

NEW CENTURY MATHS

7

NSW Stage 4

For the
**australian
curriculum**



Judy Binns
Gaspere Carrozza

Robert Yen Series Editor





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Judy Binns
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NelsonNet print resources by Kuldip Khehra
Technology sections by Sarah Hamper



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First published in 1994, *New Century Maths 7* has now been completely rewritten to reflect the spirit of the new Australian curriculum and the NSW syllabus. In this book, teachers will still find those familiar features that have made New Century Maths a leading mathematics series, such as clear worked examples, graded exercises, multiple-choice questions, Investigation, Technology, Mental skills, SkillCheck pre-chapter exercises, Power Plus extension exercises, Language of Maths, Topic overview with mind map, glossary, video tutorials and worksheets.

New features include:

- Homework sheets
- Worked solutions to selected questions
- *GeoGebra* activities in our Technology sections
- Chapter outlines showing coverage of the Working Mathematically / Proficiency strands
- *NelsonNet* student and teacher websites of print and multimedia resources
- *NelsonNetBook* web version of this book with interactive features

We always aim to write textbooks to provide flexibility for teachers to plan and teach for a variety of pathways. We thank our families and teaching colleagues for their continued support and patience, and editors Anna Pang and Alan Stewart for transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing 21st century Australian curriculum mathematics.

About the authors

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Contents

Preface		iii			Investigation: Complementary and supplementary angles	54
About the authors		iii				
Curriculum grids		viii	6MG141	2-04	Complementary and supplementary angles	55
User's guide		xii			Mental skills 2: Adding or multiplying in any order	57
Acknowledgements		1			Investigation: Angles at a point and vertically opposite angles	58
Chapter 1: Integers		2				
	SkillCheck	4	6MG141	2-05	Angles at a point and vertically opposite angles	59
6NA124	1-01 Numbers above and below zero	4	7MG163	2-06	Constructing parallel and perpendicular lines	63
6NA124	1-02 Integers on a number line	7	7MG163	2-07	Corresponding angles on parallel lines	67
	Technology: Introducing spreadsheets	9	7MG163	2-08	Alternate angles on parallel lines	69
7NA280	1-03 Ordering integers	11	7MG163	2-09	Co-interior angles on parallel lines	72
7NA280	1-04 Adding integers	13				
	Mental skills 1A: Adding 8 or 9	15	7MG163	2-10	Angles on parallel lines	74
7NA280	1-05 Subtracting integers	16			Technology: Constructing angles 2	78
	Investigation: Multiplying integers	19	7MG164	2-11	Proving parallel lines	78
8NA183	1-06 Multiplying integers	20			Power plus	81
	Technology: Daily temperatures in Thredbo	22			Chapter 2 review	82
	Investigation: Dividing integers	24				
8NA183	1-07 Dividing integers	24				
8NA183	1-08 Order of operations	26	5NA099	3-01	Rounding and estimating	91
	Mental skills 1B: Subtracting 8 or 9	27	6NA123	3-02	Multiplying numbers	93
8NA183	1-09 Integers and the calculator	28	6NA123	3-03	Dividing numbers	96
	Technology: Sorting numbers	29	5NA098,	3-04	Divisibility tests	97
7NA280	1-10 Integer problems	30	6NA122		Mental skills 3A: Multiplying by a multiple of 10	100
	Investigation: Above and below sea level	32	NSW	3-05	Long division	101
	Power plus	32	NSW	3-06	Roman numerals	103
	Chapter 1 review	34			Investigation: Aboriginal numbers	104
			7NA149	3-07	Powers and index notation	105
Chapter 2: Angles		36				
	SkillCheck	38			Technology: Powers	107
6MG141	2-01 Naming angles	39	7NA150	3-08	Square root and cube root	108
6MG141,	2-02 Measuring and drawing angles	42			Mental skills 3B: Dividing by a multiple of 10	112
7MG163		42	7NA149	3-09	Prime and composite numbers	113
6MG141	2-03 Classifying angles	50				
	Technology: Constructing angles 1	52				

7NA149	3-10 Prime factors	115
	Technology: Spreadsheet formulas	117
7NA149	3-11 Greatest common divisor	118
	Investigation: Common factor puzzle	120
	Investigation: GCD by repeated division	120
7NA149	3-12 Lowest common multiple	120
	Technology: Lowest common multiple	122
	Power plus	123
	Chapter 3 review	124

Mixed revision 1 126

Chapter 4: Fractions and percentages 128

	SkillCheck	130
7NA152	4-01 Fractions	131
7NA152	4-02 Ordering fractions	136
	Investigation: What fraction is shaded?	138
7NA153	4-03 Adding and subtracting fractions	139
7NA153	4-04 Adding and subtracting mixed numerals	142
6NA127	4-05 Fraction of a quantity	144
	Mental skills 4A: Fraction of a quantity	146
	Investigation: Netball scores	147
7NA154	4-06 Multiplying fractions	147
	Technology: More spreadsheet practice	150
7NA154	4-07 Dividing fractions	151
	Investigation: Thanh's gorgeous garden	154
7NA154	4-08 Fraction problems	154
7NA154	4-09 Fractions and the calculator	156
	Investigation: Fraction puzzles	157
7NA157	4-10 Percentages, fractions and decimals	158
7NA158	4-11 Percentage of a quantity	160
	Mental skills 4B: Percentage of a quantity	162
7NA155, 158	4-12 Expressing quantities as fractions and percentages	163
	Technology: Calculating test marks as percentages	165
	Power plus	166
	Chapter 4 review	167

Chapter 5: Algebra and equations 170

	SkillCheck	172
7NA151	5-01 The laws of arithmetic	173
7NA151	5-02 The distributive law	176
7NA175, 177	5-03 Variables	178
	Investigation: The laws of arithmetic	182
7NA177	5-04 From words to algebraic expressions	183
	Mental skills 5A: Multiplying an even number by 5	185
	Technology: Using variables and values	185
7NA176	5-05 Substitution	186
	Technology: Substitution	189
	Investigation: Equivalent expressions	190
7NA179	5-06 Equations	190
7NA179	5-07 One-step equations	191
7NA179	5-08 Two-step equations	195
	Mental skills 5B: Multiplying by 9, 11 and 12	198
7NA179	5-09 Equation problems	199
	Investigation: School concert	202
	Power plus	203
	Chapter 5 review	204

Chapter 6: Geometry 206

	SkillCheck	208
6MG142	6-01 Transformations	208
	Technology: Transformations	214
7MG181	6-02 Composite transformations	219
	Technology: Composite transformations	222
7MG181	6-03 Line symmetry	225
7MG181	6-04 Rotational symmetry	229
	Mental skills 6: Estimating answers	231
7MG165	6-05 Classifying triangles	231
	Technology: Constructing triangles	234
	Investigation: Angle sum of a triangle	236
	Technology: Angle sum of a triangle	237
7MG166	6-06 Angle sum of a triangle	242
	Investigation: Exterior angle of a triangle	244

7MG166	6-07 Exterior angle of a triangle	245
7MG165	6-08 Classifying quadrilaterals	248
	Technology: Measuring quadrilaterals	249
	Investigation: Angle sum of a quadrilateral	251
7MG166	6-09 Angle sum of a quadrilateral	252
7MG165	6-10 Properties of quadrilaterals	254
	Investigation: Shape puzzles	256
	Power plus	257
	Chapter 6 review	258

Mixed revision 2 264

Chapter 7: Decimals 268

	SkillCheck	270
5NA105	7-01 Ordering decimals	271
	Investigation: Comparing heights	274
7NA157	7-02 Decimals and fractions	276
6NA128	7-03 Adding and subtracting decimals	277
	Investigation: Multiplying and dividing decimals by powers of 10	280
6NA130	7-04 Multiplying and dividing decimals by powers of 10	281
	Mental skills 7: Calculating differences and making change	282
7NA154	7-05 Multiplying decimals by estimating	283
	Investigation: Decimal places in multiplication answers	285
7NA154	7-06 Multiplying decimals	286
7NA154	7-07 Dividing decimals by whole numbers	287
7NA154	7-08 Dividing decimals	289
	Investigation: Back-to-front problems	291
7NA157, 8NA184	7-09 Terminating and recurring decimals	291
7NA156	7-10 Rounding decimals	293
	Technology: Rounding decimals	296
7NA154	7-11 Decimal problems	296
	Technology: Fruit and vegetables	298
	Power plus	298
	Chapter 7 review	300

Chapter 8: Area and volume 302

	SkillCheck	304
6MG135, 136	8-01 The metric system	305
5MG109	8-02 Perimeter	308
8MG195	8-03 Metric units for area	312
	Investigation: The human lagoon	315
	Investigation: Standing room only!	316
	Investigation: Student space in a classroom	316
	Investigation: Area of a rectangle	316
7MG159	8-04 Area of a rectangle	318
	Mental skills 8A: Doubling and halving numbers	322
	Technology: Perimeter and area of a chicken pen	323
	Investigation: Area of a triangle	324
7MG159	8-05 Area of a triangle	324
	Investigation: Area of a parallelogram	327
7MG159	8-06 Area of a parallelogram	328
6MG136, 138,	8-07 Areas of composite shapes	330
7MG159	Investigation: Area puzzles	335
7MG161	Investigation: Local areas	337
	8-08 Drawing prisms	337
	Mental skills 8B: Multiplying and dividing by 4 or 8	344
8MG195	8-09 Metric units for volume	345
	Investigation: Volume of a rectangular prism	347
7MG160	8-10 Volume of a rectangular prism	349
	Investigation: What is your volume?	352
6MG136, 138	8-11 Volume and capacity	352
	Technology: Brogo and Windamere dams	356
	Power plus	357
	Chapter 8 review	359

Chapter 9: The number plane 364

	SkillCheck	366
5MG113	9-01 Location using coordinates	367
6MG143, 7NA178	9-02 The number plane	370

6MG143, 7NA178	9-03 The number plane with negative numbers	373	5SP117	11-03 The range of probability	450
	Investigation: Number plane picture puzzles	375		Investigation: The Greedy Pig game	452
7NA178	9-04 Graphing tables of values	376	6SP145, 146	11-04 Experimental probability	453
	Mental skills 9: Simplifying multiplication by factorising	378		Investigation: Spinning chance	456
	Investigation: Height vs head circumference	378		Technology: Rolling a die	457
7MG181	9-05 Transformations on the number plane	379		Mental skills 11: Adding 90c and 95c	457
	Technology: Transformations on the number plane	385		Investigation: Complementary events	458
	Power plus	388	8SP204	11-05 Complementary events	459
	Chapter 9 review	390		Power plus	461
				Chapter 11 review	462

Chapter 10: Analysing data 394

	SkillCheck	396
7SP170	10-01 Interpreting graphs	397
	Technology: Column graphs	401
6SP148	10-02 Misleading graphs	401
7SP170	10-03 Dot plots	405
	Technology: Olympic winning times	407
7SP170	10-04 Stem-and-leaf plots	408
	Mental skills 10: Reading linear scales	410
7SP171	10-05 The mean and mode	411
	Investigation: Finding the middle scores	415
7SP171	10-06 The median and range	416
	Technology: Mean, median and mode	419
	Investigation: Michael's family	420
7SP169, 170	10-07 Analysing dot plots and stem-and-leaf plots	421
	Technology: Dot plots and stem-and-leaf plots	424
7SP169, 172	10-08 Comparing data sets	426
	Power plus	429
	Chapter 10 review	430

Mixed revision 3 434

Chapter 11: Probability 438

	SkillCheck	440
7SP167	11-01 Sample spaces	441
7SP168	11-02 Probability	445

Chapter 12: Ratios, rates and time 464

	SkillCheck	466
7NA173	12-01 Ratios	466
7NA173	12-02 Simplifying ratios	471
	Investigation: Scale diagrams	475
7NA173	12-03 Ratio problems	475
7NA173	12-04 Rates	479
	Mental skills 12A: Multiplying by 5, 15, 25 and 50	480
7NA174	12-05 Best buys	481
7NA173	12-06 Rate problems	484
	Technology: Conversion rates	486
7NA180	12-07 Travel graphs	487
	Technology: Speed and distance	491
NSW	12-08 Time calculations	492
	Mental skills 12B: Dividing by 5, 15, 25 and 50	495
5MG110	12-09 24-hour time	496
8MG199	12-10 Time differences	498
6MG139	12-11 Timetables	502
	Power plus	506
	Chapter 12 review	507

Mixed revision 4 510

General revision 512

Appendices

1 Calculator skills	517
2 Glossary	523
Answers	534
Index	595

Curriculum grid: Australian curriculum

Strand and substrand	New Century Maths 7 chapter	New Century Maths 8 chapter
NUMBER AND ALGEBRA		
Number and place value	1 Integers 3 Whole numbers 5 Algebra and equations	2 Working with numbers
Real numbers	4 Fractions and percentages 7 Decimals 12 Ratios, rates and time	2 Working with numbers 6 Fractions and percentages 11 Ratios, rates and time
Money and financial mathematics	12 Ratios, rates and time	6 Fractions and percentages 11 Ratios, rates and time
Patterns and algebra	5 Algebra and equations	3 Algebra
Linear and non-linear relationships	5 Algebra and equations 9 The number plane 12 Ratios, rates and time	10 Equations 11 Ratios, rates and time 12 Graphing linear equations
MEASUREMENT AND GEOMETRY		
Using units of measurement	8 Area and volume 12 Ratios, rates and time	5 Area and volume 11 Ratios, rates and time
Shape	8 Area and volume	
Location and transformation	6 Geometry 9 The number plane	4 Geometry 8 Congruent figures
Geometric reasoning	2 Angles 6 Geometry	4 Geometry 8 Congruent figures
Pythagoras and trigonometry		1 Pythagoras' theorem
STATISTICS AND PROBABILITY		
Chance	11 Probability	9 Probability
Data representation and interpretation	10 Analysing data	7 Investigating data

Curriculum grid: NSW syllabus

Strand and substrand	New Century Maths 7 chapter	New Century Maths 8 chapter
NUMBER AND ALGEBRA		
Computation with integers	1 Integers 3 Whole numbers	2 Working with numbers
Fractions, decimals and percentages	4 Fractions and percentages 7 Decimals	2 Working with numbers 6 Fractions and percentages
Financial mathematics	12 Ratios, rates and time	6 Fractions and percentages 11 Ratios, rates and time
Proportion	12 Ratios, rates and time	11 Ratios, rates and time
Algebraic techniques 1	5 Algebra and equations	3 Algebra
Algebraic techniques 2	5 Algebra and equations	3 Algebra 12 Graphing linear equations
Indices	3 Whole numbers	2 Working with numbers 3 Algebra
Equations	5 Algebra and equations	10 Equations
Linear relationships	9 The number plane	12 Graphing linear equations
MEASUREMENT AND GEOMETRY		
Length		5 Area and volume
Area	8 Area and volume	5 Area and volume
Volume	8 Area and volume	5 Area and volume
Time	12 Ratios, rates and time	11 Ratios, rates and time
Right-angled triangles (Pythagoras)		1 Pythagoras' theorem
Angle relationships	2 Angles	4 Geometry
Properties of geometrical figures 1	6 Geometry	4 Geometry
Properties of geometrical figures 2		8 Congruent figures
STATISTICS AND PROBABILITY		
Single variable data analysis 1	10 Analysing data	7 Investigating data
Single variable data analysis 2	10 Analysing data	7 Investigating data
Probability 1	11 Probability	9 Probability
Probability 2		9 Probability

Curriculum grid: Year 7 content descriptions

Content description	New Century Maths 7 chapter
NUMBER AND ALGEBRA	
Number and place value	
NA149: Investigate index notation and represent whole numbers as products of powers of prime numbers	3 Whole numbers
NA150: Investigate and use square roots of perfect square numbers	3 Whole numbers
NA151: Apply the associative, commutative and distributive laws to aid mental and written computation	5 Algebra and equations
NA280: Compare, order, add and subtract integers	1 Integers
YEAR 8 NA183: Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies	1 Integers
Real numbers	
NA152: Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line	4 Fractions and percentages
NA153: Solve problems involving addition and subtraction of fractions, including those with unrelated denominators	4 Fractions and percentages
NA154: Multiply and divide fractions and decimals using efficient written strategies and digital technologies	4 Fractions and percentages 7 Decimals
NA155: Express one quantity as a fraction of another, with and without the use of digital technologies	4 Fractions and percentages
NA156: Round decimals to a specified number of decimal places	7 Decimals
YEAR 8 NA184: Investigate terminating and recurring decimals	7 Decimals
NA157: Connect fractions, decimals and percentages and carry out simple conversions	4 Fractions and percentages
NA158: Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies	4 Fractions and percentages
NA173: Recognise and solve problems involving simple ratios	12 Ratios, rates and time
Money and financial mathematics	
NA174: Investigate and calculate 'best buys', with and without digital technologies	12 Ratios, rates and time
Patterns and algebra	
NA175: Introduce the concept of variables as a way of representing numbers using letters	5 Algebra and equations
NA176: Create algebraic expressions and evaluate them by substituting a given value for each variable	5 Algebra and equations
NA177: Extend and apply the laws and properties of arithmetic to algebraic terms and expressions	5 Algebra and equations
Linear and non-linear relationships	
NA178: Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point	9 The number plane
NA179: Solve simple linear equations	5 Algebra and equations
NA180: Investigate, interpret and analyse graphs from authentic data	12 Ratios, rates and time

Content description	New Century Maths 7 chapter
MEASUREMENT AND GEOMETRY	
Using units of measurement	
YEAR 8 MG195: Choose appropriate units of measurement for area and volume and convert from one unit to another	8 Area and volume
MG159: Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving	8 Area and volume
MG160: Calculate volumes of rectangular prisms	8 Area and volume
YEAR 8 MG199: Solve problems involving duration, including using 12- and 24-hour time within a single time zone	12 Ratios, rates and time
Shape	
MG161: Draw different views of prisms and solids formed from combinations of prisms	8 Area and volume
Location and transformation	
MG181: Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries	6 Geometry 9 The number plane
Geometric reasoning	
NSW: Use the language, notation and conventions of geometry	2 Angles
NSW: Recognise the geometric properties of angles at a point	2 Angles
MG163: Classify triangles according to their side and angle properties and describe quadrilaterals	6 Geometry
MG164: Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral	6 Geometry
NSW: Use angle properties and relationships to solve problems with appropriate reasoning	6 Geometry
MG165: Identify corresponding, alternate and co-interior angles when two parallel straight lines are crossed by a transversal	2 Angles
MG166: Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning	2 Angles
STATISTICS AND PROBABILITY	
Chance	
SP167: Construct sample spaces for single-step experiments with equally likely outcomes	11 Probability
SP168: Assign probabilities to the outcomes of events and determine probabilities for events	11 Probability
YEAR 8 SP204: Identify complementary events and use the sum of probabilities to solve problems	11 Probability
Data representation and interpretation	
SP169: Identify and investigate issues involving continuous or large count data collected from primary and secondary sources	10 Analysing data
SP170: Construct and compare a range of data displays including stem-and-leaf plots and dot plots	10 Analysing data
SP171: Calculate mean, median, mode, and range for sets of data. Interpret these statistics in the context of data	10 Analysing data
SP172: Describe and interpret data displays and the relationship between the median and mean	10 Analysing data

Coverage of the Australian curriculum and NSW syllabus

- In addition to the curriculum grids on the previous pages, the table of contents includes codes for Australian Curriculum content descriptions. For example, 7NA149 refers to the Year 7 content description **ACMNA149**, which stands for Australian Curriculum Mathematics, Number and Algebra strand, content description 149.
- This book includes some Year 6 and Year 8 content as well as additional content from the NSW syllabus only. Year 8 and NSW content are highlighted in **orange**.
- There are three content strands in Mathematics:
 - NA** = Number and Algebra
 - MG** = Measurement and Geometry
 - SP** = Statistics and Probability
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiency strands covered in each section.
- There are four proficiency strands in Mathematics:
 - U** = Understanding
 - F** = Fluency
 - PS** = Problem solving
 - R** = Reasoning

Further references to the proficiency strands can be found in the teaching program and Teacher's Edition of this book.

5NA098,	3-04 Divisibility tests	97
6NA122	Mental skills 3A: Multiplying by a multiple of 10	100
NSW	3-05 Long division	101
NSW	3-06 Roman numerals	103
	Investigation: Aboriginal numbers	104
7NA149	3-07 Powers and index notation	105

Chapter outline		Proficiency strands			
5-01 The laws of arithmetic	U	F		R	
5-02 The distributive law	U	F		R	
5-03 Variables	U	F		R	
5-04 From words to algebraic expressions	U	F	PS	R	
5-05 Substitution	U	F	PS		
5-06 Equations	U	F		R	
5-07 One-step equations	U	F		R	
5-08 Two-step equations	U	F		R	
5-09 Equation problems		F	PS	R	

The Proficiency strands explained

While the three **content strands** are the 'nouns' of the mathematics curriculum, the four **proficiency strands** are the 'verbs': the doing and thinking processes that go hand-in-hand with the content being taught.

- Understanding** is 'knowing and relating' maths. It is more than just learning facts. It is deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.
- Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.
- Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.
- Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

In each chapter

- **Wordbank** is a sample glossary of 6–8 terms from the chapter.
- There is also a full glossary at the back of the book: in this book, all terms printed in **red** appear in the glossary
- Curriculum content descriptions (Australian and NSW) are listed at the beginning of each chapter
- **SkillCheck** and **StartUp assignment** review prerequisite skills and knowledge for the chapter

In this chapter you will:

- apply the associative, commutative and distributive laws to aid mental and written computation
- understand that arithmetic laws provide powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra
- introduce the concept of variables as a way of representing numbers using letters
- extend and apply the laws and properties of arithmetic to algebraic terms and expressions
- move fluently between algebraic and word representations as descriptions of the same situation
- create algebraic expressions and evaluate them by substituting a given value for each variable
- use authentic formulas to perform substitutions
- solve simple linear equations
- solve real-life problems by using pronumerals to represent unknowns

SkillCheck

Worksheet
StartUp assignment 5
MAT07NAWK10035

If $\bullet = 3$, then find the value of each expression.

a $\bullet + 5$	b $7 - \bullet$	c $2 \times \bullet$	d $12 \div \bullet$
e $\bullet - 1$	f $\bullet \times 10$	g $\bullet + \bullet$	h $27 \div \bullet$
i $-6 + \bullet$	j $\bullet \times (-1)$	k $-3 \div \bullet$	l $-4 - \bullet$

2 Find the value of $\blacksquare$ in each equation.

a $\blacksquare + 4 = 10$	b $6 \times \blacksquare = 36$	c $\blacksquare + \blacksquare = 20$
d $18 - \blacksquare = 11$	e $12 \div \blacksquare = -3$	f $\blacksquare - 6 = 1$

- Important facts and formulas are highlighted in a **Summary** box.

Summary

If a , b and c stand for numbers, then:

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications
- Worked solutions for selected exercise questions are provided.

Exercise 5-07 One-step equations

See Example 15 1 Solve each equation, showing the working.

a $w + 8 = 15$	b $x + 2 = 20$	c $m + 3 = 19$
d $p + 12 = 30$	e $x + 13 = 22$	f $k + 11 = 20$
g $5 + m = 2$	h $6 + y = 16$	i $15 + d = 6$

See Example 16 2 Solve each equation, showing the working.

a $3m = 18$	b $5n = 20$	c $4k = 44$
d $2c = 32$	e $9x = 81$	f $3x = -27$
g $7d = 42$	h $6h = -30$	i $10a = 70$

See Example 17 3 Solve each equation, showing the working.

a $p - 3 = 8$	b $m - 11 = 2$	c $x - 5 = 12$
d $y - 20 = 40$	e $k - 21 = 3$	f $n - 3 = 18$
g $7 = d - 11$	h $y - 13 = -4$	i $-5 = m - 9$

Worked solutions
Exercise 5-07
MAT07NAWS10034

- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.

Investigation: The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic. For example, if we add zero to any number, the answer is still that number. If we let N stand for any number: $N + 0 = N$

In groups of two to four students, answer the following questions.

1 Describe in words what each number rule below means.

a $N \times 1 = N$

b $N \times 0 = 0$

c $a - b \neq b - a$

d $a + b + c = b + a + c$

e $N - 0 = N$

f $ab = ba$

Just for the record

Trafalgar Square

Trafalgar Square in central London is one of Britain's great tourist attractions. It commemorates the Battle of Trafalgar in 1805 between the British navy, led by Admiral Nelson, and Napoleon's French navy. It is London's only metric square, having been designed to be one hectare in area.

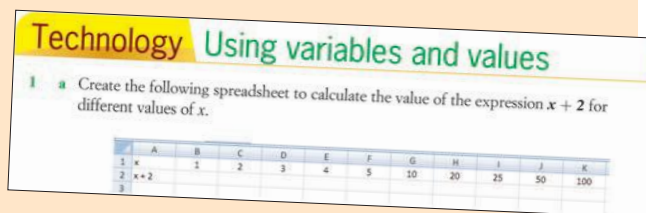
Find when the square was built and what famous British landmark stands in the centre of it.



- **Just for the record** provides interesting facts and applications of the mathematics learnt in the chapter.

New Century Maths User's guide

- **Technology** promotes ICT in the classroom, using spreadsheets, *GeoGebra* and the Internet



Mental skills 5A Maths without calculators

Multiplying an even number by 5

To multiply an even number by 5, it's easier to halve it, then multiply by 10 (by inserting a 0 at the end). This is because $\frac{1}{2} \times 10 = 5$.

Study each example.

$$14 \times 5 = 14 \times \frac{1}{2} \times 10 = 7 \times 10 = 70$$

- **Mental skills** reinforce mental calculation strategies ('maths without calculators')

NelsonNet resources

Margin icons link to print (PDF) and multimedia resources found on the NelsonNet website, www.nelsonnet.com.au. These include:

Video tutorial Angle sum of a triangle	Video tutorials of worked examples and Maths clips giving the 'big picture' view of a topic	Technology GeoGebra Transformations	Technology files are supplied for <i>GeoGebra</i> and <i>Excel</i>
Animated example Measuring angles with a protractor	Animated examples are slideshows of worked examples	Technology worksheet Excel Graphing data	Technology worksheets are extra activities that involve ICT
Worksheet Chance cards	Worksheets	Worked solutions Exercise 8-10	Worked solutions of selected exercise questions
Skillsheet Ordering numbers	Skillsheets are examples and exercises of prerequisite skills	Extra questions Index notation	Extra questions for additional practice and drill
Puzzle sheet Converting length units	Puzzle sheets of matching activities, crosswords and find-a-word puzzles	Weblink Inside a cubic metre	Weblinks to other Internet resources
Homework sheet Transformations and symmetry	Homework sheets for weekly practice and revision	TLF learning object Biscuit factory: Gear direction (L2283)	TLF Learning objects link to The Le@rning Federation interactives

There are also *ExamView* online quizzes.

Each resource can also be accessed through *NelsonNet* by its unique identifier code. For example, the video tutorial 'Solving equations' has the code **MAT07NAVT00006**, which stands for Mathematics, Year 7, Number and Algebra strand, Video Tutorial 00006.

At the end of each chapter

- **Power plus** is an extension/challenge exercise
- **Language of maths** has a chapter word list and literacy questions
- **Topic overview** has reflection questions and an incomplete mind map
- **Chapter revision** is a review exercise with links to each exercise set of the chapter
- **Mixed revision** is a review exercise after every 3–4 chapters

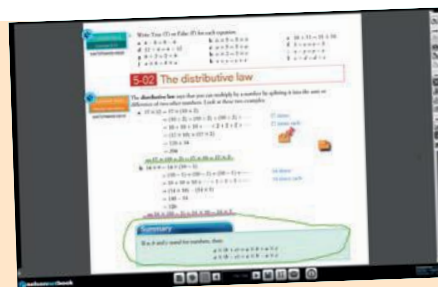
Language of maths

abbreviation	commutative law	evaluate	pronominal
algebra	consecutive	formula	solve
associative law	distributive law	guess, check and improve	solution
backtracking	equation	inverse	substitution
balancing	expanded form	one-step	two-step
check	expression		variable

- 1 What is another name for **pronominal**?
- 2 What is the difference between an **expression** and an **equation**?
- 3 Which law involves multiplying by a number by splitting that number to make the calculation easier?

NelsonNetBook

- **NelsonNetBook** is the web-based version of this textbook found on NelsonNet, containing margin icon links that go directly to NelsonNet resources
- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students



Acknowledgements

Photos

Corbis/Chen Gang/Xinhua Press: p. 108; Fairfax/Andrew Taylor: p. 504; Getty Images: pp. 88–89, 170–171, 449; iStockphoto: pp. 11, 22, 31, 36–37, 49 (bottom), 51 (left), 52, 62, 95, 103, 128, 129 (left), 197, 206–207, 226, 268–269, 295, 296, 297, 302–303, 364–365, 455; KJ Vogelius: p. 15; Masterfile: p. 420; M. C. Escher's "Symmetry Drawing E22" © 2011 The M.C. Escher Company-Holland. All rights reserved. www.mcescher.com: p. 228; Photolibary: pp. 49 (top), 129 (right), 147, 162, 454–455; Shutterstock.com: pp. 2–3, 7, 51 (right), 146, 201, 274, 275, 279, 312, 315, 347, 352, 355, 363, 372, 394–395, 402, 404, 408, 415, 438–439, 452, 454, 461, 462, 466, 468, 470, 474, 477, 485, 501, 506; © 2002–10 The University of Sydney: p. 369.

Text

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1

Number and algebra

Integers

Antarctica is the coldest continent on Earth, where the daily temperature over the year ranges from -70°C to -15°C . It also stays dark for six months of the year when the Sun does not rise between March and September. Because Antarctica is permanently covered with snow and ice, it reflects the Sun's rays rather than absorbs them, and its dry air and high altitudes contribute to its sub-zero temperatures.



Chapter outline

	Proficiency strands		
1-01 Numbers above and below zero	U	F	PS
1-02 Integers on a number line	U	F	
1-03 Ordering integers	U	F	
1-04 Adding integers	U	F	R
1-05 Subtracting integers	U	F	R
1-06 Multiplying integers	U	F	R
1-07 Dividing integers	U	F	R
1-08 Order of operations	U	F	
1-09 Integers and the calculator		F	
1-10 Integer problems	F		PS

Wordbank

- ascending** Moving upward, increasing from smallest to largest
- descending** Moving downward, decreasing from largest to smallest
- evaluate** To find the value of an expression
- integer** A positive or negative whole number or zero
- minus** To subtract or 'take-away' a number
- negative number** A number that is less than zero; the opposite of a positive number
- number line** A line that shows the position and order of numbers

In this chapter you will:

- investigate everyday situations that use positive and negative whole numbers and zero, and locate and represent these numbers on a number line
- compare, order, add and subtract integers
- carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies

SkillCheck

Worksheet

StartUp assignment 1

MAT07NAWK10001

1 Simplify each expression.

a $7 + 9$

b $12 - 4$

c 7×4

d $20 \div 5$

e $13 + 8$

f $30 - 5$

g 2×11

h $15 \div 3$

i $6 + 7$

j $14 - 6$

k 3×5

l $24 \div 4$

2 Simplify each expression.

a $12 + 37$

b $652 - 48$

c 14×5

d $66 \div 2$

e 79×3

f $384 \div 3$

g $3 + 5 \times 4$

h $18 \div 2 - 7$

i $30 - 12 \div 6$

j $20 \times (3 + 17)$

k $12 - 3 \times 2 + 6$

l $43 - [5 \times 2 + 1]$

m 3^2

n 5^2

o 8^2

p 1^2

3 State whether each statement is true (T) or false (F).

a $15 > 12$

b $8 < 11$

c $6 < 3$

d $4 + 3 < 10$

e $12 - 3 > 11$

f $7 < 3 \times 2$

g $15 \div 3 > 12 \div 4$

h $8 + 3 + 2 > 6 \times 1$

i $18 \div 3 < 2 + 2 + 2$

Skillsheet

Ordering numbers

MAT07NASS10001

4 Write each set of numbers in ascending order (smallest to largest).

a 3, 8, 2, 1, 4

b 2, 18, 5, 7, 3, 16

c 38, 51, 49, 27, 66, 54

5 Write each set of numbers in descending order (largest to smallest).

a 6, 11, 18, 17, 5, 3

b 8, 7, 14, 23, 31, 5, 2

c 89, 91, 37, 69, 41, 3, 46

1-01 Numbers above and below zero

- 'The temperature in Canberra was 3°C yesterday but overnight it dropped 5 degrees to -2°C '
- 'When Lisa withdrew \$400 from her bank account, her bank balance of \$340 went down to $-\$60$ '

Both of the above situations involve numbers above and below zero.

Numbers greater than zero, such as 3, 400, $8\frac{1}{2}$ and 14.37, are called **positive** numbers.

Numbers less than zero, such as -2 , -60 , $-10\frac{3}{4}$ and -21.6 , are called **negative** numbers.

The number -2 is the number that is 2 below 0. It is called 'negative 2', not 'minus 2'.

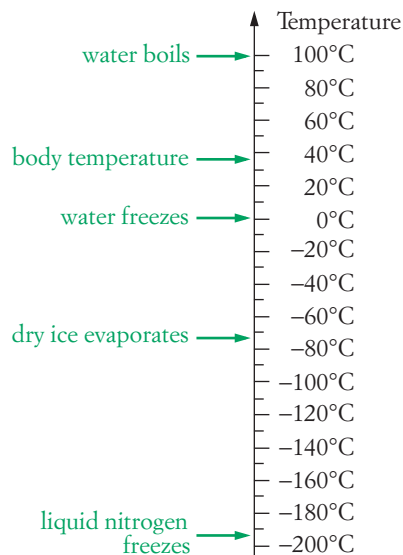
The number 3 is the number that is 3 above 0. It can also be called 'positive 3' and be written as '+3'.

The negative numbers are the opposites of the positive numbers. The opposite of 5 is -5 . The opposite of -9 is 9.

Exercise 1-01 Numbers above and below zero

- 1 The scale on the right shows temperatures in degrees Celsius ($^{\circ}\text{C}$) above and below 0°C .

- At what temperature does water boil?
- What is measured at 37°C ?
- Estimate the temperature at which dry ice evaporates.
- What is measured at 0°C ?
- Last week, it was 48°C in Algeria, Africa and -69° at the South Pole in Antarctica. What was the difference between the two temperatures?



- 2 For each number below, write its opposite number.

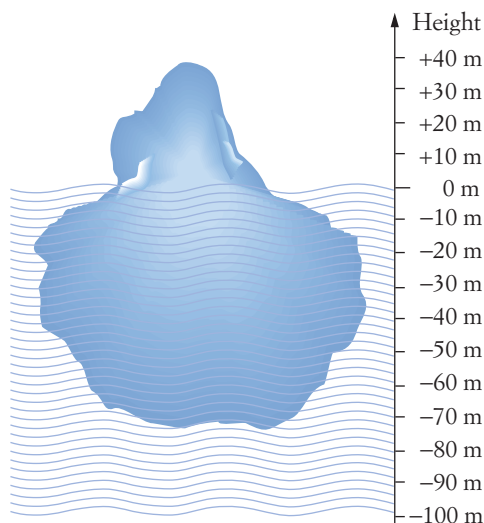
a 6	b 1	c -10
d 11	e 0	f -2

- 3 Write a word that is the opposite of:

a up	b more	c down
d left	e before	f south
g withdraw	h decrease	i west

- 4 Icebergs can be dangerous to ships because about $\frac{5}{6}$ of their mass lies under water. The following diagram shows the position of an iceberg above and below sea level.

- What is the height of the iceberg above sea level?
- What is the depth of the iceberg below sea level?
- What is the total height of the iceberg?
- Why are '+' and '-' signs useful in these measurements?

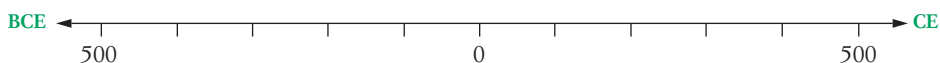


- 5 Dilshan's bank account has money coming in and going out over one day. Withdrawals have a negative sign.

Balance at the start:	\$720
	-\$50
	\$60
	-\$40
	-\$100
	\$80
	-\$260
Balance at the end of the day:	\$

Calculate the balance at the end of the day.

- 6 Write the opposite of:
- | | |
|----------------------------------|---|
| a withdrawing \$25 from the bank | b winning \$12 |
| c walking 2 steps backwards | d lowering the temperature by 3° |
| e driving 5 km west | f increasing the price by \$8 |
| g going up 3 floors | h 5 seconds before take-off |
- 7 What number's opposite is itself?
- 8 a BCE and CE are now widely used in place of the abbreviations BC and AD. Find out what BCE and CE stand for.
- b Why do some historians use BC and AD when numbering years? Find out what BC and AD stand for.
- 9 This time line starts at 500 BCE and ends at 500 CE.



Copy the time line and place each event below in the correct place.

A 100 CE	The first Chinese dictionary was compiled.
B 400 BCE	The first temple dedicated to the Greek god Zeus was built.
C 490 BCE	Greek runner Phidippides inspired the first marathon with a 26-mile run.
D 55 BCE	Roman forces under Julius Caesar invaded Britain.
E 190 CE	The abacus was invented.
F 370 CE	Hypatia, a female mathematician, was born in Alexandria, Egypt.
G 410 CE	Roman forces left Britain.
H 330 BCE	Euclid showed that an infinite number of prime numbers exist.
I 260 CE	The two-year war between Rome and Persia ended.
J 240 BCE	Eratosthenes estimated the circumference of the Earth using two sticks.
- 10 In the Jewish calendar, the year of Creation, 3760 BCE, is Year Zero, while in the Indian calendar, 78 CE is Year Zero.

- a In what year were you born, according to the:
- i Jewish calendar?
 - ii Indian calendar?
- b What is this year, according to the:
- i Jewish calendar?
 - ii Indian calendar?

Just for the record

Tutankhamen, King of Egypt

In 1922, English archaeologist Howard Carter discovered the tomb of a pharaoh (king) in the Valley of the Kings in Egypt. It turned out to be that of Tutankhamen (meaning 'living image of Amun', an ancient Egyptian god). Tutankhamen became pharaoh at the age of 9 after the death of his parents, and he died aged 18 in 1323 BCE. The cause of his death remains a mystery – many possibilities have been suggested, including murder, but none can be proved. His tomb contained over 3000 treasures, many of them covered in gold, including his coffin. Carter's discovery was significant because Tutankhamen's tomb was so well-preserved. It still exists in the Valley of the Kings today.

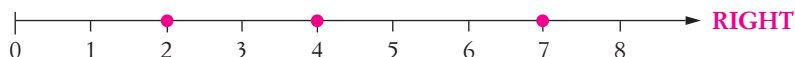


In what year did Tutankhamen become king?

How many years after his death was King Tutankhamen's tomb discovered?

1-02 Integers on a number line

A **number line** shows the position and order of numbers. The number line below shows the positions of 2, 4 and 7 marked in pink.



As we move further to the right on the number line, the numbers become bigger. The arrow on the right end of the number line shows that numbers can increase infinitely ('forever'). We can extend our number line to the left to include the negative numbers below zero.



As we move further to the left on the number line, the numbers become smaller. The arrow on the left end of the number line shows that numbers can decrease infinitely.

Positive and negative numbers have **size** and **direction**. For example, 5 (or +5) means 5 units in the positive direction, while -2 means 2 units in the negative direction.

Positive and negative **whole** numbers are called **integers**, for example, -3, 4 and -20. Zero is also an integer.

Worksheet

A page of number lines

MAT07NAWK10002

Puzzle sheet

Integer snap

MAT07NAPS10001

Words printed in red also appear in the Glossary

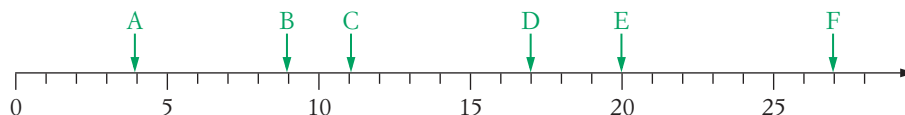
Summary

Integers are the positive and negative whole numbers and zero.

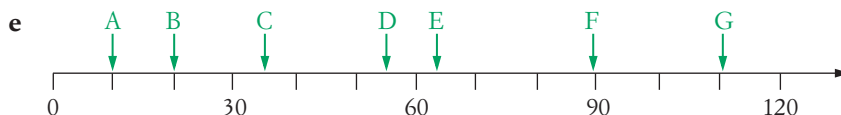
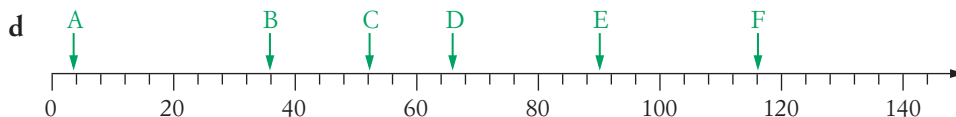
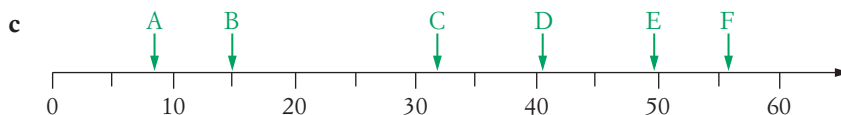
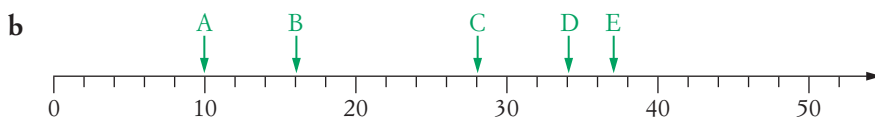
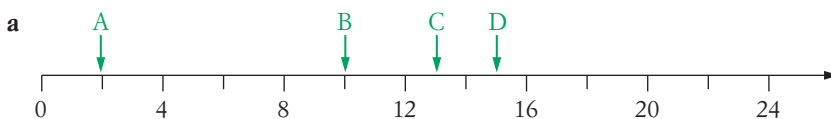
Note: Integers are **whole** numbers, not fractions or decimals, so $8\frac{1}{2}$, 14.37 , $-10\frac{3}{4}$ and -21.6 are not integers. The general name for **all** positive or negative numbers is the **directed numbers**.

Exercise 1-02 Integers on a number line

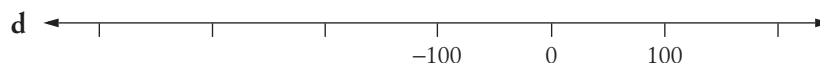
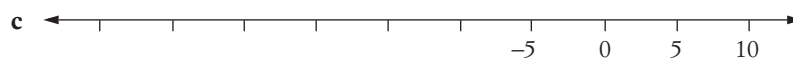
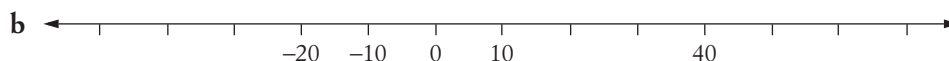
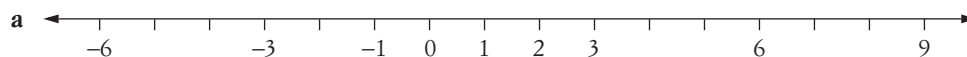
- 1 Write the positive integer indicated by each arrow on this number line.



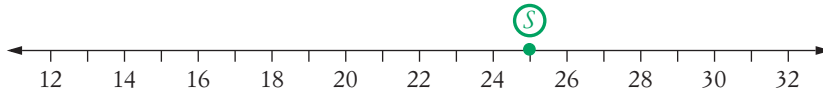
- 2 Estimate the value of the positive integer indicated by each arrow on the number lines below.



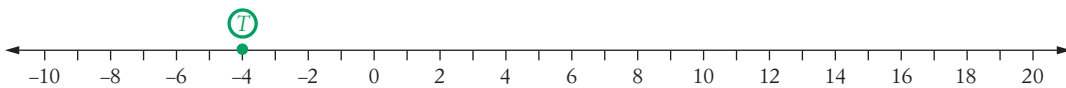
- 3 Copy and complete each number line, using a ruler to mark the positions evenly.



- 4 a Draw a number line and show all the integers between -5 and 3 .
b Draw a number line marked from -6 to 6 and show all the integers less than 1 .
- 5 On this number line, where will S be if it moves:



- a 4 places to the right? b 10 places to the right? c 7 places to the right?
d 5 places to the left? e 13 places to the left? f 20 places to the left?
- 6 On this number line, where will T be if it moves:



- a 5 places to the right? b 8 places to the right? c 10 places to the right?
d 2 places to the left? e 8 places to the left? f 11 places to the left?

Technology Introducing spreadsheets

A spreadsheet is a **calculator** in the form of a grid made up of many cells where we can enter labels and numbers and display answers to calculations. The **columns** of a spreadsheet are ordered A, B, C, D, ... while the **rows** are ordered 1, 2, 3, 4, ...

Skillsheet

Spreadsheets

MAT07NASS10002

	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								

To make a calculation, we need to write a **formula** (rule) in a cell. A spreadsheet formula uses special symbols and always starts with an equals sign '='. Here are some examples of formulas:

Formula	Meaning
=A1+A2+A3 or =sum(A1:A3)	add the values in cells A1, A2 and A3
=A5-A4	subtract the value in cell A4 from the value in cell A5
=A1*10	multiply the value in cell A1 by 10 (* is used instead of $\times$)
=A1/A2	divide the value in cell A1 by the value in cell A2 (/ is used instead of $\div$)
=average(A1:A5)	find the average of all values from cells A1 to A5

- 1 Open a new spreadsheet and enter the 5 numbers shown in column A at cells A1, A2, A3, A4 and A5.

	A	B
1	4	=A4-7
2	10	
3	6	
4	44	
5	25	
6		

- 2 In cell B1, enter the formula =A4-7 to calculate the value in A4, minus 7. You should get the answer 37, because $44 - 7 = 37$.
- 3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, $A2 + A1$	b at B3, $10 \times A5 - 6$
c at B4, $A4 - A1 \times 2$	d at B5, $A1 \times A2 + A3$
e at B6, $A1 \times (A2 + A3)$	f at B7, $3 \times A2 \div 2$
g at B8, $\frac{A5 - A2}{3}$	h at B9, sum of A1, A2, A3, A4, A5
i at B10, average of A1, A2, A3, A4, A5	j at B11, $\frac{A4}{A1} - \frac{A5}{A2}$
- 4 Now enter **new** values in cells A1 to A5. Using the formulas entered from question 3, check the new answers calculated in column B.

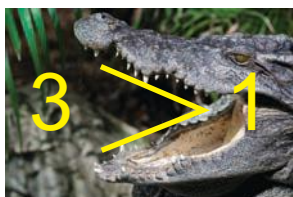
1-03 Ordering integers

Summary

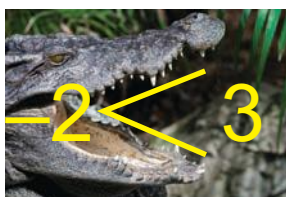
- $>$ means 'is greater than'
- $<$ means 'is less than'

Think of the $>$ and $<$ signs as being like the mouths of crocodiles. They always open towards the **bigger** number.

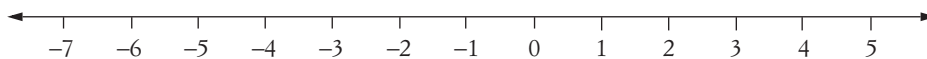
'3 is greater than 1' is written as ' $3 > 1$ '.



'-2 is less than 3' is written as ' $-2 < 3$ '.



Some more examples are shown in the table below. Remember that on a number line, the numbers on the right are larger than the numbers on the left.



Expression	Meaning	On a number line ...
$3 > -4$	3 is greater than -4	3 is to the right of -4
$-5 < -3$	-5 is less than -3	-5 is to the left of -3
$-6 > -7$	-6 is greater than -7	-6 is to the right of -7
$-5 < 2$	-5 is less than 2	-5 is to the left of 2

Example 1

Write these integers in ascending order: -5, 7, 0, 6, 3, -2, 4, -1.

Solution

Ascending order means from smallest to largest.

-5, -2, -1, 0, 3, 4, 6, 7.

Summary

- Ascending** means moving UP, from smallest to largest: think of 'A' as a ladder and going up a ladder
- Descending** means moving DOWN, from largest to smallest: think of 'D' for 'down'

Exercise 1-03 Ordering integers

- 1 Copy and complete each statement using a $>$ or $<$ symbol.

a 4 ____ 1	b 0 ____ 9	c 7 ____ -7	d 0 ____ -3
e 5 ____ -11	f -1 ____ -6	g -12 ____ 8	h -10 ____ -2
i -6 ____ 5	j -6 ____ -2	k 17 ____ 23	l 24 ____ -4
m -15 ____ -35	n -27 ____ -29	o -8 ____ 47	p -12 ____ 0

- See Example 1** 2 Here is a list of integers: 1, -2, 3, -4, 5, -6, 7, -8.

- a** Which is the biggest integer?
b Which is the smallest integer?
c Rewrite the integers in order from smallest to biggest.

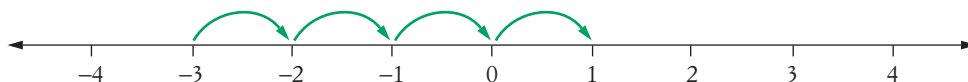
- 3 Rewrite each set of integers in ascending order.

a -3, 2, -1, 3	b 5, -5, 2, -8, -3	c -4, -6, -3, -10, 0
d 6, -3, 4, -2, -5	e -48, 36, -24, 8, 0, -11	f 15, 12, -10, -26, 3, -2

- 4 Rewrite each set of integers in descending order.

a 4, 3, -1, 5, -2	b -4, 8, -7, -2, 0	c 1, -1, 4, -5, -11, -3
d 8, -4, -18, 3, -2	e -6, -15, -48, -3, 1	f 33, 1, -100, -58, -36

- 5 The distance between -3 and 1 on a number line is 4.



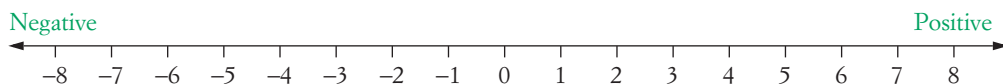
Find the distance between each pair of integers.

a 2 and 4	b -2 and -4	c 0 and 4	d 0 and -3
e -3 and 3	f -10 and 10	g -6 and 1	h -1 and 6

- 6 Which of the following statements is true? Select **A**, **B**, **C** or **D**.

A $-6 > 2$	B $4 < -5$	C $0 < -3$	D $12 > -2$

- 7 In this question, POS means positive and NEG means negative.



- a** Start at -3. Move 4 steps in the NEG direction. Move 9 steps in the POS direction. Move 6 steps in the NEG direction. Where are you now?
b Start at 8. Move 12 steps in the NEG direction. Move 5 steps in the POS direction. Move 7 steps in the NEG direction. Where are you now?
c Start at -5. Move 6 steps in the POS direction. Move 4 steps in the NEG direction. Move 7 steps in the POS direction. Where are you now?
d Start at 3. Move 5 steps in the POS direction. Move 8 steps in the NEG direction. Move 2 steps in the NEG direction. Where are you now?
e Start at -1. Move 10 steps in the NEG direction. Move 4 steps in the POS direction. Move 5 steps in the NEG direction. Where are you now?

1-04 Adding integers

Integers can be added using a number line.

Example 2

Simplify each sum.

a $-6 + 7$

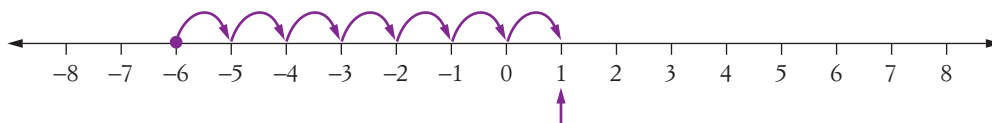
b $8 + (-6)$

c $2 + (-5)$

A **sum** is the answer to an addition

Solution

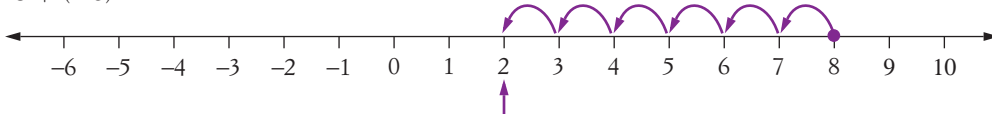
a $-6 + 7$



- -6 is where you start
- $+$ direction is right
- 7 is how far you move

$-6 + 7 = 1.$

b $8 + (-6)$

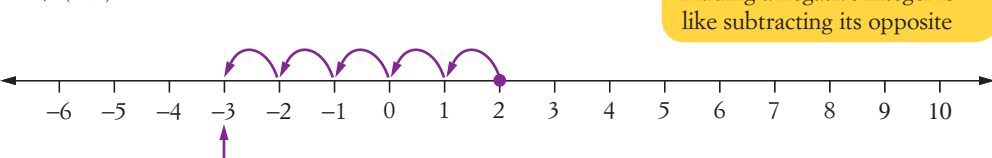


- 8 is where you start
- $+$ direction is right
- -6 is how far you move in the *opposite* direction (that is, left)

$8 + (-6) = 2.$

$8 + (-6)$ is the same as $8 - 6$

c $2 + (-5)$



- 2 is where you start
- $+$ direction is right
- -5 is how far you move in the *opposite* direction (that is, left)

$2 + (-5) = -3.$

$2 + (-5)$ is the same as $2 - 5$

Video tutorial

Adding integers

MAT07NAVT10001

Animated example

Adding directed numbers

MAT07NAAE00002

Puzzle sheet

Integers game

MAT07NAPS10002

Skillsheet

Integers using coloured squares

MAT07NASS10004

Homework sheet

Integers 1

MAT07NAHS10005

TLF learning object

Integer cruncher: Addition (L1100)

Summary

When **adding integers** on a number line:

- adding a **positive** integer means moving to the right
- adding a **negative** integer is the same as **subtracting its opposite**, so it means moving to the left: $2 + (-5) = 2 - 5$

Exercise 1-04 Adding integers

See Example 2

- 1 Simplify each sum.

a $-6 + 10$

b $8 + (-3)$

c $4 + (-5)$

d $3 + (-1)$

e $-2 + 2$

f $-1 + 5$

g $-7 + 4$

h $-2 + (-2)$

i $-8 + (-12)$

j $8 + (-12)$

k $-11 + 3$

l $10 + (-10)$

m $7 + (-9)$

n $-13 + 11$

o $-16 + 5$

p $-7 + 13$

- 2 What is the correct answer for $-2 + 7$? Select **A**, **B**, **C** or **D**.

A 9

B -9

C 5

D -5

- 3 **a** Is the answer to $-5 + 9$ the same as the answer to $9 + (-5)$?

b Can integers be added in any order? Give an example to support your answer.

- 4 Evaluate each expression.

a $-6 + (-6)$

b $-11 + 11$

c $-5 + 25$

d $-14 + (-9)$

e $23 + (-8)$

f $17 + (-17)$

g $0 + (-3)$

h $-27 + 36$

i $-4 + (-15)$

j $9 + (-19)$

k $-32 + 0$

l $-13 + 4$

m $3 + (-4) + 1$

n $-1 + 9 + (-3)$

Evaluate means 'Find the value of'.
Here it means 'to work out the answer'.

- 5 What happens when you add an integer and its opposite?

- 6 For each of the following sums, state whether it is always positive (+), always negative (-), or could be either positive or negative (E).

a The sum of two positive integers

b The sum of two negative integers

c The sum of a positive integer and a negative integer

- 7 Copy and fill in the blank for each equation.

a $7 + \underline{\quad} = 3$

b $-3 + \underline{\quad} = 4$

c $-3 + \underline{\quad} = -6$

d $\underline{\quad} + (-8) = -1$

e $\underline{\quad} + 5 = 2$

f $\underline{\quad} + (-5) = 2$

- 8 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\quad} + \underline{\quad} = 7$

b $\underline{\quad} + \underline{\quad} = -6$

c $\underline{\quad} + \underline{\quad} = 0$

d $\underline{\quad} + \underline{\quad} = 1$

e $\underline{\quad} + \underline{\quad} = -4$

f $\underline{\quad} + \underline{\quad} = -9$

- 9 Copy and fill in the blanks for each equation.

a $\underline{\quad} + \underline{\quad} + \underline{\quad} = 2$

b $\underline{\quad} + \underline{\quad} + \underline{\quad} = -1$

c $\underline{\quad} + \underline{\quad} + \underline{\quad} = -3$

d $\underline{\quad} + \underline{\quad} + \underline{\quad} = 0$

Mental skills 1A Maths without calculators

Adding 8 or 9

A quick way to mentally add 9 or 8 to a number is to add 10 and count back 1 or 2 respectively.

1 Study each example.

$$\begin{aligned} \text{a } 17 + 9 &= 17 + 10 - 1 \\ &= 27 - 1 \\ &= 26 \end{aligned}$$

Count: '17, 27, 26'

$$\begin{aligned} \text{b } 44 + 8 &= 44 + 10 - 2 \\ &= 54 - 2 \\ &= 52 \end{aligned}$$

Count: '44, 54, 52'

$$\begin{aligned} \text{c } 128 + 19 &= 128 + 20 - 1 \\ &= 148 - 1 \\ &= 147 \end{aligned}$$

Count: '128, 148, 147'

$$\begin{aligned} \text{d } 256 + 38 &= 256 + 40 - 2 \\ &= 296 - 2 \\ &= 294 \end{aligned}$$

Count: '256, 296, 294'

2 Now simplify each sum by counting.

$$\text{a } 146 + 9$$

$$\text{b } 212 + 9$$

$$\text{c } 308 + 9$$

$$\text{d } 1755 + 9$$

$$\text{e } 29 + 19$$

$$\text{f } 687 + 39$$

$$\text{g } 254 + 29$$

$$\text{h } 933 + 19$$

$$\text{i } 714 + 8$$

$$\text{j } 623 + 8$$

$$\text{k } 207 + 8$$

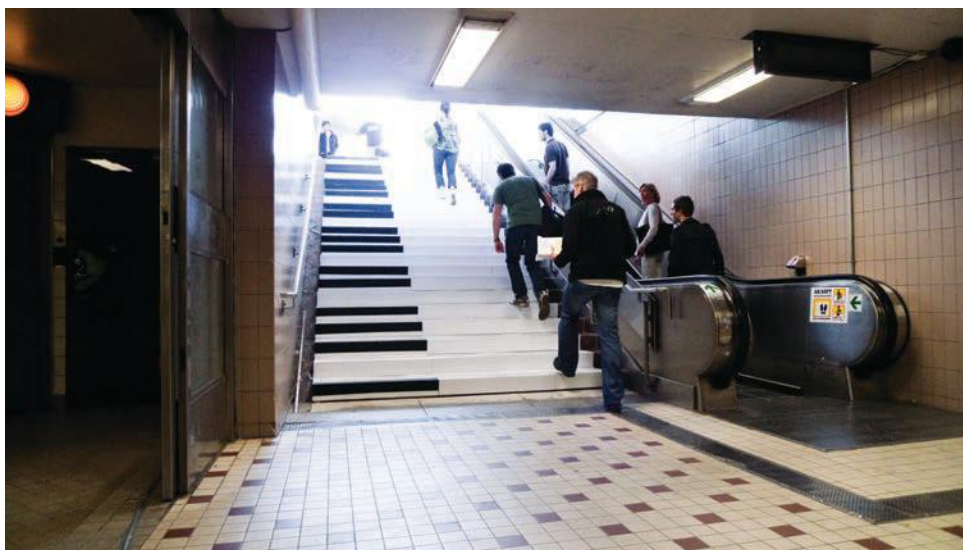
$$\text{l } 155 + 8$$

$$\text{m } 386 + 8$$

$$\text{n } 418 + 48$$

$$\text{o } 909 + 28$$

$$\text{p } 277 + 18$$



1-05 Subtracting integers

Integers can also be subtracted using a number line.

Example 3

Simplify each difference:

a $3 - 4$

b $-3 - 4$

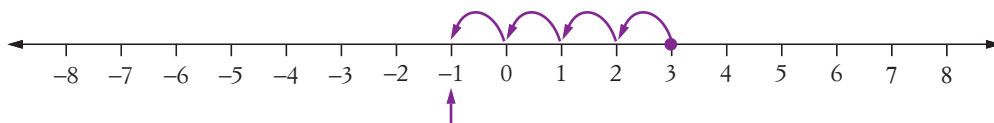
c $4 - (-3)$

d $-3 - (-1)$

A **difference** is the answer to a subtraction.

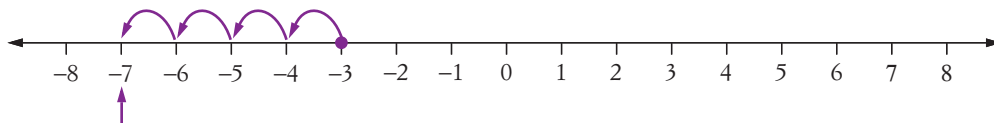
Solution

a $3 - 4$



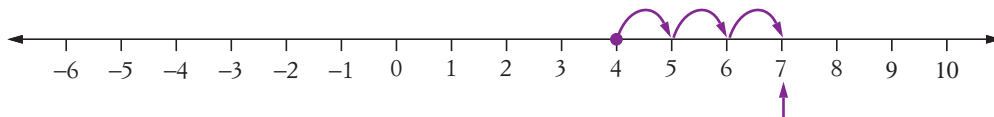
- 3 is where you start
 - $-$ direction is left
 - 4 is how far you move
- $3 - 4 = -1$.

b $-3 - 4$



- -3 is where you start
 - $-$ direction is left
 - 4 is how far you move
- $-3 - 4 = -7$.

c $4 - (-3)$



- 4 is where you start
 - $-$ direction is left
 - -3 is how far you move in the *opposite* direction (that is, right)
- $4 - (-3) = 7$.

$4 - (-3)$ is the same as $4 + 3$

Subtracting a negative integer is like adding its opposite

Animated example
Subtracting directed
numbers

MAT07NAAE00003

Video tutorial
Subtracting integers

MAT07NAVT10002

Worksheet
Adding and subtracting
integers

MAT07NAWK10003

Skillsheet
Integers

MAT07NASS10003

Skillsheet
Integers using coloured
squares

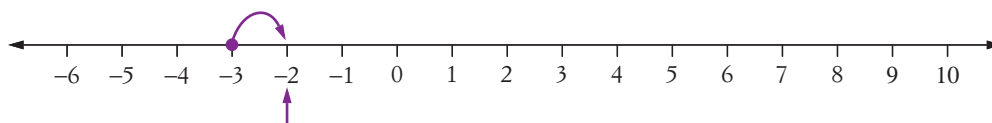
MAT07NASS10004

Homework sheet
Integers 2

MAT07NAHS10006

TLF learning object
Integer cruncher:
Subtraction (L1101)

d $-3 - (-1)$



- -3 is where you start
 - $-$ direction is left
 - -1 is how far you move in the *opposite* direction (that is, right)
- $-3 - (-1) = -2$. $-3 - (-1)$ is the same as $-3 + 1$

Note: ' $-3 - (-1)$ ' is read as 'negative 3 minus negative 1', not 'minus 3 minus minus 1'.

Summary

When **subtracting integers** on a number line:

- subtracting a **positive** integer means moving to the left
- subtracting a **negative** integer is the same as **adding its opposite**, so it means moving to the right: $4 - (-3) = 4 + 3$

Exercise 1-05 Subtracting integers

1 Simplify each difference.

a $4 - 2$

b $2 - 4$

c $3 - 13$

d $5 - (-1)$

e $7 - (-6)$

f $-3 - (-2)$

g $-5 - (-5)$

h $-5 - 5$

i $-6 - 3$

j $-4 - (-5)$

k $2 - 9$

l $4 - (-2)$

m $-6 - (-1)$

n $12 - 18$

o $9 - (-3)$

p $-2 - 7$

2 What is the correct answer for $7 - 12$? Select **A**, **B**, **C** or **D**.

A 19

B -5

C -19

D 5

3 a Is the answer to $10 - 4$ the same as the answer to $4 - 10$?

b Is $-5 - 9$ the same as $9 - (-5)$?

c Can integers be subtracted in any order?

See Example 3

Extra questions

Integers on the number line

MAT07NAEQ00012

4 Evaluate each expression.

a $20 - 9$

b $9 - 20$

c $20 - (-9)$

d $-9 - 20$

e $-4 - 27$

f $-4 - (-27)$

g $13 - (-3)$

h $-13 - (-3)$

i $0 - 50$

j $0 - (-50)$

k $17 - (-25)$

l $-17 - 25$

m $16 - 8 - 20$

n $-4 - (-10) - 1$

5 What happens when you subtract an integer from its opposite?

6 For each of the following differences, state whether it is always positive (+), always negative (-), or could be either positive or negative (E).

a A larger integer minus a smaller integer

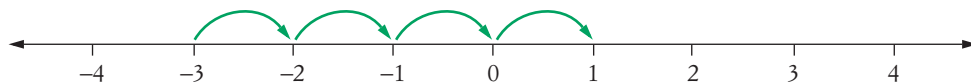
b A smaller integer minus a larger integer

c A positive integer minus a positive integer

d A positive integer minus a negative integer

e A negative integer minus a negative integer

7 The difference between 1 and -3 is also how much 1 is more than -3 on the number line.



a Find the difference between 1 and -3 .

b Evaluate $1 - (-3)$.

c Find the difference between 4 and 1.

d Evaluate $4 - 1$.

e Find the difference between -1 and -2 .

f Evaluate $-1 - (-2)$.

g Find the difference between -1 and 3.

h Evaluate $-1 - 3$.

i Find the difference between -4 and -2 .

j Evaluate $-4 - (-2)$.

8 Copy and complete the blank for each equation.

a $4 - \underline{\hspace{1cm}} = -3$

b $-1 - \underline{\hspace{1cm}} = -4$

c $-2 - \underline{\hspace{1cm}} = -10$

d $\underline{\hspace{1cm}} - (-1) = 9$

e $\underline{\hspace{1cm}} - 6 = -5$

f $\underline{\hspace{1cm}} - (-4) = -2$

9 Copy and complete the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 3$

b $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -2$

c $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 0$

d $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -5$

e $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 6$

f $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -11$

Worked solutions

Exercise 1-05

MAT07NAWS10001

Investigation: Multiplying integers

1 Copy and complete each pattern.

a $9, 6, 3, 0, -3, \_, \_, \_$

b $4, 2, 0, \_, \_, \_$

c $15, 10, 5, \_, \_, \_$

2 Copy and complete each multiplication.

a $3 \times (-7) = (-7) + (-7) + (-7) = \_$

b $2 \times (-5) = (-5) + (-5) = \_$

c $3 \times (-4) = (-4) + (-4) + (-4) = \_$

d $4 \times (-2) = (-2) + (-2) + (-2) + (-2) = \_$

e $2 \times (-6) = \_$

f $3 \times (-3) = \_$

3 a In pairs or as a group activity, copy the multiplication grid below onto a large piece of paper and complete the shaded section.

$\times$	5	4	3	2	1	0	-1	-2	-3	-4	-5
5											
4											
3											
2											
1											
0											
-1											
-2											
-3											
-4											
-5											

b Continue the pattern for the first row: 25, 20, 15, 10, 5, 0, -5, ...

c Continue the pattern for the second row: 20, 16, 12, 8, 4, 0, -4, ...

d Complete the next four rows.

e Continue the pattern for each **column**.

4 Use your completed table to simplify each product.

a $4 \times (-3)$

b -3×5

c $-4 \times (-2)$

d $5 \times (-1)$

e $0 \times (-4)$

f -3×3

g $2 \times (-4)$

h $-5 \times (-5)$

5 How do the signs (positive or negative) of the integers in the question affect the sign of the product (answer)? Try to put the rules for multiplying integers into words and write them down.

YEAR 8

1-06 Multiplying integers

Summary

When **multiplying integers**:

- positive $\times$ positive = positive
- positive $\times$ negative = negative
- negative $\times$ positive = negative
- negative $\times$ negative = positive

or

- If both numbers have the **same** sign, the answer is **positive**.
- If both numbers have **different** signs, the answer is **negative**.

$\otimes$	+	-
+	+	-
-	-	+

Example 4

Simplify each product.

a -3×2

c $4 \times (-4)$

b $-6 \times (-7)$

d $(-6)^2$

A **product** is the answer to a multiplication

Solution

a $-3 \times 2 = -6$

b $-6 \times (-7) = 42$

c $4 \times (-4) = -16$

d $(-6)^2 = (-6) \times (-6) = 36$

negative $\times$ positive = negative

negative $\times$ negative = positive

positive $\times$ negative = negative

negative $\times$ negative = positive

$(-6)^2$ is '−6 squared' and means multiplying −6 by itself

Exercise 1-06 Multiplying integers

See Example 4 1 Simplify each product.

a -3×6

e $-7 \times (-9)$

i $3 \times (-8)$

m $5 \times (-2)$

q $-12 \times (-5)$

b $6 \times (-4)$

f -9×5

j -6×1

n -7×6

r $20 \times (-3)$

c $-3 \times (-7)$

g 7×6

k $-1 \times (-1)$

o $-11 \times (-4)$

s $-10 \times (-10)$

d $4 \times (-8)$

h $-7 \times (-6)$

l -9×6

p -9×9

t $-7 \times (-8)$

2 For each expression, select the correct answer A, B, C or D.

a $8 \times (-5)$

A 13

B -13

C -40

D 40

b $-7 \times (-3)$

A 21

B -21

C -10

D 10

c $-2 \times 3 \times 7$

A 1

B -1

C 42

D -42

3 Copy and complete each multiplication grid.

a

$\otimes$	-3	5
-3		
5		

b

$\otimes$	-5	5
5		
-5		

c

$\otimes$	6	-4
-4		
6		

d

$\otimes$	-1	1
-1		
1		

e

$\otimes$	7	-3
4		
-6		

f

$\otimes$	-3	-6
8		
2		

4 a Is the answer to $6 \times (-4)$ the same as the answer to $(-4) \times 6$?

b Can integers be multiplied in any order?

5 What happens when you multiply an integer by its opposite?

6 Evaluate:

a 3^2

b $(-3)^2$

c 7^2

d $(-7)^2$

e 5^2

f $(-5)^2$

g 1^2

h $(-1)^2$

7 For each of the following products, state whether it is always positive (+), always negative (-), or could be either positive or negative (E).

a The product of two positive integers

b The product of two negative integers

c The product of a positive integer and a negative integer

d A positive integer squared

e A negative integer squared

8 Copy and complete the blank for each equation.

a $2 \times \underline{\hspace{1cm}} = -10$

b $-4 \times \underline{\hspace{1cm}} = 16$

c $9 \times \underline{\hspace{1cm}} = -27$

d $\underline{\hspace{1cm}} \times (-5) = 35$

e $\underline{\hspace{1cm}} \times 8 = -40$

f $\underline{\hspace{1cm}} \times (-11) = 22$

9 Copy and complete the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 18$

b $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = -28$

c $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 30$

d $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = -2$

e $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 6$

f $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = -45$

Worked solutions

Exercise 1-06

MAT07NAWS10002

Worked solutions

Exercise 1-06

MAT07NAWS10002

Technology Daily temperatures in Thredbo

This spreadsheet shows the minimum and maximum daily temperatures for Thredbo for the last two weeks in July 2010.

- 1 Enter the data into a spreadsheet.

	A	B	C
1	Date	Min Temp (deg C)	Max Temp (deg C)
2	Sun 18	-5	4.8
3	Mon 19	0	4
4	Tue 20	-8.5	5
5	Wed 21	-9.5	5.3
6	Thu 22	-5	4.9
7	Fri 23	-1.2	7
8	Sat 24	-4.5	8
9	Sun 25	-4	6.8
10	Mon 26	-6.4	7
11	Tue 27	-8	8.5
12	Wed 28	-6	7.5
13	Thu 29	-3.5	7.4
14	Fri 30	1.6	8.5
15	Sat 31	3	6



- 2 In cell D1, enter the label 'Difference'. To find the difference between the maximum and minimum temperatures for Sunday 18, enter the formula **=C2-B2** in cell D2.

	A	B	C	D
1	Date	Min Temp (deg C)	Max Temp (deg C)	Difference
2	Sun 18	-5	4.8	9.8
3	Mon 19	0	4	
4	Tue 20	-8.5	5	
5	Wed 21	-9.5	5.3	
6	Thu 22	-5	4.9	
7	Fri 23	-1.2	7	
8	Sat 24	-4.5	8	
9	Sun 25	-4	6.8	
10	Mon 26	-6.4	7	
11	Tue 27	-8	8.5	
12	Wed 28	-6	7.5	
13	Thu 29	-3.5	7.4	
14	Fri 30	1.6	8.5	
15	Sat 31	3	6	
16				

- 3 To copy D2's formula into cells D3 to D15, click on D2 and position the cursor over the bottom right-hand corner of this cell so that it becomes a '+' sign. Drag down to cell D15 to highlight cells D3 to D15, as shown on the right. This is called **Fill Down** and you should notice that the temperature differences for each day have been calculated. Fill Down can also be selected by highlighting cells D2 to D15 and either selecting Fill Down from the Editing toolbar in the Home menu or by pressing Ctrl-D.
- 4 a On which day was the largest difference between the maximum and minimum temperatures recorded? Enter your answer in cell E1.
b On which day was the smallest difference recorded? Enter your answer in cell E2.
- 5 Enter the following labels: 'Lowest Minimum' in cell A17, 'Highest Minimum' in cell A18, 'Lowest Maximum' in D17, 'Highest Maximum' in D18, 'Difference' in A19.
- 6 a To find the lowest minimum, in cell B17 enter the formula **=min(B2:B15)**, which finds the lowest number from cells B2 to B15.
b To find the highest minimum, in cell B18 enter **=max(B2:B15)**.
c Now, enter appropriate formulas into cells C17 and C18 for the lowest and highest maximums.
d What do you notice about the value in cell B17, compared to the other three values?
- 7 In cell B19 enter **=B18-B17** for the difference between the highest and lowest minimums. From cell B19, use **Fill Right** to copy this formula into cell C19. Alternatively, you can highlight cells B19 and C19 and either select Fill Right from the Editing toolbar in the Home menu or press Ctrl-R.
- 8 a Which column (B or C) had the greatest difference in temperature? Enter your answer in cell E3.
b In cell E4, enter a formula to find the difference between the lowest minimum temperature and the highest maximum temperature.

Investigation: Dividing integers

1 In pairs or as a group activity, copy and complete each question.

a $5 \times \underline{\hspace{1cm}} = -20$

b $-6 \times \underline{\hspace{1cm}} = 12$

c $-3 \times \underline{\hspace{1cm}} = 27$

d $\underline{\hspace{1cm}} \times (-4) = -16$

e $\underline{\hspace{1cm}} \times 2 = -10$

f $\underline{\hspace{1cm}} \times -5 = 15$

g $-20 \div 5 = \underline{\hspace{1cm}}$

h $27 \div (-3) = \underline{\hspace{1cm}}$

i $-16 \div (-4) = \underline{\hspace{1cm}}$

j $-14 \div 2 = \underline{\hspace{1cm}}$

k $-12 \div (-12) = \underline{\hspace{1cm}}$

l $21 \div (-3) = \underline{\hspace{1cm}}$

2 Write down the rules for dividing integers in words.

YEAR 8

1-07 Dividing integers

Worksheet

Integer review

MAT07NAWK10005

Worksheet

What is the question?

MAT07NAWK10006

Skillsheet

Integers

MAT07NASS10003

Because division is the opposite of multiplication, the rules for dividing integers are the same as those for multiplying them.

Summary

When **dividing integers**:

- positive $\div$ positive = positive
- positive $\div$ negative = negative
- negative $\div$ positive = negative
- negative $\div$ negative = positive

or

- If both numbers have the **same** sign, the answer is **positive**.
- If both numbers have **different** signs, the answer is **negative**.

$\div$	+	-
+	+	-
-	-	+

Example 5

Simplify each quotient:

a $-15 \div 3$

c $-24 \div (-4)$

b $18 \div (-2)$

d $\frac{14}{-7}$

A **quotient** is the answer to a division

Solution

a $-15 \div 3 = -5$

b $18 \div (-2) = -9$

c $-24 \div (-4) = 6$

d $\frac{14}{-7} = 14 \div (-7) = -2$

negative $\div$ positive = negative

positive $\div$ negative = negative

negative $\div$ negative = positive

positive $\div$ negative = negative

Exercise 1-07 Dividing integers

See Example 5

1 Simplify each quotient.

a $20 \div (-5)$

b $-8 \div 4$

c $-12 \div (-3)$

d $6 \div (-2)$

e $27 \div (-3)$

f $-18 \div (-6)$

g $24 \div (-8)$

h $40 \div (-4)$

i $-20 \div 5$

j $-21 \div (-3)$

k $45 \div (-5)$

l $-36 \div 6$

m $-45 \div (-5)$

n $-16 \div 2$

o $28 \div (-7)$

p $48 \div (-8)$

2 For each expression, select the correct answer A, B, C or D.

a $-48 \div (-2)$

A 24

B -24

C 46

D -46

b $\frac{16}{-4}$

A 12

B -4

C 4

D -12

3 Simplify:

a $\frac{12}{-2}$

b $\frac{100}{-50}$

c $\frac{-18}{-3}$

d $\frac{-32}{-2}$

e $\frac{-42}{6}$

f $\frac{45}{-5}$

g $\frac{100}{5}$

h $\frac{-81}{-9}$

4 Divide the top row by the left-hand column to complete each division grid.

a

$\div$	-6	-3	3	6
-6				
-3				

b

$\div$	8	-8	32	-32
-2				
4				

5 What happens when you divide an integer by its opposite?

6 For the statements below, are the results always positive (+), always negative (-), or could they be either (E)?

a The quotient of two positive integers

b The quotient of two negative integers

c The quotient of a positive integer and a negative integer

7 Copy and fill in the blank for each equation.

a $16 \div \underline{\hspace{1cm}} = -2$

b $-24 \div \underline{\hspace{1cm}} = -8$

c $35 \div \underline{\hspace{1cm}} = -5$

d $\underline{\hspace{1cm}} \div (-11) = 4$

e $\underline{\hspace{1cm}} \div 5 = -10$

f $\underline{\hspace{1cm}} \div (-8) = 4$

8 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 10$

b $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -4$

c $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 5$

d $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -8$

e $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 6$

f $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -3$

YEAR 8

1-08 Order of operations

Now that you can add, subtract, multiply and divide with integers, you can evaluate mixed expressions involving integers. Remember to follow the ‘order of operations’ rules.

Summary

To evaluate mixed expressions, the **order of operations** is:

- 1 Simplify any expression inside grouping symbols (brackets)
- 2 Simplify any multiplication ($\times$) and division ($\div$), from left to right
- 3 Simplify any addition (+) and subtraction ($-$), from left to right

Skillsheet

Order of operations

MAT07NASS10005

Example 6

Evaluate each expression.

a $3 \times (-4 - 6)$

b $8 + (-7) \times 4 - (-2)$

Solution

a $3 \times (-4 - 6) = 3 \times (-10)$
 $= -30$

brackets first: $-4 - 6 = -10$

b $8 + (-7) \times 4 - (-2) = 8 + (-28) - (-2)$
 $= -20 - (-2)$
 $= -18$

multiplication first: $(-7) \times 4 = -28$
left to right: $8 + (-28) = -20$
 $-20 - (-2) = -20 + 2$

Exercise 1-08 Order of operations

Extra questions

Operating with integers

MAT07NAEQ00013

- 1 Evaluate each expression.

a $9 - 12 + 10$

b $-13 - 10 - (-1)$

c $4 - 6 + 2 + (-1)$

d $1 - 6 + 7 - 9$

e $6 + (-3) - 9$

f $7 - 5 - (-4) + (-2)$

g $5 \times (-10) \div 2$

h $6 \times (-4) \div (-3)$

i $7 \times 6 \div (-2)$

j $-3 \div (-1) \times 4$

k $24 \div (-6) \times 50$

l $-56 \div (-8) \times 4$

m $-100 \div (-20) \times 3$

n $-5 \times (-9) \div 9$

o $2 \times 12 \div (-4) \div (-2)$

See Example 6

- 2 What is the value of $6 + 4 \times (-2)$? Select **A**, **B**, **C** or **D**.

A 14

B -20

C -2

D -14

Worked solutions

Exercise 1-08

MAT07NAWS10003

- 3 Evaluate each expression.

a $-5 \times 2 + 3$

b $-5 \times (2 + 3)$

c $3 \times 6 \times (-2)$

d $12 - 8 \div 4$

e $12 - 8 \div (-4)$

f $-3 \times (9 - 10)$

g $21 \div (-7) + 8$

h $[-6 - (-2)] \times (-4)$

i $(-6 - 2) \div (-4)$

j $-3 \div 3 + 9$

k $12 \times (6 - 7) + 1$

l $-4 \times 7 \div (-7 - 7)$

4 Copy and fill in the blank for each equation.

a $-5 \times 6 - \underline{\hspace{1cm}} = -38$ **b** $\underline{\hspace{1cm}} \times (6 - 8) = -10$ **c** $[8 - \underline{\hspace{1cm}}] \times 4 = 44$
d $5 + \underline{\hspace{1cm}} \times (-7) = 19$ **e** $8 - (-3 \times \underline{\hspace{1cm}}) = 17$ **f** $\underline{\hspace{1cm}} \times (-5 + 3) = -24$
g $6 \times \underline{\hspace{1cm}} + (-4) = -16$ **h** $15 \div (-3) \times \underline{\hspace{1cm}} = 10$ **i** $\underline{\hspace{1cm}} + 4 \times 6 = 5$
j $[-6 - \underline{\hspace{1cm}}] \times (-2) = -6$ **k** $(-6 - 3) \div \underline{\hspace{1cm}} = -3$ **l** $\underline{\hspace{1cm}} \div (-5) + 6 = 1$

5 Nuraan completed these 10 questions. Mark her work and correct any mistakes.

a $6 - 9 + 5 - 11 = -9$ **b** $-32 \div (-4) \times (-2) = 4$
c $-6 \times (-2) - 8 = -4$ **d** $-11 + 11 \div (-1) = -22$
e $24 \div (-4) \div (-2) = 3$ **f** $(-2 + 6) \times 2 = -8$
g $-7 - 6 + 10 = 3$ **h** $(-12 - 3) \div (-3) = 3$
i $12 - 3 \times 5 = 45$ **j** $2 - 9 \times (-3) - (-18) = 39$

6 Copy each equation and insert grouping symbols to make it true.

a $-4 \times 7 - 2 + (-3) = -8$ **b** $-4 \times 7 - 2 + (-3) = -27$
c $-4 \times 7 - 2 + (-3) = -33$ **d** $40 \div 10 \times (-2) + 15 = 13$
e $40 \div 10 \times (-2) + 15 = 52$ **f** $40 \div 10 \times (-2) + 15 = -8$

7 Use all the integers 4, -8 and 3 and grouping symbols to complete each equation.

a $\underline{\hspace{2cm}} = -20$ **b** $\underline{\hspace{2cm}} = 15$ **c** $\underline{\hspace{2cm}} = 8$

Mental skills 1B Maths without calculators

Subtracting 8 or 9

A quick way to mentally subtract 9 or 8 from a number is to subtract 10 and count forward 1 or 2 respectively.

1 Study each example.

a $66 - 9 = 66 - 10 + 1$
 $= 56 + 1$
 $= 57$
 Count: '66, 56, 57'

c $72 - 49 = 72 - 50 + 1$
 $= 22 + 1$
 $= 23$
 Count: '72, 22, 23'

b $83 - 8 = 83 - 10 + 2$
 $= 73 + 2$
 $= 75$
 Count: '83, 73, 75'

d $141 - 28 = 141 - 30 + 2$
 $= 111 + 2$
 $= 113$
 Count: '141, 111, 113'

2 Now simplify each difference by counting.

a $26 - 9$ **b** $44 - 9$ **c** $123 - 9$ **d** $270 - 9$
e $161 - 29$ **f** $187 - 59$ **g** $75 - 19$ **h** $457 - 39$
i $82 - 8$ **j** $131 - 8$ **k** $96 - 8$ **l** $120 - 8$
m $44 - 8$ **n** $577 - 28$ **o** $203 - 18$ **p** $365 - 48$

YEAR 8

1-09 Integers and the calculator

Negative numbers can be entered into a calculator using the sign change key **(-)** or **+/-**. For example, to enter '-3', type **(-)** 3 or **+/-** 3.

Example 7

Use your calculator to evaluate:

a $798 \div (-7) + 135$

b $23 \times [10 - (-6)]$

c $(-18)^2$

Solution

a $798 \div (-7) + 135 = 21$

On a calculator, enter 798 $\div$ **(-)** 7 $+$ 135 $=$

b $23 \times [10 - (-6)] = 368$

On a calculator, enter 23 $\times$ **(** 10 $-$ **(-)** 6 **)** $=$

c $(-18)^2 = 324$

On a calculator, enter **(** **(-)** 18 **)** $\times^2$ $=$

There are more examples in Appendix 1: Calculator Skills on page 519.

Exercise 1-09 Integers and the calculator

1 Evaluate each expression **without** using a calculator.

a $6 - 13$

b $4 - (-2)$

c $-7 + 3$

d $-5 - (-6)$

e $3 + (-8)$

f $11 - (-3)$

g $11 - (-15)$

h $-13 + (-15)$

i $12 - (-18)$

j -8×6

k $-11 \times (-4)$

l $-5 \times (-6)$

m $13 \times (-5)$

n $88 \div (-11)$

o $-30 \div (-2)$

p $-36 \div 9$

q $(-5)^2$

r $(-8)^2$

s $-3 \times (-4)^2$

t $18 \div (-3)^2$

See Example 7

2 Check your answers to question 1 using a calculator.

3 When using your calculator, why are there different answers for $(-5)^2$ and -5^2 ? Explain the difference in meaning between $(-5)^2$ and -5^2 .

4 Use a calculator to evaluate each expression.

a $13 + (-22) - 9$

b $64 + 23 + (-16)$

c $-200 \div (-100) \div 2$

d $-40 \times (-3) \div (-20)$

e $-44 \times (-15) \times (-6)$

f $48 \div 3 \times (-12)$

g $125 \div (-5) \div (-5)$

h $[12 - (-10)] \times (-3) + 6$

i $-3 \times (-8) \div (-4) + (-11)$

j $9 - 19 - 8 \times (-6)$

k $-12 - 30 \div (-2) + 58$

l $6 + 4 \times (-5) \div (-2)$

5 Copy and fill in the blank for each equation.

a $6 \times (\_\_ - 27) = -84$

b $\_\_ - 8 - (-14) = -4$

c $45 \div (-9) \times \_\_ = -10$

d $-2 \times \_\_ + 11 = -19$

e $\_\_ \times (-4) \div 2 = 112$

f $-14 - 6 \times \_\_ = 34$

6 Use all the integers 75, -18 and -15 and grouping symbols to complete each equation.

a $\_\_ = -1395$

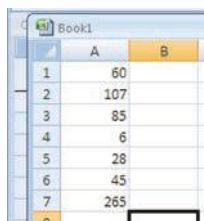
b $\_\_ = -13$

c $\_\_ = -855$

Technology Sorting numbers

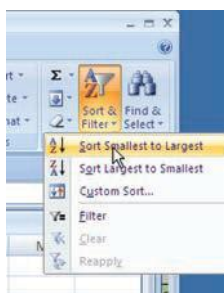
In this activity, we will sort this set of numbers {60, 107, 85, 6, 28, 45, 265} by using a spreadsheet.

- 1 a Enter the set of numbers {60, 107, 85, 6, 28, 45, 265}, in the given order, into column A of a spreadsheet.



	A	B
1	60	
2	107	
3	85	
4	6	
5	28	
6	45	
7	265	

- b Highlight cells A1 to A7 and select **Sort & Filter** from the Editing menu on the top right-hand side. Choose the option shown for ascending order: **Smallest to Largest**.



- c The data should now be sorted from the smallest number, 6 (in cell A1), to the largest number, 265 (in cell A7).
- d A set of numbers can also be sorted in descending order: choose **Largest to Smallest**.
- 2 Now enter each of the following sets of numbers in the columns given, and sort in ascending order.
- a {55, 89, 36, 21, 19, 4, 95} in column B
- b {-17, 14, -90, 58, 0, 392, -6} in column C
- c {4987, -4200, 8740, -5176, 2601, 713, -4810} in column D
- d {16 101, 12 167, 10 010, 11 412, 10 107, 10 761, 11 214} in column E
- 3 Sort each column of data from question 2 in descending order.

1-10 Integer problems

Example 8

The average temperature on the planet Mars is -63°C while the average temperature on Jupiter is -150°C . What is the difference between these temperatures?

Solution

Write a number sentence to calculate the answer.

$$\begin{aligned} -63 - (-150) &= -63 + 150 \\ &= 87 \end{aligned}$$

The difference in temperature is 87°C .

Exercise 1-10 Integer problems

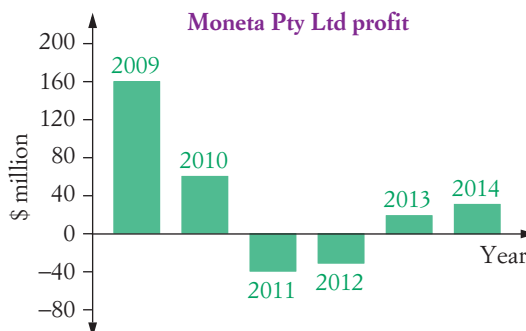
See Example 8

For each question, write a number sentence to calculate the answer.

- 1 In Bathurst, the temperature reached a maximum of 11°C . Overnight, it dropped to a minimum of -2°C . How many degrees did it drop?
- 2 Craig walked 16 kilometres east and then 20 kilometres west. How far is he from his starting point?
- 3 The heights above and below sea level of some places around the world are given below.

The Dead Sea (Jordan)	-397 metres
Death Valley (USA)	-86 metres
Mt Kosciuszko (Australia)	2230 metres
Mt Everest (Nepal)	8840 metres

 - a How much higher is Mt Everest than Mt Kosciuszko?
 - b How much higher is Mt Kosciuszko than Death Valley?
 - c How much lower is the Dead Sea than Death Valley?
 - d The highest point on Earth is Mt Everest and the lowest point is the Dead Sea. What is the difference in height between these two points?
- 4 The graph below shows the profit made by the company Moneta over 6 years.
 - a In which years did Moneta make a profit?
 - b In which years did Moneta make a loss?
 - c What was the decrease in profit between 2010 and 2011?
 - d Did the company make an overall profit or loss over the six years shown? How much profit or loss did they make?



- 5 Anya left home and walked 3 kilometres south to Brock's home. Together, they then walked north for 4 kilometres. What distance and direction are they from Anya's home now?
- 6 Karina had \$74 in her bank account yesterday but was allowed to withdraw \$121. Today, she deposited \$40 into the account. What is her account balance now?
- 7 In Hicksville, the temperature overnight dropped to -6°C . During the day it rose by 13°C but then dropped 5°C after sunset. Which of the following expressions describes the temperature after sunset? Select **A**, **B**, **C** or **D**.
- A** $-6 + 13 - 5$ **B** $-6 - 13 - 5$ **C** $-6 + 13 + 5$ **D** $-5 - 13 - 6$
- 8 In indoor cricket, each time a batting player gets out, the team's score is reduced by 5 (that is, -5 is added to the total). Liam's cricket team is midway through a match. The first 5 batters have each scored 23, 16, 19, 0 and 16 before getting out. Liam is still batting and has scored 10 so far. What is the current score for Liam's team?
- 9 Mt Etna is an active volcano in Italy, 3200 m high. The lava starts 652 m below the Earth's surface and shoots to a height of 750 m above the top of Mt Etna. How far does the lava travel?

Worked solutions

Exercise 1-10

MAT07NAWS10004



- 10 A bird dives into a river from a height of 680 cm above the water, travelling at a speed of 120 cm per second. How far below the water will it be after 8 seconds?
- 11 The submarine Nemesis descends from the surface to a depth of 955 metres to inspect a shipwreck. It then ascends 800 metres to send a message. How far below the surface is it when it sends the message?
- 12 Find the closing balance of this bank account after all deposits and withdrawals.
- | | |
|------------------|-------------------|
| Starting balance | \$ 37.85 |
| Deposit | \$ 18.20 |
| Deposit | \$120.00 |
| Withdraw | \$ 45.00 |
| Deposit | \$ 8.15 |
| Withdraw | <u>\$248.20</u> |
| Closing balance | <u> </u> |
- 13 A scuba diver was swimming at a depth of 18 metres. Her diving friend was at a depth of 7 metres. What was the vertical distance between them?

Worked solutions

Exercise 1-10

MAT07NAWS10004

- 14 Jayden's bank balance is $-\$120$ but he deposits $\$9$ each day for the next 20 days.
- On which day will his bank balance become positive?
 - What will be his bank balance after 20 days?

Investigation: Above and below sea level

- What is sea level?
- Where is the highest point in Australia? What is its height above sea level?
- Where is the lowest point in Australia? What is its height below sea level?
- What is the difference in height between the highest and lowest points in Australia? Is it more than a kilometre?
- What is the height of your town or city above sea level?
- Find five places in the world that are above sea level and five that are below sea level. Write four questions about this information for other students to answer. Swap your work with other students and answer their questions. Note: you must know the answers to your questions so you can mark other students' work!

Power plus

- Copy and complete each pattern.
 - $(-1)^2 = (-1) \times (-1) = \underline{\hspace{2cm}}$
 - $(-1)^3 = (-1) \times (-1) \times (-1) = \underline{\hspace{2cm}}$
 - $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = \underline{\hspace{2cm}}$
 - $(-1)^5 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 - $(-1)^6 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 - $(-1)^7 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- Explain how $(-1)^n$, where n is a whole number, is equal to only one of two possible numbers and which one it is depends on the type of number n is.
- Find the number (or numbers) which make each of these expressions true.
 - $\underline{\hspace{2cm}}^2 = 16$
 - $\underline{\hspace{2cm}}^2 = 100$
 - $\underline{\hspace{2cm}}^3 = -8$
- Dylan says there is no answer to $\underline{\hspace{2cm}}^2 = -25$. Is he correct?
- Find three integers that have a sum of -7 .
 - Find two negative integers that have a sum of -1 .
 - Find three integers that have a product of 36 .
 - Find three integers that have a product of -24 .
- Find the average of:
 - -8 and 12
 - -1 and -17
 - 21 and -19

7 If a and b are integers, determine whether the following equations are true or false.

a $a + b = b + a$	b $a - b = b - a$	c $a \times b = b \times a$
d $a \div b = b \div a$	e $a + (-a) = 0$	f $(-a) \times (-a) = a \times a$
g $a - (-a) = 0$	h $a \div (-a) = -1$	i $(-a) + (-a) = -2 \times a$

8 Find two integers that:

- a** have a sum of -3 and a product of -10
- b** have a sum of -8 and a product of 15
- c** have a sum of -2 and a product of -8
- d** have a sum of -1 and a product of -6
- e** have a sum of 7 and a product of -30

Chapter 1 review

Language of maths

Puzzle sheet

Integers find-a-word

MAT07NAPS10005

ascending	evaluate	number line	quotient
Celsius	greater than ($>$)	opposite	sign
degrees ($^{\circ}$)	integer	order	sum
deposit	less than ($<$)	order of operations	whole number
descending	minus	positive	withdraw
difference	negative	product	zero

- 1 What is the opposite of 'deposit'?
- 2 What number is neither positive nor negative?
- 3 Write sentences using the words 'ascending' and 'descending' to show their meanings.
- 4 Write ' $-5 < -1$ ' in words.
- 5 What is the order of operations?
- 6 What is the difference between -4 ('negative 4') and -4 ('minus 4')?

Topic overview

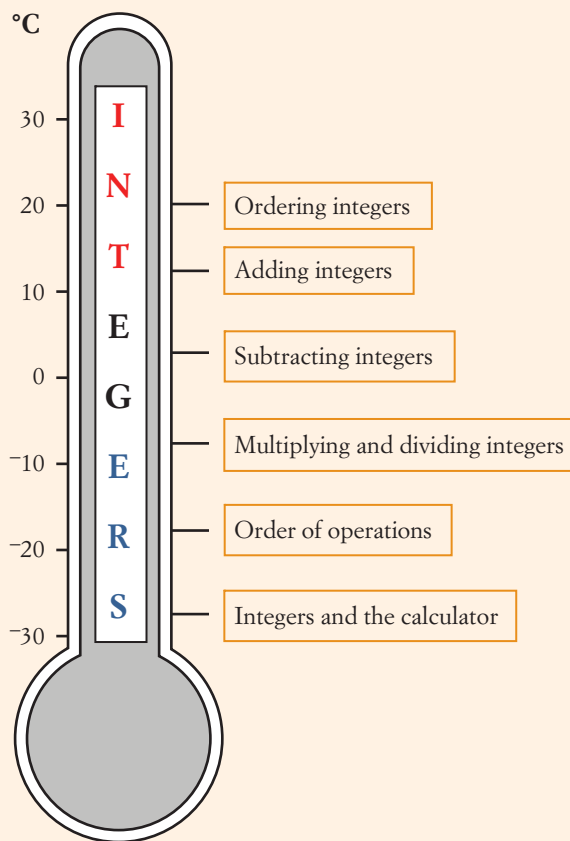
Worksheet

Mind map: Integers

MAT07NAWK10007

- Write in your own words what integers are.
- Write any rules you have learnt about integers.
- What parts of this topic did you find difficult?
- What parts didn't you understand? Discuss them with a friend or your teacher.
- Give some examples of where negative numbers are used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Write the opposite of each integer. See Exercise 1-01
a -2 **b** -23 **c** 56 **d** -10 **e** 8 **f** 0
- 2 Use a separate number line to mark each of the following sets of numbers. See Exercise 1-02
a $6, -3, 5, 1, -1, 4, 7$
b integers between -2 and 5
c integers less than 1 and greater than -6
- 3 Write true (T) or false (F) for each statement. See Exercise 1-03
a $-4 > 2$ **b** $-2 < -3$ **c** $-1 > 1$ **d** $2 > -1$
e $-11 < -10$ **f** $-3 > -4$ **g** $-8 < 11$ **h** $-4 > -3$
i $5 < -4$ **j** $-16 < 0$ **k** $1 > -5$ **l** $-9 > -2$
- 4 Write this set of integers in descending order: See Exercise 1-03
 $-9, 14, 3, 0, -5, -10, -1.$
- 5 Evaluate each expression. See Exercise 1-04
a $3 + (-4)$ **b** $-8 + (-2)$ **c** $10 + (-13)$
d $-4 + 8$ **e** $-5 + (-5)$ **f** $-3 + 6$
g $-5 + (-4) + (-2)$ **h** $-8 + 5 + 4$ **i** $-7 + 6 + (-7)$
- 6 Evaluate each expression. See Exercise 1-05
a $3 - (-4)$ **b** $-5 - (-9)$ **c** $7 - 11$
d $-4 - 8$ **e** $-5 - (-5)$ **f** $-3 - 4 - 5$
g $-8 + 5 - 4$ **h** $8 - 6 - (-4)$ **i** $3 - 8 + (-6)$
- 7 Evaluate each product. See Exercise 1-06
a $6 \times (-3)$ **b** $-8 \times (-6)$ **c** -2×5
d -11×4 **e** $7 \times (-9)$ **f** 6×8
g $-6 \times (-4) \times (-1)$ **h** $-2 \times 3 \times 7$ **i** $-4 \times 2 \times (-5)$
- 8 Evaluate each quotient. See Exercise 1-07
a $-12 \div (-4)$ **b** $15 \div (-5)$ **c** $-6 \div (-6)$
d $16 \div (-4)$ **e** $-24 \div 12$ **f** $-1000 \div 10$
g $24 \div (-6) \div (-2)$ **h** $-36 \div 3 \div 4$ **i** $-64 \div (-4) \div 2$
- 9 Evaluate each expression. See Exercise 1-08
a $-6 + (-3) \times 2$ **b** $4 - 3 \times 6$ **c** $-12 \div 4 - 10$
d $-3 \times 5 + 6 \div (-2)$ **e** $30 \div (-6) + 5$ **f** $-2 \times 7 + 15 \div (-3)$
g $-12 + (-3) \times (-7) - 2$ **h** $14 + 7 \times (-3) + 5$ **i** $-6 + -6 \div (-2) \times 2$
- 10 Use your calculator to evaluate each expression. See Exercise 1-09
a $28 - 37$ **b** $-17 + 11$ **c** $18 - 15 \times 2$
d $100 - 120 \div 4 + 17$ **e** $-16 \times 3 + (-4)$ **f** $[5 + (-20)] \div (-3)$
- 11 The minimum overnight temperature in Goulburn was -5°C . During the day, it rose to 11°C . See Exercise 1-10
By how much did the temperature rise?
- 12 Harrison starts a new job and opens a new bank account. Each week he saves \$80 to put into See Exercise 1-10
the account. Each month, however, \$210 is withdrawn from the account to pay off a car loan.
Calculate how much will be in Harrison's account after 6 months if there are 26 weeks in
6 months.

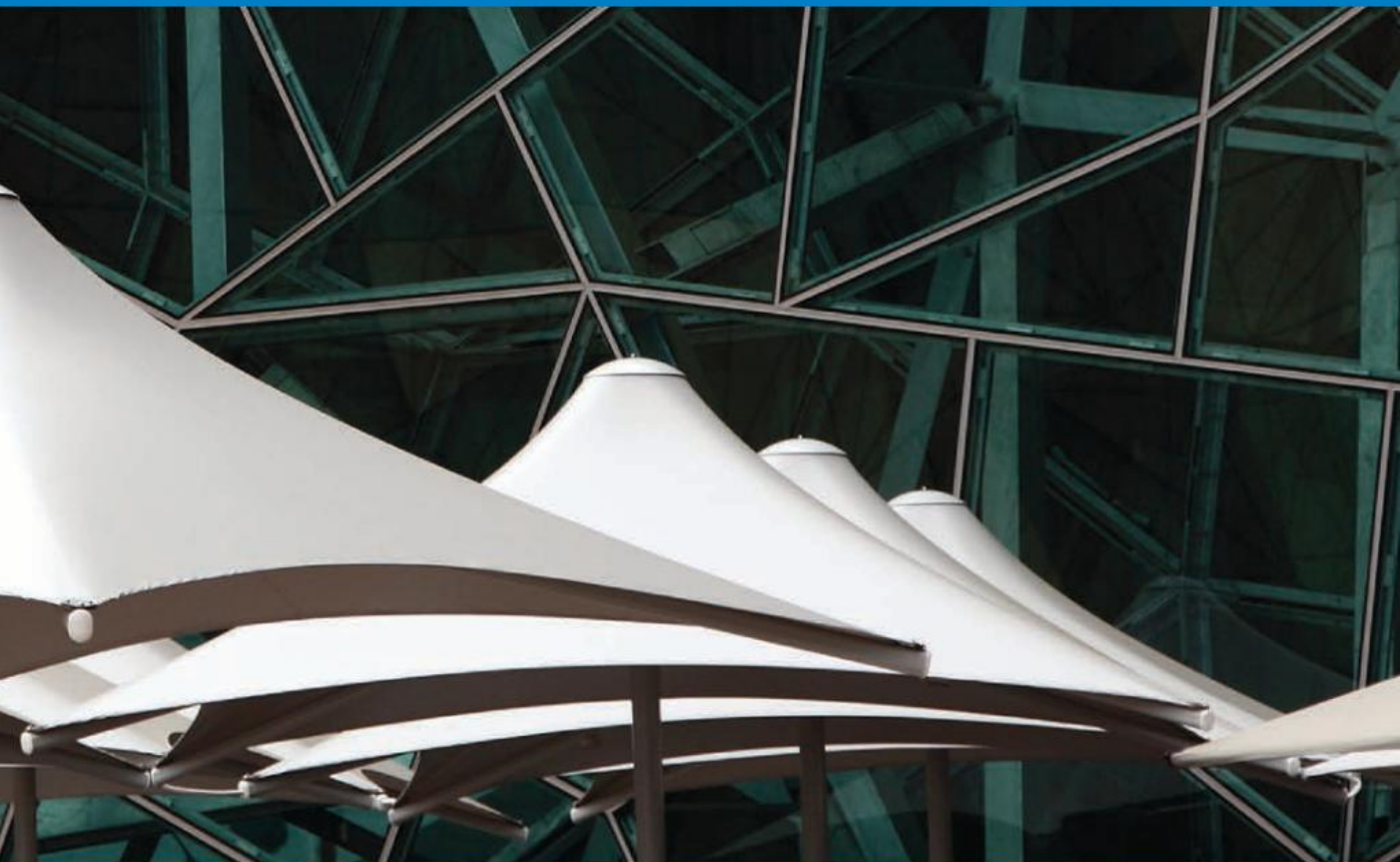


2

Measurement and geometry

Angles

Angles are used in all aspects of life. Builders and engineers use angles for buildings, roads and bridges. Sportspeople use angles when playing snooker, when shooting for goals and even when skiing. Artists use their knowledge of angles to draw portraits and paintings.



Chapter outline

	Proficiency strands			
2-01 Naming angles	U			
2-02 Measuring and drawing angles	U			
2-03 Classifying angles	U			
2-04 Complementary and supplementary angles	U	F		
2-05 Angles at a point and vertically opposite angles	U	F		
2-06 Constructing parallel and perpendicular lines	U			
2-07 Corresponding angles on parallel lines	U	F		
2-08 Alternate angles on parallel lines	U	F		
2-09 Co-interior angles on parallel lines	U	F		
2-10 Angles on parallel lines	U	F	PS	R
2-11 Proving parallel lines	U	F	PS	R

Wordbank

- complementary angles** Two angles that add to 90°
- corresponding angles** Pairs of 'matching' angles formed when a transversal crosses two or more other lines
- obtuse angle** An angle whose size is between 90° and 180°
- parallel lines** Lines that point in the same direction and do not intersect
- perpendicular lines** Lines that intersect at right angles
- supplementary angles** Two angles that add to 180°
- transversal** A line that cuts across two or more other lines

In this chapter you will:

- use the two alternate conventions for naming angles
- measure, estimate and compare angles in degrees and classify angles according to their sizes
- define and classify angles such as acute, right, obtuse, straight, reflex and revolution, and pairs of angles such as complementary, supplementary, adjacent and vertically opposite
- investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles
- use results to find unknown angles
- identify corresponding, alternate and co-interior angles when two parallel straight lines are crossed by a transversal, and the relationships between them, including using dynamic geometry software
- investigate conditions for two lines to be parallel and solve simple numerical problems
- construct parallel and perpendicular lines using their properties, a pair of compasses and a ruler, and dynamic geometry software

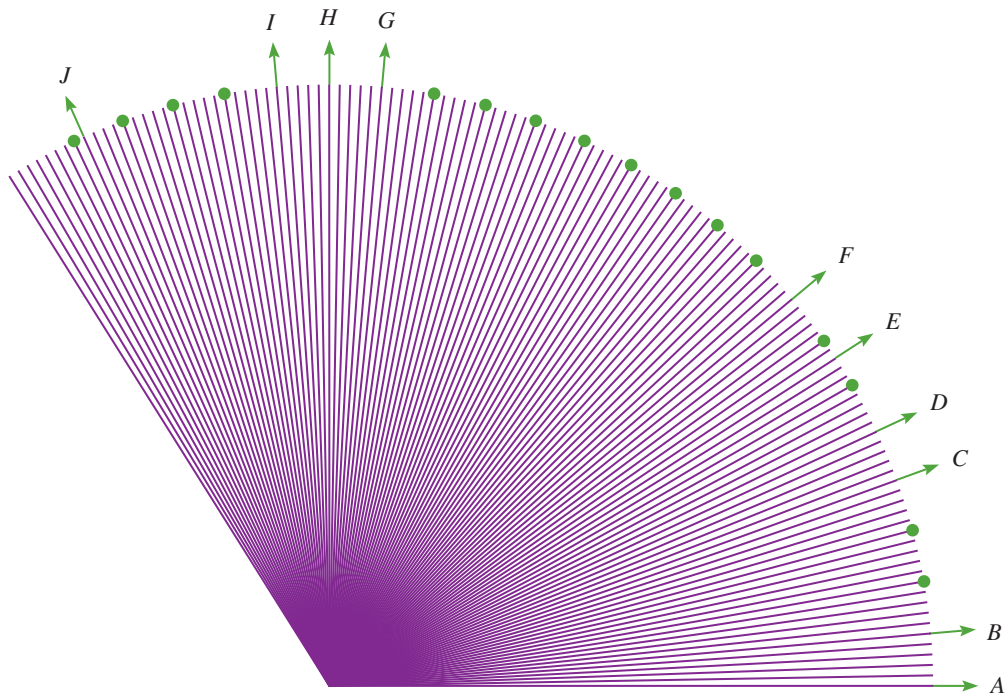
SkillCheck

Worksheet

StartUp assignment 2

MAT07MGWK10008

- 1 In this diagram, each gap represents 1° of angle size.



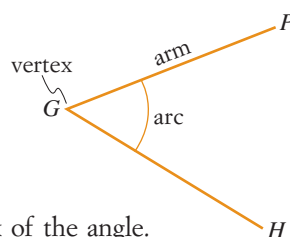
What is the angle, in degrees, between the lines labelled:

a A and C?	b A and D?	c B and C?	d C and F?
e A and F?	f B and G?	g D and G?	h E and H?
i D and I?	j C and J?	k B and E?	l E and J?

- 2 In the diagram in question 1, find:
- a one pair of labelled lines that have a 19° angle between them
 - b two pairs of labelled lines that have a 90° angle between them
- 3 In the diagram in question 1, find the pairs of labelled lines that have the following angles between them.
- a 7° b 8° c 13° d 28° e 50° f 89° g 95° h 114°

2-01 Naming angles

An angle describes the size of a turn or rotation. It is drawn with two **arms** which meet at a **vertex**. The size of the turn is normally marked with a curved line called an **arc**. The angle marked in this diagram can be written as:



$\angle G$

$\angle PGH$ or $\angle HGP$ The middle letter always labels the vertex of the angle.

Worksheet

A page of angles

MAT07MGWK10011

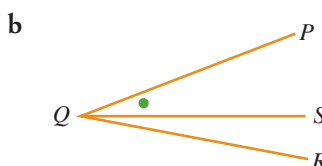
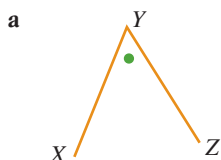
Puzzle sheet

Angle cards

MAT07MGPS10006

Example 1

Name the angle marked with $\bullet$ in each of these diagrams.



Solution

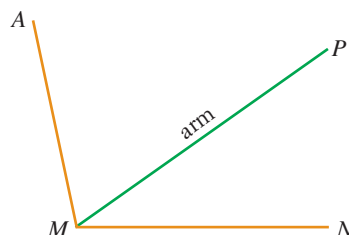
a $\angle Y$ or $\angle XYZ$ or $\angle ZYX$

b $\angle PQS$ or $\angle SQP$

Note: We cannot use $\angle Q$ in part b because it is not clear which angle that means. There are three different angles whose vertex is Q. They are $\angle PQS$, $\angle SQR$ and $\angle PQR$.

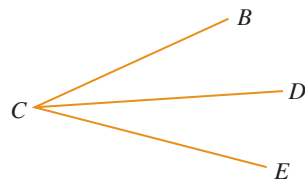
Adjacent angles

In the diagram on the right, $\angle AMP$ and $\angle PMN$ share a common arm, PM . Angles that are next to each other in this way are called **adjacent angles**.



Example 2

What angle is adjacent to $\angle DCE$?



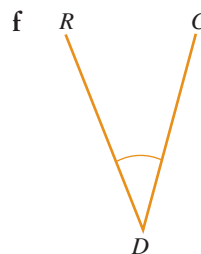
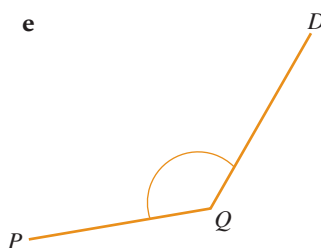
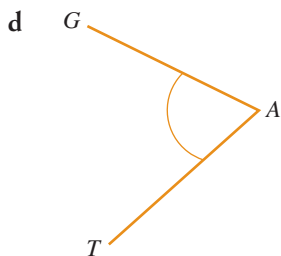
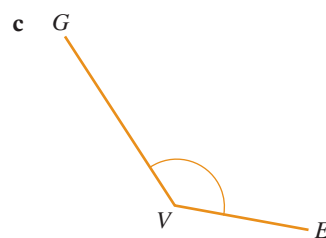
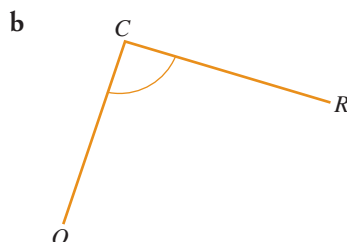
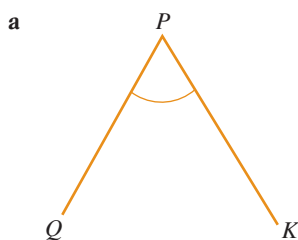
Solution

$\angle BCD$ and $\angle DCE$ share a common arm CD . So $\angle BCD$ is adjacent to $\angle DCE$.

Exercise 2-01 Naming angles

See Example 1

- 1 Name each angle in two different ways.



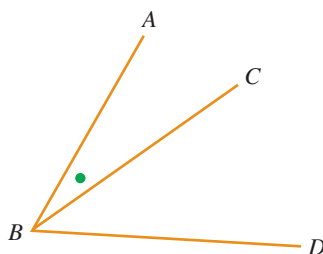
- 2 How can the angle marked • below be named? Select the correct answer A, B, C or D.

A $\angle ABD$

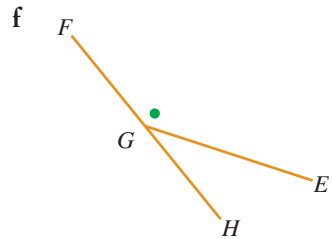
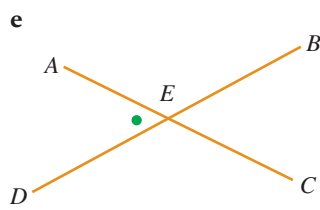
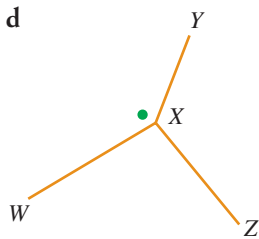
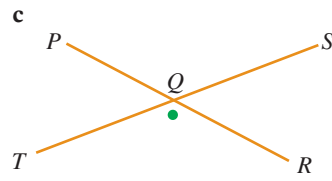
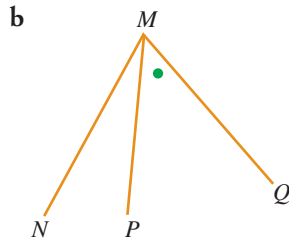
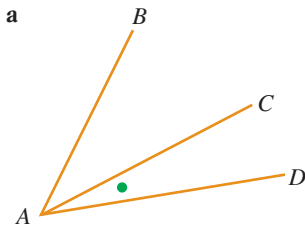
B $\angle CBD$

C $\angle ABC$

D $\angle BCA$



3 Name the angle marked • in each diagram.



4 Draw each angle named below, labelling it correctly.

a $\angle POT$

b $\angle TAF$

c $\angle AFE$

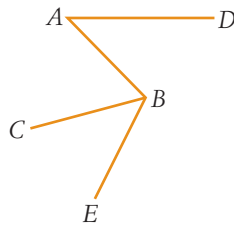
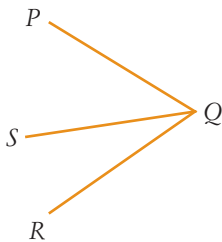
d $\angle H$

5 Name the angle adjacent to:

a $\angle RQS$

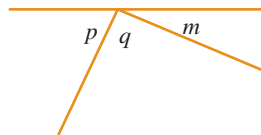
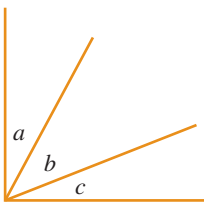
b $\angle ABC$

See Example 2



c the angle marked c

d the angle marked p

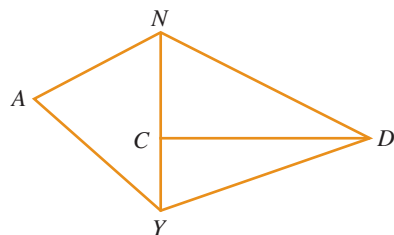


6 a There are 13 different angles inside the diagram on the right. Name them all.

b Name all 3 angles that have C as their vertex.

c What type of angle is $\angle NCY$?

d Name the angle that is adjacent to $\angle YND$.



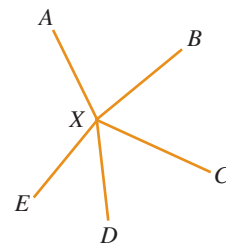
- 7 Which one of the following angles is adjacent to $\angle AXB$? Select the correct answer **A**, **B**, **C** or **D**.

A $\angle BXC$

B $\angle DXE$

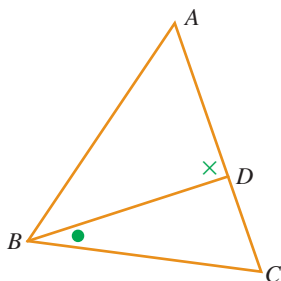
C $\angle DXC$

D $\angle CXE$

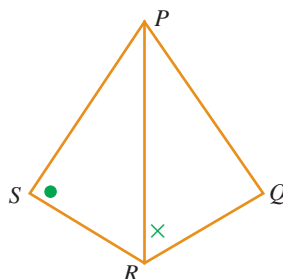


- 8 Name the angles marked $\bullet$ and $\times$ in each diagram.

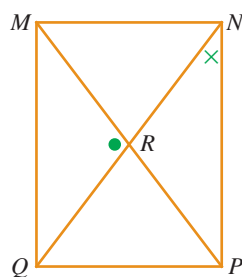
a



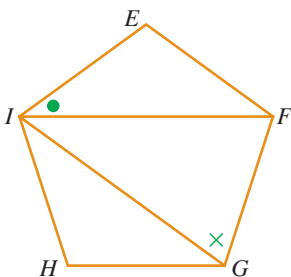
b



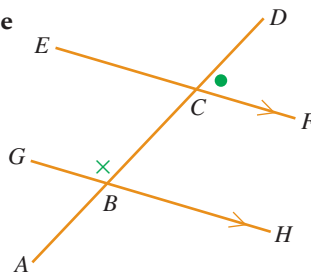
c



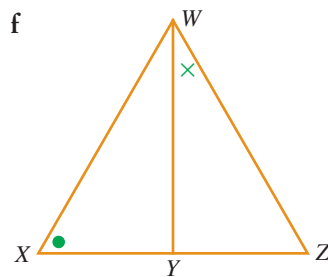
d



e



f



- 9 For each diagram in question 8, name a pair of adjacent angles.

2-02 Measuring and drawing angles

Weblink

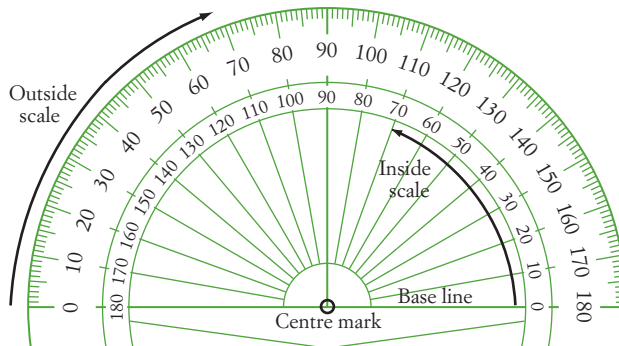
Intuitive guide to angles

Technology worksheet

Classifying angles

MAT07MGCT00009

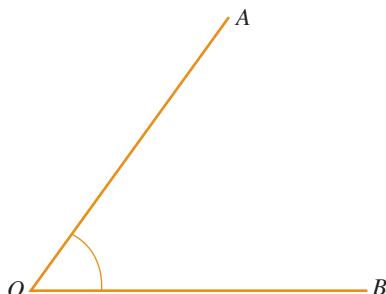
A **protractor** is an instrument used for measuring and drawing angles.



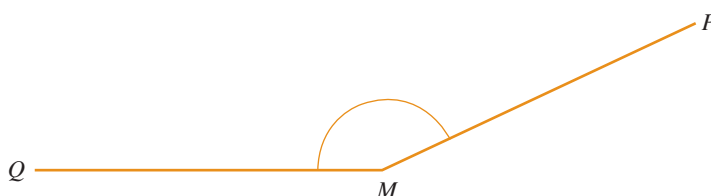
Example 3

Use a protractor to measure the size of each marked angle in degrees.

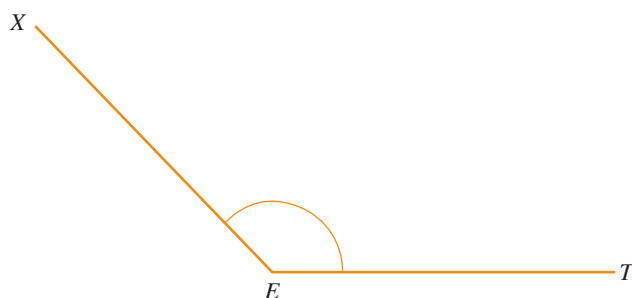
a



b



c

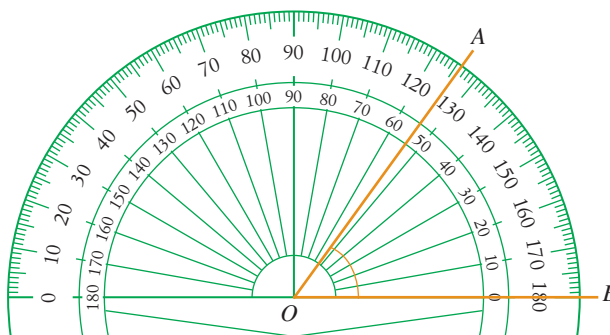


Solution

a Measuring $\angle AOB$:

- Line up OB with the base line of the protractor.
- Place the centre mark over the vertex, O .
- The angle is smaller than 90° .
- Use the inside scale, counting from 0° .

$$\angle AOB = 54^\circ$$



Video tutorial

Measuring angles

MAT07MGVT10003

Worksheet

A page of protractors

MAT07MGWK10009

Worksheet

Estimating and
measuring angles

MAT07MGWK10010

Worksheet

A page of angles

MAT07MGWK10011

Puzzle sheet

Angle cards

MAT07MGPS10006

Skillsheet

Measuring angles

MAT07MGSS10006

Worksheet

Using a protractor

MAT07MGWK00027

Animated example

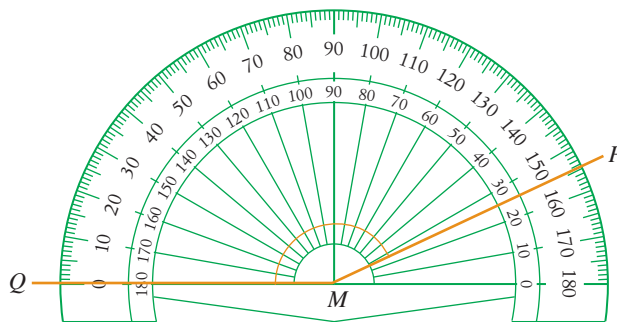
Measuring angles
with a protractor

MAT07MGAE00004

b Measuring $\angle PMQ$:

- Line up QM with the base line of the protractor.
- Place the centre mark over the vertex, M .
- The angle is greater than 90° .
- Use the outside scale, counting from 0° .

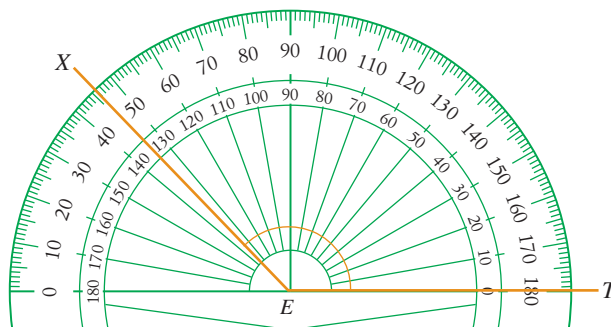
$$\angle PMQ = 155^\circ$$



c Measuring $\angle TEX$:

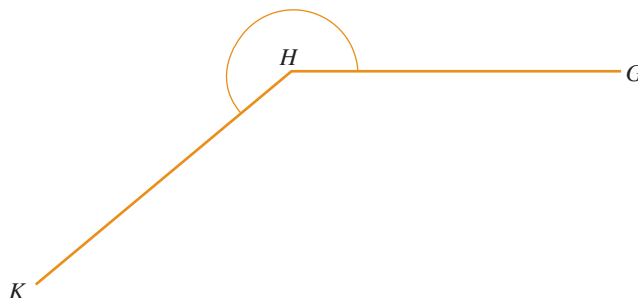
- Line up TE with the base line of the protractor.
- Place the centre mark over the vertex, E .
- $\angle TEX$ is bigger than 90° .
- Use the inside scale.

$$\angle TEX = 134^\circ$$



Example 4

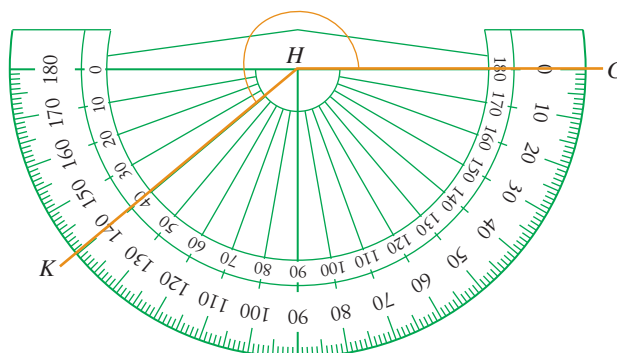
Measure the **reflex angle** GHK .



Solution

As the protractor only measures up to 180° , turn it upside down to measure the smaller adjacent angle underneath first.

Obtuse $\angle GHK = 140^\circ$



To calculate the size of reflex $\angle GHK$, subtract 140° from 360° as there are 360° in a revolution.
Reflex $\angle GHK = 360^\circ - 140^\circ = 220^\circ$

Example 5

Construct $\angle KPM$ of size 76° .

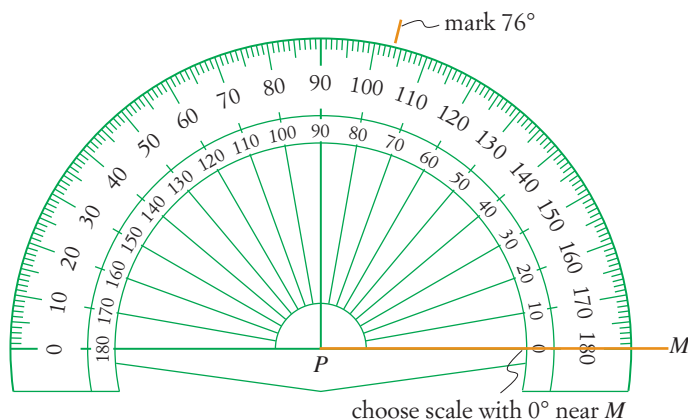
Solution

- Draw a line with endpoints P and M .

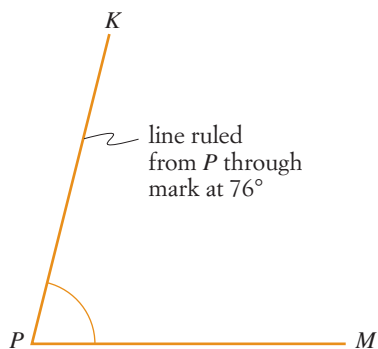
P ————— M

- Line up the base line of the protractor over PM with the centre mark on P .
Follow the inside scale around on the protractor, from 0° to 76° .

Mark this point.



- Draw a line from P through this mark and label the end of this line K .



This creates $\angle KPM$, measuring 76° .

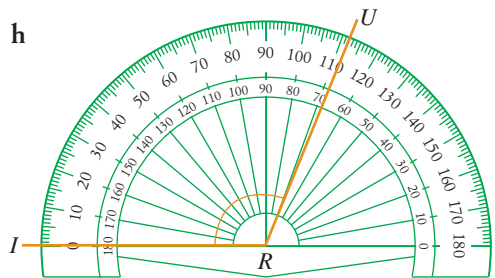
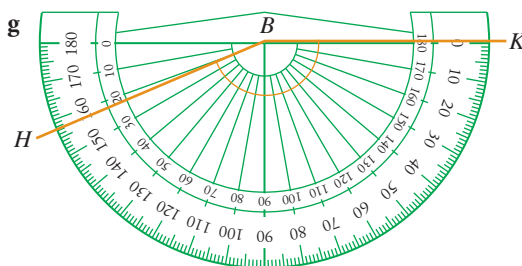
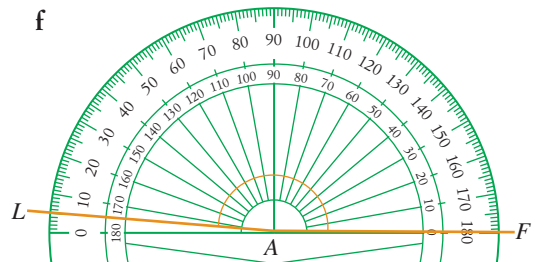
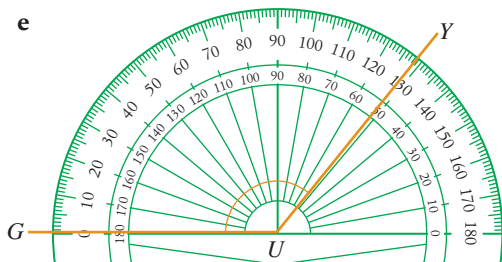
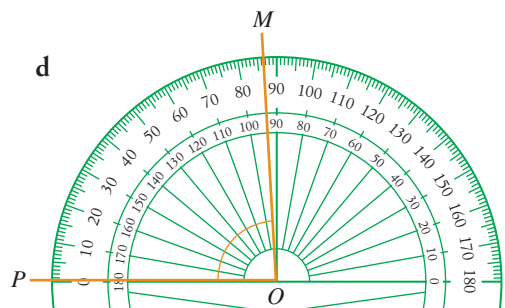
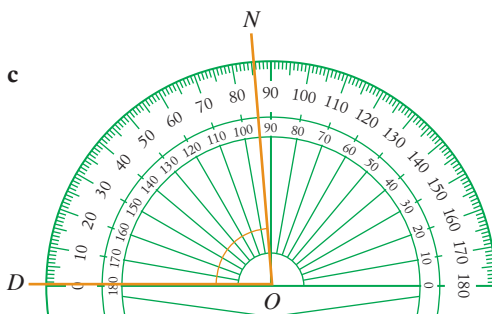
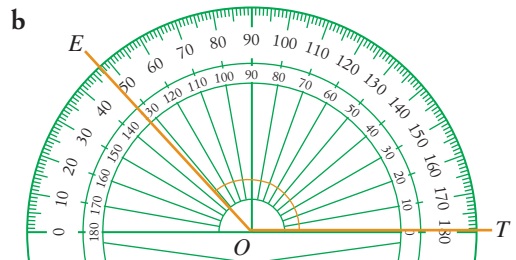
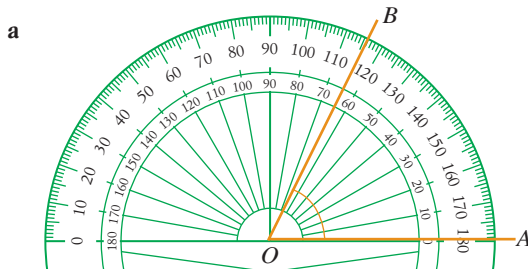
Exercise 2-02 Measuring and drawing angles

Extra questions

Points, lines and angles

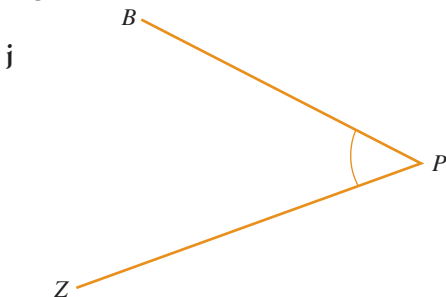
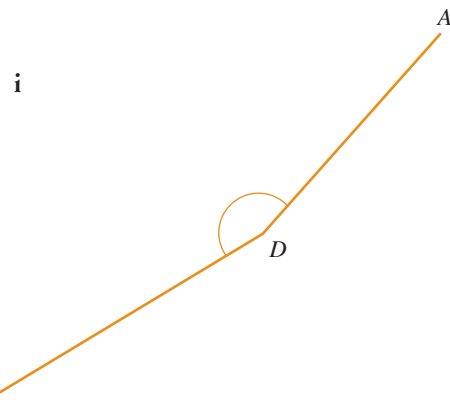
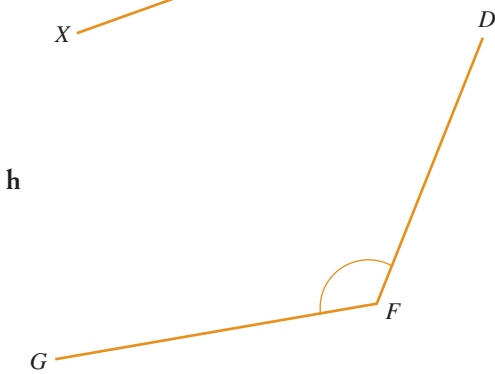
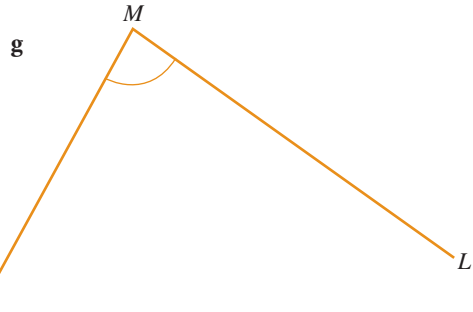
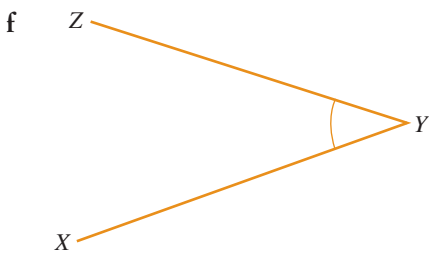
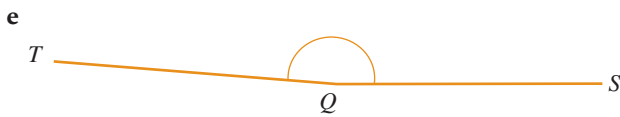
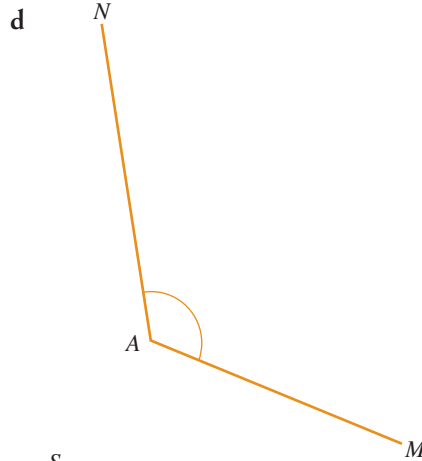
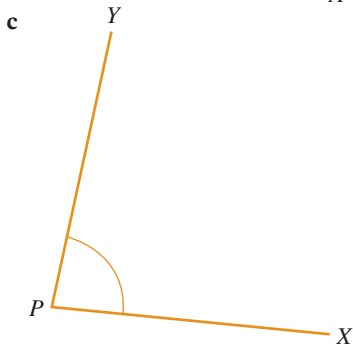
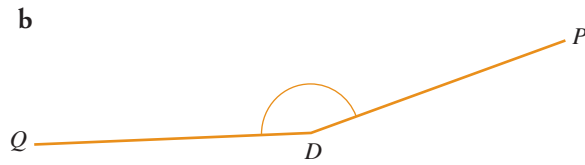
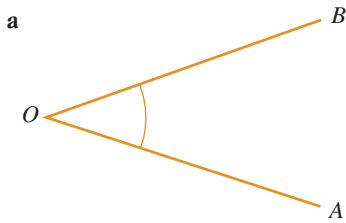
MAT07MGEQ00015

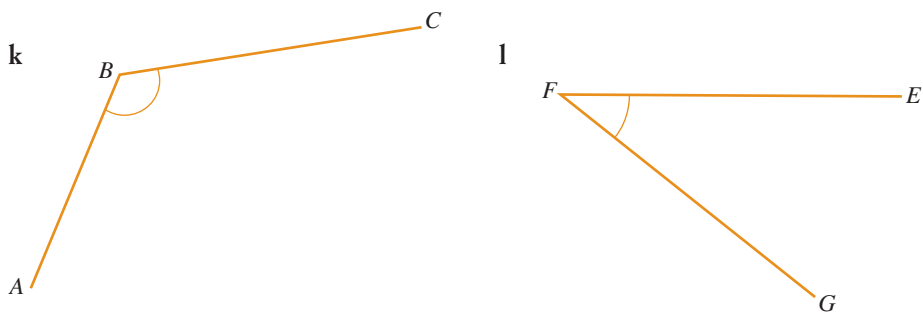
- 1 Find the size of each marked angle.



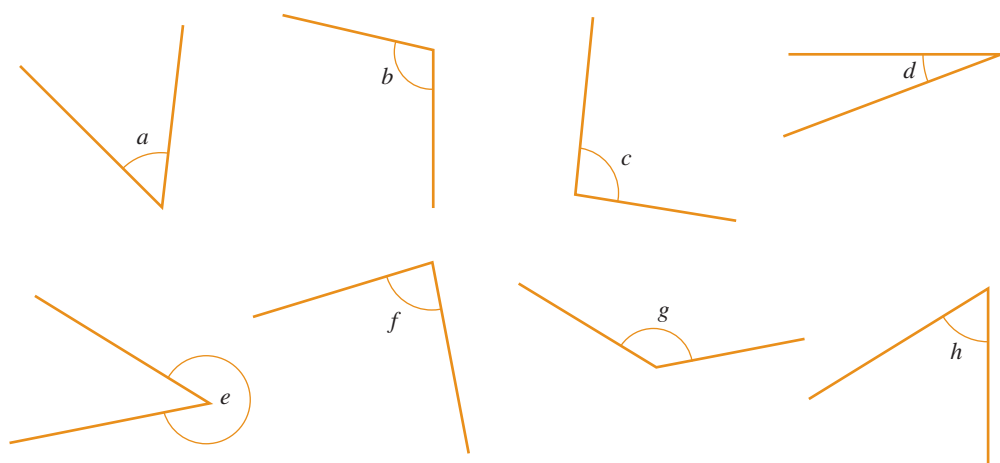
- 2 Estimate the size of each angle below, then check your estimate by measuring with a protractor accurately.

See Example 3

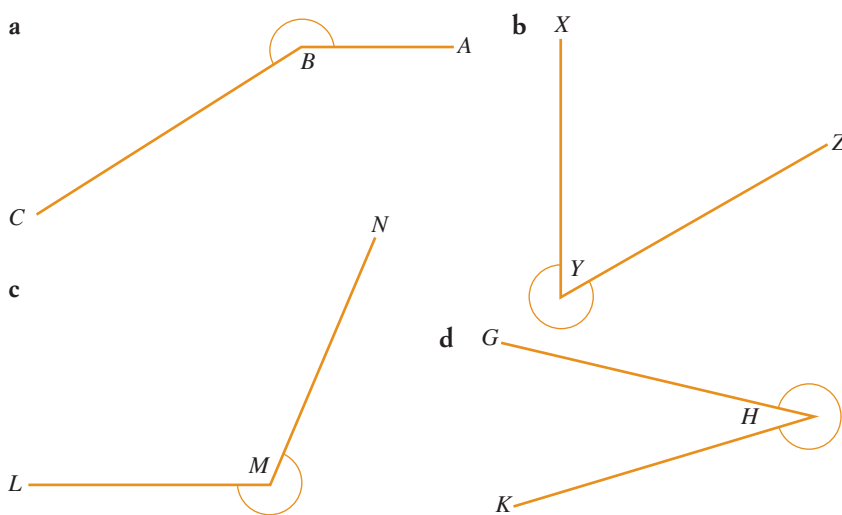




3 Sort the following angles from smallest to largest.



See Example 4 4 Estimate the size of each angle below, then check by measuring with a protractor.



- 5 The diagram at right shows Daniel shooting for goal in a game of football. His shooting angle is shown on the diagram. Estimate the size of this angle. Select A, B, C or D.

A 60° B 120°
C 150° D 240°



- 6 The word 'degree' has many meanings. Find four non-mathematical meanings for 'degree'.
- 7 Accurately construct an angle for each angle size, using your protractor.
- | | | | |
|--------------|---------------|---------------|---------------|
| a 35° | b 115° | c 150° | d 63° |
| e 17° | f 170° | g 204° | h 319° |
- 8 Use your protractor to accurately draw and label each angle named below.
- | | | |
|---------------------------|----------------------------|----------------------------|
| a $\angle DRE = 65^\circ$ | b $\angle BGH = 145^\circ$ | c $\angle GRT = 32^\circ$ |
| d $\angle ABC = 45^\circ$ | e $\angle SAQ = 110^\circ$ | f $\angle NMH = 265^\circ$ |
| g $\angle KLY = 28^\circ$ | h $\angle LMN = 180^\circ$ | i $\angle LKY = 90^\circ$ |

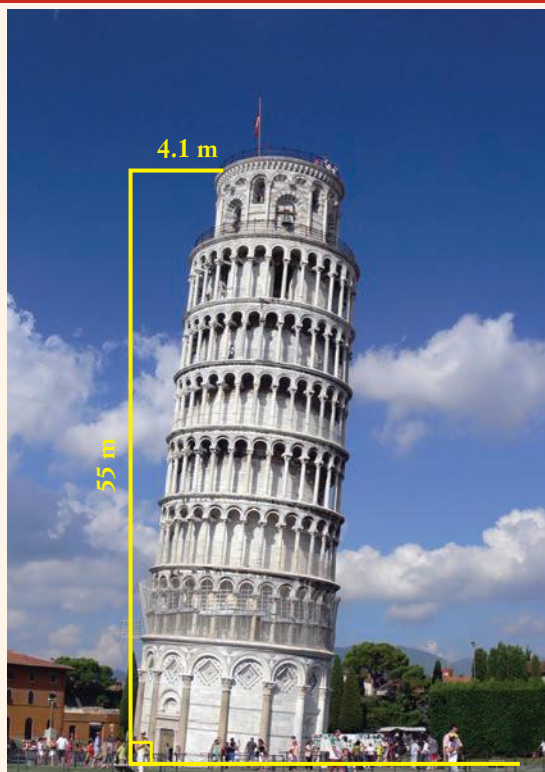
See Example 5

Just for the record

The Leaning Tower of Pisa, Italy, began leaning shortly after its construction commenced in 1173. In 1350, it was leaning at 2.5° , or 4 m, from the vertical. By 1990, its lean had grown to 5.5° , or 4.5 m, and was increasing at 1.2 mm per year. Architects estimated that the tower would have toppled over by 2020 so it was closed for 12 years to allow \$25 million worth of engineering work to take place. When it reopened in 2001, its lean had been pushed back to 5° or 4.1 m. Further restoration and cleaning occurred until 2010, when it was straightened to its 1838 position.

- 1 Draw a scale diagram of the Leaning Tower of Pisa given that its top is 55 m above the ground.
- 2 Research how engineers prevented the tower from leaning further. Use the library or the Internet to conduct your research.

Leaning tower of Pisa



2-03 Classifying angles

Worksheet

A page of angles

MAT07MGWK10011

Puzzle sheet

Angle cards

MAT07MGPS10006

Skillsheet

Types of angles

MAT07MGSS10007

Homework sheet

Angles

MAT07MGHS10028

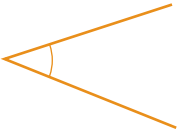
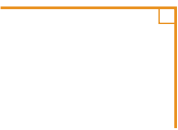
Technology

Geogebra
Classifying angles

MAT07MGCT00009

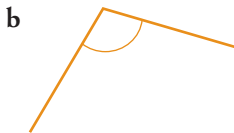
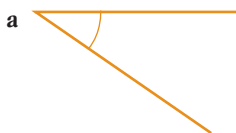
Angles may be classified into types according to their size.

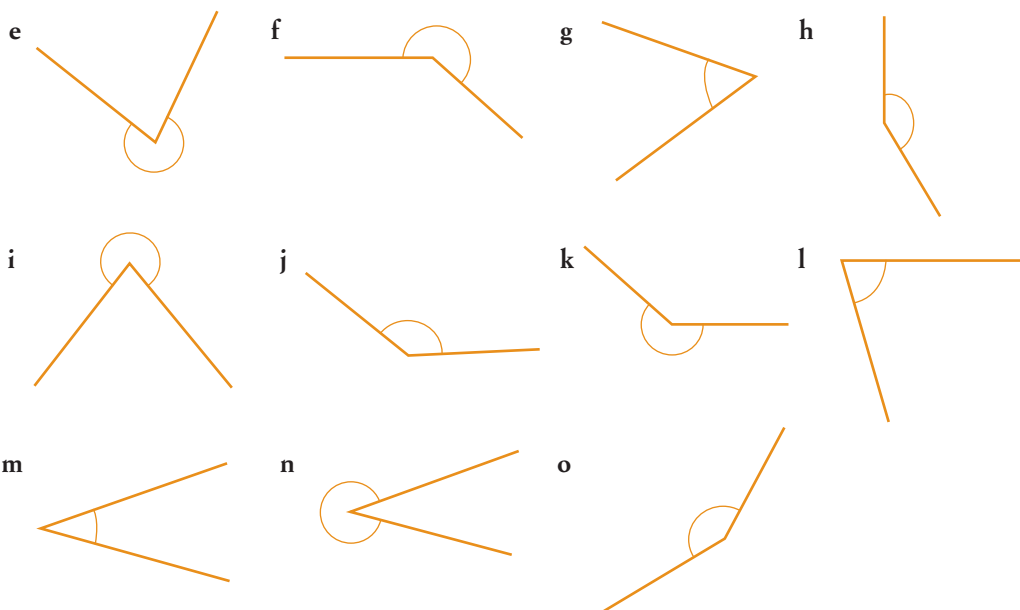
Summary

Angle	Type	Description
	acute	less than 90°
	right	90° (quarter turn) Note that a right angle is marked with a box symbol.
	obtuse	greater than 90° but less than 180°
	straight	180° (half turn)
	reflex	greater than 180° but less than 360°
	revolution	360° (complete turn)

Exercise 2-03 Classifying angles

- 1 State whether each angle is acute (A), obtuse (O) or reflex (R).





2 Draw two different examples of:

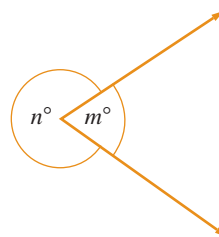
a an acute angle	b an obtuse angle	c a right angle
d a reflex angle	e a straight angle	f a revolution

3 Classify each angle into one of the six types.

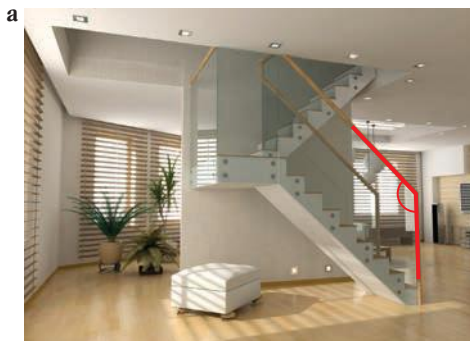
a 37°	b 107°	c 252°	d 195°	e 79°	f 180°
g 163°	h 179°	i 360°	j 5°	k 345°	l 91°
m 14°	n 299°	o 90°	p 205°	q 126°	r 44°

4 Which words best describe angles m and n respectively in the diagram at right? Select **A**, **B**, **C** or **D**.

A obtuse and reflex	B reflex and a revolution
C acute and obtuse	D acute and reflex



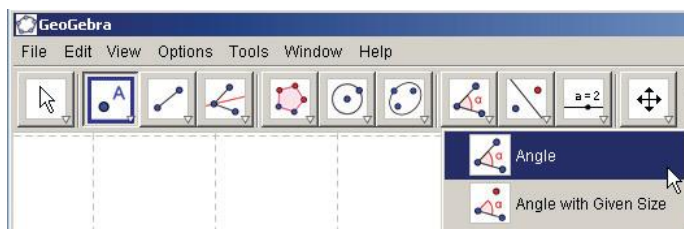
5 Decide whether each marked angle in the photos is acute, obtuse or reflex.





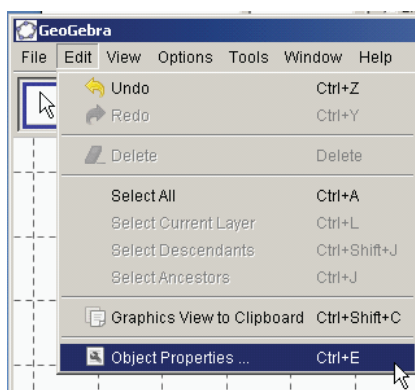
Technology Constructing angles 1

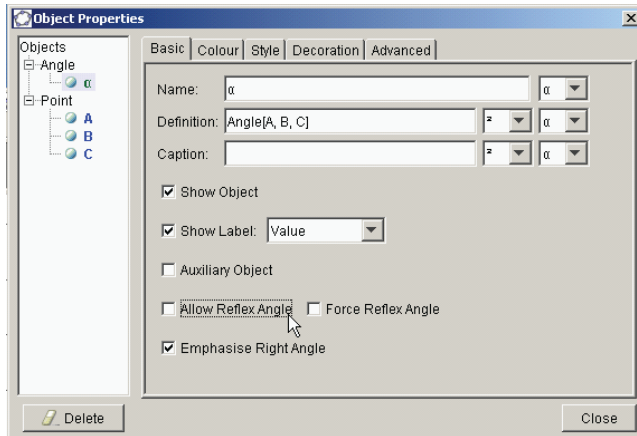
Using GeoGebra, you can construct an angle using  from the fourth-last drop-down menu.



[For instructions involving Geometer's Sketchpad, follow the link 'Technology worksheet: Geometer's Sketchpad, Constructing angles']

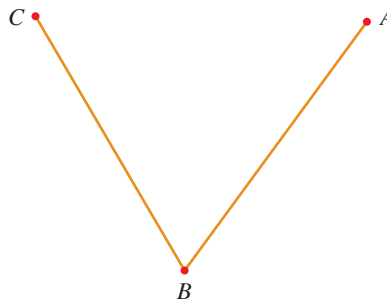
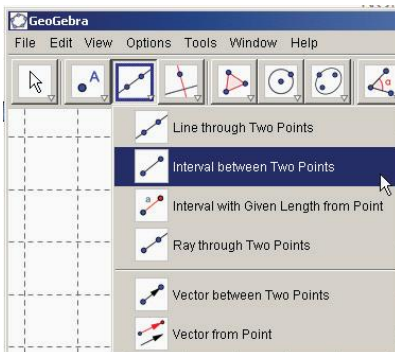
- 1 It is possible to only draw angles less than 180° . Click **Edit** and **Object Properties**. Then unselect **Allow Reflex Angle**.





- 2 To draw an angle less than 180° , it must be constructed in a **clockwise** direction.

Use  from the third drop-down menu to construct the arms of the angle, AB and BC , in order.

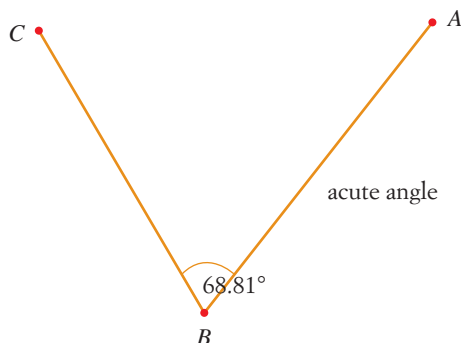


- 3 Now click  and select, in order, A , B and C to see the size of $\angle ABC$.
- 4 If the labels ABC are not showing, right-click on each point and select **Show Label**.
- 5 Reflex angles should be drawn in an **anticlockwise** direction. Click **Edit** and **Object Properties**. Then select **Allow Reflex Angle**.
- 6 a Construct each of the following 4 types of angles using  **Interval between Two Points**: acute, right, obtuse, reflex.
- b Make sure that each angle is labelled by selecting **Show Label**.
- c Measure the size of each angle (in a clockwise direction) you have drawn, using , correct to the nearest degree.

- 7 a Start a new sketch and accurately construct separate angles of the following sizes.

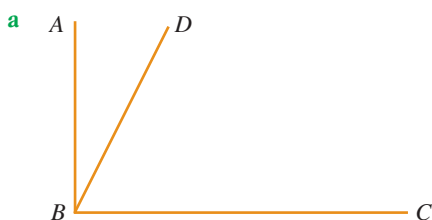
- i 72° ii 310° iii 165°
iv 98° v 236° vi 90°

- b Using  **Insert Text** from the second-last drop-down menu, classify each angle according to its type, for example, acute, reflex.

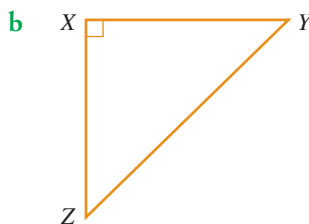


Investigation: Complementary and supplementary angles

- 1 For each diagram, measure the angles named, then copy and complete the equations.



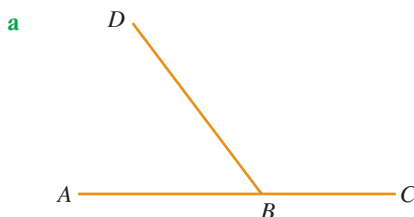
$$\begin{aligned}\angle ABD &= \underline{\hspace{2cm}} \\ \angle CBD &= \underline{\hspace{2cm}} \\ \angle ABD + \angle CBD &= \underline{\hspace{2cm}}\end{aligned}$$



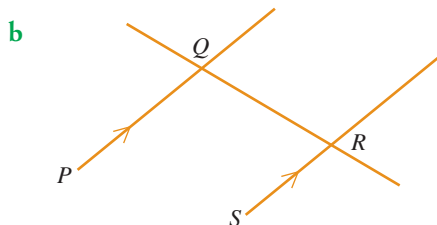
$$\begin{aligned}\angle Y &= \underline{\hspace{2cm}} \\ \angle Z &= \underline{\hspace{2cm}} \\ \angle Y + \angle Z &= \underline{\hspace{2cm}}\end{aligned}$$

- 2 The two angles you measured in each diagram of question 1 are called **complementary angles**. What do complementary angles add up to?

- 3 For each diagram, measure the angles named, then copy and complete the equations.



$$\begin{aligned}\angle ABD &= \underline{\hspace{2cm}} \\ \angle CBD &= \underline{\hspace{2cm}} \\ \angle ABD + \angle CBD &= \underline{\hspace{2cm}}\end{aligned}$$



$$\begin{aligned}\angle PQR &= \underline{\hspace{2cm}} \\ \angle SRQ &= \underline{\hspace{2cm}} \\ \angle PQR + \angle SRQ &= \underline{\hspace{2cm}}\end{aligned}$$

- 4 The two angles you measured in each diagram of question 3 are called **supplementary angles**. What do supplementary angles add up to?

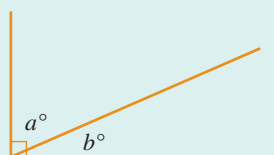
2-04 Complementary and supplementary angles

TLF learning object

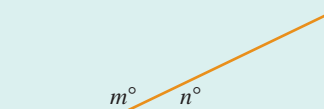
Exploring angles
(L6555)

Summary

- **Complementary angles** add to 90°
- **Supplementary angles** add to 180°
- **Angles in a right angle** are complementary (add up to 90°)
- **Angles on a straight line** are supplementary (add up to 180°)



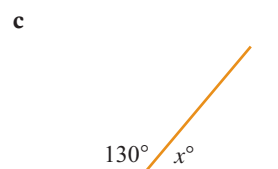
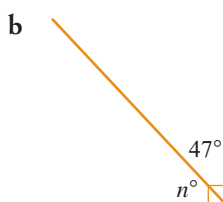
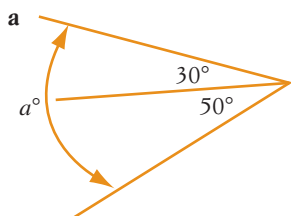
$$a + b = 90$$



$$m + n = 180$$

Example 6

In each diagram, there is an angle whose size is represented by a letter called a **pronumeral**. A pronumeral stands for a number. Find the value of each pronumeral.



Solution

a $a = 30 + 50$
 $= 80$

Adding two adjacent angles.

b $n + 47 = 90$
 $n = 90 - 47$
 $= 43$

Angles in a right angle are complementary

c $x + 130 = 180$
 $x = 180 - 130$
 $= 50$

Angles on a straight line are supplementary

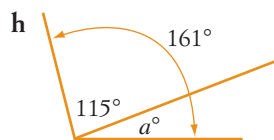
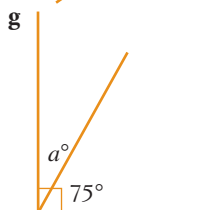
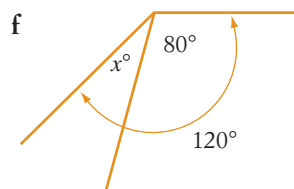
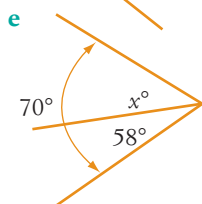
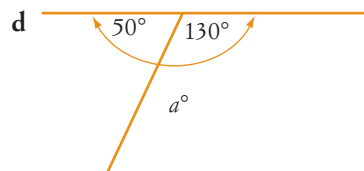
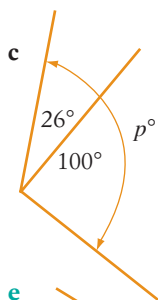
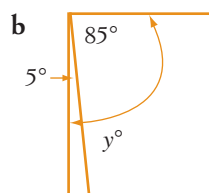
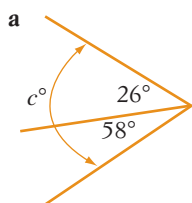
Exercise 2-04 Complementary and supplementary angles

- Why is 57° the complement of 33° ?
 - Why is 147° the supplement of 33° ?
- Find the complement of:

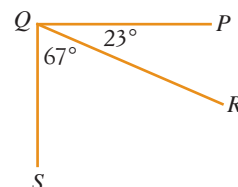
a 30°	b 70°	c 25°	d 38°
e 89°	f 57°	g 42°	h 66°
- Look up 'complement' and 'supplement' in a dictionary. Find one non-mathematical meaning of each word.
- Find the supplement of:

a 18°	b 150°	c 35°	d 125°
e 62°	f 87°	g 111°	h 4°

See Example 6 5 For each diagram, find the value of the pronumeral.



- Name the pair of complementary angles in the diagram.
 - How do you know that the angles you named are complementary?
 - What type of angle is $\angle PQS$?



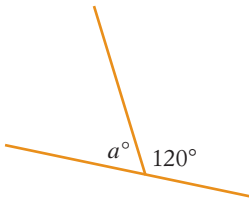
Worked solutions

Exercise 2-04

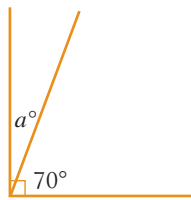
MAT07MGWS10005

7 For each diagram, find the value of the pronumeral.

a



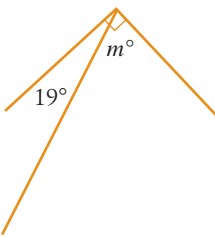
b



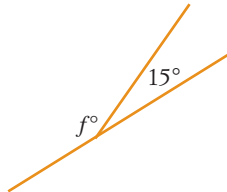
c



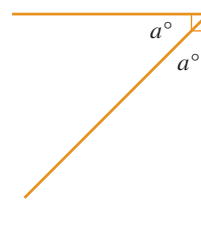
d



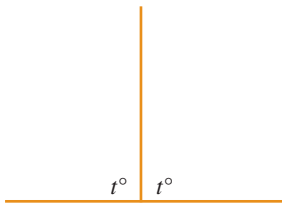
e



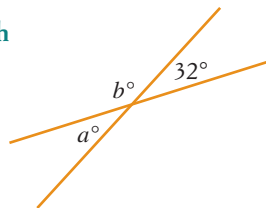
f



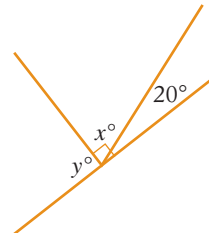
g



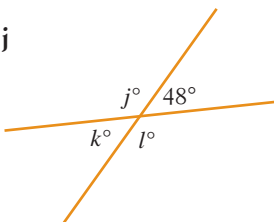
h



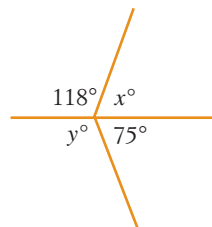
i



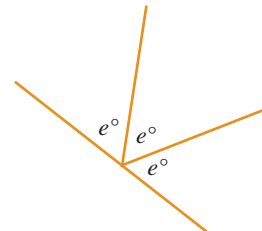
j



k



l



Worked solutions

Exercise 2-04

MAT07MGWS10005

Mental skills 2 Maths without calculators

Adding or multiplying in any order

Have you noticed that $4 + 7 = 7 + 4$?

Have you noticed that $3 \times 5 = 5 \times 3$?

Numbers can be added or multiplied in any order. We can use this property to make our calculations simpler.

1 Study each example.

$$\begin{aligned} \text{a } 19 + 5 + 5 + 1 &= (19 + 1) + (5 + 5) \\ &= 20 + 10 \\ &= 30 \end{aligned}$$

$$\begin{aligned} \text{b } 13 + 8 + 20 + 27 + 80 &= (13 + 27) + (20 + 80) + 8 \\ &= 40 + 100 + 8 \\ &= 148 \end{aligned}$$

$$\begin{aligned} \text{c } 2 \times 36 \times 5 &= (2 \times 5) \times 36 \\ &= 10 \times 36 \\ &= 360 \end{aligned}$$

$$\begin{aligned} \text{d } 25 \times 11 \times 4 \times 7 &= (25 \times 4) \times (11 \times 7) \\ &= 100 \times 77 \\ &= 7700 \end{aligned}$$

2 Now evaluate each sum.

$$\text{a } 45 + 16 + 45 + 4 + 7$$

$$\text{c } 18 + 91 + 9 + 20$$

$$\text{e } 24 + 16 + 80 + 44 + 10$$

$$\text{g } 100 + 36 + 200 + 10 + 90$$

$$\text{i } 70 + 50 + 30 + 25 + 25$$

$$\text{b } 38 + 600 + 50 + 12 + 40$$

$$\text{d } 75 + 33 + 7 + 25$$

$$\text{f } 56 + 5 + 20 + 15 + 4$$

$$\text{h } 54 + 27 + 9 + 16 + 3$$

$$\text{j } 32 + 120 + 40 + 80 + 40$$

3 Now evaluate each product.

$$\text{a } 8 \times 4 \times 5$$

$$\text{d } 5 \times 11 \times 40$$

$$\text{g } 3 \times 20 \times 7 \times 5$$

$$\text{b } 50 \times 7 \times 2$$

$$\text{e } 12 \times 2 \times 3$$

$$\text{h } 6 \times 8 \times 5 \times 2$$

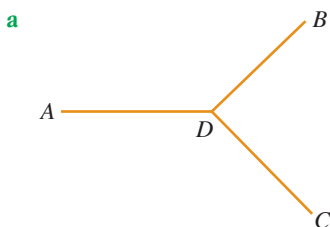
$$\text{c } 3 \times 5 \times 6$$

$$\text{f } 2 \times 4 \times 25 \times 8$$

$$\text{i } 2 \times 3 \times 2 \times 11$$

Investigation: Angles at a point and vertically opposite angles

1 In each diagram, there are adjacent angles that meet at a central point. Measure all angles, then copy and complete the equations.

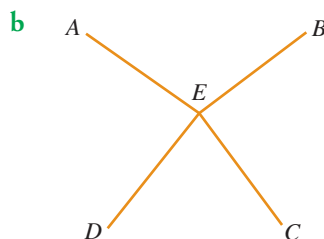


$$\angle ADB = \underline{\hspace{2cm}}$$

$$\angle ADC = \underline{\hspace{2cm}}$$

$$\angle BDC = \underline{\hspace{2cm}}$$

$$\angle ADB + \angle ADC + \angle BDC = \underline{\hspace{2cm}}$$



$$\angle AEB = \underline{\hspace{2cm}}$$

$$\angle BEC = \underline{\hspace{2cm}}$$

$$\angle CED = \underline{\hspace{2cm}}$$

$$\angle DEA = \underline{\hspace{2cm}}$$

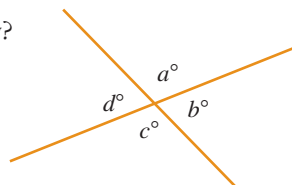
$$\angle AEB + \angle BEC + \angle CED + \angle DEA = \underline{\hspace{2cm}}$$

2 What do the angles that meet at a point add up to? Why?

3 When two lines cross, four angles are created.

a Which of these angles are equal?

b Can you prove it using supplementary angles?



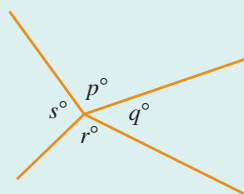
2-05 Angles at a point and vertically opposite angles

TLF learning object

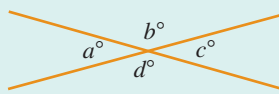
Exploring angles
(L6555)

Summary

- Angles at a point (in a revolution) add up to 360°
- Vertically opposite angles are equal



$$p + q + r + s = 360$$



$$a = c \text{ and } b = d$$

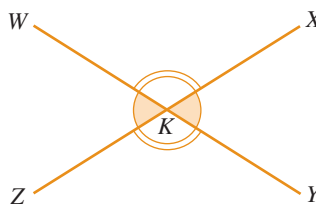
Example 7

Name the angle that is vertically opposite:

- a $\angle WKZ$ b $\angle ZKY$

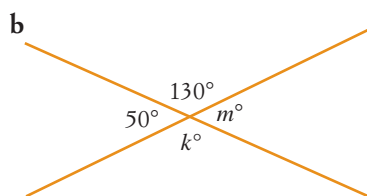
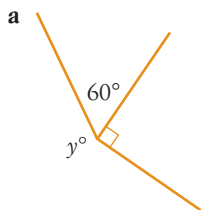
Solution

- a $\angle XKY$ is vertically opposite $\angle WKZ$
b $\angle WKX$ is vertically opposite $\angle ZKY$



Example 8

Find the value of the pronumeral in each diagram.



Solution

a $y + 60 + 90 = 360$

$y = 360 - 60 - 90$

$= 210$

b $k = 130$

$m = 50$

Angles at a point add to 360°

Vertically opposite angles are equal

Vertically opposite angles are equal

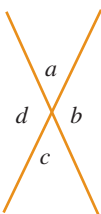
Exercise 2-05

Angles at a point and vertically opposite angles

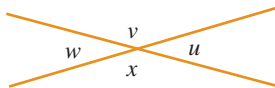
See Example 7

- 1 In each diagram, write the angle that is vertically opposite the named angle.

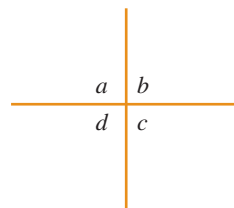
a a



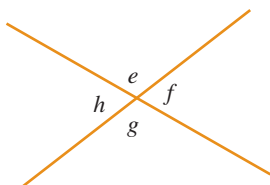
b w



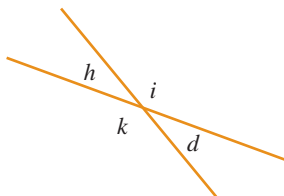
c c



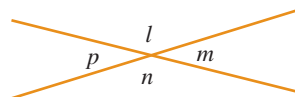
d h



e k



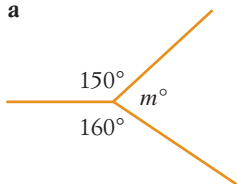
f m



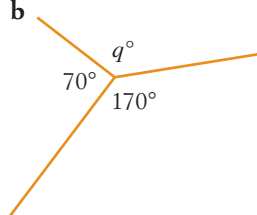
See Example 8

- 2 For each diagram, find the value of the pronumeral.

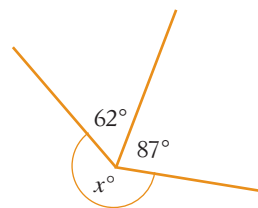
a



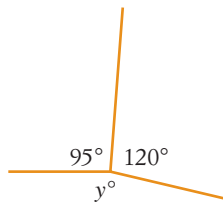
b



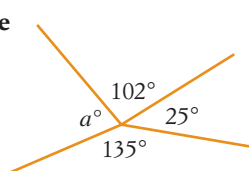
c



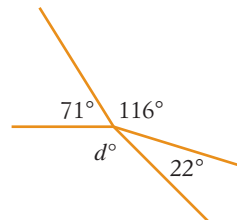
d

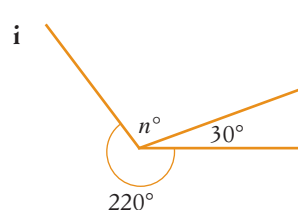
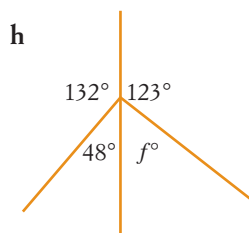
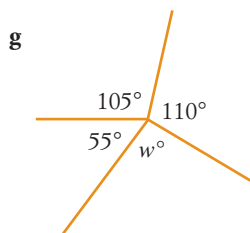


e



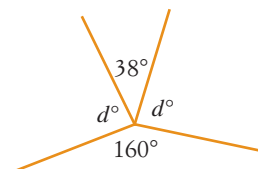
f





- 3 Find the value of d . Select the correct answer **A**, **B**, **C** or **D**.

A 122 **B** 61 **C** 142 **D** 81

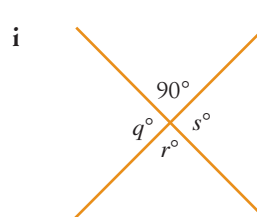
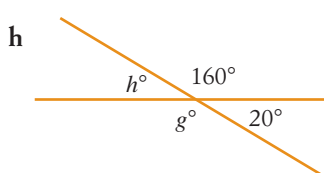
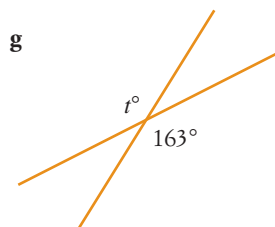
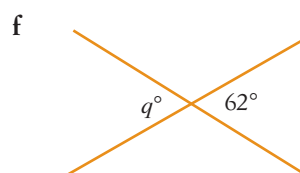
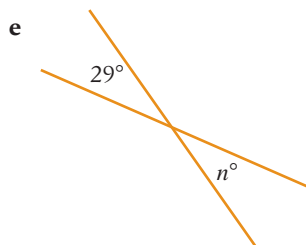
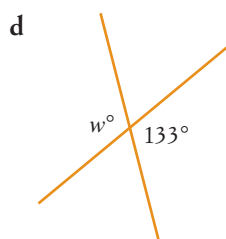
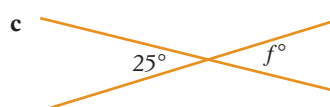
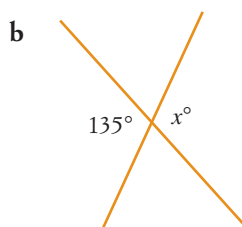
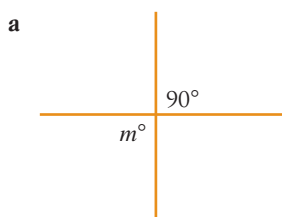
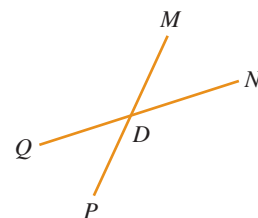


Worked solutions

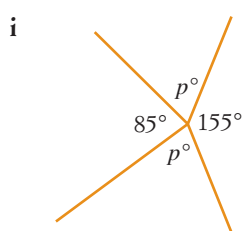
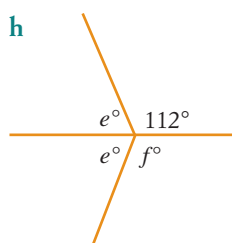
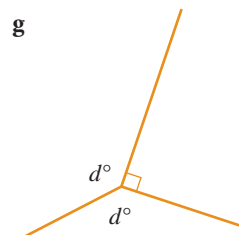
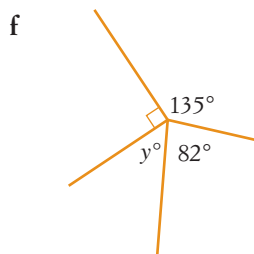
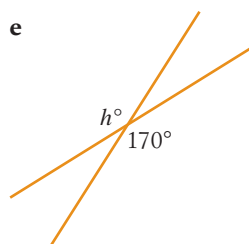
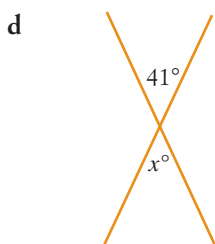
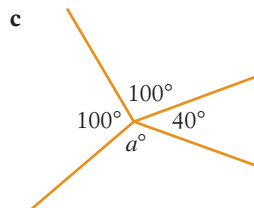
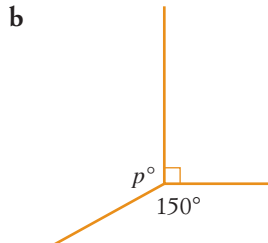
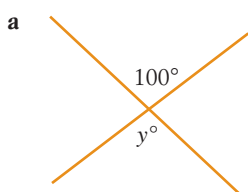
Exercise 2-05

MAT07MGWS10006

- 4 Refer to the diagram shown on the right.
- Which angle is vertically opposite $\angle NDP$?
 - Which angle is equal to $\angle MDQ$?
 - Name two straight angles.
 - Name two different pairs of supplementary angles.
- 5 For each diagram, find the value of the pronumeral.



