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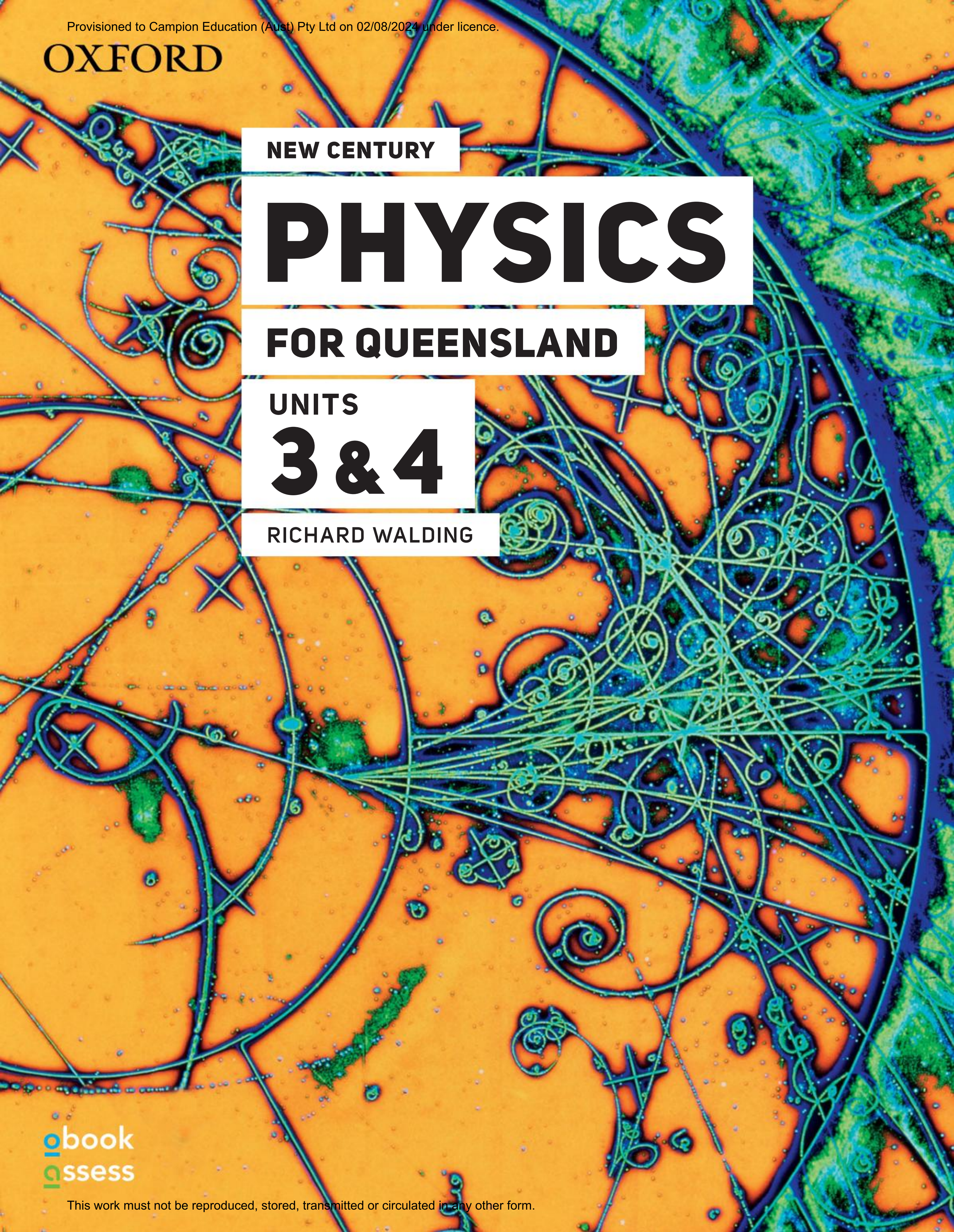

NEW CENTURY

PHYSICS

FOR QUEENSLAND

UNITS 3 & 4

RICHARD WALDING

obook
assess

NEW CENTURY

PHYSICS

FOR QUEENSLAND

UNITS 3 & 4

RICHARD WALDING

OXFORD
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Indigenous Australians and Torres Strait Islanders are advised that this publication may include images or names of people now deceased.

CONTENTS

Using New Century Physics for Queensland Units 3 & 4

New Century Physics for Queensland Units 3 & 4 has been purpose-written to meet the requirements of the QCAA Physics General Senior Syllabus. The second of a two-volume series, *New Century Physics for Queensland Units 3 & 4* offers complete support for teachers and students of Units 3 & 4 Physics, providing unparalleled depth and comprehensive syllabus coverage.

Chapter openers

Each chapter begins with a chapter opener that includes:

- subject matter from the **syllabus**
- a list of the **mandatory** and **suggested practicals** from the syllabus
- key **questions** to get students thinking about the content covered in the chapter.

Physics toolkit

The Student book begins with a stand-alone reference chapter that includes:

- assessment advice
- a step-by-step guide to preparing for your exam
- methods for presenting and analysing physics data.

Unit openers

Each unit begins with a unit opener that includes:

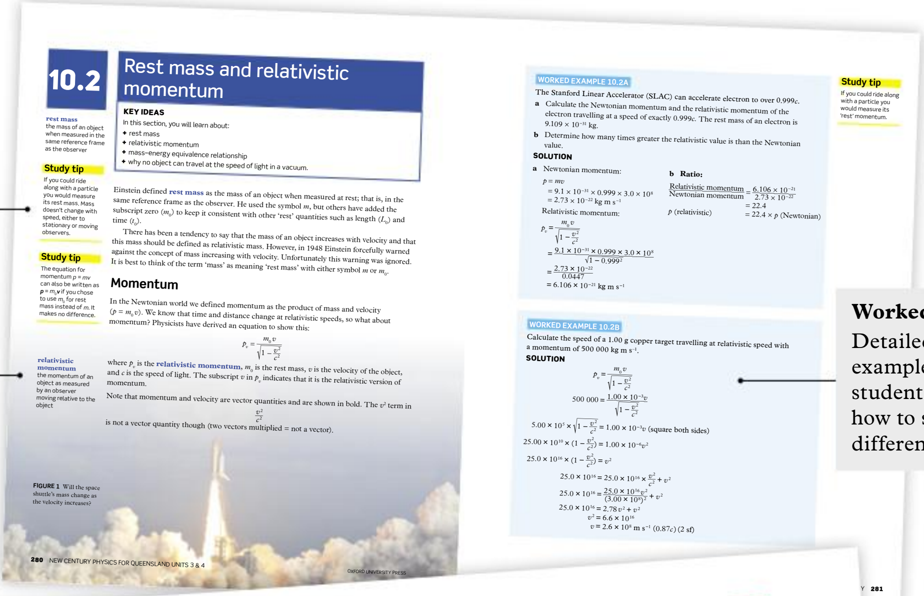
- an **overview of topics** in the unit
- **unit objectives** from the syllabus.

Section-based approach

Content is presented in clearly structured **sections**. Each section is clearly labelled and numbered to help navigation.

Check your learning

Each section ends with questions that revise the content covered in the section and allow students to practise using cognitive verbs.



10.2 Rest mass and relativistic momentum

KEY IDEAS

In this section, you will learn about:
- rest mass
- relativistic momentum
- mass–energy equivalence relationship
- why no object can travel at the speed of light in a vacuum.

Momentum

WORKED EXAMPLE 10.2A

WORKED EXAMPLE 10.2B

Study tips

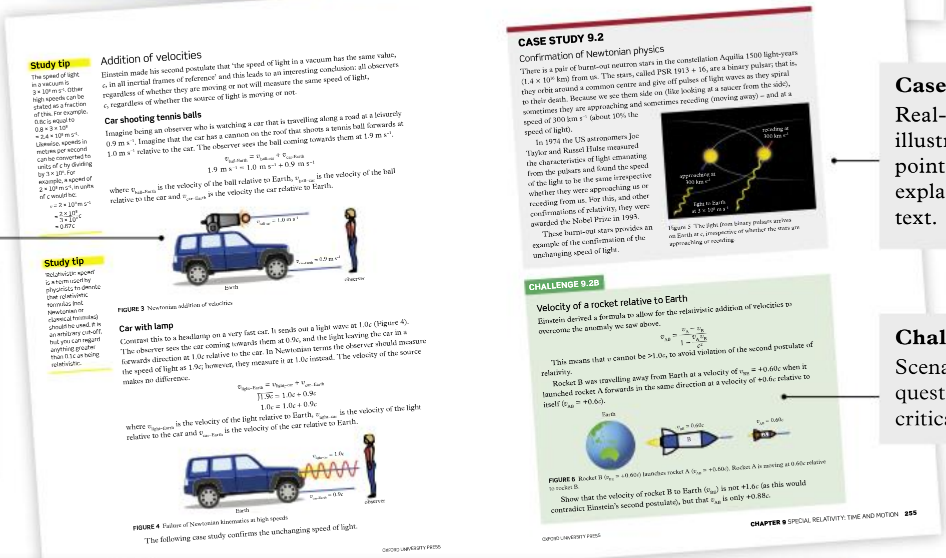
Practical advice helps students improve their performance in assessment tasks.

Margin glossary

Key terms and definitions are highlighted to help students recall important information.

Worked examples

Detailed worked examples take students through how to solve different problems.



Addition of velocities

Car shooting tennis balls

Car with lamp

CASE STUDY 9.2

Confirmation of Newtonian physics

CHALLENGE 9.2B

Velocity of a rocket relative to Earth

Case studies

Real-life examples illustrate theoretical points being explained in the text.

Full-colour diagrams and photos

Rich visual material illustrates concepts and engages students.

Challenges

Scenarios and questions encourage critical thinking.

SCIENCE AS A HUMAN ENDEAVOUR

10.5 Relativity and global positioning satellites

KEY IDEAS

In this section, you will learn about:
- technologies such as satellites that have dramatically increased the size, accuracy, and geographic and temporal scope of datasets with which scientists work
- satellites that provide experimental evidence that supports the phenomena of time dilation.

Development

Special relativity correction

CHECK YOUR LEARNING 10.5

Science as a human endeavour

Real-world contexts promote curiosity and can be used as a starting point for research investigations.

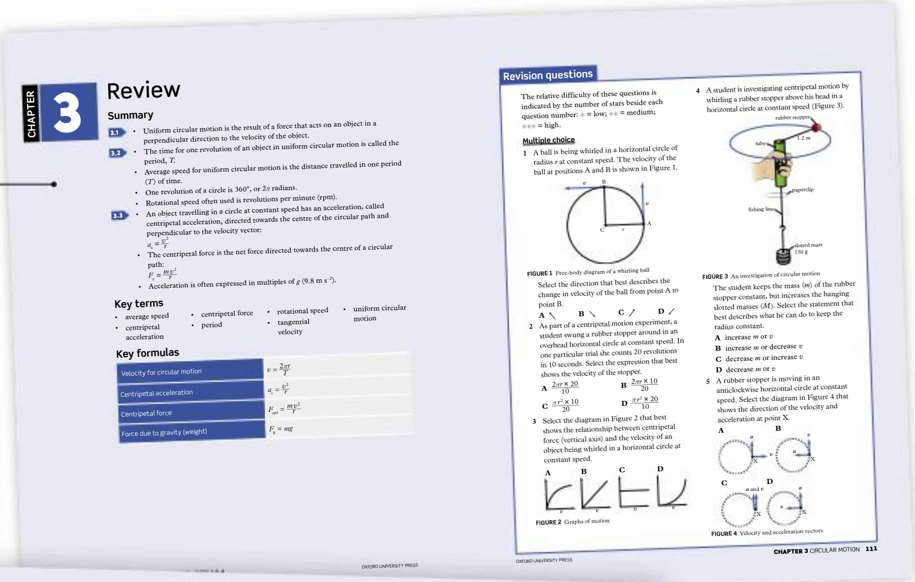
obook assess

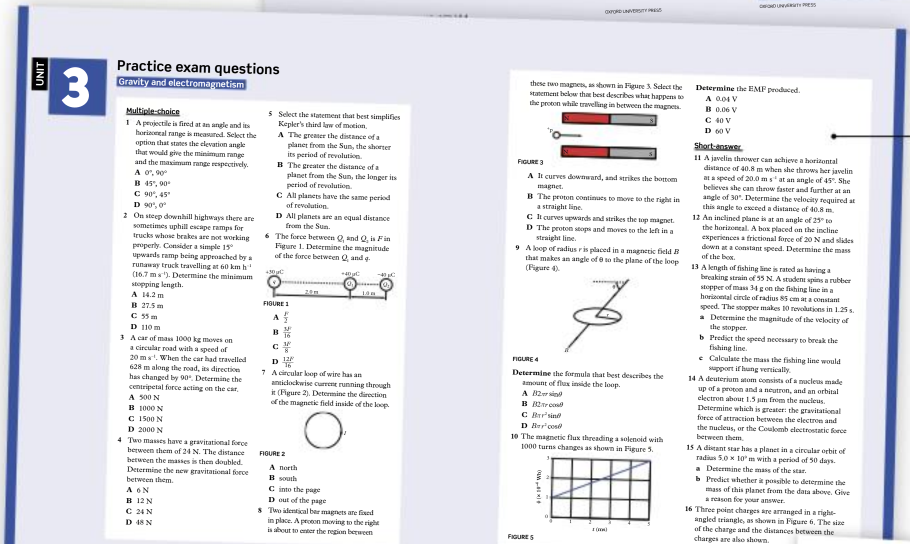
Every section includes links to a range of digital resources that support student learning and assessment.

Chapter reviews

Each chapter review includes:

- a **summary of key learning** in each chapter
- **revision questions** written to target assessment through multiple-choice and short-answer questions
- **key terms** introduced throughout the chapter
- **key formulas** used in the chapter.

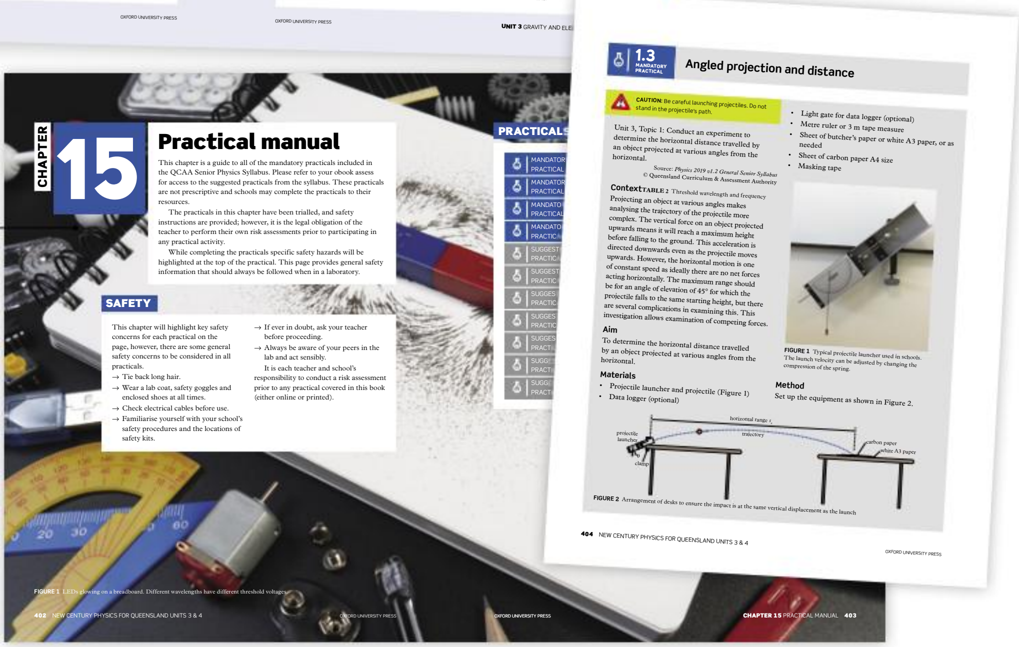




Practice exam questions

Each unit includes a set of practice questions to prepare students for their end-of-year external examination. Questions include:

- **multiple-choice questions** to consolidate learning
- **short-answer questions** with additional guidance on how long students should spend on each question.



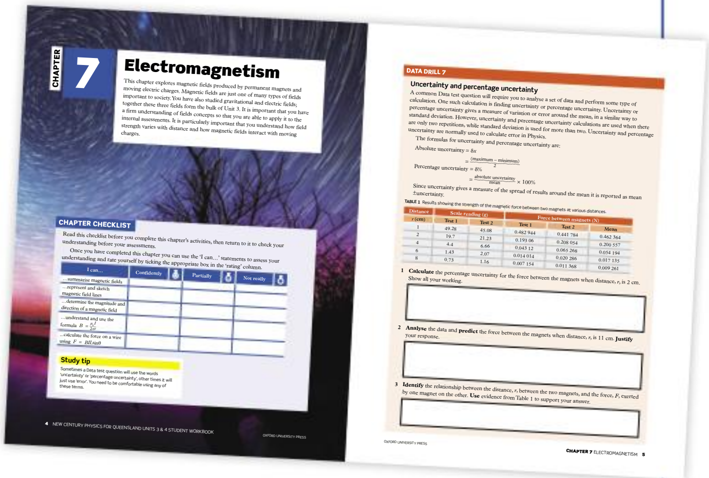
Practical manual

Each mandatory practical from the syllabus has **suggested methodologies and materials** included in the practical manual, and suggested practicals are included via obook assess. Each practical is flagged in the relevant section of the Student book.

Student workbooks

New Century Physics for Queensland Units 3 & 4 and *Units 1 & 2* are supported by two Student workbooks that follow the same structure as the Student books, ensuring that students are consolidating relevant topic knowledge and developing key assessment skills. The workbooks include:

- a **toolkit** chapter that explains each key internal assessment
- **Data drill** activities that allow students to practise analysis and interpretation skills for the Data test
- **Experiment explorer** features to support the modification of a practical as required in the Student experiment
- **Research review** activities to help students develop skills in evaluating a claim and conducting research
- **Exam excellence** questions that include multiple choice and short answer questions to prepare students for the external assessment
- **practice internal assessments** (Data test, Student experiment and Research investigation)
- write-in worksheets for all **mandatory** and **suggested practicals**
- **answers** to all activities and practice assessments.

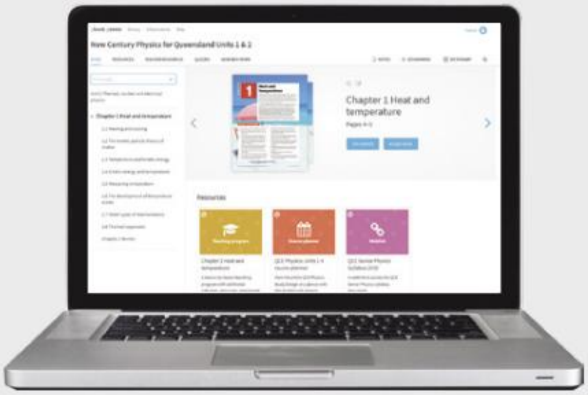


obook assess

New Century Physics for Queensland Units 3 & 4 is supported by a range of engaging and relevant digital resources via obook assess.

Students receive:

- a complete digital version of the Student book with notetaking and bookmarking functionality
- video tutorials demonstrating key skills
- write-in worksheets to accompany all mandatory and suggested practicals
- interactive auto-correcting multiple-choice quizzes
- a range of engaging weblinks to support understanding
- access to work assigned by their teacher: reading, homework, tests, assignments.



In addition to the student resources, teachers also receive:

- detailed planning resources
- Student book answers
- printable (and editable) sample assessments, including data tests and exams with answers
- the ability to set up classes, set assignments, monitor progress and graph results, and to view all available content and resources in one place.

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This syllabus forms part of a new senior assessment and tertiary entrance system in

Queensland. Along with other senior syllabuses, it is still being refined in preparation for implementation in schools from 2019. For the most current syllabus versions and curriculum information please refer to the QCAA website www.qcaa.qld.edu.au.

CHAPTER 0

The physics toolkit

The natural world is an intriguing and exciting place. Physicists, like other scientists, try to explain the natural world. Many of the greatest theories that physicists have developed have come from asking ‘why’, ‘what’, ‘how’ and ‘when’. These questions are key to being a scientist. Physicists then try to solve these by conducting research and experiments, which collect data that they then analyse using a variety of methods.

Humans have made measurements since they began questioning. These measurements mean very little without analysis and further work. Throughout your Senior Physics course you will be exposed to data collection and analysis, practical work, investigating science through research and designing your own experiment.

This chapter should be used as a toolkit. It does not need to be studied in one sitting, but is something that you can refer to throughout your Units 3 & 4 course. It has guidance on making accurate and precise measurements of physical quantities, how you can report these results and what they imply. It also has guidance for key assessment that you will come across over your course.

FIGURE 1 Physicists use devices, such as voltmeters, to measure natural phenomena.

MAKES YOU WONDER

This chapter is called the zeroth chapter to commemorate a highpoint in the history of physics. The zeroth law of thermodynamics is called the ‘zeroth’ law because it was developed after the first and second laws of thermodynamics had already been proposed and named, but was considered more fundamental and thus was given a lower number – zero. Just like this chapter.

In this chapter you will learn about the different ways physicists make measurements and interpret these measurements, and answer questions such as:

- → Is there more to a career in physics than just looking for relationships between physical quantities?
- → Do modern physicists get things named after them, or is it just those from olden days?
- → If you can’t linearise your data does this mean it’s wrong?
- → What do errors in experiments mean?

0.1

Understanding the QCAA Senior Physics course

KEY IDEAS

In this section, you will learn about:

- course structure
- key dates and assessments in Senior Physics
- QCAA Senior Physics internal and external assessment.

The requirements of the QCAA Senior Physics course are set out in the syllabus document. You can find a link to this via your obook assess. This section outlines key topics and assessments throughout Units 3 & 4 Physics.

Structure of Units 3 & 4 Physics

Physics Units 3 & 4 includes the following units and topics:

TABLE 1 Unit 3 Gravity and electromagnetism

Topic		Corresponding chapters
1	Gravity and motion	Chapter 1 Vectors and projectile motion Chapter 2 Inclined planes Chapter 3 Circular motion Chapter 4 Gravitational force and fields Chapter 5 Orbits
2	Electromagnetism	Chapter 6 Electrostatics Chapter 7 Magnetic fields Chapter 8 Electromagnetic induction and radiation

TABLE 2 Unit 4 Revolutions in modern physics

Topic		Corresponding chapters
1	Special relativity	Chapter 9 Special relativity: time and motion Chapter 10 Special relativity: length, momentum and energy
2	Quantum theory	Chapter 11 Quantum theory and light Chapter 12 Quantum theory and matter
3	The Standard Model	Chapter 13 The Standard Model Chapter 14 Particle interactions

Assessment

The QCAA has outlined three internal assessments; the data test, the student experiment and the research investigation. At the end of Unit 4, students will also sit an external examination.

Throughout the year you will complete these internal assessment as part of your coursework. Section 0.2, 0.3 and 0.4 provide guidance about each of these tasks. Before completing any of these, you should consult your teacher for their advice and if they have any preferred presentation styles. You should also familiarise yourself with the QCAA Senior Physics Syllabus, which includes ISMGs (Instrument Specific Marking Guides). When you read through each of the internal assessments you should ensure that you understand the key requirements of each.

Section 0.5 in this chapter provides guidance on exam preparation. Part of this is familiarising yourself with the syllabus, and ensuring that you follow guidance from your teacher for the exam structure.

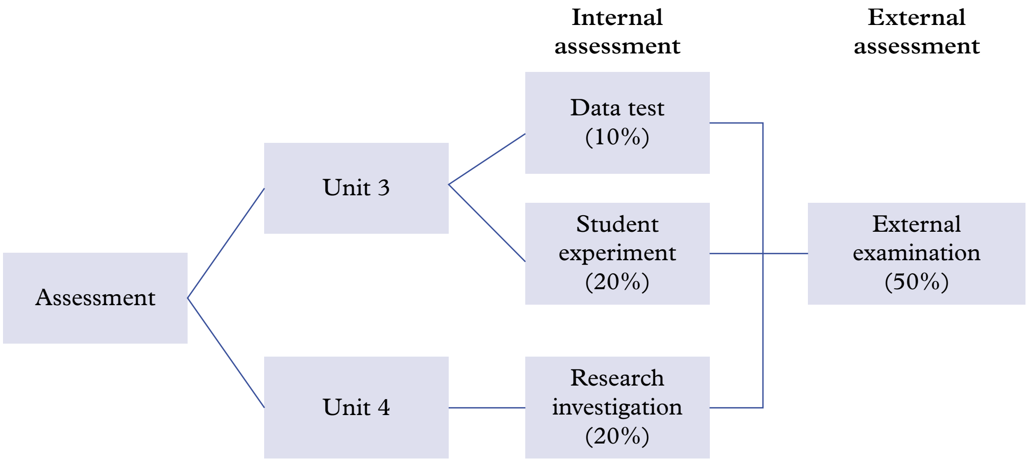

FIGURE 1 Differentiation between Internal Assessments (IA) and end of year assessment

Cognitive verbs

During your assessment tasks and examinations, questions will be presented to you as a command using **cognitive verbs**. Cognitive verbs are task words that provide information about what an answer requires.

cognitive verb
task word that provides information about what an answer requires

It is important that you understand the difference between different cognitive verbs and the level of response they require. For example, a question that asks you to **compare** is different from one that asks you to **contrast**. One requires you to show similarities and differences, while the other is only asking you to show the differences.

Understanding exactly what a cognitive verb is asking means that you will be prepared to write exactly what your teacher, or an examiner, is looking for. Examiners want to give you marks, but this can only be done if you provide the right information and make it easy to see. For example, if you describe data when the question wanted it analysed, you will not be eligible for full marks.

Table 3 on the next page provides a sample of cognitive verbs used throughout this book and in your assessments. A full list of QCAA cognitive verbs is provided on your obook assess.

TABLE 3 Cognitive verbs

Cognitive verb	Definition	Sample question
Assess	Measure, determine, evaluate, estimate or make a judgment about the value, quality, outcomes, results, size, significance, nature or extent of something	**Assess** the effect on their experiment if students noticed that some of the loops in the coil were touching each other.
Calculate	Determine or find (e.g. a number, answer) by using mathematical processes; obtain a numerical answer showing the relevant stages in the working; ascertain/determine from given facts, figures or information	**Calculate** the resistance of a 50 m length of silver wire of cross-sectional area of 0.50 mm^2 at 20°C.
Classify	Arrange, distribute or order in classes or categories according to shared qualities or characteristics	**Classify** the following units as SI or non-SI: metres, kilograms, pounds, kelvin.
Compare	Display recognition of similarities and differences and recognise the significance of these similarities and differences	**Compare** free convection and forced convection.
Consider	Think deliberately or carefully about something, typically before making a decision; take something into account when making a judgment; view attentively or scrutinise; reflect on	**Consider** what the forces would be if a very large mass of 500 kg was placed on the table.
Construct	Create or put together (e.g. an argument) by arranging ideas or items; display information in a diagrammatic or logical form, make; build	**Construct** a force–displacement graph of a school bag of mass 10 kg being raised vertically at constant speed to a height of 2.0 m.
Deduce	Reach a conclusion that is necessarily true, provided a given set of assumptions is true; arrive at, reach or draw a logical conclusion from reasoning and the information given	**Deduce** how much heat transfer occurs from a system if its internal energy decreased by 350 J while it was doing 50 J of work.
Describe	Give an account (written or spoken) of a situation, event, pattern or process, or of the characteristics or features of something	**Describe** the type of decay when there is a surplus of protons.
Design	Produce a plan, simulation, model or similar; plan, form or conceive in the mind	**Design** an experiment to investigate the intensity of light through a pair of polarisers as one of the polarisers is rotated through 180°.
Determine	Establish, conclude or ascertain after consideration, observation, investigation or calculation; decide or come to a resolution	**Determine** the amount of C-14 as a percentage of the C-14 in living tissue if the plant was living 2000 years ago.
Differentiate	Identify the difference/s in or between two or more things; distinguish, discriminate; recognise or ascertain what makes something distinct from similar things; In mathematics, obtain the derivative of the function	**Differentiate** between the scale reading uncertainty for an analogue (printed) scale and a digital scale.
Distinguish	Recognise as distinct or different; note points of difference between; discriminate; discern; make clear a difference/s between two or more concepts or items	**Distinguish** between the nuclear strong force and the electrostatic (Coulomb) force.
Evaluate	Make an appraisal by weighing up or assessing strengths, implications and limitations; make judgments about ideas, works, solutions or methods in relation to selected criteria; examine and determine the merit, value or significance of something, based on criteria	**Evaluate** this statement: 'beta positive decay and electron capture are the same thing'.
Explain	Make an idea or situation plain or clear by describing it in more detail or revealing relevant facts; give an account; provide additional information	**Explain** what it means to have a fission chain reaction.

Cognitive verb	Definition	Sample question
Identify	Distinguish; locate, recognise and name; establish or indicate who or what someone or something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature	**Identify** the formula that links the quantity symbols s, v, t and a.
Interpret	Use knowledge and understanding to recognise trends and draw conclusions from given information; make clear or explicit; elucidate or understand in a particular way; … Identify or draw meaning from, or give meaning to, information presented in various forms, such as words, symbols, pictures or graphs	**Interpret** the data and construct a force versus displacement graph for the spring.
Investigate	Carry out an examination or formal inquiry in order to establish or obtain facts and reach new conclusions; search, inquire into, interpret and draw conclusions about data and information	**Investigate** the following question: Was the strong nuclear force 'invented' in the 1970s or was it 'discovered'?
Justify	Give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable	**Justify** the conclusion that the collision was elastic.
Modify	Change the form or qualities of; make partial or minor changes to something	**Modify** the following conclusion to make it more acceptable: 'Because $V = IR$, the resistor must be ohmic'.
Predict	Give an expected result of an upcoming action or event; suggest what may happen based on available information	**Predict** the shape of the spring when two pulses interact.
Propose	Put forward (e.g. a point of view, idea, argument, suggestion) for consideration or action	**Propose** a reason for why many people die in intense bushfires even though they have not been touched by the flames.
Sketch	Execute a drawing or painting in simple form, giving essential features but not necessarily with detail or accuracy; In mathematics, represent by means of a diagram or a graph; the sketch should give a general idea of the required shape or relationship and should include features	**Sketch** the following vector quantities on a directed number line: 30 m E and 100 m W.

CHECK YOUR LEARNING 0.1

Describe and explain

1 **Identify** the three internal assessments.

2 **Recall** the topics covered in Unit 3 and those covered in Unit 4.

3 **Identify** the topics that appeal most to you, and those that you think you might find challenging.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 0.1

» Increase your knowledge
QCAA cognitive verb list

» Weblink
QCAA General Senior Physics syllabus

0.2

The data test

KEY IDEAS

In this section, you will learn about:

- the data test assessment task
- how to prepare for a data test.

The data test is the first of the four mandatory assessment tasks you will undertake in your course of study, and is worth 10% of your final grade. It is a one-hour test with 10 minute perusal time at the start that asks you, individually, to respond to questions about case studies, activities or the mandatory and suggested practical work you have done. You will be asked to provide answers to short-response items requiring single-word, sentence or short paragraph responses, and others that will require calculating using algorithms (formulas) and interpreting datasets. You will not be expected to describe and explain facts, theories and physics principles, only to apply them. It should be noted that the test is made up by your teacher and approved (endorsed) beforehand.

Preparing for a data test

The key to doing well on a data test is to make sure you do all of the practical work well. It is no good just standing back watching others do it. There is an old saying, 'I see, and I remember; I do, and I understand', which sums up the best approach to data tests. But it has to go further than just doing. During a practical or activity, you need to manipulate the equipment, collect data and analyse it to get meaning from it.

Datasets

The data test will provide you with three or more sets of data drawn from your experiments, and you will have to show that you can understand, analyse and interpret the data. It is highly likely that the datasets will be of two types: graphs and data tables.

Study tip

In your data test, you may find it useful to sketch a new graph to help you interpret data. However, always remember that graphs are not required, so any graphs you draw will not be assessed.

Graphical datasets

When you look at a graph, consider the following:

- What quantities are being plotted? (What quantities and units are shown on the x-axis and y-axis? For example, x-axis is force, F, and y-axis is velocity squared, v^2.)
- What formula links these variables (check your data booklet)? $F_c = \frac{mv^2}{r}$
- Is the graph linear, if so, what is does the gradient represent $\left(\frac{\Delta y}{\Delta x}\right)$? Gradient $= \frac{\Delta(v^2)}{\Delta F_c}$
- If the graph is linear, state the relationship in the form $y = mx + c$, where m is the gradient, and c is the intercept on the y-axis (e.g. $F_c = 1.25v^2 + 0.06$).
- Rearrange the formula to show what the gradient represents: gradient $= \frac{v^2}{F_c} = \frac{r}{m}$.
- If the gradient value is given, use the value to solve the equation for the unknown. If the gradient is not given, determine its value. For example, once you know the gradient, and the value of say r, you can solve for m.
- Use error bars (if shown on the graph) to determine the gradients for the maximum and minimum linear lines of best fit, and hence calculate the uncertainty of the gradient: $F_c = (1.25 \pm 0.11)v^2 + 0.06$.

- Does the trendline pass through the origin? If not, it may indicate a systematic error. Propose causes and solutions.
- Does the trendline pass through many of the scattered points? If not, it indicates random error (lack of precision and high uncertainty). Propose causes and solutions.
- Predict values by interpolation or extrapolation.
- Determine the impact on the data of changes to the experiment, such as changing the mass, temperature, force, distance and so on.
- State whether a given relationship is supported by the data.

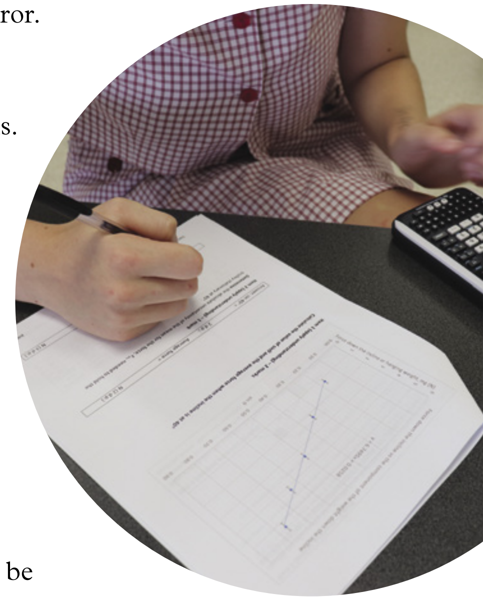

FIGURE 1 A student doing a data test

Tabular datasets (tables)

Typically, determine:

- the average
- the uncertainty of repeated results
- any missing values
- trends – check to see whether the dependent variable is doubled when the independent variable is doubled. If so, the trend is likely to be linear; if not it may be some power relationship, but you wouldn't be expected to determine the relationship
- the values of the dependent variable when independent value is halved, doubled and so on.

If you do the mandatory and suggested experiments as provided in your Student book and obook assess, and answer the questions as written, you will be well prepared for any question on a data test. But you need to do them yourself and not just read through the solutions of other students or provided by your teacher.

Summary

The data test expects you to apply understanding, and analyse and interpret evidence. Although questions are set by teachers in schools, they generally prescribe the following cognitions. These examples follow a sequence used in practical reports.

Apply understanding (calculate, determine, identify, recognise)

- Calculate the average from a set of data.
- Determine missing values from a data table by working backwards from the average.
- Determine absolute and/or percentage uncertainty from set of data.
- Identify an anomaly or outlier.

Analyse evidence (categorise, classify, contrast, distinguish, organise, sequence, identify trends, patterns, relationships limitations or uncertainty)

- Identify/recognise the relationship between two variables in a data table and use evidence to support the claim.
- Identify the relationship from a graph and justify by use of evidence.
- Identify factors affecting shape of graph.
- Analyse the data to test a claim.

Interpret evidence (compare, deduce, extrapolate, infer, justify, predict, draw conclusions)

- Predict/infer the shape of graph if variables change.
- Compare the gradient of a line of best fit for a linear or linearised graph with maximum and minimum gradients and their y-intercepts.
- Deduce absolute and/or percentage uncertainty in gradient and/or y-intercept.
- Predict values by extrapolation or interpolation.
- Propose the meaning of the gradient in terms of a physical quantity.
- Determine percentage and absolute uncertainty in physical quantity derived from graph.

Data test example

The following is an example and its worked solution.

A student set up the apparatus shown in Figure 2 to investigate the force acting on a conductor in a magnetic field. The current-carrying wire was fixed in a clamp and the magnets were placed on an electronic balance. The student varied the current through the conductor and measured the tared mass reading on the mass balance. The data was processed and plotted in Figure 3.

The effective length of the wire in the magnetic field was 2.0 cm and the wire was orientated at 90° to the magnetic field.

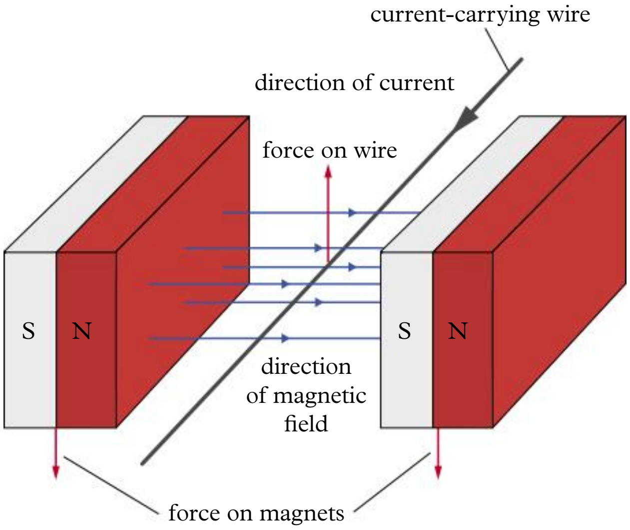

FIGURE 2 Apparatus to measure the force on a current-carrying conductor in a magnetic field

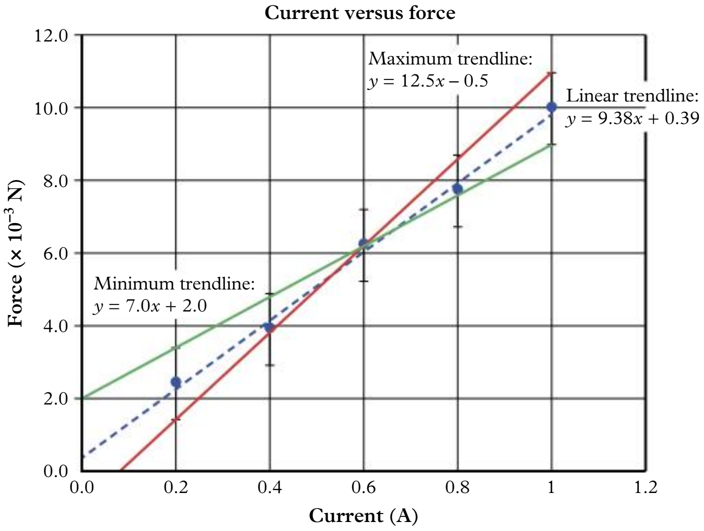

FIGURE 3 A graph of current versus force for the experiment

1 Analyse evidence

Identify a mathematical relationship between the force acting on the conductor and the current passing through the conductor, including the uncertainty of the gradient and the y-intercept. [3 marks]

Worked solution

$$\text{Uncertainty in gradient: } \delta x = \frac{x_{\max} - x_{\min}}{2}$$

$$= \frac{12.5 \times 10^{-3} - 7.0 \times 10^{-3}}{2}$$

$$= 2.75 \times 10^{-3}\ \text{N A}^{-1}$$

$$\text{Percentage uncertainty: } \delta\% = \frac{\delta x}{x_0} \times 100$$

$$= \frac{2.75 \times 10^{-3}}{9.38 \times 10^{-3}} \times 100$$

$$= 29.3\%$$

$$F = (9.38 \times 10^{-3} \pm 2.75 \times 10^{-3})I + 0.39 \times 10^{-3}\ \text{N}$$

2 Interpret evidence

Draw a conclusion that quantifies the magnitude of the magnetic field through which the wire is passing, including the absolute or percentage uncertainty in the value you determine. Show your reasoning. [4 marks]

Worked solution

$$F = BIL\sin\theta$$

$$B = \frac{F}{I \times L}\ (\sin 90° = 1)$$

$$= \frac{\text{gradient}}{L}\ [\text{as } \frac{F}{I} = \text{gradient}]$$

$$= \frac{9.38 \times 10^{-3}}{0.020}$$

$$= 0.469\ \text{T} \pm 29.3\%$$

$$= 0.47 \pm 0.14\ \text{T (2 sf)}$$

The magnetic field strength is 0.49 ± 0.14 T.

CHECK YOUR LEARNING 0.2

Describe and explain

1 **Explain** the cognitive verbs that are associated with the assessment objective ‘apply understanding’.

2 You have been asked to ‘draw a conclusion based on analyses’. **Explain** if this is associated with ‘analysing evidence’ or ‘interpreting evidence’.

Investigate, evaluate and communicate

3 The cognitive verb ‘identify’ is often used when requiring a response about uncertainty in the data. **Explain** if this means that you will need to calculate absolute or percentage error.

4 A particular data test item includes a table of data and you have been asked to ‘identify whether a relationship is present’. **Decide** if you would be expected to draw a graph to determine the relationship by visual inspection.

5 A data test item asks you to ‘justify’ your conclusion. **Determine** if it is better to support it with examples from the data or to refer to theory that you have learnt.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 0.2

» Video
The Data Test

» Increase your knowledge
Interpreting different types of graphs

0.3 The student experiment

KEY IDEAS

In this section, you will learn about:

- the student experiment assessment
- modifying an experiment.

The student experiment is part of the assessment for Senior Physics in Queensland that you will undertake in Unit 3. It is worth 20% of your final grade and involves selecting an experiment that has already been completed in class and modifying it in order to address a hypothesis or research question.

Developing a research question

The purpose of the research question is to guide the direction of the research and analysis. The research question can be developed using five steps:

- Identify the independent variable to be investigated.
- Identify the dependent variable.
- Identify the methodology to be used.
- Draft several research questions.
- Refine and focus one research question.

The statement of the research question will have to be specific and relevant, and should mention the dependent and independent variables. It must show the relationship with the original experiment. For example, consider Mandatory practical 7.2 'Strength of a magnet at various distances'. The original research question might have been: *What is the relationship between the force exerted by a bar magnet on another identical bar magnet when separated by distances between 0.5 cm and 10 cm?* This will be used as an example throughout this section.

Based on your results when completing this mandatory practical, you may pose a research question to address an error in the practical or a curious result.

Developing a rationale for the experiment

The rationale is the **purpose** of the experiment – that is, what the experiment aims to achieve and the **reasons** for conducting the experiment. The reasons for modifying an experiment should include one of:

- a *refinement* because the original experiment wasn't accurate or precise enough. It is known that the length of a permanent magnet is important because the poles on the opposite ends of the magnets interact. In fact, depending on the length, the relationship could vary between $F = \frac{1}{d^2}$ and $F = \frac{1}{d^3}$, with a typical value for school magnets of $F = \frac{1}{d^{2.9}}$). Similarly, background fields from Earth, wires in the room, and nearby steel in the bench, sink and taps are environmental factors that need to be controlled. A refined research question could be: *What effects do the lengths of permanent magnets have on the force between them as a function of separation distance?*
- an *extension* in which a relationship between two variables was observed but the data didn't extend beyond that range of parameters. There is a problem with keeping magnets perfectly aligned, particularly in the repulsive case where the magnets tend to push to one side.

The repulsive and attractive conditions produce different results unless strict controls are imposed. An extension research question could be: *Does the force between two permanent magnets vary with distance identically for both repulsion and attraction?*

- a *redirection* because a certain natural factor affected results and its relationship to the original variables needs to be assessed. The temperature of a magnet is known to affect its field strength, so an investigation of this variable is warranted. Using two solenoids as the source of the magnetic field may allow for greater control of the variables. A redirected research question could be: *How does magnetic field strength of a permanent magnet vary with temperature?*

It is a simple step to develop a hypothesis from a research question. For example, the research question 'How does magnetic field strength of a permanent magnet vary with temperature?' could become 'As the temperature of a permanent magnet is increased, the field strength will decrease'. In this hypothesis, the independent variable is the temperature of a permanent magnet and the dependent variable is the field strength. If you haven't yet researched the theoretical side of magnetism and do not know the direction of any change in your dependent variable, you could just say '… the magnetic field strength will change'.

The rationale needs to describe and explain theoretical relationships for the variables under consideration.

FIGURE 1 Arduino boards can use sensors such as magnetometers to collect field strength data. It is essential that all aspects of your experiment are 'hands-on' for all group members.

Method

The method must justify how the modifications will refine, extend or redirect the original experiment. It needs to show how the variables will be manipulated and measured, and the others controlled.

Manipulating the independent variable

The independent variable can be manipulated by **trials** and **repetitions**. Trials are variations of the independent variable. Five trials should be conducted if the relationship is expected to be linear. If the relationship is expected to be non-linear, further trials should be conducted. Repetitions are additional test measurements made under exactly the same methodology, conditions and materials.

Measuring variables

Gather information about how the variables are to be measured. This includes what instrument is to be used, how it works, how it is connected, and what the techniques are for using it and reading it accurately. Uncertainties involved in the measuring process should also be acknowledged. The only variables that should be listed are those that will have an effect on the experiment.

Using a logbook

Using a logbook, electronic or otherwise, will help you to record your data accurately as you conduct the experiment. This will make it easier to interpret your results. See your obook assess for help in setting up a logbook.

Communicating your findings

After you complete your experiment, you need to present these findings. Typically, scientific results are presented in a report, but there is also opportunity to present your results as a poster presentation or journal article.

Regardless of the medium you choose to present your findings in, you will need to include the following information.

Introduction

The introduction must address the research question and hypothesis, and explain the rationale for the experiment. Start with a statement of the research question, then the rationale. In your introduction you should discuss and explain theoretical relationships, as well as explaining the original experiment and why any modifications were required. You should address the relationship between the independent and dependent variables, and explain any theory behind this. This should include detailing any assumptions that are made about the use of formulas, and predict any expectations about the outcome. At the end of your introduction you should have a summary statement that ties everything together.

Study tip

If a table and graph represent the same data, they should share the same name.

Study tip

An appendix can be included to show anything that does not fit into a data table. If you choose to include an appendix, you must reference it in your results.

Method

The method needs to address the method used in the original experiment and how this has been modified in the student experiment. Your method also needs to include a risk assessment. This is to manage risks that may be associated with the experiment, so that anyone who repeats the experiment can avoid harm.

The method should include exact measurements that were made, equipment used and timing of measurement. This will aid anyone who repeats the experiment.

Results

The results of the experiment should be presented, but not analysed or discussed, in this section. Results should be presented using data tables and graphs. A data table should state what has been measured and show raw data that is displayed in forms appropriate for the data. For example, if data about temperature was recorded in Celsius, it should be displayed in Celsius, not kelvin.

Any sample calculations that were conducted should also be represented here. This includes any manipulation of the data, including calculating the average and calculations of quantities.

Analysis of evidence

Any trends in the data should be apparent after the results section. This is usually done using graphical analysis, including the addition of error bars. A relationship between the variables that is applicable to the research question should be identified. The relationship should not be superficial or partial. If the relationship is non-linear, then a relationship should be suggested and linearising attempted.

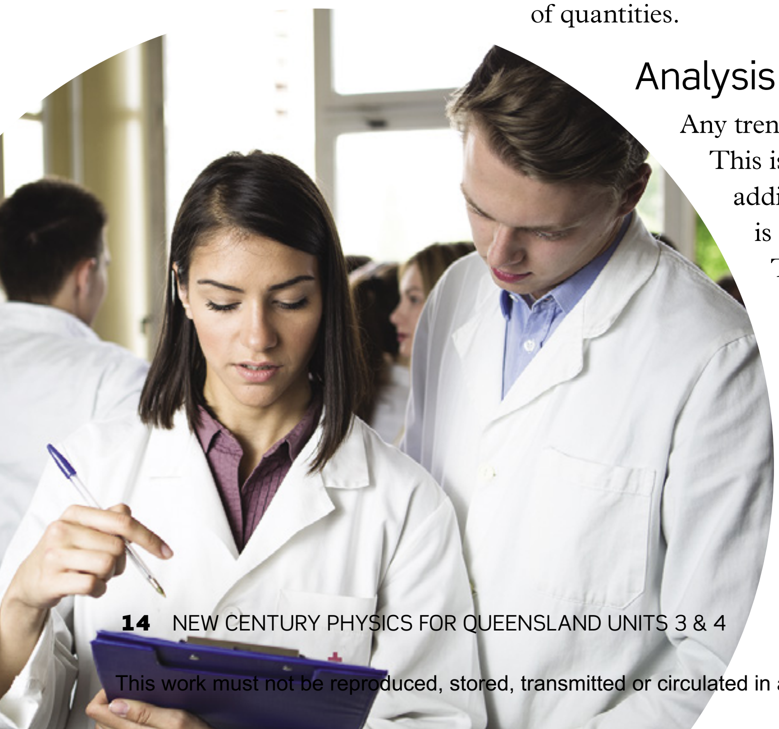

FIGURE 2 While conducting your experiment, record results in a table in your logbook and look for possible anomalies and outliers so that you can identify any problematic parts for retrials.

Any limitations of the evidence should also be described here. This will also include an error analysis of the data in terms of precision (uncertainty) and accuracy. Your findings and comparison with theoretical expectations must only be considered within the parameters of the experiment. That is, if there is no known accepted value then you would not be able to determine accuracy.

Discussion

The discussion is where you interpret the experimental evidence. A justified conclusion can be provided that links to the research question. In this section, the research question should be restated, as well as stating whether the hypothesis, if proposed, was supported or not.

A summary of the results should be presented to state whether the research question was supported or not. Any evidence or examples used should be specific and relevant. Each piece of evidence that you reference to support your conclusion needs to be explained.

Conclusion

The experiment should be summarised and given closure. It should be clear that a logical and reasonable argument has been made from valid and accurate data that is supported by trustworthy and relevant theory to generate logical conclusions.

Study tip

See Section 0.7 of *New Century Physics for Queensland Units 1 & 2* for examples of how to add custom error bars and maximum and minimum gradient linear lines of best fit.

Study tip

If linearising is achieved, lines of best fit can be added that have the maximum and minimum gradients within the error bars. From this, the uncertainty in the gradient can be expressed.

CHECK YOUR LEARNING 0.3

Describe and explain

1 **Explain** the five steps in developing a research question.

2 **Identify** the three ways a practical can be modified for use in a student experiment.

3 **Recall** whether a hypothesis is mandatory.

Apply, analyse and interpret

4 **Deduce** which one of the three ways a practical can be modified that you identified in Question 2 applies to the following scenario: 'Air resistance unexpectedly affected your earlier results and its relationship to the original variables needs to be assessed'.

Investigate, evaluate and communicate

5 **Decide** whether you would include suggestions for overcoming 'mistakes' in the evaluation section of your report. Justify whether they fit the definition of improvements or extensions.

6 You used a ping-pong gun to investigate the effect of angle of elevation on horizontal range. However, you now realise that air resistance may have been significant. **Propose** a new research question that allows you to modify your original experiment.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 0.3

» Video
The Student Experiment

» Increase your knowledge
Further information on the student experiment

0.4 The research investigation

KEY IDEAS

In this section, you will learn about:

- tackling the research investigation.

The research investigation is an assessment task that you will undertake in Unit 4. The research investigation will contribute to 20% of your final grade. It is a non-experimental task that requires you to evaluate a claim about a significant issue from your study of physics.

Your teacher will provide you with a list of claims. Select a claim from this list, and develop a research question based on the claim. There are essentially two parts to the research investigation. First, you need to obtain **secondary evidence** for your research question through scientifically credible sources. Then, based on the research you have conducted, you will have to reach a justified conclusion about the truthfulness of the claim.

secondary evidence
data that has been compiled from records of primary sources by someone not directly involved in the primary event

This assessment requires individual research and writing, and can be conducted both during class time and in your own time, to develop your response.

Selecting and researching a claim

When choosing the claim for the foundation of your research question, you should consider the following points:

- Does this topic interest me?
- Have I learnt about this during my Senior Physics study?
- Do I understand this claim and topic?

If you select a claim that meets the above criteria, it will be easier for you to research this topic.

Once you have selected a claim, you need to identify the key terms and to question these so that you can determine the relationship that exists between the variables. It can help if you identify the independent and dependent variables in your research question. After you have selected the claim, you need to ensure that you read broadly to see if the scientific literature provides evidence that supports or refutes the claim.

FIGURE 1 The research investigation is a key part of your assessment in Unit 4. It requires you to work individually throughout the task.

For example, consider the claim, ‘space goes on forever’. The key terms in this claim would be ‘space’ and ‘forever’. The relationships between these should link the cause and effect. For example, space is being created (cause) and thus we’ll never reach the end of it (effect).

You will also need to consider if you are dealing with a measurable variable. In the above example, ‘space goes on forever’, you would need to determine how to measure the end of space and if this is possible.

Credible sources for research

When conducting scientific research it is important to use credible sources. This ensures the reliability and validity of any results and studies that you may choose to read. Some good starting points are:

- Google scholar
- Physics journals such as
  - *The Physics Teacher*
  - *The American Journal of Physics*
  - *Physics Today*
- Physics databases such as The AAPT ComPADRE Digital Library
- library and university searches
- government sources.

Study tip

You can use your obook assess to find links to research databases.

Gathering scientific evidence

When researching, try to approach the scientific literature without a preconceived idea. As you read through the literature, you may find evidence that both supports and refutes your claim. A research investigation must consider the claim as a whole – this means considering and including evidence that both supports and refutes the claim. A good argument considers both sides.

Interpreting scientific evidence

You should interpret the evidence presented in the literature by drawing upon your scientific understanding. This will include separating the evidence that supports your argument and the evidence that contradicts your argument. You will then be able to use the scientific literature to construct an argument. The argument needs to directly answer your research question.

You should look at the results, the method used, and the way that the data was collected and analysed across all sources.

Presenting your findings

Once you have conducted the research, you need to present your findings. The research investigation can be presented as a report, journal article, essay, poster or presentation. You should consult with your teacher to see if there is a preferred option. Regardless of the way your report will be presented, you will need to address the following areas.

Research question

You need to state the claim and provide a rationale for how you developed the research question. This will involve explaining any critical thinking processes that were used during your research and analysis of the research.

Provide an overview of the evidence

You must also present your argument and discuss both sides with supporting and refuting evidence. You should also explain the physics behind the claim and any cause-and-effect relationships. Using the example of space, you may need to explain how space is being created by expansion, the relevant properties of the expansion of space and movement within space, how light travels and how the expansion of the universe can be faster than the speed of light. Explain this using evidence from the literature.

Analysis and interpretation

There are two parts to each argument: analysing data and interpreting the data.

When providing the analysis of data, provide a clear topic sentence that contains sufficient and directly relevant scientific evidence presented in a systematic and accurate way. Any patterns, trends and relationships in the data that support the argument should also be identified. Examine the evidence thoroughly to identify any limitations such as data that is out of date, data that may not be **reliable** and valid, or data outside of the range of the physical conditions specified in the question.

reliable
constant and dependable, or consistent and repeatable

To interpret the data, a topic sentence should include evidence that shows direct links to the claim and the research question. The argument must critically evaluate the validity and reliability of the claim showing how evidence supports the answer to the research question.

Conclusion and evaluation

Your conclusion must consider the limitations identified in the analysis of the data and how these affect the use of evidence to evaluate the claim. These limitations may include evidence of other studies, or your own investigation. The evaluation of the claim should support or refute the claim within the limitations of the evidence identified in the analysis. It should be easily understood and avoid unnecessary repetition.

Any improvements to the investigation should be presented, including limitations of the evidence. Any extensions that would complement the findings of the investigations should also be provided.

Your research investigation should finish with a strong statement explaining how you have used sound reasoning and valid and reliable evidence to support conclusions that directly respond to the claim.

CHECK YOUR LEARNING 0.4

Describe and explain

1 **Explain** whether you have to choose a claim from a list of suggestions or if you can negotiate another.

2 **Define** 'limitations' of the evidence and provide an example.

3 **Clarify** whether the claim you have selected has to be true.

Investigate, evaluate and communicate

4 **Generate** a research question from the claim 'you can't travel faster than light'.

5 **Generate** a relevant research question from the claim that 'the dream of almost limitless clean energy from nuclear power is close to being realised'.

Check your obook assess for these additional resources and more:

- » **Student book questions**
  Check your learning 0.4
- » **Video**
  The Research Investigation
- » **Increase your knowledge**
  Conducting scientific research
- » **Weblink**
  Research databases

0.5

External assessment

KEY IDEAS

In this section, you will learn about:

- an overview of the external assessment
- the structure of the external examination.

Your final assessment is the external examination, which accounts for 50% of your final Physics grade. The external examination will cover information from both Units 3 and 4, so it is important that you are continuously revising throughout the year. All Senior Physics students studying Units 3 and 4 will sit the same exam, at the same time, on the same day.

Being able to understand physics concepts is important – but being able to show it in exams requires different skills. Here we've collected together our best advice.

What does the external assessment look like?

The external assessment will consist of two papers, each of 90 minutes plus 10 minute perusal time. Paper 1 will have about 20–25 multiple-choice questions of one mark each. They will vary in difficulty from the simple 'describe and explain' to more complex 'apply, analyse and interpret' questions. Allow, on average, 2 minutes per question. Paper 1 will also include 6–8 short-answer questions of 3–6 marks each. Some of these will have multiple parts to answer such as (a), (b) and so on. The remainder will be single questions. These short-answer questions will range in difficulty, as well as including a range of questions that target retrieval and comprehension, and analytical processes.

Paper 2 will also include short-answer questions, with 5–8 questions worth 5–6 marks each. As with Paper 1, the questions will be multi-part or single question, and range from easy to difficult. Paper 2 may also include stimulus-response items that will require paragraph responses.

The overall difficulty of each paper will be the same. In general, you should allow two minutes per mark. For example, if a question is worth 5 marks, you should allocate 10 minutes to this question.

During both Paper 1 and Paper 2 you will be permitted to bring in a QCAA approved graphics calculator. You will also be provided with a formula and data booklet for reference.

Study tip

Remember, each multiple-choice question (or 'item') is worth 1 mark. If you are finding any difficult you can leave these and come back to them later. It is not worth using 10 minutes on a 1 mark question.

The assessment

Multiple-choice questions

Paper 1 includes multiple-choice questions. They are usually in a form that starts with a cognitive verb, such as 'select', 'determine' and so on. For example, 'Select the option that best describes …' Typically, multiple-choice questions follow a structure in which one answer is definitely wrong, one is partially correct but has a word that makes it incorrect, one that may be true but irrelevant to the question, and one correct answer.

Although a pattern may exist, there is no guarantee that this will be followed. It is important that you always read each option carefully to determine the correct answer.

Study tip

In short-answer questions, 1 mark = 2 minutes of time! If a question is worth 3 marks, allocate 6 minutes of time.

Short-answer questions

Both Paper 1 and Paper 2 include short-answer questions. When approaching short-answer questions, you should consider the number of marks available and then determine your time from there. Short-answer questions typically require a more thought-out response, but you should be careful to make sure you answer the question.

Stimulus questions

These will be short-answer questions that provide you with a stimulus. For example, a question may give the stimulus: 'A meteor is heading directly towards the surface of a planet at a constant speed of $0.8c$. Observers on the surface of the planet observe it at a time when it is a distance H above the surface in their reference frame. The observers calculate the time that the meteor will take to reach the surface of the planet as 784 µs.' You may then be required to 'Determine distance H'.

Developing automaticity

Expert problem-solvers often have an automatic approach to the first stages of a problem. By practising with lots of simpler one-concept and one-star questions throughout this text, you are developing an automatic approach to dealing with concepts and formulas. For example, if you see a mass resting on an inclined plane, the automatic approach would immediately think 'force down the incline, $F_P = mg \sin \theta$', and you would calculate the force. If you have already thought of this at the beginning of a question, you will have more time to focus on the more difficult aspects.

FIGURE 1
Examinations include different types of questions.

The 7F approach to problem-solving

1 Focus

It is important to have your mind focused on the right topic. For example, you may need to determine if you are working on circular motion or orbits – both of these topics have velocity, radius and time. To determine which topic you are working in, look for key words and then narrow down your thinking.

2 Facts

You should look for any **numerical data** in the question and for any **non-numerical data** in the question, such as 'above the surface of Earth', 'frictionless pulley' or 'relativistic speed'. These terms will provide information about the topic – circle these words and annotate what they mean. Finally, look for **key words** that express the limitations of the data, such as 'non-relativistic', 'uniform' or 'constant'.

3 Find

Locate the part of the question that says what you have to find. Note exactly what the physical quantity is and any units it has to be expressed in.

4 Figure

Drawing a figure can help you sort out the data. If you draw a figure, make sure it is clear so that you can sort the data easily.

5 Formula

Start by identifying the formulas you think may be appropriate. Think about what symbols are in the data and match them with the symbols from the formula. For simpler questions there may be one unknown, so you will need to find a formula that has data for everything except one. Irrelevant information is also added into questions to force you evaluate what is needed. For harder questions, you may need two or more formulas. For example, to calculate centripetal force you can use $F = \frac{mv^2}{r}$, but first you may have to calculate the velocity: $v = \frac{2\pi r}{T}$.

6 Figure it out

- Work as neatly as you can – this makes it easy to check your equations and formulas as you work through the problem.
- Rearrange the formula to have the unknown as the subject of the equation.
- Work in symbols for as long as you can – this makes calculation and transcription errors less likely.
- Keep symbols clear. For example, write u and v neatly, don't confuse m and M, and don't confuse t and T.
- Make sure your calculator is in the right setting – degrees or radians.
- Try to work in **scientific notation** – long lists of zeros after the decimal point can lead to errors.
- Don't skip steps – write each step down so it is easier to check later and, if needed, find your error.
- Remember mathematics rules; for example, if you multiply a negative by a negative, the answer will be positive.

scientific notation
a shorthand way of expressing very large or very small numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10

7 Finish

- Check your arithmetic.
- Check if your answer should be negative or positive.
- To check your answer, put it back into the original equation.
- Have you answered the question? For example, if the question asks for the magnetic field strength, don't leave the answer as $B = 10.0$ mT; finish your answer by writing 'into the page'.
- Check that you have expressed the answer in the correct units.

CHECK YOUR LEARNING 0.5

Describe and explain

1 **Create** a study timetable for yourself, including any other commitments such as sport, work, volunteering and social activities.

2 **Identify** a list of things that you need to study.

3 **Identify** what distracts you. Write these down so you can create an environment free from distraction.

4 **Explain** the 7Fs of problem-solving.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 0.5
- » Video
  Preparing for exams
- » Increase your knowledge
  Creating your study timetable
- » Weblink
  QCAA Syllabus assessment requirements

0.6

Data collection and analysis

KEY IDEAS

In this section, you will learn about:

- uncertainty
- types of experimental errors including random and systematic errors.

A part of any physics experiment is to record and analyse your measurements for quality. This is called an **error analysis**. The word 'error' is a rather vague term about the 'goodness' of your observations, but specifically it refers to the **precision** and **accuracy** of your results.

error analysis a calculation of the precision and accuracy of experimental results

precision the uncertainty of the measurement

accuracy the difference between the measured value and the true or accepted value of the observed quantity

Precision and accuracy mean different things, and it is important that they are used correctly. For a set of measurements:

- **precision** is the range of values found; that is, the *uncertainty* of the measurement
- **accuracy** is the difference between the measured value and the true or accepted value of the observed quantity.

Uncertainty

Experiments can include some uncertainty. Three readings of the time taken for a ball to drop from a certain height will give slight variations. This variation is the uncertainty of the average result.

Uncertainties are also called errors. These words can be used interchangeably, which is a problem. People tend to think of errors as mistakes, but they are not – particularly in error analysis. *Errors* are variations or fluctuations in the data. These errors are always due to either the person taking the measurement or the instrument being used.

Systematic errors

Systematic errors can occur in a measurement because of problems with the measuring instrument or the conditions under which the measurement was made.

systematic error an error that is due to the accuracy of a measurement process that causes readings to deviate from the accepted value by a consistent amount each time a measurement is made

- A zero error occurs when the instrument has not been correctly set to zero before commencing the measuring procedure.

FIGURE 1 The diameter of a steel guitar string is measured with a micrometer as part of an electromagnetism experiment.

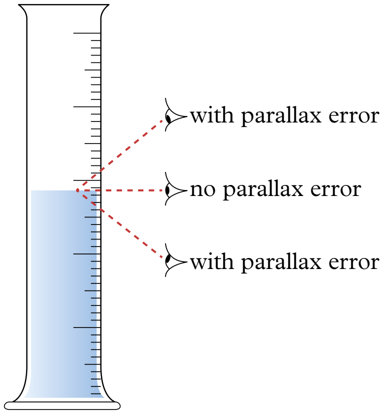

FIGURE 2 When a scale is read from the wrong angle it results in a parallax error.

- A calibration error occurs when there is a difference between the value indicated by an instrument and the actual value.
- A parallax error occurs when the scale of a measuring instrument is viewed at an angle rather than from directly in front of it (perpendicular to it), therefore using the instrument wrongly on a consistent basis (Figure 2).
- Reaction time is the time taken for a person or a system to respond to a certain event.
- Environmental causes include temperature effects, air resistance, background radiation, corrosion and so on.

Random errors

Random errors are those that tend to fluctuate above and below the true value. A common source of these errors is when you try to estimate a quantity that lies between the graduations (the lines) on an instrument such as a metre ruler, thermometer or voltmeter.

These random errors are said to be due to **scale reading limitations**. There are simple rules for reporting scale reading uncertainty:

1 Uncertainty in a printed scale (or scale measuring device) is equal to a half-scale division.

2 Uncertainty in a digital measuring device is equal to the smallest increment.

Other fluctuations come from the normal functioning of measuring devices. There is nothing you can do to prevent them. However, the best way to handle random errors is to make several repeat readings and then average them.

random error
error due to the uncertainty of the measurement equipment and the uncontrollable effects of procedure and environment on a measurement result

scale reading limitation
the inability of an instrument to resolve small measurement differences

CHECK YOUR LEARNING 0.6

Describe and explain

1 **Explain** the meaning of the following: systematic error, random error, scale reading uncertainty, zero error, calibration error, parallax error.

2 **Explain** how the scale reading uncertainty for an analogue (printed) scale is different from that for a digital scale.

3 Some voltmeters in a class had zero error problems. It seemed to be random as to which voltmeters had this error and which didn't. **Explain** why zero error is considered a systematic error rather than a random error.

Apply, analyse and interpret

4 A student said, 'You can't have a parallax error with a digital stopwatch'.
Determine if this is true.

5 **Deduce** which of the following voltage measurements is the more precise:
$V_1 = 0.55 \pm 0.01$ V or $V_2 = 6.4 \pm 0.1$ V.

Investigate, evaluate and communicate

6 A voltmeter kept giving readings 0.5 V higher than expected. **Evaluate** this situation and state what sort of error this would be and how it could be fixed.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 0.6

» Video
Error analysis

» Increase your knowledge
Other types of errors in science

» Increase your knowledge
Worked examples

0.7

Graphical analysis

KEY IDEAS

In this section, you will learn about:

- common graphs used in physics
- showing a relationship with a graph.

independent variable
a variable (often denoted by *x*) whose variation does not depend on that of another variable

dependent variable
the variable (often denoted by *y*) that responds to the independent variable; it 'depends' on the independent variable

Graphs are a useful way of showing how one quantity depends on another. On a graph, the horizontal axis is usually where the **independent variable** (or the cause) is plotted. This also includes variables that progress regardless of the experiment. An example is time elapsed, as time continues on regardless of whether any experiment is being carried out. The effect of that cause is called the **dependent variable** and plotted on the vertical axis. Graphs are designed to show relationships and tell a story about the data. Sometimes this is simplified by having the dependent variable on the horizontal axis. As long as it is clearly stated there is no problem.

There are several common relationships you should be aware of for your studies in Units 3 and 4.

Linear relationships

Linear and directly proportional

A linear relationship is a straight line when graphed. A directly proportional relationship is also linear, but it is a special case that goes through the origin (0, 0).

Linear but not directly proportional

The graph of the rubber band stretching (Figure 1) is both linear and directly proportional. The graph in Figure 2 is linear but not directly proportional as it does not go through (0, 0).

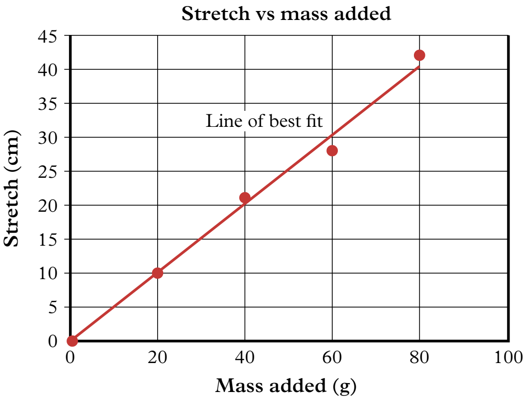

FIGURE 1 Graph of directly proportional data

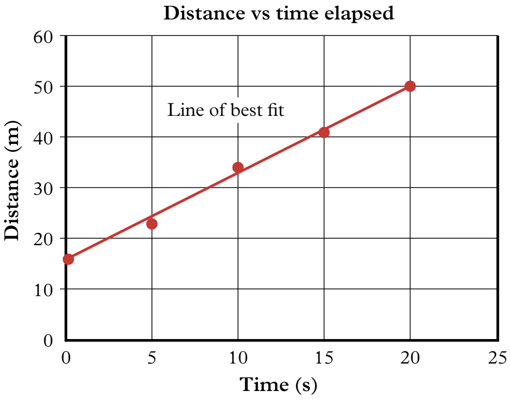

FIGURE 2 Graph of data that is linear, but not directly proportional

Non-linear relationships

A non-linear relationship is not a straight line when graphed. The most common non-linear relationships you will meet in physics are power, exponential and logarithmic relationships, as outlined in Table 1.

TABLE 1 Examples of common non-linear relationships (a is the 'power' or 'exponent' of the relationship.)

Type of relationship	Equation	Example
Power	$y \propto x^a$	parabolic ($a = 2$): $y \propto x^2$ inverse ($a = -1$): $y \propto x^{-1}$ or $y \propto \frac{1}{x}$ inverse-square ($a = -2$) : $y \propto x^{-2}$ or $y \propto \frac{1}{x^2}$ square root $\left(a = \frac{1}{2}\right)$, $y \propto x^{\frac{1}{2}}$ or $y \propto \sqrt{x}$
Exponential	$y = a^x$	natural exponential $y \propto e^{kt}$
Logarithmic	$y = \log_a x$	log y to base 10 = $\log_{10} y = x$

Power relationships

Parabolic relationship: $y \propto x^2$ where the power a is 2.

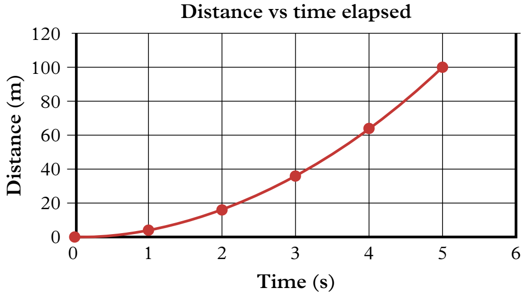

FIGURE 3 Parabolic graph (a parabola)

Inverse relationship: $y \propto x^{-1}$ or $y \propto \frac{1}{x}$ where the power a is –1.

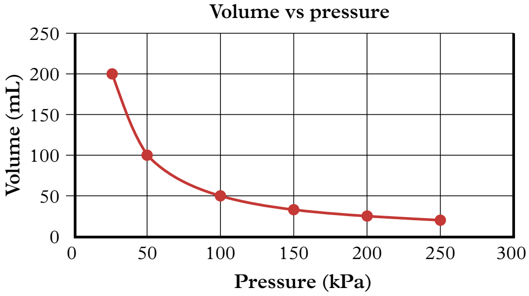

FIGURE 4 Inverse graph

Inverse-square relationship: $y \propto \frac{1}{x^2}$ where the power a is –2.

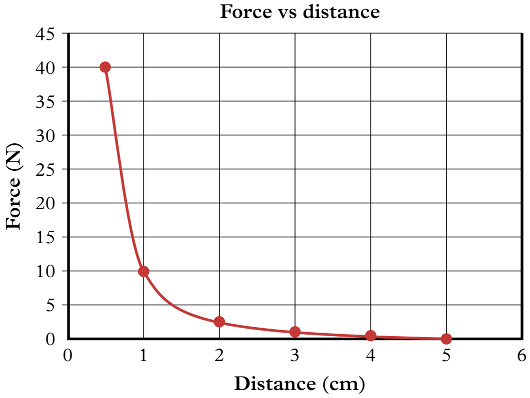

FIGURE 5 Inverse-square graph

Square root relationship: $y \propto x^{\frac{1}{2}}$ where $a = \frac{1}{2}$.

It is also written as $y \propto \sqrt{x}$ or $y \propto \sqrt[2]{x}$.

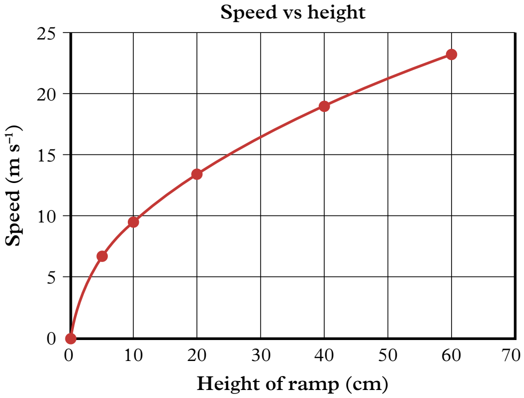

FIGURE 6 Square root graph

Study tip

Outliers and anomalies are generally regarded as the same thing but there are subtle differences.
Outlier: a value that 'lies outside' most of the other values in a set of data. It is a long way from other results. It is a legitimate data point originated from a real observation. You should note it and repeat the measurement to see if it is true.

Anomaly: something that deviates from what is standard, normal, or expected. It is a false data point, often the result of a faulty observation or equipment. You can discount this point, but make a note of its existence and track down its source.

Study tip

A summary of some common graph shapes can be found on your obook assess.

Exponential relationships

Exponential relationship: $y = a^x$

The natural exponential function is $y \propto e^{kt}$ or $y = y_0 e^{kt}$.

In $y = y_0 e^{kt}$, y is the final amount of the quantity being measured, and y_0 is the starting amount of the same quantity.

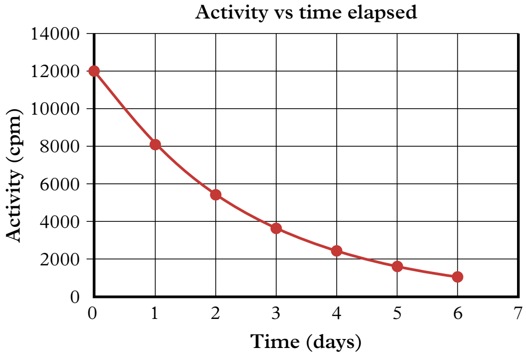

FIGURE 7 Natural exponential decay where y is proportional to e raised to the power $-kt$: $y = y_0 e^{-kt}$

Logarithmic relationships

Logarithmic relationship: $y = \log_a x$

There is an inverse relationship between logarithmic and exponential functions; i.e. if $y = a^x$, then $y = \log_a x$.

If x raised to power a is y, then the logarithm to base a of y is x.

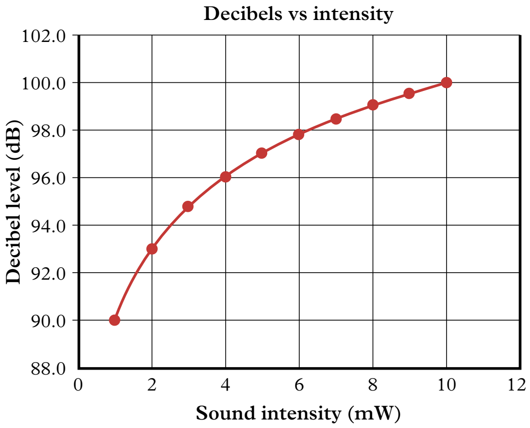

FIGURE 8 Logarithmic graph

Linearising graphs

linearising
a process of transforming non-linear data by applying a mathematical function to one of the variables so that the relationship between the variables becomes closer to a straight line

Even though a graph looks like it has a particular shape, its relationship cannot be known for sure. For example, $y \propto \frac{1}{x}$ and $y \propto \frac{1}{x^2}$ are very similar and are hard to tell apart. The only way to tell is to plot them in the form of $y = mx$. If the graph is linear, then you have confirmed the suspected relationship. This is called **linearising** a graph.

For example, consider the relationship between pressure and volume of a gas. As you increase the pressure the volume decreases. It is an inverse relationship in the form $y \propto \frac{1}{x}$.

You can try to linearise the graph by calculating $\frac{1}{x}$ $\left(\text{that is, } \frac{1}{P}\right)$ and plotting V versus $\frac{1}{P}$. If it is a straight line, then you have confirmed the relationship $y \propto \frac{1}{x}$ (which is $V \propto \frac{1}{P}$ in this case). If the $y \propto \frac{1}{x}$ graph is still curved, then it may be a $y \propto \frac{1}{x^2}$ relationship.

Study tip

Linearising your data is the most powerful way of confirming a suspected relationship. Make every attempt to linearise your data when you do any experiment, particularly the Student Experiment. It is also highly likely to appear as part of your data test.

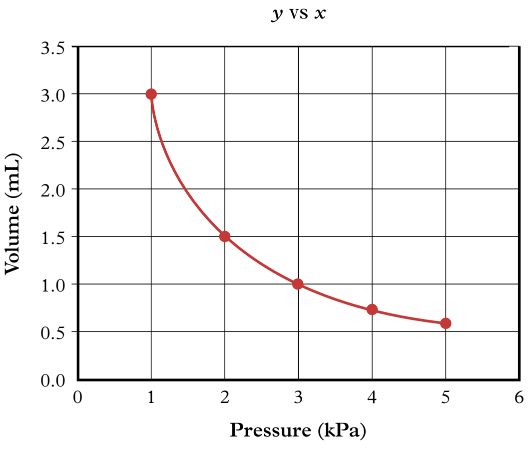

FIGURE 9 A pressure versus volume graph of the data is not linear – it appears inverse.

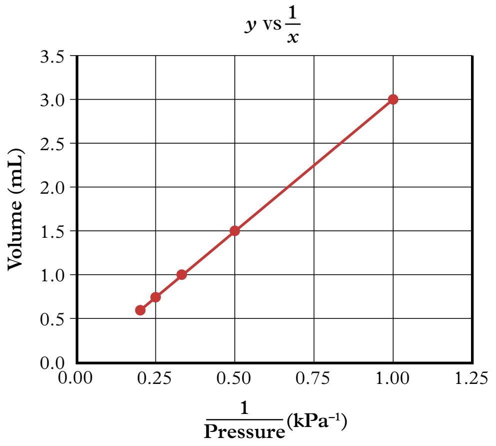

FIGURE 10 A V versus $\frac{1}{P}$ graph of the data gives a straight line, so it is a $y \propto \frac{1}{x}$ relationship.

CHECK YOUR LEARNING 0.7

Describe and explain

1 **Recall** whether it is usual to plot the independent variable on the vertical or the horizontal axis.

2 **Explain** the meaning of 'linearising' a relationship.

3 Students obtained a relationship $y \propto x$. **Explain** whether there is any need to linearise it.

4 **Construct** a graph of the data in Table 2 and draw the line of best fit. (Note: the independent variable is listed first.)

TABLE 2

Diameter of circle (cm)	0.0	4.0	8.0	12.0
Circumference of circle (cm)	0.0	12.5	25.4	37.3

Apply, analyse and interpret

5 **Classify** each graph shown in Figure 11 by the following relationships:

- **A** y is proportional to x
- **B** y is inversely proportional to x
- **C** y is independent of x
- **D** y is proportional to x^2

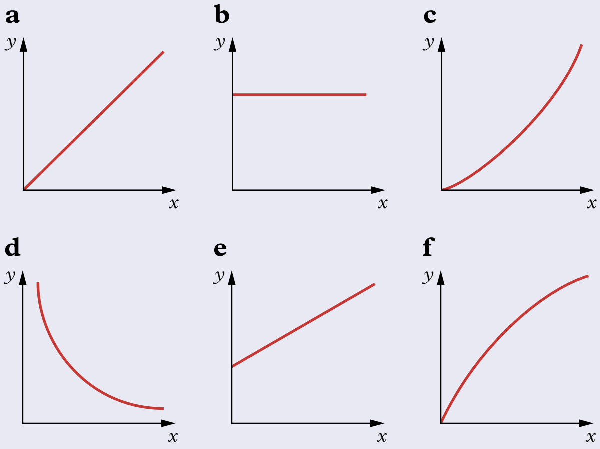

FIGURE 11 Graphs with different relationships

6 **a** A particular relationship is $W = kV$. **Determine** the effect on W of:

- **i** tripling V
- **ii** halving V.

b **Deduce** what a graph of W as a function of V looks like.

7 A plot of the variables F (vertical axis) and v (horizontal axis) gave a curve in the shape of $y \propto x^2$. **Deduce** what you would need to plot to linearise it.

Investigate, evaluate and communicate

8 Table 4 gives data from an experiment in which F and r are two related variables.

TABLE 4

F	72	48	24	18
r	2	3	6	8

The data were plotted to give the graph shown in Figure 13.

a **Comment** on the mathematical relationship suggested by this graph.

b **Construct** another graph to confirm your prediction.

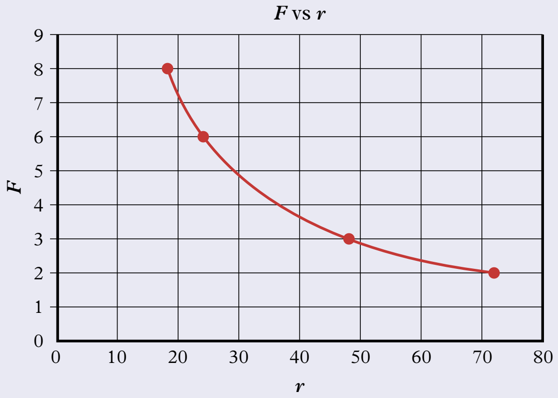

FIGURE 13 Graph of F vs r

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- » Student book questions
  Check your learning 0.7
- » Video
  Linearising a graph
- » Weblink
  Using Excel for graphing

CHAPTER 0

Review

Summary

- **0.1** During Units 3 and 4 you will complete three internal assessments and one external assessment.
- **0.2** The data test requires students to respond to questions about the mandatory or suggested practicals, activities or case studies from the unit being studied. It requires students to provide answers to short-response items requiring single-word, sentence or short paragraph responses, or others that will require calculating using algorithms (formulas) and interpreting.
- **0.3** The student experiment involves selecting an experiment or simulation that has already been completed in class and modifying it in order to address a hypothesis or research question. It includes a research question, rationale, data collection and analysis, discussion of relationships and limitations, suggestions for improvements and a conclusion.
- **0.4** The research investigation is a non-experimental task that requires you to evaluate a claim about a significant issue from your study of physics.
- **0.5** Examinations are included in the assessment program to test a student's ability to describe and explain, apply understanding, analyse and interpret evidence under supervised conditions. They are the only task that explicitly tests a student's ability to describe and explain various physics concepts.
- **0.6** Data collection for experiments can be analysed for relationships and assessment of accuracy and precision.
- All measurements include errors or uncertainties, either systematic or random.
- Accuracy is a measure of the discrepancy of experimental results when compared to accepted values.
- **0.7** Graphing is a powerful means of determining relationships between the dependent and independent variable in an experiment or dataset.
- The line of best fit for a graph should pass through as many points as possible. For the points that are off the line, there should be an equal number of points above the line as below it.
- Points a long way from the line of best fit are called anomalies or outliers.
- Common terms associated with graphs are linear, non-linear, inverse, power, exponential, logarithmic.
- Graphs of the following relationships have characteristic shapes that have been detailed in this chapter: $y \propto x$, $y = mx$, $y = mx + c$, $y \propto x^2$, $y \propto \frac{1}{x}$, $y \propto \frac{1}{x^2}$, $y \propto x^{\frac{1}{2}}$, $y = y_0 e^{-kt}$.
- Relationships between variables can be proven by plotting the predicted function of x on the x-axis and examining to see whether the graph is a straight line. This is called linearising.

Key terms

- accuracy
- cognitive verb
- dependent variable
- error analysis
- independent variable
- linearising
- precision
- random error
- reliable
- scale reading limitation
- scientific notation
- secondary evidence
- systematic error

Key formulas

Determining the slope of a graph	$\text{Slope } (m) = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
Percentage uncertainty	$\delta\% = \dfrac{\delta x}{x_0} \times 100\%$
Absolute error	$E_a = \lvert x_O - x_A \rvert$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Multiple choice

1 For practical work, you will be asked to draw a conclusion. This is not a cognitive verb, but the syllabus states that it means ‘to make a judgement based on reasoning and evidence’. Which of the following would satisfy the definition?

A drawing a graph that shows the relationships of the two variables

B interpretation of research evidence demonstrated by justified conclusions linked to the research question

C evaluation of the research processes, claims and conclusions about the research

D analysis of data by thorough identification of relevant trends, patterns or relationships

2 A student carried out an experiment about the force acting on a current-carrying wire. She plotted force vs current with custom error bars to show uncertainty. The linear trendline was $y = 9.0x - 0.1$, whereas the maximum trendline was $y = 12.0x + 0.02$, and the minimum trendline was $y = 6.0x + 0.1$. Determine the uncertainty in the gradient.

A $\delta x = \pm\dfrac{12.0 - 6.0}{2}$

B $\delta x = \pm\left|\dfrac{6.0 - 12.0}{2}\right|$

C $\delta x = \pm(12.0 - 6.0)$

D $\delta x = \pm\dfrac{12.0 - 6.0}{9.0}$

3 A graph in a data test includes error bars in the vertical direction. What do error bars show?

A the absolute uncertainty of the data point on the vertical scale

B the percentage uncertainty of the data point on the y-axis

C the accuracy of the data point

D the maximum and minimum gradients

4 A student investigated the time of flight of a tennis ball as a function of initial speed. Which of the following could be classed as a ‘refinement’?

A Use higher speeds than in the original.

B Measure time as a function of angle rather than initial speed.

C Record a video of the motion to analyse it more accurately.
D Use a marble to reduce air resistance.

5 In an experiment to measure the force between two charged objects, the task sheet specified triplicate measurements be performed in each trial. What is the purpose of doing multiple measurements for each trial?

A to reduce random error
B to reduce systematic error
C to improve accuracy
D to decrease the precision

6 Select the option that states two processes in a research investigation in the correct order.

A claim, rationale
B overview of evidence, research question
C analysis, interpretation
D conclusion, evaluation

7 Only one of the following statements is correct about external assessment. Select the correct option.

A Marks are deducted for wrong multiple-choice answers.
B Both exam papers contain short-answer questions.
C For stimulus–response questions, a stimulus will be provided ahead of time.
D Paper 2 is more difficult than Paper 1.

8 Students investigated magnetic force as a function of distance and constructed the graph in Figure 1.

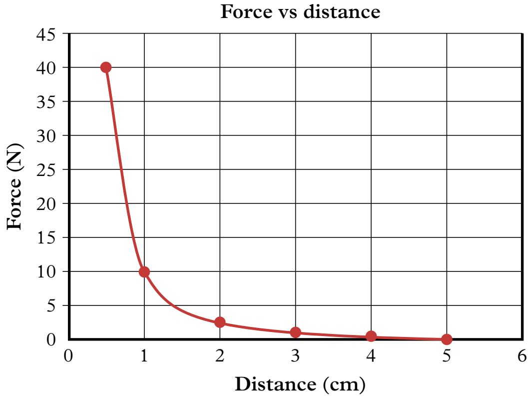

FIGURE 1

Propose what they should plot on the vertical axis and horizontal axis respectively to linearise the data.

A F, d^2
B F, $\frac{1}{d^2}$
C F^2, d
D F, $\sqrt{d}$

9 The graph in Figure 2 shows the relationship between distance and time for a box sliding down an inclined plane. Which one of the following is the correct equation for the line?

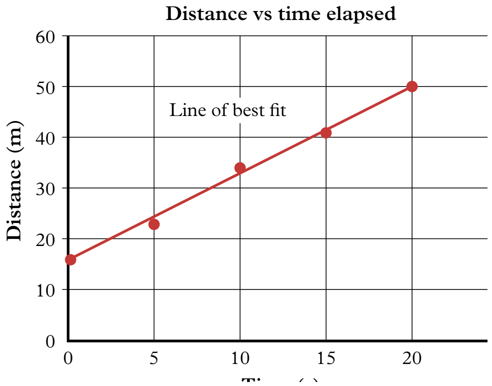

FIGURE 2

A $y = 1.7x + 16$
B $y = 1.7x - 16$
C $y = 50x + 16$
D $y = 20x - 16$

10 A student weighed a rubber stopper on an electronic balance and recorded it as 35.53 g instead of the correct value of 33.53 g. What kind of error is this?

A systematic
B random
C mistake
D outlier

Short answer

Describe and explain

★ 1 **Recall** five types of non-linear graphs.

★ 2 **Explain** what you would plot to linearise data if a graph showed a possible relationship of $y \propto \frac{1}{x}$.

Apply, analyse and interpret

★★3 **Determine** the reading and uncertainty of the measurement on the ruler in Figure 3. The numbers are centimetres and the small divisions are millimetres.

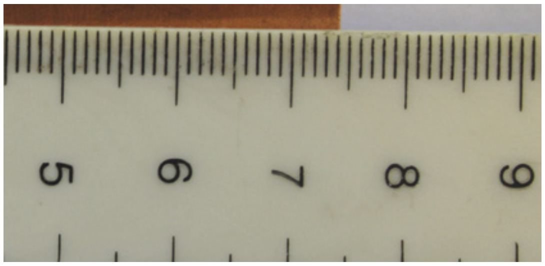

FIGURE 3

★★4 An analogue voltmeter with a scale division of 0.1 V reads 2.4 V when placed across a resistor of 47 Ω ± 5%.

a **Determine** the absolute and percentage uncertainty in the voltage reading.

b A calculation of $\frac{V^2}{R}$ was then made. **Determine** the percentage uncertainty in the result.

Investigate, evaluate and communicate

★★5 **a** **Construct** a graph of the data given in Table 1 and draw a line of best fit. (Note: the independent variable is listed first.)

TABLE 1

Time (s)	0.0	2.0	4.0	6.0
Distance (m)	0.0	12	23	37

b For the line plotted on the graph:

i calculate the gradient

ii extrapolate to 8.0 s

iii interpolate for 3.0 s.

★★★ 6 The electrostatic force between two charged objects was measured as a function of separation distance. The results are in Table 2.

TABLE 2

Distance, r (m)	1	2	3	4	5
Force, F (N)	90	23	10	6	4

a **Construct** a graph to show the relationship between force, F, in newtons, as a function of separation distance, r, in metres.

b **Describe** the relationship between F and r.

c **Construct** a linearised graph to confirm the relationship between F and r.

★★★7 The following sets of data represent common relationships in Unit 3 and 4 Physics. **Predict** the relationship by graphical means for each and confirm it where necessary by linearising.

a

x	1	2	3	5	6
y	2.00	1.10	0.67	0.41	0.33

b

x	1	2	4	6	8
y	3.00	0.75	0.19	0.08	0.05

Check your obook assess for these additional resources and more:

- Student book questions
  Chapter 0 revision questions
- Revision notes
  Chapter 0
- assess quiz
  Auto-correcting multiple-choice quiz
- Flashcard glossary
  Chapter 0

UNIT 3

GRAVITY AND ELECTROMAGNETISM

FIGURE 1 International Space Station (ISS) orbiting Earth. Gravitational and electromagnetic fields extend throughout space. The field of gravity keeps the ISS orbiting Earth.

In Unit 2, motion was examined in one dimension; now in Unit 3 you will take a more in-depth look at motion and consider it in two dimensions. You will develop an understanding of the contexts of projectile motion, inclined planes and circular motion. You will see that the horizontal and vertical motions of projectiles are independent, that acceleration down an incline depends on the parallel component of an object's weight, and that objects moving in a circle can be accelerating while travelling at constant speed. This last context takes us into the history of the laws of planetary motion and how gravity acts at a distance.

So far, this is seemingly about the laws of motion, but it is much more than that. Unit 3 is really concerned with fields. The concept of fields was devised in the 1820s by Michael Faraday and was soon applied to electromagnetism. This then introduces the second topic of the unit: magnetism, electromagnetism and the production of electromagnetic waves.

Field theory has enabled physicists to explain a vast array of natural phenomena such as gravity and electromagnetism, and has contributed to the development of technologies that have changed the world, including electrical power generation and distribution systems, artificial satellites and modern communication systems. The problem with fields, unlike the physical field on which they are modelled, is that they are not visible and can't be touched. Thus, it makes sense to have saved field theory for Unit 3, as it is quite conceptual; but the ability to represent fields with lines of force representing vector fields helps considerably.

The vast number of practical applications makes field theory very useful in modelling a number of phenomena. Practical applications include GPS navigation, motors and generators, electric cars, synchrotron science, medical imaging and astronomical telescopes, and related areas of science and engineering such as sports science, amusement parks, ballistics, forensics, black holes and dark matter.

Unit 3 topics

Topic 1 – Gravity and motion	Chapters 1–5
Topic 2 – Electromagnetism	Chapters 6–8

Unit objectives

→ Describe and explain gravity and motion, and electromagnetism.

→ Apply understanding of gravity and motion, and electromagnetism.

→ Analyse evidence about gravity and motion, and electromagnetism.

→ Interpret evidence about gravity and motion, and electromagnetism.

→ Investigate phenomena associated with gravity and motion, and electromagnetism.

→ Evaluate processes, claims and conclusions about gravity and motion and electromagnetism.

→ Communicate understanding, findings, arguments and conclusions about gravity and motion, and electromagnetism.

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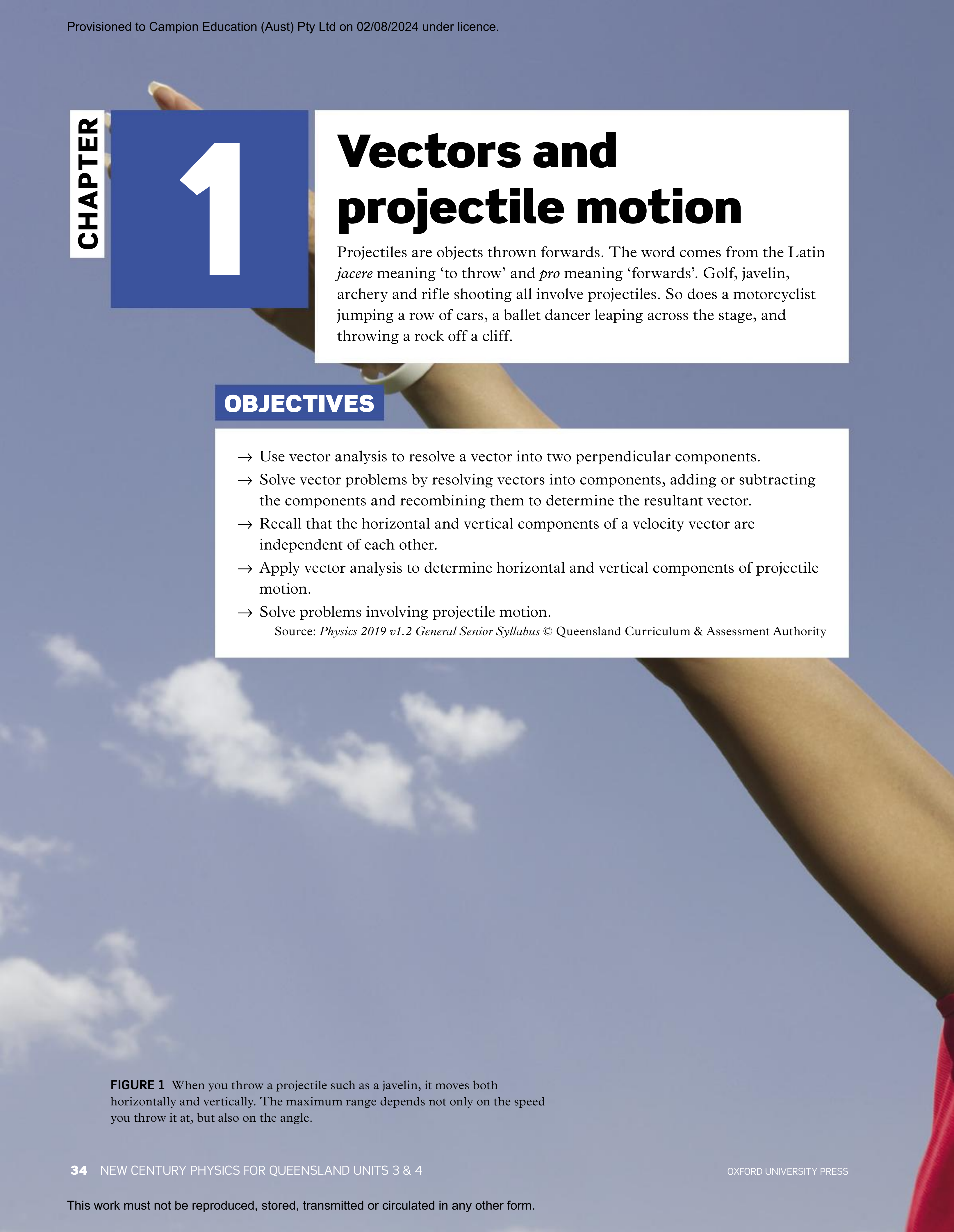

CHAPTER 1

Vectors and projectile motion

Projectiles are objects thrown forwards. The word comes from the Latin *jacere* meaning ‘to throw’ and *pro* meaning ‘forwards’. Golf, javelin, archery and rifle shooting all involve projectiles. So does a motorcyclist jumping a row of cars, a ballet dancer leaping across the stage, and throwing a rock off a cliff.

OBJECTIVES

→ Use vector analysis to resolve a vector into two perpendicular components.
→ Solve vector problems by resolving vectors into components, adding or subtracting the components and recombining them to determine the resultant vector.
→ Recall that the horizontal and vertical components of a velocity vector are independent of each other.
→ Apply vector analysis to determine horizontal and vertical components of projectile motion.
→ Solve problems involving projectile motion.

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FIGURE 1 When you throw a projectile such as a javelin, it moves both horizontally and vertically. The maximum range depends not only on the speed you throw it at, but also on the angle.

MAKES YOU WONDER

In this chapter you will be examining some aspects of projectile motion that will help to answer questions such as these:

- Before Galileo, universities taught that when a cannon ball ran out of impetus it would stop in its path and fall vertically to Earth. Were they correct?
- Soldiers in war have often reported that enemy bullets fired from miles away fell vertically into their trenches. Could this be true?
- In the Olympic hammer throw, does the hammer continue in a circular path for a fraction of a second after it is let go?
- Do bombs and bullets fired at 45° have the greatest range?
- Is it really true that the javelin design changed in 1998 so that it couldn't be thrown as far?
- Did Apollo 14 astronaut Alan Shepard play golf on the Moon in 1971 and leave three golf balls there?

PRACTICALS

MANDATORY PRACTICAL	**1.3** Angled projection and distance

1.1

Vectors and gravity

KEY IDEAS

In this section, you will learn about:

- resolving a vector into two perpendicular components
- adding and subtracting vector components
- recombining vectors.

Gravity is a force that acts down towards the centre of Earth and is responsible for the acceleration of objects in free-fall. But not all motion is straight down. A projectile such as a basketball in flight or a box sliding down an incline has both vertical and horizontal motion.

To completely specify the motion of a projectile you need to give more than just the magnitude of the motion, such as 20 m s^{-1}; you also need to specify its direction; for example, 20 m s^{-1} at 45°. In other words, you need to specify its velocity (a vector quantity), rather than just its speed (a scalar quantity). Force and acceleration need a direction too. They are also called *vector* quantities.

Vector analysis in two dimensions is hugely important in this unit and the groundwork begins here. Some of the questions that puzzle students about vectors are:

- How can something at constant speed be accelerating?
- Can a vector have zero magnitude if one of its components is not zero?

Representation of vectors

We'll do a short revision on the main points of vectors that you studied in Unit 2, which dealt with linear (one-dimensional) motion.

vector
a variable quantity, such as force, that has magnitude and direction

A **vector** is a quantity that has both magnitude and direction. A vector quantity can be represented graphically by an arrowed line segment, or symbolically by accents such as a bold typeface, tildes or overhead arrows.

For example, Table 1 shows four different ways of representing a vector quantity, in this case velocity. In this book the first and third methods will be used.

TABLE 1 Different ways of representing vector quantities

Arrowed line segment	Bold (accent)	Arrow (accent)	Tilde (accent)
$\underset{\longrightarrow}{v = 5\text{m s}^{-1}}$	$\boldsymbol{v} = 5\text{m s}^{-1}$ to the right	$\vec{v} = 5\text{m s}^{-1}$ to the right	$\tilde{v} = 5\text{m s}^{-1}$ to the right

CHALLENGE 1.A

What does an odometer measure?

Does the odometer of a car measure a scalar quantity or a vector quantity? Make a similar statement for the car's speedometer.

When vectors do not lie along the compass points (N, E, S, W), or are not horizontal or vertical, angles need to be specified. Figure 1 shows how the direction is indicated.

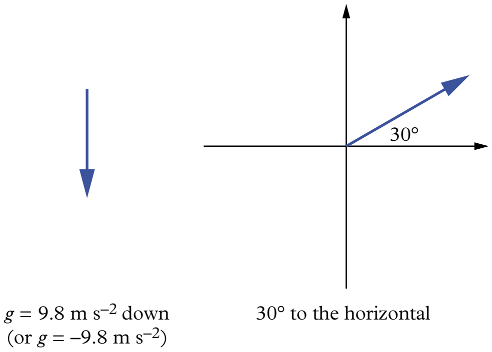

FIGURE 1 Vector direction can be represented in a number of ways.

Combining vectors

Two or more vector quantities can be combined to produce a single **resultant vector**. They can be combined in a straight line or at an angle to each other.

resultant vector
a single vector that is a combination of two or more other vectors

Combining vectors in a line (one-dimension)

Case 1: Vectors in the same direction

Consider a boat that is rowed at 5 m s^{-1} east in water that is also moving east at 1 m s^{-1}. Your actual velocity is 6 m s^{-1} east and is found by placing the two vector arrows head-to-tail. The resultant is a line drawn from the tail of the first arrow to the head of the second arrow (Figure 2). A double arrow head is usually shown on the resultant.

Study tip

The resultant vector is always drawn from the tail of the first vector (arrow) to the head of the second.

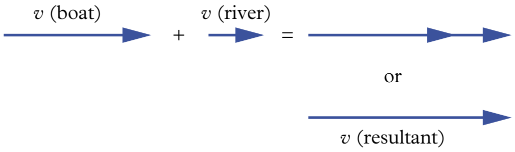

FIGURE 2 Add vectors by placing them head-to-tail.

Case 2: Vectors in opposite directions

Consider the same boat being rowed against the current. In this case the velocity of the river is 1 m s^{-1} west, which is in the direction opposite to that of the boat, and hence will slow the boat down (Figure 3).

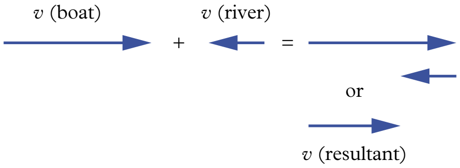

FIGURE 3 Subtract vectors by adding the negative.

The resultant velocity is 4 m s^{-1} east. Note that when two vectors in the same line are added, the direction of resultant is the same as the direction of the larger vector.

Combining vectors in two dimensions (horizontal plane)

In Unit 2 you dealt with vectors in a line, but here you will extend that to vectors at right angles. First, a reminder of the trigonometric ratios (Figure 4).

$$\sin\theta = \frac{\text{opposite side length}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side length}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side length}}{\text{adjacent}}$$

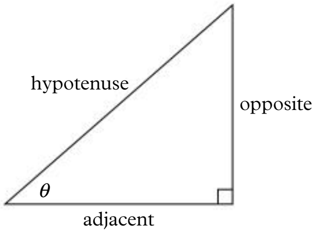

FIGURE 4 The resultant of adding vectors at right angles is the hypotenuse.

Study tip

It is useful to remember that Pythagoras' theorem states that $c^2 = a^2 + b^2$, where c is the hypotenuse, and a and b are the other sides of the right-angled triangle.

Study tip

During assessments (particularly the data test and external exam) always check that your calculator is in degrees not radians! This can be the difference between a correct and an incorrect answer.

Imagine a motor boat travelling east at 4 m s^{-1} across a river whose current is flowing south at 3 m s^{-1}, as shown in Figure 5a. The boat would be dragged off course by the current and its resultant velocity would be 5 m s^{-1} at an angle down the river. Note that the two vectors are added head-to-tail and the resultant is a line drawn from the tail of the first arrow to the head of the second arrow (Figure 5b).

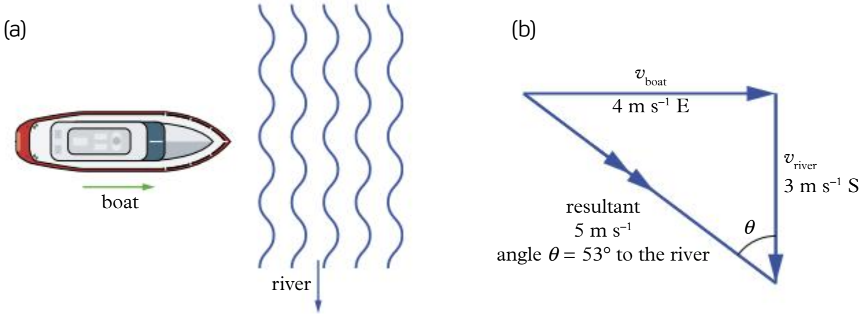

FIGURE 5 A practical example of adding vectors at right angles

The solution to this problem is in two parts – a magnitude component (5 m s^{-1}) and a direction component (53° to river). This is achieved in the following manner:

1 **Magnitude**. Because the starting vectors for the boat and river are at right angles (E and S), Pythagoras' theorem can be used:

$$\begin{aligned}\text{resultant} &= \sqrt{4^2 + 3^2}\\ &= \sqrt{25}\\ &= 5\ \text{m s}^{-1}\end{aligned}$$

2 **Direction**. Because the two vectors and the resultant form a right-angled triangle, trigonometry can be used:

$$\begin{aligned}\tan\theta &= \frac{\text{opposite side length}}{\text{adjacent}}\\ &= \frac{4}{3}\\ \text{hence } \theta &= \tan^{-1}\frac{4}{3}\\ &= 53°\end{aligned}$$

If you disagree with the values of θ shown in the solutions, check that your calculator is in degrees (shown by a small DEG in the display). A common mistake for students (and teachers) occurs when the calculator is in radians (RAD). To avoid this mistake, change the calculator to degrees.

This vector analysis technique can be applied in the vertical plane as well.

Combining vectors in two dimensions (vertical plane)

Imagine a golf ball that is struck (Figure 6) and eventually lands back on the ground. Its velocity as it strikes the ground is called its **impact velocity**. Upon impact, it has two velocity vectors, v_x (in the horizontal or x-direction) and v_y (in the vertical or y-direction). The two vectors are combined (added) to get a resultant velocity (Figure 7).

impact velocity the velocity of a projectile immediately before striking the ground; the magnitude of impact velocity is impact speed

Study tip

Horizontal velocities are labelled as u_x and v_x. Vertical velocities are labelled as u_y and v_y.

FIGURE 6 A golf ball is hit off the tee with driver. It has vertical and horizontal motion when it is hit and also when it lands.

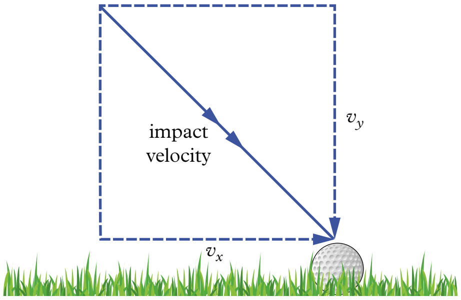

FIGURE 7 A projectile strikes the ground at an angle of impact. Its resultant impact velocity is a combination of the horizontal (v_x) and vertical (v_y) impact velocities.

WORKED EXAMPLE 1.1A

A ball makes impact with the ground with a horizontal velocity of 3 m s^{-1} and a vertical velocity of 4 m s^{-1}. Calculate its resultant (impact) velocity.

SOLUTION

Step 1 Draw a vector diagram of the information, remembering that when vectors are added they are placed head-to-tail.

Step 2 Draw the resultant (Figure 8). This is a line drawn from the tail of the first arrow to the head of the second arrow.

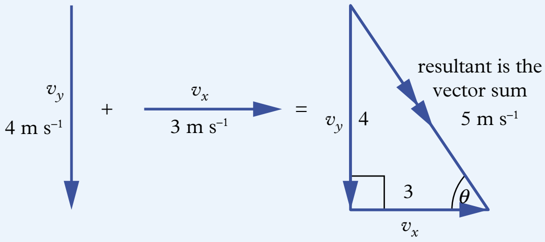

FIGURE 8 Vectors are added to give a resultant.

Step 3 Calculate the magnitude. The resultant magnitude of the impact velocity vector can be found using Pythagoras' theorem:

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$\text{Resultant (impact) velocity} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

The magnitude of the resultant vector = 5 m s^{-1}.

Step 4 Calculate the direction. This is given by the angle (theta, θ) that the resultant vector makes with the ground. Use trigonometric ratios to do this:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{4}{3}$$
$$\theta = \tan^{-1}\frac{4}{3}$$
$$= 53°\text{(to the horizontal)}$$

If you were asked to find the angle with respect to the vertical, it would just be 90°– θ; that is, 90° – 53°, which gives you 37° to the vertical. Take note of what you are asked in the question.

CHALLENGE 1.1B

As the crow flies

You go from one town to another ‘as the crow flies’. Is that always the quickest way there?

Resolving a vector into components

resolution determining the components of a vector, usually at right angles to each other

components (of a vector) a depiction of the influence of a vector in a given direction

elevation angle the angle at which a projectile is launched with respect to the horizontal

So far you have seen how two vectors can be added together to give a resultant third vector. The reverse process is called **resolution** (Latin *re* = ‘back’) – the vector is resolved into its **components**.

Why bother? In many cases it is convenient to ‘break up’ a vector into two components at right angles such as vertically and horizontally. It is then sometimes easier to apply the laws of physics to the components. In maths, you may use *i*, *j* and *k* components, but it is the same idea.

Consider an arrow being launched upwards at an angle of 30° to the horizontal. This is called the **elevation angle** and can be represented by a vector diagram. If the initial launch velocity is $\boldsymbol{u}$, and the angle θ, the vector diagram is shown in Figure 9.

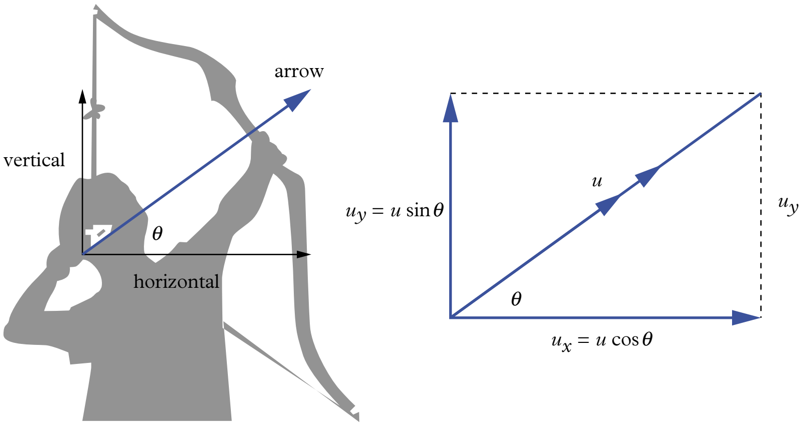

FIGURE 9 Vertical and horizontal components of the initial velocity of a projectile

The launch velocity can be resolved into two components at right angles to each other (Figure 9). It is simple to make the two components in the vertical and horizontal directions that are automatically at right angles (perpendicular) to each other. The initial velocity u is resolved into a vertical (y-axis) component u_y and a horizontal (x-axis) component u_x. All you need to do is make a rectangle with the initial vector as the diagonal. Using the trigonometric formulas you get:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{u_y}{u}\\ \text{or } u_y &= u\sin\theta\\ \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{u_x}{u}\\ \text{or } u_x &= u\cos\theta\end{aligned}$$

WORKED EXAMPLE 1.1B

A bullet is fired at a speed of 660 m s^{-1} at an angle of 36.0° to the horizontal. Calculate the horizontal and vertical components. Solve to 3 significant figures.

SOLUTION

Step 1 Prepare a vector diagram by drawing the vector arrow and then draw a rectangle around it using the vector as the diagonal (Figure 10).

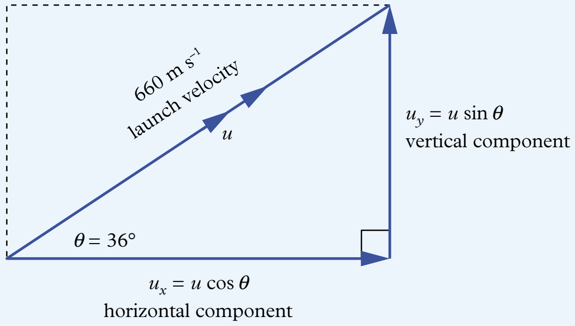

FIGURE 10 The launch velocity is resolved into its components.

Step 2 Label the horizontal and vertical components as u_x and u_y respectively.

Step 3 Calculate the magnitudes of these two components:

Horizontal component:

$$\begin{aligned}u_x &= u\cos\theta\\ &= 660 \times \cos 36°\\ &= 534\ \text{m s}^{-1}\ \text{(to 3 sf)}\end{aligned}$$

Vertical component:

$$\begin{aligned}u_y &= u\sin\theta\\ &= 660 \times \sin 36°\\ &= 388\ \text{m s}^{-1}\ \text{(to 3 sf)}\end{aligned}$$

CHECK YOUR LEARNING 1.1

Describe and explain

1 **Define** the term ‘vector’.

2 **Recall** the head and tail rule for adding vectors.

3 **Calculate** the resultant velocity for a ball with a horizontal component of 20.0 m s^{-1} to the right and a vertical component of 40.0 m s^{-1} downwards.

4 An arrow is fired to the left at an angle of elevation of 25.0° and a speed of 46.0 m s^{-1} (Figure 11). **Calculate** its horizontal and vertical components.

FIGURE 11 An archer can change the vertical and horizontal speed of an arrow by careful use of elevation (the angle) and ‘draw’ (the pull on the string to control speed).

5 **Clarify** how the magnitude of a vector quantity is shown graphically (with an arrow).

Apply, analyse and interpret

6 A 10.0 m s^{-1} horizontal velocity vector is represented by this arrow: $\rightarrow$

Determine the vector that would represent a ball projected vertically upwards at 5.0 m s^{-1}.

7 A ball strikes the ground with a resultant velocity of 150 m s^{-1}. It is known that the horizontal component of the velocity is 62 m s^{-1} to the right. **Determine** the vertical component and the angle of impact relative to the ground.

8 A football is kicked to the right at a speed of 26.6 m s^{-1} and the horizontal component is measured as 25.0 m s^{-1}. **Determine** the vertical component and the angle of elevation.

FIGURE 12 A football kick can be broken down into vertical and horizontal components.

9 **Deduce** the angle of elevation that would produce a vertical component of velocity that is twice its horizontal component.

Investigate, evaluate and communicate

10 ‘It is said that at an angle of elevation of 45° the horizontal and vertical components are equal.’ **Evaluate** this statement.

11 **Assess** whether the following statement is true: ‘When the angle of elevation is doubled, the vertical component of velocity is doubled.’

12 **Prove** that the magnitude of a component of a vector cannot be bigger than the magnitude of the vector.

Check your obook assess for these additional resources and more:

- Student book questions
  Check your learning 1.1
- Challenge worksheet
  1.1A What does an odometer measure?
- Challenge worksheet
  1.1B As the crow flies

1.2 Horizontal projection

KEY IDEAS

In this section, you will learn about:

- projectile motion
- combining (adding) vectors.

trajectory
the path taken by a projectile in flight

Projectile motion can be divided into two types: horizontal projection and projection at an angle, as shown in Figure 1. You can see the projectiles have different paths. (The path of a projectile is called a **trajectory** from the Latin *trajectus* meaning 'crossing' or 'passage'). In both cases air resistance is assumed to be negligible. Air can have a dramatic effect on the trajectory of projectiles, however you will consider projection without air first and then discuss the effect of projection through a fluid (such as air) later.

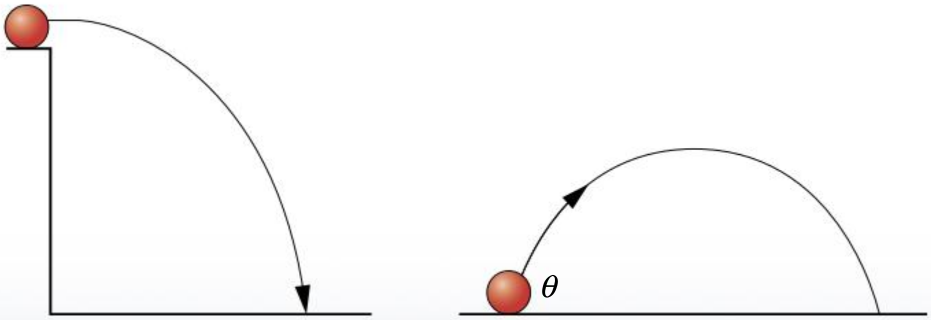

FIGURE 1 Horizontal projection, and projection at an angle

FIGURE 2 The iconic closing scene from *Thelma and Louise* shows a car as a horizontal projectile. Which will reach the ground first, and why – the car or a detatched hubcap?

Free-fall acceleration

free-fall acceleration the acceleration of a body falling freely in a vacuum near the surface of an astronomical body in the local gravitational field

If you drop a rock, it will accelerate downwards at 9.8 m s^{-2} provided that no other external forces interfere with its motion. This is called its **free-fall acceleration**.

On Earth, g (the acceleration due to gravity force) is equal to 9.8 m s^{-2}, but it does vary from location to location. The value of acceleration due to gravity on other astronomical bodies depends largely on the mass of the body, and is shown in Table 1.

TABLE 1 Mass and free-fall acceleration

Astronomical body	Free-fall acceleration (m s^{-2}) at the surface
Moon	1.6
Mars	3.7
Earth	9.8
Jupiter	24.8
Sun	274.4

Analysing motion

Horizontal projection is the simplest trajectory and an example is a rock thrown straight out off a cliff. Before you analyse the motion, you need to specify the symbols for various quantities. You will use the convention that the upwards direction is positive (+) and downwards is negative (–), and in the horizontal direction to the right is positive (+) and to the left is negative (–).

- u_x is the starting velocity in the horizontal (or x) direction.
- v_x is the final velocity in the horizontal (or x) direction.
- u_y is the starting velocity in the vertical (or y) direction.
- v_y is the final velocity in the vertical (or y) direction.
- g is the acceleration due to gravity (–9.8 m s^{-2}).
- s_x is the displacement in the horizontal (x) direction (also called the '**range**').
- s_y is the displacement in the vertical (y) direction.

range the horizontal displacement of a projectile upon impact

In this case of horizontal projection, there is no net force on the rock in the horizontal (x) direction to cause acceleration, so the final velocity equals the starting velocity ($v_x = u_x$). In the vertical direction, the initial vertical velocity equals zero ($u_y = 0$) but increases as time passes.

A graph of height (s_y) versus horizontal range (s_x) of two balls in free-fall shows this clearly (Figure 3). The graph depicts the balls at equal time intervals of 0.1 s and shows that the balls fall a greater distance each time interval. It also shows that the balls fall at the same rate whether they are dropped (red ball) or projected horizontally (blue ball). Note that the blue ball moves horizontally the same distance every time interval because it is moving at constant speed in that direction.

We can represent the vertical and horizontal components of the motion of the blue ball by using vector arrows, as shown in Figure 4.

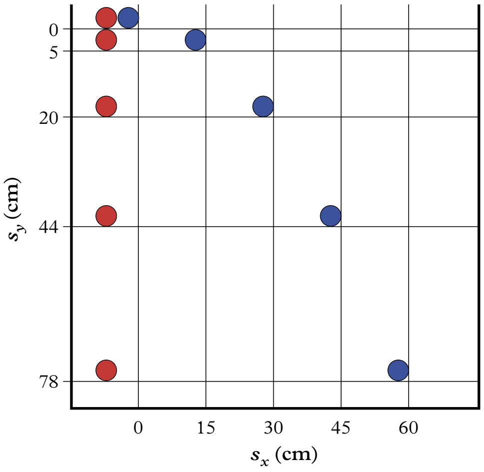

FIGURE 3 Two balls in free-fall fall at the same rate.

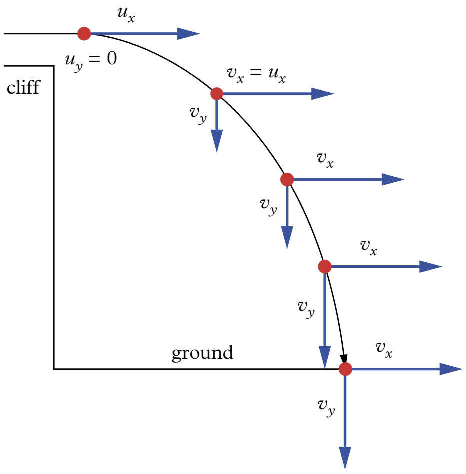

FIGURE 4 Changes in vertical and horizontal components of velocity during free-fall

Study tip

It may help you comprehend that the vertical and horizontal components of the velocity vectors are independent of each other by making a video of two balls being released simultaneously: one free-falling vertically, and the other projected horizontally. Most physics departments have a small device that projects the balls in this way. Analysis of the projectiles in slow motion is revealing.

Horizontal motion

In the horizontal direction, the acceleration (a) is zero, so the appropriate formulas will be:

i $v_x = u_x + at$ (let acceleration = zero)

$v_x = u_x$

ii $s_x = u_x t + \frac{1}{2} at^2$ (let acceleration = zero)

$s_x = u_x t$

WORKED EXAMPLE 1.2A

A golf ball is projected horizontally off a cliff at 20.0 m s^{-1} and it takes 4.0 s to reach the ground. Calculate how far out from the base of the cliff the ball will land.

SOLUTION

Facts (given): $u_x = 20.0$ m s^{-1} (= v_x); $t = 4.0$ s

Find (the answer): s_x

Formula (to use):

$$\begin{aligned} s_x &= u_x t \\ &= 20.0 \times 4.0 \\ &= 80.0 \text{ m} \end{aligned}$$

Finish (it off): The ball will land 80 m out from the base (2 sf).

Vertical motion

In the vertical direction an object will accelerate downwards. If it is given no initial downwards velocity, then it will start at rest in the vertical direction ($u_y = 0$), and its acceleration (g) will be −9.8 m s^{-1}. The appropriate formulas will be:

$$v_y = u_y + gt$$

$$v_y{}^2 = u_y{}^2 + 2gs_y$$

$$s_y = u_y t + \tfrac{1}{2}gt^2$$

It is important not to overlook the fact that the time taken in the vertical direction is the same as the time taken in the horizontal direction. The object doesn't split in two and have two different times. That's why you can just use *t* for time in both directions.

WORKED EXAMPLE 1.2B

For the golf ball in Worked example 1.2A, calculate:

a the impact velocity in the vertical direction

b the height of the cliff.

SOLUTION

Facts (given): $u_y = 0\text{ m s}^{-1}$, $t = 4.0\text{ s}$, $g = -9.8\text{ m s}^{-1}$.

Find (the answer): v_y, s_y

a *Formula (to use):*

$$\begin{aligned} v_y &= u_y + gt \\ &= 0 + -9.8 \times 4.0 \\ &= -39.2\text{ m s}^{-1} \end{aligned}$$

Impact velocity is 39 m s^{-1} in the downwards direction (2 sf).

b *Formula (to use):*

$$\begin{aligned} s_y &= u_y t + \frac{1}{2}gt^2 \\ &= 0 \times 4.0 + \frac{1}{2} \times -9.8 \times 4.0^2 \\ &= -78.4\text{ m (78 m to 2 sf)} \end{aligned}$$

The displacement is –78 m (to 2 significant figures), therefore the cliff is 78 m high.

Note: you need to state the height of the cliff, which is a scalar quantity and has no direction, rather than the displacement, which is a vector quantity and has direction (+/–).

FIGURE 5 The impact velocity of a long jumper is the combination of the vertical and horizontal components of motion.

Combining vectors

So far you have calculated the vertical and horizontal components of the velocity of a golf ball on impact. Now you need to **combine** these vectors to determine the resultant vector.

combine (vectors)
add two or more vectors to determine the resultant vector

WORKED EXAMPLE 1.2C

In Worked examples 1.2A and 1.2B, the golf ball had an impact velocity in the vertical direction of 39.2 m s^{-1} downwards (before rounding to 2 significant figures), and an impact velocity of 20.0 m s^{-1} in the horizontal direction to the right. Calculate the resultant impact velocity.

SOLUTION

Facts (given): $v_x = 20.0 \text{ m s}^{-1}$, $v_y = -39.2 \text{ m s}^{-1}$

Find (the answer): the resultant velocity (magnitude and direction)

Figure (it out): (see Figure 6)

Magnitude:

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ \text{resultant} &= \sqrt{39.2^2 + 20.0^2} \\ &= \sqrt{1937} \\ &= 44.0 \end{aligned}$$

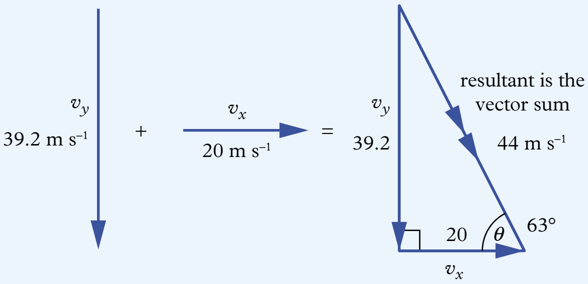

FIGURE 6 Combining vector quantities to form a resultant

The magnitude of the resultant vector = 44.0 m s^{-1} (3 sf).

Direction:

$$\begin{aligned} \theta &= \tan^{-1}\frac{39.2}{20.0} \\ &= \tan^{-1} 1.96 \\ &= 63.0°\text{(to the horizontal)} \end{aligned}$$

Finish (it off): The impact velocity is 44 m s^{-1} at 63° (to the horizontal) (2 sf).

Putting it all together

Projectile questions will not always be straightforward. You will be given different quantities and it will require critical thinking to solve problems with the data. Worked example 1.2D is on the next page.

WORKED EXAMPLE 1.2D

A rock is thrown horizontally at a speed of 35 m s^{-1} off a cliff 75 m high.

Determine the impact velocity.

SOLUTION

Facts: $u_x = 35$ m s^{-1}, $u_y = 0$ m s^{-1}, $g = -9.8$ m s^{-2}, $s_y = -75$ m (Note: this is often left as 75 m, which is incorrect. The vertical displacement of the rock when it hits the ground is in the negative (downwards) direction, so the displacement, which is a vector quantity, is written as −75 m. You will get a wrong answer if you just use 75 m.)

Find: v_{impact}

Formula:

Horizontal

$$u_y = v_y$$
$$= 35\,\text{m s}^{-1}$$

Vertical

$$v_y^{\,2} = u_y^{\,2} + 2gs_y$$
$$v_y^{\,2} = 0^2 + 2 \times -9.8 \times -75$$
$$v_y^{\,2} = 1470$$
$$v_y = \sqrt{1470}$$
$$= \pm 38.34 \text{ m s}^{-1}$$

(Recall that the square root of a number can be either positive or negative.)

You can discard the positive value, as you know the rock is heading downwards and thus in the negative direction.

Addition of vectors: (see Figure 7)

Formula:

Magnitude

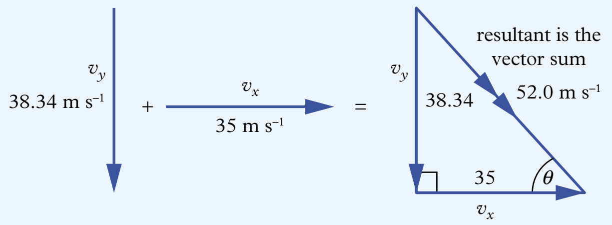

FIGURE 7 Combining vectors to form the resultant

$$c = \sqrt{a^2 + b^2}$$
$$\text{resultant} = \sqrt{38.34^2 + 35^2}$$
$$= \sqrt{2695}$$
$$= 51.9$$

The magnitude of the resultant vector = 51.9 m s^{-1}.

Direction (angle)

$$\theta = \tan^{-1}\frac{38.34}{35}$$
$$= \tan^{-1} 1.095$$
$$= 47.6°\text{(to the horizontal)}$$

Finish (it off): The impact velocity is 52 m s^{-1} at 48° (to the horizontal) (2 sf).

CHECK YOUR LEARNING 1.2

Describe and explain

1 **Explain** whether the horizontal component of a projectile's velocity is independent of the vertical component.

2 The acceleration of a projectile is usually stated as $-9.8\ \text{m s}^{-2}$. **Explain** how it can be negative when it gets faster and faster as it falls.

3 A dropped ball and a ball projected horizontally from the same height both land at the same time. **Explain** how this can be when the projected ball has to travel a longer path.

4 **Clarify** whether the initial and final components of horizontal velocity are related.

Apply, analyse and interpret

5 A skier jumps off a mountaintop at a horizontal velocity of $25\ \text{m s}^{-1}$ and takes 5.0 s to reach the ground.

FIGURE 8 A skier jumps.

Determine the:

a height of the mountaintop

b distance out from the base of the mountaintop that the skier lands

c impact velocity.

6 A rock is thrown horizontally at $8.0\ \text{m s}^{-1}$ off a 100.0 m high cliff. **Determine**:

a how long it takes the rock to hit the ground

b the impact velocity of the rock

c how far out from the cliff the rock lands.

7 A ball is thrown horizontally from the top of a cliff and hits the ground 6.0 seconds later at 30 degrees to the vertical, moving at $67.9\ \text{m s}^{-1}$. **Determine**:

a the initial velocity of the projectile

b its initial horizontal and vertical velocity

c the range

d the final horizontal and vertical velocity.

Investigate, evaluate and communicate

8 You can throw a ball horizontally on Earth at a speed of $50\ \text{m s}^{-1}$. Now imagine you threw it in the same way on the Moon. **Predict** and **justify** how the range and impact velocity would be different.

9 An astronaut on the surface of Mars decides to emulate the golf stroke of Apollo 14 astronaut Alan Shepard who hit some golf balls on the Moon while visiting in 1971. The Mars astronaut stood on the edge of the *Endeavour Crater* and hit a golf ball horizontally. It landed off in the distance. **Evaluate**, with reasoned evidence, this statement: 'Under exactly the same conditions, the ball would travel horizontally 1.6 times further on Mars than on Earth.'

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  Check your learning 1.2
- » Video
  Vector and scalar meaurements
- » Video
  Calculating vertical direction

1.3 Projection at an angle

KEY IDEAS

In this section, you will learn about:

- the horizontal and vertical components of a velocity vector
- the independence of horizontal and vertical components of a velocity vector.

Not all projectiles are launched in the horizontal direction. Arrows, cannonballs, footballs and netballs, for example, are usually projected upwards at an angle.

You can then resolve (break up) the launch velocity into horizontal and vertical components. This is the opposite of combining vectors into a resultant vector.

Motion of object projected at an angle

The motion of the projectile is a **parabola** because the vertical displacement varies as a function of t^2 (i.e. $s_y = u_y t + \frac{1}{2} gt^2$), as it is uniformly accelerated motion; however, the horizontal displacement varies with just t (i.e. $s_x = u_x t$), as it is constant velocity. Recall that the horizontal displacement is called the range.

parabola
a graph of a quadratic function for which the power a is 2, e.g. $y = x^2$

Case 1: Landing at the same height as at launch

For the simplest case of projection at an angle, the object will land at the same vertical displacement from which it was launched, as in Figure 1. These statements can be made:

- The **impact speed** will be equal to the launch speed. Recall from earlier in this chapter that for purely vertical motion, initial speed equals final speed.
- The impact velocity will have the same magnitude and angle to the ground as the **launch velocity**, but it will be in a general downwards direction, not upwards, as at launch (Figure 1).
- The **horizontal component** of velocity remains constant for the duration of the flight, hence $v_x = u_x$.
- The **vertical component** of velocity at launch equals the vertical component of velocity at impact but in the opposite direction: $v_{y\text{ (impact)}} = -u_{y\text{ (launch)}}$.

impact speed
the speed at which an object impacts a surface

launch velocity
the velocity at which a projectile is launched; is a vector quantity that has both magnitude and direction, usually specified as an angle of elevation to the horizontal

horizontal component
the resolution of a projectile's velocity in the horizontal direction

vertical component
the resolution of a projectile's velocity in the vertical direction

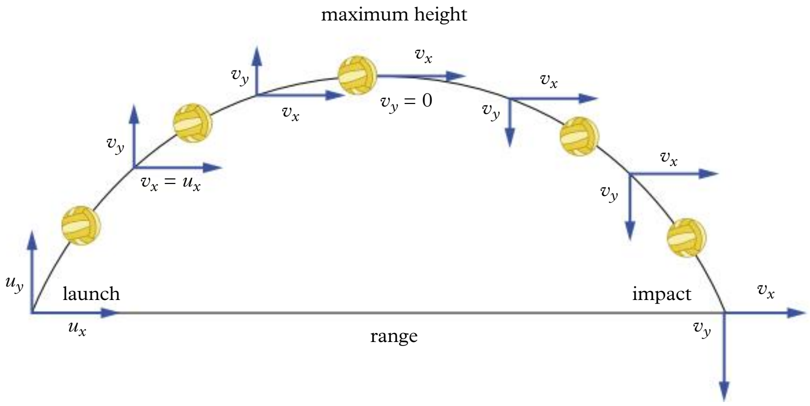

FIGURE 1 Horizontal and vertical components of velocity for the flight of a projectile

MANDATORY PRACTICAL
1.3 Angled projection and distance
Go to page 404 »

WORKED EXAMPLE 1.3A

A bullet was fired from a rifle at 200.0 m s^{-1} at an angle of 40.0° to the ground. Ignoring air resistance, calculate the:

a initial vertical and horizontal components of the velocity

b maximum height reached

c time of flight (total time taken from start to finish)

d horizontal range.

SOLUTION

Facts: $u = 200.0$ m s^{-1}, $\theta = 40.0°$, $g = -9.8$ m s^{-2} (the upwards direction is positive)

a Vertical:

$$u_y = u \sin\theta$$
$$= 200.0 \times \sin 40.0°$$
$$= +128.6$$
$$= +129 \text{ m s}^{-1}$$

in the positive direction (up)

Horizontal:

$$u_x = u \cos\theta$$
$$= 200.0 \times \cos 40.0°$$
$$= 153.2$$
$$= 153 \text{ ms}^{-1}$$

b At maximum height the vertical velocity $v_y = 0$ m s^{-1}.

$$v_y^{\,2} = u_y^{\,2} + 2gs_y$$

hence:

$$s_y = \frac{v_y^{\,2} - u_y^{\,2}}{2g}$$
$$= \frac{0^2 - (+129)^2}{2 \times (-9.8)}$$
$$= +844 \text{ m (3 sf)}$$

c Time of flight can be calculated by:

i determining the time taken to reach maximum height ($v_y = 0$) and doubling it; or

ii determining time taken until the final vertical velocity is equal and opposite to initial vertical velocity; or

iii determining the time until the vertical displacement (s_y) is zero again.

Using method **i**: $v_y = u_y + gt$

hence: $t = \dfrac{v_y - u_y}{g}$

$$= \frac{0 - (+129)}{-9.8}$$
$$= 13.12 \text{ s}$$

Using method **ii**: $v_y = u_y + gt$

hence: $t = \dfrac{v_y - u_y}{g}$

$$= \frac{-129 - (+129)}{-9.8}$$
$$= 26.24 \text{ s}$$

Using method **iii**: $s_y = u_y t + \frac{1}{2} gt^2$

hence: $0 = +128.6t + -4.9t^2$

$$4.9t = 128.6$$
$$t = 26.24 \text{ s}$$

Total time = 26.24 s

d Horizontal range = horizontal component of initial velocity × time of flight.

$$s_x = u_x \times t$$
$$= 153.2 \times 26.24$$
$$= 4020 \text{ m (3 sf)}$$

Case 2: Landing at a lower height than at launch

Landing at a lower height than at launch is similar to a person launching a shot-put at shoulder height and it hitting the ground at foot level, or a skier launching upwards off a ramp and landing further down the mountain (as in Figure 2). The main difference between this and Case 1, landing at the same height as at launch, is that the vertical displacement at impact is no longer zero but some negative quantity.

FIGURE 2 A skier jumping from a bump will land at a lower height than that from which they took off.

WORKED EXAMPLE 1.3B

A cannon is fired from the edge of a cliff, which is 60.0 m above the sea (Figure 3).

The initial velocity of the cannonball is 88.3 m s^{-1} and it is fired at an upwards angle of 34.5° to the horizontal. Determine:

a the time the ball is in the air

b the impact velocity

c the horizontal distance from the base of the cliff that the ball strikes the water.

