Adding $2a$ to both sides.

$5 = 5a$

$5a = 5$

$\frac{5a}{5} = \frac{5}{5}$ Dividing both sides by 5.

$a = 1$

Check:

LHS = $5 - 2 \times 1 = 3$

RHS = $3 \times 1 = 3$

LHS = RHS

c $9x + 10 = 7x - 6$

$9x + 10 - 7x = 7x - 6 - 7x$

$2x + 10 = -6$

$2x + 10 - 10 = -6 - 10$

$2x = -16$

$\frac{2x}{2} = \frac{-16}{2}$

$x = -8$

d Expand the LHS:

$5(p + 6) = 3p + 5$

$5p + 30 = 3p + 5$

$5p + 30 - 3p = 3p + 5 - 3p$

$2p + 30 = 5$

$2p + 30 - 30 = 5 - 30$

$2p = -25$

$\frac{2p}{2} = \frac{-25}{2}$

$p = -12\frac{1}{2}$

6.01

Example 2

STAGE 5.2

Solve $4(y + 1) + 3(y - 5) = 8$.

Solution

Expand and collect like terms.

$4(y + 1) + 3(y - 5) = 8$

$4y + 4 + 3y - 15 = 8$

$7y - 11 = 8$

$7y - 11 + 11 = 8 + 11$

$7y = 19$

$\frac{7y}{7} = \frac{19}{7}$

$y = 2\frac{5}{7}$

Check:

$\text{LHS} = 4\left(2\frac{5}{7} + 1\right) + 3\left(2\frac{5}{7} - 5\right) = 8$

RHS = 8

LHS = RHS

EXERCISE 6.01 ANSWERS ON P. 519

Equations U F

EXAMPLE 1

1 Solve each equation.

a $\frac{k}{6}=10$

b $w+3=-6$

c $5y-1=9$

d $3a+10=25$

e $2x+6=22$

f $15a-2=13$

g $12-r=18$

h $7w-10=32$

i $9y-6=-24$

j $11-6a=-10$

k $7u=u+32$

l $5a=a-7$

2 Solve $6x-3=27$. Select the correct answer **A**, **B**, **C** or **D**.

A $x=4$ **B** $x=5$ **C** $x=10$ **D** $x=18$

3 Solve $10-2a=20$. Select **A**, **B**, **C** or **D**.

A $a=-15$ **B** $a=8$ **C** $a=32$ **D** $a=-5$

4 Solve each equation.

a $5y+10=3y+30$

b $8a+20=4a+10$

c $6y-1=3y+14$

d $12a+30=5a+9$

e $5y+3=8y-21$

f $14x-20=8x-14$

g $9y+1=3y-5$

h $15x-15=8x-85$

i $8m-10=5-2m$

j $18-3y=6-2y$

k $1-7a=10+2a$

l $11-5x=3x+43$

5 Solve $4y=y-15$. Select **A**, **B**, **C** or **D**.

A $y=-3$ **B** $y=\frac{7}{4}$ **C** $y=-5$ **D** $y=11$

6 Solve each equation.

a $3(x-6)=30$

b $5(m+10)=80$

c $2(5y+3)=46$

d $3(y+2)=5y-10$

e $5(y+4)=3y+6$

f $10(x-3)=5(x+5)$

g $2(3m+6)=4(m-1)$

h $5(2a+7)=5(4-a)$

i $3(1-2y)=18-3y$

7 Solve $2(y-3)=5+4y$. Select **A**, **B**, **C** or **D**.

A $y=-9$ **B** $y=-5$ **C** $y=-\frac{11}{2}$ **D** $y=-\frac{1}{2}$

STAGE 5.2

EXAMPLE 2

8 Solve each equation.

a $3(d+3)+4(d+1)=15$

b $3(y-1)+5(y+4)=10$

c $7(k+1)+2(k-6)=3$

d $5(g-3)+2(g-2)=4$

e $6(2h+3)+5(h-3)=9$

f $2(1+p)+3(4+p)=5$

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Investigation

Make your own equation

Here are 2 equations that have the same solution, $x = 6$:

$5x - 1 = 23 + x$ and $\frac{3x+12}{10} = 3$

1 For each solution below, make up 2 equations that have that solution.

a $x = 4$ b $x = \frac{1}{2}$ c $x = 10$

d $x = 1.5$ e $x = 0$ f $x = -2$

2 Compare your answers with those of other students. Check that each equation is correct.

Equations with algebraic fractions 6.02

6.01

STAGE 5.2

Equations code puzzle

Equations order activity

Solving linear equations 1

Solving linear equations 2

Example 3

Solve each equation.

a $\frac{y+2}{5} = -1$ b $\frac{3(p-3)}{2} + 6 = 1$

Solution

a
$$\frac{y+2}{5} \times 5 = -1 \times 5$$ Multiply both sides by 5
$$y + 2 = -5$$
$$y + 2 - 2 = -5 - 2$$
$$y = -7$$

b
$$\frac{3(p-3)}{2} + 6 - 6 = 1 - 6$$
$$\frac{3(p-3)}{2} = -5$$
$$\frac{3(p-3)}{2} \times 2 = -5 \times 2$$ Multiply both sides by 2
$$3(p - 3) = -10$$
$$3p - 9 = -10$$ Expand the LHS
$$3p - 9 + 9 = -10 + 9$$
$$3p = -1$$
$$\frac{3p}{3} = \frac{-1}{3}$$
$$p = -\frac{1}{3}$$

For equations with more than one fraction, multiply both sides by a common multiple of the denominators to remove the fractions.

STAGE 5.2

Equations with algebraic fractions

Solving equations with fractions

Example 4

Solve each equation.

a $\frac{2m}{3}-\frac{m}{2}=2$

b $\frac{2a+4}{5}=\frac{2}{3}$

Solution

a $\frac{2m}{3}-\frac{m}{2}=2$

$6\left(\frac{2m}{3}-\frac{m}{2}\right)=6\times2$ — Multiply both sides by 6, the LCM of 3 and 2.

$\cancel{6}^2\times\frac{2m}{\cancel{3}_1}-\cancel{6}^3\times\frac{m}{\cancel{2}_1}=12$

$4m-3m=12$

$m=12$

b $\frac{2a+4}{5}=\frac{2}{3}$

$\frac{2a+4}{\cancel{5}_1}\times\cancel{15}^3=\frac{2}{\cancel{3}_1}\times\cancel{15}^5$ — Multiply both sides by 15, the LCM of 5 and 3.

$3(2a+4)=10$

$6a+12=10$

$6a=-2$

$a=\frac{-2}{6}$

$a=-\frac{1}{3}$

EXERCISE 6.02 ANSWERS ON P. 519

Equations with algebraic fractions U F

EXAMPLE 3

1 Solve each equation. Select the correct answer **A**, **B**, **C** or **D**.

a $\frac{3y}{4}=6$

A $y=9$ **B** $y=8$ **C** $y=7$ **D** $y=6$

b $\frac{a+1}{2}=3$

A $a=4$ **B** $a=5$ **C** $a=6$ **D** $a=7$

c $\frac{x-1}{4}+2=10$

A $x=49$ **B** $x=37$ **C** $x=33$ **D** $x=3$

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STAGE 5.2

6.02

2 Solve each equation.

a $\frac{3y}{5}=9$ **b** $\frac{2a}{9}=2$ **c** $\frac{m}{2}+5=6$ **d** $\frac{k}{5}-2=11$

e $\frac{n+5}{3}=-10$ **f** $\frac{y-1}{4}=-2$ **g** $\frac{x+1}{4}+2=10$ **h** $\frac{y-1}{5}-6=3$

i $\frac{m+2}{5}-1=3$ **j** $\frac{x-6}{5}+7=0$ **k** $\frac{2(x+1)}{5}=10$ **l** $\frac{3(m-2)}{4}=6$

m $\frac{8(n+1)}{3}+2=4$ **n** $\frac{5(1-n)}{2}-1=3$ **o** $\frac{4(1+d)}{3}+1=7\frac{1}{3}$

3 Solve each equation.

EXAMPLE 4

a $\frac{2k}{3}=\frac{5}{4}$ **b** $\frac{3w}{10}=\frac{2}{5}$ **c** $\frac{5x}{2}=-\frac{10}{3}$

d $\frac{x-1}{2}=\frac{x+1}{4}$ **e** $\frac{y+2}{5}=\frac{y-1}{2}$ **f** $\frac{a+5}{3}=\frac{a-1}{8}$

g $\frac{p+2}{5}=\frac{p-5}{2}$ **h** $\frac{2y-1}{5}=\frac{y+1}{4}$ **i** $\frac{3y+2}{3}=\frac{2y+1}{4}$

j $\frac{w}{5}+\frac{w}{2}=7$ **k** $\frac{w}{2}-\frac{w}{5}=15$ **l** $\frac{2w}{3}-\frac{w}{4}=4$

m $\frac{3a}{2}+\frac{a}{3}=1$ **n** $\frac{2y}{5}-\frac{y}{3}=4$ **o** $\frac{a}{3}+\frac{3a}{4}=2$

p $\frac{2m}{5}-\frac{m}{10}=1$ **q** $\frac{4h}{3}+\frac{h}{5}=3$ **r** $\frac{5y-2}{7}=\frac{3y+5}{3}$

4 Solve each equation. Select **A**, **B**, **C** or **D**.

a $\frac{4m}{5}-\frac{m}{3}=2$

A $m=10$ **B** $m=12$ **C** $m=\frac{30}{7}$ **D** $m=\frac{4}{3}$

b $\frac{m+1}{2}=\frac{3+2m}{5}$

A $m=1$ **B** $m=5$ **C** $m=\frac{5}{3}$ **D** $m=\frac{2}{3}$

5 Use substitution to match each solution with its equation.

a $x=18$ **b** $x=-1$ **c** $x=3$ **d** $x=4$

A $\frac{3x+1}{2}=5$ **B** $\frac{2x}{3}-\frac{x}{2}=3$ **C** $\frac{x-1}{3}=\frac{x+1}{5}$ **D** $2x-3=x-4$

Quadratic equations $x^2 + bx + c = 0$ 6.03

STAGE 5.2

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4t^2 - 3t = 8$.

The quadratic equation $x^2 = c$

The quadratic equation $x^2 = c$ (where c is a positive number) has 2 solutions:

$$x=\pm\sqrt{c} \text{ (which means } x=\sqrt{c} \text{ and } x=-\sqrt{c}\text{).}$$

STAGE 5.2

Simple quadratic equations

Example 5

Solve each quadratic equation.

a $m^2 = 16$ **b** $3x^2 = 75$ **c** $4m^2 - 12 = 0$

Solution

a $m^2 = 16$

$m = \pm\sqrt{16}$ Finding the square root of both sides.

$= \pm 4$

b $3x^2 = 75$

$x^2 = \frac{75}{3}$ Dividing both sides by 3.

$x^2 = 25$

$x = \pm\sqrt{25}$ Finding the square root of both sides.

$= \pm 5$

c $4m^2 - 12 = 0$

$4m^2 - 12 + 12 = 0 + 12$ Adding 12 to both sides.

$4m^2 = 12$

$m^2 = \frac{12}{4}$ Dividing both sides by 4.

$m^2 = 3$ Finding the square root of both sides.

$m = \pm\sqrt{3}$ Leave the answer as a surd (in exact form).

Example 6

Solve $7x^2 - 88 = 0$, writing the solution correct to one decimal place.

Solution

$7x^2 - 88 = 0$

$7x^2 = 88$

$x^2 = \frac{88}{7}$

$x = \pm\sqrt{\frac{88}{7}}$

$x = \pm 3.54562...$

$\approx \pm 3.5$

The quadratic equation $x^2 + bx + c = 0$

To solve quadratic equations of the form $x^2 + bx + c = 0$, factorise $x^2 + bx + c$ into binomial products.

STAGE 5.2

Quadratic equations by factorising

6.03

Example 7

Solve $x^2 + 5x + 6 = 0$.

Solution

To factorise $x^2 + 5x + 6$, find 2 numbers that have a sum of 5 and a product of 6.

The correct numbers are 2 and 3.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

The LHS has been factorised into 2 binomials, $(x + 2)$ and $(x + 3)$, whose product is 0.

If 2 numbers have a product of 0, then one of the numbers *must* be 0.

$\therefore x + 2 = 0$ or $x + 3 = 0$

$\therefore x = -2$ or $x = -3$

$\therefore$ The solution to $x^2 + 5x + 6 = 0$ is

$x = -2$ or $x = -3$.

Check:

When $x = -2$,

LHS $= (-2)^2 + 5 \times (-2) + 6 = 0$

RHS $= 0$

LHS $=$ RHS.

When $x = -3$,

LHS $= (-3)^2 + 5 \times (-3) + 6 = 0$

RHS $= 0$

LHS $=$ RHS.

Example 8

Quadratic equations by factorising

Solve each quadratic equation.

a $x^2 - x - 2 = 0$

b $u^2 + 3u - 28 = 0$

c $3m^2 = 6m$

d $w^2 - 10w + 25 = 0$

Solution

a $x^2 - x - 2 = 0$

Find 2 numbers that have a sum of -1 and a product of -2.

They are -2 and 1.

$(x - 2)(x + 1) = 0$

$\therefore x - 2 = 0$ or $x + 1 = 0$

$\therefore x = 2$ or $x = -1$

$\therefore$ The solution to $x^2 - x - 2 = 0$ is

$x = 2$ or $x = -1$.

Check:

When $x = 2$,

LHS $= 2^2 - 2 - 2 = 0$

RHS $= 0$

LHS $=$ RHS.

When $x = -1$,

LHS $= (-1)^2 - (-1) - 2 = 0$

RHS $= 0$

LHS $=$ RHS.

STAGE 5.2

b $u^2 + 3u - 28 = 0$

Find 2 numbers that have a sum of 3 and a product of −28.

They are 7 and −4.

$(u + 7)(u - 4) = 0$

$\therefore u + 7 = 0$ or $u - 4 = 0$

$\therefore u = -7$ or $u = 4$

$\therefore$ The solution to $u^2 + 3u - 28 = 0$ is $u = -7$ or $u = 4$.

c $3m^2 = 6m$

$3m^2 - 6m = 0$

This requires a simpler factorisation as there are only 2 terms, both involving m.

You cannot divide both sides by m because if $m = 0$, then you are dividing by 0.

$3m(m - 2) = 0$

$\therefore 3m = 0$ or $m - 2 = 0$

$\therefore m = 0$ or $m = 2$

$\therefore$ The solution to $3m^2 - 6m = 0$ is $m = 0$ or $m = 2$.

d $w^2 - 10w + 25 = 0$

Find 2 numbers that have a sum of −10 and a product of 25.

They are −5 and −5.

$(w - 5)(w - 5) = 0$

$\therefore w - 5 = 0$ or $w - 5 = 0$

$\therefore w = 5$ or $w = 5$

$\therefore$ The solution to $w^2 - 10w + 25 = 0$ is $w = 5$ (only one solution).

Note: Quadratic equations of the form $ax^2 + bx + c = 0$ can be found in Chapter 14, *Quadratic equations and the parabola*, an optional Stage 5.3 chapter.

EXERCISE 6.03 ANSWERS ON P. 519

Quadratic equations $x^2 + bx + c = 0$ U F R C

EXAMPLE 5

1 Solve each quadratic equation, writing the solutions as surds if necessary.

a $m^2 = 144$ **b** $x^2 = 400$ **c** $y^2 = 35$

d $k^2 - 169 = 0$ **e** $y^2 - 1 = 0$ **f** $w^2 - 24 = 0$

g $x^2 + 10 = 14$ **h** $t^2 - 9 = 7$ **i** $\frac{a^2}{2} = 8$

j $4k^2 = 180$ **k** $3w^2 = 300$ **l** $d^2 + 60 = 204$

m $\frac{k^2}{2} = 7$ **n** $\frac{w^2}{10} = 2.5$ **o** $4x^2 = 1$

p $\frac{m^2}{4} = 10$ **q** $5y^2 = 5$ **r** $2p^2 + 3 = 21$

s $\frac{3k^2}{2} + 1 = 13$ **t** $\frac{y^2}{5} - 2 = 18$

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STAGE 5.2

2 Solve each equation, correct to one decimal place where necessary.

EXAMPLE 6

a $5m^2 - 20 = 0$ **b** $\frac{4a^2}{9} = 36$ **c** $m^2 = 28$

d $9m^2 - 2 = 32$ **e** $9k^2 + 10 = 13$ **f** $\frac{2x^2}{5} = 23$

g $\frac{k^2}{16} = 6$ **h** $\frac{3k^2}{10} = 27$ **i** $6y^2 = 0.726$

j $3a^2 + 11 = 267$ **k** $2y^2 - 14 = 63$ **l** $\frac{2w^2}{5} - 1 = 19$

6.03

3 Solve each quadratic equation. Select the correct answer **A**, **B**, **C** or **D**.

a $x^2 = 121$

A $x = 12, -12$ **B** $x = 11, -11$ **C** $x = 10, 11$ **D** $x = 12, -11$

b $9m^2 - 1 = 35$

A $m = 3, -3$ **B** $m = 2, -2$ **C** $m = 8, -8$ **D** $m = 9, -9$

4 Solve each equation.

EXAMPLE 7

a $x^2 + 3x + 2 = 0$ **b** $y^2 + 5y + 4 = 0$ **c** $y^2 + 16y + 48 = 0$

d $x^2 + x - 12 = 0$ **e** $x^2 + 2x - 3 = 0$ **f** $x^2 + 3x - 40 = 0$

5 Solve each equation.

EXAMPLE 8

a $x^2 - x - 30 = 0$ **b** $x^2 - 8x + 16 = 0$ **c** $x^2 - 5x - 66 = 0$

d $d^2 - 2d = 0$ **e** $x^2 - 3x - 10 = 0$ **f** $n^2 + 4n = 0$

g $k^2 - 7k = 0$ **h** $y^2 = 5y$ **i** $v^2 = 12v$

6 Explain why the quadratic equation $x^2 = -25$ has no solutions. R C

7 State which of these quadratic equations have no solutions. Give reasons. R C

a $x^2 = -9$ **b** $2k^2 + 5 = 9$ **c** $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$ **e** $4 + \frac{d^2}{3} = 8$ **f** $\frac{5a^2}{2} + 3 = 2$

8 Solve each quadratic equation. Select **A**, **B**, **C** or **D**.

a $x^2 + 4x - 60 = 0$

A $x = -10, 6$ **B** $x = 12, -5$ **C** $x = 10, -6$ **D** $x = -12, 5$

b $q^2 + 3q = 0$

A $q = 3, -3$ **B** $q = 6, -3$ **C** $q = 0, -3$ **D** $q = 0, 3$

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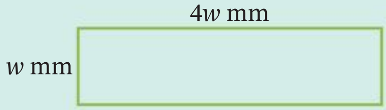
6.04 Equation problems

Example 9

A rectangle is 4 times as long as it is wide. The perimeter of the rectangle is 180 mm. Find the dimensions of the rectangle.

Solution

Let the width of the rectangle be w mm. Then the length is $4w$ mm.



$$\therefore \text{Perimeter: } w + 4w + w + 4w = 180$$
$$10w = 180$$
$$w = \frac{180}{10}$$
$$= 18$$

∴ The width of the rectangle is 18 mm.

Its length is $4w = 4 \times 18 = 72$ mm.

Check: Perimeter $= 18 + 72 + 18 + 72 = 180$ mm.

Example 10

Paris is 7 years older than Amy. 10 years from now, the sum of their ages will be 43.

How old are they now?

Solution

Let x = Amy's age now.

Then Paris' age now $= x + 7$.

Break the information into 'Now' and 'In 10 years time'

	Now	In 10 years time
Amy	x	$x + 10$
Paris	$x + 7$	$x + 7 + 10 = x + 17$

In 10 years time:

$$\text{Sum of ages: } (x + 10) + (x + 17) = 43$$
$$2x + 27 = 43$$
$$2x = 16$$
$$x = 8$$

Amy is 8 years old now.

Paris is $8 + 7 = 15$ years old now.

Check: In 10 years time, the sum of their ages will be $18 + 25 = 43$.

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EXERCISE 6.04 ANSWERS ON P. 520

Equation problems U F PS R C

For each question, write an equation and solve it to answer the problem. PS R C

1 The longer sides of an isosceles triangle are twice as long as the shorter side. The perimeter of the triangle is 90 mm. Find the lengths of the sides of the triangle.

EXAMPLE 9

6.04

2 The length of a rectangle is 3 times as long as its width. The perimeter of the rectangle is 152 mm. Find its dimensions.

3 The length of a rectangle is 3 more than twice its width. Find the dimensions of the rectangle if its perimeter is 84 cm.

4 The sum of 3 **consecutive** integers is 186. Find the integers.

EXAMPLE 10

5 Tyson is 9 times the age of his daughter, Charlotte. In 5 years, he will be 4 times the age of Charlotte. How old are they now?

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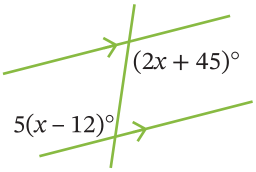
6 When 15 is subtracted from 3 times a certain number, the answer is 63. What is the number?

7 The product of 2 and a number is the same as 12 minus the number. Find the number.

8 The sum of the present ages of Vatha and Chris is 36. In 4 years time, the sum of their ages will equal twice Vatha's present age. How old are they now?

9 4 consecutive integers have a sum of 858. Find the 4 integers.

10 Find the size of x in the diagram.



11 Manori's bag contained 10-cent and 20-cent coins. She had 202 coins, with a total value of $31.90. How many 20-cent coins did Manori have?

12 If 17 more than a number is 5 more than 3 times the number, what is the number?

13 The sum of Scott's age and his mother Kait's age is 45. In 5 years time, 3 times Scott's age less 9 will be the same as Kait's age. Find the present ages of Scott and Kait.

14 One angle in a triangle is double the smallest angle, and the third angle in the triangle is 5 more than 4 times the smallest angle. Find the size of each angle in the triangle.

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Mental skills 6: Maths without calculators ANSWERS ON P. 520

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$18 \times 5 = 18 \times \frac{1}{2} \times 10$ (or $9 \times 2 \times 10$)

$= 9 \times 10$

$= 90$

b **To multiply by 50**, halve the number, then multiply by 100.

$26 \times 50 = 26 \times \frac{1}{2} \times 100$ (or $13 \times 2 \times 100$)

$= 13 \times 100$

$= 1300$

c **To multiply by 25**, quarter the number, then multiply by 100.

$44 \times 25 = 44 \times \frac{1}{4} \times 100$ (or $11 \times 4 \times 25$)

$= 11 \times 100$

$= 1100$

d **To multiply by 15**, halve the number, then multiply by 30.

$8 \times 15 = 8 \times \frac{1}{2} \times 30$ (or $4 \times 2 \times 15$)

$= 4 \times 30$

$= 120$

e **To divide by 5**, divide by 10 and double the answer. We do this because there are 2 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$

$= 14 \times 2$

$= 28$

f **To divide by 50**, divide by 100 and double the answer. This is because there are 2 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$

$= 4 \times 2$

$= 8$

g **To divide by 25**, divide by 100 and multiply the answer by 4. This is because there are 4 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$

$= 6 \times 4$

$= 24$

h **To divide by 15**, divide by 30 and double the answer. This is because there are 2 15s in every 30.

$$240 \div 15 = 240 \div 30 \times 2$$
$$= 8 \times 2$$
$$= 16$$

2 Now evaluate each expression.

a 32×5	**b** 14×5	**c** 48×5	**d** 18×50
e 52×50	**f** 36×25	**g** 28×5	**h** 12×25
i 12×15	**j** 22×35	**k** $90 \div 5$	**l** $170 \div 5$
m $230 \div 5$	**n** $1300 \div 50$	**o** $900 \div 50$	**p** $300 \div 25$
q $1000 \div 25$	**r** $360 \div 45$	**s** $210 \div 15$	**t** $360 \div 15$

6.04

Equations and formulas 6.05

STAGE 5.2

A **formula** is an equation that describes a relationship between variables. For example, the formula for the perimeter of a rectangle is $P = 2(l + w)$, where P is the perimeter, l is the rectangle's length and w is the width. Because the formula is for the perimeter, P is called the **subject** of the formula and it is the variable on its own on the left side of the '=' sign.

Getting the right formula

Equations and formulas

Formulas and equations

Example 11

The cost of a trip, C, in dollars, for a ride share company is $C = 5 + 2.4d$, where d is the distance travelled, in kilometres.

a Find the cost of a trip if the distance travelled is 15 km.

b Find the distance travelled if the cost of the trip was $78.20.

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Solution

a When $d = 15$:

$$C = 5 + 2.4 \times 15$$
$$= 41$$

The cost was $41.

b When $C = 78.20$:

$$78.20 = 5 + 2.4d$$
$$73.20 = 2.4d$$
$$d = \frac{73.20}{2.4}$$
$$= 30.5$$

The distance travelled was 30.5 km.

STAGE 5.2

Equations and formulas

Example 12

The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius.

Find (correct to one decimal place):

a the surface area of a sphere with radius 2.8 cm

b the radius of a sphere with surface area 40 m^2.

Solution

a When $r = 2.8$:

$A = 4 \times \pi \times 2.8^2$

$= 98.520...$

≈ 98.5

The surface area of the sphere is 98.5 cm^2.

b When $A = 40$:

$40 = 4\pi r^2$

$4\pi r^2 = 40$

$r^2 = \frac{40}{4\pi}$

$= 3.183...$

$r = \sqrt{3.183}$ — $r > 0$ because the radius is positive.

$= 1.784...$

≈ 1.8

The radius of the sphere is 1.8 m.

EXERCISE 6.05 ANSWERS ON P. 520

Equations and formulas U F R C

EXAMPLE 11

1 The formula for the perimeter of a rectangle is $P = 2(l + w)$. R

a Find the perimeter of a rectangle with length 10 cm and width 16 cm.

b Find the width of a rectangle whose perimeter is 58 m and length is 12 m.

2 A formula for converting speed expressed in m/s (metres/second) to a speed expressed in km/h is $k = 3.6M$, where M is the speed in m/s. Convert each speed to km/h. R C

a 10 m/s **b** 24 m/s **c** 50 m/s

3 A car is travelling at a speed of 110 km/h on a freeway. Use the formula from question **2** to calculate how fast this is in m/s.

4 The formula for converting temperatures in °F to °C is $C = \frac{5}{9}(F - 32)$.
Express each temperature in °C, correct to the nearest degree.

a 80°F **b** 32°F **c** 212°F **d** 102°F

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STAGE 5.2

5 The average of 2 numbers, m and n, is $A = \frac{m+n}{2}$. If 2 numbers have an average of 28 and one of the numbers is 13, use the formula to find the other number. R

6 Pythagoras' theorem is $c^2 = a^2 + b^2$, where c is the length of the hypotenuse in a right-angled triangle and a and b are the lengths of the other 2 sides. Find, correct to one decimal place where necessary:

EXAMPLE 12

a c, if $a = 5$ and $b = 10$ **b** a, if $c = 41$ and $b = 40$ **c** b, if $c = 20$ and $a = 10$

6.05

7 The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius. Find, correct to one decimal place: R C

a the circumference of a circle with radius 2.4 m

b the radius of a circle whose circumference is 200 cm.

8 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres. Find, correct to one decimal place: R

a the BMI of Leonie who is 1.85 m tall and has a mass of 72 kg

b the mass of Dane with a BMI of 24, who is 2.1 m tall.

9 The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. Find, correct to one decimal place: R

a the volume of a sphere with radius 3.2 cm

b the radius of a sphere with volume 500 m^3.

10 The average speed in km/h of a car is given by the formula $S = \frac{D}{T}$, where D is the distance covered in kilometres and T is the time taken in hours. Find, correct to the nearest whole number: R

a the average speed of a car that takes 4.5 hours to travel a distance of 420 km

b the distance travelled, if a car maintains a speed of 87.2 km/h for 5 hours

c the time taken, if a distance of 650 km is covered at a speed of 91 km/h.

11 The cost, C (in dollars), of a hire car is $C = 75 + 2.5d$, where d is the number of kilometres travelled. Calculate: R

a the cost of hiring a car to travel 350 km

b the distance travelled, if the cost is $135.

12 The surface area of a closed cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$. Calculate, correct to one decimal place: R

a the surface area of a cylinder with radius 2.1 m and height 3.5 m

b the height of a cylinder with surface area 1255.38 cm^2 and radius 9 cm.

6.06 Graphing inequalities on a number line

STAGE 5.2

An **inequality** looks like an equation except that the equals sign (=) is replaced by an inequality symbol $>$, $\geq$, $<$ or $\leq$.

$2x - 7 = 15$ is an **equation**. There is only one value of x that makes it true.

$2x - 7 \leq 15$ is an **inequality**. There is a range of values of x that make it true.

Inequality symbols

$>$ 'is greater than'

$<$ 'is less than'

$\geq$ 'is greater than or equal to'

$\leq$ 'is less than or equal to'

For example, '$x \geq 3$' is read 'x is greater than or equal to 3'. It includes 3 and all the numbers above 3, such as 3.01, 4, 10, 20 000. '$x > 3$' is read 'x is greater than 3' and means all the numbers above 3, but **not** 3.

Inequality	In words	Meaning
$x > 3$	x is greater than 3	Values above 3
$x < 3$	x is less than 3	Values below 3
$x \geq 3$	x is greater than or equal to 3	Values above and including 3
$x \leq 3$	x is less than or equal to 3	Values below and including 3

An inequality can be graphed on a **number line**.

Example 13

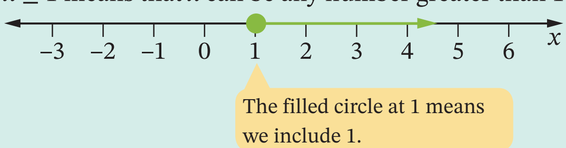
Graphing inequalities on the number line

Graph each inequality on a number line.

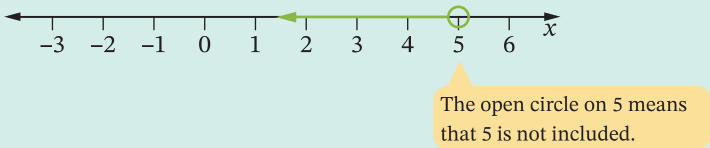
a $x \geq 1$ **b** $x < 5$ **c** $x > -3$

Solution

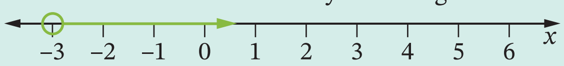
a $x \geq 1$ means that x can be any number greater than 1 or equal to 1.



b $x < 5$ means that x can be any number less than 5, but not including 5.



c $x > -3$ means that x can be any number greater than -3, but not including -3.



EXERCISE 6.06 ANSWERS ON P. 520

Graphing inequalities on a number line **U F C**

EXAMPLE 13

1 Graph each inequality on a separate number line. C

a $x \geq 2$ **b** $x < -3$ **c** $x \leq 1$ **d** $x > 7$

e $x \leq 4$ **f** $x > 0$ **g** $x \geq -2$ **h** $x < 10$

2 Write the inequality shown on each number line. C

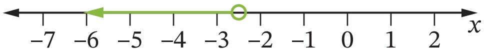
a

b

c

d

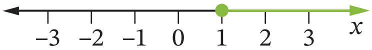
3 Which inequality is graphed below? Select the correct answer **A**, **B**, **C** or **D**. C



A $x > -2.5$ **B** $x < -2.5$ **C** $x < -3.5$ **D** $x > -3.5$

4 Write the inequality shown on each number line. C

a

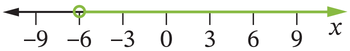


b

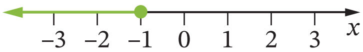
c

d

e

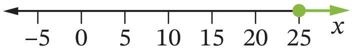


f

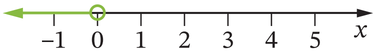


g

h



i



Investigation

The language of inequalities

Work in pairs to complete this activity.

Use inequality symbols to write each statement algebraically.

a The minimum height (H) for rides at an amusement park is 1.3 m.

b The speed limit in a school zone is 40 km/h.

c To be eligible to vote, you must be at least 18 years old (A = age).

d The overseas tour is only for people whose age (A) is from 18 to 35.

e The cost (A) of a tennis racquet will be at least \$95, but no more than \$360.

f A new flute (F) costs at least \$475.

g The price of units (U) in a new block start at \$240 000.

STAGE 5.2

Investigation

Solving inequalities

We have solved equations by doing the same thing to both sides (keeping the equation 'balanced'). Will this method work with inequalities, such as $x + 4 > 10$ or $6x < 13$?

1 Start with an inequality that is true, such as $7 > 4$.

2 Add 5 (or any number you choose) to both sides of the inequality; for example, $7 > 4$ becomes $12 > 9$. Is the new inequality true or false?

3 Subtract 9 (or any number you choose) from each side of the original inequality; for example, $7 > 4$ becomes $-2 > -5$. Is the new inequality true or false?

4 Multiply both sides of the original inequality by 4 (or any positive number you choose); for example, $7 > 4$ becomes $28 > 16$. Is the new inequality true or false?

5 Divide both sides of the original inequality by 2 (or any positive number you choose); for example, $7 > 4$ becomes $3\frac{1}{2} > 2$. Is the new inequality true or false?

6 Multiply both sides of the original inequality by -3 (or any negative number you choose); for example, $7 > 4$ becomes $-21 > -12$. Is the new inequality true or false?

7 Divide both sides of the original inequality by -4 (or any negative number you choose), for example, $7 > 4$ becomes $-1\frac{3}{4} > -1$. Is the new inequality true or false?

8 Which of the 6 operations used in questions **2** to **7** can be used on inequalities to give a true result?

9 Which of the 6 operations used in questions **2** to **7** cannot be used with inequalities because they give a false result?

10 Copy and complete each set of inequality statements.

a $6 < 8$

$6 \times 3 < 8 \times$ ___ (multiplying both sides by 3)

$\therefore 18$ __ 24

b $10 > -4$

$10 \div 2$ __ $-4 \div$ __ (dividing both sides by 2)

$\therefore$ ___________

Does the inequality sign (< or >) stay the same when multiplying or dividing by a positive number?

11 **a** Is it true that $5 < 8$?

b Multiply both sides by -2. Is it true that $-10 < -16$?

c What needs to be reversed to change $-10 < -16$ into a true inequality statement?

d Copy and complete to make a true statement: -10 _______ -16.

12 **a** Is it true that $18 > -6$?

b Divide both sides by -3. Is it true that $-6 > 2$?

c What needs to be reversed to change $-6 > 2$ into a true inequality statement?

d Copy and complete to make a true inequality statement: -6 ____ 2.

13 Copy and complete: When multiplying or d__________ both sides of an inequality by a n__________ number, the inequality sign must be r__________.

Did you know?

Film and game classification

In Australia, films and computer games are rated by the Classification Board. Films and videos are rated G, PG, M, MA15+ or R18+, with each category containing a list of guidelines related to the film's use of violence, coarse language, adult themes, sex and nudity.

General (G) means suitable for all ages. Children can watch films classified G without adult supervision.

Parental guidance (PG) means that parental guidance is recommended for persons under 15 years of age. These films contain mild content that may be confusing or upsetting to children, but not harmful or disturbing. Parents should watch the film with their children or preview it to check elements such as language used or inappropriate themes.

Mature (M) means recommended for mature audiences, 15 years and over. The film or computer game may contain moderate content that is harmful or disturbing to children, but the impact is not so strong as to require restriction.

Mature accompanied (MA15+) means persons under 15 are not allowed to see MA films unless accompanied by a parent or guardian, because they contain strong content that is likely to be harmful or disturbing to them.

Restricted (R18+) means legally restricted to adults, 18 years and over. It applies to films that deal with issues and scenes that require an adult perspective, and so are unsuitable for persons under 18. A person will be asked for proof of age before buying, hiring or viewing films or computer games in this category.

1 **Each of the classifications is represented by a logo (as shown) with the letter inside a particular shape. What shape is each logo?**

2 **Write each classification category as an inequality.**

6.07 Solving inequalities

STAGE 5.2

WS Inequalities review

Example 14

Solve each inequality and graph its solution on a number line.

a $2x - 10 \leq 16$ **b** $2(y - 1) \geq 12$ **c** $\frac{w+3}{2} > -1$

Solution

a

$2x - 10 \leq 16$

$2x - 10 + 10 \leq 16 + 10$

$2x \leq 26$

$\frac{2x}{2} \leq \frac{26}{2}$

$x \leq 13$

Check: Test a value less than 13, let $x = 11$.

LHS = $2 \times 11 - 10 = 12$

RHS = 16

$12 \leq 16$

LHS ≤ RHS

b

$2(y - 1) \geq 12$

$2y - 2 \geq 12$

$2y - 2 + 2 \geq 12 + 2$

$2y \geq 14$

$\frac{2y}{2} \geq \frac{14}{2}$

$y \geq 7$

Check: Test a value greater than 7, let $y = 8$.

LHS = $2(8 - 1) = 14$

RHS = 12

$14 \geq 12$

LHS ≥ RHS

c

$\frac{w+3}{2} > -1$

$\frac{w+3}{2} \times 2 > -1 \times 2$

$w + 3 > -2$

$w + 3 - 3 > -2 - 3$

$w > -5$

Check: Test a value greater than −5, let $w = -1$.

$\text{LHS} = \frac{-1+3}{2} = 1$

RHS = −1

$1 > -1$

LHS > RHS

STAGE 5.2

6.07

Solving inequalities

- Inequalities can be solved algebraically in the same way as equations, using inverse operations.
- However, when multiplying or dividing both sides of an inequality by a **negative** number, you must **reverse** the inequality sign.

Example 15

Solve each inequality.

a $1 - 2x \geq -11$ **b** $4 - r < 7$ **c** $\frac{a+5}{-3} > 4$

Solution

a

$1 - 2x \geq -11$

$1 - 2x - 1 \geq -11 - 1$

$-2x \geq -12$

$\frac{-2x}{-2} \leq \frac{-12}{-2}$

Dividing both sides by a negative number reverses the inequality sign.

$x \leq 6$

Check: Test a value less than 6, let $x = 5$.

LHS $= 1 - 2 \times 5 = -9$

RHS $= -11$

$-9 \geq -11$

LHS $\geq$ RHS

b

$4 - r < 7$

$4 - r - 4 < 7 - 4$

$-r < 3$

$\frac{-r}{-1} > \frac{3}{-1}$

Dividing both sides by a negative number reverses the inequality sign.

$r > -3$

Check: Test a value greater than -3, let $r = 0$.

LHS $= 4 - 0 = 4$

RHS $= 7$

$4 < 7$

LHS $<$ RHS

STAGE 5.2

c $\frac{a+5}{-3} > 4$

$\frac{a+5}{-3} \times (-3) < 4 \times (-3)$

$a + 5 < -12$

$a + 5 - 5 < -12 - 5$

$a < -17$

Multiplying both sides by a negative number reverses the inequality sign.

Check: Test a value less than −17, let $a = -20$.

$\text{LHS} = \frac{-20+5}{-3} = \frac{-15}{-3} = 5$

$\text{RHS} = 4$

$5 > 4$

LHS > RHS

EXERCISE 6.07 ANSWERS ON P. 520

Solving inequalities U F R C

EXAMPLE 14

1 Solve each inequality and graph its solution on a number line. R C

a $x - 1 > 6$ **b** $3y \geq 12$

c $m + 4 \leq 2$ **d** $\frac{x}{5} \geq -20$

e $12x < 60$ **f** $5y > -20$

g $4a \geq 2$ **h** $3w \leq -30$

i $8a + 5 \geq 45$ **j** $3a + 1 \leq 10$

k $6a + 4 \geq -2$ **l** $3w - 3 < -12$

m $5a + 3 \leq -27$ **n** $5y + 1 \leq 16$

o $4a + 5 < 15$

2 Solve $3a - 3 > -18$. Select the correct answer **A**, **B**, **C** or **D**.

A $a > -5$ **B** $a > -7$ **C** $a > 5$ **D** $a > 9$

3 Solve each inequality.

a $3(x + 2) \geq 9$ **b** $5(m - 4) \leq 10$

c $2(y + 5) \leq -6$ **d** $3(w - 2) > -6$

e $5(2w + 3) \leq 15$ **f** $4(2m - 5) \geq 8$

g $\frac{m+5}{3} \geq 1$ **h** $\frac{x-1}{2} \leq 2$

i $\frac{w-2}{5} > -1$ **j** $\frac{2a+1}{3} < 3$

k $\frac{5a+2}{4} \geq 8$ **l** $\frac{2(m+1)}{3} \leq 3$

m $\frac{5(m-1)}{4} > 3$ **n** $\frac{4(m-2)}{3} \geq -6$

o $3 + \frac{x}{5} < 10$

Foundation Standard Complex

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STAGE 5.2

EXAMPLE 15

6.07

4 Solve $\frac{x-2}{5} \geq -1$. Select **A**, **B**, **C** or **D**.

A $x \geq -7$ **B** $x \leq -3$ **C** $x \leq 10$ **D** $x \geq -3$

5 Solve each inequality and graph its solution on a number line. R C

a $5 - x \leq 2$ **b** $15 > 7 - y$ **c** $1 - k < 12$

d $7 - m \geq 7$ **e** $2 - p > 8$ **f** $-t + 6 \geq 10$

6 Solve each inequality.

a $-2x < 6$ **b** $\frac{k}{-3} \geq 4$ **c** $-5t > 12$

d $\frac{-x}{3} \leq -4$ **e** $4 - 3w > 7$ **f** $-4y + 3 \leq 11$

g $3 - 2x \geq -5$ **h** $8 - 5a < 3$ **i** $-2d - 3 > 8$

j $\frac{5+w}{-3} > 2$ **k** $\frac{x-4}{-4} \geq 3$ **l** $\frac{-p+2}{-3} < -2$

Power plus ANSWERS ON P. 521

1 Solve each equation.

a $\frac{3(1-y)}{5} = 4 - 2y$

b $\frac{50}{2y} = 10$

c $\frac{2m+5}{3} - 1 = m + 4$

2 If $y = \frac{ab+cd}{e}$, find d if $y = -12$, $a = -1$, $b = -8$, $c = 7$ and $e = 4$.

3 Randall is 10 years older than Tarni. In 3 years time, Randall will be twice as old as Tarni. How old are Randall and Tarni now?

4 One-third of a number added to one-sixth of a number is 18. What is the number?

5 Graph each inequality on a number line.

a $1 \leq x \leq 4$

b $-2 \leq x \leq 3$

c $-12 < 4x \leq 4$

Foundation Standard Complex

CHAPTER 6 REVIEW

Language of maths

Equations

brackets	check	equation
expand	factorise	formula
fraction	greater than	inequality
inverse operation	LHS	less than
lowest common multiple (LCM)	number line	quadratic equation
RHS	solution	solve
square root	subject	substitute
variable		

1 What type of equation has 2 as the highest power of x? Write an example of this type of equation.

2 True or false? $10 \geq 10$.

3 What is the difference between an **equation** and an **inequality**?

4 Why is it possible for a **quadratic equation** to have more than one solution?

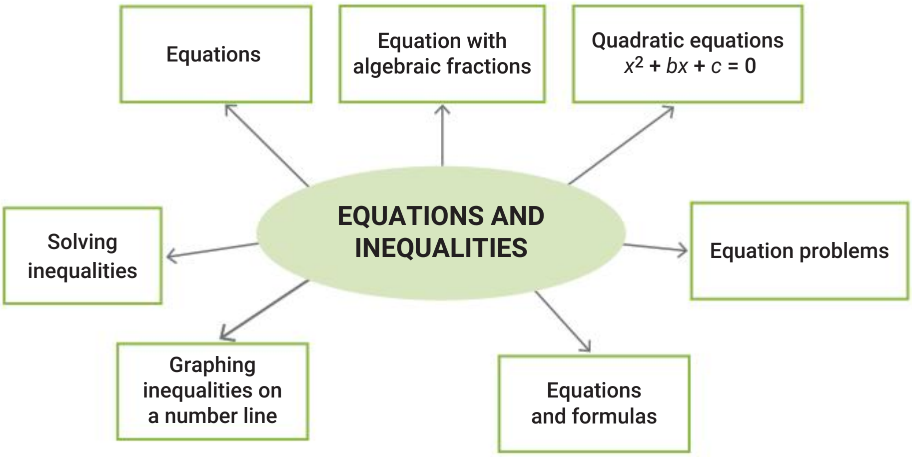
5 When checking the solution to an equation, you need to show that 'LHS = RHS'. What does that mean?

6 What does '$\leq$' mean?

Topic summary

Mind map: Equations and inequalities

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 6

ANSWERS ON P. 521

STAGE 5.2

1 Solve each equation. 6.01

a $3a + 10 = 43$ **b** $8y + 5 = 2y + 21$

c $2a - 12 = 6a$ **d** $9 - 2y = 5 + 2y$

e $3(m - 2) = 27$ **f** $2(2a + 1) = 3(a + 10)$

g $5(h + 1) + 3(h - 2) = 12$ **h** $4(2y + 1) + 3(1 + 4y) = 20$

2 Solve each equation. 6.02

a $\frac{3w+2}{5} = 4$ **b** $\frac{y}{5} = \frac{7}{4}$ **c** $\frac{2a+1}{2} = \frac{3a-1}{4}$

d $\frac{3m+5}{6} = \frac{10-m}{3}$ **e** $\frac{2s}{3} - \frac{s}{6} = 2$ **f** $\frac{x}{10} + \frac{x}{2} = 1$

3 Solve $\frac{2(p-1)}{3} = 4$. Select the correct answer **A**, **B**, **C** or **D**.

A $p = 7$ **B** $p = 11$ **C** $p = 5$ **D** $p = 4$

4 Solve each quadratic equation. 6.03

a $y^2 = 4$ **b** $p^2 - 100 = 0$ **c** $4x^2 = 40$

d $3m^2 - 3 = 0$ **e** $\frac{2w^2}{5} = 20$ **f** $x^2 + 8x + 7 = 0$

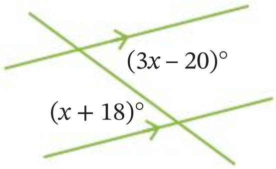
g $h^2 - 8h - 9 = 0$ **h** $u^2 + 4u - 77 = 0$ **i** $k^2 + 5k = 0$

j $m^2 - 2m = 0$ **k** $b^2 + 20b + 100 = 0$ **l** $w^2 = 9w$

5 The sum of 4 consecutive numbers is 374. Find the 4 numbers. 6.04

6 Find the value of x.

STAGE 5.2



7 The braking distance (in metres) of a bicycle travelling at a speed of v metres/second is $d = \frac{v(v+1)}{2}$. Calculate the braking distance when the speed of the bicycle is 15 m/s. 6.05

8 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid with a base area of 48 mm^2 and a perpendicular height of 10 mm

b the base area of a pyramid with a volume of 500 m^3 and a perpendicular height of 5 m

9 Graph each inequality on a number line. 6.06

a $x \geq 0$ **b** $x < 3$ **c** $x \leq -2$ **d** $x > -5$

10 Solve each inequality. 6.07

a $y - 6 \geq 10$ **b** $2y \leq -15$ **c** $3a + 10 > -5$

d $10 - 6x < 28$ **e** $\frac{a+2}{-4} > \frac{7}{2}$ **f** $\frac{3-5x}{2} \geq 9$

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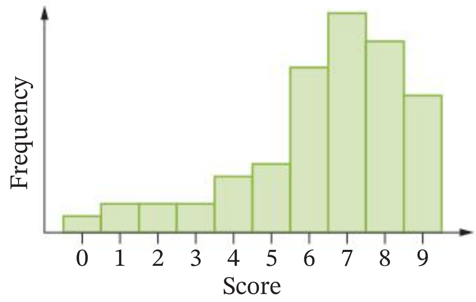
PRACTICE SET 2 ANSWERS ON P. 521

5.01

1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

4.01

2 Simplify each expression.

a $(3nm^2)^4$ **b** $5p^4q^2 \times 4qp^5$ **c** $\frac{32a^5b^6}{8ab^3}$

d $(2h)^0$ **e** 3^{-3} **f** $\left(\frac{5}{2}\right)^3$

6.01

3 Solve each equation.

a $5b + 12 = 37$ **b** $25y - 6 = 7y + 21$

c $3(m - 2) = 27$ **d** $2(2a + 1) = 3(10 - 4a)$

STAGE 5.2

5.02

4 Find the interquartile range of this set of data.

24 15 23 28 20 20 18 30 21 18

5.03

5 The maximum daily temperatures (°C) in Perth for the first 14 days in April were:

27.1 24.1 23.7 24.6 26.5 31.8 33.6 32.6 34.0 35.6 39.5 26.2 26.7 24.9

a Find the range and interquartile range for the temperatures.

b Find the five-number summary for the data.

c Draw a boxplot for the data.

d Identify any outliers.

4.01

6 Simplify each expression.

a $(3y)^{-2}$ **b** $3y^{-2}$ **c** $\left(\frac{4x^6y^6}{x^3y^2}\right)^3$

d $(2h^4k^5)^3 \times 3h^2k$ **e** $48v^5w^8 \div (-4vw^2)^2$ **f** $\left(\frac{36a^4b^2}{48ab^4}\right)^{-1}$

4.02

7 Simplify each expression.

a $\frac{3t}{8} + \frac{t}{8}$ **b** $\frac{4x}{15} - \frac{x}{3}$ **c** $\frac{3g}{4} - \frac{2g}{5}$ **d** $\frac{6k}{5} - \frac{k}{2}$

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8 Simplify each expression.

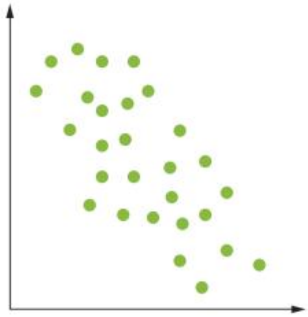
a $\frac{x}{7} \times \frac{6}{y}$ **b** $\frac{m^3}{8} \div \frac{m^2}{16}$ **c** $\frac{21e}{10d^2} \times \frac{15d}{7e}$ **d** $\frac{12pv}{15} \div \frac{4p}{10v}$

STAGE 5.2 4.03

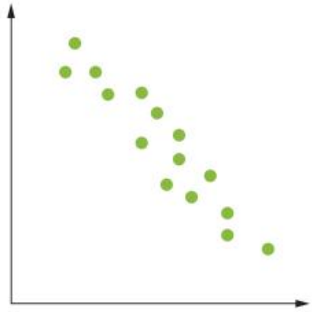
9 Describe the strength and direction of the relationship shown in each scatterplot.

5.06

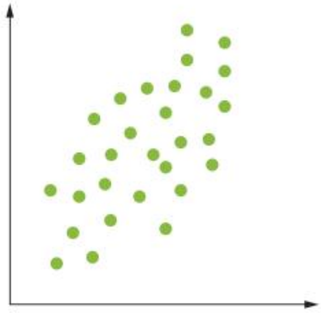
a



b



c



10 Solve each equation.

6.02

a $\frac{2a+3}{4} = 5$ **b** $\frac{k}{7} = \frac{5}{4}$

c $\frac{2w}{5} - \frac{w}{10} = 3$ **d** $\frac{1}{4}(3g+2) = \frac{1}{2}(2g-1)$

11 Expand each expression.

4.04

a $7(y - 9)$ **b** $-n(8n + m)$

c $-4(5w - 6)$ **d** $4a^2b(6a + 7b)$

12 The masses of students (in kg) was recorded before and after an exercise program.

STAGE 5.2 5.04

Before program	142	141	90	96	111	127	85	80	91
After program	99	101	77	81	85	96	79	77	70

a Find the five-number summary before and after the exercise program.

b Construct parallel boxplots for the 2 sets of data.

c Find the range and interquartile range for student weights:

i before the program **ii** after the program.

d Are there significant differences between the 2 sets of data? Justify your answer.

13 Expand and simplify each expression.

4.04

a $3ab(b - 6a) - 6a^2b$ **b** $4(9 - 3r) - 6r(8 - 3r)$

Foundation Standard Complex

4.04

14 Factorise each expression.

a $7w + 42$ **b** $p^2 - 25p$

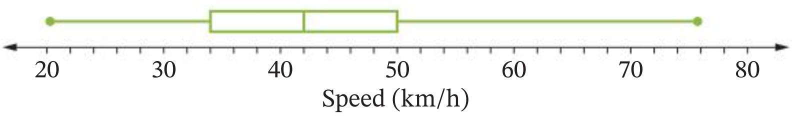
c $-4n - 44$ **d** $3(7a - 2) - 5a(7a - 2)$

STAGE 5.2

5.05

15 The speeds of cars (in km/h) were monitored during 2:30 to 4:00 p.m. in a school zone. The results are shown in the stem-and-leaf plot and boxplot below.

Stem	Leaf
2	0 3 7 9
3	0 2 2 3 5 6 8 8 9
4	0 0 1 3 5 5 7 8 9 9
5	1 1 4 4 6
6	0 1
7	6



a Which plot gives the better indication of:

i skewness? **ii** clustering and outliers?

b Find the median.

c Find the interquartile range.

6.03

16 Solve each quadratic equation, correct to one decimal place where necessary.

a $p^2 - 144 = 0$ **b** $9y^2 = 36$ **c** $5x^2 - 5 = 0$

d $\frac{t^2}{4} = 25$ **e** $q^2 = 77$ **f** $3x^2 - 70 = 0$

g $\frac{3w^2}{5} = 23$

4.05

17 Expand and simplify each expression.

a $(q + 3)(2q + 9)$ **b** $(3f + 8)(f - 7)$

c $(4g - 7)(5g - 3)$ **d** $(3x - 7)(3x + 7)$

4.06

18 Factorise each quadratic expression.

a $r^2 + 11r + 24$ **b** $y^2 - 31y + 30$

c $x^2 + 9x - 36$ **d** $t^2 - t - 72$

5.07

19 10 boxes containing 60 apples each were placed in cold storage for different periods. After each period of storage, a box was withdrawn and the number of good apples was counted.

Weeks in storage (W)	2	3	5	6	7	8	9	10	12	14
Number of good apples (N)	56	53	48	48	50	39	36	30	34	26

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the relationship between the weeks in storage and the number of good apples.

d Describe the strength and direction of the relationship between the variables W and N.

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20 Solve each quadratic equation.

STAGE 5.2

6.03

a $x^2 + 8x + 7 = 0$

b $h^2 - 8h - 9 = 0$

c $u^2 + 4u - 77 = 0$

d $w^2 = 9w$

21 Solve each problem using an equation.

6.04

a The sum of 3 consecutive numbers is 63. Find the 3 numbers.

b One angle in a triangle is double the size of the smallest angle, and the third angle in the triangle is 60° more than 3 times the size of the smallest angle. Find the size of each angle.

22 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

STAGE 5.2

6.05

a the volume of a pyramid if the base area is 96 mm^2 and the perpendicular height is 15 mm

b the base area of a pyramid if its volume is 600 cm^3 and its perpendicular height is 12 cm

c the perpendicular height of a pyramid if its volume is 48 m^3 and its base area is 40 m^2.

23 Graph each inequality on a number line.

6.06

a $x \geq 0$

b $x < 4$

c $x \leq 1$

d $x > -6$

24 Solve each inequality.

6.07

a $n - 4 \geq 1$

b $5a \leq -15$

c $4h + 25 > -7$

d $\frac{5x}{2} < 9$

PRACTICE SET 2

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NUMBER AND ALGEBRA

GRAPHING CURVES

When an object is thrown upwards, its path is a curve called a parabola. The shape and length of the path will depend on the initial speed of the object. Furthermore, car headlights, and satellite dishes all use mirrors or reflectors that have the shape of a parabola.

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Chapter outline

		Working mathematically				
7.01	Direct proportion*	U	F	PS	R	C
7.02	Inverse proportion*#	U	F	PS	R	C
7.03	Conversion graphs*	U	F	PS	R	C
7.04	The parabola $y = ax^2 + c$	U	F		R	C
7.05	The exponential curve	U	F		R	C
7.06	The circle	U	F		R	C
7.07	Identifying graphs*	U	F		R	C

*STAGE 5.2

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asymptote A line that a curve gets very close to but never touches, for example, the *x*-axis is an asymptote of the exponential curve

direct proportion A relationship between 2 variables of the form $y = kx$, where k is a constant; for example, if $y = 8.5x$, then y is directly proportional to x

exponential equation An equation involving a variable as a power, such as $y = 3^x$, whose graph is an exponential curve

inverse proportion A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.2) solve problems involving direct proportion and inverse proportion
- (STAGE 5.2) read and interpret conversion graphs
- graph parabolas of the form $y = ax^2$ and $y = ax^2 + c$
- graph exponential curves of the form $y = a^x$
- graph circles of the form $x^2 + y^2 = r^2$
- (STAGE 5.2) match graphs to their equations

SkillCheck ANSWERS ON P. 522

1 If $A = 2x^2 - 3$, find A if:

a $x = 1$ **b** $x = 4$ **c** $x = 0$ **d** $x = -6$

2 If $y = 5^x$, find y if:

a $x = 4$ **b** $x = 5$ **c** $x = 0$ **d** $x = -2$

7.01 Direct proportion

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other; when one variable changes, the other one changes by the same factor.

Direct proportion

If y is directly proportional to x, then $y = kx$, where k is a constant (number) called the constant of proportionality or constant of variation.

- A direct linear relationship exists between x and y
- If x increases (or decreases), y increases (or decreases)
- If x is doubled (or halved), y is doubled (or halved)
- Another way of saying 'y is directly proportional to x' is 'y varies directly with x'
- The graph of direct proportion is a straight line going through (0, 0) with gradient k

Direct proportion

Direct linear variation

Example 1

The distance (d) in metres travelled by a car is directly proportional to the number of rotations (r) of its tyres. After 540 rotations, a distance of 950 m is travelled.

a What distance (correct to the nearest metre) will be travelled after 800 rotations?

b How many full rotations will be needed to cover 360 m?

Solution

a d is directly proportional to r

$\therefore d = kr$ — Forming a proportion equation

To find k, substitute the information given for r and d.

When $r = 540$, $d = 950$:

$950 = k \times 540$ — Finding k.

$k = \frac{950}{540}$

$= 1.759...$ — Do not round k.

$\therefore d = 1.759...\ r$ — Rewriting the equation $d = kr$

When $r = 800$, — Solving the problem

$d = 1.759... \times 800$

$= 1407.4074...$

≈ 1407 m

After 800 rotations, the distance travelled will be 1407 m.

b When $d = 360$, — Solving the problem

$360 = 1.759... \times r$

$r = \frac{360}{1.759...}$

$= 204.661$

≈ 205 rotations — Rounding up for full rotations

For a distance of 360 m, there will be 205 rotations.

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Solving direct proportion problems

1 Identify the 2 variables (say x and y) and form a proportion equation, $y = kx$
2 Substitute values for x and y to find k, the constant of proportionality
3 Rewrite $y = kx$ using the value of k
4 Substitute a value for x or y into $y = kx$ to solve the problem

Example 2

M varies directly with n. If when $n = 6$, $M = 103.8$, find M when $n = 14.2$.

Solution

$M = kn$ — Forming a proportion equation

To find k, substitute $n = 6$ and $M = 103.8$.

$103.8 = k \times 6$ — Finding k

$6k = 103.8$

$k = \frac{103.8}{6}$

$= 17.3$

$\therefore M = 17.3n$ — Rewriting the equation $M = kn$

Substitute $n = 14.2$ to find M. — Solving the problem.

$M = 17.3 \times 14.2$

$= 245.66$

EXERCISE 7.01 ANSWERS ON P.522

Direct proportion **U F PS R C**

EXAMPLE 1

1 Match each statement with its proportion equation (k is the constant of variation). **C**

a The distance, D, travelled is directly proportional to the time, T.

b The wage, W, earned is directly proportional to the hours, h, worked.

c The wedding cost, C, varies directly with the number, n, of guests.

d The interest, I, earned varies directly with the size of the deposit, D

A $C = kn$ **B** $I = kD$ **C** $D = kT$ **D** $W = kh$

2 The distance, D, travelled by Connor, a marathon runner, varies directly with time, T. **PS** **R** **C**

Time, T (min)	1	2	3
Distance, D (m)	190	380	570

a Find the constant of proportionality and write the equation for D.

b How far, in kilometres, will Connor run in:

i 20 minutes? **ii** 45 minutes?

c How long would it take Connor to run 12.35 kilometres? Answer in hours and minutes.

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3 Mehta's earnings for working a shift at the local nursery are directly proportional to the number of hours she works. Yesterday, she earned $222.70 for working an 8.5 hour shift. PS R C

a If Mehta's earnings are represented by E, and the number of hours worked is represented by h, find the constant of proportionality and write an equation for E.

b How much will she earn for working a 7-hour shift?

c How many hours did she work today if she earned $144.10 for the shift?

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4 The amount of interest, I, earned for one year on an investment account varies directly with the size of the deposit, D. PS R C

a If Caterina earns $16 interest on an investment of $425, find the variation equation for I, including the constant of variation.

b Hence, how much will she earn on an investment of $900?

c If she doubles the size of her investment in **b**, how much will she earn in interest?

5 S varies directly with t. If when $t = 14$, $S = 106.4$, what is the value of S when $t = 0.3$? Select the correct answer **A**, **B**, **C** or **D**. R C

EXAMPLE 2

A 2.28 **B** 27.72 **C** 36.12 **D** 446.88

6 Find the linear formula for b in terms of a for this table of values.

a	4	8	12	16	20
b	10	20	30	40	50

7.01

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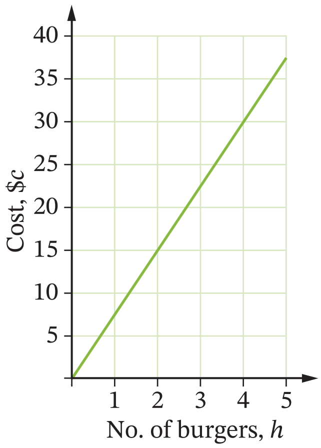
7 The line graph shows that the cost of hamburgers purchased from the local takeaway store depends directly on the number of burgers purchased. PS R C

a Copy this table and use the graph to complete it.

No. of burgers, h	1	2	3
Cost, c ($)			



b Find the variation equation to represent the relationship between the cost ($$c$) and the number of burgers (h).

c If Kendall buys 6 hamburgers, what is the total cost of the hamburgers?

d The total cost of one order of hamburgers is $82.50. How many hamburgers were ordered?

e Find the gradient of the line. How is it related to the constant of variation?

8 K varies directly with L. If $L = 9.5$ when $K = 1045$, what is the value of K when $L = 1.65$? Select **A**, **B**, **C** or **D**. R C

A 0.015 **B** 93.7 **C** 181.5 **D** 1708.575

9 A linear relationship exists between the mass of a car (m kg) and its rate of fuel consumption (F L/100 km). PS R C

a Find the variation equation for F if a 1000 kg car uses fuel at a rate of 6 L/100 km.

b Find the fuel consumption of a 2500 kg car.

10 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C? Select **A**, **B**, **C** or **D**. PS R C

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

11 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars. PS R C

a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.

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Inverse proportion#

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7.02

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WS Direct and inverse proportion

Inverse variation

WS Variation problems

Two variables are **inversely proportional** to each other if, when one variable increases, the other one decreases by the same factor.

The table below shows the different speeds of a car (s km/h), and the time it takes to travel 100 km (t min). As the speed increases, the time taken decreases.

Speed (s km/h)	50	60	80	100
Time (t min)	120	100	75	60

Inverse proportion

If y is **inversely proportional** to x, then $y = \frac{k}{x}$, where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- If x increases, y decreases ('inverse' means 'opposite')
- If x decreases, y increases
- If x is doubled, y is halved
- If x is halved, y is doubled
- Another way of saying 'y is inversely proportional to x' is 'y **varies inversely** with x'

Example 3

The time (t) in minutes taken by a car to travel 100 km is inversely proportional to the speed (s km/h) of the car, as shown in the table above. At 50 km/h, the time taken is 120 minutes.

a Find the constant of proportionality and the inverse variation equation for t.

b How long did the car take to travel 100 km at:

i 40 km/h? **ii** 110 km/h?

c Find the car's speed if it took 45 minutes to travel 100 km.

Solution

a t is inversely proportional to s.

$\therefore t = \frac{k}{s}$ Forming an inverse proportion equation

To find k, substitute the information given for s and t.

When $s = 50$, $t = 120$: Finding k, the constant of proportionality.

$120 = \frac{k}{50}$

$k = 120 \times 50$

$= 6000$

The constant of proportionality is 6000.

$\therefore t = \frac{6000}{s}$ Rewriting the equation $t = \frac{k}{s}$

STAGE 5.2

b **i** When $s = 40$, $t = \frac{6000}{40} = 150$ min

At 40 km/h, the trip takes 150 min (or 2 h 30 min).

ii When $s = 110$,

$t = \frac{6000}{110}$

$= 54.5454...$

≈ 55 min

At 110 km/h, the trip takes 55 min.

c When $t = 45$,

$45 = \frac{6000}{s}$

$45s = 6000$

$s = \frac{6000}{45}$

$= 133\frac{1}{3}$ km/h

For a travel time of 45 min, the speed must be $133\frac{1}{3}$ km/h.

Solving inverse proportion problems

1 Identify the 2 variables (say x and y) and form a proportion equation, $y = \frac{k}{x}$

2 Substitute values for x and y to find k, the constant of proportionality

3 Rewrite $y = \frac{k}{x}$ using the value of k

4 Substitute a value for x or y into $y = \frac{k}{x}$ to solve the problem

Inverse proportion

Example 4

The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 600 m above sea level, the temperature is 8°C.


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a What is the temperature at 1000 m above sea level?

b Graph the relationship between temperature and height above sea level, for heights between 0 and 5000.

Solution

a T is inversely proportional to h.

$T = \frac{k}{h}$ — Forming a proportion equation

Substitute $h = 600$ and $T = 8$ to find k.

$8 = \frac{k}{600}$ — Finding k

$k = 8 \times 600$

$= 4800$

$\therefore T = \frac{4800}{h}$ — Rewriting the equation $T = \frac{k}{h}$

When $h = 1000$, — Solving the problem

$T = \frac{4800}{1000} = 4.8$°C

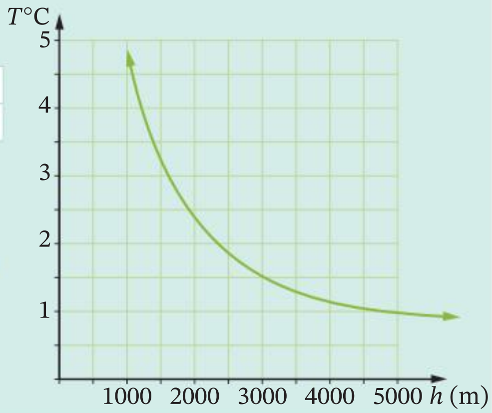
The temperature at a height of 1000 metres above sea level is 4.8°C.

b Draw a table of values for $T = \frac{4800}{h}$

h	1000	2000	3000	4000	5000
T	4.8	2.4	1.6	1.2	0.96

Note that as h increases, T decreases.

This graph is called a **hyperbola**.



Example 5

P varies inversely with q. If when $q = 4$, $P = 38$, find P when $q = 5$.

Solution

$P = \frac{k}{q}$ — Forming a proportion equation

Substitute $q = 4$, $P = 38$

$38 = \frac{k}{4}$ — Finding k

$k = 38 \times 4 = 152$

$\therefore P = \frac{152}{q}$ — Rewriting the equation $P = \frac{k}{q}$

Substitute $q = 5$ to find P. — Solving the problem

$P = \frac{152}{5} = 30.4$

EXERCISE 7.02 ANSWERS ON P.522

Inverse proportion U F PS R C

1 Match each statement with its inverse proportion equation (k is the constant of variation). C

a The travel time, T, varies inversely with the speed, S.

b The temperature, T, above sea level varies inversely with the altitude, h.

c The cost, C, per person of hiring a bus varies inversely as the number, n, of people.

d The time, T, taken to move house varies inversely with the number, n, of helpers.

A $T = \frac{k}{n}$ **B** $T = \frac{k}{S}$ **C** $T = \frac{k}{h}$ **D** $C = \frac{k}{n}$

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STAGE 5.2

EXAMPLE 3

2 The time taken, T hours, to travel from Sydney to Melbourne varies inversely with the average speed, s km/h. PS R C

a If it takes 11.5 hours at an average speed of 80 km/h, find the constant of proportionality and the variation equation for T.

b If the average speed is increased to 90 km/h, how long will the journey take? Answer in hours and minutes.

c Find the average speed needed to complete the trip in 10 hours.

EXAMPLE 4

3 The temperature, T (in degrees Celsius), of the air varies inversely with the height, h (in metres), above sea level. At 150 m above sea level, the temperature is 30°C. PS R C

a Find the constant of variation and write a variation equation for T.

b What is the temperature at:

i 300 m above sea level? **ii** 2500 m above sea level?

c What is the height above sea level when the temperature is:

i 8°C? **ii** 22.5°C?

d Graph the relationship between temperature and height above sea level. Use T on the vertical axis and h on the horizontal axis with $h = 0, 500, 1000, 1500, ..., 3000$.

4 The maximum number of diners, N, in Yen's restaurant varies inversely with the amount of floor space, S m^2, allocated to each diner. If 1.4 m^2 is allowed per diner, the restaurant can have 80 diners. PS R C

a Write a variation equation for N.

b How many diners could the restaurant contain if only 1.1 m^2 was allocated per person?

c How much space (correct to one decimal place) is allocated to each diner if the restaurant has a maximum capacity of 55 diners?

d During the COVID-19 pandemic of 2020, social distancing measures introduced to minimise the spread of the virus allocated 4 m^2 to each diner. How many diners could Yen's restaurant serve under this restriction?

5 The rate of vibration of a string varies inversely as its length. A string that is 8 cm long vibrates at 9375 Hz (hertz). What length of string will vibrate at 6250 Hz? Select the correct answer **A**, **B**, **C** or **D**. PS R C

A 5 cm **B** 7 cm **C** 12 cm **D** 73 cm

6 Which equation represents this table of values? Select **A**, **B**, **C** or **D**. R

x	2	5	8	10
y	2.5	1	0.625	0.5

A $y = \frac{10}{x}$ **B** $y = \frac{5}{x}$ **C** $y = \frac{2.5}{x}$ **D** $y = \frac{1}{x}$

EXAMPLE 5

7 K is inversely proportional to L. If $L = 2$ when $K = 7$, find K when $L = 15$. R

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8 Ankit believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 16 people waiting, the train will arrive in 2.5 minutes. PS R C

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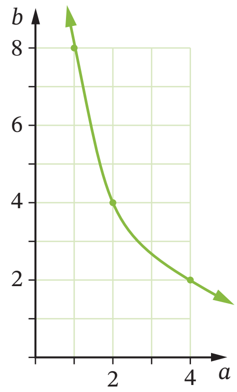
a When will the train arrive if there are 5 people waiting?

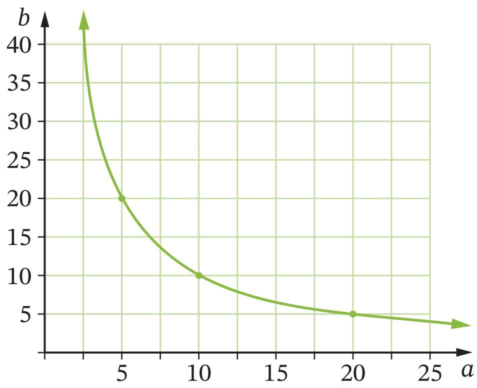
b How many people are waiting at the station 10 minutes before the train arrives?

9 Each graph shows an inverse relationship between a and b. Find each variation equation. R C

a



b



10 The frequency, F beats per second, that a bird beats its wings varies inversely as the length, L cm, of its wings. A bird with wings of length 14 cm beats them at a frequency of 8 beats per second.

a Find the variation equation for F in terms of L. PS R C

b Calculate, to the nearest whole number, the wingbeat frequency for wings of length 18 cm.

c A bird beats its wings with a frequency of 4.5 beats per second. What is the length of its wings, correct to the nearest centimetre?

STAGE 5.2

11 For a certain equation, y varies inversely with x. R

a Given $x = 0.2$ when $y = 10$, find y when $x = 32$.

b Find x when $y = 1.6$.

12 The amount of time it takes Sarah to move house is inversely proportional to the number of friends she has to help her. When she has 4 friends helping, the job takes $3\frac{3}{4}$ hours. PS R C

a How long will it take if she has 6 friends helping?

b How many friends must she have to help her to move house in 3 hours?

7.03 Conversion graphs

STAGE 5.2

WS Currency conversion graph

A **conversion graph** is used to convert from one unit to another; for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0), so it has an equation in the form $y = kx$ and is an example of **direct proportion**.

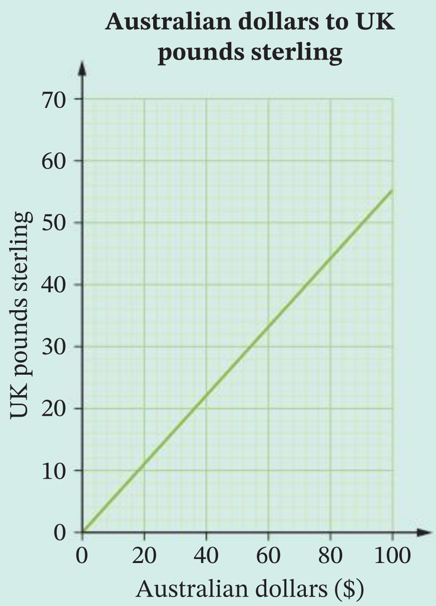
Example 6

Exchange rates change daily, but suppose the exchange rate between the Australian dollar and the UK pound sterling is \$A1 = £0.553, then \$A100 = £55.30 sterling. These values are used to draw this conversion graph.

Use the graph to convert:

a \$A50 to pounds

b £10 to Australian dollars.



Solution

Reading from the graph:

a \$A50 ≈ £28

b £10 ≈ \$A18

EXERCISE 7.03 ANSWERS ON P.523

STAGE 5.2

Conversion graphs U F R C

EXAMPLE 6

1 Use the graph on the previous page for this question.

a Convert to pounds:

i $A40

ii $A88

b Convert to Australian dollars:

i £18

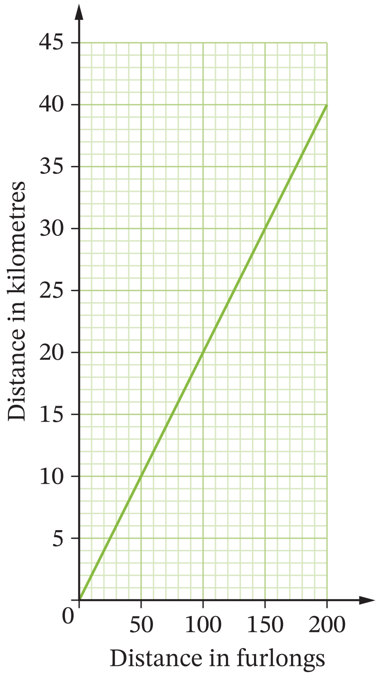
ii £60

c In 2015, $A1 = £0.50. R C

i How much less was $A40 worth in UK pounds sterling in 2015 than it is using this more recent conversion graph?

ii How much more was £60 worth in Australian dollars in 2015 for UK tourists visiting Australia, than it is using this more recent conversion graph?

2 The furlong is an imperial measure once used to measure length. This conversion graph shows distances in furlongs converted to kilometres.



a Convert to kilometres:

i 10 furlongs

ii 100 furlongs

iii 170 furlongs

b Convert to furlongs:

i 10 km

ii 25 km

iii 36 km

c Use an answer from part **a** to convert 300 furlongs to kilometres. R

d Use an answer from part **b** to convert 100 kilometres to furlongs. R

7.03

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STAGE 5.2

3 This graph converts between Australian dollars and Japanese yen (¥).

a Convert to Japanese yen:

i $200

ii $800

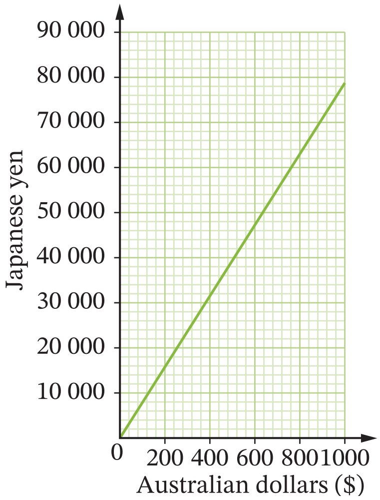
iii $1000

b Convert to Australian dollars.

i ¥20 000

ii ¥60 000

iii ¥72 000



4 This graph converts temperatures between degrees Fahrenheit and degrees Celsius. Convert:

a 0°F to °C

b 50°F to °C

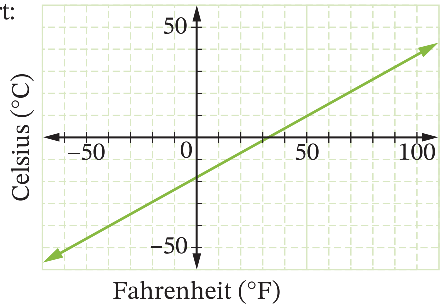
c 80°F to °C

d 0°C to °F

e −10°C to °F

f 30°C to °F

Degrees Fahrenheit to degrees Celsius



5 This conversion graph converts between acres and hectares. The acre is an imperial measure of land area.

a Use the graph to convert 12 acres to hectares.

b A garden has an area of 5 acres. What is this area in hectares?

c Use the graph to convert 4.4 hectares to acres.

d Mr Ferguson has a property with an area of 5 hectares. How big is this in acres?

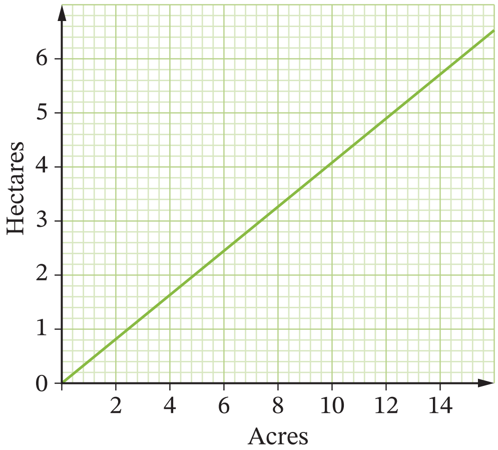
e A rectangular playing field measures 250 m by 128 m.

i What is the area of the field in square metres?

ii What is the area of the field in hectares?

iii What is the area of the field in acres?

Converting acres to hectares



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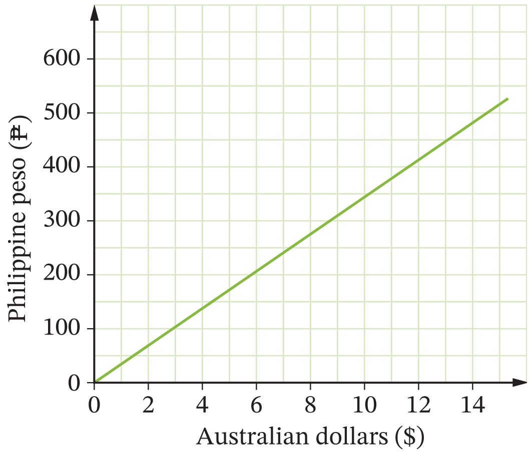
STAGE 5.2

6 This graph is used to convert Australian dollars to Philippine pesos (₱).



a Change each Australian dollar amount into Philippine pesos.

i \$15

ii \$6

iii \$9

b Change ₱350 to Australian dollars.

c How many Australian dollars would you receive for ₱200?

d Estimate the number of pesos you should get for \$120. R

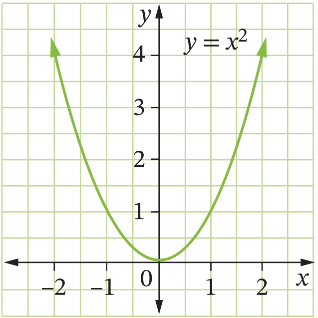
The parabola $y = ax^2 + c$ 7.04

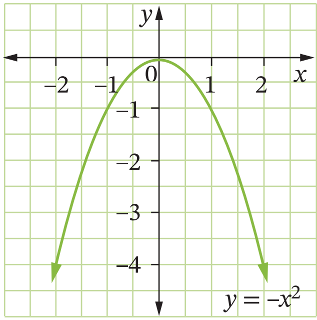
An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$. The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**.

Investigating parabolas 1

Investigating parabolas 1

The graphs of $y = x^2$ and $y = -x^2$ are shown below.





Concave up (looks like a smile ☺)

Minimum value of the parabola is 0.

Concave down (looks like a frown ☹)

Maximum value of the parabola is 0.

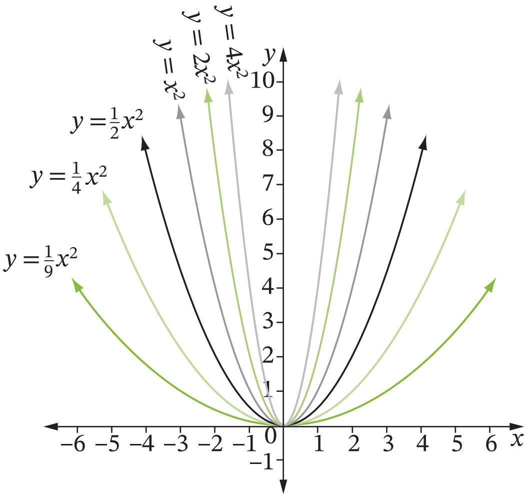
Both parabolas have one axis of symmetry, the y-axis, and a turning point, called the **vertex**, at the origin (0, 0).

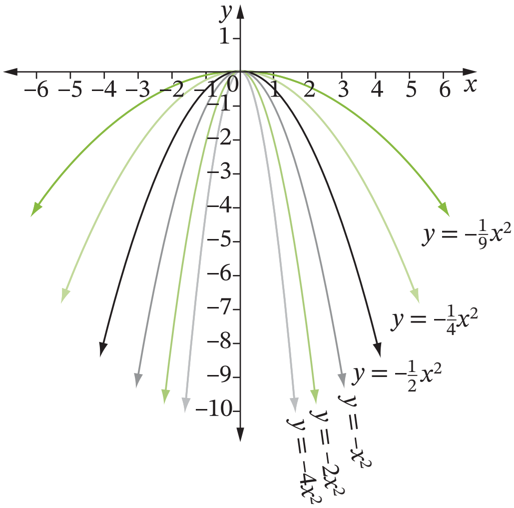
STAGE 5.2

The graph of $y = ax^2$

For the graph of $y = ax^2$, where a is a constant (number), the size of a (the **coefficient** of x^2) affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and as the size of a decreases, the parabola 'widens'. If a is negative, then the parabola is concave down.





The graph of $y = ax^2$ is a parabola with the y-axis as its **axis** (of symmetry) and (0, 0) as its **vertex**.

The graph of $y = ax^2 + c$

For the graph of $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the origin. Also, c is the y-intercept of the parabola.

Shutterstock.com/Neale Cousland

Example 7

The parabola $y = ax^2 + c$

7.04

Graph each set of quadratic equations on the same number plane, showing the vertex of each parabola.

a $y = x^2, y = x^2 - 4, y = x^2 + 2$

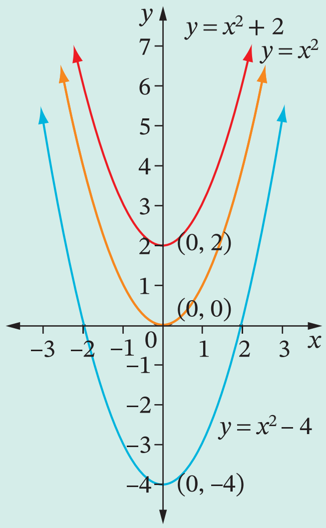
b $y = -x^2, y = -x^2 - 4, y = -x^2 + 5$

Solution

a First draw the graph of $y = x^2$. Its vertex is at (0, 0).

The graph of $y = x^2 - 4$ is like that of $y = x^2$, but moved 4 units down. Its vertex is at (0, −4).

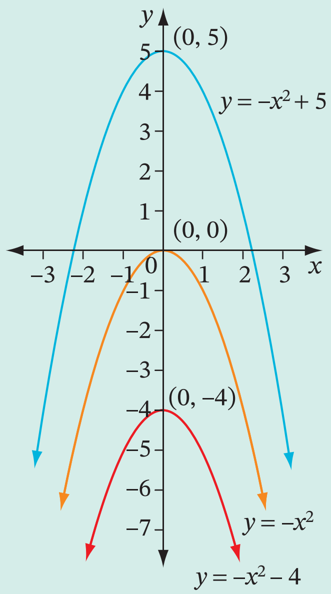
The graph of $y = x^2 + 2$ is like that of $y = x^2$, but moved 2 units up. Its vertex is at (0, 2).



b The graph of $y = -x^2$ is the graph of $y = x^2$ reflected across the x-axis. Its vertex is at (0, 0) as well.

The graph of $y = -x^2 - 4$ is like that of $y = -x^2$, but it is moved 4 units down. Its vertex is at (0, −4).

The graph of $y = -x^2 + 5$ is like that of $y = -x^2$, but it is moved 5 units up. Its vertex is at (0, 5).



Note:

- In part **a**, all parabolas are concave up, because a is positive (1)
- In part **b**, all parabolas are concave down, because a is negative (−1)
- For $y = ax^2 + c$, the y-intercept of the parabola is c

STAGE 5.2

Example 8

For the graph of each quadratic equation, state:

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 3x^2 - 1$

b $y = \frac{1}{3}x^2 + 2$

Solution

a **i** The coefficient of x^2 is 3, while the coefficient of x^2 in $y = x^2$ is 1.

$\therefore$ The parabola will be narrower than $y = x^2$.

ii The constant term is -1.

$\therefore$ The parabola has moved down.

iii The y-intercept is -1.

Vertex at $(0, -1)$

b **i** The coefficient of x^2 is $\frac{1}{3}$.

$\therefore$ The parabola will be wider than $y = x^2$.

ii The constant term is 2.

$\therefore$ The parabola has moved up.

iii The y-intercept is 2.

Vertex at $(0, 2)$

Example 9

A parabola has the equation $y = 3x^2 - 1$.

Find the x-coordinate of the point on the parabola that has a y-coordinate of:

a 11

b 191

Solution

a Substitute $y = 11$ into $y = 3x^2 - 1$.

$$\begin{aligned} 11 &= 3x^2 - 1 \\ 12 &= 3x^2 \\ 3x^2 &= 12 \\ x^2 &= \frac{12}{3} \\ &= 4 \\ x &= \pm\sqrt{4} \\ &= \pm 2 \end{aligned}$$

This means that there are 2 points on the parabola with a y-coordinate of 11.

They are $(2, 11)$ and $(-2, 11)$.

This makes sense because a parabola is symmetrical: $(2, 11)$ and $(-2, 11)$ are on either side of the y-axis.

b Substitute $y = 191$ into $y = 3x^2 - 1$

$$\begin{aligned} 191 &= 3x^2 - 1 \\ 192 &= 3x^2 \\ 3x^2 &= 192 \\ x^2 &= \frac{192}{3} \\ &= 64 \\ x &= \pm\sqrt{64} \\ &= \pm 8 \end{aligned}$$

EXERCISE 7.04 ANSWERS ON P.523

ANSWERS ON P.523

The parabola $y = ax^2 + c$ **U F PS R C**

This exercise can be completed using graphing technology.

EXAMPLE 7

1 **a** Graph each quadratic equation, showing the vertex of each parabola.

$y = x^2$ $\quad y = -x^2$ $\quad y = x^2 + 2$ $\quad y = -2x^2$ $\quad y = x^2 - 1$

b State which graphs you have drawn in part **a**: **C**

i are concave up **ii** are concave down **iii** have a turning point at (0, 0).

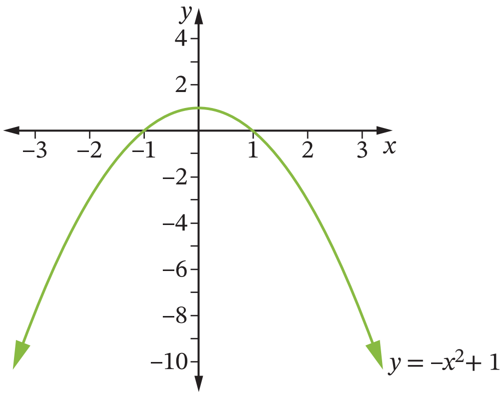
2 Which statement is false about this parabola? Select **A**, **B**, **C** or **D**. **C**

A Its axis of symmetry is the x-axis.

B It is concave down.

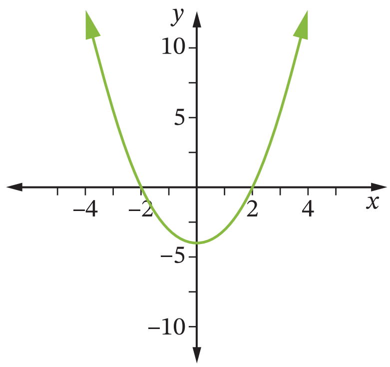
C Its vertex is (0, 1).

D It has a maximum value.

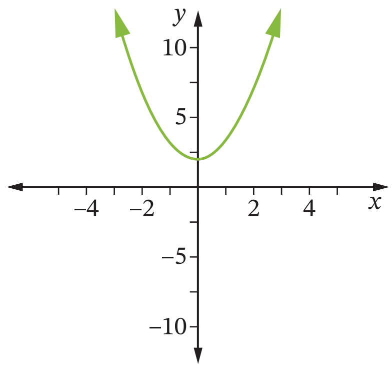


3 Which diagram shows the graph of $y = x^2 - 2$? Select **A**, **B**, **C** or **D**. **R**

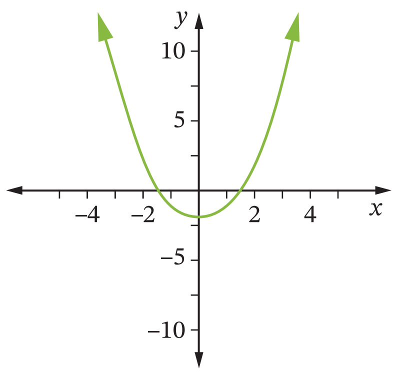
A



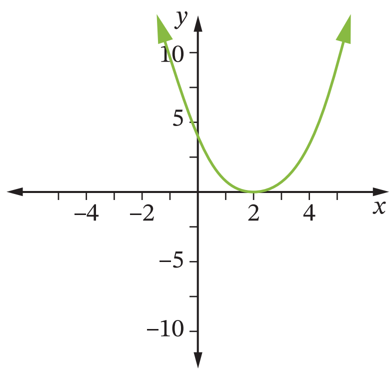
B



C



D

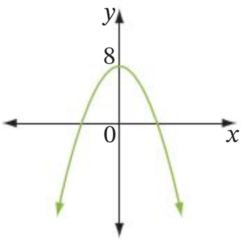


Foundation **Standard** **Complex**

STAGE 5.2

4 Match each graph with its correct quadratic equation. R C

a



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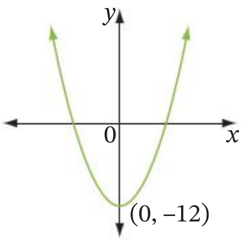

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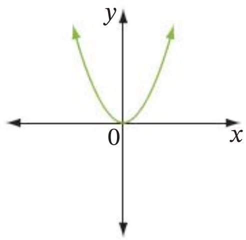

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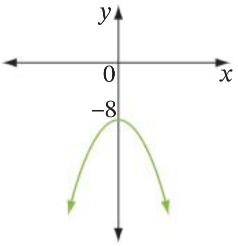
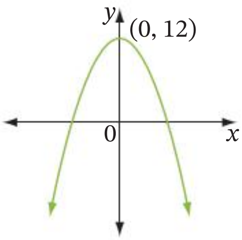


e



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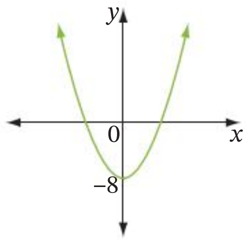

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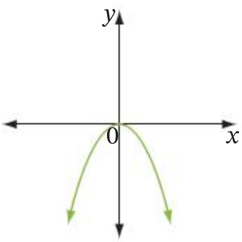

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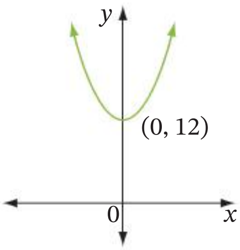
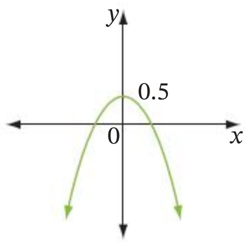


i



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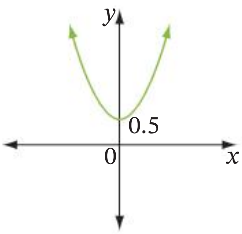

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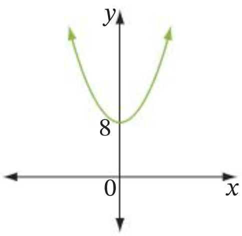

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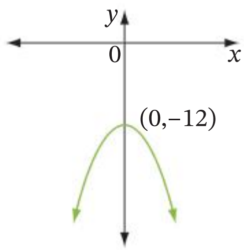


i $y = x^2$ **ii** $y = -x^2$ **iii** $y = x^2 - 8$ **iv** $y = -12 - x^2$

v $y = \frac{1}{2} + x^2$ **vi** $y = 8 - x^2$ **vii** $y = 8 + x^2$ **viii** $y = -x^2 + \frac{1}{2}$

ix $y = x^2 - 12$ **x** $y = 12 - x^2$ **xi** $y = -x^2 - 8$ **xii** $y = x^2 + 12$

5 Find the equation of each parabola described, in the form $y = x^2 + c$ or $y = -x^2 + c$ (where c is a constant). R C

a vertex (0, 0), concave down

b concave up, turning point (0, 0)

c axis of symmetry $x = 0$, maximum $y = -\frac{1}{4}$

d concave down, maximum $y = -9$

e turning point $\left(0, \frac{1}{2}\right)$, concave down

f axis of symmetry y-axis, minimum $y = 9$

Foundation Standard Complex

9780170453417

6 **a** Graph $y = 2x^2 + 1$ after copying and completing this table. R C

x	-2	-1	0	1	2
y					

b State the coordinates of the vertex.

c Is the parabola concave up or concave down?

d What is its minimum value?

7 **a** Graph $y = -3x^2 + 2$ after copying and completing this table. R C

x	-2	-1	0	1	2
y					

b Find the vertex.

c Write the equation of its axis of symmetry.

d Find its maximum value.

8 Which statement is false about the graph of $y = 4x^2 - 1$? Select **A**, **B**, **C** or **D**. R C

A Its axis of symmetry is $y = 0$.

B It is concave up.

C The vertex is $(0, -1)$.

D It has a minimum value of $y = -1$.

9 For the graph of each given quadratic equation, state: R C

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 2x^2 + 3$

b $y = \frac{1}{2}x^2 + 1$

c $y = 6x^2 - 5$

d $y = 0.2x^2 - 12$

10 A parabola has the equation $y = x^2 - 5$. Find the x-coordinates for the points on the parabola that have a y-coordinate of:

a 11

b 116.

11 A stone is dropped from a cliff and its height (h metres) at any time (t seconds) is given by $h = 80 - 4.8t^2$. PS R C

a Draw a graph of the equation for values of t from 0 to 5.

b What is the height of the cliff?

c What is the height of the stone after 3 seconds?

d When will the stone hit the ground?

e How long after it is dropped is the stone 5 metres above the ground? Answer correct to 2 decimal places.

7.04

STAGE 5.2

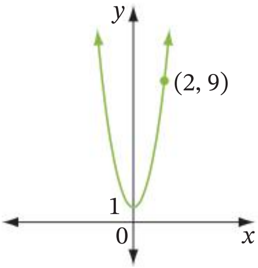
EXAMPLE 8

EXAMPLE 9

Foundation Standard Complex

STAGE 5.2

12 Match each graph with its correct quadratic equation. R C

a


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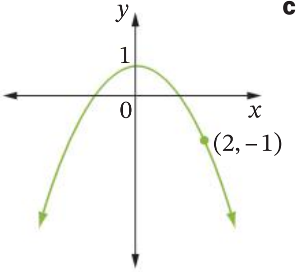

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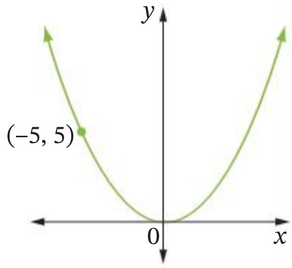

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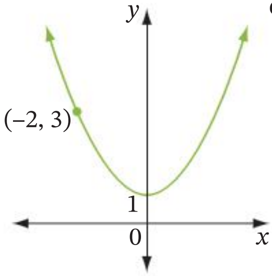

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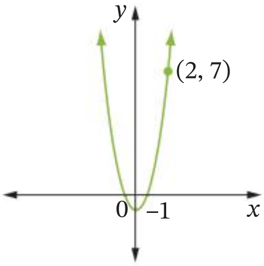

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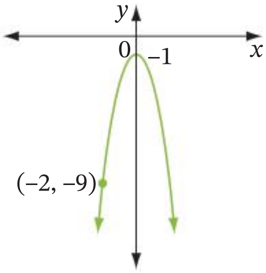

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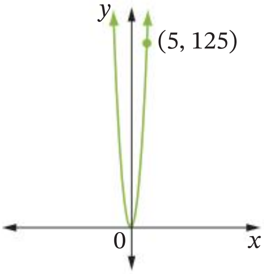

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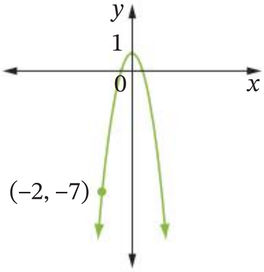

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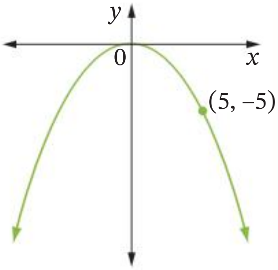

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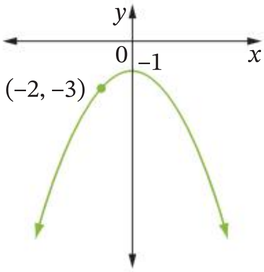

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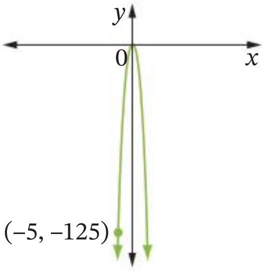

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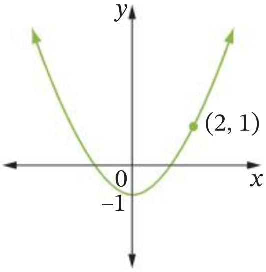


A $y = 5x^2$ **B** $y = 2x^2 + 1$ **C** $y = \frac{1}{2}x^2 - 1$

D $y = \frac{1}{5}x^2$ **E** $y = 2x^2 - 1$ **F** $y = -5x^2$

G $y = -\frac{1}{2}x^2 + 1$ **H** $y = -\frac{1}{5}x^2$ **I** $y = -2x^2 - 1$

J $y = \frac{1}{2}x^2 + 1$ **K** $y = -2x^2 + 1$ **L** $y = -\frac{1}{2}x^2 - 1$

13 A parabola has the equation $y = 2x^2 + 3$. Find the x-coordinates of the points on the parabola that have a y-coordinate of:

a 165 **b** 395

Foundation Standard Complex

Did you know?

Parabolas in architecture

There are many examples of parabolas in architecture and engineering.

The Notre Dame Cathedral in Paris, France is almost 900 years old and has flying buttresses on the outside that have the shape of parabolas.

Shutterstock.com/Bill Perry

Bridges also often use parabolic curves in their construction. One modern application is the cables used in the suspension of the Golden Gate Bridge in San Francisco.

iStock.com/MasterLu

Find 2 different uses of parabolas in real-life constructions and create a presentation involving them.

7.04

Mental skills 7: Maths without calculators ANSWERS ON P.523

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

b $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.30$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

c By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

2 Now evaluate each product.

a 0.7×5	**b** 12×0.2	**c** 0.4×0.3	**d** $(0.6)^2$
e 8×0.1	**f** 0.03×0.9	**g** 4×0.05	**h** 1.1×8
i 0.3×0.8	**j** 0.2×0.06	**k** 9×0.2	**l** 0.07×0.4

3 Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

a $1.5 \times 2.3 = 3.45$

1 dp + 1 dp = 2 dp (*Estimate* $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

0 dp + 2 dp = 2 dp

(*Estimate* $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

2 dp + 0 dp = 2 dp

(*Estimate* $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a 3.9×17	**b** 39×170	**c** 39×0.17	**d** 0.39×1.7
e 3.9×1.7	**f** 390×1.7	**g** 3.9×0.17	**h** 3.9×170
i 3900×1.7	**j** 39×1.7	**k** 39×0.017	**l** 0.39×0.17

Investigation

Graphing $y = 2^x$

This activity can be completed using graphing technology.

1 Copy and complete this table of values for $y = 2^x$.

x	−3	−2	−1	0	1	2	3	4
y								

2 Graph the points from the table and join them with a smooth curve. The equation $y = 2^x$ is called an exponential equation and its graph is called an exponential curve (exponent means 'power').

3 Graph $y = 2^{-x}$ in a similar way.

4 Compare the graphs of $y = 2^x$ and $y = 2^{-x}$. Describe any similarities and differences.

5 The y-intercept of any graph with equation $y = a^x$ (where a is a positive constant) is always 1. Explain why.

6 The graph of $y = 2^x$ is increasing. Is the graph of $y = 2^{-x}$ increasing or decreasing? Give reasons.

7 Describe what happens to the graph of $y = 2^x$ when:

a x approaches a large positive number

b x approaches a large negative number.

Technology

Exponential curves

Use graphing technology to complete this activity.

1 Graph $y = 2^x$ using the software. Adjust the colour or line thickness if you need to.

2 Now graph these exponential curves, using different colours.

$y = 2^{-x}$, $y = -2^x$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$

3 a Which graphs are similar?

b Identify any features such as y-intercepts.

c Which graphs are similar as:

i x becomes larger? ii x becomes smaller?

4 Repeat steps **1** to **3** for these exponential curves:

$y = 3^x$, $y = 3^{-x}$, $y = -3^x$, $y = -3^{-x}$, $y = 3^x + 1$, $y = 3^x - 1$

7.04

7.05 The exponential curve

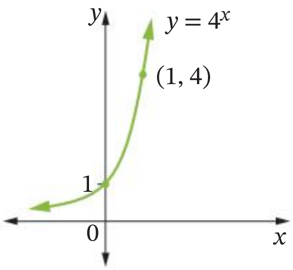
The exponential curve

WS Graphing exponentials

An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, is called an **exponential equation**, for example, $y = 5^x$, $y = 2^x$ and $y = 3^x$. The graph of an exponential equation is called an **exponential curve**.

The table of values and graph of $y = 4^x$ is shown.

x	−2	−1	0	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64	256



- The y-intercept of $y = a^x$ is always 1 since $a^0 = 1$.
- As x increases (to the right, in the positive direction), a^x becomes very large. Graphically, this means that the graph of $y = a^x$ increases sharply with a steep positive gradient.
- As x decreases (to the left, in the negative direction), a^x approaches 0. This means that the graph of $y = a^x$ flattens out and approaches the x-axis as x becomes a large negative number. The x-axis is an **asymptote** because the curve approaches it but never touches it.
- The exponential curve is always above the x-axis because the value of a^x is always positive.

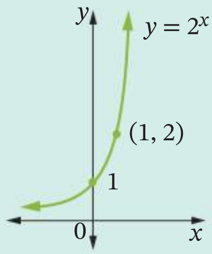
Example 10

Sketch each exponential equation and mark the y-intercept on each curve.

a $y = 2^x$ **b** $y = 3^{-x}$

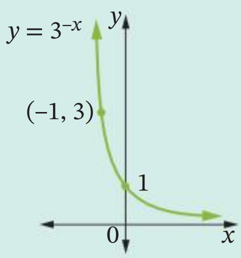
Solution

a
- The y-intercept of $y = 2^x$ is 1
- When $x = 1$, $y = 2$, so (1, 2) is a point on the curve
- As x increases (to the right in the positive direction), 2^x becomes very large (steep positive gradient)
- As x decreases (to the left in the negative direction), 2^x approaches 0.
- The x-axis is an asymptote.



STAGE 5.2

b
- The y-intercept of $y = 3^{-x}$ is 1
- When $x = -1$, $y = 3$, so (−1, 3) is a point on the curve
- As x decreases (to the left in the negative direction), 3^{-x} becomes very large (steep negative gradient)
- As x increases (to the right in the positive direction), 3^{-x} approaches 0.
- The x-axis is an asymptote.



Note that the graph of $y = 3^{-x}$ (and of $y = a^{-x}$ in general) is decreasing, and is actually a reflection of $y = 3^x$ in the y-axis.

EXERCISE 7.05 ANSWERS ON P.524

ANSWERS ON P.524

The exponential curve **U F R C**

Some of this exercise may be completed using graphing technology.

EXAMPLE 10

7.05

STAGE 5.2

1 **a** Graph each exponential equation on the same axes. R C

i $y = 2^x$ **ii** $y = 3^x$ **iii** $y = 5^x$

b What is the y-intercept of each curve?

c Describe what happens to the graph of $y = a^x$ as a increases.

2 **a** Graph $y = 4^x$ and $y = 4^{-x}$ on the same axes.

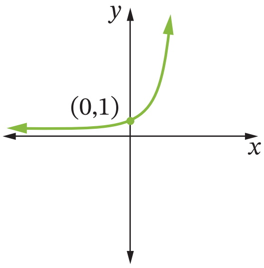
b Copy and complete:

i The reflection of $y = 4^x$ in the y-axis is ...

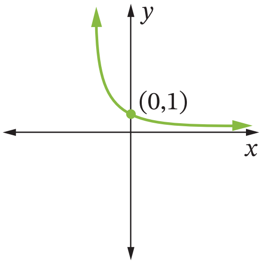
ii The reflection of $y = a^x$ in the y-axis is ...

3 Which graph represents $y = 2^{-x}$? Select the correct answer **A**, **B**, **C** or **D**. R C

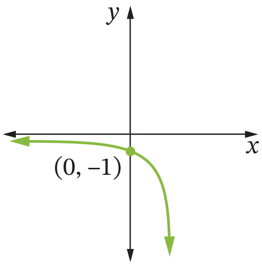
A



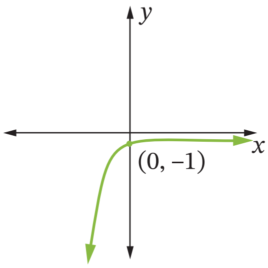
B



C



D



4 **a** Graph $y = 2^x$ and $y = -2^x$ on the same axes.

b How are the 2 graphs related? R C

c Copy and complete: The reflection of $y = a^x$ in the x-axis is ...

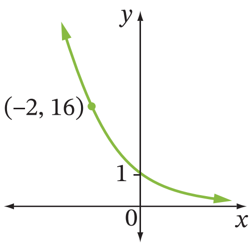
5 Graph $y = 3^x + 1$ and $y = 3^x - 1$ on the same axes and describe how they are related. R C

6 Sketch each exponential curve, showing the y-intercept.

a $y = 2^x$ **b** $y = 3^{-x}$ **c** $y = -4^x$

d $y = -2^{-x}$ **e** $y = 4^x + 1$ **f** $y = 4^x - 1$

7 Find an exponential equation for this graph. R C

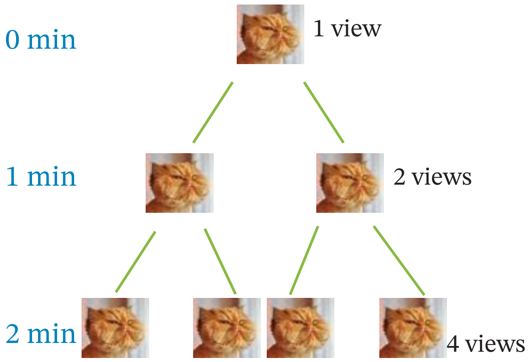


Investigation

Videos going viral

Use a spreadsheet to help you with this investigation.

With the invention of social media and social networks, we can now communicate with each other instantly. However, what often starts off as a simple message between friends or an online video post can quickly multiply at an alarming rate, be seen by the general public worldwide and 'go viral' (spread like a virus).



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Suppose a video of an angry cat is shared with a friend who one minute later shares it with 2 other friends. This occurs every minute so that every minute the number of *new* views doubles. This is an example of something growing exponentially, according to the formula $y = 2^x$, where x represents number of minutes.

1 Use a spreadsheet to calculate the number of new views after:

a 15 min **b** 30 min **c** 1 hour **d** 2 hours
e 4 hours **f** 10 hours **g** 20 hours **h** 30 hours

2 Find how long it will take until the total number of new views reaches:

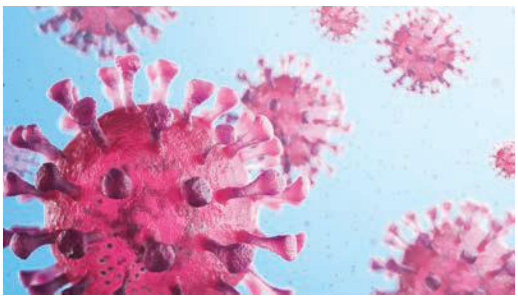
a 64 **b** 256 **c** over 500 **d** over 1000
e over 3000 **f** over 10 000 **g** over 1 million **h** over 4 million

3 Use the spreadsheet's **Graph Option** to graph the viral pattern. Describe the shape of the graph.

Did you know?

Exponential growth

When an increase can be described using an exponential equation, it is called exponential growth. Examples include the growth of a virus, population (people or animals) and compound interest investments.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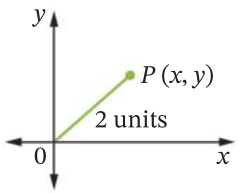
Population growth is monitored in different countries through the fertility (birth) and mortality (death) rates as well as migration. The data collected for these figures can often be modelled as an exponential function. By modelling the changes in population, predictions of future changes in population can be simulated and towns and cities can prepare for possible expansion in the numbers of schools, hospitals, housing and other necessary infrastructure.

At what rate is the population of Australia growing? What about the world's population?

The circle

7.06

Let $P(x, y)$ be any point on a number plane so that the distance $OP = 2$ units, where O is the origin.

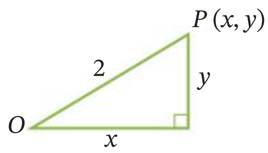


Curve sketcher

Curve sketcher

7.06

If we plotted every possible position of P, we would have the graph of a circle centred at O with a radius of 2. We can use Pythagoras' theorem to find the equation of this circle by drawing a right-angled triangle where OP is the hypotenuse.



Since P has coordinates (x, y), the triangle must have a base length of x and a height of y, so by Pythagoras' theorem: $x^2 + y^2 = 2^2 = 4$

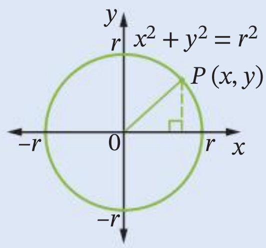
$\therefore$ The equation of a circle with centre (0, 0) and radius 2 is $x^2 + y^2 = 4$.

This can be generalised for a circle of any radius.

The equation of a circle

The equation of a circle with centre (0, 0) and radius r units is

$$x^2 + y^2 = r^2$$



Example 11

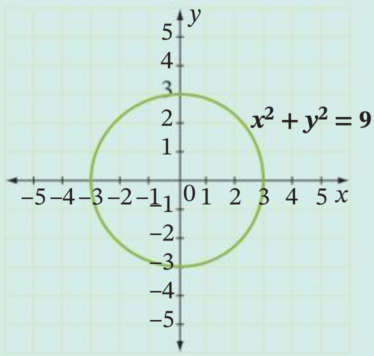
Graph the circle $x^2 + y^2 = 9$.

Solution

The centre is (0, 0). The radius is r, where $r^2 = 9$.

$r = \sqrt{9} = 3$

Radius = 3 units



Example 12

Find the equation of a circle with centre (0, 0) and diameter 14 units.

Solution

Radius $= \frac{1}{2} \times 14 = 7$ units.

The equation of the circle is

$x^2 + y^2 = 7^2$

$x^2 + y^2 = 49$

EXERCISE 7.06 ANSWERS ON P.525

ANSWERS ON P.525

The circle U F R

Some of this exercise may be completed using graphing technology.

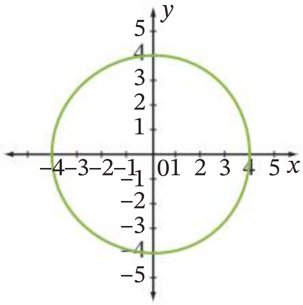
EXAMPLE 10

1 Find the centre and radius of the circle with equation:

a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 64$

d $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 81$ **f** $2x^2 + 2y^2 = 50$

2 What is the equation of this circle?
Select the correct answer **A**, **B**, **C** or **D**.



A $x^2 + y^2 = 2$

B $x^2 + y^2 = 4$

C $x^2 + y^2 = 8$

D $x^2 + y^2 = 16$

3 What is the equation of a circle with centre (0, 0) and radius 3 units? Select **A**, **B**, **C** or **D**.

A $x^2 + y^2 = -9$ **B** $x^2 + y^2 = 3$

C $x^2 + y^2 = -3$ **D** $x^2 + y^2 = 9$

EXAMPLE 12

4 Find the equation of a circle with centre (0, 0) and:

a radius 1 **b** diameter 6 **c** diameter 10 **d** radius $\frac{1}{3}$

5 Graph the circle with equation:

a $x^2 + y^2 = 16$ **b** $x^2 + y^2 = 121$ **c** $x^2 + y^2 = \frac{1}{4}$

6 Find the equation of a circle with centre (0, 0) and radius 10 units. Select **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 10$ **B** $2x^2 + 2y^2 = 20$

C $3x^2 + 3y^2 = 300$ **D** $4x^2 + 4y^2 = 14$

7 **a** Show that the point (8, 6) lies on the circle $x^2 + y^2 = 100$. R

b Show that the point (5, 9) does not lie on the circle $x^2 + y^2 = 100$.

c Does (5, 9) lie inside or outside this circle?