6 For each diagram, find the value of the pronumeral.



Worked solutions

Exercise 2-05

MAT07MGWS10006

Just for the record

Why 360 degrees?

Why are there 90° in a right angle and 360° in a revolution? Why do we use such strange numbers instead of more conventional numbers like 10 and 100?

The reason is that, in 2000 BCE, the ancient Babylonians used a base 60 system of numbers. They used a base 60 number system because 60 is a more convenient number that has more factors than 10. You can divide 60 by 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30.

Furthermore, $6 \times 60 = 360$, which was the Babylonian approximation of the number of days in a year. They defined a revolution as being 360° so that, each day, the Earth would travel 1° around the Sun. A right angle, being a quarter-revolution, thus became $360^\circ \div 4 = 90^\circ$.



Some people who prefer a base 10 system of measuring angles use **grads** instead of degrees. With this system, a right angle is 100 grads and a revolution is 400 grads.

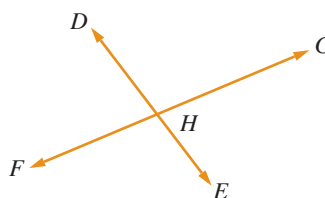
Find out more information about grads, including the exact relationship between degrees and grads.

2-06 Constructing parallel and perpendicular lines

A line is named using two points on the line.
For example, this is the line AB .



When two lines cross, we say that they **intersect**. Two lines intersect at a point. For example, in this diagram, line DE intersects line FG at the point H .



Worksheet

A page of intervals

MAT07MGWK10012

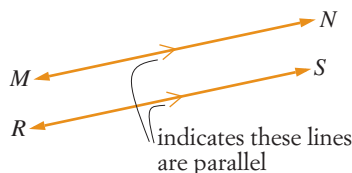
Homework sheet

Angle geometry

MAT07MGHS10029

Parallel lines

Lines that point in the same direction and never intersect are called **parallel** lines. Parallel lines are marked with identical arrowheads and are always the same distance apart. For example, in this diagram, MN is parallel to RS .



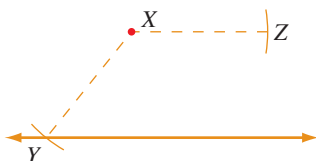
This is written as ' $MN \parallel RS$ ', where the symbol $\parallel$ stands for 'is parallel to'.

Example 9

Use compasses to construct a line through X that is parallel to the given line.

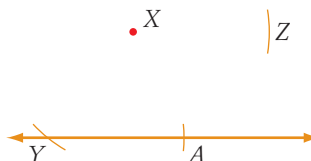
Solution

Step 1



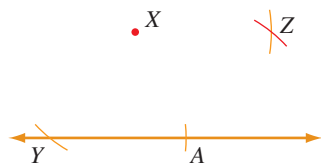
Use compasses from X to mark two large arcs at Y and Z .

Step 2



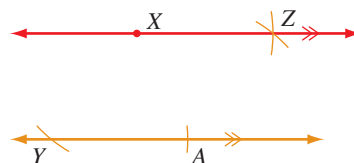
Use compasses from Y to mark an arc with the same radius at A on the line.

Step 3



Use compasses from A to mark an arc with the same radius to cross the arc at Z .

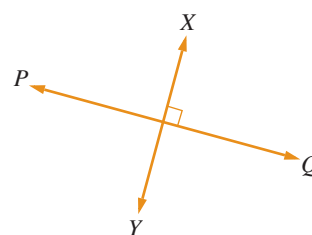
Step 4



Join XZ to construct a line parallel to AY .

Perpendicular lines

Lines that intersect at right angles (90°) are called **perpendicular** lines. For example, in this diagram, PQ is perpendicular to XY . This is written as ' $PQ \perp XY$ ', where the $\perp$ symbol stands for 'is perpendicular to'.



Example 10

Use compasses to construct a perpendicular through the point B on the line.

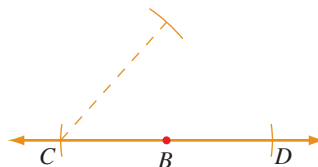
Solution

Step 1



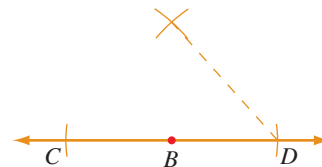
Use compasses to draw two arcs from B .

Step 2



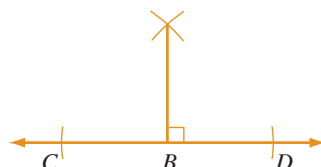
Open compasses wider to draw an arc from C .

Step 3



Use compasses to draw an arc with the same distance from D .

Step 4



Join B to where the two arcs cross.

Use a protractor or set square to check that the line is perpendicular (at 90°) to CD .

Weblink

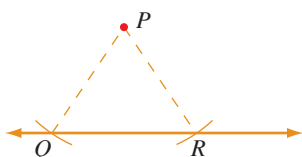
Construction
animations

Example 11

Use compasses to draw a perpendicular through the point P above the line.

Solution

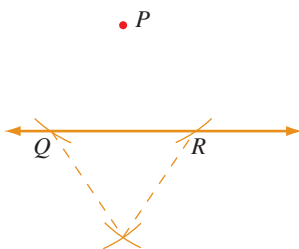
Step 1



Use compasses from P to mark two arcs with the same radius on the line.

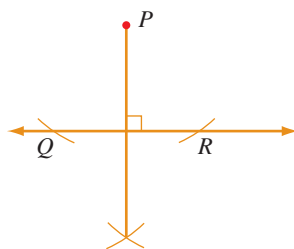
Use a protractor or set square to check that the line is perpendicular to QR .

Step 2



Use compasses from Q and R to mark two intersecting arcs with the same radius below the line.

Step 3



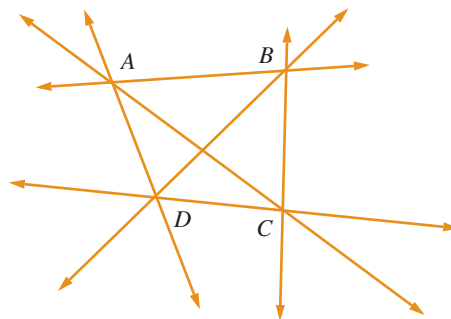
Join P to where the two arcs cross.

Weblink

Construction
animations

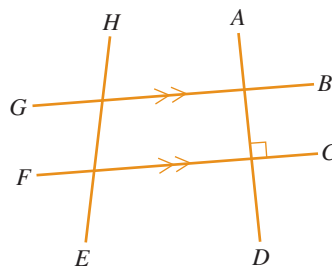
Exercise 2-06 Constructing parallel and perpendicular lines

1 Name the six different lines in this diagram.



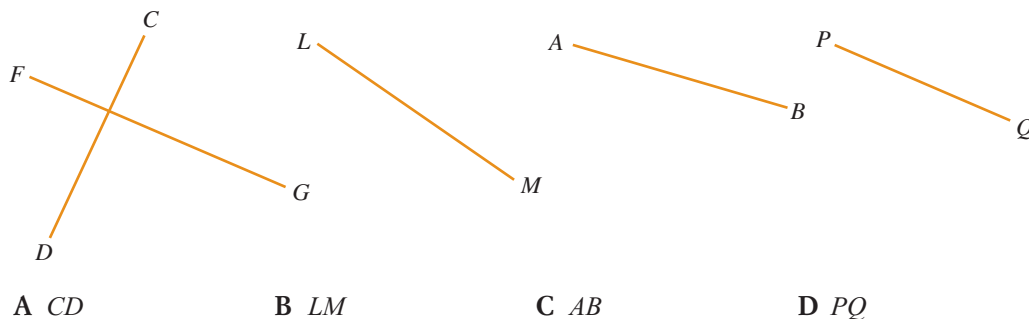
2 In this diagram, name two lines that:

- a** are perpendicular **b** are parallel **c** intersect

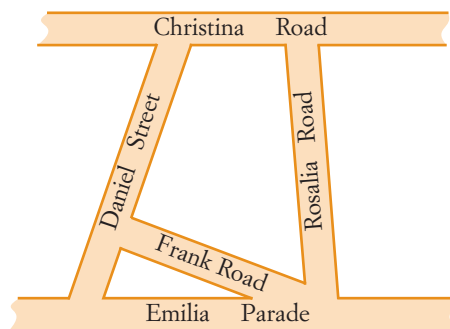


3 Rewrite your answers to question 2 parts **a** and **b** using the symbols for 'is perpendicular to' and 'is parallel to'.

- 4 Draw and label correctly:
- a line FG
 - b line AB intersecting line CD at point E
 - c line PQ parallel to line YZ
 - d line JK perpendicular to line LM .
- 5 Which interval in the diagram is parallel to FG ? Select the correct answer **A**, **B**, **C** or **D**.

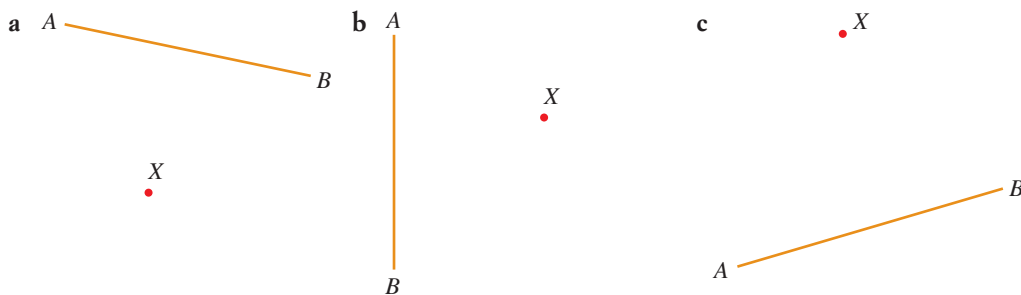


- 6 On the map, what is perpendicular to Frank Road? Select **A**, **B**, **C** or **D**.
- A Emilia Parade
 - B Rosalia Road
 - C Daniel Street
 - D Christina Road



See Example 9

- 7 Copy each diagram and construct a line parallel to AB through X .



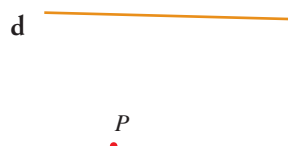
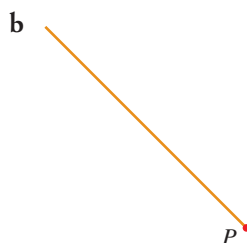
See Example 10

- 8 Draw a line and mark a point, L , on it. Construct a perpendicular line through L :
- a using compasses
 - b using a protractor

See Example 11

- 9 Draw a line and mark a point, X , above it. Construct a perpendicular line that passes through X .

10 Copy each diagram and construct a perpendicular line through P .

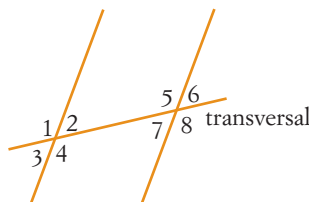
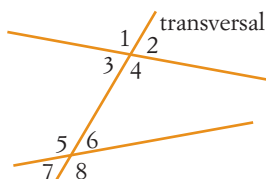


- 11 a Draw two intervals that are parallel and of different lengths.
b Join their ends to make a quadrilateral.
c What type of quadrilateral have you constructed?
- 12 a Draw an interval and mark its midpoint.
b Draw a different-sized interval through the midpoint of the first interval, perpendicular to it and with the same midpoint (as shown on the right).
c Join the ends of the interval to make a quadrilateral.
d What type of quadrilateral have you constructed?



2-07 Corresponding angles on parallel lines

A line that crosses two or more other lines is called a **transversal**. Transverse means ‘crossing’. If a transversal crosses two lines, then 8 angles are created.



These 8 angles have special properties if the transversal crosses **parallel** lines. **Corresponding angles on parallel lines** are any two angles in matching positions according to the transversal and a parallel line, as shown in the diagrams on the next page.

Worksheet

Investigating angles
on parallel lines

MAT07MGWK10013

Skillsheet

Angles and parallel
lines

MAT07MGSS10010

Technology

Geogebra
Angles on parallel lines

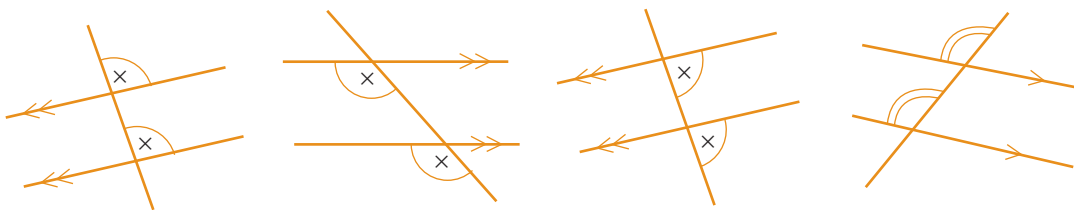
MAT07MGCT00002

Corresponding angles on parallel lines

Video tutorial

Angle relationships

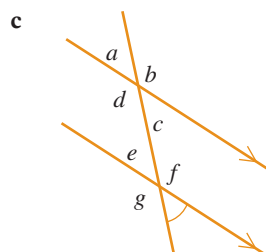
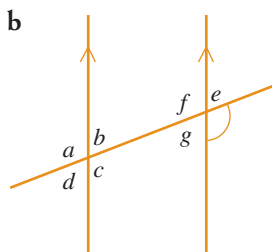
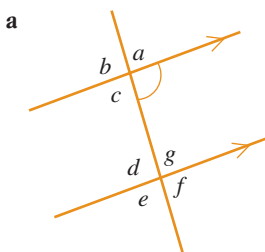
MAT07MGVT00003



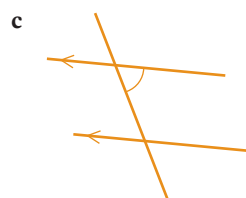
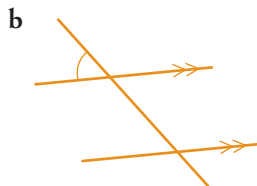
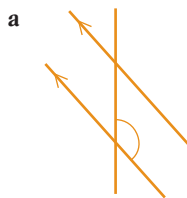
‘Corresponding’ means ‘matching’, and corresponding angles on parallel lines are equal in size.

Exercise 2-07 Corresponding angles on parallel lines

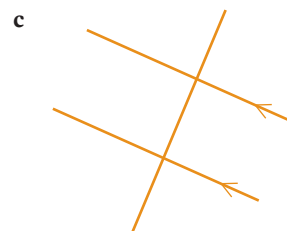
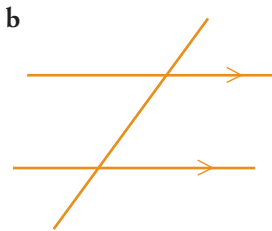
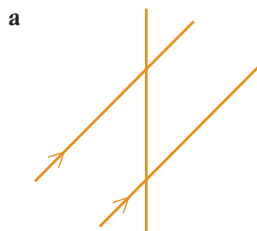
- 1 For each diagram, write the angle that corresponds to the marked angle.



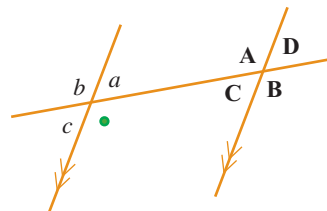
- 2 Copy each diagram and mark the corresponding angle to the marked angle.



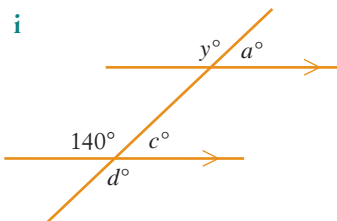
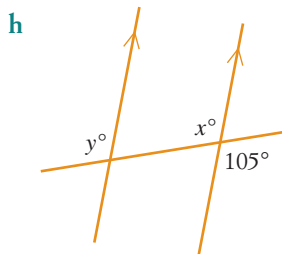
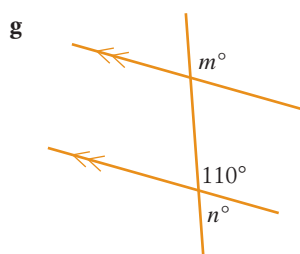
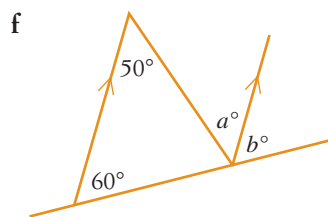
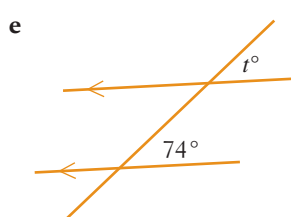
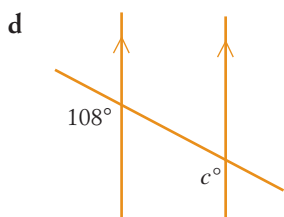
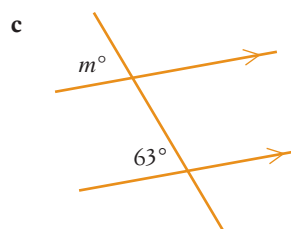
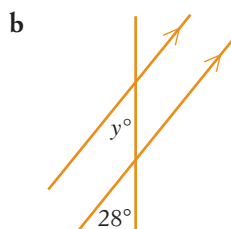
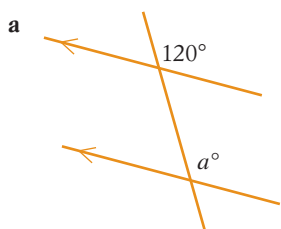
- 3 Copy each diagram and mark any pair of corresponding angles on each one.



- 4 Which angle is corresponding to the angle marked ●? Select A, B, C or D.



5 Find the value of the pronumeral in each diagram.



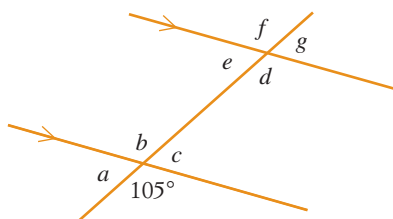
Worked solutions

Exercise 2-07

MAT07MGWS10007

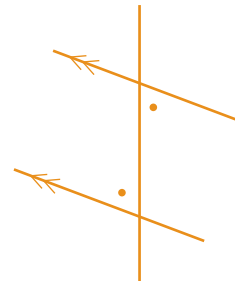
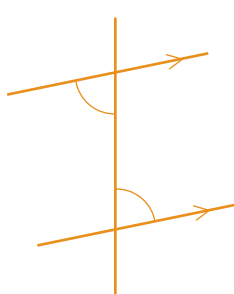
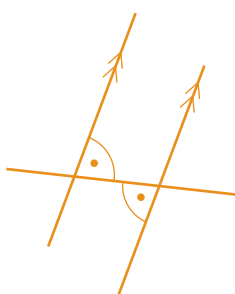
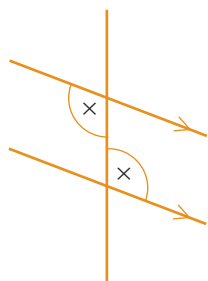
6 a How many pairs of corresponding angles are there in this diagram?

b Find the sizes of the seven unknown angles.



2-08 Alternate angles on parallel lines

Alternate angles on parallel lines are any two angles between the lines and on opposite sides of the transversal, as shown in the diagrams below.



‘Alternate’ means ‘changing direction’.

Worksheet

Investigating angles
on parallel lines

MAT07MGWK10013

Skillsheet

Angles and parallel
lines

MAT07MGSS10010

Technology

Geogebra
Angles on parallel lines

MAT07MGCT00002

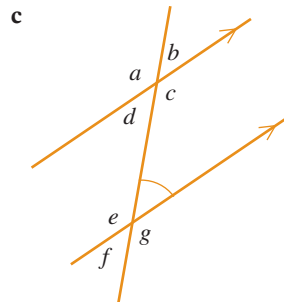
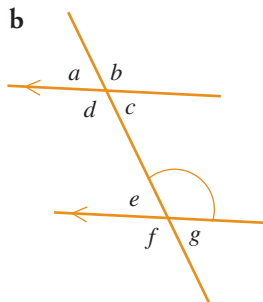
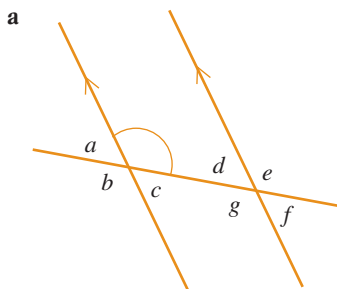
Exercise 2-08 Alternate angles on parallel lines

Video tutorial

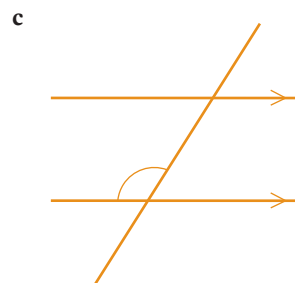
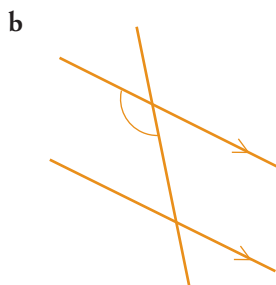
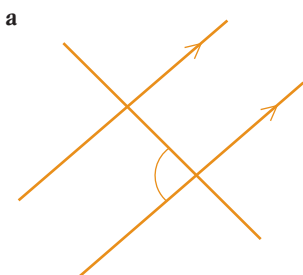
Angle relationships

MAT07MGVT00003

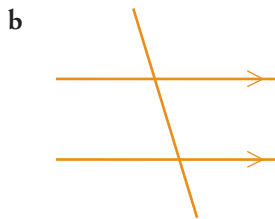
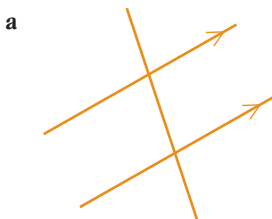
- 1 For each diagram, write the angle that is alternate to the marked angle.



- 2 Copy each diagram and mark the alternate angle to the marked angle.



- 3 Copy each diagram and mark any pair of alternate angles on each one.



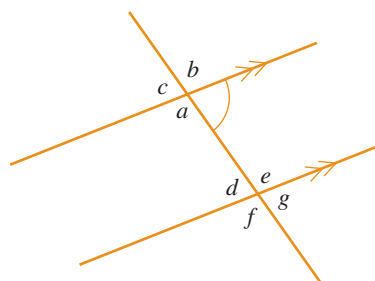
- 4 Which angle is alternate to the marked angle? Select **A**, **B**, **C** or **D**.

A d

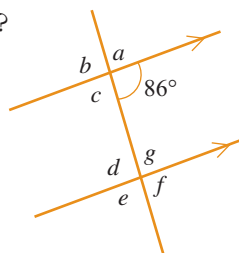
B e

C b

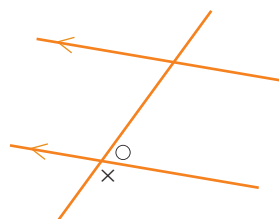
D a



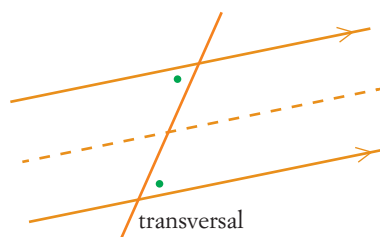
- 5 a How many pairs of alternate angles are there in this diagram?
b Find the sizes of the seven unknown angles.
c What is the relationship between the alternate angles on parallel lines?



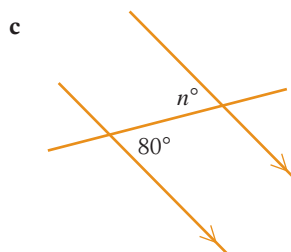
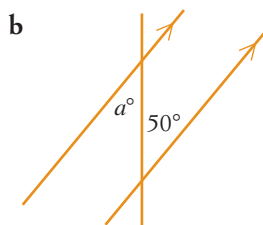
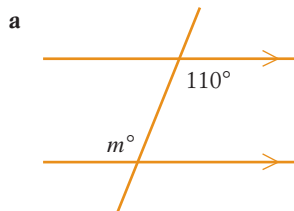
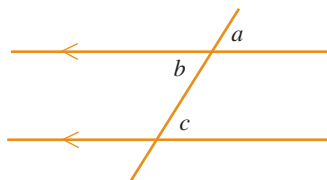
- 6 Two angles have been marked on this diagram. The other six angles are either $\bigcirc$ or $\times$.
a Copy and complete the diagram with $\bigcirc$ s or $\times$ s.
b What do you notice about the pairs of alternate angles?



- 7 a Copy the following diagram on a sheet of paper and spin it upside down to show that it has rotational symmetry. What does that mean about the alternate angles?
b Cut out your diagram and then cut along the dotted line. Spin one of the alternate angles to fit exactly on top of the other. Are they the same size?



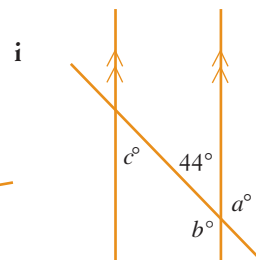
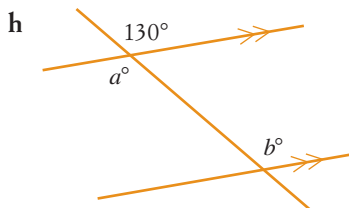
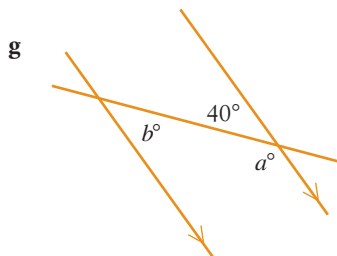
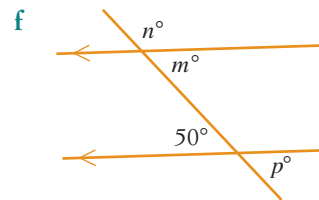
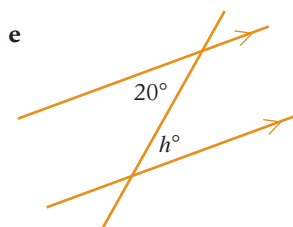
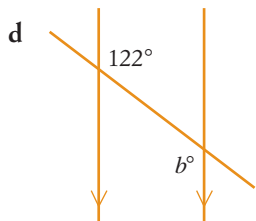
- 8 Examine this diagram and answer the following questions.
a Why is $b = a$?
b Why is $c = a$?
c So what does this mean about the values of b and c ?
d What does this prove about alternate angles on parallel lines?
9 Find the value of the pronumeral in each diagram.



Worked solutions

Exercise 2-08

MAT07MGWS10008



2-09 Co-interior angles on parallel lines

Worksheet

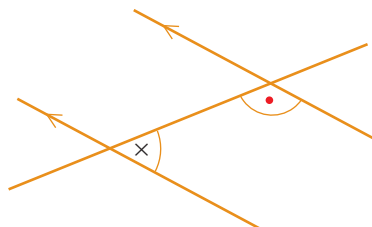
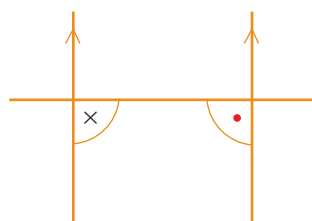
Investigating angles on parallel lines

MAT07MGWK10013

Skillsheet

Angles and parallel lines

MAT07MGSS10010



Co-interior angles on parallel lines are any two angles between the lines and on the same side of the transversal, as shown in the diagrams below.

Homework sheet

Angles on parallel lines

MAT07MGHS10030

'Co-interior' means 'together inside'.

Exercise 2-09 Co-interior angles on parallel lines

Video tutorial

Angle relationships

MAT07VT00003

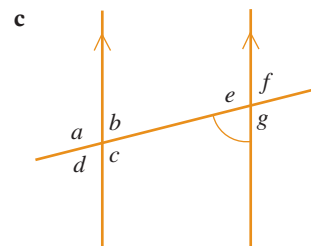
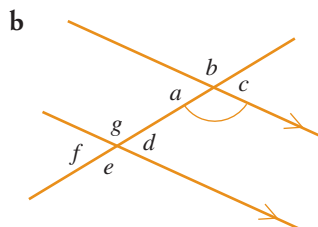
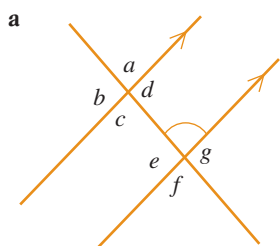
Technology

Geogebra

Angles on parallel lines

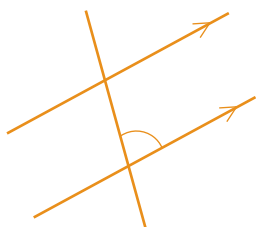
MAT07MGCT00002

- For each diagram, write the angle that is co-interior to the marked angle.

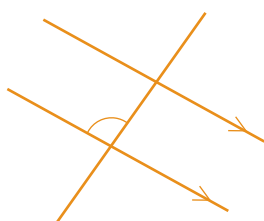


- 2 Copy each diagram and mark the co-interior angle to the marked angle.

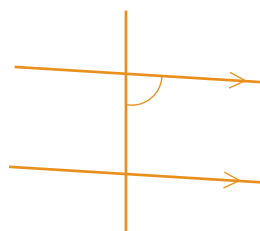
a



b

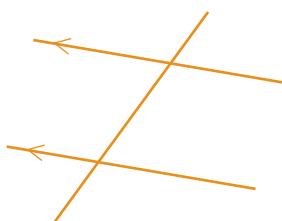


c

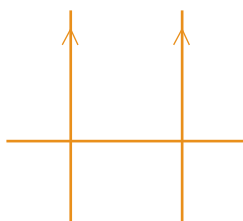


- 3 Copy each diagram and mark any pair of co-interior angles on each one.

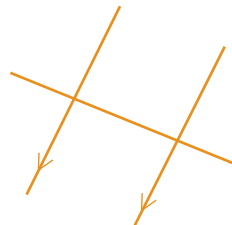
a



b



c



- 4 a How many pairs of co-interior angles are there in this diagram?
b Which angle is co-interior to the 121° angle? Select A, B, C or D.

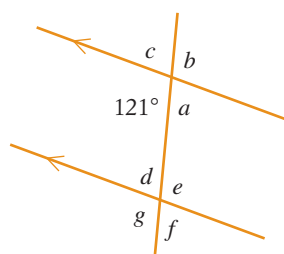
A d

B b

C e

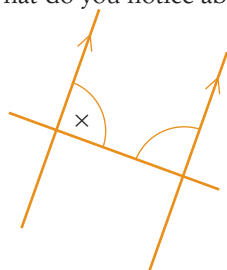
D g

- c Find the sizes of the seven unknown angles.
d What is the relationship between the co-interior angles on parallel lines?

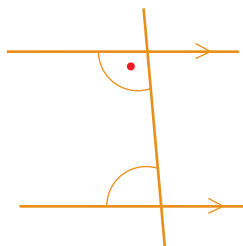


- 5 Copy each diagram, measure the marked angles and calculate their sum. What do you notice about your answer?

a

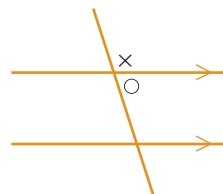


b



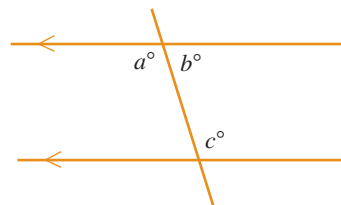
- 6 Two angles have been marked on this diagram.

- a How many degrees is $\bigcirc + \times$? Why?
b The other six angles are either $\bigcirc$ or $\times$. Copy and complete the diagram with $\bigcirc$ s or $\times$ s.
c What do you notice about the pairs of co-interior angles?

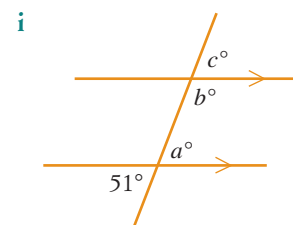
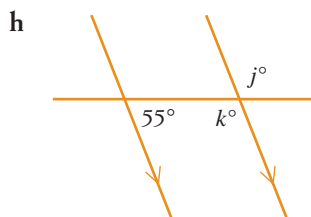
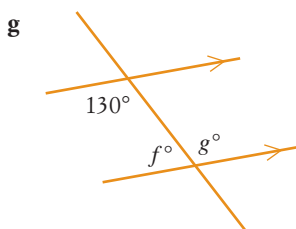
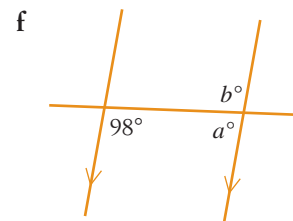
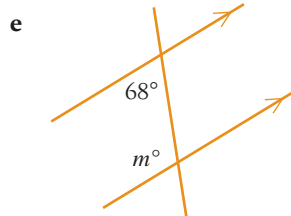
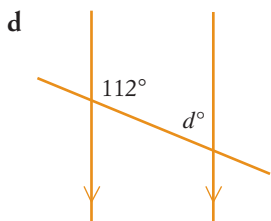
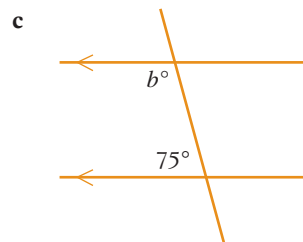
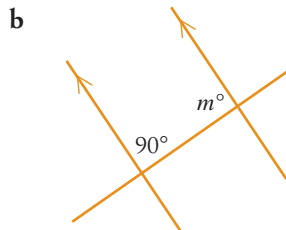
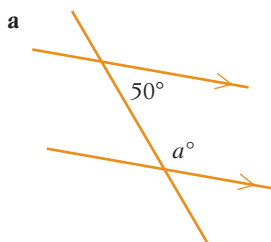


7 Examine this diagram and answer the following questions.

- What is the value of $a + b$? Why?
- Why is $a = c$?
- So what is the value of $c + b$?
- What does this prove about co-interior angles on parallel lines?



8 Find the value of the pronumeral in each diagram.



Worked solutions

Exercise 2-09

MAT07MGWS10009

2-10 Angles on parallel lines

Worksheet

Find the unknown angle

MAT07MGWK10014

Worksheet

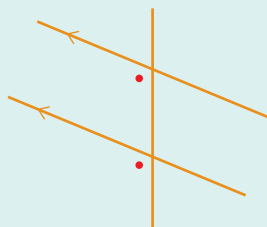
What is the diagram?

MAT07MGWK10015

Summary

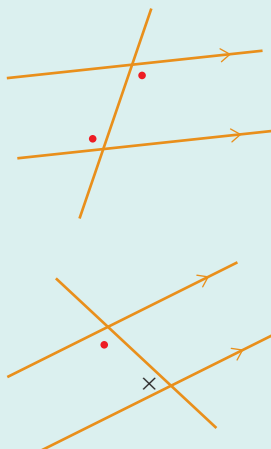
When parallel lines are crossed by a transversal:

- corresponding angles are equal



- alternate angles are equal

- co-interior angles are supplementary (add to 180°)



Worksheet

Matching angles

MAT07MGWK10016

Skillsheet

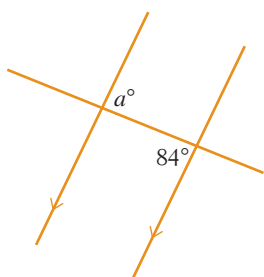
Angles and parallel lines

MAT07MGSS10010

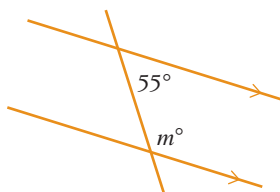
Example 12

Find the value of the pronumeral in each diagram, giving the reason.

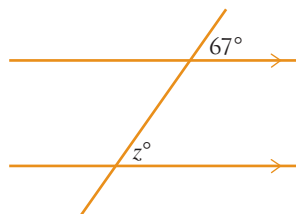
a



b



c



Solution

a $a = 84$

b $m + 55 = 180$
 $m = 180 - 55$
 $= 125$

c $z = 67$

Alternate angles on parallel lines

Co-interior angles on parallel lines

Corresponding angles on parallel lines

Video tutorial

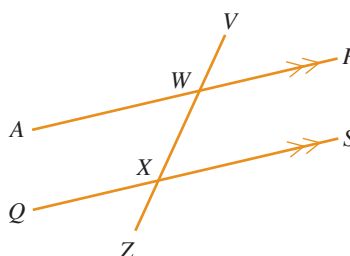
Angles on parallel lines

MAT07MGVT10004

Exercise 2-10 Angles on parallel lines

1 In the diagram on the right, name the angle that is:

- corresponding to $\angle VWA$
- alternate to $\angle QXW$
- co-interior with $\angle PWX$
- supplementary with $\angle AWX$
- alternate to $\angle SXV$
- corresponding to $\angle ZXS$.

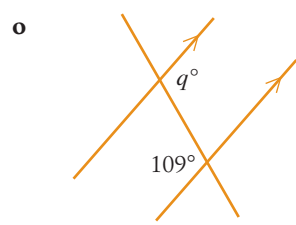
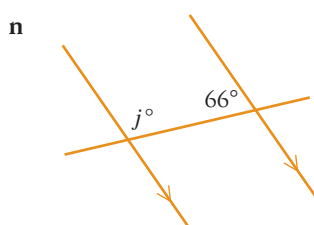
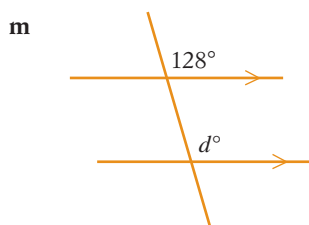
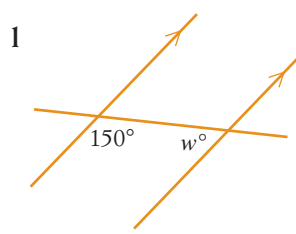
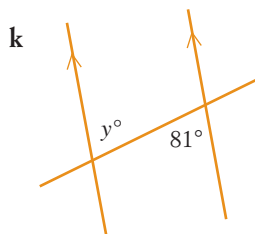
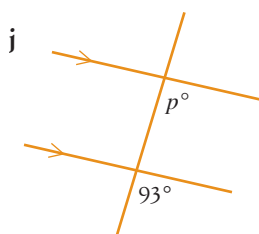
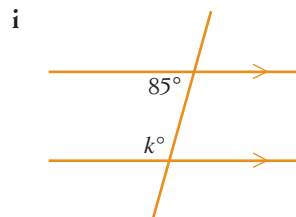
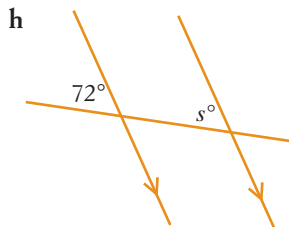
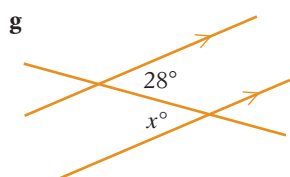
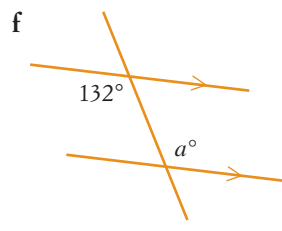
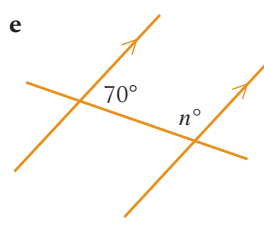
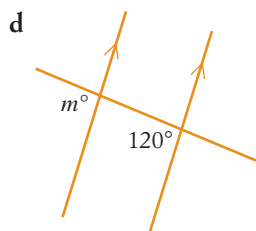
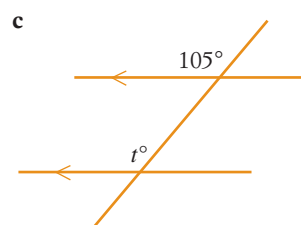
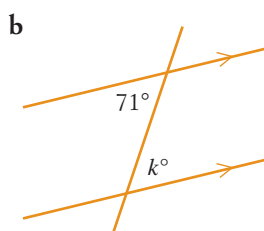
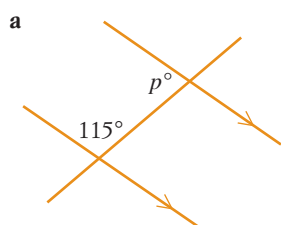


Extra questions

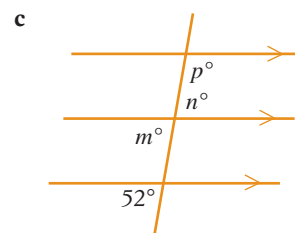
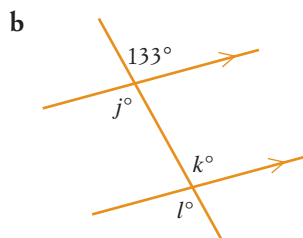
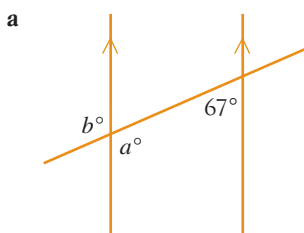
Angle relationships

MAT07MGEQ00016

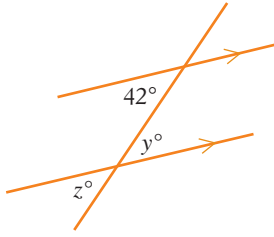
See Example 12 2 Find the value of the pronumeral in each diagram, giving a reason.



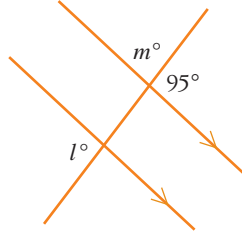
3 Find the value of the pronumeral(s) in each diagram.



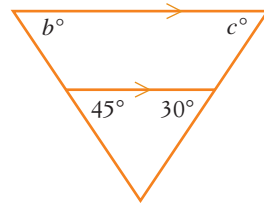
d



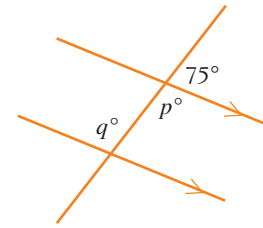
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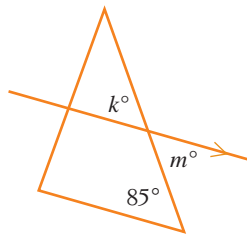
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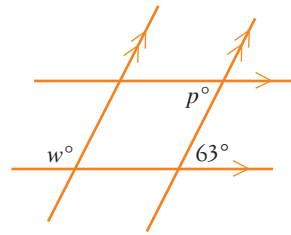
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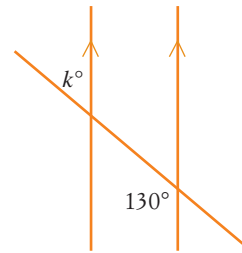
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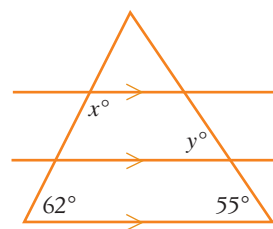
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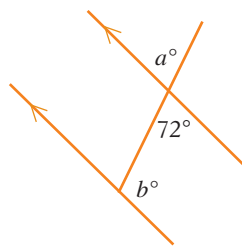
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k



l

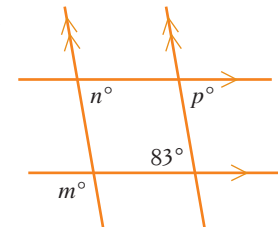


Worked solutions

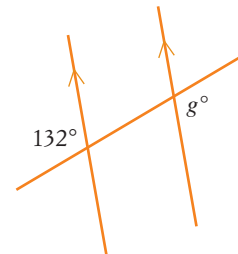
Exercise 2-10

MAT07MGWS10010

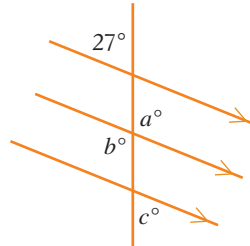
m



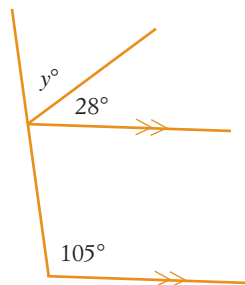
n



o



4 Which is the value of y in this diagram? Select A, B, C or D.



A 28

B 47

C 77

D 152

Worked solutions

Exercise 2-10

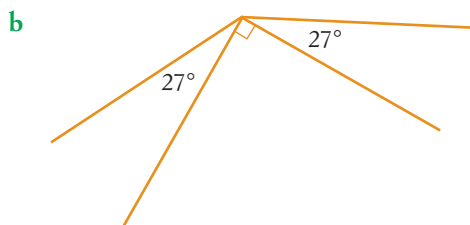
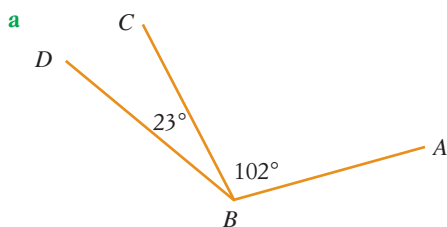
MAT07MGWS10010

Technology Constructing angles 2

- 1 Use  **Interval between Two Points** from the third drop-down menu in GeoGebra to draw three different examples of each type of angle.

a acute b reflex c obtuse

- 2 Construct the following angles accurately.



c Two complementary angles

d Two supplementary angles

- 3 To construct the following pairs of angles on parallel lines, use  **Parallel Line** from the fourth drop-down menu.

a Corresponding angles of 28°

b Alternate angles of 65°

c Co-interior angles, where one of the supplementary angles is 130°

2-11 Proving parallel lines

We can use what we know about angles and parallel lines to prove that two lines are parallel.

Homework sheet

Angles revision

MAT07MGHS10031

Summary

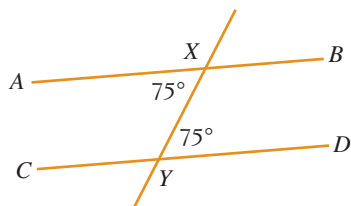
Two lines are parallel if:

- alternate angles are equal, or
- corresponding angles are equal, or
- co-interior angles are supplementary (add up to 180°)

Example 13

Prove whether:

a $AB \parallel CD$



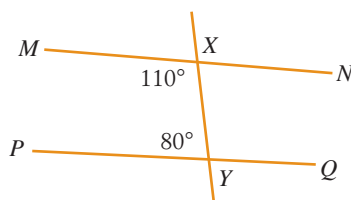
Solution

a $\angle AXY$ is alternate to $\angle DYX$.

$$\angle AXY = \angle DYX = 75^\circ$$

$\therefore AB \parallel CD$

b $MN \parallel PQ$



Alternate angles are equal

b $\angle MXY$ is co-interior to $\angle PYX$.

$$\angle MXY + \angle PYX = 110^\circ + 80^\circ$$

$$= 190^\circ$$

$$\neq 180^\circ$$

Co-interior angles *do not* add to 180°

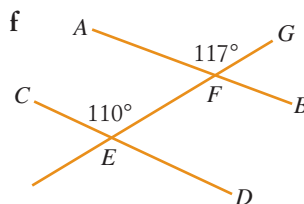
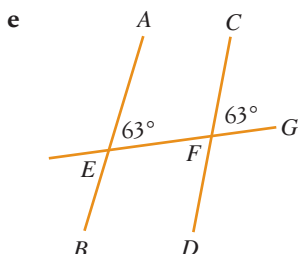
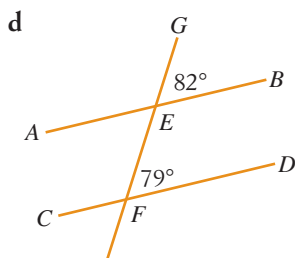
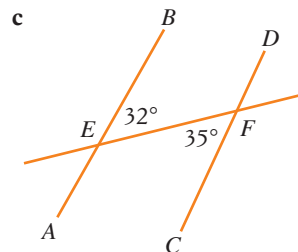
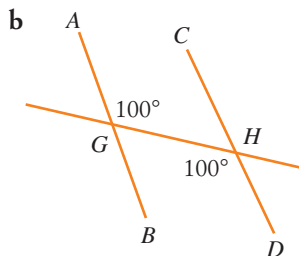
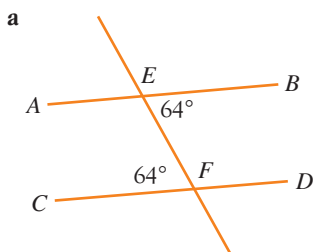
$\therefore MN$ is *not* parallel to PQ .

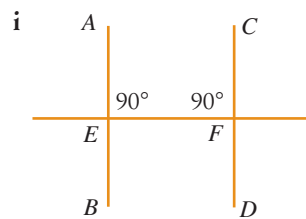
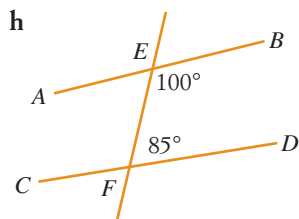
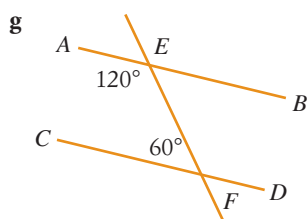
$\therefore$ means 'therefore'

Exercise 2-11 Proving parallel lines

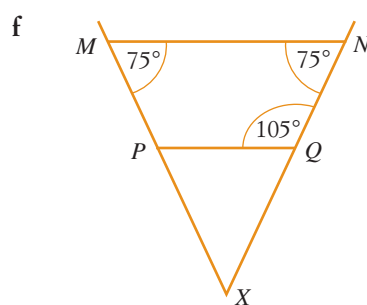
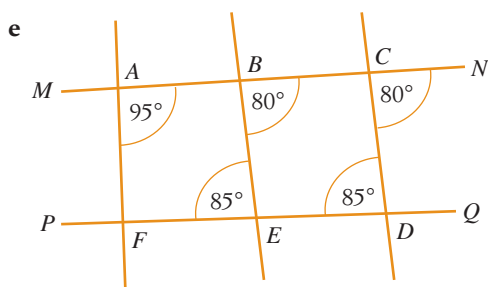
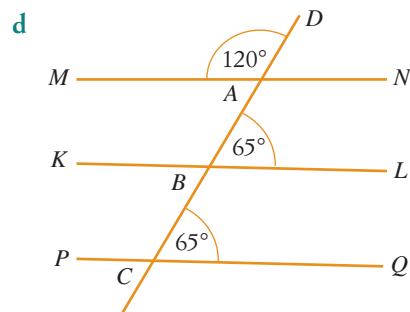
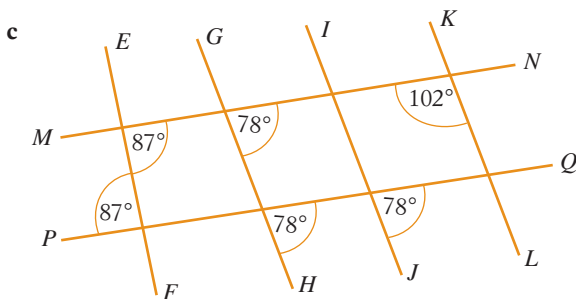
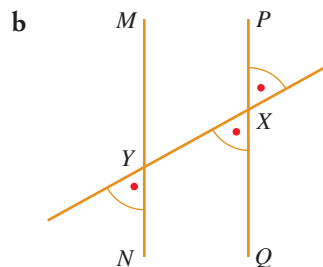
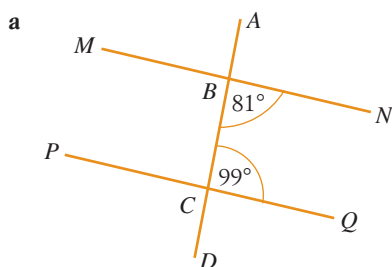
1 In each diagram below, prove whether $AB \parallel CD$.

See Example 13





2 For each diagram below, prove whether $PQ \parallel MN$, giving a reason.



3 What reason can be used to prove that $GC \parallel HE$?

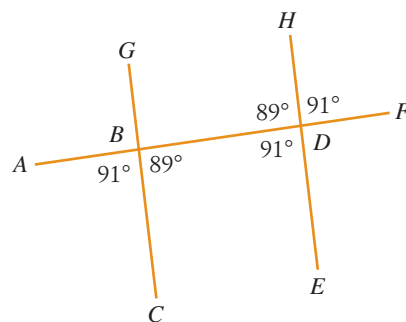
Select **A**, **B**, **C** or **D**.

A $\angle ABC = \angle HDF$ (alternate angles)

B $\angle CBD = \angle BDH$ (alternate angles)

C $\angle ADE = 91^\circ$ (corresponding angles)

D $\angle BDE = \angle FDH$ (vertically opposite angles)



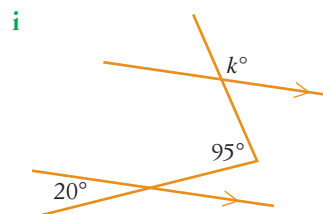
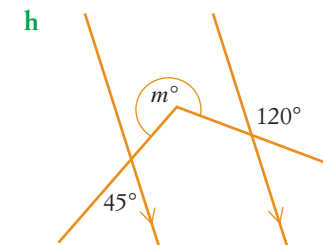
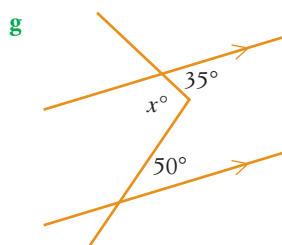
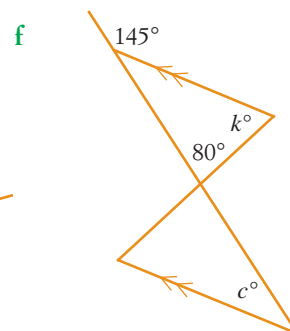
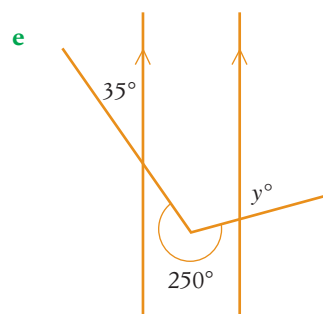
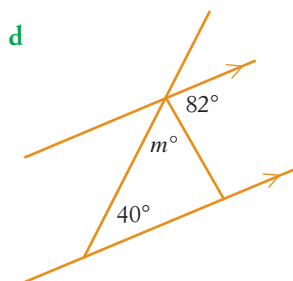
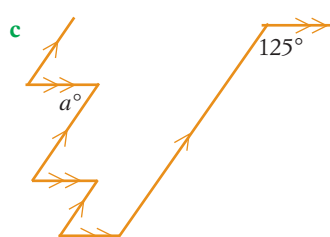
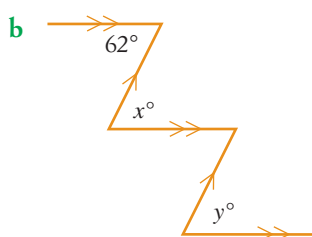
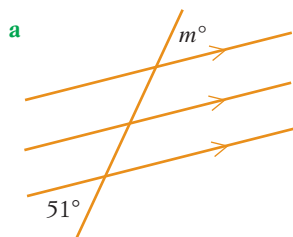
Worked solutions

Exercise 2-11

MAT07MGWS10011

Power plus

- How many degrees does the Earth spin on its axis in:
 - one day?
 - one hour?
 - 8 hours?
 - 10 minutes?
- Work out which direction (left, right, front or behind) you would be facing after making each of these series of turns.
 - Right 80° , right 240° , left 90° , right 40°
 - Left 140° , left 140° , left 140° , right 60°
 - Right 200° , left 70° , right 40° , right 10°
 - Left 240° , right 190° , right 100° , left 50°
- Find the value of the pronumeral(s) in each diagram, giving reasons for your answers.



- Draw an interval DE 6 cm in length. At D draw a line at any angle. On this line, mark a point G , also 6 cm from D . Construct a line through G parallel to DE . Construct a line through E parallel to DG . What shape have you drawn?

Chapter 2 review

Language of maths

Puzzle sheet

Angles crossword

MAT07MGPS10007

acute	complementary	obtuse	right
adjacent	construct	parallel	straight
alternate	corresponding	perpendicular	supplementary
angles at a point	degrees	pronumeral	transversal
arm	intersect	protractor	vertex
co-interior	interval	revolution	vertically opposite

- 1 What is the name given to a line that crosses two or more other lines?
- 2 Find the meaning of 'acute' when referring to a disease, for example, acute appendicitis.
- 3 What is the difference between 'complementary' and 'complimentary'?
- 4 When something happens that dramatically changes the way we think or do things, it is said to be 'revolutionary'. Why do you think this is so?
- 5 Draw 'angles at a point' and write down a property concerning it.
- 6 Mr Transversal visits his parents on **alternate** days. What does this mean? How is it similar to the mathematical meaning of 'alternate'?

Topic overview

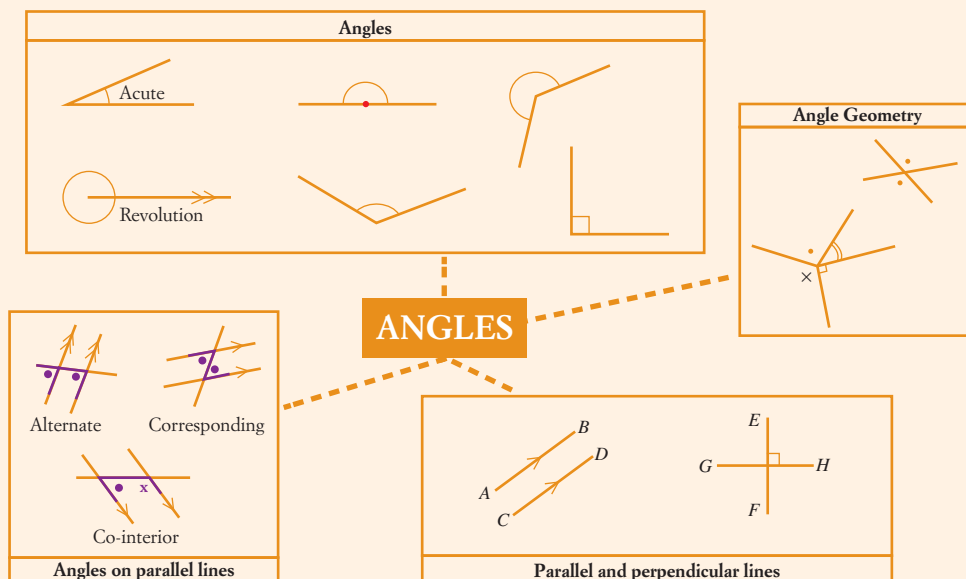
Worksheet

Mind map: Angles

MAT07MGWK10017

- Give three examples of where angles are used.
- How confident do you feel about working with angles?
- Is there anything you did not understand? Ask a friend or your teacher for help.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



1 Draw labelled diagrams of each angle.

See Exercise 2-01

a $\angle BKT$

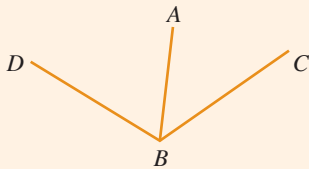
b $\angle FPR$

c $\angle MZQ$

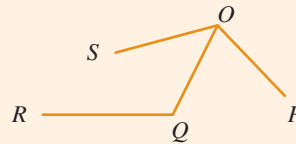
2 Name the angle adjacent to:

See Exercise 2-01

a $\angle ABC$



b $\angle POQ$



3 Use a protractor to measure each angle you drew in question 1. Name the smallest angle and the largest angle.

See Exercise 2-02

4 Use a protractor to draw these angles.

See Exercise 2-02

a $\angle JUG = 84^\circ$

b $\angle QRA = 117^\circ$

c $\angle POT = 41^\circ$

d $\angle DGE = 150^\circ$

e $\angle SAR = 96^\circ$

f $\angle XDW = 210^\circ$

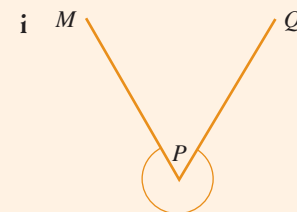
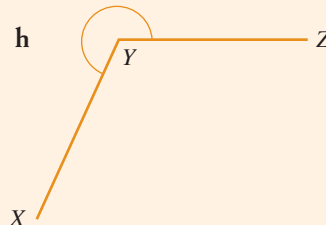
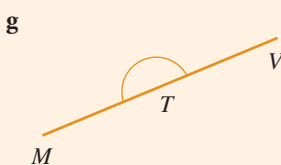
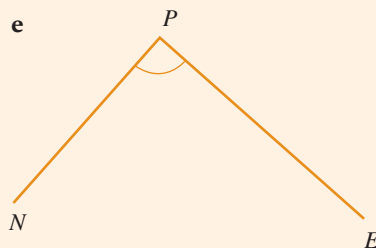
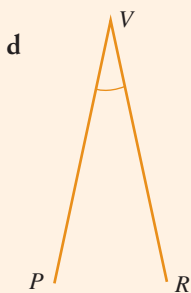
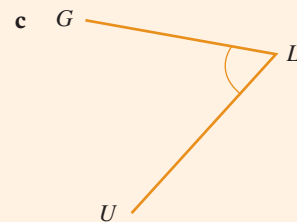
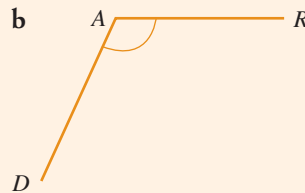
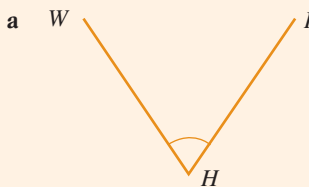
g $\angle MNB = 195^\circ$

h $\angle PLO = 270^\circ$

i $\angle AMP = 300^\circ$

5 Name each angle, then classify each as being acute, obtuse, right, reflex or straight.

See Exercise 2-03



Chapter 2 revision

See Exercise 2-04

6 a Write the complement of:

i 35°

ii 78°

iii 4°

b Write the supplement of:

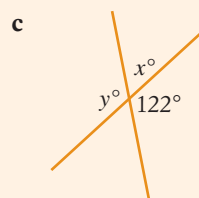
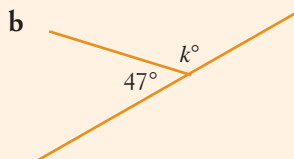
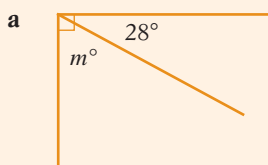
i 45°

ii 100°

iii 178°

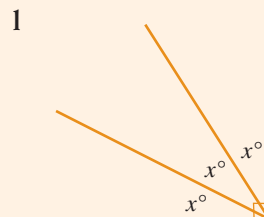
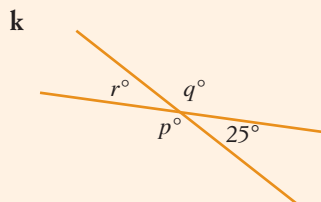
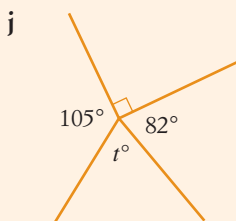
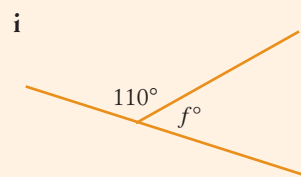
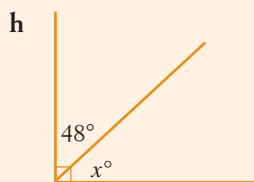
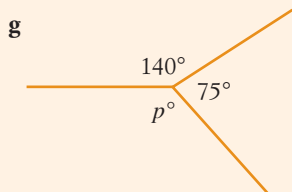
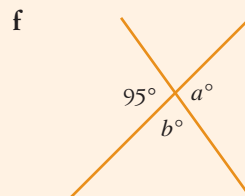
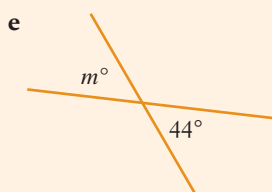
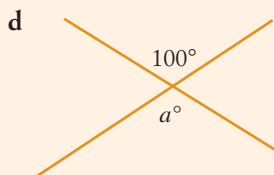
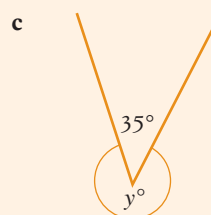
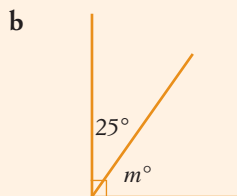
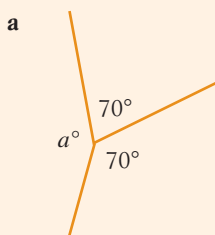
See Exercise 2-04

7 Find the value of the pronumeral(s) in each diagram.



See Exercise 2-05

8 Find the value of the pronumeral(s) in each diagram.



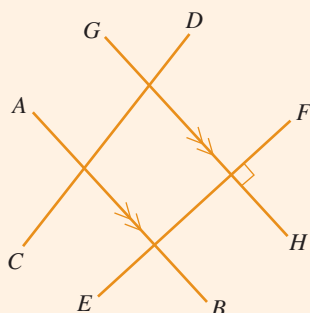
9 In this diagram, name two lines that:

See Exercise 2-06

a are parallel

b are perpendicular

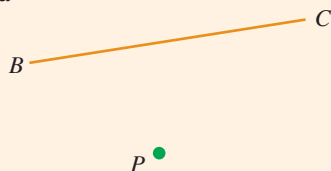
c intersect



10 Copy each diagram and construct the line parallel to BC going through P .

See Exercise 2-06

a



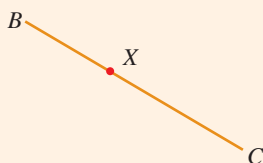
b



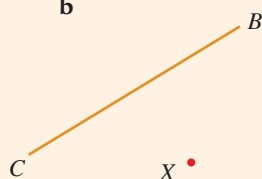
11 Copy each diagram and construct the perpendicular to BC going through X .

See Exercise 2-06

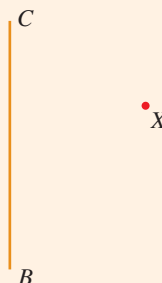
a



b



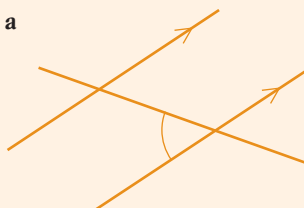
c



12 Copy each diagram and mark in the alternate angle to the one shown.

See Exercise 2-07

a



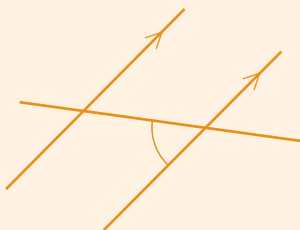
b



Chapter 2 revision

See Exercise 2-08 **13** Copy each diagram and mark in the corresponding angle to the one shown.

a

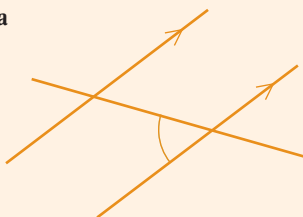


b

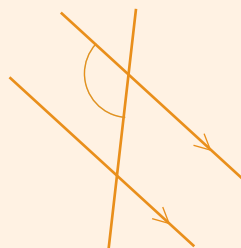


See Exercise 2-09 **14** Copy each diagram and mark in the co-interior angle to the one shown.

a

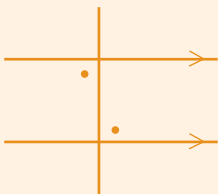


b

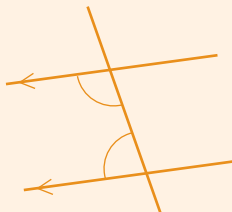


See Exercise 2-10 **15** Label the marked pairs of angles as corresponding, alternate or co-interior.

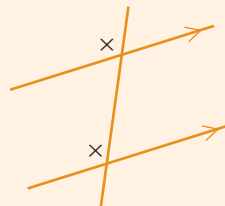
a



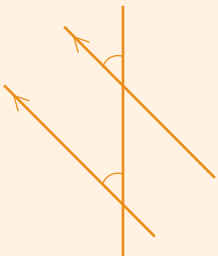
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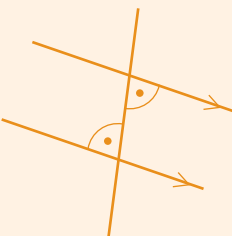
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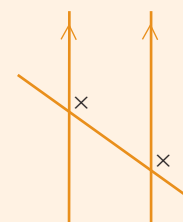
d



e

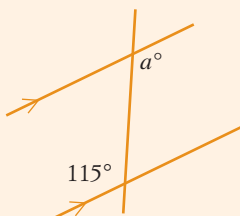


f

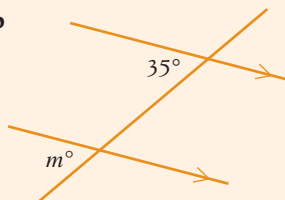


See Exercise 2-10 **16** Find the value of the pronumeral in each diagram.

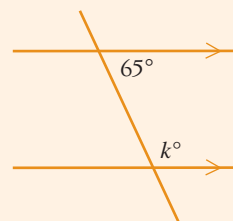
a

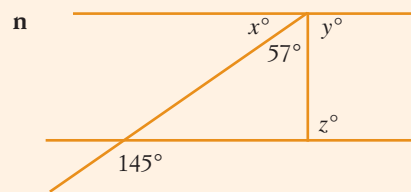
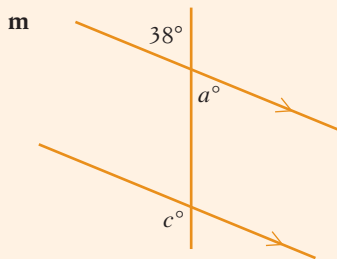
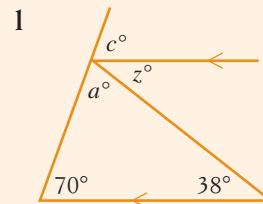
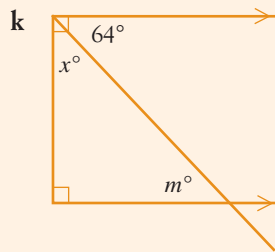
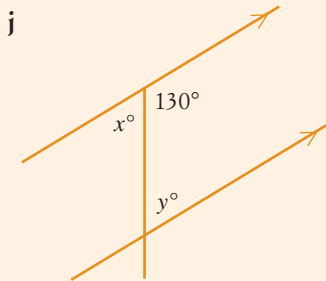
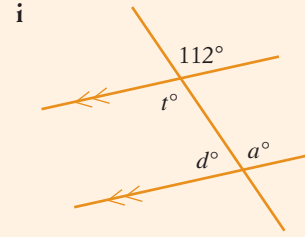
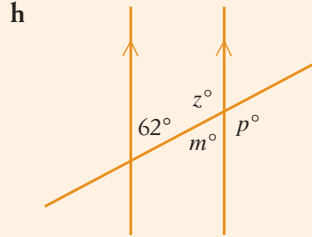
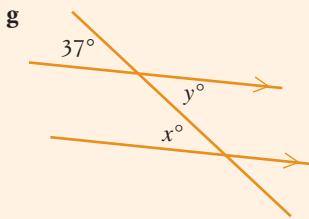
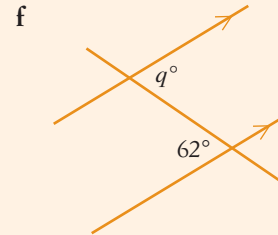
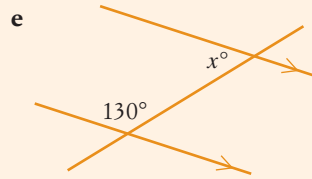
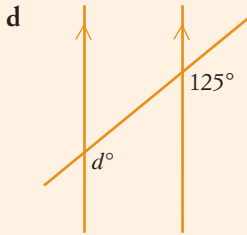


b



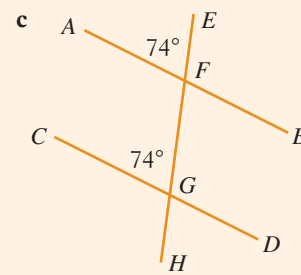
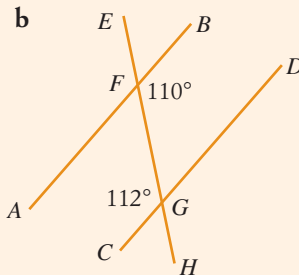
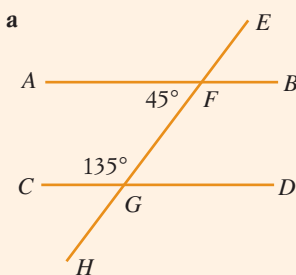
c





17 In each diagram below, determine whether $AB \parallel CD$, giving a reason.

See Exercise 2-11





3

Number and algebra

Whole numbers

The ability to use numbers is an essential part of modern living. Think of all the times during the day when you use numbers – buying food, catching buses and trains and even in sport.



Chapter outline

	Proficiency strands			
3-01 Rounding and estimating	U	F		
3-02 Multiplying numbers	U			
3-03 Dividing numbers	U			
3-04 Divisibility tests	U	F	PS	R
3-05 Long division	U			
3-06 Roman numerals	U			
3-07 Powers and index notation	U			
3-08 Square root and cube root	U	F		R
3-09 Prime and composite numbers	U	F		R
3-10 Prime factors	U			
3-11 Greatest common divisor	U	F	PS	R
3-12 Lowest common multiple	U	F	PS	R

Wordbank

- composite number** A number with more than two factors
- divisibility test** A rule for testing whether or not a number is divisible by a specific value, for example, divisible by 3
- factor** A value that divides evenly into a given number, for example, 3 is a factor of 15
- factor tree** A diagram that lists the prime factors of a number
- greatest common divisor (GCD)** The largest factor shared by two or more numbers, also called the **highest common factor (HCF)**
- index notation** A way of writing powers for the repeated multiplication of a number, for example, 3^5
- prime number** A number with only two factors, 1 and itself
- square root (of a number, symbol $\sqrt{\quad}$)** The positive value which, if squared, gives the number

In this chapter you will:

Maths clip

Whole numbers

MAT07NAVT00002

- use estimation and rounding to check the reasonableness of answers to calculations
- select and apply efficient mental strategies and appropriate digital technologies to solve problems involving multiplying and dividing whole numbers
- use simple divisibility tests
- understand that if a number is divisible by a composite number then it is also divisible by the factors of that number
- **divide by a two-digit number**
- **recognise, read and convert Roman numerals**
- investigate index notation
- investigate and use square roots of perfect square numbers, cube roots of perfect cube numbers
- define and compare prime and composite numbers and explain the difference between them
- represent whole numbers as products of powers of prime numbers
- solve problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers by comparing their prime factorisation

SkillCheck

Worksheet

1 Is 691 odd or even? How can you tell?

StartUp assignment 3

2 Is 270 divisible by 5? How can you tell?

MAT07NAWK10018

3 Evaluate each product.

Worksheet

a 7×3

b 6×5

c 4×8

d 3×6

Calculation aids

e 9×4

f 5×8

g 7×7

h 6×9

i 8×8

j 9×7

k 4×6

l 7×8

MAT07NAWK10019

m $-3 \times (-3)$

n 9×9

o 5×5

p $4 \times (-4)$

Worksheet

4 Write all the factors of:

Number grids

a 20

b 12

c 19

MAT07NAWK10020

5 Is 166 divisible by 2? How can you tell?

Puzzle sheet

6 Evaluate each quotient.

a $24 \div 3$

b $35 \div 7$

c $24 \div 4$

d $36 \div 6$

e $20 \div 5$

f $72 \div 9$

g $48 \div 8$

h $32 \div 4$

i $28 \div 7$

j $45 \div 5$

k $54 \div 9$

l $63 \div 7$

MAT07NAPS10008

7 Write the first 5 multiples of:

Puzzle sheet

a 8

b 7

c 12

Magic squares

8 Is 34 divisible by 10? How can you tell?

MAT07NAPS10009

3-01 Rounding and estimating

Rounding numbers

When calculating with numbers it is often useful to have a rough idea of the answer before actually working it out. Rounded numbers are easier to work with and to compare. For example, a town's population of 18 256 can be rounded to 18 300 (to the nearest hundred) because 18 256 is closer to 18 300 than 18 200.



Skillsheet

Place value

MAT07NASS10011

Skillsheet

Reading and writing
large numbers

MAT07NASS10012

Skillsheet

Rounding
whole numbers

MAT07NASS10013

Homework sheet

Whole numbers

MAT07NAHS10001

Summary

To **round a number**, 'cut' it at the required place and look at the digit in the next place:

- if the digit is less than 5 (that is 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is 5, 6, 7, 8 or 9), **round up**

Example 1

Round 8470 to the nearest hundred.

Solution

Counting by hundreds, 8470 is between 8400 and 8500.

- In 8470, the hundreds digit is 4
- The digit in the next (tens) place, 7, is more than 5, so **round up** to 8500.

$8470 \approx 8500$ (rounded to the nearest hundred).

The symbol ' $\approx$ ' means
'approximately equal to'.

Example 2

Round 247 182 to the nearest thousand.

Solution

In 247 182, the thousands digit is 7 and the next digit is 1.

$1 < 5$, so round down to 247 000. The thousands digit, 7, stays the same.

$247\ 182 \approx 247\ 000$ (rounded to the nearest thousand).

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

Example 3

Estimate the answer to each expression.

a $631 + 280 + 51 + 43 + 96$

c $55 + 132 - 34 + 17 - 78$

b 67×12

d $510 \div 24$

Solution

a $631 + 280 + 51 + 43 + 96$

$$\approx 600 + 300 + 50 + 40 + 100$$

$$= (600 + 300 + 100) + (50 + 40)$$

$$= 1000 + 90$$

$$= 1090$$

Estimating

(Exact answer = 1101)

b $67 \times 12 \approx 70 \times 10$

$$= 700$$

Estimating

(Exact answer = 804)

c $55 + 132 - 34 + 17 - 78$

$$\approx 60 + 130 - 30 + 20 - 80$$

$$= (60 + 20 - 80) + (130 - 30)$$

$$= 0 + 100$$

$$= 100$$

Estimating

(Exact answer = 92)

d $510 \div 24 \approx 500 \div 20$

$$= 50 \div 2$$

$$= 25$$

Estimating

(Exact answer = 21.25)

Exercise 3-01 Rounding and estimating

See Example 1

1 Round each number to the nearest hundred.

a 3148

b 49 028

c 2597

d 4 934 277

See Example 2

2 Round each number to the nearest thousand.

a 23 538

b 45 370

c 62 941

d 47 929

3 Round each number to the nearest ten.

a 45 819

b 1699

c 8314

d 71 262

4 Round 64 218 to the nearest:

a thousand

b hundred

c ten

d ten thousand

5 Round 1 327 509 to the nearest:

a thousand

b ten thousand

c ten

d million

6 Estimate the answer for each expression.

a $27 + 11 + 87 + 142 + 64$

c $684 + 903$

e $517 - 96$

g $766 - 353$

i 83×81

k $828 \div 3$

b $55 + 34 - 22 - 46 + 136$

d $35 + 81 + 110 + 22 + 7$

f $210 - 38 - 71 + 151 - 49$

h 367×2

j 984×16

l $507 \div 7$

See Example 3

7 Over the holidays, 27 792 people visited a museum. Write this figure correct to the nearest hundred.

8 The extensions on Nina's house are quoted as costing \$17 464. Write this amount correct to the nearest \$100.

9 Write a number that can be rounded to:

a 370

b 5400

c 12 900

d 6000

10 The crowd at a football Grand Final was 104 427. Round this figure to the nearest thousand.

11 The distance between Sydney and Brisbane is 998 km. Round this distance to the nearest:

a 10 km

b 100 km

c 1000 km

12 The population of Australia is 23 581 800. Round this figure to the nearest thousand.

Worked solutions

Exercise 3-01

MAT07NAWS10012

3-02 Multiplying numbers

Worksheet

Whole numbers 6

MAT07NAWK00022

Example 4

Evaluate each product.

a 243×6

b 573×36

Solution

$$\begin{array}{r} 243 \\ \times 6 \\ \hline 1458 \end{array}$$

Check by estimating: $243 \times 6 \approx 200 \times 6 = 1200$

$$\begin{array}{r} 573 \\ \times 36 \\ \hline 3438 \\ 17190 \\ \hline 20628 \end{array}$$

Check by estimating: $573 \times 36 \approx 600 \times 40 = 24\,000$

Worksheet

Calculation aids

MAT07NAWK10019

Worksheet

Number grids

MAT07NAWK10020

Puzzle sheet

Cross number puzzle

MAT07NAPS10008

Puzzle sheet

Magic squares

MAT07NAPS10009

TLF learning object

Rectangle multiplication
(L3503)

Weblink

Finger multiplication

Puzzle sheet

Find the quote 2

MAT07NAPS00013

Mental multiplication

Multiplying by:	Mental strategy
2, 4 or 8	Double once, two times or three times respectively
5	Halve, then multiply by 10 (because $\frac{1}{2} \times 10 = 5$)
9	Multiply by 10, then subtract the number
10	Insert 0 at the end of the number
100	Insert 00 at the end of the number

Example 5

Evaluate each product.

a 68×4

b 36×5

c 12×9

d 14×8

Solution

$$\begin{aligned} \text{a } 68 \times 4 &= 68 \times 2 \times 2 \\ &= 136 \times 2 \\ &= 272 \end{aligned}$$

$$\begin{aligned} \text{b } 36 \times 5 &= 36 \times \frac{1}{2} \times 10 \\ &= 18 \times 10 \\ &= 180 \end{aligned}$$

$$\begin{aligned} \text{c } 12 \times 9 &= 12 \times 10 - 12 \\ &= 120 - 12 \\ &= 108 \end{aligned}$$

$$\begin{aligned} \text{d } 14 \times 8 &= 14 \times 2 \times 2 \times 2 \\ &= 28 \times 2 \times 2 \\ &= 56 \times 2 \\ &= 112 \end{aligned}$$

Double twice

$$68 \times 2 = 60 \times 2 + 8 \times 2 = 120 + 16 = 136$$

$$136 \times 2 = 130 \times 2 + 6 \times 2 = 260 + 12 = 272$$

$$\text{Estimate: } 68 \times 4 \approx 70 \times 4 = 280$$

$$\text{Because } \frac{1}{2} \times 10 = 5$$

Insert a 0 at the end

$$\text{Estimate: } 36 \times 5 \approx 40 \times 5 = 200$$

Multiply by 10, then subtract the number

$$\text{Estimate: } 12 \times 9 \approx 12 \times 10 = 120$$

Double 3 times

$$14 \times 2 = 10 \times 2 + 4 \times 2 = 20 + 8 = 28$$

$$28 \times 2 = 20 \times 2 + 8 \times 2 = 40 + 16 = 56$$

$$56 \times 2 = 50 \times 2 + 6 \times 2 = 100 + 12 = 112$$

$$\text{Estimate: } 14 \times 8 \approx 14 \times 10 = 140$$

Exercise 3-02 Multiplying numbers

- 1 Copy and complete each multiplication.

a
$$\begin{array}{r} 34 \\ \times 7 \\ \hline \end{array}$$

b
$$\begin{array}{r} 219 \\ \times 5 \\ \hline \end{array}$$

c
$$\begin{array}{r} 28 \\ \times 16 \\ \hline \end{array}$$

d
$$\begin{array}{r} 325 \\ \times 21 \\ \hline \end{array}$$

See Example 4

- 2 Lara loves to play tennis. She pays \$7 each time she plays. How much does Lara pay to play 42 times a year?



- 3 Evaluate each product.

a 14×8

b 238×3

c 344×9

d 506×7

e 1084×3

f 45×14

g 64×25

h 107×32

- 4 A bus route is 46 kilometres long. A bus makes 12 trips in one day. Find the total distance travelled each day.
- 5 Nathan can type 76 words per minute. How many words can he type in 15 minutes?

- 6 Evaluate each product mentally.

a 85×2

b 39×2

c 64×2

d 57×4

e 28×4

f 44×4

g 16×8

h 33×8

See Example 5

- 7 Evaluate each product mentally.

a 22×5

b 14×5

c 28×5

d 36×5

e 54×5

f 82×5

g 16×5

h 48×5

- 8 Evaluate each product mentally.

a 34×9

b 51×9

c 27×9

d 19×9

e 45×10

f 18×100

g 36×10

h 41×100

- 9 How many hours are there in one week? Select the correct answer A, B, C or D.

A 84

B 151

C 168

D 240

Worked solutions

Exercise 3-02

- 10 A box contains 124 oranges. If Jessica ordered 21 boxes of oranges, how many oranges will she receive?

MAT07NAWS10013

Worksheet

Whole numbers 7

MAT07NAWK00011

3-03 Dividing numbers

Example 6

Evaluate each quotient.

a $318 \div 6$

b $1964 \div 5$

Solution

$$\begin{array}{r} 53 \\ 6 \overline{) 318} \\ \underline{318} \\ 0 \end{array}$$

$$318 \div 6 = 53$$

$$\begin{array}{r} 392 \text{ r}4 \\ 5 \overline{) 194614} \\ \underline{194614} \\ 0 \end{array}$$

Write the remainder, 4, as a fraction over the divisor, 5:

$$1964 \div 5 = 392\frac{4}{5}$$

Weblink

The Four 4s puzzle

Puzzle sheet

The accidental detective

MAT07NAPS00014

Puzzle sheet

Cross number puzzle

MAT07NAPS10008

Mental division

Dividing by:	Mental strategy
2, 4 or 8	Halve once, two times or three times respectively
Dividing a multiple of 10 by:	
5	Divide by 10, then double
10	Delete 0 from the end of the number
100	Delete 00 from the end of the number

Example 7

Evaluate each quotient.

a $520 \div 8$

b $400 \div 100$

c $260 \div 5$

d $316 \div 4$

Solution

$$\begin{aligned} \text{a } 520 \div 8 &= 520 \div 2 \div 2 \div 2 \\ &= 260 \div 2 \div 2 \\ &= 130 \div 2 \\ &= 65 \end{aligned}$$

Halve 3 times

$$\begin{aligned} 520 \div 2 &= 500 \div 2 + 20 \div 2 = 250 + 10 \\ &= 260 \\ 260 \div 2 &= 200 \div 2 + 60 \div 2 = 100 + 30 \\ &= 130 \\ 130 \div 2 &= 100 \div 2 + 30 \div 2 = 50 + 15 = 65 \\ \text{Estimate: } 520 \div 8 &\approx 520 \div 10 = 52 \end{aligned}$$

b $400 \div 100 = 4 \div 1 = 4$

Delete two 0s from the end of the number

c $260 \div 5 = 260 \div 10 \times 2$
 $= 26 \times 2$
 $= 52$

Divide by 10, then double

Estimate: $260 \div 5 \approx 300 \div 5 = 60$

d $316 \div 4 = 316 \div 2 \div 2$
 $= 158 \div 2$
 $= 79$

Halve 2 times

$316 \div 2 = 300 \div 2 + 16 \div 2 = 150 + 8$
 $= 158$

$158 \div 2 = 150 \div 2 + 8 \div 2 = 75 + 4 = 79$

Estimate: $316 \div 4 \approx 320 \div 4 = 80$

Exercise 3-03 Dividing numbers

1 Evaluate each quotient.

a $44 \div 4$

b $68 \div 2$

c $84 \div 7$

d $210 \div 3$

e $105 \div 7$

f $390 \div 5$

g $441 \div 3$

h $861 \div 7$

i $2712 \div 6$

j $1116 \div 9$

k $1980 \div 4$

l $3728 \div 8$

2 Evaluate each quotient, expressing the remainder as a fraction.

a $49 \div 4$

b $688 \div 3$

c $952 \div 6$

d $1815 \div 7$

3 At Westvale Catholic College, there are 135 students in Year 7. If they are placed evenly into five classes, how many students are in each class? Select the correct answer **A**, **B**, **C** or **D**.

A 21

B 25

C 27

D 29

4 Divide a restaurant bill of \$204 evenly among six people.

5 Evaluate each quotient mentally.

a $320 \div 10$

b $8000 \div 100$

c $60\,000 \div 100$

d $1800 \div 10$

e $410 \div 5$

f $90 \div 5$

g $230 \div 5$

h $600 \div 5$

6 Evaluate each quotient mentally.

a $144 \div 2$

b $232 \div 2$

c $648 \div 2$

d $970 \div 2$

e $216 \div 4$

f $488 \div 4$

g $872 \div 8$

h $208 \div 8$

See Example 6

See Example 7

3-04 Divisibility tests

Is 2016 divisible by 3? How do you know?

It is often useful to know whether a number is divisible by another number. Numbers that divide evenly (without a remainder) into a bigger number are called the **factors** or **divisors** of the number. **Divisibility tests** are rules for finding whether a number is divisible by specific values.

Worksheet

Divisibility tests

MAT07NAWK10021

Summary

Divisible by:	Test
2	Last digit 0, 2, 4, 6 or 8
3	Sum of digits divisible by 3
4	Last 2 digits form a number divisible by 4
	Or: Sum of double the tens digit and the units digit divisible by 4
5	Last digit 0 or 5
6	Divisible by 2 and 3
8	Last 3 digits form a number divisible by 8
9	Sum of digits divisible by 9
10	Last digit 0

Example 8

Test whether 2016 is divisible by:

a 3

b 4

c 6

d 8

Solution

- a** Sum of digits = $2 + 0 + 1 + 6 = 9$,
which is divisible by 3.

So 2016 is divisible by 3.

(Check: $2016 \div 3 = 672$)

- b** The last 2 digits of 2016 are 16.

16 is divisible by 4.

So 2016 is divisible by 4.

OR: Sum of double the tens digit and
the units digit = $2 \times 1 + 6 = 8$.

8 is divisible by 4.

So 2016 is divisible by 4.

(Check: $2016 \div 4 = 504$)

- c** 2016 ends in 6, so it is divisible by 2 (even).

2016 is divisible by 3 (from part **a**).

So 2016 is divisible by 6 (because it is
divisible by 2 and 3).

(Check: $2016 \div 6 = 336$)

- d** The last 3 digits of 2016 are $016 = 16$.

16 is divisible by 8.

So 2016 is divisible by 8.

(Check: $2016 \div 8 = 252$)

Skillsheet

Factors and divisibility

MAT07NASS10014

Video tutorial

Divisibility tests

MAT07NAVT10005

Note:

- If a number is divisible by 6, it must also be divisible by 2 and 3 (because $2 \times 3 = 6$)
- If a number is divisible by 10, it must also be divisible by 2 and 5 (because $2 \times 5 = 10$)
- If a number is divisible by 4, it must also be divisible by 2 (because $2 \times 2 = 4$)
- If a number is divisible by 9, it must also be divisible by 3 (because $3 \times 3 = 9$)
- If a number is divisible by 8, it must also be divisible by 2 and 4 (because $2 \times 4 = 8$)
- If a number is divisible by another whole number, it must also be divisible by the **factors** of that whole number

Exercise 3-04 Divisibility tests

See Example 8

1 Test whether each number is divisible by 3.

a 140	b 612	c 315	d 928
e 209	f 525	g 132	h 1652

2 Test whether each number is divisible by 2.

a 117	b 205	c 6196	d 340

3 Test whether each number is divisible by 6. Note that the numbers in parts **a** to **d** have already been tested for divisibility by 3 in question 1.

a 140	b 612	c 315	d 928
e 475	f 303	g 864	h 1278

4 Test whether each number is divisible by 4.

a 2040	b 518	c 365	d 242
e 356	f 728	g 4176	h 817

5 Test whether each number is divisible by 9.

a 812	b 309	c 567	d 243
e 837	f 462	g 6444	h 3111

Worked solutions

Exercise 3-04

MAT07NAWS10014

6 Test whether 1964 is divisible by 8.

7 Explain by just looking at the last digit why:

a 409 is not divisible by 4	b 316 is not divisible by 5
c 2015 is not divisible by 8	d 343 is not divisible by 6
e 472 is not divisible by 10	f 511 is not divisible by 12

8 Which number is divisible by both 4 and 5? Select the correct answer **A**, **B**, **C** or **D**.

A 10	B 15	C 20	D 25

9 If a number is divisible by 3 and 4, what other number must it also be divisible by?

10 Write a number between 50 and 100 which is divisible by:

a 3 and 5	b 4 and 5	c 6 and 7	d 2 and 6

Extension: Harder divisibility tests

11 There are several complicated divisibility tests for 7. Here is the most popular one:

- Remove the last digit of the number and double it
- Subtract it from the rest of the number
- If the difference is 0 or divisible by 7, the number is divisible by 7

For example, to test 112:

Last digit = 2, double 2 = 4

$11 - 4 = 7$, which is divisible by 7

So 112 is divisible by 7.

Test whether each number is divisible by 7.

a 357

b 1013

c 6258

d 901

12 Here is a divisibility test for 11.

- Add the digits in the odd places of the number
- Add the digits in the even places of the number
- Subtract the two sums
- If the difference is 0 or divisible by 11, the number is divisible by 11

For example, to test 2016:

Add digits in odd places: $2 + 1 = 3$

Add digits in even places: $0 + 6 = 6$

$6 - 3 = 3$, which is not divisible by 11

So 2016 is not divisible by 11.

Test whether each number is divisible by 11.

a 814

b 2728

c 4051

d 6105

Mental skills 3A Maths without calculators

Multiplying by a multiple of 10

Place value allows us to simply add zeros to the end of a number whenever we multiply by a power of 10.

1 Consider these examples.

a $37 \times 10 = 370$

c $16 \times 1000 = 16\,000$

e $7 \times 90 = 7 \times 9 \times 10 = 63 \times 10 = 630$

f $5 \times 400 = 5 \times 4 \times 100 = 20 \times 100 = 2000$

g $12 \times 300 = 12 \times 3 \times 100 = 36 \times 100 = 3600$

h $40 \times 800 = 4 \times 10 \times 8 \times 100 = 4 \times 8 \times 10 \times 100 = 32 \times 1000 = 32\,000$

b $45 \times 100 = 4500$

d $100 \times 1000 = 100\,000$

2 Now evaluate each product.

a 18×100

b 26×1000

c $77 \times 10\,000$

d 10×100

e 315×1000

f 1000×1000

g 294×10

h 475×100

i 3×80

j 8×200

k 6×50

l 7×30

m 2×6000

n 11×900

o 4×400

p 5×700

q 5×80

r 25×20

s 300×60

t 900×4000

3-05 Long division

NSW

Long division is a technique for dividing by number with two or more digits, that is, a number greater than 10.

Worksheet

Four operations

MAT07NAWK00020

Example 9

Evaluate each quotient.

a $312 \div 12$

b $296 \div 21$

Solution

a
$$\begin{array}{r} 26 \\ 12 \overline{)312} \\ \underline{-24} \\ 72 \\ \underline{-72} \\ 0 \end{array}$$

12 into 31 is 2, remainder 7

12 into 72 is 6

Or

$$\begin{array}{r} 26 \\ 12 \overline{)312} \\ \underline{-120} \text{ 10 times} \\ 192 \\ \underline{-120} \text{ 10 times} \\ 72 \\ \underline{-72} \text{ 6 times} \\ 0 \text{ 26 times} \end{array}$$

Guessing with 'easy' multiples of 12

$312 \div 12 = 26.$

Check by estimating: $312 \div 12 \approx 300 \div 10 = 30$

$$\begin{array}{r} \text{b} \quad 14 \\ 21 \overline{)296} \\ \underline{-21} \downarrow \\ 86 \\ \underline{-84} \\ 2 \end{array}$$

Or

$$\begin{array}{r} 14 \\ 21 \overline{)296} \\ \underline{-210} \quad 10 \text{ times} \\ 86 \\ \underline{-84} \quad 4 \text{ times} \\ 2 \quad 14 \text{ times} \end{array}$$

$$296 \div 21 = 14\frac{2}{21}$$

21 into 29 is 1, remainder 8

21 into 86 is 4, remainder 2

Write the remainder as the numerator of a fraction

Check by estimating: $296 \div 21 \approx 300 \div 20 = 15$

Exercise 3-05 Long division

See Example 9

- 1 Use long division to evaluate each quotient.

a $180 \div 15$

b $462 \div 22$

c $731 \div 17$

d $275 \div 11$

e $2352 \div 16$

f $78 \div 13$

g $900 \div 25$

h $667 \div 23$

i $6848 \div 32$

- 2 Evaluate each quotient, writing the remainder as a fraction.

a $304 \div 12$

b $505 \div 14$

c $990 \div 26$

- 3 At a party 275 lollies are shared equally among 25 children. How many lollies does each child get?
- 4 A piece of wood 390 cm in length is to be cut into 15 equal pieces. How long is each piece?
- 5 Mrs Kaur needs \$1550 to purchase a new LED television. If she can save \$62 each week, how long will it take her to save enough money to purchase the television?

3-06 Roman numerals

NSW

In 500 BCE, most of Europe was ruled by the Roman Empire, so Roman numerals were widely used until the end of the 16th century. Roman numerals use letters of the alphabet and are still often used today.



The ancient Romans used the following numerals:

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

20	30	40	50	60	90	100	400	500	1000
XX	XXX	XL	L	LX	XC	C	CD	D	M

The Romans had an unusual method of writing numbers involving 4 or 9, to avoid writing the same letter more than 3 times in a row. A smaller number written before a larger number meant a subtraction ('minus').

- Instead of writing 4 as IIII, they wrote **IV** meaning $V - I$ (that is $5 - 1 = 4$).
- Instead of writing 9 as VIIII they wrote **IX** meaning $X - I$ (that is $10 - 1 = 9$).
- For 40, they wrote **XL** (that is $50 - 10 = 40$).
- For 90, they wrote **XC** (that is $100 - 10 = 90$).

Example 10

Write each number in Roman numerals.

a 23

b 46

c 279

Solution

$$\begin{aligned}\mathbf{a} \quad 23 &= 20 + 3 \\ &= XX + III \\ &= XXIII\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 46 &= 40 + 6 \\ &= XL + VI \\ &= XLVI\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 279 &= 200 + 70 + 9 \\ &= CC + LXX + IX \\ &= CCLXXIX\end{aligned}$$

Puzzle sheet

Roman numerals dominoes

MAT07NAPS10010

Skillsheet

Roman numerals

MAT07NASS10015

Weblink

Number systems

Example 11

Write each Roman numeral as a number.

a LXXIII

b CI

c CDXXVIII

Solution

$$\begin{aligned} \text{a } \text{LXXIII} &= \text{LXX} + \text{III} \\ &= 70 + 3 \\ &= 73 \end{aligned}$$

$$\begin{aligned} \text{b } \text{CI} &= \text{C} + \text{I} \\ &= 100 + 1 \\ &= 101 \end{aligned}$$

$$\begin{aligned} \text{c } \text{CDXXVIII} &= \text{CD} + \text{XX} + \text{VIII} \\ &= 400 + 20 + 8 \\ &= 428 \end{aligned}$$

Exercise 3-06 Roman numerals**See Example 10**

- 1 Write each number using Roman numerals.

a 27

b 52

c 79

d 93

e 2600

f 314

g 928

h 3476

i 165

j 230

k 2482

l 109

See Example 11

- 2 Titus, a student in ancient Rome, wrote these numerals. Rewrite them as numbers.

a XXVI

b XL

c CCLXIV

d LIV

e LXXVII

f XLV

g CCCXXXIX

h DXXVIII

i MMCLXII

j MCMXC

k XCVIII

l MDVII

- 3 a Where do you see Roman numerals being used today?
b Why do you think Roman numerals are no longer widely used?
- 4 The Roman word for hundred was 'centum' which is why C stands for 100. List some words beginning with 'cent' that mean one hundred of something.
- 5 a Write this year in Roman numerals.
b Which year of the 21st century will use the most letters when written in Roman numerals?
- 6 Find out what the Roman numerals $\overline{\text{V}}$ and $\overline{\text{X}}$ mean.

Investigation: Aboriginal numbers

Traditionally, indigenous Australians had no need for a complicated number system in their everyday life. Aboriginal society relied on story-telling, using the spoken language rather than writing, and Aboriginal people did not have symbols for numbers. Different regions had their own names for numbers.

The **Belyando River people** of central Queensland used only two words to name their numbers:

1 = wogin

2 = booleroo

3 = booleroo wogin

4 = booleroo booleroo

The **Kamilaroi people** lived in northern New South Wales, including the regions surrounding Moree and Tamworth. They used three words to name their numbers.

1 = mal	2 = bularr	3 = guliba
4 = bularr bularr	5 = bularr guliba	6 = guliba guliba

- How did the Belyando River people form words for the numbers 3 and 4?
- How did the Kamilaroi people form words for 4, 5 and 6?
- Write the answer to each expression, using the correct Aboriginal words:

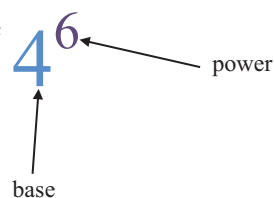
a wogin + booleroo wogin	b guliba $\times$ bularr
c bularr + bularr + mal	d booleroo $\times$ booleroo
e guliba guliba – guliba	f bularr bularr – mal
- State one advantage and one disadvantage of working with Aboriginal numbers.

3-07 Powers and index notation

The use of powers allows us to write repeated multiplication in a shorter way.

$4^2 = 4 \times 4$	'4 squared' or '4 to the power of 2'
$4^3 = 4 \times 4 \times 4$	'4 cubed' or '4 to the power of 3'
$4^4 = 4 \times 4 \times 4 \times 4$	'4 to the power of 4'
$4^5 = 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 5'
$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 6'

In 4^6 , the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.



Example 12

Write each expression using index notation.

a $6 \times 6 \times 6 \times 6 \times 6$

b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2)$

Solution

a $6 \times 6 \times 6 \times 6 \times 6 = 6^5$

b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2) = 7^3 \times (-2)^4$

Example 13

Evaluate each expression.

a 11^2

b $(-3)^3$

c 2^6

Solution

a $11^2 = 11 \times 11$
 $= 121$

b $(-3)^3 = (-3) \times (-3) \times (-3)$
 $= -27$

c $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
 $= 64$

Exercise 3-07 Powers and index notation**See Example 12** 1 Write each expression using index notation.

a $8 \times 8 \times 8 \times 8$

b $7 \times 7 \times 7 \times 7 \times 7 \times 7$

c $-5 \times (-5) \times (-5)$

d $1 \times 1 \times 1 \times 1 \times 1$

e $13 \times 13 \times 13 \times 13$

f 10

g $(-4) \times (-4) \times 9 \times 9 \times 9$

h $2 \times 2 \times 12 \times 12$

i $3 \times 3 \times 11 \times 11 \times 11 \times 11$

j $15 \times (-6) \times (-6) \times 15$

k $2 \times 8 \times 2 \times 8 \times 2$

l $10 \times 10 \times 20$

2 Write each expression in expanded form (for example, $2^4 = 2 \times 2 \times 2 \times 2$).

a 3^5

b 5^2

c 7^4

d $(-6)^3$

e $2^4 \times 4^2$

f $12^3 \times 11^6$

See Example 13 3 Evaluate each expression.

a 4^2

b 5^3

c 2^5

d 10^6

e 3^5

f 7^4

g 1^3

h $(-2)^4$

i 3^3

j $(-1)^2$

k $(-9)^5$

l 4^6

4 Copy and complete this table of powers of 10.

Power of 10	Number	Name
10^1	10	ten
10^2	100	hundred
10^3	1000	
10^6	1 000 000	
10^9		billion
10^{12}	1 000 000 000 000	
10^{15}		quadrillion
	1 000 000 000 000 000 000 000	quintillion
10^{21}		sextillion
10^{24}		septillion
	1 000 000 000 000 000 000 000 000 000 000	octillion
10^{30}		nonillion
10^{33}	1 000 000 000 000 000 000 000 000 000 000 000	decillion

Worksheet

Big numbers

MAT07NAWK00019

Technology Powers

- 1 Set up a spreadsheet to calculate powers by first entering the information shown below.

	A	B	C	D	E
1	Number:	2	3	5	7
2	1				
3	2				
4	3				
5	4				
6	5				
7	6				
8	7				
9	8				
10	9				
11	10				
12	11				
13	12				

- 2 In cell B2, enter = **\$B\$1^A2** to calculate 2^1 , where ^ in a spreadsheet formula means 'to the power of'. We also use **\$B\$1** instead of **B1** because when we copy this formula, we want it to **always** refer to the **2** in cell B1, not another cell. This is called **absolute cell referencing**. We use this technique to maintain a particular value in a cell without changing it when writing a formula. However, we use **A2** instead of **\$A\$2** because when we copy this formula, we want it to refer to the different powers shown in column A to calculate 2^1 , 2^2 , 2^3 , and so on. This is called **relative cell referencing**.
- 3 Click on cell B2 and Fill Down to cell B13. Your spreadsheet will now show the first 12 powers of 2, up to $2^{12} = 4096$.
- 4 Enter similar formulas for cells C2, D2 and E2, and **Fill Down** for columns C, D and E to show the first 12 powers of 3, 5 and 7.
- 5 If a number is too long to show in a cell, it will either look something like **2.44E+08** or **#####**. For cells that show something like **2.44E+08**, right-click, select **Format cells**, **Number** and **0** for decimal places. **#####** means the column is not wide enough to show all digits of the number. You need to widen the column until you can see all digits.

Just for the record

Googol vs Google

The number 10^{100} , the **googol**, is 1 followed by one hundred zeros. The name 'googol' was created by the 9-year-old nephew of American mathematician Dr Edward Kasner. The number 10^{googol} , that is 1 followed by a googol zeros, is called the **googolplex**. The googol is a very big number but it is rarely used for practical purposes. Even the number of particles in the observable universe, estimated at being between 10^{72} and 10^{87} , is less than a googol!



The Internet search engine Google was named after the googol, to reflect the huge size of the World Wide Web. It was created in 1996 by two Stanford University students, Larry Page and Sergey Brin. They even named Google's global headquarters in California the Googleplex. Google is a powerful search engine because it can find information from over a trillion (a million million) web pages in less than 1 second.

How many googols are there in a googolplex?

3-08 Square root and cube root

Square and square root

Squaring a number means raising it to the **power of 2**.

For example, 'squaring 7' or 'the square of 7' means $7^2 = 7 \times 7 = 49$, read '7 squared'. Also, 49 is called a **square number** or **perfect square** because it is the square of a whole number.

It is called '7 squared' because it is the area of a square of side 7 units.

The opposite of squaring is the **square root** (symbol $\sqrt{\quad}$).

For example, $\sqrt{49} = 7$, read 'the square root of 49', because $7^2 = 7 \times 7 = 49$.

Area

$$= 7 \times 7$$

$$= 7^2$$

7

7

Worksheet

Powers and roots

MAT07NAWK10022

Puzzle sheet

Square root Snap

MAT07NAPS10011

Summary

The **square root** $\sqrt{\quad}$ of a number is the positive value which, when squared, gives that number.

Example 14

Evaluate:

a the square root of 64

b $\sqrt{9}$

c $\sqrt{25}$

Solution

a The square root of 64 = $\sqrt{64} = 8$

because $8^2 = 8 \times 8 = 64$

b $\sqrt{9} = 3$

because $3^2 = 3 \times 3 = 9$

c $\sqrt{25} = 5$

because $5^2 = 5 \times 5 = 25$

Skillsheet

Square roots and
cube roots

MAT07NASS10016

Homework sheet

Powers and
square root

MAT07NAHS10002

Cube and cube root

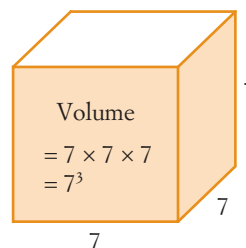
Cubing a number means raising it to the **power of 3**.

For example, 'cubing 7' or 'the cube of 7' means $7^3 = 7 \times 7 \times 7 = 343$, read '7 cubed'. Also, 343 is called a **cube number** or **perfect cube** because it is the cube of a whole number.

It is called '7 cubed' because it is the volume of a cube of side 7 units.

The opposite of cubing is the **cube root** (symbol $\sqrt[3]{}$).

For example, $\sqrt[3]{343} = 7$, read 'the cube root of 343', because $7^3 = 7 \times 7 \times 7 = 343$.



Summary

The **cube root** $\sqrt[3]{}$ of a number is the value which, when cubed, gives that number.

Example 15

Evaluate:

a the cube root of 125

b $\sqrt[3]{8}$

c $\sqrt[3]{729}$

Solution

a The cube root of 125 = $\sqrt[3]{125} = 5$

because $5^3 = 5 \times 5 \times 5 = 125$

b $\sqrt[3]{8} = 2$

because $2^3 = 2 \times 2 \times 2 = 8$

c $\sqrt[3]{729} = 9$

because $9^3 = 9 \times 9 \times 9 = 729$

Example 16

Estimate the value of $\sqrt{40}$.

Solution

There is no *exact* answer for the square root of 40, because there isn't a number which, if squared, equals 40 exactly. However, we can find a decimal whose square is close to 40.

Noting that:

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

we can tell that $\sqrt{40}$ must lie somewhere between 6 and 7, and closer to 6 because 40 is closer to 36 than 49.

So we can estimate that $\sqrt{40} \approx 6.3$.

(In fact, $6.3^2 = 39.69$).

Exercise 3-08 Square root and cube root

Extra questions

Index notation

MAT07NAEQ00011

- 1 Evaluate each square number.

a 9^2

b 17^2

c 10^2

- 2 The x^2 key on a calculator can be used to square a number. For example, to find 9^2 , press 9 x^2 =. Use your calculator to evaluate each square number.

a 7^2

b 13^2

c 18^2

d 1^2

e 5^2

f 20^2

g 16^2

h 34^2

i 15^2

- 3 Use your answers from question 2 to find:

a $\sqrt{400}$

b $\sqrt{225}$

c $\sqrt{324}$

d $\sqrt{256}$

e $\sqrt{169}$

f $\sqrt{1156}$

- 4 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2												

- 5 Which one of the following numbers is a square number? Select **A**, **B**, **C** or **D**.

A 32

B 36

C 40

D 45

- 6 Use your answers from the table in question 4 to find:

a the square root of 100

b $\sqrt{36}$

c $\sqrt{121}$

d $\sqrt{16}$

e $\sqrt{1}$

f the square root of 81

g $\sqrt{64}$

h the square root of 144

i $\sqrt{25}$

- 7 There are two numbers whose square root is itself, that is, $\sqrt{x} = x$. What are the two numbers?

See Example 14

- 8 The $\sqrt{\quad}$ key on a calculator can be used to find the square root of a number. For example, to find $\sqrt{64}$, press $\sqrt{\quad}$ 64 $=$. Use your calculator to find each square root.

a $\sqrt{49}$	b $\sqrt{81}$	c $\sqrt{625}$
d $\sqrt{484}$	e $\sqrt{1764}$	f $\sqrt{361}$
g $\sqrt{900}$	h $\sqrt{784}$	i $\sqrt{256}$
j $\sqrt{196}$	k $\sqrt{400}$	l $\sqrt{3136}$

- 9 Evaluate each cube number.

a 4^3	b 11^3	c 1^3

- 10 The x^3 key on a calculator can be used to cube a number. For example, to find 4^3 , press 4 x^3 $=$. Use your calculator to find each cube number.

a 7^3	b 16^3	c 5^3
d 10^3	e 20^3	f 17^3
g 35^3	h $(-8)^3$	i $(-5)^3$

- 11 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number cubed, x^3												

- 12 Which of the following numbers is a cube number? Select A, B, C or D.

A 192	B 512	C 625	D 800

- 13 Use your answers from question 11 to find:

a the cube root of 512	b $\sqrt[3]{729}$	c $\sqrt[3]{125}$
d $\sqrt[3]{1}$	e $\sqrt[3]{27}$	f the cube root of 1728
g $\sqrt[3]{64}$	h the cube root of 1331	i $\sqrt[3]{1000}$

See Example 15

- 14 Use your answers from question 10 to find:

a $\sqrt[3]{-512}$	b $\sqrt[3]{8000}$	c $\sqrt[3]{42\,875}$
d $\sqrt[3]{-125}$	e $\sqrt[3]{4913}$	f $\sqrt[3]{4096}$

- 15 The $\sqrt[3]{\quad}$ key on a calculator can be used to find the cube root of a number. For example, to find $\sqrt[3]{64}$, press $\sqrt[3]{\quad}$ 64 $=$. Use your calculator to find each cube root.

a $\sqrt[3]{4096}$	b $\sqrt[3]{2744}$	c $\sqrt[3]{10\,648}$
d $\sqrt[3]{64\,000}$	e $\sqrt[3]{19\,683}$	f $\sqrt[3]{9261}$

- 16 Use your answers from the table in question 4 to determine between which two consecutive whole numbers $\sqrt{27}$ must lie. Select the correct answer A, B, C or D.

A 26 and 28	B 13 and 14	C 4 and 5	D 5 and 6

See Example 16

- 17 Determine between which two consecutive whole numbers $\sqrt{15}$ must lie. Select the correct answer A, B, C or D.

A 7 and 8	B 3 and 4	C 196 and 197	D 4 and 5

18 Estimate the value of each root, then use your calculator to check.

a $\sqrt{50}$

b $\sqrt{32}$

c $\sqrt{96}$

d $\sqrt[3]{71}$

e $\sqrt[3]{900}$

f $\sqrt[3]{184}$

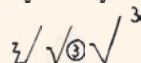
Just for the record

The radical symbol

The formal name for the square root symbol ($\sqrt{\quad}$) is the **radical symbol**. This symbol was created by the German mathematician Christoff Rudolff in 1525, but it was not until the end of the 17th century that the symbol was widely accepted. Before this, the most commonly-used symbol for square root was $\mathcal{R}_s$. It was first used in 1220 and was an abbreviation for the word **radix**, which means “root” in Latin. One theory says that Rudolff invented the symbol $\sqrt{\quad}$ to look like the letter ‘r’.

Other early symbols for square root were γ^l γ^s $\sqrt{\quad}$ $\sqrt{\quad}^3$ γ^e γ^t

The symbols on the right all represent the same thing.



What do you think they stand for?

Mental skills 3B

Maths without calculators

Dividing by a multiple of 10

Place value allows us to remove zeros from the end of a number when we divide by a power of 10.

1 Study each example.

a $2000 \div 10 = 2000 \div 10 = 200$

b $1800 \div 100 = 1800 \div 100 = 18$

c $37\,000 \div 100 = 37\,000 \div 100 = 370$

d $5\,000\,000 \div 1000 = 5\,000\,000 \div 1000 = 5000$

e $6000 \div 200 = 6000 \div 100 \div 2 = 60 \div 2 = 30$

f $350 \div 70 = 350 \div 10 \div 7 = 35 \div 7 = 5$

g $2800 \div 40 = 2800 \div 10 \div 4 = 280 \div 4 = 70$

h $40\,000 \div 5000 = 40\,000 \div 1000 \div 5 = 40 \div 5 = 8$

2 Now evaluate each quotient.

a $200 \div 10$

b $6000 \div 100$

c $45\,000 \div 100$

d $30\,000 \div 1000$

e $1900 \div 10$

f $2600 \div 100$

g $530 \div 10$

h $720\,000 \div 1000$

i $180 \div 30$

j $300 \div 50$

k $1600 \div 400$

l $45\,000 \div 5000$

m $4200 \div 60$

n $21\,000 \div 700$

o $44\,000 \div 2000$

p $1600 \div 200$

q $24\,000 \div 600$

r $15\,000 \div 3000$

s $64\,000 \div 80$

t $5400 \div 900$

3-09 Prime and composite numbers

The **factors** of a number are those whole numbers that divide exactly into it (without remainder).
For example:

- The factors of 16 are 1, 2, 4, 8 and 16
- The factors of 7 are 1 and 7

A number is **prime** if it has only **two** factors: 1 and itself. Some examples of prime numbers are 7, 3, 19, 2 and 11.

A number is **composite** if it has **more than two** factors. Some examples of composite numbers are 16, 10, 9, 15 and 4.

Worksheet

Sieve of Eratosthenes

MAT07NAWK10023

Summary

- A **prime number** has only two factors: 1 and itself
- A **composite number** has more than two factors

Note: The number 1 has only one factor so it is neither prime nor composite.

Skillsheet

Prime and composite numbers

MAT07NASS10017

Exercise 3-09 Prime and composite numbers

1 List all the factors of each number

a 17

b 21

c 24

d 11

e 35

f 4

g 18

h 23

i 25

j 9

k 3

l 19

2 List all the numbers from question 1 that are:

a prime

b composite

3 Which one of these numbers is not a factor of 45? Select the correct answer A, B, C or D.

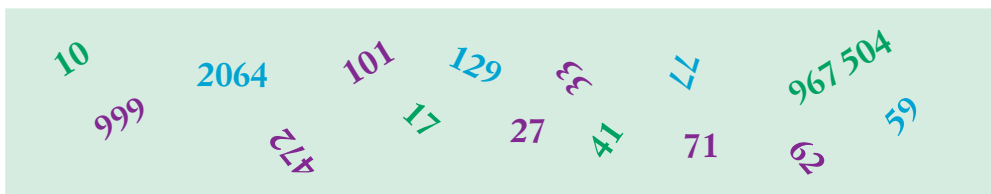
A 9

B 5

C 7

D 3

4 Sort the following numbers into primes and composites.



- 5 The ancient Greek mathematician, Eratosthenes, discovered an easy way to sort out the prime numbers from a list of numbers. It is called the Sieve of Eratosthenes (pronounced 'Siv of Era-tos-the-nees'), and works by crossing out multiples of numbers (the composite numbers).

a Copy the grid below for 1 to 120 or print out the Worksheet 'Sieve of Eratosthenes'.

Worksheet

Sieve of Eratosthenes

MAT07NAWK10023

TLF learning object

Sieve of Eratosthenes
(L3545)

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120

- b Cross out 1. It is neither prime nor composite.
- c Except for 2, colour all the multiples of 2 red and notice the pattern.
- d Except for 3, colour all the multiples of 3 green and notice the pattern.
- e Continue to colour multiples of other numbers with different colours, until there are no more multiples.
- f What do you notice about the 30 numbers left that are not coloured?
- 6 List all the:
- a prime numbers between 36 and 50 b composite numbers between 65 and 80
- c prime numbers less than 20 d composite numbers between 30 and 47
- 7 Which one of the following numbers is divisible only by prime numbers, itself and 1? Select A, B, C or D.
- A 12 B 14 C 16 D 18
- 8 Look up other meanings for the word 'composite'. Suggest why this word is used the way it is in mathematics.
- 9 a Find a number that has an odd number of factors.
- b What type of numbers have an odd number of factors?

Worked solutions

Exercise 3-09

MAT07NAWS10015

3-10 Prime factors

Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

Worksheet

Factor trees

MAT07NAWK10024

Video tutorial

Factor trees

MAT07NAVT10006

Skillsheet

Prime factors by
repeated division

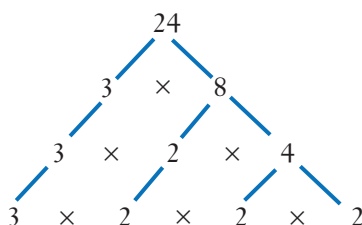
MAT07NASS10018

Example 17

Write 24 as a product of its prime factors.

Solution

Draw a factor tree for 24.

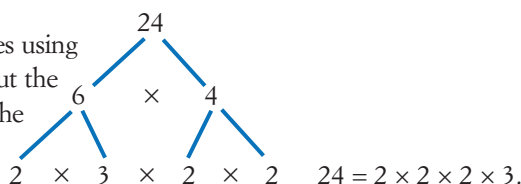


- 8 is composite, so write 8 as a product of 2 factors.
- 3 is prime, so keep writing 3 along the branches.

So 24 as a product of prime factors is:

$$24 = 2 \times 2 \times 2 \times 3.$$

Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 24:



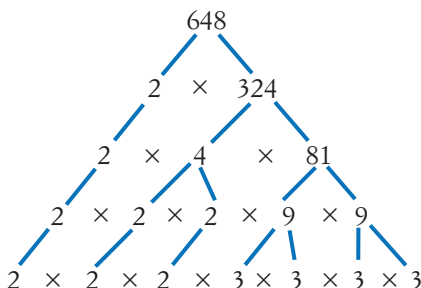
$$24 = 2 \times 2 \times 2 \times 3.$$

Example 18

Write 648 as a product of its prime factors, using index notation.

Solution

Draw a factor tree for 648.



- 2 is prime, so keep writing 2 along the branches.
- 324 is composite, so write 324 as a product of 2 factors, 4 and 81.
- Keep going until there are no composite factors left.

So 648 as a product of prime factors is:

$$\begin{aligned} 648 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= 2^3 \times 3^4 \end{aligned}$$

Finding square and cube roots using prime factors

Example 19

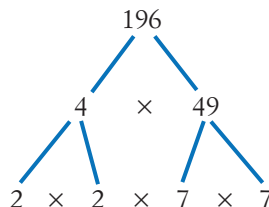
Use a factor tree to find the value of:

a $\sqrt{196}$

b $\sqrt[3]{216}$

Solution

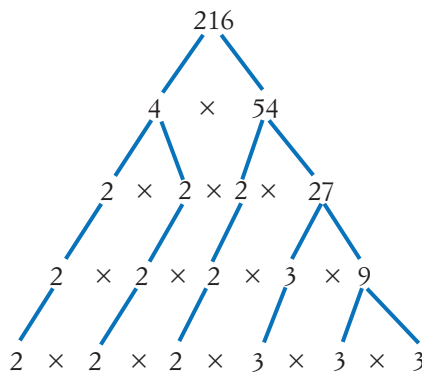
a Draw a factor tree for 196.



$$\begin{aligned}\text{So } 196 &= 2 \times 2 \times 7 \times 7 \\ \therefore \sqrt{196} &= \sqrt{2 \times 2 \times 7 \times 7} \\ &= 2 \times 7 \\ &= 14\end{aligned}$$

$$\sqrt{2 \times 2} = 2, \sqrt{7 \times 7} = 7$$

b Draw a factor tree for 216.



$$\begin{aligned}\text{So } 216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ \therefore \sqrt[3]{216} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} \\ &= 2 \times 3 \\ &= 6\end{aligned}$$

$$\sqrt[3]{2 \times 2 \times 2} = 2, \sqrt[3]{3 \times 3 \times 3} = 3$$

Exercise 3-10 Prime factors

See Example 17

1 Use a factor tree to write each number as a product of its prime factors.

a 88

b 63

c 45

d 51

e 132

f 270

g 396

h 218

i 630

j 520

k 275

l 342

2 What is 1260 expressed as a product of its prime factors? Select **A**, **B**, **C** or **D**.

A $2 \times 3 \times 3 \times 3 \times 5 \times 7$

B $2 \times 2 \times 2 \times 3 \times 5 \times 7$

C $2 \times 2 \times 3 \times 3 \times 3 \times 7$

D $2 \times 2 \times 3 \times 3 \times 5 \times 7$

3 Draw two different factor trees for 280 and show that both give the same prime factors.

- 4 Use a factor tree to write each number as a product of its prime factors, in index notation.
- a 48 b 200 c 460 d 712
e 98 f 144 g 325 h 135
- 5 484 is a perfect square.
- a Use a factor tree to write 484 as a product of its prime factors.
b Note that because 484 is a square number, the prime factors are in pairs. Use the factor tree to find $\sqrt{484}$.
- 6 Use a factor tree to evaluate each root.
- a $\sqrt{625}$ b $\sqrt{900}$ c $\sqrt{225}$ d $\sqrt{784}$
e $\sqrt[3]{9261}$ f $\sqrt[3]{3375}$ g $\sqrt[3]{1728}$ h $\sqrt[3]{4096}$

See Example 18

See Example 19

Worked solutions

Exercise 3-10

MAT07NAWS10016

Technology Spreadsheet formulas

Remember that a spreadsheet is like a calculator. We use special symbols to make calculations and a spreadsheet formula always begins with an equals (=) sign.

Formula	Meaning
=A1+A2+A3 or =sum(A1:A3)	add the values in cells A1, A2 and A3
=A5-A4	subtract the value in cell A4 from the value in cell A5
=A1*A3	multiply together the values in cells A1 and A3 (* is used instead of $\times$)
=A1/A2	divide the value in cell A1 by the value in cell A2 (/ is used instead of $\div$)
=A2^2	square the value in cell A2 (^ is used instead of $(A2)^2$)
=A5^3	cube the value in cell A5, that is, $(A5)^3$
=sqrt(A3)	find the square root of the value in cell A3, that is, $\sqrt{A3}$
=average(A1:A5)	find the average of all values from cells A1 to A5
=max(A1:A8)	find the maximum (highest) of all values from cells A1 to A8
=min(A1:A8)	find the minimum (lowest) of all values from cells A1 to A8

- 1 Open a new spreadsheet and enter the 5 numbers shown in column A. To enter the fraction $\frac{1}{2}$ in cell B4, right-click on B4 to select **Format Cells**, then choose **Fraction** and select **Up to three digits**.

	A	B
1	4	=SQRT(A1)
2	10	
3	-6	
4	$\frac{1}{2}$	
5	18	

- 2 In cell B1, enter the formula =sqrt(A1) to calculate the square root of the value in cell A1. You should get the answer 2, because $\sqrt{4} = 2$.
- 3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.
- a at B2, $A2 \div A1$ b at B3, $(A3)^2$
c at B4, $(A3)^3$ d at B5, $A1 \times A2 + A3 \times A4$

- e at B6, $A5 - A3$
 - g at B8, the product of A1 to A5
 - i at B10, $(A5)^3$
 - f at B7, the sum of A1 to A5
 - h at B9, $(A4)^2$
 - j at B11, $\sqrt{\frac{A5}{A4}}$
- 4 For the following cells in column C, enter the appropriate formula for each expression and check that the spreadsheet calculates the correct answer.
- a at C1, the average of column A
 - b at C2, the maximum value of column A
 - c at C3, the minimum value of column A
 - d at C4, $(A1 + A5)^2$
 - e at C5, the average of A1, A2 and A3
 - f at C6, $\sqrt{A2 - A3}$
- 5 Now enter **new** values in cells A1 to A5 (including a new fraction for A4) and notice the new answers calculated in columns B and C.
- 6 Create your own values and formulas. Write at least five formulas and use up to five different values. Enter the new values in column D and the new formulas in column E of your spreadsheet.

3-11 Greatest common divisor

Summary

The **greatest common divisor** (GCD) or **highest common factor** (HCF) of two (or more) numbers is the largest number that is a factor of **both** (or **all**) of these numbers.

← **Divisor** is just another name for **factor**.

Example 20

Find the greatest common divisor (GCD) of:

a 24 and 30

b 40 and 15

Solution

- a Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24
 Factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30
 The common factors are 1, 2, 3, 6
 The greatest common divisor is 6.
- b Factors of 40 = 1, 2, 4, 5, 8, 10, 20, 40
 Factors of 15 = 1, 3, 5, 15
 The greatest common divisor is 5.

Animated example

Highest common factors

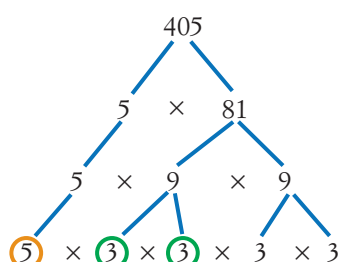
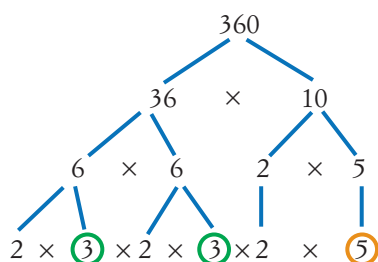
MAT07NAAE00012

Using prime factors to find the GCD

Example 21

Use factor trees to find the GCD of 360 and 405.

Solution



- Draw factor trees for 360 and 405
- Circle common prime factors: 3, 3 and 5
- Multiply 3, 3 and 5 to calculate the GCD

$$\text{GCD} = 3 \times 3 \times 5$$

$$= 45$$

45 is the highest factor of both 360 and 405

Summary

To find the greatest common divisor of two numbers using their prime factors:

- 1 Circle common prime factors
- 2 Multiply them together

Exercise 3-11 Greatest common divisor

- 1 Which one of these numbers is **not** a factor of 36? Select **A**, **B**, **C** or **D**.
A 4 **B** 6 **C** 8 **D** 9
- 2 Find all common factors for each pair of numbers.
a 27, 45 **b** 60, 32 **c** 75, 45 **d** 18, 28
- 3 Find the greatest common divisor of each pair of numbers.
a 32, 28 **b** 9, 6 **c** 6, 14 **d** 8, 12
e 14, 70 **f** 44, 64 **g** 10, 15 **h** 12, 6
i 75, 125 **j** 56, 40 **k** 60, 90 **l** 39, 26
m 132, 60 **n** 36, 84 **o** 27, 63 **p** 350, 210
- 4 On a spreadsheet, the greatest common divisor can be found using the function **=gcd()**.
a Enter **=gcd(27, 45)** into a spreadsheet cell to find the GCD of 27 and 45.
b Check your answers to question 3 using a spreadsheet.

See Example 17

See Example 18 5 Use factor trees to find the greatest common divisor of each pair of numbers.

a 324, 486

b 144, 60

c 35, 210

d 1404, 1900

Investigation: Common factor puzzle

- 1 Copy this grid, and try to move from the 200 square in the top left corner to the 100 square at the bottom right corner. You can move one step horizontally or vertically (but not diagonally), but only if both numbers have a common factor (not 1). For example, you can move from 65 to 91 (common factor 13), but not to 96 (no common factor).

200	80	65	91	143	156	195
175	32	96	71	110	77	121
35	28	15	209	87	90	21
39	169	117	95	57	37	81
63	11	29	72	76	75	51
14	98	56	132	48	78	85
105	45	44	187	112	221	100

- 2 Now try to move from the 105 square in the bottom left corner to the 195 square at the top right corner, using the same rules.
- 3 Choose different starting and finishing positions. Do they all have connecting factor paths?

Investigation: GCD by repeated division

The greatest common divisor can be found by repeated division with prime factors 2, 3, 5, 7, and so on. For example, to find the GCD of 24 and 60:

- both numbers are even so divide by 2
- both numbers are even so divide by 2 again
- both numbers are divisible by 3 so divide by 3
- Since it is not possible to divide any more, stop.
- Multiply the common prime factors to find the GCD.
- $\text{GCD of 24 and 60} = 2 \times 2 \times 3 = 12$

$$\begin{array}{r} \rightarrow 2 \overline{)24} \quad 2 \overline{)60} \\ \rightarrow 2 \overline{)12} \quad 2 \overline{)30} \\ \rightarrow 3 \overline{)6} \quad 3 \overline{)15} \\ \quad \quad 2 \quad \quad 5 \end{array}$$

Use this method to find the GCD of each set of numbers.

a 45, 120

b 18, 48

c 20, 28

d 96, 144

e 16, 48

f 15, 21, 45

Homework sheet

Prime and composite numbers

MAT07NAHS10003

3-12 Lowest common multiple

- The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, ...
- The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, ...

The **multiples** of a number are found by multiplying that number by the whole numbers 1, 2, 3, 4, ...

Homework sheet

Whole numbers review

MAT07NAHS10004

Summary

The **lowest common multiple** (LCM) of two (or more) numbers is the smallest number that is a multiple of **both** (or **all**) of these numbers.

← also called the **least** common multiple

Example 22

Find the lowest common multiple (LCM) of:

a 3 and 4

b 6 and 9

Solution

a Multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, ...

Multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24**, ...

The common multiples are **12, 24**, ...

The lowest common multiple of 3 and 4 is **12**.

b Multiples of 6 are 6, 12, **18**, 24, 30, **36**, 42, 48, **54**, ...

Multiples of 9 are 9, **18**, 27, **36**, 45, **54**, 63, 72, 81, ...

The lowest common multiple of 6 and 9 is **18**.

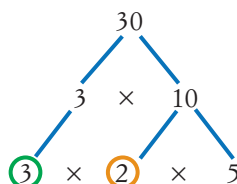
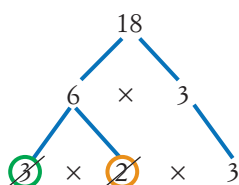
Using prime factors to find the LCM

Example 23

Use factor trees to find the LCM of 18 and 30.

Solution

- Draw factor trees for 18 and 30.



- Circle common prime factors: 3 and 2.
- Cross out one of the 3s and one of the 2s.
- Multiply the remaining factors to calculate the LCM.
- LCM of 18 and 30 = $3 \times 3 \times 2 \times 5$

$$= 90$$

Summary

To find the lowest common multiple of two numbers using their prime factors:

- 1 Circle common prime factors
- 2 Cross out **one** of each pair of common factors
- 3 Multiply the remaining factors

Exercise 3-12 Lowest common multiple

Extra questions

Special numbers

MAT07NAEQ00014

See Example 19

- 1 Write the first 5 multiples of 7.
- 2 List the multiples of 4 between 13 and 45.
- 3 How many multiples of 13 are less than 220?
- 4 Find the lowest common multiple of each set of numbers.

a 3, 5	b 6, 7	c 4, 6	d 15, 10
e 5, 8	f 4, 10	g 10, 5	h 2, 8
i 9, 6	j 3, 7	i 3, 4, 5	j 4, 12, 10
- 5 Why are some lowest common multiples easier to find? Write down your strategy for finding LCMs.
- 6 On a spreadsheet, the lowest common multiple can be found using the function `=lcm()`.
 - a Enter `=lcm(8, 12)` into a spreadsheet cell to find the LCM of 8 and 12.
 - b Check your answers to question 4 using a spreadsheet.
- 7 Use factor trees to find the lowest common multiple of each pair of numbers.

a 28, 15	b 16, 25	c 44, 20	d 18, 60

See Example 20

Technology Lowest common multiple

This activity searches for the lowest common multiple of 8, 12 and 16 by listing their multiples first. Start a new spreadsheet and enter the information shown below.

	A	B	C	D
1	Number:	8	12	16
2	1			
3	2			
4	3			
5	4			
6	5			
7	6			
8	7			
9	8			
10	9			
11	10			
12	11			
13	12			
14	13			
15	14			
16	15			

- 1 In cell B2, enter the formula $=\$B\$1*A2$. Use **Fill Down** to find the first 12 multiples of 8.
- 2 In cells C2 and D2, enter similar formulas and **Fill Down** to find the first 12 multiples of 12 and 16. [Hint: Only change the absolute cell reference.]
- 3 By examining the columns of multiples, find the lowest common multiple of 8, 12 and 16.
- 4 By using the formula $=\text{lcm}(8, 12, 16)$, find the lowest common multiple of 8, 12 and 16.
- 5 Modify your spreadsheet to find the LCM of each of the following sets of numbers. Note: You may need to extend beyond the first 12 multiples.

a 6 and 15	b 12 and 18	c 3, 7 and 15
		d 48, 60 and 75

Power plus

- 1 For each set of numbers, evaluate and write in ascending order.

a $2^3, 3^2, 3^5, 5^3, 2^5, 5^2$	b $4^4, 7^3, 3^5, 8^2, 5^2, 6^3$	c $100^2, 11^4, 2^7, 3^4, 5^3$
- 2 Evaluate each square number.

a 1^2	b 11^2	c 111^2	d 1111^2
- 3 Based on the patterns in your answers to question 2, evaluate each square number.

a $11\ 111^2$	b $1\ 111\ 111^2$	c $111\ 111\ 111^2$	d $1\ 111.1111^2$
- 4 Find the square root of:

a $3 \times 3 \times 2 \times 2$	b $5 \times 5 \times 4 \times 4 \times 3 \times 3$
- 5 Evaluate each of the following without using a calculator.

a $\sqrt{9^2}$	b $\sqrt{3^4}$	c $\sqrt{5^4 \times 2^6}$
d $\sqrt[3]{2^9}$	e $\sqrt[3]{3^3 \times 5^6}$	f $\sqrt[4]{16}$
- 6 Find the square root of each number without using a calculator.

a 2500	b 8100	c 10 000
d 1 000 000	e 1 210 000	f 100 000 000
g 640 000	h 176 400	i 10 000 000 000
- 7 Two prime numbers that differ by 2 are called twin primes. For example, 11 and 13 are twin primes, but 23 and 29 are not. Find all 8 pairs of twin primes below 100.

Chapter 3 review

Language of maths

Puzzle sheet

Number find-a-word

MAT07NAPS10012

composite	factor	lowest common	prime factor
cube	factor tree	multiple (LCM)	round down
divisibility test	greatest common	multiple	round up
divisor	divisor (GCD)	power	square
estimate	index notation	prime	square root

Quiz

Whole numbers

MAT07NAQZ00002

- What is another name for:
a divisor? **b** highest common factor?
- What is used as an abbreviation for repeated multiplication?
- What type of numbers have more than 2 factors?
- Describe the divisibility test for 3.
- Find a non-mathematical meaning of:
a prime **b** index
- What helps us to write a number as a product of its prime factors?

Topic overview

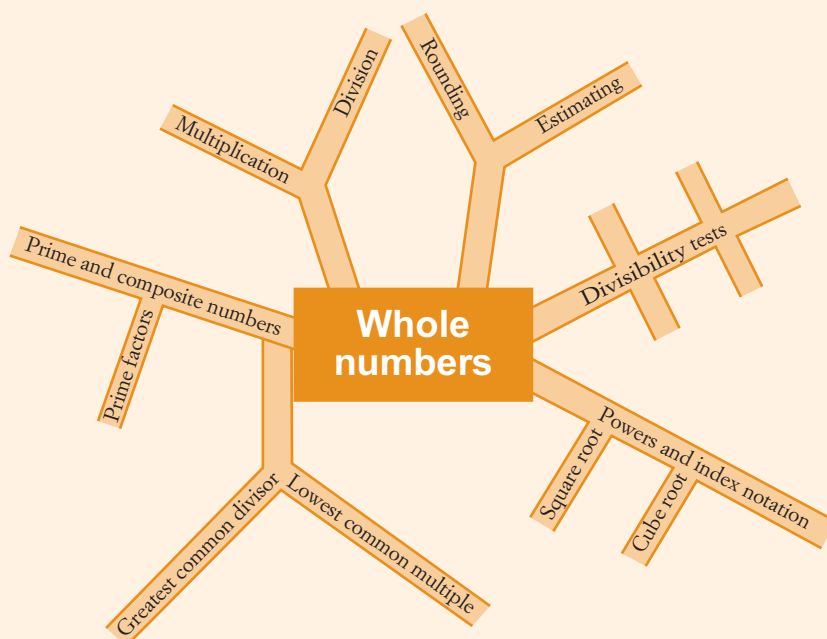
Worksheet

Mind map: Whole numbers

MAT07NAWK10025

- Write in your own words what you have learnt about whole numbers.
- What was your favourite part of this topic?
- What parts of this topic didn't you understand? Talk to your teacher or a friend about them.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Round 87 531 to the nearest:

a thousand	b hundred	c ten	d ten thousand

See Exercise 3-01
- 2 Estimate the value of each expression.

a $425 + 81 + 940 + 22$	b $864 + 250 - 417$
c 93×19	d $2402 \div 32$

See Exercise 3-01
- 3 Evaluate each product.

a 12×4	b 34×5	c 33×9
d 25×8	e 126×7	f 565×18

See Exercise 3-02
- 4 Evaluate each quotient.

a $2400 \div 10$	b $620 \div 5$	c $1296 \div 8$
d $528 \div 4$	e $144 \div 9$	f $847 \div 7$

See Exercise 3-03
- 5 Test whether 1208 is divisible by:

a 2	b 3	c 4	d 5
e 6	f 8	g 9	h 10

See Exercise 3-04
- 6 Which number is divisible by both 3 and 8? Select the correct answer **A, B, C** or **D**.

A 30	B 24	C 90	D 94

See Exercise 3-04
- 7 Evaluate each quotient.

a $3079 \div 12$	b $650 \div 25$	c $2976 \div 24$

See Exercise 3-05
- 8 Write each number using Roman numerals.

a 42	b 38	c 89	d 347

See Exercise 3-06
- 9 Write each expression using index notation.

a $3 \times 3 \times 3 \times 3$	b $6 \times 6 \times 6 \times 6 \times 6$
c $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$	

See Exercise 3-07
- 10 Evaluate each power.

a 6^2	b 5^3	c 10^5	d 2^8
- 11 Evaluate each root.

a $\sqrt{49}$	b $\sqrt{144}$	c $\sqrt[3]{8000}$	d $\sqrt[3]{729}$

See Exercise 3-08
- 12 a List the prime numbers between 26 and 50.
 b List the composite numbers between 45 and 70.
 See Exercise 3-09
- 13 Use factor trees to express each number as a product of its prime factors.

a 60	b 200	c 144

See Exercise 3-10
- 14 Use factor trees to evaluate each root.

a $\sqrt{256}$	b $\sqrt[3]{5832}$

See Exercise 3-10
- 15 Find the greatest common divisor of each pair of numbers.

a 20 and 48	b 36 and 84

See Exercise 3-11
- 16 Use factor trees to find the greatest common divisor of 243 and 162.
 See Exercise 3-11
- 17 Find the lowest common multiple of each pair of numbers.

a 6 and 8	b 18 and 10

See Exercise 3-12
- 18 Use factor trees to find the lowest common multiple of 25 and 16.
 See Exercise 3-12

Mixed revision 1

See Exercise 1-08

1 Evaluate each of the following.

a $15 - 3 \times 5$

b $7 \times 3 + 2 \times 5$

c $26 \div 2 - 14 \div 7$

d $7 + 5 \times (12 - 3)$

e $414 \div 9$

f $[(3 + 5) \times 2 - (20 \div 5)] \times 5$

See Exercise 1-03

2 Find the distance between each pair of numbers on a number line.

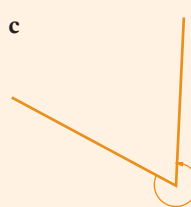
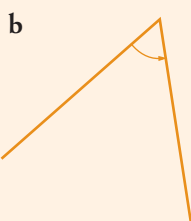
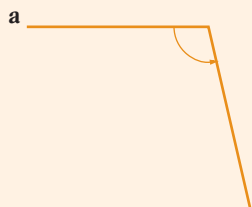
a -1 and -3

b -5 and 3

c 0 and -4

See Exercise 2-02

3 Measure each angle.



See Exercise 2-03

4 Draw an example of:

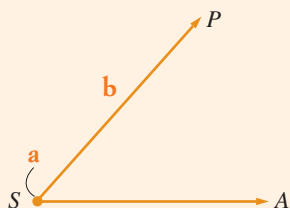
a an obtuse angle

b an acute angle

c a reflex angle

See Exercise 2-01

5 Name this angle using three letters and write the name of its parts.



See Exercise 2-04

6 Find the complement of:

a 66°

b 85°

c 12°

See Exercise 2-04

7 Find the supplement of:

a 32°

b 90°

c 105°

See Exercise 2-05

8 Draw two intersecting lines and mark in a pair of vertically opposite angles.

See Exercise 2-11

9 **a** Draw a pair of parallel lines and cut them with a transversal.

b Mark a pair of alternate angles with ($\bullet$).

c Mark a pair of corresponding angles with (*).

d Mark a pair of co-interior angles with (+).

See Exercise 3-04

10 Round these numbers to the nearest hundred:

a 2477

b 17 204

c 807 928

See Exercise 1-08

11 Simplify each expression.

a $3 - 5$

b $-2 + 8$

c $-1 - 6$

d $-3 + 5 - 8$

e $-5 \times (-3)$

f $6 \div (-2)$

g $-27 \div (-3)$

h -4×4

i $-3 \times 2 - 5$

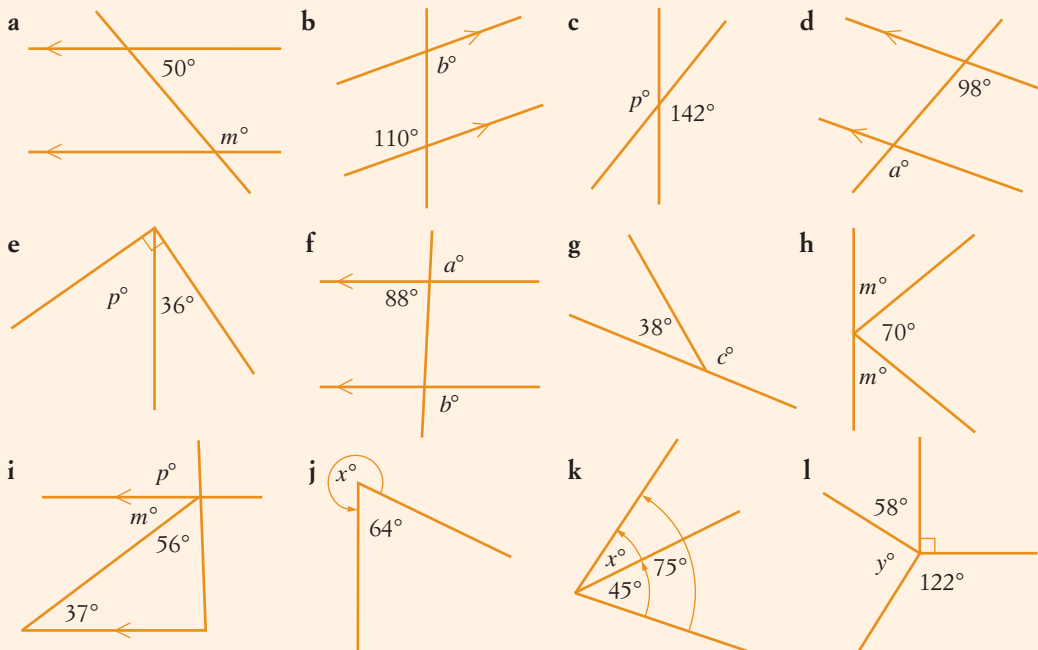
j $-2 - 3 \times 8$

k $12 - 14 \div 2 + 5$

l $5 - [2 + (-2 \times 6)]$

12 Find the value of each pronumeral.

See Exercise 2-11



13 Simplify each expression.

See Exercise 3-07

a 6^2

b 3^3

c $\sqrt{25}$

d $\sqrt[3]{1331}$

14 The average daily temperature in winter is 13°C and the average night temperature is -5°C . Find the difference between these two temperatures.

See Exercise 1-10

15 Find the factors of:

See Exercise 3-08

a 18

b 45

c 360

16 Use a factor tree to write each of the numbers in question 15 as a product of its prime factors. Write your answers using index notation.

See Exercise 3-09

17 Find the greatest common divisor for each pair of numbers:

See Exercise 3-10

a 24 and 36

b 45 and 81

18 Find the lowest common multiple for each pair of numbers:

See Exercise 3-11

a 7 and 11

b 6 and 21



4

Number and algebra

Fractions and percentages

The ancient Egyptians were one of the first civilisations to use fractions, but they only used **unit fractions** like $\frac{1}{2}$ and $\frac{1}{3}$, those with 1 as the numerator. However, they could write *any* fraction as a sum of unit fractions with different denominators, for example:

- $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$
- $\frac{3}{5} = \frac{1}{3} + \frac{1}{5} + \frac{1}{15}$

The word **fraction** comes from the Latin *fractio*, meaning 'to break'.



Chapter outline

	Proficiency strands			
4-01 Fractions	U	F		
4-02 Ordering fractions	U	F		
4-03 Adding and subtracting fractions	U	F		
4-04 Adding and subtracting mixed numerals	U	F		
4-05 Fraction of a quantity	U	F		
4-06 Multiplying fractions	U	F		R
4-07 Dividing fractions	U	F		R
4-08 Fraction problems		F	PS	
4-09 Fractions and the calculator		F	PS	
4-10 Percentages, fractions and decimals	U	F		
4-11 Percentage of a quantity	U	F		
4-12 Expressing quantities as fractions and percentages	U	F		

Wordbank

denominator The number at the bottom in a fraction

improper fraction A fraction such as $\frac{7}{3}$, in which the numerator is larger than or equal to the denominator

mixed numeral A number such as $3\frac{2}{5}$, made up of a whole number and a fraction

numerator The number at the top in a fraction

percentage A special type of fraction in which the denominator is 100

proper fraction A fraction such as $\frac{4}{10}$ in which the numerator is smaller than the denominator

reciprocal (of a fraction) The fraction 'turned upside down', for example, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$

simplify (a fraction) To reduce the numerator and the denominator of a fraction to their lowest form by dividing by the same factor

In this chapter you will:

Maths clip

Using fractions

MAT07NAVT00007

- compare fractions using equivalence
- locate and represent fractions and mixed numerals on a number line
- solve problems involving addition and subtracting of fractions, including those with unrelated denominators
- find a simple fraction of a quantity where the result is a whole number, with and without digital technologies
- multiply and divide fractions using efficient written strategies and digital technologies
- express one quantity as a fraction of another with and without the use of digital technologies
- connect fractions, decimals and percentages and carry out simple conversions
- find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies

SkillCheck

Worksheet

StartUp assignment 4

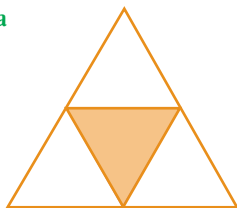
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1 For each diagram, state what fraction of the whole:

i is shaded

ii is unshaded

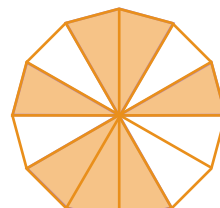
a



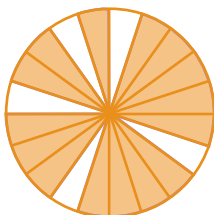
b



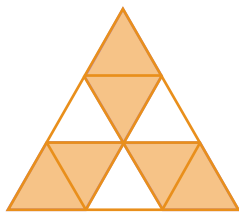
c



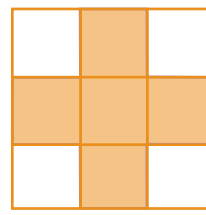
d



e



f



Worksheet

Fraction diagrams

MAT07NAWK10027

Skillsheet

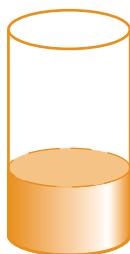
Fractions

MAT07NASS10019

2 What fraction of the month shown on this calendar has been crossed out?

JULY 2015						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

- 3 Draw a square and divide it into quarters.
- 4 Draw a rectangle and divide it into thirds.
- 5 Approximately how full is this glass?

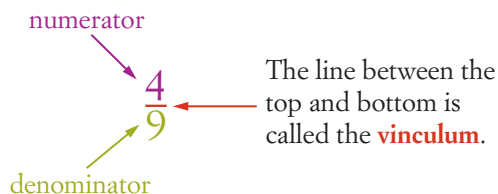


- 6 Find:
 - a $\frac{1}{2}$ of 24
 - b $\frac{1}{2}$ of 38
 - c $\frac{1}{3}$ of 21
 - d $\frac{1}{10}$ of 70
 - e $\frac{1}{4}$ of 20
 - f $\frac{1}{5}$ of 15
- 7 Draw a circle and divide it into eighths.
- 8 Find the greatest common divisor (GCD) of each pair of numbers.
 - a 15 and 18
 - b 9 and 21
 - c 16 and 40
- 9 Find the lowest common multiple (LCM) of each pair of numbers.
 - a 3 and 4
 - b 5 and 10
 - c 8 and 6
- 10 Draw any shape and divide it into fifths.
- 11 What fraction of one hour is:
 - a 13 minutes?
 - b 45 minutes?
 - d 10 minutes?
- 12 Which fraction is larger: $\frac{1}{3}$ or $\frac{1}{4}$?

4-01 Fractions

A fraction is written as $\frac{a}{b}$ where a and b are integers (but b cannot be zero). Its proper name is **common fraction** because decimals and percentages are also types of fractions.

- The top number (a) is the **numerator**
- The bottom number (b) is the **denominator**



Skillsheet

Fractions

MAT07NASS10019

TLF learning object

Exploring fractions
(L6542)

Summary

A **proper fraction** has a numerator that is less than the denominator, for example, $\frac{4}{9}, \frac{11}{13}, \frac{37}{60}, \frac{80}{100}$.

An **improper fraction** has a numerator that is more than or equal to the denominator, for example, $\frac{9}{4}, \frac{17}{3}, \frac{15}{15}, \frac{12}{10}$.

A **mixed numeral** is made up of a whole number and a proper fraction, for example, $4\frac{1}{2}, 5\frac{2}{3}, 3\frac{1}{7}, 4\frac{3}{10}$.

Worksheet

Classifying fractions

MAT07NAWK00049

Worksheet

Classifying fractions

MAT07NAWK10028

Skillsheet

Improper fractions and mixed numerals

MAT07NASS10020

Improper fractions and mixed numerals

An improper fraction is a 'top-heavy' fraction so its value is greater than 1, and it can be converted to a mixed numeral or whole number.

Summary

To convert an improper fraction to a mixed numeral, divide the numerator by the denominator and write the remainder as a proper fraction.

Example 1

Convert $\frac{38}{3}$ to a mixed numeral.

Solution

$$\frac{38}{3} = 38 \div 3$$

$$= 12 \text{ remainder } 2$$

$$= 12\frac{2}{3}$$

To find how many 'wholes' in 38 thirds, divide 38 by 3

Write the remainder in the numerator of a fraction

Example 2

Convert $5\frac{3}{7}$ to an improper fraction.

Solution

$$5\frac{3}{7} = 5 + \frac{3}{7}$$

$$= \frac{5 \times 7}{7} + \frac{3}{7}$$

$$= \frac{35}{7} + \frac{3}{7}$$

$$= \frac{38}{7}$$

Animated example

Improper fractions

MAT07NAAE00011

Puzzle sheet

Mixed numbers to improper fractions

MAT07NAPS00028

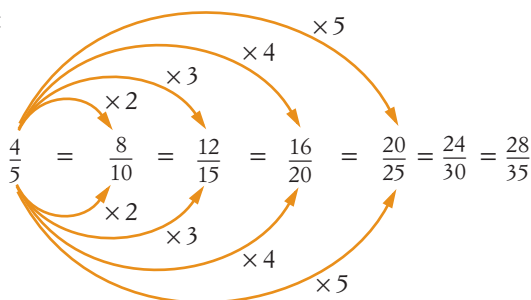
Summary

To convert a mixed numeral to an improper fraction, multiply the denominator by the whole number, then add the numerator and write the total as the new numerator of the fraction

$$5\frac{3}{7} = \frac{7 \times 5 + 3}{7} = \frac{38}{7}$$

Equivalent fractions

This diagram shows fractions that are equivalent (equal) to $\frac{4}{5}$.



Summary

Equivalent fractions are found by multiplying or dividing the numerator and the denominator by the same number.

Example 3

For each pair of equivalent fractions, find the missing number.

a $\frac{3}{4} = \frac{?}{20}$ b $\frac{5}{9} = \frac{35}{?}$ c $\frac{16}{24} = \frac{?}{6}$

Solution

- a To find the missing numerator, look at the two **denominators**, 4 and 20.

$$\frac{3}{4} = \frac{?}{20}$$

4 is multiplied by 5 to give 20, so do the same thing to the numerator 3.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

The missing number is 15.

- b To find the missing denominator, look at the **numerators** 5 and 35.

$$\frac{5}{9} = \frac{35}{?}$$

5 is multiplied by 7 to give 35, so do the same to the 9.

$$\frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}$$

The missing number is 63.

Worksheet

Fractions wall

MAT07NAWK10029

Puzzle sheet

Equivalent fractions

MAT07NAPS10013

Worksheet

Pop stick calculator

MAT07NAWK10030

Skillsheet

Equivalent fractions

MAT07NASS10021

Worksheet

Fraction strips

MAT07NAWK00050

- c To find the missing numerator, look at the **denominators** 24 and 6.

$$\frac{16}{24} = \frac{?}{6}$$

24 is **divided** by 4 to give 6, so do the same to 16.

$$\frac{16}{24} = \frac{16 \div 4}{24 \div 4} = \frac{4}{6}$$

The missing number is 4.

Simplifying fractions

We have seen the equivalent fractions $\frac{4}{5} = \frac{8}{10} = \frac{12}{15}$ described above. The most basic fraction, $\frac{4}{5}$, is called the **simplified fraction** or the fraction **reduced** to its **lowest form**. A simplified fraction has the smallest numerator and denominator possible.

Summary

- To **simplify a fraction**, keep dividing the numerator and denominator by the same number (preferably a large number such as their greatest common divisor [GCD])
- If the numerator and denominator are both even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

Example 4

Simplify each fraction.

a $\frac{20}{30}$ b $\frac{16}{48}$ c $\frac{36}{27}$

Solution

a $\frac{20}{30} = \frac{20 \div 10}{30 \div 10} = \frac{2}{3}$

b $\frac{16}{48} = \frac{16 \div 2}{48 \div 2} = \frac{8}{24} = \frac{8 \div 8}{24 \div 8} = \frac{1}{3}$ or $\frac{16}{48} = \frac{16 \div 16}{48 \div 16} = \frac{1}{3}$

c $\frac{36}{27} = \frac{36 \div 9}{27 \div 9} = \frac{4}{3} = 1\frac{1}{3}$

Exercise 4-01 Fractions

- 1 Classify each fraction as being proper (P), improper (I) or a mixed numeral (M).

a $1\frac{1}{2}$ b $\frac{3}{100}$ c $3\frac{2}{5}$ d $20\frac{1}{4}$ e $\frac{2}{3}$ f $\frac{100}{45}$
 g $\frac{150}{250}$ h $\frac{86}{87}$ i $\frac{3}{2}$ j $\frac{3}{5}$ k $4\frac{1}{20}$ l $\frac{1000}{25}$

- 2 Convert each improper fraction to a mixed numeral or whole number.

a $\frac{13}{3}$ b $\frac{13}{4}$ c $\frac{13}{10}$ d $\frac{21}{5}$ e $\frac{38}{35}$ f $\frac{35}{8}$
 g $\frac{30}{6}$ h $\frac{36}{5}$ i $\frac{28}{7}$ j $\frac{120}{11}$ k $\frac{72}{19}$ l $\frac{101}{10}$

Worksheet

Pop stick calculator

MAT07NAWK10030

Skillsheet

Simplifying fractions

MAT07NASS10022

Video tutorial

Simplifying fractions

MAT07NAVT10007

Puzzle sheet

Equivalent fractions 1

MAT07NAPS00029

Extra questions

Naming and comparing
common fractions

MAT07NAEQ00029

See Example 1

3 Convert each mixed numeral to an improper fraction.

See Example 2

- a $1\frac{1}{2}$ b $1\frac{5}{8}$ c $1\frac{2}{7}$ d $3\frac{1}{3}$
e $4\frac{1}{2}$ f $3\frac{7}{10}$ g $6\frac{1}{6}$ h $7\frac{3}{5}$

4 Which is larger: a proper fraction or a mixed numeral?

5 Complete each set of equivalent fractions.

See Example 3

- a $\frac{1}{4} = \frac{3}{?}$ b $\frac{5}{6} = \frac{?}{30}$ c $\frac{2}{3} = \frac{?}{12}$
d $\frac{7}{10} = \frac{?}{80}$ e $\frac{7}{8} = \frac{63}{?}$ f $\frac{3}{4} = \frac{?}{100}$
g $\frac{20}{24} = \frac{5}{?}$ h $\frac{15}{40} = \frac{?}{8}$ i $\frac{18}{32} = \frac{?}{16}$

6 Write three equivalent fractions for each fraction.

- a $\frac{3}{5}$ b $\frac{2}{3}$ c $\frac{1}{8}$

7 What is missing number in $\frac{12}{18} = \frac{?}{3}$? Select A, B, C or D.

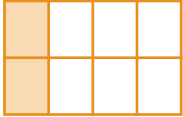
- A 6 B 1 C 5 D 2

8 Simplify each fraction.

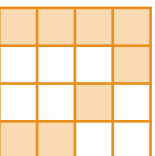
See Example 4

- a $\frac{6}{16}$ b $\frac{6}{24}$ c $\frac{40}{50}$ d $\frac{24}{32}$
e $\frac{18}{42}$ f $\frac{60}{75}$ g $\frac{5}{20}$ h $\frac{14}{21}$

9 What is the simplest form of each shaded fraction? For each part, select A, B, C or D.

- a  A $\frac{2}{8}$ B $\frac{6}{8}$ C $\frac{1}{4}$ D $\frac{1}{8}$

- b  A $\frac{1}{3}$ B $\frac{4}{6}$ C $\frac{3}{2}$ D $\frac{2}{3}$

- c  A $\frac{4}{8}$ B $\frac{2}{4}$ C $\frac{1}{2}$ D $\frac{8}{16}$

10 Reduce each improper fraction to its lowest form, converting them to mixed numerals.

- a $\frac{18}{8}$ b $\frac{30}{14}$ c $\frac{50}{25}$ d $\frac{90}{54}$
e $\frac{28}{12}$ f $\frac{75}{60}$ g $\frac{32}{24}$ h $\frac{48}{36}$

11 Write each fraction in simplest form, as a mixed numeral.

- a $1\frac{4}{8}$ b $3\frac{5}{30}$ c $4\frac{5}{20}$ d $6\frac{30}{40}$
e $17\frac{35}{100}$ f $100\frac{44}{64}$ g $1\frac{54}{100}$ h $5\frac{4}{12}$

12 Which fraction is not equivalent to $\frac{3}{4}$? Select the correct answer A, B, C or D.

- A $\frac{12}{16}$ B $\frac{6}{8}$ C $\frac{24}{32}$ D $\frac{10}{18}$

4-02 Ordering fractions

Worksheet

Fractions wall

MAT07NAWK10029

Worksheet

Pop stick calculator

MAT07NAWK10030

- Is $\frac{4}{10}$ larger than $\frac{3}{8}$?
- Is $1\frac{2}{3}$ smaller than $1\frac{5}{9}$?

The best way to **compare** fractions with different denominators is to convert them to equivalent fractions with the same denominator first. A common denominator can be found by multiplying the denominators of both fractions together or by using the lowest common multiple (LCM) of the denominators, also called the **lowest common denominator** (LCD).

Example 5

- a Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?
- b Which fraction is smaller, $1\frac{2}{3}$ or $1\frac{5}{9}$?

Solution

a Method 1

Convert both fractions so that they share a common denominator of $10 \times 8 = 80$.

$$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80} \text{ and } \frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$$

By comparing numerators, $\frac{32}{80} > \frac{30}{80}$.

$\therefore \frac{4}{10}$ is larger.

Note that we multiply the numerator and denominator of each fraction by the denominator of the **other** fraction

Method 2

Another suitable denominator is the lowest common denominator (LCD) of both denominators.

The LCD of 10 and 8 is 40.

Convert both fractions.

$$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40} \text{ and } \frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

By comparing numerators, $\frac{16}{40} > \frac{15}{40}$.

$\therefore \frac{4}{10}$ is larger.

- b Since both whole numbers are 1, we only compare the fraction parts.

Method 1

Common denominator = $3 \times 9 = 27$.

Convert both fractions so that they share a common denominator.

$$\frac{2}{3} = \frac{2 \times 9}{3 \times 9} = \frac{18}{27} \text{ and } \frac{5}{9} = \frac{5 \times 3}{9 \times 3} = \frac{15}{27}$$

By comparing numerators, $\frac{15}{27} < \frac{18}{27}$.

$\therefore 1\frac{5}{9}$ is smaller.

Homework sheet

Fractions 1

MAT07NAHS10007

TLF learning object

Fractions: Comparing (L3521)

Method 2

The LCD of 3 and 9 is 9.

We only need to convert the first fraction because the second fraction already has 9 as a denominator.

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9} \text{ and } \frac{5}{9}$$

By comparing numerators, $\frac{5}{9} < \frac{6}{9}$.

$\therefore 1\frac{5}{9}$ is smaller.

Example 6

Draw a number line and mark in the fractions $\frac{1}{5}$, $\frac{2}{3}$ and $1\frac{1}{2}$.

Solution

- Divide the interval between 0 and 1 into fifths and mark $\frac{1}{5}$
- Divide the interval between 0 and 1 into thirds and mark $\frac{2}{3}$
- Divide the interval between 1 and 2 into halves and mark $1\frac{1}{2}$



Weblink

Fraction darts

Exercise 4-02 Ordering fractions

- For each pair of fractions, find the larger fraction.
 - $\frac{7}{10}, \frac{3}{10}$
 - $\frac{5}{3}, \frac{2}{3}$
 - $\frac{3}{4}, \frac{7}{8}$
 - $\frac{2}{3}, \frac{3}{10}$
 - $2\frac{1}{5}, 2\frac{1}{6}$
 - $3\frac{4}{7}, 2\frac{5}{6}$
- For each pair of fractions, find the smaller fraction.
 - $\frac{4}{5}, \frac{3}{5}$
 - $\frac{11}{8}, \frac{9}{8}$
 - $\frac{1}{4}, \frac{3}{10}$
 - $\frac{1}{6}, \frac{2}{3}$
 - $4\frac{3}{4}, 4\frac{2}{3}$
 - $1\frac{2}{7}, 1\frac{1}{5}$
- Which set of fractions shows $\frac{19}{16}, \frac{7}{6}, \frac{5}{4}, \frac{33}{32}$ in ascending order? Select the correct answer **A**, **B**, **C** or **D**.
 - $\frac{7}{6}, \frac{5}{4}, \frac{19}{16}, \frac{33}{32}$
 - $\frac{5}{4}, \frac{7}{6}, \frac{19}{16}, \frac{33}{32}$
 - $\frac{33}{32}, \frac{7}{6}, \frac{5}{4}, \frac{19}{16}$
 - $\frac{19}{16}, \frac{33}{32}, \frac{7}{6}, \frac{5}{4}$
- Copy and complete each statement with a $<$ or $>$ symbol.
 - $\frac{1}{2} \text{ — } \frac{1}{3}$
 - $\frac{2}{3} \text{ — } \frac{3}{4}$
 - $\frac{8}{8} \text{ — } \frac{1}{2}$
 - $2\frac{3}{5} \text{ — } 2\frac{7}{10}$
 - $\frac{5}{6} \text{ — } \frac{1}{2}$
 - $\frac{5}{12} \text{ — } \frac{1}{3}$
 - $\frac{17}{20} \text{ — } \frac{6}{10}$
 - $\frac{11}{15} \text{ — } \frac{2}{3}$
 - $2\frac{3}{7} \text{ — } 5\frac{2}{5}$

See Example 5

Extra questions

Ordering fractions

MAT07NAEQ00030

Worked solutions

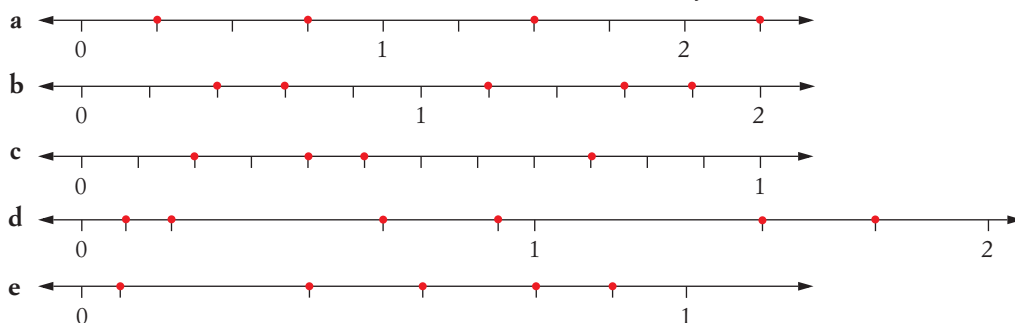
Exercise 4-02

MAT07NAWS10017

- 5 Find a proper fraction that is greater than $\frac{3}{4}$.

- a with a denominator of 10
b with any other denominator

- 6 For each number line, find the value of each fraction marked by a dot.



See Example 6

- 7 Plot each set of fractions on a separate number line.

- a $\frac{2}{3}$ $\frac{3}{3}$ $1\frac{1}{3}$ $\frac{7}{3}$ b $\frac{1}{5}$ $1\frac{3}{5}$ $\frac{5}{5}$ $2\frac{2}{5}$ $\frac{3}{5}$ $\frac{9}{5}$
c $\frac{5}{6}$ $\frac{1}{3}$ $\frac{1}{2}$ $1\frac{1}{6}$ d $\frac{7}{10}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{9}{10}$

Worked solutions

Exercise 4-02

- 8 Which one of these fractions is closest to $\frac{3}{4}$? Select the correct answer **A**, **B** or **C**.

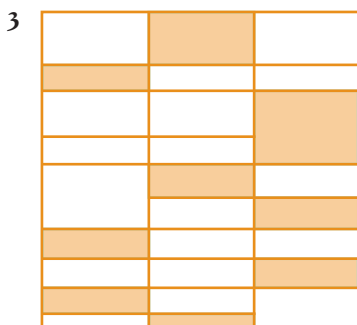
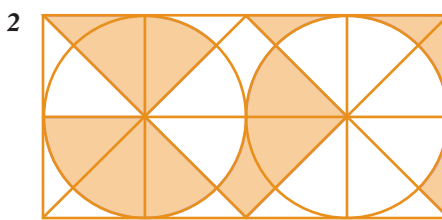
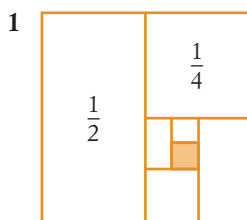
- A** $\frac{1}{2}$ **B** $\frac{7}{8}$ **C** $\frac{13}{16}$

- 9 Find a fraction that is between:

- a $\frac{3}{4}$ and $\frac{4}{5}$ b $\frac{5}{8}$ and $\frac{2}{3}$

Investigation: What fraction is shaded?

In groups or individually, decide what fraction of each diagram has been shaded. Explain how you solved each puzzle.



4-03 Adding and subtracting fractions

Fractions with the same denominator

Summary

To add or subtract fractions with the same denominator, simply add or subtract the numerators.

Worksheet

Pop stick calculator

MAT07NAWK10030

Worksheet

Magic squares

MAT07NAWK00051

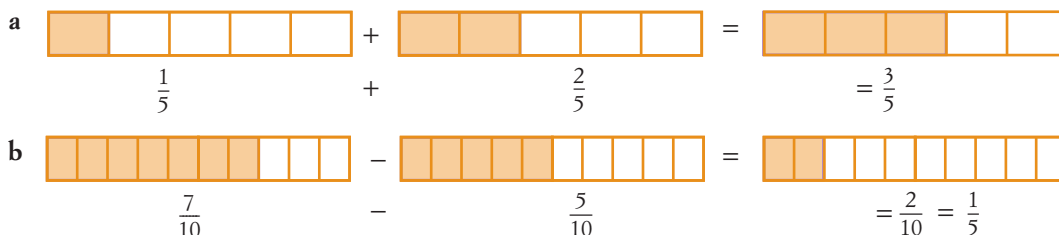
Example 7

Find:

a $\frac{1}{5} + \frac{2}{5}$

b $\frac{7}{10} - \frac{5}{10}$

Solution



Fractions with different denominators

Summary

To add or subtract fractions with different denominators, we must first convert them to equivalent fractions with the same denominator.

Example 8

Find:

a $\frac{1}{4} + \frac{5}{6}$

b $\frac{2}{5} - \frac{3}{20}$

Solution

a **Method 1**

Common denominator = $6 \times 4 = 24$.

$$\frac{1}{4} = \frac{1 \times 6}{4 \times 6} = \frac{6}{24} \quad \text{and} \quad \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Video tutorial

Adding and subtracting fractions

MAT07NAVT10008

Animated example

Adding and subtracting fractions

MAT07NAAE00013

$$\begin{aligned}
 \frac{1}{4} + \frac{5}{6} &= \frac{6}{24} + \frac{20}{24} \\
 &= \frac{26}{24} \\
 &= \frac{13}{12} \\
 &= 1\frac{1}{12}
 \end{aligned}$$

Method 2

The LCD of 4 and 6 is 12.

$$\begin{aligned}
 \frac{1}{4} &= \frac{1 \times 3}{4 \times 3} = \frac{3}{12} \quad \text{and} \quad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \\
 \frac{1}{4} + \frac{5}{6} &= \frac{3}{12} + \frac{10}{12} \\
 &= \frac{13}{12} \\
 &= 1\frac{1}{12}
 \end{aligned}$$

b Method 1

Common denominator = $5 \times 20 = 100$

$$\begin{aligned}
 \frac{2}{5} &= \frac{2 \times 20}{5 \times 20} = \frac{40}{100} \quad \text{and} \quad \frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} \\
 \frac{2}{5} - \frac{3}{20} &= \frac{40}{100} - \frac{15}{100} \\
 &= \frac{25}{100} \\
 &= \frac{1}{4}
 \end{aligned}$$

Method 2

The LCD of 5 and 20 is 20.

$$\begin{aligned}
 \frac{2}{5} &= \frac{2 \times 4}{5 \times 4} = \frac{8}{20} \quad \text{and} \quad \frac{3}{20} \\
 \frac{2}{5} - \frac{3}{20} &= \frac{8}{20} - \frac{3}{20} \\
 &= \frac{5}{20} \\
 &= \frac{1}{4}
 \end{aligned}$$

Exercise 4-03 Adding and subtracting fractions**See Example 7**

1 Evaluate each expression.

a $\frac{3}{6} + \frac{1}{6}$

b $\frac{7}{10} + \frac{2}{10}$

c $\frac{1}{2} + \frac{1}{2}$

d $\frac{2}{3} + \frac{2}{3}$

e $\frac{3}{5} - \frac{1}{5}$

f $\frac{5}{4} - \frac{3}{4}$

g $\frac{9}{10} - \frac{5}{10}$

h $\frac{7}{12} - \frac{4}{12}$

Extra questions

Adding and subtracting
fractions

MAT07NAEQ00031

2 Evaluate each expression.

a $\frac{1}{2} + \frac{1}{3}$

b $\frac{2}{3} + \frac{1}{5}$

c $\frac{3}{4} - \frac{1}{2}$

d $\frac{5}{8} - \frac{1}{2}$

e $\frac{2}{6} + \frac{1}{3}$

f $\frac{4}{5} - \frac{3}{4}$

g $\frac{3}{7} - \frac{1}{9}$

h $\frac{7}{12} + \frac{1}{6}$

i $\frac{5}{8} + \frac{1}{4}$

j $\frac{9}{10} - \frac{3}{5}$

k $\frac{7}{9} - \frac{2}{3}$

l $\frac{5}{6} + \frac{3}{8}$

m $\frac{7}{8} - \frac{2}{10}$

n $\frac{3}{4} + \frac{4}{5}$

o $\frac{5}{8} - \frac{1}{3}$

p $\frac{6}{7} + \frac{1}{5}$

3 What fraction goes in the blank? $\frac{7}{8} - \frac{\quad}{\quad} = \frac{1}{4}$. Select the correct answer A, B, C or D.