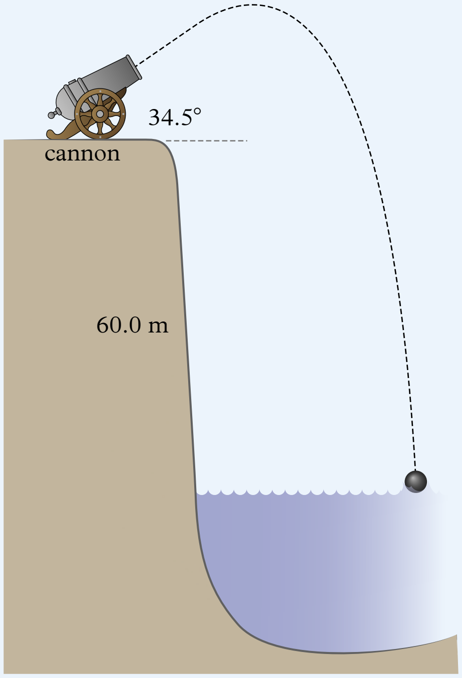

FIGURE 3 The projectile lands lower than launch.

SOLUTION

- Vertical component of initial velocity

  $u_y = u \sin\theta$

  $= 88.3 \sin 34.5°$

  $= +50.0\,\text{m s}^{-1}$ (positive is up)
- Horizontal component of initial velocity

  $u_x = u \cos\theta$

  $= 88.3 \cos 34.5°$

  $= 72.8\,\text{m s}^{-1}$
- The final vertical displacement

  $s_y = -60.0\text{ m}$
- Vertical acceleration

  $g = -9.8\text{ m s}^{-2}$

You can calculate time of flight directly, but it would involve solving a quadratic equation (and all the possibility for errors this method seems to attract). To avoid this, calculate the vertical impact velocity first:

a $v_y{}^2 = u_y{}^2 + 2gs_y$

$= +50^2 + 2 \times (-9.8 \times (-60.0))$

$= 2500 + 1176$

$= 3676$

$v_y = \sqrt{3676}$

$= \pm 60.6\,\text{m s}^{-1}$ (discard the positive value as the ball is moving down)

To calculate the time of flight:

$v_y = u_y + gt$

hence: $t = \dfrac{v_y - u_y}{g}$

$= \dfrac{-60.6 - (+50)}{-9.8}$

$= 11.3$ seconds (3 sf)

b The impact velocity is calculated by the vector addition of the final vertical velocity and the final horizontal velocity (Figure 4).

Vector sum:

$v = \sqrt{(-60.6)^2 + 72.8^2}$

$= 94.7\,\text{m s}^{-1}$

$\theta = \tan^{-1}\dfrac{60.6}{72.8}$

$= 39.8°$to the horizontal

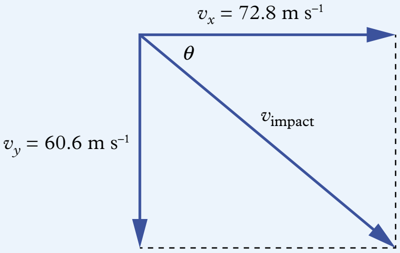

FIGURE 4 Recombining vectors for a resultant

c Horizontal distance (s_x) = horizontal component of velocity (v_x) × time of flight (t).

$s_y = 72.8 \times 11.3$

$= 823$ m out from the base of the cliff (3 sf)

FIGURE 5 When shooting for the basket, the velocity, angle and height of release have to be just right.

Case 3: Landing at higher than launch height

Study tip

This type of question is a favourite for external assessment. Pay particular attention to whether the final displacement is positive or negative.

The final variation on projectile trajectory is where it lands higher than at launch height. A good example of this is a basketball player shooting a ball into a hoop.

For example, in Figure 6 the ball leaves the player's hand at an initial vertical displacement of +1.9 m and lands in the hoop at a final vertical displacement of +3.1 m. You can simplify this by saying the initial vertical displacement is 0 m (where it leaves the player's hands), and the final vertical displacement at the hoop is +1.2 m (higher than at the start). For this example, $s_y = +1.2$ m, s_x (range) = 4.0 m, $\theta = 60°$ upwards, $g = -9.8$ m s^{-2}.

There is a question like this in Check your learning 1.3.

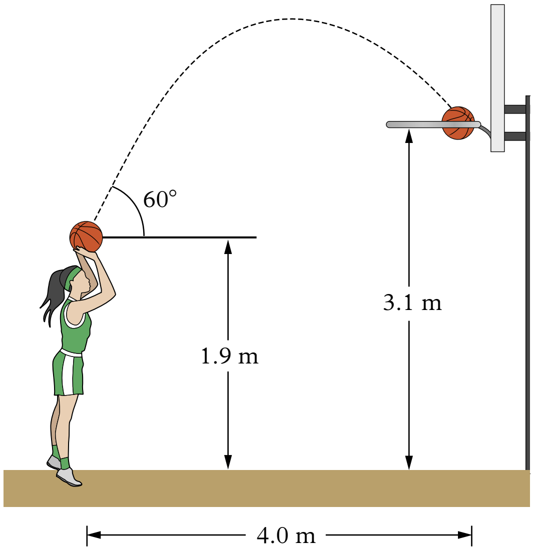

FIGURE 6 Schematic for a ball landing higher than at launch

CHALLENGE 1.3A

Cricket

Which hits the ground faster: a cricket ball thrown horizontally at 10 m s^{-1} or one thrown at 10 m s^{-1} but at 45°?

CHALLENGE 1.3B

Cannonball horizontal distance

A cannonball is fired and, after travelling 5 m horizontally, reaches half its maximum height (h_{max}). At what horizontal distance will it land?
(Beware, this is difficult and the solution for this challenge is at least three pages long!)

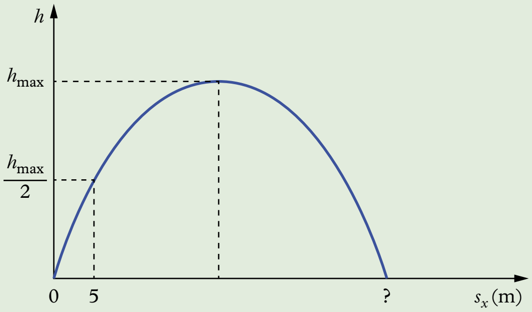

FIGURE 7 Trajectory of a cannonball

CHALLENGE 1.3C

How far will a flea go?

A flea can jump 18.4 cm high when jumping at 45°. How far horizontally will it go?

CHALLENGE 1.3D

Archery world speed record

The world speed record for an archery arrow shot over 100 m is 1.64 seconds (220 km h^{-1}). Calculate the elevation angle of the arrow so that it hits the bullseye at the same height from which it was fired (shoulder height).

Complementary angles of elevation

complementary angles
angles that add to 90°; for example, 60° is the complementary angle (complement) of 30° and 75° is the complement of 15°.

If you've ever hosed the garden, you would know that you can direct the water to fall on a particular plant by either using a high angle of elevation for the stream of water or a low angle. In both cases, the range is the same. The two angles that give the same range are complements of each other. This means they add up to 90°. For example 60° and 30° give the same range and they add up to 90°. Figure 8 shows a variety of **complementary angles** and the 'trajectory' of the projectile (including a stream of water). Mandatory practical 1.3 'Angled projection and distance' investigates this.

You can explore this further. Imagine hitting two baseballs with the same force so they both have the same starting speed but one is at 60° and the other at 30°. The following Worked example shows how this can be approached.

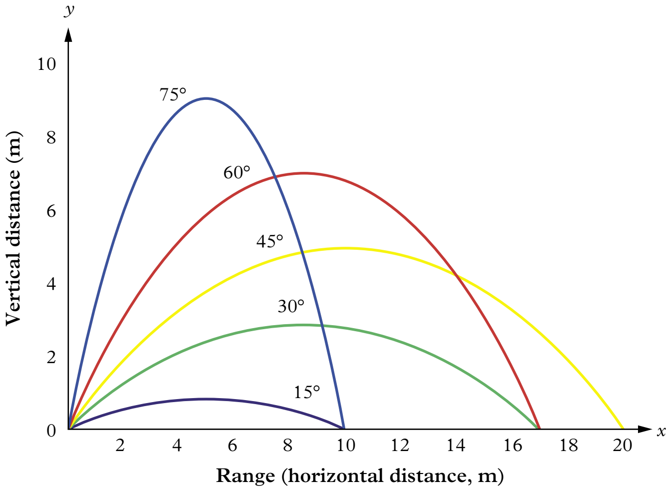

FIGURE 8 Complementary angles give the same range in the absence of air.

WORKED EXAMPLE 1.3C

A baseball is struck with an initial speed of 20 m s^{-1} at angles of elevation of 60° and 30°. Assume air resistance to be negligible. Determine the following for each angle:

a the time of flight

b the horizontal range, and how they compare.

SOLUTION

a First, calculate the components of the velocity.

For 60°:

$$u_y = 20 \sin 60°$$
$$= 17.3\,\text{m s}^{-1}$$
$$u_x = 20 \cos 60°$$
$$= 10\,\text{m s}^{-1}$$

Determine the time of flight.

$$s_y = u_y t + \frac{1}{2}gt^2$$

hence $0 = +17.3t + -4.9t^2$

$$0 = t(17.3 - 4.9t)$$

$0 = t$ (which we already knew) or $0 = 17.3 - 4.9t$

$$0 = 17.3 - 4.9t$$

$4.9t = 17.3$

$t = \dfrac{17.3}{4.9}$

$t = 3.5$ s

For 30°:

$u_y = 20 \sin 30°$

$= 10\,\text{m s}^{-1}$

$u_x = 20 \cos 30°$

$= 17.3\,\text{m s}^{-1}$

$s_y = u_y t + \dfrac{1}{2} gt^2$

hence $0 = +10t + -4.9t^2$

$0 = t(10 - 4.9t)$

$0 = t$ (which we already knew) or $0 = 10 - 4.9t$

$0 = 10 - 4.9t$

$4.9t = 10$

$t = \dfrac{10}{4.9}$

$t = 2.0$ s

b To calculate the range, let $s_y = 0$, as you want to see how long it takes to get back to the starting height.

For 60°:

$s_x = u_x t$

$= 10 \times 3.5$

$= 35$ m

For 30°:

$s_x = u_x t$

$= 17.3 \times 2.0$

$= 35$ m

A comparison of the ranges shows that they are equal (35 m), as expected for complementary angles.

Note that for the higher angle of elevation, the ball is in the air for 3.5 s compared to 2.0 s for the lower angle. You can decide which would be better in a baseball game: more time in the air means more time to run but also more time to get in position for a catch.

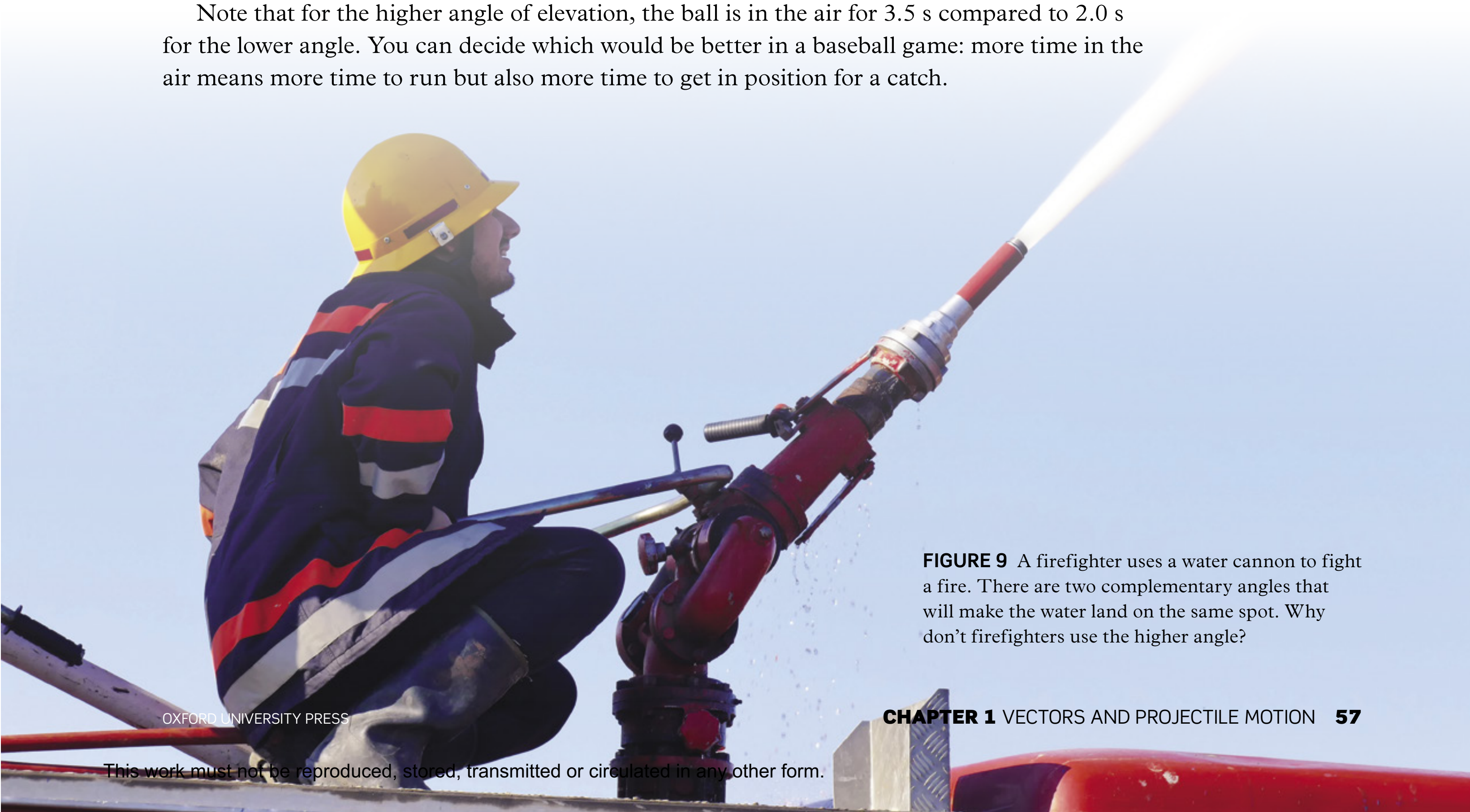

FIGURE 9 A firefighter uses a water cannon to fight a fire. There are two complementary angles that will make the water land on the same spot. Why don't firefighters use the higher angle?

CASE STUDY 1.3

The effect of air on projectiles

Aristotle argued that once a projectile ran out of impetus it would fall vertically from the sky. Galileo argued that this was wrong, and that the trajectory would be parabolic. Galileo was right – or was he?

In the discussion so far air resistance has been ignored. When air resistance is taken into account, the trajectory is different. Aristotle is almost right but for the wrong reasons. At low speeds air resistance is negligible, but at greater speeds it becomes considerable.

For instance, a fly-ball hit at an angle of elevation of 60° at 45 m s^{-1} will have different trajectories in air compared with those in a vacuum.

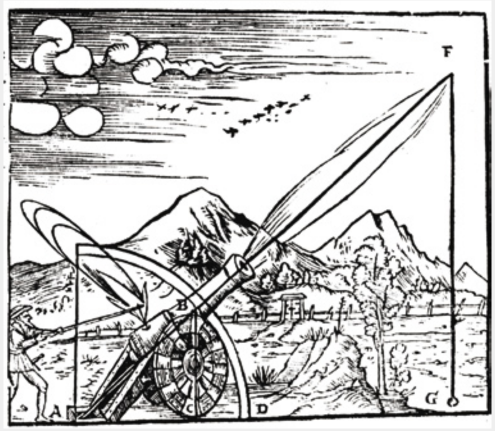

FIGURE 10 The path of the projectile according to Aristotelian physics, later contradicted by Galileo

TABLE 1 Computer model of the trajectory of a ball through air and a vacuum

	Path A (in air)	Path B (in a vacuum)
Range	100 m	177 m
Maximum height	53 m	77 m
Time of flight	6.6 s	7.9 s

Figure 11a shows the difference between the trajectory of a ball as predicted by a computer model, and Figure 11b shows that of a bullet as tracked by ballistics experts on a rifle range.

The differences come about because bullets have more complicated motions than a round ball in flight.

Trial-and-error has shown that the maximum range for a bullet fired in air is achieved at an elevation of 33°, a rough rule-of-thumb that works for most guns. As a crude approximation, the angle of descent at maximum range is about 80°, or very nearly vertical. Any greater elevation of the gun merely means that the bullet will actually drop vertically and the last part of the flight will add nothing to the range. So the war veterans were probably right – bullets did fall on them vertically from the sky (and were just as lethal).

Earlier, the question of why firefighters don't use the bigger of the two complementary angles when fighting a fire was asked. Jets of water tend to break up the further they travel. Using a higher angle makes it harder to direct the water to the right spot.

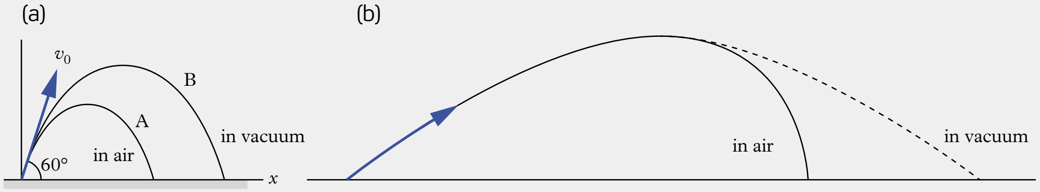

FIGURE 11 The trajectories of a ball and a bullet in air and in a vacuum

CHECK YOUR LEARNING 1.3

Describe and explain

1 **Explain** how initial velocity and final velocity are related for an object projected upwards at an angle and returning to the same height as at launch.

2 **Describe** the relationship between complementary angles of elevation and range for a projectile in the absence of air.

3 **Explain** how and why air resistance affects the range of a projectile.

Apply, analyse and interpret

4 A tennis ball close to the ground is hit by a racquet with a velocity of 30.0 m s^{-1} at an angle of 25.0° to the horizontal. **Determine**:

- **a** the initial vertical and horizontal components of the velocity
- **b** the maximum height reached by the ball
- **c** the time of flight
- **d** the horizontal range.

5 A football is kicked off the ground at an angle of 30.0° to the horizontal. It moves away at 23.0 m s^{-1}. **Determine**:

- **a** the vertical velocity after 1.00 s
- **b** the velocity of the ball after 1.00 s
- **c** the maximum height reached
- **d** the time of flight
- **e** the range of the ball.

6 A rock is thrown off a 100.0 m cliff upwards at an angle of 20.0° to the horizontal with an initial velocity of 15.0 m s^{-1} and strikes the rocks below. **Determine**:

- **a** the time of flight
- **b** the impact velocity
- **c** how far out from the base of the cliff the rock strikes the ground.

Investigate, evaluate and communicate

7 **Propose** how you would prove that the range for a projectile falling back to the same height is a maximum when the elevation angle is 45°.

8 On the Moon in February 1971, an astronaut hit a golf ball a distance of 180 m. **Evaluate** the claim that 'if the astronaut hit the same ball on Earth with the same speed and angle (assumed to be 30°), it wouldn't go as far'. Note $g_{\text{Moon}} = 1.62$ m s^{-2}.

9 The graphs in Figure 12 show how the range and altitude of a projectile change with elevation angle in the presence of air.

- **a** **Evaluate** the relationship between maximum altitude and elevation angle by graphical means.
- **b** **Predict** the maximum altitude for an angle of 90°.
- **c** **Predict**, with reasons, whether the graph would pass through the origin (0, 0).
- **d** **Describe** the shape of a graph of range versus angle and draw a justified conclusion about the best angle for maximum range.

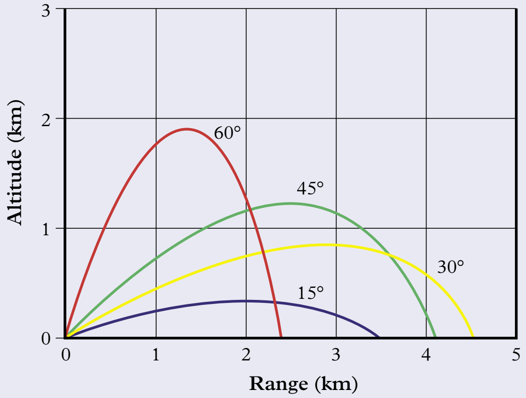

FIGURE 12 Range of a projectile as affected by the air at different altitudes

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 1.3

» Mandatory practical worksheet
1.3 Angled projection and distance

» Video
Calculating projection at an angle

» Challenge worksheet
1.3A Cricket

CHAPTER 1

Review

Summary

1.1
- A vector is a quantity that has both magnitude and direction. A vector quantity can be represented graphically by an arrowed line segment, or symbolically by a bold typeface.
- Vectors are added by placing the arrowed line segments head-to-tail.

1.2
- For a projectile, the vertical and horizontal components of the velocity vector are independent.

1.3
- The angle at which the object is thrown relative to the horizontal is called the elevation angle.
- The motion of the projectile is a parabola because the vertical displacement varies as a function t^2.
- The impact velocity will have the same magnitude as the launch velocity if returning to the same vertical displacement, but be directed down not up.
- The horizontal component of velocity is constant: $v_x = u_x$.
- The resultant (impact) velocity is the vector sum of the final horizontal and vertical components of velocity.
- The horizontal displacement is called the range.
- The range of a projectile will be the same for elevation angles of θ and $90° - \theta$. These are called complementary angles.
- The maximum range for a projectile returning to the same vertical displacement and in the absence of air resistance can be calculated by letting $\theta = 45°$.
- Air resistance affects the trajectory of a projectile by reducing its range, lowering its maximum height and making the flight path non-symmetrical.

Key terms

- combine (vectors)
- complementary angles
- components (of a vector)
- elevation angle
- free-fall acceleration
- horizontal component
- impact speed
- impact velocity
- launch velocity
- parabola
- range
- resolution
- resultant vector
- trajectory
- vector
- vertical component

Key formulas

Constancy of horizontal velocity	$v_x = u_x$
Horizontal displacement (range)	$s_x = u_x t$
Initial velocity: vertical component	$u_y = u \sin\theta$
Initial velocity: horizontal component	$u_x = u \cos\theta$
Vertical velocity as function of time	$v_y = u_y + gt$
Vertical velocity as function of (vertical) displacement	$v_y^2 = u_y^2 + 2gs_y$
Vertical displacement as a function of time	$s_y = u_y t + \frac{1}{2}gt^2$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 Figure 1 shows the vertical velocity of a projectile (a ball) with time.

Which one of the following is the best description of the ball's initial motion?

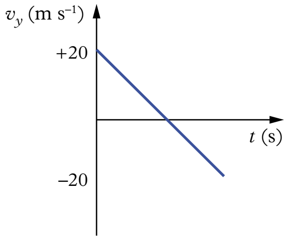

FIGURE 1

A thrown vertically upwards
B thrown vertically down
C dropped from rest
D projected horizontally off a cliff

2 A ball is fired horizontally off a cliff at different heights (h). Determine which one of the following best shows the relationship between time of fall (t) and height.

A $t \propto \sqrt{h}$
B $t \propto h$
C $t \propto h^2$
D $t \propto \frac{1}{h}$

3 A golf ball is hit into the air at an angle, reaches maximum height and falls back to the ground. Take the upwards direction as positive. Determine which one of the following best describes its vertical acceleration at maximum height.

A $+ 9.8 \text{ m s}^{-2}$
B $- 9.8 \text{ m s}^{-2}$
C 0 m s^{-2}
D -9.8^2 m s^{-2}

4 An arrow is fired with the same initial speed at angles of elevation ranging from 0° to 90°. Determine which one of the following will increase with increasing angle.

A time of flight
B range
C impact speed
D impact velocity

5 Figure 2 shows a ball being launched horizontally off a cliff at a speed of v, which gives the ball a horizontal range R at the ground below and a time of flight t to get there. If the speed is increased to $2v$, determine which one of the following best describes the new time of flight and the new range respectively.

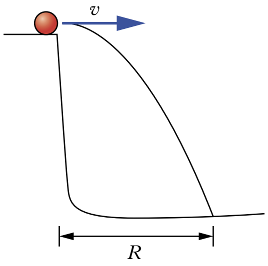

FIGURE 2

A t, R
B t, $2R$
C $2t$, R
D $2t$, $2R$

Short answer

Describe and explain

★ **6** A projectile is launched at an angle. **Recall** the magnitude and direction of the acceleration at the top of its flight.

★ **7** **Explain** whether or not the horizontal and vertical components of a projectile are dependent on each other.

★★ **8** A projectile launched at an angle follows a *parabolic* path. **Explain** what parabolic means for projectile motion.

Apply, analyse and interpret

★ **9** A projectile is fired horizontally. **Determine** the part of its trajectory for which it has the:

a highest speed
b lowest speed.

★ **10** When a wedding ring is thrown horizontally out of a fifth floor window that is 15 m off the ground, it lands 7.5 m out from the base of the building. **Determine** the:

- **a** throwing speed
- **b** impact velocity.

★★ **11** A boy sitting in a train carriage moving at constant velocity throws a ball straight up in the air. **Determine**:

- **a** whether the ball will fall behind him, in front of him or into his hands
- **b** what will happen if the train accelerates while the ball is in the air
- **c** what will happen if the train turns a corner while the ball is in the air.

★★ **12** A golf ball is hit by a club and moves off with a velocity of 30 m s^{-1} at an angle of 55° to the horizontal. **Determine** the:

- **a** initial vertical and horizontal components of the velocity
- **b** maximum height reached
- **c** time of flight
- **d** horizontal range
- **e** impact velocity.

★★ **13** A soccer ball is kicked off the ground at an angle of 20° to the horizontal. It moves away at 30.0 m s^{-1}. **Determine** the:

- **a** vertical velocity after 0.5 s
- **b** velocity of the ball after 1.0 s
- **c** maximum height reached
- **d** time of flight
- **e** range of the ball.

★★★ **14** The world record for throwing an uncooked hen's egg is 96.90 m, set in 1981. The egg was thrown at an angle of elevation of 45° and negligible air resistance. **Determine** what would have been the:

- **a** throwing speed
- **b** maximum height above the ground
- **c** time of flight
- **d** impact velocity.

★★★ **15** A dart is thrown horizontally towards the bullseye of a dartboard, but it strikes the 3 on the bottom of the board directly underneath the bullseye 0.19 s after it left the player's hand (shown by the red arrow in Figure 3). **Determine** the distance from the bullseye to the 3.

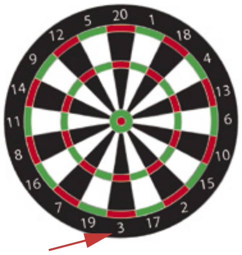

FIGURE 3

★★★ **16** A boy kicks a football off the ground and it lands on the roof of his home 57.3 m away at a height of 3.8 m. The time of flight was 3.0 seconds. **Determine**:

- **a** its launch speed
- **b** its initial elevation angle
- **c** the maximum height reached
- **d** the time of flight if he kicked the ball at the complementary angle.

★★★ **17** A plane diving at an angle of 53.0° to the vertical releases a rescue kit at an altitude of 730 m. This projectile hits the water 4.50 s after being released (Figure 4).

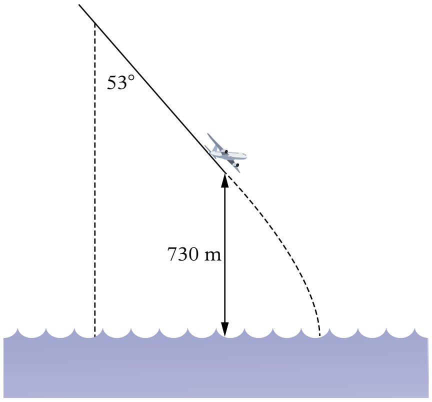

FIGURE 4

Determine:

- **a** the speed of the plane
- **b** how far the projectile travelled horizontally during its flight
- **c** the impact velocity of the projectile.

★★★ **18** A difficult one! A basketball player shoots a ball at an angle of 55° into a hoop on a post 4.3 m away (Figure 5). The ball is released from a height of 2.1 m and goes through the hoop, which is 3.0 m off the ground. **Determine** the initial speed of the ball for this foul shot to be successful.

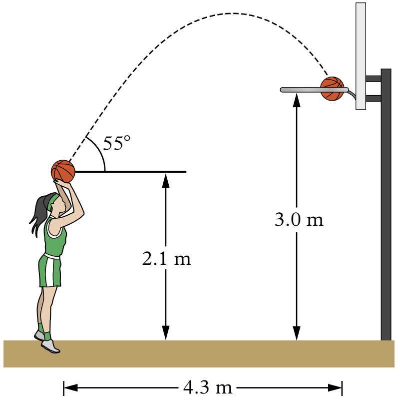

FIGURE 5

★★★ **19** Emmanuel Zacchini was a famous American ‘human cannonball’. In 1940 he attempted to clear a Ferris wheel 18 m high after being launched from a cannon at an elevation angle of 53° and a muzzle velocity of 26.5 m s^{-1}. His initial point of projection from the cannon was 3.0 m above the ground. **Determine,** with reasons and a labelled diagram:

a whether he cleared the Ferris wheel

b how far away from the cannon the net would have been placed.

★★★**20** An experiment was done to investigate the range of a steel ball launched horizontally off a bench top 0.76 m from the floor. The horizontal velocity and range were measured and shown in Table 1.

TABLE 1 Data for horizontal projectile experiment

Horizontal velocity (m s^{-1})	0.30	0.53	0.99	1.34	1.50
Horizontal range (m)	0.12	0.22	0.40	0.53	0.59

a **Construct** a graph of range (vertical axis) versus horizontal velocity (horizontal axis).

b **Interpret** the evidence to assess if range is proportional to horizontal velocity.

c **Deduce** the time of flight from the graph.

d **Calculate** the acceleration due to gravity from the data.

e **Calculate** the absolute and percentage error given $\boldsymbol{g}$ (accepted) = –9.8 m s^{-2}.

★★★ **21** Data were collected in an experiment to investigate the effect of launch angle on range and listed in Table 2.

TABLE 2 Effect of launch angle on range

Launch angle (°)	Range (m)		
	Test 1	Test 2	Test 3
15	1.30	1.06	1.31
30	2.11	2.15	2.11
45	2.39	2.50	2.47
60	2.11	2.15	2.11
75	1.30	1.06	1.31

Analyse evidence in the dataset to:

a **identify** trends in the relationship

b **justify** a conclusion about the effect of angle on range.

Investigate, evaluate and communicate

★★★ **22** In the 1968 Olympics in Mexico City, Bob Beamon shattered the world long jump record with a jump of 8.90 m. His speed on take-off was measured at 9.5 m s^{-1}, about equal to that of a sprinter. At the time, newspapers questioned how close he came to achieving maximum range for this speed in the absence of air resistance (assuming maximum range is at an initial elevation of 45°). The value of g in Mexico City is 9.78 m s^{-2}. **Propose** an answer to their question and justify your response.

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CHAPTER 2

Inclined planes

An inclined plane or ramp is a flat surface raised at one end, used as an aid for raising or lowering a load. Moving an object up an inclined plane requires less force than lifting it straight up, although it has to be moved through a longer distance. The incline used by the Ancient Egyptians for building pyramids and the sloping roads built by the Romans are examples of early inclined planes. It shows that they understood the value of this technology for moving things uphill.

OBJECTIVES

- → Solve problems involving force due to gravity (weight) and mass using the mathematical relationship between them.
- → Define the term ‘normal force’.
- → Describe and represent the forces acting on an object on an inclined plane through the use of free-body diagrams.
- → Calculate the net force acting on an object on an inclined plane through vector analysis.

Source: *Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority

FIGURE 1 Ancient Egyptian workers built the Great Pyramids by dragging blocks of limestone up inclined planes that wound around the outside.

MAKES YOU WONDER

In this chapter you will be examining some aspects of inclined planes that will help to answer questions such as these:

- → Wouldn't you need a very long incline to get to the top of a pyramid?
- → Is there a maximum legal angle for wheelchair ramps?
- → Would inclined planes be any use in space?
- → Would you still call it an inclined plane if the angle was 90°?
- → Could you have a road incline so steep that a car's brakes wouldn't hold it?
- → Does friction always act up an incline, slowing an object down?

PRACTICALS

SUGGESTED PRACTICAL	**2.3** Parallel component on an inclined plane

2.1

Forces due to gravity

KEY IDEAS

In this section, you will learn about:

- weight, normal force, applied force, friction and tension.

Scientists have identified four fundamental types of **force**: gravitational, electromagnetic (involving both electrostatic and magnetic forces), weak nuclear forces and strong nuclear forces. All interactions between matter can be explained as the action of one or a combination of these four fundamental forces. There are other forces you will come across such as weight, the normal force, tension and friction. Depending how they are used, these forces can also be labelled as the applied force or net force.

force
a push or pull between objects, which may cause one or both objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape

Weight and mass

Weight is different from mass. **Mass** is a measure of an object's resistance to motion (or inertia) and doesn't vary no matter where the object might be taken to in the universe. Weight is a force that depends on nearby astronomical bodies.

Objects near or on the surface of Earth experience a gravitational attraction towards Earth's centre by a pulling force called the force of gravity. This pulling force causes freely falling objects to move downwards with an acceleration of 9.8 m s^{-2}. This is called its free-fall acceleration, *g*.

On Earth, $g = 9.8$ m s^{-2}, but on other planets and satellites it depends largely on the mass of the planet and is shown in Table 1.

weight
a measure of the force of gravity acting on an object (symbol: F_g; SI unit: newton; unit symbol: N)

mass
an object's resistance to a change in motion; also commonly stated as the amount of matter in an object (symbol: *m*; SI unit: kilogram; unit symbol: kg)

FIGURE 1 Your weight depends on the local gravitational attraction, not what you are doing – even when free-falling. Your mass is independent of local gravity.

TABLE 1 Free-fall acceleration on selected astronomical bodies

Celestial body	Earth masses	Free-fall acceleration (m s^{-2})
Jupiter	317.8	24.8
Earth (average)	1.00	9.8
Mars	0.11	3.71
Moon	0.0123	1.62

Weight, being the force due to gravity, is given the symbol F_g. For an object of mass m in a place where free-fall acceleration is g:

$$F_g = mg$$

For example, the weight of a 10 kg object on the surface of Earth, where free-fall acceleration g is 9.8 m s^{-2}, is given by:

$$\begin{aligned} F_g &= mg \\ &= 10 \times 9.8 \\ &= 98 \text{ N.} \end{aligned}$$

On the surface of the Moon, this object of mass 10 kg would have a weight of 16 N. On Jupiter it would be nearly 250 N.

WORKED EXAMPLE 2.1A

A man has a mass of 85.0 kg. Using the information in Table 1, calculate his weight on:

a Earth

b the Moon

c Jupiter ($g = 24.8$ m s^{-2}).

SOLUTION

a $F_g = mg$

$= 85.0 \times 9.8$

$= 833$ N (833 N to 3 sf)

b $F_W = mg$

$= 85.0 \times 1.62$

$= 137.7$ N (138 N to 3 sf)

c $F_W = mg$

$= 85.0 \times 24.8$

$= 2108$ N (2110 N to 3 sf)

CHALLENGE 2.1A

The greater weight

Which one of each pair has the greater weight?

a a kilogram of lead or a kilogram of feathers

b 1 cm^3 of lead or of feathers

c 1 tonne of lead or a tonne of feathers (both in free-fall)

d a man and a child both floating in a swimming pool

CHALLENGE 2.1B

Cork

Cork is a very lightweight substance, as you'd know if you've ever put a cork in water. Imagine a 1.5 m diameter ball of cork. What is its weight? You'll be surprised.

Hint: density of cork = 240 kg m^{-3}; $V_{sphere} = \frac{4}{3}\pi r^3$.

Normal force

When a box is resting on a table, gravity pulls the box towards Earth with a force F_g called its weight. The table pushes back with an equal and opposite force that is at right angles to the tabletop (Figure 2). A right angle is also called a perpendicular or 'normal', so it is called the **normal force** (F_N).

normal force
the force acting along an imaginary line drawn perpendicular to the surface.

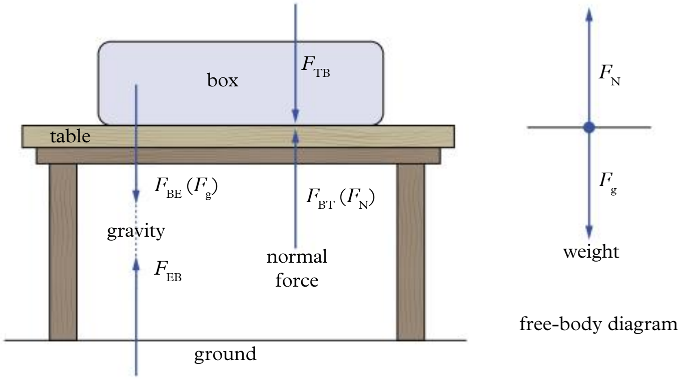

FIGURE 2 The normal force, F_N, is perpendicular to the surface.

Students often assume these two forces make up an action–reaction (or agent–receiver) pair consistent with Newton's third law. Even though these two forces are equal and opposite they are not a third law 'pair'. There are two third law pairs in the situation of the box on the table:

FIGURE 3 When the pull of gravity downwards is stronger than the supporting surface can push back up, the inevitable happens.

1 The force on the box due to Earth (F_{BE}) is also known as the weight of the box (F_g). The paired force is the pulling force on Earth due to the box (F_{EB}).

2 The downwards force on the table due to the box (F_{TB}) is paired with the upward force on the box due to the table (F_{BT}) – also known as the normal force (F_N).

One from each pair make up the action–reaction pair. The magnitudes are equal: $F_N = F_g$. This can be written in vector notation showing they are also in opposite directions: $\vec{F}_N = -\vec{F}_g$.

You could apply this to the situation in which you are standing on the ground. The gravitational attraction between you and Earth pulls you straight down towards the centre of Earth and this is called your weight, F_g. The surface of Earth pushes back on your feet and this is your normal force F_N.

WORKED EXAMPLE 2.1B

A 2.00 kg box rests on a horizontal table. There are no applied forces. Calculate the normal force.

SOLUTION

$$F_g = mg$$
$$= 2 \times 9.8$$
$$= 19.6 \text{ N down (3 sf)}$$

$$\vec{F}_N = -\vec{F}_g$$
$$= 19.6 \text{ N up (3 sf)}$$

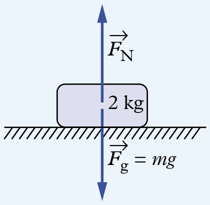

FIGURE 4 Forces acting on a box

CHECK YOUR LEARNING 2.1

Describe and explain

1 **Define** the terms ‘weight’ and ‘normal force’.

2 **Explain** how the mass and weight of an object differ on the surface of the Moon compared to the surface of Earth.

3 **Construct** a labelled vector diagram to show the relationship between the normal force and weight of an object when it is:

  a resting on the ground

  b resting on a table.

4 **Explain** whether this statement is true: ‘For an object at rest on a horizontal surface and with no other forces acting, the weight and the normal force are equal in magnitude.’

5 **Explain** why the weight of an object on the surface of Mars would be less than the weight on the surface of Earth even though the mass would not change.

6 **Calculate** the weight of a 30 kg box of vegetables on Earth.

7 A parent said their child had a weight of 36 N at birth. **Calculate** the child’s mass.

Apply, analyse and interpret

8 **Distinguish** between weight and mass.

9 A bag of apples with a mass of 3.0 kg rests on a table. **Determine** is the magnitude and direction of the normal force.

10 Imagine you could take a 1 kg rock to the centre of Earth. **Deduce** what its weight would be.

Investigate, evaluate and communicate

11 A student of mass 65 kg was standing on the ground barefoot and said ‘I am pulling Earth towards me with a force of 637 N’. A friend said, ‘No, Earth is pulling on you with a force of 637 N’. **Evaluate** both statements and **determine** which is correct.

12 A girl is standing barefoot on the ground and says ‘Earth is providing the normal force to my weight’. She then puts shoes on and says, ‘Now my shoes are providing the normal force and Earth is having a rest’. **Evaluate** these statements for accuracy.

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2.2 Applied forces: friction and tension

KEY IDEAS

In this section, you will learn about:

- applied force, friction and tension being forces that can act on an object on an inclined plane.

inclined plane
a flat surface raised at one end, used as an aid for raising or lowering a load

In this section you will look at two different types of applied forces – friction and tension – and how these forces can act on an object that is positioned on an **inclined plane**.

Friction

You may have measured frictional force by dragging an object across a desk at constant speed and noting the reading (in newtons) on a spring balance (Figure 1).

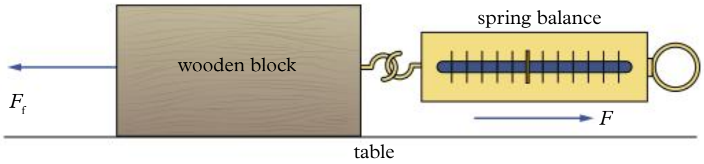

FIGURE 1 A laboratory spring balance is calibrated in both grams (g) and newton (N). A reading of 100 g (0.1 kg) corresponds to a reading of about 1.0 N. You can use the formula $F_g = mg$ to show this is true.

friction
the resistance to motion of a surface moving relative to another

Friction is the resistance to motion of two surfaces moving relative to another. It is the result of the electromagnetic attraction between the charged particles in two touching surfaces.

Unit 2 discussed what life would be like without friction – it would be impossible. Friction is absolutely necessary, but it is also a hindrance. The search for ways of altering it has gone on for thousands of years. The search to understand it has only gone on for a few hundred years – only since the birth of physics.

The properties of friction

In Unit 2, you also saw that friction has several properties, and these are worth reviewing:

- Friction is a force between two surfaces and it opposes motion.
- Friction has the symbol F_f and is measured in units of newton, with the unit symbol N.
- When there is no acceleration along the surfaces, the frictional force equals the horizontal force.

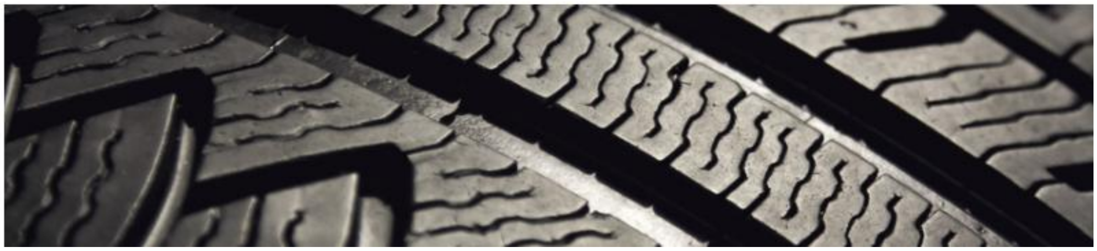

FIGURE 2 Rubber tyres have good friction with a dry road surface but less so on a wet road. The grooves in a tyre are to get rid of water between the tyre and road, but when the tyre is worn it can't do this effectively.

For this section on inclined planes, one of the most important features of friction is its relationship to net forces and acceleration (the last dot point above). If an object is being dragged along a horizontal surface by an applied force (F_H) at constant speed, there is no acceleration so the net force (F_{net}) is zero. The applied force (F_H) must then be equal to the frictional force (F_f), as shown in Figure 3a. In contrast, when the horizontal force is greater than the frictional force (Figure 3b), the object will accelerate in the direction of the larger force.

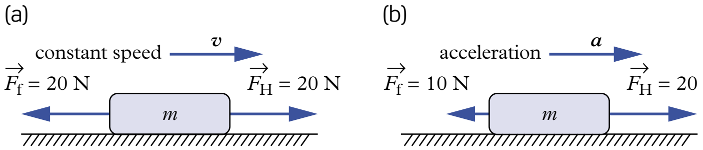

FIGURE 3 The effect of friction on the net force and acceleration

In this first worked example you will consider the effect of friction on acceleration.

WORKED EXAMPLE 2.2A

In Figure 3b, the horizontal force is 20 N (2.0×10^1) to the right and the frictional force is 10 N (1.0×10^1) to the left. The mass *m* of the block is 3.00 kg.

a Determine the weight of the block.

b Determine the normal force acting on the block.

c Determine the net force acting on the block in the horizontal direction.

d Calculate the acceleration of the block in the horizontal direction.

e Determine the net force acting on the block in the vertical direction.

SOLUTION

a $F_g = mg$

$= 3.0 \times 9.8$

$= 29.4$ N down (29.4 N to 3 sf)

b $F_N = -F_g$

$= 29.4$ N up (29.4 N to 3 sf)

c $F_{net} = F_H - F_f$

$= 20 - 10$

$= 10$ N to the right

Alternatively, using vector notation:

$\vec{F}_{net} = \vec{F}_H + \vec{F}_f$

$= 20 + (-10)$

$= +10$ N (to right)

d $F_{net} = ma$, therefore:

$$a = \frac{F_{net}}{m}$$

$$= \frac{10}{3.00}$$

$= 3.3\,\text{ms}^{-2}$ to the right (2 sf)

e There is no movement in the vertical direction, so vertical acceleration is zero and thus F_{net} (vertical) is zero.

Increasing friction

Sometimes you want to increase friction, such as for tyres on a road. At other times you want to reduce friction and this is done with a lubricant such as solid graphite, a liquid such as oil or glycerine, or a gas such as air.

Tension

tension
the pulling force transmitted along a rope, string, cable or chain on an object

Tension is the pulling force transmitted along a rope, string, cable or chain on an object. The symbol F_T is used to show that it is a force, with the subscript T to signify that it is ‘tension’ – a pulling force and not compression or any other sort of force.

Case 1: Object hanging at rest

Consider a mass, m, that is suspended at rest from a scaffold such as an overhead beam. Gravity pulls downwards on the mass with a force F_g called the weight of the object (Figure 4). To support this weight, the rope has to pull upwards with an equal and opposite force. These two forces (the upwards pull of the rope and the weight) make up the action–reaction pair (or agent–receiver pair) you learnt about in the chapter on Newton’s laws in Unit 2. As the object is not accelerating, the net force must be zero, so you can confidently say that the weight force and the tension force are equal and opposite and their net sum is also zero. This would also apply to the object moving at constant speed (zero acceleration).

Note: assume for all examples that the rope is massless, or so small compared to the mass of the object that it can be neglected, and that the string is inextensible; that is, not able to be stretched.

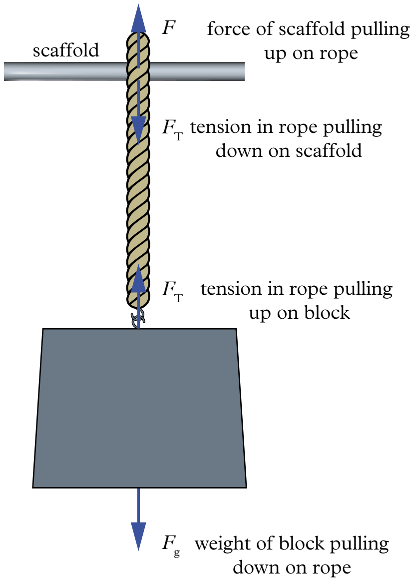

FIGURE 4 The rope is under tension from the force of gravity on a hanging object.

In Figure 4, you can see that at the top end the rope is pulling down on the scaffold with a force F_T that is the same magnitude as the tension in the rope pulling up at the lower end. These two tension forces have the same magnitude ($F_{T(upper)} = F_{T(lower)}$). At a microscopic level there are billions of these tension pairs acting on molecules in the rope for its entire length. Because they are all equal and opposite, you can just represent the ones at the ends.

The scaffold has to pull upwards on the rope with a force equal to this tension. The ultimate purpose of the rope is to transmit the weight of the block to the scaffold.

Of course, if the scaffold was accelerating upwards the tension in the rope would increase, but when it is at rest or moving at a constant speed the tension in the rope equals the weight of the object.

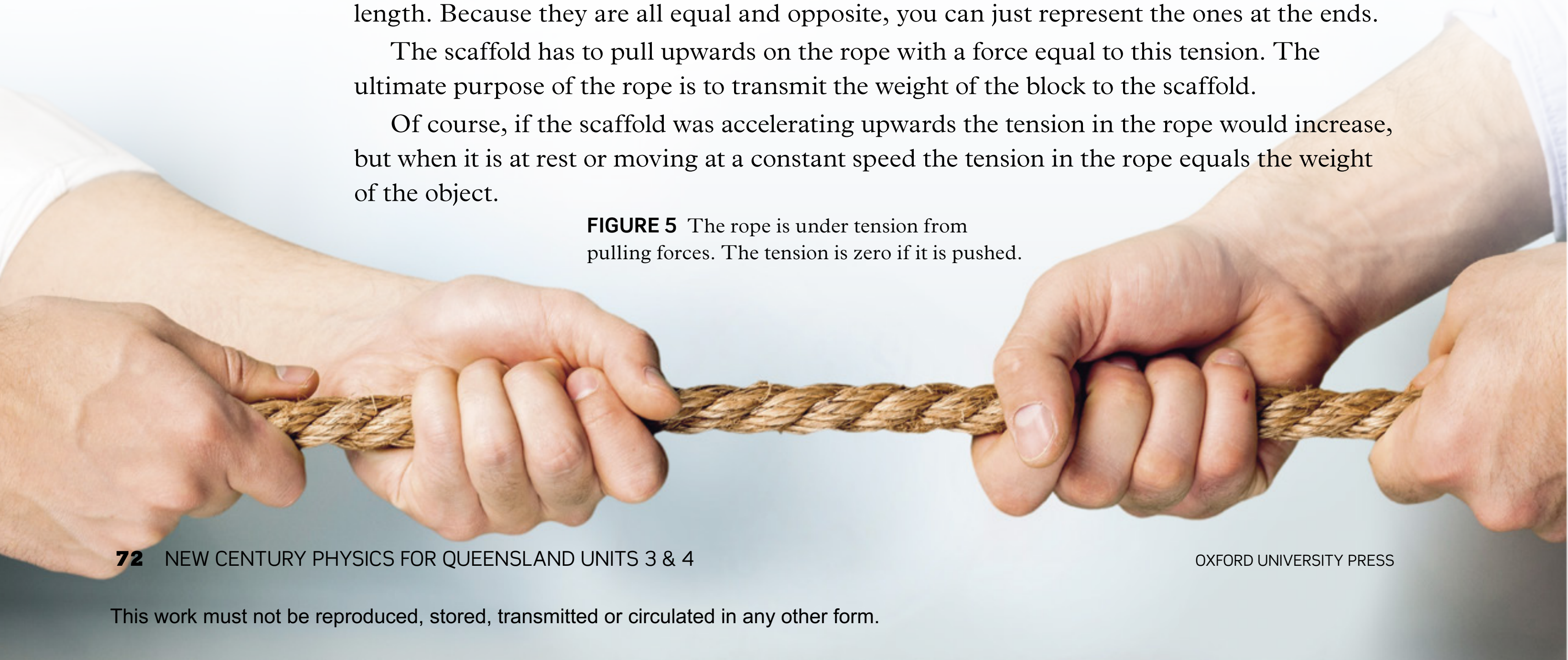

FIGURE 5 The rope is under tension from pulling forces. The tension is zero if it is pushed.

WORKED EXAMPLE 2.2B

Calculate the tension in a vertical rope needed to support a freely hanging 12.0 kg mass.

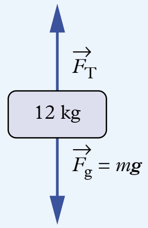

FIGURE 6 Free-body diagram of the forces acting on the object

SOLUTION

Facts: $m = 12.0$ kg, $g = 9.8$ m s^{-2}

Find: tension in the rope (F_T)

Figure 6 represents the direction and magnitude of forces with vectors.

Formula: $F_g = mg$

Figure it out: $F_g = mg$

$= 12.0 \times 9.8$

$= 117.6$ N downwards

$F_g = F_T$ (the magnitudes are equal, although opposite, as it is not accelerating)

$F_T = 117.6$ N upwards

Finish: The tension in the rope is 118 N directed upwards (3 sf).

CHALLENGE 2.2A

Pulley

If you passed a rope through an overhead pulley and tied it around your waist, could you pull on the other end and raise your body?

CHALLENGE 2.2B

Who reaches the pulley first?

The monkey in Figure 7 has the same mass as the box. He climbs a rope. Who will reach the pulley first?

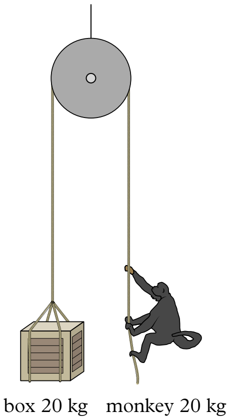

FIGURE 7 Who will reach the pulley first – the monkey or the box?

CHALLENGE 2.2C

Navy destroyers

A World War II navy destroyer (ship) had a mass typically of about 2000 tonnes. Imagine one moored at a dock and you decided to push it away with your hands. It sounds impossible, but your challenge is to work out how long it would take to shift it 30 cm with an applied force of 400 N (typical of a senior student). Assume that water offers no friction. If you want to be more accurate, put some bathroom scales on the wall and see how hard you can press (usually about 40 kg).

Case 2: Object being accelerated

If you consider Case 1, and imagine that the scaffold is a crane that is accelerating the object upwards, the tension in the rope would be greater as the net force would not be zero. The difference between the tension and the weight would be the net force (Figure 8). Flinging a rock upwards as in a trebuchet would be an example of this.

You could say that for the magnitudes of the forces:

$$F_{\text{net}} = F_{\text{T}} - F_{\text{g}}$$

which can be rearranged:

$$F_{\text{T}} = F_{\text{net}} + F_{\text{g}}$$

Alternatively, using vectors symbols (remembering that the vector arrows point in opposite directions):

$$\vec{F}_{\text{net}} = \vec{F}_{\text{T}} + \vec{F}_{\text{g}}$$

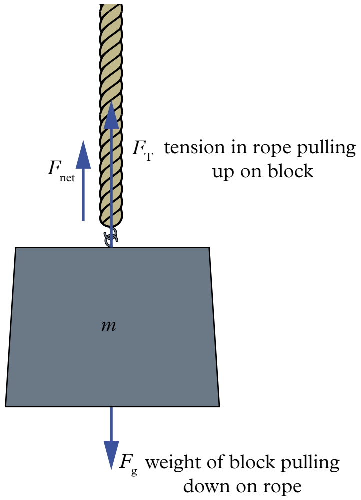

FIGURE 8 If the tension in the rope is greater than the weight, then the net force is upwards.

WORKED EXAMPLE 2.2C

A crane is lifting a 515 kg steel bin directly upwards by use of a chain hitched to the bin. It is accelerating upwards at 1.20 m s^{-2}. Calculate the tension in the chain.

SOLUTION

Facts: m = 515 kg, g = 9.8 m s^{-2} (downwards), or – 9.8 m s^{-2})

Find: Tension in the chain F_{T}

Figure 9 shows that, when adding vectors, the vector arrows are placed head-to-tail.

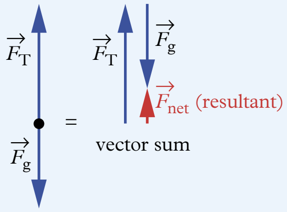

FIGURE 9 Free-body diagram for vector addition

$$\begin{aligned}
F_{\text{net}} &= F_{\text{T}} - F_{\text{g}} \text{ (for magnitudes)}\\
F_{\text{T}} &= F_{\text{net}} + F_{\text{g}}\\
&= ma + mg\\
&= 515 \times 1.20 + 515 \times 9.8\\
&= 618 + 5047\\
&= 5665 \text{ N}
\end{aligned}$$

Finish: The tension in the chain is 5670 N upwards (3 sf).

Case 3: Redirecting forces with a pulley

You don't always want to pull on a rope to move something towards the direction of the force. Sometimes you want to move it away from you, or sideways (Figure 10). A pulley can be used to redirect forces, particularly a weight force to a horizontally applied force – or through some other angle. A pulley is a frictionless wheel used for redirecting forces without loss of energy (Greek *polos* meaning 'pivot', 'axis').

Consider a block that rests on a surface and is attached to a hanging weight by a massless string (Figure 11). The block is being dragged along the horizontal surface by a weight hanging on an inextensible (non-stretching) massless cord passing over a frictionless pulley. The weight of the hanging mass is equal and opposite to the tension in the cord even though the cord is horizontal where it is attached to the block.

FIGURE 10 Pulleys are used to redirect forces. For example, to hoist a sail up a mast on a sailing boat, the rope to hoist the sail will pass through a pulley at the top of the mast, so that standing on deck you can pull down on the rope and the sail will go up.

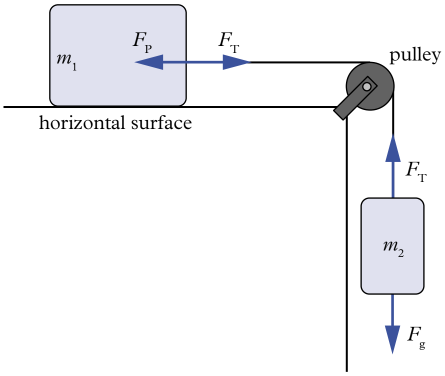

FIGURE 11 A single frictionless pulley redirects the force but doesn't change its magnitude.

You can draw a free-body diagram of the arrangement showing the magnitude and names of the forces acting on the hanging mass (Figure 12).

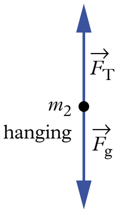

FIGURE 12 Free-body diagram for the vertical forces

You can do the same for the forces acting on the sliding mass (Figure 13). The magnitude of the tension force (F_T) is, of course, the same in both cases. The label F_P is used for the force parallel to the surface.

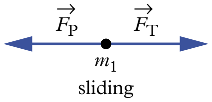

FIGURE 13 Free-body diagram for the horizontal forces

Applied force

applied force
force applied to an object by a person or another object

Applied forces (F_A) are those applied to an object by a person or another object. It can be a push or a pull. For example, if you push a desk across the room, then your hands are providing an applied force on the desk. If you pull a cart by a rope, then you are providing an applied force through the tension in the rope on the cart.

Forces applied horizontally

Forces applied horizontally have no effect on the normal force of the object on the floor. All the forces go into horizontal movement of the object (Figure 14).

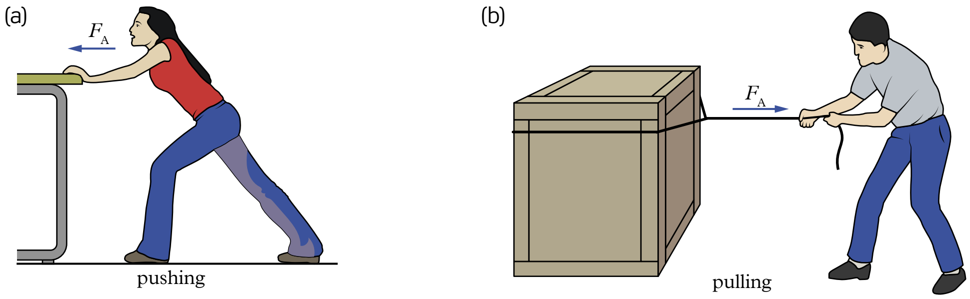

FIGURE 14 (a) Pushing or (b) pulling horizontally has no effect on the normal force.

Forces applied at an angle

Pulling

Have you noticed that is always easier to pull a pram or lawnmower backwards over rough ground? Likewise, it is easier to slide a box by pulling rather than pushing. In the case of 'pulling', the vertical component of the pulling force is directed upwards and tends to lift the object upwards. A 'free-body' diagram (Figure 15) shows how the applied force ($\vec{F}_A$) has been resolved into two components at right angles: a vertical component ($\vec{F}_V$) and a horizontal component ($\vec{F}_H$).

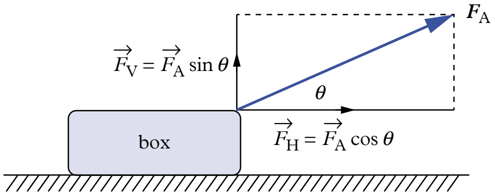

FIGURE 15 The applied force F_A is resolved into two components at right angles.

WORKED EXAMPLE 2.2D

A man drags an 80.0 kg box across a concrete floor at constant speed by means of a rope at an angle of 30° to the floor (Figure 16). Given that the applied force is 400 N (4.00×10^2), calculate the:

a vertical component of the applied force
b horizontal component of the applied force
c weight of the box
d normal force acting on the box
e frictional force.

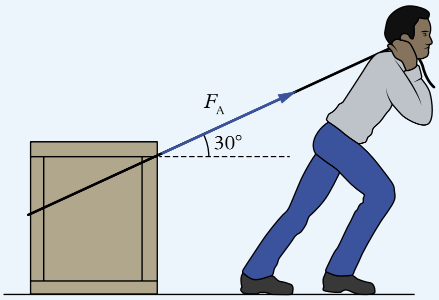

FIGURE 16 Pulling at an angle

SOLUTION

The applied force ($\vec{F}_A$) = 400 N (4.00 × 10^2) to the right at an angle θ of 30° to floor (Figure 17).

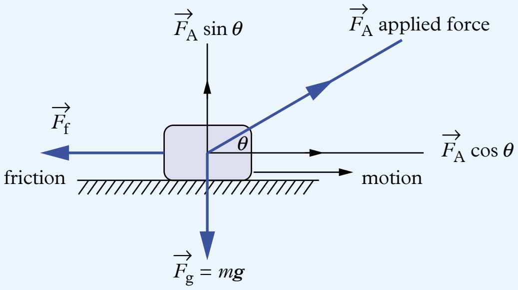

FIGURE 17 Free-body diagram

a Vertical component (F_V) = $F_A \sin\theta$

$= 400 \sin 30°$

$= 400 \times 0.5$

200 N up

b Horizontal component(F_H) = $F_A \cos\theta$

$= 400 \cos 30°$

$= 400 \times 0.866$

= 346 N (3 sf)

c Weight (F_g) = mg

$= 80.0 \times 9.8$

= 784 N down

d Normal force (F_N) = $F_g - F_V$

$= 784 - 200$

= 584 N up

e Frictional force ($\vec{F}_f$) = 346 N (because $\vec{F}_f = \vec{F}_H$ when speed is constant)

Pushing

If you've ever pushed a supermarket trolley or a lawnmower over bumpy ground, you know that it is harder to push because you seem to be pushing it into the ground as well. What is happening is that although some of your push is in the horizontal direction and moves the object along, a component of your pushing force is directed downward towards the ground, causing it to 'dig in'. For example, consider a person pushing a box across a floor (Figure 18). Some of the pushing force ($\vec{F}_A$) goes into moving the box across the floor and some of the force pushes the box down into the floor.

This free-body diagram (Figure 19) shows that the applied force has been resolved into two components at right angles: a vertical component (F_V) and a horizontal component (F_H).

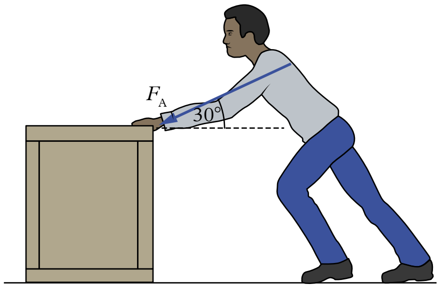

FIGURE 18 Pushing at an angle

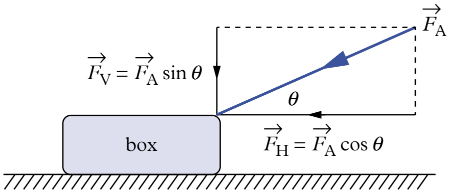

FIGURE 19 Free-body diagram

WORKED EXAMPLE 2.2E

In this example the man pushes the box instead of pulling it. He pushes the 80.0 kg box across a concrete floor at constant speed with his arms at 30° to the floor (Figure 18). He exerts an applied force of 810 N (8.10×10^2) to make it slide at constant speed.

Calculate the:

a vertical component of the applied force

b horizontal component of the applied force

c weight

d normal force

e frictional force.

SOLUTION

Facts: Applied force ($\vec{F}_A$) = 810 N to right at an angle θ of 30° to floor (Figure 20).

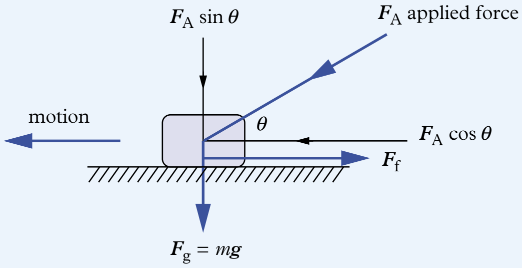

FIGURE 20 Free-body diagram

a Vertical component $(F_V) = F_A \sin\theta$

$$= 810 \sin 30°$$
$$= 810 \times 0.5$$
$$= 405 \text{ N down}$$

b Horizontal component $(F_H) = F_A \cos\theta$

$$= 810 \cos 30°$$
$$= 810 \times 0.866$$
$$= 701 \text{ N (3 sf)}$$

c Weight $(F_g) = mg$

$$= 80.0 \times 9.8$$
$$= 784 \text{ N down}$$

d Normal force $(F_N) = F_g + F_V$ (magnitudes)

$$= 784 + 405$$
$$= 1189 \text{ N up}$$

e Frictional force ($\vec{F}_f$) = 701 N (equal to the horizontal component as it is at constant speed)

CHECK YOUR LEARNING 2.2

Describe and explain

1 **Define** ‘friction’ (in one sentence).

2 An object is sliding along a horizontal surface to the right. **Describe** the direction in which the frictional force acts.

3 An object is being pulled along a horizontal surface by a rope at an upwards angle to the horizontal. **Explain** whether the normal force is greater or less than when the object is at rest.

4 **Describe** Newton’s second law without using a formula.

5 **Explain** what is meant by ‘tension’. **Identify** the symbol for tension, the unit it is measured in, and the unit symbol.

6 A 1.0 kg block of wood is dragged along a benchtop by a string to which a horizontal force of 15 N is applied. A frictional force of 10 N is acting.

- **a** **Describe** the motion of the block.
- **b** **Identify** the law you used to determine this motion.
- **c** **Calculate** the tension in the string.

7 A 1.5 kg block is made to accelerate along a horizontal surface at 2 m s^{-2} to the left by a horizontal force of 5 N.

- **a** **Construct** a free-body diagram.
- **b** **Calculate** the frictional force that is acting to oppose this motion.

Apply, analyse and interpret

8 A small block is dragged along a horizontal surface at constant speed by a force of 5.0 N. **Determine** the magnitude and direction of the frictional force.

9 **Determine** the following statement is true: ‘friction only applies between two surfaces if they are accelerating past one another’.

10 A man drags an 20 kg box across a horizontal concrete floor at constant speed by means of a rope at an angle of 30° to the floor. The applied force in the rope is 100 N. **Determine** the:

- **a** vertical component of the applied force
- **b** horizontal component of the applied force
- **c** weight of the box
- **d** normal force acting on the box
- **e** frictional force.

Investigate, evaluate and communicate

11 A student uses a spring balance to drag a wooden block along a laboratory bench at constant speed. The reading on the balance is 220 g.

- **a** **Construct** a free-body diagram.
- **b** **Calculate** the frictional force between the bench and the block.

12 A set of brass masses rests on a laboratory balance that reads 452 g. A piece of string tied to the masses at an angle of 30° to the vertical is pulled gently and the reading on the balance is reduced to 415 g.

- **a** **Propose** a reason for this observation.
- **b** **Determine** the tension in the string.

13 A set of brass masses rests on an electronic laboratory balance that reads 494 g. The balance is tilted slightly and the balance now reads 412 g.

- **a** **Propose** a cause of this change in scale reading.
- **b** **Calculate** the angle the balance makes with the benchtop.

Check your obook assess for these additional resources and more:

- » Student book questions Check your learning 2.2
- » Challenge worksheet 2.2A Pulley
- » Challenge worksheet 2.2B Who reaches the pulley first?
- » Challenge worksheet 2.2C Navy destroyers

2.3

Forces acting on an inclined plane

KEY IDEAS

In this section, you will learn about:

- describing and representing the forces acting on an object on an inclined plane by the use of free-body diagrams
- calculating the net force acting on an object on an inclined plane through vector analysis.

Inclined planes provided one of the first methods for studying accelerated motion. The Italian scientist Galileo Galilei (1564–1642) realised that measuring the rate of free-falling objects was difficult, as no accurate timing devices were available, especially for the short durations experienced in a laboratory. Galileo reasoned that free-fall was just a special case of an inclined plane for which the angle was 90°. He argued that acceleration still occurred on an inclined plane but that only the component of the acceleration along the track was effective in producing motion. The component perpendicular to the track was unable to produce motion as there was a surface in the way. By using a shallow angle, Galileo was able to study motion but at a greatly reduced rate. This gave the world an understanding of what he called 'uniformly accelerated motion'.

Resolution of forces

Galileo's analysis of the forces acting on an inclined plane are as good today as they were in 1589. When an object is placed on an inclined plane its weight still acts vertically, but this is no longer perpendicular to the surface. The weight force can be resolved into components at right angles: one parallel to the surface of the incline, and the other perpendicular to the incline (Figure 1).

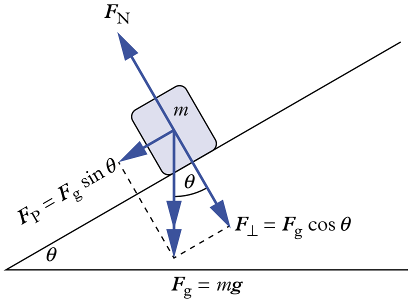

FIGURE 1 Components of forces on an incline

In the diagram, the weight of the object ($F_g = mg$) has been resolved into two components at right angles:

$$\text{Parallel component: } F_P (\text{or } F_\parallel) = F_g \sin\theta$$
$$= mg \sin\theta$$

$$\text{Perpendicular component: } F_\perp = F_g \cos\theta$$
$$= mg \cos\theta$$

Note the use of the subscripts ⊥ for perpendicular, and ∥ or P for parallel.

Note also that the normal force (F_N) is equal and opposite to the force perpendicular to the plane ($F_g \cos\theta$) because there is no acceleration in that direction; thus, $F_N = mg \cos\theta$.

Study tip

It is handy to know these two equations off by heart. They are not formulas that will appear in your data booklet. For an object on an incline, the two components of its weight are:

$F_P = mg \sin\theta$

$F_N = mg \cos\theta$.

SUGGESTED PRACTICAL 2.3 Parallel component on an inclined plane
Go to your obook assess »

CHALLENGE 2.3A

Initial acceleration

With the automatic gearbox in ‘drive’, a 1200 kg Toyota RAV 4 will remain stationary facing uphill on a 5° slope. What would its initial acceleration be on the flat (assuming the driver’s foot was not on the accelerator)?

Vector analysis

An object can move up or down an incline, and it can do so with or without friction. That gives four possible combinations and these represent four situations found in everyday life. You will examine each of these for their particular features and approaches to problem-solving.

Case 1: Sliding down – no friction

An object placed on a smooth (frictionless) inclined plane will accelerate down the plane. The accelerating force is provided by the component of the object’s weight in a direction down the plane F_P (Figure 2). There are no frictional forces to slow its motion. It is difficult to avoid friction completely, but in factories trucks are often loaded by allowing boxes to slide down an incline made of zero-friction rollers. Objects can slide down them very fast. Roller-coasters have very low friction especially when it rains. In fact, when it rains, amusement parks often have to stop roller-coasters as they get too fast (Figure 3).

FIGURE 3 The roller-coaster is about to descend an almost frictionless incline.

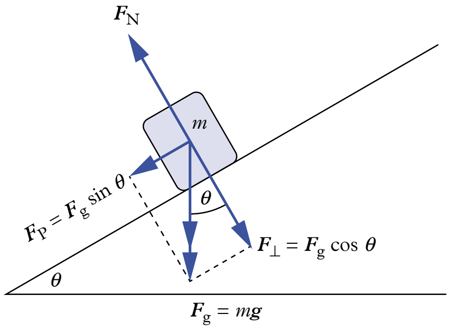

FIGURE 2 Free-body diagram – no friction

Forces parallel to the plane: The net force F_{net} is the sum of all forces parallel to the plane. However, there is just the one force, F_P, so it is the net force:

$F_{net} = F_P$, and applying Newton’s second law, $F_{net} = ma$, thus:

$ma = mg\sin\theta$, and cancelling out the m term

$a = g\sin\theta$

That is, acceleration down the incline equals the free-fall acceleration g multiplied by $\sin\theta$.

You can also say the acceleration is independent of the mass of the object in this very specific case (no friction).

Study tip

When the net forces on an object are equal there is no acceleration. This means the velocity of the object is constant, which includes being at rest. It doesn't mean the object is necessarily at rest; it could be moving at a constant speed.

WORKED EXAMPLE 2.3A

A 15.0 kg bag of fertiliser is allowed to slide freely down a smooth (frictionless) 30° incline. Find:

a the net force down the incline

b the acceleration of the object.

SOLUTION

a
$$\begin{aligned} F_{\text{net}} &= F_{\text{P}} \\ &= mg\sin\theta \\ &= 15.0 \times 9.8 \times \sin 30° \\ &= 73.5\ \text{N} \end{aligned}$$

b
$$\begin{aligned} F_{\text{net}} &= ma \\ 73.5 &= 15.0 \times a \\ a &= \frac{73.5}{15.0} \\ &= 4.9\ \text{m s}^{-2}\text{(down the incline)} \end{aligned}$$

Case 2: Sliding down – with friction

It is more usual to have friction on an incline and friction always opposes motion. A children's slippery slide is a good example of this.

The component of the child's weight down the incline (F_{P}) causes the child to move down the slide, but friction (F_{f}) acts upwards along the surface opposing this motion. If $F_{\text{f}} = F_{\text{P}}$ there will be no net force and the child will not accelerate – the child will either remain stationary or travel down the slide at constant speed (Figure 4). Students often think that if the opposing forces on an object are equal there is no motion. This is wrong – there is no acceleration but the object may continue at constant speed or stay at rest (which is a constant speed of zero anyway).

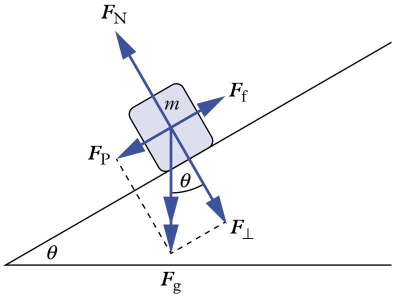

FIGURE 4 Free-body diagram with friction present

WORKED EXAMPLE 2.3B

A 34.0 kg child sits on a slippery slide that has an angle of elevation of 40.0° to the horizontal. If the frictional force is 110 N (1.10×10^2), determine the motion of the child.

SOLUTION

$$\begin{aligned} F_{\text{P}} &= mg\sin\theta \\ &= 34.0 \times 9.8 \times \sin 40.0° \\ &= 214\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}} &= F_{\text{P}} - F_{\text{f}} \\ &= 214 - 110 \\ &= 104\ \text{N(down the incline)} \end{aligned}$$

$F_{\text{net}} = ma$, therefore:

$$\begin{aligned} a &= \frac{F_{\text{net}}}{m} \\ &= \frac{104}{34.0} \\ &= 3.06\ \text{m s}^{-2}\ \text{down the incline (3 sf)} \end{aligned}$$

Case 3: Dragged up the incline – no friction

Goods can be dragged up inclines when they are being loaded into the back of a truck, and the carts in a coal mine are dragged up a rail line to the surface by a force applied to the end of a cable (Figure 5).

In a coal mine, the applied force creates a tension (F_T) in the cable that overcomes the weight component of the cart acting down the plane. The carts will invariably have some friction in the wheels, but first, consider a case without friction (Figure 6).

FIGURE 5 A Swiss 'funicular' is used to reach the top of a mountain and uses a cable to pull the carriage up the incline.

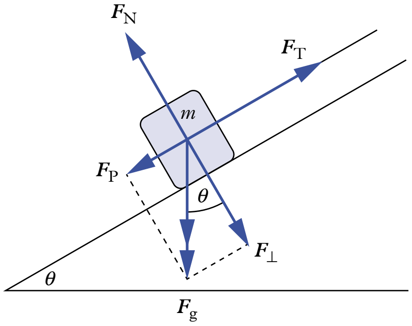

FIGURE 6 Free-body diagram of an object pulled up the incline (no friction)

The net force acting along the plane will be the larger force, the tension in the cable, F_T, up the incline minus the force F_P down the incline:

$$F_{net} = F_T - F_P$$

WORKED EXAMPLE 2.3C

A 525 kg coal truck is pulled up a 25° incline by a cable from the surface. If the cart moves up the track at constant speed, calculate the tension in the cable.

SOLUTION

(using Figure 6)

$F_{net} = 0$ (constant speed), therefore:

$F_T = F_P$ (equal and opposite)

$$F_P = mg\sin\theta \;= 525 \times 9.8 \times \sin 25°$$
$$= 2174 \text{ N down the incline}$$

Hence $F_T = 2174$ N (2170 to 3 sf) up the incline

Using a falling weight to provide the applied force

Instead of having an applied force such as a motor provide tension in the cable to drag an object up an incline, there are advantages in using the stored energy of a falling mass to do the same thing. The Egyptians developed a version of this to shift blocks of stone up inclines using a device called a 'rope-roll'. Overhead power lines for trams and railways sometimes use the same approach.

The free-body diagram in Figure 8 shows a cable connected to an object on an incline. The cable passes over a frictionless pulley at the top. A large hanging mass, m_2, provides a force up the incline. The balance between F_T and F_P will determine the direction in which the block moves, or if it moves at all.

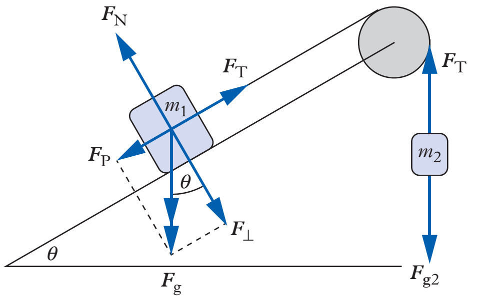

FIGURE 7 Free-body diagram – using hanging masses

Study tip

Just because the mass on the incline is heavier than the hanging mass doesn't mean the block moves down. It depends on the angle too.

The weight of the hanging mass m_2 can be calculated as $F_{g2} = m_2g$. The symbol F_{g2} is used to distinguish it from F_{g1}, which is the weight of the object on the incline. The weight of this hanging mass provides the tension force (F_T) in the cable, and if it is large enough it will pull the block up the incline. If the weight of the hanging mass just equals the force down the incline (F_P), the objects will stay at rest, or if given a small push will continue to move at constant speed in that direction. If the weight of the hanging mass is less than the force down the incline (F_P), the block will accelerate down the incline, but not as fast as if the hanging mass wasn't there. These three scenarios can be summed up thus:

- tension force > force down the incline, $F_T > F_P$: object will accelerate up incline
- tension force = force down the incline, $F_T = F_P$: object will remain stationary on incline or move at constant speed
- tension force < force down the incline, $F_T < F_P$: object will accelerate down incline.

WORKED EXAMPLE 2.3D

A 525 kg coal cart is pulled up a 25° incline by a cable from the surface. A hanging mass m_2 of 300 kg (3.00×10^2) is used to provide the applied force in the cable. Determine the motion of the cart.

SOLUTION

Forces down the incline = F_P

$$F_P = m_1 g \sin\theta$$
$$= 525 \times 9.8 \times \sin 25°$$
$$= 2174 \text{ N}$$

Forces due to hanging mass = F_{g2}

$$F_{g2} = m_2 g$$
$$= 300 \times 9.8$$
$$= 2940 \text{ N}$$

$$F_{net} = F_{g2} - F_P$$

$$F_{net} = 2940 - 2174$$

$$= 766 \text{ N up the incline (therefore cart will accelerate up the incline)}$$

$$a = \frac{F_{net}}{m_{total}}$$

$$= \frac{766}{525 + 300}$$

$$= 0.928 \text{ m s}^{-2} \text{ up the incline (3 sf)}$$

The cart will move up the incline with an acceleration of 0.928 m s^{-2}.

Study tip

The exact value of the tension force is not asked for. It is a more complex calculation and is not shown here. Nevertheless, it can be shown to be 2662 N. See 'Increase your knowledge 2.3' for the worked example. Also see worked solution for Question 20, page 90, for a further example.

Hanging mass and angle of elevation

It should be obvious that as you increase the angle of the incline, the parallel component of the object's weight down the incline increases too. That is, the steeper the incline the faster an object accelerates down it. You can measure the downward force (F_P) by connecting a hanging mass over a pulley at the top and adjusting this hanging mass until the object on the incline moves neither up nor down. At this point, $F_P = F_{g2}$ (the weight of the hanging mass).

This can be demonstrated by the apparatus shown in Figure 8.

Masses are added to the hanging pan on the left to keep the frictionless trolley stationary on the incline. As the angle is increased the component of the weight down the incline increases and thus more mass has to be added to the pan. For example, imagine a 200 g trolley on the incline (such as in Figure 8) and the results collected in Table 1.

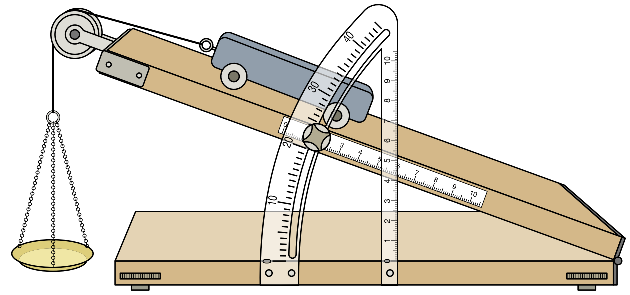

FIGURE 8 Apparatus for investigating forces on an inclined plane

TABLE 1 Parallel component of weight on an inclined plane

Angle of elevation, θ (degrees)	10°	20°	30°	40°
$\sin\theta$	0.17	0.34	0.50	0.64
Mass of hanging mass m_2 (g)	34	68	100	128
Weight of hanging mass F_{g2} (N)	0.33	0.67	0.98	1.25
Parallel component F_P (N)	0.33	0.67	0.98	1.25

Study tip

As θ gets bigger, $\sin\theta$ gets bigger, but $\cos\theta$ gets smaller.

It is immediately obvious that there is a relationship between the angle of elevation and the parallel component (represented by the hanging weight); that is, as you increase the angle, you increase the hanging weight needed. It looks almost linear but it is not. The hanging weight needed is proportional to the *sin* of the angle ($F_{g2} \propto \sin\theta$).

CHALLENGE 2.3B

Graphing elevation

Construct a graph of an angle of elevation (from 0 to 90 degrees) on the horizontal axis, and sin θ on the vertical axis. On the same graph, plot cos θ on the vertical axis. Note their shapes.

Case 4: Dragged up the incline – friction acting

A more likely scenario is one in which friction is acting to oppose motion as an object is dragged up an incline. The forces acting down the incline will now be F_P and F_f, and the forces acting up the incline will be the applied forces via the tension in the cable (Figure 9).

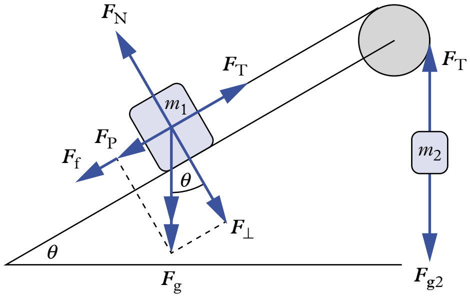

FIGURE 9 Free-body diagram. Friction acts to oppose motion – motion up, friction down

Study tip

The ideas shown here form the basis of Suggested practical worksheet 2.3 'Parallel component on an inclined plane'. It uses freely hanging weights over a pulley to provide a force equal and opposite to the parallel component of weight down the incline – a good candidate for modifying for a student experiment.

WORKED EXAMPLE 2.3E

A cart in a mine is dragged up a 20° incline by an applied force provided by a hanging mass over a frictionless pulley system. A filled cart has a mass of 750 kg (7.50×10^2) and the frictional force is 1036 N. The hanging mass, m_2, has a mass of 383 kg. Determine the motion of the cart.

SOLUTION

Using Figure 11:

$$\begin{aligned}\text{Forces down the incline} &= F_P + F_f\\ &= F_g \sin\theta + 1036\\ &= 750 \times 9.8 \times \sin 20° + 1036\\ &= 2514 + 1036\\ &= 3550 \text{ N down}\end{aligned}$$

$$\begin{aligned}\text{Forces due to hanging mass } (F_{g2}) &= m_2 g\\ &= 383 \times 9.8\\ &= 3753 \text{ N}\end{aligned}$$

$$\begin{aligned}F_{net} &= F_{up} - F_{hanging\ mass}\\ &= 3753 - 3550\\ &= 203 \text{ N up the incline}\end{aligned}$$

$$\begin{aligned}a &= \frac{F_{net}}{m_{total}}\\ &= \frac{203}{750 + 383}\\ &= 0.179 \text{ m s}^{-2} \text{ up the incline}\end{aligned}$$

The cart will move up the incline with an acceleration of 0.179 m s^{-2} (3 sf).
See Increase your knowledge 2.3 for determining tension.

CHECK YOUR LEARNING 2.3

Describe and explain

1 **Identify** the practical purpose of an inclined plane.

2 **Explain** the purpose of resolving the weight vectors into two components at right angles: one down the incline and one perpendicular to it.

3 **Recall** what happens to the magnitude of the parallel and perpendicular components of the weight as the angle of elevation of an incline increases.

Apply, analyse and interpret

4 A 20 kg object rests on a 30° inclined plane. **Determine** the:

- a parallel component of the object's weight
- b perpendicular component of the object's weight
- c the normal force on the object by the surface.

5 An object of mass 10 kg is placed on a frictionless incline of 25°. **Deduce** how many seconds it would take to travel down a 5.0 m incline from rest.

6 A 14 kg toolbox is placed on a plank of wood. When one end of the plank is raised, the toolbox begins to slide down the incline at a uniform speed when the angle reaches 40°. **Determine** the frictional force acting on the box.

7 An object of mass m_1 is placed on a frictionless incline as shown in Figure 10. It is held at rest by a hanging mass, m_2, on a cord over a frictionless pulley at the top of the incline.

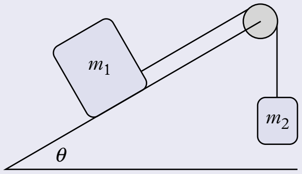

FIGURE 10 Free-body diagram

- a **Determine** the angle of elevation if m_1 is 50 kg and m_2 is 23 kg.
- b **Describe** the motion of the object m_1 if m_2 is increased to 30 kg.
- c **Describe** the motion of the object m_1 if m_2 is reduced in mass to 20 kg.

Investigate, evaluate and communicate

8 Students carried out an experiment to measure how the component of the weight parallel to the surface of an inclined plane varies with angle of elevation.

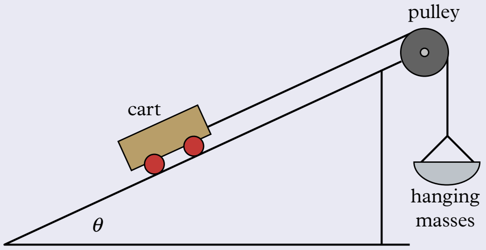

FIGURE 11 Diagram of student experiment

They set up the device shown in Figure 11 and added small masses to the hanging cup to keep the frictionless cart (mass 279 g) in place on the incline. The table shows their results.

Angle θ (degrees)	10	20	30	40
Hanging mass (g)	52.4	96.4	144.5	179.3

- a **Construct** an appropriate graph and deduce the relationship between angle of elevation and the component of the weight down the incline.
- b **Propose** how the graph would appear if there was a small amount of friction present.
- c **Interpret** the methodology to state, with reasons, whether friction would be a systematic or random error.

9 A friend lives at the top of a road that has a 5° downhill slope. When she lets her car (of mass 2000 kg) roll down the slope, it reaches 25 km h^{-1} by the time it gets to the bottom, 400 m away. **Determine** the frictional force that must be acting.

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  2.3 Parallel component on an inclined plane
- » Challenge worksheet
  2.3A Initial acceleration
- » Challenge worksheet
  2.3B Graphing elevation

CHAPTER 2

Review

Summary

2.1

- The force of gravity on an object is the object's weight.
- Mass is a measure of an object's resistance to motion, or the amount of substance in an object. It is related to weight by $F_g = mg$.
- When a body free-falls under gravity, it accelerates at g. Near the surface of Earth, $g = 9.8$ m s^{-2} downwards.
- The normal force is the force exerted on a body by a surface against which it is pressed. It is always perpendicular to the surface.

2.2

- The forces acting on an object on an inclined plane include force due to gravity (weight), the normal force, tension, frictional force and applied force.
- A force is a push or pull between objects that may cause one or both objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape.
- Tension is the pulling force transmitted along a rope, string, cable or chain on an object. It has the symbol F_T.
- Applied forces (F_A) are those applied to an object by a person or another object. They can be a push or a pull.
- Friction is the resistance to motion of a surface moving relative to another. Unlike gravity, electromagnetism or the strong and weak forces, friction is not a fundamental force. It is the result of the electromagnetic attraction between charged particles in the two touching surfaces.

2.3

- An inclined plane or ramp is a flat surface raised at one end, used as an aid for raising or lowering a load.

Key terms

- applied force
- force
- friction
- inclined plane
- mass
- normal force
- tension
- weight

Key formulas

Weight of an object	$F_g = mg$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A frictionless inclined plane is raised at one end to height *h* and the acceleration of a cart is measured at each height. Determine which diagram in Figure 1 best shows the acceleration, *a*, of the cart as a function of height.

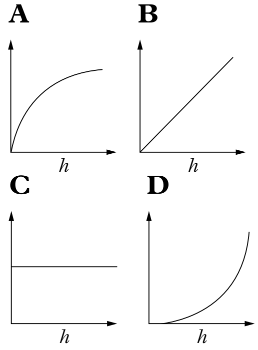

FIGURE 1 Acceleration vs height graphs

2 Determine which of the graphs in Question 1 best shows the final velocity (*v*) at the bottom of the incline, as a function of height (*h*).

3 A frictionless incline has a mass, *m*, resting on the surface and another mass, *M*, attached to a light string over a frictionless pulley (Figure 2). Mass *m* equals mass *M*. Determine which one of the following is most likely to occur.

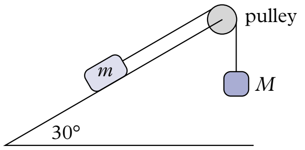

FIGURE 2 Motion on an incline

A Mass *m* moves at constant speed down the incline.

B Mass *m* moves at constant speed up the incline.

C Mass *m* accelerates down the incline.

D Mass *m* accelerates up the incline.

4 A block is placed on an incline and it accelerates down the incline even though friction was present. Select the expression that best describes the relationship.

A $F\| > F_\text{f}$ **B** $F\| = F_\text{f}$

C $F\| < F_\text{f}$ **D** $F\| = F_\text{f} = 0$

5 A 1 kg object is placed on a frictionless incline set at an angle θ. Select the expression that best describes its acceleration.

A $mg\sin\theta$ **B** $mg\cos\theta$

C $g\sin\theta$ **D** $g\cos\theta$

Short answer

Describe and explain

★ 6 **Describe** a situation in which the normal force makes a third law pair with gravity, and one in which it doesn't make a third law pair with gravity.

★ 7 **Clarify** whether you are weightless when you float in a swimming pool.

★★ 8 **Construct** a diagram showing the forces acting on an object at rest on a frictionless incline, including the tension in a rope holding it there.

★★ 9 **Construct** a diagram showing the forces acting on an object accelerating down a frictionless incline.

Apply, analyse and interpret

★ 10 **Consider** whether tension applies to pushing and/or pulling forces.

★ 11 A computer of mass 2.5 kg rests on a table.

a **Calculate** the normal reaction force exerted by the table on the computer.

b **Determine** the normal force acting on a very large mass of 500 kg that is placed on the table. **Explain** your answer.

★ 12 **Determine** how many times heavier by weight would a person be on Jupiter than on Earth. Note: $g_\text{Jupiter} = 25\ \text{m s}^{-2}$.

★★ 13 **Determine** the force necessary to uniformly accelerate the following:

a a 6.4 kg mass at 2.4 m s^{-2}

b an object weighing 25 N at 9.8 m s^{-2}

c a 0.50 kg object from rest to 5.0 m s^{-1} over 4.0 metres

d a 75 kg object from 40 m s^{-1} to 60 m s^{-1} in 5 ms.

★★ **14** A butcher pulls on a 40 kg side of beef with a horizontal force of 220 N and it slides across the boning table at constant speed.

a **Determine** the force of friction.

b The side of beef now hits a low fiction area on the table. **Determine** what the butcher would notice about the motion of the beef if the applied force is kept the same.

★★ **15** A horizontal steel cable is used to drag a bucket filled with coal along the ground at constant speed. The mass of the bucket and coal is 6.1 tonnes and the friction is 30 000 N.

a **Calculate** the normal force.

b **Determine** the tension in the cable.

c The tension in the cable is kept constant but the mass of the bucket increases as more coal is added. **Predict**, with reasons, whether the bucket would continue at constant speed.

★★ **16** The block in each diagram in Figure 3 weighs 50 N. In Figure 3b to 3d, the applied force F_A is 20 N. **Calculate** the normal force in each case.

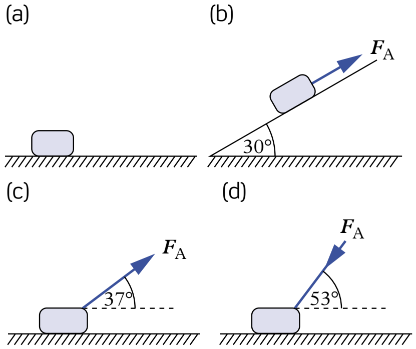

FIGURE 3 Force on a block

★★ **17** A bag of cement is sliding down a 30° incline at constant speed. **Propose** which one or more of the following relationships is false about the situation.

A $F_g \sin 30° > F_f$

B $F_f = F_g \sin 30°$

C $F_N = mg \cos 30°$

D $F_g = mg$

★★ **18** A 30 kg box of vegetables moves down a 35° frictionless incline. **Determine** the:

a component of the weight perpendicular to the incline

b component of the weight down the incline

c normal force

d acceleration down the incline.

★★ **19** A 15 kg bag of dog food is allowed to slide freely down a smooth 30° incline. **Determine** the:

a net force down the incline

b acceleration of the bag of dog food.

★★★ **20** A 20 kg object is attached by a thin cord to a 50 kg mass that hangs over a frictionless pulley at the top of a 25° incline (Figure 4).

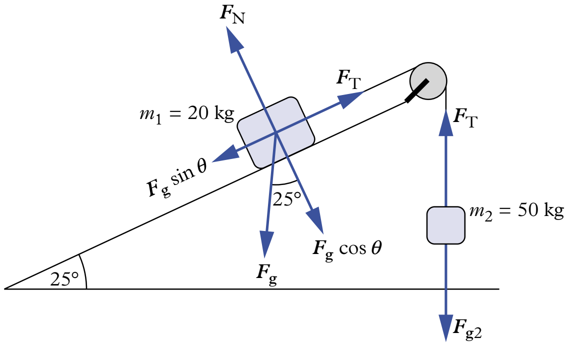

FIGURE 4 Diagram of a 20 kg attached to a 50 kg mass

Determine the:

a acceleration, if any, of the object

b tension in the string.

★★★ **21** Consider the system shown in Figure 5. The trolley has a mass of 1000 g and moves up the incline at constant speed when placed on a slope of 35° under the conditions shown.

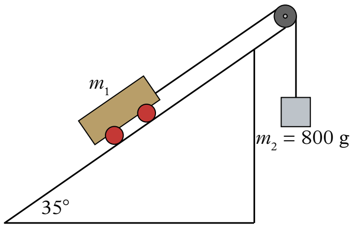

FIGURE 5 Trolley on slope

a **Determine** the frictional forces acting in this system.

b **Calculate** the tension in the rope.

★★★ **22** A crate of tiles of mass m = 14 kg moves up a 30° incline at constant speed when pulled by a rope attached to a crate of cement of equal mass hanging over a frictionless pulley at the top. The rope connecting the crates can be considered to be taut and massless. **Deduce** the frictional force.

★★★ **23** A man hauls a box of mass 100 kg up a 35° incline by a rope attached to the top of the box (Figure 6). The rope makes an angle of 20° to the incline and the friction between the box and the incline is 650 N. **Determine** the force applied by the man (F_A) to keep the box moving at constant speed.

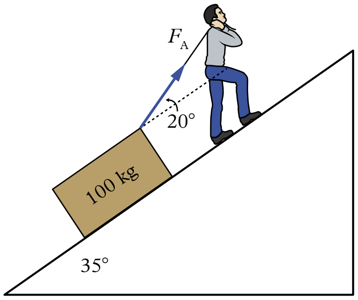

FIGURE 6 A man hauling a box up a hill

Investigate, evaluate and communicate

★★★ **24** An 8 kg carton of soft drink is being pulled up a frictionless 30° incline using a rope and an applied force (F_A) of 45 N (Figure 7). This applied force through the rope is the rope's tension, F_T.

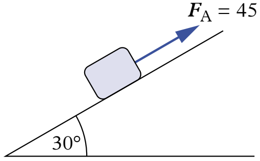

FIGURE 7

- **a** **Calculate** the acceleration, if any, up the incline.
- **b** **Decide** what the motion would be if the applied force was 40 N.

★★★ **25** Students were asked to investigate the relationship between the angle of elevation of a frictionless ramp and the parallel component of the weight of an object on the incline. They set up an experiment with a 1.2 m long aluminium ramp and varied its elevation through a range of angles from 10° to 50°. They used a Lego car of mass 75.5 g as the object on the ramp and measured the mass of a cup of coins and paper clips hanging freely over a pulley at the top of the ramp that held the car stationary. Table 1 shows their results.

TABLE 1

Angle, θ (degrees)	10.0	20.0	30.0	40.0	50.0
Hanging mass, m_2 (g)	13.10	26.30	37.90	48.80	58.80

They hypothesised that the weight of the hanging mass would equal the parallel component of the car's weight down the incline for each angle. They planned to plot a graph to help confirm their thinking. **Analyse** their data and **evaluate** their hypothesis.

★★★ **26** Students studying the acceleration of an object down a frictionless ramp used a Lego car of mass 56.0 g and let it run freely down a 1.0 m incline. They measured the time for the journey for several different angles of elevation. Table 2 shows their results.

TABLE 2

Angle, θ (degrees)	5	10	15	20	25
Time, t (s)	1.53	1.08	0.89	0.77	0.69

The students hypothesised that the acceleration down the incline should be as a result of the net force from the parallel component of the weight of the car. Draw a graph of acceleration (observed) on the vertical axis versus acceleration (theoretical) on the horizontal axis. If the formula for the trendline was $y = x$ with an R^2 of 1.00, they were going to claim their hypothesis had been confirmed. **Evaluate** their results.

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CHAPTER 3

Circular motion

It really wasn't until the 1500s that people began to believe that Earth rotates on its own axis. Until then, the rate of rotation of objects was of little consequence. Today, rotation and its measurement is of fundamental importance to society, whether it is the rotation of microwave carousels, car tyres, engines, sewing machines, nuclei, planets or orbiting satellites.

OBJECTIVES

→ Describe uniform circular motion in terms of a force acting on an object in a perpendicular direction to the velocity of the object.
→ Define the concepts of average speed and period.
→ Solve problems involving average speed of objects undergoing uniform circular motion.
→ Define the terms 'centripetal acceleration' and 'centripetal force'.
→ Solve problems involving forces acting on objects in uniform circular motion.

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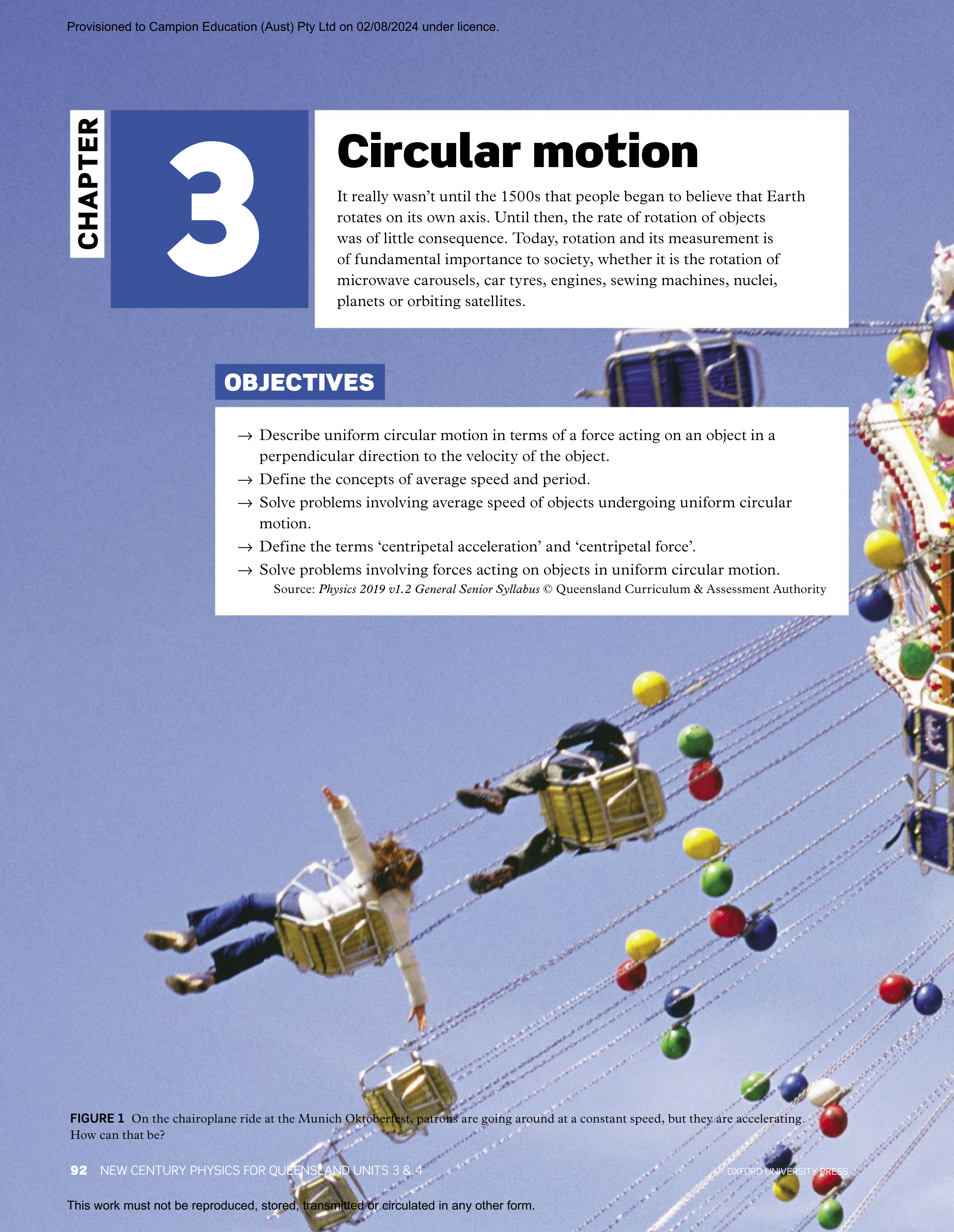

FIGURE 1 On the chairoplane ride at the Munich Oktoberfest, patrons are going around at a constant speed, but they are accelerating. How can that be?

MAKES YOU WONDER

In this chapter you will be looking at circular motion and this should help you answer questions such as these:

- → If you fell out of a chairoplane (also known as a swing-ride), would you keep going in a circle as you fell?
- → What force keeps you pressed against the wall of a Gravitron?
- → How can constant speed mean that an object can be accelerating?
- → Does high rotational speed mean high acceleration?
- → If electrons are orbiting a nucleus, where do they get their energy from?
- → Is whirling a weight on a string in a vertical circle ‘uniform’?
- → Is the average velocity of an object in circular motion equal to zero?

PRACTICALS

SUGGESTED PRACTICAL	**3.3** Centripetal force and horizontal circular motion

3.1 Uniform circular motion?

KEY IDEAS

In this section, you will learn about:

- uniform circular motion in terms of a force that acts on an object in a perpendicular direction to the velocity of the object.

uniform circular motion the motion of an object travelling at a constant speed in a circle

Circular motion is a simple idea – just moving around in a circle. To make it **uniform circular motion**, you need to be moving at constant speed as well. There are plenty of examples in daily life: whirling a toy plane around in a horizontal circle at constant speed (Figure 2), the carousel (platter) in a microwave oven, the spin dryer in a washing machine, a ceiling fan, and the second hand of a clock. It would be good to also add Earth orbiting the Sun, but it is not quite a circle and it is not quite constant in speed. The length of the seasons can vary by up to 7 days.

What keeps an object in circular motion, and what keeps the motion uniform relies on an understanding of the forces involved.

Study tip

This chapter is concerned exclusively with uniform circular motion in a horizontal circle. Objects being whirled in vertical circles are outside the scope of this chapter.

CHALLENGE 3.1A

Does a wheel ever rest?

When a wheel rolls along, is any point at rest?

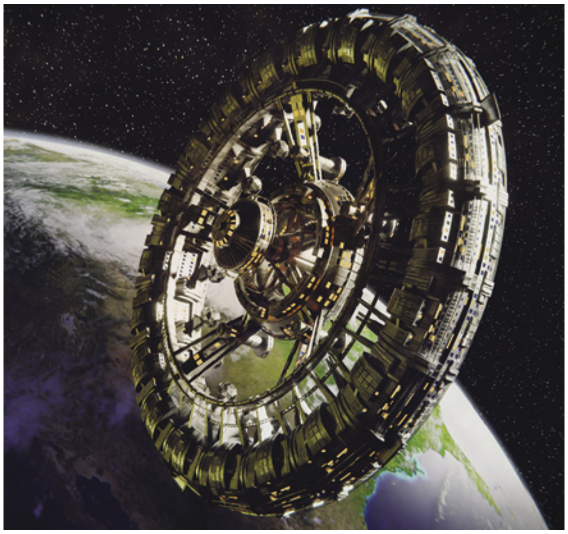

FIGURE 1 Uniform circular motion in a slowly rotating space station is one way to simulate Earth's gravity in space.

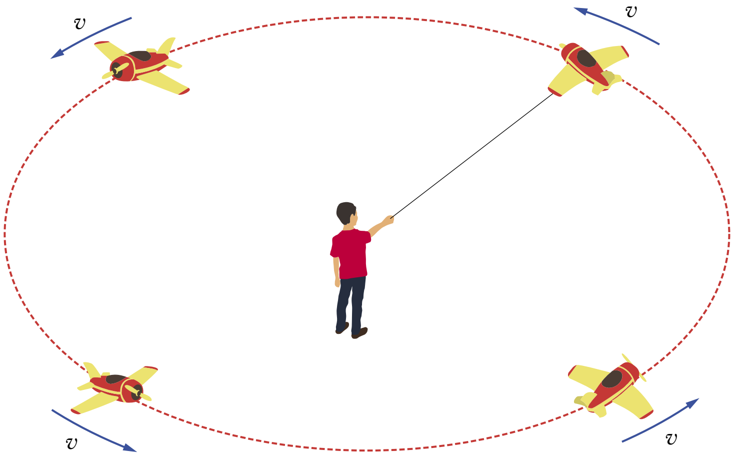

FIGURE 2 A child whirls a toy plane around in a horizontal circle.

Revolving versus rotating

Two similar terms are easy to get mixed up, and if you do this you may get the wrong answer. Since the 1660s, an object moving in a horizontal circle is said to be revolving or orbiting around the centre of the circle. So, a ball whirled on a string is said to revolve around the centre point at your hand. The similar term 'rotating' (since the 1550s) means spinning around its own axis, or 'rolling', as its origins suggest. Astronomers say Earth *rotates* on its own axis once a day, and *revolves* about the Sun once a year (Figure 3). However, a person standing on the surface of Earth can be said to be *revolving* around the centre of Earth as Earth *rotates* on its axis. In summary, something rotates around its own axis, and revolves around the axis of another object.

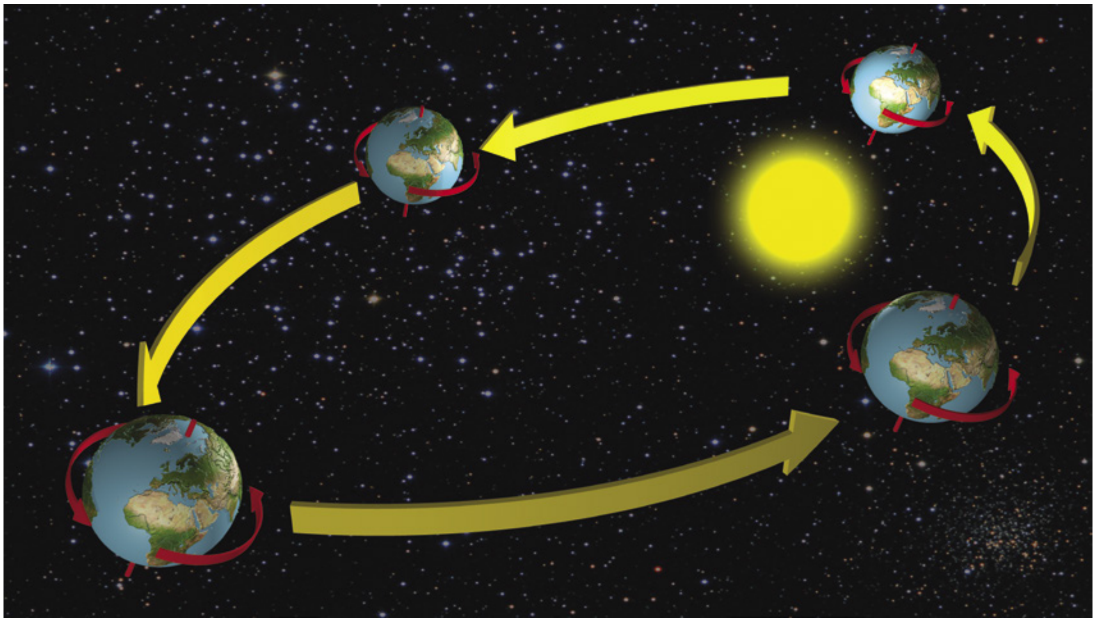

FIGURE 3 Earth revolves around the Sun, but it also rotates on its own axis.

CHALLENGE 3.1B

Where did the term 'clockwise' originate?

The hands of a clock go in a 'clockwise' direction – but how did this term originate? The Sun moves cross the sky in an anticlockwise direction as the day progresses, so why wasn't that direction 'clockwise'. Propose a solution to this dilemma.

Forces and velocity

Imagine you are whirling a ball in a horizontal circle above your head on a piece of string, much as the child with the toy plane shown in Figure 2. By Newton's first law of motion, the ball is attempting to travel in a straight line but is stopped from doing so by your pull on the string. As your hand is at the centre of the circle in which the ball moves, the force on the string, and hence on the ball, is always towards your hand and hence towards the centre. This force is called a **centripetal force** (Latin *centrum* meaning 'centre', and *petere* meaning 'seek'), a centre-seeking force, and has the symbol F_c. If you are specifying it as a vector, you use the symbol $\vec{\boldsymbol{F}}_c$.

centripetal force the force acting on an object travelling in a circle that constantly either pulls or pushes the object towards the centre of motion (symbol: F_c; SI unit: newton; unit symbol: N)

The velocity at any point on the circle is a tangent to the path at that point. For instance, at position A in Figure 4, the velocity vector $\vec{\boldsymbol{v}}_1$ points up the page. At point B, the velocity vector $\vec{\boldsymbol{v}}_2$ points to the left but is drawn with the same length because the speed remains the same. As the direction of the velocity has changed, the ball is said to be accelerating (**centripetal acceleration**). Reminder: the length of the vector arrow indicates the magnitude, so for velocity, the length is just the speed.

centripetal acceleration the acceleration experienced by any object moving in a circular path directed towards the centre of motion (symbol: a_c; SI unit: metres per second squared; unit symbol: m s^{-2})

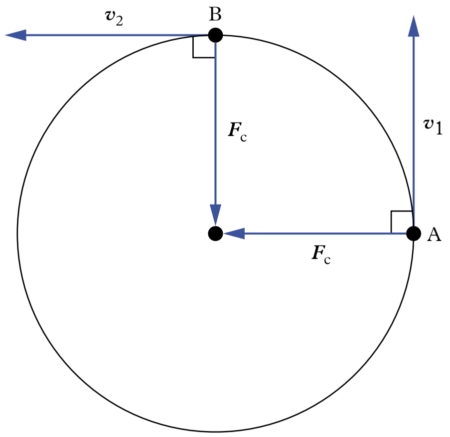

FIGURE 4 Direction of the velocity vectors in uniform circular motion

You should be able to observe that the centripetal force vector $\vec{\boldsymbol{F}}_c$ is perpendicular to the direction of the velocity vector $\boldsymbol{v}$ of the object.

Practical example: the Gravitron

In a Gravitron, or Rotor, at an amusement park, the person feels 'pressed' against the wall (Figure 5). Actually, the person is trying to travel in a straight line but the wall pushes on the person (the centripetal force) and the person pushes back. The centripetal force is the normal force directed radially inwards on the rider. At high speeds, this normal force becomes sufficiently great to provide enough friction to stop the rider sliding down the wall under the influence of gravity. It should be clear: there is no force pressing the person against the wall. The person is trying to travel in a straight line and the walls prevent this. The walls press on the rider and the rider presses back. There is no special force.

FIGURE 5 The Gravitron

CHECK YOUR LEARNING 3.1

Describe and explain

1 **Describe** uniform circular motion.

2 **Explain** how an object can be travelling at constant speed but be accelerating.

3 A ball is being whirled in a horizontal circle at uniform speed (see Figure 6, looking from above).

  a **Identify** the direction of its velocity vector at points A and B.

  b **Identify** the direction of the force vectors at A and B.

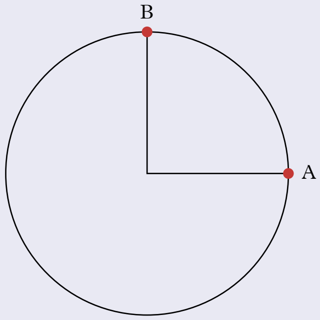

FIGURE 6 Free-body diagram of a whirling ball

4 The Moon is in an almost uniform circular motion around Earth. **Identify** what is providing the centripetal force to keep revolving around Earth.

5 **Clarify** whether Earth's revolution around the Sun is considered to be uniform circular motion.

Apply, analyse and interpret

6 When revolving in a microwave oven, a glass of water placed at the edge of the carousel (platter) doesn't fly off as the carousel turns. **Consider** what keeps it there.

Investigate, evaluate and communicate

7 Consider a spin-dryer.

  a **Propose** why clothes that come out of a spin-dryer still feel damp.

  b **Predict** whether continued spinning at the same speed will get rid of more water.

  c **Infer**, and state reasons**,** whether you could you spin the clothes completely dry.

  d **Identify** the force that pushes the water to the outside of the tub and then out through the holes.

8 Small stones get caught in the treads of car tyres. **Propose** a reason why they stay there and are not 'flung' out at high speeds.

9 Imagine you are measuring your weight on bathroom scales. It has been claimed that if Earth wasn't spinning your weight would be different. **Assess** this claim with a reasoned explanation.

10 It is claimed that if Earth wasn't spinning your height would be different. **Comment** on this claim.

11 It has been said that if you measure your weight with bathroom scales at the equator it would be less than if you measured it at the North or South Pole. The reason, it is said, is due to the lesser centripetal force at the poles due to the slower rotational speed. **Evaluate** this statement.

12 When going around a roundabout in a car, the front seat passenger feels pressed against the door. People wonder whether they are moving towards the door or whether the door moving towards them. **Analyse** this situation and **propose** an answer.

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  Explore the Gravitron ride

3.2 Objects undergoing uniform circular motion

KEY IDEAS

In this section, you will learn about:

- average speed and period
- how to solve problems that involve the average speed of objects in uniform circular motion.

Average speed and period

Calculating the **average speed**, v, of an object moving in a circle is no different from any other speed calculation: it is just the distance travelled per time of travel. The distance around a circle is equivalent to the circumference and calculated as $2\pi r$, where r is the radius of the circle. The time for one revolution around the circle is referred to as the **period** and has the symbol T. Thus the average speed of an object in uniform circular motion is given by the expression:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}, \text{ or } v = \frac{2\pi r}{T}$$

This speed is sometimes called the 'tangential speed' (Latin *tangere* meaning 'to touch', as in touches the edge of the circle) to distinguish it from 'rotational speed'. It is also called **tangential velocity**.

average speed
the rate of change of distance calculated by the formula: average speed $= \frac{\text{distance}}{\text{time}}$; a scalar quantity (symbol: v; SI unit: metres per second; unit symbol: m s^{-1})

period
the time taken to complete one revolution calculated by the formula: period $= \frac{\text{time}}{\text{no. of revolutions}}$ (symbol: T; SI unit: second; unit symbol: s)

tangential velocity
the linear velocity of an object undergoing circular motion, where the magnitude is the speed of the object, and the direction is a tangent to the circular path at that moment (directed towards the centre); for circular motion, the term is usually abbreviated to 'velocity'

FIGURE 1 A father spins his child in a horizontal circle. This is an example of uniform circular motion – unless he drops the child.

WORKED EXAMPLE 3.2A

A girl whirls a ball on a string in a horizontal circle. The length of the string is 2.2 m and the time for one revolution is 3.1 s. Calculate the:

a period

b average speed of the ball

c velocity of the ball.

SOLUTION

a
$$T = \frac{\text{time}}{\text{no. of revolutions}} = \frac{3.1}{1} = 3.1 \text{ s}$$

b
$$v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 2.2}{3.1} = 4.5 \text{ m s}^{-1}$$

c Velocity has the same magnitude as speed, so the velocity is directed perpendicular to the string in the direction of motion.

Rotational speeds

You probably don't know the speed of the Moon about Earth in metres per second or even kilometres per hour. But you know that it makes one revolution in just over 27 days. Engine speeds too are usually expressed in a number of revolutions per minute (rpm). At idle, the engine might turn at 750 rpm and at cruising speed it may reach 4000 rpm. It depends on the car. These are called **rotational speeds**.

However, when you say 'average speed' you are talking about linear speeds in metres per second and not rotational speeds in revolutions per second.

FIGURE 2 A tachometer in a car is a 'rev counter' that measures the engine speed in revolutions per minute (rpm). This one has a 'red line' (maximum advisable engine speed) of 6500 rpm.

rotational speed the number of revolutions an object does per second, as distinguished from the term 'average speed', which is the linear speed.

CHALLENGE 3.2A

Motion of a candle

A candle with a nail through the middle is supported on two glasses and lit at both ends. What would you observe about its motion?

Using rotational speeds

For now, rotational speeds are best converted into the time for one revolution. This is called the period, T. The simplest way to do this is:

$$T = \frac{\text{time}}{\text{no. of revolutions}}$$

WORKED EXAMPLE 3.2B

Calculate the period for an object with a rotational speed of 30 rpm.

SOLUTION

$$\begin{aligned}T &= \frac{\text{time}}{\text{no. of revolutions}}\\ &= \frac{60\ \text{s}}{30\ \text{revolutions}}\\ &= 2\ \text{s}\end{aligned}$$

This can then be used for any problem with rotational speeds.

Study tip

For uniform circular motion, the term 'average speed' means linear speed in metres per second, and 'velocity' has the same magnitude as speed but it also has a direction tangential to the circle at that point.

WORKED EXAMPLE 3.2C

A car tyre has a diameter of 630 mm and is being rotated at 850 rpm on a tyre-balancing machine. Determine the average speed at which the outer edge of the tyre is moving.

SOLUTION

Facts: D = 630 mm (0.63 m), rotational speed = 850 rpm

Find: average speed, v

Formulas:

$$\text{Radius of tyre} = \frac{0.630\text{ m}}{2} = 0.315\text{ m}$$

$$\text{Period, } T = \frac{\text{time}}{\text{no. of revolutions}} = \frac{60\text{ s}}{850\text{ revolutions}} = 0.07\text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 0.315}{0.07} = 28.3\text{ m s}^{-1}$$

Finish: The average speed is 28.3 m s^{-1}.

Practical example – the centrifuge

The laboratory centrifuge is a device used in chemistry, biology, biochemistry and clinical medicine for isolating and separating suspensions and immiscible liquids. A tube containing the suspension – for example, blood – is placed in a machine that spins it at a very high rate (Figure 3). Rotational speeds of 1000–5000 rpm are common. The centripetal acceleration causes denser substances and particles to move outwards to the bottom end of the tube, and at the same time, objects that are less dense are displaced and move to the centre of rotation, which is the top of the tube. Blood, for instance, separates into cells and proteins and serum.

FIGURE 3 Blood samples are separated in a centrifuge. The red blood cells are heavier and have separated to the bottom leaving the yellow plasma at the top.

WORKED EXAMPLE 3.2D

It is suggested that you can increase the speed of a dryer by doubling the diameter or doubling the rotational speed. Dryer A has a tub radius of 50.0 cm and a rotational speed of 1200 rpm, and dryer B with a radius of 100 cm and a speed of 600 rpm.

a Determine which dryer gives the higher speed.

b Propose which would be preferred for a household laundry.

SOLUTION

a Period or revolution

Dryer B: $T = \frac{1}{1200}$ minutes

$= \frac{60}{1200}$ seconds

$= 0.05$ s

Dryer B: $T = \frac{1}{600}$ minutes

$= \frac{60}{600}$ seconds

$= 0.10$ s

Average speed

Dryer A: $v = \frac{2\pi r}{T}$

$= \frac{2\pi \times 0.50}{0.05}$

$= 62.8 \text{ m s}^{-1}$

Dryer B: $v = \frac{2\pi r}{T}$

$= \frac{2\pi \times 1.00}{0.10}$

$= 62.8 \text{ m s}^{-1}$

b As they both have the same velocity, the dryer with the smaller radius is preferred. It will have a greater change of velocity (centripetal acceleration) as it is moving in a smaller circle and so the change in direction will be greater in the same period of time. Thus, the clothes are being pulled away from the water with greater (centripetal) force and allowing the water to continue to move outward from the clothes.

CHALLENGE 3.2B

Motion of a billy

An old camper's trick to make tea leaves settle to the bottom of a billy of tea is to swing the billy in a vertical circle as shown in Figure 4.

Consider the following questions about this idea.

a Explain why the leaves settle faster.

b Deduce why the water doesn't fall out when the billy is overhead at the top of the circle.

c Propose whether this is an example of uniform circular motion. If not, explain when it is moving the fastest and when it is moving the slowest.

d Graph the speed versus angle for one complete revolution of 360°, with 12 noon being 0°.

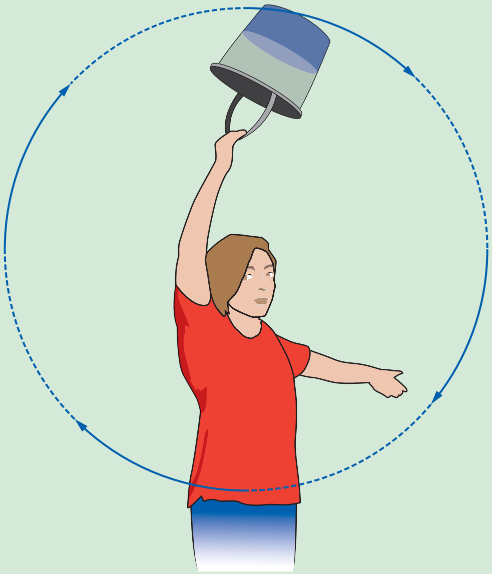

FIGURE 4 Swinging a billy of tea in a vertical circle is said to make the leaves settle faster.

CHECK YOUR LEARNING 3.2

Describe and explain

1 **Explain** how to calculate the period of revolution from the number of revolutions per minute.

2 **Describe** one practical use of a device that utilises uniform circular motion, and the typical rotational speeds involved.

3 An amusement park Gravitron moves in a horizontal circle of 15.0 m radius and completes 24 turns every minute. **Calculate** the average speed.

Apply, analyse and interpret

4 **Distinguish** between average speed and rotational speed.

5 A car of mass 2250 kg is travelling around a circular track of radius 90.0 m at a constant speed of 30.0 m s^{-1}. **Determine** how many revolutions per minute the driver makes of the track.

6 The Large Hadron Collider (LHC) in Switzerland is a device that accelerates particles such as protons to very high speed and then smashes them into each other. The LHC consists of a circular ring 4.25 km in diameter in which protons travel at close to the speed of light. **Determine** how many revolutions per minute (rpm) a proton would make at a speed of 2.0×10^8 m s^{-1}.

Investigate, evaluate and communicate

7 You are riding a bicycle at 20 km h^{-1} and the wheels are spinning very fast. Bike tyres are typically 67 cm in diameter. **Assess** whether you have sufficient information to work out the rotational speed of your tyre and, if so, justify the logic used to calculate it.

8 Spin-dryers go pretty fast – too fast to see with the naked eye. **Propose** a method to measure the speed of a spin-dryer in revolutions per minute (rpm). You don't have to build it or have the parts at home or school, just design the procedure and instrumentation.

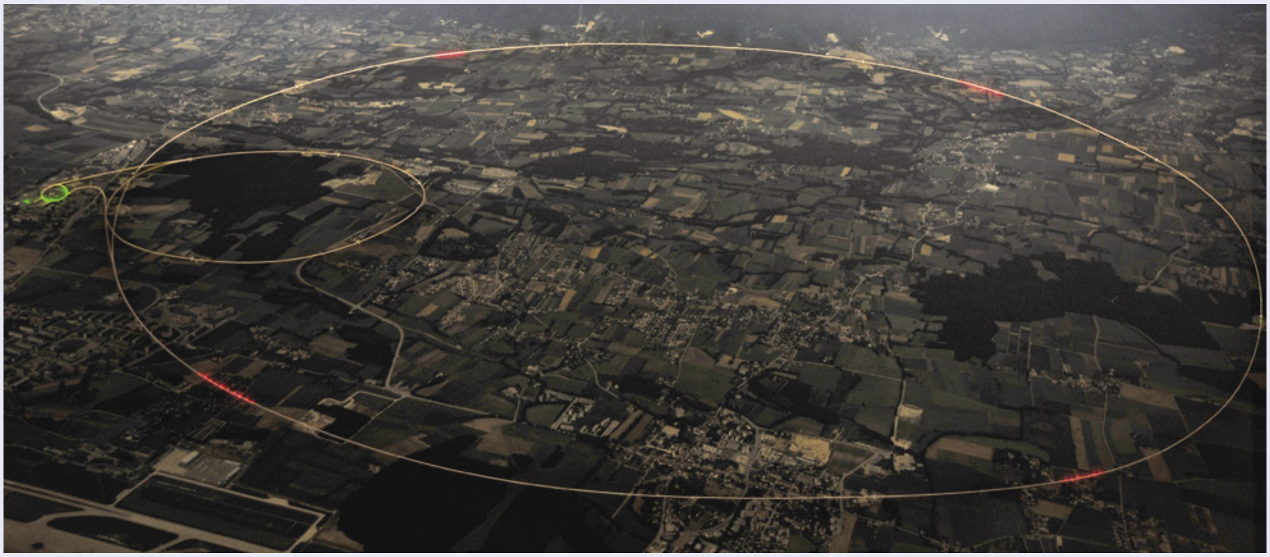

FIGURE 5 The Large Hadron Collider is mostly underground, as shown by the 23-km circle on this aerial view of the site on the France–Switzerland border. Particles undergo high speed circular motion at speeds close to that of light.

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- » Challenge worksheet
  3.2A Motion of a candle
- » Challenge worksheet
  3.2B Motion of a billy
- » Video
  Calculating average speed and period

3.3

Centripetal acceleration and force

KEY IDEAS

In this section, you will learn about:

- centripetal acceleration and centripetal force
- how to solve problems involving forces acting on objects in uniform circular motion. average speed of objects undergoing uniform circular motion.

To keep a ball revolving in a horizontal circle you have to apply a force with your hand. This is an unbalanced force directed inwards so the ball must be accelerating inwards according to Newton's second law (Figure 1). If the ball is accelerating, then, by definition, it must be undergoing a change in velocity. It can change its velocity without changing its speed if it changes direction. This section is about how a centripetal force causes centripetal acceleration, which causes a change in velocity even though the speed remains constant.

The formulas relating centripetal acceleration and force to the average speed of an object travelling in uniform circular motion are quite difficult, as they involve vectors in two dimensions. First, you need to describe how subtracting vectors is different from adding them, which you did in the previous two chapters. You need to subtract because acceleration is a 'change' in velocity, and 'change' means subtraction.

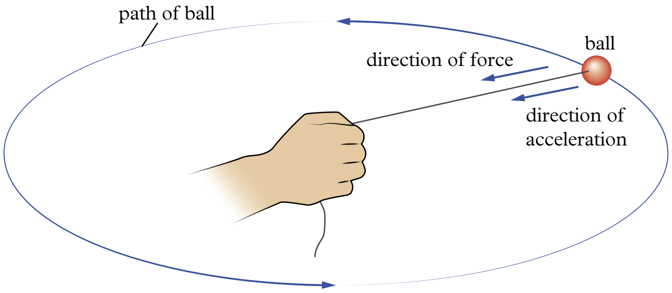

FIGURE 1 Path of a ball undergoing horizontal circular motion

Subtraction of vectors

If you were riding your bicycle with an initial speed of 10 km h^{-1} going downhill and had a final speed of 15 km h^{-1}, you would have gained 5 km h^{-1} of speed. Your 'change of speed' would be +5 km h^{-1}:

change in measurement = final measurement − initial measurement

change in speed = final speed − initial speed

$$\Delta\vec{v} = \vec{v} - \vec{u}$$

Conversely, if you slowed down by 5 km h^{-1} your change in speed would be −5 km h^{-1}. The symbol for the Greek capital 'D' (delta, Δ) is used to represent 'difference'. This is easy to remember as they both start with the letter 'D'.

This is simple for scalar quantities such as speed, mass, temperature and bank balances. But in physics it is also necessary to work with vector quantities. You've seen how to add vectors (in projectile motion), now you will learn how to subtract vectors.

SUGGESTED PRACTICAL
3.3 Centripetal force and horizontal circular motion
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In maths, you will have learnt that subtraction is the same as adding a negative; that is, 15 – 10 is equivalent to 15 + –10 and the answer is +5 either way. Thus, when performing the subtraction: vector A – vector B, the direction of vector B is changed to its opposite and then added to vector A (head to tail); that is, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ (Figure 2).

A more complicated situation arises when subtracting vectors in two-dimensions, as you have with horizontal circular motion. You will start with a familiar example of a person on a bicycle changing direction.

You will need this technique for analysing the change in velocity vectors for uniform circular motion.

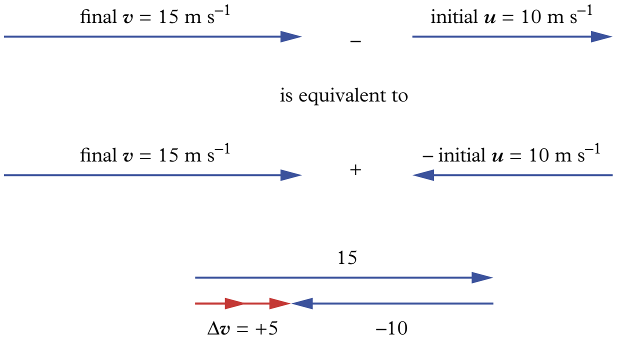

FIGURE 2 A ‘change’ means subtracting vectors.

WORKED EXAMPLE 3.3A

A girl on a bicycle is travelling north at 10 m s^{-1}. She turns left and continues at 15 m s^{-1} west. Calculate the change in velocity.

SOLUTION

$$\Delta v = v - u$$
$$= 15 \text{ m s}^{-1} \text{ W} - 10 \text{ m s}^{-1} \text{ N}$$

Change the subtraction to an addition of the negative:

$$\Delta v = 15 \text{m s}^{-1} \text{ W} + (-10 \text{ m s}^{-1} \text{ N})$$
$$= 15 \text{m s}^{-1} \text{W} + 10 \text{ m s}^{-1} \text{ S}$$

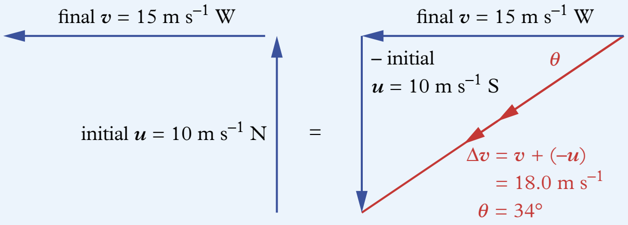

FIGURE 3 Combining vectors to give a resultant

$$\text{Change in velocity} = \sqrt{10^2 + 15^2}$$
$$= 18 \text{ m s}^{-1} \text{ (20 m s}^{-1} \text{ to 1 sf)}$$
$$\theta = \tan^{-1}\frac{10}{15}$$
$$= 34°$$

The change in velocity is 20 m s^{-1} at an angle of 34° S of W.

Centripetal acceleration

You will now use your techniques for dealing with 'change of' velocity vectors to analyse uniform circular motion for acceleration. Consider a ball being whirled around in a horizontal circle (Figure 4). Again, you use the term 'revolving' for this motion.

The velocity at any point on the circle is a tangent to the path at that point. For instance, at position A, the initial velocity vector $\vec{u}$ points up the page. At point B, the final velocity vector $\vec{v}$ points to the left but it has the same length as the initial velocity, as the speed remains the same. However, as the direction of the velocity has changed, the ball is said to be accelerating (centripetal acceleration).

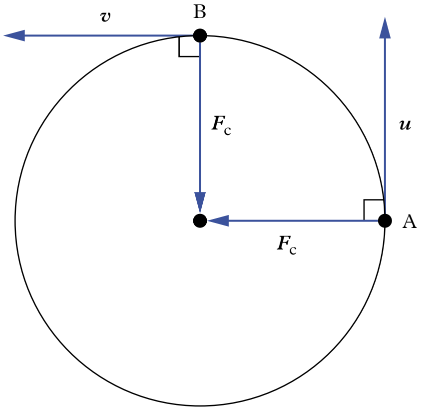

FIGURE 4 A free-body diagram showing the forces acting on a revolving object

The formula for acceleration is, as always:

$$\begin{aligned} a &= \frac{\text{change in velocity}}{\text{time}} \\ &= \frac{v-u}{t} \\ &= \frac{\Delta v}{t} \text{ (but remember these velocities are in two dimensions now)} \end{aligned}$$

The magnitude and direction of this acceleration can be calculated by determining the change in velocity:

Change in velocity (Δv) = final velocity (v) – initial velocity (u).

When you subtract a vector quantity, you turn it into an addition by reversing the direction of the initial vector and adding the arrows head-to-tail. Hence: $\Delta v = v + (-u)$. As can be seen in Figure 5, the resultant is directed towards the centre of the circle, hence 'centre seeking'.

If the time taken to go from position A to position B is Δt, then the distance s is $v\Delta t$. Let the chord $s = v\Delta t$ (if s is short enough this is a reasonable approximation). Using similar triangles:

$$\frac{\Delta v}{v} = \frac{s}{r}$$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Thus, the centripetal acceleration, a_c, is given by:

$$a_c = \frac{v^2}{r}$$

where r is the radius of the circular path in metres. The acceleration vector is towards the centre, whereas the velocity vector is perpendicular to this.

But it is a bit surprising that a_c is proportional to v^2, implying, for example, that it is four times as difficult to go around a curve at 100 km h^{-1} than at 50 km h^{-1}. A sharp corner has a small radius, so a_c is greater for tighter turns.

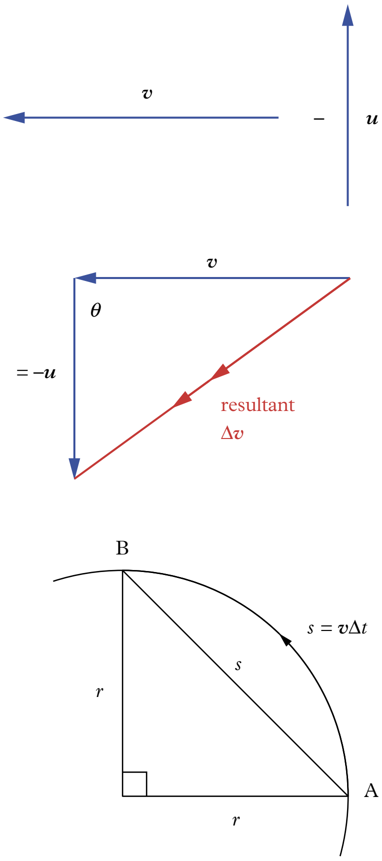

FIGURE 5 Recombining vectors to form the resultant

WORKED EXAMPLE 3.3B

A rubber stopper on a piece of string 1.5 m long is whirled in a horizontal circle at a uniform speed of 2 revolutions per second (rps).

a Calculate the average speed.

b Calculate the centripetal acceleration.

SOLUTION

a Period, $T = \frac{\text{time}}{\text{no. of revolutions}}$

$$= \frac{1\text{ s}}{2.0\text{ revolutions}}$$

$$= 0.50\text{ s}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 1.5}{0.50}$$

$$= 18.8\text{ m s}^{-1}$$

The average speed is 19 m s^{-1} (to 2 significant figures).

b $a_c = \frac{v^2}{r}$

$$= \frac{18.8^2}{1.5}$$

$$= 237\text{ m s}^{-2}\text{ (towards the centre)}$$

The centripetal acceleration is 240 m s^{-2} towards the centre (to 2 significant figures).

CHALLENGE 3.3A

Coin revolution

How many revolutions will coin A make while rotating around coin B (Figure 6)? Try it. You'll be surprised!

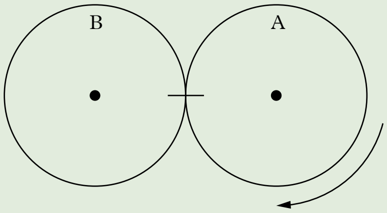

FIGURE 6 Coins rotating

CHALLENGE 3.3B

Rotational speed

The radian per second (symbol: rad s^{-1}) is the SI unit of rotational speed, commonly denoted by the Greek letter ω (omega). From your Maths classes, you may know that there are 2π radians in a circle of 360°. Engineers use the radians per second, for example, when working out the power of rotating shafts. Determine how many revolutions per second is 1 rad s^{-1}.