8 Given the equation of the circle $x^2 + y^2 = 4$, substitute each of the following points into the equation and determine whether the points are *inside*, *on* or *outside* the circle: R

a (0, 0) **b** (2, 0) **c** (3, 1) **d** (1, 1) **e** (−5, 2)

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Foundation Standard Complex

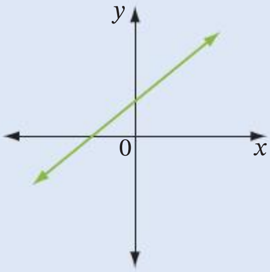
Identifying graphs 7.07

STAGE 5.2

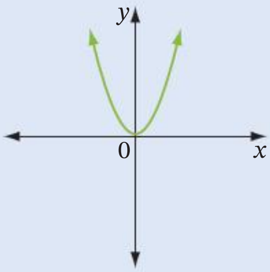
Matching graphs

Graphs

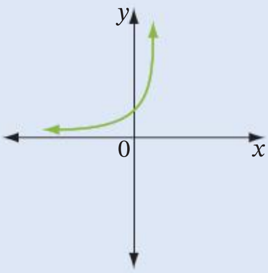
Straight line $y = mx + c$ or $ax + by + c = 0$



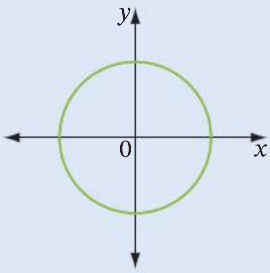
Parabola $y = ax^2 + c$



Exponential curve $y = a^x$



Circle $x^2 + y^2 = r^2$



Example 13

State whether each equation represents a straight line, a parabola, an exponential curve or a circle.

a $y = 3x^2 - 1$ **b** $y = 5x + 7$ **c** $y = -\frac{1}{2}x^2 - 10$

d $x^2 + y^2 = 4$ **e** $y = 3^x$

Solution

a $y = 3x^2 - 1$ is a parabola because it is of the form $y = ax^2 + c$.

b $y = 5x + 7$ is a straight line because it is of the form $y = mx + c$.

c $y = -\frac{1}{2}x^2 - 10$ is a parabola because it is of the form $y = ax^2 + c$.

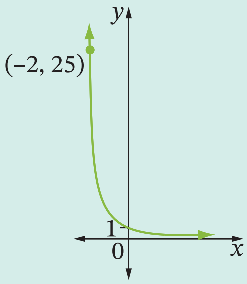
d $x^2 + y^2 = 4$ is a circle because it is of the form $x^2 + y^2 = r^2$.

e $y = 3^x$ is an exponential curve because it is of the form $y = a^x$.

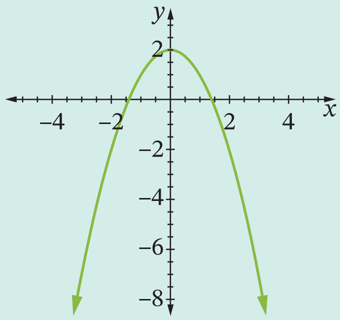
Example 14

Match each graph with its equation.

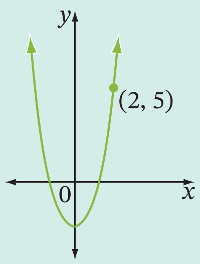
a



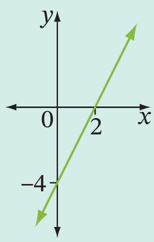
b



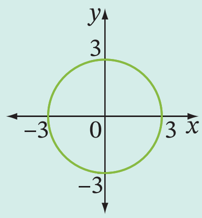
c



d



e



A $y = 2x - 4$ **B** $x^2 + y^2 = 9$ **C** $y = 2x^2 - 3$

D $y = 5^{-x}$ **E** $y = -x^2 + 2$

Solution

When matching graphs to equations, the coordinates of a point on the graph may need to be substituted into the equation to verify that the equation represents the graph.

a This is an exponential curve. The only possible match is **D**, $y = 5^{-x}$

Test point: (−2, 25)

LHS = 25

RHS $= 5^{-(-2)} = 5^2 = 25 =$ LHS

b This is a concave down parabola with a y-intercept of 2. The only possible match is **E**, $y = -x^2 + 2$

c This is a concave up parabola that matches with **C**, $y = 2x^2 - 3$

Test point: (2, 5)

LHS = 5

RHS $= 2 \times 2^2 - 3 = 5 =$ LHS

d This is a straight line with a y-intercept of −4 that matches with **A**, $y = 2x - 4$

e This is a circle with centre (0, 0) and radius 3. The only possible match is **B**, $x^2 + y^2 = 9$

EXERCISE 7.07 ANSWERS ON P.526

STAGE 5.2

Identifying graphs UFRC

EXAMPLE 13

1 For each equation state whether its graph is a straight line (L), a parabola (P), an exponential curve (E) or a circle (C). R C

a $y = 9x^2 - 4$ **b** $y = 9x$ **c** $y = 9^x$ **d** $y = 9$

e $x^2 + y^2 = 81$ **f** $y = 3x - 8$ **g** $y = 3x^2 - 8$ **h** $y = 2x + 5$

i $y = -x^2 + 6$ **j** $y = 10^{-x}$ **k** $y = 7x^2 + 2$ **l** $x^2 + y^2 = 36$

7.07

EXAMPLE 14

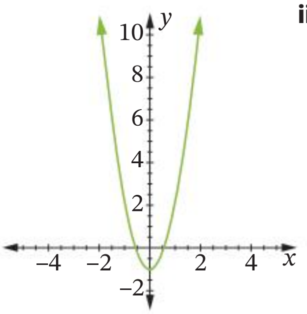
2 Match each equation with its graph. R C

a $x = 4$ **b** $y = -\frac{1}{2}x + 1$ **c** $y = 1 - x^2$ **d** $y = 5$

e $y = 3x^2 - 1$ **f** $y = 3^x$ **g** $x^2 + y^2 = 9$ **h** $y = 3^{-x}$

i $y = 2x^2$ **j** $y = 9x^2 - 4$

i



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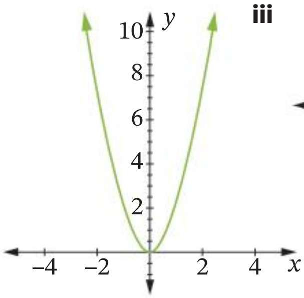

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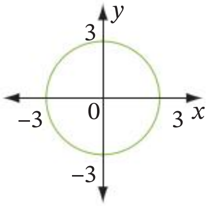
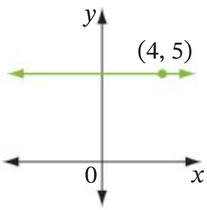


iv



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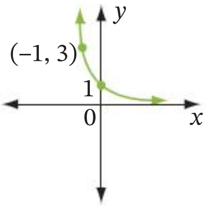

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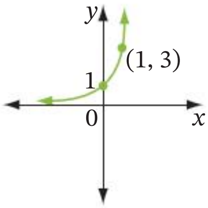

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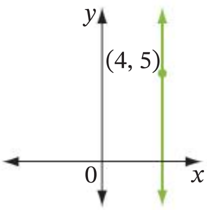
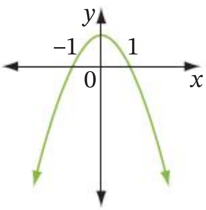


viii



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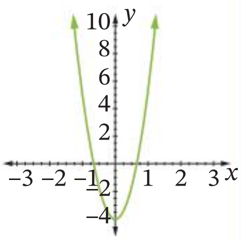

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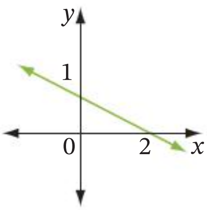


Foundation Standard Complex

STAGE 5.2

3 Sketch each equation, showing a point on the curve.

a $y = x^2 - 3$ **b** $y = 5^x$ **c** $y = -x^2 + 4$ **d** $x^2 + y^2 = 49$

e $y = \frac{1}{2}x^2$ **f** $y = -2x + 4$ **g** $x^2 + y^2 = 144$

4 Find the y-intercept of the graph of each equation.

a $y = 3^x$ **b** $y = 2x^2 + 3$ **c** $y = -7x^2 - 6$ **d** $y = 5^{-x}$

Technology

Identifying graphs

1 Use graphing technology to graph each equation and classify it as either a straight line (L), a parabola (P) or an exponential curve (E).

a $y = 2x$ **b** $y = x^2$ **c** $y = x^2 + 1$

d $y = 2^x$ **e** $y = 2x^2 + 3$ **f** $y = 4 - 2x$

g $y = 3^x$ **h** $y = 2 - x$ **i** $y = 4 - x^2$

j $y = 5^{-x}$

2 Without using graphing technology, classify each equation.

a $y = 3x - 2$ **b** $y = x^2 + 3$ **c** $y = 2^x + 1$

d $y = 3 - x^2$ **e** $y = 4x^2 - 1$ **f** $y = 3^x - 2$

g $y = 4^x - 1$ **h** $y = 3x^2 - 4$ **i** $y = 10 - 2x^2$

j $y = -2x^2$

3 Check your answers to question **2** by drawing each equation using graphing technology.

4 State briefly in words how you distinguish between each type of equation in question **2**.

5 Use graphing technology to find the x-intercepts and y-intercepts (if they exist) of the graphs in questions **1** and **3**. Provide approximate answers where necessary.

Power plus ANSWERS ON P.527

1 On the same set of axes, draw the graph of each equation.

a $y = x^2$ **b** $y = (x + 1)^2$ **c** $y = (x - 2)^2$

2 On the same set of axes, draw the graph of each equation.

a $y = -x^2$ **b** $y = -(x - 3)^2$ **c** $y = -(x + 2)^2$

3 For the graph of the parabola $y = (x + a)^2$, describe the effect on the graph of different values of a.

4 Sketch the graph of each equation and find the centre and radius of the graph.

a $y = \sqrt{16 - x^2}$ **b** $y = \sqrt{25 - x^2}$ **c** $y = -\sqrt{9 - x^2}$

5 Find the centre and radius of the circle with equation:

a $x^2 + y^2 = 5$ **b** $(x - 3)^2 + y^2 = 3$ **c** $(x + 4)^2 + (y - 2)^2 = \frac{1}{4}$

Foundation Standard Complex

9780170453417

CHAPTER 7 REVIEW

Language of maths

asymptote	axis	centre	circle
coefficient	concave down	concave up	constant
conversion graph	curve	direct proportion	exponential
inverse proportion	parabola	quadratic	radius
table of values	turning point	variable	vertex
x-intercept	y-intercept		

1 What is the coefficient of x^2 in the quadratic equation $y = 3x^2 + 10$?

2 What is the graph of a **quadratic equation** called?

3 True or false: The exponential curve $y = 2^x$ passes through the point (0, 0).

4 In the variation equation $y = \frac{k}{x}$, which is the **constant of proportionality**?

5 Write down the equation of a parabola that is concave down and has a y-intercept of 3.

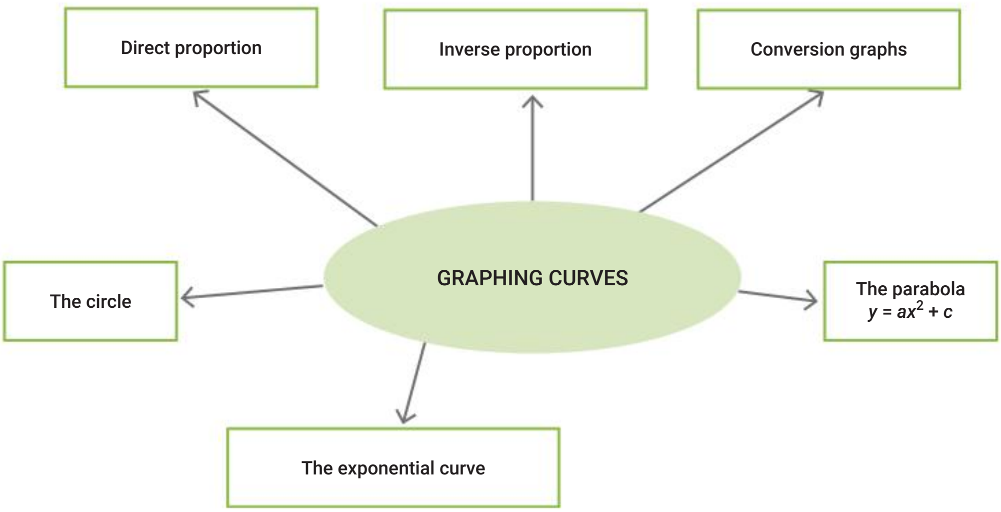
6 What is the asymptote of the exponential curve $y = a^x$?

Topic summary

WS Mind map: Graphing curves

- Which parts of this chapter were new to you?
- What is the difference between direct and inverse proportion?
- Do you know the equations of a parabola, exponential curve and circle, and how to graph them?
- Explain how the graph of $y = 2x^2 + 3$ is different from the graph of $y = -2x^2 + 3$. How are they similar?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



STAGE 5.2

TEST YOURSELF 7 ANSWERS ON P.528

7.01 **1** H is directly proportional to t. If when $t = 12$, $H = 138$, find H when $t = 27$.

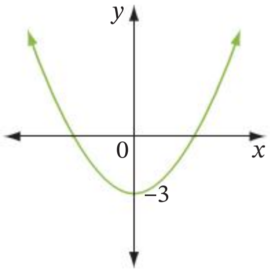
7.02 **2** The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 400 m above sea level, the temperature is 15°C. What is the temperature at 600 m above sea level?

7.03 **3** The graph in Example **6** on page 240 converts Australian dollars to UK pounds sterling. Use the graph to convert:

a \$A70 to £ **b** £56 to Australian dollars

7.04 **4** Match each graph to its correct equation.

a



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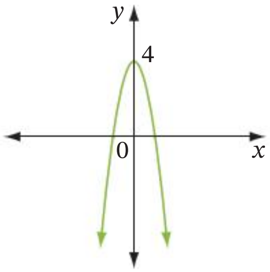

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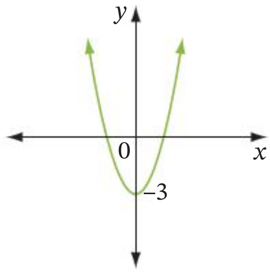

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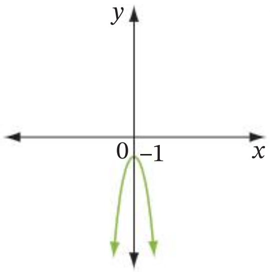

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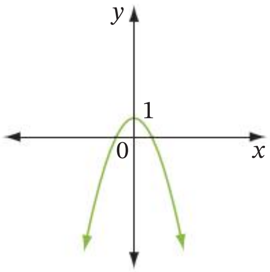

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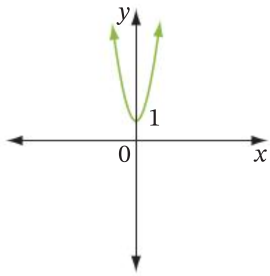

A $y = x^2 - 3$ **B** $y = 3x^2 + 1$ **C** $y = \frac{1}{2}x^2 - 3$

D $y = -x^2 + 1$ **E** $y = -4x^2 - 1$ **F** $y = 4 - 3x^2$

7.05 **5** Sketch each curve described below.

a $y = 4^x$ **b** $y = 4^{-x}$ **c** $y = -4^x$ **d** $y = -4^{-x}$

7.06 **6** Find the centre and radius of each circle described below.

a $x^2 + y^2 = 100$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 49$

7.06 **7** What is the equation of the circle with centre (0, 0) and radius 8 units?

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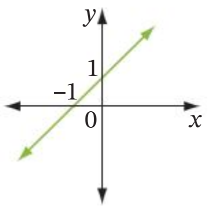
8 Match each equation with its correct graph.

a $y = \frac{1}{4}x^2$	**b** $y = 3^x$	**c** $y = -2x^2 - 1$
d $x = -5$	**e** $y = -3x^2$	**f** $y = 2x^2 - 1$
g $y = 3^{-x}$	**h** $x^2 + y^2 = 25$	**i** $y = x^2$
j $y = x + 1$	**k** $y = -5$	**l** $y = -2 - 2x$

A



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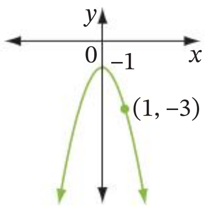

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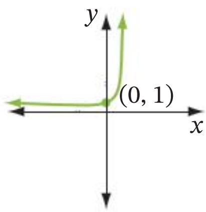

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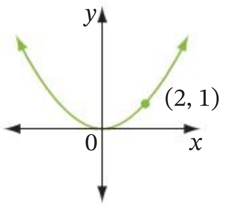

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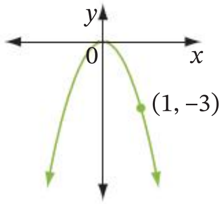

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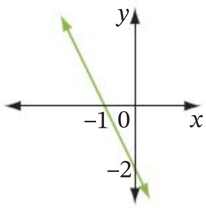

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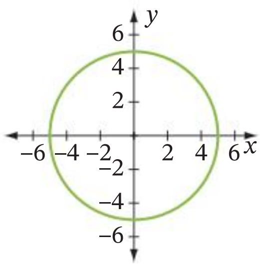

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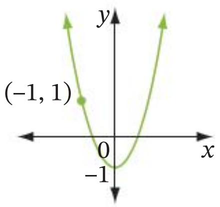

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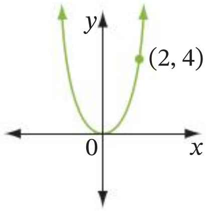
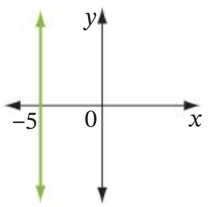


J



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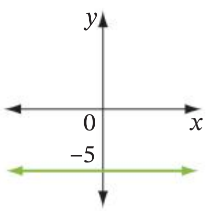

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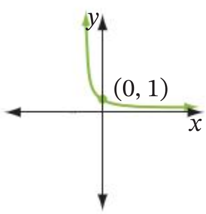

8

MEASUREMENT AND GEOMETRY

TRIGONOMETRY

Trigonometry is a branch of mathematics that uses the relationship between angles and sides of triangles. Trigonometry is used in navigation when locating the position or bearing of a destination from a known location and also finding the distances between places that cannot be physically measured.

Trigonometry helps us to understand physical phenomena that are periodic or cyclic such as tidal movements, phases of the moon, average monthly temperatures, sound waves and is also used in construction, engineering, physics and gaming.

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Chapter outline

	Working mathematically				
8.01 Pythagoras' theorem	U	F	PS	R	
8.02 The trigonometric ratios	U	F		R	C
8.03 Finding an unknown side	U	F	PS		
8.04 Finding an unknown angle	U	F	PS		
8.05 Angles of elevation and depression	U	F	PS		
8.06 Bearings*	U	F		R	C
8.07 Problems involving bearings*	U	F	PS		C

***STAGE 5.2**

Wordbank

adjacent side In a right-angled triangle, the side that is next to a given angle and pointing to the right angle

angle of depression The angle of looking down, measured from the horizontal

angle of elevation The angle of looking up, measured from the horizontal

bearing The angle used to show the direction of one location from a given point

minute (') A unit for measuring angle size, $\frac{1}{60}$ of a degree

opposite side In a right-angled triangle, the side that is facing a given angle and not one of its arms

theta (θ) A letter of the Greek alphabet used as a variable for angles

trigonometric ratio The ratio of 2 sides in a right-angled triangle; for example, sine is the ratio of the opposite side to the hypotenuse

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- use Pythagoras' theorem and trigonometry to solve problems involving right-angled triangles
- find unknown sides and angles in right-angled triangles, where the angle is measured in degrees
- (STAGE 5.2) find unknown sides and angles in right-angled triangles, where the angle is measured in degrees and minutes
- solve trigonometry problems involving angles of elevation and depression
- (STAGE 5.2) solve trigonometry problems involving bearings

SkillCheck ANSWERS ON P. 529

WS Trigonometric calculations

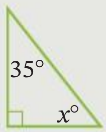
Trigonometry

1 Solve each equation.

a $\frac{x}{5} = 7$ **b** $\frac{h}{4} = 8.3$ **c** $\frac{45}{y} = 9$

2 Find the value of each variable.

a



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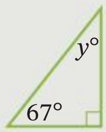

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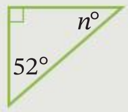

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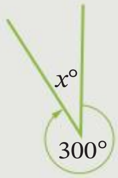

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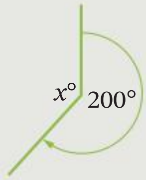
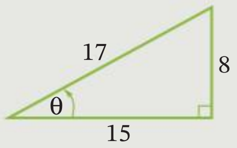
f



3 For each triangle, name the hypotenuse, opposite and adjacent sides for angle θ.

a



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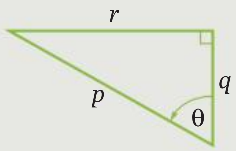

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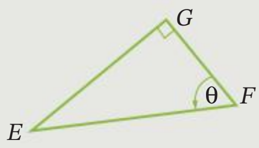


STAGE 5.2

Trigonometry on a calculator

4 Evaluate each expression correct to 4 decimal places.

a cos 32° **b** sin 50.9° **c** 200 tan 18°

d tan 8° 45′ **e** 14 sin 87° 40′ **f** $\frac{13}{\cos 18° 27'}$

5 Round each angle to the nearest degree.

a 64° 27′ **b** 25° 43′ **c** 12° 8′ 50″

6 Round each angle to the nearest minute.

a 50° 19′ 26″ **b** 31° 55′ 55″ **c** 64° 18′ 30″

7 Convert each angle to degrees and minutes, correct to the nearest minute.

a 45.8° **b** 33.175° **c** 5.346°

Pythagoras’ theorem

8.01

Pythagoras’ theorem can be used to find the length of an unknown side in a right-angled triangle, or to prove that a triangle is right-angled.

Pythagoras’ theorem

Pythagoras’ theorem 1

Applications of Pythagoras’ theorem

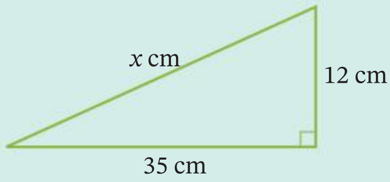
Pythagorean two-step problems

Pythagorean triads

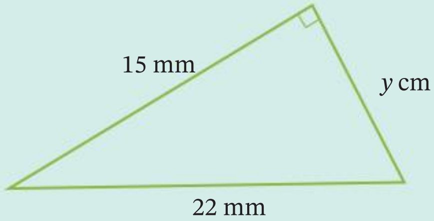
Example 1

Find the value of each variable, correct to one decimal place where necessary.

a



b



Solution

a

$$x^2 = 12^2 + 35^2$$
$$= 1369$$
$$x = \sqrt{1369}$$
$$= 37$$

b

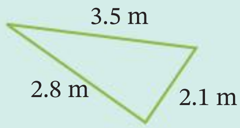
$$22^2 = y^2 + 15^2$$
$$484 = y^2 + 225$$
$$y^2 + 225 = 484$$
$$y^2 = 484 - 225$$
$$= 259$$
$$y = \sqrt{259}$$
$$\approx 16.1$$

Example 2

Testing for right-angled triangles

Testing for right-angled triangles

Test whether this triangle is right-angled.



Solution

$$3.5^2 = 12.25$$
$$2.8^2 + 2.1^2 = 12.25$$
$$\therefore 3.5^2 = 2.8^2 + 2.1^2$$

$\therefore$ The triangle is right-angled (the right angle is opposite the 3.5 m side).

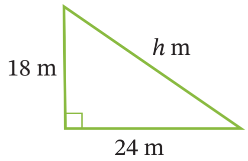
EXERCISE 8.01 ANSWERS ON P. 529

Pythagoras' theorem U F PS R

EXAMPLE 1

1 Find the value of each variable, correct to one decimal place where necessary.

a



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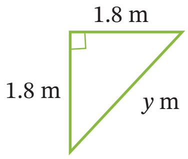

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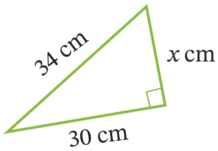

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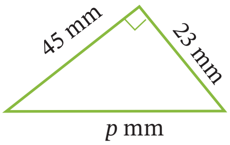

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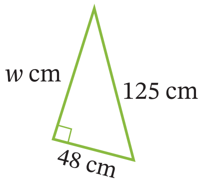

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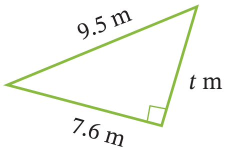
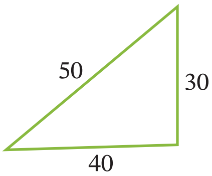


EXAMPLE 2

2 Test whether each triangle is right-angled.

a



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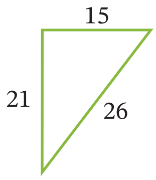

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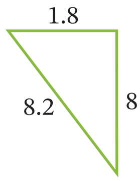

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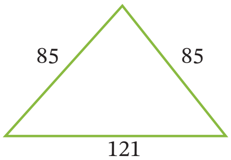

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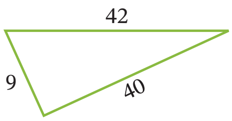

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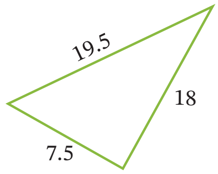
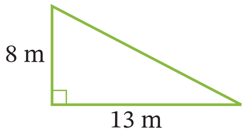


3 Find the perimeter of each shape, correct to one decimal place where necessary.

a



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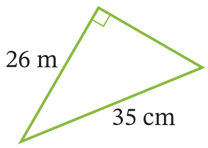

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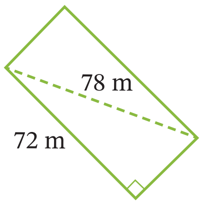

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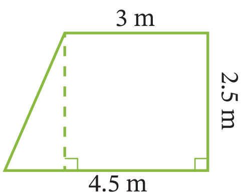

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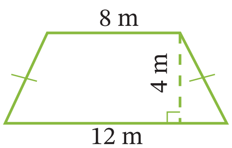

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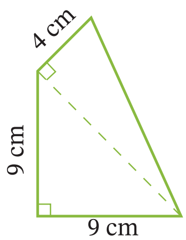

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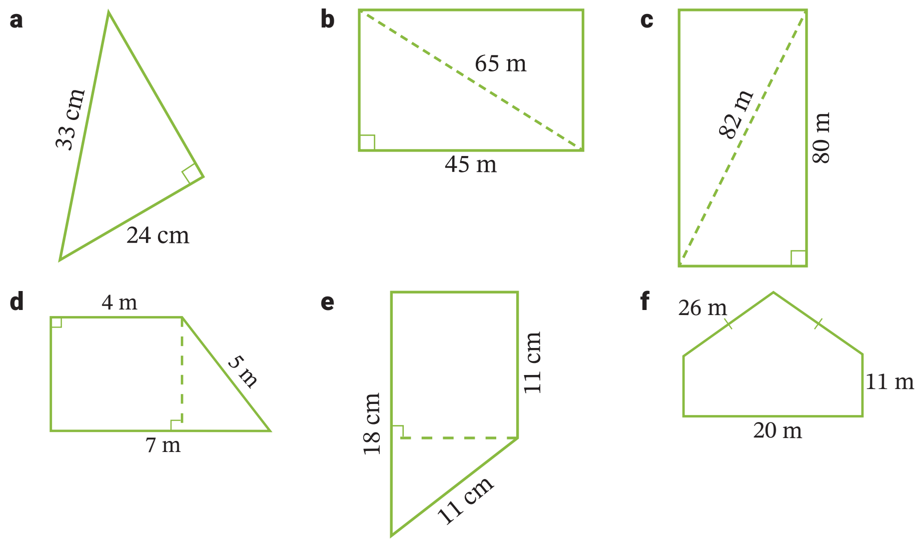
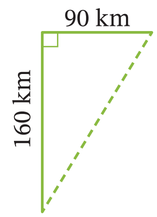
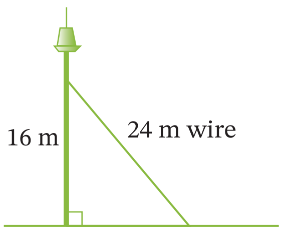
4 Find the area of each shape, correct to one decimal place where necessary.



5 A ship sails 90 km west and then 160 km south. How far is it from its starting point, correct to one decimal place? **PS**



6 A tower is supported by a wire that is 24 m long and attached to the tower at a height of 16 m. How far from the base of the tower will the wire be attached to the ground? Answer correct to the nearest 0.1 m. **PS**

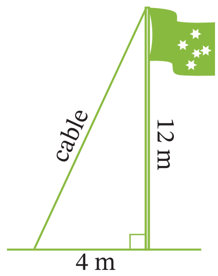


7 A park is in the shape of a rectangle with sides of 96 m and 72 m. Find the shortest distance across the park from one corner to the other. **PS**

8 What is the length of the cable used to stabilise a flagpole that is 12 metres high, if the cable is secured to the ground 4 m from the base of the flagpole? Select the correct answer **A**, **B**, **C** or **D**. **PS**

A 11.3 m   **B** 12.6 m

C 16 m   **D** 80 m



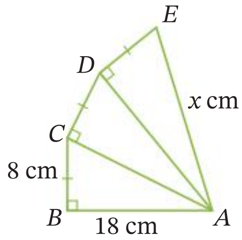
9 A triathlon course consists of 3 legs forming a right-angled triangle. If the longest leg is 11.5 km and the shortest leg is 6.9 km, find the length of the other leg. PS

10 $\triangle ABC$, $\triangle ACD$ and $\triangle ADE$ are right-angled triangles.
Find the value of x, correct to one decimal place.

Select **A**, **B**, **C** or **D**. PS R

A 24.1 **B** 22.7

C 22.4 **D** 24.3



Did you know?

The Greek alpha-bet

Here are 8 letters (in lower-case and capitals) from the Greek alphabet.

α, Α alpha β, Β beta γ, Γ gamma δ, Δ delta

θ, Θ theta φ, Φ phi σ, Σ sigma ω, Ω omega

The ancient Greeks had a great influence on the development of mathematics. It is traditional to use Greek letters as variables, particularly in geometry and trigonometry.

1 **Find out how many letters there are in the Greek alphabet, and name each one.**
2 **Compare the Greek alphabet with our Roman alphabet.**
3 **Can you see where the word *alphabet* comes from? Explain how it originated.**

8.02 The trigonometric ratios

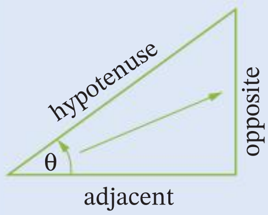
The trigonometric ratios

Trigonometry match-up

There are 3 special fractions called **trigonometric ratios** that relate the lengths of 2 sides of a right-angled triangle: **sine**, **cosine** and **tangent**.

The trigonometric ratios

Ratio	Abbreviation	Meaning
sine	sin	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
cosine	cos	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
tangent	tan	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$



You can remember the 3 ratios using **SOH-CAH-TOA** (pronounced 'so-car-toe-ah'):

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H} \quad \text{SOH}$$

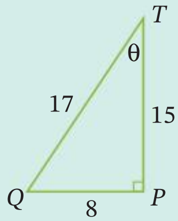
$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H} \quad \text{CAH}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A} \quad \text{TOA}$$

8.02

Example 3

In $\triangle PTQ$, find $\sin \theta$, $\cos \theta$ and $\tan \theta$.



Solution

For angle θ, opposite = 8, adjacent = 15, hypotenuse = 17.

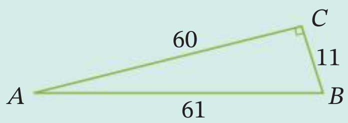
$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$ $\quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$ $\quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$

Example 4

For $\triangle ABC$, find:

a $\sin A$ **b** $\cos A$

c $\tan A$ **d** $\sin B$



Solution

For $\angle A$, opposite = 11, adjacent = 60, hypotenuse = 61

For $\angle B$, opposite = 60, adjacent = 11, hypotenuse = 61

a $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{11}{61}$ **b** $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{60}{61}$

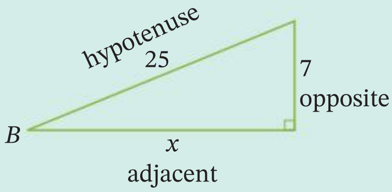
c $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{11}{60}$ **d** $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{60}{61}$

Example 5

If $\sin B = \frac{7}{25}$, find the value of cos *B* and tan *B*.

Solution

$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{25}$, so draw a right-angled triangle that has an angle *B* with opposite side 7 and the hypotenuse 25. Let *x* be the length of the adjacent side.



Find *x* using Pythagoras' theorem.

$25^2 = x^2 + 7^2$

$625 = x^2 + 49$

$x^2 = 625 - 49$

$= 576$

$x = \sqrt{576}$

$= 24$

$\therefore \cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{25}$, $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{24}$

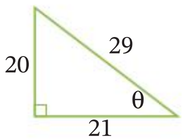
EXERCISE 8.02 ANSWERS ON P. 529

The trigonometric ratios **U F R C**

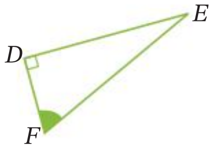
EXAMPLE 3

1 For each marked angle, find the sine, cosine and tangent ratios. **C**

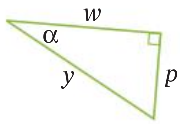
a



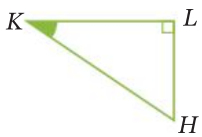
b



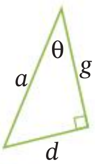
c



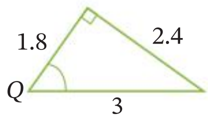
d



e



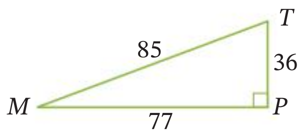
f



EXAMPLE 4

2 For the triangle shown, find:

a cos *T*
b tan *M*
c sin *T*
d sin *M*



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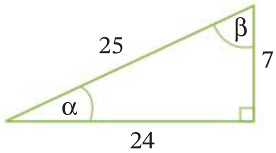
3 Copy and complete each statement below with the correct angle (α or β).



a $\sin\_\_ = \frac{7}{25}$ **b** $\tan\_\_ = \frac{24}{7}$

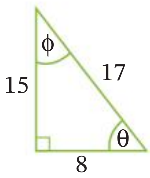
c $\sin\_\_ = \frac{24}{25}$ **d** $\cos\_\_ = \frac{7}{25}$

e $\tan\_\_ = \frac{7}{24}$ **f** $\cos\_\_ = \frac{24}{25}$

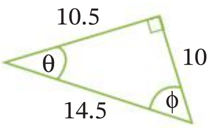
4 For each triangle below, find:

i $\tan\theta$ **ii** $\cos\theta$ **iii** $\cos\phi$ **iv** $\tan\phi$ ϕ = 'phi'

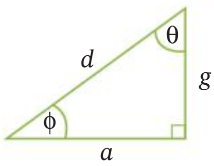
a



b

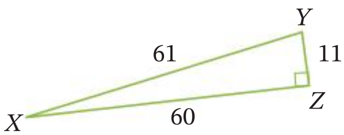


c



5 Which trigonometric ratio equals $\frac{11}{60}$?

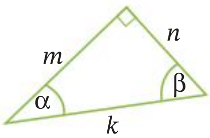
Select the correct answer **A**, **B**, **C** or **D**. C



A $\sin X$ **B** $\tan Y$

C $\cos Y$ **D** $\tan X$

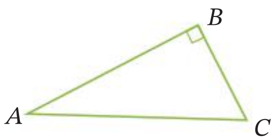
6 Which ratio equals $\frac{m}{k}$? Select **A**, **B**, **C** or **D**. C



A $\cos\beta$ **B** $\cos\alpha$

C $\tan\alpha$ **D** $\tan\beta$

7 Which equation is true for this triangle?
Select **A**, **B**, **C** or **D**. R C



A $\tan A = \cos C$ **B** $\cos A = \sin C$

C $\cos B = \sin A$ **D** $\tan B = \sin A$

8 Sketch a right-angled triangle for each trigonometric ratio, then use Pythagoras' theorem to find the length of the unknown side and the other 2 trigonometric ratios for the same angle. EXAMPLE 5

a $\cos M = \frac{8}{17}$ **b** $\tan Y = \frac{4}{3}$ **c** $\sin C = \frac{84}{85}$

9 If $\tan P = \frac{9}{40}$, find $\sin P$ and $\cos P$. Select **A**, **B**, **C** or **D**.

A $\sin P = \frac{40}{41}, \cos P = \frac{9}{41}$ **B** $\sin P = \frac{9}{41}, \cos P = \frac{9}{40}$

C $\sin P = \frac{9}{41}, \cos P = \frac{40}{41}$ **D** $\sin P = \frac{41}{9}, \cos P = \frac{41}{40}$

8.02

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8.03 Finding an unknown side

Finding an unknown side

Finding the hypotenuse

Trigonometry

Trigonometry equations 1

We can use a trigonometric ratio to calculate the length of an unknown side in a right-angled triangle if one other side and angle are known. We need to select the correct ratio that links the given angle to the unknown side and known side.

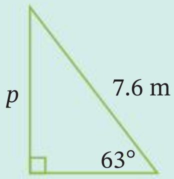
Finding an unknown side in a right-angled triangle

1 Identify the 2 labelled sides and decide whether to use sin, cos or tan
2 Write an equation using the ratio, the given angle and the variable
3 Solve the equation to find the value of the variable

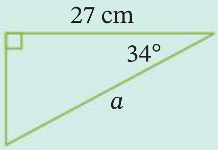
Example 6

Find the value of each variable, correct to one decimal place.

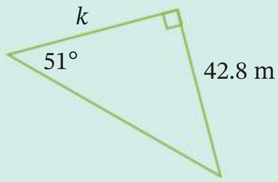
a



b



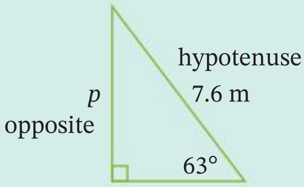
c



Solution

a SOH, CAH or TOA?

The marked sides are the opposite (O) side and the hypotenuse (H), so use sin.



$$\sin 63° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{p}{7.6}$$

$$\sin 63° \times 7.6 = \frac{p}{7.6} \times 7.6$$ Multiplying both sides by 7.6.
$$7.6 \sin 63° = p$$

$$p = 7.6 \sin 63°$$ Swapping sides.
$$= 6.7716...$$
$$\approx 6.8$$

From the diagram, a length of 6.8 m looks reasonable.

b 27 cm is adjacent, a is hypotenuse, so use cos.

$\cos 34° = \frac{27}{a}$ Note that the variable a appears in the denominator of the equation

$\cos 34° \times a = \frac{27}{a} \times a$ Multiplying both sides by a

$a \cos 34° = 27$

$\frac{a \cos 34°}{\cos 34°} = \frac{27}{\cos 34°}$ Dividing both sides by cos 34°

$a = \frac{27}{\cos 34°}$

$= 32.5678...$

≈ 32.6 cm

Note that when the unknown appears in the denominator of an equation, it can swap positions with the trigonometric ratio, so that $\cos 34° = \frac{a}{27}$ becomes $a = \frac{27}{\cos 34°}$.

c 42.8 m is opposite, k is adjacent, so use tan.

$\tan 51° = \frac{42.8}{k}$ k appears in the denominator

$k = \frac{42.8}{\tan 51°}$ Swapping the position of k with tan 51°

$= 34.6587...$

≈ 34.7 m

Alternative method

To avoid having k in the denominator, we could use tan with the third angle of the triangle.

3rd angle $= 180° - 90° - 51°$

$= 39°$

$\tan 39° = \frac{k}{42.8}$

$k = 42.8 \tan 39°$

$= 34.6587...$

≈ 34.7 m

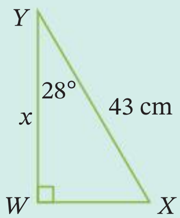
Example 7

$\triangle WXY$ is right-angled at W, $XY = 43$ cm and $\angle Y = 28°$. Find the length of WY, correct to the nearest centimetre.

Solution

Draw a diagram. Let the length of WY be x.

x is adjacent, 43 cm is hypotenuse, so use cos.



$\cos 28° = \frac{x}{43}$

$x = 43 \cos 28°$

$= 37.9667...$

$\therefore WY \approx 38$ cm

From the diagram, a length of 38 cm looks reasonable.

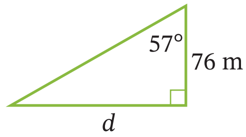
EXERCISE 8.03 ANSWERS ON P. 529

Finding an unknown side **U F**

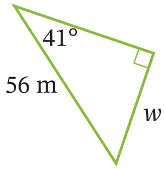
EXAMPLE 6

1 Find the value of the variable in each triangle, correct to one decimal place.

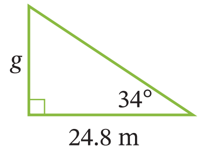
a



b



c



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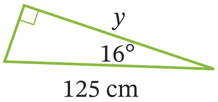
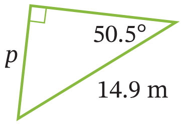


e



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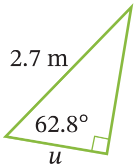
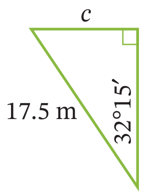


STAGE 5.2

g



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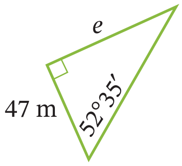
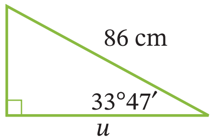
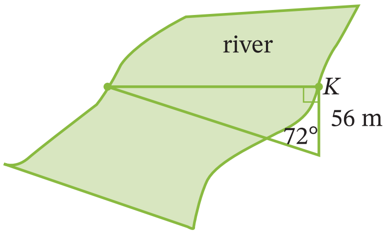


i

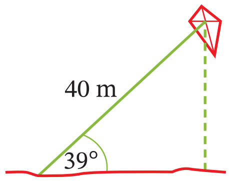


2 Find each length or distance, correct to one decimal place. **PS**

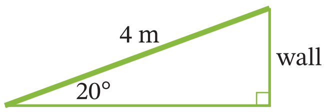
a The width of the river at the point *K*



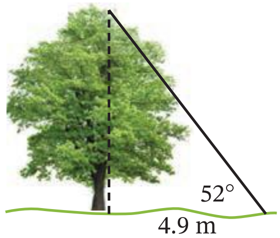
b The height of the kite



c The distance the base of the ramp is from the wall.



d The height of the tree



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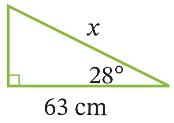
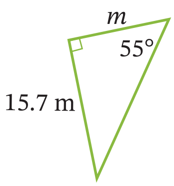
3 Find the value of each variable, correct to one decimal place.

a


b


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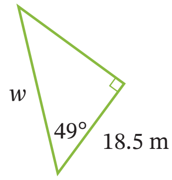
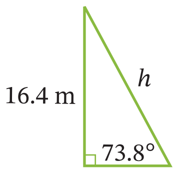
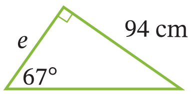


d


e


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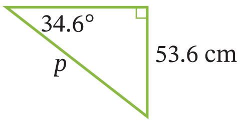

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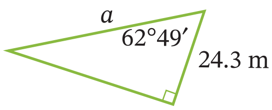

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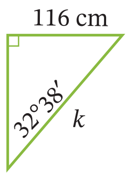

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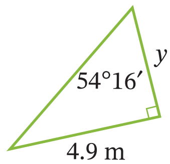
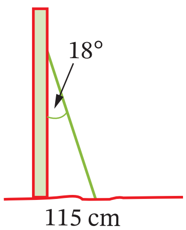
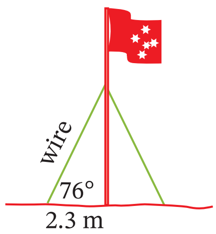


4 Find each length or distance, correct to one decimal place. PS

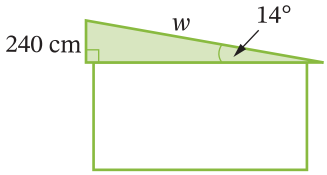
a The distance the ladder reaches up the wall.



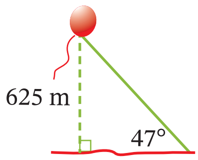
b The length of the support wire.



c The length of the sloping roof.



d The distance a balloon has drifted from its starting point.



5 A rectangular gate has a diagonal brace that makes an angle of 35° with the top of the gate. The height of the gate is 1200 mm. Calculate the length of the diagonal brace. Select the correct answer **A**, **B**, **C** or **D**. PS

A 1465 mm **B** 1714 mm **C** 2092 mm **D** 2134 mm

6 A ladder 4.3 m long is leaning against a wall. If the angle between the ladder and the wall is 28°, how far up the wall does the ladder reach? PS

8.03

STAGE 5.2

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7 The entrance to a bank has a ramp for wheelchair access that is 3.6 m long. If the ramp is inclined at an angle of 10° to the ground, what is the height of the entrance (to the nearest cm)? PS

8 A glider is descending at an angle of 25° to the horizontal. The length of its flight path until it lands is 3.5 km. What was the altitude (to the nearest 0.1 km) of the glider? PS

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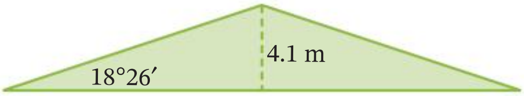
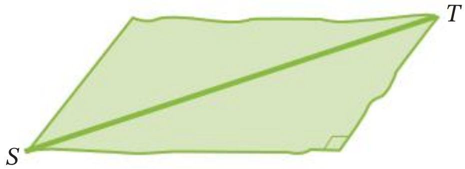
STAGE 5.2

9 A roof has a pitch of 18° 26′. The height of the roof is 4.1 m. Calculate the overall width of the roof (correct to one decimal place). PS



10 The diagonal path of a large rectangular reserve makes an angle of 32° with the longer side of the reserve. The width of the reserve is 230 m. PS



a Calculate the length of the path (correct to the nearest metre).

b How much shorter is walking along the path from S to T than walking along the 2 edges of the reserve?

EXAMPLE 7

11 $\triangle PQR$ is right-angled at Q, $PR = 15.3$ m and $\angle P = 23°$. Find the length of QR, correct to 2 decimal places.

12 $\triangle HKW$ is right-angled at K, $HK = 55$ cm and $\angle W = 67°$. Find the length of HW, correct to one decimal place.

13 In $\triangle BDC$, $\angle C = 90°$, $BC = 8.23$ m and $\angle B = 38°$. Find the length of CD, correct to 2 decimal places.

14 $\triangle TPM$ is right-angled at M, $MT = 39.3$ cm and $\angle T = 44.7°$. Find the length of PT, correct to one decimal place.

STAGE 5.2

15 In $\triangle WYX$, $\angle Y = 90°$, $\angle W = 74° \, 25'$ and $XY = 245$ mm. Find the length of WY, correct to the nearest millimetre.

16 $\triangle BFT$ is right-angled at T, $BF = 985$ mm and $\angle B = 55° \, 11'$. Find the length of FT, correct to the nearest millimetre.

17 $\triangle ABC$ is right-angled at A, $AC = 24.6$ m and $\angle C = 84° \, 56'$. Find the length of BC, correct to one decimal place.

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Finding an unknown angle

8.04

A scientific calculator can be used to evaluate a trigonometric ratio such as sin 38°, but it can also be used to find an unknown angle, θ, if the trigonometric ratio of the angle is known, for example, if sin θ = 0.9063.

An unknown angle can be found using the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ keys on the calculator. These are called the **inverse sin**, **inverse cos** and **inverse tan** functions, found by pressing the SHIFT or 2nd F key before the sin, cos or tan key.

Finding an unknown angle

Trigonometry

Trigonometry problems

Trigonometry squaresaw

Trigonometry: Finding angles

Example 8

a If $\sin\alpha = \frac{5}{8}$, find the value of angle α, correct to the nearest degree.

b If tan X = 2.1532, find the value of angle X, correct to the nearest minute.

Solution

a $\sin\alpha = \frac{5}{8}$

$\alpha = 38.6821...°$ On a calculator: SHIFT sin 5 ⊟ 8 =

$\approx 39°$ Note: ⊟ or $a^{b}/_{c}$ is the fraction key.

b tan X = 2.1572 SHIFT tan 2.1572 =

$X = 65.1292...$

$= 65° \, 7' \, 45.35''$ ° ′ ″ or DMS

$\approx 65° \, 8'$

STAGE 5.2

Finding an unknown angle in a right-angled triangle

1 Identify 2 known sides and decide whether to use the sin, cos or tan ratio.
2 Write an equation using the ratio, the angle and the 2 sides as a fraction.
3 Use the calculator's inverse trigonometric function to find the size of the angle.

Trigonometry equations 2

Trigonometry

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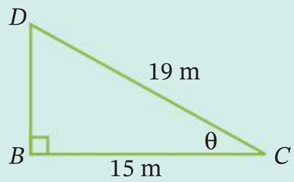
Example 9

Find the size of angle θ, correct to the nearest degree.

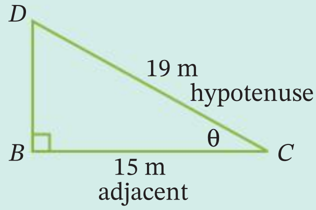


Solution

SOH, CAH or TOA?

The known sides are the adjacent (A) side and the hypotenuse (H), so use cos.

$\cos\theta = \frac{15}{19}$



$\theta = 37.8636...$

$\approx 38°$

On a calculator: SHIFT cos 15 [fraction] =

From the diagram, an angle size of 38° looks reasonable.

Example 10

$\triangle WXY$ is right-angled at X, with $XY = 72$ cm and $WX = 43$ cm. Find $\angle W$, correct to the nearest degree.

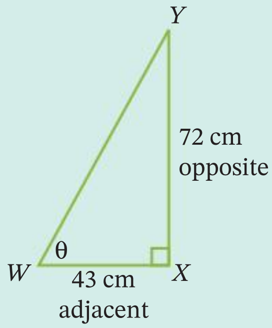
Solution

Draw a diagram.

SOH, CAH or TOA?

The known sides are the opposite (O) and adjacent (A), so use tan.

$\tan\theta = \frac{72}{43}$



$\theta = 59.1534...°$

$\approx 59°$

On a calculator: SHIFT tan 72 [fraction] 43 =

From the diagram, an angle size of 37° looks reasonable.

EXERCISE 8.04 ANSWERS ON P. 530

Finding an unknown angle **U F PS**

EXAMPLE 8

1 Find the size of angle θ, correct to the nearest degree.

a $\tan\theta = 1.85$ **b** $\sin\theta = 0.6432$ **c** $\cos\theta = \frac{\sqrt{3}}{2}$ **d** $\tan\theta = 7.1$

e $\sin\theta = \frac{3}{4}$ **f** $\cos\theta = \frac{2}{17}$ **g** $\tan\theta = \frac{1}{\sqrt{3}}$ **h** $\sin\theta = \frac{1}{\sqrt{2}}$

STAGE 5.2

8.04

2 Find the size of angle A, correct to the nearest minute.

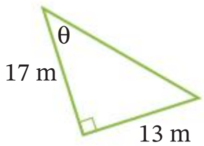
a $\cos A = \frac{15}{19}$ **b** $\tan A = 0.85$ **c** $\cos A = \frac{7}{12}$ **d** $\sin A = 0.9514$

e $\tan A = \frac{39}{40}$ **f** $\sin A = \frac{3}{5}$ **g** $\cos A = 0.5962$ **h** $\tan A = 4.406$

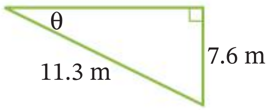
EXAMPLE 9

3 Find θ, correct to the nearest degree.

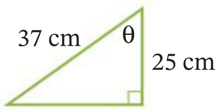
a



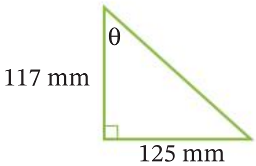
b



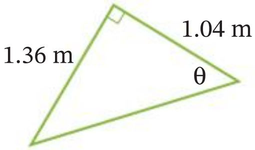
c



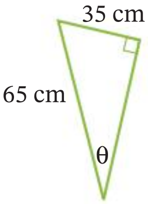
d



e



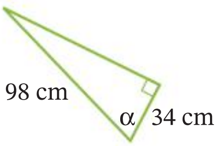
f



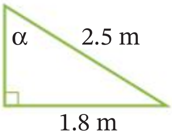
STAGE 5.2

4 Find the size of angle α, correct to the nearest minute.

a



b



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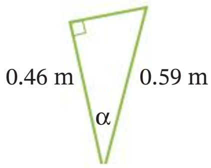
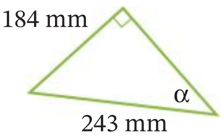
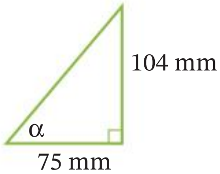


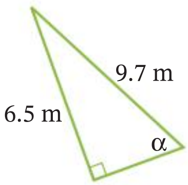
d



e



f



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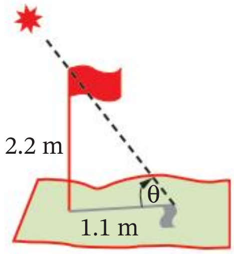
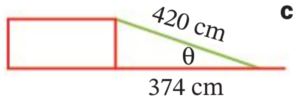
5 Find the size of angle θ, correct to the nearest degree.

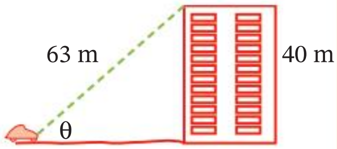
a



b



c



For questions **6** to **10**, give your answers correct to the nearest degree.

6 A ladder 6.8 m long is placed against a wall. If the foot of the ladder is 2.1 m from the base of the wall, find the angle at which the ladder is inclined to the ground. **PS**

7 A walking trail falls 7 m for every 130 m travelled along the trail. At what angle is the trail inclined to the horizontal? **PS**

8 A tree that is 15 m tall casts a shadow that is 19.6 m long. Find the angle of the sun from the ground. **PS**

9 When sand is poured from a hopper at a steady rate it forms a conical pile. The height of the pile is 3 m and its diameter is 8.6 m. Calculate the angle of repose of the sand (the angle the sloping side makes with the horizontal base) **PS**

10 Calculate the take-off angle of a passenger plane if its altitude after 30 seconds is 600 m and it has flown a distance of 2500 m. **PS**

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11 A stretch of road rises 220 m for every 3 km travelled along the road. Find the angle, correct to the one decimal place, at which the road is inclined to the horizontal. Select the correct answer **A**, **B**, **C** or **D**. **PS**

A 4.12° **B** 47° **C** 4.2° **D** 4.7°

EXAMPLE 10

12 $\triangle KLM$ is right-angled at L, $KL = 19$ cm and $LM = 21$ cm. Find $\angle K$, correct to the nearest degree.

13 In $\triangle PTW$, $\angle T = 90°$, $PW = 22.3$ m and $TW = 7.6$ m. Find $\angle W$, correct to the nearest degree.

STAGE 5.2

14 $\triangle BDH$ is right-angled at D, $BH = 2.75$ m and $BD = 1.80$ m. Find $\angle H$, correct to the nearest minute.

15 $\triangle DEG$ is right-angled at E, $DE = 15$ cm and $DG = 48$ cm. Find $\angle G$, correct to the nearest minute.

16 In $\triangle ABC$, $\angle B = 90°$, $BC = 0.8$ m and $AC = 1.3$ m. Find $\angle C$, correct to the nearest minute.

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Mental skills 8: Maths without calculators ANSWERS ON P. 530

Divisibility tests

How can you tell if a number is divisible by 2? Look at its last digit. If that digit is 2, 4, 6, 8 or 0, then the number is divisible by 2 (that is, it is even).

How can you tell if a number is divisible by 5? If its last digit is 0 or 5, then the number is divisible by 5.

These are examples of **divisibility tests**—rules for checking whether or not a number is divisible by a certain number. The table below shows some common divisibility tests.

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 2, 3 or 4.

- Last digit is 8 (even), ∴ 748 is divisible by 2
- Sum of digits = 7 + 4 + 8 = 19, which is not divisible by 3,
  ∴ 748 is not divisible by 3
- 48 is divisible by 4, ∴ 748 is divisible by 4 (748 ÷ 4 = 187)

b Test whether 261 is divisible by 5 or 9.

- Last digit is 1, not 0 or 5, ∴ 261 is not divisible by 5
- 2 + 6 + 1 = 9, which is divisible by 9, ∴ 261 is divisible by 9. (261 ÷ 9 = 29).

c Test whether 570 is divisible by 4, 6 or 10.

- 70 is not divisible by 4, ∴ 570 is not divisible by 4
- 570 is even and 5 + 7 + 0 = 12, which is divisible by 3,
  ∴ 570 is divisible by 6 (570 ÷ 6 = 95)
- Last digit is 0, ∴ 570 is divisible by 10 (570 ÷ 10 = 57)

2 Test whether each number is divisible by 2, 3, 5 or 6.

a 250 **b** 189 **c** 78 **d** 465
e 1024 **f** 840 **g** 715 **h** 627

3 Test whether each number is divisible by 4, 9 or 10.

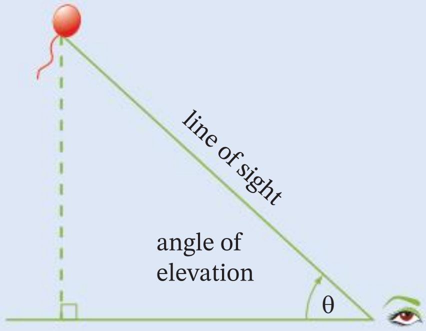
a 144 **b** 280 **c** 522 **d** 4170
e 936 **f** 726 **g** 342 **h** 5580

8.05 Angles of elevation and depression

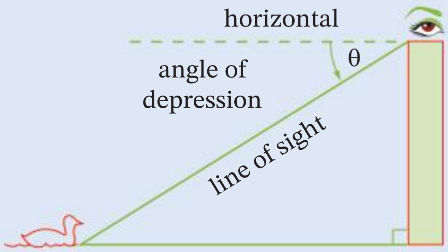
Trigonometry 2

Angles of elevation and depression

The **angle of elevation** is the angle of looking **up**, measured from the horizontal.



The **angle of depression** is the angle of looking **down**, measured from the horizontal.



Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a protractor with a sighting tube attached.

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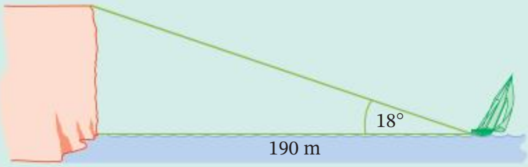
Angles of elevation and depression

Example 11

The angle of elevation from a yacht to the top of a cliff is 18°. If the yacht is 190 m from the base of the cliff, find correct to one decimal place the height of the cliff.



Solution

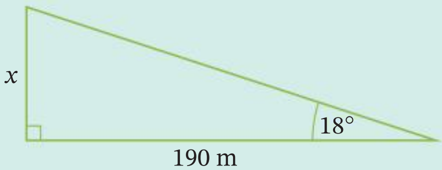
Let the height be x metres.

$\tan 18° = \frac{x}{190}$

$x = 190 \tan 18°$

$= 61.73474\ldots$

≈ 61.7 m



The height of the cliff is 61.7 m.

Angles of elevation and depression

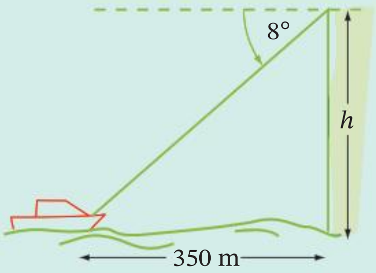
Angle of depression 1

Angle of depression 2

8.05

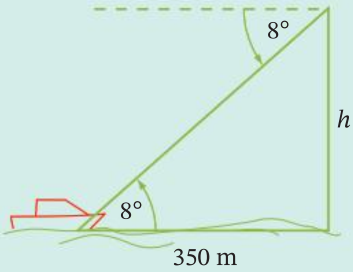
Example 12

The angle of depression of a boat from the top of a cliff is 8°. If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.



Solution

By alternate angles on parallel lines, the angle of elevation of the top of the cliff from the boat is also 8°.



$$\tan 8° = \frac{h}{350}$$

$$h = 350 \tan 8°$$

$$= 49.1892...$$

$$\approx 49$$

The height of the cliff is 49 m.

Alternative method

The third angle in the triangle (adjacent to the angle of depression) = 90° − 8° = 82°.

$$\tan 82° = \frac{350}{h}$$

$$h = \frac{350}{\tan 82°}$$

$$= 49.1892...$$

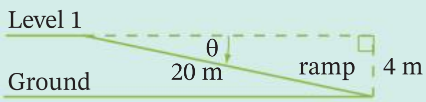
$$\approx 49$$

The height of the cliff is 49 m.

Example 13

The ramp from one level to the next in a car park is 20 m long and drops 4 m.

Find the angle of depression of the ramp, to the nearest degree.



Solution

$$\sin \theta = \frac{4}{20}$$

$$\theta = 11.5369...°$$

$$\approx 12°$$

The angle of depression of the ramp is 12°.

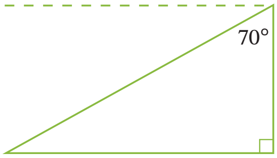
EXERCISE 8.05 ANSWERS ON P. 530

ANSWERS ON P. 530

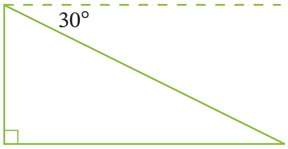
Angles of elevation and depression **U F PS**

1 Copy each diagram, mark the angle of elevation θ and find its size.

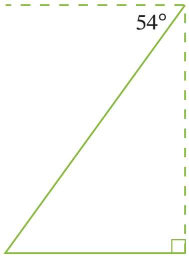
a



b

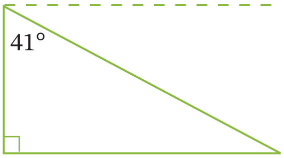


c

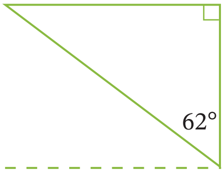


2 Copy each diagram, mark the angle of depression θ and find its size.

a



b



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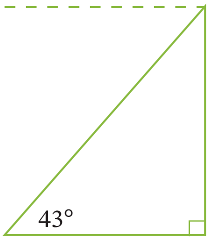
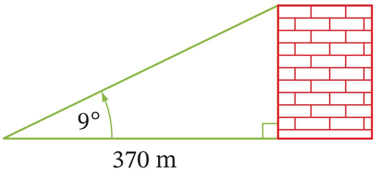
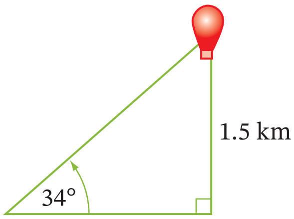


EXAMPLE 11

3 Nora stands 370 m from the base of a building. Using a clinometer, she finds that the angle of elevation of the top is 9°. Find the height of the building, correct to the nearest metre.

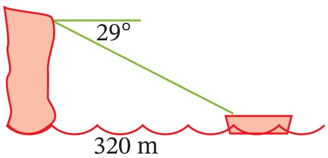


4 The angle of elevation of a weather balloon at a height of 1.5 km is 34°. How far (to the nearest metre) is the observer from being directly under the balloon?



EXAMPLE 12

5 A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29°. Find the height of the cliff, correct to the nearest metre. PS



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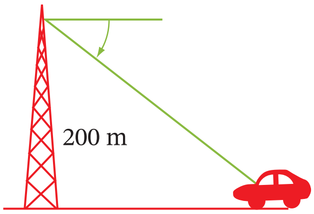
6 From the top of a 200 m tower, the angle of depression of a car is 48°. How far is the car from the foot of the tower? PS



8.05

EXAMPLE 13

7 A 275 m radio mast is 1.7 km from a school. Find, correct to the nearest degree, the angle of elevation of the top of the mast from the school. PS

8 A monument 24 m high casts a shadow 20 m long. Calculate, correct to the nearest degree, the angle of elevation of the Sun at this time of day. PS

STAGE 5.2

9 In a concert hall, Bill is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage? Answer correct to the nearest minute. PS

10 A plane is 340 m directly above one end of a 1000 km runway. Find, correct to the nearest minute, the angle of depression to the far end of the runway. PS

11 An observer 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43°. How high is the flagpole, to the nearest centimetre? PS

12 A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place. PS

13 A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15°. How far (to the nearest metre) has the fire moved between the 2 observations? PS

Shutterstock.com/Ryan Fletcher

14 The angle of elevation to the bottom of a transmission tower on a hill from an observer 1.8 km away from the base of the hill is 5°. The angle of elevation to the top of the tower from the observer is 6.8°. Find the height of the tower, correct to the nearest metre. PS

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8.06 Bearings

STAGE 5.2

WS NSW map bearings

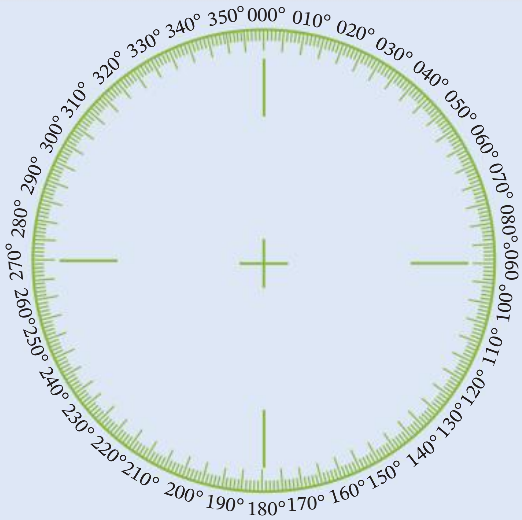
WS 16 points of the compass

WS Bearings match-up

Bearings are used in navigation. A **bearing** is an angle measurement used to precisely describe the direction of one location from a given reference point.

Three-figure bearings

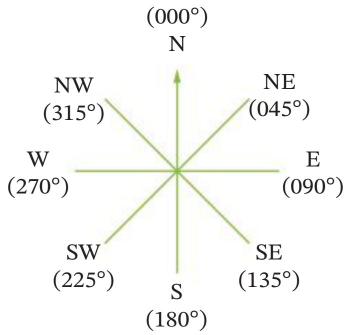
Three-figure bearings, also called **true bearings**, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°**. Note that the angles are always written with 3 digits.



True bearings

This **compass rose** shows the three-figure bearings of 8 points on the compass. A bearing of due east is 090°, while a compass direction of southwest (SW) is 225°.

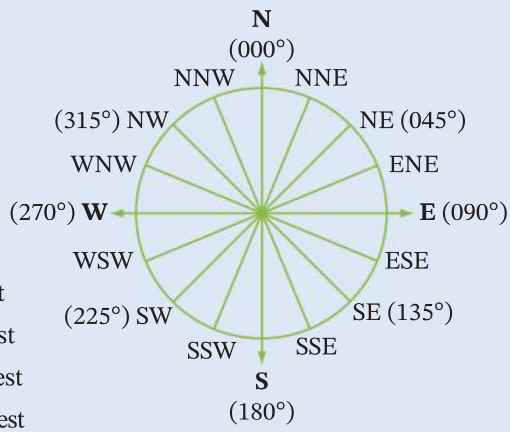
- Bearings from 000° to 090° are in the NE quadrant
- Bearings from 090° to 180° are in the SE quadrant
- Bearings from 180° to 270° are in the SW quadrant
- Bearings from 270° to 360° are in the NW quadrant



Compass bearings

Compass bearings refer to the 16 points of a mariner's compass.

N = north	NE = northeast
E = east	SE = southeast
S = south	SW = southwest
W = west	NW = northwest
NNE = north-northeast	ENE = east-northeast
ESE = east-southeast	SSE = south-southeast
SSW = south-southwest	WSW = west-southwest
WNW = west-northwest	NNW = north-northwest



STAGE 5.2

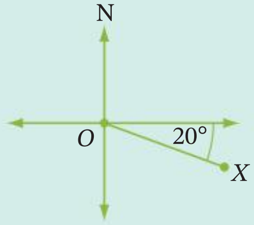
8.06

Example 14

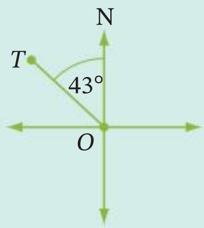
Three-figure bearings

Write the three-figure bearing of each point from O.

a



b



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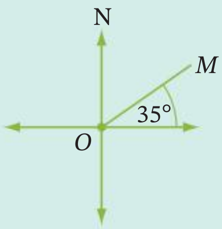


Solution

a Bearing of X from O is $90° + 20° = 110°$.

b Bearing of T from O is $360° - 43° = 317°$.

c Bearing of M from O is $90° - 35° = 055°$. Must be written as a 3-digit angle

Example 15

Three-figure bearings

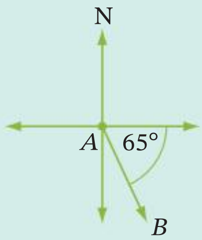
Sketch point B on a compass rose if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is measured from.

155° is between 90° and 180°, so B is in the southeast (SE) quadrant.

$155° - 90° = 65°$, so B is 65° from east (E).



Example 16

Three-figure bearings

The bearing of Y from X is 130°. What is the bearing of X from Y?

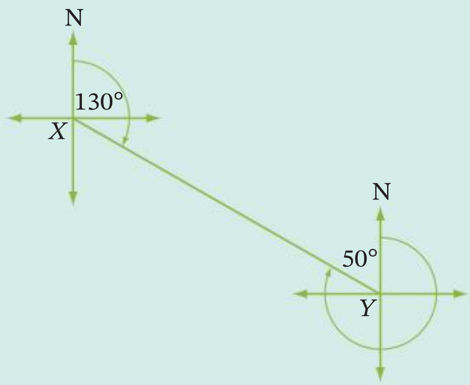
Solution

Sketch the bearing of Y from X.

On the same diagram, draw a compass rose at Y and find $\angle NYX$.

$\angle NYX = 50°$ (co-interior angles, $NX \parallel NY$)

$\therefore$ Bearing of X from $Y = 360° - 50°$
$= 310°$.



STAGE 5.2

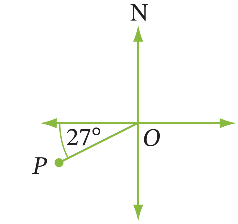
EXERCISE 8.06 ANSWERS ON P. 530

Bearings U F R C

EXAMPLE 14

1 Write the three-figure bearing of each point from O. C

a



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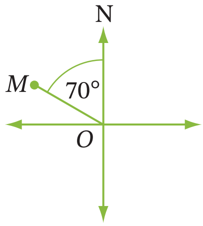

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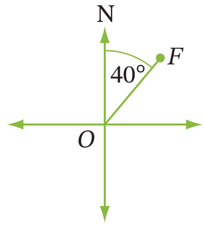

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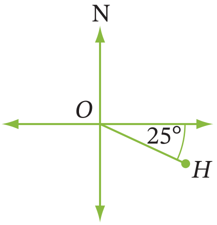

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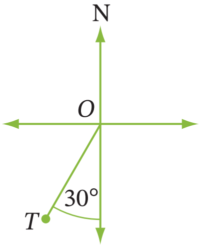

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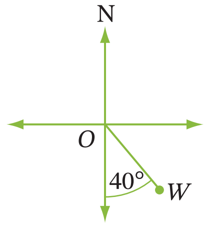

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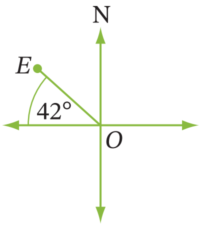

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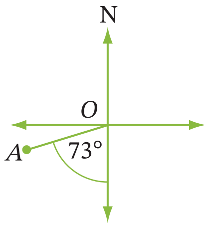
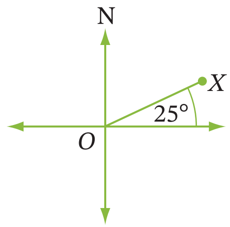
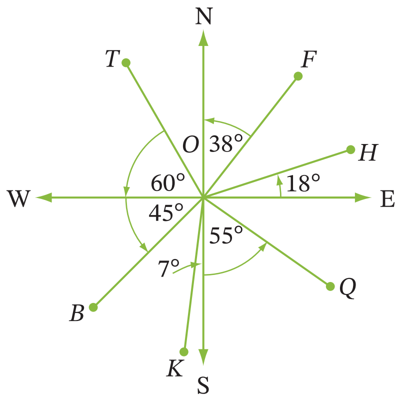


i



2 What is the bearing of each point from O? C

a N
b E
c S
d W
e F
f Q
g T
h B
i H
j K



3 What is the compass direction shown by point B in question **2**? C

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STAGE 5.2

8.06

4 What is the three-figure bearing of each compass bearing? C

a SW **b** E **c** NE **d** ESE

e WNW **f** W **g** SE **h** WSW

5 Sketch each bearing on a compass rose.

a 220° **b** 060° **c** 260° **d** 125°

e 350° **f** 267° **g** 171° **h** 032°

6 **a** What is the compass direction halfway between northwest and north?

b What is the three-figure bearing of this compass direction? C

7 Sketch *P* on a compass rose if *P* has a bearing of:

EXAMPLE 15

a 132° from *T* **b** 260° from *M* **c** 335° from *X* **d** 010° from *K*

8 If the bearing of *P* from *A* is 060°, what is the bearing of *A* from *P*? R

EXAMPLE 16

9 The bearing of *T* from *Y* is 100°. What is the bearing of *Y* from *T*? R

10 What is the (smallest) angle between:

a S and SW? **b** NE and SE? **c** E and NW?

d NE and SSW? **e** E and SSW? **f** SW and WNW?

11 The compass bearing of *H* from *M* is WNW. Find the compass bearing of *M* from *H*. R C

12 Draw a diagram for each situation described. C

a A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080°.

b A cyclist travels 15 km due east and then 20 km on a SW bearing.

13 For this diagram, find the bearing of: C

a *Y* from *Z*

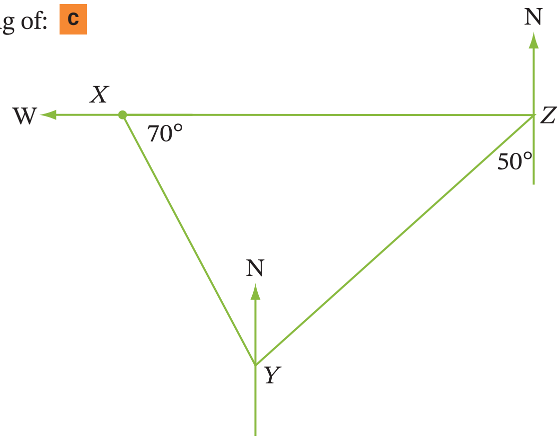
b *X* from *Z*

c *Y* from *X*

d *X* from *Y*

e *Z* from *Y*

f *Z* from *X*



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STAGE 5.2

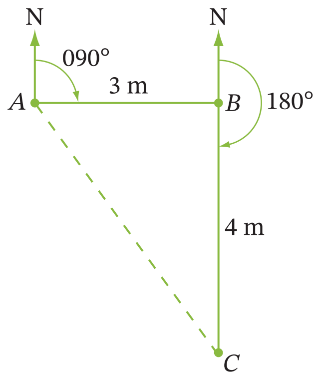
Investigation

Compass walks

You need: a directional compass and a tape measure or trundle wheel.

This activity can also be done in the classroom using scale drawings, graph paper, a ruler and a protractor.

A triangular walk



1 Starting at *A*, walk due east for 3 m to *B*.

2 From *B*, walk due south for 4 m to *C*.

3 How far is *C* from *A*?

4 What is the bearing of:

a *A* from *C*? b *C* from *A*?

A square walk

1 Starting at *P*, walk a bearing of 045° for 8 m to *Q*.

2 From *Q*, walk a bearing of 315° for 8 m to *R*.

3 From *R*, walk a bearing of 225° for 8 m to *S*.

4 How far is *S* from *P*?

5 What is the bearing of:

a *P* from *S*? b *S* from *P*?

A pentagonal walk

1 Starting at *U*, walk a bearing of 130° for 4 m to *V*.

2 From *V*, walk a bearing of 40° for 7 m to *W*.

3 From *W*, walk a bearing of 320° for 4.8 m to *X*.

4 From *X*, walk a bearing of 270° for 4.5 m to *Y*.

5 How far is *Y* from *U*?

6 What is the bearing of:

a *U* from *Y*? b *Y* from *U*?

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Problems involving bearings

8.07

STAGE 5.2

Bearings

Trigonometry 2

True bearings

Example 17

A plane leaves a town and remains on a bearing of 122° for 260 km.

a How far south of the town is the plane, correct to one decimal place?

b What is the bearing of the town from the plane?

Solution

a Let d km = distance south

$\angle SOP = 360° - 122°$ (angles on a straight line)

$= 58°$

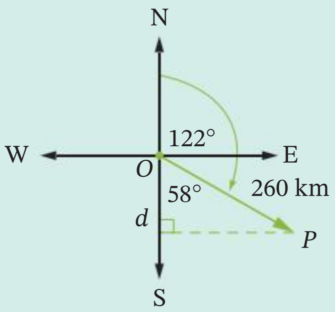
$\cos 58° = \frac{d}{260}$

$d = 260 \cos 58°$

$= 137.7790 \ldots$

≈ 137.8 km

The plane is 137.8 km south of the town.



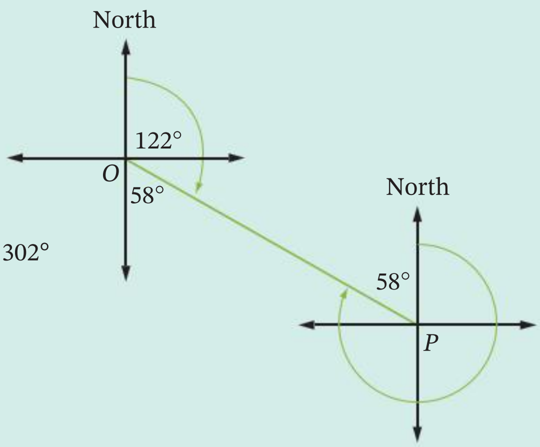
b Draw a compass rose with its centre at P.

$\angle OPN = 58°$ (alternate angles on parallel lines)

Bearing of O from $P = 360° - 58°$

$= 302°$

Bearing of the town from the plane = 302°

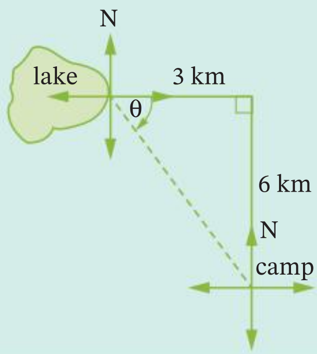


Example 18

From camp, Sumaiyaa walks due north for 6 km, then 3 km due west to a lake.

a How far is Sumaiyaa from the camp?

b What is the bearing of the camp from the lake (to the nearest minute)?



Solution

a Let x = distance from camp.

$x^2 = 6^2 + 3^2$ by Pythagoras' theorem

$= 45$

$x = \sqrt{45}$

$= 6.708...$

≈ 6.7 km

Sumaiyaa is 6.7 km from the camp.

b Note angle θ in the diagram.

$\tan\theta = \frac{6}{3}$

$\theta = 63.434...°$

$= 63° 26' 5.82''$

$\approx 63° 26'$

Bearing of camp from lake $= 90° + 63° 26'$

$= 153° 26'$

EXERCISE 8.07 ANSWERS ON P. 532

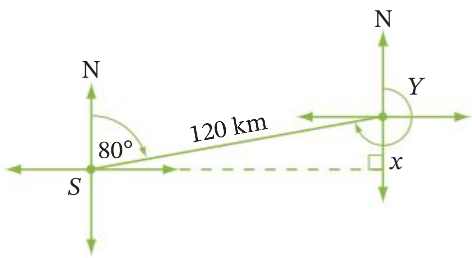
Problems involving bearings U F PS C

EXAMPLE 17

1 A yacht leaves Sydney and sails 120 km on a bearing of 080°.

a How far north (to the nearest km) of Sydney is the yacht?

b What is the bearing of Sydney from the yacht?

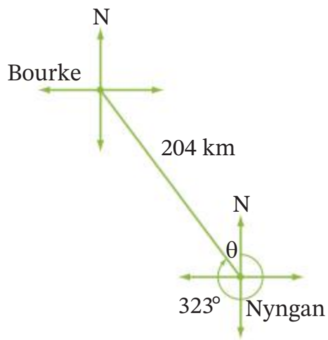


2 Declan leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323°.

a Find the value of θ.

b How far north (to the nearest km) of Nyngan is Bourke?

c What is the bearing of Nyngan from Bourke?



3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205°, calculate:

a how far south Wollongong is from Sydney (correct to one decimal place)

b how far east Sydney is from Wollongong (correct to one decimal place)

c the bearing of Sydney from Wollongong.

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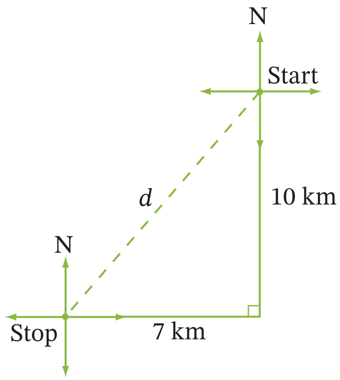
4 Jana cycles 10 km due south, then 7 km due west.

a How far (correct to one decimal place) is Jana from her starting point?

b What is her three-figure bearing from the starting point, correct to the nearest degree?

c What is the bearing of the starting point from Jana?



5 A triathlete cycles 20 km on a SSE bearing to the finish line.

a How far (to the nearest km) has the triathlete travelled in a southerly direction?

b What is the compass bearing of the starting point from the finish line? PS C

6 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) in the direction 078°. They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)? PS C

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7 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo. PS C

a Calculate, correct to 3 decimal places, the distance from Charlie to Alpha.

b Find the three-figure bearing of Alpha from Charlie, correct to the nearest degree.

8 A plane takes off from Darwin at 10:15 a.m. and flies on a bearing of 150° at 700 km/h. PS C

a How far (to the nearest km) due south of Darwin is the plane at 1:45 p.m.?

b What is the bearing (correct to the nearest degree) of Darwin from the plane?

STAGE 5.2

9 A fishing trawler sails 30 km from port on a bearing of 120° until it reaches a submerged reef. How far (to the nearest km) is the port: PS C

a north of the reef? **b** west of the reef?

10 2 racing pigeons are set free at the same time. The first bird flies on a course of 040° while the second bird flies on a course of 130°. PS C

a The first bird flies 200 km until it is due north of the second bird. Find their distance apart, correct to 2 decimal places.

b How far has the second bird flown?

11 2 horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction of 303° until he is due north of the black horse. How far did the rider of the chestnut horse travel? Answer correct to 2 significant figures. PS C

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12 2 ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.) PS C

a How far apart are the ships after 8 hours, to the nearest nautical mile?

b Calculate the bearing of the second ship from the first, to the nearest minute.

13 A plane flies 1200 km on a NE compass bearing and then changes direction to a bearing of NW and flies for a distance of 450 km. PS C

a How far, correct to the nearest km, is the plane from its starting point, to the nearest minute?

b What is the bearing of the plane from its starting point?

14 A fishing boat leaves Sydney Harbour on a compass bearing of ESE and after 15 nautical miles, changes direction to a bearing of SSW and travels a further 10 km.

Find the bearing of Sydney from the boat's final position, correct to the nearest minute. PS C

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Power plus ANSWERS ON P. 532

1 **a** Copy and complete each pair of trigonometric ratios, correct to 3 decimal places.

i sin 20° = _____, cos 70° = _____

ii sin 47° = _____, cos 43° = _____

iii sin 55° = _____, cos 35° = ______

iv sin 85° = _____, cos 5° = _____

b What do you notice about each pair of answers in part **a**?

c What do you notice about each pair of angles in part **a**?

d If cos 30° ≈ 0.8660 and sin θ ≈ 0.8660, what is the value of θ?

e Copy and complete each equation.

i sin 75° = cos ____ **ii** ___ 80° = cos 10°

iii cos ____ = sin 72° **iv** sin 30° = ____ 60°

v cos 65° = sin ____ **vi** sin ____ = cos 58°

f Copy and complete this general rule: sin x = cos (__________).

g Use a right-angled triangle with one angle x and sides a, b and c to prove that the above rule is true.

2 A plane is flying at an angle of 15° inclined to the horizontal.

a How far will the plane have to travel along its line of flight to increase its altitude (height) by 500 m?

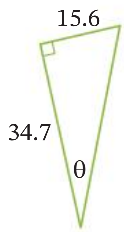
b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15°?

3 If $\sin 30° = \frac{1}{2}$, express the following as surds.

a cos 30° **b** tan 30°

4 Find the value of angle θ, correct to the nearest second.

a



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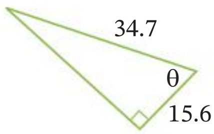


5 By drawing an appropriate triangle, prove that:

a tan 45° = 1 **b** $\sin 45° = \frac{1}{\sqrt{2}}$ **c** $\cos 45° = \frac{1}{\sqrt{2}}$

6 Sketch a right-angled triangle with side lengths a, b and c, where c is the hypotenuse.

Let θ be one of the acute angles. Prove that $(\sin \theta)^2 + (\cos \theta)^2 = 1$.

CHAPTER 8 REVIEW

Language of maths

Qz Trigonometry

adjacent	angle of depression	angle of elevation	bearing
clinometer	compass bearing	cosine (cos)	degree (°)
denominator	horizontal	hypotenuse	inverse ($^{-1}$)
minute (′)	opposite	right-angled	second (″)
sine (sin)	tangent (tan)	theta (θ)	three-figure bearing
trigonometry	trigonometric ratio	unknown	vertical

1 What is an **angle of depression**?

2 What word means ‘next to’?

3 Which side of a right-angled triangle is fixed and does not depend on the position of an angle?

4 Copy and complete: A **bearing** is an ____________ used to precisely describe the ____________ of one location from a given reference point.

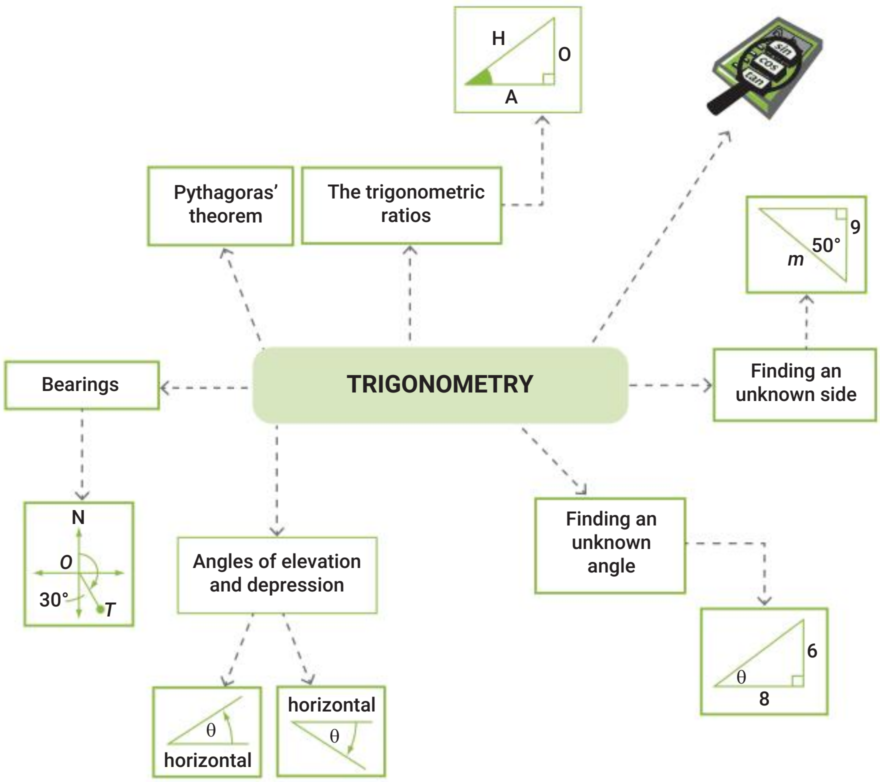
5 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?

6 What does **inverse** mean and how is it used in trigonometry?

Topic summary

WS Mind map: Trigonometry

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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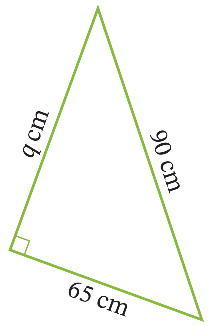
TEST YOURSELF 8 ANSWERS ON P. 532

1 Find the value of each variable, correct to one decimal place. 8.01

a



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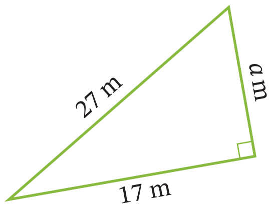
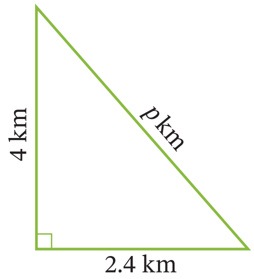
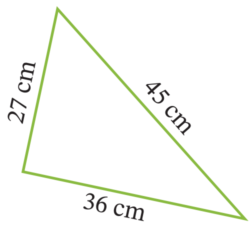


c



2 Test whether each triangle is right-angled. 8.01

a



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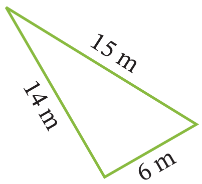

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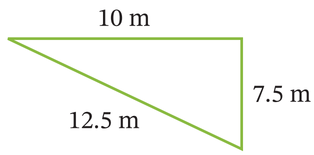
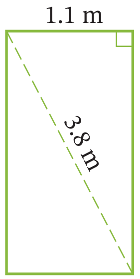
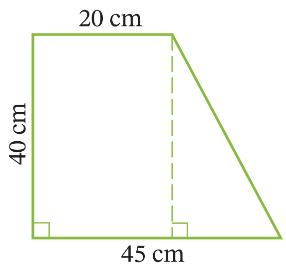


3 Calculate the perimeter of each shape, correct to one decimal place. 8.01

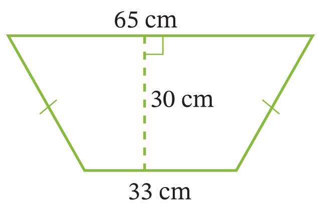
a



b



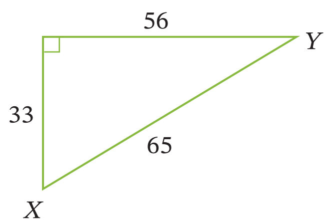
c



4 For this triangle, write as a fraction: 8.02

a sin Y **b** tan Y

c sin X **d** cos X



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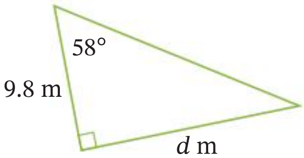
8.02

5 If $\sin \alpha = \frac{36}{85}$, express the values of $\cos \alpha$ and $\tan \alpha$ as fractions. (Draw a diagram.)

8.03

6 Find the value of each variable, correct to 2 decimal places.

a



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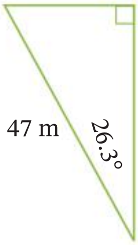

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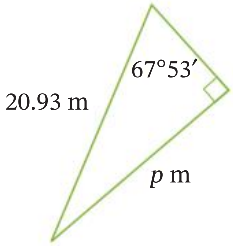
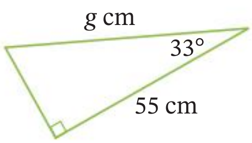


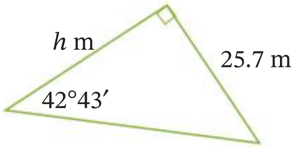
8.03

7 Find the value of each variable, correct to one decimal place.

a



b



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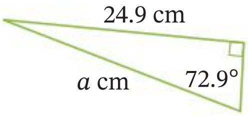


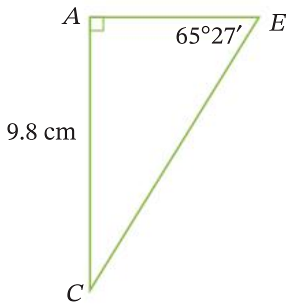
STAGE 5.2

8.03

8 Find the length of CE, correct to one decimal place.
Select the correct answer **A**, **B**, **C** or **D**.

A 23.6 cm **B** 4.5 cm

C 9.7 cm **D** 10.8 cm



8.04

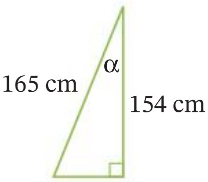
9 Find the size of angle θ, correct to the nearest degree.

a $\sin \theta = 0.8706$ **b** $\cos \theta = \frac{1}{11}$ **c** $\tan \theta = \frac{15}{7}$ **d** $\cos \theta = 0.0295$

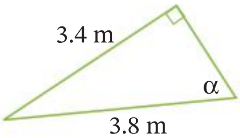
8.04

10 Find the size of angle α, correct to the nearest degree.

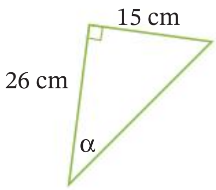
a



b



c



STAGE 5.2

8.04

11 In $\triangle XWY$, $\angle W = 90°$, $WY = 17.2$ m and $XW = 3.5$ m. Find $\angle Y$, correct to the nearest minute. Select **A**, **B**, **C** or **D**.

A 11° 30′ **B** 11° 50′ **C** 11.52° **D** 11° 31′

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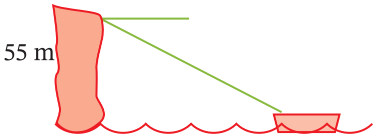
12 The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower, correct to the nearest metre. 8.05

13 Find the angle of depression (correct to the nearest degree) of a boat that is 100 m from the base of a 55 m cliff. 8.05

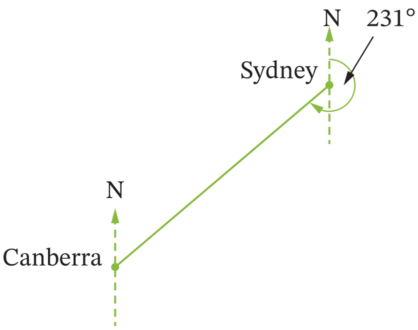


STAGE 5.2

14 What is the bearing of: 8.06

a Canberra from Sydney?

b Sydney from Canberra?



15 Sketch point *B* on a compass rose so that *B* has a bearing from *A* of: 8.06

a 220° **b** 020° **c** 120° **d** 310°

16 If the bearing of *W* from *X* is 165°, what is the bearing of *X* from *W*? Select the correct answer **A**, **B**, **C** or **D**. 8.06

A 345° **B** 315° **C** 255° **D** 195°

17 2 planes leave an airport at the same time. The first travels on a bearing of 063° at 500 km/h. The second travels on a bearing of 153° at 400 km/h. 8.07

a How far apart are the planes after 2 hours (correct to the nearest km)?

b Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.

18 A boat sails from Perth on a compass bearing of SSW for 12 nautical miles and then changes direction to a bearing of WNW and sails a further 42 nautical miles. 8.07

a How far is the boat from Perth?

b What is the bearing of the boat from Perth?

c On what three-figure bearing will it have to sail on its way directly back to Perth?

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9

NUMBER AND ALGEBRA

SIMULTANEOUS EQUATIONS

Many scientific, natural, economic and social phenomena can be modelled by equations. Often these models consist of more than one equation. For example, when manufacturing milk, equations can be written that describe relationships between quantity, cost and income. These equations can then be solved simultaneously to obtain information about pricing and the quantities that need to be produced and sold to make the most profit.

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Chapter outline

	Working mathematically				
9.01 Solving simultaneous equations graphically*	U	F		R	C
9.02 The elimination method*	U	F			
9.03 The substitution method*	U	F			
9.04 Problems involving simultaneous equations*	U	F	PS	R	C

***STAGE 5.2**

Wordbank

coefficient The numerical part of an algebraic term. For example, in $3x^2 + 7x - 1$ the coefficient of x is 7.

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables

graphical method A method of solving simultaneous equations that involves graphing them on a number plane and identifying the point(s) of intersection

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$.

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.2) solve linear simultaneous equations graphically, including using graphing technology
- (STAGE 5.2) solve linear simultaneous equations algebraically using the elimination and substitution methods
- (STAGE 5.2) solve problems using linear simultaneous equations

SkillCheck ANSWERS ON P. 533

1 Given the equation $y = 2x + 5$, find y when:

a $x = 0$ **b** $x = 4$ **c** $x = \frac{1}{2}$ **d** $x = -3$

2 Given the equation $y = 4 - 3x$, find y when:

a $x = 5$ **b** $x = 1$ **c** $x = -1$ **d** $x = -\frac{1}{2}$

3 By completing a table of values, graph each equation.

a $y = x + 1$ **b** $y = 3x$ **c** $y = \frac{x}{2} - 1$

d $y = 3 - x$ **e** $x + y = 4$ **f** $2x - y = 5$

4 Test whether the point $(-2, 3)$ lies on the line represented by each equation.

a $y = 1 - x$ **b** $x + y = 3$ **c** $2x - y = 7$

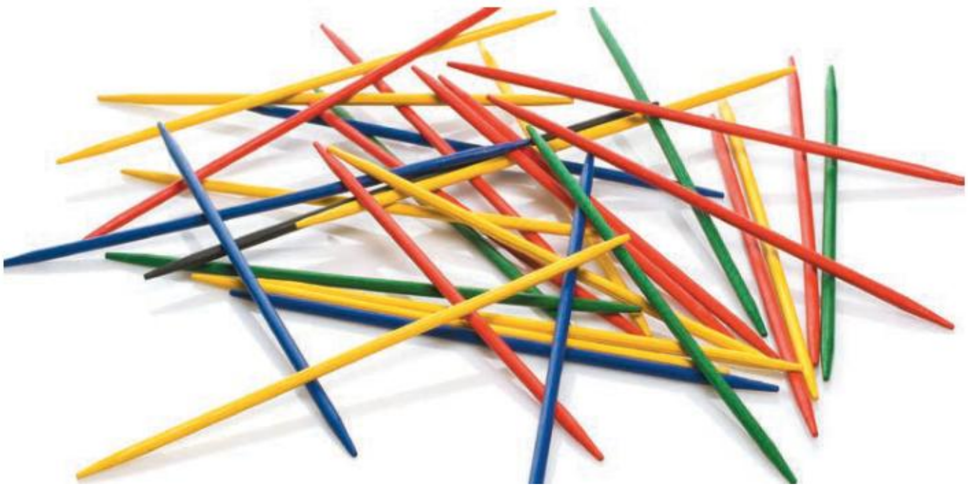
d $\frac{1}{2}x + y = 2$ **e** $y = 3x + 7$ **f** $2y = 3x$

5 **a** Show that the point $(2, 5)$ lies on both the lines $y = 2x + 1$ and $x + y = 7$.

b At what point do these 2 lines intersect?

6 Use the y-intercept and the gradient to graph each equation.

a $y = -2x + 3$ **b** $y = \frac{5}{2}x - 2$ **c** $y = -\frac{4}{3}x + 5$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Investigation

STAGE 5.2

When 2 lines meet

1 Copy and complete the table of values for each equation.

a $x + 2y = 0$

x	−2	−1	0	1	2
y					

b $y = x + 4$

x	−2	−1	0	1	2
y					

2 Which coordinates satisfy both equations?

3 On the same set of axes, draw the graphs of $x + 2y = 0$ and $y = x + 4$.

4 a Do the lines you drew in question **3** intersect?

b What are the coordinates of the point of intersection?

5 Repeat questions **1** to **4** for these pairs of equations.

a $x - y = 5$
$2x + y = 1$

b $3x + y = 8$
$x + 2y = 1$

6 Copy and complete.

a The coordinates of the p_____ of intersection between 2 lines satisfy both equations.

b The values of x and y that satisfy both equations are the coordinates of the __________.

9

Solving simultaneous equations graphically

9.01

STAGE 5.2

A linear equation such as $3x + 5 = 11$ is in one variable (x) and has only one solution ($x = 2$). However, a linear equation in **2** variables, such as $x + 3y = 5$, has more than one solution (for example, $x = 2$, $y = 1$, or $x = 5$, $y = 0$, and so on). The equation actually has an **infinite number** of solutions.

We will now look at solving 2 equations simultaneously to see if there is a solution that satisfies **both** equations. These are called **simultaneous equations**, which can be solved **graphically** or **algebraically**.

WS Testing simultaneous equations

Solving simultaneous equations

Simultaneous equations solver

Solving simultaneous equations graphically

- Linear simultaneous equations can be graphed as lines on the same number plane.
- If 2 lines are drawn, the lines will intersect (unless they are parallel).
- At the point of intersection, the x-coordinate and y-coordinate represent the solution to the simultaneous equations.

STAGE 5.2

Example 1

On the same set of axes, graph $3x + y = 4$ and $x + y = -2$, then solve the equations simultaneously.

Solution

Step 1

Construct tables of values.

$3x + y = 4$

x	0	1	2
y	4	1	−2

$x + y = -2$

x	0	1	2
y	−2	−3	−4

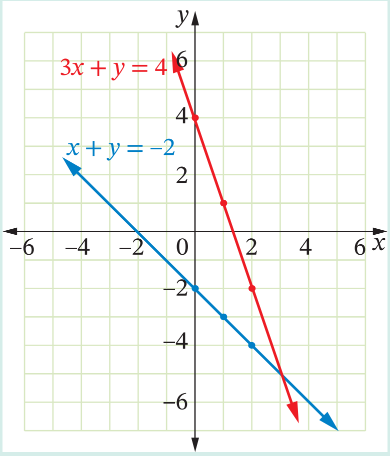
Step 2

Graph the equations.

The lines intersect at $(3, -5)$.

∴ The solution of the simultaneous equations $3x + y = 4$ and $x + y = -2$ is $x = 3, y = -5$.

Check that $x = 3, y = -5$ satisfies both equations.



EXERCISE 9.01 ANSWERS ON P. 534

ANSWERS ON P. 534

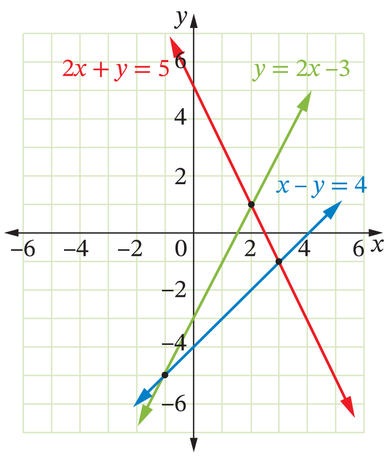
Solving simultaneous equations graphically U F R C

1 Use the graph to write the solution to each pair of simultaneous equations. C

a $x - y = 4$ and $2x + y = 5$

b $2x + y = 5$ and $y = 2x - 3$

c $x - y = 4$ and $y = 2x - 3$



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STAGE 5.2

EXAMPLE 1

9.01

2 Graph each pair of equations on the same set of axes. Then find the solution to the pair of simultaneous equations.

a $y = 2x$ and $y = 3 - x$
b $y = 2x + 1$ and $y = x - 4$
c $x + y = 3$ and $4x + y = 6$
d $y = -x + 2$ and $y = 3x + 4$
e $y = 2x - 5$ and $y = 5x + 1$
f $2x + y = 6$ and $y = 1 - x$
g $y = 7 - x$ and $y = 3x + 5$
h $x + 2y = 7$ and $2x - y = 4$
i $3x - 2y = 12$ and $x + 2y = 8$
j $y = x + 3$ and $2x - y = 2$
k $5x - y = 5$ and $x + y = 4$
l $5x + 3y = 20$ and $y = x - 4$

3 **a** On the same set of axes, draw the graphs of $y = 1 - 2x$ and $2x + y = 4$.

b Why isn't there a solution to the simultaneous equations $y = 1 - 2x$ and $2x + y = 4$? R C

Technology

Solving simultaneous equations graphically

You can use graphing technology to solve simultaneous equations graphically.
Write each answer as coordinates in the form (x, y) representing the point of intersection.

1 Use the graphing software to graph these linear equations.

$y = -x + 1$

$y = x + 3$

2 Find the coordinates of the point of intersection of the lines.

3 Repeat steps **1** and **2** to solve each pair of simultaneous equations.

a $y = 2$, $x = 3$
b $y = -2x + 4$, $y = x - 5$
c $y = 5x + 2$, $y = 3x - 1$
d $y = -1$, $x = 0$
e $y = -x - 8$, $y = -3x + 4$
f $y = 2x + 6$, $y = x + 9$
g $2x - y = 5$, $x + y = 4$
h $3x + y = 4$, $x + 2y = 3$

4 **a** Use the graphing software to graph each pair of simultaneous equations.

i $y = x + 4$, $y = x + 6$
ii $y = -2x + 2$, $y = -2x$

b What do you notice about these 2 pairs of equations? Do they intersect?

5 Graph each set of equations and find their point of intersection.

a $y = 3x$, $y = -x + 2$ and $x = 0.5$

b $y = -4x + 1$, $y = -5x$ and $y = x + 6$

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9.02 The elimination method

STAGE 5.2

Using graphs to solve simultaneous equations can be time-consuming and inaccurate.

Algebraic methods provide a better way of solving things. There are 2 algebraic methods: the **elimination method** and the **substitution method**.

In the elimination method, equations are added or subtracted to eliminate one of the variables.

Simultaneous equations

Example 2

Solve the simultaneous equations $x + 3y = 7$ and $4x - 3y = 13$ using the elimination method.

Solution

Label each equation.

$x + 3y = 7$ [1]

$4x - 3y = 13$ [2]

Since there is the same number of ys in each equation, and since they are opposite in sign ($3y$ and $-3y$), add equations [1] and [2] to eliminate the variable y.

$5x = 20$ [1] + [2]

$\frac{5x}{5} = \frac{20}{5}$

$\therefore x = 4$

Substitute $x = 4$ into equation [1] to find the y-value.

$x + 3y = 7$

$4 + 3y = 7$

$4 + 3y - 4 = 7 - 4$

$3y = 3$

$\therefore y = 1$

$\therefore$ The solution is $x = 4$, $y = 1$.

Check that this solution works for both equations [1] and [2].

Example 3

Solve the simultaneous equations $2k + 3m = 9$ and $2k - 5m = 1$.

Solution

Label each equation.

$2k + 3m = 9$ [1]

$2k - 5m = 1$ [2]

STAGE 5.2

9.02

Since there is the same number of ks in each equation, and because they have the same sign ($2k$ and $2k$), subtract equation [2] from [1] to eliminate k.

$8m = 8$ [1] – [2]

$\frac{8m}{8} = \frac{8}{8}$

$\therefore m = 1$

Substitute $m = 1$ into equation [1] to find the value of k.

$2k + 3m = 9$

$2k + 3 \times 1 = 9$

$2k + 3 = 9$

$2k + 3 - 3 = 9 - 3$

$2k = 6$

$\frac{2k}{2} = \frac{6}{2}$

$\therefore k = 3$

$\therefore$ The solution is $m = 1$, $k = 3$. Check that this solution works for both equations [1] and [2].

Example 4

Solve $3a + 4c = 8$ and $2a - 3c = 11$.

Solution

Label each equation.

$3a + 4c = 8$ [1]

$2a - 3c = 11$ [2]

In this case, neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate c. We need to make the **coefficient** of c the same in both equations ($12c$).

The **coefficient** of c is the number in front of the c in the equation

$9a + 12c = 24$ [3] Multiplying both sides of [1] by 3

$8a - 12c = 44$ [4] Multiplying both sides of [2] by 4

$17a = 68$ [3] + [4]

$\therefore a = 4$

Substitute $a = 4$ in [1] to find c.

$3a + 4c = 8$

$3 \times 4 + 4c = 8$

$12 + 4c = 8$

$4c = -4$

$c = -1$

$\therefore$ The solution is $a = 4$, $c = -1$.

STAGE 5.2

EXERCISE 9.02 ANSWERS ON P. 535

The elimination method U F

EXAMPLE 2

1 For each pair of simultaneous equations, eliminate one variable by adding the equations, then solve the equations.

a $4k + d = 5$, $2k - d = 7$
b $2x - w = 6$, $x + w = 9$
c $3g + 5h = 4$, $2g - 5h = 6$
d $7p - 4n = -20$, $3p + 4n = 10$
e $4q + 3r = 8$, $-q - 3r = 7$
f $-5k - 3x = 8$, $5k + 4x = -3$
g $-4c - 6e = -12$, $4c - 10e = -4$
h $-3y + 5k = 21$, $3y + k = -3$
i $a + 3f = 8$, $-a + 4f = 6$

EXAMPLE 3

2 For each pair of simultaneous equations, eliminate one variable by subtracting the equations, then solve the equations.

a $5k + d = 16$, $3k + d = 4$
b $4a + 3c = 7$, $a + 3c = 4$
c $4h + 3y = 24$, $4h - y = 8$
d $3x + 5e = 16$, $3x - 2e = -5$
e $4q - 2w = -1$, $7q - 2w = 8$
f $6p + 5c = 39$, $4p + 5c = 31$
g $5y + 3m = 18$, $2y + 3m = 6$
h $3a + 2r = 8$, $a + 2r = 10$
i $-x + 5w = 8$, $-x + 3w = 4$
j $4y + 7g = 2$, $4y - 3g = 22$
k $2e - 3n = 14$, $5e - 3n = -1$
l $7k - 5h = 31$, $7k + 3h = 43$

EXAMPLE 4

3 Solve each pair of simultaneous equations.

a $3w + q = 6$, $2w - 3q = 15$
b $2x + m = 5$, $3x + 2m = 3$
c $2d + 3h = 25$, $d + 4h = -5$
d $-3g + 2n = 9$, $g + 5n = 14$
e $5m - h = 10$, $m - 3h = 2$
f $2y + 3e = -6$, $5y - 2e = 23$
g $3q - 2w = 11$, $2q - 5w = 22$
h $5a + 3d = 4$, $4a + 2d = 3$
i $-2p + 3k = 19$, $7p + 4k = 6$
j $5a + 2f = -14$, $2a - 3f = 2$
k $5r - 3c = 2$, $-3r + 2c = -14$
l $5y - 4x = 1$, $2y - 3x = 6$
m $3x + 4y = 20$, $2x - 5y = 21$
n $7g + 3h = 39$, $3g + 5h = 26$
o $5w - 3k = 25$, $3w - 7k = 28$

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The substitution method

9.03

With the **substitution method**, **substitute** the x or y variables from one equation into the other equation.

STAGE 5.2

Simultaneous equations order activity

Simultaneous equations by substitution

Example 5

Solve the simultaneous equations $y = x + 4$ and $y = 3x - 2$.

Solution

Label each equation.

$y = x + 4$ [1]

$y = 3x - 2$ [2]

Use equation [1] to substitute for y in equation [2] and solve for x.

$$x + 4 = 3x - 2$$
$$x + 4 - 3x = 3x - 2 - 3x$$
$$-2x + 4 = -2$$
$$-2x + 4 - 4 = -2 - 4$$
$$-2x = -6$$
$$\frac{-2x}{-2} = \frac{-6}{-2}$$
$$x = 3$$

Now substitute $x = 3$ into equation [1] to find y.

$y = x + 4$

$y = 3 + 4$

$= 7$

∴ The solution is $x = 3$ and $y = 7$.

Example 6

Simultaneous equations

Solve the simultaneous equations $5x + 3y = 9$ and $y = 7 - 3x$.

Solution

Label each equation.

$5x + 3y = 9$ [1]

$y = 7 - 3x$ [2]

STAGE 5.2

Since y is the subject in [2], substitute equation [2] into equation [1] to give an equation using x only.

$5x + 3(7 - 3x) = 9$

$5x + 21 - 9x = 9$

$-4x = -12$

$\frac{-4x}{-4} = \frac{-12}{-4}$

$x = 3$

Now substitute $x = 3$ into equation [2] to find y.

$y = 7 - 3x$

$y = 7 - 3 \times 3$

$= -2$

$\therefore$ The solution is $x = 3$ and $y = -2$.

EXERCISE 9.03 ANSWERS ON P. 535

The substitution method U F

EXAMPLE 5

1 Use the substitution method to solve each pair of simultaneous equations.

a $y = 2x + 1$ and $y = x + 3$

b $y = 5 - 2x$ and $y = 3x + 2$

c $x = 3 + 2y$ and $x = 9 - y$

d $y = -x$ and $y = 3x - 8$

e $x = 1 - 4y$ and $x = 2y + 7$

f $x = 2y$ and $x = 6 - y$

EXAMPLE 6

2 Solve each pair of simultaneous equations.

a $y = 2x + 3$ and $3x - y = 6$

b $y = x - 2$ and $3x + y = 18$

c $y = 1 - 4x$ and $4x + 2y = 3$

d $x = 2y - 5$ and $4x - y = -13$

e $x = 3y - 4$ and $5x - 4y = 2$

f $x = 5 - 3y$ and $4y - x = 23$

g $2x - 5y = -1$ and $y = 10 - x$

h $6y - 2x = 9$ and $y = \frac{x+2}{2}$

i $x = \frac{9-y}{3}$ and $3x + 2y = 10$

j $y = 3x + 5$ and $4x - 3y = 1$

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STAGE 5.2

9.03

Investigation

Elimination or substitution method?

With 2 algebraic methods of solving simultaneous equations, often it is more efficient to use one method than the other.

1 Consider these pairs of simultaneous equations.

a $x - 2y = 9$ [1]
$3x + 2y = 11$ [2]

b $4a + 3c = 18$ [1]
$4a - 3c = -6$ [2]

c $3a - 2y = -5$ [1]
$2a + 5y = 3$ [2]

i Why might the **elimination method** be the more appropriate method to use with these equations?

ii What feature in the pairs of equations do you look for to decide if the elimination method is the best one to use?

iii Solve the 3 pairs of simultaneous equations using the elimination method.

2 Consider these pairs of simultaneous equations.

a $m = 2p$ [1]
$m + p = 15$ [2]

b $m = 4 - p$ [1]
$4m - 3p = -6$ [2]

c $p = 2m - 5$ [1]
$5m - 3p = 11$ [2]

i Why might the **substitution method** be the more appropriate method to use with these equations?

ii What feature in the pairs of equations do you look for to decide if the substitution method is the best one to use?

iii Solve the 3 pairs of simultaneous equations using the substitution method.