A $\frac{6}{4}$

B $\frac{8}{12}$

C $\frac{3}{8}$

D $\frac{5}{8}$

4 Copy and fill in each blank with the correct fraction.

a $\frac{1}{2} + \frac{\quad}{\quad} = \frac{2}{3}$

b $\frac{\quad}{\quad} - \frac{1}{4} = \frac{2}{3}$

5 A bottle is two-thirds full. One quarter of the liquid is then poured out. What fraction remains in the bottle?

6 Evaluate each expression.

a $4 + \frac{1}{2}$

b $3\frac{2}{3} - 3$

c $4\frac{1}{6} - \frac{1}{6}$

d $5\frac{3}{4} - \frac{1}{4}$

e $2\frac{3}{5} + \frac{4}{5}$

f $6\frac{5}{8} - \frac{3}{8}$

g $\frac{5}{6} + 1\frac{5}{6}$

h $3\frac{4}{5} + 2\frac{1}{5}$

7 A cash prize is shared between three girls. Cindy receives $\frac{1}{4}$, Carmen gets $\frac{2}{5}$ and Libby takes the rest. What fraction does Libby take?

8 Copy and fill in the blanks by using fractions with different denominators that will make the equations true:

a $\frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{15}{16}$

b $\frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{3}{4}$

See Example 8

Worked solutions

Exercise 4-03

MAT07NAWS10018

Just for the record

Ancient fractions

The earliest mention of fractions comes from ancient Egypt in 1800 BCE. The Egyptians wrote all their fractions as unit fractions, using 1 as their numerator. The Egyptians drew a

mouth above a number to show that it was a fraction. For example,  meant $\frac{1}{4}$.

Around 100 CE, the ancient Romans used words to describe fractions. For example:

- $\frac{1}{12}$ was called **uncia** (where the words 'ounce' and 'inch' come from)
- $\frac{6}{12}$ was called **semis**

By 500 CE, Indian mathematicians were writing fractions almost like we do, with a numerator above a denominator, but without a line. The Arabs added the line (vinculum) which we now use to separate the numerator and denominator.

$$\begin{array}{ccc} \text{Indian} & & \text{Arab} \\ \begin{array}{c} \text{8} \leftarrow \\ \text{—} \text{7} \leftarrow \end{array} & = & \frac{8}{17} \end{array}$$

What does 'semi-' mean?

Why do you think the ancient Romans used the word 'semis' for $\frac{6}{12}$?

Weblink

Egyptian fractions

4-04 Adding and subtracting mixed numerals

Summary

To add or subtract mixed numerals, add or subtract the whole numbers first and then add or subtract the fractions.

Example 9

Find:

a $6\frac{1}{3} + 4\frac{1}{2}$

b $10\frac{3}{4} - 3\frac{2}{5}$

Solution

$$\begin{aligned}\mathbf{a} \quad 6\frac{1}{3} + 4\frac{1}{2} &= 6 + 4 + \frac{1}{3} + \frac{1}{2} \\ &= 10 + \frac{2}{6} + \frac{3}{6} \\ &= 10\frac{5}{6}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 10\frac{3}{4} - 3\frac{2}{5} &= 10 - 3 + \frac{3}{4} - \frac{2}{5} \\ &= 7 + \frac{15}{20} - \frac{8}{20} \\ &= 7\frac{7}{20}\end{aligned}$$

Example 10

Find:

a $7 - \frac{3}{5}$

b $5\frac{2}{3} - 3\frac{3}{4}$

Solution

$$\begin{aligned}\mathbf{a} \quad 7 - \frac{3}{5} &= 6 + 1 - \frac{3}{5} \\ &= 6 + \frac{5}{5} - \frac{3}{5} \\ &= 6\frac{2}{5}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 5\frac{2}{3} - 3\frac{3}{4} &= 5 - 3 + \frac{2}{3} - \frac{3}{4} \\ &= 5 - 3 + \frac{2}{3} - \frac{3}{4} \\ &= 2 + \frac{8}{12} - \frac{9}{12} \\ &= 2 - \frac{1}{12} \\ &= 1 + \frac{12}{12} - \frac{1}{12} \\ &= 1\frac{11}{12}\end{aligned}$$

Exercise 4-04 Adding and subtracting mixed numerals

1 Simplify each expression.

a $1\frac{1}{2} + 1\frac{1}{2}$

b $2\frac{1}{2} + 1\frac{3}{4}$

c $5\frac{1}{3} - 2\frac{1}{5}$

d $4\frac{3}{8} - 1\frac{1}{4}$

e $1\frac{1}{3} + 2\frac{3}{4}$

f $2\frac{3}{5} + 1\frac{1}{2}$

g $4\frac{5}{6} - 2\frac{1}{3}$

h $1\frac{1}{2} + 2\frac{1}{3}$

i $3\frac{3}{8} - 1\frac{1}{4}$

j $2\frac{3}{4} + 1\frac{2}{5}$

k $3\frac{1}{3} - 1\frac{1}{4}$

l $3\frac{1}{7} + 2\frac{1}{5}$

See Example 9

2 What is the correct answer to $2\frac{3}{5} + 1\frac{1}{3}$? Select **A**, **B**, **C** or **D**.

A $3\frac{3}{8}$

B $3\frac{2}{5}$

C $3\frac{14}{15}$

D $3\frac{9}{15}$

3 Simplify each expression.

a $3 - \frac{1}{2}$

b $2 - \frac{3}{5}$

c $1 - \frac{4}{7}$

d $4 - \frac{2}{3}$

e $2 - \frac{1}{4}$

f $3 - \frac{5}{12}$

g $3\frac{1}{5} - 1\frac{1}{2}$

h $3\frac{1}{4} - 2\frac{1}{2}$

i $6\frac{2}{5} - 4\frac{7}{8}$

j $4\frac{3}{4} - 2\frac{4}{5}$

k $2\frac{1}{3} - 1\frac{1}{2}$

l $4\frac{3}{5} - 2\frac{9}{10}$

m $3\frac{1}{6} - 1\frac{2}{3}$

n $5\frac{3}{10} - 2\frac{1}{2}$

o $5\frac{2}{3} - 3\frac{3}{4}$

p $4\frac{5}{12} - 3\frac{2}{3}$

See Example 10

4 Robert drives $\frac{3}{4}$ of an hour to the airport, flies on a plane for $2\frac{1}{2}$ hours, then catches a shuttle bus to the hotel for $\frac{2}{5}$ of an hour. How long does the whole trip take?

5 Copy and complete each blank with the correct mixed numeral.

a $3\frac{1}{2} + \underline{\hspace{1cm}} = 5\frac{3}{5}$

b $4\frac{3}{4} - \underline{\hspace{1cm}} = 1\frac{1}{8}$

Worked solutions

Exercise 4-04

MAT07NAWS10019

6 In a magic square, each row, column and diagonal must add to the same total. Complete each magic square with appropriate fractions.

i

4	$1\frac{1}{2}$	
$\frac{1}{2}$		$4\frac{1}{2}$
3		

ii

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

7 On a 4-hour blank DVD, Elise has recorded one movie lasting $2\frac{1}{4}$ hours, a comedy show lasting $\frac{2}{3}$ of an hour and some music videos lasting $\frac{1}{4}$ of an hour. How many hours of recording are left?

4-05 Fraction of a quantity

What is $\frac{2}{3}$ of 12? Here are 12 lollies.



If these lollies were divided into three equal piles, each pile would be $\frac{1}{3}$ of 12.



$$\underbrace{\frac{1}{3} \text{ of } 12 = 4 \quad \quad \frac{1}{3} \text{ of } 12 = 4}_{\frac{2}{3} \times 12 = 8}$$

Note: The word 'of' usually means to multiply.

Example 11

Find:

a $\frac{1}{4} \times 28$

b $\frac{2}{3}$ of \$30

Solution

a $\frac{1}{4} \times 28 = 28 \div 4$
 $= 7$

b $\frac{2}{3}$ of 30 = $\left(\frac{1}{3} \times 30\right) \times 2$
 $= \$10 \times 2$
 $= \$20$

Example 12

Find:

a $\frac{1}{10}$ of 2 km (in metres)

b $\frac{3}{5}$ of 1 minute

Solution

a $\frac{1}{10}$ of 2 km = $\frac{1}{10} \times 2000$ m
 $= 200$ m.

Convert 2 km to metres

b $\frac{3}{5}$ of 1 min = $\frac{3}{5}$ of 60 s
 $= \left(\frac{1}{5} \times 60 \text{ s}\right) \times 3$
 $= 12 \text{ s} \times 3$
 $= 36 \text{ s}$

Convert 1 minute to 60 seconds

Exercise 4-05 Fraction of a quantity

1 Find each amount.

a $\frac{1}{4} \times 28$	b $\frac{1}{5} \times 35$	c $\frac{2}{5} \times 50$	d $\frac{1}{2} \times 16$
e $\frac{5}{12}$ of 12	f $\frac{2}{3}$ of 33	g $\frac{1}{9}$ of 27	h $\frac{6}{9}$ of 45
i $\frac{3}{10} \times 50$	j $\frac{5}{6} \times 60$	k $\frac{3}{4} \times 100$	l $\frac{2}{7} \times 42$

See Example 11

2 Find each amount.

a $\frac{1}{2}$ of \$42	b $\frac{2}{3}$ of \$12	c $\frac{1}{3}$ of \$24	d $\frac{1}{3}$ of 60c
e $\frac{1}{4}$ of \$1.20	f $\frac{1}{4}$ of \$10	g $\frac{3}{8}$ of \$360	h $\frac{2}{5}$ of 90c
i $\frac{5}{6}$ of \$3	j $\frac{3}{4}$ of \$16	k $\frac{4}{5}$ of \$4000	l $\frac{3}{10}$ of \$800

3 Find each amount.

a $\frac{1}{4}$ of 2 hours (in minutes)	b $\frac{3}{4}$ of 3 hours (in minutes)
c $\frac{3}{4}$ of 1 day (in hours)	d $\frac{3}{5}$ of half an hour (in minutes)
e $\frac{1}{5}$ of 1 year (in days)	f $\frac{1}{4}$ of 3 minutes (in seconds)
g $\frac{2}{3}$ of 5 minutes (in seconds)	h $\frac{3}{10}$ of 1 hour (in minutes)
i $\frac{2}{3}$ of 2 days (in hours)	j $\frac{1}{3}$ of 2 years (in months)

See Example 12

4 In a squad of 45 swimmers, $\frac{3}{5}$ out of every 5 were female.

- How many are female?
- How many are male?

5 Find each amount.

a $\frac{1}{4}$ of 1 km (in m)	b $\frac{3}{4}$ of 3 km (in m)	c $\frac{3}{4}$ of 6 cm (in mm)
d $\frac{1}{20}$ of 2 m (in cm)	e $\frac{1}{4}$ of 3 kg (in g)	f $\frac{7}{10}$ of 5 L (in mL)
g $\frac{1}{5}$ of 1.6 km (in m)	h $\frac{3}{10}$ of 1 cm (in mm)	i $\frac{3}{8}$ of 7.2 kg (in g)
j $\frac{5}{8}$ of 1.6 kL (in L)	k $\frac{3}{5}$ of 8 m (in mm)	l $\frac{2}{3}$ of 60 t (in kg)

6 Mr Arun began to read a 280-page book. On the weekend, he read $\frac{6}{7}$ of the book.

How many more pages does he have to read before he finishes the book? Select **A**, **B**, **C** or **D**.

A 230	B 46	C 240	D 40

7 Rana has a pencil box that is half full. She puts four extra pencils in and the box is now two-thirds full. How many pencils will fit into her pencil box?

8 Copy and fill in each blank.

a $\frac{3}{4} \times \underline{\hspace{1cm}} = 9$	b $\underline{\hspace{1cm}} \times 24 = 16$

Worked solutions

Exercise 4-05

MAT07NAWS10020

Worked solutions

Exercise 4-05

MAT07NAWS10020

- 9 Some lollies were placed on a table. Karin takes away half of the pile. Gino then takes one-third of what was left. Heather takes two and eats them. Half of those left are red and there are eight red lollies. How many lollies were there originally?



- 10 A farmer died and in his will he left 17 horses to his three children. Half of the horses would go to Ashleigh, the eldest, one-third would go to Bethany, the middle child, and one-ninth would go to Thomas, the youngest. The family was puzzled because you cannot take those fractions of 17 horses without cutting up a horse! The farmer's wife had an idea. She borrowed a horse from a neighbouring farm to make a total of 18 horses. Now it was possible to divide up the horses according to the farmer's will.
- How many horses did each child receive?
 - Why was the farmer's wife able to return the extra horse?
 - What was mathematically incorrect about the farmer's will? Can you work it out?

Mental skills 4A Maths without calculators

Fraction of a quantity

Learn these commonly-used fractions and their decimal equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Decimal	0.5	0.25	0.125	0.75	0.2	0.1	0.05	0.4

Now we will use them to find a fraction or decimal of a quantity.

- 1 Study each example.

$$\begin{aligned} \text{a } \frac{1}{4} \times 72 &= 72 \div 4 \\ &= 18 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{3}{4} \times 32 &= \left(\frac{1}{4} \times 32\right) \times 3 \\ &= 8 \times 3 \\ &= 24 \end{aligned}$$

$$\begin{aligned} \text{e } 0.05 \times 80 &= \frac{1}{20} \times 80 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b } \frac{2}{5} \times 40 &= \left(\frac{1}{5} \times 40\right) \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

$$\begin{aligned} \text{d } 0.5 \times 66 &= \frac{1}{2} \times 66 \\ &= 33 \end{aligned}$$

$$\begin{aligned} \text{f } 0.125 \times 56 &= \frac{1}{8} \times 56 \\ &= 7 \end{aligned}$$

2 Now simplify each expression.

a $\frac{1}{2} \times 28$

b $\frac{1}{4} \times 36$

c $\frac{1}{10} \times 70$

d $\frac{1}{8} \times 64$

e $\frac{1}{5} \times 15$

f $\frac{1}{10} \times 80$

g $\frac{2}{5} \times 25$

h $\frac{1}{20} \times 100$

i $\frac{3}{4} \times 44$

j $\frac{1}{8} \times 40$

k 0.25×60

l 0.4×45

m 0.1×260

n 0.125×48

o 0.75×48

p 0.05×120

q 0.2×70

r 0.5×320

s 0.25×56

t 0.125×16

Investigation: Netball scores

Elvira and Christina play netball for the Jets. A netball game is split into four quarters and each goal is worth one point. Last Saturday, Elvira and Christina scored all of the Jets' goals for the first and second quarters.

- 1 In the first quarter, Elvira shot $\frac{1}{4}$ of the goals and Christina shot 12 goals. How many points did the team score altogether?
- 2 In the second quarter, Christina scored $\frac{1}{5}$ of the team's 20 points while Elvira scored the rest. How many points did Elvira score?
- 3 What happened in the last two quarters? Finish off the story of the netball game by writing two more stories involving fractions.



4-06 Multiplying fractions

What is $\frac{1}{2}$ of $\frac{4}{5}$?

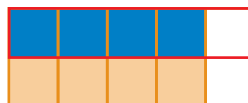
$\frac{4}{5}$ of this diagram is shaded.



If the diagram were cut in half, each fifth would also be cut in half to make a total of 10 pieces, with eight of them shaded.

So $\frac{1}{2}$ of $\frac{4}{5}$ (shaded) = $\frac{1}{2}$ of $\frac{8}{10}$ = $\frac{4}{10}$

Another way to work this out is:



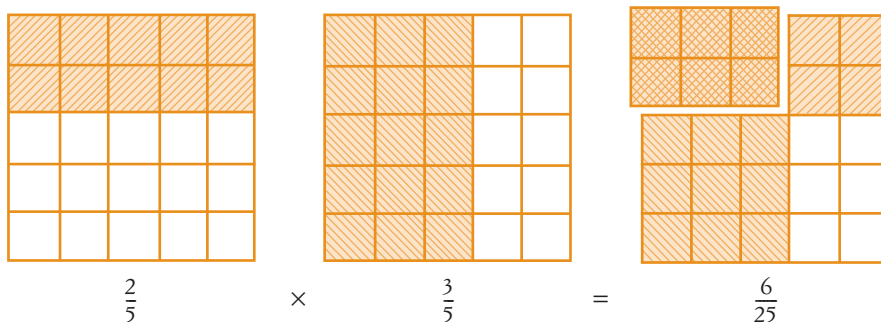
TLF learning object
Fractions: Rectangle multiplication (L3525)

Worksheet
Multiplying fractions using rectangles

MAT07NAWK00052

$$\begin{aligned}\frac{1}{2} \text{ of } \frac{4}{5} &= \frac{1}{2} \times \frac{4}{5} \\ &= \frac{1 \times 4}{2 \times 5} \\ &= \frac{4}{10}\end{aligned}$$

You can use diagrams to show other multiplications, for example $\frac{2}{5} \times \frac{3}{5}$:



Summary

To multiply fractions, multiply the numerators and multiply the denominators separately.

Example 13

Evaluate each product.

a $\frac{3}{5} \times \frac{7}{8}$

b $\frac{3}{5} \times \frac{4}{9}$

c $\frac{21}{100} \times \frac{8}{15}$

Solution

a $\frac{3}{5} \times \frac{7}{8} = \frac{3 \times 7}{5 \times 8}$
 $= \frac{21}{40}$

b $\frac{3}{5} \times \frac{4}{9} = \frac{3 \times 4}{5 \times 9}$
 $= \frac{12 \div 3}{45 \div 3}$
 $= \frac{4}{15}$

or, by simplifying first before multiplying:

$$\frac{3}{5} \times \frac{4}{9} = \frac{\overset{1}{\cancel{3}} \times 4}{5 \times \underset{3}{\cancel{9}}} \quad \text{because } \frac{3}{9} = \frac{1}{3}$$

$$= \frac{4}{15}$$

Video tutorial

Multiplying fractions

MAT07NAVT10009

- c By simplifying first before multiplying:

$$\begin{aligned}\frac{21}{100} \times \frac{8}{15} &= \frac{21^7 \times 8^2}{100^{25} \times 15^5} \\ &= \frac{14}{125}\end{aligned}$$

because $\frac{21}{15} = \frac{7}{5}$, $\frac{8}{100} = \frac{2}{25}$

Multiplying mixed numerals

Summary

To multiply mixed numerals, convert them to improper fractions first.

Example 14

Simplify $3\frac{2}{5} \times 1\frac{2}{3}$.

Solution

$$\begin{aligned}3\frac{2}{5} \times 1\frac{2}{3} &= \frac{17}{5} \times \frac{5}{3} \\ &= \frac{17}{3} \\ &= 5\frac{2}{3}\end{aligned}$$

Exercise 4-06 Multiplying fractions

- 1 Simplify each product.

a $\frac{1}{3} \times \frac{2}{5}$

b $\frac{3}{4} \times \frac{3}{5}$

c $\frac{5}{6} \times \frac{7}{8}$

d $\frac{1}{8} \times \frac{9}{10}$

e $\frac{3}{8} \times \frac{5}{6}$

f $\frac{3}{8} \times \frac{3}{8}$

g $\frac{4}{9} \times \frac{5}{7}$

h $\frac{7}{8} \times \frac{3}{20}$

- 2 What is the correct value of $\frac{2}{9} \times \frac{3}{10}$? Select the correct answer A, B, C or D.

A $\frac{1}{8}$

B $\frac{1}{15}$

C $\frac{6}{19}$

D $\frac{1}{18}$

- 3 Simplify each product.

a $\frac{1}{5} \times \frac{5}{7}$

b $\frac{3}{7} \times \frac{7}{10}$

c $\frac{1}{8} \times \frac{4}{9}$

d $\frac{3}{7} \times \frac{7}{3}$

e $\frac{3}{8} \times \frac{5}{6}$

f $\frac{3}{7} \times \frac{4}{9}$

g $\frac{2}{5} \times \frac{3}{8}$

h $\frac{3}{8} \times \frac{5}{12}$

i $\frac{3}{10} \times \frac{2}{5}$

j $\frac{9}{5} \times \frac{25}{3}$

k $\frac{8}{5} \times \frac{10}{7}$

l $\frac{1}{6} \times \frac{12}{25}$

m $\frac{5}{6} \times \frac{11}{20}$

n $\left(\frac{1}{8}\right)^2$

o $\frac{2}{3} \times \frac{9}{10}$

p $\left(\frac{3}{4}\right)^2$

- 4 Mrs Dushevina works $7\frac{3}{4}$ hours a day. She works a full day each day from Monday to Friday and works half a day on Saturday. How many hours a week does she work?

See Example 13

Extra questions

Multiplying fractions

MAT07NAEQ00032

Worked solutions

Exercise 4-06

MAT07NAWS10021

See Example 14

5 Convert each mixed numeral to an improper fraction before multiplying.

a $1\frac{2}{3} \times \frac{3}{10}$

b $4\frac{1}{3} \times \frac{2}{7}$

c $5\frac{2}{3} \times 1\frac{1}{8}$

d $3\frac{5}{6} \times 1\frac{3}{4}$

e $1\frac{2}{3} \times 1\frac{2}{3}$

f $3\frac{1}{9} \times 1\frac{3}{4}$

6 A cake recipe needs 1 egg, $2\frac{1}{2}$ cups of flour and $\frac{1}{4}$ cup of sugar. What quantities of eggs, flour and sugar will you need to make eight cakes?

7 Copy and complete each equation.

a $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$
=

b $\frac{3}{5} + \frac{3}{5} = 2 \times \frac{3}{5}$
=

c $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = 5 \times$
=

d $5 \times \frac{1}{7} =$

e $7 \times \frac{3}{20} =$
=

f $8 \times \frac{3}{5} =$
=

8 Copy and fill in each blank.

a $\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$

b $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

c $\frac{15}{16} \times \frac{3}{4} = \frac{3}{4}$

d $\frac{3}{4} \times \frac{3}{4} = \frac{3}{8}$

9 Noah wants to be a professional skater. He practises for $2\frac{1}{2}$ hours each day. How many hours of practice does he do in 7 days?

10 True or false: When a number is multiplied by a fraction, the number is decreased. Give reasons for your answer.

Technology More spreadsheet practice

1 Open a new spreadsheet and enter the five numbers shown in column A.

	A	B	C
1	10	=A1+A5	
2	-3		
3	0		
4	4		
5	-1		

2 In cell B1, enter the formula **=A1+A5** to calculate the sum of the values in cells A1 and A5. You should get the answer 9, because $10 + (-1) = 9$.

3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, $A2 + A4$

b at B3, $A2 - A4$

c at B4, $A3 - A4 - A1$

d at B5, $A3 - A4 + A1$

e at B6, $A2 - A5$

f at B7, $A1 \times A2$

g at B8, $A2 \times A4 \times A5$

h at B9, $(A2)^2$

i at B10, $(A5)^3$

j at B11, $A4 \div A5$

- 4 At C1, enter a formula for $\frac{A5}{A1}$. When you have obtained the answer, right-click on the cell, select **Format Cells** and convert the answer to a fraction.
- 5 At C2, enter a formula for $\frac{A1}{A2}$. Right-click on the cell, select **Format Cells** and convert the answer to a mixed numeral.
- 6 For the rest of the cells in column C, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer.

a at C3, the sum of column A c at C5, the minimum value of column A e at C7, $A4 \div A4$ g at C9, $(A2 - A4) \div A5$	b at C4, the average of column A d at C6, $A4 \div (-2)$ f at C8, $(A2 + A4) \times A1$ h at C10, $A1 \div (A5 - A4)$
- 7 At C11, enter a formula for $\frac{A3}{A2}$. At C12, enter a formula for $\frac{A2}{A3}$. Why doesn't the second formula work?
- 8 Now enter **new** values in cells A1 to A5 and notice the new answers calculated in columns B and C.

4-07 Dividing fractions

If we turn a fraction upside down and exchange the numerator with the denominator, we get its **reciprocal**.

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ (or $1\frac{1}{2}$)
- The reciprocal of $\frac{1}{4}$ is 4 Note: $\frac{4}{1} = 4$
- The reciprocal of 3 is $\frac{1}{3}$ Note: $3 = \frac{3}{1}$
- The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

Note that dividing by 3 is the same as multiplying by $\frac{1}{3}$.
 Similarly, dividing by $\frac{1}{3}$ is the same as multiplying by 3.
 When dividing by any fraction, we can use its reciprocal.

Summary

To divide by a fraction $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.

Example 15

Evaluate each quotient.

a $2 \div \frac{1}{3}$

b $\frac{3}{4} \div \frac{1}{2}$

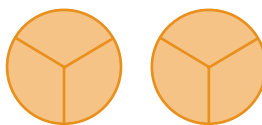
c $\frac{7}{3} \div \frac{5}{6}$

d $\frac{5}{7} \div 5$

Solution

a $2 \div \frac{1}{3} = 2 \times \frac{3}{1}$
 $= 6$

How many times does $\frac{1}{3}$ go into 2?



Worksheet

What's the fraction question?

MAT07NAWK10031

Weblink

Dividing fractions

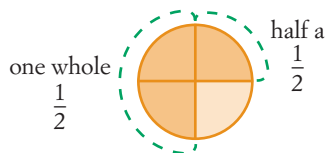
$$\begin{aligned}
 \text{b } \frac{3}{4} \div \frac{1}{2} &= \frac{3}{4} \times \frac{2}{1} \\
 &= \frac{6}{4} \\
 &= \frac{3}{2} \\
 &= 1\frac{1}{2}
 \end{aligned}$$

$\frac{3}{4}$ is '1 $\frac{1}{2}$ halves'

$$\begin{aligned}
 \text{c } \frac{7}{3} \div \frac{5}{6} &= \frac{7}{3^1} \times \frac{6^2}{5} \\
 &= \frac{14}{5} \\
 &= 2\frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \frac{5}{7} \div 5 &= \frac{5^1}{7} \times \frac{1}{5^1} \\
 &= \frac{1}{7}
 \end{aligned}$$

How many times does $\frac{1}{2}$ go into $\frac{3}{4}$?



Dividing by 5 is the same as multiplying by $\frac{1}{5}$.

Dividing mixed numerals

Summary

To divide mixed numerals, convert them to improper fractions first.

Example 16

Simplify $7\frac{1}{2} \div 5\frac{5}{6}$.

Solution

$$\begin{aligned}
 7\frac{1}{2} \div 5\frac{5}{6} &= \frac{15}{2} \div \frac{35}{6} \\
 &= \frac{15^3}{2^1} \times \frac{6^3}{35^7} \\
 &= \frac{9}{7} \\
 &= 1\frac{2}{7}
 \end{aligned}$$

Exercise 4-07 Dividing fractions

- 1 Find the reciprocal of each fraction.

a $\frac{2}{3}$

b $\frac{1}{5}$

c $\frac{5}{8}$

d $\frac{1}{10}$

e 7

f $2\frac{1}{2}$

g $1\frac{3}{4}$

h 9

- 2 a What number's reciprocal is itself?
b What number does not have a reciprocal?
c What is the reciprocal of the reciprocal of $\frac{7}{10}$?

- 3 Evaluate each quotient.

a $4 \div \frac{1}{3}$

b $5 \div \frac{1}{2}$

c $7 \div \frac{1}{3}$

d $3 \div \frac{1}{5}$

e $7 \div \frac{1}{6}$

f $8 \div \frac{1}{10}$

g $4 \div \frac{1}{4}$

h $1 \div \frac{1}{8}$

- 4 What is the correct value of $\frac{3}{8} \div \frac{1}{2}$? Select the correct answer A, B, C or D.

A $\frac{1}{12}$

B $1\frac{1}{4}$

C $\frac{3}{4}$

D $\frac{3}{16}$

- 5 Water is dripping from a tank at the rate of five-eighths of a litre per hour. How long will it take for a 24 litre tank to empty?

- 6 Evaluate each quotient.

a $\frac{1}{4} \div \frac{2}{3}$

b $\frac{8}{7} \div \frac{2}{7}$

c $\frac{3}{5} \div \frac{2}{5}$

d $\frac{5}{8} \div \frac{2}{5}$

e $\frac{7}{10} \div \frac{5}{6}$

f $\frac{3}{5} \div \frac{5}{8}$

g $\frac{11}{20} \div \frac{9}{10}$

h $\frac{5}{12} \div \frac{5}{6}$

i $\frac{5}{6} \div \frac{5}{12}$

j $\frac{1}{4} \div \frac{1}{5}$

k $\frac{1}{10} \div \frac{1}{2}$

l $\frac{1}{2} \div \frac{2}{3}$

- 7 What happens when you:
a multiply any number by its reciprocal?
b divide any number by its reciprocal?

- 8 A bag of rice weighing $\frac{4}{5}$ kg was divided among 6 friends. How much rice did each person get? Select A, B, C or D.

A 480 g

B $\frac{2}{15}$ kg

C 800 g

D $\frac{4}{15}$ kg

- 9 Evaluate each quotient.

a $\frac{5}{7} \div 5$

b $\frac{5}{7} \div 7$

c $\frac{1}{4} \div 3$

d $\frac{5}{6} \div 2$

- 10 Convert each mixed numeral to an improper fraction before dividing.

a $3\frac{1}{8} \div \frac{5}{6}$

b $3\frac{1}{2} \div 1\frac{1}{4}$

c $2\frac{3}{4} \div 3\frac{1}{7}$

d $2\frac{1}{2} \div 1\frac{7}{8}$

- 11 How many speakers were there if each speaker was allowed $3\frac{1}{4}$ minutes and the debate went for about 26 minutes?

- 12 Copy and fill in each blank.

a $\frac{\quad}{\quad} \div \frac{2}{3} = \frac{1}{7}$

b $\frac{4}{15} \div \frac{\quad}{\quad} = \frac{8}{21}$

- 13 True or false: When a number is divided by a proper fraction, the number is increased. Give reasons for your answer.

Worked solutions

Exercise 4-07

MAT07NAWS10022

Extra questions

Dividing fractions

MAT07NAEQ00033

See Example 15

See Example 16

Worked solutions

Exercise 4-07

MAT07NAWS10022

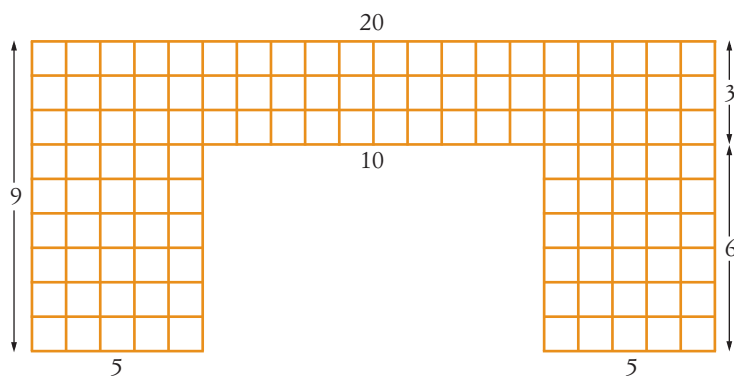
Investigation: Thanh's gorgeous garden

In this activity you will be using fractions to make a decorative pattern.

In Thanh's garden there are places for 120 flowers. He wants to plant them in the following proportions:

$\frac{1}{3}$ golden daffodils $\frac{3}{10}$ red tulips $\frac{1}{6}$ white snowdrops $\frac{1}{5}$ blue hyacinths

- How many of each type of flower should Thanh plant?
- Copy the plan below or use one provided by your teacher. Colour in squares for each flower. Make up your own pattern.



- Make up different fractions for the same garden plot. Make a list of flowers and the fractions of their proportions and colour them on a new plan (or colour the plan first and list the fractions afterwards).

4-08 Fraction problems

Exercise 4-08 Fraction problems

Worksheet

What's the fraction question?

MAT07NAWK10031

Homework sheet

Fractions 3

MAT07NAHS10009

- A class investigation found that 3 out of every 25 cars passing their school was white. If 2150 cars pass the school in a day, how many would you expect to be white?
- Three-quarters of every Year 7 class are boys. There are 24 students in 7A, 20 in 7B, 28 in 7C and 16 in 7D.
 - How many boys in Year 7?
 - How many girls in 7C?

- 3 Anthony ate $\frac{2}{3}$ of a pizza and Lisa ate $\frac{1}{3}$.
 - a Who ate more?
 - b How much did they eat together?
 - c How much is left?
- 4 Mrs Rizzo's patio measures $5 \text{ m} \times 3 \text{ m}$. How many paving stones does it contain if there are $8\frac{1}{2}$ stones per m^2 ?
- 5 A bag holds five-eighths of a kilogram of oranges. How many bags can $12\frac{1}{2}$ kg of oranges be packed into?
- 6 When Paige walks quickly to school, she takes half an hour. When she walks slowly, she takes an extra one-third of an hour.
 - a What fraction of an hour does Paige take walking at her slow pace?
 - b How many minutes is this?
- 7 Jeff bought six books and put them in his bag. His orange juice leaked and ruined two-thirds of them. How many books were not ruined?
- 8 Mrs Chan is planning a slide show. Each picture is to be shown for $2\frac{1}{4}$ seconds. How many pictures can she show in 90 seconds?
- 9 Ben was told, 'When mowing, never cut more than $\frac{1}{3}$ off the grass height'. If the lawn is $12\frac{1}{2}$ cm high, what is the minimum height to which Ben should cut the grass? Select **A**, **B**, **C** or **D**.

A $4\frac{1}{6}$ cm
B $12\frac{1}{6}$ cm
C $7\frac{1}{2}$ cm
D $8\frac{1}{3}$ cm
- 10 Each day, Mr Lanouri is at work for $7\frac{1}{2}$ hours, less $\frac{3}{4}$ of an hour for lunch. How long does he work in one day?
- 11 Two hundred sausages were required for a barbecue. One-tenth of them were donated by a parent, 25 had been left in the freezer from a previous occasion, and a local butcher gave one-quarter of the total free. The rest had to be bought at wholesale price. What fraction had to be bought?
- 12 Kylie finds $\frac{1}{3}$ of her birthday cake in the fridge. If she eats $\frac{2}{3}$ of this amount, what fraction of the whole cake will be left?
- 13 One-sixth of a class were a day late with their assignments, while the rest handed them in on time. Twenty students handed their assignments in on time.
 - a How many students were in the class?
 - b How many students were late in handing in their assignments?
- 14 The school hall has 840 seats. If it is two-thirds full, how many seats are vacant?
- 15 Four authors share royalties of $\frac{1}{8}$ of the price of a textbook. Darren receives $\frac{4}{15}$ of the author share, so what fraction of the price of each book does he receive?

Worked solutions

Exercise 4-08

MAT07NAWS10023

4-09 Fractions and the calculator

Scientific calculators have a fraction key for entering fractions: $\frac{\Box}{\Box}$ or $a^{b/c}$.

Some Casio calculators have two ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into two blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). LINE mode is used in the following examples. Ask your teacher if you need help with the MATH mode.

Example 17

Question	Calculator steps		Answer
	Casio	Sharp	
Simplify $\frac{16}{20}$.	16 $\frac{\Box}{\Box}$ 20 $=$	16 $a^{b/c}$ 20 $=$	$\frac{4}{5}$
Change $1\frac{3}{4}$ to an improper fraction	1 $\frac{\Box}{\Box}$ 3 $\frac{\Box}{\Box}$ 4 $=$ $\frac{b}{c} \leftrightarrow \frac{d}{c}$ (SHIFT S $\leftrightarrow$ D)	1 $a^{b/c}$ 3 $a^{b/c}$ 4 $=$ 2ndF $a^{b/c}$	$\frac{7}{4}$
Change $\frac{19}{4}$ to a mixed numeral	19 $\frac{\Box}{\Box}$ 4 $=$	19 $a^{b/c}$ 4 $=$	$4\frac{3}{4}$

Example 18

Question	Calculator steps		Answer
	Casio	Sharp	
$\frac{1}{2} + \frac{1}{3}$	1 $\frac{\Box}{\Box}$ 2 $+$ 1 $\frac{\Box}{\Box}$ 3 $=$	1 $a^{b/c}$ 2 $+$ 1 $a^{b/c}$ 3 $=$	$\frac{5}{6}$
$\frac{5}{7} - \frac{2}{3}$	5 $\frac{\Box}{\Box}$ 7 $-$ 2 $\frac{\Box}{\Box}$ 3 $=$	5 $a^{b/c}$ 7 $-$ 2 $a^{b/c}$ 3 $=$	$\frac{1}{21}$
$\frac{3}{5} \times \frac{2}{7}$	3 $\frac{\Box}{\Box}$ 5 $\times$ 2 $\frac{\Box}{\Box}$ 7 $=$	3 $a^{b/c}$ 5 $\times$ 2 $a^{b/c}$ 7 $=$	$\frac{6}{35}$
$\frac{4}{5} \div \frac{2}{3}$	4 $\frac{\Box}{\Box}$ 5 $\div$ 2 $\frac{\Box}{\Box}$ 3 $=$	4 $a^{b/c}$ 5 $\div$ 2 $a^{b/c}$ 3 $=$	$1\frac{1}{5}$

Exercise 4-09 Fractions and the calculator

1 Simplify each fraction.

a $\frac{14}{21}$

b $\frac{65}{130}$

c $\frac{27}{81}$

d $\frac{52}{65}$

e $\frac{168}{196}$

f $\frac{260}{325}$

2 Convert each mixed numeral to an improper fraction.

a $2\frac{3}{4}$

b $5\frac{1}{3}$

c $4\frac{2}{7}$

d $6\frac{3}{5}$

e $10\frac{1}{3}$

f $8\frac{5}{6}$

3 Convert each improper fraction to a mixed numeral.

a $\frac{18}{5}$

b $\frac{23}{7}$

c $\frac{45}{8}$

d $\frac{79}{11}$

e $\frac{123}{10}$

f $\frac{458}{24}$

4 Simplify each expression.

a $\frac{3}{5} - \frac{1}{2}$

b $\frac{4}{7} + \frac{1}{2}$

c $\frac{3}{5} + \frac{2}{3}$

d $\frac{1}{4} \times \frac{2}{7}$

e $\frac{5}{7} \div \frac{10}{21}$

f $\frac{5}{8} \div \frac{5}{6}$

g $\frac{9}{15} - \frac{1}{3}$

h $\frac{7}{8} \times \frac{3}{4}$

i $\frac{7}{8} \div \frac{3}{4}$

j $14 \div 2\frac{1}{3}$

k $2\frac{1}{2} + 1\frac{1}{3}$

l $3\frac{1}{4} - \frac{5}{8}$

m $2\frac{3}{4} \times 1\frac{1}{2}$

n $5\frac{2}{3} \div \frac{3}{8}$

o $\frac{4}{5} \div 8$

p $1\frac{1}{2} \times 2\frac{3}{10}$

5 Find:

a $\frac{2}{5}$ of 170

b $\frac{4}{7}$ of 98

c $\frac{1}{3}$ of 75

d $\frac{3}{8}$ of \$1360

e $\frac{2}{5}$ of \$4200

f $\frac{4}{9}$ of \$198

6 An echidna has a mass of $1\frac{1}{2}$ kg. A field mouse has a mass of one-twentieth that of the echidna, but 5000 ants are needed to balance the mass of one field mouse. Find the mass of one ant.

7 If a farm of 78 hectares is subdivided into lots of $4\frac{1}{2}$ hectares, how many lots are there?

8 Nikki has a part-time job mowing lawns. The table on the right shows the number of hours she worked in the last month.

- a Find the total number of hours Nikki worked.
b If Nikki charges \$12.50 per hour, how much did she earn in total?

9 Radio station WAVE-FM plays $32\frac{1}{2}$ minutes of music every hour. The average song played on the station lasts $3\frac{3}{4}$ minutes. About how many songs can be played in 24 hours?

Customer	Hours
Green	$2\frac{1}{2}$
Cook	$1\frac{3}{4}$
Verbeek	$3\frac{1}{2}$
Thompson	$2\frac{3}{4}$
Burns	$2\frac{1}{4}$

Worked solutions

Exercise 4-09

MAT07NAWS10024

Investigation: Fraction puzzles

Find strategies to solve each problem. Write down the thinking you used to work it out.

1 **How many jelly beans?**

Samantha has a bag of jelly beans to share. Ali takes $\frac{1}{4}$ of them. Duyen takes $\frac{1}{2}$ of what is left. Sharelle takes $\frac{2}{3}$ of the remainder. Samantha has 7 jelly beans left. How many jelly beans were in the bag to start with?

2 Who won the race?

- Jane finished half a minute behind Bill.
- Bill took less than half the time that Joel took.
- Joel's time was 5 minutes 23 seconds.
- Dimitra took about $3\frac{1}{4}$ minutes.
- Esther finished about $1\frac{1}{2}$ minutes after Bill.
- David finished three-quarters of a minute in front of Joel.

What is the finishing order of the six competitors?

3 How old is Zoe?

When asked her age, Zoe gave this reply:

If to my age there added be

One-half, one-third (of it) and 3 times 3,

Six score and 10 the sum you would see.

Now pray tell me what age I be.

Hint: First find how much a 'score' is.

Maths clip

Fast money

MAT07NAVT00010

Worksheet

Percentages wall

MAT07NAWK10032

Worksheet

Percentage shapes

MAT07NAWK10033

Animated example

Percentages, fractions
and decimals

MAT07NAAE00017

Puzzle sheet

Converting
percentages to
fractions

MAT07NAPS00044

4-10 Percentages, fractions and decimals

Percentages and decimals are special types of fractions. A **percentage** is a fraction whose denominator is always 100, for example, $78\% = \frac{78}{100}$. The symbol % is an abbreviation of '100' and 'per cent' which comes from the Latin *per centum*, meaning 'by the hundred'. A **decimal** is a fraction whose denominator is a **power of ten**, such as 10, 100, 1000; for example, $0.015 = \frac{15}{1000}$.

- $50\% = \frac{1}{2}$ because $50\% = \frac{50}{100} = \frac{1}{2}$
- $50\% = 0.5$ because $50\% = \frac{50}{100} = 50 \div 100 = 0.5$

Summary

- To convert a percentage to a fraction**, write the percentage with a denominator of 100 and simplify if needed
- To convert a percentage into a decimal**, divide it by 100

Example 19

Convert each percentage to a fraction in simplest form.

a 3%

b 45%

c $5\frac{1}{2}\%$

Solution

a $3\% = \frac{3}{100}$

b $45\% = \frac{45}{100}$
 $= \frac{9}{20}$

c $5\frac{1}{2}\% = \frac{5\frac{1}{2}}{100}$
 $= \frac{5\frac{1}{2} \times 2}{100 \times 2}$
 $= \frac{11}{200}$

Example 20

Convert each percentage to a decimal.

a 39% **b** 2% **c** 16.9%

Solution

a $39\% = 39 \div 100$
 $= 0.39$

b $2\% = 2 \div 100$
 $= 0.02$

c $16.9\% = 16.9 \div 100$
 $= 0.169$

← To divide a number by 100,
move the decimal point 2
places to the left

Animated example

Percentages, fractions
and decimals

MAT07NAAE00017

- $\frac{1}{4} = 25\%$ because $\frac{1}{4} \times 100\% = 25\%$
- $0.2 = 20\%$ because $0.2 \times 100\% = 20\%$

Summary

- To convert a fraction or a decimal to a percentage, multiply it by 100%

Example 21

Convert each fraction to a percentage.

a $\frac{3}{4}$ **b** $\frac{1}{8}$ **c** $\frac{34}{40}$

Solution

a $\frac{3}{4} = \frac{3}{4} \times 100\%$
 $= 75\%$

b $\frac{1}{8} = \frac{1}{8} \times 100\%$
 $= 12\frac{1}{2}\%$

c $\frac{34}{40} = \frac{34}{40} \times 100\%$
 $= 85\%$

Puzzle sheet

Converting fractions
to percentages

MAT07NAPS00042

Example 22

Convert each decimal to a percentage.

a 0.23 **b** 0.147 **c** 0.6

Solution

a $0.23 = 0.23 \times 100\%$
 $= 23\%$

b $0.147 = 0.147 \times 100\%$
 $= 14.7\%$

c $0.6 = 0.6 \times 100\%$
 $= 60\%$

← To multiply a number by 100,
move the decimal point 2
places to the right

Puzzle sheet

Converting decimals
to percentages

MAT07NAPS00043

Exercise 4-10 Percentages, fractions and decimals

See Example 19

- 1 Convert each percentage to a fraction in simplest form.

a 5% b 14% c 8% d 90%
e 7% f $3\frac{1}{2}\%$ g $10\frac{1}{3}\%$ h $5\frac{3}{4}\%$

Worksheet

Fractions, decimals
and percentages

MAT07NAWK00066

See Example 20

- 2 Convert each percentage to a decimal.

a 28% b 6% c 16.1% d 21.3%
e 2% f 20% g 78.07% h $26\frac{1}{2}\%$

See Example 21

- 3 Convert each fraction to a percentage.

a $\frac{9}{100}$ b $\frac{17}{100}$ c $\frac{5}{8}$ d $\frac{27}{30}$
e $\frac{11}{50}$ f $\frac{3}{10}$ g $\frac{1}{5}$ h $\frac{22}{40}$
i $\frac{3}{4}$ j $\frac{12}{25}$ k $\frac{17}{40}$ l $\frac{41}{50}$
m $\frac{1}{16}$ n $\frac{1}{3}$ o $1\frac{1}{2}$ p $\frac{2}{9}$

See Example 22

- 4 Convert each decimal to a percentage.

a 0.24 b 0.5 c 0.86 d 0.571
e 0.06 f 0.345 g 2.4 h 0.007
i 1.3 j 0.001 k 0.315 l 0.026

- 5 Cameron sat for a Geography test and scored 35 out of 40. What is his score as a percentage? Select the correct answer A, B, C or D.

A 87.5% B 80% C 35% D 14%

Worked solutions

Exercise 4-10

- 6 Which one of the following numbers is the smallest? Select A, B, C or D.

A $\frac{4}{5}$ B 0.885 C $\frac{7}{8}$ D 88%

MAT07NAWS10025

4-11 Percentage of a quantity

Summary

To find a percentage of a quantity, calculate:

$$\frac{\text{percentage}}{100} \times \text{quantity}$$

$$\text{or } \text{percentage} \div 100 \times \text{quantity}$$

Also see **Mental skills 4B Percentage of a quantity** on page 160 following Exercise 4-11 to learn how to mentally calculate with percentages.

Puzzle sheet

Percentages cross
number

MAT07NAPS10015

Worksheet

Percentages of an
amount

MAT07NAWK00068

Example 23

Find:

- a** 12% of 650 mL **b** 4.5% of \$2400

Solution

$$\begin{aligned} \text{a } 12\% \text{ of } 650 \text{ mL} &= \frac{12}{100} \times 650 && \text{or } 12 \div 100 \times 650 \text{ mL} \\ &= 78 \text{ mL} \end{aligned}$$

Estimate: $12\% \text{ of } 650 \approx 10\% \times 650 = 65$

Note: If you know that $12\% = 0.12$, then you can also use $0.12 \times 650 \text{ mL} = 78 \text{ mL}$.

$$\begin{aligned} \text{b } 4.5\% \text{ of } \$2400 &= \frac{4.5}{100} \times \$2400 && \text{or } 4.5 \div 100 \times \$2400 \\ &= \$108 \end{aligned}$$

Estimate: $4.5\% \text{ of } \$2400 \approx 5\% \times \$2400 = \$120$

Note: If you know that $4.5\% = 0.045$, then you can also use $0.045 \times \$2400 = \$108$.

Exercise 4-11 Percentage of a quantity

1 Find:

a 10% of 60	b 85% of \$240	c 5% of 150
d 24% of 350 mm	e 20% of 740 kg	f 12% of 60 g
g 33% of \$500	h 6% of 180	i 2% of \$4800
j 80% of 280 mL	k 20% of \$6.50	l 50% of 810 m
m 7% of \$220	n 3% of 90 hectares	o 25% of 60 minutes

2 Find 5% of 74 tonnes. Select the correct answer **A**, **B**, **C** or **D**.

- A** 370 tonnes **B** 3.7 tonnes **C** 37 tonnes **D** 3700 tonnes

3 Erika got 60% in her last mathematics test. If the test was out of 70, how many marks did she receive?

4 In a school of 650 students, 48% are girls. How many girls are there in the school?

5 Farmer Wang owns 775 animals. Of these, 16% are chickens. How many chickens does farmer Wang have?

6 Jaden planted 168 seedlings in his garden. If 12% died, how many plants survived?

7 In a city of 3 million people, 1% of the population are doctors. How many doctors are there? Select **A**, **B**, **C** or **D**.

- A** 3 **B** 3000 **C** 300 **D** 30 000

8 Copy and complete:

- a** $20\% \times \underline{\hspace{1cm}} = \$30$ **b** $35\% \times \underline{\hspace{1cm}} = \$56$

See Example 23

Worked solutions

Exercise 4-11

MAT07NAWS10026

Mental skills 4B Maths without calculators

Percentage of a quantity

Learn these commonly-used fractions and their percentage equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Percentage	50%	25%	12.5%	75%	20%	10%	5%	40%

Now we will use them to find a percentage of a quantity.

1 Study each example.

$$\begin{aligned} \text{a } 20\% \times 45 &= \frac{1}{5} \times 45 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{b } 50\% \times 28 &= \frac{1}{2} \times 28 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{c } 12.5\% \times 48 &= \frac{1}{8} \times 48 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{d } 5\% \times 120 &= \frac{1}{20} \times 120 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{e } 75\% \times 36 &= \frac{3}{4} \times 36 \\ &= \left(\frac{1}{4} \times 36\right) \times 3 \\ &= 9 \times 3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{f } 40\% \times 35 &= \frac{2}{5} \times 35 \\ &= \left(\frac{1}{5} \times 35\right) \times 2 \\ &= 7 \times 2 \\ &= 14 \end{aligned}$$

2 Now simplify each expression.

$$\text{a } 75\% \times 20$$

$$\text{b } 20\% \times 30$$

$$\text{c } 12.5\% \times 24$$

$$\text{d } 5\% \times 40$$

$$\text{e } 10\% \times 90$$

$$\text{f } 50\% \times 38$$

$$\text{g } 40\% \times 40$$

$$\text{h } 12.5\% \times 640$$

$$\text{i } 25\% \times 64$$

$$\text{j } 20\% \times 35$$

$$\text{k } 10\% \times 170$$

$$\text{l } 75\% \times 72$$

$$\text{m } 5\% \times 600$$

$$\text{n } 25\% \times 28$$

$$\text{o } 50\% \times 18$$

$$\text{p } 40\% \times 45$$

Just for the record

What fraction of the Earth is water?

Water covers most of the Earth's surface – approximately $\frac{7}{10}$ or 70%. However it is a very small fraction of the Earth's **mass**, approximately less than $\frac{1}{5000}$. Even on the surface of the Earth, known as the Earth's crust, the mass of the land is greater than that of the oceans.

If an asteroid or satellite crash-landed on Earth, would it be more likely to hit land or water?



4-12 Expressing quantities as fractions and percentages

Worksheet

Calculating percentages

MAT07NAWK00069

Puzzle sheet

Calculating percentages

MAT07NAPS00045

Homework sheet

Percentages

MAT07NAHS10010

Worksheet

Money and percentages review

MAT07NAWK00071

Daniel got 34 out of 40 for an English speech. What is this mark as a percentage?

To express one quantity as a percentage of another, we need to write it as a fraction first and then convert to a percentage by multiplying by 100%.

Summary

To write an amount as a fraction of a whole amount:

- write the amount in the numerator of the fraction
- write the whole amount (total) in the denominator
- Fraction = $\frac{\text{amount}}{\text{whole amount}}$

To write an amount as a percentage of a whole amount:

- write the amount in the numerator of a fraction
- write the whole amount (total) in the denominator
- multiply by 100%

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

Example 24

A school has 800 students. If 156 of them are in Year 7, find:

- the fraction of students that are in Year 7
- the percentage of students that are in Year 7

Solution

$$\text{a Fraction} = \frac{156}{800} = \frac{39}{200}$$

$$\begin{aligned} \text{b Percentage} &= \frac{156}{800} \times 100\% \\ &= 19.5\% \end{aligned}$$

So 156 is 19.5% of 800

Example 25

Express:

- 34 out of 40 as a percentage
- 45 minutes as a fraction of 1 hour
- 9 hours as a percentage of 1 day

Solution

$$\begin{aligned}\text{a Percentage} &= \frac{34}{40} \times 100\% \\ &= 85\%\end{aligned}$$

So 34 is 85% of 40

$$\begin{aligned}\text{b Fraction} &= \frac{45 \text{ min}}{1 \text{ h}} \\ &= \frac{45 \text{ min}}{60 \text{ min}} \\ &= \frac{3}{4}\end{aligned}$$

1 hour = 60 minutes

So 45 min is $\frac{3}{4}$ of 1 hour

$$\begin{aligned}\text{c Percentage} &= \frac{9 \text{ h}}{1 \text{ day}} \times 100\% \\ &= \frac{9 \text{ h}}{24 \text{ h}} \times 100\% \\ &= 37.5\%\end{aligned}$$

1 day = 24 hours

So 9 hours is 37.5% of 1 day

Exercise 4-12 Expressing quantities as fractions and percentages

See Example 24

Extra questions

Percentages, fractions and decimals

MAT07NAEQ00043

- 1 In a road rules test, Billal answered 20 questions correctly out of 25.
 - a What is this score as a simplified fraction?
 - b If the pass mark is 92%, did Billal pass?
- 2 In one season a soccer team scored 20 goals. If Kirstin scored 4 of them, what is this as a percentage of the team score? Select the correct answer **A**, **B**, **C** or **D**.

A 4%	B 5%	C 20%	D 40%
- 3 In a class of 30 students, 12 catch the train to school, 6 walk to school and the rest are driven by their parents.
 - a What fraction catch the train?
 - b What percentage walk to school?
 - c How many students are driven to school? What percentage is this?
- 4 Convert each test mark to a percentage.

a 74 out of 100	b 28 out of 40	c 50 out of 60
d 20 out of 32	e 13 out of 20	f 69 out of 75

See Example 25

Worked solutions

Exercise 4-12

MAT07NAWS10027

- 5 Express each quantity as a percentage.

a 24 min of 1 h	b 250 mL of 1 L	c 700 m of 1 km
d 50c of \$2	e 50 g of 1 kg	f 9 mm of 30 cm
g 6 h of 1 day	h \$1.75 of \$7	i 84 h of 5 days
j 1800 g of 4 kg	k 1500 m of 10 km	l 400 mL of 2 L

Technology Calculating test marks as percentages

Spreadsheets can easily be used to convert test marks into percentages.

Copy the data shown below into a spreadsheet.

	A	B	C	D	E	F	G	H	I	J
1	Student	Mathematics		Geography		Science		Art		
2		45	%	60	%	40	%	75	%	
3	Bond, Jeremy	33	=B3/45	45		31		60		
4	Coutler, Louise	42		50		32		55		
5	Dunlop, Nash	27		36		26		62		
6	Jones, Pip	28		52		24		70		
7	Shepherd, David	31		57		39		68		
8	Tran, Melissa	44		51		38		54		
9	Tyler, Phoebe	40		55		40		71		
10	Young, Tahlia	37		46		29		72		
11										

- In cell C3, enter the formula **=B3/45** to convert Jeremy Bond's Mathematics mark into a decimal. Click % in the toolbar to convert the decimal into a percentage.
- Use **Fill Down** to copy this formula into the cells C4 to C10. Then highlight cells C4 to C10 and right-click on the column. Choose **Format cells** and **Number**, then set **Decimal Place** to 0. This will round each percentage mark to the nearest whole number.
- Enter the formula **=D3/60** into cell E3 to convert Jeremy's Geography mark into a decimal, then click %. **Fill Down** to copy this formula into cells E4 to E10. Choose **Format cells, Number**, then set **Decimal Place** to 0.
- Complete the table by using appropriate formulas to convert the Science and Art marks into percentages, rounded to the nearest whole number.
- Now use your spreadsheet to answer each question, writing the answer in the cell shown in [square brackets].
 - What was the highest Mathematics percentage mark? [Answer in cell J1]
 - Who scored a mark of 87%? What subject was this for? [Answer in cell J2]
 - What was the highest mark? Which subject had this mark? [J3]
 - How many students achieved over 80% in Art? [J4]
 - Who scored over 85% in all subjects? [J5]
 - What was Nash's worst subject? [J6]
- Which subject do you think had the best marks overall? In cell A11, enter the label **Average** and in cell C11, enter the formula **=AVERAGE(C3:C10)** to calculate the average of the Mathematics percentage marks. Click % in the toolbar to convert the decimal to a percentage. Choose **Format cells** and **Number**, then set **Decimal Place** to 1 to round the percentage to one decimal place.

- 7 Select cell C11 to **Copy** the formula, then **Paste** it into cells E11, G11 and I11 separately. This will calculate the averages for the Geography, Science and Art marks.
- 8 Which of the 4 subjects had the highest average percentage mark? Write your answer in cell J7.

Power plus

- 1 Evaluate each expression.

a $\left(\frac{2}{5}\right)^2$ b $\left(\frac{1}{3}\right)^3$ c $\left(\frac{1}{2}\right)^3$ d $\left(\frac{7}{10}\right)^2$ e $\left(\frac{3}{4}\right)^3$
 f $\sqrt{\frac{4}{9}}$ g $\sqrt{\frac{25}{49}}$ h $\sqrt{\frac{81}{100}}$ i $\sqrt{\frac{9}{16}}$ j $\sqrt[3]{\frac{27}{64}}$

- 2 Only $\frac{2}{5}$ of an iceberg is visible above the water. If the section above the water is 30 m high, find:

- a the total height of the iceberg
b the height of the section below the water.

- 3 Todd has $\frac{3}{4}$ of a packet of biscuits. He eats half of them and has six biscuits left. How many were in the packet originally?

- 4 Rukaya sold all of her eggs. She first sold half her eggs plus half an egg to her neighbour, without having to break an egg. Then she sold half her remaining eggs plus half an egg to her uncle, again without breaking an egg. Finally she sold half her remaining eggs plus half an egg to her friend, without breaking an egg. How many eggs did Rukaya begin with?

- 5 Simplify each fraction.

a $\frac{1}{\left(\frac{1}{2}\right)}$ b $\frac{1}{\left(\frac{4}{5}\right)}$ c $\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$

- 6 Convert each fraction to a percentage.

a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{1}{7}$ d $\frac{1}{9}$
 e $\frac{1}{11}$ f $\frac{1}{12}$ g $\frac{2}{3}$ h $\frac{4}{9}$

- 7 Find:

a 87.5% of \$300 b $2\frac{1}{2}\%$ of 80 kg c $33\frac{1}{3}\%$ of 36 L

Language of maths

common	fraction	order	reduce
decimal	improper	percentage	simplify
denominator	mixed numeral	proper	vinculum
equivalent	numerator	reciprocal	

Puzzle sheet

Fractions and percentages crossword

MAT07NAPS10016

- 1 What is the bottom part of a fraction called?
- 2 What is the difference between a **proper** and **improper fraction**?
- 3 Why do you think a **mixed numeral** has that name?
- 4 What does 'simplifying' or 'reducing' a fraction mean?
- 5 Look up the words 'reciprocal' and 'reciprocate' in the dictionary. How does what you find relate to the mathematical meaning of each word?
- 6 The 'cent' in percentage means 100. Find other words which use 'cent' and are related to 100.

Topic overview

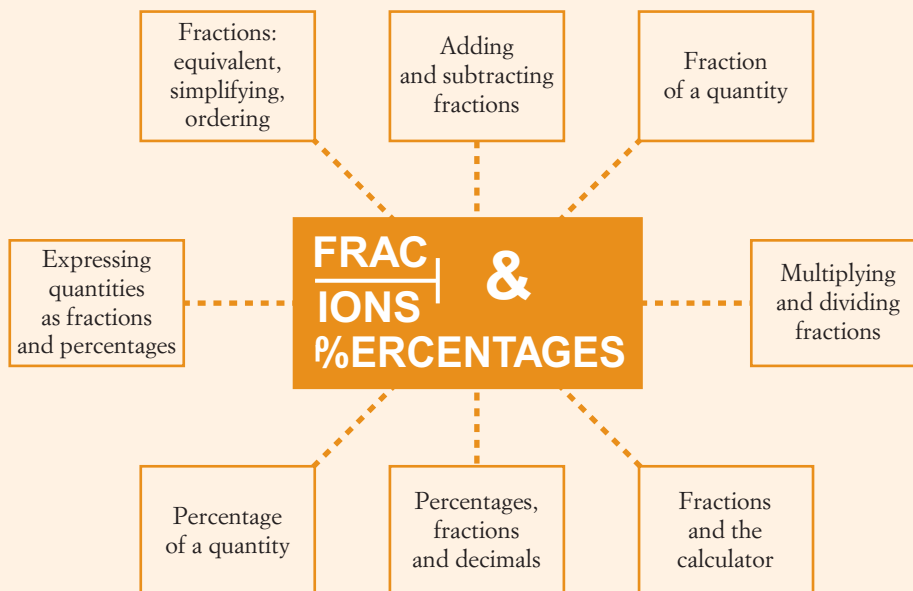
- What have you learnt about fractions and percentages?
- Is there anything you did not understand? Ask a friend or your teacher for help.
- Where are fractions used? Give at least three examples.
- Where are percentages used? Give at least three examples.

Worksheet

Mind map: Fractions and percentages

MAT07NAWK10034

Print or copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 4 revision

See Exercise 4-01

1 Write each mixed numeral as an improper fraction.

a $1\frac{3}{4}$

b $2\frac{4}{9}$

c $5\frac{3}{5}$

d $3\frac{1}{7}$

See Exercise 4-01

2 Write each improper fraction as a mixed numeral.

a $\frac{7}{4}$

b $\frac{26}{3}$

c $\frac{23}{7}$

d $\frac{36}{24}$

See Exercise 4-01

3 Copy and complete each pair of equivalent fractions.

a $\frac{1}{3} = \frac{?}{15}$

b $\frac{3}{4} = \frac{12}{?}$

c $\frac{4}{5} = \frac{?}{20}$

d $\frac{5}{12} = \frac{15}{?}$

e $\frac{1}{2} = \frac{?}{50}$

f $\frac{3}{20} = \frac{?}{100}$

See Exercise 4-01

4 Simplify each fraction.

a $\frac{25}{30}$

b $\frac{30}{40}$

c $\frac{15}{20}$

d $\frac{20}{15}$

e $\frac{24}{48}$

f $\frac{24}{42}$

See Exercise 4-02

5 Order each set of fractions, from smallest to largest.

a $\frac{4}{5}, \frac{3}{10}, \frac{2}{5}, \frac{7}{10}$

b $\frac{1}{2}, \frac{1}{3}, \frac{7}{12}, \frac{11}{12}$

See Exercise 4-03

6 Evaluate each expression.

a $\frac{3}{4} + \frac{1}{4}$

b $\frac{2}{5} + \frac{3}{10}$

c $\frac{5}{10} - \frac{1}{3}$

d $\frac{3}{4} - \frac{1}{5}$

e $\frac{1}{2} + \frac{1}{3}$

f $\frac{1}{2} - \frac{3}{8}$

See Exercise 4-04

7 Evaluate each expression.

a $10 - \frac{1}{3}$

b $2\frac{5}{6} + 1\frac{1}{2}$

c $3\frac{2}{3} - 2\frac{5}{6}$

d $3 + 2\frac{1}{2} - 1\frac{1}{5}$

e $6 - 4\frac{2}{3}$

f $2\frac{5}{6} + 4\frac{1}{2} - 3\frac{2}{3}$

See Exercise 4-05

8 Evaluate each expression. (The letters are not in order.)

A $\frac{3}{7}$ of 21

B $\frac{2}{5}$ of 20

R $\frac{4}{9}$ of 27

O $\frac{1}{4}$ of 28

T $\frac{2}{3}$ of 30

H $\frac{1}{10}$ of 35

Order the six answers, beginning with the smallest. Replace each answer by its letter. What word do you spell?

See Exercise 4-05

9 Evaluate each expression.

a $\frac{2}{3}$ of 1 day (in hours)

b $\frac{4}{5}$ of 2 km (in metres)

c $\frac{3}{4}$ of 3 hours (in min)

d $\frac{7}{10}$ of 15 cm (in mm)

e $\frac{1}{6}$ of 5 minutes (in seconds)

f $\frac{5}{8}$ of 2 m (in cm)

See Exercise 4-06

10 Evaluate each product.

a $\frac{2}{3} \times \frac{1}{5}$

b $\frac{3}{5} \times 10$

c $\frac{4}{7} \times \frac{5}{8}$

d $\frac{1}{3} \times \frac{1}{7}$

e $1\frac{1}{2} \times \frac{1}{2}$

f $\frac{1}{5} \times 2\frac{1}{2}$

g $1\frac{1}{2} \times 1\frac{1}{2}$

h $2\frac{2}{7} \times 3\frac{1}{2}$

i $3\frac{1}{5} \times \frac{3}{8}$

- 11 Evaluate each quotient.

See Exercise 4-07

a $10 \div \frac{1}{5}$

b $\frac{1}{2} \div 3$

c $\frac{1}{2} \div \frac{1}{3}$

d $\frac{3}{4} \div \frac{3}{8}$

e $\frac{3}{5} \div \frac{1}{10}$

f $\frac{4}{7} \div \frac{5}{8}$

g $1\frac{1}{2} \div \frac{1}{2}$

h $2\frac{1}{4} \div 1\frac{1}{8}$

i $1\frac{7}{8} \div 1\frac{1}{4}$

- 12 The recipe for a fruit punch is 2 litres of ginger ale, $1\frac{1}{2}$ litres of orange juice, $2\frac{3}{4}$ litres of lemonade and $\frac{1}{3}$ of a litre of raspberry juice. How many litres of punch does this recipe make?

See Exercise 4-08

- 13 A farmer owns 24 goats. One-quarter of the goats are black, two-thirds are white, and the rest are divided equally between brown and spotted. How many of the goats are spotted?

See Exercise 4-08

- 14 Evaluate each expression.

See Exercise 4-09

a $\frac{13}{18} - \frac{3}{8}$

b $\frac{5}{6} + \frac{4}{5}$

c $3\frac{1}{4} - 2\frac{3}{4}$

d $\frac{9}{16} \times \frac{2}{45}$

e $\frac{17}{20} \div \frac{3}{5}$

f $2\frac{1}{3} \times 4\frac{2}{3}$

- 15 Convert each percentage to:

See Exercise 4-10

i a fraction

ii a decimal

a 13%

b 8%

c $7\frac{1}{2}\%$

- 16 Convert each to a percentage.

See Exercise 4-10

a $\frac{7}{10}$

b $\frac{23}{25}$

c $\frac{18}{80}$

d 0.8

e 0.361

f 0.04

- 17 Copy and complete this table.

See Exercise 4-10

Fraction (simplest form)	Decimal	Percentage
$\frac{3}{5}$		
	0.8	
		82%

- 18 Find:

See Exercise 4-10

a 40% of 90

b 12% of \$600

c 5% of 120

d 11% of \$210

e 34% of 150 kg

f 65% of 360 m

- 19 In one hour of television viewing, 22% of the time was devoted to commercials. How many minutes were devoted to commercials?

See Exercise 4-11

- 20 The netball team scored 26 goals. Jessica scored 8 goals. What fraction of her team's score did Jessica make?

See Exercise 4-12

- 21 What percentage is 300 m of 1.5 km?

See Exercise 4-12

- 22 Anand scored 51 out of 60 for an Art project. What is his mark:

See Exercise 4-12

a as a simple fraction?

b as a percentage?

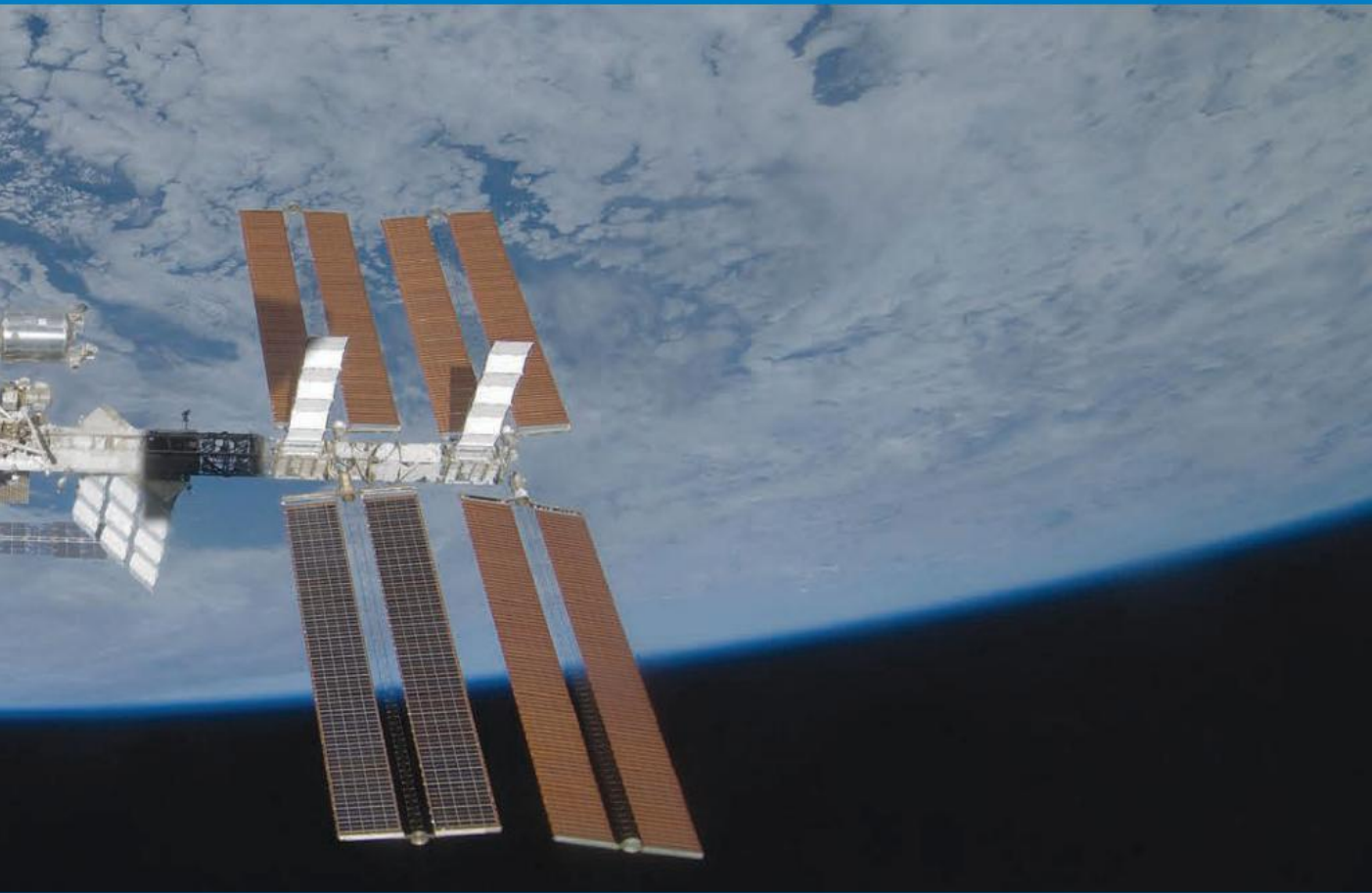


5

Number and algebra

Algebra and equations

Algebra is the branch of mathematics that uses letters of the alphabet to write general rules called **formulas**. Algebra is a powerful tool that has been used by mathematicians for over 4000 years. It was first used in ancient Babylon and Egypt in 2000 BCE and was brought to Europe from India by the Arabs during the 9th century. The word algebra comes from the Arabic *al-jabr*, meaning restoration. Algebra provides a method of solving equations and today it has a wide range of uses: from putting a satellite into space to predicting the chances that it will rain this weekend.



Chapter outline

	Proficiency strands			
5-01 The laws of arithmetic	U	F		R
5-02 The distributive law	U	F		R
5-03 Variables	U	F		R
5-04 From words to algebraic expressions	U	F	PS	R
5-05 Substitution	U	F	PS	
5-06 Equations	U	F		R
5-07 One-step equations	U	F		R
5-08 Two-step equations	U	F		R
5-09 Equation problems		F	PS	R

Wordbank

commutative law A law of arithmetic that says that numbers can be added or multiplied in reverse order, for example, $8 + 3 = 3 + 8$, $5 \times 9 = 9 \times 5$

formula A general mathematical rule written using letters and symbols

inverse operation An opposite operation used in solving an equation, for example, the inverse operation of multiplying is dividing

solve (an equation) To find the value of an unknown variable in an equation

substitute To replace a variable with a number

variable A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

In this chapter you will:

Weblink

Algebra masterclass

- apply the associative, commutative and distributive laws to aid mental and written computation
- understand that arithmetic laws provide powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra
- introduce the concept of variables as a way of representing numbers using letters
- extend and apply the laws and properties of arithmetic to algebraic terms and expressions
- move fluently between algebraic and word representations as descriptions of the same situation
- create algebraic expressions and evaluate them by substituting a given value for each variable
- use authentic formulas to perform substitutions
- solve simple linear equations
- solve real-life problems by using pronumerals to represent unknowns

SkillCheck

Worksheet

StartUp assignment 5

MAT07NAWK10035

- 1 If
- $\bullet = 3$
- , then find the value of each expression.

a $\bullet + 5$

b $7 - \bullet$

c $2 \times \bullet$

d $12 + \bullet$

e $\bullet - 1$

f $\bullet \times 10$

g $\bullet + \bullet$

h $27 \div \bullet$

i $-6 + \bullet$

j $\bullet \times (-1)$

k $-3 \div \bullet$

l $-4 - \bullet$

- 2 Find the value of
- $\blacksquare$
- in each equation.

a $\blacksquare + 4 = 10$

b $6 \times \blacksquare = 36$

c $\blacksquare + \blacksquare = 20$

d $18 - \blacksquare = 11$

e $12 \div \blacksquare = -3$

f $\blacksquare - 6 = 1$

- 3 Write the number that is:

a 4 more than 7

b double 10

c the next odd number after 9

d half of 18

e 10 less than 14

f the difference between 8 and 1

g 5 multiplied by itself

h 5 divided by itself

i the amount by which 11 is greater than 9

j the product of 4 and 8

- 4 Evaluate each product without using a calculator.

a 15×10

b 28×9

c $7 \times 4 \times 5$

d 16×5

e 13×10

f 13×2

- 5 Is
- 13×12
- the same as
- $13 \times 10 + 13 \times 2$
- ?

- 6 a If
- $\blacktriangle = -7$
- , evaluate
- $2 \times \blacktriangle + 3$

- b If
- $\bullet = 4$
- , evaluate
- $3 \times \bullet + 5$

c If $\bullet = 15$, evaluate $\bullet \div 3 - 4$ d If $\blacktriangle = 5$, evaluate $4 \times \blacktriangle - 6$ e If $\blacksquare = -2$, evaluate $9 \times 2 + \blacksquare$ f If $\blacksquare = 6$, evaluate $4 \times 3 - \blacksquare$ g If $\bullet = -4$, evaluate $\bullet \times 4 + 3$ h If $\blacktriangle = -1$, evaluate $10 \times 2 - \blacktriangle$

- 7 Evaluate each expression.

a $5 - 8$

b $-8 + 3$

c $3 \times (-4)$

d $-6 \div 3$

e $-2 + 5$

f $-2 - 9$

g $-4 \times (-9)$

h $-12 \div (-4)$

i $-3 - 7$

j $80 \div (-10)$

k $6 - 15$

l -8×9

5-01 The laws of arithmetic

Arithmetic is the branch of mathematics involved in calculating with **numbers**, involving the four operations of adding, subtracting, multiplying and dividing. We will now look at some general rules of arithmetic and use algebraic symbols to write these rules formally.

The commutative laws

The **commutative law** says that you can operate with two numbers in reverse order. For example:

$$\begin{array}{lll} 4 + 7 = 11 & \text{and} & 7 + 4 = 11, & \text{so } 4 + 7 = 7 + 4. \\ 9 \times 5 = 45 & \text{and} & 5 \times 9 = 45, & \text{so } 9 \times 5 = 5 \times 9. \end{array}$$

The commutative law works for **adding** or **multiplying** only. It does not work for subtracting or dividing. For example, $8 - 2$ is not equal to $2 - 8$ and $14 \div 2$ is not equal to $2 \div 14$.

Summary

If a and b stand for numbers, then:

$$\begin{aligned} a + b &= b + a \\ a \times b &= b \times a \end{aligned}$$

The associative laws

The **associative law** says that when operating with more than 2 numbers, you can group them in any way. For example:

$$\begin{array}{lll} 6 + 7 + 4 = (6 + 7) + 4 & \text{and} & 6 + 7 + 4 = 6 + (7 + 4) \\ = 13 + 4 & & = 6 + 11 \\ = 17 & & = 17 \end{array}$$

so $(6 + 7) + 4 = 6 + (7 + 4)$.

$$\begin{array}{lll} 5 \times 3 \times 2 \times 2 = (5 \times 3) \times 2 \times 2 & \text{and} & 5 \times 3 \times 2 \times 2 = 5 \times (3 \times 2) \times 2 \\ = 15 \times 2 \times 2 & & = 5 \times 6 \times 2 \\ = 30 \times 2 & & = 30 \times 2 \\ = 60 & & = 60 \end{array}$$

so $(5 \times 3) \times 2 \times 2 = 5 \times (3 \times 2) \times 2$.

The associative law only works for **adding** or **multiplying**, not for subtracting or dividing. For example, $10 - 3 - 1$ is not equal to $10 - (3 - 1)$ and $20 \div 5 \div 2$ is not equal to $20 \div (5 \div 2)$.

Summary

If a , b and c stand for numbers, then:

$$\begin{aligned} (a + b) + c &= a + (b + c) \\ (a \times b) \times c &= a \times (b \times c) \end{aligned}$$

The commutative and associative laws of arithmetic can be used to help us with mental calculations.

Example 1

Evaluate each sum.

a $6 + 16$

b $26 + 15 + 4$

c $58 + 55 + 45$

d $-8 + 22$

Solution

a $6 + 16 = 16 + 6$
 $= 22$

It is easier to add smaller numbers to larger ones, so swap 6 and 16 around.

b When adding, pair numbers together that add to multiples of 10, 100, and so on.
 $26 + 15 + 4 = (26 + 4) + 15$
 $= 30 + 15$
 $= 45$

26 and 4 add to 30, so put them together.

c $58 + 55 + 45 = 58 + (55 + 45)$
 $= 58 + 100$
 $= 158$

55 and 45 add to 100, so put them together.

d $-8 + 22 = 22 + (-8)$
 $= 22 - 8$
 $= 14$

It is easier to work with larger and positive numbers first, so swap -8 and 22 around.

Example 2

Evaluate each product.

a $17 \times 5 \times 2$

b $25 \times 9 \times 4$

Solution

When multiplying, pair together numbers that multiply to 10, 100, 1000 and so on.

a $17 \times 5 \times 2 = 17 \times (5 \times 2)$
 $= 17 \times 10$
 $= 170$

$5 \times 2 = 10$, so put 5 and 2 together.

b $25 \times 9 \times 4 = (25 \times 4) \times 9$
 $= 100 \times 9$
 $= 900$

$25 \times 4 = 100$, so put 25 and 4 together.

Multiplying by a multiple of 10

Example 3

Evaluate each product.

a 9×50

b 6×400

c 20×300

Solution

You can factorise multiples of 10, 100, 1000 and so on to make multiplying easier.

a $9 \times 50 = 9 \times 5 \times 10$
 $= 45 \times 10$
 $= 450$

b $6 \times 400 = 6 \times 4 \times 100$
 $= 24 \times 100$
 $= 2400$

c $20 \times 300 = 2 \times 10 \times 3 \times 100$
 $= 2 \times 3 \times 10 \times 100$
 $= 6 \times 1000$
 $= 6000$

Exercise 5-01 The laws of arithmetic

1 Evaluate each sum.

a $4 + 21$

b $19 + 99$

c $18 + 23 + 7$

d $75 + 50 + 25$

e $16 + 80 + 44 + 10$

f $56 + 5 + 15 + 4$

g $-18 + 28$

h $16 + 45 + 4$

i $-6 + 20$

j $81 + 7 + 19 + 12$

k $54 + 27 + 16 + 3$

l $44 + 12 + 16 + 12$

See Example 1

2 Evaluate each product.

a $2 \times 17 \times 5$

b $81 \times 2 \times 5$

c $50 \times 14 \times 2$

d $10 \times 32 \times 2$

e $5 \times 7 \times 4$

f $2 \times 5 \times 3 \times 5$

g $8 \times 3 \times 2$

h $25 \times 8 \times 2$

i $4 \times 4 \times 5$

j $20 \times 6 \times 5$

k $3 \times 2 \times 5 \times 3$

l $2 \times 14 \times 5 \times 10$

See Example 2

3 Evaluate each product.

a 7×1000

b 100×100

c 3×90

d 4×700

e 5×80

f 2×600

g 4×40

h 12×2000

i 4×6000

j 3×1100

k 90×20

l 5×500

m 8×40

n 20×20

o 300×70

p 700×300

See Example 3

4 Evaluate $18 \times 4 \times 5$. Select the correct answer **A**, **B**, **C** or **D**.

A 110

B 162

C 180

D 360

5 Evaluate 60×600 . Select the correct answer **A**, **B**, **C** or **D**.

A 1200

B 3600

C 12 000

D 36 000

Worked solutions

Exercise 5-01

MAT07NAWS10028

Worked solutions

Exercise 5-01

MAT07NAWS10028

6 Write True (T) or False (F) for each equation.

a $4 - 8 = 8 - 4$

b $6 \times 5 = 5 \times 6$

c $10 + 11 = 11 + 10$

d $12 \div 4 = 4 \div 12$

e $a + 5 = 5 + a$

f $3 - x = x - 3$

g $b \div 2 = 2 \div b$

h $c \times 2 = 2 \times c$

i $x - y = y - x$

j $a \times b = b \times a$

k $x + y = y + x$

l $c \div d = d \div c$

5-02 The distributive law

Homework sheet

Mental calculation

MAT07NAHS10018

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of two other numbers. Look at these two examples.

a $17 \times 12 = 17 \times (10 + 2)$

$$= (10 + 2) + (10 + 2) + (10 + 2) + \dots$$

17 times

$$= 10 + 10 + 10 + \dots + 2 + 2 + 2 + \dots$$

17 times each

$$= (17 \times 10) + (17 \times 2)$$

$$= 170 + 34$$

$$= 204$$

so $17 \times (10 + 2) = 17 \times 10 + 17 \times 2$

b $14 \times 9 = 14 \times (10 - 1)$

$$= (10 - 1) + (10 - 1) + (10 - 1) + \dots$$

14 times

$$= 10 + 10 + 10 + \dots - 1 - 1 - 1 - \dots$$

14 times each

$$= (14 \times 10) - (14 \times 1)$$

$$= 140 - 14$$

$$= 126$$

so $14 \times (10 - 1) = 14 \times 10 - 14 \times 1$

Summary

If a , b and c stand for numbers, then:

$$a \times (b + c) = a \times b + a \times c$$

$$a \times (b - c) = a \times b - a \times c$$

The distributive law of arithmetic sometimes helps us multiply numbers more easily.

Example 4

Evaluate each product.

a 25×9

b 18×8

Solution

a $25 \times 9 = 25 \times (10 - 1)$

$$= 25 \times 10 - 25 \times 1$$

$$= 250 - 25$$

$$= 225$$

b $18 \times 8 = 18 \times (10 - 2)$

$$= 18 \times 10 - 18 \times 2$$

$$= 180 - 36$$

$$= 144$$

Video tutorial

Multiplying by 8, 9, 11
and 12

MAT07NAVT10010

Summary

- To multiply a number by 9, multiply by 10 and then subtract the number
- To multiply a number by 8, multiply by 10 and then subtract double the number

Example 5

Evaluate each product.

a 13×11

b 27×12

Solution

$$\begin{aligned} \text{a } 13 \times 11 &= 13 \times (10 + 1) \\ &= 13 \times 10 + 13 \times 1 \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

$$\begin{aligned} \text{b } 27 \times 12 &= 27 \times (10 + 2) \\ &= 27 \times 10 + 27 \times 2 \\ &= 270 + 54 \\ &= 324 \end{aligned}$$

Video tutorial

Multiplying by 8, 9, 11
and 12

MAT07NAVT10010

Summary

- To multiply a number by 11, multiply by 10 and then add the number
- To multiply a number by 12, multiply by 10 and then add double the number

Exercise 5-02 The distributive law

1 Copy and complete each multiplication.

$$\begin{aligned} \text{a } 18 \times 12 &= 18 \times (10 + \underline{\quad}) \\ &= 18 \times \underline{\quad} + 18 \times \underline{\quad} \\ &= \underline{\quad} + 36 \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b } 16 \times 9 &= \underline{\quad} \times (\underline{\quad} - 1) \\ &= \underline{\quad} \times 10 - \underline{\quad} \times 1 \\ &= 160 - \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{c } 21 \times 11 &= 21 \times (\underline{\quad} + \underline{\quad}) \\ &= \underline{\quad} \times 10 + 21 \times \underline{\quad} \\ &= 210 + \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{d } 15 \times 8 &= 15 \times (10 - \underline{\quad}) \\ &= \underline{\quad} \times 10 - \underline{\quad} \times \underline{\quad} \\ &= \underline{\quad} - \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

2 Evaluate each product.

a 15×9

b 18×9

c 26×9

d 31×9

e 16×8

f 13×8

g 22×8

h 14×8

i 27×9

j 45×8

k 38×8

l 29×9

See Example 4

See Example 5 3 Evaluate each product.

a 17×11

b 24×11

c 15×11

d 29×11

e 25×12

f 14×12

g 24×12

h 33×12

i 40×12

j 12×11

k 9×12

l 19×11

4 Use the distributive law to copy and complete each multiplication.

a $6 \times 22 = 6 \times (20 + \underline{\quad})$
 $= 6 \times \underline{\quad} + 6 \times \underline{\quad}$
 $= 120 + \underline{\quad}$
 $= \underline{\quad}$

b $12 \times 19 = \underline{\quad} \times (20 - \underline{\quad})$
 $= \underline{\quad} \times 20 - \underline{\quad} \times 1$
 $= 240 - \underline{\quad}$
 $= \underline{\quad}$

c $40 \times 41 = \underline{\quad} \times (\underline{\quad} + 1)$
 $= \underline{\quad} \times 40 + \underline{\quad} \times \underline{\quad}$
 $= 1600 + \underline{\quad}$
 $= \underline{\quad}$

d $25 \times 48 = \underline{\quad} \times (50 - \underline{\quad})$
 $= \underline{\quad} \times 50 - \underline{\quad} \times \underline{\quad}$
 $= \underline{\quad} - \underline{\quad}$
 $= \underline{\quad}$

Worked solutions

Exercise 5-02

MAT07NAWS10029

5 Use the distributive law to evaluate each product.

a 5×32

b 4×51

c 12×99

d 8×102

e 6×29

f 12×21

g 15×18

h 11×49

6 Which is the correct distributive law for calculating 14×12 ? Select the correct answer **A**, **B**, **C** or **D**.

A $14 \times 10 \times 2$

B $14 \times 10 + 2$

C $10 + 4 \times 10 + 2$

D $14 \times 10 + 14 \times 2$

5-03 Variables

Algebra uses letters or symbols to write general mathematical rules called **formulas** or **equations**. We have used algebra to write the laws of arithmetic we have learnt.

The commutative laws:

- $a + b = b + a$
- $a \times b = b \times a$

The associative laws:

- $(a + b) + c = a + (b + c)$
- $(a \times b) \times c = a \times (b \times c)$

The distributive law:

- $a \times (b + c) = a \times b + a \times c$
- $a \times (b - c) = a \times b - a \times c$

In the formulas above, a , b and c represent any three numbers and are called **variables** or **pronominals**. In algebra, a variable or pronominal is a symbol, usually a letter of the alphabet, that represents a number. It is called a **variable** because its value can vary (change), and a **pronominal** because it stands in place of a numeral.

Example 6

Examine each of the following number patterns, then write the general rule using a variable.

a $4 \times 1 = 4$

$7 \times 1 = 7$

$9 \times 1 = 9$

$1 \times 1 = 1$

$6 \times 1 = 6$

$b \times 1 = \underline{\hspace{2cm}}$

c $6 - 6 = 0$

$1 - 1 = 0$

$17 - 17 = 0$

$12 - 12 = 0$

$4 - 4 = 0$

b $9 + 9 + 9 = 3 \times 9$

$4 + 4 + 4 = 3 \times 4$

$11 + 11 + 11 = 3 \times 11$

$2 + 2 + 2 = 3 \times 2$

$5 + 5 + 5 = 3 \times 5$

$x + x + x = \underline{\hspace{2cm}}$

d $2 \times (-2) = -(2^2)$

$8 \times (-8) = -(8^2)$

$5 \times (-5) = -(5^2)$

$9 \times (-9) = -(9^2)$

$1 \times (-1) = -(1^2)$

Solution

The general rule is:

a $b \times 1 = b$

Any number multiplied by 1 is itself.

b $x + x + x = 3 \times x$

The sum of the same number 3 times is 3 times that number.

c $a - a = 0$

Other variables are possible, for example, $k - k = 0$

Any number minus itself is 0.

d $r \times (-r) = -(r^2)$

Other variables are possible, for example, $y \times (-y) = -(y^2)$

Any number multiplied by its opposite is equal to the negative of that number squared.

Algebraic abbreviations

Mathematicians prefer to write expressions as simply as they can. For example:

$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 9 \times 5$ and $2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

When writing algebraic expressions, we use the following abbreviations:

• $3 \times k = 3k$

for multiplication, we leave out the '×' symbol

• $m \div 4 = \frac{m}{4}$

for division, we can write in fraction form

• $r \times r = r^2$

for powers

• $1x = x$

for multiplying by 1

• $c \times a \times 6 = 6ac$

write the number first, then the variables in alphabetical order

Example 7

Video tutorial

Algebraic expressions

MAT07NAVT10011

Puzzle sheet

Algebra bingo game

MAT07NAPS10017

Simplify each expression.

a $d \times 8$

d $u \times 9 \times w$

g $z \times z \times z$

b $m + m + m + m + m$

e $6x + 2x$

h $3n - 2n$

c $k \div 2$

f $3 \times p \times 2 \times h$

i $y \times y \times 4$

Solution

a $d \times 8 = 8d$

b $m + m + m + m + m = 5m$

c $k \div 2 = \frac{k}{2}$

d $u \times 9 \times w = 9uw$

e $6x + 2x = 8x$

f $3 \times p \times 2 \times h = 3 \times 2 \times p \times h$
 $= 6hp$

g $z \times z \times z = z^3$

h $3n - 2n = 1n$
 $= n$

i $y \times y \times 4 = 4y^2$

Write the number first.

Write the number first, then the variables in alphabetical order.

$(x + x + x + x + x + x) + (x + x) = 8 \text{ lots of } x$

$(n + n + n) - (n + n) = \text{one lot of } n$

There is no need to write the '1'.

The '2' (squared) belongs to the y only.

Example 8

Skillsheet

Order of operations

MAT07NASS10005

Use order of operations to simplify each expression.

a $2 \times r + 4$

b $15 - d \div 3$

c $7 \div (n \times n) + 1$

Solution

a $2 \times r + 4 = 2r + 4$

b $15 - d \div 3 = 15 - (d \div 3)$
 $= 15 - \frac{d}{3}$

c $7 \div (n \times n) + 1 = 7 \div n^2 + 1$
 $= \frac{7}{n^2} + 1$

 $\times$ first, then $+$ $\div$ first, then $-$ $()$ first, then $\div$, then $+$

Example 9

Write each expression in expanded form.

a $3ab$

b $-8st^2$

c $x^2 - \frac{y}{10}$

Solution

a $3ab = 3 \times a \times b$

b $-8st^2 = -8 \times s \times t \times t$

c $x^2 - \frac{y}{10} = x \times x - y \div 10$

Note that expanding is the opposite of simplifying.

Exercise 5-03 Variables

- 1 Examine each number pattern, then write the general rule using a variable. A clue has been given in parts **a** and **b**.

a $8 \div 8 = 1$

$1 \div 1 = 1$

$-2 \div (-2) = 1$

$14 \div 14 = 1$

$-7 \div (-7) = 1$

$u \div u = \underline{\hspace{2cm}}$

b $6 + 6 = 2 \times 6$

$10 + 10 = 2 \times 10$

$4 + 4 = 2 \times 4$

$9 + 9 = 2 \times 9$

$-4 + (-4) = 2 \times (-4)$

$p + p = \underline{\hspace{2cm}}$

c $3 + (-3) = 0$

$7 + (-7) = 0$

$20 + (-20) = 0$

$1 + (-1) = 0$

$16 + (-16) = 0$

d $2 \times 0 = 0$

$19 \times 0 = 0$

$-3 \times 0 = 0$

$0 \times 0 = 0$

$-24 \times 0 = 0$

e $7 + 0 = 7$

$-7 + 0 = -7$

$3 + 0 = 3$

$11 + 0 = 11$

$-100 + 0 = -100$

f $-5 \times (-5) = 5^2$

$-7 \times (-7) = 7^2$

$-12 \times (-12) = 12^2$

$-1 \times (-1) = 1^2$

$-8 \times (-8) = 8^2$

- 2 Simplify each expression.

a $w \times 3$

d $a + a + a + a + a + a$

g $5m + 2m$

j $4 \times f \times f$

m $16 \div n$

p $h + h - h$

s $6 \times p \times r \times (-2) \times r$

b $b \times b$

e $e \div 5$

h $2 \times w \times 3 \times h$

k $6d - 5d$

n $g \times 2 \times g$

q $12q - 4q$

t $a + a + a + b + b$

c $9 \times c \times a \times d$

f $f + f$

i $5m - 2m$

l $a \times c \times b \times 10$

o $3x + 6x$

r $2 \times m \times 4 \times n$

u $a \times a \times a \times b \times b$

- 3 Use order of operations to simplify each expression.

a $16 \div (a \times 5)$

d $4 \times s - 8$

g $k \div (9 + m)$

b $7 + 3 \times n$

e $c \times c + d \times d$

h $12 - r \div 2$

c $(e - 6) \div 2$

f $k \div 9 + m$

i $7 + u \times u \times 7$

See Example 6

See Example 7

Worked solutions

Exercise 5-03

MAT07NAWS10030

See Example 8

- 4 Which of the following is equal to $m \times m \times 3 + 4$? Select the correct answer **A**, **B**, **C** or **D**.
A $3m^2 + 4$ **B** $2m^3 + 4$ **C** $6m + 4$ **D** $m^2 + 7$

- 5 Explain the meaning of:

a xy

b $\frac{2n}{r}$

c $1x = x$

- 6 Which of the following is not equal to $2p$? Select the correct answer **A**, **B**, **C** or **D**.

A $3p - p$

B p^2

C $p + p$

D $p \times 2$

See Example 9

- 7 Write each expression in expanded form.

a $12rs$

b $-2n^2$

c $\frac{8}{f}$

d $r^2 + t^2$

e $9a^2b$

f $5mn - 2a$

g $\frac{x+1}{3}$

h q^3

i $14 - 2d^2$

j $\frac{-4c}{5}$

k x^2y^2

l $4j - \frac{8}{k}$

- 8 What is $4m^3$ in expanded form? Select the correct answer **A**, **B**, **C** or **D**.

A $4m \times 4m \times 4m$

B $4m + 4m + 4m$

C $4 \times m + m + m$

D $4 \times m \times m \times m$

Investigation: The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic.

For example, if we add zero to any number, the answer is still that number.

If we let N stand for any number: $N + 0 = N$

In groups of two to four students, answer the following questions.

- 1 Describe in words what each number rule below means.

a $N \times 1 = N$

b $N \times 0 = 0$

c $a - b \neq b - a$

d $a + b + c = b + a + c$

e $N - 0 = N$

f $ab = ba$

- 2 Write each of the following rules algebraically using variables.

a Any number divided by 1 equals itself.

b Multiplying a number by 8 is the same as doubling it three times.

c Any three numbers can be multiplied together in any order.

d Any number added to itself is the same as multiplying that number by 2.

e Any number subtracted from itself equals 0.

f Any number multiplied by its reciprocal equals 1.

- 3 Is each of the following equations true or false? Test your decision by substituting a number for the variable and checking.

a $a \div b = b \div a$

b $N \div N = 1$

c $4a - a = 4$

d a is a factor of a

e If N is even, then $N + 3$ is odd

f $\frac{1}{2}N = N - 2$

g $a + (-a) = 2a$

h $0 \div N = 0$

i $N \div 0 = 0$

j $N \div 1 = N$

- 4 If k is an odd number, what is an expression for:

a the previous odd number?

b the next even number?

5-04 From words to algebraic expressions

We will continue to use variables to convert worded expressions into algebraic expressions.

Example 10

If A represents any number, write an algebraic expression for:

- a three times that number
- b three less than that number
- c the next consecutive number
- d that number multiplied by itself
- e the square root of that number
- f one-third of that number

← **Consecutive** means 'following in order': for example, 7, 8, 9 are consecutive numbers

Solution

- a $3 \times A = 3A$
- b $A - 3$
- c $A + 1$

The next number after A is $A + 1$
(for example, the next number after 4 is $4 + 1 = 5$)

- d $A \times A = A^2$
- e $\sqrt{A}$
- f $\frac{1}{3}A = A \div 3 = \frac{A}{3}$

Puzzle sheet

What's the expression?

MAT07NAPS10018

Worksheet

Writing algebraic expressions

MAT07NAWK00002

Example 11

Write an expression for:

- a the sum of m and 5
- b double k
- c u increased by 10
- d the difference between p and q
- e the product of r and 12
- f how many times y divides into 50

Solution

- a $m + 5$
- b $2k$
- c $u + 10$
- d $p - q$
- e $12r$
- f $50 \div y = \frac{50}{y}$

Exercise 5-04 From words to algebraic expressions

- See Example 10**
- If N represents any number, write an algebraic expression for:

a the number added to itself	b half the number
c triple the number	d 7 less than the number
e one-tenth of the number	f the previous consecutive number
g 5 times the number	h the sum of the number and 21
i the difference between the number and 1	j the number divided by 8
k the number increased by 3	l the number cubed
m the square root of the number	
 - Suppose that in question 1, the variable X represents any number instead of N . What difference would this make to your answers? Does it matter which letter of the alphabet you choose to use?

- See Example 11**
- Write an expression for:

a the sum of all x , y and z	b the difference between b and c
c the product of u and v	d the product of u , v and v
e the quotient of C and d	f the sum of m and n , divided by 2
g the square root of the product of 10 and a	h 3 less than b
i 5 decreased by c	j c decreased by 5
k the number of times that 3 divides into e	l t squared
m the product of x , z and 9	n how much k is more than 12
o half of the difference between 20 and g	
 - Write an expression for:

a the number of students in a class if there are b boys and g girls
b the number of chicken nuggets needed for a party if there are n guests and each guest eats an average of 4 nuggets
c the number of children left in a class of d students if r of them have gone to the zone swimming carnival
d the amount of money earned selling s sushi rolls at the canteen, where each roll is sold at \$2
e the cost of one movie ticket if it costs \$ a for three tickets
f the perimeter of a rectangle of length x and width y
g the area of a rectangle of length x and width y
h the total cost of buying m milkshakes and i ice creams, where each milkshake costs \$4 and each ice cream costs \$2

Worked solutions

Exercise 5-04

MAT07NAWS10031

Worked solutions

Exercise 5-04

MAT07NAWS10031

- What is the correct expression for the total admission cost to Water World for c children at \$21 each and a adults at \$27 each? Select the correct answer **A**, **B**, **C** or **D**.

A $21a + 27c$	B $48ac$	C $27a + 21c$	D $a + c + 48$

Mental skills 5A Maths without calculators

Multiplying an even number by 5

To multiply an even number by 5, it's easier to halve it, then multiply by 10 (by inserting a 0 at the end). This is because $\frac{1}{2} \times 10 = 5$.

1 Study each example.

a $14 \times 5 = 14 \times \frac{1}{2} \times 10 = 7 \times 10 = 70$

b $36 \times 5 = 36 \times \frac{1}{2} \times 10 = 18 \times 10 = 180$

c $22 \times 5 = 11 \times 10 = 110$

d $18 \times 5 = 9 \times 10 = 90$

2 Now simplify each product.

a 32×5

b 26×5

c 12×5

d 28×5

e 42×5

f 54×5

g 38×5

h 44×5

i 60×5

j 34×5

k 16×5

l 58×5

Technology Using variables and values

- 1 **a** Create the following spreadsheet to calculate the value of the expression $x + 2$ for different values of x .

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2										
3											

- b** To evaluate $x + 2$ when $x = 1$ (column B), enter **=B1+2** into cell B2. What value should you get?

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2	=B1+2									
3											

- c** To complete the table, use **Fill Right** to copy the rule across row 2, from cell B2 until cell K2. **Centre** your answers in each of the cells. Check that the answers look correct.
- d** Type over some values in row A of the spreadsheet to find the value of $x + 2$ when:
- i** $x = 6$ **ii** $x = 17$ **iii** $x = 31$
- e** Predict what value of x will give the answer 96 and check your guess using the spreadsheet.

- 2 a Create the following spreadsheet to calculate the value of $x - 3$ for values of x .

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	$x - 3$										

- b In cell B2, enter a formula beginning with **=B1** to calculate $x - 3$.
 c Use **Fill Right** to copy this formula up to cell K2. **Centre** your answers.
 d Type over some values in row A to evaluate $x - 3$ when:
 i $x = 9$ ii $x = 28$
 e Predict what value of x will give the answer:
 i 37 ii 105
 Check your guess using the spreadsheet.

- 3 a Create the following spreadsheet and in cell B2, enter a formula to evaluate $x \div 2$.

	A	B	C	D	E	F	G	H	I	J	K
1	x	-8	-6	-4	-2	0	2	10	20	30	40
2	$x \div 2$										

- b Use **Fill Right** to copy this formula up to cell K2. **Centre** your answers.
 c Use the spreadsheet to evaluate $x \div 2$ when:
 i $x = -10$ ii $x = 6$
 d Predict what value of x will give the answer 21 and check using the spreadsheet.
 4 a Create the following spreadsheet and in cell B2, enter a formula to evaluate $10x - 1$.

	A	B	C	D	E	F	G
1	x	-10	-5	0	5	10	15
2	$10x - 1$						

- b Use **Fill Right** to copy this formula up to cell G2. Centre your answers.
 c Use the spreadsheet to evaluate $10x - 1$ when:
 i $x = 8$ ii $x = -7$
 d Predict what value of x will give the answer 199 and check using the spreadsheet.
 5 a Create a new spreadsheet to evaluate x^2 for different values of x . In cell A1, enter the label 'x', then along row A enter the following values: -2, -1, 0, and so on until 4 is in cell H1.
 b In cell A2, enter the label ' x^2 ' and in cell B2, enter a formula to evaluate x^2 .
 c **Fill Right** to complete the answers for cells C2 to H2.

Homework sheet

Algebra

MAT07NAHS10019

Worksheet

Substitution

MAT07NAWK10036

5-05 Substitution

The word **substitution** means replacing one thing with another thing. In sport, substitution means to replace one player with another during a game.

In algebra, substitution means to replace a variable with a number to evaluate an algebraic expression.

Example 12

- a Evaluate $4k - 9$ when $k = 15$
 b If $r = m^2 + 3m$, evaluate r when $m = 4$
 c If $a = 6$, $b = -4$ and $c = 10$, evaluate $\frac{c-a}{b}$

Solution

a When $k = 15$,

$$4k - 9 = 4 \times 15 - 9$$

$$= 51$$

b When $m = 4$,

$$r = 4^2 + 3 \times 4$$

$$= 28$$

c When $a = 6$, $b = -4$ and $c = 10$,

$$\frac{c-a}{b} = \frac{10-6}{-4}$$

$$= \frac{4}{-4}$$

$$= -1$$

Puzzle sheet

Substitution game

MAT07NAPS10019

Substituting into formulas

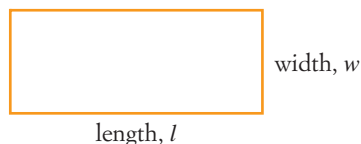
A **formula** is a general rule written as an algebraic equation showing the relationship between variables. Solving mathematical problems often involves substituting values into formulas.

Example 13

The formula for the perimeter of a rectangle is

$$P = 2l + 2w$$

where P is the perimeter, l is the length and w is the width.



Use this formula to calculate the perimeter of a rectangle with length 16 m and width 3 m.

Solution

When length $l = 16$ and width $w = 3$,

$$P = 2l + 2w$$

$$= 2 \times 16 + 2 \times 3$$

$$= 38 \text{ m}$$

Exercise 5-05 Substitution

See Example 12

- Evaluate $k + 3$ when:
 - $k = 12$
 - $k = 48$
 - $k = 119$
 - $k = -21$
- Evaluate $45 - k$ when:
 - $k = 5$
 - $k = -13$
 - $k = 28$
 - $k = 45$
- Evaluate $4k$ when:
 - $k = 2$
 - $k = 11$
 - $k = -4$
 - $k = \frac{1}{2}$
- Find the value of $5k + 1$ when:
 - $k = 3$
 - $k = -4$
 - $k = 20$
 - $k = 7$
- Find the value of $9k - 8$ if:
 - $k = 9$
 - $k = -5$
 - $k = 0$
 - $k = 2$
- Find the value of $\frac{k}{3}$ if:
 - $k = 15$
 - $k = 33$
 - $k = -9$
 - $k = 57$
- If $a = 6$, $b = 14$ and $c = -2$, find:
 - $a + b - c$
 - $2b + c$
 - $\frac{a - b}{c}$
 - $ab - 50$
- If $m = 4$, $n = -2$ and $p = 3$, find:
 - m^2
 - $3n + p$
 - mnp

Worked solutions

Exercise 5-05

MAT07NAWS10032

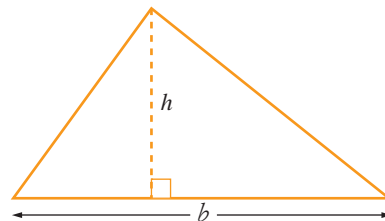
- If $d = 4c - 10$, find d when:
 - $c = 5$
 - $c = 8$
 - $c = -10$
- If $b = 3t - 1$, find b when:
 - $t = 5$
 - $t = 12$
 - $t = -9$

See Example 13

- The formula for the cost of a party is $C = 45n + 500$, where C is the cost in dollars and n is the number of guests.

Use this formula to calculate the cost of a party for 72 guests.

- The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area of the triangle, b is the length of the base and h is the height. Find the area of a triangle with base length 9 m and height 6 m.



- The number of hours of sleep recommended for children has the formula $H = 17 - \frac{A}{2}$, where H is the number of hours and A is the age of the child. Find the number of hours of sleep recommended for a 13-year-old.

Worked solutions

Exercise 5-05

MAT07NAWS10032

- The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$, where F is the temperature in Fahrenheit and C is the temperature in Celsius. Use the formula to convert 30°C to Fahrenheit.

- 15 A locksmith charges according to this formula $C = 42h + 65$, where C is the charge in dollars and h is the number of hours worked.
How much does the locksmith charge for working for 2 hours?

Technology Substitution

- 1 Create the spreadsheet below for evaluating five algebraic expressions involving two variables, a and b , whose values are entered in columns A and B.

	A	B	C	D	E	F	G
1	a	b	ab	a + b	2a + b	5(a + b)	3b ÷ 2a
2	2	1	=A2*B2				
3	10	0					
4	6	8					
5	-3	5					
6	0.4	1.7					
7	86	45					

- 2 Column C is for evaluating ab for the different values of a and b shown in rows 2 to 7. In cell C2, enter the formula for calculating ab using the values in A2 and B2.
3 **Fill Down** to copy this formula down to C7.
4 Enter appropriate formulas for the other four expressions in cells D2 to G2. Use brackets where necessary.
5 For each column, D to G, **Fill Down** to row 7.
6 Create the new spreadsheet below that shows a value for each variable, for example, $m = 5$, $n = 2$, $p = -4$.

	A	B	C	D	E	F	G
1	m	n	p	t	u	A	B
2	5	2	-4	12	0	50	31
3							

- 7 In each cell given, enter an appropriate formula to evaluate the algebraic expression shown.
- | | |
|---|---------------------------------|
| a In cell A4: $m + p$ | b In cell B4: npt |
| c In cell C4: $n + 2p$ | d In cell D4: $B - A$ |
| e In cell A5: $(p - n) \div t$ | f In cell B5: $np + tu$ |
| g In cell C5: $\frac{mt}{p}$ | h In cell D5: $u - At$ |
| i In cell A6: $A^2 - B^2$ | j In cell B6: $6p + t$ |
| k In cell C6: $\frac{m(t - n)}{A}$ | l In cell D6: $3n^2 - p$ |

Worksheet

Why aren't they the same?

MAT07NAWK10037

Investigation: Equivalent expressions

1 Determine whether each equation below is true or false by substituting different numbers for the variable and checking whether the left expression equals the right expression for each number.

a $6 + 4m + 4 = 10 + 4m$

b $5x + 3x = 8x^2$

c $4k - k = 4$

d $7g + 12 = 19g$

e $3 + 7n + 2n = 3 + 9n$

f $3p \times 7 = 21p$

g $21m \div 3 = 7m$

h $2y \times 4y = 24y$

i $2r \times 2 \times 2r = 8r^2$

2 For each of the false statements in question 1, work out what the correct statement should be.

5-06 Equations

Worksheet

Guess-and-check

MAT07NAWK10038

An **equation** is a statement involving a variable (such as x), numbers and an equals (=) sign, for example, $4x + 5 = 57$. When we find the correct value that makes the equation true, we **solve** the equation. The value is called the **solution** to the equation.

There are different ways of solving equations. The simplest method is the 'guess, check and improve' method.

The 'guess, check and improve' method

The 'guess, check and improve' method involves:

- Guessing a number for the solution
- Checking the guess by substituting the number into the equation
- Improving on the guess by testing better numbers until the correct solution is found

Example 14

Use 'guess, check and improve' to solve the equation $4x + 5 = 57$ for x .

Solution

Guess	Check	Comment
$x = 3$	$4 \times 3 + 5 = 17$	Smaller than 57. Try a bigger number.
$x = 10$	$4 \times 10 + 5 = 45$	Still smaller than 57. Try a bigger number.
$x = 20$	$4 \times 20 + 5 = 85$	Bigger than 57. Try a number between 10 and 20.
$x = 15$	$4 \times 15 + 5 = 65$	Still bigger than 57. Try a number between 10 and 15.
$x = 13$	$4 \times 13 + 5 = 57$	Correct.

The solution is $x = 13$.

Exercise 5-06 Equations

1 Solve each equation.

a $x + 3 = 6$

b $x - 3 = 6$

c $a + 12 = 17$

d $m - 5 = 15$

e $b + 1 = 11$

f $5c = 20$

g $4k = 24$

h $d - 3 = 10$

i $10 + m = 25$

j $14 - x = 9$

k $\frac{y}{3} = 5$

l $\frac{n}{5} = 5$

2 What is the solution to $2x - 6 = 24$? Select the correct answer A, B, C, or D.

A $x = 12$

B $x = 15$

C $x = 14$

D $x = 9$

3 Solve each equation.

a $2x + 4 = 14$

b $3p - 5 = 16$

c $4k + 6 = 26$

d $5x - 9 = 11$

e $7x + 6 = 27$

f $8x - 30 = 34$

g $\frac{m}{4} - 3 = 4$

h $\frac{a}{2} + 12 = 18$

i $10 - 3d = 4$

j $12 - 5n = 7$

k $\frac{r-9}{2} = 3$

l $\frac{k+1}{3} = 4$

m $2p + 3 = -1$

n $\frac{r}{5} + 4 = 6$

o $\frac{2y}{3} = 10$

p $2 \times (x + 1) = 6$

See Example 14

Worked solutions

Exercise 5-06

MAT07NAWS10033

5-07 One-step equations

The 'guess, check and improve' method is simple but slow. There are two **algebraic** methods for solving equations that are faster and more efficient:

- **balancing:** performing the same operation on both sides of the equation
- **backtracking:** undoing each operation by performing the inverse (opposite) operation

Puzzle sheet

Equations dominoes

MAT07NAPS10020

TLF learning object

Algebra balance
scales: negatives
(L3510)

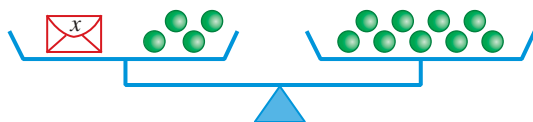
Example 15

Solve the equation $x + 4 = 9$.

Solution

Method 1: The balancing method

Let x represent an unknown number in an envelope, and $\bullet$ represent 1. We can represent the equation $x + 4 = 9$ on balance scales like this.



The two sides of the balance are equal. To find the value of x we can remove 4 balls from both sides.



The solution is $x = 5$.

The equation can also be solved algebraically without diagrams.

$$x + 4 = 9$$

$$x + 4 - 4 = 9 - 4$$

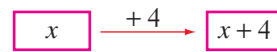
$$x = 5$$

Subtracting 4 from both sides

Check: $5 + 4 = 9$.

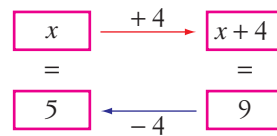
Method 2: The backtracking method

First use a flowchart to show how we get from x to $x + 4$.



In this equation, $x + 4 = 9$. To backtrack (get back) to x , we need to undo the operation 'add 4,' which is 'subtract 4'.

Use a reverse flowchart to undo what has been done to x .



The solution is $x = 5$.

The equation can also be solved algebraically without flowcharts.

$$x + 4 = 9$$

$$x = 9 - 4$$

$$x = 5$$

Undo '+ 4' by subtracting 4

Check: $5 + 4 = 9$.

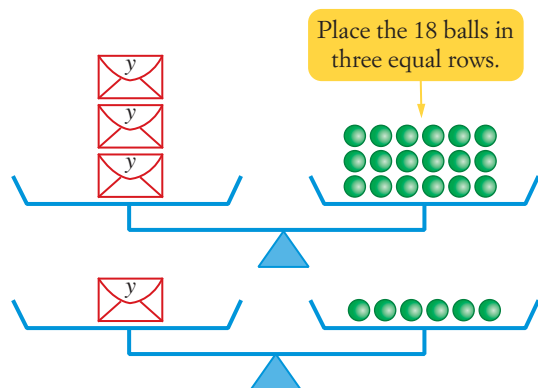
Example 16

Solve $3y = 18$.

Solution

Method 1: The balancing method

Represent the equation $3y = 18$ using balance scales.



To find the value of y , divide both sides by 3.

The solution is $y = 6$.

Solving this algebraically, we have:

$$3y = 18$$

$$\frac{3y}{3} = \frac{18}{3}$$

$$y = 6$$

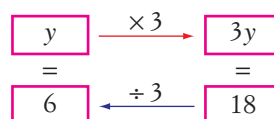
Dividing both sides by 3

$$\text{Check: } 3 \times 6 = 18$$

Method 2: The backtracking method

First use a flowchart to show how we get from y to $3y$.

In this equation, $3y = 18$. To backtrack to y , we need to undo the operation ' $\times 3$ ', which is ' $\div 3$ ', using a reverse flowchart:



The solution is $y = 6$.

Solving the problem algebraically, we have:

$$3y = 18$$

$$y = \frac{18}{3}$$

$$y = 6$$

Undo ' $\times 3$ ' in $3y$ by dividing by 3

$$\text{Check: } 3 \times 6 = 18$$

Summary

To solve an equation, aim to have the variable (such as x) on one side of the equation and a number on the other side, in the form:

$$x = \underline{\hspace{2cm}}$$

Check your solution by substituting it back into the equation.

Example 17

Solve $u - 5 = 7$.

Solution

Method 1: The balancing method

$$u - 5 = 7$$

$$u - 5 + 5 = 7 + 5$$

$$u = 12$$

Add 5 to both sides

$$\text{Check: } 12 - 5 = 7$$

Method 2: The backtracking method

$$u - 5 = 7$$

$$u = 7 + 5$$

$$u = 12$$

Undo ' $- 5$ ' by adding 5

$$\text{Check: } 12 - 5 = 7$$

Example 18

Solve $\frac{k}{7} = 2$.

Solution**Method 1: The balancing method**

$$\frac{k}{7} = 2$$

$$\frac{k}{7} \times 7 = 2 \times 7$$

$$k = 14$$

Multiply both sides by 7

Check: $\frac{14}{7} = 2$

Method 2: The backtracking method

$$\frac{k}{7} = 2$$

$$k = 2 \times 7$$

$$k = 14$$

Undo ' $\div 7$ ' by multiplying by 7

Check: $\frac{14}{7} = 2$

Summary

Operation	Inverse operation
+	−
−	+
$\times$	$\div$
$\div$	$\times$

Exercise 5-07 One-step equations**See Example 15** 1 Solve each equation, showing the working.

a $w + 8 = 15$

b $x + 2 = 20$

c $m + 3 = 19$

d $p + 12 = 30$

e $x + 13 = 22$

f $k + 11 = 20$

g $5 + m = 2$

h $6 + y = 16$

i $15 + d = 6$

See Example 16 2 Solve each equation, showing the working.

a $3m = 18$

b $5n = 20$

c $4k = 44$

d $2c = 32$

e $9x = 81$

f $3x = -27$

g $7d = 42$

h $6h = -30$

i $10a = 70$

See Example 17

3 Solve each equation, showing the working.

a $p - 3 = 8$

b $m - 11 = 2$

c $x - 5 = 12$

d $y - 20 = 40$

e $k - 21 = 3$

f $n - 3 = 18$

g $7 = d - 11$

h $y - 13 = -4$

i $-5 = m - 9$

Worked solutions

Exercise 5-07

MAT07NAWS10034

4 Solve each equation, showing the working.

a $\frac{m}{3} = 4$

b $\frac{d}{2} = 5$

c $\frac{x}{5} = 8$

d $\frac{k}{4} = 12$

e $\frac{x}{10} = 6$

f $\frac{a}{2} = -4$

g $15 = \frac{k}{2}$

h $-11 = \frac{n}{5}$

5 Solve each equation, showing the working.

a $m - 3 = 16$

b $3 + p = 11$

c $24 = p + 5$

d $n + 7 = -1$

e $3x = 30$

f $n - 14 = -5$

g $3x = 21$

h $4y = -18$

i $\frac{h}{7} = 1$

j $-2x = -6$

k $6 + k = -4$

l $\frac{n}{4} = 0$

See Example 18

Worked solutions

Exercise 5-07

MAT07NAWS10034

5-08 Two-step equations

The following 'two-step' equations require two operations in finding their solutions.

Homework sheet

Equations 1

MAT07NAHS10022

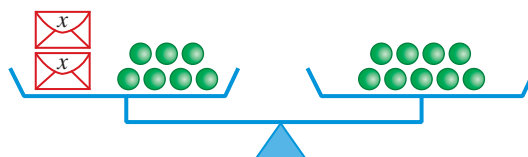
Example 19

Solve the equation $2x + 7 = 9$.

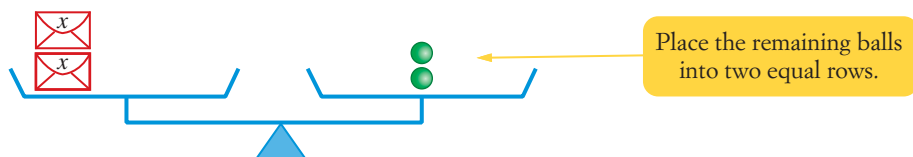
Solution

Method 1: The balancing method

We can represent $2x + 7 = 9$ using balance scales.



First subtract 7 balls from both sides.



Then divide both sides by 2.



The solution is $x = 1$.

Using algebra we have: $2x + 7 = 9$

$$2x + 7 - 7 = 9 - 7$$

$$2x = 2$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

Step 1: Subtracting 7 from both sides

Step 2: Dividing both sides by 2

Check: $2 \times 1 + 7 = 9$

Video tutorial

Two-step equations

MAT07NAVT10012

TLF learning object

Algebra balance
scales: negatives
(L3510)

Worksheet

Backtracking

MAT07NAWK10039

Worksheet

Backtracking

MAT07NAWK00003

Puzzle sheet

Solving equations

MAT07NAPS00006

Video tutorial

Solving equations

MAT07NAVT00006

Worksheet

Equations match

MAT07NAWK10040

Worksheet

Equations writing
activity

MAT07NAWK10041

Method 2: The backtracking method

Use a flowchart to go from x to $2x + 7$.

$2x + 7 = 9$. To backtrack, we need to undo the operations ' $\times 2$ ' and ' $+ 7$ ' in **reverse order**, so ' $- 7$ ' and ' $\div 2$ ' using a reverse flowchart.

The solution to $2x + 7 = 9$ is $x = 1$.

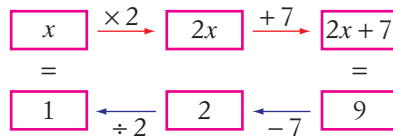
Using algebra, we have: $2x + 7 = 9$

$$2x = 9 - 7$$

$$2x = 2$$

$$x = \frac{2}{2}$$

$$x = 1$$



Step 1: Undo ' $+ 7$ ' by subtracting 7

Step 2: Undo ' $\times 2$ ' in $2x$ by dividing by 2

Check: $2 \times 1 + 7 = 9$

Note that inverse operations are performed in **reverse order**. For example, to undo putting on our socks and shoes, we take off our shoes first, then our socks.

Example 20

Solve $\frac{M}{3} + 2 = 5$.

Solution

Method 1: The balancing method

$$\frac{M}{3} + 2 = 5$$

$$\frac{M}{3} + 2 - 2 = 5 - 2$$

$$\frac{M}{3} = 3$$

$$\frac{M}{3} \times 3 = 3 \times 3$$

$$M = 9$$

Method 2: The backtracking method

$$\frac{M}{3} + 2 = 5$$

$$\frac{M}{3} = 5 - 2$$

$$\frac{M}{3} = 3$$

$$M = 3 \times 3$$

$$M = 9$$

Step 1: Subtracting 2 from both sides

Simplify.

Step 2: Multiplying both sides by 3

Check: $\frac{9}{3} + 2 = 3 + 2 = 5$

Step 1: Undo ' $+ 2$ ' by subtracting 2.

Step 2: Undo ' $\div 3$ ' by multiplying by 3.

Check: $\frac{9}{3} + 2 = 3 + 2 = 5$

Exercise 5-08 Two-step equations

1 Solve each equation showing all steps. Remember to check your answers.

a $2x + 1 = 11$

b $3x + 8 = 17$

c $4x + 5 = 13$

d $4x + 9 = 25$

e $5d + 2 = 47$

f $2x + 10 = -6$

g $-3x + 10 = 4$

h $-4x + 9 = 5$

i $-5x + 8 = -12$

j $-7x + 4 = -17$

k $-2x + 7 = 9$

l $-4x + 14 = 22$

2 The following is Liam's incorrect solution for $7x + 5 = 13$.

$$7x + 5 = 13$$

$$7x = 13 - 5 \quad \text{Line 1}$$

$$7x = 8 \quad \text{Line 2}$$

$$x = \frac{8}{7} \quad \text{Line 3}$$

$$x = 1\frac{1}{8} \quad \text{Line 4}$$

In which of the following was the error made? Select A, B, C or D.

A Line 1

B Line 2

C Line 3

D Line 4

3 Solve each equation showing all steps.

a $2x - 5 = 9$

b $3x - 1 = 23$

c $5x - 3 = 12$

d $2x - 4 = 18$

e $6x - 4 = 8$

f $5x - 19 = -9$

g $4x - 7 = 5$

h $7x - 3 = 32$

i $2x - 4 = -14$

j $5x - 2 = 43$

k $3x - 4 = -10$

l $8x - 12 = -20$

4 What is the solution for $\frac{n}{4} - 8 = 7$? Select the correct answer A, B, C or D.

A $n = 4$

B $n = 20$

C $n = 36$

D $n = 60$

5 Solve each equation.

a $\frac{x}{2} + 5 = 9$

b $\frac{m}{3} + 6 = 9$

c $\frac{k}{5} + 2 = 6$

d $\frac{a}{4} + 4 = 8$

e $\frac{n}{2} + 6 = 14$

f $\frac{h}{7} + 2 = -3$

g $\frac{m}{2} - 8 = -5$

h $\frac{k}{5} - 7 = -4$

i $\frac{x}{3} - 6 = -10$

j $\frac{a}{3} - 1 = 2$

k $\frac{h}{5} - 4 = -7$

l $\frac{x}{2} - 3 = 4$

See Example 19

Worked solutions

Exercise 5-08

MAT07NAWS10035

Worked solutions

Exercise 5-08

MAT07NAWS10035

See Example 20



Mental skills 5B Maths without calculators

Multiplying by 9, 11 and 12

To multiply a number by 9, multiply by 10 and then subtract the number.

This is because 10 times a number minus the same number equals 9 times the number.

1 Study each example.

a $14 \times 9 = 14 \times 10 - 14 = 140 - 14 = 126$

b $25 \times 9 = 25 \times 10 - 25 = 250 - 25 = 225$

c $18 \times 9 = 18 \times 10 - 18 = 180 - 18 = 162$

2 Now simplify each product.

a 12×9

b 27×9

c 46×9

d 19×9

e 34×9

f 63×9

g 21×9

h 15×9

To multiply a number by 11, multiply by 10 and then add the number.

This is because 10 times a number plus the same number equals 11 times the number.

3 Study each example.

a $26 \times 11 = 26 \times 10 + 26 = 260 + 26 = 286$

b $13 \times 11 = 13 \times 10 + 13 = 130 + 13 = 143$

c $35 \times 11 = 35 \times 10 + 35 = 350 + 35 = 385$

4 Now simplify each product.

a 17×11

b 22×11

c 38×11

d 40×11

e 25×11

f 19×11

g 54×11

h 31×11

To multiply a number by 12, multiply by 10, then add double the number.

This is because 10 times a number plus double the same number equals 12 times the number.

5 Study each example.

a $22 \times 12 = 22 \times 10 + 22 \times 2 = 220 + 44 = 264$

b $16 \times 12 = 16 \times 10 + 16 \times 2 = 160 + 32 = 192$

c $70 \times 12 = 70 \times 10 + 70 \times 2 = 700 + 140 = 840$

6 Now simplify each product.

a 44×12

b 15×12

c 29×12

d 31×12

e 52×12

f 18×12

g 26×12

h 37×12

Just for the record

Ancient Egyptian equations

The Rhind Papyrus from ancient Egypt is one of the earliest mathematical documents we have, dating back to 1650 BCE. It now belongs to the British Museum, and contains the following equation written in hieroglyphics (Egyptian picture symbols).



The algebraic translation is: $x + \frac{2}{3}x - \frac{1}{3}(x + \frac{2}{3}x) = 10$

Note that legs walking left mean 'minus' $\curvearrowright$, legs walking right mean 'add' $\curvearrowleft$.

Find out why the Rhind Papyrus has that name.

5-09 Equation problems

Many mathematical problems are stated in words. We can solve these problems by translating the problems into algebraic symbols and equations.

Homework sheet

Equations 2

MAT07NAHS10023

Example 21

When a number is doubled and then 5 is added, the answer is 69. Find the number.

Solution

Let the number be x .

Translating the problem into an equation:

$$x \times 2 + 5 = 69$$

$$2x + 5 = 69$$

Solve the equation.

$$2x + 5 - 5 = 69 - 5$$

$$2x = 64$$

$$\frac{2x}{2} = \frac{64}{2}$$

$$x = 32$$

The number is 32.

Subtracting 5 from both sides

Dividing both sides by 2

Check: $2 \times 32 + 5 = 69$

Example 22

Tom is 8 years older than Susi. If the sum of their ages is 22, find their ages.

Solution

Let Susi's age be n .

So Tom's age is $n + 8$.

$$n + n + 8 = 22$$

$$2n + 8 = 22$$

$$2n + 8 - 8 = 22 - 8$$

$$2n = 14$$

$$\frac{2n}{2} = \frac{14}{2}$$

$$n = 7$$

So Susi is 7 and Tom is $7 + 8 = 15$.

Tom is 8 years older.

Susi's age + Tom's age = 22

Simplifying $n + n$ to $2n$

Subtracting 8 from both sides

Dividing both sides by 2

Check: $2 \times 7 + 8 = 22$

Check: $7 + 15 = 22$

Summary

For word problems involving equations:

- Choose a pronumeral
- Translate the problem into an equation
- Solve the equation
- Write a sentence that answers the problem

Exercise 5-09 Equation problems

See Example 21

- 1 Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information.
 - a Five tickets for a film cost \$55. How much does each ticket cost? (Let t represent the price of a ticket.)
 - b Ten oranges cost \$4.80. How much does each orange cost? (Let x represent the cost of one orange.)
 - c A number is doubled and the result is 110. What is the number? (Let n represent the unknown number.)
 - d A number has 4 subtracted from it and the result is 6. Find the number. (Let y represent the unknown number.)



2 For each word problem, select the correct equation **A**, **B**, **C** or **D**. Then solve the equation to solve the problem.

- a Jarrad has collected 1794 beetles. This is 6 times as many beetles as Lisa has in her collection. How many beetles does Lisa have?

A $1794 - N = 6$ **B** $6N = 1794$ **C** $N = 6 \times 1794$ **D** $N + 6 = 1794$

- b Kurt mixed 590 mL of white paint with some blue paint. He mixed 1.73 L of paint altogether. How much blue paint did he use?

A $N + 590 = 1730$ **B** $N - 590 = 1730$ **C** $\frac{N}{590} = 1730$ **D** $590N = 1730$

- c Fourteen packets of chocolate biscuits are packed in a box. The supermarket sold 546 packets of biscuits. How many boxes were sold?

A $14N = 546$ **B** $546 - N = 14$ **C** $N + 14 = 546$ **D** $\frac{N}{14} = 546$

- d When a certain number is subtracted from 100, the result is 47. What is the number?

A $N - 100 = 47$ **B** $100 + 47 = N$ **C** $N - 47 = 100$ **D** $100 - N = 47$

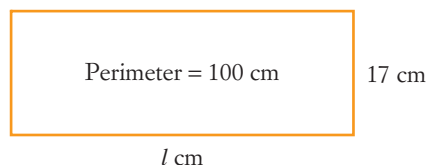
3 Translate each of the following problems into an equation, then solve the equation to solve the problem.

See Example 22

- a Mr Abdul says, 'If you multiply my age by 4 and add 12, the answer is 240.' How old is Mr Abdul? (Let a stand for his age.)

- b The perimeter of a rectangle is 100 cm and its width is 17 cm. What is the length? (Let l cm represent the length of the rectangle.)

- c I think of a number. If I multiply it by 6 and subtract 13, the answer is 95. What is the number? (Let y represent the number.)

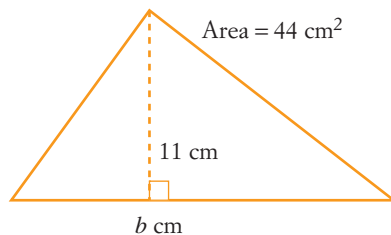


Worked solutions

Exercise 5-09

MAT07NAWS10036

- d** The area of a triangle is calculated by multiplying the base by the height and dividing by 2. If a triangle has an area of 44 cm^2 and its height is 11 cm, what is the length of its base? (Let b cm represent the length of the base.)
- e** The Student Representative Council is holding a school disco to raise money. Each ticket bought by students raises \$7 but the cost of running the disco is \$184. How many tickets must be sold to make a profit of \$2000? (Let n stand for the number of tickets sold.)
- f** Grace is a salesperson who earns \$200 per week plus one-fifth of the value of her sales for that week. If she is paid \$750 for one week, what is the value of her sales for that week? (Let x stand for the value of her sales.)
- 4** A repairman charges for fixing washing machines using the formula $C = 32h + 45$, where C is the charge in dollars and h is the number of hours the job takes. Find the number of hours worked if the charge is \$205.
- 5** The weekly profit, in dollars, made by a DVD store is given by the formula $P = 5D - 900$, where D represents the number of DVDs hired. Find the number of DVDs hired if the profit is \$1055.
- 6** The volume of a rectangular prism is given by the formula $V = lbh$. What is the value of l when $V = 340$, $b = 4$ and $h = 5$? Select the correct answer **A**, **B**, **C** or **D**.
- A** 17 **B** 68 **C** 85 **D** 425



Investigation: School concert

Year 7 wants to hold a school concert to help subsidise the cost of their camp. It wants to raise \$1500 for this.

The concert will cost \$100 for each night it is on. The hall will hold 250 people. If a band is hired for the concert this would cost an extra \$550 per night.

- a** If Year 7 decide to stage one concert only without a band, calculate how much each ticket should be to cover the costs and raise the required amount. Use an equation to help you.
- b** If they decide to hire the band, write another equation to find the new ticket price.
- c** If the Student Representative Council decides to have three concerts with the band, calculate what the ticket price should be. Consider how many people you think would attend each night. Write another equation and solve it to justify your answer.

Power plus

1 Simplify each expression.

a $2a + b + 3a - 4b$

c $3k - 2j - 5 - 4k + 6j + 9$

e $4x^2 + 6x - x^2 - 12x$

b $m - 3n - 5m - 7n$

d $a^2 + 2b^2 + 3b^2 - a^2$

f $xy + yz + xz + 4xy + 3xz + yz$

2 Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

a $p + q$

b pq

c $r^2 + t^2$

d $q^2 - 6p + rt$

e $\frac{pq}{rt}$

f $6 - 2r + p$

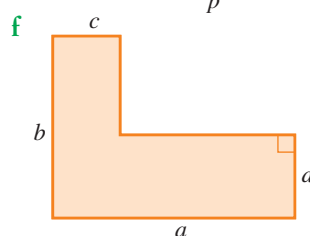
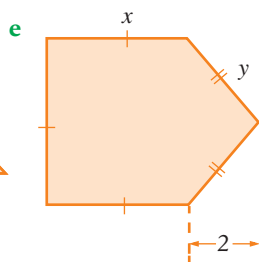
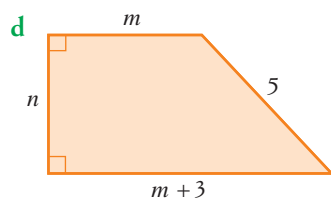
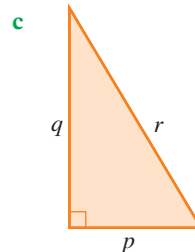
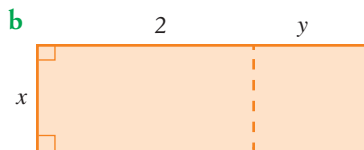
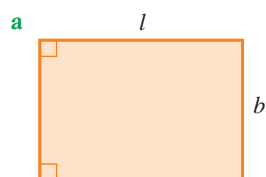
g $2p + 3q + 4r - 5t$

h $\frac{p+4}{2}$

3 For each shape, write an algebraic expression for:

i its perimeter

ii its area



4 If x is any number, then simplify each expression.

a $x + 0$

b $x \times x$

c $0 \div x$

d $x \div x$

e $x - (-x)$

f $0 \times x$

g $0 - x$

h $(-x)^2$

i $x \times \frac{1}{x}$

j $x - 2x$

5 Solve each equation.

a $6 - 3x = 9$

b $9 - 8k = 33$

c $7 - 2n = 11$

d $2(r - 4) = 10$

e $3(d + 6) = 18$

f $4(z + 1) = -6$

Chapter 5 review

Language of maths

Puzzle sheet

Algebra find-a-word

MAT07NAPS10021

abbreviation	commutative law	evaluate	pronumeral
algebra	consecutive	formula	solve
associative law	distributive law	guess, check and improve	solution
backtracking	equation	inverse	substitution
balancing	expanded form	one-step	two-step
check	expression		variable

- 1 What is another name for **pronumeral**?
- 2 What is the difference between an **expression** and an **equation**?
- 3 Which law involves multiplying by a number by splitting that number to make the calculation easier?
- 4 What are **consecutive numbers**?
- 5 Which method for solving equations is associated with 'undoing' operations?
- 6 What name is given to the value that makes an equation true?

Topic overview

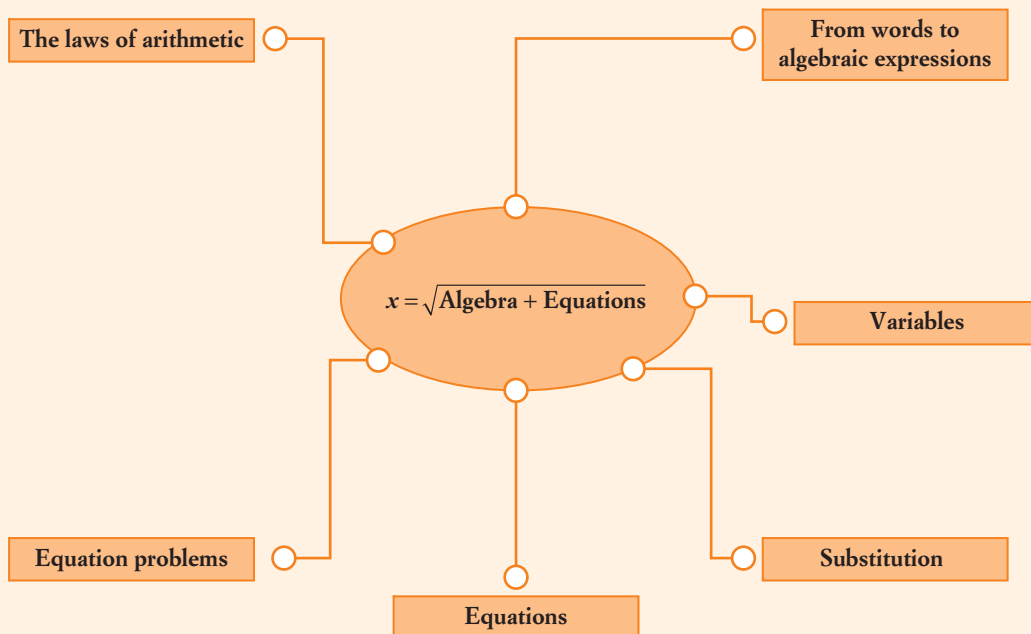
Worksheet

Mind map: Algebra and equations

MAT07NAWK10042

- Which parts of this topic did you find easy? Was there anything you already knew?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give two examples of jobs where algebra or equations might be needed.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Use the commutative and associative laws to evaluate each expression. See Exercise 5-01
 - a $7 + 18 + 3$
 - b $91 + 18 + 9$
 - c $2 \times 17 \times 5$
 - d $25 \times 31 \times 4$
 - e 13×30
 - f 7×400
- 2 Use the distributive law to evaluate each expression. See Exercise 5-02
 - a 22×9
 - b 17×8
 - c 27×11
 - d 13×12
- 3 Examine the number pattern below, then write the general rule using a variable. See Exercise 5-03

$$\begin{array}{ll} 3 - (-3) = 2 \times 3 & 11 - (-11) = 2 \times 11 \\ 5 - (-5) = 2 \times 5 & 2 - (-2) = 2 \times 2 \end{array}$$
- 4 Simplify each expression. See Exercise 5-03
 - a $x + x + x + x$
 - b $u \div 5$
 - c $2y - y$
 - c $5 \times a \times 2$
 - d $8 \times c \times b \times 3 \times a$
 - e $2 \times m \times w \times 10 \times w$
- 5 Write each of these in expanded form. See Exercise 5-03
 - a $4mnk$
 - b $\frac{6a}{b}$
 - c $9c^2$
- 6 Write an algebraic expression for each statement. Use N to represent any number. See Exercise 5-04
 - a 3 times the number
 - b the difference between the number and 5
 - c the next consecutive number
 - d one-third of the number
- 7 Write an expression for: See Exercise 5-04
 - a the sum of M and 3
 - b 5 more than B
 - c $2H$ decreased by k
- 8 Find the value of these expressions if $a = 2$, $b = 5$ and $c = 6$. See Exercise 5-05
 - a $a + b$
 - b $3a - c$
 - c $a + 2b - c$
 - d abc
- 9 If $C = 30h + 19$, find C when See Exercise 5-05
 - a $h = 3$
 - b $h = -1$
 - c $h = 5$
 - d $h = 2.5$
- 10 Solve each equation using the 'guess, check and improve' method. See Exercise 5-06
 - a $4m - 9 = 7$
 - b $\frac{k+2}{7} = 2$
 - c $\frac{d}{3} - 4 = 2$
- 11 Solve each equation algebraically, showing all steps. See Exercise 5-07
 - a $w + 17 = 52$
 - b $e - 12 = 0$
 - c $5p = 15$
 - d $\frac{x}{2} = 6$
- 12 Solve each equation algebraically, showing all steps. See Exercise 5-08
 - a $3t + 4 = 10$
 - b $2n + 6 = 12$
 - c $4y - 7 = 5$
 - d $\frac{m}{4} - 8 = 9$
 - e $\frac{x}{3} + 5 = 8$
 - f $\frac{m-4}{2} = 3$
- 13 a When a certain number is multiplied by 17, the product is 1003. Which equation can be used to solve this problem? (Let N represent the number.) Select **A**, **B**, **C** or **D**. See Exercise 5-09
 - A $N + 17 = 1003$
 - B $1003 - N = 17$
 - C $17N = 1003$
 - D $1003 + N = 17$
- b Solve the equation to solve the problem.
- 14 Caitlyn organises a charity raffle at her school. The only cost is \$180 for prizes. See Exercise 5-09
 - a If she sets the price of raffle tickets at \$5, write an equation to find out how many tickets she must sell to just cover costs. (Let N represent the number of tickets.)
 - b To make a profit of \$2100, how many tickets must she sell?



6

Measurement and geometry

Geometry

The triangle is one of the most important shapes used in construction—think of the roof in most houses. It is a strong shape because it is a rigid form, which means that it will not collapse under its own weight. Architects and engineers have used the strength of triangles to design and construct structures that can support a heavy weight over a large space, such as a bridge over a river or a roof over a wide building.



Chapter outline

	Proficiency strands			
6-01 Transformations	U			
6-02 Composite transformations	U	F		R
6-03 Line symmetry	U	F		
6-04 Rotational symmetry	U	F		
6-05 Classifying triangles	U	F		R
6-06 Angle sum of a triangle	U	F	PS	R
6-07 Exterior angle of a triangle	U	F	PS	R
6-08 Classifying quadrilaterals	U	F		R
6-09 Angle sum of a quadrilateral	U	F	PS	R
6-10 Properties of quadrilaterals	U	F		R

Wordbank

angle sum The total of the sizes of the angles in a shape, such as a triangle

axis of symmetry A line that divides a shape in half, each of which is the mirror image of the other (plural: axes)

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

isosceles triangle A triangle with two equal sides

obtuse-angled triangle A triangle with one obtuse angle

quadrilateral A shape with four straight sides

translation The process of 'sliding' a shape a certain distance and direction

trapezium A quadrilateral with one pair of parallel sides

In this chapter you will:

- investigate combinations of translations, reflections and rotations, with and without the use of digital technologies
- describe patterns and investigate different ways to produce the same transformational changes, such as using two successive reflections to provide the same result as a translation
- identify line and rotational symmetries
- classify triangles according to their side and angle properties
- describe quadrilaterals: squares, rectangles, rhombuses, parallelograms, kites and trapeziums
- demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral

SkillCheck

Worksheet

StartUp assignment 6

MAT07MGWK10043

Skillsheet

Naming shapes

MAT07MGSS10023

1 Draw:

a a pair of perpendicular lines

b a pair of parallel lines

2 Draw a rectangle and all its diagonals.

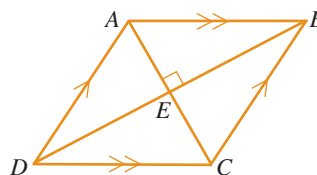
3 For the quadrilateral shown on the right:

a name two intervals that are parallel

b are the diagonals equal in length?

c what is the size of $\angle DEC$?

d if $\angle DAB$ and $\angle ABC$ are cointerior and $\angle DAB = 115^\circ$, what is the size of $\angle ABC$?



4 Draw a pair of parallel lines crossed by a transversal and mark a pair of alternate angles.

5 **a** Draw a parallelogram and label it $DEFG$.

b Mark both pairs of parallel sides.

c Name both pairs of parallel sides.

d Mark the equal sides DG and EF .

6 Draw a triangle that has:

a a right angle

b an obtuse angle

7 Draw a pair of parallel lines crossed by a transversal and mark a pair of corresponding angles.

6-01 Transformations

Worksheet

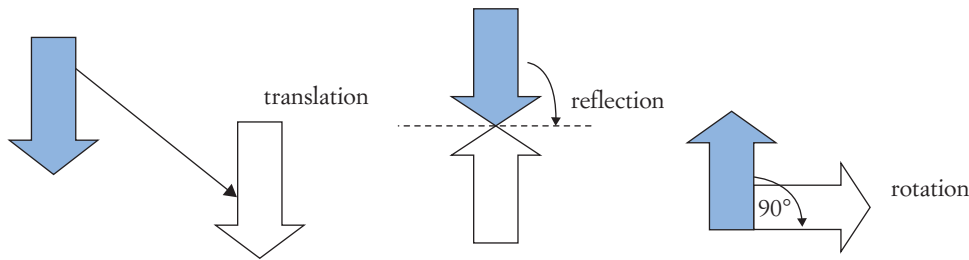
Transformations

MAT07MGWK10044

Patterns in tiles, wallpaper and paving are usually made by taking a basic shape and repeating it. A pattern can be created by **sliding**, **flipping** or **spinning** the shape.

These movements have special names.

- A 'slide' is called a **translation**
- A 'flip' is called a **reflection**
- A 'spin' is called a **rotation**

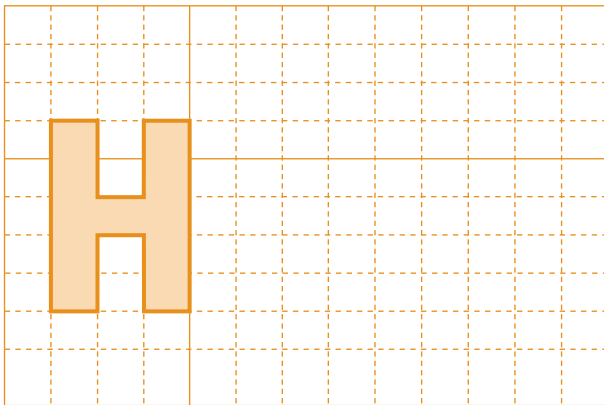


Together, these are called **congruence transformations**, where 'congruence' means identical and 'transformation' means change. Even though a shape has changed position (transformed), it still has the same shape and size (congruence).

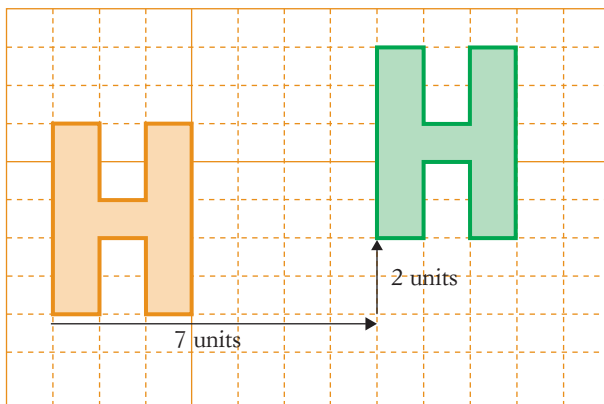
- The original shape is called the **original**
- The transformed shape is called the **image**

Example 1

Translate the H-shape below 7 units right and 2 units up.



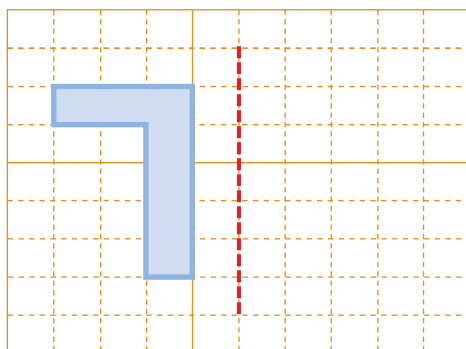
Solution



Every point on the original is shifted 7 units right and 2 units up to create the image

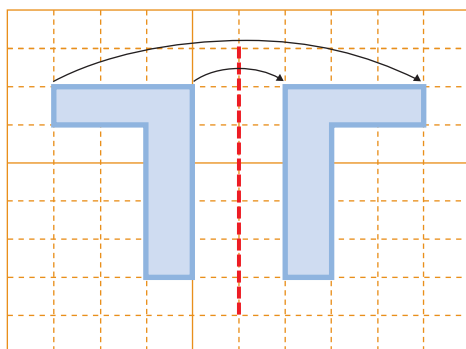
Example 2

Reflect the L-shape across the dotted line.



The dotted line is called the **line of reflection**.

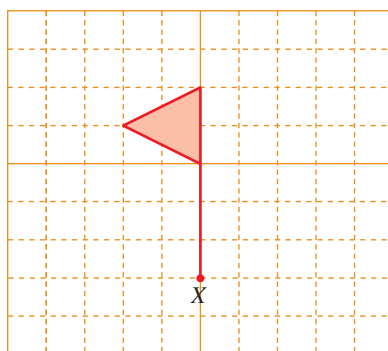
Solution



The original and its mirror-image are the same distance from the line of reflection.

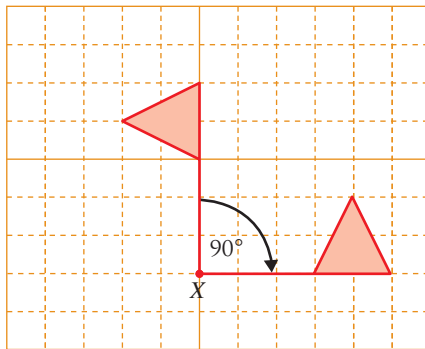
Example 3

Rotate the flag shape 90° in a clockwise direction about the point X .



X is called the **centre of rotation**.

Solution

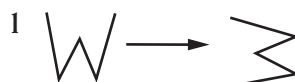
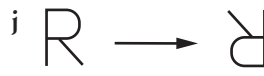
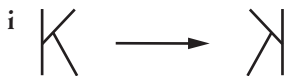
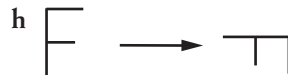
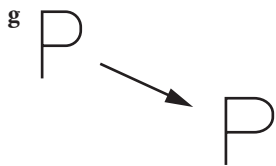


The original and image are the same distance from the centre of rotation, so the point X does not move.

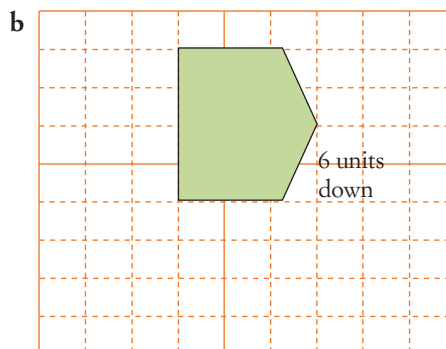
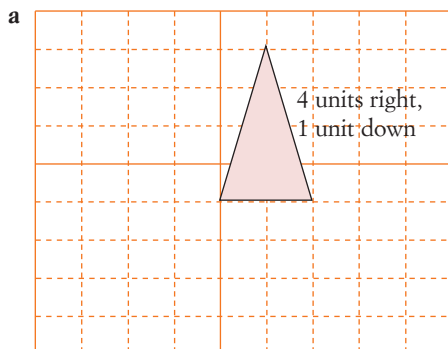
Exercise 6-01 Transformations

1 Translation, reflection or rotation?

- a To make a shape back-to-front to create a mirror-image
- b To slide or shift a shape
- c To flip a shape
- d To turn a shape upside-down
- e To move a shape up, down or diagonally
- f To spin a shape



2 Print or copy each shape onto grid paper and translate it according to the directions given.

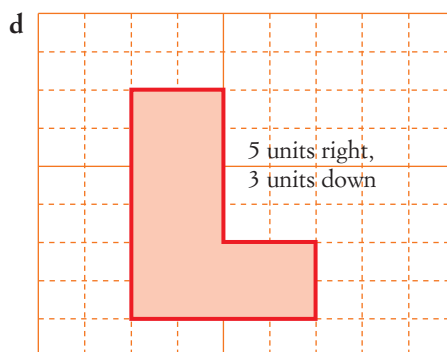
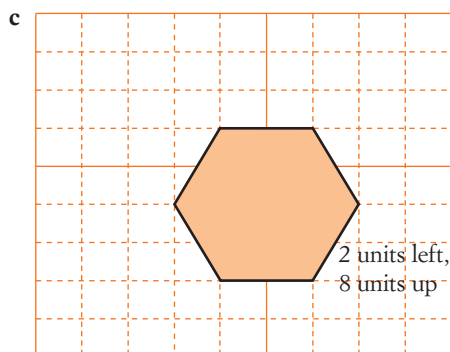


See Example 1

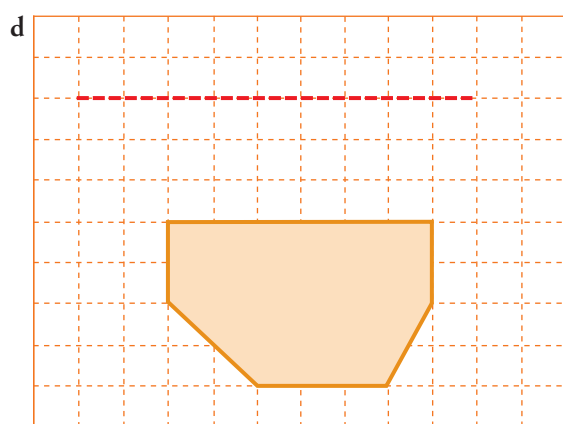
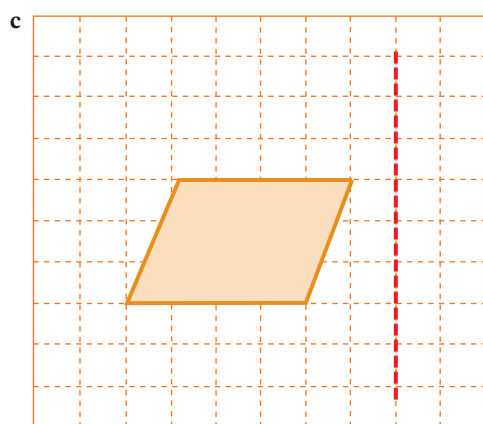
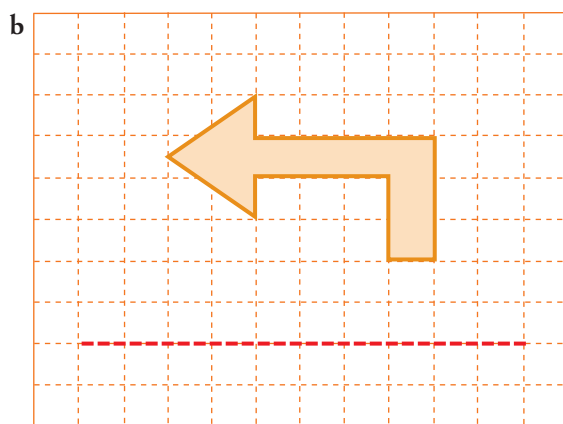
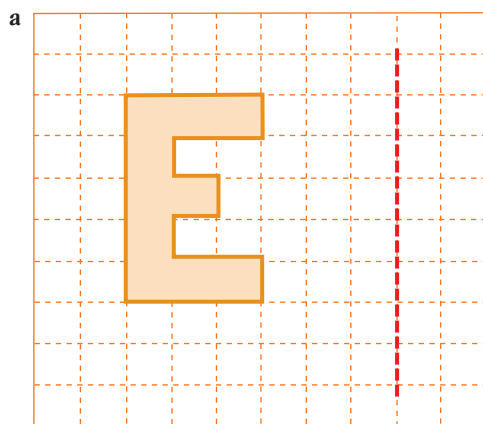
Worksheet

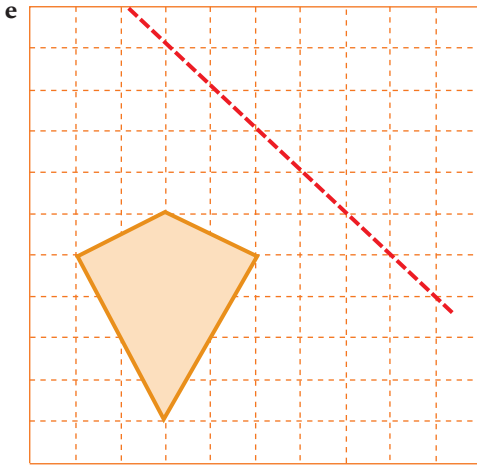
Transformations

MAT07MGWK10044



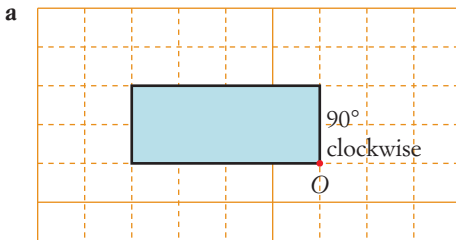
See Example 2 3 Copy each shape onto square dot paper and reflect it across the dotted line. Matching points on the original and reflected shape should be the same distance from the line of reflection.



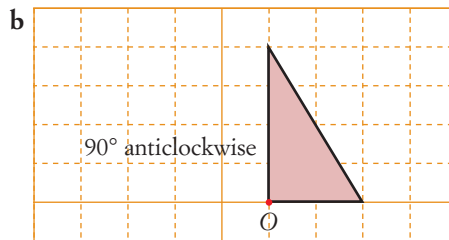


4 Rotate each shape by the given angle about the point O .

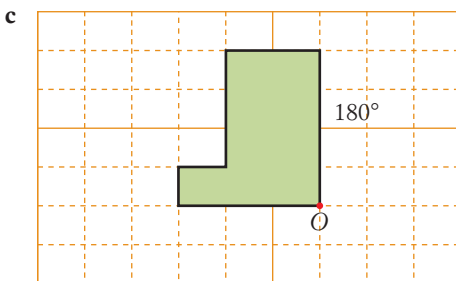
See Example 3



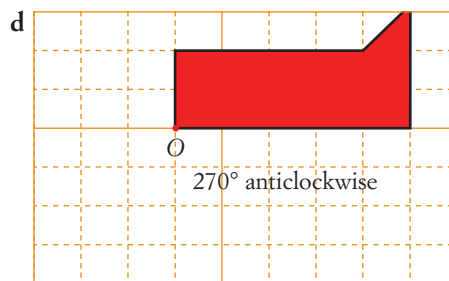
90°
clockwise



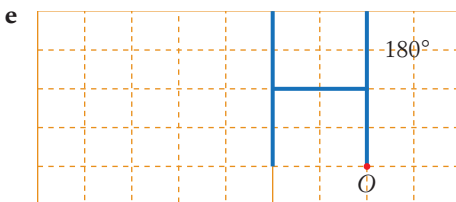
90° anticlockwise



180°



270° anticlockwise



180°

5 Why don't rotations of 180° need the label 'clockwise' or 'anticlockwise'?

- 6 What clockwise rotation gives the same result as:
 - a a 90° rotation anticlockwise
 - b a 270° rotation anticlockwise
 - c a 90° rotation anticlockwise three times
 - d a 90° rotation anticlockwise followed by a 270° rotation clockwise?
- 7 What single translation gives the same result as:
 - a a translation of 3 units right, 1 unit up followed by 4 units left, 4 units up
 - b a translation of 2 units left, 8 units down followed by 2 units right, 5 units up?

Technology Transformations

Weblink

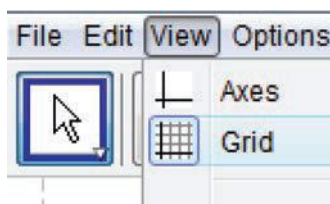
Geogebra

Technology

Geogebra
Transformations

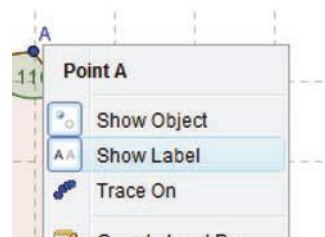
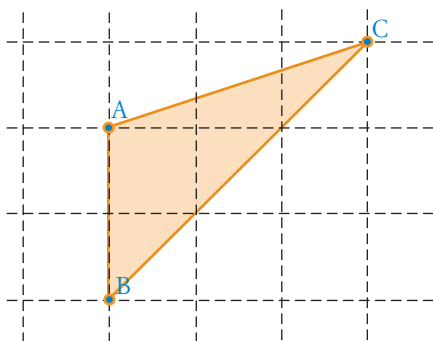
MAT07MGCT00008

This activity will use GeoGebra to explore translation, reflection and rotation. To set up your drawing page, click **View**. Check that **Grid** is ticked and **Axes** is not ticked.

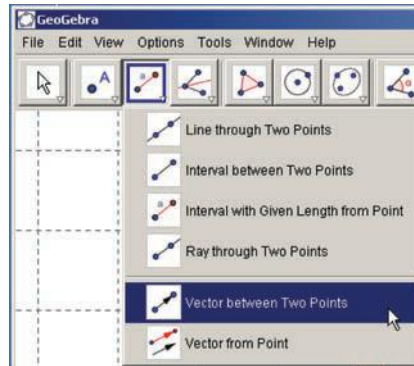


Translations

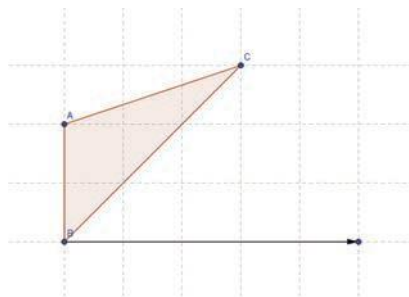
- 1 Draw the triangle shown below, using the  **Polygon** menu. Make sure that the labels are showing. If they aren't, right-click on each vertex and select **Show label**.



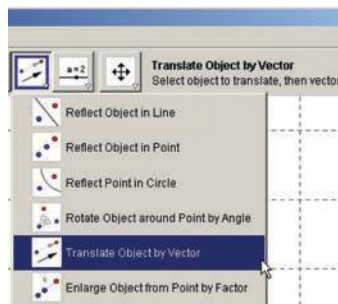
- 2 Select  **Vector between two points** from the third drop-down menu.



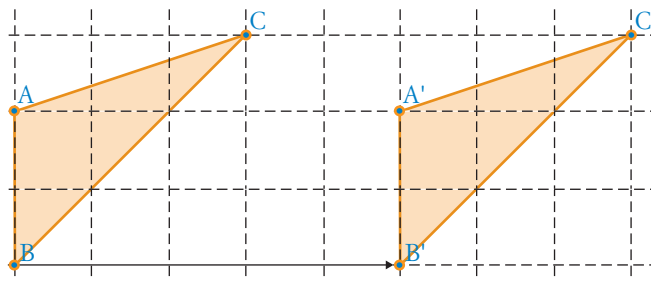
The vector represents a move of a certain size, in a certain direction. On your diagram, click on point B shown below and click 5 units to the right of point B.



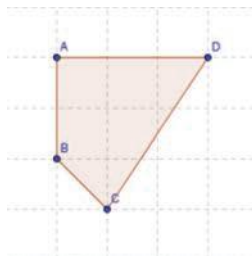
- 3 Now click on **Translate object by vector** and click on triangle ABC and the vector.



- 4 You should now see triangle A'B'C' on your screen. It is the image of the original triangle ABC translated 5 units to the right.

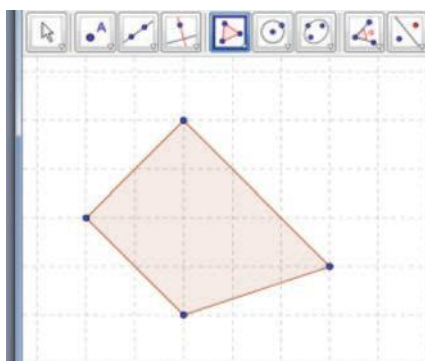


- 5 Now copy the following diagram onto your screen. Translate ABCD 3 units down and 4 units to the left of point A.

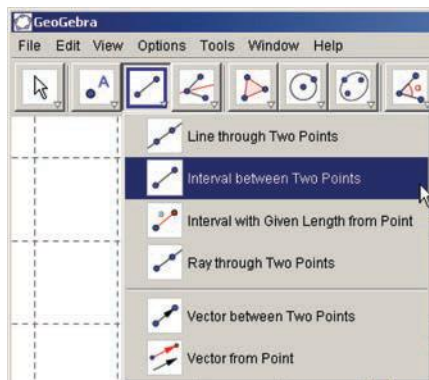


Reflections

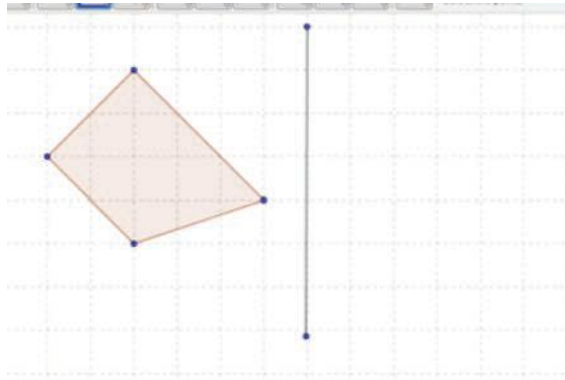
- 1 To draw the quadrilateral shown below, click on  **Polygon**.



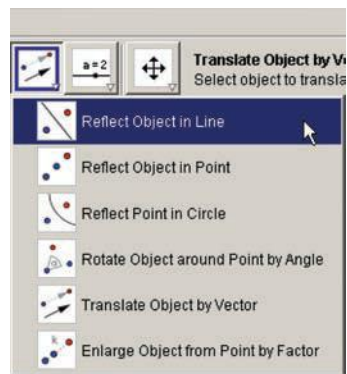
- 2 Select  **Interval between two points** from the third drop-down menu.



Draw an interval as shown below.

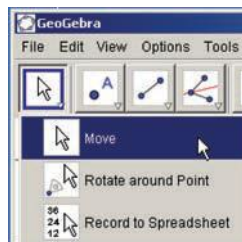


- 3 Now select  **Reflect object in line** from the third-last drop-down menu.



Click on the shape and then the line. What do you see?

- 4 Now select **Move** from the first drop-down menu.

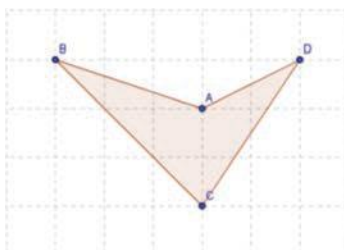


Drag any vertex of the original shape. What happens to its reflection?

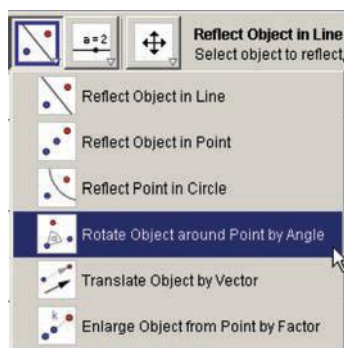
- 5 Now try to draw other shapes and reflect each one using  **Reflect object in line**.

Rotations

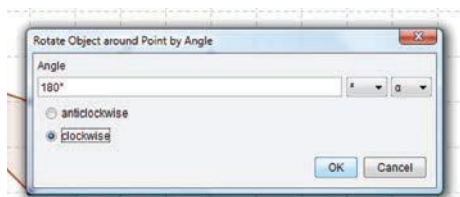
- 1 To draw the quadrilateral shown below, click on  **Polygon** from the fifth drop-down menu.



- 2 Make sure that the labels are showing. If they aren't, right-click on each vertex and select **Show label**.
- 3 Select  **Rotate object around point by angle** from the third last drop-down menu.



Click on the object and the centre of rotation, at D. Then enter the angle 180° clockwise (see below).



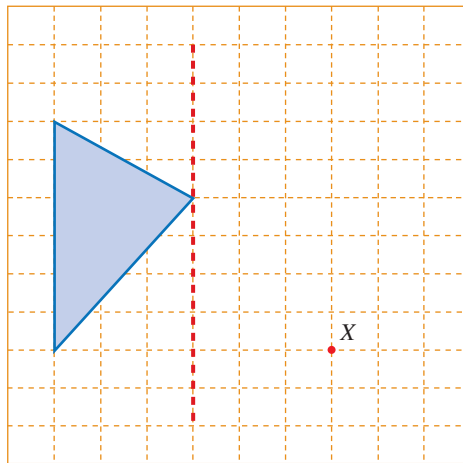
- 4 Now use **Rotate object around point by angle** using different angle sizes and choose clockwise or anticlockwise. Also, use **Move** to drag the vertices. What do you notice?
- 5 Now experiment by creating your own shapes and **Rotate object around point by angle**.

6-02 Composite transformations

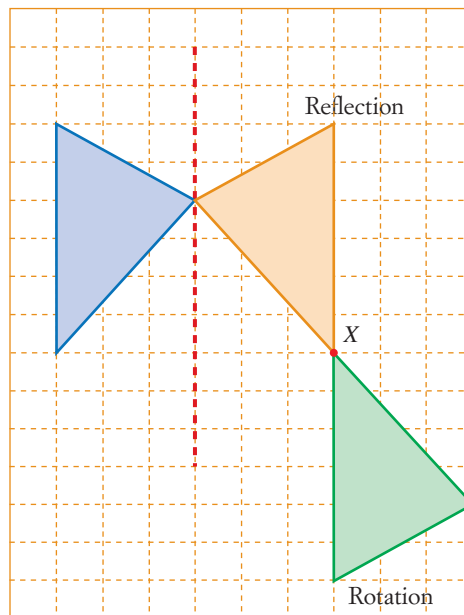
Combinations of translations, reflections and rotations can be applied one after the other. These are called **composite transformations**.

Example 4

Reflect the triangle below across the dotted line and then rotate the image 180° about X .

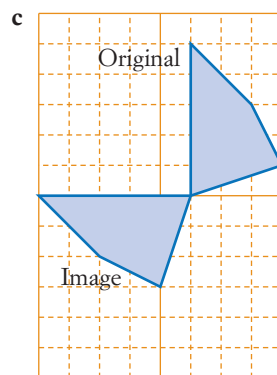
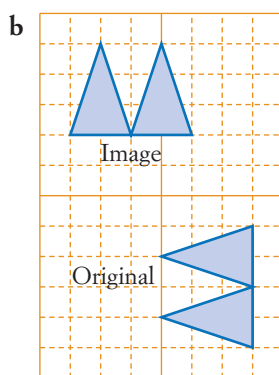
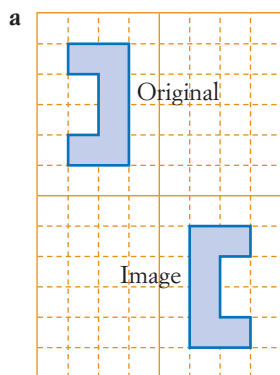


Solution

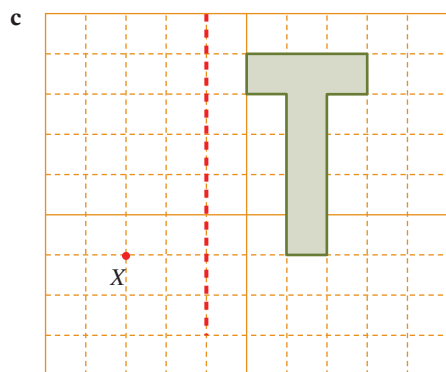
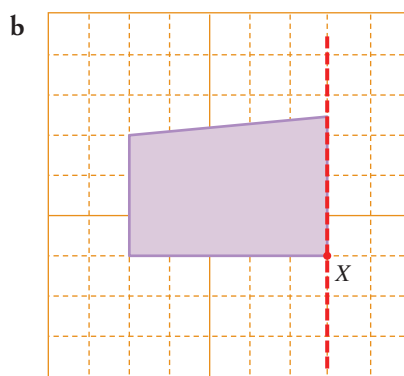
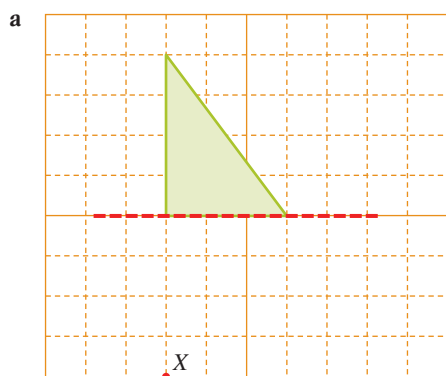


Exercise 6-02 Composite transformations

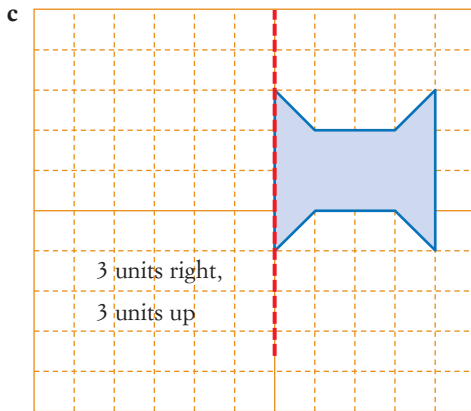
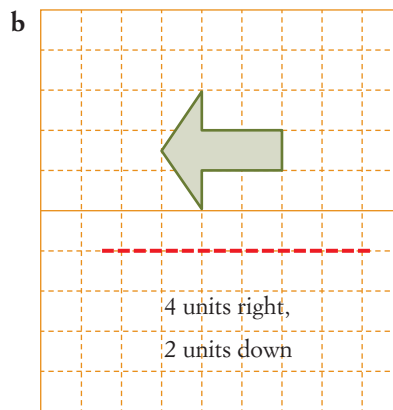
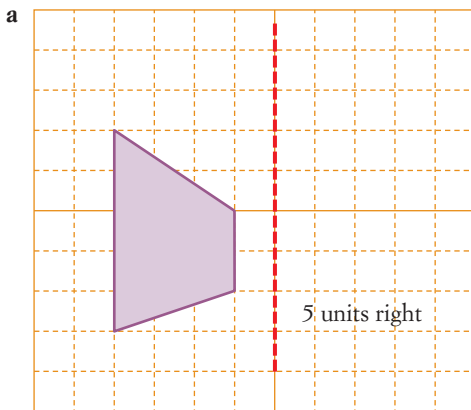
- 1 For each diagram below, state the combination of two transformations used on the original figure to form the image.