Centripetal force

The ball (described above) is experiencing a centripetal force (F_c) to keep it moving in a circle. This is provided by the tension (F_{net}) in the string. Using Newton's second law of motion ($F = ma$):

$$F_c = m \times a_c$$

$$F_{net} = \frac{mv^2}{r}$$

Isaac Newton proposed this law in his famous work *Principia* in 1687. In Book 1, Proposition 4 he wrote (in Latin): 'The centripetal forces acting on bodies undergoing uniform circular motion are proportional to the square of the arcs described in a given time, divided by the radius of the circle.'

Note that for horizontal circular motion $F_{net} = F_c$. This is different for vertical circular motion – but you are not doing that here.

WORKED EXAMPLE 3.3C

A rubber stopper of mass 20.0 g is whirled in a horizontal circle at constant speed on a string that is 1.5 m long. If the average speed (as calculated in Worked example 3.3B) is 18.8 m s^{-1}, calculate the centripetal force and hence the tension in the string.

SOLUTION

Facts: $v = 18.8$ m s^{-1} (average speed), $r = 1.5$ m, $m = 20.0$ g $= 0.0200$ kg

Find: F_c and F_{net} (that is, the tension, F_T). Note that for horizontal circular motion, $F_{net} = F_c$. This is different for vertical circular motion – but, again, you are not doing that here.

Formula:
$$\begin{aligned} F_c &= F_{net} \\ &= \frac{mv^2}{r} \\ &= \frac{0.0200 \times 18.8^2}{1.5} \\ &= 4.7 \text{ N} \end{aligned}$$

Finish: The centripetal force is 4.7 N directed towards the centre of the circle (direction is needed as it is a vector), and the tension in the string is 4.7 N (no direction as it is scalar).

CHALLENGE 3.3C

Spinning windscreen

The government steamer *Relief* attended the lighthouses along the Queensland coast from 1899 to 1952. To cope with the huge spray of seawater on the windows of the steering cabin, a novel approach was taken. The windscreen in part consisted of a circular glass disc about 40 cm diameter that was spun at high speed. How did this keep the sea spray off the window? Why couldn't they use windscreen wipers as used in a car? Propose two advantages and two disadvantages of this system compared with wipers.

Practical example – going around a curve safely

A racing car travelling around a circular track is similar to a ball being whirled around on a string. A vehicle going around a bend on a level road can be viewed also as going on a circular path. The sideways friction between the tyres and the road provides the force needed to stop the car just going straight ahead. The friction provides the centripetal force. If the car was to hit a wet patch all of a sudden, the friction would be reduced and insufficient centripetal force could be provided, so the car would tend to go straight ahead, possibly even spinning out of control.

FIGURE 7 The tyres on these racing cars have to provide the centripetal force to enable the cars to follow the circular path.

The maximum safe speed to go around a curve is when the car's average speed requires a centripetal force equal to the friction. For a car with rubber tyres on a dry bitumen road, friction is typically equal to 70% (0.7 times) of the car's weight. The mass term *m* is on both sides of the equation and so it cancels out. Big cars have the same maximum safe speed as small cars.

$$F_c = F_f$$

$$\frac{mv^2}{r} = 0.7\ mg$$

$$v^2 = 0.7\ gr$$

$$v = \sqrt{0.7\ gr}$$

Note: for a wet road the friction is closer to 50% of the weight. On an oil patch it would be as low as 5%.

WORKED EXAMPLE 3.3D

A motorcycle and rider with a total mass of 1250 kg are travelling at a constant speed around a dry circular bitumen track of radius 56 m. Calculate the maximum safe speed. Assume friction is 0.7 times the car's weight.

SOLUTION

$$v = \sqrt{0.7\ gr}$$

$$= \sqrt{0.7 \times 9.8 \times 56}$$

$$= 19.6\ \text{m s}^{-1}\ (20\ \text{m s}^{-1}\ \text{or}\ 5.6\ \text{km h}^{-1}\ \text{to 2 sf})$$

Practical example: Sun, Moon and Earth

Earth is held in orbit by a gravitational force between it and the Sun that provides sufficient centripetal force to keep it from flying away. The same is for Earth and the Moon. In the next chapter you will be making use of the relationships developed here to examine these solar and planetary forces.

FIGURE 8 The Sun, Earth and Moon use gravity to supply the centripetal forces.

CHECK YOUR LEARNING 3.3

Describe and explain

1 **Explain** the meaning of centripetal force.

2 **Describe** how the directions of the force and velocity vectors are related for an object undergoing uniform circular motion.

3 A ball is on a string 1.2 m in length and is swung in a horizontal circle at 3 rps. **Calculate** the centripetal acceleration.

4 A centripetal force of 64 N is acting on a brick of mass 2.8 kg being swung in a horizontal circle of radius 0.75 m. **Calculate**:

- **a** the velocity of the brick
- **b** its centripetal acceleration
- **c** its rotational speed in rpm.

Apply, analyse and interpret

5 A 150 g ball is tied to a pole with a rope of length 1.5 m, and spins around the pole at 20.0 m s^{-1}. **Determine** the centripetal force acting on the ball.

6 An electron ($m = 9.11 \times 10^{-31}$ kg) moves in a circle of radius 0.02 m and a centripetal force of 4.60×10^{-14} N acts on the electron. **Determine** the average speed of the electron.

7 A 1550 kg car goes round a circular path of radius 69 m, at a speed of 20 m s^{-1}. **Determine** the minimum friction required between the car and the road so that the car does not skid.

8 A string can withstand a force of 120 N before breaking. A 1.8 kg brick is tied to the string and whirled in a horizontal circle of radius of 90 cm. **Determine** the maximum speed of the brick before the string breaks.

9 In an investigation of uniform circular motion, a student whirled a 50.0 g rubber stopper above his head in a horizontal circle of radius 1.20 m (Figure 9).

The string was passed through a piece of glass tubing, and a set of slotted brass masses was suspended from the end of the string. It required a hanging mass of 150.0 g to provide enough force to keep the rubber stopper whirling in a circle at a constant speed.

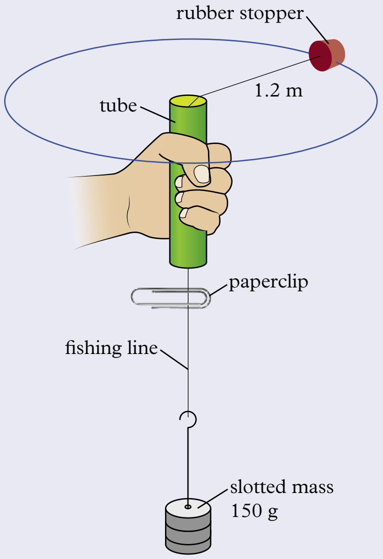

FIGURE 9 An investigation of uniform circular motion

Determine the:

- **a** tension in the string provided by the hanging mass
- **b** centripetal force
- **c** velocity of the stopper
- **d** period of the rubber stopper
- **e** time taken for 10 revolutions of the stopper.

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» Student book questions
Check your learning 3.3

» Suggested practical worksheet
3.3 Centripetal force and horizontal circular motion

» Challenge worksheet
3.3A Coin revolution

» Challenge worksheet
3.3B Rotational speed

CHAPTER 3

Review

Summary

3.1
- Uniform circular motion is the result of a force that acts on an object in a perpendicular direction to the velocity of the object.

3.2
- The time for one revolution of an object in uniform circular motion is called the period, T.
- Average speed for uniform circular motion is the distance travelled in one period (T) of time.
- One revolution of a circle is 360°, or 2π radians.
- Rotational speed often used is revolutions per minute (rpm).

3.3
- An object travelling in a circle at constant speed has an acceleration, called centripetal acceleration, directed towards the centre of the circular path and perpendicular to the velocity vector:
  $a_c = \frac{v^2}{r}$
- The centripetal force is the net force directed towards the centre of a circular path:
  $F_c = \frac{mv^2}{r}$
- Acceleration is often expressed in multiples of g (9.8 m s^{-2}).

Key terms

- average speed
- centripetal acceleration
- centripetal force
- period
- rotational speed
- tangential velocity
- uniform circular motion

Key formulas

Velocity for circular motion	$v = \frac{2\pi r}{T}$
Centripetal acceleration	$a_c = \frac{v^2}{r}$
Centripetal force	$F_{net} = \frac{mv^2}{r}$
Force due to gravity (weight)	$F_g = mg$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A ball is being whirled in a horizontal circle of radius *r* at constant speed. The velocity of the ball at positions A and B is shown in Figure 1.

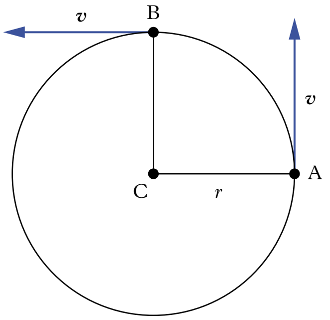

FIGURE 1 Free-body diagram of a whirling ball

Select the direction that best describes the change in velocity of the ball from point A to point B.

A ↖ **B** ↘ **C** ↗ **D** ↙

2 As part of a centripetal motion experiment, a student swung a rubber stopper around in an overhead horizontal circle at constant speed. In one particular trial she counts 20 revolutions in 10 seconds. Select the expression that best shows the velocity of the stopper.

A $\frac{2\pi r \times 20}{10}$ **B** $\frac{2\pi r \times 10}{20}$

C $\frac{\pi r^2 \times 10}{20}$ **D** $\frac{\pi r^2 \times 20}{10}$

3 Select the diagram in Figure 2 that best shows the relationship between centripetal force (vertical axis) and the velocity of an object being whirled in a horizontal circle at constant speed.

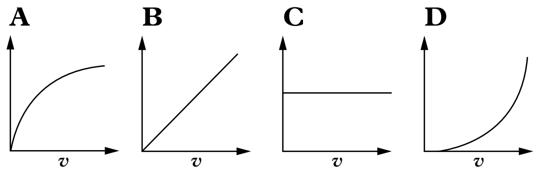

FIGURE 2 Graphs of motion

4 A student is investigating centripetal motion by whirling a rubber stopper above his head in a horizontal circle at constant speed (Figure 3).

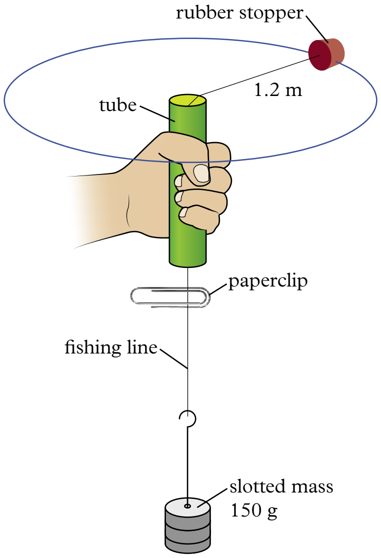

FIGURE 3 An investigation of circular motion

The student keeps the mass (*m*) of the rubber stopper constant, but increases the hanging slotted masses (*M*). Select the statement that best describes what he can do to keep the radius constant.

A increase *m* or *v*

B increase *m* or decrease *v*

C decrease *m* or increase *v*

D decrease *m* or *v*

5 A rubber stopper is moving in an anticlockwise horizontal circle at constant speed. Select the diagram in Figure 4 that shows the direction of the velocity and acceleration at point X.

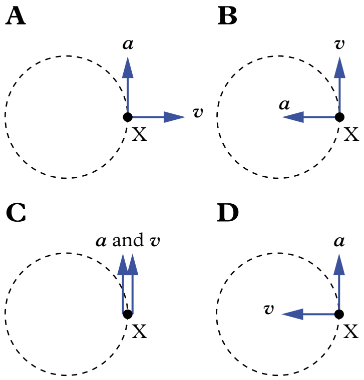

FIGURE 4 Velocity and acceleration vectors

Short answer

Describe and explain

★ **6** **Explain** the difference between uniform and non-uniform circular motion.

★ **7** **Explain** how average speed and velocity are related for uniform circular motion.

★ **8** **Determine** whether this is true: An object will travel the circumference of a circle in one period (T) of time if it is in uniform circular motion.

★ **9** A ball on a string is swirled around in a horizontal circle at a constant speed of 2.5 m s^{-1} and experiences an acceleration of 3.9 m s^{-2}. **Calculate** the radius of its motion.

★ **10** A rock is swung in a horizontal circle on a 1.2 m length of string. It has a constant speed of 3.3 m s^{-1}. **Calculate** the centripetal acceleration of the rock.

★★ **11** **Describe** the type of force necessary to keep:

- **a** the Moon in orbit around Earth
- **b** a car from sliding sideways off a circular track
- **c** a rubber stopper on a string in a horizontal circle above your head
- **d** you pressed against the walls of a Gravitron.

★★ **12** A rubber stopper tied to a string is travelling at a constant speed of 3 m s^{-1} in a circle of radius 1.5 m.

- **a** **Calculate** the magnitude of the centripetal acceleration of the stopper.
- **b** **Identify** the direction of the acceleration.

Apply, analyse and interpret

★ **13** A car of mass 2250 kg is travelling around a circular track of radius 80 m at a constant speed of 20 m s^{-1}. **Determine** how many revolutions of the track per minute the driver makes.

★ **14** A microwave carousel platter of diameter 31 cm turns at 10 rpm. **Derive** the average speed for a point on the outer rim of the carousel.

★ **15** A 2.0 m long fishing line that has a breaking strain of 20 N is attached to a lead sinker of 120 g. **Determine** the maximum speed at which the sinker can be whirled in a horizontal circle without breaking the line.

★★ **16** In a laboratory, a biochemist concentrates a DNA sample by centrifuging it at 1200 rpm. The bottom of the centrifuge tube is 15 cm from the centre spindle. **Determine** the speed that this part of the tube is experiencing.

★★ **17** You are driving a car around a corner and you are experiencing a force of 4500 N against your seatbelt. The radius of the curve is 30 m and your mass is 60 kg. **Determine** how fast you must be travelling in your car.

★★★ **18** You are on a rotating space station that has a radius of 100 m, and it is rotating at a velocity of 30 m s^{-1}. Your mass is 70 kg.

- **a** **Determine** the scale reading (in kg) on bathroom scales positioned to register centripetal force.
- **b** **Construct** a diagram of the arrangement.

★★★ **19** A 540 kg satellite is in an orbit of radius of 32 000 km from the centre of Earth. The centripetal force experienced by the satellite is 197 N. **Determine** the:

- **a** average speed
- **b** number of revolutions per day the satellite makes of Earth.

Investigate, evaluate and communicate

★ **20** A coin is placed on a record turntable and the turntable is made to rotate. As the turntable picks up speed, the coin is flung off. **Propose** why this happens.

★★ **21** You are whirling an object on a string in a horizontal circle above your head and the string breaks. **Propose**, with justification, whether the object will follow a circular path for a short time after it breaks.

★★ **22** **Determine** the shape of, and **construct** a diagram of, a graph of the following (with everything else being held constant):

- **a** F_c (vertical axis) versus v
- **b** F_c versus r
- **c** F_c versus m (mass of rotating mass).

★★ **23** A student told friends he had been to the local fair and had 'gone on the Gravitron ride – the ride that sucks you to the walls'. **Propose** where his understanding of the physics is wrong.

★★★ **24** A car of mass 1300 kg is attempting to round a circular track of radius 60.0 m at a speed of 15 m s^{-1}. It is a wet track and the friction is calculated to be 60% of the car's weight. **Propose**, with justification, whether there will be sufficient friction for the car to travel the circular path successfully.

★★★ **25** Four $1 coins are spaced equally at 5 cm centres across the radius of a record turntable, as shown in Figure 5. Coin A marks the centre. Each coin has a mass of 9.0 g. The turntable is set spinning at its normal rate of $33\frac{1}{3}$ rpm. The friction between a coin and the surface is estimated to be 10% of the coin's weight. **Evaluate** this arrangement and deduce which coins, if any, will remain on the turntable while it is rotating.

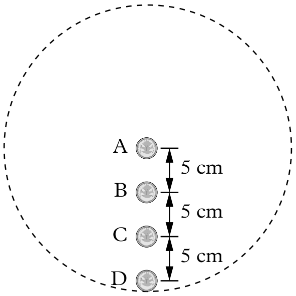

FIGURE 5 Coins on a turntable

★★★ **26** Students performed a centripetal force experiment using the apparatus shown in Figure 6. Their results are listed in Table 1.

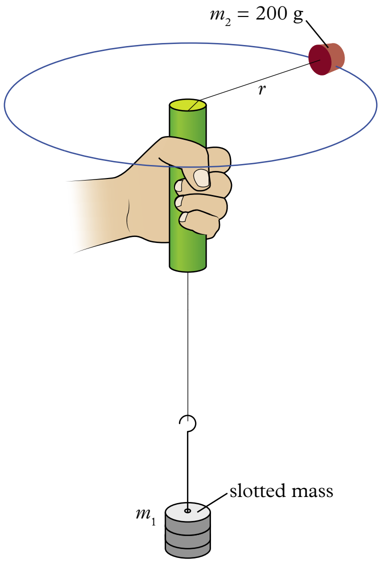

FIGURE 6 Centripetal force experiment

TABLE 1 Mass and period data

Hanging mass, m_1 (g)	Revolving mass, m_2 (g)	Radius, r (m)	Period, T (s)
50	200	1.00	4.11
100	200	1.20	3.18
150	200	1.40	2.81
200	200	1.60	2.60

a **Calculate** the centripetal acceleration and the centripetal force for each trial (use the weight of the hanging mass for F_c).

b **Plot** a graph of F_c on the vertical axis versus a_c on the horizontal axis.

c **Determine** the gradient and what it means.

d **Calculate** the observed value of the revolving mass from the gradient.

e **Compare** this to the actual value for m_2 by calculating absolute and percentage errors.

f **Assess** the result.

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CHAPTER 4

Gravitational force and fields

From the very earliest days, humans have looked into the sky and wondered what it's all about. First they would have noticed the Sun travelling across the sky during the day, and the Moon across the sky at night. They would also have seen the stars moving across the heavens, and the strange wanderers – the planets – doing loop-the-loops against the background of the stars.

Thousands of years ago priests in Babylon (now present-day Iraq) stood gazing into the night sky, not realising just how big it was but trying to make sense of it all. They developed theories that used myths and legends to explain the behaviour of these astronomical bodies. But it always started and finished with Earth being the centre of the universe. People later interpreted the Bible and sacred texts to confirm the central place of Earth in this 'geocentric' model.

It wasn't until the work of Polish astronomer Nicolaus Copernicus in 1543 that the model changed to a 'heliocentric' one with the rotating Sun at the centre and the planets revolving around it. No-one could really explain the underlying principles of how it all worked until the late 1680s when Newton proposed his magnificent synthesis of all the information – his law of universal gravitation.

OBJECTIVES

- → Recall Newton's law of universal gravitation.
- → Solve problems involving the magnitude of the gravitational force between two masses.
- → Define the term 'gravitational fields'.
- → Solve problems involving the gravitational field strength at a distance from an object.

Source: *Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority

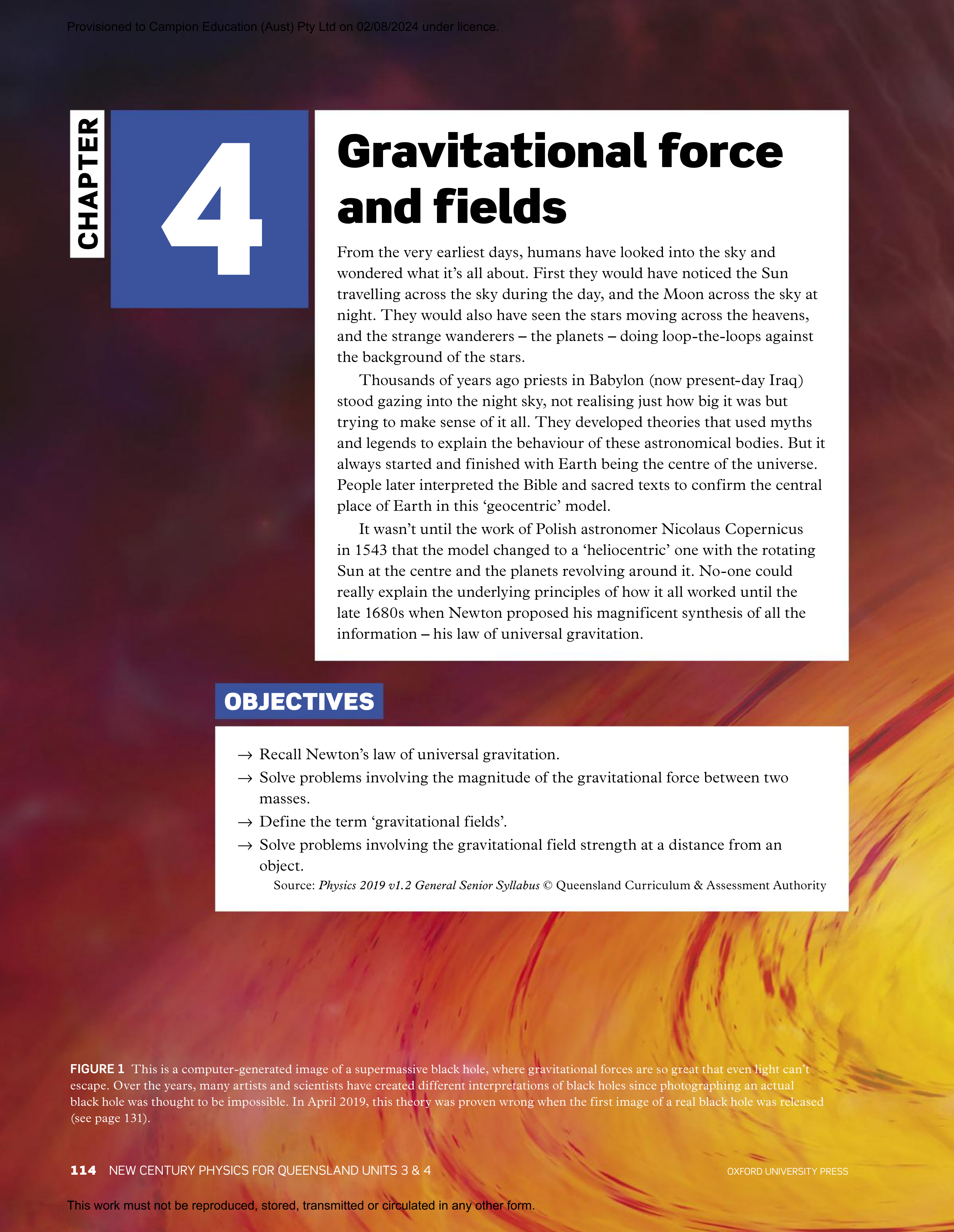


FIGURE 1 This is a computer-generated image of a supermassive black hole, where gravitational forces are so great that even light can't escape. Over the years, many artists and scientists have created different interpretations of black holes since photographing an actual black hole was thought to be impossible. In April 2019, this theory was proven wrong when the first image of a real black hole was released (see page 131).

MAKES YOU WONDER

In this chapter you will be examining some aspects of gravitational forces and orbits that will help to answer questions such as these:

→ How could anyone think the Sun revolved around Earth?

→ Why do people believe that planets affect our destiny, when the gravitational force is so small?

→ Am I weightless when I'm floating in a swimming pool?

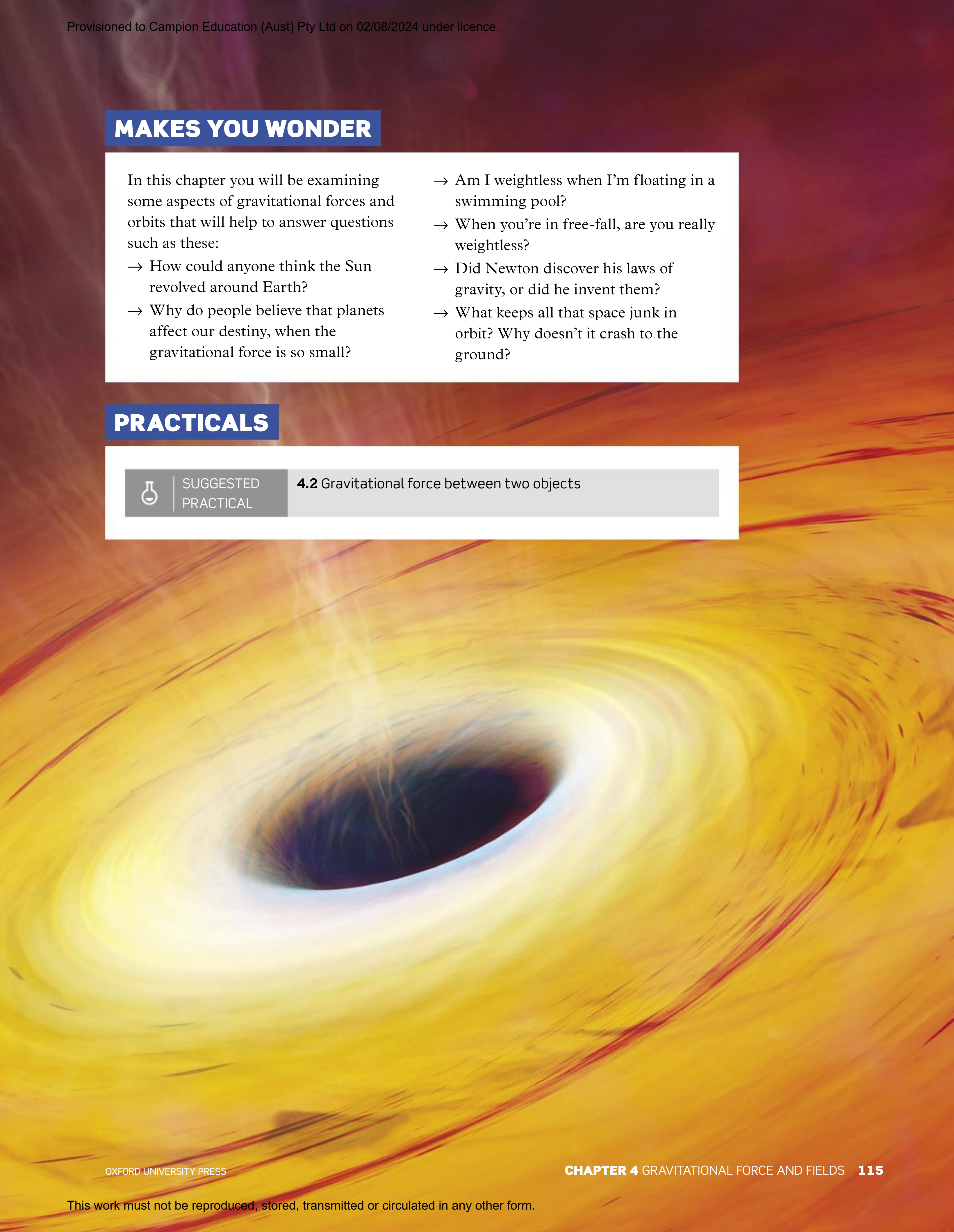
→ When you're in free-fall, are you really weightless?

→ Did Newton discover his laws of gravity, or did he invent them?

→ What keeps all that space junk in orbit? Why doesn't it crash to the ground?

PRACTICALS

SUGGESTED PRACTICAL	**4.2** Gravitational force between two objects



4.1

What is gravity?

KEY IDEAS

In this section, you will learn about:

- what gravity is
- gravitational forces between masses.

Up until 1687, the Latin word *gravitas* or gravity was used to mean 'heavy'. It referred to a quality of an object that meant it was hard to lift. No one thought that objects were being pulled to the ground. The common idea put forward by Greek philosopher Aristotle (384–322 BCE) in about 330 BCE was that things like metal and rock moved towards the ground because that was their natural place, and the heavier they were the faster they fell. There was no experimental proof of this; in fact, no one attempted to confirm this proposed relationship. Experiments were just not an approach that was considered useful in the physical sciences of the time.

In the 1500s, scientists began applying mathematics to the physical sciences, and Aristotle's work in this area was deemed hopelessly inadequate. In 1604, Italian scientist Galileo Galilei (1564–1642) experimented with balls rolling down inclines and found that they did in fact accelerate but that acceleration was independent of mass. This was contrary to Aristotle's notion and created quite a sensation in scientific circles.

Study tip

Kepler's laws will be described in the next chapter.

The astronomer Johannes Kepler (1571–1630) then showed in 1609 that the Sun was the centre of our solar system, again contrary to Aristotle's teaching that Earth was at the centre. Kepler asked: how does the Sun exert an influence over the planets? He believed that the Sun moved the planets by sending out rays like wheel spokes, which carried the planets around.

In 1687, English scientist Isaac Newton (1643–1727) united the ideas of the astronomers such as Kepler with Galileo's laws of falling bodies. Newton used the word 'gravity' to describe the force between the Sun and the planets, and extended it to mean that gravity was a force that pulled objects towards Earth. It was a subtle change, but now it meant a 'force' – the force of gravity. He extended this idea in later propositions to a force between all bodies.

FIGURE 1 Cliff diving at sunset. The diver is attracted towards Earth but Earth is also attracted to the diver. To Newton, 'gravity' applied not only to the Sun and planets, but between all bodies.

Newton's laws of motion and gravity were confirmed over and over by experimentation and were regarded as the undisputed laws of nature. That was until Albert Einstein (1879–1955) proposed his general theory of relativity in 1915. He had already proposed his special theory of relativity in 1905, which was particularly applicable at the atomic level. It survived many experimental tests of carefully collected and analysed data, and was expanded by Einstein to 'the universe at large' including planets, stars and galaxies. It was a theory of gravitation that proposed gravity as being a distortion of 'space–time' (as Einstein called it) that was so much more complicated and highly mathematical than Special Relativity. It could explain the bending of light by massive astronomical bodies that Newton's model could not. Again, experimental tests confirmed its predictions, culminating in the detection of gravitational waves in 2015. In fact, it is so well confirmed that its validity is no longer in doubt. Well, until something better comes along.

Nevertheless, whether gravity is due to the presence of mass or is a distortion of space–time, it can still be defined thus: **Gravity** is a natural phenomenon by which all things with mass are attracted to one another.

gravity
the force of attraction between objects with mass

CHALLENGE 4.1A

Is Earth pulling on the Sun?

Which is greater, Earth's pull on the Sun, or the Sun's pull on Earth?

CHALLENGE 4.1B

Flat Earth gravity

How would gravity be different if Earth was flat instead of round?

CHECK YOUR LEARNING 4.1

Describe and explain

1 **Explain** how Aristotle's explanation for the motion of heavy objects was discredited.

2 **Describe** how the word 'gravity' changed meanings in Newton's writings.

Apply, analyse and interpret

3 In his treatise on motion, Newton compared the gravitational force to electrostatic and magnetic forces. He said that gravity was an attractive force only. **Deduce** how this differs from electrostatic and magnetic forces.

Investigate, evaluate and communicate

4 It is said that gravity was invented by Newton whereas others say he discovered it. **Evaluate** these two claims.

5 **Propose**, with reasons, whether this statement is true: 'Gravitational waves didn't exist until they were discovered in 2015'.

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- Challenge worksheet
  4.1A Is Earth pulling on the Sun?
- Challenge worksheet
  4.1B Flat Earth gravity
- Weblink
  Gravity and Earth's orbit

4.2 Newton's law of universal gravitation

KEY IDEAS

In this section, you will learn about:

- Newton's law of universal gravitation
- solving problems that involve the gravitational force between two masses.

Newton's law of universal gravitation states that the force of attraction between each pair of point particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

In his most famous work, *Philosophiae Naturalis Principia Mathematica*, published in Latin in 1687, Newton wrote: 'I deduced that the forces which keep the planets in their orbs must vary reciprocally as the squares of their distances from the centres about which they revolve and in direct proportion to their masses'. This is known as **Newton's law of universal gravitation**. In the form of an equation, this becomes:

$$F_g = \frac{GMm}{r^2}$$

where F_g is the force of gravitational attraction, M and m are the masses of the attracting objects, and r is the radial distance between the centres of the objects (Figure 1). 'Radial' means a line radiating outwards from the object like the spokes of a wheel (Latin *radius* meaning 'spokes of a wheel'). G is a proportionality constant called the universal gravitational constant or more commonly known by astrophysicists as 'big' G to distinguish it from 'little' g, the acceleration due to gravity. In SI units, G has the value 6.67×10^{-11} N m^2 kg^{-2}.

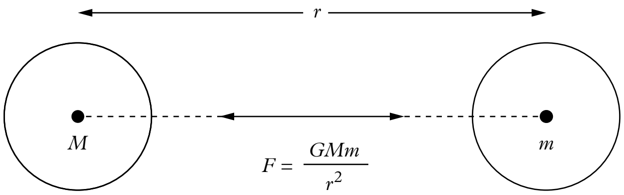

FIGURE 1 The meaning of the symbols for Newton's law of universal gravitation

- This attractive force always points inwards, from one point to the other; that is, there are two equal and opposite forces.
- It is a vector quantity as it has both magnitude and direction.
- The law applies to all objects with mass, big or small.
- Two big objects can be considered as point-like masses, if the distance between them is very large compared to their sizes or if they are spherically symmetrical. For these cases, the mass of each object can be represented as a point mass located at its centre of mass.

Newton wrote in his 'Principia' *rationem vero harum Gravitatis propietartum ex phenomenis nondrum potui decucere* ('but I have not been able to discover the reason for this property of gravitation from the phenomena'). He meant that he had discovered a law for gravitation but didn't know what made it occur. This happens in science all the time. For example, the movement of Earth's tides was well understood and tables predicting tides for the year ahead were being produced for centuries before their relationship to gravity had been worked out.

SUGGESTED PRACTICAL
4.2 Gravitational force between two objects
Go to your obook assess »

Separation distances

The formula works well when the separation distance is very large compared to the diameter of the object itself. When the two objects are close, the diameter of the object must be carefully considered. In this next section you will consider how the formula applies at large and small distances.

WORKED EXAMPLE 4.2A

Determine the force of attraction between Earth (mass = 5.97×10^{24} kg) and the Moon (mass 7.35×10^{22} kg), given that the Earth–Moon distance is 3.8×10^{8} m.

SOLUTION

Facts: $m = 7.35 \times 10^{22}$ kg, $M = 5.97 \times 10^{24}$ kg, $r = 3.8 \times 10^{8}$ m

Find: F_g

Formula: $F_g = \frac{GMm}{r^2}$, where G has the value 6.67×10^{-11} N m^2 kg^{-2}

Figure it out:
$$F_g = \frac{GMm}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{(3.80 \times 10^{8})^2}$$
$$= 2.03 \times 10^{20}\,\text{N (3 sf)}$$

Finish: The force is 2.03×10^{20} N towards Earth and the same magnitude towards the Moon (need to state direction).

Large distances

Large distances mean that the bodies are separated by several diameters, such as the distance between Earth and the Moon, other moons and their respective planets, and between stars, planets and asteroids.

FIGURE 2 The Moon is far enough away from Earth that both can be regarded as point sources of mass. Likewise, the Sun and Earth can be regarded as point masses.

Small distances

Newton's law of universal gravitation states that every point mass attracts every other point mass in the universe by a force 'inversely proportional to the square of the distance between them'. When you stand on the surface of Earth your distance to Earth is not zero, but 6.37 × 10^6 m (the mean radius of Earth, r_E) because that's how far away the centre of mass is (Figure 3). This may be significant in your calculations.

It is interesting to note that the equatorial radius (at the equator) is 6378.1 km and the polar radius is 6356.8 km. Equatorial radius is conventionally used unless the polar radius is explicitly required.

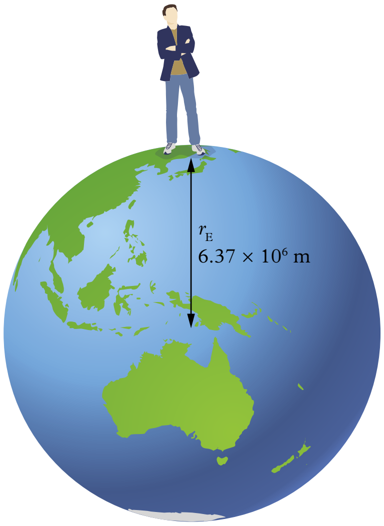

FIGURE 3 At close distances, the radius of the astronomical body must be considered.

WORKED EXAMPLE 4.2B

Determine the force of gravitational attraction between Earth (mass = 5.97 × 10^{24} kg) and a 72.0 kg physics student when the student is in an aeroplane at 10 000 m (1.0000 × 10^4 m) above Earth's surface. The radius of Earth (r_E) can be taken as 6.37 × 10^6 m.

SOLUTION

Facts: $M = 5.97 \times 10^{24}$ kg

$m = 72.0$ kg

$r = 6.37 \times 10^6$ m

$d = 6.37 \times 10^6 \text{ m} + 10\,000 = 6.38 \times 10^6$ m

Find: F_g

Formula: $F_g = \frac{GMm}{r^2}$, where G has the value 6.67 × 10^{-11}

Figure: $F_g = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 72.0}{(6.38 \times 10^6)^2}$

$= 704.3$ N (704 N to 3 sf)

Finish: There is a force of 704 N on the student acting down towards the centre of Earth and the same magnitude upwards by the student on the Earth.

CHALLENGE 4.2A

The Great Attractor

Our galaxy is being drawn towards something called 'The Great Attractor' at a rate of 610 km s^{-1}. Scientists agree that it is a beautifully named and very apt description of something that lies very far away and is pulling us ever faster, ever closer. What is it?

CHALLENGE 4.2B

How to make a swing go higher

Imagine you were writing an instruction manual for a child's swing. Without using a diagram, what instructions would you write on how to make it go higher? No one has ever written this successfully, maybe you'll be the first!

Determination of 'big *G*'

The gravitational constant G can be found experimentally by measuring the gravitational force between two spheres of known mass, separated by a known distance. The first person to do this was Henry Cavendish in 1798, more than a century after Newton proposed his law.

Cavendish fastened two small lead spheres, each of mass m, to the ends of a long wooden rod that was suspended from its midpoint by a fine fibre (Figure 4). Large lead balls (M) were brought up close to the small ones. The large and small lead balls attracted each other and caused the fibre to twist. The amount of twist was proportional to the force between the spheres. Cavendish had standardised the device beforehand by determining how much force was needed to twist the fibre by certain amounts.

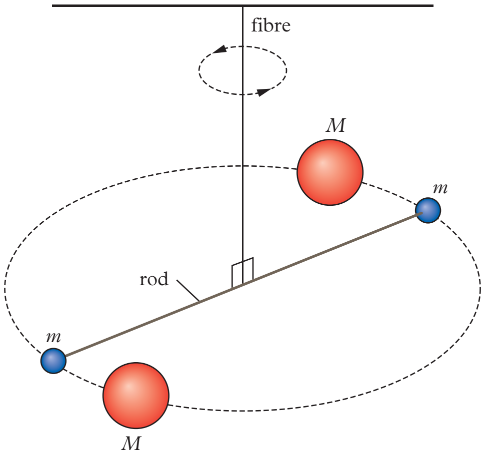

FIGURE 4 Cavendish's apparatus

Each of the two large lead spheres used by Cavendish had a mass of 158 kg and each of the two smaller spheres had a mass of 2.92 kg. Cavendish didn't actually make the apparatus he used. He inherited it from John Michell who died in 1793 before he could try the experiment himself.

The total force between the two pairs of lead masses was determined for various separation distances. The force between each pair was calculated by dividing the total force by two. Table 1 gives the results for the force on the fibre for one pair of masses (158 kg and 2.92 kg) at various distances. These are not Cavendish's results but are derived from them.

TABLE 1 Experimental data for Cavendish's apparatus

Separation distance, r (cm)	5.0	6.0	7.0	8.0	10.0	12.0	14.0
Total force, F ($\text{N} \times 10^{-6}$)	12.2	8.8	6.4	4.7	3.2	2.3	1.5

The data, when plotted, is shown in Figure 5. Note that distance has been converted to metres.

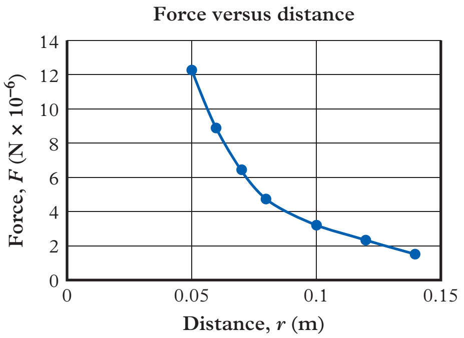

FIGURE 5 A force versus distance graph for Cavandish's experiment

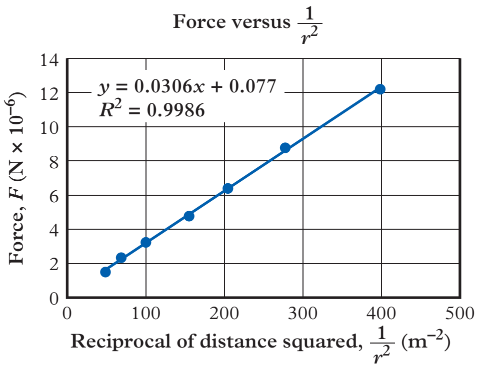

FIGURE 6 The data has been linearised by plotting force versus the reciprocal of distance squared.

The relationship appears to be inverse squared, so to linearise the data a graph of F versus $\frac{1}{r^2}$ can be plotted and a linear graph results (Figure 6). This implies that F divided by $\frac{1}{r^2}$ (that is, Fr^2) is a constant. This equals the gradient of the line.

$$\begin{aligned} Fr^2 &= G \times M \times m \\ &= 0.0306 \times 10^{-6} \text{(the gradient)} \end{aligned}$$

The total mass of the small spheres is 0.73 kg + 0.73 kg = 1.46 kg. The total mass of the two large spheres is 158 kg + 158 kg = 316 kg. Use these total masses in the calculations. Therefore:

$$\begin{aligned} G &= \frac{Fr^2}{Mm} \\ &= \frac{\text{gradient}}{Mm} \\ &= \frac{0.0306 \times 10^{-6}}{158 \times 2.92} \\ &= 6.63 \times 10^{-11} \text{ N kg}^2 \text{m}^{-2} \end{aligned}$$

Study tip

For further explanation of absolute error and percentage error, you can refer to the toolkit chapter in *New Century Physics for Queensland Units 1 & 2.*

Measurements of G have been made countless times using a variety of methods, but this value is very close to the accepted value of 6.67×10^{-11} N kg^2 m^{-2}. The accepted value is given the symbol x_A, and the observed value, the symbol x_o.

The absolute error, E_a:

$$\begin{aligned} E_a &= |x_o - x_A| \\ &= |6.63 \times 10^{-11} - 6.67 \times 10^{-11}| \\ &= 0.04 \times 10^{-11} \text{ N kg}^2 \text{m}^{-2} \end{aligned}$$

The percentage error:

$$\begin{aligned} E\,\% &= \frac{E_a}{x_A} \times 100 \\ &= \frac{0.04 \times 10^{-11}}{6.67 \times 10^{-11}} \times 100 \\ &= 0.60\% \end{aligned}$$

In the next section, gravitation is modelled in terms of a gravitational 'field' similar to electric and magnetic fields.

CASE STUDY 4.2A

Gravity and your body

When astronauts arrive back on Earth after an extended stay in space they are about 5 cm taller than when they left. This is a result of the microgravity of space, which allows the spine to elongate. When the astronauts are back on Earth, the normal force of gravity draws the spine's length back to normal.

The main compression occurs in the cartilage discs between the vertebrae of the spine. This is the main function of these discs – to act as a springy insulator against compression when standing, walking or lifting. Of course, other parts of the body undergo compression, but not as much as the discs.

FIGURE 7 The intervertebral discs of the backbone are shown in pink. Gravity causes these discs to compress.

Many tests have been done on the compression of the spine due to gravity. These tests are expressed using a quantity called Young's modulus of elasticity (E). By definition it is commonly stated as 'stress over strain' or more precisely, the force per unit area divided by change in length per unit length.

$$E = \frac{FL_{\text{spine}}}{A\Delta L}$$

Here E = Young's modulus, F = the force exerted (the person's weight); A = cross-sectional area; L_0 = initial length of person standing; L = length of person lying; ΔL = change in length; L_{spine} = length of backbone, assumed $\frac{L_0}{2}$.

This can be tested in class by measuring the height of a person standing (L_0), their height lying horizontally L, their waist circumference C, and mass m. You can calculate the force of gravity as it is just the weight of the person, $F_g = mg$. The area can be calculated from the waist circumference, $A = \frac{C^2}{4\pi}$, and $\Delta L = L - L_0$. You can assume the length of the spine is half the length of the whole body, $L_{\text{spine}} = \frac{L_0}{2}$. The results will be in N m^{-1}, which are more commonly called pascal, Pa. To convert to a megapascal, MPa, divide by 10^6. Table 2 provides the typical measurements for a Year 12 student.

TABLE 2 Measurements for a Year 12 student

Length (standing), L_0	Length (lying), L	Waist, C	Mass, m
170.5 cm	172.0 cm	70 cm	59.0 kg

$$\begin{aligned} E &= \frac{FL_{\text{spine}}}{A\Delta L} \\ &= \frac{(59.0 \times 9.8) \times \frac{1.705}{2}}{\frac{0.70^2}{4\pi} \times (1.720 - 1.705)} \\ &= \frac{492.9}{0.000585} \\ &= 842564 \text{ Pa} \\ E &= 0.843 \text{ MPa} \end{aligned}$$

The mean value and uncertainty from the lit erature is 0.974 ± 0.408 MPa.

CHALLENGE 4.2C

Drops of mercury

Two spherical drops of mercury are resting on a frictionless surface. The only force between them is that of gravitation. Propose what you would need to know to be able to calculate how much time it would take for them to touch.

CHECK YOUR LEARNING 4.2

Describe and explain

1 Newton's law of gravitation is said to be an inverse square law. **Explain** what that means.

2 **Determine** what happens to the gravitational force when:

- **a** one of the masses is doubled
- **b** the distance between the objects is halved.

3 **Explain** why Earth's radius needs to be taken into account for objects near the surface, when calculating gravitational forces.

4 **Calculate** the force between the Sun ($m = 2.0 \times 10^{30}$ kg) and Earth ($m = 5.97 \times 10^{24}$ kg) assuming their centres are 1.5×10^{8} km apart.

5 A 10 kg rock rests on the ground. **Calculate** the gravitational force acting on it, using Newton's law of universal gravitation.

Apply, analyse and interpret

6 Black holes are supermassive collapsed stars. The closest anything can get to one and still escape its gravitational force is called the 'event horizon'. **Determine** the force acting on a 15 tonne spacecraft at the event horizon of 30 km from a black hole that has a mass equal to 10 times that of the Sun.

Mass of Sun = 2.0×10^{30} kg; 1 tonne = 1000 kg

7 When a star collapses to form a black hole, the size of the star is greatly reduced although the mass remains the same. **Deduce** what would happen to the gravitational force of the Sun on our Earth if the Sun unexpectedly collapsed to form a black hole?

Investigate, evaluate and communicate

8 Jupiter is about 300 times more massive than Earth so it would be easy to deduce that an object on the surface of Jupiter would weigh 300 times more than on the surface of Earth. For example, a rover (spacecraft) with a weight of 9000 N on Earth might be expected to weigh 2 700 000 N on the surface of Jupiter. But this is not the case. A 9000 N rover on Earth weighs only about 27 000 N on the surface of Jupiter. **Evaluate** this scenario and identify any misunderstandings.

9 A 1 tonne communications satellite is orbiting a planet. A student said that 'if you double the mass of the satellite or the planet the force will double'. Another student said that doubling the mass of the planet had to have a bigger effect than just going from a 1 tonne satellite to a 2 tonne satellite. **Evaluate** both claims and decide who is correct.

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- » Challenge worksheet
  4.2A The Great Attractor
- » Challenge worksheet
  4.2B How to make a swing go higher

4.3 Gravitational fields

KEY IDEAS

In this section, you will learn about:

- gravitational field strength at a distance from an object.

Study tip

The difference between a *scalar* field and a *vector* field is worth establishing here as the distinction reappears in later chapters when the various fields affecting elementary particles are discussed.

You may have seen a demonstration in which iron filings are sprinkled over paper with a magnet underneath similar to the one in Figure 1. The patterns produced are said to reveal lines of force and show the magnetic field. The word 'field' for the region of space surrounding a magnet doesn't sound unusual today, but it was first used in this way in 1850 by English scientist Michael Faraday (1779–1867). At the time, Faraday was director of the laboratory at the Royal Institution – one of the premier laboratories in the world. He chose the word for a region of space as an analogy to a field used for farming.

A nice way to conceptualise a field is to imagine being in your physics laboratory and measuring the temperature of hundreds of points in the room and mapping them on a diagram. This would be called a temperature field. Each point has a particular magnitude (23.0°C, 24.5°C) but no direction, so is called a scalar field. Given enough time, wisdom and a computer, someone may be able to develop an equation for the field. Now, imagine you lit a Bunsen burner. The flame would be a source of temperature change that would affect the field. This is similar to mapping the light intensity in the room (an electromagnetic field) and then turning on a light, which would change the field. Or you could map the magnetic field and then turn on an electromagnet to change the field.

In these latter cases the field has a direction (up, down, E, W and so on). They would be called vector fields. They would be similar to an air–speed map of the room, where each point has a particular air speed and a direction. An air–speed field would be a vector field.

Faraday took the notion of fields further than cows in a paddock. He compared magnetic forces with electric and gravitational forces and conceptualised fields for them all. The use of a **gravitational field** as a way of depicting gravitational forces is now common, all thanks to Faraday (Figure 2).

gravitational field
the region of space surrounding a body in which another body experiences a force of gravitational attraction

In general, a field is a region of space where particular forces of nature are experienced. The force is described as mediated (brought about) by the field. You may have seen the word 'mediate' used when there is a disagreement and a person (a 'mediator') is employed to 'bring about' a resolution.

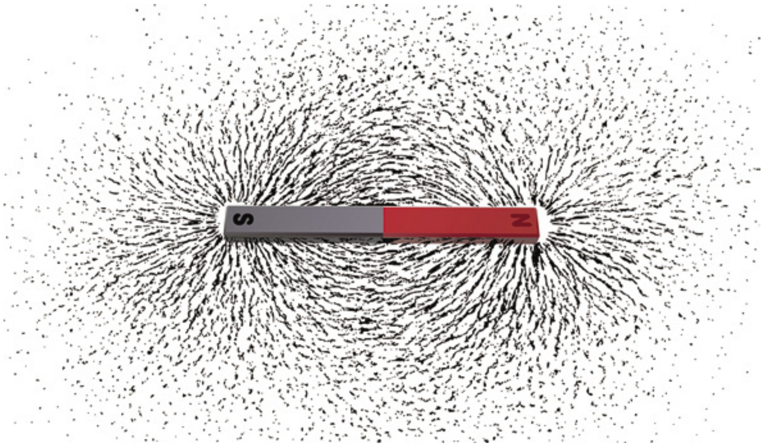

FIGURE 1 The magnetic field about a permanent magnet is shown as field lines.

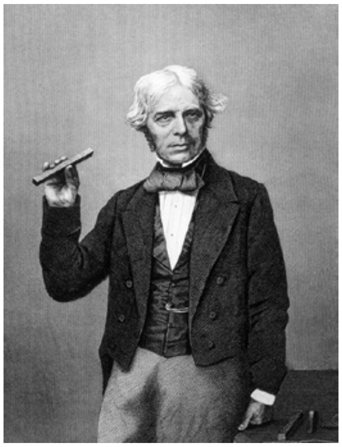

FIGURE 2 Michael Faraday holding a bar magnet (about 1850)

CHALLENGE 4.3A

Weighing less on the bathroom scales

Imagine you are standing on some bathroom scales and you bend your knees quickly. Predict what would happen to the scale reading and try to explain why.

Gravitational field direction

gravitational field direction towards the direction of the net gravitational force

Gravity is a vector field as it has both magnitude and direction. The **direction of a gravitational field** is towards the direction of the net gravitational force. For the surface of Earth, the net force acts down towards the surface. 'Down' is a local term and is from the perspective of the viewer on Earth's surface. A more general term would be 'towards Earth's centre' (Figure 3).

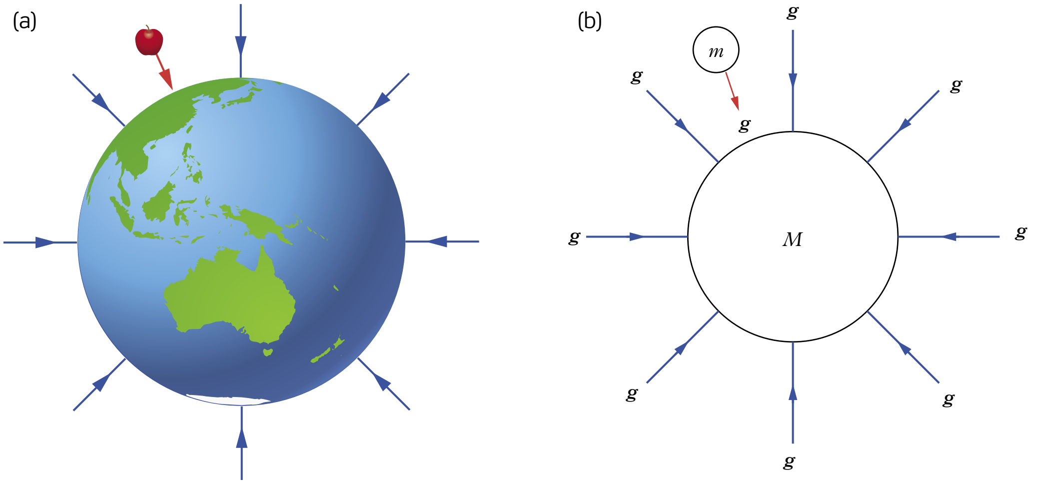

FIGURE 3 (a) All gravitational field lines point towards Earth's centre. A dropped apple would move in that direction. (b) In general, all field lines point towards the object's centre of mass. A small test mass would move in that direction.

Antigravity?

Can the field arrows point the other way? In other words, can we have antigravity – a force that pushes two objects apart? The answer seems to be 'no'. Unlike electrostatic and magnetic forces (like charges/poles repel, unlike attract), physicists have never observed a repulsive gravitational force, only an attractive one, although there is a lot of speculation about dark matter and dark energy having repulsive properties.

Einstein produced a comprehensive theory of gravity in 1915, his general theory of relativity. This theory argues that gravitational force is different from forces such as the magnetic and electrostatic forces, even though the mathematical relationships are identical. Einstein said that gravity is not so much something that happens in space; rather, it is a distortion or a warp of space itself. His theory encompasses all of Newton's laws, and takes them further. Einstein's theory has been confirmed by research thousands of times and physicists accept his theory as being the best current model for forces in the universe.

Gravitational field strength

You know that gravity is stronger on the surface of Earth than on the Moon, and that at the surface of a black hole gravity is so strong it will rip you apart. They have different field strengths as a measure of gravity. Field strength is a measure of the force acting on 1 unit of a given substance in the field. For an electric field it is the force experienced by a 1 coulomb (+1 C) positive test charge in the field. Likewise, for gravitation, it is the gravitational force experienced by a 1 kg 'test mass' of matter in the gravitational field. There's no such thing as negative mass, so a test charge is just 1 kg (not +1 kg).

Field forces

Gravitational field strength is described as force per unit mass. So it is written as $\frac{F}{m}$ and Newton's second law of motion for forces on an object in a gravitational field, $F_g = mg$, can be rearranged to give $g = \frac{F_g}{m}$. This says that the gravitational field strength (g) is the net force (F_g) per unit mass (m) at a particular point in the gravitational field.

gravitational field strength
(g) is the net force (F) per unit mass (m) at a particular point in the gravitational field

As F_g is a vector quantity and m is a scalar quantity, it follows that g is also a vector quantity and points in the same direction as the force. This is obvious; the force of gravity is towards Earth so that objects accelerate in the direction of the field, which is also towards Earth's centre – the field points down.

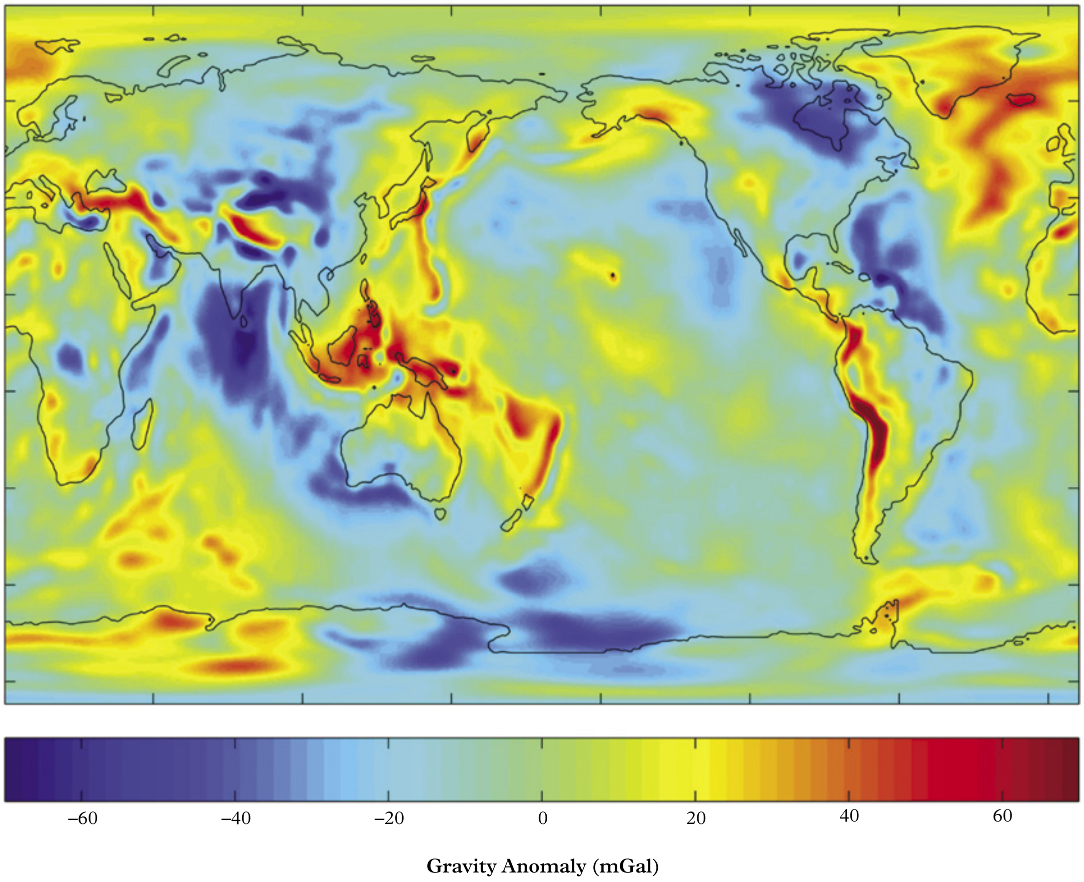

FIGURE 4 High-resolution gravity maps – such as this one of the world – were constructed from three billion calculations. The more red the greater the value of g; more blue means lower gravity.

Field sources

The strength of the gravitational field can now be determined – not by measuring experimentally the force acting on a 1 kg test object – but by using Newton's law of universal gravitation to calculate the field strength g at a distance r from a body of mass M.

To do this, F_g in the formula $g = \frac{F_g}{m}$ is replaced with $\frac{GMm}{r^2}$, where G is the universal gravitational constant, 6.67×10^{-11} N m² kg⁻², M (kg) is the mass of the larger object, m (kg) is the mass of the smaller object and r (m) is the distance between their centres. Using Newton's law:

$$g = \frac{F_g}{m}$$

$$= \frac{\frac{GMm}{r^2}}{m}$$

$$= \frac{GM}{r^2}$$

This allows g to be calculated if m_E and r are known. The units will be N kg^{-1} but can be shown to be the same as the units for acceleration, m s^{-2}.

WORKED EXAMPLE 4.3A

Calculate the gravitational field strength at:

a a point 10 000 km above Earth's centre

b Earth's surface (mean radius of Earth $r_E = 6.37 \times 10^6$ m).

SOLUTION

a *Facts:* $r = 10\,000 \text{ km} = 10\,000 \times 10^3 \text{ m} = 10^7 \text{ m}$

M = mass of Earth = 5.97×10^{24} kg

Find: g

Formula: $g = \frac{GM}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(10^7)^2}$$

$$= 3.98 \text{ m s}^{-2} \text{ (or N kg}^{-1}\text{)}$$

Finish: The field strength is 3.98 m s^{-2} directed downwards.

b *Facts:* r = mean radius of Earth (r_E) = 6.37×10^6 m

Find: g

Formula: $g = \frac{GM}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2}$$

$$= 9.81 \text{ m s}^{-2}$$

Finish: The field strength is 9.81 m s^{-2} directed downwards.

In the above example the value for the radius has been the mean radius of 6.37×10^6 m, although the radius varies from the equator (6.37×10^6 m) to the poles (6.36×10^6 m). Thus g will vary; for example, in Peru it is 9.76 and at the Arctic it is 9.83. You should note that at the surface of Earth, on average g is given a nominal value = 9.8 m s^{-2}.

The value of *g* decreases with distance following an inverse square law. At two Earth radii from the centre of Earth, *g* would be one-quarter of 9.8 m s^{-2} (2.45 m s^{-2}), and at three Earth radii from the centre, *g* would be one-ninth of 9.8 m s^{-2} (1.09 m s^{-2}). Further values are shown in Table 1.

TABLE 1 Acceleration due to gravity at distances from the surface of Earth

Acceleration due to gravity (m s^{-2})	Distance from Earth's surface (km)	Distance from Earth's centre (km)
9.8	0	6400
9.0	270	6670
8.0	670	7050
7.0	1160	7560
5.0	2540	8940
3.0	5150	11550
2.0	7740	14140
1.0	13590	19990

Gravitational field strength inside Earth

In theory, at the centre of Earth the gravitational fields would all point outwards and cancel out. The field strength would be zero. You can show this mathematically, as the gravitational field strength decreases linearly as you go from the surface to the centre. However, Earth's core is substantially denser than the outer layers (mantle and crust), and gravity actually increases slightly as you descend, reaching a maximum at the boundary between the outer core and the lower mantle. Within the core, gravity rapidly drops to zero as you approach the centre, where the planet's entire mass is exerting a gravitational pull from all directions. It was this mathematical treatment of gravity within a sphere that lead to the inverse square law years before it was confirmed experimentally by Cavendish.

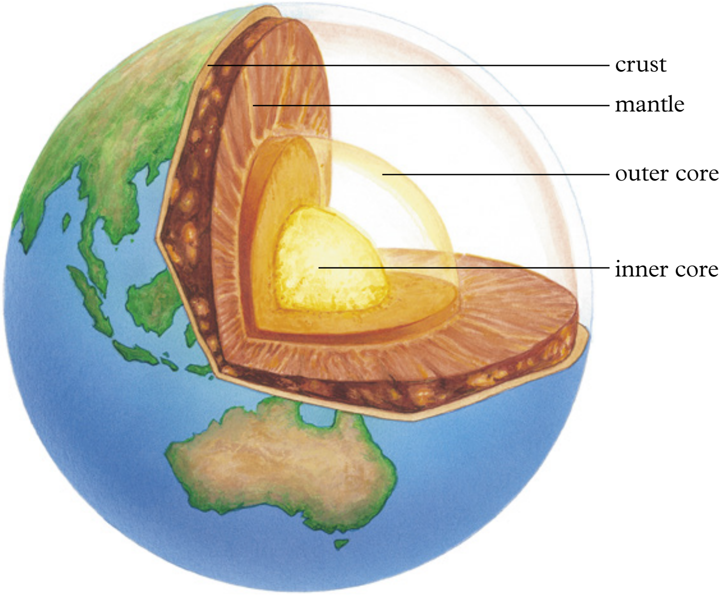

FIGURE 5 Earth's layers

CHALLENGE 4.3B

Scale readings in a mine

If you took a set of bathroom scales to the bottom of a deep mine shaft would your scale reading be less, the same or greater than on the surface?

Weightlessness in space

In his 1867 novel *From the Earth to the Moon*, science fiction writer Jules Verne predicted that there would be a spot between Earth and the Moon where a space traveller would be weightless. We now know that you can achieve a state of so-called 'weightlessness' in a space station close to Earth, or even just jumping off a diving board.

What Jules Verne was trying to say was that there was a point somewhere on the line between Earth and the Moon where the gravitational acceleration towards Earth and towards the Moon would cancel out, as shown in Figure 6. This point can be found by equating $g = \frac{Gm_E}{r^2}$ for both bodies.

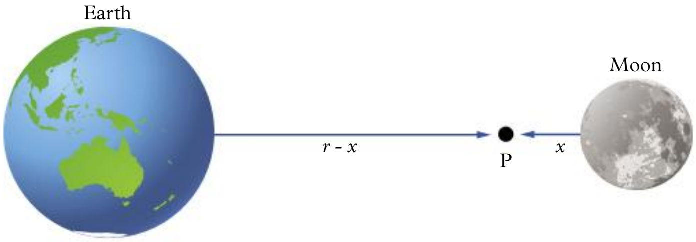

FIGURE 6 P is the point where g is zero.

It is known that Earth's mass = $m_E = 5.97 \times 10^{24}$ kg, the Moon's mass = $m_M = 7.35 \times 10^{22}$ kg, and the Earth–Moon distance is 3.8×10^8 m.

$$g_{\text{towards Earth}} = g_{\text{towards Moon}}$$

$$\frac{Gm_E}{(r-x)^2} = \frac{Gm_M}{x^2}$$

$$\frac{5.97 \times 10^{24}}{(3.8 \times 10^8 - x)^2} = \frac{7.35 \times 10^{22}}{x^2}$$

So, solving for x: distance to Moon (x) = 3.8×10^7 m, distance to Earth ($r - x$) = 3.42×10^8 m. That's a ratio of 1:9 for where $g_{\text{towards Earth}}$ and $g_{\text{towards Moon}}$ are equal. However, it cannot be said that this is a point where the gravitational field strength is zero, as there are contributions from the Sun, planets and other stars to add in. It is a very complex equation and best left to the experts.

What does weightless mean?

'Weightless' is a pretty meaningless term. Your weight is just your mass multiplied by the local value of gravitational field strength, g, at that location ($F_g = mg$). When you jump off a diving board or float in a swimming pool your weight remains the same as if you were walking around. The same is true for astronauts who float around in the space station. They are just falling as if they were jumping off a diving tower. They are not without weight, so they are not really weightless; they are just in free-fall.

Gravitational fields and black holes

black hole
a region of space where the gravitational field is so intense that no matter or radiation can escape

The accepted model of our solar system has slowly shifted under the influence of carefully collected and analysed data. Such data has helped scientists formulate the notion of **black holes** in our universe.

A 'black hole' is a term used to describe a region of space that contains matter so dense that even light cannot escape its grip. It was coined by John Wheeler of Princeton University (USA) in 1967. A black hole is thought to come about from the gravitational collapse of a star. The first tentative identification of a black hole was announced in December 1972 in the binary-star X-ray source Cygnus X-1. It seems there are supermassive black holes at the centres of all currently known galaxies. They are hundreds of thousands to billions of solar masses (one solar mass = mass of the Sun). In the case of our galaxy, the Milky Way, there appears to be a 4.3 million solar mass black hole in the region of Sagittarius A.

For a black hole with a mass ten times that of our Sun, the point at which light cannot escape (that is, the event horizon, the point at which it becomes 'black') is within 30 km of the centre. This is called the **Schwarzschild radius**.

Schwarzschild radius
a point near a black hole where the gravity is so powerful that nothing, not even light, can escape

FIGURE 7 This image, released in April 2019, is the first ever image of a black hole. It was created using data collected by eight telescopes around the world. Access the Increase your knowledge on your obook assess to find out more.

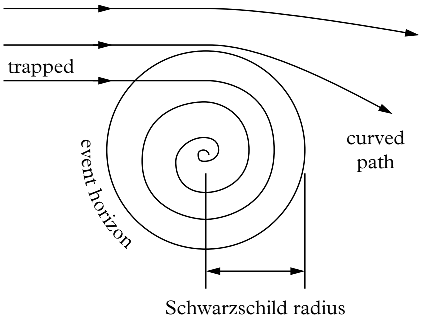

FIGURE 8 Black hole gravitation

CHECK YOUR LEARNING 4.3

Describe and explain

1 **Explain** how the unit symbol N kg^{-1} is equivalent to m s^{-2}.

2 **Describe** a gravitational field.

3 **Define** 'gravitational field strength' and state its units.

4 **Explain** what is meant by 'the gravitational field strength does not depend on the mass of the object in the field'.

5 **Calculate** the gravitational field strength at a point 100 000 km above Earth's centre.

6 **Clarify** how the direction of a gravitational field is defined.

Apply, analyse and interpret

7 **Determine** the gravitational field strength at a point whose distance from Earth's surface is equal to three Earth radii.

8 **Determine** the altitude above Earth's surface where the gravitational field strength is one-eighth the value on the surface.

9 **Derive** a location between the Sun and Earth where the gravitational field between them due to their mass is equal to zero.

10 **Construct** a graph of gravitational field strength versus distance from Earth's centre, using the data from Table 1. **Determine** the relationship. **Construct** a second graph to confirm your prediction.

11 The gravitational field strength 100 km from Earth's centre is 3.99 × 10^{4} m s^{-2}. That's 40 000 m s^{-2}. **Deduce** why this nonsensical.

Investigate, evaluate and communicate

12 Jupiter is about 300 times more massive than Earth, so it would be easy to deduce that an object on the surface of Jupiter would weigh 300 times more than on the surface of Earth. But this is not the case. It is just three times as heavy. **Evaluate** this scenario and identify any misunderstandings.

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CHAPTER 4

Review

Summary

4.1
- The heliocentric (Sun-centred) view of the heavens is based on the work of Copernicus, Kepler, Galileo, Brahe and Newton. Ptolemy and Aristotle favoured a geocentric (Earth-centred) view. Modern theories also rely on Einstein's theories of gravitation.

4.2
- Newton's law of universal gravity: the force of attraction between each pair of point particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them: $F_g = \frac{GMm}{r^2}$, where G is called the universal gravitational constant.

4.3
- The region of space surrounding a body in which another body experiences a force of gravitational attraction is called the gravitational field.
- Gravitational field strength is a vector quantity that has an inverse square relationship with distance and a direct relationship with mass: $g = \frac{GM}{r^2}$. It points towards the mass.

Key terms

- black hole
- gravitational field
- gravitational field direction
- gravitational field strength
- gravity
- Newton's law of universal gravitation
- Schwarzschild radius

Key formulas

Gravitational force	$F_g = mg$
Universal law of gravitation	$F_g = \frac{GMm}{r^2}$
Gravitational field strength	$g = \frac{F}{m}$ $= \frac{GM}{r^2}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

For questions that follow, use the following data:

Constant	Value
• g (on surface of Earth) = 9.8 m s^{-2}	• r_{Mo} (radius of Moon's orbit around Earth) = 3.8 × 10^{8} m
• G = 6.67 × 10^{-11} N m^{2} kg^{-2}	• r_{Eo} (mean radius of Earth's orbit around Sun) = 1.5 × 10^{11} m

Body	Mass	Radius
Earth	$m_E = 5.97 \times 10^{24}$ kg	$r_E = 6.37 \times 10^{6}$ m
Moon	$m_M = 7.34 \times 10^{22}$ kg	$r_M = 1.74 \times 10^{6}$ m
Sun	$m_S = 2.0 \times 10^{30}$ kg	$r_S = 6.96 \times 10^{8}$ m

Multiple choice

1 Select the diagram in Figure 1 that cannot be used as a diagram for a gravitational field.

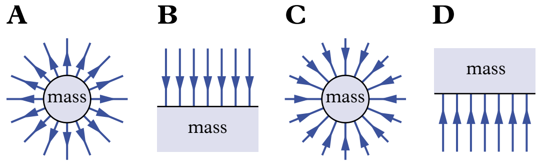

FIGURE 1

2 There are many planets in the universe orbiting a star other than our own Sun. Most of these planets are quite huge – as big as Jupiter. The planet Kepler-7b has a mass 0.433 times that of Jupiter and a radius 1.614 times that of Jupiter. If the gravitational field strength at the surface of Kepler-7b is g_K, and the gravitational field strength at the surface of Jupiter is g_J, the ratio $\frac{g_K}{g_J}$ is:

A 0.17 **B** 0.27

C 0.69 **D** 1.1

3 Two equal masses, m, separated by a distance, r, exert a force, F, on each other due to their gravitational attraction. Calculate the magnitude of the gravitational force between an object of mass m and an object of mass $2m$ separated by the same distance r.

A $\frac{1}{4}F$ **B** $\frac{1}{2}F$

C $2F$ **D** $4F$

4 Select the diagram in Figure 2 that best describes the forces acting on the Moon (M) due to Earth (E).

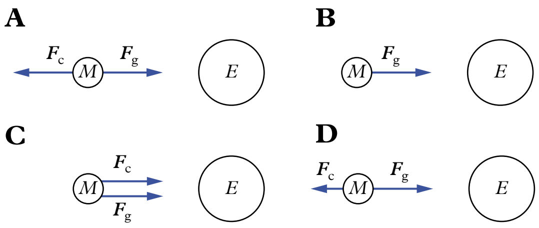

FIGURE 2

5 Select the statement that best defines gravitational field strength.

A It is the force acting on a small object in the field.

B It is the force acting on a 1 kg object in the field.

C It is the work done on a 1 kg object to shift it 1.0 m in the field.

D It is the work done by a field to shift a 1 kg object 1.0 m in the field.

6 On the surface of a planet with a radius r_P, a mass experiences a gravitational force of attraction of g. At a height of $4r_P$ above the planet, the mass experiences a force of:

A $\frac{g_P}{4}$ **B** $\frac{g_P}{5}$

C $\frac{g_P}{16}$ **D** $\frac{g_P}{25}$

7 The force of gravity on a 1 kg object on the surface of Earth is 9.8 N directed downwards. Select the answer that best states the force on a 1 kg object on the surface of Saturn given that Saturn is 100 times the mass of Earth and 10 times the diameter.

A $\frac{1}{10}$ N **B** 1 N

C 10 N **D** 100 N

8 Imagine that Earth preserved its mass but occupied a sphere only half its current diameter. Determine the weight of a person who normally weighs 600 N on the surface of this alternative Earth.

A 300 N
B 600 N
C 1200 N
D 2400 N

9 Two planets have the same gravitational field strength at their surfaces. Select the ratio that must also be the same for the two planets.

A $\frac{\text{radius}^3}{\text{mass}}$
B $\frac{\text{radius}^2}{\text{mass}}$
C $\frac{\text{radius}}{\text{mass}}$
D radius

10 Planet X has mass M and radius R. Planet Y has mass $10M$ and radius $4R$. Determine the ratio $\frac{\text{gravitational field strength at the surface of planet X}}{\text{gravitational field strength at the surface of planet Y}}$.

A 0.5
B 1.6
C 2.0
D 2.5

Short answer

Describe and explain

★ 11 **Explain** the difference between weight and mass.

★ 12 A satellite is in a circular orbit around Earth. **Explain** why an astronaut aboard the satellite feels weightless if the value of g at that altitude is 6.5 m s^{-2}.

★ 13 **Recall** the formula for Newton's law of universal gravitation and **explain** the meaning of the symbols.

★ 14 **Explain** the link between acceleration due to gravity and gravitational field strength.

★ 15 **Calculate** the gravitational force between:

- **a** two 60 kg people 1.0 m apart
- **b** two protons ($m_p = 1.7 \times 10^{-27}$ kg) at a distance of 5×10^{-15} m apart.

★ 16 **Clarify** the difference between heliocentric and geocentric models of the solar system.

Apply, analyse and interpret

★ 17 A satellite is orbiting Earth at a distance of 35 kilometres. The satellite has a mass of 500 kg. **Determine** the force between the planet and the satellite.

★ 18 An astronaut is on the Moon.

- **a** **Calculate** the acceleration due to gravity on its surface.
- **b** **Determine** the time it would it take a spanner to fall from rest from a height of 1.5 m to the surface of the Moon.

★ 19 A 15 kg object has a weight of 8000 N. **Determine** the gravitational field strength at this point.

★ 20 **Determine** the gravitational field strength of the Sun (mass 2.0×10^{30} kg) at a distance of 15×10^{10} m from its centre.

★ 21 **Determine** the distance between two objects of mass 5.0×10^4 kg and 2.5×10^4 kg respectively, if the gravitational force between them is 2.00×10^{-8} N.

★★ 22 **Determine** the gravitational field strength at a point:

- **a** on Earth's surface (see data table at start of questions)
- **b** 1.5 Earth radii above Earth's surface
- **c** 3.0 Earth radii above Earth's surface
- **d** 1000 m above the surface of the Moon
- **e** on the surface of the Sun.

★★ 23 A hydrogen atom consists of an electron of mass 9×10^{-31} kg and a proton of mass 1.9×10^{-27} kg separated by an average distance of 6×10^{-11} m. **Determine** the gravitational force between them.

★★ 24 A satellite is orbiting Earth in a circular orbit at a distance of two Earth radii from Earth's surface.

- **a** **Determine** the acceleration due to gravity aboard the satellite.
- **b** **Calculate** the weight of a 70 kg astronaut aboard the satellite.

★★ 25 **Determine** the distance between two objects of mass 6×10^5 kg and 3×10^5 kg, if the gravitational force between them is 1.5×10^8 N.

★★ 26 **Deduce** the gravitational field strength in the region of a satellite orbiting 8000 km above Earth's surface.

★★ 27 **Determine** the height above the surface of Earth at which the gravitational field strength is equal to 2.0 N kg^{-1}.

★★ 28 **Determine** the altitude above Earth's surface at which the gravitational field strength is one twenty-fifth the value on the surface of Earth.

★★★ 29 **Determine** the gravitational force of attraction between an electron and a nucleus 1.5 μm apart. The mass of the electron is 9.11×10^{-31} kg. Consider the nucleus to be made up of a proton and neutron, each of mass 1.67×10^{-27} kg.

★★★ 30 **Deduce** the acceleration due to gravity at the surface of the Moon if its mass is $\frac{1}{80}$ times the mass of Earth and its diameter $\frac{1}{4}$ times that of Earth.

★★★ 31 Earth's gravitational field strength at its surface is g, and Earth's radius is r. **Determine** the gravitational field strength at a distance of $33r$ from the centre of Earth.

★★★ 32 Figure 3 shows a satellite orbiting Earth. The satellite is part of the network of global positioning satellites that transmit radio signals used to locate the position of receivers on Earth.

When the satellite is directly overhead, the signal reaches the receiver 67 ms after it leaves the satellite. Radio waves travel at the speed of light ($c = 3 \times 10^8$ m s^{-1}).

a **Calculate** the height of the satellite above the surface of Earth.

b **Explain** why the satellite is accelerating towards the centre of Earth even though its orbital speed is constant.

c **Determine** the gravitational field strength due to Earth at the position of the satellite.

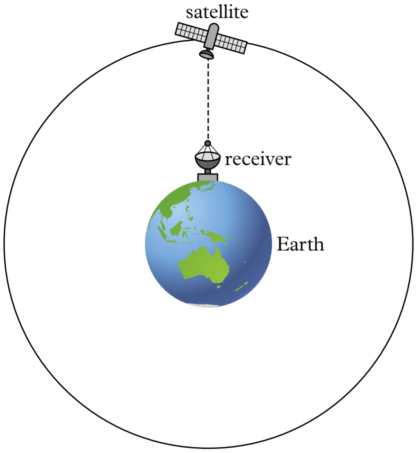

FIGURE 3

Investigate, evaluate and communicate

★★ 33 **Predict** how far would you have to travel upwards from Earth's surface to notice a 1 N kg^{-1} difference in gravitational field strength. (Earth has a radius of 6400 km.)

★★ 34 **Predict** the height at which the force of gravity on a satellite will be half that on it at ground level.

★★ 35 **Predict** the gravitational field strength at a point whose distance from Earth's surface is equal to four Earth radii.

★★★ 36 **Evaluate** whether the gravitational field strength of Earth at height h above the surface is given by $g = g_s\left(\frac{R}{R+h}\right)^2$, where g_s is the gravitational field strength at the surface and R is the radius of Earth.

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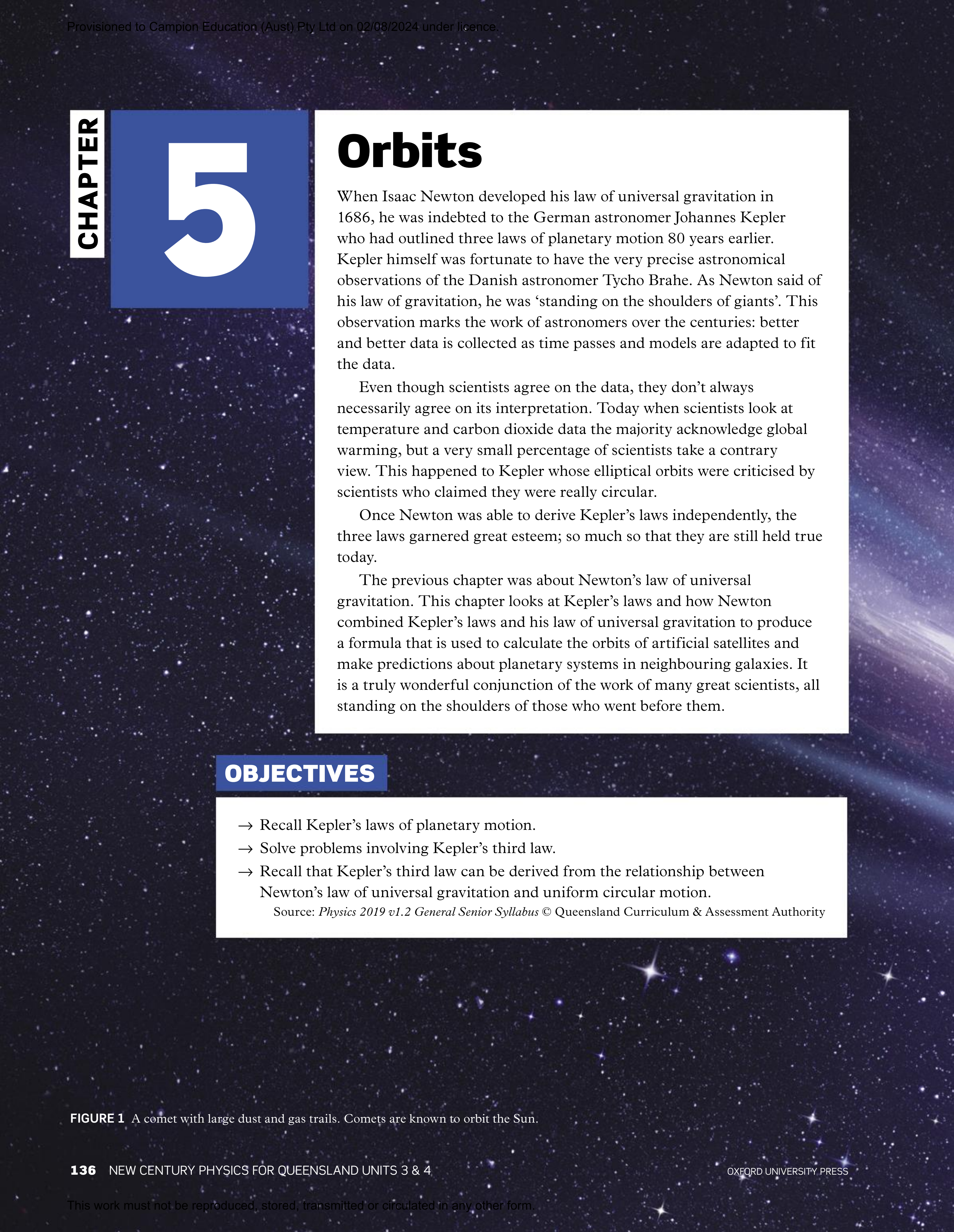

CHAPTER 5

Orbits

When Isaac Newton developed his law of universal gravitation in 1686, he was indebted to the German astronomer Johannes Kepler who had outlined three laws of planetary motion 80 years earlier. Kepler himself was fortunate to have the very precise astronomical observations of the Danish astronomer Tycho Brahe. As Newton said of his law of gravitation, he was 'standing on the shoulders of giants'. This observation marks the work of astronomers over the centuries: better and better data is collected as time passes and models are adapted to fit the data.

Even though scientists agree on the data, they don't always necessarily agree on its interpretation. Today when scientists look at temperature and carbon dioxide data the majority acknowledge global warming, but a very small percentage of scientists take a contrary view. This happened to Kepler whose elliptical orbits were criticised by scientists who claimed they were really circular.

Once Newton was able to derive Kepler's laws independently, the three laws garnered great esteem; so much so that they are still held true today.

The previous chapter was about Newton's law of universal gravitation. This chapter looks at Kepler's laws and how Newton combined Kepler's laws and his law of universal gravitation to produce a formula that is used to calculate the orbits of artificial satellites and make predictions about planetary systems in neighbouring galaxies. It is a truly wonderful conjunction of the work of many great scientists, all standing on the shoulders of those who went before them.

OBJECTIVES

→ Recall Kepler's laws of planetary motion.

→ Solve problems involving Kepler's third law.

→ Recall that Kepler's third law can be derived from the relationship between Newton's law of universal gravitation and uniform circular motion.

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FIGURE 1 A comet with large dust and gas trails. Comets are known to orbit the Sun.

MAKES YOU WONDER

In this chapter you will examine some aspects of gravitational forces and orbits that will help to answer questions such as these:

→ If planets speed up and slow down, a lot of energy must be used. Where does this energy come from?

→ Pluto takes 284 years to orbit the Sun once. How do they know, as this hasn't actually been measured?

→ The distance between Earth and the Sun gets longer and shorter during the year. Is it summer when it is close, and winter when it's further away?

→ If you launched a rocket to Mars would you just aim it ahead of where Mars is now so that they meet up in the future?

PRACTICALS

SUGGESTED PRACTICAL	**5.1** Orbital radius and mass

5.1

Kepler's laws of planetary motion

KEY IDEAS

In this section, you will learn about:

- Kepler's laws
- how Kepler's third law can be derived from the relationship between Newton's third law of universal gravitation and uniform circular motion.

In 1618, Johannes Kepler (1571–1630) published the first two of his three laws and was hailed as a hero by those who wanted to do away with the old geocentric (Earth-centred) universe of Aristotle and Ptolemy – a model in which the Sun, Moon and planets revolved around a stationary Earth (Figure 1). Instead, Kepler showed that a heliocentric (Greek *helios* meaning 'the Sun') model fitted the data so much better. In a heliocentric universe, the planets (including Earth) revolve around the Sun, the model in use today. Kepler developed three laws that have stood the test of time and are still in use. Isaac Newton showed that these laws were consistent with his (Newtonian) model of the universe.

Kepler's first law: the law of orbits

The wandering orbit of Mars could not be explained by astronomers using circular orbits for planets with Earth at the centre. Kepler proposed an elliptical orbit and a Sun-centred focus to account for this.

The **first law of planetary motion (law of orbits)** states that all planets move about the Sun in elliptical orbits that have the Sun as one of the foci.

first law of planetary motion (law of orbits) all planets move about the Sun in elliptical orbits, having the Sun as one of their foci

What are elliptical orbits?

An ellipse is produced if you make a sloping cut through a conical pyramid (Figure 2).

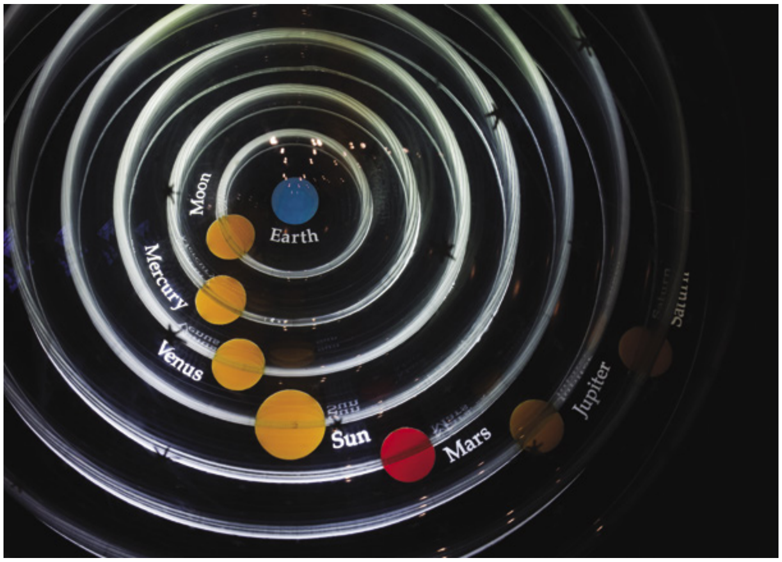

FIGURE 1 This early geocentric (Earth-centred) model of the solar system became less and less sustainable as astronomers collected more data.

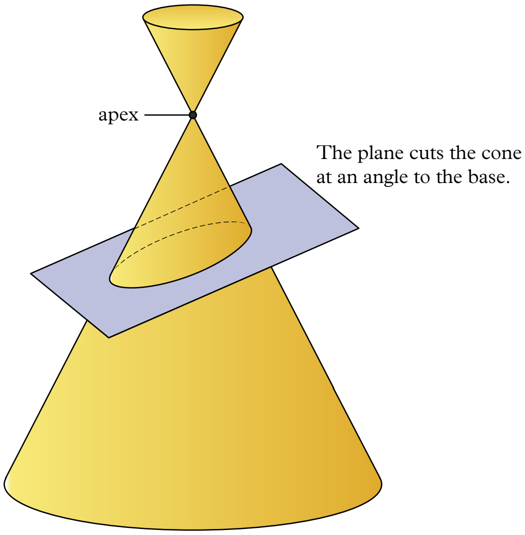

FIGURE 2 An ellipse is an angled cut by a plane through a conical pyramid.

SUGGESTED PRACTICAL

5.1 Orbital radius and mass

Go to your obook assess »

A planet of mass *m* moves in an elliptical orbit around the Sun. The Sun, of mass *M*, is at one focus, F, of the ellipse (Figure 3). The other, or 'empty' focus is F′. The point closest to the Sun is called the perihelion and the opposite point farthest from the Sun is called the aphelion. When referring to an elliptical Earth orbit, these points are called the perigee and apogee respectively. The words come from the Greek *peri* meaning 'around', *apo* meaning 'away' and *helios* meaning 'Sun'. The suffix *-gee* is derived from the Greek *geo*, meaning 'Earth'.

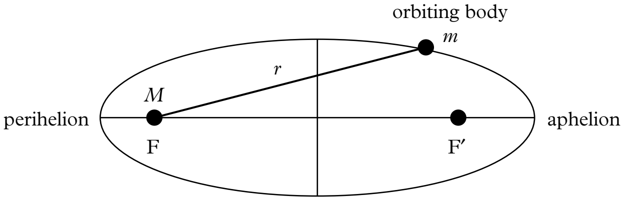

FIGURE 3 An orbit such as that of the Earth around the Sun is an ellipse.

The shape of the ellipse is almost circular and is exaggerated in the drawings shown here (Figure 3). It should look more like a circle. At perihelion, Earth is 147 097 800 km from the Sun; at aphelion it is 152 098 200 km, a difference of 5 million km. The difference in radius between the two is about 1.5%, which means the orbit is almost circular. In a circle of diameter 100 cm, this difference corresponds to the centre being about 1.5 cm off-centre; not much!

CHALLENGE 5.1A

Trajectory of a marble

A marble is fired into a circular tube lying flat on a table (see Figure 4). When it exits at the end, which trajectory does it follow: A, B or C?

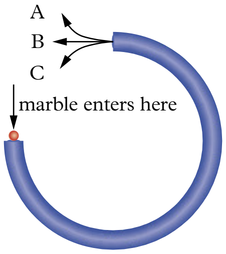

FIGURE 4 The trajectory of a marble

CHALLENGE 5.1B

Signal to Jupiter and back

A marathon runner starts off at the same time as a radar signal leaves Earth for Jupiter, a distance of 588 million km. He stops when the echo of the signal is received back on Earth. How many kilometres does he run? Note: radar signals are a form of electromagnetic radiation and travel at the speed of light (3×10^8 m s^{-1}).

Kepler's second law: the law of areas

Kepler's analysis of Tycho Brahe's planetary data showed that planets changed speed during their orbit. When a planet was closer to the Sun it sped up and when further away it slowed down. Kepler developed a law about this, but had no explanation. Newton's explanation was still 60 years away.

This effect is used today when trying to speed up a spacecraft. It is called the 'gravitational slingshot' effect or 'gravity assist' and increases the speed of the spacecraft without using any fuel. The gravitational attraction of the planet is used to transfer some of its momentum to the spacecraft or probe. That is, the planet slows down ever so slightly (really, really slightly because the probe is so much less massive).

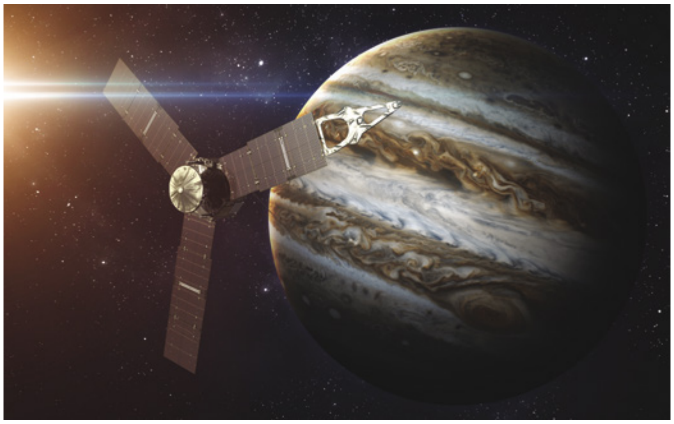

FIGURE 5 The gravity assist (slingshot) manoeuvre was used by the Juno mission to get to Jupiter. It used Earth's gravity to boost Juno's velocity and allowed it to escape the Sun's orbit.

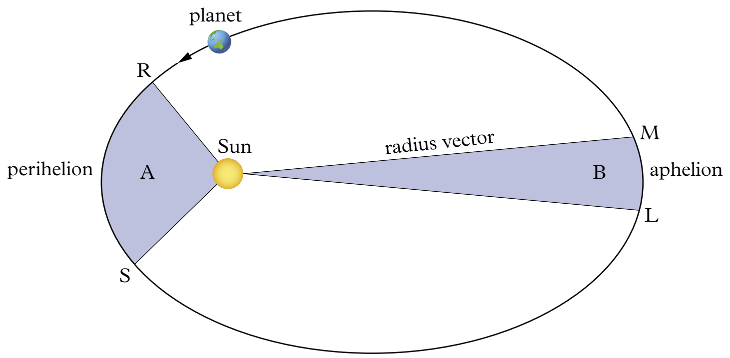

FIGURE 6 From Kepler's second law, if the time taken for the planet to travel from R to S is the same as that to travel from L to M, then area A will equal area B.

Kepler found that a planet's higher speed at a short distance was equivalent to a slower speed at a larger distance, so the areas swept out were the same. He framed his law around this idea.

Kepler's **second law of planetary motion (the law of areas)** states that a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time. A radius vector is any line from the Sun to the ellipse.

In Figure 6, the planet takes the same time to move from L to M as it does to move from R to S. The shaded areas A and B are equal (Kepler's second law).

Both these laws were in contradiction to conventional wisdom of the time. Kepler was the first astrophysicist.

second law of planetary motion (law of areas)
a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time

third law of planetary motion (law of periods)
the square of the sidereal period of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$

sidereal period
the time it takes for a planet to complete one orbit of another body relative to the stars

synodic period
time taken for a planet to appear in front of the same constellation of stars as seen from Earth

Kepler's third law: the harmonic law or the law of periods

In 1619, a more remarkable hypothesis followed. The **third law of planetary motion (law of periods)** states that the square of the sidereal periods of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$.

The **sidereal period** is the true orbital period of a planet; it is the time it takes the planet to complete one full orbit of the Sun. It should not be confused with **synodic period**, which is the time taken for a planet to appear in front of the same constellation of stars as seen from Earth. More generally, it is the time taken for one body to orbit another relative to the stars. These two periods are wildly different. By 'period' we will mean the true period. Kepler's third law takes the mathematical form: $T^2 \propto r^3$, or:

$$\frac{T^2}{r^3} = \text{constant (Kepler's ratio), or } \frac{T_A{}^2}{r_A{}^3} = \frac{T_B{}^2}{r_B{}^3}$$

where T_A is the period of planet A, and T_B the period of planet B. Likewise r_A is the orbital radius of planet A (between planet and Sun), and r_B is the orbital radius for planet B.

Table 1 shows the $\frac{T^2}{r^3}$ values for Earth and its neighbouring planets. Note how constant the Kepler's ratio is (average = 2.97×10^{-19} s^2 m^{-3}).

TABLE 1 Law of periods data for selected planets

	Radius of orbit, r ($\times 10^{10}$ m)	Period of revolution, T ($\times 10^7$ s)	Kepler's ratio, $\frac{T^2}{r^3}$ ($\times 10^{-19}$ s^2 m^{-3})
Mercury	5.79	0.761	2.98
Venus	10.8	1.94	2.99
Earth	15.0	3.16	2.96
Mars	22.8	5.93	2.97
Jupiter	77.8	37.4	2.97
Saturn	143	93.1	2.96
Uranus	287	265	2.97
Neptune	450	521	2.98

WORKED EXAMPLE 5.1A

Triton and Nereid are the two moons of Neptune. Triton is 353 000 km from the surface of Neptune (r_t) and has a period of 5.87 Earth days (T_t). Nereid is 5 560 000 km from the surface of Neptune (r_n) and its period is 359.9 Earth days (T_n). Determine if these data are consistent with Kepler's third law. Neptune has a radius of 24 750 km (r_N).

SOLUTION

Facts: r_t (above surface) = 353 000 km, r_n (above surface) = 5 560 000 km,
r_N = 24 750 km
T_t = 5.87 days, T_n = 359.9 days

Formula: Kepler's ratio $\frac{{T_t}^2}{{r_t}^3} = \frac{{T_n}^2}{{r_n}^3}$, to be consistent with Kepler's third law

Figure: For Triton: $\frac{T^2}{r^3} = \frac{5.87^2}{(353\,000 + 24\,750)^3}$

$= 6.39 \times 10^{-16}$

(Add on the radius of Neptune to find the distance from the centre of Neptune.)

For Nereid: $\frac{T^2}{r^3} = \frac{359.9^2}{(5\,560\,000 + 24\,750)^3}$

$= 7.346 \times 10^{-16}$

Finish: Values are close, so Kepler's law is confirmed.

WORKED EXAMPLE 5.1B

A satellite is launched into orbit around Earth. Knowing that the natural satellite of Earth (the Moon) has a period of 28 days and an orbiting radius of 3.8×10^8 m, calculate the desired radius for the artificial satellite so that it has a period of 1 day.

SOLUTION

Facts: T_s = 1.0 day, T_M = 28 days, $r_M = 3.8 \times 10^8$ m

Find: r_s

Formula: $\frac{{T_M}^2}{{r_M}^3} = \frac{{T_s}^2}{{r_s}^3}$ to be consistent with Kepler's third law

Figure: ${r_s}^3 = \frac{{r_M}^3 \times {T_s}^2}{{T_M}^2}$

$= \frac{(3.8 \times 10^8)^3 \times 1.0^2}{28^2}$

$= 7.0 \times 10^{22}$

$r_s = \sqrt[3]{37.0 \times 10^{22}}$

$= 4.1 \times 10\ 7$ m (41 000 km)

Finish: The orbital radius for the artificial satellite should be 41 000 km.

CHALLENGE 5.1C

A galactic year

The period of our solar system is called one galactic year. That is the time it takes for our solar system to return to the same place in the Milky Way. How long is a galactic year (in Earth years): 1 million, 230 million, 500 million or 5 billion?

Kepler’s and Newton’s third laws combined

Kepler’s third law can be derived from Newton’s universal gravitation law. We also need to incorporate Newton’s law of centripetal motion, which was developed for circular motion. Although planets have elliptical orbits about the Sun, we can assume that they are close enough to being circular paths. There are, however, some motions that are circular:

- The natural satellite of Earth (the Moon) has a circular path.
- Most artificial Earth-orbiting satellites have circular paths, thus keeping their speed constant.
- Earth rotates on its own axis, so a point on its surface travels in a circular path too.

The development of Kepler’s third law starts by equating Newton’s formula for universal gravity with his formula for centripetal motion. This makes sense, as the centripetal force needed to keep a planet in circular motion around the Sun is provided by the gravitational force between the planet and the Sun. The logic is thus:

Study tip

A useful formula is $v = \sqrt{\frac{GM}{r}}$. You can work out the velocity of an orbiting object if you know the mass of the central body (planet or star) and the orbital radius.

$$\begin{aligned}
\text{gravitational force, } F_g &= F_c\text{, centripetal force} \\
\frac{GMm}{r^2} &= \frac{mv^2}{r} \text{ (transfer } m \text{ and } r \text{ to the other side)} \\
\text{(cancel down) } \frac{GMmr}{r^2 m} = \frac{GM}{r} &= v^2 \\
\text{(multiply both sides by } r\text{) } GM &= v^2 r \\
&= \left(\frac{2\pi r}{T}\right)^2 r \text{ (by letting } v = \frac{s}{t} = \frac{2\pi r}{T}\text{)} \\
&= \frac{4\pi^2 r^2 r}{T^2} \text{ (by expanding the above)} \\
\text{Kepler's third law } \frac{T^2}{r^3} &= \frac{4\pi^2}{GM} \text{ (a constant)}
\end{aligned}$$

WORKED EXAMPLE 5.1C

A satellite moves in a circular orbit around Earth at an altitude of 4690 km. The radius of Earth is 6378 km. Determine the satellite’s:

a period

b velocity.

SOLUTION

a Radial distance r = radius of Earth + altitude

$$\begin{aligned}
&= 6378 \text{ km} + 4690 \text{ km} \\
&= 11\,068 \text{ km } (1.107 \times 10^7 \text{ m})
\end{aligned}$$

$$\begin{aligned}
\frac{T^2}{r^3} &= \frac{4\pi^2}{GM} \\
T^2 &= \frac{4\pi^2 r^3}{GM} \\
T &= \sqrt{\frac{4\pi^2 r^3}{GM}} \\
&= \sqrt{\frac{4\pi^2 (1.107 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}} \\
&= 11\,597 \text{ s} \\
&= 3.22 \text{ h}
\end{aligned}$$

b

$$\begin{aligned}
v &= \frac{s}{t} \\
&= \frac{2\pi r}{T} \\
&= \frac{2\pi \times 1.107 \times 10^7}{11\,597} \\
&= 6000 \text{ m s}^{-1}
\end{aligned}$$

CHECK YOUR LEARNING 5.1

Describe and explain

1 **Identify** Kepler's three laws.

2 **Define** 'elliptical' as applied to planetary orbits.

3 **Explain** why the notion of elliptical orbits was rejected for so long.

4 **Explain** the difference between the heliocentric and geocentric models of the solar system.

5 The average radius of the orbit of Uranus is 2.87×10^{12} m. **Calculate** the period of Uranus using the average value for $\frac{T^2}{r^3}$ from Table 1 on page 140. Give the answer in:

a seconds b years.

6 **Clarify** whether all orbits of natural astronomical bodies are elliptical.

7 **Identify** one of the contributions made by astronomer Johannes Kepler to the study of planetary motion. **Clarify** whether his explanation for the motion was correct.

Apply, analyse and interpret

8 Neptune takes 164.8 years to orbit the Sun. **Determine** the average radius of Neptune's orbit using the average value of $\frac{T^2}{r^3}$ from Table 1 on page 140.

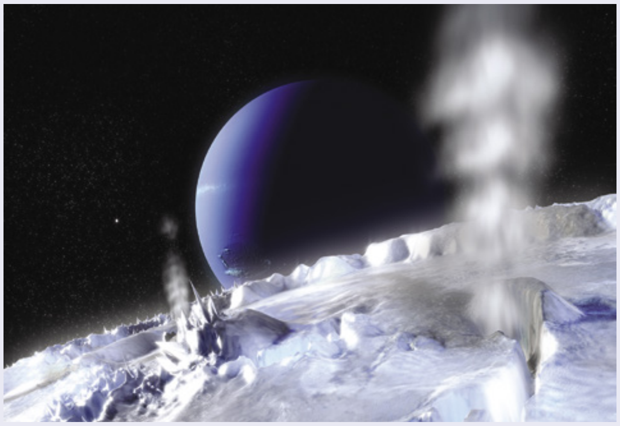

FIGURE 7 The planet Neptune (blue) as seen from Triton, one of its moons, with the moon Nereid in the distant background.

9 It was once thought that the planet Vulcan existed between Mercury and the Sun. **Determine** what the period would have been if it orbited at a mean radius of 40 million km from the Sun.

10 The mean radius of the orbit of planet X in another solar system is 4×10^{12} m and the average value of $\frac{T^2}{r^3}$ for this system is 2.86×10^{-19} s^2 m^{-3}. **Determine** the period of planet Y in the same solar system, given it has an orbital radius of 3×10^{10} m.

11 A satellite moves in a circular orbit around Earth with a speed of 5.0 km s^{-1}.

a **Determine** the satellite's altitude above Earth's surface.

b **Calculate** the period of the satellite's orbit.

Investigate, evaluate and communicate

12 In 1610, Galileo used his telescope to discover the four most prominent moons of Jupiter (the Galilean moons). Their mean orbital radii and periods are given in Table 2.

TABLE 2 Radius and period data

Name	r ($\times 10^8$ m)	T (days)
Io	4.22	1.77
Europa	6.71	3.55
Ganymede	10.70	7.16
Callisto	18.80	16.70

a **Construct** a graph of r^3 (horizontal axis) against T^2 (vertical axis).

b **Comment** on what this graph shows about the relationship between r and T.

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  Check your learning 5.1
- » Suggested practical worksheet
  5.1 Orbital radius and mass
- » Challenge worksheet
  5.1A Trajectory of a marble
- » Video
  Calculating Kepler's ratio

SCIENCE AS A HUMAN ENDEAVOUR

5.2 Gravitational waves

KEY IDEAS

In this section, you will learn about:

- the international collaboration required in the discovery of gravity waves and associated technologies, e.g. Laser Interferometer Gravitational Wave Observatory (LIGO).

FIGURE 1 This is a computer simulation of the collision of two black holes, which created gravitational waves that took one billion years to get to Earth.

Scientists have wondered how gravitation force is transmitted over the vastness of space. Isaac Newton first described the nature of gravitational force, but did not propose what was responsible for it. Eventually, the notion of gravity being some sort of wave took hold.

Einstein's prediction

Gravitational waves were predicted by Einstein about in 1916, as part of his theory of general relativity. The logic was that oscillating electric charges produced electromagnetic radiation, so an oscillating (accelerating) mass should produce gravitational waves. He predicted that gravitational waves would travel at the speed of light.

It was clearly a complicated concept and it was well known that detection of the waves would be extremely difficult. It requires an enormous amount of accelerating mass to produce even a small wave. More recently, scientists thought that the collision of two black holes might produce waves big enough to detect.

FIGURE 2 The LIGO detector site in Livingston, Louisiana, USA

When were they discovered? More than 1000 scientists across many countries, including Australia, were involved in the search for the gravitational waves. Success came to the Laser Interferometer Gravitational Wave Observatory (LIGO) project in the USA on 15 September 2015, and confirmed the following year. Since then, international collaboration has seen these waves detected many times.

How are they detected? As a gravitational wave passes an observer, objects are stretched and shrunk by the wave. It is a matter of measuring the object to see if it has changed size. If it has, then it could be due to a gravitational wave. If there are two detectors, then both should measure the same changes and background effects can be eliminated. The trouble is the changes are extremely small – a change of 1 part in 10^{22}. The only way to get this resolution is with a laser interferometer.

What were they detected on? The most sensitive interferometer in 2016 was LIGO, which had two detectors: one in Livingston, Louisiana, and one at the Hanford site in Washington about 3030 km away.

Where did the waves come from? The gravitational waves that arrived on 15 September 2015 had been travelling for over a billion years, coming from a pair of black holes of mass greater than 30 times the solar mass.

What were the signals like? The most interesting plots are those shown in Figure 3. Together they really confirm the existence of the gravitational wave.

Why do we want to detect them? Detecting and analysing the information carried by gravitational waves allows scientists to observe the universe in a way never before possible. In the first 380 000 years after the Big Bang the universe was so hot that it did not allow electromagnetic radiation to pass through it – but gravitational waves could – so there are now parts of the universe that can only be seen with gravitational wave detectors, and they may tell the story of those first 380 000 years.

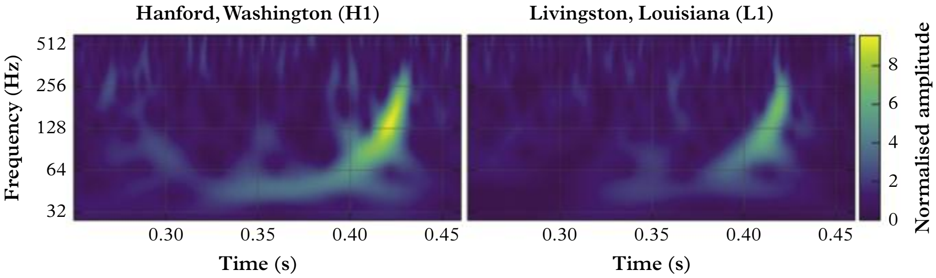

FIGURE 3 The gravitational waves received at the LIGO Hanford and Livingston detectors. The two plots show the frequency (in Hz) sweeping sharply upwards (in green), from 35 Hz to about 150 Hz over 0.2 s. The wave arrived first at Livingston and then at Hanford about 0.007 s later – consistent with the time taken for light, or gravitational waves, to travel between the two detectors.

CHECK YOUR LEARNING 5.2

Describe and explain

1 **Describe** the EM wave principle on which the interferometer relies.

2 **Explain** whether Michelson and Morley could have detected gravitational waves on their instrument.

3 **Explain** why the LIGO tubes needed to be 4 km long.

4 A valid conclusion requires correct interpretation of evidence. **Identify** the evidence LIGO scientists used to draw a conclusion.

Apply, analyse and interpret

5 It took a gravitational wave 0.007 s to travel from Livingstone to Hanford. **Analyse** these data to confirm that the wave must have been travelling at the speed of light.

6 **Deduce** the one crucial difference about the ‘aether wind’ theory that could be disproven using a LIGO interferometer.

7 **Deduce** what link Einstein was trying to make between oscillating electric charge and oscillating masses.

Investigate, evaluate and communicate

8 **Propose** a relationship between the patterns in the frequency graphs from the LIGO detectors.

9 **Evaluate** the claim that ‘black holes were not around in Einstein’s time otherwise he would have found gravity waves earlier’.

10 **Propose** a question about the LIGO experiment that requires a correct calculation of quantities using a mathematical formula. Supply an answer.

11 **Evaluate** this claim: ‘Without electromagnetic wave theory, gravitational waves could never have been confirmed experimentally.’

12 **Evaluate** this claim: ‘It was the limitations of the detecting system that made EM waves and gravitational waves so hard to detect.’

13 Michelson and Morley used an interferometer in 1887 to confirm the electromagnetic wave theory, and LIGO used an interferometer in 2015 to confirm the gravitational wave theory. **Propose** a way in which the experiments were similar and one in which were they different.

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» Video
The Research Investigation

» Weblink
Exploring the LIGO

SCIENCE AS A HUMAN ENDEAVOUR

5.3

Artificial satellites

KEY IDEAS

In this section, you will learn about:

- the importance of the position of artificial satellites for weather observations, traffic and military movements.

Isaac Newton proposed firing a cannonball from the top of a very high mountain. He reasoned that if its speed was low, it would simply fall back on Earth, but if its speed was high enough the rate of fall of the cannon ball would be the same rate as that at which Earth fell away, and so go into orbit. We now know that the speed Newton referred to is 7 km s^{-1}; it is called the escape velocity.

The term ‘satellite’ refers to an object in space that orbits or circles around a bigger object. There are natural satellites such as the Moon, or artificial ones such as communications satellites.

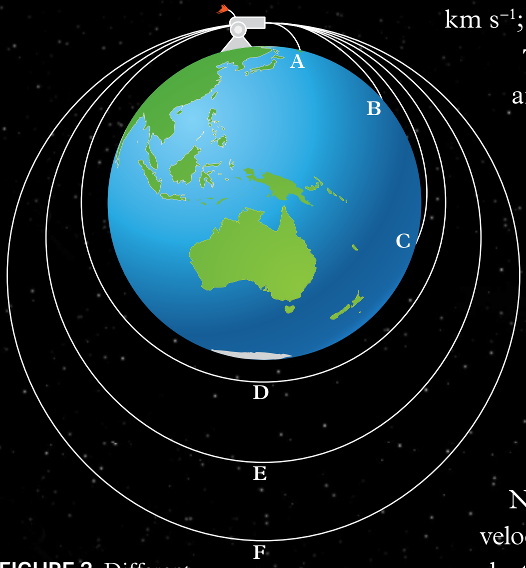

FIGURE 2 Different motions of a cannonball: A, B, C fall to the ground; D is a circular orbit, E and F are ellipses.

Satellite forces

Objects in orbit are undergoing circular motion and have to obey the relationship $F_c = \frac{mv^2}{r}$, where F_c is the centripetal force provided by the gravitational force between Earth and the satellite, m is the mass of the satellite, v is its linear speed and r is the radial distance from Earth’s centre.

Earlier in this chapter, the equation $GM = v^2r$ was developed, using Newton’s law of gravity and centripetal force. This equation says that velocity is inversely related to the orbital radius of the satellite. In other words, to get a certain value of v, there is a corresponding value for radius r. Note that the radius of the orbit is from the centre of Earth. The term ‘altitude’ refers to the distance above the surface. Orbital radius = Earth’s radius (6.37×10^3 km) + altitude.

Types of orbits

As shown in Figure 2, the orbits can be either circular or elliptical. The altitudes at which satellites can orbit Earth divide these orbits into three categories: low Earth orbit (LEO), medium Earth orbit (MEO), high Earth orbit (HEO). Satellites can orbit around the equator or the poles, though technically they can orbit Earth on any elliptical or circular path. When a satellite’s orbit matches the rotation of Earth, it is called a geosynchronous orbit. If it’s position over Earth remains fixed, it’s called a geostationary orbit.

The actual satellite orbit that is chosen will depend on factors including its function, and the area it is to serve. This is shown in Table 1.

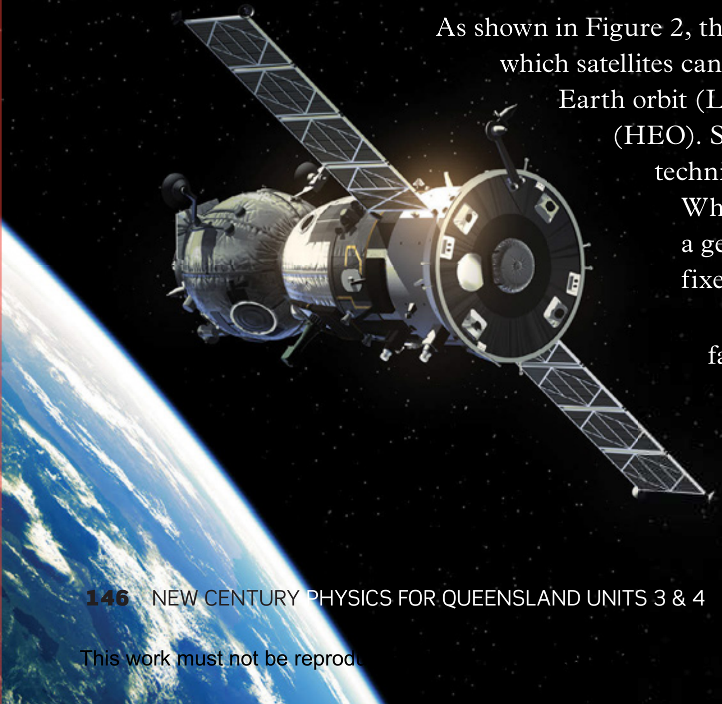

FIGURE 1 The Hubble Space Telescope orbits at 569 km. This orbit is high enough to avoid the distortions of the Earth's atmosphere.

TABLE 1 Satellite orbits and their characteristics

Orbit name	Altitude (km)	Period (h)	Uses	Description
Low Earth orbit, LEO	200–1200	1.5–2	Vast majority of satellites (800): communications, Earth-monitoring, International Space Station (320–400 km), Hubble Space Telescope (560 km), military uses, satellite phone	• Short round-trip time (RTT) for signals • Low solar radiation • Less energy to get it there • Lower transmission energy • Some air drag below 160 km • Risks from 8500 pieces of space debris (junk). • Close, so easier to fix • Passes overhead very quickly
Medium Earth orbit, MEO	1200–35790	2–24	Global positioning satellites (GPS) at 20 000 km, microgravity experiments	• With a 12 h period, it passes same spot twice a day • Higher transmission power than LEO • Has RTT of 100 ms • More solar radiation, so lower life
Geosynchronous orbit, GEO	35 790	24	Direct broadcast, communications relay	• Orbits once a day, but not necessarily in the same direction as the rotation of the Earth • Not necessarily stationary
Geostationary orbit, GEO	35 790	24	Radio, TV, weather; cover 42% of Earth's surface; congested with several hundred satellites	• Orbits once a day and moves in the same direction as Earth and therefore appears stationary above the same point on Earth's surface • Can only be above the equator • Longer RTT for signals • Higher transmission energy
High Earth orbit, HEO	Above 35 790	Varies	Astronomical observations	• Can be a high elliptical orbit as well • Useful for astronomical work • High costs, high transmission energy, long RTT

CHECK YOUR LEARNING 5.3

Describe and explain

1 **Recall** the two types of orbits.

2 **Explain** the relationship between velocity and radius.

Apply, analyse and interpret

3 **Deduce** why military satellites use a low Earth orbit.

Investigate, evaluate and communicate

4 **Propose** an argument for which type of orbit is best suited for transmitting live TV interviews.

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- » Weblink
  The Hubble Space Telescope
- » Weblink
  Satellite forces

CHAPTER 5 Review

Summary

5.1

- Kepler proposed three laws of planetary motion:
  1. The first law (law of orbits) states that all planets move about the Sun in elliptical orbits, having the Sun as one of the foci.
  2. The second law (law of areas) states that a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.
  3. The third law (law of periods) states that the squares of the sidereal periods of the planets are directly proportional to the cubes of their mean distance from the Sun: $T^2 \propto r^3$, or $\frac{T^2}{r^3}$ = constant.
- For an object travelling at uniform speed v in a circle of radius r, the distance travelled during one full circuit $s = 2\pi r$ and the time taken to complete one full circuit is called the period, T: $v = \frac{s}{t} = \frac{2\pi r}{T}$.
- Kepler's third law can be derived from the relationship between Newton's law of universal gravitation and uniform circular motion: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$.
- A satellite is a small body held in orbit around a larger body by gravitational attraction. Satellites can be natural or artificial.

5.2

- Black holes are regions of space with huge mass and intense gravitational fields. When they collide produce gravitational waves.

5.3

- Artificial Earth satellites generally have a circular orbit and are positioned at specific altitudes depending on their application. Their period, speed and altitude are related by Newton's law of universal gravitation and uniform circular motion.

Key terms

- first law of planetary motion (law of orbits)
- second law of planetary motion (law of areas)
- sidereal period
- synodic period
- third law of planetary motion (law of periods)

Key formulas

Kepler's third law (law of periods)	$\frac{T^2}{r^3}$ = constant
Newton's derivation of Kepler's third law	$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 Select the statement that best describes one of Kepler's laws of planetary motion.

- **A** Planets move on elliptical orbits with the Sun at one focus.
- **B** The gravitational force between two objects decreases with the distance squared.
- **C** Gravitational field strength on the surface of inner planets is less than on the surface of outer planets.
- **D** Inner planets orbit in the opposite direction to outer ones.

2 Suppose a planet has an elliptical orbit. The speed of the planet is 15 km s^{-1} when it is at its average distance from its star. Select one of the following that is most likely to be the planet's speed when it is nearest the star.

- **A** 5 km s^{-1}
- **B** 10 km s^{-1}
- **C** 15 km s^{-1}
- **D** 20 km s^{-1}

3 A comet from the outer part of the solar system has a very long orbit. Select one of the following that is correct about the orbit according to Kepler's second law.

- **A** As the comet approaches the Sun, it speeds up.
- **B** As the comet approaches the Sun, it slows down.
- **C** The speed of the comet is constant.
- **D** The comet is moving in a circle.

4 The mean distance of Mars from the Sun is 1.524 times the distance of Earth from the Sun. Determine the period of revolution of Mars around Sun.

- **A** 1.23 year
- **B** 1.32 year
- **C** 1.88 year
- **D** 3.53 years

5 Jupiter's period of revolution around the Sun is 12 times that of Earth's. Determine how many times further the Sun is from Jupiter than Earth.

- **A** 2.29
- **B** 3.46
- **C** 5.24
- **D** 41.57

6 Newton's laws of motion and law of gravity can be used to find the mass of Earth by using a relationship between period (T) and average distance (r) of the Moon from Earth's centre, and mass (M) of Earth. Determine the theoretical mass of Earth M, using $T = 27.4$ days, $r = 385\,000$ km.

- **A** 5.8×10^{24} kg
- **B** 5.9×10^{24} kg
- **C** 6.0×10^{24} kg
- **D** 6.1×10^{24} kg

7 Uranus's orbital radius around the Sun is 2.87 billion km whereas Saturn's orbital radius is just 1.43 billion km. Saturn's orbital period is 29.5 years. Determine how long Uranus takes to go once around the Sun.

- **A** 15 years
- **B** 42 years
- **C** 84 years
- **D** 107 years

8 Each of the following lists two facts. Determine which pair can be used with Newton's version of Kepler's third law to determine the mass of the Sun. Note: AU = 1 astronomical unit, or the distance from Earth to the Sun.

- **A** Mercury is 0.387 AU from the Sun and Earth is 1 AU from the Sun.
- **B** The mass of Earth is 6×10^{24} kg and Earth orbits the Sun in 1 year.
- **C** Earth rotates in 1 day and orbits the Sun in 1 year.
- **D** Earth is 150 million km from the Sun and orbits the Sun in 1 year.

9 A satellite of mass m is in a circular orbit above Earth (mass m_E) at a distance h above the surface where $h = r$ (the Earth's radius). Select the expression that best states the velocity the satellite must have in order to maintain its orbit.

- **A** $v = \sqrt{\frac{Gm_E}{r}}$
- **B** $v = \frac{Gm_E}{2r}$
- **C** $v = \sqrt{\frac{Gm_E m}{2r}}$
- **D** $v = \sqrt{\frac{Gm_E}{2r}}$

10 Imagine if the Sun became a black hole of the same mass but one-tenth of its diameter. Select the statement that could be said about Earth's orbit about the Sun.

A Earth would spiral into the Sun.
B Earth's orbit would be smaller.
C Earth's orbit would stay the same.
D Earth's orbit would get larger.

Short answer

Describe and explain

★ **11** **Describe** an elliptical orbit and explain the terms associated with it.

★ **12** **Recall** Kepler's three laws and **describe** their main features using diagrams.

Apply, analyse and interpret

★★ **13** It was once thought that the planet Vulcan existed between Mercury and the Sun. **Determine** the mean orbital radius of Vulcan from the Sun if its period was estimated to be 50 days.

★★ **14** A satellite is in a geostationary orbit about Earth at a distance of 42 200 km above the surface.

a **Determine** the height above Earth's surface of a satellite whose circular orbit takes 90 minutes.

b **Determine** how many times a day this satellite would orbit Earth.

★★ **15** Earth satellites in low orbit have orbital periods typically in the range of 90 to 105 minutes. **Determine** the range of heights above the surface to which this corresponds.

★★ **16** The period of the Moon is 27.3 days and its orbital radius is 3.82×10^8 m. **Determine** the period of a satellite 6.90×10^6 m from Earth's centre.

★★ **17** **Determine** how long it takes a satellite to orbit Earth if its average orbital radius is 18 000 km.

★★ **18** A satellite already in orbit around a planet is put into a new orbit whose radius is four times as large as the old radius. **Determine** how many times longer the new period is than the old.

★★ **19** **Determine** the average distance of a satellite from the centre of Earth if its orbital period is 15 days.

★★ **20** Io, one of the moons of Jupiter, orbits Jupiter every 1.8 Earth days at a distance of 4.2 units from its centre. Ganymede, another of Jupiter's moons, is 10.7 units from Jupiter's centre. **Determine** the period of Ganymede.

★★★ **21** **Determine** the time required for Venus to complete an orbit around the Sun (using Kepler's law of periods), given that the radius of the orbit is 108.2 million km, the mass of Venus is 4.9×10^{24} kg, and the mass of the Sun is 2.0×10^{30} kg.

★★★ **22** Satellite X orbits a planet $v = 12\,000$ m s^{-1}. Satellite Y is three times more massive than satellite X but orbits at a distance only half the distance of X from the centre of the planet. **Determine** the speed of satellite Y.

★★★ **23** Suppose there is a small planet orbiting our Sun at a distance 14 times the Sun–Earth distance of 1.5×10^{11} m. **Determine** the orbital period of such a planet, given Kepler's ratio is 2.97×10^{-19} s^2 m^{-3}.

★★★ **24** A satellite moves in a circular orbit around Earth at a speed of 6100 m s^{-1}. **Determine**:

a the altitude of the satellite above the surface of Earth

b the period of the satellite's orbit.

★★ **25** In the early days of artificial Earth satellites, one particular satellite began with an elliptical orbit but slowly lost energy due to the effects of atmospheric friction and crashed into the sea. **Calculate** the missing values to complete the data in Table 1 and then answer the questions.

TABLE 1

Data	January 1968	March 1968	May 1968
Orbital period (min)	96.2	95.4	91.0
Min. height (km)	219	216	196
Max. height (km)	941	866	463
Mean height (km)			
Mean radius (km)			
Kepler's ratio $\frac{T^2}{r^3}$			

a **Determine** whether the orbit became less elliptical as time passed and justify your answer.

b **Calculate** approximately how many orbits of Earth the satellite made in the 90 days it was in space.

Investigate, evaluate and communicate

★★**26** The orbital distance of Mars from the Sun is 1.52 times the orbital distance of Earth. **Predict** how many days it takes Mars to orbit the Sun.

★★**27** Military reconnaissance satellites have a period of 90 minutes. Civilian Earth observation satellites have a period of 100 minutes. **Propose** a reason for this difference.

★★★**28** Jupiter's four most significant moons are called the Galilean moons because they were discovered by Galileo in 1610. Their names derive from the lovers of Zeus. The orbital data for these moons is listed in Table 2.

TABLE 2

Moon	Period (s)	Radius (m)	Kepler's ratio $\frac{T^2}{r^3}$
Io	1.5×10^5	4.2×10^8	
Europa	3.1×10^5	6.7×10^8	
Ganymede	6.2×10^5	10.7×10^8	
Callisto	14.4×10^5	18.8×10^8	
		Average:	

a **Solve** the column labelled Kepler's ratio.

b **Determine** the absolute uncertainty in Kepler's ratio.

c **Construct** a graph of r^3 (horizontal axis) versus T^2 (vertical axis) and determine the gradient.

d **Discuss** the relationship between the gradient of the graph and the average of Kepler's ratio.

e **Propose** which one of Kepler's laws the data supports.

f **Determine** how the average for Kepler's ratio compares with Newton's derivation of the same quantity using the formula $\frac{T^2}{r^3} = \frac{4\pi}{GM}$ where M, the mass of Jupiter, is 1.9×10^{27} kg.

★★★ **29** Another new planet outside our solar system was proposed earlier this century. It was thought to lie within the Oort Cloud and had an orbital radius of 0.5 ly (where 1 light-year (ly) = 9.47×10^{15} m). It has a mass 1.5 to 6 times of Jupiter and a period of 6 million years. **Evaluate** these data by analysing whether the $\frac{T^2}{r^3}$ ratio is the same as that of our solar system.

★★★ **30** **Investigate** the following:

a 'Planet' in Greek means 'wanderer'. **Explain** how a planet wanders.

b 'Galaxy' comes from the Greek *galas* meaning 'milk'. **Explain** what our galaxy has got to do with milk.

c Einstein made predictions about gravity and pulsars but this has more recently been applied to black holes.

- **i** **Explain** the difference between a pulsar and a black hole.
- **ii** **Identify** a pulsar and a black hole and **determine** their distance from Earth.

d Pluto has been declassified as a planet, but it does have some properties similar to planets.

- **i** **Determine** T and r values for Pluto.
- **ii** **Determine** its $\frac{T^2}{r^3}$ value.
- **iii** **Compare** the value for Pluto with those of the planets in Table 1 on page 140.
- **iv** **Evaluate** the data to show whether it conforms to Kepler's third law.

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CHAPTER 6

Electrostatics

Electric charge is very important in nature. Electric forces and charges control many natural effects, and are seen in dramatic circumstances such as lightning strikes and the magnificent southern lights display of the aurora australis.

Much of our modern technology relies on controlling electric charges, either trying to eliminate their effects or making use of their attracting or repelling properties. A knowledge of charge will help us to understand the operation of application devices such as electrostatic generators, photocopiers, lightning arrestors and even the various forms of biological electrostatic defences possessed by animals such as electric eels and rays.

Electric forces are responsible for most of the interactions in chemistry between atoms and molecules either holding them together to form liquids and solids, or in reactions to make new substances.

Electric forces are also involved in the metabolic processes that occur within our cells. They have an important role to play in large biological molecules such as proteins and nucleic acids. These molecules – so important to life – are usually electrically charged.

Physicists regard the force of electricity as a fundamental force of nature that is ultimately responsible for other forces such as friction, contact pushes, adhesion and cohesion.

OBJECTIVES

- → Define Coulomb's law and recognise that it describes the force exerted by electrostatically charged objects on other electrostatically charged objects.
- → Solve problems involving Coulomb's law.
- → Define the terms 'electric fields', 'electric field strength' and 'electrical potential energy'.
- → Solve problems involving electric field strength.
- → Solve problems involving the work done when an electric charge is moved in an electric field.

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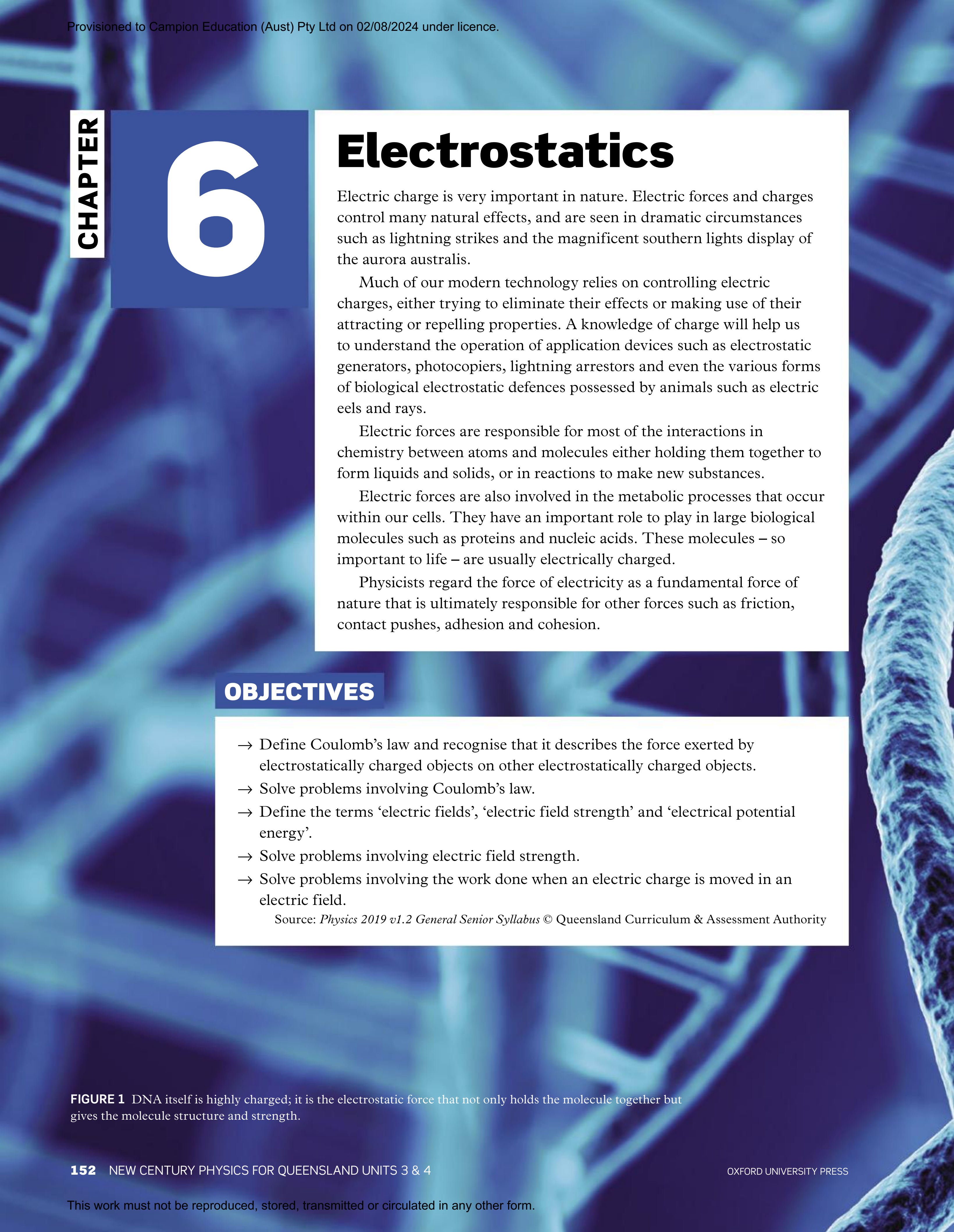

FIGURE 1 DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength.

MAKES YOU WONDER

In this chapter you will be examining some aspects of electric charges that will help to answer questions such as these:

- → Can you only have positive and negative charge? What if a third type of charge is discovered – what would it be named?
- → Air is a non-conductor, so why can it conduct electricity in lightning?
- → Coulomb's law and Newton's law of gravitation are both inverse square laws. Is this some universal law of nature?
- → How can protons sit side-by-side in the nucleus? Shouldn't the electrostatic repulsion make them fly apart?

PRACTICALS

SUGGESTED PRACTICAL	**6.1** Effects of electrostatic charge on various materials

6.1

Coulomb's law

KEY IDEAS

In this section, you will learn about:

- Coulomb's law
- solving Coulomb's law problems in one and two dimensions.

In Unit 1, you saw that when electrically charged objects are brought into close proximity, there is a force between them that is either attractive or repulsive, depending on the nature of the charge (Figure 1). This had been known for hundreds of years, but the mathematical relationship was not proven until 1775 by Henry Cavendish, an English scientist. In 1885, French military engineer and physicist Charles-Augustin de Coulomb used a very sensitive electrostatic torsion bar balance system to finally determine the nature of this force experimentally (Figure 2). From careful measurements made on the quantity of charge, the distances between charges and the forces acting on the charges, Coulomb was able to show that the:

- magnitude of the force was proportional to the product of the charges
- magnitude of the force was inversely proportional to the square of the distance separating the charges
- direction of the force was along a line joining the centres of the charges
- magnitude of the force was dependent on the medium in which the charges were placed.

These points can be summarised mathematically as:

$$F \propto \frac{Qq}{r^2} = k\frac{Qq}{r^2}$$

$$\text{where } k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

where Q and q are electric charges (C), r is the distance between their centres (m) and ε_0 is the permittivity of free space (8.85×10^{-12} C V^{-1} m^{-1}).

This last equation is referred to as **Coulomb's law** and you should note that the force involved is a vector quantity.

Coulomb's law states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them, expressed by the formula $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$

FIGURE 1 A negatively charged rod deflects a stream of water.

FIGURE 2 A Coulomb torsion balance. Two charged spheres are visible on the right.

FIGURE 3 A coulomb meter can measure positive or negative charge up to 1.999 nC.

SUGGESTED PRACTICAL
6.1 Effects of electrostatic charge on various materials
Go to your obook assess »

Newton's third law

A pair of charged objects makes a good example of Newton's third law in action. In Figure 4, the force on A due to B (F_{AB}) is equal and opposite to the force on B due to A (F_{BA}). The forces are shown as vectors of equal length and facing in opposite directions. These vectors can also be shown using algebraic notation:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

The forces meet the conditions for Newton's third law. The forces are:

- equal in magnitude (vectors are equal length)
- opposite in direction (vectors point in opposite directions)
- the same type of force (both electrostatic)
- acting on different objects (not both acting on a third object).

Coulomb's law constant is equivalent to $\frac{1}{4\pi\varepsilon_0}$, and has the value of 9×10^9 N m^2 C^{-2} for electric charges in a vacuum. It is often given the symbol k. The symbol ε_0 (epsilon zero) is called the permittivity of free space (vacuum) and has a magnitude of 8.85×10^{-12} C V^{-1} m^{-1}. For other media, such as plastic, the value is twice this. Air is almost the same as a vacuum, only differing in the fourth decimal place. You are not expected to use the permittivity value separately from the overall constant.

Coulomb's law constant, k
a constant of proportionality relating the force between charged objects to the magnitude of their charge and separation distance;
$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

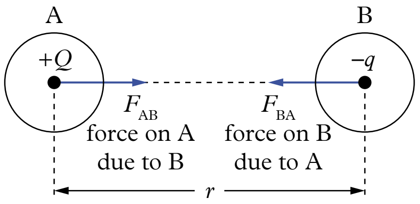

FIGURE 4 A system of two point charges is depicted in this manner. Opposite charges attract.

Several of the ideas developed in Unit 1 on electrical circuits are needed again. These include:

- charges can be positive or negative; unlike charges attract and like repel
- negative charge is an excess of electrons, and positive charge is a deficiency of electrons
- electric charge has the symbol q and the unit coulomb, symbol C
- the elementary charge q_e is 1.60×10^{-19} C
- conventional current is the flow of positive charge
- potential difference is measured in volts, symbol V.

Study tip

For two charges in isolation, the direction of the force will be repulsive if the charges are the same sign and attractive if the charges are opposite in sign (Figure4).

CHALLENGE 6.1A

Which is bigger?