3 Using whichever method is more efficient, solve each pair of simultaneous equations.

a $7c + 2y = 13$ [1]
$3c + 2y = 1$ [2]

b $m = 5 - k$ [1]
$2m - k = 4$ [2]

c $3x + 8y = 10$ [1]
$x = 3 - 2y$ [2]

d $4h - 3w = 8$ [1]
$4h + 7w = 12$ [2]

e $3d = q$ [1]
$q + 4d = 14$ [2]

f $3h + 5r = 7$ [1]
$2h - 3r = -8$ [2]

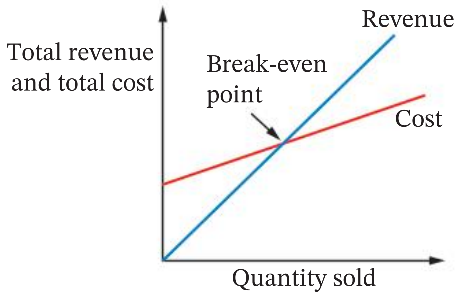
STAGE 5.2

Did you know?

Break-even point

Manufacturers use simultaneous equations to make decisions about how many products they should make and sell. Linear equations can be formed to determine total revenue (the amount made from selling products) and total costs (the cost of making the products).

Total revenue = cost per item × number of items made, while total costs includes rent and production costs.



The equations can be graphed as shown. The point where the 2 lines intersect is called the **break-even point** and occurs when total revenue is equal to total cost.

A publisher receives \$35 per book sold. There are fixed costs of \$110 000 and production costs per book are \$8.50.

a **Determine the equations for total revenue and total costs.**

b **Graph the equations to find the break-even point.**

c **How many books must be sold before the publisher makes a profit?**

9.04 Problems involving simultaneous equations

STAGE 5.2

WS Simultaneous equations problems

Simultaneous equations

Sometimes, worded problems can be solved using simultaneous equations. In these situations, following these steps:

- Read the problem carefully
- Identify the variables to be used
- Use the variables to write simultaneous equations from the information given in the problem
- Solve the equations
- Solve the problem by answering in words

Example 7

At an art show there were 520 guests. If there were 46 more women than men, how many women attended the show?

Solution

Let the number of women attending be w.

Let the number of men attending be m.

$w + m = 520$ [1] 520 people altogether.

$w = m + 46$ [2] 46 more women than men.

Use equation [2] to substitute for w in equation [1].

$m + 46 + m = 520$

$2m + 46 = 520$

$2m + 46 - 46 = 520 - 46$

$2m = 474$

$m = 237$

Substitute $m = 237$ into equation [2] to find w.

$w = 237 + 46$

$= 283$

∴ There were 283 women who attended the art show.

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STAGE 5.2

Simultaneous equations problems

Example 8

Masi and Amir spent \$1582 on shrubs and trees to plant on their large block of land. Altogether they bought 73 plants. The shrubs cost \$19 each while the trees cost \$32 each. How many of each plant did they buy?

Solution

Let x be the number of shrubs.

Let y be the number of trees.

$\therefore x + y = 73$ [1]

and $19x + 32y = 1582$ [2]

Neither adding nor subtracting equations [1] and [2] will eliminate a variable.
Let's choose to eliminate x.

We will need to make the coefficient of x the same in both equations ($19x$).

$19x + 32y = 1582$ [2]

$19x + 19y = 1387$ [3] Multiplying both sides of equation [1] by 19.

$13y = 195$ [2] – [3]

$y = 15$

Substitute $y = 15$ in [1] to find the value of x.

$x + y = 73$

$x + 15 = 73$

$x = 58$

So Masi and Amir bought 58 shrubs and 15 trees.

EXERCISE 9.04 ANSWERS ON P. 535

Problems involving simultaneous equations U F PS R C

EXAMPLE 7

1 At a school concert there were 640 guests. There were 70 more women than men. How many of the audience were men? R C

2 At a circus, there were twice as many children as there were adults in attendance. Altogether, 1020 attended the circus. How many were children? R C

EXAMPLE 8

3 Tickets to a concert cost \$15 for children and \$28 for adults. Altogether, 650 people attended the concert and ticket sales totalled \$13 052. Let a stand for the number of adults and c stand for the number of children in the audience. R C

a Explain why the equations $a + c = 650$ and $28a + 15c = 13\,052$ correctly match the information.

b Solve the equations simultaneously to find the number of children that attended the concert.

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4 Ashleigh bought a total of 130 meat pies and sausage rolls for the canteen of the local football club. Each meat pie cost her $3 and the sausage rolls were $2 each. Altogether, Ashleigh spent $335. How many sausage rolls did she buy? PS R C

5 Tayyab is 3 times as old as Sejuti. The sum of their ages is 48. How old are Tayyab and Sejuti? PS R C

6 The sum of the ages of Hayley and her mother is 70. The difference between their ages is 38 years. How old is Hayley? PS R C

7 A business bought a total of 60 ink cartridges. Some of them were black, costing $35 each. The others were colour, each costing $49. How many of each type did the business buy if the total cost of the ink cartridges was $2422? PS R C

8 The cost of going to the movies for 3 adults and 5 children is $142, while the cost for 5 adults and 8 children is $231.50. Find the cost for an adult and the cost for a child. PS R C

9 Pete's Pizzas sells Supreme pizzas for $15.90 each and vegetarian pizzas for $13.50 each. If 45 pizzas were sold at lunchtime, totalling $684.30, how many of each pizza were sold? PS R C

10 Ronel bought 4 punnets of strawberries and 7 punnets of blueberries for $40.30 and Nikita bought 5 punnets of strawberries and 2 punnets of blueberries for $26.75. What was the cost of each punnet of strawberries and blueberries? PS R C

11 A money box contains only 20-cent coins and 50-cent coins. Altogether, there are 853 coins in the money box and they amount to $281. Let x be the number of 20c coins and y be the number of $2 coins. R C

- **a** Explain why the equations $x + y = 853$ and $20x + 50y = 28\,100$ correctly match the information.
- **b** Solve the equations to determine the number of 20-cent and 50-cent coins in the money box.

12 The initial cost for producing bottles of fresh orange juice is $135 plus $1.20 for each bottle. The bottles of juice are sold for $3 each. C is the cost in dollars, R is the total sales in dollars and n is the number of bottles produced and/or sold. PS R C

- **a** Explain why the equations $C = 135 + 1.2n$ and $R = 3n$ correctly match the information.
- **b** Copy and complete the tables of values below for both equations.

$C = 135 + 1.2n$

n	0	50	100
C			

$R = 3n$

n	0	50	100
R			

- **c** Draw the graphs of both equations on the same axes for values of 0 to 100 for n on the horizontal axis and values of $0 to $300 on the vertical axis.
- **d** For what value of n is total sales equal to total cost (the break-even point)?

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Mental skills 9: Maths without calculators ANSWERS ON P. 535

Simplifying fractions and ratios

When simplifying a fraction or a ratio, look for a common factor to divide into both the numerator and the denominator, preferably the highest common factor (HCF).

1 Study each example.

a Simplify $\frac{27}{45}$

$\frac{\cancel{27}^{9}}{\cancel{45}^{15}} = \frac{9}{15}$ Dividing numerator and denominator by 3

$\frac{\cancel{9}^{3}}{\cancel{15}^{5}} = \frac{3}{5}$ Dividing numerator and denominator by 3 again

Note: This fraction could be simplified in one step if you divided by 9, the highest common factor (HCF) of 27 and 45.

b Simplify $\frac{160}{400}$

$\frac{\cancel{160}^{16}}{\cancel{400}^{40}} = \frac{16}{40}$ Dividing numerator and denominator by 10

$\frac{\cancel{16}^{2}}{\cancel{40}^{5}} = \frac{2}{5}$ Dividing numerator and denominator by 8

Note: This fraction could be simplified in one step if you divided by 80, the HCF of 160 and 400.

c Simplify 24 : 36.

$\cancel{24}^{4} : \cancel{36}^{6} = 4 : 6$ Dividing both terms by 6

$\cancel{4}^{2} : \cancel{6}^{3} = 2 : 3$ Dividing both terms by 2

Note: This fraction could be simplified in one step if you divided by 12, the HCF of 24 and 36.

d Simplify 135 : 90.

$\cancel{135}^{27} : \cancel{90}^{18} = 27 : 18$ Dividing both terms by 5

$\cancel{27}^{3} : \cancel{18}^{2} = 3 : 2$ Dividing both terms by 9

e Calculate $\frac{3}{8} \times \frac{2}{15}$ in simplest form.

$\frac{3}{\cancel{8}^{4}} \times \frac{\cancel{2}^{1}}{15} = \frac{3}{4} \times \frac{1}{15}$ Dividing 2 and 8 by 2

$\frac{\cancel{3}^{1}}{4} \times \frac{1}{\cancel{15}^{5}} = \frac{1}{20}$ Dividing 3 and 15 by 3

f What fraction is 36 minutes of 1 hour?

$\frac{36}{1 \text{ hour}} = \frac{36 \text{ min}}{60 \text{ min}} = \frac{3}{5}$

2 Now simplify each fraction or ratio.

a $\frac{10}{15}$ b $\frac{16}{20}$ c $\frac{30}{42}$ d $\frac{8}{16}$

e $\frac{20}{80}$ f $\frac{6}{36}$ g $\frac{20}{24}$ h $\frac{12}{30}$

i 20 : 36 j 25 : 45 k 18 : 40 l 28 : 35

m 27 : 21 n 16 : 12 o $\frac{5}{6}\times\frac{18}{25}$ p $\frac{12}{50}\times\frac{10}{21}$

3 Express each as a simplified fraction.

a 425 g of 1 kg b 8 months of 1 year c 64 cm of 1 m

d 750 mL of 3 L e 10 hours of 2 days f 80c of $10

9.04

Power plus ANSWERS ON P. 535

1 With simultaneous equations in 2 variables, we have 2 equations to solve.
With simultaneous equations in 3 variables, we have 3 equations to solve.

Step 1: Take 2 of the equations and eliminate one of the variables.

Step 2: Take another 2 of the equations and eliminate the same variable.

Step 3: Solve the 2 new simultaneous equations from Steps **1** and **2**.

Step 4: Use substitution to find the values of the other 2 variables.

Use the above steps to solve the following sets of simultaneous equations.

a $2x + y - 3w = -16$
$x - y + 4w = 25$
$3x - y + 2w = 19$

b $3a - 2c + d = 5$
$5a + 2c + d = 25$
$4a + 3c - d = 10$

c $2m + 3n - p = 9$
$3m - 2n + 5p = 27$
$4m + 3n + 2p = 13$

2 a Show that the solutions to the simultaneous equations $ax + by = c$ and $dx + ey = f$ are $x = \frac{ce - bf}{ae - bd}$ and $y = \frac{af - cd}{ae - bd}$.

b The above solutions do not work when $ae = bd$. Explain why.

c Solve the equations $3x - 2y = 11$ and $5x + y = 14$ by either the substitution or elimination method. Check that the results in part **a** give the same answer.

d Set up a spreadsheet to solve simultaneous equations of the form $ax + by = c$ and $dx + ey = f$ using the solutions $x = \frac{ce - bf}{ae - bd}$ and $y = \frac{af - cd}{ae - bd}$.

Use your spreadsheet to solve each pair of simultaneous equations.

i $3x + y = 4$
$2x - y = 6$

ii $3x - 5y = 4$
$2x - 3y = 8$

iii $15x + 6y = 17$
$2x + 3y = 8$

CHAPTER 9 REVIEW

Language of maths

algebraic	axes	coefficient	elimination method
graphical	linear	point of intersection	satisfy
simultaneous equations	solution	substitution method	variable

1 How do you think **simultaneous equations** got their name?

2 What are the 2 algebraic methods for solving simultaneous equations?

3 Which algebraic method involves cancelling one of the variables?

4 What word means the answer to an equation or problem?

5 What does '**linear**' mean?

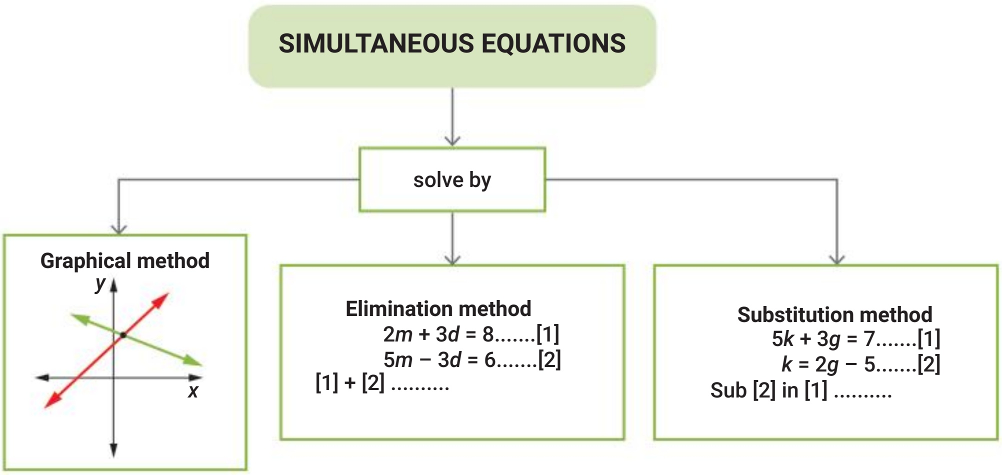
6 Which method of solving simultaneous equations involves finding the point of intersection of lines on a number plane?

Topic summary

Mind map: Simultaneous equations

- In your own words, write down the new things you have learnt about simultaneous equations.
- What parts of this topic did you like?
- What parts of the topic did you find difficult or not understand?
- Copy and complete the following topic overview, and refer to the **Language of maths** word list for keywords you might like to include.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 9 ANSWERS ON P. 536

STAGE 5.2

TEST YOURSELF 9

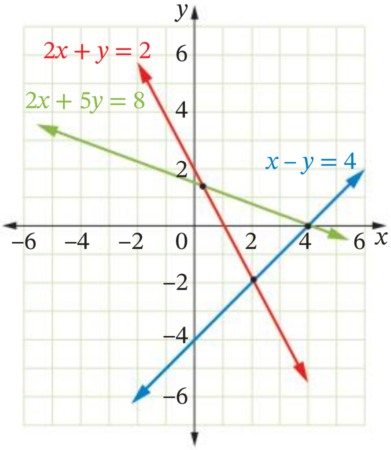
1 Use the graph to write the solution to each pair of simultaneous equations.

9.01

a $x - y = 4$ and $2x + y = 2$

b $2x + 5y = 8$ and $x - y = 4$

c $2x + 5y = 8$ and $2x + y = 2$ (fractional answers)



2 Graph each pair of simultaneous equations on the same set of axes. By finding their point of intersection, write the solution to each pair of equations.

9.01

a $y = x + 2$
$y = 6 + 2x$

b $y = 3 - \frac{x}{2}$
$y = 2x - 7$

c $y = 4 - 3x$
$y = x$

d $y = 2x + 3$
$y = 9 - x$

e $x + y = 7$
$y = 2x + 1$

f $y = 5 - 2x$
$y = -1 - x$

3 Use the elimination method to solve each pair of simultaneous equations.

9.02

a $5m + 2c = 6$
$3m + 2c = -4$

b $2x + 3y = 5$
$5x - 3y = 9$

c $3a + 4d = 7$
$3a + d = 4$

d $4x - y = 9$
$x - y = -9$

e $x - 4y = 3$
$x + 2y = -9$

f $3d - 2w = 11$
$2d - 5w = 44$

4 Use the substitution method to solve each pair of simultaneous equations.

9.03

a $y = 7x - 3$
$y = x + 9$

b $m = 4 - p$
$m = -2 + p$

c $h = 3t - 2$
$h = t + 6$

d $a = 4 - 2c$
$a = 6c$

e $x + 2y = 3$
$y = 2 - x$

f $p = 4 - 2q$
$p = 3q + 24$

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TEST YOURSELF 9

STAGE 5.2

9.04

5 Solve each problem using simultaneous equations.

a In an audience of 2500, there were 700 more adults than children. Find the number of adults and the number of children that were in the audience.

b Rosanna bought 30 movie passes as raffle prizes at a school fete for a total cost of \$322.50. Movie passes for children cost \$8.25 each and the adult price was \$14.50.

How many of each did she buy?

c It costs 2 adults and 5 children \$191 to go to a football game, while the cost for 3 adults and 2 children is \$160. Find the cost of an adult ticket.

d At the cake stall, the student council sell 2 types of cakes: cheesecakes for \$4 each and mudcakes for \$3 each. In total, they sold 75 cakes for a total of \$253. How many of each cake did they sell?

e In Year 10, there are 213 students. There are 27 more boys than girls. Find the number of boys and girls in Year 10.

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PRACTICE SET 3 ANSWERS ON P. 537

STAGE 5.2

1 This conversion graph shows the exchange rate between the Australian dollar and the Euro. 7.03

Use the graph to convert:

a A\$25 to € **b** €60 to A\$ **c** A\$140 to €

2 Solve each pair of simultaneous equations graphically. 9.01

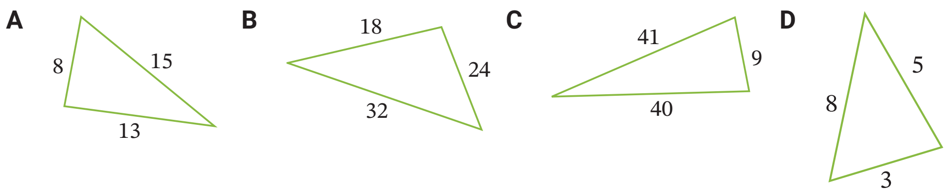
a $y = 2x - 3$ and $x + y = 6$ **b** $2x + y = 1$ and $y = 3x - 4$

3 T varies directly with h. If $T = 48$ when $h = 5$, find T when $h = 16.5$. 7.01

4 Which of the following points lies on the circle with equation $x^2 + y^2 = 9$? Select the correct answer **A**, **B**, **C** or **D**. 7.06

A (0, 3) **B** (2, 2) **C** (4, 1) **D** (1.5, 2)

5 Which of the following is a right-angled triangle? Select **A**, **B**, **C** or **D**. 8.01



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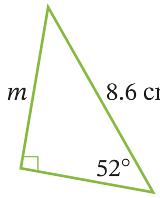
PRACTICE SET 3

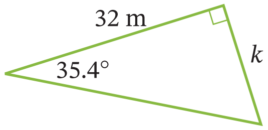
8.03

6 Find the value of each variable, correct to one decimal place.

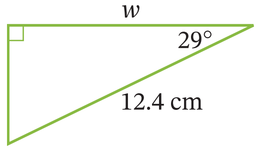
a



b



c



7.04

7 **a** Graph each quadratic equation, showing the vertex of each parabola.

i $y = x^2$ **ii** $y = 4 - x^2$ **iii** $y = 3x^2 + 1$ **iv** $y = 3 - 2x^2$

b State which graphs you have drawn in part **a**:

i are concave up **ii** are concave down **iii** have a turning point at (0, 1).

STAGE 5.2

7.02

8 The time taken, t seconds, to complete a race is inversely proportional to the speed, S m/s, of the sprinter.

a If it takes Leilani 9.8 seconds to complete a race at a speed of 10.2 m/s, find the variation equation for t.

b If it took Leilani 10.5 seconds to complete a similar race, what was her speed?

c If Leilani runs at a speed of 10.3 m/s, how long will it take her to complete the race?

9.02

9 Solve each pair of simultaneous equations using the elimination method.

a $3g - 2w = 8$
$g - 2w = 4$

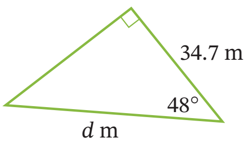
b $2y + 3f = 15$
$5y - 2f = 9$

c $3a - 4c = 5$
$5a - 3c = 1$

8.03

10 Calculate the value of each variable, correct to one decimal place.

a



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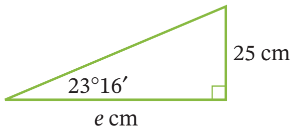

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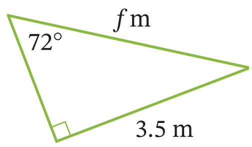
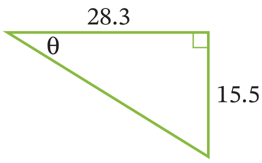


8.04

11 Find θ, correct to the nearest degree.

a



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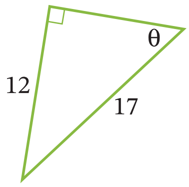

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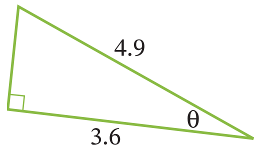

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12 Graph each equation. 7.06

a $x^2 + y^2 = 6.25$ **b** $x^2 + y^2 = 1$ **c** $x^2 + y^2 = 49$

13 A kite is flying at the end of a string that is 85 m long. The string makes an angle of 57° with the ground. At what height is the kite flying? Answer correct to the nearest metre. 8.03

STAGE 5.2

14 The amount of time (in hours) it takes to paint a house varies inversely with the number of painters used to paint the house. It takes 5 painters 28 hours to paint the house. 7.02

a How long will it take 8 painters to paint the house?

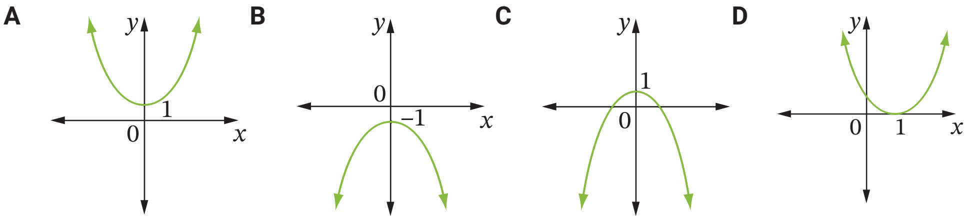
b How many painters are required to paint the house in 24 hours?

15 Tickets to the school play cost \$20 for adults and \$15 for children. Altogether, 395 people attended and ticket sales totalled \$6700. Let A stand for the number of adults and C for the number of children that attended the school play. 9.04

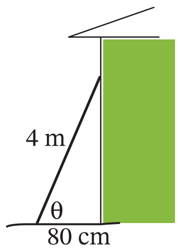
a Write a pair of simultaneous equations to represent this situation.

b Solve the simultaneous equations to find the number of children who attended the play.

16 Which of these could be a graph of $y = 1 - 2x^2$? Select **A**, **B**, **C** or **D**. 7.04



17 A 4 m ladder is placed against the side of a house. The foot of the ladder is 80 cm from the base of the house. Find the angle between the ladder and the ground, correct to the nearest degree. 8.04



18 Which equation represents a circle with centre (0, 0) and radius 4 units? Select **A**, **B**, **C** or **D**. 7.06

A $x^2 + y^2 = 4$ **B** $2x^2 + 2y^2 = 32$ **C** $x^2 + y^2 = 8$ **D** $4x^2 + 4y^2 = 32$

STAGE 5.2

19 Solve each pair of simultaneous equations using the substitution method. 9.03

a $y = x + 3$
$y = 5x - 7$

b $2w + p = 5$
$p = 2w - 3$

c $3k - 2g = 8$
$k = 4g + 1$

STAGE 5.2

7.04

7.05

7.07

20 A parabola has the equation $y = 4x^2 - 3$. Find the x-coordinates of the points on the parabola that have a y-coordinate of 13.

21 Sketch each exponential curve, showing the y-intercept.

a $y = 10^x$ **b** $y = 2^x - 3$ **c** $y = 4^{-x}$ **d** $y = 5^x + 2$

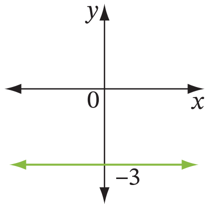
22 Match each equation to its graph.

a $y = 2x^2 - 2$ **b** $x = -3$ **c** $y = 2^{-x}$ **d** $x + y = 1$

e $y = 2 - x^2$ **f** $x^2 + y^2 = 1$ **g** $y = 2x^2$ **h** $y = 3^x$

i $3x^2 + 3y^2 = 75$ **j** $y = x$ **k** $y = -3$ **l** $y = 4^x - 1$

A



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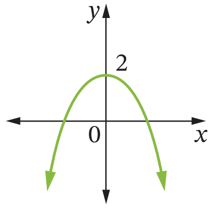

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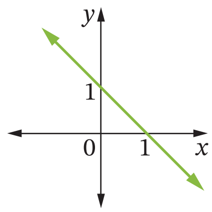
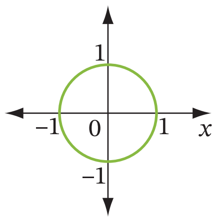


D



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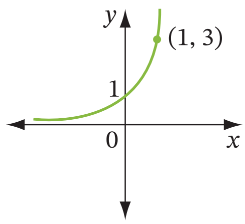

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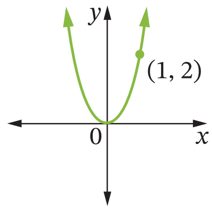

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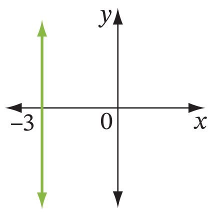

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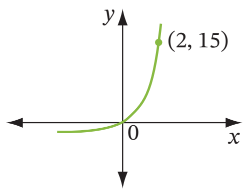

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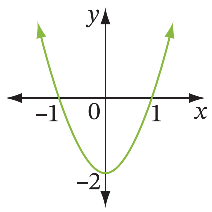

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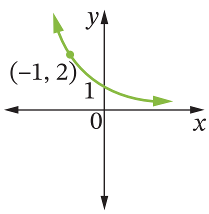

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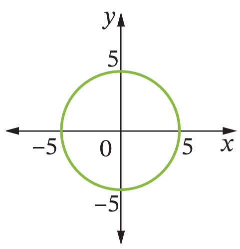

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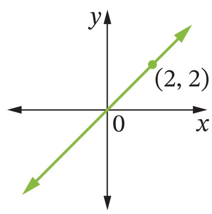

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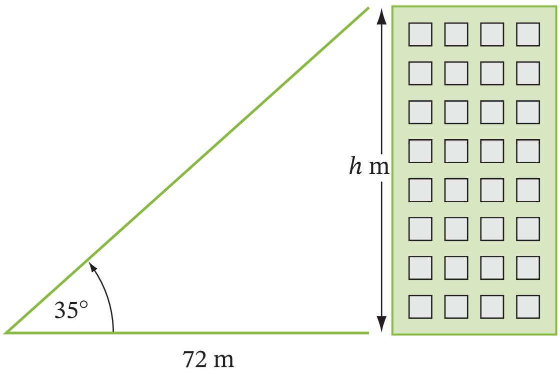
23 The angle of elevation of the top of a building is 35° at a distance of 72 m from its base. Find the height of the building, correct to the nearest metre.



8.05

STAGE 5.2

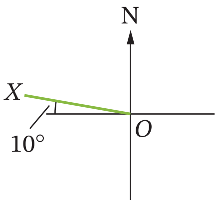
24 In $\triangle ABC$, $\angle C = 90°$, $AC = 56$ mm and $AB = 72$ mm. Find the size of $\angle B$, correct to the nearest minute.

8.04

25 Write the bearing of point X from O in each diagram.

8.06

a



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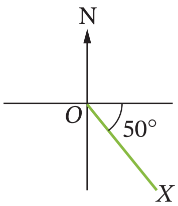

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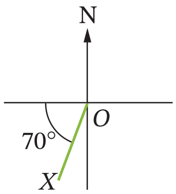
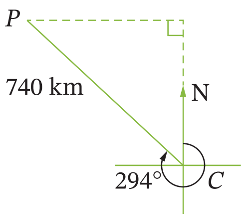


26 A plane flies on a bearing of 294° from Canberra for a distance of 740 km.

a How far north has the plane travelled (correct to the nearest kilometre)?

b What is the bearing of Canberra from the plane's position?



8.07

27 R varies directly with M. If $R = 80$ when $M = 50$, find the value of M when $R = 110$. Select **A**, **B**, **C** or **D**.

A 176 **B** 128 **C** 68.75 **D** 58.25

7.01

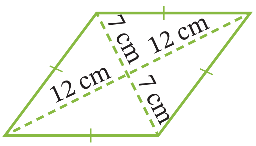
28 Lord Howe Island is 781 km from Sydney on a bearing of 073°. How far east of Sydney, to the nearest kilometre, is Lord Howe Island?

8.07

29 Find the perimeter of each shape, correct to one decimal place.

8.01

a



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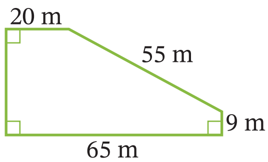

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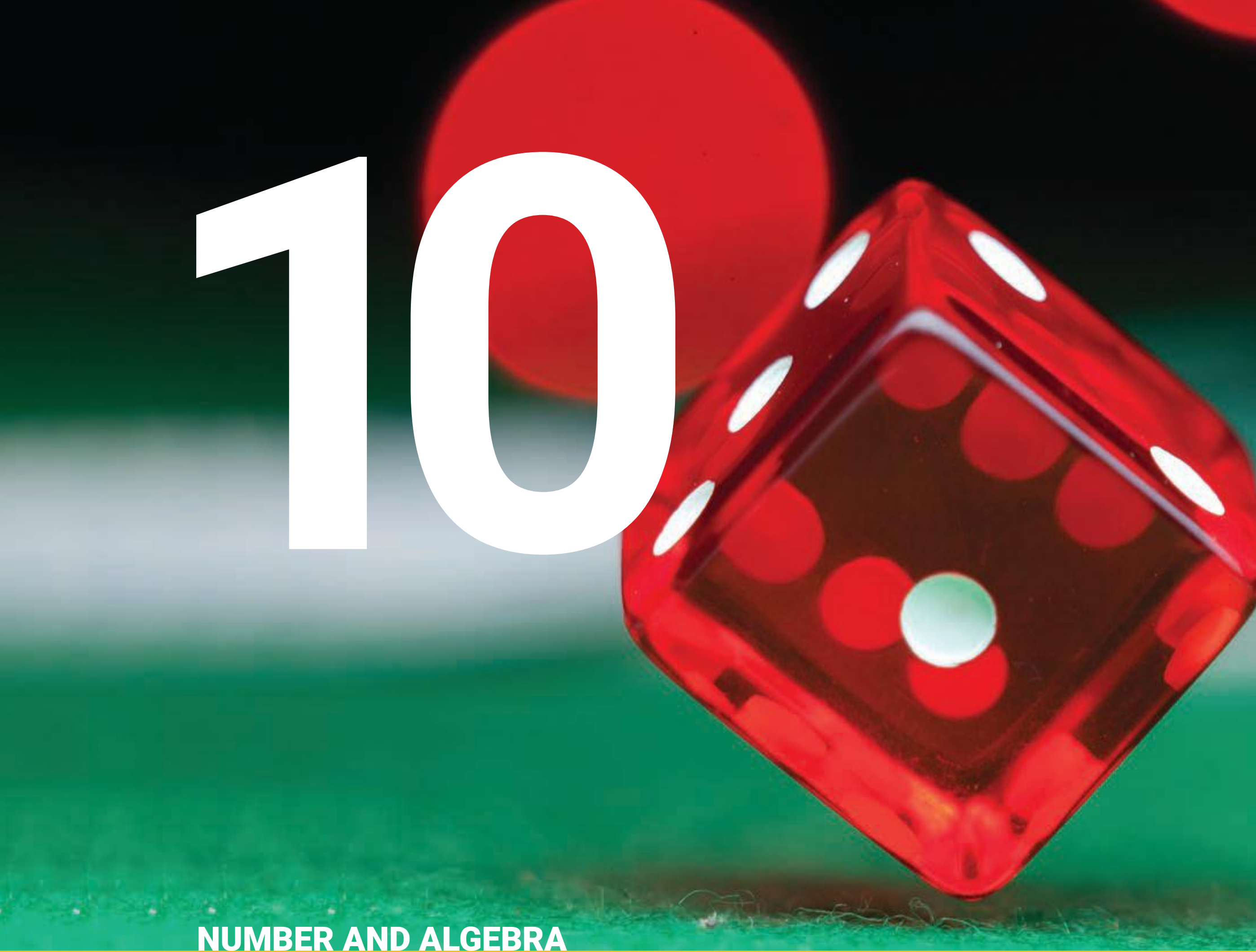

10

NUMBER AND ALGEBRA

PROBABILITY

Probability theory, the study of chance, began in the 17th century when 2 great mathematicians, Blaise Pascal and Pierre de Fermat, wrote to each other to discuss mathematical problems arising from games of chance. Since then, probability has become an essential branch of mathematics that is used widely in fields such as weather forecasting, finance, insurance, politics and medical testing.

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Chapter outline

	Working mathematically				
10.01 Relative frequency	U	F		R	C
10.02 Venn diagrams	U	F		R	C
10.03 Two-way tables	U	F		R	C
10.04 Tree diagrams*	U	F	PS	R	C
10.05 Selecting with and without replacement*	U	F	PS	R	C
10.06 Dependent and independent events*	U	F	PS	R	C
10.07 Conditional probability*	U	F	PS	R	C

***STAGE 5.2**

Wordbank

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

tree diagram A diagram of branches for listing all the possible outcomes of a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with 2 steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into 2 overlapping categories, such as gender and the ability to drive a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate the probabilities and relative frequencies of events
- describe compound events using terminology such as 'and', exclusive 'or', inclusive 'or' and 'at least'
- use Venn diagrams and two-way tables to represent sample spaces and compound events to solve probability problems
- (STAGE 5.2) use tree diagrams and tables (arrays) to represent the sample space of two- and three-step chance experiments, with and without replacement, to solve probability problems
- (STAGE 5.2) identify dependent and independent events, and use the product rule for independent events
- (STAGE 5.2) solve problems involving conditional probability

WS Probability review

SkillCheck ANSWERS ON P. 538

1 If a die is rolled, which of the following is more likely? Select the correct answer **A**, **B**, **C** or **D**.

A a number less than 3 **B** an even number

C a number that is 3 or more **D** a number that is a prime number

2 A bag contains 5 10-cent coins, 4 20-cent coins and 3 50-cent coins. A coin is drawn at random from the bag.

a How many outcomes are in the sample space?

b Are the outcomes equally likely?

3 For this spinner, the red sector is twice as large as each of the other sectors. Find the probability that when the spinner is spun, the arrow lands on:

a red

b purple or blue

c not green

4 A normal die is rolled. What is the probability of rolling:

a a 7? **b** a number less than 7?

5 The probability of Danica hitting a bullseye when playing darts is 0.6. What is the probability of Danica not hitting the bullseye?

6 Aditi bought 10 tickets in a raffle in which 400 tickets were sold and there is only one prize. What is the probability that Aditi will win the prize? Select **A**, **B**, **C** or **D**.

A $\frac{1}{400}$ **B** 2.5% **C** 0.0025 **D** 0.975

7 The probability of cloudy skies tomorrow is 0.85. What is the probability of clear skies?

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Relative frequency

10.01

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability**.

We can also determine probability based on the results of a trial that has been repeated many times, such as testing the effectiveness of 100 light globes, or rely on past statistics, such as weather patterns or the ages of drivers having car accidents. This type of probability is called **experimental probability** or **observed probability**, which is based on **relative frequency**, the number of times an **event** occurred as a fraction of the total frequency of outcomes.

Experimental probability

Relative frequencies

Dice probability

Long-run proportion

Long-run proportion

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

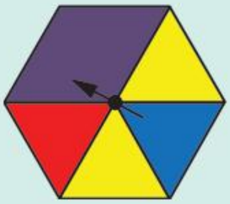
Expected frequency

Expected frequency = theoretical probability × number of trials.

Example 1

This spinner was spun 160 times and the results are shown.

Outcome	Frequency
Red	30
Blue	32
Yellow	50
Purple	48



a Calculate, as a decimal:

i the experimental probability that the arrow stops on blue

ii the theoretical probability that the arrow stops on blue.

b Are the experimental and theoretical probabilities similar?

c If the spinner is spun 500 times, calculate the expected frequency of spinning purple based on the theoretical probability.

Solution

a **i** Experimental probability:

$P(\text{Blue}) = \frac{32}{160}$

$= 0.2$

ii Theoretical probability:

$P(\text{Blue}) = \frac{1}{6}$

$= 0.1666...$

≈ 0.17

b By comparing the decimals for the 2 answers, we see that the experimental and theoretical probabilities are similar.

c Expected frequency of purple $= \frac{1}{3} \times 500$ — probability × number of trials

$= 166.6666...$

≈ 167

Example 2

James rolled a die 100 times and recorded the results in a table.

Outcome	Frequency
1	23
2	19
3	11
4	12
5	18
6	17

a Find the experimental probability of rolling:

i an even number

ii an even number or a number greater than 4

'Even number or a number greater than 4' is an example of a **compound event**.

iii an even number less than or equal to 4.

b Calculate the probability of rolling a 2 or 3

i as an experimental probability **ii** as a theoretical probability.

c If the die is rolled 100 times, what is the expected frequency of rolling a 2 or a 3? How does this compare with James' observed frequency?

Solution

a **i** Rolls of even numbers $= 19 + 12 + 17$ — Frequencies of 2, 4, 6

$= 48$

Experimental $P(\text{even}) = \frac{48}{100} = \frac{12}{25}$

ii Rolls of even numbers or numbers greater than 4 — Frequencies of 2, 4, 6, 5

$= 19 + 12 + 17 + 18$

$= 66$

Experimental $P(\text{even or} > 4) = \frac{66}{100} = \frac{33}{50}$

iii Rolls of even numbers less than or equal to 4 — Frequencies of 2 and 4

$= 19 + 12$

$= 31$

Experimental $P(\text{even and} \le 4) = \frac{31}{100}$

b **i** Rolls of 2 or 3 = 19 + 11 — Frequencies of 2 and 3

= 30

Experimental $P(2 \text{ or } 3) = \frac{30}{100} = \frac{3}{10}$

ii Theoretical $P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$

c Expected frequency of 2 or 3 $= \frac{1}{3} \times 100$ — Probability × number of trials

$= 33.333...$

≈ 33

From the table, the observed frequency = 19 + 11 = 30, which is close to 33.

EXERCISE 10.01 ANSWERS ON P. 538

Relative frequency U F R C

1 Aashima spun this spinner 200 times and recorded the results. R C

EXAMPLE 1

Event	Frequency
Red	85
Green	42
Blue	28
Yellow	15
Purple	30

a Calculate, as a decimal, the experimental probability that the arrow points to:

i red **ii** blue **iii** green

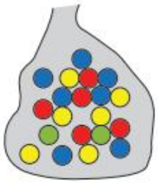
b Calculate, as a decimal, the theoretical probability that the arrow points to:

i red **ii** blue **iii** green

c Are the experimental and theoretical probabilities similar?

d For 200 spins, what is the expected frequency of red or purple based on the theoretical probability? How does this compare with the observed frequency?

2 A bag contains 7 blue, 6 yellow, 5 red and 2 green marbles. Lamisa selects a marble at random, records its colour and then returns it to the bag. Lamisa repeats this process 100 times and the results are shown. R C



Outcome	Frequency
Red	20
Blue	38
Yellow	33
Green	9

a Find the relative frequency of selecting a marble that is:

i red **ii** blue

iii yellow **iv** green

b What is the theoretical probability of selecting a marble that is:

i red? **ii** blue?

iii yellow? **iv** green?

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c Are the experimental and theoretical probabilities similar?

d If the process is repeated 100 times, what is the expected frequency of a selecting a yellow or green marble? How does this compare with the observed frequency?

3 A coin is tossed. R C

a What is the expected number of heads if the coin is tossed 100 times?

b Toss a coin 100 times. Copy this table and record your results in it.

Outcome	Frequency
Head	
Tail	

c Calculate, as a decimal:

i the experimental probability of tossing a head

ii the theoretical probability of tossing a tail.

d Are the experimental and theoretical probabilities similar?

EXAMPLE 2

4 A die was repeatedly rolled and the results are shown in the table. R C

Outcome	Frequency
1	95
2	119
3	108
4	87
5	78
6	113

a How many times was the die rolled?

b Find the experimental probability (as a decimal) of rolling:

i an odd number

ii a number less than 4

iii a 2 or a 3

iv a number less than 4 or an even number.

c Find the theoretical probability (as a decimal) of rolling:

i an odd number

ii a number less than 4

iii a 2 or a 3

iv a number less than 4 or an even number.

d Compare the experimental probabilities to the theoretical probabilities.

5 Place 5 blue counters, 2 red counters and 3 yellow counters in a bag. Select a counter at random from the bag, note its colour and return the counter to the bag. Repeat this 60 times. Copy this table and record your results in it. R C

Event	Tally	Frequency
Blue		
Red		
Yellow		

a What is the experimental probability of selecting:

i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

b What is the theoretical probability of selecting:

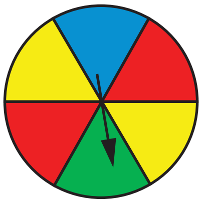
i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

c Are the experimental probabilities similar to the theoretical probabilities?

10.01

6 Tara spun this spinner 50 times and the results are shown. R C



Event	Frequency
Red	15
Blue	6
Yellow	24
Green	5

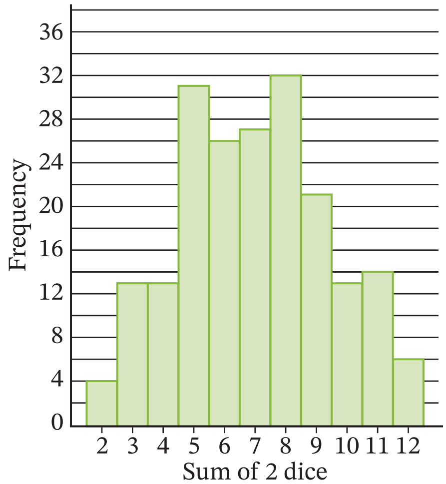
a What is the experimental probability (as a decimal) of the arrow stopping on
i red? **ii** blue? **iii** yellow? **iv** green?

b What is the theoretical probability (as a decimal) of the arrow stopping on
i red? **ii** blue? **iii** yellow? **iv** green?

c Are the experimental and theoretical probabilities similar?

d What is the expected frequency of the arrow stopping on a colour that is not yellow? How does this compare with Tara's observed frequency?

7 2 dice are rolled and the sum of the numbers rolled was recorded in the frequency histogram.



a How many times were the dice rolled?

b Based on these results, what is the experimental probability of rolling a sum:

i of 2? **ii** of 7?
iii of 10? **iv** greater than 7?
v less than 7?
vi of 7 or 8?
vii that is even and greater than 6?

8 Children at a shopping mall were asked how they travelled to school. R C

Mode of transport	Frequency
Walk	27
Bus	80
Car	62
Train	21
Bicycle	5
Skateboard	1
Other	4

a How many students were surveyed?

b Based on these results, find the probability that a student chosen at random will:

i walk to school
ii be driven to school
iii catch a bus to school
iv catch a train to school
v ride a skateboard to school

c What mode of transport could 'Other' include?

d Survey 100 students at your school and make up a table showing the results. How do the results from your school compare with the results from the survey?

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9 A die is rolled 100 times. R C

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a What is the probability of rolling a 6? (Express your answer as a fraction and as a decimal.)

b How many times would you expect a 6 to appear if the die was rolled 100 times?

c Roll a die 100 times and record your results in a table similar to the one shown.

d What is the relative frequency of rolling a 6? (Express your answer as a fraction and as a decimal.)

e How does the theoretical probability of rolling a 6 compare with the experimental probability?

10 'If there are 10 horses in a race, the probability of each horse winning is 1 in 10'. Explain why this statement is not true. Give at least 2 reasons. R C

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10.02 Venn diagrams

Venn diagrams 1

Venn diagrams 2

Venn diagrams matching activity

A **Venn diagram** is a diagram of circles (usually overlapping) that is used to group items into categories. A rectangle represents the whole group, while the circles represent categories. Items common to 2 or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

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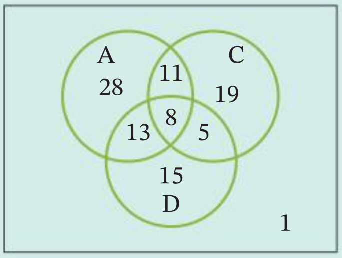
Example 3

This Venn diagram shows the results of a survey on what type of movies – action (A), comedy (C) or drama (D) – students prefer to watch.



a How many students were surveyed?

b How many students preferred to watch 2 types of movies only?

c Calculate, as a decimal, the probability of selecting a student who prefers to watch:

i action movies only **ii** action or comedy movies, but not dramas

iii action and drama movies **iv** all types.

d A student is chosen from those who like action and comedy movies. What is the probability that they also like to watch drama movies?

e What is the probability of selecting a student who does not like watching any of the 3 movie types?

Solution

a Number of students = 28 + 11 + 8 + 13 + 5 + 19 + 15 + 1
= 100

b 29 students preferred 2 types of movies only. 11 + 13 + 5 = 29

c **i** Students preferring action movies only = 28 The region of A that doesn't overlap C or D.

$P(\text{action only}) = \frac{28}{100} = 0.28$

ii Students preferring action or comedy only
= 28 + 19 + 11 = 58 The regions of A and C that don't overlap with D.

$P(\text{action or comedy only}) = \frac{58}{100} = 0.58$

'Action or comedy only' is an example of a **compound event**.

iii Students preferring action and drama = 13 + 8
= 21 The regions where A and D intersect.

$P(\text{action and drama}) = \frac{21}{100} = 0.21$

iv Students preferring all types = 8 The region where the 3 circles intersect.

$P(\text{all types}) = \frac{8}{100} = 0.08$

d Students preferring action and comedy = 11 + 8
= 19

Students preferring action and comedy and drama = 8

$P(\text{drama if preferring action and comedy}) = \frac{8}{19}$
≈ 0.42

And/or problems

Venn diagrams

10.02

e There is one student who doesn't prefer action, comedy or drama.

$P(\text{not action, comedy or drama}) = \frac{1}{100}$

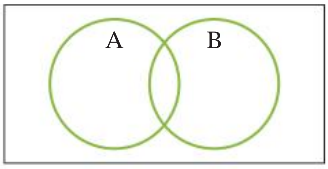
$= 0.01$

When we combine 2 or more simple events, we get a **compound event**. In the above example, 'action and drama' and 'all types' are compound events.

'And' vs 'or'

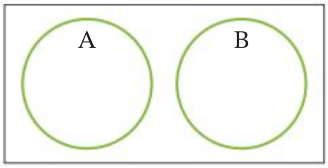
For 2 categories or events A and B, the compound event '**A and B**' means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

If A and B are **overlapping**, the compound event '**A or B**' means to have A or B or both. For example, 'to drive a car' or 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.



Overlapping events:
'A or B' means A or B or both

If A and B are **mutually exclusive**, this means that they are **not overlapping** and on a Venn diagram they appear as 2 separate circles. For mutually exclusive categories or events, the phrase '**A or B**' means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.



Mutually exclusive events:
'A or B' means A or B but not both

Example 4

A survey of 110 students at Hamper Valley College showed that 34 students study Art, 65 students study Chemistry, and 23 students study both Chemistry and Art.

a Represent this information on a Venn diagram.

b How many students study Art and Chemistry, but not both?

c What is the probability of randomly selecting a student from this group who takes:

- **i** Chemistry?
- **ii** Art and Chemistry?
- **iii** Art or Chemistry?
- **iv** neither Art nor Chemistry?

Alamy Stock Photo/Ian Shaw

Solution

a S = students surveyed

A = students doing Art

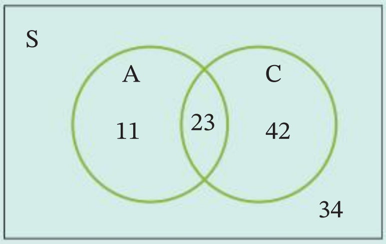
C = students doing Chemsitry

There are 23 students who take both Art and Chemistry.

∴ Students doing Art only = 34 – 23 = 11

∴ Students doing Chemistry only = 65 – 23 = 42

∴ Students who take neither Art nor Chemistry = 110 – 11 – 42 – 23 = 34



b Number of students studying Art and Chemistry only = 11 + 42 = 53

'Art and Chemistry only' is a compound event.

c **i** 65 students take Chemistry.

$P(\text{Chemistry}) = \frac{65}{110} = \frac{13}{22}$

ii $P(\text{Art and Chemistry}) = \frac{23}{110}$

iii Number of students who take Art or Chemistry = 11 + 23 + 42 = 76

$P(\text{Art or Chemistry}) = \frac{76}{110} = \frac{38}{55}$

iv $P(\text{neither Art nor Chemistry}) = \frac{34}{110} = \frac{17}{55}$

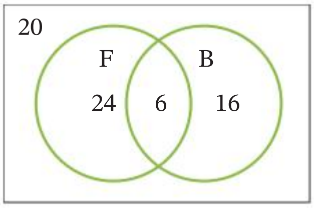
EXERCISE 10.02 ANSWERS ON P. 539

Venn diagrams U F R C

10.02

EXAMPLE 3

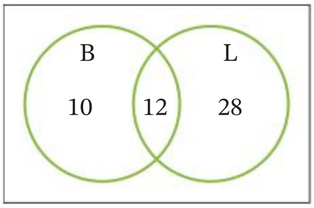
1 This Venn diagram shows the number of competitors who swam in freestyle (F) and butterfly (B) events at the school swimming carnival. How many students swam in Freestyle or Butterfly, but not both?



Select the correct answer **A**, **B**, **C** or **D**.

A 28 **B** 34 **C** 40 **D** 60

2 50 people were asked whether they had breakfast (B) or lunch (L) today. The results are shown in the Venn diagram. C



a What is the probability of selecting a person from this group who had:

i breakfast? **ii** lunch?

iii breakfast but not lunch? **iv** breakfast and lunch?

v breakfast or lunch only?

b Of the people who had lunch, find the probability that a person also had breakfast.

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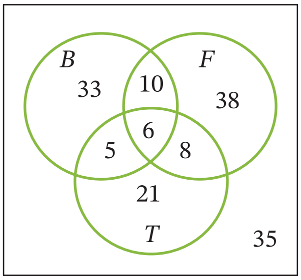
3 The Venn diagram shows the number of Year 10 students who play basketball (B), touch football (F) or tennis (T). C



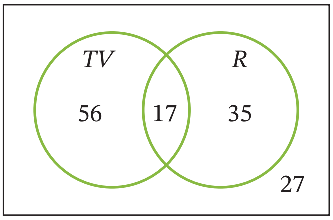
a How many students are in Year 10?

b Find the probability of selecting a student who plays:

- **i** basketball only
- **ii** tennis only
- **iii** touch football and tennis
- **iv** touch football or tennis
- **v** basketball but not touch football
- **vi** all 3 sports.

c Of the students that play touch football, find the probability of selecting a student who also plays tennis.

4 This Venn diagram shows the results of a survey asking people whether they watch TV or read (R) at home. R



a How many people were surveyed?

b Find the probability of selecting a person who watches TV only.

c What is the probability of selecting a person who doesn't watch TV or read?

d Of the people who read, find the probability that they also watch TV.

EXAMPLE 4

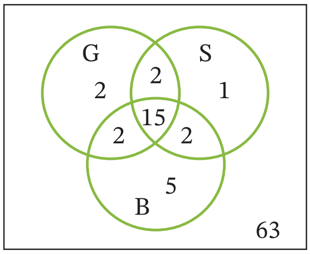
5 Of the 54 Year 10 Music students, 23 students sing (S), 43 students play a musical instrument (P) and 12 students sing and play a musical instrument. R C

a Show this information on a Venn diagram.

b Find the probability of selecting a Music student who:

- **i** sings or plays an instrument
- **ii** sings only
- **iii** plays a musical instrument only
- **iv** sings or plays an instrument, but not both.

6 This Venn diagram shows the number of countries that won medals at the Winter Olympic Games in Pyeongchang, South Korea in 2018. R



a How many countries competed at these games?

b What is the probability of randomly selecting a country that:

- **i** won only gold medals?
- **ii** won gold, silver and bronze medals?
- **iii** won gold or silver medals, but not bronze?
- **iv** did not win a gold or silver medal?

c Of the countries that won medals, what is the probability of selecting a country that

- **i** won gold medals?
- **ii** won bronze, but no gold or silver?

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7 At Riverside College, Year 10 students are asked what language they are studying. 64 students take French (F), 47 students take Japanese (J), 15 students take both French and Japanese, and 27 do not study a language. R C

a How many students are in Year 10?

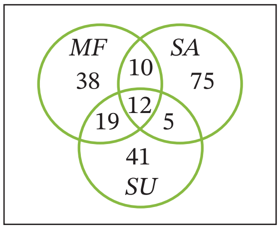
b Show the information on a Venn diagram.

c How many students studied only one language?

d Find the probability of selecting a Year 10 student at random who studies:

- **i** French but not Japanese
- **ii** Japanese but not French
- **iii** no languages
- **iv** only one language.

8 People were surveyed on the day they preferred to shop: Monday to Friday (MF), Saturday (SA) or Sunday (SU). The results are shown in the Venn diagram. R C



a How many people were surveyed?

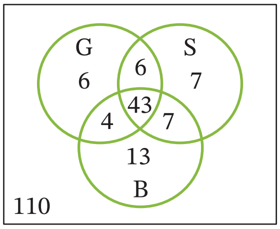
b What is the probability of selecting a person who prefers to shop on:

- **i** Monday to Friday?
- **ii** Saturday?
- **iii** Sunday?
- **iv** on the weekend only?
- **v** on Saturday or Sunday?
- **vi** any day (has no preference)?

c Find the probability of selecting a person who only prefers Saturday or Sunday, but not both.

d Is it necessary to include the rectangle in this Venn diagram? Give reasons.

9 This Venn diagram shows the number of countries that won gold, silver, bronze or no medals at the 2016 Olympic Games in Rio de Janeiro, Brazil.



a Find the total number of countries that competed at these games.

b What is the probability of randomly selecting a country that:

- **i** won a silver medal only?
- **ii** won one medal only?
- **iii** won at least 2 medals?
- **iv** won at most one medal only?

c Out of the countries that won gold medals, find the probability of selecting a country that:

- **i** won gold and silver, but not bronze
- **ii** won gold, silver and bronze.

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10.03 Two-way tables

WS Two-way probability tables

Combined events: Two-way tables

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by a Venn diagram.

Example 5

Year 11 students at Southbank College were surveyed on whether they had part-time jobs.

	Part-time work	No part-time work
Male	43	27
Female	35	31

a How many students are in Year 11 at Southbank College?

b How many students had part-time work?

c How many male students were in Year 11?

d What is the probability of selecting a student at random that:

i works part-time?

ii is female and works part-time?

iii is male and doesn't work?

iv doesn't work?

e What is the probability of selecting a student working part-time given that:

i the student is male?

ii the student is female?

Solution

a Number of Year 11 students = 43 + 27 + 35 + 31 = 136

b Students with part-time work = 43 + 35 = 78

c Male students in Year 11 = 43 + 27 = 70

d **i** $P(\text{student works part-time}) = \frac{78}{136} = \frac{39}{68}$

ii There are 35 female students who work part-time.

$P(\text{female and part-time}) = \frac{35}{136}$

'Female and works part-time' is a compound event.

iii There are 27 males who don't work.

$P(\text{male and not working}) = \frac{27}{136}$

iv Number of students not working = 27 + 31 = 58

$P(\text{not working}) = \frac{58}{136} = \frac{29}{68}$

e **i** There are 70 male students and 43 of them work part-time.

$P(\text{working part-time given that student is male}) = \frac{43}{70}$

ii There are 66 female students and 35 of them work part-time.

$P(\text{working part-time given that student is female}) = \frac{35}{66}$

EXERCISE 10.03 ANSWERS ON P. 539

ANSWERS ON P. 539

Two-way tables U F R C

EXAMPLE 5

10.03

1 People attending the Staying Alive Fitness Centre early on a Saturday morning either went swimming or did a workout in the gym. The numbers are shown in the table. R C

	Swimming	Gym
Male	32	53
Female	24	41

a How many people went to the fitness centre?

b Find the probability that a person selected at random:

i was female and went swimming

ii was male and did a workout in the gym

iii went swimming.

c Find the percentage (to the nearest whole number) of females who did a workout in the gym.

2 Year 10 students at Nusentry High School were given a choice of 2 activities on a wet sports afternoon: bowling or indoor soccer. Their selections are shown in the table. R C

	Bowling	Indoor soccer
Boys	25	48
Girls	43	12

a How many students are in Year 10?

b How many students:

i went bowling? **ii** played indoor soccer?

c What is the probability of randomly selecting a student who went bowling?

d What is the probability of randomly selecting a girl who played indoor soccer?

3 The composition of the Legislative Assembly (the lower house) in the NSW State Parliament is shown in the table. R C

	Liberal/Nationals	Labor	Greens/Other
Male	41	20	5
Female	11	14	6

The Liberal/Nationals are in government and the others are in opposition.

a How many members of parliament (MPs) are there in the Legislative Assembly?

b Find the percentage probability of randomly selecting an MP who is:

i female **ii** male and in the Opposition **iii** Greens/Other

c What percentage probability of:

i Government MPs are female? **ii** Opposition MPs are female?

d Compare your answers to part **c** and comment on the differences between the 2 results.

4 People were asked to name their favourite takeaway food. The results are shown in the table. R C

	Pizza	Hamburger	Fish and Chips
Men	22	35	18
Women	43	26	6

a How many people were surveyed?

b Find the probability (as a decimal) that a person selected at random:

i is male

ii is female and likes fish and chips

iii likes pizza

iv is male and likes hamburgers.

c If a male is selected at random, what is the probability that his favourite takeaway food is pizza?

5 Year 7 students were asked about their favourite drink. The results are in the table. R C

	Boys	Girls
Water	21	35
Milk	11	12
Juice	15	17
Soft drink	31	18

a How many students were in Year 7?

b What is the probability of randomly selecting a student that:

i prefers water?

ii is a boy and likes milk?

iii is a girl and likes soft drinks?

c What is the probability that if a girl is randomly selected, she prefers water?

6 A survey looked at whether people ate breakfast and whether they exercised regularly. R C

	Exercise	No exercise
Ate breakfast	72	27
Did not eat breakfast	38	63

Shutterstock.com/Ddaxiao Productions

a How many people were surveyed?

b What percentage of people exercised?

c Find the percentage probability of picking a person at random who:

i eats breakfast

ii does not exercise regularly

iii eats breakfast and exercises regularly

iv does not eat breakfast and does not exercise.

d Of the people who exercise regularly, what is the probability of picking someone who eats breakfast?

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7 Students at Mt Badger College were asked to indicate their preference for dark or milk chocolate in a survey. R C

		Milk chocolate	
		Like	Dislike
Dark chocolate	Like	545	134
	Dislike	157	42

a How many students attended the college?

b What is the probability (correct to 3 decimal places) of selecting a student at random who:

i likes dark chocolate?

ii likes both milk chocolate and dark chocolate?

iii likes dark chocolate, but dislikes milk chocolate?

iv dislikes both dark and milk chocolate?

10.03

Tree diagrams 10.04

STAGE 5.2

Tables and tree diagrams

Combined events: Tree diagrams

A **two-** or **three-step experiment** is a **chance experiment** that has 2 or 3 parts or stages, for example:

- rolling 2 or 3 dice
- drawing 2 or 3 prizes in a raffle
- observing the weather each day over a weekend or a long weekend
- throwing 2 or 3 coins together

A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each step or stage. An outcomes column at the end of the diagram lists the sample space.

The sample space for two-step experiments can be displayed using lists, tables or tree diagrams, but the sample space for three-step experiments is best displayed using a tree diagram.

Example 6

A coin is tossed and a die is rolled.

a Use a table to display the sample space.

b Find the probability of rolling:

i a tail and a 3 **ii** a head and an even number.

STAGE 5.2

Solution

a The sample space of a coin is a head (H) and a tail (T).

The sample space for a die is 1, 2, 3, 4, 5 and 6.

The sample space of tossing a coin and rolling a die is shown in the table below.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

Using a table ensures that all outcomes are counted

b **i** There are 12 outcomes in the sample space.

$\therefore P(\text{a tail and a 3}) = P(\text{T3}) = \frac{1}{12}$

'Tail and 3' is a compound event.

ii There are 3 outcomes that make up the event a head and an even number: H2, H4, H6

$\therefore P(\text{a head and an even number}) = \frac{3}{12}$

$= \frac{1}{4}$

Tree diagrams 3

Tree diagrams 1

Tree diagrams 2

Example 7

2 coins are tossed.

a Use a tree diagram to list the sample space.

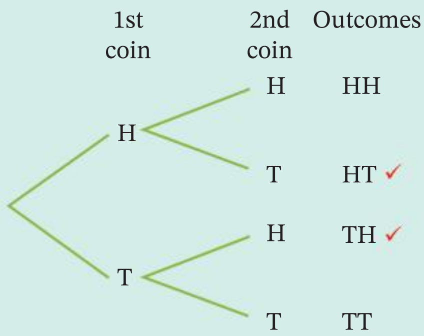
b Find the probability of tossing:

i 2 heads **ii** a head and a tail (in any order).

Solution

a There are 2 outcomes for the first coin, followed by 2 outcomes for the second coin. There are $2 \times 2 = 4$ possible outcomes.

Using a tree diagram ensures that all outcomes are counted.



b **i** There is one outcome out of a possible 4 for 2 heads.

$\therefore P(\text{2heads}) = \frac{1}{4}$

ii There are 2 outcomes for a head and a tail (✓ on the tree diagram).

$\therefore P(\text{a head and a tail}) = \frac{2}{4}$

$= \frac{1}{2}$

EXERCISE 10.04 ANSWERS ON P. 540

STAGE 5.2

Tree diagrams U F PS R C

EXAMPLE 6

10.04

1 A boy and a girl are to be chosen from this shortlist to represent the school at a conference:

Boys: Ben, Christian, Ewan, William.

Girls: Becky, Cassandra, Emily, Marianne, Roz, Siana. R C

a List all the possible pairs of a boy and a girl.

b Find the probability of selecting:

i Christian and Roz

ii a boy and a girl whose names begin with a B or a C

iii a pair that includes Emily.

2 2 coins are tossed together. R C

a Copy and complete the table to find all the outcomes in the sample space.

		1st coin	
		H	T
2nd coin	H		
	T		

b What is the probability of tossing:

i 2 tails? **ii** a head and a tail?

iii at least one head?

3 2 dice are rolled.

a Copy and complete this table to list the sample space. PS R C

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1				
	2						
	3						
	4			3, 4			
	5						
	6						

b How many possible outcomes are there?

c Find the probability of rolling:

i two 6s

ii at least one 6

iii doubles

iv 2 even numbers

v at least one 2

vi 2 numbers greater than 3

vii one odd and one even number

viii 2 numbers where the first number is greater than the second number.

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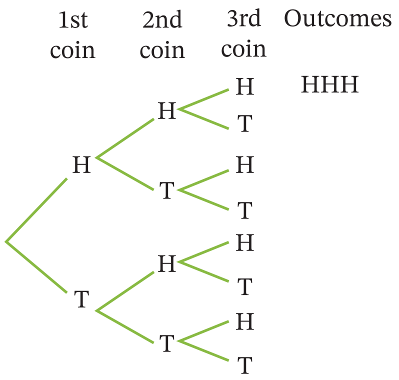
STAGE 5.2

EXAMPLE 7

4 3 coins are tossed. PS R C



a Copy and complete the tree diagram.

b How many outcomes are there in the sample space?

c Use the tree diagram to find:

i *P*(3 heads) **ii** *P*(2 heads)

iii *P*(3 tails)

iv *P*(head, then tail and then head)

v *P*(2 heads or 3 heads)

d Find the probability of tossing:

i at least 1 tail **ii** at most 2 tails.

e If 3 coins are tossed 200 times, find the expected frequency of:

i tossing 2 heads **ii** tossing no tails.

5 Use a tree diagram to display all possible outcomes when a coin and die are tossed together. R C

6 A 6-sided die and a 4-sided die (numbered 1, 2, 3 and 4) are rolled together. PS R C

a Construct a table to list the outcomes in the sample space.

b How many outcomes are in the sample space?

c Find the probability of rolling:

i doubles

ii 2 even numbers

iii one even and one odd number

iv a pair of numbers that are both less than 4

v a pair of numbers that are both greater than 4.

7 2 dice are rolled and the sum of the 2 numbers is calculated. PS R C

a Copy and complete this table to show all possible sums.

		1st die					
		1	2	3	4	5	6
2nd die	1	2					
	2						
	3				7		
	4						
	5						
	6					11	

b Find the of probability of rolling a sum:

i of 5 **ii** of 12 **iii** of 7

iv that is even **v** less than 2 **vi** more than 7

vii at least 7 **viii** between 4 and 8.

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8 4 coins are tossed. PS R C

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

i 4 heads **ii** 1 head **iii** 2 tails

iv at least 1 tail **v** 2 heads and then 2 tails **vi** not more than 1 tail

c If 4 coins are tossed 1000 times, find the expected frequency of having:

i 4 heads **ii** 2 heads and 2 tails **iii** at least one tail.

9 The weather on a long weekend will either be fine or rain each day, with each outcome being equally likely. PS R C

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that:

i it rains on all 3 days?

ii it is fine on 2 of the 3 days?

iii it is fine on Saturday and Sunday, but rains on Monday?

iv it rains on at least one day of the long weekend?

Mental skills 10: Maths without calculators ANSWERS ON P. 541

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

1 Study each example.

a Increase 360 by 25%.

$$25\% \text{ of } 360 = \frac{1}{4} \times 360$$
$$= 360 \div 4$$
$$= 90$$
$$360 + 90 = 450$$

b Increase \$80 by 5%.

$$5\% \text{ of } \$80 = \frac{1}{20} \times \$80$$
$$= \$80 \div 20$$
$$= \$4$$

or

$$10\% \text{ of } \$80 = \$8$$
$$\therefore 5\% \text{ of } \$80 = \$8 \div 2$$
$$= \$4$$

$$\$80 + \$4 = \$84$$

2 Now increase:

a \$340 by 20% **b** 66 by 50% **c** 150 by $33\frac{1}{3}\%$

d \$400 by 1% **e** 640 by 5% **f** \$72 by $12\frac{1}{2}\%$

g \$470 by 10% **h** 180 by 25% **i** 420 by $33\frac{1}{3}\%$

j \$80 by 5% **k** \$280 by 25% **l** 70 by 20%

3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$.

$33\frac{1}{3}\%$ of \$225 $= \frac{1}{3} \times \$225$

$= \$225 \div 3$

$= \$75$

\$225 − \$75 = \$150.

b Decrease \$70 by 15%

10% of \$70 $= \frac{1}{10} \times \$70 = \$7$

$\therefore$ 5% of \$70 $= \frac{1}{2} \times \$7 = \$3.50$

$\therefore$ 15% of \$70 = (10% × \$70) + (5% × \$70)

= \$7 + \$3.50

= \$10.50

4 Now decrease:

a 440 by 25% **b** \$300 by 20% **c** 2400 by $33\frac{1}{3}\%$

d \$500 by 15% **e** \$250 by 10% **f** \$120 by 50%

g \$72 by $12\frac{1}{2}\%$ **h** 80 by 5% **i** \$85 by 20%

j \$3800 by 1% **k** \$440 by 15% **l** \$150 by $33\frac{1}{3}\%$

Investigation

The birth month paradox

A **paradox** is a statement or proposition that seems impossible but is actually true.

1 Copy this table.

Group	Outcome (Y or N)
1	
2	
3	
4	
5	

2 Randomly select a group of 5 people and ask them what month they were born in. If 2 or more people have the same birth month, record a Y in the table for Group 1, otherwise write N.

3 Repeat this process 4 more times, recording your results in the table.

4 Combine your results with those of 6 other students so that you have the outcomes for 30 groups.

5 What fraction of the groups had repeated birth months?

6 Collect the results of another group of 6 students. What fraction of the groups had a repeated birth month?

7 The birth month paradox is that in any randomly selected group of 5 people, the probability that at least 2 people have the same birth month is greater than 0.5. Have your results shown this to be true?

8 Can you show the following?

a For every 23 people selected at random, the probability that at least 2 people will share the same birthday is 50%.

b If 30 people are selected at random, this probability is 70%.

c If 50 people are selected at random, this probability is 97%

Selecting with and without replacement 10.05

STAGE 5.2

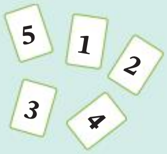
WS Multi-step experiments

10.05

In two- and three-step experiments where an item is selected repeatedly, the outcome of the second or third step may be affected by the outcome of the previous step. This depends on whether each selected item is **returned** to the set of items before the next item is selected. If it is, then this is called **selecting 'with replacement'**. If it isn't, then it is called **selecting 'without replacement'**.

Example 8

2 cards are selected at random from a set of cards numbered 1 to 5, to form a 2-digit number.



a Make a list of all possible outcomes if the cards are drawn:

i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that the number formed is:

i even **ii** greater than 30 **iii** divisible by 5.

c If the first card is not replaced, find the probability that the number formed is:

i even **ii** odd **iii** less than 20

Solution

a **i** The possible outcomes, **with replacement**, are:

		1st digit				
		1	2	3	4	5
2nd digit	1	11	21	31	41	51
	2	12	22	32	42	52
	3	13	23	33	43	53
	4	14	24	34	44	54
	5	15	25	35	45	55

There are $5 \times 5 = 25$ different outcomes possible.

ii The possible outcomes, **without replacement**, are:

		1st digit				
		1	2	3	4	5
2nd digit	1	-	21	31	41	51
	2	12	-	32	42	52
	3	13	23	-	43	53
	4	14	24	34	-	54
	5	15	25	35	45	-

There are fewer outcomes for 'without replacement' because numbers with repeated digits such as 11 and 44 are not allowed.

There are $5 \times 4 = 20$ different outcomes possible.

b **i** With replacement, there are 10 even numbers.

$P(\text{even number}) = \frac{10}{25} = \frac{2}{5}$

ii There are 15 numbers greater than 30.

$P(\text{number} > 30) = \frac{15}{25} = \frac{3}{5}$

iii There are 5 numbers divisible by 5. 15, 25, 35, 45 and 55

$P(\text{number divisible by 5}) = \frac{5}{25} = \frac{1}{5}$

c **i** Without replacement, there are 8 even numbers.

$P(\text{even}) = \frac{8}{20} = \frac{2}{5}$

ii There are 12 odd numbers.

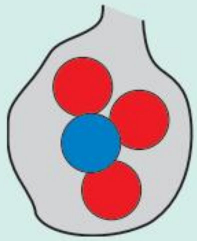
$P(\text{odd}) = \frac{12}{20} = \frac{3}{5}$

iii There are 4 numbers less than 20.

$P(\text{number} < 20) = \frac{4}{20} = \frac{1}{5}$

Example 9

A bag contains 3 red badges and one blue badge.
3 badges are drawn at random without replacement.



a Use a tree diagram to display all possible outcomes.

b Find the probability of drawing:

i 2 red badges

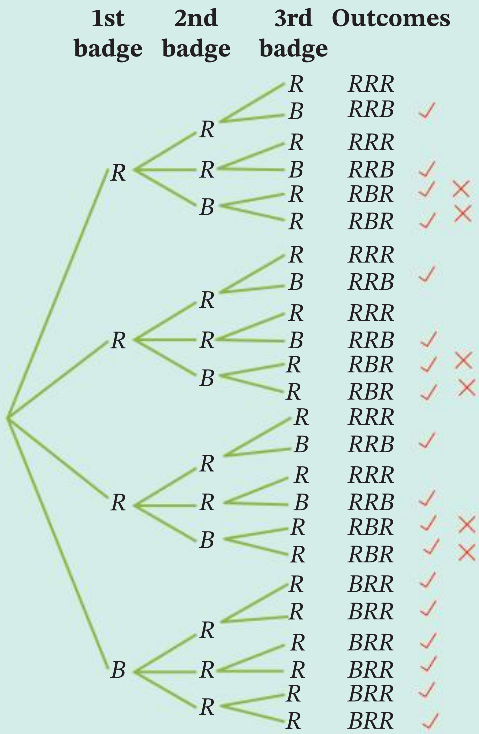
ii a red, blue and red in that order

iii at least one red badge.

Solution

a The tree diagram will have 4 branches for the first step or stage, followed by 3 branches for the second step, followed by 2 branches for the third step.

So there are $4 \times 3 \times 2 = 24$ outcomes in the sample space.



b **i** There are 18 outcomes with 2 red badges (✓ on the tree diagram)

$\therefore P(\text{2 red badges}) = \frac{18}{24} = \frac{3}{4}$

ii Red, blue, red (RBR) occurs 6 times (× on the tree diagram).

$\therefore P(\text{red, blue, red}) = \frac{6}{24} = \frac{1}{4}$

iii All outcomes contain at least one red badge.

$\therefore P(\text{at least one red badge}) = \frac{24}{24} = 1$

EXERCISE 10.05 ANSWERS ON P. 541

Selecting with and without replacement U F PS R C

1 The positions of captain and vice-captain of a netball team are to be selected from Cate, Amal, Gemma, Josie, Evangeline and Rukshana. R C

EXAMPLE 8

- **a** List the possible pairings of captain and vice-captain.
- **b** What is the probability of Amal being captain or vice-captain?
- **c** What is the probability of Evangeline becoming vice-captain?

Alamy Stock Photo/Tom Lindsey

2 2 cards are drawn from a set of cards labelled A, B, C, D and E. PS R C

- **a** Make a list of all possible outcomes if the cards are drawn:
  - **i** with replacement
  - **ii** without replacement.
- **b** If the first card is replaced before the second card is drawn, find the probability that:
  - **i** both letters are the same
  - **ii** both letters are vowels
  - **iii** one letter is a vowel and the other is a consonant.
- **c** If the first card is not replaced, find the probability that:
  - **i** both letters are vowels
  - **ii** one letter is a vowel and the other is a consonant
  - **iii** the first letter is a B or a D
  - **iv** the last letter is not A.

STAGE 5.2

3 When staying at a hotel, David and Sarah can select one item from each course of a breakfast menu. R C

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Fruit (F)	Ham and cheese croissants (H)
Yoghurt (Y)	Pancakes (P)
	Sausages and tomatoes (S)
	Toast and jam (T)

a Copy and complete the table to list all the different 2-course breakfasts available.

		2nd course				
		B	H	P	S	T
1st course	C					
	F					
	Y					

b If one of the combinations of breakfasts is chosen at random, what is the probability that it includes:

i fruit?

ii cereal but not bacon and eggs?

iii fruit and croissants?

EXAMPLE 9

4 The numbers 3, 4, 6 and 7 are written on separate cards and placed in a bag. 3 cards are drawn at random without replacement to form a 3-digit number. PS R C

a Copy and complete the tree diagram to show all 24 possible outcomes.

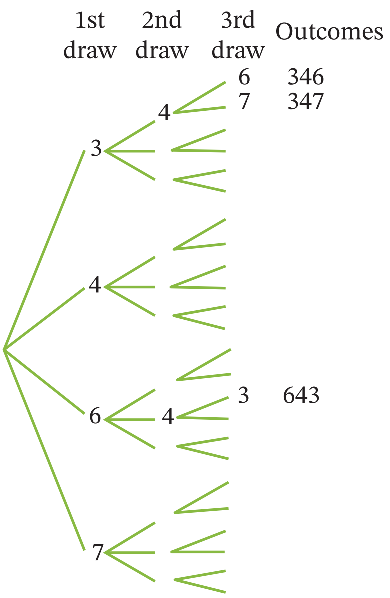
b Find the probability of forming a number:

i that is even

ii greater than 400

iii between 400 and 700

iv that is even and greater than 400.



5 The numbers 4, 5 and 9 are written on separate cards and placed in a bag. 3 cards are drawn at random with replacement to form a 3-digit number. PS R C

a Use a tree diagram to show all 27 possible outcomes.

b Find the probability of forming a number:

i with all digits the same

ii that is odd

iii less than 500

iv with all digits different.

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6 A bag contains 2 red marbles, 1 green marble and 1 yellow marble. 3 marbles are drawn from the bag at random without replacement. PS R C

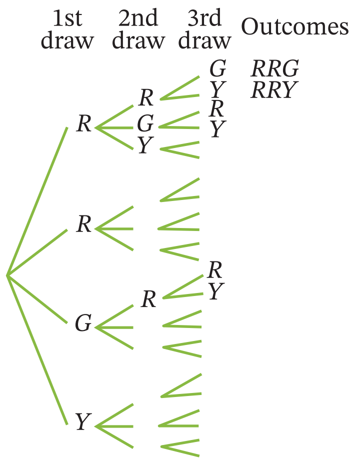
a Copy and complete the tree diagram to list the sample space.

b Find the probability of drawing:

i 2 red marbles

ii a red, green, and red marble in that order

iii at least one red marble.



7 A family has 3 children. PS R C

a Use a tree diagram to list all possible outcomes in the sample space.

b What is the probability that the family consists of:

i 3 boys? **ii** 3 girls?

iii 2 girls and a boy? **iv** a girl and then 2 boys?

8 2 ice cream flavours are selected from vanilla, strawberry and chocolate. PS R C

a Draw a tree diagram to show the sample space if the flavours are selected:

i with replacement (repetition allowed) **ii** without replacement.

b If the flavours are selected with replacement, find the probability of selecting:

i 2 identical flavours **ii** 2 different flavours

iii no vanilla **iv** at least one strawberry flavour.

c If the flavours are selected without replacement, find the probability of selecting:

i 2 identical flavours **ii** 2 different flavours

iii no vanilla **iv** at least one strawberry flavour.

Investigation

Dependent or independent?

Work in pairs.

You will need: a coin, 3 blue pens and 2 red pens.

1 **a** **i** Toss a coin and record the outcome (head or tail).

ii What is the probability of obtaining your outcome?

b **i** Toss the coin a second time and record the outcome.

ii What is the probability of obtaining the second outcome?

c Is the outcome of the second toss affected by the outcome of the first toss? Is the probability of the second outcome independent or dependent on the first outcome? Justify your answer.

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STAGE 5.2

2 a Copy this table.

With replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Put 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Put back the pen you drew and shake the bag. Draw a pen again and record its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i *P*(blue pen drawn first) ii *P*(blue pen drawn second)

f i Are your 2 results for part **e** the same?

ii Would you expect the results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw?

3 a Copy this table.

Without replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Again, place 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Do not return the pen, shake the bag and draw a second pen, recording its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i *P*(blue pen drawn first) ii *P*(blue pen drawn second)

f i Are your 2 results for part **e** the same?

ii Would you expect your results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw? Compare your results with those of other students in your class.

10.06 Dependent and independent events

STAGE 5.2

Two events are **independent** if the outcome of one event **does not affect** the outcome of the other event. So, one event occurring does not change the probability of the other event. For example, if a coin and a die are tossed together, the 2 events are independent as the outcome on the coin does not affect the outcome on the die. Wearing blue socks and passing a driving test are also independent events.

Two events are **dependent** if the outcome of one event **does affect** the outcome of the other event. So, one event occurring changes the probability of the other event occurring. For example, when selecting 2 coloured pencils from a pencil case without replacement, the 2 events are dependent because the outcome of the second draw is affected by the outcome of the first draw. Raining on a school sports day and sport being cancelled are also dependent events.

Example 10

A coin and a die are tossed together.

a List the outcomes in the sample space.

b Find:

i P(head on the coin) **ii** P(even number on the die)

iii P(head and even number)

c Is P(head and even number) = P(head) × P(even number)?

d Are the 2 events dependent or independent?

Solution

a The outcomes are H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5 and T6.

b **i** $P(\text{head}) = \frac{1}{2}$ **ii** $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

iii $P(\text{head and even}) = \frac{3}{12} = \frac{1}{4}$ H2, H4 and H6

c Yes, since $P(\text{head and even}) = \frac{1}{4}$ and $P(\text{head}) \times P(\text{even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

d The 2 events are independent since the outcome when tossing a coin does not affect the outcome when rolling a die.

The product rule for independent events

Two events are **independent** if the outcome of one event does not affect the outcome of the other event.

If A and B are 2 independent events, then $P(A \text{ and } B) = P(A) \times P(B)$.

Example 11

A bag contains 3 brown chocolates and 1 white chocolate. 2 chocolates are drawn from the bag, without replacement.

a Find the probability of:

i selecting a brown chocolate with the first draw

ii selecting a brown chocolate with the second draw if the first chocolate was brown.

b Are selecting a brown chocolate with the first and second draws dependent or independent?

Solution

a **i** $P(\text{brown on the first draw}) = \frac{3}{4}$

ii After drawing a brown chocolate, there are 3 chocolates left, of which 2 are brown.

$\therefore P(\text{brown on the second draw}) = \frac{2}{3}$

b The bag contains 2 brown chocolates and 1 white chocolate for the second draw, so P(brown) decreases from $\frac{3}{4}$ to $\frac{2}{3}$. The second event is dependent on the first event.

EXERCISE 10.06 ANSWERS ON P. 543

Dependent and independent events U F PS R C

1 State whether each pair of events are dependent or independent. R C

- **a** Rolling 4 on a die and rolling an even number on another die
- **b** Rolling 6 on a die and rolling 6 again on the same die
- **c** Training hard at soccer and winning a soccer match
- **d** Drawing a red ball from a bag containing red and blue balls, replacing it and then drawing a blue ball from the bag
- **e** Electing a team captain from a group of players and then electing a vice-captain from the same group
- **f** Tossing 2 coins and obtaining a head on the first coin and a head on the second coin
- **g** Finding \$20 on the street and getting a phone call from your parent

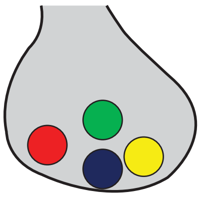
2 In Lotto, 6 balls are drawn without replacement. Are the events of drawing each of the balls dependent or independent events? Give reasons. R C

3 A coin is tossed 3 times and the result is heads each time. R C

- **a** Are each of the 3 coin tosses dependent or independent events?
- **b** The coin is tossed a 4th time. What is the probability of obtaining a head on the 4th toss?

EXAMPLE 10

4 A normal die is rolled and a marble is drawn from a bag containing a yellow marble, green marble, blue marble and red marble. R C



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- **a** Find the probability of:
  - **i** rolling a number less than 3 with the die
  - **ii** drawing a green marble from the bag.
- **b** List the outcomes for rolling the die and drawing a marble from the bag.
- **c** What is the probability of rolling a number less than 3 and drawing a green marble?
- **d** Is P(rolling a number less than 3) × P(drawing a green marble) = P(a number less than 3 and a green marble)?
- **e** Are the events of rolling a number less than 3 and drawing a green marble dependent or independent?

5 A red die and a blue die are rolled. R

- **a** Find:
  - **i** P(5 on the red die)
  - **ii** P(an even number on the blue die)
- **b** Hence find P(5 on the red die AND an even number on the blue die).

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STAGE 5.2

EXAMPLE 11

10.06

6 A bag contains 5 red balls and 4 yellow balls. 2 balls are drawn at random without replacement. R C

a What is the probability of drawing a red ball first?

b What is the probability of drawing a red ball on the second draw if the first ball was red?

c Are the 2 events dependent or independent? Give reasons.

7 A bag contains 5 yellow counters and 3 red counters. 2 draws are made with no replacement. Find the probability of drawing: R C

a **i** a yellow counter on the first draw

ii a yellow counter on the second draw after a yellow counter was drawn with the first draw

b **i** a red counter on the first draw

ii a yellow counter on the second draw after a red counter was drawn on the first draw

c **i** a yellow counter on the first draw

ii a red counter on the second draw after a yellow counter was drawn on the first draw

d **i** a red counter on the first draw

ii a red counter on the second draw after a red counter was drawn on the first draw.

8 3 children in a family are all girls. What is the probability that the next child in this family will be a girl? R

9 A pair of dice are rolled. Use the product rule to find the probability of rolling: PS R

a a 6 on the first die and an odd number on the second die

b a 1 on both dice

c a prime number on the first die and a factor of 6 on the second die

d an even number on the first die and a number between 2 and 5 on the second die

10 The probability that Natalie wins her tennis match is $\frac{13}{20}$. The chance that she will pass her Science exam is $\frac{4}{5}$. What is the probability that Natalie: PS R

a wins her tennis match and passes her Science exam?

b loses her tennis match and passes her Science exam?

11 The probability that it will rain on Saturday is 0.6. The probability that it will rain on Sunday is 0.25. What is the probability that it will rain: PS R

a on both Saturday and Sunday?

b on Sunday only?

c on neither Saturday nor Sunday?

12 A main road has 3 sets of traffic lights, each with a probability of 0.7 of being green. Use the product rule to find the probability of: PS R

a green on the first 2 lights, red (or amber) on the 3rd light

b green on all 3 lights

c red (or amber) on all 3 lights

d only one of the 3 lights being green

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STAGE 5.2

Did you know?

Lotteries and Lotto

A lottery is a game of chance in which numbered tickets are drawn from tickets that have been sold. Lotteries were introduced by the State Government to raise money for hospitals. The first lottery was drawn on 20 August, 1931 with a first prize of £5000.

Lotteries have been used to celebrate special events and to help finance special projects. The Opera House Lottery, which commenced selling on 25 November 1957, was used to finance the construction of the Sydney Opera House.

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Other games of chance have been introduced, including Lotto (1979), Instant Scratchies (1982), OzLotto (1994) and Powerball (1996).

Research the probability of winning Lotto, OZ lotto and Powerball.

10.07 Conditional probability

STAGE 5.2

WS Conditional probability

Conditional probability: Two-way tables

In many practical situations, events are **dependent**. For example, the probability of a student arriving at school on time when catching a bus may be dependent on the amount of traffic.

Conditional probability is used to calculate probabilities for dependent events.

The **conditional probability** of an event B *given event* A, also written as $P(B|A)$, is the probability that event B occurs, given that event A has already occurred.

Example 12

A bag contains 3 red marbles and 2 yellow marbles. 2 marbles are drawn at random from the bag without replacement. What is the probability that the second marble is yellow, given that the first marble was also yellow?

Solution

If the first marble is yellow, then there are 3 red marbles and 1 yellow marble left in the bag.

$\therefore$ P(second marble yellow, given the first marble is yellow) $= \frac{1}{4}$

Example 13

STAGE 5.2

Conditional probability

10.07

2 dice are rolled and their total is calculated.

a Use a table to show all possible totals.

b Given that the total is 7, what is the probability that one of the dice shows a 3?

c Given that one of the dice shows a 4, what is the probability that the total is 10?

d Given that the total is 6, what is the probability of a double?

e Given that a double is rolled, what is the probability of:

i a total of 12? **ii** a total less than 10?

Solution

a

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

36 possible outcomes in the sample space

b There are 6 outcomes that give a total of 7. (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 2)

If one of the dice shows a 3, the possible outcomes are (3, 4) and (4, 3).

The conditional event 'total of 7' reduces the sample space from 36 to 6 outcomes.

$P(\text{one die is 3, given total is 7}) = \frac{2}{6} = \frac{1}{3}$

c There are 11 outcomes that have 4 showing on one die. (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)

Of these outcomes, only 2 have a total of 10. (6, 4) and (4, 6)

$P(\text{total is 10, given one die is 4}) = \frac{2}{11}$

The conditional event 'one die is 4' reduces the sample space to 11 outcomes.

d There are 5 outcomes that give a total of 6. (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)

There is only 1 double. (3, 3)

$P(\text{double} \mid \text{total is 6}) = \frac{1}{5}$

'|' means 'given that'; 'total of 6' reduces the sample space to 5 outcomes.

e **i** There are 6 doubles that can be rolled. (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

(6, 6) is the only double with a total of 12.

The conditional event 'a double' reduces the sample space to 6 outcomes.

$P(\text{total is 12} \mid \text{double}) = \frac{1}{6}$

STAGE 5.2

ii (1, 1), (2, 2), (3, 3) and (4, 4) are the doubles with a total less than 10.

$P(\text{total} < 10 \mid \text{double}) = \frac{4}{6} = \frac{2}{3}$

EXERCISE 10.07 ANSWERS ON P. 543

Conditional probability U F PS R C

EXAMPLE 12

1 A bag contains 4 yellow and 3 red marbles. 2 marbles are drawn from the bag without replacement. R

a Given that the first marble was red, what is the probability that the second marble is also red?

b Given that the first marble was yellow, what is the probability that the second marble is red?

2 10 remote-controlled cars are delivered to a toy store, 3 of which are defective. 2 cars are randomly selected and tested. What is the probability that the second car tested is defective, given that the first car tested was defective and was not replaced? R C

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3 In a physics class, there are 5 boys and 7 girls. 2 students are randomly selected to help with an experiment. If the first student was a boy, what is the probability that: R

a both students chosen were boys?

b a boy and a girl were chosen?

4 In Aiden's pencil case, there are 3 red pens, 4 blue pens and 5 black pens. Aiden takes out 2 pens at random. Find: R C

a $P(\text{2nd pen red} \mid \text{1st pen red})$

b $P(\text{2nd pen blue} \mid \text{1st pen red})$

c $P(\text{2nd pen black} \mid \text{1st pen black})$

d $P(\text{2nd pen black} \mid \text{1st pen blue})$

5 A die is rolled and a number less than 4 is the result. What is the probability that the number is even?

6 A coin is tossed and a die is rolled at the same time. Knowing that an even number has been rolled, what is the probabililty of the result being a head and a 4? R C

EXAMPLE 13

7 2 dice are rolled and the sum of the numbers is calculated. PS R C

a Draw up a table to show all possible sums.

b Given that the sum is 9, find the probability that:

i one of the dice shows a 4

ii one of the dice shows an even number.

c Knowing that one of the dice shows a 6, find the probability that the sum is 11.

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d Given that one of the dice shows an even number, find the probability that:

i the sum is even **ii** the sum is 7.

e If the dice show a double, what is the probability of a sum of 4?

8 A drawer contains 5 different pairs of coloured socks: black, blue, red, brown and white.

a Haylee randomly takes 2 socks from the drawer. Find: R C

i P(2nd sock blue | 1st sock blue)

ii P(2nd sock not blue | 1st sock blue)

b Haylee selects a red sock and a blue sock. What is the probability that the third sock she selects will form a matching pair?

c What is the maximum number of socks that Haylee will need to take from the drawer before she has at least one matching pair of socks?

9 Lotto is a game of chance in which 6 balls are selected at random from a barrel containing balls numbered 1 to 45. What is the chance of Yuri winning Lotto with the 6th ball, given that he has the first 5 numbers? R

10 3 cards are chosen at random from a normal deck of 52 cards. Given that the 3 cards are hearts, what is the probability that the 4th card is also a heart? R

11 A card is drawn at random from a deck of 52 cards. What is P(Queen | heart card)? R C

12 2 dice are rolled. What is P(2nd dice = 6 | 1st die = 6)? R C

13 2 dice are rolled and the difference between the numbers is calculated. PS R C

a Copy and complete this table to show all possible outcomes.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					3	
	3						
	4			1			
	5						
	6						

6 − 1 = 5

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The difference between these numbers is 5.

b What is the probability of rolling a difference:

i of 0? **ii** of 5? **iii** greater than 3?

c Knowing that the difference is 4, what is the probability that one of the dice shows:

i a 1? **ii** a 3?

d If one of the dice shows a 4, find the probability that the difference is:

i 0 **ii** 1

e Given that the difference is odd, find the probability that one of the dice shows:

i a 3 **ii** an even number.

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Power plus ANSWERS ON P. 544

1 Students at Nelson Secondary College were surveyed about which sport they like to watch and what type of movies they like to see.

	Horror/Drama	Fantasy	Comedy	Action
Football	23	34	30	48
Cricket	25	12	45	34
Tennis	8	12	32	17

a How many students were surveyed?

b If a student is selected at random, what is the probability that the student likes to watch:

i horror/drama movies?

ii comedy and football?

iii tennis, but not fantasy?

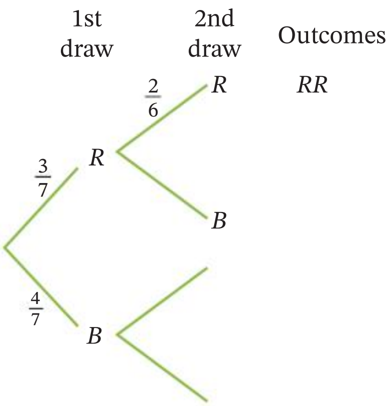
iv action, but not cricket or tennis?

c Given that a student likes to watch football, find the probability that the student also likes to watch action movies.

d Of the students who like comedy, what is the probability that they also like to watch cricket?

2 A bag contains 3 red and 4 blue marbles. 2 marbles are taken out of the bag without replacement.

a A probability tree is a tree diagram that has the probability of each step or stage listed on the branches. Copy and complete the probability tree shown to list all possible outcomes.



b Use the probability tree diagram to find the probability of drawing:

i 2 red marbles

ii 2 blue marbles

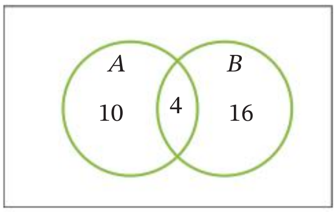
iii a blue and a red marble

iv at least one blue marble.

3 **a** Find:

i $P(A)$ **ii** $P(B)$ **iii** $P(A \text{ and } B)$

iv $P(A \mid B)$ **v** $P(B \mid A)$



b **i** Find the value of $\frac{P(A \text{ and } B)}{P(B)}$

ii Is $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$?

c Show, by calculation, that $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$.

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CHAPTER 10 REVIEW

Language of maths

at least	compound event	conditional probability
dependent event	die/dice	event
expected frequency	experimental probability	independent event
mutually exclusive	observed frequency	overlapping
random	relative frequency	sample space
table	theoretical probability	three-step experiment
tree diagram	trial	two-step experiment
two-way table	Venn diagram	with/without replacement

1 What is the meaning of **expected frequency**?

2 What term from the above list is another name for **experimental probability**?

3 On a **Venn diagram**, what does the rectangle represent?

4 Give an example of **dependent events**.

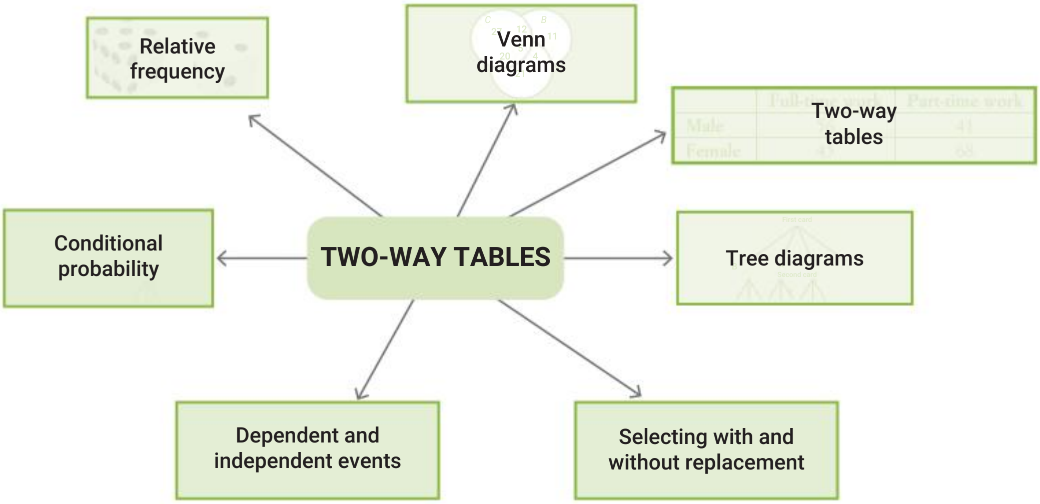
5 When are **tree diagrams** used in probability?

6 For 2 events A and B, what is the difference between '**A or B**' and '**A and B**'?

Topic summary

WS Mind map: Probability

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

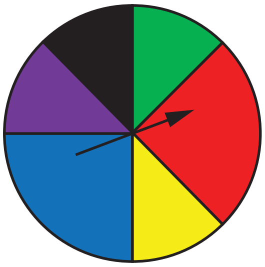


TEST YOURSELF 10

ANSWERS ON P. 544

10.01

1 For the spinner shown, the red and blue sectors are twice as large as the other sectors. Rafiya spun the spinner 100 times and her results are shown in the table.



Outcome	Frequency
Red	22
Blue	29
Yellow	10
Purple	12
Black	13
Green	14

a What is the experimental probability (as a decimal) of the arrow stopping on:

i red? **ii** blue? **iii** yellow? **iv** green?

b What is the theoretical probability (as a decimal) of the arrow stopping on:

i red? **ii** blue? **iii** yellow? **iv** green?

c Are the experimental and theoretical probabilities similar?

d What is the experimental probability of the arrow stopping at purple or black?

e What is the expected frequency of a colour that is not purple or black? How does this compare with Rafiya's observed frequency?

10.01

2 3 coins are tossed 150 times and the number of heads at each trial is recorded in the table.

Number of heads	Frequency
0	20
1	53
2	64
3	13

a Find correct to 3 decimal places the relative frequency of tossing:

i one head **ii** 2 heads

iii 3 heads **iv** at least 2 heads.

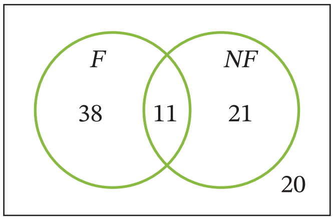
b Find correct to 3 decimal places the experimental probability of:

i at least one head **ii** 3 tails.

c Are the answers in part **b** the same or different? Explain why.

10.02

3 People at a train station were asked whether they prefer to read fiction (F) or non-fiction (NF) books. The results are shown in the Venn diagram.



a How many people were surveyed?

b Find the probability of selecting a person from this group who only reads fiction books.

c What is the probability of selecting a person who doesn't read fiction or non-fiction books?

d Of the people who read fiction books, find the probability that they read non-fiction books.

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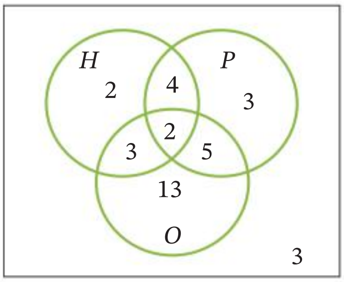
4 The Venn diagram shows the results of a survey on the types of music that school students listen to – Hip Hop/Rap (H), Pop (P) and Other (O), which includes R&B, Dance, Metal, Rock and Independent.



10.02

a How many students were surveyed?

b Find the probability of selecting a student who likes to listen to all types of music.

c What is the probability of selecting a student who listens to

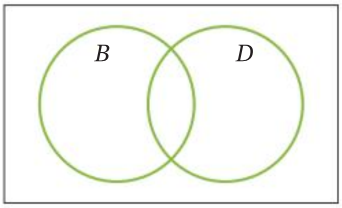
i Hip Hop/Rap and Pop?

ii Hip Hop/Rap or Pop?

iii Pop music only?

d Why are 3 students in the rectangle, but not in the circles?

5 Of 20 people in an office, 6 have blue eyes (B), 8 have dark hair (D) and 3 have blue eyes and dark hair.



10.02

a Copy and complete the Venn diagram to show the given information.

b What is the probability of selecting a person at random from the office who has:

i blue eyes only? **ii** dark hair?

iii blue eyes and dark hair? **iv** hair that is not dark?

c What is the probability of selecting a person at random who has neither blue eyes nor dark hair?

6 Students were asked what type of activities they would like to do on a camp.

10.03

	Hiking	Rock climbing	Kayaking
Boys	25	38	47
Girls	45	23	22

a How many students were surveyed?

b Find the probability (as a decimal) that a student selected at random:

i likes rock climbing

ii likes kayaking and is a girl

iii is a girl who likes hiking

iv is a boy who likes rock climbing or kayaking

c If a boy is selected at random, what is the probability that his favourite activity is hiking?

d If a student who likes kayaking is chosen, what is the probability that the student is:

i a boy? **ii** a girl?

7 A pair of dice are rolled and the sum of the 2 numbers is calculated. Find the probability of rolling a sum of 6. Select the correct answer **A**, **B**, **C** or **D**.

STAGE 5.2

10.04

A $\frac{1}{5}$ **B** $\frac{1}{9}$ **C** $\frac{5}{6}$ **D** $\frac{5}{36}$

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STAGE 5.2

8 A pair of 4-sided dice (numbered 1, 2, 3 and 4) are rolled.

a Copy and complete this table.

		1st die			
		1	2	3	4
2nd die	1				
	2		2, 2		
	3				
	4			3, 4	

b How many possible outcomes are there?

c Find the probability of rolling:

i one odd and one even number

ii 2 even numbers

iii at least one 3

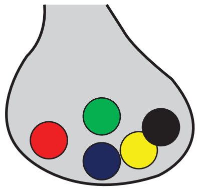
iv 2 numbers less than 3

v a double

vi 2 numbers so that the first number is odd.

10.05

9 2 marbles are drawn from a bag containing a red, a blue, a green, a yellow and a black marble.



a Make up a list to show all the possible outcomes if the marbles are taken:

i with replacement

ii without replacement.

b If the first marble is replaced before the second marble is drawn, find the probability of drawing:

i 2 red marbles

ii 2 marbles of the same colour

iii a yellow and a black marble

iv at least one green marble.

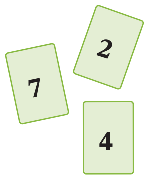
c If the first marble is not replaced, find the probability of drawing:

i a green marble and a yellow marble

ii no red marbles.

10.05

10 The numbers 2, 4, and 7 are written on separate cards and placed in a bag. 3 cards are drawn at random to form a 3-digit number.



a Make up a tree diagram to list the sample space if the cards are drawn:

i with replacement

ii without replacement.

b If the cards are drawn with replacement, find the probability of forming:

i an even number

ii a number less than 400

iii the numbers 222, 444, or 777

iv an odd number greater than 400.

c If the cards are drawn without replacement, find the probability of forming:

i an odd number

ii a number greater than 400

iii a number beginning with 7

iv a number divisible by 4.

10.06

11 State whether the following pairs of events are dependent or independent.

a Tossing a tail from a coin and then a head from the same coin.

b Drawing a ticket in a raffle and winning a first prize and then drawing a second ticket and winning a second prize.

c Electing a president for a cricket club and then electing the vice-president of the cricket club.

d A family's first 3 children are girls and then the 4th child is also a girl.

e Rain in your town or city today and a car accident.

f The Roosters winning a football match and you winning Lotto.

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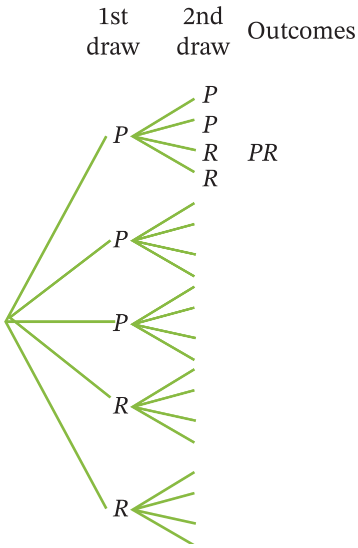
12 2 hair ties are drawn without replacement from a bag containing 3 purple and 2 red hair ties.



STAGE 5.2

10.07

- **a** Copy and complete the tree diagram to show the sample space.
- **b** Given that the first hair tie drawn is purple, what is the probability that the second hair tie will be:
  - **i** red? **ii** purple?
- **c** Given that the first hair tie drawn is red, find the probability that the second hair tie is:
  - **i** red **ii** purple.
- **d** Given that the drawn hair ties are the same colour, what is the probability that they are:
  - **i** both red? **ii** both purple?

10.07

13 The pair of 4-sided dice from question **8** are rolled and the sum of the numbers is calculated.

- **a** Draw up a table to show all possible sums.
- **b** Given that the sum is 5, find the probability that one of the dice shows:
  - **i** a 2 **ii** an odd number.
- **c** Find P(sum of 7 | one die is 4).
- **d** Given that one of the dice shows an even number, find the probability that the sum:
  - **i** is even **ii** is 5.
- **e** Find P(sum of 2 | dice show a double).

TEST YOURSELF 10

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11

MEASUREMENT AND GEOMETRY

GEOMETRY

The word 'geometry' comes from the Greek word *geometria*, which means 'earth measuring'. The principles and ideas of geometry are evident in many aspects of our lives. For example, geometry can be seen in the design of buildings, bridges, roads and transport networks, such as this footbridge over the Yarra River in central Melbourne.

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Chapter outline

		Working mathematically				
11.01	Angle sum of a polygon*#	U	F		R	C
11.02	Exterior angle sum of a polygon*#	U	F		R	
11.03	Congruent triangle proofs*	U	F		R	C
11.04	Proving properties of triangles and quadrilaterals*	U	F		R	C
11.05	Similar figures	U	F		R	C
11.06	Finding unknown sides in similar figures	U	F	PS	R	
11.07	Tests for similar triangles*	U	F		R	C

***STAGE 5.2**

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Wordbank

congruence test One of 4 tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical, exactly the same (symbol: ≡)

enlargement An increase in the size of a shape

included angle The angle between 2 given sides of a shape

convex polygon A polygon whose vertices all point outwards

regular polygon A polygon with all angles equal and all sides equal, such as an equilateral triangle or a square

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||)

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.2) solve geometrical problems involving the angle sum of a polygon and the exterior angle sum of a convex polygon
- (STAGE 5.2) write formal proofs for congruent triangles
- investigate properties of special triangles and quadrilaterals using congruent triangles and solve related geometry problems
- (STAGE 5.2) formulate proofs involving congruent triangles and angle properties
- use scale factors to find unknown sides in similar figures
- (STAGE 5.2) identify and use the 4 tests for similar triangles

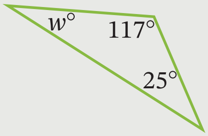
SkillCheck ANSWERS ON P. 545

Geometry

Finding angles

1 Find the value of each variable.

a



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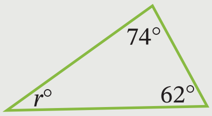

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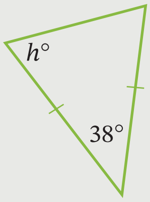

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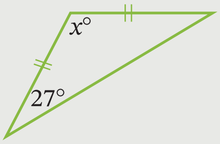

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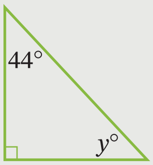

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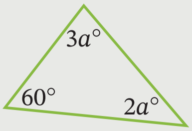
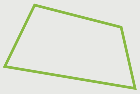


2 Match shapes that are congruent.

A



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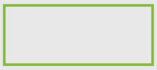

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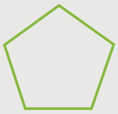

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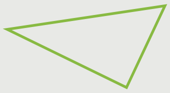

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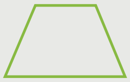

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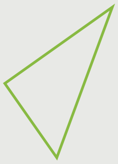
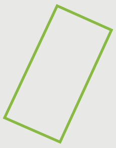
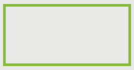


G

H



I



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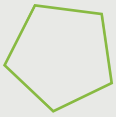

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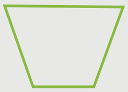
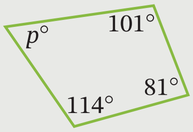
L



3 Find the value of p in each diagram.

a



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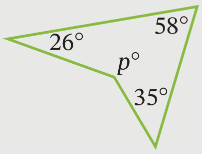

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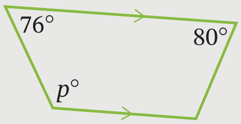

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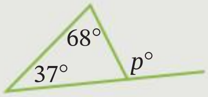
d



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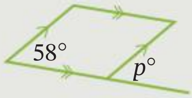

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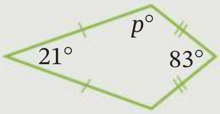
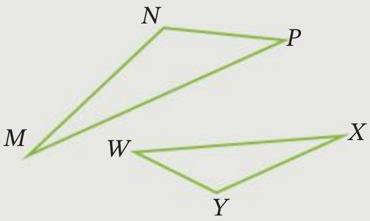


4 Triangles *MNP* and *WXY* are similar.

a List all pairs of matching angles.

b List all pairs of matching sides.

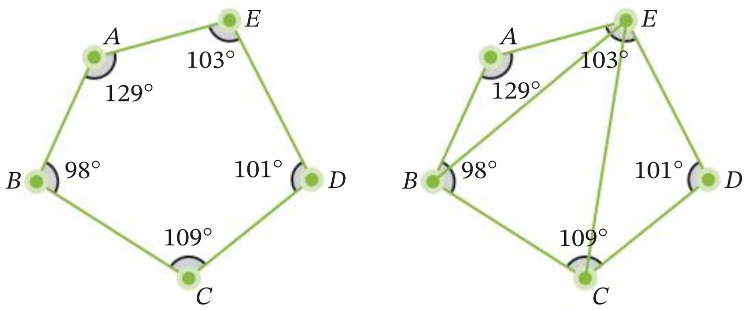


Technology

Angle sum of a polygon

In this activity, you will use dynamic geometry software to find the angle sum of a polygon.

1 Construct a pentagon. Measure the size of each angle in the pentagon (the **interior angles**). Find the sum of the 5 interior angles. Write your answer in your book.



2 From one vertex, draw as many triangles as possible in your pentagon, as shown above.

3 Copy and complete the first row of this table for the pentagon.

No. of sides in polygon	Angle sum of each triangle	Angle sum of all triangles in polygon	$\frac{\text{Angle sum of polygon}}{\text{Angle sum of triangle}}$
5	180°		

4 Repeat Steps **1** to **3** for each shape:

- regular hexagon
- regular pentagon
- octagon
- nonagon (9 sides)

5 What is the relationship between the number of sides in a polygon (n) and the angle sum of a triangle?

6 Copy and complete:

$$\text{Angle sum of a polygon with } n \text{ sides} = 180 \times (n - \_\_\_\_)^\circ$$

11.01 Angle sum of a polygon#

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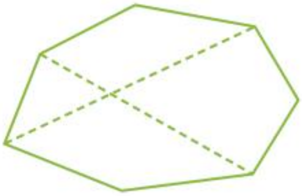
STAGE 5.2

WS Angle sum of a polygon

WS Equal angles

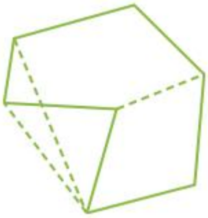
A **polygon** is any shape with straight sides. A polygon may be **convex** or **non-convex** (**concave**).

Convex polygon



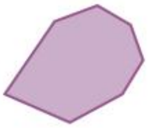
- All vertices point outwards.
- All diagonals lie within the shape.
- All angles are less than 180°.

Non-convex polygon

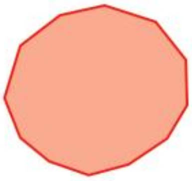


- Some vertices point inwards.
- Some diagonals lie outside the shape.
- Some angles are more than 180° (reflex angles).

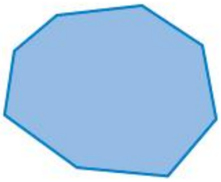
Name	Number of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Hendecagon	11
Dodecagon	12



heptagon



dodecagon



octagon

Angle sum of a polygon

The angle sum of a polygon with n sides is given by the formula

$$A = 180(n - 2)^\circ.$$

This formula applies to both convex and non-convex polygons.

STAGE 5.2

11.01

Example 1

Find the angle sum of a 15-sided polygon.

Solution

Angle sum = $180\,(15 - 2)°$ $\quad$ $n = 15$

$= (180 \times 13)°$

$= 2340°$

Example 2

Find the number of sides in a polygon that has an angle sum of 1080°.

Solution

$180\,(n - 2) = 1080$ $\quad$ $A = 1080$

$180n - 360 = 1080$

$180n = 1440$

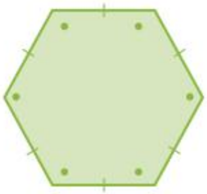
$n = \frac{1440}{180}$

$= 8$

∴ The polygon has 8 sides (octagon).

Regular polygons

A **regular polygon** has all its angles and sides equal. For example, a regular hexagon has 6 equal angles and 6 equal sides. A square is a regular polygon, but a rhombus is not.



Angle in a regular polygon

The size of each angle in a **regular polygon** with n sides is:

$$\frac{\text{Angle sum}}{\text{Number of sides}} = \frac{180(n-2)°}{n}$$

Example 3

Find the size of one angle in a regular pentagon.

Solution

A pentagon has 5 sides ($n = 5$).

$$\text{Size of one angle} = \frac{180(5-2)°}{5}$$

$$= \frac{(180 \times 3)°}{5}$$

$$= 108°$$

Each angle in a regular pentagon is 108°.

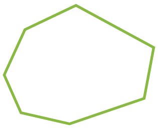
STAGE 5.2

EXERCISE 11.01 ANSWERS ON P. 545

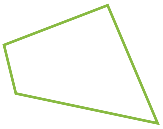
Angle sum of a polygon U F R C

1 Name each polygon. R C

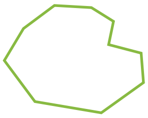
a



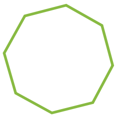
b



c

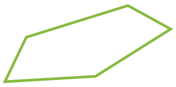


d



e

f



2 Which polygons from question **1** are: R C

a convex? **b** regular?

3 What is the more common name for a regular triangle?
Select the correct answer **A**, **B**, **C** or **D**. R C

A isosceles **B** scalene **C** equilateral **D** acute

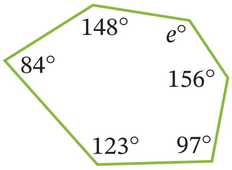
EXAMPLE 1

4 Find the angle sum of a polygon with:

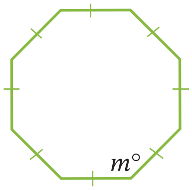
a 12 sides **b** 10 sides **c** 9 sides **d** 20 sides **e** 15 sides

5 Find the value of each variable. R

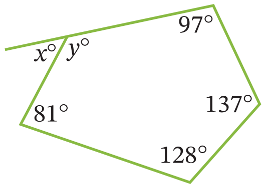
a



b



c



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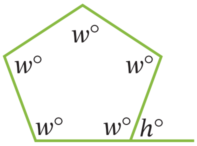

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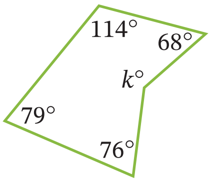

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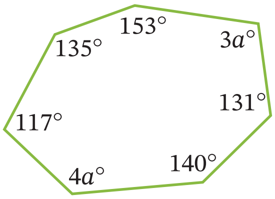

EXAMPLE 2

6 Find the number of sides in a polygon that has an angle sum of: R

a 720° **b** 3420° **c** 1980° **d** 5040° **e** 1260°.

7 The angle sum of a regular polygon is 2520°. R

a How many sides does the polygon have?

b Find the size of each angle.