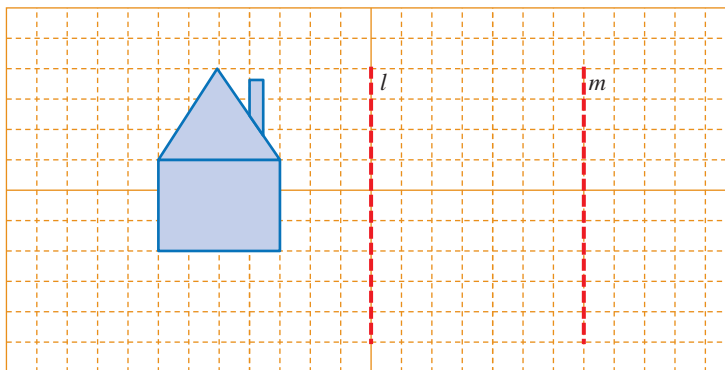
- See Example 4** 2 Draw the final image when each shape is reflected in the given line and then rotated 90° clockwise about the point X .



- 3 Draw the final image when each shape is reflected across the given line and then translated by the given distance and direction.

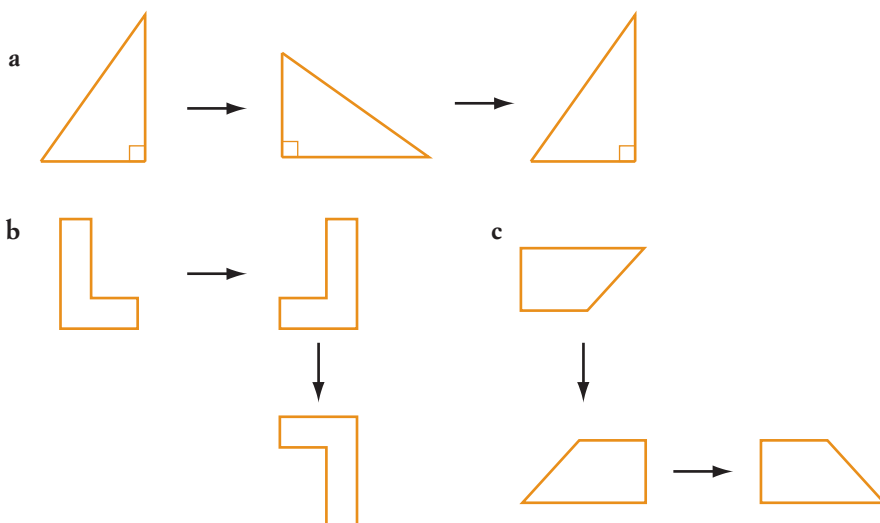


- 4 a Reflect the house shape across line l and then across line m .



- b Which single transformation would give the same result as the two reflections in part a?

- 5 Each diagram below has been transformed twice.
- Name the two transformations that have been performed.
 - Name one transformation that would give the same result as the two transformations.

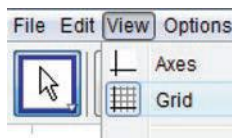


Technology Composite transformations

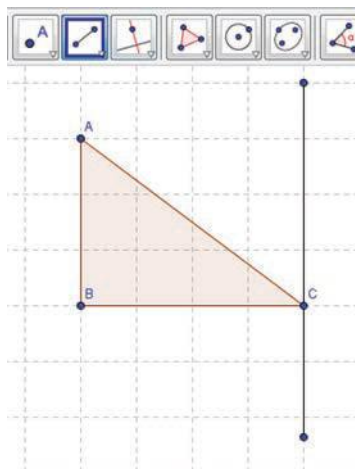
This activity will use GeoGebra to explore composite transformations.

Triangle transformation

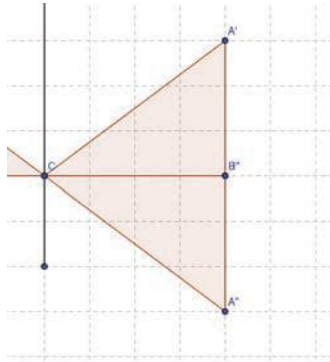
- 1 To set up your drawing page, click **View**. Check that **Grid** is ticked and **Axes** is not ticked.



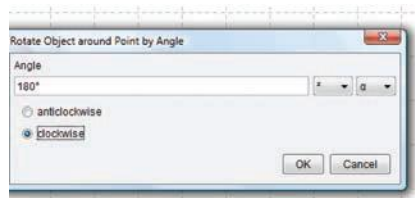
- 2 Draw the triangle shown below, using **Polygon**. Make sure that the labels for each vertex are showing. Draw an interval as shown as a line of reflection and **Reflect object in line**.



- 3 Now reflect triangle $A'B'C$ in line $B'C$ using **Reflect object in line**.



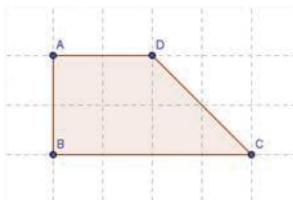
- 4 There can be more than one way to represent the same transformation. Open a **New window** and draw triangle ABC shown in step 2. Select **Rotate object around point by angle**. Select triangle ABC and Point C. Choose a rotation of 180° and 'clockwise' (shown below).



- 5 Compare this with the method used in steps 2 and 3 above. What do you notice?

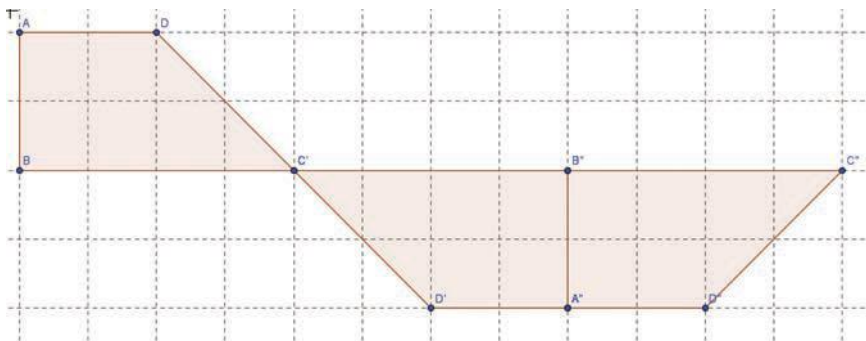
Trapezium transformation

- Set up your drawing page by clicking **View**. Make sure that **Grid** is ticked and **Axes** is not ticked.
- To draw the trapezium shown below, click on **Polygon**. Select **Show label** for each vertex if the labels are not already displayed.

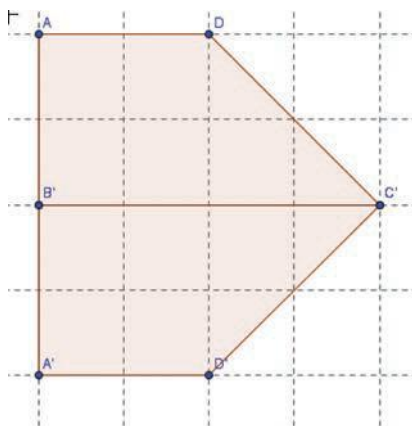


- 3 Select **Rotate object around point by angle**. Select trapezium ABCD and Point C. Rotate ABCD clockwise, 180° .

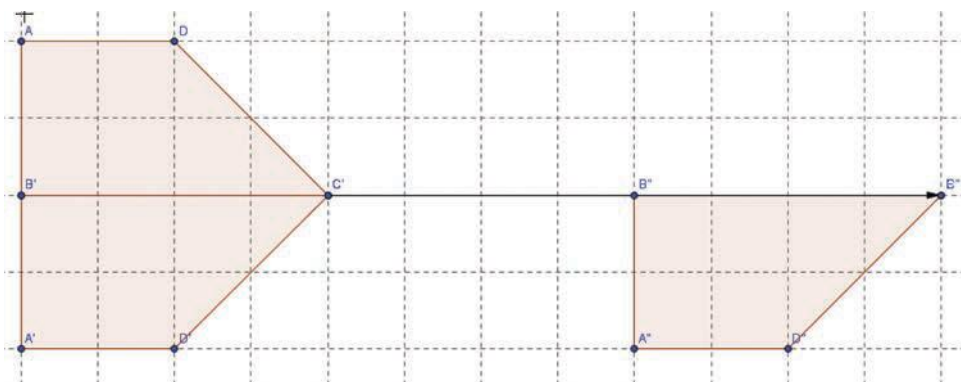
- 4 Now select **Reflect object in line**. Click on trapezium $A'B'C'D'$ and line $A'B'$.



- 5 Remember, there can be more than one way to represent the same transformation. Open a **New window** and draw trapezium ABCD shown in 1. Select **Reflect object in line** and click on ABCD and line BC.



- 6 Select **Vector between two points**. Click on Point C and count 8 horizontal units to the right. Select **Translate object by vector** and click on $A'B'C'D'$ and the vector.



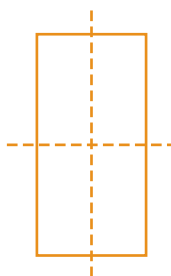
- 7 Compare this method of reflection and translation with the rotation and reflection used in steps 2 to 4. What do you notice?

- 8 Now use GeoGebra to create your own composite transformation in a total of 2 to 3 steps, using any combination of translation, reflection and rotation.
- 9 Swap your composite transformation with your peers and try to find an alternative method (or more if possible) to the composite transformation that they have created. Remember, only use a total of 2 to 3 steps.

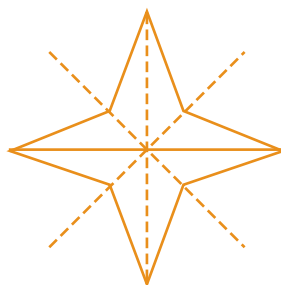
6-03 Line symmetry

Symmetrical objects appear balanced and are usually pleasing to look at. The two main types of symmetry are **line symmetry** and **rotational symmetry**.

The shapes below have line symmetry. If you fold along the line, one half of the shape will fit exactly on top of the other half. One half is the reflection or mirror-image of the other half. The fold line is called the **axis of symmetry**. The plural of 'axis' is **axes**.



This rectangle has 2 axes of symmetry.



This star shape has 4 axes of symmetry.

Worksheet

Symmetry

MAT07MGWK10045

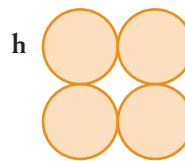
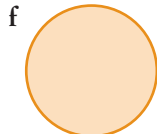
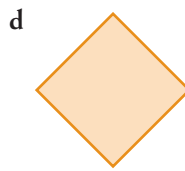
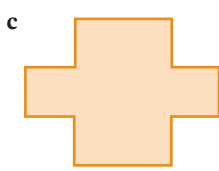
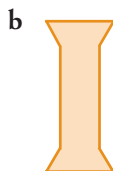
Skillsheet

Line and rotational
symmetry

MAT07MGSS10024

Exercise 6-03 Line symmetry

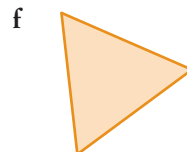
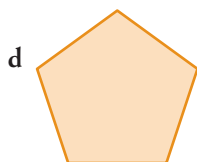
- 1 Copy each shape and mark in its axes of symmetry.



- 2 Which shapes in question 1 have:

a one axis of symmetry?	b four axes of symmetry?
c an infinite number of axes of symmetry?	d two axes of symmetry?

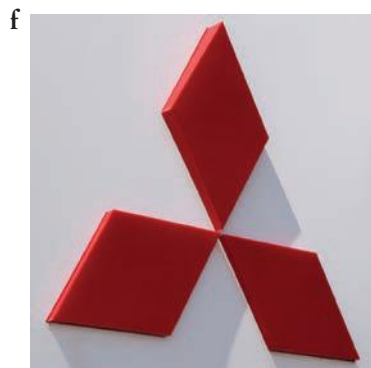
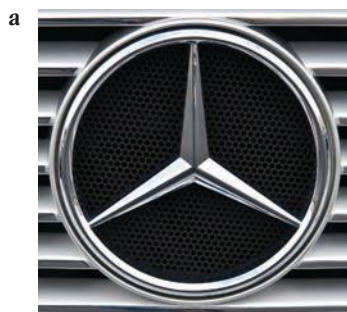
3 Count the number of axes of symmetry in each shape.



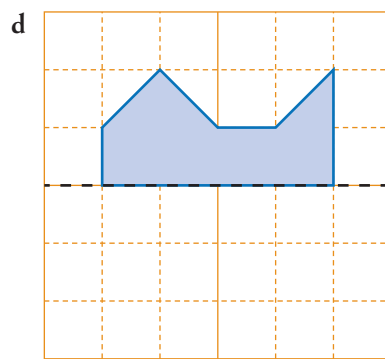
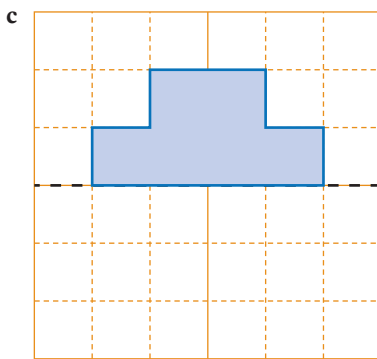
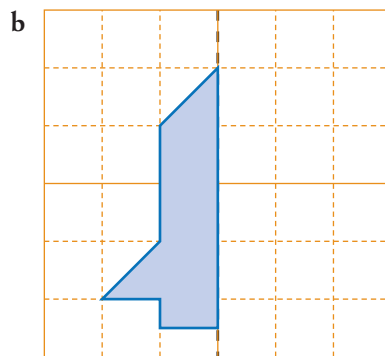
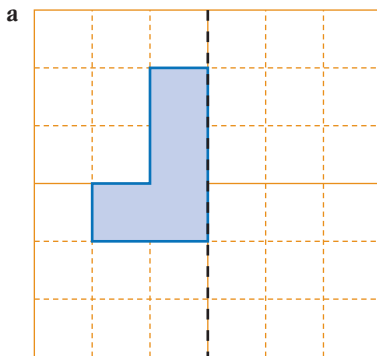
4 Copy the capital letters below that have line symmetry and draw their axes of symmetry.

D H I K M N O R S V X Z

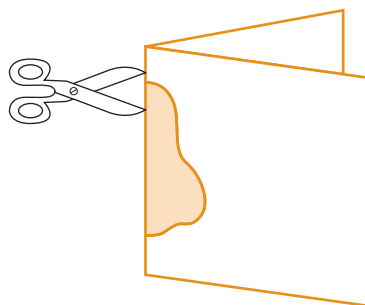
5 How many axes of symmetry has each car logo?



- 6 Search in a newspaper or magazine for an example of any logo that has line symmetry.
- 7 Copy each figure onto grid paper and complete it so that the dotted line is an axis of symmetry.



- 8 Fold a piece of paper in half and draw a figure on one side of the folded edge, as shown in the diagram. Cut your shape out. What do you notice?

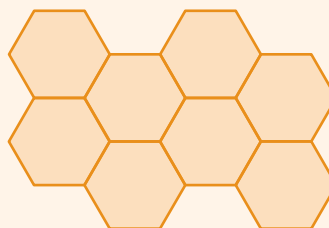
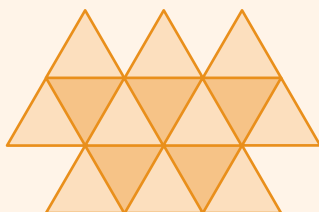


- 9 Fold a piece of paper in half and place a few drops of ink in the fold. What pattern do you get when you press the sides together?
- 10 Draw a shape that has:
- a** no axes of symmetry
 - b** one axis of symmetry
 - c** two axes of symmetry
 - d** three axes of symmetry

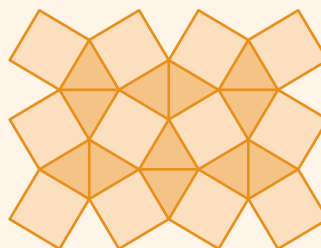
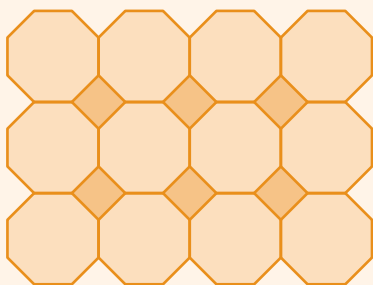
Just for the record

Tessellations

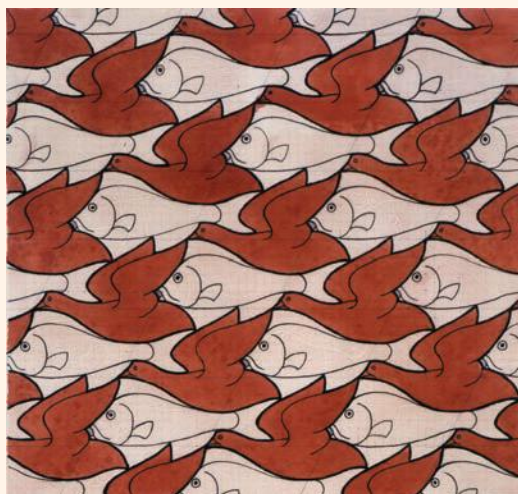
A tessellation is created when a shape is repeated over and over again. The word “tessellation” comes from the Latin word *tessella*, which is the small square tile used in producing mosaics. All of the shapes fit together without any gaps or overlaps. Think of tiles, where the same shape is used to cover the whole floor leaving no gaps. These are examples of tessellations:



Sometimes it is necessary to use more than one shape to make sure that there are no gaps.



Maurits Cornelis Escher (1898–1972) was a Dutch graphic artist who created some interesting tessellations, such as those shown below.



Which of the following shapes tessellate?

kite
hexagon

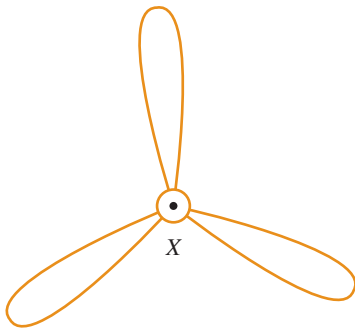
oval
rectangle

parallelogram
octagon

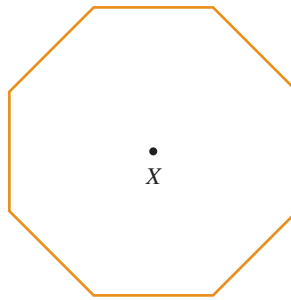
rhombus
pentagon

6-04 Rotational symmetry

The two shapes below have **rotational symmetry**. If you rotate each shape about the point X , it will fit exactly on top of itself at least once before one full revolution (360°). The centre point, X , is called the **centre of symmetry**. The number of times that a shape fits on itself in one revolution is called its **order of rotational symmetry**.



This propeller fits on itself three times in a full turn, so it has rotational symmetry of **order 3**.



This regular octagon fits on itself eight times in a full turn, so it has rotational symmetry of **order 8**.

Worksheet

Symmetry

MAT07MGWK10045

Skillsheet

Line and rotational
symmetry

MAT07MGSS10024

Homework sheet

Transformations and
symmetry

MAT07MGHS10027

Worksheet

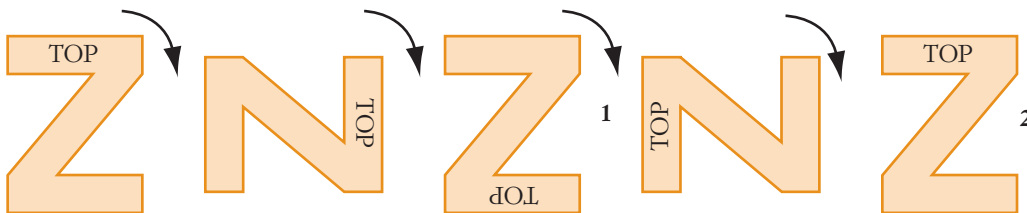
Symmetry of flat
shapes

MAT07MGWK00031

Example 5

What order of rotational symmetry does the letter Z have?

Solution

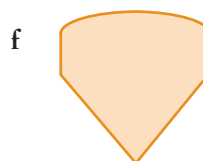
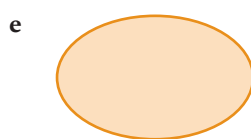
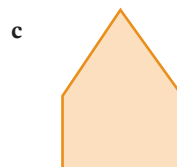
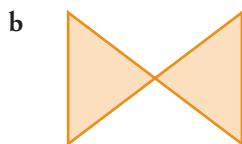
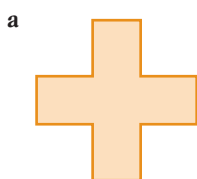


Z has rotational symmetry of order 2.

Exercise 6-04 Rotational symmetry

See Example 5

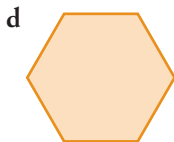
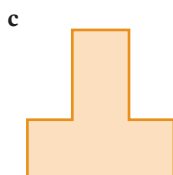
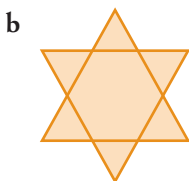
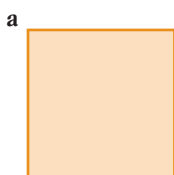
- 1 For each shape, state whether or not it has rotational symmetry, and if it does, copy the shape, mark its centre of symmetry and find its order of rotational symmetry.



- 2 Copy the capital letters below that have rotational symmetry and mark their centre of symmetry.

D H I K M N O R S V X Z

- 3 For the car logos in question 5 of Exercise 6-03 on page 226, state which ones have rotational symmetry and their order of rotational symmetry.
- 4 Search in a newspaper or magazine for an example of any logo that has rotational symmetry.
- 5 For each shape:
- find how many axes of symmetry it has
 - state whether it has rotational symmetry and if it does, state the order



- 6 Draw a shape that has:
- no rotational symmetry
 - rotational symmetry of order 2
 - rotational symmetry of order 3
 - rotational symmetry of order 4
- 7 What shape has an *infinite* order of rotational symmetry?

Mental skills 6 Maths without calculators

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

$$\begin{aligned} \text{a } 631 + 280 + 51 + 43 + 96 &\approx 600 + 300 + 50 + 40 + 100 \\ &= (600 + 300 + 100) + (50 + 40) \\ &= 1000 + 90 \\ &= 1090 \text{ (Actual answer = 1101)} \end{aligned}$$

$$\begin{aligned} \text{b } 55 + 132 - 34 + 17 - 78 &\approx 60 + 130 - 30 + 20 - 80 \\ &\approx (60 + 20 - 80) + (130 - 30) \\ &= 0 + 100 \\ &\approx 100 \text{ (Actual answer = 92)} \end{aligned}$$

$$\begin{aligned} \text{c } 67 \times 13 &\approx 70 \times 12 \\ &= 840 \text{ (Actual answer = 871)} \end{aligned}$$

$$\begin{aligned} \text{d } 78 \times 7 &\approx 80 \times 7 \\ &= 560 \text{ (Actual answer = 546)} \end{aligned}$$

$$\begin{aligned} \text{e } 929 \div 5 &\approx 1000 \div 5 \\ &= 200 \text{ (Actual answer = 185.8)} \end{aligned}$$

$$\begin{aligned} \text{f } 510 \div 24 &\approx 500 \div 20 \\ &= 50 \div 2 \\ &= 25 \text{ (Actual answer = 21.25)} \end{aligned}$$

2 Now estimate each answer.

$$\text{a } 27 + 11 + 87 + 142 + 64$$

$$\text{c } 684 + 903$$

$$\text{e } 517 - 96$$

$$\text{g } 766 - 353$$

$$\text{i } 83 \times 81$$

$$\text{k } 828 \div 3$$

$$\text{b } 55 + 34 - 22 - 46 + 136$$

$$\text{d } 35 + 81 + 110 + 22 + 7$$

$$\text{f } 210 - 38 - 71 + 151 - 49$$

$$\text{h } 367 \times 2$$

$$\text{j } 984 \times 16$$

$$\text{l } 507 \div 7$$

6-05 Classifying triangles

Triangles can be classified in two ways:

- by their **sides**: equilateral, isosceles or scalene
- by their **angles**: acute-angled, obtuse-angled, right-angled

Puzzle sheet

Classifying triangles

MAT07MGPS10022

Technology

Geogebra

Classifying triangles by angle and side length

MAT07MGCT00003

Worksheet

Properties of triangles

MAT07MGWK10046

Worksheet

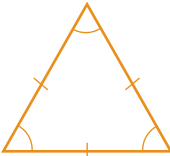
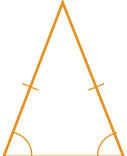
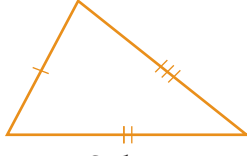
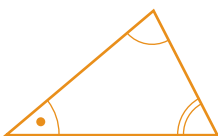
Constructing triangles and quadrilaterals

MAT07MGWK10048

TLF learning object

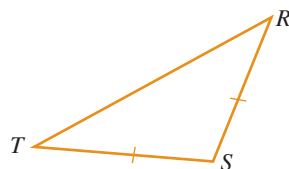
Exploring triangles (L6558)

Summary

Sides	 Equilateral Three equal sides	 Isosceles Two equal sides	 Scalene No equal sides
Angles	 Acute-angled All three angles acute	 Right-angled One right angle	 Obtuse-angled One obtuse angle

Example 6

Classify this triangle by sides and angles.



Solution

The triangle has two equal sides so it is **isosceles**.

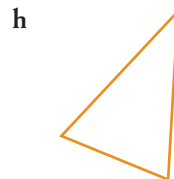
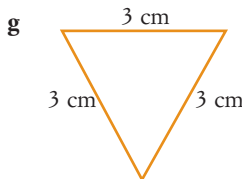
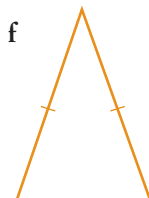
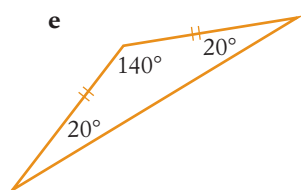
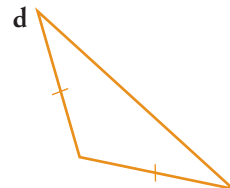
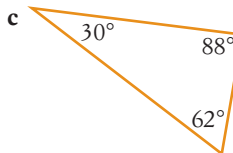
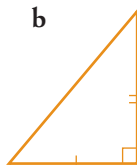
The triangle has an obtuse angle, $\angle S$, so it is **obtuse-angled**.

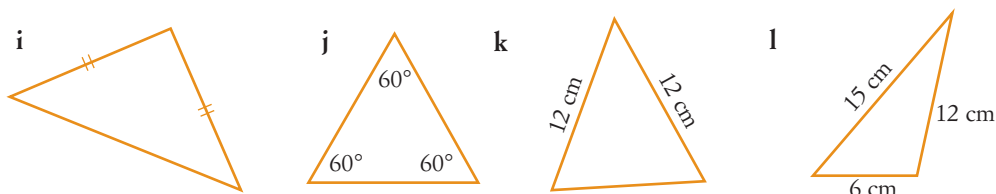
The triangle is **isosceles** and **obtuse-angled**.

Exercise 6-05 Classifying triangles

See Example 6

1 Classify each triangle by sides and angles.



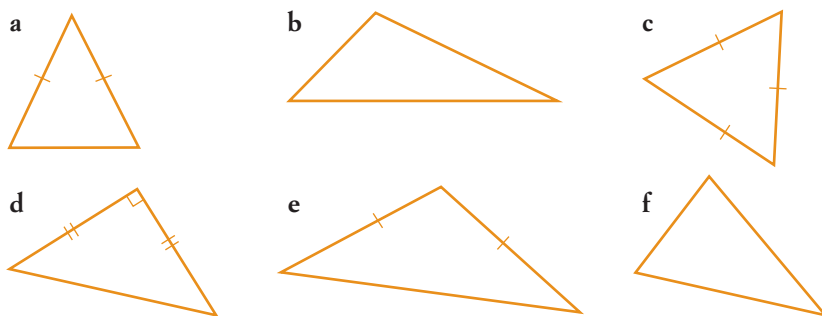


2 Draw an example of each type of triangle described.

a a scalene triangle	b a right-angled scalene triangle
c an isosceles triangle	d an equilateral triangle
e a right-angled isosceles triangle	f an acute scalene triangle

3 Is it possible to draw an equilateral right-angled triangle? Why?

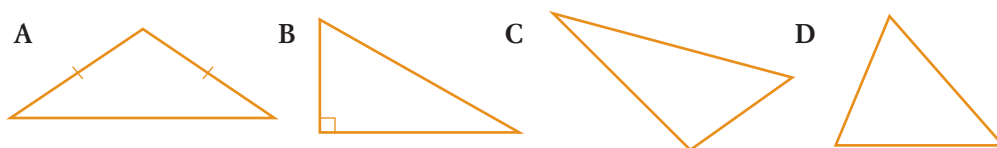
4 Copy each triangle into your book and draw in all axes of symmetry.



5 Do any triangles have rotational symmetry? Give examples to support your answer.

6 Is it possible to draw a triangle with two obtuse angles? Why?

7 Which triangle is both obtuse-angled and scalene? Select A, B, C or D.



8 Draw and cut out an isosceles triangle.

- How many axes of symmetry has an isosceles triangle?
- By folding, mark the equal angles in an isosceles triangle.
- Describe in words where the equal angles in an isosceles triangle are.

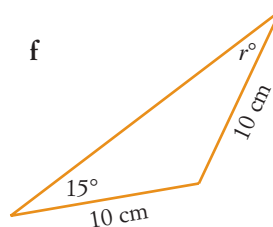
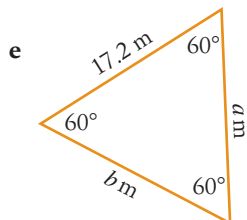
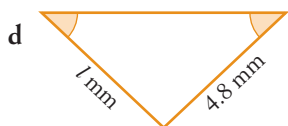
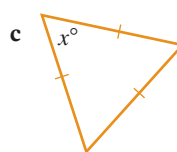
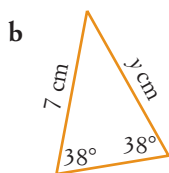
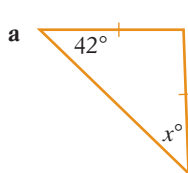
9 Draw and cut out an equilateral triangle.

- How many axes of symmetry has an equilateral triangle?
- By folding, mark the equal angles in an equilateral triangle.
- Describe in words where the equal angles in an equilateral triangle are.

10 a Does an isosceles triangle have rotational symmetry? If so, state the order.

b Does an equilateral triangle have rotational symmetry? If so, state the order.

11 Find the value of the pronumeral(s) in each diagram, giving a reason.



12 The prefix 'tri' means 'three'. Find the meaning of these mathematical 'tri' words.

a trisect

b trilateral

c triangulate

Technology Constructing triangles

Worksheet

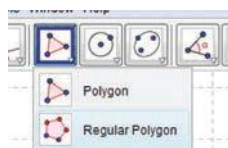
Constructing triangles and quadrilaterals

MAT07MGWK10048

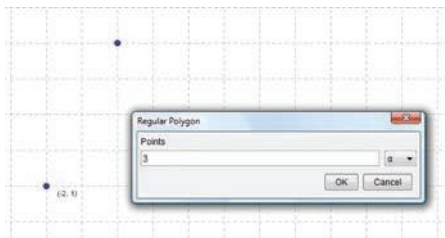
This activity will use GeoGebra to draw a variety of triangles. To set up your drawing page, click **View** and **Grid**. Also click **View** and **Axes** (to remove the axes).

Equilateral triangle

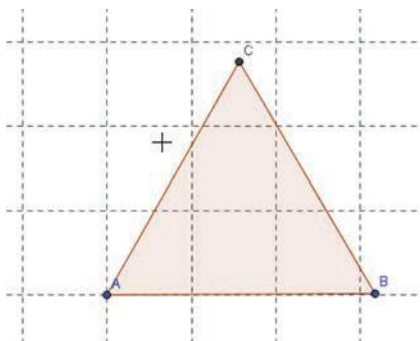
1 To draw an equilateral triangle, click on **Regular Polygon**.



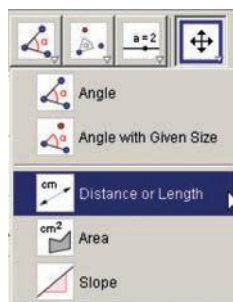
2 Draw two points at least 5 cm apart and then you will be asked for the number of vertices required. Type in '3'. Click OK.



- 3 If you can't see any labels, right-click on each point and select **Show Label**. Select **Move** from the first drop-down menu and try to move any of the three vertices.



- 4 Select **distance or length** from the fourth-last drop-down menu.

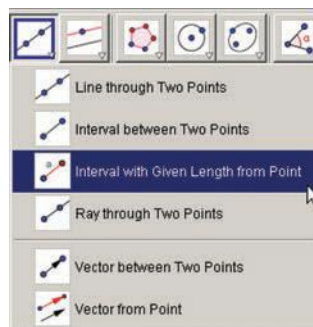


Click each side of the triangle to calculate the length of each side. What do you notice about each side length?

- 5 Select **Angle** from the same menu as above and click on each vertex in a clockwise direction to find the interior angle size. The angle you want to measure must be the middle vertex you click on. Measure every angle. What do you notice about the size of each angle?
- 6 Select **Move** from the first drop-down menu and click on any vertex, you will notice that you can make the triangle larger or smaller. Do the side lengths change? What happens to the size of the angles?

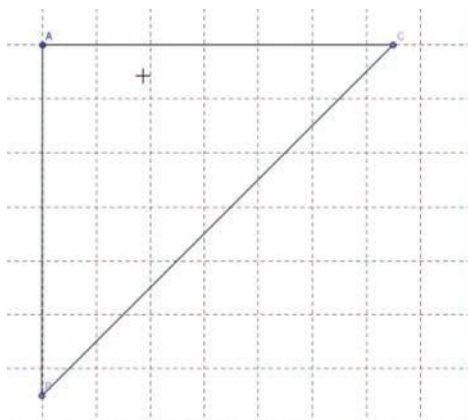
Isosceles triangle

- 1 Select **Interval with given length from point** from the third drop-down menu.



Draw two sides of a triangle with length 6.5 cm. It will draw one interval directly on top of the other, so use **Move** to drag the second side down at one end after you have drawn it.

- 2 Now join the two remaining vertices with **Interval between two points**.



- 3 Right-click on each point and select **Show Label** if you can't see any labels. Select **distance or length** and click each side of the triangle to show the length of each side.
- 4 Now select **angle** and click on each vertex in a clockwise direction to find each interior angle size. What do you notice about the size of each angle?
- 5 Click on any vertex. You will notice that you can only change parts of the triangle. What do you notice about the side lengths and the size of each angle?
- 6 Try to draw an obtuse-angled isosceles triangle. Measure the sides and angles of your triangle.

Scalene triangle

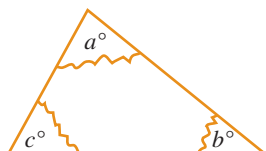
- 1 Select  **Interval between two points** to draw any triangle.
- 2 Right-click on each point and select **Show Label** if the labels are not showing.
- 3 Measure the size of each angle (in a clockwise direction) and measure the length of each side in the triangle.
- 4 Use **Move** to draw a right-angled scalene triangle. Use **Angle** and **Distance or length** to show that the triangle is right-angled and scalene.

Investigation: Angle sum of a triangle

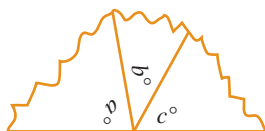
What is the sum of the three angles in a triangle? In groups of 2 to 4, complete the following activity.

Paper-cutting activity

- 1 Draw a large triangle on paper, cut it out and label its three angles a , b and c .



- 2 Tear off the three angles and arrange them next to each other, so that their points meet.

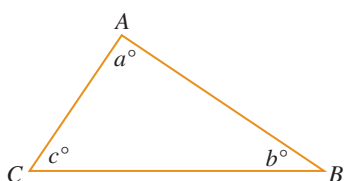


- 3 What type of angle do they form? How many degrees are in this type of angle?
4 To see if this works for all triangles, repeat the above steps for different triangles.

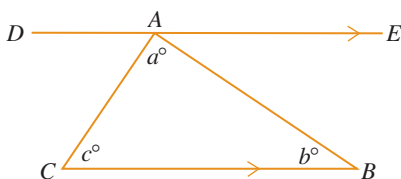
Formal proof

We can use parallel lines to prove that the angle sum of a triangle is 180° .

- 1 Draw any triangle ABC , with angles of size a° , b° and c° .



- 2 Draw a line DE parallel to CB through A .



- 3 Which angle in $\triangle ABC$ is equal to $\angle DAC$? Why?
4 Which angle in $\triangle ABC$ is equal to $\angle EAB$? Why?
5 What is $\angle DAC + a^\circ + \angle EAB$? Why?
6 What does this show about $a^\circ + b^\circ + c^\circ$?
7 What does this show about the angle sum of a triangle?
8 Check by measuring with a protractor.

Technology Angle sum of a triangle

This activity will use GeoGebra to find the angle sum of a triangle. To set up your drawing page, click **View** and **Grid**. Also click **View** and **Axes** (to remove the axes).

Measuring angles in a triangle

- 1 To draw any triangle, click on **Polygon**.

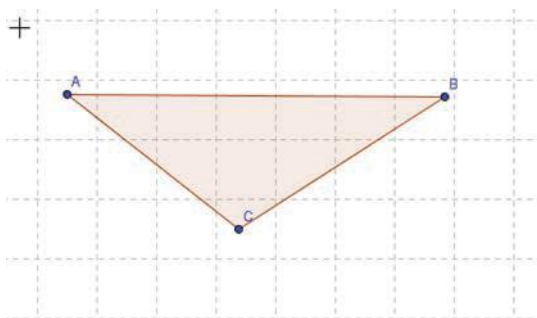


Technology

Geogebra
Angle sum of a triangle

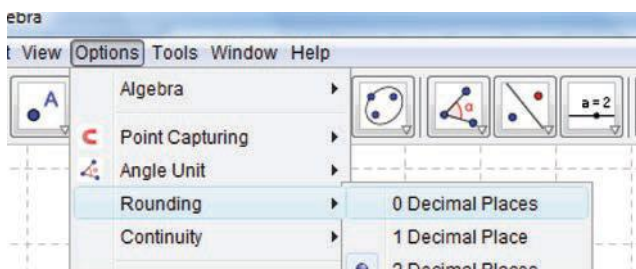
MAT07MGCT00001

Draw any triangle ABC.

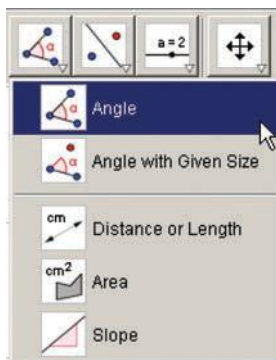


If your diagram doesn't have any labels, display them by right-clicking on each vertex and selecting **Show Label**.

- 2 Select **Options** then **Rounding** and **0 decimal places**.



- 3 Choose the **Angle** menu to measure the size of each angle (Remember: click 3 vertices in a clockwise direction with the angle you want to measure as the middle vertex).



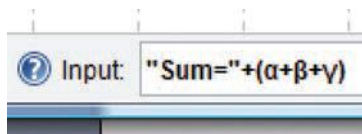
- 4 To find the angle sum of the triangle, use the input bar at the bottom of the screen.

Note: If you can't see an input bar at the bottom of your screen, then go to **View** and select **Input Bar**.

Choose **Sum** from **Command** on the bottom right-hand side.



- 5 Enter the formula shown below into **Input:** and hit Enter on your keyboard.

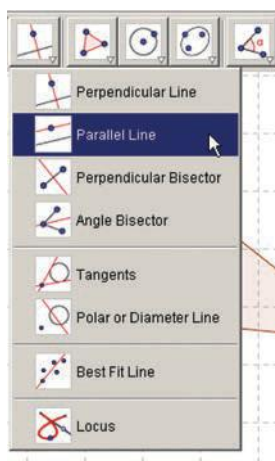


- 6 Look for the sum of the angles of your triangle near your diagram. Select **Move** from the first drop-down menu and click on each vertex in the triangle that you have drawn and watch the size of each angle change. What is the angle sum of any triangle?

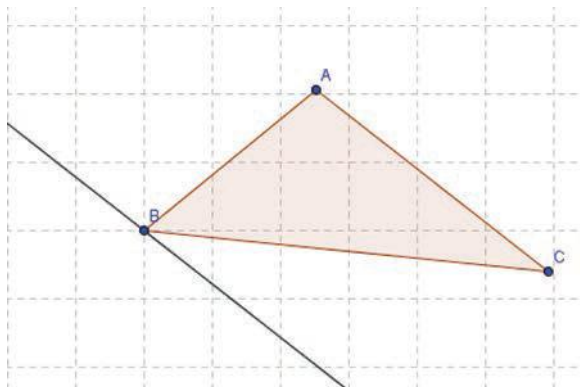
Moving a triangle

To set up your drawing page, click **View** and **Grid**. Also click **View** and **Axes** (to remove the axes).

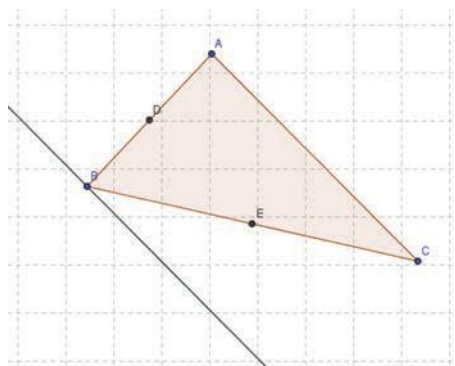
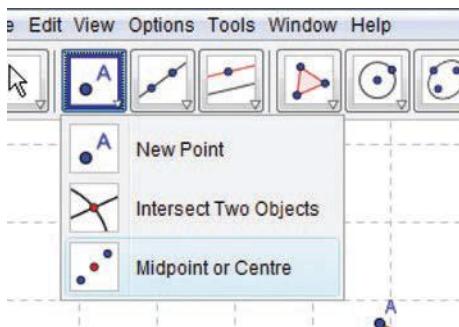
- 1 Use **Polygon** to draw any triangle and select **Show Label**.
- 2 Select **Parallel line**.



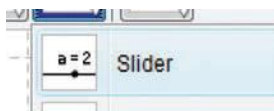
Select AC and Point B opposite. The line that you create is parallel to AC and passes through Point B.



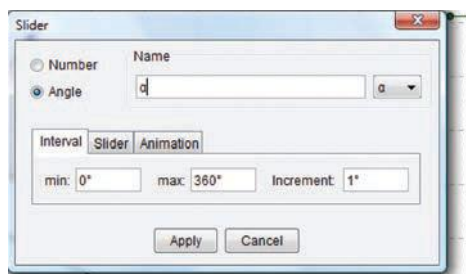
- 3 Select **Midpoint or centre** and click on AB and BC. Label the two midpoints (centres of AB and BC) D and E.



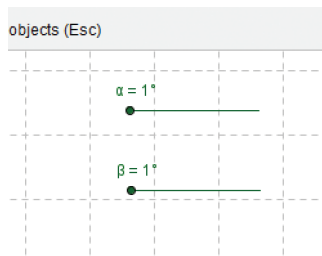
- 4 Next, create two sliders. Select **Slider**.



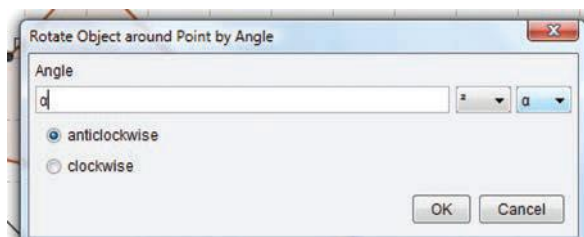
Click in triangle ABC. A slider screen will pop up. Select α from the menu and then **Angle**.



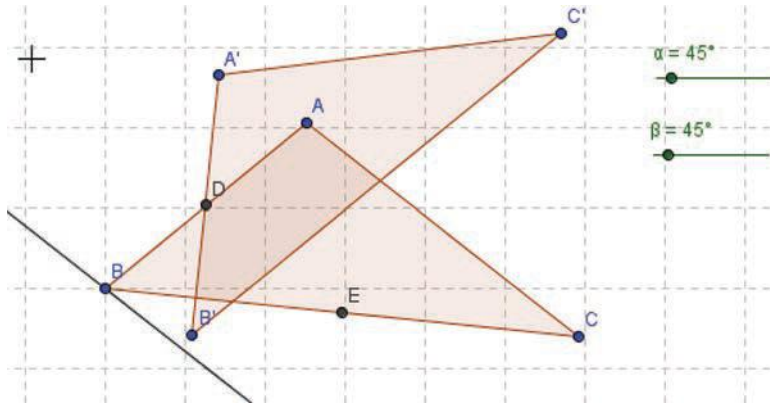
- 5 Make another slider with Name β . They will appear on your screen like this.



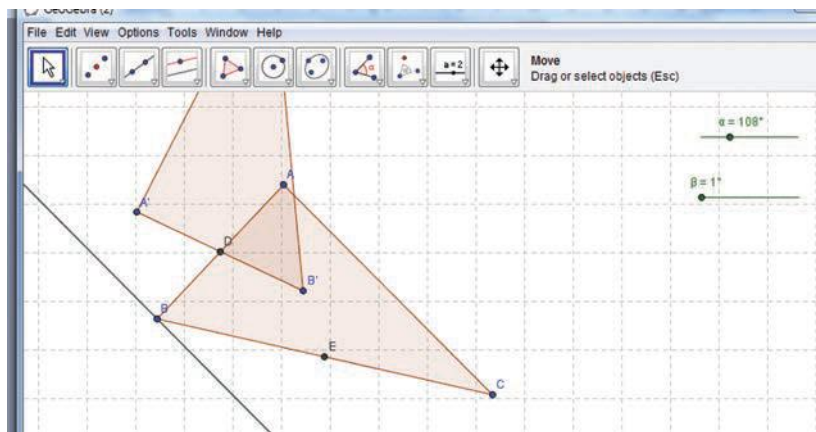
- 6 Select **Rotate Object around Point by Angle**. Click on triangle ABC and point D. Remove the 45° and click on α as shown below.



- 7 A new triangle named $A'B'C'$ is now positioned on top of triangle ABC .

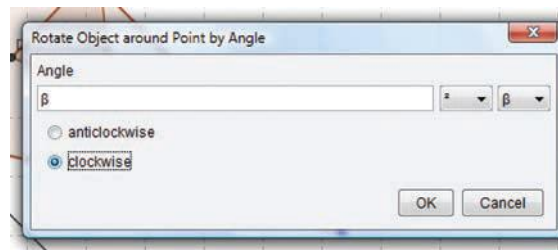


- 8 Select  **Move** from the first drop-down menu. To move the α slider, click on the slider, hold down your mouse button and drag it.



Use the slider to rotate the triangle until $\angle A'$ is aligned with $\angle B$ and parallel.

- 9 Repeat this procedure for triangle ABC and point E . Note that the angle is changed to β and select **clockwise** as shown below.

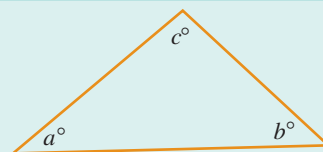


- 10 Rotate the triangle until $\angle C'$ in triangle $A'B'C'$ is aligned with $\angle B$.
- 11 Explain how these steps demonstrate that the angle sum of a triangle is 180° .

6-06 Angle sum of a triangle

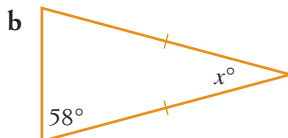
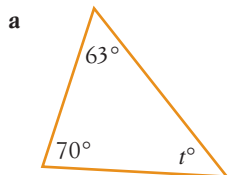
Summary

The **angle sum of a triangle** is 180°
 $a + b + c = 180$



Example 7

Find the value of each pronumeral, giving reasons.



Solution

a $t + 70 + 63 = 180$ (angle sum of a triangle)

$$t + 133 = 180$$

$$t = 180 - 133$$

$$= 47$$

b $x + 58 + 58 = 180$ (angle sum of an isosceles triangle)

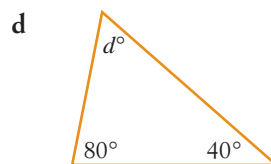
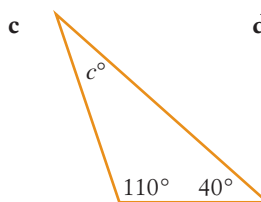
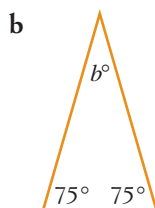
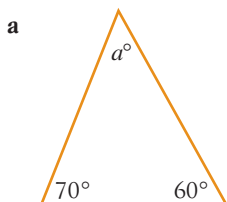
$$x + 116 = 180$$

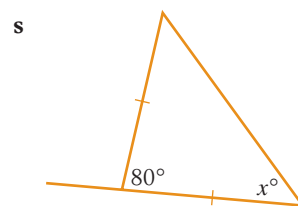
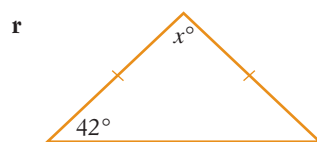
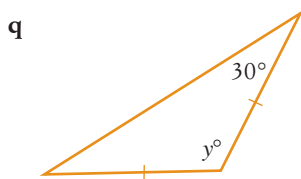
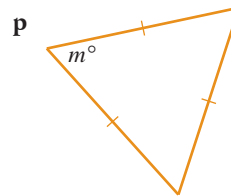
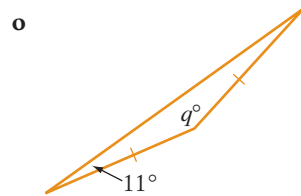
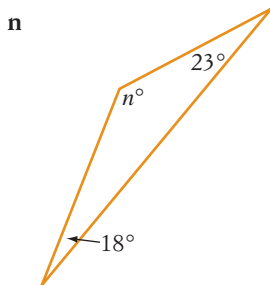
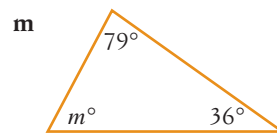
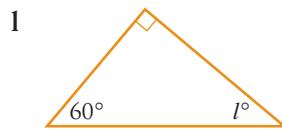
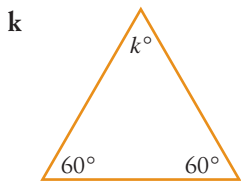
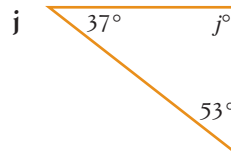
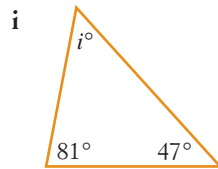
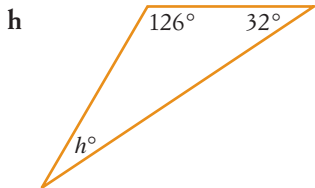
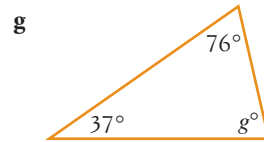
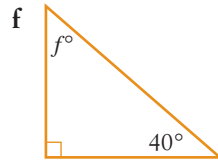
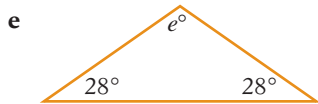
$$x = 180 - 116$$

$$= 64$$

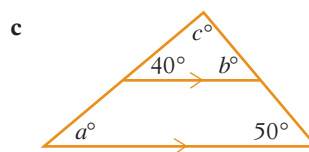
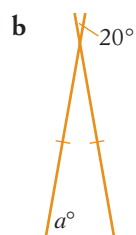
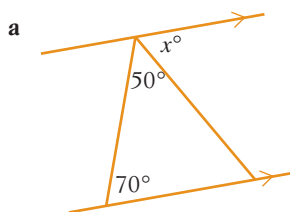
Exercise 6-06 Angle sum of a triangle

See Example 7 1 Find the value of each pronumeral.





2 Using what you know about angles, find the value of each pronumeral.

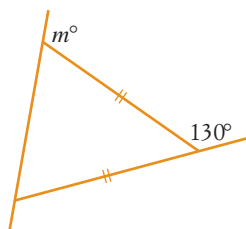


Worked solutions

Exercise 6-06

MAT07MGWS10037

- 3 What is the value of m in this diagram? Select the correct answer **A**, **B**, **C** or **D**.



A 115

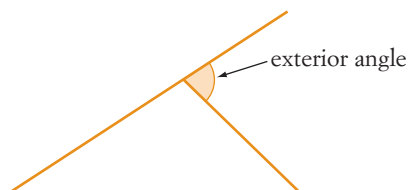
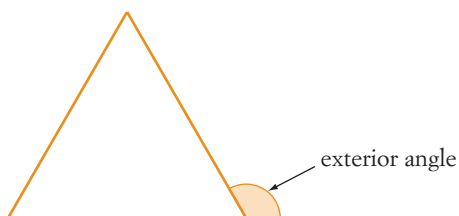
B 100

C 50

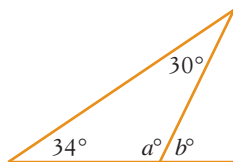
D 130

Investigation: Exterior angle of a triangle

An **exterior angle** of a triangle is created by extending one side of the triangle. 'Exterior' means 'outside', while 'interior' means 'inside'.



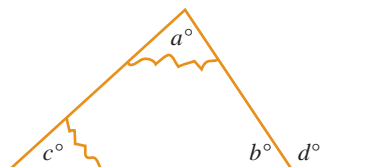
What is the relationship between the exterior angle of a triangle and the interior angles?



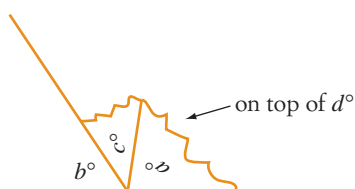
- Find the value of a and b in this triangle.
- Which is the exterior angle: a or b ?
- How is the exterior angle related to **two** of the interior angles of this triangle?
- Copy and complete: An exterior angle of a triangle is equal to the _____ of the interior opposite _____.

Paper cutting activity

- Draw a large triangle on paper and label its three angles a° , b° and c° .
- Extend one of the sides of the triangle and label the exterior angle d° . Cut out the triangle as well as angle d° .



- 3 Tear off the two angles a° and c° that are not adjacent to d° and arrange them next to each other on top of angle d° .



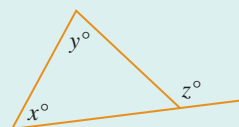
- 4 Do the two interior angles fit exactly on top of the exterior angle?
5 Check by measuring with a protractor.
6 To see if this works for all triangles, repeat the above steps for different triangles.

6-07 Exterior angle of a triangle

Summary

An **exterior angle of a triangle** is equal to the sum of the two interior opposite angles.

$$z = x + y$$



Worksheet

Triangle geometry

MAT07MGWK10047

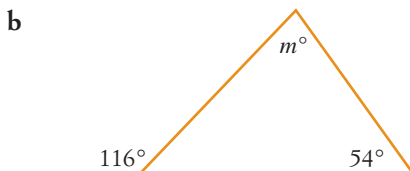
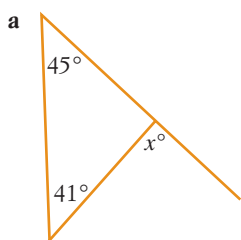
Homework sheet

Triangles

MAT07MGHS10032

Example 8

Find the value of each pronumeral, giving reasons.



Solution

a $x = 45 + 41$ (exterior angle of a triangle)
 $= 86$

b $m + 54 = 116$ (exterior angle of a triangle)
 $m = 116 - 54$
 $= 62$

Video tutorial

Exterior angle of a triangle

MAT07MGVT10014

Technology worksheet

Geogebra
Exterior angle of a triangle

MAT07MGCT10002

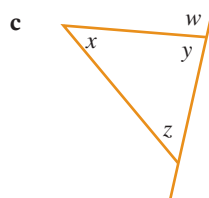
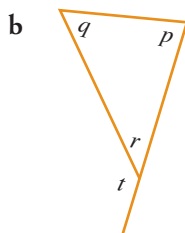
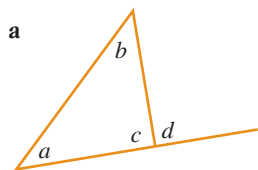
Technology

Geogebra
Exterior angle of a triangle

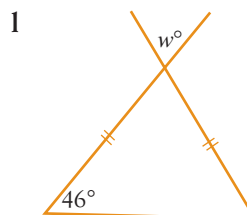
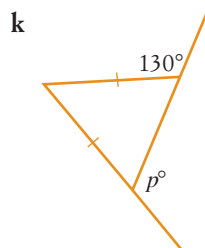
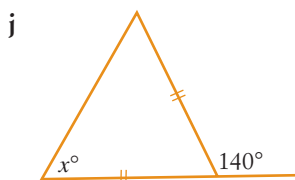
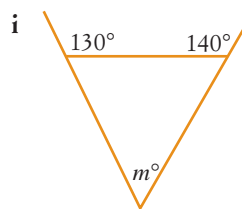
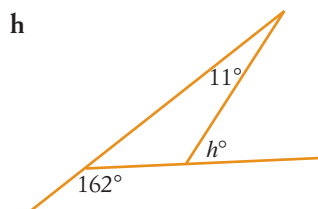
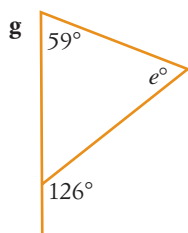
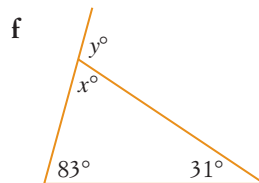
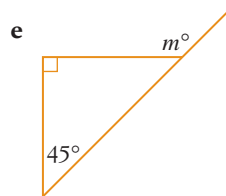
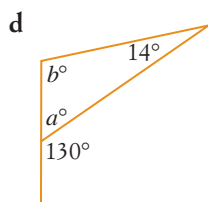
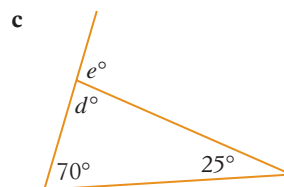
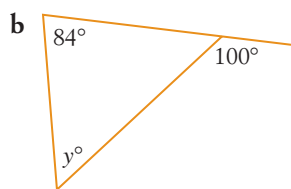
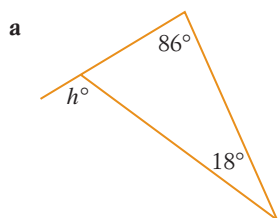
MAT07MGCT00004

Exercise 6-07 Exterior angle of a triangle

- 1 For each triangle below, name:
i the exterior angle
ii the two interior angles opposite the exterior angle.

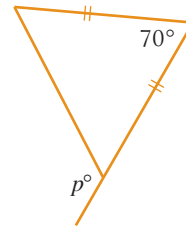


- See Example 8 2 Find the value of each pronumeral.



3 What is the value of p ? Select A, B, C or D.

- A 110
- B 125
- C 55
- D 140

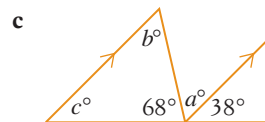
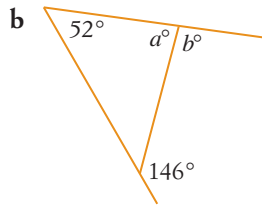
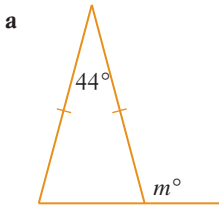


Worked solutions

Exercise 6-07

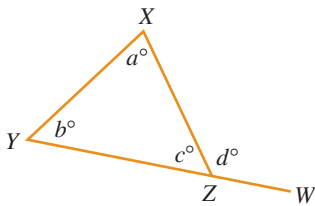
MAT07MGWS10038

4 Find the value of each pronumeral.



5 **Another exterior angle proof**

Copy and complete the missing reasons in this proof. Consider any triangle XYZ in which the angles are a° , b° and c° . Extend the interval YZ to the point W .



$a^\circ + b^\circ + c^\circ = 180^\circ$ because _____.

$\therefore a^\circ + b^\circ = 180^\circ - c^\circ$

But $d^\circ = 180^\circ - c^\circ$ because _____.

$\therefore d^\circ = a^\circ + b^\circ$

6-08 Classifying quadrilaterals

Worksheet

Constructing triangles and quadrilaterals

MAT07MGWK10048

Puzzle sheet

Constructions group clues

MAT07MGPS10023

Skillsheet

Naming shapes

MAT07MGSS10023

Technology

Geogebra
Making quadrilaterals

MAT07MGCT00005

TLF learning object

Exploring quadrilaterals (L6562)

TLF learning object

Exploring kites (L11104)

Weblink

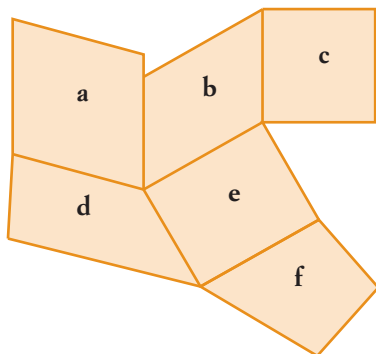
Tangrams

A **quadrilateral** is any shape with four sides, but there are six special quadrilaterals whose definitions are listed in the table below.

Name	A quadrilateral with:	Diagrams
Trapezium	one pair of opposite sides parallel	
Parallelogram	two pairs of opposite sides parallel	
Rhombus (or diamond)	four equal sides	
Rectangle	four right angles	
Square	four equal sides and four right angles	
Kite	two pairs of adjacent sides equal	

Exercise 6-08 Classifying quadrilaterals

- 1 Name each quadrilateral in this diagram.



2 Draw each of the special quadrilaterals and mark all axes of symmetry.

- a rectangle b square c parallelogram
d rhombus e trapezium f kite

3 Copy and complete this table.

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
	4	
Parallelogram		2
Trapezium		No rotational symmetry
Rhombus		2
	1	
	2	

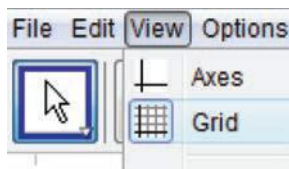
4 Name all quadrilaterals that have:

- a all angles equal to 90° b opposite sides parallel
c opposite sides equal d one pair of parallel sides
e four equal sides f two pairs of adjacent sides equal
g no axes of symmetry h rotational symmetry of order 2

5 Find what the prefix 'quad' means. Find other words beginning with 'quad' and their meanings.

Technology Measuring quadrilaterals

This activity will use GeoGebra to construct quadrilaterals. For each quadrilateral, set up your drawing page by clicking **View**. Make sure that **Grid** is ticked and **Axes** is not ticked.



Worksheet

Constructing triangles
and quadrilaterals

MAT07MGWK10048

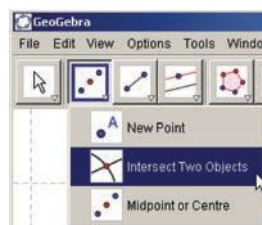
Square

- To draw a square, click on **Regular Polygon**. Use the grid to draw an interval of length 7.5 cm. Select 4 points. Make sure that the labels are showing.
- Use **interval between two points** to construct the two diagonals for the square. Select **distance or length** to measure the length of each diagonal in the square. What do you notice?
- Now measure the **Angle** of each vertex of the square. What do you notice?
- Draw another square with side length 9 cm. Repeat steps 2 and 3. List two properties of the square.

Rectangle

- To draw a rectangle, click on **Polygon**. Use the grid to draw a rectangle with sides 6 cm by 3 cm.
- Use **distance or length** to measure each side length of the rectangle.

- 3 Now measure the **Angle** of each vertex to check your accuracy. Correct any inaccurate sides using **Move**. What symbol is shown on each vertex when the angle is shown as exactly 90° ?
- 4 Draw another rectangle with length 5 cm and breadth 8.4 cm. Use the instructions given in step 2 above to measure the side lengths and angles and also to correct any inaccurate sides and/or angles.
- 5 Use **interval between two points** to construct the two diagonals for each of your rectangles. Select **distance or length** to measure the length of each diagonal in every rectangle. What do you notice?
- 6 Complete this property: The _____ in a rectangle are _____.
- 7 In one rectangle, select  **Intersect two objects** and click on the two diagonals.

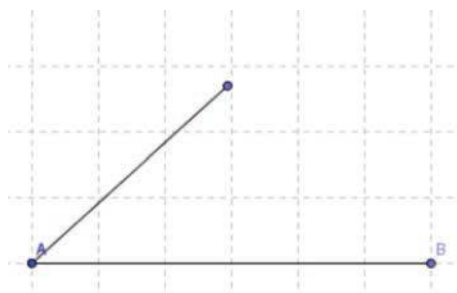


Use **distance or length** to measure the distance from the intersection point to the vertex for each diagonal. Repeat for the second rectangle. What do you notice?

- 8 Complete this property: The _____ of a rectangle _____ each other.

Parallelogram

- 1 Select  **Interval with given length from point**. Make the interval 6 cm long. Click **Show label**.
- 2 Now select **Interval with given length from point** and click point A. Make the interval 4 cm. Use **Move** and drag the new interval as shown below. Label the new point, C.



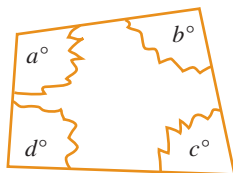
- 3 Click  **Parallel line** and select line AB and point C. Now select AC and point B.
- 4 Use **Intersect two objects** to create the missing vertex of the parallelogram. Label the vertex.

- 5 Use **distance or length** to show the length of each side of the parallelogram. What do you notice?
- 6 Complete the following: The _____ sides of a parallelogram are _____.
- 7 Now use **Angle** to find the size of $\angle CAB$ and $\angle CDB$. Repeat for $\angle ACD$ and $\angle ABD$. What do you notice?
- 8 Complete the following: The _____ angles of a parallelogram are _____.
- 9 Now draw the diagonals of the parallelogram. Use **distance or length** to show the length of each diagonal. What do you notice?
- 10 Complete the following: The diagonals of a parallelogram are _____.
- 11 Do the diagonals of a parallelogram bisect each other? Repeat step 7 from 'Rectangle' to help you.
- 12 Complete the following: The diagonals of a parallelogram _____ bisect each other.
- 13 Drag any vertices of the parallelogram that you can. Is it possible to draw other parallelograms with the same dimensions, 6 cm by 4 cm? What do you notice?
- 14 Use your GeoGebra skills to **accurately** construct other quadrilaterals such as a rhombus, kite or trapezium.

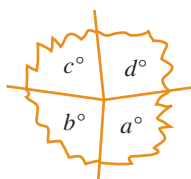
Investigation: Angle sum of a quadrilateral

What is the sum of the four angles in a quadrilateral? In groups of two to four, complete the following activity.

- 1 Draw any large quadrilateral on paper, cut it out and label its four angles a , b , c and d .



- 2 Tear off the four angles and arrange them so that their points meet.



- 3 What type of angle do they form? How many degrees does this type of angle have?
- 4 To see if this works for all quadrilaterals, repeat the above steps for different quadrilaterals.

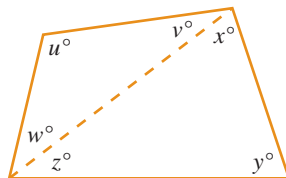
6-09 Angle sum of a quadrilateral

Puzzle sheet

Any quadrilateral can be divided into two triangles along one of its diagonals.

Find the unknown angle

MAT07MGPS10024



Homework sheet

Quadrilaterals

MAT07MGHS10033

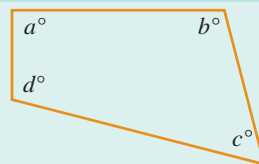
Because the angles in each triangle add to 180° , the angles in both triangles add to $2 \times 180^\circ = 360^\circ$.
 $u^\circ + v^\circ + w^\circ = 180^\circ$ and $x^\circ + y^\circ + z^\circ = 180^\circ$

$$\begin{aligned}\therefore \text{Angle sum of a quadrilateral} &= 180^\circ + 180^\circ \\ &= 360^\circ\end{aligned}$$

Summary

The **angle sum of a quadrilateral** is 360° .

$$a + b + c + d = 360$$



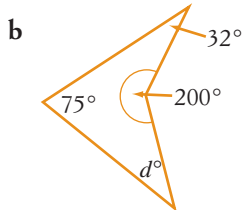
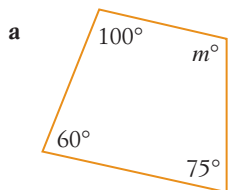
Homework sheet

Geometry revision

MAT07MGHS10034

Example 9

Find the value of each pronumeral, giving reasons.



Solution

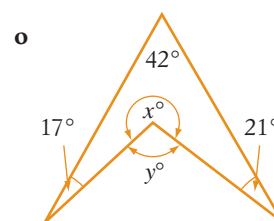
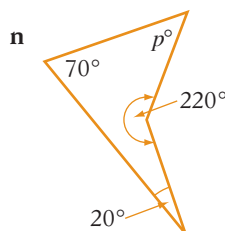
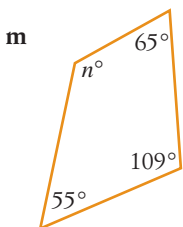
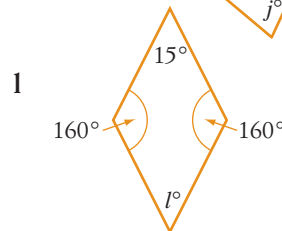
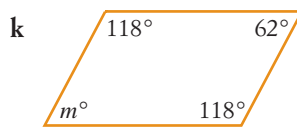
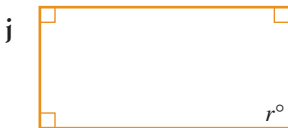
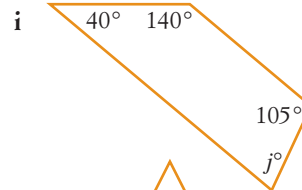
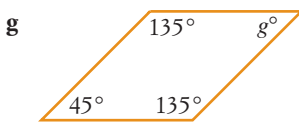
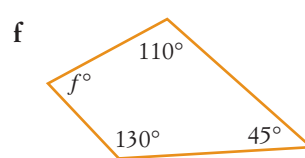
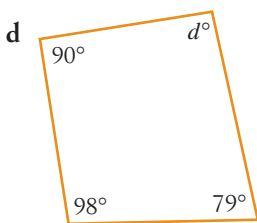
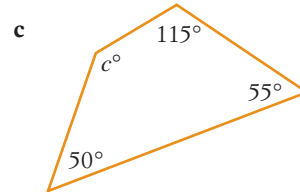
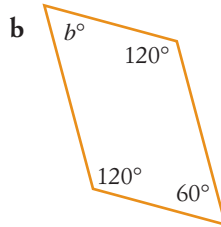
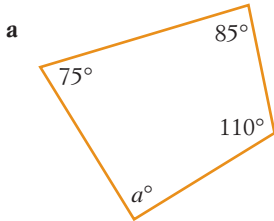
$$\begin{aligned}\mathbf{a} \quad m + 75 + 60 + 100 &= 360 \text{ (angle sum of a quadrilateral)} \\ m + 235 &= 360 \\ m &= 360 - 235 \\ &= 125\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad d + 75 + 32 + 200 &= 360 \text{ (angle sum of a quadrilateral)} \\ d + 307 &= 360 \\ d &= 360 - 307 \\ &= 53\end{aligned}$$

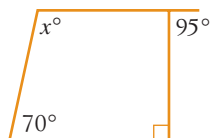
Exercise 6-09 Angle sum of a quadrilateral

1 Find the value of each pronumeral.

See Example 9



2 What is the value of x ? Select the correct answer A, B, C or D.



A 95

B 105

C 85

D 115

Worked solutions
Exercise 6-09

MAT07MGWS10039

- 3 Sketch a rhombus. If one of its angles is 75° , which of the following are the sizes of the other three angles? Select **A**, **B**, **C** or **D**.
- A** $75^\circ, 105^\circ, 105^\circ$ **B** $95^\circ, 95^\circ, 95^\circ$ **C** $75^\circ, 75^\circ, 75^\circ$ **D** $75^\circ, 15^\circ, 15^\circ$
- 4 Robert's favourite quadrilateral is equiangular, meaning that all of its angles are equal.
- a** What is the size of each angle?
- b** What is the most general name for Robert's favourite quadrilateral?

6-10 Properties of quadrilaterals

Exercise 6-10 Properties of quadrilaterals

Worksheet
Properties of
quadrilaterals

MAT07MGWK10049

Technology
Geogebra
Sides and angles of
quadrilaterals

MAT07MGCT00006

Technology
Geogebra
Quadrilaterals and their
diagonals

MAT07MGCT00007

- 1 **a** Draw and cut out an example of each special quadrilateral listed in the table below, or print out the worksheet 'Properties of quadrilaterals'.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides are equal						
Opposite sides are parallel						
Opposite angles are equal						
All angles are						
Diagonals are equal						
Number of axes of symmetry						
Order of rotational symmetry						

- b** Copy the table and test the properties of each quadrilateral listed by folding and measuring with a ruler, protractor and set square. If the listed property is true, then place a tick in the appropriate space. Write appropriate numbers in the last two rows.
- c** You should have noticed that there are no ticks for the kite. Write two properties of the kite (that is, two things that are always true about its sides, angles or diagonals).
- 2 Use a ruler and protractor with the quadrilaterals you cut out in question 1 to discover the properties of the diagonals of each one, as listed in the table below. Copy or print out this table. Place ticks in the appropriate spaces.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Diagonals bisect angles of quadrilateral						

- 3 I am a quadrilateral with opposite sides equal and parallel. My diagonals are equal and I have four axes of symmetry. Which quadrilateral am I? Select **A**, **B**, **C** or **D**.

A rectangle **B** square **C** parallelogram **D** rhombus

- 4 Which quadrilateral am I? (There may be more than one answer.)

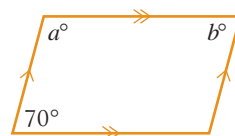
- My diagonals are equal.
- My diagonals bisect each other.
- I have four right angles.
- I have two pairs of parallel opposite sides.
- I have rotational symmetry, but no axes of symmetry.
- My diagonals cross each other at right angles.
- My diagonals bisect each other at right angles
- My diagonals are equal and bisect each other.

- 5 Copy and complete the blanks to find the values of a and b .

$a + 70 = 180$ (_____ angles on _____ lines)

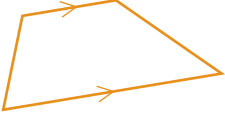
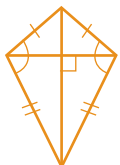
$a =$ _____

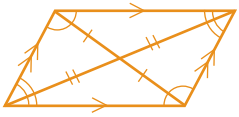
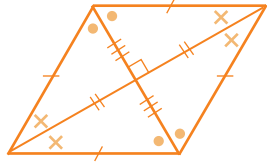
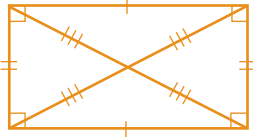
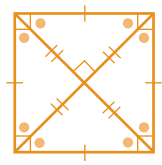
$b =$ _____ (opposite _____ of a parallelogram)



- Does a square have all the properties of a rectangle? Why?
- Does a rhombus have all the properties of a parallelogram? Why?

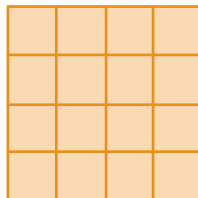
- 7 Copy this table of properties of quadrilaterals into your book and complete it.

Shape	Properties
Trapezium 	- One pair of _____ sides - No axes of symmetry
Kite 	_____ pairs of adjacent sides are equal - One pair of opposite angles are _____ - Has _____ axis of symmetry - Diagonals intersect at _____ angles

Shape	Properties
Parallelogram 	• _____ sides are equal and parallel • Opposite angles are _____ • No axes of symmetry • Diagonals _____ each other
Rhombus 	• All four sides are _____ • _____ sides are parallel • Opposite angles are _____ • Has _____ axes of symmetry • _____ bisect at right angles • Diagonals bisect the _____ of the rhombus
Rectangle 	• All four angles measure _____ • Opposite sides are _____ and _____ • Has two axes of _____ • Diagonals are _____ • _____ bisect each other
Square 	• All four sides are _____ • All four angles measure _____ • Has _____ axes of symmetry • Diagonals are equal and _____ each other at right angles • _____ bisect the angles of the square

Investigation: Shape puzzles

- 1 a How many squares can you find in this diagram? The answer is not 16!
- b How many rectangles can you find?



- 2 Can you trace this shape without going over any line twice and without lifting your pencil from the paper?

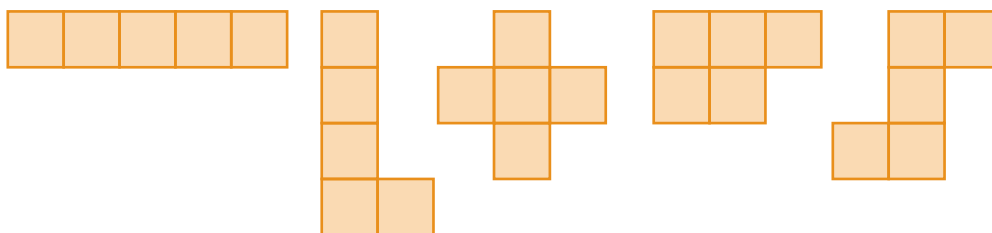


Puzzle sheet

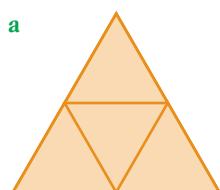
Shapes puzzle 1

MAT07MGPS00019

- 3 There are 12 different ways to join five squares edge to edge. These shapes are called pentominoes. Here are five of them. Draw the other seven.



- 4 How many triangles can you find in each of these shapes?



Power plus

- 1 Use the definitions of the quadrilaterals in the table on page 246 to decide whether:
 - a** a square is a special type of rhombus
 - b** a rhombus is a special type of square
 - c** a parallelogram is a special type of trapezium
 - d** a rectangle is a special type of parallelogram
 - e** a parallelogram is a special type of kite
 - f** a rectangle is a special type of square
- 2
 - a** What additional property makes a parallelogram a rectangle?
 - b** What makes a kite a rhombus?
 - c** What makes a rectangle a square?
- 3 Name the most general quadrilateral in which:

a opposite angles are equal	b diagonals intersect at 90°
c diagonals are equal	d all angles are 90°
e opposite sides are parallel	f diagonals bisect each other
- 4 Investigate the angle sum of a pentagon.
- 5
 - a** Draw any quadrilateral and extend each side to create four exterior angles.
 - b** Investigate the sum of the exterior angles.

Language of maths

Puzzle sheet
Geometry find-a-word
MAT07MGPS10025

acute-angled	image	quadrilateral	rotational symmetry
angle sum	isosceles	rectangle	scalene
axis/axes of symmetry	kite	reflection	square
centre of symmetry	line symmetry	rhombus	transformation
equilateral	obtuse-angled	right-angled	translation
exterior angle	parallelogram	rotation	trapezium

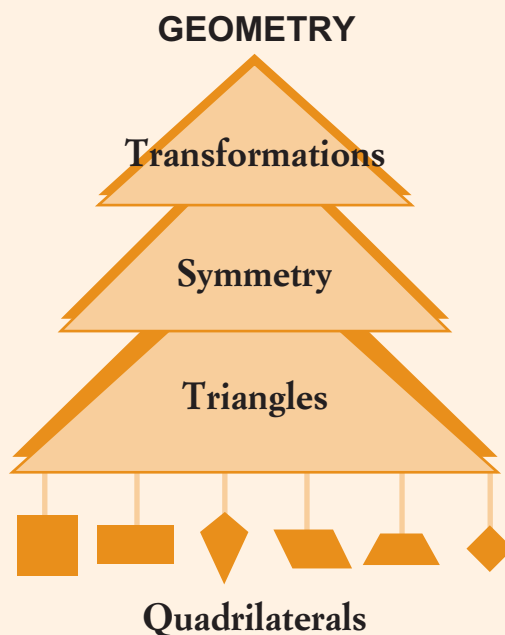
- What are the two types of symmetry?
- When you look into a mirror, you see your reflection. Is this meaning of 'reflection' related to its meaning in this chapter?
- A **regular** shape has equal sides and equal angles. What is the more common name for a **regular quadrilateral**?
- The word 'isosceles' comes from Greece. Use a dictionary to find out what it means in Greek.
- Use a dictionary to find the different meanings of:
 - translation
 - axis
- Name the shape that has two pairs of equal adjacent sides.

Topic overview

Worksheet
Mind map: Geometry
MAT07MGWK10050

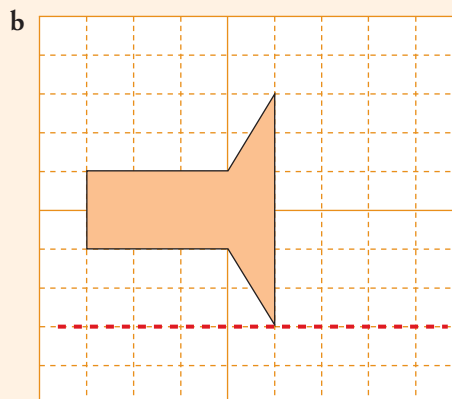
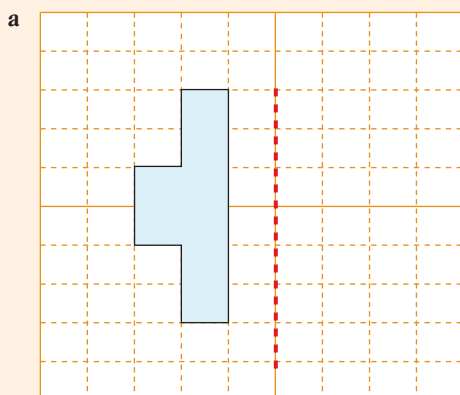
- How useful do you think this chapter will be to you in the future?
- Can you name any jobs which use some of the concepts covered in this chapter?
- Did you have any problems with any sections of this chapter? Discuss any problems with a friend or your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



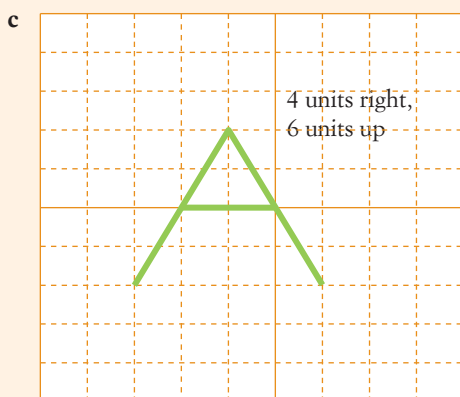
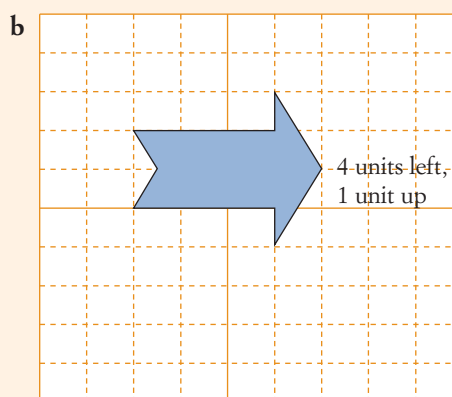
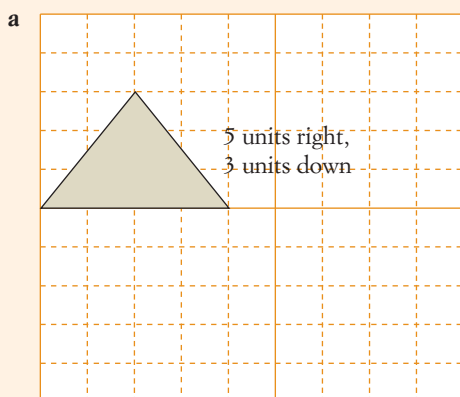
1 Copy and reflect each figure about the dotted line.

See Exercise 6-01



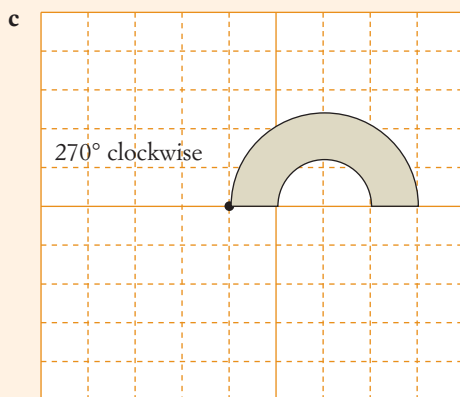
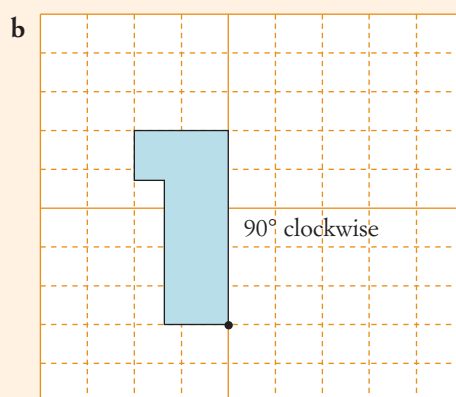
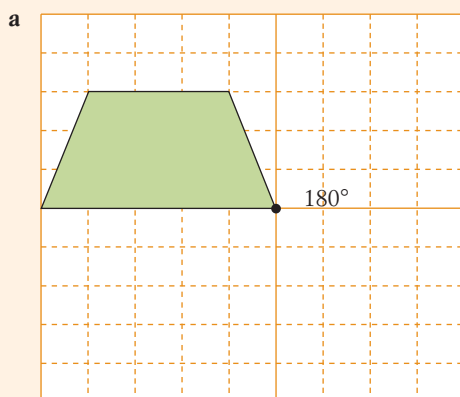
2 Copy and translate each figure by the given distance and direction.

See Exercise 6-01

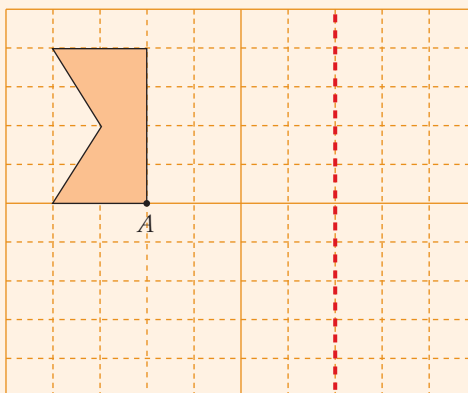


Chapter 6 revision

See Exercise 6-01 3 Copy and rotate each figure about the given point by the stated angle.



See Exercise 6-02 4 Copy and translate the shape below 5 units to the right and 2 units up. Then rotate it 90° clockwise about the vertex A and reflect it across the given dotted line.

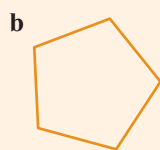


5 Draw the axes of symmetry of the following figures.

See Exercise 6-03

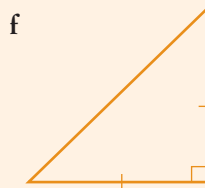
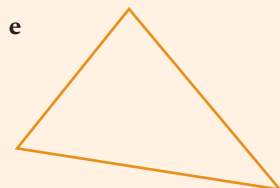
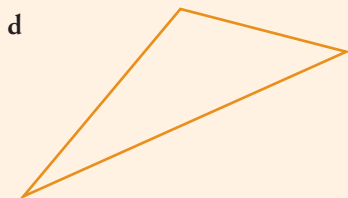
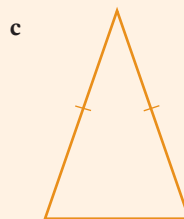
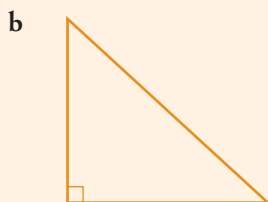
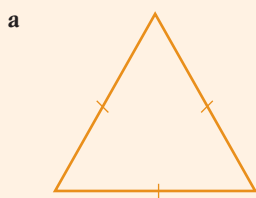


6 Which of the following have rotational symmetry? For those that do, write the order of rotational symmetry. See Exercise 6-04



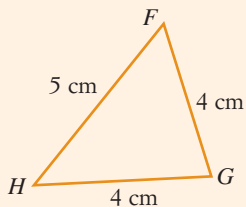
7 Classify each triangle by its sides and angles.

See Exercise 6-05



8 **a** Classify $\triangle FGH$ by sides and angles.

See Exercise 6-05

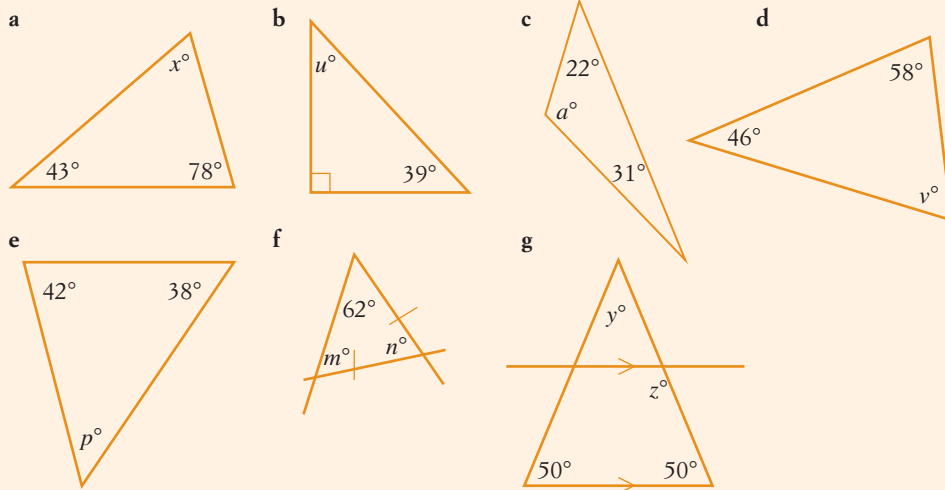


b Which angles in $\triangle FGH$ are equal?

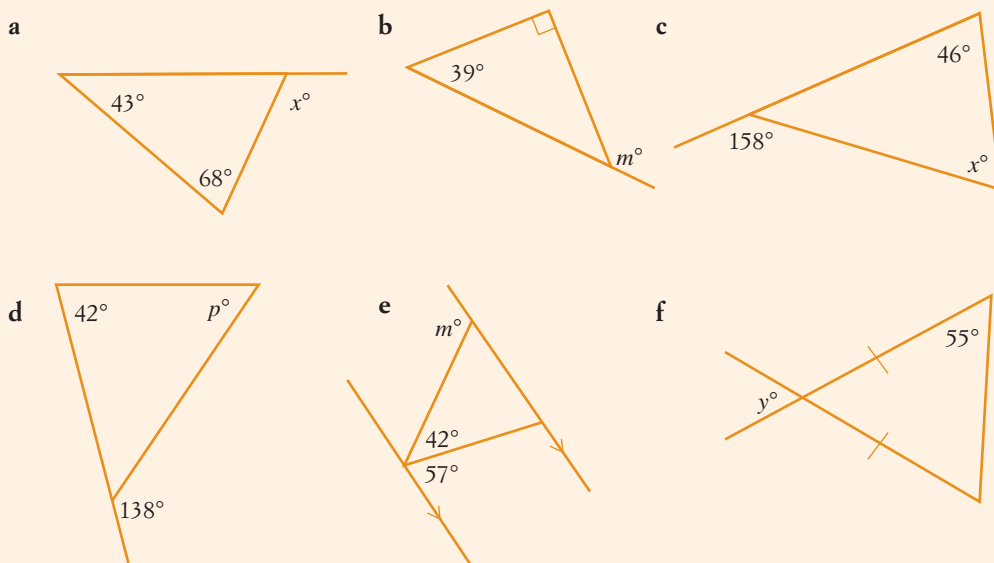
Chapter 6 revision

- See Exercise 6-05 9 Classify each triangle by sides and angles.
 a All of my angles are equal.
 b My angles are 60° , 80° and 40° .

- See Exercise 6-06 10 Find the value of each pronumeral.



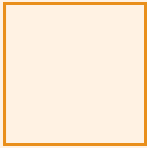
- See Exercise 6-07 11 Find the value of each pronumeral.



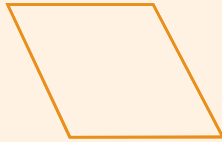
12 Name each quadrilateral.

See Exercise 6-08

a



b



c



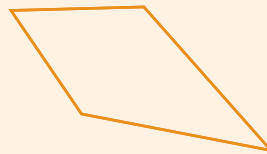
d



e



f



13 a Copy each shape in question 12 and mark all the axes of symmetry.

See Exercise 6-08

b List the special quadrilaterals that have rotational symmetry, and state the order of rotational symmetry of each one.

14 What is the definition of a rhombus?

See Exercise 6-08

15 What quadrilateral am I?

See Exercise 6-08

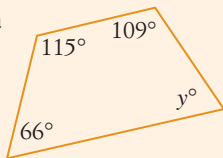
a I have opposite sides parallel.

b I have one pair of parallel sides.

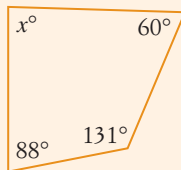
16 Find the value of each pronumeral.

See Exercise 6-09

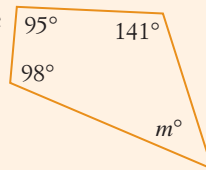
a



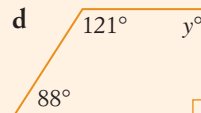
b



c



d



17 List two properties of a parallelogram.

See Exercise 6-10

18 What quadrilateral am I?

See Exercise 6-10

a All of my angles are equal.

b My diagonals are equal and bisect each other.

c My diagonals bisect each other.

d My opposite angles are equal.

Mixed revision 2

See Exercise 4-03

- 1 Evaluate each expression.

a $\frac{1}{2} + \frac{1}{3}$

b $\frac{1}{3} + \frac{2}{5}$

c $\frac{1}{2} - \frac{3}{8}$

See Exercise 4-10

- 2 Convert each fraction to a percentage.

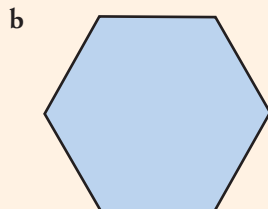
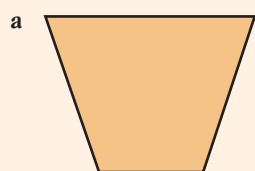
a $\frac{1}{4}$

b $\frac{3}{5}$

c $\frac{23}{40}$

See Exercise 6-03

- 3 Copy each shape and mark in its axes of symmetry.



See Exercise 5-03

- 4 Simplify each expression.

a $w \times 2$

b $a \times a$

c $4 \times a \times b \times k$

d $p + p + p + p$

e $m \div 2$

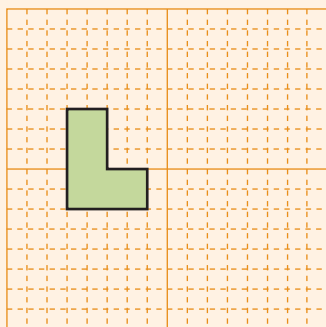
f $h - h$

g $6k + 2k$

h $5 \times m \times 2$

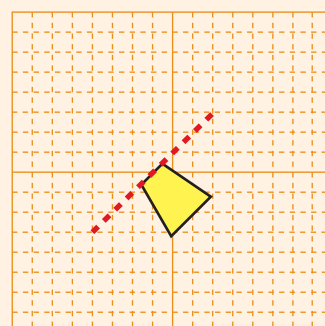
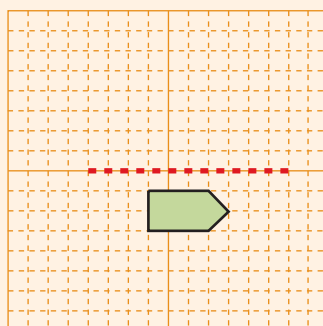
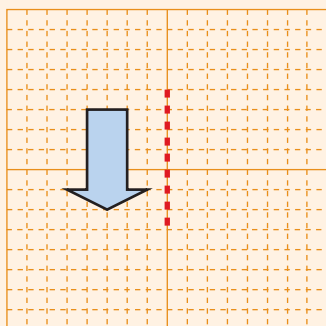
See Exercise 6-01

- 5 Copy and translate the L-shape 5 units to the right and 2 units down.



See Exercise 6-01

- 6 Copy each shape on square dot paper and reflect it across the dashed line.



- 7 Change each improper fraction to a mixed numeral.

See Exercise 4-01

a $\frac{12}{5}$

b $\frac{7}{2}$

c $\frac{5}{3}$

d $\frac{11}{6}$

- 8 Simplify each fraction.

See Exercise 4-01

a $\frac{8}{10}$

b $\frac{12}{18}$

c $\frac{15}{24}$

d $\frac{70}{100}$

- 9 Evaluate the following:

See Exercise 5-01

a $32 + 24 + 6$

b 2×900

c $-12 + 22$

d $31 \times 5 \times 4$

- 10 Evaluate each product.

See Exercise 5-02

a 18×11

b 24×8

c 15×9

d 22×12

- 11 Copy and complete each statement with a $>$ or $<$ symbol.

See Exercise 4-02

a $\frac{1}{3} \square \frac{1}{4}$

b $\frac{2}{5} \square \frac{5}{10}$

c $\frac{5}{7} \square \frac{3}{5}$

- 12 Evaluate each expression.

See Exercise 4-04

a $5\frac{3}{4} + 1\frac{2}{5}$

b $4\frac{3}{5} - 1\frac{1}{2}$

c $1\frac{2}{3} - \frac{7}{8}$

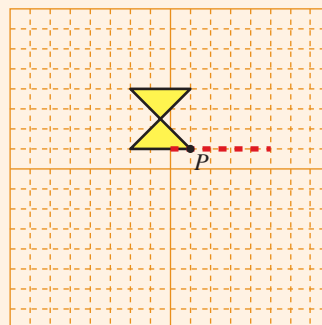
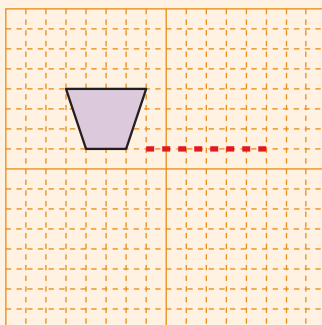
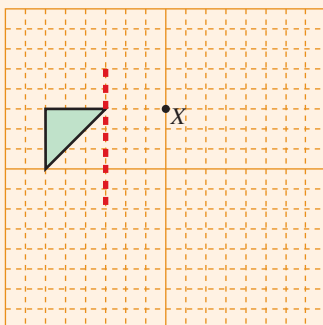
- 13 Copy each shape and draw the image after performing the stated composite transformations.

See Exercise 6-02

- a Reflect across the line, rotate 90° clockwise about X

- b Translate 5 units right, then reflect across the line

- c Rotate 90° clockwise about P , then reflect across the line, then translate 3 units left



- 14 For each shape:

See Exercise 6-04

- i find how many axes of symmetry it has
ii state whether it has rotational symmetry and if it does, state the order

a



b



c



Mixed revision 2

See Exercise 4-05 **15** Find $\frac{3}{8}$ of 2 m (in cm)

See Exercise 4-06 **16** Evaluate each product.

a $\frac{3}{5} \times \frac{1}{2}$

b $\frac{2}{3} \times \frac{7}{12}$

c $3\frac{1}{3} \times 2\frac{1}{2}$

See Exercise 5-05 **17** Evaluate each expression.

a $m - 3$ if $m = -2$

b $2p - 1$ if $p = 10$

c $\frac{2d}{3}$ if $d = 12$

See Exercise 5-07 **18** Solve each equation.

a $x + 11 = 34$

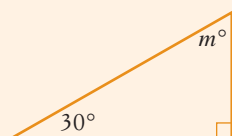
b $3x = -27$

c $d - 3 = -9$

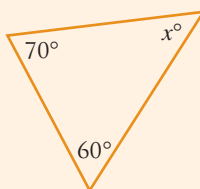
d $\frac{m}{5} = 1$

See Exercise 6-05 **19** Classify each triangle by sides and by angles.

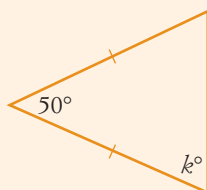
a



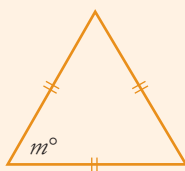
b



c



d



See Exercise 6-06 **20** Find the value of the pronumeral in each triangle in question 19.

See Exercise 4-10 **21** Convert each decimal to a percentage.

a 0.4

b 0.05

c 0.12

See Exercise 4-11 **22** Find:

a 5% of \$200

b 30% of 1 hour (in mins)

See Exercise 5-04 **23** Write an expression for each statement.

a the product of p and q

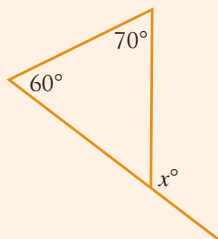
b 4 less than x

c d decreased by 7

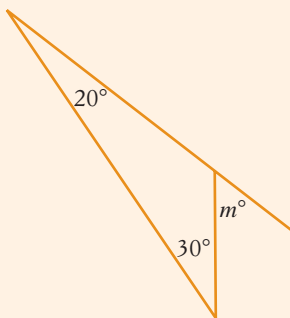
d the number of girls if there are b boys in a class of 28 students

See Exercise 6-07 **24** Find the value of each pronumeral.

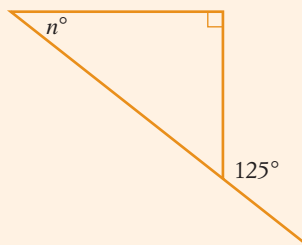
a



b



c



- 25 Evaluate each quotient.

See Exercise 4-07

a $\frac{3}{4} \div \frac{1}{2}$

b $\frac{2}{10} \div 5$

c $2\frac{1}{2} \div \frac{3}{5}$

- 26 Chloe owns 40 pairs of shoes. Two pairs are black.

See Exercise 4-12

a What fraction of Chloe's shoes are black?

b What percentage of Chloe's shoes are black?

- 27 Solve each equation.

See Exercise 5-08

a $5m - 3 = 22$

b $6n + 3 = 12$

c $-3x - 11 = -20$

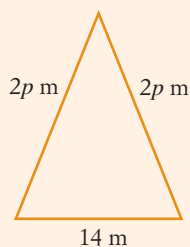
d $\frac{x}{2} + 5 = 9$

- 28 Solve each problem by writing an equation and then solving it.

See Exercise 5-09

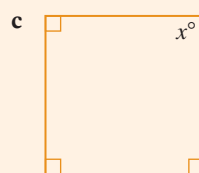
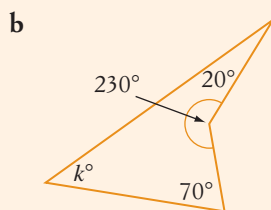
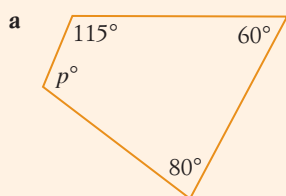
a If you multiply Mr Ha's age by 4 and add 16 you obtain 200. How old is Mr Ha?
(Let h be his age.)

b The perimeter of this isosceles triangle is 56 cm. Find the value of p .



- 29 Find the value of each pronumeral.

See Exercise 6-09



- 30 Name each quadrilateral whose definition is given below.

See Exercise 6-08

a The quadrilateral with opposite sides parallel.

b The quadrilateral with four right angles.

c The quadrilateral with four equal sides and four equal angles.

d The quadrilateral with two pairs of equal adjacent sides.



7

Number and algebra

Decimals

Since the introduction of the metric system, decimals have become more commonly-used than fractions to describe parts of a whole.

Metric units follow a decimal system, meaning that they are related by powers of ten, for example: $1 \text{ km} = 1000 \text{ m}$, $1 \text{ m} = 100 \text{ cm}$, $1 \text{ cm} = 10 \text{ mm}$. However, before the metric system was adopted in Australia in 1974, we used the imperial system of measurement that was not decimal-based: $1 \text{ mile} = 1760 \text{ yards}$, $1 \text{ yard} = 3 \text{ feet}$, $1 \text{ foot} = 12 \text{ inches}$. In those days, measurement calculations were more difficult.



Chapter outline

	Proficiency strands			
7-01 Ordering decimals	U	F		
7-02 Decimals and fractions	U	F		
7-03 Adding and subtracting decimals	U	F	PS	
7-04 Multiplying and dividing decimals by powers of 10	U	F		
7-05 Multiplying decimals by estimating	U	F		
7-06 Multiplying decimals	U	F	PS	R
7-07 Dividing decimals by whole numbers	U	F	PS	R
7-08 Dividing decimals	U	F	PS	R
7-09 Terminating and recurring decimals	U	F		R
7-10 Rounding decimals	U	F	PS	
7-11 Decimal problems		F	PS	

Wordbank

ascending Increasing from smallest to largest

decimal places The places after the decimal point in a number

recurring decimal A decimal that has one or more digits that repeat endlessly

round To write a number to a given number of places

tenth The fraction $\frac{1}{10}$ or the decimal 0.1, the first decimal place

terminating decimal A decimal that is not recurring, but comes to an end

In this chapter you will:

Maths clip

Naming decimals

MAT07NAVT00004

- compare and order decimals
- connect fractions and decimals and carry out simple conversions
- add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers
- multiply and divide decimals by powers of 10
- multiply and divide decimals using efficient written strategies and digital technologies
- round decimals to a specified number of decimal places
- **investigate terminating and recurring decimals**

SkillCheck

Worksheet

StartUp assignment 7

MAT07NAWK10051

Skillsheet

Decimals

MAT07NASS10025

- 1 Copy this place value table so that it has 8 blank rows.

hundreds	tens	units	•	tenths	hundredths	thousandths

Write each decimal in the table, with the digits in their correct columns.

- a** 14.82 **b** 6.014 **c** 931.02 **d** 70.8
e 0.375 **f** 0.19 **g** 8.592 **h** 715.3

- 2 Write the value of the digit 4 in each decimal.

- a** 431.70 **b** 31.047 **c** 761.04
d 114.37 **e** 3.734 **f** 907.431

- 3 Simplify each fraction.

- a** $\frac{18}{100}$ **b** $\frac{4}{10}$ **c** $\frac{350}{1000}$ **d** $\frac{24}{100}$

- 4 List each set of numbers in ascending order.

- a** 21, 32, 34, 30, 20 **b** 213, 216, 234, 206, 230

- 5 Evaluate each expression.

- a** 32×10 **b** 578×100 **c** 325×1000
d 400×10 **e** $400 \div 10$ **f** $1200 \div 100$
g $1\ 000\ 000 \div 1000$ **h** $81\ 000 \div 100$ **i** $640 \div 10$

- 6 List each set of numbers in descending order.

- a** 44, 39, 42, 45, 38, 40 **b** 505, 556, 513, 520, 549, 522

- 7 Round:

- a** 27 to the nearest ten **b** 752 to the nearest hundred
c 9079 to the nearest thousand **d** 16 837 to the nearest thousand

Skillsheet

Multiplying by 10, 100, 1000

MAT07NASS10026

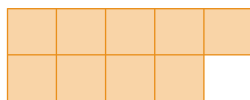
8 Estimate the value of each expression.

- a $140 + 32 + 381$ b $432 + 45 + 2341 + 7$ c $746 - 29$
 d $596 - 338$ e $1138 - 374$ f 58×3
 g 126×25 h $5920 \div 8$ i $2233 \div 7$

9 Evaluate each expression in question 8 without using a calculator.

7-01 Ordering decimals

A **decimal** is a special type of fraction where the denominator is a **power of ten** such as 10, 100 or 1000. This shape has been divided into 10 equal parts, of which 9 have been shaded.



- As a fraction, $\frac{9}{10}$ or 'nine-tenths' of the shape is shaded.
 - As a decimal, 0.9 or 'zero-point-nine' of the shape is shaded.
- You are familiar with numbers that have decimal points, for example:
- money \$532.81
 - measurements 6.2 km.

The position of a digit in a number shows its size. This is called the **place value**.

In a decimal, the digits after the decimal point indicate a part of a whole. For example, the meaning of the decimal 532.81 is shown on the place value table below:

hundreds	tens	units	tenths	hundredths
100	10	1	$\frac{1}{10}$	$\frac{1}{100}$
5	3	2	8	1

decimal point

$$532.81 \text{ is } 5 \times 100 + 3 \times 10 + 2 \times 1 + 8 \times \frac{1}{10} + 1 \times \frac{1}{100}.$$

Summary

The number of digits after the decimal point tells us the number of **decimal places** in a decimal.

Worksheet

Decimals wall

MAT07NAWK10052

Worksheet

Dewey decimals

MAT07NAWK10053

Skillsheet

Decimals

MAT07NASS10025

TLF learning object

Swamp survival:
thousandths challenge
(L7906)

TLF learning object

Design a farm (L124)

Worksheet

Decimals 1

MAT07NAWK00035

Worksheet

Decimals 10

MAT07NAWK00039

Example 1

How many decimal places are there in:

a 3.6567?

b 15.801?

Solution

a 3.6567 has 4 decimal places.

1234

b 15.801 has 3 decimal places.

123

Example 2

Arrange these decimals in ascending order: 67.41, 67.14, 6.714, 67.04.

Solution

To compare decimals more easily, place them in a column with the decimal points in a vertical column and make them all have the same number of decimal places by inserting 0s at the end.

For example, 67.41 becomes 67.410.

67.410

67.140

6.714

67.040

All numbers now have three decimal places.

From smallest to largest: 6.714, 67.040, 67.140, 67.410.

In ascending order: **6.714, 67.04, 67.14, 67.41**

Example 3

Arrange these numbers in descending order: 0.5, 0.08, 1.7, 0.85.

Solution

Make them all have the same number of decimal places by inserting 0s at the end.

0.5 becomes 0.50 and 1.7 becomes 1.70.

0.50

0.08

1.70

0.85

All numbers now have two decimal places.

From largest to smallest: 1.70, 0.85, 0.50, 0.08.

In descending order: **1.7, 0.85, 0.5, 0.08**

Exercise 7-01 Ordering decimals

1 How many decimal places does each number have?

a 1.65	b 3.881	c 15.3062
d 0.005	e 7.045 73	f 814.3
g 9.597 684	h 203.602 39	i 0.0424 706

See Example 1

2 Which of these is the smallest decimal? Select the correct answer A, B, C or D.

- A 1.07 B 1.7 C 1.077 D 1.77

See Example 2

3 Arrange each set of numbers in ascending order.

- a 43.89, 56.324, 9.998, 80.879, 400, 23.89, 56.314
b 0.568, 0.684, 0.099, 1.002, 0.586, 5.608, 0.0586
c 1.23, 0.891, 1.814, 0.222, 7.007, 0.89
d 0.5, 0.05, 0.005
e 3.441, 3.404, 3.4, 3.44, 3.004, 3.044
f 0.2, 0.202, 0.22, 0.022

4 Arrange each set of numbers in descending order.

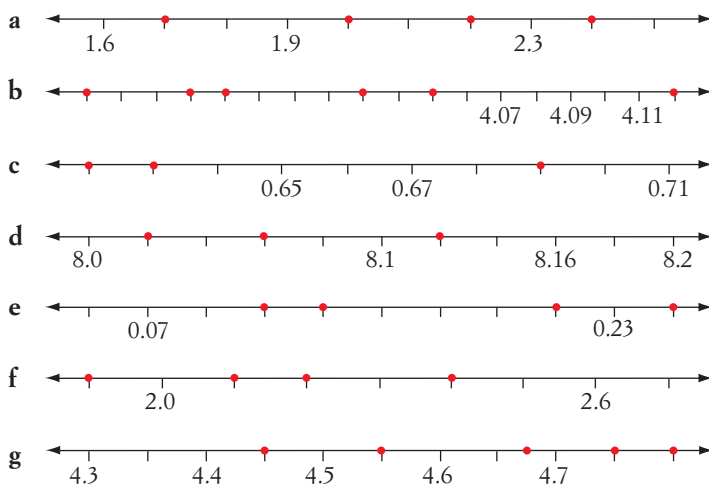
- a 570.25, 125.63, 0.9899, 4000.99, 1256.3, 400.099
b 5.37, 6.539, 5.639, 5.367, 3.659, 3.66, 5.369
c 1.6, 1.61, 1.599, 1.601, 1.509
d 6, 0.06, 0.6, 6.6
e 0.7, 0.07, 0.707, 0.77, 0.007, 7.07
f 0.4004, 0.044, 0.404, 0.44

See Example 3

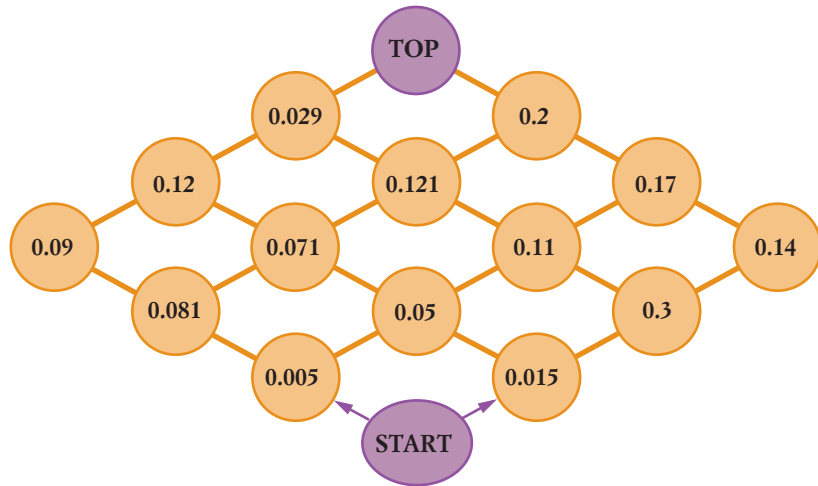
5 Insert $<$ or $>$ to make each statement true.

a 0.2 _____ 0.25	b 0.731 _____ 0.73
c 0.035 _____ 0.305	d 0.007 _____ 0.070
d 1.59 _____ 1.059	f 0.099 _____ 0.99
g 44.44 _____ 4.444	h 0.7932 _____ 0.7239

6 Copy each number line and write the values of the points marked with dots.



- 7 Write a decimal that is:
- a less than 1.3
 - b greater than 2.07
 - c between 8.25 and 8.5
 - d 6.401 and 6.41
- 8 Find a path from START to TOP. You can make your move from START in any direction, but then you can only move to a larger decimal. Try to find the shortest path, then try to find the longest path.



Investigation: Comparing heights

- 1 Use the clues below to find each girl's name and height.

- Mandy is taller than Sarah.
- Sarah is shorter than Yin.
- Kelly is taller than Sarah but shorter than Mandy.
- Mandy is not the tallest.

The heights of the girls are 168.5 cm, 166.3 cm, 164.2 cm and 160.7 cm.



2 Use the clues below to find each boy's name and height.

- Steve is 164.7 cm tall.
- Mike is 14.7 cm taller than Milof.
- Steve is 3.9 cm shorter than Milof.
- Ganesh is 1.6 cm taller than Mike.

A



B



C



D



3 Use the clues below to find each student's name and height.

- Yoko is 15.1 cm taller than Peter.
- Jade is 13.7 cm shorter than Yoko.
- Karl is 20.6 cm taller than Jade.
- Yoko is 6.9 cm shorter than Karl.
- Peter is 163 cm tall.

A



B



C



D



4 Create your own problem using four students from your class. Estimate their heights and height differences. Write a set of clues. (Don't forget to change their names!)

7-02 Decimals and fractions

Example 4

Convert each decimal to a simplified fraction.

a 0.09

b 0.274

c 1.45

Solution

a $0.09 = \frac{9}{100}$

2 decimal places means hundredths ($\frac{\quad}{100}$)

b $0.274 = \frac{274}{1000}$
 $= \frac{137}{500}$

3 decimal places means thousandths ($\frac{\quad}{1000}$)

Simplify the fraction

c $1.45 = 1 + \frac{45}{100}$
 $= 1\frac{9}{20}$

2 decimal places means hundredths ($\frac{\quad}{100}$)

Simplify the fraction

Note: The number of decimal places in the decimal gives the number of zeros in the denominator of the fraction.

Summary

- tenths have one decimal place ($\frac{1}{10}$ has 1 zero)
- hundredths have two decimal places ($\frac{1}{100}$ has 2 zeros)
- thousandths have three decimal places ($\frac{1}{1000}$ has 3 zeros)
- ten thousandths have four decimal places ($\frac{1}{10000}$ has 4 zeros)

Example 5

Convert each fraction to a decimal.

a $\frac{22}{100}$

b $3\frac{7}{10}$

c five thousandths

Solution

a $\frac{22}{100} = 0.22$

2 zeros $\rightarrow$ 2 decimal places

b $3\frac{7}{10} = 3.7$

1 zero $\rightarrow$ 1 decimal place

c five thousandths $= \frac{5}{1000} = 0.005$

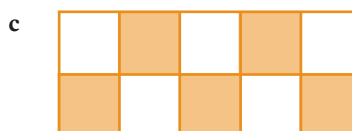
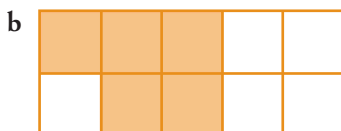
3 zeros $\rightarrow$ 3 decimal places

Exercise 7-02 Decimals and fractions

1 What part of each shape has been shaded? Write the answer as:

i a fraction

ii a decimal



2 Convert each decimal to a simplified fraction.

a 0.7	b 0.4	c 0.39	d 0.572
e 0.003	f 0.05	g 0.11	h 0.328
i 0.3	j 0.999	k 0.013	l 0.0004
m 0.0471	n 0.321	o 0.5005	p 0.91
q 0.082	r 1.6	s 27.25	t 2.007

See Example 4

3 Which decimal is equal to $\frac{23}{1000}$? Select the correct answer A, B, C or D.

- A 0.23 B 0.023 C 0.0023 D 0.000 23

See Example 5

4 Convert each fraction to a decimal.

a $\frac{9}{10}$	b $\frac{15}{100}$	c $\frac{79}{100}$	d $\frac{60}{100}$
e four-tenths	f $\frac{23}{100}$	g $\frac{6}{10}$	h $\frac{411}{1000}$
i $\frac{704}{1000}$	j eighty-seven hundredths	k $\frac{7}{100}$	
l $\frac{14}{100}$	m $\frac{8}{10}$	n $\frac{235}{1000}$	o $\frac{247}{1000}$
p $\frac{17}{100}$	q $\frac{368}{1000}$	r $\frac{9345}{10\ 000}$	s $\frac{493}{1000}$
t $1\frac{67}{100}$	u $4\frac{47}{100}$	v $23\frac{9}{10}$	w $6\frac{8}{10}$
x twenty-three thousandths	y forty-five hundredths		

7-03 Adding and subtracting decimals

Summary

When adding or subtracting decimals:

- keep decimal points below one another
- check your answer by estimating

Homework sheet

Decimals 1

MAT07NAHS10011

TLF learning object

Wishball: tournament
(L8460)

Example 6

TLF learning object

Circle 3 (L3507)

Find the sum of 10.92, 0.89, 32 and 0.6.

Solution

10.92

0.89

32.00

+0.60

44.41

Fill in any spaces with zeros.

Remember that 32 is the same as 32.00.

Estimate: $10.92 + 0.89 + 32 + 0.6 \approx 11 + 1 + 32 + 1 = 45$

Example 7

Evaluate each difference.

a $76.029 - 8.914$

b $4.31 - 2.183$

Solution

a 76.029

-8.914

67.115

Estimate: $76.029 - 8.914 \approx 76 - 9 = 67$

b 4.310

Fill in any spaces with zeros.

-2.183

2.127

Estimate: $4.31 - 2.183 \approx 4 - 2 = 2$

Exercise 7-03 Adding and subtracting decimals

See Example 6

1 Copy and complete each sum.

a $\begin{array}{r} 5.3 \\ 6.2 \\ +0.5 \\ \hline \end{array}$

b $\begin{array}{r} 4.723 \\ 0.01 \\ +12.2 \\ \hline \end{array}$

c $\begin{array}{r} 43.5 \\ 116.29 \\ 7.3 \\ + 0.227 \\ \hline \end{array}$

d $\begin{array}{r} 0.0076 \\ 1.23 \\ +0.09 \\ \hline \end{array}$

See Example 7

2 Copy and complete each difference.

a $\begin{array}{r} 57.703 \\ -16.21 \\ \hline \end{array}$

b $\begin{array}{r} 6.1 \\ -0.2 \\ \hline \end{array}$

c $\begin{array}{r} 23.57 \\ -16.88 \\ \hline \end{array}$

d $\begin{array}{r} 22.6 \\ -13.54 \\ \hline \end{array}$

3 Evaluate $0.61 + 12.345$. Select the correct answer A, B, C or D.

A 73.345

B 18.445

C 12.955

D 12.406

- 4 An electrician needed these lengths of cable to complete a wiring job: 12.3 m, 4.8 m, 18.7 m, 7.98 m, 13.65 m and 23.6 m.
- How many metres of cable did the electrician use?
 - If the full spool of cable was 100 m long, how many metres of cable were left on the spool after the electrician completed the job?
- 5 To keep fit, Angela runs each day. Last week she ran 3.8 km, 4.1 km, 2.3 km, 2.6 km, 3 km, 1 km and 1.8 km. How far did she run last week?
- 6 A truck carrying sand had a total mass of 13 248 kg. If the truck alone had a mass of 5210.8 kg, what is the mass of the sand?
- 7 Five runners in the school's 100 m race recorded the following times: 13.5 s, 13.81 s, 12.7 s, 14.62 s, 12.45 s.
- Place these times in order, from fastest to slowest.
 - What is the time difference between the fastest and slowest runners?
 - If the runner in second place had run 0.3 seconds faster, would she have won the race? Explain your answer.
- 8 Dion's expenses for one week are shown in the table below.

Food	\$128.80
Clothing	\$ 88.45
Car	\$ 58.35
Rent	\$185.00
Entertainment	\$ 78.95
Savings	\$ 66.00

- How much did he spend?
 - How much did he have left out of his weekly pay of \$620.80?
- 9 A new block of wood is 11.27 cm thick. If it has 0.34 cm shaved off one side and 0.55 cm shaved off the other side, how thick is the block of wood? Select the correct answer **A**, **B**, **C** or **D**.
- A** 10.38 cm **B** 11.06 cm **C** 11.16 cm **D** 11.48 cm
- 10 Wendy was making teddy bears. She needed these amounts of material for five bears: 2.6 m, 0.8 m, 1.2 m, 0.75 m and 0.88 m. How much material did she need altogether?



Worked solutions

Exercise 7-03

MAT07NAWS10040

11 Copy and complete each equation with the correct decimals.

a $4.8 + \underline{\hspace{2cm}} = 7.34$ b $\underline{\hspace{2cm}} - 6.6 = 2.14$ c $8.75 - \underline{\hspace{2cm}} = 3.3$

d $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 10.719$ e $\underline{\hspace{2cm}} - \underline{\hspace{2cm}} = 3.016$

Investigation: Multiplying and dividing decimals by powers of 10

When a number is multiplied by 10, 100, 1000 or other powers of 10, every digit in the number increases in place value by moving to the left.

When a number is divided by a power of 10, every digit decreases in place value by moving to the right.

1 Copy the place value table below, then use a calculator to evaluate each expression involving multiplying or dividing by a power of 10. Notice the pattern in your answers.

Calculation	Ten thousands	Thousands	Hundreds	Tens	Units	tenths	hundredths	thousandths	ten thousandths
36.7				3	6	7			
36.7×10									
36.7×100									
36.7×1000									
2.35					2	3	5		
2.35×10									
2.35×100									
2.35×1000									
$36.7 \div 10$									
$36.7 \div 100$									
$36.7 \div 1000$									
$2.35 \div 10$									
$2.35 \div 100$									

2 Copy and complete each statement.

- a To multiply by 10, move the decimal point _____ place to the _____.
- b To multiply by 100, move the decimal point _____ places to the _____.
- c To multiply by 1000, move the decimal point _____ places to the _____.
- d To divide by 10, move the decimal point _____ place to the _____.
- e To divide by 100, move the decimal point _____ places to the _____.
- f To divide by 1000, move the decimal point _____ places to the _____.

7-04 Multiplying and dividing decimals by powers of 10

Summary

To multiply a decimal by:	Move the decimal point:
10	1 place to the right
100	2 places to the right
1000	3 places to the right
To divide a decimal by:	Move the decimal point:
10	1 place to the left
100	2 places to the left
1000	3 places to the left

Note that the number of 0s in the power of 10 is how many places the decimal point is moved.

Example 8

Evaluate each expression.

a 13.69×100

b 2.54×1000

c $13.69 \div 10$

d $2.54 \div 100$

Solution

a $13.69 \times 100 = 13. \overset{\curvearrowright}{\underset{\curvearrowright}{69}}$
 $= 1369$

Move the decimal point 2 places to the right

Moving right makes the decimal bigger

b $2.54 \times 1000 = 2. \overset{\curvearrowright}{\underset{\curvearrowright}{54}} \overset{\curvearrowright}{0}$
 $= 2540$

Move the decimal point 3 places to the right:
add a 0 to the end to allow this.

c $13.69 \div 10 = 1 \overset{\curvearrowright}{3.69}$
 $= 1.369$

Move the decimal point 1 place to the left

Moving left makes the decimal smaller

d $2.54 \div 100 = 0 \overset{\curvearrowright}{\underset{\curvearrowright}{2.54}}$
 $= 0.0254$

Move the decimal point 2 places to the left:
add two 0s to the start to allow this

Exercise 7-04 Multiplying and dividing decimals by powers of 10

See Example 8

1 Evaluate each product.

a 2.49×100

b 0.81×10

c 37.42×1000

d 3.416×100

e 7.25×100

f 2.196×1000

g 6.043×100

h 0.032×100

i 0.065×10

j 45.213×100

k 10.64×1000

l 63.04×100

m 5.98×1000

n 847.612×100

o 0.0592×10

p 36.2×100

q 219.4×1000

r 40.075×10

2 Evaluate $18.5 \div 1000$. Select the correct answer **A**, **B**, **C** or **D**.

A 0.00 185

B 1.85

C 0.0185

D 0.185

3 Evaluate each quotient.

a $46.3 \div 10$

b $507 \div 100$

c $1203 \div 1000$

d $36.4 \div 100$

e $381.5 \div 1000$

f $55.02 \div 10$

g $11.46 \div 1000$

h $694.3 \div 1000$

i $4.28 \div 1000$

j $67 \div 100$

k $21.31 \div 1000$

l $5.72 \div 10$

m $81.348 \div 1000$

n $50.2 \div 100$

o $4.91 \div 100$

p $0.4 \div 1000$

q $1.73 \div 100$

r $125.3 \div 10\ 000$

4 Copy and complete each equation.

a $14.219 \times \underline{\hspace{2cm}} = 14\ 219$

b $892.5 \times \underline{\hspace{2cm}} = 8.925$

c $742 \div \underline{\hspace{2cm}} = 7.42$

d $205.7 \div \underline{\hspace{2cm}} = 20.57$

e $\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = 560.1$

f $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = 0.382$

Mental skills 7 Maths without calculators

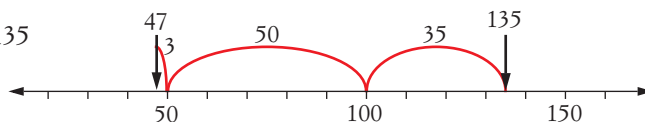
Calculating differences and making change

In every subtraction problem, for example $135 - 47$, think of finding the 'gap' between the two numbers. That is, find the number that must be added to 47 to get 135.

1 Study each example.

a $135 - 47$

Think: $47 + \underline{\hspace{2cm}} = 135$



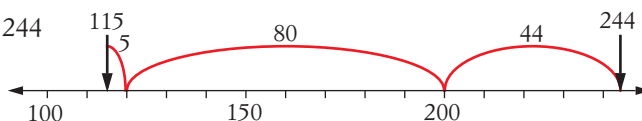
Count: '47, 50, 100, 135'

Add: $3 + 50 + 35 = 88$

Answer: $135 - 47 = 88$

b $244 - 115$

Think: $115 + \underline{\hspace{2cm}} = 244$

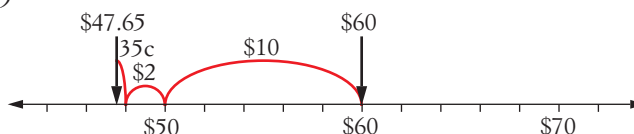


Count: '115, 120, 200, 244'

Add: $5 + 80 + 44 = 129$

Answer: $244 - 115 = 129$

c $\$60 - \$47.65$

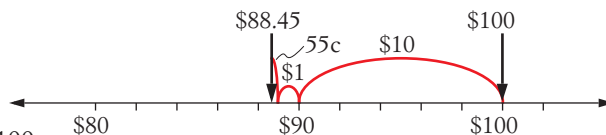


Count: '\$47.65, \$48, \$50, \$60'

Add: $\$0.35 + \$2 + \$10 = \$12.35$

Answer: $\$60 - \$47.65 = \$12.35$

d $\$100 - \$88.45$



Count: '\$88.45, \$89, \$90, \$100'

Add: $\$0.55 + \$1 + \$10 = \$11.55$

Answer: $\$100 - \$88.45 = \$11.55$

2 Now evaluate each difference.

a $176 - 88$

b $221 - 54$

c $670 - 356$

d $425 - 340$

e $518 - 389$

f $199 - 78$

g $\$70 - \$58.40$

h $\$80 - \$73.25$

i $\$45 - \$40.30$

j $\$100 - \$69.95$

k $\$30 - \$22.90$

l $\$50 - \$17.10$

7-05 Multiplying decimals by estimating

If you know the answer to a whole number multiplication, then you can use estimation to find the answer to a related decimal multiplication by correctly positioning the decimal point.

Example 9

a Given that $17 \times 12 = 204$, find:

i 1.7×12

ii 1.7×1.2

b Given that $23 \times 47 = 1081$, find:

i 2.3×4.7

ii 230×4.7

iii 23×0.47

Solution

a $17 \times 12 = 204$

	Multiplication	Estimate	Answer	
i	1.7×12	$\approx 2 \times 10 = 20$	20.4	← use the digits 204 to make a number near 20
ii	1.7×1.2	$\approx 2 \times 1 = 2$	2.04	← use the digits 204 to make a number near 2

b $23 \times 47 = 1081$

	Multiplication	Estimate	Answer	
i	2.3×4.7	$\approx 2 \times 5 = 10$	10.81	← use the digits 1081 to make a number near 10
ii	230×4.7	$\approx 200 \times 5 = 1000$	1081	← use the digits 1081 to make a number near 1000
iii	23×0.47	$\approx 20 \times 0.5 = 10$	10.81	← use the digits 1081 to make a number near 10

Exercise 7-05 Multiplying decimals by estimating

See Example 9 1 Use the result of the whole number multiplication to copy and complete each table.

a $69 \times 18 = 1242$

Multiplication	Estimate	Answer
0.69×18		
6.9×180		
6.9×1.8		
690×1.8		

b $104 \times 42 = 4368$

Multiplication	Estimate	Answer
10.4×42		
1.04×4.2		
104×4.2		
0.104×4.2		

c $38 \times 92 = 3496$

Multiplication	Estimate	Answer
3.8×92		
0.38×92		
38×0.92		
380×0.92		

2 Given that $63 \times 34 = 2142$, use estimates to find:

- a 6.3×3.4 b 0.63×3.4 c 0.63×3400
d 630×3.4 e 6.3×34 f 63×0.34

3 Given that $1.7 \times 1.2 = 2.04$, use estimates to find:

- a 1.7×12 b 17×0.12 c 0.17×1.2
d 0.17×12 e 17×12 f 17×1.2

4 Given that $7.2 \times 3.4 = 24.48$, use estimates to find:

- a 7.2×34 b 72×3.4 c 0.72×3.4
d 72×34 e 7.2×0.34 f 0.72×34

- 5 Given that $1.26 \times 6 = 7.56$, use estimates to find:
- a 12.6×6 b 126×6 c 1.26×0.6
d 0.126×6 e 0.126×0.6 f 126×0.6
- 6 Use the fact that $0.3 \times 0.24 = 0.072$ to find:
- a 3×0.24 b 0.3×2.4 c 0.3×24
d 3×24 e 3×2.4 f 0.03×24
- 7 Write an estimate for each product.
- a 2.7×3.1 b 6.2×0.44 c 8.6×19

Worked solutions

Exercise 7-05

MAT07NAWS10041

Investigation: Decimal places in multiplication answers

- What happens when you multiply by a number less than one?
 - Is the answer to 12×0.8 more or less than 12? Why?
 - Estimate the answer to 12×0.8 .
 - How many decimal places do 12 and 0.8 each have?
 - Use a calculator to evaluate 12×0.8 . How many decimal places does the answer have?
- Is the answer to 0.7×0.3 more or less than 0.7? Why?
 - Estimate the answer to 0.7×0.3 .
 - How many decimal places do 0.7 and 0.3 each have?
 - Use a calculator to evaluate 0.7×0.3 . How many decimal places does the answer have?
- Is the answer to 2.5×4.1 more or less than 2.5? Why?
 - Estimate the answer to 2.5×4.1 .
 - How many decimal places do 2.5 and 4.1 each have?
 - Use a calculator to evaluate 2.5×4.1 . How many decimal places does the answer have?
- What is the relationship between the number of decimal places in the question and the number of decimal places in the answer?
- If $82 \times 6 = 492$, what do you think is the answer to 82×0.6 ? Where does the decimal point go?
 - If $4 \times 17 = 68$, what do you think is the answer to 0.4×1.7 ?
 - If $367 \times 51 = 18\,717$, what do you think is the answer to 3.67×5.1 ?
 - What is the answer to 0.5×0.9 ?

7-06 Multiplying decimals

Worksheet

Shopping and change

MAT07NAWK10054

Worksheet

Decimal cards

MAT07NAWK10055

Worksheet

Where's the point?

MAT07NAWK10056

Video tutorial

Multiplying decimals

MAT07NAVT10015

Puzzle sheet

Multiplication
estimation game

MAT07NAPS10027

Puzzle sheet

Which decimals?

MAT07NAPS10028

Summary

When **multiplying decimals**, the number of decimal places in the answer equals the total number of decimal places in the question.

Example 10

Evaluate each product.

a 3.06×4.8

b 0.6×4.1

Solution

- a** Complete the multiplication without decimal points.

$$\begin{array}{r} 306 \\ \times 48 \\ \hline 2448 \\ 12240 \\ \hline 14688 \end{array}$$

Method 1

Decide where to place the decimal point by counting decimal places.

3.06 has 2 decimal places.

4.8 has 1 decimal place.

So we need $2 + 1 = 3$ decimal places.

Insert the decimal point so that the answer has 3 decimal places.

$$3.06 \times 4.8 = \mathbf{14.688}$$

Method 2

Estimate the answer.

$$3.06 \times 4.8 \approx 3 \times 5 = 15$$

Insert the decimal point so that the answer is near 15.

$$3.06 \times 4.8 = \mathbf{14.688}$$

- b** Complete the multiplication without decimal points.

$$\begin{array}{r} 41 \\ \times 6 \\ \hline 246 \end{array}$$

Method 1

Decide where to place the decimal point by counting decimal places.

0.6 has 1 decimal place.

4.1 has 1 decimal place.

So the answer has 2 decimal places.

Insert the decimal point so that the answer has 2 decimal places.

$$0.6 \times 4.1 = \mathbf{2.46}$$

Method 2

Estimate the answer.

$$0.6 \times 4.1 \approx 1 \times 4 = 4$$

Insert the decimal point so that the answer is near 4.

$$0.6 \times 4.1 = 2.46$$

Exercise 7-06 Multiplying decimals

1 How many decimal places will each product have?

a 0.25×11	b 10.2×4	c 0.5×10
d 7×2.193	e 0.9×0.75	f 8.06×4.1
g 0.11×1.01	h 6.3×0.04	i 2.95×5.3
j 0.237×1.2	k 0.023×0.042	l 321.2×8.1

2 Evaluate each product.

a 3.05×4	b 1.02×7	c 2.001×9	d 17.1×2
e 10×2.25	f 3×4.20	g 6.95×5	h 1.004×8
i 0.18×5	j 0.4×12	k 6×0.002	l 3×4.2
m 0.4×0.8	n 3.9×0.5	o 0.8×0.6	p 0.3×0.24

3 Estimate each product, then evaluate the product.

a 47.9×0.3	b 6.43×7.2	c 83.4×6.3

4 Dress material costs \$31.24 per metre. How much does 5.2 metres of material cost?

5 Sean earns \$18.45 per hour working at a cinema. How much will he earn for working a 39-hour week?

6 A flower bed is 2.6 m long by 0.8 m wide. Calculate its area.

7 Calculate the total of this grocery shopping bill. Select A, B, C or D.

- 3 kg of butter at \$2.50 per kilogram
- 500 g of cheese at \$12.88 per kilogram
- 2 kg of meat at \$6.90 per kilogram
- 2 dozen eggs at \$4.26 per dozen

A \$30.47	B \$36.26	C \$64.70	D \$129.98

See Example 10

Worked solutions

Exercise 7-06

MAT07NAWS10042

7-07 Dividing decimals by whole numbers

Summary

When dividing a decimal by a whole number:

- rewrite the question in 'short division' form
- make the decimal point in the answer line up with the decimal point in the question
- add zeros to the end of the decimal being divided, if needed

Example 11

Evaluate each quotient.

a $10 \div 4$

b $0.018 \div 6$

c $2.66 \div 8$

Solution

a
$$\begin{array}{r} 2.5 \\ 4 \overline{)10.20} \end{array}$$

Write 10 as 10.0 so that you can complete the division

$10 \div 4 = 2.5$

b
$$\begin{array}{r} 0.003 \\ 6 \overline{)0.018} \end{array}$$

$0.018 \div 6 = 0.003$

c
$$\begin{array}{r} 0.3325 \\ 8 \overline{)2.662000} \end{array}$$

Write two 0s after 2.66 so that you can complete the division

$2.66 \div 8 = 0.3325$

Exercise 7-07 Dividing decimals by whole numbers

See Example 11

1 Evaluate each quotient.

a $4.8 \div 2$

b $18.6 \div 3$

c $20.8 \div 5$

d $32.8 \div 8$

e $29.3 \div 2$

f $8.79 \div 4$

g $0.056 \div 7$

h $10.71 \div 4$

i $195.6 \div 8$

j $7.35 \div 2$

k $4.15 \div 8$

l $0.318 \div 3$

m $12 \div 5$

n $13.56 \div 4$

o $23 \div 8$

p $256.84 \div 4$

q $107.1 \div 9$

r $82.5 \div 6$

s $0.732 \div 6$

t $2075.6 \div 8$

2 Yesterday, Anh earned \$137.12 for working 8 hours at the chemist. How much was he paid per hour?

3 Trent took 5 hours to travel 163.5 km on his bike. What was his average speed in km/h? Select **A**, **B**, **C** or **D**.

A 30.6

B 32.6

C 8.175

D 32.7

4 At the restaurant, the bill for 6 friends totalled \$168.96. How much should each friend pay if the bill is to be split evenly?

5 Nadine ran 100 metres in the following times (in seconds): 11.4, 12.1, 11.5, 11.3, 11.6, 11.7.

a Calculate the sum of Nadine's six times.

b Calculate Nadine's average time.

6 Copy and complete each equation with the correct decimals.

a _____ $\div 8 = 3.1$ **b** _____ $\div 5 = 2.46$ **c** _____ $\div$ _____ $= 6$

Worked solutions

Exercise 7-07

MAT07NAWS10043

7-08 Dividing decimals

Look at this pattern:

$$18 \div 3 = 6$$

$$180 \div 30 = 6$$

$$1800 \div 300 = 6$$

This is like writing them as equivalent fractions: $\frac{18}{3} = \frac{180}{30} = \frac{1800}{300}$.

When dividing numbers, if we multiply both numbers by the **same number** first, the answer stays the same. We can use this property to help us divide decimals. For example:

$$\begin{aligned} 9.8 \div 0.08 &= 980 \div 8 \quad (\text{multiplying both numbers by } 100) \\ &= 122.5 \end{aligned}$$

$980 \div 8$ is easier to evaluate than $9.8 \div 0.08$ because 8 is a whole number.

Worksheet

Decimals writing activity

MAT07NAWK10057

Homework sheet

Decimals 2

MAT07NAHS10012

Worksheet

Decimals 7

MAT07NAWK00040

Technology worksheet

Excel
Calculating with
decimals

MAT07NACT10003

Summary

To divide a decimal by a decimal:

- make the second decimal a whole number by moving the decimal point the required number of places to the right
- move the point in the first decimal the same number of places to the right
- divide the new first number by the whole number

This works because we multiply both decimals by the same power of 10 before dividing.

Example 12

Evaluate each of the following:

a $0.4 \div 0.2$

b $1.75 \div 0.5$

c $122.4 \div 0.03$

Solution

$$\begin{aligned} \text{a } 0.4 \div 0.2 &= 0.\overset{\curvearrowright}{4} \div 0.\overset{\curvearrowright}{2} \\ &= 4 \div 2 \\ &= 2 \end{aligned}$$

Move both decimal points one place to the right so that 0.2 becomes a whole number

← multiplying both decimals by 10

$$\begin{aligned} \text{b } 1.75 \div 0.5 &= 1.\overset{\curvearrowright}{7}5 \div 0.\overset{\curvearrowright}{5} \\ &= 17.5 \div 5 \\ &= 3.5 \end{aligned}$$

Move both decimal points one place to the right

$$\begin{aligned} \text{c } 122.4 \div 0.03 &= 122.\overset{\curvearrowright}{4}0 \div 0.\overset{\curvearrowright}{0}3 \\ &= 12\,240 \div 3 \\ &= 4080 \end{aligned}$$

Move both decimal points two places to the right so that 0.03 becomes a whole number

← multiplying both decimals by 100

Video tutorial

Dividing decimals

MAT07NAVT10016

Exercise 7-08 Dividing decimals

Extra questions

Multiplying and
dividing decimals

MAT07NAEQ00023

- 1 a $18 \div 0.5$ means 'how many times does 0.5 go into 18?'. Is the answer more or less than 18? Why?
 b Estimate the answer to $18 \div 0.5$.
 c Evaluate $18 \div 0.5$.
 d Show that $18 \div \frac{1}{2}$ gives the same answer.
- 2 a $20.4 \div 0.3$ means 'how many times does 0.3 go into 20.4?'. Is the answer more or less than 20.4?
 b Estimate the answer to $20.4 \div 0.3$.
 c Find the exact answer to $20.4 \div 0.3$.
- 3 What happens when you divide by a number less than 1? Is the answer more or less than the number? (Check your answers to questions 1 and 2).
- 4 Which of the following is the answer to $13.59 \div 0.03$? Select **A**, **B**, **C** or **D**.
 A 45.3 B 453 C 4.53 D 4530
- 5 Rewrite each of the following divisions so that the second decimal is a whole number.
 a $508.8 \div 0.8$ b $17.92 \div 0.07$ c $333 \div 0.9$
 d $1.725 \div 0.05$ e $129.2 \div 0.4$ f $49.5 \div 0.06$
 g $168 \div 0.7$ h $14.823 \div 0.09$ i $0.66 \div 0.3$
- 6 Evaluate each of these, and check that your answers seem reasonable by estimating.
 a $3.48 \div 0.4$ b $7.32 \div 0.2$ c $2.94 \div 0.6$
 d $16.28 \div 0.08$ e $27 \div 0.9$ f $10.08 \div 0.8$
 g $10.4 \div 0.05$ h $5.6 \div 0.07$ i $1.71 \div 0.3$
 j $40.82 \div 0.02$ k $0.532 \div 0.5$ l $0.7812 \div 0.006$
- 7 A drink bottle holds 0.8 litres. How many drink bottles can be filled from a container that holds 12.8 litres?
- 8 A square wall tile has length 0.3 m. If a bathroom wall has length 5.2 m, how many whole tiles are needed for one row along the wall?
- 9 James can cycle 0.7 km in one minute.
 a How far will he travel in 20 minutes?
 b How long will it take him to travel 26.6 km?
- 10 Copy and complete each blank with the correct decimal.
 a $27.2 \div \underline{\hspace{2cm}} = 8$
 b $\underline{\hspace{2cm}} \div 5 = 2.46$
 c $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = 6.2$

Worked solutions

Exercise 7-08

MAT07NAWS10044

Investigation: Back-to-front problems

The cards for this set of questions have been printed without any decimal points. Insert the decimal points so that the numbers on the cards fit the clues.

- The difference between these two numbers is 53.7.
The sum of the numbers is 58.5.
- The difference between the numbers is 178.8. When you divide the greater number by the smaller number, the quotient is between 43 and 44.
- The sum of the three numbers is 5.36.
The product of the numbers is 1.2096.
- The sum of the three numbers is 4.61.
The product of the numbers is 0.9135.
- The sum of the four numbers is 2.55.
The product of two of the numbers is 0.196. The product of the other two numbers is 0.217.

561	24		
183	42		
8	42	36	
15	29	21	
7	31	14	14

7-09 Terminating and recurring decimals

When fractions are converted to decimals, the decimals can be **terminating** or **recurring**.

Terminating decimals have a definite number of decimal places, such as 0.625, while **recurring decimals** are decimals whose digits repeat endlessly, such as 0.272727 ... 'Terminate' means 'to stop' while 'recurring' means 'repeating'.

Terminating decimals

Example 13

Convert each fraction into a decimal.

a $\frac{3}{5}$

Solution

a $\frac{3}{5}$ means $3 \div 5$

$$\begin{array}{r} 0.6 \\ 5 \overline{) 3.0} \end{array}$$

b $\frac{5}{8}$

b $\frac{5}{8}$ means $5 \div 8$

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \end{array}$$

Puzzle sheet

Decimals squaresaw 2

MAT07NAPS10029

Weblink

Master class on decimals

Skillsheet

Fractions and decimals

MAT07NASS10027

Remember to add zeros, if necessary, to complete the division.

$$\frac{3}{5} = 0.6$$

$$\frac{5}{8} = 0.625$$

Note also that $\frac{3}{5} = \frac{6}{10} = 0.6$.

YEAR 8

Recurring decimals

Example 14

Convert each fraction to a decimal.

a $\frac{1}{3}$

b $\frac{5}{6}$

c $\frac{2}{11}$

Solution

a $\frac{1}{3}$ means $1 \div 3$

$$\begin{array}{r} 0.333\dots \\ 3 \overline{)1.000\dots} \end{array}$$

$$\frac{1}{3} = 0.333\dots = 0.\dot{3} \text{ or } 0.\overline{3}$$

b $\frac{5}{6}$ means $5 \div 6$

$$\begin{array}{r} 0.833\dots \\ 6 \overline{)5.0000\dots} \end{array}$$

$$\frac{5}{6} = 0.8333\dots = 0.8\dot{3} \text{ or } 0.8\overline{3}$$

c $\frac{2}{11}$ means $2 \div 11$

$$\begin{array}{r} 0.18181\dots \\ 11 \overline{)2.00000\dots} \end{array}$$

$$\frac{2}{11} = 0.181818\dots = 0.1\dot{8} \text{ or } 0.1\overline{8}$$

The answers are all **recurring decimals**, where one or more of the digits in the decimal repeat endlessly. To show this, we use dots or a line to mark the repeating section: for example, $0.259259259\dots = 0.2\dot{5}9$ or $0.2\overline{5}9$.

Exercise 7-09 Terminating and recurring decimals

See Example 13 1 Copy and complete this table. Use a calculator if you need to.

	Common fraction	Meaning as division	Decimal
a	$\frac{3}{5}$	$3 \div 5$	0.6
b	$\frac{1}{2}$	$1 \div 2$	
c	$\frac{1}{4}$		0.25
d	$\frac{4}{5}$		
e	$\frac{2}{5}$		
f		$3 \div 4$	0.75
g		$1 \div 5$	
h		$1 \div 8$	

2 Use your answers from question 1 to help you convert each fraction into a decimal.

a $\frac{2}{5}$

b $\frac{3}{8}$

c $\frac{3}{4}$

d $\frac{2}{2}$

e $\frac{2}{4}$

f $\frac{6}{8}$

g $\frac{3}{5}$

h $\frac{2}{8}$

i $\frac{5}{8}$

j $\frac{7}{8}$

k $\frac{4}{8}$

l $\frac{5}{5}$

3 Explain why some of the fractions in question 2 have the same decimal value.

4 Write each mixed numeral as a decimal.

a $4\frac{8}{10}$

b $23\frac{3}{4}$

c $12\frac{5}{8}$

d $6\frac{3}{5}$

e $57\frac{2}{5}$

f $19\frac{1}{8}$

g $110\frac{7}{8}$

h $80\frac{1}{4}$

5 Rewrite each recurring decimal using the dot notation.

a 0.41666666 ...

b 0.27272727...

c 0.111111 ...

d 0.1027510275 ...

6 Rewrite each recurring decimal showing the repeated pattern.

a 0.858 $\dot{3}$

b 0.04 $\dot{5}$

c 0. $\dot{7}$

d 0. $\dot{4}$ 6153 $\dot{8}$

7 Convert each fraction to a recurring decimal.

a $\frac{1}{9}$

b $\frac{1}{6}$

c $\frac{5}{6}$

d $\frac{1}{7}$

e $\frac{2}{3}$

f $\frac{2}{7}$

g $\frac{2}{9}$

h $\frac{3}{7}$

i $\frac{4}{7}$

j $\frac{4}{9}$

k $\frac{4}{6}$

l $\frac{5}{9}$

m $\frac{6}{7}$

n $\frac{7}{9}$

o $\frac{8}{9}$

8 Copy and complete the table and notice the pattern.

Fraction	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$
Decimal								

Worked solutions

Exercise 7-09

MAT07NAWS10045

See Example 14

7-10 Rounding decimals

Sometimes, to approximate an answer with many decimal places, we round to fewer decimal places. We need to be able to round when working with money, measuring quantities or writing answers to division calculations.

Summary

To round a decimal, 'cut' it at the required decimal place and look at the digit in the next place:

- if the digit is less than 5 (that is 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is 5, 6, 7, 8 or 9), **round up**

Video tutorial

Rounding decimals

MAT07NAVT10017

Example 15

- a** Round 86.246 to:
- i** one decimal place
 - ii** two decimal places
- b** Write 0.087 1245 correct to:
- i** one decimal place
 - ii** the nearest thousandth

Solution

a i 86.246

cut the next digit is 4 (less than 5), so round **down**
So $86.246 \approx 86.2$ (correct to one decimal place).

ii 86.246

cut the next digit is 6 (more than 5), so round **up** to 86.25
So $86.246 \approx 86.25$ (correct to two decimal places).

b i 0.087 1245

cut the next digit is 8 (more than 5), so round **up** to 0.1
 $0.087\ 1245 \approx 0.1$

ii 0.087 1245

cut the next digit is 1 (less than 5), so round **down**
'the nearest thousandth' means 3 decimal places
 $0.087\ 1245 \approx 0.087$

Summary

Rounding to:

- the nearest tenth = one decimal place
- the nearest hundredth = two decimal places
- the nearest thousandth = three decimal places

Exercise 7-10 Rounding decimals

See Example 15

- 1** Write each decimal correct to one decimal place.

a 0.35	b 0.47	c 0.81	d 0.69
e 2.55	f 0.32	g 0.90	h 2.88

- 2** Round each decimal to two decimal places.

a 0.481	b 0.736	c 0.069	d 0.293
e 0.309	f 0.655	g 2.096	h 3.995

- 3** Write each decimal correct to two decimal places.

a 25.3759	b 341.6143	c 420.8971
d 734.6541	e 27.359 94	f 1314.210 47

Extra questions

Decimals, fractions
and rounding

MAT07NAEQ00021

- 4 Copy and complete this table. Use a calculator to help you.

	Question	Calculator display	Rounded to the nearest tenth	Rounded to the nearest hundredth
a	$12.19 \div 3$	4.0633333333	4.1	4.06
b	$12.32 \div 6$			
c	$19.82 \div 9$			
d	$56.85 \div 11$			
e	$17.13 \div 4$			
f	$12.65 \div 12$			
g	$4.875 \div 21$			
h	$27.45 \div 8$			
i	$17 \div 12$			
j	$254.678 \div 32$			

- 5 Round each number to four decimal places.

- a 10.33374 b 431.54327 c 1.444 95
d 3217.654 061 e 4.67089 f 0.888 88

- 6 The answers to the following problems are whole numbers but, for particular reasons, some need to be rounded up and some need to be rounded down. Find each rounded answer, giving the reason why you rounded up or down.

- a A box of chocolates with 44 chocolates is shared among a family of 5 people. How many chocolates does each person receive?
- b A new bathroom requires 32 square metres of tiles. A box of tiles covers 0.8 square metres of area. How many boxes are needed to tile the bathroom?
- c A team of four golfers wins 27 new golf balls in a competition. How many does each person receive?
- d Some timber comes in 0.9 m lengths. How many lengths are needed to build a chicken coop needing 23 m of timber?
- e One dress requires 1.3 m of material. How many dresses can be made from a 5 m length of material?



- 7 Write a decimal that could be rounded to:

- a 2.718 b 0.8 c 25.6

- 8 Australia had 1c coins and 2c coins until 1990. After that, supermarket prices had to be rounded to the nearest 5 cents. Find out how prices are rounded this way.

Technology Rounding decimals

This activity will show you how to format a cell so that its value is rounded to a specific number of decimal places. Start a new spreadsheet.

- 1 Enter **748.61** into cell A1, **x** (for multiply) into cell A2 and **3.75** into cell A3.
- 2 Enter a formula into cell A4 for the product of A1 and A3 and **bold** your answer.
- 3 To round this answer to 1 decimal place, **right-click** on cell A4, choose **Format Cells**, **Number**, and 1 decimal place.
- 4 Enter **748.61** into cell B1, **/** (for divide) into cell B2 and **3.75** into cell B3.
- 5 Enter a formula into cell B4 for B1 divided by B3 and **bold** your answer.
- 6 To round this answer to 2 decimal places, **right-click** on cell B4, choose **Format Cells**, **Number**, and 2 decimal places.
- 7 Complete each calculation using your spreadsheet and creating appropriate formulas.
 - a 284.796×32.4 (round to 2 decimal places)
 - b $1217.9 \div 45.6$ (round to 3 decimal places)
 - c $1.15 \div 1.5$ (to 2 decimal places)
 - d $1604.12 \div 0.02 \times 4.578$
 - e $(8756.32 - 9025.198 + 1023.5697) \div 1.444$ (to 4 decimal places)
 - f $(12.3 + 6.59) \div (56.4 \times 0.04)$ (to 5 decimal places)

Just for the record

The salami technique

When banks started using computers to keep track of customers' accounts, they left themselves open to a new type of crime: computer theft. One such crime employs the **salami technique**, where computer hackers steal a cent or a fraction of a cent from many bank accounts. They round down the decimal amount of an account balance (for example \$234.6523 would become \$234.65) and the stolen fraction of a cent (\$0.0023) is deposited into the hacker's account, with no one noticing it missing. When this is done to thousands of bank customers over a number of years, a considerable amount of money can be accumulated. **Why do you think this type of crime is difficult to detect?** **Why do you think it is called the 'salami technique'?**



7-11 Decimal problems

Worked solutions

Exercise 7-11

MAT07NAWS10046

Worksheet

Shopping and change

MAT07NAWK10054

Exercise 7-11 Decimal problems

- 1 Find the cost of 352 units of electricity at 25.3 cents per unit.
- 2 A farmer wants to fence a rectangular paddock. The paddock is 35.6 metres long and 20.85 metres wide. How many metres of fencing will be needed?
- 3 Mark buys golf balls for \$4.85 each and sells them for \$5.15 each. How much money does he make if he sells 30 golf balls?

9780170188777

- 4 A drink bottle holds 0.6 litres. How many drink bottles can be filled from a tub that holds 4.5 litres?
- 5 A car travels 110.3 kilometres on 7 litres of petrol. How many kilometres would the car travel on one litre of petrol? (Give the answer to one decimal place.)
- 6 Anja runs 3.8 kilometres each day of the week. How far does she run in one week?
- 7 A long distance train is made up of a diesel engine, two dining cars and 15 passenger carriages. The engine has a mass of 20.2 tonnes, each dining car has a mass of 14.35 tonnes and each passenger carriage has a mass of 13.96 tonnes. How heavy is the entire train?



- 8 Samir is cutting shelves from a board which is 4.6 metres long. Each shelf needs to be 0.9 metres long. How many shelves can be cut?
- 9 Holly walks to work and back each day. She works six days a week and, in one week, walks 16.8 kilometres. How far is Holly's apartment from work?
- 10 The table lists some Sydney FM radio stations and their allocated frequencies measured in megahertz (MHz).

Station	Frequency (MHz)	Station	Frequency (MHz)
C91.3	91.3	Hope	103.2
ABC Classic	92.9	2DAY	104.1
Classic Rock	95.3	Triple M	104.9
The Edge	96.1	Triple J	105.7
Nova	96.9	Mix	106.5
SBS	97.7	2SER	107.3
WS-FM	101.7		

- a Copy this number line, then locate the stations on it according to their frequencies.



- b What is the frequency difference in megahertz between 2DAY and WS-FM?
 - c Find the smallest frequency difference between adjacent stations. ('Adjacent' means side-by-side.)
 - d What is the largest difference in frequency between adjacent stations?
- 11 The following are calculator displays for amounts in dollars and cents. Rewrite each amount in dollars and cents, to the nearest cent.

a b c d

- 12 A sheet of cardboard is 0.03 cm thick. How many sheets would be in a stack that is 3.6 cm high?

Worksheet

Calculating change

MAT07NAWK00067

Homework sheet

Decimals revision

MAT07NAHS10014

Homework sheet

Decimals 3

MAT07NAHS10013

TLF learning object

Design a city (L123)

Extra questions

Money transactions

MAT07NAEQ00042

Technology Fruit and vegetables

In this activity, a spreadsheet is used to show a shopping list of items and to complete calculations involving their cost.

- 1 Zoe did her fruit and vegetables shopping for the week. Enter the items into a spreadsheet, as shown below. (Centre values, bold headings, include cell borders and \$ signs for column B values.)

	A	B	C	D
1	Item	Cost per kg	kg bought	Cost
2	Oranges	\$3.99	1	
3	Apples	\$4.99	0.5	
4	Kiwi fruit	\$2.99	0.4	
5	Bananas	\$5.60	1.2	
6	Tomatoes	\$7.49	1.5	
7	Grapes	\$3.50	1	
8	Carrots	\$1.55	2	
9	Potatoes	\$2.45	5	
10	Beans	\$6.95	0.355	
11	Pumpkins	\$3.20	0.562	
12	Zucchinis	\$2.85	0.38	

- 2 Write a formula in cell D2 to calculate the cost of the oranges.
- 3 Use **Fill Down** to calculate the cost of each item purchased.
- 4 In cell C13, enter the label 'Total cost'.
- 5 In cell D13, write a **sum** formula to find the cost of Zoe's shopping.
- 6 In cell C14, enter the label 'Cash'. In cell C15, enter the label 'Change'.
- 7 If Zoe paid \$60 for her shopping, enter this value into cell D14 and in cell D15 write a formula to calculate the amount of change Zoe will receive.
- 8 Zoe paid cash, but because the change is an irregular amount, she could not be given the amount in cell D15 in coins. In cell C16, enter the label 'Rounded change'. In cell D16, enter the amount of change Zoe will actually receive.
- 9 Create a spreadsheet with similar formatting to calculate a different shopping list.

Power plus

- 1 Copy and complete the table and notice the pattern.

Fraction	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$
Decimal						

- 2 Investigate the value of $0.\dot{9}$.
- 3 Evaluate each expression.

a $(0.02)^2$

b $(0.02)^3$

c $(1.1)^3$

d $\sqrt{0.04}$

e $\sqrt{0.36}$

f $\sqrt[3]{0.027}$

- 4 Wolf, the warrior, is trying to break a decimal code that will open the dungeon doors. He has to free the prisoners before midnight so that they will not be turned into frogs by an evil spell. Each castle door is operated by a combination lock. Use the following clues to match the various combinations with the doors to the different rooms in the castle.

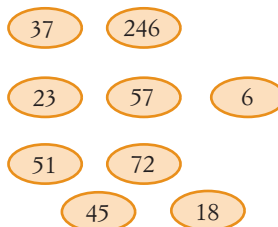
Combinations	Rooms
8.262	Queen's chamber
9.24	armoury
9.96	throne room
8.07	banquet hall
8.16	kitchen
8.79	dungeon

Clues

- Combination 9.24 opens a door to a room that deals with food.
- The combination to the armoury has a 6 in the hundredths place.
- The combinations of the throne room and the banquet hall add to 18.03.
- The Queen's chamber has a combination that is bigger than 3.78×2.1 but smaller than $25.11 + 3.1$.
- The kitchen combination is one of the three largest combination numbers.

(Hint: The doors in the dungeon and the throne room remained locked when Wolf tried 9.96 and 8.262.)

- 5 Evaluate $28.98 \div 1.2$ without using a calculator.
- 6 Decide where the decimal points should be so that the numbers in the ovals fit the clues.
- The product of the two numbers is 91.02.
The sum of the numbers is 28.3.
 - The sum of the three numbers is 14.
The product of the three numbers is 78.66.
 - The sum of the four numbers is 28.32.
The product of two of the numbers is 3.672.
The product of the other two is 81.



Language of maths

Puzzle sheet
Decimals crossword
MAT07NAPS10030

ascending	descending	power of 10	tenth
decimal	estimate	recurring decimal	terminating decimal
decimal place	fraction	round down	thousandth
decimal point	hundredth	round up	

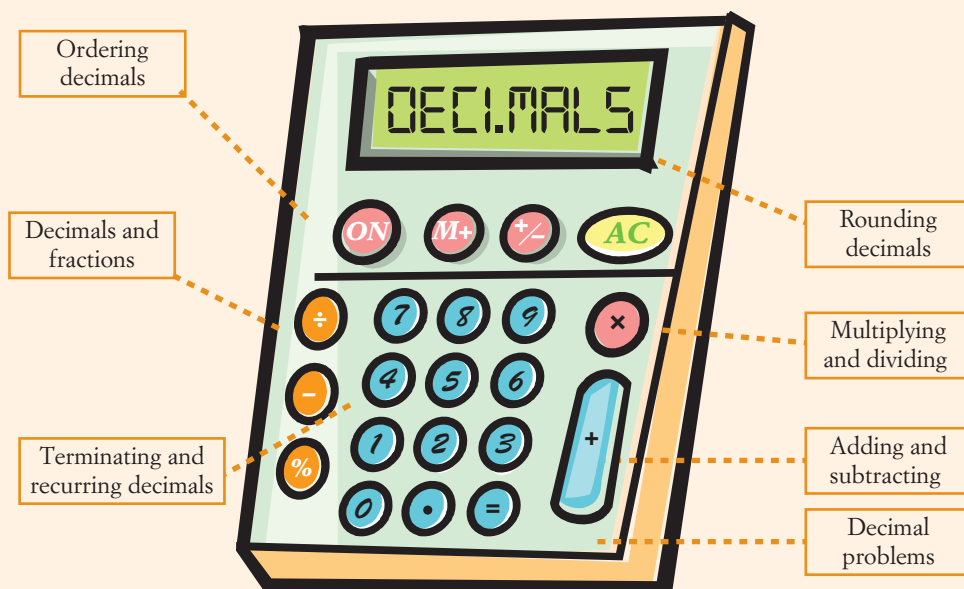
- 1 What is the collective name for numbers such as 10, 100 and 1000?
- 2 'The bushfire decimated the possum population of the forest.' Look up the meaning of the word '**decimate**'.
- 3 What is a **recurring** dream or a **recurring** back pain? What does 'recurring' mean?
- 4 What happens when a train **terminates** at a station?
- 5 What is a 'decimetre'?
- 6 How many decimal places do you round to if you are rounding to the nearest hundredth?

Topic overview

Worksheet
Mind map: Decimals
MAT07NAWK10058

- What parts of this topic were new to you? What parts did you already know?
- Write any rules you have learnt about working with decimals.
- What parts of this topic did you not understand? Be specific. Talk to a friend or your teacher about them.
- Give three examples of where decimals are used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 **a** Arrange 34.98, 36.86, 3.998, 38.141, 340, 34.89 in ascending order. See Exercise 7-01
b Arrange 1.015, 1.293, 1.1015, 1.239, 1.006 in descending order.
- 2 Convert each decimal to a simplified fraction. See Exercise 7-02
a 0.5 **b** 0.89 **c** 0.09 **d** 0.444
- 3 Convert each fraction to a decimal. See Exercise 7-02
a $\frac{4}{10}$ **b** $\frac{13}{100}$ **c** $\frac{7}{100}$ **d** $\frac{11}{1000}$
- 4 Evaluate each expression. See Exercise 7-03
a $12.35 + 4.53 + 0.56 + 3.125 + 24.7 + 20.09$ **b** $214.33 - 109.84$
c $0.568 + 23 + 4.027 - 16.28$ **d** $1600.8 - 562.9$
e $1453.6 + 1287.31 - 2344.4$ **f** $9.23 - 6.851$
- 5 Evaluate each expression. See Exercise 7-04
a 7.54×10 **b** 7.54×100 **c** 7.54×1000
d $13.9 \div 10$ **e** $13.9 \div 100$ **f** $13.9 \div 1000$
- 6 Given that $42 \times 76 = 3192$, find: See Exercise 7-05
a 4.2×76 **b** 4.2×7.6 **c** 0.42×760
d 4.2×0.76 **e** 42×7.6 **f** 0.42×0.76
- 7 Evaluate each product. See Exercise 7-06
a 2.75×6 **b** 0.5×1.2 **c** 72.23×4
d 6.1×1.2 **e** 0.92×5 **f** 3×9.7
g 3.25×0.41 **h** 0.05×0.02 **i** 4.67×1.1
- 8 Evaluate each quotient. See Exercise 7-07
a $762.4 \div 2$ **b** $97.6 \div 8$ **c** $2.75 \div 4$ **d** $195.6 \div 3$
- 9 Evaluate each quotient. See Exercise 7-08
a $12.5 \div 0.5$ **b** $12.72 \div 0.4$ **c** $6.9 \div 0.03$ **d** $0.508 \div 0.02$
- 10 Convert each fraction to a decimal. See Exercise 7-09
a $\frac{4}{5}$ **b** $\frac{3}{8}$ **c** $\frac{5}{9}$ **d** $\frac{2}{3}$
- 11 Round each number to the given number of decimal places. See Exercise 7-10
a 406.28 to the nearest tenth **b** 125.724 to 2 decimal places
c 2345.876 to 1 decimal place **d** 3.8967 to the nearest hundredth
e 78 654.056 to 2 decimal places **f** 678.4309 to the nearest thousandth
- 12 The Liverpool Women's Cricket Club is having a pizza night. They order 16 Super Supreme pizzas at \$13.70 each and 10 Hawaiian pizzas at \$12.10 each. How much will the club need to spend? See Exercise 7-11
- 13 Ray bought 800 bricks for \$572.80. How much did one brick cost? See Exercise 7-11
- 14 Maria saved \$90 to go to a rock concert. Her return fare cost \$5.60, her concert ticket cost \$48.95, the program cost \$11 and food cost \$8.70. She did not have enough to buy the band's latest compact disc (priced \$24) after the concert. How much did she need to borrow from her friend Sam to buy the disc? See Exercise 7-11



8

Measurement and geometry

Area and volume

The largest city square in the world is Tiananmen Square in the centre of Beijing, China. It measures 880 m by 500 m, so it has an area of 440 000 square metres. It is the size of 62 soccer fields and can contain a million people. The largest city square in Australia is Adelaide's Victoria Square, measuring 322 m by 161 m, with an area of 51 842 square metres, or 7 soccer fields. This means one Tiananmen Square = 8.5 Victoria Squares!



Chapter outline

	Proficiency strands			
8-01 The metric system	U	F		
8-02 Perimeter	U	F	PS	R
8-03 Metric units for area	U	F	PS	R
8-04 Area of a rectangle	U		PS	R
8-05 Area of a triangle	U		PS	R
8-06 Area of a parallelogram	U		PS	R
8-07 Areas of composite shapes	U	F	PS	
8-08 Drawing prisms	U	F		
8-09 Metric units for volume	U	F	PS	R
8-10 Volume of a rectangular prism	U	F	PS	R
8-11 Volume and capacity	U	F	PS	

Wordbank

- area** The amount of surface enclosed by a shape
- capacity** The amount of fluid (liquid or gas) in a container
- cubic metre** The volume of a cube that measures 1 m by 1 m by 1 m
- kilo** A prefix that means one thousand of something
- perimeter** The distance around the outside of a shape
- perpendicular height** The height of a shape taken at right angles to its base
- square metre** The area of a square that measures 1 m by 1 m
- volume** The amount of space inside a solid

In this chapter you will:

- connect decimal representations to the metric system
- convert between common metric units of length, mass and capacity
- recognise the significance of the metric prefixes in units of measurement
- identify and use the correct operations when converting units, including millimetres, centimetres, metres, kilometres, milligrams, grams, kilograms, tonnes, millilitres, litres, kilolitres and megalitres
- calculate the perimeter and area of rectangles using familiar metric units
- choose appropriate units of measurement for area and convert from one unit to another
- establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving
- draw different views of prisms and solids formed from combinations of prisms
- choose appropriate units of measurement for volume and convert from one unit to another
- calculate volumes of rectangular prisms
- connect volume and capacity and their units of measurement
- recognise that 1 mL is equivalent to 1 cm³

SkillCheck

Worksheet

StartUp assignment 8

MAT07MGWK10059

Skillsheet

Multiplying by 10, 100, 1000

MAT07NASS10026

1 Copy and complete each equation.

a 1 kg = _____ g

b 1 min _____ s

c 1 L = _____ mL

d 1 cm = _____ mm

e 1 g = _____ mg

f 1 day = _____ hours

g 1 kL = _____ L

h 1 t = _____ kg

i 1 m = _____ mm

j 1 h = _____ min

k 1 m = _____ cm

l 1 km = _____ m

2 Evaluate each expression.

a 10.32×100

b 9.45×10

c 0.4×1000

d 2.7×1000

e 0.9×100

f 6.3×10

g $2000 \div 100$

h $43\,000 \div 1000$

i $650 \div 10$

j $3750 \div 100$

k $95 \div 10$

l $8200 \div 1000$

3 What metric unit would you use to measure:

a the distance from the street to your front door?

b the length of your foot?

d the length of your bed?

f the depth of a river?

h the length of a long-jump pit?

j the length of a fingernail?

c the length of an ant?

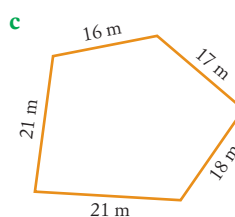
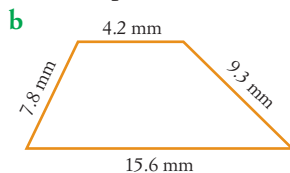
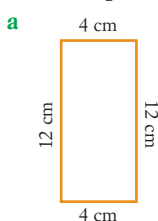
e the width of a basketball court?

g the distance around a football field?

i the height of a door?

k an athlete's time in a race?

4 Find the perimeter of each shape.



8-01 The metric system

In Australia we measure using the **metric system**. This system began in France in the 1790s after the French revolution and, because it is based on powers of 10, it is logical and easy to use. The word 'metric' comes from the Greek word *metron* meaning 'to measure'.

In 1970, the Metric Conversion Board was established in Australia to start the change to metric units. The following metric units are most commonly used.

Worksheet

Metric units

MAT07MGWK10060

Quantity	Name of unit	Abbreviation
Length	metre	m
	millimetre	mm
	centimetre	cm
	kilometre	km
Mass	kilogram	kg
	gram	g
	milligram	mg
	tonne	t
Time	second	s
	minute	min
	hour	h
	day	d
Area	square metre	m ²
	square centimetre	cm ²
	hectare	ha
	square kilometre	km ²
Volume	cubic metre	m ³
	cubic centimetre	cm ³
Capacity	litre	L
	millilitre	mL
	kilolitre	kL
	megalitre	ML
Temperature	degree Celsius	°C

The metric system uses the following **prefixes** for metric units.

Prefix	Abbreviation	Meaning	Example
micro-	μ	one-millionth: $\frac{1}{1\,000\,000}$	1 μm = $\frac{1}{1\,000\,000}$ m
milli-	m	one-thousandth: $\frac{1}{1\,000}$	1 mg = $\frac{1}{1\,000}$ g
centi-	c	one hundredth: $\frac{1}{100}$	1 cm = $\frac{1}{100}$ m
kilo-	k	one thousand times: 1000 ×	1 km = 1000 m
Mega-	M	one million times: 1 000 000 ×	1 ML = 1000 L

The following table shows the relationships between commonly-used units of measurement.