Which is the bigger charge of each pair?

a 3.5×10^{-6} C or 7.5×10^{-6} C
b 4×10^{-6} C or 7×10^{-5} C
c 1 mC or 1 μC

CHALLENGE 6.1B

A charged rod

When a rod becomes positively charged does its mass increase or decrease? What if it became negatively charged?

Solving problems

Forces in a line

The simplest problems involving Coulomb's law are for objects in a straight line. This can be two charged objects such as two protons in a nucleus, or three or more charged objects in a straight line. The vector sum for linear arrangements is much simpler than for two-dimensional set-ups.

WORKED EXAMPLE 6.1A

Consider the system of two charges as shown in Figure 5. If $Q = +10\ \mu C$ and $q = -5\ \mu C$, and the separation distance is 30 cm, calculate the force between them.

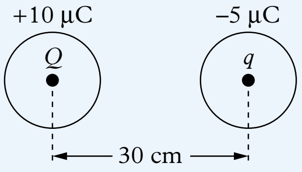

FIGURE 5 Forces between point charges

SOLUTION

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

$$= 9 \times 10^9 \times \frac{Qq}{r^2}\ \text{N m}^2\ \text{C}^{-2}$$

$$= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 5 \times 10^{-6}}{0.30^2}\ \text{N m}^2\ \text{C}^{-2} \text{(the + and – signs on the charges are omitted)}$$

$$= 5\ \text{N}$$

The charges are unlike, so $F = 5$ N (attraction).

The vector nature of this law is also very important if a system of more than two charges is considered. In this case it is necessary to determine the resultant electrostatic force in both magnitude and direction using vector addition techniques. In problems dealing with Coulomb's law, it is often convenient to consider the charges as point charges. This is possible whenever the separation distance of the charges is very large compared with the size of the charges themselves.

Labelling forces

Labelling the forces can be tricky, but it is good to be consistent. Consider a pair of charged objects A and B (Figure 6). Each experiences a force due to the other. The force on A due to B is labelled as F_{AB}. It represents $F_{\text{on A due to B}}$. Using this order for the subscripts helps you keep track of the various forces and what objects are being considered. It is important to recognise that both objects are responsible for the force (agent and receiver) and the magnitudes are equal.

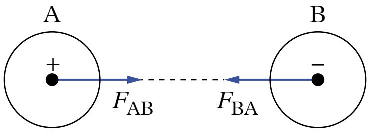

FIGURE 6 Labelling the forces

WORKED EXAMPLE 6.1B

A system of three charges in a line consists of object A with Q_A= +10 μC, Q_B = −10 μC, and Q_C = +10 μC, separated by distances to their centres as shown in Figure 7. Distances are measured to 2 significant figures.

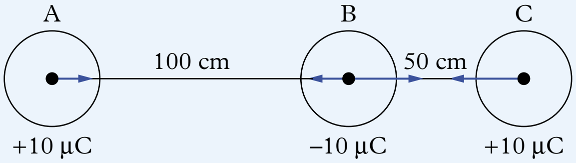

FIGURE 7 Multiple charges in one dimension

Calculate the net force on B due to A and C.

SOLUTION

Step 1: Calculate the force on B due to A (F_{BA}).

$$F_{BA} = 9 \times 10^9 \times \frac{Q_A Q_B}{r^2} \text{ N m}^2 \text{ C}^{-2}$$
$$= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{1.0^2} \text{ N m}^2 \text{ C}^{-2}$$
$$= 0.90 \text{ N attraction (to the left)}$$

Step 2: Calculate the force on B due to C.

$$F_{BC} = 9 \times 10^9 \times \frac{Q_B Q_C}{r^2} \text{ N m}^2 \text{ C}^{-2}$$
$$= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{0.50^2} \text{ N m}^2 \text{ C}^{-2}$$
$$= 3.6 \text{ N attraction (to the right)}$$

Step 3: Calculate the net force by adding vectors.

$$F_{net} = 0.90 \text{ N} \leftarrow +3.6 \text{ N} \rightarrow$$
$$= 2.7 \text{ N} \rightarrow$$

Answer: F_{net} = 2.7 N to the right

Forces in two-dimensions

For three or more objects not in a line, the vector sum becomes more difficult.

Study tip

You will be examined on a range of scenarios involving Coulomb's law and this means from a simple pair of charged objects up to the more difficult system of three objects not in a line. Use of the sine and cosine rules can make solutions less arduous. If all else fails, you can try a graphical solution using ruler and protractor, but be warned that the question may ask for an algebraic solution.

WORKED EXAMPLE 6.1C

Consider a system of three charges $Q_A = +5\ \mu C$, $Q_B = +5\ \mu C$ and $Q_C = -5\ \mu C$, and separation distances as shown in Figure 8. The lines between the charges make a right-angled triangle with interior angles of 45°. Calculate the force acting on charge object Q_C.

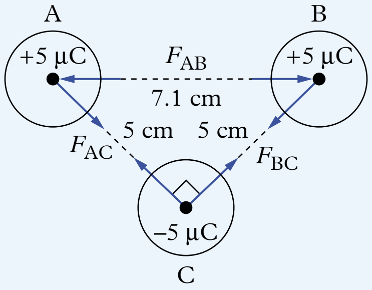

FIGURE 8 Charges in two dimensions

SOLUTION

You need to calculate the force acting on Q_C by Q_A and Q_B.

Step 1: Calculate the force on C due to A (written as F_{CA}).

$$F_{CA} = 9 \times 10^9 \frac{Q_A Q_C}{r^2}\ \text{N m}^2\ \text{C}^{-2}$$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.05^2}\ \text{N m}^2\ \text{C}^{-2}$$

$$= 90\ \text{N attraction}$$

Step 2: Calculate the force on C due to B (written as F_{CB}).

$$F_{CB} = 9 \times 10^9 \times \frac{Q_B Q_C}{r^2}\ \text{N m}^2\ \text{C}^{-2}$$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.05^2}\ \text{N m}^2\ \text{C}^{-2}$$

$$= 90\ \text{N attraction}$$

Note: as it is an isosceles triangle (two equal angles), CA = CB.

Step 3: Determine the net force by adding the vectors head-to-tail (Figure 9).

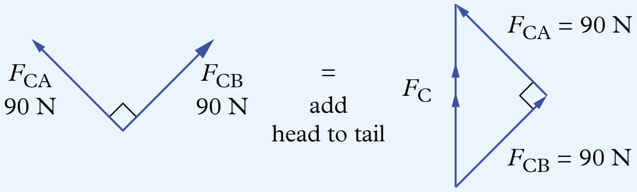

FIGURE 9 Combining vectors to produce a resultant

Answer: $F_C = \sqrt{90^2 + 90^2}$

$= 127\ \text{N}$

Direction is vertically up the page.

WORKED EXAMPLE 6.1D

For the system of charges shown in Worked example 6.1C on the previous page, calculate the net force on object B.

SOLUTION

The net force on B is the vector sum of the forces on B due to the presence of A and the presence of C.

$\vec{F}_{\mathrm{B}}\ (\text{or}\ \vec{F}_{\mathrm{net}}) = \vec{F}_{\mathrm{BC}} + \vec{F}_{\mathrm{BA}}$ (vector sum)

You know that $F_{\mathrm{CB}} = 90$ N, from previous worked example.

So $F_{\mathrm{BC}} = 90$ N but towards C.

$F_{\mathrm{BC}} = -90$ N

$$F_{\mathrm{BA}} = 9 \times 10^9 \times \frac{Q_{\mathrm{B}} Q_{\mathrm{A}}}{r^2}\ \mathrm{N\ m^2\ C^{-2}}$$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 5 \times 10^{-6}}{0.071^2}\ \mathrm{N\ m^2\ C^{-2}}$$

$$= 44.6\ \mathrm{N\ repulsion}$$

Now draw the vector diagram and calculate the resultant net force.

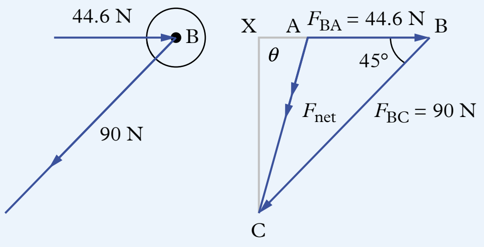

FIGURE 10 Combining vectors to produce a resultant

Here is one way to solve it (only because the angle is 45°).

$$(XB)^2 = \frac{1}{2} \times 90^2$$

$$XB = 63.6\ \mathrm{N}$$

$$XC = 63.6\ \mathrm{N}\ (45°\ \text{triangle})$$

$$XA = 63.6 - 44.6$$

$$= 19.0\ \mathrm{N}$$

$$AC = \sqrt{63.6^2 + 19.0^2}$$

$$= 66.4\ \mathrm{N}\ (\text{Pythagoras' theorem})$$

Therefore $F_{\mathrm{net}} = 66.4$ N

$$\theta = \tan^{-1}\frac{63.6}{19.0}$$

$$= 74.0°$$

Alternative solution is to use the cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$$F_{\mathrm{AC}}{}^2 = F_{\mathrm{AB}} + F_{\mathrm{BC}}{}^2 - \left(2 \times F_{\mathrm{AB}} \times F_{\mathrm{BC}} \times \cos 45°\right)$$

$$F_{\mathrm{net}}{}^2 = (44.6)^2 + (90)^2 - (2 \times 44.6 \times 90 \times 0.707)$$

$$= 4412$$

$$F_{\mathrm{net}} = 66.4\ \mathrm{N}$$

Study tip

Most problems involving forces in two-dimensions may also be solved by dividing the diagram up into smaller right-angled triangles and using simpler Pythagorean calculations.

WORKED EXAMPLE 6.1E

Consider the system of charges in Figure 11. Deduce the magnitude and direction of the force on charge C. Each charge is $+4.0 \times 10^{-8}$ C and they are arranged in an equilateral triangle of side 20 cm (2.0×10^1).

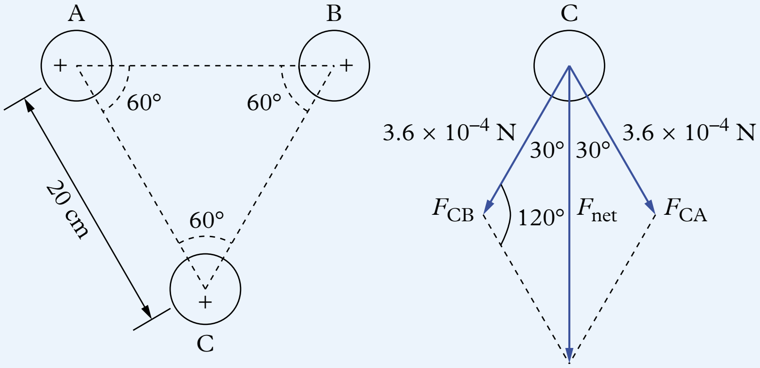

FIGURE 11 Charges in equilateral triangle

SOLUTION

The net force on charge C (F_C or F_{net}) will be the vector resultant of both forces labelled F_{CA} and F_{CB} as shown, which are equal to each other in magnitude.

Step 1: Calculate the magnitude of each force vector.

$$F_{CA} = 9 \times 10^9 \times \frac{Q_A Q_C}{r^2} \text{N m}^2 \text{C}^{-2}$$

$$= 9 \times 10^9 \times \frac{4 \times 10^{-8} \times 4 \times 10^{-8}}{0.2^2} \text{ N m}^2 \text{ C}^{-2}$$

$F_{CA} = 3.6 \times 10^{-4}$ N repulsion

$F_{CB} = 3.6 \times 10^{-4}$ N repulsion

Step 2: Force (F_{net}) may now be calculated using the cosine rule for triangles.

$$F^2 = F_{CA}{}^2 + F_{CB}{}^2 - \left(2 \times F_{CA} \times F_{CB} \times \cos 120°\right)$$

$$F_{net}{}^2 = (3.6 \times 10^{-4})^2 + (3.6 \times 10^{-4})^2 - (2 \times 3.6 \times 10^{-4} \times 3.6 \times 10^{-4} \times -0.5)$$

$$= 3.89 \times 10^{-7}$$

$$F_{net} = 6.2 \times 10^{-4} \text{ N}$$

The force is repulsive and will be directed vertically down the page or at an angle of 30° to the line of the force F_{CA}.

CHALLENGE 6.1C

Increasing magnitude?

Arrange these electric charges in order of increasing magnitude:

a 4.3×10^{-6} C, 6 µC, 0.000 005 C

b 500 µC, 400 nC, 4.5×10^{-6} C

c 0.001 C, 100 µC, 1.5 mC

d +10 µC, –15 µC, 12.5 µC

CHECK YOUR LEARNING 6.1

Describe and explain

1 **Define** 'Coulomb's law' and state the numerical value of the constant to 1 significant figure, including units.

2 **Describe** how you can predict whether two charged objects have like charges or opposite charges.

3 **Calculate** the size of the Coulomb repulsion force between two protons in a helium nucleus, given that they are separated by a distance of 1×10^{-14} m. Recall that the magnitude of the charge on a proton is the elementary charge.

4 **Clarify** what is meant by an 'elementary charge'.

Apply, analyse and interpret

5 Two point charges A and B each with a charge $+Q$ coulombs are separated by a distance of r metres. The force acting between them is 6×10^{-4} N. **Determine:**

- **a** whether the force is attractive or repulsive
- **b** the magnitude of the force if distance r is doubled
- **c** the magnitude of the force if charge Q on both A and B is doubled
- **d** the magnitude of the force if charge Q on both is halved, and so is the distance between them.

6 Consider the system of charges in Figure 12.

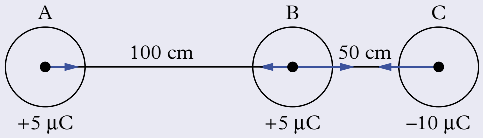

FIGURE 12 Three charges in one dimension

Determine the net force on:

a B **b** C

7 Four point charges A, B, C and D are arranged on corners of a square of side 25 cm (Figure 13). Charges A and B each have a charge of +1 μC, while C and D each have a charge of +2 μC. **Determine** the resultant force on a charge P of +1 μC that is placed at the centre of the square.

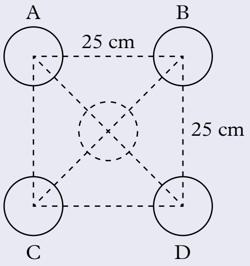

FIGURE 13 Four charges in two dimensions

8 An electron ($m_e = 9.1 \times 10^{-31}$ kg) is in a chamber on Earth's surface. Another electron is placed directly underneath it so that the weight of the first electron is balanced by the electrostatic force between the two. The charge on an electron is -1.60×10^{-19} C. **Determine** the distance between the electrons.

9 Three charges A (+20 μC), B (−20 μC) and C (+10 μC) are arranged as shown in Figure 14. **Determine** the net force on A due to B and C.

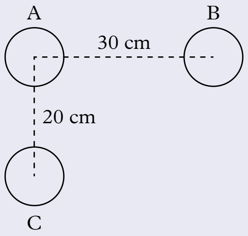

FIGURE 14 Charges in two dimensions

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6.2 Electric fields and field strength

KEY IDEAS

In this section, you will learn about:

- electric fields
- electric field strength, electric potentials and electrical potential energy
- work done when an electric charge is moved in an electric field.

Like other fields you've met in physics, such as gravitational fields, you can visualise an electric field as a region of space in which any electrified object will experience a force. This makes it similar to the concept of a mass experiencing a gravitational force of attraction to another mass, such as Earth.

electric field
a region of space near an electrically charged particle or object within which a force would be exerted on other electrically charged particles or objects

An **electric field** exists in space around an electrically charged particle or object within which a force would be exerted on other electrically charged particles or objects.

CHALLENGE 6.2A

True or false?

Consider whether the following statement is true or false: 'We are usually not aware of the electric force acting between two everyday objects because there are as many positive charges in a substance as negative charges.'

Electric field lines

Michael Faraday introduced the idea of electric field lines in 1845, although he called them lines of force. They were a graphical way of depicting the field around a charged body, and at the time were believed to be real things. Of course, they are not real but they are a good way of visualising patterns about charges. It is worth remembering that electric fields are in three dimensions around a charged object but are represented as being in two dimensions on paper. This is called a cross-sectional electric field diagram.

FIGURE 1 Hair strands on a charged head follow the electric field lines radiating outwards.

The electric field diagrams in Figure 2 show the two-dimensional field lines present in several situations in which the influence of more than one charge may be involved. It is important to realise that:

- electric field lines never cross
- electric field lines are directed from positive charge to negative charge
- electric field lines will enter or leave any charged surface at right angles
- the number of lines per unit area represents the electric field strength at that point; that is, if the lines are close together the field strength is high
- electric fields radiate in three-dimensional space but we represent them as ‘cross-sectional’ diagrams in two dimensions on paper or on the screen.

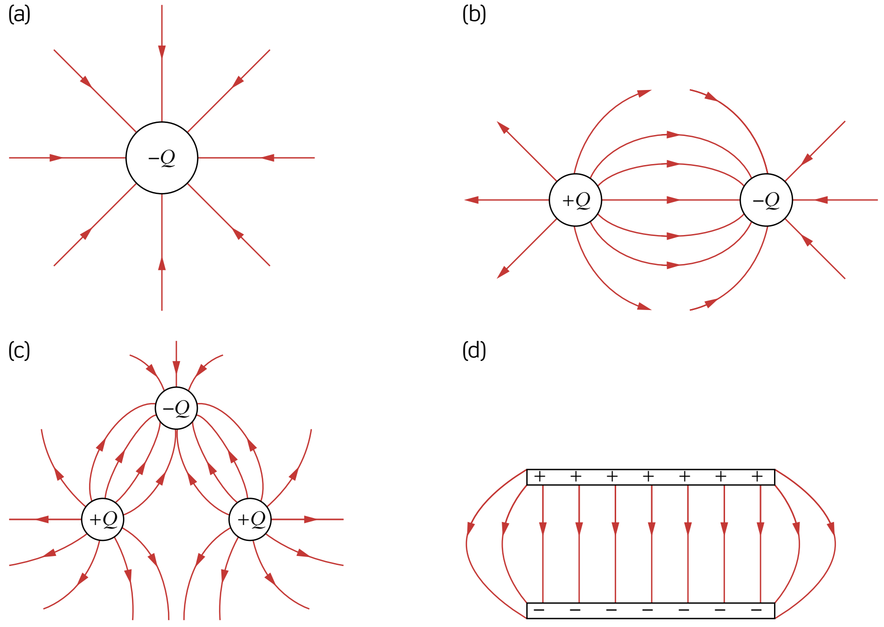

FIGURE 2 Cross-sectional electric field diagrams

CHALLENGE 6.2B

True or false?

Are the following true or false about the field at point A compared to point B (Figure 3)?

a E_A is twice as strong as E_B because it is half as far.

b E_A is weaker than E_B because it is not on a field line.

c E_A is stronger than E_B because the field lines are closer at A.

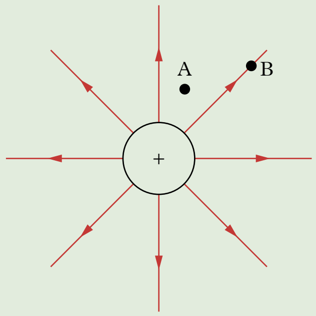

FIGURE 3

Electric field strength

electric field strength
the intensity of an electric field at a particular location (symbol: *E*; SI unit: newton per coulomb; unit symbol: N C^{-1})

We can have strong electric fields, such as a bolt of lightning, or weak electric fields, such as when you rub a plastic comb on a piece of wool. As force is a vector quantity, **electric field strength** is also a vector quantity. It has both magnitude and direction.

The main difference between electric fields and gravitational fields is that gravitational fields have only attractive forces, whereas electric fields can provide both attractive and repulsive forces. The intensity of the electric field – its electric field strength – at any point is defined as the force acting on a test unit charge, q, placed at that point in the field. The direction of the electric field at any point is given by the resultant force direction acting on a positive test charge placed at that point in the field. Thus mathematically, electric field strength is defined by the following equation:

$$\vec{E} = \frac{\vec{F}}{q}$$

where E is the electric field strength in units of N C^{-1} (newton per coulomb), F (N) is the force acting on a charge in the field, and q is the size of the test charge in coulomb (C). Refer to Figure 4.

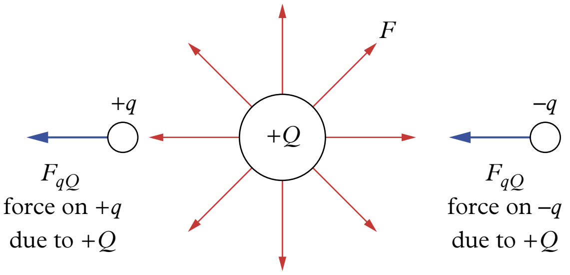

FIGURE 4 An electric field surrounds $+Q$ and test charges ($+q$ and $-q$) experience forces in the field.

WORKED EXAMPLE 6.2A

A short length of nichrome wire is held vertically under an overhead power wire where the field strength is 5000 N C^{-1} downwards. Calculate the force experienced by an electron in the wire, $q_e = -1.60 \times 10^{-19}$ C.

SOLUTION

The question can be simplified to: calculate the force (F) acting on an electron of charge (q) = -1.60×10^{-19} C in a field (E) of 5000 N C^{-1} downwards.

$E = 5000$ N C^{-1}, $q = -1.60 \times 10^{-19}$ C

$$E = \frac{F}{q}$$

$$F = Eq$$

$$= 5000 \times 1.60 \times 10^{-19}$$

$$= 8 \times 10^{-16} \text{ N}$$

The field is directed downwards so the force on a negative charge acts upwards.

Uniform electric fields

Notice that in Figure 5 the electric field between parallel charged plates is very uniform in nature. This **uniform electric field** will produce a constant force on any test charge held between the plates, independent of position. In 1909, the American physicist Robert Millikan used the very uniform electric field between charged parallel metallic plates to investigate the nature of electric charge itself. Millikan was able to balance the electric force on charged oil droplets sprayed between the plates with the droplets' own gravitational weight. By carefully measuring the mass of these oil droplets and changing their electric charge with X-rays, Millikan was able to calculate a value for the charge on an electron. He won the Nobel Prize in Physics in 1923 for his work on the electrostatics of elementary charges.

uniform electric field
a field that has constant field strength, as found between charged parallel plates

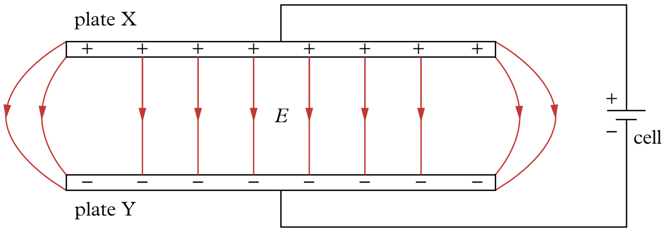

FIGURE 5 A uniform electric field between oppositely charged parallel plates

Examples of field strengths

To give you a feel for the strength of an electric field, some common appliances and situations are listed in Table 1.

TABLE 1 Examples of electric field strengths

Electric field	Electric field strength (N C^{-1})
220 kV transmission lines	500–7800
Electric razor	200–2600
Hair dryer	300–800
Electric eel	600–800
Electric blanket	60–600
Normal day (at ground level)	100 down
Thunderstorm cloud (ground to base of cloud)	50 up
Background (home/school/office)	3–20

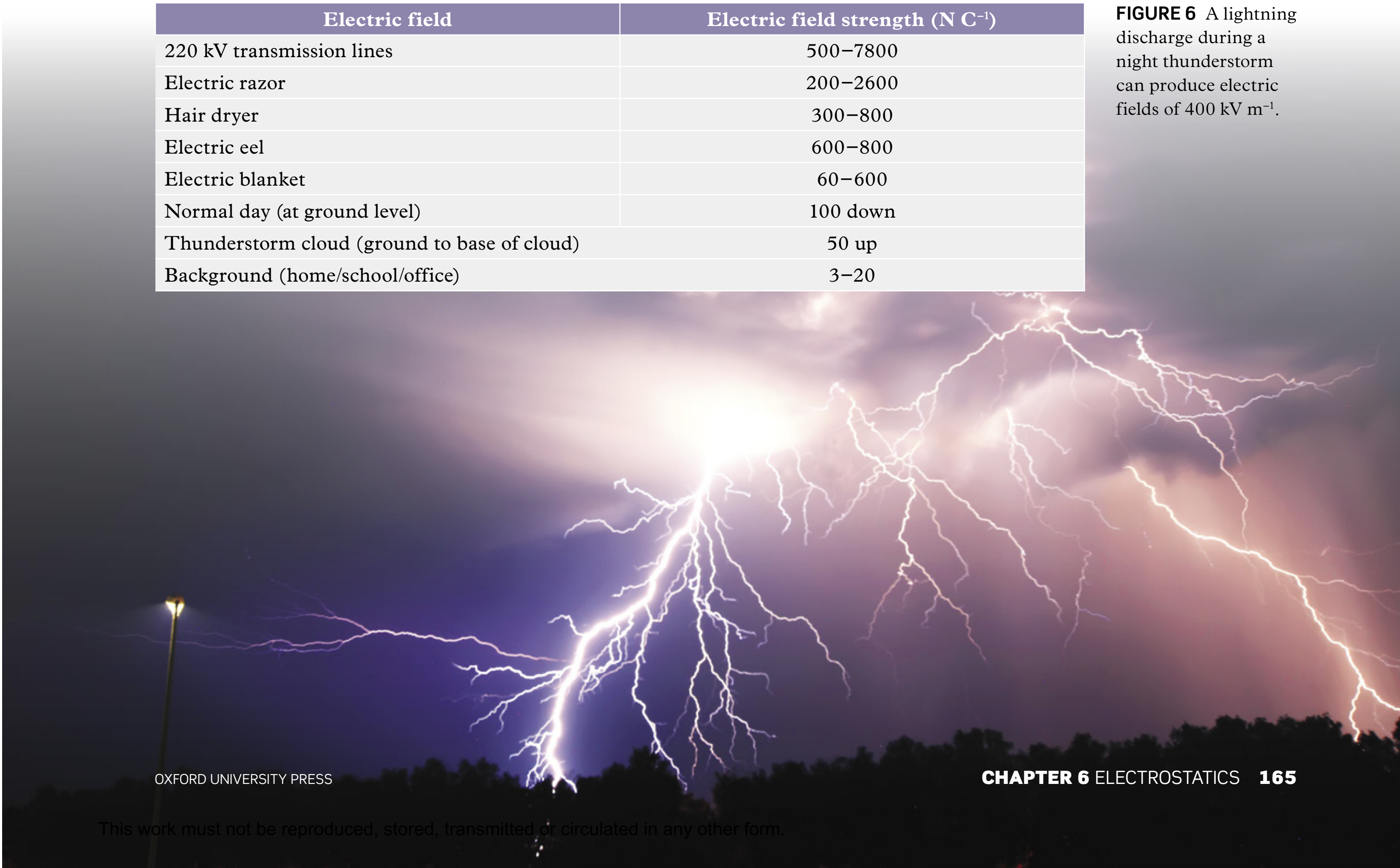

FIGURE 6 A lightning discharge during a night thunderstorm can produce electric fields of 400 kV m^{-1}.

Newton's laws of motion and electric fields

FIGURE 7 Electric eels can generate an electric field of up to 800 N C^{-1} in order to stun prey.

The electric field strength has been defined as the force acting on a charged particle in the field. You saw that if the field strength and the charge on a particle are known, the force acting can be calculated.

Newton's laws of motion describe the effect of forces acting on matter. You have already considered Newton's third law, so now consider the first and second laws in turn to assess how they relate to electrostatic forces.

Newton's first law

A charged particle, Q, is fired at constant speed into a uniform electric field between two charged plates (Figure 8).

The direction of the electric field vector is pointing down from the positive plate to the negative plate, as shown by the arrows. This means that the direction of a positively charged particle in the field is also down.

Note: The field strength vector and force vector on a positive test charge are both in the same direction.

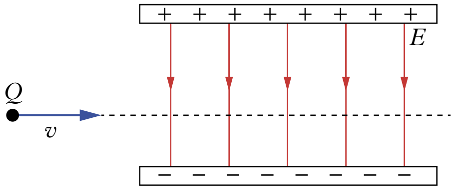

FIGURE 8 Charged particle in a uniform field

If a positive particle is fired into the field at constant speed from the left, it would continue at a constant velocity unless acted on by an outside force (Newton's first law). In this case, there is a force and it will cause a positive particle to move downwards, thereby changing its velocity. The bigger the force, the greater the deflection away from the horizontal path. If the particle was negative it would move upwards.

Newton's second law

The magnitude of the force can be determined by using Newton's second law:

$$\vec{a} = \frac{\vec{F}}{m}$$

This is illustrated in Worked example 6.2B.

WORKED EXAMPLE 6.2B

A proton ($q = +1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) is introduced at rest into the electric field of a mass spectrometer that has a constant electric field strength of 4.00×10^3 N C^{-1}, as shown in Figure 9.

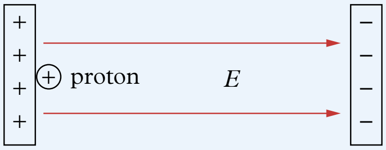

FIGURE 9 Electric field in a mass spectrometer

Calculate:

a the force acting on the proton due to the electric field

b the acceleration of the proton in the direction of the field

c the time for the proton to reach a speed of 2.00×10^5 m s^{-1}

d the distance travelled in going from rest to this speed.

SOLUTION

a
$$E = \frac{F}{q}$$
$$F = Eq$$
$$= 4.00 \times 10^3 \times 1.60 \times 10^{-19}$$
$$F = 6.40 \times 10^{-16} \text{ N (to the right) (3 sf)}$$

b
$$a = \frac{F}{m} \text{ (Newton's second law)}$$
$$= \frac{6.40 \times 10^{-16}}{1.67 \times 10^{-27}}$$
$$a = 3.83 \times 10^{11} \text{ m s}^{-2} \text{ (3 sf)}$$

c
$$v = u + at$$
$$t = \frac{v - u}{a}$$
$$= \frac{2.00 \times 10^5 - 0}{3.83 \times 10^{11}}$$
$$t = 5.22 \times 10^{-7} \text{ s (3 sf)}$$

d
$$s = ut + \frac{1}{2}at^2$$
$$= \frac{1}{2}\,at^2 \text{ (as } u = 0)$$
$$= 0.5 \times 3.83 \times 10^{11} \times (5.22 \times 10^{-7})^2$$
$$s = 0.0522 \text{ m (5.22 cm) (3 sf)}$$

Electric field strength formula

Consider a point source of positive charge, $+Q$. If a small positive test charge, $+q$, was near the charge $+Q$, then forces of repulsion would be felt in radial lines directed outwards. The further away $+q$ was moved along these lines the less intense the electric field strength would be. At all times the magnitude of the force would be given by Coulomb's law and thus the electric field strength, E, at some distance, r, from the charge, $+Q$, would be:

$$\vec{E} = \frac{\vec{F}}{q}$$
$$= F \times \frac{1}{q}$$
$$= \frac{kQq}{r^2} \times \frac{1}{q}$$
$$E = \frac{kQ}{r^2}$$

and its direction is radially outwards.

This equation therefore describes the electric field strength at a point r from a large point charge $+Q$. The electric field strength E is defined as having a value of zero at an infinite distance from the source. Note that diagrams such as that in Figure 5 on page 164 represent the nature of the electric field in a two-dimensional cross-section only and the actual zone of influence is always in three dimensions around point charges.

WORKED EXAMPLE 6.2C

Calculate the electric field strength 1 nm (1.0×10^{-9} m) from a proton ($q_p = +1.60 \times 10^{-19}$ C).

SOLUTION

$$
\begin{aligned}
E &= \frac{kQ}{r^2} \\
&= \frac{9 \times 10^9 \times 1.60 \times 10^{-19}}{(1.0 \times 10^{-9})^2} \\
&= 1.4 \times 10^9 \text{ N C}^{-1} \text{ directed away from the proton}
\end{aligned}
$$

WORKED EXAMPLE 6.2D

Figure 10 shows two point charges A and B separated by a distance of 15.0 cm.

a Determine the magnitude and direction of the electric field at point X midway between the two charges.

b Determine the point between the charges at which the electric field strength is zero.

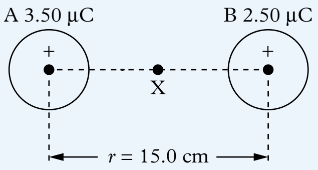

FIGURE 10

SOLUTION

a The electric field at X is the vector sum of the fields due to each point charge.

$$
\begin{aligned}
E_A &= \frac{kQ_A}{r^2} \\
&= \frac{9 \times 10^9 \times 3.50 \times 10^{-6}}{(7.50 \times 10^{-2})^2} \\
&= 5.59 \times 10^6 \text{ N C}^{-1} \text{ (to the right)}
\end{aligned}
$$

$$
\begin{aligned}
E_B &= \frac{kQ_B}{r^2} \\
&= \frac{9 \times 10^9 \times 2.50 \times 10^{-6}}{(7.50 \times 10^{-2})^2} \\
&= 4.00 \times 10^6 \text{ N C}^{-1} \text{ (to the left)}
\end{aligned}
$$

$$
\begin{aligned}
\vec{E}_{net} &= \vec{E}_A + \vec{E}_B \text{ (vector sum)} \\
&= 5.59 \times 10^6 - 4.00 \times 10^6 \\
&= 1.59 \times 10^6 \text{ N C}^{-1} \text{ (to the right, towards B)}
\end{aligned}
$$

b The point at which the electric field is zero is the point where the electric field E_A is numerically equal to the electric field E_B, but opposite in direction. Let this take place at a distance s from A.

Thus:

$$|E_A| = |E_B|$$

$$\frac{9 \times 10^9 \times 3.50 \times 10^{-6}}{s^2} = \frac{9 \times 10^9 \times 2.50 \times 10^{-6}}{(15.0 - s)^2}$$

$$\frac{3.50}{s^2} = \frac{2.50}{(15.0 - s)^2}$$

$$\frac{3.50}{2.50} = \frac{s^2}{(15.0 - s)^2}$$

$$\sqrt{\frac{3.50}{2.50}} = \sqrt{\frac{s^2}{(15.0 - s)^2}}$$

$$1.183 = \frac{s}{(15.0 - s)}$$

$$1.183 \times 15.0 - 1.183s = s$$

$$17.745 = 2.183s$$

$$s = 8.13 \text{ cm from A}$$

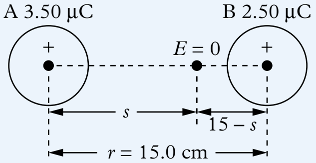

FIGURE 11

CHECK YOUR LEARNING 6.2

Describe and explain

1 **Sketch** the cross-sectional electric field diagram for a system of three negative charges situated at the corners of an equilateral triangle.

2 **Calculate** the magnitude and direction of the electric field strength 0.2 m from a point charge of –6 μC.

Apply, analyse and interpret

3 **Distinguish** between electric field and electric field strength.

4 Two charged spheres are placed 2.0 m apart as shown in Figure 12.

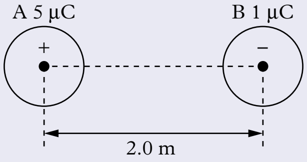

FIGURE 12 Two charges in a line

Determine:

a the electric field strength midway between the two charges

b the electric field strength at a point 50 cm to the right of object B

c the location of a point P (besides infinity) at which the field strength is zero.

5 An electron ($q_e = -1.60 \times 10^{-19}$ C, $m_e = 9.109 \times 10^{-31}$ kg) is introduced at rest into an electric field with a constant electric field strength of 50 N C^{-1}.

Determine:

a the force acting on the electron due to the electric field

b the acceleration of the electron in the direction of the field

c the time to reach 80% of the speed of light (use $c = 3 \times 10^8$ m s^{-1})

d how far it moved in the time taken to go from rest to the final speed.

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6.3 Electric potential and energy

KEY IDEAS

In this section, you will learn about:

- electrical potential energy
- electric potential
- equipotential lines.

The concepts of electric field and electric field strength are just the beginning of our understanding of electrical phenomena. You are probably more familiar with other aspects of electricity, namely energy and voltage. Energy can be stored in common devices such as batteries, and those batteries have a voltage. But energy and voltage are different things. A computer battery and a car battery might be both 12 V but you couldn't start a car with the computer battery as it would be too small. It is the stored electrical energy that makes the difference, not the voltage. In this section, you will learn about the energy and voltage of charged objects.

Electrical potential energy

The gravitational field analogy can be used again when discussing the energy stored in an electric field. When a mass moves in the direction of the gravitational field (towards the ground) it goes from high gravitational potential energy to low. That is, it goes from a region of high potential to a region of lower potential. Similarly, when a positive charge moves in the direction of an electric field it too goes from high potential energy to lower potential energy.

Conversely, work was needed to lift a mass against the direction of the gravitational field and, as this work was done, gravitational potential energy was stored in the combined masses and the gravitational field. The energy stored was equivalent to the work done, W, and calculated using $W = mgh$. Similarly, it requires work to be done to move a positive electric charge against the direction of the field. The charge and the field acquire **electrical potential energy** (U) by the work done. For an electric field, the work done is given by $W = qEd$. You should be able to see the similarity in Table 1.

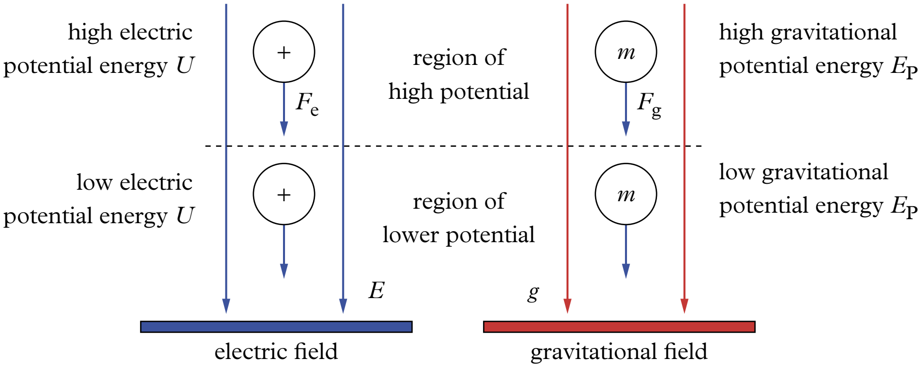


FIGURE 1 A gravitational field analogy for electric fields

electrical potential energy the capacity of electric charge carriers to do work due to their position in an electric field (symbol: W; SI unit: joule; unit symbol: J)

TABLE 1 Gravitational and electrical quantities compared

Quantity	Gravitational field	Electric field
Work done	work, W (in J) $W = m\mathbf{g}h$	work, W (in J) $W = q\mathbf{E}d$
Property of the objects	mass, m (in kg)	charge, q (in C)
Field strength	gravitational, g (N kg^{-1})	electric, E (in N C^{-1})
Distance moved	height, h (in m)	distance, d (in m)

This relationship for electrical work done is true even though the nature of the coulomb force of repulsion in the field is not uniform. By using very small incremental distances and mathematical calculus techniques, it is possible to verify that the total work done is the same as the sum of all incremental force times distance calculations.

If the charge is moved perpendicular to the field lines then no work at all will be done, just as a rolling a ball on a frictionless horizontal desk requires no energy.

The work done in moving charge $+q$ will subsequently store electrostatic potential energy in the charge $+q$. If the repulsive force between the charges is allowed to act by itself, then the stored electrical potential energy will be converted back into kinetic energy of motion as the particles fly away from each other. This effect can be used in an electrostatic charge accelerator such as the cyclotron, which produces a beam of charged particles that can be used for medical, industrial and research processes.

Electric potential

Calculating the work done is difficult as the field is not uniform. It is better to have a physical quantity that is independent of the test charge, which is defined as **electric potential** (V). It is a point in space within any given electric field. That is, it is not the energy of a system of charges but merely a point in space.

electric potential the electrostatic potential energy stored per unit charge at any given point (symbol: V; SI unit: volt; unit symbol: V)

The electric potential, V, at that point would be the work done if you moved a unit positive charge (+1 C) from infinity to that point. At infinity, the electric potential and electrical potential energy are defined as being zero ($V_\infty = 0$, $U_\infty = 0$), so ΔV and ΔU are the same as V and U respectively. This is shown in Figure 2.

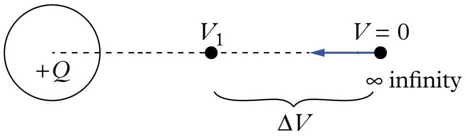


FIGURE 2 Electric potential changes when a charge is brought in from infinity.

Hopefully, you can see the difference: electrical potential energy is the actual energy residing in a charged object and the field at that point; the electric potential is just a measure of the energy per coulomb of an actual charged object at that point.

The electric potential, V, is thus the electrical potential energy stored per unit charge at any given point. Thus:

$$\Delta V = \frac{\Delta U}{q}$$

Electrical potential difference

electrical potential difference the work done in moving a unit charge between the final and the initial positions in an electric field (symbol: ΔV; SI unit: volt; unit symbol: V)

Electric potential at a point is a scalar quantity and is measured in units of joules per coulomb (J C^{-1}). One joule per coulomb is also known as a volt: 1 volt (V) = 1 J C^{-1}. Of course, moving unit positive charges from infinity to a point within the field of a point charge Q is not particularly realistic, and hence the term **electrical potential difference** is more useful, in that it describes the difference in electric potential between two positions within the field (V_1 to V_2), neither of which is at infinity (Figure 3).

$$\Delta V = V_2 - V_1 = \frac{\Delta U}{q}$$

The potential difference ΔV is often given the symbol V, the same unit as potential. This is not strictly correct, but it is used for convenience. For example, a 9 V battery isn't labelled as a potential difference of 9 V, it just says 9 V and it is assumed that the negative terminal is zero volts. The potential difference formula is usually written as:

$$V = \frac{\Delta U}{q}$$

but should be interpreted as:

$$\Delta V = \frac{\Delta U}{q}$$

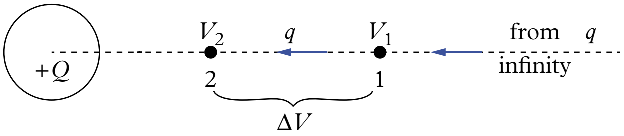


FIGURE 3 Electrical potential difference between two points in a field

WORKED EXAMPLE 6.3A

Calculate the potential difference when 15 J of energy is used to move 2.5 C of charge through an electric field in a wire.

SOLUTION

$$V = \frac{\Delta U}{q}$$
$$= \frac{15}{2.5}$$
$$= 6.0 \text{ V}$$

The potential difference is 6.0 V (2 sf).

CHALLENGE 6.3A

Electric potential

The formula for electric potential at a point has been worked out by physicists. It is: $V = \frac{kQ}{r}$, where r is the radial distance from an object with a charge of Q (C). It is not a vector quantity but an algebraic one. Show that the electric potential at a point P midway between a charge Q_1 of +10 μC and a charge Q_2 of −5 μC, which are 70 cm apart, is +128 571 V.

CHALLENGE 6.3B

The charge of table tennis balls

If table tennis balls each had a charge of +1 C, and they could be brought together so that they were touching, what would be the charge on a suitcase full of table tennis balls?

CHALLENGE 6.3C

Rocket launch

You've invented a new way to shoot a rocket into space. Get 1 mole of hydrogen ions (6×10^{23} positive H^+ particles) and put them in the base of a spaceship standing on a launch pad. Get another mole of hydrogen ions (H^+) and put them in a wheelbarrow and wheel them under the rocket. The repulsive force will force the rocket into the air.

a Calculate the force between these two moles of H^+ ions when placed 1 metre apart under a Saturn V rocket of mass 3 million kg.

b Calculate the initial acceleration.

c Propose what is implausible about this suggestion.

Equipotential lines

Points in space that all have the same potential make up what is called an equipotential line, or, if they are in three dimensions, an equipotential surface. A series of equipotential lines with each line representing a different value of potential can be used to represent the electric field. Figure 4 is an example of this.

No net work is done on a charge by an electric field when the charge moves along the equipotential lines. This is a consequence of $V_f - V_i = \frac{\Delta U}{q}$. That is, if there is no potential difference, no work is done when a charge is moved from one point to another. However, work is done when shifting from one equipotential line to another.

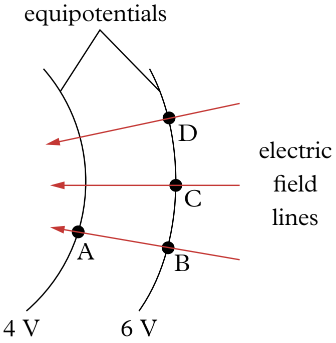


FIGURE 4 No work is done along an equipotential line (B, C, D).

WORKED EXAMPLE 6.3B

In Figure 4, what are the differences in electric potential for:

a $V_B - V_A$ **b** $V_C - V_A$ **c** $V_C - V_B$

SOLUTION

a
$$\begin{aligned}\Delta V &= V_B - V_A\\ &= 6 - 4\\ &= 2\,\text{V}\end{aligned}$$

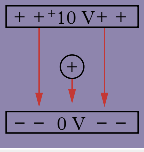
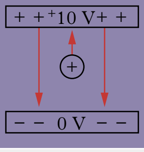
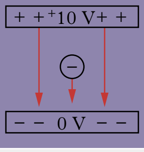
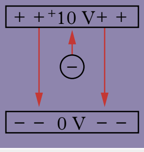
b
$$\begin{aligned}\Delta V &= V_C - V_A\\ &= 6 - 4\\ &= 2\,\text{V}\end{aligned}$$

c
$$\begin{aligned}\Delta V &= V_C - V_B\\ &= 6 - 6\\ &= 0\,\text{V}\end{aligned}$$

Transfers and transformations of energy

When a charged object moves in a field, it can either gain or lose potential energy if there is a net force acting on it. This change is associated with a change in electric potential. The following description details what happens when a positive or negative charge moves with or against a field. That means there are four possible scenarios and the changes are detailed in Table 2.

TABLE 2 Energy changes in moving charges in an electric field

	CASE A positive charge moving with the field 	CASE B positive charge moving against the field 	CASE C negative charge moving with the field 	CASE D negative charge moving against the field 
Motion of charge in field	with (down)	against (up)	with (down)	against (up)
Force on charge, $\boldsymbol{F}_{net}$	down	down	up	up
Acceleration, $\boldsymbol{a}$	down (speeds up)	down (slows down)	up (slows down)	up (speeds up)
Kinetic energy, ΔE_k	increases (+)	decreases (−)	decreases (–)	increases (+)
Potential energy, ΔU	decreases (ΔU is −)	increases (ΔU is +)	increases (ΔU is +)	decreases (ΔU is –)
Potential difference, $\Delta V = V_f - V_i$	$0 - +10 = -10$ V decreases	$+10 - 0 = +10$ V increases	$0 - +10 = -10$ V decreases	$+10 - 0 = +10$ V increases
Work done	by the field	on the field	on the field	by the field

- **Case A:** When a positively charged object enters an electric field, it will experience a net force in the direction of the field (by definition). Therefore, it will accelerate in that direction. This means its kinetic energy (E_k) will change. If the force is in the direction of motion, as in Case A, the object will speed up and its kinetic energy will increase (ΔE_k is positive). By the law of conservation of energy, the gain in kinetic must be offset by a loss in potential energy (U); therefore, U decreases and thus ΔU ($U_{final} - U_{initial}$) is negative. The field (U) has lost energy, so work has been done *by* the field (on the object).
- **Case B:** When the positively charged object enters the electric field it experiences a net force downwards in the direction of the field (again, by definition). Therefore, the direction of the acceleration will be down, but as it is moving upwards it will slow down and its kinetic energy will decrease (ΔE_k is negative). The loss in kinetic energy is offset by a gain in potential energy (U), therefore U increases and thus ΔU is positive. The field (U) has gained energy so work is done *on* the field.
- **Case C:** When a negatively charged object enters the electric field it experiences a net force upwards in the direction opposite to the field (by definition). Therefore, the direction of acceleration will be up, and as it is moving downwards it will slow down and its kinetic energy will decrease (ΔE_k is negative). The loss in kinetic energy means a gain in potential energy (U), therefore ΔU is positive. The field gains energy so work is done *on* the field.

- **Case D:** When a negatively charged object enters the electric field it experiences a net force upwards in the direction opposite to the field (by definition). Therefore, the direction of the acceleration is up, and as it is moving upwards it will speed up and its kinetic energy will increase (ΔE_k is positive). The gain in kinetic energy is offset by a loss in potential energy (U), therefore ΔU is negative. The field loses energy so work is done *by* the field.

CHECK YOUR LEARNING 6.3

Describe and explain

1 **Describe** what it means to say that when work is done on a field, U increases.

2 **Explain** whether an electron entering an electric field naturally wants to travel with the field or against it.

3 **Calculate** the work done by an electron ($q_e = -1.60 \times 10^{-19}$ C, $m_e = 9.109 \times 10^{-31}$ kg) in travelling from the +6 V end to the 0 V end of a piece of nichrome wire 20 cm long and connected across the terminals of a 6 V battery.

Apply, analyse and interpret

4 a **Calculate** the kinetic energy of an electron as it enters an electric field at a speed of 0.6*c* (where *c* = the speed of light = 3×10^8 m s^{-1}).

  b **Determine** the potential difference the electron will experience when it is brought to rest by transferring all of its kinetic energy to the field.

5 Two points in space are at electric potentials of +18 V and −6 V respectively.

  a **Calculate** the potential difference between these two points.

  b **Determine** the work done in moving a charge of 5.5 μC from one point to the other.

6 **Determine** the energy of a lightning flash when there is a potential difference of 2×10^9 V between the cloud and the ground, and 35 C of charge is transferred during the strike.

7 **Determine** the electric potential in an electric field at the points listed below. Note that Figure 5 shows two equipotential lines about a charged object. It takes 0.2 J of work to move a +0.1 C charge at point C to point E.

  a point E

  b point D

  c point B

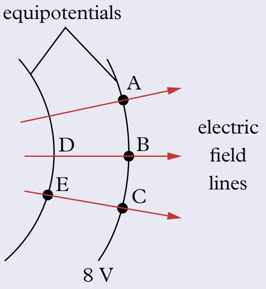


FIGURE 5

Investigate, evaluate and communicate

8 **Evaluate** whether the statement ‘electric field lines never cross’ is true.

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6.3A Electric potential

» Challenge worksheet
6.3B The charge of table tennis balls

» Weblink
Understanding energy transfers and transformations

CHAPTER 6 Review

Summary

6.1

- Two states of charge exist for any material: positive if it has lost electrons and negative if it has gained electrons. Opposite charges attract and like charges repel.
- Charge is a quantity of electricity measured in coulombs. The charge on one electron is 1.60×10^{-19} C.
- Coulomb's (inverse square) law describes the nature of the force between electrically charged objects. The force is proportional to the charges Q_1, Q_2 and inversely proportional to the square of the separation distance (r). The constant of proportionality (k) in Coulomb's law has a value of 9×10^9 N m^2 C^{-2}.

6.2

- An electric field is a region in three-dimensional space surrounding an electrically charged object. Electric field lines define the direction and strength of the electric field in terms of the force on a test positive charge.
- Electric field strength (E) is a vector quantity defined as the force per unit charge at a point in the electric field. It is measured in units of newton per coulomb (N C^{-1}).
- The electric field surrounding point charges or systems of point charges is not uniform, whereas the electric field between a set of parallel metallic plates is uniform.

6.3

- The potential difference between any two points in an electric field is defined as the work done per unit charge as the charge is moved from one point to the other. Potential difference $V = \frac{\Delta U}{q}$. It is measured in the unit of volts (V). One volt (V) equals one joule per coulomb (J C^{-1}).

Key terms

- Coulomb's law
- Coulomb's law constant, *k*
- electric field
- electric field strength
- electric potential
- electrical potential difference
- electrical potential energy
- uniform electric field

Key formulas

Coulomb's law	$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$
Coulomb's law constant	$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Electric field strength	$E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
Electric potential difference	$V = \frac{\Delta U}{q}$

Revision questions

The relative difficulty of these items is indicated by the number of stars beside each one: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 Two small spheres have equal charges q and are separated by a distance r, as shown in Figure 1. The force exerted on each sphere by the other has magnitude F. If the distance between them, r, is doubled, the force on each sphere has a magnitude:

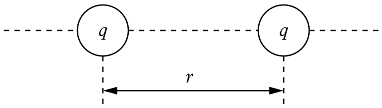

FIGURE 1 Two separated charges

A $2F$

B $\frac{F}{2}$

C $\frac{F}{4}$

D $\frac{F}{8}$

2 Two oppositely charged particles experience an attractive force as shown in Figure 2. If the force on Q_1 is 10 N, the force, F_2, acting on Q_2 (and the reason) is:

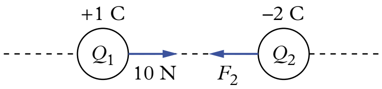

FIGURE 2 Forces between two charges

A 5 N, as Q_1 is half the charge and can only pull half as hard as Q_2.

B 10 N, as Newton's third law says action–reaction pairs are equal and opposite.

C 10 N, as they are the same distance apart.

D 20 N, as Q_2 has twice the charge of Q_1.

3 The electric potentials near a charged plate are shown in Figure 3.

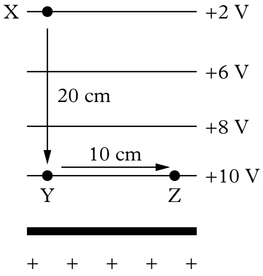

FIGURE 3 Electric potentials in a field

A particle with a net charge of +10 μC is moved from point X to point Y and then to point Z. The work done in the process from X to Y and Y to Z respectively is:

A 8×10^{-5} J, 0 J

B 8×10^{-5} J, 1×10^{-5} J

C 8 J, 0 J

D 8 J, 10 J

4 How does the electrostatic repulsive force between two electrons compare to the gravitational force between them?

A The electrostatic force is greater.

B The electrostatic force is smaller.

C They are about the same.

D It will depend on the separation distance.

5 Two identical charges repel each other with a force of 16 N. When the distance between the charges is doubled, the force will be:

A 8 N

B 4 N

C 32 N

D 64 N

6 Figure 4 shows four charges placed at the corners of a square. Which arrow most accurately shows the direction of the force on a negatively charged particle at the centre point C?

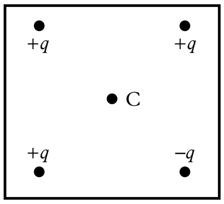

FIGURE 4 Arrangement of charges

A ↑

B ↓

C ↖

D ↘

7 If the electric field strength is 18 N C^{-1} at point R in Figure 5, what is it at point S?

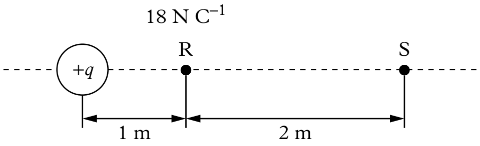

FIGURE 5 Electric field about a point charge

A 9 N C^{-1}

B 4.5 N C^{-1}

C 3 N C^{-1}

D 2 N C^{-1}

8 For Figure 5 in the previous question, the direction of the electric field at point S is:

A up the page.
B to the right.
C down the page.
D to the left.

9 A negatively charged particle is fired into an electric field from the right and undergoes motion as shown in Figure 6.

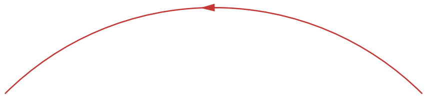

FIGURE 6 Motion of charge in field

Which diagram in Figure 7 best represents the electric field?

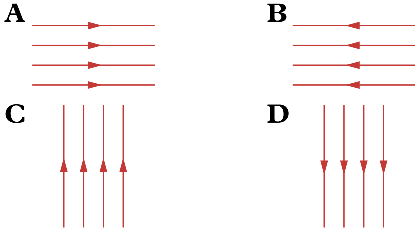

FIGURE 7

10 Three positions between a pair of positive and negative plates are shown in Figure 8.

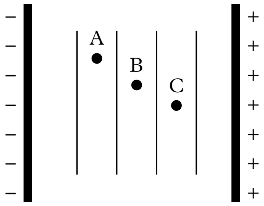

FIGURE 8 Positions between charged plates

Which position has the highest electric potential?

A A
B B
C C
D They are all at the same electric potential.

Short answer

Describe and explain

★ **11** **Explain** the difference between U and ΔU.

★★ **12** **Calculate** the magnitude of the charge of an oil droplet if the droplet experiences an electrostatic force of 5.6×10^{-14} N when placed into a uniform electric field of 4000 N C^{-1}.

★★★ **13** **Calculate** the net force on charged object B for the set-up as shown in Figure 9.

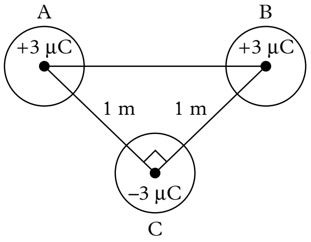

FIGURE 9 The three charges make a right-angled triangle.

★★★ **14** Figure 10 shows a particle of charge $+8.0 \times 10^{-19}$ C and mass 9.60×10^{-26} kg entering a uniform electric field of strength 20 N C^{-1} at a right angle to the field. The initial speed of the particle is 1200 m s^{-1}.

a **Calculate** the magnitude and direction of the force on the charged particle.
b **Calculate** the acceleration of the particle in the field.
c **Calculate** the time of travel of the particle in the field.
d **Calculate** the final displacement and velocity in the direction of the field.
e **Describe** the probable path of the particle in the field.

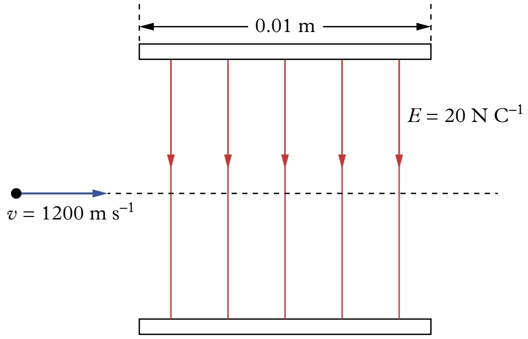

FIGURE 10

Apply, analyse and interpret

★ **15** It takes 2.30 J of work done on a +0.5 C charged object to move it in an electric field.

a **Determine** the change in energy of the charged object.
b **Calculate** the potential difference experience by the object.
c **Determine** the final electric potential if the starting electrical potential was +6.6 V.

★★**16** A system of three charges in a line, Q_A = +10 μC, Q_B = −5 μC and Q_C = +10 μC, are separated by distances to their centres as shown in Figure 11. **Determine** the net force:

a of B due to A and C

b on C due to A and B.

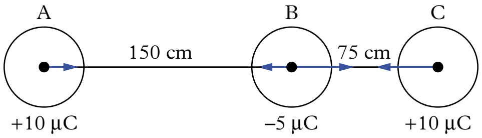

FIGURE 11 A system of three charges

★★★**17** **Consider** the system of charges as illustrated in Figure 12. Each charge is +2.0 × 10^{-6} C and they are arranged in an equilateral triangle of side 30 cm. **Deduce** the magnitude and direction of the force on charge C.

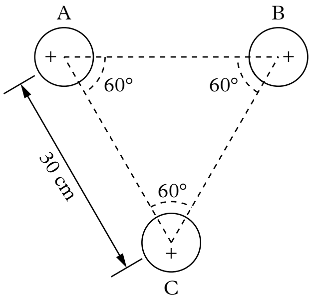

FIGURE 12 Charges in an equilateral triangle

Investigate, evaluate and communicate

★★★**18** Students conducted an experiment to replicate Coulomb's original experiment with electrostatic charges to derive a value for the electrostatic force of repulsion between the two spheres. They set up a torsional balance as shown in Figure 13. The apparatus enabled the students to measure the twist in the suspension wire as the 'free' charge tried to rotate away from the fixed charge. From the value of the twist, the students could determine a relationship between force and distance.

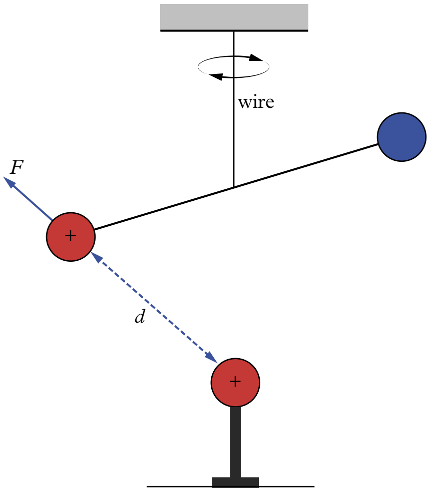

FIGURE 13 Experimental set-up for student experiment

Part of the experimental data is shown in Table 1. Unfortunately, two of the data points were not recorded.

a **Construct** a graph to show the relationship between the electrostatic force (vertical axis) and the separation distance (horizontal axis).

b **Predict** the two missing readings.

c **Comment** on the relationship between the force and the distance as suggested by your graph. Explain your answer.

d **Construct** a linearised graph to confirm your answer.

TABLE 1 Force versus separation data

Average force (units)	Separation (cm)
28.4	0.75
10.2	1.25
	1.50
	1.75
4.00	2.00
1.80	3.00
1.00	4.00

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CHAPTER 7

Magnetic fields

In 1269, the French scholar Pelerin de Maricourt, also known by his Latin name of Petrus Peregrinus de Maricourt, was taking part in the battle siege of an Italian city. As the action was very slow and dull, he wrote a letter to a friend describing his study of magnets. In this letter he described the existence of magnetic poles – regions on the magnets where the force seemed to be most intense – and explained how to determine the north and south poles of magnets, using the fact that the same poles always repelled. He also described how a single pole could not be isolated, for if a magnet was broken in two, then each piece would have both a north and a south pole. In the same letter, Peregrinus explained that a compass would work better if the magnetic sliver was placed onto a pivot rather than being floated on a cork, and that a graduated scale placed under the sliver would allow more accurate directions to be read. He had described a navigation compass.

OBJECTIVES

- → Define the term ‘magnetic field’.
- → Recall how to represent magnetic field lines, including sketching magnetic field lines due to a moving electric charge, electric currents and magnets.
- → Recall that a moving electric charge generates a magnetic field.
- → Determine the magnitude and direction of a magnetic field around electric current-carrying wires and inside solenoids.
- → Solve problems involving the magnitude and direction of magnetic fields around a straight electric current-carrying wire and inside a solenoid.
- → Recall that electric current-carrying conductors and moving electric charges experience a force when placed in a magnetic field.
- → Solve problems involving the magnetic force on an electric current-carrying wire and moving charge in a magnetic field.

Source: *Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority

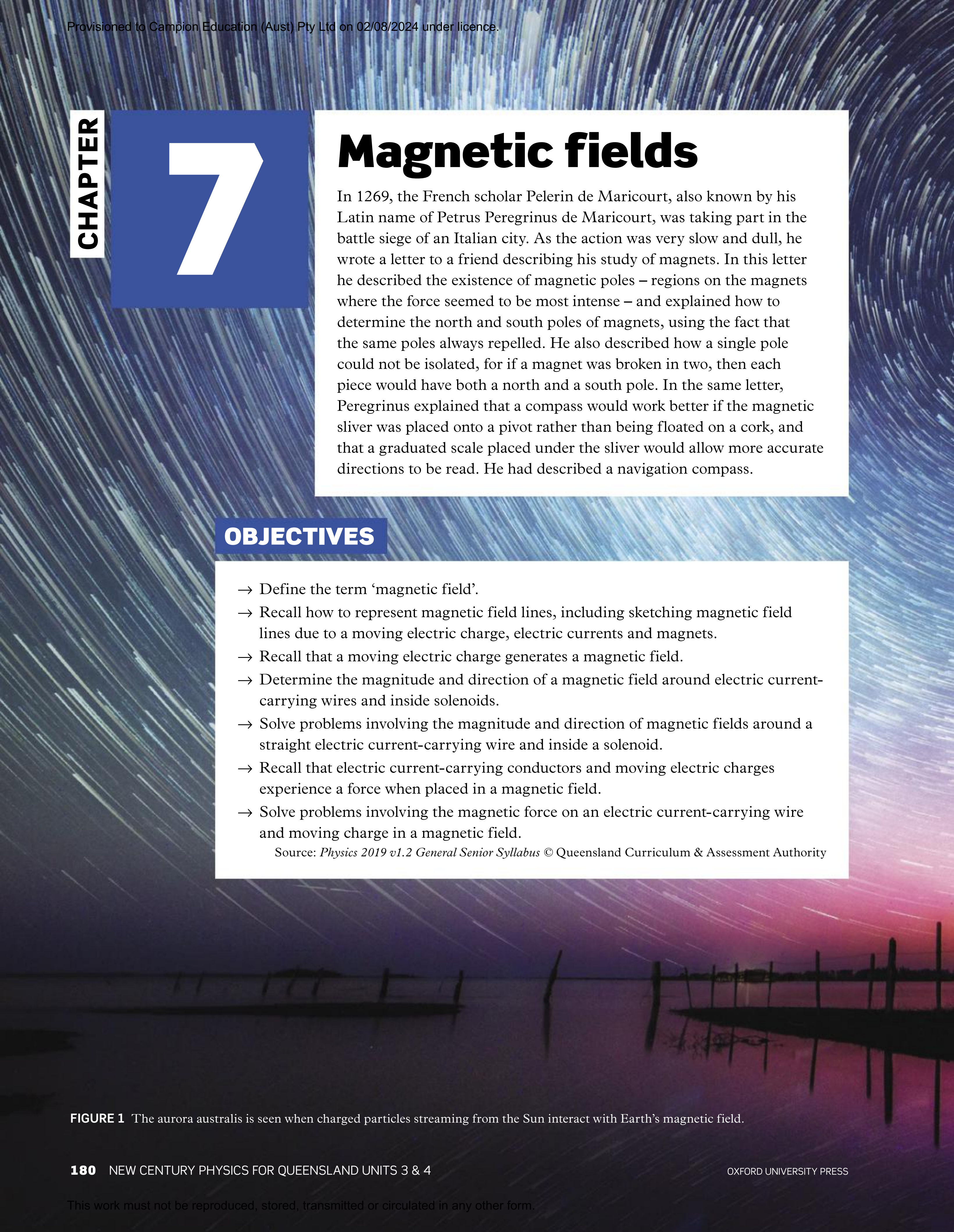
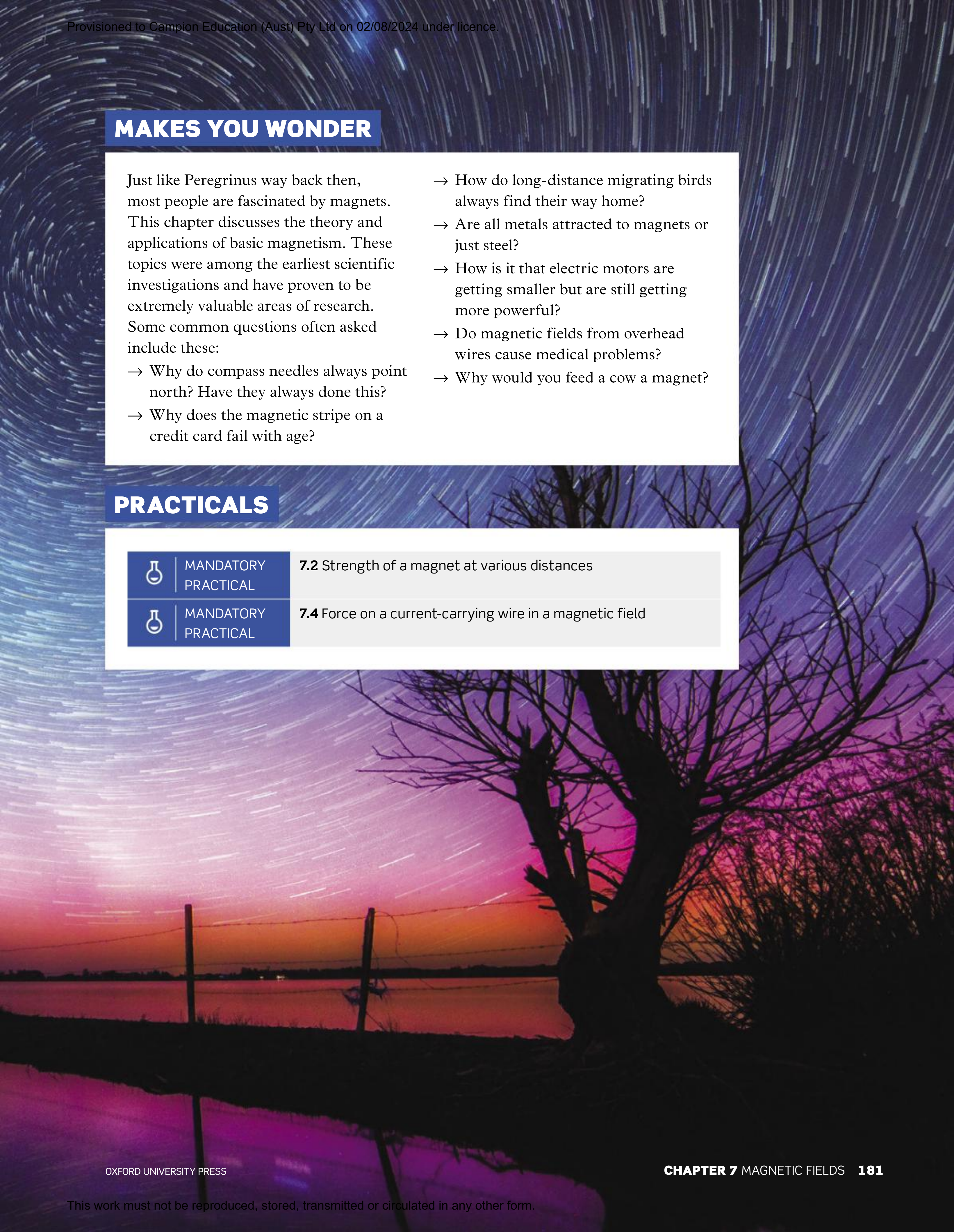


FIGURE 1 The aurora australis is seen when charged particles streaming from the Sun interact with Earth’s magnetic field.



MAKES YOU WONDER

Just like Peregrinus way back then, most people are fascinated by magnets. This chapter discusses the theory and applications of basic magnetism. These topics were among the earliest scientific investigations and have proven to be extremely valuable areas of research. Some common questions often asked include these:

→ Why do compass needles always point north? Have they always done this?

→ Why does the magnetic stripe on a credit card fail with age?

→ How do long-distance migrating birds always find their way home?

→ Are all metals attracted to magnets or just steel?

→ How is it that electric motors are getting smaller but are still getting more powerful?

→ Do magnetic fields from overhead wires cause medical problems?

→ Why would you feed a cow a magnet?

PRACTICALS

MANDATORY PRACTICAL	**7.2** Strength of a magnet at various distances
MANDATORY PRACTICAL	**7.4** Force on a current-carrying wire in a magnetic field

7.1 What is a magnetic field?

KEY IDEAS

In this section, you will learn about:

- the origin and direction of magnetic fields
- ways to depict a magnetic field with directed lines.

magnetism a phenomenon associated with magnetic fields, which arise from the motion of electric charges

Magnetism is something you come across every day; it might be the *permanent* magnet that keeps your refrigerator closed, or the fridge magnets on the door, or the *electromagnets* that use electricity, such as the magnets in headphones, electric toothbrushes or motors of any sort. All magnets transmit their effects through magnetic fields. In the previous chapters gravitational fields and electric fields were introduced. Now you will look at magnetic fields. All three have much in common. They are all defined in terms of a force acting on an object in the field. For the gravitational field the object is a 1 kg mass, and for the electric field the object is a +1 C charge. Magnetic fields are a little more complicated, but the quantity is a combination of charge and velocity.

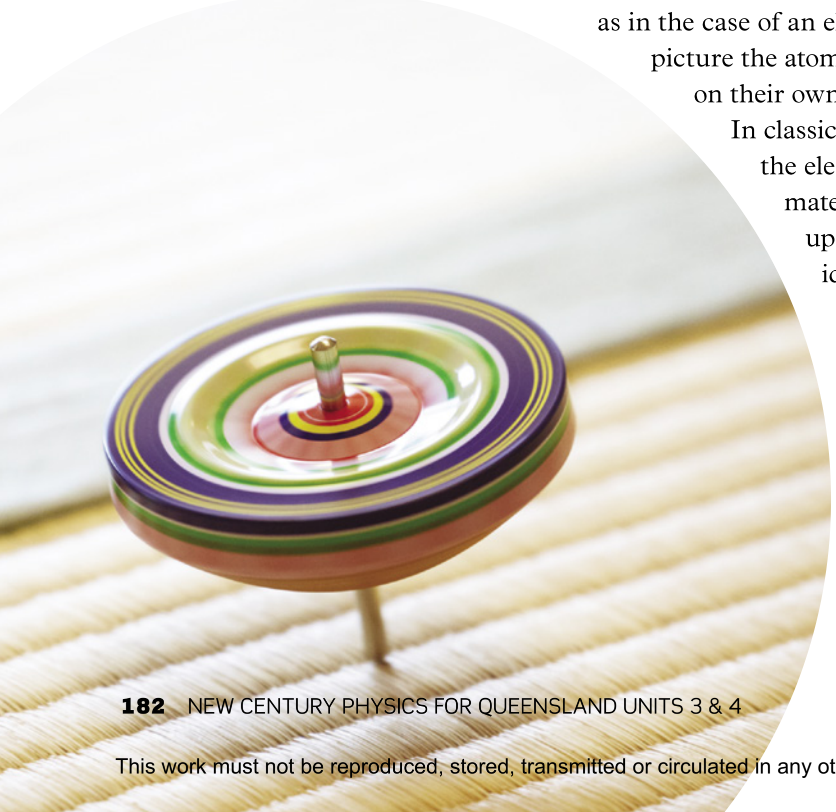
Electric and magnetic fields are both due to electric charges; but although electric fields act between stationary 'static' charges, magnetic fields act between moving 'electromotive' charges.

The properties of magnets have been investigated for 2000 years, but it was the work of English scientist William Gilbert (1544–1603) that put magnetism on a scientific footing. However, it was Michael Faraday's conception of magnetic fields and magnetic lines of force that provided the world with a model for magnetism that enabled dramatic progress from 1830 onwards.

Origins of the magnetic force

Magnetism arises from the motion of charges, namely electrons. There are various motions of electrons that give rise to magnetic fields. It can be an electron current in a wire, as in the case of an electromagnet. But in the case of permanent magnets, picture the atom being made up of rotating electrons either spinning on their own axes (like a spinning top) or orbiting the nucleus. In classical physics, the motions spinning and orbiting turn the electrons and atoms into tiny bar magnets. In certain materials, such as iron, these tiny bar magnets all line up and the result is a permanent magnet. However, the idea of spinning electrons is really just a model. In quantum physics the electron is considered a point particle and the idea of spinning makes no sense. Quantum physics retains the idea of spin because the electron produces a magnetic field as if it were really spinning; but it doesn't. Magnetism is just an intrinsic property of an electron.

FIGURE 1 You can picture an atom as a spinning top with electrons rotating on their own axes.



Representing the magnetic field

First, a **magnetic field** can be defined in a similar way to the others: it is a region of space where a magnetic force is experienced.

You've probably seen a demonstration in which iron filings are sprinkled over a piece of paper that has a magnet underneath. The filings line up in patterns that represent the magnetic field of the magnet. Like the gravitational field and electric field of earlier chapters, this magnetic field is a vector field that has both magnitude and direction. The magnitude is called the magnetic field strength and arrows are used to indicate the direction. This field is, in fact, three dimensional and the iron filings represent the cross-section through the full three-dimensional field.

magnetic field
a region of space where a magnetic force is experienced

magnetic field line
the direction an isolated north pole would move in the field

Representing the field direction

Magnetic field diagrams are drawn with **magnetic field lines** and directional arrows indicating the direction of the force that a small test magnetic north pole would experience if placed into the field.

You know that north poles of magnets repel each other, so the lines will consequently always be oriented from north pole to south pole for a typical bar magnet. The direction of the force at any point in a magnetic field diagram is given by the tangent to a field line at that point. You can't really have an isolated north pole (north pole); they can't exist – it is just used as a model.

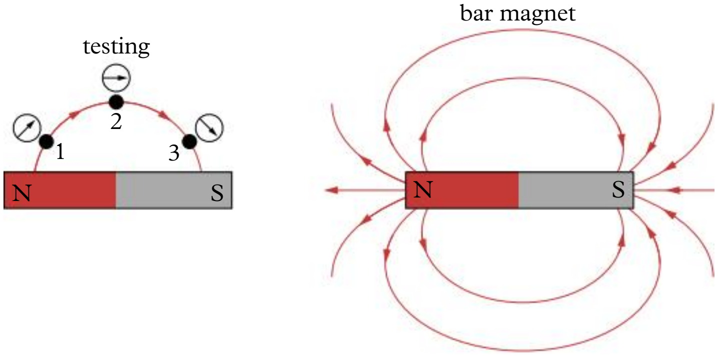


FIGURE 2 The field direction around a bar magnet can be mapped with a small compass.

Earth's magnetic poles

Magnetic poles come in pairs – for every north (N) pole there is a south (S) pole – and so the idea of an 'isolated' north pole is an imaginary device used to determine the direction of a magnetic field.

A confusing situation arises when you consider Earth's magnetic field. How is it that the field lines point towards the Geographic North Pole? If it is a north pole, shouldn't the field lines point away from it? To get the answer, imagine that Earth has the equivalent of a big bar magnet inside it with its south pole in the Northern Hemisphere, as shown in Figure 3. An isolated north pole held in your hand would tend to move north towards the geographic north pole because there really is a magnetic south pole inside Earth at this point. The source of Earth's magnetic field will be discussed in Chapter 8.

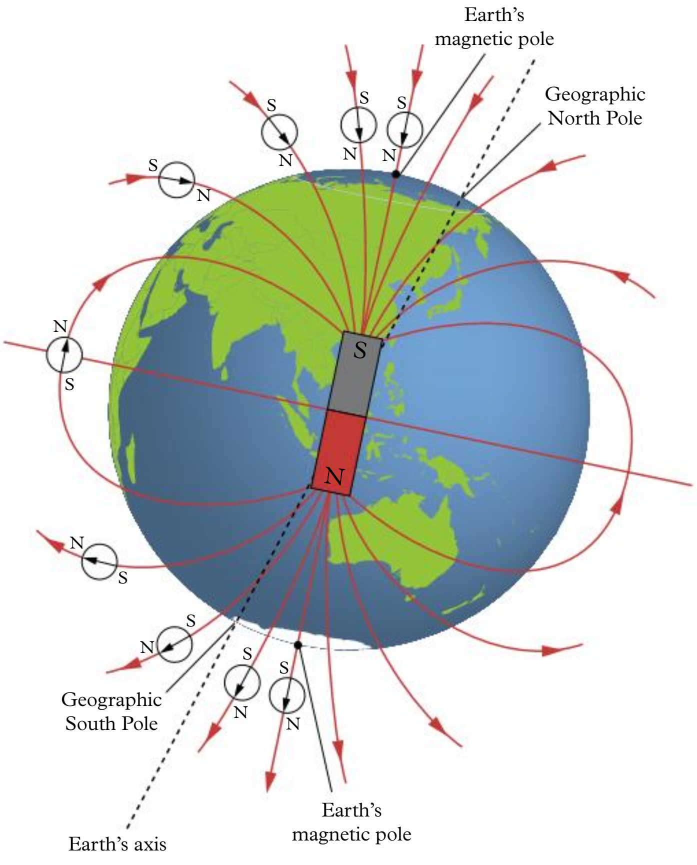


FIGURE 3 Earth's magnetic field

Testing the field direction

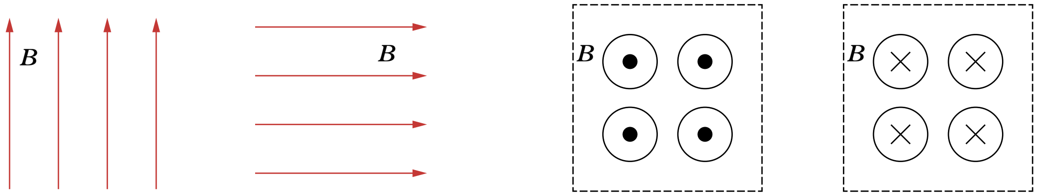
A small compass needle can be used to test the direction of magnetic field lines. The north pole of the compass points away from the north pole of the magnet and towards the south pole. The red end of a compass is the north pole and it will always point towards Earth's magnetic north pole (which is really a south pole).

Flux lines

The other word for field lines is 'flux' lines. The word flux comes from the Latin *fluere*, meaning 'to flow' (the old idea about the flow of magnetism). You may note that the lines of magnetic flux never cross over because at such a crossing point the force would be acting in two different directions, which doesn't make sense. Flux lines are shown as pointing away from the north pole or towards the south pole.

Study tip

When you see circles with dots, you can imagine there is a north pole behind the page with flux lines coming towards you. When you see a circle with a cross, you can think you are the north pole and the south pole is behind the page.



magnetic field up the page

magnetic field to the right

magnetic field out of the page

magnetic field into to the page

FIGURE 4 Representing field lines in two ways

The direction of a magnetic field can be shown in two ways:

- directed line segments (arrows) along the page where the arrows point from north to south.
- circles showing the lines perpendicular to the page, these can show flux lines coming towards you (drawn as a circle with a point, like the tip of an arrow), or away from you (drawn as a circle with a cross representing the tail feathers of the arrow).

Representing field strength

Magnets come in different strengths. Some are strong, some are weak. As well as indicating the direction of the magnetic field, a way of specifying magnetic field strength is needed, as with electric field strength.

Study tip

Note that the symbol for magnetic field strength, *B*, is shown with an arrow over it, $\vec{B}$, when field strength as a vector (magnitude and direction) is considered. If you are just talking separately about the magnitude of the field strength, or the direction of the field, then the symbol is *B*.

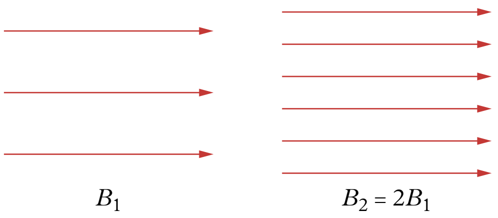


FIGURE 5 The strength of a magnetic field is shown by how close the field lines are together. The field on the right is twice as strong as the field on the left.

Using vectors

Magnetic field strength is a vector quantity and is represented by the vector symbol $\vec{B}$. The magnitude of the magnetic field can be represented by the flux lines being drawn closer together or further apart, as shown in the field representation of Figure 5. Its direction is given by the arrowhead showing the direction of the force on an isolated north pole.

CHALLENGE 7.1

Breaking a magnet in half

You can lift 50 paper clips with both ends of a permanent magnet. If it was broken in half could you lift more? If you said yes, explain where the extra energy comes from.

CHECK YOUR LEARNING 7.1

Describe and explain

1 **Explain** whether this statement true: magnetism is the result of a moving electric charge.

2 **Describe** how the direction of a magnetic field is determined.

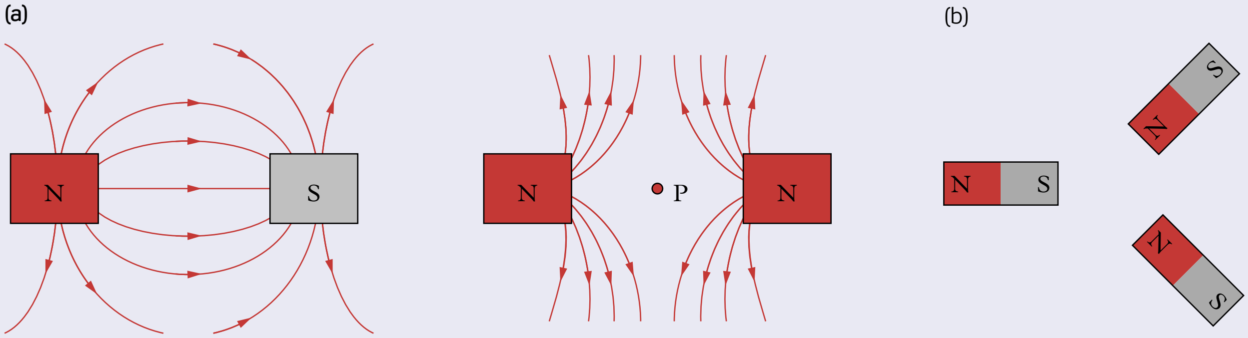
3 **Define** 'magnetic field'.

4 The magnetic field lines about two magnets with unlike poles together and like poles together is shown in Figure 6a below. **Sketch** the magnetic field lines about the arrangement of permanent bar magnets in Figure 6b.

5 **Clarify** whether the direction of a field line is from N to S, or from S to N.

Investigate, evaluate and communicate

6 **Propose** how you might tell which of two identical looking steel rods is a magnet. One rod is a magnet, but the other is unmagnetised steel.



unlike pole attraction

like pole repulsion

FIGURE 6 Magnetic field lines around magnets

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7.2 Defining magnetic field strength

KEY IDEAS

In this section, you will learn about:

- magnetic fields and current-carrying wires.

The strength (or intensity) of a magnetic field can be defined by the force acting on charged particles moving in the field. The simplest way to do this is by considering the charged particle moving in a length of wire – that is, an electric current. This is called a 'current-carrying wire'.

The previous chapter was about electrostatics. The 'statics' means stationary, so electric fields around stationary charges were discussed. You saw that a stationary electric charge produced an electric field around itself and that its strength decreased with distance. The question is how does magnetic field strength depend on distance and current?

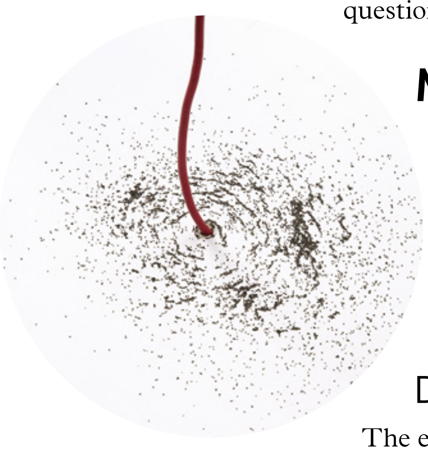

FIGURE 1 Circular magnetic field lines make a pattern in iron filings about a current-carrying wire.

Magnetic field around a current-carrying wire

When you pass a current through a wire, a magnetic field develops perpendicular to the wire. This field can easily be shown by sprinkling iron filings on paper around a current-carrying wire (Figure 1).

As with any physical phenomena, scientists want to know more about its properties. For this magnetic field, physicists experimented and theorised about the factors affecting the magnitude and direction of this field.

Direction of the magnetic field surrounding a wire

The earliest recorded experience of this phenomenon was by the Danish physicist Hans Christian Ørsted (1777–1851). He was giving a lecture on electricity and noticed, quite by chance, that when a current flowed in a wire a nearby compass needle would move. In 1820 he published *Experiments on the effect of electricity on the magnetic needle*. In this work he described the way the compass needle follows the almost circular pattern of magnetic field lines around the current-carrying wire (Figure 2).

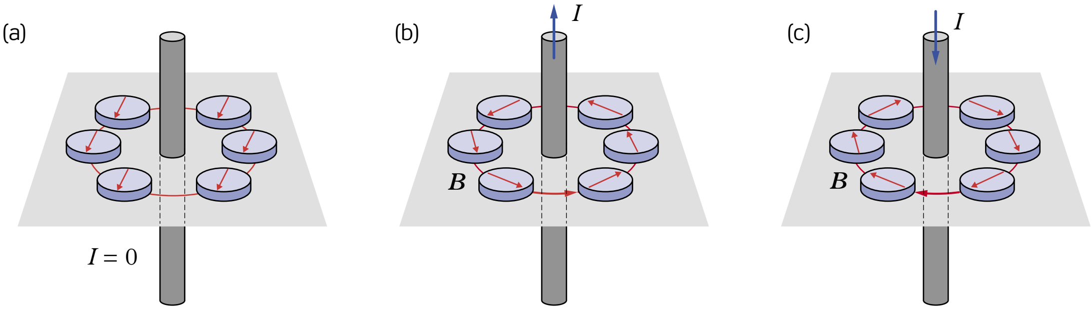

FIGURE 2 Compass directions around a wire. (a) There is no current, so the compass points N–S. (b) Current up the page gives an anticlockwise circular field. (c) Current down the page gives a clockwise field.

MANDATORY PRACTICAL
7.2 Strength of a magnet at various distances
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Ampere's right-hand rule

The magnetic field direction can be easily remembered by making use of Ampere's right-hand rule. Ampere's rule makes use of the right hand and conventional flow of current. Point your thumb along the direction of the current and then curl your fingers around the wire. The direction in which your fingers are pointing represents the direction of rotation of the magnetic field lines (Figure 3).

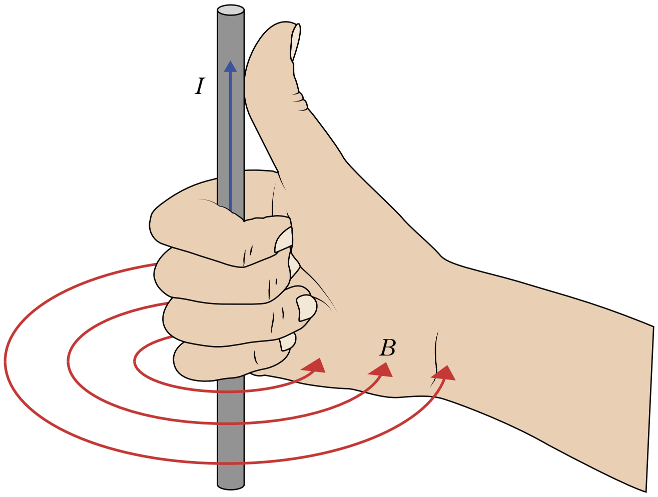


FIGURE 3 Ampere's right-hand rule shows current up the page and the field is circulating around the wire in an anticlockwise direction.

Representing fields around a wire

The simplest way of representing the direction of a field around a wire is to imagine you are looking at the wire from above, so that you are just looking at the end of the wire. From this perspective, if the current is coming out of the page towards you it can be represented by a circle with a dot that resembles the pointy tip of the arrow. If the current is moving into the page away from you, it is shown as a circle with a cross to represent the tail feathers of the arrow (Figure 4).

The main characteristics of the magnetic field in Figure 5 are:

- the field lines are circular and concentric around the wire
- the strength of the field decreases away from the wire, so the lines get further apart
- the direction of the field reverses if the direction of the current is reversed.

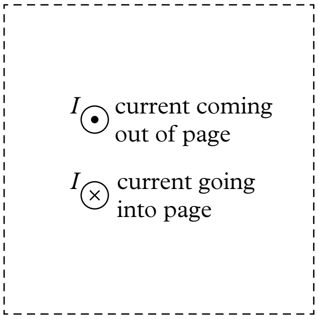
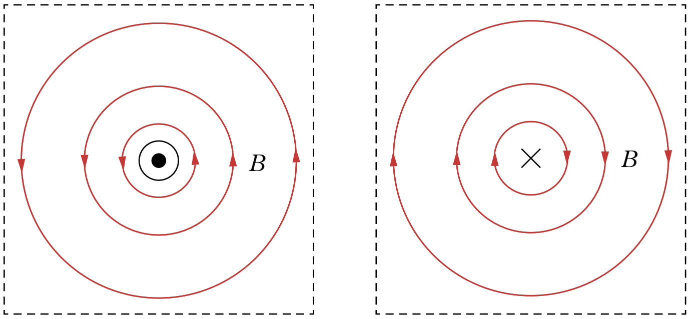


FIGURE 4 Convention for indicating the direction of the current when it is perpendicular to the page



current coming out of page; field is anticlockwise

current going into page; field is clockwise

FIGURE 5 The circular field about a current-carrying wire

Magnitude of magnetic field strength

The unit of magnetic field strength is **tesla**; symbol T, named after the Serbian–American engineer and physicist Nicola Tesla (1856–1943). It is defined by the force acting on a moving charge, either as a particle in a magnetic field, or as a current-carrying wire in a field.

tesla
the SI unit of magnetic field strength;
$1\ \text{T} = 1\ \text{N}\ \text{C}^{-1}\ \text{m}^{-1}\ \text{s}$

Typical examples of magnetic field strengths are:

- 5×10^{-5} T – Earth's magnetic field
- 5×10^{-3} T – fridge magnet
- 0.3 T – sunspot
- 3.25 T – surface of a neodymium magnet
- 3 T – magnetic resonance imaging (MRI) scanner
- 10^{10} T – a magnetar (special type of neutron star).

The formula relating the magnetic force to field strength will be given in Section 7.4. For the time being just accept it and use a formula derived from these ideas.

As with the electric field formula, development of the magnetic field formula is not simple. It requires use of calculus techniques (integration) to come to a final formula.

TABLE 1 Electric and magnetic field constants

Electric fields	Magnetic fields
$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$	$B = \frac{\mu_0}{2\pi} \times \frac{I}{r}$
Directly proportional to charge	Directly proportional to charge
Inverse square proportion with distance	Inversely proportional to distance
ε_0 is the electric permittivity constant $= 8.85 \times 10^{-12}$ C V^{-1} m^{-1}	μ_0 is the magnetic permeability constant $= 4\pi \times 10^{-7}$ T mA^{-1}
k is the Coulomb constant $k = \frac{1}{4\pi\varepsilon_0}$ $= 9 \times 10^9$ N m^2 C^{-2}	$k = \frac{\mu_0}{2\pi}$ $= \frac{4\pi \times 10^{-7} \text{ T mA}^{-1}}{2\pi}$ $= 2 \times 10^{-7}$ T mA^{-1}

You can use the formula to show, for instance, that the magnetic field strength at a distance of 5 cm from a wire carrying a current of 20 A is 8×10^{-5} T. This is about twice Earth's magnetic field strength. If there is more than one current-carrying wire, then the total magnetic field is the vector sum of the individual fields.

There are three common situations that can be analysed:

- fields in one dimension, single wire
- fields in one dimension, two wires
- fields in two dimensions.

WORKED EXAMPLE 7.2A

Calculate the magnetic field at point P, which is 5 cm directly above a wire carrying a current of 20 A to the left (Figure 6).

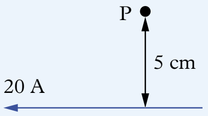
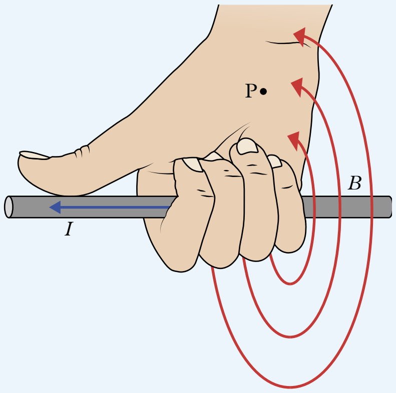


FIGURE 6 Field about a wire

SOLUTION

Magnitude:

$$\begin{aligned} B_P &= \frac{\mu_0 I}{2\pi r} \text{ (substitute } \mu_0) \\ &= \frac{(4\pi \times 10^{-7}) \times I}{2\pi r} \\ &= \frac{2 \times 10^{-7} \times 20}{0.05} \\ &= 8 \times 10^{-5} \text{ T} \end{aligned}$$



Direction: It is into the page (using Ampere's right-hand rule, as shown in Figure 7).

FIGURE 7 Ampere's right-hand rule applied

WORKED EXAMPLE 7.2B

In Figure 8, two separate parallel wires are in close proximity. Calculate the value of the magnetic field strength at a point X between the two wires, given that wire A carries a current of 1.5 A and wire B carries a current of 2.5 A.

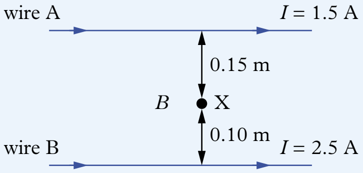


FIGURE 8 Field between two wires

SOLUTION

The magnetic field at point X due to wire A (written as B_{XA}) is given by:

$$\begin{aligned} B_{XA} &= \frac{\mu_0}{2\pi} \times \frac{I}{r} \\ &= \frac{1.26 \times 10^{-6}}{2\pi} \times \frac{1.5}{0.15} \\ &= 2 \times 10^{-6} \text{ T into the page (using Ampere's right-hand rule)} \end{aligned}$$

The magnetic field at point X due to wire B is:

$$\begin{aligned} B_{XB} &= \frac{\mu_0}{2\pi} \times \frac{I}{r} \\ &= \frac{1.26 \times 10^{-6}}{2\pi} \times \frac{2.5}{0.10} \\ &= 5 \times 10^{-6} \text{ T out of the page (using Ampere's right-hand rule)} \end{aligned}$$

$$\vec{B}_{tot} = \vec{B}_{XA} + \vec{B}_{XB} \text{ (vector sum)}$$

If you choose out of the page as the positive direction:

$$\begin{aligned} B_{tot} &= {}^{-}2 \times 10^{-6} \text{ T} + {}^{+}5 \times 10^{-6} \text{ T} \\ &= {}^{+}3.2 \times 10^{-6} \text{ T (}^{+}\text{ sign means 'out of the page')} \end{aligned}$$

Alternatively:

$$\begin{aligned} B_{tot} &= 2 \times 10^{-6} \text{ T into page} + 5 \times 10^{-6} \text{ T out of page (opposite directions so subtract)} \\ &= 3.2 \times 10^{-6} \text{ T out of the page (the greater value 'wins')} \end{aligned}$$

FIGURE 9 The magnetic field strength at ground level underneath these 500 kV power lines is over the 0.1 μT safe level. You have to be 150 m away for it to be safe.

Combining magnetic fields in two dimensions

It becomes so much more difficult when you have to calculate the resultant magnetic field in two dimensions. You are expected to be able to do this and the process is similar to using vector addition in two dimensions in electrostatics.

WORKED EXAMPLE 7.2C

Two long parallel wires X and Y are positioned 28.3 cm apart and perpendicular to the page, as shown in Figure 10. They carry currents of 8 A and 6 A out of the page respectively. There is a point P directly under the wires and 20 cm away from each that makes a 90° angle to them. Calculate the magnetic field strength (magnitude and angle) at point P.

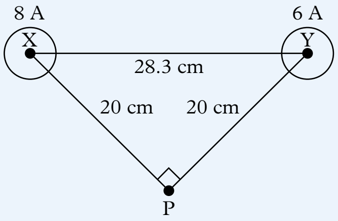


FIGURE 10 Fields in 2D

SOLUTION

First, calculate the field strength at point P due to X (B_{PX}), and to Y (B_{PY}) separately, as shown in Figure 11.

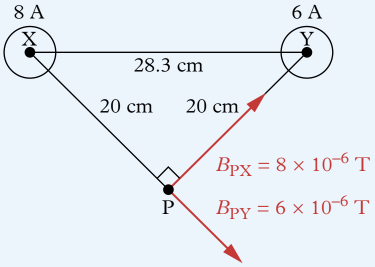


FIGURE 11 Determining the component vectors

$$B_{PX} = \frac{\mu_0 I_X}{2\pi r_{PX}}$$
$$= \frac{(4\pi \times 10^{-7}) \times I_X}{2\pi r_{PX}} \text{ (substitute } \mu_0)$$
$$= \frac{2 \times 10^{-7} \times 8}{0.20}$$
$$= 8 \times 10^{-6} \text{ T, perpendicular to line PX}$$

$$B_{PY} = \frac{\mu_0 I_Y}{2\pi r_{PY}}$$
$$= \frac{(4\pi \times 10^{-7}) \times I_Y}{2\pi r_{PY}} \text{ (substitute } \mu_0)$$
$$= \frac{2 \times 10^{-7} \times 6}{0.20}$$
$$= 6 \times 10^{-6} \text{ T, perpendicular to line PY}$$

Place the two vectors head-to-tail and calculate the resultant vector, as shown in Figure 12.

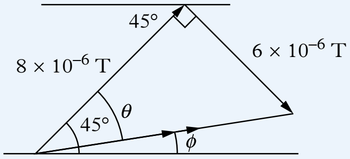


FIGURE 12 Combining component vectors to form a resultant vector

$$\theta = \tan^{-1}\frac{6 \times 10^{-6}}{8 \times 10^{-6}}$$
$$= 36.9°$$
$$\phi = 45 - 36.9$$
$$= 8.1° \text{ (to line joining the two wires)}$$

$$B_{net} = \sqrt{(6 \times 10^{-6})^2 + (8 \times 10^{-6})^2}$$
$$= 1 \times 10^{-5} \text{ T}$$

CHECK YOUR LEARNING 7.2

Describe and explain

1 **Explain** whether this is true: 'When you double the distance from a wire, you halve the field strength.'

2 **Clarify** whether the field strength at a fixed distance from a wire is proportional to the current.

Apply, analyse and interpret

3 **Determine** the magnetic field strength at a distance of 15 cm to the left of a wire that carries an electric current of:

- a 5.5 A north
- b 25 A west.

4 **Determine** the magnitude and direction of the current in the wire AB in Figure 13. The magnetic field at point P is 1.5 × 10^{-3} T and it is 1.0 cm from the wire.

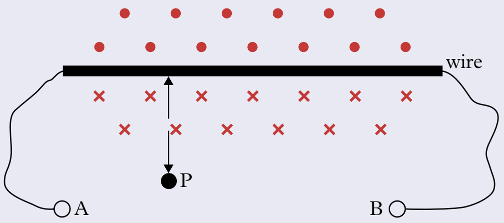


FIGURE 13 Field near wire

5 **Determine** the direction and magnitude of the magnetic field at points P_1 and P_2 in the diagram shown in Figure 14.

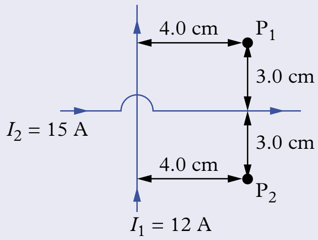


FIGURE 14 Field due to two wires

Investigate, evaluate and communicate

6 Two long parallel wires R and S are positioned 42.4 cm apart and perpendicular to the page (Figure 15). They carry currents of 3 A out of the page and 5 A into the page respectively. There is a point P directly under the wires and 30 cm away from each that makes a 90° angle to them. **Determine** the magnetic field strength (magnitude and angle) at point P.

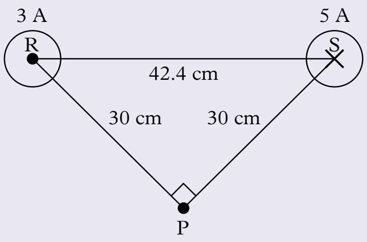


FIGURE 15 Field in two dimensions

7 **Consider** two conducting wires in a guitar amplifier that are producing unwanted magnetic fields. The engineers are trying to work out the magnetic field strength at various positions near the wires. The wires carry 2 A and 3 A respectively as shown in Figure 16.

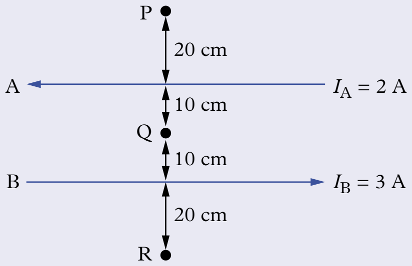


FIGURE 16 Wires in a guitar amp

a **Calculate** the magnetic field strength due to both wires at these positions.

i P ii Q iii R

b **Determine** the position (besides infinity) where the field strength would be zero.

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7.3 Solenoids

KEY IDEAS

In this section, you will learn about:

- solenoids.

FIGURE 1 Solenoids are used in many home appliances, such as washing machines.

In your earlier studies of electricity and magnetism, you probably made an electromagnet by winding lots of turns of wire into a coil on a cardboard tube, and when connected to a power supply the coil could pick up paperclips. What you made was a **solenoid** and it consisted of dozens of loops of wire. Solenoids are very useful in home appliances and in industry. They are used for car door locks, water pressure valves in washing machines, electromagnets, speakers and microphones, to name just a few. The properties of a solenoid can be understood by starting with a single loop (Figure 2).

Figure 2 shows a wire bent into a single circular loop. This loop can be considered as being made up of many small, straight segments each adding its individual magnetic field together at the centre of the loop where the field will be the strongest and will be directed through the loop as shown. The direction is once again determined by the right-hand rule.

If you take a plastic or cardboard tube and wind hundreds of turns of wire side by side, as shown in Figure 3, you will have produced a device called a solenoid. The word solenoid comes from the Greek *solen*, meaning 'tube'. This concentrates the magnetic field lines into a region of space that produces an almost perfectly uniform magnetic field within the hollow body of the solenoid.

solenoid
a long straight coil of wire used to generate a controlled and almost uniform magnetic field

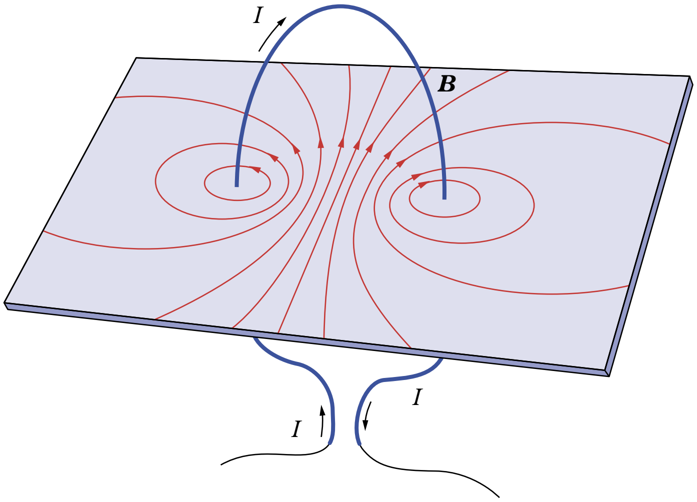

FIGURE 2 Representation of the field about a single loop

FIGURE 3 An air-cored solenoid of 200 turns

The magnetic field at the centre of a very long solenoid is constant, and is found to depend only on the current flowing in the coil as well as the number of turns per unit length of the solenoid. This type of field is illustrated in Figure 4, and the formula for the magnitude of the field strength in the solenoid's centre is:

$$B = \mu_0 nI$$

where B is the magnitude of the field strength in the solenoid's centre, μ_0 is the permeability constant of 1.26×10^{-6} T m A^{-1}, n is the number of turns per metre of length, and I is the current in ampere. To calculate the number of turns per metre, use the formula:

$$\text{no. of turns per metre} = \frac{\text{number of turns}}{\text{length of solenoid}}$$

$$n = \frac{N}{L}$$

The polarity of the solenoid's magnetic field is predicted with the right-hand rule for solenoids (Figure 4b). This states that if you grip the solenoid in the right hand so that your fingers naturally curl around the solenoid in the direction of conventional current flow, then the extended thumb will point to the effective north pole of the solenoid's magnetic field. The field lines are then drawn in such a way that they flow externally from the north pole towards the south pole at the coil's opposite end. Externally, the solenoid field has a very similar shape to that of a bar magnet. The magnetic field lines are continuous and extend down through the centre of the solenoid to create the uniform field.

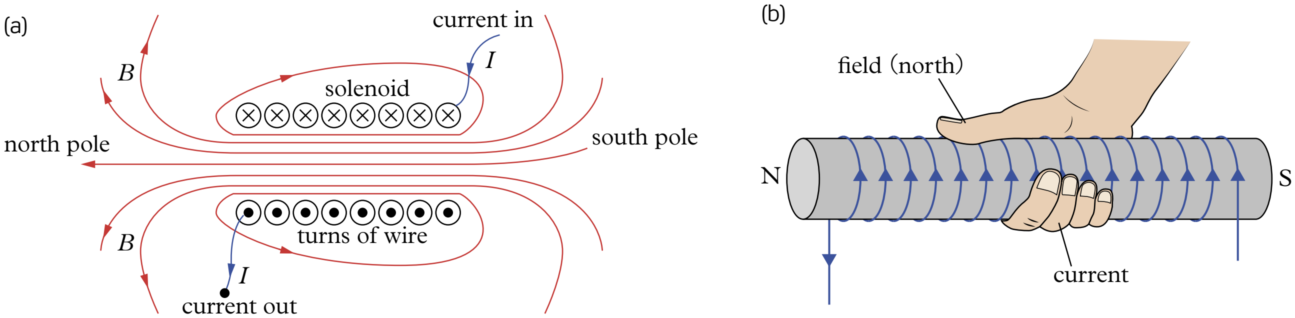

FIGURE 4 (a) Current flow and polarity for a solenoid; (b) using the right-hand rule for a solenoid

The solenoid can be made into an electromagnet if the hollow core contains a magnetically soft material. (Magnetically soft materials are those that become magnetised when a current flows but lose their magnetic properties when the current is turned off.) The core concentrates the lines of force and increases the magnetic strength through the induction principle. Iron–nickel alloys are the most commonly used material in the physical construction of electromagnet cores, in which they can increase magnetic field strengths several hundred times above that produced by the solenoid itself. The greatest advantage of electromagnet assemblies is that the magnetic field can be switched on or off simply by breaking the flow of current through the coil turns.

CHALLENGE 7.3

The hand rule

Devise a hand rule for the magnetic field due to an electron current in a solenoid.

WORKED EXAMPLE 7.3

A solenoid has a length of 25 cm and contains 600 turns arranged as shown in Figure 5. It carries a current of 3.0 A.

a Determine the magnetic field strength in the centre of the solenoid.

b Predict which end will be a north pole.

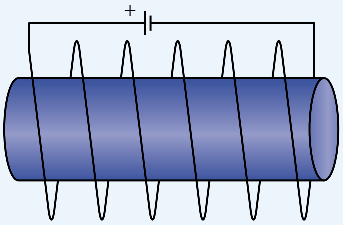

FIGURE 5 Current in a solenoid

SOLUTION

a

$$n = \frac{N}{L}$$
$$= \frac{600}{0.25}$$
$$= 2400 \text{ turns per metre}$$
$$B = \mu_0 nI$$
$$= 1.26 \times 10^{-6} \times 2400 \times 3.0$$
$$= 0.0091 \text{ T}$$

b Current is flowing down the front of the solenoid, therefore the thumb points to the right. The right end is north.

CHECK YOUR LEARNING 7.3

Describe and explain

1 **Clarify** whether the field strength inside a solenoid is proportional to the current.

2 **Identify** the magnetic polarity of ends A and B, for the diagrams of Figure 6.

Apply, analyse and interpret

3 **Determine** the magnetic field strength at the centre of a solenoid that has a length of 20 cm and contains 8000 turns. It carries a current of 15 A.

4 A 200 turn solenoid of length 50 cm has a magnetic field of 2.3×10^{-5} T. **Determine** the current flowing through it.

5 A solenoid has a length 70 cm and when a current of 12 A passes through it a magnetic field strength of 6.91×10^{-4} T is produced. **Determine** the number of turns in the solenoid.

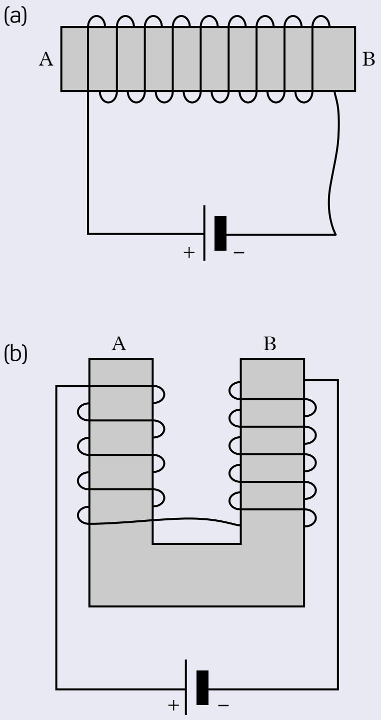

FIGURE 6 Field inside coil

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7.4 Magnetic forces on a moving charge

KEY IDEAS

In this section, you will learn about:

- forces on a charged particle
- forces on a current-carrying wire.

Recall that, in general, the strength (or intensity) of a field is defined by the size of the force acting on an object in the field. For a gravitational field it is the force on a mass, for an electric field it is a force on a charge, and for a magnetic field it is the force on a moving charge. There are two ways of defining the strength of the magnetic field: by the force on a moving charged particle, and by the force on a current-carrying wire.

Forces on a charged particle

If a moving particle with charge of +1 C enters a magnetic field it will experience a force that is dependent on the field strength and the velocity of the particle.

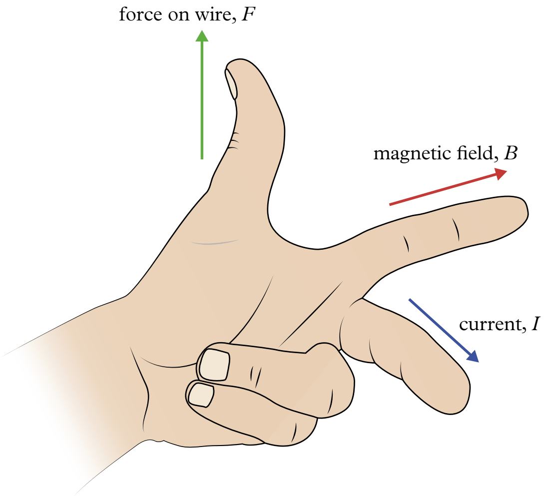

FIGURE 1 Fleming's left-hand rule

Direction of the force on a moving charged particle in a magnetic field

To determine the direction of the force on a positively charged particle, use Fleming's left-hand rule (Figure 1).

1. Hold your left hand so that your thumb, index finger and middle finger are at right angles to each other.
2. Place your hand so that your index finger is pointing in the direction of the magnetic field.
3. Point your middle finger in the direction of the motion of a positive charge.
4. Your thumb will now be pointing in the direction of the force.

You can now apply the rule to a situation in which a positive charge is being fired from the left into a magnetic field *B* that is 'out of the page' (as shown by the small dots in the circles in Figure 2a on the next page). Fleming's left-hand rule shows that the force is down the page, so the charge moves down the page.

For a negative charge, just reverse the direction of the electron current and pretend it is a conventional current (Figure 2b on the next page).

MANDATORY PRACTICAL
7.4 Force on a current-carrying wire in a magnetic field
Go to page 408 »

Study tip

When dealing with negative charges, you can use Fleming's left-hand rule but you should reverse the direction of the electron current.

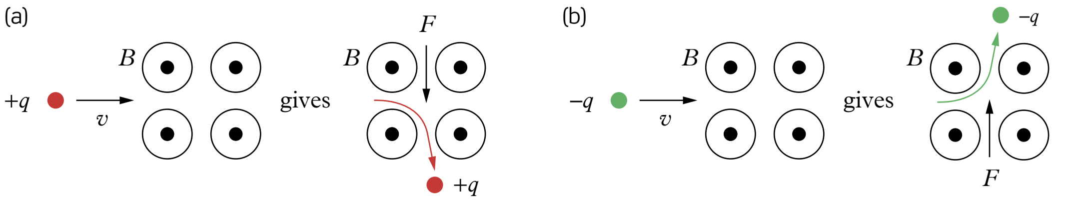

FIGURE 2 Use Fleming's left-hand rule to see how the positive and negative changes move in the situations above.

Cathode ray tubes

Study tip

To remember which finger is which in Fleming's left-hand rule, start with the thumb and label them as FBI (as in the US law enforcement agency).

A cathode ray tube (like the one shown in Figure 3), produces a beam of electrons when a high voltage is placed across the terminals. 'Cathode rays' was the name for electrons, before physicists knew what they were. In a cathode ray tube, the electrons are produced at the negative terminal (the cathode) on the left and are attracted to the positive terminal (the anode) on the right. The electrons skim along a zinc sulfide screen, which lights up (green) when the electron beam strikes it. A magnet brought up to the beam will deflect the beam. In this case, Fleming's left-hand rule indicates that the beam should be deflected downwards. Remember that it is a beam of negatively charged particles, so you assume the conventional current is from the right and thus the force is downwards.

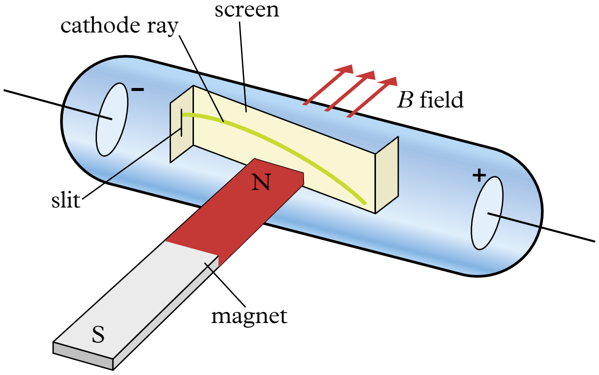

FIGURE 3 Cathode ray tube showing the deflection of an electron in a field that is facing into the page

Magnitude of the force on a moving charged particle in a magnetic field

As you learnt in Section 7.2, the SI unit of magnetic field strength is the tesla, symbol T. One tesla (1 T) is defined as the field intensity (field strength) generating one newton (N) of force per coulomb (C) of charge per metre per second (m s^{-1}) of speed at right angles to the field.

$$B = \frac{F}{qv}$$

$$F = qvB \text{ (at 90° to the field)}$$

$$F = qvB\sin\theta \text{ (at angle } \theta \text{ to the field)}$$

where B is the magnetic field strength, F is the force, q is the charge and v is the velocity of the charge.

Note that:

- if the direction of motion is parallel to the field ($\theta = 0°$), $F = 0$
- if the direction is perpendicular to the field ($\theta = 90°$), F is a maximum
- the magnitude of the force is directly proportional to q and v; that is, the greater the value of q and v the bigger the force.

WORKED EXAMPLE 7.4A

A +10 μC (1.0×10^{1}) charge moves at a speed of 250 m s^{-1} in a magnetic field of strength 1.5 T, as shown in Figure 4. Give your answer to 2 significant figures.

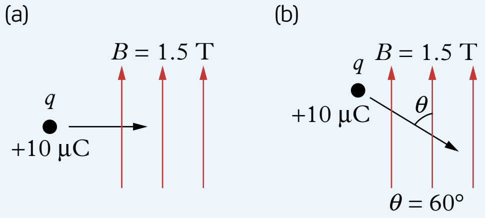

FIGURE 4 Charge moving in magnetic field

Calculate the force (including direction) acting on the charge if it makes an angle to the field as shown of:

a 90°

b 60°.

SOLUTION

a $q = 10 \times 10^{-6}$ C, $v = 250$ m s^{-1},

$B = 1.5$ T, $\theta = 90°$

$$F = qvB \sin 90°$$
$$= 10 \times 10^{-6} \times 250 \times 1.5 \times \sin 90°$$
$= 0.003\,75$ N out of the page (3.8×10^{-3} N to 2 sf)

b $q = 10 \times 10^{-6}$ C, $v = 250$ m s^{-1}, $B = 1.5$ T, $\theta = 60°$

$$F = qvB \sin 60°$$
$$= 10 \times 10^{-6} \times 250 \times 1.5 \times \sin 60°$$
$= 0.003\,284$ N out of the page (3.2×10^{-3} N to 2 sf)

Forces on a current-carrying wire

The second definition of magnetic field strength comes from simply considering the positive charges entering the field as being contained in a wire.

Magnitude of the force on a current-carrying wire in a magnetic field

The formula can easily be developed from the $F = qvB \sin\theta$ formula. Let the length of the wire in the field be L, and the time taken for charge to move this distance be t, then the velocity, v, of the charge is L divided by t:

$$F = qvB \sin\theta$$

Replace v by $\frac{L}{t}$:

$$F = \frac{qLB \sin\theta}{t}$$

Replace $\frac{q}{t}$ by I (the current in the wire):

$$F = BIL \sin\theta$$

Magnetic field strength, B, can therefore also be defined in terms of force, length, current, velocity and angle: one tesla (1 T) is the field intensity (or field strength) generating one newton (N) of force per ampere (A) of current per metre (m) of conductor.

Direction of the force on a current-carrying wire in a magnetic field

As discussed earlier, Fleming's left-hand rule can be used for determining the direction of the force on charged particles moving in a magnetic field. The rule can also be used for charges in a wire. If the particles are contained in a wire, you simply use your middle finger to point in the direction of the current in the wire (Figure 5). Additionally, if your index finger points in the direction of the field, your thumb will show the direction of the force on the wire.

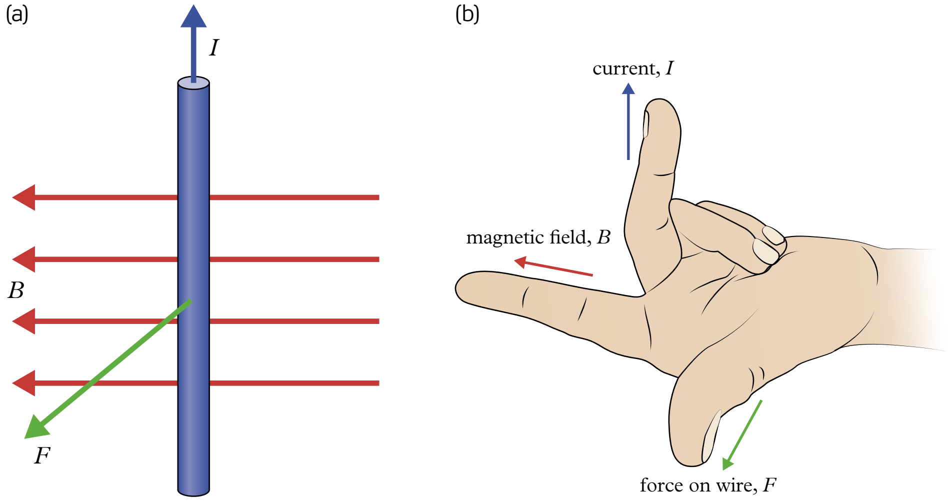

FIGURE 5 Fleming's left-hand rule used to determine the direction of the force on a current-carrying wire

WORKED EXAMPLE 7.4B

A 3.0 A current flows in a wire of length 1.5 m in a magnetic field of strength 5.0×10^{-5} T. Calculate the magnitude and direction of the force on the wire, given that:

a the wire makes an angle of 55° to the field, as shown in Figure 6a

b the wire is at 90° to the field, as shown in Figure 6b.

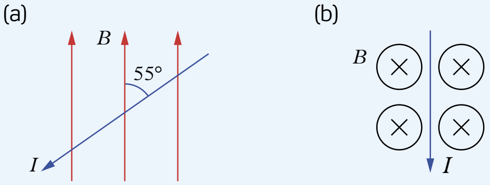

FIGURE 6 Current-carrying wire in magnetic field

SOLUTION

a $F = BIL \sin\theta$
$= 5 \times 10^{-5} \times 3.0 \times 1.5 \times \sin 55°$
$= 1.8 \times 10^{-4}$ N into the page

b $F = BIL \sin\theta$
$= 5 \times 10^{-5} \times 3.0 \times 1.5 \times \sin 90°$
$= 2.25 \times 10^{-4}$ N to the right

Loudspeakers: using magnetic fields

A moving-coil loudspeaker, as is commonly found in small radios, headphones or home stereo systems, is designed to change electrical signals from the output of an amplifier back into sound waves. The device relies on the force produced by a flowing current in a conductor within a magnetic field. A movable coil attached to a strengthened paper cone is placed over the central shaft of a permanent magnet. The magnetic field is radial, so that any movement of the coil (the 'voice coil') produced will be backwards and forwards, as shown in the diagram. The amplifier supplies variable frequency currents into the loudspeaker, and as the currents flow through the speaker voice coil it is forced to vibrate at the same rate as the current. The paper cone also vibrates backwards and forwards, moving the air and producing sound waves that match the amplitude and frequency of the original electric current signals.

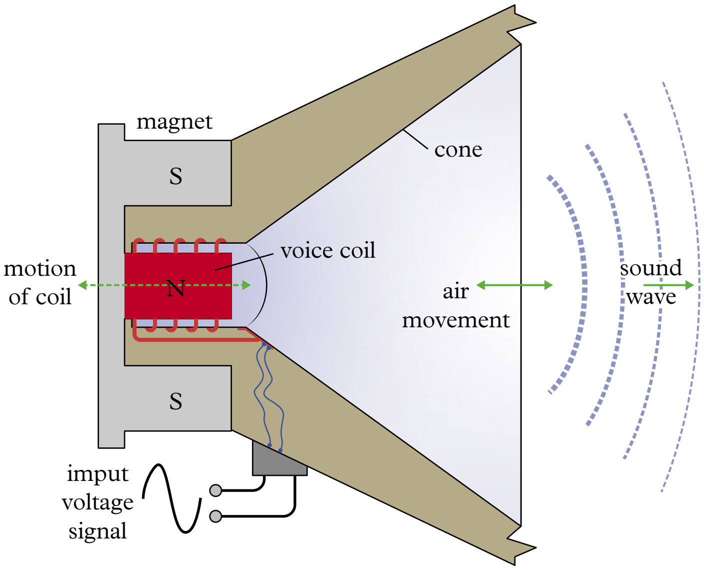

FIGURE 7 A loudspeaker turns electricity in a magnetic field into the motion of air particles

Electric meters

Both ammeter and voltmeter electric circuit measurement meters make use of the motor principle. They are a form of electrical meter called the moving-coil galvanometer, which uses the current flowing through a coil placed in a magnetic field to move a pointer along a calibrated scale; that is, current + magnetic field → motion.

Motors

Motors work on the principle of a current-carrying coil being forced to move when in a magnetic field. In the simplest case, the current in a coil is reversed every half rotation so there is a force that keeps the coil spinning in the same direction. In Figure 8, the current is moving in a clockwise direction. If you apply Fleming's left-hand rule, you will see that the force on the left-hand side of the coil side (labelled 1) is up, and that on the right-hand side (labelled 2) is down. This provides a turning force that rotates the coil. When the coil has moved through 180° and side 1 of the coil is over at the north pole, the direction of the current is reversed by a 'split-ring', which makes the coil keep rotating in the same direction. It is a clever idea and first put into practice in 1837.

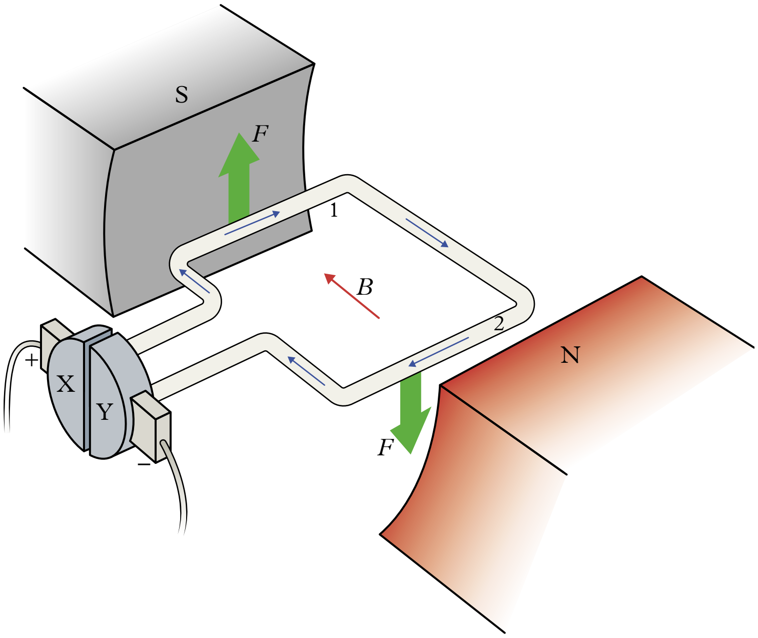

FIGURE 8 A simple DC motor

CHALLENGE 7.4A

Magnetic crystals in tuna

In 1986, scientists discovered that the yellowfin tuna has 10 million magnetic crystals in its skull. Propose how you could test whether tuna use these crystals to aid navigation, as has been suggested.

Study tip

When a wire is parallel to the field, the angle to the field is zero, so the force is zero. When the angle is 90°, the force is at a maximum.

CHALLENGE 7.4B

Strange properties of a magnet

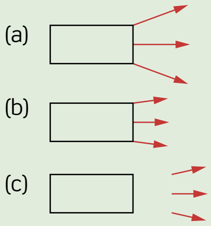

FIGURE 9 Possible changes to a magnetic field

1. Magnets are fed to cows to attract the bits of wire and nails that they eat with the grass. This seems a bit farfetched when you consider how long wire would last in a cow's acidic stomach. Design a test to see if this is possible. The acid in a cow's stomach is 0.17 M HCl (pH = 0.8).
2. Figure 9a shows a part of the field about an electromagnet. The current is now turned off. Propose whether the field lines would change as in Figure 9b or 9c.
3. People who sell magnetic pillows and wristbands don't seem to offer any scientific evidence for their healing claims. Perhaps it is just a placebo. Some horse magazines offer magnetic rugs for the comfort and protection of the animal. Evaluate this claim by reference to the literature.
4. Several species of aquatic bacteria swim along magnetic field lines. They have tiny chains of magnetite crystals of one domain each. When stirred, the bacteria swim north, which is towards the bottom (in the Northern Hemisphere where they live). Propose what they would do if this experiment was carried out at the equator or in the Southern Hemisphere. Justify your predictions if you can.

CASE STUDY 7.4

Nuclear fusion

One of the most active areas of research today is in the field of nuclear fusion. Scientists try to create and maintain the nuclear fusion reaction that drives the Sun. In order to do this, physicists need to hold extremely hot deuterium plasma (10^9 K) inside a closed container. Not an easy job!

Some success has been gained with devices such as the Joint European Torus (JET) experimental fusion reaction, which is basically a magnetic container in which the hot charged plasma is confined within a highly evacuated toroidal chamber (a donut-like chamber) by extremely powerful superconducting electromagnets.

These types of reactors are based on the tokomak field shape in which the plasma circulates around the torus. Presently these reactors require more energy input than is released during the brief periods of actual fusion that take place; however, they could prove to be an extremely valuable energy resource in the future.

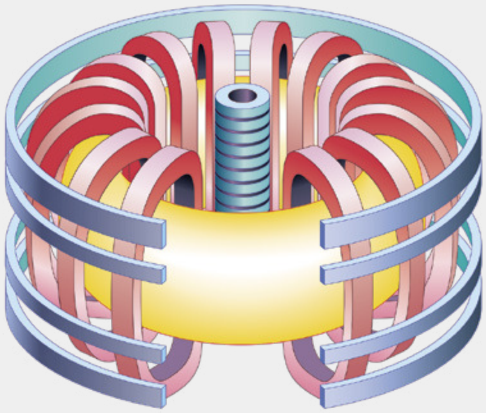

FIGURE 10 A tokamak nuclear fusion reactor in which hyper-warm plasma at a billion degrees Celsius (yellow) is contained by a magnetic bottle so that it doesn't melt the wall of the container

CHECK YOUR LEARNING 7.4

Describe and explain

1 **Explain** what it means when you say that 'force, field and motion of charge are all perpendicular to each other'.

Apply, analyse and interpret

2 **Analyse** Figure 11 to establish the direction of the charged particle *q* after it enters the magnetic field in each case.

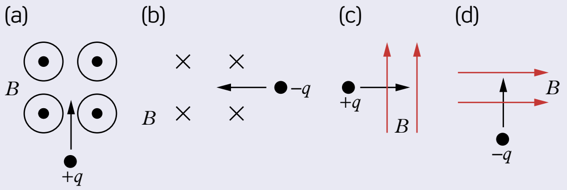

FIGURE 11 A charged particle in a field

3 **Determine** the magnitude and direction of the force on a current-carrying wire placed in a uniform magnetic field, as shown in Figure 12. The length of wire in the field is 50.0 cm.

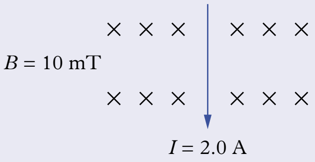

FIGURE 12 A 50 cm wire in a field

4 **Determine** the magnitude and direction of the force on a current-carrying wire that passes perpendicularly through a magnetic field as shown in Figure 13. The magnetic field strength is 1.5×10^{-3} T and the wire carries a current of 8.0 A.

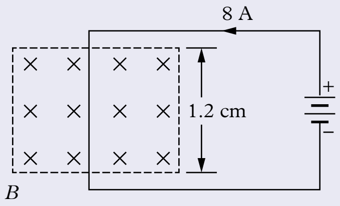

FIGURE 13 Wire moving in a magnetic field

5 **Determine** the force on a conductor of length 8.5 cm that is placed between the poles of a large magnet as shown in Figure 14. The wire conductor carries a current 25 A in the direction shown.

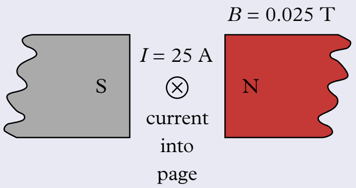

FIGURE 14 Current-carrying wire between poles of a magnet

Evaluate, investigate and communicate

6 Figure 15 is a diagram of a motor consisting of a rotating coil between the poles of a permanent magnet. The forces acting on the coil are shown by the arrows labelled ***F*** on the sides of the coil. **Decide**, with reasons, whether terminal X or Y is the positive terminal.

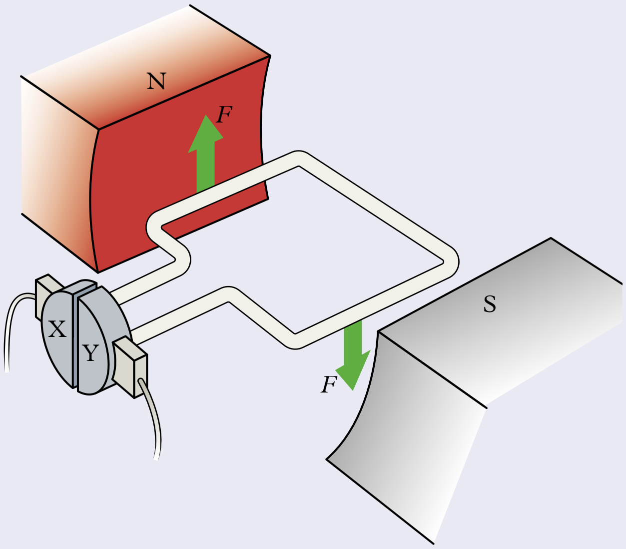

FIGURE 15 A motor consisting of a rotating coil between the poles of a permanent magnet

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SCIENCE AS A HUMAN ENDEAVOUR

7.5

The Square Kilometre Array (SKA)

KEY IDEAS

In this section, you will learn about:

- the Square Kilometre Array (SKA).

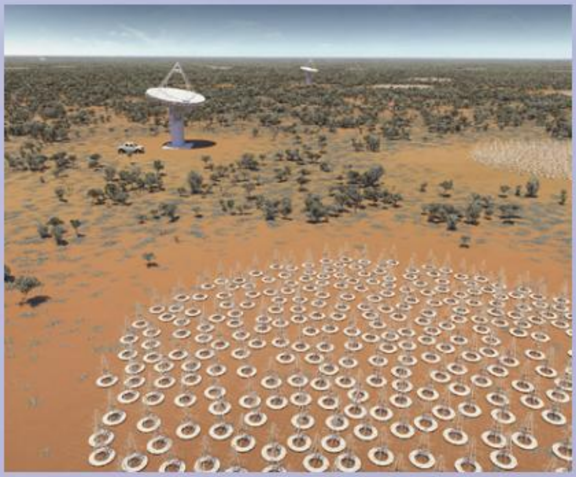


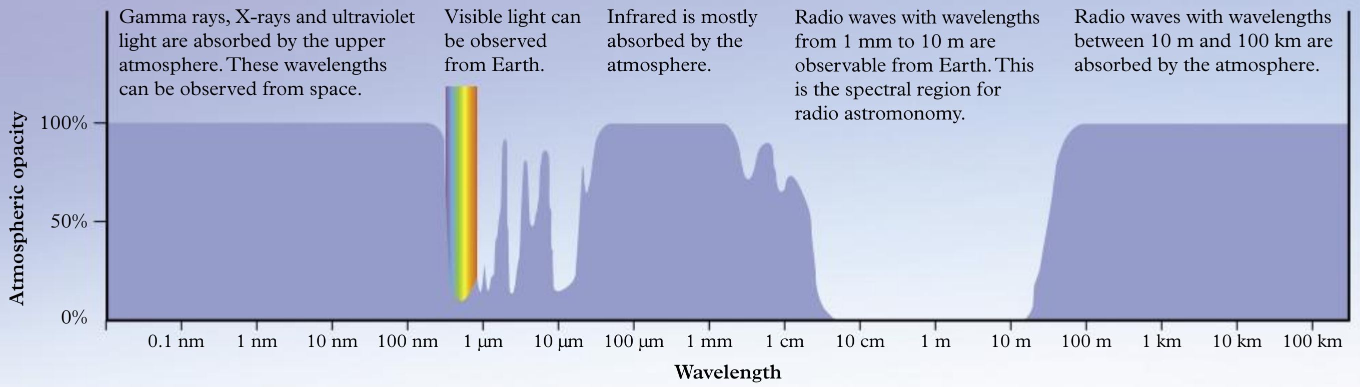
FIGURE 1 The Square Kilometre Array (artist's impression).

Scientists have always had the goal of being able to look further and further into deep space, and locate objects that can provide cues about the formation of the universe. Physicists have made a huge leap in this research with the Square Kilometre Array (SKA), a large multi-radio telescope project built in Australia and South Africa. The SKA will combine the signals received from thousands of small antennas spread over a distance of several thousand kilometres to simulate a single giant radio telescope capable of extremely high sensitivity with a total collecting area of approximately one square kilometre.

Principles of radio astronomy

Electromagnetic radiation has a vast range of wavelengths from a microscopic 1 nm up to an incredibly long 1000 km. Stars, galaxies and gas clouds generate radiation over this whole spectrum and detecting them allows physicists and astronomers to build up a picture of the universe out to the distant edges. The problem is that when this radiation arrives on Earth not all of it can penetrate the atmosphere. The two spectral regions that can pass through the atmosphere are the visible and radio waves (Figure 2). Visible light (400–800 nm) does get through, but there is some distortion. The atmosphere is transparent to radio waves in the region of 1 mm up to 10 m, so land-based radio telescopes are possible. Because radio waves can also pass through clouds of dust and gas in space, radio telescopes are also able to reveal objects that are not visible to optical telescopes.