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8 Find the size of one angle in a regular: R

a decagon b octagon c hexagon d dodecagon.

9 Find the size of one angle in a regular polygon with 15 sides. Select **A**, **B**, **C** or **D**. R

A 142° B 150.5° C 156° D 165°

10 How many sides does a regular polygon have if each of its angles is: R

a 168°? b 172°? c 165.6°?

STAGE 5.2

EXAMPLE 3

11.01

Did you know?

The Beijing 'Water Cube'

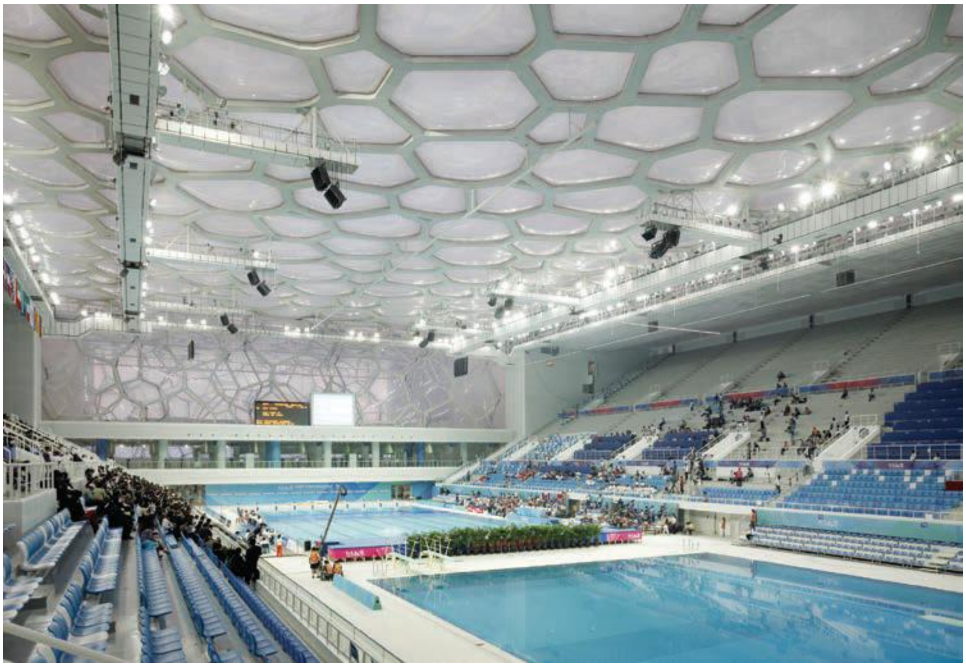
The National Aquatic Centre in Beijing, China, was specifically designed by Australian architects for the 2008 Summer Olympic Games. Also called the 'Water cube', it held the Olympic swimming, diving and synchronized swimming events.

This aquatic centre was built using a groundbreaking geometric and sustainable design for its time, inspired by the geometry of the soap bubble, which has the properties of a polygon. The polygonal panels are made from ETFE and are semi-translucent. They allow natural daylight into the centre and save on lighting energy.

Beijing is a city that suffers from water scarcity and water conservation is also a key feature in the geometric design. Pipes in the polygon structure allow water to be harvested from roof catchment areas and reused throughout the building.

1 **Find out why the geometry of the polygon was useful for the design of the aquatic centre, and the special properties of the material ETFE.**

2 **The polygons are what gives the building its name 'Water cube', find out why.**



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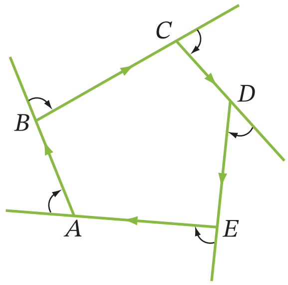
STAGE 5.2

Investigation

Exterior angle sum of a convex polygon

1. Draw any convex polygon and extend the sides as shown. Label the vertices A, B, C, etc.
2. Use a protractor to measure all of the exterior angles of the polygon.



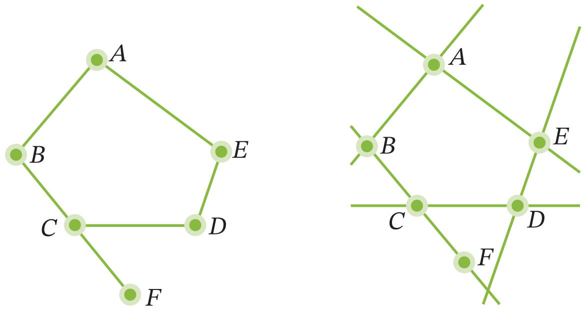
3. What is the sum of the exterior angles of the polygon?
4. Start at A and move around the polygon, turning in the direction indicated at each vertex, until you return to A, facing the same direction that you started from.
5. What must be the sum of the turns in any round trip of a convex polygon?
6. Test whether this rule works for a non-convex polygon.

Technology

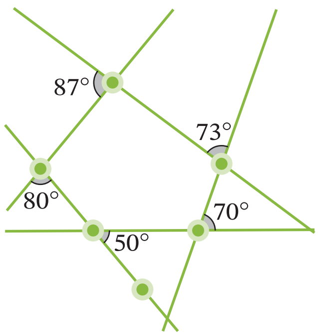
Exterior angle sum of a polygon

In this activity, you will use dynamic geometry software to find the exterior angle sum of a polygon.

1. Construct any pentagon, $ABCDE$.
2. To produce side BC, draw a segment from C to F.



3. Now produce the remaining 4 sides of the pentagon, as shown above.
4. Measure the size of each of the angles outside the pentagon (the **exterior angles**). For example, $\angle FCD = 50°$.



5. Calculate the total sum of the 5 exterior angles. Write the answer in your book.
6. Repeat steps **1** to **5** for each shape.
   - octagon
   - regular hexagon
   - regular heptagon

7 Copy and complete the rule for the exterior angle sum of any polygon:

Exterior angle sum of a polygon = ________

8 Copy and complete the rule for each exterior angle of any regular polygon:

Exterior angle of a regular polygon with n sides = ________

STAGE 5.2

Exterior angle sum of a convex polygon#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

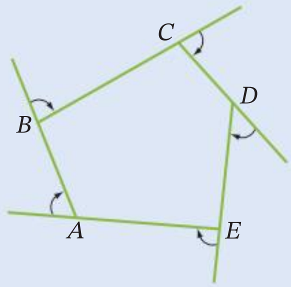
11.02

Exterior angle sum of a convex polygon

STAGE 5.2

WS Angles in polygons

The sum of the exterior angles of a convex polygon is 360°.

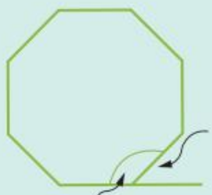


Example 4

For a regular octagon, find the size of:

a each exterior angle

b each (interior) angle.



Solution

a Sum of exterior angles = 360°

One exterior angle = 360° ÷ 8 = 45°

b Each angle = 180° – 45° (angles on a straight line)

= 135°

OR : Each angle = $\frac{180(8-2)^\circ}{8}$

= 135°

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STAGE 5.2

Example 5

Find the number of sides of a regular polygon if:

a each exterior angle is 24°

b each (interior) angle is 140°.

Solution

a Number of exterior angles = 360 ÷ 24

= 15

∴ The regular polygon has 15 sides.

b Exterior angle = 180° – 140° (angles on a straight line)

= 40°

Sum of exterior angles = 360°

Number of exterior angles = 360° ÷ 40°

= 9

OR: $\frac{180(n-2)}{n} = 140$

$180(n-2) = 140n$

$180n - 360 = 140n$

$40n = 360$

$n = \frac{360}{40}$

$= 9$

∴ The regular polygon has 9 sides.

EXERCISE 11.02 ANSWERS ON P. 546

Exterior angle sum of a polygon U F R

EXAMPLE 4

1 Find the size of each exterior angle in a regular:

a pentagon **b** dodecagon

c 18-sided polygon **d** hexagon.

2 Find the size of each angle in a regular: R

a nonagon **b** 20-sided polygon

c decagon **d** 30-sided polygon.

EXAMPLE 5

3 Find the number of sides in a regular polygon if each exterior angle is:

a 15° **b** 72° **c** 20° **d** 40° **e** 5°

4 Find the number of sides in a regular polygon with an exterior angle of 12°. Select the correct answer **A**, **B**, **C** or **D**.

A 12 **B** 15 **C** 30 **D** 45

5 Find the number of sides in a regular polygon if each angle is: R

a 135° **b** 144° **c** 156°

d 178° **e** 165° **f** 150°.

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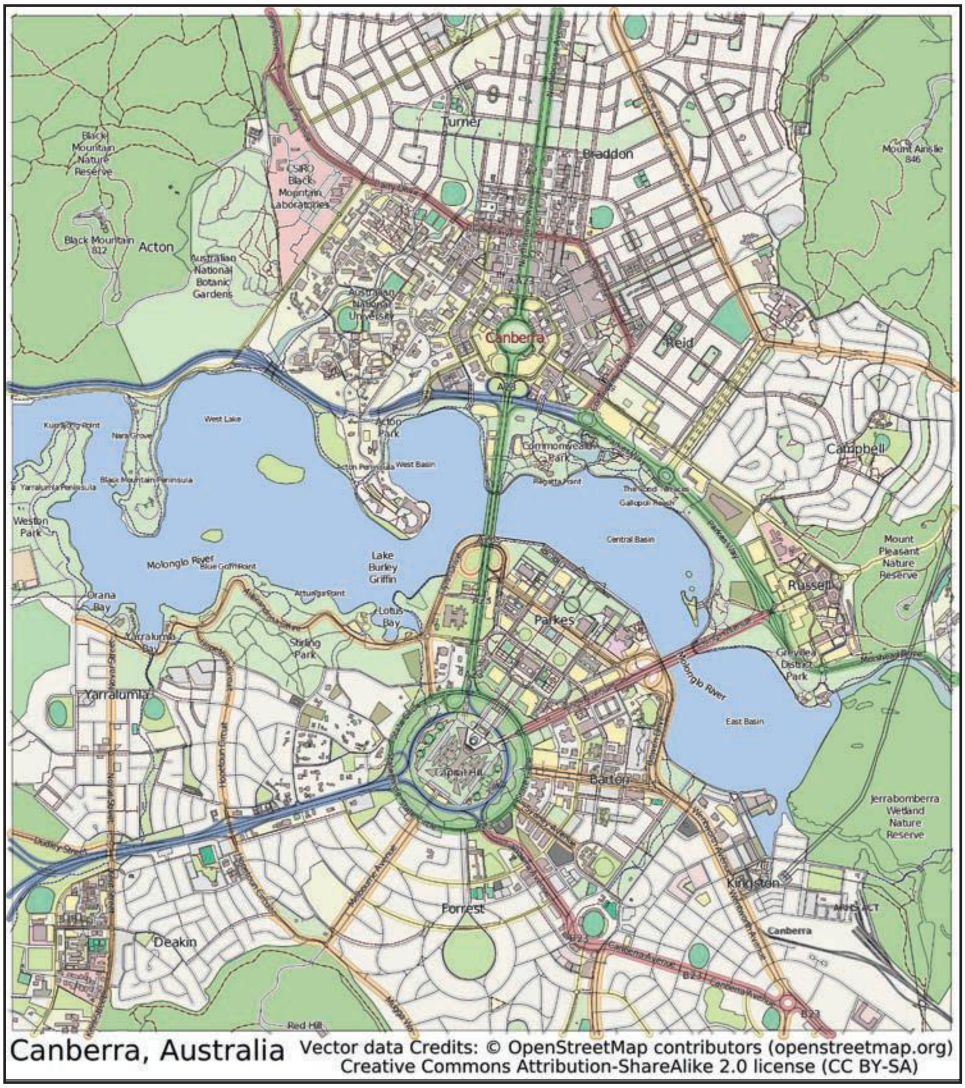
Did you know?

The geometry of Canberra

Canberra is located 300 km south-west of Sydney and was designed by the American architect Walter Burley Griffin. Construction of Australia's capital city began in 1913. The 'centre' of Canberra is based on an equilateral triangle, bounded by the 'sides' Commonwealth Avenue, Kings Avenue and Constitution Avenue. The smaller 'Parliamentary triangle' is bounded by Commonwealth Avenue, Kings Avenue and King Edward Terrace. The axis of symmetry of the triangle runs from Parliament House, across Lake Burley Griffin, directly along Anzac Parade to the Australian War Memorial.



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What other geometrical features can you see in Canberra's design?

11.02

11.03 Congruent triangle proofs

STAGE 5.2

WS Congruent triangles proofs

WS Congruent triangles

WS Congruent and similar triangle proofs

Congruent triangles proofs

Two figures are **congruent** if they are identical in shape and size. For congruent figures, **matching sides** are equal and **matching angles** are equal.

There are 4 sets of conditions that can be used to determine if **2 triangles** are congruent.

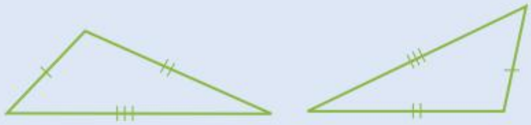
These are called the **tests for congruent triangles** or **congruence tests**.

Congruence tests

There are 4 tests for congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

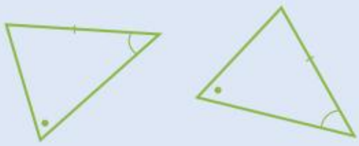
Two triangles are congruent if:

- the 3 sides of one triangle are respectively equal to the 3 sides of the other triangle (**SSS** rule)

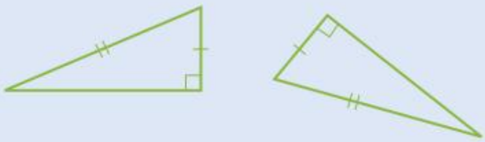


- 2 sides and the **included angle** of one triangle are respectively equal to 2 sides and the **included angle** of the other triangle (**SAS** rule)

- 2 angles and one side of one triangle are respectively equal to 2 angles and the matching side of the other triangle (**AAS** rule)

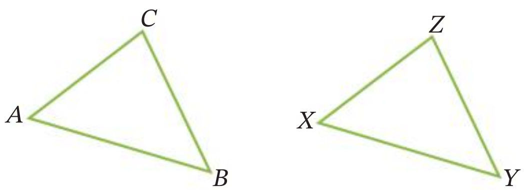


- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle (**RHS** rule).



The congruence symbol ≡

The symbol for 'is congruent to' is a special equals sign, written as '≡' (which also means 'is identical to'). The 2 triangles below are congruent, so we can write $\triangle ABC \equiv \triangle XYZ$, which is read as 'triangle *ABC* is congruent to triangle *XYZ*'.



When using this notation, we must make sure that the vertices (angles) of the congruent figures are written in matching order:

$\triangle ABC \equiv \triangle XYZ$ means $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

To formally prove that 2 triangles are congruent, we need to use one of the 4 tests for congruence SSS, SAS, AAS or RHS.

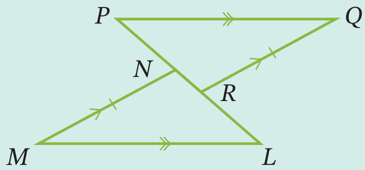
STAGE 5.2

11.03

Example 6

In the diagram, $PQ \parallel LM$, $QR \parallel MN$ and $QR = MN$.

Prove that $\triangle PQR \equiv \triangle LMN$.



Solution

In $\triangle PQR$ and $\triangle LMN$: — *Identifying the triangles in matching order of vertices.*

$QR = MN$ (given) — *Stating each part of the congruence test, giving reasons.*

$\angle P = \angle L$ (alternate angles, $PQ \parallel LM$)

$\angle QRP = \angle MNL$ (alternate angles, $QR \parallel MN$)

$\therefore \triangle PQR \equiv \triangle LMN$ (AAS) — *Concluding the congruence proof, stating the test used.*

EXERCISE 11.03 ANSWERS ON P. 546

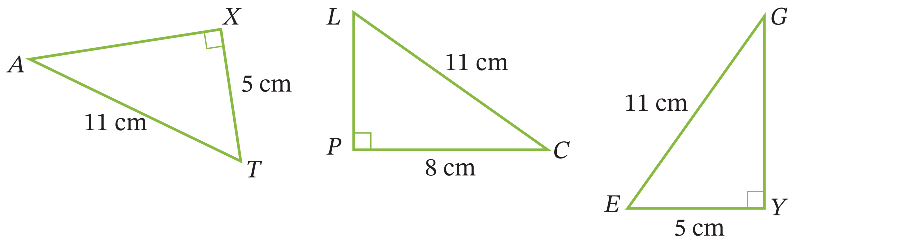
Congruent triangle proofs **U F R C**

1 For each set of triangles: **R C**

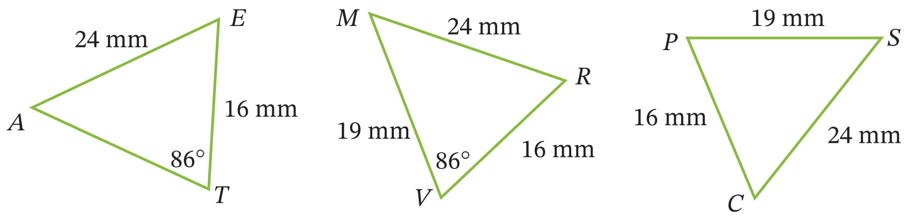
i decide which 2 are congruent and state the congruence test used

ii use the correct notation to write a congruency statement relating those 2 triangles.

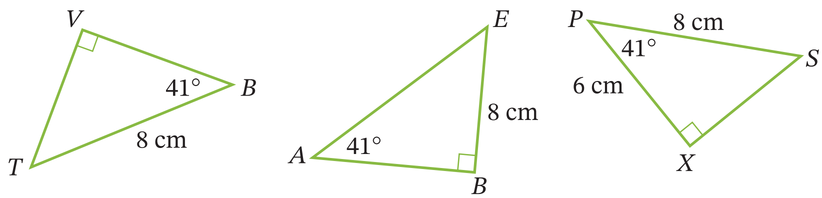
a



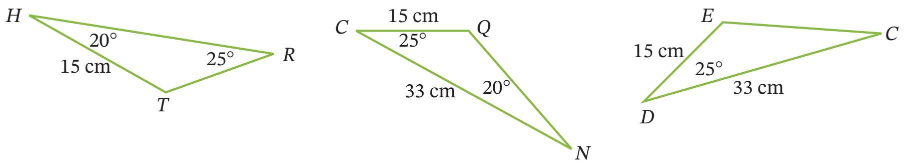
b



c

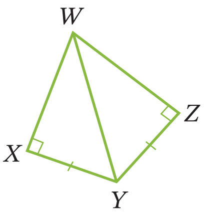


d

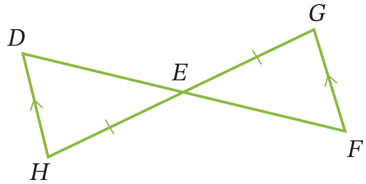


EXAMPLE 6

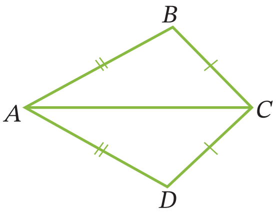
2 Prove that $\triangle WXY \equiv \triangle WZY$. R C



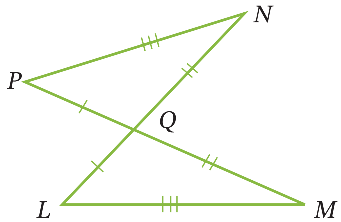
3 In the diagram, $EG = EH$ and $DH \parallel FG$. Show that $\triangle DEH \equiv \triangle FEG$. R C



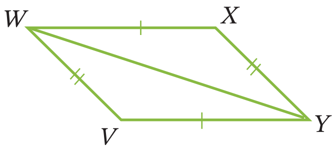
4 For this kite $ABCD$, prove that $\triangle ABC \equiv \triangle ADC$. R C



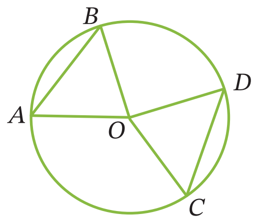
5 If $PQ = LQ$ and $NQ = MQ$, prove that $\triangle PQN \equiv \triangle LQM$. R C



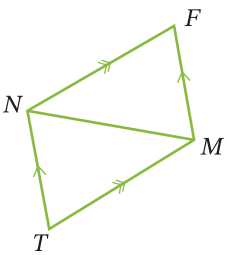
6 Prove that $\triangle WXY \equiv \triangle YVW$. R C



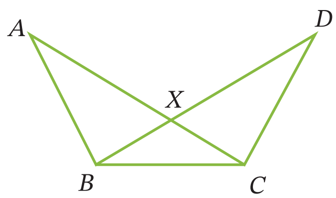
7 O is the centre of the circle and $AB = CD$. Prove that $\triangle AOB \equiv \triangle COD$. R C



8 Prove that $\triangle FNM \equiv \triangle TMN$. R C



9 If $\angle ABC = \angle DCB$ and $AB = DC$ in the diagram, prove that $\triangle ABC \equiv \triangle DCB$. R C



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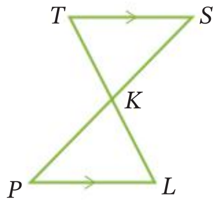
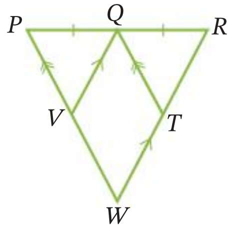
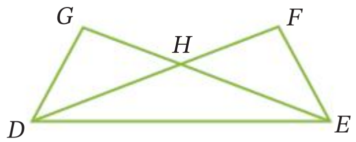
10 $TS \parallel PL$ and K is the midpoint of TL. Prove that $\triangle TSK \equiv \triangle LPK$. **R** **C**



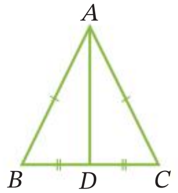
11 $PW \parallel QT$, $RW \parallel QV$ and $PQ = QR$. Prove that $\triangle PVQ \equiv \triangle QTR$. **R** **C**



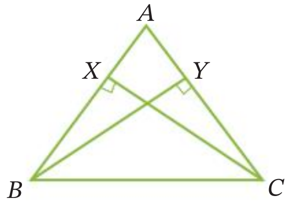
12 If $\angle DEG = \angle EDF$ and $GE = FD$, prove that $\triangle DEG \equiv \triangle EDF$. **R** **C**



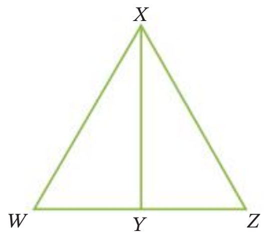
13 In $\triangle ABC$, $AB = AC$ and $AD \perp BC$. Prove that $\triangle ABD \equiv \triangle ACD$ and hence AD bisects $\angle BAC$. **R** **C**



14 If $CX \perp AB$, $BY \perp AC$ and $XC = YB$, prove that $\triangle BCX \equiv \triangle CBY$. **R** **C**



15 $XW = XZ$ in this isosceles triangle and Y is the midpoint of WZ. Prove that $\triangle WYX \equiv \triangle ZYX$. **R** **C**



11.03

Proving properties of triangles and quadrilaterals 11.04

Properties of triangles and quadrilaterals can be proved using the congruence tests.

STAGE 5.2

WS Quadrilaterals: True or false?

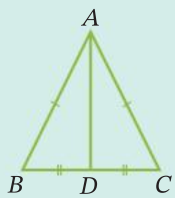
Example 7

$\triangle ABC$ is an isosceles triangle with $AB = AC$. D is the midpoint of BC.

a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?

b Explain why $\angle B = \angle C$.

c What geometrical result about isosceles triangles does this prove?



WS
Congruent and similar triangle proofs

Geometric problems and proofs

Proving properties of a rectangle

Solution

a For $\triangle ABD$ and $\triangle ACD$:

$AB = AC$ (given)

AD is common.

$BD = CD$ (D is the midpoint of BC)

$\therefore$ The congruence test is SSS.

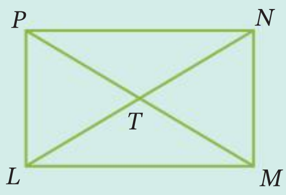
b $\angle B = \angle C$ because they are matching angles of congruent triangles.

c The angles opposite the equal sides of an isosceles triangle are equal.

Example 8

a If $LMNP$ is a rectangle, prove that $\triangle PNT \equiv \triangle MLT$.

b Prove that the diagonals of a rectangle bisect each other.



Solution

a In $\triangle PNT$ and $\triangle MLT$:

$PN = ML$	(opposite sides of a rectangle)
$\angle PNT = \angle MLT$	(alternate angles, $PN \parallel ML$ for a rectangle)
$\angle PTN = \angle MTL$	(vertically opposite angles)

$\therefore \triangle PNT = \triangle MLT$ (AAS)

b $\therefore PT = MT$ and $NT = LT$ (matching sides of congruent triangles)

$\therefore T$ is the midpoint of the diagonals LN and MP.

$\therefore$ The diagonals of a rectangle bisect each other.

EXERCISE 11.04 ANSWERS ON P. 546

Proving properties of triangles and quadrilaterals U F R C

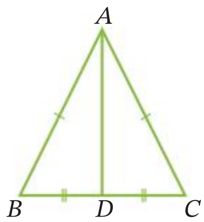
EXAMPLE 7

1 $\triangle ABC$ is an isosceles triangle, with $AB = AC$. D is the midpoint of BC. R C

a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?

b Explain why $\angle ADB = \angle ADC$.

c Hence prove that $AD \perp BC$.



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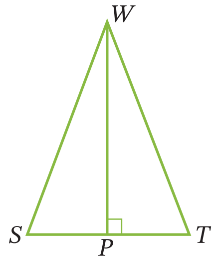
STAGE 5.2

11.04

2 In the diagram, $\angle S = \angle T$ and $WP \perp ST$. R C

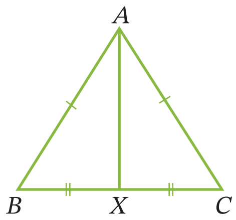


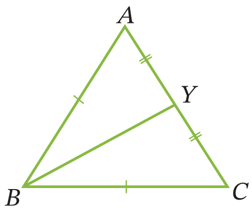
a Which congruence test can be used to prove that $\triangle SPW \equiv \triangle TPW$?

b Explain why $WS = WT$.

c What geometrical result about triangles does this prove?

3 $\triangle ABC$ is an equilateral triangle ($AB = BC = AC$). X is the midpoint of BC. R C





a Which congruence test can be used to prove that $\triangle ABX \equiv \triangle ACX$?

b Explain why $\angle B = \angle C$.

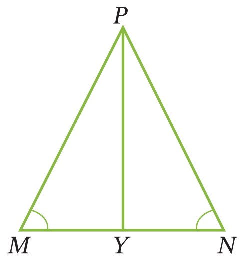
c In the second diagram, $\triangle ABC$ is redrawn so that Y is the midpoint of AC.
Which congruence test can be used to prove that $\triangle BAY \equiv \triangle BCY$?

d Is $\angle A = \angle C$? Why?

e Calculate the sizes of the 3 angles of $\triangle ABC$.

f What geometrical property of equilateral triangles does this prove?

4 In $\triangle PMN$, $\angle M = \angle N$ and YP bisects $\angle MPN$. R C



a Explain why $\angle MPY = \angle NPY$.

b Which congruence test can be used to prove that $\triangle PMY \equiv \triangle PNY$?

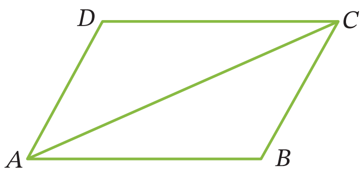
c Explain why $MY = NY$.

d Is $\angle PYM = \angle PYN$? Why?

e Prove that $PY \perp MN$.

EXAMPLE 8

5 $ABCD$ is a quadrilateral whose opposite sides are equal. R C



a Prove that $\triangle ABC \equiv \triangle CDA$.

b Explain why $\angle BAC = \angle DCA$ and $\angle BCA = \angle DAC$.

c Hence state why $AB \parallel CD$ and $AD \parallel CB$.

d What type of quadrilateral is $ABCD$?

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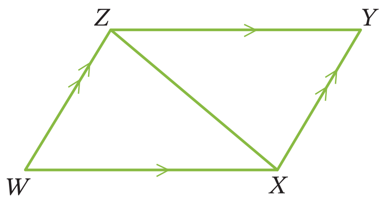
STAGE 5.2

6 *WXYZ* is a parallelogram whose opposite sides are parallel. R C



a Copy the diagram into your book.

b On your diagram, show 2 pairs of equal alternate angles.

c Prove that $\triangle WXZ \equiv \triangle YZX$.

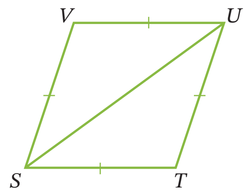
d Explain why $\angle W = \angle Y$.

e Draw the other diagonal *WY* and prove that $\triangle WXY \equiv \triangle YZW$.

f Explain why $\angle WXY = \angle YZW$.

g What angle property of a parallelogram does this prove?

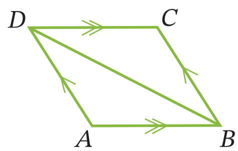
7 *STUV* is a rhombus, so all sides are equal. R C



a Prove that $\triangle VUS \equiv \triangle TUS$.

b Prove that the diagonal *US* bisects $\angle VUT$ and $\angle VST$.

8 *ABCD* is a parallelogram with opposite sides parallel. R C

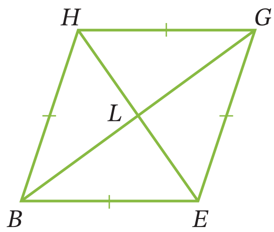


a Prove that $\triangle ABD \equiv \triangle CDB$.

b Explain why $AB = CD$ and $AD = CB$.

c What property of a parallelogram does this prove?

9 *BEGH* is a rhombus (a parallelogram with equal sides) whose diagonals *BG* and *EH* intersect at *L*. R C



a Prove that $\triangle BEL = \triangle GHL$.

b Prove that the diagonals of a rhombus bisect each other.

c $\triangle BEH$ is isosceles, so which equation is true? Select **A**, **B**, **C** or **D**.

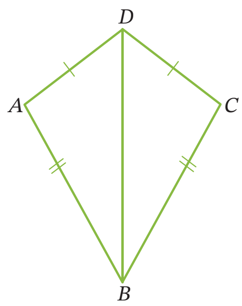
A $\angle BEH = \angle BHE$ **B** $\angle BLE = \angle BEH$

C $\angle HBE = \angle BEH$ **D** $\angle BEH = \angle EBH$

d Hence prove that $\triangle BEL \equiv \triangle BHL$.

e Hence prove that the diagonals of a rhombus cross at right angles.

10 *ABCD* is a kite, so pairs of adjacent sides are equal. R C



a Prove that $\triangle ABD \equiv \triangle CBD$.

b Prove that $\angle A = \angle C$.

c Prove that diagonal *DB* bisects $\angle ADC$ and $\angle ABC$.

d Copy the diagram and draw the other diagonal *AC*, intersecting *DB* at point *X*.

e Prove that $\triangle DAX \equiv \triangle DCX$.

f Prove that diagonal *DB* bisects diagonal *AC*.

g Prove that $DB \perp AC$.

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Mental skills 11: Maths without calculators ANSWERS ON P. 548

Dividing a quantity in a given ratio

1 Study this example.

Divide \$5600 between Alice and Peter in the ratio 5 : 3.

Total number of parts = 5 + 3 = 8.

1 part = \$5600 ÷ 8 = \$700

Alice's share = 5 × \$700 = \$3500

Peter's share = 3 × \$700 = \$2100

Check: \$3500 + \$2100 = \$5600 (original amount)

2 Now divide each of these quantities in the given ratio.

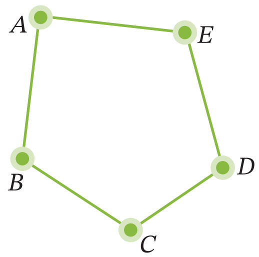
- **a** Divide \$150 between Mark and Jenni in the ratio 2 : 1.
- **b** Divide \$2100 between Simon and Sunil in the ratio 4 : 3.
- **c** Divide \$720 between Lisa and Bree in the ratio 2 : 7.
- **d** Divide \$2000 between William and Adriana in the ratio 1 : 3.
- **e** Divide \$4500 between Anne and Pete in the ratio 3 : 2.
- **f** Divide \$3000 between Sharanya and Asam in the ratio 3 : 7.
- **g** Divide \$3600 between Cindy and Carmen in the ratio 5 : 1.
- **h** Divide \$1600 between Nancy and John in the ratio 3 : 5.
- **i** Divide \$990 between Carol and Louis in the ratio 5 : 4.
- **j** Divide \$4000 between Yvette and Andre in the ratio 1 : 4.
- **k** Divide \$4900 between Arden and Ivan in the ratio 3 : 4.
- **l** Divide \$3200 between Tan and Mai in the ratio 5 : 3.

Technology

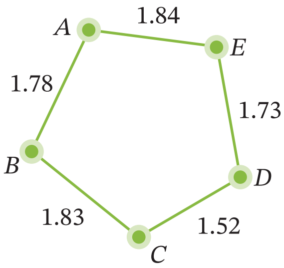
Properties of similar figures

We will use dynamic geometry software to look at the properties of similar figures.

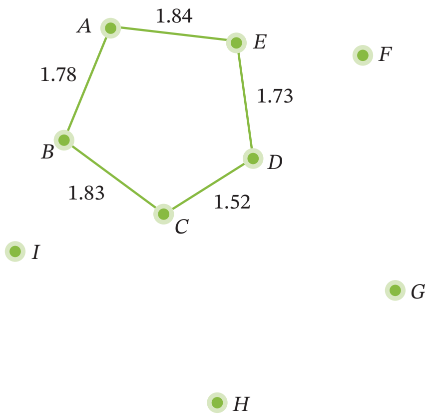
1 Construct a 5-sided polygon as shown.



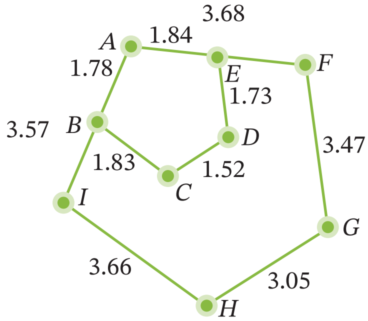
2 Show the lengths of the sides of the polygon as well.



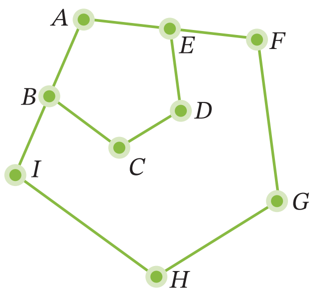
3 Enlarge the polygon by a scale factor of 2 to obtain the image *AFGHI* as shown.

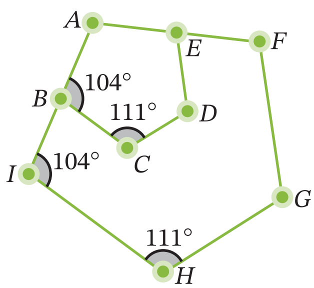


4 The lengths of the sides of the image are shown. Is the ratio of matching sides the same for all sides?



5 Measure the angles of the polygon and its image. An example is shown of some of the matching angles on the right. Are the matching angles equal?





6 Repeat Steps **1** to **5** for:

a a triangle **b** a quadrilateral.

7 For similar figures:

a are matching angles equal? **b** are matching sides in the same ratio?

Similar figures 11.05

Similar figures have the same shape, but are not necessarily the same size.

When a figure is enlarged or reduced, a **similar figure** is created. The original figure is called the **original**, while the enlarged or reduced figure is called the **image**.

The **scale factor** describes by how much a figure has been enlarged or reduced.

Scale factor

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

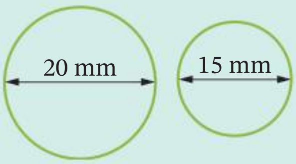
- If the scale factor is greater than 1, then the image is an enlargement.
- If the scale factor is between 0 and 1, then the image is a reduction.

Example 9

Find the scale factor for each pair of similar figures.

In all questions, assume the left figure is the original and the right figure is the image.

a



b

Solution

a $\text{Scale factor} = \frac{15}{20}$ $\qquad \frac{\text{image length}}{\text{original length}}$

$= \frac{3}{4}$

b $\text{Scale factor} = \frac{45}{27} \left(\text{or } \frac{20}{12}\right)$ $\qquad \frac{\text{image length}}{\text{original length}}$

$= \frac{5}{3}$

The symbol for 'is similar to' is '|||'. As with congruence notation, we must make sure that the vertices (angles) of similar figures are written in matching order.

WS A page of similar figures

WS Enlargements and reductions

WS Enlarging a logo

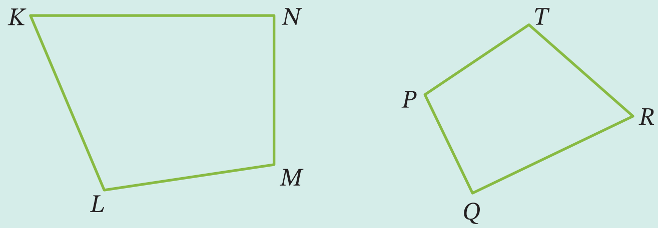
Cartoon enlargement

Properties of similar figures

- Matching angles are equal
- Matching sides are in the same ratio

Example 10

The 2 quadrilaterals *KLMN* and *PQRT* are similar.



a List all pairs of matching sides and matching angles.

b Use the correct notation to write a similarity statement relating these 2 quadrilaterals.

Solution

a By rotating the figure *KLMN*, its shape can be matched with *PQRT*.

The pairs of matching sides are:	The pairs of matching angles are:
KN and *QR*	$\angle K$ and $\angle R$
MN and *PQ*	$\angle N$ and $\angle Q$
ML and *PT*	$\angle M$ and $\angle P$
LK and *TR*.	$\angle L$ and $\angle T$.

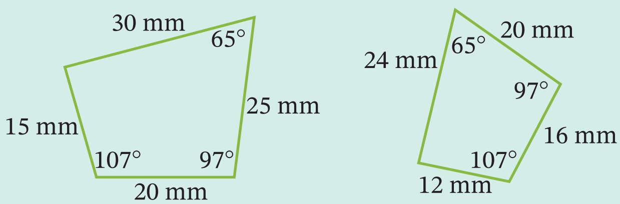
b $\angle K$ matches with $\angle R$, $\angle L$ matches with $\angle T$,
$\angle M$ matches with $\angle P$, $\angle N$ matches with $\angle Q$.

$\therefore$ *KLMN* ||| *RTPQ* Matching order of vertices.

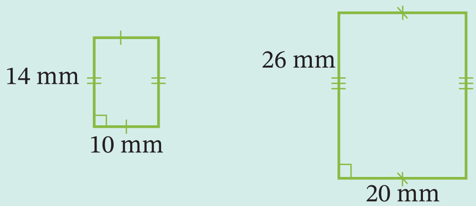
Example 11

Test whether each pair of figures are similar.

a



b



Solution

a For the 2 quadrilaterals, matching angles are equal and the ratios of matching sides are equal.

$\frac{20}{16} = \frac{5}{4}, \frac{25}{20} = \frac{5}{4}, \frac{30}{24} = \frac{5}{4}, \frac{15}{12} = \frac{5}{4}$

∴ The quadrilaterals are similar.

b For the 2 rectangles, matching angles are equal (90°), but the ratios of matching sides are not equal.

$\frac{10}{20} = \frac{1}{2}$ but $\frac{14}{26} = \frac{7}{13}$

∴ The rectangles are not similar.

11.05

EXERCISE 11.05 ANSWERS ON P. 548

Similar figures **U F R C**

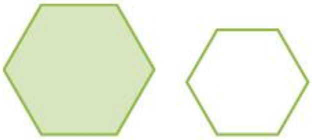
EXAMPLE 9

1 By measurement, find the scale factor for each pair of similar figures.

a

b

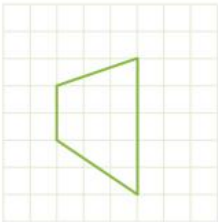
c



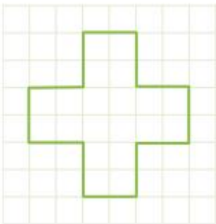
d

2 Copy each figure onto graph paper and draw its image using the given scale factor.

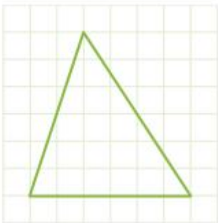
a Scale factor = 2



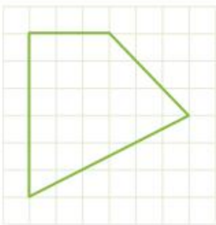
b Scale factor = 2.5



c Scale factor = $\frac{1}{2}$



d Scale factor = $\frac{2}{3}$



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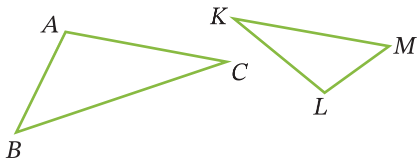
3 For each pair of similar figures: R C

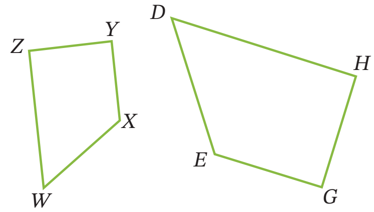
EXAMPLE 10

- **i** list all pairs of matching angles
- **ii** list all pairs of matching sides
- **iii** use the correct notation to write a similarity statement relating them.

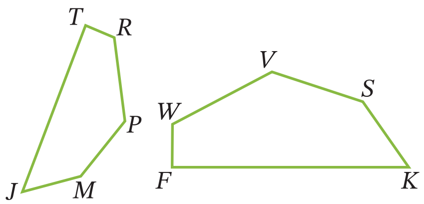
a



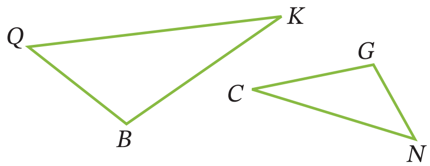
b



c



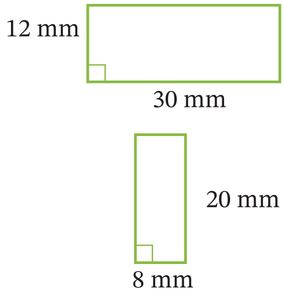
d



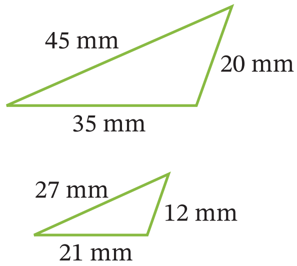
4 Test whether each pair of figures are similar. R C

EXAMPLE 11

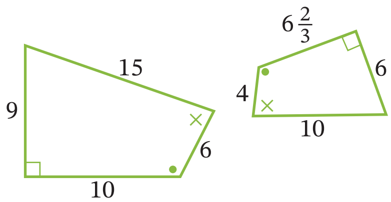
a



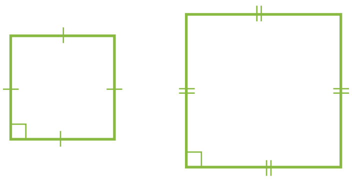
b



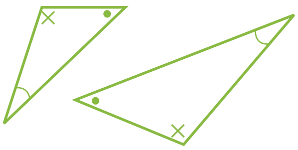
c



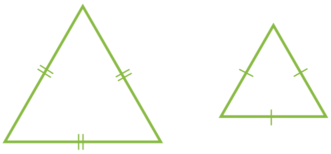
d



e



f



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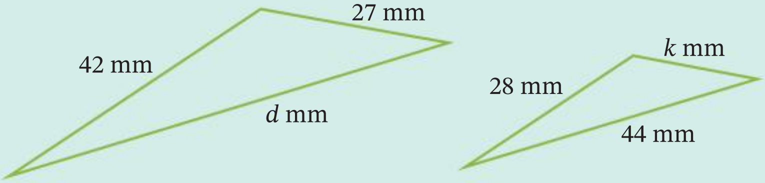
Finding unknown sides in similar figures

11.06

Example 12

The 2 triangles are similar. Find the values of d and k.



Solution

Since the triangles are similar, the ratios of matching sides are equal.

$$\frac{d}{44} = \frac{42}{28}$$

$$d = \frac{42}{28} \times 44$$

$$= 66$$

$$\frac{k}{27} = \frac{28}{42}$$

$$k = \frac{28}{42} \times 27$$

$$= 18$$

Alternative method:

Scale factor $= \frac{28}{42} = \frac{2}{3}$

$$d = 44 \div \frac{2}{3}$$

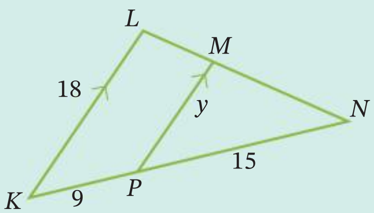
$$= 66$$

$$k = 27 \times \frac{2}{3}$$

$$= 18$$

Example 13

$\triangle KLN \,|||\, \triangle PMN$. Find the value of y.



Solution

$\frac{MP}{LK} = \frac{PN}{KN}$ — Ratios of matching sides are equal.

$\frac{y}{18} = \frac{15}{24}$ — $KN = 9 + 15 = 24$

$$y = \frac{15}{24} \times 18$$

$$= 11\frac{1}{4}$$

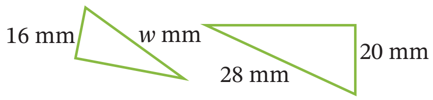
EXERCISE 11.06 ANSWERS ON P. 549

Finding unknown sides in similar figures **U F PS R**

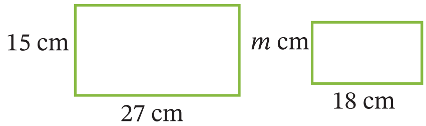
EXAMPLE 12

1 Find the value of each variable in each pair of similar figures. **R**

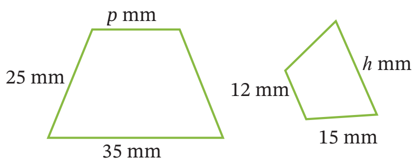
a



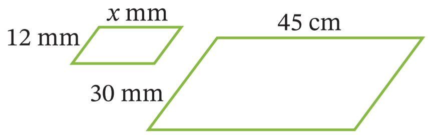
b



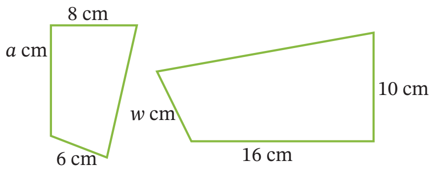
c



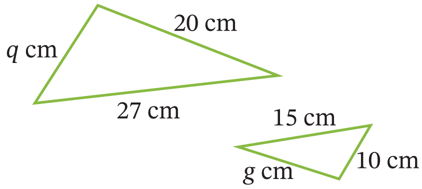
d



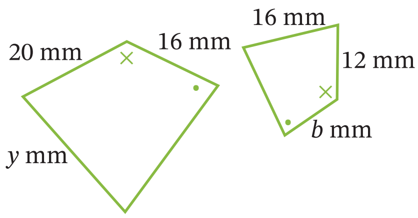
e



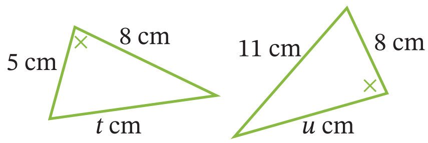
f



g

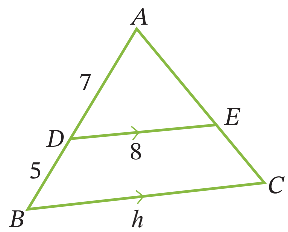


h

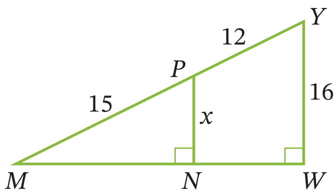


EXAMPLE 13

2 $\triangle ABC \mid\mid\mid \triangle ADE$. Find the value of h. **R**



3 $\triangle MNP \mid\mid\mid \triangle MWY$. Find the value of x. **R**



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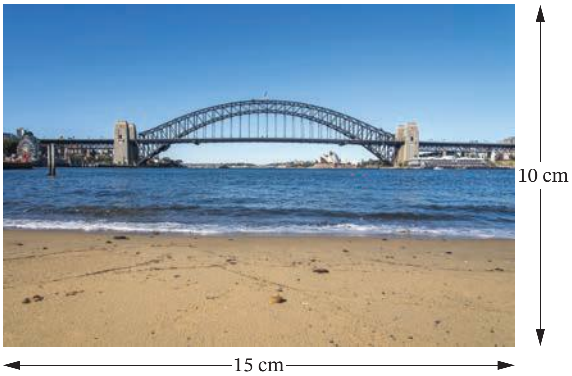
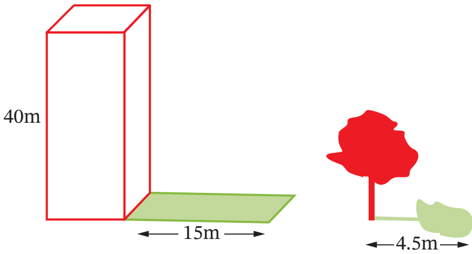
4 This photograph of the Sydney Harbour Bridge has been enlarged so that its length is 24 cm. If the dimensions of the original photo were 15 cm × 10 cm, what is the width of the enlargement? R



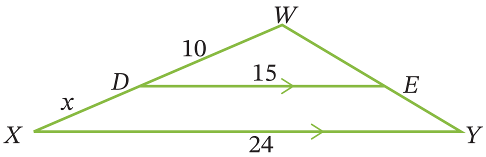
Shutterstock.com/Taki O

5 A building that is 40 m high casts a shadow 15 m long. At the same time, the shadow of a tree is 4.5 m long. What is the height of the tree? PS R

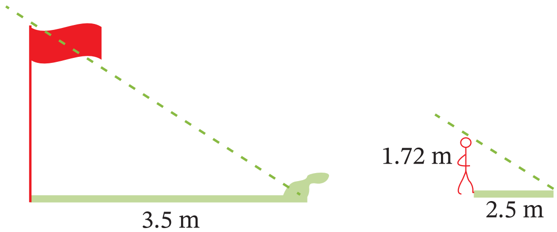


6 $\triangle WXY \,|||\, \triangle WDE$. What is the value of x? Select the correct answer **A**, **B**, **C** or **D**. R

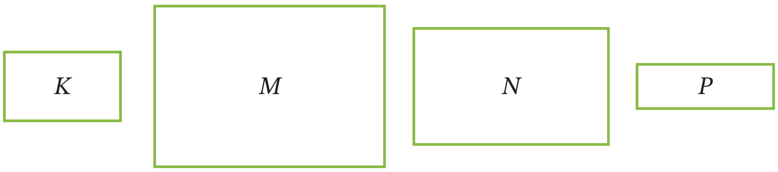
A 4 **B** 6
C 8 **D** 10



7 Katrina is 1.72 m tall and casts a shadow that is 2.5 m long. At the same time, a flagpole casts a shadow that is 3.5 m long. How long is the flagpole? PS R



8 Which 2 rectangles are similar? Select **A**, **B**, **C** or **D**.



A *M* and *N* **B** *K* and *P* **C** *M* and *P* **D** *K* and *N*

9 A 2 m high fence casts a shadow of 1.4 m. How long is the shadow cast by a pole that is 3.2 m high at the same time? PS R

11.07 Tests for similar triangles

STAGE 5.2

WS Congruent and similar triangle proofs

WS Congruence and similarity review

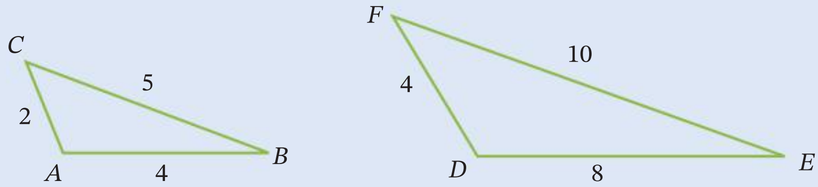
There are 4 sets of conditions that can be used to determine if 2 triangles are similar. These are called the tests for similar triangles or **similarity tests**.

Similarity tests

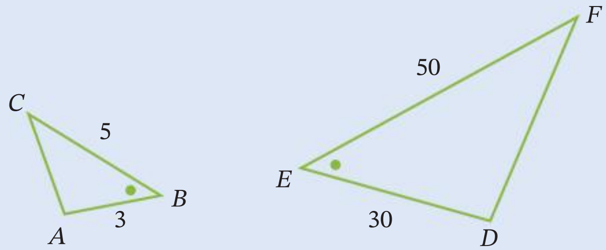
There are 4 tests for similar triangles.

Two triangles are similar if:

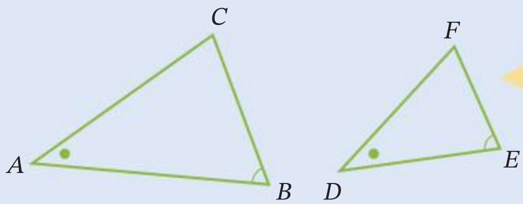
- the 3 sides of one triangle are proportional to the 3 sides of the other triangle (**'SSS'**)



- 2 sides of one triangle are proportional to 2 sides of the other triangle, and the **included angles** are equal (**'SAS'**)

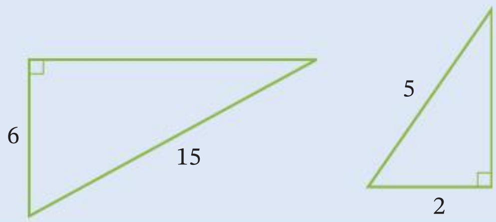


- 2 angles of one triangle are equal to 2 angles of the other triangle (**'AA'** or **'equiangular'**)



Equiangular means 'equal angles'

- they are right-angled and the hypotenuse and a second side of one triangle are proportional to the hypotenuse and a second side of the other triangle (**'RHS'**).



STAGE 5.2

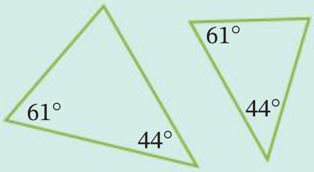
Tests for similar triangles

11.07

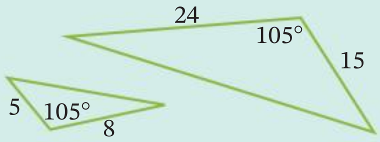
Example 14

Which test can be used to prove that each pair of triangles are similar?

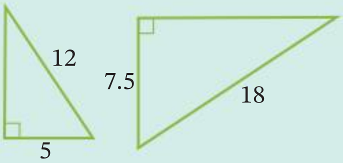
a



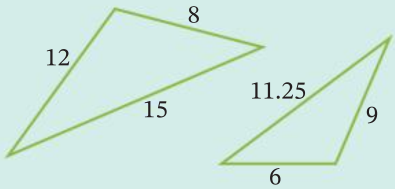
b



c



d



Solution

a 2 pairs of angles are equal, or equiangular ('AA').

b 2 pairs of matching sides are in the same ratio and the included angles in both triangles are equal ('SAS').

$\frac{15}{5} = \frac{24}{8} = 3$

c Both have right angles, and the pairs of hypotenuses and second sides are in the same ratio ('RHS').

$\frac{7.5}{5} = \frac{18}{12} = \frac{3}{2}$

d All 3 pairs of matching sides are in the same ratio ('SSS').

$\frac{11.25}{15} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$

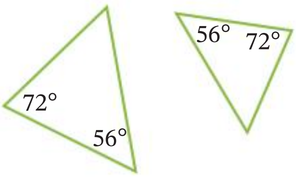
EXERCISE 11.07 ANSWERS ON P. 549

Tests for similar triangles U F R C

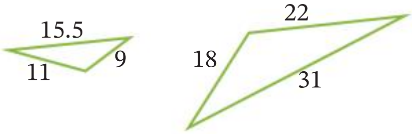
EXAMPLE 14

1 Which test can be used to prove that each pair of triangles are similar? R C

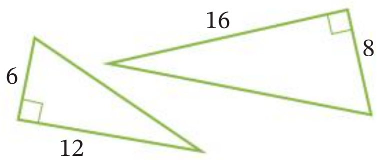
a



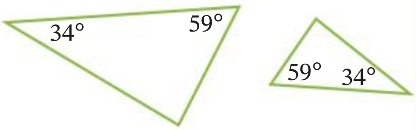
b



c



d



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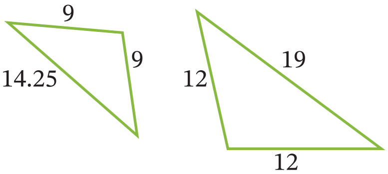
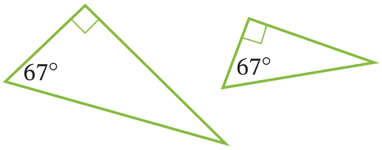
e

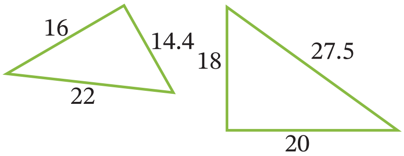


f

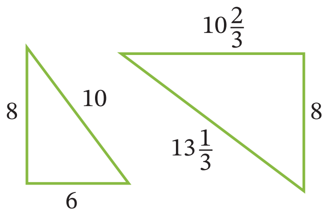
g



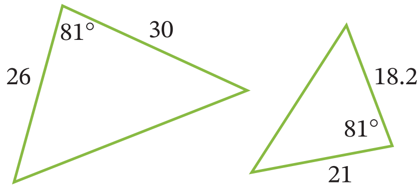
h



i

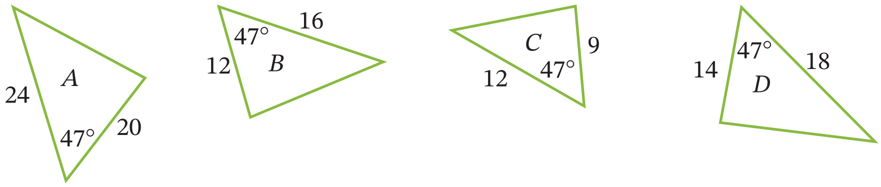


j

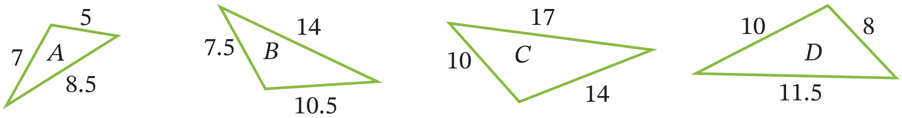


2 For each set of triangles, find the pair of similar triangles. R

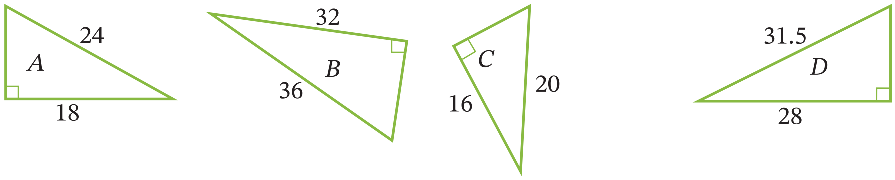
a



b



c

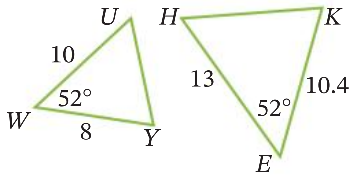


STAGE 5.2

11.07

3 Use the correct notation to write a similarity statement relating each pair of similar triangles. R C

a



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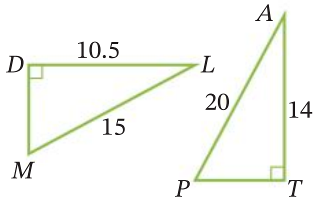
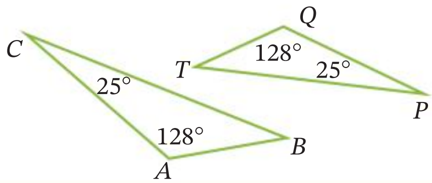
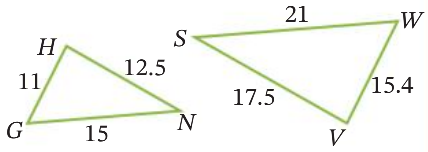


c



d

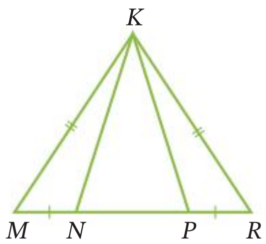


Power plus ANSWERS ON P. 549

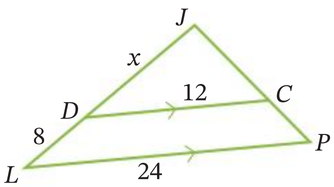
1 **a** Explain why $\angle KMN = \angle KRP$.

b Prove that $\triangle KMN \equiv \triangle KRP$.

c Hence prove that $KN = KP$ and that $\triangle KNP$ is an isosceles triangle.



2 $\triangle JDC \mathbin{|||} \triangle JLP$. Find the value of x.

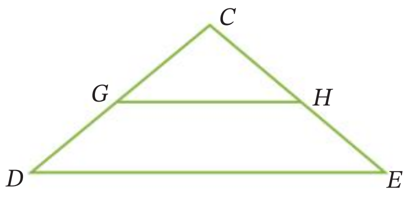


3 G and H are the midpoints of CD and CE respectively. Prove that:

a $\triangle CGH \mathbin{|||} \triangle CDE$

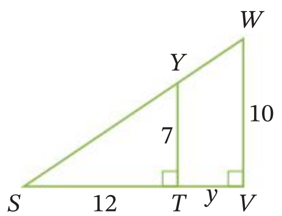
b $GH \parallel DE$

c $GH = \frac{1}{2}DE$



4 **a** Which similarity test proves that $\triangle STY \mathbin{|||} \triangle SVW$?

b Find the value of y.



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CHAPTER 11 REVIEW

Qz Geometry

Qz Congruence and similarity

Language of maths

AAS	angle sum	congruence test
congruent (≡)	convex polygon	enlargement
equiangular	exterior angle	hypotenuse
image	included angle	matching
original	polygon	proof
proportional	reduction	regular polygon
RHS	SAS	scale factor
similar (|||)	similarity test	SSS

1 What is a **convex polygon**?

2 Explain the difference between the **interior** and **exterior** angles of a polygon.

3 What is the symbol and meaning of 'is similar to'?

4 What happens to a figure that is changed by a scale factor of $\frac{1}{2}$?

5 What are the 4 tests for similar triangles?

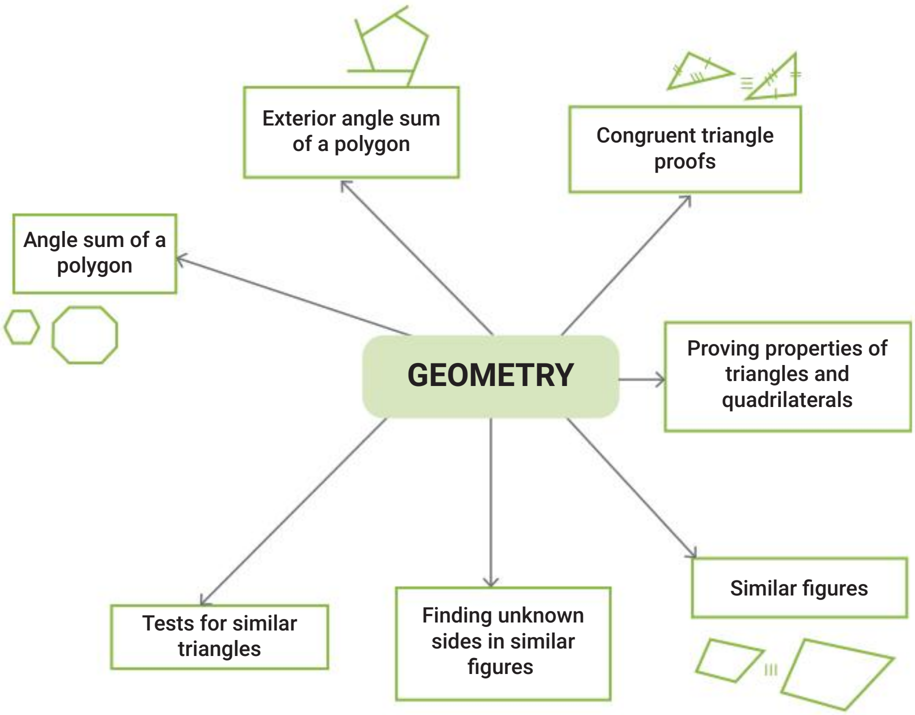
6 What is the meaning of the 'A' in the SAS test for congruent triangles?

7 What does **equiangular** mean in the similarity tests?

WS Mind map: Geometry

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 11

ANSWERS ON P. 549

STAGE 5.2

1 Find the angle sum of a polygon with: 11.01

a 15 sides **b** 24 sides **c** 12 sides **d** 48 sides

2 Find the size of one angle in a regular 15-sided polygon. 11.01

3 The angle sum of a polygon is 6120°. How many sides does the polygon have? 11.01

4 Find the number of sides in a regular polygon if each exterior angle is: 11.02

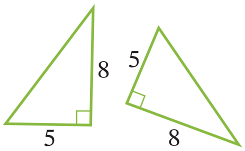
a 10° **b** 24° **c** 45° **d** 15°

5 Each angle of a regular polygon is 162°. How many sides does the polygon have? Select the correct answer **A**, **B**, **C** or **D**. 11.02

A 18 **B** 20 **C** 22 **D** 24

6 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent? 11.03

a



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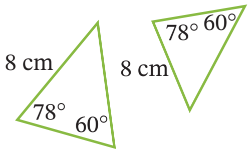
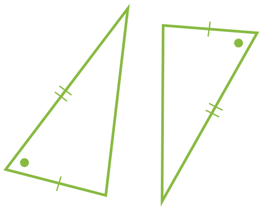
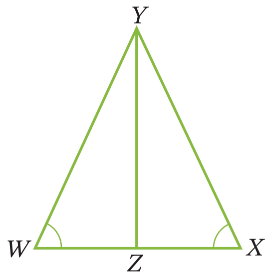


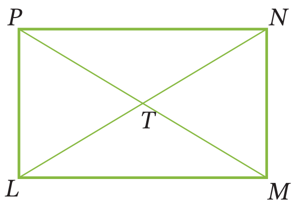
c



7 In $\triangle WXY$, $\angle W = \angle X$ and $YZ \perp WX$. 11.03

Prove that $\triangle WZY \equiv \triangle XZY$.





8 *PNML* is a rectangle. 11.04

a Which congruence test can be used to prove that $\triangle PML \equiv \triangle NLM$?

b Hence explain why $PM = NL$.

c What geometrical result about rectangles does this prove?

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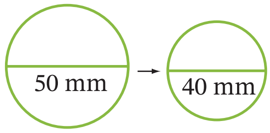
11.05

9 Calculate the scale factor between each pair of similar figures.

a



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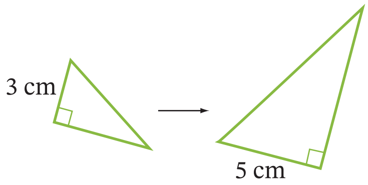
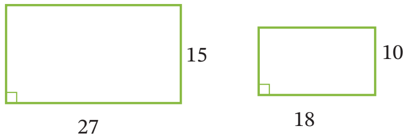


11.05

10 Test whether each pair of figures are similar.

a



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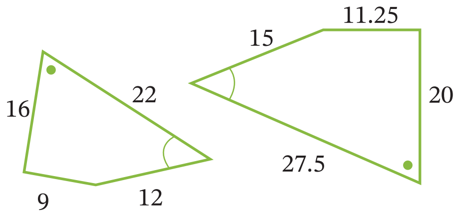
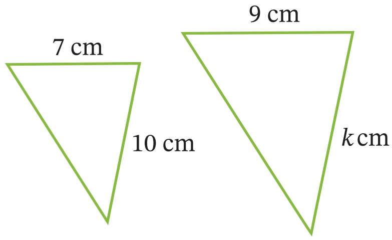


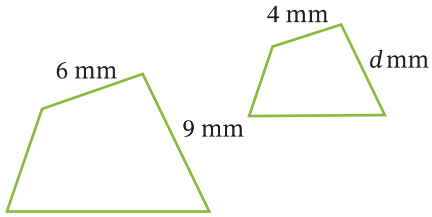
11.06

11 Find the value of the variable in each pair of similar figures.

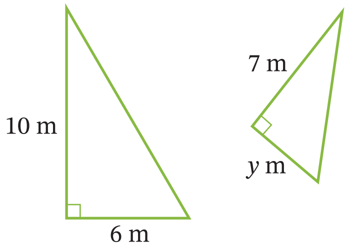
a



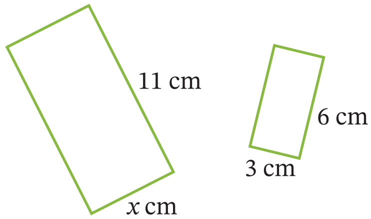
b



c

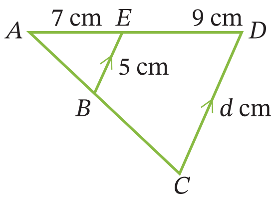


d



11.06

12 If $\triangle ABE \mathbin{|||} \triangle ACD$, find the value of d.

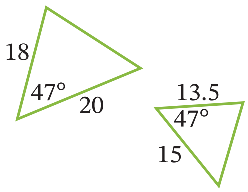


STAGE 5.2

11.07

13 Which test can be used to prove that each pair of triangles are similar?

a



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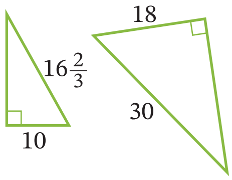
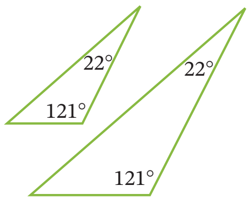


c



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PRACTICE SET 4 ANSWERS ON P. 550

STAGE 5.2

1 2 dice are rolled. 10.05

a How many outcomes are possible?

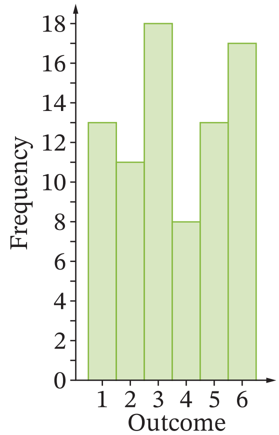
b What is the probability of rolling:

i double 1s?

ii any doubles?

iii 2 numbers both less than 4?

2 A die was rolled and the outcomes were recorded in this frequency histogram. 10.01



a How many times was the die rolled?

b Find the relative frequency of rolling:

i a 1

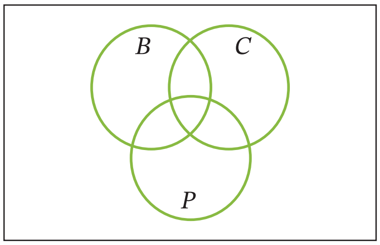
ii an even number

iii a number less than 4

iv at least a 3.

c What is the theoretical probability of rolling a 6? How does this compare with the experimental probability of rolling a 6?

3 Of 160 Year 11 students at Westvale High, 54 do Biology (B), 75 take Chemistry (C) and 68 study Physics (P). 55 students take both Chemistry and Physics, 20 do Biology and Chemistry and 10 students take all three. 10.02



a Copy and complete the Venn diagram to show this information.

b Find the probability of selecting a student who:

i only takes Physics

ii does not do Biology, Chemistry or Physics

iii does Chemistry and Physics but not Biology

iv studies Chemistry or Biology

v only does one Science subject

c From the students who do Biology, what is the probability that the student also takes Physics?

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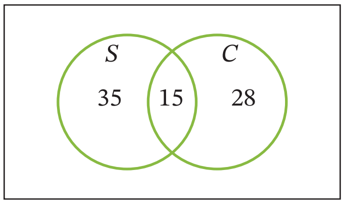
10.02

4 This Venn diagram shows the results of a survey on whether people prefer to watch movies by streaming (S) or at the cinema (C).



a How many people were surveyed?

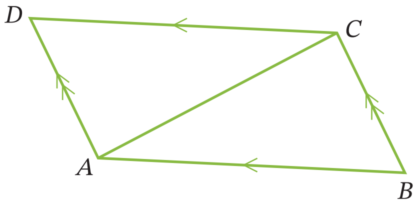
b Find the probability of selecting a person from this survey who prefers to watch movies:

i by streaming **ii** only at the cinema

c What is the probability of selecting a person who likes to stream movies only?

11.03

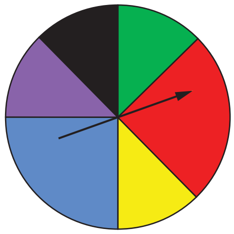
5 Prove that $\triangle ABC \equiv \triangle CDA$.



STAGE 5.2

10.06

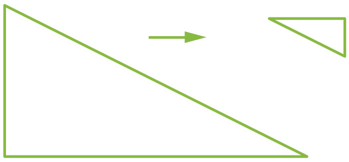
6 Olivia spun this spinner twice, landing on red both times. Is the second outcome dependent or independent of the outcome of the first spin?



11.05

7 By measurement, calculate the scale factor for each pair of similar figures.

a



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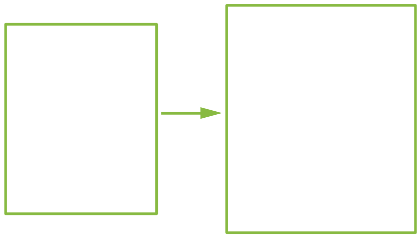

10.01

8 A bag contains 4 red, 5 black and 3 green marbles. Sadiya selects a marble at random, records its colour and then returns the marble to the bag. Sadiya does this process 150 times. The results are shown in the table.

Outcome	Frequency
red	58
black	65
green	27

a Find, correct to 2 decimal places, the relative frequency of selecting a marble that is:

i red **ii** black **iii** green.

b Find, correct to 2 decimal places, the theoretical probability of selecting a marble that is

i red **ii** black **iii** green.

c For which outcome are the experimental and theoretical probabilities most similar?

d What is the expected frequency of selecting a red or green marble? How does this compare with the observed frequency?

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9 The angle sum of a regular polygon is 6120°. 11.01

a How many sides does the polygon have?

b Find the size of each angle.

10 A 6-sided die and a 4-sided die are rolled together and the product of the 2 numbers is calculated. 10.04

	1	2	3	4	5	6
1						
2						
3						
4						

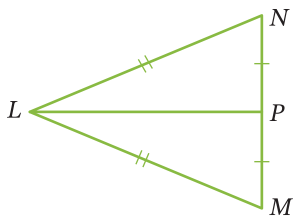
a Copy and complete this table to show all possible products.

b Find the probability of rolling a product:

i of 20 **ii** of 6 **iii** that is odd

iv of at least 12 **v** less than 10 **vi** from 11 to 23.

11 $LM = LN$ and P is the midpoint of MN. Prove that $\triangle LMP \equiv \triangle LNP$ and hence that $\angle LPM = \angle LPN = 90°$. 11.03



12 Shoppers at a mall were asked whether they had a pet dog or cat. The results are shown in the 2-way table. 10.03

	Cat	No cat
Dog	28	35
No dog	32	40

a How many shoppers were surveyed?

b Find the probability of randomly selecting a shopper from the survey who does not have a cat or dog.

c What is the probability of randomly selecting a shopper who:

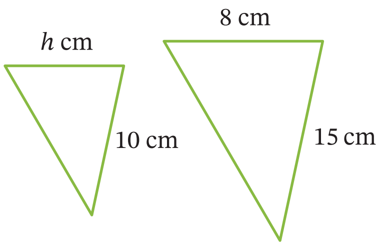
i has only a dog or cat (not both)?

ii has a cat?

iii does not have a dog?

13 Find the value of the variable in each pair of similar figures. 11.06

a



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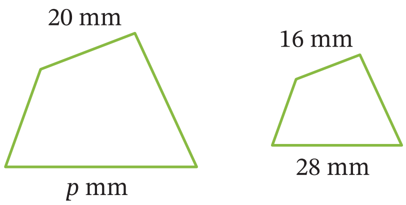

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PRACTICE SET 4

STAGE 5.2

10.05

14 3 cards are drawn from a set of cards numbered 2, 3, 4 and 5, without replacement, to form a 3-digit number.

a Copy and complete the tree diagram to list all possible outcomes.

b Find the probability of forming:

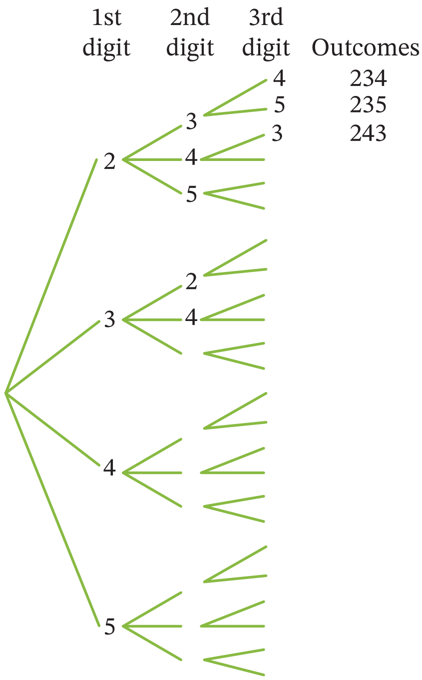
i an even number

ii a number ending in 3

iii a number greater than 400

iv a number between 200 and 500

v a number divisible by 5.



11.01

15 Find the angle sum of a polygon that has:

a 15 sides **b** 20 sides **c** 8 sides **d** 48 sides.

11.02

16 Find the number of sides in a regular polygon if each exterior angle is 10°.

10.06

17 A bag contains 3 yellow and 2 red marbles. 2 marbles are drawn from the bag, without replacing the marble from the first draw.

a Find the probability of:

i selecting a red marble with the first draw

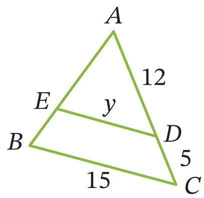
ii selecting a red marble with the second draw if the first marble was yellow.

b Are the 2 draws dependent or independent? Justify your answer.

c If the 2 draws are made with replacement of the marble after the first draw, are the draws dependent or independent? Justify your answer.

11.06

18 If $\triangle ABC \,|||\, \triangle AED$, find the value of y (correct to one decimal place).



Foundation Standard Complex

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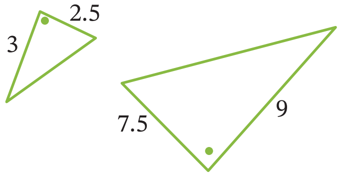
STAGE 5.2

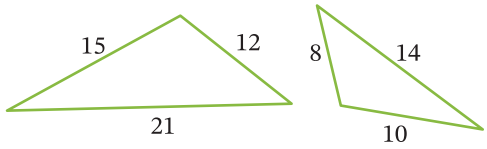
19 Which similarity test can be used to prove that each pair of triangles are similar?

11.07

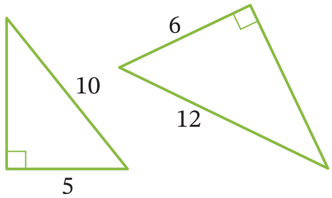
a



b



c



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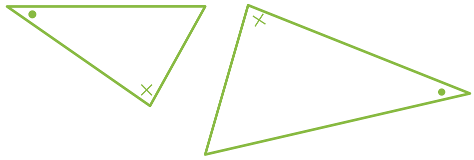


20 2 dice are rolled and the product of the 2 numbers is calculated.

10.07

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

a Copy and complete the table to show all possible outcomes.

b Given that one of the numbers was 3, what is the probability of obtaining a product that is odd?

c Given that one of the numbers is 4, what is the probability of obtaining a product:

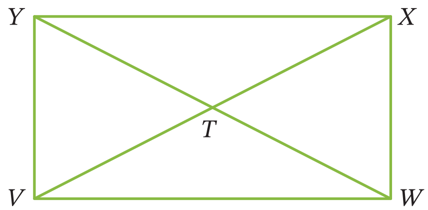
i that is even? **ii** that is odd?

d Find P(an even product | the first number is odd)

e Find P(an even product | an even number and an odd number are obtained).

21 $VWXY$ is a rectangle with diagonals intersecting at T.

11.04



a Since $YX \parallel VW$, list the 2 pairs of equal alternate angles between them.

b Why is $YX = VW$?

c Which congruence test can now be used to prove that $\triangle YXT \equiv \triangle WVT$?

d Explain why $YT = WT$ and $XT = VT$.

e Which property of a rectangle has been proved?

GENERAL PRACTICE

ANSWERS ON P. 551

CHAPTER 1

1 Kim is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of \$1250. How much will Kim earn in a week if she sells an apartment for \$578 000?

STAGE 5.2

CHAPTER 2

2 Find the gradient, m, and y-intercept, c, of each linear equation.

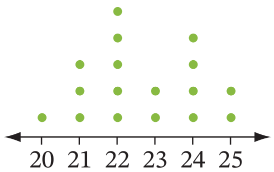
a $y = 3x - 4$ **b** $y = 5 - 2x$ **c** $4x + 3y - 9 = 0$

CHAPTER 5

3 Find the interquartile range for each set of data.

a 4 7 8 12 5 8 10 7 13 6 9 2

b



CHAPTER 4

4 Factorise each expression.

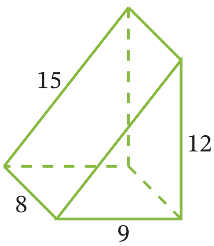
a $8x^2 + 16x$ **b** $y^2 - 25$

c $a^2 + 6a + 8$ **d** $5p + 3 - 2p^2$

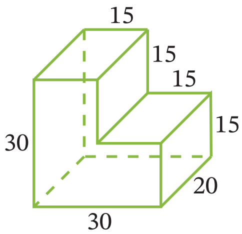
CHAPTER 3

5 Find the surface area of each solid, correct to the nearest mm². All measurements shown are in millimetres.

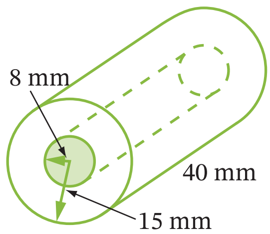
a



b



c



CHAPTER 11

6 **a** Calculate the size of each interior angle in a regular nonagon (9 sides).

b Find the size of each exterior angle.

CHAPTER 4

7 Simplify each expression, writing your answer with a positive index where necessary.

a $7x^5 \times 8x^7$ **b** $4x^2 \div 16x^{-3}$

c $(3y)^{-2}$ **d** $m^{-6}n^3 \times mn^{-1} \div m^2 n$

Foundation Standard Complex

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8 Students in Year 10 at Nelson Secondary College were asked if they had studied Japanese.

CHAPTER 10

	Male	Female
Japanese	35	87
No Japanese	67	21

a How many students are in Year 10 at the school?

b What is the probability of selecting a Year 10 student at random who is:

i male and has studied Japanese?

ii female or has studied Japanese?

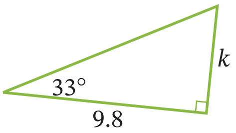
c Find the probability, expressed as a percentage to the nearest whole number, of randomly selecting a male in Year 10 who has not studied Japanese.

d Given that a student is a Year 10 female, what is the probability that she has studied Japanese?

9 Find the value of the variable in each diagram, correct to one decimal place.

CHAPTER 8

a



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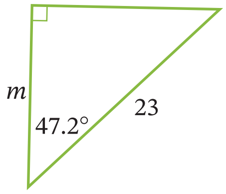

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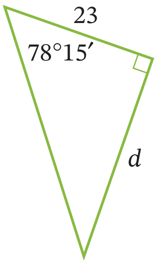

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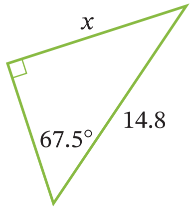

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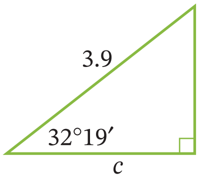

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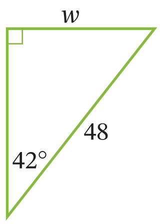


10 Solve each equation.

CHAPTER 6

a $2k - 5 = 8$

b $3(m + 7) = 12$

c $4(x - 3) - 2(x - 1) = 5$

GENERAL PRACTICE

CHAPTER 8

11 The angle of elevation of the top of a tree is 65° at a distance of 12 m from its base. Find the height of the tree, to the nearest metre.

STAGE 5.2

CHAPTER 6

12 Solve each equation.

a $\frac{a-2}{3}=7$

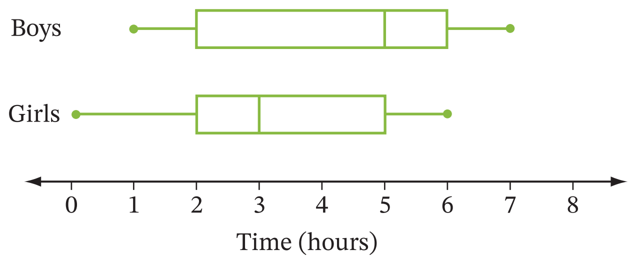
b $\frac{4y}{5}-2=3$

c $\frac{3m+8}{2}=-1$

d $\frac{5k}{3}-\frac{k}{2}=4$

CHAPTER 5

13 Students were surveyed about the amount of time they typically spend on the Internet over a weekend. The results for boys and girls are displayed in these parallel box plots.



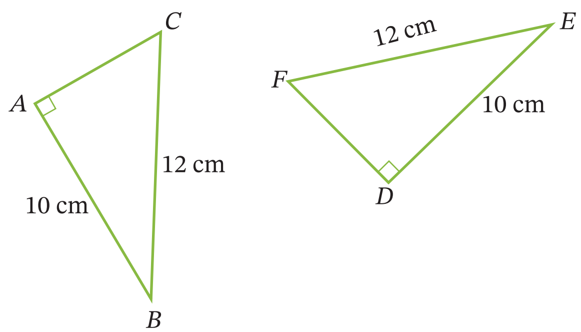
a Calculate the interquartile range for the boys.

b What is the median amount of time spent on the Internet by the girls surveyed?

c What percentage of girls usually spend fewer than 5 hours on the Internet over the weekend?

CHAPTER 11

14 Prove that $\triangle ABC \equiv \triangle DEF$.



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15 Solve each inequality and graph its solution on a number line.

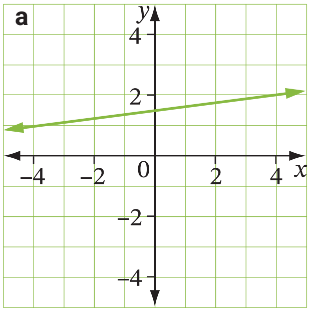
a $5y + 3 \geq -2$

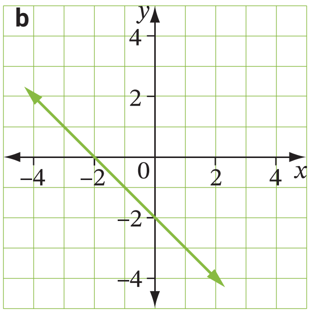
b $\frac{2x+5}{2} < 4$

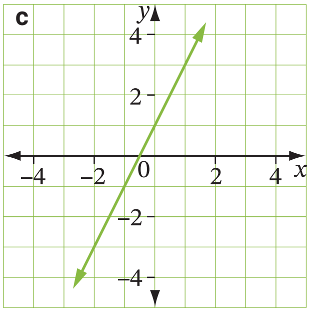
c $5 - 4x > 17$

16 For each line, find:

i the gradient **ii** the y-intercept **iii** the equation of the line.





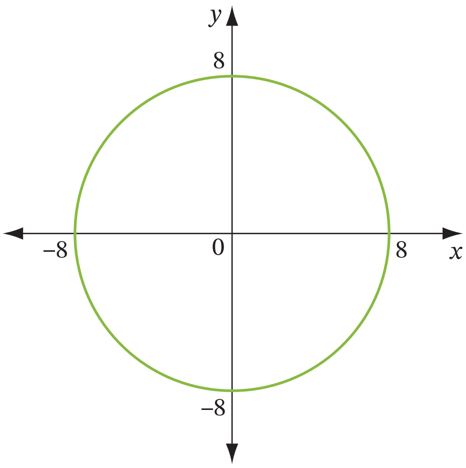


17 Solve each pair of simultaneous equations.

a $3x - y = 4$
$2x + y = 6$

b $2m - 3p = 5$
$5m - 2p = 7$

18 Find the equation of this circle.



19 Calculate the simple interest on each investment.

a \$500 invested at 4% p.a. for 2 years

b \$280 invested at 2.5% p.a. for 7 months

20 Find θ, correct to the nearest degree, if:

a $\sin\theta = \frac{3}{7}$

b $\tan\theta = 6$

c $\cos\theta = 0.816$

STAGE 5.2

CHAPTER 6

CHAPTER 2

CHAPTER 9

CHAPTER 7

CHAPTER 1

CHAPTER 8

Foundation Standard Complex

GENERAL PRACTICE

STAGE 5.2
CHAPTER 1

21 Calculate the final amount for each investment.

a \$800 invested at 4% p.a. for 4 years, compounded annually

b \$1260 invested at 8% p.a. for 3 years, compounded quarterly

CHAPTER 2

22 For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place

ii the midpoint of the interval

iii the gradient of the interval

a $C(2, 1)$ and $D(6, 9)$ **b** $X(-7, -2)$ and $Y(5, 4)$

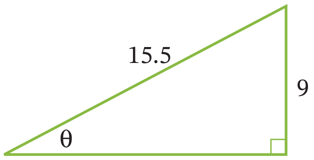
STAGE 5.2
CHAPTER 8

23 Find θ:

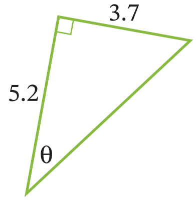
i correct to the nearest degree

ii correct to the nearest minute

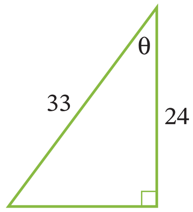
a



b



c



CHAPTER 7

24 Graph each equation on a number plane.

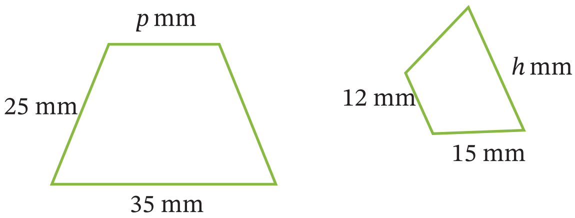
a $y = x^2$

b $y = -x^2 + 3$

c $y = 3^x$

CHAPTER 11

25 Find the value of each variable in this pair of similar figures.



STAGE 5.2
CHAPTER 8

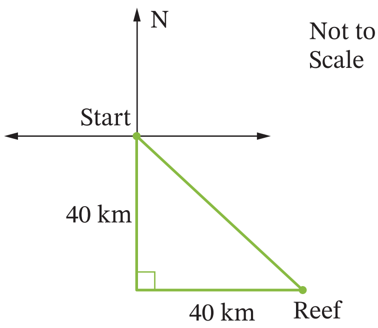
26 Imogen sailed due south for 40 km. Then she sailed due east for 40 km to a reef. What is the bearing of the reef from Imogen's starting point?
Select the correct answer **A**, **B**, **C** or **D**.

A 225°

B 045°

C 090°

D 135°



27 The weather on a long weekend will either be fine or rainy each day, with each outcome being equally likely.

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that it is fine:

i on all 3 days?

ii on exactly 2 of the days?

iii on at least one of the days?

28 For an object that is cooling, the drop in temperature is directly proportional to the time. The temperature drops 5°C in 12 minutes. How long will it take to drop 8°C?

STAGE 5.2

CHAPTER 10

CHAPTER 7

GENERAL PRACTICE

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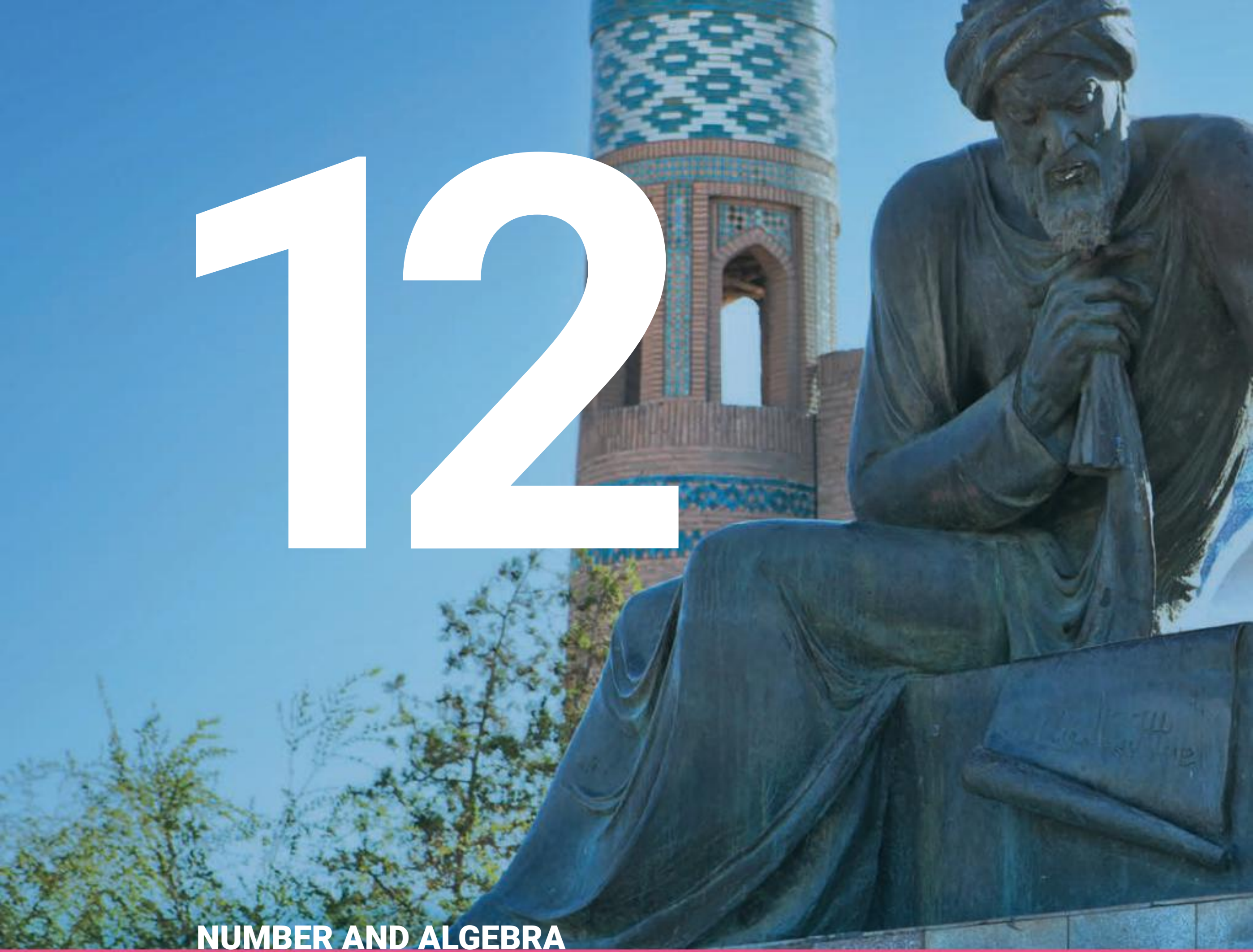

12

NUMBER AND ALGEBRA

PRODUCTS AND FACTORS

OPTIONAL STAGE 5.3 TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

In 825 CE, the Persian mathematician al-Khwarizmi used the Arabic word 'al-jabr' to describe the process of adding equal quantities to both sides of an equation. When al-Khwarizmi's book was translated into Latin and introduced to Europe, 'al-jabr' became 'algebra' and the word was adopted as the name for the branch of mathematics that uses formulas to describe patterns and relationships in our world. The photo above shows a monument of al-Khwarizmi located in Khiva, Uzbekistan.

Chapter outline

STAGE 5.3	Working mathematically				
12.01 Perfect squares	U	F			
12.02 Difference of 2 squares	U	F		R	C
12.03 Mixed expansions	U	F		R	C
12.04 Factorising special binomial products	U	F			
12.05 Factorising quadratic expressions $ax^2 + bx + c$	U	F		R	
12.06 Mixed factorisations	U	F		R	
12.07 Factorising algebraic fractions	U	F		R	

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$.

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor; factorising is the opposite of expanding, for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

highest common factor (HCF) The largest term that is a factor of 2 or more terms, for example, the HCF of $9r^2 + 36r$ is $9r$

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$

quadratic expression An algebraic expression in which the highest power of the variable is 2, for example, $2x^2 + 5x - 3$ or $x^2 + 2$

quadratic trinomial An algebraic expression that consists of 3 terms, for example, $x^2 + 2x + 6$

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) recognise and expand special binomial products that are perfect squares or the difference of 2 squares
- (STAGE 5.3) factorise an expression with 4 terms by grouping in pairs
- (STAGE 5.3) factorise non-monic quadratic expressions, including those that are perfect squares or the difference of 2 squares
- (STAGE 5.3) factorise and simplify algebraic fractions

SkillCheck ANSWERS ON P. 552

1 Expand each binomial product.

a $(x + 1)(x + 7)$ **b** $(d + 3)(d - 4)$

c $(t - 5)(t + 5)$ **d** $(y + 2)(5y - 6)$

e $(7y - 3)(7y - 3)$ **f** $(5p + 4)(2p - 7)$

2 Factorise each quadratic expression.

a $y^2 + 10y + 25$ **b** $x^2 - 21x + 20$

c $n^2 + 8n - 33$ **d** $a^2 - 11a + 28$

e $m^2 - 5m - 84$ **f** $p^2 + 3p - 54$

12.01 Perfect squares

STAGE 5.3

Factorominoes

We know how to expand **binomial products** such as $(k + 3)(k - 7)$. Now we will examine the special binomial product where a binomial is multiplied by itself, for example, $(x + 4)(x + 4) = (x + 4)^2$ or $(2a - 9)(2a - 9) = (2a - 9)^2$.

16, 49, v^2 and $(y + 5)^2$ are called **perfect squares** because they are square numbers.

Perfect squares

The formulas for expanding the perfect square of a binomial are:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Proof

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a(a - b) - b(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

STAGE 5.3

12.01

Example 1

Copy and complete the expansion of each perfect square.

a $(x + 7)^2 = x^2 + \_\_\_\_ + 49$

b $(y - 6)^2 = y^2 - 12y + \_\_\_\_$

c $(5g + 9)^2 = \_\_\_\_ + 90g + 81$

d $(3d - 5)^2 = 9d^2 - \_\_\_\_ + 25$

Solution

a In the expansion,

$\_\_\_\_ = 2 \times x \times 7$ — Doubling the product of the 2 terms

$= 14x$

$\therefore (x + 7)^2 = x^2 + 14x + 49$

b In the expansion,

$\_\_\_\_ = 6^2$ — The second term squared

$= 36$

$\therefore (y - 6)^2 = y^2 - 12y + 36$

c $\_\_\_\_ = (5g)^2$ — The first term squared

$= 25g^2$

$\therefore (5g + 9)^2 = 25g^2 + 90g + 81$

d $\_\_\_\_ = 2 \times 3d \times 5$ — Doubling the product of the 2 terms

$= 30d$

$\therefore (3d - 5)^2 = 9d^2 - 30d + 25$

Example 2

Special binomial products

Expand each perfect square.

a $(n - 5)^2$

b $(k + 4)^2$

c $(3y - 8)^2$

Solution

a $(n - 5)^2 = n^2 - 2 \times n \times 5 + 5^2$ — 1st term squared – double product + 2nd term squared

$= n^2 - 10n + 25$

b $(k + 4)^2 = k^2 + 2 \times k \times 4 + 4^2$

$= k^2 + 8k + 16$

c $(3y - 8)^2 = (3y)^2 - 2 \times 3y \times 8 + 8^2$

$= 9y^2 - 48y + 64$

STAGE 5.3

EXERCISE 12.01 ANSWERS ON P. 552

Perfect squares U F

EXAMPLE 1

1 Copy and complete the expansion of each perfect square.

a $(x + 10)^2 = x^2 +$ _____ $+ 100$

b $(m - 8)^2 =$ _____ $- 16m + 64$

c $(p - t)^2 = p^2 - 2pt +$ _____

d $(h + 4)^2 = h^2$ _____ $+ 16$

e $(k - 9)^2 = k^2$ _____ $+ 81$

f $(8 + 5f)^2 = 64$ _____ $+ 25f^2$

g $(2d + 3)^2 =$ _____ $+$ _____ $+ 9$

h $(6a + 1)^2 =$ _____ $+ 12a +$ _____

EXAMPLE 2

2 Expand each perfect square.

a $(m + 9)^2$
b $(u + 3)^2$
c $(y - 6)^2$
d $(8 + k)^2$
e $(5 - h)^2$
f $(7 + k)^2$
g $(f + 20)^2$
h $(q - 11)^2$
i $(10 + t)^2$
j $(x - w)^2$
k $(a + g)^2$
l $(2m - 3)^2$
m $(5x - 6)^2$
n $(9a + 2)^2$
o $(3e - 4)^2$
p $(5 + 7b)^2$
q $(4 - 5p)^2$
r $(11 - 2c)^2$
s $(10g + 3)^2$
t $(3k + 11)^2$
u $(5 + 2v)^2$

3 Expand each perfect square.

a $(7h + 2k)^2$
b $(8a - 3y)^2$
c $(xy + z)^2$
d $\left(1+\frac{1}{y}\right)^2$
e $\left(k-\frac{1}{k}\right)^2$
f $\left(w+\frac{3}{w}\right)^2$

4 Use expansion to evaluate each square number without using a calculator.

a $21^2 = (20 + 1)^2$
b $45^2 = (40 + 5)^2$
c $29^2 = (30 - 1)^2$
d $59^2 = (60 - 1)^2$
e $102^2 = (100 + 2)^2$
f $98^2 = (100 - 2)^2$

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Difference of 2 squares

12.02

STAGE 5.3

Enough time

The binomial product $(a + b)(a - b)$ has a special pattern when expanded.

Difference of 2 squares

$$(a + b)(a - b) = a^2 - b^2$$

The answer is called the **difference of 2 squares**.

Proof:

$$\begin{aligned}(a - b)(a + b) &= a(a + b) - b(a + b)\\ &= a^2 + ab - ba - b^2\\ &= a^2 - b^2 \qquad + ab - ba = 0\end{aligned}$$

When the sum of 2 terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of 2 squares).

Example 3

Difference of 2 squares

Special binomial products

Expand each expression.

a $(d + 3)(d - 3)$

b $(2 + r)(2 - r)$

c $(7x + 2)(7x - 2)$

d $(4k - 5p)(4k + 5p)$

Solution

a $(d + 3)(d - 3) = d^2 - 3^2$
$= d^2 - 9$

b $(2 + r)(2 - r) = 2^2 - r^2$
$= 4 - r^2$

c $(7x + 2)(7x - 2) = (7x)^2 - 2^2$
$= 49x^2 - 4$

d $(4k - 5p)(4k + 5p) = (4k)^2 - (5p)^2$
$= 16k^2 - 25p^2$

STAGE 5.3

EXERCISE 12.02 ANSWERS ON P. 552

Difference of 2 squares U F R C

EXAMPLE 3

1 Expand each expression.

a $(m + 5)(m - 5)$ **b** $(c - 10)(c + 10)$

c $(a + 12)(a - 12)$ **d** $(6 - y)(6 + y)$

e $(8 - m)(8 + m)$ **f** $(p + 1)(p - 1)$

g $(5 + e)(5 - e)$ **h** $(v + 11)(v - 11)$

i $(w - 3)(w + 3)$ **j** $(x - 10)(x + 10)$

k $(q + 7)(q - 7)$ **l** $(9 - g)(9 + g)$

m $(b - 2)(b + 2)$ **n** $(15 - r)(15 + r)$

o $(d + 13)(d - 13)$

2 Expand each expression.

a $(2h - 3)(2h + 3)$ **b** $(5r + 4)(5r - 4)$

c $(5b + 8)(5b - 8)$ **d** $(4p - 7)(4p + 7)$

e $(3 - 8k)(3 + 8k)$ **f** $(7x - 5)(7x + 5)$

g $(2 + 9m)(2 - 9m)$ **h** $(9k - 4l)(9k + 4l)$

i $(7n + 8m)(7n - 8m)$ **j** $(4g - 5h)(4g + 5h)$

k $(7u + 3w)(7u - 3w)$ **l** $(11a + 3b)(11a - 3b)$

m $\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$ **n** $\left(\frac{w}{3}-2\right)\left(\frac{w}{3}+2\right)$

o $\left(1-\frac{1}{r}\right)\left(1+\frac{1}{r}\right)$

3 Natalie's age is p years. R C

a What was Natalie's age last year?

b What will Natalie's age be next year?

c Write an expression for (Natalie's age last year) × (Natalie's age next year).

d If (Natalie's age last year) × (Natalie's age next year) is equal to 48, what is Natalie's age?

4 By expressing 31 × 29 as (30 + 1)(30 – 1), use the difference of 2 squares to find the value of 31 × 29. R C

5 Use the method of question **4** to evaluate each expression. R C

a 21 × 19 **b** 51 × 49 **c** 89 × 91 **d** 78 × 82

6 Use 'sum by difference' to evaluate each expression.

a $15^2 - 14^2$ **b** $24^2 - 23^2$ **c** $65^2 - 35^2$

d $101^2 - 99^2$ **e** $23^2 - 17^2$ **f** $50^2 - 48^2$

Foundation Standard Complex

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Mixed expansions

12.03

STAGE 5.3

WS Mixed expansions

Products and factors squaresaw

Expanding binomials

WS Algebra 6

Example 4

Expand and simplify each expression.

a $(4r + 5)(1 - 2r)$ **b** $(7 + 9x)^2$ **c** $(3d - 10)(3d + 10)$

d $(a + 6)(a - 6) + (a + 12)(a + 3)$ **e** $(m - 2)^2 - (m - 2)(m + 2)$

Solution

a $(4r + 5)(1 - 2r) = 4r(1 - 2r) + 5(1 - 2r)$ Expanding

$= 4r - 8r^2 + 5 - 10r$

$= -8r^2 - 6r + 5$ Rearranging and putting the higher order terms first.

b $(7 + 9x)^2 = 7^2 + 2 \times 7 \times 9x + (9x)^2$ Expand the perfect square.

$= 49 + 126x + 81x^2$

$= 81x^2 + 126x + 49$ Rearranging and putting the higher order terms first.

c $(3d - 10)(3d + 10) = (3d)^2 - 10^2$ Expand into the difference of 2 squares

$= 9d^2 - 100$

d $(a + 6)(a - 6) + (a + 12)(a + 3) = a^2 - 36 + a^2 + 3a + 12a + 36$

$= 2a^2 + 15a$

e $(m - 2)^2 - (m - 2)(m + 2) = m^2 - 4m + 4 - (m^2 - 4)$

$= m^2 - 4m + 4 - m^2 + 4$

$= -4m + 8$

EXERCISE 12.03 ANSWERS ON P. 552

Mixed expansions U F R C

EXAMPLE 4

1 Expand and simplify each expression.

a $(2m - 1)(2m + 1)$ **b** $(y + 4)(5y - 3)$

c $(2k - 7)^2$ **d** $(d + 9)^2$

e $(2e - 1)(e + 1)$ **f** $(5a + 4)(5a - 4)$

g $(2 - p)(p - 2)$ **h** $(10 - 6y)(10 + 6y)$

i $(h - 3m)^2$ **j** $(2x - 3)(y + 3)$

k $(11a - 4b)(11a + 4b)$ **l** $\left(u - \frac{1}{u}\right)^2$

Foundation Standard Complex

STAGE 5.3

2 Expand and simplify each expression. R C

a $(m - 5)(m + 5) + 25$

b $6y + (y - 3)^2 + 9$

c $(3x + 1)(2 - x) + 2x + 4$

d $(d + 4)^2 - 8d + 5$

e $16 + (4k - 8)(4k + 8)$

f $(x - y)^2 - (x + y)^2$

g $20a - (a - 2)(a - 5) + a^2$

h $2(f - 2)(f + 2)$

i $(2h + 3)^2 - (2h - 3)(2h + 3)$

j $7xy - (2x - 3)(y + 3)$

3 Expand and simplify each expression. R C

a $(8a - 1)(8a + 1) - 4a^2 + 1$

b $(n + 1)^2 + 2n + 3$

c $3(4 - u)(4 + u) + (u - 12)(u + 4)$

d $(2m - n)^2 + (2m + n)^2$

e $(x - 2)(x + 3) - (x - 2)(x + 2)$

f $2(b - 1)^2 - (2b - 1)^2$

g $(y + 1)^2 + (y + 2)^2 + (y + 3)^2$

h $(x - 3)(x + 3) + (x + 3)^2 + (x - 3)^2$

i $(5n + 3)(5n - 3) + (3n - 5)(3n + 5)$

j $2(a - b)(a + b) - (a + b)^2 - (a - b)^2$

12.04 Factorising special binomial products

STAGE 5.3

Factorising by grouping in pairs

Grouping

An **algebraic expression with 4 terms** can often be factorised in pairs, that is, 2 terms at a time, to make a binomial product.

Example 5

Factorise each expression.

a $3ac + 2bd + 2bc + 3ad$

b $4km + 6mn - 6kp - 9np$

c $10xw - 6yw - 10xp + 6yp$

Solution

a $3ac + 2bd + 2bc + 3ad = 3ac + 3ad + 2bd + 2bc$ — Grouping into pairs for factorising

$= 3a(c + d) + 2b(d + c)$ — Factorising each pair

$= (c + d)(3a + 2b)$ — Factorising again

b $4km + 6mn - 6kp - 9np = 2m(2k + 3n) - 3p(2k + 3n)$ — Factorising each pair

$= (2k + 3n)(2m - 3p)$ — Factorising again

c $10xw - 6yw - 10xp + 6yp = 2(5xw - 3yw - 5xp + 3yp)$ — Factorising all terms first

$= 2[w(5x - 3y) - p(5x - 3y)]$ — Factorising each pair

$= 2(5x - 3y)(w - p)$ — Factorising again

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Factorising the difference of 2 squares

We have learned the formula $(a + b)(a - b) = a^2 - b^2$.

If we use this rule in reverse, then the factors of $a^2 - b^2$ are $(a + b)$ and $(a - b)$.

The difference of 2 squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 6

Factorise each expression.

a $x^2 - 4$ **b** $9 - 16b^2$ **c** $20d^2 - 5a^2$ **d** $y^3 - y$

Solution

a $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

b $9 - 16b^2 = 3^2 - (4b)^2$
$= (3 + 4b)(3 - 4b)$

c $20d^2 - 5a^2 = 5(4d^2 - a^2)$
$= 5[(2d)^2 - a^2]$
$= 5(2d + a)(2d - a)$

d $y^3 - y = y(y^2 - 1)$
$= y(y + 1)(y - 1)$

STAGE 5.3

Difference of 2 perfect squares

Difference of 2 squares

12.04

EXERCISE 12.04 ANSWERS ON P. 552

Factorising special binomial products U F

1 Factorise each expression. EXAMPLE 5

a $4ab + 5bc + 4ad + 5cd$ **b** $2xy - 5wy + 2xt - 5wt$

c $9ac + 6bc + 12ad + 8bd$ **d** $10x^2 + 30 + x^3 + 3x$

e $3a^2 + 3ab + 3ac + 3bc$ **f** $6rt - 18wt + 6rp - 18wp$

g $14e - 21 + 2de - 3d$ **h** $hk - h^2 - 2k + 2h$

i $3mn - 6m + pn - 2p$ **j** $9p^2 - 27 + qp^2 - 3q$

k $fg - fh - 10g + 10h$ **l** $9kl - 12ml + 9kn - 12mn$

m $2p - 2c - p^2 + pc$ **n** $l^3 + lm^2 - 3l^2 - 3m^2$

o $a(x + 1) + y(x + 1) - ka - ky$ **p** $p(a - b) - 2q(a - b) + 3qp - 6q^2$

2 Factorise each expression. EXAMPLE 6

a $w^2 - 9$ **b** $y^2 - 36$ **c** $k^2 - 1$ **d** $m^2 - 121$

e $p^2 - 64$ **f** $c^2 - 100$ **g** $4e^2 - f^2$ **h** $a^2 - 9b^2$

i $16y^2 - 1$ **j** $4 - b^2$ **k** $25 - e^2$ **l** $1 - 16x^2$

m $k^2 - u^2$ **n** $49 - 16m^2$ **o** $b^2 - 121d^2$ **p** $36c^2 - 25k^2$

q $16 - 81h^2$ **r** $25a^2 - 64m^2$ **s** $100 - 49n^2$ **t** $121p^2 - 144q^2$

u $\frac{1}{4} - 25c^2$ **v** $4w^2 - \frac{1}{9}$ **w** $64h^2 - 2\frac{1}{4}$ **x** $1 - m^2n^2$

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STAGE 5.3

3 Factorise each expression.

a $2a^2 - 2b^2$ b $7k^2 - 28$ c $3 - 75u^2$ d $x^3 - 49x$

e $k - 16k^3$ f $50q^2 - 2$ g $3d^2 - 12v^2$ h $5t^5 - 125t^3$

i $2a^2b^2 - 2$ j $x^2y^2 - x^2w^2$ k $192f^2 - 108g^2$ l $45d^2 - \frac{5}{4}$

m $2x^2 - 8a^2$ n $100 - 25w^2$ o $1\frac{1}{4} - 80e^2$ p $9c^2 - 6\frac{1}{4}$

4 Factorise each expression.

a $\frac{p^2}{16} - \frac{x^2}{25}$ b $x^2 - \frac{1}{9}$ c $\frac{v^2}{64} - \frac{u^2}{81}$

d $\frac{2y^2}{9} - \frac{2m^2}{121}$ e $\frac{16a^2}{49} - \frac{25b^2}{4}$ f $g^4 - 81$

g $10\,000 - n^4$ h $(x + y)^2 - x^2$ i $(p - 2q)^2 - (2p + q)^2$

j $\frac{x^2}{4} - \frac{y^2}{36}$ k $(a + b)^2 - (a - b)^2$ l $25 - (4 - x)^2$

12.05 Factorising quadratic expressions $ax^2 + bx + c$

STAGE 5.3

Products and factors squaresaw

Perfect squares

Trinominoes

Factorising trinomials

Factorising trinomials

In Chapter 4, Algebra, we factorised **quadratic trinomials** of the type $x^2 + bx + c$. For example, the factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

We will now factorise quadratic expressions of the type $ax^2 + bx + c$, such as $6x^2 + 19x + 15$, where x^2 has a **coefficient**.