Length 1 cm = 10 mm 1 m = 100 cm = 1000 mm 1 km = 1000 m	Capacity 1 L = 1000 mL 1 kL = 1000 L 1 ML = 1000 kL = 1 000 000 L
Mass 1 g = 1000 mg 1 kg = 1000 g 1 t = 1000 kg	Time 1 min = 60 s 1 h = 60 min = 3600 s 1 day = 24 h

Example 1

Convert:

- a 13 km to m b 5.8 t to kg c 16 000 mL to L d 1500 min to h

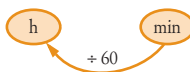
Solution

- a $13 \text{ km} = 13 \times 1000 \text{ m} = 13\,000 \text{ m}$ b $5.8 \text{ t} = 5.8 \times 1000 \text{ kg} = 5800 \text{ kg}$



To convert large to small units, multiply ($\times$).

- c $16\,000 \text{ mL} = 16\,000 \div 1000 \text{ L} = 16 \text{ L}$ d $1500 \text{ min} = 1500 \div 60 \text{ h} = 25 \text{ h}$



To convert small to large units, divide ($\div$).

Exercise 8-01 The metric system

See Example 1

1 Copy and complete:

a 1 h = _____ min

b 1 km = _____ m

c 1 kg = _____ g

d 1 cm = _____ mm

e 1 day = _____ h

f 1 kL = _____ L

g 1 t = _____ kg

h 1 min = _____ s

i 1 L = _____ mL

j 1 g = _____ mg

k 1 ML = _____ L

l 1 h = _____ s

Extra questions

The metric system

MAT07MGEQ00035

2 Convert:

a 6.4 m to cm	b 4 t to kg	c 3 days to hours
d 2.4 ML to L	e 0.9 m to mm	f 12.7 kg to g
g 8 h to min	h 5.6 L to mL	i 42 cm to mm
j 72 g to mg	k 3.8 km to cm	l 4.5 h to s

3 Convert each length to centimetres.

a 2 m	b 1500 mm	c 0.5 m	d 3.5 m
e 0.25 m	f 2 000 000 μm	g 15.8 km	h 0.4 km

4 Convert each length to metres.

a 200 cm	b 3000 mm	c 850 cm	d 9800 mm
e 75 cm	f 325 mm	g 6 km	h 18 km

5 Write the metric prefix that means:

a 1000	b $\frac{1}{100}$	c $\frac{1}{1000}$

6 Find other prefixes that are used in the metric system. Write their meanings.

7 Copy and complete each statement.

a 4.5 kg = _____ g	b 3 hours = _____ minutes	c 750 mL = _____ L
d 240 mm = _____ cm	e 12 kL = _____ L	f 5.2 m = _____ cm
g 420 s = _____ min	h 2 days = _____ hours	i 8.65 t = _____ kg
j 72 hours = _____ days	k 65 mm = _____ m	l 0.96 kL = _____ L

8 Which length is the shortest? Select the correct answer **A**, **B**, **C** or **D**.

A 0.3 m	B 29 cm	C 300 mm	D 160 mm

9 Karla and Cassie measured these objects in the classroom.

- length of pencil 12 cm
- width of computer screen 33 cm
- width of door 80 cm
- width of ruler 3.5 cm
- length of room 740 cm

- a** Karla wrote her measurements in millimetres. What did she write?
- b** Cassie wrote her measurements in metres. What did she write?
- c** What unit would you use (cm, mm or m)? Would you use different units for different objects? If so, what would they be?

10 Copy and complete each statement.

a 2.2 L = _____ mL	b 800 mm = _____ m	c 235 cm = _____ m
d 360 mg = _____ g	e $5\frac{1}{2}$ hours = _____ min	f 4900 L = _____ kL
g 270 kg = _____ t	h 4 years = _____ months	i 10.8 km = _____ m
j 16.9 L = _____ mL	k 1 year = _____ days	l 740 cm = _____ m

8-02 Perimeter

Worksheet

A page of composite shapes

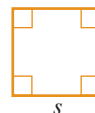
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Summary

The **perimeter** of a shape is the distance around the shape.
It is the sum of the lengths of the sides of the shape.

The **perimeter of a square** can be found by adding all of the sides:

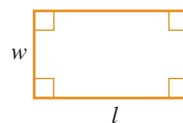
$$\begin{aligned}\text{Perimeter} &= s + s + s + s \\ &= 4s\end{aligned}$$



The general formula is $P = 4s$, where s is the length of one side.

The **perimeter of a rectangle** can be found by adding all the sides:

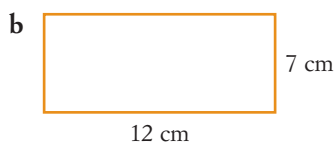
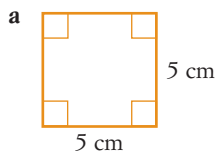
$$\begin{aligned}\text{Perimeter} &= l + w + l + w \\ &= 2l + 2w \text{ or } 2(l + w) \text{ using the distributive law.}\end{aligned}$$



The general formula is $P = 2l + 2w$ or $P = 2(l + w)$, where l is the length and w is the width (breadth).

Example 2

Find the perimeter of each shape.



Solution

a Perimeter of a square $= 4s$

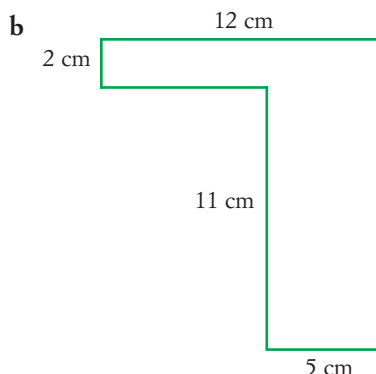
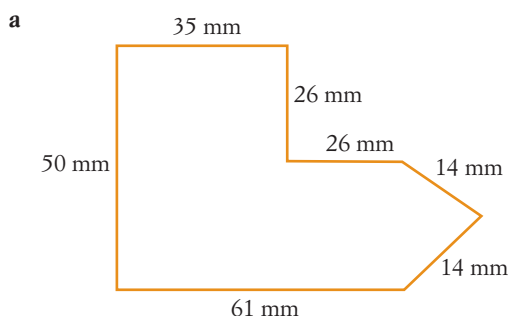
$$\begin{aligned}&= 4 \times 5 \\ &= 20 \text{ cm}\end{aligned}$$

b Perimeter of a rectangle $= 2l + 2w$ or $2(l + w)$

$$\begin{aligned}&= 2 \times 12 + 2 \times 7 \text{ or } 2 \times (12 + 7) \\ &= 38 \text{ cm}\end{aligned}$$

Example 3

Find the perimeter of each shape.

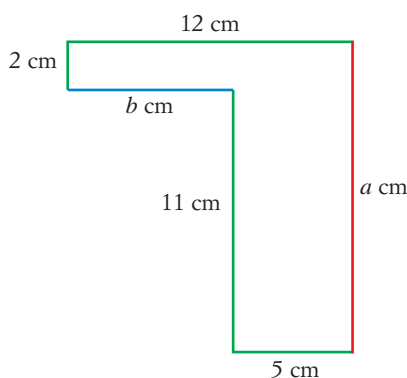


Solution

a Perimeter = $50 + 35 + 26 + 26 + 14 + 14 + 61$
= 226 mm

- b** First, we need to find the lengths of the unknown sides, a and b .

Adding side lengths



$$a = 2 + 11 = 13 \text{ cm}$$

$$b = 12 - 5 = 7 \text{ cm}$$

$$\begin{aligned} \text{Perimeter} &= 12 + 13 + 5 + 11 + 7 + 2 \\ &= 50 \text{ cm} \end{aligned}$$

The side shown by the red line
The side shown by the blue line

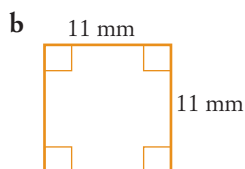
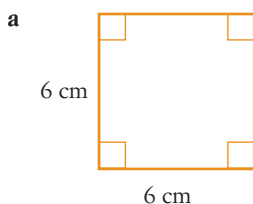
Animated example

Calculating the perimeter

MAT07MGAE00014

Exercise 8-02 Perimeter

- 1 Find the perimeter of each shape.

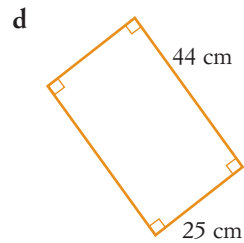
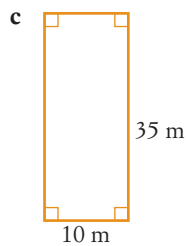


See Example 2

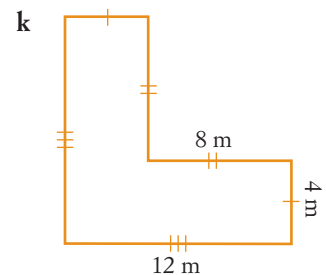
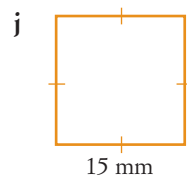
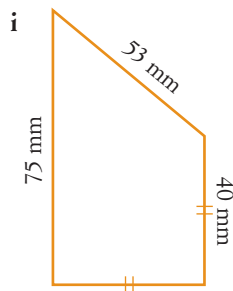
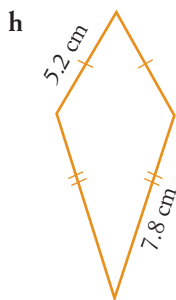
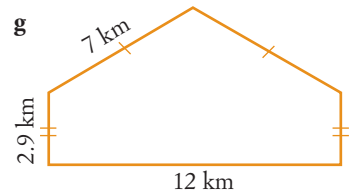
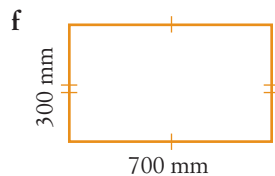
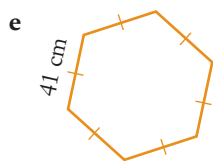
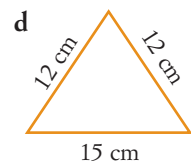
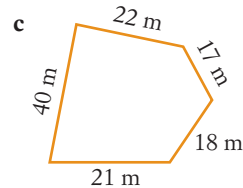
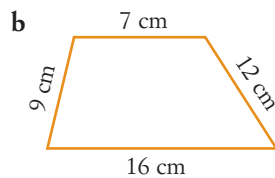
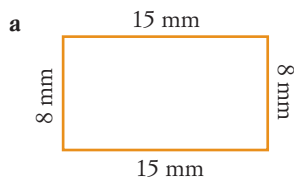
Extra questions

Calculating perimeter

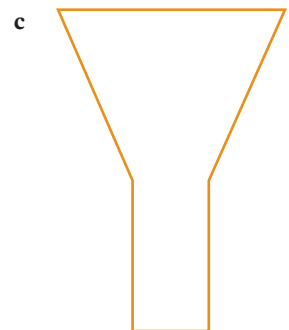
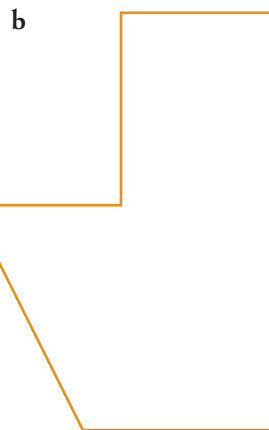
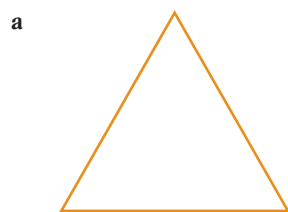
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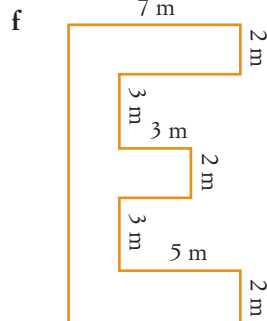
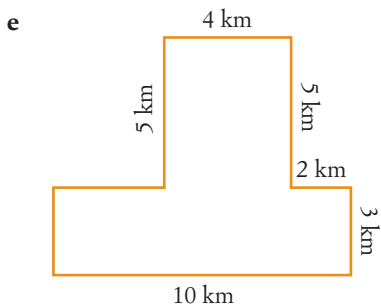
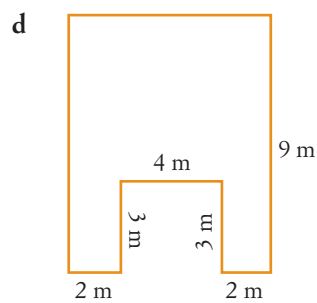
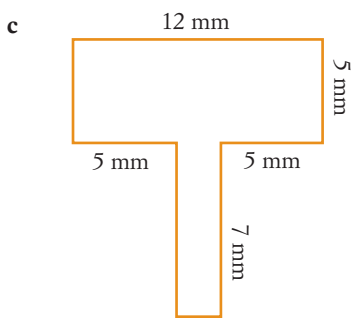
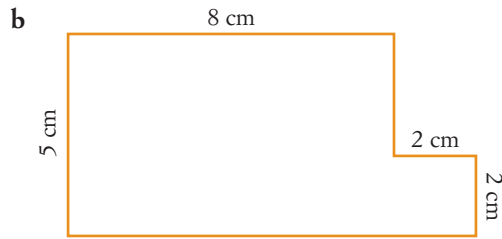
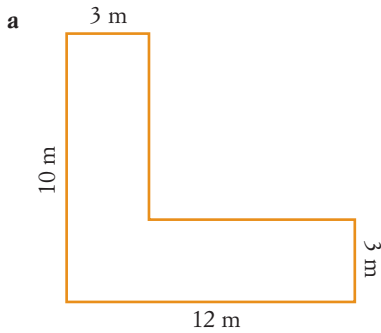
See Example 3 2 Find the perimeter of each shape.



3 Measure the sides of each shape in mm and find its perimeter.



- 4 Find the missing lengths and then calculate the perimeter of each figure.

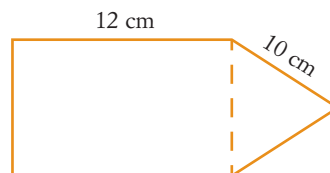


- 5 a A square has a perimeter of 16 cm. Find the length of one side.
b A rectangle has a perimeter of 40 cm. If its length is 12 cm, find its width.
c A square has a perimeter of 51 cm. Find the length of one side.
d A rectangle has a perimeter of 62 cm. If its width is 13 cm, find its length.
- 6 A rectangle has a perimeter of 30 cm. What could its length and width be?
- 7 The figure on the right is made up of a rectangle and an equilateral triangle. Find its perimeter. Select the correct answer **A**, **B**, **C** or **D**.
A 44 cm **B** 64 cm **C** 54 cm **D** 22 cm

Worked solutions

Exercise 8-02

MAT07MGWS10047



YEAR 8

8-03 Metric units for area

Skillsheet

What is area?

MAT07MGSS10028

Worksheet

Areas on a grid

MAT07MGWK10062

Worksheet

Australian areas

MAT07MGWK10063

A person estimating the number of tiles needed to cover the floor of a room needs to know the size of a tile and the size of the room. In this case, the **area** of the floor gives its 'size'.



Summary

The **area** of a shape is the amount of surface that is enclosed by the shape.

For example, the area of this trapezium has been shaded.



The table shows metric units for area.

Area unit	Abbreviation	The size of a square with each side measuring
square millimetre	mm ²	1 mm (Actual size: ■)
square centimetre	cm ²	1 cm (Actual size: ■)
square metre	m ²	1 m
hectare	ha	100 m

A **square centimetre** is about the area of a fingernail.

A **square metre** is approximately the size of the floor of a large shower recess.

A **hectare** is about the area of two rugby league fields, side-by-side.

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$$

(double the number of zeros)

10 mm



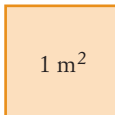
10 mm

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$$

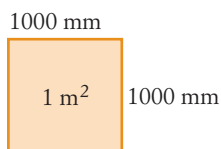
(double the number of zeros)

100 cm

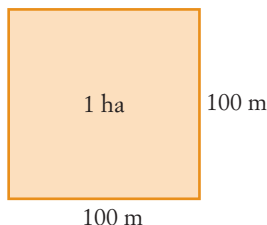


100 cm

1 m = 1000 mm
 $1 \text{ m}^2 = 1000 \text{ mm} \times 1000 \text{ mm} = 1\,000\,000 \text{ mm}^2$
 (double the number of zeros)



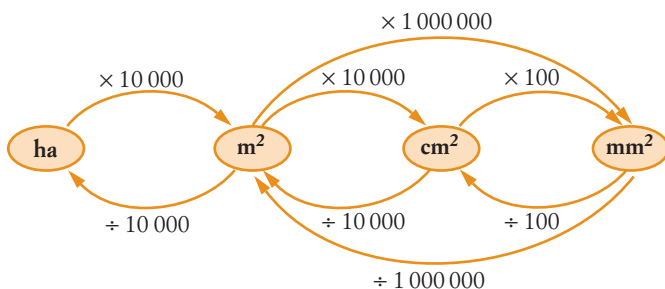
1 ha = 100 m × 100 m = 10 000 m²



Summary

$1 \text{ cm}^2 = 100 \text{ mm}^2$
 $1 \text{ m}^2 = 10\,000 \text{ cm}^2 = 1\,000\,000 \text{ mm}^2$
 $1 \text{ ha} = 10\,000 \text{ m}^2$

This diagram shows how to convert between different units of area.



Example 4

Convert

a 3 cm^2 to mm^2

b 4000 mm^2 to m^2

c $81\,000 \text{ m}^2$ to ha

Solution

a $3 \text{ cm}^2 = 3 \times 100 \text{ mm}^2$
 $= 300 \text{ mm}^2$

cm^2 to mm^2 : $\times 100$.

b $4000 \text{ mm}^2 = 4000 \div 1\,000\,000 \text{ m}^2$
 $= 0.004 \text{ m}^2$

mm^2 to m^2 : $\div 1\,000\,000$.

c $81\,000 \text{ m}^2 = 81\,000 \div 10\,000 \text{ ha}$
 $= 8.1 \text{ ha}$

m^2 to ha : $\div 10\,000$.

Exercise 8-03 Metric units for area

Extra questions

Area and its measurement

MAT07MGEQ00037

See Example 4

- What unit of area would you use when measuring the area of:
 - a farm?
 - a classroom?
 - a shirt?
 - the school oval?
 - a sheet of paper?
 - your eardrum?
- What is the approximate area of a calculator? Select the correct answer **A, B, C** or **D**.
 - 80 cm^2
 - 120 cm^2
 - 160 cm^2
 - 200 cm^2

- Copy and complete each statement.

a $15 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	b $1\,500\,000 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ m}^2$
c $690 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$	d $6.5 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
e $0.5 \text{ m}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	f $12 \text{ ha} = \underline{\hspace{2cm}} \text{ m}^2$
g $12\,200 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$	h $1250 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
i $0.32 \text{ ha} = \underline{\hspace{2cm}} \text{ m}^2$	j $7.9 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ mm}^2$
k $0.75 \text{ m}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	l $2\,450\,000 \text{ m}^2 = \underline{\hspace{2cm}} \text{ ha}$
m $865\,000 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$	n $51\,300 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ m}^2$
o $0.47 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$	p $9000 \text{ m}^2 = \underline{\hspace{2cm}} \text{ ha}$

- A square tile has an area of 1024 cm^2 . Write this area in:

- mm^2
- m^2

Worked solutions

Exercise 8-03

MAT07MGWS10048

- Arrange 6.5 m^2 , $114\,000 \text{ cm}^2$ and $25\,050\,000 \text{ mm}^2$ in ascending order.
 - Arrange 990 mm^2 , 54 cm^2 , $0.000\,032 \text{ m}^2$ in descending order.
- Sydney's central business district (CBD) is approximately a rectangle bounded by George Street, Circular Quay, Macquarie and College Streets and Liverpool Street, an area of $875\,000$ square metres. What is this area:
 - in hectares?
 - in square centimetres?
- What is the approximate area of a 5-cent coin? Select the correct answer **A, B, C** or **D**.
 - 25 mm^2
 - 2.5 cm^2
 - 25 cm^2
 - 2.5 m^2
- Sydney Airport has an area of 881 hectares. Sydney Harbour has an area of about 5500 hectares.
 - What is the area of Sydney Airport in square metres?
 - How many 'Sydney Airports' would fit onto Sydney Harbour?
- A postage stamp has an area of 8.36 cm^2 . What is this area in square millimetres?
- The theme park, Dreamworld, on the Gold Coast, has an area of one square kilometre.
 - How many square metres are there in a square kilometre?
 - How many hectares are there in a square kilometre?
 - Name something that might be measured in square kilometres.

Just for the record

Trafalgar Square

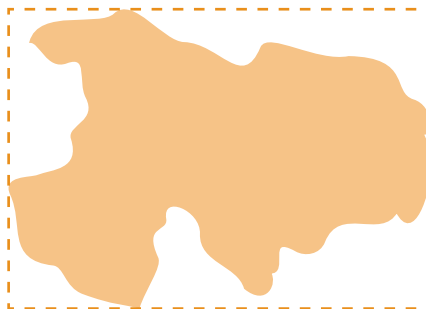
Trafalgar Square in central London is one of Britain's great tourist attractions. It commemorates the Battle of Trafalgar in 1805 between the British navy, led by Admiral Nelson, and Napoleon's French navy. It is London's only metric square, having been designed to be one hectare in area.

Find when the square was built and what famous British landmark stands in the centre of it.



Investigation: The human lagoon

- 1
 - a Find a level area of concrete or bitumen. Carefully pour a litre of water over it to form a puddle.
 - b Use a metre ruler or tape measure to measure the dimensions of the smallest rectangle that surrounds the puddle.
 - c On graph paper marked in 1 cm squares, make a scale drawing of the rectangle that surrounds the puddle.
 - d Sketch the shape of your puddle inside the rectangle on the graph paper.
 - e Use the method of counting squares to find the area of the puddle formed by the litre of water.
- 2
 - a About 80% of the human body is made up of water. If 1 litre of water has a mass of 1 kilogram, measure your own mass and use it to calculate the number of litres of water in your body. Use this result, together with those from question 1, to find the area of the puddle the water in your body would make.
 - b Go outside and mark out a rectangle that has the same area as the puddle you would make. Record the dimensions of this rectangle.
 - c Using the scale 1 cm = 1 m, draw a rectangle on graph paper that has the same area as the puddle you would make. Write a brief report of what this rectangle represents and how it was found.



Investigation: Standing room only!

- 1 As a group, mark out a square metre in your classroom or school yard.
- 2 Estimate how many students could stand in the square.
- 3 Check your estimate by getting as many students as possible to stand in the square.
- 4 How could you estimate whether all the Year 7 students in your school could stand in your classroom?
- 5 How many Year 7 students could stand in your room?

Investigation: Student space in a classroom

- 1 As a group, shift all the furniture to the sides of the classroom.
- 2 Use a set of metre-long sticks and connectors to lay out a grid of square metres on the floor.
- 3 Calculate or count the area of the floor in metres.
- 4 Sit two people to a square metre. Are you comfortable enough to stay within your square for a whole class?
- 5 Sit one person to a square metre. How comfortable are you?
- 6 Replace the furniture over the grid and sit at your desk.
- 7 How much area do you and your desk occupy?
- 8 Copy and complete each sentence.
 - a My class has _____ students.
 - b Today my maths class was held in a room with floor dimensions _____ by _____.
 - c The area of the floor is approximately _____.
 - d Each student is allocated _____ of floor space.
 - e The floor space left over for aisles and the teacher is _____.

Worksheet

Areas on a grid

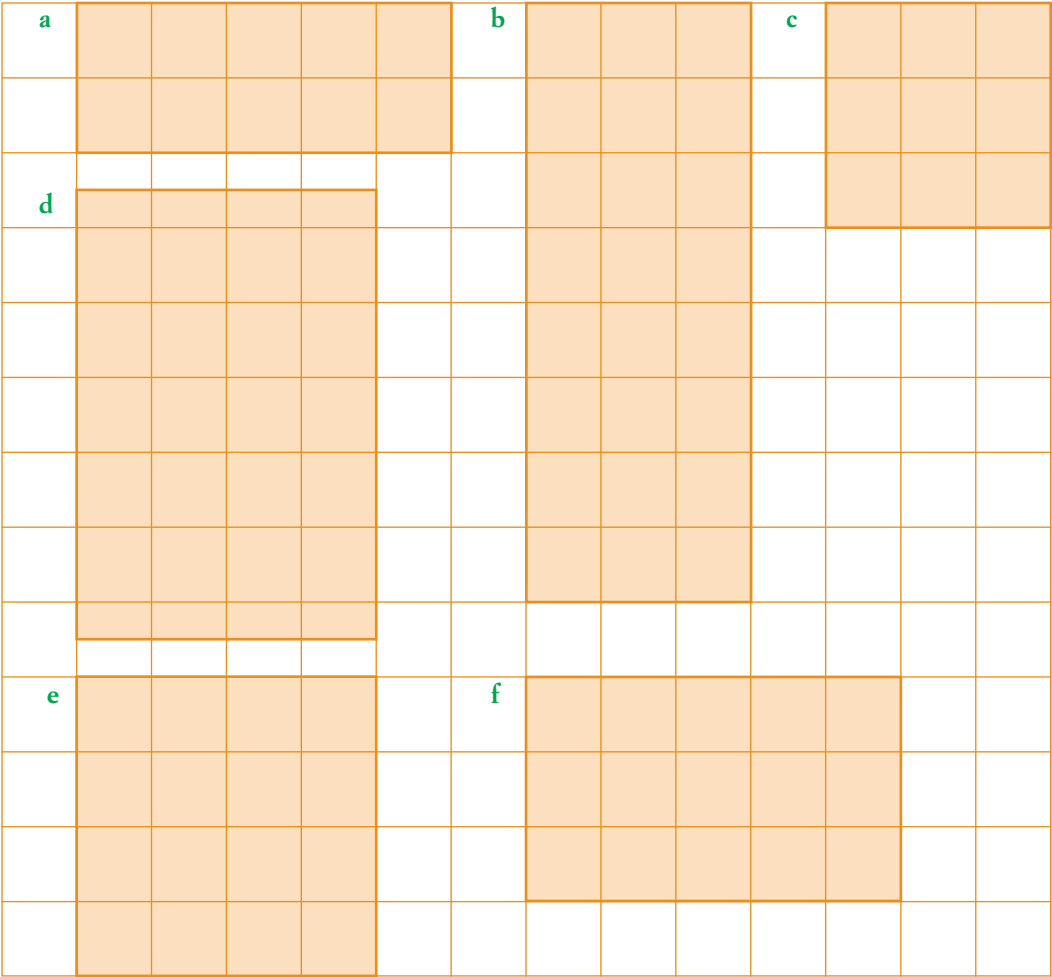
MAT07MGWK10062

TLF learning object

Exploring areas of rectangles and squares (L10569)

Investigation: Area of a rectangle

- 1 The rectangles and squares on the next page are drawn on a grid of 1-cm squares. Find the area of each rectangle and square (in square centimetres) by counting the squares in it. Copy and complete the table. For rectangles, we usually call the *shorter* measurement the width.



Rectangle	Length (cm)	Width (cm)	Area (cm ²)
a	5	2	
b	8		
c			
d			
e			
f			

2 Look at your results for question 1. Write down a rule for finding the area of a rectangle or square from its length and width.

8-04 Area of a rectangle

Worksheet

Areas on a grid

MAT07MGWK10062

Worksheet

Australian areas

MAT07MGWK10063

Puzzle sheet

Area group clues

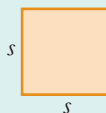
MAT07MGPS10031

Summary

Square

$$\text{Area} = \text{side} \times \text{side} = s \times s$$

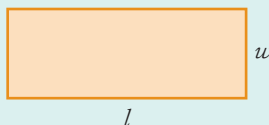
$$A = s^2$$



Rectangle

$$\text{Area} = \text{length} \times \text{width}$$

$$A = lw$$

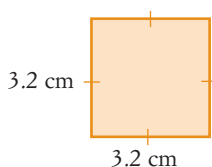


Note: This formula can also be written as $A = lb$, where b stands for breadth, another name for width.

Example 5

Find the area of each shape.

a



b



Solution

$$\begin{aligned} \text{a Area} &= s^2 \\ &= 3.2^2 \\ &= 3.2 \times 3.2 \\ &= 10.24 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{b Area} &= l \times w \\ &= 60 \times 5 \\ &= 300 \text{ mm}^2 \end{aligned}$$

Change 6 cm to
60 mm to make
units the same

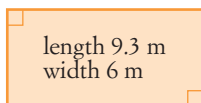
It is also possible to change
5 mm to 0.5 cm to have an
answer of 3 cm² instead

Exercise 8-04 Area of a rectangle

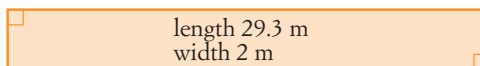
See Example 5

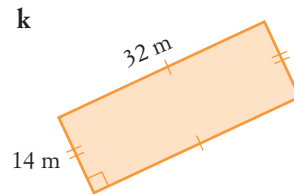
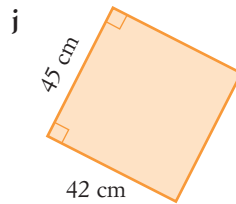
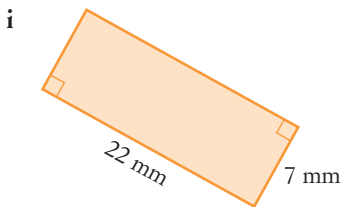
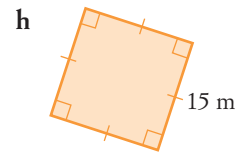
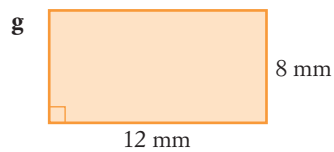
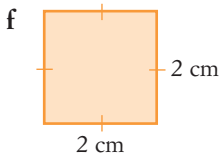
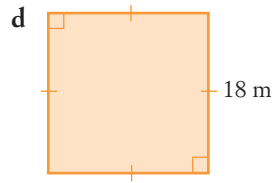
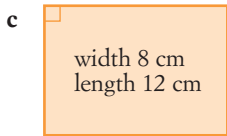
1 Find the area of each rectangle.

a



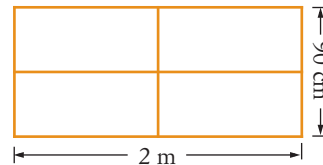
b





- 2 A window is 2 metres long and 90 centimetres wide.
Find, in square metres, the area of the window. Select
A, B, C or D.

A 180 m^2 B 0.18 m^2 C 1.8 m^2 D 18 m^2



- 3 Find the area of each item described below:

i in m^2

ii in cm^2

iii in mm^2

a a square bowling green of length 5 m

b a desk top measuring 95 cm by 54 cm.

- 4 Measure the sides of each rectangle below and calculate its area:

i in mm^2

ii in cm^2





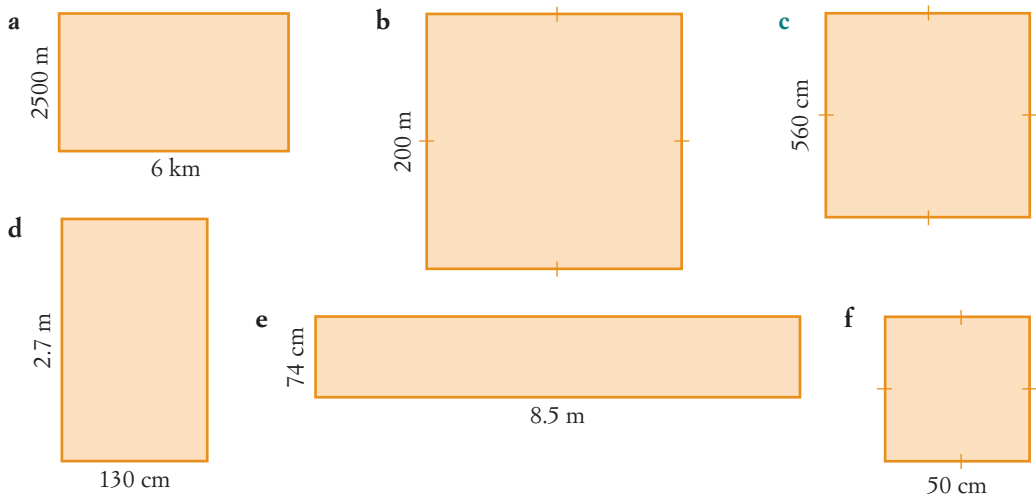
5 A square has an area of 49 cm^2 . What is the length of one side?

6 Find the area of each rectangle in square metres.

Worked solutions

Exercise 8-04

MAT07MGWS10049



7 The front yard of a house measures 20 m by 8 m while the backyard measures 35 m by 7.5 m. Calculate the cost of covering both yards with grass if it costs \$18.60 per square metre.

8 A rectangle has a width of 4 m and an area of 18 m^2 . What is its length?

9 Sandra's bedroom floor measures 3.6 m by 2.8 m. She buys carpet squares of length 40 cm.

a What is the area of Sandra's bedroom floor?

b What is the area of a carpet square in square metres?

c How many carpet squares are needed to cover Sandra's bedroom floor?

10 A rectangle has an area of 35 cm^2 . What could its length and width be?

11 A $4 \text{ m} \times 5 \text{ m}$ basement has a ceiling height of 3.2 m.

a Find the area of:

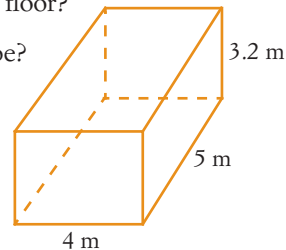
i the floor

ii the ceiling

iii one of the smaller walls

b How many square tiles measuring 50 cm on each side are needed to cover the floor of the basement?

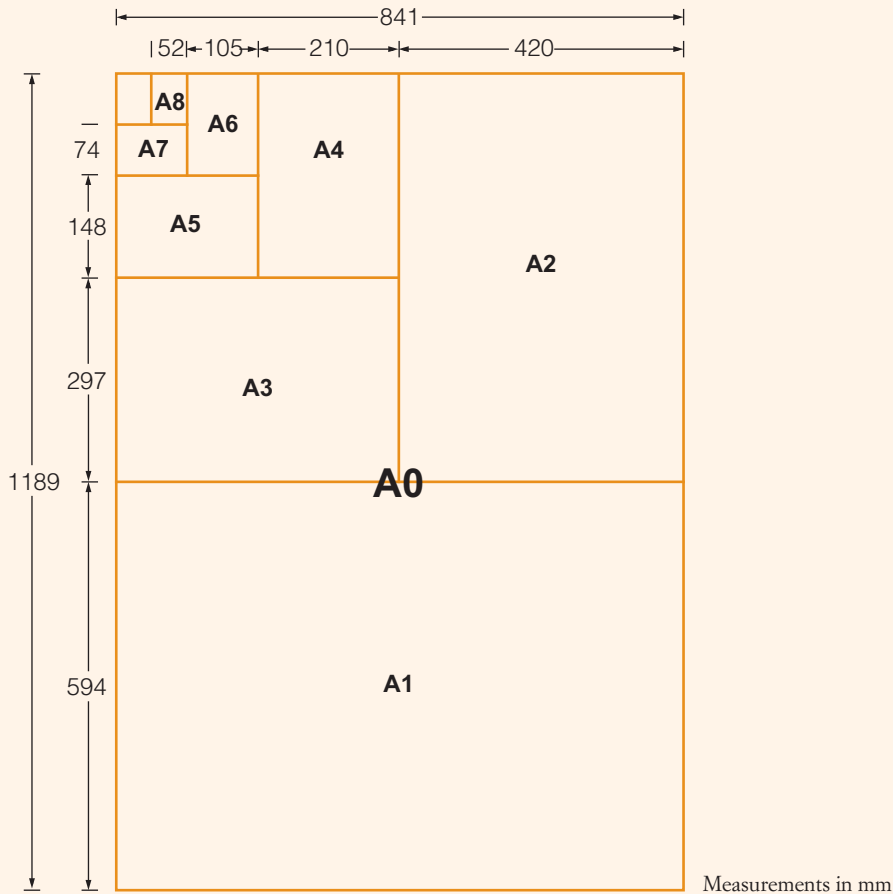
c How many litres of paint are needed to cover the 4 walls of the basement if one litre of paint covers 16 m^2 ?



Just for the record

Paper sizes

The size of a sheet of paper is labelled by the letter A and a number. An A4 sheet has a length of 297 mm and a width of 210 mm. If you cut an A4 sheet crosswise into halves, you will get two A5 sheets. If you cut an A5 sheet crosswise into halves, you will get two A6 sheets, and so on. Similarly, an A3 sheet is double the area of an A4 sheet, like two A4 sheets side-by-side. The largest size is A0. A diagram showing the A series of paper sizes from A0 to A8 is shown below.



- Find the area of a sheet of A3 paper and a sheet of A5 paper.
- How many A4 sheets of paper is equal to one A0 sheet?
- What fraction of an A4 sheet is an A8 sheet?

Mental skills 8A Maths without calculators

Doubling and halving numbers

You can double or halve a number by splitting it up first, then doubling or halving.

1 Study each example.

a 92×2

Think: Double 92 = double 90 + double 2
 $= 180 + 4$
 $= 184$

b 37×2

Think: Double 37 = double 30 + double 7
 $= 60 + 14$
 $= 74$

c $\frac{1}{2} \times 86$

Think: Half of 86 = half of 80 + half of 6
 $= 40 + 3$
 $= 43$

d $\frac{1}{2} \times 244$

Think: Half of 244 = half of 240 + half of 4
 $= 120 + 2$
 $= 122$

If the tens number is **odd**, then:

e $\frac{1}{2} \times 78$

Think: Half of 78 = half of 60 + half of 18
 $= 30 + 9$
 $= 39$

f $\frac{1}{2} \times 132$

Think: Half of 132 = half of 120 + half of 12
 $= 60 + 6$
 $= 66$

2 Now simplify each product.

a 54×2

b 77×2

c 83×2

d 105×2

e 26×2

f 41×2

g 98×2

h 162×2

i $\frac{1}{2} \times 182$

j $\frac{1}{2} \times 274$

k $\frac{1}{2} \times 92$

l $\frac{1}{2} \times 138$

m $\frac{1}{2} \times 506$

n $\frac{1}{2} \times 76$

o $\frac{1}{2} \times 48$

p $\frac{1}{2} \times 170$

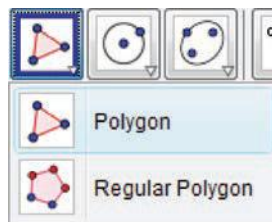
Technology Perimeter and area of a chicken pen

Farmer Jones has 36 m of fencing to make a rectangular chicken pen.

- 1 Open a new spreadsheet and enter the labels, values and formula for calculating the perimeter of a rectangle as shown below.

	A	B	C	D
1	Chicken Pens			
2				
3	Length (m)	Width (m)	Perimeter (m)	Area (m)
4	10	8	=2*A4+2*B4	
5				

- 2 In cell D4, enter a formula for the area of the rectangle.
- 3 Now use GeoGebra to help you solve this problem. To set up your drawing page, click **View**. Check that **Grid** is ticked and **Axes** is not ticked.
- 4 Click the **Polygon** tool and use the grid to construct a rectangle 10 m by 8 m.



- 5 Now click the **Area** tool and **select** the rectangle. You will see the area of this rectangle as 80.



- 6 Now repeat steps 4 and 5 and find at least 3 pairs of dimensions (whole numbers only) for this rectangle and enter them into your spreadsheet.
- 7 Return to the spreadsheet and enter the dimensions for length and width that you found. Also, **Fill Down** from cells C4 and D4 to complete the perimeter and area calculations.
- 8 There are more pairs of whole-number dimensions (lengths and widths) for the rectangular chicken pen. Use GeoGebra and fill in your spreadsheet to find any missing pairs of dimensions.
- 9 Which dimensions make the largest area for the chicken pen?

Investigation: Area of a triangle

Worksheet

1 cm grid paper

MAT07MGWK10108

TLF learning object

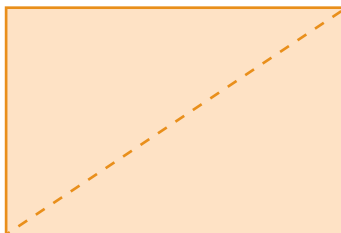
Area of triangles
(L145)

Worksheet

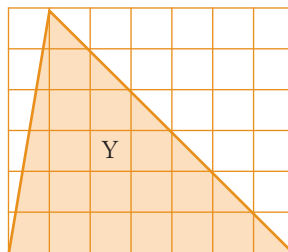
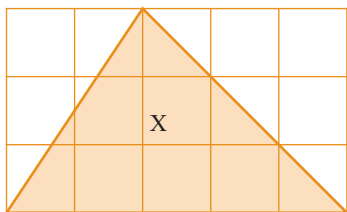
Triangle areas

MAT07MGWK00059

You will need 1-cm grid paper and scissors.



- 1 On your grid paper, draw a rectangle 6 cm by 4 cm.
 - a Cut the rectangle in half along a diagonal. What shape have you made?
 - b Area of rectangle = $\_\_\_ \times \_\_\_ = \_\_\_ \text{cm}^2$
 - c So what is the area of each triangle?
 - d Draw two other rectangles and repeat steps 2 to 4. What do you notice regarding the area of the triangle and the area of the rectangle?
- 2 On your grid paper, draw triangle X as shown, and cut out the rectangle.



- a Area of rectangle = $\_\_\_\_\_\_$
- b Cut out triangle X.
- c Place the leftover pieces on triangle X. Do they cover triangle X exactly?
- d Area of triangle = $\_\_\_\_\_\_$
- e Repeat the above steps for triangle Y.

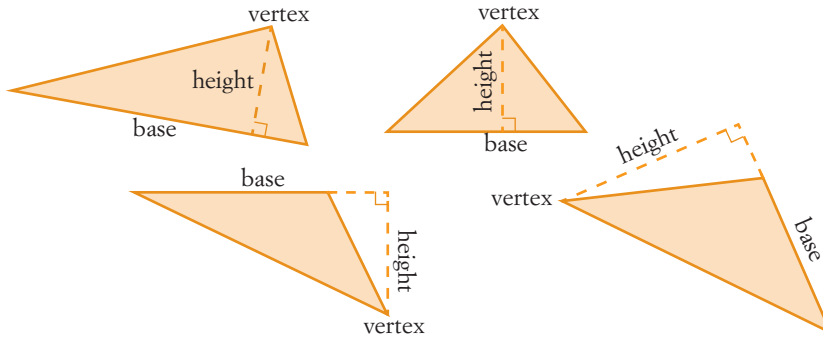
8-05 Area of a triangle

Technology

Geogebra
Area of a triangle

MAT07MGCT00010

Any side of a triangle can be called its **base**. The **height** is the distance from the base to the opposite vertex. This distance is measured at right angles to the base, so it is also called the **perpendicular height**. Here are some examples:

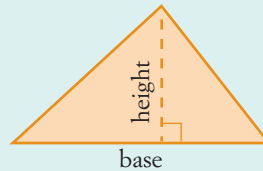


Note that with the last two examples involving obtuse-angled triangles, the perpendicular height is **outside** the triangle.

Summary

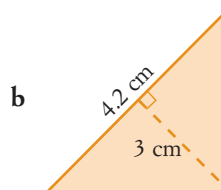
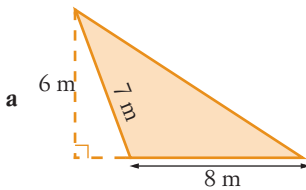
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A = \frac{1}{2}bh$$



Example 6

Find the area of each triangle.



Solution

a $\text{Area} = \frac{1}{2}bh$

$$= \frac{1}{2} \times 8 \times 6$$

$$= 24 \text{ m}^2$$

Choosing 8 m as the base

The length of 7 m is not required to find this triangle's area

b $\text{Area} = \frac{1}{2}bh$

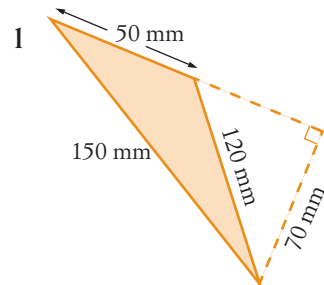
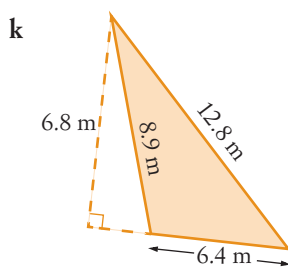
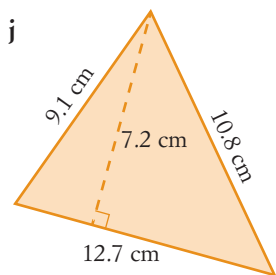
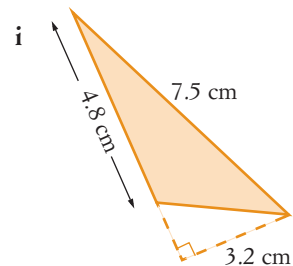
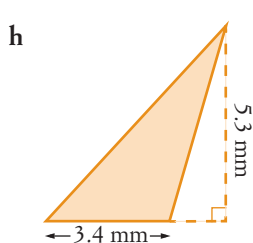
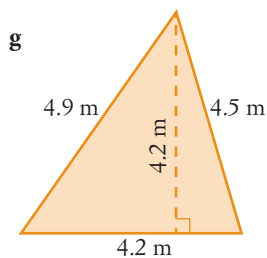
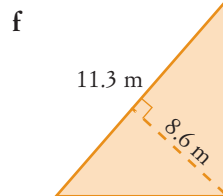
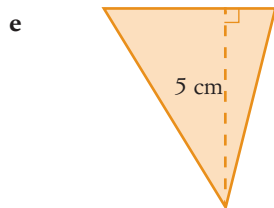
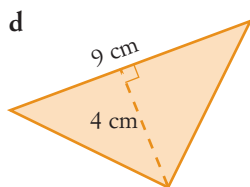
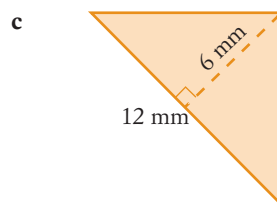
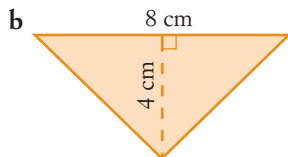
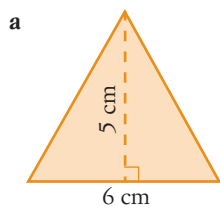
$$= \frac{1}{2} \times 4.2 \times 3$$

$$= 6.3 \text{ cm}^2$$

Choosing 4.2 cm as the base

Exercise 8-05 Area of a triangle

See Example 6 1 Find the area of each triangle.

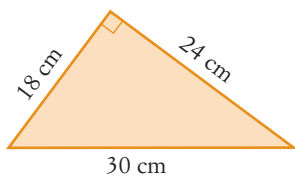


Worked solutions

Exercise 8-05

MAT07MGWS10050

2 What is the area of this triangle? Select the correct answer A, B, C or D.



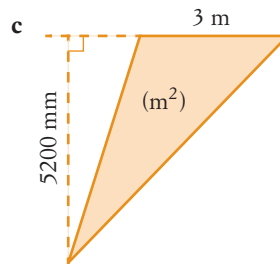
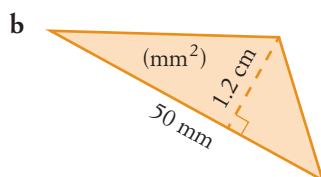
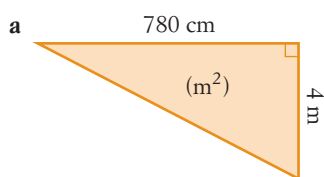
A 360 cm^2

B 432 cm^2

C 270 cm^2

D 216 cm^2

- 3 Find the area of each triangle, using the units given in brackets.



- 4 A triangle has a base of length 4 m and an area of 40 m^2 . What is its perpendicular height?
- 5 A triangle with an area of 27 cm^2 has a perpendicular height of 18 cm. How long is its base?
- 6 The fly-screen of a tent is to be made into the shape of a triangle 0.8 m long and 1.4 m high. Calculate the cost of the screen if one square metre costs \$9.10.
- 7 A triangle has an area of 36 m^2 . What could its base length and perpendicular height be?
- 8 A triangle's perpendicular height is the same as its base length. If its area is 32 cm^2 , what is its perpendicular height?

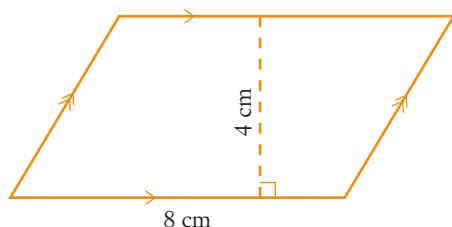
Worked solutions

Exercise 8-05

MAT07MGWS10050

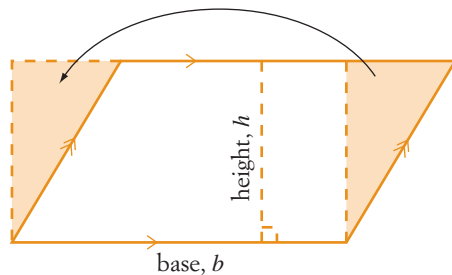
Investigation: Area of a parallelogram

A **parallelogram** is a quadrilateral with opposite sides parallel and equal in length.



- On a sheet of paper, draw a parallelogram with a base length of 8 cm and a perpendicular height of 4 cm.
- Draw an identical parallelogram in your book.
- Cut out the parallelogram from the sheet of paper.
- Cut along the dotted line (perpendicular height) and rearrange the pieces to form a rectangle.
- Measure the length and width of the rectangle to the nearest millimetre, and then find its area.
- Why is the area of the rectangle the same as the area of the parallelogram?
- Suggest a general formula for finding the area of any parallelogram. (Check your answer with your teacher.)

- 8 Paste the rectangle in your book under your diagram of the parallelogram.



8-06 Area of a parallelogram

Technology worksheet

Excel
Area formulas

MAT07MGCT10004

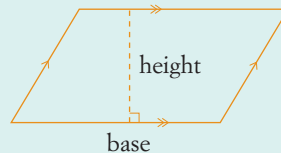
The area of a parallelogram has the same area as the rectangle with the same base, b , and perpendicular height, h . It is found by multiplying the base, b , by the height, h .

$$\text{Area} = b \times h = bh$$

Summary

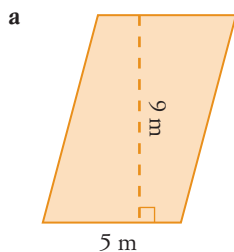
Area = base $\times$ height

$$A = bh$$



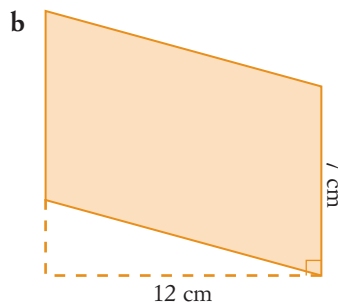
Example 7

Find the area of each parallelogram.



Solution

$$\begin{aligned} \text{a Area} &= bh \\ &= 5 \times 9 \\ &= 45 \text{ m}^2 \end{aligned}$$



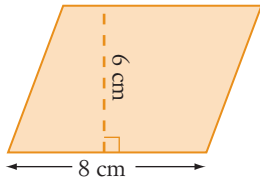
$$\begin{aligned} \text{b Area} &= bh \\ &= 7 \times 12 \\ &= 84 \text{ cm}^2 \end{aligned}$$

Exercise 8-06 Area of a parallelogram

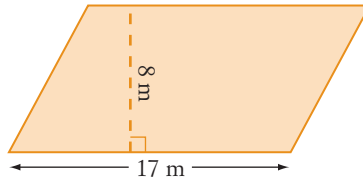
1 Find the area of each parallelogram.

See Example 7

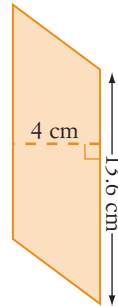
a



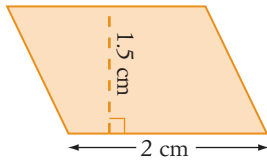
b



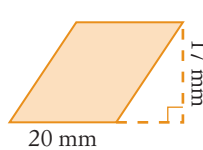
c



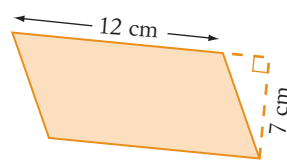
d



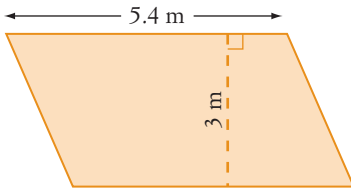
e



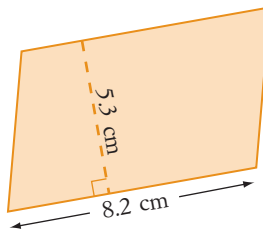
f



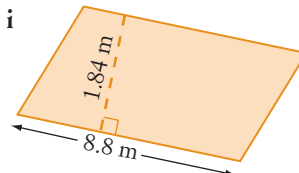
g



h



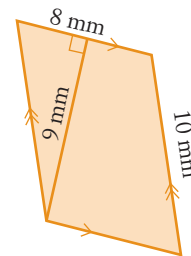
i



2 What is the area of this parallelogram?

Select the correct answer A, B, C or D.

- A 72 mm^2
- B 36 mm^2
- C 45 mm^2
- D 80 mm^2



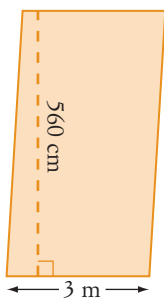
Worked solutions

Exercise 8-06

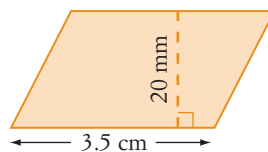
MAT07MGWS10051

3 Find the area of each parallelogram in square centimetres. Make sure you change both lengths to centimetres first.

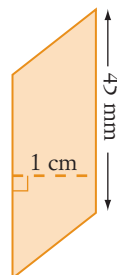
a

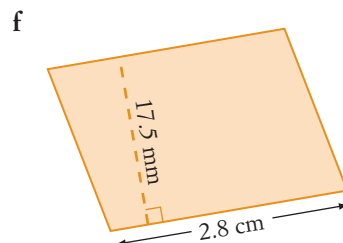
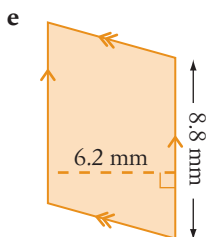
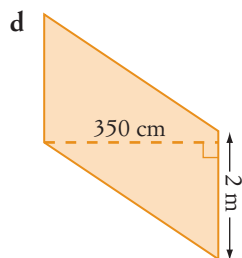


b



c





- 4 The courtyard of a house is in the shape of a parallelogram with a base length of 4.3 m and an area of 24.94 m^2 . What is its perpendicular height? Select **A**, **B**, **C** or **D**.
A 5.8 m^2 **B** 10.6 m^2 **C** 53.621 m^2 **D** 107.24 m^2
- 5 A parallelogram has an area of 245 cm^2 . What could be its base length and perpendicular height?

8-07 Areas of composite shapes

Worksheet

A page of composite shapes

MAT07MGWK10061

Worksheet

Composite areas

MAT07MGWK10064

Video tutorial

Areas of composite shapes

MAT07MGVT10018

Puzzle sheet

Area puzzles

MAT07MGPS10032

Homework sheet

Area

MAT07MGHS10025

TLF learning object

Exploring area and perimeter (L6557)

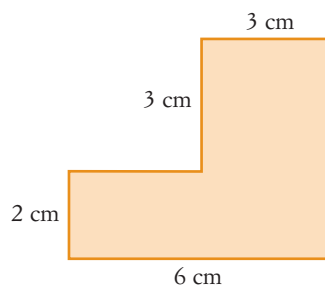
TLF learning object

Compound shapes (L153)

A **composite shape** is made up of two or more shapes.

Example 8

Find the area of this shape.



Solution

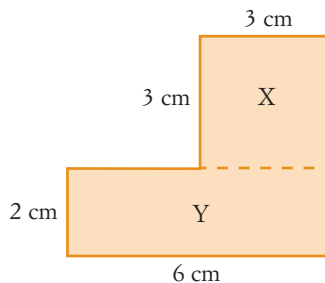
Method 1

Separate the shape into a square (X) and a rectangle (Y).

Area of shape = area of square X + area of rectangle Y

$$= 3 \times 3 + 6 \times 2$$

$$= 21 \text{ cm}^2$$



Method 2

Separate the shape into two rectangles P and Q.

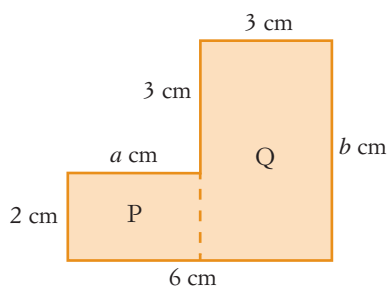
$$a = 6 - 3 = 3$$

$$b = 3 + 2 = 5$$

Area of shape = area of P + area of Q

$$= 3 \times 2 + 5 \times 3$$

$$= 21 \text{ cm}^2$$



Method 3

The area can also be found by subtracting areas.

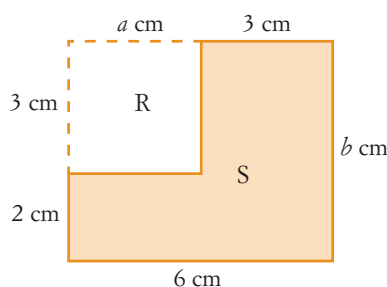
$$a = 6 - 3 = 3$$

$$b = 3 + 2 = 5$$

Area of shape = area of big rectangle – area of R

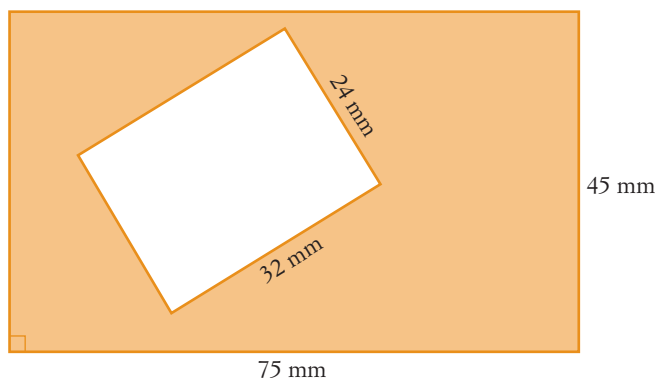
$$= 6 \times 5 - 3 \times 3$$

$$= 21 \text{ cm}^2$$



Example 9

Find the shaded area.



Solution

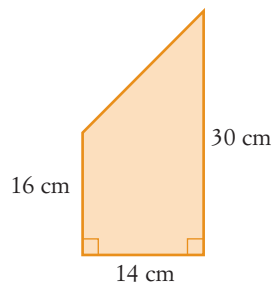
Shaded area = area of big rectangle – area of small rectangle

$$= 75 \times 45 - 32 \times 24$$

$$= 2607 \text{ mm}^2$$

Example 10

Find the area of this shape.

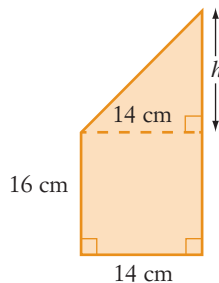
**Solution**

Divide the shape into a triangle and a rectangle.

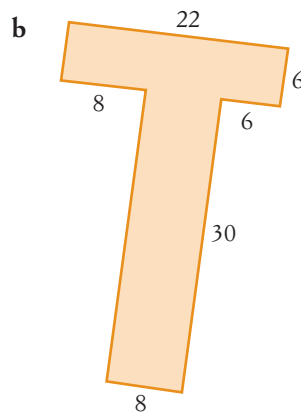
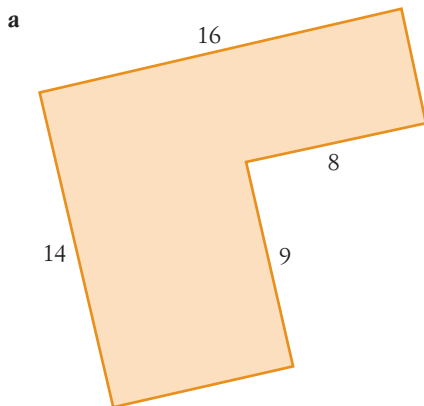
$$\begin{aligned}\text{Height of triangle, } h &= 30 - 16 \\ &= 14 \text{ cm}\end{aligned}$$

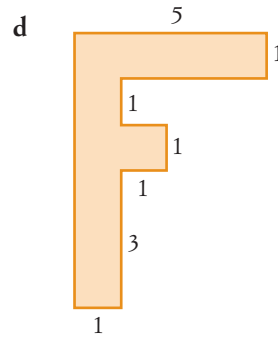
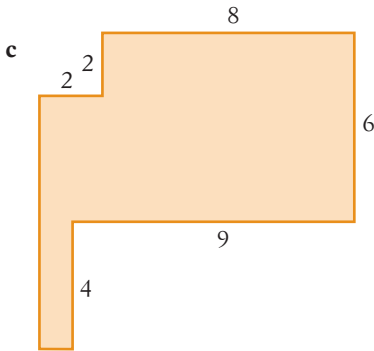
Area of shape = area of rectangle + area of triangle

$$\begin{aligned}&= 16 \times 14 + \frac{1}{2} \times 14 \times 14 \\ &= 224 + 98 \\ &= 322 \text{ cm}^2\end{aligned}$$

**Exercise 8-07 Areas of composite shapes**

See Example 8 1 Find the area of each shape. All measurements are in centimetres.





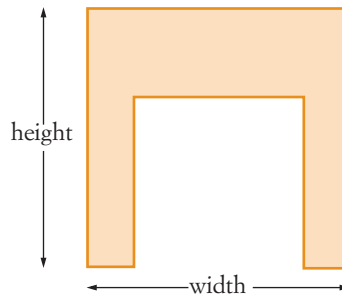
2 Find the area of each shape.

See Example 9

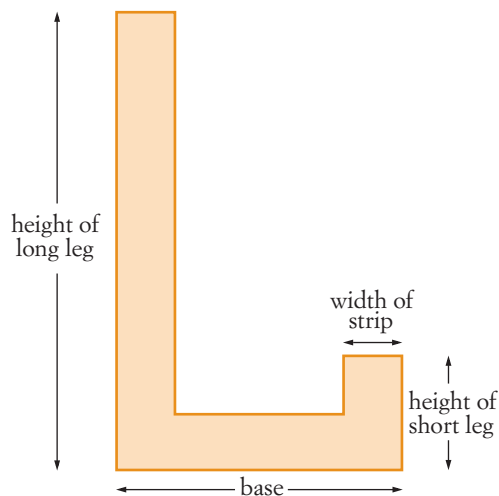
a Outside dimensions: $230 \text{ cm} \times 55 \text{ cm}$. Hole dimensions: $40 \text{ cm} \times 35 \text{ cm}$



b Total height: 30 cm, total width: 30 cm
Dimensions of cut-out: $20 \text{ cm} \times 20 \text{ cm}$



c This shape is formed from strips of metal 10 cm wide. The height of the long leg is 80 cm. The base is 50 cm wide. The height of the short leg is 20 cm.



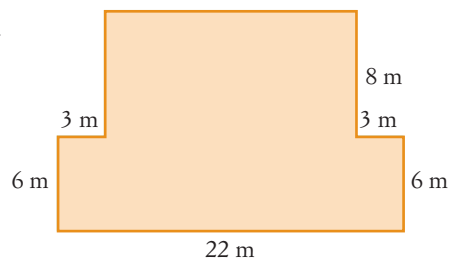
- 3 The diagram on the right shows a swimming pool in a holiday resort. What is the area of the floor of the pool? Select the correct answer **A**, **B**, **C** or **D**.

A 156 m^2

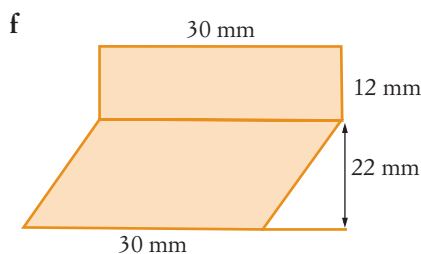
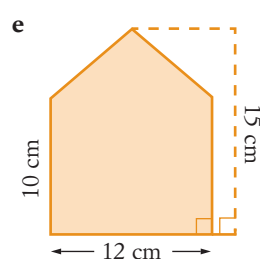
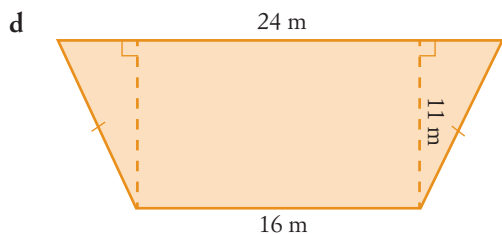
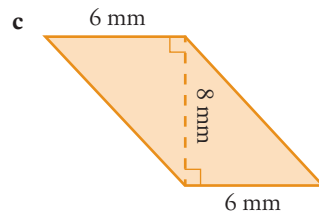
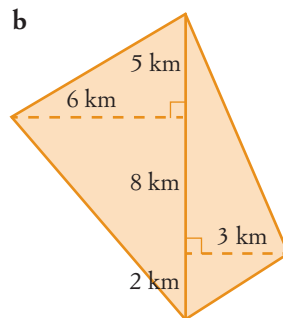
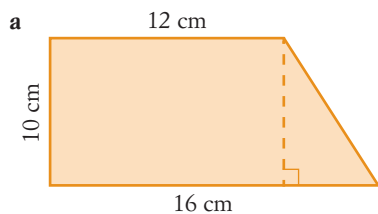
B 180 m^2

C 72 m^2

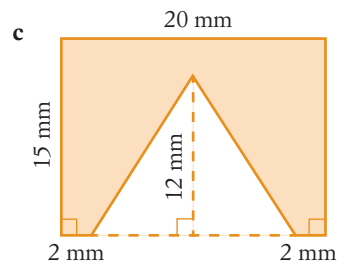
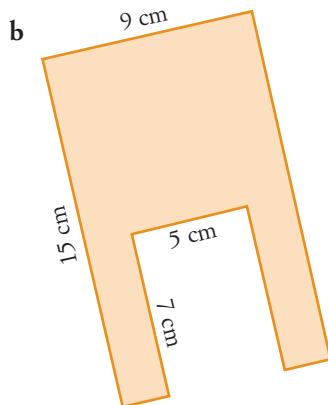
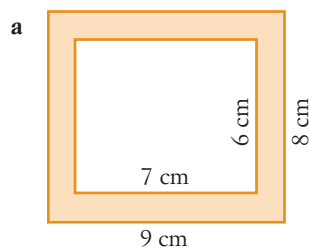
D 260 m^2

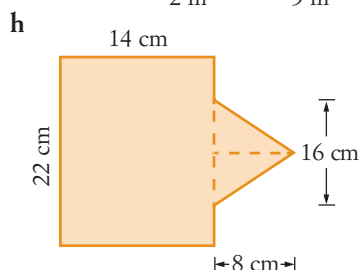
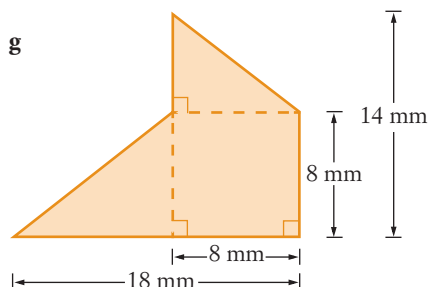
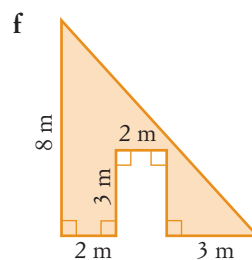
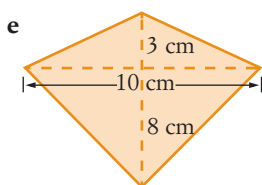
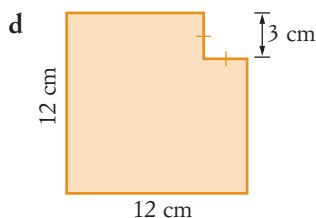


See Example 10 4 Find the area of each shape.



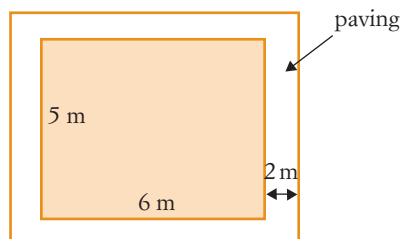
- 5 Find each shaded area.



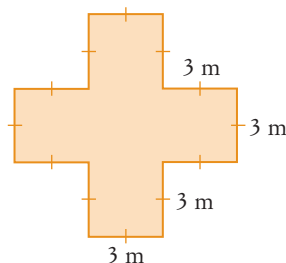


6 A rectangular lawn contains two square flower gardens, each measuring 3 m by 3 m. If the lawn measures 9.5 m by 6.4 m, find the area of the grass.

7 A garden measuring 6 m by 5 m has a 2 m wide strip of paving around its border. What is the area of the paving?



8 The floor of a children's pool is in the shape of a cross as shown. Each side is 3 m long. How many tiles are required to tile this floor if one tile covers 1600 cm^2 ?



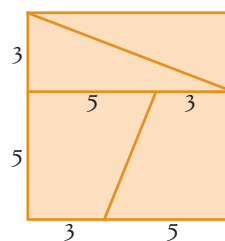
Worked solutions

Exercise 8-07

MAT07MGWS10052

Investigation: Area puzzles

1 Draw this square on 1 cm dot paper or print out the worksheet 'Area puzzles'. All measurements are in centimetres. What is its area in cm^2 ?



2 a Cut out the square, then cut along the marked lines.

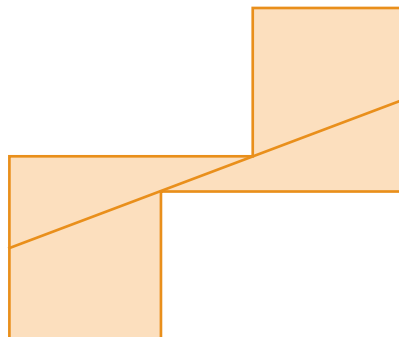
b Arrange the four pieces to make a rectangle. Find its length and width. Use these to find its area.

Puzzle sheet

Area puzzles

MAT07MGPS10032

- 3 Is the calculated area of the rectangle the same as the area of the original square? Should it be? Explain this mystery.
- 4 Use the same four pieces to make this shape. Calculate its area. Is this the same as the area of the original square? Should it be? Why?

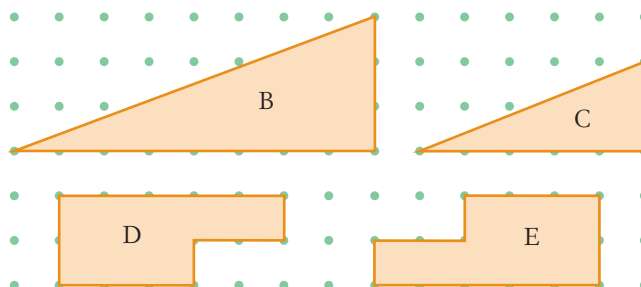


Worksheet

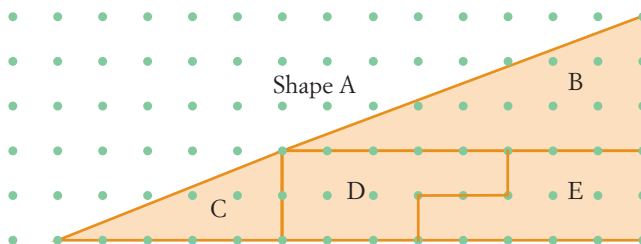
Square dot paper

MAT07MGWK10109

- 5 a Use square dot paper to draw each of the shapes B, C, D and E as shown, or access the worksheet on NelsonNet and print them out.



- b Find the area of each shape in cm^2 , then find the sum of the areas.
- c Cut out the shapes and combine them to form Shape A, as shown.



- 6 a Shape A looks like a right-angled triangle. Find its height and the length of its base. Use these to find its area.
- b Does the area of the triangle equal the sum of the areas of B, C, D and E from question 1? Explain the mystery about the area of the triangle, the area of Shape A and the sum of the areas of B, C, D and E.

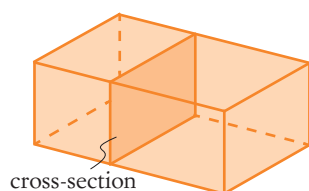
Investigation: Local areas

- 1 **a** Use a map or street directory to find a recreation area (park, garden or reserve) near where you live.
- b** Find the scale for measuring distance used on your map or street directory.
- c** Estimate the area of the park, garden or reserve.
- 2 **a** Rule lines on a sheet of tracing paper to make a grid of squares scaled like your map or street directory. Trace your park, garden or reserve from question 1 onto the paper.
- b** Approximate the area (in m^2 and ha) of your park, garden or reserve. Use the method of counting squares. (It may be more accurate to subdivide each square on the grid into smaller shapes.)

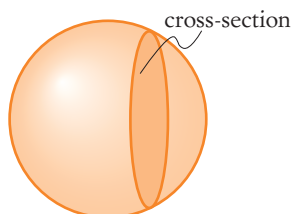
8-08 Drawing prisms

Cross-sections

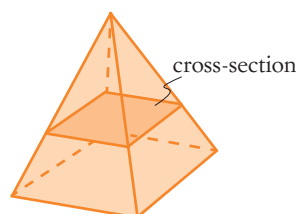
A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it. These diagrams show cross-sections of a rectangular prism, a sphere and a square pyramid.



Rectangular prism



Sphere



Square pyramid

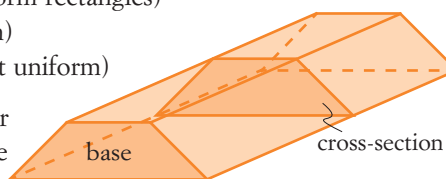
Note that the cross-sections of a rectangular prism are congruent (identical) rectangles, the cross-sections of a sphere are circles (but of different sizes), and the cross-sections of a square pyramid are squares (again, of different sizes).

Prisms

If a solid has the same (uniform) cross-section along its length, and each cross-section has straight sides (not rounded), then the solid is called a **prism**. From the above diagrams:

- a rectangular prism is a prism (its cross-sections are uniform rectangles)
- a sphere is not a prism (its cross-sections are not uniform)
- a square pyramid is not a prism (its cross-sections are not uniform)

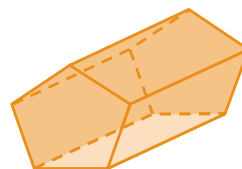
The end faces of a prism are called its **base**. Prisms take their names from their base. For example, the prism shown on the right is called a **trapezoidal prism** because its cross-sections are all trapeziums, identical and parallel to its base.



Trapezoidal prism

Example 11

- Draw a cross-section of the prism shown.
- What shape is the prism's cross-section?
- What shape are the side faces?
- What is the name of this prism?



Solution

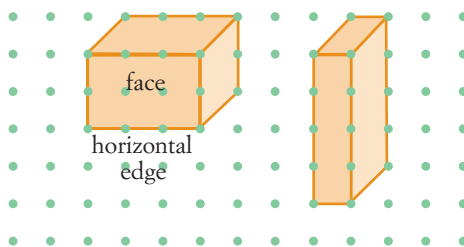
a



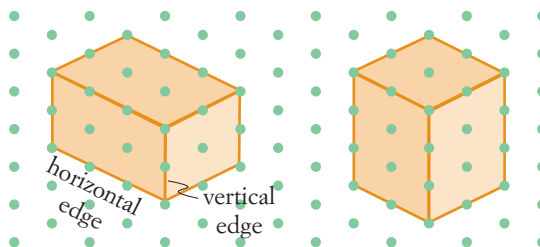
- The cross-section is a pentagon.
- The side faces are all rectangles.
- This is a pentagonal prism.

Drawing different views of solids

Prisms can be difficult to draw. Sometimes it is easier to use square or isometric dot paper to draw them. These rectangular prisms are drawn on **square dot paper**, a square grid of dots.



These rectangular prisms are drawn on **isometric dot paper**, a '3D' triangular grid of dots arranged as equilateral triangles. A 2D (flat) diagram of a 3D figure (solid) looks more natural when drawn on isometric paper. On isometric paper, all horizontal edges are shown at an angle of 30° .

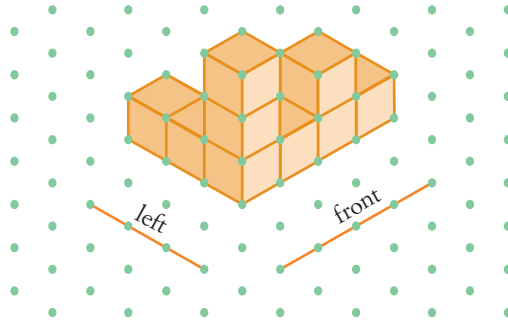


- When drawing on **square** paper, one of the prism's **faces** is shown at the forefront.
- When drawing on **isometric** paper, one of the prism's **vertical edges** is shown at the forefront.

Example 12

For the solid shown, assume that there are no hidden cubes. Draw:

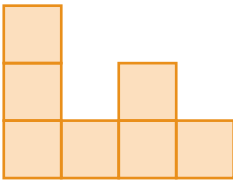
- a the front view
- b the left view
- c the top view



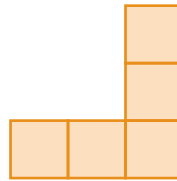
Solution

For each view, there will be some cubes hidden behind the cubes closest to you.

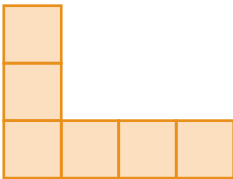
- a Viewed from the front



- b Viewed from the left



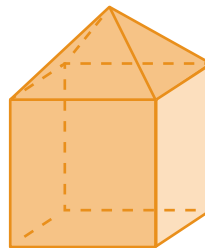
- c Viewed from the top



Example 13

For the solid shown, draw:

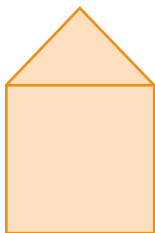
- a the front view
- b the top view



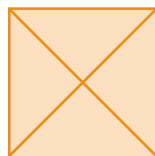
front

Solution

a Viewed from the front



b Viewed from the top



Exercise 8-08 Drawing prisms

Worksheet

Square dot paper

MAT07MGWK10109

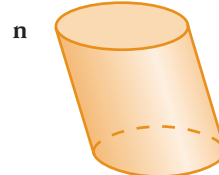
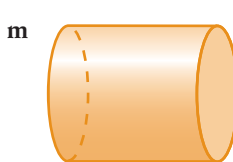
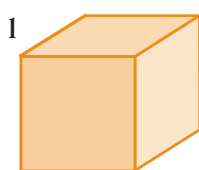
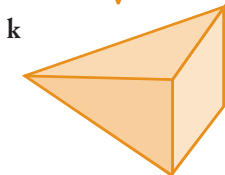
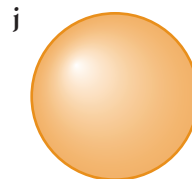
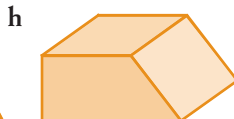
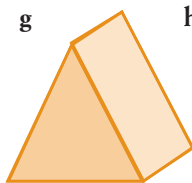
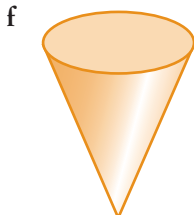
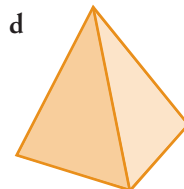
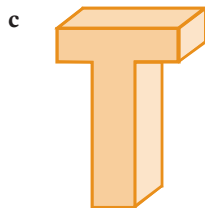
Worksheet

Isometric dot paper

MAT07MGWK10110

For this exercise you will need square dot paper, isometric dot paper and construction cubes. Both types of dot paper can be printed out from NelsonNet. You can also draw shapes using dynamic geometry software, using a square grid or an isometric grid.

1 State whether each solid is a prism or not.

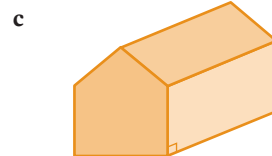
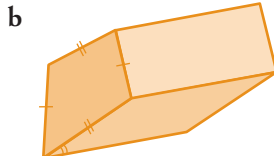
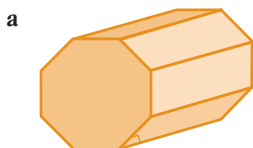


See Example 11

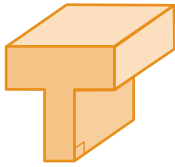
2 For each of the prisms below,

i draw its cross-section

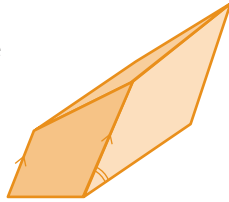
ii write the name of the prism



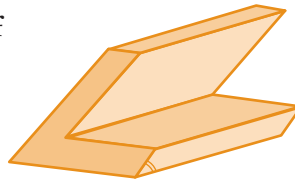
d



e



f



3 Draw each prism listed below and shade its base.

a square prism

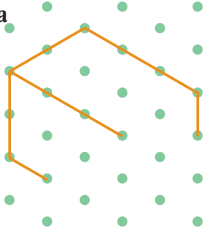
b isosceles triangular prism

c trapezoidal prism

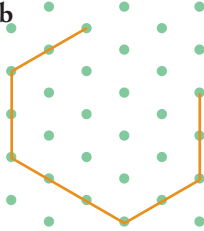
d hexagonal prism

4 Each diagram is the start of a drawing of a rectangular prism. Copy each drawing onto isometric dot paper and complete it.

a



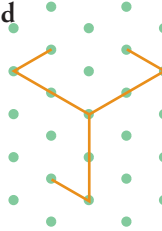
b



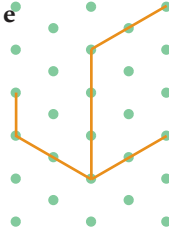
c



d



e



5 Draw a triangular prism:

a on square dot paper

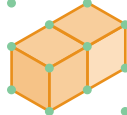
b on isometric dot paper.

6 Use cubes to make each solid, then draw them on isometric dot paper.

a



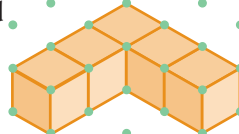
b



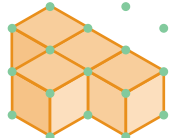
c



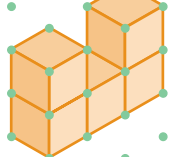
d



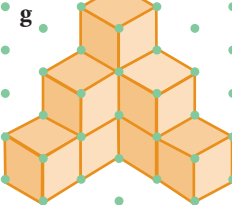
e



f

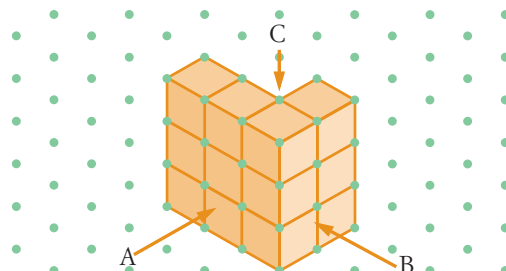


g

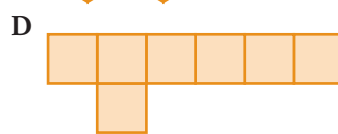
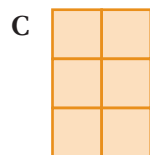
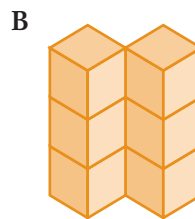
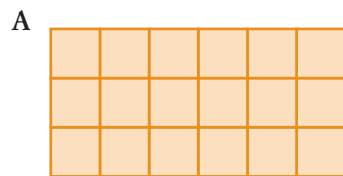
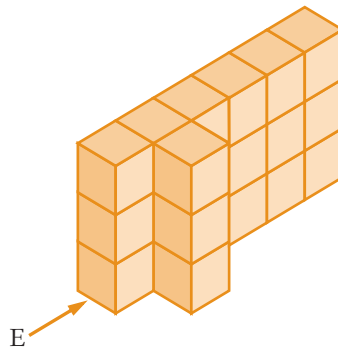


7 Draw what you would see of this prism if you were looking from A, then from B, and then from C.

See Example 12



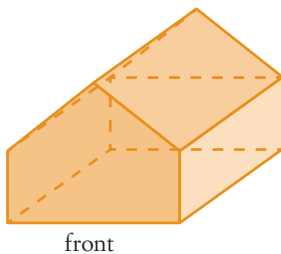
- 8 What would Pooja see if she were looking at this prism from point E? Select the correct answer A, B, C or D.



See Example 13

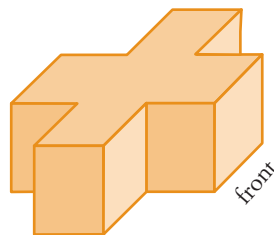
- 9 For each prism, draw each view requested.

a



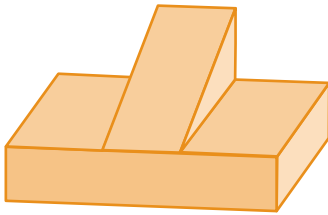
- i front view
- ii right view

b



- i left view
- ii top view
- iii front view

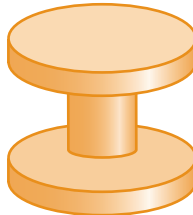
c



front

- i front view
- ii left view
- iii right view
- iv top view

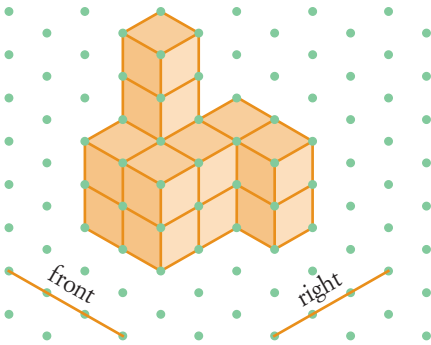
d



front

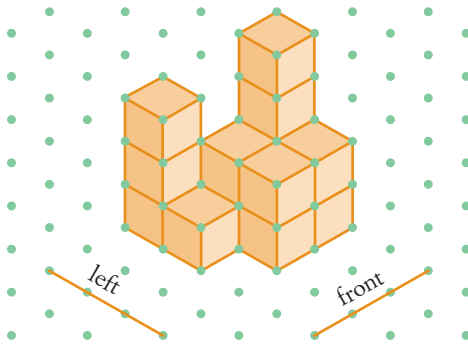
- i top view
- ii front view

e



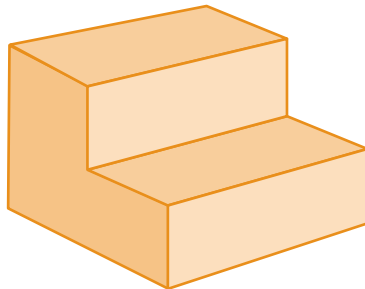
- i front view
- ii right view
- iii top view

f



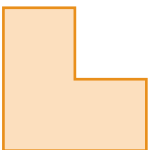
- i back view
- ii left view
- iii top view

10 Damien flew over the top of this building.

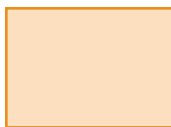


Which view would he see when directly overhead? Select A, B, C or D.

A



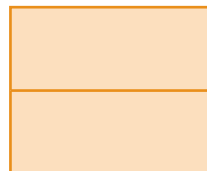
B



C



D



Mental skills 8B Maths without calculators

Multiplying and dividing by 4 or 8

Multiplying and dividing by 4 or 8 involves repeated doubling or halving.

1 Study each example.

a	65×4	<i>Think:</i>	Double twice Double $65 = 130$ Double $130 = 260$
b	27×4	<i>Think:</i>	Double twice. Double $27 = 54$ Double $54 = 108$
c	14×8	<i>Think:</i>	Double three times. Double $14 = 28$ Double $28 = 56$ Double $56 = 112$
d	$236 \div 4$	<i>Think:</i>	Halve twice. $\frac{1}{2} \times 236 = 118$ $\frac{1}{2} \times 118 = 59$
e	$564 \div 4$	<i>Think:</i>	Halve twice. $\frac{1}{2} \times 564 = 282$ $\frac{1}{2} \times 282 = 141$
f	$392 \div 8$	<i>Think:</i>	Halve three times $\frac{1}{2} \times 392 = 196$ $\frac{1}{2} \times 196 = 98$ $\frac{1}{2} \times 98 = 49$

2 Now evaluate each expression.

a	14×4	b	27×4	c	16×4	d	105×4
e	43×8	f	16×8	g	28×8	h	33×8
i	$184 \div 4$	j	$272 \div 4$	k	$560 \div 4$	l	$432 \div 4$
m	$624 \div 8$	n	$312 \div 8$	o	$256 \div 8$	p	$152 \div 8$

8-09 Metric units for volume

YEAR 8

Summary

The **volume** of a solid is the amount of space occupied by the solid.

Skillsheet

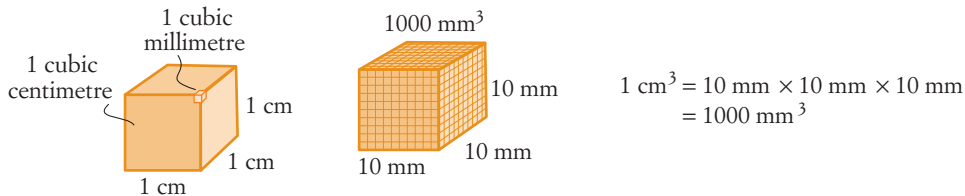
What is volume?

MAT07MGSS10029

The table shows metric units of volume.

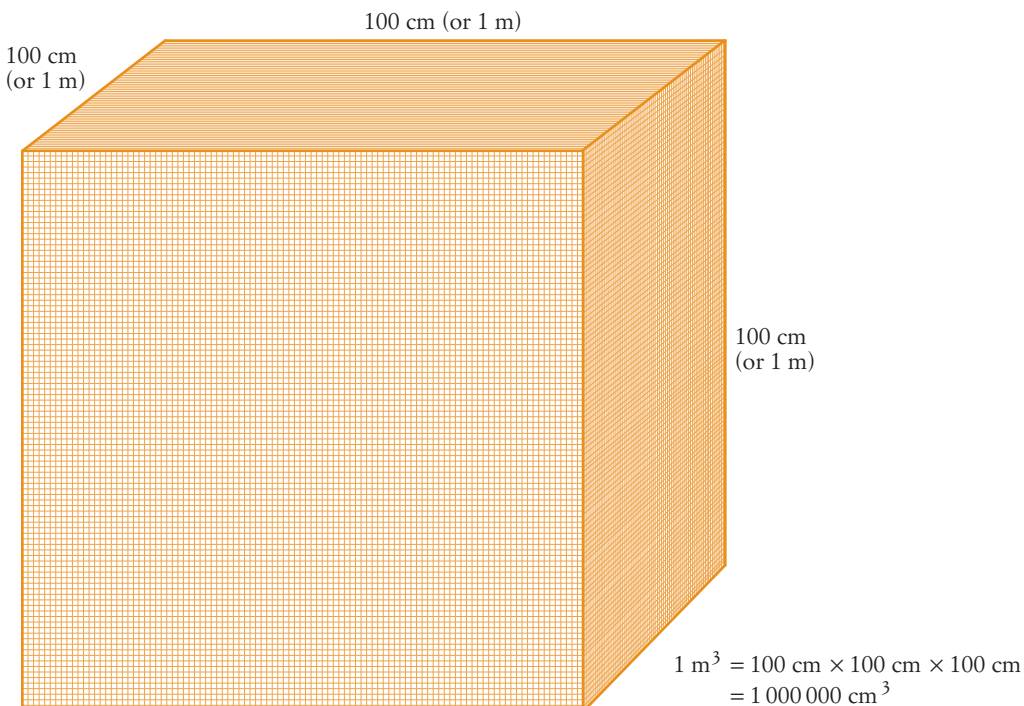
Volume unit	Abbreviation	The size of a cube with each side measuring ...
cubic millimetre	mm ³	1 mm
cubic centimetre	cm ³	1 cm
cubic metre	m ³	1 m

A **cubic millimetre** is about the size of a grain of raw sugar or rock salt. A **cubic centimetre** is about the size of a person's tooth or a medical pill.



There are 1000 cubic millimetres in one cubic centimetre.

A **cubic metre** is about the size of two washing machines. There are 1 000 000 cubic centimetres in one cubic metre.



TLF learning object

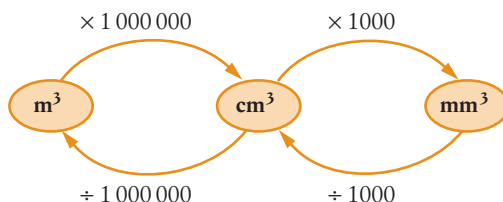
Inside a cubic metre (L164)

Summary

$$1 \text{ cm}^3 = 1000 \text{ mm}^3$$

$$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$

This diagram shows how to convert between different units of volume.



Example 14

Convert:

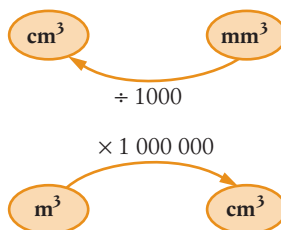
a $12\,000 \text{ mm}^3$ to cm^3

b 48 m^3 to cm^3

Solution

a $12\,000 \text{ mm}^3 = (12\,000 \div 1\,000) \text{ cm}^3$
 $= 12 \text{ cm}^3$

b $48 \text{ m}^3 = (48 \times 1\,000\,000) \text{ cm}^3$
 $= 48\,000\,000 \text{ cm}^3$



Exercise 8-09 Metric units for volume

- What unit of volume would you use when measuring the volume of:
 - a textbook?
 - a backpack?
 - the carton for a large TV?
 - a large suitcase?
 - a matchbox?
 - a room?
- What is the approximate volume of a brick? Select the correct answer **A**, **B**, **C** or **D**.
A 1000 cm^2 **B** 20 cm^2 **C** 1600 cm^3 **D** 2100 cm^3

See Example 14 3 Copy and complete the following.

a $3 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$	b $5 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$	c $2.6 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$
d $4000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$	e $7.2 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$	f $66\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
g $1 \text{ m}^3 = \underline{\hspace{2cm}} \text{ mm}^3$	h $2300 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$	i $126\,000\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
j $3450 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$	k $25 \text{ m}^3 = \underline{\hspace{2cm}} \text{ mm}^3$	l $78\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
m $63\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$	n $1.4 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$	o $5000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$

- 4 The volume of a chest of drawers is $306\,000\text{ cm}^3$. Convert this to cubic metres.
- 5 Match the correct volume (A to G) with each of the items (a to g) listed.

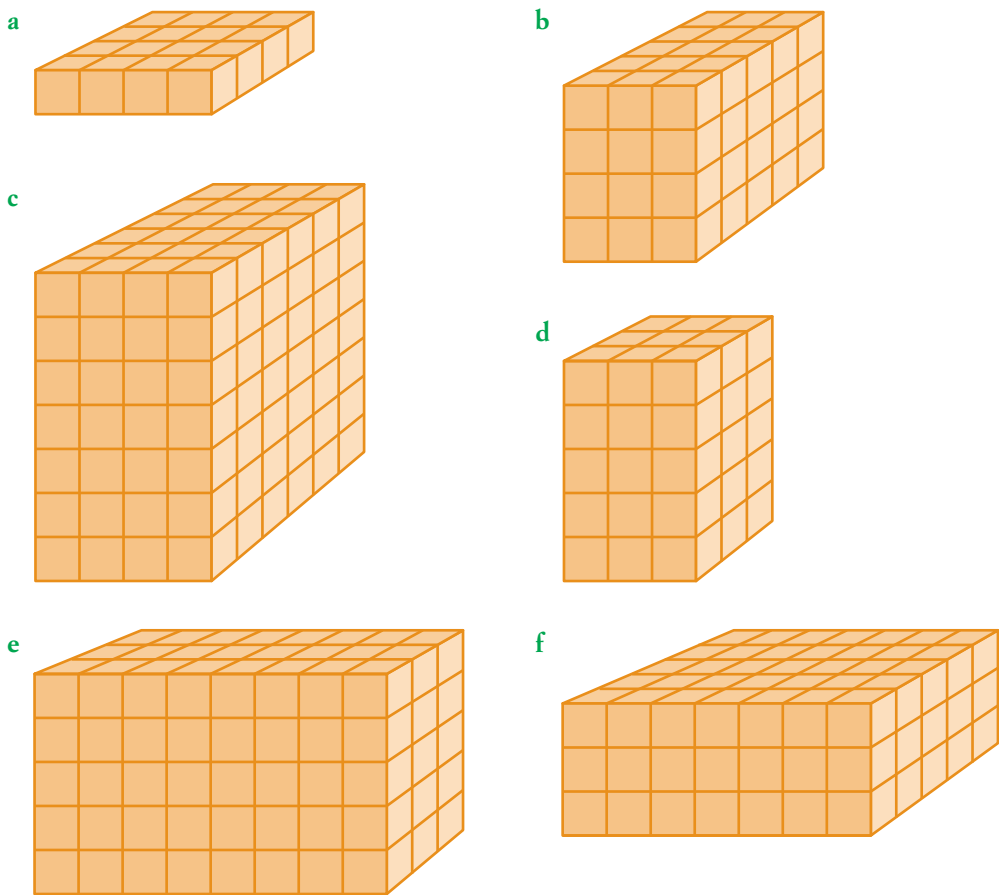


a a bottle of liquid paper	A 200 m^3
b a box of tissues	B 3890 m^3
c a glass of water	C 1250 cm^3
d a bottle of lemonade	D 5000 cm^3
e a classroom	E $20\,000\text{ mm}^3$
f a school hall	F 250 cm^3
g a cereal package	G 2200 cm^3
- 6 The volume of a lunchbox is 2520 cm^3 . Convert this to cubic millimetres.
- 7 A swimming pool has a volume of 38 m^3 . Convert this to cubic centimetres.

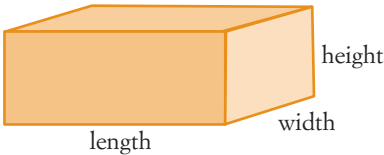
Investigation: Volume of a rectangular prism

- 1 The rectangular prisms on the next page are made up of 1 cm cubes. Copy and complete this table.

Shape	Number of cubes in one layer	Number of layers	Volume (cm^3)
a			
b			
c			
d			
e			
f			



2 Copy and complete this table for the rectangular prisms in question 1.



Shape	Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)
a	4	4	1	16
b				
c				
d				
e				
f				

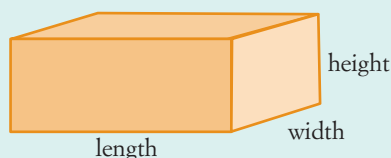
- 3 What is the relationship between the length, width and height of a rectangular prism and its volume?
- 4 Write the relationship as a rule: Volume of a rectangular prism = ____ × ____ × ____

8-10 Volume of a rectangular prism

Summary

Volume = length $\times$ width $\times$ height

$$V = lwh$$



Worksheet

Volume

MAT07MGWK10065

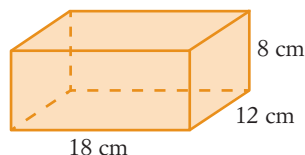
TLF learning object

Measures: Volumes
(L2316)

Note: This formula is sometimes written as $V = lbh$, where b = breadth, another name for width.

Example 15

Find the volume of the rectangular prism.



Solution

$$V = lwh$$

$$= 18 \times 12 \times 8$$

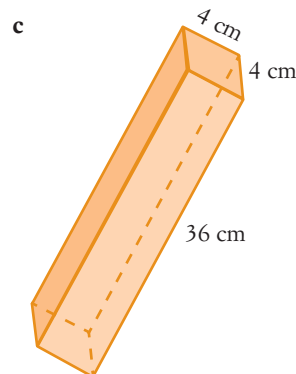
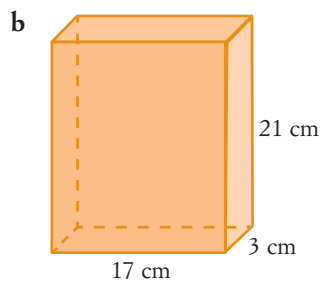
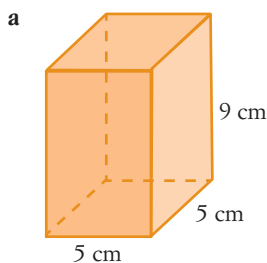
$$= 1728$$

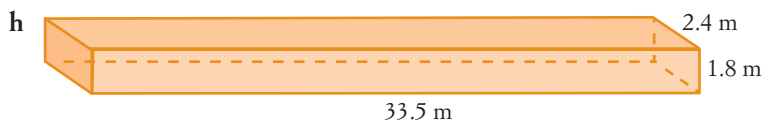
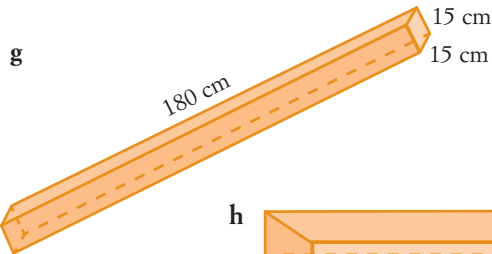
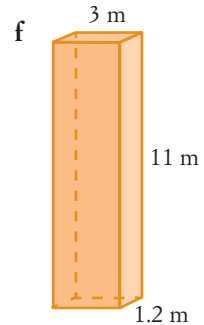
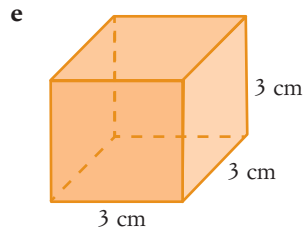
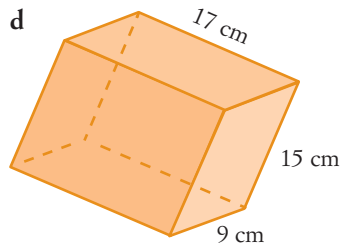
The volume is 1728 cm^3 .

Exercise 8-10 Volume of a rectangular prism

1 Find the volume of each prism.

See Example 15





2 Calculate the volume of each rectangular prism, given its dimensions shown in this table.

	Length	Width	Height
a	50 cm	50 cm	50 cm
b	5 cm	10 cm	18 cm
c	4 m	2.5 m	1.4 m
d	24 mm	16 mm	11 mm

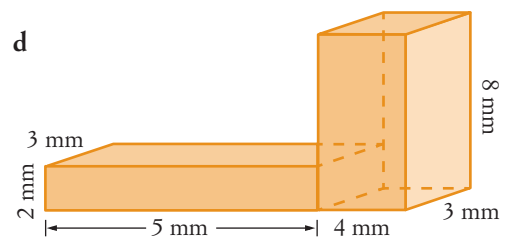
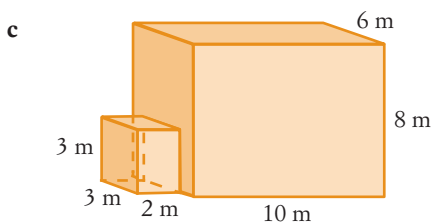
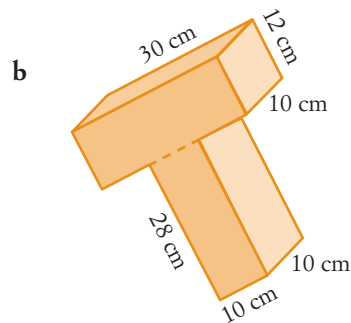
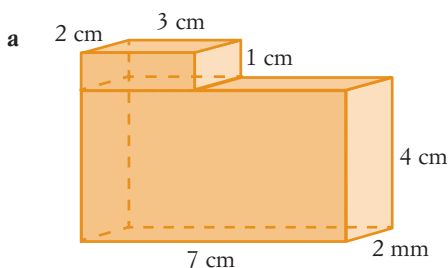
3 A rectangular prism has length 5 mm, width 3 mm and volume 100 mm^3 . What is its height?

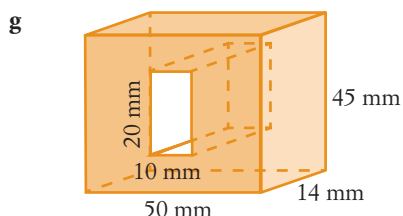
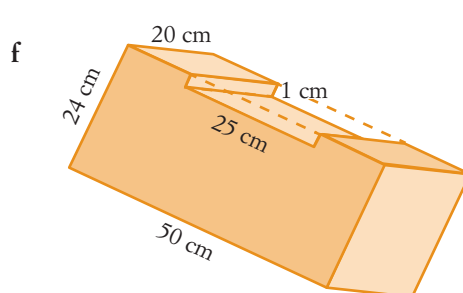
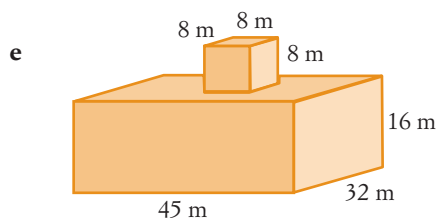
4 A tissue box has dimensions of 21 cm by 9 cm by 12 cm. Calculate its volume:

a in cubic centimetres

b in cubic millimetres

5 Find the volume of each composite prism.





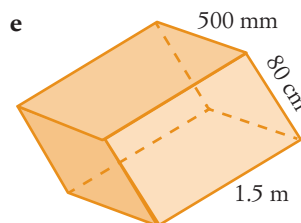
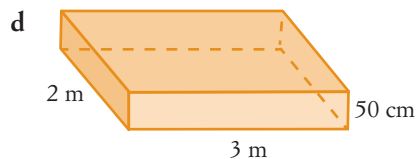
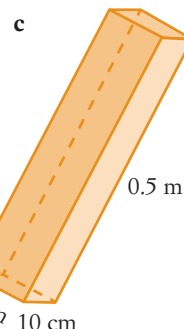
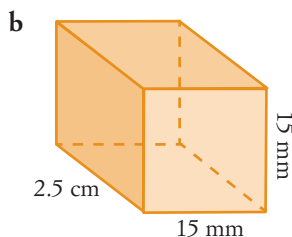
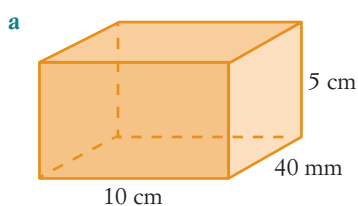
- 6 A refrigerator has dimensions of 68 cm by 166 cm by 65 cm. Calculate its volume:

a in cubic centimetres

b in cubic metres

- 7 A rectangular prism has width 1.5 m, height 3 m and volume 27 m^3 . What is its length?

- 8 Find the volume of each rectangular prism in cubic centimetres.



Worked solutions

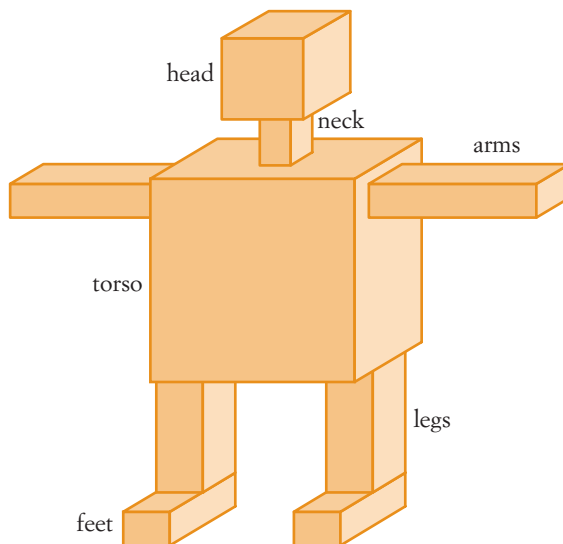
Exercise 8-10

MAT07MGWS10053

- 9 A rectangular prism has a volume of 880 cm^3 . What could its length, width and height be?
- 10 A rectangular garden is 12 m long and 4 m high. It is filled with soil to a depth of 15 cm. Calculate the volume of the soil. Select the correct answer **A**, **B**, **C** or **D**.
A 0.72 m^2 **B** 7.2 m^2 **C** 72 m^2 **D** 720 m^2
- 11 A cube has a volume of 125 m^3 . What is its side length?

Investigation: What is your volume?

Imagine that you are made up of rectangular prisms.



- 1 With the help of a partner, make measurements of your body. Use them to find dimensions (to the nearest centimetre) for each of the prism body parts.
- 2 Sketch each body part prism and label its dimensions.
- 3 Use the prisms to find your volume, in cm^3 .
- 4 Write a report of what you did, showing all diagrams and calculations. Explain how you found the dimensions (length, width and height) for the prisms. Do you believe that you found a good approximation of your volume? Why?

8-11 Volume and capacity

Worksheet

Volume and capacity cards

MAT07MGWK10066

‘What is the capacity of the water tank?’

Capacity is the amount of fluid (liquid or gas) in a container.

The metric units of capacity are the litre (L), the millilitre (mL), the kilolitre (kL) and the megalitre (ML).

Worksheet

Volume and capacity

MAT07MGWK10067

- A large drop of water is about 1 mL
- A teaspoon holds about 5 mL
- A tall standard carton of milk holds 1 L
- A small rainwater tank holds about 1 kL
- It takes 40 minutes to pump 1 kL of water out of a garden hose
- Half an Olympic-sized swimming pool holds about 1 ML

Homework sheet

Volume and capacity

MAT07MGHS10026



Summary

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1000 kL = 1 000 000 L

Puzzle sheet

Capacity puzzle

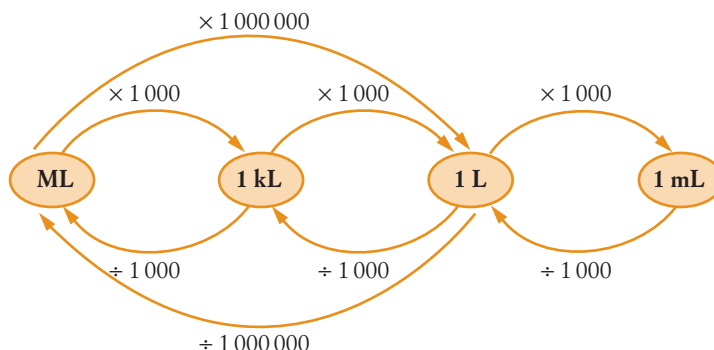
MAT07MGPS00048

Worksheet

Capacity

MAT07MGWK00078

This diagram shows how to convert between different units of capacity.

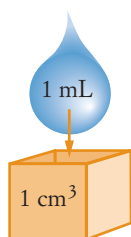


It is also useful to know the relationship between **volume** and **capacity**.

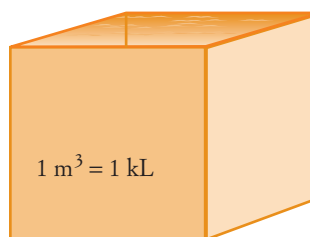
Summary

1 cm³ contains 1 mL
1 m³ contains 1 kL or 1000 L

This means that a cubic centimetre can hold 1 mL of liquid, while a cubic metre can hold 1000 L of liquid.



$\times 1\,000\,000 =$



Exercise 8-11 Volume and capacity

1 State what unit of capacity you would use when measuring:

a a glass of milk	b a dam	c a car's petrol tank
d a bottle of medicine	e an office water cooler	f a backyard pool

2 Copy and complete the following.

a 7000 mL = ____ L	b 2.7 kL = ____ L	c 3.5 L = ____ mL
d 10 000 L = ____ kL	e 5200 kL = ____ ML	f 1.4 ML = ____ L

Extra questions

Volume and capacity

MAT07MGEQ00048

g 420 L = _____ kL

h 8.9 L = _____ mL

i 6.2 ML = _____ kL

j 1750 mL = _____ L

k 5 kL = _____ L

l 9000 kL = _____ ML

m 2500 kL = _____ L

n 520 L = _____ kL

o 23 mL = _____ L

- 3 What is the capacity of a regular bottle of cough medicine? Select the closest answer **A**, **B**, **C** or **D**.

A 200 mL

B 500 mL

C 1500 mL

D 2000 mL

- 4 Match the correct capacity (**A** to **J**) with the items (**a** to **j**) listed.

a car petrol tank

A 200 mL

b a cup of flour

B 23 kL

c bath tub

C 5 mL

d bucket of water

D 70 L

e can of drink

E 1250 mL

f glass of water

F 1.875 ML

g Olympic swimming pool

G 250 mL

h bottle of lemonade

H 9 L

i teaspoon

I 375 mL

j water storage tank

J 180 L

- 5 A jug holds 2 L of water. How many 250 mL glasses could be filled from it?

- 6 James is inviting 30 friends to a party. He calculates that each person will drink 1800 mL of soft drink.

a How many litres of soft drink must he buy?

b If James intends to buy large 2 L bottles of drink, how many bottles must he buy?

- 7 Find out the name of the metric unit of capacity that is equal to 1000 megalitres.

- 8 A tap leaks 10 mL of water every 50 seconds. How much water will be lost in:

a 1 second?

b 1 minute?

c 3 hours?

d 1 day?

- 9 Your skin releases moisture as a way of controlling body temperature. On average 200 mL is released per hour. If all this moisture was captured, how long would it take to fill a 1.25 L soft drink bottle?

- 10 A lunch box is made in the shape of a rectangular prism. Its dimensions are 20 cm, 15 cm and 9 cm.

a Find the volume of the lunch box, in cm^3 .

b How many mL of water would fit in the lunch box?

- 11 Gina's swimming pool is a rectangular prism 8 m long, 4 m wide and 1.5 m deep.

a Find the volume of the swimming pool.

b How many litres of water would be needed to fill the pool? (Hint: 1 m^3 holds 1 kL)

- 12 A fish tank is a rectangular prism 60 cm long, 40 cm high and 30 cm wide.

a Find the volume of the tank.

b How many litres of water will it hold?

Worked solutions

Exercise 8-11

MAT07MGWS10054

Just for the record

Water, water, everywhere

To help you better understand the size of a litre and a kilolitre, here are some examples of water use in and around the home:



• Washing your hands/face	5 L
• Brushing your teeth (tap running)	5 L
• Brushing your teeth (tap not running)	1 L
• Cooking and making coffee/tea	8 L per day
• Flushing the toilet	9 L to 13 L
• Flushing the toilet (half flush)	4.5 L to 6 L
• Household tap	18 L per minute
• Washing the dishes (hand)	18 L
• Washing the dishes (dishwasher)	25 L per cycle
• Bath	85 L to 150 L
• Shower (8 minutes)	80 L to 120 L
• Washing machine (front loading)	120 L per cycle
• Washing machine (top loading)	180 L per cycle
• Washing the car (with hose)	100 L to 300 L
• Garden sprinkler	1 kL to 1.5 kL per hour
• Garden hose	1.8 kL per hour
• Swimming pool (backyard)	20 kL to 55 kL
• Campbelltown swimming pool (Olympic 50 m)	1870 kL

On average, a four-person Sydney house (with garden) uses 936 litres of water per day. Half of it is used by outside taps or is flushed in a toilet.

How much water does your household use each day? Find out by asking your parents to show you the water bill.

Technology Brogo and Windamere dams

Brogo Dam is situated near Bega on the south coast of NSW. Windamere Dam is between Dubbo and Newcastle. In this activity, we will examine the volume of water in the dams for each month of 2010.

- 1 Start a new spreadsheet and enter only the labels and values in rows 1 and 3.
(Note: ML = Megalitres.)

	A	B	C	D	E	F	G	H	I
1	Brogo Dam	Capacity=	8989 ML			Windamere Dam	Capacity =	368120 ML	
2									
3	Date/Year	Volume (ML)	% Capacity			Date/Year	Volume (ML)	% Capacity	
4	1-Jan-10	1440				1-Jan-10	71607		
5	1-Feb-10	1068				1-Feb-10	69253		
6	1-Mar-10	9076				1-Mar-10	68456		
7	1-Apr-10	9018				1-Apr-10	67161		
8	1-May-10	8992				1-May-10	67164		
9	1-Jun-10	10258				1-Jun-10	67234		
10	1-Jul-10	9055				1-Jul-10	67738		
11	1-Aug-10	9032				1-Aug-10	71906		
12	1-Sep-10	9001				1-Sep-10	85247		
13	1-Oct-10	8898				1-Oct-10	98282		
14	1-Nov-10	8990				1-Nov-10	102291		
15	1-Dec-10	9157				1-Dec-10	110630		
16	Source: www.waterinfo.nsw.gov.au								

- 2 Enter **1 Jan 2010** in cell A4 and **1 Feb 2010** in cell A5. **Highlight** cells A4 and A5, right-click and **Fill Down** until you finish at **1 Dec 2010**.
- 3 Repeat (or copy) this for column F, cells F4 to F15.
- 4 Enter the volume amounts in columns B and G.
- 5 'Capacity' refers to the total capacity of a dam. '% Capacity' calculates the volume as a percentage of the total capacity. To find the percentage capacity on 1 January 2010, enter **=B4/\$C\$1** in cell C4 and click %.
- 6 **Fill Down** the formula from cell C4 to C15. Notice that, by using an absolute cell reference (\$C\$1), each volume is divided by the same number (the dam's total capacity).

	A	B	C	D
1	Brogo Dam	Capacity=	8989 ML	
2				
3	Date/Year	Volume (ML)	% Capacity	
4	1-Jan-10	1440	16%	
5	1-Feb-10	1068	12%	
6	1-Mar-10	9076	101%	
7	1-Apr-10	9018	100%	

- 7 Repeat the steps above in cells H4 to H15 to calculate Windamere Dam's percentage capacity.
- 8 In cell A18, explain (in one sentence) why it is inappropriate to graph the % capacity for both dams on a single graph.
- 9 State the driest month for:
 - a Brogo Dam (answer in cell A19)
 - b Windamere Dam (answer in B19).

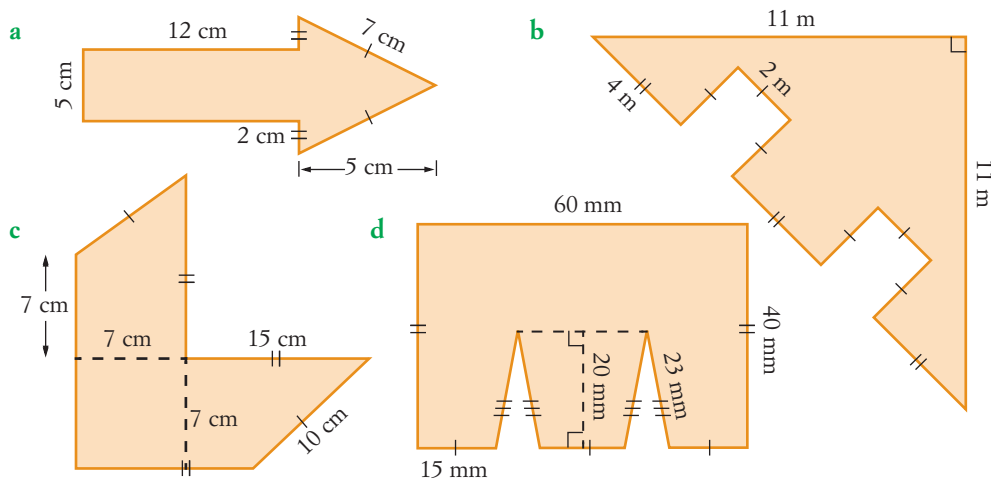
- 10 State the wettest month for:

a Brogo Dam (A20)

b Windamere Dam (B20).
- 11 Use the '% Capacity' column to find the month which shows the lowest dam level for each dam. Answer in cell A21 (Brogo) and B21 (Windamere).
- 12 Describe what happened at Brogo Dam from February to March. (A22)
- 13 What do the % capacity values from March to December show? (A23)
- 14 Do you think this data can be used to find out which area is more suitable for the development of a new town in the region? Begin your answer in cell A24. Give reasons to support your answer.
- 15 Which dam seems to be located in an area more likely to receive rain? Give reasons to support your answer. Answer in cell A25.

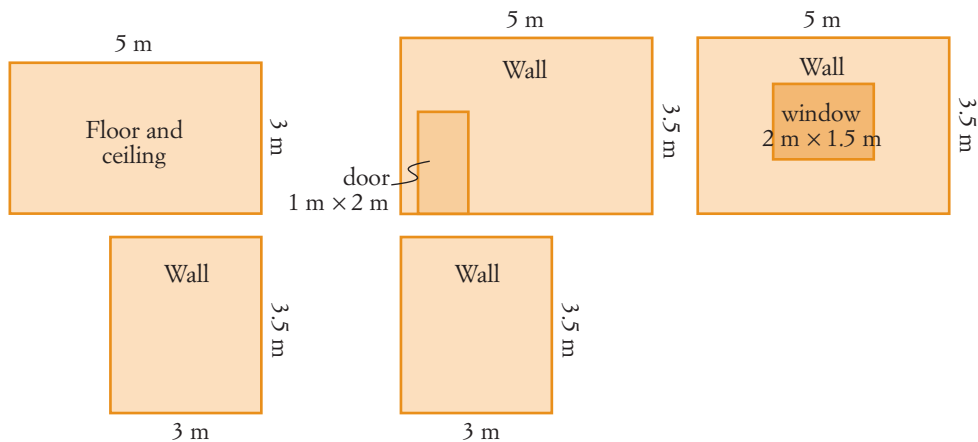
Power plus

- 1** Find the perimeter and area of each shape.

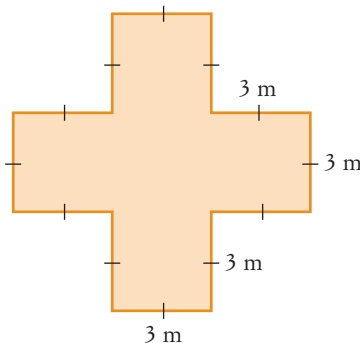


- 2 A rectangle has a perimeter of 30 cm.
 - a List some possible dimensions for this rectangle.
 - b For each pair of dimensions, calculate the area of the rectangle.
 - c Which dimensions give the greatest area?
- 3 Find the length and width of a rectangle whose perimeter value is equal to its area value.

- 4 A room has dimensions as shown.



- The four walls and the ceiling are to be painted (not including the door and window). Calculate how many square metres are to be painted
 - Each surface requires two coats of paint, and 1 L of paint covers approximately 12 m^2 . How many litres of paint are needed?
 - Paint comes in 4 L cans that cost \$45.95. Calculate how many 4 L cans are needed and calculate the cost of the paint.
 - The floor is to be carpeted. How many square metres of carpet are needed?
- 5 A children's pool is in the shape of a cross as shown on the right. Each side is 3 m long. The pool is filled with water to a depth of 300 mm.



- Find the area of the pool's surface.
- Calculate the volume of water in cubic metres (m^3).
- If water is charged for at \$0.80 per kL, how much does it cost to fill the pool?

■ Language of maths

area	cubic	Mega-	perpendicular height
breadth	height	metric	prefix
capacity	isometric	micro-	prism
centi-	kilo-	milli-	square
composite shape	length	parallelogram	volume
cross-section	mass	perimeter	width

MAT07MGPS10033

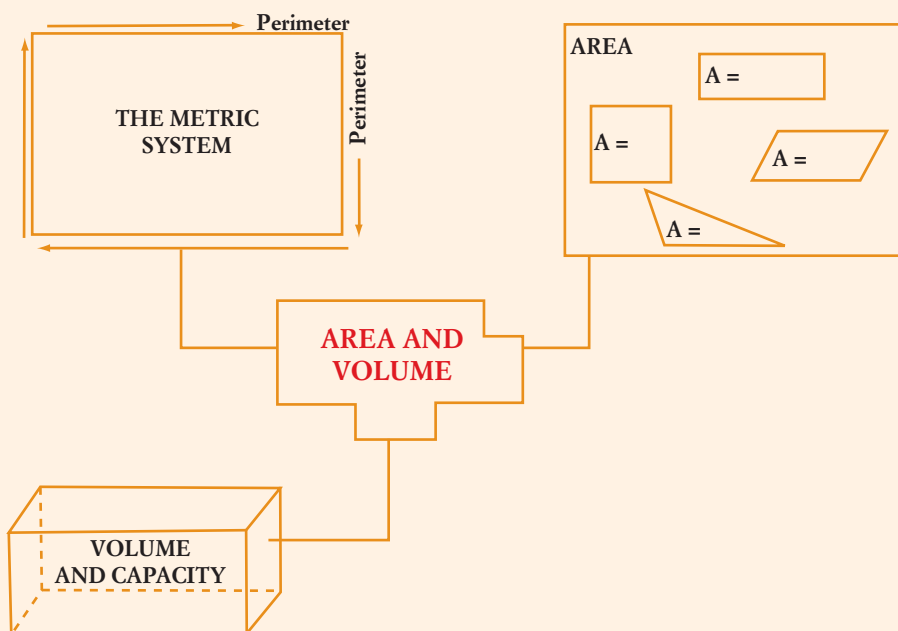
- 1 Which Latin prefix means:
a one thousandth? **b** one thousand?
- 2 What is another name for **width**?
- 3 In your own words, explain how you would calculate the perimeter of a square.
- 4 What does **perpendicular height** mean?
- 5 Complete: A prism has a uniform _____.
- 6 What is the difference between volume and capacity?

■ Topic overview

- What have you learnt about measurement?
- What types of people use area as part of their jobs?
- List at least two measuring situations in which accuracy is important.
- Is there anything that you did not understand about the topic? Ask a friend or your teacher for help.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

MAT07MGWK10068



Chapter 8 revision

See Exercise 8-01

1 Convert:

a 240 min to h

d 3200 m to km

g 3.5 kg to t

j 7150 mm to m

b 0.042 kg to g

e 8.3 L to mL

h 15 h to min

k 4.2 kL to mL

c 125 L to kL

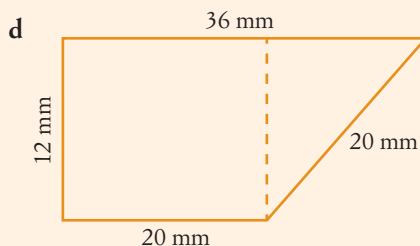
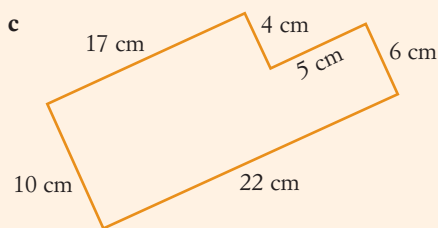
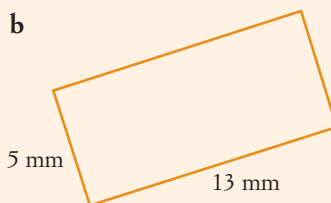
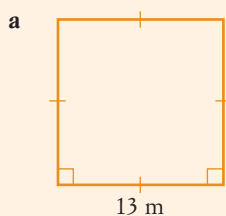
f 904 cm to mm

i 15 h to s

l 17 000 000 mg to kg

See Exercise 8-02

2 Find the perimeter of each shape.



See Exercise 8-03

3 How many:

a cm^2 in 9.1 m^2 ?

d m^2 in $240\,000 \text{ mm}^2$?

b mm^2 in 2.5 cm^2 ?

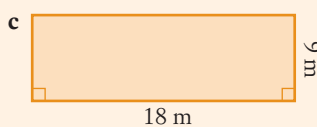
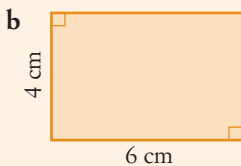
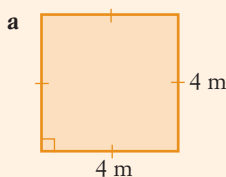
e cm^2 in 240 mm^2 ?

c m^2 in $175\,000 \text{ cm}^2$?

f mm^2 in 8.6 m^2 ?

See Exercise 8-04

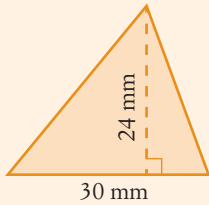
4 Find the area of each shape.



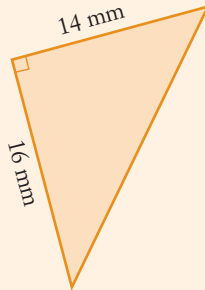
5 Find the area of each shape.

See Exercise 8-05

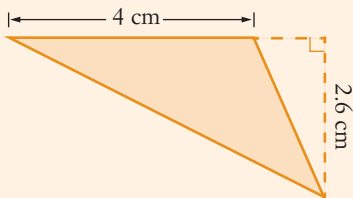
a



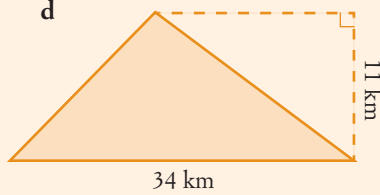
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c



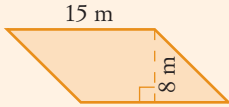
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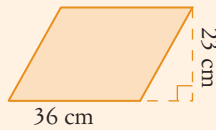
6 Find the area of each parallelogram.

See Exercise 8-06

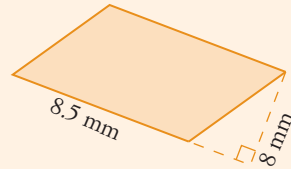
a



b



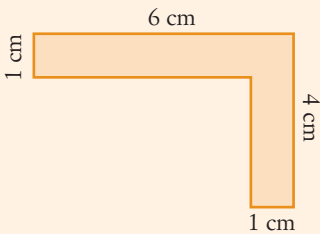
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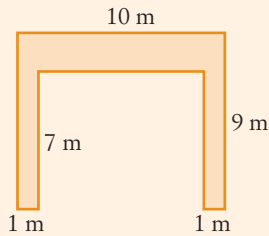
7 Find the shaded area in each shape.

See Exercise 8-07

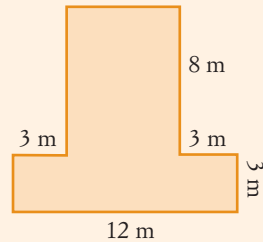
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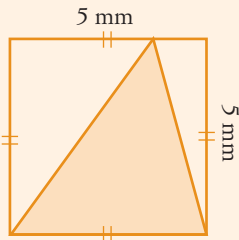
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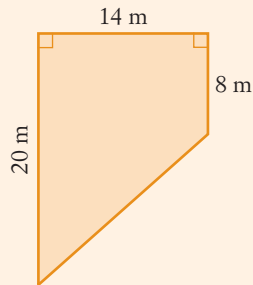
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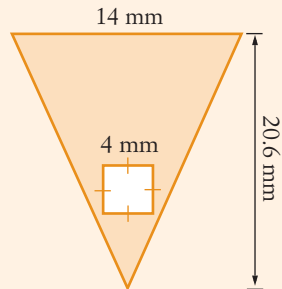
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e



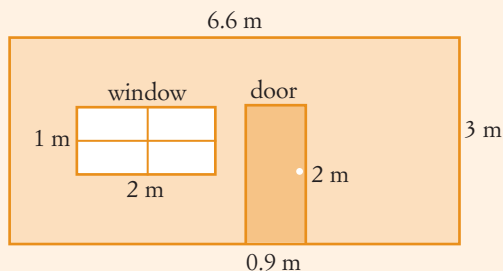
f



Chapter 8 revision

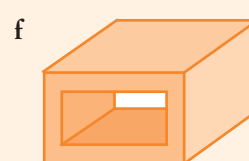
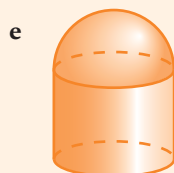
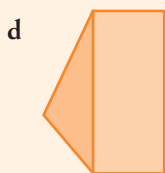
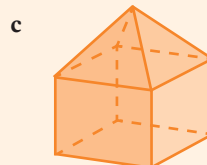
See Exercise 8-07

- 8 The wall shown in the diagram needs painting. If one litre of paint covers 3 m^2 of wall, how many whole litres should be bought?



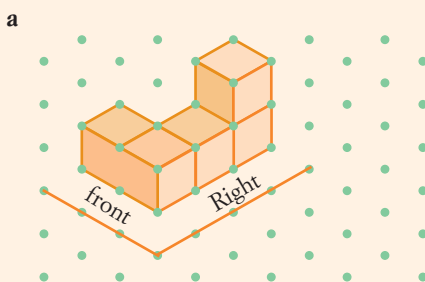
See Exercise 8-08

- 9 Draw a cross-section of each solid and state whether it is a prism.

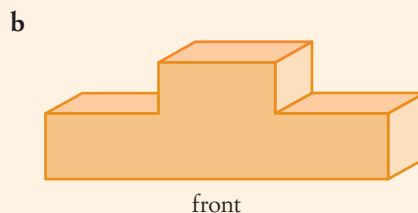


See Exercise 8-08

- 10 For each solid, sketch each view requested.



- i front view
ii left view
iii top view



- i right view
ii front view
iii top view

See Exercise 8-09

- 11 Copy and complete the following.

a $20 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$

c $7500 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$

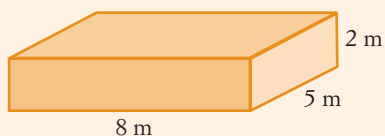
b $0.5 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$

d $230\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ m}^3$

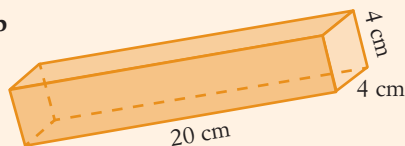
12 Find the volume of each prism.

See Exercise 8-10

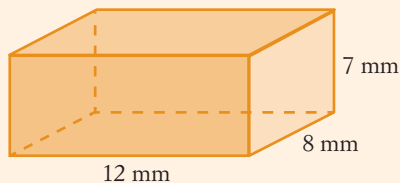
a



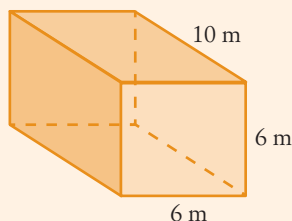
b



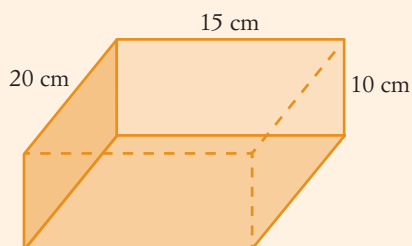
c



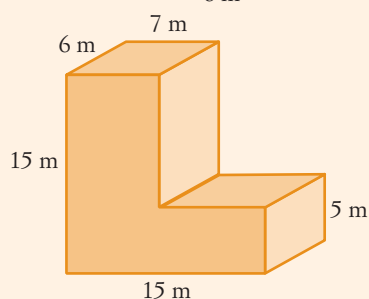
d



e



f



13 Find the capacity, in litres, of each prism in question 12.

See Exercise 8-11

14 The biggest iceberg on record was called B9. It had the same volume as a rectangular prism with dimensions 160 km long, 50 km wide and 250 metres high. When B9 melted, how many megalitres of water were contained in B9?

See Exercise 8-11





9

Number and algebra,
Measurement and geometry

The number plane

Any place on the Earth's surface can be described using two numbers called **coordinates**. For example, the position coordinates of Uluru (Ayers Rock) are 25° south, 131° east. Coordinates describe the location of an object or place and their use is important in navigation and surveying. When searching for a place on a street map or a shopping centre map, you usually use a letter and a number to find the correct square on the grid. In mathematics, coordinates are used to locate a point on a grid called a **number plane**.



Chapter outline

	Proficiency strands		
9-01 Location using coordinates	U	F	
9-02 The number plane	U	F	R
9-03 The number plane with negative numbers	U	F	R
9-04 Graphing tables of values	U	F	R
9-05 Transformations on the number plane	U	F	R

Wordbank

- coordinates** The two values that give the position of a point on a number plane
- number plane** A grid that is made up of two number lines, meeting at right angles
- ordered pair** Another name for coordinates
- origin** The centre point of a number plane, with coordinates (0,0)
- quadrant** One quarter of the number plane
- x-axis** The horizontal number line on the number plane
- y-axis** The vertical number line on the number plane

In this chapter you will:

- use a grid reference system to describe locations
- introduce the Cartesian coordinate system using all four quadrants
- given coordinates, plot points on the Cartesian plane, and find coordinates for a given point
- plot points from a table of integer values and recognise simple patterns, such as points that lie on a straight line
- describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates

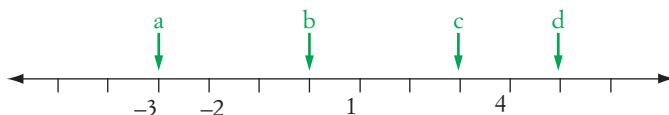
SkillCheck

Worksheet

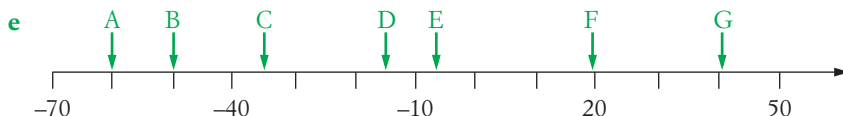
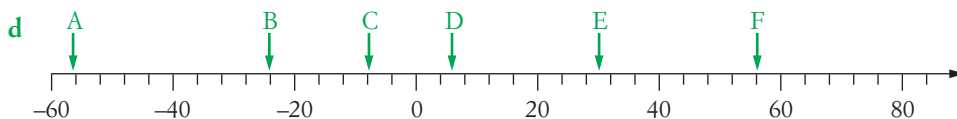
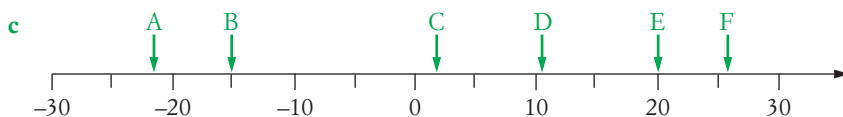
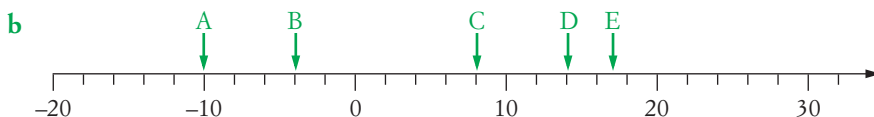
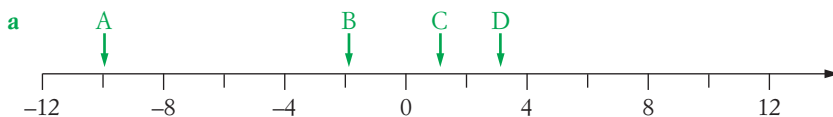
StartUp assignment 9

MAT07NAWK10069

- 1 Write the integer shown by each arrow on this number line.



- 2 Write the integer shown by each arrow on each number line

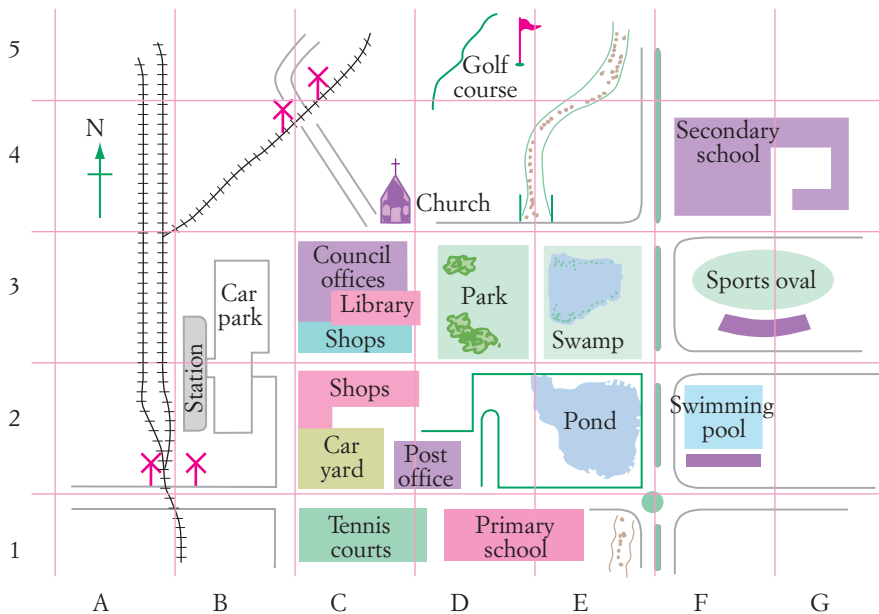


- 3 Draw a number line for each set of numbers listed or described and mark the numbers with dots:
- a $-2, 4, 8, -3, 0$
 - b the integers between -5 and 1
 - c the negative and positive integers between -10 and 4 that are divisible by 3
 - d $-1.5, 5.5, 2.5, -4.5, -2.5$

9-01 Location using coordinates

To locate a position on a map or a theatre seating plan, we usually use a grid system of two coordinates – one for the horizontal and one for the vertical. On a map this may be a letter and a number or two numbers; on a theatre plan it may be a row letter and a number.

This map of a country town uses a coordinate grid system where every position can be described by a column letter and row number. For example, the position of the golf course is D5, because it is in the square in column D, row 5.

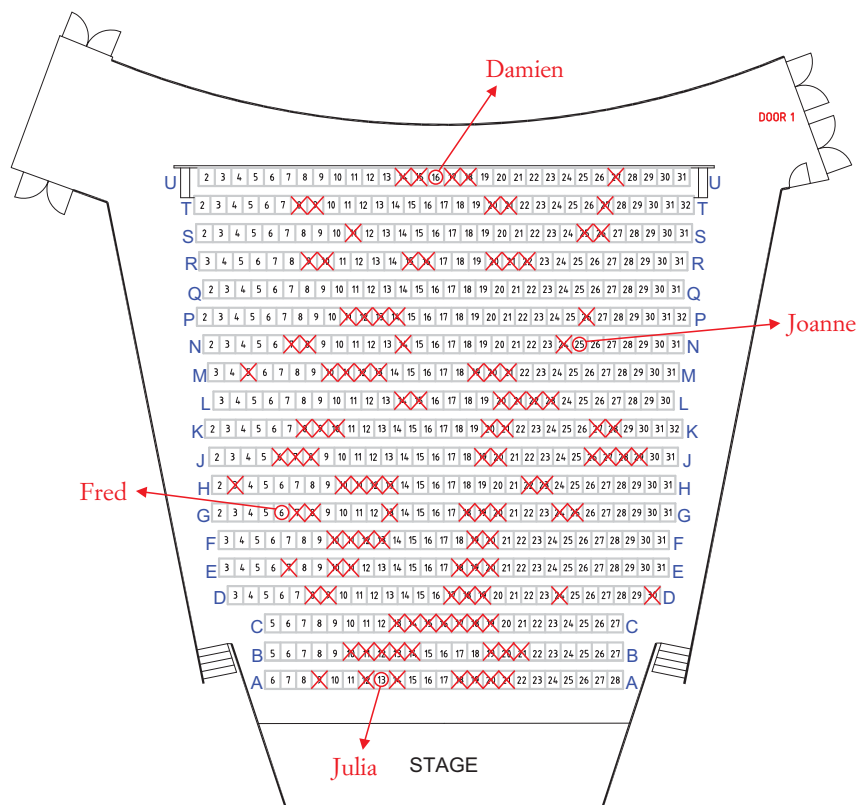


Exercise 9-01 Location using coordinates

- Use the above map to write the location reference for:

a the swamp	b the library	c the church
d the secondary school	e the sports oval	f the council offices
- In the above map, describe what can be found at:

a F2	b D1	c D3
d E2	e B3	f C1
- The map below shows the seating plan for the Drama Theatre at the Sydney Opera House. Each seat in the theatre has a row number and a position number, for example, seat A17 is in the middle of the front row.

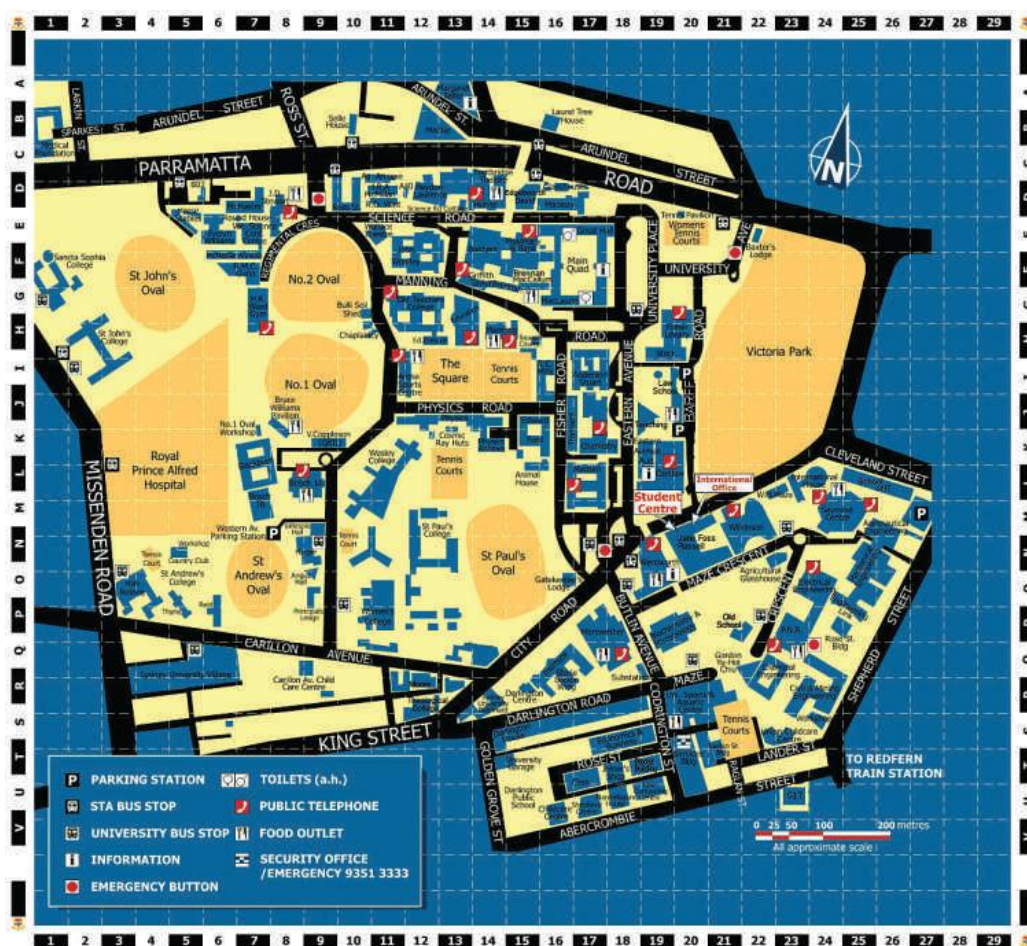


Write the seat location of each person.

a Julia	b Fred
c Joanne	d Damien
- How many rows are there in the Drama Theatre?
 - Are there more seats in the front rows or the back rows?
 - A seat marked with a cross has already been booked. State whether each seat has been booked:

a D4	b H15	c R11	d T9

- 7 Tara, Zoe and Asha want to sit together at the Drama Theatre shown on the opposite page. Give two sets of possible locations where they could sit.
- 8 What type of pattern would be seen if:
- all the people sitting in a seat with number 15 stood up?
 - all the people sitting in a seat with letter K stood up?
- 9 This is a map of the University of Sydney's main campus.



Write down the feature that can be found at:

- F4
 - I13
 - E20
 - F17
 - P19
 - F9
- 10 From the university map, write the location of:
- St Paul's Oval
 - Shepherd St car park
 - St John's College
 - Fisher Library

9-02 The number plane

Worksheet

Plane grid 1

MAT07NAWK10070

Puzzle sheet

Big top employee

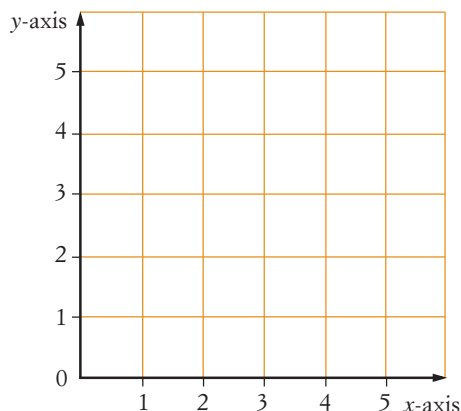
MAT07NAPS10034

Skillsheet

The number plane

MAT07NASS10030

A **number plane** is a grid made from a horizontal number line called the **x-axis**, and a vertical number line called the **y-axis**. Axis means ‘line’, and the plural of **axis** is ‘axes’.

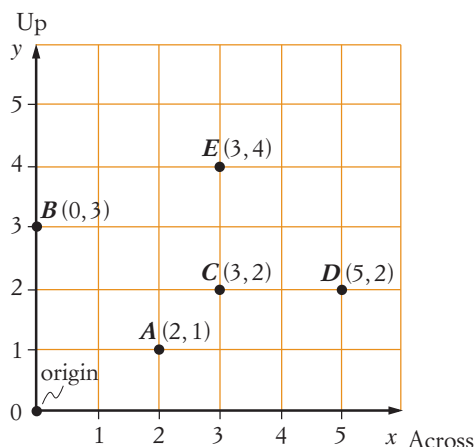


The number plane is also called a **Cartesian plane**, named after René Descartes (pronounced ‘Ren-ay Day-cart’), a French mathematician and philosopher who developed the idea.

Points on the number plane are located using a pair of coordinates, for example (3, 2) means 3 on the x-axis (‘3 across’) and 2 on the y-axis (‘2 up’).

A pair of coordinates is also called an **ordered pair**. On the number plane on the right:

- A is the point (2, 1)
- B is the point (0, 3).



Summary

For coordinates on a number plane, the first number is called the **x-coordinate** and the second number is called the **y-coordinate**.

The point (0, 0) is called the **origin**.

To locate a point, always start from (0, 0): the first coordinate tells you how far to move across, the second tells you how far to move up.

(2, 1) means (2 across, 1 up)

- start from (0, 0)
- move 2 units across
- then 1 unit up

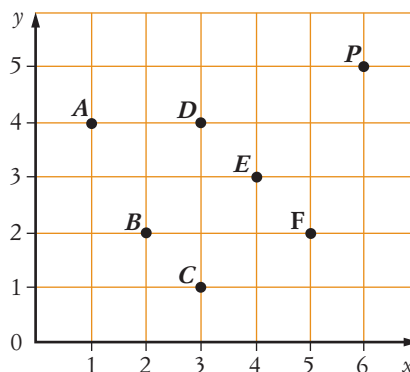
(0, 3) means (0 across, 3 up)

- start from (0, 0)
- do not move across
- move 3 units up

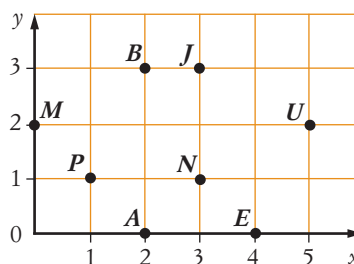
Note that order is important with coordinates: (0, 3) is not the same as (3, 0).

Exercise 9-02 The number plane

- 1 Write the coordinates of each point shown on this number plane.



- 2 Write the coordinates of the eight points shown on this number plane.

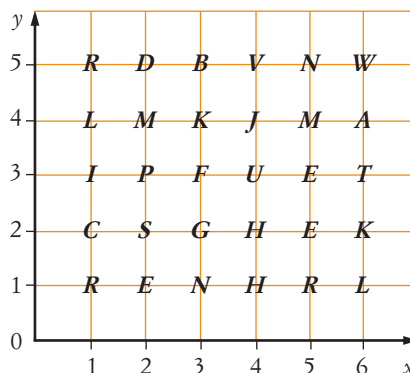


- 3 Draw a number plane with the numbers from 0 to 6 on each axis.

- Start at the point (1, 1) and draw a line joining it to (4, 1).
- Join (4, 1) to (6, 2), then continue through (3, 2) $\rightarrow$ (1, 1) $\rightarrow$ (1, 4) $\rightarrow$ (3, 5) $\rightarrow$ (6, 5) $\rightarrow$ (4, 4) and finish at (1, 4).
- Now join (3, 2) to (3, 5).
- Join (4, 1) to (4, 4).
- Join (6, 2) to (6, 5).
- What have you drawn?

- 4 Write the letter shown at each point on the number plane on the right, to spell some words.

- (3, 1), (4, 3), (2, 4), (3, 5), (5, 2), (1, 5)
- (2, 3), (6, 1), (6, 4), (5, 5), (2, 1)
- (6, 5), (6, 4), (6, 3), (5, 3), (5, 1)
- (2, 4), (5, 2), (6, 3), (5, 1), (5, 3)
- (3, 1), (4, 3), (5, 1), (2, 2), (2, 1)
- (4, 3), (2, 4), (2, 3), (1, 3), (1, 1), (2, 1)



Worksheet

1 cm grid paper

MAT07MGWK10108

Worksheet

1 cm grid paper

MAT07MGWK10108

- 5 a Draw a number plane. On the x -axis, mark 0 to 20. On the y -axis mark 0 to 15.
- b Plot the following points and connect them as you go. Colour appropriately.
- (0, 9), (8, 11), (10, 14), (15, 10), (16, 12), (17, 9), (19, 7), (19, 5), STOP
 - (19, 7), (15, 5), (11, 9), (13, 5), (11, 0), (15, 4), (19, 5), STOP
 - (17, 6), (13, 7), STOP
 - (12, 7), (10, 7), (11, 3), (7, 7), (3, 7), (5, 8), (2, 8), (0, 9), STOP
 - (3, 9), STOP
- c What did you draw?

Worksheet

Plane grid 1

MAT07NAWK10070

- 6 a Draw a number plane or use the link to print Plane grid 1. On the x -axis, mark 0 to 11. On the y -axis mark 0 to 16.
- b Plot the following points, joining them as you go.
- (1, 13), (3, 14), (4, 15), (5, 15), (6, 14), (6, 13), (4, 10), (7, 11), (9, 10), (10, 9), (11, 6), (11, 4), (8, 8), (9, 2), (9, 1), (7, 1), (8, 2), (7, 6), (6, 6), (7, 4), (7, 1), (5, 1), (6, 2), (6, 4), (5, 6), (3, 7), (2, 9), (2, 10), (4, 13), (1, 13)
- c Draw an eye at (4, 14).
- d What did you draw?

Just for the record

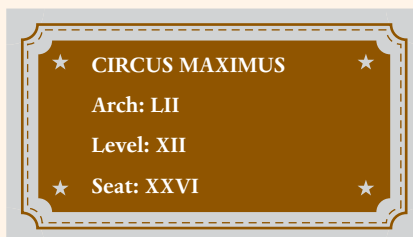
The Colosseum

The Colosseum or Flavian Amphitheatre in the centre of Rome, Italy, was built in 82 CE by the ancient Romans. It was used by Roman emperors for gladiatorial contests and public spectacles such as mock sea battles, animal hunts and re-enactments of famous battles. The amphitheatre was a large oval shape with tiers of seating for 50 000 spectators around a central arena. Below the wooden arena floor, there was a complex set of rooms and passageways for wild beasts.

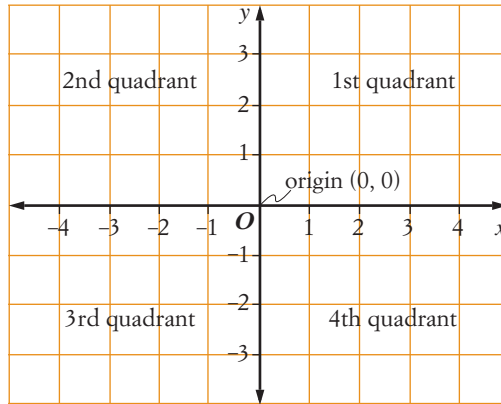


It was believed that an entire crowd could be seated in the Colosseum in 15 minutes, because the Romans invented numbered seating. Each admission ticket had a number that corresponded to one of the 79 entrance arches, a level number and a seat number. The seats were made of marble, numbered with lines inscribed on the marble. The basic structure of the Colosseum and its ticketing system is still used today for many sporting arenas.

Convert the Roman numerals on this ticket into modern numerals.



9-03 The number plane with negative numbers



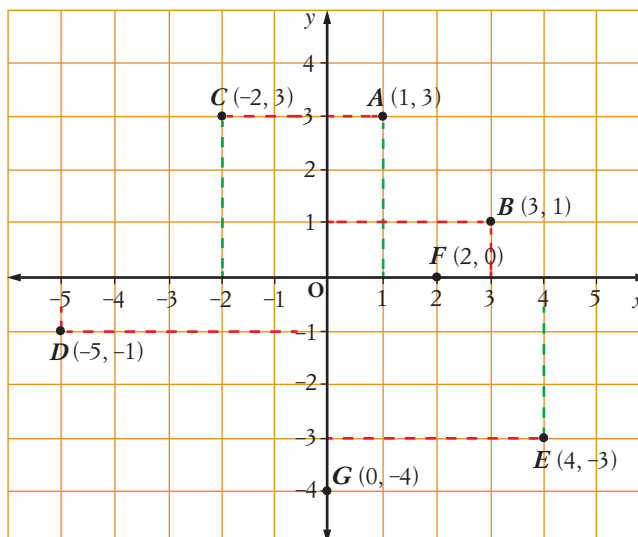
We can extend the number plane to use both positive and negative coordinates, by including negative numbers on the x - and y -axes.

The number plane is divided evenly into four regions called **quadrants**.

The centre point $(0, 0)$ is called the **origin**.



A few points are labelled on this number plane as examples.



Animated example

Graph coordinates

MAT07NAAE00019

Worksheet

Plane grid 2

MAT07NAWK10071

Worksheet

The number plane

MAT07NAWK10072

Worksheet

Number plane writing activity

MAT07NAWK10073

Worksheet

Number plane review

MAT07NAWK10074

Puzzle sheet

Girt by sea

MAT07NAPS10035

Puzzle sheet

Quadrilateral person

MAT07NAPS10036

Puzzle sheet

Coordinates code puzzle

MAT07NAPS10037

Skillsheet

The number plane

MAT07NASS10030

Video tutorial

The number plane

MAT07NAVT10019

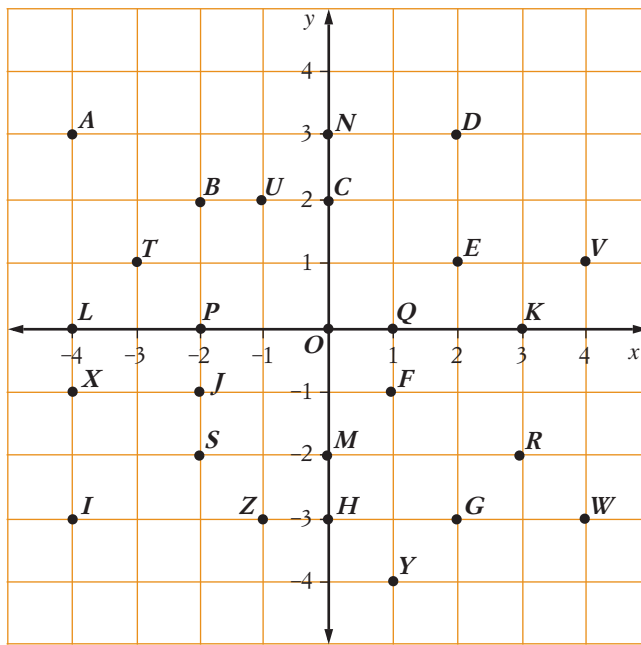
Homework sheet

The number plane 1

MAT07NAHS10020

Exercise 9-03 The number plane with negative numbers

- 1 Write the coordinates of each of the 26 points shown on the number plane.



- 2 a Draw a number plane with both axes extending from -6 to 6 .

- b Mark these points.

$A(3, 1)$

$B(-4, 3)$

$C(-3, 4)$

$D(-2, -2)$

$E(0, -2)$

$F(1, -5)$

$G(4, -4)$

$H(-3, 0)$

$I(-6, 0)$

$J(-1, -6)$

$K(-3, -4)$

$L(0, 5)$

$M(6, 2)$

$N(-3, 5)$

$P(5, -5)$

$Q(-1, 6)$

- 3 List all of the points from question 2 that are:

- a in the 1st quadrant

- b in the 2nd quadrant

- c in the 3rd quadrant

- d in the 4th quadrant

- e on the border of two quadrants

- 4 In which quadrant would you find each of the following points?

- a $(3, -5)$

- b $(-2, -4)$

- c $(-8, 1)$

- 5 Copy this table and complete it by placing a $+$ or a $-$ sign in each blank space.

Points in:	x-coordinate	y-coordinate
1st quadrant	+	
2nd quadrant	-	
3rd quadrant		
4th quadrant		

- 6 a Draw a number plane, with both axes extending from -3 to 3 , or access NelsonNet and print worksheet Plane grid 2.

- b Mark these points:

$A(0, 1)$

$B(2, -1)$

$C(-1, 2)$

$D(-2, 3)$

$E(3, -2)$

$F(1, 0)$

- c What do you notice about the points? Check this with your ruler.

- d Points that lie on the same straight line are called **collinear**. Write down four collinear points from your number plane in part a.

- 7 a Draw a number plane with both axes extending from -10 to 10 , or print worksheet Plane grid 2.

- b Start at the first point and join the points in the order given:

- $(4, 0), (5, 1), (8, 1), (9, 0), (9, -3), (8, -4), (5, -4), (4, -3), (4, 0)$, STOP

- $(-2, 0), (-3, 1), (-6, 1), (-7, 0), (-7, -3), (-6, -4), (-3, -4), (-2, -3), (-2, 0)$, STOP

- $(7, -2), (4, 5), (1, -1), (-2, 5), (-5, -2)$, STOP

- $(3, 6), (5, 6), (4, 5), (-2, 5), (-2, 6), (-4, 6), (-4, 4), (-3, 4)$, STOP

- $(1, 1), (1, -3)$, STOP

- $(0.5, 1), (1.5, 1)$, STOP

- $(0.5, -3), (1.5, -3)$, STOP

- c What did you draw?

Worksheet

1 cm grid paper

MAT07MGWK10108

Worksheet

Plane grid 2

MAT07NAWK10071

Investigation: Number plane picture puzzles

- Draw a number plane from -10 to 10 on both x - and y -axes.
- Create your own number plane drawing using all four quadrants.
- Write out the set of points so that someone else could draw it.

9-04 Graphing tables of values

Worksheet

Graphing tables of values

MAT07NAWK10075

x	y
-3	-2
-2	-1
0	2
1	0

means

 $(-3, -2)$ $(-2, -1)$ $(0, 2)$ $(1, 0)$

x	0	1	2
y	5	7	3

means

 $(0, 5)$ $(1, 7)$ $(2, 3)$

Exercise 9-04 Graphing tables of values

- 1 Write down the ordered pairs shown in each table of values.

a

x	y
0	2
2	3
4	4
7	9

b

x	y
-2	-1
-1	3
2	4
5	-2

c

x	0	1	2
y	2	5	8

d

x	-4	-2	1	5
y	-1	-3	0	2

- 2 Plot the coordinates shown in each table of values on a separate number plane, with both axes extending from -4 to 4 . Use a ruler to join each set of points, and describe the pattern of points.

a

x	0	1	2	3
y	2	2	2	2

b

x	-1	0	1	2
y	-1	-1	-1	-1

c

x	3	-1	1	-3
y	1	1	1	1

d

x	-3	2	0	4
y	-3	-3	-3	-3

- 3 Graph this table of values on a number plane, with both axes extending from -4 to 4 , and describe the pattern of points.

x	-2	-1	0	1
y	0	0	0	0

- 4 Graph each table of values on a number plane. Use a ruler to join each set of points and describe their pattern.

a

x	1	1	1	1
y	-2	-1	0	1

b

x	-2	-2	-2	-2
y	-2	-1	0	1

c

x	3	3	3	3
y	-2	1	-1	3

d

x	-1	-1	-1	-1
y	-3	-1	0	2

- 5 Graph each table of values on a number plane. Join each set of points and describe their pattern.

a

x	-2	-1	1	2
y	-1	0	2	3

b

x	-2	0	1	3
y	-4	-2	-1	1

c

x	-4	-2	2	4
y	-1	0	2	3

d

x	-3	-1	0	1
y	-3	1	3	5

- 6 a What do you notice about the patterns of points in question 5?
b What happens to the y -values as the x -values increase?
- 7 A line is drawn through the points (0, 4), (2, 0) and (-1, 6). Which of the following points also lies on the line? Select **A**, **B**, **C** or **D**.
A (1, -1) **B** (1, 2) **C** (-2, 7) **D** (3, -3)

- 8 Graph each table of values on a separate number plane and describe the pattern of the points.

a

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

b

x	-2	-1	0	1	2
y	-8	-1	0	1	8

c

x	-2	-1	0	1	2	3	4	5	6
y	0	-2	0	2	0	-2	0	2	0

- 9 The following table shows the height of a tree as it ages. Plot each point on a number plane where the x -coordinate is the age of the tree and the y -coordinate is the height of the tree. After plotting each (age, height) point, explain whether there is a pattern between the age and height of the tree.

Age of tree (years)	5	10	15	20	25
Height of tree (metres)	10	12.5	17.5	21	26

Mental skills 9

Maths without calculators

Simplifying multiplication by factorising

Sometimes, one big multiplication may be split into many simpler little multiplications if we break each term into two 'easier' factors. We use the property that numbers can be multiplied in any order.

1 Study each example.

$$\begin{aligned} \text{a } 15 \times 6 &= 3 \times 5 \times 3 \times 2 \\ &= 3 \times 3 \times 5 \times 2 \\ &= 9 \times 10 \\ &= 90 \end{aligned}$$

$$\begin{aligned} \text{b } 30 \times 22 &= 3 \times 10 \times 2 \times 11 \\ &= 3 \times 2 \times 10 \times 11 \\ &= 6 \times 11 \times 10 \\ &= 660 \end{aligned}$$

$$\begin{aligned} \text{c } 24 \times 8 &= 6 \times 4 \times 2 \times 4 \\ &= 4 \times 4 \times 6 \times 2 \\ &= 16 \times 6 \times 2 \\ &= 96 \times 2 \\ &= 192 \end{aligned}$$

$$\begin{aligned} \text{d } 14 \times 36 &= 7 \times 2 \times 6 \times 6 \\ &= 6 \times 7 \times 6 \times 2 \\ &= 42 \times 6 \times 2 \\ &= 252 \times 2 \\ &= 504 \end{aligned}$$

Look for different pairs of numbers that you can multiply easily and rearrange.

Note: For each question, there are many different possible ways of arriving at the correct result.

2 Now evaluate each product by factorising first.

$$\text{a } 30 \times 24$$

$$\text{b } 18 \times 27$$

$$\text{c } 25 \times 33$$

$$\text{d } 16 \times 8$$

$$\text{e } 28 \times 20$$

$$\text{f } 12 \times 18$$

$$\text{g } 21 \times 9$$

$$\text{h } 36 \times 12$$

$$\text{i } 16 \times 35$$

$$\text{j } 22 \times 28$$

$$\text{k } 9 \times 15$$

$$\text{l } 27 \times 25$$

Investigation: Height vs head circumference

Is your height related to the circumference of your head?

A number plane can be used to graph different types of information.

a Draw a table to collect data about the students from your class:

Name	Head circumference (C)	Height (H)

b For each student in the class, measure (in cm):

- Height without shoes
- Circumference of head

c Draw a number plane using the first quadrant only. Place head Circumference (C) on the horizontal axis and Height (H) on the vertical axis.

d For each person, plot a point using your data.

e By considering your graph, do you think there is a relationship between head circumference and height?

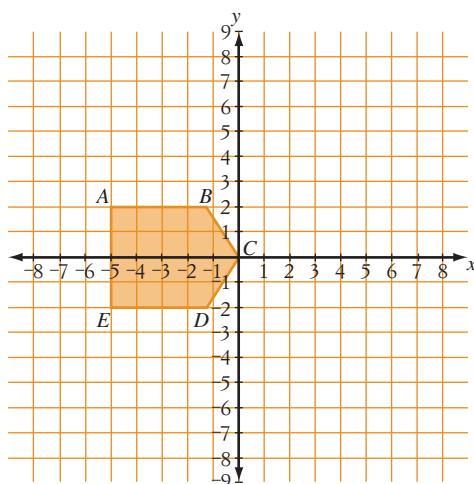
9-05 Transformations on the number plane

In Chapter 6, Geometry, we looked at the three congruence transformations: **translation** (slide), **reflection** (flip) and **rotation** (spin). The original shape is called the **original**, while the transformed shape is called the **image**.

When a point or vertex P of the original shape is transformed, the corresponding point or vertex on the image is labelled P' , pronounced 'P-dash' or 'P-prime'.

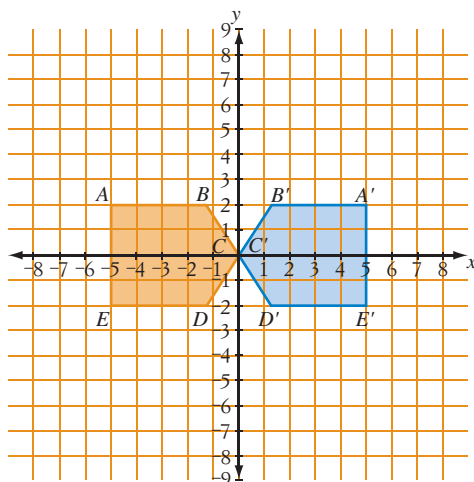
Example 1

- Reflect the pentagon $ABCDE$ across the y -axis to create the image $A'B'C'D'E'$.
- Compare the coordinates of each vertex of $ABCDE$ to its matching vertex in $A'B'C'D'E'$.



Solution

a



- $A(-5, 2) \rightarrow A'(5, 2)$
 $B(-1, 2) \rightarrow B'(1, 2)$
 $C(0, 0) \rightarrow C'(0, 0)$

Homework sheet

The number plane 2

MAT07NAHS10021

$$D(-1, -2) \rightarrow D'(1, -2)$$

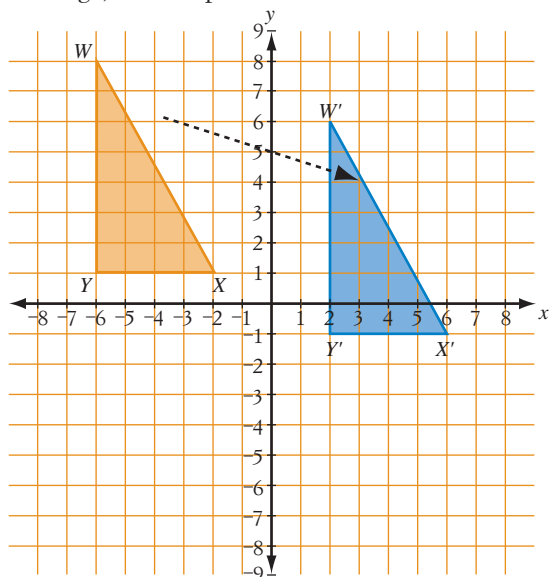
$$E(-5, -2) \rightarrow E'(5, -2)$$

When reflected in the y -axis, the x -coordinate of each vertex changes sign while the y -coordinate stays the same.

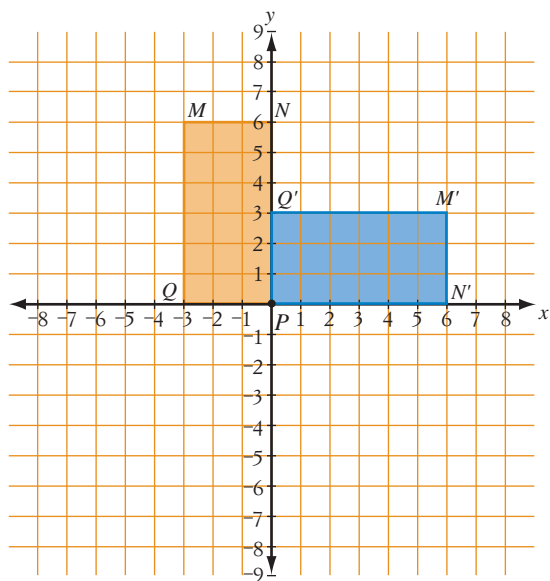
Example 2

Accurately describe the transformation that has been performed on each orange figure to produce the blue image, and compare the coordinates of the vertices of the original to those of the image.

a



b



Solution

- a $\triangle WXY$ has been translated 8 units right and 2 units down.

$$W(-6, 8) \rightarrow W'(2, 6)$$

$$X(-2, 1) \rightarrow X'(6, -1)$$

$$Y(-6, 1) \rightarrow Y'(2, -1)$$

The x -coordinate of each vertex increases by 8 units, while the y -coordinate decreases by 2 units.

- b Rectangle $MNPQ$ has been rotated 90° clockwise about the point $P(0, 0)$.

$$M(-3, 6) \rightarrow M'(6, 3)$$

$$N(0, 6) \rightarrow N'(6, 0)$$

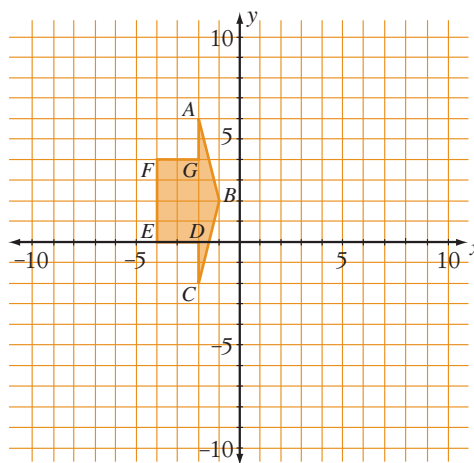
$$P(0, 0) \rightarrow P'(0, 0)$$

$$Q(-3, 0) \rightarrow Q'(0, 3)$$

The x -coordinate of each vertex changes sign and becomes the y -coordinate of the image vertex, while the y -coordinate of each vertex becomes the x -coordinate of the image vertex.