A single telescope does not work well for radio waves. A single antenna (dish) provides a limited snapshot of the radio signals from space. Mathematically, the ability of a radio telescope to distinguish fine detail in the sky depends on the wavelength of the radio waves divided by the size of the antenna. In other words, to get finer detailed views of the sky, you need a small wavelength and a big antenna.



Gamma rays, X-rays and ultraviolet light are absorbed by the upper atmosphere. These wavelengths can be observed from space.

Visible light can be observed from Earth.

Infrared is mostly absorbed by the atmosphere.

Radio waves with wavelengths from 1 mm to 10 m are observable from Earth. This is the spectral region for radio astromonomy.

Radio waves with wavelengths between 10 m and 100 km are absorbed by the atmosphere.

FIGURE 2 Absorption of electromagnetic radiation by Earth's atmosphere

Radio telescopes observe long wavelengths (1 mm to 10 m), so even when you divide the shortest radio wavelengths (1 mm) by the diameter of the largest antennas, the resolution is still only similar to that of the unaided eye observing the sky. To have a resolution as good as that of optical telescopes, the size of the antenna of a radio telescope needs to be very large.

In 1974, British physicist Martin Ryle developed a new system for detecting faint and distant stars. He proposed that the views of a group of antennas spread over a large area could be combined to operate together as one gigantic telescope. This innovation won him a Nobel Prize in Physics.

This is the basis of the SKA, which has antennae scattered throughout the Western Australia desert as well as South Africa.

Aims of the SKA

There are five big questions physicists are trying to resolve with the SKA. These are the same questions students always ask about 'what's out there'. They make ideal research questions.

- *How were the first black holes and stars formed?* Being able to detect the radiation given off from black holes and stars formed 13 billion years ago will give a picture of the early universe.
- *How do galaxies evolve and what is dark energy?* The universe expands at an ever-increasing rate due to dark energy – and nobody knows what it is.
- *What generates giant magnetic fields in space?* Cosmic magnetism exists throughout the universe and affects how objects form, age and evolve.
- *Are we alone in the universe?* Detecting very faint radio transmissions might provide evidence for intelligent life among the stars.
- *Was Einstein right?* By studying pulsars and black holes, scientists will test Einstein's general theory of relativity and the laws of physics.

CHECK YOUR LEARNING 7.5

Describe and explain

1 **Recall** the two main principles used by the SKA to achieve high sensitivity.

2 **Explain** why it is not necessary to have a radio telescope in orbit around Earth to get a clear image.

Apply, analyse and interpret

3 It is said there are two windows in the atmosphere to view distant objects. **Infer** what is meant by this statement.

4 A long baseline is a key principle of the SKA. **Critique** the assertion that it would be better to have the second set of antennae as far away from Australia as possible, such as England.

Investigate, evaluate and communicate

5 A claim is made that 'the SKA will see the Big Bang'. **Propose** a research question that could support or refute this.

6 The following research question has been proposed: 'How does the SKA allow us to see 'further and fainter' than any other radio telescopes?' **Propose** one significant piece of evidence you would use to justify this.

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Australia's role in the Square Kilometre Array

CHAPTER 7

Review

Summary

7.1
- The region of influence in which a magnet exerts a force is called a magnetic field.
- The direction of magnetic flux can be represented conventionally as lines traversing from the north pole of a magnet to its south pole.

7.2
- Magnitudes of magnetic field strength vectors can be defined for basic electromagnetic applications such as single current-carrying wires, flat coils and solenoid coils, and are measured in units called tesla (T).
- Directions of electromagnetic forces and effects can be predicted using hand rules.

7.3
- A solenoid is a straight coil of wire used to create a controlled and almost uniform magnetic field.
- The polarity of a solenoid's magnetic field can be predicted using Ampere's right-hand rule, where you hold the solenoid in your right hand. The way your fingers curl is the direction of current flow, and the extended thumb will point to the north pole of the solenoid's magnetic field.

7.4
- Charged particles are influenced by magnetic fields in such a way that their path of travel is a curve of radius r by a force whose magnitude is $F = qvB\sin\theta$.
- The motor principle states that the force acting on a current-carrying conductor within a magnetic field is perpendicular to both the field and the direction of magnetic flux, and is given by $F = BIL\sin\theta$.
- The motor principle is the basis for the operation of simple DC electric motors.
- Many technological applications exist for basic magnets, electromagnets and their effects on charged particles.

7.5
- Radio astronomy allows scientists to view to the edge of the universe and it relies on two principles: using radiation that is not absorbed in the atmosphere, and using many widely separated antennae.

Key terms

- magnetic field
- magnetic field line
- magnetism
- solenoid
- tesla

Key formulas

Force on a charged particle	$F = qvB\sin\theta$
Force on a wire	$F = BIL\sin\theta$
Field about a wire	$B = \frac{\mu_0 I}{2\pi r}$
Field inside a solenoid	$B = \mu_0 nI$
Magnetic constant	$\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A straight wire carries a current into the page as shown in Figure 1. What is the direction of the magnetic field at a point east of the wire?

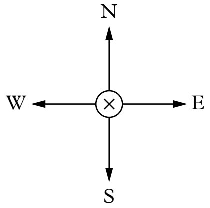

FIGURE 1 Field about a wire

A north **B** south

C west **D** east

2 A circular loop of wire is facing you and has an anticlockwise current running through it. What is the direction of the magnetic field inside the loop?

A north **B** south

C into the page **D** out of the page

3 A wire with mass m and length L has a current I flowing to the left of the page (Figure 2).

FIGURE 2 Wire in magnetic and gravitational field

What would be the direction of a magnetic field that provides a magnetic force that could cancel out the gravitational force on the wire?

A up **B** down

C out of the page **D** into the page

4 If the magnitude of the magnetic field is B in Question 3, what must the magnitude of this field be to cancel out the gravitational force on the wire?

A $\frac{mg}{IL}$ **B** $\frac{m}{IL}$ **C** $\frac{IL}{mg}$ **D** $\frac{mgL}{I}$

5 The diagram in Figure 3 shows a current-carrying wire in a magnetic field. Which one of the following best describes the direction of the force on the wire?

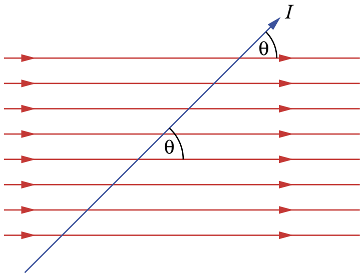

FIGURE 3 Wire in magnetic field

A out of the page

B into the page

C perpendicular to angle θ

D at angle θ to the direction of the field

Short answer

Describe and explain

★ 6 **Describe** a similarity and a difference between a gravitational field and a magnetic field.

★ 7 **Recall** whether the force is a maximum or a minimum when an electric current and a magnetic field are parallel.

★★ 8 **Calculate** the field strength inside a solenoid made up of 800 turns of wire on a 20 cm length of plastic tube if the wire carries a current of 700 mA.

★★ 9 A proton of mass 1.67×10^{-27} kg and charge $+1.6 \times 10^{-19}$ C enters a magnetic field of strength $B = 3.0 \times 10^{-2}$ T at right angles to the field and with a velocity of 2.5×10^{5} m s^{-1}.

a **Calculate** the magnitude of the force on the proton.

b **Calculate** its acceleration.

★★ 10 **Clarify** whether the direction of a magnetic field is defined in terms of the force on a positive charge or an electron.

Apply, analyse and interpret

★★ **11** **Determine** the direction of an electric charge after it enters each of the magnetic fields shown in Figure 4.

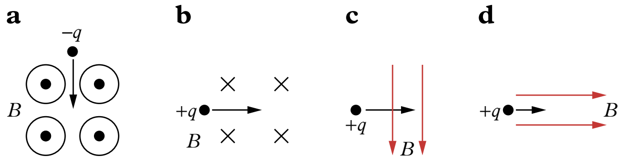

FIGURE 4 Motion of a charged particle in a field

★★ **12** An electron ($q_e = -1.6 \times 10^{-19}$ C) is fired at right angles into a 1.2 T uniform magnetic field at 3×10^7 m s^{-1}. **Determine** the magnitude of the force acting on it.

★★ **13** A wire of length 1.4 m is placed entirely in a magnetic field of strength 0.85 T and carries a current of 2.37 A (Figure 5). The wire experiences a force of 1.19 N.

- **a** **Determine** the angle θ it must make with the field.
- **b** **Determine** the direction of the force.

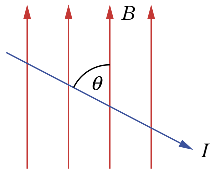

FIGURE 5 Wire at angle to field

★★ **14** Two parallel conductors 40.0 mm apart carry currents of 3.0 A (left wire) and 2.0 A (right wire), as shown in Figure 6.

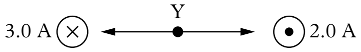

FIGURE 6 Field between two wires

- **a** **Calculate** the magnitude and direction of the magnetic field at point Y midway between the wires.
- **b** **Determine** where, other than infinity, the magnetic field strength would be zero.

★★★ **15** A coil of wire is suspended from a spring balance between the poles of two magnets. The rectangular coil is 80 cm high and 10 cm wide, and has 100 turns of wire. In an experiment, the spring balance readings were recorded for different currents. The apparatus is shown in Figure 7 and the results in Table 1.

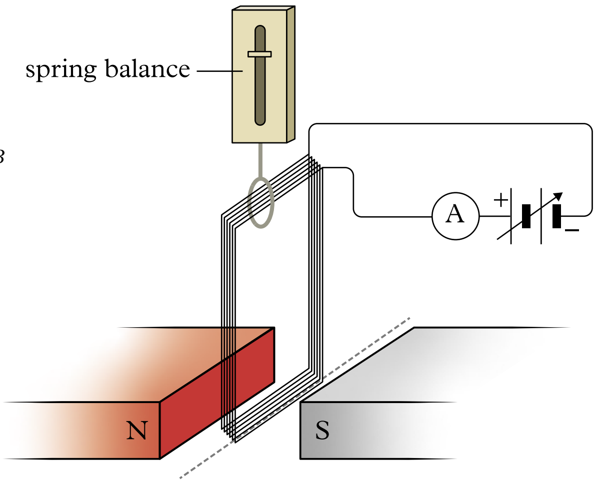

FIGURE 7 Square loop in a magnetic field

TABLE 1 Experimental results

Current (A)	Force (N)
0.5	3.5
1.5	5.0
2.0	5.6
3.0	6.8
3.5	7.5
4.0	8.3
5.0	9.5

- **a** **Construct** a graph of force (N, vertical axis) versus current (A, horizontal axis).
- **b** **Determine** the weight of the coil.
- **c** **Calculate** the magnetic field strength.
- **d** **Determine** the current direction as clockwise or anticlockwise.
- **e** The current is adjusted so that the balance reads zero. **Determine** the current that now flows in the coil and the direction in which it flows.

Investigate, evaluate and communicate

★★ **16** A wire is connected to a battery and allowed to hang between the poles of a horseshoe magnet as shown in Figure 8. **Predict** the motion, if any, of the wire. **Explain** your answer.

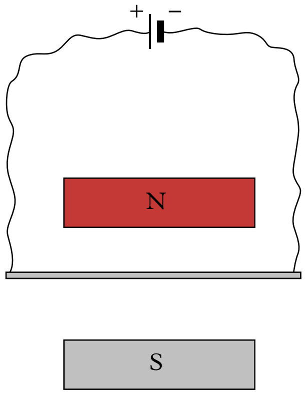

FIGURE 8 Wire suspended between poles of a magnet

★★★ **17** The diagrams in Figure 9 show a current-carrying wire near the poles of an electromagnet. For each diagram, **predict** the direction of the induced force acting on the conductor if the direction of current flow is determined by the battery.

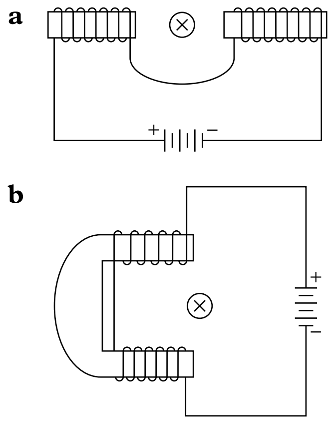

FIGURE 9 Wire near an electromagnet

★★★ **18** Figure 10 shows a 3500 turn circular solenoid of diameter 3.0 cm and length 70.0 cm carrying an input current of 75 mA flowing east. The current circulates clockwise as shown. **Predict** the field strength (magnitude and direction) at point X, the centre of the loop.

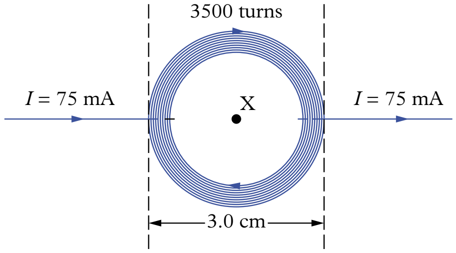

FIGURE 10 Field inside a solenoid

★★★ **19** A metal rod XY is 5.0 cm long. It lies on two metal rails connected to a DC supply. The rod and rails are balanced on a flat insulator base in a magnetic field of strength 0.20 T. A current is then passed through the rod causing a downwards movement. A mass, m, of 1.0 g is needed to restore the system to a level position, as shown in Figure 11. **Determine** the direction and magnitude of the current in rod XY. Use $g = 9.8\ \text{m s}^{-2}$.

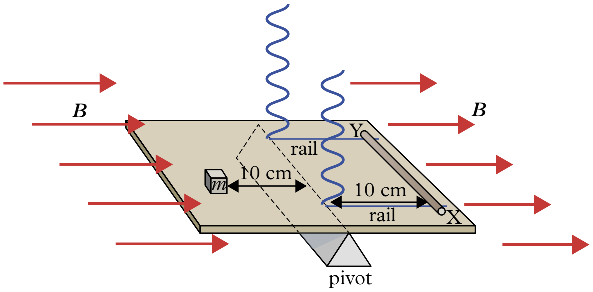

FIGURE 11 Rod in a field

★★★ **20** Two parallel wires carry the same current and in the same direction (Figure 12).

a **Calculate** the magnetic field strength at positions A, B and C.

b **Determine** the position, if any, where the field strength would be zero.

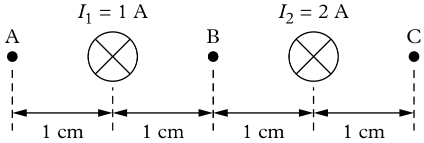

FIGURE 12 Force between parallel wires

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CHAPTER 8

Electromagnetic induction and radiation

One of the greatest achievements of physics was the discovery of electricity generation in 1830. The principles that were discovered then remain the basis for most of our electricity production today. The idea is quite simple: rotate a coil of wire in a magnetic field and the kinetic energy is transformed into electrical energy. You may have seen it in those little torches that you turn by hand and the bulb lights up, or the bike light generator that rests against the tyre and turns when the wheel turns. You put kinetic energy in and get electrical energy out. It really comes down to what Faraday discovered in 1830 – cut a magnetic field with a wire and you get electricity. However, it was Faraday's mental model of 'lines of magnetic flux' that was so crucial to the successful development of motors and generators, which were to dominate the 19th century.

OBJECTIVES

- → Define the terms 'magnetic flux', 'magnetic flux density', 'electromagnetic induction', 'electromotive force (EMF)', 'Faraday's law' and 'Lenz's law'.
- → Solve problems involving the magnetic flux in an electric current-carrying loop.
- → Describe the process of inducing an EMF across a moving conductor in a magnetic field.
- → Solve problems involving Faraday's law and Lenz's law.
- → Explain how Lenz's law is consistent with the principle of conservation of energy.
- → Explain how transformers work in terms of Faraday's law and electromagnetic induction.

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FIGURE 1 Electric guitar pickups as on this Fender Stratocaster generate an electric current from the movement of the steel strings. This is electromagnetic induction in action.

MAKES YOU WONDER

In this chapter you will be examining some aspects of electric charges in motion that will help to answer questions such as these:

- → Where is the moving coil or magnet in a guitar pickup?
- → How can a transformer turn 240 V into 10 V? Where does the energy go?
- → Why is it that Australia has 240 V power outlets but the USA has 110 V? Which is better?
- → If a 240 V shock can electrocute you, why doesn't 10 000 V?

PRACTICALS

SUGGESTED PRACTICAL	**8.2** Induction of current using a magnet and coil
SUGGESTED PRACTICAL	**8.4** Induced EMF from an AC generator

8.1

Magnetic flux

KEY IDEAS

In this section, you will learn about:

- magnetic flux and magnetic flux density
- solving problems involving magnetic flux and flux density.

When Michael Faraday developed his model for magnetic fields around current-carrying wires, he conceived the idea of lines of magnetic flux. Faraday was using the word 'flux', which is derived from the Latin *fluere*, meaning 'to flow'. Faraday thought of the wire as giving out a flow of magnetic force.

The term **magnetic flux** has stuck, even though it is now used as a visual representation of the magnetic field.

Physicists began defining magnetic field strength in terms of flux lines and how close they were together, namely **magnetic flux density**. They stated that the magnetic flux had the symbol, ϕ ('phi, which rhymes with pie), as it sounds like 'f' for flux.

magnetic flux
a measurement of the total magnetic field that passes through a given area; a measure of the number of magnetic field lines passing through the given area (symbol: ϕ; SI unit: weber; unit symbol: Wb)

magnetic flux density
the strength of a magnetic field or the number of magnetic field lines per unit area (symbol: B; SI unit: weber per square metre; unit symbol: Wb m^{-2} or T)

Study tip

In Chapter 7, magnetic field strength (tesla, T) is defined in terms of the force acting on a moving charge, either as a particle in a magnetic field, or as a current-carrying wire in a field. The new definition above (magnetic field lines per unit area, or weber per square metre, Wb m^{-2}) is just a visual way of conceptualising the same quantity.

Flux through a loop perpendicular to the field

Magnetic flux is measured in the unit weber (pronounced 'vey-ber'), after the German physicist Wilhelm Weber (1804–1891). Weber has the symbol Wb and is equal to the amount of flux flowing through an area of 1 m^2 in a magnetic field of strength 1 tesla (T):

$$\text{magnetic field strength} = \text{flux density}$$
$$1\ \text{T} = 1\ \text{Wb m}^{-2}$$

If a single arrow was used to represent 1 line of flux (1 Wb), then 4 arrows flowing through an area of 1 m^2 would be a flux density of 4 Wb m^{-2} or 4 T (Figure 1). The quantities 'flux density' and 'field strength' can be used interchangeably. The formula is thus:

$$\text{magnetic field strength} = \frac{\text{magnetic flux}}{\text{area}}$$
$$B = \frac{\phi}{A}$$

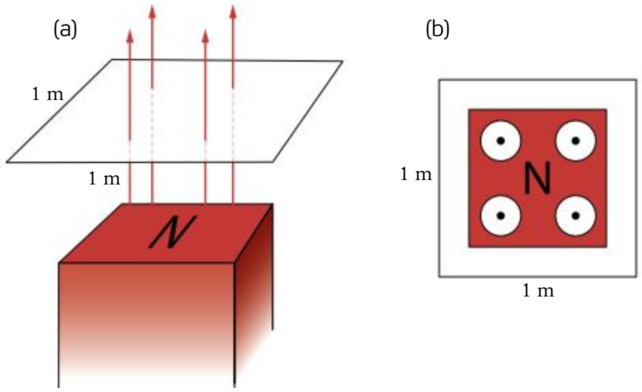

FIGURE 1 (a) Four lines of flux through an area of 1 m^2 represents a magnetic field strength of 1 Wb m^{-2} or 1 T. (b) B = 4 T as seen from above with flux lines pointing towards you.

Earth's magnetic field strength

The strength of Earth's magnetic field is about 5×10^{-5} T. In Brisbane, Earth's magnetic field is angled at 57° to the horizontal. An area of Brisbane's CBD (about 1.9 km^2) would contain just one flux line, as shown in Figure 2.

It is unrealistic to represent 1 Wb by one line, so the lines are used to show the relative amount of flux 'flowing' through an area. Note that the term is that flux 'flows' through an area. Another term is flux 'threading' an area such as a loop. A loop can be round, square, rectangular, oval and so on. Simply, in a loop a wire goes around something and the two ends come together.

FIGURE 2 An area of 1.9 km^2 of Brisbane's CBD is marked out to show the area enclosing 1 Wb of flux due to Earth's magnetic field.

WORKED EXAMPLE 8.1A

Calculate the flux threading a circular loop of diameter 24 cm with the plane of the loop at right angles to a field of strength 5.0 mT.

SOLUTION

Diameter, $d = 24 \text{ cm} = 0.24 \text{ m}$

Radius, $r = \frac{d}{2} = \frac{0.24}{2} = 0.12 \text{ m}$

$$\begin{aligned} \text{Area, } A &= \pi r^2 \\ &= \pi \times 0.12^2 \\ &= 0.045 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \phi &= BA \\ &= 5.0 \times 10^{-3} \times 0.045 \\ &= 2.3 \times 10^{-4} \text{ Wb (2 sf)} \end{aligned}$$

Flux at an angle to a field

When the plane of the loop is not at 90° to the field, fewer flux lines will thread the same size loop. When the plane of the loop is parallel to the field for instance, no lines will thread the loop, so the flux in the loop must be zero. A formula is needed that takes the angle into account. The area is a vector quantity and points in the direction perpendicular to its surface (as shown in Figure 3). Because of that the cosine function is used. The product of two vectors such as B and A (the 'dot' product) is given by the product of the two magnitudes multiplied by the cosine of the angle between them.

This is a tricky concept and might take a while to digest. The important part to remember is that the angle a loop makes with a field is the angle between the perpendicular to the loop and the field. That is, when the plane of the loop is at 90° to the field, the perpendicular is at an angle $\theta = 0°$ to the field.

$$\phi = BA\cos\theta \text{ (magnetic flux formula)}$$

where ϕ is the magnetic flux, B is magnetic field strength, A is the area of the loop and θ is the angle that the perpendicular to the plane of the loop makes with a field.

Study tip

When completing questions on loops in magnetic fields, you should check the wording to see if it says the angle is to 'the plane of the loop' or 'perpendicular to the loop'. You should be able to convert from one to the other. If θ is in the equation, use perpendicular.

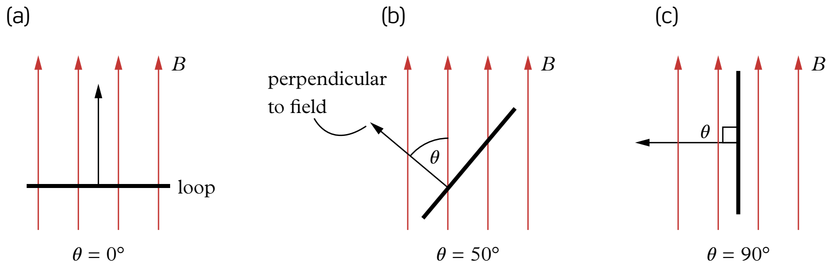

FIGURE 3 A loop rotating in a magnetic field. Note that θ is the angle between the perpendicular to the loop, and the direction of the field.

Consider a loop of 1.0 m^2 with four lines of flux threading it at right angles (Figure 3). This is a magnetic field strength of 4.0 T. The angle θ is the angle perpendicular to the angle the loop (black arrow) makes with the direction of the field (red arrows). In Figure 3a, the loop's perpendicular is parallel to the field, so the angle $\theta = 0°$. In Figure 3b, the loop's perpendicular is at 50° to the field, so $\theta = 50°$. In Figure 3c, the loop's perpendicular is at right angles to the field, so the angle $\theta = 90°$. The calculations for this are shown in Worked example 8.1B.

WORKED EXAMPLE 8.1B

A loop of area 1.0 m^2 is placed in a magnetic field of strength 4.0 T, as shown in Figure 3. Calculate the magnetic flux threading the loop when its perpendicular makes an angle to the field of:

a 50°

b 90°

SOLUTION

a $\phi = BA\cos\theta$

$= 4.0 \times 1.0 \times \cos 50°$

$= 2.6\ \text{Wb}$ (2 sf)

b $\phi = BA\cos\theta$

$= 4.0 \times 1.0 \times \cos 90°$

$= 0.0\ \text{Wb}$ (2 sf)

Flux inside a solenoid

A solenoid that is carrying an electric current will have a magnetic field inside running from one end to the other, so there will be lines of magnetic flux from one end to the other. To determine the amount of flux, you need to know the magnetic field strength, B, inside the solenoid, and the cross-sectional area, A. Recall from Chapter 7, the formula for the magnitude of the field strength in the solenoid's centre is $B = \mu_0 nI$, where B is the magnitude of the field strength in the solenoid's centre, μ_0 is the permeability constant of 1.26×10^{-6} T m A^{-1}, n is the number of turns per metre of length of the solenoid, and I is the current in ampere.

WORKED EXAMPLE 8.1C

A 25 cm long solenoid of diameter 5.4 cm with 1200 turns of wire carries a current of 1.5 A. Determine the flux inside the solenoid.

SOLUTION

You know the solenoid formula from Chapter 7, so you need to calculate the number of turns per metre (n), the field strength (B), and area (A), thus:

$$\begin{aligned} d &= 5.4 \text{ cm} \\ &= 0.054 \text{ m}, \\ L &= 25 \text{ cm} \\ &= 0.25 \text{ m}, \\ N &= 1200, \\ I &= 1.5 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Radius of solenoid, } R &= \frac{0.054}{2} \\ &= 0.027 \text{ m (Note: use } R \text{ for radius of a solenoid or loop.)} \end{aligned}$$

$$\begin{aligned} \text{Number of turns per metre, } n &= \frac{N}{L} \\ &= \frac{1200}{0.25} \\ &= 4800 \text{ turns per metre} \end{aligned}$$

$$\begin{aligned} \text{Cross-sectional area, } A_\perp &= \pi R^2 \\ &= \pi \times (0.027)^2 \\ &= 0.0023 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Field strength, } B &= \mu_0 nI \\ &= 1.26 \times 10^{-6} \times 4800 \times 1.5 \\ &= 0.0091 \text{ T} \end{aligned}$$

$$\begin{aligned} \text{Flux, } \phi &= BA_\perp \\ &= 0.0091 \text{ T} \times 0.0023 \text{ m}^2 \\ &= 2.1 \times 10^{-5} \text{ Wb (2 sf)} \end{aligned}$$

CHALLENGE 8.1

Solenoids vs loops

A solenoid is made up of many loops placed together, so you should also be able to calculate the flux inside a single loop. The field strength, B, at the centre of a loop of radius R and carrying current I is given by the field in a loop formula: $B_{\text{loop}} = \frac{\mu_0 I}{2R}$. Capital R is used for devices such as loops and solenoids, and lower case r for the radial distance to a point in a magnetic or electric field. Your challenge is to show that the flux inside a single loop of radius 0.025 m when a current of 1.5 A flows is 7.40×10^{-8} Wb.

CHECK YOUR LEARNING 8.1

Describe and explain

1 **Explain** the difference between the terms ‘magnetic flux’ and ‘magnetic flux density’.

2 **Calculate** the flux threading a rectangular loop measuring 10 cm × 20 cm that is placed in a field of strength 4 mT, as shown in Figure 4.

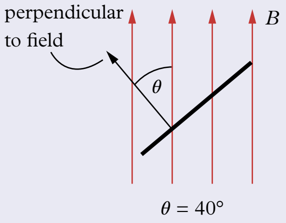

FIGURE 4 Loop at an angle in a field

Apply, analyse and interpret

3 **Determine** the flux inside a solenoid of diameter 4 cm, given that the solenoid has 800 turns in its length of 15 cm and carries a current of 1.5 A.

4 A loop is placed in a magnetic field, as shown in red in Figure 5. **Deduce** the angle between the perpendicular to the loop and the field.

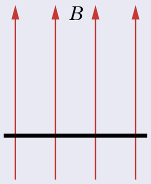

FIGURE 5 Field through a loop

Investigate, evaluate and communicate

5 **Propose** the maximum flux you can achieve through a loop of wire with a perimeter of 100 cm in a 5 mT magnetic field. **Justify** your claim with a reasoned explanation.

6 A loop of area 0.10 m^2 has its perpendicular parallel to the magnetic field. A student claims that the change in flux will be the same when the plane is rotated from 0° to 30° as when it is rotated from 30° to 60°. **Evaluate** this claim.

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8.2

Electromagnetic induction

KEY IDEAS

In this section, you will learn about:

- the process of inducing an EMF across a moving conductor in a magnetic field
- solving problems involving Faraday's law.

In the early 1800s, there were three researchers working on electromagnetism: Andre Marie Ampere, Michael Faraday and Joseph Henry. They were all following up Ørsted's finding that an electric current produces a magnetic field. It seemed that there should symmetry: if electricity produces magnetism, then magnetism should be able to produce electricity.

In 1831, Michael Faraday conducted a simple experiment in which he brought a bar magnet to the end of a coil connected to a centre-reading microammeter. As the magnet approached, the needle deflected to the left (Figure 1a). When the magnet was stationary (Figure 1b), the needle read zero. But, when the magnet was pulled away from the coil (Figure 1c), the needle deflected to the right. Faraday published these results and he is now considered to be the discoverer of **electromagnetic induction**.

electromagnetic induction
the production of an electromotive force (EMF) or voltage across an electrical conductor due to its dynamic interaction with a magnetic field

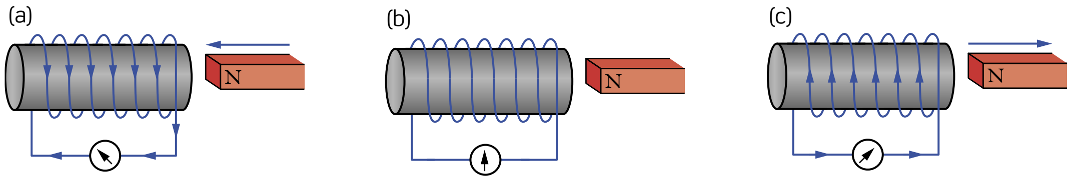

FIGURE 1 Faraday's experiment from 1831 is easily repeated in the classroom. (a) The magnet moves towards the solenoid. (b) The magnet is stationary. (c) The magnet moves away from the solenoid.

Study tip

The symbol for electromotive force is either emf or EMF; both can be used.

When there is relative motion between the magnet and the coil, there is electromagnetic induction; that is, an **electromotive force** (EMF) or voltage is induced in the coil. If the coil is part of a circuit, then the induced EMF produces a current.

The direction of current flow in the coil can be determined and will be discussed later when Lenz's law is covered. Now, you will look at what affects the voltage.

electromotive force
(EMF) a difference in potential that tends to give rise to an electric current: it is measured in volts (V)

Factors affecting voltage

When you move a magnet in and out of a coil, there are five variables that you can change to increase the EMF (voltage):

TABLE 1

Variable	Increase in EMF
Magnetic field strength (B)	Use a stronger magnet or two magnets held together with a rubber band (with their like poles side by side) and the voltage will increase.
Time interval (Δt)	Decrease the time taken for one back-and-forth oscillation of the magnet. This means increasing the speed.
Area (A)	This is difficult, but a bigger coil area means more lines of flux fit inside and the voltage will be greater.
Angle (θ)	As the angle between the direction of the magnet and the axis of the coil decreases, the voltage increases. That is, the closer the angle between the perpendicular to the loops of the coil and the direction of the incoming magnetic field is to zero, the greater the voltage.
Number of turns (N)	The greater the number of turns the greater the voltage. For a coil wound so that the turns are close to each other, the change in flux through each turn is the same, and an EMF appears across each turn, so for n turns, the total EMF is n times the EMF of one turn.

SUGGESTED PRACTICAL
8.2 Induction of current using a magnet and coil
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Faraday's law
a law stating that when the magnetic flux linking a circuit changes, an electromotive force (EMF) is induced in the circuit proportional to the rate of change of the flux linkage

These factors all come together in **Faraday's law**:

$$\text{EMF} = -\frac{n\Delta(BA_{\perp})}{\Delta t} \qquad \text{(Faraday's law, first version)}$$

$$= -\frac{n\Delta(BA\cos\theta)}{\Delta t} \qquad \text{(for any angle } \theta\text{)}$$

$$\text{but } \phi = BA\cos\theta$$

$$\text{EMF} = -n\frac{\Delta\phi}{\Delta t} \qquad \text{(Faraday's law, second version)}$$

That is, the EMF equals the rate of change of magnetic flux.

The negative sign indicates that the induced EMF acts to oppose any change in magnetic flux.

Calculating the magnitude of the EMF

As you have seen, there are a number of scenarios that will generate an EMF. You can look at these in turn by changing one of the factors B, A, θ, n, or Δt and keeping the others constant.

This gives rise to these main situations:

1 changing field strength; but keeping area and angle constant

$$\text{EMF} = \frac{-n\Delta(BA\cos\theta)}{\Delta t} = \frac{-nA\cos\theta(B_f - B_i)}{\Delta t}$$

2 changing area; but keeping field strength and angle constant

$$\text{EMF} = \frac{-n\Delta(BA\cos\theta)}{\Delta t} = \frac{-nB\cos\theta(A_f - A_i)}{\Delta t}$$

3 changing angle; but keeping field strength and area constant

$$\text{EMF} = \frac{-n\Delta(BA\cos\theta)}{\Delta t} = \frac{-nBA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

For changes to time (Δt) and number of turns (n), the value can just be substituted in.

Study tip

The negative sign in Faraday's law can be omitted if the EMF is stated as the absolute value:

$|\text{EMF}| = n\frac{\Delta\phi}{\Delta t}$

Study tip

You are not expected to be able to use Ohm's law ($R = \frac{V}{I}$) in Units 3 & 4 external assessment.

WORKED EXAMPLE 8.2A

A magnetic field of strength 4.0 mT threads (passes through) a loop of area 1.5 m^2 at right angles to the plane of the loop (Figure 2). The field strength is suddenly reduced to 1.0 mT in 0.50 s.

a Calculate the change in flux.

b Calculate the EMF produced.

SOLUTION

In this case, the value of magnetic field strength (B) is changing, while the angle (θ) and area (A) are held constant.

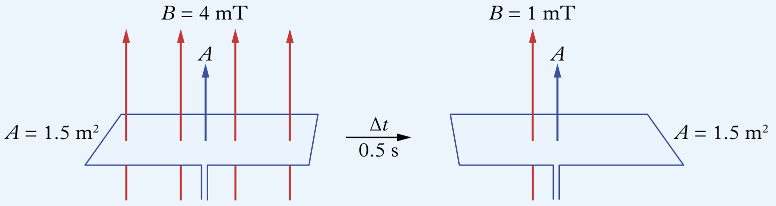

FIGURE 2 Changing flux in stationary loop

a $B_i = 4 \times 10^{-3}$ T, $B_f = 1 \times 10^{-3}$ T, $A = 1.5$ m², $\Delta t = 0.50$ s

$$\begin{aligned}\Delta\phi &= \Delta(BA\cos\theta)\\ &= A \times \Delta B \text{ (as the angle } \theta = 0°\text{, thus } \cos\theta = 1)\\ &= A(B_f - B_i)\\ &= 1.5 \times (1 \times 10^{-3} - 4.0 \times 10^{-3})\end{aligned}$$

$\Delta\phi = -4.5 \times 10^{-3}$ Wb (the negative means it decreased) (2 sf)

b $\text{EMF} = -n\dfrac{\Delta\phi}{\Delta t}$

$$\begin{aligned}&= -1 \times \frac{-4.5 \times 10^{-3}}{0.50} \quad (n = 1 \text{ for a single loop})\\ &= +9.0 \times 10^{-3}\text{ V}\end{aligned}$$

$|\text{EMF}| = 9.0$ mV (2 sf)

WORKED EXAMPLE 8.2B

Consider the same set-up as in Worked example 8.2A, in which a magnetic field of strength 4.0 mT threads a loop of area 1.5 m² at right angles to the field. The loop is now rotated through 20° in 0.10 s, as shown in Figure 3. Determine the EMF generated in the loop.

SOLUTION

In this question, the angle of the loop (θ) is changing, and magnetic field strength (B) and area (A) are being held constant.

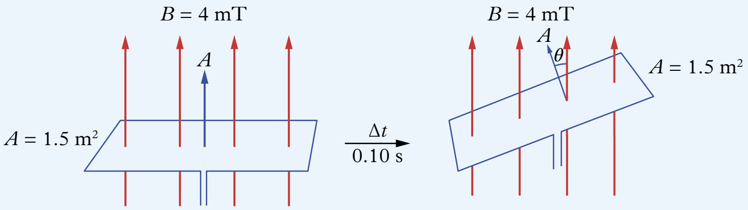

FIGURE 3 Changing the angle θ of loop

If the loop is at right angles to the field, the perpendicular to the loop is at 0° to the field.

Therefore, $\theta_i = 0°$, $\theta_f = 20°$

$$\begin{aligned}\text{EMF} &= -\frac{n\Delta(BA_\perp)}{\Delta t}\\ &= -\frac{n\Delta(BA\cos\theta)}{\Delta t}\\ &= -\frac{nBA \times \Delta\cos\theta}{\Delta t} \quad (B \text{ and } A \text{ are not changing})\\ &= -\frac{nBA \times (\cos\theta_f - \cos\theta_i)}{0.10}\\ &= -\frac{1 \times 4.0 \times 10^{-3} \times 1.5 \times (\cos 20° - \cos 0°)}{0.1}\\ &= -\frac{6.0 \times 10^{-3} \times (0.94 - 1.0)}{0.10}\\ &= -\frac{-0.36 \times 10^{-4}\text{Wb}}{0.10}\end{aligned}$$

$|\text{EMF}| = 3.6$ mV (2 sf)

WORKED EXAMPLE 8.2C

Consider the same set-up as in Worked example 8.2A, in which a magnetic field of strength 4.0 mT threads (passes through) a loop of area 1.50 m^2 at right angles to the field. The loop now shrinks in area to 0.60 m^2 in 0.10 s, as shown in Figure 4. Calculate the EMF induced in the loop.

SOLUTION

In this case, the area (A) is changing, while the angle (θ) and magnetic field strength (B) and are being held constant.

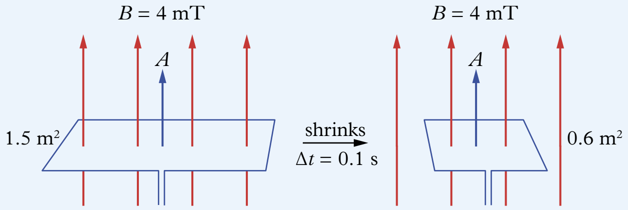

FIGURE 4 Changing the area, A, of the loop

$$\text{EMF} = -\frac{nB\cos\theta \times \Delta A}{\Delta t} \quad (B \text{ and } \theta \text{ are not changing})$$

$$= -\frac{nB\cos\theta \times (A_f - A_i)}{0.10}$$

$$= -\frac{1 \times 4.0 \times 10^{-3} \times (0.60 - 1.50)}{0.10}$$

$$|\text{EMF}| = 0.036 \text{ V (2 sf)}$$

Determining the direction of EMF

You have seen that when the flux threading a loop changes, an EMF is produced. The electrons in the wire that are free to move will experience a force along the length of the wire. As electrons shift to one end of the wire, there is a net excess of negative charge at that end and a net excess of positive charge at the other. This leads to a potential difference or EMF across the ends of the wire and current will flow in any external circuit. This can be shown with an ammeter. How to determine the direction of the EMF, in other words, which end becomes positive and which end becomes negative, is dealt with in the next section.

CHALLENGE 8.2A

Single coil motor

A single coil motor would have a very uneven rotation, as the force is a maximum when the plane of the loop is cutting the magnetic field fastest and that is when the loop is parallel to the field. How would a heavy loop even this out? Wouldn't a heavy loop slow it down more?

CHALLENGE 8.2B

Copper skipping rope

If you used a skipping rope made out of copper wire, would you feel the biggest voltage between your hand when you faced north or faced west? Why is this?

CHECK YOUR LEARNING 8.2

Describe and explain

1 **Define** 'electromagnetic induction'.

2 **Recall** the terms contained in Faraday's law.

3 **Explain** how magnetic flux can be zero even though the magnetic field is not zero.

4 **Explain** why a voltage is induced in this coil when it is pulled by the sides, as shown in Figure 5.

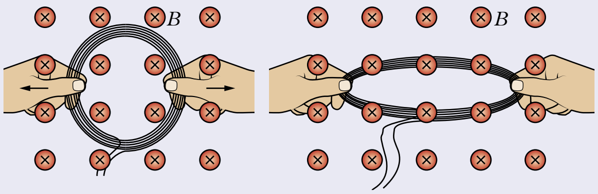

FIGURE 5 Changing the shape of a loop

5 A square loop of side 25 cm has a magnetic field of strength 2.2 T threading through it. **Calculate** the magnitude of the EMF produced when the field strength falls to 1.0 T in a time of 0.20 s.

6 A single proton is fired down a long tube that has a coil of wire wrapped around it to detect the passing of individual particles.

   a **Sketch** a graph of the voltage output of the coil as the proton passes through it.

   b **Explain** your reasoning.

Apply, analyse and interpret

7 **Determine** the order from smallest to largest: 0.050 Wb, 1.6×10^{-3} Wb, 15 m Wb.

8 **Determine** the magnitude of the EMF produced in this scenario. A 10-turn coil lies in the plane of this page and a uniform magnetic field of strength 0.90 T is directed into this page. The coil has an area of 0.150 m^2 and is stretched to have zero area in a time of 0.20 s.

9 **Determine** the number of turns in a coil that is pulled between the poles of a magnet so that the magnetic flux decreases from 3.1×10^{-4} Wb to 1.0×10^{-5} Wb in 0.020 s. The induced EMF is measured as 0.75 V.

Investigate, evaluate and communicate

10 A person who works in a strong magnetic field feels dizzy when they quickly turn their head. **Discuss** how this could be associated with induction.

11 A permanent magnet is tied to the end of a piece of string and set up like a pendulum. A loop of wire is placed on the floor directly under the pivot point. The magnet is drawn to one side and allowed to make one oscillation (out and back). **Propose** the number of times the loop experiences an increase in flux, and the number of times the loop experiences a decrease in flux in that one oscillation.

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  Check your learning 8.2
- » Suggested practical worksheet
  8.2 Induction of current using a magnet and coil
- » Challenge worksheet
  8.2A Single coil motor
- » Challenge worksheet
  8.2B Copper skipping rope

8.3

Lenz's law

KEY IDEAS

In this section, you will learn about:

- problems involving Faraday's law and Lenz's law
- the consistency of Lenz's law and the principle of conservation of energy.

Recall that when a magnet is moved in and out of a coil, the charge flows in one direction as the magnet goes in, and flows in the opposite direction as the magnet goes out. The question is how do you work out the direction? That is, looking at the coil from the magnet end, when does the charge move clockwise, and when does it move anticlockwise?

CHALLENGE 8.3A

Rare earth magnets

Drop a rare earth magnet through an aluminium tube and it falls slowly. Drop a similar but unmagnetised object such as a steel nut, and it will fall fast. Why is this?

Direction of current in a coil

It is easy to push the magnet in and out when it is an ordinary school bar magnet and a small coil. However, if a very powerful magnet and a very large solenoid coil with thousands of turns are used, you would notice that a huge force would be needed to push the magnet into the solenoid. It appears that nature is trying to prevent, or oppose, the induced current in the coil. The Russian physicist Heinrich Lenz (1804–1864) first explained the direction of the induced current in a solenoid coil as being the result of a changing magnetic field. He used the notion of nature trying to oppose any applied force. **Lenz's law**, as it is referred to today, states that nature does not provide something for nothing! In the field of electromagnetics, Lenz's law states:

> 'The direction of an induced electric current always opposes the change in the circuit or the magnetic field that produces it.'

Lenz's law states that the direction of an induced electric current is such that it produces a current whose magnetic field opposes the change in the circuit or the magnetic field that produces it

law of conservation of energy energy can neither be created nor destroyed, but only changed from one form to another or transferred from one object to another

When an induced current flows through the solenoid, the magnetic field produced by the solenoid has a polarity that repels the incoming permanent magnet pole. It repels the magnet and forces you to do work. If you didn't have to do work you would get electricity for no effort. Lenz's law is one expression of a fundamental law of nature – the **law of conservation of energy**:

> 'The total energy of a system remains constant; energy can neither be created nor destroyed, rather, it transforms from one form to another.'

In Figure 1, you see that in the first case the north pole of a magnet is approaching so the coil produces a north pole at its top end to repel the incoming pole (like poles repel). To generate a north pole at that end, a current has to flow anticlockwise as shown (as though you are looking from above). To determine the north pole, you can use Ampere's right-hand grip rule (discussed in Chapter 7). Point your fingers in the direction of the positive current in the coil and your thumb will point to the north pole.

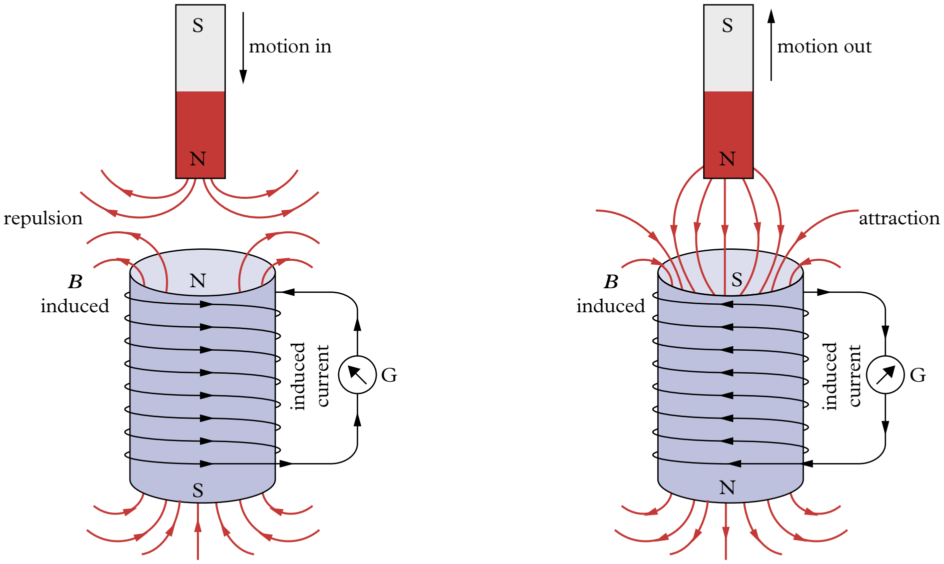


FIGURE 1 Direction of the induced field inside a coil as a magnet approaches and recedes

As the magnet moves away, the coil produces a south pole to try to keep the magnet there (unlike poles attract). To get a south pole on the top end of the coil, you need an induced clockwise current (as viewed from above). Using Ampere's right-hand grip rule, your thumb should point to the bottom of the coil, indicating the location of the north pole.

In both situations, a force needs to be exerted and work needs to be done in order to continue moving the magnet. It is this work done that is the origin of the induced electrical energy – mechanical energy is transformed into electrical energy – and this principle is the basis of all electric generators. The big problem is how to produce a continuous flow of electrical energy from a generator, and not just single current pulses.

Direction of current in a straight conductor

You can also generate a current in a single wire. After all, a loop of wire is just a lot of short segments joined end to end, so you should also look at this simple case. If you have a wire between the poles of a horseshoe magnet as shown in Figure 2, and pull the wire towards you, there is opposition to this motion and positive charge flows to the end on the right, leaving the end on the left with an excess of negative charge. (It is really the negative electrons moving, but everything is done in terms of positive charge moving.)

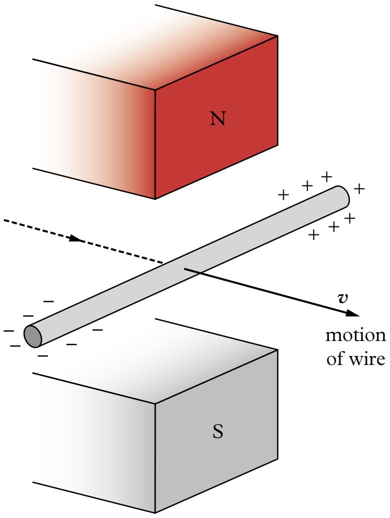
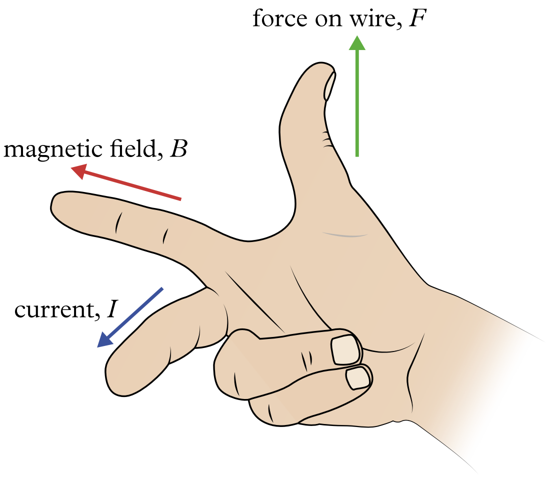


FIGURE 2 Pulling a wire between the poles of a magnet induces charge separation.

Study tip

You can remember the order for Fleming's right- hand rule starting with the thumb as FBI. It should remind you of the US Federal Bureau of Investigation.



Fleming's right-hand rule

Fleming's right-hand rule can be used to determine the direction of the induced current. To use the rule, hold your thumb, index finger and middle finger at right angles to each other, as shown in Figure 3. Your index finger should point in the direction of the magnetic field (B) and your thumb should point in the direction the wire is moving (A). Your middle finger will then point in the direction of the induced current (I).

FIGURE 3 Fleming's right-hand rule

Using Lenz's law on a straight conductor

Consider the apparatus depicted simply in Figure 4. It shows a metal rod AB resting on conducting rails connected to a galvanometer, G, on CD. The apparatus is sitting in a magnetic field, directed into the page, as shown. The conductor ABCD forms a loop through which the magnetic field passes (into the page).

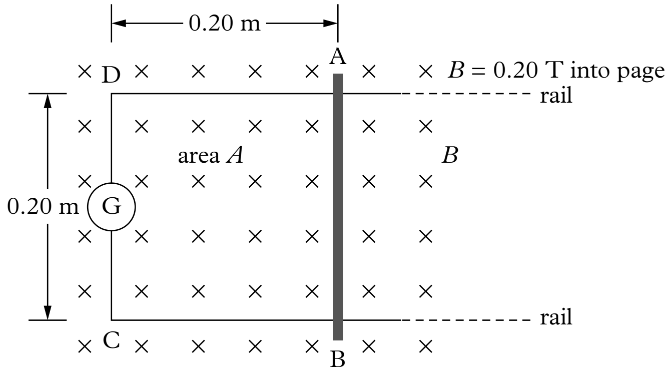


FIGURE 4 A metal rod on conducting rails in a magnetic field forms a loop.

Inducing an EMF by changing the field strength

Imagine the magnetic field strength in Figure 4 changing from a value of 0.20 T to zero in a time of 5.0 ms.

- **Magnitude of the EMF:** The value of the EMF produced is given by Faraday's law:

$$\begin{aligned}\text{EMF} &= -\frac{n\Delta(BA_{\perp})}{\Delta t}\\ &= -\frac{nA_{\perp}\Delta B}{\Delta t} \quad (A \text{ is not changing})\\ &= -\frac{nA_{\perp}\times(B_{\text{f}}-B_{\text{i}})}{\Delta t}\\ &= -\frac{1\times 0.20\times 0.20\times(0-0.20)}{5.0\times 10^{-3}}\\ |\text{EMF}| &= 1.6\,\text{V}\end{aligned}$$

- **Direction of the induced EMF and current:** The direction of the induced current around the loop ABCD can be determined using Lenz's law. The logic is that field strength (B) is decreasing, so the loop wants to oppose this change and keep field strength the same. It does this by generating a field in the same direction as the original field to make up for the losses. The right-hand for loops and solenoids (see Section 7.3, Figure 4b, page 193) shows that the current must flow clockwise from D to A to B to C.

Inducing an EMF by moving the rod

Imagine now, that the rod is moved to the right at a velocity of, say, 5.0 cm s^{-1}, with the magnetic field remaining constant at its initial value. The induced EMF from D to C can be calculated.

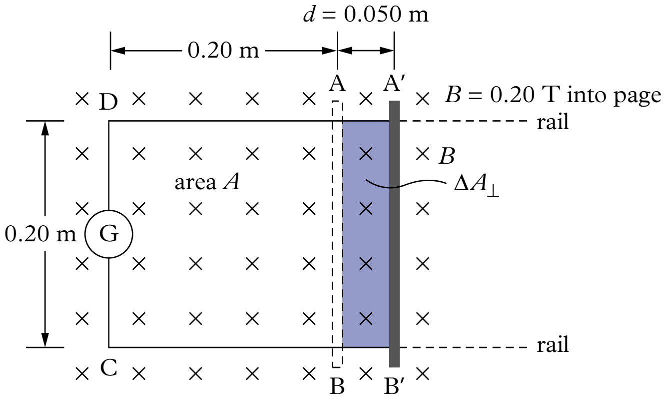


FIGURE 5 The shaded area contains the flux cut by the moving rod.

- **Magnitude of the induced EMF:** In one second of time, the rod cuts (or 'sweeps') through 5.0 cm (0.050 m) of the magnetic field (shown in blue as the rod AB moves to new position A'B' in Figure 5). This is an increase in area of 0.050 × 0.20 = 0.010 m^2, so the change in area (ΔA) is 0.010 m^2.

$$\text{EMF} = -\frac{n\Delta(BA_{\perp})}{\Delta t}$$
$$= -\frac{nB\Delta A_{\perp}}{\Delta t} \text{ (}B\text{ is not changing)}$$
$$= -\frac{1 \text{ turn} \times 0.20\,\text{T} \times 0.010\,\text{m}^2}{1.0\,\text{s}}$$
$$|\text{EMF}| = 2.0 \times 10^{-3}\,\text{V (2 sf)}$$

- **Direction of the induced EMF and current:** To determine the direction, use Lenz's law. When the rod slides to the right, the amount of flux enclosed by the loop increases (Figure 5). The loop opposes this change and generates a field that is in the opposite direction to the existing field to get it back to the original value. Using the right-hand rule for loops and solenoids (see Section 7.3, Figure 4b, page 193), an anticlockwise current (from B' to A') is induced to achieve this.

Alternatively, you can use Fleming's right-hand generator rule. With index finger (magnetic field, B) pointing into the page, thumb (force, F) pushes wire (AB) to the right, and your middle finger (current, I) points up the page (from B' to A').

Study tip

Most Physics teachers adapt Faraday's law to incorporate the speed of a moving wire or rod:
Area swept by wire = length × distance travelled

$$\text{EMF} = \frac{B\Delta A_{\perp}}{\Delta t} = \frac{B \times l \times d}{\Delta t}$$
$$= Bl \times \frac{d}{\Delta t} = Blv$$

This formula is not mandatory in Senior Physics, so you are not expected to be able to use it. However, it is useful for checking the answers produced by the earlier method.

CHALLENGE 8.3B

Falling magnet

When you drop a rare earth magnet through an aluminium tube it falls slowly and doesn't touch the sides of the tube. If you hang the tube from a scale balance, will the scale reading increase or stay the same as the magnet falls through it?

CHECK YOUR LEARNING 8.3

Describe and explain

1 **Clarify** whether an induced voltage always produces a current.

2 **Summarise** how Lenz's law applies to electromagnetic induction.

Apply, analyse and interpret

3 The area if a loop halves as shown in Figure 6.

  a **Deduce** whether an induced current is produced.

  b **Determine** whether X or Y becomes positive if a current is produced.

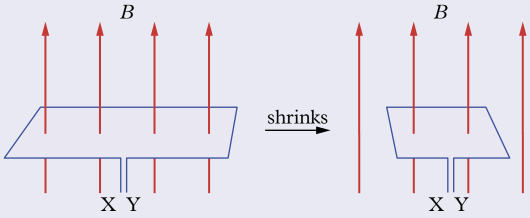


FIGURE 6 Changing area

4 A magnet is brought up to a solenoid as shown in Figure 7. **Determine** the direction of the induced current through the voltmeter (from A to B, or from B to A) in each case.

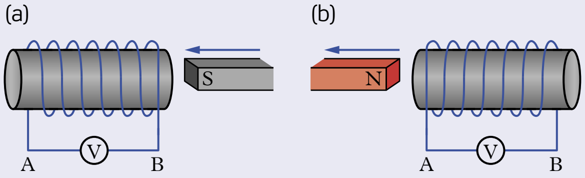


FIGURE 7 Induced current in a solenoid

5 Figure 8 illustrates a conductive metal square coil positioned within a magnetic field of strength 150 mT. The coil has side AB = 5.0 cm and is moved sideways at 8.5 m s^{-1}. **Determine:**

  a the voltages induced across each of the sides of the coil AB, BC, CD, DA

  b whether an induced current will flow around this coil as it is moved. **Explain** your answer.

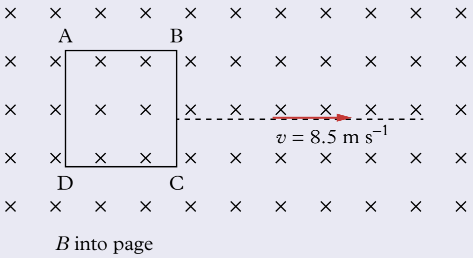


FIGURE 8 Moving loop in a magnetic field

Investigate, evaluate and communicate

6 Consider the apparatus of Figure 9.

  a **Predict** which way the current will flow through the galvanometer (if at all) when switch S is closed.

  b **Propose** what would occur if switch S is opened and closed repeatedly.

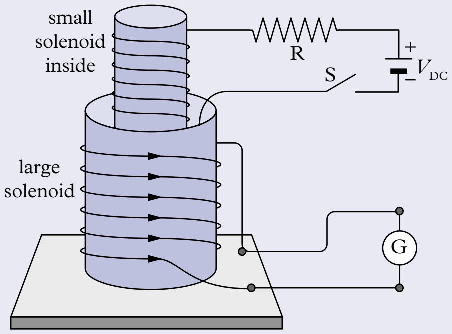


FIGURE 9 Loop inside a loop

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  Check your learning 8.3
- Challenge worksheet
  8.3A Rare earth magnets
- Challenge worksheet
  8.3B Falling magnet
- Weblink
  The right-hand motor rule

8.4 Transformers

KEY IDEAS

In this section, you will learn about:

- transformers and Faraday's law
- transformers and electromagnetic induction.

mutual induction the production of an EMF in a circuit by a change in the current in an adjacent circuit that is linked to the first by the flux lines of a magnetic field

Electromagnetic induction will also occur in a situation where the expanding or collapsing magnetic field of an electromagnetic solenoid coil cuts through a stationary conductor or second coil. This type of induction between closely separated coils is called **mutual induction**. The very first experiment on electromagnetic induction done by Michael Faraday was this type of experiment. Just 6 weeks before his discovery of electromagnetic induction, Faraday wrapped coils of wire around two side of a big steel ring. A sketch from his diary shows this ring (similar to Figure 1), which is a simplified sketch of the apparatus shown in Figure 2.

Figure 2 shows two solenoid coils wound onto a common soft iron core. When the switch is closed in the left-hand circuit, an expanding magnetic flux is produced that cuts the right-hand circuit. This induces a voltage pulse across the ends of the right-hand coil in such a way as to oppose this expanding flux, according to Lenz's law. No voltage is induced when the current in the left-hand coil is constant. However, if the switch is opened again, a collapsing magnetic flux now cuts the right-hand coil and again an induced voltage, opposite in direction to the original pulse, is produced. You may have seen an induction coil (a 'spark coil') in your laboratory at school.

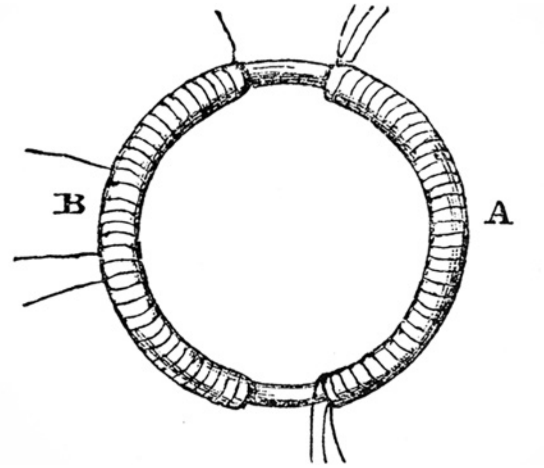

FIGURE 1 An engraving of the first transformer, which was created from a sketch in Faraday's notebook

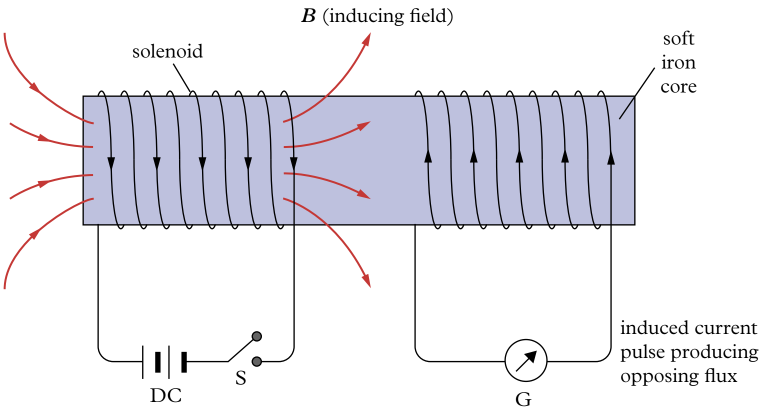

FIGURE 2 A schematic of a simple induction coil

Another obvious method of producing continuously expanding and collapsing magnetic fields is to use a primary circuit driven by an alternating current (AC) similar to that produced by a magnet moving in and out of a coil.

SUGGESTED PRACTICAL
8.4 Induced EMF from an AC generator
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transformer
a device that transfers an alternating current from one circuit to another, usually with an increase (step-up transformer), or decrease (step-down transformer) in voltage

Study tip

As a possible topic for a research investigation, students could investigate how scientific knowledge has been used to develop methods of renewable energy production (e.g. wind and wave power generation). Knowledge of electromagnetic induction and transformers is essential.

This produces a **transformer** device that uses mutual induction to vary the alternating voltages. Two coils – called the primary and secondary coils – are wound onto a common soft iron core (Figure 3). The soft iron core concentrates the magnetic flux lines threading both coils. If the primary coil is fed with alternating (AC) voltages at a particular frequency, an induced alternating (AC) voltage of equal frequency will occur across the secondary coil. It won't work with a battery as the input, as the voltage is not changing. Notice that there is no physical electrical connection between the two sets of coils. If the ratio of turns in the windings is varied, either a step-up or a step-down transformer is produced. For any transformer operating under ideal conditions (that is, no loss of energy), the following relationships hold:

$$V_\text{p} = -n_\text{p}\left(\frac{\Delta\phi}{\Delta t}\right) \quad \text{and} \quad V_\text{s} = -n_\text{s}\left(\frac{\Delta\phi}{\Delta t}\right) \quad \text{Faraday's law}$$

The amount of flux threading in both coils is the same if no energy is lost, therefore

$$\left(\frac{\Delta\phi}{\Delta t}\right) = \frac{V_\text{p}}{n_\text{p}}$$
$$= \frac{V_\text{s}}{n_\text{s}}$$

which can be rearranged to:

$$\frac{V_\text{p}}{V_\text{s}} = \frac{n_\text{p}}{n_\text{s}}$$

where V_p = alternating voltage across the primary coil; V_s = alternating voltage across the secondary coil; n_p = number of primary turns; n_s = number of secondary turns.

- If $n_\text{p} > n_\text{s}$, then the transformer is a step-down transformer, which reduces the alternating voltage.
- If $n_\text{p} < n_\text{s}$, then the transformer is a step-up transformer, which increases the alternating voltage.

Electricity for Australian homes is normally generated at an AC voltage of 23 kV, then stepped up to 330 kV for long range transmission. It is then gradually stepped down at power pole transformers in your street to 415 V to then provide the 240 V used in household appliances.

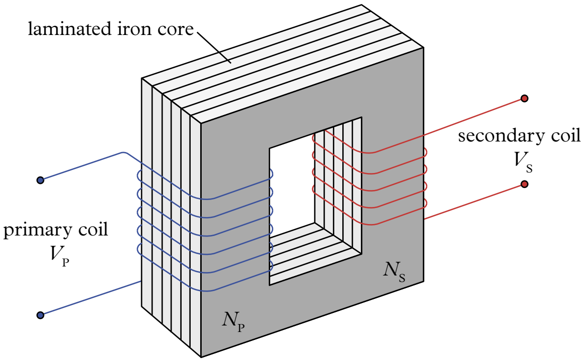


FIGURE 3 Soft iron core of a transformer

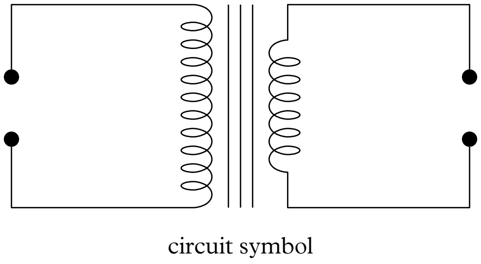


FIGURE 4 The circuit symbol for a transformer

Many household appliances contain miniature transformers. Appliances are designed to operate on a specific voltage, AC or DC. Not operating at the correct voltage can cause overheating and endanger the user. For example, your computer is likely to have an adapter that uses a step-down transformer for power, and further step-down and step-up transformers inside to power particular circuitry.

Without transformers we couldn't have many of the electrical devices as we have today, and we couldn't use as much energy. Case study 8.4 (page 228) investigates the efficiency of transformer devices.

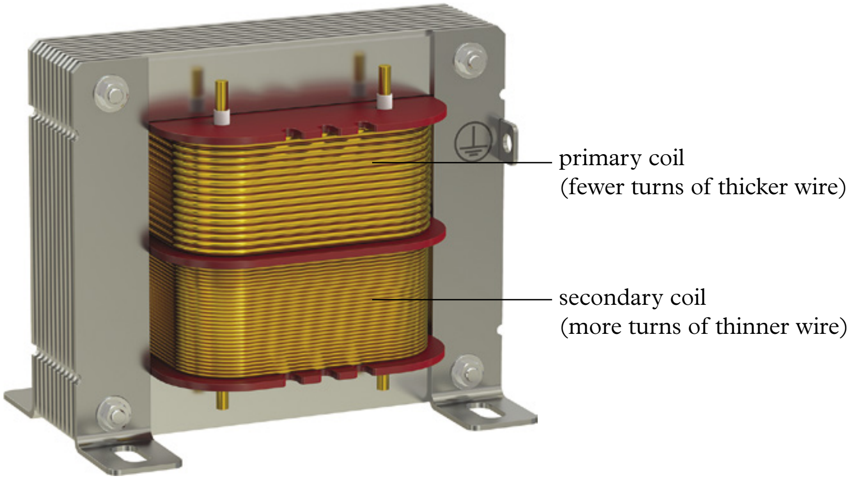


FIGURE 5 A transformer showing the two coils. The top coil has fewer turns of thicker wire, than the lower coil, which has more turns of thinner wire. To act as a step-up transformer, the top coil is made the primary and the bottom coil is the secondary. To act as a step-down transformer the reverse is true.

Study tip

Analysing the role of transformers in electrical power production makes a good research question, particularly, evaluating the impact of transformers. A claim worthy of evaluation might be: 'The development of the AC transformer has improved our quality of life in many ways, but the negative impacts on the environment cannot be ignored'.

CHALLENGE 8.4A

Step-up transformer

In a step-up transformer the voltage in the secondary coil is higher than in the primary coil. Where does the extra voltage come from?

WORKED EXAMPLE 8.4A

A transformer purchased from an electronics store is labelled as 240 V AC input, 50 V AC output. It has 400 turns of wire on its primary.

a Calculate the number of turns on the secondary.

b Determine whether this is a step-up or step-down transformer.

SOLUTION

a $\frac{V_p}{V_s} = \frac{n_p}{n_s}$

$$n_s = \frac{n_p V_s}{V_p}$$

$$= \frac{400 \times 50}{240}$$

$$= 83 \text{ turns (2 sf)}$$

b As $V_s < V_p$, it is a step-down transformer.

Study tip

One of the suggested experiments is to investigate the induced EMF in an AC transformer. One interesting modification would be to research the effect of changing frequency on the efficiency, with particular emphasis on efficiency peaks. This could lead to a discussion of mutual induction (*back* EMF) between conductors in the transformer winding.

CASE STUDY 8.4

Transformers

The discussion so far makes transformers sound like marvellous devices for varying AC voltages, and they are. However, energy is not created in these devices – the electrical power available at the output is never greater than the electrical power supplied to the input of the transformer. In an ideal transformer (no energy loss):

$$\text{input power in the primary coil} = \text{output power in the secondary coil}$$
$$P_p = P_s$$
$$I_p V_p = I_s V_s$$

In essence, when the voltage in the secondary is greater than in the primary (step up), the current in the secondary must be less than in the primary (step down).

In practice, although transformers are very efficient devices, there is always some energy loss. Where does the energy go? The answer is in the core. When an alternating current passes through the primary coil, tiny circulating currents called eddy currents are set up in the soft iron core. This causes heating and represents lost energy.

This is obviously not a good thing, and to minimise it, laminated cores are used. Insulated laminations are used to separate the core into smaller sections, significantly reducing the size of eddy currents and thus reducing the energy loss due to heat.'

The coil conductors also lose heat through ohmic heating due to coil resistances.

The efficiency (η) of a transformer is measured by the ratio of power produced in the secondary coil to the power input from the primary coil:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\,\%$$
$$= \frac{V_s I_s}{V_p I_p} \times 100\%$$

Frequency and resonance

It is also known that the frequency of the alternating current affects the efficiency, with some frequencies being favoured but with a general decline in efficiency at higher values. Typical electromagnetic transformers are about 90–95% efficient in operation. The point at which efficiency is a maximum is called 'resonance' and is a point at which there is a maximum transfer of energy.

FIGURE 6 Transformer core being assembled with sheets of silicon steel laminations

Study tip

The term 'turns ratio' is often used to describe a transformer. It is the ratio of the number of turns in the primary coil to the number of turns in the secondary coil.

Turns ratio = $\frac{n_p}{n_s}$

A step-down transformer would have a ratio greater than 1, e.g. 10 : 1. A step-up transformer would have a ratio less than 1, e.g. 1 : 10.

CHALLENGE 8.4B

Oil, heat and transformers

Small transformers are air cooled, but the big transformers on power poles along the street often have a jacket of oil around them. Why do they get hot and what is the purpose of the oil?

CHECK YOUR LEARNING 8.4

Describe and explain

1 **Explain** whether a transformer will operate if a 1.5 V battery is used as the primary input.

2 An iron core is used in most transformers. **Explain** whether it true to say that without it, the transformer output produces a lower power than with it.

3 **Explain** whether it is true to say that a step-up transformer steps-up the voltage but it also steps-down the number of turns.

4 Figure 7 shows the primary and secondary waveforms of a step-up transformer. **Calculate:**

  a the approximate turns ratio

  b the frequency of the AC signal.

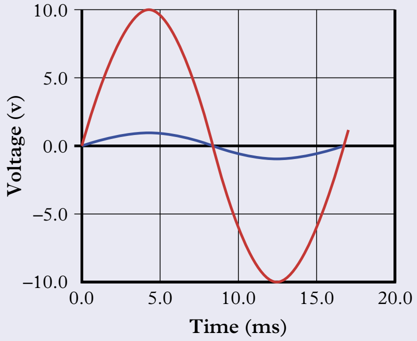


FIGURE 7 Input and output waveforms

Apply, analyse and interpret

5 A transformer with a turns ratio ($n_p : n_s$) of 26 : 1 is used to charge a portable drill from a domestic supply of 240 V. The charger draws 0.30 A from the household outlet.

  a **Determine** the voltage output of the charger.

  b **Determine** whether the transformer is step-up or step-down.

Investigate, evaluate and communicate

6 A street transformer converts 12 kV in the overhead wires to 415 V for household use.

  a **Determine** the turns ratio required of the transformer used.

  b **Propose** whether it would be a step-up or a step-down transformer.

7 Neon lights require at least 12 kV for their operation, and big advertising signs operate from a 240 V line.

  a **Calculate** the turns ratio required of the transformer used.

  b **Predict** whether it would be a step-up or a step-down transformer.

Check your obook assess for these additional resources and more:

- » Student book questions Check your learning 8.4
- » Suggested practical worksheet 8.4 Induced EMF from an AC generator
- » Challenge worksheet 8.4A Step-up transformer
- » Challenge worksheet 8.4B Oil, heat and transformers

8.5 Electromagnetic radiation

KEY IDEAS

In this section, you will learn about:

- electromagnetic radiation in terms of electric fields and magnetic fields
- examples of electromagnetic radiation.

In the previous sections, you learnt that electric fields can produce magnetic fields, and that magnetic fields can produce electric fields. If there was no loss of energy during this process, we could have a self-perpetuating (self-maintaining) system of fields in sync with each other. We do have this – it is called electromagnetic radiation.

In the 1860s, James Clerk Maxwell (1831–79), a Scottish mathematician and theoretical physicist, first proposed that light consisted of transverse **electromagnetic waves** propagating through space as changing electric and magnetic fields that are in phase and at right angles to each other (Figure 1). This model is still accepted today.

electromagnetic wave
waves produced by an oscillating electric charge that radiate out at the speed of light as mutually perpendicular electric and magnetic fields

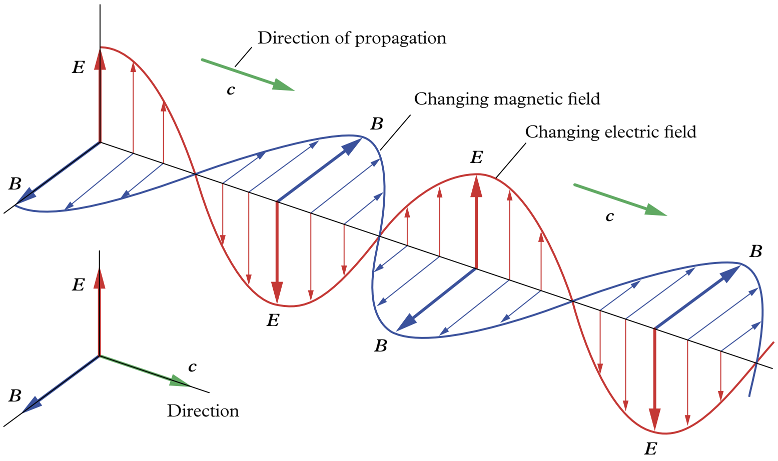


FIGURE 1 An electromagnetic wave has the changing magnetic and electric fields at right angles to each other and to the direction of propagation. The waves propagating in one direction only are shown here. In reality, they would be in all directions. The ***E*** and ***B*** arrows represent the electric and magnetic fields.

Producing electromagnetic waves

A simple way to understand the production of electromagnetic waves is to imagine a short conducting rod that can be connected to a battery. This battery can be turned on and off, and its polarity can be reversed. In fact, a generator could be used that produced an alternating current like the output of a coil in which a magnet is moved in and out. Figure 2 provides an example of this.

Imagine the set-up in Figure 2 part A. There is no battery connected so there is no electric field and no magnetic field. In Figure 2 part B, the top of the rod is positive so the current flows towards the bottom and the field (E) is down the page. Associated with this current is a magnetic field (B) at right angles to the field. Using the right-hand rule, you can see that the field is out of the page, as shown by the circle with the dot.

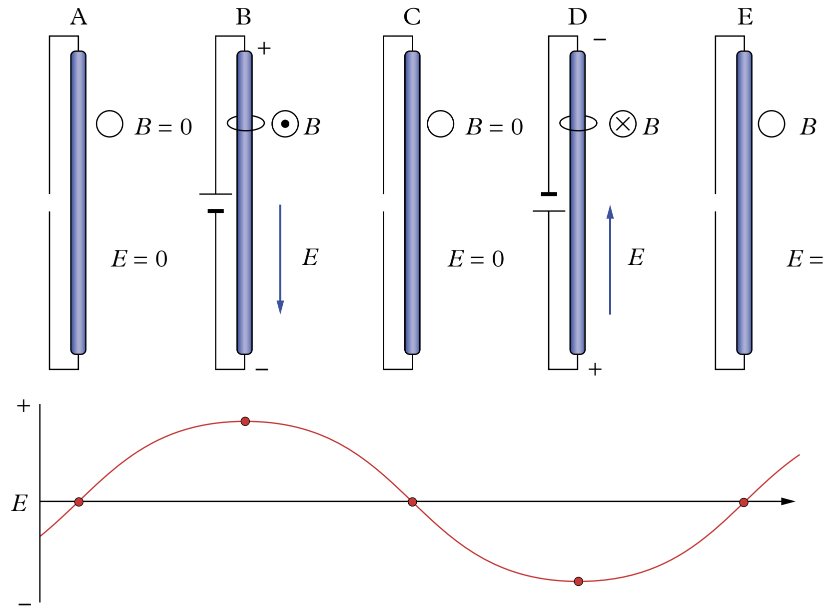


FIGURE 2 Oscillating charges produce electric and magnetic fields. When a charge moves up or down the rod, a magnetic field is produced. When there is no movement of charge there is no magnetic field.

In Figure 2 part C, there is no battery connected so there is no electric or magnetic field. In Figure 2 part D, the battery is reversed, so the electric field (E) is up the page and the magnetic field (B) is into the page. Lastly, Figure 2 part E shows the battery disconnected, so no fields are produced.

What is interesting is that while the electric field oscillates up and down, the magnetic field oscillates into and out of the page, but they are acting in phase.

How the fields are linked

A steadily changing magnetic field can induce a constant voltage, while an oscillating magnetic field, such as a magnet moving in and out of a solenoid, can induce an oscillating voltage.

In summary:

- An oscillating electric field E generates an oscillating magnetic field B.
- An oscillating magnetic field B generates an oscillating electric field E.
- The planes of two fields are at 90° to each other.
- The E and B fields, as well as being perpendicular to each other, are perpendicular to the direction of travel of the wave, meaning that an electromagnetic wave is a **transverse wave**.
- The oscillations are in phase.
- The energy of the wave is stored in the electric and magnetic fields.
- Electromagnetic waves travel away from the source at the speed of light, c.
- They do not need a medium for their propagation (they can travel in a vacuum).
- When the source is turned off, the waves still keep travelling.

transverse wave
a wave where the direction of oscillation of particles is perpendicular to the direction of energy transfer

CHALLENGE 8.5

Speed of light – theoretical value

When Maxwell developed his equations for electromagnetic waves, he found that the velocity was dependent on two constants you used in previous chapters: for electric field strength he used the permittivity of free space (ε_0) equal to

$$8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$$

and for magnetic field strength he used the permeability of free space (μ_0) equal to $4\pi \times 10^{-7}$ T mA^{-1}. Maxwell showed that the speed of light:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Your challenge is to compare this value to the accepted value of 299 792 458 m s^{-1}.

The electromagnetic spectrum

Wave equation

The energy carried by an electromagnetic wave is proportional to the frequency of the wave. The wavelength and frequency of the wave are connected via the speed of light, c:

$$v = f\lambda$$

where v is the speed of light, c ($c = 3 \times 10^8$ m s^{-1} in air or a vacuum), f is the frequency and λ is the wavelength.

Spectral regions

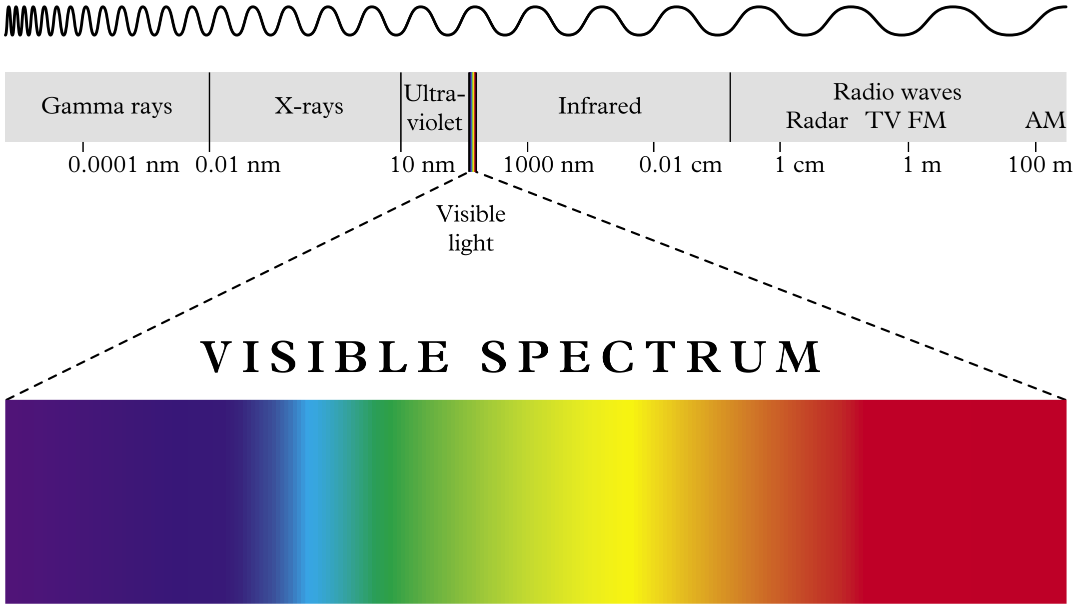


FIGURE 3 The spectral regions of the electromagnetic spectrum

Electromagnetic waves are split into different categories or ‘spectral regions’, based on their frequency or wavelength, as shown in Figure 3. Visible light, for example, ranges from violet to red. Violet light has a wavelength of 400 nm, corresponding to a frequency of 7.5×10^{14} Hz.

Red light has a wavelength of 700 nm, and a frequency of 4.3×10^{14} Hz. Any electromagnetic wave with a frequency (or wavelength) between those extremes can be seen by humans.

Visible light makes up a very small part of the full electromagnetic spectrum. Electromagnetic waves that are of higher energy (higher frequency, shorter wavelength) than visible light include ultraviolet light, X-rays, and gamma rays. Lower energy waves (lower frequency, longer wavelength) include infrared light, microwaves, and radio and television waves.

CHECK YOUR LEARNING 8.5

Describe and explain

1 An electromagnetic wave shown is shown in Figure 4.

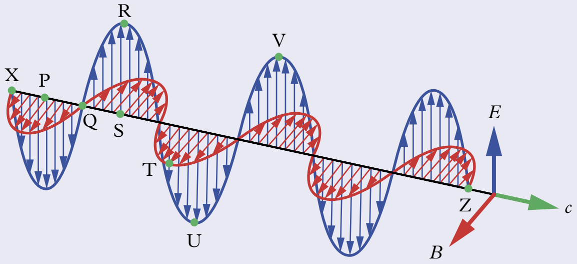


FIGURE 4 Electromagnetic wave

- **a** **Identify** the colour that represents the electric field and the colour that represents the magnetic field. Give reasons for your answer.
- **b** **Identify** the direction in which the wave is travelling and explain your reasoning.
- **c** **Explain** how you can tell that the two waves are in phase.
- **d** **Explain** whether the wave is transverse or longitudinal.
- **e** **Identify** two points that are one wavelength apart.

2 **Identify** the direction of the electric field and of the magnetic field at the following points in Figure 4:

a P **b** Q **c** S

3 **Identify** which pair of points on the wave in Figure 4 are in phase: P and U, or T and U. **Explain** your reasoning.

4 **Calculate** the wavelength of a microwave of frequency 5×10^{10} Hz.

Apply, analyse and interpret

5 **Compare** the magnitude and direction of the field strengths at points R and U in Figure 4.

6 **Consider** the wave in Figure 4. Imagine the distance between points X and Z is 1.0 m.

- **a** **Determine** the frequency of the wave.
- **b** **Identify** the spectral region to which it belongs (visible, radio etc.).

Investigate, evaluate and communicate

7 The wave in Figure 4 represents green light. **Evaluate** the approximate distance between X and Z.

8 **Decide** which wave crest came first in time: V or R. **Justify** your reasoning.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 8.5
- » Challenge worksheet
  8.5 Speed of light – theoretical value
- » Weblink
  Producing electromagnetic waves
- » Weblink
  The visible spectrum

SCIENCE AS A HUMAN ENDEAVOUR

8.6 Mobile phone radiation

KEY IDEAS

In this section, you will learn about:

- the potential risks of electromagnetic phenomena associated with mobile phone use.

Study tip

A good research investigation may be to evaluate these claims. Some data is presented here but this offers an opportunity to develop a research question and gather secondary scientific evidence.

Two stories that keep doing the rounds are that mobile phones can cook eggs and that overhead power lines will give you cancer. Some people have claimed that the radiation from a mobile phone cooks the proteins in an egg, and they then ask 'imagine what it can do to the proteins in our brains …'. However, the average power from a mobile phone is about 0.25 W – and that is when the phone is a long distance from the base station. When near the base station, the power drops to about 2 mW. Is that enough to cook and egg – or do anything for that matter? It turns out that it is not possible, but now many people are convinced it is true. The same goes for cancer from wi-fi or under power lines. Is there enough transmitted radiation to do harm?

You could ask whether there is any scientific evidence about the risks. The problem is that the effect is likely to be small if at all because of the low power, and any damage might not show up for a long time. It is sometimes difficult to get a definite answer as the threshold for damage is a matter of definition and what is 'damage' to one researcher is acceptable to another. The best way to tackle this for high school Physics is to ask a group of experts for their opinion and to see if there is any consensus. See what some say below.

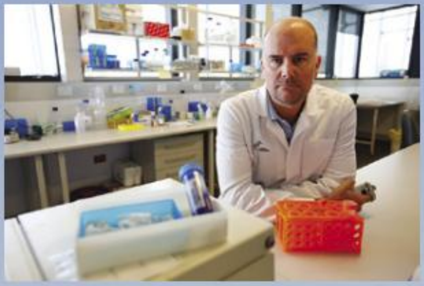
Some quotes to get you thinking

Professor Rodney Croft, Director of the National Health and Medical Research Council of Australia's Centre for Research Excellence in Electromagnetic Energy. ICNIRP Commissioner, and Professor of Health Psychology at University of Wollongong

'In fact, the scientific consensus is strong, and is that there is no substantiated evidence that the low levels of radiofrequency emissions encountered by mobile telecommunications can cause any harm.'

Simon Chapman, Emeritus Professor in Public Health at the University of Sydney

'There is no evidence of any increase in the rate per 100 000 population of brain cancer in any age group in Australia from 1982 to the present, other than for the very oldest age group where the increase started well before mobile phones were introduced in Australia and so cannot be explained by mobile phones. All cancer in Australia is notifiable, and over 85% of brain cancer is histologically verified: it is not just a doctor's opinion.'



Dr Darren Saunders, cancer biologist at the University of NSW and visiting fellow at the Kinghorn Cancer Centre, Garvan Institute

'The really frustrating aspect is that rebuttals and factchecks won't undo the damage. There are very real public health effects of scaremongering like this, creating anxiety and fear.

The two main flaws in the argument that stand out scientifically are:

1 the lack of any demonstrable increase in brain cancer incidence over time. We have been exposed to the same kind of non-ionising electromagnetic radiation long before mobile phones and wi-fi became commonplace, and
2 the absence of a plausible biological mechanism for how this kind of radiation can cause cancer. There were very poor analogies made with microwave ovens and smoking, which are purely emotive and not based on actual science. Comparing a microwave to a mobile phone is like comparing a Saturn V rocket to your lawnmower.'

Dr Sarah Loughran, researcher at the National Health and Medical Research Council of Australia's Centre for Research Excellence in Electromagnetic Energy

'Indeed there is currently no scientific evidence that exposure to low level radiofrequency, such as emitted by mobile phones and wi-fi, has an impact on health.'

Source: Do Wi-Fi and mobile phones really cause cancer? Experts respond, https://theconversation.com/do-wi-fi-and-mobile-phones-really-cause-cancer-experts-respond-54881

CHECK YOUR LEARNING 8.6

Describe and explain

1 **Recall** whether the radiation from mobile phones has a higher or lower frequency than visible light.

2 **Clarify** whether the photons from mobile phone radiation have more or less energy than UV photons.

Apply, analyse and interpret

3 **Deduce** what additional evidence the researchers would like to be able to make a more definitive conclusion.

Investigate, evaluate and communicate

4 On balance, the researchers seem to have reached a consensus on the dangers of mobile phone radiation. **Appraise** their comments and draw your own conclusion.

5 A claim was made that 'we have been exposed to the same kind of non-ionising electromagnetic radiation long before mobile phones became commonplace'. **Propose** what sort of radiation the researcher meant and from what source.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 8.6
- » Weblink
  Mobile phone use
- » Weblink
  Do mobile phones cause cancer?

CHAPTER 8 Review

Summary

8.1
- a visual representation of the magnetic field strength as the number of lines of magnetic flux passing through a unit area.

8.2
- Electromagnetic induction is the process of producing an EMF within a conductor or wire coil as a result of a changing magnetic flux.

8.3
- Faraday's law of electromagnetic induction states that the EMF induced in a conductive loop is proportional to the rate of change of flux.
- Lenz's law states that the direction of an induced electric current always opposes the change in the circuit or the magnetic field that produces it.

8.4
- A transformer is an electromagnetic device that operates on mutual induction principles to vary AC voltages.
- An efficient transformer can be wired in either step-up or step-down mode.
- Electromagnetic waves are composed of two plane waves of equal amplitude, in phase, and at 90° to each other.

8.5
- Electromagnetic radiation is a transverse wave made up of oscillating electric and magnetic fields at right angles to each other and to the direction of propagation.
- Examples of electromagnetic radiation are X-rays, UV light, visible light and infrared light.

8.6
- Electromagnetic radiation poses risks to human health but the intensity of radiation from devices such as mobile phones is low and there appears to be no credible evidence to say otherwise.

Key terms

- electromagnetic induction
- electromagnetic wave
- electromagnetic force (EMF)
- Faraday's law
- law of conservation of energy
- Lenz's law
- magnetic flux
- magnetic flux density
- mutual induction
- transformer
- transverse wave

Key formulas

Flux in a loop	$\phi = BA\cos\theta$
Faraday's law of induction (1st version)	$EMF = -\frac{n\Delta\left(BA_{\perp}\right)}{\Delta t}$
Faraday's law of induction (2nd version)	$EMF = -n\frac{\Delta\phi}{\Delta t}$
Transformer – energy conservation relationship	$I_pV_p = I_sV_s$
Transformer formula	$\frac{V_p}{V_s} = \frac{n_p}{n_s}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A bar magnet with the N-pole facing downwards is held above a horizontal circular coil and allowed to fall through the loop (Figure 1). Which of the following statements about the induced current is true (viewed from above)?

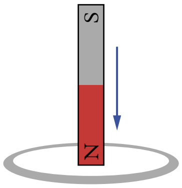


FIGURE 1 Bar magnet falling through a loop

A It flows in a clockwise direction.

B It flows in an anticlockwise direction.

C It flows first in a clockwise and then in an anticlockwise direction.

D It flows first in an anticlockwise and then in a clockwise direction.

2 Coil A is connected to a circuit that includes a battery, a switch, and a resistor. Coil B lies in the same plane as coil A (Figure 2).

Determine the direction of the induced current in coil B at the moment when the switch is closed.

A clockwise

B anticlockwise

C clockwise to anticlockwise

D anticlockwise to clockwise

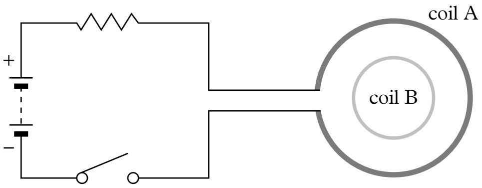


FIGURE 2 Two coils linked by magnetic flux

3 Select one of the following statements about a transformer that is false for a step-up transformer.

A the primary current > secondary current

B the primary voltage > secondary voltage

C the primary frequency = secondary frequency

D the primary power > secondary power (for a non-ideal transformer)

4 The E and B fields in an electromagnetic wave are oriented:

A parallel to the direction of travel of the wave, as well as to each other.

B parallel to the direction of travel of the wave, and perpendicular to each other.

C perpendicular to the direction of travel of the wave, and parallel to each other.

D perpendicular to the direction of travel of the wave, and also to each other.

5 Magnetic flux density has the units:

A Wb m^{-2}

B Wb A^{-1} m^{-1}

C A m^{-1}

D T m

6 Figure 3 shows a uniform magnetic field of field strength B threading a circular loop of area A. The field makes an angle θ with a perpendicular to the plane of the loop. The strength of B is increasing at a uniform rate of R. Which one of the following best describes the EMF induced in the loop?

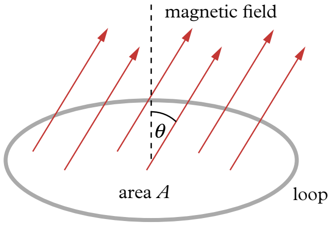


FIGURE 3 Changing field in a loop

A $RA\cos\theta$

B $RA\sin\theta$

C $\dfrac{RA}{\cos\theta}$

D $\dfrac{RA}{\sin\theta}$

7 In order to reduce power losses in the transmission lines between a power station and a factory, two transformers are used. One is located at the power station and the other at the factory. Which one of the following gives the correct types of transformer used?

A step-down, step-up
B step-down, step-down
C step-up, step-up
D step-up, step-down

8 Which one or more of the following statements *must* be true about an ideal transformer?

I The power output exceeds the power input.
II The magnetic flux produced by the primary coil entirely links the secondary coil.
III There are more turns on the secondary coil than on the primary coil.

A I and II only **B** I and III only
C II only **D** III only

9 An ideal transformer has n_p primary turns, and n_s secondary turns. The electrical power input into the primary is P_p. **Determine** the power output of the secondary, P_s.

A $\frac{n_p}{n_s}P_p$ **B** $\frac{1}{P_p}$ **C** $\frac{n_s}{n_p}P_p$ **D** P_p

10 All electromagnetic waves travel through a vacuum at:

A the same speed.
B speeds that are proportional to their frequency.
C speeds that are inversely proportional to their frequency.
D none of the above.

Short answer

Describe and explain

★ 11 A magnet is brought up to the end of a solenoid and an EMF is induced. **Describe** how the EMF would change if:

a a similar coil with fewer turns was used
b the coil was moved faster
c a stronger magnetic field was used.

★ 12 **Explain** why the operation of a transformer depends on AC rather than DC voltage.

★ 13 Figure 4 shows lines of flux (B) cutting (threading) a horizontal plane. The formula $\phi = BA\cos\theta$ is used to calculate the flux. **Explain** whether angle X or angle Y represents θ.

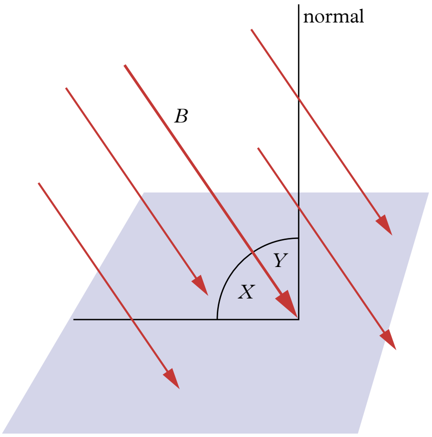


FIGURE 4 Lines of flux

★ 14 **Identify** the direction in which a wire would have to move in a magnetic field for there to be no induced current produced.

★ 15 **Explain** why the solenoids used in electromagnets and transformers need to have soft iron cores, and state why the cores are laminated.

★ 16 **Describe** the factors affecting the output EMF of any practical transformer. If these transformers are not ideal, state where the energy is lost between input and output.

★ 17 A transformer is designed to step-down the 240 V mains voltage used in Australia to a voltage of 6.3 V AC. **Calculate** the number of turns on the secondary coil given that the primary coil has 2000 turns.

★★ 18 Figure 5 is the graph of the output of a generator that is rotating at 50 Hz in a magnetic field of 0.40 T. **Describe** the changes to the output waveform when:

a the magnetic field changes to 0.80 T
b the field remains the same but the rate of rotation increases to 100 Hz.

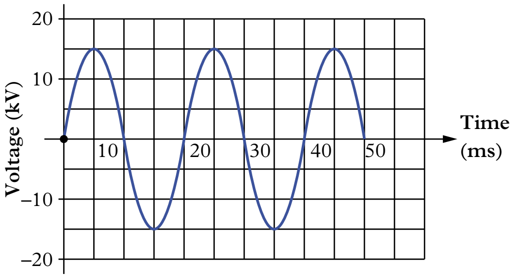


FIGURE 5 Waveform

★★★ **19** Figure 6 illustrates two solenoid coils connected in series.

- **a** **Describe** the behaviour of the suspended magnet in the right-hand coil after a bar magnet is dropped through the left-hand coil as shown.
- **b** **Explain** your reasoning.

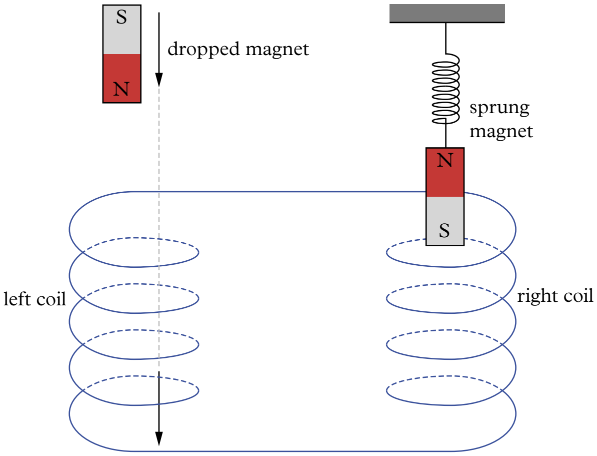


FIGURE 6 Two solenoids in series

★★★ **20** A galvanometer is connected across a 200-turn coil of area 50 cm², as shown in Figure 7. This assembly is perpendicular to the field and its intensity varies from 30 mT to 10 mT in a time of 0.02 s. **Calculate** the voltage reading on the galvanometer.

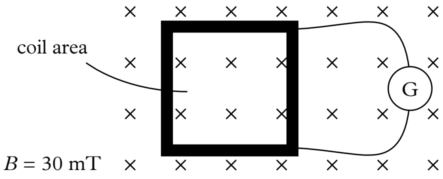


FIGURE 7 Coil in changing magnetic field

Apply, analyse and interpret

★ **21** **Distinguish** between magnetic flux and magnetic field.

★ **22** **Determine** which is greater: 15 mT or 0.016 T.

★ **23** **Categorise** the following field strengths from smallest to largest:

0.011 T, 1.8×10^{-3} T, 15 mT, 16 500 nT

★★ **24** **Determine** the magnitude of magnetic flux in the following situations:

- **a** A 1 cm² loop is placed perpendicular to a magnetic field of strength 4.5×10^{-10} T.
- **b** A 0.5 cm² loop is placed perpendicular to the magnetic field 5 cm from a wire carrying a steady current of 5 A.
- **c** A 60 mm² loop is placed in a magnetic field of strength 6.0×10^{-9} T such that the plane of the loop makes an angle of 60° to the field lines.
- **d** A loop of diameter 20 cm is placed in magnetic field of strength 5 mT at an angle of 35° between the plane of the loop and the field direction.

★★ **25** **Determine** the direction of the current (in terms of X and Y) when the following change is made to the field threading a loop (Figure 8).

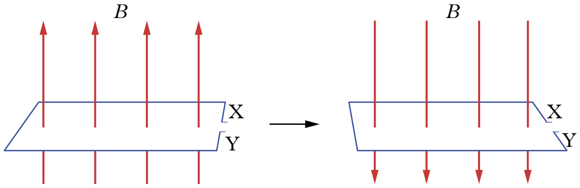


FIGURE 8 Reversing the field direction

★★ **26** **Determine** which terminal, X or Y becomes negative when the following change is made to the magnetic field strength in a loop of constant area (Figure 9).

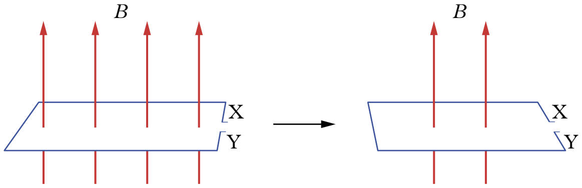


FIGURE 9 Changing the field strength

★★ 27 **Determine** the direction of the current (if any) when a magnet is moved relative to a solenoid as shown in Figures 10a and 10b.

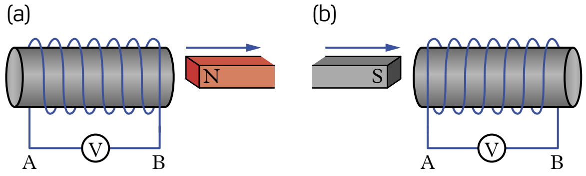


FIGURE 10 Magnet moving into a solenoid

★★ 28 **Determine** the direction of motion of the magnet (towards the solenoid, away from the solenoid, or no movement) in the two situations shown in Figure 11.

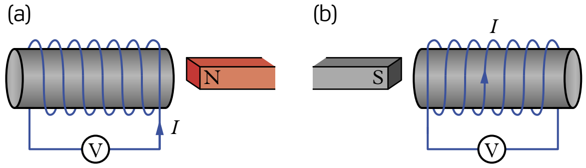


FIGURE 11 Current produced in a solenoid

★★★ 29 **Determine** the EMF induced in a coil that has 1200 evenly spaced turns and a radius of 2.5 cm, when a magnet is brought near its end so that the magnetic field inside it is increased from 0 to 0.060 T in 20 ms.

★★ 30 A copper bar 50 cm long is moved at right angles to a magnetic field of induction 0.80 T at a speed of 130 cm s $^{-1}$. **Determine** the EMF induced in the bar.

★★ 31 A train is moving due north at a constant speed of 100 km h^{-1} in which the vertical component of Earth's magnetic field is 5.4 × 10 $^{-5}$ T. **Determine** the EMF induced in a carriage axle 1.25 m long.

★★★ 32 An aircraft is flying through Earth's magnetic field at a steady horizontal speed of 800 km h^{-1}. The vertical component of Earth's magnetic field is 5.0 × 10^{-5} T. **Determine** the EMF generated between the wing tips if they are 25 m apart.

★★★ 33 Figure 12 on page 240 shows a circular spring loop perpendicular to a magnetic field. The spring is released and its area changes from 0.55 m^2 to 0.15 m^2 in 0.40 s. **Determine** the EMF induced between the points X and Y shown on the loop.

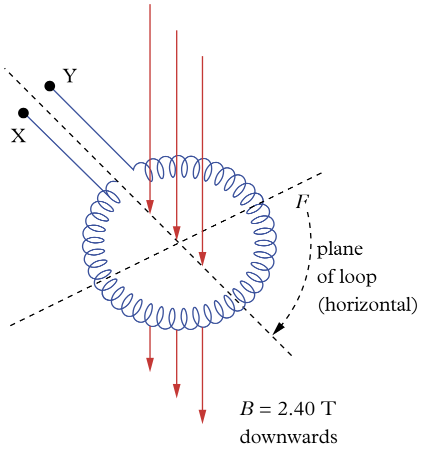


FIGURE 12 Spring loop

Investigate, evaluate and communicate

★★ 34 **Propose** the direction in which the magnet must have been moved (towards or away from the solenoid) in the two situations in Figure 13. The current is shown on the diagram for each case.

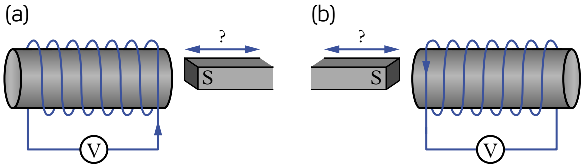


FIGURE 13 Magnet moving in or out

★★ 35 **Predict** the direction in which the magnet must have been moved (to the left or the right) in the two situations shown in Figure 14, to give rise the induced current as shown.

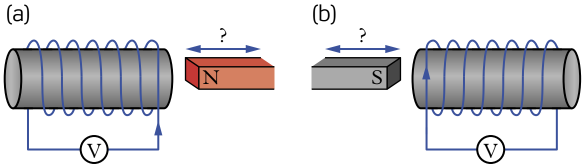


FIGURE 14 Magnet moving in or out

★★★ 36 **Propose** what the effect (if any) there will be on the solenoid to the left, if the switch for the solenoid to the right is closed (Figure 15).

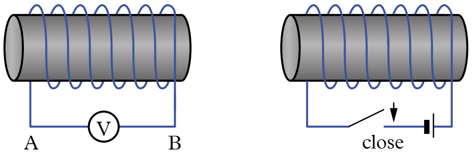


FIGURE 15 Switch closing

★★★ **37** In Figure 16, the solenoid to the right is moved as shown. **Predict** the effect (if any) on the solenoid to the left.

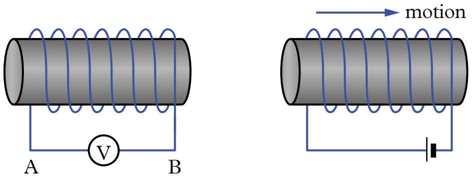


FIGURE 16 Solenoid moving away

★★★ **38** A permanent magnet is tied to a length of string and allowed to swing freely (like a pendulum) over the top of a solenoid (Figure 17). The magnet is released from position A and allowed to swing to position B and back to A.

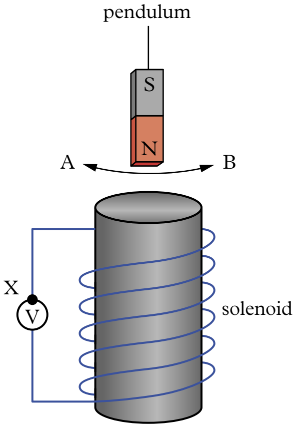


FIGURE 17 Magnetic pendulum

a **Sketch** the shape of the resultant voltage waveform.

b **Justify** your proposed shape by referring to physics principles.

c **Evaluate** the claim: 'the frequency of the voltage waveform is equal to the frequency of the pendulum's displacement'.

★★★ **39** A flexible loop of wire is held between the poles of an electromagnet that provides a uniform field $\boldsymbol{B}$ in the region of the loop. At time $t = 0$, the current through the electromagnet is turned off and the field $\boldsymbol{B}$ falls to zero at time t_1, as shown in the accompanying graph (Figure 18).

a **Sketch** a corresponding graph of the nature of the induced EMF across the ends of the loop as a function of time.

b **Determine** the average EMF induced if time $t_1 = 2.0$ s, given that the loop has an area of 0.04 m^2, and $B = 0.6$ T at time $t = 0$ s.

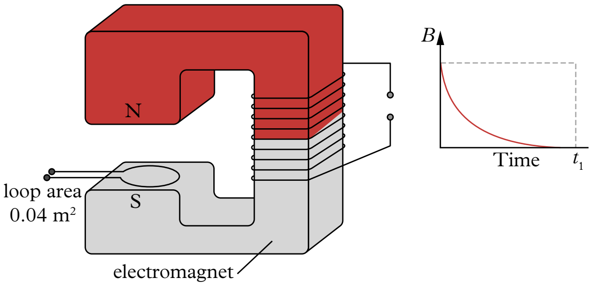


FIGURE 18 Loop inside poles of an electromagnet

★★★ **40** A variable resistor is connected into circuit with solenoid A and a battery, as shown in Figure 19. The resistor is varied from position X to Y, which reduces the size of the current.

a **Predict** the direction of induced current in solenoid B.

b **Explain** your analysis.

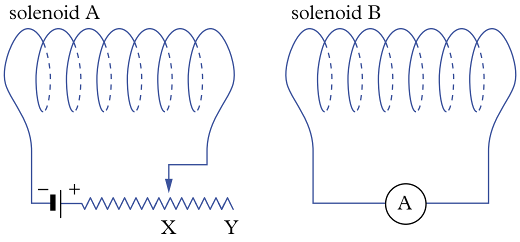


FIGURE 19 Variable resistor and solenoid

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UNIT 3

Practice exam questions

Gravity and electromagnetism

Multiple choice

1 A projectile is fired at an angle and its horizontal range is measured. Select the option that states the elevation angle that would give the minimum range and the maximum range respectively.

A 0°, 90°
B 45°, 90°
C 90°, 45°
D 90°, 0°

2 On steep downhill highways there are sometimes uphill escape ramps for trucks whose brakes are not working properly. Consider a simple 15° upwards ramp being approached by a runaway truck travelling at 60 km h^{-1} (16.7 m s^{-1}). Determine the minimum stopping length.

A 14.2 m
B 27.5 m
C 55 m
D 110 m

3 The force between Q_1 and Q_2 is F in Figure 1. Determine the magnitude of the force between Q_1 and q.

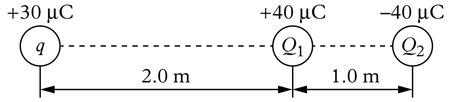


FIGURE 1

A $\frac{F}{2}$
B $\frac{3F}{16}$
C $\frac{3F}{8}$
D $\frac{12F}{16}$

4 A circular loop of wire has an anticlockwise current running through it (Figure 2). Determine the direction of the magnetic field inside of the loop.

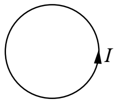


FIGURE 2

A north
B south
C into the page
D out of the page

5 The magnetic flux threading a solenoid with 1000 turns changes as shown in Figure 3.

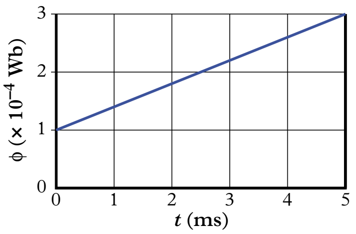


FIGURE 3

Determine the EMF produced.

A 0.04 V
B 0.06 V
C 40 V
D 60 V

Short answer

6 A javelin thrower can achieve a horizontal distance of 40.8 m when she throws her javelin at a speed of 20.0 m s^{-1} at an angle of 45°. She believes she can throw faster and further at an angle of 30°. Determine the velocity required at this angle to exceed a distance of 40.8 m.

7 An inclined plane is at an angle of 25° to the horizontal. A box placed on the incline experiences a frictional force of 20 N and slides down at a constant speed. Determine the mass of the box.

8 A length of fishing line is rated as having a breaking strain of 55 N. A student spins a rubber stopper of mass 34 g on the fishing line in a horizontal circle of radius 85 cm at a constant speed. The stopper makes 10 revolutions in 1.25 s.

a Determine the magnitude of the velocity of the stopper.

  b Predict the speed necessary to break the fishing line.
  c Calculate the mass the fishing line would support if hung vertically.

9 A deuterium atom consists of a nucleus made up of a proton and a neutron, and an orbital electron about 1.5 μm from the nucleus. Determine which is greater: the gravitational force of attraction between the electron and the nucleus, or the Coulomb electrostatic force between them.

10 A distant star has a planet in a circular orbit of radius 5.0×10^9 m with a period of 50 days.
  a Determine the mass of the star.
  b Predict whether it possible to determine the mass of this planet from the data above. Give a reason for your answer.

11 Three point charges are arranged in a right-angled triangle, as shown in Figure 4. The size of the charge and the distances between the charges are also shown.

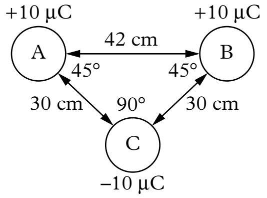


FIGURE 4

  a Determine the magnitude and direction of the force between charges A and B.
  b Determine the magnitude and direction of the net force acting on charge C.

12 An electron with a velocity of $3.0 \times 10^7\,\mathrm{m\,s^{-1}}$ passes through Earth's magnetic field of 25×10^{-6} T at an angle of 40° to the direction of the field. Calculate the force experienced by the electron.

13 A 30 cm long solenoid has 300 turns of wire and carries a current of 3 A, as shown in Figure 5. A copper wire carrying a current of 2 A up the page is placed near the end of the solenoid.

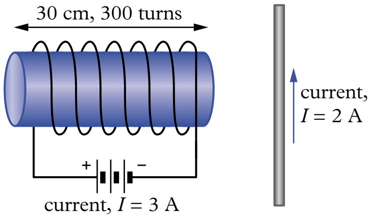


FIGURE 5

  a Determine the magnitude and direction of the magnetic field strength at the end of the coil.
  b Determine the direction of the force on the wire due to the solenoid's magnetic field.
  c A 10 cm segment of the wire is in the solenoid's field. Determine the magnitude of the force acting on the wire.

14 A bar magnet is brought up to a short coil, as shown in Figure 6.

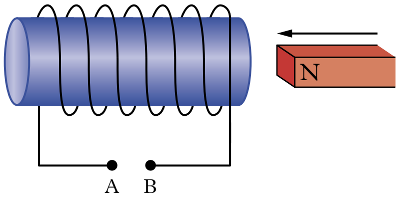


FIGURE 6

  a Determine which of the terminals, A or B, will become positive.
  b Explain your answer in terms of Lenz's law.

15 A circular loop of wire has a radius of 16 cm and a resistance of 4 Ω. It is placed in a 1.5 T magnetic field into the page, as shown in Figure 7. The magnetic field is reduced uniformly to 1.0 T over a period of 0.20 s.

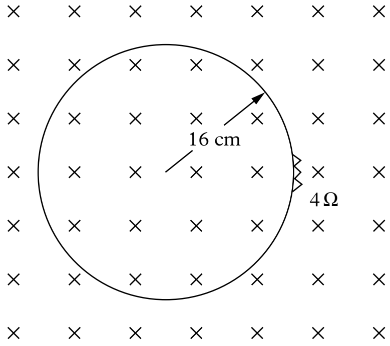
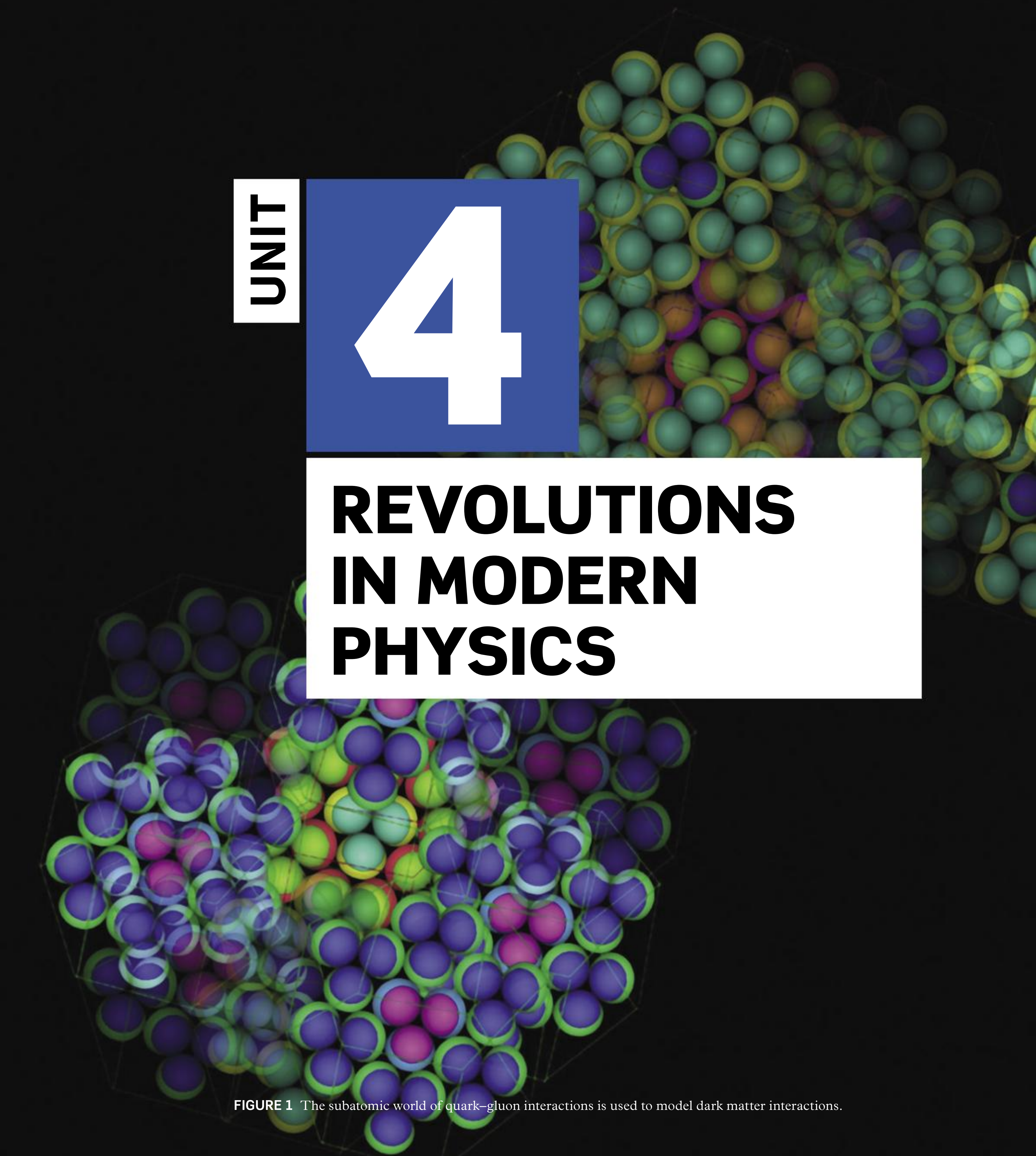


FIGURE 7

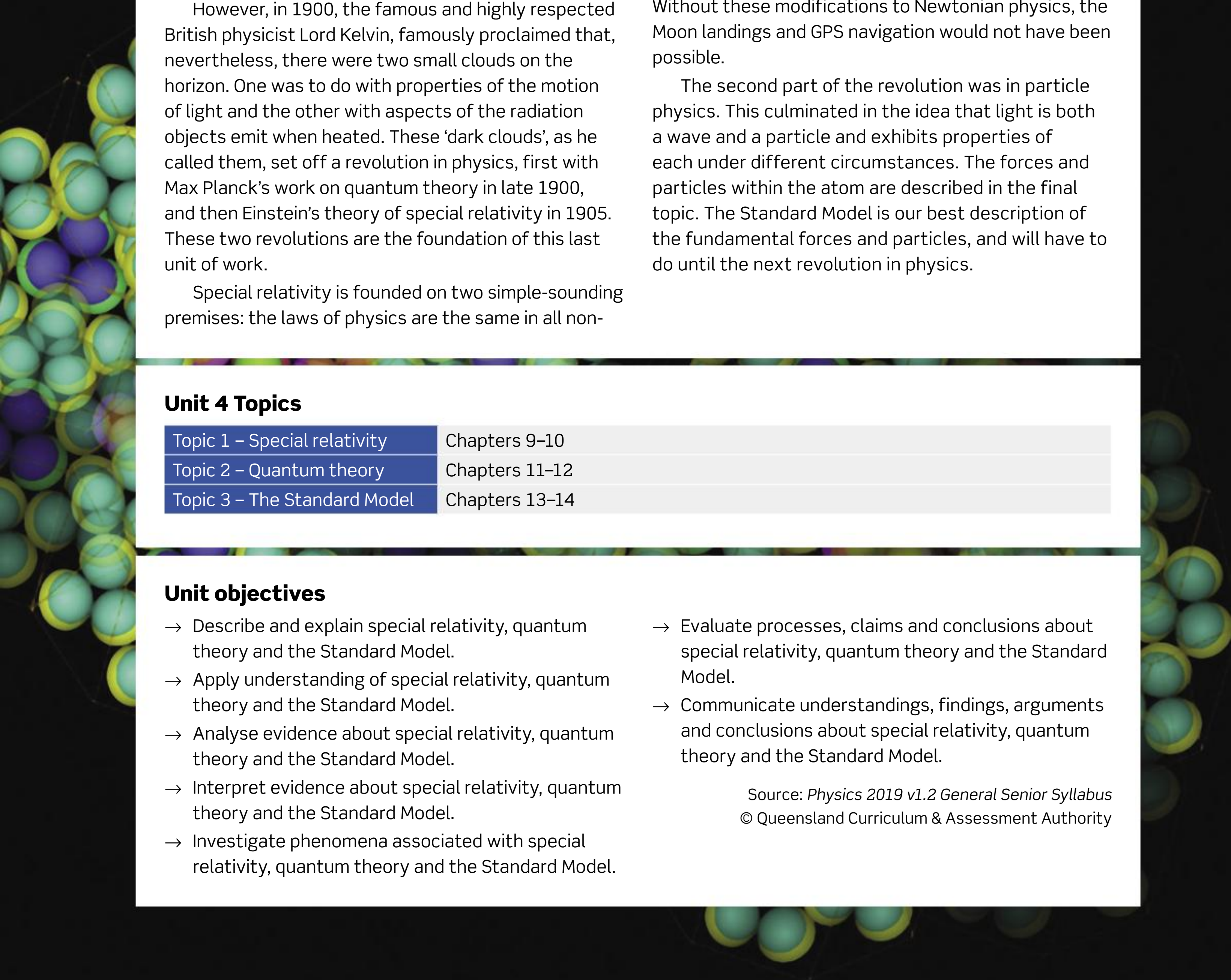
  a Determine the initial magnetic flux in the loop.
  b Determine the EMF generated by the collapsing field.
  c Determine the direction of the induced current in the loop as the field collapses.
  d Explain how you worked out the direction of the current in the loop.



UNIT 4

REVOLUTIONS IN MODERN PHYSICS

FIGURE 1 The subatomic world of quark–gluon interactions is used to model dark matter interactions.



At the beginning of the 20th century, physics had solved nearly all of the problems to do with heat and light. Physicists were quite satisfied that there was not much more to discover.

However, in 1900, the famous and highly respected British physicist Lord Kelvin, famously proclaimed that, nevertheless, there were two small clouds on the horizon. One was to do with properties of the motion of light and the other with aspects of the radiation objects emit when heated. These 'dark clouds', as he called them, set off a revolution in physics, first with Max Planck's work on quantum theory in late 1900, and then Einstein's theory of special relativity in 1905. These two revolutions are the foundation of this last unit of work.

Special relativity is founded on two simple-sounding premises: the laws of physics are the same in all non-accelerated frames of reference, and the speed of light is constant. This led to a number of predictions, namely, that time and length are not fixed but can vary depending on the relative motion of the observers. Without these modifications to Newtonian physics, the Moon landings and GPS navigation would not have been possible.

The second part of the revolution was in particle physics. This culminated in the idea that light is both a wave and a particle and exhibits properties of each under different circumstances. The forces and particles within the atom are described in the final topic. The Standard Model is our best description of the fundamental forces and particles, and will have to do until the next revolution in physics.

Unit 4 Topics

Topic 1 – Special relativity	Chapters 9–10
Topic 2 – Quantum theory	Chapters 11–12
Topic 3 – The Standard Model	Chapters 13–14

Unit objectives

- → Describe and explain special relativity, quantum theory and the Standard Model.
- → Apply understanding of special relativity, quantum theory and the Standard Model.
- → Analyse evidence about special relativity, quantum theory and the Standard Model.
- → Interpret evidence about special relativity, quantum theory and the Standard Model.
- → Investigate phenomena associated with special relativity, quantum theory and the Standard Model.
- → Evaluate processes, claims and conclusions about special relativity, quantum theory and the Standard Model.
- → Communicate understandings, findings, arguments and conclusions about special relativity, quantum theory and the Standard Model.

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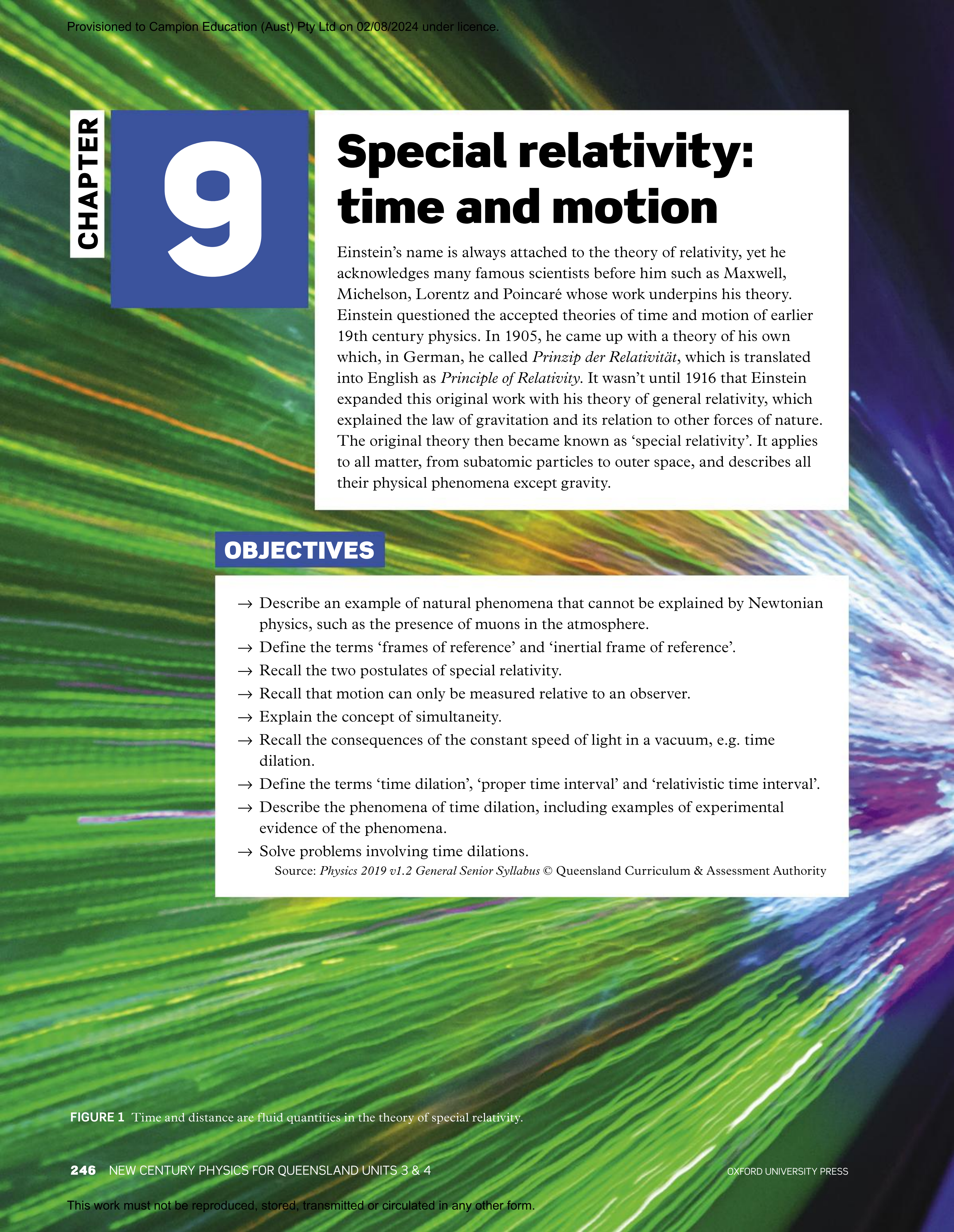

CHAPTER 9

Special relativity: time and motion

Einstein's name is always attached to the theory of relativity, yet he acknowledges many famous scientists before him such as Maxwell, Michelson, Lorentz and Poincaré whose work underpins his theory. Einstein questioned the accepted theories of time and motion of earlier 19th century physics. In 1905, he came up with a theory of his own which, in German, he called *Prinzip der Relativität*, which is translated into English as *Principle of Relativity*. It wasn't until 1916 that Einstein expanded this original work with his theory of general relativity, which explained the law of gravitation and its relation to other forces of nature. The original theory then became known as 'special relativity'. It applies to all matter, from subatomic particles to outer space, and describes all their physical phenomena except gravity.

OBJECTIVES

- → Describe an example of natural phenomena that cannot be explained by Newtonian physics, such as the presence of muons in the atmosphere.
- → Define the terms 'frames of reference' and 'inertial frame of reference'.
- → Recall the two postulates of special relativity.
- → Recall that motion can only be measured relative to an observer.
- → Explain the concept of simultaneity.
- → Recall the consequences of the constant speed of light in a vacuum, e.g. time dilation.
- → Define the terms 'time dilation', 'proper time interval' and 'relativistic time interval'.
- → Describe the phenomena of time dilation, including examples of experimental evidence of the phenomena.
- → Solve problems involving time dilations.

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FIGURE 1 Time and distance are fluid quantities in the theory of special relativity.

MAKES YOU WONDER

Special relativity was a revolution in theoretical physics and astronomy, superseding the 200-year-old theory of Newtonian mechanics. It introduced concepts such as relativity of simultaneity, time dilation and length contraction. You will ask the same questions Einstein asked:

→ If I ran at the speed of light with a mirror in my hand, could I see my own reflection?

→ If a torch was moving, wouldn't its light travel faster than if the torch was at rest?

→ Who is really moving?

9.1

Special relativity

KEY IDEAS

In this section, you will learn about:

- frames of reference
- inertial frames of reference
- the postulates of the theory of special relativity.

For more than two centuries after its inception, the Newtonian view of the world dominated, to the point that scientists developed an almost blind faith in this theory. Almost all problems could be explained using the Newtonian approach. Nevertheless, by the end of the 19th century new experimental data began to accumulate that was difficult to explain using Newtonian theory. New theories soon replaced the old ones. In 1900, Lord Kelvin said that there were '19th century clouds' hanging over the physics of the time, referring to problems that had resisted explanation using the Newtonian approach:

- Light appeared to be a wave, but the medium for its propagation (the 'aether') was undetectable.
- The equations describing electricity and magnetism were inconsistent with Newton's descriptions of space and time.

From the 1940s, the problem of accounting for the lifetimes of muons (radioactive particles) at different altitudes stimulated interest in how the theory of relativity could be incorporated to explain the results. This provides a good case study on how the theory of relativity was confirmed.

The problem of muon decay

A muon is a subatomic particle that is created in particle collisions. When cosmic ray protons from the Sun collide with oxygen and nitrogen atoms in the upper atmosphere, subatomic particles called **pions** are created. These decay within a few metres into **muons** and **muon neutrinos**. The muons generally continue in the same direction as the original proton, at a velocity near the speed of light ($>0.99c$). But the muons are short-lived with an average life span of 2.2 microseconds and quickly decay into an electron and two neutrinos. The generated muons rain down at high speed, but in the few microseconds it takes them to reach the ground some of them will have decayed.

In 1962, the physicists David Frisch and James Smith at the Massachusetts Institute of Technology were experimenting to prove to their students that relativity is real and that it is needed to solve the riddle of muons. They were testing one of the consequences of high-speed travel – that time slows and distances contract.

The scientists set up equipment on the top of the nearby Mount Washington and measured how many muons were present in the atmosphere by counting the number that passed through their counter in one hour at that altitude (1910 m). Their equipment counted 563 per hour. They then travelled back down to sea level and repeated the count. It should have taken the muons 6.4 μs to travel the intervening distance, which meant that most of them should have decayed. Using Newtonian (non-relativistic) calculations only 27 per hour should have been detected at sea level. However, they counted 409 per hour, as predicted by the **theory of special relativity**.

pion
a subatomic particle produced in the atmosphere as a result of the collision of cosmic ray protons with nitrogen and oxygen atoms

muon
an elementary particle similar to the electron but with a greater mass; it is the product of the decay of pions

muon neutrino
an almost massless and neutral elementary particle produced in radioactive decay of pions

theory of special relativity
a theory that all motion must be defined relative to a frame of reference; it consists of two principal postulates. It explains how space and time are linked for objects moving at constant speed in a straight line, and forms part of the basis of modern physics

FIGURE 1 Cosmic ray protons from the Sun strike atoms in the atmosphere to produce high-energy pions that decay to muons.

This was a major confirmation of Einstein's theory. To the observers, the distance from mountaintop to sea level was 1910 m. That is, the distance was 1910 m in the scientists' 'frame of reference', but to the muons the distance was much shorter – just 183 m. Thus, in the muon's frame of reference the distance was 183 m. This is a consequence of the relativity postulate for very high speeds and is called 'length contraction' (that is, the path length contracted from 1910 m to 183 m as measured in the muon's frame of reference).

The other measurement that changed was the muon's lifetime. The mean lifetime in the muon's frame of reference is 2.2 μs but in the scientists' frame of reference it became 23 μs, so to the scientists the muons took longer to decay. This too is a consequence of relativity and is known as 'time dilation' ('dilation' means enlargement; that is, time lengthened from 2.2 μs to 23 μs).

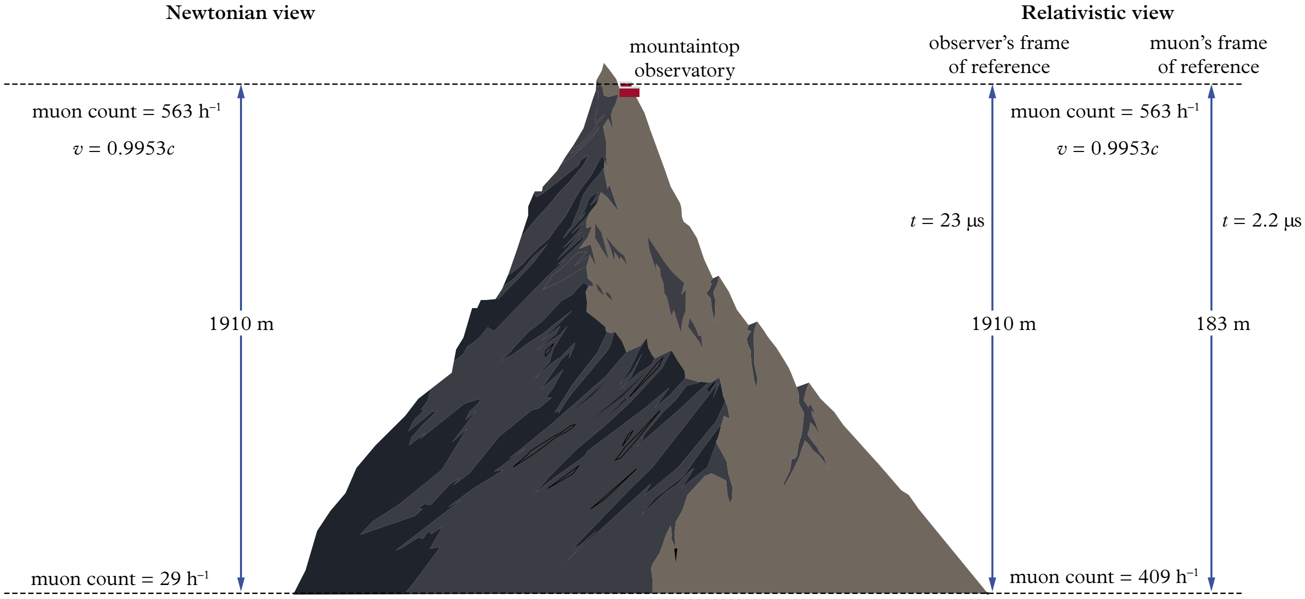

FIGURE 2 The muon experiment confirms aspects of special relativity.

Frames of reference

The term '**frame of reference**' used in the description of the muon experiment is one of the cornerstones of any study of modern physics. When you describe the motion of an object, you need something to compare it against. This is the central idea of **relative motion**. When a car drives past at 60 km h^{-1}, this speed is relative to the ground or Earth. That is your frame of reference. To the person inside the car, it could be they who are stationary and the Earth going by at 60 km h^{-1}. The car becomes their frame of reference.

A frame of reference is an arbitrary or abstract coordinate system that defines the location of the observer. It is also called a reference frame.

frame of reference
an arbitrary set of axes with reference to which the position or motion of something is described or physical laws are formulated

relative motion
the motion of an object with regard to some other moving object; the motion is not calculated with reference to Earth, but is the velocity of the object in reference to the other moving object as if it were in a static state

Inertial versus non-inertial frames of reference

Let us consider how the laws of physics hold for the different frames of reference. Consider three simple experiments you have done:

- timing the swing of a pendulum to confirm g, the acceleration due to gravity
- pulling a block along the bench with hanging weights to confirm Newton's second law $F = ma$
- swinging a rubber stopper in a circle above your head to confirm the centripetal motion formula $F_c = \frac{mv^2}{r}$.

Your results will not be different from those of other students who used a different coordinate system. For example, students at other schools in Queensland will achieve the same results. This is because they are all at rest in your frame of reference and, because the laws of inertia hold, you can say you are all in an inertial frame of reference.

But what if you were travelling aboard a ship and doing Physics classes while on holidays. Imagine the ship is moving at constant speed. Will you get the same results as you would on land? The answer to this is yes, of course you will. If the laws of inertia hold, then the ship is said to be an **inertial frame of reference**.

inertial frame of reference
a non-accelerating frame of reference in which Newton's laws of motion hold

In general, a frame of reference that is stationary or moving at constant velocity is an inertial frame. However, in a frame of reference that is accelerating, things are different. Imagine doing a pendulum experiment while accelerating. The pendulum will make an angle to the vertical even when it is not swinging, in the same way that something hanging from the rear view mirror of a car moves when the car accelerates away at the lights.

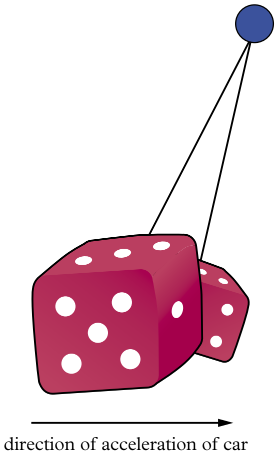

FIGURE 3 As the car accelerates forwards the fluffy dice seem to be pushed back. But what is pushing them?

Table 1 provides examples of inertial and non-inertial frames of reference.

TABLE 1 Frames of reference

Inertial reference frames	Non-inertial reference frames
At rest on Earth	Accelerating upwards in an elevator
In a car at constant velocity	Going around a corner
In a spaceship at constant velocity	In free-fall

Note: the surface of Earth is considered an inertial reference frame even though it is undergoing centripetal acceleration. The acceleration is considered small enough to be disregarded.

CHALLENGE 9.1

The Gravitron

Is riding in a Gravitron that spins at constant speed an example of an inertial reference frame?

Postulates of special relativity

In 1905, Albert Einstein put forward a revolutionary theory that explained and predicted these phenomena. His theory of special relativity changed the way nature is understood.

Special relativity is a deceptively simple theory and has only two assumptions or 'postulates' – **the two postulates of special relativity**.