Example 7

Factorise each quadratic expression.

a $3g^2 + 12g - 36$ b $48 - 8p - p^2$

Solution

a $3g^2 + 12g - 36 = 3(g^2 + 4g - 12)$ Taking out the HCF of 3 first.

$= 3(g - 2)(g + 6)$ Product = −12, sum = 4

b $48 - 8p - p^2 = -p^2 - 8p + 48$ Rearranging the terms to make the p^2 term first.

$= -1(p^2 + 8p - 48)$ Taking out a common factor of −1

$= -(p + 12)(p - 4)$ Product = −48, sum = 8

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STAGE 5.3

Factorising quadratic expressions (Advanced)

12.05

Example 8

Factorise $3x^2 + 8x + 4$.

Solution

There is no HCF, so we need to split up the middle term $8x$.

Find 2 numbers that have a product of 12 and a sum of 8.

sum of 8

$3x^2 + 8x + 4 = 3x^2 + 8x + 4$

product of 12 (3 × 4)

The 2 numbers are +6 and +2, so we will split $8x$ into $6x$ and $2x$.

$\therefore 3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$

$= 3x(x + 2) + 2(x + 2)$ Factorising by grouping in pairs

$= (x + 2)(3x + 2)$ Factorising again

Factorising quadratic trinomials of the form $ax^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of ac
- Use these 2 numbers to split the middle term bx into 2 terms
- Factorise by grouping in pairs

Factorising quadratic expressions 2

Advanced algebra

Example 9

Factorise each quadratic expression.

a $5k^2 - 12k + 4$ **b** $9m^2 - 9m - 4$ **c** $6t^2 + t - 12$

Solution

a $5k^2 - 12k + 4$

$5 \times 4 = 20$.

Find 2 numbers that have a product of 20 and a sum of –12.

Since the sum is negative, one of the numbers must be negative.

Since the product is positive, both of the numbers must be negative.

Try –5, –4: $-5 \times (-4) = 20$, $-5 + (-4) = -9$ ✗

Try –10, –2: $-10 \times (-2) = 20$, $-10 + (-2) = -12$ ✓

They are –10 and –2. Split $-12k$ into $-10k$ and $-2k$.

$5k^2 - 12k + 4 = 5k^2 - 10k - 2k + 4$

$= 5k(k - 2) - 2(k - 2)$ Factorising by grouping in pairs

$= (k - 2)(5k - 2)$

STAGE 5.3

b $9m^2 - 9m - 4$

$9 \times (-4) = -36$

Find 2 numbers with a product of −36 and a sum of −9.

Since the product is negative, one of the numbers must be negative.

Try 12, −3: $12 \times (-3) = -36,\ 12 + (-3) = 9$ ✗

Try −12, 3: $-12 \times 3 = -36,\ -12 + 3 = -9$ ✓

They are −12 and 3.

$$\begin{aligned} 9m^2 - 9m - 4 &= 9m^2 - 12m + 3m - 4 \\ &= 3m(3m - 4) + 1(3m - 4) \\ &= (3m - 4)(3m + 1) \end{aligned}$$

c $6t^2 + t - 12$

$6 \times (-12) = -72$

Find 2 numbers with a product of −72 and a sum of 1.

Try 8, −9: $8 \times (-9) = -72,\ 8 + (-9) = -1$ ✗

Try −8, 9: $-8 \times (9) = -72,\ -8 + 9 = 1$ ✓

They are −8 and +9.

$$\begin{aligned} 6t^2 + t - 12 &= 6t^2 - 8t + 9t - 12 \\ &= 2t(3t - 4) + 3(3t - 4) \\ &= (3t - 4)(2t + 3) \end{aligned}$$

Example 10

Factorise each quadratic expression.

a $18a^2 - 18a - 8$

b $10 - 7x - 12x^2$

Solution

a

$18a^2 - 18a - 8 = 2(9a^2 - 9a - 4)$ — Taking out the HCF of 2 first

$= 2(9a^2 - 12a + 3a - 4)$ — Product = −36, sum = −9

$= 2[3a(3a - 4) + 1(3a - 4)]$

$= 2(3a - 4)(3a + 1)$

b

$10 - 7x - 12x^2 = -12x^2 - 7x + 10$ — Rearranging the terms to make the x^2 term first

$= -(12x^2 + 7x - 10)$ — Taking out a common factor of −1

$= -(12x^2 + 15x - 8x - 10)$ — Product = −120, sum = −7

$= -[3x(4x + 5) - 2(4x + 5)]$

$= -(4x + 5)(3x - 2)$ — This can also be written as $(4x + 5)(2 - 3x)$

EXERCISE 12.05 ANSWERS ON P. 553

STAGE 5.3

Factorising quadratic expressions $ax^2 + bx + c$ U F R

1 Factorise each quadratic expression. Look for the highest common factor first. R

EXAMPLE 7

a $3m^2 + 9m + 6$	**b** $2y^2 + 2y - 4$	**c** $5t^2 - 10t - 400$
d $5e^4 + 25e^3 - 120e^2$	**e** $x^3 - x^2 - 110x$	**f** $4b^2 - 4b - 168$
g $4w^2 + 4w - 48$	**h** $3a^3 - 9a^2 - 12a$	**i** $2e^2 + 18e + 40$
j $24 - 5t - t^2$	**k** $42 + u - u^2$	**l** $28 + 3x - x^2$
m $12 - b - b^2$	**n** $7k - 12 - k^2$	**o** $12x - 35 - x^2$

12.05

2 Factorise each quadratic expression. R

EXAMPLE 8

a $2x^2 + 11x + 5$	**b** $4m^2 + 13m + 3$	**c** $5y^2 + 17y + 6$
d $6u^2 + 19u + 10$	**e** $2w^2 + 31w + 15$	**f** $4e^2 + 15e + 9$
g $8f^2 + 14f + 3$	**h** $3d^2 + 5d + 2$	**i** $2b^2 + 9b + 7$
j $5y^2 + 16y + 11$	**k** $8g^2 + 26g + 15$	**l** $6a^2 + 23a + 21$

3 Factorise each quadratic expression. R

EXAMPLE 9

a $2y^2 - 11y + 12$	**b** $10k^2 - 19k + 6$	**c** $6e^2 - 13e + 6$
d $4b^2 - 13b + 3$	**e** $6w^2 - 23w + 15$	**f** $12f^2 - 25f + 12$
g $15m^2 - 26m + 8$	**h** $9x^2 - 12x + 4$	**i** $2a^2 - 23a + 45$
j $12y^2 - 35y + 8$	**k** $8d^2 - 34d + 21$	**l** $4h^2 - 36h + 81$

4 Factorise each quadratic expression. R

a $5y^2 - 6y - 11$	**b** $4d^2 - d - 5$	**c** $2m^2 - 3m - 9$
d $3t^2 - t - 30$	**e** $6h^2 - h - 7$	**f** $2y^2 - 5y - 12$
g $8a^2 - 2a - 3$	**h** $15u^2 - 7u - 4$	**i** $9c^2 - 12c - 5$
j $6c^2 - 7c - 24$	**k** $20n^2 - 27n - 14$	**l** $12x^2 - 7x - 10$

5 Factorise each quadratic expression. R

a $5m^2 + 2m - 7$	**b** $6g^2 + g - 12$	**c** $3p^2 + 4p - 4$
d $7w^2 + 6w - 1$	**e** $5y^2 + 14y - 3$	**f** $3n^2 + 10n - 8$
g $4b^2 + 9b - 9$	**h** $8m^2 + 10m - 3$	**i** $3x^2 + 2x - 16$
j $8u^2 + 26u - 15$	**k** $24e^2 + 31e - 15$	**l** $20h^2 + 11h - 42$

6 Factorise each expression given it is a perfect square. R

a $81w^2 - 180w + 100$	**b** $4y^2 + 8y + 4$	**c** $25h^2 - 40h + 16$

7 Factorise each quadratic expression by first taking out a common factor. R

EXAMPLE 10

a $6y^2 + 10y - 4$	**b** $6g^2 + 15g - 36$	**c** $24e^2 - 28e - 12$
d $8a^2 - 10a - 12$	**e** $12u^2 + 20u - 8$	**f** $-25q^2 - 5q + 6$
g $-12m^2 + 14m - 4$	**h** $20 - h - 12h^2$	**i** $24c^2 + 48c + 18$
j $15 + 9w - 6w^2$	**k** $12d^2 + 2d - 30$	**l** $22x - 12 - 6x^2$

Foundation Standard Complex

STAGE 5.3

8 Factorise each quadratic expression. R

a	$2a^2 + 5a + 3$	**b**	$12m^2 - 32m + 5$	**c**	$4x^2 + 11x - 3$
d	$7w^2 - 8w + 1$	**e**	$4h^2 - 7h - 15$	**f**	$8x^2 - 2x - 3$
g	$5r^2 + 26r + 5$	**h**	$2d^2 - 15d + 7$	**i**	$6n^2 - 7n - 3$
j	$8 - 6m - 9m^2$	**k**	$3 - 2c - 5c^2$	**l**	$15g^2 + 19g + 6$
m	$15 + 14q - 8q^2$	**n**	$3x^2 - 13x + 14$	**o**	$16 - 8d - 3d^2$
p	$42 - 10y - 12y^2$	**q**	$28d^2 - 44d - 24$	**r**	$100k^2 + 80k + 16$

12.06 Mixed factorisations

STAGE 5.3

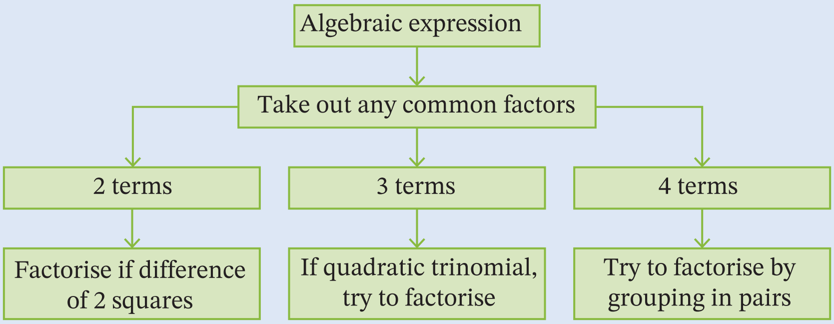
Mixed factorisations

Factorominoes

Products and factors squaresaw

Factorisation strategies

- Look for any common factors and factorise first.
- If there are 2 terms, try factorising using the difference of 2 squares.
- If there are 3 terms, try factorising as a quadratic trinomial.
- If there are 4 terms, try factorising by grouping in pairs.



Example 11

Factorise each quadratic expression.

a $3a^2 - 27$ **b** $5a^2 + 100$ **c** $20b^2 - 52b + 24$ **d** $d^3 - d^2 - d + 1$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$ Taking out the HCF of 3 first

$= 3(a + 3)(a - 3)$ Difference of 2 squares

b $5a^2 + 100 = 5(a^2 + 20)$ 2 terms but not a difference of 2 squares

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c $20b^2 - 52b + 24 = 4(5b^2 - 13b + 6)$

$= 4(5b^2 - 10b - 3b + 6)$ Product = 30, sum = –13

$= 4[5b(b - 2) - 3(b - 2)]$

$= 4(b - 2)(5b - 3)$

d $d^3 - d^2 - d + 1 = d^2(d - 1) - 1(d - 1)$ Factorising by grouping in pairs

$= (d - 1)(d^2 - 1)$

$= (d - 1)(d + 1)(d - 1)$ Difference of 2 squares

$= (d - 1)^2(d + 1)$

STAGE 5.3

12.06

EXERCISE 12.06 ANSWERS ON P. 553

Mixed factorisations U F R

EXAMPLE 11

1 Factorise each expression. R

a $m^2 - 16m + 64$
b $3d^2 - 3d$
c $3w^2 - 4w - 15$
d $3k - 15 - 5h + hk$
e $25y^2 - 64$
f $100f^2 - 64$
g $q^2 + 3q - 3pq$
h $3 + 2g - g^2$
i $24n^2 + 44n - 40$
j $25r^2 - 1$
k $b^3 + b^2 + b + 1$
l $4x^2 - 20x + 25$
m $4 - d - 5d^2$
n $a^3 - a^2 - a + 1$
o $8 - 2v^2$
p $mn^2 + mnp + 3mn + 3mp$
q $2w^2 - 24w + 72$
r $36h^2 + 12h + 1$

2 Factorise each expression. R

a $15r^2 - 31rt - 24t^2$
b $4d^2 + 4d + 1$
c $9g^2 - 36k^2$
d $e^3 - 3e^2 - 10e$
e $5(p + q)^2 - 125(p - q)^2$
f $28x^2 - 7$
g $a^2 - b^2 + 4a - 4b$
h $c^3 - 2c^2 - 4c + 8$
i $6a^2 + 13a - 5$
j $y^2 - 3y + 5y - 35$
k $18p^2 + 24p + 8$
l $1 - 2a - 24a^2$
m $9x^2 - 27x + 18x - 54$
n $2a^2b - 6ab - 3a + 9$
o $2a^2 + 12a + 18$
p $25u^2 - 10u + 1$
q $4k^2 - 5k - 21$
r $48 - 3w^2$
s $3 - 27c^2$
t $k^3 + 4k^2 - 16k - 64$
u $5y^3 - 10y^2 + 15y$
v $m^3n - 4mn$
w $8 - 2a^2$
x $32c^2 - 40c - 12$

3 Factorise each expression. R

a $12e^2 + 13e - 35$
b $10y^2 + 19ky - 15k^2$
c $16h^4 - 81$
d $w^2 - (6 - w)^2$
e $d^3 - 16d + 4 - d$
f $u^2 - 8u + 16 - k^2$

Foundation Standard Complex

12.07 Factorising algebraic fractions

STAGE 5.3

WS Simplifying algebraic fractions

Example 12

Simplify each expression.

a $\frac{10a+25b}{5}$ **b** $\frac{9y^2-16}{6y+8}$ **c** $\frac{x^2+x}{-4x-4}$ **d** $\frac{t^2-3t+2}{3t^2-5t-2}$

Solution

a $\frac{10a+25b}{5} = \frac{5^1(2a+5b)}{5^1}$

$= 2a+5b$

b $\frac{9y^2-16}{6y+8} = \frac{(3y+4)(3y-4)}{2(3y+4)}$

$= \frac{3y-4}{2}$

c $\frac{x^2+x}{-4x-4} = \frac{x(x+1)}{-4(x+1)}$

$= -\frac{x}{4}$

d $\frac{t^2-3t+2}{3t^2-5t-2} = \frac{(t-2)(t-1)}{(t-2)(3t+1)}$

$= \frac{t-1}{3t+1}$

Example 13

Simplify each expression.

a $\frac{4}{x^2+x}+\frac{3}{x^2-1}$ **b** $\frac{3m-6}{4}\times\frac{8m}{m^2-2m}$ **c** $\frac{d^2+3d+2}{d^2-9}\div\frac{d^2+d}{3d+9}$

Solution

a $\frac{4}{x^2+x}+\frac{3}{x^2-1} = \frac{4}{x(x+1)}+\frac{3}{(x+1)(x-1)}$ Factorising denominators

$= \frac{4(x-1)}{x(x+1)(x-1)}+\frac{3x}{x(x+1)(x-1)}$ Using common denominators

$= \frac{4x-4+3x}{x(x+1)(x-1)}$

$= \frac{7x-4}{x(x+1)(x-1)}$

b $\frac{3m-6}{4}\times\frac{8m}{m^2-2m} = \frac{3(m-2)}{4_1}\times\frac{{}^2 8m}{m(m-2)}$

$= 6$

c $\frac{d^2+3d+2}{d^2-9}\div\frac{d^2+d}{3d+9} = \frac{d^2+3d+2}{d^2-9}\times\frac{3d+9}{d^2+d}$

$= \frac{(d+2)(d+1)}{(d+3)(d-3)}\times\frac{3(d+3)}{d(d+1)}$

$= \frac{3(d+2)}{d(d-3)}$

EXERCISE 12.07 ANSWERS ON P. 553

Factorising algebraic fractions U F R

STAGE 5.3

1 Simplify each expression. R

EXAMPLE 12

a $\frac{3x+3y}{3}$ b $\frac{5}{10t-10r}$ c $\frac{ab-ac}{a^2}$

d $\frac{y-1}{1-y}$ e $\frac{w^2-16}{w+4}$ f $\frac{5d-5w}{d^2-w^2}$

g $\frac{(k+5)^2}{k^2-25}$ h $\frac{6c^2-6}{2c+2}$ i $\frac{am-an+m-n}{m^2-n^2}$

j $\frac{y^2+9y+20}{2y+10}$ k $\frac{k^2-3k-4}{k^2-16}$ l $\frac{16a^2-25c^2}{4a^2-9ac+5c^2}$

m $\frac{x^2+4x+4}{x^2-x-6}$ n $\frac{1-c-2c^2}{3c^2+2c-1}$ o $\frac{ap+4p-2a-8}{2p^2-8}$

p $\frac{c^2-9c+20}{cu-4+c-4u}$ q $\frac{h^2+3h+2}{2h^2+4h}$ r $\frac{8e^2-14e-15}{4e^3-25e}$

2 Simplify each expression. R

EXAMPLE 13

a $\frac{5}{m(m+1)}+\frac{2}{(m+1)(m+2)}$ b $\frac{6}{(w+5)(w+3)}-\frac{4}{w(w+3)}$

c $\frac{3}{(b+2)(b-1)}+\frac{1}{(b-1)(b-3)}$ d $\frac{2}{k^2+k}-\frac{3}{k^2-1}$

e $\frac{5}{4h+4}+\frac{3}{h^2+h}$ f $\frac{3}{d^2+3d+2}-\frac{4}{d+2}$

g $\frac{3}{r^2-36}-\frac{5}{4r+24}$ h $\frac{3}{d^2+2d}+\frac{d}{d^2-4}$

i $\frac{5}{k^2-3k-4}-\frac{k}{k^2-1}$ j $\frac{2}{q^2-1}+\frac{3}{q+1}$

k $\frac{4}{a^2-8a+15}+\frac{3}{a^2+4a-21}$ l $\frac{2}{y^2+5y+6}-\frac{7}{y^2-y-12}$

m $\frac{5}{e^2+3e+2}-\frac{8}{e^2+9e+14}$ n $\frac{3}{6h^2-h-2}+\frac{4}{12h^2-23h+10}$

3 Simplify each expression. R

a $\frac{3m+9}{2}\times\frac{4m}{m+3}$ b $\frac{5d-10}{3d-9}\times\frac{5d-15}{8d-16}$ c $\frac{4}{e+2}\times\frac{e^2+2e}{8e}$

d $\frac{3k+6}{5}\times\frac{10k}{k+2}$ e $\frac{5h}{3h+9}\times\frac{6h+18}{h^2+h}$ f $\frac{4}{a^2-b^2}\times\frac{3a+3b}{8}$

g $\frac{r+t}{t^2-r^2}\times\frac{r^2-rt}{5r+5t}$ h $\frac{20m+16}{7m-7}\times\frac{7m}{5m+4}$ i $\frac{p^2+2p+1}{p^2-1}\times\frac{4p-4}{p^2+p}$

j $\frac{y+2}{5y}\div\frac{7y+14}{15y}$ k $\frac{5}{x^2-4}\div\frac{15}{2x+4}$ l $\frac{4n+8}{n+5}\div\frac{6n+12}{5n+25}$

m $\frac{d^2+d}{d+3}\div\frac{6d}{d^2-9}$ n $\frac{1}{f^2-6f+9}\div\frac{4}{f^2-9}$ o $\frac{3f+6}{f^2+f-6}\div\frac{f^2-2f-8}{f^2-f-12}$

12.07

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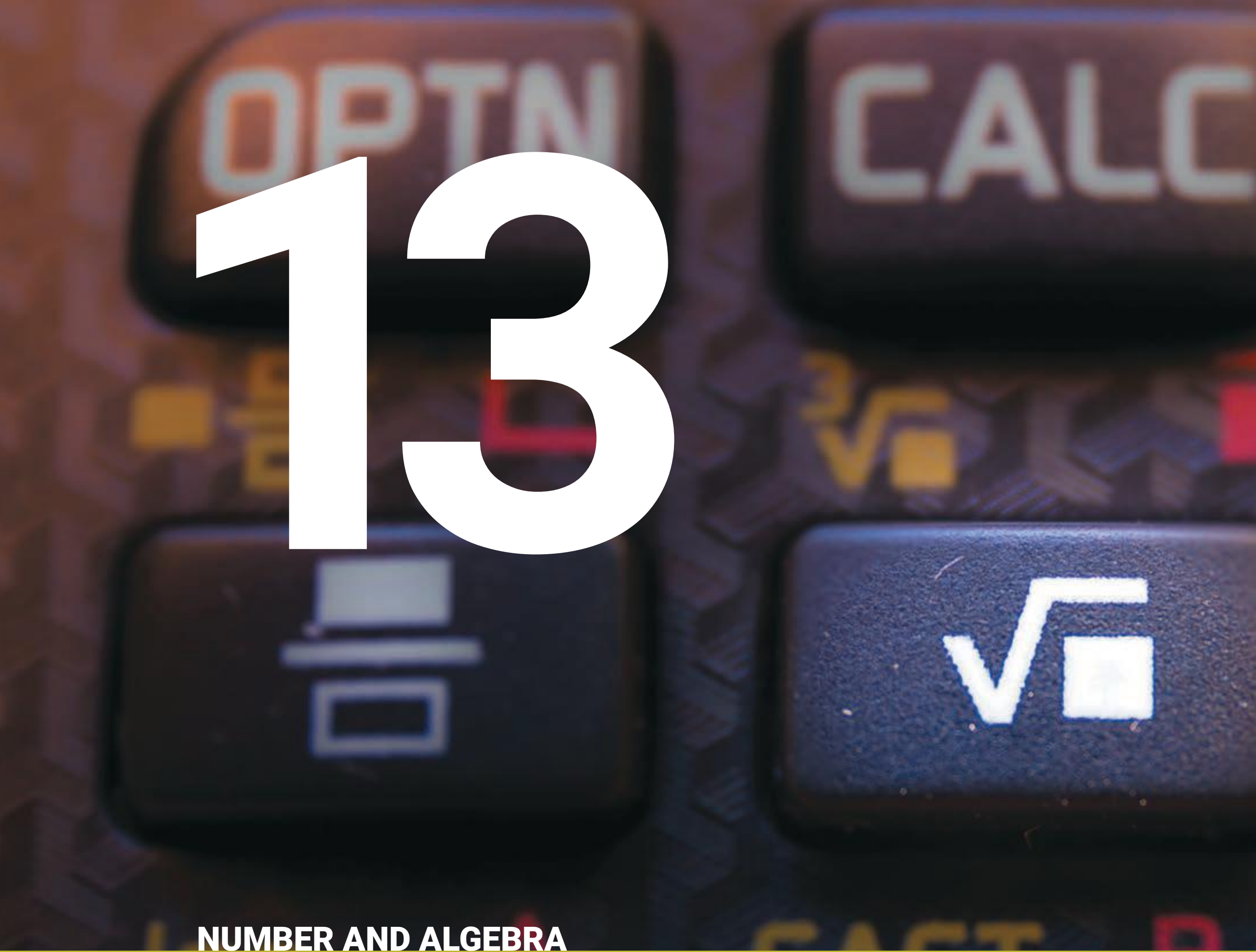

13

NUMBER AND ALGEBRA

SURDS

OPTIONAL STAGE 5.3 TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

When applying Pythagoras' theorem, we have found lengths that cannot be expressed as an exact rational number. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit. A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found.

The Pythagoreans explained nature, the universe — in fact everything — in terms of whole numbers. Apparently, they were so upset about the discovery of surds that they tried to keep the discovery a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

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Chapter outline

STAGE 5.3	Working mathematically				
13.01 Surds and irrational numbers	U	F		R	C
13.02 Simplifying surds	U	F		R	
13.03 Adding and subtracting surds	U	F			
13.04 Multiplying and dividing surds	U	F			
13.05 Binomial products involving surds	U	F			
13.06 Rationalising the denominator	U	F			

Note: Chapter 12, Products and factors, must be completed before this chapter.

Wordbank

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction $\frac{a}{b}$

rational number Any number that can be written in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$

rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd)

real number A number that is either rational or irrational and whose value can be graphed on a number line

simplify a surd To write a surd $\sqrt{x}$ in its simplest form so that *x* has no factors that are perfect squares

surd A square root (or other root) whose exact value cannot be found

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) describe real, rational and irrational numbers and surds
- (STAGE 5.3) simplify, add, subtract, multiply and divide surds
- (STAGE 5.3) expand and simplify binomial products involving surds
- (STAGE 5.3) rationalise the denominator of expressions of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

SkillCheck ANSWERS ON P. 554

1 Simplify each expression.

a $(5y)^2$ **b** $(4m)^3$ **c** $(-3x)^2$

2 Expand each expression.

a $5(x + 2)$ **b** $4(y - 3)$ **c** $3(1 + 2w)$

d $2(5 - y)$ **e** $-5(2a + 3)$ **f** $k(1 + 2k)$

3 Select the square numbers from the following list of numbers.

44 81 25 100 75

72 16 50 64 32

4 Expand and simplify each expression.

a $(m + 3)(m + 7)$ **b** $(y + 1)(y - 4)$ **c** $(n - 2)(n - 3)$

d $(2d + 3)(1 + 3d)$ **e** $(1 - 5p)(4 + 3p)$ **f** $(3a + 2f)(a + 5f)$

g $(x + 4)^2$ **h** $(y - 3)^2$ **i** $(2k + 1)^2$

j $(a - 5)(a + 5)$ **k** $(t + 7)(t - 7)$ **l** $(3m + 4)(3m - 4)$

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Surds and irrational numbers

13.01

STAGE 5.3

A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

$\sqrt{7}$ is read as 'the square root of 7' or simply 'root 7'.

$\sqrt{7}$ is called the **exact value**, $\sqrt{7} \approx 2.64575$ is the **approximate value**.

Rational numbers such as fractions, decimals and percentages, can be expressed in the form $\frac{a}{b}$, where a and b are integers ($b \neq 0$), but surds are **irrational numbers** because they cannot be expressed in this form.

Rational numbers

can be expressed in the form $\frac{a}{b}$

Integers

$\frac{4}{1} = 4, \frac{26}{1} = 26, \frac{-3}{1} = -3$

Recurring decimals

$\frac{2}{3} = 0.666\ldots$

$\frac{5}{6} = 0.833\ldots$

$\frac{4}{11} = 0.3636\ldots$

Terminating decimals

$0.5, 7\frac{1}{8} = 7.125,$

$16\% = 0.16, 1.32$

Irrational numbers

cannot be expressed in the form $\frac{a}{b}$

Surds

$\sqrt{5}, -\sqrt{2}, \frac{\sqrt{11}}{3}, 8\sqrt{6}$

Transcendental numbers

Have no pattern and are non-recurring

e.g. $\pi = 3.14159\ldots$,

$\cos 38° = 0.78801\ldots$, $e = 2.71828\ldots$

Example 1

Select the surds from this list of square roots:

$$\sqrt{72} \quad \sqrt{121} \quad \sqrt{64} \quad \sqrt{90} \quad \sqrt{28}$$

Solution

$\sqrt{72} = 8.4852\ldots$ $\quad \sqrt{121} = 11$ $\quad \sqrt{64} = 8$

$\sqrt{90} = 9.4868\ldots$ $\quad \sqrt{28} = 5.2915\ldots$

So the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

STAGE 5.3

Example 2

Is each number rational or irrational?

a $4\frac{2}{5}$ **b** $\sqrt[3]{-8}$ **c** $-\sqrt{7}$ **d** $0.\dot{6}$ **e** 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$ which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

b $\sqrt[3]{-8} = -2$ which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645751311\ldots$ The digits run endlessly without repeating.

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666\ldots$ which is a recurring decimal

$= \frac{2}{3}$ which is a fraction

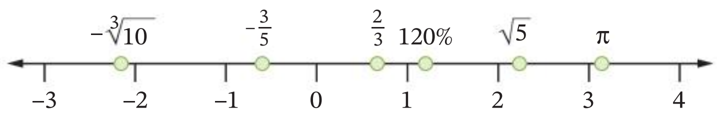
$\therefore 0.\dot{6}$ is a rational number.

e $5\pi = 15.707\,963\,27\ldots$ The digits run endlessly without repeating

$\therefore 5\pi$ is an irrational number.

Surds on a number line

The rational and irrational numbers together make up the **real numbers**. Any real number can be represented by a point on the number line.



$-\sqrt[3]{10} \approx -2.1544\ldots$	irrational (surd)
$-\frac{3}{5} = -0.6$	rational (fraction)
$\frac{2}{3} \approx 0.6666\ldots$	rational (fraction)
$120\% = 1.2$	rational (percentage)
$\sqrt{5} \approx 2.2360\ldots$	irrational (surd)
$\pi \approx 3.1415\ldots$	irrational (pi)

STAGE 5.3

WS Surds on the number line

13.01

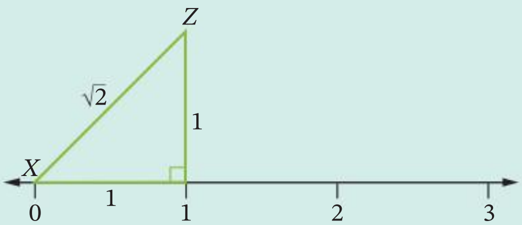
Example 3

Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

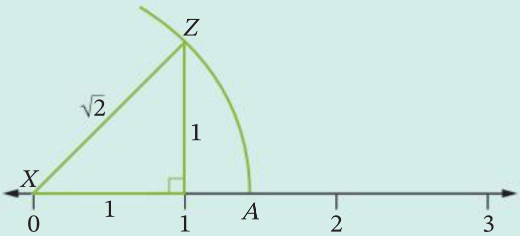
Solution

Step 1: Using a scale of 1 unit to 2 cm, draw a number line as shown.

Step 2: Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3: With 0 as the centre, use compasses with radius $XZ\left(\sqrt{2}\right)$ to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142 ...



EXERCISE 13.01 ANSWERS ON P. 554

Surds and irrational numbers U F R C

EXAMPLE 1

1 Which one of the following is a surd? Select the correct answer **A**, **B**, **C** or **D**. C

A $\sqrt{9}$ **B** $\sqrt{225}$ **C** $\sqrt{160}$ **D** $\sqrt{81}$

2 Which one of the following is NOT a surd? Select the correct answer **A**, **B**, **C** or **D**. C

A $\sqrt{77}$ **B** $\sqrt{144}$ **C** $\sqrt{18}$ **D** $\sqrt{200}$

3 Select the surds from the following list of square roots. C

$\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$

$\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

STAGE 5.3

EXAMPLE 2

4 Is each number rational (R) or irrational (I)? R C

a $5.\dot{6}$ **b** $\sqrt{8}$ **c** $\sqrt{4}$ **d** $3\frac{1}{7}$

e $\sqrt[3]{27}$ **f** $1.3\dot{5}$ **g** $\sqrt[3]{-64}$ **h** $27\frac{1}{2}\%$

i $\sqrt{5\times10^3}$ **j** $\frac{3}{11}$ **k** $\frac{\sqrt{50}}{3}$ **l** $\sqrt{\sqrt{4}}$

5 Arrange each set of numbers in descending order.

a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ **b** $\sqrt[3]{20}, 2.\dot{6}, 2\frac{7}{9}$

6 Express each real number correct to one decimal place and graph them on a number line.

a $-1\frac{4}{5}$ **b** 74% **c** $\frac{4}{11}$ **d** $-\sqrt{12}$

e $-\sqrt[3]{15}$ **f** $2\frac{5}{9}$ **g** $\frac{\pi}{2}$ **h** 187%

EXAMPLE 3

7 Use the method from Example **3** to estimate the value of $\sqrt{2}$ on a number line.

8 **a** Use the method from Example **3** to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.

b Use a similar method to estimate the following surds on a number line:

i $\sqrt{10}$ **ii** $\sqrt{17}$

13.02 Simplifying surds

STAGE 5.3

Simplifying surds quiz

Simplifying surds quiz

The square of any real number is always positive (except for $0^2 = 0$), so it is not possible to give the square root of a *negative* number.

The radical symbol $\sqrt{\ }$ stands for the *positive* square root of a number. For example $\sqrt{4} = 2$ (not –2).

The square root of *x*

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x} = 0$.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of x.

For $x \geq 0$, $\left(\sqrt{x}\right)^2 = \sqrt{x}\times\sqrt{x} = x$ and $\sqrt{x^2} = x$.

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STAGE 5.3

13.02

Example 4

Simplify each expression.

a $\left(\sqrt{7}\right)^2$ **b** $\left(3\sqrt{5}\right)^2$ **c** $\left(-2\sqrt{3}\right)^2$

Solution

a $\left(\sqrt{7}\right)^2 = 7$

b $\left(3\sqrt{5}\right)^2 = 3\sqrt{5} \times 3\sqrt{5}$ — $3\sqrt{5}$ means $3 \times \sqrt{5}$

$= 3^2 \times \left(\sqrt{5}\right)^2$

$= 9 \times 5$

$= 45$

c $\left(-2\sqrt{3}\right)^2 = (-2)^2 \times \left(\sqrt{3}\right)^2$

$= 4 \times 3$

$= 12$

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into 2 factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, ...

Example 5

Simplify each surd.

a $\sqrt{8}$ **b** $\sqrt{108}$ **c** $4\sqrt{45}$ **d** $\frac{\sqrt{288}}{3}$

Solution

a $\sqrt{8} = \sqrt{4} \times \sqrt{2}$ — 4 is a square number

$= 2 \times \sqrt{2}$

$= 2\sqrt{2}$

b *Method 1*:

$\sqrt{108} = \sqrt{36} \times \sqrt{3}$

$= 6 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2:

$\sqrt{108} = \sqrt{4} \times \sqrt{27}$

$= 2 \times \sqrt{9} \times \sqrt{3}$

$= 2 \times 3 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2 involves simplifying surds *twice* ($\sqrt{108}$ and $\sqrt{27}$). Method 1 shows that when simplifying surds, look for the highest square factor possible.

Simplifying surds 2

Simplifying surds 1

c $4\sqrt{45} = 4\times\sqrt{9}\times\sqrt{5}$
$= 4\times 3\times\sqrt{5}$
$= 12\sqrt{5}$

d $\frac{\sqrt{288}}{3} = \frac{\sqrt{144}\times\sqrt{2}}{3}$
$= \frac{12\sqrt{2}}{3}$
$= \frac{\cancel{12}^4\sqrt{2}}{\cancel{3}^1}$
$= 4\sqrt{2}$

EXERCISE 13.02 ANSWERS ON P. 554

Simplifying surds U F R

EXAMPLE 4

1 Simplify each expression.

a $\left(\sqrt{2}\right)^2$ **b** $\left(\sqrt{5}\right)^2$ **c** $\left(3\sqrt{3}\right)^2$ **d** $\left(5\sqrt{10}\right)^2$

e $\left(\sqrt{0.09}\right)^2$ **f** $\left(-2\sqrt{7}\right)^2$ **g** $\left(-3\sqrt{5}\right)^2$ **h** $\left(-5\sqrt{2}\right)^2$

EXAMPLE 5

2 Simplify each surd.

a $\sqrt{50}$ **b** $\sqrt{27}$ **c** $\sqrt{24}$ **d** $\sqrt{54}$

e $\sqrt{243}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{200}$

i $\sqrt{96}$ **j** $\sqrt{63}$ **k** $\sqrt{288}$ **l** $\sqrt{108}$

m $\sqrt{75}$ **n** $\sqrt{147}$ **o** $\sqrt{32}$ **p** $\sqrt{242}$

q $\sqrt{162}$ **r** $\sqrt{245}$ **s** $\sqrt{125}$ **t** $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$ **b** $3\sqrt{8}$ **c** $4\sqrt{27}$ **d** $8\sqrt{98}$

e $\frac{\sqrt{40}}{2}$ **f** $\frac{\sqrt{243}}{9}$ **g** $\frac{\sqrt{28}}{6}$ **h** $3\sqrt{24}$

i $9\sqrt{68}$ **j** $\frac{\sqrt{3125}}{10}$ **k** $\frac{1}{2}\sqrt{72}$ **l** $\frac{3}{4}\sqrt{48}$

m $10\sqrt{160}$ **n** $3\sqrt{75}$ **o** $7\sqrt{68}$ **p** $\frac{\sqrt{52}}{6}$

4 Which surd below is equivalent to $4\sqrt{50}$? Select **A**, **B**, **C** or **D**.

A $8\sqrt{5}$ **B** $20\sqrt{2}$ **C** $8\sqrt{2}$ **D** $20\sqrt{5}$

5 Which surd below is equivalent to $\frac{\sqrt{250}}{10}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{5}}{10}$ **B** $\frac{\sqrt{10}}{2}$ **C** $2\sqrt{10}$ **D** $5\sqrt{10}$

6 Decide whether each statement is true (T) or false (F). R

a $3\sqrt{7} = \sqrt{21}$ **b** $\sqrt{12} = 6$

c $\left(\sqrt{9.4}\right)^2 = 9.4$ **d** $\sqrt{75} = 5\sqrt{3}$

e $\sqrt{3} \approx 1.7$ **f** The exact value of $\sqrt{10}$ is 3.162 277 8

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Adding and subtracting surds

13.03

STAGE 5.3

Surds code puzzle

13.03

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'. You may first need to express all the surds in their simplest forms.

Example 6

Simplify each expression.

a $4\sqrt{2}+5\sqrt{2}$

b $7\sqrt{3}-2\sqrt{3}$

c $5\sqrt{2}-3\sqrt{3}+\sqrt{2}$

d $\sqrt{50}+\sqrt{32}$

e $\sqrt{8}-\sqrt{27}+\sqrt{18}$

f $5\sqrt{20}-3\sqrt{125}$

Solution

a $4\sqrt{2}+5\sqrt{2}=9\sqrt{2}$

b $7\sqrt{3}-2\sqrt{3}=5\sqrt{3}$

c $5\sqrt{2}-3\sqrt{3}+\sqrt{2}=6\sqrt{2}-3\sqrt{3}$

d $$\begin{aligned}\sqrt{50}+\sqrt{32}&=\sqrt{25}\sqrt{2}+\sqrt{16}\sqrt{2}\\&=5\sqrt{2}+4\sqrt{2}\\&=9\sqrt{2}\end{aligned}$$

Simplifying each surd

e $$\begin{aligned}\sqrt{8}-\sqrt{27}+\sqrt{18}&=\sqrt{4}\sqrt{2}-\sqrt{9}\sqrt{3}+\sqrt{9}\sqrt{2}\\&=2\sqrt{2}-3\sqrt{3}+3\sqrt{2}\\&=5\sqrt{2}-3\sqrt{3}\end{aligned}$$

f $$\begin{aligned}5\sqrt{20}-3\sqrt{125}&=5\sqrt{4}\sqrt{5}-3\sqrt{25}\sqrt{5}\\&=5\times2\sqrt{5}-3\times5\sqrt{5}\\&=10\sqrt{5}-15\sqrt{5}\\&=-5\sqrt{5}\end{aligned}$$

STAGE 5.3

EXERCISE 13.03 ANSWERS ON P. 554

Adding and subtracting surds U F

EXAMPLE 6

1 Simplify each expression.

a $5\sqrt{7}+2\sqrt{7}$
b $3\sqrt{2}-8\sqrt{2}$
c $7\sqrt{5}-\sqrt{5}$
d $\sqrt{5}+3\sqrt{5}$
e $5\sqrt{17}-5\sqrt{17}$
f $3\sqrt{10}-2\sqrt{10}$
g $4\sqrt{15}-3\sqrt{15}+7\sqrt{15}$
h $5\sqrt{6}-2\sqrt{6}-4\sqrt{6}$
i $3\sqrt{3}+4\sqrt{3}-5\sqrt{3}$
j $4\sqrt{5}+7\sqrt{5}-\sqrt{5}$
k $8\sqrt{10}-5\sqrt{10}+3\sqrt{10}$
l $10\sqrt{3}-3\sqrt{3}-12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5}-8+2\sqrt{5}$
b $11\sqrt{10}+3\sqrt{2}+2\sqrt{10}$
c $-4\sqrt{3}+5\sqrt{2}-5\sqrt{3}$
d $3\sqrt{15}+3\sqrt{2}+4\sqrt{15}+5\sqrt{2}$
e $\sqrt{7}-3\sqrt{5}-4\sqrt{7}+\sqrt{5}$
f $4\sqrt{6}-3\sqrt{3}-2\sqrt{6}-5\sqrt{3}$
g $10\sqrt{11}-5\sqrt{3}+3\sqrt{11}+4\sqrt{3}$
h $\sqrt{13}+8\sqrt{7}-7\sqrt{13}+3\sqrt{7}$
i $2\sqrt{5}-3\sqrt{7}-2\sqrt{5}-3\sqrt{7}$
j $4\sqrt{10}-3\sqrt{5}-4\sqrt{10}$

3 For each expression, select the correct simplified answer **A**, **B**, **C** or **D**.

a $\sqrt{3}+\sqrt{12}$

A $5\sqrt{3}$ **B** $\sqrt{15}$ **C** $2\sqrt{6}$ **D** $3\sqrt{3}$

b $4\sqrt{5}-2\sqrt{125}$

A $-6\sqrt{5}$ **B** $\sqrt{5}$ **C** $-\sqrt{45}$ **D** $-46\sqrt{5}$

4 Simplify each expression.

a $\sqrt{8}+\sqrt{32}$
b $\sqrt{108}-\sqrt{27}$
c $\sqrt{20}-\sqrt{80}$
d $\sqrt{28}-\sqrt{63}$
e $3\sqrt{6}+\sqrt{24}$
f $2\sqrt{5}+\sqrt{125}$
g $\sqrt{40}-\sqrt{90}$
h $5\sqrt{11}+\sqrt{99}$
i $3\sqrt{2}+\sqrt{18}$
j $\sqrt{27}+5\sqrt{3}$
k $\sqrt{200}-7\sqrt{2}$
l $\sqrt{50}+\sqrt{32}$
m $5\sqrt{3}+2\sqrt{27}$
n $3\sqrt{20}-\sqrt{245}$
o $7\sqrt{12}-5\sqrt{48}$
p $4\sqrt{27}+2\sqrt{243}$
q $3\sqrt{63}-2\sqrt{28}$
r $2\sqrt{98}+3\sqrt{162}$
s $-5\sqrt{6}+2\sqrt{150}$
t $4\sqrt{50}+3\sqrt{18}$
u $5\sqrt{27}-6\sqrt{75}$
v $3\sqrt{112}-2\sqrt{252}$
w $\sqrt{32}+\sqrt{8}+\sqrt{12}$
x $\sqrt{27}+\sqrt{54}+\sqrt{243}$
y $\sqrt{98}-3\sqrt{20}-2\sqrt{8}$
z $3\sqrt{96}-2\sqrt{150}+\sqrt{24}$

Foundation Standard Complex

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Multiplying and dividing surds

13.04

STAGE 5.3

Surds

Multiplying and dividing surds

13.04

The square root of products and quotients

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Example 7

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$

b $\sqrt{10} \times \sqrt{6}$

c $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$

e $\sqrt{54} \div (-\sqrt{2})$

f $\frac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b
$$\begin{aligned}\sqrt{10} \times \sqrt{6} &= \sqrt{60} \\ &= \sqrt{4} \times \sqrt{15} \\ &= 2\sqrt{15}\end{aligned}$$

c
$$\begin{aligned}3\sqrt{7} \times 5\sqrt{7} &= 3 \times 5 \times \sqrt{7} \times \sqrt{7} \\ &= 15 \times 7 \\ &= 105\end{aligned}$$

d
$$\begin{aligned}5\sqrt{27} \times 3\sqrt{6} &= 5 \times 3 \times \sqrt{27} \times \sqrt{6} \\ &= 15\sqrt{162} \\ &= 15 \times \sqrt{81} \times \sqrt{2} \\ &= 15 \times 9\sqrt{2} \\ &= 135\sqrt{2}\end{aligned}$$

e
$$\begin{aligned}\sqrt{54} \div (-\sqrt{2}) &= -\frac{\sqrt{54}}{\sqrt{2}} \\ &= -\sqrt{27} \\ &= -\sqrt{9} \times \sqrt{3} \\ &= -3\sqrt{3}\end{aligned}$$

f
$$\begin{aligned}\frac{15\sqrt{32}}{5\sqrt{8}} &= 3\sqrt{4} \\ &= 3 \times 2 \\ &= 6\end{aligned}$$

Example 8

Simplify $\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$$\begin{aligned}\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}} &= \frac{20\sqrt{24}}{10\sqrt{8}} \\ &= 2\sqrt{3}\end{aligned}$$

STAGE 5.3

EXERCISE 13.04 ANSWERS ON P. 554

Multiplying and dividing surds U F

EXAMPLE 7

1 Simplify each expression.

a $\sqrt{7}\times\sqrt{2}$ **b** $-\sqrt{5}\times\sqrt{7}$ **c** $\sqrt{6}\times\sqrt{8}$

d $\sqrt{12}\times\sqrt{3}$ **e** $\sqrt{10}\times(-\sqrt{5})$ **f** $3\sqrt{3}\times5\sqrt{3}$

g $5\sqrt{10}\times3\sqrt{3}$ **h** $-2\sqrt{7}\times5\sqrt{3}$ **i** $7\sqrt{5}\times4\sqrt{5}$

j $2\sqrt{3}\times(-5\sqrt{6})$ **k** $4\sqrt{3}\times\sqrt{27}$ **l** $-3\sqrt{5}\times4\sqrt{10}$

m $-7\sqrt{2}\times4\sqrt{8}$ **n** $\sqrt{18}\times8\sqrt{3}$ **o** $10\sqrt{2}\times2\sqrt{8}$

p $3\sqrt{18}\times5\sqrt{12}$ **q** $3\sqrt{44}\times(-2\sqrt{99})$ **r** $5\sqrt{8}\times4\sqrt{40}$

s $8\sqrt{3}\times3\sqrt{54}$ **t** $-8\sqrt{32}\times\sqrt{27}$ **u** $\sqrt{90}\times\sqrt{72}$

v $-5\sqrt{20}\times3\sqrt{8}$ **w** $7\sqrt{18}\times3\sqrt{24}$ **x** $3\sqrt{48}\times2\sqrt{12}$

2 Simplify each expression.

a $\sqrt{15}\div\sqrt{3}$ **b** $\sqrt{18}\div(-\sqrt{6})$ **c** $\frac{6\sqrt{48}}{2\sqrt{8}}$

d $10\sqrt{54}\div5\sqrt{27}$ **e** $-3\sqrt{98}\div6\sqrt{14}$ **f** $\frac{7\sqrt{18}}{\sqrt{2}}$

g $2\sqrt{24}\div4\sqrt{6}$ **h** $\frac{\sqrt{128}}{\sqrt{2}}$ **i** $15\sqrt{18}\div3\sqrt{6}$

j $\frac{-20\sqrt{10}}{-4\sqrt{5}}$ **k** $36\sqrt{24}\div9\sqrt{8}$ **l** $16\sqrt{30}\div8\sqrt{5}$

m $12\sqrt{14}\div6$ **n** $\frac{3\sqrt{2}}{-12}$ **o** $\sqrt{80}\div4\sqrt{5}$

p $5\sqrt{60}\div\sqrt{15}$ **q** $6\sqrt{8}\div3\sqrt{2}$ **r** $\frac{-42\sqrt{54}}{6\sqrt{3}}$

s $12\sqrt{63}\div3\sqrt{7}$ **t** $\frac{8\sqrt{50}}{2\sqrt{200}}$ **u** $6\sqrt{3}\div\sqrt{243}$

3 Simplify:

a $\sqrt{6}\times\sqrt{6}$ **b** $\sqrt{7}\times\sqrt{7}$ **c** $2\sqrt{3}\times\sqrt{3}$

d $5\sqrt{y}\times3\sqrt{y}$ **e** $\sqrt{x}\times\sqrt{x}$ **f** $\sqrt{a^2}\times\sqrt{a}$

4 Simplify $3\sqrt{2}\times\sqrt{6}$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** $6\sqrt{2}$ **C** $6\sqrt{3}$ **D** $12\sqrt{2}$

5 Simplify $20\sqrt{10}\div5\sqrt{2}$. Select **A**, **B**, **C** or **D**.

A $4\sqrt{5}$ **B** $15\sqrt{5}$ **C** 10 **D** 20

Foundation Standard Complex

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6 Simplify each expression.

a $\dfrac{3\sqrt{5}\times 4\sqrt{2}}{3\sqrt{40}}$ **b** $\dfrac{3\sqrt{12}\times 8\sqrt{6}}{4\sqrt{27}}$ **c** $\dfrac{5\sqrt{8}\times 2\sqrt{90}}{10\sqrt{24}}$

d $\dfrac{4\sqrt{5}}{2\sqrt{15}\times 5\sqrt{27}}$ **e** $\dfrac{10\sqrt{686}\times 3\sqrt{12}}{5\sqrt{28}\times\sqrt{18}}$ **f** $\dfrac{8\sqrt{80}\times 3\sqrt{2}}{4\sqrt{5}\times 6\sqrt{8}}$

STAGE 5.3

EXAMPLE 8

Binomial products involving surds

13.05

STAGE 5.3

Surd expressions involving brackets can be expanded in the same way as algebraic expressions of the form $a(b + c)$ and $(a + b)(c + d)$.

Example 9

Expand and simplify each expression.

a $\sqrt{3}(\sqrt{5}+\sqrt{7})$ **b** $2\sqrt{11}(3\sqrt{11}-5\sqrt{2})$

Solution

a
$$\sqrt{3}(\sqrt{5}+\sqrt{7}) = \sqrt{3}\times\sqrt{5}+\sqrt{3}\times\sqrt{7}$$
$$= \sqrt{15}+\sqrt{21}$$

b
$$2\sqrt{11}(3\sqrt{11}-5\sqrt{2}) = 2\sqrt{11}\times 3\sqrt{11}-2\sqrt{11}\times 5\sqrt{2}$$
$$= 6\times 11-10\times\sqrt{22}$$
$$= 66-10\sqrt{22}$$

Example 10

Expand and simplify each binomial product.

a $(\sqrt{7}+\sqrt{5})(3\sqrt{2}-\sqrt{3})$ **b** $(3-2\sqrt{10})(\sqrt{5}-3\sqrt{2})$

Solution

a
$$(\sqrt{7}+\sqrt{5})(3\sqrt{2}-\sqrt{3}) = \sqrt{7}(3\sqrt{2}-\sqrt{3})+\sqrt{5}(3\sqrt{2}-\sqrt{3})$$
$$= \sqrt{7}\times 3\sqrt{2}-\sqrt{7}\times\sqrt{3}+\sqrt{5}\times 3\sqrt{2}-\sqrt{5}\times\sqrt{3}$$
$$= 3\sqrt{14}-\sqrt{21}+3\sqrt{10}-\sqrt{15}$$

b
$$(3-2\sqrt{10})(\sqrt{5}-3\sqrt{2}) = 3(\sqrt{5}-3\sqrt{2})-2\sqrt{10}(\sqrt{5}-3\sqrt{2})$$
$$= 3\times\sqrt{5}-3\times 3\sqrt{2}-2\sqrt{10}\times\sqrt{5}+2\sqrt{10}\times 3\sqrt{2}$$
$$= 3\sqrt{5}-9\sqrt{2}-2\sqrt{50}+6\sqrt{20}$$
$$= 3\sqrt{5}-9\sqrt{2}-2(5\sqrt{2})+6(2\sqrt{5})$$
$$= 3\sqrt{5}-9\sqrt{2}-10\sqrt{2}+12\sqrt{5}$$
$$= 15\sqrt{5}-19\sqrt{2}$$

Foundation Standard Complex

STAGE 5.3

Special binomial products (see Chapter 12)

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Difference of 2 squares

$(a + b)(a - b) = a^2 - b^2$

Binomial products involving surds

Example 11

Expand and simplify each expression.

a $\left(\sqrt{7}-\sqrt{5}\right)^2$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)$

Solution

a $\left(\sqrt{7}-\sqrt{5}\right)^2=\left(\sqrt{7}\right)^2-2\times\sqrt{7}\times\sqrt{5}+\left(\sqrt{5}\right)^2$ using $(a - b)^2 = a^2 - 2ab + b^2$

$=7-2\sqrt{35}+5$

$=12-2\sqrt{35}$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2=\left(2\sqrt{3}\right)^2+2\times2\sqrt{3}\times3\sqrt{5}+\left(3\sqrt{5}\right)^2$ using $(a + b)^2 = a^2 + 2ab + b^2$

$=(4\times3)+12\sqrt{15}+(9\times5)$

$=12+12\sqrt{15}+45$

$=57+12\sqrt{15}$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$ using $(a + b)(a - b) = a^2 - b^2$

$= 5 - 2$

$= 3$

Note that because of the 'difference of 2 squares', the answer is not a surd but a rational number.

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)=\left(3\sqrt{11}\right)^2-4^2$ using $(a + b)(a - b) = a^2 - b^2$

$= (9 \times 11) - 16$

$= 83$

EXERCISE 13.05 ANSWERS ON P. 555

STAGE 5.3

Binomial products involving surds U F

1 Expand and simplify each expression.

EXAMPLE 9

a $\sqrt{5}(\sqrt{3}+\sqrt{2})$ **b** $\sqrt{6}(\sqrt{2}-1)$ **c** $\sqrt{2}(\sqrt{3}+\sqrt{7})$

d $\sqrt{5}(3\sqrt{2}-\sqrt{5})$ **e** $3\sqrt{2}(\sqrt{2}+2\sqrt{3})$ **f** $-\sqrt{11}(4-\sqrt{5})$

g $2\sqrt{7}(3\sqrt{7}-4)$ **h** $5\sqrt{5}(1+3\sqrt{5})$ **i** $3\sqrt{2}(4\sqrt{2}+\sqrt{3})$

13.05

2 Expand and simplify $(\sqrt{3}+2\sqrt{5})(5\sqrt{2}+\sqrt{3})$. Select the correct answer **A**, **B**, **C** or **D**.

EXAMPLE 10

A $20\sqrt{10}$ **B** $2\sqrt{15}+5\sqrt{6}$

C $5\sqrt{6}+3+10\sqrt{10}+2\sqrt{15}$ **D** $5\sqrt{5}+\sqrt{3}+7\sqrt{7}+4\sqrt{2}$

3 Expand and simplify each expression.

a $(\sqrt{5}-3)(2\sqrt{5}+\sqrt{2})$ **b** $(\sqrt{7}-\sqrt{3})(\sqrt{7}+2)$

c $(7\sqrt{3}+2)(4\sqrt{2}+\sqrt{3})$ **d** $(3\sqrt{2}-\sqrt{5})(5\sqrt{2}+2\sqrt{5})$

e $(\sqrt{7}+2\sqrt{11})(3\sqrt{7}+4\sqrt{11})$ **f** $(5\sqrt{3}-2\sqrt{2})(4\sqrt{3}-3\sqrt{2})$

g $(6+2\sqrt{10})(3\sqrt{10}-1)$ **h** $(\sqrt{7}-2\sqrt{5})(3\sqrt{5}+2\sqrt{7})$

4 Expand $(5+\sqrt{7})^2$. Select **A**, **B**, **C** or **D**.

EXAMPLE 11

A 12 **B** 32 **C** $32+10\sqrt{7}$ **D** $32+5\sqrt{7}$

5 Expand and simplify each expression.

a $(\sqrt{5}-\sqrt{3})^2$ **b** $(\sqrt{7}+\sqrt{2})^2$

c $(\sqrt{5}-2)^2$ **d** $(3+\sqrt{10})^2$

e $(5\sqrt{2}+3\sqrt{3})^2$ **f** $(5\sqrt{7}-2)^2$

g $(3\sqrt{2}+2\sqrt{5})^2$ **h** $(2\sqrt{5}+\sqrt{3})^2$

6 Expand and simplify each expression.

a $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ **b** $(5+\sqrt{3})(5-\sqrt{3})$

c $(6+2\sqrt{7})(6-2\sqrt{7})$ **d** $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

e $(\sqrt{11}-\sqrt{10})(\sqrt{11}+\sqrt{10})$ **f** $(5\sqrt{7}+3)(5\sqrt{7}-3)$

g $(3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$ **h** $(4\sqrt{2}-5\sqrt{3})(4\sqrt{2}+5\sqrt{3})$

Foundation Standard Complex

STAGE 5.3

7 Expand and simplify $(5\sqrt{2}-4\sqrt{3})(5\sqrt{2}+4\sqrt{3})$. Select **A**, **B**, **C** or **D**.

A $25\sqrt{2}-16\sqrt{3}$ **B** $10\sqrt{2}+10\sqrt{6}$ **C** 2 **D** 26

8 Expand and simplify each expression.

a $(3\sqrt{7}-5)^2$ **b** $(5\sqrt{2}-4)(\sqrt{2}+5)$

c $(2\sqrt{7}+3\sqrt{5})(\sqrt{5}+\sqrt{7})$ **d** $(4\sqrt{3}+5)^2$

e $(4\sqrt{2}+\sqrt{3})(4\sqrt{2}-\sqrt{3})$ **f** $(3\sqrt{10}-\sqrt{2})^2$

13.06 Rationalising the denominator

STAGE 5.3

If $\sqrt{2} \approx 1.4142$, what is the value of $\frac{3}{\sqrt{2}}$? Fractions containing surds in the denominator are difficult to work with. When approximating the value of $\frac{3}{\sqrt{2}}$, it is difficult to mentally divide by 1.4142. We can overcome this by making the denominator **rational** (that is, **not** a surd).

Numbers that have irrational denominators, such as $\frac{1}{\sqrt{5}}, \frac{3}{2\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{2}}, \frac{5\sqrt{7}}{\sqrt{3}}$ can be rewritten with a rational denominator by multiplying both the numerator and denominator by the surd that appears in the denominator. This method is called **rationalising the denominator**.

Example 12

Rationalise the denominator of each surd.

a $\frac{3}{\sqrt{2}}$ **b** $\frac{5}{4\sqrt{3}}$ **c** $\frac{8\sqrt{2}}{3\sqrt{5}}$ **d** $\frac{\sqrt{2}+1}{\sqrt{3}}$

Solution

a $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ because $\frac{\sqrt{2}}{\sqrt{2}} = 1$

$= \frac{3\sqrt{2}}{2}$

Note: Now it is easier to approximate $\frac{3}{\sqrt{2}}$ by approximating $\frac{3\sqrt{2}}{2}$ and mentally multiplying $\frac{3}{2}$ by 1.4142 than by dividing 3 by 1.4142.

b $\frac{5}{4\sqrt{3}} = \frac{5}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{3}}{4 \times 3}$

$= \frac{5\sqrt{3}}{12}$

Foundation Standard Complex

c $\frac{8\sqrt{2}}{3\sqrt{5}} = \frac{8\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{8\sqrt{10}}{3 \times 5}$

$= \frac{8\sqrt{10}}{15}$

d $\frac{\sqrt{2}+1}{\sqrt{3}} = \frac{\sqrt{2}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}+\sqrt{3}}{3}$

STAGE 5.3

13.06

EXERCISE 13.06 ANSWERS ON P. 555

ANSWERS ON P. 555

Rationalising the denominator U F

EXAMPLE 12

1 By rationalising the denominator, which surd is equivalent to $\frac{2}{\sqrt{6}}$? Select the correct answer **A**, **B**, **C** or **D**.

A $2\sqrt{6}$ **B** $\frac{\sqrt{6}}{3}$ **C** $\frac{\sqrt{6}}{6}$ **D** $\frac{2\sqrt{6}}{3}$

2 Rationalise the denominator of each surd.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{7}}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{3}{\sqrt{2}}$

e $\frac{2}{\sqrt{7}}$ **f** $\frac{1}{3\sqrt{2}}$ **g** $\frac{1}{2\sqrt{3}}$ **h** $\frac{1}{4\sqrt{7}}$

i $\frac{7}{3\sqrt{5}}$ **j** $\frac{\sqrt{2}}{3\sqrt{5}}$ **k** $\frac{3\sqrt{2}}{2\sqrt{6}}$ **l** $\frac{5\sqrt{3}}{4\sqrt{5}}$

3 Which surd is equivalent to $\frac{\sqrt{3}}{2\sqrt{5}}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{15}}{10}$ **B** $2\sqrt{15}$ **C** $\frac{\sqrt{15}}{3}$ **D** $\sqrt{5}$

4 Which surd is equivalent to $\frac{\sqrt{27}}{3\sqrt{18}}$? Select **A**, **B**, **C** or **D**.

A $\frac{1}{2}$ **B** $\frac{\sqrt{2}}{2}$ **C** $\frac{\sqrt{5}}{6}$ **D** $\frac{\sqrt{6}}{6}$

5 Rationalise the denominator of each expression.

a $\frac{\sqrt{2}-1}{\sqrt{2}}$ **b** $\frac{1-\sqrt{5}}{\sqrt{5}}$ **c** $\frac{5+\sqrt{3}}{2\sqrt{2}}$ **d** $\frac{\sqrt{2}-\sqrt{3}}{3\sqrt{6}}$

6 Simplify each expression, giving the answer with a rational denominator.

a $\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{2}}$ **b** $\frac{\sqrt{2}}{\sqrt{5}}+\frac{3}{\sqrt{3}}$ **c** $\frac{3}{2\sqrt{3}}-\frac{1}{\sqrt{2}}$

Foundation Standard Complex

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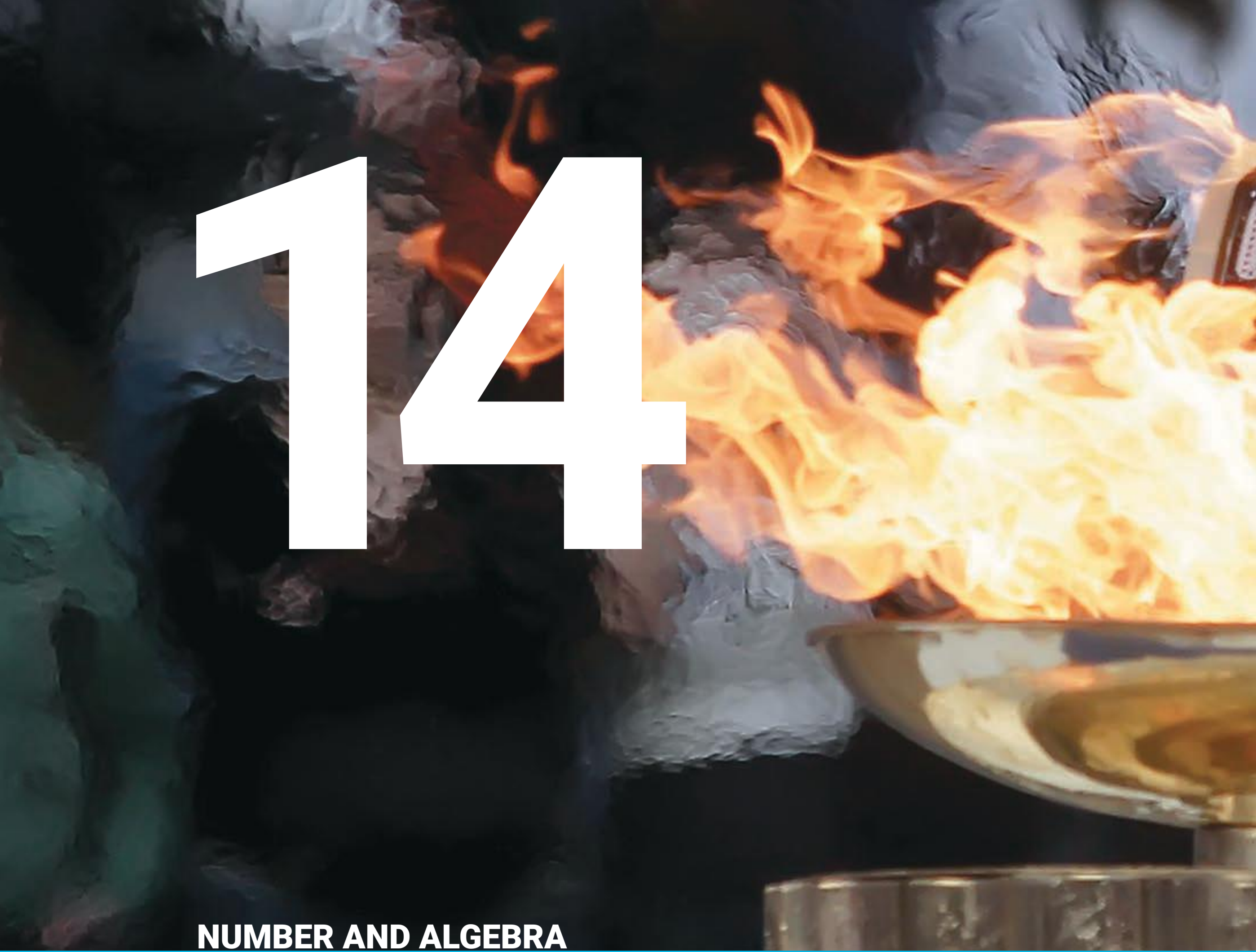

14

NUMBER AND ALGEBRA

QUADRATIC EQUATIONS AND THE PARABOLA

OPTIONAL STAGE 5.3 TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

Every 4 years, a few months before the Olympic Games Opening Ceremony, the Olympic flame is lit in a symbolic ceremony near the Temple of Hera, at the sacred site of the Ancient Olympics in Greece.

The flame is lit by traditional means, using the Sun's rays, to demonstrate its purity. To ensure it is lit by natural means, a parabolic mirror is used, which acts as a giant magnifying glass.

The Olympic flame has been used to light the Olympic cauldron for every Olympic Games Opening ceremony, since the Summer Olympic Games held in Berlin, Germany in 1936.

Chapter outline

STAGE 5.3	Working mathematically				
14.01 Quadratic equations $ax^2 + bx + c = 0$	U	F	PS	R	C
14.02 Completing the square	U	F		R	C
14.03 The quadratic formula	U	F			C
14.04 Quadratic equation problems	U	F	PS	R	C
14.05 The parabola $y = ax^2 + bx + c$	U	F		R	C
14.06 The axis of symmetry and vertex of a parabola	U	F		R	C
14.07 Non-linear simultaneous equations#	U	F		R	C

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Note: Chapter 12, *Products and factors*, must be completed before this chapter.

Wordbank

axis of symmetry The vertical line that divides a parabola in half and passes through the parabola's vertex

coefficient The number in front of a variable; for example, in $y = 3x^2 + 4x - 6$, the coefficient of x^2 is 3

quadratic equation An equation in which the highest power of the variable is 2; for example, $2x^2 - 12x + 10 = 0$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $3x^2 + 4x - 6$

quadratic formula A formula for solving quadratic equations of the form $ax^2 + bx + c = 0$

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true

vertex A turning point of a curve; a parabola has one vertex

***x*-intercept** The *x*-value(s) at which a graph intersects the *x*-axis

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) solve quadratic equations of the form $ax^2 + bx + c = 0$ by factorising, completing the square and using the quadratic formula
- (STAGE 5.3) solve problems involving quadratic equations
- (STAGE 5.3) graph parabolas of the form $y = ax^2 + bx + c$ and identify their features
- (STAGE 5.3) find the axis of symmetry and the vertex of a parabola
- (STAGE 5.3) find the point of intersection of a line with a parabola, circle or hyperbola by solving linear and non-linear simultaneous equations

SkillCheck ANSWERS ON P. 555

1 Solve each quadratic equation.

a $4x^2 = 100$ **b** $3m^2 = 12$ **c** $x^2 - 81 = 0$

d $12k^2 = 3$ **e** $4u^2 - 49 = 0$ **f** $7w^2 - 63 = 0$

2 Factorise each expression.

a $16 - m^2$ **b** $d^2 - 121$ **c** $14y - 2y^2$

d $10p^2 + 25p$ **e** $5x^2 - 320$ **f** $18w^2 - 50$

g $k^2 + 5k + 4$ **h** $y^2 - 10y + 16$ **i** $m^2 - m - 56$

j $u^2 + 8u - 65$ **k** $w^2 - 10w + 21$ **l** $x^2 - 2x - 24$

3 If $y = x^2 + 4x - 7$, find the value of y if:

a $x = 1$ **b** $x = -1$ **c** $x = 2$ **d** $x = -\frac{1}{2}$

4 Factorise each expression.

a $3a^2 + 10a + 3$ **b** $5x^2 - 13x - 6$ **c** $6y^2 + y - 40$

d $15t^2 + 7t - 4$ **e** $5v^2 - 32v - 21$ **f** $8y^2 + 34y + 35$

g $15h^2 - 23h + 4$ **h** $12p^2 + 11p - 15$ **i** $16d^2 + 40d + 25$

14.01 Quadratic equations $ax^2 + bx + c = 0$

STAGE 5.3

Factorising quadratic equations

In Chapter 6, *Equations and inequalities*, we learnt that in a **quadratic equation**, the highest power of the variable is 2; for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4y^2 - 3y = 8$.

The general form of a quadratic equation is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b and c are constants (numbers) and $a \neq 0$.

Solving a quadratic equation means finding those values of the variable that satisfy the equation (or make the statement true). When checking solutions, substitute the possible solutions into the equation and show that the left-hand side (LHS) of the equation is equal to the right-hand side (RHS) of the equation.

Solving $ax^2 + bx + c = 0$ by factorising

STAGE 5.3

14.01

To solve quadratic equations of the form $ax^2 + bx + c = 0$, we need to factorise the quadratic expression on the LHS (see Chapter 12, *Products and factors*).

Example 1

Solve each quadratic equation.

a $4w(3w + 2) = 0$ **b** $(5m + 2)(2m - 7) = 0$

Solution

a $4w(3w + 2) = 0$

$\therefore 4w = 0$ or $3w + 2 = 0$

$w = 0$ or $3w = -2$

$w = 0$ or $w = -\frac{2}{3}$

b $(5m + 2)(2m - 7) = 0$

$5m + 2 = 0$ or $2m - 7 = 0$

$5m = -2$ or $2m = 7$

$m = -\frac{2}{5}$ or $m = \frac{7}{2}$

$m = -\frac{2}{5}$ or $m = 3\frac{1}{2}$

Example 2

Quadratic equations by factorising

Solve each quadratic equation. Use substitution to check your solutions.

a $6p^2 - 9p = 0$ **b** $2x^2 - x - 15 = 0$ **c** $6y^2 = -7y + 5$

Solution

a $6p^2 - 9p = 0$ Factorising

$3p(2p - 3) = 0$

$3p = 0$ or $2p - 3 = 0$

$p = 0$ or $2p = 3$

$p = 0$ or $p = \frac{3}{2}$

$p = 0$ or $p = 1\frac{1}{2}$

Check: When $p = 0$, LHS $= 6 \times 0^2 - 9 \times 0 = 0 =$ RHS

When $p = 1\frac{1}{2}$, LHS $= 6 \times \left(1\frac{1}{2}\right)^2 - 9 \times 1\frac{1}{2} = 0 =$ RHS

b $2x^2 - x - 15 = 0$

$2x^2 - 6x + 5x - 15 = 0$

$2x(x - 3) + 5(x - 3) = 0$

$(x - 3)(2x + 5) = 0$

STAGE 5.3

$$x - 3 = 0 \quad \text{or} \quad 2x + 5 = 0$$
$$x = 3 \quad \text{or} \quad 2x = -5$$
$$x = 3 \quad \text{or} \quad x = -\frac{5}{2}$$
$$x = 3 \quad \text{or} \quad x = -2\frac{1}{2}$$

Check: When $x = 3$, LHS $= 2 \times 3^2 - 3 - 15 = 0 =$ RHS

When $x = -2\frac{1}{2}$, LHS $= 2 \times \left(-2\frac{1}{2}\right)^2 - \left(-2\frac{1}{2}\right) - 15 = 0 =$ RHS

c
$$6y^2 = -7y + 5$$
$$6y^2 + 7y - 5 = 0$$
$$6y^2 - 3y + 10y - 5 = 0$$
$$3y(2y - 1) + 5(2y - 1) = 0$$
$$(2y - 1)(3y + 5) = 0$$

$$2y - 1 = 0 \quad \text{or} \quad 3y + 5 = 0$$
$$2y = 1 \quad \text{or} \quad 3y = -5$$
$$y = \frac{1}{2} \quad \text{or} \quad y = -\frac{5}{3}$$
$$y = \frac{1}{2} \quad \text{or} \quad y = -1\frac{2}{3}$$

Check: When $y = \frac{1}{2}$:

LHS $= 6 \times \left(\frac{1}{2}\right)^2 = 1\frac{1}{2}$

RHS $= -7 \times \left(\frac{1}{2}\right) + 5 = 1\frac{1}{2}$

LHS = RHS

When $y = -1\frac{2}{3}$:

LHS $= 6 \times \left(-1\frac{2}{3}\right)^2 = 16\frac{2}{3}$

RHS $= -7 \times \left(-1\frac{2}{3}\right) + 5 = 16\frac{2}{3}$

LHS = RHS

EXERCISE 14.01 ANSWERS ON P. 555

Quadratic equations $ax^2 + bx + c = 0$ U F PS R C

EXAMPLE 1

1 Solve each quadratic equation.

a $(m + 7)(m + 3) = 0$ **b** $(d - 3)(d - 7) = 0$ **c** $(y - 3)(y + 5) = 0$

d $k(k - 3) = 0$ **e** $t(t + 7) = 0$ **f** $2p(p - 3) = 0$

g $w(3w - 2) = 0$ **h** $(2n + 1)(n - 3) = 0$ **i** $(5a - 3)(2a - 1) = 0$

j $(3x + 1)(2x + 3) = 0$ **k** $(2c - 5)^2 = 0$ **l** $(1 - 2f)^2 = 0$

m $(3c + 1)(4c + 1) = 0$ **n** $(1 - 2h)(h + 1) = 0$ **o** $(5 - 7e)(1 - e) = 0$

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2 Solve each quadratic equation.

a	$2y^2 + 7y + 6 = 0$	**b**	$2g^2 + 5g + 3 = 0$	**c**	$3d^2 + 5d + 2 = 0$
d	$5t^2 + 16t + 11 = 0$	**e**	$2p^2 - 11p + 12 = 0$	**f**	$10x^2 - 19x + 6 = 0$
g	$8y^2 - 2y - 3 = 0$	**h**	$6a^2 - 5a - 4 = 0$	**i**	$4w^2 - 7w - 15 = 0$
j	$5c^2 + 2c - 7 = 0$	**k**	$8e^2 + 10e - 3 = 0$	**l**	$3q^2 + 4q - 15 = 0$
m	$4g^2 - 20g + 25 = 0$	**n**	$18m^2 - 3m - 10 = 0$	**o**	$16 - 8w - 3w^2 = 0$
p	$36 + 3y - 3y^2 = 0$	**q**	$2f^2 - 24f + 72 = 0$	**r**	$12h^2 + 3h - 9 = 0$

3 Express each quadratic equation in the form $ax^2 + bx + c = 0$ and solve.

a	$2x^2 = x + 15$	**b**	$4t(t + 2) = 5$	**c**	$41u = -8u^2 - 5$
d	$7m^2 = 8m - 1$	**e**	$p(p - 3) = 28$	**f**	$(e - 2)^2 = 9$
g	$t(2t - 13) = -15$	**h**	$7 = 6d^2 + 11d$	**i**	$5h^2 = 125$
j	$8f^2 = 4f$	**k**	$6w^2 + 3 = 19w$	**l**	$4a(3a + 5) - 8 = 0$

4 A certain positive number, plus its square, minus 72, equals 0. Find the number. PS R C

STAGE 5.3

EXAMPLE 2

14.01

Completing the square 14.02

STAGE 5.3

Complete the square order activity

Solving quadratic equations

Quadratic equations can be solved by using a method called **completing the square**. We try to make the LHS of the equation a perfect square (see Chapter 12, *Products and factors*).

The method of **completing the square** is based on the following results for perfect squares.

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

We note that the last term, a^2, is the square of 'half the coefficient of x'.

Example 3

Find the numbers that complete the square in each equation.

a $x^2 + 10x + ... = (x + ...)^2$

b $x^2 - 14x + ... = (x - ...)^2$

Solution

a The coefficient of x is 10.
Half of 10 is 5, and $5^2 = 25$.
The perfect square is
$x^2 + 10x + 25 = (x + 5)^2$

b The coefficient of x is -14.
Half of -14 is -7, and $(-7)^2 = 49$.
The perfect square is
$x^2 - 14x + 49 = (x - 7)^2$

Foundation Standard Complex

STAGE 5.3

Simple quadratic equations

Example 4

Solve $(k + 3)^2 = 7$.

Solution

$(k + 3)^2 = 7$

$k + 3 = \pm\sqrt{7}$ — Taking the square root of both sides.

$k + 3 - 3 = \sqrt{7} - 3$ **or** $k + 3 - 3 = -\sqrt{7} - 3$

This solution is usually written as:

$k = -3 + \sqrt{7}$ **or** $k = -3 - \sqrt{7}$

Notice that the 2 solutions are surds (not rational), of the form $a + \sqrt{b}$ and $a - \sqrt{b}$.

Example 5

Solve $x^2 + 6x - 2 = 0$ by completing the square.

Solution

Step 1

Move the constant term to the RHS.

$x^2 + 6x = 2$

Step 2

Halve the coefficient of x, square it and then add the square to both sides.

$x^2 + 6x + 3^2 = 2 + 3^2$

$x^2 + 6x + 9 = 11$

Step 3

Express the LHS as a perfect square.

$(x + 3)^2 = 11$

Step 4

Solve the resulting equation.

$x + 3 = \pm\sqrt{11}$ — Taking the square root of both sides.

$x = -3 \pm \sqrt{11}$ — This answer in surd form is called the **'exact answer'**.

$x = -3 + \sqrt{11}$ or $x = -3 - \sqrt{11}$

Example 6

Solve $2x^2 - 3x - 4 = 0$, writing the solution correct to 2 decimal places.

Solution

$2x^2 - 3x - 4 = 0$

$2x^2 - 3x = 4$ — Moving the constant term to the RHS.

$x^2 - \frac{3}{2}x = 2$ — Dividing both sides by 2, the coefficient of x^2.

$$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = 2 + \left(-\frac{3}{4}\right)^2$$ Completing the square.

$$\left(x - \frac{3}{4}\right)^2 = \frac{41}{16}$$ Expressing the LHS as a perfect square and leaving the RHS as an improper fraction.

$$x - \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$$ Taking the square root of both sides.

$$x - \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$$ Simplifying the RHS.

$$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$$ This is the exact answer in surd form.

$$x = \frac{3 \pm \sqrt{41}}{4}$$

$x = \frac{3+\sqrt{41}}{4}$ or $x = \frac{3-\sqrt{41}}{4}$ This is an approximate answer in decimal form.

$x = 2.35078...$ or $x = -0.85078...$

$x \approx 2.35$ or $x \approx -0.85$

EXERCISE 14.02 ANSWERS ON P. 556

Completing the square U F R C

1 Find the numbers that 'complete the square' in each equation. R C EXAMPLE 3

a $x^2 + 2x + \_\_\_ = (x + \_\_\_)^2$ **b** $p^2 - 6p + \_\_\_ = (p - \_\_\_)^2$

c $m^2 - 8m + \_\_\_ = (m - \_\_\_)^2$ **d** $k^2 + 4k + \_\_\_ = (k + \_\_\_)^2$

e $y^2 - 7y + \_\_\_ = (y - \_\_\_)^2$ **f** $w^2 - 3w + \_\_\_ = (w - \_\_\_)$

g $x^2 + x + \_\_\_ = (x + \_\_\_)^2$ **h** $h^2 - 5h + \_\_\_ = (h - \_\_\_)^2$

i $a^2 + \frac{7}{2}a + \_\_\_ = (a + \_\_\_)^2$ **j** $v^2 + \frac{5}{3}v + \_\_\_ = (v + \_\_\_)^2$

2 Solve each equation, writing the solution in surd form. R C EXAMPLE 4

a $(d + 3)^2 = 7$ **b** $(x - 5)^2 = 5$ **c** $(p + 1)^2 = 10$

d $(y - 1)^2 = 2$ **e** $\left(m - \frac{1}{2}\right)^2 = 5$ **f** $\left(t + \frac{2}{3}\right)^2 = 3$

g $(c + 1)^2 = \frac{21}{2}$ **h** $(w - 3)^2 = \frac{41}{2}$ **i** $\left(n + \frac{2}{3}\right)^2 = \frac{7}{9}$

j $\left(e - \frac{3}{2}\right)^2 = \frac{71}{4}$ **k** $(d - 2)^2 = 5$ **l** $\left(x - \frac{3}{4}\right)^2 = 2$

3 Solve each equation by completing the square. Leave your answers in exact form. R C EXAMPLE 5

a $h^2 + 2h - 5 = 0$ **b** $r^2 - 2r - 1 = 0$ **c** $m^2 + 6m + 2 = 0$

d $w^2 - 4w - 60 = 0$ **e** $a^2 - 10a - 5 = 0$ **f** $y^2 + 8y - 3 = 0$

g $p^2 + 12p - 5 = 0$ **h** $x^2 - 4x + 2 = 0$ **i** $u^2 + 9u + 14 = 0$

j $d^2 + d - 7 = 0$ **k** $c^2 - 9c + 2 = 0$ **l** $e^2 + 5e + 2 = 0$

m $y^2 - 3y - 8 = 0$ **n** $b^2 - b - 5 = 0$ **o** $q^2 - 3q + 1 = 0$

p $2g^2 + 7g = 3$ **q** $2x^2 = 7 - 5x$ **r** $3f(f + 4) = 6$

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EXAMPLE 6

4 Solve each of these by completing the square. Give your answers correct to 2 decimal places if irrational. R C

a $x^2 + 12x + 9 = 0$ **b** $m^2 - 16m - 7 = 0$ **c** $g^2 + 4g - 3 = 0$

d $2h^2 + 3h - 7 = 0$ **e** $5w^2 - 4w - 3 = 0$ **f** $3y^2 + y - 5 = 0$

g $p(3p + 2) = 8$ **h** $4e^2 - 4 = e$ **i** $2n^2 = 5 - 3n$

14.03 The quadratic formula

STAGE 5.3

WS Investigating quadratic equations

The quadratic formula

The quadratic formula

The quadratic formula

The quadratic formula

Solving quadratic equations

There is a formula for solving a quadratic equation of the form $ax^2 + bx + c = 0$ that involves the coefficients a, b and c.

The quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 7

Solve each quadratic equation using the quadratic formula.

a $x^2 - 3x + 2 = 0$ **b** $6x^2 + x - 2 = 0$ **c** $3x^2 + 11x + 2 = 0$

Solution

a For $x^2 - 3x + 2 = 0$:

$a = 1$, $b = -3$, and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{1}}{2}$$

$$= \frac{3 \pm 1}{2}$$

$$x = \frac{3+1}{2} \quad \text{or} \quad x = \frac{3-1}{2}$$

$$x = 2 \quad \text{or} \quad x = 1$$

b For $6x^2 + x - 2 = 0$:

$a = 6$, $b = 1$, and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 6 \times (-2)}}{2 \times 6}$$

$$= \frac{-1 \pm \sqrt{49}}{12}$$

$$= \frac{-1 \pm 7}{12}$$

$$x = \frac{-1+7}{12} \quad \text{or} \quad x = \frac{-1-7}{12}$$

$$x = \frac{6}{12} \quad \text{or} \quad x = \frac{-8}{12}$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -\frac{2}{3}$$

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c For $3x^2 + 11x + 2 = 0$:

$a = 3$, $b = 11$ and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{-11 \pm \sqrt{97}}{6}$$

$x = \frac{-11 \pm \sqrt{97}}{6}$ or $x = \frac{-11 - \sqrt{97}}{6}$ In exact surd form.

$x = -0.1918...$ or $x = -3.4748...$

$x \approx -0.19$ or $x \approx -3.47$ Rounded to 2 decimal places.

Example 8

Solve $2x^2 - 6 = 9x$ using the quadratic formula, expressing the answer correct to 2 decimal places.

Solution

$2x^2 - 6 = 9x$

$2x^2 - 9x - 6 = 0$

Writing the quadratic equation in the form $ax^2 + bx + c = 0$

Substitute $a = 2$, $b = -9$, $c = -6$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times (-6)}}{2 \times 2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{9 \pm \sqrt{129}}{4}$$

$x = \frac{9 + \sqrt{129}}{4}$ or $x = \frac{9 - \sqrt{129}}{4}$

$x = 5.0894...$ or $x = -0.5894...$

$x \approx 5.09$ or $x \approx -0.59$

EXERCISE 14.03 ANSWERS ON P. 556

The quadratic formula U F C

EXAMPLE 7

1 Solve each quadratic equation using the quadratic formula. Write each solution in exact form. C

a	$x^2 + 6x + 2 = 0$	**b**	$m^2 - 5m - 3 = 0$	**c**	$w^2 - 8w + 3 = 0$
d	$k^2 + 3k - 5 = 0$	**e**	$y^2 - 4y - 1 = 0$	**f**	$p^2 + p - 5 = 0$
g	$u^2 - 7u - 3 = 0$	**h**	$2a^2 + 3a - 7 = 0$	**i**	$5q^2 - 6q + 1 = 0$
j	$3c^2 + 2c - 2 = 0$	**k**	$4e^2 - 5e - 2 = 0$	**l**	$3x^2 + 8x + 2 = 0$
m	$2d^2 - 4d - 5 = 0$	**n**	$3a^2 - 10a - 2 = 0$	**o**	$2t^2 + 3t - 5 = 0$
p	$3y^2 + 8y + 4 = 0$	**q**	$6k^2 - 11k + 5 = 0$	**r**	$2n^2 - 5n - 11 = 0$

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STAGE 5.3

2 Solve each quadratic equation using the quadratic formula, expressing the solution as a surd. C

a $5y^2 - 9y = 3$	**b** $3m^2 = 7 - 2m$	**c** $4x^2 = 3x + 2$
d $1 - 4k - k^2 = 0$	**e** $3m^2 - 1 - 3m = 0$	**f** $1 - 2g - 5g^2 = 0$
g $8 = 9h - 2h^2$	**h** $2w + 2 = 3w^2$	**i** $4p - 3p^2 = -1$
j $2 - 4u - 5u^2 = 0$	**k** $6a^2 = 9 - 4a$	**l** $10 = 3y + 2y^2$

EXAMPLE 8

3 Solve each quadratic equation, writing the solutions correct to 2 decimal places. C

a $k^2 - 9k + 1 = 0$	**b** $c^2 - 2 = 0$	**c** $m^2 - 5 = 2$
d $2n^2 + 2 = 7n$	**e** $2p^2 + 3p - 4 = 0$	**f** $6w^2 + 5w - 2 = 0$
g $3x^2 + 2 - 8x = 0$	**h** $h^2 = 7 + 2h$	**i** $1 + x - x^2 = 0$
j $36 = 13a - a^2$	**k** $5v^2 - 11 = 0$	**l** $5c^2 + 8 = 15c$
m $t^2 = 5(t + 5)$	**n** $(x - 6)^2 = 3$	**o** $12 = 2d^2 - 3d$

14.04 Quadratic equation problems

STAGE 5.3

WS Problems involving quadratic equations

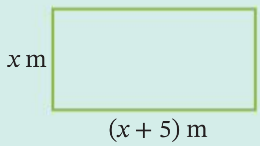
Example 9

A rectangular garden is 5 m longer than it is wide. The area of the rectangle is 84 m².
Find the dimensions of the garden.

Solution

Let the width of the rectangular garden be x cm.

∴ The length of the garden is $(x + 5)$ cm.



Area = length × width

$= x(x + 5)$ m²

$\therefore x(x + 5) = 84$

$x^2 + 5x = 84$

$x^2 + 5x - 84 = 0$

$(x + 12)(x - 7) = 0$

$x + 12 = 0$ or $x - 7 = 0$

$\therefore x = -12$ or $x = 7$

Since x represents a measurement of width, $x \neq -12$.

∴ The width is 7 m and the length is 7 + 5 = 12 m

Check: Area = 7 m × 12 m = 84 m²

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STAGE 5.3

Quadratic equation problems

14.04

Example 10

A ball is thrown upwards and its height, h metres, after t seconds is given by the formula

$$h = 30t - 5t^2$$

At what times did the ball reach a height of 24 m? Express the answer correct to 2 decimal places.

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Solution

The equation is $h = 30t - 5t^2$

When $h = 24$,

$24 = 30t - 5t^2$

Rearranging the equation,

$5t^2 - 30t + 24 = 0$

Using the quadratic formula,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \times 5 \times 24}}{2 \times 5}$$

$$= \frac{30 \pm \sqrt{420}}{10}$$

$t = \frac{30 + \sqrt{420}}{10}$ or $t = \frac{30 - \sqrt{420}}{10}$

$t = 5.049...$ or $t = 0.950...$

$t \approx 5.05$ or $t \approx 0.95$

The ball reaches a height of 24 m at 0.95 s and 5.05 s.

Why are there 2 answers to this problem? Why is the ball at a height of 24 m at 2 different times?

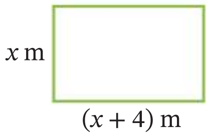
EXERCISE 14.04 ANSWERS ON P. 556

Quadratic equation problems U F PS R C

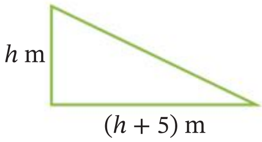
To answer each problem, first form a quadratic equation and then solve it.

EXAMPLE 9

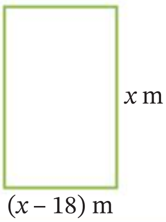
1 A garden is in the shape of a rectangle and its length is 4 m longer than its width. If the area of the garden is 96 m², find the dimensions of the garden. PS R C



2 A park is in the shape of a right-angled triangle, with its base 5 m longer than its perpendicular height. If the area of the park is 700 m², find the dimensions of the park. PS R C



3 A rectangular block of land has its width 18 m shorter than its length. If the area of the block is 1008 m², find the dimensions of the block of land. PS R C



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STAGE 5.3

4 A block of land is in the shape of a rectangle with its width 20 m shorter than its length. If the area of the block is 2204 m², find the length and width of the block of land. PS R C

5 A rectangular garden bed is twice as long as it is wide. Its area is 84.5 m². Find the length of the garden bed. PS R C

6 Find the dimensions of a rectangle whose length is 3 m longer than its width and its area is 180 m². PS R C

EXAMPLE 10

7 A golf ball is thrown upwards and its height, h metres, after t seconds is given by the formula $h = 18t - 5t^2$. At what times did the ball reach a height of 8 m? Answer correct to 2 decimal places. PS R C

8 After jumping from a plane, the height (in metres) of a skydiver above the ground is given by $h = 4000 - 5t^2$, where t is the time (in seconds) after jumping. R C


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- **a** How high was the plane at the moment the skydiver jumped?
- **b** What was the skydiver's height after 20 seconds?
- **c** The skydiver opened his parachute at 1000 m. How long did it take the skydiver to reach this height? (Give your answer correct to one decimal place.)

9 A ball is thrown from a balcony and its height (in metres) after t seconds is given by the formula $h = 30 + 12t - 5t^2$. R C

- **a** What is the height of the ball after 2 seconds?
- **b** When does the ball hit the ground? Answer correct to one decimal place.
- **c** At what time (correct to one decimal place) is the ball at a height of:
  - **i** 35 m?
  - **ii** 10 m?

10 The sum of a number and its square is 72. What is the number? PS R C

11 The product of 2 consecutive integers is 600. Find the integers. PS R C

12 When a number is subtracted from its square, the result is 1190. What is the number? PS R C

13 The difference between 2 positive integers is 12 and their product is 405. Find the integers. PS R C

14 The production costs, \$$C$, of a factory producing n toy boats each week is given by:

$$C = 0.05n^2 - 12n + 2700$$

Find: R C

- **a** the cost of producing 500 toy boats
- **b** the profit made on 500 toy boats if each one sells for \$29.50
- **c** the (whole) number of toy boats that can be produced at a production cost of \$8150.

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The parabola $y = ax^2 + bx + c$

14.05

STAGE 5.3

Investigating parabolas 1

Investigating parabolas 1

The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**. We have already graphed simple parabolas of the form $y = ax^2 + c$ in Chapter 7, *Graphing curves*. Now we will graph parabolas of the form $y = ax^2 + bx + c$.

Example 11

For each quadratic equation:

- **i** complete a table of values and draw the graph
- **ii** find the x-intercepts
- **iii** find the y-intercept
- **iv** solve $y = 0$ and compare the solutions to the x-intercepts.

a $y = x^2 + 6x + 5$ **b** $y = 2x^2 - x - 10$

Solution

a **i** $y = x^2 + 6x + 5$

x	−6	−5	−4	−3	−2	−1	0	1
y	5	0	−3	−4	−3	0	5	12

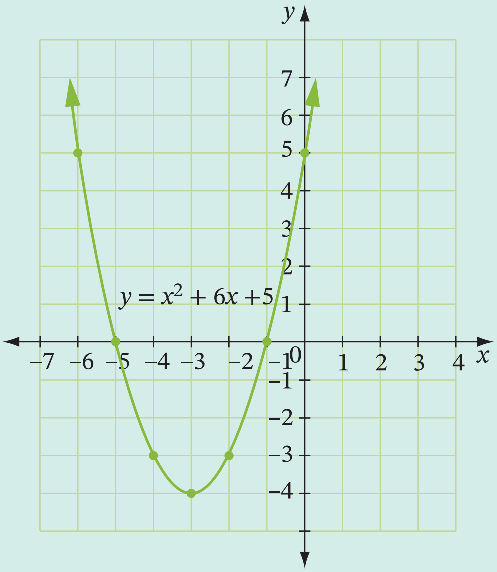
ii The x-intercepts are −5 and −1.

iii The y-intercept is 5.

Note that this is the constant term, $c = 5$, in $y = x^2 + 6x + 5$.

iv
$$x^2 + 6x + 5 = 0$$
$$(x + 5)(x + 1) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -5 \quad \text{or} \quad x = -1$$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = x^2 + 6x + 5$.



STAGE 5.3

b **i** $y = 2x^2 - x - 10$

x	−3	−2	−1	0	1	2	3
y	11	0	−7	−10	−9	0	5

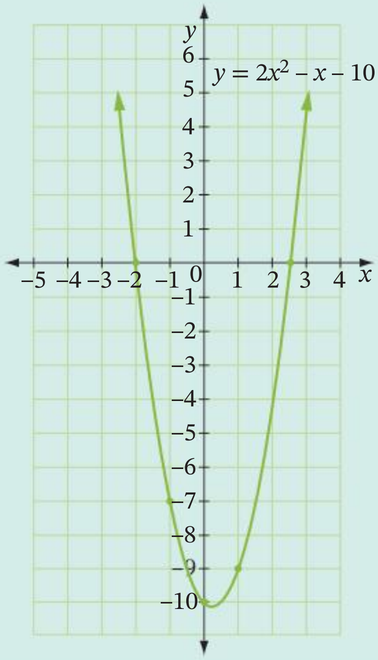
ii The x-intercepts are −2 and $2\frac{1}{2}$.

iii The y-intercept is −10.

Note that this is the constant term, $c = -10$, in $y = 2x^2 - x - 10$.

iv
$$2x^2 - x - 10 = 0$$
$$(2x - 5)(x + 2) = 0$$
$$2x - 5 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 2\frac{1}{2} \quad \text{or} \quad x = -2$$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = 2x^2 - x - 10$.



The graph of the parabola $y = ax^2 + bx + c$

- If $a > 0$ (positive), the parabola is **concave up**
- If $a < 0$ (negative), the parabola is **concave down**
- The y-**intercept** of the parabola is c
- The x-**intercepts** of the parabola are the solutions to the quadratic equation $ax^2 + bx + c = 0$

Example 12

Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(2x - 5)$ **b** $y = 2x^2 + x - 6$ **c** $y = -2x^2 - 3x + 9$

Solution

a $y = x(2x - 5)$

$= 2x^2 - 5x$

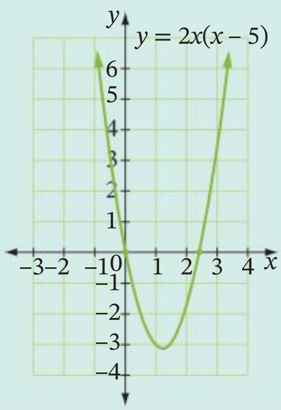
$a = 2 > 0$, so the parabola is concave up

x-intercepts: $x(2x - 5) = 0$

$x = 0 \quad \text{and} \quad 2x - 5 = 0$

$x = 0 \quad \text{and} \quad x = 2\frac{1}{2}$

y-intercept: $c = 0$



b $y = 2x^2 + x - 6$

$a = 2 > 0$, so the parabola is concave up

x-intercepts:

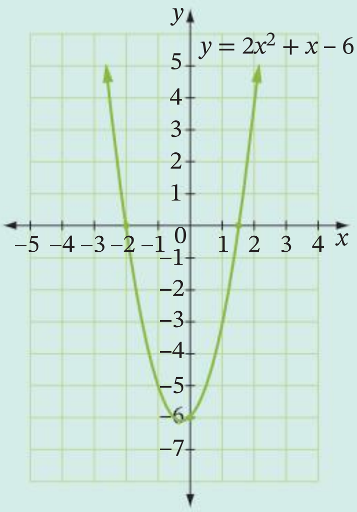
$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ and $x + 2 = 0$

$x = 1\frac{1}{2}$ and $x = -2$

y-intercept: $c = -6$



c $y = -2x^2 - 3x + 9$

$a = -2 < 0$, so the parabola is concave down

x-intercepts:

$-2x^2 - 3x + 9 = 0$ Dividing by −1

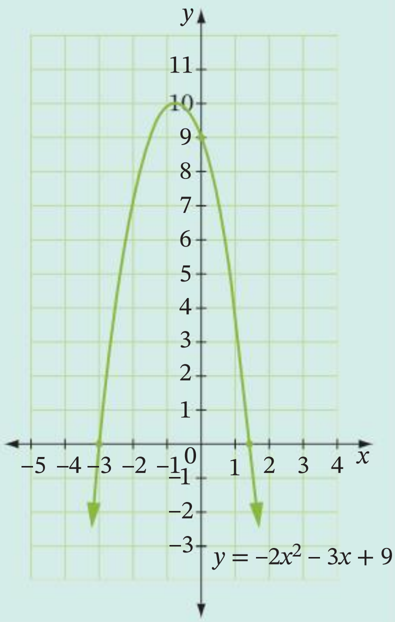
$2x^2 + 3x - 9 = 0$

$(2x - 3)(x + 3) = 0$

$2x - 3 = 0$ and $x + 3 = 0$

$x = 1\frac{1}{2}$ and $x = -3$

y-intercept: $c = 9$



STAGE 5.3

EXERCISE 14.05 ANSWERS ON P. 557

ANSWERS ON P. 557

The parabola $y = ax^2 + bx + c$ U F R C

EXAMPLE 11

1 For each quadratic equation: R C

- **i** complete a table of values and draw the graph
- **ii** find the x-intercepts
- **iii** find the y-intercept
- **iv** solve $y = 0$ and compare the solutions to the x-intercepts

a $y = x^2 + 4x + 3$ **b** $y = 2x^2 + 7x + 3$

c $y = 2x^2 - 3x - 9$ **d** $y = x^2 - 2x$

2 Find the y-intercept of the parabolas with equation:

a $y = 3x^2 - 2x - 5$ **b** $y = 2x^2 + 6x + 3$ **c** $y = 5x^2 - 10x$

EXAMPLE 12

3 Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(x - 4)$ **b** $y = 3x(x + 2)$

c $y = (x + 3)(x - 5)$ **d** $y = (x - 2)(2x - 5)$

e $y = -(x + 1)(3x + 2)$ **f** $y = x^2 + 6x + 8$

g $y = -x^2 + 4x + 5$ **h** $y = 3x^2 + 11x - 20$

i $y = -2x^2 + 5x - 2$

14.06 The axis of symmetry and vertex of a parabola

STAGE 5.3

WS Features of a parabola

WS A page of parabolas

WS Parabolas

The axis of symmetry and vertex of $y = ax^2 + bx + c$

- The **axis of symmetry** of the parabola is the vertical line: $x = -\dfrac{b}{2a}$
- The **axis of symmetry** passes through the point halfway between the 2 x-intercepts of the parabola
- The **vertex** of the parabola lies on the axis of symmetry, so its ***x*-coordinate** is $x = -\dfrac{b}{2a}$ and its ***y*-coordinate** can be found by substituting the x-coordinate into the equation of the parabola

Foundation Standard Complex

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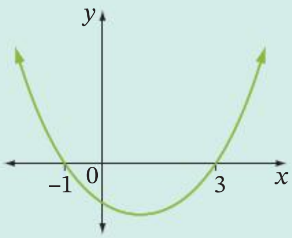
STAGE 5.3

14.06

Example 13

Find the equation of the axis of symmetry of this parabola.



Solution

The x-intercepts are -1 and 3.

The equation of the axis of symmetry is:

$$x = \frac{-1+3}{2}$$
$$= 1$$

Finding the average of the x-intercepts.

Example 14

A parabola has the equation $y = 2x^2 - 4x + 3$.

a Find the equation of its axis of symmetry

b Find the coordinates of the vertex of the parabola.

Solution

a For $y = 2x^2 - 4x + 3$, $a = 2$, $b = -4$ and $c = 3$.

The axis of symmetry is:

$$x = -\frac{b}{2a}$$
$$= -\frac{-4}{2 \times 2}$$
$$= 1$$

$x = 1$ is the equation of the axis of symmetry.

b The vertex lies on the axis of symmetry.

Substitute $x = 1$ in $y = 2x^2 - 4x + 3$

$y = 2 \times 1^2 - 4 \times 1 + 3$

$= 1$

The vertex is (1, 1).

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STAGE 5.3

Example 15

Sketch the parabola $y = 2x^2 + 5x + 1$.

Solution

$a = 2 > 0$, so the parabola is concave up.

y-intercept $= 1$.

$y = 2x^2 + 5x + 1$ cannot be factorised, so we cannot find the x-intercepts precisely.

For the vertex:

$x = -\frac{b}{2a}$ where $a = 2$, and $b = 5$

$= -\frac{5}{2 \times 2}$

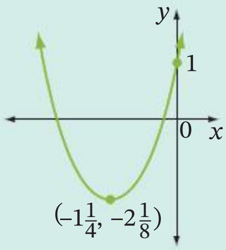
$= -1\frac{1}{4}$

Substitute $x = -1\frac{1}{4}$ into $y = 2x^2 + 5x + 1$

$y = 2 \times \left(-1\frac{1}{4}\right)^2 + 5 \times \left(-1\frac{1}{4}\right) + 1$

$= -2\frac{1}{8}$

$\therefore$ The vertex is $\left(-1\frac{1}{4}, -2\frac{1}{8}\right)$.



Graphing the parabola $y = ax^2 + bx + c$

- Use the sign of a (the coefficient of x^2) to determine whether the parabola is **concave up** or **down**
- Find the y-**intercept**, c.
- Find the x-**intercepts** where possible by solving a quadratic equation
- Use $x = -\frac{b}{2a}$ to find the **axis of symmetry** and the **vertex**

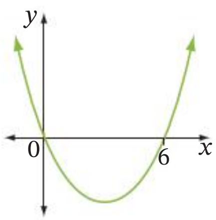
EXERCISE 14.06 ANSWERS ON P. 558

The axis of symmetry and vertex of a parabola U F R C

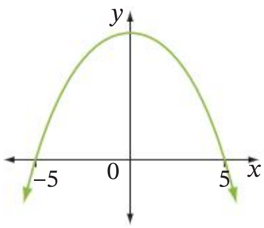
EXAMPLE 13

1 Write the equation for the axis of symmetry in each parabola. R C

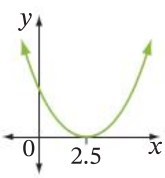
a



b



c



Foundation Standard Complex

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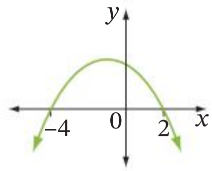
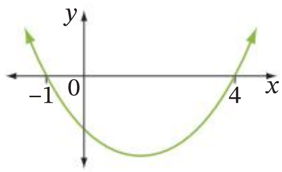
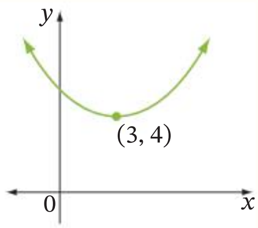
d



e



f



STAGE 5.3

2 For each parabola with the equation given below:

i find the equation of its axis of symmetry

ii find the coordinates of its vertex

EXAMPLE 14

a	$y = x^2 - 6x + 8$	**b**	$y = -x^2 + 10x - 9$	**c**	$y = x^2 - 2x + 10$
d	$y = -x^2 + 8x + 9$	**e**	$y = -x^2 + x - 25$	**f**	$y = 5x^2 + 40x$
g	$y = 24x - 3x^2$	**h**	$y = 4x^2 + 2x - 1$	**i**	$y = 1 - 3x - 9x^2$

3 Graph each quadratic equation, showing:

EXAMPLE 15

i the x-intercepts (correct to one decimal place where necessary)

ii the y-intercept

iii the equation of the axis of symmetry

iv the coordinates of the vertex

v the concavity

a	$y = x^2 - 6x - 40$	**b**	$y = x^2 - 3x$	**c**	$y = 2x^2 + 3x + 4$
d	$y = -x^2 + 6x + 5$	**e**	$y = -4x^2 - 12x + 21$	**f**	$y = x^2 - 8x + 3$
g	$y = 5x^2 + 7x + 4$	**h**	$y = 8x - 2x^2$	**i**	$y = -2x^2 + 7x - 6$

Non-linear simultaneous equations#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

14.07

STAGE 5.3

The points of intersection of a line with a parabola, circle or hyperbola may be found either graphically or algebraically. The algebraic method involves solving simultaneous equations using the substitution method, which you learned about in Chapter 9, *Simultaneous equations*.

Example 16

Find the points of intersection of:

a the line $y = 2x - 3$ and the parabola $y = x^2 - 3x + 1$

b the line $y = x + 2$ and the circle $x^2 + y^2 = 4$

c the line $y = x + 5$ and the hyperbola $y = \frac{6}{x}$

STAGE 5.3

Solution

a $y = 2x - 3$ [1]

$y = x^2 - 3x + 1$ [2]

Use [2] to substitute for y in [1].

$x^2 - 3x + 1 = 2x - 3$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4$ or $x = 1$

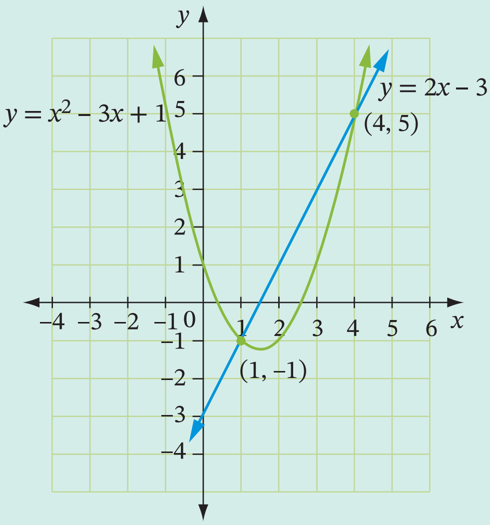
Substitute $x = 4$ and $x = 1$ into [1] to find y.

$y = 2 \times 4 - 3$ and $y = 2 \times 1 - 3$

$= 5$ $= -1$

The solutions are $x = 4, y = 5$ and $x = 1, y = -1$

The points of intersection are (4, 5) and (1, −1).



This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.

b $y = x + 2$ [1]

$x^2 + y^2 = 4$ [2]

Use [1] to substitute for y in [2].

$x^2 + (x + 2)^2 = 4$

$x^2 + x^2 + 4x + 4 = 4$

$2x^2 + 4x = 0$

$2x(x + 2) = 0$

$2x = 0$ or $x + 2 = 0$

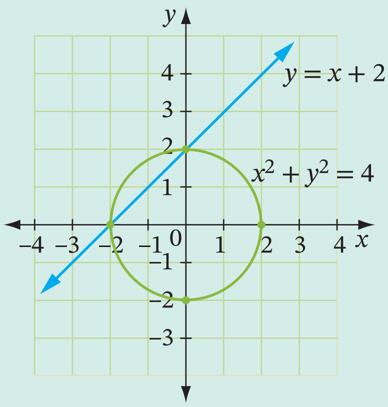
$x = 0$ or $x = -2$

Substitute $x = 0$ and $x = -2$ into [1] to find y.

$y = 0 + 2$ and $y = -2 + 2$

$y = 2$ and $y = 0$

The points of intersection are (0, 2) and (−2, 0).



This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.

c $y = x + 5$ [1]

$y = \frac{6}{x}$ [2]

Use [1] to substitute for y in [2].

$$x + 5 = \frac{6}{x}$$
$$x(x + 5) = x \times \frac{6}{x}$$
$$x^2 + 5x = 6$$
$$x^2 + 5x - 6 = 0$$
$$(x + 6)(x - 1) = 0$$

$x + 6 = 0$ or $x - 1 = 0$

$x = -6$ or $x = 1$

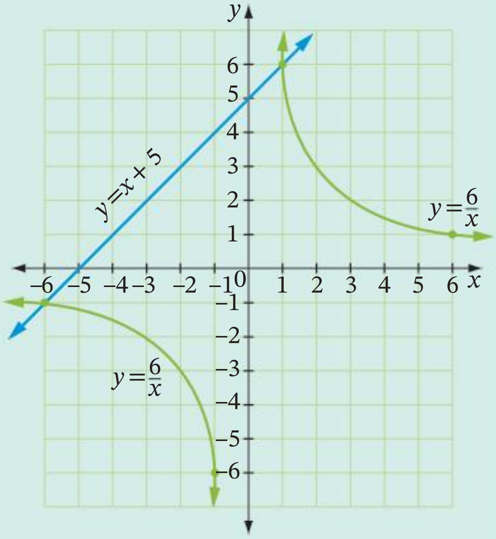
Substitute $x = -6$ and $x = 1$ into [1] to find y.

$y = -6 + 5$ and $y = 1 + 5$

$y = -1$ and $y = 6$

The points of intersection are $(-6, -1)$ and $(1, 6)$.

(Also, see their graphs).



EXERCISE 14.07 ANSWERS ON P. 559

ANSWERS ON P. 559

Non-linear simultaneous equations U F R C

1 Find the points of intersection of the line and the parabola with equations: R

EXAMPLE 16

a $y = x^2$ and $y = 5x - 6$

b $y = 2x^2$ and $y = 8x$

c $y = x^2 + 10$ and $y = 2x + 18$

d $y = 5x^2$ and $y = x + 6$

e $y = 3x^2 + 2x + 10$ and $y = 12 - 3x$

f $y = 9x - x^2$ and $y = 6x - 10$

g $y = 6x - 3$ and $y = x^2 - 8x + 46$

h $x + y = 2$ and $y = x^2 - 2x$

2 Find the points of intersection of the line: R

a $y = x - 3$ and the hyperbola $y = \frac{2}{x}$

b $y = 1 - x$ and the circle $x^2 + y^2 = 1$

c $x + y = 3$ and the circle $x^2 + y^2 = 9$

d $y = x + 5$ and the circle $x^2 + y^2 = 25$

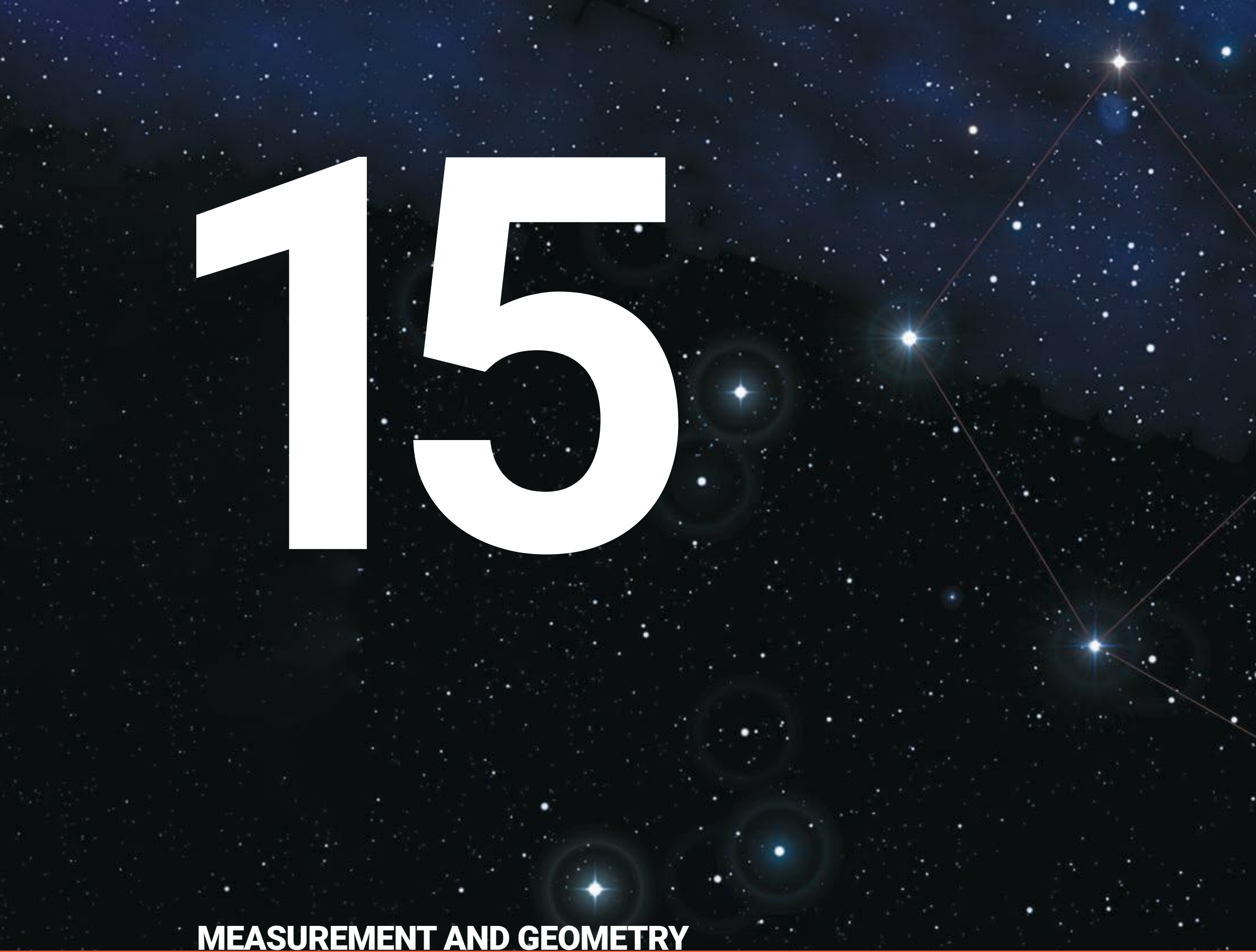
e $y = 5x + 2$ and the hyperbola $y = \frac{7}{x}$

f $2x - y = 1$ and the hyperbola $y = \frac{15}{x}$

g $y = 2x + 5$ and the hyperbola $y = -\frac{3}{x}$

3 Explain why there are no points of intersection between the equations $y = -3x - 1$ and $y = \frac{2}{x}$. Show working to help justify your answer. R C

Foundation Standard Complex



15

MEASUREMENT AND GEOMETRY

FURTHER TRIGONOMETRY

OPTIONAL STAGE 5.3 TOPIC RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED AND MATHEMATICS STANDARD 2

The word **trigonometry** comes from the Greek language: *trigonon*, meaning triangle, and *metron*, meaning measure. Trigonometry uses triangles to find unknown lengths and angles that cannot be measured physically. It has wide applications in engineering, surveying, navigation, astronomy, electronics and construction.