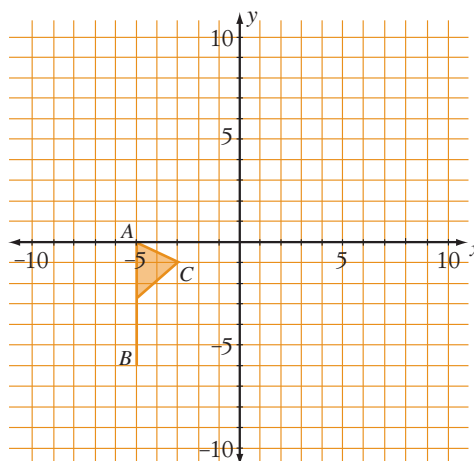
Exercise 9-05 Transformations on the number plane

- 1 a Copy and translate arrow $ABCDEFGF$ 6 units left and 2 units down to create the image $A'B'C'D'E'F'G'$.
- b Compare the coordinates of each vertex in the original arrow to its matching vertex in the image arrow.

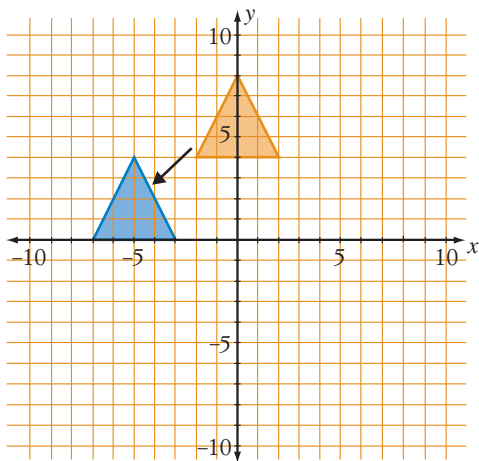


See Example 1

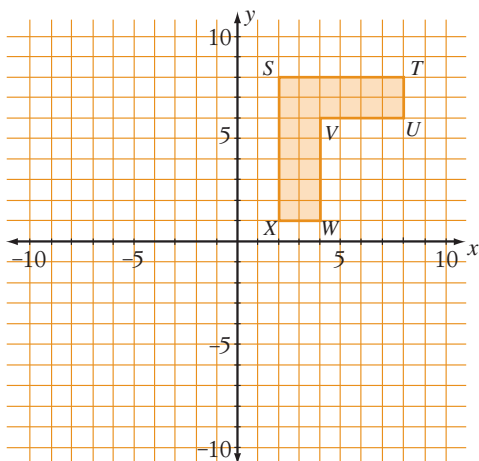
- 2 Copy and translate this flag 6 units right and 3 units up. Write down the coordinates of the new positions of A , B and C .



See Example 2 3 Accurately describe the transformation that has been performed on the triangle.

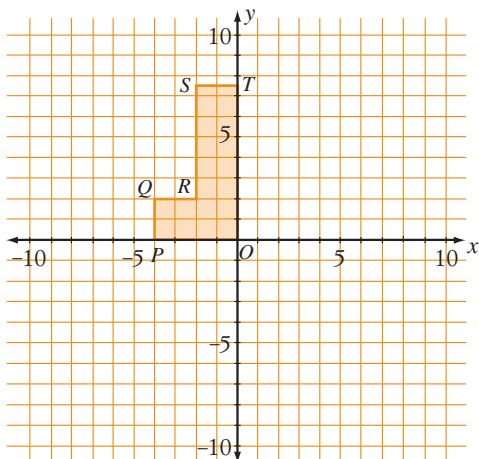


4 a Copy and reflect the L-shape across the x -axis to create the image $S'T'U'V'W'X'$.



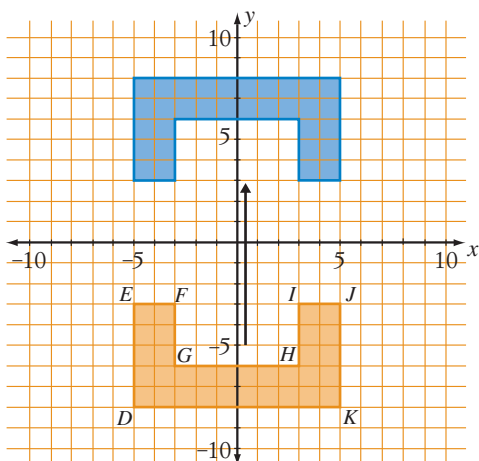
b Compare the coordinates of each vertex in the original figure to its matching vertex in the image figure.

5 a Copy and rotate $OPQRST$ about O through an angle of 180° to produce the image $O'P'Q'R'S'T'$.

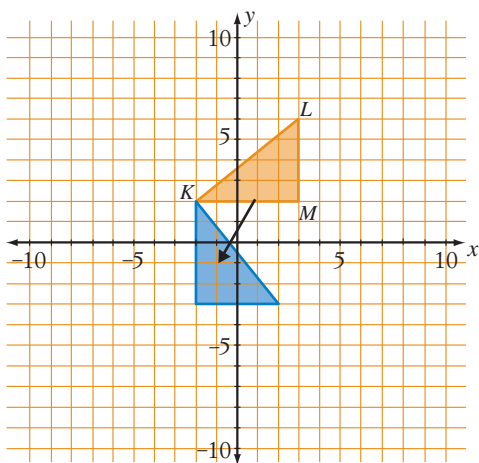


b Compare the coordinates of each vertex of $OPQRST$ to its matching vertex in $O'P'Q'R'S'T'$.

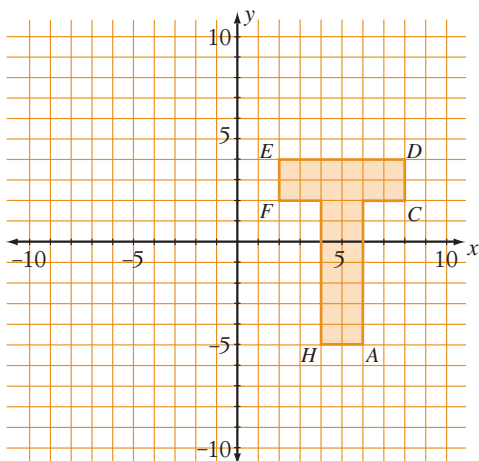
- 6 a Accurately describe the transformation that has been performed on shape $DEFGHIJK$.



- b Write the coordinates of the new position of D and compare them to its original coordinates.
- 7 a Accurately describe the transformation that has been performed on $\triangle LMK$.

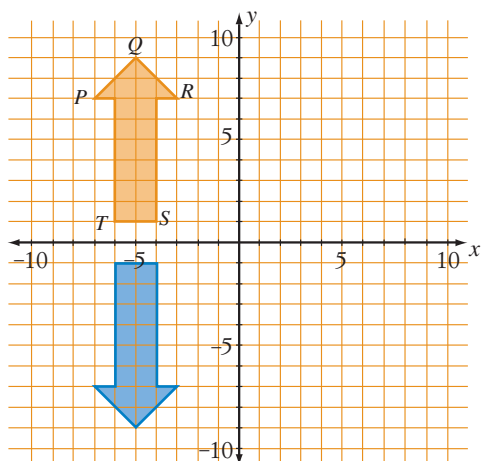


- b Write the coordinates of the new position of L .
- 8 a Reflect the T-shape across the y -axis.

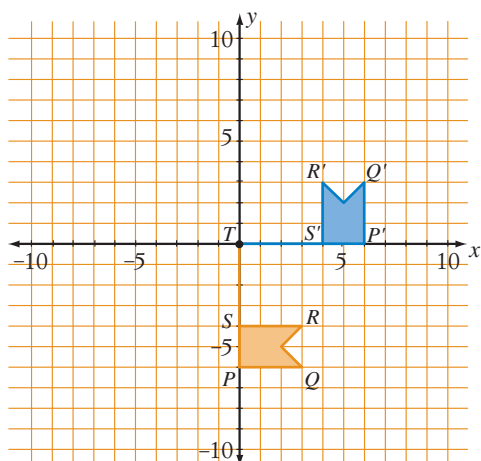


- b Write the coordinates of the new position of A and compare them to its original coordinates.

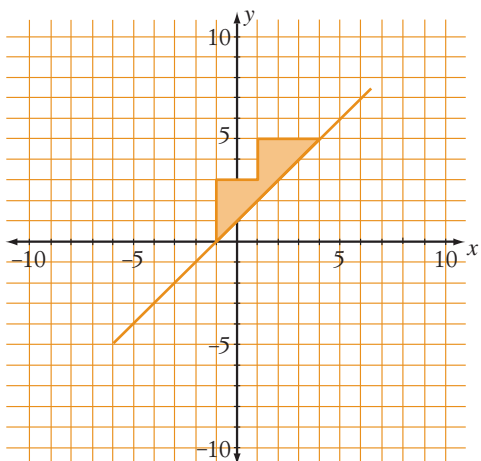
- 9 a Accurately describe the transformation that has been performed on this arrow.



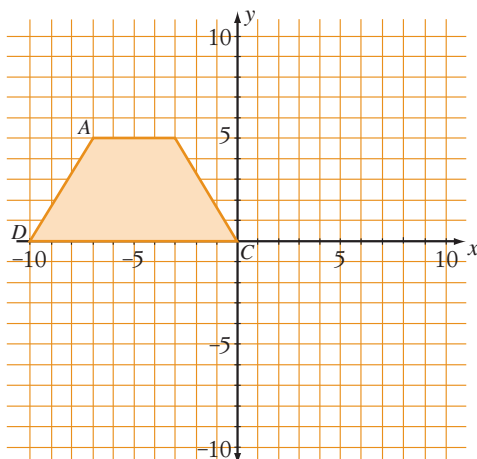
- b Write the coordinates of the new position of R .
- 10 a Accurately describe the transformation that has been performed on this flag.



- b Write the coordinates of R' and compare them to the coordinates of R .
- 11 Copy and reflect the shape across the coloured line.



- 12 Point A of this trapezium is transformed to Point A' . Accurately describe the transformation, write the new coordinates of vertex D and compare them to its original coordinates.

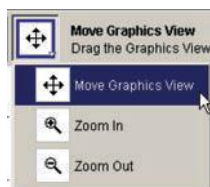


Technology Transformations on the number plane

In this activity you will create reflections, rotations and translations of various shapes using GeoGebra. Follow the instructions, as shown below.

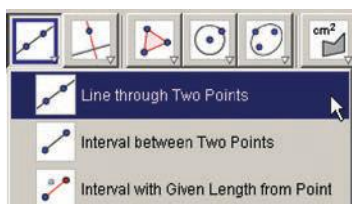
To set up your drawing page, click **View**. Check that **Grid** and **Axes** are both ticked.

Click  **Move Graphics View** (the last drop-down menu) until the axes are centred in the middle of the screen. [Approx: x : -10 to 10 and y : -10 to 10]



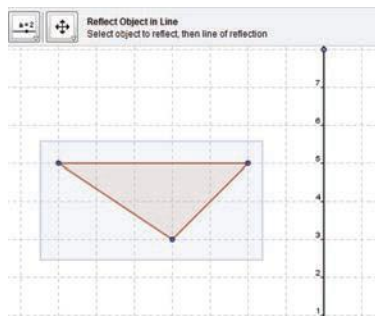
Activity 1: Reflection

- Select  **Polygon** from the fifth drop-down menu and click these coordinates $(-7, 5)$, $(-2, 5)$, $(-4, 3)$ and $(-7, 5)$.
- Select  **Line through two points** from the third drop-down menu.



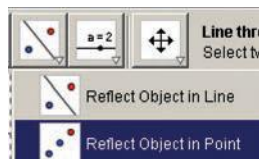
Select $(0, 8)$ and $(0, -3)$.

Your diagram should be shaded as shown below and the vertical axis should appear darker (see diagram)



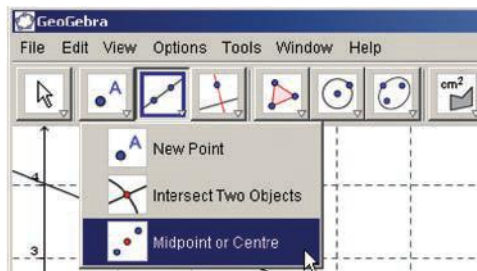
Click  **Reflect Object in Line** from the third-last drop-down menu. Click on the object to reflect and then the line.

- c Now click , grab a vertex and drag it. What happens?
- d Click the third-last drop-down menu button and choose . Select the shape and the point $(-2, 5)$. What happens?



Activity 2: Reflection

- a Using the diagram from Activity 1, click .
- b Now select the points $(2, 7)$ and $(-5, -2)$. Click  and choose this new line through $(2, 7)$ and $(-5, -2)$ as the line of reflection.

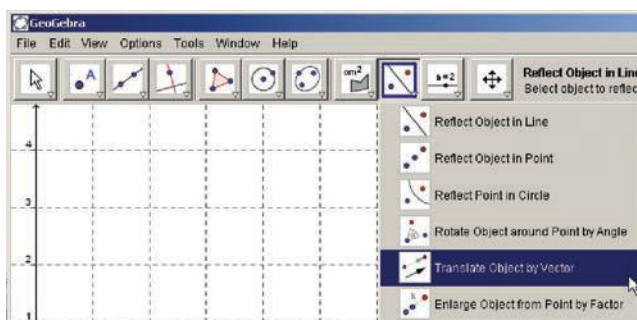


Activity 3: Rotation

- To set up your drawing page, click **View**. Check that **Grid** and **Axes** are both ticked.
Click  **Move Drawing Pad** and move the axes until they are in the middle of the screen.
[Approx: x : -10 to 10 and y : -10 to 10].
- Click **Polygon** and these points, in the order shown: $(0, 3)$, $(-2, 3)$, $(-3, 2)$, $(-2, 0)$, $(0, 3)$.
- Click  from the third-last drop down menu. Select object and point $(0, 3)$. Enter: 45° clockwise.
- Repeat . Select object and point $(0, 3)$. Enter: 180° clockwise. What happens?
- Try other points as the centre point and enter different angles. What happens?

Activity 4: Translation

- To set up your drawing page, click **View**. Check that **Grid** and **Axes** are both ticked.
Click  **Move Drawing Pad** and move the axes until they are in the middle of the screen.
[Approx: x : -10 to 10 and y : -10 to 10]
- Click **Polygon** and these points, in the order shown: $(-3, 4)$, $(-3, -1)$, $(1, -1)$, $(1, 1)$, $(-2, 1)$, $(-2, 4)$, $(-3, 4)$.
- Click  from the third drop-down menu. Click $(1, 1)$ and $(4, 3)$. Click  from the third last drop-down menu. What happens?



- Now translate the shape by another two different vector values.

Activity 5

- To set up your drawing page, click **View**. Check that **Grid** and **Axes** are both ticked.
Click  **Move Drawing Pad** and move the axes until they are in the middle of the screen.
[Approx: x : -10 to 10 and y : -10 to 10]
- Construct your own shape, using **Polygon**.
- Now complete each of the following tasks:
 - reflect the object about a point
 - reflect the object about a line
 - rotate the object by an angle
 - translate the object by a vector

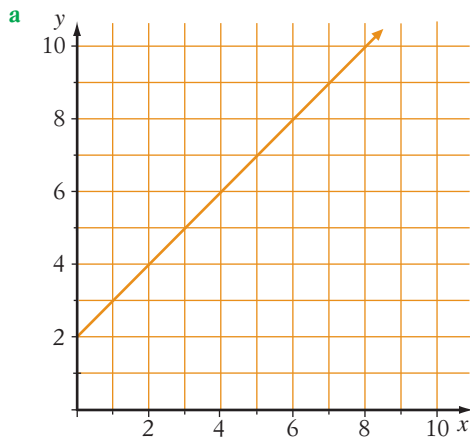
Power plus

Worksheet

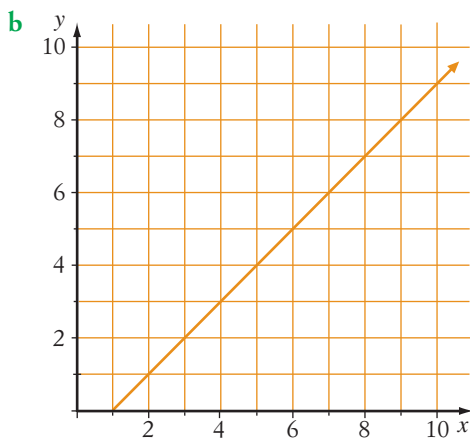
Plane grid 2

MAT07NAWK10071

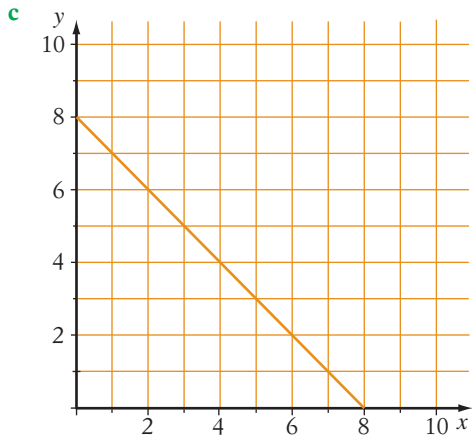
- 1
 - a Draw a number plane extending from -7 to 7 on the x -axis, and from 12 to -8 on the y -axis, or print out Worksheet Plane grid 2.
 - b Start at the first point and join the points in the order given.
 - $(3\frac{1}{2}, 0), (4, 2), (4, 4), (3\frac{1}{2}, 7), (4\frac{1}{2}, 8), (4\frac{1}{2}, 9), (5, 10), (4\frac{1}{2}, 11\frac{1}{2}), (3\frac{1}{2}, 10), (2, 10), (1\frac{1}{2}, 11\frac{1}{2}), (1, 10), (1, 8), (1\frac{1}{2}, 7), (2, 6\frac{1}{2}), (1\frac{1}{2}, 4\frac{1}{2}), (0, 2\frac{1}{2}), (-2\frac{1}{2}, 1), (-3, 0), (-3\frac{1}{2}, -4), (-4\frac{1}{2}, 6), (-4, -7), (0, -8), (2, -7\frac{1}{2}), (2, -6\frac{1}{2}), (0, -7), \text{STOP}$
 - $(-2, -7\frac{1}{2}), (1, -6), (3, -6), (4, -6\frac{1}{2}), (4\frac{1}{2}, -5\frac{1}{2}), (3, -4\frac{1}{2}), (0, -5), (-3, -6\frac{1}{2}), \text{STOP}$
 - $(\frac{1}{2}, -2), (1, -3), (0, -5), \text{STOP}$
 - $(1\frac{1}{2}, 0), (1, -2), (1\frac{1}{2}, -4\frac{1}{2}), \text{STOP}$
 - $(2, -6), (2\frac{1}{2}, -7\frac{1}{2}), (3\frac{1}{2}, -7\frac{1}{2}), (3\frac{1}{2}, -6\frac{1}{2}), (3, -6), \text{STOP}$
 - $(2\frac{1}{2}, -4\frac{1}{2}), (2\frac{1}{2}, -2), (3\frac{1}{2}, 0), \text{STOP}$
 - $(1\frac{1}{2}, -3), (1, -5), \text{STOP}$
 - c Complete the picture as you wish.
 - d What have you drawn?
- 2 Complete each table by reading values from each graph. Find an algebraic formula $y = \underline{\hspace{2cm}}$ that relates each y -value with the x -value.



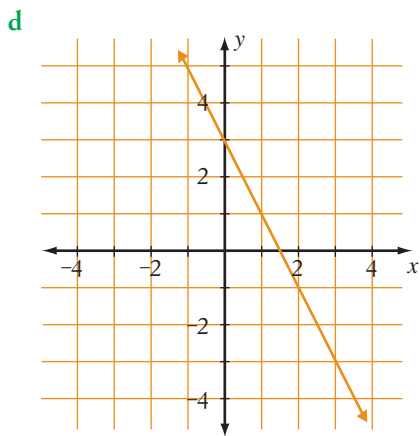
x	y
0	
1	
3	
6	
8	



x	y
1	
4	
	6
8	
10	



x	y
0	
	6
	4
6	
	0



x	y
-1	
0	
	-1
3	
	-5

Chapter 9 review

Language of maths

Puzzle sheet

Number plane
crossword

MAT07NAPS10038

axis/axes

Cartesian

collinear

coordinates

number plane

ordered pair

origin

point

quadrant

x-axis

y-axis

- 1 Look up the word 'coordinate' in the dictionary. Find both its mathematical and non-mathematical meanings.
- 2 Why do you think 'ordered pair' is another name for coordinates?
- 3 What is the **origin** and why does it have that name?
- 4 What word means 'lying in a line'?
- 5 What is another name for the number plane?

Topic overview

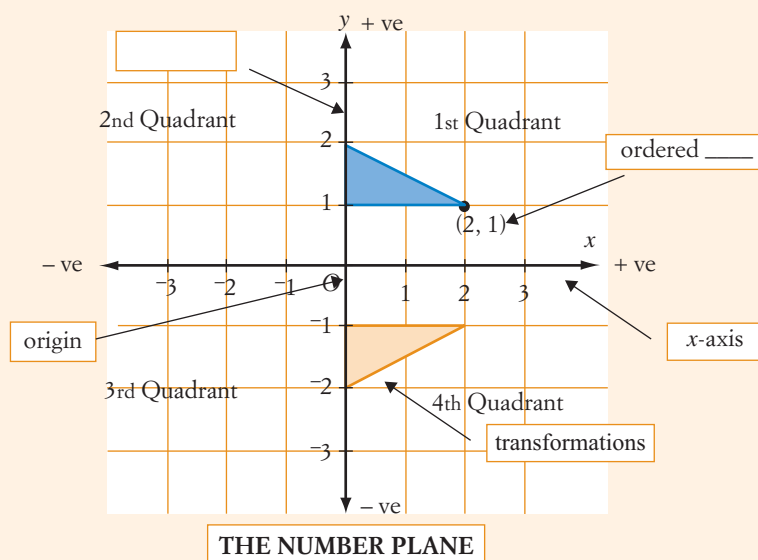
Worksheet

Mind map: The number
plane

MAT07NAWK10076

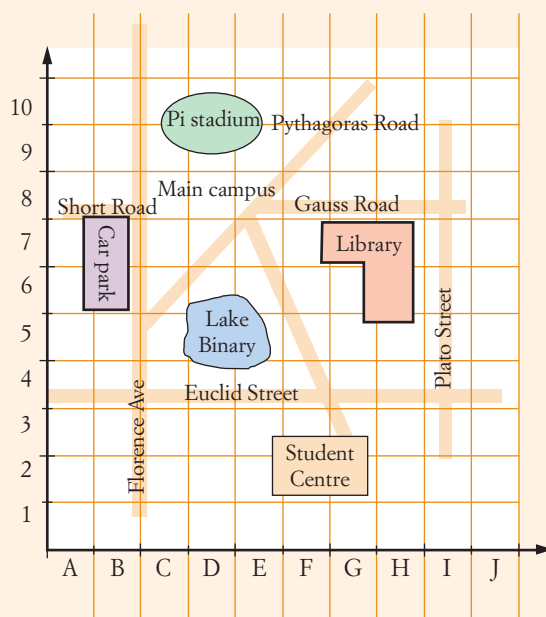
- In your own words, write about how points are plotted on the number plane.
- In your own words, write about how coordinates are used to locate a position on the number plane.
- What parts of this topic did you find difficult? What parts didn't you understand? Discuss them with a friend or your teacher.
- Give some examples of where the number plane is used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



1 This is a map of the Cartesian University campus.

See Exercise 9-01



State the feature that appears at:

a F2

b D9

c D8

d H7

2 Use the map in question 1 to write the reference for:

See Exercise 9-01

a Lake Binary

b the car park

c Gauss Rd

3 Plot each point on the same number plane.

See Exercise 9-02

A (2, 3)

B (1, 6)

C (3, 0)

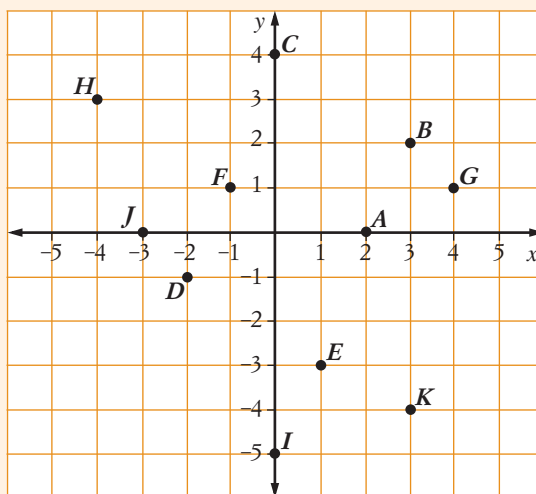
D (0, 4)

E (5, 2)

F (6, 5)

4 Write the coordinates of each labelled point on this number plane.

See Exercise 9-03



Chapter 9 revision

See Exercise 9-03

- 5 a In which quadrant is the point $(-3, -3)$?
 b Write the coordinates of a point on the y -axis.
 c In which quadrant is a point with a positive x -coordinate and a negative y -coordinate?
 d Write the coordinates of a point in the 2nd quadrant.
 e What are the coordinates of the origin?

See Exercise 9-03

- 6 a Plot these points on a number plane, joining them with straight lines.
 $(1, 3), (2, 3), (2, 2), (3, 1), (2, 0), (2, -1), (1, -1), (0, -2), (-1, -1), (-2, -1), (-2, 0),$
 $(-3, 1), (-2, 2), (-2, 3), (-1, 3), (0, 4), (1, 3).$
 b What shape have you drawn?

See Exercise 9-04

- 7 Plot the following sets of points on a number plane. Use a ruler to join each set of points.

a

x	-2	-2	-2	-2
y	-2	-1	0	1

b

x	-2	-1	0	2
y	3	3	3	3

See Exercise 9-04

- 8 Graph each table of values on a separate number plane. Use a ruler to join up the points and describe their pattern.

a

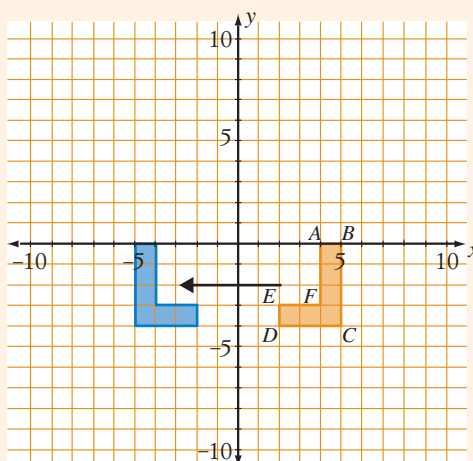
x	y
0	5
1	4
2	3
5	0

b

x	-1	0	1	2
y	-1	1	3	5

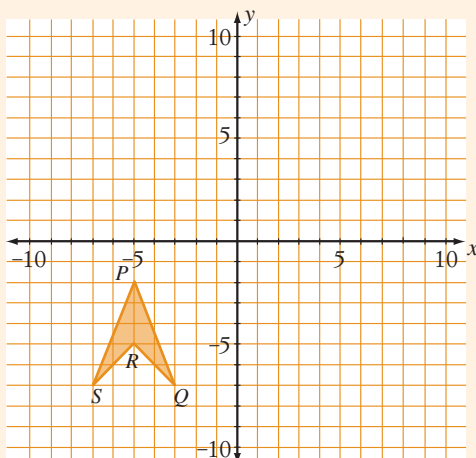
See Exercise 9-05

- 9 a Accurately describe the transformation that has been performed on the orange L-shape.



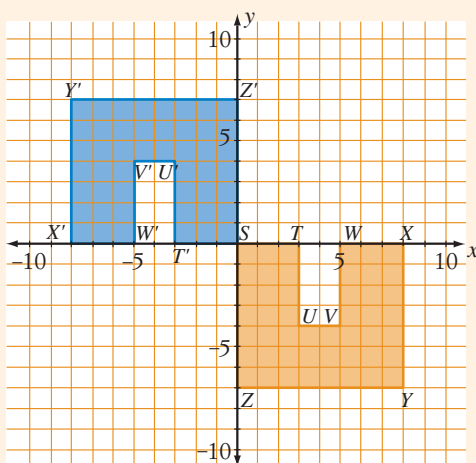
- b Write the coordinates of the new position of F and compare them to its original coordinates.

- 10 a Copy and translate this arrowhead $PQRS$ 4 units right and 1 unit up to create the image $P'Q'R'S'$.



- b Compare the coordinates of each vertex in $PQRS$ to its matching vertex in $P'Q'R'S'$.
- 11 a Accurately describe the transformation that has been performed on $STUVWXYZ$.

See Exercise 9-05



- b Compare the coordinates of V and V' .



10

Statistics and probability

Analysing data

A motor company is releasing a new model of car next year. How do they decide which colours and features to offer? They could conduct a statistical survey to determine which features are popular.

Statistics is the branch of mathematics concerned with collecting, organising, presenting and analysing data (information). Governments, businesses, research organisations, financial institutions and sporting groups all use statistics in their planning and decision-making.



Chapter outline

	Proficiency strands			
10-01 Interpreting graphs	U	F	PS	R
10-02 Misleading graphs	U	F	PS	R
10-03 Dot plots	U	F		
10-04 Stem-and-leaf plots	U	F		
10-05 The mean and mode	U	F	PS	R
10-06 The median and range	U	F	PS	R
10-07 Analysing dot plots and stem-and-leaf plots	U	F	PS	R
10-08 Comparing data sets	U	F	PS	R

Wordbank

data set A collection of data or scores about the same subject

dot plot A special column graph of dots for a small set of numerical data

mean The average value of a data set, found by dividing the sum of scores by the number of scores

median The middle score of a data set when the scores are arranged in order, or the average of the two middle scores

mode The most common score(s) of a data set

outlier An extreme score that is very different from the other scores in a data set

range The difference between the highest score and the lowest score in a set of data

stem-and-leaf plot A 'sideways column graph' used to list numerical data

In this chapter you will:

Video tutorial

Statistics

MAT07SPVT00001

Weblink

Census At School

Weblink

Worldometer

Weblink

Earth's population
chain letter

- understand that some data representations are more appropriate than others for particular data sets, and answer questions about those data sets
- identify potentially misleading data representations in the media
- construct and compare a range of data displays including stem-and-leaf plots and dot plots
- calculate mean, median, mode and range for sets of data, and interpret these statistics in the context of the data
- describe and interpret data displays and the relationship between the median and mean
- use mean and median to compare data sets and explain how outliers may affect the comparison
- identify and investigate issues involving continuous or large amounts of data collected from primary and secondary sources

SkillCheck

Worksheet

StartUp assignment 10

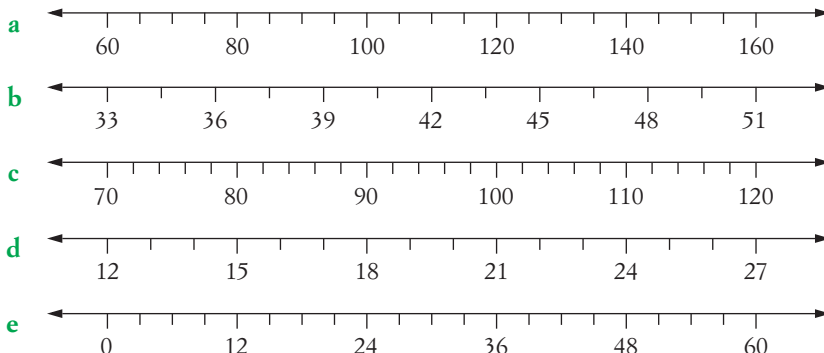
MAT07SPWK10077

Skillsheet

Reading linear scales

MAT07SPSS10031

1 Copy and complete each of the following scales and state the size of one interval.



2 Find the average of:

- a** 42 and 58 **b** 30 and 39

3 Some students were asked to state their favourite soft drink. The results are displayed in this column graph.

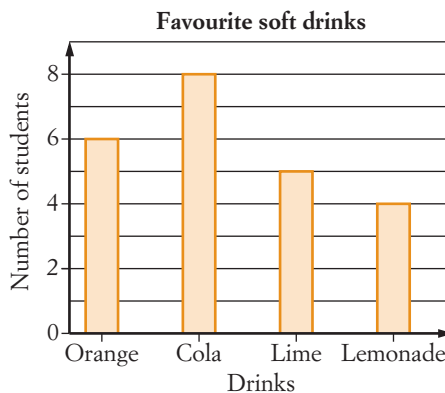
- a** List the drinks in order of preference, starting with the most popular.
b How many students preferred Lime?
c How many students were in the group?

4 Write each set of numbers in ascending order.

- a** 31, 28, 22, 11, 24 **b** 21, 18, 15, 17, 13, 16

5 Write each set of numbers in descending order.

- a** 6, 11, 18, 17, 5 **b** 8, 17, 14, 23, 31, 5, 2



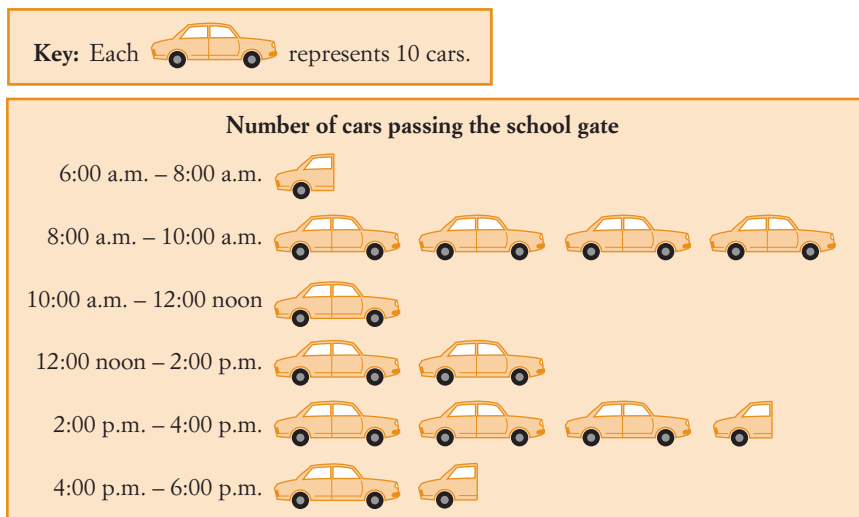
10-01 Interpreting graphs

In statistics, many types of graphs are used to present data (information).

- **picture graphs**
- **column graphs**, also called **bar charts**
- **divided bar graphs**
- **sector graphs**, also called **pie charts** or **circle graphs**
- **line graphs**

Exercise 10-01 Interpreting graphs

- 1 **Picture graphs** are used to show data about things that can be counted.
This picture graph shows the number of cars passing a school at different times during a day.



- How many cars does each  represent?
- What does  represent? Can you see a disadvantage with this symbol?
- What is the busiest time of day for traffic?
- What is the quietest time?
- List each time period and the number of cars at that time.
- Suggest possible reasons for the flow of traffic at:
 - 8:00–10:00 a.m.
 - 6:00–8:00 a.m.
 - 2:00–4:00 p.m.

Technology worksheet

Graphing data

MAT07SPCT10005

Worksheet

Every picture tells a story

MAT07SPWK10078

Worksheet

Car survey

MAT07SPWK10080

Worksheet

Student survey form

MAT07SPWK10081

Skillsheet

Displaying data

MAT07SPSS10032

Weblink

Nation master: country comparisons

TLF learning object

Home internet survey (L3150)

TLF learning object

Healthy life survey (L3158)

TLF learning object

Leisure survey (L3154)

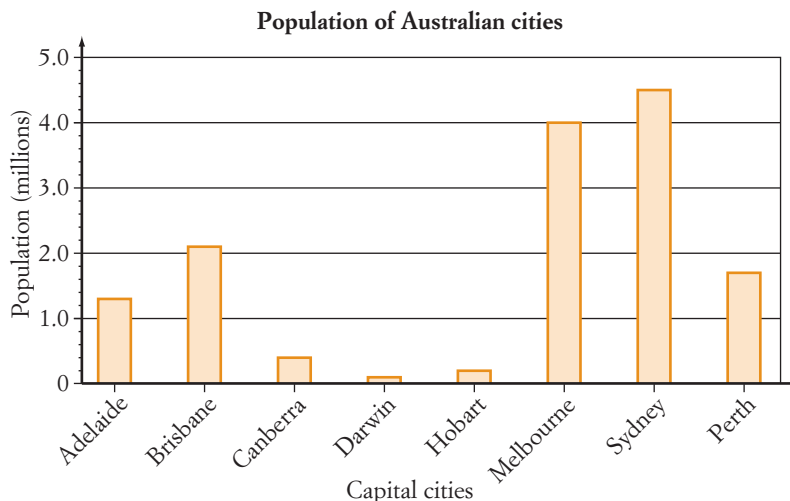
Puzzle sheet

Where all the cars are red

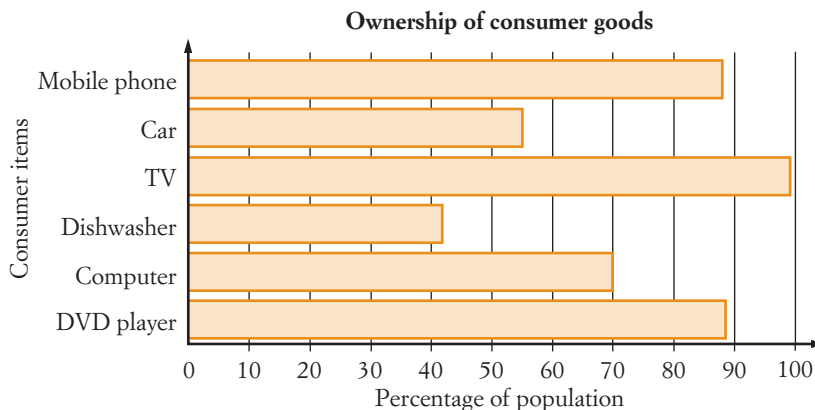
MAT07SPSS00007

- 2 **Column graphs** are mostly used for data that are in categories.

This column graph shows the populations of the eight Australian capital cities in 2010.

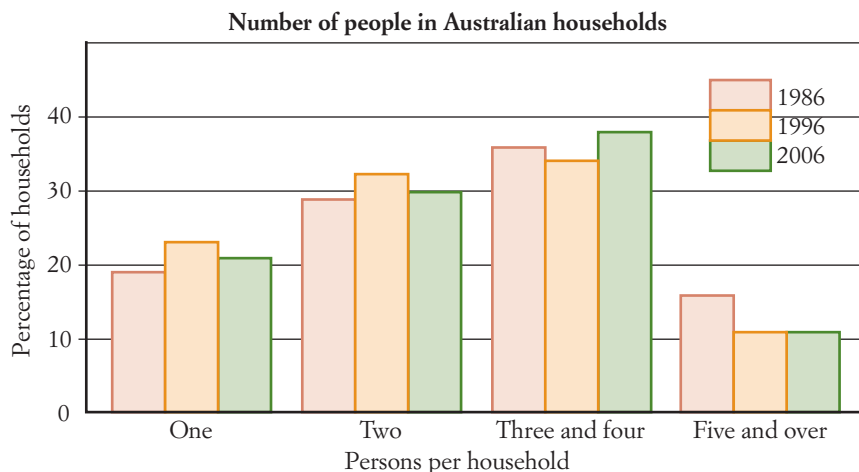


- Which city has the biggest population?
 - Which city has the smallest population?
 - What does one interval on the vertical axis (the 'Population' axis) represent?
 - What is the population of Brisbane?
 - Which city has a population of 1.3 million?
 - How many times Hobart's population is Melbourne's population?
- 3 Sometimes a column graph is presented sideways. The following graph shows the percentage of the population that owned various consumer items in 2008.

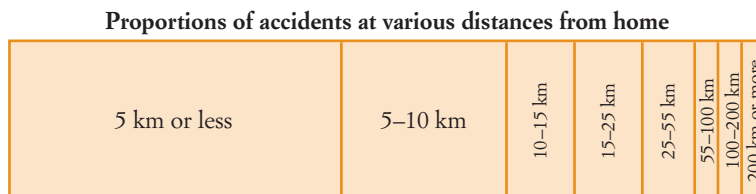


- What percentage of the population owned:
 - a television set?
 - a mobile phone?
 - a DVD player?
- What item was owned by 42% of the population?
- Did more people own mobile phones or computers?
- What was the percentage difference between people owning a car and people owning a computer?
- How might this column graph be different if it described the ownership of consumer items *this year*?

- 4 This **clustered column graph** or **side-by-side column graph** compares the number of people living in Australian households in 1986, 1996 and 2006.



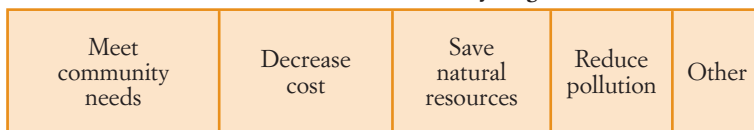
- What scale is used on the vertical axis?
 - What percentage of households were made up of more than two persons in 1996?
 - In which year were 30% of households made up of two people?
 - Which category was:
 - most common?
 - least common?
 - Which category had the greatest difference between 1996 and 2006?
 - 'There was a higher percentage of one-person households in 1996 than in 1986.' True or false?
- 5 **Divided bar graphs** are rectangular graphs used to compare parts of a whole. This divided bar graph shows the relative proportions of motor vehicle accidents that happen at various distances from home.



- What distances from home are accidents most likely to occur?
- What fraction of accidents happen within 5 km of home?
- Altogether, what fraction of accidents happen 15 km or less from home?
- What happens as you get farther from home? Suggest a reason why this may be so.
- What fraction of accidents happen 100 km or more from home?
- 'More than $\frac{3}{4}$ of all accidents occur within 25 km of home.' True or false?
- 'Smart Alec' says that to avoid having a car accident he should make all of his trips 100 km or more from home. Why is Alec wrong in saying this?

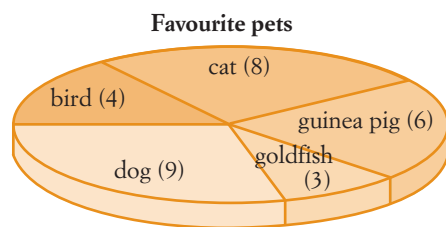
- 6 This divided bar graph shows the relative proportions of reasons that various councils gave for involving themselves in recycling.

Council's reasons for recycling



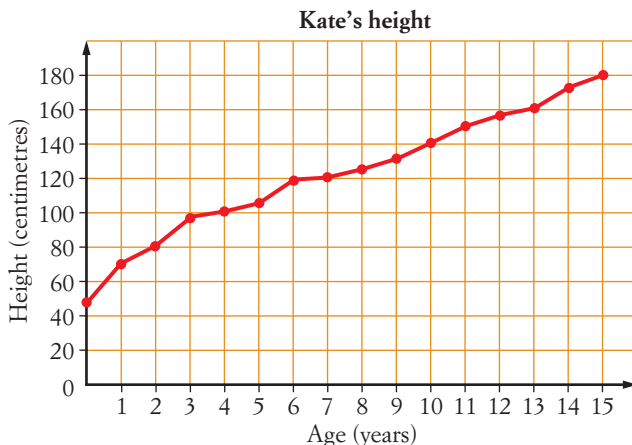
- What is the total length of this bar graph, in millimetres?
- What is the most common reason given? What fraction of the graph is this?
- What fraction of the graph represents 'Save natural resources'?
- List the reasons in order of popularity (according to the graph).
- What are the disadvantages of using a divided bar graph to illustrate this data?

- 7 **Sector graphs** are circle graphs that are also used to compare parts of a whole. This graph shows the results of a survey of 30 students regarding their favourite pet, with the circle being divided into five sectors of different sizes.



- What is the most popular pet in this survey?
- What is the least popular pet?
- What fraction of students:
 - prefer goldfish
 - prefer birds?
- How many degrees are there in a revolution?
- How many degrees are there in the angle of the cat sector?
- Do cats and guinea pigs together make up more than half of the preferred pets?
- If the figures from this survey are typical for all students, calculate the number of each type of pet you would expect to have if 90 students were surveyed.

- 8 **Line graphs** are usually drawn to show data measured over time. This line graph shows Kate's height over her first 15 years. At birth she is 48 cm tall, and at age 10 she is about 140 cm tall. Use the graph to answer the following questions.



- What was Kate's height:
 - on her first birthday?
 - at age 7?
- At what age did Kate reach:
 - 1 metre?
 - 150 cm?
- Between which two birthdays did Kate grow the most? Give a possible reason.
- Between which two birthdays did Kate grow the least? Give a possible reason.

- e How long did it take Kate to double her height from birth?
- f How long did it take her to triple her height from birth? Compare this to your answer for the previous question. Why is there a difference?
- g What do you think the graph will look like:
 - i after 15 years?
 - ii after 20 years?
- h How might a graph of a boy's height differ from this one?

Technology Column graphs

Pet survey

- Survey the students in your class on their pets. Keep a record of how many of each type of pet students have.
- Enter your results into a spreadsheet, as shown on the right.
- Create a column graph. Select **Insert** and **2-D clustered column graph**. Give the graph an appropriate title and label the axes.

	A	B
1	Pet	Number
2	Dog	
3	Cat	
4	Bird	
5	Fish	
6		

Technology worksheet

Excel
Creating a graph

MAT07SPCT00012

Travel to school

- Survey the students in your class on how they travel to school. Count the types of transport in your survey.
- Enter your results into a spreadsheet, as shown on the right.
- Create a **bar chart** (horizontal column graph). Select **Insert** and create a **2-D clustered bar chart**. Give the graph an appropriate title and label the axes.

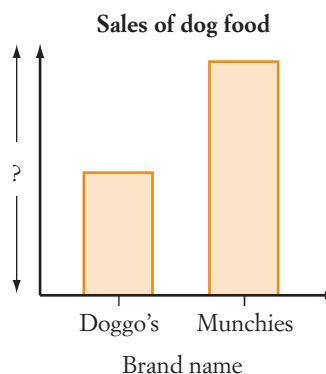
	A	B
1	Transport	Number
2	Bus	
3	Train	
4	Walk	
5	Car	
6	Bicycle	
7		

10-02 Misleading graphs

When used incorrectly, graphs can give a false or misleading impression. The following three graphs were used by *Munchies* dog food to compare its sales figures to those of *Doggo's* dog food, but each one is misleading in some way.

No scale

This graph does not have a scale on the vertical axis. You cannot tell how big the difference is between *Munchies*' sales and *Doggo's* sales.



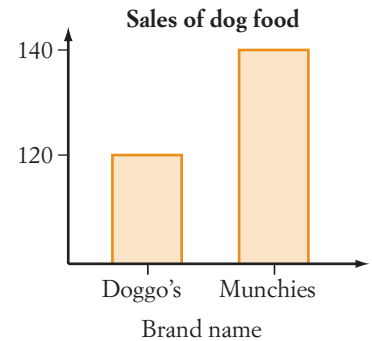
Worksheet

Misleading graphs

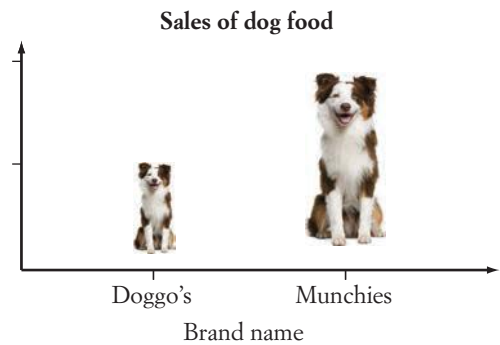
MAT07SPWK10079

Uneven scale

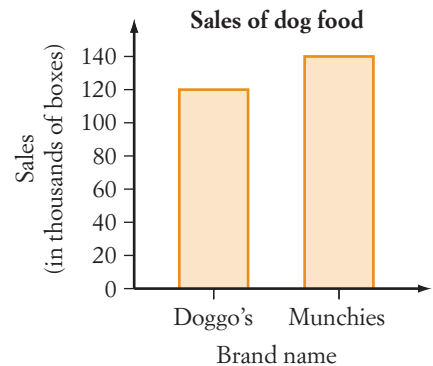
This graph shows **only part** of a scale, and the scale is not regular and does not start at 0. This makes the difference between *Munchies*' sales and *Doggo's* sales look much greater.

**Incorrect use of pictures**

This graph uses pictures instead of columns. The *Munchies* dog is twice as tall as the *Doggo* dog, but it is also twice as wide, making it seem much bigger (4 times bigger in area).

**A correct graph**

This graph shows the information correctly. The scale is even and begins at 0. Notice that the difference in sales figures is not as large as it seemed in the 3 misleading graphs above.

**Summary**

A **misleading graph** can give a wrong impression by:

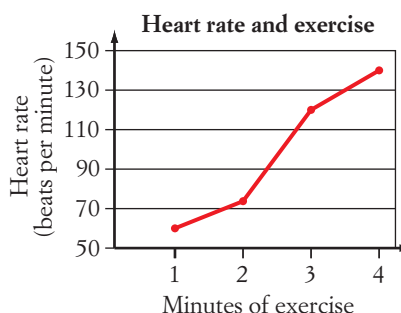
- not having a scale
- showing only part of the scale or an irregular scale
- not showing the position of zero on the scale
- using pictures instead of columns to exaggerate the differences

Exercise 10-02 Misleading graphs

- 1
 - a What is the actual difference in sales figures between Munchies and Doggo's in the previous example?
 - b How does the last graph illustrate this difference correctly?
 - c Why is it misleading to not show the position of 0 on the vertical axis?
 - d Why is it misleading to use pictures or diagrams on graphs instead of columns?

- 2 This line graph shows Lisa's heart rate while she is exercising on a treadmill.

- a What is Lisa's heart rate after 3 minutes?
 - b What is the size of one unit on the vertical axis?
 - c What is misleading about this graph?
 - d Redraw this graph correctly so that it is not misleading.



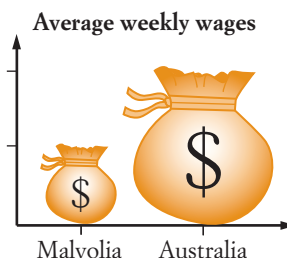
- 3
 - a Draw a line graph for the profit information in this table. Place 'Year' on the horizontal axis, and use a scale of 1 cm = \$2 million on the vertical axis.

Year	Profit in \$ millions
2010	2.5
2011	2.1
2012	3.2
2013	3.5
2014	3.6
2015	3.9

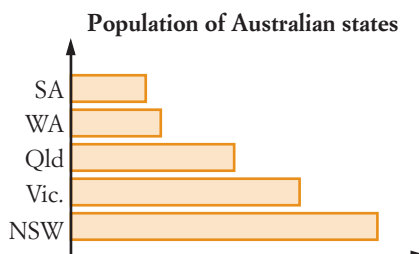
- b Draw another line graph using the same information, this time using a scale of 1 cm = \$0.5 million, but show only from \$2 million to \$4 million on it.
 - c Which graph looks more impressive? Why?

- 4 This graph compares the average weekly wages in Malvolia (\$700) and Australia (\$1400).

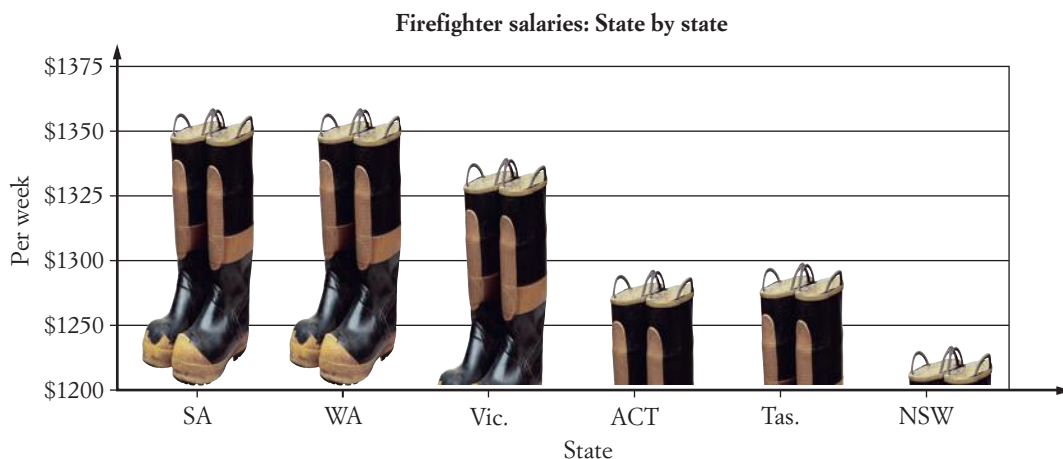
- a What misleading impression does this graph give?
 - b Explain two things that are wrong about this graph.
 - c What should be drawn on the graph instead of pictures?
 - d Redraw this graph correctly.



- 5 Why is this graph difficult to interpret?



- 6 This graph compares the weekly salaries of firefighters in six states.



- a Describe two ways in which this graph is misleading.
- b Redraw this graph correctly.
- 7 Draw two graphs using the figures below for school sport, one that is correct and accurate, and one that gives a misleading impression.

School sport	Hockey	Netball	Soccer
Number of players	18	20	15

Just for the record

Top 10 baby names

This table lists the 10 most popular baby names in New South Wales for boys and girls born in 2000 and 2010:

	Boys				Girls			
	2000		2010		2000		2010	
1	Joshua	846	William	631	Jessica	653	Isabella	543
2	Jack	738	Jack	541	Emily	614	Olivia	530
3	Thomas	700	Joshua	495	Sarah	538	Chloe	525
4	Lachlan	683	Thomas	490	Georgia	473	Ruby	519
5	Matthew	633	Lachlan	484	Olivia	434	Charlotte	479
6	James	620	Noah	479	Emma	395	Mia	465
7	Daniel	597	Oliver	478	Chloe	389	Lily	447
8	Nicholas	528	Cooper	473	Sophie	375	Emily	437
9	Benjamin	518	Ethan	453	Hannah	346	Ella	433
10	William	513	Lucas	445	Isabella	337	Amelia	421

Source: NSW Registry of Births, Deaths and Marriages.

What is the most common first name for the Year 7 students at your school?

10-03 Dot plots

A **dot plot** is a simple type of column graph that uses dots to display the frequency of each score. It is easy to draw and is useful for small sets of data. A dot plot shows:

- any gaps in the data
- any **clusters**, where scores are grouped or bunched together
- any **outliers**: extremely high or low scores that are very different from the other scores
- how the scores are spread out.

An **outlier** 'lies outside' the other scores in the data set.
Outlier is pronounced 'out-ly-er'

Homework sheet

Statistics 1

MAT07SPHS10037

Example 1

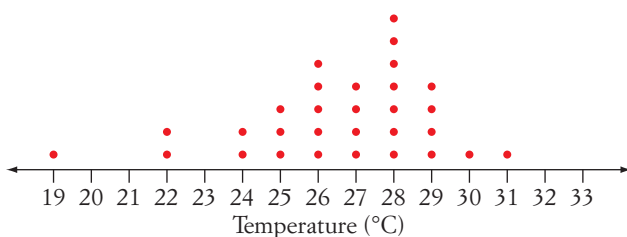
The data below show the daily maximum temperatures (in °C) in Tamworth during November:

27	28	26	25	26	27	22	30	28	29
28	26	24	22	28	24	27	29	28	27
19	25	26	29	29	26	28	28	31	25

- Construct a dot plot for the data.
- What was the highest temperature?
- On how many days was the temperature 25°C?
- Where were the temperatures clustered?
- What was the outlier temperature?

Solution

a



- The highest temperature was 31°C.
- The temperature was 25°C on 3 days.
- Temperatures were clustered between 26°C and 29°C.
- The outlier temperature was 19°C.

3 dots at 25

The most dots are bunched here

Extremely low, and different from the rest of the scores.

Exercise 10-03 Dot plots

See Example 1

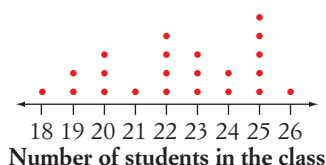
- 1 a The following information shows the number of goals that Daniel kicked in the soccer games he played last year. Draw a dot plot to represent this data.

4 5 1 2 3 0 3 8 4 6 5
4 1 1 4 4 2 5 3 1 1 0

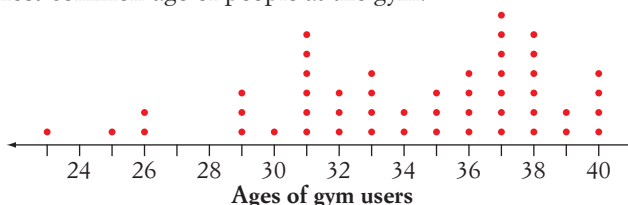
- b Are there any outliers in this data? If so, what are they?
c Where is the data clustered?
d How many games did Daniel play?

- 2 This dot plot shows the number of students in each class at Westvale Primary School.

- a How many students are in the smallest class?
b How many classes have more than 20 students in them?
c Copy and complete: Class sizes range from _____ to _____.
d What is the most common number of students in a class?
e Are there any outliers in this data?
f How many classes are there at Westvale Primary School?



- 3 a The ages of the people exercising at a gym one evening are shown in the dot plot below. What is the most common age of people at the gym?



- b Between which ages are the data clustered?
c Identify any outliers.
d Copy and complete: Ages range from _____ to _____.
e How many people were at the gym that evening?

Worked solutions

Exercise 10-03

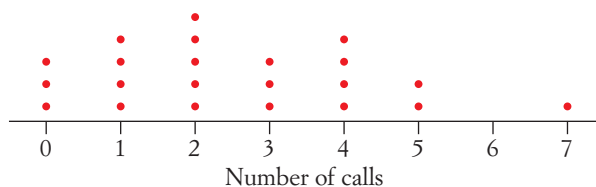
MAT07SPWS10055

- 4 The number of motor accidents that occurred on the M5 motorway each day was recorded:

1 0 0 2 0 3 5
1 0 1 0 2 3 0

- a Draw a dot plot for this data.
b What is the most common number of daily accidents?
c Calculate to one decimal place the average number of accidents per day.
d What is the outlier? Why?

- 5 This dot plot shows the number of phone calls made by a group of students on Sunday. Which of the following is the number of students who made phone calls? Select the correct answer A, B, C or D.



A 7

B 19

C 22

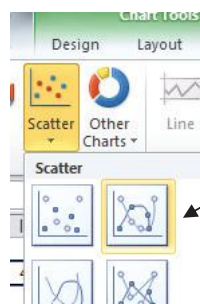
D 8

Technology Olympic winning times

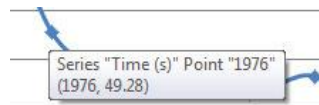
The table below shows the gold medal winning times of the women's 400 m track event for the Olympic Games held from 1972 to 2008.

Year	1972	1976	1980	1984	1988	1992	1996	2000	2004	2008
Time (s)	51.08	49.28	48.88	48.83	48.65	48.83	48.25	49.11	49.41	49.62

- Copy the data, as shown in the table above, into an Excel spreadsheet.
- To create a scatter graph choose **Insert** and **Scatter (with smooth lines and markers)**.



- Give the graph an appropriate title and axes labels. Save the file.
- On the graph, position the mouse over a data point. (Do not click on it.) You can view the specific details of the Olympic year and winning time. For example (see below), the winning time was 49.28 seconds in 1976.



- Use your spreadsheet, graph and the formulas below to answer these questions.
 - In what year was the fastest gold medal winning time run? In cell A5, enter **=min(B2:K2)**. In cell B5, enter the year that corresponds to this time.
 - In cell A6, type the label 'Average'. In cell B6, use the formula **=average(B2:K2)** to calculate the average winning time for this event, from 1972 to 2008.
 - In cell A7, enter **=max(B2:K2)** to find the slowest winning time in this event. In cell B7, enter the year that corresponds to this time.
 - In cell A8, enter a formula to find the difference between the fastest and slowest winning times.
 - Predict the gold medal time at the 2012 Olympic Games for this event. Justify your answer, then find out what the actual gold medal time was in 2012.
 - In cell A9, enter a formula to calculate the speed, in metres per second, of the fastest women's 400 m runner, from 1972 to 2012.
 - Starting in cell A10, write a paragraph describing the changes in winning times for this event between 1972 and 2012.
 - In cell A15, suggest reasons why the pattern of gold medal times has changed between 1972 and 2012.

10-04 Stem-and-leaf plots

Worksheet

Stem-and-leaf plots 1

MAT07SPWK10082

TLF learning object

Stem-and-leaf graphs
(L5911)

A **stem-and-leaf plot** (or **stemplot**) is like a sideways column graph but one that lists the actual scores on the horizontal columns. An example is shown on the right, with the scores listed in the leaf column.

The name ‘stem-and-leaf’ comes from the structure of a plant, where the stem is a branch or a vine on which the leaves hang.

An **ordered** stem-and-leaf plot shows:

- all the scores, listed from smallest to largest
- the most common scores
- any clusters or outliers
- how the scores are spread out

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 4 8
7	4
8	0 1 3 4
9	2 5 7 8

This leaf means 83.



Example 2

Animated example

Stem-and-leaf plot

MAT07SPAE00001

Use a stem-and-leaf plot to show the following ages of the 21 musicians in an orchestra:

25	36	28	47	52	33	50	28	33	35	48
55	29	50	39	41	32	29	56	26	35	

Solution

The ages range from 25 to 56.

We write 2, 3, 4 and 5 down the first column to make the stem.

Stem	Leaf
2	
3	
4	
5	

The leaves are the single digits written next to the stem, for example:

- 25 is shown by writing a 5 next to the 2 stem.
- 36 is shown by writing a 6 next to the 3 stem.

Stem	Leaf
2	5
3	6
4	
5	

Continue in the same manner until all the data has been entered.

Stem	Leaf
2	5 8 8 9 9 6
3	6 3 3 5 9 2 5
4	7 8 1
5	2 0 5 0 6

It is more useful to rearrange the scores in ascending order.
This results in an **ordered** stem-and-leaf plot.

Stem	Leaf
2	5 6 8 8 9 9
3	2 3 3 5 5 6 9
4	1 7 8
5	0 0 2 5 6

Exercise 10-04 Stem-and-leaf plots

- 1 This stem-and-leaf plot shows the heights, in centimetres, of 22 women at a gym.

Stem	Leaf
14	7 8 8
15	0 1 2 3 6 6 8
16	1 5 8 9 9 9
17	2 3 3 5
18	1 1

See Example 2

Extra questions

Data display

MAT07SPEQ00007

What is the most common score? Select the correct answer **A**, **B**, **C** or **D**.

A 15 cm **B** 156 cm **C** 181 cm **D** 169 cm

- 2 A survey shows that the number of students served at the school canteen daily over a 3-week period is:

105 76 97 88 114 86 124 101
112 98 95 105 117 81 112

- a** Show this information in an ordered stem-and-leaf plot.
b What are the least number and the greatest number of students served over the period?
c On how many days were over 100 students served?
d Find the middle score.
- 3 The point scores of all teams in the first four rounds of one season of the Australian Football League (AFL) are shown in this stem-and-leaf plot.

- a** How many scores are there altogether?
b How many scores are below 100 points?
c What was the most frequent score?
d Copy and complete: The team scores range from _____ to _____.
e Which stem had the most scores?
f Where are the scores clustered?

Stem	Leaf
6	4 4 6 6 6 8
7	0 0 2 2 3 3 7 8 8 9
8	0 2 3 4 4 6 9 9
9	3 4 4 4 5 7 7 8 8 9 9 9 9
10	0 3 4 8 9
11	0 1 1 2 3 4 8 9
12	0 7
13	4 4 5 5 7
14	2 3
15	0 0 2 3 4

- 4 The heights of the students in a PE class, recorded in centimetres, are:

155	153	157	166	163	162	154	175
159	157	137	162	171	140	145	168
158	141	170	166	143	175	157	177

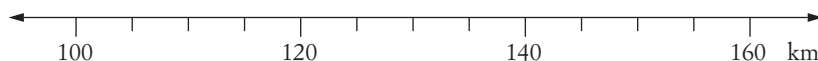
- Make a stem-and-leaf plot of these heights.
- What are the shortest height and the tallest height?
- How many students were under 150 cm tall?
- What was the most common height?
- What were the two middle heights?
- What fraction of the class had heights in the 160s?
- What percentage of the class had heights in the 170s? Answer correct to one decimal place.

Mental skills 10 Maths without calculators

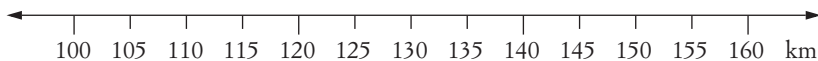
Reading linear scales

Understanding and reading the scale on a measuring instrument, on a number line or on the axis of a graph is an important mathematical skill.

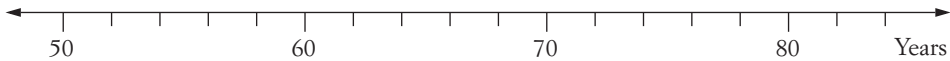
- Study each example.
 - Complete the missing values on this scale.



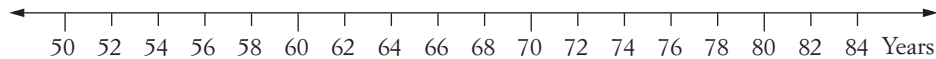
- First, choose two values on the scale, say 100 and 120.
- Count the number of intervals ('spaces') between the two values. There are four intervals between 100 and 120.
- To find the size of each interval, divide the difference between the two values by the number of intervals:
 $\text{Difference} = 120 - 100 = 20 \text{ km}$
 $\text{Number of intervals} = 4$
 $\text{Size of an interval} = 20 \div 4 = 5 \text{ km}$
- Use the calculated size of an interval to complete the missing values.



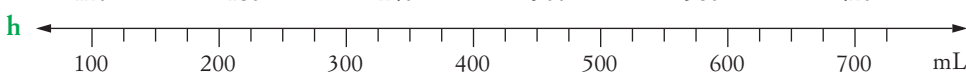
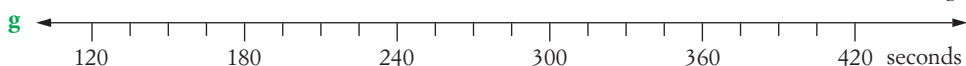
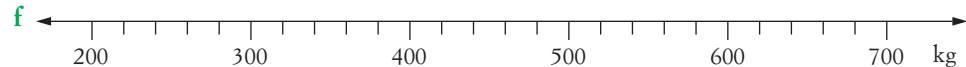
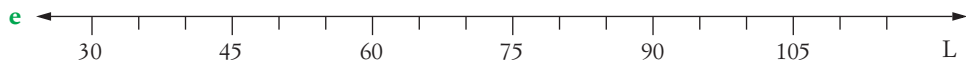
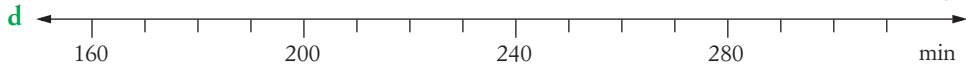
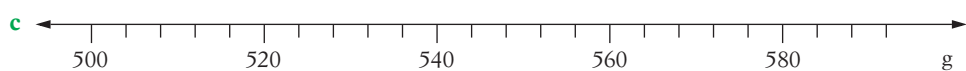
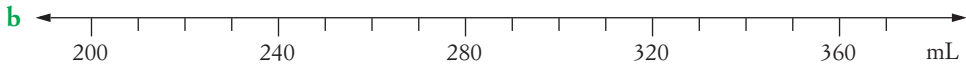
b Complete the values on this scale.



- Choose 50 and 60 on the scale.
- Number of intervals (between 50 and 60) = 5
- Difference (between 50 and 60) = $60 - 50 = 10$ years
- Size of an interval = $10 \div 5 = 2$ years.



2 Now copy and complete the following scales.



10-05 The mean and mode

How do you use the word **average**?

- 'That was an average film'
- 'The average person in the street thinks ...'
- 'My average score this year is ...'

In statistics, we use the word 'average' to mean a **typical or central value** of a set of data.

Skillsheet

Statistical measures

MAT07SPSS10033

Weblink

Census At School

Puzzle sheet

Looking for gold

MAT07SPPS00008

The mean

The best-known average is the **mean**. The mean is found by adding all the scores and dividing by the number of scores.

Summary

The symbol for the **mean** is $\bar{x}$.

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

The mode

The **mode** is the most common or frequent score (or scores). A set of data may have more than one mode, or no mode at all.

Summary

The **mode** is the score (or scores) that occurs most often; the score with the highest frequency.

Think: mode = 'most often'

Example 3

Video tutorial

The mean, mode, median and range

MAT07SPVT10020

The residents in a street were surveyed about the number of children living in each household. The results were:

2 2 1 2 0 3 2 1 1 4 1 0

- Find the mean, correct to one decimal place.
- Find the mode.

Solution

$$\begin{aligned} \text{a Sum of scores} &= 2 + 2 + 1 + 2 + 0 + 3 + 2 + 1 + 1 + 4 + 1 + 0 \\ &= 19 \end{aligned}$$

$$\text{Number of scores} = 12$$

$$\begin{aligned} \text{Mean: } \bar{x} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{19}{12} \\ &= 1.58333 \dots \\ &\approx 1.6 \text{ (rounding to one decimal place)} \end{aligned}$$

The mean is about 1.6.

Note that the value of the mean is at the centre of the scores.

- The modes are 1 and 2. Both occur most often (with a frequency of 4 each).

Example 4

Alex's scores in 8 games of ten-pin bowling were:

88 149 153 147 156 168 135 122

- a Find the mean.
- b Find the mode.

Solution

$$\begin{aligned} \text{a Sum of scores} &= 88 + 149 + 153 + 147 + 156 + 168 + 135 + 122 \\ &= 1118 \end{aligned}$$

$$\begin{aligned} \bar{x} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{1118}{8} \\ &= 139.75 \end{aligned}$$

The mean is 139.75.

- b There is no mode, because every score occurs the same number of times (once).

Exercise 10-05 The mean and mode

- 1 A group of 8 children were surveyed about the amount of pocket money (in dollars) that they received each week. The results were:

See Example 3

20 32 32 40 18 32 18 50

- a Find the mean of this set of data.
 - b Find the mode.
- 2 For each set of data, find:
- i the mean (rounded to two decimal places, if needed)
 - ii the mode(s).
- a 1 2 3 3 5 3 2 3 1
 - b 6 9 2 1 2 9 2
 - c 67 43 89 65 54 86 45 76 53
 - d 45.1 45.0 45.4 45.1 45.8 44.6
 - e 3 3 4 5 5 6 7 9 10
- 3 a Find the mode of this set of data:
blue, green, yellow, green, blue, red, green, yellow, red, green, red, blue
- b Why is it not possible to find the **mean** of this set of data?
- 4 a Find 5 scores that have a mean of 7 and a mode of 4.
- b Find 8 scores that have a mean of 10 and a mode of 12.

See Example 4

- 5 In a gymnastics competition, the judges awarded the following scores out of 10:

7.0 6.1 8.2 8.8 6.1 9.7 6.1 8.8

- Calculate the mean of these scores.
- Find the mode.
- Which measure (mean or mode) describes this set of scores better? Give a reason for your answer.

- 6 The ages of the members of the Phuong family (in years) are:

19 31 21 3 6 14 19 24 11

The ages of the members of the Arteri family (in years) are:

19 31 21 3 6 14 19 24 91

- What is the only difference between these two sets of data?
- Which family should have a higher mean age?
- Find the mean age (to one decimal place) for each family.
- What effect does the difference identified in part **a** have on the means?

Worked solutions

Exercise 10-05

MAT07SPWS10056

- 7 A group of six students was surveyed on the number of phones owned in their households. The results were

3 2 4 3 2 $\square$

where $\square$ represents a missing score.

What is the value of $\square$ if the mean of the results is 2.5?

Select the correct answer **A**, **B**, **C** or **D**.

- A** 1 **B** 2 **C** 2.5 **D** 3

- 8 Tanika scored 68%, 73%, 80% and 75% in her last four maths tests.

- Calculate her mean maths mark.
- Find how much Tanika needs to score in her next maths test to increase her mean to 75%.

- 9 Jamie, Sam, Karly and Tess all work at the mall on Saturdays. Jamie earns \$48 while Tess earns \$90. If the mean of the four wages is \$75, find possible values for Sam's and Karly's wages.
- 10 The mean point score of a basketball team for the 30 games they played during the season is 85. What is the total number of points the team scored for the season?

Investigation: Finding the middle scores

- 1 a Arrange these scores in ascending order:

5 4 3 8 7 1 7

- b Cross out the first and last score from your sorted list.

- c Cross out the second and second-last score.

- d Keep crossing out pairs of scores from both ends until you find the middle score.

- 2 a Arrange these scores in ascending order:

10 3 6 2 6 8 2 10 9 8

- b Keep crossing out pairs of scores at both ends of your sorted list until you find the two middle scores.

- 3 a Arrange these scores in ascending order:

9 5 5 10 4 6 6 3

- b Are there one or two middle scores? How can you tell?

- c Find the middle score(s).

- 4 a Does this set of data have one or two middle scores? How can you tell?

8 11 15 18 20 24 27 39 44

- b Find the middle score(s).

- 5 If a set of data has an odd number of scores, how many middle scores does it have?

- 6 a How many scores are there in this sorted data set?

2 5 9 12 17 18 27 35 39 41 45

- b Is your answer to part a an odd number or an even number?

- c What number is half of the number of scores?

- d Is the middle score the 5th, 6th or 7th score?



10-06 The median and range

The median

Worksheet

Mean, median, mode 1

MAT07SPWK10083

Puzzle sheet

Mean, median, mode 2

MAT07SPPS10039

Skillsheet

Statistical measures

MAT07SPSS10033

Homework sheet

Statistics 2

MAT07SPHS10038

Like the mean and the mode, the **median** is a measure of the centre of a set of data. It is the middle score or the average of the two middle scores.

Summary

When scores are ordered, the **median** is:

- the middle score if there is an odd number of scores
- the average of the two middle scores if there is an even number of scores.

Think: 'Median' sounds like 'medium', which is halfway between small and large.

A 'median' strip is the strip of grass down the *middle* of a highway.

The range

The **range** is a measure of the spread of a set of data, and is simply the difference between the highest and lowest scores.

Summary

Range = highest score – lowest score

Example 5

Tahir scored the following number of runs in a series of cricket matches:

35 98 17 54 2 22 51 45 86

Find:

- a** the median **b** the range.

Solution

- a** First, rewrite the scores in order.



The median of the data is 45.

- b** Range = Highest score – lowest score
 $= 98 - 2$
 $= 96$

Note that the value of the median is at the centre of the scores.

Example 6

The lap times (in seconds) of six cyclists were:

13.5 23.1 10.2 18.4 11.9 9.3

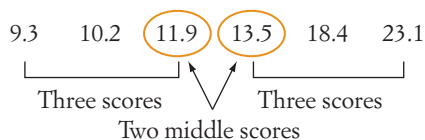
Find:

a the median

b the range

Solution

a Rewrite the scores in order.



$$\begin{aligned}\text{Median} &= \frac{11.9 + 13.5}{2} \\ &= 12.7\end{aligned}$$

12.7 is halfway between 11.9 and 13.5

b Range = Highest score – lowest score

$$\begin{aligned}&= 23.1 - 9.3 \\ &= 13.8\end{aligned}$$

Exercise 10-06 The median and range

1 For each set of data, find:

i the median **ii** the range

a 23 20 25 22 20 21 22

b 5.5 4.5 3.4 5.3 4.9

c 7 8 3 6 5 3 5 5 4

2 The ages of the members of the Carrozza family are:

7 10 12 42 47

The ages of the members of the Binns family are:

7 10 12 38 47

a What is the only difference between these two sets of data?

b Find the median age for each family.

c Find the range of the ages for each family.

d What effect does the difference identified in part **a** have on the medians and ranges?

3 For each set of data, find:

i the median **ii** the range

a 10 8 6 4

b 36 40 38 37 40 30

c 12 13 11 14 10 15 11 12

See Example 5

See Example 6

- 4 The favourite party food for a group of three-year-old children was recorded:

popcorn	fruit	chocolate	fruit	popcorn
chocolate	chocolate	fruit	fruit	fruit

Which is the only statistical measure that can be found for this data?

Select the correct answer **A**, **B**, **C** or **D**.

A mean **B** median **C** mode **D** range

- 5 Which of the following is the median of the scores below?

Select the correct answer **A**, **B**, **C** or **D**.

9 2 4 9 5 3 10

A 6 **B** 9 **C** 5 **D** 4

- 6 **a** Find 5 scores that have a median of 7 and a range of 16.

b Find 8 scores that have a median of 12 and a range of 9.

- 7 Eleven houses were sold in Keswick Street. The selling prices are listed below:

\$620 000	\$625 000	\$700 500	\$738 000	\$625 000
\$1 800 000	\$598 000	\$612 000	\$696 500	\$720 000
\$705 000				

a Find the median price.

b Find the range.

c Calculate the mean price, correct to the nearest dollar.

d Find the mode.

e Which measure (mean, mode or median) best describes this set of house prices? Give a reason for your answer.

Worked solutions

Exercise 10-06

- 8 A group of 8 friends counted the number of letters in their surnames. The results were:

4 6 7 5 4 6 9 □

where □ represents a missing score.

Find a possible value of □ if the median is 6 and the range is 5.

- 9 Alf's golf scores were (in order):

75 75 75 75 76 76 76 77 77 77

Mike's golf scores were (in order):

73 73 74 75 75 76 76 77 79 79

a Calculate the mean score for each golfer.

b Which golfer should have a higher range of scores?

c Calculate the range for each golfer.

d In golf, the lower score is the better score. Who is the more talented golfer? Explain your answer.

e Who is the more consistent golfer? Explain your answer.

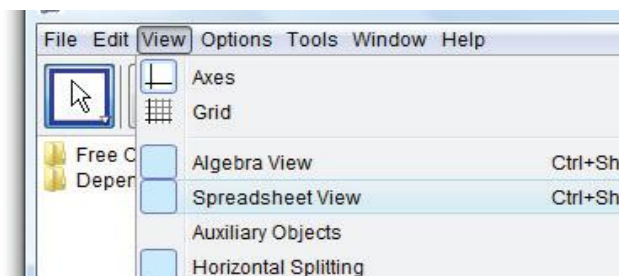
f Find the median score for each golfer.

MAT07SPWS10057

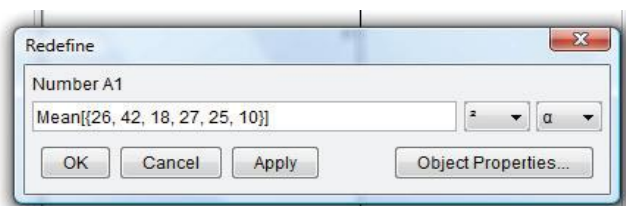
Technology Mean, median and mode

GeoGebra can be used to determine the mean, median and mode of a set of data.

- 1 Click **View** and **Spreadsheet view**.



- 2 To calculate the **mean**, enter **me** in cell A1. You should see the complete word **Mean[]**. Enter the data set, with commas: **Mean[26,42,18,27,25,10]**. Click **Enter**.
- 3 Double-click on cell A1. You will see what you entered.



- 4 To calculate the **median**, in cell A2 enter **median[26,42,18,27,25,10]**. Click **Enter**. Double-click on A2 to see the formula entry in full.
- 5 To calculate the **mode**, in cell A3 enter **mode[1,2,2,3,3,6]**. Press **Enter**. It allows for more than one mode. *Note:* If there is no mode, such as in **mode[5,6,7,8]**, it will return an empty set, written as {}.
- 6 Survey the students in your class and collect data such as:
 - height
 - number of hours slept last night
 - number of children in family
 - number of letters in surname
 Use GeoGebra to calculate the mean, median and mode for each set of data.
- 7 Analyse each data set from question 6.
 - a Are there any outliers?
 - b Is the data clustered around specific values?
 - c What other conclusions can you make from the mean, median and mode for each set of data?

Investigation: Michael's family

Suppose that Michael's family decided to have a family photograph and to record the ages and heights of everyone at that time.

	Age	Height
Father	35	177 cm
Mother	33	170 cm
Big brother	11	150 cm
Big sister	10	145 cm
Michael	6	131 cm
Little brother	4	118 cm



Answer the following questions about Michael's family. (Round your mean answers to two decimal places.)

- Find the mean age of Michael's whole family.
- Find the mean age of the children in Michael's family.
- What is the median age of Michael's whole family? Who is this age?
- What is the median age of the children in Michael's family? Who is this age?
- Is the mean or the median affected more when the parents' ages are not counted? What is the difference in each case?
- Predict what would happen to the mean age if Grandpa (aged 75) came to live with the family. Test your prediction.
- What was the family's mean age two years ago? How does this compare to the family's mean age now?
- Compare the mean height with the median height of the whole family. Are there any outliers?
- Compare the mean height and the median height of the children in the family. Are there any outliers?
- Why is the mode not useful in this case?
- Cousin Lee has come to stay with the family, and the mean height is now 148 cm. What is Lee's height?

10-07 Analysing dot plots and stem-and-leaf plots

The mean, mode, median and range can be found from data displayed on dot plots and stem-and-leaf plots.

TLF learning object
Stem-and-leaf graphs
(L5911)

Video tutorial

Dot plots

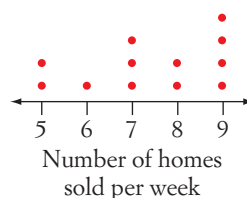
MAT07SPVT10021

Example 7

This dot plot shows the number of homes sold per week by a real estate agency over 12 weeks.

Find:

- the range
- the mode
- the median
- the mean, correct to one decimal place.



Solution

a Range = Highest score – lowest score
 $= 9 - 5$
 $= 4$

b The mode is the score with the most dots.
 Mode = 9

c There are 12 scores (12 dots). This is an even number, so there are two middle scores (the 6th and 7th scores). By counting the dots, or by crossing out pairs of dots at each end, we can see that the 6th and 7th scores (circled on the right) are 7 and 8 respectively.

$$\text{Median} = \frac{7 + 8}{2}$$

$$= 7.5$$

d Mean: $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$

$$= \frac{2 \times 5 + 1 \times 6 + 3 \times 7 + 2 \times 8 + 4 \times 9}{12}$$

$$= \frac{10 + 6 + 21 + 16 + 36}{12}$$

$$= \frac{89}{12}$$

$$= 7.41666 \dots$$

$$\approx 7.4$$



The mean is approximately 7.4.

Note that the mean and the median are close to each other and at the centre of the scores.

Example 8

Video tutorial

Stem-and-leaf plots

MAT07SPVT10022

This stem-and-leaf plot shows the number of people joining the MyFace website each day over 20 days.

Find:

- a** the range **b** the mode **c** the median
d the mean, correct to one decimal place.

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 4 8
7	4
8	0 1 3 4
9	2 5 7 8

Solution

a Range = Highest – lowest
 $= 98 - 53$
 $= 45$

b Mode = 61

- c** There are 20 scores, so the median is between the 10th and 11th scores (64 and 68).

$$\text{Median} = \frac{64 + 68}{2}$$

$$= 66$$

d Mean: $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$

$$= \frac{53 + 54 + 57 + 57 + \dots + 95 + 97 + 98}{20}$$

$$= \frac{1437}{20}$$

$$= 71.85$$

The most frequent score

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 (4) (8)
7	4
8	0 1 3 4
9	2 5 7 8

Note that the mean and the median are close to each other and at the centre of the scores.

The mean is 71.85.

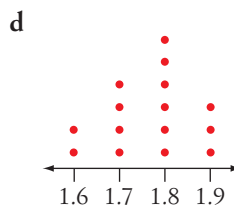
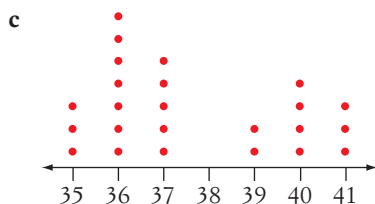
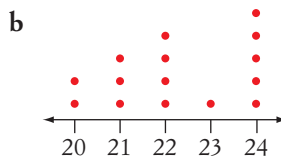
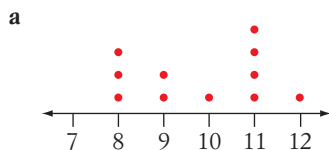
Exercise 10-07 Analysing dot plots and stem-and-leaf plots

In this exercise, round mean values to one decimal place where necessary.

See Example 7

- 1 For each dot plot, find:

- i** the range **ii** the mode **iii** the median **iv** the mean



Extra questions

Analysing data

MAT07SPEQ00008

2 For each stem-and-leaf plot below, find:

See Example 8

- i the range ii the mode iii the median iv the mean

a

Stem	Leaf
1	0 2 3
2	1 4 4 5 6
3	3 3 3 7
4	1 2 3 5 9

b

Stem	Leaf
7	3 4 5 7 7
8	1 2 2 3 8 9
9	0 4 4 4 4 6 7 9

c

Stem	Leaf
10	0 1 1 2
11	7 8 8 8 9
12	3 6 6 7
13	1 1 1 1 1 8
14	0

d

Stem	Leaf
0	5 5 6
1	4 4 7 7 7 7 7
2	0 3 8 8
3	
4	9 9 9

3 The maximum daily temperatures (in °C) in Cowra over a fortnight were:

10 12 10 15 14 15 11
10 19 14 11 10 11 15

Illustrate this data on a dot plot and use it to find:

- a** the median **b** the mode **c** the mean **d** the range

4 The quiz marks out of 10 for two Year 8 classes are shown below:

8 Huxley: 3 2 0 1 5 8 6 7 6 3
 5 4 5 6 7 9 2 5 7
8 Crancher: 7 6 3 7 8 1 9 4 6 7 2
 7 2 8 10 9 9 5 7 8 9 10

- a** Draw a dot plot for the data of each class.
b What is the mode of the marks for 8 Huxley?
c What is the median of the marks for 8 Crancher?
d What is the range of the marks for 8 Huxley?
e Calculate the mean for 8 Crancher, correct to one decimal place.
f Which do you think is the 'better' class? Give a reason for your answer.

5 This stem-and-leaf plot shows the number of students buying from the school canteen each day over a three-week period.

- a** What is the range for this data?
b Find the median.
c What are the modes?
d Calculate the mean.

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

6 This stem-and-leaf plot shows the heart rates (in beats per minute) of people riding on a rollercoaster at a theme park.

Find:

- a** the mode
b the range
c the median
d the mean

Stem	Leaf
4	0
5	2 3 5 8 9
6	0 1 1 2 4 4 6 7 7 7 8 8 9
7	0 1 3 3 7 9
8	2 5 6 7
9	1

Worked solutions

Exercise 10-07

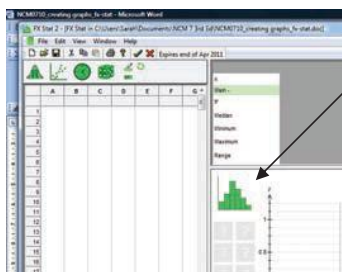
MAT07SPWS10058

Technology Dot plots and stem-and-leaf plots

Graphing software such as *Fx-Stat* can be used to analyse dot plots and stem-and-leaf plots.

Dot plots

- 1 Open *Fx-Stat* and click on the graph in the middle of the screen.



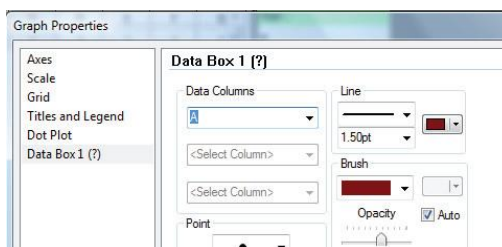
Click on the diagram of the **dot plot**.

- 2 The data to be entered are the marks (out of 10) of 15 students in a History quiz.

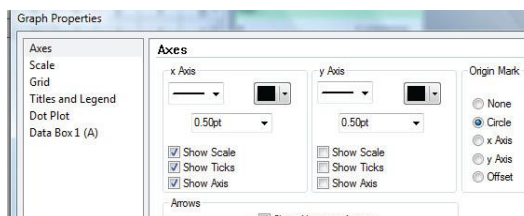
8 5 6 7 7 8 9 10
5 4 6 7 8 9 8

Enter the marks into column A. Then click on the question mark symbol (under the dot plot symbol in the middle of the screen) and then **Data Columns**. Choose column A (where the marks have been entered).

	A	B
	x	x
1	8	
2	5	
3	6	
4	7	
5	7	
6	8	
7	5	



- 3 Now click on **Scale** and enter x -values min:0, max:10 and y -values min:0, max:10. Also click on **Axes** and remove ticks on y -Axis for 'Show Scale' 'Show Ticks' and 'Show Axis' (as shown below).



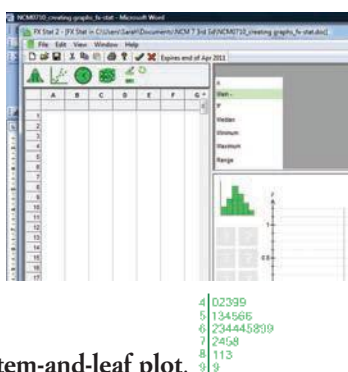
- 4 Click on '**Titles and Legend**' and enter an appropriate title for the dot plot.
- 5 The mean (shown by $\bar{x}$), median and range are displayed above the graph.

	A
n	15
Main -	
$\bar{x}$	7.13333333
Median	7
Minimum	4
Maximum	10
Range	6

- 6 Click on the green tick  to finish your graph.
- 7 Now return to Exercise 10-07 and use *Fx-Stat* to graph any dot plots in it.

Stem-and-leaf plots

- 1 Open *Fx-Stat* and click on the graph in the middle of the screen.



Click on the diagram of the **stem-and-leaf plot**.

- 2 The data to be entered are the daily temperatures for the first 20 days in January 2011 in Adelaide. *Source:* bom.gov.au

29	26	26	30	29	35	36	28	32	32
26	27	22	28	32	27	24	27	29	37

- 3 Enter the marks into column A. Then click on the question mark symbol (under the stem-and-leaf plot symbol in the middle of the screen) and then **Data Columns**. Choose column A (where the marks have been entered).
- 4 Now click on **Stem and Leaf** and tick **Reverse Stem Order**. Enter **Max. Number of stems** as 4.
- 5 Click on **Titles and Legend** and enter an appropriate title for the stem-and-leaf plot.

- 6 The mean (shown by $\bar{x}$), median and range are displayed above the graph. Answer these questions:
- What is the mode?
 - What is the mean?
 - What is the median temperature?
 - Are there any outliers? If so, write down their values.
- 7 Using your analysis of the data in question 6, write a summary describing the temperatures experienced for the 20 days.
- 8 Click on the green tick  to finish your graph.
- 9 Now return to Exercise 10-07 and use *Fx-Stat* to graph any stem-and-leaf plots from it.

10-08 Comparing data sets

Homework sheet

Statistics 3

MAT07SPHS10039

When we analyse data, we try to describe or summarise the information. This allows us to notice patterns and trends and to draw some conclusions from them. We can use the mean, mode, median and the range to make comparisons between sets of data.

Homework sheet

Statistics revision

MAT07SPHS10040

Technology worksheet

Excel
Daily rainfall

MAT07SPCT10006

Statistical measure	Features	When is it appropriate to use?
Mean $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$	- Depends on all the scores in the data set. - Affected by outliers.	When the data set does not have many extreme scores (outliers).
Mode Most common score(s)	- There may be more than one mode, or no mode at all. - Not affected by outliers.	When the most common score or category is needed.
Median Middle score, or average of the two middle scores, when scores are arranged in order	- Can be one of the scores. - Not affected by outliers.	When the data set has extreme scores (outliers).
Range Highest score – lowest score	Depends on the highest and lowest scores only.	When a measure of spread is needed.

The mean, mode and median are called **measures of location** (averages) while the range is a **measure of spread**.

Example 9

The results of a survey investigating the number of boys and girls who visited a shopping mall each day over a fortnight are as shown below.

Boys: 105, 76, 97, 88, 114, 86, 124, 102, 111, 97, 96, 81

Girls: 78, 102, 99, 89, 113, 116, 99, 108, 98, 116, 114, 97

- Show this information on a back-to-back stem-and-leaf plot.
- Calculate the range for each set of data.
- Find the median for each set of data.
- Comment on the differences between the data for boys and girls.

Solution

- A back-to-back stem-and-leaf plot combines two stem-and-leaf plots, sharing the same stem.

Boys		Girls
6	7	8
8 6 1	8	9
7 7 6	9	7 8 9 9
5 2	10	2 8
4 1	11	3 4 6 6
4	12	

- Boys: Range = $124 - 76$
= 48

$$\text{Girls: Range} = 116 - 78 \\ = 38$$

- Boys: Median = 97

$$\text{Girls: Median} = \frac{99 + 102}{2} \\ = 100.5$$

- The boys' data is more spread out but the median for the girls is higher.

Exercise 10-08 Comparing data sets

- Two groups of students were given the following marks out of 10 for their PE assignment:

Group A: 5 5 5 5 6 6 6 7 7 7 8

Group B: 3 3 4 5 5 6 6 7 9 9 10

- Calculate the mean (to one decimal place) and the median of each group's marks.
- Draw a dot plot for each group and hence describe the differences between the way each group has its marks spread out.
- Group C was given the following marks, but one person was away.

Group C: 4 5 5 6 6 7 7 7 10

What mark would the absent student need in order to give this group the same mean as the other two groups? (Hint: What total does each group need?)

See Example 9

- 2 Two different brands of batteries were tested in the same toy to determine which lasted longer. The back-to-back stem-and-leaf plot shows the two sets of data, recorded to the nearest hour.

- a How many batteries of each brand were tested?

- b Find the mean, median and range for each set of data.

- c Comment on the differences between Dynamo batteries and Energy Plus batteries.

- d Which brand do you think is better? Explain your answer.

	Dynamo		Energy Plus
	4 1	1	3 3 7 7 8 8 9
	8 8 6 3	2	5 6 9 9 9 9 9
8 7 7 6	6 5 4 4	3	2 4 6 7 7 8 8 9 9
6 5 5	4 3 2 2	4	0 1
	3 2 0 0	5	

- 3 The 3D-TV MegaStore recorded the weekly number of sales at two stores over a 20-week period.

Hurstville: 34, 43, 45, 55, 66, 71, 78, 35, 83, 86, 94, 81, 75, 68, 66, 96, 34, 66, 71, 83

Penrith: 96, 36, 86, 81, 35, 46, 38, 33, 56, 66, 66, 48, 54, 71, 81, 37, 48, 56, 55, 40

- a Display this information on a back-to-back ordered stem-and-leaf plot.

- b Calculate the mean, mode, median and range for each store.

- c Which store performed better? Explain your answer.

- d Which sales figure from the Penrith store is an outlier? Which of the measures calculated in part b are most affected by this outlier?

- 4 The marks out of 50 for the same English test, scored by two different classes of Year 7 students are listed below.

7 Murray marks: 35, 44, 40, 48, 47, 42, 47, 45, 38, 38, 31, 32, 38, 50, 43, 31, 49, 31, 47, 37, 48, 46, 29

7 Winton marks: 40, 41, 46, 47, 47, 36, 33, 32, 26, 39, 48, 44, 31, 35, 31, 29, 45, 48, 45, 29

- a Draw a back-to-back stem-and-leaf plot for these two data sets.

- b What is the highest mark overall?

- c What is the lowest mark overall?

- d In class 7 Murray where are the marks clustered?

- e Copy and complete: In 7 Murray the marks range from _____ to _____.

In 7 Winton the marks range from _____ to _____.

- f Find the mean and the median for each class.

- g Which do you think is the 'better' class? Give a reason why you think this.

- h Identify any outliers.

- 5 This back-to-back stem-and-leaf plot shows the number of goals scored in each match by two basketball teams during last season.

- a How many games did these teams play in one season?

- b Find the mean, median, mode and range for each team.

- c Comment on the differences between the Cobar Cougars and the Tilba Tigers.

- d Which team do you think is better? Explain your answer.

	Cobar Cougars		Tilba Tigers
	6 6 5 4 3	4	4 9
	8 8 3 0	5	2 3 3 6 8
8 8 6	6 3 1 1	6	5 6 8 9
	7 4 3 0	7	0 0 1 3 6
	6 6 5	8	2 5 7 7 9 9
	2 2	9	0 3 4

- 6 A basketball coach kept a record of the number of points scored by Maria and Stacey in each match during the first half of the season. The results were as shown below:

Maria: 41 38 25 19 53 35 30 32 39 45 46 37
Stacey: 35 30 30 24 37 25 37 44 20 29 29 35

- Display this data on a back-to-back ordered stem-and-leaf plot.
- Calculate the mean (correct to one decimal place), mode, median and range for each player.
- Who do you think is the more consistent player? Explain.
- Who is the higher scorer?
- Who do you think is the better player? Why?

Worked solutions

Exercise 10-08

MAT07SPWS10059

Power plus

- The mean of five numbers is 24. If a sixth number is added, the new mean of the six numbers is 21. Find the sixth number.
- A small class obtained these results in an exam:
66 68 74 76 82 79
 - Find the mean of these marks.
 - The teacher realised there was an error in the marking and added three more marks to each score. Find the mean of the new scores.
 - What effect did the extra three marks have on the mean?
 - What effect does adding or subtracting the same number for all of the data have on the mean?
- Melissa sat five exams. Her average mark was 74%. What mark should Melissa obtain in the sixth exam if she wishes her average mark for the six exams to be 77%?
- Five data scores were collected, but the figures were lost. The mean of the data was 8 and the median was 9.
 - What was the total of the data?
 - What could the data have been?
 - If you are now told that the range is 7, what could the set of data have been?
- Four sisters work at the same bank. One earns \$500 per week and another earns \$800 per week. The mean weekly wage of the four sisters is \$2000. Is this possible? How?

Chapter 10 review

Language of maths

Puzzle sheet

Data crossword

MAT07SPPS10040

analyse	dot plot	measure of location	outlier
average	extreme	middle	range
cluster	mean	mode	spread
data	median	ordered	stem-and-leaf plot

- 1 Look up the word 'average' in the dictionary. What does it mean?
- 2 What is another name for an extreme score?
- 3 Why are the dot plots and stem-and-leaf plots called 'plots' rather than 'graphs'?
- 4 Which word is an example of a measure of **spread**?
- 5 Explain what is meant by a **cluster** of scores.
- 6 What is a 'median strip'? Why do you think it has this name?

Topic overview

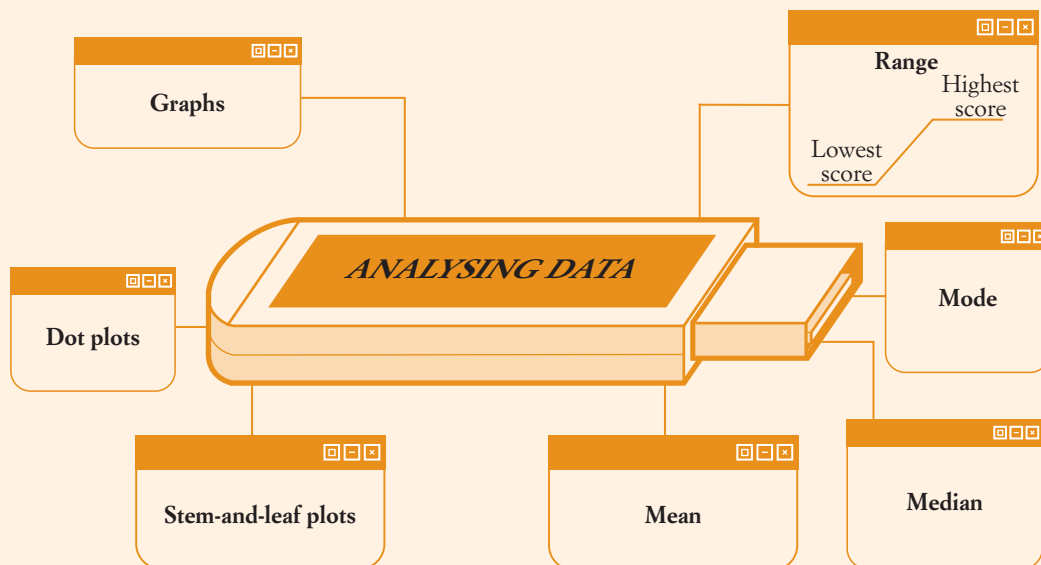
Worksheet

Mind map: Analysing data

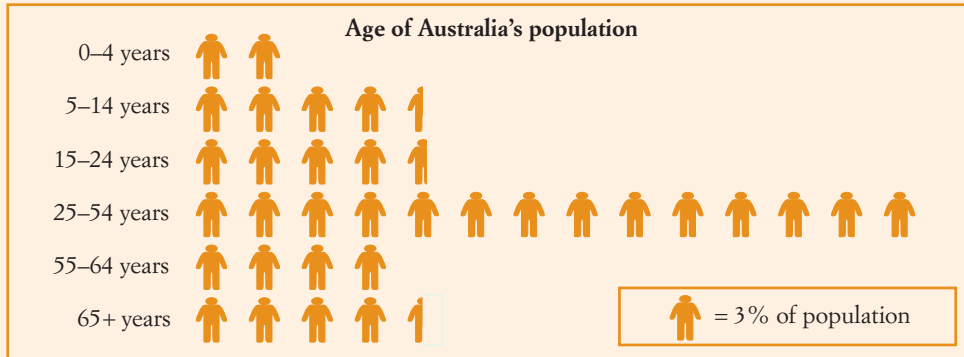
MAT07SPWK10085

- Write about what you have learnt in this chapter.
- Was this work new to you? If not, in what subject have you studied it?
- Did you have any difficulties? Discuss them with a friend or your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



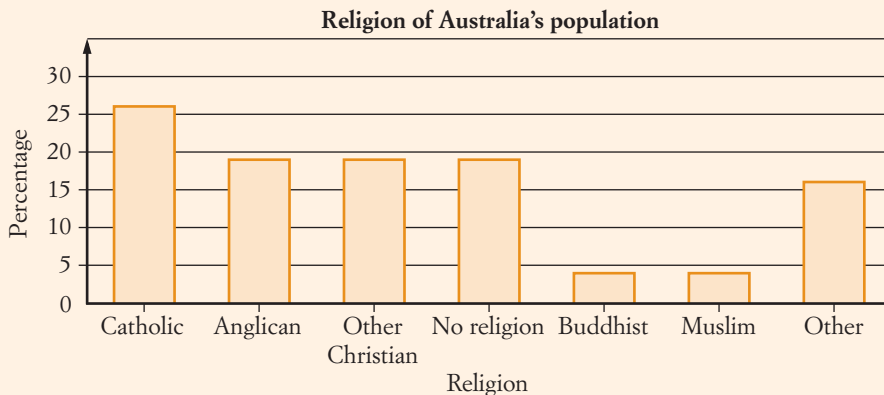
1



See Exercise 10-01

- What age group has the smallest percentage of the population?
- What percentage are in the 15–24 years age group?
- Why might the 25–54 years group be the largest?

2

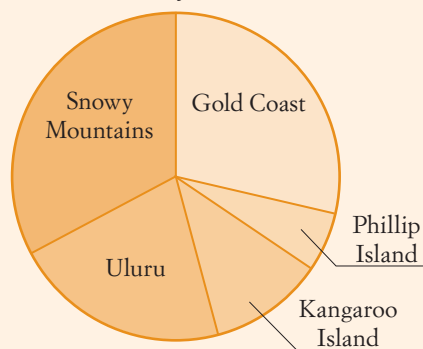


See Exercise 10-01

- What percentage of the population is Catholic?
- Which religions are followed by 19% of the population?
- How might this graph have been different in 1950?

3 This sector graph shows the favourite holiday destinations of 80 people surveyed at a city shopping centre.

Favourite holiday destinations



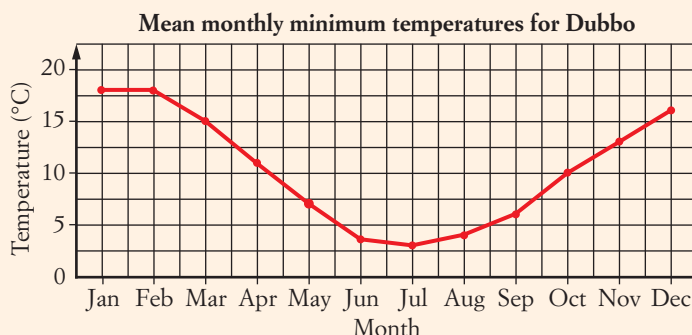
See Exercise 10-01

- What is the most popular destination?
- Which holiday destination was preferred by 15 people?
- Estimate how many people preferred the Gold Coast.
- True or false: Fewer than 10 people preferred Phillip Island.

Chapter 10 revision

See Exercise 10-01

- 4 The graph below shows temperature data for Dubbo.



- Which month(s) has the highest mean minimum temperature? What is that temperature?
- Name two months with the same mean minimum temperature.
- Which month has a mean minimum temperature of 10°C?
- Between which two months is the smallest drop in mean minimum temperature?

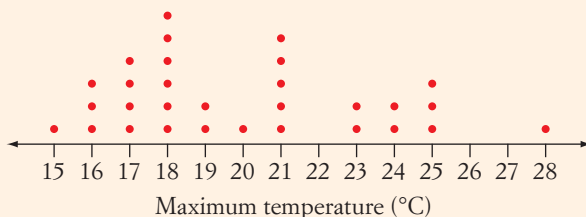
See Exercise 10-02

- 5 This line graph shows the yearly profits of a company over a period of six years.
- What is incorrect about this graph?
 - What misleading impression does this give?
 - Redraw the graph correctly.



See Exercise 10-03

- 6 The daily maximum temperatures (in °C) at Bega during April are shown in this dot plot:



- What is the mode?
- What is the outlier?
- On how many days was the temperature 25°C?
- On what fraction of days did the temperature drop below 20°C?

- 7 This stem-and-leaf plot shows the masses of the players of a rugby league team, in kilograms.

See Exercise 10-04

Stem	Leaf
7	9 5 8 1 6 9
8	4 2 5 0 0 8 0 3 2 4 2 8
9	0 8 4 2 2 0 0 8 0 5 6
10	5 4

- How many players are on the squad?
- Present this information as an ordered stem-and-leaf plot.
- Which mass occurs most often?
- Which is the lowest mass?

For the rest of this exercise, round mean values to two decimal places where necessary.

- 8 For each set of data, find:

See Exercise 10-05

- the mean
 - the mode
- a 4 3 2 5 6 4 4 b 6 12 11 12 10 6 6 10 6 c 8 4 1 1 4 1 3 6

- 9 For each set of data in question 8, find:

See Exercise 10-06

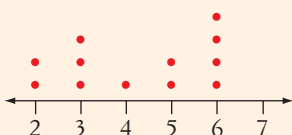
- the median
- the range

- 10 For each plot below, find:

See Exercise 10-07

- the range
- the median
- the mode
- the mean

a



b

Stem	Leaf
4	0 0 1 2
5	1 3 4
6	6 6 6 7 8
7	4 5

- 11 The assignment marks for 20 girls and 20 boys are as follows:

See Exercise 10-08

Girls	75	28	37	35	60	73	69	52	94	66
	55	39	48	51	53	18	29	76	59	83
Boys	88	29	38	72	50	74	73	30	85	10
	28	93	66	17	75	40	55	62	73	58

- Construct a back-to-back ordered stem-and-leaf plot for the data.
- What was the highest mark? Who scored it, a boy or a girl?
- Find the mean, median, mode and range for each group.
- Comment on the differences between the girls and the boys on this assignment.
- Which group of students seemed to be more consistent in the marks they scored, the girls or the boys? Give reasons for your answer.

Mixed revision 3

See Exercise 9-03

- 1 Draw a number plane and mark the following points.

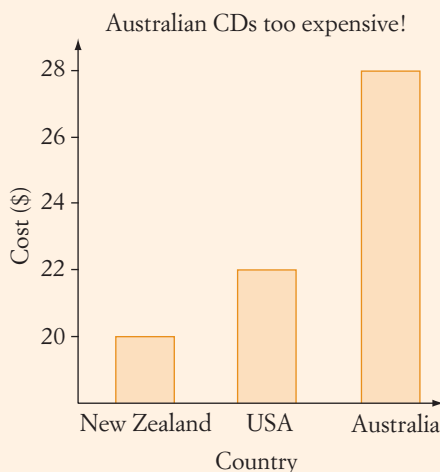
$A(1, 3)$ $B(-2, 2)$ $C(3, -4)$ $D(-2, -3)$ $E(0, 7)$ $F(-3, 0)$

See Exercise 7-01

- 2 Draw a number line labelled from 0 to 3, and mark these decimals on it.
0.5, 2.25, 1.3, 2.9, 0.75, 1.6

See Exercise 10-02

- 3 a Explain why the graph below gives a misleading impression.
b Redraw the graph correctly.



See Exercise 8-01

- 4 Copy and complete the following.

a 2000 mg = _____ g	b 2 t = _____ kg
c 56 days = _____ weeks	d 6.5 L = _____ mL
e 3 days = _____ hours	f 4 kg = _____ g
g 1500 g = _____ kg	h 3000 mL = _____ L
i 480 s = _____ min	j 2500 kg = _____ t
k 9.5 g = _____ mg	l 12000 000 mg = _____ kg

See Exercise 10-03

- 5 a Construct a dot plot for the following scores, which are marks in a test out of 25:

11	8	12	12	15	13	10	12	13	15	16
22	12	11	11	10	13	16	12	15	12	11

- b Where are the scores clustered?
c Identify any outliers.

6 Organise this set of scores into an ordered stem-and-leaf plot:

See Exercise 10-04

9	5	15	22	11	14	19	28	18
13	17	6	24	16	15	4	10	21

7 Find:

See Exercise 7-03

a $1.54 + 7.9$	b $9.2 - 6.1$	c $22.6 - 13.55$
d $6.31 + 0.2 + 15.38$	e $0.003 + 1.01 + 4.394$	f $9.803 - 4.52$

8 Evaluate each product.

See Exercise 7-06

a 6.2×0.7	b 4.21×0.3	c 0.05×1.4

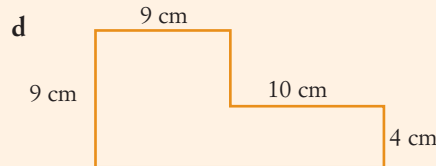
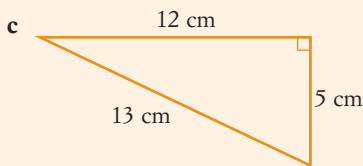
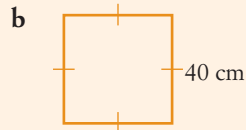
9 Evaluate each quotient.

See Exercise 7-07

a $60.138 \div 6$	b $1.672 \div 0.02$	c $4.032 \div 0.3$

10 Find the perimeter of each shape.

See Exercise 8-02



11 Find the area of each shape in question 10.

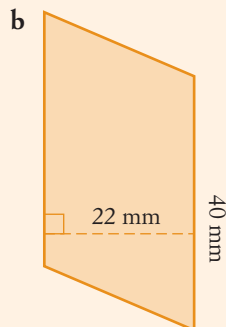
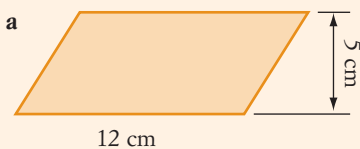
See Exercise 8-07

12 A sheet of A4 paper has a width of 21.1 cm and a length of 29.7 cm. Find its perimeter.

See Exercise 8-02

13 Find the area of each parallelogram.

See Exercise 8-06



Mixed revision 3

- See Exercise 9-04 **14** Plot the ordered pairs for this table of values on a number plane, with both axes extending from -4 to 4 . Use a ruler to join the points.

x	-1	0	2	3
y	-4	-2	2	4

- See Exercise 7-09 **15** Convert each fraction to a decimal.

a $\frac{1}{5}$

b $\frac{3}{8}$

- See Exercise 7-09 **16** Convert $\frac{5}{9}$ to a recurring decimal.

- See Exercise 7-10 **17** Round:

a 0.753 to one decimal place

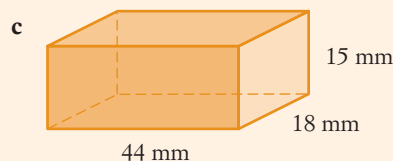
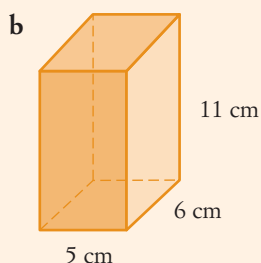
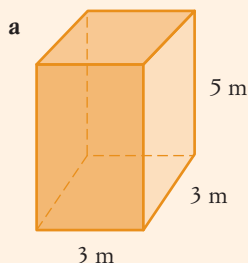
b 12.064 to two decimal places

c $\$9.8217$ to the nearest cent

d 3.795 to the nearest tenth

- See Exercise 7-11 **18** **a** If the temperature at Penrith was 20.6°C at 9:00 a.m. and was 29.3°C at 3:00 p.m., by how much did it rise?
b A piece of cloth 2.5 m long was cut into 10 equal pieces. How long was each piece?
c Sandra bought 40 litres of LPG at 79.9c per litre. Catherine bought 40 litres at 87.9c per litre. How much more than Sandra did Catherine pay?

- See Exercise 8-10 **19** Find the volume of each rectangular prism.



- See Exercise 8-11 **20** Convert 7500 L:

a to kL

b to m^3

- See Exercise 8-11 **21** Find the capacity of a box measuring 48 cm by 33 cm by 17 cm:

a in mL

b in L

22 For each of the following sets of scores, find:

See Exercise 10-05

- i the mean
- ii the mode

- a** 32 30 31 30 35 30 29
b 3.3 3.5 3.3 3.8 3.4 3.5 3.9

23 For each set of scores in question 22, find:

See Exercise 10-06

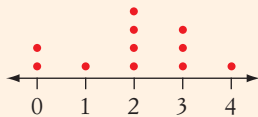
- i the median
- ii the range

24 For each of the sets of data below, find:

See Exercise 10-07

- i the range
- ii the mode
- iii the median
- iv the mean (to two decimal places).

- a** Number of goals scored **b** Ages of soccer players



Stem	Leaf
1	5 7 9 9
2	1 1 1 2
3	0

- 25 a** A rectangle has an area of 20 cm^2 . If its length is 5 cm, how wide is it?
b A square has an area of 64 cm^2 . Find its perimeter.

See Exercise 8-04



11

Statistics and probability

Probability

Many people suffer from aviophobia, the fear of flying. However, we are more likely to be involved in a car accident than a plane crash. There is a 1-in-5000 chance of being killed on a car trip compared to 1-in-11 million on a plane flight. This is amazing when you consider that each day worldwide there are 27 000 planes in the sky carrying over 4 million passengers. Why do you think there are so few plane accidents compared to road accidents? And why are people more afraid of flying than travelling in a car?



Chapter outline

	Proficiency strands			
11-01 Sample spaces	U			
11-02 Probability	U	F	PS	
11-03 The range of probability	U	F	PS	R
11-04 Experimental probability	U	F	PS	R
11-05 Complementary events	U	F	PS	R

Wordbank

certain Sure to happen; the opposite of impossible

complementary event The 'opposite' event; for example, the complementary event to selecting an Ace from a deck of cards is *not* selecting an Ace

equally likely To have exactly the same chance of occurring; for example, when tossing a coin, heads or tails are equally likely

event A result involving one or more outcomes; for example, the event of rolling an even number on a die contains the three outcomes {2, 4, 6}

outcome A single result from a situation or experiment; for example, one outcome when rolling a die is to roll a 6

random Describes a situation in which every possible outcome has an equal chance

sample space The set of all possible outcomes of a situation or experiment

In this chapter you will:

MAT07SPVT00009

Master class on probability

- construct sample spaces for single-step experiments with equally likely outcomes
- assign probabilities to the outcomes of events and determine probabilities for events
- recognise that probabilities range from 0 to 1
- compare observed frequencies across experiments with expected frequencies
- identify complementary events and use the sum of probabilities to solve problems

SkillCheck

StartUp assignment 11

MAT07SPWK10086

MAT07SPWK10087

Describing probabilities

MAT07SPWK00061

- 1** Rate each event as being impossible, unlikely, even chance, likely or certain.

- a Choosing a pink ball from a bag of blue balls
- b It will rain next month.
- c A coin shows tails when you toss it.
- d You obtain a driver's licence tomorrow.
- e You will have your driver's licence in 20 years time.
- f Rolling two sixes on a pair of dice
- g You are over 10 years old.
- h You draw a red card from a deck of playing cards.
- i You will use a computer today.
- j It will snow in your town tomorrow

- 2** Simplify each fraction.

- a** $\frac{2}{10}$ **b** $\frac{8}{18}$ **c** $\frac{27}{30}$ **d** $\frac{15}{80}$

- 3** Convert each number to a decimal.

- a** $\frac{7}{8}$ **b** $\frac{3}{5}$ **c** 26% **d** 70%

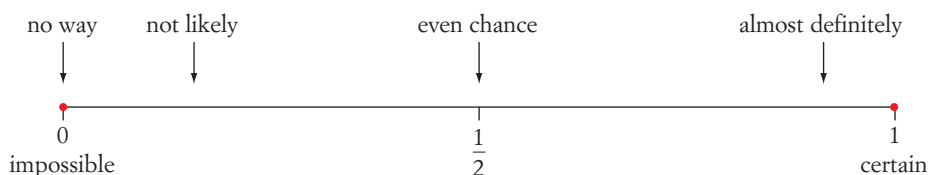
- 4** Convert each number to a percentage:

- a** 0.6 **b** 0.31 **c** $\frac{3}{4}$ **d** $\frac{1}{20}$

- 5** Is the chance of each event more than or less than $\frac{1}{2}$?

- a** You having another brother or sister one day
- b** You going overseas this year
- c** You being at school next Monday
- d** You being a parent in 20 years time
- e** You getting a good report for English this year
- f** Your home phone ringing today

- 6 a** Draw an interval 15 cm long and use it to make a probability scale for the chances from impossible to certain.



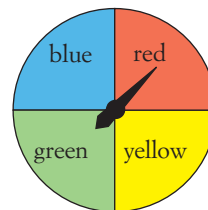
- b** On your scale, mark the position of each event described below.
- A** It will be hot tomorrow.
 - B** The Sun will rise tomorrow.
 - C** There will be floods in your town this year.
 - D** There will be no car accidents in Sydney tomorrow.
 - E** There are more than five people at your home at the same time today.
 - F** Someone at school has a birthday today.
 - G** It will rain tomorrow.
 - H** The Sharks will win their next game.
 - I** The next person to visit your classroom is male.
 - J** Your favourite song comes on your radio station in the next hour.
- 7** Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer **A**, **B**, **C**, or **D**.
- A** certain **B** definite **C** even chance **D** probable
- 8** Convert each number to a simplified fraction.
- a** 0.35 **b** 0.2 **c** 48% **d** 6%
- 9** Evaluate each expression.
- a** $1 - \frac{1}{6}$ **b** $1 - \frac{3}{4}$ **c** $1 - \frac{4}{10}$

11-01 Sample spaces

Probability is the branch of mathematics that studies the chances of different events occurring. In any situation, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

Example 1

- a** Write the sample space for this spinner.
- b** How many outcomes are possible?
- c** Is each outcome equally likely?



Solution

- a** The sample space is {red, yellow, green, blue}.
- b** There are 4 possible outcomes.
- c** Each coloured region is equal in size ($\frac{1}{4}$ of the circle), so each outcome is equally likely.

The language of probability

This table lists some important probability terms, and how they apply to Example 1.

Probability term	In Example 1
An experiment is a situation involving chance that leads to results called outcomes.	Spinning a spinner
A trial is one go or run of the experiment.	One spin of the spinner
An outcome is the result of an experiment.	The arrow landing on green
The sample space is the set of all possible outcomes.	{red, yellow, green, blue}
An event is one or more outcomes of an experiment.	The arrow landing on a 'traffic light' colour: red, yellow or green
In a random experiment, every possible outcome has the same chance of occurring.	All spins on this spinner are random because every colour has the same chance

Example 2

- How many outcomes are in the sample space when a die is rolled?
- Is each outcome equally likely?

'Die' is the singular word for 'dice': one 'die', two or more 'dice'



Solution

- The sample space is {1, 2, 3, 4, 5, 6}, so there are 6 possible outcomes.
- Each outcome is equally likely, because each number has the same chance of coming up.

Example 3

A jar contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random.

'At random' means that each lolly has an equal chance of being chosen



- How many lollies are in the jar?
- How many outcomes are in the sample space for the colour of the lolly?
- Is each colour equally likely?

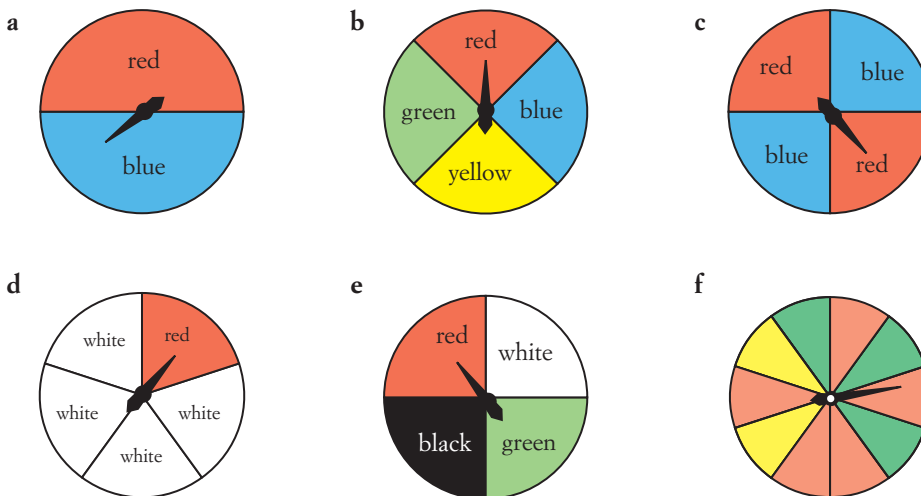
Solution

- Number of lollies = $5 + 3 + 6 + 2 = 16$
- 4 outcomes {red, green, yellow, blue}
- Each colour is not equally likely because there are different amounts of each colour. Some colours occur more frequently and have higher chances of being chosen.

Exercise 11-01 Sample spaces

- 1 For each spinner, write down the sample space and count the number of possible outcomes.

See Example 1



- 2 For each experiment, count the number of possible outcomes and state whether each outcome is equally likely.

See Example 2

- tossing a coin
- the result of a soccer game when Australia plays South Korea
- the first letter of a person's name
- the gender of a baby
- the last digit of a phone number
- the result of a driving test

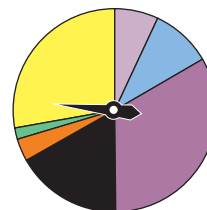
- 3 List the outcomes in each event.

- rolling an odd number on a die
- selecting a vowel from the letters of the alphabet
- having a house number greater than 4 but less than 10
- having a birthday in a month beginning with M
- living in a state capital city
- being in a primary school grade

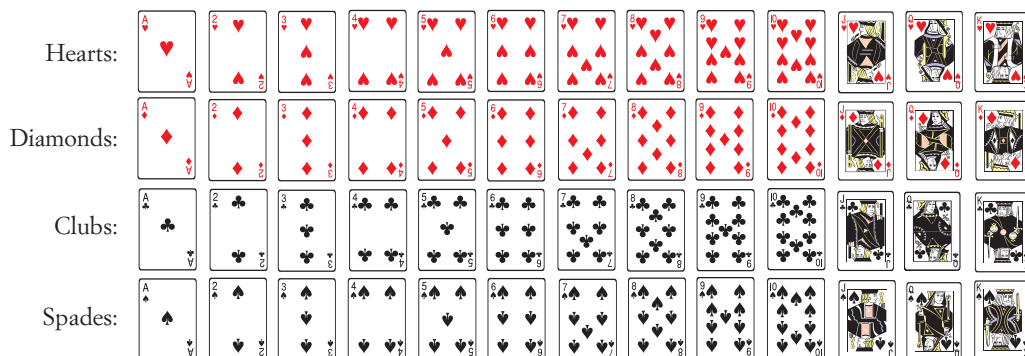
- 4 This spinner is spun.

- How many outcomes are possible?
- Is each outcome equally likely? Explain your answer.
- Which outcome is most likely to occur?
- Which outcome is least likely?

See Example 3

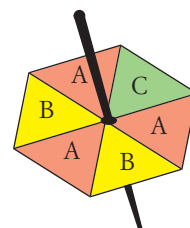


- 5 A money box contains four \$2 coins, three \$1 coins, two 50c coins, six 20c coins and five 10c coins. It is shaken and one coin falls out at random.
- How many coins are in the money box?
 - List the outcomes in the sample space for the type of coin.
 - Is each type of coin equally likely?
 - Which type of coin is most likely to fall out?
- 6 The 52 cards in a standard deck of playing cards are shown below, divided evenly into four suits: hearts, diamonds, clubs, spades.



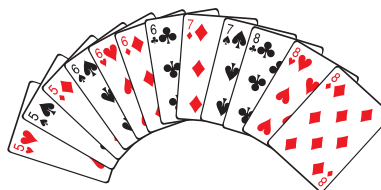
The cards are shuffled and one is taken out at random.

- How many outcomes are in the sample space?
 - Is each outcome equally likely?
 - How many cards are in each suit?
 - How many red cards are in the deck?
 - How many 7s?
 - How many black Kings?
 - How many 8 of diamonds?
 - How many cards with an even number?
 - How many picture cards (J, K, Q)?
 - How many red Aces?
- 7 A 3-digit number is to be formed from the digits 1, 2, 5. Which of the following answers shows the sample space? Select **A**, **B**, **C** or **D**.
- A** {1, 2, 5} **B** {125}
C {125, 152, 215} **D** {125, 152, 215, 251, 512, 521}
- 8 This spinner is spun.
- Write down the sample space of possible letters.
 - Is each letter equally likely?
 - Which letter is most likely?



9 Daniel is holding these 12 playing cards. Elvira picks a card without looking.

- How many 5s are held in the 12 cards?
- List the sample space of possible numbers.
- Which number is Elvira most likely to pick?
- List the sample space of possible colours.
- Which colour is Elvira more likely to pick?
- List the sample space of possible suits.
- Which suit is Elvira least likely to pick?



11-02 Probability

We can calculate the chance or probability of an event occurring as a fraction, percentage or a decimal.

Summary

$P(E)$ means 'the probability of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

or $P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

Worksheet

Probability problems

MAT07SPWK10088

Worksheet

Games of chance

MAT07SPWK10089

Puzzle sheet

Spinner game

MAT07SPPS10042

Example 4

A die is rolled. Find the probability of rolling:

- a a 5 b a 7 c an odd number d a number less than 3

Solution

The sample space is {1, 2, 3, 4, 5, 6}. There are six possible outcomes and each outcome is equally likely.

a $P(5) = \frac{1}{6}$	One chance in 6
b $P(7) = \frac{0}{6} = 0$	No chance because we cannot roll a 7 on a die
c $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$	3 odd numbers out of 6: {1, 3, 5}
d $P(\text{less than 3}) = \frac{2}{6} = \frac{1}{3}$	2 numbers out of 6: {1 and 2}

Homework sheet

Probability 1

MAT07SPHS10035

Puzzle sheet

Theoretical probabilities

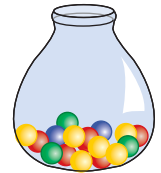
MAT07SPPS00041

Example 5

TLF learning object

The foul food maker
(L215)

A jar of lollies contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random and notes its colour. Calculate each probability below.



- a $P(\text{yellow})$
- b $P(\text{green or blue})$
- c $P(\text{traffic light colour}), \text{ as a percentage}$
- d $P(\text{not red}), \text{ as a decimal}$

Solution

Total number of lollies in the jar = $5 + 3 + 6 + 2 = 16$

$$\begin{aligned} \text{a } P(\text{yellow}) &= \frac{\text{Number of yellow lollies}}{\text{Total number of lollies}} \\ &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} \text{b } P(\text{green or blue}) &= \frac{\text{No. of green lollies} + \text{No. of blue lollies}}{\text{Total no. of lollies}} \\ &= \frac{3 + 2}{16} \\ &= \frac{5}{16} \end{aligned}$$

$$\begin{aligned} \text{c } P(\text{traffic light colour}) &= P(\text{red or green or yellow}) \\ &= \frac{5 + 3 + 6}{16} \\ &= \frac{14}{16} \end{aligned}$$

Multiply by 100 to convert the fraction into a percentage.

$$\frac{14}{16} \times 100 = 87.5\%$$

$$\begin{aligned} \text{d } P(\text{not red}) &= P(\text{green or yellow or blue}) \\ &= \frac{3 + 6 + 2}{16} \\ &= \frac{11}{16} \\ &= 0.6875 \text{ converting to a decimal} \end{aligned}$$

Exercise 11-02 Probability



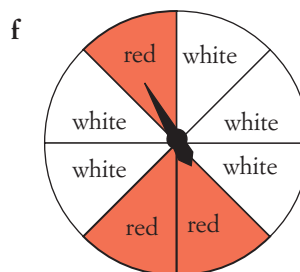
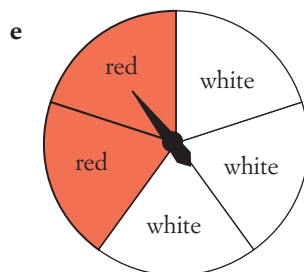
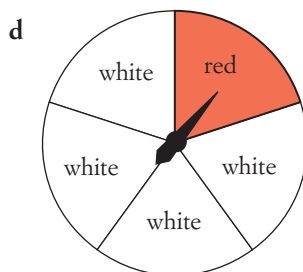
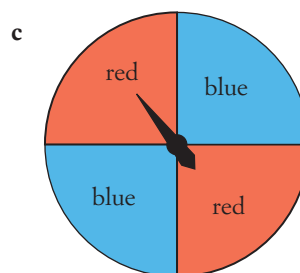
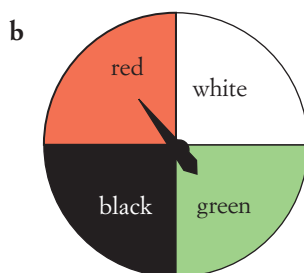
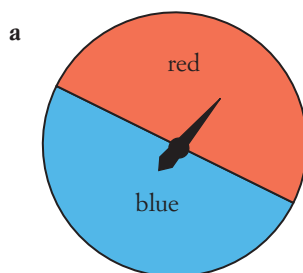
See Example 4

Extra questions

Describing probability

MAT07SPEQ00039

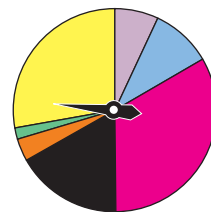
- 1
 - a List all the possible outcomes when a coin is tossed.
 - b How many different outcomes are possible?
 - c What is the probability of getting a tail?
 - d Write the probability of getting a tail as a percentage.
- 2 A die is rolled. Find the probability of rolling:
 - a a 4
 - b an even number
 - c a 10
 - d a number greater than 1
 - e a prime number
 - f a number less than 7
- 3 For each spinner, calculate the probability of the pointer landing on red, as a percentage.



- 4 A money box contains four \$2 coins, three \$1 coins, two 50c coins, six 20c coins and five 10c coins. It is shaken and one coin falls out at random. Calculate each probability below.
 - a $P(50c \text{ coin})$
 - b $P(\$1 \text{ coin})$, as a decimal
 - c $P(10c \text{ or } 20c \text{ coin})$, as a percentage
 - d $P(\text{not a } 10c \text{ coin})$
 - e $P(\text{gold coin})$, as a decimal
 - f $P(\text{a coin under } \$1)$, as a percentage
- 5 A packet of jellybeans has 4 yellow, 3 red, 6 green and 3 black jellybeans remaining. You tip the packet and one jellybean rolls out at random. What is the probability that it is not a red jellybean? Select the correct answer **A**, **B**, **C** or **D**.
 - A $\frac{4}{12}$
 - B $\frac{3}{16}$
 - C $\frac{3}{13}$
 - D $\frac{13}{16}$
- 6 A letter is selected at random from the alphabet. Find the probability that it is:
 - a M
 - b R
 - c M or R
 - d a vowel
 - e not a vowel
 - f a letter before K in the alphabet

See Example 5

- 7 The needle on the spinner shown is spun.
- Which colour has a probability of $\frac{1}{3}$?
 - Which colour has a probability closest to $\frac{1}{4}$?
 - Which colour has a probability of $\frac{1}{6}$?
 - Which three colours have a combined probability of 50%?



Worked solutions

Exercise 11-02

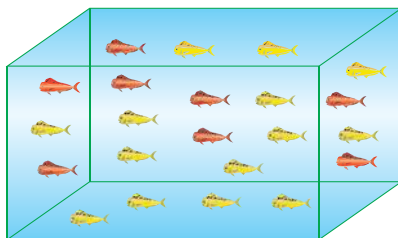
MAT07SPWS10060

- 8 A bag contains 6 white, 9 green and 5 blue marbles. Hannah takes one from the bag at random. Which colour has a probability of:
- 0.25?
 - 45%?
 - $\frac{3}{10}$?
- 9 A computer generates a random number from 1 to 10. Find the following probabilities.
- $P(\text{a square number})$
 - $P(\text{a number less than 4}), \text{ as a decimal}$
 - $P(\text{a prime number}), \text{ as a percentage}$
 - $P(\text{a factor of 10})$
 - $P(\text{a number greater than 6}), \text{ as a decimal}$
 - $P(\text{a 5 or 7}), \text{ as a percentage}$
- 10 What is the probability that a baby is born on a weekday rather than a weekend, if each day is equally likely?
- 11 Draw and label a spinner that would produce the following probabilities:
 $P(\text{red}) = \frac{1}{8}$, $P(\text{yellow}) = 24\%$, $P(\text{blue}) = \frac{1}{3}$, $P(\text{black}) = 20\%$.
- 12 Ryan bought 6 raffle tickets at the school fete. If a total of 800 tickets was sold, what is the probability that Ryan wins first prize? Express your answer:
- as a fraction
 - as a decimal
- 13 Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:
- your name?
 - a girl's name?
 - someone aged 12?
 - someone with blond hair?
- 14 A deck of 52 playing cards is shuffled and one is taken out at random. Find the probability that it is:
- a red card
 - a club card
 - the King of spades
 - a Queen
 - a card with an odd number
 - a black picture card
 - a red 7
 - a 4

You may refer to the diagram from question 6, Exercise 11-01, on page 444.

- 15 An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting:
- a can of lemonade is 50%?
 - an orange drink is 0.25?
 - a lime drink is $\frac{1}{12}$?
 - a cola can is $\frac{2}{5}$?
- 16 What is the probability that a person randomly chosen has a birthday in a month beginning with the letter J? Select the correct answer **A**, **B**, **C** or **D**.
- $\frac{1}{12}$
 - $\frac{1}{6}$
 - $\frac{1}{4}$
 - $\frac{1}{3}$
- 17 There are 16 teams in a football competition. Paula, therefore, believes that her favourite team, The Bulldogs, has a probability of $\frac{1}{16}$ of winning the competition. Discuss with your friends whether Paula is correct and write your answer, giving reasons.

- 18 A student council is made up of a number of boys and girls. One member of council is selected at random to attend the regional conference. How many boys and girls could there be in the council if the probability of selecting a boy is $\frac{5}{8}$?
- 19 A fish tank has 8 red and 14 yellow fish. One fish is randomly selected from the tank.
- Write the probability that the fish is yellow as a decimal correct to 2 decimal places.
 - Write the probability that the fish is red as a percentage.
 - How many red fish must be removed from the tank to make the probability of selecting a red fish 0.3?



- 20 a What is the lowest probability value? Name an event that could have this value.
- b What is the highest probability value? Name an event that could have this value.

Just for the record

Insurance

Insurance is a financial scheme that offers money in times of loss or damage. People can insure their houses against fire, flood or theft. You pay an amount each month or year, and if the loss or damage happens, the insurance company pays you a large sum of money to replace the insured item.

Insurance was invented in ancient Babylon around 3000 BCE, when voyagers and traders insured their ships against damage or loss. Modern mathematicians working for insurance companies use the statistics of past losses and deaths to predict the chances of future losses and deaths. They have found some interesting probabilities, such as those shown in the table.



Event	Chance
Being in a house fire	1 chance in 800
Going to prison	Male: 1 chance in 800 Female: 1 chance in 24 000
Dying in a traffic accident	1 in 5000
Dying at age 5	1 chance in 10 000
Being struck by lightning	1 chance in 280 000
Dying in a plane crash	1 chance in 11 million

An actuary works in the insurance industry. Find out what an actuary does.

11-03 The range of probability

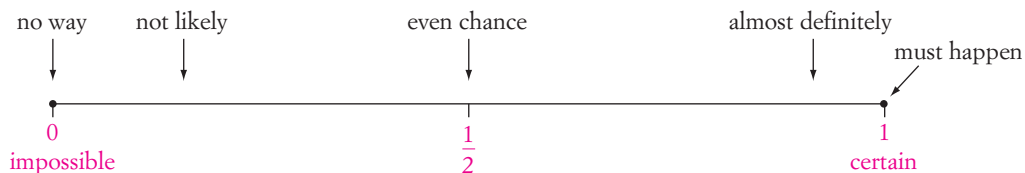
TLF learning object

Because probability is a fraction

Exploring probability
(L6567)

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

its value must range from 0 to 1, or as a percentage, from 0% to 100%.



Summary

- The probability of an **impossible** event is 0.
- The probability of a **certain** event is 1.
- The probability of any event ranges from 0 to 1.

A probability close to 1 indicates an event that is very **likely** or probable.

A probability close to zero indicates an event that is very **unlikely** or improbable.

Example 6

Five cards numbered 11, 5, 9, 7 and 3 are shuffled and Jo chooses one at random. Find the probability that the number chosen is:

- a** divisible by 3 **b** less than 12 **c** a factor of 45 **d** even

Solution

a $P(\text{divisible by } 3) = \frac{2}{5}$

2 numbers {9, 3}

b $P(\text{less than } 12) = \frac{5}{5} = 1$

All 5 numbers are less than 12: a certain event.

c $P(\text{a factor of } 45) = \frac{3}{5}$

3 numbers {5, 9, 3}

d $P(\text{even}) = \frac{0}{5} = 0$

None are even: an impossible event

Exercise 11-03 The range of probability

See Example 6

- 1 If a six-sided die is rolled, find the probability of rolling:

a a 5 b an 8 c a factor of 12
d a prime number e a number less than 7 f a multiple of 3

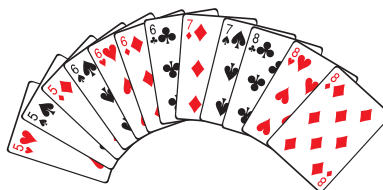
- 2 One letter is selected at random from the word EVACUATION. Find the percentage probability that it is:

a V b A c X d a vowel

- 3 Daniel is holding these 12 playing cards. Elvira picks a card without looking.

Find the probability that it:

a is red b shows an even number
c is a picture card d has the number 7
e is a hearts card f not an Ace



- 4 A basketball team captain is to be chosen randomly from four candidates, Carl, Lee, Su and Aldo. What is the probability that the captain chosen is:

a Aldo? b Su? c Manjeet?

- 5 Match each probability value to its correct description:

a $\frac{1}{2}$ b 0 c 90% d 1
e $\frac{3}{4}$ f 0.1 g 0.6 h 2%

A cannot happen B better than average chance
C even chance D good chance
E very likely F almost impossible
G slim chance H must happen

- 6 A coin falls out of a piggy bank containing four 5-cent, five 10-cent, six 20-cent and three \$1 coins. Find the probability that it is:

a a 5c coin b a \$1 coin c a 50c coin
d a 10c or 20c coin e not a \$1 coin f not a 20c coin

- 7 A traffic light shows red for 63 seconds, green for 99 seconds and yellow for 2 seconds. Find as a decimal the probability that it shows:

a green b yellow c red or green d not red, yellow or green

- 8 A computer generates a random number from 1 to 20. Find as a percentage the probability that the number is:

a a multiple of 4 b a factor of 20 c a number less than 30
d 25 e a prime number f divisible by 5

- 9 A die has 2 faces blue, 1 face red and the other faces green. It is rolled once. Match each event to its correct probability:

a red	b blue	c yellow
d a colour that is not red	e blue, red or green	f a traffic light colour
A $\frac{2}{3}$	B 50%	C $\frac{5}{6}$
E 0	F $\frac{1}{6}$	G 100%
		D $\frac{1}{3}$

Worked solutions

Exercise 11-03

MAT07SPWS10061

- 10 A letter is chosen at random from the words 'NEW CENTURY MATHS'. Find the following probabilities.

a $P(T)$	b $P(M)$	c $P(N \text{ or } E)$
d $P(\text{a consonant})$	e $P(\text{a letter also found in PROBABILITY})$	f $P(K)$

- 11 Write an event that could have a probability of:

a $\frac{1}{2}$	b 0	c 90%	d 1
e $\frac{3}{4}$	f 0.1	g 0.6	h 2%

Investigation: The Greedy Pig game

This is a game for four or more players. You need a playing die and a score sheet (use the link to print one out).

- 1 All players stand at the start of a round. A die is rolled twice and the total is each player's starting score.
- 2 Before the next roll of the die, each player has the option of sitting down and keeping their score, or remaining standing and 'being greedy' with the chance of increasing their score from the next roll of the die.
- 3 The die is rolled and the number is added to the score of each player standing, but if 2 comes up, all standing players lose their points.
- 4 A round ends when a 2 is rolled or all players are seated. Each player then records their score for the round.
- 5 The round is repeated four more times and the overall winner is the player with the highest total score for the five rounds.
- 6 Play this game several times. Is there any strategy to winning?



Puzzle sheet

Greedy Pig game

MAT07SPPS10041

Weblink

The Monty Hall problem

Just for the record

Buckley's chance

When someone says you have 'Buckley's chance' of something happening, it means that you have little or no chance at all. This is old Australian slang based on the name of a Melbourne department store in the 1850s called 'Buckley and Nunn'. The original expression was 'You have two chances: Buckley's and none'.

English is often a funny and illogical language. For example, 'slim chance' and 'fat chance' sound like opposites but they both mean the same thing.

Find another slang expression that describes a chance.

11-04 Experimental probability

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called the **theoretical probability** (or **calculated probability**).

We can also determine probability based on the results of an experiment or trial that has been repeated many times, such as testing the safety of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** or **relative frequency**, and has the following formula.

Summary

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Frequency means the number of times something happens

Worksheet

A page of spinners

MAT07SPWK10090

Worksheet

Coins probability

MAT07SPWK10091

Homework sheet

Probability 2

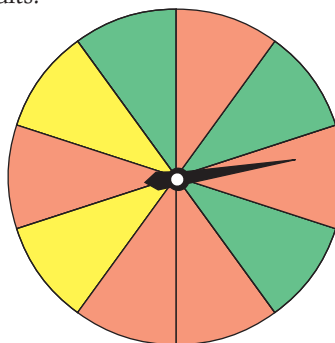
MAT07SPHS10036

Example 7

Selina spun this spinner 80 times and found the following results:

Outcome	Red	Green	Yellow
No. of times	44	25	11

- What is the theoretical probability of spinning red?
- For 80 spins, what is the expected number of times of spinning red? How does this compare with the actual number of times?
- What is the experimental probability of spinning red?



Solution

a $P(\text{red}) = \frac{4}{10} = \frac{2}{5}$

b Expected number of reds = $\frac{2}{5} \times 80 = 40$.

From the table, the observed number of reds = 44, which is close to the expected number.

c Experimental $P(\text{red}) = \frac{44}{80} = \frac{11}{20}$

Animated example

Theoretical probability

MAT07SPAE00016

Puzzle sheet

Experimental probability

MAT07SPPS00039

Summary

The expected number of times an event will occur over repeated trials is called the **expected frequency**.

Expected frequency = theoretical probability $\times$ number of trials

Exercise 11-04 Experimental probability

See Example 7

Extra questions

Experimental probability

MAT07SPEQ00040

Worked solutions

Exercise 11-04

MAT07SPWS10062

Puzzle sheet

Duelling dice

MAT07SPPS00038

Worksheet

Coin toss experiment

MAT07SPWK00062

Worksheet

Experimental probabilities

MAT07SPWK00063

- 1 For the spinner in Example 7, find:
 - a the theoretical probability of spinning yellow
 - b the expected number of times of spinning yellow and how this compares with the observed number of times
 - c the experimental probability of spinning yellow.

- 2 A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

- a Find, as a decimal, the experimental probability of rolling a sum:
 - i of 10
 - ii of 7 or 11
 - iii that is even
 - iv less than 6
 - v greater than 10
 - vi that is a prime number
- b Which sum:
 - i was most likely?
 - ii had a probability of $\frac{2}{25}$?
 - iii was least likely?
 - iv had a probability of 10%?
 - v was second-most likely?
 - vi had a probability of $\frac{4}{25}$?

- 3 a Copy this table.

Outcome	Tally	Frequency
Head		
Tail		
	Total	

- b Toss a coin 50 times and record the result for each toss in the table.
- c What is the theoretical probability of tossing a tail?
- d In 50 tosses, what is the expected frequency of tails? How does this compare with the actual frequency?
- e What is the experimental probability of tossing a tail?
- f What do you think would happen to the experimental probability of a tail if the coin was tossed 1000 times rather than 50?



- 4 The exact amounts of soft drink in 50 cans of drink were measured and the results are shown below.

Amount (mL)	373	374	375	376	377
Number of cans	2	6	38	3	1

Using these results, if you bought a can of drink, what is the probability that it will contain exactly 375 mL of drink? Select the correct answer **A**, **B**, **C** or **D**.

- A $\frac{375}{50}$ B $\frac{19}{25}$ C $\frac{6}{25}$ D $\frac{3}{50}$

- 5 a Copy this table.

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total	



- b Roll a die 50 times and record the result for each roll in the table.
- c Find the theoretical probability and expected frequency of rolling:
- i 3 ii 6
- iii an even number iv a number below 6
- d Find the experimental probability of rolling:
- i 3 ii 6
- iii an even number iv a number below 6
- e How do the observed frequencies compare with the expected frequencies?
- f What would you expect to happen if the die was rolled 1000 times?

- 6 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

- a Is each outcome equally likely?
- b Do you think this die is loaded (unfair)? Give a reason for your answer.
- c Write the experimental probability of rolling a 1 on this die, as a percentage.
- d If this die was rolled 100 times, how many times would you expect 3 to come up?

- 7 Tamara tossed a coin many times and got 140 heads and 110 tails. Calculate the experimental probability of heads with this coin as a percentage.

Worked solutions

Exercise 11-04

MAT07SPWS10062

Investigation: Spinning chance

Worksheet

Spinning chance

MAT07SPWK10092

Worksheet

A page of spinners

MAT07SPWK10090

Puzzle sheet

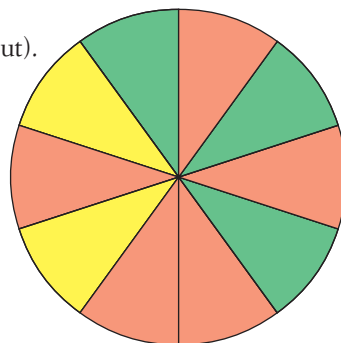
Spinner game

MAT07SPPS10042

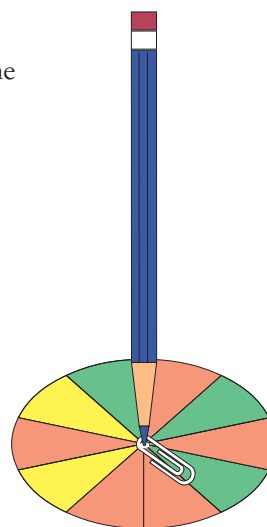
You will need: A pencil, a paper clip and an enlarged copy of the spinner used in Example 7 (use the link to print one out).

- 1 Which colour is most likely to be spun? Why?
- 2 Which colour is least likely?
- 3 Copy this table.

Outcome	Tally	Frequency
Red		
Yellow		
Green		
	Total	



- 4 Work with a partner. Place a pencil on the centre of the spinner and a paper clip around the point. Spin the paper clip around the spinner, and note which colour it lands on.
- 5 Take turns to spin the spinner 100 times and use the table to record your results.
- 6 Find the theoretical probability and expected frequency of spinning:
 - i red
 - ii yellow
 - iii green
- 7 How did your results compare with the expected number?
- 8 Find the experimental probability of spinning:
 - i red
 - ii yellow
 - iii green
- 9 What would happen if 1000 spins were made? How many of each colour would be expected?
- 10 Combine the results of your class on a whiteboard or projector to calculate the experimental probability for each colour. For such a large number of trials, are the experimental probabilities closer to the theoretical probabilities?
- 11 The spinner can also be simulated (imitated) using the random number generator on a calculator, which outputs a random decimal between 0 and 1 with three decimal places. Let 0, 1, 2, 3 and 4 represent red, 5 and 6 represent yellow and 7, 8 and 9 represent green. Now each decimal generated can represent three spins of the spinner. For example, 0.561 represents yellow, yellow, red.
- 12 Copy the blank table from question 3 again and use the 'Random' function on your calculator to simulate 100 spins.
- 13 Use the table to record your results.
- 14 How did your results compare with the expected frequencies?



Technology Rolling a die

Simulations can be used to model or replicate real-life events. Mathematicians, actuaries, scientists and statisticians can often simulate events on a computer, which saves time and money because large amounts of data can be obtained in a relatively short period of time.

In this activity, a graphics calculator is used to simulate the rolling of a normal die with sides 1, 2, 3, 4, 5 and 6.

Note: This activity uses a Casio graphics calculator.

- Copy the table shown below.

Outcome	Number of times rolled
1	
2	
3	
4	
5	
6	

- Using the **RUN** mode, enter the following formula: **Int(Ran# × 6 + 1)** as shown below.

- Repeat the simulation 20 times and record the results in your table.
- Are certain numbers more likely to be rolled than others? (For example is a 2 more likely to be rolled than a 5?) Do your results reflect this?
- Compare your results with the simulated results of your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

Technology worksheet

Excel
Coin tossing experiment

MAT07SPCT00022

TLF learning object

Dice duels (L2641)

TLF learning object

Dice duels: tool (L2645)

Mental skills 11 Maths without calculators

Adding 90c and 95c

A quick way to mentally add monetary amounts ending in 90c or 95c is to round up to whole dollars and then subtract 10c or 5c respectively.

- Study each example.

$$\begin{aligned}
 \text{a } \$8.95 + \$17.95 + \$11.95 &= \$9 + \$18 + \$12 - 5\text{c} - 5\text{c} - 5\text{c} \\
 &= \$39 - 15\text{c} \\
 &= \$38.85
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \$14.90 + \$5.95 + \$12.90 &= \$15 + \$6 + \$13 - 10\text{c} - 5\text{c} - 10\text{c} \\
 &= \$34 - 25\text{c} \\
 &= \$33.75
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \$18.90 + \$24.90 + \$8.80 &= \$19 + \$25 + \$9 - 10\text{c} - 10\text{c} - 20\text{c} \\
 &= \$53 - 40\text{c} \\
 &= \$52.60
 \end{aligned}$$

2 Now simplify each expression.

- a $\$12.95 + \$7.95 + \$14.95$
 c $\$9.95 + \$11.90 + \$13.90$
 e $\$13.95 + \$19.95 + \$8.95$
 g $\$27.95 + \$16.95 + \$12.90$
 i $\$24.95 + \$34.95$

- b $\$20.90 + \$4.95 + \$13.95$
 d $\$11.95 + \$15.90 + \$6.95$
 f $\$34.90 + \$34.90$
 h $\$11.90 + \$21.95 + \$19.80$
 j $\$33.90 + \$49.90 + \$49.90$

Investigation: Complementary events

In everyday life, we use the word ‘complementary’ to describe things that go together and ‘complete the picture’ when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that ‘complementary’ angles add to 90° .

In probability, **complementary events** are events that together make up all the possible outcomes.

Event	Complementary event
Tossing a tail on a coin	Tossing a head on a coin
Rolling a 6 on a die	Rolling any of the other numbers, from 1 to 5, on a die
Raining	Not raining
Being born on a Monday	Being born on a day other than Monday

1 Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.

- a What is $P(\text{Tues})$, the probability of being born on a Tuesday?
 b What is $P(\text{not Tues})$, the probability of being born on a day other than Tuesday?
 c What do you notice about $P(\text{Tues}) + P(\text{not Tues})$?

2 A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.

- a Find $P(\text{orange})$ b Find $P(\text{not orange})$
 c What do you notice about $P(\text{orange}) + P(\text{not orange})$?

3 A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.

- a Find $P(\text{boy})$ b Find $P(\text{girl})$
 c What do you notice about $P(\text{boy}) + P(\text{girl})$?

4 Copy and complete the following sentence:

The probability of an event _____ the probability of its complementary event must always equal _____.

11-05 Complementary events

YEAR 8

Worksheet

Theoretical probabilities

MAT07SPWK00064

In any situation, the probabilities of all possible outcomes must add to 1.

Complementary events are events that together make up all the possible outcomes, such as a head and a tail when tossing a coin. The **complement** of an event E means all of those outcomes that are *not* E , or that are the 'opposite' of E .

Because an event and its complement covers all possible outcomes, the sum of their probabilities must equal 1.

Summary

$$P(E) + P(\text{not } E) = 1$$

$$\text{or } P(\text{not } E) = 1 - P(E)$$

$$\text{or } P(\text{complementary event}) = 1 - P(\text{event})$$

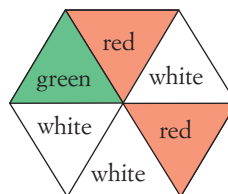
$$\text{or } P(\text{event not occurring}) = 1 - P(\text{event occurring})$$

If the probability is written in percentage form, then $P(\text{not } E) = 100\% - P(E)$.

Example 8

On this spinner, what is the probability of spinning:

- a red?
- b a colour that is not red?
- c a colour that is not green?



Solution

$$\text{a } P(\text{red}) = \frac{2}{6} = \frac{1}{3}$$

$$\begin{aligned} \text{b } P(\text{not red}) &= 1 - P(\text{red}) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

Note that $P(\text{red}) + P(\text{not red}) = \frac{1}{3} + \frac{2}{3} = 1$, which covers all possible outcomes.

$$\begin{aligned} \text{c } P(\text{green}) &= \frac{1}{6} \\ P(\text{not green}) &= 1 - P(\text{green}) \\ &= 1 - \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$

Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{6} + \frac{5}{6} = 1$, which covers all possible outcomes.

The complement of 'red' is 'not red', which is 'white or green'.

The complement of 'green' is 'not green', which is 'red or white'.

Exercise 11-05 Complementary events

- Write the complement for each event.

a The sex of a new baby being female	b Rain tomorrow
c Losing a soccer match	d Being left-handed
e Being over 18 years of age	f Choosing a heart from a deck of cards
g A traffic light showing green	h A train arriving late

See Example 8

- A die is rolled. What is the probability that the result is:

a 3?	b less than 3?	c prime?
d not a 3?	e not less than 3?	f not prime?
- Kylie buys a ticket in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Kylie:

a winning the prize?	b not winning the prize?
- A jar contains jellybeans in the following colours and amounts: red 40, blue 25, black 50, white 15. One jellybean is selected at random. What is the probability that the jellybean is:

a white?	b not white?	c yellow?
d not yellow?	e not red?	f not blue or black?
- Write the probability of the event that is **complementary** to each of the following events.

a The probability of choosing a Jack from a pack of cards is $\frac{1}{13}$.
b The chance of shooting a basketball hoop is 55%.
c The probability of winning a prize is 0.07.



- In a bag of toy cars there are only three colours: red, blue and white. If you take out a car at random, the chance of it being red is 0.5, and the chance of it being white is 0.2.

a What is the chance of selecting 'red or white'?
b What is the chance of the car you select being blue?
c If the bag holds 30 cars, how many of each colour would you expect to find?
d What is the chance of the car you select being pink?
- What is the probability of the next person you meet being born in a month:

a beginning with the letter A?	b that does not begin with the letter A?
- Four students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.

a Find the probability that Emily was not chosen.
b Find the probability that the captain is a boy.
c What is the chance that the captain is not a boy?
d What is the chance the captain is the teacher?
- Which of the following is the complementary event to 'winning a race'? Select the correct answer A, B, C, or D.

A coming last	B coming second or third
C not winning the race	D coming second

Worked solutions

Exercise 11-05

MAT07SPWS10063

- 10 In a football match, the Eels have a 43% chance of winning, while the Tigers have a 49% chance of winning.
a What other outcome is possible? b What is the probability of this outcome?
- 11 The probability that a man has skin cancer is 0.03. What is the probability that a man doesn't have skin cancer? Select the correct answer **A**, **B**, **C** or **D**.
A 0.2 **B** 0.7 **C** 0.07 **D** 0.97
- 12 The letters of the word PROBABILITY are written on separate cards and one is randomly selected. What is the probability that a letter drawn out is:
a not P? b not a vowel? c not I? d not A or B?
- 13 The probability that it will rain this weekend is 85%. What is the probability that it won't rain?
- 14 What is the decimal probability that a mobile number selected at random doesn't end in 0 or 1?

Worked solutions

Exercise 11-05

MAT07SPWS10063

Power plus

- 1 Two dice are rolled and the sum of the numbers is calculated.
a Copy and complete this table to show all possible sums.
b How many different sums are possible?
c Why isn't each sum equally likely?
d Which sum is most likely?
e Which sum is least likely?
f What is the probability of a sum of 2?
g What is the probability of a sum of 10?
h Which sum has a probability of $\frac{1}{9}$?
- | | | First die | | | | | |
|------------|---|-----------|---|---|---|---|---|
| Second die | + | 1 | 2 | 3 | 4 | 5 | 6 |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | | | | |
| | 3 | 4 | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| | 6 | | | | | | |
- 2 Three coins are tossed together. One possible outcome is HHT, that is, heads on the first coin, heads on the second coin, tails on the third coin.
a How many other possible outcomes are there?
List them.
b Use your answers from part a to calculate the probability of:
i three heads ii two heads iii one head
iv no heads v at least one head vi at most one head
- 3 At Harry's takeaway food shop, you can order a special meal deal for \$8 that consists of one main meal, one drink and one dessert.

Harry's Takeaway Menu

Main meals	Drinks	Desserts
Hot dog	Cola	Cake
Pizza	Lemonade	Ice cream
Hamburger		



Chapter 11 review

Language of maths

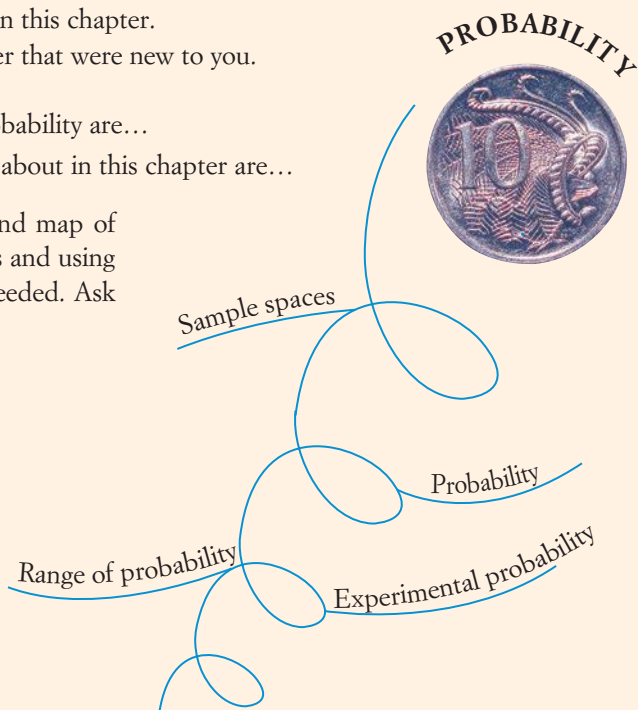
certain	expected frequency	improbable	probable
chance	experiment	likely	random
complementary event	experimental probability	observed	sample space
die/dice	favourable	outcome	theoretical probability
equally likely	frequency	possible	trial
event	impossible	probability	unlikely

- 1 What is the opposite of **impossible** and what is its probability value?
- 2 What is the difference between **impossible** and **improbable**?
- 3 What is the name given to the set of all possible outcomes for a situation?
- 4 What is the **complementary event** to a baby being born on a weekend?
- 5 What does it mean when a name is drawn out of a box '**at random**'?
- 6 What word means the number of times something happens?

Topic overview

- Write about what you have learnt in this chapter.
- Write down the parts of the chapter that were new to you.
- Copy and complete:
The things I understand about probability are...
The things I am still not confident about in this chapter are...

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

Mind map: Probability

MAT07SPWK10093

1 List the outcomes for the sample space in each situation.

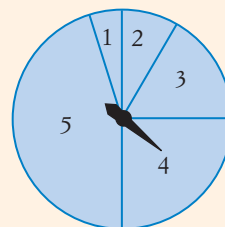
See Exercise 11-01

- a The result of a driving test
- b The classification of a new film (for example, PG)
- c The winner of a tennis match when Katrina plays Biljana
- d The type of public transport that goes to your school

2 a For this spinner, how many outcomes are there in the sample space?

See Exercise 11-01

- b Why isn't each outcome equally likely?
- c Which number is most likely?
- d Which number is least likely to be spun?
- e Which number has a probability of 25%?



3 A die is rolled. What is the probability of rolling:

See Exercise 11-02

- a a 1?
- b an odd number?
- c a number less than 5?

4 A truck carries 325 boxes of exercise books, 210 boxes of rulers, 360 boxes of paper and 145 boxes of pens. If a box is taken at random from the truck, what is the probability (as a percentage) that the box contains:

See Exercise 11-02

- a rulers?
- b no pens?
- c exercise books or paper?
- d erasers?

5 A jar contains 1 red, 6 yellow, 2 white and 5 black jellybeans. If a jellybean is selected at random, find the probability that it is:

See Exercise 11-03

- a yellow
- b white or red
- c blue
- d not green

6 Which word best describes an event that has a probability of 0.29? Select the correct answer A, B, C, or D.

See Exercise 11-03

- A unlikely
- B highly probable
- C almost impossible
- D good chance

7 Which event below could be described as a certain event? Select the correct answer A, B, C, or D.

See Exercise 11-03

- A choosing a 4 from a hat containing the numbers 1, 2, 3 and 4
- B choosing a red ball from a bag of green balls
- C rolling an odd or even number on a die
- D a new baby being a girl

8 A coin was tossed 80 times and landed heads 24 times.

See Exercise 11-04

- a What is the theoretical probability of obtaining a head?
- b What is the expected frequency of heads from 80 tosses of a coin? How does the observed frequency compare to this?
- c What is the experimental probability of obtaining a head?

9 Write the complementary event for each of the following events and its probability.

See Exercise 11-05

- a Choosing a queen from a standard deck of cards.
- b Rolling a multiple of 3 on a die.
- c Buying the winning ticket out of 550 tickets sold.

10 There are 8 brown marbles, 4 black marbles and 3 white marbles in a bag. One marble is taken at random from the bag. What is the probability that a marble that is not brown is selected?

See Exercise 11-05



12

**Number and algebra,
Measurement and geometry**

Ratios, rates and time

How long would it take to travel from Sydney to Perth if you:

- walked at 5 km/h?
- jogged at 9 km/h?
- cycled at 25 km/h?
- took the Indian Pacific train at 85 km/h?
- drove at 100 km/h?
- flew at 900 km/h?

The distance is 3972 km by road and 3276 km by air.



Chapter outline

	Proficiency strands			
12-01 Ratios	U			
12-02 Simplifying ratios	U	F		
12-03 Ratio problems	U	F	PS	
12-04 Rates	U			
12-05 Best buys	U	F	PS	R
12-06 Rate problems	U	F	PS	
12-07 Travel graphs	U	F	PS	R
12-08 Time calculations	U	F		
12-09 24-hour time	U	F		
12-10 Time differences	U	F	PS	
12-11 Timetables	U	F	PS	

Wordbank

- **24-hour time** Time of day written using 4 digits (instead of a.m. and p.m.), for example, 1820
- **best buy** When comparing different brands or sizes during shopping, this is the item with the lowest unit cost and best value for money
- **rate** A measurement comparing quantities of different types, expressed using two units
- **ratio** A relationship between two or more quantities measured in the same units; for example, the ratio of 3 teachers to 40 students is 3 : 40
- **speed** A rate that compares distance travelled with time taken
- **unit cost** The price of one item or unit, such as 1 mL or 1 g
- **unitary method** A method of finding a quantity by finding one part first

In this chapter you will:

- recognise and solve problems involving simple ratios
- understand that rate and ratio problems can be solved using fractions or percentages and choose the most efficient form to solve a particular problem
- investigate and calculate 'best buys'
- use travel graphs to investigate and compare the distance travelled to and from school
- interpret features of travel graphs such as the slope of lines and the meaning of horizontal lines
- **add and subtract time mentally and by using a calculator**
- compare 12- and 24-hour time systems and convert between them
- **solve problems involving duration, including using 12- and 24-hour time within a single time zone**
- interpret and use timetables

SkillCheck

Worksheet

StartUp assignment 12

MAT07NAWK10094

1 Copy and complete:

a $2000 \text{ kg} = \underline{\hspace{2cm}} \text{ t}$

b $360 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$

c $18 \text{ g} = \underline{\hspace{2cm}} \text{ mg}$

d $300 \text{ s} = \underline{\hspace{2cm}} \text{ min}$

e $6.5 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

f $480 \text{ min} = \underline{\hspace{2cm}} \text{ h}$

g $7500 \text{ mL} = \underline{\hspace{2cm}} \text{ L}$

h $8.15 \text{ km} = \underline{\hspace{2cm}} \text{ m}$

i $850 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$

2 Find the greatest common divisor (GCD) of each pair of numbers.

a 12 and 18

b 8 and 36

c 20 and 64

3 Complete each pair of equivalent fractions.

a $\frac{7}{10} = \frac{\square}{30}$

b $\frac{3}{5} = \frac{12}{\square}$

c $\frac{18}{45} = \frac{\square}{5}$

4 Simplify each fraction.

a $\frac{6}{24}$

b $\frac{40}{50}$

c $\frac{9}{36}$

12-01 Ratios

TLF learning object

In proportion: ratios
(L8098)

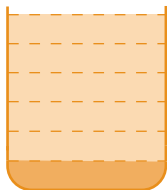
A **ratio** consists of two or more numbers that compare the parts or shares of things of the same type, in the same units. For example, if a cake recipe uses sugar and flour in a ratio of 1 to 2, written '1 : 2,' it means that for every one part of sugar we need two parts of flour. Each number in a ratio is called a **term** of the ratio.



Summary

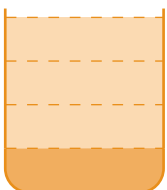
A **ratio** compares quantities of the same type given in the same units.
We say ' a to b ' and write $a : b$, where a and b are numbers.

Jerry mixes cordial and water in the ratio $1 : 5$, meaning that for every 1 part of cordial there are 5 parts of water.



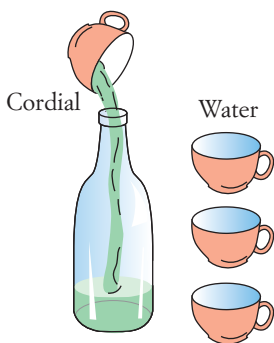
$1 : 5$ (read as '1 to 5')
1 part cordial to 5 parts water

Nina prefers a stronger flavour, so she uses a cordial to water ratio of $1 : 3$.



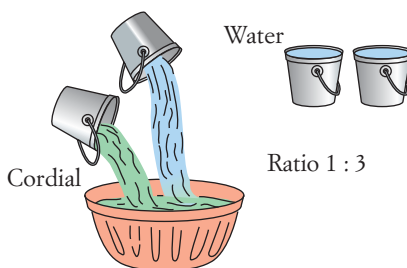
$1 : 3$ (read as '1 to 3')
1 part cordial to 3 parts water

For a picnic, Nina can make a bottle of her drink by mixing one **cup** of cordial to 3 **cups** of water.



Ratio $1 : 3$

For a party, Nina can make a tub of her drink by mixing 1 **bucket** of cordial to 3 **buckets** of water.



Ratio $1 : 3$

Example 1

For this box of chocolates, write each of the ratios below:



- a dark brown chocolates to light brown chocolates
- b rectangular chocolates to circular chocolates
- c white-topped chocolates to all chocolates

Solution

- a dark chocolates to light chocolates = 4 : 10 4 dark, 10 light
- b rectangular chocolates to circular chocolates = 6 : 2 6 rectangular, 2 circular
- c white-topped chocolates to all chocolates = 6 : 20 6 white, 20 in total

Example 2

Express each pair of quantities as a ratio.

- a A mass of 69 kg to a mass of 80 kg
- b A distance of 37 cm to a distance of 1 metre

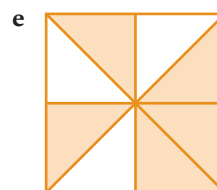
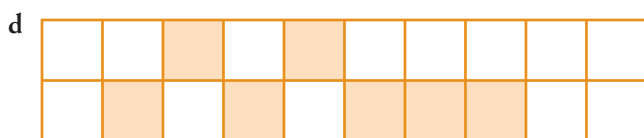
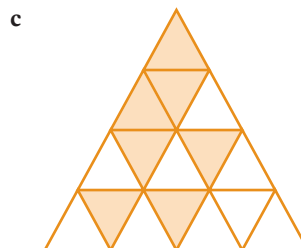
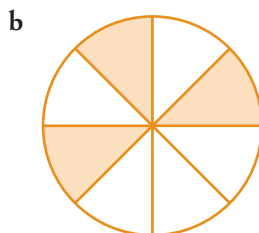
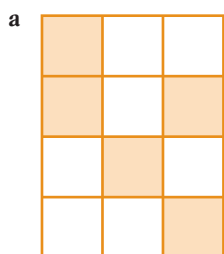
Solution

- a Since the quantities are in the same units (kg) the ratio is simply 69 : 80.
69 kg : 80 kg = 69 : 80
- b Change 1 metre to 100 centimetres so that both quantities are in the same units.
37 cm : 1 m = 37 cm : 100 cm
= 37 : 100

Exercise 12-01 Ratios

- 1 For each diagram, write the ratio of shaded to unshaded parts.

See Example 1



- 2 **a** Johnny likes to make a drink with a ratio of 2 parts cordial to 5 parts water (or 2 : 5). Draw a diagram to show this ratio in a glass.
- b** Melanie made a drink mixed with the ratio shown in the diagram. Estimate the ratio of cordial to water used.
- 3 In your classroom, find the ratio of:
- | | |
|-------------------------------|-----------------------------|
| a teachers to students | b girls to boys |
| c chairs to tables | d students to chairs |
- 4 In the 1950s, the average ratio of teachers to students in classrooms was 1 : 45, in the 1970s it was 1 : 35, and in the 1980s the ratio was set at 1 : 30 for most classes. Today, it is 1 : 24. What does it mean when we say that the teacher to student ratio has improved?
- 5 In a class of 25 students, Hadid found that 13 students in his class have brown eyes, 8 students have blue eyes, 3 students have green eyes and 1 student has hazel eyes. Find the ratios of the student eye colours below:
- | | | | |
|------------------------|------------------------|-------------------------|-----------------------------|
| a blue to brown | b green to blue | c brown to hazel | d hazel to the total |
|------------------------|------------------------|-------------------------|-----------------------------|
- 6 Out of every 100 people at the football, 39 were men, 23 were women, and the rest were children. Find each ratio:
- | | |
|--------------------------|-----------------------------|
| a men to women | b women to the total |
| c children to men | d women to children |
- 7 Express each pair of quantities as a ratio.
- | | | |
|-------------------------------|----------------------------|--------------------------------|
| a 13 g to 25 g | b \$17 to \$100 | c 5 litres to 12 litres |
| d 150 km to 1 km | e 0.5 kg to 43 g | f \$3.50 to 83 cents |
| g 3 kL to 1421 L | h 27 mm to 4 cm | i 59 minutes to 3 hours |
| j 15 years to 7 months | k 9 days to 7 weeks | l 77 m to 17 cm |

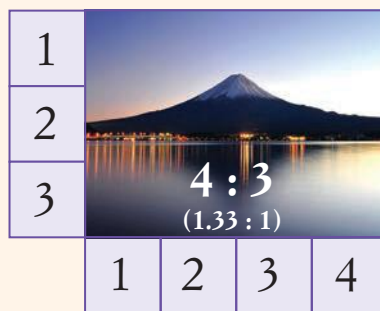
See Example 2

- 8 What is 39 kg to 41 g expressed as a ratio? Select the correct answer **A**, **B**, **C**, or **D**.
A 39 : 41 **B** 41 : 39 **C** 39 : 41 000 **D** 39 000 : 41
- 9 Write each statement as a ratio.
- For each teacher there were 25 students.
 - To cook steamed rice, you need 1 cup of rice to 2 cups of water.
 - Adventure cycling activities in schools require one teacher for every four students.
 - There are 11 girls for every 9 boys in the school.
 - No matter how much they won on Lotto, they agreed to share the prize equally.
 - She was twice as tall as he was.

Just for the record

Screen ratios

Old analogue TV screens had length and width in the ratio 4 : 3 (approximately 1.33 : 1). This was called its **aspect ratio**. The ratio means that the screen is 4 units across and 3 units up (a 'squarish'-looking screen).



Digital TV screens have an aspect ratio of 16 : 9 (approximately 1.77 : 1), giving a wider picture, while cinema screens show films with a widescreen format of 1.85 : 1 or 2.35 : 1.

When films and digital TV programs are shown on a 4 : 3 television set, the wide picture needs to be 'squeezed', using a 'pan-and-scan' (full frame) format, a 'letterbox' format or 'pillarbox' format. Some TVs and DVD players allow you to choose the way you view the picture.

Find out how the 'pan-and-scan', 'letterbox' and 'pillarbox' formats work.

12-02 Simplifying ratios

Equivalent ratios are *equal* ratios and can be found in a similar way to that used for finding equivalent fractions.