- The laws of physics are the same in all inertial (uniformly moving) frames of reference.
- The speed of light in a vacuum has the same value, *c*, in all inertial frames of reference.

Most physicists agree that the second postulate is redundant as it is a logical consequence of the first.

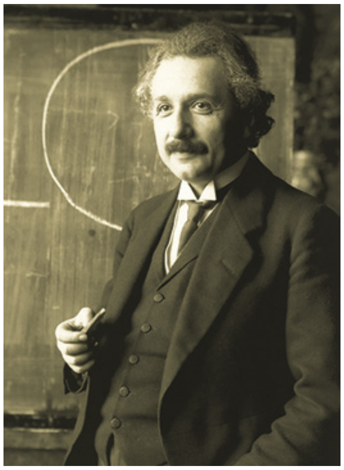

FIGURE 4 Albert Einstein, 1921

the two postulates of special relativity the two assumptions of the theory of special relativity: 1, that the laws of physics are the same in all inertial frames of reference; 2, that the speed of light in a vacuum has the same value, *c*, in all inertial frames of reference

CHECK YOUR LEARNING 9.1

Describe and explain

1 **Explain** briefly how the muon experiment confirmed the theory of relativity.

2 **Recall** the two postulates of special relativity.

3 **Clarify** whether the surface of Earth is an inertial reference frame.

Apply, analyse and interpret

4 **Distinguish** between inertial and non-inertial frames of reference.

5 **Deduce** which of the following is an inertial reference frame (or a very good approximation).

  a your bedroom
  b a car rolling down a steep hill
  c a train coasting along a level track
  d a rocket being launched
  e a roller-coaster going over the top of a hill
  f a skydiver falling at terminal speed

Investigate, evaluate and communicate

6 You are in a spaceship travelling in outer space. **Evaluate** the claim that there is no way you can measure your speed without looking outside.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 9.1
- » Challenge worksheet
  9.1 The Gravitron
- » Video
  Introduction to special relativity
- » Weblink
  Einstein's theory of relativity

9.2

Relative motion

KEY IDEAS

In this section, you will learn about:

- motion being measured relative to an observer
- the constant speed of light in a vacuum
- simultaneity.

For Newton, there was a 'master' or absolute inertial frame: a frame that was stationary relative to absolute space. However, subsequent investigations showed that there is no master frame and that all motion is relative.

What this means is that the idea of being stationary or of moving is in the eyes of the observer. For example, you are travelling in a train at 60 km h^{-1} along a railway. You see houses go past and you imagine that you are stationary and the houses (Earth) are moving at 60 km h^{-1}. But to people in the houses looking at the train, it is you who is moving.

'Who is really moving?' The answer is that you are moving relative to the stationary observer outside the train, but the stationary observer is moving relative to you. So you are each moving relative to the other. Both your inertial frame and the observer's inertial frame are equally valid. There is no way of working out who is really moving because any experiments in the train would give the same results as on the platform. The platform and the train are both inertial frames. There is one frame in which the train is moving, another in which the observer is moving, and another in which both frames are moving. However, the one thing all observers agree on is the relative speed.

FIGURE 1 Who is really moving: the people on the platform or the people on the train?

Cannonball and the boat

An old favourite example to illustrate this further is a cannonball dropped from the mast of a yacht sailing past an observer on the shore (Figure 2). For a sailor on the yacht the cannonball appears to fall straight down (Figure 2a). From the point of view of an observer on shore, the cannonball falls with a uniform acceleration downwards while moving with constant speed in the horizontal direction; that is, the cannonball follows a parabolic path relative to the shore, just like a rock thrown horizontally off a cliff (Figure 2b). For both observers the cannonball lands at the base of the mast, and the laws of inertia are the same in both reference frames even though the paths are different.

However, in frames moving relative to each other, the velocity of an object will appear different.

Not all things change when viewed in different reference frames. For example, the number of atoms in an object doesn't change. If you time your pulse rate at home as 60 beats per minute, then you will also time it as 60 beats per minute aboard a moving train. But if you are sitting down on the train as it travels along the railway line at 5 m s^{-1}, you could say your speed is zero relative to the train ($v_{\text{person–train}}$ = 0 m s^{-1}), and the speed of the train relative to the Earth ($v_{\text{train–Earth}}$) = 5 m s^{-1}. Your speed relative to the Earth ($v_{\text{person–Earth}}$) would then also be 5 m s^{-1}.

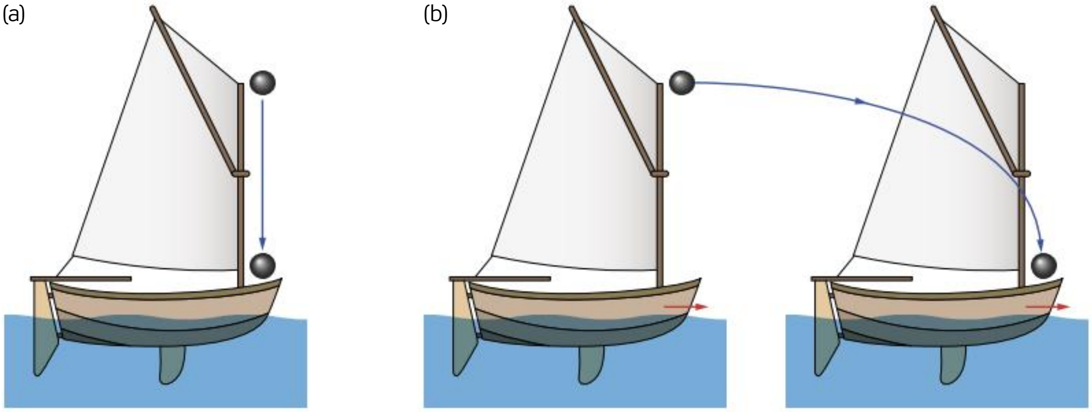

FIGURE 2 A falling cannonball travels different paths depending on the frame of reference: (a) from aboard the yacht; (b) from the shore as the yacht sails past you.

CHALLENGE 9.2A

Where does the pen land?

If you were running along and dropped a pen would it land at your feet, in front of you, or behind you?

Constancy of the speed of light

As you saw in earlier chapters, changing magnetic fields produce electricity; conversely, changing electric fields produce magnetism. In the mid-19th century, the great Scottish physicist James Clerk Maxwell deduced that, as each field could create the other, a 'wondrous new phenomenon' would result. Once a changing field of one type appears, self-perpetuating systems of electric and magnetic fields take on an independent existence that is no longer associated with what started them, and which would propagate through space as an electromagnetic wave. Maxwell explored the properties of these waves theoretically and calculated their speed as 3.00×10^8 m s^{-1} (equal to the speed of light). The symbol 'c' was chosen to represent the speed of light. It was the initial letter in the Latin word *celeritas* meaning 'swiftness' (as in accelerate). Today, the accepted value is 299 792 458 m s^{-1}, which is rounded off to two decimal places as 3.00×10^8 m s^{-1}. So there was no doubt that the speed of Maxwell's electromagnetic waves was the speed of light and his brilliant conclusion was inescapable: light is an electromagnetic wave.

Nineteenth-century physicists were familiar with the properties of water waves, sound waves and waves on springs. These waves all needed a medium for their propagation, be it water, air or steel. There was no reason to think that Maxwell's electromagnetic waves should be any different. They called this transparent medium the 'aether' and assumed it permeated all space.

It was therefore presumed that the velocity of light given by Maxwell's equations must be with respect to this 'aether'. Ultimately, scientists found that there was no aether; no material medium was needed for the propagation of light, and the velocity of light had a constant value in free space irrespective of the motion of the source or the motion of the observer. Einstein used these findings to develop his theory of relativity.

Study tip

In this unit, the speed of light of 299 792 458 m s^{-1} is rounded off to one significant figure as 3×10^8 m s^{-1}.

Addition of velocities

Einstein made his second postulate that 'the speed of light in a vacuum has the same value, *c*, in all inertial frames of reference' and this leads to an interesting conclusion: all observers regardless of whether they are moving or not will measure the same speed of light, *c*, regardless of whether the source of light is moving or not.

Car shooting tennis balls

Imagine being an observer who is watching a car that is travelling along a road at a leisurely 0.9 m s^{-1}. Imagine that the car has a cannon on the roof that shoots a tennis ball forwards at 1.0 m s^{-1} relative to the car. The observer sees the ball coming towards them at 1.9 m s^{-1}.

$$v_{\text{ball-Earth}} = v_{\text{ball-car}} + v_{\text{car-Earth}}$$
$$1.9\ \text{m s}^{-1} = 1.0\ \text{m s}^{-1} + 0.9\ \text{m s}^{-1}$$

where $v_{\text{ball–Earth}}$ is the velocity of the ball relative to Earth, $v_{\text{ball–car}}$ is the velocity of the ball relative to the car and $v_{\text{car–Earth}}$ is the velocity the car relative to Earth.

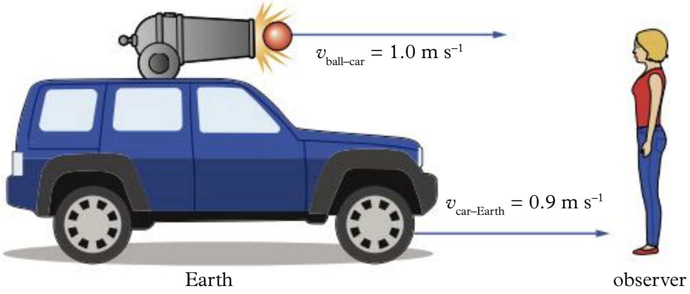

FIGURE 3 Newtonian addition of velocities

Study tip

The speed of light in a vacuum is 3×10^8 m s^{-1}. Other high speeds can be stated as a fraction of this. For example, 0.8*c* is equal to $0.8 \times 3 \times 10^8 = 2.4 \times 10^8$ m s^{-1}. Likewise, speeds in metres per second can be converted to units of *c* by dividing by 3×10^8. For example, a speed of 2×10^8 m s^{-1}, in units of *c* would be:

$$v = 2 \times 10^8\,\text{m s}^{-1}$$
$$= \frac{2 \times 10^8}{3 \times 10^8}c$$
$$= 0.67c$$

Study tip

'Relativistic speed' is a term used by physicists to denote that relativistic formulas (not Newtonian or classical formulas) should be used. It is an arbitrary cut-off, but you can regard anything greater than 0.1*c* as being relativistic.

Car with lamp

Contrast this to a headlamp on a very fast car. It sends out a light wave at 1.0*c* (Figure 4). The observer sees the car coming towards them at 0.9*c*, and the light leaving the car in a forwards direction at 1.0*c* relative to the car. In Newtonian terms the observer should measure the speed of light as 1.9*c*; however, they measure it at 1.0*c* instead. The velocity of the source makes no difference.

$$v_{\text{light–Earth}} = v_{\text{light–car}} + v_{\text{car–Earth}}$$
$$1.9c \neq 1.0c + 0.9c$$
$$1.0c = 1.0c + 0.9c$$

where $v_{\text{light–Earth}}$ is the velocity of the light relative to Earth, $v_{\text{light–car}}$ is the velocity of the light relative to the car and $v_{\text{car–Earth}}$ is the velocity of the car relative to Earth.

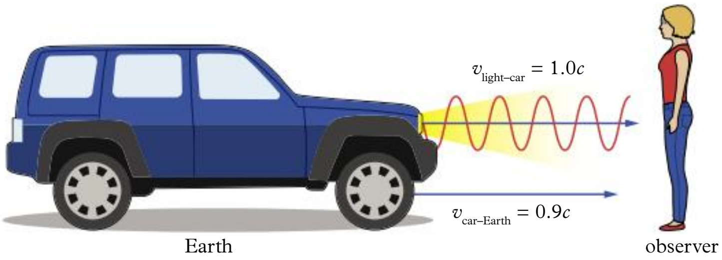

FIGURE 4 Failure of Newtonian kinematics at high speeds

The following case study confirms the unchanging speed of light.

CASE STUDY 9.2

Confirmation of special relativity

There is a pair of burnt-out neutron stars in the constellation Aquilia 1500 light-years (1.4×10^{16} km) from us. The stars, called PSR 1913 + 16, are a binary pulsar; that is, they orbit around a common centre and give off pulses of light waves as they spiral to their death. Because we see them side on (like looking at a saucer from the side), sometimes they are approaching and sometimes receding (moving away) – at a speed of 300 km s^{-1} (about 10% the speed of light).

In 1974, the US astronomers Joe Taylor and Russel Hulse measured the characteristics of light emanating from the pulsars and found the speed of the light to be the same irrespective of whether they were approaching us or receding from us. For this, and other confirmations of relativity, they were awarded the Nobel Prize in Physics in 1993.

These burnt-out stars provide an example of the confirmation of the unchanging speed of light.

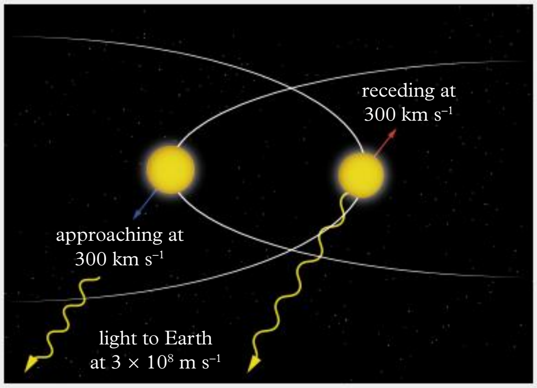

FIGURE 5 The light from binary pulsars arrives on Earth at c, irrespective of whether the stars are approaching or receding.

CHALLENGE 9.2B

Velocity of a rocket relative to Earth

Einstein derived a formula to allow for the relativistic addition of velocities to overcome the anomaly above.

$$v_{AB} = \frac{v_A - v_B}{1 - \frac{v_A v_B}{c^2}}$$

This means that v cannot be $>1.0c$, to avoid violation of the second postulate of relativity.

Rocket B was travelling away from Earth at a velocity of $v_{BE} = +0.60c$ when it launched rocket A forwards in the same direction at a velocity of $+0.6c$ relative to itself ($v_{AB} = +0.6c$).

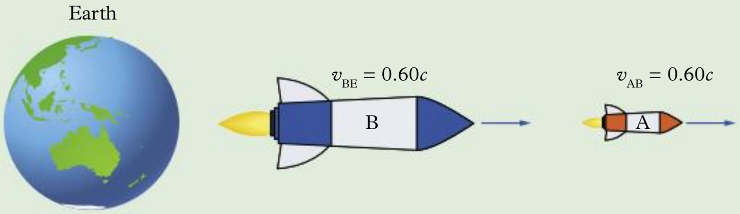

FIGURE 6 Rocket B ($v_{BE} = +0.60c$) launches rocket A ($v_{AB} = +0.60c$). Rocket A is moving at $0.60c$ relative to rocket B.

Show that the velocity of rocket A relative to Earth (v_A) is not $+1.2c$ (as this would contradict Einstein's second postulate), but that v_A is only $+0.88c$.

CHALLENGE 9.2C

Can you add velocities?

Is it now wrong to say that you can't just simply add velocities together?

CHECK YOUR LEARNING 9.2

Describe and explain

1 **Describe** what is meant by 'relativistic speed'.

2 **Explain** what it means to travel at 0.5*c*.

3 **Describe** a simple example of two observers and relative motion.

Apply, analyse and interpret

4 A very fast car travels at 0.6*c* relative to Earth and fires a particle forwards at +0.5*c* relative to the car. **Determine** which of the following is the speed of the particle relative to Earth: <1.0*c*, 1.0*c*, >1.0*c*.

5 A mirror moves towards you at speed *v*. You shine a light towards it and the light beam bounces back at you. **Determine** the speed of the reflected beam.

Investigate, evaluate and communicate

6 The speed of light in glass is only 2×10^8 m s^{-1}. Write a short paragraph to **evaluate** the proposition that 'Einstein made a blunder in saying that the speed of light is constant'.

7 **Propose** a method for measuring the speed of light in a laboratory.

8 a **Predict** what the path in Figure 7b would look like if gravity was:

  i less than 9.8 m s^{-1}

  ii more than 9.8 m s^{-1}

  iii zero

b **Predict** how the path in Figure 7b would differ if the cannonball was half the original mass.

c The mast was 20 m high, and the boat sailed at a speed of 20 m s^{-1} relative to the shoreline. **Determine** how many seconds the cannonball would take to hit the deck in Figure 7a and Figure 7b.

d **Calculate** how far to the right the boat would have travelled in this time.

e **Calculate** the displacement and average velocity of the cannonball relative to the shoreline in its journey shown in Figure 7b.

f **Determine** how far the cannonball would have travelled in Figure 7b relative to the shoreline. You will need to work out the 'arc length' of the parabola. Caution – a very difficult question and you will need to use calculus!

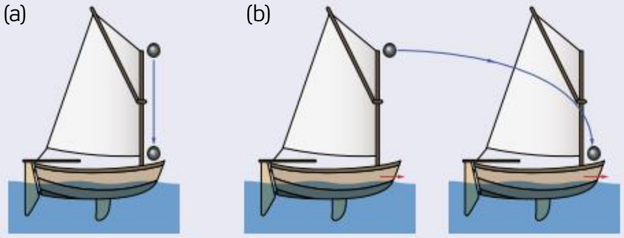

FIGURE 7

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- » Student book questions Check your learning 9.2
- » Challenge worksheet 9.2A Where does the pen land?
- » Challenge worksheet 9.2B Velocity of a rocket relative to Earth
- » Challenge worksheet 9.2C Can you add velocities?

9.3 Simultaneity

KEY IDEAS

In this section, you will learn about:

- events that occur simultaneously
- the relativity of simultaneity.

Imagine that at your school there are two bells, one at each end of the school. You hear both bells sound at the same time. Could there be a situation in which another observer hears one bell before the other? In other words, can an event (the sounding of the bells) be simultaneous to one observer but not to another?

What does 'simultaneous' mean? Two **events** are simultaneous if they occur at the same time. But how can you tell if the two bells rang at the same time? If the bells were side by side there would be no problem; but when events are separated in space it gets difficult. If you were midway between the two bells and you heard them ring at the same time, you could say they were simultaneous. But what if you were closer to one bell than the other? If you still heard them at the same time, the more distant one must have sounded first because the sound had to travel further to your ears.

Does this apply to light as well? If you were looking out your window at dusk and two street lights came on at the same time, you would say the events were simultaneous if you were midway between them (Figure 1). If you were not midway, you would have to calculate the time it took to get from each event to your position so that you could work out when the events actually occurred. If both lights appeared to turn on at the same time but one was closer to you than the other, the closer one must have occurred after the more distant one. They were not simultaneous. **Simultaneity** can be defined thus: two events are simultaneous in a given reference frame if light signals from the events reach an observer who is midway between them at the same time.

event
an act or action with a distinct beginning and end, e.g. the swing of a pendulum, ticking of a watch, motion of an object (including a light pulse) from one place to another, decay of a nucleus, propagation of an energy wave (light, sound, water)

simultaneity
the relation between two events assumed to be happening at the same time in a frame of reference

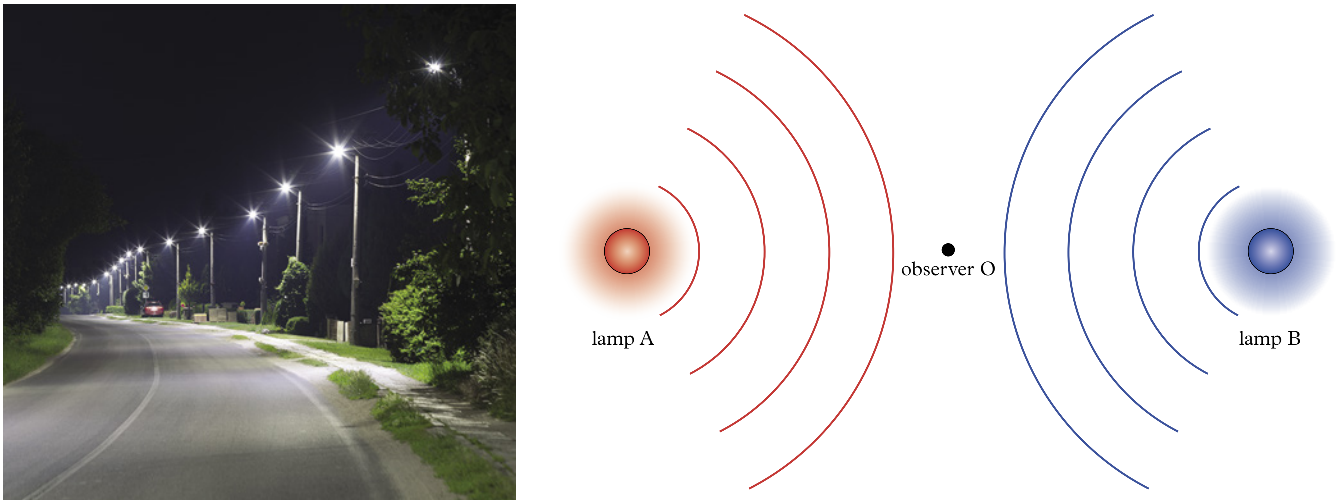

FIGURE 1 A moment after street lights A and B turn on, light waves travel outwards. If they arrive at observer O at the same time, she can say the events are simultaneous because she is midway between the two lights.

relativity of simultaneity
events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial

The relativity of simultaneity

To show that two events that are simultaneous in one frame, S, are not simultaneous in another frame, S', moving relative to S, you will use a paradox introduced by Einstein. He called such paradoxes *Gedunken* ('thinking') experiments and they were true premises that seemed to result in self-contradictory conclusions (the definition of a paradox).

Flashlights on a train paradox

Imagine an observer sitting on an embankment at the side of a train track. A train is the moving frame of reference and the embankment is the stationary frame.

Imagine that in the centre of the train carriage there is a person who is holding a flashlight that can send out a pulse of light in the forwards direction and at the same time a pulse of light in the backwards direction, as shown in Figure 3. Also, imagine that the rear and front doors are opened automatically when the light pulses arrive.

To the person holding the device, the doors of the carriage will open simultaneously (Figure 4a). But to a person on the embankment, the rear door will open before the front door (Figure 4b). This is because the stationary observer sees the back door move forwards to meet the light pulse while the front door moves away from the light pulse, so the light gets to the rear door before the other pulse can get to the front door.

Hence, the opening of the doors may be simultaneous to one observer (on the train) but not simultaneous to another (on the embankment). Students often say, 'Who is right? Do the doors really open together or not?' The answer is that both are right. It depends on your frame of reference. Remember, there is no best frame of reference; some are just more useful than others. You, as an observer, will decide which is the more useful frame.

FIGURE 2 The flashlights on a train paradox is one of Einstein's 'thinking' experiments.

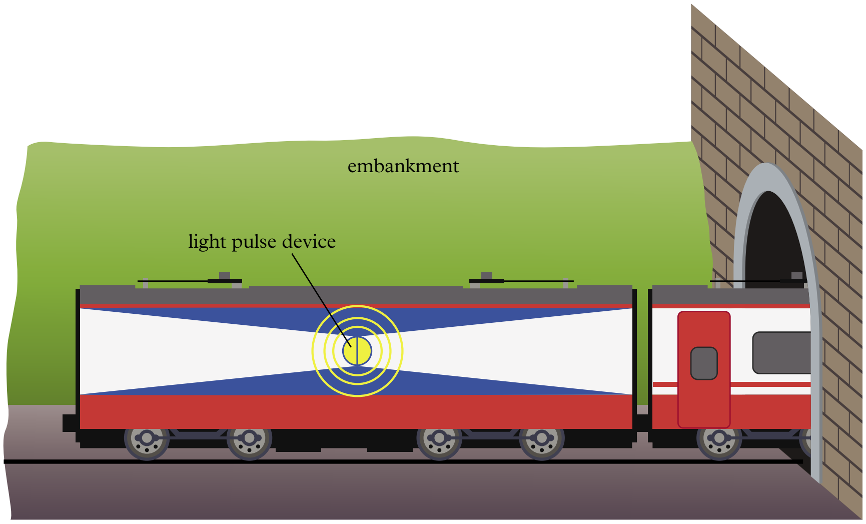

FIGURE 3 The train with the light pulse device

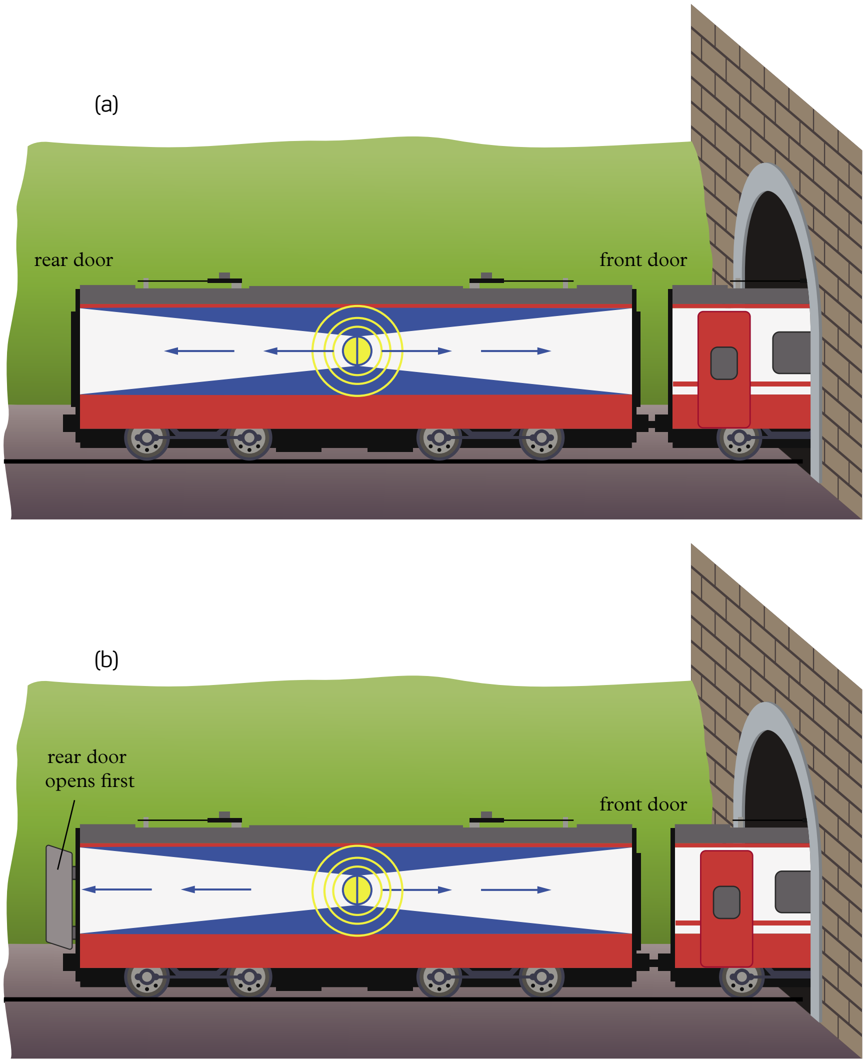

FIGURE 4 (a) Motion of the light pulse as seen by an observer inside the train. (b) Motion of the light pulse as seen by the observer on the embankment — it gets to the rear door first and opens it.

CHALLENGE 9.3

Image in a moving mirror

When Einstein was a boy he wondered about the following question. A runner holds a mirror at arm's length in front of his face. Can he see himself in the mirror if he runs at the speed of light?

When you look at yourself in a mirror, light travels from your face to the mirror and then is reflected back to your eyes as shown in Figure 5a.

Einstein wondered how light could ever get from your face to the mirror if the mirror is travelling away from the light beam at the speed of light (Figure 5b). The light would never catch up to the mirror! He soon realised the flaw in the logic. Can you? Propose your reasons.

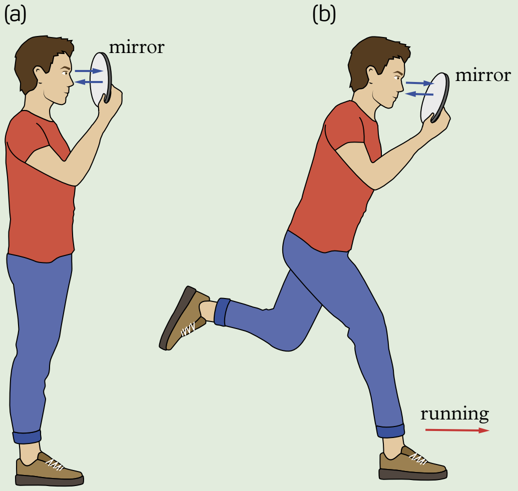

FIGURE 5 (a) The motion of light when you are at rest. (b) Your face and mirror are moving at speed *c*. Can you see yourself now?

CHECK YOUR LEARNING 9.3

Describe and explain

1 **Explain** the meaning of 'simultaneity'.

2 **Describe** how the 'flashlight on a train' paradox is resolved.

3 **Recall** the term that states that 'events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial'.

Apply, analyse and interpret

4 A light in the middle of a train carriage is used to open doors at each end of the carriage. One door is coloured red, the other is black. To an observer in the carriage the doors open at the same time. To an observer on the platform the red door opens first. **Determine** the direction in which the train is travelling and state this clearly.

5 Two bells in a school are rung and a person midway between them hears them at the same time. However, a person standing closer to one bell hears it ring first. **Determine** whether the bells rang simultaneously or not.

Investigate, evaluate and communicate

6 **Evaluate** this claim: 'Events that are simultaneous in one frame will be simultaneous in another frame travelling at constant speed with respect to it, but not if it is accelerating with respect to the first frame.' Write a paragraph showing your evaluation.

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  Check your learning 9.3
- » Challenge worksheet
  9.3 Image in a moving mirror
- » Video
  What is simultaneity?
- » Weblink
  Flashlights on a train paradox

9.4

Relativity of time

KEY IDEAS

In this unit, you will learn about:

- proper time intervals
- relativistic time intervals
- time dilation
- mean lifetime of a particle.

What is time? When Einstein was asked this he said 'what a clock measures' and left it at that. You usually think of time marching on, oblivious to anything you may be doing. Although you may think time drags when you are doing something boring and goes fast when you are having fun, this is not time in a technical sense, only psychological time.

Variability of time

The following example should convince you that the time interval between two events cannot be the same for two observers in motion with respect to each other. The first observer is sitting on the bus. This bus has a light source on the floor and a mirror directly above it on the ceiling. A brief flash of light leaves the source and travels upwards to hit the mirror, reflect and return to the source, as shown in Figure 1a.

To a second observer who is sitting at a bus stop watching the bus pass by, the flash occurs when the bus is located to the left, strikes the ceiling mirror when the bus is directly in front of the second observer, and then returns to the source when the bus is to the right of the observer, as shown in Figure 1b.

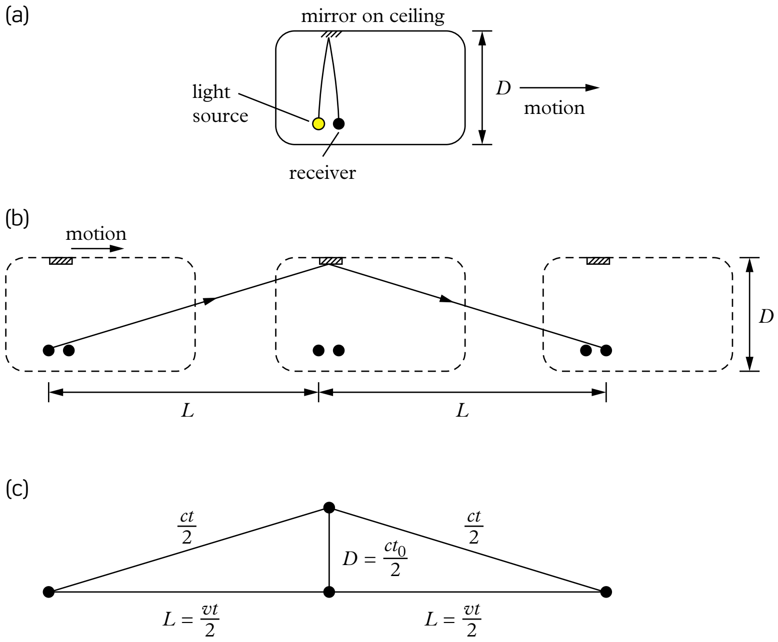

FIGURE 1 (a) Path of light as viewed by the observer aboard the bus. (b) Path of light as viewed by the observer at the bus-stop — the path looks much longer. (c) The derivation of Einstein's formula

Figure 1's diagram labelling is as follow:

- The distance from the source to the ceiling mirror is D.
- To the second observer (at the bus stop) the bus is travelling to the right at velocity v and moves a distance L in the time between the flash and the light striking the ceiling.
- The bus moves another distance L by the time the light goes from the mirror back to the source. This makes the total distance moved by the bus equal to $2L$.
- Remember, light travels at a speed c for both observers (Einstein's second postulate).

proper time interval
the time between two events measured be an observer at rest to the events

To the first observer aboard the bus, the light travels a distance of $2D$ (up to the mirror and back to the source) in a time t_0. This time is called the **proper time interval** because the start and finish occur in the same place in space. They have not moved relative to the observer. In other words, they were at rest relative to the observer. Using our velocity formula:

$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v}, \text{ hence } t_0 = \frac{2D}{c}.$$

When rearranged, the distance is given by $D = \frac{c t_0}{2}$, where D is the distance from the source to the ceiling mirror, c is the speed of light and t_0 is the proper time as measured by the observer on the bus. To the second observer (at the bus stop), the light has travelled a triangular path in time t (t with no subscript, as distinct from proper time, t_0). As the speed of light is the same for both observers, the light actually travelled a longer path from the viewpoint of the observer at the bus stop, so it must have taken a longer time. This is called the **relativistic time interval** or 'dilated' time (Latin *dilatare* meaning 'to spread out'; that is, to bring them away from each other or make them bigger). Relativistic time is the time between two events that occur in two different places in space. In this case the two events (the flash leaving the source and the flash arriving back at the source) are separated by a distance of $2L$. They have moved relative to the outside observer.

relativistic time interval
the time between two events measured by an observer moving with respect to the events; also known as dilated time (interval)

Looking at Figure 1(c), as seen by the observer at the bus stop, the total distance s travelled by the light is calculated by using the formula $v = \frac{s}{t}$, or $s = vt$, where v is the velocity of light and t is the time taken by the flash to strike the mirror and return to the source. In this case, the distance travelled by the light will be ct. In the diagram, the length of the hypotenuse will equal $\frac{ct}{2}$. The time taken (t) for the bus to go from start to finish will be given by $t = \frac{2L}{v}$, hence $L = \frac{vt}{2}$ (the base of each triangle).

Using Pythagoras' theorem on one of the right-angled triangles:

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{c t_0}{2}\right)^2 + \left(\frac{vt}{2}\right)^2$$

$$(ct)^2 = \left(c t_0\right)^2 + \left(vt\right)^2$$

$$t^2 = t_0^{\ 2} + \frac{v^2}{c^2} t^2$$

$$t_0^{\ 2} = t^2 - \frac{v^2}{c^2} t^2$$

$$t_0^{\ 2} = t^2\left(1 - \frac{v^2}{c^2}\right)$$

$$t^2 = \frac{t_0^{\ 2}}{1 - \frac{v^2}{c^2}}$$

Take square roots of both sides:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where t is the time as measured by an observer at the bus stop (relativistic time interval), t_0 is the time as measured by the observer on the bus at rest to the event (proper time), v is the velocity of the bus relative to the ground and c is the speed of light.

Because v is always less than c, the value of $\sqrt{1 - \frac{v^2}{c^2}}$ must always be less than 1, so $t > t_0$. That is, the time between the two events (the sending of the light and its arrival at the receiver) is greater for the second observer (at the bus stop) than for the first observer on the bus).

This is a general result of the theory of relativity, and is known as **time dilation**. Sometimes this is stated simply as *moving clocks are measured to run slow*, but you need to be careful with the interpretation.

time dilation the difference in the time interval between two events as measured by observers moving with respect to each other

CHALLENGE 9.4

Can you use your pulse to measure time?

Galileo measured the period of swinging lamps using his pulse.

a Why didn't he use a watch?

b Suggest two advantages and disadvantages of using your pulse as a timekeeper.

Moving clocks run slow

One evening in May 1905, Einstein was on his way home from his work at the Swiss Patent Office in Bern, Switzerland. He was riding in a tram along Kramgasse Street in the old city centre when he passed the big medieval clock tower. He gazed up at the tower and suddenly imagined what would happen if the tram raced away from the tower at the speed of light.

Einstein reasoned that while he was stopped beside the clock he would be able to see the hands tick over. The light from the clock's hand would travel to his eyes and he would see movement. He also reasoned that if he was to travel at the speed of light, the light from the hand of the clock would never catch him and he would not see the hands tick over. It would seem as through the clock – and time – had stopped. With this, Einstein realised that his wristwatch would continue to show that time was really passing. Within six weeks of this moment Einstein had completed a paper outlining the principles of relativity.

You can think of the principles of relativity as *moving clocks run slow*. If you choose to do this, you need to remember that you are referring to clocks moving with respect to you. The clocks will be stationary to someone 'moving' along with the clock – that is to someone in the same reference frame as the clock. Moving is relative! Time really does appear to pass more slowly in a frame of reference moving with respect to you (in Einstein's case, the tram).

FIGURE 2 The clock in Bern, Switzerland, that inspired Albert Einstein in 1905

For example, you could measure the time between two events in a frame moving relative to you as taking 2 minutes (the time between hand movements on the clock). To someone stationary with respect to the clock, the time between the events may be 1 minute. If they say 1 minute and you say 2 minutes, it will seem longer to you. This is the inevitable outcome of the two postulates of the theory of relativity! Equally, you see your wristwatch tick over every minute as you are stationary with respect to that watch, but to an observer on the ground as you drive past, your clock would be moving and would take 2 minutes to tick over. They are the ones moving relative to your watch.

You might ask whether it is really true or whether it just appear to be true. All you can say is that it does not violate the laws of physics and that it has been confirmed by many experiments, so it can be called a 'fact'. However, like all 'facts' in science, it can be replaced if better theories and experiments come along. For now, special relativity works beautifully well.

The concept of time dilation is hard to accept, as it violates common sense, but you can see from the equation that the time dilation effect is negligible unless v is reasonably close to c. Table 1 shows the ratio of $\frac{v}{c}$ (called the speed factor, beta β) for different speeds, and the ratio of $\frac{t}{t_0}$ (called the Lorentz factor, or gamma γ, after the Dutch physicist H.A. Lorentz who developed the formula before Einstein, but didn't realise its significance). It is also written as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \beta^2}}$$

TABLE 1 Time dilation factors at various speeds

Speed	$\beta = \frac{v}{c}$	$\gamma = \frac{t}{t_0}$
Car at 60 km h^{-1}	0.0000	1.000 000
Jet at 8000 km h^{-1}	0.000 006 8	1.000 000
Parker space probe 343 100 km h^{-1}	0.000 317 7	1.000 000
Electron in a cathode ray tube.	0.1	1.005
Lightning strike	0.3	1.05
Speed of light in optic fibre	0.7	1.40
1 GeV electron	0.999 999 88	2000
20 GeV electron	0.999 999 999 67	40 000
Light pulse	1.000	infinite

Study tip

The speed factor, β, equal to $\frac{v}{c}$ is not required for the external assessment but is a shorthand way of problem-solving. An object travelling at 0.8c has a speed factor $\beta = 0.8$.

Table 1 shows that at ordinary speeds (for example, the car), relativistic effects are negligible, but at speeds approaching the speed of light, the effects are dramatic. An experimenter working with 1 MeV electrons travelling at 0.94 times the speed of light (0.94c) would have to realise that 1 second of time in the electron's frame of reference is the same as 2.9 seconds of time in the laboratory frame of reference. Imagine that the electron orbited a nucleus every second when viewed from a frame of reference stationary to the electron (that is, if you rode along with the electron). To an observer in the laboratory, the electron would take 2.9 s to orbit once. The electron's 'clock' appears to run slow.

Thousands of experiments have confirmed the theory of relativity. For example, in 1971 extremely accurate clocks were flown around the world on commercial aircraft, and when they were compared to the clocks left back in the laboratory a time dilation effect was confirmed.

Which clock is moving?

The thing that confuses students most of all in this work is defining which is the moving clock and which is the stationary clock. Einstein said that motion is relative so you could say either is stationary or either is moving. So which is 't' and which is 't_0' in the formula?

Synchronised clocks

Before you can compare clocks you need to ensure they are synchronised. This means they agree on the time of day. If clocks are side by side, it is easy to check if they are synchronised; however, if they are on different planets it is harder. The simplest way to have synchronised clocks at a distance is to synchronise them when they are together and then move them apart.

Proper versus dilated time

A good demonstration that may help you understand the difference between proper and dilated time is to consider a toy car running across the front bench of the laboratory (Figure 3).

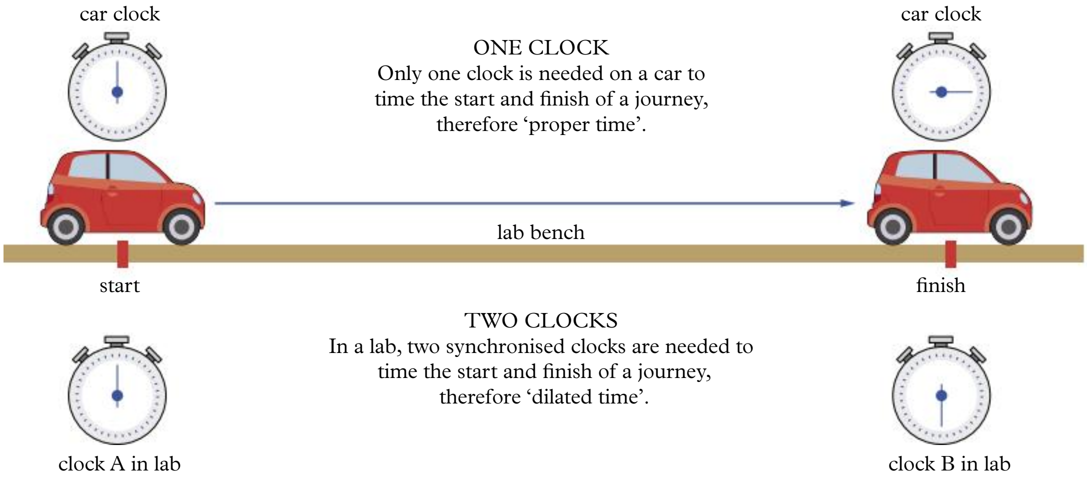

FIGURE 3 A toy car travels across the bench.

The simplest way to work out who is measuring proper time is to ask 'How many clocks are needed by each observer to measure the time interval for the journey across the bench?' The observer riding along with the car needs just one clock (fixed to the car). The start and finish line will appear in the same place (under the clock on the car). So the observer in the car measures proper time. The events (start and finish) occur in one place to the car driver. However, the observer sitting in the laboratory will need two clocks – one at the start and one at the finish. This observer is measuring dilated (relativistic) time, as the events are in two different places to this observer.

Ultimately, it all depends on what event you are timing. Imagine that you are in fact timing a rocket flight to the Moon. There is a clock on board the rocket for the space travellers and there are synchronised clocks on Earth and on the Moon.

Study tip

The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

can be used as shorthand in writing equations. For example,

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

becomes $t = t_0\gamma$.

Study tip

If you could ride along with a muon or any other particle you would measure its rest lifetime.

The two events – take-off and landing – are measured by the space travellers with a *single* clock on the spacecraft, but the observers on Earth need *two clocks* for their timing – one on Earth to register the time of take-off and a synchronised one on the Moon to register the time of landing. Thus the space travellers have measured the proper time interval, t_0, because the two events were measured in the one place by one clock at rest with respect to both events. The people on Earth measured the relativistic time interval, t, or the dilated time interval. Relativistic time (interval) is longer than proper time (interval): $t > t_0$.

WORKED EXAMPLE 9.4A

A very fast train travelling at $0.80c$ passes through a very long railway station. A woman on board the train measures the time interval to go from one end of the station to the other as 1.0 s. Calculate the length of time a man on the platform would measure the interval.

SOLUTION

The passenger on the train measures proper time t_0 as she is in the same position (her carriage seat) for the start and finish of the events (beginning and end of the platform). She only needs one clock as the two events are happening in the same place and are at rest to her.

The observer on the platform measures the relativistic time interval t – commonly known as dilated time interval – as the start and finish of the time interval occur in two places – one at the start of the platform and one at the other end. He sees the events as moving. He therefore needs two clocks – one at each end.

Let t = time measured by observer on platform, t_0 is proper time as measured by passenger on the train, v is velocity of the train and c is the speed of light.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.0}{\sqrt{1 - \frac{(0.80c)^2}{c^2}}} \text{ (cancel out the } c^2 \text{ terms)}$$

$$= \frac{1.0}{\sqrt{1 - (0.80)^2}}$$

$$= \frac{1.0}{\sqrt{0.36}}$$

$$= 1.7c \text{ (2 sf)}$$

Mean lifetime

mean lifetime
the average time before decay of an elementary particle as measured by an observer at rest to the particle; also known as rest life

A term commonly used when describing extremely high-speed subatomic particles is 'rest life' or **mean lifetime**. Many of these elementary particles decay in short periods of time. Recall that the muon decays into an electron and two neutrinos in an average time of 2.2 μs (2.2 microseconds) or 2.2×10^{-6} s as measured by an observer at rest to the muon. This is called its 'mean lifetime' or 'rest life', which is an average value of all muons. Some are short-lived and some live longer. It is a time interval for an event at rest with respect to the object, so is proper time, t_0.

WORKED EXAMPLE 9.4B

Determine the mean lifetime of a pion (an elementary particle) as measured in the laboratory if it is travelling 0.669*c* with respect to the laboratory. Its mean lifetime at rest is 3.5×10^{-8} s. Calculate the answer to 3 significant figures.

SOLUTION

The question is asking what is the relativistic time interval *t* for the life of a pion. You are given the proper time interval t_0 for the mean lifetime of a pion as 3.5×10^{-8} s. To an observer in the laboratory, the pion lives longer because of time dilation:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{3.5 \times 10^{-8}}{\sqrt{1 - \frac{(0.669c)^2}{c^2}}} \text{(cancel out the } c^2 \text{ terms)}$$

$$= \frac{3.5 \times 10^{-8}}{\sqrt{1 - (0.669)^2}}$$

$$= \frac{3.5 \times 10^{-8}}{0.743}$$

$$= 4.71 \times 10^{-8} \text{ s (3 sf)}$$

CHECK YOUR LEARNING 9.4

Describe and explain

1 **Explain** what is meant by ‘moving clocks run slowly’.

2 **Recall** whether the need for two clocks to time an event indicates proper or relativistic time.

3 **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 3.1×10^{-8} s.

4 **Clarify** whether proper time is longer or shorter than relativistic time

Apply, analyse and interpret

5 **Distinguish** proper time interval and relativistic time interval.

6 **Determine** the rest lifetime of an elementary particle travelling at a speed of 2.85×10^8 m s^{-1} if its average lifetime at this speed is measured to be 2.50×10^{-8} s.

7 Sarah boards a spaceship, and flies past Earth at 0.800 times the speed of light. Her twin sister, Emma, stays on Earth. At the instant Sarah’s ship passes Earth, they both start timers. Sarah stops her timer after 60.0 s have passed on her timer. **Determine** how much time has passed on Emma’s timer at that instant.

8 **Determine** whether the ratio $t : t_0$ doubles when comparing speeds of 0.4*c* and 0.8*c*.

Investigate, evaluate and communicate

9 **Predict** how *t* and t_0 compare at a speed of 1.0*c*.

10 **Evaluate** the claim that a moving clock that runs at half the rate of a clock at rest must be moving at 0.5*c*.

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CHAPTER 9

Review

Summary

9.1
- A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame. All inertial reference frames are equivalent for the description of mechanical phenomena. No one inertial reference frame is any better than another.
- There is no absolute frame of reference.
- There are two postulates of the theory of special relativity:
  1. The laws of physics have the same form in all inertial reference frames.
  2. Light propagates through empty space with a definite speed, *c*, independent of the speed of the source or the observer.

9.2
- All observers in inertial frames of reference agree on the relative speed.

9.3
- Two events in a reference frame are simultaneous if light signals from the events reach an observer halfway between the events at the same time.
- Two events that are simultaneous to one observer are not necessarily simultaneous to a second observer moving with respect to the first.

9.4
- Moving clocks are measured to run slowly:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  where t_0 is proper time and t is dilated or relativistic time interval. Relativistic (dilated) time interval is longer than proper time interval.
- A light-year (ly) is the distance travelled by light in one year.
- Relativistic speeds are arbitrarily chosen as those greater than $0.1c$.

Key terms

- event
- frame of reference
- inertial frame of reference
- mean lifetime
- muon
- muon neutrino
- pion
- proper time interval
- relative motion
- relativistic time interval
- relativity of simultaneity
- simultaneity
- the two postulates of special relativity
- theory of special relativity
- time dilation

Key formulas

Time dilation	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A spaceship moves towards you at a speed of $\frac{1}{3}c$, where c is the speed of light. The spaceship emits a beam of light in your direction. As measured in your frame of reference, the speed of the light emitted by the spaceship is:

A $\frac{4}{3}c$

B c

C $\frac{2}{3}c$

D $\frac{1}{3}c$

2 Select the statement that describes one of the key postulates of Einstein's theory of special relativity.

A Nothing can travel faster than the speed of light.

B The laws of physics are the same in all inertial (non-accelerated) frames of reference.

C All inertial observers obtain the same result when measuring the time and position of an event.

D Time dilates for a moving observer.

3 Select the option about which observers in different inertial systems will agree.

A the simultaneity of events at separate locations

B the rate at which their clocks run

C the number of seconds elapsed for an event

D none of the above

4 The theory of special relativity treats problems involving:

A an inertial frame of reference

B a non-inertial frame of reference

C a fast-moving frame of reference

D an accelerated frame of reference.

5 The muon detection experiment helped confirm one aspect of special relativity, namely that:

A to a muon distance was contracted

B muons are created in the upper atmosphere

C muons are relatively short-lived

D fewer muons reach the ground than expected.

6 If you approach a lamp while travelling at seven-tenths the speed of light ($0.7c$), you will measure the speed of light from the lamp to be:

A $0.49c$

B $0.70c$

C $1.0c$

D $1.7c$

7 A train is travelling at $0.9c$ along a straight, horizontal track at a constant speed approaches a rail crossing that flashes a light once per second. An observer on the train will measure the time between flashes to be:

A somewhat greater than one second

B equal to one second

C somewhat less than one second

D much greater than one second.

8 Proper time interval is the elapsed time between two events measured in a frame of reference in which the two events:

A are simultaneous

B occur at the same time

C are measured with synchronised clocks

D occur at the same point in space.

9 A meteor is heading for a planet at high speed. Select the statement that best describes the time of a meteor's descent to the planet's surface as measured aboard the meteor, and the time as measured by the observers on the surface of the planet.

A Only observers on the meteor measure the proper time.

B Both measure proper time in their own reference frames.

C Neither measure proper time.

D Only the observers on the planet measure proper time.

10 Select one of the following natural phenomena that cannot be explained by Newtonian physics.

A the absence of muons anywhere in the atmosphere

B the presence of muons in the atmosphere

C the relatively high number of muons close to Earth

D the relatively low number of muons close to Earth

Short answer

Describe and explain

★ **11** **Explain** whether if you ran at the speed of light you could see your own reflection in a mirror in your hand.

★ **12** **Clarify** whether can you tell which rocket is really moving when two rockets are moving relative to each other.

★ **13** **Explain** whether the light from a moving torch would be faster than light from a torch at rest.

★ **14** **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 5.2×10^{-8} s.

Apply, analyse and interpret

★ **15** **Deduce** whether anything travels faster than light.

★ **16** **Determine** the speed at which sunlight would pass you if you were on a spacecraft travelling at 0.6*c* away from the Sun.

★★ **17** A particular elementary particle travels at a speed of 2.6×10^8 m s^{-1}. At this speed, the average lifetime is measured to be 2.20×10^{-7} s.

a **Derive** its speed in units of *c*.

b **Determine** the particle's lifetime at rest.

★★★ **18** A train travelling at relativistic speed passes through a station. An observer on the platform measures the time for the train to pass. The driver of the train has synchronised clocks at the front and back of the train, and measures a time for the same event but disagrees with the value measured by the observer on the platform. One time is twice that of the other. **Determine** the speed of the train and which observer had the higher figure.

Investigate, evaluate and communicate

★ **19** You are standing on top of a moving train and threw a rock straight up (as it appeared to you).

a **Deduce** how the motion of the rock would appear to your mother, who is standing beside the train.

b **Deduce** whether the rock would land behind the carriage or on top of it.

c **Predict** if you would be in trouble when your mother got hold of you.

★ **20** **Assess** the claim that if a torch was moving, its light would travel faster than if the torch was at rest.

★ **21** One of the earliest attempts to measure the speed of light was by Italian scientist Galileo in 1638. He attempted to measure the speed of light by measuring the time lag between one observer turning on a lamp and another observer noting this and turning on a second lamp on a distant hill 1.7 km away. His results were inconclusive. **Propose** a reason for why this might be.

★★ **22** You may have learned that 'moving clocks run slow'. **Discuss** what this statement is supposed to mean.

★★ **23** A rocketship is travelling at relativistic speed past an observer R on a runway on a distant planet. Observer R is located mid-way between two landing lights X and Y. The astronaut S inside the rocketship is directly opposite observer R and notes that lights X and Y give out a flash of light simultaneously. **Propose** whether the lamps flash simultaneously according to observer S.

★★ **24** You have a powerful laser and shine it on the left-hand side of the moon, then move it to the right-hand side of the moon so that it draws a track on the surface. The moon is

3474 km wide and it takes one-hundredth of a second for the laser spot to travel this distance.

a **Evaluate** this claim: the spot is travelling faster than the speed of light.

b **Propose** a response.

★★★ 25 Because of the rotational motion of Earth about its axis, a point on the equator moves with a speed 460 km h^{-1} relative to a point on the North Pole. **Propose** whether this means that a clock placed on the equator runs more slowly than a similar clock placed at the North Pole.

★★★ 26 Pions have a half-life of 1.8×10^{-8} s, which means that every 1.8×10^{-8} s, the number of pions present halves. A target is struck by extremely high-energy protons, which results in a beam of pions exploding out of the target at a speed of 0.98 c. The number of pions present is half the original after they have travelled just 26.59 m. However, you would think that particles travelling at 0.98 c would travel just 5.29 m in the 1.8×10^{-8} s. **Propose** a reason for this anomaly.

★★★ 27 The Superluminal Scissors: You have a very large pair of scissors and as you close them, you notice that the point of intersection travels along the blades. You find that the quicker you close them, the faster the point of intersection moves. When you close them really quickly you find that the thepoint of intersection travels along the blades faster than the speed of light (that is, 'superluminal'). It seems to have disproven Einstein's claim that nothing can travel faster than light. **Assess** the evidence to explore whether your experiment does disprove his claim.

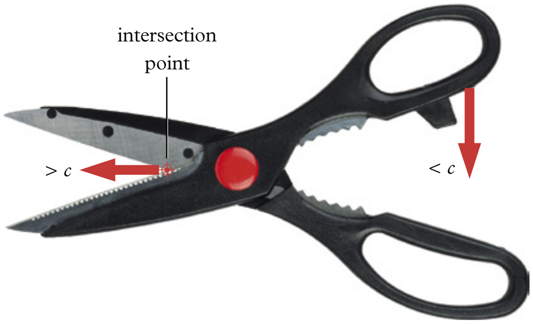

FIGURE 1 The intersection point can travel faster than c. Is that possible?

★★★ 28 The driver in a relativistic high-speed train passes two synchronised clocks on a station platform placed 200.0 m apart. The clocks register a time interval of 7.326×10^{-7} s for the front of the train to pass by the two clocks. The driver and an observer on the platform agree that the speed is 0.910 c but disagree about the time interval.

a **Evaluate** the claim that the speed is 0.910 c.

b **Propose** what time the train driver would say for the time interval.

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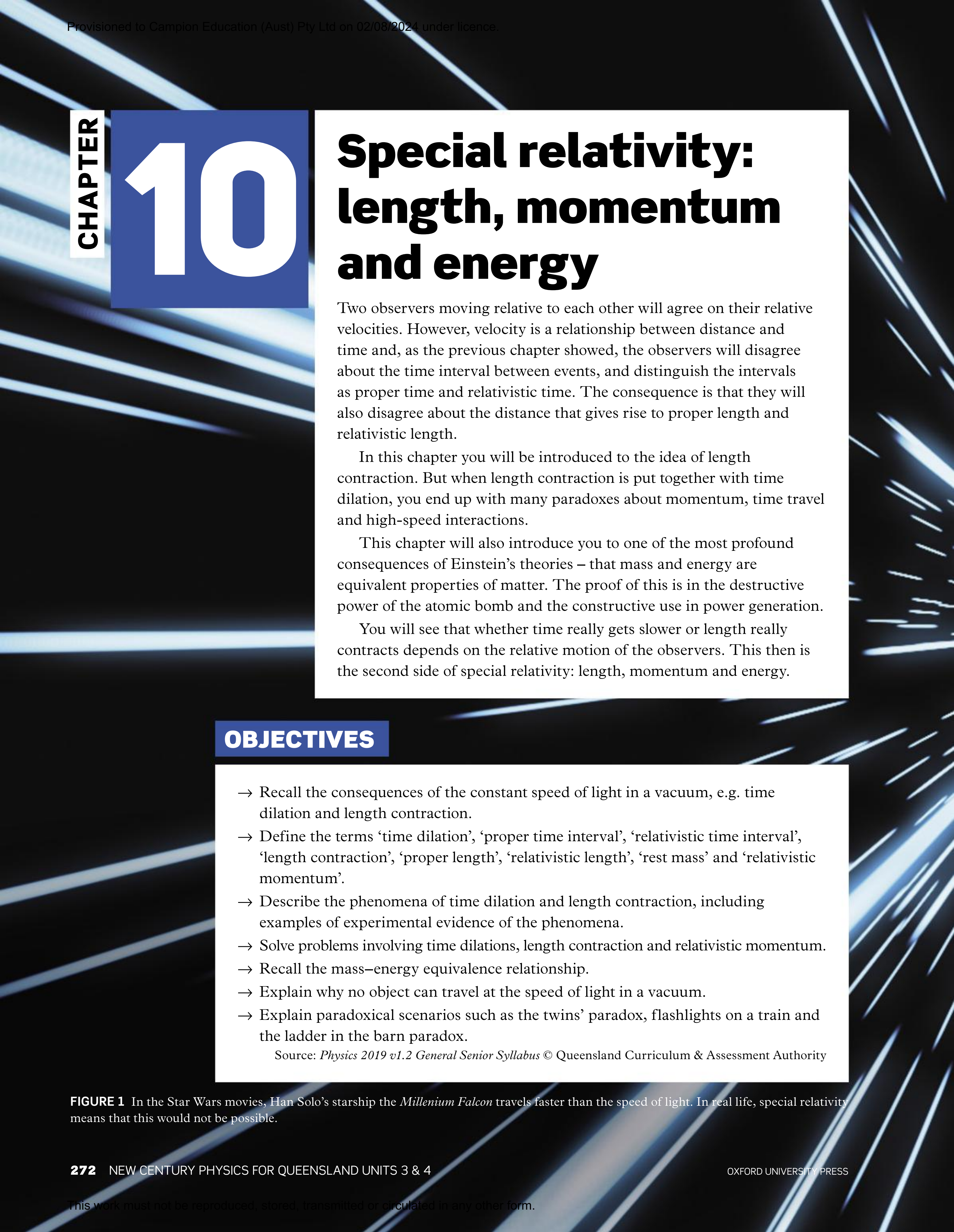

CHAPTER 10

Special relativity: length, momentum and energy

Two observers moving relative to each other will agree on their relative velocities. However, velocity is a relationship between distance and time and, as the previous chapter showed, the observers will disagree about the time interval between events, and distinguish the intervals as proper time and relativistic time. The consequence is that they will also disagree about the distance that gives rise to proper length and relativistic length.

In this chapter you will be introduced to the idea of length contraction. But when length contraction is put together with time dilation, you end up with many paradoxes about momentum, time travel and high-speed interactions.

This chapter will also introduce you to one of the most profound consequences of Einstein's theories – that mass and energy are equivalent properties of matter. The proof of this is in the destructive power of the atomic bomb and the constructive use in power generation.

You will see that whether time really gets slower or length really contracts depends on the relative motion of the observers. This then is the second side of special relativity: length, momentum and energy.

OBJECTIVES

→ Recall the consequences of the constant speed of light in a vacuum, e.g. time dilation and length contraction.

→ Define the terms 'time dilation', 'proper time interval', 'relativistic time interval', 'length contraction', 'proper length', 'relativistic length', 'rest mass' and 'relativistic momentum'.

→ Describe the phenomena of time dilation and length contraction, including examples of experimental evidence of the phenomena.

→ Solve problems involving time dilations, length contraction and relativistic momentum.

→ Recall the mass–energy equivalence relationship.

→ Explain why no object can travel at the speed of light in a vacuum.

→ Explain paradoxical scenarios such as the twins' paradox, flashlights on a train and the ladder in the barn paradox.

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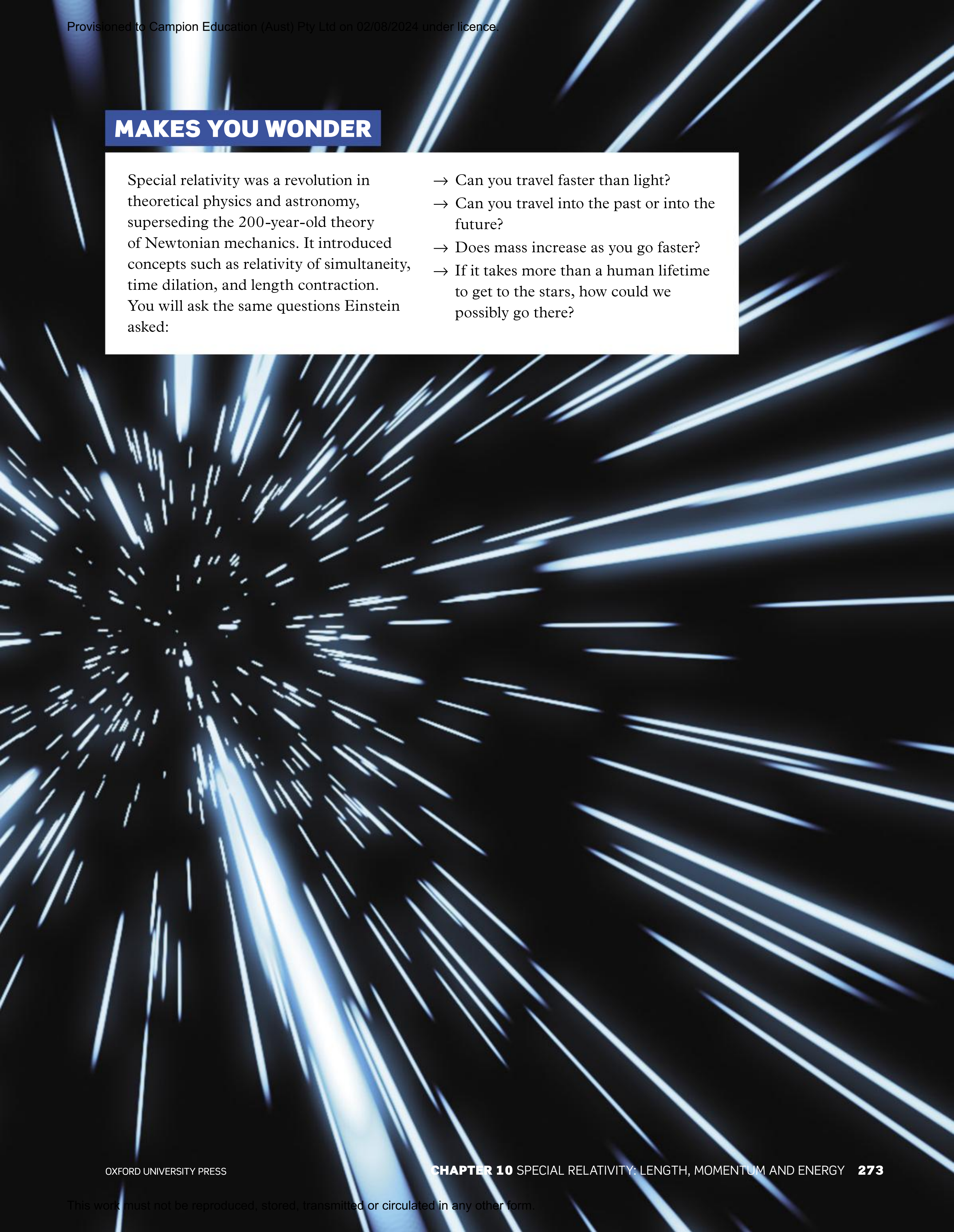
FIGURE 1 In the Star Wars movies, Han Solo's starship the *Millenium Falcon* travels faster than the speed of light. In real life, special relativity means that this would not be possible.

MAKES YOU WONDER

Special relativity was a revolution in theoretical physics and astronomy, superseding the 200-year-old theory of Newtonian mechanics. It introduced concepts such as relativity of simultaneity, time dilation, and length contraction. You will ask the same questions Einstein asked:

- → Can you travel faster than light?
- → Can you travel into the past or into the future?
- → Does mass increase as you go faster?
- → If it takes more than a human lifetime to get to the stars, how could we possibly go there?



10.1

Length contraction

KEY IDEAS

In this section, you will learn about:

- space travel
- length contraction
- proper length
- relativistic length.

length contraction the shorter measurement made by an observer moving relative to the object in the direction of the length being measured

One of the strange consequences of special relativity is **length contraction**. An object moving at relativistic speeds ($> 0.1c$) will contract along the direction in which it is travelling. For example, a 1 m ruler travelling lengthwise at $0.8c$ will be measured by an observer watching the ruler go by as being only 60 cm long. Its width and height remain the same.

Students often ask whether the ruler really contracts or whether it just appears to contract. The answer is that it really does contract – as measured by someone who sees the ruler moving. If you rode along with the ruler you would still measure it to be 1.0 m long.

The amount of contraction of the object depends on the speed of the object relative to the observer. Good confirmation of this is obtained at the Stanford Linear Accelerator (SLAC) in California (Figure 1). The SLAC is a 3.2 km long tube in which particles are accelerated to over $0.999\,999c$. At this speed the tube appears to be only about a metre long to the particles.

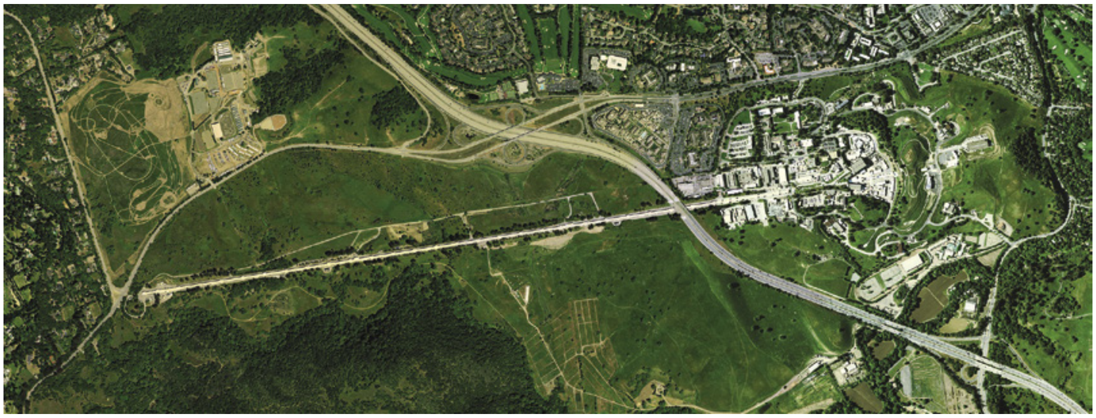


FIGURE 1 The Stanford Linear Accelerator (SLAC) at Stanford University

CHALLENGE 10.1A

'Mr Tompkins in Wonderland'

Physicist George Gamow wrote a science fantasy book called *Mr Tompkins in Wonderland* (1940). In the book the speed of light was walking pace. Mr Tompkins had studied relativity and when he began speeding on a bicycle, he thought he should be immediately contracted in length. But to his great surprise nothing happened to him or to his cycle. However, the streets got shorter and the windows of the shops looked like narrow slits. A policeman walking past looked really thin. Mr Tompkins exclaimed 'By Jove! This is where the word relativity comes in!'.

Explain Mr Tompkins's observations.

Space travel

You can now look at another practical example. The nearby star Rigel is 800 light-years away, which means it takes light 800 years to reach Earth from Rigel. If you could travel at the speed of light it would take 800 years to get there. We can't travel at that speed, but let's imagine that we can travel at $0.999c$. Using our relativistic time formula, $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, the 800 years becomes 35.8 years to someone aboard the spacecraft, which is more reasonable.

You might wonder how it can do this in such a short time if nothing can go faster than light, and even it takes 800 years to get here. The answer is that, although time gets stretched in different reference frames, length gets squeezed. This is called contraction of length. An example involving a rocket departing Earth for the star Rigel may help.

The Earth–Rigel frame of reference

Imagine that an observer on Earth watches a rocket take off for the star Rigel at a speed of v. Both the astronauts and the Earth observer will agree on the speed of the rocket. Earth and Rigel are at rest to one another so they form a single frame of reference (Figure 3). The distance between Earth and Rigel is 800 light-years and has the symbol L_0. This is called the rest length or **proper length** (hence the subscript 0) because it is the length measured by an observer at rest relative to both Earth and Rigel.

proper length the length as measured by an observer at rest with respect to the object

The time taken for the journey according to observers on Earth or Rigel is the dilated time (t), because the departure and arrival events are separated in space and require two separate clocks for their measurement.

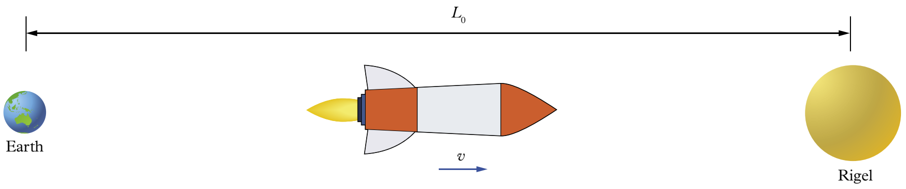


FIGURE 2 As viewed from Earth, the rocket is the moving frame.

The spaceship frame of reference

Figure 3 shows the journey from Earth to Rigel from the astronauts' perspective. They can picture themselves as being stationary and therefore assume that Earth is rushing away from them and Rigel is approaching them. This frame is moving with respect to the astronauts, so they measure the distance between Earth and Rigel as the contracted or **relativistic length** L. The time taken for the journey as measured by the astronauts is t_0 (proper time), because they are measuring the departure and arrival events in the same place in space (inside their rocket ship) and can use one clock to do this.

relativistic length the length as measured by an observer moving with respect to the object in the direction of motion

The space travellers and the Earth observers do, however, agree on the relative velocity, v, between the two frames of reference.

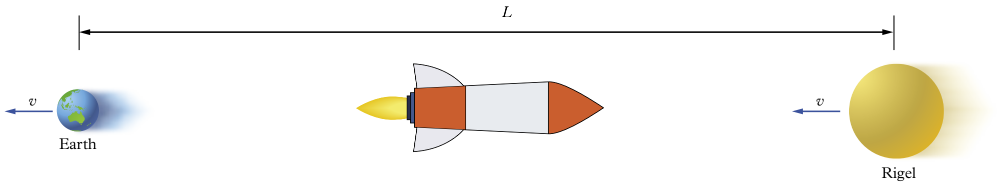


FIGURE 3 As viewed from the rocket, Earth–Rigel is the moving frame.

Relationships between the frames

The time for the journey is t for the Earth observers and t_0 for the astronauts. The distance is L_0 for the Earth observers and L for the astronauts. They both agree that the velocity of the spaceship is v. As the relationship between t and t_0 is given by $t_0 = t\sqrt{1 - \frac{v^2}{c^2}}$, the time measured by the astronauts (t_0) is less than that measured by Earth observers (t), hence $t_0 < t$. But as they agree on the velocity of the spaceship (v), the distance travelled by the astronauts must also be less than that measured by Earth observers. In other words, $L < L_0$.

We now have two relationships:

$$v = \frac{L_0}{t} = \frac{L}{t_0}$$

If we rearrange the second part we get $L = vt_0$, and if we replace the t_0 with the earlier equation we get:

$$\begin{aligned} L &= v \times t_0 \\ &= v \times t\sqrt{1 - \frac{v^2}{c^2}} \\ &= L_0\sqrt{1 - \frac{v^2}{c^2}} \end{aligned}$$

Study tip

The Lorentz factor, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, can be used as shorthand in writing equations. For example, $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$ becomes $L = \frac{L_0}{\gamma}$. Alternatively, using $\beta = \frac{v}{c}$, it can be written as $L = L_0\sqrt{1 - \beta^2}$.

Study tip

If you want to see the derivation of the length contraction formula from first principles, access your obook.

Length contraction

This length contraction applies not only to distances between heavenly bodies but also between atoms – so objects shrink as they speed up. But this contraction occurs only along the direction of motion. For example, if a car travelled forwards at high speed, it would appear to shrink in length (from say 4 m to 2 m) but its height would remain the same at 1.5 m and its width the same at 2 m. If you could run as fast, your height would remain the same but you'd get thinner, but stay just as wide (Figure 4).

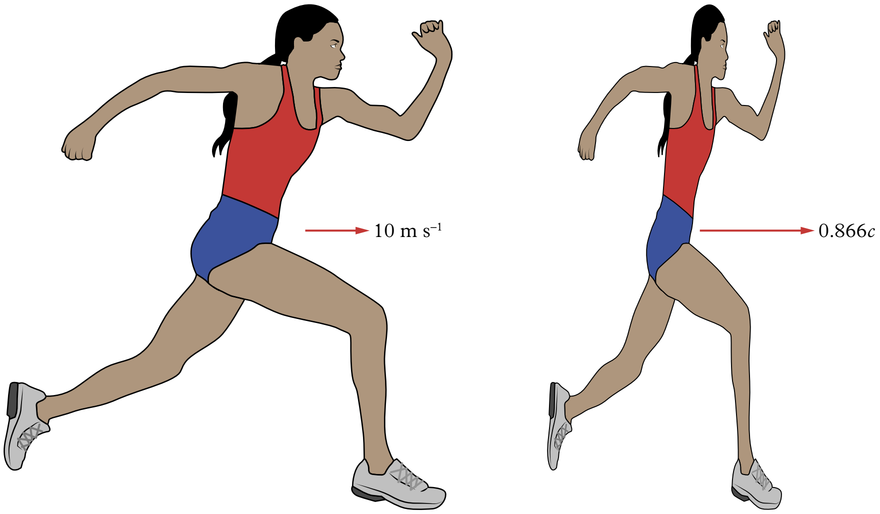


FIGURE 4 Running at 0.866c will halve your length in the direction of motion but leave your height unchanged.

Summary of relationships for length contraction (Table 1):

$$v = \frac{L_0}{t} = \frac{L}{t_0}$$

TABLE 1 Relationships between the frames

Frame of reference	Earth–Rigel	Spacecraft
Time for journey	t	t_0
Distance travelled	L_0	L
Velocity	v	v

WORKED EXAMPLE 10.1A

A spaceship passes you at a speed of $0.80c$. You measure its length to be 90 m (9.0×10^1). Calculate the length it would be to observers on board the spaceship.

SOLUTION

$v = 0.80c$

relativistic length $L = 90$ m

proper length, $L_0 = ?$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{90}{\sqrt{1 - 0.8^2}}$$

$$= \frac{90}{0.6}$$

$$= 150 \text{ m}$$

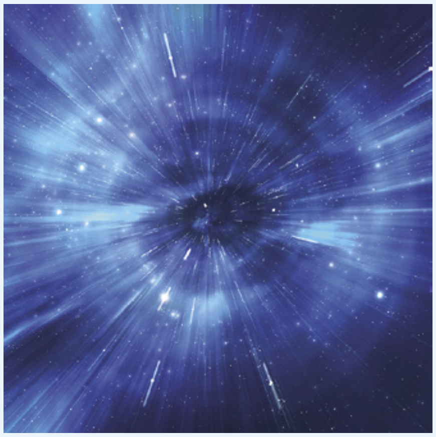


FIGURE 5 A representation of the speed of light in space

CHALLENGE 10.1B

Running a red light

A physicist driving a very fast sports car is booked for travelling through a red traffic light. The physicist argues that because he was travelling fast with respect to the light, the colour of the light had its wavelength altered and appeared green to him. The judge said that he would let him off the charge of running a red light but would fine him 1 cent for every metre per second he was travelling over 100 km h^{-1}.

How much was the physicist fined?

Note: the frequency of red light is 4.5×10^{14} Hz and the green light is 6.0×10^{14} Hz. The transverse frequency shift formula is:

$$f_0 = f\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

where f_0 is the frequency of the light with respect to an observer in the frame of reference of the source (i.e. the police).

Light years – a unit of distance

light-year (ly)
the distance travelled by light in one year (9.5×10^{15} m)

A **light-year** (ly) is the distance travelled by light in one year. Numerically it is equal to $3 \times 10^8 \times 60 \times 60 \times 24 \times 365$ or 9.5×10^{15} m. The distance to the red star Betelgeuse in the constellation Orion is 650 ly, while our Sun is only 500 light-seconds away from us.

Calculating travel time

When a distance in light-years (ly) is divided by a speed in units of *c*, the answer is time in years (y). For example, to calculate the time it would take to travel 100 ly at 0.5*c*:

$$\begin{aligned} t &= \frac{\text{distance in ly}}{\text{speed in } c} \\ &= \frac{100 \text{ ly}}{0.5\ c} \\ &= 200 \text{ y} \end{aligned}$$

The logic behind this is that light travels at a speed of *c*, but it also travels at 1 ly per year. So *c* has the units ly y^{-1}. Thus, the above equation can be written as:

$$\begin{aligned} t &= \frac{\text{distance in ly}}{\text{speed in } c} \\ &= \frac{100 \text{ ly}}{0.5 \text{ ly y}^{-1}} \\ &= 200 \text{ y} \end{aligned}$$

Study tip

- Proper length is the measurement you make when you ride along with the object being measured.
- Distance between planets is always proper length L_0, unless specifically stated otherwise.
- Moving objects (rods) appear short. Students learn this as 'moving rods appear short, they must measure L_0' (pronounced L-nought or L-zero).

WORKED EXAMPLE 10.1B

A certain star is 36 light-years away.

a Calculate how many years would it take a spacecraft travelling at 0.98*c* to reach that star from Earth as measured by an observer:

  i on Earth

  ii on the spacecraft.

b Calculate the distance travelled according to an observer on the spacecraft.

c Determine what the spacecraft occupants will calculate their speed to be from the results of (aii) and (b).

SOLUTION

$v = 0.98c$; distance between the star and Earth is proper length, $L_0 = 36$ ly

a **i** An observer on Earth measures dilated time, *t*:

$$\begin{aligned} v &= \frac{L_0}{t} \\ t &= \frac{L_0}{v} \\ &= \frac{36 \text{ ly}}{0.98\ c} \\ &= 36.73 \text{ y} \end{aligned}$$

ii Space traveller measures proper time, t_0:

$$\begin{aligned} t_0 &= t\sqrt{1 - \frac{v^2}{c^2}} \\ &= 36.73 \times \sqrt{1 - 0.98^2} \\ &= 7.3 \text{ y (2 sf)} \end{aligned}$$

b Space traveller measures relativistic length:

$$\begin{aligned} L &= vt_0 \\ &= 0.98c \times 7.31 \text{ y} \\ &= 7.16 \text{ ly} \end{aligned}$$

Alternatively, $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$$\begin{aligned} &= 36 \text{ ly} \times \sqrt{1 - 0.98^2} \\ &= 7.2 \text{ ly (2 sf)} \end{aligned}$$

c

$$\begin{aligned} v &= \frac{L}{t_0} \\ &= \frac{7.16 \text{ ly}}{7.31 \text{ y}} \\ &= 0.98c \text{ (same as for Earth observer)} \end{aligned}$$

CHECK YOUR LEARNING 10.1

Describe and explain

1 A 1 cm cube of copper is seen passing at a speed of 0.8c. **Describe** changes in its dimensions (in all three directions) and its density.

2 Two people, A and B, have metre rulers with the dimensions 100 cm × 2 cm × 0.5 cm. Person A measures B's ruler as it passes at high speed to be 100 cm × 1 cm × 0.5 cm. **Explain** how this is possible.

3 **Calculate**:

- a 1.8×10^7 m s^{-1} as units of c
- b 0.95c as m s^{-1}
- c 30 ly as km.

4 An aeroplane that has a rest length of 40.0 m is moving at a uniform velocity with respect to Earth at a speed of 630 m s^{-1}. **Calculate** the length of the aeroplane as measured from Earth.

Apply, analyse and interpret

5 You decide to travel to a star that is 85 light-years away. **Determine** how fast would you have to travel so the distance would only be 20 light-years to you.

6 A friend travels past you in her fast sports car at a speed of 0.760c. The car appears to be 5.80 m long and 1.45 m high.

- a **Calculate** its length and height at rest.
- b **Determine** how many seconds you saw elapse on your friend's watch when 20.0 s passed in Earth's frame of reference.
- c **Deduce** how fast you appeared to be travelling to your friend.

7 After the Sun, the nearest star visible to the naked eye is Rigel Centaurus, which is 4.35 light-years away. A spacecraft is sent there from Earth at a speed of 0.80c.

- a **Determine** how many years it would take to reach that star from Earth as measured by observers:
  - i on Earth
  - ii on the spacecraft.
- b **Calculate** the distance travelled according to observers on the spacecraft.

FIGURE 6 Betelgeuse and Rigel are both part of the constellation Orion.

Investigate, evaluate and communicate

8 An astronaut leaves Earth and travels at 0.95c to another galaxy 5 light-years away as measured by the astronaut. When she arrives, she immediately sends a radio signal back to Earth to confirm her arrival. **Determine** how long after her departure observers on Earth would receive her radio signal. **Discuss** why it would take this time.

Check your obook assess for these additional resources and more:

- » Student book questions Check your learning 10.1
- » Challenge worksheet 10.1A 'Mr Tompkins in Wonderland'
- » Challenge worksheet 10.1B Running a red light

10.2

Rest mass and relativistic momentum

KEY IDEAS

In this section, you will learn about:

- rest mass
- relativistic momentum
- mass–energy equivalence relationship
- why no object can travel at the speed of light in a vacuum.

rest mass
the mass of an object when measured in the same reference frame as the observer

Einstein defined **rest mass** as the mass of an object when measured at rest; that is, in the same reference frame as the observer. He used the symbol *m*, but others have added the subscript zero (m_0) to keep it consistent with other 'rest' quantities such as length (L_0) and time (t_0).

There has been a tendency to say that the mass of an object increases with velocity and that this mass should be defined as relativistic mass. However, in 1948 Einstein forcefully warned against the concept of mass increasing with velocity. Unfortunately this warning was ignored. It is best to think of the term 'mass' as meaning 'rest mass' with either symbol m or m_0.

Study tip

The equation for momentum $p = mv$ can also be written as $\mathbf{p} = m_0\mathbf{v}$ if you chose to use m_0 for rest mass instead of *m*. It makes no difference.

Momentum

In the Newtonian world, momentum is defined as the product of mass and velocity ($p = m_0 v$). You have learnt that time and distance change at relativistic speeds, so what about momentum? Physicists have derived an equation to show this:

$$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

relativistic momentum
the momentum of an object as measured by an observer moving relative to the object

where p_v is the **relativistic momentum**, m_0 is the rest mass, v is the velocity of the object, and c is the speed of light. The subscript v in p_v indicates that it is the relativistic version of momentum.

Note that momentum and velocity are vector quantities and are shown in bold. The v^2 term in $\frac{v^2}{c^2}$ is not a vector quantity though (two vectors multiplied = not a vector).

FIGURE 1 Will the space shuttle's mass change as the velocity increases?

WORKED EXAMPLE 10.2A

The Stanford Linear Accelerator (SLAC) can accelerate electrons to over 0.999*c*.

a Calculate the Newtonian momentum and the relativistic momentum of the electron travelling at a speed of exactly 0.999*c*. The rest mass of an electron is 9.109×10^{-31} kg.

b Determine how many times greater the relativistic value is than the Newtonian value.

SOLUTION

a Newtonian momentum:

$$\begin{aligned} p &= mv \\ &= 9.109 \times 10^{-31} \times 0.999 \times 3.00 \times 10^{8} \\ &= 2.73 \times 10^{-22} \text{ kg m s}^{-1} \end{aligned}$$

Relativistic momentum:

$$\begin{aligned} p_v &= \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{9.109 \times 10^{-31} \times 0.999 \times 3.00 \times 10^{8}}{\sqrt{1 - 0.999^2}} \\ &= \frac{2.73 \times 10^{-22}}{0.0447} \\ &= 6.11 \times 10^{-21} \text{ kg m s}^{-1} \text{ (3 sf)} \end{aligned}$$

b Ratio:

$$\begin{aligned} \frac{\text{Relativistic momentum}}{\text{Newtonian momentum}} &= \frac{6.11 \times 10^{-21}}{2.73 \times 10^{-22}} \\ &= 22.4 \\ p \text{ (relativistic)} &= 22.4 \times p \text{ (Newtonian)} \end{aligned}$$

Study tip

If you could ride along with a particle you would measure its 'rest' momentum.

Study tip

If you could ride along with a particle you would measure its rest mass. Mass doesn't change with speed, either to stationary or moving observers.

WORKED EXAMPLE 10.2B

Calculate the speed of a 1.00 g copper target travelling at relativistic speed with a momentum of 500 000 kg m s^{-1} (5.0×10^5).

SOLUTION

$$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$500\,000 = \frac{1.00 \times 10^{-3} v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$5.0 \times 10^5 \times \sqrt{1 - \frac{v^2}{c^2}} = 1.00 \times 10^{-3} v \text{ (square both sides)}$$

$$25 \times 10^{10} \times (1 - \frac{v^2}{c^2}) = 1.00 \times 10^{-6} v^2$$

$$25 \times 10^{16} \times (1 - \frac{v^2}{c^2}) = v^2$$

$$\begin{aligned} 25 \times 10^{16} &= 25.0 \times 10^{16} \times \frac{v^2}{c^2} + v^2 \\ 25 \times 10^{16} &= \frac{25.0 \times 10^{16} v^2}{(3 \times 10^8)^2} + v^2 \\ 25 \times 10^{16} &= 2.78 v^2 + v^2 \\ v^2 &= 6.61 \times 10^{16} \\ v &= 2.57 \times 10^8 \text{ m s}^{-1} \text{ (0.857}c\text{) (2 sf)} \end{aligned}$$

Conservation of relativistic momentum

As with Newtonian (or 'classical') momentum, relativistic momentum is conserved in all interactions. This comes with the caution of 'as long as no external force is applied to the objects' (as for Newtonian momentum). The use of **conservation of momentum** principles is the fundamental theoretical basis for the analysis of collisions in atom-smashing colliders such as the Large Hadron Collider (LHC) and the SLAC.

conservation of momentum for a collision occurring between object 1 and object 2 in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision

mass–energy equivalence relationship relates change in mass to change in energy, given by $\Delta E = \Delta mc^2$

Mass and energy

The famous equation $\Delta E = \Delta mc^2$ is called 'Einstein's **mass–energy equivalence relationship**'. According to this equation, 1 kg mass of any matter is equivalent to 9×10^{16} J of energy. This is a huge amount of energy and is same irrespective whether the matter is uranium, hydrogen, wood, oil or anything else, including radioactive elements. It does not depend on atomic number, mass number, electronic configuration and so on. It means that 'mass' is the connecting link between energy and matter. However, it seems that Einstein was somewhat loose in his use of symbols in his popular writings. In formal writings he used $E_0 = mc^2$ with the zero subscript to show that he was referring to rest energy (E_0). This was to distinguish it from any kinetic energy a particle could have due to its motion.

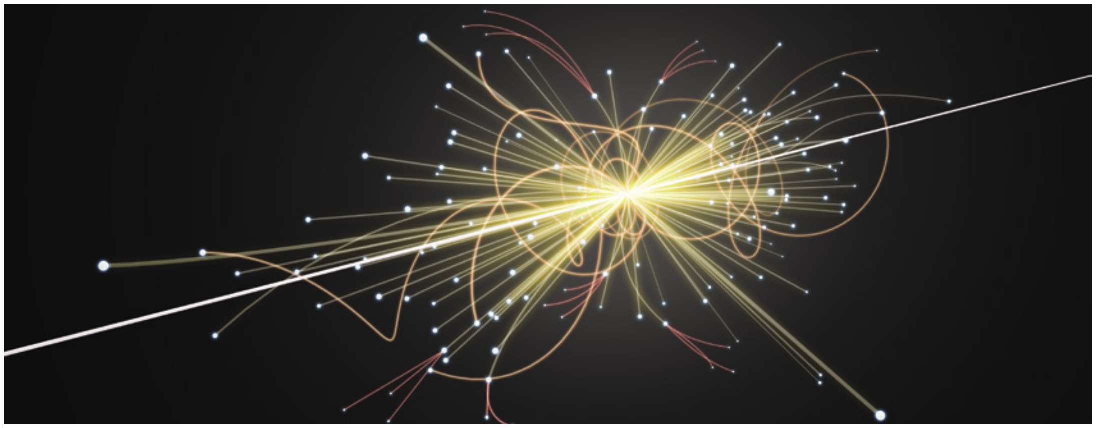


FIGURE 2 When protons approach from opposite sides and meet during a collision in the LHC, they break apart and quarks and gluons are released. The angle and the length of the path give clues to each particle's momentum, and hence its identity.

CHALLENGE 10.2A

Rest or relativistic mass?

Does an electronic balance in the lab measure rest mass or relativistic mass? Explain.

CHALLENGE 10.2B

Speed

An unstable particle is at rest and suddenly decays into two fragments. One of the fragments has a speed of $0.60c$ and a mass of 2×10^{-27} kg; the other has a mass of 8×10^{-27} kg. Determine the speed of the other fragment.

Einstein's mass–energy equivalence relationship

To avoid the complications introduced by having to consider whether it is rest energy or total energy, you will look at *change* in energy. The formula is quite simple:

$$\Delta E = \Delta m c^2$$

where ΔE is change in energy, Δm is the change in mass and c is the speed of light.

This formula works both ways. Energy and mass are equivalent: mass can appear as energy, and energy can appear as mass. It is common to say mass is *converted to* energy, but Einstein made it clear that mass and energy were just two different expressions of the same thing. Since the value of c is very high, a very small mass is equivalent to a large amount of energy. Conversely, even a large amount of energy appears as a very small change in mass. This was discussed in detail in the study of nuclear physics in Unit 1.

Energy to mass

Recall that binding energy is related to mass defect. When you use energy to pull a nucleus apart, the constituent particles have a total mass greater than that of the nucleus. The energy you used to overcome the binding energy appears as mass (Δm). But the energy doesn't have to involve nuclear particles. When you change the energy of any system, that energy appears as mass. Consider the charging of your phone. It takes 2 hours at a current of 2.4 A to charge a 3.6 V phone battery. That equates to energy given by:

$$\begin{aligned} W &= VIt \\ &= 3.6 \times 2.4 \times 2.0 \times 60 \times 60 \\ &= 62\ 208\ \text{J} \end{aligned}$$

This is a change in energy (ΔE) equivalent to a change in mass (Δm):

$$\begin{aligned} \Delta E = \Delta m c^2 \Rightarrow \Delta m &= \frac{\Delta E}{c^2} \\ &= \frac{62\ 208}{(3 \times 10^8)^2} \\ &= 6.9 \times 10^{-13}\ \text{kg (2 sf)} \end{aligned}$$

That's one-billionth of a gram, or a nanogram, and undetectable on an electronic balance no matter how expensive.

WORKED EXAMPLE 10.2C

Calculate the binding energy in joules for a carbon-12 nuclide that has a mass defect of 0.098 940 u.
Note: 1 u = 1.66×10^{-27} kg.

SOLUTION

$$\begin{aligned} \Delta m &= 0.098\ 940\ \text{u} \times 1.66 \times 10^{-27}\ \text{kg u}^{-1} \\ &= 1.643 \times 10^{-28}\ \text{kg} \end{aligned}$$

$$\begin{aligned} \Delta E &= \Delta m c^2 \\ &= 1.643 \times 10^{-28} \times (3 \times 10^8)^2 \\ &= 1.48 \times 10^{-11}\ \text{J (3 sf)} \end{aligned}$$

WORKED EXAMPLE 10.2D

Calculate the amount of energy that would be released if a pi-meson (π°) of rest mass of 2.4×10^{-28} kg decayed and its entire rest mass was transformed completely into electromagnetic radiation.

SOLUTION

$$\begin{aligned} \Delta E &= \Delta m c^2 \\ &= 2.4 \times 10^{-28} \times (3 \times 10^8)^2 \\ &= 2.2 \times 10^{-11}\ \text{J (2 sf)} \end{aligned}$$

WORKED EXAMPLE 10.2E

A typical U-235 fission reaction is given by $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{94}_{38}\text{Sr} + ^{139}_{54}\text{Xe} + 3\,^{1}_{0}\text{n}$ + energy.

Masses of the nuclides:

m(U-235) = 235.043 930 u;
$m(^{1}_{0}\text{n})$ = 1.008 665 u;
m(Sr-94) = 93.915 356 u;
m(Xe-139) = 138.918 792 u

Determine:

a the mass defect

b the energy released in joules per fission of an atom of U-235 reacted.

SOLUTION

a First, calculate the sum of the masses of all of the reactants (m_r) and the sum of the masses of all the products (m_p).

Mass of reactants:

$$\begin{aligned} m_r &= m(\text{U-235}) + m(\text{neutron}) \\ &= 235.043\,930 \text{ u} + 1.008\,665 \text{ u} \end{aligned}$$

Total m_r = 236.052 595 u

Mass of products:

$$\begin{aligned} m_p &= m(\text{Sr-94}) + m(\text{Xe-139}) + m(3 \text{ neutrons}) \\ &= 93.915\,356 \text{ u} + 138.918\,792 \text{ u} + (3 \times 1.008\,665) \text{ u} \end{aligned}$$

Total m_p = 235.860 143 u

Calculate the change in mass (Δm) by subtracting the combined mass of the reactants from the combined mass of the products ($m_p - m_r$).

Mass defect, $\Delta m = \left| m_p - m_r \right| = 235.860\,143 \text{ u} - 236.052\,595 \text{ u} = 0.192\,452 \text{ u}$

Convert the change in mass (the mass defect, Δm) into kg (1 u = 1.66×10^{-27} kg).

Mass defect = $0.192\,452 \text{ u} \times 1.66 \times 10^{-27} \text{ kg u}^{-1} = 3.195 \times 10^{-28}$ kg

b Convert the change in mass (Δm in kg) into its equivalent change in energy.

$$\begin{aligned} \Delta E &= \Delta m c^2 \\ &= 3.195 \times 10^{-28} \times (3 \times 10^8)^2 \text{ J} \\ &= 2.88 \times 10^{-11} \text{ J (for one atom of U-235)} \end{aligned}$$

Universal speed limit

In terms of Newtonian mechanics, if a rocket engine was used to propel a spacecraft through space, then its speed would continue to increase forever. Newton's second law of motion implies that as long as there is an unbalanced force, acceleration will continue ($F = ma$). For example, if a 10 000 N force was applied to a 100 kg satellite, then its acceleration would be 100 m s^{-2}. To go from rest to the speed of light could be calculated thus:

$$\begin{aligned} a &= \frac{v-u}{t} \\ t &= \frac{v-u}{a} \\ &= \frac{3 \times 10^8 - 0}{100} \\ &= 3 \times 10^6 \text{ s (about 1 month)} \end{aligned}$$

If the force was continued for another month, then the speed would be twice that of light, and so on. Clearly, this is wrong. It turns out that as speed approaches *c*, the momentum approaches infinity (Figure 3).

The amount of effort needed for each increment of velocity becomes larger and larger until the added effort makes no difference. It just won't get any faster. This implies a cosmic speed limit.

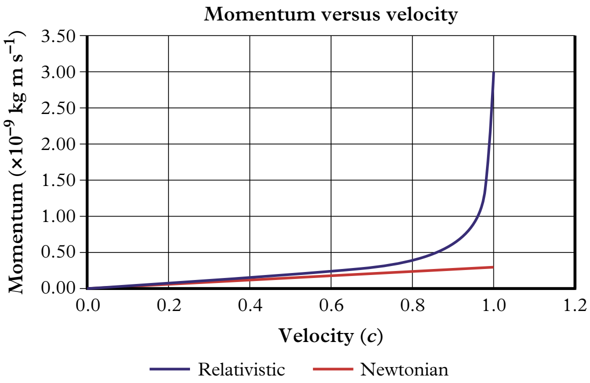


FIGURE 3 Comparison of relativistic and Newtonian momentum as velocity increases. Note that there is little difference until about 0.5*c*.

CHECK YOUR LEARNING 10.2

Describe and explain

1 **Recall** whether or not relativistic momentum is conserved in collisions of subatomic particles when it is known that Newtonian momentum is conserved.

2 **Explain** whether relativistic momentum is conserved when an external force acts during an interaction.

3 **Explain** whether all 100 tons of U-235 fuel in a nuclear reactor is converted to an equivalent amount of energy.

Apply, analyse and interpret

4 **Determine** the momentum of a neutral pi-meson $\pi^0(m_0 = 2.4 \times 10^{-28}$ kg) travelling at a velocity of 0.992*c* using the:

   a relativistic formula

   b Newtonian formula.

5 **Deduce** the velocity (in units of *c*) for a kaon, K^0, of mass $8.870\,77 \times 10^{-28}$ kg that has a relativistic momentum of $5.339\,57 \times 10^{-19}$ kg m s^{-1}.

6 **Determine** the increase in mass when 1.0 kg of water is placed on an electronic balance and heated from 20°C to 100°C, and identify whether the scale reading on the balance therefore would get bigger. Recall that the energy to heat water is given by $Q = mc\Delta T$, where m = rest mass of water, and c is the specific heat capacity of water = 4180 J kg^{-1} K^{-1}.

Investigate, evaluate and communicate

7 Imagine the ocean increased in temperature by 1°C this century as scientists fear. It is claimed that the mass of the ocean would increase by a measurable amount if this happened. **Evaluate** this claim by using Einstein's mass–energy equivalence postulate $\Delta E = \Delta mc^2$. Assume the ocean has a mass of 1.4×10^{21} kg and a specific heat constant of 4000 J kg^{-1} K^{-1}.

8 Australia's OPAL nuclear reactor is for experimental and industrial purposes and not for power generation. Because of this it has a small energy output of 20 MW (20 MJ s^{-1}). **Evaluate** the claim that the mass of the reactor would change by an insignificant amount if it ran every second for a full 365-day year.

9 **Evaluate** the claim that 'special relativity leads to the idea of mass–energy equivalence, which has been applied in nuclear fission reactors'.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 10.2

» Challenge worksheet
10.2A Rest or relativistic mass?

» Challenge worksheet
10.2B Speed

» Video
Calculating mass-energy equivalence

SCIENCE AS A HUMAN ENDEAVOUR

10.3

Mass to energy

KEY IDEAS

In this section, you will learn about:

- mass and energy
- nuclear fission.

Mass–energy equivalence

Special relativity leads to the idea of mass–energy equivalence, which has been applied in nuclear fission reactors. Nuclear energy from the neutron bombardment of uranium is a good example of mass appearing as energy. The reactants, a U-235 nucleus and a slow neutron, turn into fission fragments that have a lower total mass than the reactants. This mass deficit appears as a large amount of energy. For example, when 1 kg of U-235 undergoes fission, there is a mass defect of 0.8 g, which is equivalent to 10^{14} J – a staggering amount.

The energy for nuclear fission and fusion can be put to peaceful uses, such as in a power station. A typical thermonuclear reactor produces energy at the rate of 3000 MW (3×10^9 J s^{-1}) but, because of energy losses, the electrical output is only 1000 MW. To produce this, about 1 kg of U-235 is 'converted' to energy in a year.

Power reactors

The most common form of nuclear fission power reactors is the thermal reactor. There are hundreds of thermal power reactors in countries throughout the world. To appreciate how the energy contained in mass can be converted into thermal energy, you need to look at the design of a fission power reactor and some of its components.

In essence, it consists of a containment vessel in which the fuel rods are housed. The U-235 fuel is in the form of rods immersed in water to take away the heat. The fuel is constantly undergoing fission, but to stop the reactor overheating the neutrons produced by the reaction are 'soaked up' by control rods. To allow the fuel to react more vigorously the control rods are adjusted so the energy from the fission is released at a manageable level. This is called a self-sustaining chain reaction. If the control rods are removed too far the reactor could explode. Thermal energy produced by the fission reaction is passed to a working fluid (usually heavy water), which in turn runs through steam turbines. These turn the electrical generators to produce electrical energy that gets transmitted to an electrical distribution network for industry and our homes.

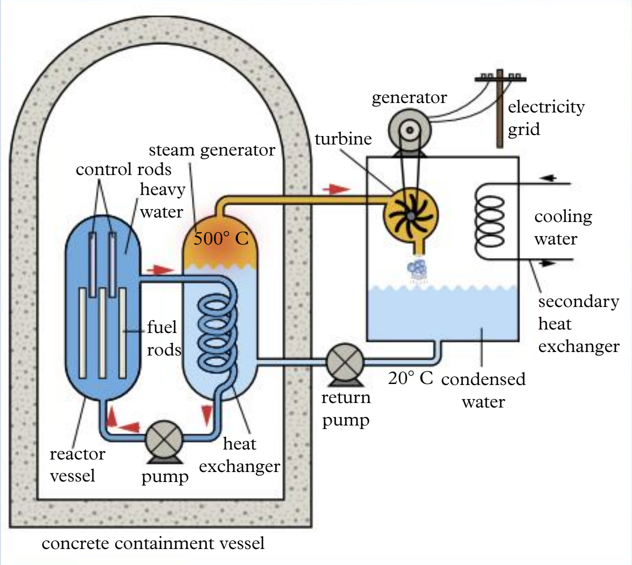


FIGURE 1 Schematic diagram of a typical pressurised water reactor in a thermal nuclear power station

An Australian reactor

There are no nuclear-power reactors in Australia. However, there is a multipurpose fission reactor at Lucas Heights, in Sydney. The Australian Nuclear Science and Technology Organisation (ANSTO) manages the Open Pool Australian Lightwater (OPAL) reactor, which is used for:

- irradiation of target materials to produce radioisotopes for medical and industrial applications
- research in the field of materials science using neutron beams and associated instruments
- analysis of minerals and samples using neutron-activation techniques and delayed-neutron activation techniques
- irradiation of silicon ingots (termed neutron transmutation doping or NTD) for use in the manufacture of electronic semiconductor devices.

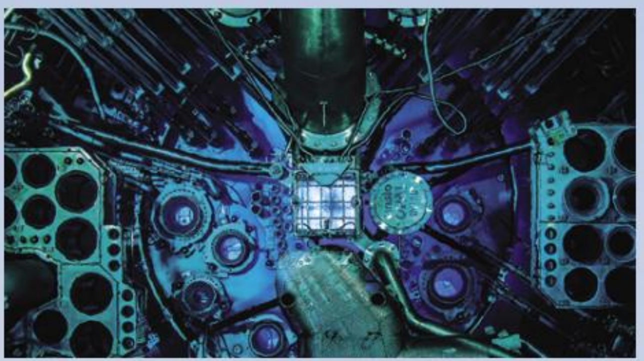


FIGURE 2 The OPAL reactor core is housed within a facility designed to withstand an aeroplane impact.

Fission versus fusion

Nuclear fusion reactions also produce vast amounts of energy as a result of the mass–energy equivalence principle. However, a sustainable fusion reaction suitable for power generation has yet to be achieved. There are promising results and the dream of fusion for electrical power generation always seems to be just around the corner but never quite attained. There are several points for and against:

- Fusion is known to produce about eight times the energy of fission per kilogram of reactant, even though for a single atom of reactant fission produces eight times the energy of fusion.
- Fusion doesn't produce the harmful radioisotopes that need hundreds of years of dedicated storage as is the case for fission.
- Fusion bombs are almost impossibly difficult to make by terrorists, whereas fission bombs – the so-called 'dirty fission bombs' – can be loaded into a briefcase.

The claim that unlimited power from fusion is just around the corner seems – at the moment – just a dream.

CHECK YOUR LEARNING 10.3

Describe and explain

1 **Explain** what the problem is for a nuclear fission reactor when too few neutrons are released per fission event.

2 **Interpret** the statement that 'nuclear energy is based on Einstein's mass–energy equivalence principle'.

Apply, analyse and interpret

3 **Determine** in what way fission is better than fusion, and in what way is fusion is better than fission.

Investigate, evaluate and communicate

4 One problem with nuclear fusion is that high temperatures close to 100 million degrees C are needed to sustain it. **Propose** how you would contain the fuel, since most containers would melt well before that.

5 A claim is sometimes made that 'unlimited clean energy from fusion power is just around the corner'. **Propose** two arguments supporting the claim, and two opposing it.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 10.3

» Weblink
ANSTO

» Weblink
The OPAL reactor

10.4

Paradoxical scenarios

KEY IDEAS

In this section, you will learn about:

- the twin's paradox
- why no object can travel at the speed of light in a vacuum
- ladder in the barn paradox
- time travel.

paradox
a self-contradictory conclusion from true premises

The consequences of Einstein's theory of special relativity do seem rather weird and unbelievable. You have to accept clocks running slow, things getting shorter, disagreements about who is moving, and unbelievable amounts of energy coming from a tiny nucleus. These ideas give rise to some strange **paradoxes** – self-contradictory conclusions from true premises. In most cases it is because proper time and rest length are wrongly applied. Let's look at some examples. More examples can be found on your obook.

Study tip

'Moving clocks run slow, they must measure t_0.' (t-oh) 'Moving rods appear short, they must measure L_0.' (L-nought)

Twins' paradox

Not long after Einstein proposed his theory, an apparent paradox was pointed out, the twins' paradox. Suppose that there is a pair of 25-year-old twins, and one of them takes off in a spaceship travelling at very high speeds to a distant star and back again, while the other twin remains on Earth. When the travelling twin returns they have aged differently according to the concept of time dilation (Figure 1). The question is: which twin has aged more? In the examples below, assume the travelling twin travelled at an average speed of 0.80*c* for the journey and had aged 30 years aboard the spacecraft before returning.

1 The travelling twin leaves the Earth twin for space.

2 The travelling twin travels at higher speeds than the Earth twin.

3 The travelling twin returns to Earth.

4 The Earth twin finds that his travelling brother has not aged as much.

FIGURE 1 The twins' paradox: one twin travels through space and returns to find his twin is older in appearance.

Scenario 1: Earth's reference frame

According to the Earth twin (the twin who remained on Earth), the travelling twin will have aged less.

- Proper time (aboard spacecraft): $t_0 = 30$ years
- Velocity: $v = 0.8c$
- Dilated time: $t = \dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$$= \frac{30 \text{ years}}{\sqrt{1 - 0.8^2}}$$

$$= 50 \text{ years}$$

Hence, the time elapsed on Earth is 50 years; and this is how much the Earth twin will have aged. The Earth twin will be 75 years old (25 + 50). The travelling twin (who went into space) will be 55 years old (25 + 30).

Scenario 2: spaceship's reference frame

Since 'everything is relative', all inertial reference frames are equally as good as each other. The travelling twin could make all the same claims as the Earth twin, only in reverse. The travelling twin could claim that since Earth is moving away at high speed, time passes more slowly on Earth and so the Earth twin who will have aged less. In this case, proper time will be time measured aboard the 'moving' Earth (30 years) and dilated time will be measured by the 'stationary' spacecraft (50 years). This is the opposite of what the Earth twin would say. They both can't be right, can they? When the spacecraft returns to Earth they can stand beside one another and compare ages and clocks. Only one of the above scenarios will be correct – but which one?

Resolution

The problem can be resolved by deciding who is travelling with **uniform motion**. The travelling twin must change velocity at the beginning and end of the trip and also when turning around in space, so he must be really moving, even if these acceleration periods occupy only a tiny portion of the total time. So the Earth twin measures proper length and the travelling twin measures the contracted (shortened) length. But as both twins agree on the relative velocity, the travelling twin must measure a shorter time (to cover the shorter length) and thus returns to Earth having aged less than the Earth twin. Even when the acceleration periods are considered, Einstein's theory of general relativity, which deals with accelerating reference frames, confirms this result.

The ultimate judge, of course, is experiment. In 1971, precise clocks were sent around the world in jet planes and confirmed that less time would pass for the travelling twin.

uniform motion
motion of an object that is not undergoing acceleration

CHALLENGE 10.4

Infinite speed

Imagine that the speed of light was infinite. What would happen to the ideas about relativistic length contraction and time dilation?

Travelling at the speed of light

It is established that no material object can travel at the speed of light as it would require infinite energy to get to that speed. Some things do travel faster than light – but they are not material objects.

For example, since the Big Bang, space has expanded much faster than the speed of light. However, it is not matter that is travelling faster than the speed of light, but the space between the matter. This Big Bang paradox doesn't violate the theory of special relativity as nothing is breaking the light barrier.

The pole and the barn

Imagine a barn, 40 m wide, with a door on each end. A runner has a pole that is exactly as long as the barn (at rest), as shown in Figure 2a. The plan is for the runner to run into the barn and a farmer inside will close both doors for a brief time to demonstrate that the pole fits.

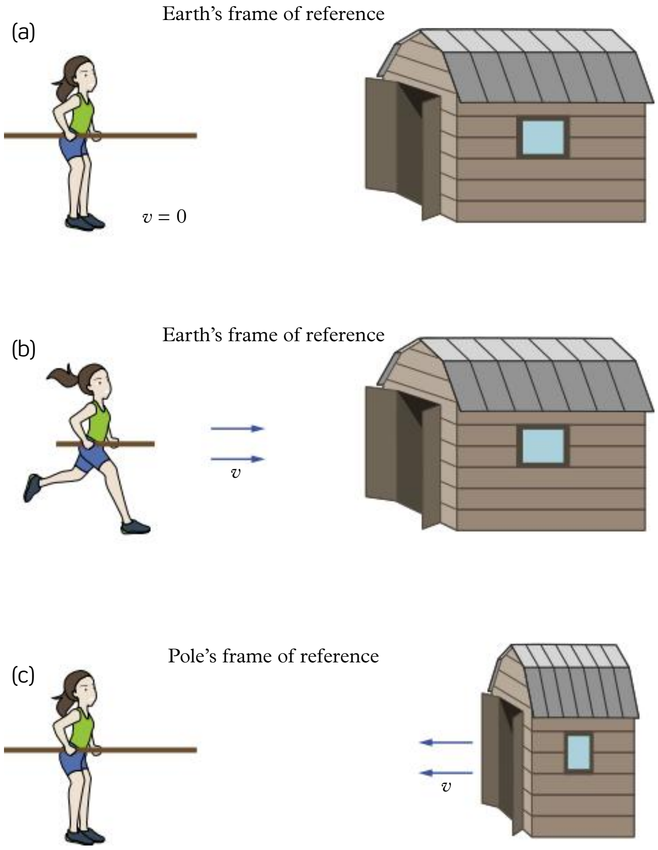


FIGURE 2 The pole in the barn

The runner runs into the barn with the pole (Figure 2b). According to relativity, in the reference frame of the barn (and Earth), the pole is moving and experiences length contraction and gets shorter so it fits easily inside the barn, and the farmer can shut both doors momentarily.

But from the pole's frame of reference (Figure 2c), it is the barn that is rushing towards it, so the barn suffers length contraction and gets smaller. The pole is too long for the barn. There is no way that the farmer can close both doors with the pole inside.

In one reference frame, the pole fits in the barn, in the other it does not. Does the pole get caught in the door or not?

Resolution

This apparent paradox results from the mistaken assumption of absolute simultaneity (see Chapter 9). The pole fits into the barn only if both of its ends are simultaneously inside the barn. The paradox is resolved when it is considered that in relativity, simultaneity is relative to each observer, making the answer of whether the pole fits inside the barn also relative to each observer.

The runner's idea of simultaneity is when she sees both doors close at the same time while inside the barn. But because she is approaching the exit door and retreating from the entrance door, she will see the exit door close first then open, and then the entrance door close and then open. That will not be simultaneous, so the pole doesn't fit.

The farmer's idea of simultaneity is the same but as he is at rest relative to the doors he can judge whether the doors open and close simultaneously. He can do this and finds that the pole fits.

CHECK YOUR LEARNING 10.4

Describe and explain

1 **Describe** the twins' paradox.

2 **Explain** why you can't have faster-than-light movement.

3 **Summarise** the barn door paradox.

Apply, analyse and interpret

4 A young-looking female astronaut has just arrived home from a long space trip. She rushes up to an old grey-haired man and in the ensuing conversation refers to him as her son. **Consider** how this is possible.

5 As you are travelling away from Earth at a speed of 0.6c, your pulse rate is being measured by a monitor on your wrist. The results are automatically radioed to Earth. **Determine** if observers on Earth would find the pulse rate increased when compared to normal.

Investigate, evaluate and communicate

6 **Consider** the following scenario as shown in Figure 3. Albert Einstein is on an ultrafast railway carriage and his wife Mileva Einstein is standing in a nearby field watching her high-speed husband go past travelling at close to light speed relative to her. As he passes, he drops Mileva's favourite potted plant, and to him the pot appears to shatter as it hits the floor of his ultrafast railway carriage.

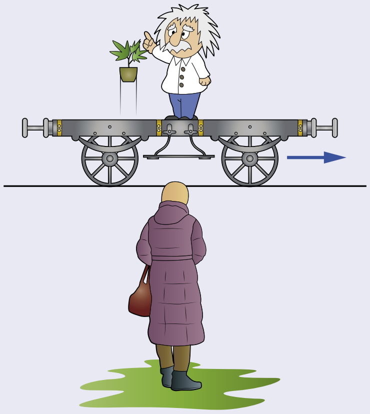


FIGURE 3

Determine what Mrs Einstein observes with respect to the pot's:

- motion (momentum and velocity in the vertical plane)
- physical characteristics (size and shape etc.) and
- fate on impact.

Justify your answer by identifying and explaining the relevant principles of special relativity. Present these ideas in an essay.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 10.4
- » Challenge worksheet
  10.4 Infinite speed
- » Increase your knowledge
  More paradoxes
- » Weblink
  The twin's paradox

SCIENCE AS A HUMAN ENDEAVOUR

10.5

Relativity and global positioning satellites

KEY IDEAS

In this section, you will learn about:

- technologies such as satellites that have dramatically increased the size, accuracy, and geographic and temporal scope of datasets with which scientists work
- satellites that provide experimental evidence that supports the phenomena of time dilation.

One of the great benefits of understanding special relativity is the accuracy of the array of global position satellites (GPS) orbiting Earth. By using a light signal from Earth these satellites make use of an extremely accurate on-board clock and can pinpoint a location on Earth to within a few metres.

You may wonder where relativity comes in, but consider this: the satellite clocks are moving at 14 000 km h^{-1} with an orbital time of 12 hours. This means they are travelling much faster than clocks on Earth, and you know that ‘moving clocks run slow’ at this speed by about 7 μs (7×10^{-6} s) per day. The on-board clocks adjust for this loss of time and work out the real time at which the signal was received. These GPS satellites are constantly transmitting signals with their location and the time the signal was sent back out. There are 24 of these satellites active at any one time, so any place on Earth can get a signal from at least four satellites (and usually several more).

By knowing the transmitted time and the speed of light, it is possible for your GPS device to calculate the distance between it and each of the four satellites. Using simple geometry it can work out where on Earth you are. The GPS system has an accuracy of about 50 ns (50 nanoseconds = 50×10^{-9} s) and in that time light can travel about 15 m, so that is the best navigational accuracy you could hope for. However, with land-based compensation systems accuracy can be increased to less than a metre.

Development

Global Positioning Satellites have an extremely valuable role in today’s society. Perhaps the most important is for critical positioning capabilities to military, civil and commercial users around the world. The United States Department of Defense created the system in 1973 for use by the US military. They maintain it and make it freely accessible to anyone with a GPS receiver.

Originally, it was to get an accurate position of enemy targets in the battlefield, but it was made available to the public in the 1980s mainly for civilian aircraft, which were spending a small fortune trying to maintain a rival system. So that the enemy didn’t use it against the USA, some ‘fuzziness’ was introduced into the system so that accuracy was limited to several 100s of metres. This made it less useful than it could have been to civilian users. In 2000 the ‘selective availability’ (fuzziness) was turned off. However, the US Air Force has alternative ways of blocking signals in specific locations in war zones.

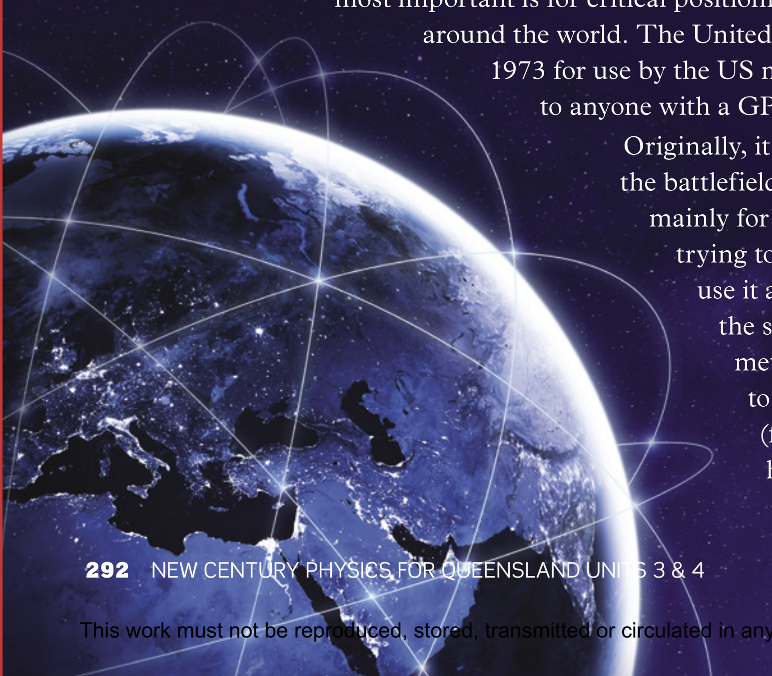


FIGURE 1 Thirty-two satellites provide complete GPS coverage of Earth.

Special relativity correction

There are a number of inaccuracies that creep into the signals. Some are random errors such as interference from the ionosphere or random errors in the clocks (±0.1 μs). The most important error is due to relativistic effects as defined by Einstein's theory of special relativity (1905) and his theory of general relativity (1915). The effects of gravity on time forms a part of the general theory and that is beyond the scope of this chapter. However, it is known to add an extra 45.9 μs to the ground-based time. This is due to the gravitational effect on time in which the stronger the gravitational field the greater the effect on time. Special relativity also accounts for a drift in time – but it is 7.2 μs in the opposite direction and together add to an error of 38.7 μs per day. Calculations for this are in your obook.

Relativity is not just an abstract mathematical theory. Our global navigation system would not work without an understanding of it.

CHECK YOUR LEARNING 10.5

Describe and explain

1 **Describe** how a difference in time signals can be used to calculate a position on Earth.

2 **Explain** which theory of relativity, special or general, has the bigger impact on the calculation of position.

Apply, analyse and interpret

3 A GPS satellite makes two orbits in 24 hours.

- **a** **Determine** orbital distance above the surface of the Earth. Hint: you will need to use formulas from Chapter 5. (Radius of the Earth, R_E = 6378 km; mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg; $G = 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$).
- **b** **Calculate** the time it takes light to go from Earth to the satellite.

4 The ratio of relativistic time to proper time, $\frac{t}{t_0}$, has been shown to equal:

$$\frac{t}{t_0} = \frac{GM_E}{c^2}\left(\frac{1}{R_E} - \frac{1}{R_{GPS}}\right)$$

where the gravitational constant $G = 6.674 \times 10^{-11}$, the mass of the Earth $m_E = 5.974 \times 10^{24}$ kg; Earth's radius, R_E = 6 357 000 m; and $c = 2.998 \times 10^8\ \text{m s}^{-1}$. The satellites have an altitude of 20 184 000 m, making their orbital radius R_{GPS} = 26 541 000 m. Substitute these values into the equation to show that this works out to 45.850 μs per day.

Investigate, evaluate and communicate

5 If the satellites orbited in the opposite direction would the calculations change?

Investigate and **assess** the following claims in a brief essay for each:

- **a** GPS is less accurate in the vertical direction than in the horizontal direction.
- **b** Without special relativistic correction, the navigational error of a GPS system would be much greater than 15 m.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 10.5
- » Weblink
  Uses of satellites
- » Weblink
  Global Positioning Satellites

CHAPTER 10

Review

Summary

10.1
- Moving objects contract: $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$, where L_0 is the proper length or the rest length and L is the relativistic length.
- Summary of relationships for space travel: $v = \frac{L_0}{t} = \frac{L}{t_0}$

10.2
- Relativistic momentum is given by the formula: $p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$
- Relativistic momentum is conserved in all interactions, providing no external force acts.
- Einstein's mass–energy equivalence relationship, $\Delta E = \Delta m c^2$, shows the relationship between energy and mass.

10.3
- Nuclear power stations rely on the interchange between mass and energy as the source of power.

10.4
- You can travel into the future – but only someone else's future. You can travel into the past – but only someone else's past.

10.5
- There are many applications where relativity theory is required, such as GPS navigation.

Key terms

- conservation of momentum
- length contraction
- light-year (ly)
- mass–energy equivalence relationship
- paradox
- proper length
- relativistic length
- relativistic momentum
- rest mass
- uniform motion

Key formulas

Contraction of length	$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$
Space travel relationships	$v = \frac{L_0}{t} = \frac{L}{t_0}$
Relativistic momentum	$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$
Mass–energy equivalence relationship	$\Delta E = \Delta mc^2$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 The resolution to the twins' paradox lies in the observation that:

- **A** no two people can be exactly the same age.
- **B** the twin who stays on Earth ages differently because of Earth's gravitational field.
- **C** the predictions of special relativity only apply to subatomic particles.
- **D** one of the twins must accelerate in order to leave Earth and come back.

2 Select the statement about which all observers in different inertial systems will agree.

- **A** the simultaneity of events at separate locations
- **B** the rate at which their clocks run
- **C** the lengths they measure along the direction of their relative travel
- **D** they will agree on none of the above

3 A meteor is heading directly towards the surface of a planet at a constant speed of 0.8*c*. Observers on the surface of the planet observe it at a time when it is a distance *h* above the surface in their reference frame. The observers calculate the time that the meteor will take to reach the surface of the planet as 784 µs.

The value that is closest to the distance *h* is:

- **A** 105 km
- **B** 200 km
- **C** 235 km
- **D** 380 km

4 The relativistic momentum of an atomic particle of mass 1.62×10^{-27} kg moving at 0.92*c* is:

- **A** 3.80×10^{-27} kg m s^{-1}
- **B** 9.70×10^{-27} kg m s^{-1}
- **C** 1.14×10^{-18} kg m s^{-1}
- **D** 2.91×10^{-18} kg m s^{-1}

5 The rest energy, in joules, of an electron of mass 9.11×10^{-31} kg is:

- **A** 3.0×10^{-39} J
- **B** 2.7×10^{-22} J
- **C** 8.2×10^{-14} J
- **D** 1.1×10^{-11} J

Short answer

Describe and explain

★ 6 **Explain** whether you can travel into the past or the future.

★ 7 **Explain** why you can't just keep applying a force to an object to increase its speed to be faster than light.

★ 8 **Calculate** the speed of a pion if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 5.2×10^{-8} s.

★ 9 **Clarify** whether anything can travel faster than light.

Apply, analyse and interpret

★ 10 You are travelling away from Earth at a speed of 0.5*c*. **Deduce** whether your mass, height, or waistline would change. **Infer** what observers on Earth using telescopes would say about these things.

★ 11 Consider the piece of paper on which one page of this book is printed. **Deduce** which of the following properties of the piece of paper are absolute, that is, which are independent of whether the paper is at rest or in motion relative to you:

- **a** thickness of the paper
- **b** mass of the paper
- **c** volume of the paper
- **d** number of atoms in the paper
- **e** chemical composition of the paper
- **f** speed of the light reflected by the paper
- **g** colour of the coloured print on the paper.

★★ 12 **Determine** the rest length of a spaceship given that it passes you at a speed of 0.75*c* and you measure its length to be 120 m.

★★ 13 **Determine** how much time it would take to travel to a star 24 light-years from Earth at a speed of 2.4×10^{8} m s^{-1}, according to:

- **a** the spaceship travellers
- **b** an observer on Earth.

★★ **14** Electrons travel through a linear accelerator 1.5 km long at a speed of 0.999 999 218c. **Determine** the length of this tube in the electrons' reference frame.

★★ **15** **Determine** how much energy can be obtained from conversion of 1.0 mg of mass.

★★★ **16** A 100 MeV electron travelling at 0.999 987c moves along the axis of an evacuated tube that has a length of 3.00 m as measured by a laboratory observer S with respect to whom the tube is at rest. An observer S' moving with the electron, however, would see this tube moving past her. **Determine** the length that the tube would appear to the observer S'.

★★★ **17** The star Alpha Centauri is 4.0 light-years away. **Determine** at what constant velocity a spacecraft would have to travel from Earth if it is to reach the star in 3.0 years, as measured by travellers on the spacecraft.

★★★ **18** After the Sun, our next nearest star is Proxima Centauri, which is 4.225 light-years away.

- **a** **Determine** how many years it would take a spacecraft travelling 0.80c to reach that star from Earth, as measured by an observer:
  - **i** on Earth
  - **ii** on the spacecraft.
- **b** **Calculate** the distance travelled according to an observer on the spacecraft.

★★★ **19** Imagine a rocket takes off for a distant planet and can travel at many times the speed of light. (You know that this is impossible, but just say it is for this question.) Observers on the planet are viewing the incoming spaceship through a powerful telescope. **Deduce** what they will see from the moment the rocket leaves Earth until it lands on their planet.

★★★ **20** Plutonium-239 is a nuclide that undergoes neutron-induced fission. A typical reaction is:

$$^{239}_{94}\mathrm{Pu} + ^{1}_{0}\mathrm{n} \rightarrow ^{133}_{55}\mathrm{Cs} + ^{104}_{46}\mathrm{Pd} + 3^{1}_{0}\mathrm{n}$$

The exact masses of the nuclides are:

m (Pu-239) = 239.052 162 u

m (Cs-133) = 132.905 452 u

m (Pd-104) = 103.904 030

m (neutron) = 1.008 665 u

Deduce the energy released in joules per atom of plutonium reacted.

Investigate, evaluate and communicate

★ **21** **Propose** the speed at which sunlight would pass you if you were on a rocket travelling at 0.6c away from the Sun.

★★★ **22** You want to get to a star 100 light-years away and have calculated you could do it in 4.5 years if you travelled at 0.999c. The problem is that the human body can't withstand accelerations greater than 5g. **Predict** the time, as measured by an Earth observer, it would it take you to reach 0.999c from rest at an acceleration of 5g.

★★★ **23** **Predict** how much time it would take in the spacecraft's frame of reference for the above.

★★★ **24** Imagine you have slid into a parallel universe in which the year is 1840. A US marshal (sheriff) is travelling by in a train at 0.75c and approaches two gunslingers about to have a duel. The gunslingers are both the same distance from the railway line. Gunslinger A is closer to the marshal. The marshal sees both men draw their guns at the same time. **Decide** who actually drew first in:

- **a** the marshal's frame of reference
- **b** the gunslingers' frame of reference.

★★★ **25** The Andromeda galaxy is 100 light-years away and astronauts plan to travel there at 0.98c to get there in a reasonable time.

- **a** **Predict** how long would it take in their frame of reference.
- **b** Once they arrive the astronauts send a radio signal back to Earth. **Determine** how long after the astronauts left Earth would observers on Earth have to wait to receive the radio signal.

★★★ **26** The Large Hadron Collider track is a circular track exactly 26 659 m long. Protons have been made to whizz around the track at speeds of 0.999 999 999 999 999 999 999 995 1c (there are 23 nines in the number). Most calculators can't work with that many decimal places so a slower proton can be used as an example. Imagine a proton moving at 0.999 99c around the track.

a **Determine** the length of the track as seen by this proton.

b **Propose** how many seconds the scientists would measure for the proton to complete one revolution of the track.

c **Deduce** how long would it take in the reference frame of the proton.

d **Determine** the relativistic momentum of the proton, given that its rest mass is 1.67×10^{-27} kg.

★★★ **27** Because of the rotational motion of Earth about its axis, a point on the equator moves with a speed 460 km h^{-1} relative to a point on the North Pole. **Propose** whether this means that a clock placed on the Equator runs more slowly than a similar clock placed on the North Pole.

★★★ **28** Suppose that a special breed of cat (*Felix schrödingerus*) lives for exactly 7.00 years according to its own body clock. When such a cat is born, it is put aboard a spaceship and sent off at a speed of $0.80c$ toward the star Alpha Centauri.

a **Determine** how far from Earth (reckoned in the reference frame of Earth) the cat would be when it dies.

b As soon as the cat dies, a radio signal announcing the death of the cat will be sent from the spaceship. **Decide** how many years after the departure of the spaceship will the signal reach Earth (radio signals travel at the speed of light).

★★★ **29** A spacecraft of rest mass 20 000 kg has been accelerated to $0.25c$.

a **Derive** its momentum.

b **Determine** by what percentage you would you be in error if you used the classical formula for momentum.

★★★ **30** You are sitting in your car when a very fast sports car passes you at a speed of $0.18c$. The driver of the sports car says his car is 6.00 m long and yours is 6.15 m long. **Predict** what you would measure for the length of:

a the sports car

b your car.

★★★ **31** Train carriages A and B each have a rest length of 10 m and are directly approaching each other at high speed. Ann is in carriage A and as she passes the front of carriage B she starts her clock. She stops her clock when she passes the rear end of carriage B. Ann's clock reading shows it took carriage B a time of 2×10^{-7} s to pass her. **Propose** what the speed of the carriage B will be.

★★★ **32** Trains A and B are about to pass each other at extremely high speeds from opposite directions. Train A has a rest length of 100 m and train B measures 50 m at rest. Albert is in train A and as he passes the front of train B he starts his clock. He stops his clock when he passes the rear end of train B. Albert's clock reading shows it took train B exactly 1.00 μs to pass him. **Determine** the speed of train B.

★★★ **33** A 20-year-old astronaut leaves Earth in a rocket for a distant planet 9 light-years away (as measured by Earth observers). When he lands on the planet he sends a radio signal back to Earth which arrives at Earth 19 years after he left (Earth).

a **Deduce** the speed at which he travelled.

b A radio signal was sent back to the planet confirming that Earth had received the signal from him. **Predict** how old he would be when the return signal was received on the planet.

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  Chapter 10

CHAPTER 11

Quantum theory and light

Until the end of the 19th century, classical physics, based on Newton's laws, successfully explained all our natural surroundings of matter, space and time. It was at the turn of the century that the experimental observations by physicists and subsequent theoretical explorations began to question the validity of the Newtonian laws, especially at very small distances, at very high speeds and within the world of the emerging atom. For example, lines had been noticed in the spectra of light emitted by heated gases or gas discharges. Light itself was difficult to explain, as it seemed to have both a particle nature and a wave nature, and the field of thermodynamics did not seem to be related to molecules and atoms at all.

OBJECTIVES

- → Explain how Young's double slit experiment provides evidence for the wave model of light.
- → Describe light as an electromagnetic wave produced by an oscillating electric charge that produces mutually perpendicular oscillating electric fields and magnetic fields.
- → Explain the concept of black-body radiation.
- → Identify that black-body radiation provides evidence that electromagnetic radiation is quantised into discrete values.
- → Describe the concept of a photon.
- → Solve problems involving the energy, frequency and wavelength of a photon.
- → Describe the photoelectric effect in terms of the photon.
- → Define the terms 'threshold frequency', 'Planck's constant' and 'work function'.
- → Solve problems involving the photoelectric effect.
- → Recall that photons exhibit the characteristics of both waves and particles.

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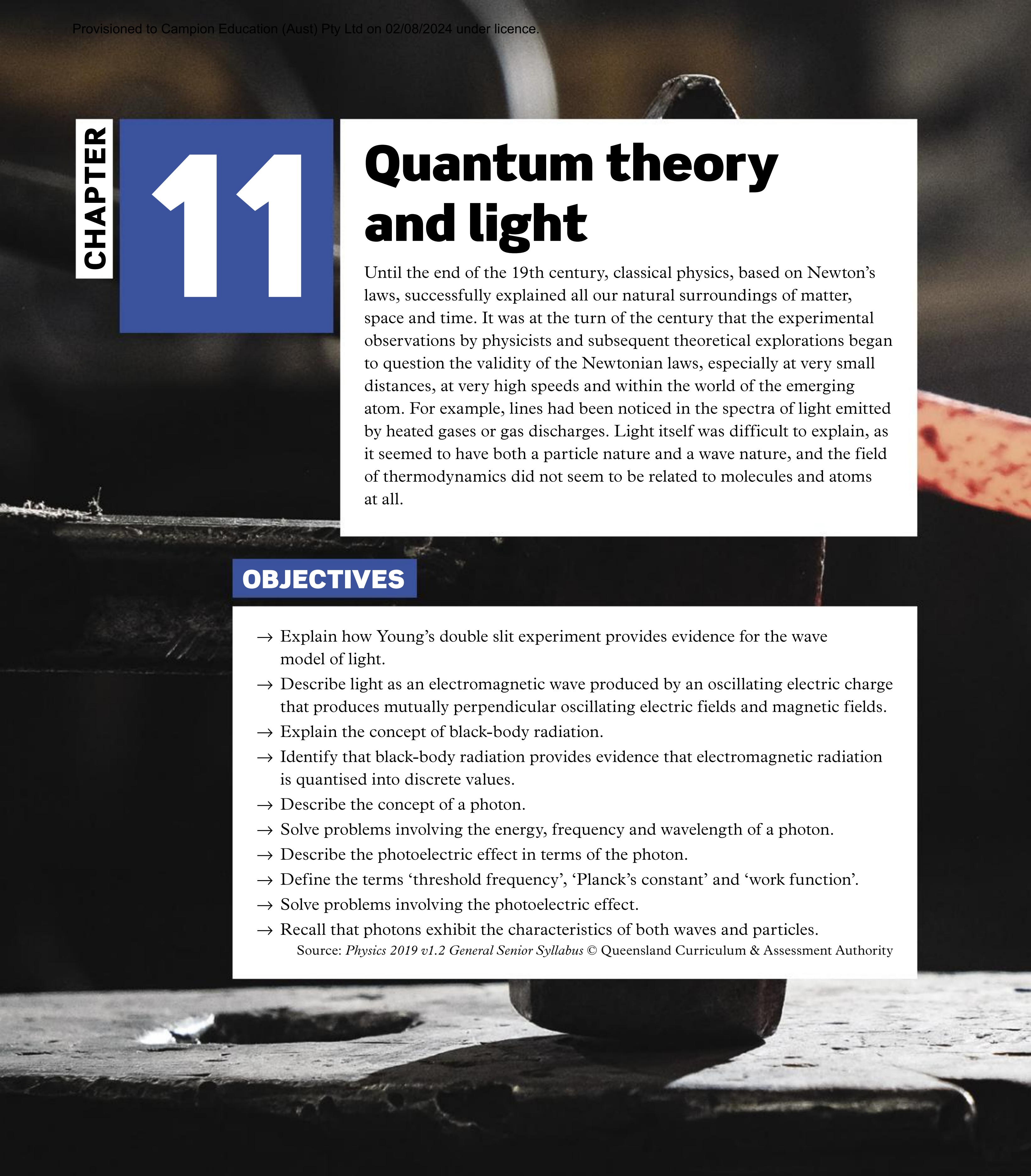

FIGURE 1 A blacksmith works with heated metals. As it is heated the metal glows red then yellow and then white, as the peak wavelength gets shorter and shorter. Classical physics could not explain this.

MAKES YOU WONDER

In this chapter you will be examining some aspects of the quantum theory that will help to answer questions such as:

→ Why is the Sun yellow?
→ What is hotter than white hot?
→ UV lights are called ‘black lights’ – but how can they be black?
→ How does light decide whether it’s going to be a wave or a particle?
→ Can you see an atom?

PRACTICALS

MANDATORY PRACTICAL **11.5** The photoelectric effect

11.1

Wave model for light

KEY IDEAS

In this section, you will learn about:

- Young's double slit experiment
- light as an electromagnetic wave produced by an oscillating electric charge

By the end of the 19th century the dispute over competing theories of light was just about over. Wave theory had won the battle and the particle theory was put on hold. Even Newton's support for the particle theory couldn't withstand the amazing success of the wave model. Waves and particles were just as successful for explaining reflection, rectilinear (straight line) propagation, speed, and shadows. But the particle model had little success in explaining polarisation, or the diffraction (bending) of light as it passes around objects and through slits. The particle theory was completely unable to deal with interference of light which produced bright and dark fringes when it passed through thin slits, or even the source of the beautiful colours in beetles' wings.

FIGURE 1 The beautiful iridescent colours in this jewel bug's wings are produced by the interference of light and can only be explained by the wave model of light.

Young's double slit experiment

One of the most resounding demonstrations in support of the wave model was provided in 1801 by English scientist Thomas Young (1773–1829), a contemporary of Newton. His was the first serious challenge to the particle theory of Newton, which proposed the existence of 'corpuscles' of light. These 'corpuscles' could be likened to the modern notion of photon particles. Newton was aware of diffraction but failed to recognise its significance.

Origins of Young's theory

Young wrote a thesis on the physical and mathematical properties of sound and, in 1800, he presented a paper to the Royal Society in London in which he argued that light was also a wave. His idea was greeted with some scepticism because it contradicted Newton's corpuscular theory. Young knew that sound was a wave phenomenon and that if two sound waves of equal intensity were 180° out of phase they would experience destructive interference and cancel out. Young argued that if light was a wave phenomenon then similar interference effects should occur. This proposal inspired Young to perform an experiment that is referred to as **Young's double slit experiment**.