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Chapter outline

STAGE 5.3	Working mathematically				
15.01 The trigonometric functions	U	F		R	C
15.02 Trigonometric equations	U	F			
15.03 The sine rule	U	F	PS	R	
15.04 The sine rule for angles	U	F		R	C
15.05 The cosine rule	U	F	PS	R	C
15.06 The cosine rule for angles	U	F	PS	R	
15.07 The area of a triangle	U	F	PS	R	
15.08 Problems involving the sine and cosine rules	U	F	PS	R	C

Note: Students intending to study Mathematics Standard 2 should skip 15.01 and 15.02.

Wordbank

cosine rule A rule that relates the 3 sides and one of the angles of any triangle: $c^2 = a^2 + b^2 - 2ab \cos C$

included angle The angle between 2 known sides

quadrant A quarter of the number plane with the *x*- and *y*-axes as borders

sine rule A rule that relates the sides of any triangle to the sine of their opposite angles: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

unit circle A circle on the number plane with centre (0, 0) and radius 1

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) use the unit circle to define and graph trigonometric functions
- (STAGE 5.3) solve trigonometric equations
- (STAGE 5.3) use the sine and cosine rules to find an unknown side or angle in any triangle
- (STAGE 5.3) use the formula $A = \frac{1}{2}ab \sin C$ to find the area of a triangle with side lengths a, b and included angle C

SkillCheck ANSWERS ON P. 559

WS Trigonometric calculations

1 Evaluate each expression correct to 4 decimal places.

a cos 32° **b** sin 50.9° **c** tan 8°45′

d 200 tan 18° **e** 14 sin 87°40′ **f** $\frac{13}{\cos 18°27'}$

2 Find the size of angle A, correct to the nearest minute.

a $\cos A = \frac{3}{7}$ **b** $\tan A = 2.7$ **c** $\sin A = 0.4716$

15.01 The trigonometric functions

STAGE 5.3

WS The sine and cosine curves

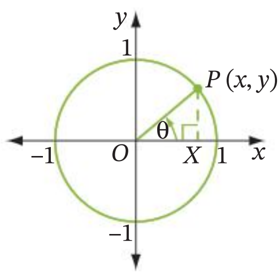
WS Unit circle investigation

Trigonometric graphs

Trigonometric graphs

Trigonometric ratios of any angle

The sine, cosine and tangent ratios can be extended to include angles that are over 90°, that is, obtuse and reflex angles. The trigonometric ratios for angles of any size can be best explained using a **unit circle**.



A unit circle is a circle of radius 1 drawn on a number plane, with the origin as the centre of the circle. Starting from the positive direction of the x-axis, angles can be measured around this circle in an anti-clockwise direction.

Let $P(x, y)$ be any point on the unit circle as shown and θ the angle that PO makes with the positive x-axis.

Let the vertical interval from P meet the x-axis at X to make the right-angled triangle OXP.

Since P has coordinates (x, y), $OX = x$ and $XP = y$.

In $\triangle XOP$, $\cos\theta = \frac{OX}{OP}$ — $OP = 1$ because it is the radius of the unit circle

$= \frac{x}{1}$

$\therefore \cos\theta = x$ — The x-coordinate of point P on the unit circle

Also, $\sin\theta = \frac{XP}{OP}$

$= \frac{y}{1}$

$\therefore \sin\theta = y$ — The y-coordinate of the point P on a unit circle

and $\tan\theta = \frac{XP}{OX}$

$\therefore \tan\theta = \frac{y}{x}$

STAGE 5.3

15.01

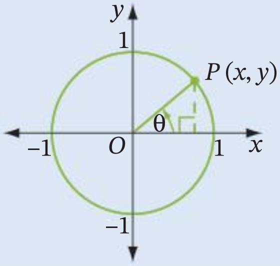
Trigonometric ratios on the unit circle

If $P(x, y)$ is any point on the unit circle, and θ is the angle that PO makes with the positive x-axis, then:

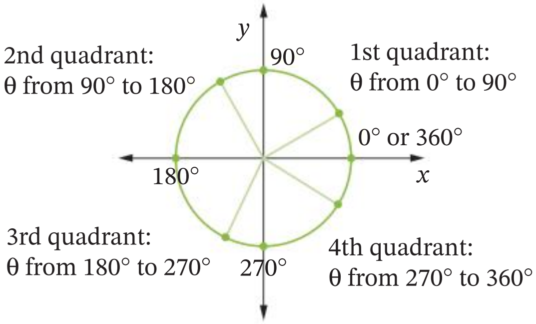
$$\sin\theta = y\text{-coordinate of } P$$

$$\cos\theta = x\text{-coordinate of } P$$

$$\tan\theta = \frac{y\text{-coordinate of } P}{x\text{-coordinate of } P}$$



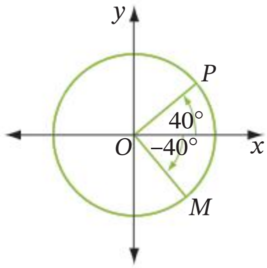
The x- and y-axes divide the number plane into 4 equal **quadrants**. Now we can investigate the trigonometric ratios for all angles from $\theta = 0°$ to $360°$, by looking at $P(x, y)$ on the unit circle in the 1st, 2nd, 3rd and 4th quadrants.



The unit circle can also be used to define the trigonometric ratios for angles below 0° and above 360°.

Negative angles (below 0°) are measured in a clockwise direction on the unit circle. In this diagram, M represents $-40°$ but it could also represent $360° - 40° = 320°$.

Angles above 360° are measured on the unit circle by going around the circle more than once. In the diagram, P represents $40°$, but it could also represent $360° + 40° = 400°$.



The tangent ratio

The tangent ratio can be expressed in terms of the sine and cosine ratios.

Since $\sin\theta = y$ and $\cos\theta = x$,

$$\frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

But $\tan\theta = \dfrac{y}{x}$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

The tangent ratio

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

STAGE 5.3

Example 1

Given that $\sin\alpha = \frac{2}{\sqrt{13}}$ and $\cos\alpha = \frac{3}{\sqrt{13}}$, find $\tan\alpha$.

Solution

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$
$$= \frac{2}{\sqrt{13}} \div \frac{3}{\sqrt{13}}$$
$$= \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{3}$$
$$= \frac{2}{3}$$

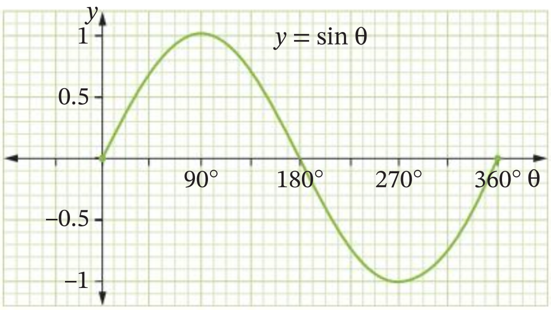
The sine curve

$\sin\theta = y$-coordinate of P (the height of P above the x-axis), so note its value as θ moves around the unit circle (in an anti-clockwise direction) from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
sin θ	0	from 0 to 1	1	from 1 to 0	0	from 0 to −1	−1	from −1 to 0	0

Note that the value of $\sin\theta$ always lies between 1 and −1.

The graph of $y = \sin\theta$ for θ from 0° to 360° is a 'wave curve' that repeats itself after 360°.



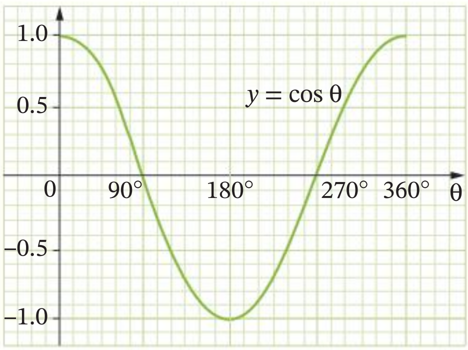
The cosine curve

$\cos\theta = x$-coordinate of P (the length of P from the y-axis), so note its value for θ from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
cos θ	1	from 1 to 0	0	from 0 to −1	−1	from −1 to 0	0	from 0 to 1	1

Note that the value of $\cos\theta$ always lies between 1 and −1.

The graph of $y = \cos\theta$ for θ from 0° to 360° is another 'wave curve' that repeats itself after 360°.



The tangent curve

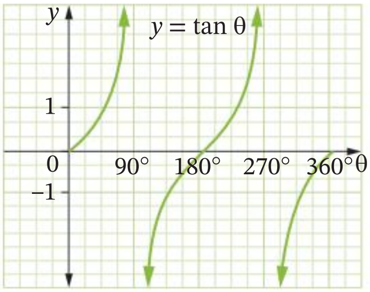
$\tan\theta = \frac{y\text{-coordinate of } P}{x\text{-coordinate of } P} = \frac{\sin\theta}{\cos\theta}$, so note its value for θ from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°
tan θ	$\frac{0}{1} = 0$	$\frac{+}{+}$ = positive	$\frac{1}{0}$ = undefined	$\frac{+}{-}$ = negative	$\frac{0}{-1} = 0$

θ	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
tan θ	$\frac{-}{-}$ = positive	$\frac{-1}{0}$ = undefined	$\frac{-}{+}$ = negative	$\frac{0}{1} = 0$

Note that the value of tan θ has no value at 90° and 270°.

The graph of $y = \tan\theta$ for θ from 0° to 360° is a curve that repeats itself after 180°, with asymptotes at θ = 90° and 270°.

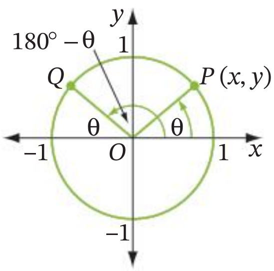


Trigonometric ratios of supplementary angles

For obtuse angles (between 90° and 180°, represented by the 2nd quadrant in the unit circle), we can use this diagram where *Q* is a reflection of *P* across the *y*-axis.

QO makes an angle of 180° − θ with the positive *x*-axis.

In the 2nd quadrant, *Q* has a negative *x*-coordinate and a positive *y*-coordinate, so if the coordinates of *P* are (*x*, *y*), then the coordinates of *Q* are (−*x*, *y*).



$$\therefore \cos(180° - \theta) = -x$$
$$= -\cos\theta$$

$$\therefore \sin(180° - \theta) = y$$
$$= \sin\theta$$

$$\therefore \tan(180° - \theta) = \frac{y}{-x}$$
$$= -\frac{y}{x}$$
$$= -\tan\theta$$

STAGE 5.3

Trigonometric ratios of obtuse angles

For obtuse angles (in the second quadrant), sine is positive while cosine and tangent are negative.

$$\sin(180° - A) = \sin A$$

The sine of an obtuse angle is equal to the **sine** of its supplement.

$$\cos(180° - A) = -\cos A$$

The cosine of an obtuse angle is equal to the **negative cosine** of its supplement.

$$\tan(180° - A) = -\tan A$$

The tangent of an obtuse angle is equal to the **negative tangent** of its supplement.

Example 2

If θ is acute, find θ if:

a $\tan 140° = -\tan\theta$ **b** $\sin 100° = \sin\theta$ **c** $\cos 120° = -\cos\theta$

Solution

a $\theta = 180° - 140°$
$= 40°$
$\therefore \tan 140° = -\tan 40°$

b $\theta = 180° - 100°$
$= 80°$
$\therefore \sin 100° = \sin 80°$

c $\theta = 180° - 120°$
$= 60°$
$\therefore \cos 120° = -\cos 60°$

EXERCISE 15.01 ANSWERS ON P. 559

The trigonometric functions U F R C

EXAMPLE 1

1 **a** If $\sin A = \frac{60}{109}$ and $\cos A = \frac{91}{109}$, find $\tan A$

b If $\sin Y = 0.2$ and $\cos Y = 0.15$, find $\tan Y$

c If $\sin X = \frac{2}{\sqrt{13}}$ and $\cos X = \frac{3}{\sqrt{13}}$, find $\tan X$

d If $\cos P = \frac{40}{41}$ and $\sin P = \frac{9}{41}$, find $\tan P$

e If $\cos Q = \frac{3}{7}$ and $\sin Q = \frac{\sqrt{40}}{7}$, find $\tan Q$

f If $\cos X = \frac{60}{61}$ and $\tan X = \frac{11}{60}$, find $\sin X$

g If $\tan X = \frac{24}{7}$ and $\sin X = \frac{24}{25}$, find $\cos X$

h If $\tan X = \frac{2}{\sqrt{5}}$ and $\cos X = \frac{\sqrt{5}}{3}$, find $\sin X$

2 State whether the trigonometric function of each acute or obtuse angle is positive (P) or negative (N).

a sin 95° **b** cos 46° **c** tan 153° **d** cos 171°
e sin 142° **f** tan 91° **g** tan 130° **h** cos 87°

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STAGE 5.3

15.01

3 Evaluate, correct to 2 decimal places, each trigonometric expression.

a cos 153° **b** tan 349° **c** sin 230° **d** tan 173° 42′

e cos 300.9° **f** sin 324.8° **g** sin 176° 54′ **h** cos 245° 23′

i tan (−38°) **j** sin (−61°) **k** tan 370° **l** cos 434°

4 **a** Copy and complete this table of values for $y = \sin\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	…	360°
y	0	0.5	0.87	…	0

b Graph $y = \sin\theta$, either by using graphing technology or on paper using a scale of 1 cm = 30° on the θ-axis and a scale of 4 cm = 1 unit on the y-axis.

c Comment on the shape of the graph $y = \sin\theta$. What are the maximum and minimum values of the graph and when do they occur?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is sin θ:

i positive **ii** negative?

5 **a** Copy and complete this table of values for $y = \cos\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	…	360°
y	1	0.87	0.5	…	1

b Graph $y = \cos\theta$, either by using graphing technology or on paper.

c Comment on the shape of the graph $y = \cos\theta$. What are the maximum and minimum values of the graph and when do they occur?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is cos θ:

i positive **ii** negative?

g Comment on the similarities and differences between the graphs of $y = \sin x$ and $y = \cos x$.

6 If θ is acute, find θ if:

EXAMPLE 2

a cos 170° = −cos θ **b** sin 110° = sin θ **c** tan 130° = −tan θ

d tan 97° = −tan θ **e** cos 115° = −cos θ **f** sin 168° = sin θ

7 Write each expression in terms of sin A, cos A or tan A, where A is acute.

a cos 142° **b** sin 105° **c** cos 155° **d** tan 102°

e cos 172.7° **f** sin 115.5° **g** cos 139° 35′ **h** tan 170.8°

i sin 120° 35′ **j** tan 160° 10′ **k** sin 95.5° **l** tan 139.5°

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STAGE 5.3

8 **a** Copy and complete this table of values for $y = \tan\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	...	360°
y	0	0.58	1.73	...	0

b Graph $y = \tan\theta$, either by using graphing technology or on paper.

c Comment on the shape of the graph $y = \tan\theta$. When does the graph start to repeat itself?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is $\tan\theta$:

i positive **ii** negative?

15.02 Trigonometric equations

STAGE 5.3

WS Trigonometric equations

Example 3

Solve each trigonometric equation, giving all possible acute and obtuse solutions correct to the nearest degree.

a $\sin\theta = 0.7538$ **b** $\tan\theta = -2.5$

Solution

a $\sin\theta = 0.7538$

$\theta = 48.9206...$

$\approx 49°$

On a calculator: SHIFT sin 0.7538 =

On some calculators, 2nd F instead of SHIFT

(Check: sin 49° = sin 131° = 0.7547...)

But θ could be obtuse, because $\sin\theta$ is also positive in the second quadrant.

$\theta \approx 180° - 49°$

$= 131°$

$\therefore \theta \approx 49°$ or $131°$.

b $\tan\theta = -2.5$

On a calculator: SHIFT tan (−) 2.5 =

$\theta = -68.1985...$

$\approx -68°$

But θ is obtuse, because $\tan\theta$ is negative in the second quadrant.

$\theta \approx 180° - 68°$

$= 112°$

On a calculator: 180 + Ans =

(Check: tan 112° = −2.4750...)

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STAGE 5.3

15.02

Example 4

Solve each trigonometric equation correct to the nearest minute, if x is obtuse.

a $\cos x = -0.09$ **b** $\sin x = 0.64$

Solution

SHIFT cos automatically gives the obtuse angle when you enter a negative value

a $\cos x = -0.09$ On a calculator: SHIFT cos (−) 0.09 =

$x = 95.1636...$

$= 95°\ 9'48.9\ 9''$ On a calculator: °'" or DMS

$\approx 95°10'$

b $\sin x = 0.64$ On a calculator: SHIFT sin 0.64 =

$x = 39.7918...$

But x is obtuse, so:

$x = 180 - 39.7918...$ 180 − Ans =

$= 140.2081...$

$= 140°\ 12'\ 29.45''$ °'" or DMS

$\approx 140°\ 12'$

EXERCISE 15.02 ANSWERS ON P. 560

Trigonometric equations U F

1 Solve each trigonometric equation, giving all possible acute and obtuse solutions, correct to the nearest degree. EXAMPLE 3

a $\sin\theta = 0.84$ **b** $\tan\theta = -\frac{3}{4}$ **c** $\cos\theta = -0.342$

d $\cos\theta = -\frac{7}{11}$ **e** $\sin\theta = 0.1164$ **f** $\tan\theta = -1$

g $\tan\theta = -5.8671$ **h** $\sin\theta = \frac{3}{7}$ **i** $\cos\theta = -0.4$

j $\sin\theta = \frac{3.8}{7}$ **k** $\cos\theta = -\frac{21}{80}$ **l** $\tan\theta = -\frac{15}{8}$

2 Solve each trigonometric equation, correct to the nearest minute, if x is obtuse. Note: some equations have no solution. EXAMPLE 4

a $\sin x = \frac{4}{7}$ **b** $\sin x = 0.7438$ **c** $\sin x = 0.3514$

d $\sin x = 0.108$ **e** $\sin x = \frac{5}{11}$ **f** $\sin x = 0.9$

g $\cos x = 0.6$ **h** $\cos x = -0.6$ **i** $\tan x = 0.3$

j $\tan x = -0.3$ **k** $\sin x = 0.8$ **l** $\sin x = \frac{3}{7}$

3 Solve each trigonometric equation correct to the nearest degree, if A is between 0° and 180°.

a $\cos x = \frac{-8}{11}$ **b** $\tan x = -0.95$ **c** $\sin x = \frac{7}{8}$

d $4\cos x = \sqrt{2}$ **e** $3\sin x = 2$ **f** $-4\tan x = 3$

g $\tan x = 1$ **h** $\cos x = \frac{1}{2}$ **i** $\sin x = \frac{1}{\sqrt{2}}$

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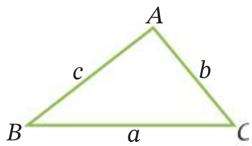
15.03 The sine rule

STAGE 5.3

WS Discovering the sine rule

The angles of a triangle are labelled with capital letters while the sides are labelled with lowercase letters. By convention, we use a to label the side opposite $\angle A$, b to label the side opposite $\angle B$, and so on.



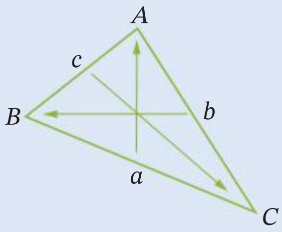
There is a relationship between each angle in a triangle and its opposite side. The longest side is always opposite the largest angle, the next smallest side is opposite the next smallest angle and so on. This relationship is called the **sine rule**.

The sine rule

For any triangle ABC:

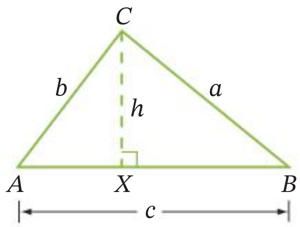
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The ratios of the sides in a triangle to the sine of their opposite angles are equal.



Proof:

In $\triangle ABC$, draw CX for the perpendicular height, h, of the triangle.
CX divides $\triangle ABC$ into 2 right-angled triangles.



In $\triangle AXC$, $\sin A = \frac{h}{b}$

$\therefore h = b \sin A$

In $\triangle BXC$, $\sin B = \frac{h}{a}$

$\therefore h = a \sin B$

$\therefore a \sin B = b \sin A$

$$\frac{a \sin B}{\sin B} = \frac{b \sin A}{\sin B}$$ Dividing both sides by sin B

$$a = \frac{b \sin A}{\sin B}$$

$$\frac{a}{\sin A} = \frac{b \sin A}{\sin B \sin A}$$ Dividing both sides by sin A

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

By drawing the perpendicular from A to BC, it can be shown that $\frac{b}{\sin B} = \frac{c}{\sin C}$

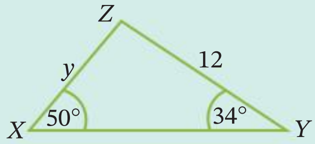
$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine rule allows us to apply trigonometry to any triangle, not just right-angled triangles.

The sine rule is used in problems involving 2 sides of a triangle and the 2 angles opposite them.

Example 5

Find y, correct to one decimal place.



Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$ Sides and opposite angles

$$\frac{y}{\sin 34^\circ} = \frac{12}{\sin 50^\circ}$$

$$y = \frac{12 \sin 34^\circ}{\sin 50^\circ}$$

$$= 8.7596\ldots$$

$$\approx 8.8\text{ cm}$$ From the diagram, an answer of 8.8 cm looks reasonable.

STAGE 5.3

The sine rule

15.03

EXERCISE 15.03 ANSWERS ON P. 560

The sine rule U F PS R

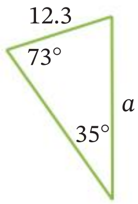
1 Evaluate each expression, correct to one decimal place.

a $\frac{14.7 \sin 64^\circ}{\sin 46^\circ}$ **b** $\frac{34.5 \sin 33.4^\circ}{\sin 115.7^\circ}$ **c** $\frac{69 \sin 107^\circ 33'}{\sin 38^\circ 47'}$

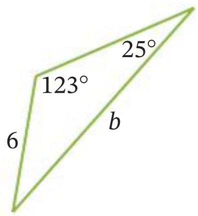
EXAMPLE 5

2 Find the value of each variable, correct to 2 decimal places.

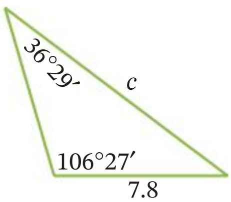
a



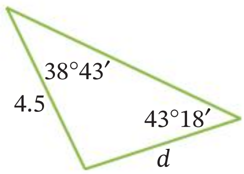
b



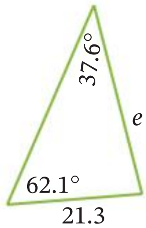
c



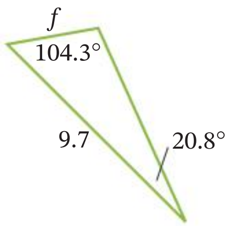
d



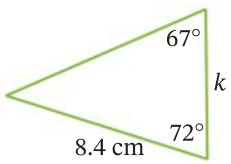
e



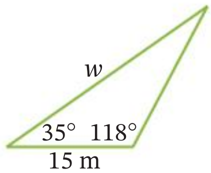
f



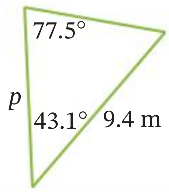
g



h

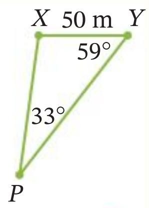


i



STAGE 5.3

3 X and Y are 2 light towers 50 m apart on one side of a park. P is a light tower on the other side of the park. If $\angle Y = 59°$ and $\angle P = 33°$, find PX correct to the nearest metre. PS R

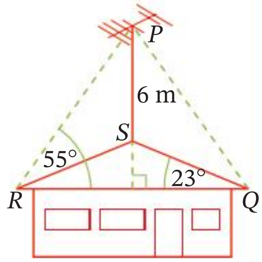


4 A golfer drives a ball 275 m at an angle of 5° off centre. The ball lands at an angle of 107° from the hole. Calculate the distance of the ball from the hole, correct to the nearest metre. PS R

5 A 6 m television antenna is mounted on a roof pitched at an angle of 23°. It is supported by 2 wires, PQ and PR, inclined at 55° to the horizontal. PS R

a Show that $\angle PSR = 113°$.

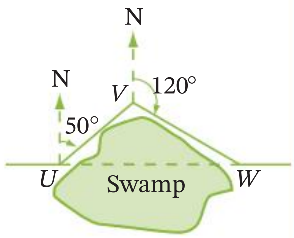
b Calculate the length of the wire PR, correct to the nearest centimetre.



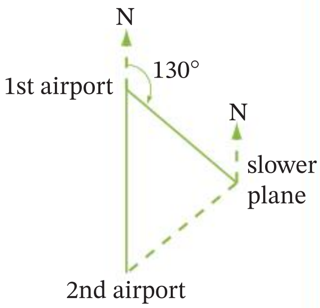
6 To avoid a swamp, Jesinta runs 70 m on a bearing of 050° to V. She then turns and runs to W on a bearing of 120°. If W is directly east of U: PS R

a find $\angle UVW$

b calculate UW, correct to one decimal place.



7 2 planes leave the airport at the same time. One flies due south at 400 km/h and lands at a second airport after $1\frac{1}{2}$ hours. The other flies on a bearing of 130° and after $1\frac{1}{2}$ hours is at a bearing of 075° from the second airport. How far (to the nearest km) is the slower plane from the second airport? PS R



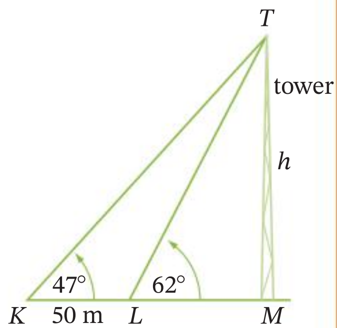
8 The angle of elevation of a tower from a point L is 62°. From a point K, 50 m further from the tower, the angle of elevation is 47°. PS R

a Use the sine rule in $\triangle KTL$ to show that $TL = \dfrac{50 \sin 47°}{\sin 15°}$.

b Let the height of the tower be h. In the right-angled $\triangle LMT$, show that $TL = \dfrac{h}{\sin 62°}$.

c Hence show that $h = \dfrac{50 \sin 47° \sin 62°}{\sin 15°}$

d Hence calculate the height of the tower, correct to one decimal place.



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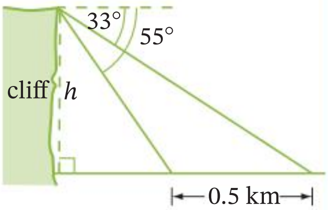
9 From the top of a cliff, the angles of depression of 2 boats at sea that are 0.5 km apart are 55° and 33°. PS R



a Let the height of the cliff be h. Show that $h = \frac{0.5 \sin 33° \sin 55°}{\sin 22°}$.

b Hence calculate the height, correct to the nearest metre.

STAGE 5.3

The sine rule for angles

15.04

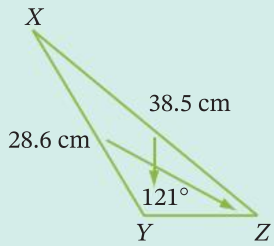
Example 6

STAGE 5.3

Sine rule problems

Obtuse angles using the sine rule

Find angle Z, correct to the nearest degree.



Solution

$\frac{28.6}{\sin Z} = \frac{38.5}{\sin 121°}$ Sides and opposite angles

$\frac{\sin Z}{28.6} = \frac{\sin 121°}{38.5}$ Inverting both sides so that Z is in the numerator.

$\sin Z = \frac{28.6 \sin 121°}{38.5}$ On a calculator: SHIFT sin Ans =

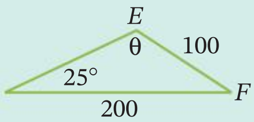
$= 0.636\ldots$

$Z = 39.55\ldots$

$\approx 40°$ From the diagram, an answer of 40° looks reasonable.

Example 7

Find θ correct to the nearest minute if it is an obtuse angle.



Solution

$\frac{200}{\sin \theta} = \frac{100}{\sin 25°}$

$\frac{\sin \theta}{200} = \frac{\sin 25°}{100}$

$\sin \theta = \frac{200 \sin 25°}{100}$

$= 0.845\ldots$

$\theta = 57.697\ldots$

But θ is obtuse, so:

$\theta = 180 - 57.697\ldots$

$= 122.3027\ldots$

$= 122° 18' 9.77''$

$\approx 122° 18'$

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STAGE 5.3

The ambiguous case (when there are 2 possible answers)

When we use the sine rule to find an angle, it is possible to find both an acute angle and an obtuse angle as solutions. Likewise, there could be 2 possible triangles: one acute-angled, the other obtuse-angled. However, the obtuse-angled triangle may not be possible. We need to check that the sum of the angles in the triangle is not greater than 180°.

The sine rule

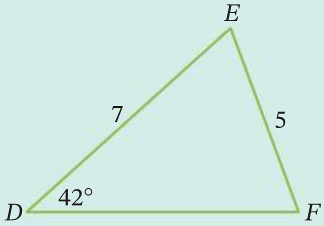
Example 8

a In $\triangle DEF$, $\angle D = 42°$, $d = 5$ cm and $f = 7$ cm. Find $\angle F$ correct to the nearest degree.

b In $\triangle LMN$, $\angle M = 130°$, $LN = 15$ cm and $LM = 7$ cm. Find $\angle N$, correct to the nearest degree.

Solution

a Draw a rough diagram.



$$\frac{7}{\sin F} = \frac{5}{\sin 42°}$$

$$\frac{\sin F}{7} = \frac{\sin 42°}{5}$$

$$\sin F = \frac{7 \sin 42°}{5}$$

$$= 0.93678\ldots$$

$$F = 69.5181\ldots .$$

$$\approx 70°$$

But F could be obtuse.

$$F = 180° - 70°$$

$$= 110°$$

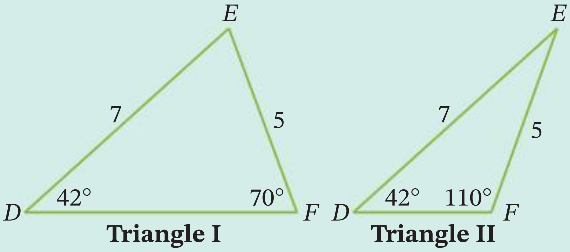
Checking the third angle of the obtuse-angled triangle:

$$\angle E = 180° - 42° - 110°$$

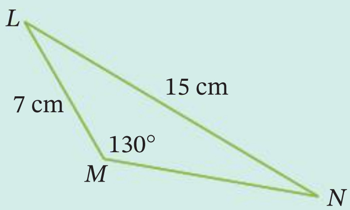
$$= 28°$$

∴ The obtuse-angled solution is possible.

$$\therefore \angle F = 70° \text{ or } 110°$$



b Draw a rough diagram.



$$\frac{\sin N}{7} = \frac{\sin 130°}{15}$$

$$\sin N = \frac{7 \sin 130°}{15}$$

$$= 0.3574\ldots$$

$$N = 20.9459\ldots$$

$$\approx 21°$$

But N could be obtuse.

$N = 180° - 21°$

$\quad = 159°$

Checking the third angle of the obtuse-angled triangle:

$\angle L = 180° - 42° - 159°$ Impossible

$\quad = -21°$

$\therefore$ The obtuse-angled solution is not possible.

$\therefore \angle N = 21°$

STAGE 5.3

15.04

EXERCISE 15.04 ANSWERS ON P. 560

ANSWERS ON P. 560

The sine rule for angles U F R C

1 Find the acute angle X in each equation, correct to the nearest degree.

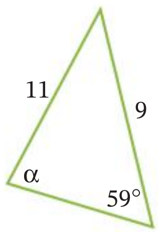
a $\sin X = \dfrac{5.3 \sin 123°}{9.7}$ **b** $\sin X = \dfrac{39 \sin 85°29'}{64}$

c $\sin X = \dfrac{467 \sin 63.8°}{518}$

EXAMPLE 6

2 Find α in each triangle if α is acute, correct to the nearest 0.1 degree.

a



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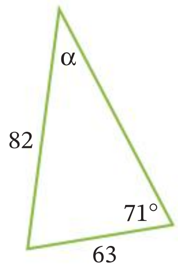

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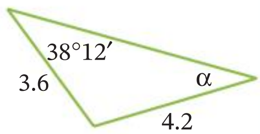

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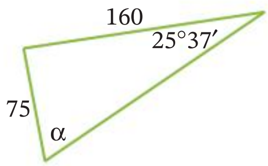

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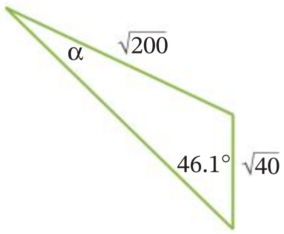

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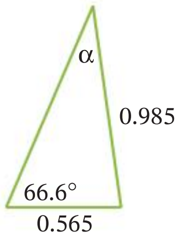
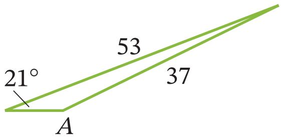


STAGE 5.3

EXAMPLE 7

3 Find the size of ∠A to the nearest minute if ∠A is obtuse.

a



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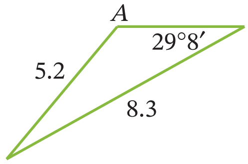

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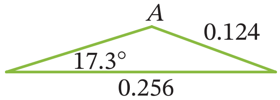

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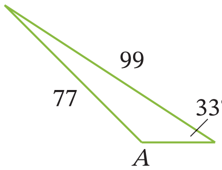

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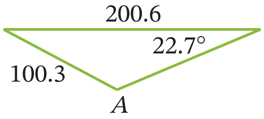

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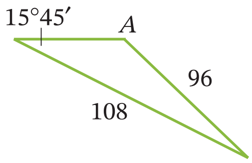


EXAMPLE 8

4 Find all possible angles for each triangle, correct to the nearest degree, after sketching a diagram. R C

a In △PQR, ∠P = 35°, p = 8 cm, and q = 10 cm. Find ∠Q.

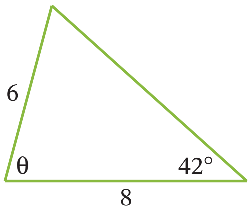
b In △UVW, ∠W = 95°, w = 16 cm, and v = 10 cm. Find ∠V.

c In △XYZ, ∠Y = 24°, y = 3.4 km, and z = 5.7 km. Find ∠Z.

d In △DEF, ∠E = 37°, e = 107 mm, and d = 121 mm. Find ∠D.

5 Find θ in each triangle correct to the nearest degree, given that θ is acute.

a



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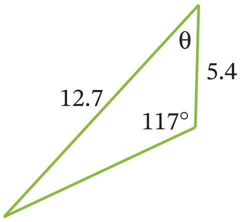

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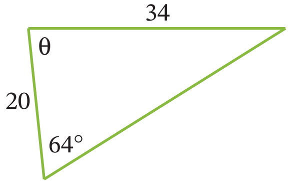


15.05 The cosine rule

STAGE 5.3

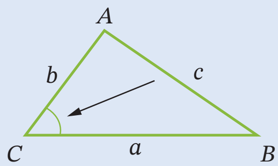
The **cosine rule** is a relationship between the 3 sides of a triangle and one of its angles. It is an extension of Pythagoras' triangle that can be applied to any triangle, not just right-angled ones.

The cosine rule

For any triangle ABC:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

where c is the unknown side, C is the angle opposite c, and a and b are the other 2 sides.



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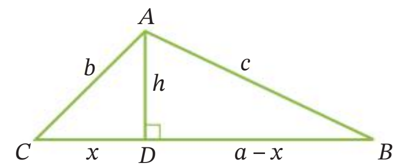
Proof:



In $\triangle ABC$, draw AD for the perpendicular height, h, of the triangle. AD divides $\triangle ABC$ into 2 right-angled triangles.

Let $CD = x$, $\therefore DB = a - x$

In $\triangle ADC$, $b^2 = h^2 + x^2$

$\therefore h^2 = b^2 - x^2$ [1]

In $\triangle ABD$, $c^2 = h^2 + (a - x)^2$

$\therefore h^2 = c^2 - (a - x)^2$

$\therefore h^2 = b^2 - x^2$ From [1]

$= c^2 - (a - x)^2$

$b^2 - x^2 = c^2 - (a^2 - 2ax + x^2)$

$b^2 - x^2 = c^2 - a^2 + 2ax - x^2$

$b^2 = c^2 - a^2 + 2ax$

$\therefore c^2 = a^2 + b^2 - 2ax$ [2] Making c^2 the subject

In $\triangle ADC$, $\cos C = \frac{x}{b}$

$\therefore x = b \cos C$

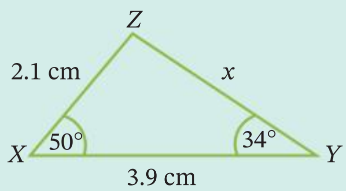
Substituting for x in [2]:

$c^2 = a^2 + b^2 - 2ab \cos C$

The cosine rule can be used in problems involving 3 sides of a triangle and one of the angles.

Example 9

Find x correct to 2 decimal places.



The cosine rule

Solution

$c^2 = a^2 + b^2 - 2ab \cos C$

$x^2 = 2.1^2 + 3.9^2 - 2 \times 2.1 \times 3.9 \cos 50°$ 50° is the angle opposite x.

$= 9.091138...$

$x = \sqrt{9.091138...}$

$= 3.01515...$

≈ 3.02 cm

From the diagram, an answer of 3.02 cm looks reasonable.

STAGE 5.3

EXERCISE 15.05 ANSWERS ON P. 560

The cosine rule U F PS R C

1 Solve each equation for x, correct to one decimal place.

a $x^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 38°$

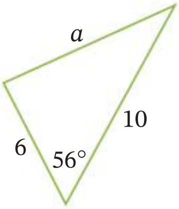
b $x^2 = 11.3^2 + 9.7^2 - 2 \times 11.3 \times 9.7 \times \cos 76.9°$

c $x^2 = 17^2 + 20.1^2 - 2 \times 17 \times 20.1 \times \cos 149° 45'$

EXAMPLE 9

2 Find, correct to 2 decimal places, the value of each variable.

a



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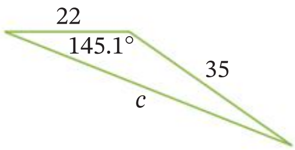

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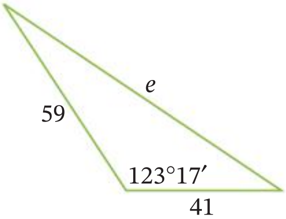

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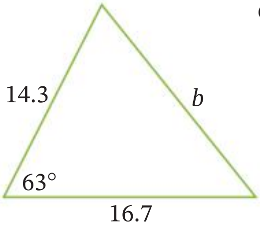

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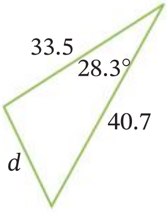

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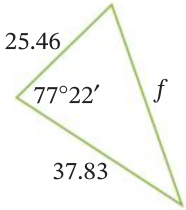
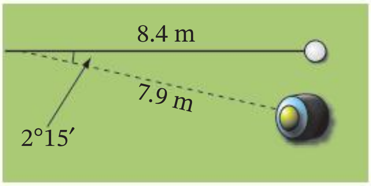
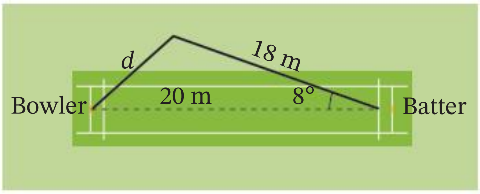


3 In a game of lawn bowls, Jayden is aiming to hit the jack (target ball) 8.4 m away. If he bowls 2° 15′ off centre and his bowl travels 7.9 m, how far is his bowl from the jack? Answer correct to one decimal place. PS R



4 In a cricket match, the distance between the bowler and the batter was 20 m. During one bowl, the batter hit the ball at an angle of 8° to the line of the pitch and the bowler ran and caught the ball after it had travelled 18 m. How far did the bowler run to catch the ball? Select the correct answer **A**, **B**, **C** or **D**. PS R



A 1.1 m **B** 2.0 m **C** 3.3 m **D** 4.0 m

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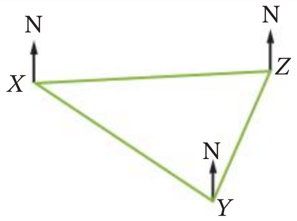
5 A yacht sails from X to Y on a bearing of 130° for 4.2 km. It then turns and travels to Z on a bearing of 025° for 2.9 km. PS R C

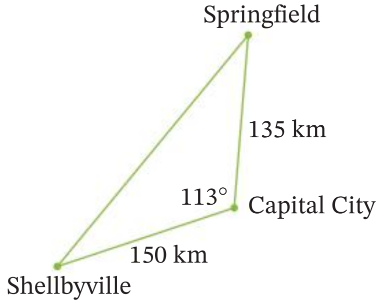


a Copy the diagram and mark the given information on it.

b Explain why $\angle XYZ = 75°$.

c Calculate the distance XZ, correct to one decimal place.

6 3 towns are joined by straight roads. What distance (correct to the nearest kilometre) is saved by going directly from Springfield to Shellbyville instead of travelling via Capital City? PS R C



7 **a** What is the value of cos 90°?

b What does $c^2 = a^2 + b^2 - 2ab \cos C$ simplify to if $C = 90°$?

c Hence what happens to the cosine rule when it is applied to a right-angled triangle?

STAGE 5.3

15.05

The cosine rule for angles 15.06

STAGE 5.3

WS Cosine rule problems

If we rewrite the cosine rule so that cos A is the subject, then we will have a formula for finding an unknown angle when the 3 sides of a triangle are known.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 + 2ab \cos C = a^2 + b^2 \quad \text{Adding } 2ab \cos C \text{ to both sides so that } \cos C \text{ appears on the LHS}$$

$$2ab \cos C = a^2 + b^2 - c^2$$

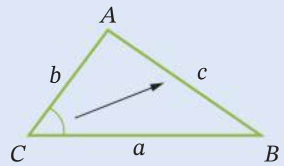
$$\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The cosine rule for angles

For any triangle ABC:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

where C is the unknown angle, c is the side opposite C, and a and b are the other 2 sides.



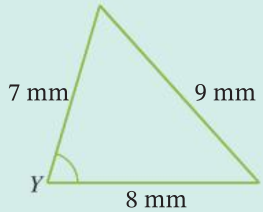
STAGE 5.3

The cosine rule can be used to find an unknown angle if the lengths of the 3 sides are known.

The cosine rule for angles 2

Example 10

Find the size of the marked angle Y, correct to the nearest degree.



Solution

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos Y = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8}$$

$$= \frac{32}{112}$$

9 mm is opposite angle Y

$$Y = 73.398...$$

$$\approx 73°$$

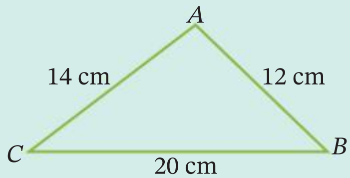
From the diagram, an answer of 73° looks reasonable.

The cosine rule

The cosine rule for angles 1

Example 11

Calculate, correct to the nearest minute, the size of the largest angle in this triangle.



Solution

The largest angle is opposite the longest side, so it is $\angle A$.

$$\cos A = \frac{14^2 + 12^2 - 20^2}{2 \times 14 \times 12}$$

Cos is negative so the angle will be obtuse.

$$= \frac{-60}{336}$$

$$A = 100.28656...$$

$$= 100°17'11.6''$$

$$\approx 100°17'$$

From the diagram, an answer of 100°17′ looks reasonable.

EXERCISE 15.06 ANSWERS ON P. 560

STAGE 5.3

The cosine rule for angles U F PS R

15.06

1 Solve each equation for X, correct to the nearest degree.

EXAMPLE 10

a $\cos X = \dfrac{12^2 + 14^2 - 15^2}{2 \times 12 \times 14}$

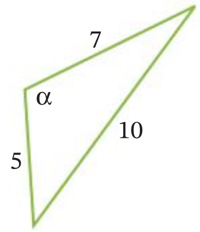
b $\cos X = \dfrac{5.7^2 + 6.8^2 - 3.7^2}{2 \times 5.7 \times 6.8}$

c $\cos X = \dfrac{5^2 + 6^2 - 9^2}{2 \times 5 \times 6}$

d $\cos X = \dfrac{9.2^2 + 4.7^2 - 12.8^2}{2 \times 9.2 \times 4.7}$

2 Find the size of α in each triangle, correct to the nearest degree.

a



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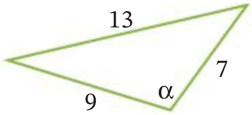

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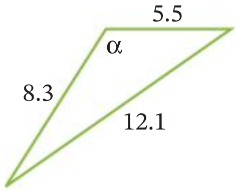

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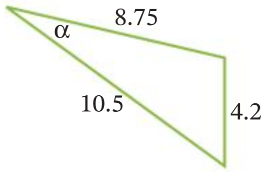

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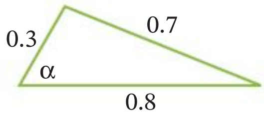

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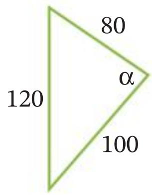
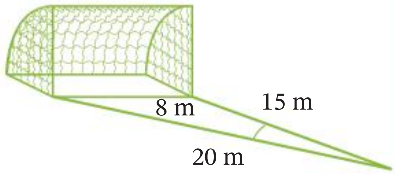
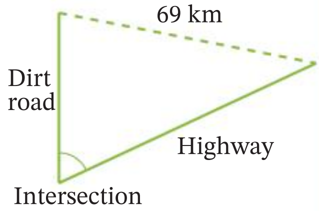


EXAMPLE 11

3 A soccer goal is 8 m wide. A player shoots for goal (along the ground) when 20 m from one post and 15 m from the other post. Within what angle (correct to the nearest 0.1 degree) must the shot be made for the player to have a chance of scoring a goal? PS R



4 2 cars leave an intersection at the same time. Car A drives down the dirt road at 60 km/h and car B drives down the highway at 100 km/h. After 45 minutes they are 69 km apart. Find the angle between the 2 roads, correct to the nearest minute. PS R



5 A triangle has sides of 21 m, 17 m and 10 m. Find the size of the largest angle, correct to the nearest degree. R

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15.07 The area of a triangle

STAGE 5.3

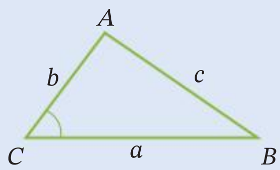
We already know that the formula for the area of a triangle is $A = \frac{1}{2}bh$, but there is also a trigonometric formula if we know the lengths of 2 sides of the triangle and the size of the **included angle** between them.

We've learned about **included angles** with the congruent and similar triangles tests.

The area of a triangle

$A = \frac{1}{2}ab\sin C$

where C is the included angle between sides a and b.



Proof:

In $\triangle ABC$, draw AX for the perpendicular height, h, of the triangle.

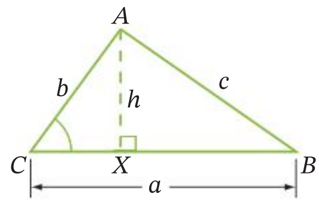
Area of $\Delta ABC = \frac{1}{2}ah$ [1]

But in $\triangle AXC$, $\sin C = \frac{h}{b}$

$\therefore h = b \sin C$

Substituting this into [1]:

$A = \frac{1}{2}ab\sin C$



Example 12

Find, correct to one decimal place, the area of this triangle.

Solution

$A = \frac{1}{2}ab \sin C$

$= \frac{1}{2} \times 5.7 \times 9 \times \sin 83°$

$= 25.458...$

$\approx 25.5 \text{ cm}^2$

83° is the included angle between 5.7 cm and 9 cm.

EXERCISE 15.07 ANSWERS ON P. 560

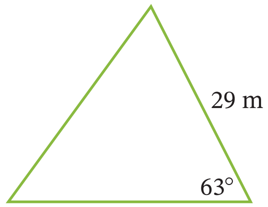
The area of a triangle **U F PS R**

EXAMPLE 12

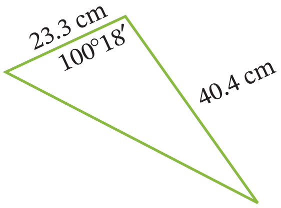
15.07

1 Find, correct to one decimal place, the area of each triangle.

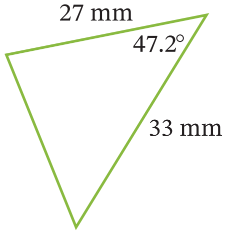
a



b



c



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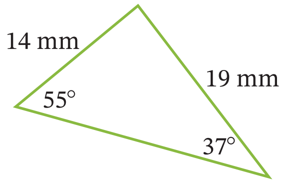

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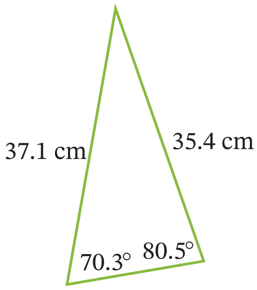

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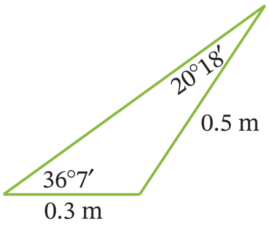
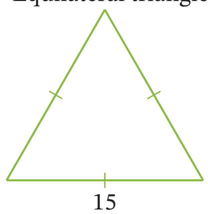
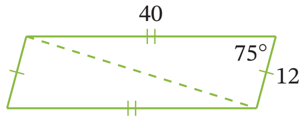


2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres. **R**

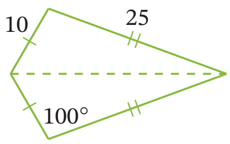
a Equilateral triangle



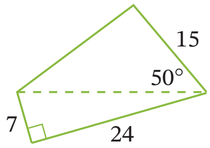
b Parallelogram



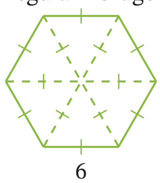
c Kite



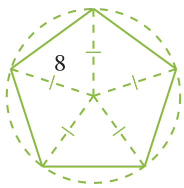
d Quadrilateral



e Regular hexagon



f Regular pentagon inscribed in a circle of radius 8



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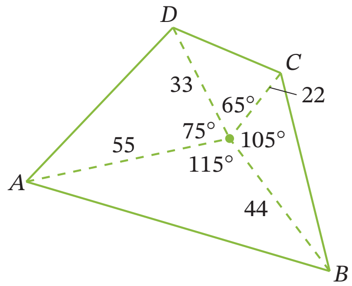
STAGE 5.3

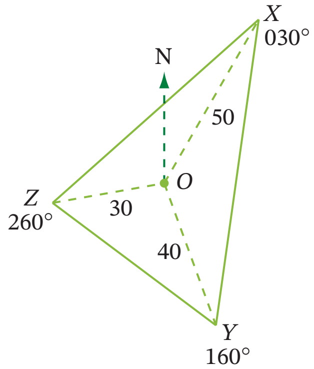
3 The diagram shows the results of a radial survey of a block of land. All distances are in metres. PS R



a Use the cosine rule to find the lengths of *AB*, *BC*, *CD* and *AD* and, hence, find the perimeter of the block of land (to the nearest metre).

b Use the area formula to find the area of each triangle and, hence, calculate the area of the block of land (to the nearest m²).

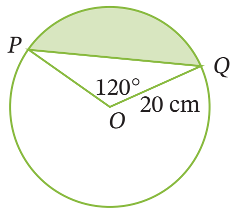
4 The results of a radial survey are shown in the diagram. All measurements are in metres. PS R



a Find the size of $\angle XOY$.

b Calculate, correct to 2 decimal places, the area of $\triangle XOY$.

5 *O* is the centre of a circle of radius 20 cm. Calculate, correct to one decimal place, the area of: PS R

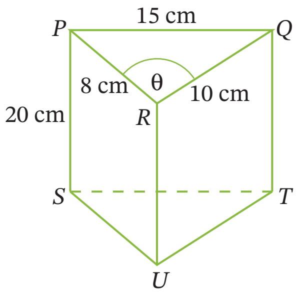


a sector *POQ*

b triangle *OPQ*

c the shaded segment.

6 A triangular prism has base edges of 8 cm, 10 cm and 15 cm, and a height of 20 cm. PS R



a Calculate the size $\angle PRQ$, correct to nearest degree.

b Find the area of $\triangle PQR$, correct to the nearest cm².

c Find the volume of the prism.

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Problems involving the sine and cosine rules

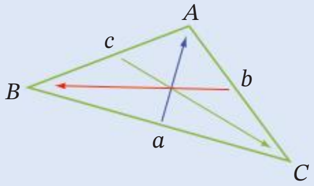
15.08

STAGE 5.3

The sine and cosine rules

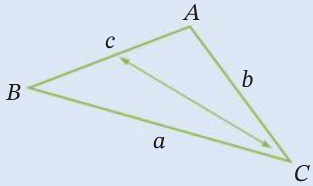
The sine rule is used for triangle problems involving 2 sides and the 2 angles opposite them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The cosine rule is used for triangle problems involving 3 sides and one angle.

$c^2 = a^2 + b^2 - 2ab \cos C$ and $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



WS Finding an unknown side

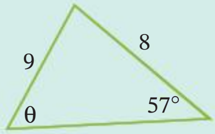
WS The sine and cosine rules

The sine and cosine rules

Example 13

a Find the value of k, correct to one decimal place.

b Find the value of θ, correct to the nearest minute.



Solution

a The problem involves 3 sides and one angle so use the cosine rule.

$k^2 = 8.4^2 + 12.9^2 - 2 \times 8.4 \times 12.9 \times \cos 37°$

$= 63.889...$

$k = \sqrt{63.889...}$

$= 7.993...$

≈ 8.0 m

b The problem involves 2 sides and the 2 angles opposite them, so use the sine rule.

$$\frac{\sin\theta}{8} = \frac{\sin 57°}{9}$$

$$\sin\theta = \frac{8\sin 57°}{9}$$

$$= 0.7454...$$

$\theta = 48.2007...$

$= 48°12'\,2.77''$

$\approx 48°12'$

STAGE 5.3

EXERCISE 15.08 ANSWERS ON P. 560

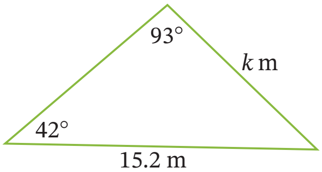
Problems involving the sine and cosine rules

U F PS R C

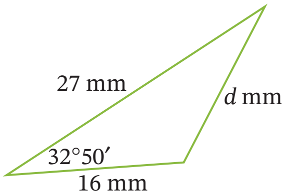
EXAMPLE 13

1 Find, correct to one decimal place, the value of each variable.

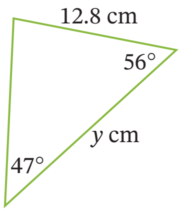
a



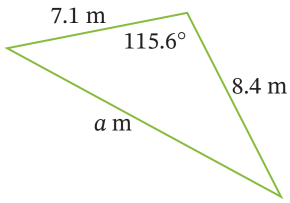
b



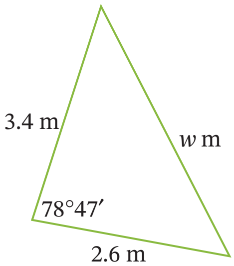
c



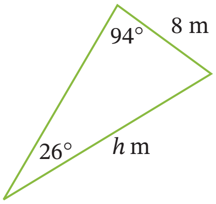
d



e

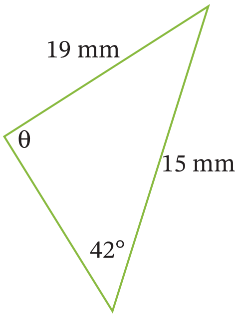


f

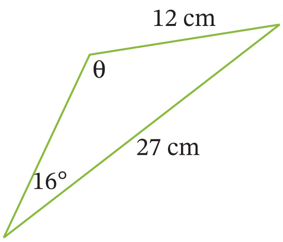


2 Find the value of θ to the nearest degree. Use diagrams to note whether θ is acute or obtuse.

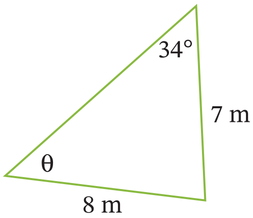
a



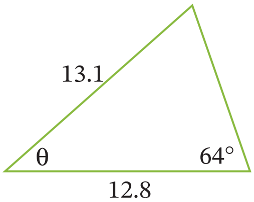
b



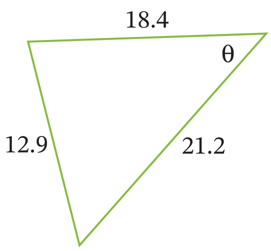
c



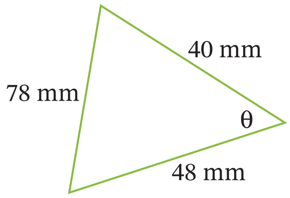
d



e



f



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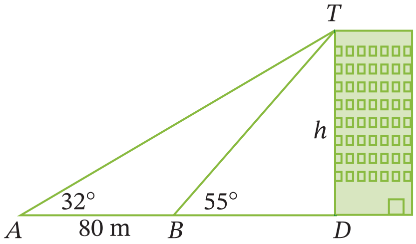
STAGE 5.3

15.08

3 The angles of elevation of a building measured from 2 positions 80 m apart are 32° and 55°. PS R C



a Explain why $\angle ATB = 23°$.

b Find, correct to 2 decimal places, the length of BT.

c Hence find the height, h, of the building, correct to the nearest metre.

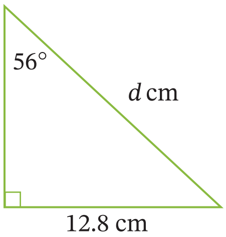
4 **a** What is the value of sin 90°? PS R C

b Find, correct to one decimal place, the value of d using:

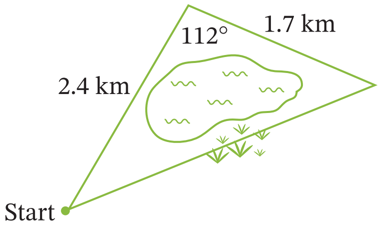
i the sine rule

ii the sine ratio for right-angled triangles.

c What do you notice about your results? Give reasons.



5 Mikayla needs to run around a cross-country course as shown. What is the length of the course, correct to one decimal place? PS R



6 A plane flew on a bearing of 150° for 370 km. It then changed direction and flew another 285 km on a bearing of 235°. How far, correct to the nearest kilometre, is the plane from its starting point? PS R

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ANSWERS

Chapter 1

SkillCheck

1 a 0.04 b 0.22 c 0.183 d 0.047
e 0.095 f 0.0675 g 0.1525 h 0.2
2 a $72 b $116.25 c $4494
3 a $7350 b $4034.10 c $8737.60
4 a 36 b 24 c 60
5 a 52 b 26
c 365 d 4
e 12 f 8 years and 4 months
6 a 1152 b 50 c 0.06
7 a $5962.59 b $33 433.46
c $18 481.63 d $64937.10

Exercise 1.01

1 a $874 b $938.80 c $734.40
2 Greta earns more per week by $27.48.
3 a $3461.86 b $6923.73 c $15 053.33
4 Job 1: $1104.64; Job 2: $1160; Job 2 by $55.36
5 $1107.40 6 $851.18 7 $780.86
8 A 9 $13 312.50 10 $1394.40
11 $2115 12 54 13 $63.95
14 a $854 b $700 c $956.87 d $625.55
15 a $972.12 b $680.48 c $4568.96

Exercise 1.02

1 a $45 697 b $6398.53
2 a $114 719 b $29 943.03
3 a $90 904 b $21 131.48
4 C 5 $19 635.78 6 $45 006.10
7 $696.42 8 $623.52
9 a $462 b $1699.10 c 25.4%
10 a $468 b $1732.65 c 24.5%
11 a $2296 b $468 c $1634.73
12 a $2304.91 b $470 c $1543.71
13 Gross weekly income = $825.30;
Total deductions = $374.10; Net income = $451.20

Exercise 1.03

1 a $5040 b $1047.15 c $102.50
d $263.60 e $37.40 f $1192.32
2 a $87.50 b $5925.15 c $207 000
d $690 e $1404 f $723.04
3 A
4 a $11 200 b $1551.30
c $9392.50 d $10 695.31
5 a $1440 b $7440
6 4.5%
7 a $6400 b 5.82%
8 5.85% p.a. 9 approx. 2 years
10 30 weeks 11 137 days
12 C
13 31 months 14 2.6% p.a.
15 a $18.90 b $1063.90
16 B

Exercise 1.04

1 a Check with your teacher. Investment after 1st year = $24 150; Investment after 2nd year = $25 357.50
b Compound interest = $2357.50
2 a $16 153.36 b $1153.36
3 a $38 459.48 b $4359.48
4 a $5408, $408 b $30 245.29, $2445.29
c $11 113.20, $1513.20
d $41 905.55, $2405.55
e $19 337.39, $937.39
5 a $4791.80 b $1642.38
c $308.93 d $3913.84
e $6834.42

Mental skills 1

2 a 11 b 40 c 7 d 24
e 23 f 6 g 43 h 80
i 18 j 15 k 40 l 65
m 11 n 14 o 12 p 135

Exercise 1.05

1 B
2 a i $9754.75 ii $3254.75
b i $13 858.59 ii $3858.59
c i $12 634.81 ii $394.81
d i $43 949.46 ii $9349.46
e i $8427.39 ii $427.39
3 D 4 $1 301 018.83
5 B 6 A
7 a i $11 273.62 ii $1273.62
b i $52 751.13 ii $17 251.13
c i $9448.23 ii $548.23
d i $48 063.26 ii $6063.26
e i $17 829.56 ii $1329.56
f i $4990.24 ii $90.24
8 C
9 a $600 b $615 c Tegan by $15.
10 a D b A c C d B
11 a $17 807.64 b $124 less
12 a i $5746.85 ii $5793.89
iii $5817.79 iv $5833.87
b Monthly, because it earns the most interest.
13 D

Exercise 1.06

1 **a** \$175.50 **b** \$1579.50 **c** \$328.14 **d** \$1907.64 **e** \$105.98 **f** \$2083.14
2 **a** \$1275 **b** \$24 225 **c** \$10 416.75 **d** \$34 641.75 **e** \$577.36 **f** \$35 916.75
3 **a** \$1379 **b** \$2316.72 **c** \$217.58
4 **a** \$3420 **b** \$720 **c** \$1500 **d** 48%
5 **a** \$2080 **b** \$8320 **c** \$13 200 **d** \$4880 **e** 14.7%
6 **a** \$104.40 **b** \$1536 **c** \$596.40 **d** 63.5%
7 **a** \$1073.40 **b** \$273.40 **c** 34.2%
8 **a** \$2599 **b** \$3576 **c** \$677 **d** 10.4%
9 **a** \$262.50 **b** 7.4%

Exercise 1.07

1 \$933.89
2 **a** \$20 429.69 **b** \$29 560.31
3 **a i** \$659.66 **ii** 60.0%
b i \$17 406.69 **ii** 45.2%
c i \$5073.42 **ii** 60.0%
d i \$1024 **ii** 41.0%
e i \$14 020.37 **ii** 51.0%
f i \$1073.44 **ii** 37.0%
g i \$403.03 **ii** 46.3%
h i \$1782.95 **ii** 68.6%
4 **a i** 63% **ii** 25.00% **iii** 6.25%
b By trial and error, in approx. 1.5 years.
5 **a i** \$10 000 **ii** \$8000 **iii** \$4096
b 32.8%
6 **a** \$17 969.66 **b** \$7521
7 **a** \$71 680 **b** \$103 320
c Approx. 5 years and 8 months.
d 13.4%
8 Yes, it will lose approximately 52% after 7 years.
9 **a** \$1800 **b** 5 years **c** \$798.67
d Yes, in the 30th year. **e** No

Power plus

1 4 years and 61 days 2 \$4444.44
3 \$12 838.71 4 \$63 367.50
5 \$2276.87 6 790 000
7 **a** 18 years. **b** 18 years.
c No. The size of the interest rate and the number of compounding periods determine how quickly the principal takes to double in value.

Test yourself 1

1 \$13 045.75 2 \$1349.18
3 **a** \$879.45 **b** \$1115.40
4 **a** \$1052.51 **b** \$736.76 **c** \$4946.80
5 **a** \$67 725 **b** \$13 557.63
6 **a** \$2400 **b** \$245.31 **c** \$45.90 **d** \$238.19
7 **a** \$5360.85 **b** \$360.85
8 \$36 282.78 9 \$12 107.19
10 \$291.98 11 \$13 145.47
12 **a** \$487.50 **b** \$4387.50 **c** \$1908.56 **d** \$6296.06 **e** \$174.89 **f** \$6783.56
13 **a** \$14 756 **b** \$10 234 **c** 59.0%

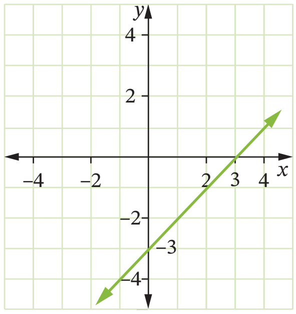
Chapter 2

SkillCheck

1 **a** (6, 1) **b** (−5, −4)
c 6 **d** 6
e $AC = BC = 4.5$ **f** isosceles
g $\frac{1}{3}$ **h** $-\frac{2}{3}$

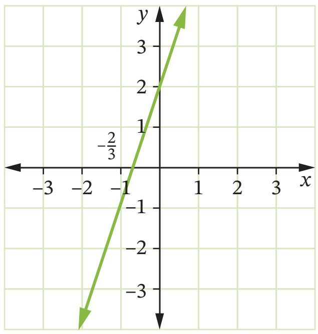
2 **a**

x	0	1	2	3
y	−3	−2	−1	0



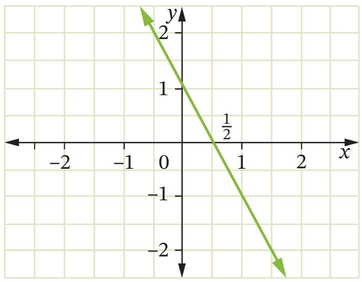
b

x	−2	−1	0	1
y	−4	−1	2	5



c

x	−1	0	1	2
y	3	1	−1	−3



3 **a** negative **b** neither **c** positive
d positive **e** negative **f** neither

Exercise 2.01

1 B **2** C **3** A

4 **a** $\frac{1}{3}$ **b** -2 **c** $\frac{7}{3}$

5 **a** **i** 2.2 **ii** (6, 2.5) **iii** $-\frac{1}{2}$

b **i** 10.8 **ii** (3.5, 3) **iii** $\frac{2}{3}$

c **i** 7.1 **ii** (−2.5, −0.5) **iii** −1

d **i** 7.6 **ii** (0.5, −7.5) **iii** $-\frac{3}{7}$

e **i** 10.2 **ii** (−6, −3) **iii** −5

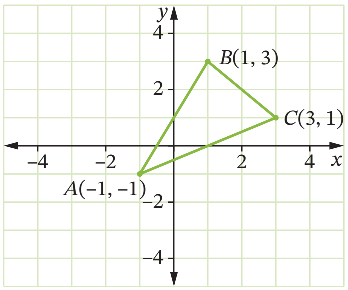
f **i** 5.7 **ii** (5, 0) **iii** 1

6 **a** $\sqrt{89}$ **b** $\sqrt{194}$ **c** $\sqrt{82}$

7 k: $m = \frac{1}{5}$; l: $m = -\frac{1}{2}$

8 B

9 **a**



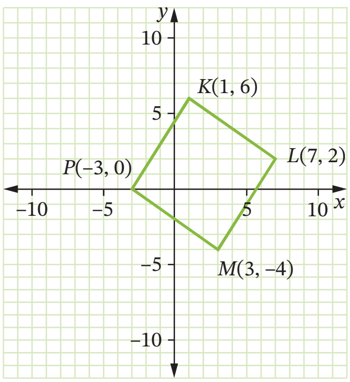
b $AB = AC = \sqrt{20}$ or $2\sqrt{5}$, $BC = \sqrt{8}$ or $2\sqrt{2}$

c $AB = AC = \sqrt{20}$ or $2\sqrt{5}$

d isosceles

e 11.8

10 **a**



b square

c $m_{KL} = -\frac{2}{3}$, $m_{PM} = -\frac{2}{3}$

d $m_{KP} = \frac{3}{2}$, $m_{LM} = \frac{3}{2}$

e the gradients are equal, they are parallel

f $KL = LM = PM = KP = \sqrt{52}$ or $2\sqrt{13}$

g 28.8 **h** 52 sq. units

11 **a** $P(-2, 1)$, $Q(1, 3)$

b $PQ = 3.6$, $AC = 7.2$, $AC = 2 \times PQ$

c $m_{PQ} = \frac{2}{3}$, $m_{AC} = \frac{2}{3}$; the gradients are equal.

Exercise 2.02

1 **a** neither **b** perpendicular **c** parallel
d neither **e** parallel **f** neither

2 **a** 4 **b** −2 **c** $\frac{1}{3}$ **d** −0.2

3 **a** −1 **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $-\frac{2}{5}$

4 D **5** B **6** A

7 **a** $m_{AB} = -\frac{4}{3}$, $m_{CD} = -\frac{4}{3}$; $\therefore AB \parallel CD$

b $m_{PQ} = \frac{3}{4}$, $m_{CD} = -\frac{4}{3}$; $\therefore PQ \perp CD$

8 **a** $\frac{1}{3}$ **b** −3

Exercise 2.03

Teacher to check graphs.

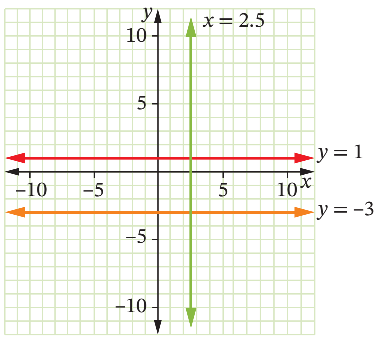
1 **a** **i** $\frac{1}{3}$ **ii** −1
b **i** −2.5 **ii** 5
c **i** 4 **ii** 4
d **i** −1 **ii** −2
e **i** 0 **ii** 0
f **i** −6 **ii** 3

2 **a** no **b** yes **c** yes
d yes **e** no **f** no

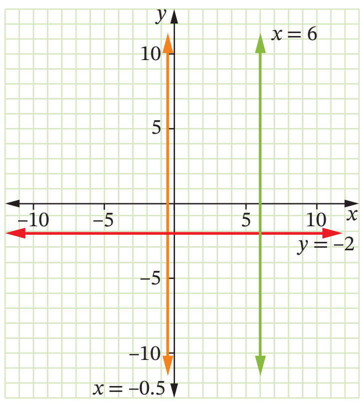
3 C

4 **a** $x = -4$ **b** $x = 1$ **c** $y = 5$ **d** $y = -3$

5 **a**



b



6 **a** $y = 2$ **b** $x = 4$ **c** $x = -1$ **d** $y = -2$
e $y = 3$ **f** $x = -1$ **g** $y = 6$ **h** $x = -1$

7 A

8 C

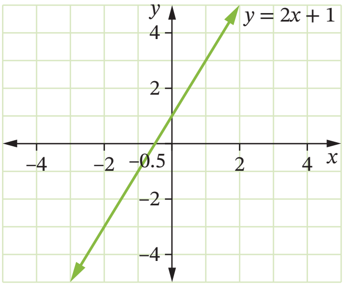
9 **a** x-axis **b** y-axis

Exercise 2.04

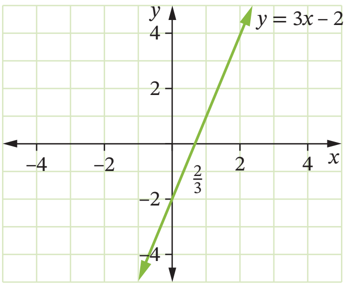
1 **a** $m = 3, c = -2$ **b** $m = -2, c = 7$
c $m = 1, c = 4$ **d** $m = -1, c = 9$
e $m = \frac{3}{4}, c = 6$ **f** $m = 1, c = 0$
g $m = \frac{1}{2}, c = -11$ **h** $m = \frac{2}{3}, c = 6$
i $m = -\frac{1}{3}, c = -8$ **j** $m = 2, c = -6$
k $m = -3, c = 11$ **l** $m = 1, c = -\frac{7}{2}$

2 **a** $y = 2x + 1$ **b** $y = \frac{3}{4}x + 2$
c $y = -7x + 5$ **d** $y = -\frac{2}{5}x + 3$
e $y = -2x - 3$ **f** $y = -3x + \frac{1}{2}$

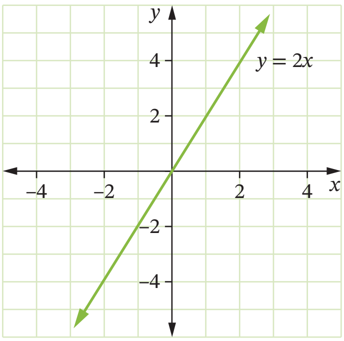
3 **a** $m = 2, c = 1$



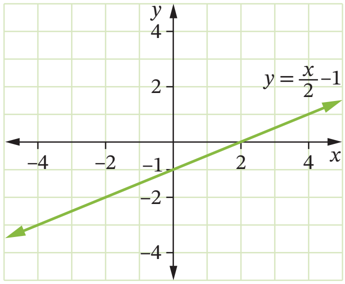
b $m = 3, c = -2$



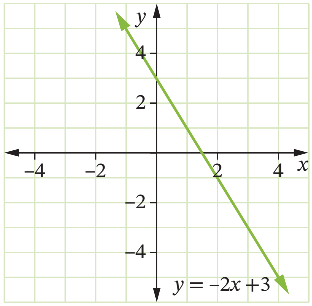
c $m = 2, c = 0$



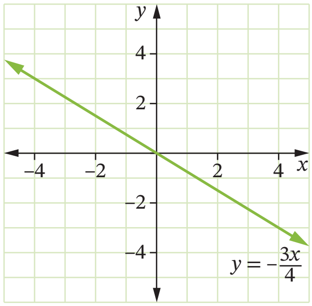
d $m = \frac{1}{2}, c = -1$



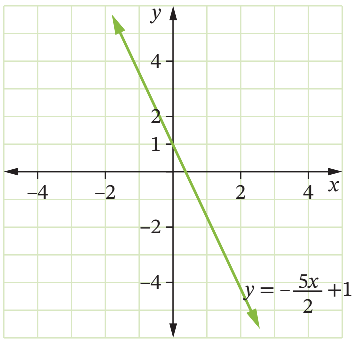
e $m = -2, c = 3$



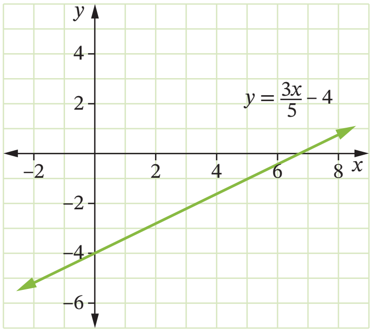
f $m = -\frac{3}{4}, c = 0$



g $m = -\frac{5}{2}, c = 1$



h $m = \frac{3}{5}, c = -4$



4 $y = 2x$

5 **a** C **b** B **c** D **d** A

6 **a** C **b** B, D **c** B
d C, D **e** A, B **f** D

7 **a** $y = 4x + 3, y = 4x - 6$
b $3x - y + 7 = 0, y = 3x - 2$

Mental skills 2

2 **a** 18 **b** \$126 **c** 39 **d** \$30.30
e \$7.50 **f** 10.8 **g** \$27 **h** 60
i \$240 **j** \$3.30 **k** 900 **l** \$52.50

4 **a** 10 **b** 166 **c** \$50 **d** \$22
e 37.5 **f** \$5.80 **g** 135 **h** \$22.60

6 **a** 500 **b** \$20 **c** 4.5 **d** \$6.25
e 81 **f** \$35 **g** 16.5 **h** 74.5
i \$195 **j** \$425 **k** \$31.50 **l** 290

8 **a** 160 **b** \$1.50 **c** 7.5 **d** \$32.50
e \$67.50 **f** \$31.25 **g** 38 **h** 170

Exercise 2.05

1 **a** $x - y + 2 = 0$ **b** $3x - y - 1 = 0$
c $5x - y + 8 = 0$ **d** $x + 2y - 3 = 0$
e $x - 2y - 6 = 0$ **f** $8x - y + 2 = 0$
g $6x - y - 3 = 0$ **h** $x - 2y - 6 = 0$
i $3x - 5y + 10 = 0$

2 **a** $m = -2, c = 6$ **b** $m = 4, c = -5$
c $m = \frac{3}{2}, c = 2$ **d** $m = -2, c = 1$
e $m = -2, c = -5$ **f** $m = -\frac{4}{3}, c = 4$

3 B **4** B

Exercise 2.06

1 **a** $y = 2x + 5$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -3x + 6$
d $y = -x + 3$ **e** $y = \frac{1}{2}x + 3$ **f** $y = -3x - 3$

2 **a** $y = \frac{1}{2}x + 2$ **b** $y = x$ **c** $y = -\frac{1}{2}x + 5$
d $y = -\frac{1}{2}x + 3$ **e** $y = -3x - 3$ **f** $y = -x - 2$
g $y = 3x - 10$ **h** $y = \frac{2}{5}x + 2$ **i** $y = 2x - 3$

Exercise 2.07

1 **a** $y = 2x + 4$ **b** $y = 3x + 6$ **c** $y = -\frac{1}{2}x + \frac{11}{2}$
d $y = 2x - 12$ **e** $y = -5x - 13$ **f** $y = \frac{1}{2}x - 10$

2 **a** $y = -2x - 2$ **b** $y = \frac{1}{5}x - \frac{1}{5}$ **c** $y = -\frac{1}{3}x + \frac{4}{3}$
d $y = -3x - 3$ **e** $y = x + 6$ **f** $y = -\frac{1}{3}x - \frac{31}{3}$

3 **a** $m = 2$ **b** $M(0, 2)$
c $-\frac{1}{2}$ **d** $y = -\frac{1}{2}x + 2$

4 **a** $y = \frac{1}{3}x + 1$ **b** -3 **c** $y = -3x + 11$

5 **a** $y = -\frac{4}{5}x + 8$ **b** $A(10, 0)$ **c** $\frac{5}{4}$
d $y = \frac{5}{4}x - \frac{25}{2}$ **e** $(0, -12.5)$

Power plus

1 **a** $\frac{2}{3}$ **b** $y = \frac{2}{3}x - 2$ **c** $y = 4$

2 $k = 5$ **3** $B(2, -1)$ **4** $X(-2, 3)$

5 Teacher to check, see worked solutions.

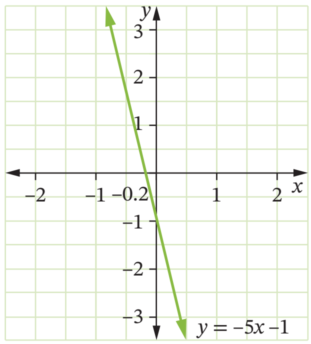
Test yourself 2

1 **a** 12.6 **b** $M(-1, 4)$ **c** $\frac{1}{3}$

2 **a** $HJ = JK = KL = HL = \sqrt{58}$
b $m_{HJ} = \frac{3}{7}, m_{JK} = \frac{7}{3}, m_{KL} = \frac{3}{7}, m_{HL} = \frac{7}{3}$
c $HK = \sqrt{200}$ or $10\sqrt{2}$, $JL = \sqrt{32}$ or $4\sqrt{2}$
d rhombus

3 **a** $-\frac{1}{2}$ **b** 2

4



5 C **6** D

7 **a** $m = 2, c = -10$ **b** $m = 4, c = 3$
c $m = -\frac{3}{8}, c = \frac{1}{2}$

8 **a** C **b** B **c** A **d** D

9 **a** $3x - y + 5 = 0$ **b** $2x - 5y - 50 = 0$
c $x - 3y - 6 = 0$

10 **a** $m = 1, c = 2$ **b** $m = \frac{1}{4}, c = 1$
c $m = -3, c = 9$

11 a $y = 2x + 3$ b $y = -\frac{1}{2}x - 4$

The product of their gradients, 2 and $-\frac{1}{2}$, is equal to −1, so they are perpendicular.

12 a $y = 3x - 6$ b $y = -2x$

Chapter 3

SkillCheck

1 a 198 cm² b 187.5 cm² c 86.1 cm² d 108.8 cm² e 273 cm² f 240.25 cm² g 696 cm² h 252 cm² i 336.96 cm²

2 a 28.7 cm b 36.9 cm c 18.3 cm

3 a i 40.2 cm ii 128.7 cm²
b i 69.1 cm ii 380.1 cm²
c i 219.9 mm ii 3848.5 mm²

4 a 105 m³ b 27 m³ c 308 m³ d 480 m³ e 640 m³ f 560 m³

Exercise 3.01

1 a 897.5 m² b 127.5 cm² c 283 cm² d 486 cm² e 70.5 cm² f 364 cm² g 2093 m² h 287 cm² i 174.08 cm² j 252 cm² k 900 m² l 356.5 m²

2 a 1399.8 m² b 2.5 m² c 145.1 m² d 1.1 m² e 88.8 m² f 814.2 m² g 50.3 m² h 131.7 m² i 78.5 m² j 601.9 m² k 159.0 m² l 54.9 m²

3 a 35.3 cm² b 40.8 m² c 1.7 m²

4 a 2.25 m b 2226

5 D

6 a 1.50 m² b 72.8%

7 a 0.16 m² b 750 c $9514.80

8 a 7694 m² b 154 × $29.50 = $4543

9 a 414 m² b 240 m² c 58% d $2044.50

Exercise 3.02

1 a cube, 8.64 m² b triangular prism, 75.6 m² c trapezoidal prism, 295 m² d rectangular prism, 27.76 m²

2 a 282 m² b 298 cm² c 2720 mm² d 204 m² e 1288 mm² f 165 m²

3 A

4 a 80 m², $4400 b 171.4 m²

5 a 1036 cm² b 1020 mm² c 204 m² d 390 cm² e 672 cm² f 5672 mm²

6 B

7 a 60 336 cm² b Need 7 m², cost =$175

Exercise 3.03

1 a 31.7 m² b 12 370.0 cm² c 805.8 cm² d 41.7 m²

2 a 49 m² b 4623 cm²

3 587.2 cm²

4 D

5 a 1009 m² b 3054 cm² c 1355 cm² d 7 m² e 905 cm² f 39 m² g 17 m² h 5 m² i 1253 cm²

6 a 64.4 m² b 8 L

7 a 27.3 m² b 56.7 m² c Cost = 28 × $18.50 + 57 × $21.75 = $1757.75

Exercise 3.04

1 a 446.96 cm² b 49 270 cm² c 864 cm² d 11 064 cm² e 45 160 cm² f 40 270.7 cm²

2 464 cm²

3 a 352 cm² b 76 cm²

4 a 9721.7 cm² b 14 031.4 cm² c 14 778.1 cm² d 2858.8 cm² e 2793.5 cm² f 394.7 cm²

5 B

6 a 854.51 cm² b 20 slices c 138 cm²

7 2953 cm²

8 a 26.14 m² b 19 m²

9 1028.32 cm²

10 a 75.4 m² b 50.3 m

Mental skills 3

2 a 8 hours 30 mins b 5 hours 40 mins c 3 hours 25 mins d 8 hours 15 mins e 11 hours 25 mins f 1 hour 40 mins g 5 hours 10 mins h 5 hours 45 mins i 7 hours 55 mins j 7 hours 40 mins

Exercise 3.05

1 a 576 m³ b 1330 cm³ c 92.4 m³

2 a 1399.6 cm³ b 46.6 m³ c 56 160 cm³ d 17 066 cm³ e 33 931.8 cm³ f 192.4 m³ g 1.2 m³ h 21 756 cm³ i 208.8 m³

3 a i 1539 m³ ii 1539 kL
b i 14 432 cm³ ii 14 432 mL
c i 226 m³ ii 226 kL
d i 5 cm³ ii 5 mL

4 a 251.3 cm³ b 320 cm³ c 21.5%

5 350.4 m³

6 a 118 800 L b 113 040 L

7 2 500 000 L

8 a 182.83 m³ b $21.94 per day

9 1415.7 cm³

10 a 4825.49 cm³ b 5026.55 cm³ c 1989.38 cm³ d 6375 cm³ e 5301.44 cm³ f 3084.96 cm³ g 536.19 cm³ h 1884.96 cm³ i 167.33 cm³ j 12 900 cm³ k 167.55 cm³ l 794.12 cm³

Power plus

1 1728 cm³

2 8 cm

3 a $2p^2 + 4pr$ b $\frac{3\pi x^2}{2}$

4 36 mm (to the nearest mm)

5 0.5 m/h or 50 cm/h

6 40 mm

Test yourself 3

1 a 133 cm^2 b 2340 mm^2 c 326.7 cm^2
d 1318.2 cm^2 e 151.7 m^2 f 1039.1 cm^2

2 a 1.08 m^2 b 3150 mm^2 c 5236 cm^2
d 277.6 m^2 e 216 cm^2 f 434 mm^2

3 a 354 371.7 cm^2 b 4863.2 cm^2 c 17.4 m^2

4 a 3180 cm^2 b 1268 cm^2 c 395 cm^2
d 14 294 cm^2 e 5871 cm^2 f 4428 cm^2

5 a 10 125 m^3 b 11 084 m^3 c 11 m^3
d 36 816 m^3 e 20 160 m^3 f 10 016 m^3

6 127.875 L

7 a 59.11 kL b 1.44 m

Practice set 1

1 \$2199.12

2 a 13.9 b $\left(-1\frac{1}{2}, 3\right)$ c $-\frac{12}{7}$

3 a 616 m^2 b 280.5 m^2 c 935 m^2
d 452.4 m^2 e 1268.3 cm^2 f 457.1 cm^2

4 a \$1505.94 b \$1054.16 c \$7077.92

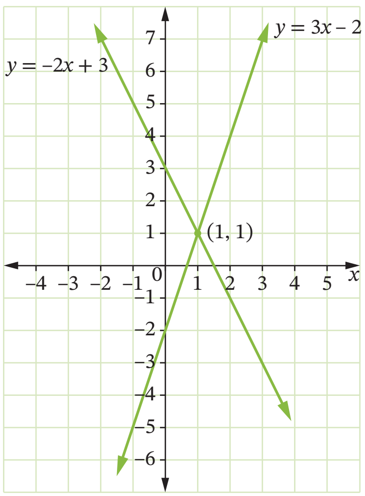
5 a 117.5 m^2 b 480 m^2 c 324 m^2

6 a \$575 b \$1164.38 c \$1495

7 a $-\frac{4}{3}$ b $\frac{3}{4}$

8 a \$71 904 b \$14 915.80

9



10 a \$3000 b \$5.63 c \$306

11 a 9349.4 cm^2 b 1979.2 cm^2 c 279.4 cm^2

12 a \$12 979.20 b \$979.20

13 B

14 B

15 a \$768 b \$6912 c \$4008.96
d \$10 920.96 e \$227.52 f \$11 688.96

16 a 1352 m^2 b 22 m^2 c 2822 m^2

17 a \$52 042.84 b \$52 142.72 c \$52 165.50

18 44 791.90

19 a $m = -3, c = 8$ b $m = 1, c = 7$
c $m = -\frac{2}{3}, c = 1$

20 a $2x - y - 3 = 0$ b $2x - y - 5 = 0$
c $3x - 4y + 24 = 0$

21 a $y = 5x + 3$ b $y = -4x + 8$ c $y = \frac{1}{3}x + \frac{4}{3}$

22 line A: $y = -\frac{3x}{2} - 3$; line B: $y = \frac{2x}{3}$

23 a $y = 4x + 32$ b $y = 5x$

24 a \$26 237.44 b \$13 752.56 c 65.6%

25 C

26 a 6600 cm^3 b 3308 cm^3 c 10 175 cm^3

27 a $y = -3x + 7$ b $y = \frac{2}{3}x - 5$ c $y = 4$

28 a \$34.70 b \$5413.20

29 a 65 m^3 b \$137.15

30 A

Chapter 4

SkillCheck

1 a g^9 b r^6 c d^{15} d k^2
e h^{10} f m^4 g a h 1
i $6e^6$ j $3n^4$ k $1000w^9$ l 25
m v^5w^5 n $\frac{c^3}{p^3}$ o $\frac{1}{y}$ p $\frac{1}{k^2}$
q $\frac{7d}{4a}$ r $\frac{3}{2}y^5$ s $24g^2$ t $\frac{4b^2}{9h^2}$

2 a $\frac{19a}{20}$ b $\frac{p}{6}$ c $\frac{5}{3}$ d $\frac{x}{7y}$

3 a $18m^2 + 66m$ b $-15g + 40$ c $24wy - 12w^2$

4 a $4(x + 6)$ b $5(4 - 3a)$ c $q(q + 1)$
d $6a(3a - 2)$ e $-2(y + 15)$ f $-6(3w - 4)$

5 a 3 and 6 b −2 and −4
c 4 and −5 d 8 and −2

Exercise 4.01

1 a $15k^{11}$ b $10y^3$ c $9p^{11}$
d w^{15} e $32n^{15}$ f $3h^4$
g $12a^7d^5$ h $9q^6$ i $4w^2y^4$
j $-128c^{14}$ k $\frac{4a^2cd^3}{3}$ l $125y^{18}$
m $24h^4k^5w^5$ n $32d^{15}g^{10}$ o $\frac{2m^2p^4}{3}$

2 a $l^{18}m^{30}$ b $\frac{n^3}{8}$ c 7 d $\frac{w^{10}}{k^{15}}$
e 1 f $64k^2y^{10}$ g −15 h $\frac{16b^4}{81d^4}$
i 1 j $625d^4y^8$ k $-\frac{27k^{12}}{1000}$ l −9
m $16p^8q^{12}r^{16}$ n 64 o $9g^{12}k^2$ p 4

3 a 1 b 1 c 7 d 1
e −8 f 9 g 625 h 2
i 128 j 1 k 0 l 64
m $\frac{1}{216}$ n 1 o −7 p 25
q 2 r $\frac{1}{81}$ s 1 t 1

4 a $\frac{1}{25}$ b $\frac{1}{32}$ c $\frac{1}{20}$ d $\frac{1}{1000}$ e $\frac{1}{81}$

5 D

6 C

7 a $\frac{1}{8^7}$ b $\frac{1}{3^5}$ c $\frac{1}{y}$ d $\frac{1}{x^3}$
e $\frac{1}{25b^2}$ f $\frac{8}{h^3}$ g $\frac{1}{ab}$ h $-\frac{p}{q}$

9780170453417

i $\frac{11}{w^3}$ **j** $\frac{1}{216x^3}$ **k** $\frac{a^3}{b^5}$ **l** $\frac{m}{w^3}$

m $\frac{8}{u^3v^4}$ **n** $\frac{-2r^6}{y^5}$ **o** $\frac{10f^3}{e}$ **p** $\frac{g^4}{2h^3}$

q $\frac{3d^7}{4n^2}$ **r** $\frac{1}{16c^2}$ **s** $\frac{5x^2}{yw^2}$ **t** $\frac{2}{mp}$

8 **a** $\frac{10}{r^6}$ **b** $\frac{2}{5}$ **c** 3 **d** x

e $\frac{3}{k}$ **f** $\frac{5}{a^2}$ **g** $\frac{1}{36w^2}$ **h** $\frac{-2}{m}$

i $\frac{3g^3}{5}$ **j** $\frac{3h}{2r}$ **k** $\frac{m^3n^2}{p^2}$ **l** $\frac{4a^2}{5b}$

9 **a** $20x^6y^9$ **b** $81m^{24}n^4$ **c** $\frac{4w^4}{9k^2}$

d $\frac{5g^4y^{12}}{2}$ **e** $\frac{27a^3}{8x^6}$ **f** $48a^{11}d^{15}$

g $\frac{1}{12q^7r}$ **h** $64h^{11}k^{16}$ **i** $9b^4$

10 **a** $\frac{16}{9}$ **b** $3\frac{3}{8}$ **c** 1 000 000 **d** $\frac{8}{125}$

e $-\frac{243}{1024}$ **f** $\frac{256}{625}$ **g** $\frac{16}{81}$ **h** $\frac{125}{343}$

i $\frac{9}{k^2}$ **j** $-\frac{x^3}{27}$ **k** $\frac{625}{a^8}$ **l** $\frac{9g^6}{16}$

Exercise 4.02

1 **a** $\frac{5k}{7}$ **b** $\frac{3x}{11}$ **c** $\frac{2m}{5}$ **d** $\frac{13}{5g}$

e $\frac{7w}{20}$ **f** $\frac{16a}{21}$ **g** $\frac{83c}{60}$ **h** $\frac{7u}{3}$

i $\frac{1}{2k}$ **j** $\frac{2}{x}$ **k** $\frac{2}{h}$ **l** $\frac{17}{2m}$

m $\frac{2}{3a}$ **n** $\frac{29}{6h}$ **o** $\frac{87}{40k}$ **p** $\frac{13}{21p}$

2 **a** $\frac{5m+3n}{15}$ **b** $\frac{6k-7w}{42}$ **c** $\frac{9x-4p}{36}$ **d** $\frac{15+8h}{24}$

e $\frac{7n-40}{70}$ **f** $\frac{25y-4c}{10}$ **g** $\frac{9q+22d}{33}$ **h** $\frac{9a+20e}{24}$

i $\frac{19b}{45}$ **j** $\frac{15y}{28}$ **k** $\frac{49c}{33}$ **l** $\frac{26g}{15}$

m $\frac{35h}{24}$ **n** $\frac{78x}{35}$ **o** $\frac{36-40k}{15}$ **p** $\frac{9m}{20}$

Exercise 4.03

1 **a** $\frac{2m}{35}$ **b** $\frac{dh}{12}$ **c** $\frac{24}{yq}$ **d** $\frac{5x}{11y}$

e $\frac{2v}{k}$ **f** $\frac{8}{9h}$ **g** $\frac{8}{3v}$ **h** $\frac{2}{3}$

i $\frac{7}{15}$ **j** $\frac{2h}{5}$ **k** $\frac{12a^2}{k^2}$ **l** $\frac{2a}{9}$

2 B **3** D

4 **a** $\frac{9h}{20}$ **b** $\frac{7x}{12p}$ **c** $\frac{11k}{6a}$ **d** $\frac{w}{4}$

e $\frac{6}{5}$ **f** $\frac{1}{6a}$ **g** $\frac{5e^2}{6}$ **h** $\frac{20}{9x^2}$

i $\frac{2y}{3}$ **j** $\frac{4}{9h^2}$ **k** $\frac{4dn}{5}$ **l** $\frac{3g}{20y}$

5 **a** $\frac{4w}{3}$ **b** 1 **c** $\frac{3q}{7}$ **d** $\frac{2m}{3g}$

e $\frac{15w}{14}$ **f** $\frac{25g}{3}$ **g** $\frac{8m}{45d}$ **h** $\frac{ad}{5}$

i $\frac{1}{9}$ **j** $20x$ **k** $\frac{1}{q}$ **l** $3a$

6 C

Exercise 4.04

1 **a** $5d + 55$ **b** $-3r - 30$
c $7x - 63y$ **d** $-4a + 20w$
e $-2 + p^2$ **f** $-20e^3 - 30e$
g $6y + 42y^2$ **h** $12x^2y^2 - 4xy$
i $16rq^2 - 8r^2q$ **j** $12ab^2 - 21a^2b$
k $-6h^2 + 18h^3$ **l** $-25x^3 - 20xy$
m $-3 - 8a$ **n** $-6m^3 + 8m^2n$
o $15g + 35g^3$ **p** $-5e + 12$

2 C

3 **a** Yes **b** No **c** Yes

4 **a** $3k^2 - 20k$ **b** $-23h + 7h^2$
c $3w^3 - 15w$ **d** $49x^3 - 10x^4$
e $2 + 21d$ **f** $12n - 26n^2$
g $4y^2 - 6y + 5$ **h** $5 - 13a - 4a^2$
i $16 + 50w$ **j** $20y^3 - 8y^2 + 8$
k $-2v^2 - v - 6$ **l** $8 - 9a + a^2$
m $10c^2 - 30c + 20$ **n** $2m^2 + 18m^3$
o $20x + 26xy - 60y$

5 **a** $5(3y - 4)$ **b** $7(3 + 5w)$
c $p(2 + p)$ **d** $10y(3 - 2y)$
e $12d(3d + 2)$ **f** $7k(4k - 3)$
g $(c - 5)(8 - c)$ **h** $(3 + 2m)(m + 7)$
i $-q(q + 36)$ **j** $-4x(2 - 3x)$
k $(3b + 5)(b - 2)$ **l** $-4cd(3d - 2)$
m $-hn(n - h)$ **n** $-3g(5g + 6)$
o $6q(8q - 9)$

6 B

7 **a** $4my(2my - 3)$ **b** $9bc(4ab + 3)$
c $12mn(2m - 9n)$ **d** $5g(4dg - 7a)$
e $8wy^2(5y + 3w)$ **f** $25gh(3g^2h - 5)$
g $p(1 - 8p - 4p^2)$ or $-p(4p^2 + 8p - 1)$
h $3mn(2n + 1 + 16m)$
i $8pg(4p^2 + g - 1)$ **j** $3a^2(6a^3 - 4 + 5a^2)$
k $7mh^2(4m^2 - 3)$ **l** $3w(5kp - 8p^2 - 3k)$

Mental skills 4

Exact answers shown

2 **a** 331 **b** 157 **c** 1587 **d** 255
e 421 **f** 203 **g** 413 **h** 734
i 6723 **j** 15 744 **k** 276 **l** $72\frac{3}{7}$

4 **a** 28.231 **b** 14.187 **c** 177.4967
d 416.752 **e** 2.4156 **f** 5.0237
g 3.6890 **h** 5.8065 **i** 23.9121

Exercise 4.05

1 **a** $m^2 + 7m + 12$ **b** $w^2 + 10w + 25$
c $y^2 - 144$ **d** $h^2 - 2h - 63$
e $a^2 - 2a - 15$ **f** $x^2 - 15x + 44$

g $p^2 + 11p + 24$ h $c^2 - 19c + 84$
i $g^2 - 3g + 2$ j $u^2 - u - 56$
k $m^2 - 6m - 40$ l $q^2 - 5q - 66$
m $d^2 + 3d - 40$ n $-e^2 + 17e - 70$
o $h^2 - 14h + 45$

2 D 3 B

4 a $3y^2 + 22y + 7$ b $12k^2 + 35k + 18$
c $3m^2 + 4m - 15$ d $10p^2 - 19p - 15$
e $2w^2 - 19w + 24$ f $14x^2 + 29x + 12$
g $9b^2 - 24b + 16$ h $25a^2 + 60a + 36$
i $10q^2 - 21q - 49$ j $5p^2 - 9p - 18$
k $12d^2 - 29d - 11$ l $-8r^2 + 38r - 45$
m $-14y^2 + 15y + 9$ n $64h^2 - 9$
o $-49w^2 + 126w - 81$ p $16d^2 - 1$
q $3f^2 - 2f - 1$ r $-36u^2 + 60u - 25$

5 B

6 a $(3x - 10)(2x + 15)$ b $6x^2 + 25x - 150$
c $-6x^2 - 25x + 750$

7 a $3k(k + 5) = 3k^2 + 15k$
b length $= 3k - 8$, width $= k + 1$
c $(3k - 8)(k + 1)$
d $3k^2 - 5k - 8$
e $20k + 8$

8 a length $= a + 3$, width $= b + 1$
b Area $= (a + 3)(b + 1)$
c Area $= ab + a + 3b + 3$
d Increase in area $= a + 3b + 3$

9 Proof: see worked solutions

Exercise 4.06

1 a $-3, 5$ b $-10, -4$ c $6, 7$ d $-3, 2$
e $5, 9$ f $6, -4$ g $6, 9$ h $-4, 4$
i $-2, 5$ j $-7, -5$

2 a $(y + 6)(y + 2)$ b $(m + 8)(m + 7)$
c $(g + 2)(g + 7)$ d $(w + 9)(w + 4)$
e $(p + 8)(p + 3)$ f $(a + 6)(a + 7)$
g $(e + 3)(e + 9)$ h $(n + 3)^2$
i $(c + 3)(c + 7)$

3 a $(x - 5)(x - 4)$ b $(h - 3)(h - 10)$
c $(p - 3)(p - 8)$ d $(e - 6)(e - 5)$
e $(w - 9)(w - 8)$ f $(k - 9)(k - 1)$
g $(m - 8)^2$ h $(u - 3)(u - 2)$
i $(d - 5)(d - 7)$

4 a $(q - 10)(q + 2)$ b $(h - 9)(h + 4)$
c $(y + 11)(y - 4)$ d $(x - 9)(x + 7)$
e $(u + 10)(u - 1)$ f $(e + 10)(e - 3)$
g $(a - 11)(a + 10)$ h $(y + 9)(y - 3)$
i $(m - 7)(m + 1)$ j $(c + 9)(c - 2)$
k $(k + 9)(k - 6)$ l $(r - 11)(r + 2)$
m $(p - 8)(p + 4)$ n $(u + 15)(u - 3)$
o $(b - 8)(b + 2)$

5 D

6 a $(h - 1)^2$ b $(x + 5)(x + 10)$
c $(r + 8)(r + 12)$ d $(a + 4)(a - 7)$
e $(u + 5)(u - 12)$ f $(y - 9)^2$
g $(v - 8)(v + 7)$ h $(w - 15)(w + 4)$
i $(g + 6)(g - 3)$ j $(p + 6)(p + 8)$
k $(e + 8)(e - 1)$ l $(x - 7)(x - 12)$

Power plus

1 a 13 b 1 c 7 d 1 e $-\frac{125}{8}$ f 0

2 a $\frac{wy + wx - xy}{wxy}$ b $\frac{w}{xy}$ c $\frac{w + y}{wxy}$

3 a $a^2 - b^2 - ac + bc$
b $x^2 + y^2 - 2xy + 2x - 2y + 1$
c $3t + 4 - \frac{1}{t}$

4 a $(x - 8)(x - 125)$ b $(y + 50)(y - 36)$
c $(b + 41)^2$ d $(n - 50)(n + 50)$

Test yourself 4

1 a $6v^5 w^7$ b $8t^7 h^6$ c $25x^2 y^4$
d 1 e $\frac{1}{4k}$ f $\frac{125y^3}{8}$
g 0 h $1\frac{1}{3}$ i $\frac{8a^2 g}{9}$

2 a $\frac{1}{16m^2}$ b $\frac{4}{m^2}$ c $625b^{24} y^{12}$
d $512t^{14} u^{16}$ e $5c^4 d^4$ f $\frac{6a}{5b}$

3 a $\frac{-3t}{20}$ b $\frac{19g}{6}$ c $\frac{x}{16}$ d $\frac{3b + 2}{7}$
e $\frac{5w - 12}{8}$ f $\frac{35 - 12y}{60}$ g $\frac{20m + 21n}{28}$ h $\frac{17p}{24}$

4 a $\frac{3p}{4m}$ b $\frac{3}{q^2}$ c $\frac{5y}{7}$ d $\frac{3}{2}$
e $\frac{35}{8a^2}$ f $8d^2$ g $\frac{6y}{5}$ h $4ac$

5 a $9m - 72$ b $10ab + b^2$
c $-12x^2 y + 15y^2$ d $56tp^2 - 40t^2 p$
e $-3n + 10$ f $-15h^3 - 35h^2$
g $20y^2 - 28hy$ h $-3wx^2 + 7w^2x$

6 a $-13g + 21g^2$ b $4fg^2 - 30f^2 g$ c $93 - 22n$
d $8x^3 + 7x^4$ e $10y^2 - 41y + 21$

7 a $8(t - 9)$ b $b(b + 36)$ c $-3(m + 11)$
d $4wr(9r + 7w)$ e $-6(4p - 3q)$
f $(5x - 1)(2 - 3x)$

8 a $15xy^2(1 - 2x^2 y)$ b $6p(t^2 + 2pt - 8p^2)$
c $4r^2 s^3(8s + 3r^2)$ d $25x^3 y^3(2x - 3y)$
e $-8p^3 q^3(1 - 6q^3)$ f $(n^2 + 6)(n - 1)$

9 a $b^2 + 13b + 30$ b $d^2 + d - 56$
c $15t - 54 - t^2$ d $20x^2 + 13x - 21$
e $49y^2 - 25$ f $21p^2 - 62p + 16$
g $9m^2 + 42m + 49$ h $8 + 10x - 3x^2$
i $10d^2 - 27d + 18$

10 a $(y + 5)^2$ b $(x - 20)(x - 1)$
c $(n + 11)(n - 3)$ d $(a - 7)(a - 4)$
e $(m - 12)(m + 7)$ f $(p + 9)(p - 6)$

Chapter 5

SkillCheck

1	**a i**	10	**ii** 16.5	**iii** 15	**iv** 15
	b i	13	**ii** 1.8	**iii** 2.5	**iv** 3
	c i	7	**ii** 11.5	**iii** 11	**iv** 11
	d i	6	**ii** 43.6	**iii** 43	**iv** 43
	e i	48	**ii** 34.3	**iii** 34.5	**iv** 24, 35
	f i	5	**ii** 2.2	**iii** 2	**iv** 2
2	**a i**	31	**ii** 33.3	**iii** 62	

b 78

c i Median = 30, mean = 28.3, range = 25.

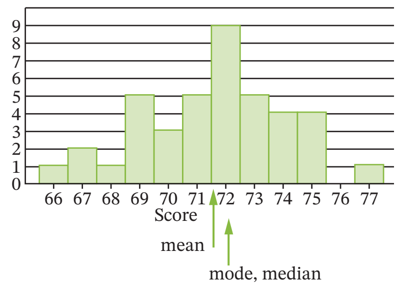
ii The outlier has increased the median (by 1), the mean (by 5), and the range (by 37).

Exercise 5.01

1 a i symmetrical
ii clustering at 9, no outliers
b i bimodal
ii clustering in the 30s and 60s, no outliers
c i positively skewed
ii clustering at 1, 8 is an outlier
d i negatively skewed
ii clustering at 23–24, no outliers
e i positively skewed
ii clustering at 130–150, no outliers
f i symmetrical
ii clustering at 4 and 5, no outliers
g i positively skewed
ii clustering at 13, 23 is an outlier
h i bimodal
ii clustering at 50s and 100s, 136 is an outlier

2 a

Score	Frequency
66	1
67	2
68	1
69	5
70	3
71	5
72	9
73	5
74	4
75	4
76	0
77	1



b no outliers
c negatively skewed
d In golf scoring, the lower the score, the better. In the final round of a tournament you would expect good players, so more low scores.
e clustering at 72
f mode = 72, $\bar{x}$ = 71.6, median = 72

3 a 46 **b** 1–9 hours (stem of 0)
c no outliers **d** positively skewed
e Most students spend limited time on their computers, and have other commitments and do other activities such as sport. Only a few students spend many hours on the computer during the week.
f Mode = 1, $\bar{x}$ = 13.9, median = 11

4 a

Stem	Leaf
12	3
13	
14	2 3
15	0 1 3 3 3 5 5
16	0 0 1 2 2 2 2 3 4 5 5 7 8 9
17	0 0 1 2 3
18	2

b Symmetrical
c 123 is an outlier
d Clustering occurs in the 160s
e mode = 162, median = 162, $\bar{x}$ = 160.2

5 a Bimodal **b** No **c** 29.3
d 28.4 **e** 28.45 **f** 8.7
g Yes, because there aren't any outliers.

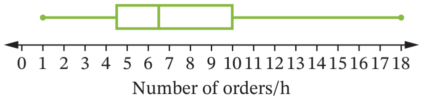
6 B

Exercise 5.02

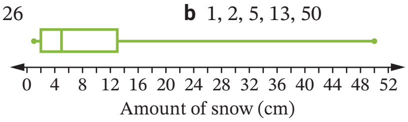
1 a 5, 6.5, 8 **b** 18, 20, 26.5 **c** 32, 34.5, 38
2 a range = 7, IQR = 3 **b** range = 22, IQR = 8.5
c range = 16, IQR = 6
3 a 7.5 **b** 3
4 a 283 **b** 128
5 a 3 **b** 2.5 **c** 17.5
d 19 **e** 21.5 **f** 1.5
6 a 34 **b** 13
c i 68, 72, 72, 75, 77, 78, 79, 80 **ii** 50%
d 75%
7 a i 28 **ii** 9.5
b The interquartile range, as it is not affected by the value of 35.
c 48, 48, 48, 49, 51, 53, 55; 54%

Exercise 5.03

1 a 1, 4.5, 6.5, 10, 18
b

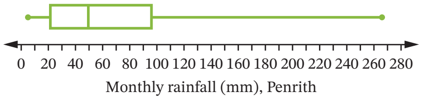


2 a 26 **b** 1, 2, 5, 13, 50
c



3 **a** 261 **b** 5, 21.5, 49.5, 96, 266

c



4 **a** 27.5 h **b** 26 h **c** 30 h **d** 4 h **e** 50%

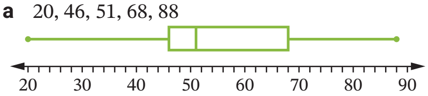
5 **a** 26 **b** 21 **c** 14

d **i** 25% **ii** 75%

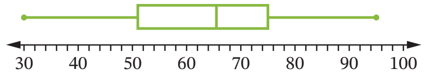
6 **a** 6, 10, 19, 23, 29 **b** 13

c **i** 14 **ii** 7 **iii** 7 **iv** 21

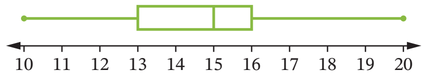
7 **a** 20, 46, 51, 68, 88



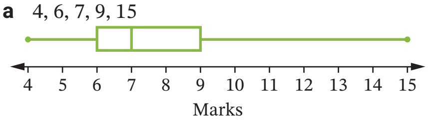
b 30, 51, 65.5, 75, 95



c 10, 13, 15, 16, 20



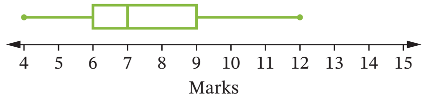
8 **a** 4, 6, 7, 9, 15



b Dot plot is positively skewed. The length of the box plot from the median to the highest value is greater than the length from the median to the lowest value.

c 15

d 4, 6, 7, 9, 12



e **i** The box plots are the same up to Q_3.

ii The whisker from Q_3 is reduced without the outlier.

Exercise 5.04

1 **a** **i** Year 10: 3.5; Year 8: 8

ii Year 10: 7.5; Year 8: 8.5

iii Year 10: 1; Year 8: 2

b **i** 25% **ii** 75%

c **i** 10 **ii** 0

2 **a** **i** 32 **ii** 34

b Swifts: 59.5, Thunderbirds: 51.5

c Swifts: 8, Thunderbirds: 11

d The range for the Swifts is slightly smaller (32) and the IQR (8) for the Swifts is less than the IQR (11) for the Thunderbirds, indicating that the Swifts are more consistent in their performance.

e The position of the Swifts box plot shows that they scored more points in games, they have a higher median of 59.5 than the Thunderbirds who had a median of 51.5, so the Swifts performed better in the season.

3 **a** 10K: 9; 10N: 10 **b** 10K: 6.5; 10N: 5.5

c 10K: 3; 10N: 4

d 10K – lower range and IQR.

e 75%

4 C

5 **a** Brisbane: 26.9, 9.3, 4.7

Sydney: 23.5, 8.5, 4.9

Melbourne: 21.4, 13, 8.6

Hobart: 18.6, 11.2, 7

b Melbourne – it has the highest range and IQR.

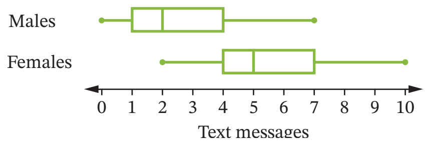
c Brisbane, more than half of the mean monthly temperatures are higher than most of the mean monthly temperatures of the other cities.

d Sydney's median temperature is significantly higher than Melbourne's, so Sydney is the warmer city.

e Sydney has the smaller range and IQR of mean monthly temperatures, so it is more consistent.

6 **a** Male: 0, 1, 2, 4, 7; Female: 2, 4, 5, 7, 10

b

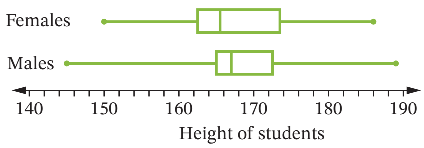


c Male: 3; Females: 3

d Males: 7; Females: 8

e Both are positively skewed, the interquartile range is the same, and the range of females is one more than that of the males. Females do receive more text messages, as the box plot shows that 75% of females receive more messages than 75% of males.

7 **a** Male: 145, 165, 167, 172.5, 189; Female: 150, 162.5, 165.5, 173.5, 186

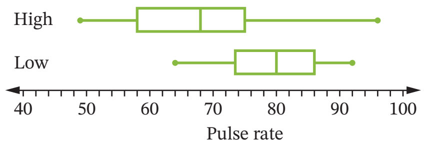


b Male: range = 44 IQR = 7.5

Female: range = 36 IQR = 11

c Male students have a greater range (44 compared to 36), but a smaller interquartile range (7.5 compared to 12).