Worksheet

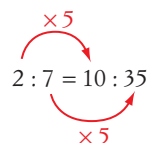
Ratios match

MAT07NAWK10095

Summary

To find an **equivalent ratio**, multiply or divide each term by the same number.

For example, $2 : 7$ and $10 : 35$ are equivalent ratios.



Example 3

Complete each pair of equivalent ratios.

a $6 : 4 = 18 : \underline{\hspace{2cm}}$

b $3 : 4 = \underline{\hspace{2cm}} : 24$

c $12 : 8 = 3 : \underline{\hspace{2cm}}$

Solution

- a** To find the missing term, look at the two known matching terms, 6 and 18.

6 is *multiplied* by 3 to give 18, so do the same thing to the 4.

$$4 \times 3 = 12$$

$$\therefore 6 : 4 = 18 : 12$$

- b** The two known matching terms are 4 and 24.

4 is *multiplied* by 6 to give 24, so do the same thing to the 3.

$$3 \times 6 = 18$$

$$\therefore 3 : 4 = 18 : 24$$

- c** The two known matching terms are 12 and 3.

12 is *divided* by 4 to give 3, so do the same thing to the 8.

$$8 \div 4 = 2$$

$$\therefore 12 : 8 = 3 : 2$$

Summary

- To **simplify** a ratio, keep dividing both terms by the same number, preferably a large number such as their greatest common divisor (GCD), until each term is as small as possible
- If all terms are even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

Example 4

Worksheet

Ratios match

MAT07NAWK10095

Simplify each ratio.

a $5 : 10$

b $27 : 18$

c $12 : 20 : 16$

Solution

$$\begin{aligned} \text{a } 5 : 10 &= \frac{5}{5} : \frac{10}{5} \\ &= 1 : 2 \end{aligned}$$

Or enter $5 : 10$ as a fraction $\frac{5}{10}$ on the calculator:

5 $\text{a}^{\text{b/c}}$ 10 $=$

Divide both terms by their GCD, 5

On a Casio calculator, use the $\frac{\square}{\square}$ key instead of $\text{a}^{\text{b/c}}$

$$\begin{aligned} \text{b } 27 : 18 &= \frac{27}{9} : \frac{18}{9} \\ &= 3 : 2 \end{aligned}$$

Or enter $27 : 18$ as an improper fraction $\frac{27}{18}$ on the calculator:

27 $\text{a}^{\text{b/c}}$ 18 $=$

Divide both terms by their GCD, 9

Then to change the mixed numeral to an improper

fraction, press: $\text{d/c} \left(\begin{array}{c} \text{SHIFT} \\ \text{2ndF} \end{array} \text{a}^{\text{b/c}} \right)$

On a Casio calculator, press

$$\frac{a}{c} \frac{b}{c} \leftrightarrow \frac{d}{c} \left(\text{SHIFT} \text{S} \leftrightarrow \text{D} \right)$$

$$\begin{aligned} \text{c } 12 : 20 : 16 &= \frac{12}{4} : \frac{20}{4} : \frac{16}{4} \\ &= 3 : 5 : 4 \end{aligned}$$

Divide all terms by their GCD, 4

Because there are more than 2 terms in this ratio, the calculator cannot be used here

Example 5

Simplify 10 hours : 1 day.

Solution

10 hours : 1 day = 10 hours : 24 hours

Change to the same units

$$= 10 : 24$$

$$= \frac{10}{2} : \frac{24}{2}$$

$$= 5 : 12$$

Puzzle sheet

Simplifying ratios

MAT07NAPS00031

Exercise 12-02 Simplifying ratios

1 Copy and complete each pair of equivalent ratios.

a $2 : 3 = 8 : \underline{\hspace{1cm}}$

b $1 : 5 = 2 : \underline{\hspace{1cm}}$

c $3 : 5 = \underline{\hspace{1cm}} : 15$

d $4 : 7 = \underline{\hspace{1cm}} : 35$

e $5 : 8 = 20 : \underline{\hspace{1cm}}$

f $7 : 12 = 49 : \underline{\hspace{1cm}}$

g $5 : 11 = \underline{\hspace{1cm}} : 66$

h $3 : 4 = \underline{\hspace{1cm}} : 100$

i $2 : 1 = 10 : \underline{\hspace{1cm}}$

j $\underline{\hspace{1cm}} : 9 = 20 : 36$

k $12 : \underline{\hspace{1cm}} = 3 : 1$

l $\underline{\hspace{1cm}} : 45 = 6 : 9$

m $24 : 12 = 4 : \underline{\hspace{1cm}}$

n $16 : \underline{\hspace{1cm}} = 2 : 5$

o $\underline{\hspace{1cm}} : 20 = 15 : 60$

2 Which of the following ratios is not equivalent to the ratio 32 : 48? Select the correct answer **A**, **B**, **C**, or **D**.

A $16 : 24$

B $4 : 6$

C $2 : 3$

D $6 : 8$

3 Simplify each ratio.

a $10 : 100$

b $8 : 16$

c $12 : 30$

d $35 : 49$

e $18 : 12$

f $56 : 24$

g $1000 : 100$

h $45 : 99$

i $7 : 7$

j $20 : 48$

k $56 : 14$

l $3 : 36$

m $8 : 12 : 20$

n $15 : 20 : 30$

o $27 : 9 : 36$

p $9 : 6 : 24 : 12$

4 Simplify each ratio.

a $80 \text{ cm} : 2 \text{ m}$

b $300 \text{ g} : 1.2 \text{ kg}$

c $6 \text{ days} : 4 \text{ weeks}$

d $45 \text{ min} : 2 \text{ hours}$

e $70\text{c} : \$2.10$

f $2 \text{ years} : 15 \text{ months}$

g $15 \text{ hours} : 2 \text{ days}$

h $20 \text{ mm} : 1 \text{ m}$

i $4 \text{ t} : 350 \text{ kg}$

j $250 \text{ mL} : 3 \text{ L}$

k $1800 \text{ m} : 1 \text{ km}$

l $8 \text{ kg} : 400 \text{ g}$

m $2 \text{ min} : 8 \text{ s}$

n $750 \text{ L} : 5 \text{ kL}$

o $\$18 : \$4.50$

5 Simplify 4 m to 25 cm. Select the correct answer **A**, **B**, **C** or **D**.

A $4 : 25$

B $400 : 25$

C $8 : 5$

D $16 : 1$

See Example 3

See Example 4

See Example 5

- 6 The following amounts of books were sold at a garage sale: 48 crime novels, 20 science fiction (SF) novels, 35 non-fiction books, 28 romance novels.

Express each ratio in simplest form:

- a crime to romance b SF to non-fiction
c non-fiction to romance d SF to crime
e romance to non-fiction f crime to all books



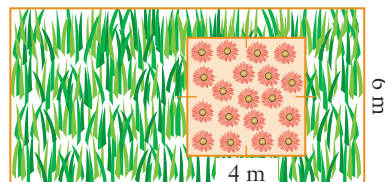
Worked solutions

Exercise 12-02

MAT07NAWS10064

- 7 A backyard lawn contains a square garden, as shown on the right. What is the ratio of garden area to lawn area? Select the correct answer A, B, C or D.

- A 2 : 7 B 2 : 9 C 7 : 2 D 9 : 2



- 8 At Westvale Catholic College there are 72 teachers and 1440 students. Of these, 624 students are girls.

- a Find each ratio.

i teachers : students

ii boys : girls

iii boys : teachers

- b The government states that the teacher to student ratio must be no less than 1 : 25. Does the school have enough teachers? Give reasons for your answer.

- 9 When comparing the rate at which babies are born in different countries (that have different population sizes), we use a base number of 1000. The annual birth rate in Australia is approximately 12 per 1000 people (a ratio of 12 : 1000). The rate in Pakistan is approximately 25 per 1000 people, or 25 : 1000. Express each ratio in simplest form.

- 10 Tom and Ahmed bought a scooter together, but Tom paid \$70 and Ahmed paid \$55. Write the ratio of their shares of the scooter in simplest form.

- 11 To make a 2 L jug of cordial drink, 375 mL of cordial is mixed with water.

Find:

- a the amount of water used
b the ratio of cordial to water
c the ratio of cordial to drink



Worked solutions

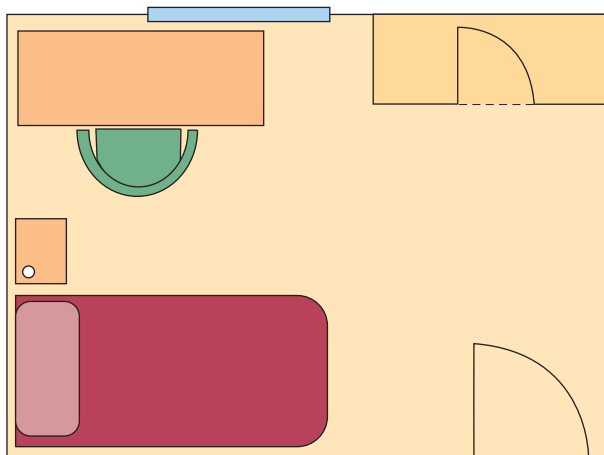
Exercise 12-02

MAT07NAWS10064

- 12 Three Year 7 classes were surveyed to find the ratio of left-handed to right-handed students. In 7N the ratio was 1 : 5, in 7P the ratio was 4 : 15, and in 7D the ratio was 1 : 6. Which class had the highest proportion of left-handed students?

Investigation: Scale diagrams

Here is a scale plan of a bedroom. The scale ratio is 1 : 50, meaning that 1 cm on the plan represents 50 cm on the actual bedroom. The actual bedroom is 50 times larger.



- a By measuring the plan, find in metres:
 - i the width of the bedroom
 - ii the length of the bedroom
- b Calculate:
 - i the perimeter of the bedroom in metres
 - ii the area of the bedroom in square metres
- c By measuring the plan, find in centimetres:
 - i the length of the bed
 - ii the width of the desk
 - iii the width of the door
 - iv the length of the window

12-03 Ratio problems

Problems involving ratios can be solved using **equivalent ratios** or the **unitary method**. With the unitary method, we find the size of **one part** first.

Example 6

The ratio of boys to girls in a Year 7 class is 2 : 3. If there are 10 boys in the class, how many girls are there?

Worksheet

Ratio calculations

MAT07NAWK10096

Video tutorial

Ratio problems

MAT07NAVT10023

Homework sheet

Ratios

MAT07NAHS10015

Solution**Method 1: Equivalent ratios**

Write the problem as a pair of equivalent ratios.

$$\text{Boys : girls} = 2 : 3 = 10 : \underline{\quad}$$

10 boys

2 is multiplied by 5 to give 10, so do the same thing to 3.

Number of girls = $3 \times 5 = 15$

There are 15 girls in the class.

Method 2: Unitary method

Boys : girls = 2 : 3

2 parts (boys) = 10

1 part = $10 \div 2 = 5$

3 parts (girls) = $3 \times 5 = 15$

There are 15 girls in the class.

Unitary means 'one'

Finding one part first

Example 7

To make concrete, a builder mixes sand and cement in the ratio 5 : 4. If a mix of concrete contains 24 kg of cement, find:

- the amount of sand in the mix
- the total mass of the mix.

Solution**a Method 1: Equivalent ratios**

$$\text{sand : cement} = 5 : 4 = \underline{\quad} : 24$$

24 kg of cement

4 is multiplied by 6 to give 24, so do the same thing to 5.

Amount of sand = $5 \times 6 = 30$ kg

Method 2: Unitary method

Sand : cement = 5 : 4

4 parts (cement) = 24 kg

1 part = $24 \div 4 = 6$ kg

5 parts (sand) = $5 \times 6 = 30$ kg

- Total mass = 24 kg + 30 kg = 54 kg

So the total mixture was 54 kg.

Cement and sand

Exercise 12-03 Ratio problems

- 1 In Year 7, the ratio of boys to girls is 4 : 5. If there are 64 boys in Year 7, find the number of girls.
- 2 Simon and Joshua's heights are in the ratio 8 : 9. If Joshua is 171 cm tall, how tall is Simon?
- 3 In a rectangle, the ratio of the width to the length is 5 : 12. If the length of the rectangle is 48 cm, find the width.
- 4 A car dealer finds that the ratio of sedans to hatchbacks sold last week was 15 : 4. If 105 sedans were sold, how many hatchbacks were sold?
- 5 A tiler uses 4 green tiles to every 3 white ones. How many tiles are used altogether if 140 green tiles are used?
- 6 When making concrete, sand and cement are mixed together in the ratio 4 : 1. If 120 kg of cement has been delivered, what mass of sand is needed?

See Example 6

See Example 7



- 7 Alison and Elena buy a length of material and divide it between them in the ratio 2 : 3. If Alison has 3.6 m, what length of material does Elena have? Select the correct answer **A**, **B**, **C** or **D**.
A 1.8 m **B** 2.4 m **C** 5.4 m **D** 9 m
- 8 An alloy contains copper and iron in the ratio 2 : 5. A quantity of alloy contains 20 kg of iron. What mass of copper does it contain?
- 9 The ratio of adults to children at the cinema was 3 : 4. If there were 96 children, how many adults were there?

Worked solutions

Exercise 12-03

MAT07NAWS10065

- 10 Enzo's Produce Store buys fruit and vegetables in the ratio 5 : 7. The mass of fruit is 8.5 tonnes. What is the total mass of produce ordered?



- 11 Ali, Bree and Felicity share a Lotto prize in the ratio 10 : 8 : 7. If Bree received \$72,
 a how much did the other two receive?
 b what was the total prize money shared?
- 12 In a triangle, the lengths of the sides are in the ratio 3 : 4 : 5. If the longest side is 30 cm long, find the perimeter of the triangle.
- 13 In an outback mining town, the ratio of women to men is 2 : 5. If there are 240 women, how many men and women are in the town altogether? Select **A**, **B**, **C** or **D**.
A 840 **B** 600 **C** 96 **D** 1680
- 14 To make Superglue, the contents of Tube A and Tube B are mixed in the ratio 3 : 1.
 a If 15 mL of Tube A is used, how much of Tube B is needed?
 b How much glue is made altogether if 15 mL of Tube B is used?
- 15 The heights of two buildings are in the ratio 7 : 6. If the shorter building is 150 m tall, how high is the taller building?
- 16 To make concrete, a bricklayer mixes 1 part cement to 2 parts gravel to 4 parts sand. If Jo uses 50 kg of gravel, how much cement and sand does she need?

Extension

Worked solutions

Exercise 12-03

MAT07NAWS10065

- 17 Flour and butter are mixed in the ratio 3 : 1 to make short-crust pastry. If Adam makes 800 g of pastry, how much flour does he need?
- 18 Sonya and Xanthe share a prize of \$495 in the ratio of 4 : 5. How much prize money does each person receive?

12-04 Rates

While a ratio compares two or more quantities measured in the same units, a **rate** compares two quantities measured in **different units**.

A rate shows how one quantity changes with another quantity. We write a rate using a '/' symbol in the form 'something per something else'. For example:

- the price of petrol is stated in cents per litre, or c/L
- your heart beats at a rate measured in beats per minute, or beats/min
- the speed of a car is measured in kilometres per hour, or km/h

The word 'per' means 'for each' so we express a rate 'per single unit'. For instance, travelling at a rate of 50 km/h means travelling 50 km in *each* hour.

Worksheet

What's my speed?

MAT07NAWK10097

Example 8

Write each statement as a simplified rate.

- An Internet connection costs \$88.50 for 3 months.
- A car travels 820 km in 8 hours.
- A garden bed needs 6 kg of fertiliser for 5 square metres.

Solution

- The cost rate would be expressed in dollars per month.

$$\begin{aligned}\text{Connection rate} &= \frac{\$88.50}{3 \text{ months}} \\ &= \$29.50 \text{ month}\end{aligned}$$

We can write a rate as a fraction: divide the number of dollars by the number of months

- The car's speed would be expressed in kilometres per hour.

$$\begin{aligned}\text{Speed} &= \frac{820 \text{ km}}{8 \text{ hours}} \\ &= 102.5 \text{ km/h}\end{aligned}$$

Divide the number of kilometres by the number of hours

- The rate would be expressed in kg per m².

$$\begin{aligned}\text{Fertiliser cover} &= \frac{6 \text{ kg}}{5 \text{ m}^2} \\ &= 1.2 \text{ kg/m}^2\end{aligned}$$

Divide the number of kilograms by the number of square metres

Exercise 12-04 Rates

- 1 Write the units suitable for each rate below, in the form ____/____.
- | | | |
|-----------------------------------|----------------------------|--|
| a typing speed | b heart rate | c cost of a mobile phone call |
| d cost of meat | e a worker's wage | f water filling a swimming pool |
| g cost of posting a parcel | h population growth | i the cost of water |

See Example 8

- 2 Write each statement as a simplified rate.
- | | |
|---|-----------------------------------|
| a 180 km in 3 hours | b \$26.40 for 4 hours |
| c 100 m in 8 seconds | d \$12 for 750 mL |
| e 76 goals in 4 games | f \$48 for 2.5 kg |
| g 180 minutes for 10 questions | h 3000 words in 50 minutes |
| i 36 children in 20 families | j 9.8 m in 1 second |
| k 9000 revolutions in 60 seconds | l 36 runs in 8 overs |
| m \$1.60 for 32 grams | n 12 L to travel 100 km |
| o 50 sheep in 2.5 hours | p 98 containers in 7 days |
- 3 A lift should carry no more than 1600 kg or 20 people. What is this weight allowance, in kg/person? Select the correct answer **A**, **B**, **C** or **D**.
- | | | | |
|------------|-------------|----------------|-----------------|
| A 8 | B 80 | C 0.125 | D 32 000 |
|------------|-------------|----------------|-----------------|
- 4 In your own words, explain what is meant by each rate.
- | | |
|---|--|
| a a postage cost of \$1.40/kg | b a farmer keeping 60 sheep/hectare |
| c a factory manufacturing 920 toys/day | d a pump supplying 230 L/hour |

Worked solutions

Exercise 12-04

MAT07NAWS10066

- 5 The cost of 38 litres of petrol is \$55.86. Express this cost in c/L.
- 6 A tree grew 27 metres in 60 years. Calculate its growth rate in cm/year.
- 7 A complaints hotline took 852 calls in one year. Calculate the number of calls per month.
- 8 A pulp mill clears 27 000 hectares of forest in 15 years. At what rate in hectares/year does the mill clear the forest?

Mental skills 12A Maths without calculators

Multiplying by 5, 15, 25 and 50

It is easier to multiply a number by 10 than by 5, so to multiply by 5, we can halve the number, then multiply by 10. This is because $\frac{1}{2} \times 10 = 5$. We can use a similar strategy to multiply by 15, 25 and 50.

- 1 Study each example.

- a** To multiply by 5, halve the number, then multiply by 10.

$$\begin{aligned}
 18 \times 5 &= 18 \times \frac{1}{2} \times 10 \text{ (or factorise 18 into } 9 \times 2\text{)} \\
 &= 9 \times 10 \\
 &= 90
 \end{aligned}$$

- b** To multiply by 50, halve the number, then multiply by 100.

$$\begin{aligned} 26 \times 50 &= 26 \times \frac{1}{2} \times 100 \text{ (or factorise 26 into } 13 \times 2) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

- c** To multiply by 25, quarter the number, then multiply by 100.

$$\begin{aligned} 44 \times 25 &= 44 \times \frac{1}{4} \times 100 \text{ (or factorise 44 into } 11 \times 4) \\ &= 11 \times 100 \\ &= 1100 \end{aligned}$$

- d** To multiply by 15, halve the number, then multiply by 30.

$$\begin{aligned} 8 \times 15 &= 8 \times \frac{1}{2} \times 30 \text{ (or factorise 8 into } 4 \times 2) \\ &= 4 \times 30 \\ &= 120 \end{aligned}$$

- 2** Now simplify each product.

a 32×5

b 14×5

c 48×5

d 18×50

e 52×50

f 36×25

g 44×50

h 12×25

i 12×15

j 16×25

k 22×50

l 14×15

m 28×25

n 36×50

o 26×15

p 22×35

12-05 Best buys

When shopping, it is important to compare the prices of different brands or sizes of items and calculate which is the **best buy** ('best value for money'). This can be done by comparing the **unit cost** (cost of one item or unit) of each brand or size and choosing the cheapest one. A unit could be one gram or one millilitre.

Summary

Unit cost = cost $\div$ number of items or units

If you look carefully at price tags on supermarket shelves, you will notice unit prices displayed in small print under the main prices.

Example 9

Which brand of corn chips is the best buy?

- Cruncho: 50 g for \$1.02
- Hombre: 125 g for \$2.20
- Mexicana: 200 g for \$4.81

Solution

Calculating the unit cost (cost per gram) for each brand:

$$\text{Cruncho} = \$1.02 \div 50 = \$0.0204/\text{g}$$

$$\text{Hombre} = \$2.20 \div 125 = \$0.0176/\text{g}$$

$$\text{Mexicana} = \$4.81 \div 200 = \$0.024/\text{g}$$

Hombre has the lowest unit cost, so it is the best buy.

Example 10

Which size of fruit juice is the better buy?

- Small bottle: \$3.20 for 600 mL
- Large carton: \$8.05 for 2.25 L

Solution

Calculating the unit cost (cost per mL) for each size:

$$\text{Small bottle} = 320\text{c} \div 600 = 0.533333\dots \text{c/mL}$$

$$\text{Large carton} = 805\text{c} \div 2250 = 0.357777\dots \text{c/mL}$$

$$2.25 \text{ L} = 2250 \text{ mL}$$

The large carton of fruit juice has the lower unit cost, so it is the better buy.

Exercise 12-05 Best buys

1 Calculate the unit cost of each item correct to the nearest cent.

a 5 razors for \$4.50	b 6 eggs for \$3.19	c 8 juice packs for \$4.45
d 80 teabags for \$3.90	e 200 g biscuits for \$2.79	f 40 g lollies for \$1.20

See Example 9

2 Find the better buy in each pair of items.

- a** Block ices: 18-pack for \$5.00, Frooty Licious: 8-pack for \$2.40
- b** Large salad: 400 g for \$4.80, Medium salad: 250 g for \$2.80
- c** Crunchy chips single pack: 45 g for \$2.10, party pack: 150 g for \$6.50
- d** Moo Milk: 400 mL for \$0.78, or 3 L for \$5.20
- e** Peanut butter: 80 g for \$2.20, or 280 g for \$6.50
- f** Super DVDs: 6 for \$15.50, Mega DVDs: 8 for \$20

3 Find the best brand for each item.

See Example 10

	Sols	Baldy	GIA	Bullworths
a Salt	500 g for \$1.89	200 g for 76 c	100 g for 39 c	1 kg for \$3.74
b Juice	250 mL for \$1	50 mL for 17 c	1 L for \$2	200 mL for 70 c
c Bananas	750 g for \$5.20	400 g for \$2.40	250 g for \$1.70	2 kg for \$12.20
d Aluminium foil	20 m for \$3.29	5 m for \$0.80	30 m for \$4.90	18 m for \$2.85

4 For each item, find which size is the best value for money.

a

375 g \$1.90 750 g \$3.90

b

2 L \$3.25 375 mL × 6 \$3.85

c

735 g \$2.78 440 g \$2.21

d

235 g \$4.20 175 g \$3.20 115 g \$1.80

5 Which brand of rice is the best buy? Select the correct answer **A**, **B**, **C** or **D**.

- A** Super Grain: 400 g for \$1.15
B Wonder Bowl: 600 g for \$1.74
C Hearty White: 1 kg for \$2.95
D The Rice Stuff: 20 kg for \$59.80

6 For each item, which size gives the better value for money?

	Small	Large
a Premium mince	450 g for \$5.35	600 g for \$6.80
b Spring water	300 mL for \$1.84	420 mL for \$2.20
c Rice crackers	350 g for \$4.20	500 g for \$5.90
d Mixed nuts	450 g for \$7.60	750 g for \$11.90
e Coffee	150 g for \$6.65	60 g for \$3.30

12-06 Rate problems

Worksheet

What's my speed?

MAT07NAWK10097

Problems involving rates can usually be solved by multiplying or dividing. The following strategy may help.

- Write the units of the rate x/y as a fraction: $\frac{x}{y}$
- To find the quantity in the numerator, x , **multiply** by the rate.
- To find the quantity in the denominator, y , **divide** by the rate.

Worksheet

Rate problems

MAT07NAWK10098

Homework sheet

Rates

MAT07NAHS10016

Example 11

A bus travels at an average speed of 88 km/h. How far will it travel in $3\frac{1}{2}$ hours?

Solution

The units of the rate expressed as a fraction is $\frac{\text{km}}{\text{h}}$.

To find the distance travelled in kilometres (the numerator), **multiply** by the rate.

$$\begin{aligned}\text{Distance travelled} &= 3\frac{1}{2} \times 88 \\ &= 308 \text{ km}\end{aligned}$$

← No. of hours $\times$ distance
travelled per hour

Example 12

A soft drink factory makes cans at a rate of 150 cans/min.

- How long (to the nearest minute) would it take to produce 1000 cans?
- How many cans are produced in the 4-hour shift before lunch?

Solution

The units of the rate expressed as a fraction is $\frac{\text{cans}}{\text{min}}$.

- To find the number of minutes (the denominator), **divide** by the rate.

$$\text{Time taken} = 1000 \div 150 = 6\frac{2}{3} \text{ minutes} \approx 7 \text{ minutes}$$

← How many times 150 divides
into 1000

- To find the number of cans (the numerator), **multiply** by the rate.

$$\begin{aligned}\text{No. of cans} &= 4 \times 60 \times 150 & 4 \text{ h} &= 4 \times 60 \text{ min} \\ &= 360\,000\end{aligned}$$

← No. of mins $\times$ cans per min

Exercise 12-06 Rate problems

- 1 Ben types 55 words per minute. How many words can he type in 20 minutes?
- 2 Marisol earns \$23.50 per hour as a publicist. How much will she earn if she works 38 hours in a week?
- 3 A cricket batsman scores 35 runs per innings. How many runs will he score in 12 innings?
- 4 An aeroplane travelled at 920 km/h. How far did it travel in 4 hours?
- 5 A tap drips water at a rate of 15 mL/h.
How much water will be wasted:
 - a in one day?
 - b in one week?

See Example 11



- 6 A plane can carry a total of 450 passengers per flight. How many flights will it take to carry 2 700 passengers?
- 7 Suyen runs 100 metres in 12 seconds. How far will she run in one minute at the same rate?
- 8 Mia cycles at 12 km per hour for 4 hours. How far does she travel?
- 9 A farmer can graze 15 sheep per hectare. If he has 65 hectares set aside for sheep, how many sheep can he graze?
- 10 The flywheel of an engine rotates at a rate of 1200 revolutions per minute. How many revolutions does it make in 20 seconds? Select the correct answer **A**, **B**, **C** or **D**.
A 400 **B** 60 **C** 4000 **D** 24 000
- 11 The temperature in Mittagong at 7 a.m. is 8°C . If the temperature rises at a rate of 3°C/h , find:
 - a the temperature at 10 a.m.
 - b the temperature at 2 p.m.
 - c when the temperature will be 23°C
- 12 A bus travels at an average speed of 76 km/h. Find:
 - a how far the bus travels in 8 hours
 - b how long the bus will take to travel 380 km
- 13 Zac has 504 cows in a 42 hectare paddock.
 - a How many cows per hectare does he have?
 - b How many cows could he keep in a 50 hectare paddock at the same rate of cows per hectare?
 - c How many hectares would he need to keep 840 cows at the same rate?

See Example 12

Worked solutions

Exercise 12-06

MAT07NAWS10067

- 14 Petrol costs \$1.48/L. If Robert has \$50 to spend, how much petrol could he buy (correct to the nearest litre)?
- 15 Dominic needs to buy new carpet for the family home. The carpet costs \$125/m² and the carpet layers charge \$480 to lay it. How much will it cost to carpet the house if Dominic needs 28 square metres of carpet?
- 16 Zdena uses fertiliser on her farm at a rate of 30 kg/ha. How many hectares can she cover if she has 145 kg of fertiliser? Answer to the nearest 0.1 ha.
- 17 Best Mince costs \$9/kg on special. Find:
- the cost of buying 1.5 kg of Best Mince
 - the amount of mince that could be bought for \$19.80.
- 18 The distance between Sydney and Perth is 3972 km by road. How long would it take to travel this distance if you:
- walked at 5 km/h?
 - jogged at 9 km/h?
 - cycled at 25 km/h?
 - took the Indian Pacific train at 85 km/h?
 - drove at 100 km/h?
 - flew 3276 km by air at 900 km/h?

Worked solutions

Exercise 12-06

MAT07NAWS10067

Technology Conversion rates

In this activity you will use a spreadsheet to convert between two different units.

Miles to kilometres

An Imperial unit of length still used in some countries is the **mile**. It is approximately equal to 1.6 kilometres.

- Set up the spreadsheet shown below to convert between miles and kilometres. In cell C2, enter the formula **=C1*1.6**.

	A	B	C	D	E	F	G	H
1	Number of Miles	0	2	4	6	8	10	
2	Number of Kilometres	0	=C1*1.6					
3								

- Use **Fill Right** from cell C2 to convert from miles to kilometres for each of the cells from D2 to G2.
- Extend your table to calculate the equivalent number of kilometres for:
 - 12 miles
 - 20 miles
 - 30 miles
- Now use your table to find the (approximate) equivalent number of miles for:
 - 16 kilometres
 - 22 kilometres
 - 40 kilometres

Australian dollars to Hong Kong dollars

Another commonly-used rate is currency conversion. We will convert between Australian dollars and Hong Kong dollars using a spreadsheet.

	A	B
1	Australian Dollars	Hong Kong Dollars
2	0	0
3	50	
4	100	
5	150	
6	200	
7	250	
8		

- Set up the spreadsheet shown below to convert between Australian dollars and Hong Kong dollars. One Australian dollar is equal to about 7.5 Hong Kong dollars. In cell B3, enter the formula $=A3*7.5$.
- Use **Fill Down** from cell B3 to convert from Australian to Hong Kong dollars for each of the cells down to B7.
- Extend your table to calculate the equivalent number of Hong Kong Dollars for:
 - A\$300
 - A\$450
 - A\$700
- Now use your table to find the (approximate) Australian dollars for:
 - HK\$1000
 - HK\$1500
 - HK\$2500
- Kiah wants to go on a holiday to Hong Kong after she finishes school. She intends to save 1500 Australian dollars from her part-time job to spend on her holiday. Extend your table to find what amount this is equal to in Hong Kong dollars.

12-07 Travel graphs

A **travel graph** is a line graph that describes a journey, by comparing distance with time (on the vertical and horizontal axes respectively). The slope or steepness of the graph indicates speed (how fast you are travelling).

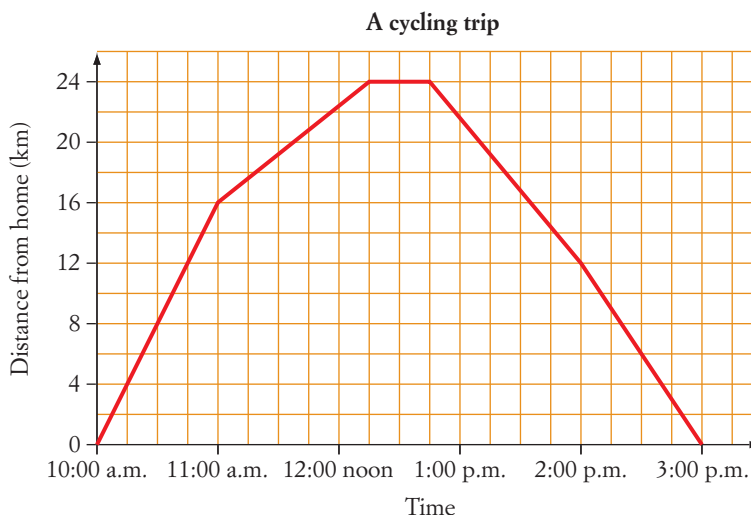
Puzzle sheet

Jane's diary

MAT07NAPS10044

Example 13

This graph shows Thuy's cycling trip.



Worksheet

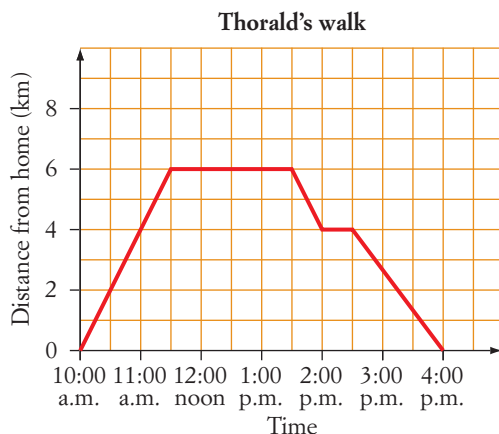
Travel graph stories

MAT07NAWK10099

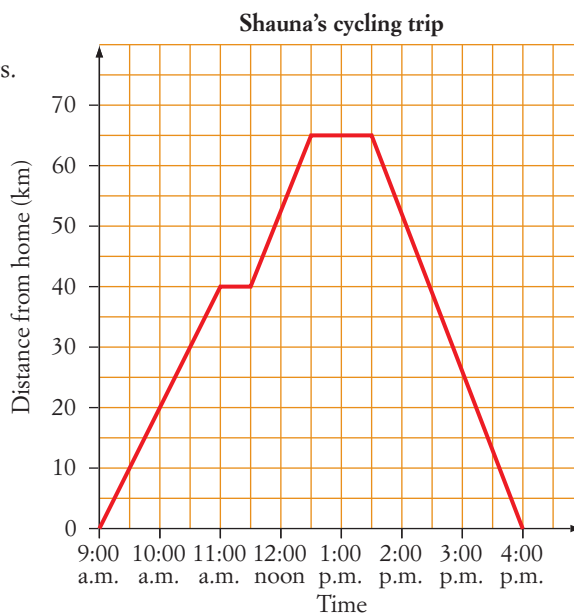
Exercise 12-07 Travel graphs

- 1 This travel graph shows Thorald's return trip to his friend Emil's house.

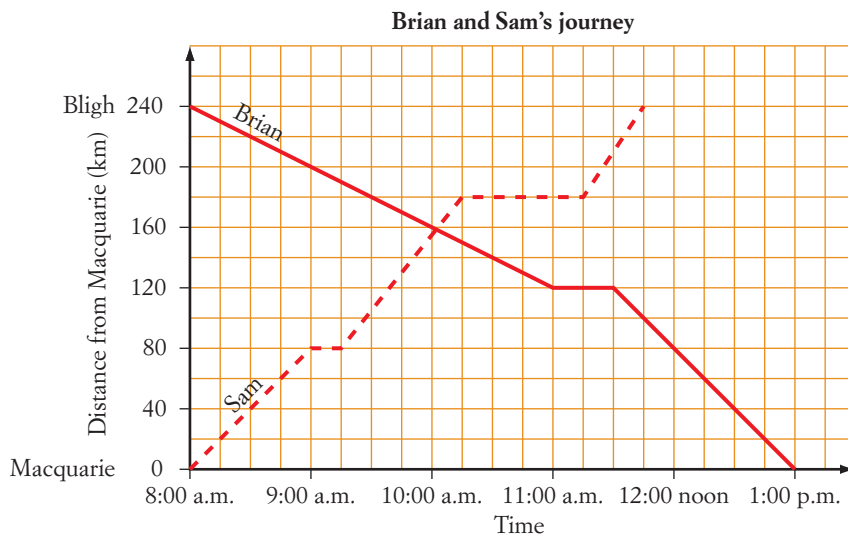
See Example 13



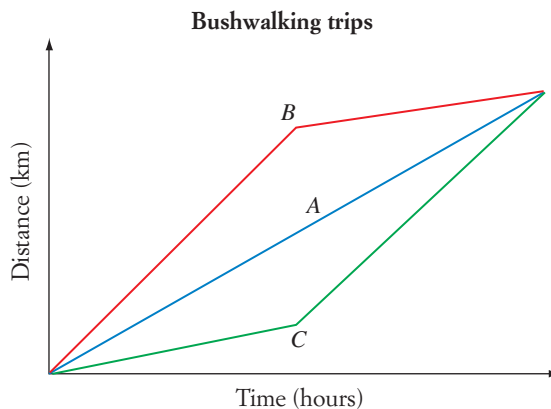
- a At what time did Thorald begin his trip?
 - b How far is Emil's house from Thorald's house?
 - c How long did it take Thorald to walk to Emil's house?
 - d Use the formula $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$ to calculate his speed in km/h during this time.
 - e How long did Thorald spend at Emil's house?
 - f What was the total distance that Thorald travelled?
 - g When and at what distance did Thorald stop on his way home?
 - h What was the total time of his trip, including rests?
- 2 This graph shows Shauna's cycling trip.
- a Calculate her speed in the first 2 hours.
 - b When was her first stop? How far from home was she?
 - c When did Shauna start to return home?
 - d How long did she take to return home?
 - e Calculate her speed for her journey home.
 - f How far from home was Shauna at 3 p.m.?
 - g How far did Shauna travel altogether on this day?
 - h Did Shauna travel faster between 9 a.m. and 11 a.m., or between 11:30 a.m. and 12:30 a.m.? How does the graph illustrate this?



- 3 Brian travels from Bligh to Macquarie, while Sam travels from Macquarie to Bligh.

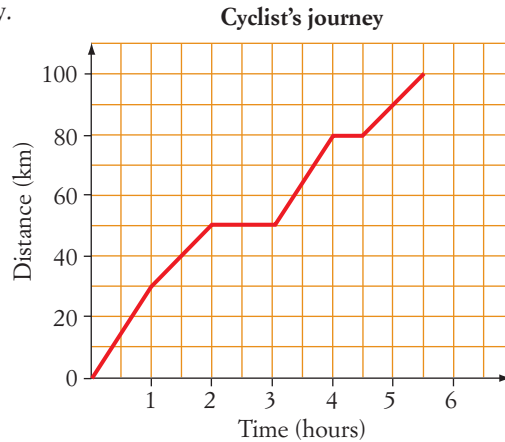


- How far is it between the two towns?
 - How far is Sam from Macquarie at 10:30 a.m.?
 - When does Sam reach Bligh?
 - At what speed is Brian travelling before he stops?
 - When do Brian and Sam pass each other? How far are they from Macquarie when they pass?
 - Is Brian or Sam travelling faster during the day? How can you tell?
 - Is Brian's speed lower before 11 a.m. or after 11:30 a.m.? How can you tell?
 - For how long did Sam stop altogether on the trip? Select the correct answer **A**, **B**, **C** or **D**.
A 60 minutes **B** 75 minutes **C** 5 minutes **D** 90 minutes
- 4 Match each bushwalker described below with the correct travel graph.



- Jeremy was fast at first, but then got slower after lunch.
- Richard kept a constant speed all day.
- James was slow at first, but got faster after lunch.

- 5 This travel graph shows Caitlin's cycling journey. Write a story about her ride, based on the information in the graph.



- 6 Jack goes on a trip to visit his grandparents.
- Draw a travel graph to represent Jack's journey:
 - Jack starts at 9 a.m.
 - He travels 180 km in 2 hours.
 - He stops for $\frac{1}{2}$ an hour.
 - He travels 120 km in $1\frac{1}{2}$ hours.
 - He stops at his grandparents' house for 6 hours.
 - He travels back home at 100 km/h.
 - How far from Jack's home do his grandparents live?
 - At what time does Jack arrive there?
 - In which part of the journey is Jack travelling the fastest?
 - How long does the journey home take?
 - What is the total time taken for the trip?
 - How far from home is Jack at 9 p.m.?

Technology Speed and distance

Jana and Sunil travel in separate cars from the same starting point. They travel at different constant speeds to the same destination. In hours, how much more quickly did Sunil arrive at their destination, compared to Jana who reached the destination in 8 hours?

- Set up a spreadsheet as shown below.

Worksheet

What's my speed?

MAT07NAWK10097

	A	B	C
1	Person:	Jana	Sunil
2	Speed (km/h):	70	80
3			
4	Time (h)	Distance travelled (km)	
5	1		
6	2		
7	3		
8	4		
9	5		
10	6		
11	7		
12	8		
13			

- 2 Use the formula $\text{Distance} = \text{speed} \times \text{time}$. Convert this into a spreadsheet formula in cell B5 to calculate the distance that Jana has travelled after each hour has passed.
- 3 Use the Fill Down function to copy the formula from cell B5 down to cell B12.
- 4 Modify the formula entered from Step 2 in cell C5 to calculate the distance that Sunil travels in the first hour.
- 5 Use the Fill Down function to copy the formula from cell C5 down to cell C12.
- 6 By comparing the values in columns B and C, find how far Jana and Sunil's starting point was from their final destination.
- 7 In hours, how much more quickly did Sunil arrive at their destination, compared to Jana?

NSW

12-08 Time calculations

Example 14

Weblink

Time for time

Maths clip

Time

MAT07NAVT00005

Worksheet

Converting time units

MAT07MGWK00045

Puzzle sheet

Converting time units

MAT07MGPS00025

- a** Round each amount of time to the nearest hour.
- i 7.83 h ii 12 h 19 min iii 2 h 43 min 6 s
- b** Round each amount of time to the nearest minute.
- i 11.4 min ii 25 min 37 s iii 3 h 6 min 30 s

Solution

- a i** $7.83 \text{ h} \approx 8 \text{ h}$
ii $12 \text{ h } 19 \text{ min} \approx 12 \text{ h}$

Rounding down because $19 \text{ min} < 30 \text{ min}$

← When rounding to the nearest hour, use 30 minutes as the halfway mark because there are 60 minutes in an hour

- iii** $2 \text{ h } 43 \text{ min } 6 \text{ s} \approx 3 \text{ h}$

Rounding up because $43 \text{ min} > 30 \text{ min}$

← When rounding to the nearest minute, use 30 seconds as the halfway mark because there are 60 seconds in a minute

- b i** $11.4 \text{ min} \approx 11 \text{ min}$
ii $25 \text{ min } 37 \text{ s} \approx 26 \text{ min}$
iii $3 \text{ h } 6 \text{ min } 30 \text{ s} \approx 3 \text{ h } 7 \text{ min}$

Rounding up because $37 \text{ s} > 30 \text{ s}$
 Rounding up 30 s

Example 15

Complete:

- a 3 years = _____ months
- b 91 days = _____ weeks
- c 275 min = _____ h _____ min

Solution

$$\text{a } 3 \text{ years} = 3 \times 12 \text{ months} \\ = 36 \text{ months}$$

$$1 \text{ year} = 12 \text{ months}$$

$$\text{b } 91 \text{ days} = (91 \div 7) \text{ weeks} \\ = 13 \text{ weeks}$$

$$1 \text{ week} = 7 \text{ days}$$

$$\text{c } 275 \text{ min} = (275 \div 60) \text{ h} \\ = 4.58333 \dots \text{ h} \\ = 4 \text{ h } 35 \text{ min}$$

$$(\text{or } 4\frac{7}{12} \text{ h})$$

Shown on a calculator as $4^{\circ}35'$
after entering $\circ' ''$ or **DMS**.

Most scientific calculators have a degrees-minutes-seconds key ($\circ' ''$ or **DMS**) that is useful for calculations involving minutes and seconds (base 60). This key converts decimal answers for time to hours-and-minutes or minutes-and-seconds.

Scientific calculator
exercise

Time

MAT07MGSC00004

Example 16

Simplify each expression.

- a 9 h 25 min + 6 h 48 min
- b 7 h 5 min - 3 h 24 min

Solution

$$\begin{aligned} \text{a } 9 \text{ h } 25 \text{ min} + 6 \text{ h } 48 \text{ min} &= (9 + 6) \text{ h} + (25 + 48) \text{ min} \\ &= 15 \text{ h} + 73 \text{ min} \\ &= 15 \text{ h} + 1 \text{ h } 13 \text{ min} \\ &= 16 \text{ h } 13 \text{ min} \end{aligned}$$

OR On a calculator, enter 9 $\circ' ''$ 25 $\circ' ''$ + 6 $\circ' ''$ 48 $\circ' ''$ =

$$\begin{aligned} \text{b } 7 \text{ h } 5 \text{ min} - 3 \text{ h } 24 \text{ min} &= (7 - 3) \text{ h} + (5 - 24) \text{ min} \\ &= 4 \text{ h} + (-19) \text{ min} \\ &= 3 \text{ h} + (60 - 19) \text{ min} \\ &= 3 \text{ h } 41 \text{ min} \end{aligned}$$

OR On a calculator, enter 7 $\circ' ''$ 5 $\circ' ''$ - 3 $\circ' ''$ 24 $\circ' ''$ =

See Example 14

Exercise 12-08 Time calculations

Extra questions

Using time lines

MAT07MGEQ00026

- 1 Round each amount of time to the nearest hour.
- | | | |
|-------------------|----------|---------------|
| a 1 h 28 min | b 20.7 h | c 5 h 11 min |
| d 3 h 40 min 18 s | e 9.15 h | f 10 h 30 min |

Extra questions

Converting units of time

MAT07MGEQ00027

- 2 Round each amount of time to the nearest minute.
- | | | |
|-------------------|-------------|-------------------|
| a 21 min 43 s | b 7.5 min | c 8 min 8 s |
| d 2 h 15 min 30 s | e 16.31 min | f 4 h 20 min 39 s |

See Example 15

- 3 Copy and complete.
- | | |
|-------------------------|---------------------------------------|
| a 5 weeks = _____ days | b $2\frac{1}{2}$ years = _____ months |
| c 3 days = _____ h | d 2 years = _____ weeks |
| e 1 week = _____ h | f 3 fortnights = _____ days |
| g 96 h = _____ days | h 480 s = _____ min |
| i 84 days = _____ weeks | j 60 months = _____ years |
| k 60 h = _____ days | l 56 days = _____ fortnights |

Worked solutions

Exercise 12-08

MAT07NAWS10068

- 4 Calculate the number of seconds in:
- | | | |
|------------|-----------|------------|
| a one hour | b one day | c one week |
|------------|-----------|------------|
- 5 Convert each time to hours and minutes.
- | | | |
|-----------|-----------|---------------------------------------|
| a 340 min | b 186 min | c 227 min |
| d 405 min | e 262 min | f $499 \text{ min} + 105 \text{ min}$ |
- 6 Are you over a billion seconds old? Find your age in seconds to answer this question.
- 7 Convert each time to minutes and seconds.
- | | | |
|---------|---------|-----------------------------------|
| a 372 s | b 545 s | c 238 s |
| d 422 s | e 500 s | f $216 \text{ s} + 308 \text{ s}$ |

See Example 16

- 8 Simplify each expression.
- | | |
|--|---|
| a $2 \text{ h } 15 \text{ min} + 4 \text{ h } 32 \text{ min}$ | b $9 \text{ h } 20 \text{ min} + 3 \text{ h } 47 \text{ min}$ |
| c $6 \text{ h } 36 \text{ min} - 2 \text{ h } 11 \text{ min}$ | d $10 \text{ h } 31 \text{ min} - 7 \text{ h } 6 \text{ min}$ |
| e $5 \text{ h } 12 \text{ min} + 1 \text{ h } 48 \text{ min}$ | f $3 \text{ h } 45 \text{ min} + 3 \text{ h } 29 \text{ min}$ |
| g $12 \text{ h } 32 \text{ min} - 8 \text{ h } 19 \text{ min}$ | h $4 \text{ h } 10 \text{ min} - 42 \text{ min}$ |
| i $7 \text{ h } 46 \text{ min} + 2 \text{ h } 17 \text{ min}$ | j $5 \text{ h } 33 \text{ min} + 4 \text{ h } 18 \text{ min}$ |
| k $8 \text{ h } 15 \text{ min} - 6 \text{ h } 24 \text{ min}$ | l $2 \text{ h } 29 \text{ min} - 1 \text{ h } 40 \text{ min}$ |

Just for the record**Minutes and seconds**

In Chapter 2, you learnt that there are 360° in a revolution because the ancient Babylonians used a base 60 number system and believed that a year lasted 360 days. The Babylonians, who lived where Iraq is today, invented the units for measuring angles and time in 2000 BCE. That is why there are 60 minutes in an hour and 60 seconds in a minute.

The word 'minute' has another meaning. When pronounced '*my-newt*', it means tiny but this meaning is also related to the minute as a unit of time. A minute is a tiny fraction of an hour, and comes from the Latin '*pars minuta prima*', meaning the first division (or part) of an hour. Find out how the two meanings of the word 'second' are similarly related.

Mental skills 12B Maths without calculators

Dividing by 5, 15, 25 and 50

It is easier to divide a number by 10 than by 5, so to divide by 5, we can divide by 10, then double the answer. We can use a similar strategy to divide by 15, 25 and 50.

1 Study each example.

- a** To divide by 5, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$$\begin{aligned} 140 \div 5 &= 140 \div 10 \times 2 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

- b** To divide by 50, divide by 100 and double the answer. This is because there are two 50s in every 100.

$$\begin{aligned} 400 \div 50 &= 400 \div 100 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

- c** To divide by 25, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$$\begin{aligned} 600 \div 25 &= 600 \div 100 \times 4 \\ &= 6 \times 4 \\ &= 24 \end{aligned}$$

- d** To divide by 15, divide by 30 and double the answer. This is because there are two 15s in every 30.

$$\begin{aligned} 240 \div 15 &= 240 \div 30 \times 2 \\ &= 80 \times 2 \\ &= 160 \end{aligned}$$

2 Now simplify each quotient.

a $90 \div 5$

b $170 \div 5$

c $230 \div 5$

d $330 \div 5$

e $1300 \div 50$

f $800 \div 50$

g $400 \div 50$

h $2200 \div 50$

i $300 \div 25$

j $1000 \div 25$

k $700 \div 25$

l $600 \div 15$

m $270 \div 15$

n $360 \div 45$

o $900 \div 25$

p $7000 \div 50$

12-09 24-hour time

Video tutorial

24-hour time

MAT07MGVT10024

Worksheet

TV times

MAT07MGWK10100

Puzzle sheet

12- and 24- hour time

MAT07MGPS00024

Skillsheet

Units of time

MAT07NASS10034

Skillsheet

Telling the time

MAT07MGSS10035

Skillsheet

24-hour time

MAT07MGSS10036

TLF learning object

Time tools: 24-hour to the minute (L9642)

Scientific calculator exercise

Time

MAT07MGSC00004

Worksheet

12- and 24- hour time

MAT07MGWK00043

Worksheet

24-hour time on an analogue clock

MAT07MGWK00044

There are two ways to show the time of day:

- **12-hour time:** the conventional way using a.m. and p.m. and the hours 1 to 12
- **24-hour time:** a more formal way that uses the hours 0 to 23 and no a.m./p.m.

24-hour time is commonly used around the world, especially in the armed forces, navigation, transport timetables and hospital records. This table shows the relationship between 12-hour time and 24-hour time.

24-hour time	12-hour time
0000	12 a.m. (midnight)
0100	1 a.m.
0200	2 a.m.
0300	3 a.m.
0400	4 a.m.
0500	5 a.m.
0600	6 a.m.
0700	7 a.m.
0800	8 a.m.
0900	9 a.m.
1000	10 a.m.
1100	11 a.m.

24-hour time	12-hour time
1200	12 p.m. (midday)
1300	1 p.m.
1400	2 p.m.
1500	3 p.m.
1600	4 p.m.
1700	5 p.m.
1800	6 p.m.
1900	7 p.m.
2000	8 p.m.
2100	9 p.m.
2200	10 p.m.
2300	11 p.m.

Summary

To convert from 24-hour time to 12-hour time

- If it begins with '00', then it is the 12:00 a.m. (midnight) hour
- If it begins with '12', then it is the 12:00 p.m. (midday) hour
- If it is less than 1200, then it is a.m. (morning time)
- If it is 1300 or more, then it is p.m. (afternoon/evening) time: subtract 12 from the hour.

Example 17

Convert each 24-hour time to 12-hour time.

a 1020

b 2130

c 0050

d 1240

Solution

a $1020 < 1200$, so it is a.m. time.

$$1020 = 10:20 \text{ a.m.}$$

b $2130 > 1300$, so it is p.m. time.

$$\text{Subtract 12 from the hours: } 21 - 12 = 9$$

$$2130 = 9:30 \text{ p.m.}$$

- c 0050 begins with 00, so it is 12:00 a.m. time.
0050 = 12:50 a.m.
- d 1240 begins with 12, so it is 12:00 p.m. time.
1240 = 12:40 p.m.

Summary

To convert from 12-hour time to 24-hour time

- If it is the 12:00 a.m. (midnight) hour, begin with '00'
- If it is a.m. time or the 12:00 p.m. (midday) hour, write as is but make sure the hour has two digits (for example, 02, 09)
- If it is 1:00 p.m. or later, then add 12 to the hour

Example 18

Convert each 12-hour time to 24-hour time.

- a 8:15 a.m. b 2:40 p.m. c 12:20 p.m.

Solution

- a 8:15 a.m. is a.m. time, so write as a 4-digit number and rename 8 as 08.
8:15 a.m. = 0815
- b 2:40 p.m. is after 1:00 p.m., so add 12 to the hour: $2 + 12 = 14$
2:40 p.m. = 1430
- c 12:20 p.m. is in the 12:00 p.m. (midday) hour, so write it as a 4-digit number.
12:20 p.m. = 1220

Video tutorial

24-hour time

MAT07MGVT10024

Exercise 12-09 24-hour time

- 1 Convert each 24-hour time to 12-hour time.

a 0845	b 1320	c 1750	d 0017
e 2105	f 1832	g 1115	h 0238
i 1440	j 0320	k 1655	l 2331
m 0108	n 1045	o 2000	p 0643

- 2 Convert each 12-hour time to 24-hour time.

a 6:35 p.m.	b 8:05 a.m.	c 11:44 a.m.	d 11:20 p.m.
e 2:51 a.m.	f 12:30 p.m.	g 3:38 p.m.	h 7:11 a.m.
i 9:08 a.m.	j 9:54 p.m.	k 12:42 a.m.	l 7:39 p.m.
m 1:59 a.m.	n 10:18 p.m.	o 8:46 a.m.	p 5:23 p.m.

See Example 17

Extra questions

12-hour and 24-hour
time

MAT07MGEQ00025

See Example 18

- 3 Write the times shown on each clock in 12-hour time.

a 11:23

b 17:54

c 13:24

- 4 Write the times shown on each clock in 24-hour time.

a AM
PM 5:17b AM
PM 10:06c AM
PM 2:45

- 5 This plane flight timetable shows flights from eastern Australia to Christchurch in New Zealand.

	Flight code	Depart	Arrive
Sydney - Christchurch	SYCH	0855	1355
		1710	2210
		1945	0045
Melbourne - Christchurch	MECH	0845	1405
		0905	1425
		1755	2315
Brisbane - Christchurch	BNCH	0750	1415
		1630	2255

Express the answers to each question in 12-hour time.

- a When does the first flight leave Sydney and when does it arrive in Christchurch?
 b When does the last flight leave Melbourne and when does it arrive in Christchurch?
 c At what time between 12 midnight and 1 a.m. does a flight arrive in Christchurch?
 d What is the earliest departure time?
 e What is the latest departure time?

YEAR 8

12-10 Time differences

Animated example

Time calculations

MAT07MGAE00006

Worksheet

Time calculations

MAT07MGWK10101

Worksheet

World time zones

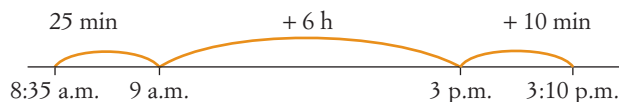
MAT07MGWK10102

Example 19

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

Solution

Method 1



From 8:35 a.m. to 9:00 a.m. = 25 minutes

From 9:00 a.m. to 3:00 p.m. = 6 hours

From 3:00 p.m. to 3:10 p.m. = 10 minutes

$$\begin{aligned}\text{Total time difference} &= 6 \text{ h} + 25 \text{ min} + 10 \text{ min} \\ &= 6 \text{ h } 35 \text{ min}\end{aligned}$$

Method 2

Convert to 24-hour time first, then use the calculator's $\odot, \text{''}$ or **DMS** key to enter hours and minutes, and subtract the times.

Puzzle sheet

Australian times

MAT07MGPS00027

Puzzle sheet

Time calculations

MAT07MGPS00026

Worksheet

Time

MAT07MGWK00047

Example 20

What is the difference in time between 1050 and 1930?

Solution

Method 1

From 1050 to 1100 = 10 min

From 1100 to 1900 = 8 h

$$19 - 11 = 8$$

From 1900 to 1930 = 30 min

$$\begin{aligned}\text{Total time difference} &= 8 \text{ h} + 10 \text{ min} + 30 \text{ min} \\ &= 8 \text{ h } 40 \text{ min}\end{aligned}$$

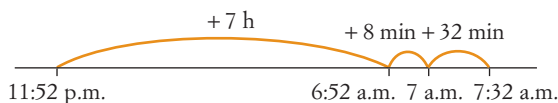
Method 2

Use the calculator's $\odot, \text{''}$ or **DMS** key to enter hours and minutes, and subtract the times.

Example 21

What is the time 7 hours and 40 minutes after 11:52 p.m.?

Solution



7 hours after 11:52 p.m. is 6:52 a.m.

40 minutes after 6:52 a.m. is 7:32 a.m.

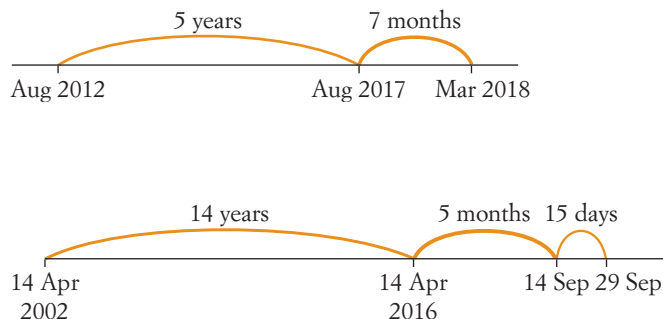
Example 22

- a Ms Hossain bought a new car in August 2012. How old is her car in March 2018?
 b Boris was born on 14 April 2002. How old is he on 29th September 2016?

Solution

- a From August 2012 to August 2017 = 5 years
 From August 2017 to March 2018 = 7 months
 Age of car = 5 years 7 months

- b From 14/4/02 to 14/4/16 = 14 years
 From 14/4/16 to 14/9/16 = 5 months
 From 14/9/16 to 29/9/16 = 15 days
 Boris' age = 14 years 5 months 15 days



Exercise 12-10 Time differences

See Example 19

- 1 What is the difference in time between 10:42 a.m. and 2:13 p.m.? Select the correct answer A, B, C or D.
 A 3 h 31 min B 4 h 55 min C 8 h 29 min D 12 h 55 min
- 2 Calculate the time difference between each pair of times.
- | | | |
|----------------------------|-----------------------------|---------------------------|
| a 7:15 p.m. and 8:20 p.m. | b 10:16 a.m. and 12:06 p.m. | c 4:09 a.m. and 9:53 a.m. |
| d 11:15 p.m. and 3:08 a.m. | e 7:27 a.m. and 1:12 p.m. | f 9:36 p.m. and 9:14 a.m. |
| g 7:45 p.m. and 10:10 p.m. | h 2:24 a.m. and 3:07 a.m. | i 4:15 p.m. and 6:02 p.m. |
| j 10:25 a.m. and 2:33 p.m. | k 8:40 a.m. and 4:19 p.m. | l 6:45 a.m. and 8:10 p.m. |

See Example 20

- 3 Calculate the time difference between each pair of times.
- | | | |
|-----------------|-----------------|-----------------|
| a 0800 and 1100 | b 0500 and 1500 | c 0450 and 1930 |
| d 0630 and 1045 | e 0940 and 1455 | f 1120 and 2005 |
| g 0510 and 1215 | h 0740 and 1310 | i 1245 and 2125 |
| j 1340 and 2150 | k 1035 and 1630 | l 0945 and 1825 |

See Example 21

- 4 What time will it be:
- | | |
|---|--|
| a 5 hours after 3:00 p.m.? | b 8 hours after 11:00 a.m.? |
| c 28 minutes after 7:15 p.m.? | d 3 hours 32 minutes after 9:45 a.m.? |
| e 3 hours 19 minutes after 10:49 p.m.? | f 4 hours after 9:32 a.m.? |
| g 9 hours after 5:14 p.m.? | h 45 minutes after 3:30 p.m.? |
| i $2\frac{1}{4}$ hours after 4:02 a.m.? | j 12 hours 40 minutes after 2:45 a.m.? |

- 5 A marathon began at 10:20 a.m. Here are six of the competitors and the times they ran. Write the runners in their order of finishing and the time each crossed the finishing line.

Mike 3:11 (3 h 11 min)

Anna 2:54

Ken 2:59

Joe 2:23

Pathena 3:01

Gail 3:42



- 6 A plane left Perth at 0900 and arrived in Sydney at 1345 (Perth time). How long was the journey?
- 7 At Westvale Islamic School, the school day begins at 8:45 a.m. and ends at 3:10 p.m. How long is the school day?
- 8 What time will it be:
- a 4 hours before 6:15 p.m.?
 - c 45 minutes before 3:20 p.m.?
 - e 3 hours 30 minutes before 11:25 a.m.?
 - b 7 hours before 9:30 a.m.?
 - d 2 hours 10 minutes before 1:35 p.m.?
 - f 1 hour 25 minutes before 7:40 p.m.?
- 9 Jamie bought a computer in June 2014. How old is it in:
- a August 2018?
 - b January 2019?
- 10 Diana was born on 15th March 2006. How old is she in years, months, days on:
- a 20th April 2012?
 - b 9th September 2021?
- 11 A movie starts at 1:40 p.m. and ends at 3:55 p.m. What is the length of the movie?
- 12 South Australia is $1\frac{1}{2}$ hours ahead of Western Australia. Anne is flying from Perth to Port Augusta in South Australia. If the flight takes $2\frac{1}{2}$ hours and it leaves Perth at 10 a.m., what will be the local time in Port Augusta when the plane lands?
- 13 A box of chocolates bought in November 2012 has a 'Use by' date of March 2014. How long will it last?
- 14 Mr Badger has been teaching at Westvale High School since February 1998. How long has he been at the school when it is December 2015?
- 15 Karen works at the office from 8:10 a.m. till 4:30 p.m. How much will she earn for the day if her hourly wage is \$18.60?
- 16 Will and Kate were married on 29th April 2011. How long have they been married in years, months and days on 10th May 2025?

Worked solutions

Exercise 12-10

MAT07NAWS10069

See Example 22

12-11 Timetables

Example 23

This timetable is part of a schedule for a bus travelling between Sydney and Wagga Wagga.

Sydney to Wagga Wagga

Sydney	2:30 p.m.
Strathfield	3:00 p.m.
Yagoona	3:20 p.m.
Liverpool	3:45 p.m.
Mittagong	4:40 p.m.
Goulburn*	5:40 p.m.
Yass	7:10 p.m.
Jugiong	7:55 p.m.
Gundagai	8:20 p.m.
Wagga Wagga	9:30 p.m.

Wagga Wagga to Sydney

Wagga Wagga	7:15 a.m.
Gundagai	8:25 a.m.
Jugiong	8:54 a.m.
Yass	9:41 a.m.
Goulburn*	10:41 a.m.
Mittagong	12:10 p.m.
Liverpool	1:05 p.m.
Yagoona	1:20 p.m.
Strathfield	1:35 p.m.
Sydney	2:05 p.m.

*30 minute meal stop at Goulburn

- At what time does the bus from Sydney arrive in Goulburn?
- Where is this bus at 8:20 p.m.?
- How long does it take to travel from Liverpool to Mittagong?
- Tim is waiting at Yass at 9:15 a.m. for the bus to Sydney. How long will he have to wait before the bus arrives?

Solution

- The bus from Sydney arrives in Goulburn at 5:40 p.m.
- At 8:20 p.m., the bus arrives in Gundagai.
- From Liverpool at 3:45 p.m. to Mittagong at 4:40 p.m., the journey takes 55 minutes.
- Tim needs to wait from 9:15 a.m. to 9:41 a.m., a total of 26 minutes.

Exercise 12-11 Timetables

See Example 23

- Refer to the bus timetable from the previous example.
 - How long does the trip from Sydney to Wagga Wagga take?
 - How long would the trip take without a meal break?
 - At what time does the bus from Sydney arrive in Yagoona?
 - Ali joins the return bus at Jugiong and gets off at Liverpool. How long is his trip?
 - How long does Cherie have to wait for the bus to Sydney if she arrives in Strathfield at 1:22 p.m.?
 - Find the time taken from Liverpool to Sydney and from Sydney to Liverpool. Suggest a reason for the difference.

- 2 Daniel's volleyball team flies from Sydney to Brisbane for a tournament. Daniel visited the Cengage Air website and found the following daily flight schedule.

Worked solutions

Exercise 12-11

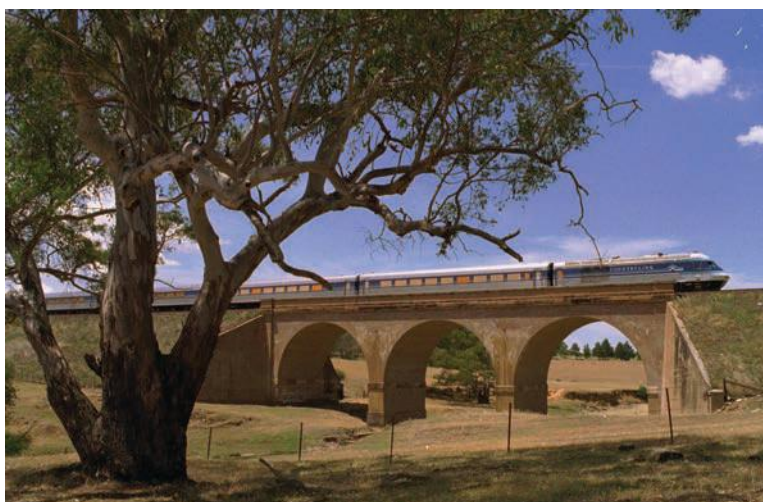
MAT07NAWS10070

Flight no.	Sydney	Brisbane
TH503	0905	1030
TH511	0935	1100
TH038	1005	1130
TH114	1040	1210
TH514	1105	1230
TH051	1135	1300

- a When does flight TH511 leave Sydney and how long is the journey?
- b What and when is the latest flight to leave before 11 a.m.?
- c The team plans to meet at Sydney airport at 9:40 a.m. How long will they need to wait for the next available flight?
- d The team needs to be at Sydney airport 40 minutes before a flight takes off. When should they be at the airport to catch the TH038 flight?
- e The team needs to be at their hotel in Brisbane by 12:30 p.m. If it takes 30 minutes to drive from the airport to the hotel, what is the latest flight the team can catch from Sydney?
- 3 This part of a Countrylink train timetable shows the Goulburn to Sydney run.

Goulburn to Sydney — Monday to Friday								
	am	am	am	pm	pm	pm	pm	pm
GOULBURN	5:08	7:27	8:17	1:47	2:45	4:26	6:47	7:45
MARULAN	5:26	7:45			3:03			8:03
TALLONG	5:32	7:51	Bookings	Bookings	3:09	Bookings	Bookings	8:09
WINGELLO	5:39	7:58	essential	essential	3:16	essential	essential	8:16
PENROSE	5:44	8:03			3:21			8:21
BUNDANOON	5:50	8:09	8:52	2:22	3:27		7:22	8:27
EXETER	5:55	8:14			3:32			8:32
MOSS VALE	6:05	8:24	9:05	2:35	3:42	5:13	7:35	8:42
BURRADOO	6:10				3:47			8:47
BOWRAL	6:13	8:30	9:11	2:41	3:50		7:41	8:50
MITTAGONG	6:17	8:34	9:16	2:46	3:54		7:46	8:54
YERRINBOOL	6:30				4:07			9:07
BARGO	6:41				4:18			9:18
TAHMOOR	6:48				4:25			9:25
PICTON	6:56	9:08			4:33			9:33
CAMPBELLTOWN	7:23	9:30	10:11	3:41	5:00	6:13	8:42	10:00
STRATHFIELD			10:42	4:17		7:00	9:12	
SYDNEY	8:12	10:12	10:54	4:29	6:20	7:13	9:24	11:04

- a Michael has an interview in Sydney on Tuesday at 10:45 a.m. At what time must he catch the train in Goulburn?
- b How long does the train take to travel from Goulburn to Sydney on the 5:08 a.m. train and the 8:17 a.m. train?
- c Georgina travels from Penrose to Yerrinbool, arriving at 4:07 p.m. How long did the trip take?
- d You have been visiting friends in Moss Vale and are returning to Sydney. Decide which train you would catch and explain why.
- e A new train is added to the timetable, leaving Goulburn at 11:12 a.m. Write out a timetable for this train if it stops at the same stations as the 6:47 p.m. train.



- 4 An Explorer Bus is a shuttle bus that takes tourists on a tour of a city and allows them to visit places of interest.

Explorer depot	10:00	10:25	10:50	11:15	11:45	12:00	12:25	12:50	1:15
City cathedral	10:08	10:33	10:58	11:23	11:53	12:08	12:33	12:58	1:23
Railway station	10:15	10:40	11:05	11:30	12:00	12:15	12:40	1:05	1:30
Parliament	10:24	10:49	11:14	11:39	12:09	12:24	12:49	1:14	1:39
Museum	10:35	11:00	11:25	11:50	12:20	12:35	1:00	1:25	1:50
City square	10:45	11:10	11:35	12:00	12:30	12:45	1:10	1:35	2:00
Zoo	11:00	11:25	11:50	12:15	12:45	1:00	1:25	1:50	2:15
Town Hall	11:12	11:37	12:02	12:27	12:57	1:12	1:37	2:02	2:27
Arts centre	11:19	11:44	12:09	12:34	1:04	1:19	1:44	2:09	2:34
Water gardens	11:30	11:55	12:20	12:45	1:15	1:30	1:55	2:20	2:45
Hall of fame	11:38	12:03	12:28	12:53	1:23	1:38	2:03	2:28	2:53
Explorer depot	11:50	12:15	12:40	1:05	1:35	1:50	2:15	2:40	3:05

- a The bus leaves the railway station at 11:30 a.m. When will it arrive at the Arts centre and how long will the journey take?
- b Harrison needs to be at the Hall of fame by 2:30 p.m. What is the latest time he needs to get to Parliament to achieve this?
- c Vo, Binh and Vicki came to the city by train, arriving at the station at 11:42 a.m. They caught the Explorer Bus to the zoo. What is the earliest time they could expect to arrive at the zoo?
- d What is the smallest number of buses needed to meet this Explorer Bus timetable? Explain how you arrived at your answer.
- e Manuel and Sofia arrived at the City cathedral at 10:25 a.m. They arrange to meet their hosts at the Hall of fame at 2:45 p.m. They want to spend at least half an hour at the museum, photograph the City square and have lunch at the Town Hall. Plan a list of times for them to catch the Explorer Bus to do these things and meet their hosts on time.
- f In summer, an extra Explorer Bus tour leaves the depot at 1:30 p.m. Make a list of departure times that appear in the timetable for this bus.



Power plus

- 1 A spider moves at 1 cm/s. If it starts at the back left-hand corner of your classroom, find how long it will take to reach:
 - a the nearest person
 - b you
 - c the whiteboard
 - d the door
- 2 Simplify each ratio.

a $0.4 : 0.7$	b $1.3 : 0.8$	c $0.5 : 0.05$
d $0.49 : 1.4$	e $\frac{1}{3} : \frac{1}{8}$	f $\frac{3}{5} : \frac{1}{2}$



- 3 Kobe and Hashim renovated their gardens. Kobe planted 19 native plants and 14 non-native plants. Hashim put in 24 native plants and 20 non-native plants. Kobe believes that her garden is more Australian native than Hashim's. Is Kobe right? Explain your answer.
- 4 Australia's annual birth rate in 2010 was approximately 12.4 per 1000. Given that the population then was about 21 885 000, how many babies were born in 2010?
- 5 Singapore is a small island but it has a population of 5 076 700, making it the third-most densely populated country in the world. Its population density is 7148 persons/km².
 - a What is the area of Singapore, to the nearest square kilometre?
 - b Australia is the seventh-least densely populated country in the world, at a rate of only 2.9 persons/km² and an area of 7 682 300 km². If Australia was as densely populated as Singapore, what would its population be?
- 6 Diane invents a decimal system of time, where 1 minute = 100 seconds and 1 hour = 100 minutes.
 - a How many of our minutes will there be in a decimal hour?
 - b How many decimal hours will there be in one of our days?

Language of maths

12-hour time	per	speed	travel graph
24-hour time	rate	steepness	unit cost
best buy	ratio	term	unitary method
equivalent ratios	simplify	timetable	

Puzzle sheet

Ratios and rates
find-a-word

MAT07NAPS10045

- What is **unit cost** and what is it used for?
- Which feature of a travel graph indicates the speed of the traveller?
- What is the difference between a **ratio** and a **rate**?
- How do you simplify:
 - a ratio?
 - a rate?
- Why does the **unitary method** have that name?
- What are the advantages of using 24-hour time?

Topic overview

- Give examples of places in which ratios, rates or time are used.
- Give some examples of situations where you would use what you learnt in this topic.
- What parts of this chapter were new to you?
- What did you find most difficult about this topic? Discuss any problems with your teacher or with a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet

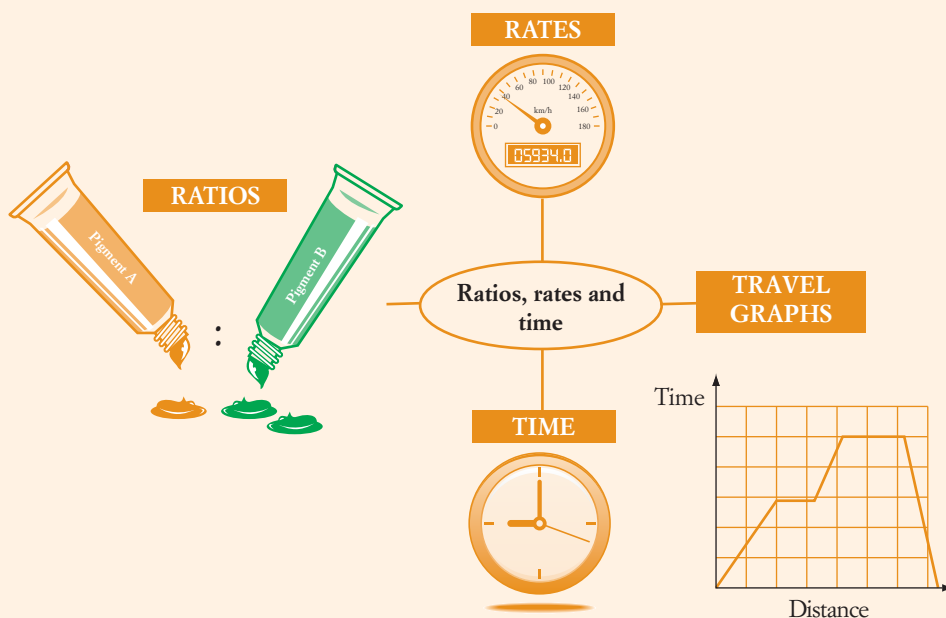
Mind map: Ratios,
rates and time

MAT07NAWK10104

Quiz

Time

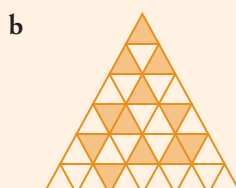
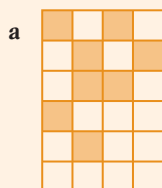
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Chapter 12 revision

See Exercise 12-01

- 1 In each diagram, find the ratio of shaded parts to unshaded parts.



See Exercise 12-01

- 2 In a construction business, there are 17 women and 38 men. Find the ratio of:

a men to women

b women to the total

See Exercise 12-02

- 3 Copy and complete each pair of equivalent ratios.

a $2 : 3 = 8 : \underline{\hspace{1cm}}$

b $1 : 5 = 2 : \underline{\hspace{1cm}}$

c $2 : 3 : 4 = 6 : \underline{\hspace{1cm}} : \underline{\hspace{1cm}}$

See Exercise 12-02

- 4 Simplify each ratio.

a $12 : 21$

b $25 : 75$

c $6 : 36$

d $25 : 45$

e $18 : 6 : 24$

f $5 : 25 : 100$

g $2 \text{ cm} : 10 \text{ mm}$

h $5 \text{ dozen eggs} : 6 \text{ eggs}$

i $3 \text{ L} : 600 \text{ mL}$

j $50 \text{ m} : 1 \text{ km}$

k $20 \text{ minutes} : 1 \text{ hour}$

l $40 \text{ cents} : \$2$

m $18 \text{ months} : 3 \text{ years}$

n $12 \text{ days} : 3 \text{ weeks}$

o $4 \text{ days} : 10 \text{ hours}$

See Exercise 12-03

- 5 **a** The ratio of girls to boys in Year 7 is $4 : 3$. If there are 75 boys in Year 7, find how many girls there are.
b Donald and Evelyn invested in a business in the ratio $5 : 7$. If Evelyn invested \$63 000, how much did Donald invest?

See Exercise 12-04

- 6 Write each statement as a rate.

a 150 000 voters for 50 politicians

b 1000 sheep over 5 hectares

c 10 tickets for \$8

d 810 stitches to make 3 balls

See Exercise 12-05

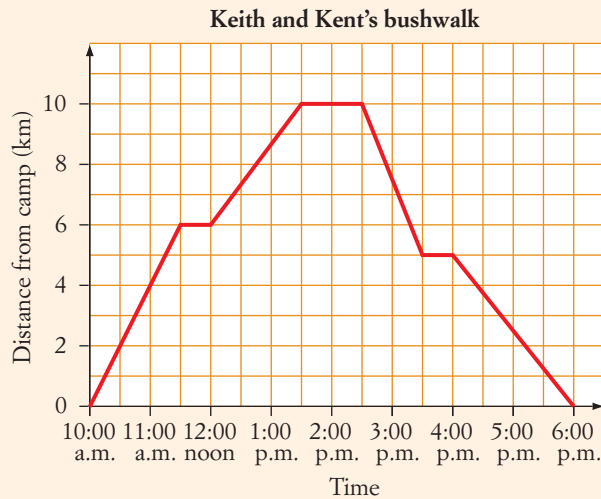
- 7 For each pair, determine which is the better buy.
a Dog biscuits: 400 g of Woof for \$5.20 or 1 kg of Chow for \$11.10.
b Milk: 1 L of Dairy Farm for \$2.85 or 1250 mL of Milky for \$3.90.

See Exercise 12-06

- 8 **a** Daria earns \$13.25 per hour. How much is she paid for 38 hours work?
b How many litres of petrol can you buy with \$35 if petrol costs \$1.45/L? Answer correct to one decimal place.
c Fertiliser is used at 20 kg/ha. How many hectares can be covered with 150 kg of fertiliser?

9 Keith and Kent decided to go bushwalking one day. This travel graph shows their journey.

See Exercise 12-07



- a How far did Keith and Kent walk?
 - b How many stops did they make?
 - c At what time did they start back?
 - d Between what times were Keith and Kent walking fastest?
 - e Calculate their speed in the last two hours.
- 10** Round each time to the nearest hour.
- a 9 h 50 min
 - b 3.2 h
 - c 4 h 12 min 49 s
- 11** Convert:
- a 316 minutes to hours and minutes
 - b 208 seconds to minutes and seconds
- 12** Simplify each expression.
- a $6 \text{ h } 45 \text{ min} + 3 \text{ h } 20 \text{ min}$
 - b $3 \text{ h } 16 \text{ min} + 1 \text{ h } 26 \text{ min}$
 - c $4 \text{ h } 33 \text{ min} - 2 \text{ h } 24 \text{ min}$
 - d $2 \text{ h } 19 \text{ min} - 1 \text{ h } 50 \text{ min}$
- 13** Convert to 12-hour time.
- a 0148
 - b 0624
 - c 1550
 - d 1230
- 14** Convert to 24-hour time.
- a 2:10 p.m.
 - b 5:33 a.m.
 - c 12:19 a.m.
 - d 1:50 p.m.
- 15** Calculate the time difference between each pair of times.
- a 5:25 a.m. and 9:45 a.m.
 - b 11:55 p.m. and 7:30 a.m.
 - c 1:15 p.m. and 8:10 p.m.
 - d 0750 and 1420
 - e 2345 and 0015
 - f 1530 and 0325
- 16** Samantha was born on 3rd June 2004. Calculate her age in years, months and days on 16th August 2018.
- 17** Refer to the Countrylink train timetable from Exercise 12-10, question 3 on page 503.
- a How long does the 7:45 p.m. train take to get to Sydney?
 - b At what time would you need to catch the train at Moss Vale to be in Sydney by 11 a.m.?

See Exercise 12-08

See Exercise 12-08

See Exercise 12-08

See Exercise 12-09

See Exercise 12-09

See Exercise 12-10

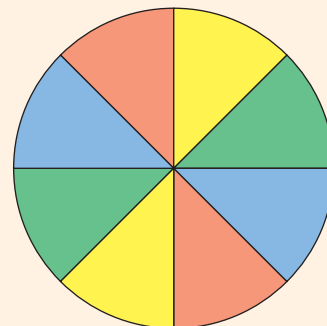
See Exercise 12-10

See Exercise 12-11

Mixed revision 4

See Exercise 11-01

- 1 List the sample space for each experiment.
 - a Rolling a die
 - b Spinning the spinner shown



See Exercise 12-01

- 2 There are 18 girls in a class of 30 students. Write each ratio in simplest form:
 - a girls to students
 - b girls to boys
 - c students to boys

See Exercise 12-04

- 3 Express each of these as a simplified rate:
 - a 200 km in 8 hours
 - b \$24 for 6 metres
 - c 1000 revolutions in 2 minutes
 - d 2100 cars in 7 days

See Exercise 12-10

- 4 How many hours and minutes between:
 - a 4:30 a.m. to 9:55 a.m.?
 - b 7:15 p.m. to 8:25 p.m.?
 - c 9:24 p.m. to 9:05 a.m.?
 - d 1505 to 2210?

See Exercise 12-09

- 5 Write in 24-hour time:
 - a 8:25 a.m.
 - b 4:45 p.m.

See Exercise 12-09

- 6 Write in 12-hour time:
 - a 2115
 - b 0430

See Exercise 12-02

- 7 Simplify each ratio.
 - a 2 : 6
 - b 96 : 36
 - c 24 : 72
 - d 2 m to 15 cm
 - e 10 minutes to 2 hours
 - f 12 : 30 : 42

See Exercise 11-02

- 8 A standard die is rolled. Find the probability of rolling:
 - a an even number
 - b a number greater than 2

See Exercise 11-04

- 9 At a vehicle checkpoint, the following types of vehicles were counted in a 2-hour period.

Cars 30	Trucks 7	Motorcycles 11	Buses 2

 - a How many vehicles passed the checkpoint in this time period?
 - b Based on this data, what is the probability the next vehicle to pass the checkpoint will be a car?

See Exercise 12-03

- 10 The ratio of boys to girls at the public speaking competition is 4 : 3. If there are 15 girls, find the number of boys.

See Exercise 11-03

- 11 A pack of 20 cards contains 10 blue, 6 purple and 4 pink cards. One card is drawn from the pack at random. Find the probability that this card is:
 - a purple
 - b not purple

12 In 20 hours, a train travels 800 km.

See Exercise 12-06

- What is its average speed in km/h?
- How far would it travel in 11 hours at the same rate?

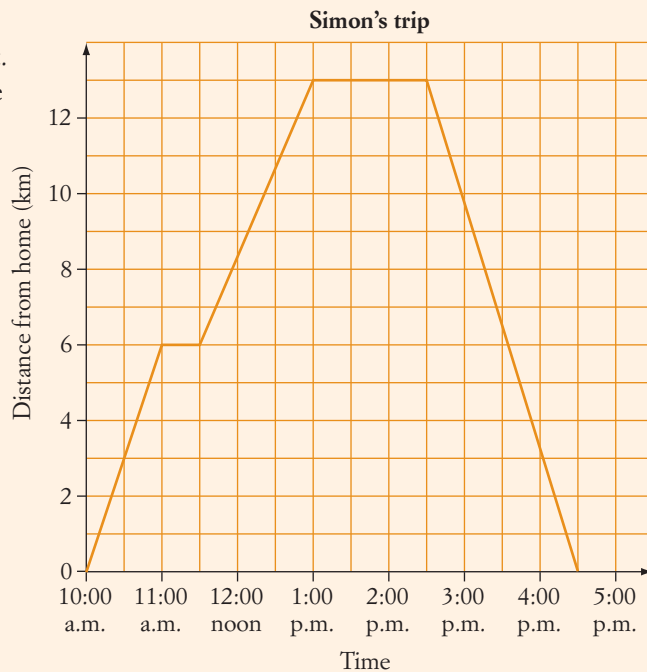
13 Water flows into a tank at the rate of 15 litres per hour. How long will it take to fill the tank if the tank holds 750 litres?

See Exercise 12-06

14 Simon goes on a cycling trip as shown in the graph on the right.

See Exercise 12-07

- How far was Simon from home at 11 a.m.?
- When, and for how long, did Simon stop for lunch?
- How far did Simon cycle in total?
- At what times was Simon 3 km from home?
- Was Simon's speed greater in the first hour or between 11:30 a.m. and 1 p.m.?



15 Choc-chip ice-cream is sold in three sizes:

See Exercise 12-05

- 1 litre for \$3.25
- 2 litres for \$6.30
- 1.5 litres for \$4.80.

Which size is the best buy?

General revision

See Chapter 3

- 1 **a** Explain using a divisibility test why 195 is divisible by 3.
b Evaluate $195 \div 3$

See Chapter 3

- 2 Write $8 \times 8 \times 8 \times 8 \times 8$ using index notation (powers).

See Chapter 3

- 3 Evaluate each expression.

a 11^2

b $\sqrt{49}$

c 10^3

See Chapter 5

- 4 Evaluate each expression.

a $3 \times 5 \times 4$

b 20×18

c 45×9

d 16×12

See Chapter 5

- 5 Write an algebraic expression for 8 more than double x .

See Chapter 5

- 6 Simplify each expression.

a $2x + x$

b $m \times m \times 5$

c $7k \div 2$

d $2y \times 3$

e $10 - k \times n$

f $2 \times p + 4$

See Chapter 8

- 7 Copy and complete:

a $7.2 \text{ m} = \underline{\hspace{2cm}} \text{ mm}$

b $425 \text{ mL} = \underline{\hspace{2cm}} \text{ L}$

c $96 \text{ hours} = \underline{\hspace{2cm}} \text{ days}$

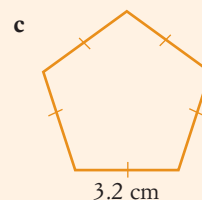
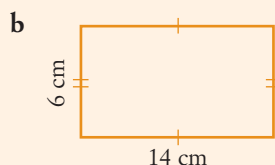
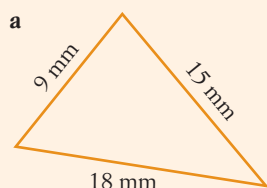
d $240 \text{ kg} = \underline{\hspace{2cm}} \text{ t}$

e $5000 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$

f $6.7 \text{ m}^3 = \underline{\hspace{2cm}} \text{ L}$

See Chapter 8

- 8 Find the perimeter of each shape.



See Chapter 12

- 9 Write 6:10 p.m. in 24-hour time.

See Chapter 12

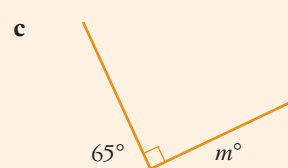
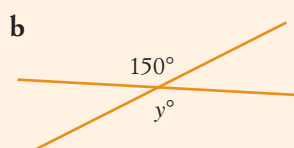
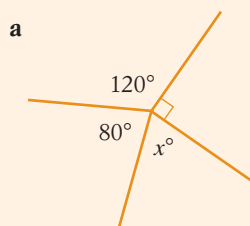
- 10 How many hours and minutes between 0635 and 1310 on the same day?

See Chapter 12

- 11 The ratio of parents to children at the Wiggles concert was 4 : 5. If there were 376 parents, how many children were there?

See Chapter 2

- 12 Find the size of each pronumeral.



See Chapter 1

- 13 Write this set of integers in ascending order: 6, -5, -3, 2, 4, 11.

14 Evaluate each expression.

See Chapter 1

a $-5 + 3$

b $-8 - 4$

c $10 - (-3)$

d -7×8

e $-12 \times (-10)$

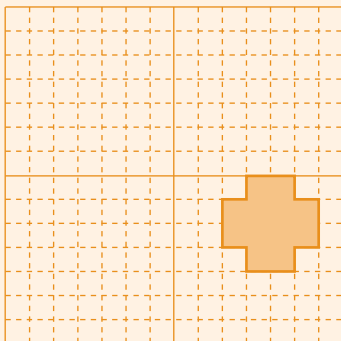
f $(-10)^2$

g $\frac{36}{-4}$

h $10 + 4 \times (-3)$

15 Translate this cross shape 8 units left and 5 units up.

See Chapter 6



16 If a coin is tossed, what is the probability that heads will come up? Write the answer as:

See Chapter 11

a a decimal

b a percentage

17 Write the sample space for the colours of a traffic light.

See Chapter 11

18 Copy and complete each pair of equivalent fractions.

See Chapter 4

a $\frac{8}{12} = \frac{\square}{3}$

b $\frac{4}{25} = \frac{\square}{100}$

c $\frac{15}{20} = \frac{3}{\square}$

d $\frac{2}{5} = \frac{10}{\square}$

19 Evaluate each expression.

See Chapter 4

a $\frac{2}{5} + \frac{1}{2}$

b $\frac{2}{3} - \frac{1}{4}$

c $\frac{1}{2} \times \frac{3}{4}$

d $\frac{3}{4} \div \frac{1}{2}$

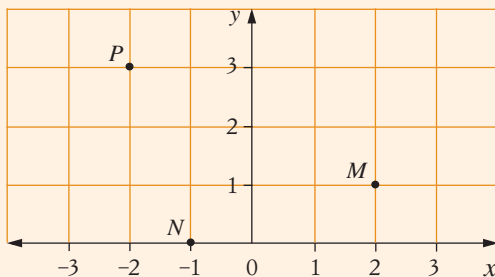
20 Write the coordinates of:

See Chapter 9

a M

b N

c P



21 Represent this set of data on a dot plot:

See Chapter 10

5 3 2 0 3 2 1 2 4 5

22 For the data set in question 21, find:

See Chapter 10

a the mean

b the median

c the mode

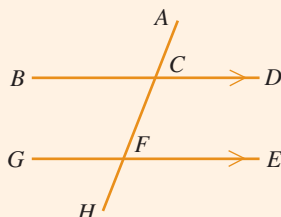
d the range

See Chapter 2

23 For this diagram, name:

- a** a pair of corresponding angles
- c** the transversal.

b a pair of vertically opposite angles



24 If $\angle AFE = 75^\circ$ in the diagram above, find the value of:

a $\angle FCD$

b $\angle BCF$

c $\angle ACD$

See Chapter 11

25 What is the probability that a person selected at random was born in a month beginning with J?

See Chapter 11

26 a Give an example of an event that is certain.

b What is the probability of an event that is certain?

See Chapter 12

27 Simplify each ratio.

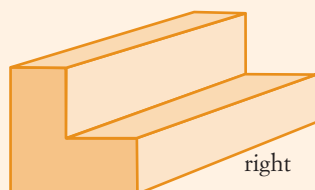
a $8 : 40$

b $12 : 9$

c $45\text{s} : 3\text{ min}$

See Chapter 8

28 For this prism, sketch each view.



a front view

b top view

c right view

See Chapter 9

29 Plot these points on a number plane and label them.

$R(3, 2)$ $S(0, 4)$ $T(-1, 3)$

See Chapter 3

30 Select the prime numbers from this list: 3, 12, 33, 11, 9, 40, 14, 18.

See Chapter 3

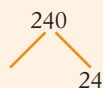
31 Write:

a the factors of 27

b the greatest common divisor (GCD) of 27 and 45.

See Chapter 3

32 Copy and complete this factor tree and express 240 as a product of its prime factors.



See Chapter 6

33 Draw a quadrilateral with one pair of parallel sides and name it.

See Chapter 6

34 a Draw a rhombus.

b How many axes of symmetry does a rhombus have?

c What order of rotational symmetry does a rhombus have?

35 Convert each decimal to a fraction in simplest form.

See Chapter 7

a 0.35

b 0.1

c 0.012

36 Round each decimal to one decimal place.

See Chapter 7

a 1.736

b 0.69

c 0.98

37 True or false?

See Chapter 7

a $5.92 \times 3 = 0.1776$

b $\frac{3}{100} = 0.03$

c $0.65 = \frac{12}{20}$

38 Solve each equation.

See Chapter 5

a $3b = 21$

b $r + 4 = 11$

c $2n + 3 = 11$

d $\frac{5p}{2} = 15$

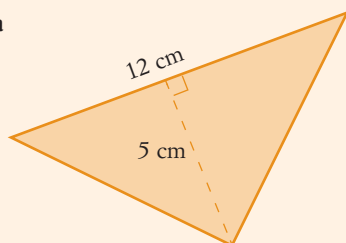
39 If $t = -1$, evaluate $3t - 4$.

See Chapter 5

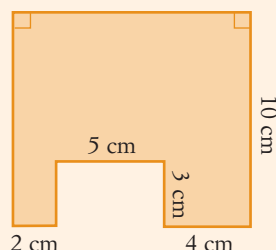
40 Find the area of each shape.

See Chapter 8

a



b



41 What is the volume of a room 7 m by 8 m by 3 m?

See Chapter 8

42 Find:

See Chapter 4

a $\frac{3}{4}$ of \$12

b $\frac{2}{3}$ of 24 m

c $\frac{1}{10}$ of 2 L (in mL)

d 10% of \$200

e 25% of 16 kg

43 Convert each to a percentage.

See Chapter 4

a $\frac{73}{100}$

b $\frac{3}{4}$

c 0.6

d 0.47

44 This stem-and-leaf plot shows the ages of people waiting at a medical centre.

See Chapter 10

Stem	Leaf
0	3
1	2 6 9
2	4
3	5 5 6 8
4	2 8
5	0 7
6	3 4

a How many people were waiting at the centre?

b Find the median age.

c How old was the oldest person?

d Find the mode.

45 Over 30 days, the number of sunny days in Westvale was 18.

See Chapter 11

a What is the experimental probability of having a sunny day in Westvale?

b Based on this probability, how many sunny days would be expected over a period of 100 days?

- See Chapter 12* **46** Petrol costs \$1.49 per litre.
 a Calculate the cost of 30 litres of petrol.
 b How much petrol can be bought for \$50? Answer correct to the nearest 0.1 L.
- See Chapter 7* **47** Evaluate each expression.
 a 6.9×4.3 **b** $15.35 \div 5$ **c** $7.3 - 1.986$
- See Chapter 7* **48** Convert each fraction to a decimal.
 a $\frac{5}{8}$ **b** $\frac{11}{20}$ **c** $\frac{1}{6}$
- See Chapter 6* **49** **a** Draw an isosceles right-angled triangle.
 b Write the size of each angle in the above triangle.
- See Chapter 12* **50** Write 2045 in 12-hour time.

Calculator skills

Basic keys

Key	Use	Key	Use
+ , - , × , ÷	Basic operations	(-) or +/-	Enters negative numbers
=	Equals sign, gives the answer	$\frac{\Box}{\Box}$ or a^{b/c}	Enters fractions
.	Decimal point	()	Enters parentheses
DEL	Deletes previous entry	x²	Squares a number
ANS	Retrieves previous answer	√	Finds the square root of a number
	Moves cursor around the screen	x³	Cubes a number
MODE or SHIFT or 2ndF	Accesses other operations	$\sqrt[3]{\Box}$	Finds the cube root of a number

1 Basic operations

Example 1

Performing a calculation

Question	Calculator steps	Answer
$34 + 6 - 16$	34 + 6 - 16 =	24
7.4×9.1	7.4 × 9.1 =	67.34
$234 \div 1.5$	234 ÷ 1.5 =	156
$4569 \div (1.7 + 4.3)$	4569 ÷ (1.7 + 4.3) =	761.5

Example 2

Correcting a wrong entry

Calculating $34 + 6 - 16$

a Wrong entry: 34 **+** 8

To delete the 8 and continue, enter

DEL 6 **-** 16 **=**

b Wrong entry: 34 **+** 6 **+** 16 **=**

To correct this, press **←** repeatedly to go back to the second **+**, press **DEL** and then **-**, then press **=**

Exercise 1**Basic operations**

1 Calculate each expression.

a 362×14

b $26.3 - 11.8$

c $810 \div 18$

d 5.3×1.08

e $51.1 \div 14.6$

f $19.3 + 8.8 - 1.2$

g $2.4 \div 0.2 + 11$

h $5.6 \times (0.1 + 0.2)$

i $(56 - 24) \div (1.2 - 1)$

j $12.6 \times 3.8 - 18.4$

k $(16.07 + 9.02) - (38.3 - 5.126)$

l $(24.8 + 3.06 + 9.4) \div 3$

2 Bronte purchased a loaf of bread for \$2.75, a packet of cornflakes for \$4.55 and 3 kg of tomatoes at \$3.95 per kg.

a Find the total cost.

b Find the change Bronte will receive from a \$20 note.

3 Mark started the month with \$1650 in his bank account. He made deposits of \$380 and \$725.50 and withdrew \$880. How much was left in Mark's account?

4 An unloaded truck has a mass of 6550 kg. It is loaded with eight boxes, each weighing 124 kg. What is the total mass of the truck and the boxes?

5 Salma needs to cut a 9.6 m length of rope into four equal pieces. Find the length of each piece.

6 Cameron mailed 25 Christmas cards to his friends. If the stamps were 45c each, how much did he spend on stamps?

7 An orchard has 1288 trees arranged in 56 rows. How many trees are in each row?

8 A train takes 15 hours to travel 1128 km. What is its average speed in km/h?

9 Chloe bought 60 L of petrol at 142.9c per litre. How much did she pay for the petrol?

10 Wombat High School has the following numbers of students in each Year level.

Year 7	Year 8	Year 9	Year 10	Year 11	Year 12
122	128	123	131	117	94

The whole school is going on an excursion. If each bus can carry 55 people, how many buses will be needed?

11 The manager of a sports store purchased 32 tennis racquets for \$79.60 each and sold them for \$120.80 each.

a Find the profit made on each racquet.

b Find the total profit.

12 A batsman scored the following runs over five cricket innings.

27 4 123 66 47

Find the batsman's average score.

Calculator skills

2 Integers

Example 3

Question	Calculator steps		Answer
	Casio	Sharp	
$-4 + 9$	$(-)$ 4 + 9 =	$+/-$ 4 + 9 =	5
$17 \times (-2)$	17 $\times$ $(-)$ 2 =	17 $\times$ $+/-$ 2 =	-34
$18 - (-3) \times 4$	18 - $(-)$ 3 $\times$ 4 =	18 - $+/-$ 3 $\times$ 4 =	30

Exercise 2 Integers

1 Calculate each expression.

a $-5 + 12$

b $-6 - 3$

c -3×4

d $16 \div (-2)$

e $12 - (-3)$

f $-19 + 4$

g $-25 \times (-3)$

h $-55 \div (-11)$

i $8 - 12 + (-1)$

j $-3 + 7 - 2 - (-5)$

k $6 \times (-2) \times 8$

l $-18 \div (-2) \times 5$

m $-3 \times 7 - 4$

n $(-3 + 9) \times 4$

o $80 - (-2) \times 8$

p $12 \times (-3) + 50$

q $64 \div (-8) \times 2$

r $27 - (-3) \times (-4)$

s $18 \times (-5 + 3)$

t $-14 - 3 \times (-8)$

3 Powers and roots

Check with your teacher to locate the following keys:



Example 4

Question	Calculator steps	Answer
16^2	16 x^2 =	256
11^3	11 x^3 =	1331
$\sqrt{196}$	$\sqrt{}$ 196 =	14
$\sqrt[3]{4913}$	$\sqrt[3]{}$ 4913 =	17

Exercise 3

Powers and roots

1 Calculate each expression.

a 14^2

b 23^2

c 6^3

d 1.2^3

e 5.13^2

f 101^2

g 2.04^2

h 19^3

i $\sqrt{676}$

j $\sqrt{21904}$

k $\sqrt[3]{729}$

l $\sqrt{6^2 \times 7^2}$

2 Evaluate each expression correct to one decimal place.

a $\sqrt{24}$

b $\sqrt{101}$

c $\sqrt{11 \times 2}$

d $\sqrt[3]{100}$

3 Calculate each expression.

a $5^2 - 3^2$

b $16^2 - 10^2$

c $\sqrt{6} \times \sqrt{6}$

d $\sqrt{25 \times 4}$

4 Fractions

Some calculators have two ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into two blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). LINE mode is used in the following examples. Ask your teacher if you need help with the MATH mode.

Example 5

Converting fractions

Question	Calculator steps		Answer
	Casio	Sharp	
Simplify $\frac{16}{20}$	16 $\frac{\square}{\square}$ 20 $=$	16 $\text{a}^{\text{b/c}}$ 20 $=$	$\frac{4}{5}$
Change $1\frac{3}{4}$ to an improper fraction	1 $\frac{\square}{\square}$ 3 $\frac{\square}{\square}$ 4 $=$ $\frac{b}{c} \leftrightarrow \frac{d}{c}$ (SHIFT $\text{S} \leftrightarrow \text{D}$)	1 $\text{a}^{\text{b/c}}$ 3 $\text{a}^{\text{b/c}}$ 4 $=$ 2ndF $\text{a}^{\text{b/c}}$	$\frac{7}{4}$
Change $\frac{19}{4}$ to a mixed numeral	19 $\frac{\square}{\square}$ 4 $=$	19 $\text{a}^{\text{b/c}}$ 4 $=$	$4\frac{3}{4}$
Change $\frac{3}{4}$ to a decimal	3 $\frac{\square}{\square}$ 4 $=$ $\text{S} \leftrightarrow \text{D}$ or 3 $\div$ 4 $=$	3 $\text{a}^{\text{b/c}}$ 4 $=$ $\text{a}^{\text{b/c}}$ or 3 $\div$ 4 $=$	0.75
Change 1.7 to a fraction	1.7 $=$ $\frac{\square}{\square}$	1.7 $=$ $\text{a}^{\text{b/c}}$	$1\frac{7}{10}$

$\text{S} \leftrightarrow \text{D}$ is a key that converts between standard fractions and decimals.

Calculator skills

Example 6

Operations with fractions

Question	Calculator steps		Answer
	Casio	Sharp	
$\frac{1}{2} + \frac{1}{3}$	1 $\frac{\square}{\square}$ 2 + 1 $\frac{\square}{\square}$ 3 =	1 a ^{b/c} 2 + 1 a ^{b/c} 3 =	$\frac{5}{6}$
$\frac{5}{7} - \frac{2}{3}$	5 $\frac{\square}{\square}$ 7 - 2 $\frac{\square}{\square}$ 3 =	5 a ^{b/c} 7 - 2 a ^{b/c} 3 =	$\frac{1}{21}$
$\frac{3}{5} \times \frac{2}{7}$	3 $\frac{\square}{\square}$ 5 × 2 $\frac{\square}{\square}$ 7 =	3 a ^{b/c} 5 × 2 a ^{b/c} 7 =	$\frac{6}{35}$
$\frac{4}{5} \div \frac{2}{3}$	4 $\frac{\square}{\square}$ 5 ÷ 2 $\frac{\square}{\square}$ 3 =	4 a ^{b/c} 5 ÷ 2 a ^{b/c} 3 =	$1\frac{1}{5}$

Example 7

Mixed operations with fractions

Question	Calculator steps	Answer
$\frac{13 + 7}{9 \times 2}$	(13 + 7) ÷ (9 × 2) = or (13 + 7) $\frac{\square}{\square}$ (or a ^{b/c}) (9 × 2) =	1.1111... $1\frac{1}{9}$

Exercise 4 Fractions

1 Simplify each fraction.

- a $\frac{20}{32}$ b $\frac{40}{85}$ c $\frac{45}{72}$
d $\frac{112}{126}$ e $\frac{84}{100}$ f $\frac{48}{192}$

2 Convert each mixed numeral into an improper fraction.

- a $1\frac{7}{8}$ b $5\frac{2}{9}$ c $10\frac{3}{4}$

3 Convert each improper fraction into a mixed numeral.

- a $\frac{20}{9}$ b $\frac{44}{5}$ c $\frac{18}{8}$

4 Convert each fraction into a decimal.

- a $\frac{1}{2}$ b $\frac{4}{5}$ c $\frac{7}{8}$
d $\frac{9}{25}$ e $\frac{34}{200}$ f $\frac{5}{9}$

5 Convert each decimal into a fraction in its simplest form.

- a** 0.35 **b** 2.04 **c** 0.001
d 0.365 **e** 1.06 **f** 0.325

6 Simplify each expression.

- a** $\frac{3}{5} + \frac{1}{4}$ **b** $\frac{4}{7} + \frac{1}{2}$ **c** $\frac{2}{3} - \frac{3}{5}$ **d** $\frac{1}{3} \times \frac{4}{7}$
e $\frac{5}{7} \div \frac{15}{28}$ **f** $\frac{5}{6} - \frac{5}{8}$ **g** $\frac{9}{10} + \frac{2}{3}$ **h** $\frac{7}{8} \times \frac{5}{12}$
i $\frac{7}{8} \div \frac{1}{2}$ **j** $24 \div 1\frac{1}{4}$ **k** $5\frac{1}{4} + 1\frac{2}{5}$ **l** $9\frac{7}{8} - 1\frac{1}{3}$
m $5\frac{1}{4} \times 2\frac{3}{7}$ **n** $4\frac{2}{3} \div \frac{2}{3}$ **o** $\frac{8}{9} \div 6$ **p** $10\frac{1}{6} \times 1\frac{9}{10}$

7 Simplify each expression.

- a** $\frac{12 - 4 \times 2}{18 + 14}$ **b** $\frac{6^2}{100 \div 4 + 23}$ **c** $\frac{9 \times 9 - 2}{\sqrt{30} - 5}$ **d** $\frac{20 \div 2 + 8}{3 \times 9 \times 4}$

8 Mr Lanouri is at his office for $7\frac{1}{2}$ hours each day. He takes $\frac{3}{4}$ of an hour for lunch each day. How long does he work:

- a** in one day? **b** in 5 days?

Glossaries

Instructional glossary

Mathematical 'doing' words

bisect To cut in half.

calculate To find the value of a numerical expression. See also **Evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or whole.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction to a decimal, or convert dollars to cents.

decrease To make smaller by subtracting or dividing.

estimate To make an educated guess about the value of an answer; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2 , or evaluate $4x + 1$ when $x = 5$.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using previous answers or any information supplied.

increase To make larger by adding or multiplying.

measure To find the size of something using an instrument. For example, to find a length using a ruler.

prove/show that In questions where the answer is given, to use mathematical reasoning to prove that the answer is true.

reduce (a fraction) to its lowest terms See **Simplify (a fraction)**.

round (a number) To find the nearest approximation for a number. For example, round 4.3 to the nearest whole number (4), \$12.9598 to the nearest cent (\$12.96), 0.166 66 to three decimal places (0.167).

show working To show the steps you used to find an answer.

simplify To give the answer in its simplest, shortest, neatest form.

simplify (a fraction) To reduce the numerator and denominator of a fraction by dividing by their greatest common divisor (GCD). When the numerator and denominator are as small as possible, the fraction has been simplified or reduced to its lowest terms.

simplify (a ratio or rate) To reduce the terms or units of a ratio or rate by dividing by a common factor, in a similar way to simplifying a fraction.

sketch To draw a rough diagram. Less accurate than to construct.

solve To find the value of the unknown variable in an equation.

substitute To replace a variable with a number and evaluate.

write correct to See **Round (a number)**.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

Mathematical glossary

Some common symbols

+	plus
−	minus
×	multiplied by (times)
÷	divided by
%	per cent
()	parentheses, round brackets
[]	(square) brackets
{ }	braces
=	is equal to
≈	is approximately equal to
≠	is not equal to
<	is less than
>	is greater than
≤	is less than or equal to
≥	is greater than or equal to
∴	therefore

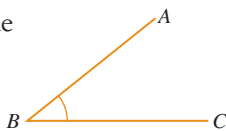
−3	negative 3
$\sqrt{\quad}$	the square root of
$\sqrt[3]{\quad}$	the cube root of
$5 \overline{)855}$	855 divided by 5, or $855 \div 5$
$0.\dot{6}$	the recurring decimal 0.666 666 ...
$3x$	3 multiplied by x , $3 \times x$
$\frac{x}{2}$	x divided by 2, $x \div 2$
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\bar{x}$	the mean (average)
$\angle ABC$	angle ABC
$\triangle ABC$	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$P(E)$	The probability of an event, E

12-hour time Time of day written using a.m. or p.m. notation and the hours 1 to 12. For example, 6:20 p.m.

24-hour time Time of day written using four digits (instead of a.m. or p.m.) and the hours 0 to 23. For example, 1820 is the 24-hour time for 6:20 p.m.

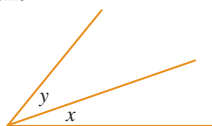
A

acute angle A 'sharp' angle between 0° and 90° , for example, the marked angle in the diagram.



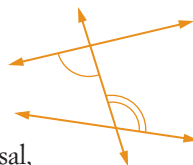
acute-angled triangle A triangle with three acute angles.

adjacent angles Angles next to each other that share a common arm, for example, angles x and y in the diagram.

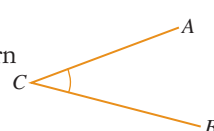


algebraic expression A number written in algebraic form using variables, for example, $2xy + 4y - 5$.

alternate angles A pair of angles between two lines crossed by a transversal, on opposite sides of the transversal, for example, the two angles marked in the diagram.



angle A description or measure of the size of a turn or rotation. The angle marked in the diagram is named $\angle C$ or $\angle ACB$.



angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180° .

area The amount of surface enclosing a shape, measured in square units.

ascending order Going up, increasing, from smallest to largest (1-2-3). The opposite of **descending order**.

Glossaries

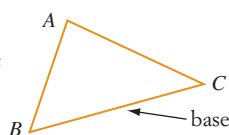
associative law A law of arithmetic that says that when operating with more than two numbers, you can group them in any way. For example, the associative law of addition says that $(a + b) + c = a + (b + c)$ for any three numbers a , b and c .

axis of symmetry See **line symmetry**.

B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5 , the 3 is called the base.

base (of a shape) The bottom side of a flat shape such as a triangle.



best buy When comparing different brands or sizes of items during shopping, the best buy is the item with the lowest unit cost and the best value for money.

bisect To cut in half.

braces See **Grouping symbols**.

brackets See **Grouping symbols**.

breadth Another word for 'width'; how wide or broad something is.

C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). See also **volume**.

cartesian plane Another name for **number plane**. Named after French mathematician and philosopher, René Descartes.

centi- A prefix meaning 'one-hundredth of'.

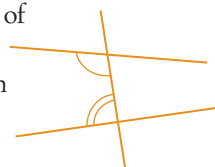
centre of symmetry See **rotational symmetry**.

certain Must happen; has a probability of 1 or 100%.

classify To group into categories or types.

cluster A group of data scores that are bunched or close together.

co-interior angles A pair of angles between two lines crossed by a transversal, on the same side of the transversal (co-interior means 'together inside'), for example, the two angles marked in the diagram.



collinear points Points that lie in a line.

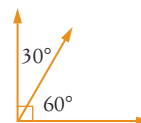
common fraction See **Fraction**.

commutative law A law of arithmetic that says that you can operate with 2 numbers in any order. For example, the commutative law of multiplication says that $a \times b = b \times a$ for any two numbers a and b .

compasses Geometrical instrument for constructing circles and equal lengths.



complementary angles Two angles whose sum is 90° . The angles 30° and 60° are complementary.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

composite number A number with more than two factors. 12 is a composite number since it has six factors: 1, 2, 3, 4, 6 and 12. (See also **prime number**.)

composite shape A shape made up of two or more basic shapes.

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10.

construct To draw a geometrical figure accurately.

coordinates An ordered pair of numbers used to locate a point or position, for example, $A(2, 5)$ tells us that the point A is located 2 units to the right and 5 units up from the origin.

corresponding angles A pair of angles in matching positions when two lines are crossed by a transversal. They are on the same side of the transversal and the lines, for example, the two angles marked on the diagram.



cross-section A 'slice' of a solid cut across it rather than along it.



cube (of a number) The number raised to the power of 3.

For example, 7 cubed = $7^3 = 7 \times 7 \times 7 = 343$.

cube root (of a number) The value which, if cubed, gives the number.

For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.

cubic metre A metric unit of volume, the volume of a 1 m cube.

D

data Information, a collection of facts.

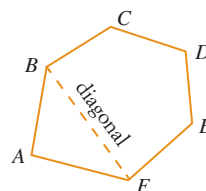
decimal places The places after the decimal point in a number. For example, 3.1416 has four decimal places.

degree (symbol $^\circ$) A unit for measuring angle size. There are 360° in a complete revolution and 90° in a right angle.

denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3.

descending order Going down, decreasing, from largest to smallest (3-2-1). The opposite of **ascending order**.

diagonal An interval joining two non-adjacent vertices of a shape.



difference The result of a subtraction. The difference between 65 and 10 is 55.

digit A single figure; the whole numbers 0, 1, 2, 3, ..., 9.

For example, the number 129 has three digits: 1, 2 and 9.

distributive law A law of arithmetic that says that you can multiply by a number by splitting it into the sum or difference of two other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. More generally, $a \times (b + c) = a \times b + a \times c$ for any three numbers a , b and c .

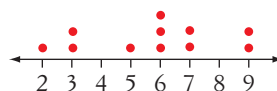
divided bar graph A rectangular graph which is divided into proportionately-sized sections to represent parts of a whole.



divisibility test A rule for testing whether a number is divisible by a specific value. For example, the divisibility test for 3 is to add the digits of the number and if the answer is a multiple of 3, then the number is divisible by 3.

divisor See **factor**.

dot plot A graph that uses dots above a number line to show frequencies of data scores.



Glossaries

E

equation A mathematical statement that two quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$.

equilateral triangle A triangle with three equal sides.



equivalent Having the same value. For example, 0.75 and $\frac{3}{4}$ are equivalent.

equivalent fraction An equal fraction created by multiplying or dividing the numerator and the denominator of a fraction by the same number.

equivalent ratio An equal ratio created by multiplying or dividing each term of a ratio by the same number.

even chance Equally likely to happen or not happen, '50-50 chance', having a probability of $\frac{1}{2}$.

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}.

expected frequency The expected number of times an event will occur over repeated trials, found by multiplying the probability of the event by the number of trials.

experiment A situation involving chance that leads to outcomes, for example, rolling a die.

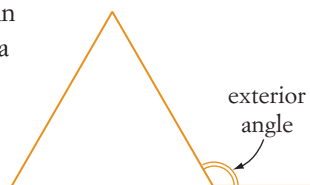
experimental probability (or relative frequency) An estimate of theoretical probability; the relative frequency of an event in repeated trials of an experiment, found using the formula

$$P(E) = \frac{\text{number of times } E \text{ happened}}{\text{total number of trials}} \\ = \frac{\text{frequency of } E}{\text{total frequency}}$$

expression A mathematical description of one or more operations. For example, $2 \times 4 + 5$ and

$3b - 7$ are mathematical expressions (but $3b - 7 = 5$ is an equation.) See also **Algebraic expression**.

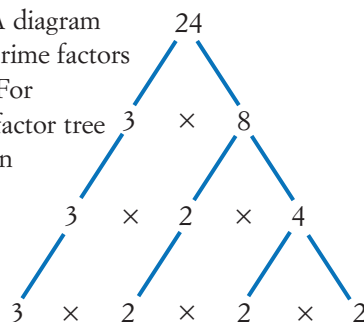
exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape. (Angles inside a shape are *interior angles*.)



F

factor (of a number) A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. Also called **divisor**.

factor tree A diagram that lists the prime factors of a number. For example, the factor tree for 24 is shown below.



formula (Plural: **formulas** or **formulae**). A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area, b is the length of the base and h is the perpendicular height of the triangle.

fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. More precisely called a **common fraction**, unlike other types of fractions such as decimals, percentages and ratios.

frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data.

G

greatest common divisor (GCD) Also called the **highest common factor (HCF)**. The largest factor shared by two or more numbers. For example, the factors of 36 are 1, 2, 3, 4, 9, 12, 18 and 36, and the factors of 8 are 1, 2, 4 and 8, so the GCD of 36 and 8 is 4.

grouping symbols The collective name for parentheses (), square brackets [] and braces { }.

H

hectare A large metric unit of area, equivalent to 10 000 m² or to the area of a square 100 m × 100 m.

height How tall or high something is, the vertical distance between its top and its base.

highest common factor (HCF) See **Greatest common divisor**.

horizontal Going across, sideways, flat.



I

image A transformed shape after it has been translated, reflected or rotated.

impossible Cannot happen, no chance, has a probability of 0.

improbable See **Unlikely**.

improper fraction A fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$.

index (Plural: **indices**, pronounced 'in-da-sees'). See **Power**.

index notation A way of writing repeated multiplication using indices (powers), in the form a^n . Index notation for $2 \times 2 \times 2 \times 2$ is 2^4 .

integer A number that is a positive or negative whole number or zero. The numbers { ..., -3, -2, -1, 0, 1, 2, 3, ... } are integers.

intersect To cross.

interval A section of a line with a definite length, such as AB below.



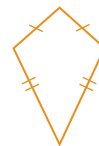
isosceles triangle A triangle with two equal sides.



K

kilo- A prefix meaning one thousand (1000), represented by the symbol k. For example, one kilogram is 1000 grams.

kite A quadrilateral with two pairs of equal adjacent sides.



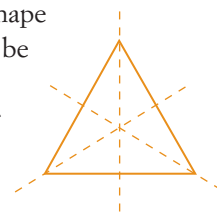
L

LHS The left-hand side (of an equation).

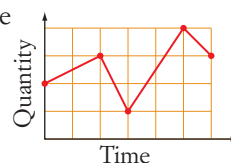
likely A high chance it will happen, **probable**.

line A straight edge that extends indefinitely in both directions.

line symmetry A plane shape has line symmetry if it can be folded so that one half fits exactly on top of the other half; that is, one half is the mirror-image of the other. The folding line is called the **axis of symmetry** (plural: **axes**). An equilateral triangle has three axes of symmetry. See also **rotational symmetry**.



line graph A graph made up of a line or several line intervals, often showing how a quantity is changing over time.



lowest common multiple (LCM) The smallest multiple that is shared by two or more numbers. For example, the multiples of 4 are 4,

Glossaries

8, 12, 16, 20, ... and the multiples of 10 are 10, 20, 30, 40, ..., so the LCM of 4 and 10 is 20.

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the scores by the number of scores.

measure of location An average, middle or typical value of a set of data. The three measures of location are the **mean**, **median** and **mode**.

median The middle score when the scores are arranged in order. If the number of values is even, then the median is the average of the two middle values.

mega- A prefix meaning one million (1 000 000), represented by the symbol M. A megalitre is one million litres.

mental Using the mind.

micro- A prefix meaning one-millionth, represented by the Greek letter μ . One microsecond is one-millionth of a second.

milli- A prefix meaning one-thousandth ($\frac{1}{1000}$), represented by the symbol m. One millimetre is one-thousandth of a metre.

mixed expression An expression with more than one arithmetic operation, such as $(17 - [3 + 1]) \times 2$.

mixed numeral A number written as a whole number and a fraction, for example $5\frac{3}{4}$.

mode The most common or frequent score(s) in a set of data.

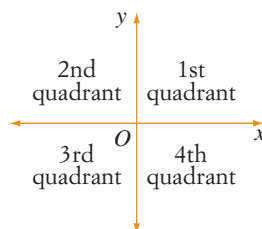
multiple (of a number) The product of the number and a whole number. For example, the multiples of 6 are 6, 12, 18, 24, ...

N

negative number A number less than 0, written with a negative sign ($-$), for example, -3 , -10 and -7 .

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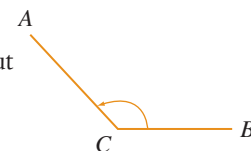
number plane A coordinate grid system based on two number lines that cross at right angles: a horizontal line called the x -axis and a vertical line called the y -axis. Also called **Cartesian plane**.



numerator The number above the line in a fraction. The numerator of $\frac{2}{5}$ is 2.

O

obtuse angle A 'wide' angle greater than 90° but less than 180° .



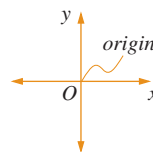
obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°).

order of operations The order in which a mixed expression such as $(17 - [3 + 1]) \times 2$ is evaluated. Work from left to right, first performing any operations in grouping symbols, then multiplication and division, and finally addition and subtraction.

order of rotational symmetry See **rotational symmetry**.

ordered pair A pair of numbers (x, y) that can be used as coordinates to plot a point on the number plane.

origin The point $O(0, 0)$ at the centre of the number plane, where the x -axis and y -axis cross.



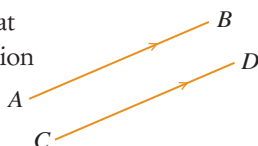
outcome In probability, a result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4.

outlier An extreme data value that is very different from the other values in a set.

P

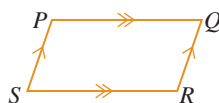
positive number A number greater than 0, for example, 11, 6 and 4.

parallel lines Lines that point in the same direction and do not intersect.



$AB \parallel CD$ means 'AB is parallel to CD'.

parallelogram A quadrilateral in which the opposite sides are parallel.



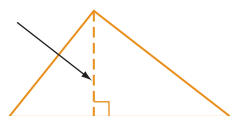
parentheses (Pronounced 'pa-ren-th-sees'.) See **Grouping symbols**.

percentage A fraction with a denominator of 100 that is written in a special way. For example, 7% means $\frac{7}{100}$.

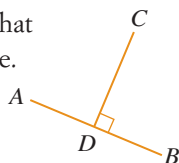
perimeter The distance around the outside of a shape. The sum of the lengths of its sides.

perpendicular height The height of a shape, measured at right angles to the base.

perpendicular height



perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means AB is perpendicular to CD.



place value The way that the position of a digit in a number tells us its value. For example, the value of the 4 in 2417 is 400.

plane A flat surface that extends in all directions.

polygon Any flat shape that is made up of straight sides.

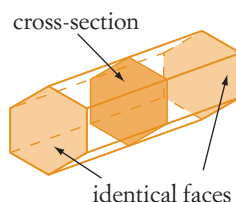


power/index The number of times a base is multiplied by itself. In 2^5 , the power is 5. Also called the *exponent*.

power of 10 The numbers 10, 100, 1000 and so on, formed by multiplying 10 by itself repeatedly.

prime number A number that has only two factors, 1 and itself. For example, 2 and 7 are prime numbers, but 15 is not. (See also **composite number**.)

prism A solid shape with identical cross-sections that have straight sides.



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1.

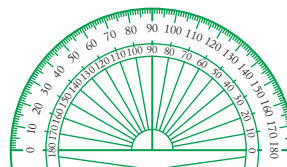
probable See **Likely**.

product The result of a multiplication. The product of 7 and 3 is 21.

pronumeral Another name for **variable**.

proper fraction A fraction whose numerator is less than its denominator, such as $\frac{3}{8}$.

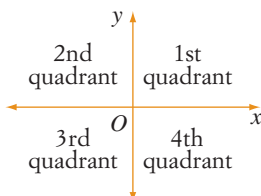
protractor A geometrical instrument that measures the size of an angle in degrees.



Glossaries

Q

quadrant (of a number plane) A quarter of the number plane created by the x -axis and y -axis crossing at right angles.



quadrilateral Any polygon with four sides, for example, rectangle, parallelogram, kite and trapezium.



quotient The result of a division. If 36 is divided by 3, the quotient is 12.

R

random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely.

range In a set of data, the difference between the highest and lowest scores.

rate A relationship between two quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours).

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3 : 40 (read '3 to 40').

reciprocal The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The product of any number and its reciprocal is 1.

rectangle A quadrilateral with four right angles.

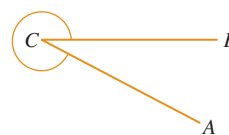


recurring (or repeating) decimal A decimal with one or more digits that repeat endlessly. For example 0.1666..., abbreviated as 0.1 $\dot{6}$.

reflection The process of 'flipping' a shape across a line to give a mirror image that is back-to-front.



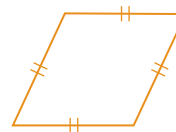
reflex angle A 'bent-back' angle greater than 180° but less than 360° .



revolution An angle of 360° .

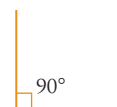


rhombus A quadrilateral with four equal sides.

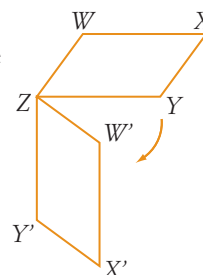


RHS The right-hand side (of an equation).

right angle A 90° angle, a 'square corner', a quarter-turn.

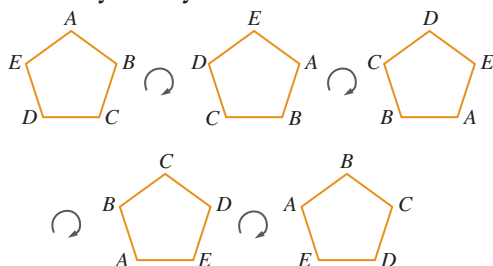


rotation The process of 'spinning' or turning a shape about or around a fixed point at a certain angle and direction (clockwise or anti-clockwise).



rotational symmetry A plane shape has rotational symmetry if it can be spun around its centre so that it fits onto itself again before making a complete revolution. The centre point is called the **centre of symmetry**. A regular pentagon (pictured) has rotational

symmetry, and because it can fit onto itself five times during one complete revolution we say that it has *rotational symmetry of order 5*. See also **line symmetry**.



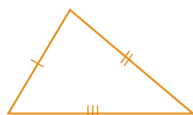
round To give an approximate answer. For example, rounding 3.14159 to three decimal places gives 3.142.

S

sample space In a probability situation, the set of all possible outcomes.

scale The amount represented by one unit or interval on a measuring instrument or on the axis of a graph.

scalene triangle A triangle with no equal sides.



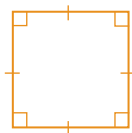
sector graph Also called a pie chart, a circular graph representing parts of a whole by being divided into proportionately-sized sectors.



solution In algebra, the value(s) that make an equation true.

speed A rate that compares distance travelled with time taken. Usually measured in kilometres per hour (km/h) or metres per second (m/s).

square A quadrilateral with four equal sides and four right angles.



square (of a number) The number multiplied by itself. For example, 7 squared = $7^2 = 7 \times 7 = 49$.

square root (of a number) The positive value which, if squared, gives the number. For example, $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$.

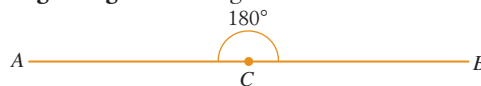
stem-and-leaf plot A 'number graph' that lists all the data scores, in groups. Each score is split into a 'stem' and a 'leaf'.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

This stem-and-leaf plot shows 12 test scores, from 42 to 82.

Key: 5|8 stands for 58

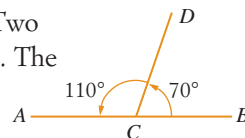
straight angle An angle of 180° .



substitute To replace a variable with a number, to evaluate an expression. For example, substituting $t = 5$ in the expression $t^2 + 6$ gives 31.

sum The result of an addition. The sum of 5 and 7 is 12.

supplementary angles Two angles whose sum is 180° . The angles 70° and 110° are supplementary.



symmetry See **line symmetry** and **rotational symmetry**.

T

terminating decimal A decimal whose digits do not run endlessly, for example, 0.125.

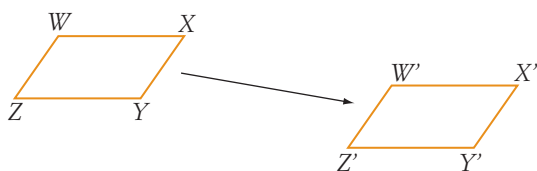
terms of an expression The parts of an algebraic expression. For example, $b^2 + 6b - 9$ has three terms: b^2 , $6b$ and -9 .

Glossaries

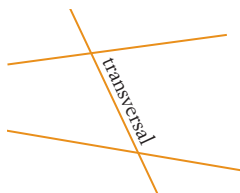
terms of a ratio The numbers in a ratio, for example, in the ratio 14 : 9, the terms are 14 and 9.

transformation In geometry, the process of moving or changing a shape. See **Reflection**, **Rotation** and **Translation**.

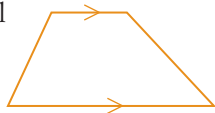
translation The process of 'sliding' a shape a certain distance and direction.



transversal A straight line that crosses two or more lines at different points.



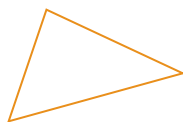
trapezium A quadrilateral with one pair of opposite sides parallel.



travel graph A line graph that describes a journey and shows the distance travelled over time.

trial One go or run of a repeated probability experiment, for example, one roll of a die.

triangle A polygon with three sides.



tonne A metric unit of mass for heavy objects, equal to 1000 kg.

U

unit cost The cost of one item or unit, found by dividing the cost of the item by the number of items or units.

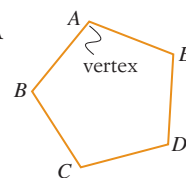
unitary method A method for finding a quantity by finding the size of one part first.

unlikely A low chance it will happen, improbable, probably won't happen.

V

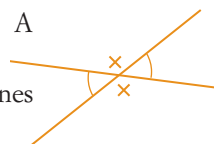
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral**.

vertex (Plural: **vertices**). A corner of a shape or angle.



vertical Going up and down, at a right angle to the **horizontal**.

vertically opposite angles A pair of opposite and equal angles formed when two lines cross.

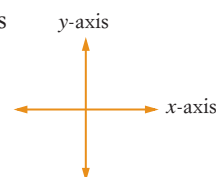


vinculum In a fraction, the horizontal line that separates the numerator from the denominator.

volume The amount of space occupied by a solid object, measured in cubic units.

X

x-axis The horizontal axis of a number plane (running across).



Y

y-axis The vertical axis of a number plane (running up and down).

Chapter 1

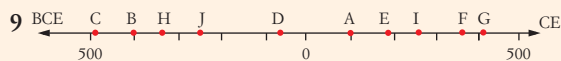
SkillCheck

- 1 a 16 b 8 c 28 d 4
 e 21 f 25 g 22 h 5
 i 13 j 8 k 15 l 6
- 2 a 49 b 604 c 70 d 33
 e 237 f 128 g 23 h 2
 i 28 j 400 k 12 l 32
 m 9 n 25 o 64 p 1
- 3 a T b T c F d T
 e F f F g T h T
 i F
- 4 a 1, 2, 3, 4, 8
 b 2, 3, 5, 7, 16, 18
 c 27, 38, 49, 51, 54, 66
- 5 a 18, 17, 11, 6, 5, 3
 b 31, 23, 14, 8, 7, 5, 2
 c 91, 89, 69, 46, 41, 37, 3

Exercise 1-01

- 1 a 100°C b body temperature
 c -74°C d water freezes
 e 117°C
- 2 a -6 b -1 c 10
 d -11 e 0 f 2
- 3 a down b less c up
 d right e after f north
 g deposit h increase i east
- 4 a i about 38 metres
 ii about 73 metres
 iii about 111 metres
 b '+' signs measure height above water level and
 '-' signs measure depth below water level
- 5 \$410
- 6 a depositing \$25 in the bank
 b losing \$12
 c walking 2 steps forward
 d increasing the temperature by 3°
 e driving 5 km east
 f decreasing the price by \$8
 g going down 3 floors
 h 5 seconds after take-off

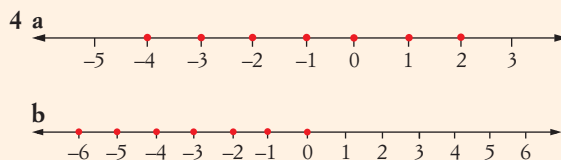
- 8 a BCE stands for Before the Common Era and CE stands for the Common Era.
 b Traditionally, the zero for time is when Jesus Christ was born. BC, meaning Before Christ, refers to the time *before* Christ. AD, meaning Anno Domini ('in the year of our Lord' in Latin), refers to the time *after* Christ was born.



- 10 a Teacher to check. b Teacher to check.

Exercise 1-02

- 1 A = 4, B = 9, C = 11, D = 17, E = 20, F = 27
- 2 a A = 2, B = 10, C = 13, D = 15
 b A = 10, B = 16, C = 28, D = 34, E = 37
 c A = 8, B = 15, C = 32, D = 41, E = 49, F = 56
 d A = 4, B = 36, C = 52, D = 66, E = 90, F = 116
 e A = 10, B = 20, C = 35, D = 55, E = 63, F = 89, G = 111
- 3 a Missing values are -5, -4, -2, 4, 5, 7, 8
 b Missing values are -50, -40, -30, 20, 30, 50, 60, 70
 c Missing numbers are -35, -30, -25, -20, -15, -10
 d Missing numbers are -400, -300, -200, 200
 e Missing numbers are -33, -30, -27, -24, -21, -18, -15, -12, -9, -6



- 5 a 29 b 35 c 32
 d 20 e 12 f 5
- 6 a 1 b 4 c 6
 d -6 e -12 f -15

Exercise 1-03

- 1 a > b < c > d >
 e > f > g < h <
 i < j < k < l >
 m > n > o < p <
- 2 a 7 b -8 c -8, -6, -4, -2, 1, 3, 5, 7

- 3 a $-3, -1, 2, 3$
 b $-8, -5, -3, 2, 5$
 c $-10, -6, -4, -3, 0$
 d $-5, -3, -2, 4, 6$
 e $-48, -24, -11, 0, 8, 36$
 f $-26, -10, -2, 3, 12, 15$

- 4 a $5, 4, 3, -1, -2$
 b $8, 0, -2, -4, -7$
 c $4, 1, -1, -3, -5, -11$
 d $8, 3, -2, -4, -18$
 e $1, -3, -6, -15, -48$
 f $33, 1, -36, -58, -100$

- 5 a 2 b 2 c 4 d 3
 e 6 f 20 g 7 h 7

6 D

- 7 a -4 b -6 c 4
 d -2 e -12

Exercise 1-04

- 1 a 4 b 5 c -1 d 2
 e 0 f 4 g -3 h -4
 i -20 j -4 k -8 l 0
 m -2 n -2 o -11 p 6

2 C

- 3 a Yes, 4
 b Yes, integers can be added in any order.

- 4 a -12 b 0 c 20 d -23
 e 15 f 0 g -3 h 9
 i -19 j -10 k -32 l -9
 m 0 n 5

5 The answer is always zero.

- 6 a (+) b (−) c (E)

- 7 a -4 b 7 c -3
 d 7 e -3 f 7

8 Teacher to check.

9 Teacher to check.

Mental skills 1A

- 2 a 155 b 221 c 317 d 1764
 e 48 f 726 g 283 h 952
 i 722 j 631 k 215 l 163
 m 394 n 466 o 937 p 295

Exercise 1-05

- 1 a 2 b -2 c -10 d 6
 e 13 f -1 g 0 h -10
 i -9 j 1 k -7 l 6
 m -5 n -6 o 12 p -9

2 B

- 3 a No, $10 - 4 = 6$, $4 - 10 = -6$
 b No, $-5 - 9 = -14$, $9 - (-5) = 14$
 c No, they give different answers.

- 4 a 11 b -11 c 29 d -29
 e -31 f 23 g 16 h -10
 i -50 j 50 k 42 l -42
 m -12 n 5

5 You get the negative of double the integer.

- 6 a (+) b (−) c (E)
 d (+) e (E)

- 7 a 4 b 4 c 3 d 3
 e 1 f 1 g 4 h -4
 i 2 j -2

- 8 a 7 b 3 c 8
 d 8 e 1 f -6

9 Teacher to check.

Exercise 1-06

- 1 a -18 b -24 c 21 d -32
 e 63 f -45 g 42 h 42
 i -24 j -6 k 1 l -54
 m -10 n -42 o 44 p -81
 q 60 r -60 s 100 t 56

- 2 a C b A c D

3 a $\begin{array}{r|rr} \otimes & -3 & 5 \\ \hline -3 & 9 & -15 \\ \hline 5 & -15 & 25 \end{array}$ b $\begin{array}{r|rr} \otimes & -5 & 5 \\ \hline 5 & -25 & 25 \\ \hline -5 & 25 & -25 \end{array}$

c $\begin{array}{r|rr} \otimes & 6 & -4 \\ \hline -4 & -24 & 16 \\ \hline 6 & 36 & -24 \end{array}$ d $\begin{array}{r|rr} \otimes & -1 & 1 \\ \hline -1 & 1 & -1 \\ \hline 1 & -1 & 1 \end{array}$

e $\begin{array}{r|rr} \otimes & 7 & -3 \\ \hline 4 & 28 & -12 \\ \hline -6 & -42 & 18 \end{array}$ f $\begin{array}{r|rr} \otimes & -3 & -6 \\ \hline 8 & -24 & -48 \\ \hline 2 & -6 & -12 \end{array}$

4 a Yes, both give -24 . b Yes

5 You always get the negative of the integer squared.

6 a 9 b 9 c 49 d 49
e 25 f 25 g 1 h 1

7 a (+) b (+) c (-)
d (+) e (+)

8 a -5 b -4 c -3
d -7 e -5 f -2

9 Teacher to check.

Exercise 1-07

1 a -4 b -2 c 4 d -3
e -9 f 3 g -3 h -10
i -4 j 7 k -9 l -6
m 9 n -8 o -4 p -6

2 a A b B

3 a -6 b -2 c 6 d 16
e -7 f -9 g 20 h 9

4 a $\begin{array}{r|rrrr} \otimes & -6 & -3 & 3 & 6 \\ \hline -6 & 1 & \frac{1}{2} & -\frac{1}{2} & -1 \\ \hline -3 & 2 & 1 & -1 & -2 \end{array}$ b $\begin{array}{r|rrrr} \otimes & 8 & -8 & 32 & -32 \\ \hline -2 & -4 & 4 & -16 & 16 \\ \hline 4 & 2 & -2 & 8 & -8 \end{array}$

5 a You get -1

6 a (+) b (+) c (-)

7 a -8 b 3 c -7
d -44 e -50 f -32

8 Teacher to check.

Exercise 1-08

1 a 7 b -22 c -1 d -7
e -6 f 4 g -25 h 8
i -21 j 12 k -200 l 28
m 15 n 5 o 3

2 C

3 a -7 b -25 c -36
d 10 e 14 f 3
g 5 h 16 i 2
j 8 k -11 l 2

4 a 8 b 5 c -3
d -2 e 3 f 12
g -2 h -2 i -19
j -9 k 3 l 25

5 a Correct
b Incorrect, $-32 \div (-4) \times (-2) = -16$
c Incorrect, $-6 \times (-2) - 8 = 4$
d Correct
e Correct
f Incorrect, $(-2 + 6) \times 2 = 8$
g Incorrect, $-7 - 6 + 10 = -3$
h Incorrect, $(-12 - 3) \div (-3) = 5$
i Incorrect, $12 - 3 \times 5 = -3$
j Incorrect, $2 - 9 \times (-3) - (-18) = 47$

6 a $-4 \times [7 - 2 + (-3)] = -8$
b $(-4 \times 7) - [2 + (-3)] = -27$
c $(-4 \times 7) - 2 + (-3) = -33$
d $40 \div [10 \times (-2)] + 15 = 13$
e $(40 \div 10) \times [(-2) + 15] = 52$
f $40 \div [10 \times (-2) + 15] = -8$

7 Teacher to check.

Mental skills 1B

2 a 17 b 35 c 114 d 261
e 132 f 128 g 56 h 418
i 74 j 123 k 88 l 112
m 36 n 549 o 185 p 317

Exercise 1-09

1 a -7 b 6 c -4 d 1
e -5 f 14 g 26 h -28
i 30 j -48 k 44 l 30
m -65 n -8 o 15 p -4
q 25 r 64 s -48 t 2

2 Same answers as question 1.

3 $(-5)^2$ means $-5 \times -5 = 25$, -5^2 means $-(5^2) = -25$.

4 a -18 b 71 c 1
 d -6 e -3960 f -192
 g 5 h -60 i 5
 j 38 k 61 l 16

5 a 13 b -10 c 2
 d 15 e -56 f -8

6 Teacher to check.

Exercise 1-10

1 13°C 2 4 km
 3 a 6610 m b 2316 m c 311 m d 9237 m

4 a 2009, 2010, 2013, 2014
 b 2011, 2012
 c \$100 million
 d Profit, \$200 million

5 1 km north 6 -\$7 7 A
 8 59 9 4602 m 10 280 cm

11 155 m below the surface

12 -\$109.00 13 11 m

14 a 14th day b \$60

Power plus

1 a 1 b -1 c 1
 d $(-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
 e $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$
 f $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

2 $(-1)^n = 1$ when n is even, $(-1)^n = -1$ when n is odd

3 a 4 or -4 b 10 or -10 c -2

4 Yes, no number squared gives a negative answer.

5 Teacher to check.

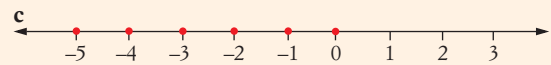
6 a 2 b -9 c 1

7 a True b False c True d False
 e True f True g False h True
 i True

8 a -5, 2 b -3, -5 c -4, 2
 d -3, 2 e -3, 10

Chapter 1 revision

1 a 2 b 23 c -56
 d 10 e -8 f 0



3 a F b F c F d T
 e T f T g T h F
 i F j T k T l F

4 14, 3, 0, -1, -5, -9, -10

5 a -1 b -10 c -3
 d 4 e -10 f 3
 g -11 h 1 i -8

6 a 7 b 4 c -4 d -12
 e 0 f -12 g -7 h 6
 i -11

7 a -18 b 48 c -10 d -44
 e -63 f 48 g -24 h -42
 i 40

8 a 3 b -3 c 1
 d -4 e -2 f -100
 g 2 h -3 i 8

9 a -12 b -14 c -13
 d -18 e 0 f -19
 g 7 h -2 i 0

10 a -9 b -6 c -12
 d 87 e -52 f 5

11 16°C 12 \$820

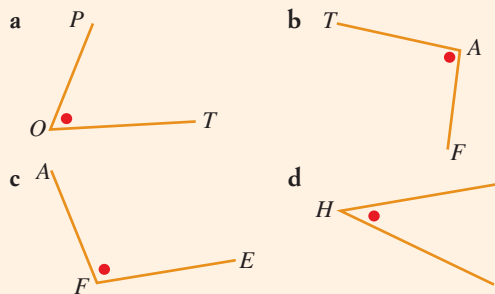
Chapter 2

SkillCheck

- 1 a 20° b 25° c 15° d 20°
 e 40° f 80° g 60° h 57°
 i 70° j 94° k 28° l 81°
- 2 a I and J b A and H , B and I
- 3 a E and F b D and E c C and E
 d B and E e F and H f D and J
 g A and I h A and J

Exercise 2-01

- 1 a $\angle QPK$, $\angle P$ or $\angle KPQ$
 b $\angle OCR$, $\angle C$ or $\angle RCO$
 c $\angle GVE$, $\angle V$ or $\angle EVG$
 d $\angle TAG$, $\angle A$ or $\angle GAT$
 e $\angle PQD$, $\angle Q$ or $\angle DQP$
 f $\angle RDC$, $\angle D$ or $\angle CDR$
- 2 C
- 3 a $\angle CAD$ or $\angle DAC$ b $\angle PMQ$ or $\angle QMP$
 c $\angle TQR$ or $\angle RQT$ d $\angle WXY$ or $\angle YXW$
 e $\angle AED$ or $\angle DEA$ f $\angle FGE$ or $\angle EGF$
- 4 The following are examples, the sizes of the angles will differ.



- 5 a $\angle SQP$ b $\angle CBE$ c angle b d angle q
- 6 a $\angle NAY$, $\angle AYN$ or $\angle AYC$, $\angle ANY$ or $\angle ANC$, $\angle AND$, $\angle AYD$, $\angle NCD$, $\angle NDY$, $\angle NCY$, $\angle DNC$ or $\angle DNY$, $\angle DCY$, $\angle CDY$, $\angle NYD$ or $\angle CYD$, $\angle NDC$ (Remember that $\angle NAY$ and $\angle YAN$ are two ways of naming the same angle.)

- b $\angle NCD$, $\angle NCY$, $\angle DCY$
 c A straight angle
 d $\angle ANY$ or $\angle YNA$

7 A

- 8 a • $\angle DBC$ or $\angle CBD \times \angle ADB$ or $\angle BDA$
 b • $\angle PSR$ or $\angle RSP \times \angle PRQ$ or $\angle QRP$
 c • $\angle MRQ$ or $\angle QRM \times \angle RNP$ or $\angle PNR$
 d • $\angle EIF$ or $\angle FIE \times \angle FGI$ or $\angle IGF$
 e • $\angle DCF$ or $\angle FCD \times \angle GBC$ or $\angle CBG$
 f • $\angle WXY$ or $\angle YXW \times \angle YWZ$ or $\angle ZWY$

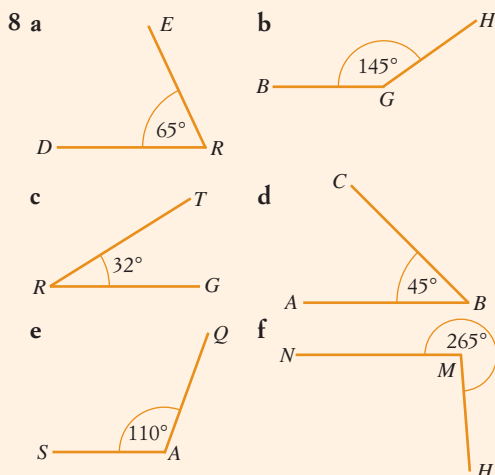
9 Teacher to check

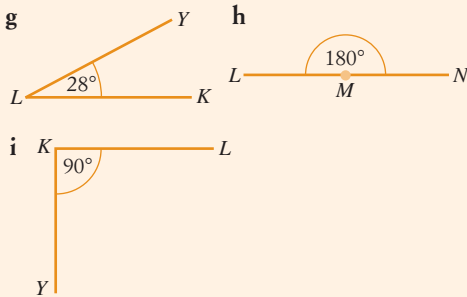
Exercise 2-02

- 1 a 64° b 133° c 85° d 87°
 e 129° f 175° g 157° h 112°
- 2 a $\angle AOB = 38^\circ$ b $\angle PDQ = 162^\circ$
 c $\angle XPY = 84^\circ$ d $\angle MAN = 121^\circ$
 e $\angle TQS = 175^\circ$ f $\angle XYZ = 38^\circ$
 g $\angle LMN = 82^\circ$ h $\angle GFD = 121^\circ$
 i $\angle ADM = 162^\circ$ j $\angle BPZ = 47^\circ$
 k $\angle ABC = 122^\circ$ l $\angle EFG = 38^\circ$
- 3 d, a, h, f, c, b, g, e
- 4 a $\angle ABC = 213^\circ$ b $\angle XYZ = 300^\circ$
 c $\angle LMN = 248^\circ$ d $\angle GHK = 330^\circ$
- 5 B

6 Teacher to check

7 Teacher to check





Exercise 2-03

1 a acute	b obtuse	c obtuse
d acute	e reflex	f reflex
g acute	h obtuse	i reflex
j obtuse	k reflex	l acute
m acute	n reflex	o obtuse

2 Teacher to check.

3 a acute	b obtuse	c reflex
d reflex	e acute	f straight
g obtuse	h obtuse	i revolution
j acute	k reflex	l obtuse
m acute	n reflex	o right
p reflex	q obtuse	r acute

4 D

- 5 a obtuse b acute c acute d acute

Exercise 2-04

1 a $57 + 33 = 90$	b $147 + 33 = 180$
2 a 60°	b 20°
d 52°	e 1°
g 48°	h 24°
c 65°	f 33°

3 Teacher to check

4 a 162°	b 30°	c 145°	d 55°
e 118°	f 93°	g 69°	h 176°
5 a $c = 84$	b $y = 90$	c $p = 126$	d $a = 180$
e $x = 12$	f $x = 40$	g $a = 15$	h $a = 46$
6 a $\angle PQR$ and $\angle RQS$	b $67 + 23 = 90$		
c right angle			
7 a $a = 60$	b $a = 20$		
c $m = 135$	d $m = 71$		

e $f = 165$	f $a = 45$
g $t = 90$	h $a = 32, b = 148$
i $x = 70, y = 90$	j $j = 132, k = 48, l = 132$
k $x = 62, y = 105$	l $c = 60$

Mental skills 2

2 a 117	b 740	c 138	d 140
e 174	f 100	g 436	h 109
i 200	j 312		
3 a 160	b 700	c 90	d 2200
e 72	f 1600	g 2100	h 480
i 132			

Exercise 2-05

1 a c	b u	c a
d f	e i	f p
2 a $m = 50$	b $q = 120$	c $x = 211$
d $y = 145$	e $a = 98$	f $d = 151$
g $w = 90$	h $f = 57$	i $n = 110$
3 D		
4 a $\angle MDQ$	b $\angle NDP$	
c $\angle NDQ, \angle MDP$	d Teacher to check	
5 a $m = 90$	b $x = 135$	
c $f = 25$	d $w = 133$	
e $n = 29$	f $q = 62$	
g $t = 163$	h $h = 20, g = 160$	
i $q = 90, r = 90, s = 90$		
6 a $y = 100$	b $p = 120$	
c $a = 120$	d $x = 41$	
e $h = 170$	f $y = 53$	
g $d = 135$	h $e = 68, f = 112$	
i $p = 60$		

Exercise 2-06

- 1 AB, AC, AD, BD, BC, CD
- | | |
|-----------------------|---------------------|
| 2 a AD and CF, GB | b BG and CF |
| c Teacher to check | |
| 3 a $AD \perp CF, GB$ | b $BG \parallel CF$ |
| 4 Teacher to check | |
| 5 D | 6 C |
| 7-12 Teacher to check | |

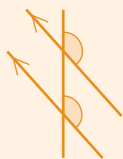
Exercise 2-07

1 a f

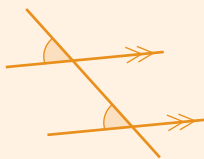
b c

c c

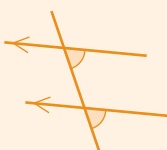
2 a



b

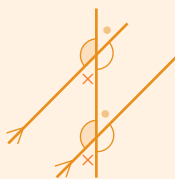


c

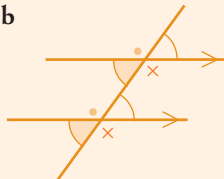


3 Pairs of corresponding angles are marked in the same way.

a



b



c



4 B

5 a $a = 120$

b $y = 28$

c $m = 63$

d $c = 108$

e $t = 74$

f $a = 50$ and $b = 60$

g $m = 110, n = 70$

h $x = 105, y = 105$

i $a = 40, c = 40, d = 140, y = 140$

6 a 4

b $a = 75, b = 105, c = 75, d = 105, e = 75, f = 105, g = 75$

Exercise 2-08

1 a g

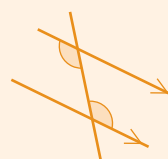
b d

c d

2 a



b

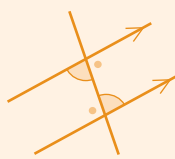


c

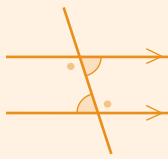


3 Pairs of alternate angles are marked in the same way.

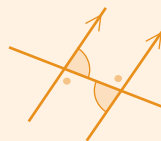
a



b



c



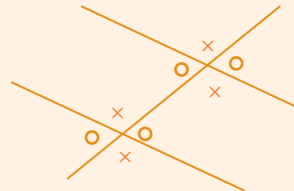
4 A

5 a 2

b $a = 94, b = 86, c = 94, d = 86, e = 94, f = 86, g = 94$

c They are equal

6 a



b They are equal

7 Teacher to check

8 a Vertically opposite angles

b Corresponding angles

c Both b and c equal a

d They are equal

9 a $m = 110$

b $a = 50$

c $n = 80$

d $b = 122$

e $h = 20$

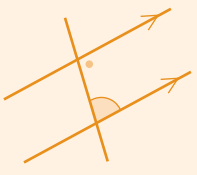
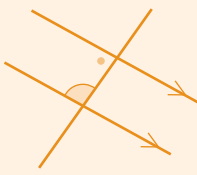
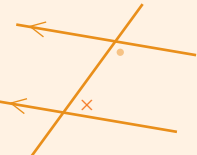
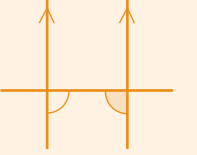
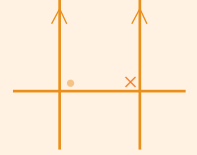
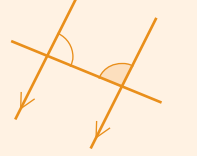
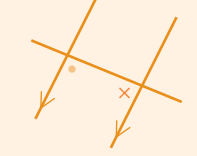
f $m = 50, n = 130, p = 50$

g $a = 140, b = 40$

h $a = 130, b = 130$

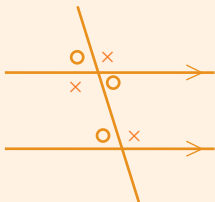
i $a = 136, b = 136, c = 44$

Exercise 2-09

- 1 a d b d c c
- 2 a  b 
- c 
- 3 a  or 
- b  or 
- c  or 
- 4 a 2 b A
 c $a = 59, b = 121, c = 59, d = 59, e = 121, f = 59, g = 121$
 d They add up to 180°

5 Both sums are 180°

- 6 a 180° , Angles on a straight line
 b



- c They add up to 180°
- 7 a 180° , Angles on a straight line
 b Alternate angles
 c 180°
 d They add up to 180°

- 8 a $a = 130$ b $m = 90$
 c $b = 105$ d $d = 68$
 e $m = 112$ f $a = 82, b = 98$
 g $f = 50, g = 130$ h $j = 125, k = 125$
 i $a = 51, b = 129, c = 51$

Exercise 2-10

- 1 a $\angle WXQ$ b $\angle XWP$ c $\angle WXS$
 d $\angle AWV$ or $\angle XWP$ e $\angle XWA$ f $\angle XWP$
- 2 a 65, co-interior b 71, alternate
 c 105, corresponding d 120, corresponding
 e 110, co-interior f 132, alternate
 g 28, alternate h 72, corresponding
 i 95, co-interior j 93, corresponding
 k 81, alternate l 30, co-interior
 m 128, corresponding n 114, co-interior
 o 109, alternate
- 3 a $a = 113, b = 113$
 b $j = 133, k = 133, l = 133$
 c $m = 52, n = 52, p = 128$
 d $y = 42, z = 42$
 e $l = 95, m = 85$
 f $b = 45, c = 30$
 g $p = 105, q = 105$
 h $k = 85, m = 85$
 i $p = 63, w = 117$
 j $k = 50$
 k $x = 118, y = 55$
 l $a = 72, b = 108$
 m $m = 97, n = 83, p = 83$
 n $g = 132$
 o $a = 153, b = 153, c = 27$
- 4 C

Exercise 2-11

- 1 a $\angle BEF$ and $\angle CFE$
 $AB \parallel CD$, alternate angles are equal
 b $\angle AGH$ and $\angle GHD$
 $AB \parallel CD$, alternate angles are equal
 c $\angle BEF$ and $\angle EFC$
 AB is not parallel to CD , alternate angles are not equal
 d $\angle GEB$ and $\angle DFE$
 AB is not parallel to CD , corresponding angles are not equal
 e $\angle AEF$ and $\angle CFG$
 $AB \parallel CD$, corresponding angles are equal

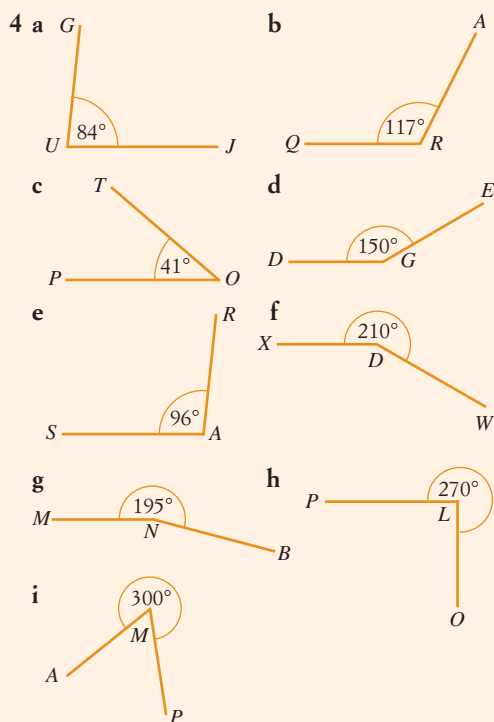
- f $\angle CEF$ and $\angle AFG$
 AB is not parallel to CD , corresponding angles are not equal
- g $\angle AEF$ and $\angle EFC$
 $AB \parallel CD$, co-interior angles add to 180°
- h $\angle BEF$ and $\angle EFD$
 AB is not parallel to CD , co-interior angles do not add to 180° .
- i $\angle AEF$ and $\angle EFC$
 $AB \parallel CD$, co-interior angles add to 180°
- 2 a $MN \parallel PQ$, co-interior angles add to 180°
 b $PQ \parallel MN$, corresponding angles are equal
 c $MN \parallel PQ$, alternate angles are equal, corresponding angles are equal
 d PQ is not parallel to MN , corresponding angles are not equal, $\angle PCB = 115^\circ$ and $\angle MAD = 120^\circ$
 e MN is not parallel to PQ , alternate angles are not equal
 f $MN \parallel PQ$, co-interior angles add to 180°
- 3 B

Power plus

- 1 a 360° b 15° c 120° d 2.5°
- 2 a left b front c behind d front
- 3 a $m = 51$, vertically opposite, corresponding angles
 b $x = 62$, alternate angles; $y = 62$, alternate angles
 c $a = 55$, alternate, co-interior
 d $m = 58$, alternate, straight angle, or co-interior
 e $y = 75$, teacher to check (corresponding)
 f $k = 65$, straight angle, angle sum of a triangle; $c = 35$, alternate angles
 g $x = 85$, alternate angles
 h $m = 255$, teacher to check (corresponding angles)
 i $k = 105$, teacher to check (corresponding, co-interior, and vertically opposite)
- 4 Teacher to check

Chapter 2 revision

- 1 Teacher to check.
- 2 a $\angle ABD$ b $\angle QOS$
- 3 Teacher to check.



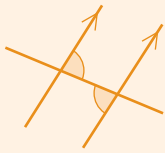
- 5 a $\angle WHI$ or $\angle IHW$, acute angle
 b $\angle DAR$ or $\angle RAD$, obtuse angle
 c $\angle GLU$ or $\angle ULG$, acute angle
 d $\angle PVR$ or $\angle RVP$, acute angle
 e $\angle NPE$ or $\angle EPN$, right angle
 f $\angle SAM$ or $\angle MAS$, obtuse angle
 g $\angle MTV$ or $\angle VTM$, straight angle
 h $\angle XYZ$ or $\angle ZYX$, reflex angle
 i $\angle MPQ$ or $\angle QPM$, reflex angle
- 6 a i 55° ii 12° iii 86°
 b i 135° ii 80° iii 2°
- 7 a $m = 62$ b $k = 133$ c $x = 58, y = 122$
- 8 a $a = 220$ b $m = 65$ c $y = 325$
 d $a = 100$ e $m = 44$ f $a = 95, b = 85$
 g $p = 145$ h $x = 42$ i $f = 70$
 j $t = 83$ k $p = 155, q = 155, r = 25$
 l $x = 30$

- 9 a AB and GH
 b EF and GH , EF and AB
 c AB and CD , AB and EF , GH and CD , GH and EF

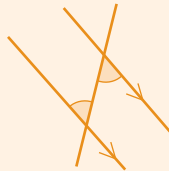
- 10 Teacher to check

11 Teacher to check

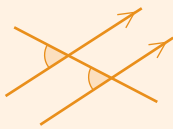
12 a



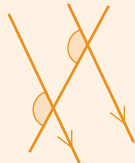
b



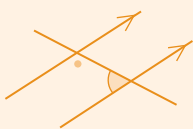
13 a



b



14 a



b



- 15 a alternate
c corresponding
e alternate
- b co-interior
d corresponding
f corresponding
- 16 a $a = 115$
c $k = 115$
e $x = 50$
g $x = 37, y = 37$
h $m = 62, p = 118, z = 118$
i $a = 112, d = 68, t = 112$
j $x = 50, y = 50$
k $x = 26, m = 64$
l $a = 72, c = 70, z = 38$
m $a = 38, c = 142$
n $x = 35, y = 88, z = 92$
- b $m = 35$
d $d = 125$
f $q = 62$

- 17 a $AB \parallel CD$, co-interior angles add to 180°
b AB is not parallel to CD , alternate angles are not equal
c $AB \parallel CD$, corresponding angles are equal

Chapter 3

SkillCheck

- 1 Odd, the last digit is an odd number.
2 Yes, the last digit is a zero.

- 3 a 21 b 30 c 32 d 18
e 36 f 40 g 49 h 54
i 64 j 63 k 24 l 56
m 9 n 81 o 25 p -16

- 4 a 1, 2, 4, 5, 10, 20 b 1, 2, 3, 4, 6, 12
c 1, 19

5 Yes, the last digit is even.

- 6 a 8 b 5 c 6 d 6
e 4 f 8 g 6 h 8
i 4 j 9 k 6 l 9

- 7 a 8, 16, 24, 32, 40 b 7, 14, 21, 28, 35
c 12, 24, 36, 48, 60

8 No, it does not end with a zero.

Exercise 3-01

- 1 a 3100 b 49 000 c 2600 d 4 934 300
2 a 24 000 b 45 000 c 63 000 d 48 000
3 a 45 820 b 1700 c 8310 d 71 260
4 a 64 000 b 64 200 c 64 220 d 60 000
5 a 1 328 000 b 1 330 000
c 1 327 510 d 1 000 000
- 6 Your answers should be close to the following exact answers.
a 331 b 157 c 1587
d 255 e 421 f 203
g 413 h 734 i 6723
j 15 744 k 276 l 72.43 (to two decimal places)

7 27 800 8 \$17 500

9 Teacher to check. 10 104 000

11 a 1000 km b 1000 km c 1000 km

12 23 582 000

Exercise 3-02

- 1 a 238 b 1095 c 448 d 6825
2 \$294

Answers

3 a 112 b 714 c 3096 d 3542
e 3252 f 630 g 1600 h 3424

4 552 km 5 1140

6 a 170 b 78 c 128 d 228
e 112 f 176 g 128 h 264

7 a 110 b 70 c 140 d 180
e 270 f 410 g 80 h 240

8 a 306 b 459 c 243 d 171
e 450 f 1800 g 360 h 4100

9 C 10 2604

Exercise 3-03

1 a 11 b 34 c 12 d 70
e 15 f 78 g 147 h 123
i 452 j 124 k 495 l 466

2 a $12\frac{1}{4}$ b $229\frac{1}{3}$ c $158\frac{2}{3}$ d $259\frac{2}{7}$

3 C 4 \$34 each.

5 a 32 b 80 c 600 d 180
e 82 f 18 g 46 h 120

6 a 72 b 116 c 324 d 485
e 54 f 122 g 109 h 26

Exercise 3-04

1 a No b Yes c Yes d No
e No f Yes g Yes h No

2 a No b No c Yes d Yes

3 a No b Yes c No d No
e No f No g Yes h Yes

4 a Yes b No c No d No
e Yes f Yes g Yes h No

5 a No b No c Yes d Yes
e Yes f No g Yes h No

7 a 9 is not divisible by 2.
b Last digit is not a 0 or 5.
c 5 is not divisible by 2.
d 3 is not divisible by 2.
e Last digit is not a 0.
f 1 is not divisible by 2.

8 C 9 12

10 a 75, 90 b 60, 80 c 84
d 54, 60, 66, 72, 78, 84, 90, 96

11 a Yes b No c Yes d No

12 a Yes b Yes c No d Yes

Mental skills 3A

2 a 1800 b 26 000 c 770 000
d 1000 e 315 000 f 1 000 000
g 2940 h 47 500 i 240
j 1600 k 300 l 210
m 12 000 n 9900 o 1600
p 3500 q 400 r 500
s 18 000 t 3 600 000

Exercise 3-05

1 a 12 b 21 c 43
d 25 e 147 f 6
g 36 h 29 i 214

2 a $25\frac{1}{3}$ b $36\frac{1}{14}$ c $38\frac{1}{13}$

3 11 4 26 cm 5 25 weeks

Exercise 3-06

1 a XXVII b LII c LXXIX
d XCH e MMDC f CCCXIV
g CMXXVIII h MMMCDLXXVI i CLXV
j CCXXX k MMCDLXXXII l CIX

2 a 26 b 40 c 264 d 54
e 77 f 45 g 339 h 528
i 2162 j 1990 k 98 l 1507

3 a Teacher to check.
b They are cumbersome to use, and they take too long to write.