FIGURE 2 Thomas Young

Young's method

Young split a beam of sunlight to produce two **coherent** (in phase) beams that he let fall onto a screen where they overlapped to produce the familiar banded diffraction pattern (Figure 3). The explanation was that when crests met crests or troughs met troughs there would be constructive interference and bright bands would be produced. In positions where a crest and a trough met there would be dark bands. The wave model was confirmed and the particle supporters had no answer for it. After that, the corpuscular theory of light was vanquished, not to be heard of again till the 20th century, some 100 years later.

Coherent beams are needed for this experiment, but Young knew that the usual light sources at the time (candles and lanterns) could not serve as coherent light sources. He made a pinhole in his window shutter and directed the beam of sunlight across the room to strike a piece of card edge-on. Some light went on the left side, some went on the right. Since the two beams were both from the same source (the pinhole), Young considered them as coming from two coherent sources.

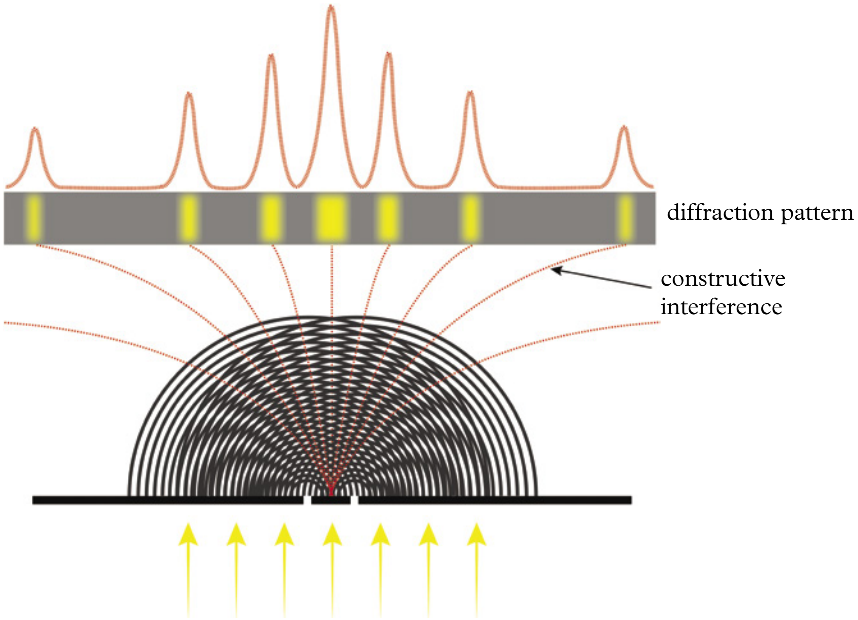

FIGURE 3 A stylised version of Young's double slit experiment showing the bright and dark bands on the screen.

Study tip

A great aid in visualising double slit interference is to view the pattern produced by a laser beam passed through a diffraction grating. Your school may have them both.

Young's double slit experiment demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern

coherent waves with the same frequency and amplitude, and a constant phase relationship. Conventional light sources such as candles and incandescent bulbs are incoherent sources; laser beams are coherent

Study tip

'Coherent' doesn't have to mean in phase. As long as the waves are in a constant phase relationship they are said to be coherent. A good analogy is found in the motion of loudspeakers. If the cones are both going in and out together, they are coherent and in phase. If one goes in as the other comes out they are coherent but a half-wavelength out of phase.

Wave model for light

A light wave is an electromagnetic (EM) wave – it is an oscillation of the electromagnetic field. Examples include gamma rays, ultraviolet light, infrared light, X-rays and radio waves. They all share the same characteristics. They do not need a medium for transmission, so **electromagnetic waves** can get to us from distant stars by passing through the vacuum of interstellar space.

Students ask in terms of the **wave model for light**: 'if they are waves, what is doing the waving?' The simplest answer is that the strength of the **magnetic fields** and **electric fields** that make up the electromagnetic waves are oscillating back and forth as they travel out into space (Figure 4). The original source is an **electric charge** that oscillates and sends these waves out into space. Both fields oscillate in phase and the rate of oscillation is a measure of their frequency. The spacing between successive crests is their wavelength, and they travel at a speed of 3×10^8 m s^{-1} in a vacuum as **electromagnetic radiation**. In material media such as glass, plastic and water the speed is less. The following diagram best depicts the characteristics of an electromagnetic wave.

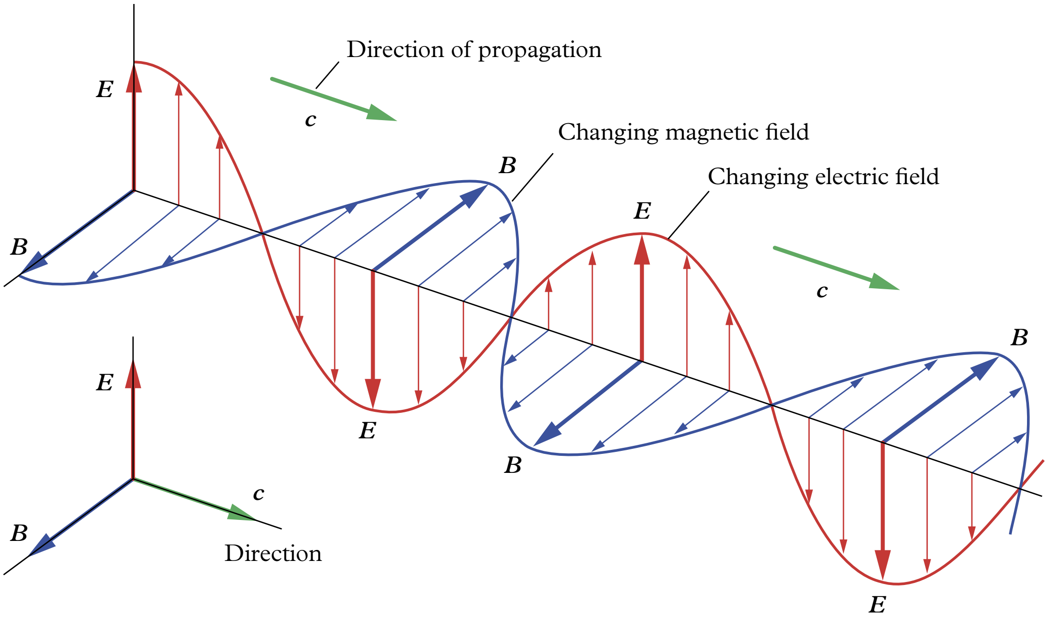

FIGURE 4 In an electromagnetic wave, the changing magnetic and electric fields are at right angles to each other and to the direction of propagation. The waves propagating in one direction only are shown here. In reality, they would be in all directions. The *E* (red) and *B* (blue) arrows represent the electric and magnetic fields respectively.

Spectral regions

As you saw in an earlier chapter, electromagnetic waves can be divided into 'spectral regions' based on their **frequency** or **wavelength**. This is shown in Figure 5. Visible light, for example, ranges from violet to red. Violet light has a wavelength of 400 nm, and a frequency of 7.5×10^{14} Hz. Red light has a wavelength of 700 nm, and a frequency of 4.3×10^{14} Hz. Any electromagnetic wave with a frequency (or wavelength) between those extremes can be seen by humans, but visible light makes up a very small part of the full electromagnetic spectrum. Electromagnetic waves that are of higher **energy** than visible light (higher frequency, shorter wavelength) include ultraviolet light, X-rays and gamma rays. Lower energy waves (lower frequency, longer wavelength) include infrared light, microwaves, and radio and television waves.

electromagnetic wave
a wave produced by an oscillating electric charge that radiates out at the speed of light as mutually perpendicular electric and magnetic fields

wave model of light
uses wave characteristics such as wavelength, frequency and speed to describe the behaviour of light such as polarisation, interference and diffraction

magnetic field
a region of space in which a magnetic force is experienced

electric field
a region of space near an electrically charged particle or object within which a force would be exerted on other electrically charged particles or objects

electric charge
a physical property of an object that causes it to experience a force when placed in an electromagnetic field

electromagnetic radiation
energy radiating out as synchronised oscillations of electric and magnetic fields, or electromagnetic waves, propagated at the speed of light in a vacuum

frequency
the number of waves that move past a given point in one second (symbol: *f*; SI unit: hertz; unit: Hz)

Wave equation

You will have used the wave equation in earlier years:

$$v = f\lambda$$

where v is the speed (velocity), f is the frequency and λ is the wavelength.

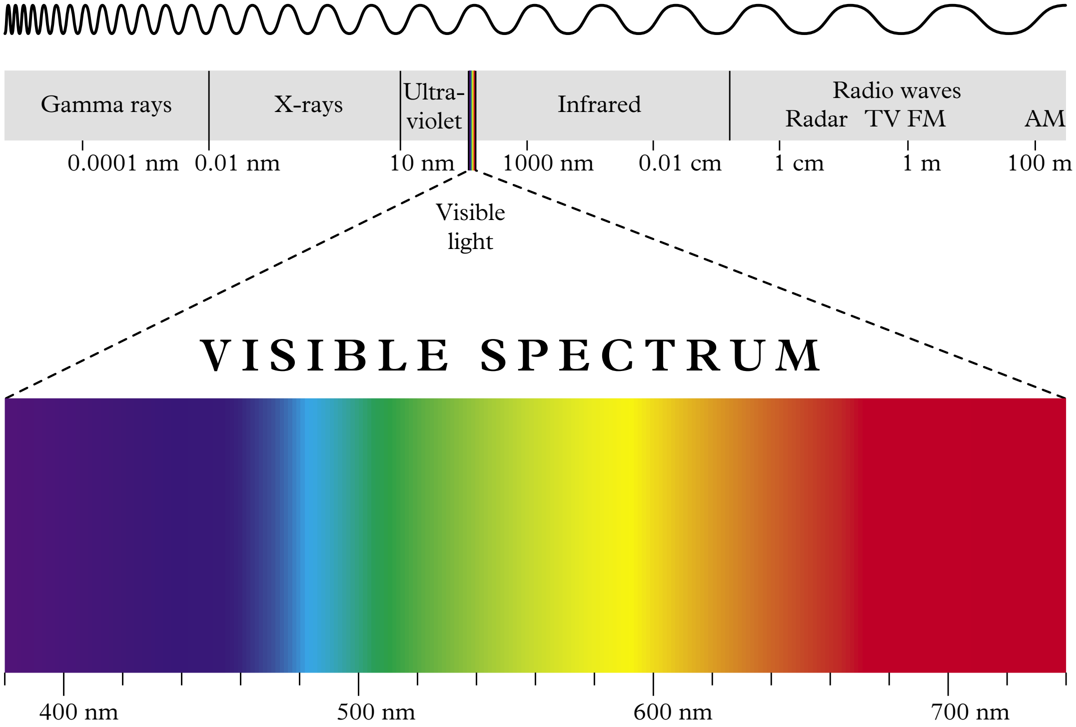

FIGURE 5 The electromagnetic spectrum

WORKED EXAMPLE 11.1A

A beam of light has a wavelength of 7.00×10^{-7} m.

a Calculate its wavelength in nanometres.

b Calculate its frequency.

c Identify its spectral region.

SOLUTION

a $\lambda = 7.00 \times 10^{-7}\ \text{m}$

$$= \frac{7.00 \times 10^{-7}\ \text{m}}{10^{-9}\ \text{m nm}^{-1}}$$

$$= 700\ \text{nm}$$

b $f = \frac{v}{\lambda}$

$$= \frac{3 \times 10^{8}\ \text{m s}^{-1}}{7 \times 10^{-7}\ \text{m}}$$

$$= 4.28 \times 10^{14}\ \text{Hz}$$

c It is in the visible region, red.

wavelength
the distance between corresponding points on successive waves with the same velocity (symbol: λ; SI unit: metre; unit symbol: m)

energy
the capacity to do work in which it is transformed or transferred

CHALLENGE 11.1A

Micro

Does the ‘micro’ prefix in the word microwave mean that they have wavelengths smaller than visible light?

CHALLENGE 11.1B

Double slit underwater

If Young’s double slit experiment was performed underwater, how would the distance between bright fringes change?

Study tip

For much of the work that follows, the wavelength will be expressed in nanometres (nm, equal to 10^{-9} m). For example, the wavelength of mid-green is about 550 nm or 550 × 10^{-9} m. To convert nm to m, just multiply the nm by 10^{-9}. Sometimes you will be asked to express your answer in nm, so you divide your value in metres by 10^{-9}. Frequencies are expressed in Hz, but are often stated as terahertz (THz) where the prefix *tera* = 10^{12}. Unless otherwise stated you should assume the wave speed, c, is 3×10^8 m s^{-1}.

Limitations of the wave model

The electromagnetic wave model for light was very successful in explaining many phenomena such as diffraction, interference and polarisation. However, it stumbled when it came to some phenomena being investigated in the late 1800s, namely black-body radiation and the photoelectric effect. This conundrum, and its solution, will be discussed in the following sections.

CHECK YOUR LEARNING 11.1

Describe and explain

1 **Explain** how Young’s double slit experiment provides evidence for the wave model of light.

2 **Describe** the relationship between the electric and magnetic fields in an electromagnetic wave.

3 **Construct** a diagram to show two waves undergoing constructive and destructive interference.

4 **Identify** the colour of visible light that has the longest wavelength.

5 **Clarify** what is oscillating and produces an electromagnetic wave.

6 **Construct** a diagram to represent two ‘coherent’ electromagnetic waves of the same frequency, wavelength and amplitude.

Apply, analyse and interpret

7 **Determine** the colour of light of frequency 400 THz (tera, T = 10^{12}).

8 **Determine** the frequency range of visible light, given the range of wavelengths is from 390 nm to 700 nm (nano, n = 10^{-9}).

9 **Determine** the wavelength in picometres for a gamma wave of frequency 5×10^{17} Hz (pico, p = 10^{-12}).

Check your obook assess for these additional resources and more:

- » Student book questions Check your learning 11.1
- » Challenge worksheet 11.1A Micro
- » Challenge worksheet 11.1B Double slit underwater
- » Weblink Young’s double slit experiment

11.2

Black-body radiation

KEY IDEAS

In this section, you will learn about:

- the concept of black-body radiation
- black-body radiation providing evidence that electromagnetic radiation is quantised into discrete values.

black body
an object that absorbs all radiation falling on it, at all wavelengths; it is a perfect absorber or emitter of radiation

black-body radiation
the radiation emitted by a black body from the conversion of thermal energy, and which has a characteristic frequency distribution that depends on the temperature

Wien's displacement law
states that the black-body radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature

While many scientists were investigating the structure of the atom, a number were looking at the light produced from heated bodies. The object being heated was called a **black body**, which is an object that absorbs all radiation; that is, it doesn't reflect any light shining onto it. A big block of hollowed-out charcoal was commonly used. So as the black body is heated, any light radiation that it emits is solely coming from within itself. This is called **black-body radiation**.

Wien's displacement law

German physicist Wilhelm Wien (pronounced *veen*; 1864–1928) worked on the problem of black-body radiation through the late 1880s and early 1890s and ended up making one of the most significant contributions to our understanding of the inner workings of an atom.

Wien grew up on a farm to the south-west of Berlin and was well aware of the effect of heat on the colour of objects. For example, he knew how the colour of steel changed when it was heated for horseshoes. It went from black to red, to yellow and then to white at its maximum temperature. His PhD in Berlin on the emission of light from metals led him to investigate the distribution of wavelengths of emitted light as an object is heated. Wien published his law in 1893 to much acclaim. **Wien's displacement law** states that the black-body radiation curve for different temperatures peaks at a wavelength that is inversely proportional to the temperature. It is shown graphically in Figure 1.

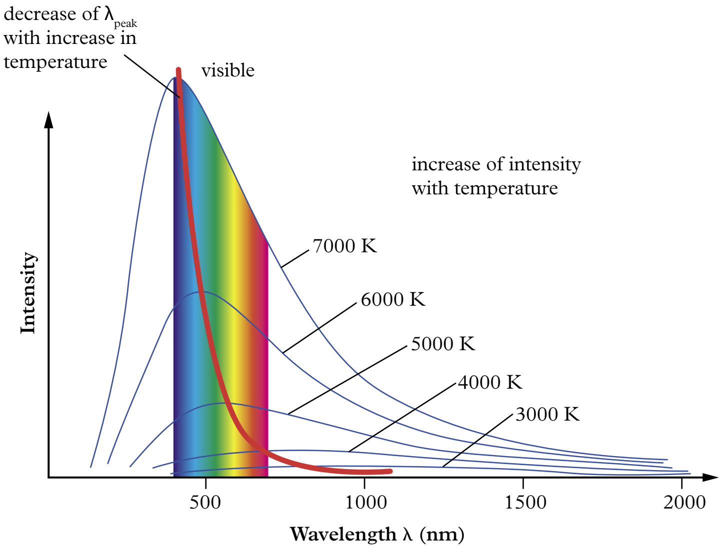

FIGURE 1 Wien's black-body radiation graph. The red line shows how the peak wavelength changes with temperature.

Study tip

A good way to visualise the changes in the spectrum of light with temperature is to use the bulb from a raybox kit at voltages from 2 V to 12 V and notice how it goes from yellow to white.

Features of Wien's law

Wien's displacement law has four important features:

1. All black bodies at the same temperature produce the same spectrum, that is, the spectrum depends on the temperature and not the substance from which the body is made.
2. Making an object hotter produces more radiation (of all wavelengths) across the whole spectrum.
3. The higher the temperature the shorter the wavelength of the peak of the spectrum.
4. The visible 'rainbow' is only a small region of the whole emission spectrum, which also includes IR, and some UV (if very hot).

Wien was awarded a Nobel Prize in Physics in 1911 for his work on black-body radiation.

Wien's law formula

The equation he developed for the law is:

$$\lambda_{\text{max}} = \frac{b}{T}$$

where λ_{max} is the peak wavelength in metres (m), T is the temperature in kelvin, and b is the constant of proportionality equal to 2.898×10^{-3} m K. Recall that $T_K = T_C + 273$.

WORKED EXAMPLE 11.2A

a Calculate the peak wavelength for radiation from the Sun and Earth given that the effective surface temperature of the Sun is 5800 K, and the Earth 16.0°C.

b Identify the spectral regions for each peak wavelength.

SOLUTION

a

$$\lambda_{\text{max}} = \frac{b}{T}$$

$$\lambda_{\text{max(Sun)}} = \frac{2.898 \times 10^{-3}\ \text{mK}}{5800\ \text{K}}$$

$$= 5.00 \times 10^{-7}\ \text{m (500 nm)}$$

$$\lambda_{\text{max(Earth)}} = \frac{2.898 \times 10^{-3}\ \text{mK}}{(273 + 16.0)\ \text{K}}$$

$$= 1.00 \times 10^{-5}\ \text{m (10 μm, 10 000 nm)}$$

b The wavelength of 500 nm for the Sun's radiation is in the blue-green spectral region in visible light, whereas the wavelength of 10 000 nm for Earth's radiation is in the infrared region.

Black-body radiation and the greenhouse effect

greenhouse effect
the process by which certain gases in the atmosphere, known as greenhouse gases, warm Earth by absorbing the long-wavelength radiation from Earth's surface

The consequences of the results from Worked example 11.2A are important to our survival. Visible light passes through our atmosphere easily and is not absorbed. It strikes the surface of Earth and is absorbed by the ground and buildings, which causes them to warm. However, the wavelengths radiated by Earth's surface and objects on it in return are in the infrared (IR) and are much longer than those from the Sun. These IR waves are strongly absorbed by the atmospheric gases that cause a blanket effect to keep Earth warm – the **greenhouse effect**. An increased concentration of these 'greenhouse gases' (mainly CO_2, methane and water) will cause too much heating and result in dire consequences from global warming. The next section explores this in more detail.

CHALLENGE 11.2A

Maximum intensity

The temperature of a radiating object is increased. State whether the maximum intensity increases or decreases and whether the peak wavelength increases or decreases.

CHALLENGE 11.2B

Crookes radiometer

A Crookes radiometer consists of four paddles suspended on a needle point in a low-pressure glass container (Figure 2). It was invented by English scientist William Crookes in 1873. One side of the paddle is painted black, and the other side white. When placed in sunlight the radiometer turns around. Explain whether the black side moves away from the Sun or towards it, and why this happens. It will turn in the opposite direction when placed near a block of dry ice (–44°C). Most people (even scientists) give the wrong explanation.

FIGURE 2 Crookes radiometer when light is on it

Study tip

Black-body radiation and the greenhouse effect make a good Research Investigation as there are many claims made and there is a great deal of secondary evidence in the literature available to evaluate the claim.

CHECK YOUR LEARNING 11.2

Describe and explain

1 **Explain** how the frequency of light is related to temperature.

2 **Describe** the way in which Wien's law shows an inverse relationship.

3 **Explain** what is meant by the claim that a black body is a perfect absorber and emitter.

Apply, analyse and interpret

4 Earth radiates in the infrared, which is invisible to our eyes. **Propose** how it is that we can see Earth from space.

5 The Moon has an average temperature during its daytime of 100°C and at its night an average temperature of –173°C. **Determine** the peak wavelength and spectral regions of the radiation in the daytime and at night.

6 **Determine** the peak wavelength for a steel horseshoe that has been heated to 350°C.

7 **Determine** the temperature in °C that would produce radiation with a peak wavelength of 1.0 μm ($\mu = 10^{-6}$).

Investigate, evaluate and communicate

8 Radiation produced 180 000 years after the Big Bang has been detected by University of Arizona physicists on a spectrometer in outback Western Australia. It indicated the temperature of space at that time was just 1.5 K.

  a **Determine** the peak wavelength of the light, expressed in cm.

  b **Propose** how this energy gets to Earth without warming up when it is so far away.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 11.2
- » Challenge worksheet
  11.2A Maximum density
- » Challenge worksheet
  11.2B Crookes radiometer
- » Weblink
  What is black-body radiation?

SCIENCE AS A HUMAN ENDEAVOUR

11.3

Black-body radiation and the greenhouse effect

KEY IDEAS

In this section, you will learn about:

- the greenhouse effect and black-body radiation.

Scientists have produced several models to explain the greenhouse effect and how it is intensified by changes in human activity. They look at climate change in the past and try to predict future changes. Their models are tested against the evidence and modified to find a better fit. These models are based largely on several laws you are now familiar with: Wien's law, and the laws of conservation of mass and conservation of energy.

The idealised greenhouse model is based on the fact that atmospheric gases are transparent to incoming radiation from the Sun, which is mostly in the IR–visible–UV spectral region, but more opaque to that re-radiated IR from Earth's surface. This is commonly summed up as 'heat is easily let in but gets trapped by the gases as it tries to leave'.

Incoming radiation

The surface of the Sun is about 5614°C (5887 K) and it radiates light mostly in the visible spectral region with some either side in the UV and IR. Earth is cooler, so it radiates at longer wavelengths – mainly in the IR. Sunlight arriving at the top of the atmosphere has an energy density of 1364 W m^{-2}. The amount of this energy absorbed by Earth depends on the **albedo** (reflectivity) of the clouds and surface. Snow and sea ice, being white, are good reflectors and contribute to a high albedo. As this polar ice melts to form sea water or to expose the underlying rocks the albedo drops. Water and rocks are darker so don't reflect as much so the radiation is absorbed and leads to greater heating.

albedo
the fraction of light that is reflected by a body or surface; it is a measure of the relative brightness of the surface

Outgoing radiation

For Earth to maintain a constant temperature, the outgoing radiant energy must be the same as the incoming. It doesn't have to be the same wavelength to do this as long as the energies are equal. Very little of the radiation from the surface (land, sea water, ice) goes directly back to outer space (see Figure 1). Most of it gets absorbed by gases in the atmosphere such as carbon dioxide CO_2, methane CH_4, water H_2O, and nitrogen oxides NO_x. These are the commonly known greenhouse gases. These gases occupy a layer about 5 km from the surface, but at this altitude they are cooler than the surface and don't radiate as much. You can assume that the atmosphere is now like a black-body emitter that radiates some of the absorbed radiation out into space. However, the gases emit in all directions and much of the emitted radiation finds its way back to Earth's surface, which is warmed to an average of about 15°C (288 K).

FIGURE 1 Radiation pathways on Earth. Sunlight (yellow) is partially reflected by the clouds and by the surface (mainly ice and snow). The rest goes into heating the surface. The red arrow shows it being emitted as IR. Some goes directly into space but the rest gets absorbed and re-emitted either out to space or back to Earth to further warm the surface.

If there were no greenhouse gases at all, you could use the **Stefan–Boltzmann law** to determine the temperature of Earth's surface from the incoming radiation. The equation is:

$$S\varepsilon = 4\sigma T^{4}$$

where S is the solar radiation falling on Earth (1364 W m^{-2}), ε is the emissivity of the Earth (assumed to be 0.7), σ is the Stefan–Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}), T is temperature (K). Substitute the known values:

$$1364 \times 0.7 = 4 \times 5.67 \times 10^{-8}\, T^{4}$$

$$T = \sqrt[4]{\frac{1364 \times 0.7}{4 \times 5.67 \times 10^{-8}}}$$

$$= 255 \text{ K } (-18°\text{C})$$

Stefan–Boltzmann law states that the energy radiated by an ideal black body is proportional to the fourth power of the temperature in kelvin

The fact that Earth's average temperature is +15°C is attributed to the greenhouse gas effect, and without it we wouldn't be alive. We would have trouble surviving at −18°C.

Changing greenhouse gas levels

The levels of greenhouse gases in the atmosphere are affected by human activity. The production of increased amounts of greenhouse gases has increased energy absorption by the atmosphere and makes Earth hotter than it otherwise would have been. This is called the enhanced greenhouse effect.

Scientists make models to see if they can account for historical temperatures and rainfall. If the models are successful, they can then attempt to predict the future some 50–100 years from now. A collaboration of tens of thousands of scientists constantly look for any anomalies the model can't explain, and ask 'how perfectly does the model fit'? There are always disputes, but the fundamental principles of the greenhouse model are overwhelmingly accepted by the scientific community.

CHECK YOUR LEARNING 11.3

Describe and explain

1 **Describe** the difference between the incoming radiation and the radiation from Earth emitted back into space.

Apply, analyse and interpret

2 **Determine** the temperature of Earth without greenhouse gases using the Stefan–Boltzmann law, but assuming the emissivity (ε) was 0.5 instead of 0.7 (indicating that a smaller fraction of solar radiation reaches Earth's surface)

Investigate, evaluate and communicate

3 **Assess** the claim that 'accurate data about past climate enables scientists to test their models of Earth's energy balance'.

4 A claim is sometimes made that studying the past enables us to predict the future. **Evaluate** this claim in terms of changes in Earth's energy balance due to the enhanced greenhouse effect.

5 **Evaluate** the claim that using the concept of black-body radiation to study the Earth energy balance enables scientists to monitor changes in global temperature, assess the evidence for changes in climate due to the enhanced greenhouse effect and evaluate the risk posed by anthropogenic climate change.

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 11.3

» Increase your knowledge
The coldest place in the universe

» Weblink
The effects of greenhouse gas

11.4

What is a photon?

KEY IDEAS

In this section, you will learn about:

- the concept of photons
- the Planck equation
- solving problems involving the energy, frequency and wavelength of a photon.

Wien's equation and the shapes of his graphs were accurate for a wide range of temperatures but ran into two major problems. One was that classical physics could not explain the shape. Secondly, there was a discrepancy at low temperatures. Other physicists developed equations based on Maxwell's electromagnetic theories which predicted that at very high temperatures, a black body should emit enormous amounts of UV, X-rays and gamma rays – but in practice it did not. More of these high energy particles were supposed to be produced than the energy used to make them. This is clearly impossible and was described as an 'ultraviolet catastrophe'. That is, a failure to account for the lack of UV, X-rays and gamma rays at high temperatures was a failure of classical physics. The equations were not valid at the extremes.

Planck's quanta

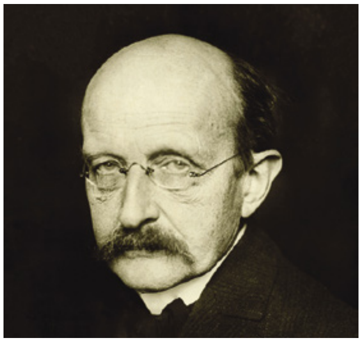

FIGURE 1 Max Planck

The German physicist Max Planck (1858–1947) finally produced the equation that did describe black-body distribution and, in doing so, he proposed a revolutionary theory of subatomic matter. Planck proposed that the energy released by a black body was, in fact, emitted by atoms, and that these atoms could only vibrate at certain frequencies that were multiples of a smallest value. He had to assume that the energy released by the atoms was not given off continuously, but in small energy packets that he called **quanta** (singular quantum), from the Latin *quantus*, meaning 'how much'. Each frequency, *f*, of light emitted by the atoms is proportional to the change in energy of the atom, so that, for example, as violet light is twice the frequency of red light, the energy quanta of violet light are twice the size of those of red light.

quanta
the smallest discrete packets of energy of electromagnetic waves, also later known as photons

Planck's constant
a fundamental constant used in quantum mechanics that relates frequency to energy (symbol: *h*; SI unit: joule second; unit symbol: J s), equal to 6.626×10^{-34} J s. Also known as the Planck constant

Planck equation
a relationship that relates frequency to energy: $E = hf$. It accounts for the quantised nature of light and plays a key role in understanding phenomena such as the photoelectric effect and Planck's law of black-body radiation

Planck's constant

Mathematically, the quanta energy is given by $E = hf$, where the constant h is called **Planck's constant** and has a value of 6.626×10^{-34} J s.

If light radiation is governed by the wave equation for velocity, c, frequency, f, and wavelength, λ, namely $c = f\lambda$, or $f = \frac{c}{\lambda}$, then light quanta will have energy given by the **Planck equation**:

$$E = hf$$
$$= \frac{hc}{\lambda}$$

where E is the energy of the light quanta (the photon), h is Planck's constant, f is the frequency of the light, c is the velocity of electromagnetic radiation (light) and equals 3×10^8 m s^{-1}, and λ is the wavelength of the light.

Black-body radiation provides evidence that electromagnetic radiation is **quantised** into **discrete** values.

As the Planck constant is extremely small in magnitude, energy quanta are not noticeable in most everyday circumstances. A typical light source such as an incandescent bulb releases millions of quanta per second.

With this idea in place, Planck was able to describe the reason for the absence of high-energy emissions from black bodies. The vibrating atoms were simply not large enough to provide the necessary energy changes. Also, certain states of vibration of the atoms were more likely and this accounted for the peak in the frequency distribution curves. As you will see later, Planck's idea that the whole atom vibrates is, in fact, not quite correct. Energy emissions are due to electron movements (transitions) within the atom. Quantum theory today shows that electrons in atoms can only move between defined energy levels within the atom. Planck himself did not have any evidence for energy quanta, but it was an excellent idea that perfectly described solutions to several problems in physics at the time. Not many physicists accepted Planck's interpretation as it didn't relate to classical physics. However, Albert Einstein did and used it as the basis for a revolution in physics. Planck's work is regarded as the beginning of modern quantum physics and he received the 1918 Nobel Prize in Physics.

quantised
form into quanta with certain discrete energies

discrete
individually separate and distinct

CHALLENGE 11.4A

Energy per photon

Which laser light contains more energy per photon: a red laser ($\lambda = 635$ nm) or a blue laser ($\lambda = 442$ nm)?

CHALLENGE 11.4B

Can you shut a door with photons?

It is said you can shut a door by firing photons at it. If it takes 5 J to close a door, how many photons of wavelength 635 nm are needed?

Study tip

This will give you a feeling for the energy of a photon. The number of photons streaming from a fluorescent lamp in your lab is about 10^{18} per second. However, the dark-adapted eye (stand in complete darkness for a few minutes) can see a single photon.

The electron volt

The joule as a unit of energy is useful in thermodynamics where thousands of joules are needed to heat a beaker of water, or megajoules to boil water. But for binding energy in nuclear physics and black-body radiation in quantum mechanics the values are very small, such as 10^{-19} J, and it becomes tedious to have to talk in those amounts. Physicists introduced a new unit for energy – the **electron volt (eV)** – which is the energy needed to accelerate an electron through an electrical potential difference of 1 volt. The formula $W = qV$ tells you how much energy that is. The charge on an electron is 1.60×10^{-19} C, so $W = 1.60 \times 10^{-19} \times 1 = 1.60 \times 10^{-19}$ J.

$$1 \text{ electron volt (eV)} = 1.60 \times 10^{-19} \text{ J}$$

The prefixes keV (kilo, k = 10^3 eV) and MeV (mega, M = 10^6 eV) are used.

electron volt (eV)
a unit of energy equal to the work done on an electron in accelerating it through an electrical potential difference of 1 volt (unit: eV); equivalent to 1.60×10^{-19} J

WORKED EXAMPLE 11.4A

Calculate the energy, in J and eV, of a quantum (photon) of violet electromagnetic radiation of wavelength 450 nm. Give your answer to 2 significant figures.

SOLUTION

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{450 \times 10^{-9}} \\ &= 4.417 \times 10^{-19}\ \text{J} \\ &= \frac{4.417 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} \\ &= 2.8\ \text{eV (2 sf)} \end{aligned}$$

CHECK YOUR LEARNING 11.4

Describe and explain

1 **Describe** the problems with Wien's theory of black-body radiation and **explain** how Planck's model overcame these.

2 **Clarify** what experimental evidence Planck had for the existence of quanta.

Apply, analyse and interpret

3 **Determine** the energy in joules of a 2.49 eV quantum of green light.

4 **Determine** the energy in both joules and eV for a single quantum of green light of wavelength 500 nm.

5 A particular light wave has a frequency of 4.3×10^{14} Hz.
   - **a** **Determine** its wavelength.
   - **b** **Determine** the energy of one quantum.
   - **c** **Deduce** what colour it would appear to our eyes.

6 Using Planck's formula **determine**:
   - **a** the energy in eV of a quantum (photon) of light of wavelength 750 nm
   - **b** the wavelength of X-ray quanta of energy 3 keV.

7 **Deduce** whether red or blue light has more energetic quanta (photons).

Investigate, evaluate and communicate

8 **Predict** the spectral region (UV, gamma, etc.) to which the following quanta belong.
   - **a** 0.50 eV
   - **b** 100 eV
   - **c** 20 keV

9 **Investigate** information that enables you to discuss the term 'ultraviolet catastrophe' in relation to quantum theory.

Check your obook assess for these additional resources and more:

- » Student book questions Check your learning 11.4
- » Challenge worksheet 11.4A Energy per photon
- » Challenge worksheet 11.4B Can you shut a door with photons?
- » Weblink Planck's constant

11.5 The photoelectric effect

KEY IDEAS

In this section, you will learn about:

- the photoelectric effect in terms of the photon
- the terms 'threshold frequency', 'Planck's constant' and 'work function'
- solving problems involving the photoelectric effect.

photoelectric effect
the emission of electrons (or other free carriers) when light shines on a material

In the 1880s, the German physicist Heinrich Hertz (1857–1894) attempted the experimental verification of an aspect of Maxwell's proposals on electromagnetism. He was trying to produce electromagnetic waves that would travel through space to the other side of the lab and induce a spark in an induction coil. Not only did Hertz get sparks, but he also noticed that when exposed to UV light the sparks got bigger. He called this phenomenon the **photoelectric effect**. However, as the structure of the atom was not known at the time, the explanations just concluded that the energy originates inside the atom.

FIGURE 1 Heinrich Hertz 1894

Photoelectric experiments

Study tip

Try the demonstration in Figure 2 in your lab. It will help you appreciate the meaning of the photoelectric effect. UV bulbs are sold as UV aquarium lights, or UV LED Fishing Black Torches.

A simple demonstration to Physics students at the time of Hertz's experiments was to shine different lights on a negatively charged electroscope that had a square of zinc metal sitting on the top (Figure 2). With ordinary incandescent light from a light bulb there was no change, but when ultraviolet light was shone on the electroscope the leaves collapsed, indicating that the zinc was losing electrons and they were being pulled from the electroscope.

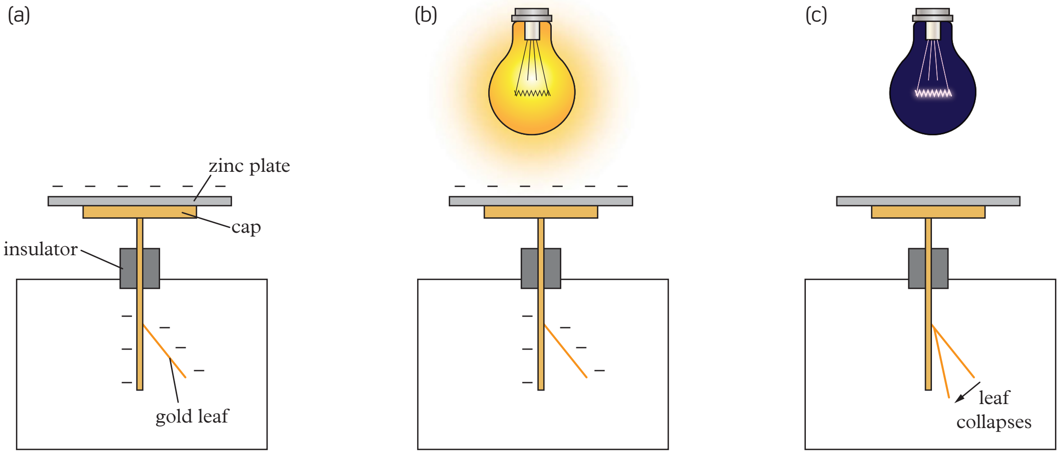

FIGURE 2 Typical demonstration of the photoelectric effect. (a) The electroscope is negatively charged with the gold leaf extended. (b) In the presence of incandescent bulb there is no change. (c) When a UV light is introduced, the gold leaf collapses indicating loss of the negative charge.

MANDATORY PRACTICAL
11.5 The photoelectric effect
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anode
the positive terminal, which attracts electrons

cathode
the negative electrode from which electrons are emitted

In 1902, one of Hertz's students, Philipp Lenard (1862–1947), constructed a glass tube with a metal plate at each end (Figure 3). He removed the air to create an evacuated chamber so there was nothing, not even air, between the two metal plates, and connected a battery. The metal plate connected to the positive terminal was termed the **anode**, and the plate connected to the negative terminal termed the **cathode**. The tube itself was termed a cathode ray tube. No electron current was registered on the ammeter, as would be expected since a vacuum is a poor conductor.

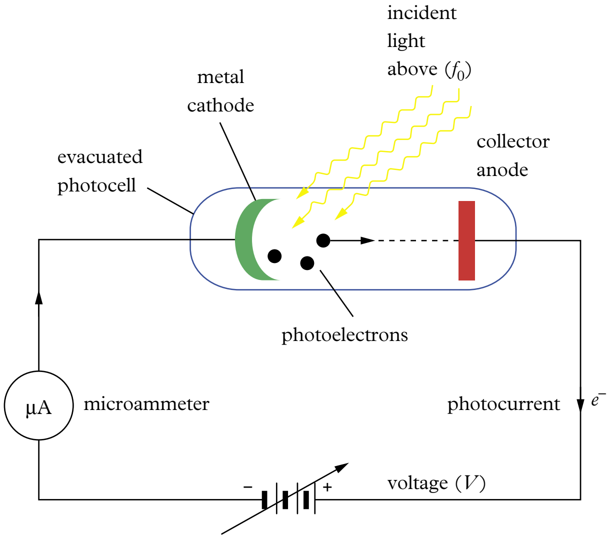


FIGURE 3 Lenard's photoelectric apparatus

threshold frequency
the minimum frequency of a photon that can eject an electron from a surface

Experiment 1: Varying the frequency

Lenard shone light on the metal in the cathode ray tube and varied the frequency. Nothing happened until the frequency was in the ultraviolet region, then a current started to flow. The UV light had caused electrons to be ejected from the metal and pass through the vacuum to the anode. He called this the **threshold frequency**, f_0. As he increased the frequency past this minimum, the electrons became faster and faster with more and more kinetic energy.

Lenard then repeated the experiment with increased intensity (brightness). Again, nothing happened until he reached the threshold frequency. However, once the threshold was reached, the more intense the light the bigger the current. He then tried different metals and found different threshold frequencies for each. This is a summary of his findings:

- Electrons were ejected from the metal only if the frequency of the incident light exceeded a minimum value called the threshold frequency, f_0.
- The threshold frequency was different for various metals.
- If the frequency was below the threshold value, electrons were not ejected and there was no flow of photocurrent, not even when very intense light was used.
- Once a photocurrent is registered, increasing the incident light intensity increased the amount of photocurrent flowing.
- Light of a frequency higher than the threshold increased the kinetic energy of the ejected electrons.

Study tip

If you study Chemistry as well as Physics, it's easy to get these terms mixed up. For Chemistry, in a voltaic cell the anode is negative and the cathode is positive. However, in an electrolytic cell, the anode is positive while the cathode is negative. Keep the Chemistry and Physics definitions separate. Remember – in Physics the cathode is negative and the anode is positive.

Experiment 2: Varying the voltage

Lenard then experimented with the voltage. He left the light shining on the tube at a frequency greater than threshold so that a current was flowing. When he increased the voltage, *V*, the current didn't change. He reversed the battery and made the anode negative with respect to the cathode. Thus, the voltage was now negative. This was causing the anode to repel the ejected electrons. Lenard kept increasing the voltage in the negative direction and noticed that the current decreased. At a certain voltage the current reached zero. The reverse voltage was now large enough to stop the photocurrent completely. He called this the **stopping potential**, also known as the reverse cut-off voltage, V_c. In symbol form: $V = -V_c$. This behaviour is shown in Figure 4a.

stopping potential the negative potential on the collector at which the photoelectric current becomes zero

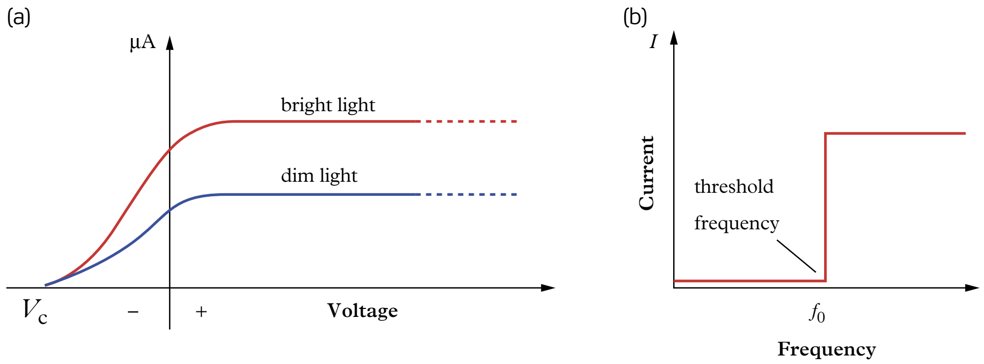


FIGURE 4 Results of Lenard's photoelectric experiment

Each of these experimental observations was impossible to explain using conventional wave theories of light.

A youthful Albert Einstein wondered how it could be that light, which is considered a wave, could interact with an atom that exists at only a point. He took the quantum idea of Planck seriously. His thoughts along these lines culminated in his famous paper of 1905. In it, Einstein made one of the most revolutionary statements in the history of physics:

> '…the energy…consists of a finite number of energy quanta which … can only be produced and absorbed as complete units.'

Energy quanta eventually became known as **photons**. Einstein predicted that his light quanta each had energy that was a product of the frequency, *f*, and Planck's constant, *h*.

photon a quantum of all forms of electromagnetic radiation

CHALLENGE 11.5A

Origin of the name photon

The photon was named by US physicist Gilbert Lewis in 1926 using the Greek *phos*, meaning 'light'. Verify the following:

a All words beginning with the prefix *phos* are related to the concept of light.

b The planet Venus used to be called Phosphor when appearing as the morning star.

Work function and kinetic energy

Einstein assumed that the light quanta, now called photons, interacted with the surface electrons in the metal so that a single photon ejects a single electron. The photon will give either all of its energy to the electron or none of it. Each electron can only absorb the energy of one photon and the collision interactions between photons and electrons in the metal are totally elastic and obey the law of conservation of energy. Einstein defined three forms of energy in the system, namely:

- photon energy, $E = hf$, which is frequency dependent
- **work function** or energy of binding of the electron to the metal, which is measured as $W = hf_0$, where f_0 is the threshold frequency
- maximum kinetic energy of the electron, E_k, of the ejected electrons from the metal surface.

work function
the minimum energy required to remove an electron from a solid (symbol: *W*; SI unit: joule; unit symbol: J). Formula: $W = hf_0$

Einstein's photoelectric equation relates these energy values together such that conservation occurs, namely:

$$E_k = hf - W$$

or

$$\begin{aligned} E_k &= hf - hf_0 \\ &= qV_c \\ \frac{1}{2}mv^2 &= hf - hf_0 \\ &= qV_c \end{aligned}$$

where V_c is the (reverse) cut-off voltage necessary to reduce the flowing photocurrent to zero, and v is the velocity of the ejected electron. It is also called the stopping voltage, V_s.

Figure 5 represents a typical set of graphs obtained from photoelectric experiments carried out on various metals (see Table 1 for some work functions). Notice that the gradient of the straight lines of the graph are equal to Planck's constant. This experimental determination of h was first performed by Robert Millikan in 1916. It can also be shown that the intercept on the x-axis is the threshold frequency (f_0), and the intercept on the y-axis is the work function W (stated as a positive value).

TABLE 1 Work functions for some common metals

Metal	W (eV)
Rubidium	2.05
Sodium	2.36
Calcium	2.87
Zinc	3.63
Aluminium	4.28
Copper	4.64
Gold	5.10
Platinum	5.63

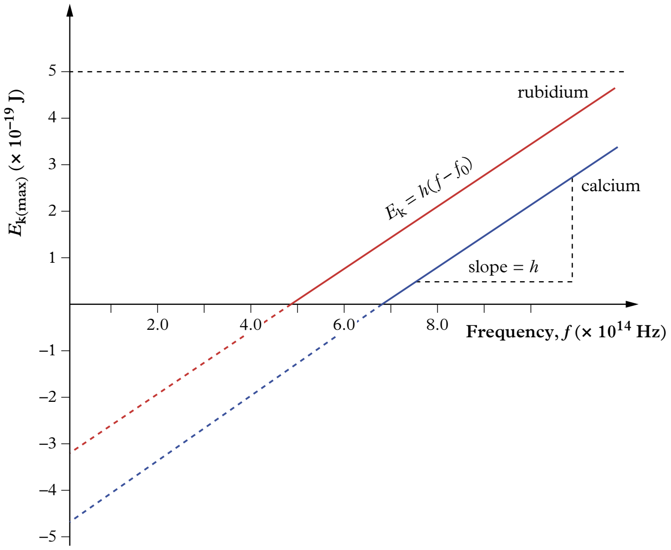


FIGURE 5 Results from photoelectric experiments: E_k versus frequency

Study tip

The type of graph in Figure 5 almost always appears in external exams, often in multiple-choice questions. Try to familiarise yourself with it. Learn that: gradient = h, x-intercept = f_0, y-intercept = W.

WORKED EXAMPLE 11.5A

The threshold frequency for a metal illuminated by violet light is 6.93×10^{14} Hz.

a Determine the work function, W.

b Identify the likely metal from the values in Table 1.

SOLUTION

a $f_0 = 6.93 \times 10^{14}$ Hz

$$W = hf_0 = 6.626 \times 10^{-34} \times 6.93 \times 10^{14}$$
$$= 4.59 \times 10^{-19}\ \text{J}$$
$$= \frac{4.59 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = 2.87\ \text{eV}$$

b The metal is mostly likely calcium ($W = 2.87$ eV).

WORKED EXAMPLE 11.5B

A barium metal cathode is illuminated with 420 nm violet light. Determine:

a the maximum kinetic energy, E_k

b the velocity of a photoelectron if the work function is 2.52 eV. The mass of an electron, $m_e = 9.1{\times}10^{-31}$ kg.

SOLUTION

$W = 2.52$ eV; $\lambda = 420$ nm

This problem can be done in eV or in joules.

In joules:

a $1\ \text{eV} = 1.60 \times 10^{-19}$ J

$$W = 2.52\ \text{eV}$$
$$= 2.52 \times 1.60 \times 10^{-19}$$
$$= 4.03 \times 10^{-19}\ \text{J}$$

$$E_k = hf - W$$
$$= \frac{hc}{\lambda} - W$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{420 \times 10^{-9}}\ \text{J} - 4.03 \times 10^{-19}\ \text{J}$$
$$= 4.73 \times 10^{-19}\ \text{J} - 4.03 \times 10^{-19}\ \text{J}$$
$$= 0.70 \times 10^{-19}\ \text{J}$$

b

$$E_k = \frac{1}{2}mv^2$$
$$0.70 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31}\,v^2$$
$$v^2 = \frac{2 \times 0.70 \times 10^{-19}}{9.1 \times 10^{-31}}$$
$$= 1.54 \times 10^{11}$$
$$v = 3.9 \times 10^{5}\ \text{m s}^{-1}$$

In electron-volts, eV:

a $E_k = hf - W$

$$= \frac{hc}{\lambda} - W \text{ (convert J to eV by dividing by } 1.6 \times 10^{-19} \text{ J eV}^{-1}\text{)}$$
$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{420 \times 10^{-9} \times 1.60 \times 10^{-19}} \text{ eV} - 2.52 \text{ eV}$$
$$= 2.96 \text{ eV} - 2.52 \text{ eV}$$
$$= 0.44 \text{ eV}$$

b Convert 0.44 eV to J.

$$E_k = \frac{1}{2}mv^2$$
$$0.44 \times 1.60 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} v^2$$
$$0.704 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} v^2$$
$$v^2 = \frac{2 \times 0.704 \times 10^{-19}}{9.1 \times 10^{-31}}$$
$$= 1.547 \times 10^{11}$$
$$v = 3.9 \times 10^{5} \text{ m s}^{-1}$$

Applications of the photoelectric effect

A familiar use for photoelectricity is in light-sensitive detector circuits, and light-activated switching circuits.

If the incoming photons make the electrons more mobile without ejecting them, the conductivity of the material varies with light intensity. This is a type of photoelectric device called a photoconductor. For example, inexpensive cadmium sulfide cells (Figure 6) can be found in many consumer items such as camera light meters, clock radios, alarm devices (as the detector for a light beam), nightlights, outdoor clocks, solar street lamps. Photoelectric devices can be placed in streetlights to control when the light is on. Daylight keeps the photoelectric device producing current and this keeps the light switch turned off so street lights are off. When it gets dark, the photoelectricity stops and this triggers the switch to turn the light on.

Study tip

A good way to appreciate the photoelectricity is to measure the resistance of an LDR in darkness and in bright light. You'll see that it conducts well when light is shining on it (and knocks out electrons).

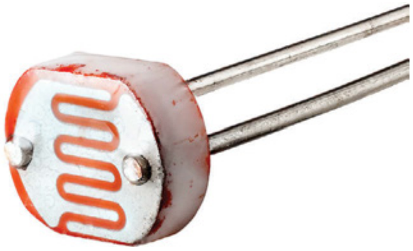


FIGURE 6 A light-dependent resistor (LDR) is a photoelectric device that uses cadmium sulfide as the photon absorber.

CHALLENGE 11.5B

Power per photon

In a dark room let your eyes become dark-adapted. Give your eyelid a sharp tap with your finger and you should see a flash. A single rod (light receptor in your eye) will detect a single photon, but your visual system only responds when between two and ten photons are absorbed by your rods within 0.1 seconds; you will then see the flash. Estimate how much power two visible photons will give to your eyes in 0.1 seconds.

CHECK YOUR LEARNING 11.5

Describe and explain

1 **Explain** the difference between frequency and threshold frequency.

2 **Define** the terms 'Planck's constant' and 'work function'.

3 **Describe** a photon.

Apply, analyse and interpret

4 **Determine** the work function of a particular metal if photoelectrons are produced when the metal is illuminated with light of wavelength of less than 400 nm.

5 **Determine** the wavelength of light that produces electrons with a maximum kinetic energy of 1.32 eV from a copper surface.

6 Rubidium metal, which has a threshold frequency of 5.0×10^{14} Hz, is illuminated by light photons of frequency 8.1×10^{14} Hz.

  a **Determine** the value of the work function.

  b **Determine** the maximum velocity of the photoelectrons ($m_e = 9.11 \times 10^{-31}$ kg).

7 **Deduce** how the graph in Figure 7 would appear if classical physics was a good model for the photoelectric effect. Explain your reasoning.

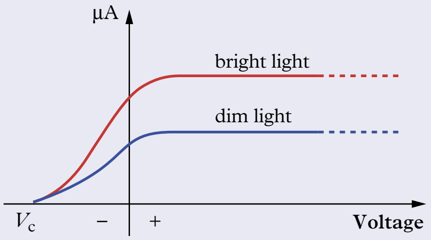


FIGURE 7 Photocurrent and stopping voltage

8 **Determine** whether light of frequency 5.25×10^{14} Hz would allow for the photoelectric effect to be demonstrated in rubidium and sodium metals.

Investigate, evaluate and communicate

9 **Predict** the order of the stopping potentials for the following metals A, B, and C. Base your response on their work functions of 2.0 eV, 2.5 eV and 3.0 eV respectively, when UV light is shone on them and they produce photoelectrons.

10 A photoelectric effect experiment was performed in which a metal surface was illuminated by light of varying wavelengths and the photoelectric current measured. For each metal the threshold wavelength was noted. The results were as follows (Table 2):

TABLE 2 Threshold wavelengths

Metal target	Threshold wavelength (nm)
Copper	264
Platinum	197
Calcium	428

  a **Determine**, for each metal, (i) the threshold frequency in Hz; (ii) the work function in J; (iii) the work function in eV.

  b **Assess** the percentage error in the work function for each metal by comparing it to the accepted value. The accepted value for platinum is 6.35 eV.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 11.5
- » Mandatory practical worksheet
  11.5 The photoelectric effect
- » Challenge worksheet
  11.5A Origin of the name photon
- » Challenge worksheet
  11.5B Power per photon

11.6 The Compton effect and momentum

KEY IDEAS

In this section, you will learn about:

- the wave and particle characteristics of photons.

Even though Einstein had received the Nobel Prize in 1922, physicists did not accept his 1905 photon concept. However, further evidence for the particle nature of electromagnetic radiation came in 1923 with the discovery of X-ray scattering by Arthur Holly Compton (1892–1962). Compton used the apparatus shown in Figure 1 to show that the X-ray photons behaved like particles with definite **momentum** characteristics. The X-rays collided with the electrons in the graphite target. After the collision, the scattered X-ray photons had reduced energy and longer wavelengths than the photons that had not been scattered. In a Compton collision between an X-ray photon and an electron, the change in energy is not complete and the reduction in energy and wavelength is dependent on the angle of scattering. The electron involved is scattered or ejected from the graphite in such a way that both energy and momentum are conserved in the collision. Remember that momentum is a vector quantity. Compton's collision calculations correctly predicted the speed and direction of the recoil electrons. For this work, as well as further X-ray spectra analysis, Compton shared the 1927 Nobel Prize in Physics with British physicist Charles Wilson.

momentum
the product of an object's mass and its velocity (Newtonian); it is a vector quantity and is conserved in interactions (symbol: *p*; unit symbol: kg m s^{-1})

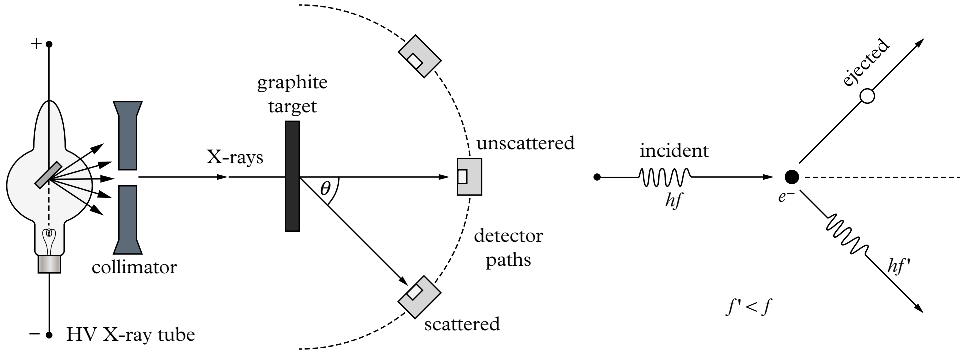


FIGURE 1 Compton's photon momentum apparatus

Compton combined the two ideas: the mass–energy equivalence relationship $E = mc^2$, and Planck's quantum relationship $E = hf$, to derive the photon's momentum:

$$\begin{aligned} E &= mc^2 \\ &= hf \\ mc^2 &= \frac{hc}{\lambda} \text{ (because } f = \frac{c}{\lambda}\text{)} \\ mc &= \frac{h}{\lambda} \end{aligned}$$

But *mc* is the definition of a particle's momentum, *p*, hence:

$$p = \frac{h}{\lambda} \text{ (the photon's momentum)}$$

$$\lambda = \frac{h}{p} \text{ (the photon's wavelength)}$$

WORKED EXAMPLE 11.6A

A source of green light has a frequency of 560 THz (tera, T = 10^{12}).

a Calculate the momentum of a photon. **b** Determine the speed of a photon.

SOLUTION

a
$$\lambda = \frac{c}{f}$$
$$= \frac{3 \times 10^{8}}{560 \times 10^{12}}$$
$$= 5.357 \times 10^{-7}\,\text{m}$$
$$p = \frac{h}{\lambda}$$
$$= \frac{6.626 \times 10^{-34}}{5.357 \times 10^{-7}}$$
$$= 1.2 \times 10^{-27}\ \text{kg m s}^{-1}\ (2\ \text{sf})$$

b The speed will be 3×10^{8} m s^{-1} (all photons travel at *c*).

Wave–particle duality

Einstein was awarded the Nobel Prize in Physics in 1921 for his explanation of the photoelectric effect. Although Planck was the first to suggest the idea of quantisation, Einstein showed that energy is quantised and that light comes in particle-like packets of energy later called photons. Einstein acknowledged that there was a conflict here: if light is made of particle-like photons then how can they exhibit wave-like properties in Young's double slit experiment?

A classical (Newtonian) particle, when faced with Young's double slit would have to go through one slit or the other. If light consisted of classical particles, you would see two bright spots on the screen – but instead you see an interference pattern. Einstein proposed that a photon must somehow go through both slits and interfere with itself. Photons thus have both wave-like and particle-like characteristics at the same time. When its properties are tested, it behaves either as a wave or as a particle depending on the apparatus used for the test. It is the testing that forces light to adopt either of its two personalities as wave or particle. This is called the **wave–particle duality** of light.

wave–particle duality
every particle or quantum entity may be partly described in terms not only of particles, but also of waves

Study tip

Think of light this way: it is both a wave and a particle until you observe it – then it becomes one or the other.

CHECK YOUR LEARNING 11.6

Describe and explain

1 **Describe** the wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.

Apply, analyse and interpret

2 Red light from a source has a frequency of 460 THz (tera, T = 10^{12}).
  a **Calculate** the momentum of a photon.
  b **Determine** the speed of a photon.

3 A photon has a momentum of 1.5×10^{-27} kg m s^{-1}.
  a **Calculate** its frequency in terahertz.
  b **Determine** its wavelength in nanometres.

Investigate, evaluate and communicate

4 A claim is made that the double slit interference of light supports the wave model of light. **Evaluate** this claim by referring to the evidence.

5 **Propose** whether it fair to say that Compton scattering between photons and electrons is like billiard balls colliding.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 11.6
- » Increase your knowledge
  Beyond violet
- » Video
  Calculating momentum

CHAPTER 11

Review

Summary

11.1
- Wien's displacement law shows the maximum wavelength of black-body radiation at various temperatures.

11.2
- Max Planck postulated the quantum nature of energy absorbed and released by atoms.

11.3
- Earth's energy balance is explained by the greenhouse effect and black-body radiation.

11.4
- Electromagnetic energy quanta are called photons and possess energy directly related to their frequency, $E = hf$.

11.5
- The photoelectric effect is the emission of electrons by metals illuminated by light. Its correct interpretation, in terms of quantum theory, was described by Albert Einstein.

11.6
- Einstein's interpretation of the photoelectric effect is an extension of the law of conservation of energy.
- Photons and electrons may interact by Compton scattering, with the photon being considered to have particle momentum.
- Light exhibits wave–particle duality, with Young's double slit experiment supporting the wave model, and with the photoelectric effect and Compton scattering supporting the particle model.

Key terms

- albedo
- anode
- black body
- black-body radiation
- cathode
- coherent
- discrete
- electric charge
- electric field
- electromagnetic radiation
- electromagnetic wave
- electron volt (eV)
- energy
- frequency
- greenhouse effect
- magnetic field
- momentum
- photoelectric effect
- photon
- Planck equation
- Planck's constant
- quanta
- quantised
- Stefan–Boltzmann law
- stopping potential
- threshold frequency
- wave model of light
- wavelength
- wave–particle duality
- Wien's displacement law
- work function
- Young's double slit experiment

Key formulas

Wien's displacement law	$\lambda_{\text{max}} = \frac{b}{T}$
Black-body radiation	$E = hf = \frac{hc}{\lambda}$
Kinetic energy and work function	$E_k = hf - W$
Photon momentum	$\lambda = \frac{h}{p}$

Key constants

Electron-volt	1 eV = 1.60×10^{-19} J
Elementary charge	$q_e = 1.60 \times 10^{-19}$ C
Planck's constant	$h = 6.626 \times 10^{-34}$ J s
Speed of light in a vacuum	$c = 3 \times 10^8$ m s^{-1}
Wien's displacement constant	$b = 2.898 \times 10^{-3}$ m K

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 An aluminium cathode is illuminated with ultraviolet light in a photoelectric experiment. Figure 1 is a graph of the stopping potential versus frequency. Planck's constant can be determined by multiplying the charge on an electron by:

A the gradient.
B the *x*-intercept.
C the *y*-intercept.
D the inverse of the gradient.

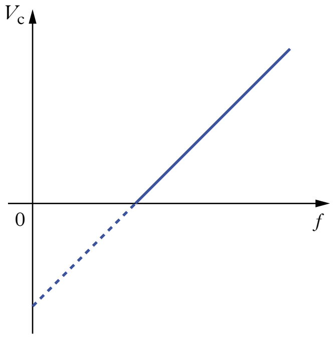


FIGURE 1 Stopping potential versus frequency

2 A photon has:

A no momentum and no mass.
B momentum but no mass.
C momentum and mass.
D speed and mass.

3 A metal is illuminated with light of increasing frequency and at a particular minimum value electrons start being ejected. If the work function is given the symbol *W*, which one of the following best describes the maximum kinetic energy of the electrons:

A $hf - W$
B $hf + W$
C $\frac{W}{hf}$
D $h(f - W)$

4 When a metal is illuminated by light, electrons can only be photo-emitted if the light:

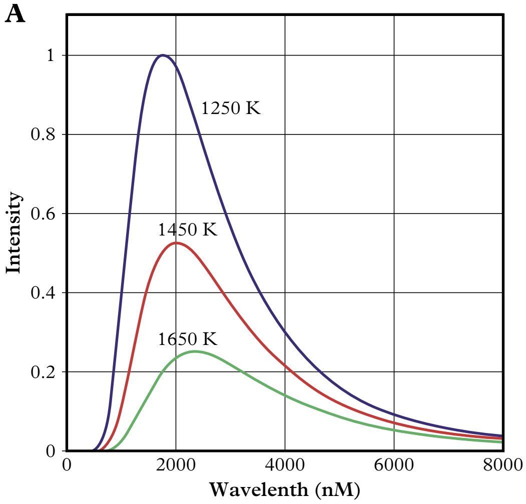
A is in the visible region of the spectrum.
B has a frequency in the UV or X-ray range.
C has a certain minimum wavelength.
D has a certain minimum frequency.

5 Light is described as an electromagnetic wave produced by an oscillating charge that generates:

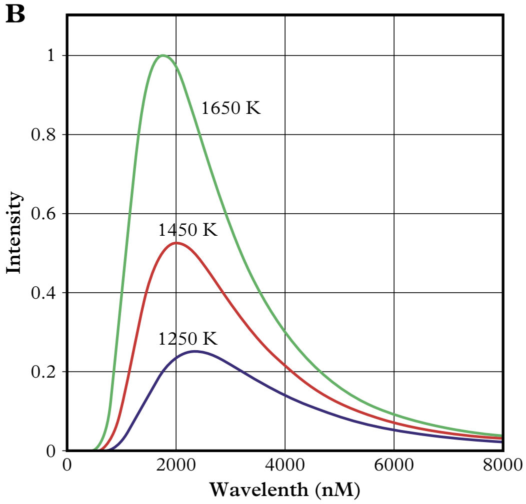
A photons that travel at the speed of light.
B mutually perpendicular oscillating magnetic and electric fields.
C electric and magnetic fields 90° out of phase to each other.
D fields that oscillate parallel to the direction of propagation.

6 Select the diagram bvelow that best represents the distribution of energies of a black-body radiator.

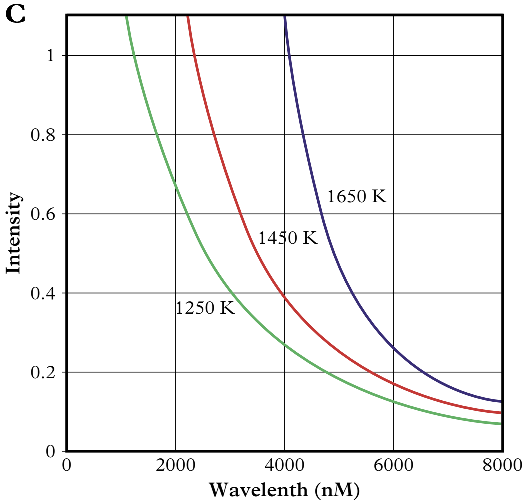
A



B



C



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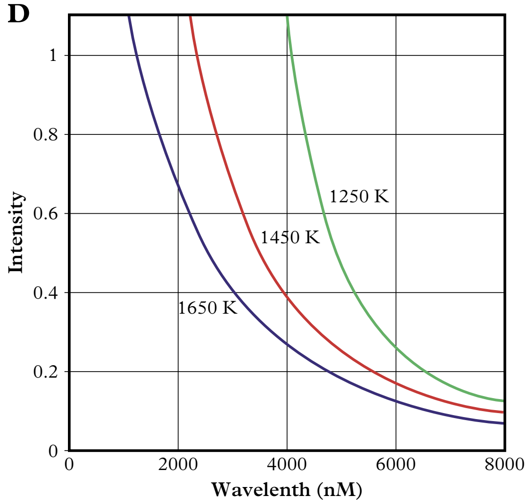


7 Wien's law predicts that for a black-body radiator, the frequency at which maximum radiation of energy occurs is proportional to:

A T^{-4}
B T^{-1}
C T
D T^4

8 The frequency of a photon with a wavelength of 550 nm is:

A 5.45×10^{-4} Hz
B 5.45×10^{5} Hz
C 5.45×10^{12} Hz
D 5.45×10^{14} Hz

9 The energy of a photon with a wavelength of 550 nm is:

A 0.1 eV
B 1.0 eV
C 2.3 eV
D 3.6 eV

10 Figure 2 shows the photoelectric results for metals A and B. Select the statement that is true about the graph in Figure 2.

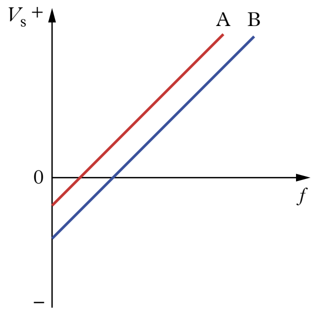


FIGURE 2 Work function graph

A There is significant systematic error as the lines don't pass through the origin.

B Metals A and B have the same gradient so are likely to be the same metal.

C Metal A intercepts the x-axis earlier than metal B so metal A has the smaller work function.

D The outer electron of metal A is less strongly held than the outer electron of metals B.

Short answer

Describe and explain

★ **11** **Explain** why it is necessary to have a coherent light source for Young's double slit experiment.

★ **12** **Explain** whether it is minimum energy or minimum frequency that determines if an electron is photo-ejected in the photoelectric effect on metals.

★ **13** **Clarify** whether the statement that 'hot' objects glow red, very hot glow blue' is true for Wien's law.

★ **14** **Clarify** whether Planck's relationship $E = hf$ applies to all transverse waves.

Apply, analyse and interpret

★ **15** **Determine** the temperature, in °C, of an object emitting light with a wavelength maximum of 15 cm.

★ **16** **Determine** the energy, in both joules and electron volts, of a photon with wavelength 5.5×10^{-7} m.

★ **17** **Determine** the wavelength of a 6 keV photon, and **identify** what spectral region it would be in.

★ **18** **Determine** the colour and wavelength (in nm) of light with a frequency of 500 THz ($T = 10^{12}$).

★ **19** **Determine** the energy in J and eV of a quanta of blue light with a wavelength of 450 nm.

★★ **20** **Determine** the momentum of a 22.00 cm wavelength photon.

★★ **21** The threshold frequency for a particular metal is 2.5×10^{14} Hz and light of frequency 6.0×10^{14} Hz falls onto the surface.

- **a** **Identify** the colour of the incident light.
- **b** **Determine** the energy of the incident photon.
- **c** **Apply** the work function equation to determine the work function of the metal.
- **d** **Determine** the maximum kinetic energy of the photoelectrons.
- **e** **Deduce** the maximum velocity of the photoelectrons.

★★ **22** A photon has a momentum of 4.00×10^{-29} kg m s^{-1}.

- **a** **Determine** its wavelength.
- **b** **Determine** its energy in eV.

★★ **23** A photon has a wavelength of 2.5 μm.

- **a** **Determine** its momentum.
- **b** **Determine** the velocity of an electron with the same momentum and state whether it is relativistic (i.e. $>0.1c$).
- **c** **Deduce** the kinetic energy of the electron.

★★★ **24** It is found that a neutron travelling at 1.98×10^4 m s^{-1} has the same energy as a light photon of frequency 5×10^{14} Hz. **Deduce** the mass of the neutron.

★★★ **25** A proton (mass 1.6727×10^{-27} kg) has the same momentum as a 100 keV photon. **Deduce**:

- **a** the velocity of the proton
- **b** the kinetic energy of the proton.

Investigate, evaluate and communicate

★★ **26** A photon has an energy of 100 keV.

- **a** **Determine** its momentum.
- **b** **Deduce** its speed.
- **c** **Predict** the equivalent velocity of a neutron with the same momentum.
- **d** **Determine** the neutron's kinetic energy in keV.

★★★ **27** Table 1 contains data obtained from a photoelectric experiment.

TABLE 1 Photoelectric data

f (× 10^{14}) (Hz)	3.75	4.5	5.5	7.0	8.0	8.9
$E_{k\,(max)}$ (eV)	0.50	0.80	1.02	1.75	2.3	2.5

- **a** **Construct** a graph of $E_{k\,(max)}$ (vertical axis) versus frequency.
- **b** **Propose** values for:
  - **i** Planck's constant (in J s)
  - **ii** the threshold frequency for the metal (in Hz)
  - **iii** the work function of the metal (in eV).

★★★ **28** The graph in Figure 3 is used to measure the work function of two unknown metals.

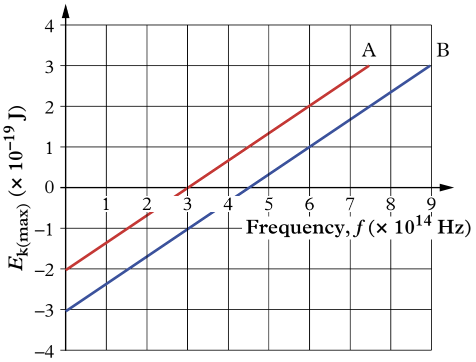
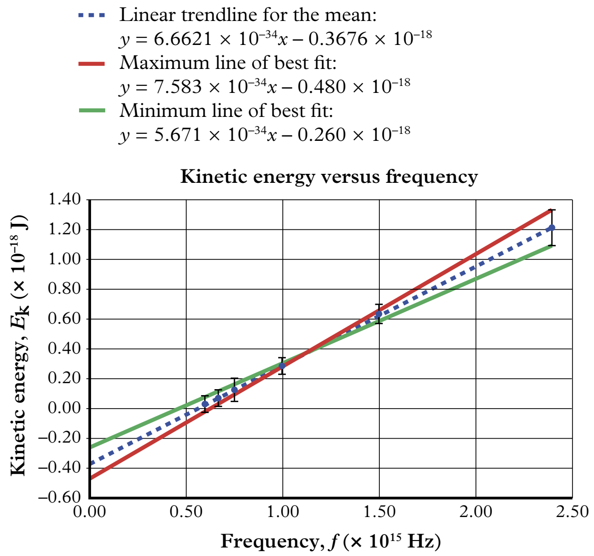


FIGURE 3 Work function data

- **a** **Deduce** the work function of each metal.
- **b** **Calculate** the value of Planck's constant.
- **c** **Predict** the shape of the curve for metal A if a less intense light was used.
- **d** **Propose** a reasoned interpretation of the data if the gradient for metal B was steeper than the gradient for metal A.

★★★ **29** Students conducted a photoelectric experiment to determine the kinetic energy being given to electrons in a sodium target by the incident photons of light. A set of parallel plates was set up in a vacuum. One plate consisted of sodium metal that would be exposed to different wavelengths or frequencies of light. The plates were also connected to a battery to provide a potential difference. If the light was able to eject electrons from the sodium, it would cause a current to flow in the circuit, as shown by an ammeter. The source of potential difference could be adjusted to 'stop' the current and cause the current to fall to zero.

A graph of kinetic energy versus frequency was plotted, as shown in Figure 4. A linear trendline was found and the formula determined. Error bars were added and maximum and minimum linear lines of best fit were added. Formulas for these lines were also determined. Note that all y-values are in joules. The trendlines are shown:



Linear trendline for the mean:
$y = 6.6621 \times 10^{-34}x - 0.3676 \times 10^{-18}$
Maximum line of best fit:
$y = 7.583 \times 10^{-34}x - 0.480 \times 10^{-18}$
Minimum line of best fit:
$y = 5.671 \times 10^{-34}x - 0.260 \times 10^{-18}$

FIGURE 4 Kinetic energy versus frequency

- **a** **Determine** the experimental value for Planck's constant, and its absolute and percentage uncertainty.
- **b** **Assess** the experimental value for Planck's constant for percentage error by comparing it to the accepted value.
- **c** **Determine** the experimental work function for sodium, and its absolute uncertainty.
- **d** **Assess** the results for percentage error in the work function by comparing the result to the accepted value of 2.36 eV.

★★★ **30** A photoelectric experiment similar to the one outlined in Question 29 provided the results listed in Table 2. A metal target was illuminated with light of wavelength from 150 nm to 250 nm, and the voltage required to stop any photocurrent was recorded for each trial wavelength.

TABLE 2 Stopping voltages

Wavelength, λ (nm)	Stopping voltage, ΔV (V)
150	4.00
175	2.85
200	1.90
225	1.20
250	0.65

a **Determine** the maximum kinetic energy of the electrons for each wavelength.

b **Construct** a graph of kinetic energy (vertical axis) versus frequency (horizontal axis).

c **Develop** a formula for the linear trendline.

d **Discuss** the meaning of the gradient, the x-intercept and the y-intercept.

e **Determine** the experimental value for Planck's constant.

f **Assess** the accuracy of the experiment for determining Planck's constant.

g **Determine** the experimental value for the work function of the metal (in eV).

h **Predict** which metal target may have been used given the data in Table 3.

TABLE 3

Metal	W (eV)
Rubidium	2.05
Sodium	2.36
Calcium	2.87
Zinc	3.63
Aluminium	4.28
Copper	4.64
Gold	5.10
Platinum	5.63

★★★ **31** A propulsion method suggested for interstellar travel is a large plastic sail that is bombarded by photons from the Sun. The argument is that if photons really do have momentum they will push the sail along. **Propose** whether this would work and deduce which would be a better surface coating: black paint or a shiny metallic foil (based on maximum transfer of momentum to the sail).

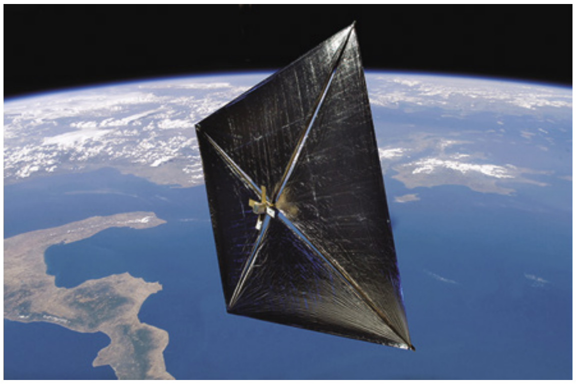


FIGURE 5 A solar sail

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CHAPTER 12

Quantum theory and matter

For much of 19th century, the atom was thought to be nothing more than a tiny indivisible sphere. But towards the end of the century and into the first half of 20th century various models were proposed and discarded as more evidence was obtained. By the mid-1900s, the model we have today was conceptualised by scientists by building on each other's work.

Although the idea of an atom goes back to the Ancient Greeks, only five basic atomic models have been proposed. Each model has contributed to how we perceive the structure of atom today. The first model was Dalton's billiard ball model in 1808, then in 1897 we had J.J. Thomson's 'plum pudding' model, followed by Rutherford's planetary model in 1898, Bohr's atomic model in 1913, and finally, in 1926, the quantum mechanical model.

This chapter describes how a series of discoveries in the fields of chemistry, electricity and magnetism, radioactivity, and quantum mechanics all contributed to the modern model.

OBJECTIVES

→ Describe Rutherford's model of the atom including its limitations.

→ Describe the Bohr model of the atom and how it addresses the limitations of Rutherford's model.

→ Explain how the Bohr model of the hydrogen atom integrates light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum.

→ Solve problems involving the line spectra of simple atoms using atomic energy states or atomic energy level diagrams.

→ Describe wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.

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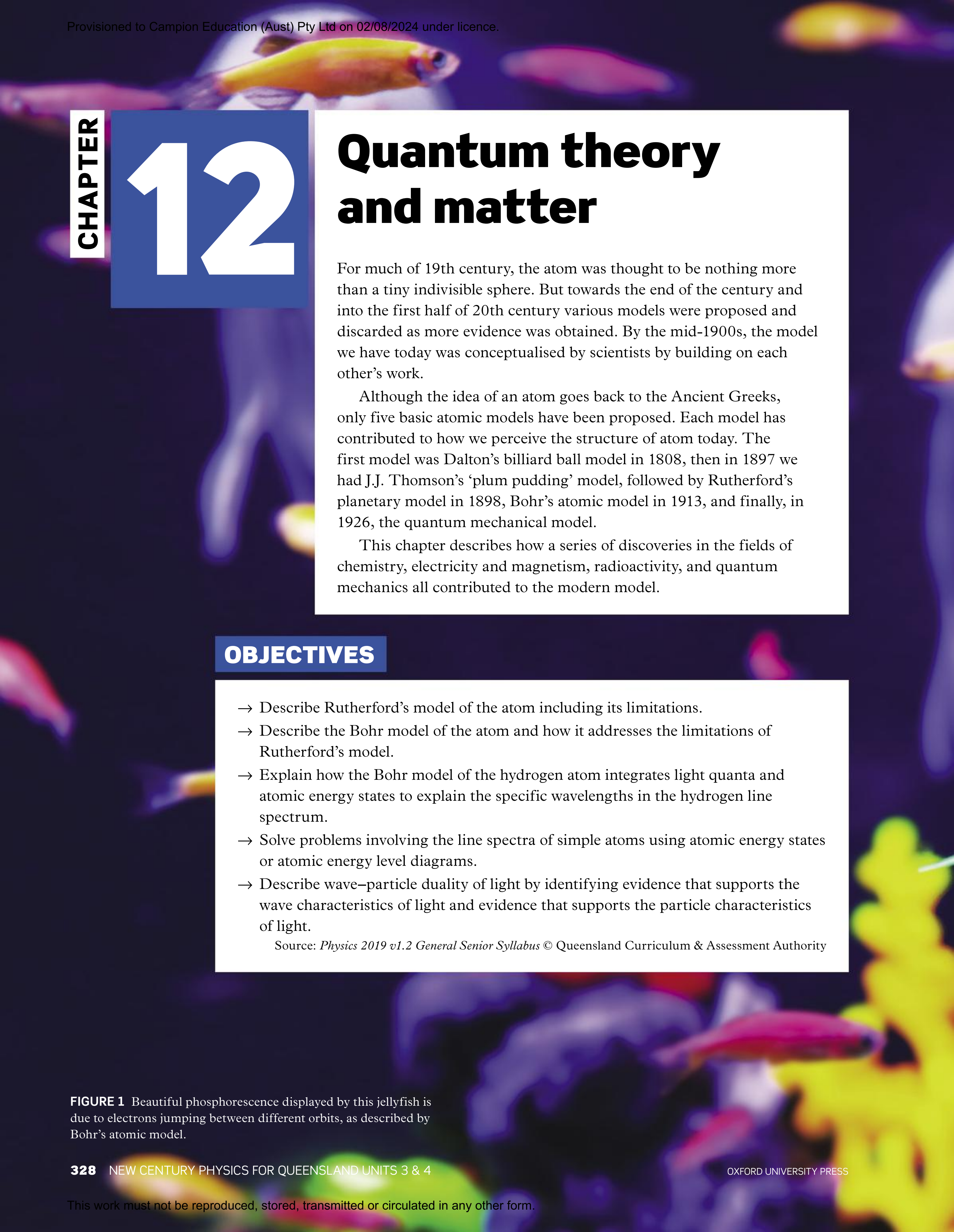
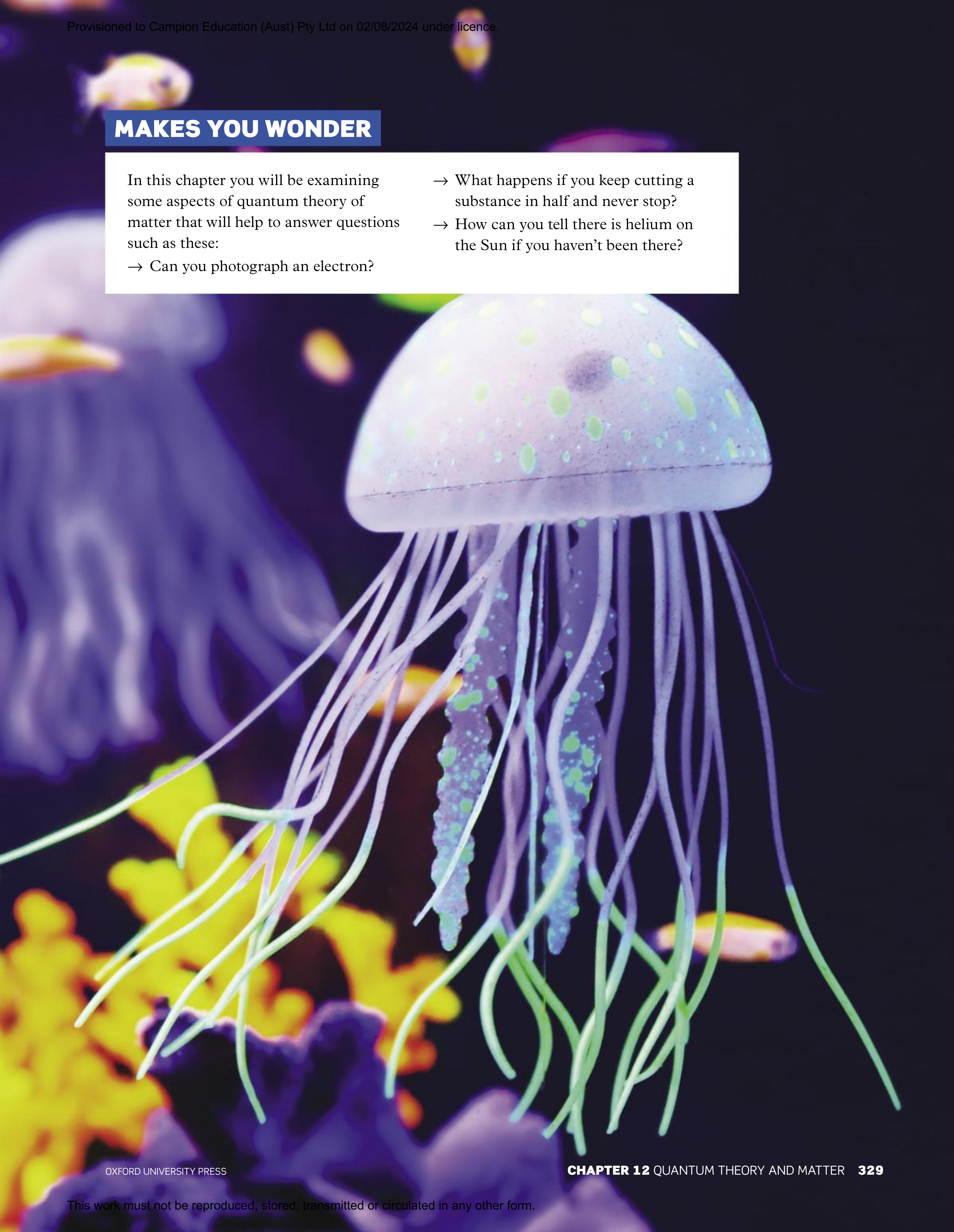


FIGURE 1 Beautiful phosphorescence displayed by this jellyfish is due to electrons jumping between different orbits, as described by Bohr's atomic model.



MAKES YOU WONDER

In this chapter you will be examining some aspects of quantum theory of matter that will help to answer questions such as these:

→ Can you photograph an electron?

→ What happens if you keep cutting a substance in half and never stop?

→ How can you tell there is helium on the Sun if you haven't been there?

12.1

Rutherford's model of the atom

KEY IDEAS

In this section, you will learn about:

- Rutherford's model of the atom (1898)
- the limitations of Rutherford's model.

In 1911, the New Zealand-born British physicist Ernest Rutherford (1871–1937) carried out one of the most important physics experiments into the structure of the atom of the 20th century.

He was aware of the so-called 'plum pudding model' developed by J.J. Thomson, which depicted the atom as consisting of tiny, negatively charged electrons embedded in a positively charged substrate 'dough', giving a neutral atom overall. Rutherford expected to find results consistent with Thomson's atomic model when he began a series of tests from 1908 onwards. He began by testing the passage of alpha rays through gold foil (Figure 1).

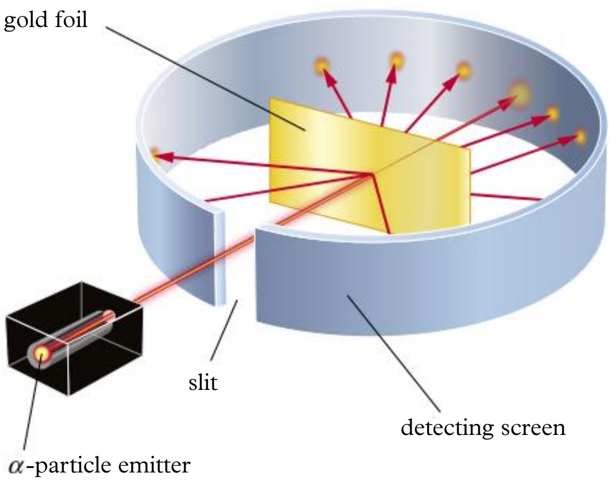


FIGURE 1 Schematic of Rutherford's gold foil apparatus. Note the big deflections for some particles.

Rutherford was inspired to look for alpha particles with very high deflection angles. He had his assistants Hans Geiger and Ernest Marsden perform the work under his guidance. Rutherford wasn't really expecting anything major, as he pictured the alpha bullets passing right through the 'plum pudding'. But huge deflections were found and it was his interpretation of this data that led him to formulate a new model of the atom – **Rutherford's model** – that a very small charged nucleus containing much of the atom's mass was orbited by low-mass electrons.

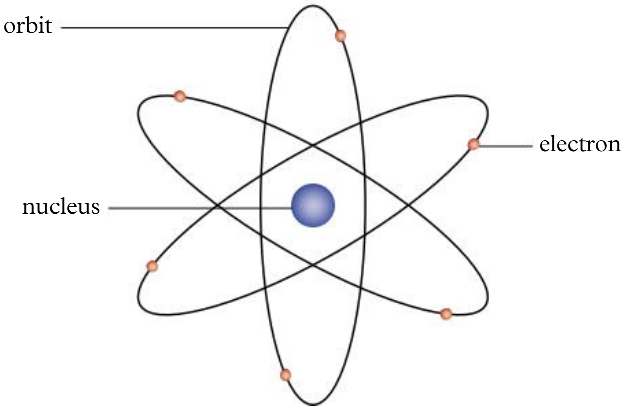


FIGURE 2 The Rutherford planetary model of the atom

Rutherford's model
a small, central positively charged nucleus with negatively charged electrons orbiting around it

Rutherford proposed the simplest atom of hydrogen as a single positive charge with a single negative electron circling it in planetary fashion (Figure 2). This atomic model had two serious flaws or 'limitations'. The first was that, according to the electromagnetic equations of Maxwell, any electron revolving in circular fashion around a nucleus is undergoing centripetal acceleration and should continuously radiate electromagnetic energy. The electron would therefore continuously lose energy, which cause it to spiral in towards the nucleus. These equations predicted that the Rutherford atom should be highly unstable and not exist for any length of time. If the electron was to lose its energy and collapse, then the atom should collapse in about 10^{-8} seconds. But this is not what happens: atoms are stable. This indicates that there is something wrong with the Rutherford nuclear model of the atom.

Atomic spectra

The second limitation of Rutherford's model was that it could not explain the line spectra of the hydrogen atom. If hydrogen gas is ionised by subjecting it to 20 000 V in a discharge tube, it emits a characteristic purple glow. When this glowing light is passed through a prism, it is split into four very distinct lines, as shown in Figure 3.

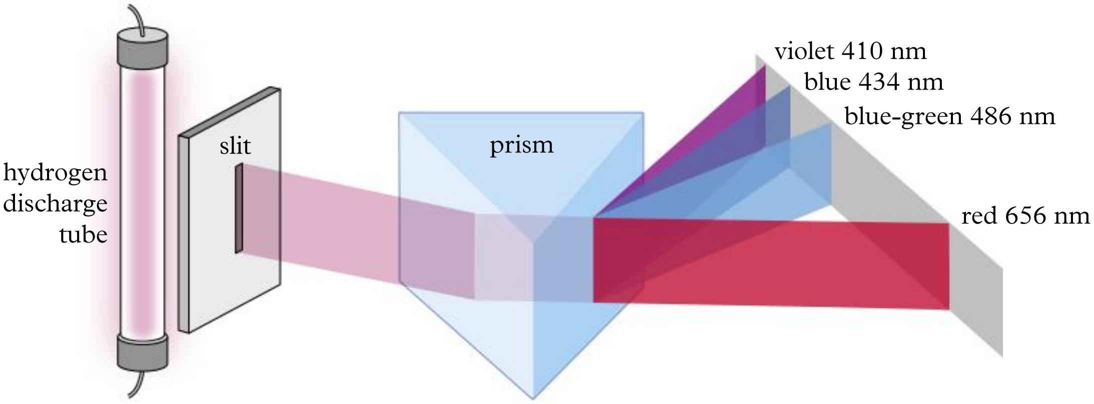


FIGURE 3 A prism is used to split up the different colours that make up the hydrogen emission spectrum. The wavelengths of the four lines are shown.

A second type of spectrum can also be produced. It is an absorption spectrum and is observed when white light is passed through cold atomic hydrogen gas. The gas removes some of the wavelengths from the white light and leaves blacks lines in their place in the spectrum. The black lines have the same wavelengths as the coloured lines of the emission spectrum.

This spectrum was not new. Spectroscopy was being used in analytical processes at the time to definitively identify individual atoms or molecules. The word 'spectrum' comes from the Latin *specere* meaning 'to look at', so scientists were looking at the light given off by heated or ionised samples of all sorts of materials. The first prism spectroscope (as shown in Figure 3) was designed by Gustav Kirchhoff and Robert Bunsen in 1859 while working on chemical analysis.

In 1885, Johann Balmer, a Swiss physicist, worked out a mathematical link between the wavelengths of the light colours emitted. His formula describes the four visible spectral lines of a hydrogen atom:

$$\lambda = K_B \frac{m^2}{m^2 - n^2} \quad \text{(with } n = 2\text{, and } m > n\text{)}$$

where λ is the wavelength of the absorbed or emitted light, K_B is Balmer's constant of 364.5 nm (the value for λ will also be in nm) and m and n are integers with $n = 2$, and $m > n$. These four lines are referred to as the 'Balmer series' (Figure 4).

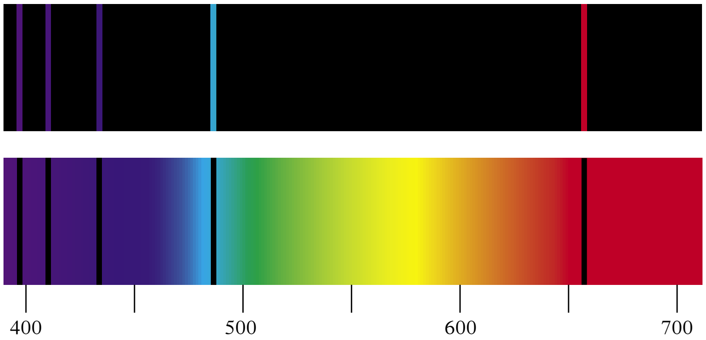


FIGURE 4 The hydrogen emission spectrum (top) and the absorption spectrum (bottom) show the four lines in the visible region, with the wavelength scale in nanometres along the bottom. The leftmost violet line is in the ultraviolet and not able to be seen with the eye.

Table 1 lists the wavelength and colour of the four emission lines for hydrogen.

TABLE 1 Wavelength and colour of the four emission lines of the Balmer series

	$m = 3$	$m = 4$	$m = 5$	$m = 6$
$n = 2$	656.3 nm	486.1 nm	434.1 nm	410.2 nm
Colour	Red	Aqua	Blue	Violet

In 1888, the Swedish physicist Johannes Rydberg (1854–1919) expanded Balmer's result in the Rydberg equation:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \text{ for } n = 3, 4, 5, \ldots$$

where λ is the wavelength of the absorbed or emitted light and R_H is the Rydberg constant (1.097×10^7 m^{-1}). No-one was able to suggest what these numbers meant in terms of hydrogen and they remained a puzzle. Rydberg found that the formula worked, not only for hydrogen, but for other atoms that were hydrogen-like; that is, atoms with just one electron: He^+, Li^{2+}, Be^{3+} and so on. But the Rydberg formula also proved correct for distant electrons, for which the effective nuclear charge can be estimated as being the same as that for hydrogen, since all but one of the nuclear charges have been screened (cancelled out) by other electrons, and the core of the atom has an effective positive charge of +1.

Balmer was so confident of his formula that he predicted other groups of spectral lines for hydrogen with different n and m values. These additional lines, however, remained undiscovered until the early 1900s.

CHALLENGE 12.1

Electrons in space

In Rutherford's planetary model of the atom, what keeps the electrons from flying off into space?

Limitations of the Rutherford model

Ultimately, there were two major limitations of the Rutherford model:

- the failure to account for the stability of the atom
- the failure to account for emission lines in the hydrogen spectrum.

Obviously, a revised model was needed. Rutherford also had to account for the fact that his positive nuclear particles would repel and fly apart. Any new model had to keep them together, as well as account for the two limitations above. In 1920, Rutherford theorised about the existence of neutrons, which would somehow compensate for the repelling effect of the positive charges of protons by causing an attractive nuclear force.

But by then the Danish physicist Niels Bohr (1885–1962) had already theorised that electrons travel in orbits about the nucleus, and was about to make a momentous leap with a new model of the atom. One that is still much the same today.

CHECK YOUR LEARNING 12.1

Describe and explain

1 **Describe** the major features of the Rutherford model of the atom.

2 **Explain** Rutherford's proposal for overcoming the repulsion of positive nuclear particles and whether he had experimental evidence to support it.

3 **Clarify** the two major limitations of his model.

Apply, analyse and interpret

4 **Determine** the wavelength of the violet spectral line of hydrogen using Balmer's formula and compare it to the experimental value.

5 **Deduce** whether the wavelength of light is related to the amount of bending of light as it passes through a prism. Analyse the spectrometer diagram in Figure 3 to make your deduction.

6 Balmer applied his formula only to the four lines in the visible spectrum. However, Figure 4 shows there is another 'Balmer series' line at the far left in the near-UV spectral region. **Deduce** the m, n and λ values for this line.

Investigate, evaluate and communicate

7 **Prove** that an electron transition from $n = 4$ to $n = 2$ in a hydrogen atom gives a wavelength of 486.1 nm consistent with the Rydberg equation.

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12.2 Bohr model of the atom

KEY IDEAS

In this section, you will learn about:

- the Bohr model of the atom (1913)
- how the Bohr model addresses the limitations of the Rutherford model
- light quanta and atomic energy states to explain the specific wavelengths of the hydrogen line spectrum
- the line spectra of simple atoms using atomic energy states or atomic energy level diagrams
- limitations of the Bohr model.

In 1911, Bohr was invited by Rutherford to undertake his postdoctoral research with him at Manchester University in England. After returning to Denmark, Bohr applied Planck's quantum theory to Rutherford's nuclear model to create his **Bohr model** of the atom. In 1913, Bohr proposed a revolutionary hypothesis with three major postulates.

Bohr model
electrons orbit the nucleus in particular circular orbits called stationary states with fixed angular momentum and energy, their distance from the nucleus (their radius) being proportional to their energy. When an electron moves between these stationary states, it is accompanied by the emission or absorption of a photon

Bohr's postulates

1 **Electrons in an atom exist in stationary states**.

Bohr stated that normal atoms exist with their electrons in fixed 'stationary states'. While in these stationary states, electrons orbit the nucleus without emitting energy.

2 **Transition between stationary states absorbs or emits electromagnetic radiation.**

- Any permanent change in an electron's motion must be accompanied by a complete transition from one stationary state to another.
- When an electron moves between stationary states, it is accompanied by the emission or absorption of a photon.
- This photon's energy is given by $\Delta E = hf$.
- If energy is added to any atom, such as by particle bombardment or sufficient heating, then the electrons are forced into higher energy or 'excited' states temporarily by absorbing discrete light quanta.
- As the atom restabilises, the electron transitions back down to a stationary state in one jump or a series of allowed steps. Each orbital jump results in the emission of a light quantum of electromagnetic energy of discrete predictable value.
- If an atom absorbs too much energy, then the outermost electron will be promoted completely away from the attraction of the nucleus and will be removed from the atom; the atom is ionised. The energy required is called the *ionisation energy* and for the simplest hydrogen atom is equal to 2.17×10^{-18} J.

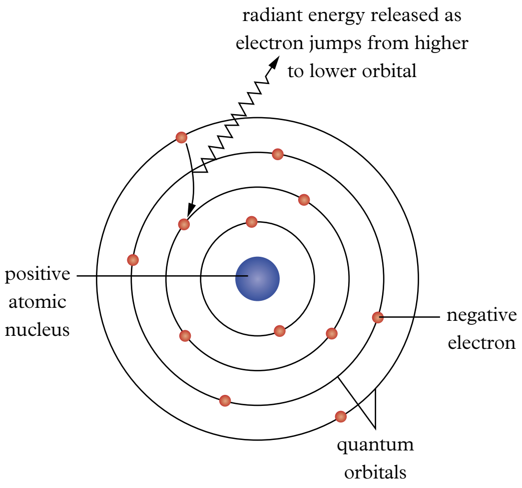

FIGURE 1 Bohr's atomic model. The blue centre is the nucleus and the red dots are the electrons on different levels orbiting the nucleus.

3 **Angular momentum of an electron in a stationary state is quantised.**

In the Rutherford model of the atom, the electrons could orbit in circular orbits at any distance from the nucleus (that is, at any radius). This would mean that every element could emit a full spectrum as any transition would be possible. But this is not the case, so electrons must orbit at certain fixed radii.

Bohr made his third postulate not about the radius of an electron's orbit but about the related quantity, **angular momentum**: $L = mvr$.

$$mvr = \frac{nh}{2\pi}$$

where m is the mass of the electron (9.109×10^{-31} kg), v is velocity of the electron, r is the radius of the electron's orbit, n = energy level (1, 2, 3, ...), and h = Planck's constant = 6.626×10^{-34} J s.

The formula equates angular momentum (mvr) to integer multiples (n) of the constant $\frac{h}{2\pi}$, which therefore means angular momentum is quantised. For example, the first energy level ($n = 1$), has an angular momentum $= \frac{1h}{2\pi}$.

angular momentum
for circular motion, the momentum of a particle in which the velocity vector points along the radius of the circular path and is equal to *mvr* (symbol: *L*; unit: kg m^2 s^{-1})

CHALLENGE 12.2

Multiple lines in hydrogen spectra

a When you look at the spectrum of hydrogen in the lab you only see the Balmer series. Why don't you observe the other series?

b Why is it that the spectrum of hydrogen contains so many lines when hydrogen contains only one electron?

Study tip

A handy way of recalling Bohr's three postulates is STA: stationary states, transitions, angular momentum.

Calculating angular momentum

The question of what 'radius' means intrigues physicists to this day. In terms of a planetary model, which isn't far off from what Bohr postulated, the radius is simple: it is equivalent to a radius such as Earth's (almost circular) orbit around the Sun. However, it is now known that the motion of an electron is not a distinct orbit but a fuzzy cloud around the nucleus. It is more like measuring the radius of the orbit of a fruit fly around a honey pot. The probability of finding the electron close to the nucleus is high, and the further away it is from the nucleus the lower the probability. To calculate a radius you need a velocity and you don't have that. However, you can calculate angular momentum.

WORKED EXAMPLE 12.2A

Calculate the angular momentum (L) for the first and second energy levels of a hydrogen atom.

SOLUTION

$L = mvr$ (angular momentum)

$$= \frac{nh}{2\pi}$$

For $n = 1$:

$$L = \frac{1 \times 6.626 \times 10^{-34}}{2\pi}$$
$$= 1.055 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

For $n = 2$:

$$L = \frac{2 \times 6.626 \times 10^{-34}}{2\pi}$$
$$= 2.109 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

Interpreting Balmer's equation

Bohr's model thus addressed the problem of accounting for the absorption and emission spectra of elements. So, now it became possible to relate the electron transitions and the wavelengths quantitatively.

Bohr could now choose orbits for the hydrogen electron that would yield exactly the required wavelengths for the emitted spectral lines of the hydrogen spectrum, according to the generalised Balmer equation:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_{\mathrm{f}}^{2}} - \frac{1}{n_{\mathrm{i}}^{2}}\right) \text{ for } n = 3, 4, 5, \ldots$$

He relabelled the n_1 and n_2 terms as n_i and n_f and called them the initial and final **principal quantum numbers**.

principal quantum number
n, is a discrete variable assigned to each electron in an atom to describe the energy level of the electron, with higher numbers representing higher potential energy (further from the nucleus)

Figure 2 represents the energy level diagram for hydrogen that correlates the Bohr orbits and their corresponding energies with the series of spectral lines present in the hydrogen spectrum. The spectral series are named after their discoverers.

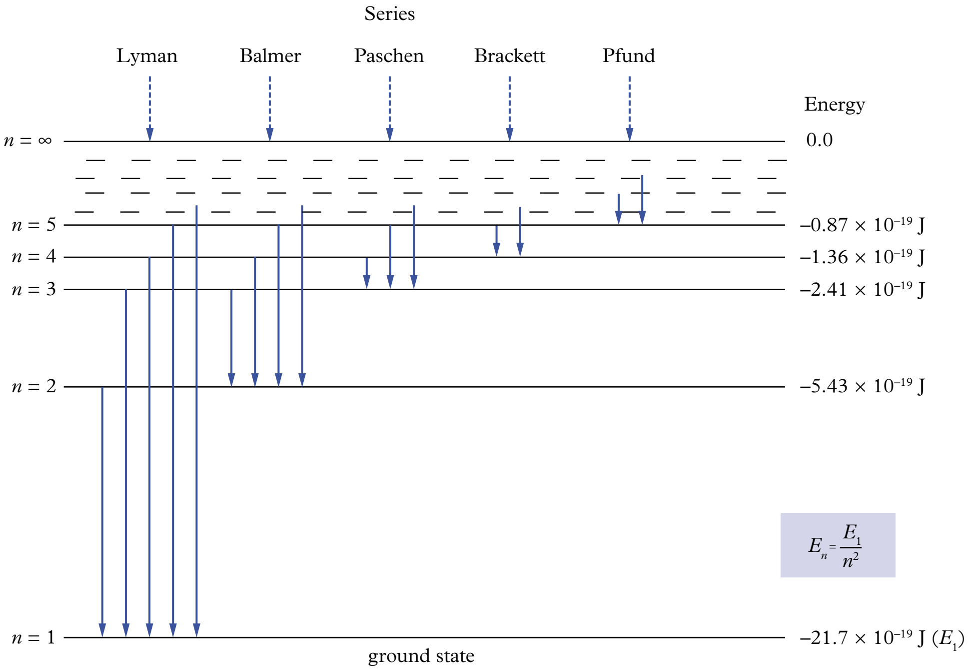

FIGURE 2 Energy level diagram for hydrogen

Mathematically, Bohr's theory used a quantum condition that specified that the angular momentum of the electron was restricted to allowed orbits, given by:

$$mvr = \frac{nh}{2\pi}$$

where $n = 1, 2, 3, \ldots$ integers; r_n is the radius of the nth orbit.

Bohr was able to show that, at least for the simplest hydrogen atom, the radius of the electron orbit (the Bohr radius) for each quantum orbital possible is given by:

$$r_n = \frac{n^2 h^2}{4\pi^2 kmq^2}$$

where n is the principal quantum number, k is the Coulomb constant (9×10^9 N m^2 C^{-2}), m is the mass of the electron (kg), and q is the electron charge (C). Rutherford and Einstein saw this as a great breakthrough.

The radius of the orbit of an electron can be calculated for $n = 1$:

$$\begin{aligned} r_n &= \frac{n^2 h^2}{4\pi^2 kmq^2} \\ &= \frac{1^2 \times (6.626 \times 10^{-34})^2}{4\pi^2 \times (9 \times 10^9) \times (9.109 \times 10^{-31}) \times (1.60 \times 10^{-19})^2} \\ &= \frac{4.390 \times 10^{-67}}{8.285 \times 10^{-57}} \\ &= 5.3 \times 10^{-11}\text{ m} \end{aligned}$$

Thus, for $n = 1$, the Bohr radius takes a value of 5.3×10^{-11} m, and represents the orbital radius for the electron in the lowest energy state or ground state of the hydrogen atom.

Notice that in the energy level diagram of atomic hydrogen (Figure 2), the energy associated with the ground state is a negative value of -21.7×10^{-19} J, which represents the ionisation energy for hydrogen. This amount of energy is required to remove an electron out to infinity. Because we can say that the electron at infinity has zero energy, by definition all possible energy states of the hydrogen atom can be regarded as negative. According to the Bohr theory, the energy of each quantum orbit is given by the series:

$$E_n = E_1\left(\frac{1}{n^2}\right), \text{ where } E_1 \text{ is the ground state energy}$$

$$E_1 = -2.17 \times 10^{-18}\text{ J or } -13.6\text{ eV}$$

The four of the lines of the Balmer series are in the visible region. They are labelled alpha, beta, gamma and delta and correspond to the transitions in Table 1.

TABLE 1 Transitions in hydrogen

Label	Transition	Energy of photon (J)	Frequency (Hz)	Wavelength (nm)	Colour
H_α	3 → 2	3.02×10^{-19}	4.57×10^{14}	656	Red
H_β	4 → 2	4.09×10^{-19}	6.17×10^{14}	486	Aqua
H_γ	5 → 2	4.58×10^{-19}	6.91×10^{14}	434	Blue
H_δ	6 → 2	4.85×10^{-19}	7.31×10^{14}	410	Violet

WORKED EXAMPLE 12.2B

a Determine the energy of an electron in both the fourth and second quantum orbitals of the hydrogen atom.

b Calculate the frequency of the energy emitted when an electron jumps between these orbitals.

c Calculate the wavelength of this emitted light in:

i metres

ii nanometres.

SOLUTION

From Figure 2 on page 336, $E_1 = -21.7 \times 10^{-19} = -2.17 \times 10^{-18}$.

a Fourth level $n = 4$:

$$E_n = E_1\left(\frac{1}{n^2}\right)$$

$$E_4 = \frac{-2.17 \times 10^{-18}}{4^2}$$

$$= -1.36 \times 10^{-19}\,\text{J}$$

Second level $n = 2$:

$$E_2 = \frac{-2.17 \times 10^{-18}}{2^2}$$

$$= -5.43 \times 10^{-19}\ \text{J}$$

b Energy change is in transition:

$$\Delta E = E_f - E_i$$

$$= -5.43 \times 10^{-19} - (-1.36 \times 10^{-19})$$

$$= -4.07 \times 10^{-19}\ \text{J (the negative means lost or emitted)}$$

Use the equation $E = hf$ or $f = \frac{E}{h}$.

$$f = \frac{4.07 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= 6.14 \times 10^{14}\ \text{Hz}$$

which represents the blue line of the Balmer series.

c Use the equation $v = f\lambda$ or $\lambda = \frac{c}{f}$

i $\lambda = \frac{3 \times 10^8}{6.14 \times 10^{14}}$

$$= 4.88 \times 10^{-7}\ \text{m}$$

ii Convert to nanometres: $\lambda = \frac{4.88 \times 10^{-7}\ \text{m}}{10^{-9}\ \text{m nm}^{-1}}$

$$= 488\ \text{nm}$$

A cautionary note

Electrons in atoms will only jump to higher energy levels if the energy of an incoming photon exactly matches the difference in energy between the two levels. For example, in Worked example 12.2B, the energy difference between level 2 and 4 in hydrogen is 4.07×10^{-19} J. So a photon with that energy (λ = 488 nm) will cause a 2 → 4 transition. If light with photons of different energy, for example, 4.5×10^{-19} J of energy (442 nm) bombards hydrogen, it won't cause a level 2 → 4 jump because the photon energy is not an equal match. It may cause a different transition where it does match. Don't ever think that some of the energy of the photon can be used, leaving the photon with the leftover energy. This does *not* happen. However, if the atom is bombarded with electrons instead of photons, then the incoming electrons can give up some of their energy and keep the rest. Electrons can do it, photons cannot.

Limitations of Bohr's model of the atom

Bohr used quantised angular momenta and quantised energy levels in his model. This was a significant step towards the understanding of electrons in atoms and the development of quantum mechanics. However, Bohr's model had a few limitations. It could not explain the:

- **spectra of large atoms**. The Bohr model could only successfully explain the spectra of hydrogen and hydrogen-like ions with one electron.

- **relative spectra intensity**. Bohr could not explain why the intensity of the spectral lines were not all equal. This suggests that some transitions are favoured more than others.
- **hyperfine spectral lines**. It was found that with better equipment there were previously undiscovered spectral lines that accompanied the other more visible lines. These were call 'hyperfine lines'.
- **Zeeman effect**. When hydrogen gas was excited in a magnetic field, the emission spectrum showed a splitting of lines. This is now known to be due to the magnetic field of the electron.
- **stationary states**. Bohr proposed that the electrons were in stationary states, but he could not explain why.

Nevertheless, his application of the quantum theory to atomic structure was very important, and for his work, Bohr was awarded the Nobel Prize in Physics in 1922. Today, the artificial radioactive element of atomic number 107 is called bohrium (Bh).

Study tip

A useful mnemonic for remembering the limitations of Bohr's model is: **S**ome **R**oads **H**ave **Z**ebra **S**tripes.

CHECK YOUR LEARNING 12.2

Describe and explain

1 **Explain** these terms as applied to quantum atomic theory:
   - a quantum orbital
   - b excitation energy
   - c ionisation energy
   - d principal quantum number

2 **Explain** why the spectrum of hydrogen contains several very bright lines but the atom itself contains only one electron and one proton.

3 **Explain** how the Bohr model of the hydrogen atom uses the concepts of light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum.

4 Figure 3 shows the energy levels, in eV, for mercury. **Demonstrate** that when an electron drops from its first excitation energy level to the ground state an ultraviolet photon is emitted.

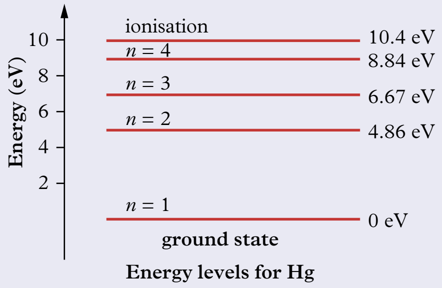

FIGURE 3 Energy levels for mercury

Apply, analyse and interpret

5 **Determine** the wavelengths of the first three lines of the Lyman series in the spectrum of hydrogen (see Figure 2 on page 336). To what part of the electromagnetic spectrum do they belong? See Chapter 11 for a table of spectral regions.

Investigate, evaluate and communicate

6 **Evaluate** the proposition that the attraction for an outer electron in mercury is lower than the attraction for the outer electron in hydrogen.

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12.3 Wave–particle duality

KEY IDEAS

In this section, you will learn about:

- the wave–particle duality
- the evidence for wave–particle duality.

Einstein had led the revolution in blurring the distinction between a wave and a particle by introducing the notion of wave–particle duality and you saw in the previous chapter that Compton had verified that light does indeed have particle-like momentum. In 1924, the French physicist Louis-Victor de Broglie (pronounced 'de-broy') (1892–1987) took the relationship for the momentum of a photon a little further. He thought that if light was a particle and particles can now have a wavelength, then why shouldn't other particles such as electrons and protons have a wavelength.

When applied to particles other than photons de Broglie derived a similar equation to Compton, although approaching it from different viewpoints. His formula is the familiar:

$$\lambda = \frac{h}{p}$$

where λ is the wavelength, h is the Planck constant and p is the momentum of the particle.

The wavelength became known as the de Broglie wavelength. He was awarded the Nobel Prize in Physics for this work in 1929.

WORKED EXAMPLE 12.3A

An electron (mass 9.11×10^{-31} kg) has a velocity of 6.00×10^{5} m s^{-1}.

a Calculate its momentum.

b Determine the wavelength of the electron.

SOLUTION

a $p = mv$

$= 9.11 \times 10^{-31} \times 6.00 \times 10^{5}$

$= 5.47 \times 10^{-25}$ kg m s^{-1}

Note that this speed is equal to 0.002c and is therefore non-relativistic (<0.1c). The relativistic momentum, p_v is the same as the classical momentum.

b $\lambda = \frac{h}{p}$

$= \frac{6.626 \times 10^{-34}}{5.47 \times 10^{-25}}$

$= 1.21 \times 10^{-9}$ m (1.21 nm)

This notion that particles such as electrons could also exhibit wave-like properties led to a better understanding of the behaviour of electrons in atoms. If electrons had wave-like properties, then, it was argued, they should form standing waves as a result of constructive and destructive interference. Right on cue, in 1927 the wave-like diffraction and interference of electron waves was discovered (by accident).

Wave–particle duality for light

The revolution in thinking caused by the quantum theory and its successful application to black-body radiation, the photoelectric effect and Compton scattering caused electromagnetic energy to be given a dual nature by physicists. The **wave–particle duality** concept for light and other forms of electromagnetic energy is our current explanation. If we are describing what light is, then we need to consider what we are explaining. In general, if light energy is interacting with other forms of light energy (for example, in optical effects such as interference and diffraction), then the wave behaviour model is the best explanation. If light is interacting with matter (for example, in Compton collisions), then the particle behaviour model is the best explanation.

wave–particle duality
every particle or quantum entity may be partly described in terms not only of particles, but also of waves

Wave–particle duality for matter

The problem of how an electron could exist in quantum orbitals without losing energy was solved in 1924 when de Broglie suggested that matter could also exhibit wave-like characteristics. He called these matter waves. Louis de Broglie postulated that an electron particle could have a wavelength $\lambda = \frac{h}{mv}$, just as the photon has $\lambda = \frac{h}{p}$ as a result of its momentum.

This idea allowed Bohr's quantum orbits (now called **orbitals** to distinguish them from planetary-type orbits) to be considered as electron wave orbits whose circumference contained an integral number of wavelengths, as shown in Figure 1a. The standing waves of the electrons in orbitals would not require any loss of energy, and the angular momentum of the electrons in their orbitals is quantised. This de Broglie prediction was experimentally verified by the American team of Clinton Davisson and Lester Germer, as well as the British physicist George Thomson. They showed that a beam of electrons scattered by crystals does, in fact, produce a characteristic wave diffraction pattern.

orbital
region of space around the nucleus of an atom where an electron is likely to be found

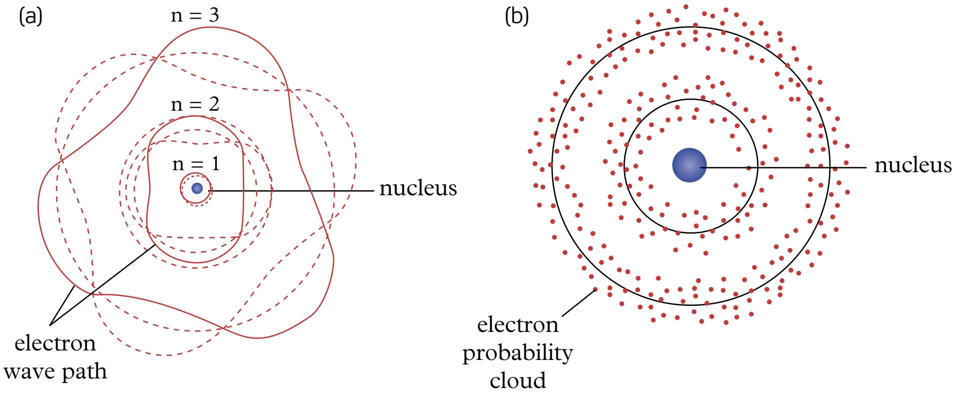

FIGURE 1 (a) Orbital wavelengths and (b) electron clouds. Note the similarity.

CHALLENGE 12.3A

Photon or electron?

What are the differences between a photon and an electron? Make a list.

CHALLENGE 12.3B

Same speed, different wavelength

For an electron and a proton travelling at the same speed, explain which has the shorter wavelength.

De Broglie wavelengths and the Bohr model

The de Broglie wavelengths of anything other than subatomic particles are very short. It makes little sense, for instance, to think of the de Broglie wavelength of a car driving down the road at 100 km h^{-1} even though a wavelength can be calculated.

Assuming a mass of 1500 kg for the car, it can be shown that the de Broglie wavelength is 1.6×10^{-38} m. In practice, this value is immeasurably small and can be neglected.

When the formula for the quantised angular moment and the de Broglie wavelength formula are incorporated, the result is a value for the radius for the stationary states. This explanation treats the stationary states like standing waves on a string. There has to be a whole number of wavelengths that fit into the orbit. The condition for a stable orbit is calculated like this:

$$\begin{aligned} mvr &= \frac{nh}{2\pi} \\ r &= \frac{nh}{2\pi mv} \\ &= \frac{n}{2\pi}\frac{h}{mv} \quad \left(\text{but } \lambda = \frac{h}{mv}\right) \\ r &= \frac{n\lambda}{2\pi} \\ n\lambda &= 2\pi r \end{aligned}$$

where λ is the de Broglie wavelength (m), n is the energy level and r is the radius of the orbital (m).

Using this formula, the wavelength of the ground state electron in hydrogen is 0.166 nm and successive energy levels will have wavelengths that are integer multiples of that value.

WORKED EXAMPLE 12.3B

The first three energy levels in the single-electron helium ion (n = 1, 2, 3) have orbital radii of 0.026 nm, 0.106 nm and 0.238 nm respectively.

Determine the wavelength of the electron standing wave for each level in nanometres.

SOLUTION

$$\begin{aligned} n\lambda &= 2\pi r \\ \lambda &= \frac{2\pi r}{n} \\ \lambda_1 &= \frac{2\pi \times 0.026 \times 10^{-9}}{1} = 0.166 \times 10^{-9} \text{ m (0.166 nm)} \\ \lambda_2 &= \frac{2\pi \times 0.106 \times 10^{-9}}{2} = 0.332 \times 10^{-9} \text{ m (0.332 nm)} \\ \lambda_3 &= \frac{2\pi \times 0.238 \times 10^{-9}}{3} = 0.500 \times 10^{-9} \text{ m (0.500 nm)} \end{aligned}$$

Wave mechanics

The wave–particle concept has led to very complex mathematical models of the nature of atomic structure, called wave mechanics. Wave equations, developed by the Austrian physicist Erwin Schrödinger, describe the wave properties of electrons in both hydrogen and helium atoms. The solutions of Schrödinger's wave equations also indicate that no two electrons can possess the same set of characteristics defined by quantum numbers. This verified the exclusion principle established by Wolfgang Pauli in 1925. Further mathematical refinements by German theorists Max Born, Ernst Jordan and Werner Heisenberg led to the 'matrix mechanics' theory, which is very successful in making predictions about atomic behaviour.

FIGURE 2 Werner Heisenberg, who developed the uncertainty principle

Although quantum mechanics describes an atom in purely mathematical terms, a verbal description and a visual model can be constructed for our modern view of the atom. Surrounding the dense nucleus of any atom is a series of standing wave electron orbitals with wave crests at certain points. The square of the wave amplitude at any point is a measure of the probability that an electron can be found at that point at any given time. This gives us a picture of an electron cloud around the nucleus. (See Figure 1b on page 341.) This probability is as close as scientists can get to defining the position of any electron, and is a result of the uncertainty principle developed by Werner Heisenberg in 1927. His work pointed out that any measurement made on a physical system will, in fact, change the system itself and introduce a fundamental uncertainty into measurements of all other properties of that system. Heisenberg was awarded the 1932 Nobel Prize in Physics for his contribution to quantum mechanics.

The principle states:

> 'It is impossible to measure the position and the corresponding momentum of a particle simultaneously with complete accuracy.'

Again, it might be obvious that this effect is really only important in the subatomic domain. For example, if an electron is measured with a velocity of 4.4×10^6 m s^{-1} with an uncertainty of 0.1%, then it can be calculated (not shown here) that the value 1.3×10^{-8} m represents the positional uncertainty of the electron. This uncertainty is, in fact, about 100 times the diameter of the hydrogen atom, so this principle will not even allow us to determine if the electron is within the atom. The uncertainty principle places large limits on measurement of atomic properties.

> 'Particles do not oscillate because they are a wave. Their wave nature means they are spread out in a fuzzy ball as they travel.'

Quantum mechanics has solved a lot of the great scientific problems that have troubled physicists. It is interesting to note, however, that even Albert Einstein had difficulties with the ideas of quantum mechanics and had many famous arguments with Niels Bohr on the subject. It was Einstein, however, who proposed Heisenberg for the Nobel prize with the endorsement: 'I am convinced that this theory undoubtedly contains part of the ultimate truth.' Perhaps one of the most striking features of quantum physics that has only recently been discovered is that it is not possible in general to say when things 'actually happen'. Time itself is very peculiar indeed in quantum physics!

CASE STUDY 12.3

Electron microscopy

Scientists have found that good resolution in microscopy is limited to about half of the wavelength of the lamp used to illuminate the object. As shown by Young's double slit experiment, the longer the wavelength the greater the amount of diffraction (bending), and hence the greater the interference (blurring of the image). Because our eyes can only detect light of wavelength greater than 400 nm, the best resolution that can be achieved by light microscopes is about 200 nm. A gold atom, with a size of 0.174 nm, is much too small to be seen, so you need a much smaller wavelength.

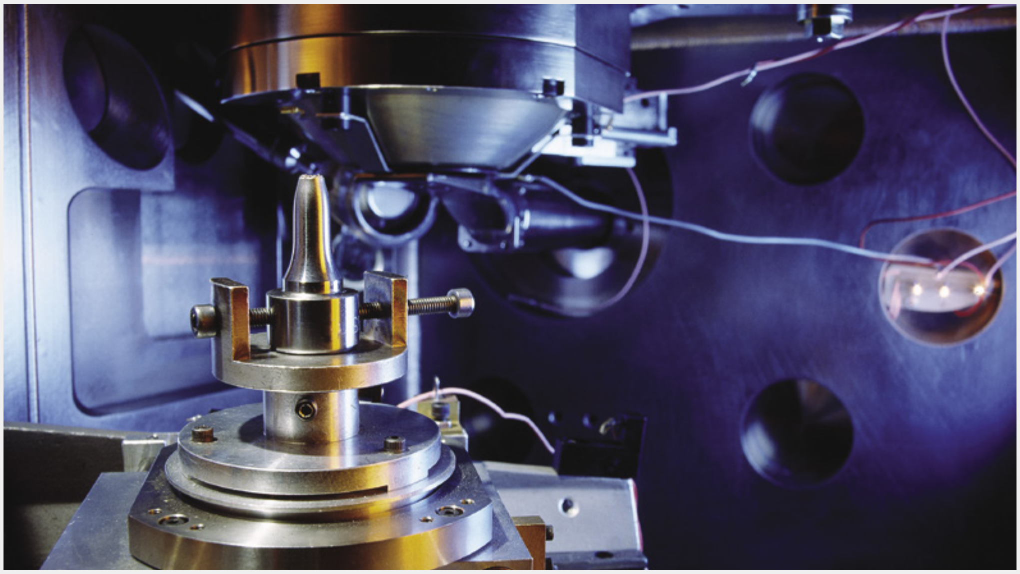

FIGURE 3 A scanning electron microscope uses magnets rather than lenses to focus electron beams.

When it was discovered that electrons had a wave-like nature and that their wavelength was extremely small, new possibilities opened up. At high energies, electrons could have wavelengths as small as 1 pm (picometre = 1×10^{-12} m, or 0.001 nm). It was then proposed that electron waves could be used in microscopy. The practical limit for electrons used in microscopes is a wavelength of about 1 nm, which means clear viewing (resolution) of objects of less than 0.5 nm is possible. A gold atom (Figure 4), with a diameter of 0.174 nm can be seen, although with some fuzziness. This clearly supports the wave nature of matter.

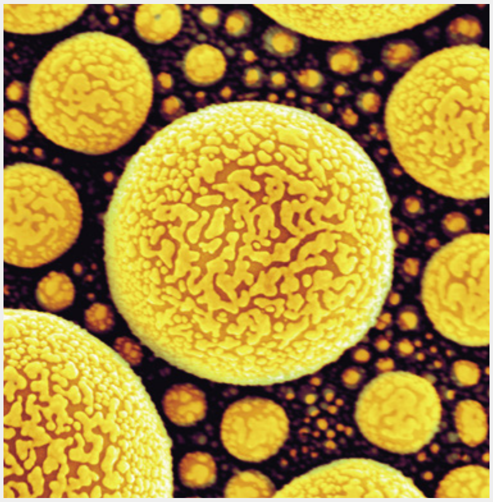

FIGURE 4 Individual gold atoms as seen by a scanning electron microscope. The low diffraction of high-frequency electron waves is clear confirmation of the wave nature of matter.

CHECK YOUR LEARNING 12.3

Describe and explain

1 **Describe** one piece of evidence that supports the wave model for light, and one that supports the wave model for matter.

2 An electron is travelling at 0.01*c*. **Calculate** its:
  a de Broglie wavelength
  b momentum
  c kinetic energy.

3 **Summarise** one piece of evidence that supports the particle model for light, and one that supports the particle model for matter.

Apply, analyse and interpret

Use the energy level diagram for hydrogen in Figure 5 to answer Questions 4 and 5.

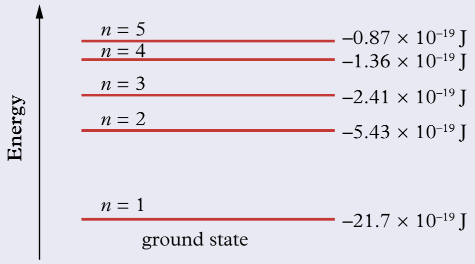

FIGURE 5 Energy level diagram for hydrogen

4 **Determine** the energy of a photon emitted when an electron transitions from $n = 3$ to $n = 1$ in a hydrogen atom.

5 a **Calculate** the energy that must be supplied to raise the atom from quantum state $n = 2$ to $n = 5$.
  b **Determine** the frequency of the photon emitted in an electron transition from $n = 5$ to $n = 2$.
  c The energy of the electron at a particular energy level (n) in hydrogen is given as $E_n = \frac{E_1}{n^2}$ where E_1 is the energy of the electron in level 1 (-21.7×10^{-19} J). **Determine** the energy of the electron in level 7.
  d Ionisation is defined as the energy to remove the electron to infinity ($n = \infty$). **Prove** that the ionisation energy for hydrogen is 21.7×10^{-19} J.
  e An electron transitions in two stages: from level 3 to 2, and then from level 2 to level 1. **Determine** if this would produce the same coloured light as a single transition from level 3 to 1.

6 It is found that a proton travelling at 1.0149×10^4 m s^{-1} has the same energy as a light photon of frequency 1.300×10^{14} Hz. **Determine** the mass of the proton.

7 **Deduce** the three longest wavelengths of an electron in the outer shell of a uranium atom, given that the electron is at a radius of 225.9 pm (pico, p = 10^{-12}).

8 **Determine** the radius of the outer shell of a caesium atom, given that the longest wavelength is 1.77 nm.

9 **Determine** the de Broglie wavelength of a neutron ($m = 1.6749 \times 10^{-27}$ kg) travelling at these speeds:
  a 0.02*c*
  b 20 000 m s^{-1}

10 A coccus bacterium is 0.5 µm in length. **Deduce** with reasoning whether an electron microscope wavelength should be half this or twice this to achieve a clear image.

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CHAPTER 12

Review

Summary

- **12.1** Rutherford's atomic model comprised a dense, charged nucleus and planetary electrons.
- **12.1** Limitations of Rutherford's model were the failure to account for the stability of the atom and failure to account for spectral lines in hydrogen.
- **12.2** Postulates of the Bohr model are the existence of stationary states for electron orbits, that transitions between energy levels produced absorption or emission of radiation and angular momentum is quantised.
- **12.2** Atomic emission spectra of simple atoms can be described mathematically by applying the quantum theory to atomic structure in what is called the Bohr model of the atom.
- **12.2** The excitation energy states of any atom can be illustrated with a quantum energy level diagram.
- **12.3** All particles have an associated wavelength.

Key terms

- angular momentum
- Bohr model
- orbital
- principal quantum number
- Rutherford's model
- wave–particle duality

Key formulas

Electron wavelength and radius	$n\lambda = 2\pi r$
Angular momentum (*L*)	$mvr = \frac{nh}{2\pi}$
Rydberg's equation	$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$
Energy levels	$E_n = \frac{E_1}{n^2}$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 Select the statement that describes why the Bohr model of the atom was able to explain the Balmer series.

A Larger orbits required electrons to have more negative energy in order to match the angular momentum.

B Differences between the energy levels of the orbits matched the difference between energy levels of the line spectra.

C Electrons could exist only in allowed orbits and nowhere else.

D Electrons with different energies passed through the prism to produce different spectral lines.

2 An electron and a proton both moving at non-relativistic speeds have the same de Broglie wavelength. Which of the following are also the same for the two particles?

A speed

B kinetic energy

C momentum

D frequency

3 A rock of mass 0.100 kg is thrown with a speed of 50.0 m s^{-1}. Determine its de Broglie wavelength.

A 6.63×10^{-34} m

B 5.30×10^{-34} m

C 1.33×10^{-34} m

D 1.66×10^{-31} m

4 The de Broglie wavelength applies to:

A photons only.

B photons and electrons.

C electrons, protons and neutrons.

D all particles.

5 Figure 1 shows four energy levels in an atom.

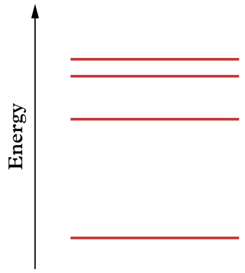

FIGURE 1 Energy levels in an atom

Deduce how many transitions are possible within these four lines.

A 3 **B** 4 **C** 6 **D** 7

6 The energy level diagram for an atom is shown in Figure 2.

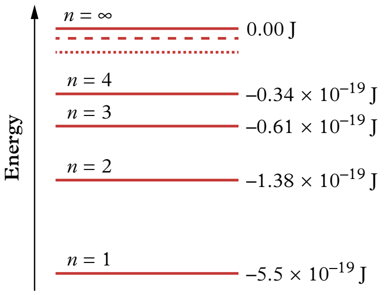

FIGURE 2 Energy levels in a particular atom

Select the statement that best indicates the transitions that would give the highest wavelength radiation and the highest energy photons respectively.

A 3 → 2, 3 → 1

B 3 → 1, 3 → 1

C 4 → 3, ∞ → 1

D ∞ → 2, 3 → ∞

7 Figure 3 represents the energy level diagram for a mercury atom. The values are in eV.

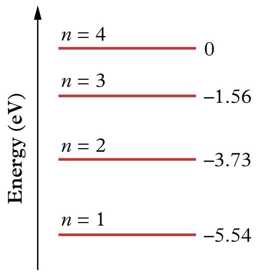

FIGURE 3 Energy levels for mercury

The expression that best states the energy of the photon in joules for the 3 → 1 transition is:

A $\frac{5.54}{1.6 \times 10^{-19}}$ J

C $5.54 \times 1.6 \times 10^{-19}$ J

B $\frac{3.98}{1.6 \times 10^{-19}}$ J

D $3.98 \times 1.6 \times 10^{-19}$ J

8 One of the key limitations of the Rutherford model of the atom was a failure to account for:

A the production of a spectrum.

B orbiting electrons giving off electromagnetic radiation.

C a charged nucleus.

D repulsion of electrons.

9 In Rutherford's experiment, a thin gold foil was bombarded with alpha particles. According to the Thomson 'plum-pudding' model of the atom:

A all the alpha particles would have been deflected by the foil.

B all the alpha particles should have bounced straight back from the foil.

C alpha particles should have passed through the foil with little or no deflection.

D alpha particles should have become embedded in the foil.

10 Rutherford's alpha-particle scattering experiment established that:

A protons are not evenly distributed throughout an atom.

B electrons have a negative charge.

C protons are 1840 times heavier than electrons.

D atoms are made of protons, neutrons, and electrons.

Short answer

Describe and explain

★ **11** **Describe** Rutherford's model of the atom and its limitations.

★ **12** **Explain** the importance of the large deflection angle in Rutherford's gold foil experiment.

★ **13** **Explain**, in fewer than 50 words, the three postulates of the Bohr model.

★ **14** **Describe** how the Bohr model addresses the limitations of the Rutherford model.

★ **15** **Explain** how the Bohr model of the hydrogen atom integrates light quanta and atomic energy states to explain the specific wavelengths in the hydrogen spectra.

★ **16** **Describe** the wave–particle duality of light.

★★ **17** **Identify** evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.

★★ **18** **Explain**, for the Rydberg equation, whether it is true to say that the bigger the difference between the n_i and n_f values the longer the wavelength.

★★ **19** **Clarify** whether angular momentum of an electron refers to the electron spinning on its own axis, or revolving around the nucleus.

★★★ **20** **Construct** a line on the energy level diagram in Question 7 to represent a transition in which an electron absorbs a photon of 2.17 eV.

Apply, analyse and interpret

★ **21** **Determine** the energy of a photon emitted when an electron transitions from level 2 to 1 in a hydrogen atom. Use the energy level diagram for hydrogen in Figure 2 on page 336.

★★ **22** An electron is travelling at 0.05c. **Determine** its:

a de Broglie wavelength

b momentum

c kinetic energy.

★★ **23** It is found that a neutron travelling at 1.99×10^4 m s^{-1} has the same energy as a light photon of frequency 5×10^{14} Hz. **Determine** the mass of the neutron.

★★ **24** **Determine** the three longest wavelengths of an electron in the outer shell of a sodium atom, given that it is at a radius of 174 pm (pico, p = 10^{-12}).

★★ **25** **Determine** the radius of the outer shell of a francium atom given that the longest wavelength is 469 pm (p = 10^{-12}).

★★★ **26** **Determine** the de Broglie wavelength of a neutron ($m = 1.6749 \times 10^{-27}$ kg) travelling at these speeds:

a 0.08c

b 30 000 m s^{-1}

★★★ **27** A proton has the same momentum as an electron travelling at 0.09c.

a **Compare** their speeds.

b **Compare** their wavelengths.

★★★ **28** **Determine** the de Broglie wavelength of an electron in a cathode ray tube that uses a gun accelerating potential of 750 V.

Investigate, evaluate and communicate

★★ **29** Use the energy level diagram for hydrogen in Figure 2 on page 336 to answer this question.

a **Deduce** the amount of energy that must be supplied to raise the atom from quantum state $n = 1$ to $n = 4$.

b **Determine** the energy needed to ionise the atom.

c **Predict** the frequency of the photon emitted in an electron transition from $n = 5$ to $n = 1$.

★★ **30** Consider the energy level diagram for mercury given in Figure 3 on page 339.

a **Determine** the energy of the photon emitted in a transition from $n = 4$ to $n = 2$.

b **Predict** the transition that will have to take place for an electron to completely absorb a 1.81 eV photon.

★★ **31** The energies of possible quantum states for a gas are listed below. Assume that the ground state energy has been included in this list:

-8.64×10^{-19} J
-5.76×10^{-19} J
-16.6×10^{-19} J
-11.5×10^{-19} J
-6.72×10^{-19} J

a **Construct** an appropriate energy level diagram by reorganising and assessing these values.

b **Propose** the shortest and longest wavelength expected in the emission spectra of this gas under excitation.

c **Determine** how much energy is required to cause the gas atoms to transition from energy level 3 to energy level 4.

★★ **32** An electron at rest is accelerated through a potential difference of 100.0 V, where it acquires substantial kinetic energy. A student claims that because its energy is increasing, its wavelength must also be increasing. **Evaluate** this claim. Your answer should include a calculation of its wavelength after being accelerated.

★★★ **33** **Construct** a graph of the energy (eV, on the vertical axis) versus principal quantum number (horizontal axis) for the first four levels of the hydrogen atom. **Create** a linearised version of this graph. **Propose** the relationship between E and n.