8 **a** Low: 64, 73.5, 80, 86, 92; High: 49, 58, 68, 75, 96

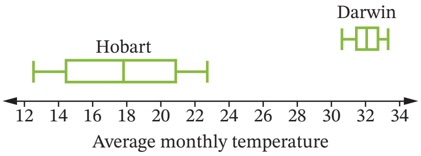


b **i** 28, 12.5 **ii** 47, 17

c The range and interquartile range of the High Frequency group are both greater than the Low Frequency group.

d The High Frequency group.

9 a Hobart: 12.5, 14.4, 17.8, 20.85, 22.7; Darwin: 30.6, 31.45, 32.05, 32.7, 33.3



b Hobart: 10.2°, 6.45° Darwin: 2.7°, 1.25°

c Darwin average monthly temperatures were more consistent than Hobart, since its range (2.7) was much smaller than Hobart's (10.2), while Hobart's IQR was much larger (6.5) than Darwin's (1.25).

10 a Simone

b Simone: 12 Amal: 10

c Amal, smaller range.

d Simone: 10; Amal: 9

e Simone: 4; Amal: 5

f Not enough information given to make a valid decision. The interquartile range and range only differ by 1.

g 25%

h 25%

Mental skills 5

2 **a** 176	**b** 363	**c** 261	**d** 405
e 682	**f** 707	**g** 1818	**h** 3564
i 152	**j** 540	**k** 2142	**l** 588
m 288	**n** 693	**o** 3939	**p** 852

Exercise 5.05

1 a Boys: \$34.58 Girls: \$31.78

b Boys: \$33.50 Girls: \$28

c Boys: Range = 72, IQR = 25

Girls: Range = 69, IQR = 30.5

d i Boys are positively skewed slightly, girls are positively skewed.

ii There are no outliers, clustering occurs for the boys in the 20–30s and for the girls in the 10–20s.

e Boys generally carry more cash – they have a higher mean than the girls and the shape of the data for girls is more positively skewed.

2 a 21 games

b i 34 ii 51

c Scorpions: $\bar{x}$ = 1.6 goals; Vale United: $\bar{x}$ = 2.4

d Scorpions 5, Vale United 6

e The shape of both teams' results is positively skewed. Clustering for Scorpions occurs at 1 and 2 and for Vale United it occurs at 2.

f Vale United performed better as its mean was 2.4 goals/game compared to Scorpions 1.6 goals/game.

3 a Sydney: $\bar{x}$ = 26.2, median = 26.5, mode = 28

Perth: $\bar{x}$ = 34.3, median = 35, mode = 38

b Sydney: Range = 9°, IQR = 3

Perth: Range = 16°, IQR = 8

c The temperatures for Sydney and Perth are both negatively skewed, there are no outliers. Sydney's temperatures are clustered from 26 to 28, while Perth's have no distinct cluster.

d Sydney's temperatures are lower than Perth's, as evidenced by the significantly lower mean, median and mode. The range and interquartile range for Perth are greater than the range and interquartile range for Sydney, indicating greater spread.

4 a 30

b Quiz 1: $\bar{x}$ = 5.6, mode = 6; Quiz 2: $\bar{x}$ = 6.3, mode = 7

c Quiz 1: 6; Quiz 2: 7

d Quiz 1: i Range = 7 ii IQR = 2

Quiz 2: i Range = 8 ii IQR = 2

e Quiz 1: Results are symmetrical with clustering 5–6, no outliers.

Quiz 2: Results are bimodal with clustering at 5 and 7–8, no outliers.

f Scores for Quiz 2 are better than Quiz 1, as the mean of Quiz 2 is higher than the mean of Quiz 1.The spread for both quizzes are similar as there is only a difference of 1 between the ranges and the IQRs are equal.

5 a 39

b i mode = 2 ii median = 2

iii range = 6 iv IQR = 1.5

c positively skewed, no outliers

d 50%

e i By the highest columns.

ii By the short length of the box when compared to the whole length of the box plot.

f i The shape of the distribution, the frequency for each household size and the mode. The mean can also be calculated from the histogram.

ii The shape of the distribution, the median and the quartiles Q_1 and Q_2.

6 a i 5 ii 16

b i mode = 22 ii range = 18

iii IQR = 24 – 16 = 8

c Negatively skewed.

i The tail of the dot plot goes to the left.

ii The length of the box plot from the lowest value to the median is longer than from the median to the highest value.

d i dot plot ii box plot

iii dot plot iv box plot

7 a Sunbeam Valley: range = 24, median = 71, IQR = 8

Bentley's Beach: range = 30, median = 73, IQR = 15

b Sunbeam Valley: negatively skewed (slight)

Bentley's Beach: positively skewed

c Sunbeam Valley's speeds are clustered in the 70s.

d 25%

ANSWERS

e Bentley's Beach – higher median, positively skewed. 25% of drivers drive faster than all drivers in Sunbeam Valley. This may be due to more main roads with higher speed limits.

8 **a** 36

b Lamissa: mode = 7, median = 7
Anneka: mode = 7, median = 6

c Lamissa: range = 8, IQR = 8 − 6 = 2
Anneka: range = 9, IQR = 7 − 4 = 3

d Lamissa's distribution of values is negatively skewed with clustering at 7. Anneke's distribution is negatively skewed with clustering at 6 and 7.

e **i** 25% **ii** 50%

f Lamissa is the better archer. Her median score is higher than Anneke's. According to the box plot, roughly 25% of Lamissa's scores are less than 6 compared to Anneke's 50%, and 50% of Lamissa's scores are equal to or better than 75% of Anneke's. Lamissa's range and IQR are also slightly smaller than Anneke's, which suggests she is the more consistent archer.

9 **a** The range (47) is too large.

b Women: 31 Men: 37

c Women: Range = 38, IQR = 40 − 24 = 16
Men: Range = 47, IQR = 46 − 25 = 21

d Distribution for women is positively skewed. Distribution for men is symmetrical.

e Men have the greater spread in the number of sit-ups completed, as the range and IQR are both greater than those for women.

10 **a** **i** 56 **ii** 38

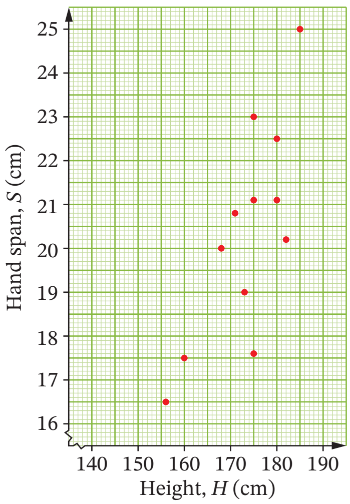
b **i** 10 Blue, 10 Yellow **ii** 10 Green
iii 10 Red

c **i** 10 Green **ii** 10 Yellow
iii 10 Red, 10 Blue

d 10 Blue. It shares the highest median with 10 Red, but its lowest score is still higher than 25% of 10 Red's scores.

Exercise 5.06

1 **a**



b linear

c As the heights of students increase, their hand spans tend to increase.

2 **a** weak negative relationship

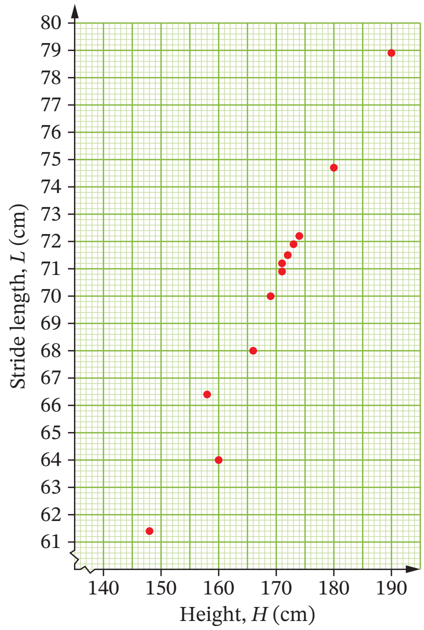
b no relationship

c strong positive relationship

3 Weak positive.

4 **a** Stride length depends on a person's height; the taller the person is, the longer their legs are.

b



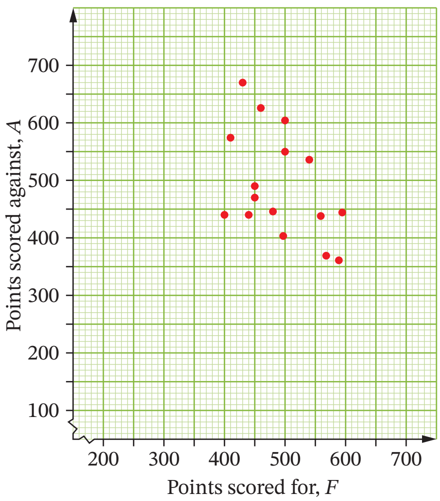
c linear

d Students' stride length increases with height.

e strong positive relationship

f About 72.5–73 cm

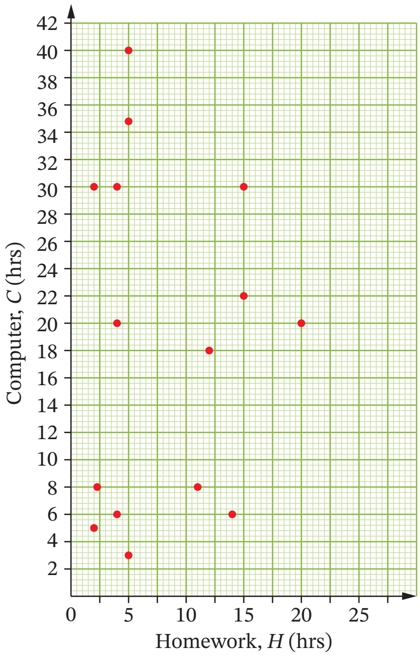
5 **a**



b Yes

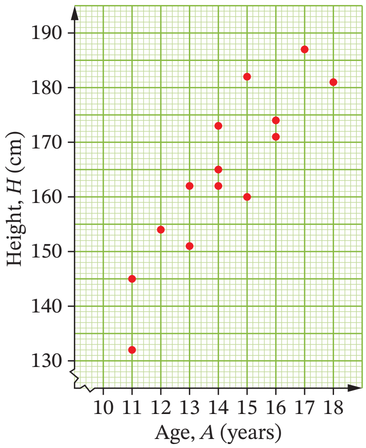
c Weak negative relationship

6 a



b no relationship

7 a



b Height, because its value depends on age (not the other way around).

c weak positive relationship

Exercise 5.07

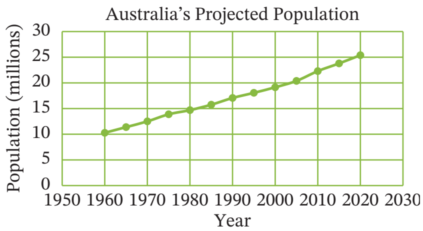
1 a i 25 **ii** 42 **iii** 15

b December, more customers due to summer and Christmas holiday season.

c June, fewer customers due to winter, busy end-of-financial year season.

d Number of people employed peaks in December, then falls, only to increase in March, April (the Easter holiday period). It then falls again to a low in June, July and then slowly the number of people employed rises to a peak in December. From 2017 to 2020, the number of people employed is showing a slow increase.

2 a



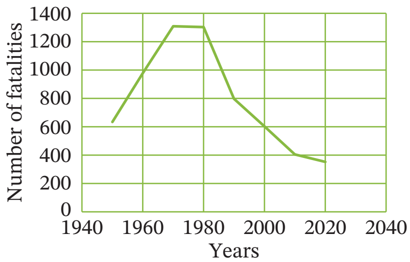
b 2005–2010

c Australia's population increased at 1.1–1.2 million every 5 years up to 1975. The population growth then slowed down for 5 years. From 1980, the population grew at a steady rate of just over a million people every 5 years, but in 2005–2010, the population then increased by 2 million for this 5-year period and this decreased slightly for 2010–2020.

d i approx. 28 million

ii approx. 35 million

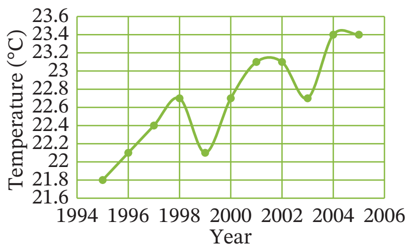
3 a



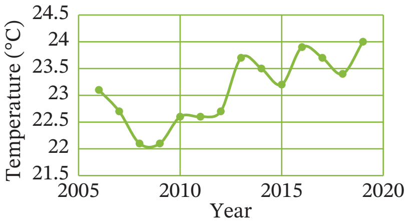
b From 1960, road fatalities rose at a steady rate, reaching a peak of approximately 1300 in 1970–1980. Road fatalities then fell at a rapid rate between 1980–1990 and continued to fall at a slower rate until 2020.

c Improved safety in cars, with seat belts being compulsory, then drink driving laws introduced and regular police presence on the roads and demerit points.

4 a i



ii

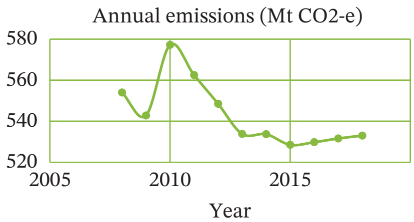


b **i** Starting at 21.8° in 1995, the temperature has seen a series of increases of less than 0.5° up to 1998, followed by a decrease of less than about 1° to 1999. The increase from 1999 to 2001 is about 1°, remaining the same for 2002, then dropping by about 0.4° in 2003, increasing by about 0.7° in 2004 and remaining at 23.4° in 2005.

ii Starting at 23.1° in 2006, the temperature falls to 22.7° in 2008 and then increases to a high of 23.7° in 2013, before falling to 23.2° in 2015. From 2015, the temperature rose to 23.9°, then dropped to 23.4° in 2018, before increasing to 24° in 2019.

c Both time periods have 3 periods of big increases with smaller decreases in between. The lengths of these periods of increase and decrease are different between the 2 time periods. The range of annual temperatures for both periods is under 2°, but the minimum and maximum temperatures for 2006–2019 are about 0.6° higher than for 1995–2005.

5 **a**



b There was an increase of 35 Mt of Carbon emissions from 2009 to 2010, but then there was a decrease of 45 Mt overall, to 2018.

c Carbon emissions stabilised.

d More environmentally-friendly policies and practices in Australia.

e **i**, **ii** eacher to check and discuss.

6 **a** 3 800 000 **b** 20 400 000

c 325 000 persons per year **d** 30 000 000

7 **a** Gradual increase in passenger movements with peaks in October and troughs in February.

b **i** approx. 4.7 million

ii approx. 5.0 million

iii approx. 5.0 million

iv approx. 5.1 million

c approx. 8.5%

Exercise 5.08

1 **a** Just surveying 300 people between 9 a.m. and 11 p.m. in shopping centres only targets a narrow group of people in certain areas.

b The sample needs to be more random and over a large area, not just in shopping centres. A telephone survey should produce more accurate feedback.

2 The report does not say what conditions are needed for the hot water system to work effectively. The temperature in Queensland is much warmer than in NSW and Victoria. Consequently, with the cooler climate in NSW and Victoria, especially in winter, the heat pump system may not provide the savings that people in Queensland obtain.

3 **a** **i** The price of petrol has shown little increase from December to February.

ii The price of petrol has shown marked rises and falls over the period from December to February.

b Both graphs could be improved by starting the vertical scale at 0 cents/litre.

4 **a** That there is a marked difference between the fuel consumption of the different cars.

b **i** 0.2 L/100 km

ii 1 L/100 km **iii** 0.2 L/100 km

c Begin the scale on the vertical axis with 0 and use a scale of 1 cm = 0.5 L/100 km instead of 1 cm = 0.2 L/100 km.

5 Yes, as there is no option for a customer to rate the app as unsatisfactory or poor.

6 **a** An example of a biased question could be: Which of these colours do you prefer – red, black, silver, blue?

b Apart from surveying people, they need to look at the sales figures of all cars. This will give information about the most popular car colour.

7, 8 Teacher to check.

Power plus

1 **a** −1 and 1

b There is no relationship between the variables.

c **i** 1 **ii** −0.2 **iii** −0.8

Other answers possible, teacher to check.

2 **b, d, f**

3 **a** $\bar{x} = 13.35$, median = 14, mode = 14

b Range = 10, IQR = 15 − 12.5 = 2.5

c The mean, median, and mode will increase by 4, the range and the interquartile range remain unchanged.

Test yourself 5

1 **a** **i** negatively skewed

ii clustering at 7 and 8, no outliers

b **i** negatively skewed

ii clustering at 16 and 17, 10 is an outlier

c **i** positively skewed

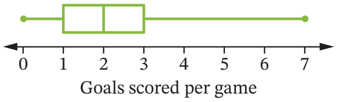
ii clustering at 40s and 50s, no outliers

2 **a** 6.5 **b** 6 **c** 2.5 **d** 12.5 **e** 2

3 **a** Range = 7, IQR = 3 − 1 = 2

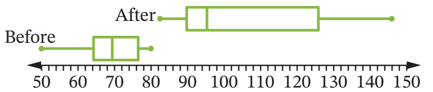
b 0, 1, 2, 3, 7

c



4 **a** Before: 50, 64, 69, 76, 80; After: 82, 89, 95.5, 126, 146

b



c **i** Range = 30, IQR = 12

ii Range = 64, IQR = 37

d The pulse rates for after exercise are significantly higher. In fact, all the rates for after exercise are above all the rates for before exercise. The median pulse for after exercise is 95.5 compared to the median pulse of 69 for before exercise. The range and interquartile range are also greater for the after exercise pulse rate.

5 **a** **i** Both **ii** Stem-and-leaf plot

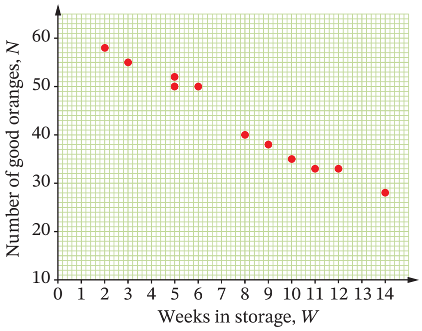
b The range (126 − 70 = 56) is too large.

c **i** median = 92 **ii** IQR = 99.5 − 84 = 15.5

d 50%

6 **a** Weeks in storage – this determines how many oranges stay good.

b



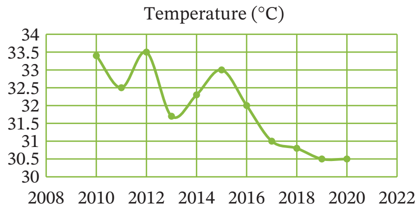
c linear

d The longer the oranges remain in storage, the fewer good oranges there are in the box.

e strong negative correlation

7 **a** Year

b



c For the last 10 years, the mean maximum temperatures for Perth, after starting at 33.4°, have dropped to 32.5°in 2011 and then after an increase dropped again to 31.7° in 2013, rising to 33° in 2015 and then dropping consistently to a finishing temperature of 30.5° in 2020. This shows there has been some change, approx. 3° overall, in the temperature for the month of January over the last 10 years.

8 **a** That the product is healthy.

b There is no data given on the actual fat content in the product. This should also be stated in terms of the daily percentage requirement of fat or in mg of fat.

Chapter 6

SkillCheck

1 **a** $y = 2$ **b** $x = 12$ **c** $m = 10$
d $w = 4$ **e** $m = 15$ **f** $n = -10$

2 **a** $5x + 50$ **b** $4y - 4$ **c** $10 - 6y$

3 **a** $x = 10$ **b** $x = 8$ **c** $a = -12$
d $x = 6$ **e** $x = 8$

Exercise 6.01

1 **a** $k = 60$ **b** $w = -9$ **c** $y = 2$
d $a = 5$ **e** $x = 8$ **f** $a = 1$
g $r = -6$ **h** $w = 6$ **i** $y = -2$
j $a = 3\frac{1}{2}$ **k** $u = 5\frac{1}{3}$ **l** $a = -1\frac{3}{4}$

2 B **3** D

4 **a** $y = 10$ **b** $a = -2\frac{1}{2}$ **c** $y = 5$
d $a = -3$ **e** $y = 8$ **f** $x = 1$
g $y = -1$ **h** $x = -10$ **i** $m = 1\frac{1}{2}$
j $y = 12$ **k** $a = -1$ **l** $x = -4$

5 C

6 **a** $x = 16$ **b** $m = 6$ **c** $y = 4$
d $y = 8$ **e** $y = -7$ **f** $x = 11$
g $m = -8$ **h** $a = -1$ **i** $y = -5$

7 C

8 **a** $d = \frac{2}{7}$ **b** $y = -\frac{7}{8}$ **c** $k = \frac{8}{9}$
d $g = 3\frac{2}{7}$ **e** $h = \frac{6}{17}$ **f** $p = -1\frac{4}{5}$

Exercise 6.02

1 **a** B **b** B **c** C

2 **a** $y = 15$ **b** $a = 9$ **c** $m = 2$
d $k = 65$ **e** $n = -35$ **f** $y = -7$
g $x = 31$ **h** $y = 46$ **i** $m = 18$
j $x = -29$ **k** $x = 24$ **l** $m = 10$
m $n = -\frac{1}{4}$ **n** $n = -\frac{3}{5}$ **o** $d = 3\frac{3}{4}$

3 **a** $k = 1\frac{7}{8}$ **b** $w = 1\frac{1}{3}$ **c** $x = -1\frac{1}{3}$
d $x = 3$ **e** $y = 3$ **f** $a = -8\frac{3}{5}$
g $p = 9\frac{2}{3}$ **h** $y = 3$ **i** $y = -\frac{5}{6}$
j $w = 10$ **k** $w = 50$ **l** $w = 9\frac{3}{5}$
m $a = \frac{6}{11}$ **n** $y = 60$ **o** $a = 1\frac{11}{13}$
p $m = 3\frac{1}{3}$ **q** $h = 1\frac{22}{23}$ **r** $y = -6\frac{5}{6}$

4 **a** C **b** A

5 **a** B **b** D **c** A **d** C

Exercise 6.03

1 **a** $m = \pm 12$ **b** $x = \pm 20$
c $y = \pm\sqrt{35}$ **d** $k = \pm 13$
e $y = \pm 1$ **f** $w = \pm\sqrt{24}$ $\left(\text{or } \pm 2\sqrt{6}\right)$
g $x = \pm 2$ **h** $t = \pm 4$
i $a = \pm 4$ **j** $k = \pm\sqrt{45}$ $\left(\text{or } \pm 3\sqrt{5}\right)$
k $w = \pm 10$ **l** $d = \pm 12$
m $k = \pm\sqrt{14}$ **n** $w = \pm 5$

o $x = \pm\frac{1}{2}$ **p** $m = \pm\sqrt{40}$ $\left(\text{or } \pm 2\sqrt{10}\right)$

q $y = \pm 1$ **r** $p = \pm 3$

s $k = \pm\sqrt{8}$ $\left(\text{or } \pm 2\sqrt{2}\right)$

t $y = \pm 10$

2 **a** $m = \pm 2$ **b** $a = \pm 9$ **c** $m \approx \pm 5.3$
d $m \approx \pm 1.9$ **e** $k \approx \pm 0.6$ **f** $x \approx \pm 7.6$
g $k \approx \pm 9.8$ **h** $k \approx \pm 9.5$ **i** $y \approx \pm 0.3$
j $a \approx \pm 9.2$ **k** $y \approx \pm 6.2$ **l** $w \approx \pm 7.1$

3 **a** B **b** B

4 **a** $x = -2, -1$ **b** $y = -4, -1$ **c** $y = -4, -12$
d $x = -4, 3$ **e** $x = -3, 1$ **f** $x = -8, 5$

5 **a** $x = 6, -5$ **b** $x = 4$ **c** $x = 11, -6$
d $d = 0, 2$ **e** $x = 5, -2$ **f** $n = 0, -4$
g $k = 0, 7$ **h** $y = 0, 5$ **i** $v = 0, 12$

6 You cannot take the square root of a negative number.

7 **a, c, f:** cannot find the square root of a negative number.

8 **a** A **b** C

Exercise 6.04

1 18 mm, 36 mm, 36 mm **2** 57 mm, 19 mm
3 13 cm, 29 cm **4** 61, 62, 63
5 Tyson is 27, Charlotte is 3 **6** 26
7 4 **8** Vatha is 22, Chris is 14
9 213, 214, 215, 216 **10** $x = 35$
11 117 **12** 6
13 Scott is 11, Kait is 34 **14** 25°, 50°, 105°

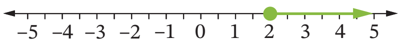
Mental skills 6

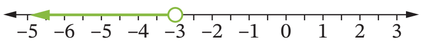
2 **a** 160 **b** 70 **c** 240 **d** 900
e 2600 **f** 900 **g** 140 **h** 300
i 180 **j** 770 **k** 18 **l** 34
m 46 **n** 26 **o** 18 **p** 12
q 40 **r** 8 **s** 14 **t** 24

Exercise 6.05

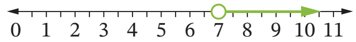
1 **a** 52 cm **b** 17 m
2 **a** 36 km/h **b** 86.4 km/h **c** 180 km/h
3 $30.\dot{5}$ km/h
4 **a** 27°C **b** 0°C **c** 100°C **d** 39°C
5 43
6 **a** 11.2 **b** 9 **c** 17.3
7 **a** 15.1 m **b** 31.8 cm
8 **a** 21.0 **b** 105.8 kg
9 **a** 137.3 cm^3 **b** 4.9 m
10 **a** 93 km/h **b** 436 km **c** 7 h
11 **a** $950 **b** 24 km
12 **a** 73.9 m^2 **b** $h = 13.2$ cm

Exercise 6.06

1 **a**


b


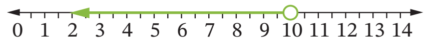
c


d


e


f


g

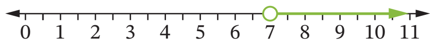

h


2 **a** $x < 4$ **b** $x \geq 2$ **c** $x > -6$ **d** $x \leq 1$

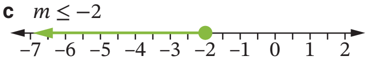
3 B

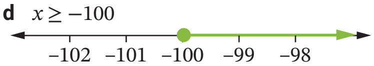
4 **a** $x \geq 1$ **b** $x < 4$ **c** $x > 6$
d $x \leq 2$ **e** $x > -6$ **f** $x \leq -1$
g $x \geq -4$ **h** $x \geq 25$ **i** $x < 0$

Exercise 6.07

1 **a** $x > 7$


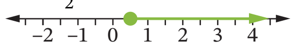
b $y \geq 4$

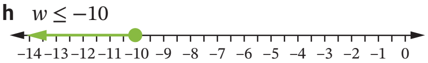

c $m \leq -2$


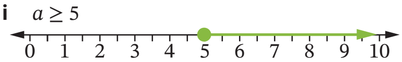
d $x \geq -100$


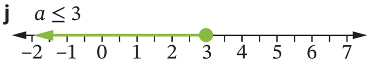
e $x < 5$

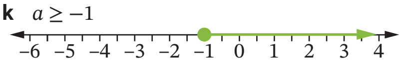

f $y > -4$


g $a \geq \frac{1}{2}$


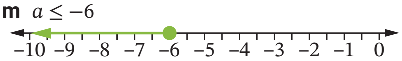
h $w \leq -10$


i $a \geq 5$


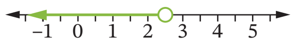
j $a \leq 3$


k $a \geq -1$


l $w < -3$


m $a \leq -6$


n $y \leq 3$


o $a < 2\frac{1}{2}$


2 A

3 **a** $x \geq 1$ **b** $m \leq 6$ **c** $y \leq -8$

d $w > 0$ **e** $w \leq 0$ **f** $m \geq 3\frac{1}{2}$

g $m \geq -2$ **h** $x \leq 5$ **i** $w > -3$

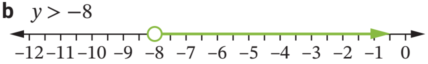
j $a < 4$ **k** $a \geq 6$ **l** $m \leq 3\frac{1}{2}$

m $m > 3\frac{2}{5}$ **n** $m \geq -2\frac{1}{2}$ **o** $x < 35$

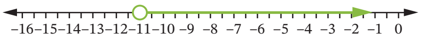
4 D

5 **a** $x \geq 3$

b $y > -8$

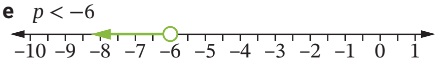


c $k > -11$

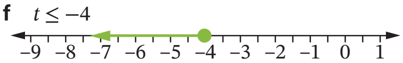


d $m \leq 0$

e $p < -6$



f $t \leq -4$



6 **a** $x > -3$ **b** $k \leq -12$ **c** $t < -2\frac{2}{5}$

d $x \geq 12$ **e** $w < -1$ **f** $y \geq -2$

g $x \leq 4$ **h** $a > 1$ **i** $d < -5\frac{1}{2}$

j $w < -11$ **k** $x \leq -8$ **l** $p < -4$

Power plus

1 **a** $y = 2\frac{3}{7}$ **b** $y = 2\frac{1}{2}$ **c** $m = -10$

2 $d = -8$

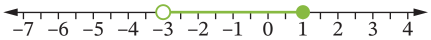
3 Randall is 17, Tarni is 7.

4 The number is 36.

5 **a**

b

c



Test yourself 6

1 **a** $a = 11$ **b** $y = 2\frac{2}{3}$ **c** $a = -3$

d $y = 1$ **e** $m = 11$ **f** $a = 28$

g $h = 1\frac{5}{8}$ **h** $y = \frac{13}{20}$

2 **a** $w = 6$ **b** $y = 8\frac{3}{4}$ **c** $a = -3$

d $m = 3$ **e** $s = 4$ **f** $x = 1\frac{2}{3}$

3 A

4 **a** $y = \pm 2$ **b** $p = \pm 10$

c $x = \pm\sqrt{10}$ **d** $m = \pm 1$

e $w = \pm\sqrt{50}$ $\left(\text{or } \pm 5\sqrt{2}\right)$ **f** $x = -7, -1$

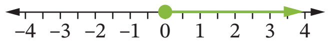
g $h = 9, -1$ **h** $u = 7, -11$ **i** $k = 0, -5$

j $m = 0, 2$ **k** $b = -10$ **l** $w = 9, 0$

5 92, 93, 94, 95 **6** $x = 19$ **7** 120 m

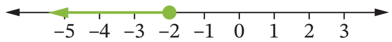
8 **a** 160 mm^3 **b** 300 m^2

9 **a** $x \geq 0$



b $x < 3$

c $x \leq -2$



d $x > -5$

10 **a** $y \geq 16$ **b** $y \leq -7\frac{1}{2}$ **c** $a > -5$

d $x > -3$ **e** $a < -16$ **f** $x \leq -3$

Practice set 2

1 **a** **i** negatively skewed

ii no outliers, clusters at 7 and 8

b **i** positively skewed

ii outlier 98, clustering in 40s and 50s

2 **a** $81n^4 m^8$ **b** $20p^9 q^3$ **c** $4a^4 b^3$

d 1 **e** $\frac{1}{27}$ **f** $\frac{125}{8}$

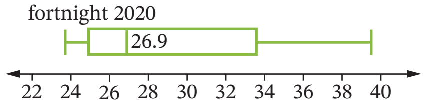
3 **a** $b = 5$ **b** $y = 1\frac{1}{2}$ **c** $m = 11$ **d** $a = 1\frac{3}{4}$

4 6

5 **a** range = 15.8, interquartile range = 8.7

b 23.7, 24.9, 26.9, 33.6, 39.5

c April's maximum temperature first fortnight 2020



d 39.5°C

6 **a** $\frac{1}{9y^2}$ **b** $\frac{3}{y^2}$ **c** $64x^9 y^{12}$

d $24h^{14} k^{16}$ **e** $3v^3 w^4$ **f** $\frac{4b^2}{3a^3}$

7 **a** $\frac{t}{2}$ **b** $\frac{-x}{15}$ **c** $\frac{7g}{20}$ **d** $\frac{7k}{10}$

8 **a** $\frac{6x}{7y}$ **b** $2m$ **c** $\frac{9}{2d}$ **d** $2v^2$

9 **a** weak negative **b** strong negative

c weak positive

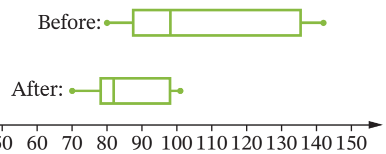
10 **a** $a = 8\frac{1}{2}$ **b** $k = 8\frac{3}{4}$ **c** $w = 10$ **d** $g = 4$

11 **a** $7y - 63$ **b** $-8n^2 - nm$

c $-20w + 24$ **d** $24a^3 b + 28a^2 b^2$

12 **a** Before: 80, 87.5, 96, 134, 142; After: 70, 77, 81, $97\frac{1}{2}$, 101

b



c Before: range = 62, interquartile range = 46.5

After: range = 31, interquartile range = 20.5

d Yes, there is a significant difference with the students' weights decreasing and having less spread after the program. According to the box plot, 75% of the masses after the program are below 50% of the masses before the program. The median has dropped significantly, as well as both the range and the interquartile range, meaning the students have reached more consistent weights.

13 **a** $3ab^2 - 24a^2b$ **b** $18r^2 - 60r + 36$

14 **a** $7(w + 6)$ **b** $p(p - 25)$
c $-4(n + 11)$ **d** $(7a - 2)(3 - 5a)$

15 **a** **i** stem-and-leaf plot **ii** stem-and-leaf plot
b median = 41 **c** interquartile range = 18

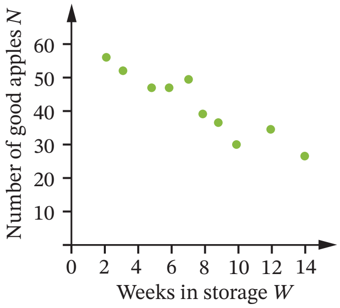
16 **a** $p = \pm 12$ **b** $y = \pm 2$ **c** $x = \pm 1$ **d** $t = \pm 10$
e $q = \pm 8.8$ **f** $x = \pm 4.8$ **g** $w = \pm 6.2$

17 **a** $2q^2 + 15q + 27$ **b** $3f^2 - 13f - 56$
c $20g^2 - 47g + 21$ **d** $9x^2 - 49$

18 **a** $(r + 8)(r + 3)$ **b** $(y - 30)(y - 1)$
c $(x + 12)(x - 3)$ **d** $(t - 9)(t + 8)$

19 **a** The independent variable is W, the weeks in storage. The number of weeks in storage is set first, after which time the number of good apples is counted.

b



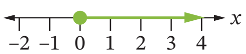
c The number of good apples decreases the longer the apples are in storage.
d There is a strong and negative relationship between the variables W and N.

20 **a** $x = -7, x = -1$ **b** $h = 9, h = -1$
c $u = -11, u = 7$ **d** $w = 0, w = 9$

21 **a** 20, 21, 22 **b** 20, 40, 120

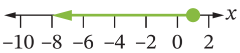
22 **a** 480 mm³ **b** 150 cm² **c** 3.6 m

23 **a**



b

c



d

24 **a** $n \geq 5$ **b** $a \leq -3$ **c** $h > -8$ **d** $x < 3\frac{3}{5}$

Chapter 7

SkillCheck

1 **a** −1 **b** 29 **c** −3 **d** 69

2 **a** 625 **b** 3125 **c** 1 **d** $\frac{1}{25}$

Exercise 7.01

1 **a** C **b** D **c** A **d** B

2 **a** 190, $D = 190T$
b **i** 3.8 km **ii** 8.55 km
c 1 h 5 min

3 **a** 26.2, $E = 26.2h$
b $183.40 **c** 5.5 h

4 **a** $I = \frac{16D}{425}$ **b** $33.88 **c** $67.76

5 A **6** $b = 2.5a$

7 **a**

h	c
1	$7.50
2	$15
3	$22.50

b $c = 7.5$ h **c** $45 **d** 11
e 7.5. It is the same.

8 C

9 **a** $F = 0.006m$ **b** 15 L/100 km

10 A

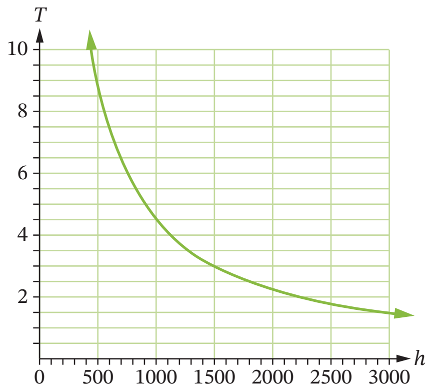
11 **a** 22.8 kg **b** 84.1 kg

Exercise 7.02

1 **a** B **b** C **c** D **d** A

2 **a** 920, $T = \frac{920}{s}$ **b** 10 h 13 min **c** 92 km/h

3 **a** 4500, $T = \frac{4500}{h}$
b **i** 15°C **ii** 1.8°C
c **i** 562.5 m **ii** 200 m
d



4 **a** $N = \frac{112}{S}$ **b** 101 **c** 2.0 m² **d** 28

5 C **6** B **7** $\frac{14}{15}$

8 **a** 8 min **b** 4 people

9 **a** $b = \frac{8}{a}$ **b** $b = \frac{100}{a}$

10 **a** $F = \frac{112}{L}$ **b** 6 beats/sec **c** 25 cm

11 **a** $y = \frac{1}{16}$ **b** $x = 1\frac{1}{4}$

12 **a** 2.5 h **b** 5 friends

Exercise 7.03

1 **a** **i** £22 **ii** £49
b **i** \$33 **ii** \$108
c **i** £22 – £20 = £2 **ii** \$120 – \$108 = \$12

2 **a** **i** 2 km **ii** 20 km **iii** 34 km
b **i** 50 furlongs **ii** 125 furlongs **iii** 180 furlongs
c 60 km **d** 500 furlongs

3 **a** **i** ¥16 000 **ii** ¥63 000 **iii** ¥78 000
b **i** \$250 **ii** \$760 **iii** \$920

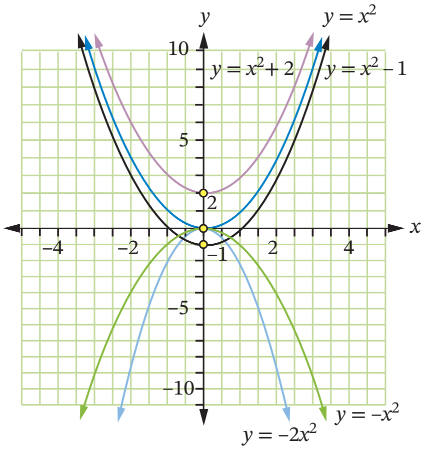
4 **a** −18°C **b** 10°C **c** 26°C
d 32°F **e** 14°F **f** 86°F

5 **a** 4.9 ha **b** 2 ha **c** 10.8 acres **d** 12.2 acres
e **i** 32 000 m^2 **ii** 3.2 ha **iii** 7.8 acres

6 **a** **i** ₱520 **ii** ₱210 **iii** ₱310
b \$10.20 **c** \$5.80 **d** ₱4200

Exercise 7.04

1 **a**



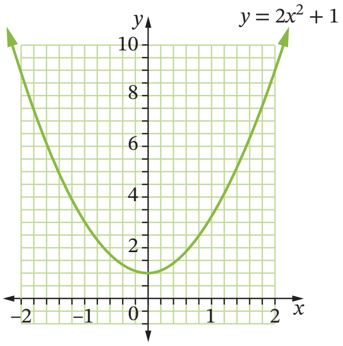
b **i** $y = x^2, y = x^2 + 2, y = x^2 - 1$
ii $y = -x^2, y = -2x^2$
iii $y = x^2, y = -x^2, y = -2x^2$

2 A **3** C

4 **a** vi **b** ix **c** i **d** xi
e x **f** iii **g** ii **h** xii
i viii **j** v **k** vii **l** iv

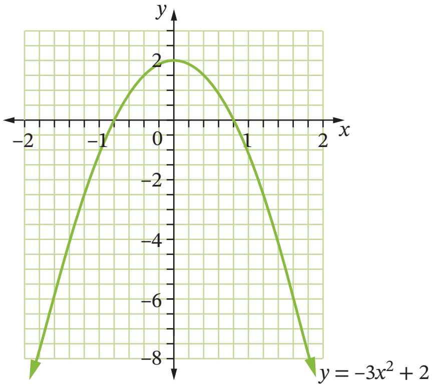
5 **a** $y = -x^2$ **b** $y = x^2$ **c** $y = -x^2 - \frac{1}{4}$
d $y = -x^2 - 9$ **e** $y = \frac{1}{2} - x^2$ **f** $y = x^2 + 9$

6 **a**



b (0, 1) **c** concave up **d** $y = 1$

7 **a**



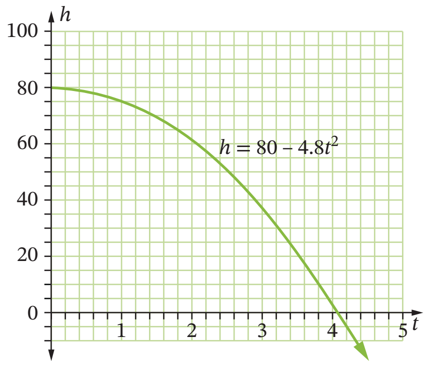
b (0, 2) **c** $x = 0$ **d** $y = 2$

8 A

9 **a** **i** narrower **ii** up **iii** (0, 3)
b **i** wider **ii** up **iii** (0, 1)
c **i** narrower **ii** down **iii** (0, −5)
d **i** wider **ii** down **iii** (0, −12)

10 **a** $x = \pm 4$ **b** $x = \pm 11$

11 **a**



b 80 m **c** 36.8 m **d** 4.1 s **e** 3.95 s

12 **a** B **b** G **c** D **d** J
e E **f** I **g** A **h** K
i H **j** L **k** F **l** C

13 **a** $x = \pm 9$ **b** $x = \pm 14$

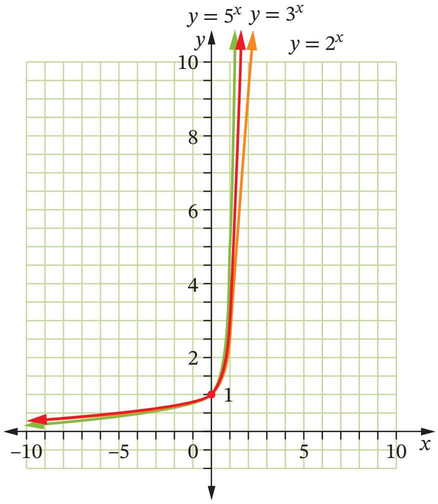
Mental skills 7

2 **a** 3.5 **b** 2.4 **c** 0.12 **d** 0.36
e 0.8 **f** 0.027 **g** 0.2 **h** 8.8
i 0.24 **j** 0.012 **k** 1.8 **l** 0.028

4 **a** 66.3 **b** 6630 **c** 6.63 **d** 0.663
e 6.63 **f** 663 **g** 0.663 **h** 663
i 6630 **j** 66.3 **k** 0.663 **l** 0.0663

Exercise 7.05

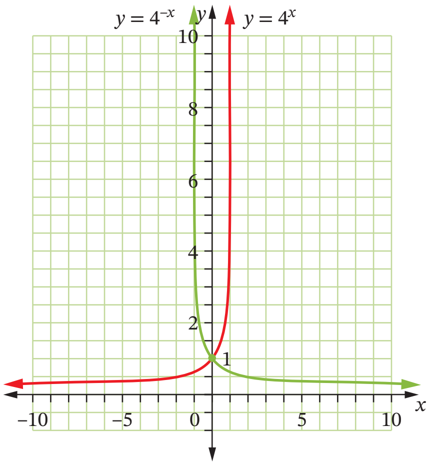
1 a



b 1

c becomes steeper, in 1st quadrant

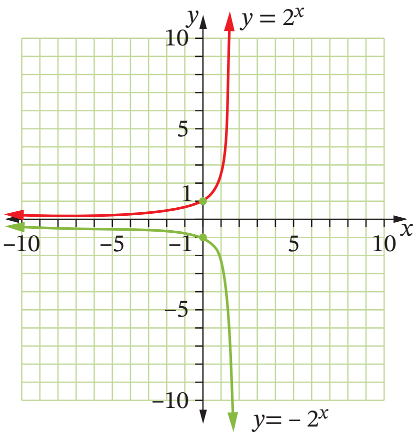
2 a



b i $y = 4^{-x}$ **ii** $y = a^{-x}$

3 B

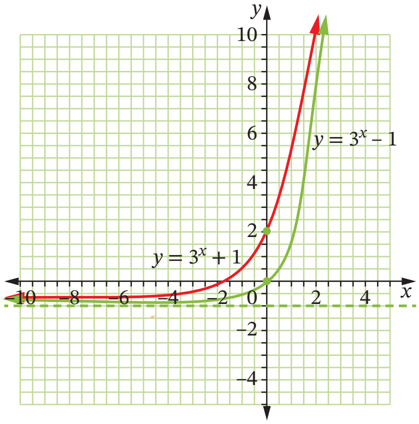
4 a



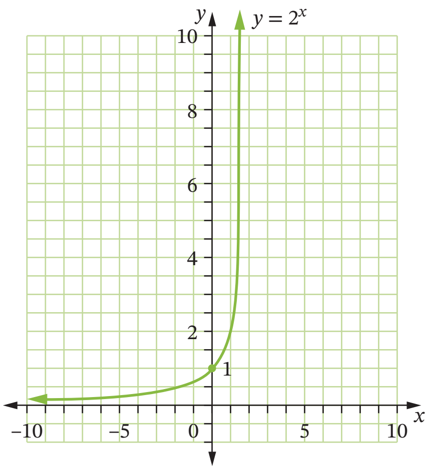
b one is the reflection of the other in the x-axis

c $y = -a^x$

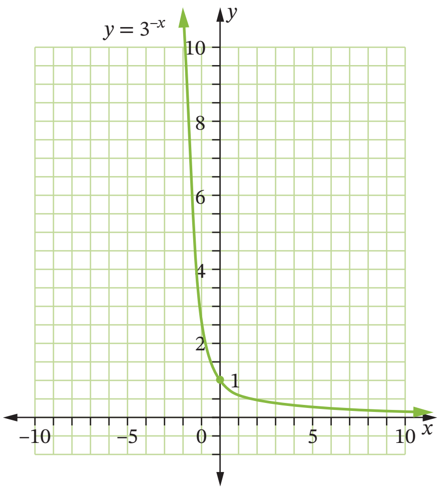
5 Same shape, shifted down 2 units.



6 a

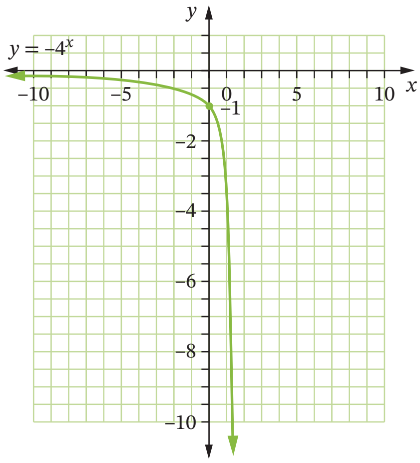


b

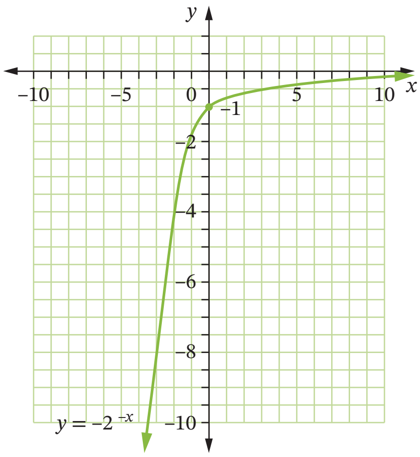


9780170453417

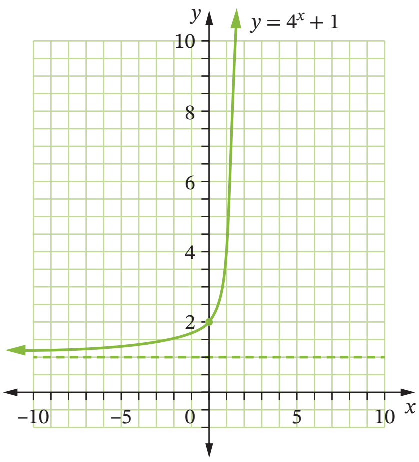
c



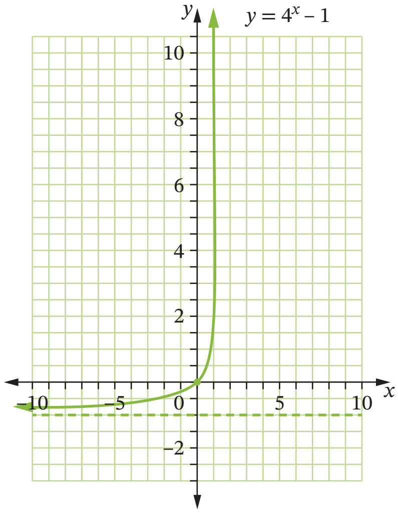
d



e



f



7 $y = 4^{-x}$

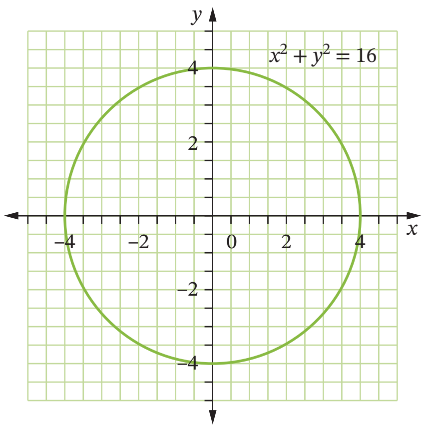
Exercise 7.06

1 **a** centre (0, 0), $r = 2$ **b** centre (0, 0), $r = 6$
c centre (0, 0), $r = 8$ **d** centre (0, 0), $r = 10$
e centre (0, 0), $r = 9$ **f** centre (0, 0), $r = 5$

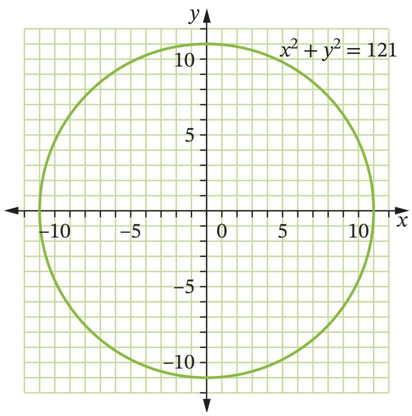
2 D **3** D

4 **a** $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 9$
c $x^2 + y^2 = 25$ **d** $x^2 + y^2 = \frac{1}{9}$

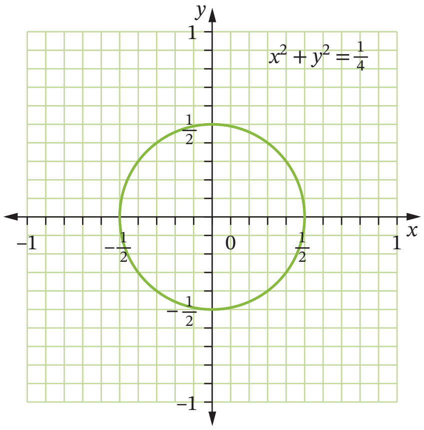
5 **a**



b



c



6 C

7 **a** LHS = $8^2 + 6^2$

= $64 + 36$

= 100

= RHS

$\therefore$ (8, 6) lies on $x^2 + y^2 = 100$

b LHS = $5^2 + 9^2$

= $25 + 81$

= 106

$\neq$ RHS

$\therefore$ (5, 9) does not lie on $x^2 + y^2 = 100$

c outside

8 **a** inside **b** on **c** outside

d inside **e** outside

Exercise 7.07

1 **a** P **b** L **c** E **d** L

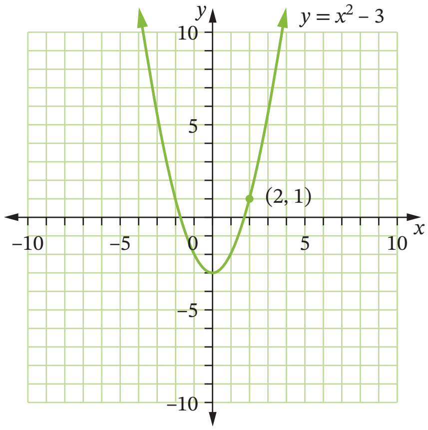
e C **f** L **g** P **h** L

i P **j** E **k** P **l** C

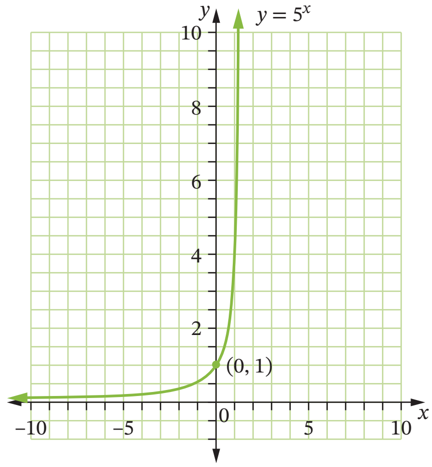
2 **a** vii **b** x **c** viii **d** iv **e** i

f vi **g** iii **h** v **i** ii **j** ix

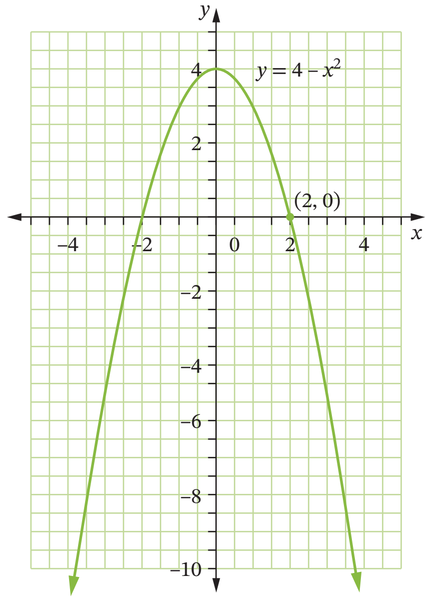
3 **a**



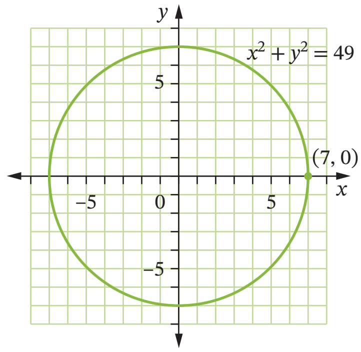
b



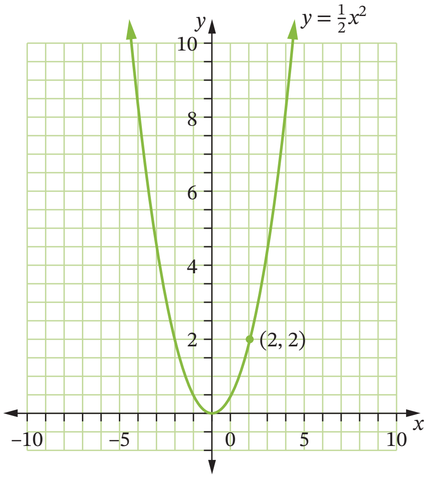
c



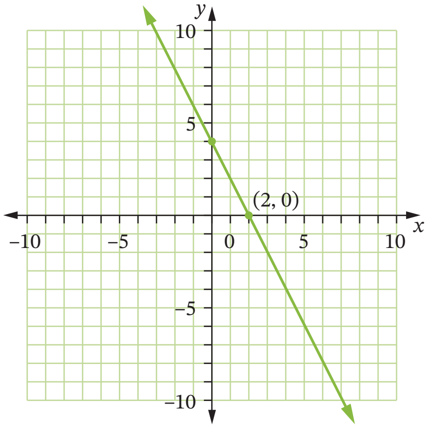
d



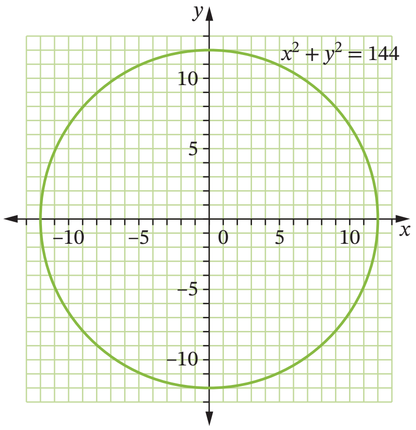
e



f



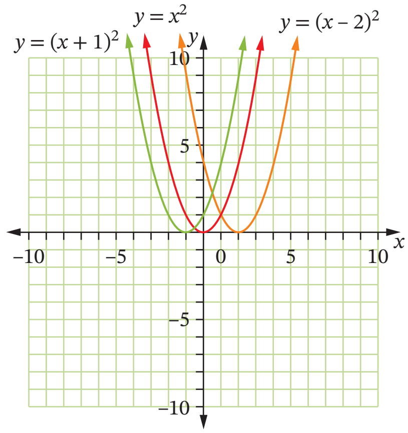
g



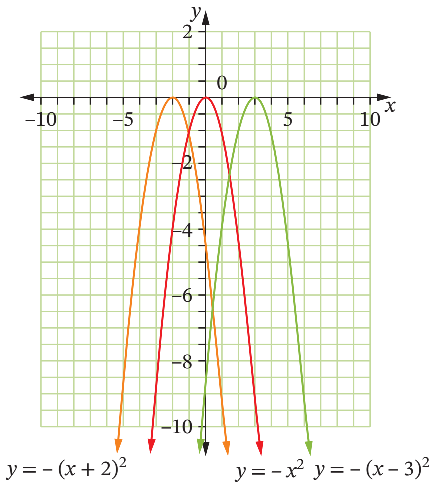
4 **a** 1 **b** 3 **c** −6 **d** 1

Power plus

1



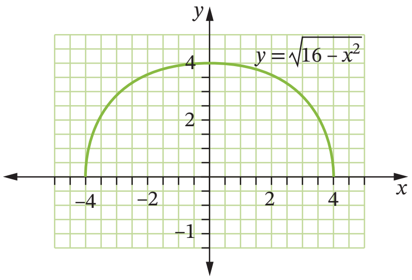
2



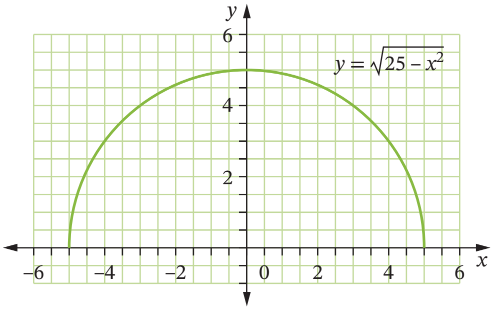
3 If a is positive, the parabola $y = x^2$ is shifted left a units.

If a is negative, the parabola $y = x^2$ is shifted right a units.

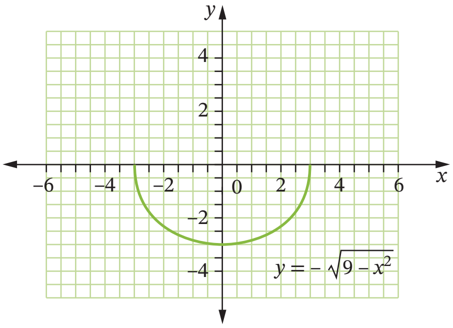
4 **a** centre (0, 0) and $r = 4$



b centre (0, 0) and $r = 5$



c centre (0, 0) and $r = 3$



5 **a** centre (0, 0), radius $\sqrt{5}$

b centre (3, 0), radius $\sqrt{3}$

c centre (–4, 2), radius $\frac{1}{2}$

Test yourself 7

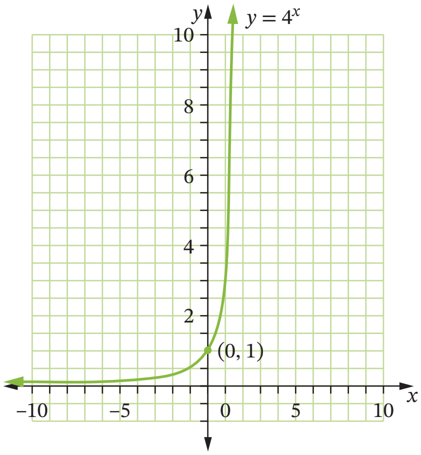
1 $H = 310.5$ **2** 10°C

3 **a** £39 **b** $A101

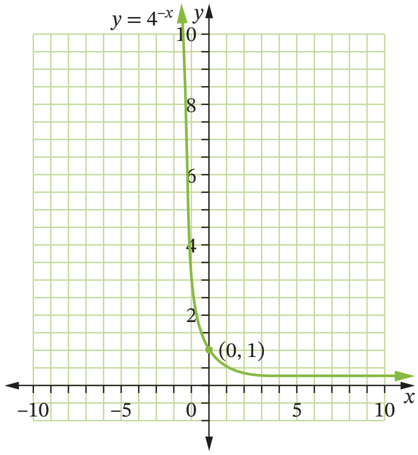
4 **a** C **b** F **c** A

d E **e** D **f** B

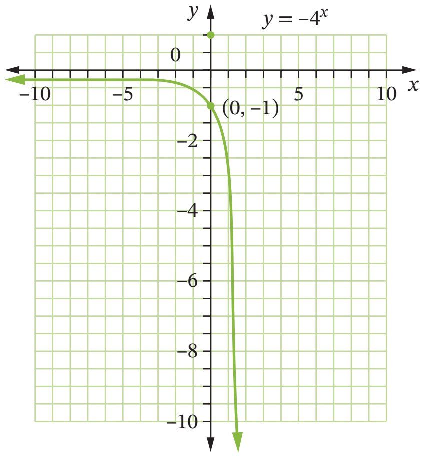
5 **a**



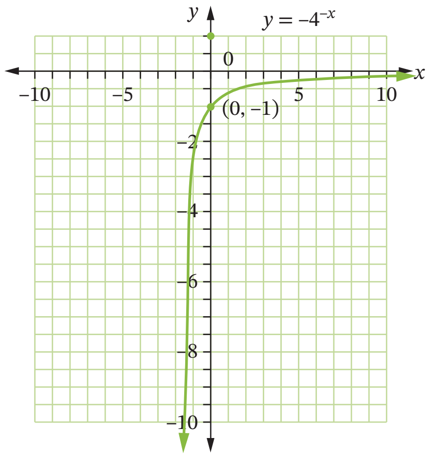
b



c



d



6 **a** centre (0, 0), $r = 10$ **b** centre (0, 0), $r = 6$

c centre (0, 0), $r = 7$

7 $x^2 + y^2 = 64$

8 **a** D **b** C **c** B **d** J

e E **f** H **g** L **h** G

i I **j** A **k** K **l** F

Chapter 8

SkillCheck

1 **a** $x = 35$ **b** $h = 33.2$ **c** $y = 5$

2 **a** $x = 55$ **b** $y = 23$ **c** $n = 38$
d $x = 60$ **e** $x = 50$ **f** $x = 160$

3 **a** hyp: 17, opp: 8, adj: 15
b hyp: p, opp: r, adj: q
c hyp: EF, opp: EG, adj: GF

4 **a** 0.8480 **b** 0.7760 **c** 64.9839
d 0.1539 **e** 13.9884 **f** 13.7044

5 **a** 64° **b** 26° **c** 12°

6 **a** 50°19′ **b** 31°56′ **c** 64°19′

7 **a** 45°48′ **b** 33°11′ **c** 5°21′

Exercise 8.01

1 **a** $h = 30$ **b** $y = 2.5$ **c** $x = 16$
d $p = 50.5$ **e** $w = 115.4$ **f** $t = 5.7$

2 **a** Yes **b** No **c** Yes
d No **e** No **f** Yes

3 **a** 36.3 m **b** 84.4 cm **c** 204 m
d 12.9 m **e** 28.9 m **f** 35.3 cm

4 **a** 271.8 cm² **b** 2110.7 m² **c** 1440 m²
d 22 m² **e** 123.0 cm² **f** 460 m²

5 183.6 km **6** 17.9 m **7** 120 m

8 B **9** 9.2 km **10** B

Exercise 8.02

1 **a** $\sin\theta = \frac{20}{29}$ $\cos\theta = \frac{21}{29}$ $\tan\theta = \frac{20}{21}$
b $\sin F = \frac{DE}{FE}$ $\cos F = \frac{FD}{FE}$ $\tan F = \frac{DE}{FD}$
c $\sin\alpha = \frac{p}{y}$ $\cos\alpha = \frac{w}{y}$ $\tan\alpha = \frac{p}{w}$
d $\sin K = \frac{LH}{KH}$ $\cos K = \frac{KL}{KH}$ $\tan K = \frac{LH}{KL}$
e $\sin\theta = \frac{d}{a}$ $\cos\theta = \frac{g}{a}$ $\tan\theta = \frac{d}{g}$
f $\sin Q = \frac{2.4}{3} = \frac{4}{5}$ $\cos Q = \frac{1.8}{3} = \frac{3}{5}$ $\tan Q = \frac{4}{3}$

2 **a** $\frac{36}{85}$ **b** $\frac{36}{77}$ **c** $\frac{77}{85}$ **d** $\frac{36}{85}$

3 **a** α **b** β **c** β **d** β **e** α **f** α

4 **a i** $\frac{15}{8}$ **ii** $\frac{8}{17}$ **iii** $\frac{15}{17}$ **iv** $\frac{8}{15}$
b i $\frac{10}{10.5} = \frac{20}{21}$ **ii** $\frac{10.5}{14.5} = \frac{21}{29}$
iii $\frac{10}{14.5} = \frac{20}{29}$ **iv** $\frac{10.5}{10} = \frac{21}{20}$
c i $\frac{a}{g}$ **ii** $\frac{g}{d}$ **iii** $\frac{a}{d}$ **iv** $\frac{g}{a}$

5 D **6** B **7** B

8 **a** $\sin M = \frac{15}{17}$, $\tan M = \frac{15}{8}$
b $\sin Y = \frac{4}{5}$, $\cos Y = \frac{3}{5}$
c $\cos C = \frac{13}{85}$, $\tan C = \frac{84}{13}$

9 C

Exercise 8.03

1 **a** 117.0 cm **b** 36.7 m **c** 16.7 m
d 120.2 cm **e** 11.5 m **f** 1.2 m
g 9.3 m **h** 61.4 m **i** 71.5 cm

2 **a** 172.4 m **b** 25.2 m **c** 3.8 m **d** 6.3 m

3 **a** 71.4 cm **b** 11.0 m **c** 28.2 m
d 17.1 m **e** 39.9 cm **f** 94.4 cm
g 53.2 m **h** 215.1 cm **i** 3.5 m

4 **a** 353.9 cm **b** 9.5 m **c** 992.1 cm **d** 582.8 m

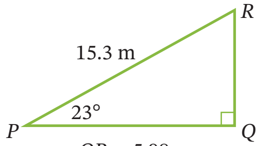
5 C **6** 3.8 m

7 0.63 m or 63 cm

8 1.5 km **9** 24.6 m

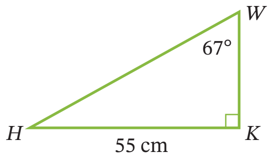
10 **a** 434 m **b** 164 m

11



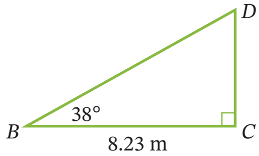
$QR = 5.98$ m

12



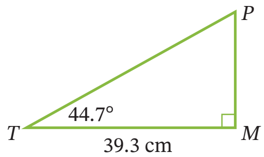
$HW = 59.7$ cm

13



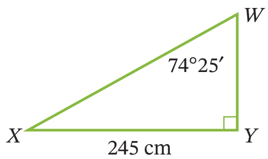
$CD = 6.43$ cm

14



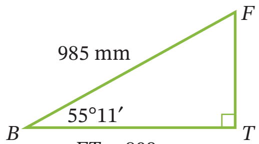
$PT = 55.3$ cm

15



$WY = 68$ mm

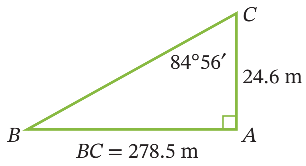
16



$FT = 809$ mm

ANSWERS

17



Exercise 8.04

1 **a** 62° **b** 40° **c** 30° **d** 82°
e 49° **f** 83° **g** 30° **h** 45°

2 **a** 37°52′ **b** 40°22′ **c** 54°19′ **d** 72°4′
e 44°16′ **f** 36°52′ **g** 53°24′ **h** 77°13′

3 **a** 37° **b** 42° **c** 47°
d 47° **e** 53° **f** 33°

4 **a** 69°42′ **b** 46°3′ **c** 38°46′
d 49°13′ **e** 54°12′ **f** 42°5′

5 **a** 63° **b** 27° **c** 39°

6 72° **7** 3° **8** 37° **9** 35°

10 14° **11** C **12** 48° **13** 70°

14 40° 53′ **15** 18° 13′ **16** 52° 1′

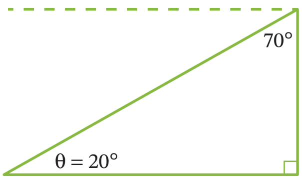
Mental skills 8

2 **a** 2, 5 **b** 3 **c** 2, 3, 6 **d** 3, 5
e 2 **f** 2, 3, 5, 6 **g** 5 **h** 3

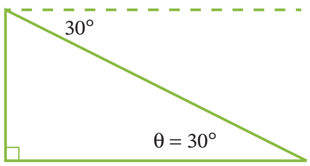
3 **a** 4, 9 **b** 4, 10
c 9 **d** 10
e 4, 9 **f** none of these
g 9 **h** 4, 9, 10

Exercise 8.05

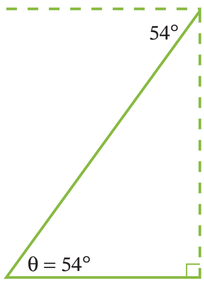
1 **a**



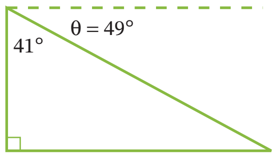
b



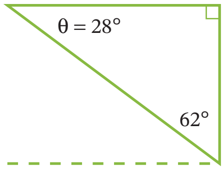
c



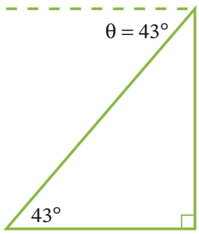
2 **a**



b



c



3 59 m

4 2224 m or 2.224 km

5 177 m

6 180 m

7 9°

8 50°

9 14°29′

10 18°47′

11 12.56 m or 1256 cm

12 14.5 m

13 970 m

14 57 m

Exercise 8.06

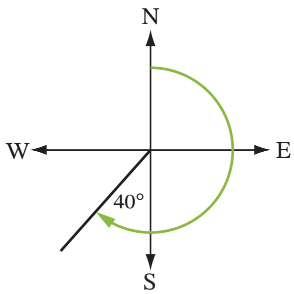
1 **a** 243° **b** 290° **c** 040°
d 115° **e** 210° **f** 140°
g 312° **h** 253° **i** 065°

2 **a** 000° **b** 090° **c** 180° **d** 270°
e 038° **f** 125° **g** 330° **h** 225°
i 072° **j** 187°

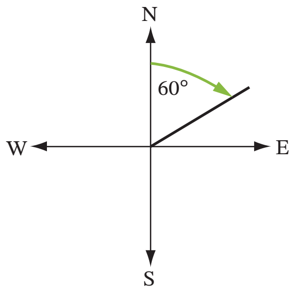
3 SW

4 **a** 225° **b** 090° **c** 045° **d** 112.5°
e 292.5° **f** 270° **g** 135° **h** 247.5°

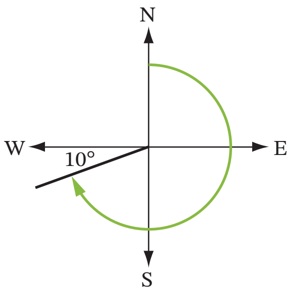
5 **a**



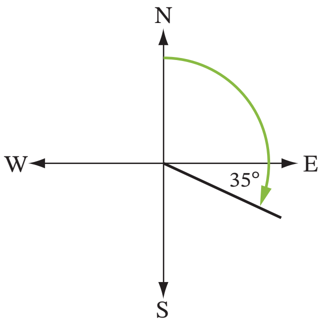
b



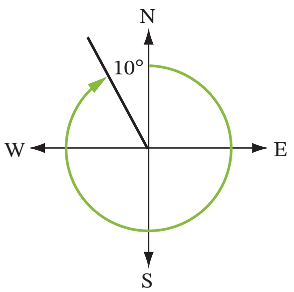
c



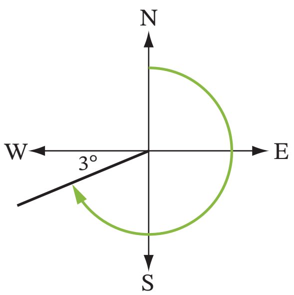
d



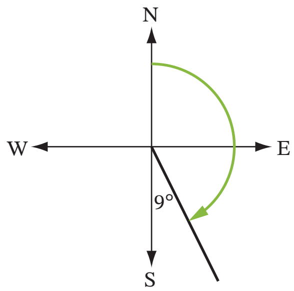
e



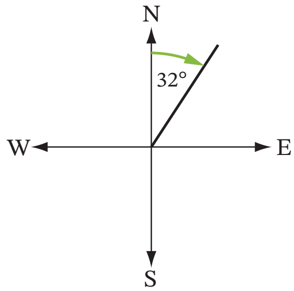
f



g

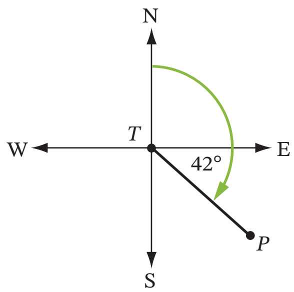


h

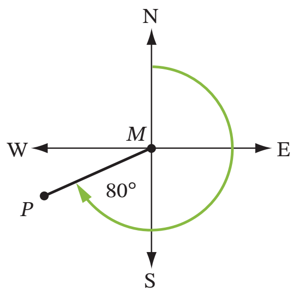


6 **a** NNW **b** 337.5°

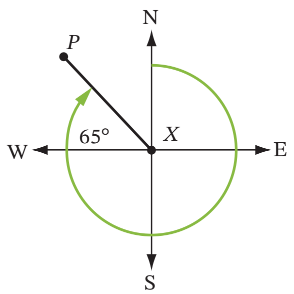
7 **a**



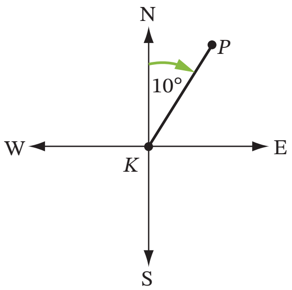
b



c



d

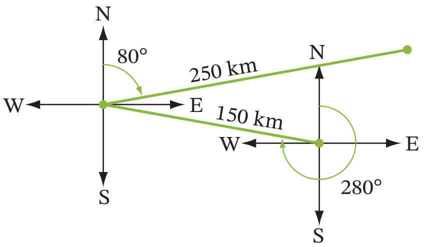


8 240° **9** 280°

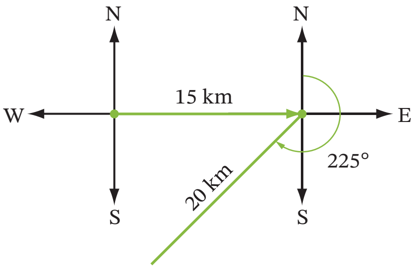
10 **a** 45° **b** 90° **c** 135°
d 157.5° **e** 112.5° **f** 67.5°

11 ESE

12 **a**



b



13 **a** 230° **b** 270° **c** 160°
d 340° **e** 050° **f** 090°

Exercise 8.07

1 **a** 21 km **b** 260°
2 **a** 37° **b** 163 km **c** 143°
3 **a** 62.5 km **b** 29.2 km **c** 025°
4 **a** 12.2 km **b** 215° **c** 035°
5 **a** 18 km **b** NNW
6 45.7 km
7 **a** 13.509 km **b** 321°
8 **a** 2122 km **b** 330°
9 **a** 15 km **b** 26 km
10 **a** 261.08 km **b** 167.82 km
11 6.6 km
12 **a** 322 nautical miles **b** 276° 45′
13 **a** 1282 km **b** 024° 27′
14 326° 11′

Power plus

1 **a** **i** 0.342, 0.342 **ii** 0.731, 0.731
iii 0.819, 0.819 **iv** 0.996, 0.996
b Each pair of trigonometric ratios has the same value.
c The pairs of angles are complementary (add to 90°).
d 60°
e **i** 15° **ii** sin **iii** 18°
iv cos **v** 25° **vi** 32°
f $\sin x = \cos(90° - x)$
g Teacher to check.

2 **a** 1932 m **b** 31°

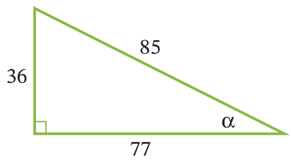
3 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{3}}$

4 **a** 24° 12′ 26″ **b** 63° 17′ 3″

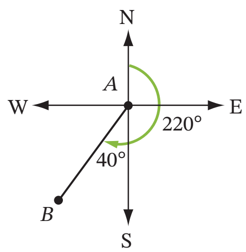
5, 6 Teacher to check.

Test yourself 8

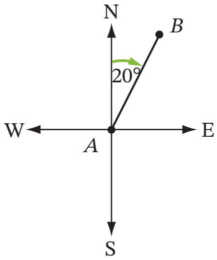
1 **a** 62.2 **b** 21.0 **c** 4.7
2 **a** Yes **b** No **c** Yes
3 **a** 9.5 m **b** 152.2 cm **c** 166 cm
4 **a** $\frac{33}{65}$ **b** $\frac{33}{56}$ **c** $\frac{56}{65}$ **d** $\frac{33}{65}$
5 $\cos \alpha = \frac{77}{85}$ $\tan \alpha = \frac{36}{77}$



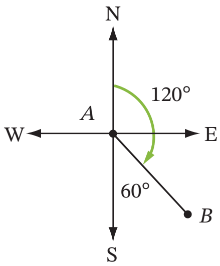
6 **a** 15.68 **b** 42.13 **c** 19.39
7 **a** 65.6 **b** 27.8 **c** 26.1
8 D
9 **a** 61° **b** 85° **c** 65° **d** 88°
10 **a** 21° **b** 63° **c** 30°
11 A **12** 195 m **13** 29°
14 **a** 231° **b** 051°
15 **a**



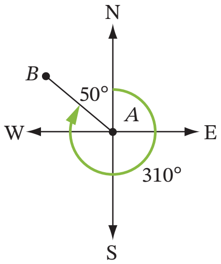
b



c



d



16 A

17 **a** 1281 km **b** 024°

18 **a** 43.7 nautical miles

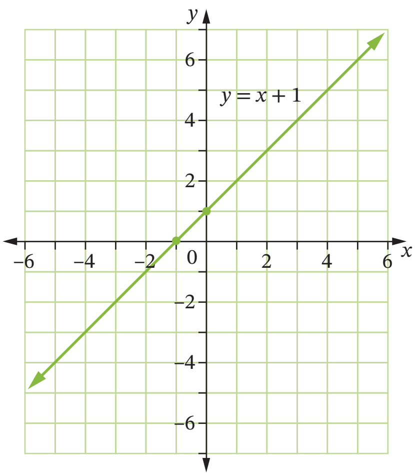
b 276.6° **c** 096.6°

Chapter 9

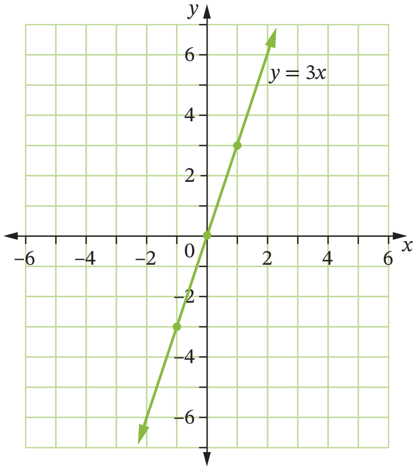
SkillCheck

1	**a** 5	**b** 13	**c** 6	**d** −1
2	**a** −11	**b** 1	**c** 7	**d** $5\frac{1}{2}$

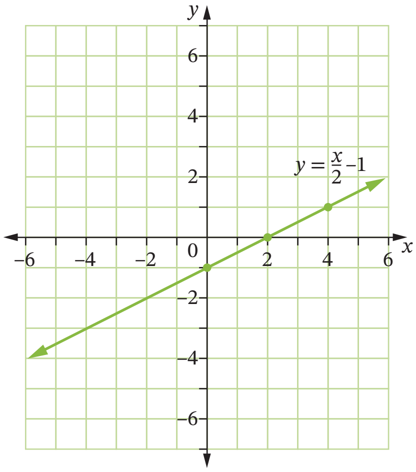
3 **a**



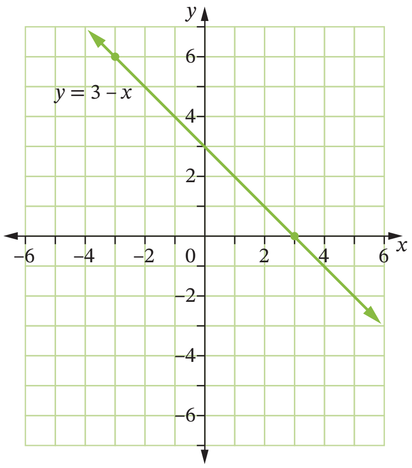
b



c

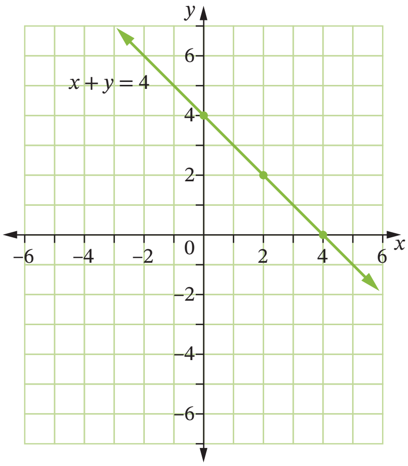


d

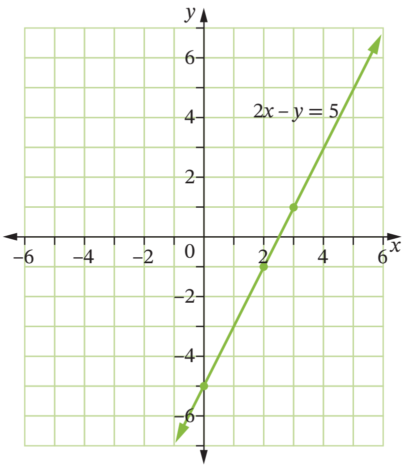


ANSWERS

e



f



4 $(-2, 3)$ lies on a, d

5 **a** For $y = 2x + 1$, when $x = 2$, $y = 2 \times 2 + 1 = 5$

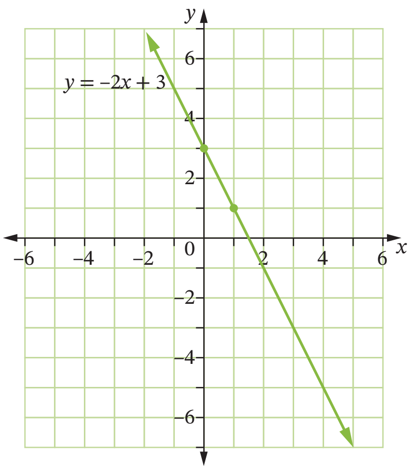
$\therefore (2, 5)$ lies on $y = 2x + 1$

For $x + y = 7$, when $x = 2$, $y = 5$, $2 + 5 = 7$

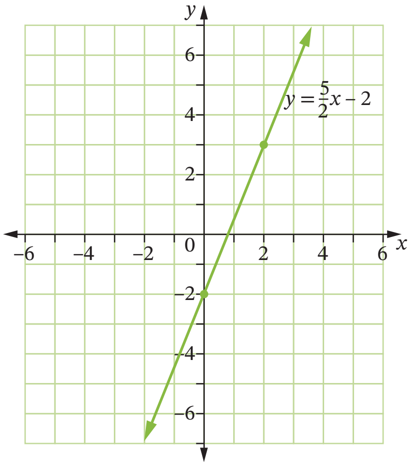
$\therefore (2, 5)$ lies on $x + y = 7$

b $(2, 5)$

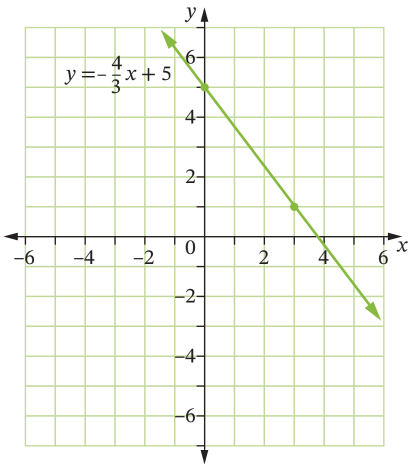
6 **a** $m = -2, c = 3$



b $m = \frac{5}{2}, c = -2$



c $m = -\frac{4}{3}, c = 5$



Exercise 9.01

1 **a** $x = 3, y = -1$

b $x = 2, y = 1$

c $x = -1, y = -5$

2 **a** $x = 1, y = 2$

b $x = -5, y = -9$

c $x = 1, y = 2$

d $x = -\frac{1}{2}, y = 2\frac{1}{2}$

e $x = -2, y = -9$

f $x = 5, y = -4$

g $x = \frac{1}{2}, y = 6\frac{1}{2}$

h $x = 3, y = 2$

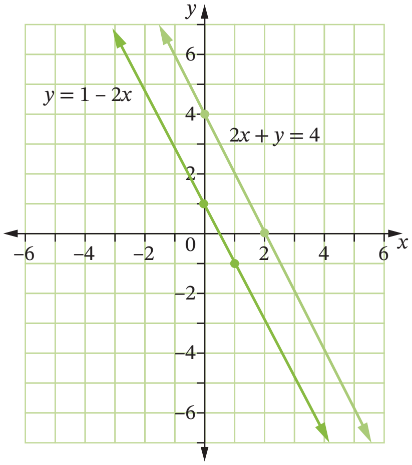
i $x = 5, y = 1\frac{1}{2}$

j $x = 5, y = 8$

k $x = 1\frac{1}{2}, y = 2\frac{1}{2}$

l $x = 4, y = 0$

2 **a**



b The lines are parallel.

Exercise 9.02

1 **a** $d = -3, k = 2$ **b** $x = 5, w = 4$

c $g = 2, h = -\frac{2}{5}$ **d** $n = 3\frac{1}{4}, p = -1$

e $q = 5, r = -4$ **f** $k = -4\frac{3}{5}, x = 5.$

g $c = 1\frac{1}{2}, e = 1$ **h** $k = 3, y = -2$

i $a = 2, f = 2$

2 **a** $d = -14, k = 6$ **b** $a = 1, c = 1$

c $h = 3, y = 4$ **d** $e = 3, x = \frac{1}{3}$

e $q = 3, w = 6\frac{1}{2}$ **f** $c = 3, p = 4$

g $m = -\frac{2}{3}, y = 4$ **h** $a = -1, r = 5\frac{1}{2}$

i $x = 2, w = 2$ **j** $g = -2, y = 4$

k $e = -5, n = -8$ **l** $k = 5\frac{1}{2}, h = 1\frac{1}{2}$

3 **a** $q = -3, w = 3$ **b** $m = -9, x = 7$

c $d = 23, h = -7$ **d** $g = -1, n = 3$

e $h = 0, m = 2$ **f** $e = -4, y = 3$

g $q = 1, w = -4$ **h** $a = \frac{1}{2}, d = \frac{1}{2}$

i $k = 5, p = -2$ **j** $a = -2, f = -2$

k $c = -64, r = -38$ **l** $x = -4, y = -3$

m $x = 8, y = -1$ **n** $g = 4\frac{1}{2}, h = 2\frac{1}{2}$

o $k = -2\frac{1}{2}, w = 3\frac{1}{2}$

Exercise 9.03

1 **a** $x = 2, y = 5$ **b** $x = \frac{3}{5}, y = 3\frac{4}{5}$ **c** $x = 7, y = 2$

d $x = 2, y = -2$ **e** $x = 5, y = -1$ **f** $x = 4, y = 2$

2 **a** $x = 9, y = 21$ **b** $x = 5, y = 3$ **c** $x = -\frac{1}{4}, y = 2$

d $x = -3, y = 1$ **e** $x = 2, y = 2$ **f** $x = -7, y = 4$

g $x = 7, y = 3$ **h** $x = 3, y = 2\frac{1}{2}$ **i** $x = 2\frac{2}{3}, y = 1$

j $x = -3\frac{1}{5}, y = -4\frac{3}{5}$

Exercise 9.04

1 285 **2** 680

3 **a** Teacher to check **b** 396

4 55 **5** Tayyab 36, Sejuti 12

6 16 **7** black 37, colour 23

8 Children: $15.50, Adult: $21.50

9 Supreme 32, Vegetarian 13

10 Strawberries $3.95; Blueberries $3.50

11 **a** Teacher to check.

b 20-cent coins: 485, 50-cent coins: 368

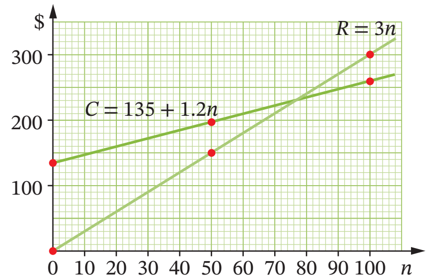
12 **a** Teacher to check.

b $C = 135 + 1.2n$ $R = 3n$

n	0	50	100
C	135	195	255

n	0	50	100
R	0	150	300

c



d $n = 75$

Mental skills 9

2 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{5}{7}$ **d** $\frac{1}{2}$

e $\frac{1}{4}$ **f** $\frac{1}{6}$ **g** $\frac{5}{6}$ **h** $\frac{2}{5}$

i 5 : 9 **j** 5 : 9 **k** 9 : 20 **l** 4 : 5

m 9 : 7 **n** 4 : 3 **o** $\frac{3}{5}$ **p** $\frac{4}{35}$

3 **a** $\frac{17}{40}$ **b** $\frac{2}{3}$ **c** $\frac{16}{25}$

d $\frac{1}{4}$ **e** $\frac{5}{24}$ **f** $\frac{2}{25}$

Power plus

1 **a** $x = 1\frac{1}{2}, y = -5\frac{1}{2}, w = 4\frac{1}{2}$

b $a = 1\frac{7}{13}, c = 4\frac{3}{13}, d = 8\frac{11}{13}$

c $p = -11\frac{3}{13}, m = 18\frac{11}{13}, n = -13\frac{4}{13}$

2 **a** Teacher to check.

b $ae - bd = 0$ and a fraction cannot have denominator 0.

c $x = 3, y = -1$

d Teacher to check.

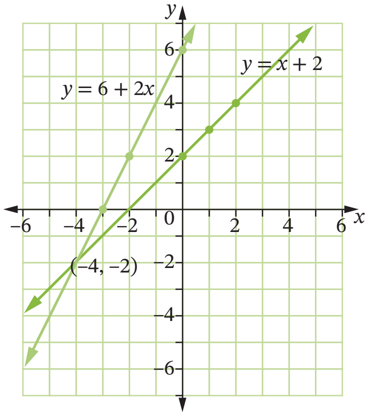
i $x = 2, y = -2$ **ii** $x = 28, y = 16$

iii $x = \frac{1}{11}, y = 2\frac{20}{33}$

Test yourself 9

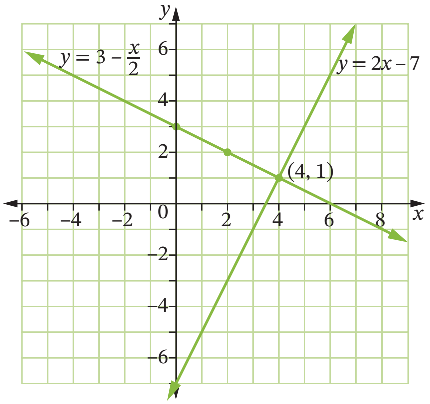
1 **a** $x = 2, y = -2$ **b** $x = 4, y = 0$ **c** $x = \frac{1}{4}, y = 1\frac{1}{2}$

2 **a**



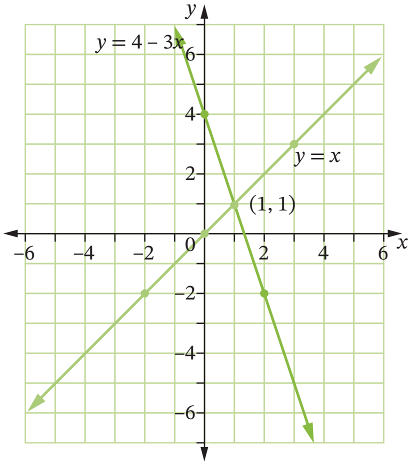
$x = -4, y = -2$

b



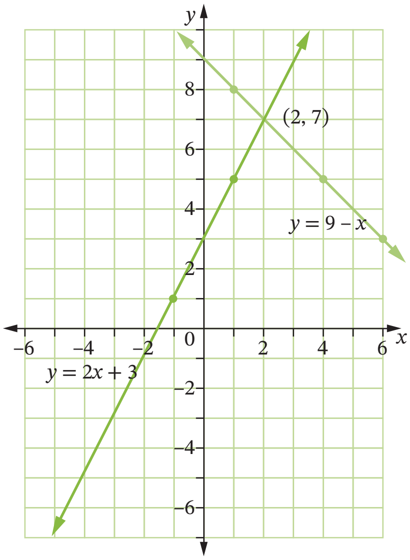
$x = 4, y = 1$

c



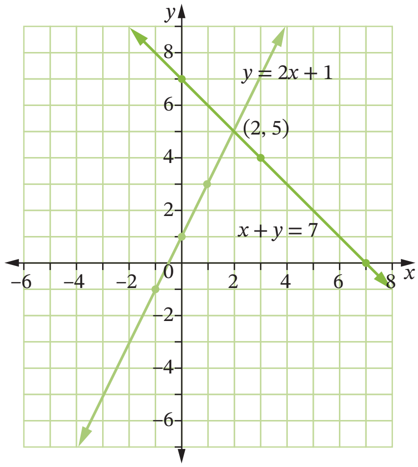
$x = 1, y = 1$

d



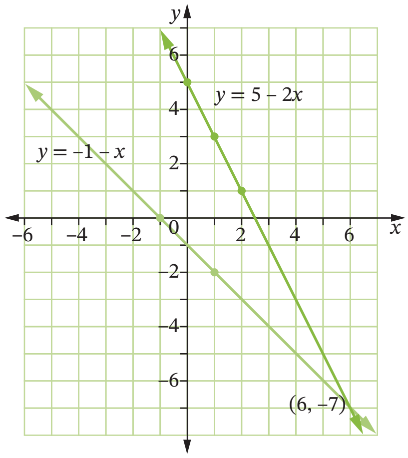
$x = 2, y = 7$

e



$x = 2, y = 5$

f



$x = 6, y = -7$

3 **a** $m = 5, c = -9\frac{1}{2}$ **b** $x = 2, y = \frac{1}{3}$
c $a = 1, d = 1$ **d** $x = 6, y = 15$
e $x = -5, y = -2$ **f** $d = -3, w = -10$

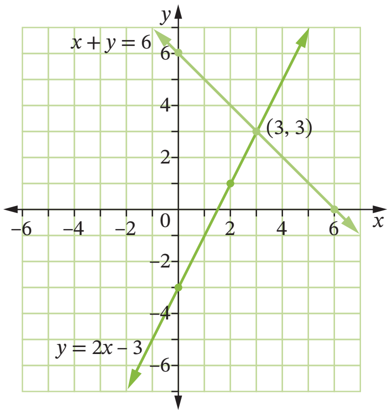
4 **a** $x = 2, y = 11$ **b** $m = 1, p = 3$
c $h = 10, t = 4$ **d** $a = 3, c = \frac{1}{2}$
e $x = 1, y = 1$ **f** $p = 12, q = -4$

5 **a** 1600 adults, 900 children
b 18 children, 12 adults
c \$38
d 28 cheesecakes, 47 mudcakes
e 120 boys, 93 girls

Practice set 3

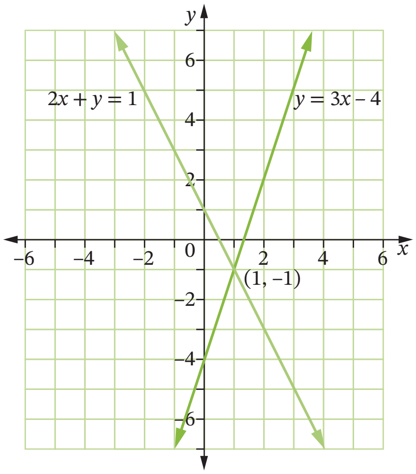
1 **a** € 17.50 **b** A\$86 **c** € 98

2 **a**



$x = 3, y = 3$

b

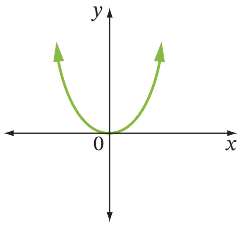


$x = 1, y = -1$

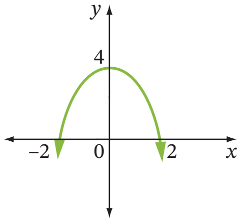
3 158.4 **4** A **5** C

6 **a** 6.8 cm **b** 22.7 m **c** 10.8 cm

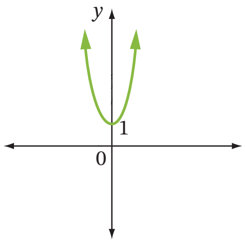
7 **a** **i**



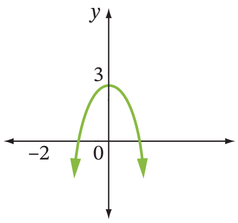
ii



iii



iv



b **i** $y = x^2, y = 3x^2 + 1$
ii $y = 4 - x^2, y = 3 - 2x^2$
iii $y = 3x^2 + 1$

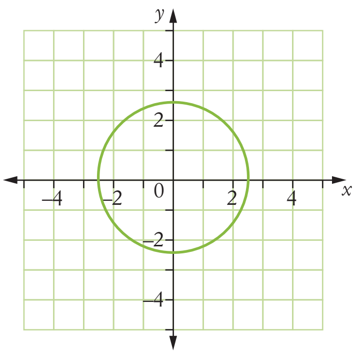
8 **a** $t = \frac{99.96}{S}$ **b** 9.52 m/s **c** 9.70 s

9 **a** $g = 2, w = -1$ **b** $f = 3, y = 3$
c $a = -1, c = -2$

10 **a** $d = 51.9$ **b** $e = 58.1$ **c** $f = 3.7$

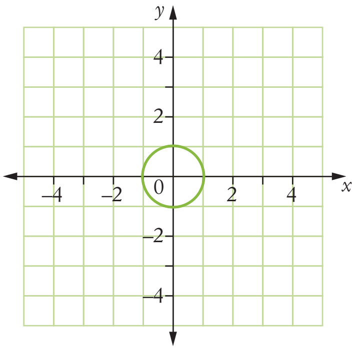
11 **a** 29° **b** 45° **c** 43°

12 **a**



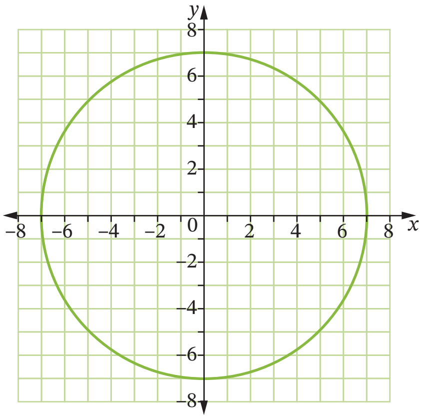
centre (0, 0), radius = 2.5

b



centre (0, 0), radius = 1

c



centre (0, 0), radius = 7

13 71 m

14 **a** 17.5 hours **b** 6 painters

15 **a** $A + C = 395, 20A + 15C = 6700$

b 240 children

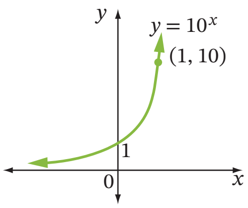
16 C **17** 78° **18** B

19 **a** $x = 2\frac{1}{2}, y = 5\frac{1}{2}$ **b** $w = 2, p = 1$

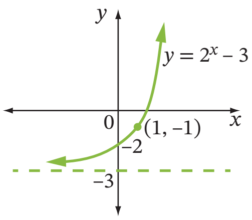
c $g = \frac{1}{2}, k = 3$

20 $x = \pm 2$

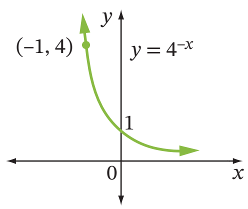
21 **a**



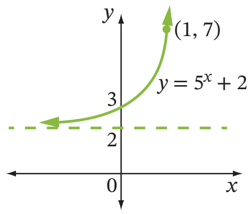
b



c



d



22 **a** I **b** G **c** J **d** C

e B **f** D **g** F **h** E

i K **j** L **k** A **l** H

23 50 m

24 51°3′

25 **a** 280° **b** 140° **c** 200°

26 **a** 301 km **b** 114°

27 C **28** 747 km

29 **a** 55.6 cm **b** 189.6 m

Chapter 10

SkillCheck

1 C

2 **a** 3

b No, $P(\text{10c coin}) = \frac{5}{12}$,

$P(\text{20c coin}) = \frac{1}{3}, P(\text{50c coin}) = \frac{1}{4}$

3 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{5}{6}$

4 **a** 0 **b** 1

5 0.4 **6** B **7** 0.15

Exercise 10.01

1 **a** **i** 0.425 **ii** 0.14 **iii** 0.21

b **i** 0.375 **ii** 0.125 **iii** 0.25

c Yes

d Expected frequency = 100. The observed frequency of red or purple is 115, which is more than the expected frequency.

2 **a** **i** $\frac{1}{5} = 0.2$ **ii** $\frac{19}{50} = 0.38$

iii $\frac{33}{100} = 0.33$ **iv** $\frac{9}{100} = 0.09$

b **i** $\frac{1}{4} = 0.25$ **ii** $\frac{7}{20} = 0.35$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{1}{10} = 0.1$

c Yes

d Expected frequency = 40. The expected frequency compares very favourably with the observed frequency of 42.

9780170453417

3 **a** 50 **b** Teacher to check.

c i Teacher to check. **ii** $\frac{1}{2}$

d Teacher to check.

4 **a** 600

b i $\frac{281}{600} \approx 0.468$ **ii** $\frac{322}{600} \approx 0.537$

iii $\frac{227}{600} \approx 0.378$ **iv** $\frac{522}{600} = 0.87$

c i 0.5 **ii** 0.5 **iii** 0.33 **iv** 0.83

d The probabilities are similar.

5 **a** Teacher to check.

b i $\frac{1}{2} = 0.5$ **ii** $\frac{1}{5} = 0.2$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{7}{10} = 0.7$

c Teacher to check.

6 **a i** $\frac{3}{10} = 0.3$ **ii** $\frac{3}{25} = 0.12$

iii $\frac{12}{25} = 0.48$ **iv** $\frac{1}{10} = 0.1$

b i 0.33 **ii** 0.17 **iii** 0.33 **iv** 0.17

c No, the experimental probability of yellow was higher and the experimental probability of green was lower.

d Expected frequency of not yellow is 33. This is more than the observed frequency of 26.

7 **a** 200

b i $\frac{4}{200} = 0.02$ **ii** $\frac{27}{200} = 0.135$

iii $\frac{13}{200} = 0.065$ **iv** $\frac{86}{200} = 0.43$

v $\frac{87}{200} = 0.435$ **vi** $\frac{59}{200} = 0.295$

vii $\frac{51}{200} = 0.255$

8 **a** 200

b i $\frac{27}{200} = 0.135$ **ii** $\frac{62}{200} = 0.31$

iii $\frac{80}{200} = 0.4$ **iv** $\frac{21}{200} = 0.105$

v $\frac{1}{200} = 0.005$

c Ferry, light rail (tram)

d Teacher to check.

9 **a** $\frac{1}{6} \approx 0.17$ **b** 16 or 17 times

c, d, e Teacher to check.

10 Teacher to check. The probability of winning would be different for each horse, due to methods of training, strength of horse and conditions of the racetrack, so it is not a simple probability of 1 in 10.

Exercise 10.02

1 C

2 **a i** $\frac{11}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{1}{5}$ **iv** $\frac{6}{25}$ **v** $\frac{19}{25}$

b $\frac{3}{10}$

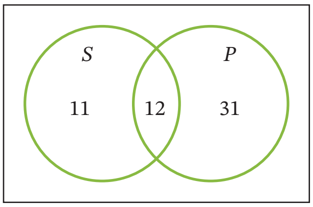
3 **a** 156

b i $\frac{11}{52}$ **ii** $\frac{7}{52}$ **iii** $\frac{7}{78}$ **iv** $\frac{22}{39}$ **v** $\frac{19}{78}$ **vi** $\frac{1}{26}$

c $\frac{7}{31}$

4 **a** 135 **b** $\frac{56}{135}$ **c** $\frac{1}{5}$ **d** $\frac{17}{52}$

5 **a**



b i 1 **ii** $\frac{11}{54}$ **iii** $\frac{31}{54}$ **iv** $\frac{7}{9}$

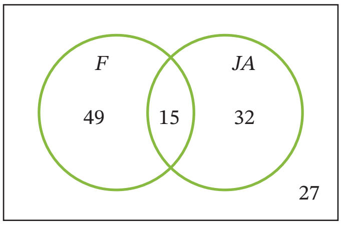
6 **a** 92

b i $\frac{1}{46}$ **ii** $\frac{15}{92}$ **iii** $\frac{5}{92}$ **iv** $\frac{17}{23}$

c i $\frac{21}{29}$ **ii** $\frac{5}{29}$

7 **a** 123

b



c 81

d i $\frac{49}{123}$ **ii** $\frac{32}{123}$ **iii** $\frac{9}{41}$ **iv** $\frac{27}{41}$

8 **a** 200

b i $\frac{79}{200}$ **ii** $\frac{51}{100}$ **iii** $\frac{77}{200}$

iv $\frac{121}{200}$ **v** $\frac{81}{100}$ **vi** $\frac{3}{50}$

c $\frac{29}{40}$

d No, because all the people surveyed indicated a day on which they preferred to shop.

9 **a** 196

b i $\frac{7}{196}$ **ii** $\frac{13}{98}$ **iii** $\frac{15}{49}$ **iv** $\frac{34}{49}$

c i $\frac{6}{59}$ **ii** $\frac{43}{59}$

Exercise 10.03

1 **a** 150

b i $\frac{4}{25}$ **ii** $\frac{53}{150}$ **iii** $\frac{28}{75}$

c 63%

2 **a** 128

b i 68 **ii** 60

c $\frac{17}{32}$ **d** $\frac{3}{32}$

3 **a** 97

b i 32.0% **ii** 25.8% **iii** 11.3%

c i 21.2% **ii** 44.4%

d The percentage of females in the opposition is just more than double that of females in the government.

4 **a** 150

b **i** 0.5 **ii** 0.04 **iii** 0.43 **iv** 0.23

c $\frac{22}{75} = 0.293$

5 **a** 160

b **i** $\frac{7}{20} = 0.35$ **ii** $\frac{11}{160} \approx 0.069$ **iii** $\frac{9}{80} \approx 0.113$

c $\frac{35}{82} \approx 0.43$

6 **a** 200 **b** 55%

c **i** 49.5% **ii** 45%

iii 36% **iv** 31.5%

d $\frac{72}{110} \approx 65.5\%$

7 **a** 878

b **i** $\frac{679}{878} = 0.773$ **ii** $\frac{545}{878} = 0.621$

iii $\frac{67}{439} = 0.153$ **iv** $\frac{21}{439} = 0.048$

Exercise 10.04

1 **a**

		Girls					
		Be	**Ca**	**Em**	**M**	**R**	**S**
Boys	**Be**	Ben, Be	Ben, Ca	Ben, Em	Ben, M	Ben, R	Ben, S
	C	C, Be	C, Ca	C, Em	C, M	C, R	C, S
	Ew	Ew, Be	Ew, Ca	Ew, Em	Ew, M	Ew, R	Ew, S
	W	W, Be	W, Ca	W, Em	W, M	W, R	W, S

b **i** $\frac{1}{24}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$

2 **a**

	H	*T*
H	HH	HT
T	TH	TT

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

3 **a**

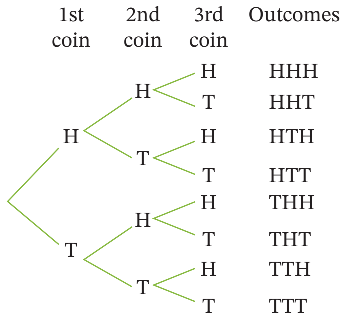
		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

b 36

c **i** $\frac{1}{36}$ **ii** $\frac{11}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{4}$

v $\frac{11}{36}$ **vi** $\frac{1}{4}$ **vii** $\frac{1}{2}$ **viii** $\frac{5}{12}$

4 **a**



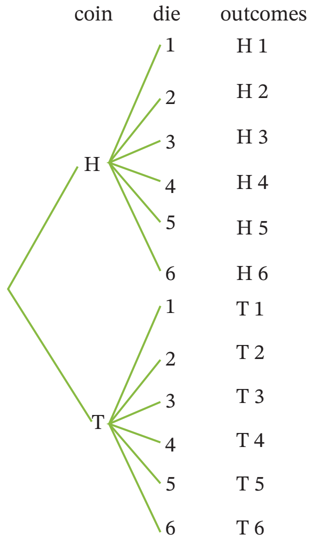
b 8

c **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{1}{2}$

d **i** $\frac{7}{8}$ **ii** $\frac{7}{8}$

e **i** 75 **ii** 25

5 **a**



6 **a**

		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4

b 24

c **i** $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$ **v** 0

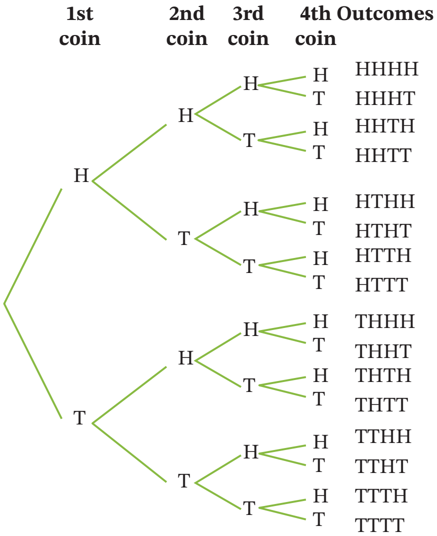
7 **a**

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b **i** $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

v 0 **vi** $\frac{5}{12}$ **vii** $\frac{7}{12}$ **viii** $\frac{5}{12}$

8 **a**

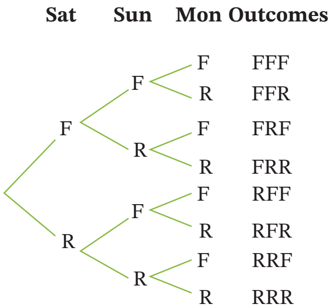


b **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{8}$

iv $\frac{15}{16}$ **v** $\frac{1}{16}$ **vi** $\frac{5}{16}$

c **i** 63 **ii** 375 **iii** 938

9 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

Mental skills 10

2 **a** $408 **b** 99 **c** 200 **d** $404
e 672 **f** $81 **g** $517 **h** 225
i 560 **j** $84 **k** $350 **l** 84

4 **a** 330 **b** $240 **c** 1600 **d** $425
e $225 **f** $60 **g** $63 **h** 76
i $68 **j** $3762 **k** $374 **l** $100

Exercise 10.05

1 **a** C = Cate, A = Amal, G = Gemma, J = Josie, E = Evangeline, R = Rukshana

		Captain					
		C	A	G	J	E	R
Vice-captain	C	-	AC	GC	JC	EC	RC
	A	CA	-	GA	JA	EA	RA
	G	CG	AG	-	JG	EG	RG
	J	CJ	AJ	GJ	-	EJ	RJ
	E	CE	AE	GE	JE	-	RE
	R	CR	AR	GR	JR	ER	-

30 possible pairings

b $\frac{1}{3}$ **c** $\frac{1}{6}$

2 **a** **i**

	A	B	C	D	E
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

ii

	A	B	C	D	E
A		AB	AC	AD	AE
B	BA		BC	BD	BE
C	CA	CB		CD	CE
D	DA	DB	DC		DE
E	EA	EB	EC	ED	

b **i** $\frac{1}{5}$ **ii** $\frac{4}{25}$ **iii** $\frac{12}{25}$

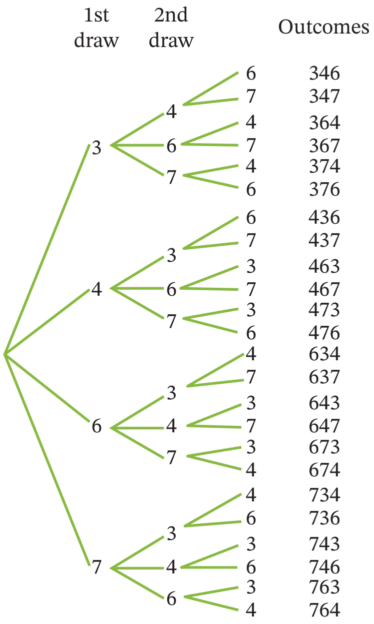
c **i** $\frac{1}{10}$ **ii** $\frac{3}{5}$ **iii** $\frac{2}{5}$ **iv** $\frac{4}{5}$

3 **a**

		2nd course				
		B	H	P	S	T
1st course	C	CB	CH	CP	CS	CT
	F	FB	FH	FP	FS	FT
	Y	YB	YH	YP	YS	YT

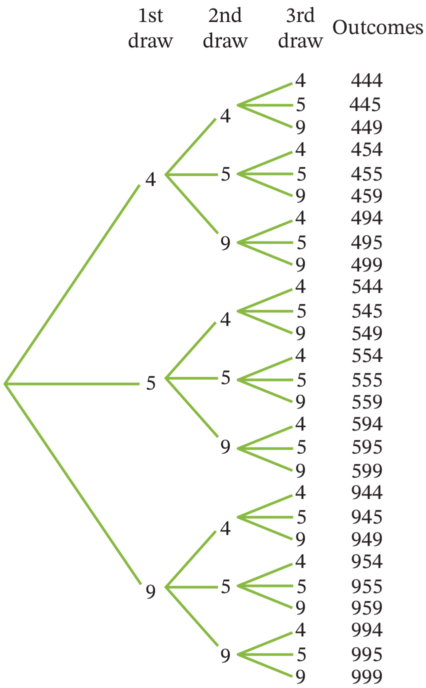
b **i** $\frac{1}{3}$ **ii** $\frac{4}{15}$ **iii** $\frac{1}{15}$

4 **a**



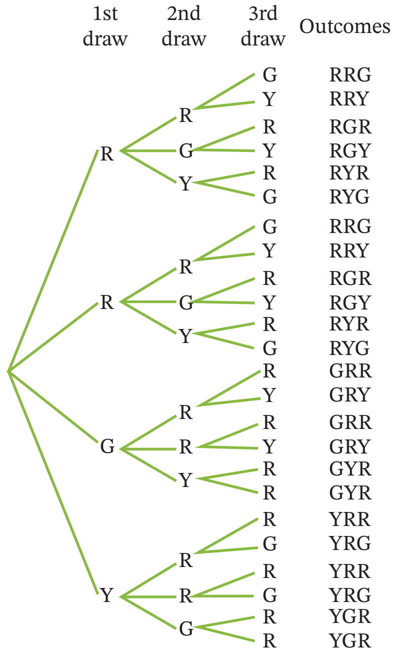
b **i** $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

5 **a**



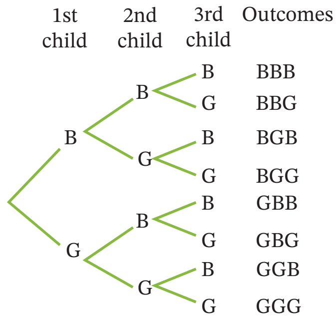
b **i** $\frac{1}{9}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{9}$

6 **a**



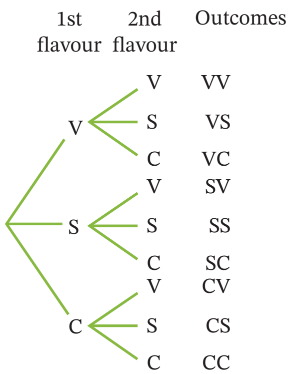
b **i** $\frac{1}{2}$ **ii** $\frac{1}{12}$ **iii** 1

7 **a**

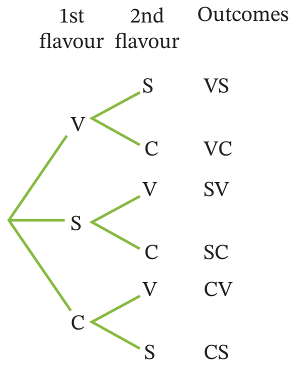


b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{8}$

8 **a** **i**



ii



b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{4}{9}$ **iv** $\frac{5}{9}$

c **i** 0 **ii** 1 **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

Exercise 10.06

1 **a** independent **b** independent **c** dependent **d** independent **e** dependent **f** independent **g** independent

2 Dependent, as the balls are not replaced when drawn.

3 **a** independent **b** $\frac{1}{2}$

4 **a** **i** $\frac{1}{3}$ **ii** $\frac{1}{4}$

b 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 1G, 2G, 3G, 4G, 5G, 6G, 1B, 2B, 3B, 4B, 5B, 6B, 1R, 2R, 3R, 4R, 5R, 6R,

c $\frac{1}{12}$ **d** Yes, $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

e independent

5 **a** **i** $\frac{1}{6}$ **ii** $\frac{1}{2}$

b $\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$

6 **a** $\frac{5}{9}$ **b** $\frac{4}{8}=\frac{1}{2}$

c Dependent, as the first draw changes the contents of the bag.