4 Teacher to check.

5 Teacher to check.

6 5000 and 10 000

Exercise 3-07

1 a 8^4 b 7^6 c $(-5)^3$
d 1^5 e 13^4 f 10^1
g $(-4)^2 \times 9^3$ h $2^2 \times 12^2$ i $3^2 \times 11^4$
j $15^2 \times (-6)^2$ k $2^3 \times 8^2$ l $10^2 \times 20^1$

- 2 a $3 \times 3 \times 3 \times 3 \times 3$
 b 5×5
 c $7 \times 7 \times 7 \times 7$
 d $(-6) \times (-6) \times (-6)$
 e $2 \times 2 \times 2 \times 2 \times 4 \times 4$
 f $12 \times 12 \times 12 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$

- 3 a 16 b 125 c 32
 d 1 000 000 e 243 f 2401
 g 1 h 16 i 27
 j 1 k -59 049 l 4096

4 See table below.

Exercise 3-08

- 1 a 81 b 289 c 100

- 2 a 49 b 169 c 324
 d 1 e 25 f 400
 g 256 h 1156 i 225

- 3 a 20 b 15 c 18
 d 16 e 13 f 34

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2	1	4	9	16	25	36	49	64	81	100	121	144

5 B

- 6 a 10 b 6 c 11 d 4 e 1
 f 9 g 8 h 12 i 5

7 0 and 1

- 8 a 7 b 9 c 25
 d 22 e 42 f 19
 g 30 h 28 i 16
 j 14 k 20 l 56

- 9 a 64 b 1331 c 1

- 10 a 343 b 4096 c 125
 d 1000 e 8000 f 4913
 g 42 875 h -512 i -125

11

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number cubed, x^3	1	8	27	64	125	216	343	512	729	1000	1331	1728

12 B

- 13 a 8 b 9 c 5 d 1
 e 3 f 12 g 4 h 11
 i 10

- 14 a -8 b 20 c 35
 d -5 e 17 f 16

- 15 a 16 b 14 c 22
 d 40 e 27 f 21

16 D

17 B

- 18 a 7.1 b 5.7 c 9.8
 d 4.1 e 9.7 f 5.7

Exercise 3-07

Power of 10	Number	Name
10^1	10	ten
10^2	100	hundred
10^3	1000	thousand
10^6	1 000 000	million
10^9	1 000 000 000	billion
10^{12}	1 000 000 000 000	trillion
10^{15}	1 000 000 000 000 000	quadrillion
10^{18}	1 000 000 000 000 000 000	quintillion
10^{21}	1 000 000 000 000 000 000 000	sextillion
10^{24}	1 000 000 000 000 000 000 000 000	septillion
10^{27}	1 000 000 000 000 000 000 000 000 000	octillion
10^{30}	1 000 000 000 000 000 000 000 000 000 000	nonillion
10^{33}	1 000 000 000 000 000 000 000 000 000 000 000	decillion

Mental skills 3B

2 a 20	b 60	c 450	d 30
e 190	f 26	g 53	h 720
i 6	j 6	k 4	l 9
m 70	n 30	o 22	p 8
q 40	r 5	s 800	t 6

Exercise 3-09

- 1 a** 1, 17 **b** 1, 3, 7, 21
c 1, 2, 3, 4, 6, 8, 12, 24 **d** 1, 11
e 1, 5, 7, 35 **f** 1, 2, 4
g 1, 2, 3, 6, 9, 18 **h** 1, 23
i 1, 5, 25 **j** 1, 3, 9
k 1, 3 **l** 1, 19
- 2 a** 3, 11, 17, 19, 23 **b** 4, 9, 18, 21, 24, 25, 35
- 3 C**
- 4** Primes: 17, 41, 59, 71, 101 Composites: 10, 27, 33, 62, 77, 129, 472, 999, 2064, 967 504
- 5 f** They are all prime numbers.
- 6 a** 37, 41, 43, 47
b 66, 68, 69, 70, 72, 74, 75, 76, 77, 78
c 2, 3, 5, 7, 11, 13, 17, 19
d 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46
- 7 B** **8** Teacher to check.
- 9 a** Some possible answers: 1, 25, 9, 36, 4.
b Square numbers

Exercise 3-10

- 1** (Different factor trees are possible.)
- 2 D**
- 3** (Different factor trees are possible.)
- 4** (Different factor trees are possible.)
a $2^4 \times 3$ **b** $2^3 \times 5^2$ **c** $2^2 \times 5 \times 23$
d $2^3 \times 89$ **e** 2×7^2 **f** $2^4 \times 3^2$
g $5^2 \times 13$ **h** $3^3 \times 5$
- 5 a** $2^2 \times 11^2$ **b** 22
- 6 a** 25 **b** 30 **c** 19 **d** 28
e 21 **f** 15 **g** 12 **h** 16

Exercise 3-11

- 1 C**
- 2 a** 1, 3, 9 **b** 1, 2, 4 **c** 1, 3, 5, 15 **d** 1, 2
- 3 a** 4 **b** 3 **c** 2 **d** 4
e 14 **f** 4 **g** 5 **h** 6
i 25 **j** 8 **k** 30 **l** 13
m 12 **n** 12 **o** 9 **p** 70
- 4 a** 9
- 5 a** 162 **b** 12 **c** 35 **d** 4

Exercise 3-12

- 1 a** 7, 14, 21, 28, 35
2 16, 20, 24, 28, 32, 36, 40, 44
3 16
- 4 a** 15 **b** 42 **c** 12 **d** 30
e 40 **f** 20 **g** 10 **h** 8
i 18 **j** 21 **k** 60 **l** 60
- 5** Teacher to check.
- 6 a** 24
- 7** (Different factor trees are possible.)
a 420 **b** 400 **c** 220 **d** 180

Power plus

- 1 a** 8, 9, 25, 32, 125, 243
b 25, 64, 216, 243, 256, 343
c 81, 125, 128, 10 000, 14 641
- 2 a** 1 **b** 121 **c** 12 321 **d** 1 234 321
- 3 a** 123 454 321
b 1 234 567 654 321
c 12 345 678 987 654 321
d 1 234 567.876 543 21
- 4 a** 6 **b** 60
- 5 a** 9 **b** 9 **c** 200
d 8 **e** 75 **f** 2

- 6 a 50 b 90 c 100
 d 1000 e 1100 f 10 000
 g 800 h 420 i 100 000

7 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31,
 41 and 43, 59 and 61, 71 and 73

Chapter 3 revision

- 1 a 88 000 b 87 500 c 87 530 d 90 000

2 Teacher to check

- 3 a 48 b 170 c 297
 d 200 e 882 f 10 170

- 4 a 240 b 124 c 162
 d 132 e 16 f 121

- 5 a Yes b No c Yes d No
 e No f Yes g No h No

6 B

- 7 a $256\frac{7}{12}$ b 26 c 124

- 8 a XLII b XXXVIII
 c LXXXVIX d CCCXLVII

- 9 a 3^4 b 6^5 c 11^7

- 10 a 36 b 125 c 100 000 d 256

- 11 a 7 b 12 c 20 d 9

- 12 a 29, 31, 37, 41, 43, 47
 b 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 62,
 63, 64, 65, 66, 68, 69

- 13 a $60 = 2^2 \times 3 \times 5$ b $200 = 2^3 \times 5^2$
 c $144 = 2^4 \times 3^2$

- 14 a 16 b 18

- 15 a 4 b 12

16 (Different factor trees are possible.) 81

- 17 a 24 b 90

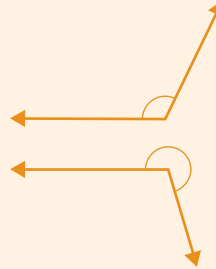
18 (Different factor trees are possible.) 400

Mixed revision 1

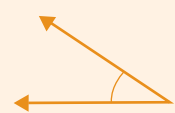
- 1 a 0 b 31 c 11
 d 52 e 46 f 60

- 2 a 2 b 8 c 4
 3 a 103° b 57° c 295°

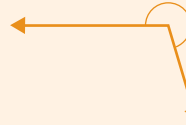
4 a



b



c

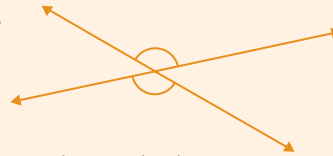


- 5 $\angle PSA$ a vertex b arm

- 6 a 24° b 5° c 78°

- 7 a 148° b 90° c 75°

8



9 Teacher to check

- 10 a 2500 b 17 200 c 807 900

- 11 a -2 b 6 c -7 d -6
 e 15 f -3 g 9 h -16
 i -11 j -26 k 10 l 15

- 12 a $m = 130$ b $b = 110$
 c $p = 142$ d $a = 98$
 e $p = 54$ f $a = 88, b = 92$
 g $c = 142$ h $m = 55$
 i $m = 37, p = 87$ j $x = 296$
 k $x = 30$ l $y = 90$

- 13 a 36 b 27 c 5 d 11

14 18°C

- 15 a 1, 2, 3, 6, 9, 18 b 1, 3, 5, 9, 15, 45
 c 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30,
 36, 40, 45, 60, 72, 90, 120, 180, 360

- 16 a $18 = 2 \times 3^2$ b $45 = 3^2 \times 5$
 c $360 = 2^3 \times 3^2 \times 5$

- 17 a 12 b 9

- 18 a 77 b 42

Chapter 4

SkillCheck

- 1 a i $\frac{1}{4}$ ii $\frac{3}{4}$
 b i $\frac{3}{6}$ ii $\frac{3}{6}$
 c i $\frac{7}{12}$ ii $\frac{5}{12}$
 d i $\frac{15}{20}$ ii $\frac{5}{20}$
 e i $\frac{6}{9}$ ii $\frac{3}{9}$
 f i $\frac{5}{9}$ ii $\frac{4}{9}$
- 2 $\frac{12}{31}$ 3 Teacher to check
- 4 Teacher to check 5 $\frac{1}{3}$
- 6 a 12 b 19 c 7
 d 7 e 5 f 3
- 7 Teacher to check
- 8 a 3 b 3 c 8
- 9 a 12 b 10 c 24
- 10 Teacher to check
- 11 a $\frac{13}{60}$ b $\frac{3}{4}$ c $\frac{1}{6}$
- 12 $\frac{1}{3}$

Exercise 4-01

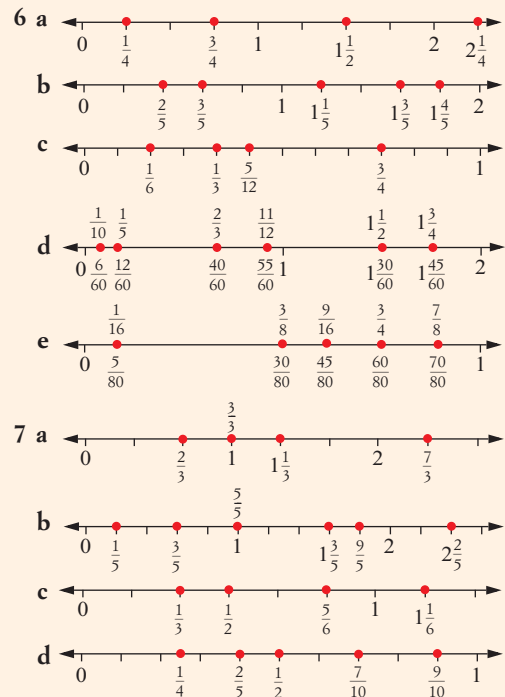
- 1 a M b P c M
 d M e P f I
 g P h P i I
 j P k M l I
- 2 a $4\frac{1}{3}$ b $3\frac{1}{4}$ c $1\frac{3}{10}$ d $4\frac{1}{5}$
 e $1\frac{3}{35}$ f $4\frac{3}{8}$ g 5 h $7\frac{1}{5}$
 i 4 j $10\frac{10}{11}$ k $3\frac{15}{19}$ l $10\frac{1}{10}$
- 3 a $\frac{3}{2}$ b $\frac{13}{8}$ c $\frac{9}{7}$ d $\frac{10}{3}$
 e $\frac{9}{2}$ f $\frac{37}{10}$ g $\frac{37}{6}$ h $\frac{38}{5}$
- 4 Mixed numeral
- 5 a 12 b 25 c 8
 d 56 e 72 f 75
 g 6 h 3 i 9
- 6 Teacher to check 7 D
- 8 a $\frac{3}{8}$ b $\frac{1}{4}$ c $\frac{4}{5}$ d $\frac{3}{4}$
 e $\frac{3}{7}$ f $\frac{4}{5}$ g $\frac{1}{4}$ h $\frac{2}{5}$

- 9 a C b D c C
- 10 a $2\frac{1}{4}$ b $2\frac{1}{7}$ c 2 d $1\frac{2}{3}$
 e $2\frac{1}{3}$ f $1\frac{1}{4}$ g $1\frac{1}{3}$ h $1\frac{1}{3}$
- 11 a $1\frac{1}{2}$ b $3\frac{1}{6}$ c $4\frac{1}{4}$ d $6\frac{3}{4}$
 e $17\frac{7}{20}$ f $100\frac{11}{16}$ g $1\frac{27}{50}$ h $5\frac{1}{3}$
- 12 D

Exercise 4-02

- 1 a $\frac{7}{10}$ b $\frac{5}{3}$ c $\frac{7}{8}$
 d $\frac{2}{3}$ e $2\frac{1}{5}$ f $3\frac{4}{7}$
- 2 a $\frac{3}{5}$ b $\frac{9}{8}$ c $\frac{1}{4}$
 d $\frac{1}{6}$ e $4\frac{2}{3}$ f $1\frac{1}{5}$
- 3 C
- 4 a $\frac{1}{2} > \frac{1}{3}$ b $\frac{2}{3} < \frac{3}{4}$ c $\frac{3}{8} < \frac{1}{2}$
 d $2\frac{3}{5} < 2\frac{7}{10}$ e $\frac{5}{6} > \frac{1}{2}$ f $\frac{5}{12} > \frac{1}{3}$
 g $\frac{17}{20} > \frac{6}{10}$ h $\frac{11}{15} > \frac{2}{3}$ i $2\frac{3}{7} < 5\frac{2}{5}$

5 Teacher to check



- 8 C 9 Teacher to check

Exercise 4-03

- 1 a $\frac{2}{3}$ b $\frac{9}{10}$ c 1 d $\frac{4}{3}$
 e $\frac{2}{5}$ f $\frac{1}{2}$ g $\frac{2}{5}$ h $\frac{1}{4}$
- 2 a $\frac{5}{6}$ b $\frac{13}{15}$ c $\frac{1}{4}$ d $\frac{1}{8}$
 e $\frac{2}{3}$ f $\frac{1}{20}$ g $\frac{20}{63}$ h $\frac{3}{4}$
 i $\frac{7}{8}$ j $\frac{3}{10}$ k $\frac{1}{9}$ l $1\frac{5}{24}$
 m $\frac{27}{40}$ n $1\frac{11}{20}$ o $\frac{7}{24}$ p $1\frac{2}{35}$
- 3 D
- 4 a $\frac{1}{6}$ b $\frac{11}{12}$
- 5 $\frac{1}{2}$
- 6 a $4\frac{1}{2}$ b $\frac{2}{5}$ c 4 d $5\frac{5}{4} = 5\frac{1}{2}$
 e $3\frac{2}{5}$ f $6\frac{3}{8}$ g $2\frac{2}{3}$ h 6
- 7 $\frac{7}{20}$ 8 Teacher to check

Exercise 4-04

- 1 a 3 b $4\frac{1}{4}$ c $3\frac{2}{15}$ d $3\frac{1}{8}$
 e $4\frac{1}{12}$ f $4\frac{1}{10}$ g $2\frac{1}{2}$ h $3\frac{5}{6}$
 i $2\frac{1}{8}$ j $4\frac{3}{20}$ k $2\frac{1}{12}$ l $5\frac{10}{21}$
- 2 C
- 3 a $2\frac{1}{2}$ b $1\frac{2}{5}$ c $\frac{3}{7}$ d $3\frac{1}{3}$
 e $1\frac{3}{4}$ f $2\frac{7}{12}$ g $1\frac{7}{10}$ h $\frac{3}{4}$
 i $1\frac{21}{40}$ j $1\frac{19}{20}$ k $\frac{5}{6}$ l $1\frac{7}{10}$
 m $1\frac{1}{2}$ n $2\frac{4}{5}$ o $1\frac{11}{12}$ p $\frac{3}{4}$
- 4 $3\frac{11}{12}$ or 3h 55 mins
- 5 a $2\frac{1}{10}$ b $3\frac{5}{8}$

6 a

4	$1\frac{1}{2}$	
$\frac{1}{2}$		$4\frac{1}{2}$
3		

b

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

Add into empty cells in grid i: top row 2, second row $2\frac{1}{2}$, third row $3\frac{1}{2}$ and 1

Add into empty cells in grid ii: top row 1 and $\frac{5}{6}$, second row $1\frac{1}{3}$ and 0, third row $\frac{1}{2}$

7 $\frac{5}{6}$ hour

Exercise 4-05

- 1 a 7 b 7 c 20 d 8
 e 5 f 22 g 3 h 30
 i 15 j 50 k 75 l 12
- 2 a \$21 b \$8 c \$8 d 20c
 e \$0.30 f \$2.50 g \$135 h 36c
 i \$2.50 j \$12 k \$3200 l \$240
- 3 a 30 mins b 135 mins c 18 h
 d 18 mins e 73 days f 45 secs
 g 200 secs h 18 mins i 32 h
 j 8 months
- 4 a 27 b 18
- 5 a 250 m b 2250 m c 45 mm
 d 10 cm e 750 g f 3500 mL
 g 320 m h 3 mm i 2700 g
 j 1000 L k 4800 mm l 40 000 kg
- 6 D 7 24 pencils
- 8 a 12 b $\frac{2}{3}$
- 9 54
- 10 a Ashleigh 9, Bethany 6, Thomas 2
 b Only 17 horses are given to the children
 c Teacher to check

Mental skills 4A

- 2 a 14 b 9 c 7 d 8
 e 3 f 8 g 10 h 5
 i 33 j 5 k 15 l 18
 m 26 n 6 o 36 p 6
 q 14 r 160 s 14 t 2

Exercise 4-06

- 1 a $\frac{2}{15}$ b $\frac{9}{20}$ c $\frac{35}{48}$ d $\frac{9}{80}$
 e $\frac{5}{16}$ f $\frac{9}{64}$ g $\frac{20}{63}$ h $\frac{21}{160}$
- 2 B
- 3 a $\frac{1}{7}$ b $\frac{3}{10}$ c $\frac{1}{18}$ d 1
 e $\frac{5}{16}$ f $\frac{4}{21}$ g $\frac{3}{20}$ h $\frac{5}{32}$
 i $\frac{3}{25}$ j 15 k $2\frac{2}{7}$ l $\frac{2}{25}$
 m $\frac{11}{24}$ n $\frac{1}{64}$ o $\frac{3}{5}$ p $\frac{9}{16}$

4 $42\frac{5}{8}$ hours

Answers

5 a $\frac{1}{2}$ b $1\frac{5}{21}$ c $6\frac{3}{40}$
 d $6\frac{17}{24}$ e $2\frac{7}{9}$ f $5\frac{4}{9}$

6 8 eggs, 20 cups of flour, 2 cups of sugar

7 a $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ b $\frac{3}{5} + \frac{3}{5}$
 $= 3 \times \frac{1}{4}$ $= 2 \times \frac{3}{5}$
 $= \frac{3}{4}$ $= \frac{6}{5} = 1\frac{1}{5}$

c $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$ d $5 \times \frac{1}{7} = \frac{5}{7}$
 $= 5 \times \frac{3}{8}$
 $= \frac{15}{8} = 1\frac{7}{8}$

e $7 \times \frac{3}{20} = \frac{21}{20}$ f $8 \times \frac{3}{5} = \frac{24}{5}$
 $= 1\frac{1}{20}$ $= 4\frac{4}{5}$

8 a $\frac{7}{8}$ b $\frac{2}{3}$ c $\frac{4}{5}$ d Teacher to check

9 $17\frac{1}{2}$ hours

10 Teacher to check

Exercise 4-07

1 a $\frac{3}{2}$ b 5 c $\frac{8}{5}$ d 10
 e $\frac{1}{7}$ f $\frac{2}{5}$ g $\frac{4}{7}$ h $\frac{1}{9}$

2 a 1 b 0 c $\frac{7}{10}$

3 a 12 b 10 c 21 d 15
 e 42 f 80 g 16 h 8

4 C 5 $38\frac{2}{5}$ hours

6 a $\frac{3}{8}$ b 4 c $1\frac{1}{2}$
 d $1\frac{9}{16}$ e $\frac{21}{25}$ f $\frac{24}{25}$
 g $\frac{11}{18}$ h $\frac{1}{2}$ i 2
 j $1\frac{1}{4}$ k $\frac{1}{5}$ l $\frac{3}{4}$

7 a It equals 1
 b Your answer is the number squared

8 B

9 a $\frac{1}{7}$ b $\frac{5}{49}$ c $\frac{1}{12}$ d $\frac{5}{12}$

10 a $3\frac{3}{4}$ b $2\frac{4}{5}$ c $\frac{7}{8}$ d $1\frac{1}{3}$

11 8

12 a $\frac{2}{21}$ b $\frac{7}{10}$

13 Teacher to check

Exercise 4-08

1 258

2 a 66 b 7

3 a Anthony b $\frac{11}{15}$ c $\frac{4}{15}$

4 $127\frac{1}{2}$ 5 20

6 a $\frac{5}{6}$ b 50 minutes

7 2 8 40 9 D

10 $6\frac{3}{4}$ hours 11 $\frac{21}{40}$ 12 $\frac{1}{9}$

13 a 24 b 4

14 280 15 $\frac{1}{30}$

Exercise 4-09

1 a $\frac{2}{3}$ b $\frac{1}{2}$ c $\frac{1}{3}$
 d $\frac{4}{5}$ e $\frac{6}{7}$ f $\frac{4}{5}$

2 a $\frac{11}{4}$ b $\frac{16}{3}$ c $\frac{30}{7}$
 d $\frac{33}{5}$ e $\frac{31}{3}$ f $\frac{53}{6}$

3 a $3\frac{3}{5}$ b $3\frac{2}{7}$ c $5\frac{5}{8}$
 d $7\frac{2}{11}$ e $12\frac{3}{10}$ f $19\frac{1}{12}$

4 a $\frac{1}{10}$ b $1\frac{1}{14}$ c $1\frac{4}{15}$ d $\frac{1}{14}$
 e $1\frac{1}{2}$ f $1\frac{11}{24}$ g $\frac{4}{15}$ h $\frac{21}{32}$
 i $1\frac{1}{6}$ j 6 k $3\frac{5}{6}$ l $2\frac{5}{8}$
 m $4\frac{1}{8}$ n $15\frac{1}{9}$ o $\frac{1}{10}$ p $3\frac{9}{20}$

5 a 68 b 56 c 25
 d \$510 e \$1680 f \$88

6 0.015g 7 $17\frac{1}{3}$

8 a $12\frac{3}{4}$ h b \$159.38

9 208

Exercise 4-10

1 a $\frac{1}{20}$ b $\frac{7}{50}$ c $\frac{2}{25}$ d $\frac{9}{10}$
 e $\frac{7}{100}$ f $\frac{7}{200}$ g $\frac{31}{300}$ h $\frac{23}{400}$

2 a 0.28 b 0.06 c 0.161 d 0.213
 e 0.02 f 0.2 g 0.7807 h 0.265

- 3 a 9% b 17% c $62\frac{1}{2}\%$ d 90%
 e 22% f 30% g 20% h 55%
 i 75% j 48% k $42\frac{1}{2}\%$ l 82%
 m $6\frac{1}{4}\%$ n $33\frac{1}{3}\%$ o 150% p $22\frac{2}{5}\%$
- 4 a 24% b 50% c 86% d 57.1%
 e 6% f 34.5% g 240% h 0.7%
 i 130% j 0.1% k 31.5% l 2.6%

5 A

6 C

Exercise 4-11

- 1 a 6 b \$204 c 7.5
 d 84 mm e 148 kg f 7.2 g
 g \$165 h \$10.80 i \$96
 j 224 mL k \$1.30 l 405 m
 m \$15.40 n 2.7 ha o 15 mins
- 2 B 3 42 4 312 girls
- 5 124 chickens 6 148 7 D
- 8 a \$150 b \$160

Mental skills 4B

- 2 a 15 b 6 c 3 d 2
 e 9 f 19 g 16 h 80
 i 16 j 7 k 17 l 54
 m 30 n 7 o 9 p 18

Exercise 4-12

- 1 a $\frac{4}{5}$ b No (only got 80%)
- 2 C
- 3 a $\frac{2}{5}$ b 20% c 12, 40%
- 4 a 74% b 70% c $83\frac{1}{3}\%$
 d $62\frac{1}{2}\%$ e 65% f 92%
- 5 a 40% b 25% c 70% d 25%
 e 5% f 3% g 25% h 25%
 i 70% j 45% k 15% l 20%

Power plus

- 1 a $\frac{4}{25}$ b $\frac{1}{27}$ c $\frac{1}{8}$ d $\frac{49}{100}$
 e $\frac{27}{64}$ f $\frac{2}{3}$ g $\frac{5}{7}$ h $\frac{9}{10}$
 i $\frac{3}{4}$ j $\frac{3}{4}$
- 2 a 75 m b 45 m

3 16

4 7 eggs

- 5 a 2 b $1\frac{1}{4}$ c $\frac{3}{5}$
- 6 a $33\frac{1}{3}\%$ b $16\frac{2}{3}\%$ c $14\frac{2}{7}\%$ d $11\frac{1}{9}\%$
 e $9\frac{1}{11}\%$ f $8\frac{1}{3}\%$ g $66\frac{2}{3}\%$ h $44\frac{4}{9}\%$
- 7 a \$262.50 b 2 kg c 12 L

Chapter 4 revision

- 1 a $\frac{7}{4}$ b $\frac{22}{9}$ c $\frac{28}{5}$ d $\frac{22}{7}$
- 2 a $1\frac{3}{4}$ b $8\frac{2}{3}$ c $3\frac{2}{7}$ d $1\frac{12}{24} = 1\frac{1}{2}$
- 3 a $\frac{1}{3} = \frac{5}{15}$ b $\frac{3}{4} = \frac{12}{16}$ c $\frac{4}{5} = \frac{16}{20}$
 d $\frac{5}{12} = \frac{15}{36}$ e $\frac{1}{2} = \frac{25}{50}$ f $\frac{3}{20} = \frac{15}{100}$
- 4 a $\frac{5}{6}$ b $\frac{3}{4}$ c $\frac{3}{4}$
 d $\frac{4}{3}$ e $\frac{1}{2}$ f $\frac{4}{7}$
- 5 a $\frac{3}{10}, \frac{2}{5}, \frac{7}{10}, \frac{4}{5}$ b $\frac{1}{3}, \frac{1}{2}, \frac{7}{12}, \frac{11}{12}$
- 6 a 1 b $\frac{7}{10}$ c $\frac{1}{6}$
 d $\frac{11}{20}$ e $\frac{5}{6}$ f $\frac{1}{8}$
- 7 a $9\frac{2}{3}$ b $4\frac{1}{3}$ c $\frac{5}{6}$
 d $4\frac{3}{10}$ e $1\frac{1}{3}$ f $3\frac{2}{3}$
- 8 a A 9, B 8, R 12, O 7, T 20, H $3\frac{1}{2}$
 b $3\frac{1}{2}, 7, 8, 9, 12, 20$; H O B A R T
- 9 a 16 hours b 1600 m
 c $2\frac{1}{4}$ hours or 135 mins d 105 mm
 e 50 seconds f 125 cm
- 10 a $\frac{2}{15}$ b 6 c $\frac{5}{14}$
 d $\frac{1}{21}$ e $\frac{3}{4}$ f $\frac{1}{2}$
 g $2\frac{1}{4}$ h 8 i $1\frac{1}{5}$
- 11 a 50 b $\frac{1}{6}$ c $1\frac{1}{2}$
 d 2 e 6 f $\frac{32}{35}$
 g 3 h 2 i $1\frac{1}{2}$
- 12 $6\frac{7}{12}$ litres 13 1 spotted
- 14 a $\frac{25}{72}$ b $1\frac{19}{30}$ c $\frac{1}{2}$
 d $\frac{1}{40}$ e $1\frac{5}{12}$ f $10\frac{8}{9}$
- 15 a i $\frac{13}{100}$ ii 0.13
 b i $\frac{2}{25}$ ii 0.08
 c i $\frac{3}{40}$ ii 0.075

Answers

- 16 a 70% b 92% c $22\frac{1}{2}\%$
 d 80% e 36.1% f 4%

17	Fraction (simplest form)	Decimal	Percentage
	$\frac{3}{5}$	0.6	60%
	$\frac{4}{5}$	0.8	80%
	$\frac{41}{50}$	0.82	82%

- 18 a 36 b \$72 c 6
 d \$23.10 e 51 kg f 234 m
- 19 13.2 mins 20 $\frac{4}{13}$ 21 20%
- 22 a $\frac{17}{20}$ b 85%

Chapter 5

SkillCheck

- 1 a 8 b 4 c 6 d 15
 e 2 f 30 g 6 h 9
 i -3 j -3 k -1 l -7
- 2 a 6 b 6 c 10
 d 7 e -4 f 7
- 3 a 11 b 20 c 11 d 9
 e 4 f 7 g 25 h 1
 i 2 j 32
- 4 a 150 b 252 c 140
 d 80 e 130 f 26
- 5 Yes
- 6 a -11 b 17 c 1 d 14
 e 16 f 6 g -13 h 21
- 7 a -3 b -5 c -12 d -2
 e 3 f -11 g 36 h 3
 i -10 j -8 k -9 l -72

Exercise 5-01

- 1 a 25 b 118 c 48
 d 150 e 150 f 80
 g 10 h 65 i 14
 j 119 k 100 l 84

- 2 a 170 b 810 c 1400
 d 640 e 140 f 150
 g 48 h 400 i 80
 j 600 k 90 l 1400
- 3 a 7000 b 10 000 c 270 d 2800
 e 400 f 1200 g 160 h 24 000
 i 24 000 j 3300 k 1800 l 2500
 m 320 n 400 o 21 000 p 210 000
- 4 D 5 D
- 6 a F b T c T d F
 e T f F g F h T
 i F j T k T l F

Exercise 5-02

- 1 a $18 \times 12 = 18 \times (10 + 2)$
 $= 18 \times 10 + 18 \times 2$
 $= 180 + 36$
 $= 216$
- b $16 \times 9 = 16 \times (10 - 1)$
 $= 16 \times 10 - 16 \times 1$
 $= 160 - 16$
 $= 144$
- c $21 \times 11 = 21 \times (10 + 1)$
 $= 21 \times 10 + 21 \times 1$
 $= 210 + 21$
 $= 232$
- d $15 \times 8 = 15 \times (10 - 2)$
 $= 15 \times 10 - 15 \times 2$
 $= 150 - 30$
 $= 120$
- 2 a 135 b 162 c 234 d 279
 e 128 f 104 g 176 h 112
 i 243 j 360 k 304 l 261
- 3 a 187 b 264 c 165 d 319
 e 300 f 168 g 288 h 396
 i 480 j 132 k 108 l 209
- 4 a $6 \times 22 = 6 \times (20 + 2)$
 $= 6 \times 20 + 6 \times 2$
 $= 120 + 12$
 $= 132$

b $12 \times 19 = 12 \times (20 - 1)$
 $= 12 \times 20 - 12 \times 1$
 $= 240 - 12$
 $= 228$

c $40 \times 41 = 40 \times (40 + 1)$
 $= 40 \times 40 + 40 \times 1$
 $= 1600 + 40$
 $= 1640$

d $25 \times 48 = 25 \times (50 - 2)$
 $= 25 \times 50 - 25 \times 2$
 $= 1250 - 50$
 $= 1200$

5 a 160 **b** 204 **c** 1188 **d** 816
e 174 **f** 252 **g** 270 **h** 539

6 D

Exercise 5-03

1 a $u \div u = 1$ **b** $p + p = 2 \times p$
c $a + (-a) = 0$ **d** $p \times 0 = 0$
e $u + 0 = u$ **f** $-a \times (-a) = a^2$

2 a $3w$ **b** b^2 **c** $9acd$ **d** $6a$
e $\frac{e}{5}$ **f** $2f$ **g** $7m$ **h** $6wh$
i $3m$ **j** $4f^2$ **k** d **l** $10abc$
m $\frac{16}{n}$ **n** $2g^2$ **o** $9x$ **p** h
q $8q$ **r** $8mn$ **s** $-12pr^2$ **t** $3a + 2b$
u a^3b^2

3 a $\frac{16}{5a}$ **b** $7 + 3n$ **c** $\frac{e-6}{2}$
d $4s - 8$ **e** $c^2 + d^2$ **f** $\frac{k}{9} + m$
g $\frac{k}{9+m}$ **h** $12 - \frac{r}{2}$ **i** $7 + 7u^2$

4 A

5 a $x \times y$ **b** $2 \times n \div r$ **c** $1 \times x = x$

6 B

7 a $12 \times r \times s$ **b** $-2 \times n \times n$
c $8 \div f$ **d** $r \times r + t \times t$
e $9 \times a \times a \times b$ **f** $5 \times m \times n - 2 \times a$
g $(x + 1) \div 3$ **h** $q \times q \times q$
i $14 - 2 \times d \times d$ **j** $-4 \times c \div 5$
k $x \times x \times y \times y$ **l** $4 \times j - 8 \div k$

8 D

Exercise 5-04

1 a $N + N$ **b** $\frac{N}{2}$ **c** $3N$ **d** $N - 7$
e $\frac{N}{10}$ **f** $N - 1$ **g** $5N$ **h** $N + 21$
i $N - 1$ **j** $\frac{N}{8}$ **k** $N + 3$ **l** N^3
m $\sqrt{N}$

2 No difference except the N would be replaced by the X . The letter used makes no difference to the expression.

3 a $x + y + z$ **b** $b - c$ or $c - b$ **c** uv
d uv^2 **e** $\frac{C}{d}$ **f** $\frac{(m+n)}{2}$
g $\sqrt{10a}$ **h** $b - 3$ **i** $5 - c$
j $c - 5$ **k** $e \div 3$ **l** t^2

m $9xz$ **n** $k - 12$

o $\frac{1}{2}(20 - g)$ or $\frac{1}{2}(g - 20)$

4 a $b + g$ **b** $4n$ **c** $d - r$
d $\$2s$ **e** $\$a \div 3$ **f** $2x + 2y$ or $2(x + y)$
g xy **h** $\$(4m + 2t)$

5 C

Mental skills 5A

2 a 160 **b** 130 **c** 60 **d** 140
e 210 **f** 270 **g** 190 **h** 220
i 300 **j** 170 **k** 80 **l** 290

Exercise 5-05

1 a 15 **b** 51 **c** 122 **d** -18

2 a 40 **b** 58 **c** 17 **d** 0

3 a 8 **b** 44 **c** -16 **d** 2

4 a 16 **b** -19 **c** 101 **d** 36

5 a 73 **b** -53 **c** -8 **d** 10

6 a 5 **b** 11 **c** -3 **d** 19

7 a 22 **b** 26 **c** 4 **d** 34

8 a 16 **b** -3 **c** -24

9 a 10 **b** 22 **c** -50

10 a 14 **b** 35 **c** -28

11 \$3740 **12** 27 m² **13** 10 $\frac{1}{2}$

14 86°F **15** \$149

Exercise 5-06

1 a $x = 3$	b $x = 9$	c $a = 5$
d $m = 20$	e $b = 10$	f $c = 4$
g $k = 6$	h $d = 13$	i $m = 15$
j $x = 5$	k $y = 15$	l $n = 25$

2 B

3 a $x = 5$	b $p = 7$	c $k = 5$
d $x = 4$	e $x = 3$	f $x = 8$
g $m = 28$	h $a = 12$	i $d = 2$
j $n = 1$	k $r = 15$	l $k = 11$
m $p = -2$	n $r = 10$	o $y = 15$
p $x = 2$		

Exercise 5-07

1 a $w = 7$	b $x = 18$	c $m = 16$
d $p = 18$	e $x = 9$	f $k = 9$
g $m = -3$	h $y = 10$	i $d = -9$
2 a $m = 6$	b $n = 4$	c $k = 11$
d $c = 16$	e $x = 9$	f $x = -9$
g $d = 6$	h $h = -5$	i $a = 7$
3 a $p = 11$	b $m = 13$	c $x = 17$
d $y = 60$	e $k = 24$	f $n = 21$
g $d = 18$	h $y = 9$	i $m = 4$
4 a $m = 12$	b $d = 10$	c $x = 40$
d $k = 48$	e $x = 60$	f $a = -8$
g $k = 30$	h $n = -55$	
5 a $m = 19$	b $p = 8$	c $p = 19$
d $n = -8$	e $x = 10$	f $n = 9$
g $x = 7$	h $y = -4\frac{1}{2}$	i $h = 7$
j $x = 3$	k $k = -10$	l $n = 0$

Exercise 5-08

1 a $x = 5$	b $x = 3$	c $x = 2$
d $x = 4$	e $d = 9$	f $x = -8$
g $x = 2$	h $x = 1$	i $x = 4$
j $x = 3$	k $x = -1$	l $x = -2$
2 D		
3 a $x = 7$	b $x = 8$	c $x = 3$
d $x = 11$	e $x = 2$	f $x = 2$
g $x = 3$	h $x = 5$	i $x = -5$
j $x = 9$	k $x = -2$	l $x = -1$

4 D

5 a $x = 8$	b $m = 9$	c $k = 20$
d $a = 16$	e $n = 16$	f $h = -35$
g $m = 6$	h $k = 15$	i $x = -12$
j $a = 9$	k $h = -15$	l $x = 14$

Mental skills 5B

2 a 108	b 243	c 414	d 171
e 306	f 567	g 189	h 135
4 a 187	b 242	c 418	d 440
e 275	f 209	g 594	h 341
6 a 528	b 180	c 348	d 372
e 624	f 216	g 312	h 444

Exercise 5-09

1 a $t = \$11$	b $x = \$0.48$	c $n = 55$	d $y = 10$
2 a B, 299	b A, 1140 mL	c A, 39	d D, 53
3 a $4a + 12 = 240, a = 57$			
b $2l + 17 + 17 = 100, l = 33$			
c $6y - 13 = 95, y = 18$			
d $\frac{11b}{2} = 44, b = 8$			
e $7n - 184 = 2000, n = 312$			
f $200 + \frac{x}{5} = 750, x = \$2750$			
4 5	5 391	6 A	

Power plus

1 a $5a - 3b$	b $-4m - 10n$	c $4j - k + 4$
d $5b^2$	e $3x^2 - 6x$	f $5xy + 2yz + 4xz$
2 a -2	b -24	c 10
e 8	f 4	g 7
		d 9
		h 4
3 a i $P = 2l + 2b$ or $2(l + b)$		
ii $A = lb$		
b i $P = 2 + y + x + y + 2 + x$ or $2(x + 2 + y)$		
$= 4 + 2x + 2y$		
ii $A = x(2 + y)$		
c i $P = p + q + r$		
ii $A = \frac{pq}{2}$		
d i $P = m + 5 + m + 3 + n$		
$= 8 + 2m + n$		
ii $A = mn + \frac{1}{2} \times 3 \times n = mn + \frac{3n}{2}$		

e i $P = x + y + y + x + x$
 $= 3x + 2y$

ii $A = x^2 + \frac{1}{2} \times x \times 2$
 $= x^2 + x$

f i $P = a + b + c + (b - d) + (a - c) + d$
 $= 2a + 2b$

ii $A = ad + c(b - d)$
 or $bc + d(a - c)$
 or $ab - [(a - c) \times (b - d)]$

4 a x	b x^2	c 0	d 1
e $2x$	f 0	g $-x$	h x^2
i 1	j $-x$		

5 a $x = -1$	b -3	c $n = -2$
d $r = 9$	e $d = 0$	f $z = -2\frac{1}{2}$

Chapter 5 revision

1 a 28	b 118	c 170
d 3100	e 390	f 2800

2 a 198	b 136	c 297	d 156

3 $n - (-n) = 2 \times n$

4 a $4x$	b $\frac{u}{5}$	c y
d $10a$	e $24abc$	f $20mw^2$

5 a $4 \times m \times n \times k$	b $6 \times a \div b$
c $9 \times c \times c$	

6 a $3N$	b $N - 5$ or $5 - N$	c $N + 1$
d $\frac{1}{3}N$		

7 a $M + 3$	b $B + 5$	c $2H - k$

8 a 7	b 0	c 6	d 60

9 a 109	b -11	c 169	d 94

10 a $m = 4$	b $k = 12$	c $d = 18$

11 a $w = 35$	b $e = 12$	c $p = 3$	d $x = 12$

12 a $t = 2$	b $n = 3$	c $y = 3$
d $m = 68$	e $x = 9$	f $m = 10$

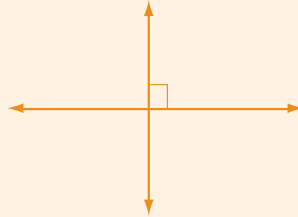
13 a C	b $N = 59$

14 a $5N = 180$	b 456

Chapter 6

SkillCheck

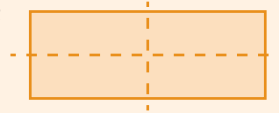
1 a



b



2

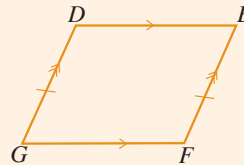


3 a AB and DC , or AD and BC

b No **c** 90° **d** 65°

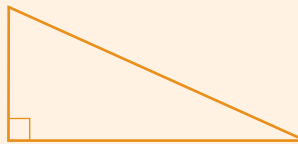
4 Teacher to check

5 a, b, d

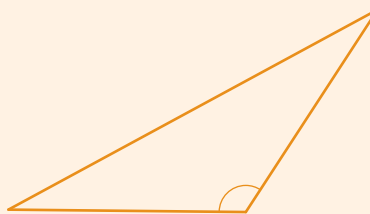


c DE and FG , DG and EF

6 a



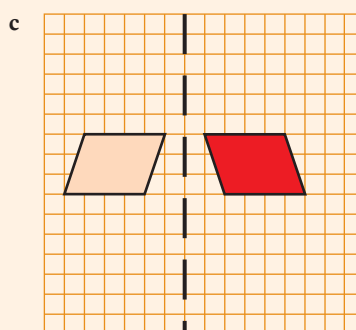
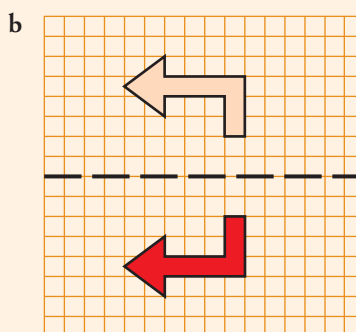
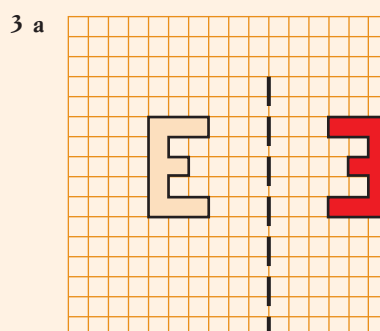
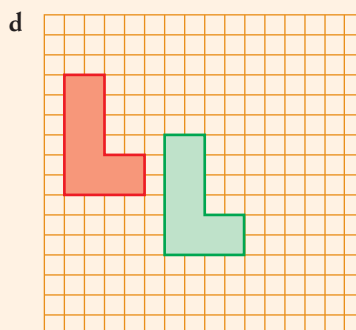
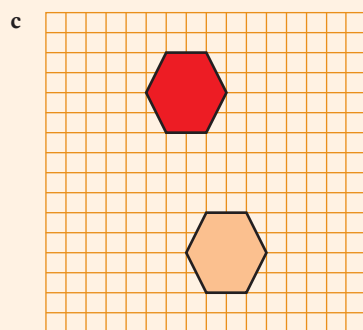
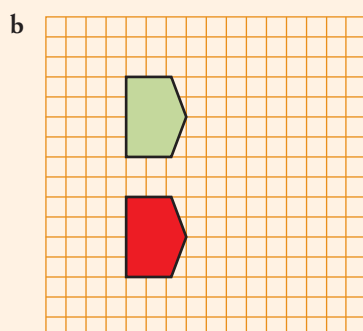
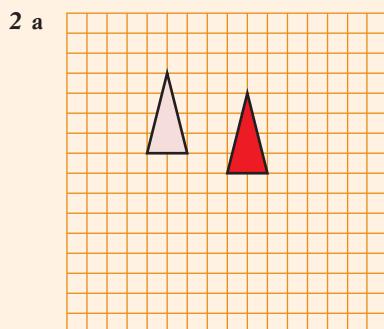
b

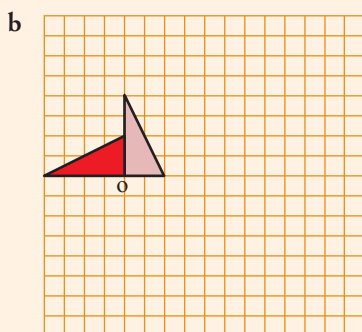
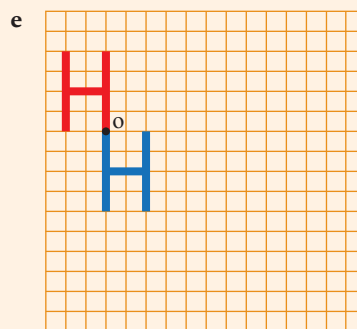
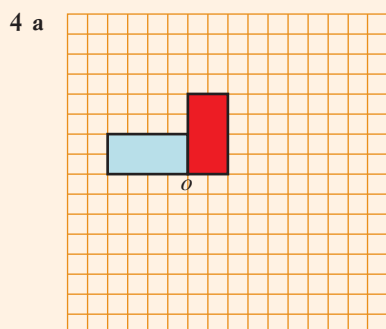
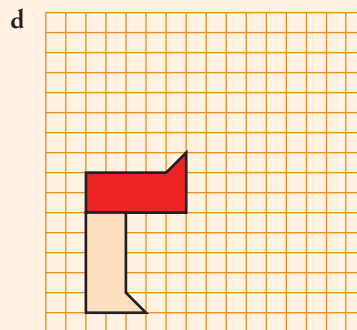
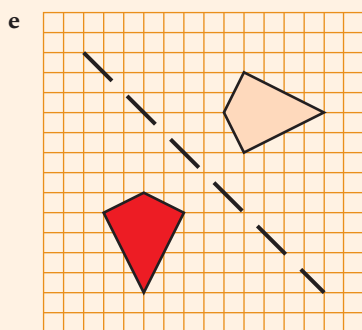
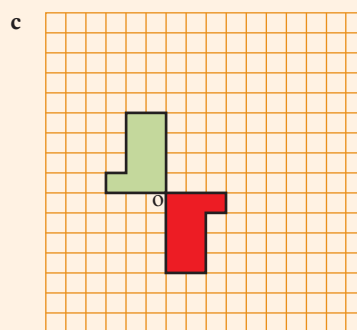
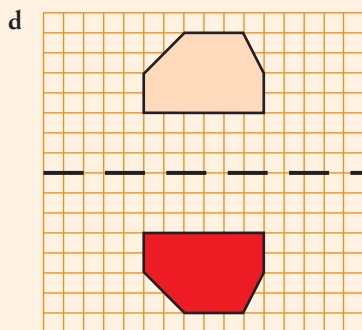


7 Teacher to check

Exercise 6-01

1 a	reflection	b	translation
c	reflection	d	rotation
e	translation	f	rotation
g	translation	h	rotation
i	reflection	j	rotation
k	reflection	l	rotation





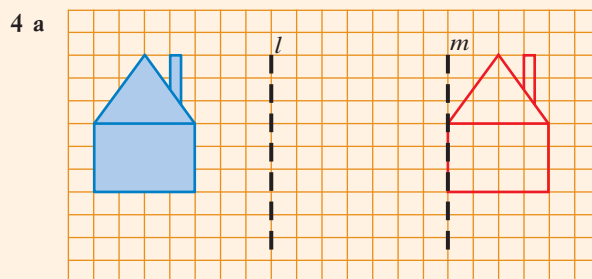
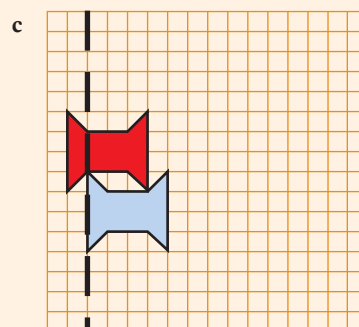
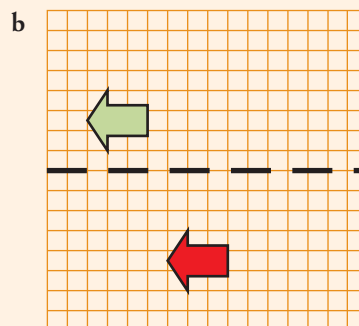
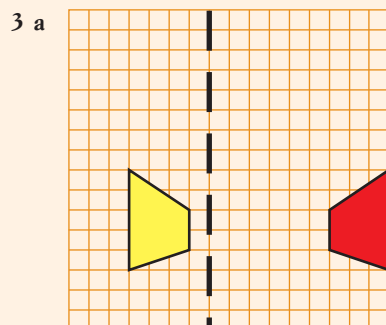
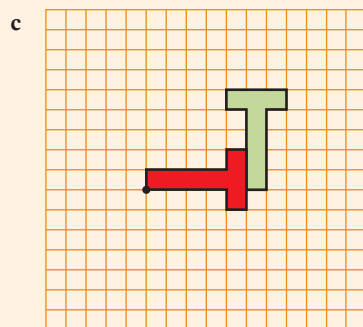
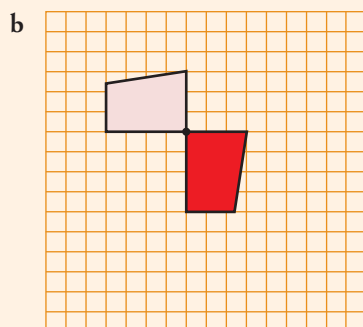
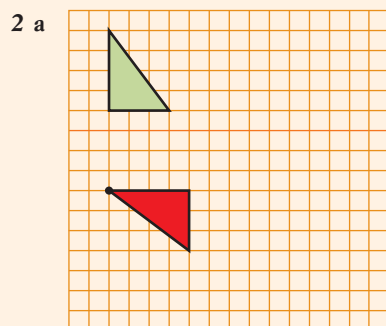
5 The answer is the same.

6 a 270° b 90° c 270° d 180°

7 a 1 left, 5 up b 3 down

Exercise 6-02

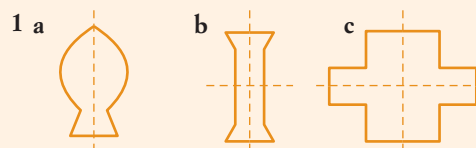
- 1 a rotation and translation, or two reflections, or reflection and translation
 b rotation and translation
 c rotation and reflection

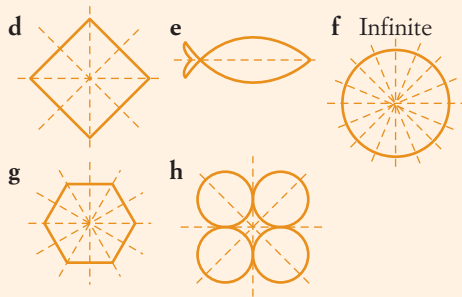


b Translation

- 5 a
- | | | | | |
|---|--------------------|------------------------|-------------|------------|
| i | rotation, rotation | ii | 0° rotation | |
| b | i | reflection, reflection | ii | rotation |
| c | i | rotation, reflection | ii | reflection |

Exercise 6-03





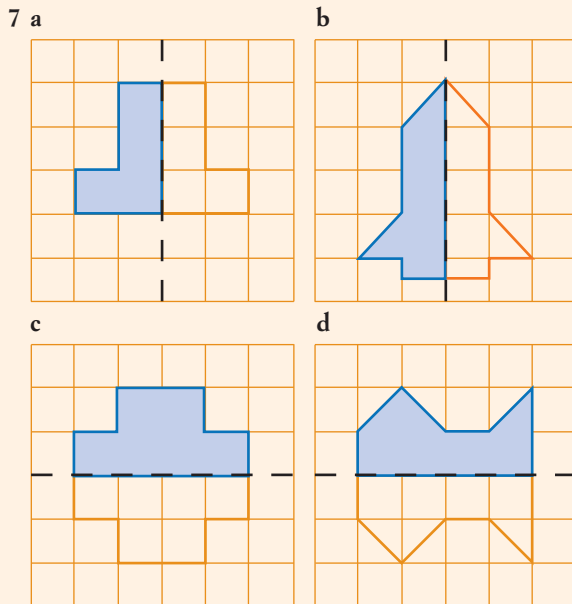
2 a a, e b c, d, h c f d b

3 a 1 b 1 c 0
d 5 e 0 f 3

4 D H I M O V X

5 a 3 b 1 c 1
d 1 e 1 f 3

6 Teacher to check



8 Teacher to check

9 Teacher to check

10 Teacher to check

Exercise 6-04

1 a Yes, order 4 b Yes, order 2 c No
d Yes, order 6 e Yes, order 2 f No

2 H, I, N, O, S, X, Z

3 a and f, order 3

4 Teacher to check

5 a i 4 ii Yes, order 4
b i 6 ii Yes, order 6
c i 1 ii No
d i 6 ii Yes, order 6
e i 5 ii Yes, order 5
f i 1 ii No

6 Teacher to check

7 Circle

Mental skills 6

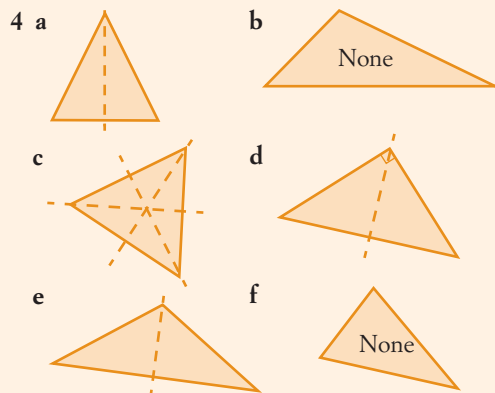
2 a 330 b 160 c 1600 d 260
e 400 f 290 g 400 h 800
i 6400 j 16 000 k 300 l 70

Exercise 6-05

1 a scalene, right b scalene, right
c scalene, acute d isosceles, obtuse
e isosceles, obtuse f isosceles, acute
g equilateral, acute h isosceles, acute
i isosceles, right j equilateral, acute
k isosceles, acute l scalene, obtuse

2 Teacher to check.

3 No. In an equilateral triangle, all angles are equal.



5 Yes, equilateral triangles.

6 No, two obtuse angles can't be joined to form a triangle. The sum of the angles in a triangle is equal to 180° , and two obtuse angles would sum to more than that.

7 C

- 8 a 1 b, c Teacher to check
 9 a 3 b, c Teacher to check
 10 a no b yes, order 3
 11 a $x = 42$, equal angles of an isosceles triangle
 b $y = 7$, equal sides of an isosceles triangle
 c $x = 60$, angles in an equilateral triangle equal 60°
 d $l = 4.8$, equal sides of an isosceles triangle
 e $a = b = 17.2$, equal sides of an equilateral triangle
 f $r = 15$, equal angles of an isosceles triangle

12 Teacher to check.

Exercise 6-06

- 1 a $a = 50$ b $b = 30$ c $c = 30$
 d $d = 60$ e $e = 124$ f $f = 50$
 g $g = 67$ h $h = 22$ i $i = 52$
 j $j = 90$ k $k = 60$ l $l = 30$
 m $m = 65$ n $n = 139$ o $q = 158$
 p $m = 60$ q $y = 120$ r $x = 96$
 s $x = 50$

- 2 a $x = 60$ b $a = 80$
 c $a = 40, b = 50, c = 90$

3 A

Exercise 6-07

- 1 a i d ii a and b
 b i t ii q and p
 c i w ii x and z
 2 a $h = 104$ b $y = 16$
 c $e = 95, d = 85$, d $b = 116, a = 50$
 e $m = 135$ f $y = 114, x = 66$
 g $e = 67$ h $h = 29$
 i $m = 90$ j $x = 70$
 k $p = 130$ l $w = 88$

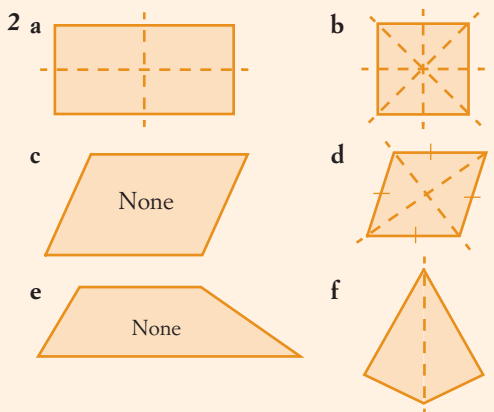
3 B

- 4 a $m = 112$ b $a = 94, b = 86$
 c $a = 74, b = 74, c = 38$

5 the angle sum of a triangle is 180° , angles on a straight line add to 180°

Exercise 6-08

- 1 a rhombus b parallelogram c square
 d trapezium e rectangle f kite



Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Square	4	4
Parallelogram	0	2
Trapezium	0	No rotational symmetry
Rhombus	2	2
Kite	1	No rotational symmetry
Rectangle	2	2

- 4 a square, rectangle
 b parallelogram, rhombus, square, rectangle
 c parallelogram, rhombus, square, rectangle
 d trapezium
 e rhombus, square
 f kite, rhombus, square
 g parallelogram, trapezium
 h parallelogram, rhombus, rectangle

5 Teacher to check

Exercise 6-09

- 1 a $a = 90$ b $b = 60$ c $c = 140$
 d $d = 93$ e $e = 90$ f $f = 75$
 g $g = 45$ h $i = 130$ i $j = 75$
 j $r = 90$ k $m = 62$ l $l = 25$
 m $n = 131$ n $p = 50$ o $x = 280, y = 80$

2 D

3 A

4 a 90°

b Rectangle

Exercise 6-10

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides are equal		✓	✓	✓	✓	
Opposite sides are parallel		✓	✓	✓	✓	
Opposite angles are equal		✓	✓	✓	✓	
All angles are 90°				✓	✓	
Diagonals are equal				✓	✓	
Number of axes of symmetry	0	0	2	2	4	1
Order of rotational symmetry	0	2	2	2	4	0

(Note: An isosceles trapezium has line symmetry, but other trapeziums do not.)



c Pairs of adjacent sides equal, diagonals intersect at right angles. (Other answers possible.)

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal				✓	✓	
Diagonals bisect each other		✓	✓	✓	✓	
Diagonals intersect at right angles			✓		✓	✓
Diagonals bisect the angles of the quadrilateral			✓		✓	

3 B

4 a rectangle, square

b square, rhombus

c rectangle, square

d parallelogram, rhombus, rectangle, square

e parallelogram

f rectangle, square, rhombus, kite

g square

h square

5 cointerior, parallel, 110, 70, angles

6 a Yes, a square is a rectangle

b Yes, a rhombus is a parallelogram.

7

Shape	Properties
Trapezium	- One pair of parallel sides - No axes of symmetry
Kite	- Two pairs of adjacent sides are equal - One pair of opposite angles are equal - Has one axis of symmetry - Diagonals intersect at right angles
Parallelogram	- Opposite sides are equal and parallel - Opposite angles are equal - No axes of symmetry - Diagonals bisect each other
Rhombus	- All four sides are equal - Opposite sides are parallel - Opposite angles are equal - Has two axes of symmetry - Diagonals bisect at right angles - Diagonals bisect the angles of the rhombus
Rectangle	- All four angles measure 90° - Opposite sides are equal and parallel - Has two axes of symmetry - Diagonals are equal - Diagonals bisect each other
Square	- All four sides are equal - All four angles measure 90° - Has four axes of symmetry - Diagonals are equal and bisect each other at right angles - Diagonals bisect the angles of the square

Power plus

- 1 a Yes b No c Yes
d Yes e No f No

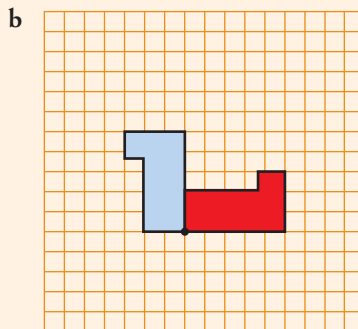
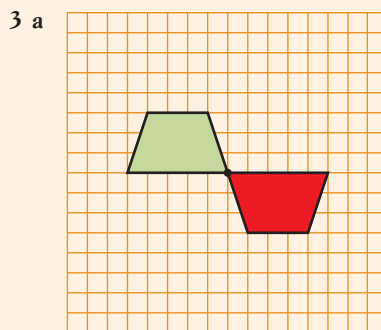
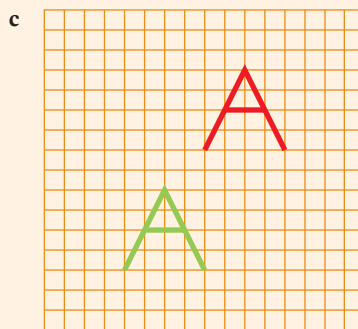
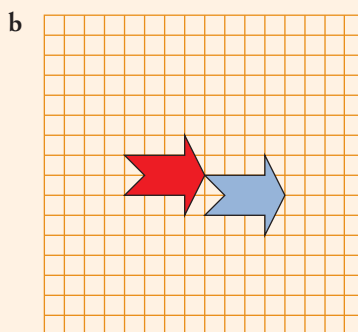
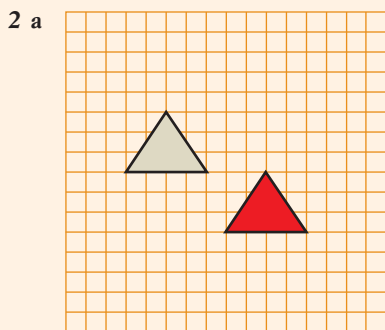
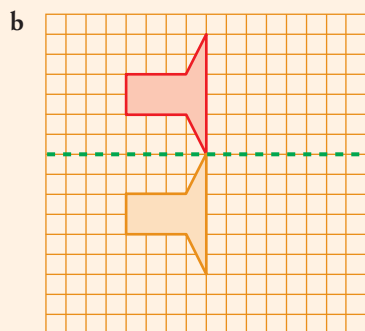
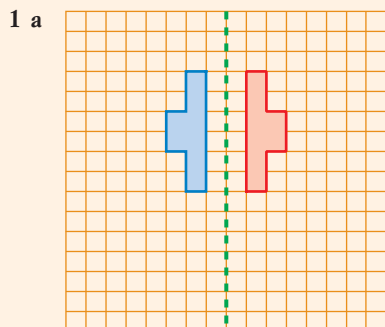
- 2 a all angles equal 90° b all sides equal
c all sides equal

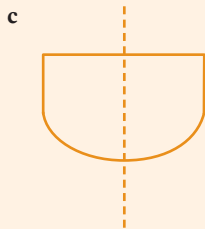
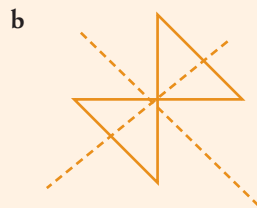
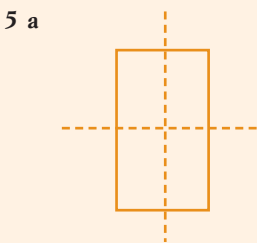
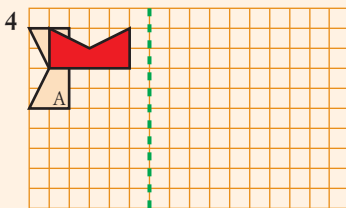
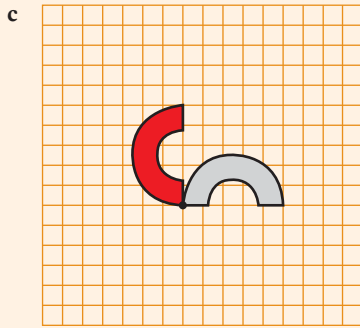
- 3 a parallelogram b kite
c rectangle d rectangle
e parallelogram f parallelogram

4 Teacher to check

5 Teacher to check

Chapter 6 revision





6 a Yes, order 2 b Yes, order 5 c No

- 7 a equilateral, acute-angled
 b scalene, right-angled
 c isosceles, acute-angled
 d scalene, obtuse-angled
 e scalene, acute-angled
 f isosceles, right-angled

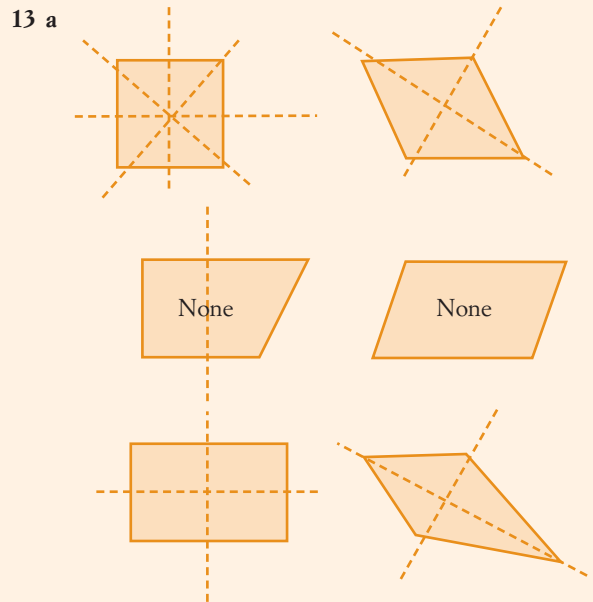
8 a isosceles, acute-angled
 b $\angle F$ and $\angle H$

9 a equilateral, acute angled
 b scalene, acute angled

10 a $x = 59$ b $u = 51$ c $a = 127$
 d $v = 76$ e $p = 100$ f $m = 62, n = 56$
 g $y = 100, z = 130$

11 a $x = 111$ b $m = 129$ c $x = 112$
 d $p = 96$ e $m = 99$ f $y = 70$

12 a square b rhombus c trapezium
 d parallelogram e rectangle f kite



b square 4, rhombus 2, parallelogram 2, rectangle 2

14 A quadrilateral with all sides equal.

15 a parallelogram b trapezium

16 a $y = 70$ b $x = 81$ c $m = 26$ d $y = 61$

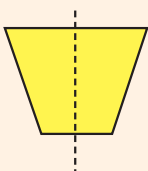
17 Teacher to check.

18 a square, rectangle
 b square, rectangle
 c square, rectangle, parallelogram, rhombus
 d square, rectangle, parallelogram, rhombus

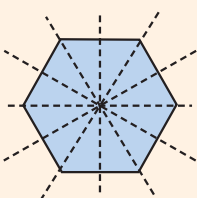
Mixed revision 2

1 a $\frac{5}{6}$ b $\frac{11}{15}$ c $\frac{1}{8}$
 2 a 25% b 60% c 57.5%

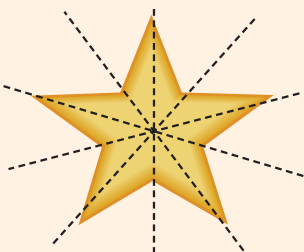
3 a



b



c



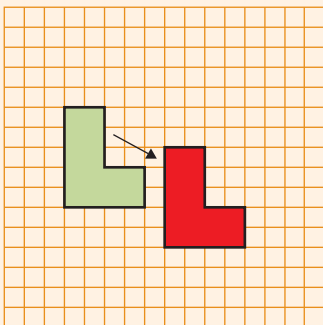
4 a $2w$
e $\frac{m}{2}$

b a^2
f 0

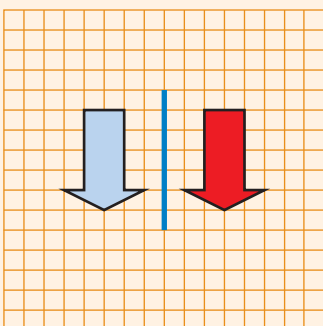
c $4abk$
g $8k$

d $4p$
h $10m$

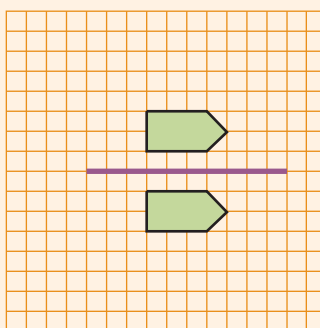
5



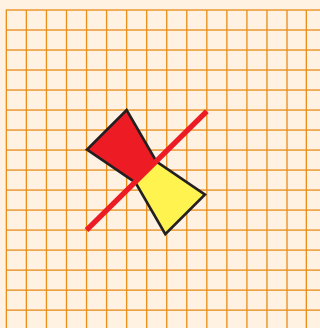
6 a



b



c



7 a $2\frac{2}{5}$

b $3\frac{1}{2}$

c $1\frac{2}{3}$

d $1\frac{5}{6}$

8 a $\frac{4}{5}$

b $\frac{2}{3}$

c $\frac{5}{8}$

d $\frac{7}{10}$

9 a 62

b 1800

c 10

d 620

10 a 198

b 192

c 135

d 264

11 a >

b <

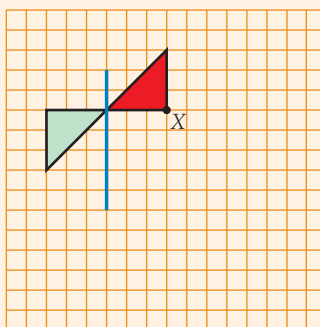
c >

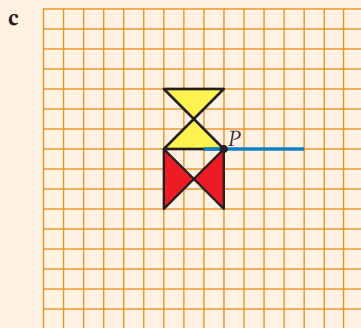
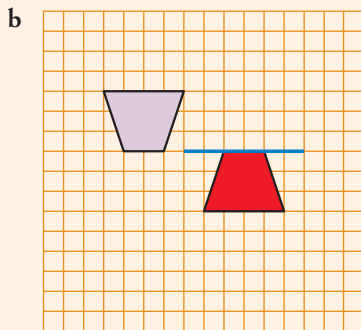
12 a $7\frac{5}{12}$

b $3\frac{1}{10}$

c $\frac{19}{24}$

13 a





- 14 **a** **i** 5 **ii** Yes, order 5
b **i** 4 **ii** Yes, order 4
c **i** 3 **ii** Yes, order 3

15 75 cm

16 **a** $\frac{3}{10}$ **b** $\frac{7}{18}$ **c** $8\frac{1}{3}$

17 **a** -5 **b** 19 **c** 8

18 **a** $x = 23$ **b** $x = -9$ **c** $d = -6$ **d** $m = 5$

- 19 **a** scalene, right-angled
b scalene, acute-angled
c isosceles, acute-angled
d equilateral, acute-angled

20 **a** $m = 60$ **b** $x = 50$ **c** $k = 65$ **d** $m = 60$

21 **a** 40% **b** 5% **c** 12%

22 **a** \$10 **b** 18 mins

23 **a** pq **b** $x - 4$ **c** $d - 7$ **d** $28 - b$

24 **a** $x = 130$ **b** $m = 50$ **c** $n = 35$

25 **a** $1\frac{1}{2}$ **b** $\frac{1}{25}$ **c** $4\frac{1}{6}$

26 **a** $\frac{1}{20}$ **b** 5%

27 **a** $m = 5$ **b** $n = 1.5$ **c** $x = 3$ **d** $x = 8$

28 **a** $4x + 16 = 200, x = 46$
b $4p + 14 = 56, p = 10.5$

29 **a** $p = 105$ **b** $k = 40$ **c** $x = 90$

- 30 **a** parallelogram, rhombus, rectangle, square
b rectangle, square
c square
d kite

Chapter 7

SkillCheck

1

	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
a		1	4	8	2	
b			6	0	1	4
c	9	3	1	0	2	
d		7	0	8		
e			0	3	7	5
f			0	1	9	
g			8	5	9	2
h	7	1	5	3		

- 2 **a** 4 hundreds **b** 4 tenths
c 4 hundredths **d** 4 units
e 4 thousandths **f** 4 tenths

3 **a** $\frac{9}{50}$ **b** $\frac{2}{5}$ **c** $\frac{7}{20}$ **d** $\frac{6}{25}$

4 **a** 20, 21, 30, 32, 34 **b** 206, 213, 216, 230, 234

5 **a** 320 **b** 57 800 **c** 325 000
d 4000 **e** 40 **f** 12
g 1000 **h** 810 **i** 64

6 **a** 45, 44, 42, 40, 39, 38
b 556, 549, 522, 520, 513, 505

7 **a** 30 **b** 800 **c** 9000 **d** 17 000

8 Teacher to check

9 **a** 553 **b** 2825 **c** 717
d 258 **e** 764 **f** 174
g 3150 **h** 740 **i** 319

Exercise 7-01

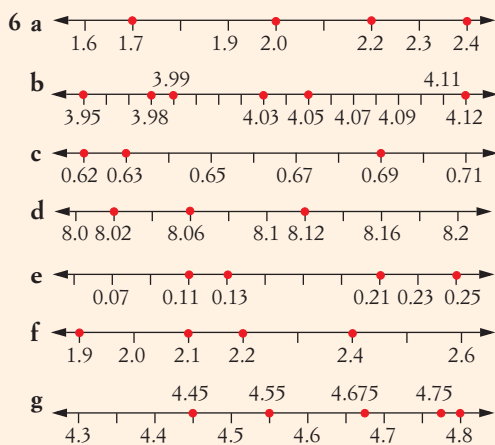
- 1 a 2 b 3 c 4 d 3
e 5 f 1 g 6 h 5
i 7

2 A

- 3 a 9.998, 23.89, 43.89, 56.314, 56.324, 80.879, 400
b 0.0586, 0.099, 0.568, 0.586, 1.002, 5.608
c 0.222, 0.89, 0.891, 1.23, 1.814, 7.007
d 0.005, 0.05, 0.5
e 3.004, 3.044, 3.4, 3.404, 3.441, 3.44, 3.441
f 0.022, 0.2, 0.202, 0.22

- 4 a 4000.99, 1256.3, 570.25, 400.099, 125.63, 0.9899
b 6.539, 5.639, 5.37, 5.369, 5.367, 3.66, 3.659
c 1.61, 1.601, 1.6, 1.599, 1.509
d 6.6, 6, 0.6, 0.06
e 7.07, 0.77, 0.707, 0.7, 0.07, 0.007
f 0.44, 0.404, 0.4004, 0.044

- 5 a $0.2 < 0.25$ b $0.731 > 0.73$
c $0.035 < 0.305$ d $0.007 < 0.070$
e $1.59 > 1.059$ f $0.099 < 0.99$
g $44.44 > 4.444$ h $0.7932 > 0.7239$



7 Teacher to check.

8 Teacher to check.

Exercise 7-02

- 1 a i $\frac{7}{10}$ ii 0.7
b i $\frac{5}{10} = \frac{1}{2}$ ii 0.5
c i $\frac{5}{10} = \frac{1}{2}$ ii 0.5
d i $\frac{4}{10} = \frac{2}{5}$ ii 0.4

- 2 a $\frac{7}{10}$ b $\frac{2}{5}$ c $\frac{39}{100}$ d $\frac{143}{250}$
e $\frac{3}{1000}$ f $\frac{1}{20}$ g $\frac{11}{100}$ h $\frac{41}{125}$
i $\frac{3}{10}$ j $\frac{999}{1000}$ k $\frac{13}{1000}$ l $\frac{1}{2500}$
m $\frac{471}{10000}$ n $\frac{321}{1000}$ o $\frac{1001}{2000}$ p $\frac{91}{100}$
q $\frac{41}{500}$ r $1\frac{3}{5}$ s $27\frac{1}{4}$ t $2\frac{7}{1000}$

3 B

- 4 a 0.9 b 0.15 c 0.79 d 0.6
e 0.4 f 0.23 g 0.6 h 0.411
i 0.704 j 0.87 k 0.07 l 0.14
m 0.8 n 0.235 o 0.247 p 0.17
q 0.368 r 0.9345 s 0.493 t 1.67
u 4.47 v 23.9 w 6.8 x 0.023
y 0.45

Exercise 7-03

- 1 a 12 b 16.933 c 167.317 d 1.3276

- 2 a 41.493 b 5.9 c 6.69 d 9.06

3 C

- 4 a 81.03 m b 18.97 m

- 5 18.6 km

- 6 8037.2 kg

- 7 a 12.45 s, 12.7 s, 13.5 s, 13.81 s, 14.62 s

b 2.17 s

c Yes, $12.7 - 0.3 = 12.4$ s, which is faster than 12.45 s.

- 8 a \$605.55

b \$15.25

9 A

- 10 6.23 m

- 11 a 2.54 b 8.74 c 5.45
d Teacher to check. e Teacher to check.

Exercise 7-04

- 1 a 249 b 8.1 c 37 420
d 341.6 e 725 f 2196
g 604.3 h 3.2 i 0.65
j 4521.3 k 10 640 l 6304
m 5980 n 84761.2 o 0.592
p 3620 q 219 400 r 400.75

2 C

- 3 a 4.63 b 5.07 c 1.203
 d 0.364 e 0.3815 f 5.502
 g 0.01146 h 0.6943 i 0.00428
 j 0.67 k 0.02131 l 0.572
 m 0.081 348 n 0.502 o 0.0491
 p 0.0004 q 0.0173 r 0.012 53

- 4 a 1000 b 0.01
 c 100 d 10
 e Teacher to check. f Teacher to check.

Mental skills 7

- 2 a 88 b 167 c 314
 d 85 e 129 f 121
 g \$11.60 h \$6.75 i \$4.70
 j \$30.05 k \$7.10 l \$32.90

Exercise 7-05

1 a

Multiplication	Estimate	Answer
0.69×18	$0.7 \times 20 = 14$	12.42
6.9×180	$7 \times 200 = 1400$	1242
6.9×1.8	$7 \times 2 = 14$	12.42
690×1.8	$700 \times 2 = 1400$	1242

b

Multiplication	Estimate	Answer
10.4×42	$10 \times 40 = 400$	436.8
1.04×4.2	$1 \times 4 = 4$	4.368
104×4.2	$100 \times 4 = 400$	436.8
0.104×4.2	$0.1 \times 4 = 0.4$	0.4368

c

Multiplication	Estimate	Answer
3.8×92	$4 \times 100 = 400$	349.6
0.38×92	$0.4 \times 100 = 40$	34.96
38×0.92	$40 \times 1 = 40$	34.96
380×0.92	$400 \times 1 = 400$	349.6

- 2 a 21.42 b 2.142 c 2142
 d 2142 e 214.2 f 21.42

- 3 a 20.4 b 2.04 c 0.204
 d 2.04 e 204 f 20.4

- 4 a 244.8 b 244.8 c 2.448
 d 2448 e 2.448 f 24.48

- 5 a 75.6 b 756 c 0.756
 d 0.756 e 0.0756 f 75.6

- 6 a 0.72 b 0.72 c 7.2
 d 72 e 7.2 f 0.72

7 Teacher to check.

Exercise 7-06

- 1 a 2 b 1 c 1 d 3
 e 3 f 3 g 4 h 3
 i 3 j 4 k 6 l 2

- 2 a 12.2 b 7.14 c 18.009
 d 34.2 e 22.5 f 12.6
 g 34.75 h 8.032 i 0.9
 j 4.8 k 0.012 l 12.6
 m 0.32 n 1.95 o 0.48
 p 0.072

- 3 a 14.37 b 46.296 c 525.42

- 4 \$162.45 5 \$719.55 6 2.08 m^2 7 B

Exercise 7-07

- 1 a 2.4 b 6.2 c 4.16
 d 4.1 e 14.65 f 2.1975
 g 0.008 h 2.6775 i 24.45
 j 3.675 k 0.51875 l 0.106
 m 2.4 n 3.39 o 2.875
 p 64.21 q 11.9 r 13.75
 s 0.122 t 259.45

- 2 \$17.14 3 D 4 \$28.16

- 5 a 69.6 b 11.6

- 6 a $24.8 \div 8 = 3.1$
 b $12.3 \div 5 = 2.46$
 c Teacher to check

Exercise 7-08

- 1 a More, we are dividing by a number less than 1
 b Teacher to check
 c 36

- 2 a more
 b Teacher to check
 c 68

- 3 The answer is more than the number.

Answers

4 B

5 a $5088 \div 8$ b $1792 \div 7$ c $3330 \div 9$
 d $172.5 \div 5$ e $1292 \div 4$ f $4950 \div 6$
 g $1680 \div 7$ h $1482.3 \div 9$ i $6.6 \div 3$

6 a 8.7 b 36.6 c 4.9 d 203.5
 e 30 f 12.6 g 208 h 80
 i 5.7 j 2041 k 1.064 l 130.2

7 16 8 18

9 a 14 km b 38 min

10 a $27.2 \div 3.4 = 8$ b $12.3 \div 5 = 2.46$
 c Teacher to check

Exercise 7-09

	Fraction	Meaning as division	Decimal
a	$\frac{3}{5}$	$3 \div 5$	0.6
b	$\frac{1}{2}$	$1 \div 2$	0.5
c	$\frac{1}{4}$	$1 \div 4$	0.25
d	$\frac{4}{5}$	$4 \div 5$	0.8
e	$\frac{2}{5}$	$2 \div 5$	0.4
f	$\frac{3}{4}$	$3 \div 4$	0.75
g	$\frac{1}{5}$	$1 \div 5$	0.2
h	$\frac{1}{8}$	$1 \div 8$	0.125

2 a 0.4 b 0.375 c 0.75 d 1
 e 0.5 f 0.75 g 0.6 h 0.25
 i 0.625 j 0.875 k 0.5 l 1

3 The fractions are equivalent, for example, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

4 a 4.8 b 23.75 c 12.625
 d 6.6 e 57.4 f 19.125
 g 110.875 h 80.25

5 a 0.416 b 0.27 c 0.1 d 0.10275

6 a 0.8583333 ... b 0.0454545 ...
 c 0.777 ... d 0.461538461538 ...

7 a 0.1 b 0.16 c 0.83
 d 0.142857 e 0.6 f 0.285714
 g 0.2 h 0.428571 i 0.571428
 j 0.4 k 0.6 l 0.5
 m 0.857142 n 0.7 o 0.8

8 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8

Exercise 7-10

1 a 0.4 b 0.5 c 0.8
 d 0.7 e 2.6 f 0.3
 g 0.9 h 2.9

2 a 0.48 b 0.74 c 0.07 d 0.29
 e 0.31 f 0.66 g 2.10 h 4.00

3 a 25.38 b 341.61 c 420.90
 d 734.65 e 27.36 f 1314.21

	Calculator display	Rounded (1 decimal place)	Rounded (2 decimal places)
a	4.063333333	4.1	4.06
b	2.053333333	2.1	2.05
c	2.202222222	2.2	2.20
d	5.168181818	5.2	5.17
e	4.2825	4.3	4.28
f	1.054166666	1.1	1.05
g	0.232142857	0.2	0.23
h	3.43125	3.4	3.43
i	1.416666667	1.4	1.42
j	7.9586875	8.0	7.96

5 a 10.3337 b 431.5433 c 1.4450
 d 3217.6541 e 4.6709 f 0.8889

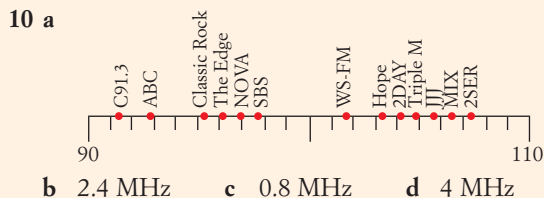
6 a 8 b 40 c 6
 d 26 m e 3

7 Teacher to check.

8 Teacher to check.

Exercise 7-11

1 \$89.06 2 112.9 m 3 \$9
 4 7 5 15.8 km 6 26.6 km
 7 258.3 t 8 5 9 1.4 km



11 a \$15.24 b \$6.84 c \$487.76 d \$1234.05
 12 120 sheets

Power plus

1 $0.\dot{1}4285\dot{7}$, $0.\dot{2}8571\dot{4}$, $0.\dot{4}2857\dot{1}$, $0.\dot{5}7142\dot{8}$, $0.\dot{7}1428\dot{5}$, $0.\dot{8}5714\dot{2}$; the same set of six digits are repeated in order, but with a different digit starting each time.

2 $0.\dot{9} = \frac{9}{9} = 1$

- 3 a 0.0004 b 0.000 008 c 1.331
d 0.2 e 0.6 f 0.3

4	Queen's chamber	8.07
	armoury	8.262
	throne room	8.79
	banquet hall	9.24
	kitchen	9.96
	dungeon	8.16

5 24.15

- 6 a 3.7, 24.6
b 2.3, 5.7, 6
c 5.1, 0.72, 4.5, 18

Chapter 7 revision

- 1 a 3.998, 34.89, 34.98, 36.86, 38.141, 340
b 1.293, 1.239, 1.1015, 1.015, 1.006

- 2 a $\frac{1}{2}$ b $\frac{89}{100}$ c $\frac{9}{100}$ d $\frac{111}{250}$

- 3 a 0.4 b 0.13 c 0.07 d 0.011

- 4 a 65.355 b 104.49 c 11.315
d 1037.9 e 396.51 f 2.379

- 5 a 75.4 b 754 c 7540
d 1.39 e 0.139 f 0.0139

- 6 a 319.2 b 31.92 c 319.2
d 3.192 e 319.2 f 0.3192

- 7 a 16.5 b 0.6 c 288.92
d 7.32 e 4.6 f 29.1
g 1.3325 h 0.001 i 5.137

- 8 a 381.2 b 12.2 c 0.6875 d 65.2

- 9 a 25 b 31.8 c 230 d 25.4

- 10 a 0.8 b 0.375 c $0.\dot{5}$ d $0.\dot{6}$

- 11 a 406.3
d 3.90

- b 125.72 c 2345.9
e 78 654.06 f 678.431

- 12 \$340.20

- 13 \$0.72

- 14 \$8.25

Chapter 8

SkillCheck

- 1 a 1000 b 60 c 1000
d 10 e 1000 f 24
g 1000 h 1000 i 1000
j 60 k 100 l 1000

- 2 a 1032 b 94.5 c 400
d 2700 e 90 f 63
g 20 h 43 i 65
j 37.5 k 9.5 l 8.2

- 3 a metres b centimetres
c millimetres d metres
e metres f metres
g metres h centimetres
i centimetres j millimetres
k seconds

- 4 a 32 cm b 36.9 mm c 93 m

Exercise 8-01

- 1 a 60 b 1000 c 1000
d 10 e 24 f 1000
g 1000 h 60 i 1000
j 1000 k 1 000 000 l 3600

- 2 a 640 cm b 4000 kg c 72 h
d 2 400 000 L e 900 mm f 12 700 g
g 480 mins h 5600 mL i 420 mm
j 72 000 mg k 380 000 cm l 16 200 secs

- 3 a 200 b 150 c 50
d 350 e 25 f 200
g 1 580 000 h 40 000

- 4 a 2 b 3 c 8.5
d 9.8 e 0.75 f 0.325
g 6000 h 18 000

Answers

5 a kilo b centi c milli

6 Teacher to check

7 a 4500 b 180 c 0.75
 d 24 e 12 000 f 520
 g 7 h 48 i 8650
 j 3 k 0.065 l 960

8 D

9 a 120, 330, 800, 35, 7400
 b 0.12, 0.33, 0.8, 0.035, 7.4
 c Teacher to check

10 a 2200 b 0.8 c 2.35
 d 0.36 e 330 f 4.9
 g 0.27 h 48 i 10 800
 j 16900 k 365 l 7.4

Exercise 8-02

1 a 24 cm b 44 mm c 90 m d 138 cm

2 a 46 mm b 44 cm c 118 m
 d 39 cm e 246 cm f 2000 mm
 g 31.8 km h 26 cm i 208 mm
 j 60 mm k 48 m

3 a 90 mm b 178 mm c 130 mm d 116 mm

4 a 44 m b 30 cm c 48 mm
 d 40 m e 36 km f 54 mm

5 a 4 cm b 8 cm c 12.75 cm d 18 cm

6 Any two numbers that add up to 15

7 C

Exercise 8-03

1 a hectares b m^2 c cm^2
 d m^2 e cm^2 f mm^2

2 B

3 a 1500 mm^2 b 1.5 m^2
 c 6.9 cm^2 d $65\,000 \text{ cm}^2$
 e $500\,000 \text{ mm}^2$ f $120\,000 \text{ m}^2$
 g 1.22 m^2 h 12.5 cm^2
 i 3200 m^2 j 790 mm^2
 k $750\,000 \text{ mm}^2$ l 245 ha
 m 86.5 m^2 n 0.0513 m^2
 o 4700 cm^2 p 0.9 ha

4 a $102\,400 \text{ mm}^2$ b 0.1024 m^2

5 a 6.5 m^2 , $114\,000 \text{ cm}^2$, $25\,050\,000 \text{ mm}^2$
 b 54 cm^2 , 990 mm^2 , $0.000\,032 \text{ m}^2$

6 a 87.5 ha b $8\,750\,000\,000 \text{ cm}^2$

7 B

8 a $8\,810\,000 \text{ m}^2$
 b 6.24 (to 2 decimal places)

9 836 mm^2

10 a 1 000 000
 b 100
 c Teacher to check

Exercise 8-04

1 a 55.8 m^2 b 58.6 m^2 c 96 cm^2
 d 324 m^2 e 1.44 cm^2 f 4 cm^2
 g 96 mm^2 h 225 m^2 i 154 mm^2
 j 1890 cm^2 k 448 m^2

2 C

3 a i 25 m^2
 ii $250\,000 \text{ cm}^2$
 iii $25\,000\,000 \text{ mm}^2$
 b i 0.513 m^2
 ii 5130 cm^2
 iii $513\,000 \text{ mm}^2$

4 a i 625 mm^2 ii 6.25 cm^2
 b i 1100 mm^2 ii 11 cm^2
 c i 1050 mm^2 ii 10.5 cm^2
 d i 2250 mm^2 ii 22.5 cm^2

5 7 cm

6 a $15\,000\,000 \text{ m}^2$ b $40\,000 \text{ m}^2$ c 31.36 m^2
 d 3.51 m^2 e 6.29 m^2 f 0.25 m^2

7 \$7858.50

8 4.5 m

9 a 10.08 m^2 b 0.16 m^2 c 63

10 Teacher to check

11 a i 20 m^2 ii 20 m^2 iii 12.8 m^2
 b 80 tiles
 c 3.6 L

Mental skills 8A

2 a 108	b 154	c 166	d 210
e 52	f 82	g 196	h 324
i 91	j 137	k 46	l 69
m 253	n 38	o 24	p 85

Exercise 8-05

1 a 15 cm^2	b 16 cm^2	c 36 mm^2
d 18 cm^2	e 10 cm^2	f 48.59 m^2
g 8.82 m^2	h 9.01 mm^2	i 7.68 cm^2
j 45.72 cm^2	k 21.76 m^2	l 1750 mm^2

2 D

3 a 15.6 m^2	b 300 mm^2	c 7.8 m^2

4 20 m	5 3 cm	6 \$5.10

7 Any 2 values whose product is 72.

8 8 cm

Exercise 8-06

1 a 48 cm^2	b 136 m^2	c 54.4 cm^2
d 3 cm^2	e 340 mm^2	f 84 cm^2
g 16.2 m^2	h 43.46 cm^2	i 16.192 m^2

2 A

3 a $168\,000 \text{ cm}^2$	b 7 cm^2	c 4.5 cm^2
d $70\,000 \text{ cm}^2$	e 0.5456 cm^2	f 4.9 cm^2

4 A

5 Teacher to check

Exercise 8-07

1 a 152 cm^2	b 372 cm^2	c 60 cm^2	d 11 cm^2
2 a $11\,250 \text{ cm}^2$	b 500 cm^2	c 1300 cm^2	

3 D

4 a 140 cm^2	b 67.5 km^2	c 48 mm^2
d 220 m^2	e 150 cm^2	f 1020 mm^2

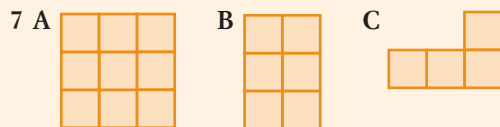
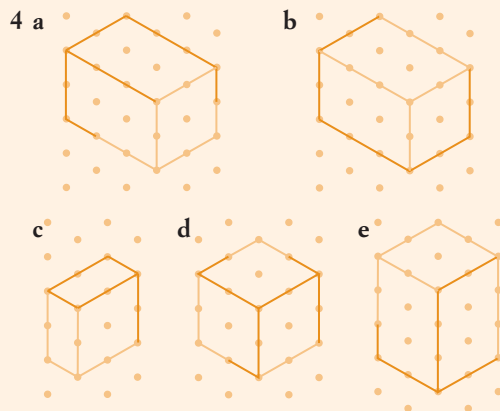
5 a 30 cm^2	b 100 cm^2	c 204 mm^2
d 135 cm^2	e 55 cm^2	f 22 m^2
g 128 mm^2	h 372 cm^2	

6 42.8 m^2	7 60 m^2	8 282 tiles

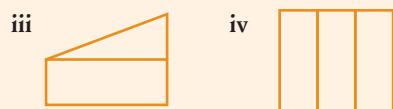
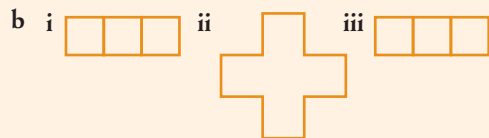
Exercise 8-08

1 Prisms are **a, c, e, g, h, i, l**

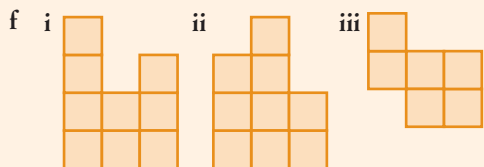
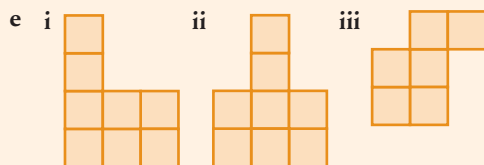
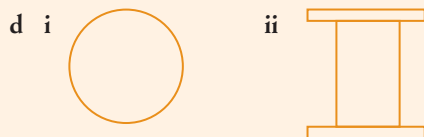
- 2 a** **i** octagon **ii** octagonal prism
b **i** kite **ii** kite prism
c **i** pentagon **ii** pentagonal prism
d **i** T-shape or octagon **ii** T-prism or octagonal prism
e **i** trapezium **ii** trapezoidal prism
f **i** L-shape or hexagon **ii** L-prism or hexagonal prism



8 C



Answers



10 D

Mental skills 8B

- 2 a 56 b 108 c 64
d 420 e 344 f 128
g 224 h 264 i 46
j 68 k 140 l 108
m 78 n 39 o 32
p 19

Exercise 8-09

- 1 a cm^3 b cm^3 c m^3 d m^3
e mm^3 f m^3 (Other answers possible.)

2 D

- 3 a 3000 mm^3 b $5\,000\,000 \text{ cm}^3$
c 2600 mm^3 d 4 cm^3
e $7\,200\,000 \text{ cm}^3$ f 66 cm^3
g $1\,000\,000\,000 \text{ mm}^3$ h 0.0023 m^3
i 126 m^3 j 3.45 cm^3
k $25\,000\,000\,000 \text{ mm}^3$ l $0.000\,078 \text{ m}^3$
m 0.063 m^3 n 0.0014 cm^3
o 0.005 m^3

4 0.306 m^3

- 5 a E b G c F d C
e A f B g D

6 $2\,520\,000 \text{ mm}^2$ 7 $38\,000\,000 \text{ cm}^3$

Exercise 8-10

- 1 a 225 cm^3 b 1071 cm^3 c 576 cm^3
d 2295 cm^3 e 27 cm^3 f 39.6 m^3
g $40\,500 \text{ cm}^3$ h 144.72 m^3

	Length	Breadth	Height	Volume
a	50 cm	50 cm	50 cm	$125\,000 \text{ cm}^3$
b	5 cm	10 cm	18 cm	900 cm^3
c	4 m	2.5 m	1.4 m	14 m^3
d	24 mm	16 mm	11 mm	4224 mm^3

3 $6\frac{2}{3} \text{ mm}$

4 a 2268 cm^3 b $2\,268\,000 \text{ mm}^3$

- 5 a 62 cm^3 b 6400 cm^3 c 498 m^3
d 126 mm^3 e $23\,552 \text{ m}^3$ f $23\,500 \text{ cm}^3$
g $28\,700 \text{ mm}^3$

6 a $733\,720 \text{ cm}^3$ b $0.73\,372 \text{ m}^3$

7 6 m

- 8 a 200 cm^3 b 5.625 cm^3 c 5000 cm^3
d $3\,000\,000 \text{ cm}^3$ e $600\,000 \text{ cm}^3$

9 Teacher to check

10 B 11 5 m

Exercise 8-11

- 1 a mL b ML c L
d mL e L f kL
(Other answers possible.)

- 2 a 7 b 2700 c 3500
d 10 e 5.2 f $1\,400\,000$
g 0.42 h 8900 i 6200
j 1.75 k 5000 l 9
m $2\,500\,000$ n 0.52 o 0.023

3 A

- 4 a D b G c J d H
e I f A g F h E
i C j B

5 8

6 a 54 L b 27

7 gigalitre

- 8 a 0.2 mL b 12 mL
c 2160 mL or 2.16 L d $17\,280 \text{ mL}$ or 17.28 L

9 $6\frac{1}{4}$ h

10 a 2700 cm^3 b 2700 mL

11 a 48 m^3 b $48\,000 \text{ L}$

12 a $72\,000 \text{ cm}^3$ b 72 L

Power plus

- 1 a $P = 47 \text{ cm}$ $A = 82.5 \text{ cm}$
 b $P = 46 \text{ cm}$ $A = 52.5 \text{ m}$
 c $P = 78 \text{ cm}$ $A = 203 \text{ cm}$
 d $P = 277 \text{ mm}$ $A = 2250 \text{ mm}$

- 2 a and b Teacher to check.
 c $15 \text{ cm} \times 15 \text{ cm}$

3 $l = w = 4 \text{ cm}$

4 a 66 m b 11 L c $3, \$137.85$ d 15 m

5 a 45 m^2 b 13.5 m^2 c $\$10.80$

Chapter 8 revision

- 1 a 4 h b 42 g c 0.125 kL
 d 3.2 km e 8300 mL f 9040 mm
 g 0.0035 t h 900 mins i 54000 s
 j 7.15 m k $4\,200\,000 \text{ mL}$ l 17 kg

2 a 52 m b 36 mm c 64 cm d 88 mm

3 a $91\,000 \text{ cm}^2$ b 250 mm^2 c 17.5 m^2
 d 0.24 m^2 e 2.4 cm^2 f $8\,600\,000 \text{ mm}^2$

4 a 16 m^2 b 24 cm^2 c 162 m^2

5 a 360 mm^2 b 112 mm^2 c 5.2 cm^2
 d 187 km^2

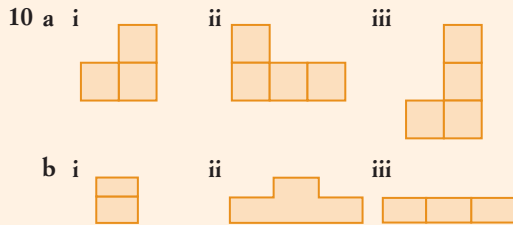
6 a 120 m^2 b 828 cm^2 c 68 mm^2

7 a 9 cm^2 b 34 m^2 c 84 m^2
 d 12.5 mm^2 e 196 m^2 f 128.2 mm^2

8 6 L

- 9 a  Prism
 b  Not a prism
 c  Not a prism

- d  Prism
 e  Not a prism
 f  Prism



11 a $20\,000$ b $500\,000$ c 7.5 d $0.00\,023$

12 a 80 m^3 b 320 cm^3 c 672 mm^3
 d 360 m^3 e 3000 cm^3 f 870 m^3

13 a $80\,000 \text{ L}$ b 0.32 L c $0.000\,672 \text{ L}$
 d $360\,000 \text{ L}$ e 3 L f $870\,000 \text{ L}$

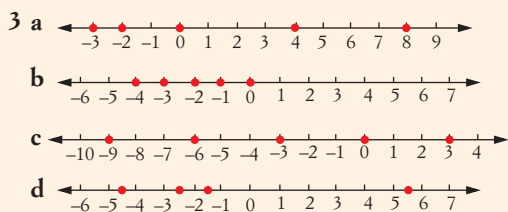
14 $2 \times 10^{15} = 2\,000\,000\,000\,000\,000 \text{ L}$

Chapter 9

SkillCheck

1 a -3 b 0 c 3 d 5

- 2 a A -10 B -2 C 1 D 3
 b A -10 B -4 C 8 D 14 E 17
 c A -22 B -15 C 2 D 11 E 20 F 26
 d A -56 B -24 C -8 D 6 E 30 F 56
 e A -60 B -50 C -35 D -15 E -6
 F 19 G 41



Exercise 9-01

- 1 a E3 b C3 c C4
 d F4 and G4 e F3 and G3 f C3

- 2 a swimming pool b primary school
 c park d pond
 e car park f tennis courts

3 a A13 b G6 c N25 d U16

4 19

Answers

5 Back rows

6 a No b No c No d Yes

7 Teacher to check

8 a Wavy vertical line or column

b Horizontal line or row

9 a St John's oval b The Square

c Women's Tennis Centre d Main Quad

e Biochemistry and Microbiology

10 a N14, N15, O14 or O15 b M27

c H2 or H3 d H19 or H20

Exercise 9-02

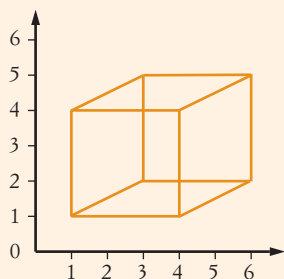
1 $A(1, 4)$ $B(2, 2)$ $C(3, 1)$ $D(3, 4)$ $E(4, 3)$

$F(5, 2)$ $G(6, 5)$

2 $M(0, 2)$ $P(1, 1)$ $A(2, 0)$ $B(2, 3)$ $N(3, 1)$

$J(3, 3)$ $E(4, 0)$ $U(5, 2)$

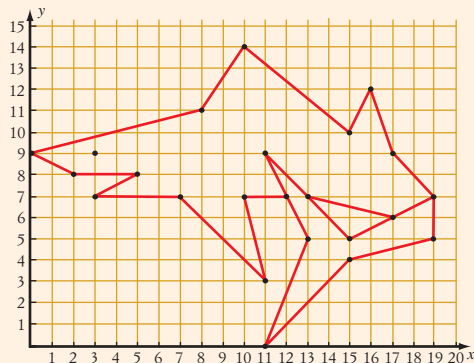
3 a cube



4 a NUMBER b PLANE c WATER

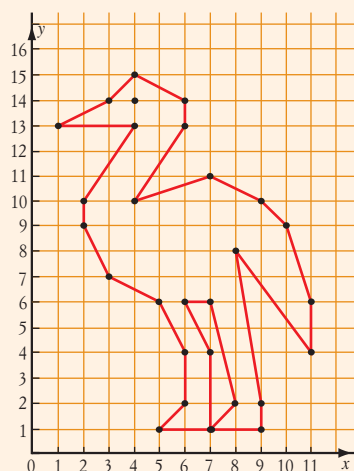
d METRE e NURSE f UMPIRE

5 b



c a shark

6 b



c an ostrich

Exercise 9-03

1 $A(-4, 3)$

$D(2, 3)$

$G(2, -3)$

$J(-2, -1)$

$M(0, -2)$

$P(-2, 0)$

$S(-2, -2)$

$V(4, 1)$

$Y(1, -4)$

$B(-2, 2)$

$E(2, 1)$

$H(0, -3)$

$K(3, 0)$

$N(0, 3)$

$Q(1, 0)$

$T(-3, 1)$

$W(4, -3)$

$Z(-1, -3)$

$C(0, 2)$

$F(1, -1)$

$I(-4, -3)$

$L(-4, 0)$

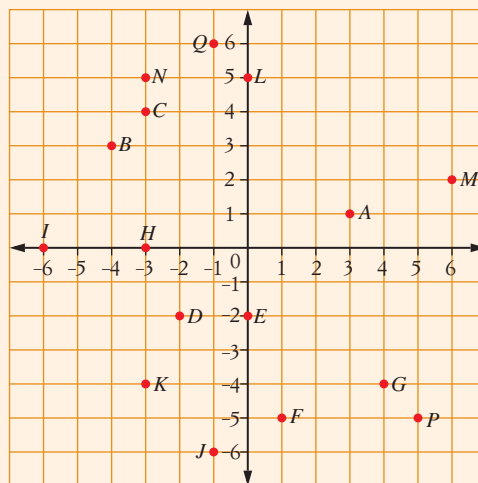
$O(0, 0)$

$R(3, -2)$

$U(-1, 2)$

$X(-4, -1)$

2



3 a A, M

d F, G, P

b B, C, N, Q

e E, I, H, L

c D, K, J

4 a 4th

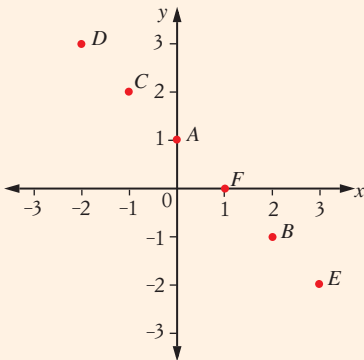
b 3rd

c 2nd

5

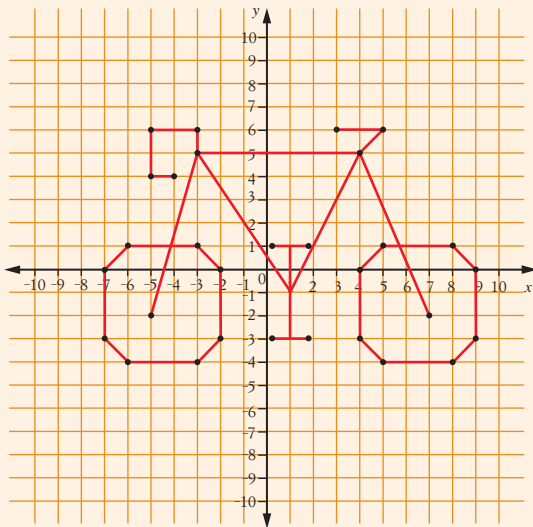
+	+
-	+
-	-
+	-

6 a, b



- c They lie in a straight line.
d Teacher to check.

7 a, b

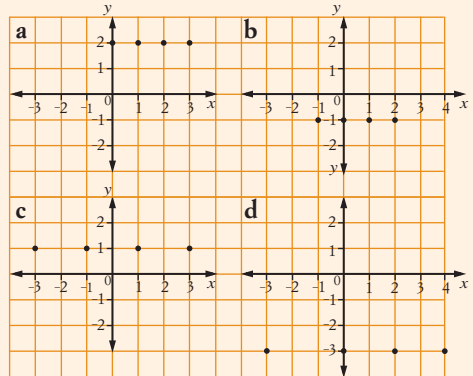


- c a bicycle

Exercise 9-04

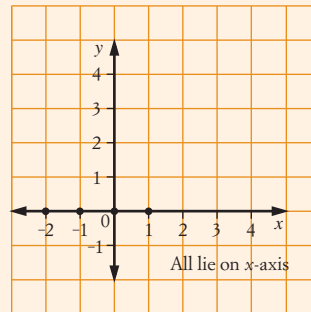
- 1 a (0, 2), (2, 3), (4, 4), (7, 9)
b (-2, -1), (-1, 3), (2, 4), (5, -2)
c (0, 2), (1, 5), (2, 8)
d (-4, -1), (-2, -3), (1, 0), (5, 2)

2

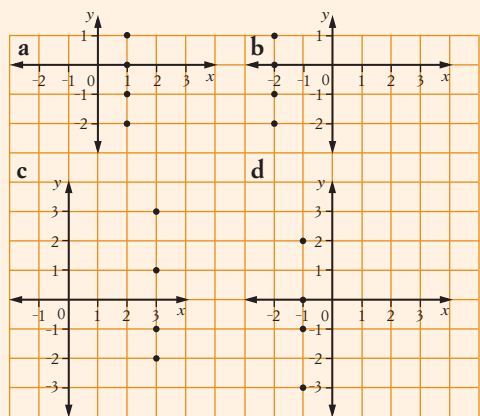


All points lie on a horizontal line.

3

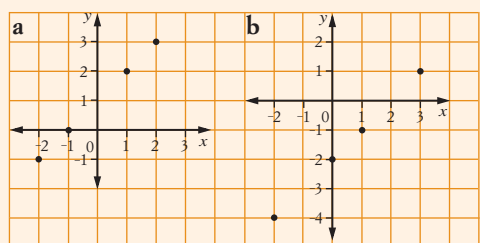


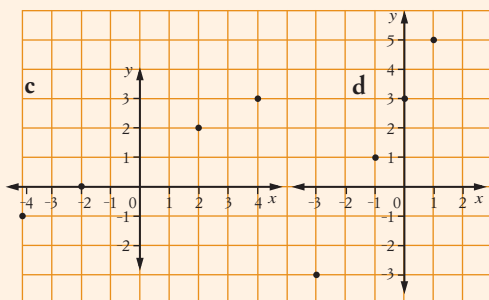
4



All points lie on a vertical line.

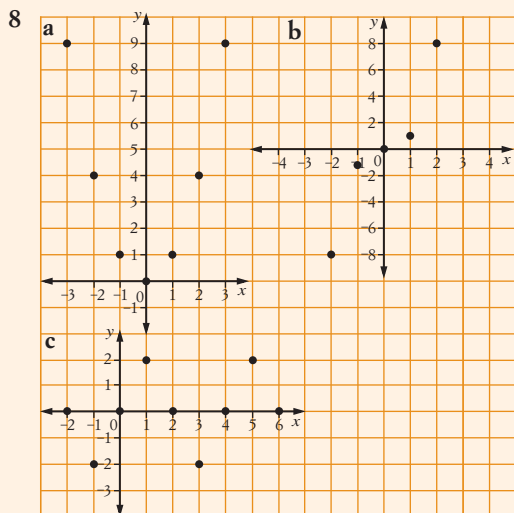
5





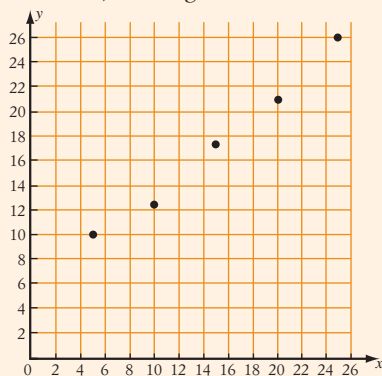
- 6 a They are collinear
b They increase by the same amount

7 B



The dots in parts **a** and **b** lie on a curve, not in a straight line. The dots form a zig-zag line in part **c**.

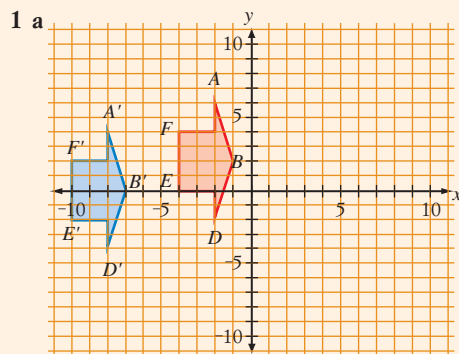
- 9 There is a rough pattern; as the age of the tree increases, the height increases at a steady rate



Mental skills 9

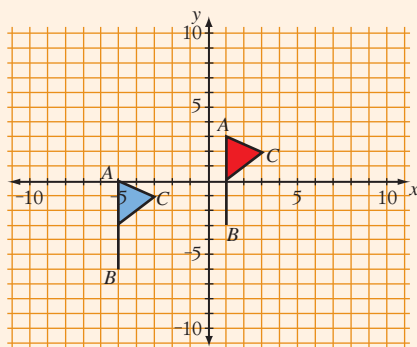
2 a	720	b	486	c	825	d	128
e	560	f	216	g	189	h	432
i	560	j	616	k	135	l	675

Exercise 9-05



- b $A(-2, 6) \rightarrow A'(-8, 4)$
 $B(-1, 2) \rightarrow B'(-7, 0)$
 $C(-2, -2) \rightarrow C'(-8, -4)$
 $D(-2, 0) \rightarrow D'(-8, -2)$
 $E(-4, 0) \rightarrow E'(-10, -2)$
 $F(-4, 4) \rightarrow F'(-10, 2)$
 $G(-2, 4) \rightarrow G'(-8, 2)$
 The x -coordinate of each vertex decreases by 6 while the y -coordinate decreases by 2.

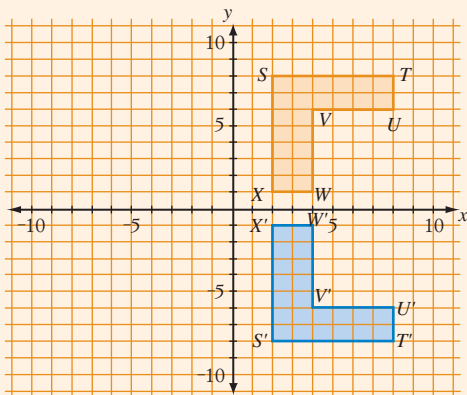
2



$A(1, 3)$, $B(1, -3)$, $C(3, 2)$

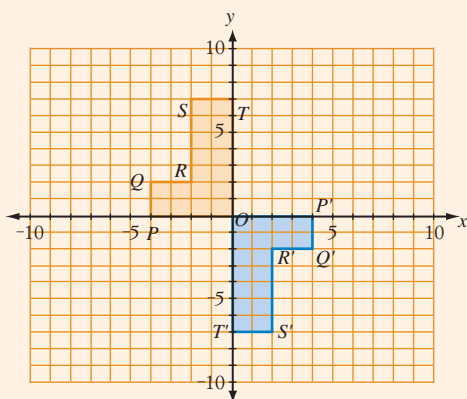
- 3 Translated 5 units left, 4 units down

4 a



- b $S(2, 8) \rightarrow S'(2, -8)$
 $T(8, 8) \rightarrow T'(8, -8)$
 $U(8, 6) \rightarrow U'(8, -6)$
 $V(4, 6) \rightarrow V'(4, -6)$
 $W(4, 1) \rightarrow W'(4, -1)$
 $X(2, 1) \rightarrow X'(2, -1)$
 The x -coordinate of each vertex does not change while the y -coordinate changes sign.

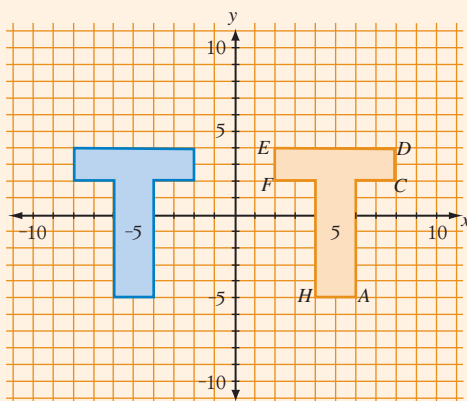
5 a



- b $O(0, 0) \rightarrow O'(0, 0)$
 $P(-4, 0) \rightarrow P'(4, 0)$
 $Q(-4, 2) \rightarrow Q'(4, -2)$
 $R(-2, 2) \rightarrow R'(2, -2)$
 $S(-2, 7) \rightarrow S'(2, -7)$
 $T(0, 7) \rightarrow T'(0, -7)$
 The x - and y -coordinate of each vertex changes sign.
- 6 a Reflection across the x -axis
 b $D(-5, -8) \rightarrow D'(-5, 8)$
 The x -coordinate does not change but the y -coordinate changes sign.

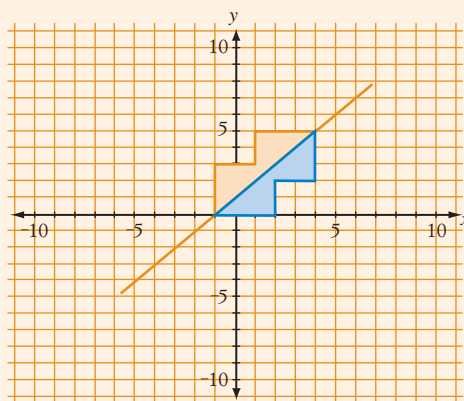
- 7 a Rotation 90° clockwise about $K(-2, 2)$
 b $(2, -3)$

8 a



- b $A(6, -5) \rightarrow A'(-6, -5)$
 The x -coordinate changes sign but the y -coordinate does not change.
- 9 a Reflection across the x -axis b $(-3, -7)$
- 10 a Rotation 90° clockwise about $S(-3, 3)$
 b $R(3, -4) \rightarrow R'(4, 3)$
 The x -coordinate of R becomes the y -coordinate of R' and the y -coordinate of R changes sign and becomes the x -coordinate of R' .

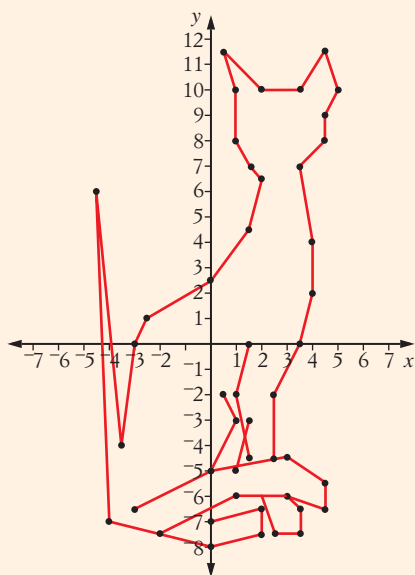
11

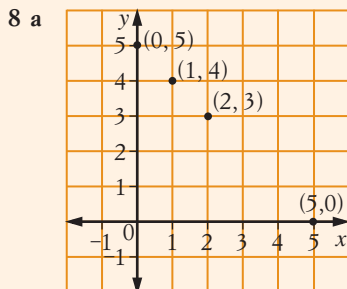
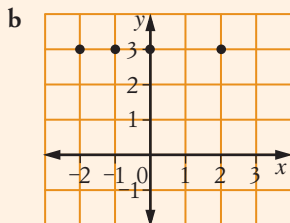


- 12 Translated 10 units right.
 $D: (-10, 0) \rightarrow (0, 0)$
 The x -coordinate of D increased by 10 while the y -coordinate did not change.

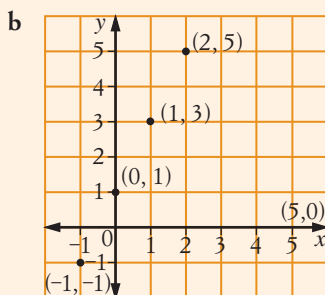
Power plus

1 b



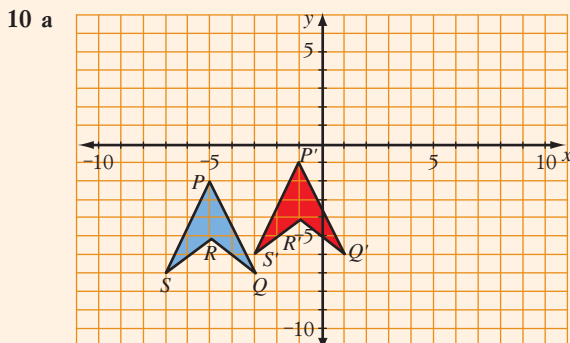


Collinear, decreasing by the same amount



Collinear, increasing by same amount

- 9 a** Reflected across the y -axis.
b $F(4, -3) \rightarrow (-4, -3)$; the x -coordinate changes sign while the y -coordinate does not change.



- b** $P(-5, -2) \rightarrow P'(-1, -1)$
 $Q(-3, -7) \rightarrow Q'(1, -6)$
 $R(-5, -5) \rightarrow R'(-1, -4)$
 $S(-7, -7) \rightarrow S'(-3, -6)$
 The x -coordinate of each vertex increases by 4 units and the y -coordinate increases by one unit.

- 11 a** Rotated 180° about point S . Or 2 reflections
b $V(5, -4) \rightarrow V'(-5, 4)$
 The x - and y -coordinates of V both changed sign when transformed to V' .

Chapter 10

SkillCheck

- 1 a** 5 **b** 1.5 **c** 2
d 1 **e** 3
 (Teacher to check scale diagrams.)

- 2 a** 50 **b** 34.5
3 a Cola, Orange, Lime, Lemonade
b 5 **c** 23
4 a 11, 22, 24, 28, 31 **b** 13, 15, 16, 17, 18, 21
5 a 18, 17, 11, 6, 5 **b** 31, 23, 17, 14, 8, 5, 2

Exercise 10-01

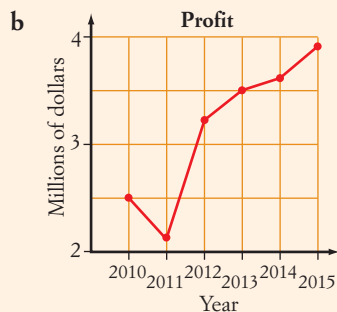
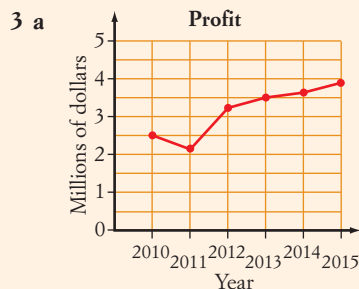
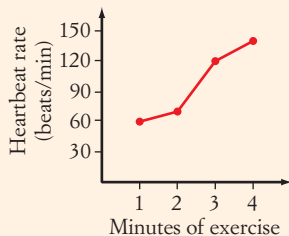
- 1 a** 10 cars
b 5 cars. It is difficult to represent the whole range of values accurately.
c 8:00–10:00 a.m.
d 6:00–8:00 a.m.
e 6:00–8:00 a.m., 5; 8:00–10:00 a.m., 40; 10:00 a.m.–12:00 noon, 10; 12:00 noon–2:00 p.m., 20; 2:00–4:00 p.m., 35; 4:00–6:00 p.m., 15
f i 8:00–10:00 a.m. dropping children at school, going to work
ii 6:00–8:00 a.m. not many people around yet, a few going to work
iii 2:00–4:00 p.m. parents picking up children.
2 a Sydney **b** Darwin
c 200 000 people **d** 2.1 million or 2 100 000
e Adelaide **f** 20 times
3 a i 99% **ii** 88% **iii** 88%
b Dishwasher **c** Mobile phones
d About 15% **e** Teacher to check.

Answers

- 4 a 1 unit = 10% b 45% c 2006
 d i 3 and 4 ii 5 and over
 e 3 and 4 f True
- 5 a Within 5 km of home
 b $\frac{44}{100} = \frac{11}{25}$ c $\frac{75}{100} = \frac{3}{4}$
 d Fewer accidents occur. Most travelling is done close to home.
 e $\frac{6}{100} = \frac{3}{50}$ f True
 g Teacher to check
- 6 a 100 mm
 b Meet community needs, $\frac{28}{100} = \frac{7}{25}$
 c $\frac{21}{100}$
 d Meet community needs, decrease cost, save natural resources, reduce pollution, and others.
 e Teacher to check
- 7 a dog b goldfish
 c i $\frac{3}{30} = \frac{1}{10}$ ii $\frac{4}{30} = \frac{2}{15}$
 d 360° e 96° f No
 g Dog 27, Bird 12, Cat 24, Guinea pig 18, Goldfish 9
- 8 a i 70 cm ii 120 cm
 b i 4 ii 11
 c 0 and 1, babies grow quickly
 d 6 and 7, children's growth slows down
 e 3 years
 f 10 years, growth slows down
 g i fairly flat ii possibly horizontal
 h Teacher to check.

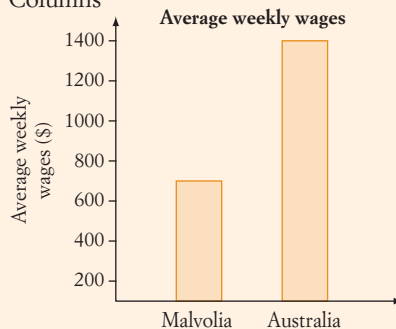
Exercise 10-02

- 1 a 20 000 boxes
 b It has a correct even scale for the sales axis
 c It is difficult to see where the graph starts, it exaggerates the amount of difference.
 d Bigger pictures/diagrams can appear larger than the actual amount and exaggerate any difference.
- 2 a 120 beats/min b 20 beats/min
 c Heart rate scale does not begin at 0
 d Heart rate and exercise

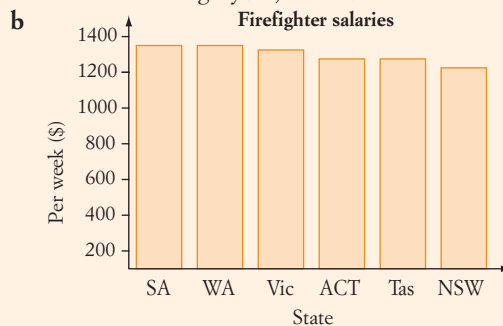


- c Graph in part b because it is much steeper.

- 4 a Australian weekly wages are four times bigger than Malvolia's.
 b No scale on vertical axis, use of pictures instead of columns.
 c Columns



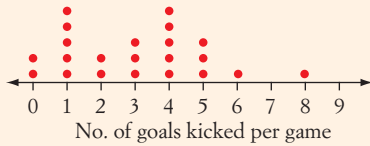
- 5 No scale on the horizontal population axis.
 6 a The scale does not begin at 0, the scale is not even: increasing by 50, then 25.



- 7 Teacher to check

Exercise 10-03

1 a

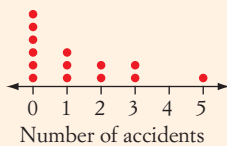


b 8 c 0 to 5 d 22

2 a 18 b 16 c 18 to 26
d 25 e No f 22

3 a 37 b 31 to 40 c 23
d 23 to 40 e 49

4 a Accidents on motorway each day



b 0 c 1.3 d 5, it's unusually large

5 B

Exercise 10-04

1 D

2 a

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

b 76, 124 c 8 d 101

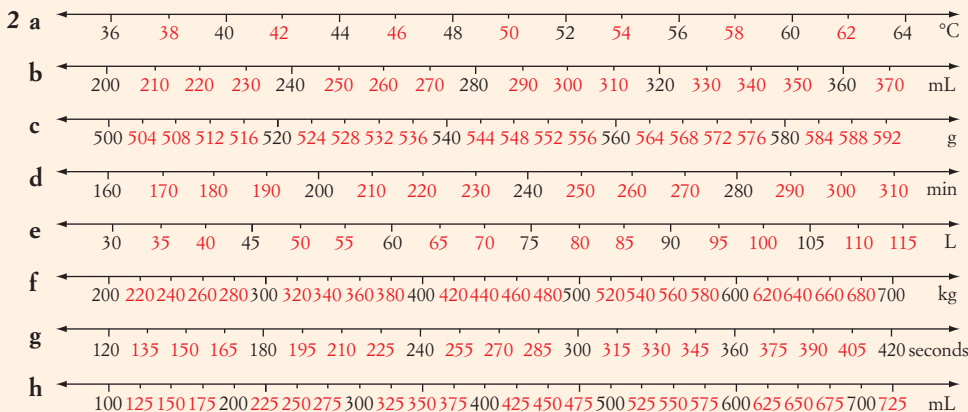
3 a 64 b 37 c 99
d 64 to 154 e 9 f 60 to 110

4 a

Stem	Leaf
13	7
14	0 1 3 5
15	3 4 5 7 7 7 8 9
16	2 2 3 6 6 8
17	0 1 5 5 7

b 137 cm, 177 cm c 5 d 157 cm
e 158 cm, 159 cm f $\frac{1}{4}$ g 20.8%

Mental skills 10



Exercise 10-05

1 a 30.25 b 32
2 a i 2.56 ii 3
b i 4.43 ii 2
c i 64.22 ii all values unique
d i 45.17 ii 45.1
e i 5.78 ii 3, 5

3 a green b They are not numbers

4 Teacher to check

5 a 7.6 b 6.1
c Mean, takes into account all scores.

6 a The last score. b Arteri
c Phuong 16.44, Arteri 25.33
d Arteri family with a higher last score has the higher mean.

Answers

7 A

8 a 74%

b 79%

9 Teacher to check

10 2550

Exercise 10-06

- 1 a i 22 ii 5
b i 4.9 ii 2.1
c i 5 ii 5

- 2 a The 4th score
b Carozza 12, Binns 12
c Carozza 40, Binns 40
d No effect

- 3 a i 7 ii 6
b i 37.5 ii 10
c i 12 ii 5

- 4 C 5 C 6 Teacher to check

- 7 a \$696 500 b \$1 202 000
c \$767 273 d \$625 000
e Median, not affected by outlier price (\$1 800 000)

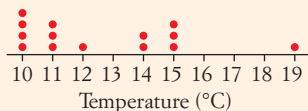
- 8 6, 7, 8 or 9

- 9 a Alf 75.9, Mike 75.7 b Mike
c Alf 2, Mike 6 d Mike, lower average
e Alf, smaller range f Alf 76, Mike 75.5

Exercise 10-07

- 1 a i 4 ii 11 iii 10 iv 9.8
b i 4 ii 24 iii 22 iv 22.3
c i 6 ii 36 iii 37 iv 37.6
d i 0.3 ii 1.8 iii 1.8 iv 1.8
- 2 a i 39 ii 33 iii 33 iv 30.1
b i 26 ii 94 iii 88 iv 86.3
c i 40 ii 131 iii 124.5 iv 121.45
d i 44 ii 17 iii 17 iv 22.1

- 3 Maximum daily temperature in Cowra

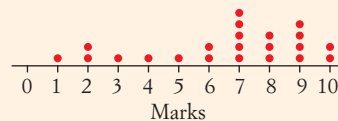


- a 11.5°C b 10°C c 12.6°C d 9°C

- 4 a 8 Huxley quiz marks



- 8 Crancher quiz marks



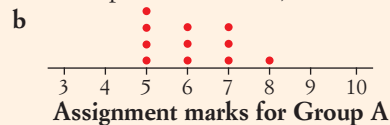
- b 5 c 7 d 9 e Mean = 6.5
f 8 Crancher, greater proportion of higher marks

- 5 a 48 b 101 c 105, 112 d 100.73

- 6 a 67 b 51 c 67 d 67.8

Exercise 10-08

- 1 a Group A: mean = 6.1, median = 6
Group B: mean = 6.1, median = 6



Assignment marks for Group B

Group B marks are more spread out (from 3 to 10 and spread evenly) than Group A (clustered around 5 to 7)

- c 4

- 2 a 25
b Dynamo: mean = 37.16, median = 37, range = 42
EnergyPlus: mean = 28.88, median = 29, range = 28

- c, d Teacher to check

3 a

	Penrith	Hurstville
8	7 6 5 3	3 4 4 5
8	8 6 0	4 3 5
6	6 5 4	5
6	6 6	6 6 6 8
1	7	1 1 5 8
6	1 1	8 1 3 3 6
6	9	4 6

- b Hurstville: mean = 66.5, mode = 66, median = 69.5, range = 62

Penrith: mean = 56.45, mode = 48, 66, 81,
median = 54.5, range = 63

- c** Hurstville, scores are clustered in the 60s, 70s and 80s, it has a higher median and mean
d 96, mean and range

4 a

7 Murray	7 Winton
9 8 8 7 5 2 1 1 1	2 6 9 9
9 8 8 7 7 6 5 4 3 2 0	3 1 1 2 3 5 6 9
0 5	4 0 1 4 5 5 6 7 7 8 8

- b** 50 **c** 26 **d** in the 30s and 40s
e 29 to 50, 26 to 48.
f 7 Murray: mean = 40.7, median = 42,
7 Winton: mean = 38.6, median = 39.5
g Teacher to check
h No outliers

5 a 25

- b** Cougars: mean = 65.24, median = 66,
mode = 46, 58, 61, 66, 68, 86, 92, range = 49
Tigers: mean = 71.56, median = 70,
mode = 53, 70, 87, 89, range = 50
c,d Teacher to check

6 a

Maria	Stacey
9 5 9 8 7 5 2 0 6 5 1 4 3 5	1 2 0 4 5 9 9 0 0 5 5 7 7 4

- b** Maria: mean = 36.7, median = 37.5,
mode = no mode, range = 34
Stacey: mean = 31.3, median = 30,
mode = 29, 30, 35, 37, range = 24
c Stacey, smaller range **d** Maria **e** Maria

Power plus

1 6

2 a 74.16 **b** 77.16

- c** It also went up by 3 marks
d It changes the mean by the amount added or subtracted

3 92%

4 a 40 **b, c** Teacher to check

5 Yes, the other two sisters earn a total of \$6700 per week

Chapter 10 revision

1 a 0–4 years **b** 13.5%

- c** It covers more years than each of the other groups.

2 a 26% **b** Anglican, Other Christian

- c** There would have been almost no Buddhist, Muslim or Other.

3 a Snowy Mountains **b** Uluru

- c** 21 to 25 **d** True

4 a January and February

- b** January and February

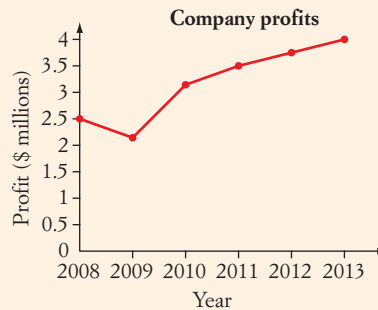
c October

d January and February

5 a Profit scale does not begin at 0.

- b** Sales have increased greatly

c



6 a 18 **b** 28 **c** 3 **d** $\frac{8}{15}$

7 a 31

b

Stem	Leaf
7	1 5 6 8 9 9
8	0 0 0 2 2 2 3 4 4 5 8 8
9	0 0 0 0 2 2 4 5 6 8 8
10	4 5

c 90 kg

d 71 kg

8 a i 4

ii 4

b i 8.78

ii 6

c i 3.5

ii 1

9 a i 4

ii 4

b i 10

ii 6

c i 3.5

ii 7

10 a i 4

ii 4.5

iii 6

iv 4.25

b i 35

ii 60

iii 66

iv 57.36

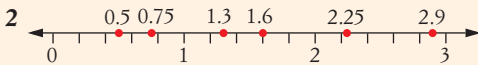
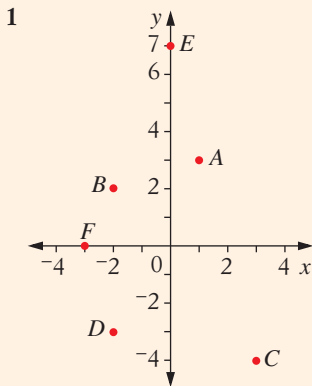
Answers

11 a

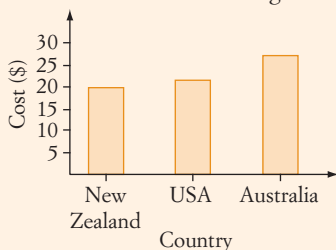
Boys	Girls
7 0	1 8
9 8	2 8 9
8 0	3 5 7 9
0	4 8
8 5 0	5 1 2 3 5 9
6 2	6 0 6 9
5 4 3 3 2	7 3 5 6
8 5	8 3
3	9 4

- b** 94, girl
c Girls: mean = 55, median = 54, mode = 66, range = 76
 Boys: mean = 55.8, median = 60, mode = 73, range = 83
d Overall the boys performed better than the girls with a higher median and mode.
e The girls seem to be more consistent, clustered from 50 to 80, whereas the boys are evenly spread across the whole range.

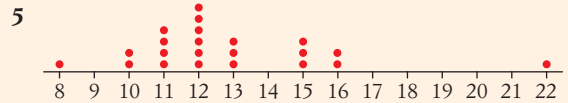
Mixed revision 3



- 3 a** The cost axis does not begin at 0.
b



- 4 a** 2 g **b** 2000 kg **c** 8 weeks
d 6500 mL **e** 72 hours **f** 4000 g
g 1.5 kg **h** 3 L **i** 8 min
j 2.5 t **k** 9500 mg **l** 12 kg



- a** Between 10 and 13 **b** 22

6

Stem	Leaf
0	4 5 6 9
1	0 1 3 4 5 5 6 7 8 9
2	1 2 4 8

- 7 a** 9.44 **b** 3.1 **c** 9.05
d 21.89 **e** 5.407 **f** 5.283

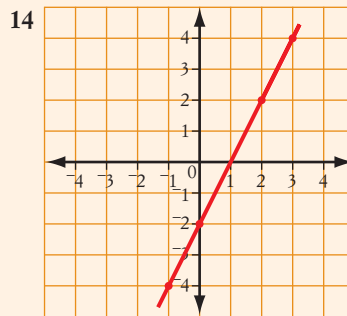
- 8 a** 4.34 **b** 1.263 **c** 0.07
9 a 10.023 **b** 83.6 **c** 13.44

- 10 a** 24 mm **b** 160 cm **c** 30 cm **d** 56 cm

- 11 a** 32 mm² **b** 1600 cm²
c 30 cm² **d** 121 cm²

- 12** 101.6 cm

- 13 a** 60 cm² **b** 880 mm²



- 15 a** 0.2 **b** 0.375

- 16** 0.5

- 17 a** 0.8 **b** 12.06 **c** \$9.82 **d** 3.8

- 18 a** 8.7°C **b** 0.25 m or 25 cm **c** \$3.20

- 19 a** 45 m³ **b** 330 cm³ **c** 11 880 mm³

- 20 a** 7.5 kL **b** 7.5 m³

- 21 a** 26 928 mL **b** 26.928 L

22 a i 31
b i 3.529

ii 30
ii 3.3, 3.5

23 a i 30
b i 3.5

ii 6
ii 0.6

24 a i 4 ii 2 iii 2 iv 2
b i 15 ii 21 iii 21 iv 20.56

25 a 4 cm b 32 cm

Chapter 11

SkillCheck

1 a impossible b likely
c even chance d impossible
e likely f unlikely
g certain h even chance
i likely or certain j teacher to check

2 a $\frac{1}{5}$ b $\frac{4}{9}$ c $\frac{9}{10}$ d $\frac{3}{16}$

3 a 0.875 b 0.6 c 0.26 d 0.7

4 a 60% b 31% c 75% d 5%

5 a teacher to check b teacher to check
c more d more
e teacher to check f more

6 teacher to check 7 C

8 a $\frac{7}{20}$ b $\frac{1}{5}$ c $\frac{12}{25}$ d $\frac{3}{50}$

9 a $\frac{5}{6}$ b $\frac{1}{4}$ c $\frac{6}{10} = \frac{3}{5}$

Exercise 11-01

1 a {red, blue}, 2
b {red, green, blue, yellow}, 4
c {red, blue}, 2
d {red, white}, 2
e {red, black, white, green}, 4
f {orange, yellow, green}, 3
2 a 2, equally likely b 3, not equally likely
c 26, not equally likely d 2, equally likely
e 10, equally likely f 2, not equally likely
3 a {1, 3, 5} b {a, e, i, o, u}
c {5, 6, 7, 8, 9} d {March, May}
e {Adelaide, Brisbane, Hobart, Melbourne, Perth, Sydney}

f Grade 1, Grade 2, Grade 3, Grade 4, Grade 5, Grade 6

4 a 7
b No, some coloured regions are bigger than others.

c pink d green

5 a 20 b {\$2, \$1, 50c, 20c, 10c}

c No d 20c

6 a 52 b Yes c 13

d 26 e 4 f 2

g 1 h 20 i 12

j 2

7 D

8 a {A, B, C} b No, there are more As
c A

9 a 3 b {5, 6, 7, 8} c 6

d {black, red} e red likely

f {hearts, spades, clubs, diamonds}

g clubs

Exercise 11-02

1 a heads, tails b 2
c $\frac{1}{2}$ d 50%

2 a $\frac{1}{6}$ b $\frac{1}{2}$ c 0
d $\frac{5}{6}$ e $\frac{1}{2}$ f 1

3 a 50% b 25% c 50%
d 20% e 40% f 37.5%

4 a $\frac{1}{10}$ b 0.15 c 55%
d 75% e 0.35 f 65%

5 D

6 a $\frac{1}{26}$ b $\frac{1}{26}$ c $\frac{2}{26} = \frac{1}{13}$
d $\frac{5}{26}$ e $\frac{21}{26}$ f $\frac{10}{26} = \frac{5}{13}$

7 a pink b yellow c black
d purple, blue, pink

8 a blue b green c white

9 a $\frac{3}{10}$ b 0.3 c 40%
d $\frac{2}{5}$ e 0.4 f 20%

10 $\frac{5}{7}$

11 Teacher to check

12 $\frac{6}{800} = \frac{3}{400}$

13 Teacher to check

14 a $\frac{26}{52} = \frac{1}{2}$ b $\frac{13}{52} = \frac{1}{4}$ c $\frac{1}{52}$
 d $\frac{4}{52} = \frac{1}{13}$ e $\frac{16}{52} = \frac{4}{13}$ f $\frac{6}{52} = \frac{3}{26}$
 g $\frac{2}{52} = \frac{1}{26}$ h $\frac{4}{52} = \frac{1}{13}$

15 a 1 b 5
 c 9 d Teacher to check

16 C

17 Teacher to check

18 Teacher to check

19 a 0.64 b $36\frac{4}{11}\%$ or $36.\dot{3}\dot{6}\%$
 c 2

20 a 0, Teacher to check b 1, Teacher to check

Exercise 11-03

1 a $\frac{1}{6}$ b 0 c $\frac{5}{6}$
 d $\frac{3}{6} = \frac{1}{2}$ e 1 f $\frac{2}{6} = \frac{1}{3}$
 2 a $\frac{1}{10}$ b $\frac{2}{10} = \frac{1}{5}$ c 0 d $\frac{6}{10} = \frac{3}{5}$
 3 a $\frac{7}{12}$ b $\frac{7}{12}$ c 0
 d $\frac{2}{12} = \frac{1}{6}$ e $\frac{3}{12} = \frac{1}{4}$ f 1
 4 a $\frac{1}{4}$ b $\frac{1}{4}$ c 0
 5 a C b A c E
 d H e D f G
 g B h F
 6 a $\frac{4}{18} = \frac{2}{9}$ b $\frac{3}{18} = \frac{1}{6}$ c 0
 d $\frac{11}{18}$ e $\frac{15}{18} = \frac{5}{6}$ f $\frac{12}{18} = \frac{2}{3}$
 7 a 0.604 b 0.012 c 0.988 d 0
 8 a 25% b 30% c 100%
 d 0% e 40% f 20%
 9 a F b D c E
 d C e G f A
 10 a $\frac{2}{15}$ b $\frac{1}{15}$ c $\frac{4}{15}$
 d $\frac{11}{15}$ e $\frac{5}{15} = \frac{1}{3}$ f O

11 Teacher to check

Exercise 11-04

1 a $\frac{2}{10} = \frac{1}{5}$
 b 16, the expected value is higher than the observed value
 c $\frac{11}{80}$
 2 a i 0.16 ii 0.16 iii 0.54
 iv 0.24 v 0.1 vi 0.32
 b i 8 ii 4 iii 2
 iv 6, 7 v 10 vi 10
 3 a Teacher to check b Teacher to check
 c $\frac{1}{2}$ d 25 e Teacher to check
 f Experimental probability would get closer to $\frac{1}{2}$
 4 B
 5 a Teacher to check b Teacher to check
 c i $\frac{1}{6}$, 8 ii $\frac{1}{6}$, 8 iii $\frac{1}{2}$, 25 iv $\frac{5}{6}$, 41
 d Teacher to check e Teacher to check
 f The experimental probability would get closer to the theoretical probability
 6 a Yes b Teacher to check
 c 13.75% d 11
 7 $\frac{140}{250} = \frac{14}{25}$

Mental skills 11

2 a \$35.85 b \$39.80 c \$35.75
 d \$34.80 e \$42.85 f \$69.80
 g \$57.80 h \$53.65 i \$59.90
 j \$133.70

Exercise 11-05

1 a The sex of a new baby is male
 b No rain tomorrow
 c Winning or drawing a soccer match
 d Being right-handed
 e Being 18 years or under
 f Choosing a card that is not hearts from a deck of playing cards
 g A traffic light showing red or amber
 h A train arriving on time or early
 2 a $\frac{1}{6}$ b $\frac{2}{6} = \frac{1}{3}$ c $\frac{3}{6} = \frac{1}{2}$
 d $\frac{5}{6}$ e $\frac{4}{6} = \frac{2}{3}$ f $\frac{3}{6} = \frac{1}{2}$
 3 a $\frac{1}{1000}$ b $\frac{999}{1000}$
 4 a $\frac{15}{130} = \frac{3}{26}$ b $\frac{23}{26}$ c 0
 d 1 e $\frac{90}{130} = \frac{9}{13}$ f $\frac{55}{130} = \frac{11}{26}$

5 a $\frac{12}{13}$ b 45% c 0.93

6 a 0.7 b 0.3
c 15 red, 6 white, 9 blue d 0

7 a $\frac{2}{12} = \frac{1}{6}$ b $\frac{5}{6}$

8 a $\frac{3}{4}$ b $\frac{2}{4} = \frac{1}{2}$ c $\frac{1}{2}$ d 0

9 C

10 a draw b 8%

11 a D

12 a $\frac{10}{11}$ b $\frac{7}{11}$ c $\frac{9}{11}$ d $\frac{8}{11}$

13 15% 14 0.8

Power plus

1 a

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 11

c There are more ways of obtaining some sums than others

d 7 e 2 f $\frac{1}{36}$ g $\frac{3}{36} = \frac{1}{12}$ h 5, 9

2 a HHH, HHT, HTH, HTT, TTT, TTH, THT, THH

b i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$
iv $\frac{1}{8}$ v $\frac{7}{8}$ vi $\frac{4}{8} = \frac{1}{2}$

3 a hot dog, cola, cake
hot dog, cola, ice cream
hot dog, lemonade, cake
hot dog, lemonade, ice cream
pizza, cola, cake
pizza, cola, ice cream
pizza, lemonade, cake
pizza, lemonade, ice cream
hamburger, cola, cake
hamburger, cola, ice cream
hamburger, lemonade, cake
hamburger, lemonade, ice cream
12

b $\frac{1}{12}$ c $\frac{8}{12} = \frac{2}{3}$ d $\frac{6}{12} = \frac{1}{2}$

Chapter 11 revision

1 a pass, fail b G, PG, M, MA15+, R
c Katrina, Biljana d train, bus

2 a 5 b sections are not of equal size
c 5 d 1 e 4

3 a $\frac{1}{6}$ b $\frac{3}{6} = \frac{1}{2}$ c $\frac{4}{6} = \frac{2}{3}$

4 a $\frac{210}{1040} = \frac{21}{104}$ b $\frac{895}{1040} = \frac{179}{208}$
c $\frac{685}{1040} = \frac{137}{208}$ d 0

5 a $\frac{6}{14} = \frac{3}{7}$ b $\frac{3}{14}$ c 0 d 1

6 A 7 C

8 a $\frac{1}{2}$
b 40, the observed frequency is lower than the expected frequency
c $\frac{24}{80} = \frac{3}{10}$

9 a Choosing a card that is not a queen from a deck of cards, $\frac{48}{52} = \frac{12}{13}$
b Rolling a number that is not a multiple of 3 on a die, $\frac{4}{6} = \frac{2}{3}$
c Buying a ticket that does not win out of 550 tickets sold, $\frac{549}{550}$

10 $\frac{7}{15}$

Chapter 12

SkillCheck

1 a 2 b 3.6 c 18 000
d 5 e 65 f 8
g 7.5 h 8150 i 85

2 a 6 b 12 c 4

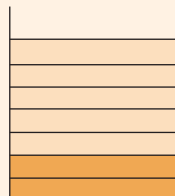
3 a 21 b 20 c 2

4 a $\frac{1}{4}$ b $\frac{4}{5}$ c $\frac{1}{4}$

Exercise 12-01

1 a 5 : 7 b 3 : 5 c 7 : 9
d 7 : 13 e 5 : 3

2 a b 1 : 4



Answers

3 Teacher to check

4 Fewer students for each teacher

5 a 8 : 13 b 3 : 8 c 13 : 1 d 1 : 25

6 a 39 : 23 b 23 : 100
c 38 : 39 d 23 : 28

7 a 13 : 25 b 17 : 100 c 5 : 12
d 150 : 1 e 500 : 43 f 350 : 83
g 3000 : 1421 h 27 : 40 i 59 : 180
j 180 : 7 k 9 : 49 l 7700 : 17

8 D

9 a 1 : 25 b 1 : 2 c 1 : 4
d 11 : 9 e 1 : 1 f 2 : 1

Exercise 12-02

1 a 12 b 10 c 9
d 20 e 32 f 84
g 30 h 75 i 5
j 5 k 4 l 30
m 2 n 40 o 5

2 D

3 a 1 : 10 b 1 : 2 c 2 : 5
d 5 : 7 e 3 : 2 f 7 : 3
g 10 : 1 h 5 : 11 i 1 : 1
j 5 : 12 k 4 : 1 l 1 : 12
m 2 : 3 : 5 n 3 : 4 : 6 o 3 : 1 : 4
p 3 : 2 : 8 : 4

4 a 80 : 200 = 2 : 5 b 300 : 1200 = 1 : 4
c 6 : 28 = 3 : 14 d 45 : 120 = 3 : 8
e 1 : 3 f 24 : 15 = 8 : 5
g 15 : 48 = 5 : 16 h 20 : 1000 = 1 : 50
i 4000 : 350 = 80 : 7 j 250 : 3000 = 1 : 12
k 9 : 5 l 20 : 1
m 30 : 2 = 15 : 1 n 3 : 20
o 1800 : 450 = 4 : 1

5 D

6 a 48 : 28 = 12 : 7 b 20 : 35 = 4 : 7
c 35 : 28 = 5 : 4 d 20 : 48 = 5 : 12
e 28 : 35 = 4 : 5 f 48 : 131

7 A

8 a i 72 : 1440 = 1 : 20
ii 816 : 624 = 17 : 13
iii 816 : 72 = 34 : 3
b Teachers: students = 1 : 20 > 1 : 25 so schools has enough teachers

9 Australia 3 : 250, Pakistan 1 : 40

10 14 : 11

11 a 1625 mL b 375 : 1625 = 3 : 13
c 375 : 2000 = 3 : 16

12 7P

Exercise 12-03

1 80 2 152 cm 3 20 cm 4 28

5 245 6 480 kg 7 C

8 8 kg 9 72 10 20.4 t

11 a Ali \$90, Felicity \$63
b \$225

12 72 cm 13 A

14 a 5 mL b 60 mL

15 175 m

16 25 kg cement, 100 kg sand

17 600 g

18 Sonya \$220, Xanthe \$275

Exercise 12-04

1 a words/min b beats/min
c cents/min or cents/s d \$/kg
e \$/week f L/min
g \$/kg h people/year
i \$/kL

2 a 60 km/h b \$6.60/h
c 12.5 m/s d \$16/L or 1.6 c/mL
e 19 goals/game f \$19.20/kg
g 18 min/question h 60 words/min
i 1.8 children/family j 9.8 m/s
k 150 revolutions/s l 4.5 runs/over
m \$0.05/g or 5 c/g n 0.12 L /km or 120 ml/km
o 20 sheep/h p 14 containers/day

3 B

4 a It costs \$1.40 for every kilogram of mass
b An average of 60 sheep in each hectare of land
c An average of 920 toys produced each day
d 230 L pumped out every hour

5 147 c/L 6 45 cm/year

7 71 calls/month 8 1800 hectares/month

Mental skills 12A

- 2 a 160 b 70 c 240
 d 900 e 2600 f 900
 g 2200 h 300 i 180
 j 400 k 1100 l 210
 m 700 n 1800 o 390
 p 770

Exercise 12-05

- 1 a \$0.90/razor b \$0.53/egg
 c \$0.56/pack d \$0.05/teabag
 e \$0.01/g f \$0.03/g
- 2 a Block Ices b Medium salad
 c Party pack d 3L
 e 280 g f Mega DVDs
- 3 a Bullworts b GIA
 c Baldy d Bullworts
- 4 a 375 g b 2 L c 735 g d 115 g
- 5 A
- 6 a Large b Large c Large
 d Large e Small

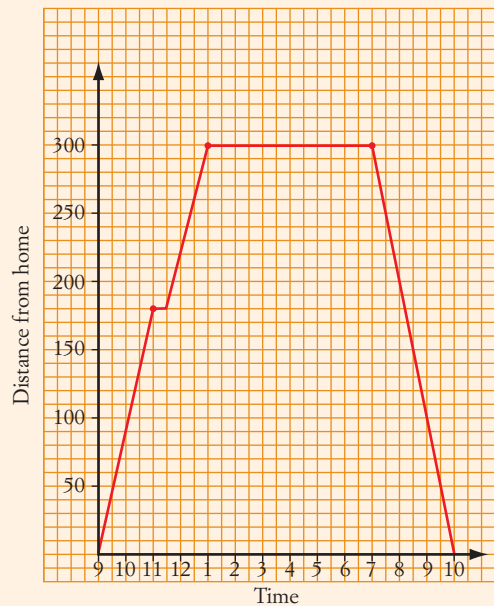
Exercise 12-06

- 1 1100 words 2 \$893 3 420 4 3680 km
- 5 a 360 mL b 2520 mL
- 6 6 7 500 m 8 48 km
- 9 975 10 A
- 11 a 17°C b 29°C c 12 noon
- 12 a 608 km b 5 hours
- 13 a 12 b 600 c 70
- 14 34 L 15 \$3980 16 4.8
- 17 a \$13.50 b 2.2 kg
- 18 a 794.4 h (= 33.1 days) b $444\frac{1}{3}$ h ($\approx$ 18.3 days)
 c 158.88 h (= 6.62 days) d 46.7 h ($\approx$ 1.9 days)
 e 39.72 h (=1.655 days) f 3.64 h

Exercise 12-07

- 1 a 10 a.m. b 6 km c 1.5 h
 d 4 km/h e 2 h f 12 km
 g 2 p.m., 4 km h 6 h
- 2 a 20 km/h b 11 a.m., 40 km
 c 1:30 p.m. d 2.5 h
 e 26 km/h f 25 km
 g 130 km
 h 11:30 a.m. – 12:30 p.m., graph is steeper
- 3 a 240 km b 180 km c 11:45 a.m.
 d 40 km/h e 10 a.m., 160 km
 f Sam, finishes journey earlier
 g Before 11 a.m., graph less steep h B
- 4 a B b A c C
- 5 Caitlin travelled 30 km in the first hour, then 20 km in the second hour. She stopped for an hour, travelled another 30 km in one hour, stopped for $\frac{1}{2}$ an hour, then travelled another 20 km in one hour.

6 a



- b 300 km c 1 p.m.
 d When he is returning home at 100 km/h from 7 p.m. to 10 p.m.
 e 3 hours f 13 h g 100 km

Exercise 12-08

- 1 **a** 1 h **b** 21 h **c** 5 h
d 4 h **e** 9 h **f** 11 h
- 2 **a** 22 min **b** 8 min **c** 8 min
d 2 h 16 min **e** 16 min **f** 4 h 21 min
- 3 **a** 30 **b** 30 **c** 72 **d** 104
e 168 **f** 42 **g** 4 **h** 8
i 12 **j** 5 **k** 2.5 **l** 4
- 4 **a** 3600 **b** 86 400 **c** 604 800
- 5 **a** 5 h 40 min **b** 3 h 6 min **c** 3 h 47 min
d 6 h 45 min **e** 4 h 22 min **f** 10 h 4 min
- 6 No. A 13-year-old is 410 248 800 seconds old, which is under 1 000 000 000 (a billion) seconds.
- 7 **a** 6 min 12 s **b** 9 min 5 s **c** 3 min 58 s
d 7 min 2 s **e** 8 min 20 s **f** 8 min 44 s
- 8 **a** 6 h 47 min **b** 13 h 7 min **c** 4 h 25 min
d 3 h 25 min **e** 7 h **f** 7 h 14 min
g 4 h 13 min **h** 3 h 28 min **i** 10 h 3 min
j 9 h 51 min **k** 51 min **l** 49 min

Mental skills 12B

- 2 **a** 18 **b** 34 **c** 46
d 66 **e** 26 **f** 16
g 8 **h** 44 **i** 12
j 40 **k** 28 **l** 40
m 18 **n** 8 **o** 36
p 140

Exercise 12-09

- 1 **a** 8:45 a.m. **b** 1:20 p.m. **c** 5:50 p.m.
d 12:17 a.m. **e** 9:05 p.m. **f** 6:32 p.m.
g 11:15 a.m. **h** 2:38 a.m. **i** 2:40 p.m.
j 3:20 a.m. **k** 4:55 p.m. **l** 11:31 p.m.
m 1:08 a.m. **n** 10:45 a.m. **o** 8:00 p.m.
p 6:43 a.m.
- 2 **a** 1835 **b** 0805 **c** 1144 **d** 2320
e 0251 **f** 1230 **g** 1538 **h** 0711
i 0908 **j** 2154 **k** 0042 **l** 1939
m 0159 **n** 2218 **o** 0846 **p** 1723
- 3 **a** 11:23 a.m. **b** 5:54 p.m. **c** 1:24 p.m.
d 10:03 a.m. **e** 11:17 p.m.

- 4 **a** 1717 **b** 2206 **c** 0245 **d** 0421
- 5 **a** 8:55 a.m., 1:55 p.m. **b** 5:55 p.m., 11:15 p.m.
c 12:45 a.m. **d** 7:50 a.m.
e 7:45 p.m.

Exercise 12-10

- 1 A
- 2 **a** 1 h 5 min **b** 1 h 50 min **c** 5 h 44 min
d 3 h 53 min **e** 5 h 45 min **f** 11 h 38 min
g 2 h 25 min **h** 43 min **i** 1 h 47 min
j 4 h 8 min **k** 7 h 39 min **l** 13 h 25 min
- 3 **a** 3 h **b** 10 h **c** 14 h 40 min
d 4 h 15 min **e** 5 h 15 min **f** 8 h 45 min
g 7 h 5 min **h** 5 h 30 min **i** 8 h 40 min
j 8 h 10 min **k** 5 h 55 min **l** 8 h 40 min
- 4 **a** 8:00 p.m. **b** 7:00 p.m. **c** 7:43 p.m.
d 1:17 p.m. **e** 2:08 p.m. **f** 1:32 p.m.
g 2:14 a.m. **h** 4:15 p.m. **i** 6:17 a.m.
j 3:25 p.m.
- 5 Joe 12:43 p.m., Anna 1:14 p.m., Ken 1:19 p.m.,
Pathena 1:21 p.m., Mike 1:31 p.m., Gail 2:02 p.m.
- 6 4 h 45 min 7 6 h 25 min
- 8 **a** 2:15 p.m. **b** 2:30 a.m. **c** 2:35 p.m.
d 11:25 a.m. **e** 7:55 a.m. **f** 6:15 p.m.

- 9 **a** 4 years 2 months **b** 4 years 6 months
- 10 **a** 6 years 1 month 6 days
b 15 years 5 months 25 days
- 11 2 h 15 min 12 2 p.m.
- 13 1 year 4 months 14 17 years 8 months
- 15 \$155 16 14 years 0 months 11 days

Exercise 12-11

- 1 **a** 7 h **b** 6 h 30 min **c** 3:20 p.m.
d 4 h 11 min **e** 13 min
f 1 h, 1 h 15 min, Liverpool-Sydney trip occurs at
the start of peak traffic period

- 2 a 9:35 a.m., 1 h 25 min
b TH114 (10:40 a.m.)
c 25 min d 9:25 a.m.
e TH038, 10:05 a.m.
- 3 a 7:27 a.m.
b The 5:08 train takes 3 h 4 min and the 8:17 train takes 2 h 37 min.
c 46 min
d Teacher to check
e Goulburn 11:12 a.m., Bundanoon 11:47 a.m., Moss Vale 12:00 noon, Bowral 12:06 p.m., Mittagong 12:11 p.m., Campbelltown 1:07 p.m., Strathfield 1:37 p.m., Sydney 1:49 p.m.
- 4 a 12:34, 1 h 4 min b 1:14 c 12:45
d 5, the first bus finishes its run at 11:50, ready to start the 6th run at 12:00
e 10:33 City cathedral, 11:00 Museum, 12:30 City square, 1:12 Town Hall, 2:28 Hall of Fame (other answers possible)
f 1:30, 1:38, 1:45, 1:54, 2:05, 2:15, 2:30, 2:42, 2:49, 3:00, 3:08, 3:20.

Power plus

- 1 Teacher to check
- 2 a 4 : 7 b 13 : 8 c 10 : 1
d 7 : 20 e 8 : 3 f 6 : 5
- 3 Kobe is right (38 : 66 vs 36 : 66)
- 4 271 374
- 5 a 710 km² b 54 913 080 400
- 6 a $166\frac{2}{3}$ min b 8.64 h

Chapter 12 revision

- 1 a 8 : 16 = 1 : 2 b 12 : 24 = 1 : 2
- 2 a 38 : 17 b 17 : 55
- 3 a 12 b 10 c 9 : 12
- 4 a 4 : 7 b 1 : 3 c 1 : 6
d 5 : 9 e 3 : 1 : 4 f 1 : 5 : 20
g 2 : 1 h 10 : 1 i 5 : 1
j 1 : 20 k 1 : 3 l 1 : 5
m 1 : 2 n 4 : 7 o 48 : 5
- 5 a 100 b \$45 000

- 6 a 3000 voters/politician b 200 sheep/ha
c \$0.80/ticket d 270 stitches/ball
- 7 a Chow b Dairy Farm
- 8 a \$503.50 b 24.1 L c 7.5 ha
- 9 a 20 km b 3
c 2:30 p.m. d 2:30 p.m. – 3:30 p.m.
e 2.5 km/h
- 10 a 10 h b 3 h c 4 h
- 11 a 5 h 16 min b 3 h 28 min
- 12 a 10 h 5 min b 4 h 42 min
c 2 h 9 min d 29 min
- 13 a 1:48 a.m. b 6:24 a.m.
c 3:50 p.m. d 12:30 p.m.
- 14 a 1410 b 0533 c 0019 d 1350
- 15 a 4 h 20 min b 7 h 35 min
c 6 h 55 min d 6 h 30 min
e 30 min f 8 h 55 min

16 14 years 2 months 13 days

17 a 3 h 19 min b 9:05 a.m.

Mixed revision 4

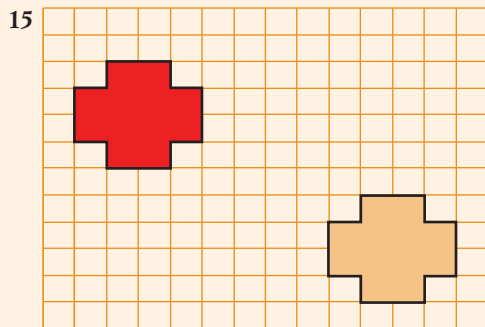
- 1 a {1, 2, 3, 4, 5, 6}
b {red, blue, green, yellow}
- 2 a 3 : 5 b 3 : 2 c 5 : 2
- 3 a 25 km/h b \$4/m
c 500 revolutions/min d 300 cans/day
- 4 a 5 h 25 min b 1 h 10 min
c 11 h 41 min d 7 h 5 min
- 5 a 0825 b 1645
- 6 a 9:15 p.m. b 4:30 a.m.
- 7 a 1 : 3 b 8 : 3 c 1 : 3
d 40 : 3 e 1 : 12 f 2 : 5 : 7
- 8 a $\frac{1}{2}$ b $\frac{2}{5}$
- 9 a 50 b $\frac{3}{5}$
- 10 20

Answers

- 11 a $\frac{3}{10}$ b $\frac{7}{10}$
 12 a 40 km/h b 440 km
 13 50 h
 14 a 6 km b 1 p.m. for 1.5 h
 c 26 km d 10:30 a.m. and 4 p.m.
 e first hour
 15 2 L

General revision

- 1 a Sum of digits = $1 + 9 = 5 = 15$, which is divisible by 3. So 195 is divisible by 3
 b 65
 2 8^5
 3 a 121 b 7 c 1000
 4 a 60 b 360 c 405 d 192
 5 $2x + 8$
 6 a $3x$ b $5m^2$ c $\frac{7k}{2}$
 d $6y$ e $10 - kn$ f $2p + 4$
 7 a 7200 b 0.425 c 4
 d 0.24 e 50 f 6700
 8 a 42 mm b 40 cm c 16 cm
 9 1810 10 6 h 35 min 11 470
 12 a 70° b 150° c 25°
 13 -5, -3, 2, 4, 6, 11
 14 a -2 b -12 c 13 d -56
 e 120 f 100 g -9 h -2

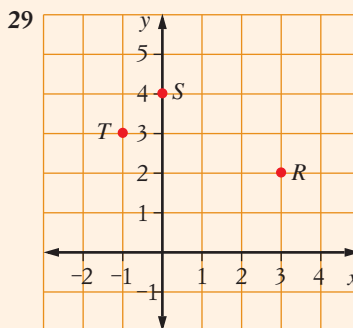
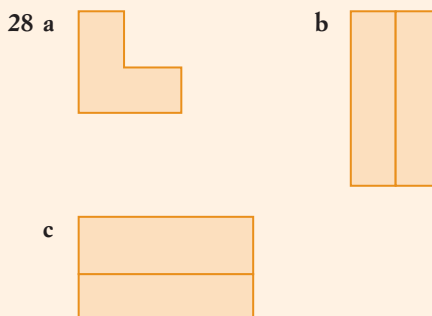


- 16 a 0.5 b 50%

- 17 {red, amber, green}
 18 a 2 b 16 c 4 d 25
 19 a $\frac{9}{10}$ b $\frac{5}{12}$ c $\frac{3}{8}$ d $1\frac{1}{2}$
 20 a (2, 1) b (-1, 0) c (-2, 3)

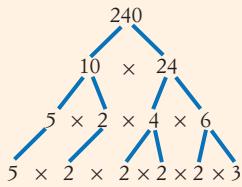


- 22 a 2.7 b 2.5 c 2 d 5
 23 a $\angle ACD$ and $\angle CFE$ (other answers possible)
 b $\angle ACD$ and $\angle BCF$ (other answers possible)
 c AH
 24 a 105° b 75° c 75°
 25 $\frac{1}{4}$
 26 a Teacher to check b 1
 27 a 1 : 5 b 4 : 3 c 1 : 4



- 30 3, 11
 31 a 1, 3, 9, 27 b 9

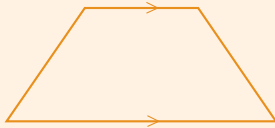
32 a



Other factor trees possible.

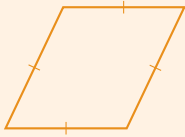
b $240 = 2^4 \times 3 \times 5$

33



Trapezium

34 a



b 2 c 2

35 a $\frac{7}{20}$ b $\frac{1}{10}$ c $\frac{3}{250}$

36 a 1.7 b 0.7 c 1.0

37 a False b True c False

38 a $b = 7$ b $r = 7$ c $n = 4$ d $p = 6$

39 -7

40 a 30 cm^2 b 95 cm^2

41 168 m^3

42 a \$9 b 16 m c 200 mL
d \$20 e 4 kg

43 a 73% b 75% c 60% d 47%

44 a 15 b 36 c 64 d 35

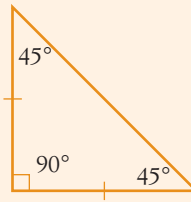
45 a $\frac{3}{5}$ b 60

46 a \$44.70 b 33.6 L

47 a 29.67 b 3.07 c 5.314

48 a 0.625 b 0.55 c 0.16

49



50 8:45 p.m.

Appendix 1 Calculator skills

Exercise A-01

1 a 5068 b 14.5 c 45
d 5.724 e 3.5 f 26.9
g 23 h 1.68 i 160
j 29.48 k -8.084 l 12.42

2 a \$19.15 b 85 cents

3 \$1875.50 4 7542 kg 5 2.4 m

6 \$11.25 7 23 trees 8 75.2 km/h

9 \$85.74 10 13 buses

11 a \$41.20 b \$1318.40

12 53.4

Exercise A-02

1 a 7 b -9 c -12
d -8 e 15 f -15
g 75 h 5 i -5
j 7 k -96 l 45
m -25 n 24 o 96
p 14 q -16 r 15
s -36 t 10

Exercise A-03

1 a 196 b 529 c 216
d 1.728 e 26.3169 f 10 201
g 4.1616 h 6859 i 26
j 148 k 9 l 42

Answers

2 a 4.9 b 10.0 c 4.7 d 4.6
3 a 16 b 156 c 6 d 10

Exercise A-04

1 a $\frac{5}{8}$ b $\frac{8}{17}$ c $\frac{5}{8}$
d $\frac{8}{9}$ e $\frac{21}{25}$ f $\frac{1}{4}$
2 a $\frac{15}{8}$ b $\frac{47}{9}$ c $\frac{43}{4}$
3 a $2\frac{2}{9}$ b $8\frac{4}{5}$ c $2\frac{1}{4}$
4 a 0.5 b 0.8 c 0.875
d 0.36 e 0.17 f 0.5

5 a $\frac{7}{20}$ b $2\frac{1}{25}$ c $\frac{1}{1000}$
d $\frac{73}{200}$ e $1\frac{3}{50}$ f $\frac{13}{40}$

6 a $\frac{17}{20}$ b $1\frac{1}{14}$ c $\frac{1}{15}$ d $\frac{4}{21}$
e $1\frac{1}{3}$ f $\frac{5}{24}$ g $1\frac{17}{30}$ h $\frac{35}{96}$
i $1\frac{3}{4}$ j $19\frac{1}{5}$ k $6\frac{13}{20}$ l $8\frac{13}{24}$
m $12\frac{3}{4}$ n 7 o $\frac{4}{27}$ p $19\frac{19}{60}$

7 a $\frac{1}{8}$ b $\frac{3}{4}$
c $15\frac{4}{5}$ or 15.8 d $\frac{1}{6}$ or 0.1 $\dot{6}$
8 a $6\frac{3}{4}$ hours b $33\frac{3}{4}$ hours

- Aboriginal numbers 104–5
- absolute cell referencing 107
- acute angle 50
- acute-angled triangle 232
- adding
 - 8 or 9 (mental skill) 15
 - 90c and 95c (mental skill) 457–8
 - in any order 57–8
 - associative law 173–4, 178
 - commutative law 173, 178
 - decimals 277–8
 - fractions 139–40
 - integers 13–14
 - mixed numerals 142
- adjacent angles 39–40
- algebraic abbreviations 179–81
- algebraic expressions 183
 - substituting into 187
- alternate angles on parallel lines 69, 75
- ancient Egyptian equations 199
- ancient fractions 141
- angle sum
 - of a quadrilateral 251–2
 - of a triangle 236–42
- angles
 - adjacent 39–40
 - alternate 69, 75
 - at a point 59, 60
 - classifying 50
 - co-interior 72, 75
 - complementary 54, 55
 - constructing 52–4, 78
 - corresponding 67–8, 74
 - measuring and drawing 43–5
 - naming 39–40
 - supplementary 54, 55
 - vertically opposite 59, 60
- area
 - composite shapes 330–2
 - metric units 312–13
 - parallelogram 327–8
 - rectangle 316–18
 - triangle 324–5
- arithmetic, laws of 173–7
- ascending order 11
- aspect ratio (screens) 470
- associative laws 173–4, 178
- average 411–12, 426
- axis of symmetry 225
- backtracking method 192, 193, 194, 196
- balancing method 191–3, 194, 195, 196
- bar charts 397, 401
- base 105, 324, 337
- best buys 481–2
- calculated probability 453
- calculator skills 517–22
- capacity
 - metric units 306, 352–3
 - and volume 352, 353
- Cartesian plane 370
- centre of symmetry 229
- certain event 450
- circle graphs 397
- clustered bar charts 401
- clustered column graphs 399
- clusters 405
- co-interior angles on parallel lines 72, 75
- collinear points 375
- column graphs 397, 398–9, 401, 405
- common fractions 131–4
- commutative laws 173, 178
- complement 459
- complementary angles 54, 55
- complementary events 459
- composite numbers 113
- composite shapes, area 330–2
- composite transformations 219
- congruence transformations 209, 379
- consecutive numbers 183
- constructing
 - angles 52–4, 78
 - parallel lines 63–4
 - perpendicular lines 64–5
 - quadrilaterals 249–51
 - triangles 234–6
- conversion rates 486–7
- coordinates for location 367
- corresponding angles on parallel lines 67–8, 74
- cross-sections 337
 - prisms 337–8
- cube (power) 109
- cube root 109–10, 116
- data sets, comparing 426–7
- decimal places 271
- decimals 271
 - adding and subtracting 277–8
 - dividing 289
 - dividing by whole numbers 287–8
 - and fractions 276
 - multiplying 286–7
 - multiplying by estimating 283–4
 - multiplying and dividing by powers of 10 281
 - ordering 271–2
 - and percentages 158, 159
 - problems 296–7
 - recurring 291, 292
 - rounding 293–4, 296
 - terminating 291–2
- denominator 131
- descending order 11
- directed numbers 8
- distributive law 176–7, 178
- divided bar graphs 399–400
- dividing
 - by 4 or 8 (mental skill) 344
 - by 5, 15, 25 and 50 (mental skill) 495
 - by a multiple of 10 (mental skill) 112
 - decimals 289
 - decimals by powers of 10 281
 - decimals by whole numbers 287–8
 - divisibility tests 97–9
 - fractions 151–2
 - integers 24
 - long division 101–2
 - mental skill 96
 - numbers 96–7

- dot plots 405
 - analysing 421, 424–5
- drawing
 - different views of solids 338–40
 - prisms 337–8
- equally likely outcomes 441, 445
- equations 178
 - backtracking method 192, 193, 194, 196
 - balancing method 191–3, 194, 195, 196
 - 'guess, check and improve' method 190
 - one-step 191–5
 - problems 199–200
 - solving 190–4
 - two-step 195–6
- equilateral triangle 232
 - constructing 234–5
- equivalent fractions 133–4, 471
- equivalent ratios 471, 476
- estimating
 - answers 92, 231
 - multiplying decimals 283–4
- event 442
- expected frequency 454
- experiment 442
- experimental probability 453–4
- exterior angle of a triangle 244–5
- factor tree 114
- factorising, in multiplication 378
- factors 97, 99, 113
- favourable outcome 445
- formulas 178
 - substituting into 187
- fractions 131–2
 - adding and subtracting 139–40
 - ancient 141
 - and the calculator 156
 - and decimals 276
 - dividing 151–2
- equivalent 133–4
 - expressing quantities as 163–4
 - improper 132–3
 - mixed numerals *see* mixed numerals
 - multiplying 147–9
 - ordering 136–7
 - and percentages 158, 159
 - problems 154–5
 - proper 132
 - of a quantity 144, 146
 - simplifying 132
- frequency 453
- glossary 523–33
- googol 107
- googolplex 107
- graphing, tables of values 376
- graphs
 - interpreting 397–401
 - misleading 401–2
 - travel 487–8
- greater than ($>$) 11
- greatest common divisor (GCD) 118–19
- 'guess, check and improve' method 190
- height 324
- highest common factor (HCF) 118
- image (transformed shape) 379
- impossible event 450
- improper fractions 132–3
- index notation 105–6
- integers 7–8
 - adding 13–14
 - and the calculator 28
 - dividing 24
 - multiplying 20
 - on a number line 7
 - ordering 11
 - problems 30
 - subtracting 16–17
- interpreting graphs 397–401
- intersect 63
- isometric dot paper 338
- isosceles triangle 232
 - constructing 235–6
- kite 248, 254–6
- length, metric units 306
- less than ($<$) 11
- line graphs 400–1
- line symmetry 225
- linear scales, reading 410–11
- location, using coordinates 367
- long division 101–2
- lowest common denominator (LCD) 136
- lowest common multiple 120–2
- mass, metric units 306
- mathematical symbols 524
- mean 412, 413, 419, 426
- measures of location 426
- measures of spread 426
- median 416, 417, 419, 426
- metric system 305–6
 - prefixes 305
- metric units
 - area 312–13
 - capacity 306, 352–3
 - length 306
 - mass 306
 - volume 345–6
- misleading graphs 401–2
- mixed numerals 132–3
 - adding and subtracting 142
 - dividing 152
 - multiplying 149
- mode 412–13, 419, 426
- monetary amounts, adding 457–8
- multiplying
 - an even number by 5 (mental skill) 185
 - in any order 57–8
 - associative law 173–4, 178
 - by 4 or 8 (mental skill) 344
 - by 5, 15, 25 and 50 (mental skill) 480–1
 - by 9, 11 and 12 (mental skill) 198
 - by factorising (mental skill) 378
 - by a multiple of 10 175

- commutative law 173, 178
- decimals 286–7
- decimals by estimating 283–4
- decimals by powers of 10 281
- doubling and halving numbers (mental skill) 322
- fractions 147–9
- integers 20
- mental skill 93
- mixed numerals 149
- numbers 93–4
- negative numbers 4–5
 - on the number plane 373
- number line 7
- number plane 370
 - with negative numbers 373
 - transformations 379–81, 385–7
- numbers
 - composite 113
 - dividing 96–7
 - estimating 92
 - multiplying 93–4
 - prime 113
 - rounding 91
- numerator 131
- obtuse angle 50
- obtuse-angled triangle 232
- one-step equations 191–5
- order of operations 26
- order of rotational symmetry 229
- ordered pairs 370, 376
- ordered stem-and-leaf plot 408
- ordering
 - decimals 271–2
 - fractions 136–7
- origin 370, 373
- original (shape) 379
- outcome 442
- outliers 405
- paper sizes 321
- parallel lines 63
 - alternate angles 69, 75
 - angles on 74–5
- co-interior angles 72, 75
- constructing 63–4
- corresponding angles 67–8, 74
- proving 78–9
- parallelogram 248, 250–1, 254–6
 - area 327–8
- percentages 158, 165–6
 - and decimals 158, 159
 - expressing quantities as 163–4
 - and fractions 158, 159
 - of a quantity 160–1, 162
- perfect cube 109
- perfect square 108
- perimeter 308–9
- perpendicular height 324
- perpendicular lines 64
 - constructing 64–5
- picture graphs 397
- pie charts 397
- place value 271
- positive numbers 4
- powers 105–6, 107
- prime factors 114–16, 119, 121–2
- prime numbers 113
- prisms
 - cross-sections 337, 338
 - drawing 338–40
 - volume 349
- probability 441, 445–6
 - complementary events 459
 - experimental 453–4
 - language of 442
 - range of 450
 - theoretical 453
- pronumerals 55, 178
- proper fractions 132
- protractors 42–5
- quadrants 373
- quadrilaterals
 - angle sum of 251–2
 - classifying 248
 - constructing 249–51
 - properties 254–6
- quantities
 - expressed as fractions and percentages 163–4
 - fractions of 144, 146
 - percentage of 160–1, 162
- radical symbol 112
- random experiment 442
- range 416–17, 426
- rates 479
 - conversion 486–7
 - problems 484
- ratios 466–8
 - equivalent 471
 - problems 475–6
 - simplifying 472–3
- rectangle 248, 249–50, 254–5
 - area 316–18
 - perimeter 308
- rectangular prism 337
 - volume 349
- recurring decimals 291, 292
- reflections 208, 209, 210, 216–17, 379–80, 385–6
- reflex angle 44, 50
- relative cell referencing 107
- reverse order (backtracking method) 196
- revolution 50, 62
- rhombus 248, 254–6
- right angle 50, 62
- right-angled triangle 232
- Roman numerals 103–14
- rotational symmetry 225, 229
- rotations 208, 209, 210–11, 218, 379, 380, 381, 387
- rounding
 - decimals 293–4, 296
 - numbers 91
- sample spaces 441, 442
- scale diagrams 475
- scalene triangle 232
 - constructing 236
- sector graphs 397, 400
- side-by-side column graphs 399
- simplifying
 - fractions 134
 - ratios 472–3

- solids
 - cross-sections 338–9
 - drawing different views of 338–40
- sphere 337
- spreadsheets 9–10, 107, 117
- square (geometry) 248, 249, 254–5
 - perimeter 308
- square (power) 108
- square dot paper 338
- square pyramid 337
- square root 108–9, 116
- stem-and-leaf plots 408–9
 - analysing 422, 425–6
- straight angle 50
- substitution 186–7
- subtracting
 - 8 or 9 (mental skill) 27
 - calculating differences (mental skill) 282–3
 - decimals 277–8
 - fractions 139–40
 - integers 16–17
 - mixed numerals 142
- supplementary angles 54, 55
- symmetry
 - line 225
 - rotational 225, 229
- tables of values, graphing 376
- terminating decimals 291–2
- tessellations 228
- theoretical probability 453
- time
 - 12-hour 496–7
 - 24-hour 496–7
 - calculations 492–3
 - converting units 306
- time differences 498–500
- timetables 502
- transformations 208–11, 214–18
 - composite 219, 222–5
 - on number plane 379–81, 385–7
- translations 208, 209, 214–16, 379, 380–1, 387
- transversal 67, 74
- trapezium 248, 254–6
- trapezium transformation 223–4
- trapezoidal prism 337
- travel graphs 487–8
- trial 442
- triangle transformation 222–3
- triangles
 - angle sum of 236–42
 - area 324–5
 - classifying 231–2
 - constructing 234–6
 - exterior angle 244–5
- 12-hour time 496–7
- 24-hour time 496–7
- two-step equations 195–6
- unit cost 481–2
- unitary method 475, 476
- variables 178–81
- vertex 39
- vertically opposite angles 59, 60
- volume
 - and capacity 352, 353
 - metric units 345–6
 - rectangular prism 349
- x-axis 370
- x-coordinate 370
- y-axis 370
- y-coordinate 370

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7

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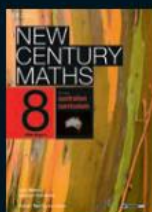
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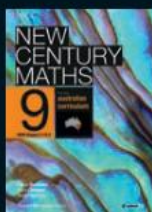
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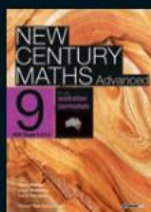
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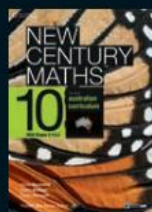
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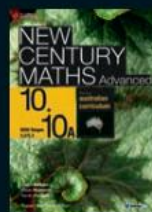
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