7 **a** **i** $\frac{5}{8}$ **ii** $\frac{4}{7}$

b **i** $\frac{3}{8}$ **ii** $\frac{5}{7}$

c **i** $\frac{5}{8}$ **ii** $\frac{3}{7}$

d **i** $\frac{3}{8}$ **ii** $\frac{2}{7}$

8 $\frac{1}{2}$

9 **a** $\frac{1}{12}$ **b** $\frac{1}{36}$ **c** $\frac{1}{3}$ **d** $\frac{1}{6}$

10 **a** $\frac{13}{25}$ **b** $\frac{7}{25}$

11 **a** 0.15 **b** 0.1 **c** 0.3

12 **a** 0.147 **b** 0.343 **c** 0.027 **d** 0.189

Exercise 10.07

1 **a** $\frac{2}{6}=\frac{1}{3}$ **b** $\frac{3}{6}=\frac{1}{2}$

2 $\frac{2}{9}$

3 **a** $\frac{4}{11}$ **b** $\frac{7}{11}$

4 **a** $\frac{2}{11}$ **b** $\frac{4}{11}$ **c** $\frac{4}{11}$ **d** $\frac{5}{11}$

5 $\frac{1}{3}$ **6** $\frac{1}{6}$

7 **a**

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b **i** $\frac{1}{2}$ **ii** 1

c $\frac{2}{11}$

d **i** $\frac{9}{27}=\frac{1}{3}$ **ii** $\frac{6}{27}=\frac{2}{9}$

e $\frac{1}{6}$

8 **a** **i** $\frac{1}{9}$ **ii** $\frac{8}{9}$

b $\frac{2}{8}=\frac{1}{4}$ **c** 6

9 $\frac{1}{40}$ **10** $\frac{10}{49}$ **11** $\frac{1}{13}$ **12** $\frac{1}{6}$

13 **a**

		1st die					
		1	2	3	4	5	6
2nd die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b **i** $\frac{1}{6}$ **ii** $\frac{1}{18}$ **iii** $\frac{1}{6}$

c **i** $\frac{1}{2}$ **ii** 0

d **i** $\frac{1}{11}$ **ii** $\frac{4}{11}$

e **i** $\frac{6}{18}=\frac{1}{3}$ **ii** 1

ANSWERS

Power plus

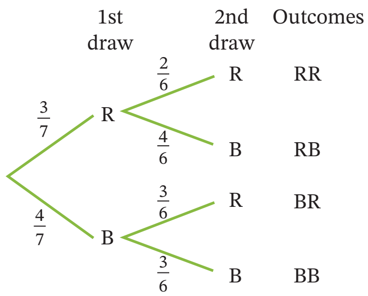
1 **a** 320

b **i** $\frac{7}{40}=0.175$ **ii** $\frac{3}{32}\approx0.094$

iii $\frac{57}{320}\approx0.178$ **iv** $\frac{3}{20}=0.15$

c $\frac{16}{45}$ **d** $\frac{45}{107}$

2 **a**



b **i** $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{6}{7}$

3 **a** **i** $\frac{14}{30}=\frac{7}{15}$ **ii** $\frac{20}{30}=\frac{2}{3}$ **iii** $\frac{4}{30}=\frac{2}{15}$

iv $\frac{4}{20}=\frac{1}{5}$ **v** $\frac{4}{14}=\frac{2}{7}$

b **i** $\frac{\frac{2}{15}}{\frac{2}{3}}=\frac{1}{5}$ **ii** Yes

c $P(B|A)=\frac{2}{7}, \frac{P(A \text{ and } B)}{P(A)}=\frac{\frac{2}{15}}{\frac{7}{15}}=\frac{2}{7}$

Test yourself 10

1 **a** **i** 0.22 **ii** 0.29 **iii** 0.1 **iv** 0.14

b **ii** 0.25 **ii** 0.25 **iii** 0.125 **iv** 0.125

c Yes **d** 0.25

e The expected number of times the arrow stops at a colour not purple or black is 75, which is the same as the observed number of times.

2 **a** **i** 0.353 **ii** 0.427 **iii** 0.087 **iv** 0.513

b **i** 0.867 **ii** 0.133

c Different – at least one head occurring excludes 0 heads occurring, which is the same as 3 tails occurring. The events are complementary.

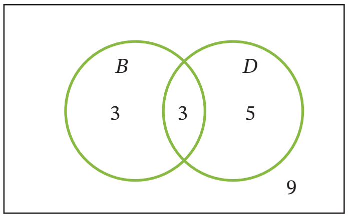
3 **a** 90 **b** $\frac{19}{45}$ **c** $\frac{2}{9}$ **d** $\frac{11}{49}$

4 **a** 35 **b** $\frac{2}{35}$

c **i** $\frac{6}{35}$ **ii** $\frac{19}{35}$ **iii** $\frac{3}{35}$

d They don't like the types of music mentioned in the survey.

5 **a**



b **i** $\frac{3}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{20}$ **iv** $\frac{3}{5}$

c $\frac{9}{20}$

6 **a** 200

b **i** 0.305 **ii** 0.11 **iii** 0.225 **iv** 0.425

c $\frac{25}{110}\approx0.227$

d **i** $\frac{47}{69}\approx0.681$ **ii** $\frac{22}{69}\approx0.319$

7 D

8 **a**

		1st die			
		1	2	3	4
2nd die	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4

b 16

c **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{16}$

iv $\frac{1}{4}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{2}$

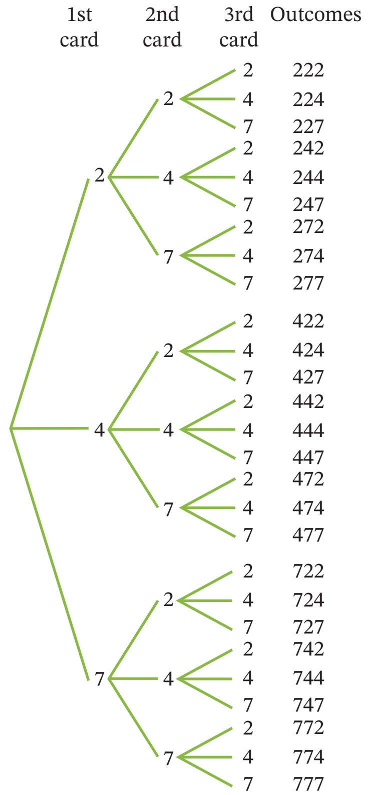
9 **a** **i** RR, RB, RG, RY, RBla, BR, BB, BG, BY, BBla, GR, GB, GG, GY, GBla, YR, YB, YG, YY, YBla, BlaR, BlaB, BlaG, BlaY, BlaBla

ii RB, RG, RY, RBla, BR, BG, BY, BBla, GR, GB, GY, GBla, YR, YB, YG, YBla, BlaR, BlaB, BlaG, BlaY

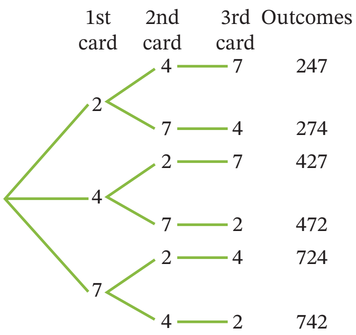
b **i** $\frac{1}{25}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{25}$ **iv** $\frac{9}{25}$

c **i** $\frac{1}{10}$ **ii** $\frac{3}{5}$

10 a i



ii



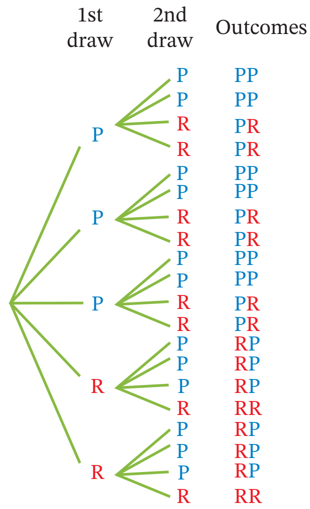
b i $\frac{2}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{2}{9}$

c i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$

11 a independent **b** dependent **c** dependent

d independent **e** dependent **f** independent

12 a



b i $\frac{1}{2}$ **ii** $\frac{1}{2}$

c i $\frac{1}{4}$ **ii** $\frac{3}{4}$

d i $\frac{1}{4}$ **ii** $\frac{3}{4}$

13 a

		1st die			
		1	**2**	**3**	**4**
2nd die	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b i $\frac{1}{2}$ **ii** 1 **c** $\frac{2}{7}$

d i $\frac{1}{3}$ **ii** $\frac{1}{3}$ **e** $\frac{1}{4}$

Chapter 11

SkillCheck

1 a $w = 38$ **b** $r = 44$ **c** $h = 71$

d $x = 126$ **e** $y = 46$ **f** $a = 24$

2 **A** and **G**, **C** and **J**, **D** and **F**, **H** and **I**

3 a $p = 64$ **b** $p = 241$ **c** $p = 104$

d $p = 105$ **e** $p = 58$ **f** $p = 128$

4 a $\angle M$ and $\angle X$, $\angle N$ and $\angle Y$, $\angle P$ and $\angle W$

b MN and XY, NP and YW, MP and XW

Exercise 11.01

1 a heptagon **b** quadrilateral **c** decagon

d octagon **e** triangle **f** pentagon

2 a **a, b, d, e, f** **b** **d**

3 C

4 **a** 1800° **b** 1440° **c** 1260°
d 3240° **e** 2340°

5 **a** $e = 112$ **b** $m = 135$
c $x = 83, y = 97$ **d** $w = 108, h = 72$
e $k = 203$ **f** $a = 32$

6 **a** 6 **b** 21 **c** 13 **d** 30 **e** 9

7 **a** 16 **b** 157.5°

8 **a** 144° **b** 135° **c** 120° **d** 150°

9 C

10 **a** 30 **b** 45 **c** 25

Exercise 11.02

1 **a** 72° **b** 30° **c** 20° **d** 60°

2 **a** 140° **b** 162° **c** 144° **d** 168°

3 **a** 24 **b** 5 **c** 18 **d** 9 **e** 72

4 C

5 **a** 8 **b** 10 **c** 15 **d** 180 **e** 24 **f** 12

Exercise 11.03

1 **a** $\triangle AXT \equiv \triangle GYE$ (RHS) **b** $\triangle MVR \equiv \triangle SPC$ (SSS)
c $\triangle BTV \equiv \triangle PSX$ (AAS) **d** $\triangle CNQ \equiv \triangle DCE$ (SAS)

2 In $\triangle WXY$ and $\triangle WZY$
$\angle X = \angle Z = 90°$ (given)
$XY = ZY$ (given)
WY is common.
$\therefore \triangle WXY \equiv \triangle WZY$ (RHS)

3 In $\triangle DEH$ and $\triangle FEG$
$\angle D = \angle F$ (alternate angles, $HD \parallel GF$)
$\angle H = \angle G$ (alternate angles, $HD \parallel GF$)
$EH = EG$ (given)
$\therefore \triangle DEH \equiv \triangle FEG$ (AAS)

4 In $\triangle ABC$ and $\triangle ADC$
$BC = DC$ (given)
$AB = AD$ (given)
AC is common.
$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

5 In $\triangle PQN$ and $\triangle LQM$
$\angle PQN = \angle LQM$ (vertically opposite angles)
$PQ = LQ$ (given)
$NQ = MQ$ (given)
$\therefore \triangle PQN \equiv \triangle LQM$ (SAS)

6 In $\triangle WXY$ and $\triangle YVW$
$XY = VW$ (given)
$WX = YV$ (given)
WY is common.
$\therefore \triangle WXY \equiv \triangle YVW$ (SSS)

7 In $\triangle AOB$ and $\triangle COD$
$OA = OC$ (equal radii)
$OB = OD$ (equal radii)
$AB = CD$ (given)
$\therefore \triangle AOB \equiv \triangle COD$ (SSS)

8 In $\triangle FNM$ and $\triangle TMN$
$\angle FNM = \angle TMN$ (alternate angles, $FN \parallel TM$)
$\angle FMN = \angle TNM$ (alternate angles, $TN \parallel FM$)
NM is common.
$\therefore \triangle FNM \equiv \triangle TMN$ (AAS)

9 In $\triangle ABC$ and $\triangle DCB$
$\angle ABC = \angle DCB$ (given)
$AB = DC$ (given)
BC is common.
$\therefore \triangle ABC \equiv \triangle DCB$ (SAS)

10 In $\triangle TSK$ and $\triangle LPK$
$\angle T = \angle L$ (alternate angles, $TS \parallel PL$)
$\angle S = \angle P$ (alternate angles, $TS \parallel PL$)
$TK = LK$ (K is the midpoint of TL)
$\therefore \triangle TSK \equiv \triangle LPK$ (AAS)

11 In $\triangle PVQ$ and $\triangle QTR$
$\angle VPQ = \angle TQR$ (corresponding angles, $PW \parallel QT$)
$\angle VQP = \angle TRQ$ (corresponding angles, $RW \parallel VQ$)
$PQ = QR$ (given)
$\therefore \triangle PVQ \equiv \triangle QTR$ (AAS)

12 In $\triangle DEG$ and $\triangle EDF$
$\angle DEG = \angle EDF$ (given)
$GE = FD$ (given)
ED is common.
$\therefore \triangle DEG \equiv \triangle EDF$ (SAS)

13 In $\triangle ABD$ and $\triangle ACD$
$AB = AC$ (given)
AD is common.
$\angle ADB = \angle ADC = 90°$ ($AD \perp BC$)
$\therefore \triangle ABD \equiv \triangle ACD$ (RHS)
$\therefore \angle BAD = \angle CAD$ (matching angles of congruent triangles)
$\therefore AD$ bisects $\angle BAC$

14 In $\triangle BCX$ and $\triangle CBY$
$\angle BXC = \angle CYB = 90°$ ($CX \perp AB$, $BY \perp AC$)
$XC = YB$ (given)
BC is common.
$\therefore \triangle BCX \equiv \triangle CBY$ (RHS)

15 In $\triangle WYX$ and $\triangle ZYX$
$XW = XZ$ (given)
$WY = ZY$ (Y is midpoint of WZ)
XY is common.
$\therefore \triangle WYX \equiv \triangle ZYX$ (SSS)

Exercise 11.04

1 **a** SSS
b $\angle ADB = \angle ADC$ (matching angles of congruent triangles)
c $\angle ADB + \angle ADC = 180°$ (angles on a line)
$\therefore \angle ADB = \angle ADC = 90°$
$\therefore AD \perp BC$

2 **a** AAS
b $WS = WT$ (matching sides of congruent triangles)
c A triangle with 2 equal angles is isosceles (has 2 equal sides) and the equal sides are the sides opposite the equal angles.

3 **a** $\triangle ABX \equiv \triangle ACX$ (SSS)
b $\angle B = \angle C$ (matching angles of congruent triangles)
c $\triangle BAY \equiv \triangle BCY$ (SSS)

d $\angle A = \angle C$ (matching angles of congruent triangles)

e Since $\angle B = \angle C$, $\angle A = \angle C$,

$\angle A = \angle B = \angle C$

$\angle A + \angle B + \angle C = 180°$ (angle sum of a triangle)

$\therefore \angle A = \angle B = \angle C = 60°$

f Each angle in an equilateral triangle is 60°.

4 a $\angle MPY = \angle NPY$ (PY bisects $\angle MPN$)

b $\triangle PMY \equiv \triangle PNY$ (AAS)

c $MY = NY$ (matching sides of congruent triangles)

d $\angle PYM = \angle PYN$ (matching angles of congruent triangles)

e $\angle PYM + \angle PYN = 180°$ (angles on a straight line)

$\therefore \angle PYM = \angle PYN = 90°$

$\therefore PY \perp MN$

5 a In $\triangle ABC$ and $\triangle CDA$

$CB = AD$ (given)

$AB = CD$ (given)

AC is common.

$\therefore \triangle ABC \equiv \triangle CDA$ (SSS)

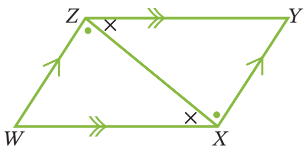
b $\angle BAC = \angle DCA$ (matching angles of congruent triangles)

$\angle BCA = \angle DAC$ (matching angles of congruent triangles)

c $\therefore AB \parallel CD$ and $AD \parallel BC$ (alternate angles are equal)

d $ABCD$ is a parallelogram since opposite sides are parallel.

6 b



c In $\triangle WXZ$ and $\triangle YZX$

$\angle WXZ = \angle YZX$ (alternate angles, $WX \parallel YZ$)

$\angle WZX = \angle YXZ$ (alternate angles, $WZ \parallel YX$)

XZ is common.

$\therefore \triangle WXZ \equiv \triangle YZX$ (AAS)

d $\angle W = \angle Y$ (matching angles of congruent triangles)

e In $\triangle WXY$ and $\triangle YZW$

$\angle XYW = \angle ZWY$ (alternate angles, $XY \parallel ZW$)

$\angle XWY = \angle ZYW$ (alternate angles, $WX \parallel YZ$)

WY is common.

$\therefore \triangle WXY \equiv \triangle YZW$ (AAS)

f $\angle WXY = \angle YZW$ (matching angles of congruent triangles)

g Opposite angles of a parallelogram are equal.

7 a In $\triangle VUS$ and $\triangle TUS$

$VU = TU$ (given)

$VS = TS$ (given)

SU is common.

$\therefore \triangle VUS \equiv \triangle TUS$ (SSS)

b $\angle VUS = \angle TUS$ (matching angles of congruent triangles)

$\angle VSU = \angle TSU$ (matching angles of congruent triangles)

$\therefore VS$ bisects $\angle VUT$ and $\angle VST$.

8 a In $\triangle ABD$ and $\triangle CDB$

$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)

$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)

BD is common.

$\therefore \triangle ABD \equiv \triangle CDB$ (AAS)

b $AD = CB$ (matching sides of congruent triangles)

$AB = CD$ (matching sides of congruent triangles)

c $\therefore$ opposite sides of a parallelogram are equal.

9 a In $\triangle BEL$ and $\triangle GHL$

$\angle BEL = \angle GHL$ (alternate angles, $BE \parallel GH$)

$\angle BLE = \angle GLH$ (vertically opposite angles are equal)

$BE = GH$ (given)

$\therefore \triangle BEL \equiv \triangle GHL$ (AAS)

b $BL = GL$ (matching sides of congruent triangles)

$EL = HL$ (matching sides of congruent triangles)

$\therefore$ diagonals of a rhombus bisect each other.

c A

d In $\triangle BEL$ and $\triangle BHL$

$\angle BEL = \angle BHL$ (proven in **c**)

$BE = BH$ (given)

$EL = HL$ (proved in **b**)

$\therefore \triangle BEL \equiv \triangle BHL$ (SAS)

e $\angle BLE = \angle BLH$ (matching angles of congruent triangles)

and $\angle BLE + \angle BLH = 180°$ (angles on a straight line)

$\therefore \angle BLE = \angle BLH = 90°$

$\therefore$ The diagonals of a rhombus are at right angles.

10 a In $\triangle ABD$ and $\triangle CBD$

$AB = CB$ (given)

$AD = CD$ (given)

BD is common.

$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

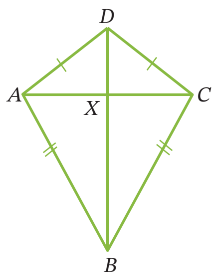
b $\angle A = \angle C$ (matching angles of congruent triangles)

c $\angle ADB = \angle CDB$ (matching angles of congruent triangles)

$\angle ABD = \angle CBD$ (matching angles of congruent triangles)

$\therefore DB$ bisects $\angle ADB$ and $\angle ABC$.

d



e In $\triangle DAX$ and $\triangle DCX$

$AD = CD$ (given)

DX is common.

$\angle ADX = \angle CDX$ (matching angles of congruent triangles proved in **a**)

$\therefore \triangle ADX \equiv \triangle CDX$ (SAS)

f $AX = CX$ (matching sides of congruent triangles)

$\therefore X$ is the midpoint of AC so diagonal DB bisects diagonal AC.

g $\angle AXD = \angle CXD$ (matching angles of congruent triangles)

and $\angle AXD + \angle CXD = 180°$ (angles on a straight line)

$\therefore \angle AXD = \angle CXD = 90°$

$\therefore DX \perp AC$

$\therefore BD \perp AC$

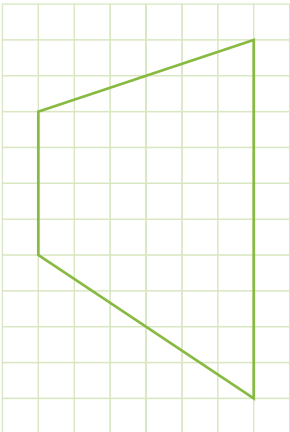
Mental skills 11

2 **a** \$100, \$50 **b** \$1200, \$900 **c** \$160, \$560
d \$500, \$1500 **e** \$2700, \$1800 **f** \$900, \$2100
g \$3000, \$600 **h** \$600, \$1000 **i** \$550, \$440
j \$800, \$3200 **k** \$2100, \$2800 **l** \$2000, \$1200

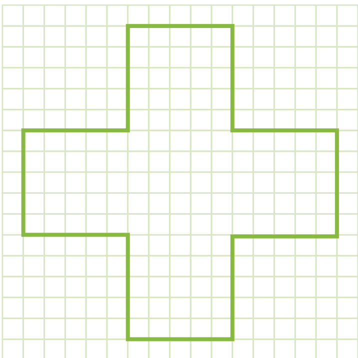
Exercise 11.05

1 **a** 2 **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $2\frac{1}{2}$

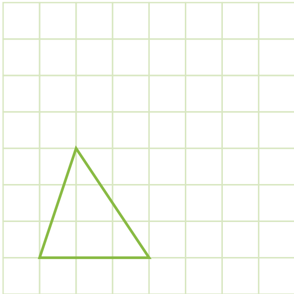
2 **a**



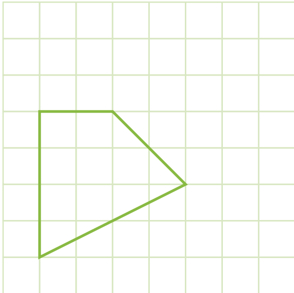
b



c



d



3 **a** **i** $\angle A$ and $\angle L$, $\angle B$ and $\angle M$, $\angle C$ and $\angle K$.
ii AC and LK, AB and LM, BC and MK.
iii $\triangle ABC \parallel\!| \triangle LMK$

b **i** $\angle W$ and $\angle D$, $\angle X$ and $\angle E$, $\angle Y$ and $\angle G$, $\angle Z$ and $\angle H$.
ii WX and DE, XY and EG, YZ and GH, WZ and DH.
iii $WXYZ \parallel\!| DEGH$

c **i** $\angle T$ and $\angle F$, $\angle R$ and $\angle W$, $\angle P$ and $\angle V$, $\angle M$ and $\angle S$, $\angle J$ and $\angle K$.
ii TR and FW, RP and WV, PM and VS, MJ and SK, TJ and FK.
iii $TRPMJ \parallel\!| FWVSK$

d **i** $\angle Q$ and $\angle N$, $\angle K$ and $\angle C$, $\angle B$ and $\angle G$.
ii QB and NG, BK and GC, KQ and CN.
iii $\triangle QKB \parallel\!| \triangle NCG$

4 **a** Yes, $\frac{8}{12} = \frac{20}{30} = \frac{2}{3}$

b Yes, $\frac{12}{20} = \frac{27}{45} = \frac{21}{35} = \frac{3}{5}$

c Yes, $\frac{4}{6} = \frac{6}{9} = \frac{6\frac{2}{3}}{10} = \frac{10}{15} = \frac{2}{3}$

d Yes, all squares are similar.

e Yes, matching angles are equal.

f Yes, all equilateral triangles are similar.

Exercise 11.06

1 **a** $w = 22.4$ **b** $m = 10$
c $p = 20, h = 21$ **d** $x = 18$
e $a = 12.8, w = 7.5$ **f** $g = 11\frac{1}{9}, q = 18$
g $y = 26\frac{2}{3}, b = 9\frac{3}{5}$ or 9.6
h $u = 12\frac{4}{5}$ or 12.8, $t = 6\frac{7}{8}$ or 6.875

2 $h = 13\frac{5}{7}$ **3** $x = 8\frac{8}{9}$

4 $w = 16$ cm 5 12 m 6 B
7 $h = 2.408$ m 8 D 9 2.24 m

Exercise 11.07

1 a 2 pairs of angles are equal (AA).
b All 3 pairs of matching sides are in the same ratio, $\frac{9}{18}=\frac{11}{22}=\frac{15.5}{31}=\frac{1}{2}$ (SSS).
c 2 pairs of matching sides are in the same ratio $\frac{6}{8}=\frac{12}{16}=\frac{3}{4}$ and the included angles are equal (SAS).
d 2 pairs of angles are equal (AA).
e All 3 pairs of matching sides are in the same ratio $\frac{9}{12}=\frac{9}{12}=\frac{14.25}{19}=\frac{3}{4}$ (SSS).
f In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{12}{15}=\frac{20.8}{26}=\frac{4}{5}$ (RHS).
g 2 pairs of angles are equal (AA).
h All 3 pairs of matching sides are in the same ratio $\frac{18}{14.4}=\frac{27.5}{22}=\frac{20}{16}=\frac{5}{4}$ (SSS).
i All 2 pairs of matching sides are in the same ratio $\frac{6}{8}=\frac{8}{10\frac{2}{3}}=\frac{10}{13\frac{1}{3}}=\frac{3}{4}$ (SSS).
j 2 pairs of matching sides are in the same ratio $\frac{26}{18.2}=\frac{30}{21}=\frac{10}{7}$, and the included angles are equal (SAS).

2 a B and C (SAS) b A and C (SSS)
c B and D (RHS)

3 a $\triangle UWY \mid\mid\mid \triangle HEK$ (SAS)
b $\triangle DML \mid\mid\mid \triangle TPA$ (RHS)
c $\triangle ABC \mid\mid\mid \triangle QTP$ (AA) d $\triangle GHN \mid\mid\mid \triangle WVS$ (SSS)

Power plus

1 a Angles opposite equal sides of isosceles $\triangle KMR$ are equal.
b In $\triangle KMN$ and $\triangle KRP$:
$KM = KR$ (given)
$MN = PR$ (given)
$\angle KMN = \angle KRP$ (angles opposite equal sides of isosceles triangle KMR)
$\therefore \triangle KMN \equiv \triangle KRP$ (SAS)
c $KN = KP$ (matching sides of congruent triangles)
$\therefore \triangle KNP$ is isosceles (2 sides equal)

2 $x = 8$

3 a In $\triangle CGH$ and $\triangle CDE$
$\frac{CG}{CD}=\frac{1}{2}$ (G is the midpoint of CD)
$\frac{CH}{CE}=\frac{1}{2}$ (H is the midpoint of CE)
$\angle C$ is common.
$\therefore \triangle CGH \mid\mid\mid \triangle CDE$ (SAS)
b Since $\triangle CGH \mid\mid\mid \triangle CDE$
$\angle CGH = \angle CDE$ (matching angles of similar triangles)
$\therefore GH \parallel DE$ (corresponding angles are equal)
c $\frac{CG}{CD}=\frac{GH}{DE}$ (pairs of matching sides of similar triangles in the same ratio)
But $\frac{CG}{CD}=\frac{1}{2}$
$\therefore \frac{GH}{DE}=\frac{1}{2}$
$\therefore GH = \frac{1}{2}DE$

4 a 2 pairs of angles are equal (AA).
b $y = 5\frac{1}{7}$

Test yourself 11

1 a 2340° b 3960° c 1800° d 8280°
2 156° 3 36
4 a 36 b 15 c 8 d 24
5 B
6 a SAS b AAS c SAS
7 In $\triangle WYZ$ and $\triangle XYZ$
$\angle W = \angle X$ (given)
$\angle WZY = \angle XZY = 90°$ ($YZ \perp WX$)
YZ is common.
$\therefore \triangle WYZ \equiv \triangle XYZ$ (AAS)
8 a $\triangle PML \equiv \triangle NLM$ (SAS)
b $PM = NL$ (matching sides of congruent triangles)
c The diagonals of a rectangle are equal.
9 a $\frac{4}{5}$ b $\frac{5}{3}$
10 a Yes, $\frac{18}{27}=\frac{10}{15}=\frac{2}{3}$
b Yes $\left(\frac{9}{11.25}=\frac{12}{15}=\frac{22}{27.5}=\frac{16}{20}=\frac{4}{5}\right)$
11 a $k = 12\frac{6}{7}$ b $d = 6$
c $y = 4\frac{1}{5}$ or 4.2 d $x = 5\frac{1}{2}$ or 5.5
12 $d = 11\frac{3}{7}$
13 a 2 pairs of matching sides are in the same ratio $\frac{18}{13.5}=\frac{20}{15}=\frac{4}{3}$ and the included angles are equal (SAS).
b In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{16\frac{2}{3}}{30}=\frac{10}{18}=\frac{5}{9}$ (RHS)
c 2 pairs of angles are equal (AA).

Practice set 4

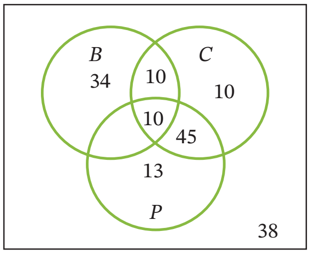
1 a 36
b i $\frac{1}{36}$ ii $\frac{1}{6}$ iii $\frac{1}{4}$

2 **a** 80

b **i** $\frac{13}{80}$ **ii** $\frac{36}{80}=\frac{9}{20}$ **iii** $\frac{42}{80}=\frac{21}{40}$ **iv** $\frac{56}{80}=\frac{7}{10}$

c $\frac{1}{6}=0.1\dot{6}$, which is lower than the experimental probability of $\frac{17}{80}=0.2125$

3 **a**



b **i** $\frac{13}{160}$ **ii** $\frac{19}{80}$ **iii** $\frac{9}{32}$

iv $\frac{109}{160}$ **v** $\frac{57}{160}$

c $\frac{5}{27}$

4 **a** 78

b **i** $\frac{25}{39}$ **ii** $\frac{14}{39}$

c $\frac{35}{78}$

5 In $\triangle ABC$ and $\triangle CDA$,
$\angle CAB = \angle ACD$ (alternate angles, $AB \| CD$)
$\angle BCA = \angle DAC$ (alternate angles, $AD \| CB$)
AC is common.
$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)

6 independent

7 **a** $\frac{1}{4}$ or 0.25 **b** $\frac{5}{4}$ or 1.25

8 **a** **i** 0.39 **ii** 0.43 **iii** 0.18

b **i** 0.33 **ii** 0.42 **iii** 0.25

c drawing a black marble

d 88, which is close to the observed frequency of 85

9 **a** 36 **b** 170°

10 **a**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

b **i** $\frac{1}{24}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$

iv $\frac{1}{3}$ **v** $\frac{5}{8}$ **vi** $\frac{7}{24}$

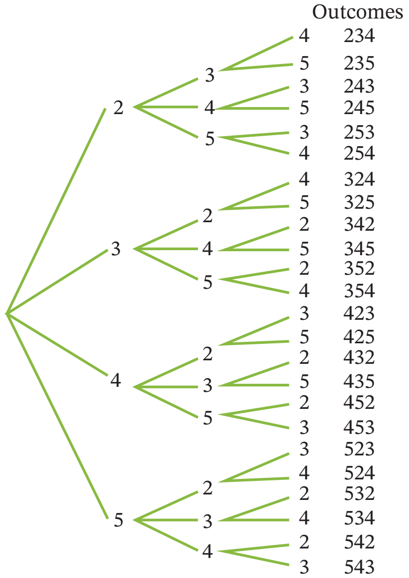
11 In $\triangle LMP$ and $\triangle LNP$,
$LM = LN$ (given)
$MP = NP$ (P is the midpoint of MN)
LP is common
$\therefore \triangle LMP \equiv \triangle LNP$ (SSS)
$\therefore \angle LPM = \angle LPN$ (matching angles of congruent triangles)
But $\angle LPM + \angle LPN = 180°$ (angles on a straight line)
$\therefore \angle LPM = \angle LPN = 90°$

12 **a** 135 **b** $\frac{40}{135}=\frac{8}{27}$

c **i** $\frac{67}{135}$ **ii** $\frac{60}{135}=\frac{4}{9}$ **iii** $\frac{72}{135}=\frac{8}{15}$

13 **a** $h=5\frac{1}{3}$ **b** $p = 35$

14 **a**



b **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$ **v** $\frac{1}{4}$

15 **a** 2340° **b** 3240° **c** 1080° **d** 8280°

16 36

17 **a** **i** $\frac{2}{5}$ **ii** $\frac{1}{2}$

b Dependent, as the sample space has reduced from 5 to 4.

c Independent, as the number of marbles in the bag remains the same.

18 10.6

19 **a** SAS **b** SSS **c** RHS **d** AA

20 **a**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b $\frac{5}{11}$

c **i** 1 **ii** 0

d $\frac{1}{2}$ **e** 1

21 **a** $\angle YXV = \angle WVX$, $\angle XYW = \angle VWY$
b Opposite sides of a rectangle are equal.
c AAS
d Matching sides of congruent triangles are equal.
e The diagonals of a rectangle bisect each other.

General practice

1 \$15 700

2 **a** $m = 3, c = -4$
b $m = -2, c = 5$
c $m = -\frac{4}{3}, c = 3$

3 **a** 4 **b** 2.5

4 **a** $8x(x + 2)$ **b** $(y + 5)(y - 5)$
c $(a + 4)(a + 2)$ **d** $-(2p + 1)(p - 3)$

5 **a** 396 mm^2 **b** 3750 mm^2 **c** 6792 mm^2

6 **a** 140° **b** 40°

7 **a** $56x^{12}$ **b** $\frac{x^5}{4}$ **c** $\frac{1}{9y^2}$ **d** $\frac{n}{m^7}$

8 **a** 210
b **i** $\frac{1}{6}$ **ii** $\frac{143}{210}$
c 32% **d** $\frac{29}{36}$

9 **a** 6.4 **b** 15.6 **c** 110.6
d 13.7 **e** 3.3 **f** 32.1

10 **a** $k = 6\frac{1}{2}$ **b** $m = -3$ **c** $x = 7\frac{1}{2}$

11 26 m

12 **a** $a = 23$ **b** $y = 6\frac{1}{4}$ **c** $m = -3\frac{1}{3}$ **d** $k = 3\frac{3}{7}$

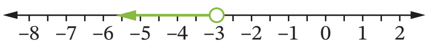
13 **a** 4 **b** 3 h **c** 75%

14 In $\triangle ABC \equiv \triangle DEF$:
$AB = DE = 10$ cm (given)
$CB = FE = 12$ cm (given)
$\angle A = \angle D = 90°$ (given)
$\therefore \triangle ABC \equiv \triangle DEF$ (RHS)

15 **a** $y \geq -1$

b $x < 1\frac{1}{2}$

c $x < -3$



16 **a** **i** $\frac{1}{8}$ **ii** $\frac{3}{2}$ **iii** $y = \frac{x}{8} + \frac{3}{2}$
b **i** -1 **ii** -2 **iii** $y = -x - 2$
c **i** 2 **ii** 1 **iii** $y = 2x + 1$

17 **a** $x = 2, y = 2$ **b** $m = 1, p = -1$

18 $x^2 + y^2 = 64$

19 **a** \$40 **b** \$4.08

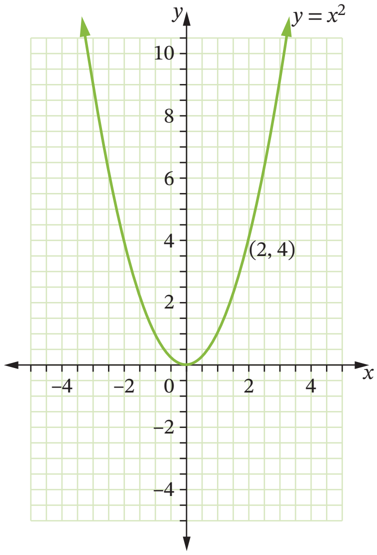
20 **a** 25° **b** 81° **c** 35°

21 **a** \$935.89 **b** \$1597.98

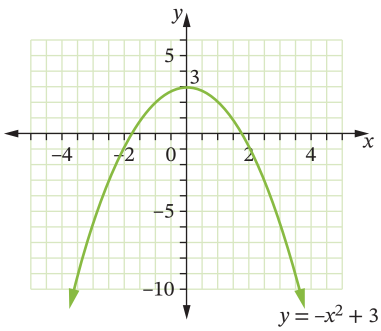
22 **a** **i** 8.9 **ii** (4, 5) **iii** 2
b **i** 13.4 **ii** (−1, 1) **iii** $\frac{1}{2}$

23 **a** **i** 35° **ii** 35° 30′
b **i** 35° **ii** 35° 26′
c **i** 43° **ii** 43° 21′

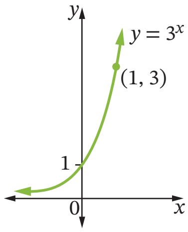
24 **a**



b

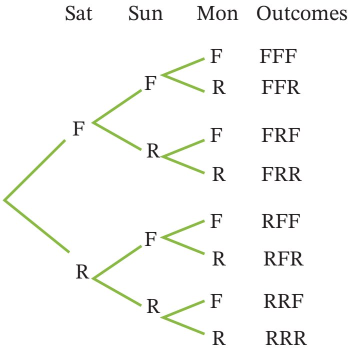


c



25 $h = 21, p = 20$ **26** D

27 a F = fine, R = rain



b i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{7}{8}$

28 19.2 min

Chapter 12

SkillCheck

1 a $x^2 + 8x + 7$ b $d^2 - d - 12$
c $t^2 - 25$ d $5y^2 + 4y - 12$
e $49y^2 - 42y + 9$ f $10p^2 - 27p - 28$

2 a $(y + 5)^2$ b $(x - 20)(x - 1)$
c $(n + 11)(n - 3)$ d $(a - 7)(a - 4)$
e $(m - 12)(m + 7)$ f $(p + 9)(p - 6)$

Exercise 12.01

1 a $20x$ b m^2 c t^2 d $+ 8h$
e $- 18k$ f $+ 80f$ g $4d^2 + 12d$ h $36a^2$, 1

2 a $m^2 + 18m + 81$ b $u^2 + 6u + 9$
c $y^2 - 12y + 36$ d $64 + 16k + k^2$
e $25 - 10h + h^2$ f $49 + 14k + k^2$
g $f^2 + 40f + 400$ h $q^2 - 22q + 121$
i $100 + 20t + t^2$ j $x^2 - 2xw + w^2$
k $a^2 + 2ag + g^2$ l $4m^2 - 12m + 9$
m $25x^2 - 60x + 36$ n $81a^2 + 36a + 4$
o $9e^2 - 24e + 16$ p $25 + 70b + 49b^2$
q $16 - 40p + 25p^2$ r $121 - 44c + 4c^2$
s $100g^2 + 60g + 9$ t $9k^2 + 66k + 121$
u $25 + 20v + 4v^2$

3 a $49h^2 + 28hk + 4k^2$ b $64a^2 - 48ay + 9y^2$
c $x^2y^2 + 2xyz + z^2$ d $1+\frac{2}{y}+\frac{1}{y^2}$
e $k^2-2+\frac{1}{k^2}$ f $w^2+6+\frac{9}{w^2}$

4 a 441 b 2025 c 841
d 3481 e 10 404 f 9604

Exercise 12.02

1 a $m^2 - 25$ b $c^2 - 100$ c $a^2 - 144$
d $36 - y^2$ e $64 - m^2$ f $p^2 - 1$
g $25 - e^2$ h $v^2 - 121$ i $w^2 - 9$
j $x^2 - 100$ k $q^2 - 49$ l $81 - g^2$
m $b^2 - 4$ n $225 - r^2$ o $d^2 - 169$

2 a $4h^2 - 9$ b $25r^2 - 16$ c $25b^2 - 64$
d $16p^2 - 49$ e $9 - 64k^2$ f $49x^2 - 25$
g $4 - 81m^2$ h $81k^2 - 16l^2$ i $49n^2 - 64m^2$
j $16g^2 - 25h^2$ k $49u^2 - 9w^2$ l $121a^2 - 9b^2$
m $a^2-\frac{1}{a^2}$ n $\frac{w^2}{9}-4$ o $1-\frac{1}{r^2}$

3 a $p - 1$ b $p + 1$ c $p^2 - 1$ d 7

4 899

5 a 399 b 2499 c 8099 d 6396

6 a 29 b 47 c 3000
d 400 e 240 f 196

Exercise 12.03

1 a $4m^2 - 1$ b $5y^2 + 17y - 12$
c $4k^2 - 28k + 49$ d $d^2 + 18d + 81$
e $2e^2 + e - 1$ f $25a^2 - 16$
g $-p^2 + 4p - 4$ h $100 - 36y^2$
i $h^2 - 6hm + 9m^2$ j $2xy + 6x - 3y - 9$
k $121a^2 - 16b^2$ l $u^2-2+\frac{1}{u^2}$

2 a m^2 b $y^2 + 18$ c $-3x^2 + 7x + 6$
d $d^2 + 21$ e $16k^2 - 48$ f $-4xy$
g $27a - 10$ h $2f^2 - 8$ i $12h + 18$
j $5xy - 6x + 3y + 9$

3 a $60a^2$ b $n^2 + 4n + 4$ c $-2u^2 - 8u$
d $8m^2 + 2n^2$ e $x - 2$ f $1 - 2b^2$
g $3y^2 + 12y + 14$ h $3x^2 + 9$ i $34n^2 - 34$
j $-4b^2$

Exercise 12.04

1 a $(b + d)(4a + 5c)$ b $(y + t)(2x - 5w)$
c $(3c + 4d)(3a + 2b)$ d $(10 + x)(x^2 + 3)$
e $3(a + b)(a + c)$ f $6(t + p)(r - 3w)$
g $(7 + d)(2e - 3)$ h $(h - 2)(k - h)$
i $(3m + p)(n - 2)$ j $(9 + q)(p^2 - 3)$
k $(f - 10)(g - h)$ l $3(l + n)(3k - 4m)$
m $(2 - p)(p - c)$ n $(l - 3)(l^2 + m^2)$
o $(a + y)(x + 1 - k)$ p $(a - b + 3q)(p - 2q)$

2 a $(w + 3)(w - 3)$ b $(y + 6)(y - 6)$
c $(k + 1)(k - 1)$ d $(m + 11)(m - 11)$
e $(p + 8)(p - 8)$ f $(c + 10)(c - 10)$
g $(2e + f)(2e - f)$ h $(a + 3b)(a - 3b)$
i $(4y + 1)(4y - 1)$ j $(2 + b)(2 - b)$
k $(5 + e)(5 - e)$ l $(1 + 4x)(1 - 4x)$
m $(k + u)(k - u)$ n $(7 + 4m)(7 - 4m)$
o $(b + 11d)(b - 11d)$ p $(6c + 5k)(6c - 5k)$
q $(4 + 9h)(4 - 9h)$ r $(5a + 8m)(5a - 8m)$
s $(10 + 7n)(10 - 7n)$ t $(11p + 12q)(11p - 12q)$
u $\left(\frac{1}{2}+5c\right)\left(\frac{1}{2}-5c\right)$ v $\left(2w+\frac{1}{3}\right)\left(2w-\frac{1}{3}\right)$
w $\left(8h+\frac{3}{2}\right)\left(8h-\frac{3}{2}\right)$ x $(1 + mn)(1 - mn)$

3 a $2(a + b)(a - b)$ b $7(k + 2)(k - 2)$
c $3(1 + 5u)(1 - 5u)$ d $x(x + 7)(x - 7)$
e $k(1 + 4k)(1 - 4k)$ f $2(5q + 1)(5q - 1)$
g $3(d + 2v)(d - 2v)$ h $5t^3(t + 5)(t - 5)$

i $2(ab + 1)(ab - 1)$ **j** $x^2(y + w)(y - w)$

k $12(4f + 3g)(4f - 3g)$

l $5\left(3d+\frac{1}{2}\right)\left(3d-\frac{1}{2}\right)$ or $\frac{5}{4}(6d+1)(6d-1)$

m $2(x + 2a)(x - 2a)$ **n** $25(2 + w)(2 - w)$

o $5\left(\frac{1}{2}+4e\right)\left(\frac{1}{2}-4e\right)$ or $\frac{5}{4}(1+8e)(1-8e)$

p $\left(3c+2\frac{1}{2}\right)\left(3c-2\frac{1}{2}\right)$ or $\frac{1}{4}(6c+5)(6c-5)$

4 **a** $\left(\frac{p}{4}+\frac{x}{5}\right)\left(\frac{p}{4}-\frac{x}{5}\right)$ **b** $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$

c $\left(\frac{v}{8}+\frac{u}{9}\right)\left(\frac{v}{8}-\frac{u}{9}\right)$ **d** $2\left(\frac{y}{3}-\frac{m}{11}\right)\left(\frac{y}{3}+\frac{m}{11}\right)$

e $\left(\frac{4a}{7}+\frac{5b}{2}\right)\left(\frac{4a}{7}-\frac{5b}{2}\right)$ **f** $(g + 3)(g - 3)(g^2 + 9)$

g $(10 + n)(10 - n)(100 + n^2)$

h $y(2x + y)$

i $-(p + 3q)(3p - q)$ **j** $\left(\frac{x}{2}+\frac{y}{6}\right)\left(\frac{x}{2}-\frac{y}{6}\right)$

k $4ab$ **l** $(1 + x)(9 - x)$

Exercise 12.05

1 **a** $3(m + 1)(m + 2)$ **b** $2(y + 2)(y - 1)$
c $5(t - 10)(t + 8)$ **d** $5e^2(e + 8)(e - 3)$
e $x(x - 11)(x + 10)$ **f** $4(b - 7)(b + 6)$
g $4(w + 4)(w - 3)$ **h** $3a(a - 4)(a + 1)$
i $2(e + 5)(e + 4)$ **j** $-(t + 8)(t - 3)$
k $-(u - 7)(u + 6)$ **l** $-(x - 7)(x + 4)$
m $-(b + 4)(b - 3)$ **n** $-(k - 3)(k- 4)$
o $-(x - 5)(x - 7)$

2 **a** $(x + 5)(2x + 1)$ **b** $(m + 3)(4m + 1)$
c $(y + 3)(5y + 2)$ **d** $(2u + 5)(3u + 2)$
e $(w + 15)(2w + 1)$ **f** $(e + 3)(4e + 3)$
g $(2f + 3)(4f + 1)$ **h** $(d + 1)(3d + 2)$
i $(b + 1)(2b + 7)$ **j** $(y + 1)(5y + 11)$
k $(4g + 3)(2g + 5)$ **l** $(3a + 7)(2a + 3)$

3 **a** $(y - 4)(2y - 3)$ **b** $(2k - 3)(5k - 2)$
c $(2e - 3)(3e - 2)$ **d** $(b - 3)(4b - 1)$
e $(w - 3)(6w - 5)$ **f** $(3f - 4)(4f - 3)$
g $(3m - 4)(5m - 2)$ **h** $(3x - 2)^2$
i $(2a - 5)(a - 9)$ **j** $(3y - 8)(4y - 1)$
k $(4d - 3)(2d - 7)$ **l** $(2h - 9)^2$

4 **a** $(y + 1)(5y - 11)$ **b** $(4d - 5)(d + 1)$
c $(2m + 3)(m - 3)$ **d** $(3t - 10)(t + 3)$
e $(6h - 7)(h + 1)$ **f** $(y - 4)(2y + 3)$
g $(2a + 1)(4a - 3)$ **h** $(5u - 4)(3u + 1)$
i $(3c + 1)(3c - 5)$ **j** $(3c - 8)(2c + 3)$
k $(4n - 7)(5n + 2)$ **l** $(4x - 5)(3x + 2)$

5 **a** $(5m + 7)(m - 1)$ **b** $(3g - 4)(2g + 3)$
c $(3p - 2)(p + 2)$ **d** $(7w - 1)(w + 1)$
e $(5y - 1)(y + 3)$ **f** $(3n - 2)(n + 4)$
g $(4b - 3)(b + 3)$ **h** $(4m - 1)(2m + 3)$
i $(3x + 8)(x - 2)$ **j** $(4u + 15)(2u - 1)$
k $(3e + 5)(8e - 3)$ **l** $(4h + 7)(5h - 6)$

6 **a** $(9w - 10)^2$ **b** $4(y + 1)^2$ **c** $(5h - 4)^2$

7 **a** $2(y + 2)(3y - 1)$ **b** $3(g + 4)(2g - 3)$
c $4(2e - 3)(3e + 1)$ **d** $2(a - 2)(4a + 3)$
e $4(u + 2)(3u - 1)$ **f** $-(5q + 3)(5q - 2)$
g $-2(2m - 1)(3m - 2)$ **h** $-(3h + 4)(4h - 5)$
i $6(2c + 3)(2c + 1)$ **j** $-3(w + 1)(2w - 5)$
k $2(2d - 3)(3d + 5)$ **l** $-2(x - 3)(3x - 2)$

8 **a** $(a + 1)(2a + 3)$ **b** $(2m - 5)(6m - 1)$
c $(4x - 1)(x + 3)$ **d** $(w - 1)(7w - 1)$
e $(h - 3)(4h + 5)$ **f** $(4x - 3)(2x + 1)$
g $(r + 5)(5r + 1)$ **h** $(d - 7)(2d - 1)$
i $(3n + 1)(2n - 3)$ **j** $-(3m - 2)(3m + 4)$
k $-(5c - 3)(c + 1)$ **l** $(3g + 2)(5g + 3)$
m $-(4q + 3)(2q - 5)$ **n** $(x - 2)(3x- 7)$
o $-(3d - 4)(d + 4)$ **p** $-2(3y + 7)(2y - 3)$
q $4(d - 2)(7d + 3)$ **r** $4(5k + 2)^2$

Exercise 12.06

1 **a** $(m - 8)^2$ **b** $3d(d - 1)$
c $(w - 3)(3w + 5)$ **d** $(3 + h)(k -5)$
e $(5y + 8)(5y - 8)$ **f** $4(5f + 4)(5f - 4)$
g $q(q + 3 - 3p)$ **h** $(3 - g)(g + 1)$
i $4(2n + 5)(3n - 2)$ **j** $(5r + 1)(5r - 1)$
k $(b^2 + 1)(b + 1)$ **l** $(2x - 5)^2$
m $-(5d - 4)(d + 1)$ **n** $(a - 1)^2(a + 1)$
o $2(2 + v)(2 - v)$ **p** $m(n + 3)(n + p)$
q $2(w - 6)^2$ **r** $(6h + 1)^2$

2 **a** $(3r - 8t)(5r + 3t)$ **b** $(2d + 1)^2$
c $9(g + 2k)(g - 2k)$ **d** $e(e - 5)(e + 2)$
e $-20(2p - 3q)(3p - 2q)$ **f** $7(2x + 1)(2x - 1)$
g $(a - b)(a + b + 4)$ **h** $(c - 2)^2(c + 2)$
i $(3a - 1)(2a + 5)$ **j** $(y + 7)(y - 5)$
k $2(3p + 2)^2$ **l** $-(6a - 1)(4a + 1)$
m $9(x + 2)(x - 3)$ **n** $(a - 3)(2ab - 3)$
o $2(a + 3)^2$ **p** $(5u - 1)^2$
q $(k - 3)(4k + 7)$ **r** $3(4 + w)(4 - w)$
s $3(1 + 3c)(1 - 3c)$ **t** $(k + 4)^2(k - 4)$
u $5y(y^2 - 2y + 3)$ **v** $mn(m + 2)(m - 2)$
w $-2(a + 2)(a - 2)$ **x** $4(2c - 3)(4c + 1)$

3 **a** $(3e + 7)(4e - 5)$ **b** $(2y + 5k)(5y - 3k)$
c $(2h - 3)(2h + 3)(4h^2 + 9)$ **d** $12(w - 3)$
e $(d - 4)(d^2 + 4d - 1)$ **f** $(u - 4 - k)(u - 4 + k)$

Exercise 12.07

1 **a** $x + y$ **b** $\frac{1}{2(t-r)}$ **c** $\frac{b-c}{a}$
d -1 **e** $w - 4$ **f** $\frac{5}{d+w}$
g $\frac{k+5}{k-5}$ **h** $3(c - 1)$ **i** $\frac{a+1}{m+n}$
j $\frac{y+4}{2}$ **k** $\frac{k+1}{k+4}$ **l** $\frac{4a+5c}{a-c}$
m $\frac{x+2}{x-3}$ **n** $\frac{1-2c}{3c-1}$ **o** $\frac{a+4}{2(p+2)}$
p $\frac{c-5}{u+1}$ **q** $\frac{h+1}{2h}$ **r** $\frac{4e+3}{e(2e+5)}$

2 **a** $\frac{7m+10}{m(m+1)(m+2)}$ **b** $\frac{2w-20}{w(w+3)(w+5)}$

c $\frac{4b-7}{(b-1)(b+2)(b-3)}$ **d** $\frac{-k-2}{k(k+1)(k-1)}$

e $\frac{5h+12}{4h(h+1)}$ **f** $\frac{-4d-1}{(d+2)(d+1)}$

g $\frac{42-5r}{4(r+6)(r-6)}$ **h** $\frac{d^2+3d-6}{d(d+2)(d-2)}$

i $\frac{-k^2+9k-5}{(k+1)(k-1)(k-4)}$ **j** $\frac{3q-1}{(q+1)(q-1)}$

k $\frac{7a+13}{(a-3)(a-5)(a+7)}$ **l** $\frac{-5y-22}{(y+3)(y+2)(y-4)}$

m $\frac{-3e+27}{(e+2)(e+1)(e+7)}$ **n** $\frac{20h-11}{(3h-2)(2h+1)(4h-5)}$

3 **a** $6m$ **b** $1\frac{1}{24}$ **c** $\frac{1}{2}$

d $6k$ **e** $\frac{10}{h+1}$ **f** $\frac{3}{2(a-b)}$

g $\frac{-r}{5(r+t)}$ **h** $\frac{4m}{m-1}$ **i** $\frac{4}{p}$

j $\frac{3}{7}$ **k** $\frac{2}{3(x-2)}$ **l** $3\frac{1}{3}$

m $\frac{(d+1)(d-3)}{6}$ **n** $\frac{f+3}{4(f-3)}$ **o** $\frac{3}{f-2}$

Chapter 13

SkillCheck

1 **a** $25y^2$ **b** $64m^3$ **c** $9x^2$

2 **a** $5x + 10$ **b** $4y - 12$ **c** $3 + 6w$

d $10 - 2y$ **e** $-10a - 15$ **f** $k + 2k^2$

3 81, 25, 100, 16, 64

4 **a** $m^2 + 10m + 21$ **b** $y^2 - 3y - 4$

c $n^2 - 5n + 6$ **d** $6d^2 + 11d + 3$

e $4 - 17p - 15p^2$ **f** $3a^2 + 17af + 10f^2$

g $x^2 + 8x + 16$ **h** $y^2 - 6y + 9$

i $4k^2 + 4k + 1$ **j** $a^2 - 25$

k $t^2 - 49$ **l** $9m^2 - 16$

Exercise 13.01

1 C **2** B

3 $\sqrt{32}, \sqrt{33}, \sqrt{4.9}, \sqrt{52}, \sqrt{200}$

4 **a** R **b** I **c** R **d** R

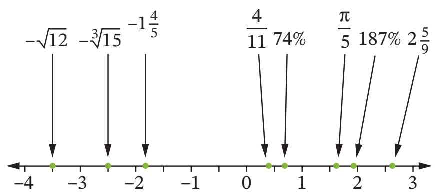
e R **f** R **g** R **h** R

i I **j** R **k** I **l** I

5 **a** $1\frac{4}{7}, \frac{\pi}{2}, \sqrt{2}$ **b** $2\frac{7}{9}, \sqrt[3]{20}, 2.\dot{6}$

6 **a** −1.8 **b** 0.7 **c** 0.4 **d** −3.5

e −2.5 **f** 2.6 **g** 1.6 **h** 1.9



7 1.4

8 Teacher to check, $\sqrt{5} \approx 2.24, \sqrt{10} \approx 3.16, \sqrt{17} \approx 4.12$

Exercise 13.02

1 **a** 2 **b** 5 **c** 27 **d** 250

e 0.09 **f** 28 **g** 45 **h** 50

2 **a** $5\sqrt{2}$ **b** $3\sqrt{3}$ **c** $2\sqrt{6}$ **d** $3\sqrt{6}$

e $9\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $10\sqrt{2}$

i $4\sqrt{6}$ **j** $3\sqrt{7}$ **k** $12\sqrt{2}$ **l** $6\sqrt{3}$

m $5\sqrt{3}$ **n** $7\sqrt{3}$ **o** $4\sqrt{2}$ **p** $11\sqrt{2}$

q $9\sqrt{2}$ **r** $7\sqrt{5}$ **s** $5\sqrt{5}$ **t** $16\sqrt{2}$

3 **a** $25\sqrt{2}$ **b** $6\sqrt{2}$ **c** $12\sqrt{3}$ **d** $56\sqrt{2}$

e $\sqrt{10}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{7}}{3}$ **h** $6\sqrt{6}$

i $18\sqrt{17}$ **j** $\frac{5\sqrt{5}}{2}$ **k** $3\sqrt{2}$ **l** $3\sqrt{3}$

m $40\sqrt{10}$ **n** $15\sqrt{3}$ **o** $14\sqrt{17}$ **p** $\frac{\sqrt{13}}{3}$

4 B **5** B

6 **a** false **b** false **c** true

d true **e** true **f** false

Exercise 13.03

1 **a** $7\sqrt{7}$ **b** $-5\sqrt{2}$ **c** $6\sqrt{5}$ **d** $4\sqrt{5}$

e 0 **f** $\sqrt{10}$ **g** $8\sqrt{15}$ **h** $-\sqrt{6}$

i $2\sqrt{3}$ **j** $10\sqrt{5}$ **k** $6\sqrt{10}$ **l** $-5\sqrt{3}$

2 **a** $5\sqrt{5}-8$ **b** $13\sqrt{10}+3\sqrt{2}$ **c** $5\sqrt{2}-9\sqrt{3}$

d $7\sqrt{15}+8\sqrt{2}$ **e** $-2\sqrt{5}-3\sqrt{7}$ **f** $2\sqrt{6}-8\sqrt{3}$

g $13\sqrt{11}-\sqrt{3}$ **h** $11\sqrt{7}-6\sqrt{13}$ **i** $-6\sqrt{7}$

j $-3\sqrt{5}$

3 **a** D **b** A

4 **a** $6\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-2\sqrt{5}$

d $-\sqrt{7}$ **e** $5\sqrt{6}$ **f** $7\sqrt{5}$

g $-\sqrt{10}$ **h** $8\sqrt{11}$ **i** $6\sqrt{2}$

j $8\sqrt{3}$ **k** $3\sqrt{2}$ **l** $9\sqrt{2}$

m $11\sqrt{3}$ **n** $-\sqrt{5}$ **o** $-6\sqrt{3}$

p $30\sqrt{3}$ **q** $5\sqrt{7}$ **r** $41\sqrt{2}$

s $5\sqrt{6}$ **t** $29\sqrt{2}$ **u** $-15\sqrt{3}$

v 0 **w** $6\sqrt{2}+2\sqrt{3}$ **x** $12\sqrt{3}+3\sqrt{6}$

y $3\sqrt{2}-6\sqrt{5}$ **z** $4\sqrt{6}$

Exercise 13.04

1 **a** $\sqrt{14}$ **b** $-\sqrt{35}$ **c** $4\sqrt{3}$ **d** 6

e $-5\sqrt{2}$ **f** 45 **g** $15\sqrt{30}$ **h** $-10\sqrt{21}$

i 140 **j** $-30\sqrt{2}$ **k** 36 **l** $-60\sqrt{2}$

- **m** -112 **n** $24\sqrt{6}$ **o** 80 **p** $90\sqrt{6}$
- **q** -396 **r** $160\sqrt{5}$ **s** $216\sqrt{2}$ **t** $-96\sqrt{6}$
- **u** $36\sqrt{5}$ **v** $-60\sqrt{10}$ **w** $252\sqrt{3}$ **x** 144

2 **a** $\sqrt{5}$ **b** $-\sqrt{3}$ **c** $3\sqrt{6}$ **d** $2\sqrt{2}$
e $-\frac{\sqrt{7}}{2}$ **f** 21 **g** 1 **h** 8
i $5\sqrt{3}$ **j** $5\sqrt{2}$ **k** $4\sqrt{3}$ **l** $2\sqrt{6}$
m $2\sqrt{14}$ **n** $-\frac{\sqrt{2}}{4}$ **o** 1 **p** 10
q 4 **r** $-21\sqrt{2}$ **s** 12 **t** 2
u $\frac{2}{3}$

3 **a** 6 **b** 7 **c** 6
d $15y$ **e** x **f** $a\sqrt{a}$

4 C **5** A

6 **a** 2 **b** $4\sqrt{6}$ **c** $\sqrt{30}$
d $\frac{2}{45}$ **e** $14\sqrt{3}$ **f** 2

Exercise 13.05

1 **a** $\sqrt{15}+\sqrt{10}$ **b** $2\sqrt{3}-\sqrt{6}$ **c** $\sqrt{6}+\sqrt{14}$
d $3\sqrt{10}-5$ **e** $6+6\sqrt{6}$ **f** $\sqrt{55}-4\sqrt{11}$
g $42-8\sqrt{7}$ **h** $5\sqrt{5}+75$ **i** $24+3\sqrt{6}$

2 C

3 **a** $10+\sqrt{10}-6\sqrt{5}-3\sqrt{2}$ **b** $7+2\sqrt{7}-\sqrt{21}-2\sqrt{3}$
c $28\sqrt{6}+21+8\sqrt{2}+2\sqrt{3}$ **d** $20+\sqrt{10}$
e $109+10\sqrt{77}$ **f** $72-23\sqrt{6}$
g $16\sqrt{10}+54$ **h** $-16-\sqrt{35}$

4 C

5 **a** $8-2\sqrt{15}$ **b** $9+2\sqrt{14}$ **c** $9-4\sqrt{5}$
d $19+6\sqrt{10}$ **e** $77+30\sqrt{6}$ **f** $179-20\sqrt{7}$
g $38+12\sqrt{10}$ **h** $23+4\sqrt{15}$

6 **a** 1 **b** 22 **c** 8 **d** 2
e 1 **f** 166 **g** 13 **h** -43

7 C

8 **a** $88-30\sqrt{7}$ **b** $21\sqrt{2}-10$ **c** $5\sqrt{35}+29$
d $73+40\sqrt{3}$ **e** 29 **f** $92-12\sqrt{5}$

Exercise 13.06

1 B

2 **a** $\frac{\sqrt{2}}{2}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{3}$ **d** $\frac{3\sqrt{2}}{2}$
e $\frac{2\sqrt{7}}{7}$ **f** $\frac{\sqrt{2}}{6}$ **g** $\frac{\sqrt{3}}{6}$ **h** $\frac{\sqrt{7}}{28}$
i $\frac{7\sqrt{5}}{15}$ **j** $\frac{\sqrt{10}}{15}$ **k** $\frac{\sqrt{3}}{2}$ **l** $\frac{\sqrt{15}}{4}$

3 A **4** D

5 **a** $\frac{2-\sqrt{2}}{2}$ **b** $\frac{\sqrt{5}-5}{5}$
c $\frac{5\sqrt{2}+\sqrt{6}}{4}$ **d** $\frac{2\sqrt{3}-3\sqrt{2}}{18}$

6 **a** $\frac{2\sqrt{7}+7\sqrt{2}}{14}$ **b** $\frac{\sqrt{10}+5\sqrt{3}}{5}$ **c** $\frac{\sqrt{3}-\sqrt{2}}{2}$

Chapter 14

SkillCheck

1 **a** $x=\pm 5$ **b** $m=\pm 2$ **c** $x=\pm 9$
d $k=\pm\frac{1}{2}$ **e** $u=\pm 3\frac{1}{2}$ **f** $w=\pm 3$

2 **a** $(4-m)(4+m)$ **b** $(d-11)(d+11)$
c $2y(7-y)$ **d** $5p(2p+5)$
e $5(x-8)(x+8)$ **f** $2(3w-5)(3w+5)$
g $(k+1)(k+4)$ **h** $(y-8)(y-2)$
i $(m-8)(m+7)$ **j** $(u+13)(u-5)$
k $(w-3)(w-7)$ **l** $(x-6)(x+4)$

3 **a** $y=-2$ **b** $y=-10$
c $y=5$ **d** $y=-8\frac{3}{4}$

4 **a** $(3a+1)(a+3)$ **b** $(5x+2)(x-3)$
c $(2y-5)(3y+8)$ **d** $(3t-1)(5t+4)$
e $(5v+3)(v-7)$ **f** $(2y+5)(4y+7)$
g $(3h-4)(5h-1)$ **h** $(4p-3)(3p+5)$
i $(4d+5)^2$

Exercise 14.01

1 **a** $m=-7$ or -3 **b** $d=3$ or 7
c $y=-5$ or 3 **d** $k=0$ or 3
e $t=-7$ or 0 **f** $p=0$ or 3
g $w=0$ or $\frac{2}{3}$ **h** $n=-\frac{1}{2}$ or 3
i $a=\frac{1}{2}$ or $\frac{3}{5}$ **j** $x=-\frac{1}{3}$ or $-1\frac{1}{2}$
k $c=2\frac{1}{2}$ **l** $f=\frac{1}{2}$
m $c=-\frac{1}{3}$ or $-\frac{1}{4}$ **n** $h=-1$ or $\frac{1}{2}$
o $e=\frac{5}{7}$ or 1

2 **a** $y=-2$ or $-1\frac{1}{2}$ **b** $g=-1$ or $-1\frac{1}{2}$
c $d=-1$ or $-\frac{2}{3}$ **d** $t=-2\frac{1}{5}$ or -1
e $p=1\frac{1}{2}$ or 4 **f** $x=\frac{2}{5}$ or $1\frac{1}{2}$
g $y=\frac{3}{4}$ or $-\frac{1}{2}$ **h** $a=-\frac{1}{2}$ or $1\frac{1}{3}$
i $w=-1\frac{1}{4}$ or 3 **j** $c=1$ or $-1\frac{2}{5}$
k $e=\frac{1}{4}$ or $-1\frac{1}{2}$ **l** $q=-3$ or $1\frac{2}{3}$
m $g=2\frac{1}{2}$ **n** $m=-\frac{2}{3}$ or $\frac{5}{6}$
o $w=-4$ or $1\frac{1}{3}$ **p** $y=-3$ or 4
q $f=6$ **r** $h=-1$ or $\frac{3}{4}$

3 **a** $x=-2\frac{1}{2}$ or 3 **b** $t=-2\frac{1}{2}$ or $\frac{1}{2}$
c $u=-\frac{1}{8}$ or -5 **d** $m=\frac{1}{7}$ or 1
e $p=-4$ or 7 **f** $e=-1$ or 5

ANSWERS

g $t=1\frac{1}{2}$ or 5 **h** $d=-2\frac{1}{3}$ or $\frac{1}{2}$

i $h = \pm 5$ **j** $f=0$ or $\frac{1}{2}$

k $w=\frac{1}{6}$ or 3 **l** $a=-2$ or $\frac{1}{3}$

4 8

Exercise 14.02

1 **a** $x^2 + 2x + 1 = (x + 1)^2$
b $p^2 - 6p + 9 = (p - 3)^2$
c $m^2 - 8m + 16 = (m - 4)^2$
d $k^2 + 4k + 4 = (k + 2)^2$
e $y^2-7y+\frac{49}{4}=\left(y-\frac{7}{2}\right)^2$
f $w^2-3w+\frac{9}{4}=\left(w-\frac{3}{2}\right)^2$
g $x^2+x+\frac{1}{4}=\left(x+\frac{1}{2}\right)^2$
h $h^2-5h+\frac{25}{4}=\left(h-\frac{5}{2}\right)^2$
i $a^2+\frac{7}{2}a+\frac{49}{16}=\left(a+\frac{7}{4}\right)^2$
j $v^2+\frac{5}{3}v+\frac{25}{36}=\left(v+\frac{5}{6}\right)^2$

2 **a** $d=-3+\sqrt{7},-3-\sqrt{7}$
b $x=5+\sqrt{5},5-\sqrt{5}$
c $p=-1+\sqrt{10},-1-\sqrt{10}$
d $y=1+\sqrt{2},1-\sqrt{2}$
e $m=\frac{1+2\sqrt{5}}{2},\frac{1-2\sqrt{5}}{2}$
f $t=\frac{-2+3\sqrt{3}}{3},\frac{-2-3\sqrt{3}}{3}$
g $c=\frac{-2+\sqrt{42}}{2},\frac{-2-\sqrt{42}}{2}$
h $w=\frac{6+\sqrt{82}}{2},\frac{6-\sqrt{82}}{2}$
i $n=\frac{-2+\sqrt{7}}{3},\frac{-2-\sqrt{7}}{3}$
j $e=\frac{3+\sqrt{71}}{2},\frac{3-\sqrt{71}}{2}$
k $d=2+\sqrt{5},2-\sqrt{5}$
l $x=\frac{3+4\sqrt{2}}{4},\frac{3-4\sqrt{2}}{4}$

3 **a** $h=-1\pm\sqrt{6}$ **b** $r=1\pm\sqrt{2}$
c $m=-3\pm\sqrt{7}$ **d** $w = -6, 10$
e $a=5\pm\sqrt{30}$ **f** $y=-4\pm\sqrt{19}$
g $p=-6\pm\sqrt{41}$ **h** $x=2\pm\sqrt{2}$
i $u = -7, -2$ **j** $d=\frac{-1\pm\sqrt{29}}{2}$
k $c=\frac{9\pm\sqrt{73}}{2}$ **l** $e=\frac{-5\pm\sqrt{17}}{2}$
m $y=\frac{3\pm\sqrt{41}}{2}$ **n** $b=\frac{1\pm\sqrt{21}}{2}$

o $q=\frac{3\pm\sqrt{5}}{2}$ **p** $g=\frac{-7\pm\sqrt{73}}{4}$
q $x=-3\frac{1}{2},1$ **r** $f=-2\pm\sqrt{6}$

4 **a** $x = -11.20$ or -0.80
b $m \approx -0.43$ or 16.43
c $g \approx -4.65$ or 0.65
d $h \approx 1.27$ or -2.77
e $w \approx 1.27$ or -0.47
f $y \approx 1.14$ or -1.47
g $p = -2$ or $1\frac{1}{3}$
h $e \approx 1.13$ or -0.88
i $n = 1$ or -2.5

Exercise 14.03

1 **a** $x=-3\pm\sqrt{7}$ **b** $m=\frac{5\pm\sqrt{37}}{2}$
c $w=4\pm\sqrt{13}$ **d** $k=\frac{-3\pm\sqrt{29}}{2}$
e $y=2\pm\sqrt{5}$ **f** $p=\frac{-1\pm\sqrt{21}}{2}$
g $u=\frac{7\pm\sqrt{61}}{2}$ **h** $a=\frac{-3\pm\sqrt{65}}{4}$
i $q=\frac{1}{5},1$ **j** $c=\frac{-1\pm\sqrt{7}}{3}$
k $e=\frac{5\pm\sqrt{57}}{8}$ **l** $x=\frac{-4\pm\sqrt{10}}{3}$
m $d=\frac{2\pm\sqrt{14}}{2}$ **n** $a=\frac{5\pm\sqrt{31}}{3}$
o $t=-2\frac{1}{2},1$ **p** $y=-\frac{2}{3},-2$
q $k=\frac{5}{6},1$ **r** $n=\frac{5\pm\sqrt{113}}{4}$

2 **a** $y=\frac{9\pm\sqrt{141}}{10}$ **b** $m=\frac{-1\pm\sqrt{22}}{3}$
c $x=\frac{3\pm\sqrt{41}}{8}$ **d** $k=-2\pm\sqrt{5}$
e $m=\frac{3\pm\sqrt{21}}{6}$ **f** $g=\frac{-1\pm\sqrt{6}}{5}$
g $h=\frac{9\pm\sqrt{17}}{4}$ **h** $w=\frac{1\pm\sqrt{7}}{3}$
i $p=\frac{2\pm\sqrt{7}}{3}$ **j** $u=\frac{-2\pm\sqrt{14}}{5}$
k $a=\frac{-2\pm\sqrt{58}}{6}$ **l** $y=\frac{-3\pm\sqrt{89}}{4}$

3 **a** $k = 8.89, 0.11$ **b** $c = 1.41, -1.41$
c $m = 2.65, -2.65$ **d** $n = 3.19, 0.31$
e $p = 0.85, -2.35$ **f** $w = 0.30, -1.13$
g $x = 2.39, 0.28$ **h** $h = 3.83, -1.83$
i $x = 1.62, -0.62$ **j** $a = 4, 9$
k $v = 1.48, -1.48$ **l** $c = 2.31, 0.69$
m $t = 8.09, -3.09$ **n** $x = 4.27, 7.73$
o $d = 3.31, -1.81$

Exercise 14.04

1 12 m by 8 m **2** 40 m by 35 m
3 42 m by 24 m **4** Length 58 m, width 38 m
5 Length 13 m **6** 15 m by 12 m

7 0.52 s, 3.08 s

8 **a** 4000 m **b** 2000 m **c** 24.5 s

9 **a** 34 m **b** 3.9 s

c **i** 0.5 s and 1.9 s **ii** 3.5 s

10 8 or −9 **11** 24, 25 or −24, −25

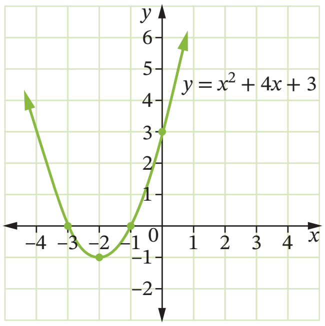
12 35 or −34 **13** 27 and 15

14 **a** $9200 **b** $5550 **c** 471

Exercise 14.05

1 **a** **i**

X	-3	-2	-1	0	1	2	3
Y	0	-1	0	3	8	15	24

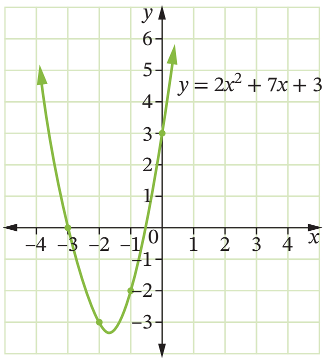


ii −3 and −1 **iii** 3

iv $x = -3$ and -1, same as x-intercepts

b **i**

X	-3	-2	-1	0	1	2	3
Y	0	-3	-2	3	12	25	42

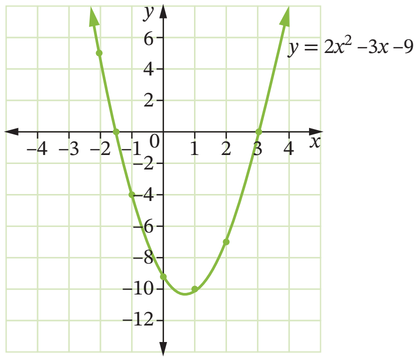


ii -3 and $-\frac{1}{2}$ **iii** 3

iv $x = -3$ and $-\frac{1}{2}$, same as x-intercepts

c **i**

X	-3	-2	-1	0	1	2	3
Y	18	5	-4	-9	-10	-7	0

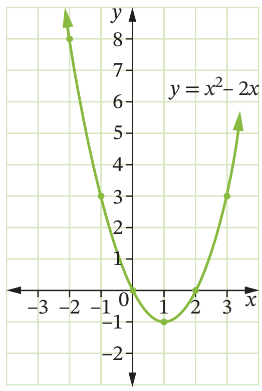


ii $-1\frac{1}{2}$ and 3 **iii** −9

iv $x = -1\frac{1}{2}$ and 3, same as x-intercepts

d **i**

X	-3	-2	-1	0	1	2	3
Y	15	8	3	0	-1	0	3

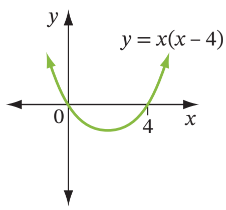


ii 0 and 2 **ii** 0

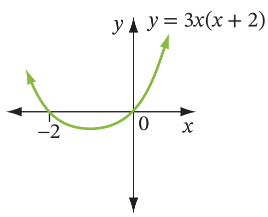
iv $x = 0$ and 2, same as x-intercepts

2 **a** −5 **b** 3 **c** 0

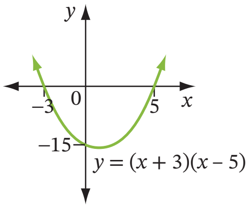
3 **a**



b



c



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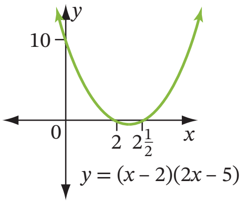

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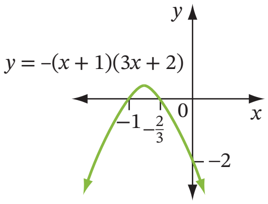

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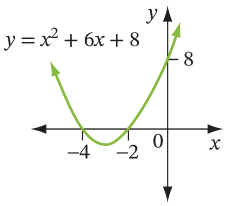

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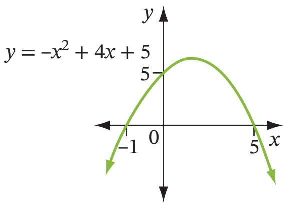
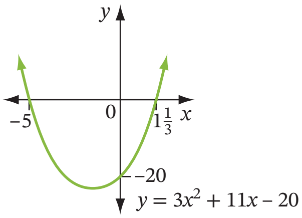
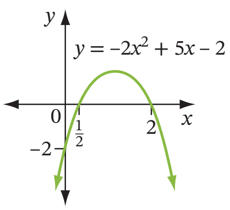


h



i



Exercise 14.06

1 **a** $x = 3$ **b** $x = 0$ **c** $x = 2\frac{1}{2}$

d $x = -1$ **e** $x = 1\frac{1}{2}$ **f** $x = 3$

2 **a** **i** $x = 3$ **ii** $(3, -1)$

b **i** $x = 5$ **ii** $(5, 16)$

c **i** $x = 1$ **ii** $(1, 9)$

d **i** $x = 4$ **ii** $(4, 25)$

e **i** $x = \frac{1}{2}$ **ii** $\left(\frac{1}{2}, -24\frac{3}{4}\right)$

f **i** $x = -4$ **ii** $(-4, -80)$

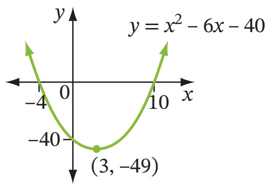
g **i** $x = 4$ **ii** $(4, 48)$

h **i** $x = -\frac{1}{4}$ **ii** $\left(-\frac{1}{4}, -1\frac{1}{4}\right)$

i **i** $x = -\frac{1}{6}$ **ii** $\left(-\frac{1}{6}, 1\frac{1}{4}\right)$

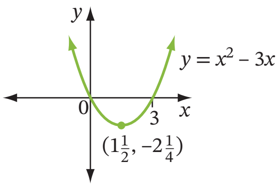
3 **a** **i** $-4, 10$ **ii** -40 **iii** $x = 3$

iv $(3, -49)$ **v** concave up



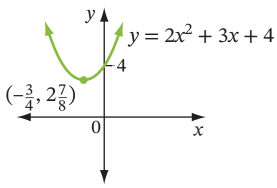
b **i** $0, 3$ **ii** 0 **iii** $x = 1\frac{1}{2}$

iv $\left(1\frac{1}{2}, -2\frac{1}{4}\right)$ **v** concave up



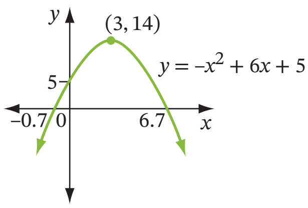
c **i** No x-intercepts. **ii** 4 **iii** $x = -\frac{3}{4}$

iv $\left(-\frac{3}{4}, 2\frac{7}{8}\right)$ **v** concave up



d **i** $-0.7, 6.7$ **ii** 5 **iii** $x = 3$

iv $(3, 14)$ **v** concave down



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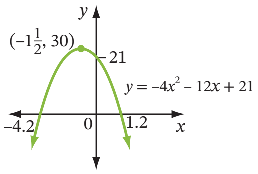
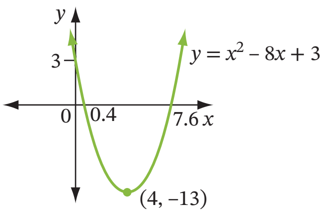
e **i** −4.2, 1.2 **ii** 21 **iii** $x = -1\frac{1}{2}$

iv $(-1\frac{1}{2}, 30)$ **v** concave down



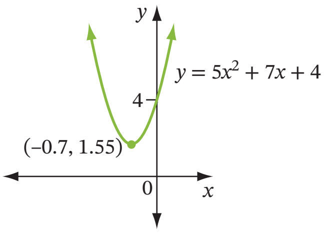
f **i** 0.4, 7.6 **ii** 3 **iii** $x = 4$

iv (4, −13) **v** concave up



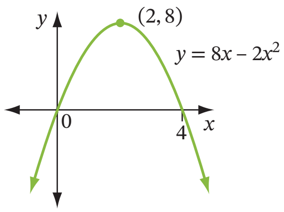
g **i** No x-intercepts. **ii** 4 **iii** $x = -0.7$

iv (−0.7, 1.55) **v** concave up



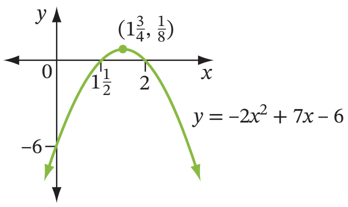
h **i** 0, 4 **ii** 0 **iii** $x = 2$

iv (2, 8) **v** concave down



i **i** $1\frac{1}{2}, 2$ **ii** −6 **iii** $x = 1\frac{3}{4}$

iv $\left(1\frac{3}{4}, \frac{1}{8}\right)$ **v** concave down



Exercise 14.07

1 **a** (3, 9), (2, 4) **b** (0, 0), (4, 32)

c (4, 26), (−2, 14) **d** $(-1, 5), (1\frac{1}{5}, 7\frac{1}{5})$

e $(-2, 18), (\frac{1}{3}, 11)$ **f** (−2, −22), (5, 20)

g (7, 39) **h** (2, 0), (−1, 3)

2 **a** (1, −2), (2, −1) **b** (0, 1), (1, 0)

c (0, 3), (3, 0) **d** (0, 5), (−5, 0)

e $(1, 7), \left(-1\frac{2}{5}, -5\right)$ **f** $(3, 5), \left(-2\frac{1}{2}, -6\right)$

g $(-1, 3), \left(-1\frac{1}{2}, 2\right)$

3 There are no solutions for x when solving $-3x - 1 = \frac{2}{x}$, which becomes the quadratic equation $3x^2 + x + 2 = 0$, as $x = \frac{-1 \pm \sqrt{-23}}{6}$. Also, when both equations are graphed, the line $y = -3x - 1$ with gradient −3 and y-intercept −1 does not cross the hyperbola $y = \frac{2}{x}$.

Chapter 15

SkillCheck

1 **a** 0.8480 **b** 0.7760 **c** 0.1539

d 64.9839 **e** 13.9884 **f** 13.7044

2 **a** 64° 37′ **b** 69° 41′ **c** 28° 8′

Exercise 15.01

1 **a** $\tan A = \frac{60}{91}$ **b** $\tan Y = 1.\dot{3}$ **c** $\tan X = \frac{2}{3}$

d $\tan P = \frac{9}{40}$ **e** $\tan Q = \frac{\sqrt{40}}{3}$ **f** $\sin X = \frac{11}{61}$

g $\cos X = \frac{7}{25}$ **h** $\sin X = \frac{2}{3}$

2 **a** P **b** P **c** N **d** N

e P **f** N **g** N **h** P

3 **a** −0.89 **b** −0.19 **c** −0.77 **d** −0.11

e 0.51 **f** −0.58 **g** 0.05 **h** −0.42

i −0.78 **j** −0.87 **k** 0.18 **l** 0.28

4 **a, b** Teacher to check.

c The graph has a wave shape that repeats itself after 360°.

Maximum $y = 1$ at $\theta = 90°$; Minimum $y = -1$ at $\theta = 270°$

d No

e Yes, centre of symmetry at (180°, 0).

f **i** $0° < \theta < 180°$ (1st and 2nd quadrants)

ii $180° < \theta < 360°$ (3rd and 4th quadrants)

5 **a, b** Teacher to check.

c The graph has a wave shape that repeats itself after 360°.

Maximum $y = 1$ at $\theta = 0°$ and $\theta = 360°$; Minimum $y = -1$ at $\theta = 180°$

d Yes, axis of symmetry $\theta = 180°$

e No

f **i** $0° < \theta < 90°$ and $270° < \theta < 360°$ (1st and 4th quadrants)

ii $90° < \theta < 270°$ (2nd and 3rd quadrants)

g Similarities: Both graphs have the same wave shape that runs between $y = -1$ and $y = 1$ and repeats itself after 360°.
Differences: The graphs have different y- and θ-intercepts

6 a 10° b 70° c 50° d 83° e 65° f 12°

7 a −cos 38° b sin 75° c −cos 25°
d −tan 78° e −cos 7.3° f sin 64.5°
g −cos 40°25′ h −tan 9.2° i sin 59°25′
j −tan 19°50′ k sin 84.5° l −tan 40.5°

8 a, b Teacher to check.
c The tan graph is broken into 3 sections and repeats itself after 180°. It has asymptotes at 90° and 270°.
d No
e centre of symmetry at (180°, 0).
f i 0° < θ < 90° and 180° < θ < 270° (1st and 3rd quadrants).
ii 90° < θ < 180° and 270° < θ < 360° (2nd and 4th quadrants).

Exercise 15.02

1 a 57°, 123° b 143° c 110° d 130°
e 7°, 173° f 135° g 100° h 25°, 155°
i 114° j 33°, 147° k 105° l 118°

2 a 145° 9′ b 131° 57′ c 159° 26′ d 173° 48′
e 152° 58′ f 115° 51′ g no solution
h 126° 52′ i no solution j 163° 18′
k 126° 52′ l 154° 37′

3 a 137° b 136° c 61°, 119°
d 69° e 42°, 138° f 143°
g 45° h 60° i 45°, 135°

Exercise 15.03

1 a 18.4 b 21.1 c 105.0

2 a $a = 20.51$ b $b = 11.91$ c $c = 12.58$
d $d = 4.10$ e $e = 30.85$ f $f = 3.55$
g $k = 5.99$ cm h $w = 29.17$ m i $p = 8.29$ m

3 79 m 4 25 m

5 b 1042 cm

6 a 110° b 131.6 m

7 561 km

8 d 124.7 m

9 b 595 m

Exercise 15.04

1 a 27° b 37° c 54°

2 a 44.5° b 46.6° c 32.0°
d 67.3° e 18.8° f 31.8°

3 a 149° 7′ b 129° 0′ c 142° 8′
d 135° 33′ e 129° 29′ f 162° 13′

4 a 46° or 134° b 39°
c 43° or 137° d 43° or 137°

5 a 75° b 41° c 84°

Exercise 15.05

1 a 5.6 b 13.1 c 35.8

2 a $a = 8.30$ b $c = 54.52$ c $e = 88.41$
d $b = 16.33$ e $d = 19.44$ f $f = 40.72$

3 0.6 m 4 C

5 a Teacher to check.
b $\angle XYN = 180° - 130° = 50°$ (co-interior angles on parallel lines)
$\angle XYZ = 50° + 25° = 75°$
c 4.4 km

6 47 km

7 a 0 b $c^2 = a^2 + b^2$
c With cos 90° = 0, the cosine rule reverts to Pythagoras' theorem.

Exercise 15.06

1 a 70° b 33° c 109° d 131°

2 a 112° b 108° c 121°
d 23° e 60° f 83°

3 20.8° 4 64°40′ 5 99°

Exercise 15.07

1 a 413.4 m² b 463.1 cm² c 326.9 mm²
d 132.9 mm² e 320.4 cm² f 0.1 m²

2 a 97.4 m² b 463.6 m² c 246.2 m²
d 227.6 m² e 93.5 m² f 152.2 m²

3 a 225 m b 2770 m²

4 a 130° b 766.04 m²

5 a 418.9 cm² b 173.2 cm² c 245.7 cm²

6 a 112° b 37 cm² c 740 cm³

Exercise 15.08

1 a 10.2 b 16.1 c 17.1
d 13.1 e 3.9 f 18.2

2 a 32° b 142° c 29°
d 55° e 37° f 125°

3 a $\angle ATB = 55° - 32°$ (exterior angle of a triangle)
b 108.50 m c 89 m

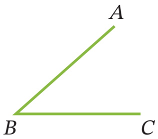
4 a 1 b i 15.4 ii 15.4
c The results are the same. The sine rule $\frac{d}{\sin 90°} = \frac{12.8}{\sin 56°}$ becomes $d = \frac{12.8}{\sin 56°}$ (since sin 90° = 1), which is the same result when using the sine ratio.

5 7.5 km 6 486 km

GLOSSARY AND INDEX

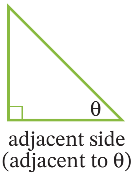
A

acute angle An angle between 0° and 90°, such as the marked angle in the diagram.



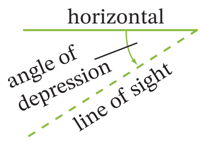
acute-angled triangle A triangle with all 3 angles acute.

adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle. (p. 272)

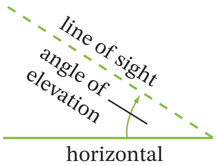


allowable (tax) deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**. (p. 11)

angle of depression The angle of looking **down**, measured from the horizontal. (p. 286)



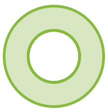
angle of elevation The angle of looking **up**, measured from the horizontal. (p. 286)



angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180°. (p. 376)

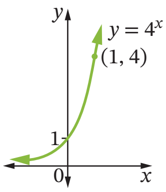
annual leave loading (or **holiday loading**) Extra payment to a worker during annual leave based on 17.5% of 4 weeks' pay. (p. 6)

annulus A ring shape between 2 different-sized circles with the same centre. (p.82)



ascending order Going up, increasing, from smallest to largest (1-2-3). The opposite of **descending order**.

asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve. (p. 254)



average See **mean**.

B

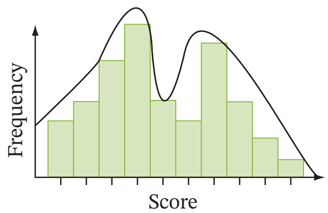
base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base. (p. 121)

bearing The angle used to show the direction of one location from a given point. See also **compass bearing** and **three-figure bearing**. (p. 290)

bias In statistics, something that causes a sample to not truly represent the population. (p. 186)

bisect To cut in half. (p. 388)

bimodal distribution A statistical distribution that has 2 peaks. (p. 148)



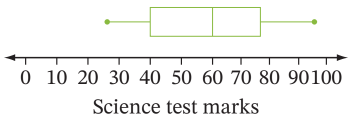
binomial expression An algebraic expression with 2 terms, for example, $x + 9$, $2y - 12$. (p. 134)

binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$. (p. 134)

bivariate data Data that measures 2 variables, such as a person's height and arm span, represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis. (p. 177)

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the 'box' contains the middle 50% of values while the 'whiskers' extend to the 2 extremes. (p. 157)



C

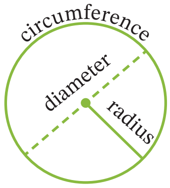
capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). (p. 102) See also **volume**.

categorical data Data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not **numerical**.

census A survey of the entire **population** of people or items, not just a survey of a **sample**.

chance experiment An activity or process that involves chance, for example, rolling a die or tossing a coin. (p. 347)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius. (p. 91)



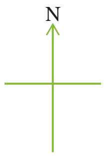
cluster A group of data values that are bunched or close together. (p. 148)

coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2. (pp. 138, 244, 311, 428)

commission Pay earned by salespeople and agents, calculated as a percentage of the value of items sold or income made. (p. 6)

compass bearing A bearing that refers to one of the 16 points of a mariner's compass; for example, north-northwest (NNW). (p. 290) See also **bearing** and **three-figure bearing**.

compass rose A cross-shaped diagram that shows the direction of north. (p. 290) See also **compass bearing**.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

composite shape A shape made up of 2 or more basic shapes. (p. 81)

compound event A chance event that is a combination of 2 or more simple events, for example, 'female or left-handed'. (p. 334, 340)

compound interest Interest that is calculated as a percentage of the original principal and the accumulated interest. (p. 20) See also **simple interest**.

conditional probability The probability that an event occurs given that another event occurs. (p. 362)

congruent Identical, exactly the same. The symbol '≡' means 'is congruent to' or 'is identical to'. (p. 384)

congruent figures Identical figures, having the same shape and size. (p. 384)

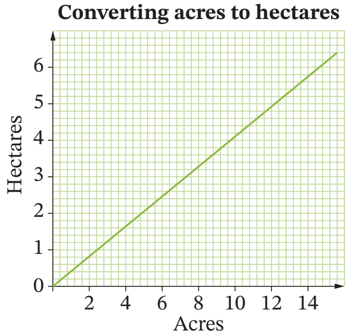
congruence test One of 4 tests for proving that 2 triangles are congruent: SSS, SAS, AAS and RHS. (p. 384)

consecutive numbers Any series of integers that follow each other in order; for example, 8, 9 and 10. (p. 209)

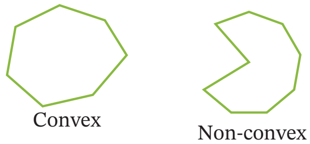
constant term The term in an algebraic expression that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6. (p. 138)

continuous data Numerical data that can be measured on a smooth scale without any gaps, and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**.

conversion graph A line graph for converting between different units or currencies, for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0). (p. 240)



convex polygon A polygon whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180°. (p. 376)



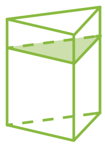
cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 272) See also **sine** and **tangent**.

cosine rule A rule that relates the 3 sides and one of the angles of any triangle: $c^2 = a^2 + b^2 - 2ab \cos C$ (p. 492)

cross-section A 'slice' of a solid cut across it rather than along it. (p. 86)



cumulative frequency A progressive or running total of frequencies, the sum of frequencies of a particular data value and all values below it.

D

data Information, a collection of facts. (p. 147)

denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3. (p. 125)

dependent event An event whose outcome (and probability) depends upon the outcome of another event; for example, the colour of the second marble drawn from a bag depends on the colour of the first marble drawn. (p. 358)

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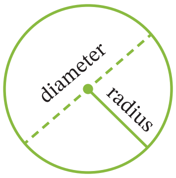
dependent variable A variable that depends on another variable for its value. For example, if y depends on x, then the dependent variable is y and the **independent variable** is x. (p. 179)

depreciation The decrease in the value of items over time due to ageing or use. (p. 32)

descending order Going down, decreasing, from largest to smallest (3-2-1). The opposite of **ascending order**.

diameter An interval joining 2 points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**. (p. 84)

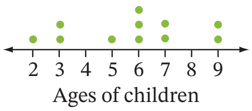


difference of 2 squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$. (p. 423)

discrete data Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**.

direct proportion (or **direct variation**) A relationship between 2 variables of the form $y = kx$, where k is a constant called the **constant of proportionality**. For example, if $y = 8.5x$, then y is directly proportional to x. (p.230)

dot plot A graph that uses dots above a number line to show the frequencies of data values. (p. 147)



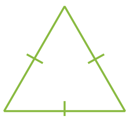
double time Overtime pay that is calculated at 2 times the normal pay rate. (p. 5)

E

equation A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 198)

equiangular All angles equal. (p. 400)

equilateral triangle A triangle with all 3 sides equal (and all angles 60°). (p. 378)



elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables. (p. 310)

expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials. (p. 333)

experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula $P(E) = \dfrac{\text{frequency of } E}{\text{total frequency}}$ (p. 333)

exponential curve The graph of an exponential equation $y = a^x$. (p.254) See **asymptote** for a diagram.

exponential equation An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, for example, $y = 4^x$. (p. 254)

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 333)

F

factor or **divisor (of a number)** A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. (p. 126)

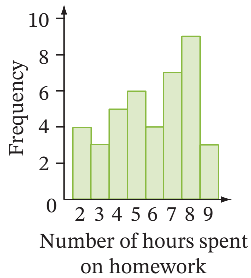
five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value; used to draw a **boxplot**. (p. 158)

formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 211)

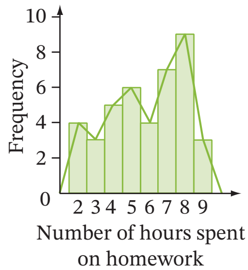
fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data. (p. 148)

frequency histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings. (p. 147)



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the column tops of a histogram. The graph looks like a mountain. (p. 148)

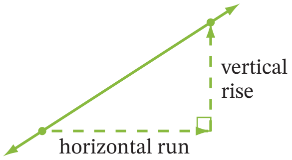


frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx. (p. 150)

G

general form of a linear equation The equation of a straight line $ax + by + c = 0$, where a, b and c are integers and a is positive. (p. 66)

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$. (p. 43)



gradient–intercept form of a linear equation The equation of a straight line $y = mx + c$, where m is the **gradient** and c is the y-intercept. (p. 60)

greatest common divisor (GCD) See **highest common factor**. (p. 129)

gross pay Pay received before tax and other deductions are taken out. (p. 12)

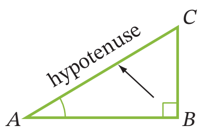
H

highest common factor (HCF) or **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$. (p. 129)

hire-purchase See **term payments**. (p. 28)

horizontal Going across, sideways, flat. (p. 57)

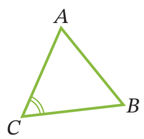
hypotenuse The longest side of a right-angled triangle, opposite the right angle. (p. 272)



I

image A transformed shape after it has been enlarged or reduced. (p. 393)

included angle The angle between 2 given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$. (p. 384, 498)



income tax A tax paid to the government based on the size of a person's income. (p. 11)

independent event An event whose outcome (and probability) does not depend upon the outcome of another event; for example, the number rolled on the second die does not depend on the number rolled on the first die. (p. 358)

independent variable A variable whose value does not depend on another variable. For example, if y depends on x, then the **dependent variable** is y and the independent variable is x. (p. 179)

inequality A mathematical statement that 2 quantities are not equal, involving algebraic expressions and an inequality sign ($>$, $\geq$, $<$, or $\leq$), for example, $-3 > -10$ or $2x - 7 \leq 15$. (p. 214)

index (Plural: **indices**, pronounced 'in-da-sees') See **power**. (p. 121)

index law An algebraic rule for simplifying expressions involving powers of the same base, for example, $a^m \times a^n = a^{m+n}$. (p.121)

instalment See **repayment**. (p. 28)

interquartile range (IQR) The difference between the upper quartile and lower quartiles,

$$\text{IQR} = Q_3 - Q_1$$

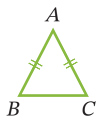
representing the middle 50% of values. (p. 153)

interval A section of a line with a definite length, such as AB shown. (p. 43)

inverse proportion (or **inverse variation**) A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x. (p. 235)

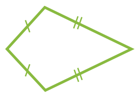
irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. (p. 439) See also **rational number** and **real number**.

isosceles triangle A triangle with 2 equal sides (and 2 equal angles opposite those sides). (p. 378)



K

kite A quadrilateral with 2 pairs of equal adjacent sides. (p. 386)



L

LHS The left-hand side (of an equation). (pp. 56, 198)

like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms. (p. 129)

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (pp. 55, 198)

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the values by the number of values. (p. 150)

measure of central tendency or **measure of location** An average, middle or typical value of a set of data. The 3 measures of central tendency are the **mean**, **median** and **mode**. (p. 172)

measure of spread A statistical value that describes how the values in a data set are spread, for example, **range** or **interquartile range**. (p. 153)

median The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 150)

midpoint The point in the middle of an interval or halfway between 2 given points. (p. 43)

minute (symbol ′) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60′$. (p. 281)

mode The most common or frequent value(s) in a set of data. (p. 150)

mutually exclusive events Events or categories that have no items in common. (p. 340)

N

negatively skewed See **skewed distribution**. (p. 148)

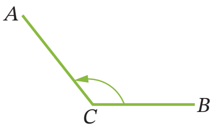
net pay Pay received after deductions from gross pay; 'take-home' pay. (p. 12)

numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2. (p. 125)

numerical data Data that can be measured or counted, such as a person's height or the number of goals scored. Data that is not **categorical**.

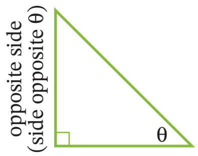
O

obtuse angle An angle greater than 90° but less than 180°. (p. 478)



obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°). (p. 490)

opposite side In a right-angled triangle, the side directly facing the given angle. (p. 272)



outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 333)

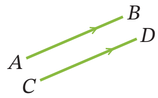
outlier An extreme data value that is much different to the other values in a set. (p. 148)

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay. (p. 5)

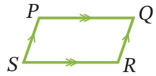
P

parabola A U-shaped curve that is the graph of a quadratic equation such as $y = x^2$. (pp. 243, 467)

parallel lines Lines that point in the same direction and do not intersect. $AB \parallel CD$ means 'AB is parallel to CD'. (p. 41)



parallelogram A quadrilateral in which the opposite sides are parallel. (p. 390)



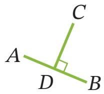
PAYG (Pay As You Go) tax Income tax deducted from your pay each payday by your employer. (p. 12)

perfect square A square number or an algebraic expression that represents one; for example, 64, $(x + 9)^2$, $(a - b)^2$. (p. 420)

perimeter The distance around the outside of a shape. The sum of the lengths of its sides.

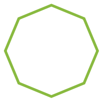
per annum (p.a) Per year. (p. 15)

perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means 'AB is perpendicular to CD'. (p. 41)



piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item rather than on the number of hours worked. (p. 6)

polygon Any flat shape made up of straight sides. (p. 86, 376)



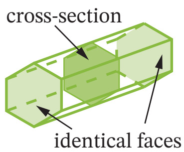
population In statistics, all of the items being studied, the entire group. (p. 186)

positively skewed See **skewed distribution**. (p. 148)

power (or **index**) The number of times a base is multiplied by itself. In 2^5, the power is 5. Also called the *exponent*. (p. 121)

principal An amount of money invested or borrowed, on which interest is calculated. (p. 15)

prism A solid shape with identical cross-sections and straight sides. (p. 86)

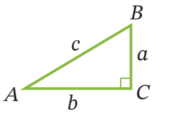


probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (p. 333)

product The result of a multiplication. The product of 7 and 3 is 21.

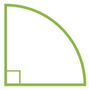
pronumeral Another name for **variable**. (p. 55)

Pythagoras' theorem The relationship $c^2 = a^2 + b^2$ for a right-angled triangle, where c is the length of the hypotenuse and a and b are the lengths of the other 2 shorter sides. (pp. 43, 269)



Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle. (p. 82)



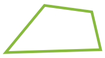
quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$. (p. 137)

quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared; for example, $3x^2 - 6 = 69$ or an equation such as $y = 3x^2 - 6$, whose graph is a **parabola**. (pp. 203, 243, 456)

quadratic formula The formula for solving quadratic equations of the form $ax^2 + bx + c = 0$, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (p. 462)

quadratic trinomial (p. 137) See **trinomial**.

quadrilateral Any polygon with 4 sides. (p. 387)

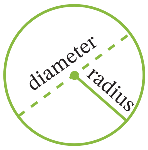


quarterly Occurring regularly 4 times a year, that is, every 3 months. (p. 25)

quartile The values Q_1, Q_2, Q_3 that divide a set of data into 4 equal parts. The 1st quartile Q_1 is the lower quartile, the 2nd quartile Q_2 is the **median**, the 3rd quartile Q_3 is the upper quartile. (p. 152)

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**. (p. 82)



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 335)

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population. (p. 186)

range In a set of data, the difference between the highest and lowest values. (p. 153)

rational number A number that can be written as a fraction in the form $\dfrac{a}{b}$, where a and b are integers and $b \neq 0$. (p. 439) See also **irrational number** and **real number**.

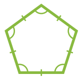
rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd). (p.452)

real number A **rational** or **irrational number** that can be ordered on a number line. (p. 440)

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\dfrac{1}{5}$ and the reciprocal of $\dfrac{2}{3}$ is $\dfrac{3}{2}$. (pp. 122, 126)

rectangle A quadrilateral with 4 right angles. (p. 388)

regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has 5 equal sides and 5 equal angles. (p. 377)

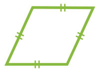


relative frequency The number of times an event or data value occurred, written as a fraction of the total number of events or data values. (p. 333) See also **experimental probability**.

repayment (or **instalment**) The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan. (p. 28)

retainer A fixed amount paid to a salesperson before **commission** is added. (p. 6)

rhombus A quadrilateral with 4 equal sides. (p. 390)



RHS The right-hand side (of an equation). (pp. 56, 196) See also **similarity test.** (p. 400)

right-angled triangle A triangle with one 90° angle. (p. 269)

rise Short for 'vertical rise', this is the change in vertical position between 2 points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. (p. 43) See **gradient**.

run Short for 'horizontal run', this is the change in horizontal position between 2 points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. (p. 43) See **gradient**.

S

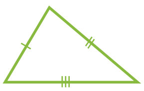
salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked. (p. 5)

sample In statistics, a group of people or items selected from a population for study. (p. 186)

sample space In a probability situation, the set of all possible outcomes. (p. 347)

scale factor The amount by which a shape has been enlarged or reduced, equal to $\dfrac{\text{image length}}{\text{original length}}$. (p. 393)

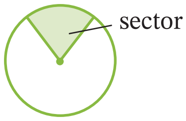
scalene triangle A triangle with no equal sides. (p. 378)



scatterplot A graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern. (p. 177)

second (″) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$.

sector A region of a circle cut off by 2 radii. (p. 82)



shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, positively **skewed** or negatively **skewed**. (p. 147)

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits; for example, 31 487 000 has 5 significant figures.

similar To have the same shape but not necessarily the same size, an enlargement or reduction. The symbol '|||' means 'is similar to'. (p. 393)

similarity test One of 4 tests for proving that 2 triangles are similar. (p. 400)

simple interest Interest that is calculated as a percentage of the original principal. (p. 15) See also **compound interest**.

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$. (p. 307)

sine A ratio in a right-angled triangle:

$$\sin\theta = \frac{\text{side opposite to }\theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 272) See also **cosine** and **tangent**.

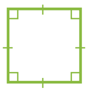
sine rule A rule that relates the 3 sides of any triangle to the sine of their opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ (p. 486)}$$

skewed distribution A distribution in which most of the values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. (p.148) See also **symmetrical distribution**.

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true. (p. 201)

square A quadrilateral with 4 equal sides and 4 right angles.



stem-and-leaf plot A 'number graph' that lists all the data values, in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test scores, from 42 to 82. (p. 147)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key, 5|8 stands for 58

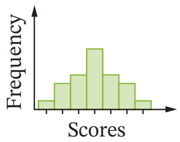
subject of a formula The variable for which a formula is written, the variable on the left-hand side of a formula. The subject of the formula $A = \frac{1}{2}bh$ is A. (p. 211)

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation. (p. 313)

surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (p. 439)

surface area The total area of all the faces of a solid shape. (p. 86)

symmetrical distribution A distribution in which all values are distributed equally on both sides of the centre, its shape having line symmetry. (p. 147) See also **skewed distribution**.



T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to }\theta}{\text{side adjacent to }\theta}$$

where θ is an angle. (p. 272) See also **sine** and **cosine**.

tax deduction See **allowable deduction**. (p. 11)

taxable income The part of a person's income that is taxed, equal to yearly income minus allowable deductions. (p. 11)

term payments Paying for an expensive item through a loan in which regular instalments are made over time. Also called **hire-purchase**. (p. 28)

term (of an expression) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has 3 terms: b^2, $6b$ and -9. (p. 121)

theoretical probability Probability calculated using the formula:

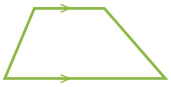
$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \text{ (p. 333)}$$

three-figure bearing (or **true bearing**) A bearing that uses 3-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. (p. 290) See also **bearing** or **compass bearing**.

three-step experiment (or **three-stage experiment**) A chance experiment with 3 steps or stages, such as tossing 3 coins together. (p. 347)

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate. (p. 5)

trapezium A quadrilateral with one pair of opposite sides parallel. (p. 86)



tree diagram A diagram of branches for listing all of the possible outcomes of a multi-step chance experiment. (p. 347)

trial One go or run of a repeated probability experiment; for example, one roll of a die. (p. 333)

trinomial An algebraic expression with 3 terms, for example, $3x + 2y - 5$. In a **quadratic trinomial** such as $x^2 + 4x + 6$, the highest power of the variable is 2. (p.137). See also **binomial** and **quadratic expression**.

trigonometric ratios The ratio of 2 sides in a right-angled triangle; for example, sine is the ratio of the opposite side to the hypotenuse. (p. 272)

two-step experiment (or **two-stage experiment**) A chance experiment with 2 steps or stages, such as rolling a pair of dice. (p. 347)

two-way table A table that shows the number of items belonging to overlapping categories. (p. 344)

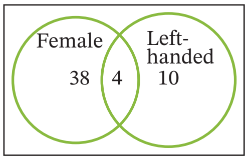
	Can swim	Cannot swim
Boys	13	2
Girls	9	3

V

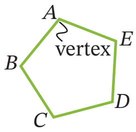
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**. (p. 55)

variation See **direct proportion**. (p. 230)

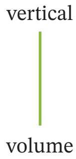
Venn diagram A diagram of circles (usually overlapping) for grouping items into categories. (p. 338)



vertex (plural: **vertices**) A corner of a shape, angle or curve. (p. 243)



vertical Going up and down, at a right angle to the **horizontal**. (p. 57)



volume The amount of space taken up by a solid object, measured in cubic units. (p. 101)

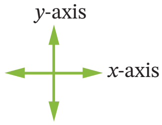
W

wage An amount of money paid to people for work, calculated on the number of hours worked. (p. 5)

X

***x*-axis** The horizontal axis of a number plane (running across). (p. 56)

***x*-intercept** The x-value at which a graph cuts the x-axis. (p. 55)



Y

***y*-axis** The vertical axis of a number plane (running up and down, see diagram above). (p. 56)

***y*-intercept** The y-value at which a line cuts the y-axis. (p. 55) nelson net.

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