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CHAPTER 13

The Standard Model

Up until the end of the 19th century, the atom was thought to be a single hard indivisible particle, like a billiard ball. This was Dalton's model introduced in 1808. Little changed until Thomson discovered the electron in 1897, and Rutherford identified a nucleus in 1898. But, to probe deeper inside an atom, the atom had to be smashed apart. This idea was proposed by Rutherford during his presidential address to the Royal Society of London in 1938. He said that to find out the secrets off an atom he needed a beam of charged particles more energetic than those produced by natural radioactivity. Physicists at the Cavendish Laboratory at the University of Cambridge in England took up the challenge and invented the first particle accelerator. Since then physicists have discovered strange antimatter particles such as positrons and antiprotons as well as muons, and well over a 100 other new particles. Eighty years after the first accelerator, a new generation of high-energy accelerators is shared by research groups from numerous laboratories and funds from all over the world. Trying to classify all of these particles has sent physics on a wild goose chase, but there is now a model that ties them all together. It is not perfect but the Standard Model is the best physicists have come up with (for the time being).

OBJECTIVES

- → Define the concept of an elementary particle and antiparticle.
- → Recall the six types of quarks.
- → Define the terms 'baryon' and 'meson'.
- → Recall the six types of leptons.
- → Recall the four gauge bosons.
- → Describe the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons.
- → Contrast the fundamental forces experienced by quarks and leptons.

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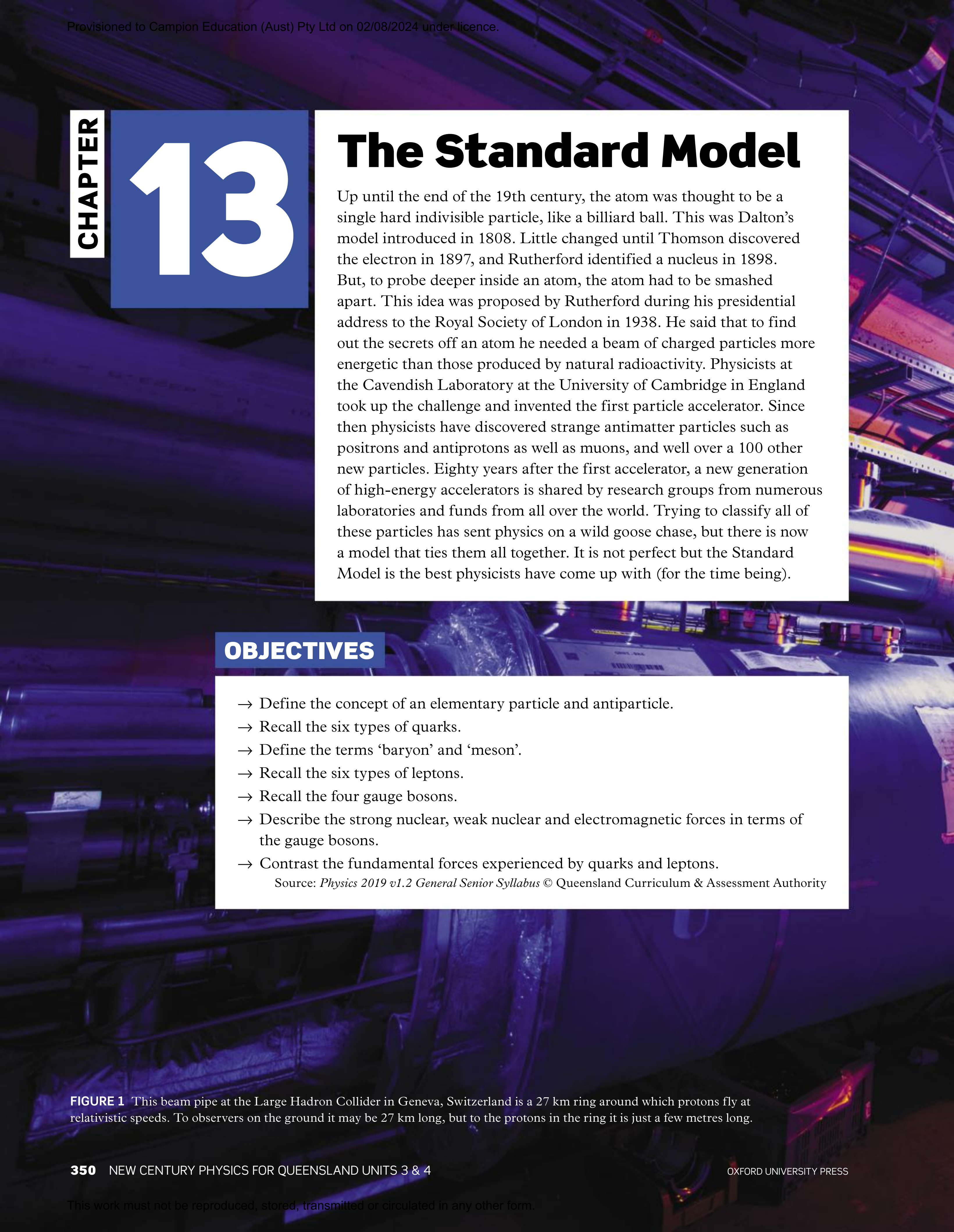

FIGURE 1 This beam pipe at the Large Hadron Collider in Geneva, Switzerland is a 27 km ring around which protons fly at relativistic speeds. To observers on the ground it may be 27 km long, but to the protons in the ring it is just a few metres long.

MAKES YOU WONDER

In this chapter you will be examining the fundamental particles that make up matter, and answer questions such as:

→ What is a quark made of?

→ Why is the Higgs boson called a 'God particle'?

→ How can a neutrino travel at the speed of light if it has mass?

→ Is gravity a wave or a particle?

13.1

Matter and antimatter

KEY IDEAS

In this section, you will learn about:

- the concept of an elementary particle and antiparticle
- the six types of quarks
- baryons, mesons and leptons
- the six types of leptons.

Matter is made up of hundreds of different particles that move around and interact with each other in a particular manner. Physicists have developed a theory called the **Standard Model** that says that all the known matter particles are composites of quarks and leptons, and they interact by exchanging force-carrier particles called bosons. It is a good theory but it doesn't explain everything. For example, gravity is not included in the model. Nevertheless, experiments have verified its predictions to incredible precision, and all the particles predicted by this theory have been found.

Standard Model
a theory describing three of the four known fundamental forces in the universe, as well as classifying all known elementary particles

elementary particle
a particle with no substructure, and thus not composed of other particles

Higgs boson
an elementary particle in the Standard Model that acts as a 'force carrier' for the Higgs field – a field that pervades the universe and is responsible for giving certain elementary particles mass. It is similar to the way a photon is a force carrier for the electromagnetic field

particle zoo
a term used in particle physics to describe the relatively extensive list of particles by comparison to the variety of species in a zoo

antiparticle of matter
a particle that has the same mass and opposite charge and/or spin as a corresponding particle; for example, positron and electron

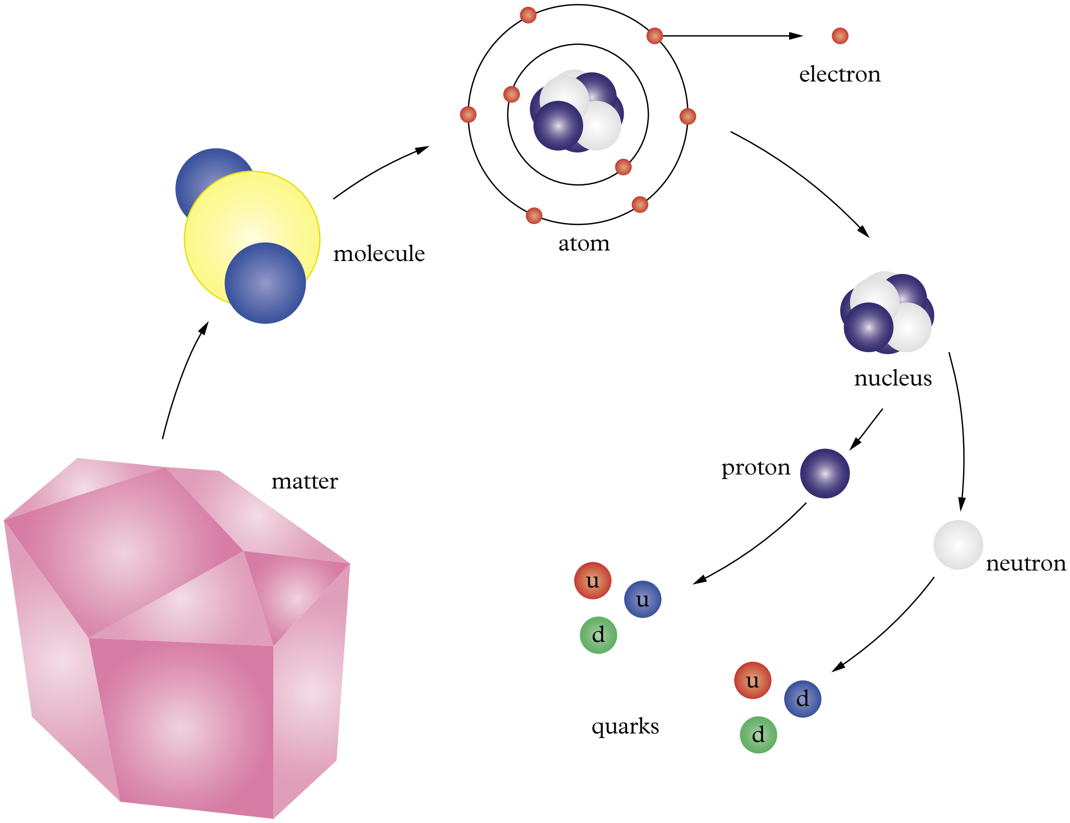

FIGURE 1 The structure of matter shown to subatomic levels

Elementary particles

In physics, an **elementary particle** is a particle with no underlying structure, which means it is not composed of other particles. The current model views the truly fundamental particles as consisting of leptons, quarks, gauge bosons (photon, W and Z, and the gluons), and the **Higgs boson**.

You've learnt that matter is composed of atoms, which used to be thought of as the elementary particles. Then the electron and proton were discovered, followed by the photon, neutron, neutrino, strong and weak nuclear force bosons and a whole avalanche of other particles. It certainly became a **particle zoo** as it became known. Once the elementary quarks and leptons were identified, the problem became how to show combinations that help us understand matter.

As well as having particles of matter, physicists said there should also be **antiparticles of matter**. In the 1920s, an antiparticle to the electron was proposed. It was called a positron – that is, a positive electron. It was discovered in 1932. The suggestion was that all particles should have a corresponding antiparticle, and the **antiproton** (a negatively charged proton) was finally discovered in 1955.

Antiparticles do not live very long in the presence of matter. For example, a positron is stable when it is by itself, but it doesn't survive long. When it meets an electron, the two annihilate each other and give off two gamma rays with their combined energy. This annihilation also occurs for all other particle–antiparticle pairs. During the first second of the Big Bang, the hot and dense universe was filled with particle–antiparticle pairs coming in and out of existence. For each particle formed, an antiparticle was also formed (called 'symmetry'). These annihilated each other in a burst of gamma rays. If **matter** and **antimatter** are created and destroyed together, it seems the universe should contain nothing else but leftover energy.

However, everything around us – from bacteria to galaxies – is made almost entirely of matter. There is not much antimatter around, which means that something must have happened to tip the balance towards matter. What happened in that first second of the Big Bang was that for every one billion antimatter particles produced, there were one billion and one matter particles produced. That extra one matter particle per billion was enough to build the universe as it is today. There are still antimatter particles around, but they are difficult to find.

You'll now look at elementary particles in detail. This first section looks at quarks and leptons, and the next section introduces the force carriers (the gauge bosons).

Quarks and antiquarks

Quarks really do seem to be fundamental particles. There is no evidence that they can be broken up, nor is there anything to suggest that they have an internal structure (an atom has internal structure).

There are six types (or 'flavours') of quarks. They were discovered two at a time, so the first two were called the first **generation** (or family), then the next two were the second generation and then the third generation. Each quark has a corresponding **antiquark**. An antiquark is an example of an antiparticle: a particle that has the same mass but opposite charge. Antiquarks are shown with a bar above the symbol. This makes 12 quarks and antiquarks in total. They have whimsical names, but don't get mislead – this is serious business. Particles made up of quarks are called hadrons (such as the proton, neutron and meson). Table 1 on page 354 shows the 12 quarks and some of their properties.

antiproton
the antiparticle of the proton, with an electric charge of −1e. It is relatively stable but it is typically short-lived because any collision with a proton causes both particles to be annihilated in a burst of energy

matter
a physical substance that has mass and takes up space by having volume, especially as distinct from energy

antimatter
matter that is composed of the antiparticles of those particles that constitute ordinary matter

quark
subatomic particles governed by the strong nuclear force that constitute hadrons; there are six quarks in the Standard Model

generation
a division of the elementary particles according to the Standard Model. There are three generations, or families, of elementary particles grouped according to mass and charge

antiquark
a particle with the same mass and opposite charge to a corresponding quark; for example, a strange quark and an antistrange quark are said to be antiparticles

TABLE 1 Generations of quarks and their properties

Generations	Quarks				Antiquarks			
	Flavour	Symbol	Charge	Colour	Flavour	Symbol	Charge	Colour
First	up	u	$+\frac{2}{3}e$	r, g, b	antiup	$\bar{u}$	$-\frac{2}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$
	down	d	$-\frac{1}{3}e$	r, g, b	antidown	$\bar{d}$	$+\frac{1}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$
Second	charm	c	$+\frac{2}{3}e$	r, g, b	anticharm	$\bar{c}$	$-\frac{2}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$
	strange	s	$-\frac{1}{3}e$	r, g, b	antistrange	$\bar{s}$	$+\frac{1}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$
Third	top	t	$+\frac{2}{3}e$	r, g, b	antitop	$\bar{t}$	$-\frac{2}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$
	bottom	b	$-\frac{1}{3}e$	r, g, b	antibottom	$\bar{b}$	$+\frac{1}{3}e$	$\bar{r}$, $\bar{g}$, $\bar{b}$

Electric charge

Quarks have fractional electric charge values, either $+\frac{2}{3}e$ or $-\frac{1}{3}e$, where e is the elementary charge. Three quarks – the up, charm and top quarks (the first members of each of the three generations) – have a charge of $+\frac{2}{3}e$, while down, strange and bottom quarks (second member of each generation) have charge of $-\frac{1}{3}e$. The six antiquarks have the opposite charge to their corresponding quarks. This is shown in Table 1.

Colour charge

Physicists found there was a need for an additional quantity to fully describe a quark. Quarks had to have three values, so they devised the term ‘colour charge’. A quark can take one of three colour charges: red (r), green (g), or blue (b). An antiquark can take one of three anticolours: antired ($\bar{r}$), antigreen ($\bar{g}$), and antiblue ($\bar{b}$), which are usually represented as cyan, magenta and yellow respectively). They have nothing to do with optical colour such as the red of a rose – it is just another whimsical word.

hadron
particle composed of quarks and gluons (optional term)

gluon
the fundamental exchange particle that operates between quarks and hence underlies the strong nuclear force between nucleons (protons and neutrons) in a nucleus

meson
subatomic particle composed of one quark and one antiquark, held together by the strong nuclear force

baryon
composite subatomic particle made up of three quarks (or three antiquarks) held together by the strong nuclear force

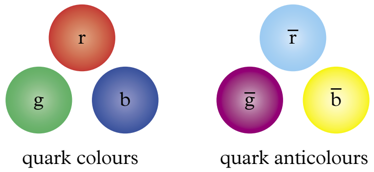

FIGURE 2 Example of the colours. The quark colours are red, green and blue, and the antiquark colours are antired (cyan), antigreen (magenta) and antiblue (yellow).

The purpose of colour charge will be seen in the next section. It is not required for external assessment.

Hadrons – quark composites

Particles composed of quarks are called **hadrons** (from the Greek *hadros* ‘bulky’, which is what they are). As well as the quarks, hadrons need to have force-carrying particles called **gluons** to hold the quarks together. Use of the term ‘hadron’ is not required for external assessment.

Quark composites are formed in two ways: in pairs to form **mesons**, or in threes to form **baryons**.

Mesons

Mesons are named from the Greek *mesos* meaning ‘middle’ and are so called because they are intermediate in mass between protons and electrons. Mesons are part of the hadron particle family, and are defined simply as particles composed of a quark and an antiquark bound together by the strong nuclear force. All mesons are unstable, with the longest-lived lasting for just under a second. Some examples of mesons are shown in Figure 3. The first meson is called a pion (π^+), and is shown in Figure 3a. It is made up of an up quark and an antidown quark ($u\bar{d}$) and an overall electric charge of +1e. In this example the colours are red and antired, which add to white when combined. This is called colour neutral (white).

The second meson, shown in Figure 3b, is a charmed eta-c meson made up of a charm and an anticharm quark ($c\bar{c}$); it has a net electric charge of zero. The colours used are green and antigreen and again add to white. The third example (Figure 3c) is a pion π^- made up of a down quark and an antiup quark and has a net electric charge of −1e. It is the antiparticle to the π^+. The top quark can’t form a meson because it has an extremely short lifetime of 5×10^{-25} s.

Study tip

Some mesons are their own antiparticle. For example, the charmed eta meson $c\bar{c}$ has an antiparticle $\bar{c}c$, which is identical. However, the pion meson $u\bar{d}$ has the antiparticle $\bar{u}d$, which is different.

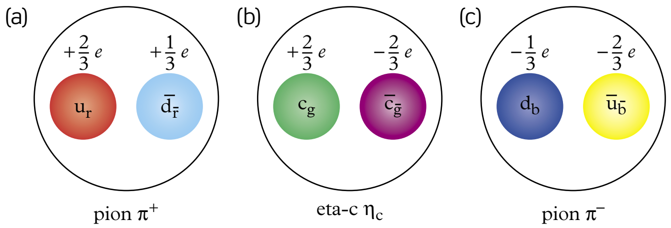

FIGURE 3 Mesons are composed of a quark and an antiquark bound together by the strong nuclear force.

Baryons

Baryons are named from the Greek *barys* meaning ‘heavy’ because at the time of their naming, baryons were the heaviest of the known nuclear particles. They are composites of three quarks (qqq), or three antiquarks ($\overline{qqq}$). If it has three antiquarks it is called an **antibaryon**. The most famous baryons are the proton and the neutron. The quark or antiquark can be of any flavour – any one of u, d, c, s, t, b. Some examples of baryons and antibaryons are shown in Table 2. Note that the electric charge can be −1, 0, or +1. The quarks or antiquarks are held together by the strong nuclear force. Again, top quarks are not likely to form mesons or baryons because of their short lifetime.

antibaryon
a composite subatomic particle made up of three antiquarks held together by the strong nuclear force

TABLE 2 Properties of the baryons and quark composition

Baryon	symbol	quarks	electric charge
Neutron	n	udd	$+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} = 0$
Proton	p	uud	$+\frac{2}{3}, +\frac{2}{3}, -\frac{1}{3} = +1$
Antiproton	$\bar{p}$	$\overline{uud}$	$-\frac{2}{3}, -\frac{2}{3}, +\frac{1}{3} = -1$
Omega minus	Ω^-	sss	$-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} = -1$
Lambda	Λ	uds	$+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} = 0$

Study tip

Colour is a very important property of quarks. It is sometimes called ‘colour charge’ but has nothing to do with colours as we know them (red, orange, yellow …). While it is hugely important to particle physicists it will not be examined in any external assessment.

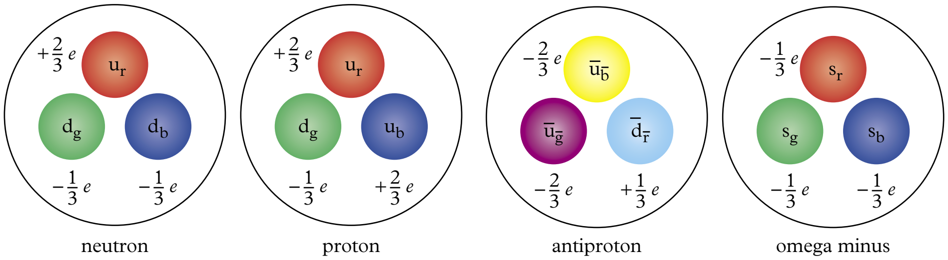

FIGURE 4 Examples of baryons. The colours have been randomly chosen, but note that they all add to white (colour neutral).

CHALLENGE 13.1A

Mesons or baryons?

Can any of the following combinations of quarks exist as particles?

a $uu\bar{d}$

b ss

c $u\bar{d}$

If so, are they mesons or baryons?

CHALLENGE 13.1B

Colour charge rules

Using colour charge rules, decide what the colour of the last quark in each of the following combinations must be to make each particle colour neutral.

a sigma baryon (Σ^0): $u_r d_b s$

b pi meson (ϕ): $s_r\bar{s}$

c strange D meson ($D_s^{\star +}$): $c_b\bar{s}$

d double charmed Xi baryon ($\Xi_{cc}^{\star +}$): $d_b c_g c_r$

e antiproton ($\bar{p}$): $\bar{u}_{\bar{r}}\bar{u}_{\bar{b}}\bar{d}$

lepton
a class of elementary particles that respond only to the weak force and the gravitational force. They can carry one unit of electric charge or are neutral, and those that are charged experience the electromagnetic force. There are six leptons in the Standard Model

Leptons

As you have seen, one type of elementary particle is the quark. The other is the **lepton**.

It comes from the Greek *leptos* meaning ‘small, delicate, or peeled’. One difference between quarks and leptons is that quarks form composite particles (hadrons) such as baryons and mesons. Leptons do not – they don’t form composite particles at all. The first lepton to be discovered was the electron, which was discovered by J.J. Thomson in 1897. It is indeed small, but the tau lepton is about as twice as heavy as a proton.

There are six leptons in the Standard Model: the electron, muon, and tau particles and their associated neutrinos – the electron neutrino, the muon neutrino and the tau neutrino. As well as these six, there are the corresponding six antileptons. Table 3 sets out their properties.

Unlike the quarks, which interact via the strong nuclear force, leptons interact via the weak nuclear force. Leptons that have an electric charge also interact via the electromagnetic force. They are stable particles except for the muon and the tau**,** which have mean lifetimes of 2.2×10^{-6} s and 2.9×10^{-13} s respectively. They have too much energy and tend to break down into the stable first family particles such as the electron and electron neutrino. Leptons can be divided up into charged and uncharged particles: the charged leptons (also known as the electron-like leptons), and the neutral leptons, which are better known as neutrinos.

TABLE 3 Properties of the leptons

Generation	Lepton			Antilepton		
	Flavour	Symbol	Charge	Flavour	Symbol	Charge
First	electron	e^-	1–	positron	e^+	1+
	electron neutrino	ν_e	0	electron antineutrino	$\bar{\nu}_e$	0
Second	muon	μ^-	1–	antimuon	μ^+	1+
	muon neutrino	ν_μ	0	muon antineutrino	$\bar{\nu}_\mu$	0
Third	tau	τ^-	1–	antitau	τ^+	1+
	tau neutrino	ν_τ	0	tau antineutrino	$\bar{\nu}_\tau$	0

CHECK YOUR LEARNING 13.1

Describe and explain

1 **Recall** the names of the six quarks.

2 **Define** the terms 'baryon' and 'meson'.

3 **Select** the correct term from this list for each of the following composite particles: hadron, baryon, meson, muon, antimuon, lepton, antilepton.

  a $u\bar{d}$

  b dds

  c ccc

  d $\bar{d}\overline{uc}$

4 **Describe** how you can tell whether a particle is a meson or a baryon by its quark content.

5 **Clarify** the meaning of the colour charge.

Apply, analyse and interpret

6 **Distinguish** between a lepton and a quark.

7 **Distinguish** between an elementary particle and its antiparticle.

8 **Deduce** whether there are any quark–antiquark combinations that result in a non-integer electric charge.

9 A particle is made up of quarks of the same flavour. Two of the quarks and their colours are s_r and s_b. **Deduce**, using the Standard Model, what else is in the composite particle.

Investigate, evaluate and communicate

10 **Decide** what conditions are necessary for a particle and its antiparticle to be identical.

11 A hadron has a charge of +1e. **Propose** the possible composition of the hadron, and describe it as a baryon or a meson.

12 A particle is identified with a charge of $+\frac{2}{3}e$. **Evaluate** the possibility that it could be either an up quark or a meson made up of an antistrange quark and an antibottom quark.

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Check your learning 13.1

» Challenge worksheet
13.1A Mesons or baryons?

» Challenge worksheet
13.1B Colour charge rules

» Video
Considering matter and antimatter

13.2

Gauge bosons – the force carriers

KEY IDEAS

In this section, you will learn about:

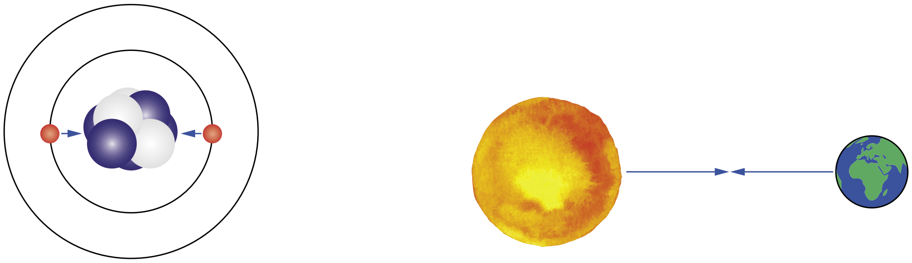
- the four fundamental forces
- four gauge bosons
- the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons
- the fundamental forces experienced by quarks and leptons.

fundamental forces those that act between bodies of matter and are mediated by one or more particles. In order from strongest to weakest: the strong nuclear force, electromagnetic force, the weak force, the gravitational force

There are four **fundamental forces** in the universe. They act between bodies of matter and are mediated by one or more particles. The four fundamental forces going from strongest to weakest are the strong nuclear force, electromagnetic force, the weak force, and the gravitational force. They act over different ranges and have different strengths. This section contrasts the four forces as experienced by quarks and leptons.

The strong nuclear force holds the nucleus together.

The weak nuclear force is responsible for the radioactive decay of atoms.



Electromagnetic force holds electrons in their atoms and binds matter together as molecules.

The gravitational force holds the universe together.

FIGURE 1 The four fundamental forces of the universe are the strong nuclear force, the weak nuclear force, the electromagnetic force and the gravitational force. At the time of the Big Bang they were all combined but gradually separated out.

The fundamental forces

Strong nuclear force

strong nuclear force the strongest of the four fundamental forces; binds quarks together to make subatomic particles such as protons and neutrons and underlies interactions between all particles containing quarks; also called the strong force

mediating particle a descriptive name for the gauge bosons, which govern the interaction of the four fundamental forces; also known as carrier particles or exchange particles

Only quarks within nucleons experience the **strong nuclear force**. It is also called the fundamental strong nuclear force or more commonly the ‘strong force’. It acts only over extremely small distances: about 10^{-15} for these quark–quark interactions. It has the peculiar property of increasing in strength as the particle separation increases. The **mediating particle** for the strong nuclear force is the gluon.

Electromagnetic force

Electrically charged objects such as all quarks and leptons with charge experience the **electromagnetic force**. As a reminder, the charged leptons are the electrons, muons, tau particles (also called 'tauons'), and their antiparticles. Neutrinos are not charged and so are not affected. The electromagnetic force holds electrons in their atoms and binds matter together as molecules. But it also causes protons in the nucleus to repel each other. Like gravity it operates to the edges of the universe, but because there can be both attraction and repulsion its effect over vast distance is not as pronounced as that of gravity. Nevertheless, it is the second strongest force, being just one-hundredth that of the strong nuclear force. You learnt about this force as Coulomb's law. The mediating particle for the electromagnetic force is the photon.

electromagnetic force
the second strongest of the four fundamental forces; the electromagnetic force is mediated by photons

Study tip

The syllabus does not use the term 'strong force' but refers to it by the overarching name 'strong nuclear force'. To be completely safe, you should learn the exact definition from the syllabus: one of the four fundamental forces; the strong nuclear force acts over small distances in the nucleus to hold the nucleons together against the repulsive electrostatic forces exerted between the protons; the strong nuclear force is mediated by gluons. Source: *Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority

CASE STUDY 13.2A

The naming of the strong force

FIGURE 2 Hideki Yukawa

There is a lot of confusion about the naming of the strong force. In 1937, Japanese physicist Hideki Yukawa proposed that the attractive force between nucleons be called the 'strong nuclear force' and that is was due to the exchange of particles. These particles were discovered in 1947 and were named pi mesons (π) or just pions. With the development of quark theory in 1963, it was realised that protons and neutrons (nucleons) were not fundamental particles because within in them were the quarks, and quarks were the fundamental particles. The 'strong nuclear force' was no longer considered a fundamental force. The new fundamental force was the force between quarks and was to be called the 'strong force'. Note that the word 'nuclear' has been removed. However, this 'strong force' between quarks also extends out from any nucleon and is responsible for the force between nucleons that stops protons flying apart – so we end up with a confusion of terms.

Today, in the Standard Model, we think of the fundamental force being the 'strong force' between quarks, but the overarching term 'strong nuclear force' is still used for this interaction. The fact that this force has a residual component that acts between nucleons is confusing and this aspect is sometimes called the 'residual strong nuclear force'.

Weak nuclear force

All quarks and all leptons experience this force. It is independent of electric charge. The **weak nuclear force** is primarily responsible for slow nuclear processes such as the radioactive decay of atoms, and seems to control the energy-producing fusion reactions going on in stars. It is the third strongest force, at about 10^{-5} times that of the strong nuclear force. The mediating particles for the weak nuclear force are the W and Z bosons.

weak nuclear force
the third strongest force of the four fundamental forces; it is responsible for radioactive decay and is mediated by W (W^+, W^-) and Z^0 bosons

CHALLENGE 13.2A

Femtometre

The range of the strong nuclear force is one femtometre (10^{-15} m). How many femtometres are in 1 metre?

Gravitational force (gravity)

Gravity is one of the four fundamental forces but is yet to be incorporated into the Standard Model and so cannot be considered in any discussion of the model. It is included here for completeness, but is obviously not a part of the Standard Model in the syllabus.

All objects with mass experience the gravitational force. Hence, quarks and leptons, and any composite of these will feel the force. You will have learnt this force as Newton's law of gravitation and that it is the product of the masses and the inverse square of distance. The gravitational force keeps planets in orbit, controls the expansion of the universe and stops us from falling off Earth. It is the weakest of all forces, at about 10^{-39} of the strong nuclear force, but operates over an infinite distance. Gravity is this last point that makes gravity so different from the other forces. Gravity is an attractive force only. No-one has found negative mass, so every particle in the universe experiences an attractive force. Gravity can range from one side of the universe to the other with no opposition. However, it can be ignored inside the atom as it is weak compared to the other forces. The mediating particle for gravity is proposed to be the 'graviton', but so far it remains undiscovered.

Contrasting the four fundamental forces

TABLE 1 The four fundamental forces and their mediating particles (plus the Higgs field). A femtometre (fm) = 10^{-15} m

	Fundamental forces				Higgs field (Standard Model)
	Strong nuclear force (Standard Model)	Electromagnetic force (Standard Model)	Weak nuclear force (Standard Model)	Gravitational force	
Strength	1	10^{-2}	10^{-5}	10^{-39}	
Range	10^{-15} m (1 fm)	infinite	10^{-18} m (10^{-3} fm)	infinite	infinite

The Standard Model includes only three of the four fundamental forces. The gravitational force is still outside the model, although scientists have been trying hard for decades to include it in what they call 'the grand unification'. Gravity, gravitational waves, and gravitons are dealt with separately in Case study 13.2B later in this chapter.

CHALLENGE 13.2B

The 'Oh-My-God!' particle

What is the 'Oh-My-God' particle? Is it a particle that produces a field that interacts with particles and gives them mass, or a particle that is so fast it almost travels at the speed of light?

CHALLENGE 13.2C

Why do you notice gravity so much?

Gravity is the weakest force, so why do you notice it so much? Decide whether the following are true or false and explain your reasoning:

1. Gravity acts only on charged particles.
2. Gravity can act only on elementary particles not composites such as atoms.
3. Gravitational forces have a short range.

The four gauge bosons

The three forces of the Standard Model operate by exchanging particles between the objects they operate on. These force-carrier particles are called **gauge bosons** (photons, the W and Z bosons, and gluons). The word 'gauge' means 'measurement', and refers to the idea that you can't measure the properties of these bosons directly but only indirectly by other measurements. There is another type of boson called a 'scalar boson' of which the Higgs boson is the only present example. It is not a force carrier and will be dealt with in Chapter 14. As well as the bosons described above, there are also composite or molecular bosons. Any composite particle containing an even number of quarks is a boson and is said to be 'bosonic'. These include composite particles such as the meson (2 quarks), deuterium H-2 (6 quarks), He-4 (12 quarks), C-12 (36 quarks) and so on. This chapter will only consider the non-composite (elementary) bosons.

gauge bosons
mediating particles that govern particle interaction and the mediation of the four fundamental forces. There are four gauge bosons in the Standard Model

The exchange of gauge bosons produces the force or – in the language of physics – 'mediates' the force. 'Mediate' comes from the Latin *mediates* meaning 'in the middle', which is where these bosons are – in the middle between the particles. Each of the three fundamental forces of the Standard Model has its own corresponding boson:

- The electromagnetic force is mediated by the photon.
- The strong nuclear force is mediated by the gluon.
- The weak force is mediated by the W (W^+, W^-) and Z gauge bosons.

These four gauge bosons, as well as the unconfirmed graviton (a suspected gauge boson) and the scalar Higgs boson are summarised in Table 2.

TABLE 2 The four gauge bosons and the Higgs boson of the Standard Model, and the non-Standard Model proposed boson – the graviton

Force	Standard Model					non-Standard Model
	Strong nuclear force	Weak nuclear force		Electromagnetic force	Higgs field	Gravitational force
Particles experiencing the force	quarks	quarks, leptons		quarks and electrically charged leptons	W^+, W^-, Z^0	all
Mediating particle	gluon, g	W^+, W^-	Z^0	photon γ	Higgs boson, H^0	graviton, G^0 (unconfirmed)
Type of boson	Gauge bosons				Scalar boson	possibly a gauge boson

Study tip

Composite bosons and the graviton are not a part of the Senior Physics curriculum. You will consider only the non-composite (elementary) bosons – the four confirmed gauge bosons, and the one scalar boson (Higgs).

The recently discovered Higgs boson is a fundamental particle within the Standard Model, but is not one of the force mediators responsible for the three fundamental forces. Although not yet found, the graviton should be the corresponding mediating boson of gravity.

Gluons

A gluon is an elementary particle that acts as the exchange particle for the strong (nuclear) force between quarks. It is similar to the exchange of photons in the electromagnetic force between two electrically charged particles. So when quarks (which have special colour charge) are close to each other there is an exchange of gluons that creates a very strong nuclear force that binds the quarks together. The other more common term for this is the strong nuclear force. You could say the gluons 'glue' quarks together, forming baryons such as protons and neutrons. However, isolated gluons have never been observed experimentally, only their effects.

Study tip

Quarks and lepton are usually grouped together by particle physicists as the convenient term 'fermions' (pronounced fermi-ons) in honour of the Italian physicist Enrico Fermi. The term is not included in the syllabus. Another overarching term is 'hadron', which is a particle made up of quarks, such as protons, neutrons, mesons; hence the naming of the Large Hadron Collider, which makes protons (hadrons) collide at extremely high speeds.

W and Z bosons

The W and Z bosons act between quarks and leptons. The W bosons are weak, hence the symbol W, and they can have either a positive or negative electric charge they can be W^+ or W^-. The W and Z bosons are almost 100 times as massive as the proton – heavier, even, than entire iron atoms. They acquire their mass by interacting with the Higgs field.

Photons

The photon is gauge boson that mediates the electromagnetic force (and its composites the electrostatic force, and the magnetic force). You learnt about the photon as a particle in the previous chapter. It is a massless particle that travels at the speed of *c* in free space. Photons are considered to be their own antiparticle (which becomes important in Chapter 14).

Study tip

It would be extremely wise to be able to reproduce this chart of the Standard Model from memory.

TABLE 3 Summary

<table>
<tr><th colspan="9">STANDARD MODEL</th></tr>
<tr><th colspan="9">Elementary particles</th></tr>
<tr><th colspan="4">Quarks and leptons</th><th colspan="5">Elementary bosons</th></tr>
<tr><td colspan="2">Quarks and antiquarks strong interactions</td><td colspan="2">Leptons and antileptons electroweak interactions</td><td colspan="4">*Gauge bosons: force carriers</td><td>*Scalar boson: mass carrier</td></tr>
<tr><td>quarks</td><td>antiquarks</td><td>leptons and neutrinos</td><td>antileptons and antineutrinos</td><td rowspan="2">γ photon</td><td rowspan="2">W^+, W^-, Z bosons</td><td rowspan="2">g gluon (8 types)</td><td rowspan="2">G^0 graviton (proposed: not Standard Model)</td><td rowspan="2">H^0 Higgs boson</td></tr>
<tr><td>u, d</td><td>$\bar{u}$, $\bar{d}$</td><td>e^-, ν_e</td><td>$e^+, \bar{\nu}_e$</td></tr>
<tr><td>c, s</td><td>$\bar{c}$, $\bar{s}$</td><td>μ^-, ν_μ</td><td>$\mu^+, \bar{\nu}_\mu$</td><td rowspan="2">carries electro-magnetic force</td><td rowspan="2">carry weak force</td><td rowspan="2">carries strong force</td><td rowspan="2">carries gravity (proposed)</td><td rowspan="2">carries mass</td></tr>
<tr><td>t, b</td><td>$\bar{t}$, $\bar{b}$</td><td>τ^-, ν_τ</td><td>$\tau^+, \bar{\nu}_\tau$</td></tr>
</table>

*These terms do not have to be recalled.

CASE STUDY 13.2B

The graviton

In an attempt to develop an overarching theory that combines gravity into the Standard Model, physicists came up with a hypothetical particle, the graviton.

According to physicists, it will be a long time before gravitons are considered part of the established subatomic 'zoo' of particles. The graviton is the hypothetical elementary particle that mediates the force of gravity. It has yet to be detected but it is predicted to be massless because the gravitational force is very long range and appears to propagate at the speed of light. You know from the theory of special relativity that particles with mass can't travel at the speed of light as their momentum would become infinite. Although gravitational waves have been detected, the graviton has not. It has been calculated that to capture a graviton, a detector the size of Jupiter would have to be placed in close orbit around a neutron star, and even then gravitons would only be detected every 10 years, even under the most favourable conditions.

The detection of gravitational waves, however, provides information about certain properties of the graviton. For example, if gravitational waves were observed to propagate slower than c, that would imply that the graviton has mass. The LIGO observations of gravitational waves put an upper bound of 1.2×10^{-22} eV c^{-2} ($= 2.1 \times 10^{-58}$ kg) on the graviton's mass; very close to zero! Astronomical observations of the rotation of galaxies might give conclusive evidence that gravitons have non-zero mass.

CHECK YOUR LEARNING 13.2

Describe and explain

1 **Explain** which particle is involved in the strong nuclear force between nucleons.

2 **Recall** the names of the four gauge bosons.

3 **Describe** the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons.

Apply, analyse and interpret

4 **Contrast** the fundamental forces experienced by quarks and leptons.

5 **Determine** which is the faster: a photon or a Z^0 boson.

Investigate, evaluate and communicate

6 A boson mediates between two fundamental particles. **Propose** what you would need to know about the particles to decide which boson it was.

7 The table below lists fundamental particles and possible forces between them. **Modify** the table so the two columns match correctly.

Particles	Feel the
a charged leptons	**i** weak force
b all quarks	**ii** weak and EM forces
c all leptons and all quarks	**iii** weak, EM and strong forces

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Check your learning 13.2
- » Challenge worksheet 13.2A Femtometre
- » Challenge worksheet 13.2B The 'Oh-My-God!' particle
- » Challenge worksheet 13.2 C Why do you notice gravity so much?

SCIENCE AS A HUMAN ENDEAVOUR

13.3

The Big Bang theory

KEY IDEAS

In this section, you will learn about:

- the evidence that supports the Big Bang theory (cosmic background radiation, abundance of light elements, Hubble's Law).

The big questions

Like those before us, you may often ask the same questions. Here are the top five questions that are asked:

1. What is out there?
2. What is the universe made of?
3. Are we alone?
4. How did it all start, and when?
5. How will it end?

Until the 1920s, the general opinion was that the universe was infinite in size and in a steady unchanging 'static' state made up of fixed stars that had always shone and would continue to shine forever. This view was well entrenched – even Einstein believed it! But in 1929 the astronomer Edwin Hubble made a finding that was to shake the foundations of this steady state model forever.

Cosmological red shift

During the 1920s, astronomers looked at starlight through spectrometers and noticed that the spectral lines of elements such as hydrogen and helium seemed to be occurring at longer wavelengths than normal. You will recall from the previous chapter that electron transitions between energy levels produce light of specific wavelengths. The shift in wavelength was towards the red end of the spectrum and so the term 'red shift' was coined for this phenomenon. Physicists deduced that the shift in wavelength meant that the star was moving relative to an observer on Earth. But a red shift from a distant light source (generally more than a few million light years away) is a **cosmological red shift** that is due to the expansion of the universe (Figure 1).

cosmological red shift
shift in the wavelengths of sufficiently distant light sources (galaxies, quasars) due to the expansion of the universe

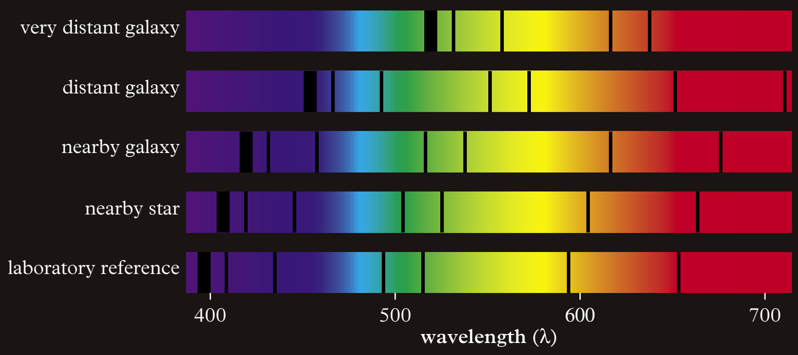


FIGURE 1 A comparison of red shifts for the spectra of starlight. The further the galaxies are away from us the bigger the red shift.

Hubble's law

In 1929, US astronomer Edwin Hubble used his 100 inch (2.5 m) diameter telescope to show that the universe is expanding. This was a monumental breakthrough. And if galaxies are moving away from each other, there must have been a time when they were all together. That was nearly 14 billion years ago – the time of the Big Bang – when the whole universe was the size of the full stop at the end of this sentence.

Hubble combined his knowledge of galaxy red shifts with an estimate of the distance to these galaxies, and determined that galaxies more distant from us were moving away from us more rapidly than closer galaxies. This relationship has become known as **Hubble's law**.

Hubble's law
there is a direct proportion between the distance to a galaxy and its recessional velocity as determined by the red shift

The Big Bang

Some key stages in the history of the universe are as follows.

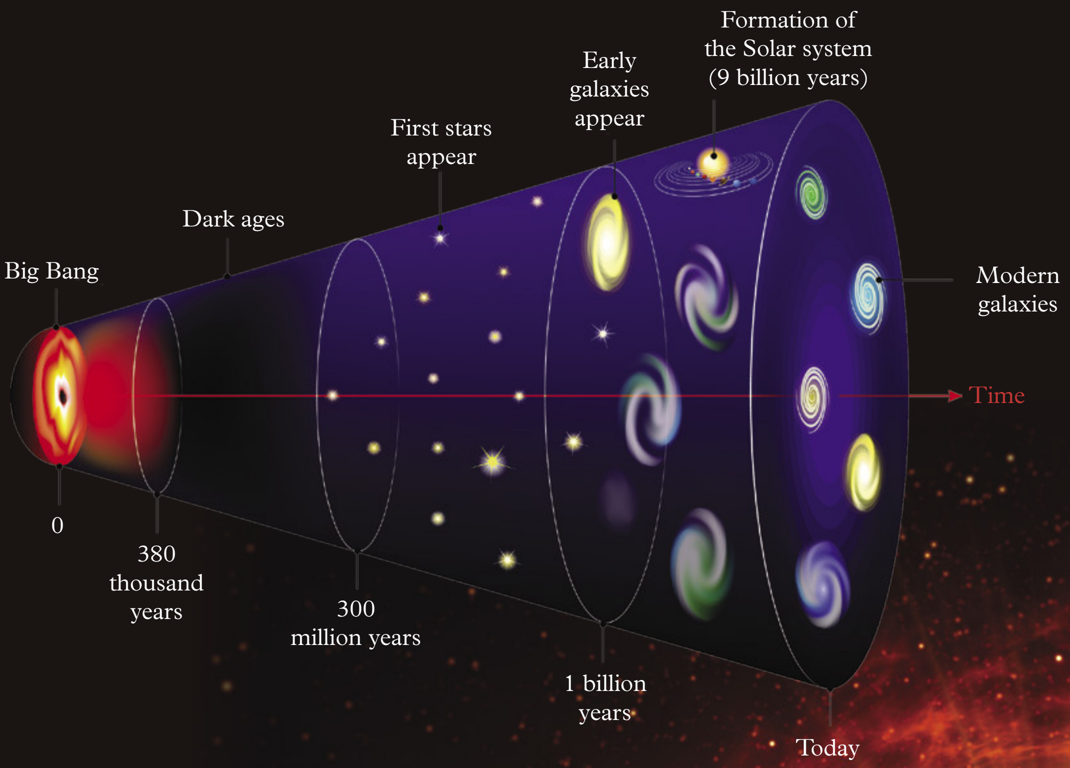
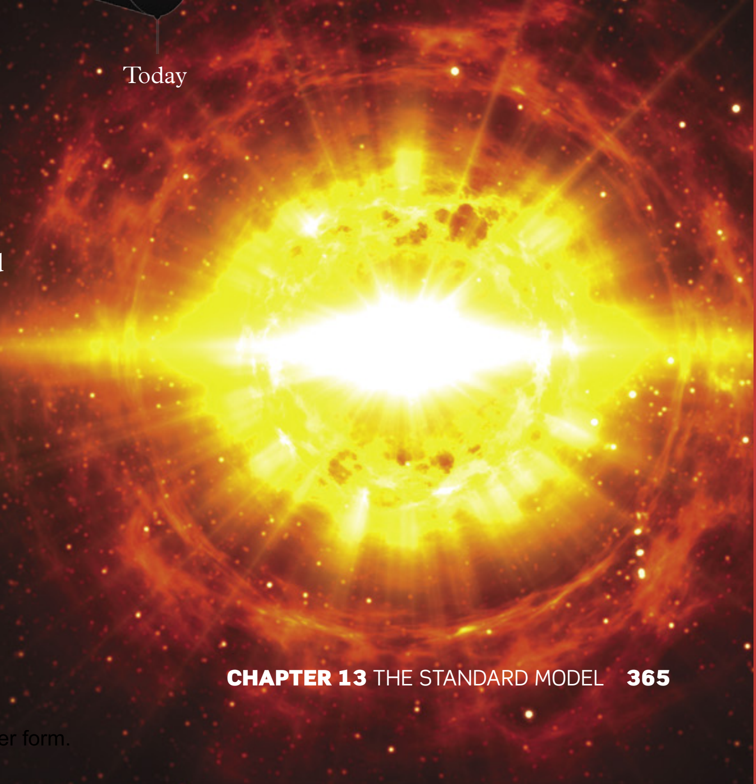


FIGURE 2 Big Bang theory – description of past, present and future

The Primordial Era: $t = 0$ to 3 minutes

About 13.8 billion years ago, the universe was just a tiny speck in space, a billion times smaller than a proton. It had almost infinite density and temperature. This was the beginning of time ($t = 0$) as we know it.

An enormous explosion (the Big Bang) occurred at $t = 0$. At 10^{-43} s after the Big Bang, the universe began to expand and cool at a fantastic rate. In this very short time the temperature fell from 10^{32} K to 10^{20} K.



Physicists don't know what particles were present at the Big Bang but they do know that whatever they were these particles soon decayed into lighter particles such as quarks, electrons, gluons, photons, neutrinos and dark matter. For every one billion antimatter particles produced there were one billion and one matter particles produced. As the universe cooled, the matter and antimatter quarks annihilated each other, leaving the excess fraction of matter quarks to survive.

Because there were more quarks than antiquarks, more protons and neutrons formed than antiprotons and antineutrons. These particles and antiparticles continued to annihilate each other; but as the temperature dropped further, fewer and fewer particles and their antiparticles were created, which just left an excess of protons and neutrons.

You are made up of quarks from the Big Bang.

Nucleosynthesis

The next major achievement of the early universe was the production of small compound nuclei. About one second after the Big Bang, when the temperature had dropped to 10^{10} K, the universe was cool enough to allow the fusing of protons and neutrons to synthesise (Greek *syn* meaning 'together', *tithenai* meaning 'to place') light atomic nuclei, mainly helium ($^{4}_{2}\mathrm{He}$) with some traces of deuterium ($^{2}_{1}\mathrm{H}$) and lithium ($^{7}_{3}\mathrm{Li}$). This nucleosynthesis continued for about three minutes until the temperature of the ever-expanding universe dropped to a mere 10^9 K. Nuclear reactions abruptly stopped and nucleosynthesis came to an end.

The Stelliferous Era: $t = 3$ minutes to 10^{14} years

'Stelliferous' comes from the Latin and means 'star-producing' (*stella* meaning 'star', *fero* meaning 'bring forth' or 'produce'). We are in the middle of this era right now. Our Sun ignited some 4.6 billion years ago (4.6×10^9 years) and has enough hydrogen to last another 6 billion years. We are all children of the stars because 10 billion years ago every atom in our bodies was once near the centre of a star.

Once the explosive first three minutes ended, the universe settled into a much calmer phase. For the next 300 000 years, the universe consisted of a sea of hydrogen and helium nuclei, photons, free electrons and the mysterious dark matter.

Gravity started pulling the hydrogen and helium together and this collapse produced the vast aggregations of gas and other matter we now call galaxies.

The future

This future of the universe is believed to be one of continued expansion. It could be slow (Big Freeze) or fast (Big Rip). In either case – the Big Rip is currently favoured – it is believed that the universe will go through a number of different stages.

- **Degenerate Era ($t = 10^{15}$ to 10^{39} years):** The stars will have burnt out, leaving all stellar-mass objects as stellar remnants such as white dwarfs, neutron stars, and black holes.
- **Black Hole Era ($t = 10^{40}$ to 10^{100} years):** The white dwarfs, neutron stars, and other smaller astronomical objects will have been destroyed by proton decay, leaving only black holes.
- **Dark Era, ($t = 10^{100}$ years to eternity):** Even black holes will have disappeared, leaving only a dilute gas of photons and leptons.
- **The end:** The final solution sees two possible scenarios in the future: the Big Freeze, in which the universe dies a cold lonely death, or the Big Rip, in which the expansion proceeds at an increasing rate. At present, the evidence is in for the Big Rip, but physics knowledge proceeds fast and in your lifetime our model of the universe will continue to be examined. No doubt, it will be called the 'Big' something.

CHECK YOUR LEARNING 13.3

Describe and explain

1 **Explain** how each of the following terms relates to the creation or future of our universe:
   - a before the Big Bang
   - b inflation in first 3 minutes after Big Bang
   - c Edwin Hubble's discoveries
   - d how matter beat antimatter
   - e the end of the universe scenarios

Apply, analyse and interpret

2 The four diagrams in Figure 3 represent different models for the fate of the universe from the Big Bang onwards.
   - a **Assess** what each diagram is predicting.
   - b **Propose** which model is most accepted by scientists.
   - c **Justify** this choice by evaluating recent evidence that supports or refutes each model.

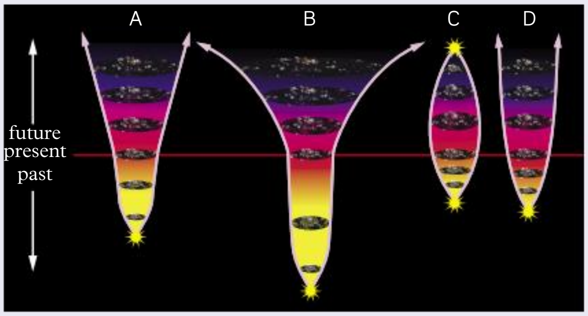


FIGURE 3 Possible fates of the universe

Investigate, evaluate and communicate

3 A student claimed that the universe is accelerating in its rate of expansion. **Discuss** whether you agree and justify your position by evaluating the evidence for and against this statement and making a conclusion.

4 Figure 4 shows the spectra of three Type 1a supernovae labelled star A, star B and star C. The black lines are 'hydrogen lines' that represent the presence of hydrogen in the stars. The spectrum of a hydrogen sample in a laboratory on Earth is shown as the reference diagram.

Use your understanding of astronomy to answer these questions.
   - a **Propose** what the lines represent.
   - b **Deduce** which supernova would be the brightest.
   - c **Justify** your choice by evaluating the evidence for and against this statement in terms of speed (towards us or away) and distance from us.

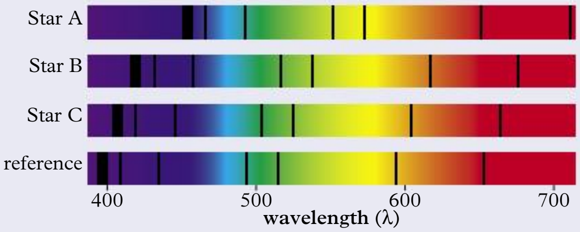


FIGURE 4 Spectra of supernovae

5 It has been claimed that the Deep Field image by Hubble can see back to 1 million years after the Big Bang.

A student made the following statements about this claim:

It means that astronomer Edwin Hubble used a telescope to search for the Big Bang.

It implies that the Big Bang happened 1 million years ago.
   - a **Express** a view on whether each of the student's comments are correct.
   - b **Justify** your statements for part (a) by **evaluating** the truthfulness of each of these two statements by considering points for and again each one.

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  What was the Big Bang?
- » Weblink
  Background radiation

CHAPTER 13

Review

Summary

13.1

- There are two types of elementary particles: leptons and quarks.
- Particles and their antiparticles have identical masses but opposite values of charge.
- Elementary particles have no internal structure.
- There are six quarks, called up (u), down (d), strange (s), charmed (c), top (t) and bottom (b).
- Hadrons are composite particles that are made up of quarks. There are two types of hadrons: baryons and mesons.
- Baryons, which include the neutron and the proton, consists of three quarks.
- Mesons, which include pions and kaons, are made up of a quark and antiquark.
- There are six leptons in the Standard Model: the electron, the muon, and the tau particles and their associated neutrinos; namely, the electron neutrino, the muon neutrino and the tau neutrino. There are also the corresponding six antileptons.

13.2

- In addition to the six fundamental leptons and six fundamental quarks, there are four mediating particles, called gauge bosons, that are associated with the four fundamental forces.
- The four fundamental forces are strong nuclear, electromagnetic, weak, and gravitational.
- The gauge bosons that act as mediating particles are the gluon for quarks, the W^+, W^-, Z^0 for the weak force, and photons for the electromagnetic force.
- Photons are their own antiparticle (this is important for crossing symmetry in Chapter 14).
- Hadrons (baryons and mesons) experience a residual force resulting from the fundamental strong nuclear force between the quarks that make up the hadrons.
- All particles with electric charge experience the force due to the electromagnetic interaction.
- Quarks and leptons have flavour and experience the weak interaction.
- All particles with mass experience the force due to the gravitational interaction.
- The electron, muon and tau have mass and electric charge, and participate in the gravitational, electromagnetic and weak interactions, but not the strong interaction.
- The neutrinos have no electric charge.

13.3

- Evidence for the Big Bang includes cosmic background radiation, the abundance of light elements, and the red shift of light from galaxies that obey Hubble’s law.

Key terms

- antibaryon
- antimatter
- antiparticle of matter
- antiproton
- antiquark
- baryon
- cosmological red shift
- electromagnetic force
- elementary particle
- fundamental forces
- gauge bosons
- generation
- gluon
- hadron
- Higgs boson
- Hubble's law
- lepton
- matter
- mediating particle
- meson
- particle zoo
- quark
- Standard Model
- strong nuclear force
- weak nuclear force

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 The elementary particles known at the moment are:

A baryons and leptons.
B baryons and quarks.
C leptons and quarks.
D photons and baryons.

2 The muon and the electron are both:

A baryons.
B hadrons.
C leptons.
D mesons.

3 The gluon is the force carrier for the:

A electromagnetic force.
B gravity.
C strong nuclear force.
D weak force.

4 Identify the false statement.

A The mediating particle for gravity is the graviton, but it hasn't been discovered yet.
B Gluons have such a small mass that they can move very fast, and this is why the strong force has such a long range.
C The interactions between subatomic particles are mediated by the exchange of particles called gauge bosons.
D The strong nuclear force is mediated by the exchange of gluons.

5 The carrier particle/s that is/are transmitted solely between nucleons is/are the:

A gluon.
B pion.
C photon.
D W and Z bosons.

6 The quarks that make up most of the universe are:

A u, d
B c, s
C t, b
D all of the above

7 Which one of the following is not composed of quarks?

A neutron
B muon
C baryon
D hadron

8 The strong nuclear force between protons and neutrons is due to:

A exchange of gravitons between protons and neutrons.
B exchange of gluons between protons and neutrons.

C exchange of photons between protons and neutrons.

D electrons cancelling out the proton's positive charge.

9 The particle that is massless is the:

A electron.

B Higgs boson.

C photon.

D up quark.

10 The force that is not described by the Standard Model is:

A electromagnetism.

B gravitation.

C strong force.

D weak force.

Short answer

Describe and explain

★ **11** **Recall** the names and charges of the six types of leptons and their antiparticles.

★ **12** **Clarify** which of the following are elementary particles: proton, neutron, quark, muon, electron, gluon, Higgs boson, atom, photon.

★ **13** **Clarify** whether a positron and an anti-electron are identical particles.

★★ **14** Baryons and mesons are similar in many ways but have two major differences in their composition. **Identify** these two differences.

★★ **15** **Identify** similarities and differences between the first family of quarks and the first family of leptons.

★★ **16** **Describe** the properties of the particles made up of the following quarks.

a $u\bar{d}$

b $\bar{d}u$

c ddc

d uss

Apply, analyse and interpret

★ **17** **Compare** the force between leptons (such as electrons) with the force between antileptons (such as positrons).

★ **18** **Sequence** the six quarks into their three families (generations) and distinguish between each family.

★ **19** **Classify** the four types of gauge bosons by:

a the force they mediate

b the particles they mediate between.

★★ **20** **Identify** similarities and differences between particles and antiparticles.

★★ **21** **Determine** the charge on these baryons.

	Name	Symbol	Quarks
a	charmed lambda	Λ_c^+	udc
b	sigma	Σ^+	uus
c	charmed sigma	Σ_c^{++}	uuc
d	xi	Ξ^-	dss

★★ **22** **Determine** the charge on these mesons.

	Name	Symbol	Quarks
a	charmed eta meson	η_c	$c\bar{c}$
b	strange B meson	B_s^0	$s\bar{b}$
c	strange D meson.	D_s^+	$c\bar{s}$

★★ **23** The symbols for a D meson and its antiparticle are D^0 and $\overline{D}^0$, but not necessarily in that order. **Infer** which is the particle and which is the antiparticle, with reasons.

Investigate, evaluate and communicate

★★ **24** **Decide** whether the force between leptons is the same as the force between quarks.

★★ **25** **Create** three lists that divide the fundamental particles of the Standard Model (including bosons) into groups based on their charge (+, −, neutral).

★★ **26** Physicists have defined a category of particles known as *hadrons*. Any composite particle containing quarks (or antiquarks) is a hadron. **Assess** which of the following are hadrons: proton, neutron, electron, muon, meson, gluon, graviton, Higgs boson, alpha particle, beta particle, positron, hydrogen nucleus, deuterium nucleus, ${}^{7}_{3}\text{Li}^{3+}$ ion, tauon, antineutron.

★★★ **27** **Propose** the total number of elementary particles in the Standard Model. You should not include quark colours in your answer but you will need to provide the names of all particles.

★★★ **28** The Standard Model proposes that the forces between quarks and between leptons are transmitted by particles known as gauge bosons. What puzzled physicists was that some particles, like W and Z bosons, had mass but others, such as gluons and photons, didn't.

To account for this, they proposed a particle – the Higgs boson, and its associated field – that causes other particles to have mass. The Higgs field uses Higgs boson particles to transmit its effects. Think of it in the same way as a gravitational field uses gravitons (proposed) to transmit the force of gravity. The Higgs boson is difficult to detected directly like an electron or proton as it has a tiny lifetime – about 10^{-23} seconds – but nevertheless it was discovered in 2013 the Large Hadron Collider in Switzerland.

a The Higgs field uses Higgs bosons to transmit its force. **Propose** what bosons are used by the electromagnetic field to transmit its force?

b **Compare** a photon and a Higgs boson for one similarity and one difference.

c **Deduce** one similarity and one difference between a graviton and a Higgs boson.

d **Draw a conclusion** about whether quarks are affected by the Higgs field.

★★★ **29** In early 2015, scientists at the Large Hadron Collider (LHC) announced the discovery of a new kind of particle called the pentaquark, solving a 50-year-old puzzle about the building blocks of matter.

Physicists had theorised the existence of the pentaquark since the 1960s, but had never been able to prove it until its detection at the LHC, the world's most powerful particle smasher.

A spokesman for the LHC said the pentaquark represented a way to combine five quarks – the subatomic particles that make up protons and neutrons – 'in a pattern that has never been observed before in over 50 years of experimental searches'.

Synthesise the information from this chapter, including Tables 1 and 3 in Section 13.1 to propose one possible combination of quarks and/or antiquarks to make the pentaquark with a charge of +1. There may be many, but you should show just one.

You should use the normal principles of quark combination to list the quarks present and their properties.

a **Discuss** the principles you used to decide which quarks were selected for the pentaquark.

b **Explain** how they are used to decide the final property of the pentaquark.

c **Infer**, with justification, whether the following terms apply to the pentaquark:

- meson
- baryon
- boson
- hadron (optional term)

d **Comment** on the likelihood that the pentaquark is made up of five antiquarks.

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Chapter 13

CHAPTER 14

Particle interactions

There is an old saying in physics that 'everything that can happen without violating a conservation law does happen'. It doesn't say 'might happen', it says 'does happen'. So, if a physicist knows a particular event must be happening somewhere in the universe, they set themselves the task of observing it and quantifying it. That's just one of the things that drives science along. But you will see that some interactions are ruled out by energy and momentum conservation laws – so there is no point looking for those. This chapter looks at a variety of particle interactions and shows you how to work out what's allowable and what is not allowable.

OBJECTIVES

- → Define the concept of lepton number and baryon number.
- → Recall the conservation of lepton number and baryon number in particle interaction.
- → Explain the following interactions of particles using Feynman diagrams:
  - – electron and electron
  - – electron and positron
  - – a neutron decaying into a proton.
- → Describe the significance of symmetry in particle interactions.

Source: *Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority

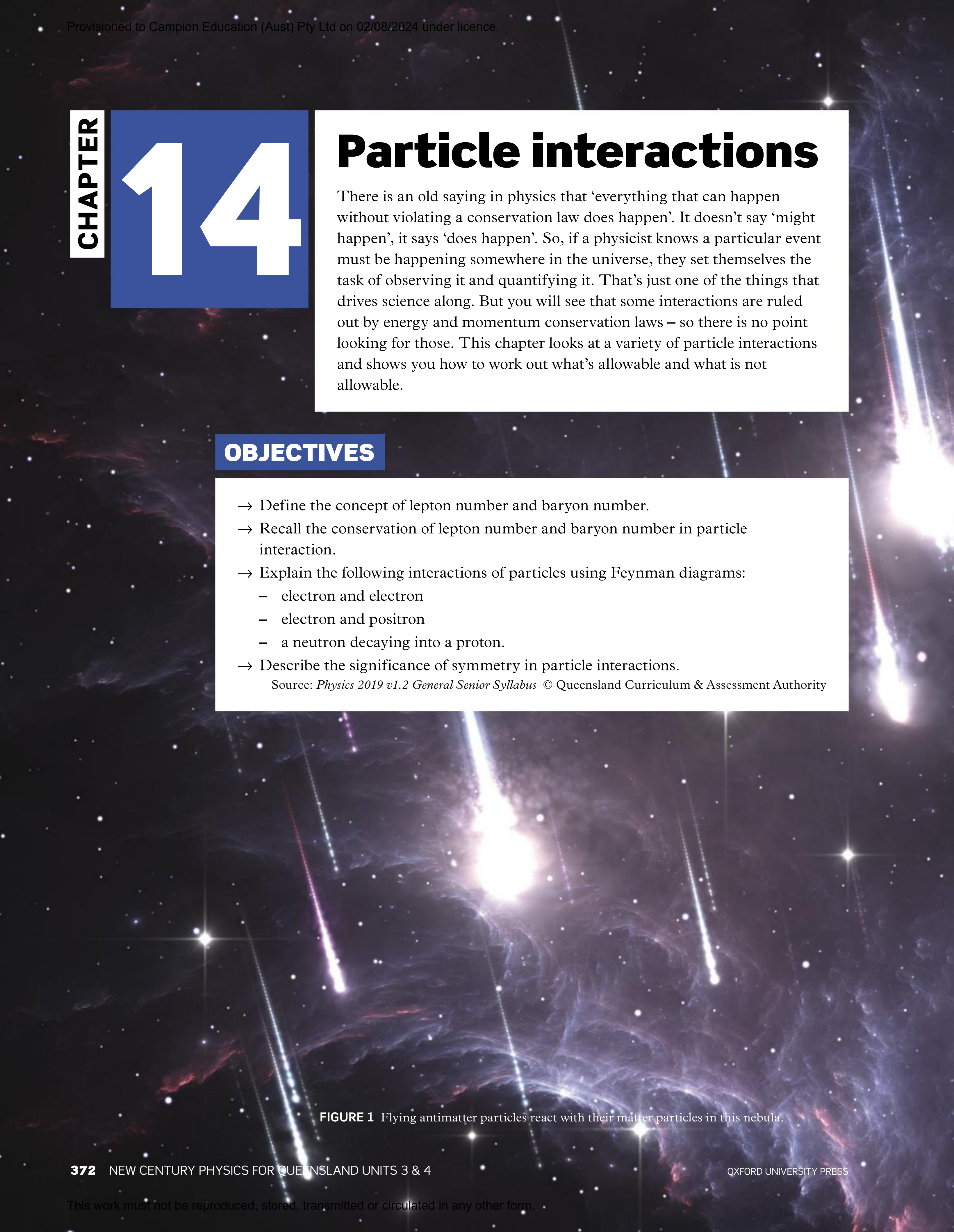
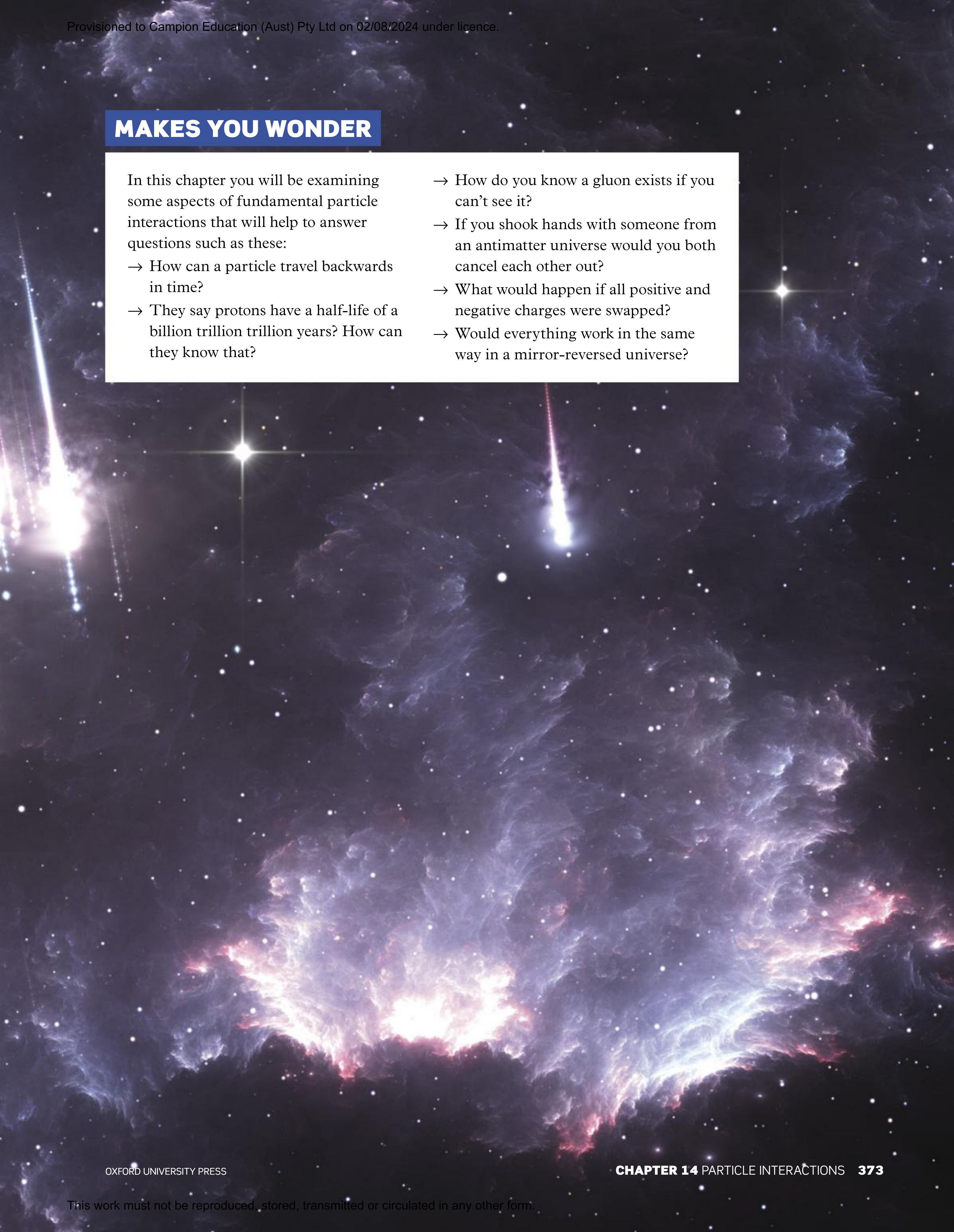


FIGURE 1 Flying antimatter particles react with their matter particles in this nebula.



MAKES YOU WONDER

In this chapter you will be examining some aspects of fundamental particle interactions that will help to answer questions such as these:

→ How can a particle travel backwards in time?

→ They say protons have a half-life of a billion trillion trillion years? How can they know that?

→ How do you know a gluon exists if you can't see it?

→ If you shook hands with someone from an antimatter universe would you both cancel each other out?

→ What would happen if all positive and negative charges were swapped?

→ Would everything work in the same way in a mirror-reversed universe?

14.1

Conservation in interactions

KEY IDEAS

In this section, you will learn about:

- lepton and baryon number
- the principles of fundamental particle conservation.

One of the great principles of physics is conservation. You have already learnt about conservation of energy, of momentum, and of electric charge. These all still hold for particle interactions. However, there are a number of new conservation laws that help explain why some reactions occur and others do not.

In physics, a conservation law states that a particular measurable property of an isolated physical system does not change as the system changes over time. Some were mentioned above. There are also many approximate conservation laws, which apply to such quantities as mass, charge, lepton number, baryon number and strangeness. These quantities are conserved in certain classes of physics processes, but not in all. This section will look at two of the most fundamental predictions of conservation laws in particle physics – the laws of conservation of **baryon number** and lepton number.

baryon number a quantum number of a system defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks; it is strictly conserved and additive

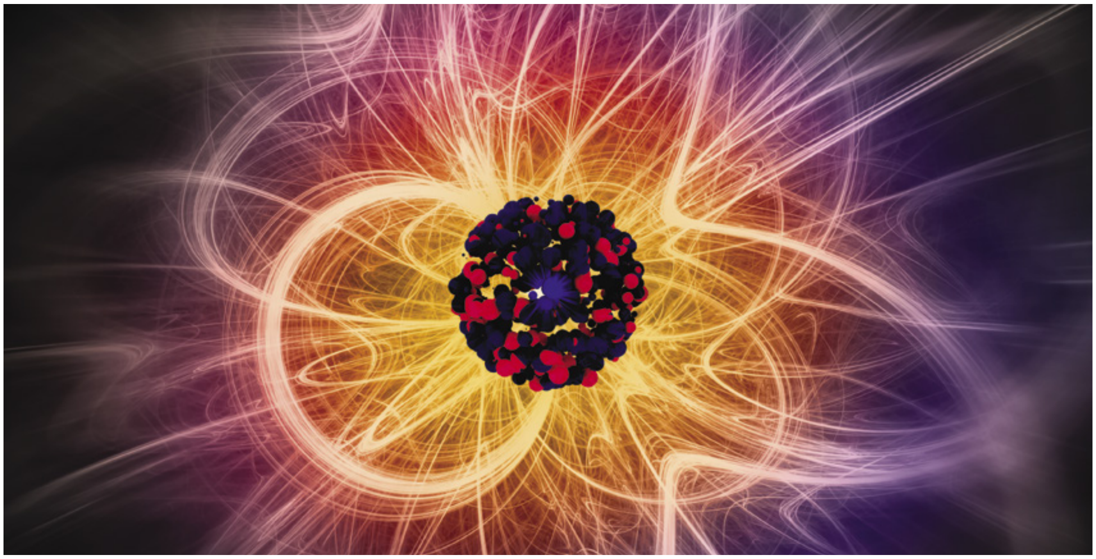


FIGURE 1 An abstract illustration of particles in collision

Baryon number

conservation of baryon number in any interaction baryon number is conserved

Before you can look at **conservation of baryon number**, you need to consider what baryon number is. (Recall from Chapter 13 that baryons are composite subatomic particles made up of three quarks (or in the case of antibaryons, three antiquarks) held together by the strong nuclear force.) Put simply, if a particle is a baryon its baryon number, B, is +1. If it is an antibaryon its baryon number is –1. Mesons, which are made up of a quark and antiquark pair and are not baryons, must have a baryon number of 0. That sums it up well, but let's dig a little deeper. For instance, what is the baryon number of a quark? Well, as there are three quarks in a baryon, each quark has a baryon number of $+\frac{1}{3}$.

Baryon number is a strictly conserved additive quantum number of a system. That means baryon number is the same before and after an interaction (conserved); and that you can add and subtract the number of baryons (additive). It is defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks. Some examples are shown in Table 1. Note: you are not expected to determine baryon number quantitatively in any external assessment, but examples are provided on your obook so that you can understand what the number means and how it is calculated.

TABLE 1 Baryon numbers of some common particles

Particle	Quark composition	$B = \frac{1}{3}(n_q - n_{\bar{q}})$	Baryon no. *B*
quark	q	$B = \frac{1}{3}(1 - 0)$	$+\frac{1}{3}$
antiquark	$\bar{q}$	$B = \frac{1}{3}(0 - 1)$	$-\frac{1}{3}$
proton	uud	$B = \frac{1}{3}(3 - 0)$	+1
neutron	udd	$B = \frac{1}{3}(3 - 0)$	+1
antiproton	$\bar{u}\bar{u}\bar{d}$	$B = \frac{1}{3}(0 - 3)$	–1
antineutron	$\bar{u}\bar{d}\bar{d}$	$B = \frac{1}{3}(0 - 3)$	–1
meson	$q\bar{q}$	$B = \frac{1}{3}(1 - 1)$	0
lepton (e.g. electron)	e^-	$B = \frac{1}{3}(0 - 0)$	0

Study tip

Baryon numbers are as follows:

- quarks (q) = $+\frac{1}{3}$
- antiquarks ($\bar{q}$) = $-\frac{1}{3}$
- baryons (qqq) = +1 (e.g. protons, neutrons)
- antibaryons $\bar{q}\bar{q}\bar{q}$ = −1 (e.g. antiprotons, antineutrons)
- mesons ($q\bar{q}$) = 0 (e.g. pion, kaon)
- leptons and antileptons = 0 (e.g. electrons, positrons, neutrinos, antineutrinos)

Conservation of baryon number

You will recall that when you balanced a nuclear equation in Unit 1, the mass numbers of the reactants had to equal the mass number of the products. For example, consider the alpha bombardment of beryllium: ${}^{4}_{2}\text{He} + {}^{9}_{4}\text{Be} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$

You would have to check that the mass numbers (top numbers) were equal on both sides. In this example 4 + 9 = 12 + 1, so the mass number is balanced (or conserved). As protons and neutrons are baryons (composites of three quarks), this could also be referred to as baryon conservation. However, you have seen that there are more baryons than just protons and neutrons, so the idea of baryon conservation means something broader in this chapter.

In Worked example 14.1A on page 376, the baryon numbers of all species are given and you have to check whether the baryon number of the reactants, *B*(reactants), equals the baryon number of the products, *B*(products).

Study tip

You are not expected to calculate the baryon number of an individual particle, but you should be able to determine if the total baryon number is conserved.

WORKED EXAMPLE 14.1A

The equation $\Omega^- \rightarrow \Xi^0 + K^-$ shows the possible decay of an omega particle.

The baryon numbers for the particles involved are as follows:

Ω^-, omega (sss), $B = +1$ Ξ^0, Xi (uss), $B = +1$ K^-, kaon ($s\bar{u}$), $B = 0$

Predict whether the decay is allowable or whether it violates the law of conservation of baryon number.

SOLUTION

$B(\text{reactants}) = B\ (\Omega^-)$

$= +1$

$B(\text{products}) = B\ (\Xi^0) + B\ (K^-)$

$= +1 + 0$

$= +1$

The baryon number for reactants and products is the same, so this reaction would be allowable in terms of conservation of baryon number. In other words, it does not violate the law of conservation of baryon number.

A more complex situation arises when leptons are involved. This is shown in Worked example 14.1B below.

WORKED EXAMPLE 14.1B

Determine whether baryon number is conserved in this decay, given that a proton is a uud baryon, and a neutron is a udd baryon:

$p \rightarrow n + e^+ + \nu_e$

SOLUTION

Reactants:

- Proton: $B = +1$

Products:

- Neutron: $B = +1$

  A positron is a lepton: $B = 0$

  A neutrino is a lepton: $B = 0$

  Baryon sum of products $= +1 + 0 + 0$

  $= +1$

As $B(\text{reactants}) = B(\text{products})$, baryon number is conserved ($B = +1$ for reactants and products) therefore the reaction is allowable.

Note: The underlying calculations for baryon number in Worked example 14.1B above is shown below to aid comprehension, but is not required for external assessment.

- A proton is made of 3 quarks (uud), thus $B = \frac{1}{3}\left(n_q - n_{\bar{q}}\right) = \frac{1}{3}(3 - 0) = +1$.
- A neutron is made of 3 quarks (udd), thus $B = \frac{1}{3}\left(n_q - n_{\bar{q}}\right) = \frac{1}{3}(3 - 0) = +1$.
- A positron is a lepton (0 quarks), thus $B = \frac{1}{3}(0 - 0) = 0$.
- A neutrino has 0 quarks, thus $B = \frac{1}{3}(0 - 0) = 0$.

Now consider a case involving baryons, mesons and leptons, for which the reaction is found to be not allowable.

WORKED EXAMPLE 14.1C

Consider this decay reaction in which a proton decays into a neutral pion and a positron.

$p \rightarrow \pi^0 + e^+$

Determine whether baryon number is conserved, given that a proton is a uud baryon ($B = +1$), and a pion is a meson ($B = 0$), and the positron is a lepton ($B = 0$).

SOLUTION

Reactants:

- Proton: $B = +1$

Products:

- Pion (pi meson): $B = 0$
- Positron is a lepton: $B = 0$
- $B(\text{products}) = 0 + 0$
  $= 0$

Compare: $B(\text{reactants}) \neq B(\text{products})$.

Baryon number is not conserved and this reaction violates the law of conservation of baryon number, and so is not allowable.

Note: The underlying calculations for baryon number in Worked example 14.1C above is shown below to aid comprehension, but is not required for external assessment.

- A proton is made of 3 quarks (uud) or (qqq), thus $B = \frac{1}{3}\left(n_q - n_{\bar{q}}\right) = \frac{1}{3}(3 - 0) = +1$.
- A pion (pi meson) is a quark–antiquark pair ($q\bar{q}$) thus $B = \frac{1}{3}\left(n_q - n_{\bar{q}}\right) = \frac{1}{3}(1 - 1) = 0$.
- A positron is a lepton (0 quarks), thus $B = \frac{1}{3}(0 - 0) = 0$.

conservation of lepton number
in any interaction lepton number is conserved

lepton number
a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction: $L = \left(n_l - n_{\bar{l}}\right)$

Lepton numbers

Another useful particle conservation law is the law of **conservation of lepton number**. In particle physics, **lepton number** is a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction. Lepton number is an additive quantum number, so its sum is preserved in interactions. Mathematically, the lepton number is defined as:

$$L = \left(n_l - n_{\bar{l}}\right)$$

where n_l is the number of leptons, and $n_{\bar{l}}$ is the number of antileptons.

However, a lepton number is commonly assigned to individual particles by using the rule: lepton ($L = +1$), antilepton ($L = -1$) and non-leptons ($L = 0$).

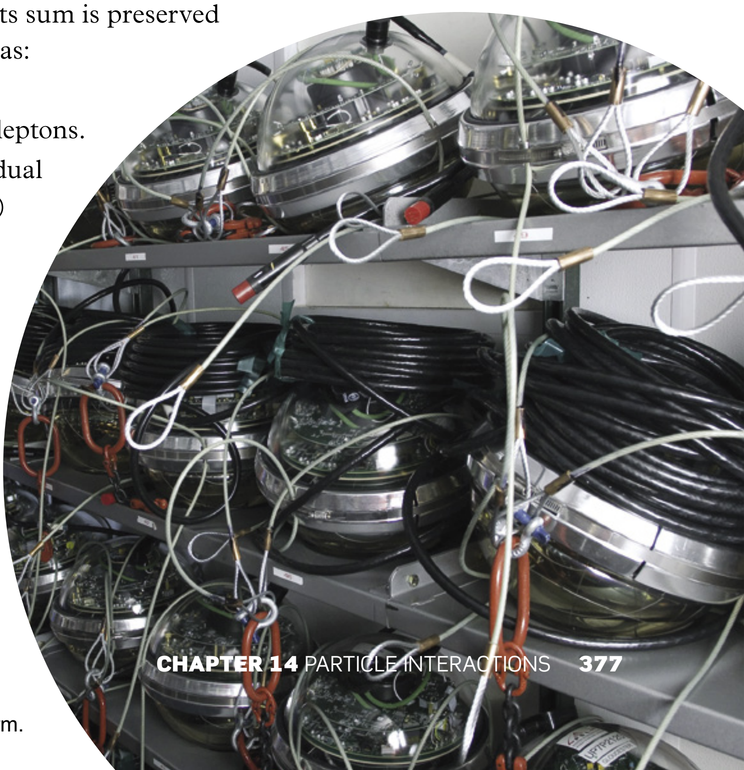


FIGURE 2 Neutrino detectors at the IceCube neutrino observatory waiting to be installed in the ice under the Amundsen–Scott South Pole Station. Conservation of lepton number is violated in beta emission interactions without the elusive neutrino.

Study tip

Lepton number is not the same as electric charge. For example, an electron e^- has an electric charge of −1 because it has a negative charge, but it has a lepton number of +1 because it is a lepton.

TABLE 2 Lepton numbers of some lepton and antileptons

Family	Lepton No. $L = +1$		Antilepton No. $L = -1$	
	Flavour	Symbol	Flavour	Symbol
First	electron	e^-	positron	e^+
	electron neutrino	ν_e	electron antineutrino	$\bar{\nu}_e$
Second	muon	μ^-	antimuon	$\bar{\mu}^+$
	muon neutrino	ν_μ	muon antineutrino	$\bar{\nu}_\mu$
Third	tau	τ^-	antitau	$\bar{\tau}^+$
	tau neutrino	ν_τ	tau antineutrino	$\bar{\nu}_\tau$

Non-leptons ($L = 0$) are baryons (qqq) such as protons and neutrons, and mesons ($q\bar{q}$) such as kappa-zero (K^0) and pions (π^+, π^0, π^-).

Study tip

You are not expected to calculate lepton number, but you should be able to determine if it is conserved. There is a short description after each Worked example of how lepton number is calculated to aid your understanding. The calculation is not for external assessment.

WORKED EXAMPLE 14.1D

Determine the lepton number for:

a tau

b tau neutrino

c tau antineutrino

d proton.

SOLUTION

a A tau is a lepton: $L = +1$

b A tau neutrino is a lepton: $L = +1$

c A tau antineutrino is an antilepton: $L = -1$

d A proton contains no leptons or antileptons: $L = 0$

Note: To demonstrate how the lepton number formula can be applied to these simple cases, the following three calculations from Worked example 14.1D are given to aid your comprehension:

- A tau is a lepton: $L = \left(n_l - n_{\bar{l}}\right) = (1 - 0) = +1$
- A tau neutrino is a lepton: $L = \left(n_l - n_{\bar{l}}\right) = (1 - 0) = +1$
- A tau antineutrino is an antilepton: $L = \left(n_l - n_{\bar{l}}\right) = (0 - 1) = -1$
- A proton contains no leptons or antileptons: $L = \left(n_l - n_{\bar{l}}\right) = (0 - 0) = 0$

Conservation of lepton number

Like baryon number, lepton number is also conserved in particle interactions. The method of testing for conservation is similar to the process for baryons.

WORKED EXAMPLE 14.1E

Determine whether lepton number is conserved for:

a the decay of a neutron: $n \rightarrow p + e^- + \bar{\nu}_e$

b the decay of a muon: $\mu^- \rightarrow e^- + \bar{\nu}_e$.

SOLUTION

a Reactants:

A neutron is not a lepton, so $L = 0$.

Total: L(reactants) = 0

Products:

A proton is not a lepton, so $L = 0$.

An electron is a lepton, so $L = +1$.

An electron antineutrino is an antilepton so $L = -1$.

Total: L(products) = 0 + (+1) + (–1) = 0

Therefore the lepton number (L) is conserved as L(reactants) = L(products) and this reaction is allowable.

Lepton number is conserved

b Reactants:

A muon μ^- is a lepton, so $L = +1$.

Total: L(reactants) = +1

Products:

An electron, e^-, is a lepton, so $L = +1$.

An electron antineutrino, $\bar{\nu}_e$, is an antilepton, so $L = -1$.

Total: L(products) = +1 + (–1) = 0.

Result: L(reactants) ≠ L(products). Lepton number is not conserved, so the law is violated and the reaction is not possible.

Note: To demonstrate how the lepton number formula can be applied to these simple cases, the following calculations from Worked example 14.1D are given to aid your comprehension:

a A neutron is not a lepton, so $L = \left(n_l - n_{\bar{l}}\right) = (0 - 0) = 0$.

A proton is not a lepton, so $L = \left(n_l - n_{\bar{l}}\right) = (0 - 0) = 0$.

An electron is a lepton, so $L = \left(n_l - n_{\bar{l}}\right) = (1 - 0) = +1$.

An electron antineutrino, $\bar{\nu}_e$, is an antilepton, so $L = \left(n_l - n_{\bar{l}}\right) = (0 - 1) = -1$.

b A muon μ^- is a lepton, so $L = \left(n_l - n_{\bar{l}}\right) = (1 - 0) = +1$.

An electron, e^-, is a lepton, so $L = \left(n_l - n_{\bar{l}}\right) = (1 - 0) = +1$.

An electron antineutrino, $\bar{\nu}_e$, is an antilepton, so $L = \left(n_l - n_{\bar{l}}\right) = (0 - 1) = -1$.

Conservation of baryon number and lepton number together

For a particle reaction to be able to occur, it has to obey the various conservation laws. As well as conservation of electrical charge and momentum, you now see there is conservation of baryon number and lepton number. Physicists have divided lepton conservation up into three subcategories – electron lepton (L_e), muon lepton (L_μ) and tau lepton (L_τ) – all of which have to be conserved. However, our rule for leptons in general will suffice for this level of work. Physicists have discovered that there are other conservation rules that have to be applied, namely conservation of charm and strangeness (for baryons and mesons), but you do not need to be concerned with that either.

Study tip

Neutrinos are uncharged but they still have a lepton number of +1. Antineutrinos have a lepton number of –1.

WORKED EXAMPLE 14.1F

The baryon sigma-plus Σ^+ is composed of uus quarks and decays into a proton (uud) and a pi-zero meson (π^0) according to the following equation: $\Sigma^+ \rightarrow p + \pi^0$. Prove that the reaction does not violate the baryon and lepton conservation laws.

SOLUTION

	Reactant	→	Products		Result
	Σ^+	→	p	π^0	
Baryon No. *B*	+1 (baryon)	=	+1 (baryon)	0 (meson)	*B* is conserved
Lepton No. *L*	0 (non-lepton)	=	0 (non-lepton)	0 (non-lepton)	*L* is conserved

Conclusion:

As both baryon number and lepton number are conserved, the reaction is allowable.

CHALLENGE 14.1

Quick now!

Is it possible for a tau lepton (whose mass is almost twice that of a proton) to decay into only baryons or mesons?

CHECK YOUR LEARNING 14.1

Describe and explain

1 **Explain** what the conservation of baryon and lepton numbers means for the likelihood of a given reaction occurring.

2 **Recall** the baryon number for these particles: muon, proton, baryon, meson.

3 **Recall** the lepton number for these particles: muon, proton, baryon, meson.

Apply, analyse and interpret

4 **Determine** the baryon number for these particles: a uus quark composite, an $\bar{s}\bar{s}\bar{s}$ quark composite, a $u\bar{d}$ quark composite.

5 **Determine** the lepton number for these particles: a uud quark composite, a $\bar{c}\bar{c}\bar{s}$ quark composite, a $d\bar{s}$ quark composite.

6 The pi-negative meson (π^-) has the quark composition of $u\bar{d}$. It reacts with a proton (uud) to produced a kappa-zero meson K^0 ($d\bar{s}$) and a lambda-zero baryon Λ^0 (uds) according to the following equation: $\pi^- + p \rightarrow K^0 + \Lambda^0$. **Determine** whether the reaction would violate the baryon and lepton conservation laws.

Investigate, evaluate and communicate

7 Lepton number was introduced in 1953 to explain the Cowan–Reines neutrino experiment, which investigated these two possible reactions:

$$\bar{\nu} + n \rightarrow p + e^- \text{ and } \bar{\nu} + p \rightarrow n + e^+$$

Predict which reaction violates conservation of lepton number and did not occur.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 14.1
- » Challenge worksheet
  14.1 Quick now!
- » Video
  Calculating baryon and lepton numbers

14.2

Feynman diagrams

KEY IDEAS

In this section, you will learn about:

- the following interactions of particles using Feynman diagrams
  - electron and electron scattering
  - electron and positron scattering and annihilation
  - a neutron decaying into a proton.

Over this course of study you have learnt about objects interacting with each other. You have seen how energy and momentum are conserved in collisions and explosions, and more recently how charge, baryon number and lepton number are conserved. You have also seen how leptons and baryons interact via mediating particles such as gluons and photons, and W and Z bosons. Trying to illustrate all of this in simple diagrams is not easy.

For example, how do you show two electrons approaching and colliding with each other, exchanging a photon as the mediating particle and then rebounding away from each other? The interaction of subatomic particles can be complex and difficult to understand intuitively. The US physicist Richard Feynman devised a method in 1948 that proved exceedingly useful and popular so that it remains in use today.

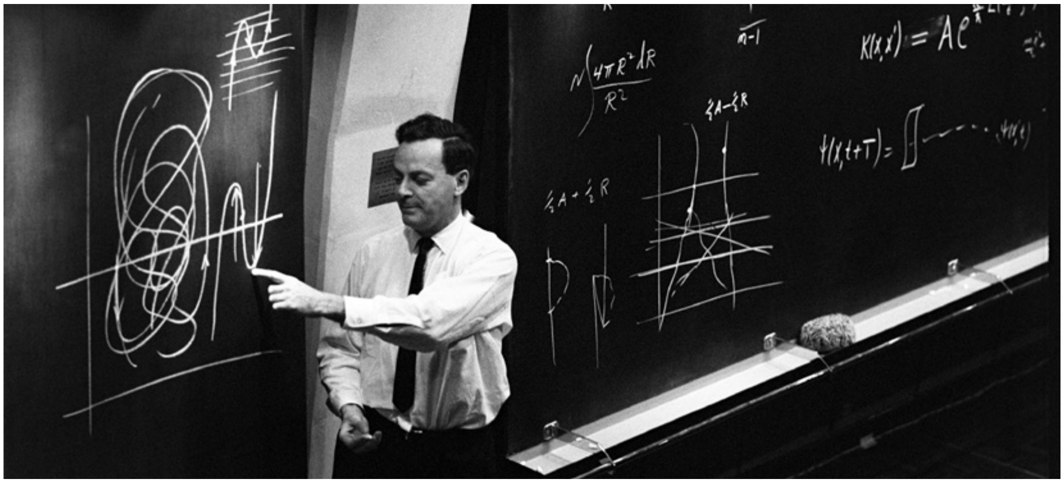


FIGURE 1 Feynman diagrams were named after their inventor Richard Feynman, physicist and 1965 Nobel laureate.

Feynman diagrams

There are several conventions adopted by physicists for drawing **Feynman diagrams**. You should learn these.

Feynman diagram
the graphical representation of particle interactions with time along the horizontal axis and space along the vertical axis

Convention 1: space–time graph

We represent space on the vertical axis and time on the horizontal axis.

Feynman diagrams can be thought of as space and time diagrams that outline the 'trajectory' of particles. These diagrams are read from left to right. This is much the same as the displacement–time graphs used for linear motion.

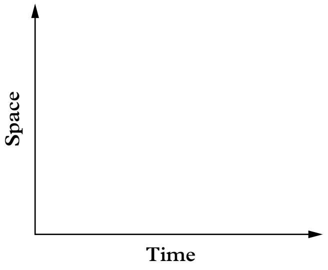


FIGURE 2 Feynman diagram axes

Convention 2: particle motion

We represent particles as straight-line arrows in the direction of time, with a letter indicating the type of particle represented. For example, an electron and an electron neutrino are shown in Figure 3. Parallel lines would indicate similar speeds.

FIGURE 3 Electron and electron neutrino travelling through space

Convention 3: antiparticle motion

The motion of an antiparticle is represented as pointing in the opposite direction to time (Figure 4).

FIGURE 4 Antiparticles are shown as if travelling backwards in time. Remember that the arrow for time points to the right →.

It appears that antiparticles travel backwards in time. There is dispute about whether this really happens or if it is just a mathematical idea. Physicists liken it to the way we think of conventional electric current as being electron current travelling backwards. It is just a graphical convenience. The path particles take through space is determined not only by the interactions (which are shown on Feynman diagrams), but also the kinematics (which are not). For example, how do you factor in momentum and energy conservation? The point of the Feynman diagram is to understand the interactions along a particle's path, not the actual trajectory of the particle in space.

Convention 4: Representing gauge bosons

We represent gauge bosons as wiggly lines, with a letter indicating the type of boson represented. Gauge bosons are the force carriers and these lines are usually placed between the particles.

FIGURE 5 Representations of gauge bosons

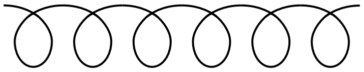


FIGURE 6 Alternative representation of a gluon

The gluon, which is the gauge boson for strong nuclear force between quarks and nucleons, is more commonly represented by a string of loops.

Note: you will not be using Feynman diagrams involving gluons, so it can be disregarded. If it is mentioned, a wiggly line with the letter 'g' would still be suitable.

By way of clarification, there are three classes of bosons:

1 gauge bosons (photon, W^+, W^-, Z, gluon g, and the hypothetical graviton g^0)
2 the scalar boson (Higgs)
3 composite bosons (particles made up of equal numbers of quarks and antiquarks, e.g. meson). In this section on Feynman diagrams you are concerned solely with gauge bosons.

Summary

The following conventions should be adhered to when constructing Feynman diagrams:

1. Represent space on the vertical axis and time on the horizontal axis.
2. Represent particles as straight-line arrows in the direction of time, with a letter indicating the type of particle represented.
3. Represent antiparticle arrows pointing in the opposite direction to time.
4. Represent gauge bosons as wiggly lines, with a letter indicating the type of gauge boson represented.

Study tip

'Time' will always be on the horizontal axis in this chapter, and in any external assessment. Some sources choose to have the axes reversed with time on the vertical axis. Be careful if you use these sources.

Study tip

You will not be expected to use Feynman diagrams containing gluons.

Particle interactions

Feynman diagrams consist of lines representing particles, but they also need to show how the particles interact.

To do this, there are several terms and conventions that you will need to understand.

Convention 1: Vertex

The point at which particles interact is called a **vertex** (plural, vertices). At the vertex they will emit or absorb new particles, deflecting one another, or changing type.

vertex
the point where particles interact (plural, *vertices*). At the vertex they will emit or absorb new particles, deflecting one another, or changing type

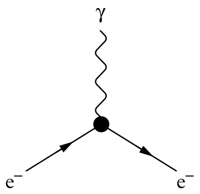


FIGURE 7 A vertex is sometimes shown as a dot, but it is just the point where they are all together. Here it shows where an electron (e^-) emits a photon (γ).

Convention 2: Vertex location – external particles

For external particles (real particles), if the particle is incoming, the vertex is on the right, and if it is outgoing the vertex is on the left. It is not always shown with a dot. The lines are drawn horizontally here to save space.

	Incoming	Outgoing
Fermion (electron, quark etc.)		
Antifermion (positron, antiquark)		
Gauge boson		

Convention 3: Vertex location – internal lines

For internal lines (representing virtual particles) the vertices are at both ends:

Gauge bosons	

There are other types of internal lines, but they are not to be dealt with here.

Convention 4: Vertex – number of lines

In Feynman diagrams, a vertex always has three lines attached to it: one gauge boson line, and two fermion lines: one in and one out.

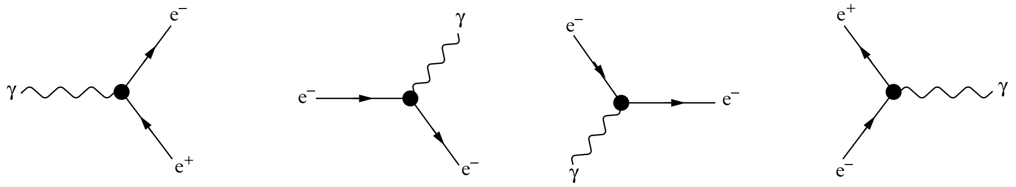


FIGURE 8 Four examples showing how each vertex has three lines attached. Can you identify the two fermion lines? Does the gamma symbol (γ) represent a fermion or a gauge boson?

Examples of particle interactions

There are three sets of particle interactions that are important in depicting how fundamental particle behave. These are the only ones required by the syllabus.

1 Electron and electron

One of the things electrons do when they collide is to scatter off in various directions, unchanged other than in their velocity. The original particle colliders were designed specifically for electron–electron collisions, but we also see this occurring naturally in the repulsion of electrons in the helium atom, for example. The process, known as Møller scattering, can be followed in a Feynman diagram and represented in equation form as follows:

$$e^-e^- \rightarrow e^-e^-$$

Study tip

Learn Figure 9 as an electron–electron interaction. You do not have to use the term 'Møller scattering'. Note that the photon line (the wiggle) is almost vertical, showing that the electrons interacted ('bounced') but didn't change into something else.

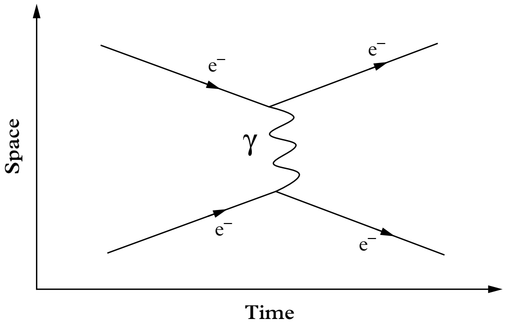


FIGURE 9 An electron–electron interaction in Møller scattering. Can you identify the vertices and state how many lines are attached to each?

As an electron approaches another electron they move closer. This is shown by the lines on the left getting closer together as they move towards the right as time passes.

The electrostatic repulsion between the two negative charges gets stronger and stronger as they invade each other's space, so they begin to slow down. At a certain distance they exchange a photon, which pushes each electron away from the other. With electrons, the force carrier (the photon) is created, then acts between them and disappears. It is called a 'virtual' photon as it only exists as long as the interaction takes place. It doesn't have an independent existence of its own.

After the interaction the two electrons move away from each other, having changed velocities (magnitude and direction). This is shown as the two lines of the Feynman 'tree' moving apart.

2 Electron and positron

In particle physics, the interaction between an electron and a positron can be represented in equation form as:

$$e^-e^+ \rightarrow e^-e^+$$

but this hardly tells us anything about what is happening. The interaction can actually take two forms.

Bhabha scattering

The first is called Bhabha scattering after the Indian physicist Homi J. Bhabha (1909–1966). You need to know this as electron–positron interaction.

Here you see the use of the 'time-reversed' arrow for the positron at the bottom left. It is really approaching the electron at the top left and their lines move closer together. At the moment of interaction they exchange a virtual photon (γ) and scatter off each other with just their velocities changed. Bhabha gained international prominence after deriving a correct expression for the probability of this type of scattering.

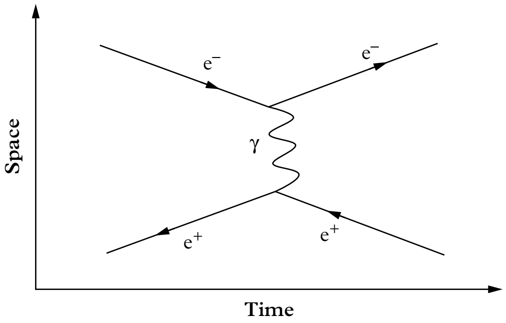


FIGURE 10 Bhabha scattering in an electron–positron interaction. There are two vertices; can you identify them? Does this diagram obey the convention of three lines at each vertex?

Study tip

Learn Figure 10 as an electron–positron interaction. It is also known by the term 'Bhabha scattering'. Note again that the photon wiggly line is almost vertical, showing that the electrons interacted ('bounced') but didn't change into something else.

Electron–positron annihilation

This is the second process that can happen during an electron–positron interaction.

By now you should be able to interpret this. Again, the electron and positron approach each other on the left. The time-reverse arrow indicates the positron (e^+). As they get closer they do something rather strange. Rather than scatter (bounce) off each other they **annihilate** (wipe out) each other. They go up in a puff of smoke in a manner of speaking. Actually they form a virtual photon (γ). Being virtual the photon lives for a very short amount of time but then out of the virtual photon's energy a new electron–positron pair is created. This is a great example of energy (γ) being expressed as matter (e^-, e^+). This is the same as the way in which matter was created in the Big Bang: energy was converted into particle–antiparticle pairs.

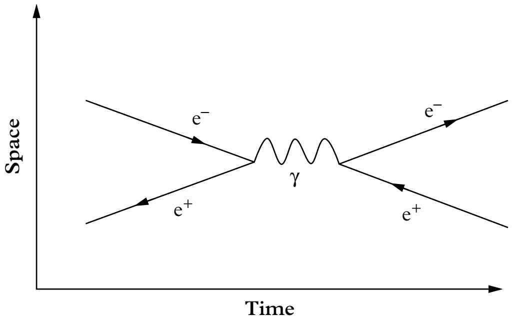


FIGURE 11 Electron–positron annihilation, again with two vertices. Each vertex has three lines.

annihilation the process that occurs when a subatomic particle collides with its respective antiparticle to produce other particles such as photons

Study tip

Learn Figure 11 as an electron–positron annihilation. Note this time that the wiggly line representing the photon is **horizontal**, showing that the electron and positron actually combined (annihilated) in this interaction to form the photon. The electron and positron effectively disappeared by combining to form this photon, which then turned back into a new electron and positron.

3 Neutron decaying into a proton

For this third and final type of interaction, consider the beta decay of carbon-14:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_e$$

Carbon-14 is unstable and has too many neutrons for the number of protons. It can shed some of its excess energy by undergoing beta decay in which one of the neutrons turns into a proton. It uses some of its excess energy to form a temporary (virtual) intermediate particle called the W^- boson that disappears by forming an electron and an electron antineutrino:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

This is a process called quark flavour changing. The neutron is an udd quark composite, and a proton is a uud composite. All that is happening is that a *down* quark turns into an *up* quark and produces a W^- boson: $d \rightarrow u + W^-$. The W^- boson immediately decays into an electron and an electron antineutrino: $W^- \rightarrow e^- + \bar{\nu}_e$. This can be represented in a simple diagram (Figure 12).

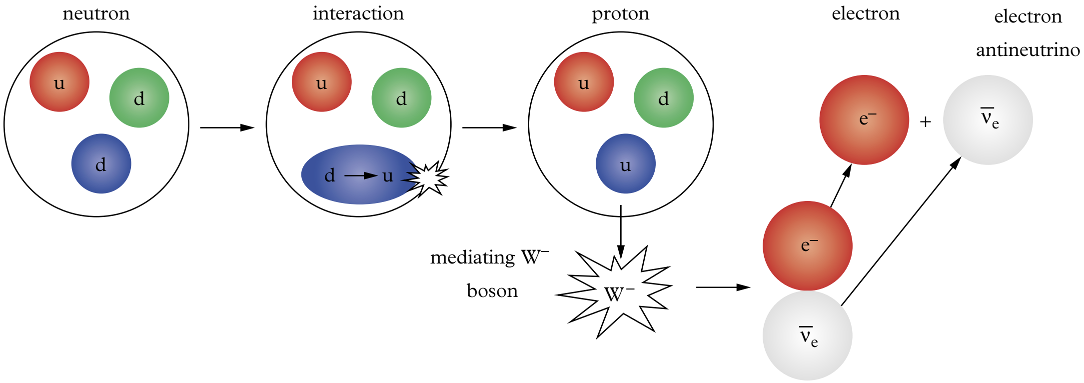


FIGURE 12 A neutron decays into a proton by emitting a beta negative particle.

This sort of diagram is messy and doesn't convey as much information as a Feynman diagram, which is shown in Figure 13.

The flavour of the down quark changing into an up quark can also be shown (Figure 14).

Study tip

Ensure you can understand, construct and interpret Feynman diagrams of a neutron decaying into a proton. They are complex and highly likely to appear in the external assessment, possibly in multiple-choice questions.

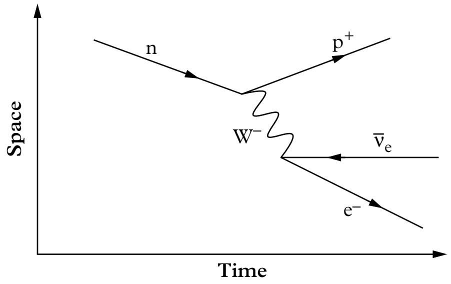


FIGURE 13 Beta negative decay of a neutron as depicted by a Feynman diagram

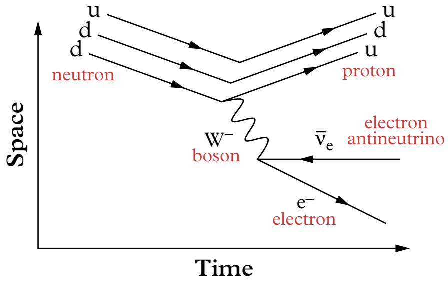


FIGURE 14 Beta negative decay of a neutron showing flavour change of a down quark

CHALLENGE 14.2A

Time on the horizontal axis

In some books the axes for Feynman diagrams are reversed so that time appears on the vertical axis, and position along the horizontal axis. Redraw Figure 15 so that it follows the usual convention with time along the bottom.

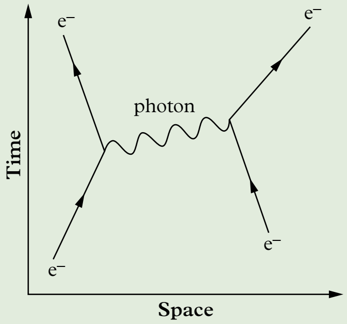


FIGURE 15 Feynman diagram with axes reversed

CHALLENGE 14.2B

Travelling backwards in time

Antielectrons (positrons) are said to be electrons travelling backwards in time. Figure 16 is for a muon. Draw a diagram for an antimuon.

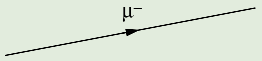


FIGURE 16 Feynman diagram for a muon

4 Proton decaying into a neutron

As a comparison, consider an example of beta positive decay in which a positron and an electron neutrino are formed. In this case, it is a proton that changes into a neutron by quark flavour changing. An up quark transforms into a down quark.

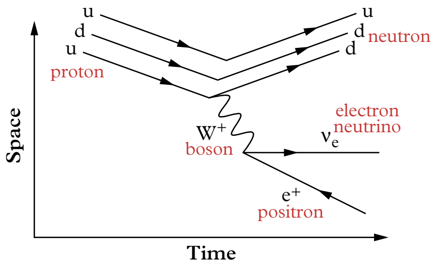


FIGURE 17 Beta positive decay of a proton showing flavour change of an up quark

Study tip

You will not be asked about a proton decaying into a neutron in any external assessment. It is included here as another example. Understand it, but there is no need to learn it off by heart.

CHECK YOUR LEARNING 14.2

Describe and explain

1 **Explain** whether the purpose of a Feynman diagram is to show the trajectory of the particles or the nature of the interaction.

2 **Recall** the convention about time and space used in constructing Feynman diagrams.

3 **Sketch** the Feynman symbol for a gauge boson.

4 Consider the beta decay of a neutron.

  a **Sketch** a Feynman diagram that indicates what happens at the quark level.

  b **Recall** whether baryon number and lepton number are conserved.

5 **Explain** what the arrow for a positron represents in an electron–positron interaction (Bhabha scattering).

Apply, analyse and interpret

6 **Interpret** the interaction in which an electron and positron annihilate to form a photon, to suggest where the new electron and positron came from.

7 **Analyse** the following interaction and draw a Feynman diagram:

An up quark interacts with a antidown quark to form a W^+ boson. The W^+ boson immediately decays into an antimuon and a muon neutrino.

Investigate, evaluate and communicate

8 **Evaluate** this claim by referring to the appropriate Feynman diagram: ‘When a neutron decays into a proton, a positive charge is created out of nothing.’

Check your obook assess for these additional resources and more:

- Student book questions
  Check your learning 14.2
- Challenge worksheet
  14.2A Time on the horizontal axis
- Challenge worksheet
  14.2B Travelling backwards in time
- Weblink
  Examples of particle interactions

14.3

Symmetry in particle interactions

KEY IDEAS

In this section, you will learn about:

- the significance of symmetry in particle interactions.

symmetry
when a particle interaction is subjected to a certain operation and it appears exactly the same after the operation

One of the features of modern physics is the use of **symmetry**. For example, an experiment performed in one location should have the same result as an identical experiment performed elsewhere or at a different time. During the 20th century, particle physicists believed that the laws of physics governing elementary particles would not change when tested under various transformations such as the reversal of direction in space or time. Physicists hoped to answer the question of why the Big Bang produced more particles than antiparticles when theory suggested they should be exactly equal: for every billion antimatter particles produced there was a billion and one matter particles. This 'symmetry violation' is one of the big unanswered questions of modern physics.

There are many different forms of symmetry that physicists deal with, but there are three simple ones that play a unique role: charge reversal, time-reversal, and crossing. You will consider examples of each of these to understand what symmetry means and to determine whether it has been violated experimentally.

Charge-reversal symmetry (C)

charge-reversal symmetry
that interactions are not affected if all charges are swapped (i.e. positive for negative and vice versa)

Charge-reversal symmetry says that interactions are not affected if all charges are swapped. That is, there is nothing special about what we call positive charge and nature treats it equal and opposite to negative charge. But this idea also includes switching from particle to antiparticle and vice versa, such as neutrinos to antineutrinos, and neutrons to antineutrons. A good example is the decay of a pion:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Its charge-reversed equivalent has its particles swapped for the antiparticle (negative and positive charges are swapped for their opposites, and neutrinos and antineutrinos are swapped for their opposites). Hence, the π^+ becomes π^- and ν_μ becomes $\bar{\nu}_\mu$.

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Both have been seen experimentally, so you can say charge reversal is symmetrical. Note that uncharged bosons such as the photon are unchanged during charge reversal symmetry operations.

WORKED EXAMPLE 14.3A

Predict the outcome of charge reversal symmetry on the decay of a muon:

$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

SOLUTION

Change all particles to their antiparticles

$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

Time-reversal symmetry (T)

Richard Feynman said that the fundamental physical laws, on a microscopic and fundamental level, are completely reversible in time. Yet our everyday experience tells us that there are many situations in which a system seems to go in one direction and not the reverse, suggesting that there actually is a preferred direction in time. For example, a glass falling to the ground and smashing into tiny pieces is a process that always seems to occur in the same way and never in the reverse; the broken glass never reforms into an unbroken glass. However, Feynman used the words 'on a microscopic and fundamental level' and therein lies the difference. At an atomic scale time is reversible. In other words, products become reactants, and reactants become products.

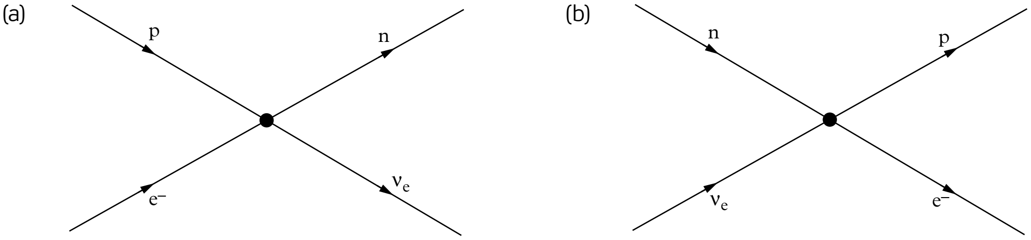


FIGURE 1 Time-reversal symmetry of an inverse beta decay (a), to a beta negative decay (b), as depicted in a Feynman diagram

Consider a particle interaction called inverse beta decay in which a proton in a nucleus captures an orbital electron to form a neutron and an electron neutrino (Figure 1a). This process is also known as electron capture and is a well known way in which radioisotopes gain stability by getting rid of protons.

Time is shown along the horizontal axis going from past (on the left) to the future (on the right). To run this backwards in time, just flip the Feynman diagram about a vertical axis as in Figure 1b. Now you would see a neutron grab an electron neutrino and form a proton and an electron. This is known as beta decay and is a well-known way in which a nucleus gets rid of a neutron. So time is reversible, as shown by this example of **time-reversal symmetry**.

time-reversal symmetry
an interaction looks the same if the flow of time is reversed (products become reactants, and reactants become products)

WORKED EXAMPLE 14.3B

Figure 2 shows another version of inverse beta decay.

a Draw a Feynman diagram to show time-reversal symmetry.

b Describe the particle interaction process for the original and time-reversed interactions.

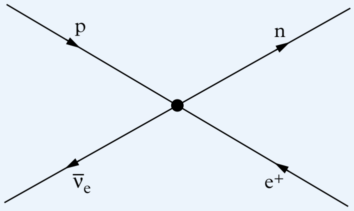


FIGURE 2

SOLUTION

In Figure 2, a proton interacts with an electron antineutrino to form a neutron and a positron. In the time-reversed diagram (Figure 3), a neutron interacts with a positron to form a proton and an electron antineutrino.

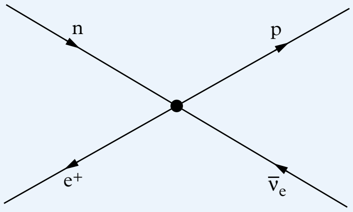


FIGURE 3

Crossing symmetry (X)

crossing symmetry if a particle interaction is observed to occur, any of the particles can be replaced by its antiparticle on the other side of the interaction

If a particular particle interaction is known, you can anticipate other possible interactions by replacing a particle with its antiparticle on the other side of the interaction. This is commonly known as '**crossing symmetry**'. For example, consider the interaction:

$$A + B \rightarrow C + D$$

You can expect the following interactions:

$$A + B + \overline{C} \rightarrow D$$
$$A + \overline{C} \rightarrow \overline{B} + D$$
$$B \rightarrow \overline{A} + C + D$$

WORKED EXAMPLE 14.3C

Propose one other example of crossing symmetry in which two particles cross, using the reaction shown above: A + B → C + D

SOLUTION

There are many more, but one example would be $B + \overline{C} + \overline{D} \rightarrow \overline{A}$.

Study tip

It is highly likely that you will have a double-crossing symmetry operation in which a photon is shown as forming its own antiparticle. Be prepared for this by ensuring you do Questions 26 and 27 on page 399.

A real-life example is the capture of a positron by a neutron (Figure 4).

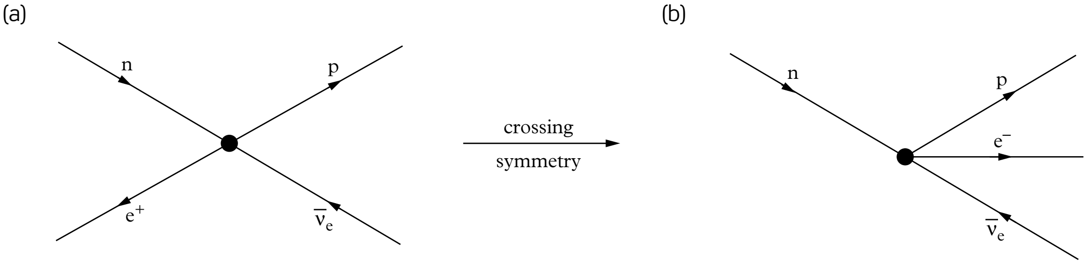


FIGURE 4 Crossing symmetry changes a positron capture to beta negative decay

The incoming positron (e^+) is replaced by an electron on the other side. This has become the familiar beta negative decay of a neutron. There are two points to be made about the crossing operation: (i) the photon is its own antiparticle so its symbol (γ) remains unchanged after a crossing operation; (ii) more than one crossing operation can be done on a particular interaction at the one time.

Probability of a reaction

The most powerful use of Feynman diagrams is to determine the probability of a particular reaction occurring. This is far too complex to detail here, but it is fair to say that every line in a Feynman diagram can be assigned a measure of probability (by looking up a table) and then summed for the whole diagram. Physicists refer to it as measuring the 'scattering amplitude' and regard it as the diagram's most important job. For example, there are nine different ways two electrons can interact, but some are highly unlikely (low amplitudes).

FIGURE 5 Time-reversal symmetry applied to neutron decay has a lower probability of occurring.

But you can try to predict which of two interactions may be more likely by considering what the interaction involves. For example, consider the two reactions shown in Figure 5. They are time-reversed versions of the same interaction. In the first case a neutron is decaying into a proton, electron and electron antineutrino. In the second case, a proton must collide with an electron and an electron antineutrino *at the same time*. The probability of three particles coming together at the same time is fairly low, so the first reaction is more likely to occur.

Significance of symmetry and its violation

In conclusion, there are many forms of symmetry and just three have been considered here. Symmetry reversal holds for the majority of interactions, but there are cases in which violations do occurs. Whatever the type of symmetry proposed, energy and momentum must also be conserved for such a reaction to be possible.

The violation of symmetries is not a bad thing. True, it stops a particular symmetry from being called a general law of nature, but it gives even more clues to the inner working of the atom. And that surely is a good thing!

Summary

One of the key objectives of this section is for you to be able to 'describe the significance of symmetry in particle interactions' (*Physics 2019 v1.2 General Senior Syllabus* © Queensland Curriculum & Assessment Authority). Here is a suitable response:

1 Three important symmetries in particle interactions are charge-reversal (C), time-reversal (T), and crossing (X) symmetry.
2 Symmetry operations are generally upheld by nature, and this enables physicists to predict new reactions although the probability of the new interaction may be unknown.
3 In any symmetry operation, conservation of energy and momentum must be obeyed.
4 In some cases symmetry is violated, which means that form of symmetry cannot be a universal law of nature.
5 Violation of a symmetry (symmetry-breaking) provides physicists with additional data with which to investigate interactions further.

Study tip

The syllabus doesn't mention any particular type of symmetry operation, so the best thing to do is learn the five points in the summary and be able to explain one operation as an example.

CASE STUDY 14.3

Parity – another form of symmetry

Consider a ‘mirror image’ of our universe, one in which all objects have their positions reflected by an arbitrary plane. This is said to be parity inversion.

For a long while, left and right were believed to be just conventions, and that nature had no preference for one over the other. Physicists believed that the laws of physics would work the same way in a mirror universe. They said that there was no experiment you could do that would tell you whether you were in your own world or a mirror world. This was verified for particle interactions involving the strong nuclear force and the electromagnetic force. The world was said to obey parity conservation, and this became a well-established principle in the field.

However, during the 1950s, physicists puzzled over the decay of the kaon, a meson made up of a quark and an antiquark. There seemed to be two types of kaons, one which decayed into two pions, and the other which decayed into three pions. One possible explanation was that it was the same particle undergoing two different decay modes. If this were the case, for reasons outside the scope of this chapter, parity could not be conserved.

In 1956, Tsung-Dao (T. D.) Lee and Chen Ning (Frank) Yang, while they were visiting scientists at Brookhaven National Laboratory in New York, predicted the non-conservation of parity in the weak interaction. This was very controversial, so they examined experimental data from the Brookhaven particle accelerator as well as the literature from almost 40 years of research. They found that although there was a great deal of support for parity conservation in the strong and electromagnetic interactions, there was no evidence for it in the weak interaction.

Their prediction was quickly tested when C. S. Wu and collaborators confirmed the theory when they examined the beta negative decay of cobalt-60 in 1957. They oriented the nuclei in cobalt to all spin one way (e.g. anticlockwise when viewed from above). They counted the number of electrons emitted upwards and downwards and found them equal. However, when the spin of the nuclei was reversed (that is, the mirror image) they found that more electrons were emitted downwards than upwards. This meant that the decay process varied between the original and mirror image – parity was not conserved.

Lee and Yang went on to win the 1957 Nobel Prize in Physics for setting the stage for this revolutionary discovery.

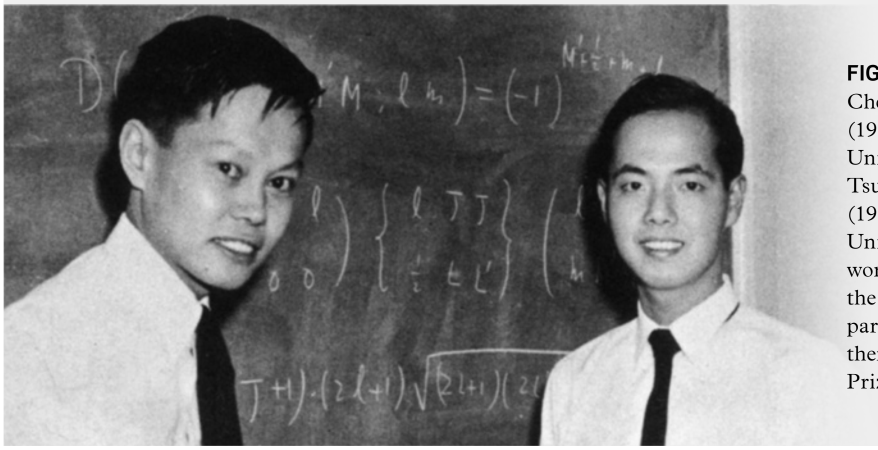


FIGURE 6
Chen Ning Yang (1922–) of Princeton University and Tsung-Dao Lee (1926–) of Columbia University, whose work in disproving the conservation of parity principle won them the 1957 Nobel Prize in Physics

CHALLENGE 14.3

Time reversal and the Big Bang

In the Big Bang model of the universe time only goes forwards, and therefore time-reversal (T) may not be a valid symmetry of the universe. It is also believed that matter and antimatter were originally created in equal amounts in the Big Bang. However, we observe that we live in a matter-dominated universe, so baryons outnumber antibaryons.

Does this mean that charge, time or crossing symmetry must have been violated to explain the difference between matter and antimatter?

CHECK YOUR LEARNING 14.3

Describe and explain

1 **Recall** the three types of symmetry.

2 **Explain** the changes that occur for time-reversal symmetry.

3 **Recall** the name given to the operation in which particles are replaced with their antiparticles.

Apply, analyse and interpret

4 **Derive** a time-reversed symmetry Feynman diagram based on the diagram shown in Figure 7.

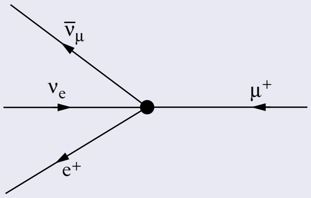


FIGURE 7

Investigate, evaluate and communicate

5 **Modify** the Feynman diagram of electron capture by a proton shown in Figure 8 to represent the process after crossing symmetry is applied.

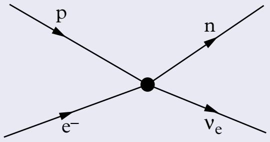


FIGURE 8

6 The following reaction shows a possible hypothetical decay of a muon:

$$\mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e$$

  a **Develop** a Feynman diagram to represent this reaction

  b **Modify** the diagram from part (a) to represent the process after charge-reversal symmetry has been applied.

7 **Evaluate** this statement about particle interactions: ‘Symmetry-breaking is not a good thing to do.’

8 **Assess** this claim about symmetry: ‘Our understanding of the atomic world would have progressed faster if violations of symmetry had not been found.’

Check your obook assess for these additional resources and more:

» Student book questions
Check your learning 14.3

» Challenge worksheet
14.3 Time reversal and the Big Bang

» Video
Calculating symmetry

» Weblink
What is parity?

SCIENCE AS A HUMAN ENDEAVOUR

14.4

Particle accelerators – the synchrotron

KEY IDEAS

In this section, you will learn about:

- the history of particle physics models and theories through the development of particle accelerators and contributions from notable physicists
- the construction of the Australian Synchrotron.

In the study of atomic particles, or high-energy physics, scientists require atom-smashing machines. These large accelerators, such as the 13.5 TeV Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) in Geneva, are used to study the collisions of highly energetic particles in order to learn about the structure of matter. Many of these large accelerators use powerful magnetic fields to deflect the charged particles into circular paths. It was at the LHC that the Higgs boson was finally confirmed in 2012.

One of the earliest devices, developed in 1930, was called a cyclotron. More modern versions of the original cyclotron are called synchrocyclotrons, synchrotrons, tevatrons and supercolliders.

Australian Synchrotron

Australia has a synchrotron in Melbourne that opened in 2007. Its construction involved collaboration between Australian and New Zealand science organisations, as well as state and federal governments. To ensure it was what the science world needed, Australian scientists collaborated with international organisations and committees, including the International Science Advisory Committee and the International Machine Advisory Committee.

The synchrotron provides researchers access to the specialised tools and techniques needed for research in areas such as advanced manufacturing, food, health and medical research, materials and textile science.

The electrons begin their journey in the electron gun, which is similar to the cathode ray tubes in old television sets. A hot metal cathode produces free electrons that are fired into the linear accelerator in packets of about 100 million spaced two nanoseconds apart. This means they are travelling at more than 640 million km h^{-1}, almost 60% of the speed of light. After 600 milliseconds and 1.38 million laps, the packets are travelling at 99.999 98% of the speed of light. Their energy is immense.

These electrons in motion generate an associated magnetic field. As these charged particles accelerate, they create fluctuations in the electric field that propagate outwards from the path of the beam. These variations in the electric field, in turn, generate an associated magnetic field. These variations produce electromagnetic waves such as radio waves, microwaves, infrared, visible light, ultraviolet, X-rays or gamma rays.

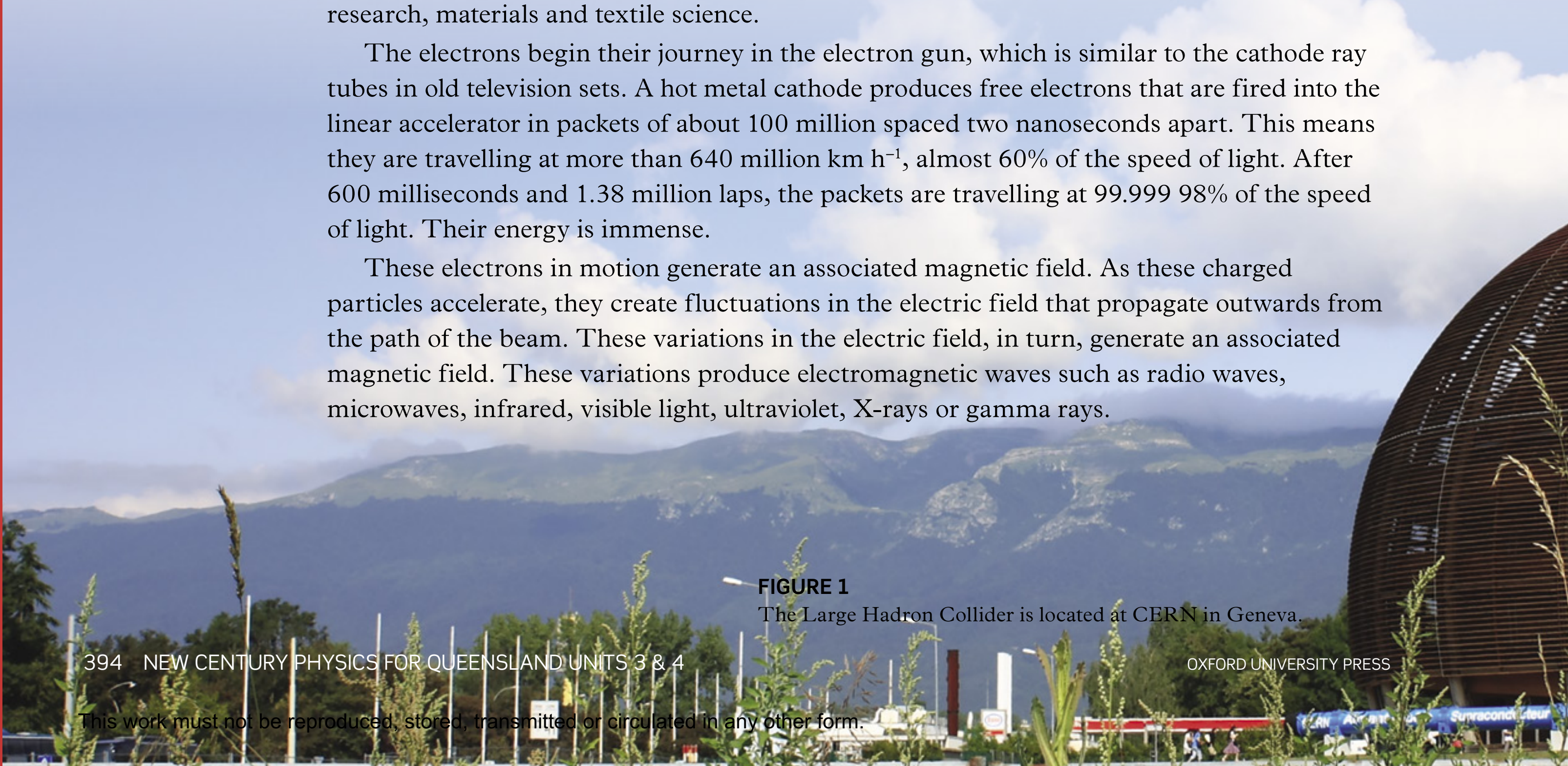


FIGURE 1
The Large Hadron Collider is located at CERN in Geneva.

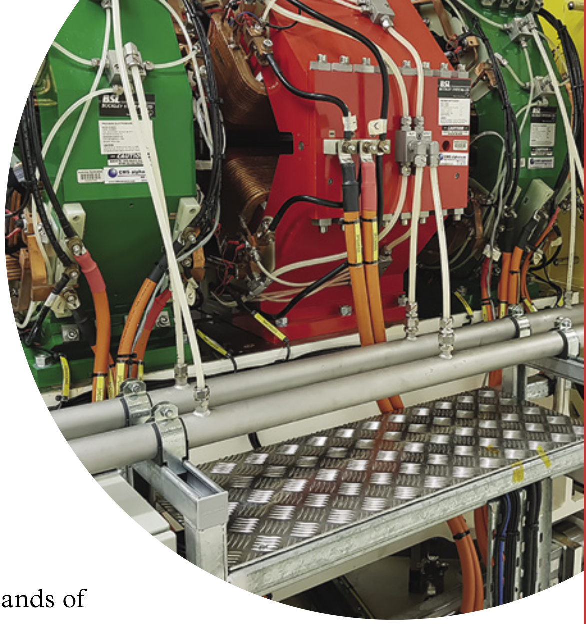


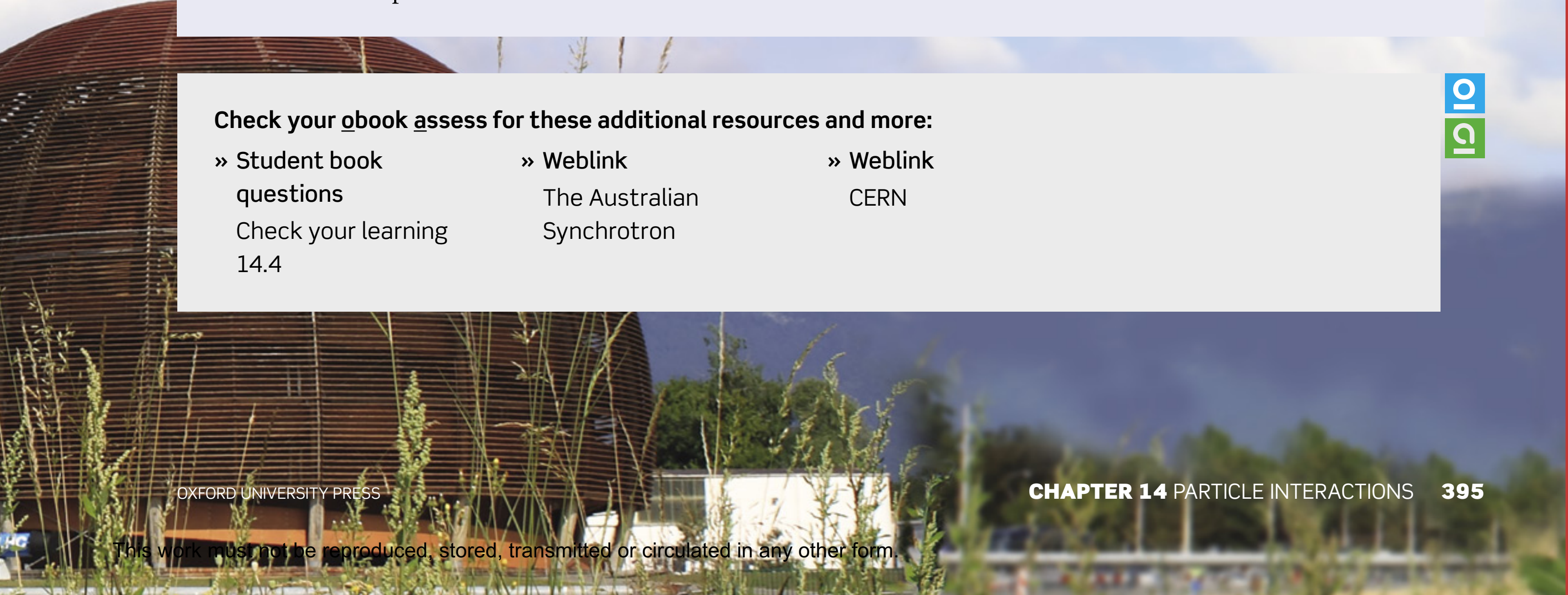
FIGURE 2 The storage ring tunnel of the Australian Synchrotron. The magnetic coils are clearly visible.

However, because the electrons are moving at velocities very close to the speed of light, relativistic effects apply, so the path length appears shortened to the electrons, meaning that the frequency of the radiation is increased. Most of the light is in the X-ray part of the spectrum, with some ultraviolet, visible and infrared light as well. The electrons speed around the ring for about 30–40 hours, travelling the equivalent of more than seven laps around the Earth every second.

The accelerating charged particles emit extremely intense light known as synchrotron radiation. This radiation gives scientists a mighty toolkit of advanced analytical and imaging techniques. It also gives a consistent and reliable supply of X-ray and infrared photons to the thousands of national and international researchers who use the synchrotron's unique capabilities each year.

One example of research using the synchrotron beamlines is the international efforts to beat the malaria parasite, which infects about half a billion people around the world every year and kills more than one million, many of them children. Researchers have been able to obtain high-resolution crystal structures of a key malarial enzyme in the hope of inhibiting its action.

A collaboration between Monash University, the University of Technology Sydney, and the University of Queensland on protein crystallography is aimed at producing a new class of antimalarial drugs that are able to bind to the enzyme's active site, stopping it from functioning and effectively starving the parasite to death.



CHECK YOUR LEARNING 14.4

Describe and explain

1 **Recall** the purpose of having charged particles travel at such high speeds.

Apply, analyse and interpret

2 To use the electrons, a magnetic field is applied to the clockwise beam of electrons to make them move to the outside of the ring and into openings called beamlines. **Determine** the direction a magnetic field would have to be applied to make the electron path bend to the outside.

Investigate, evaluate and communicate

3 In the Australian Synchrotron, the storage ring is 216 m in circumference and electrons travel around it at a speed of 0.999 999 8c (where c = the speed of light). To an electron travelling at such speeds, the ring appears to be only a few metres long. **Evaluate** this statement by using the special relativity formula for contraction of length.

Check your obook assess for these additional resources and more:

- » Student book questions
  Check your learning 14.4
- » Weblink
  The Australian Synchrotron
- » Weblink
  CERN

CHAPTER 14

Review

Summary

- 14.1 Baryons have a baryon number (B) of +1, and antibaryons have a baryon number of –1. Leptons and antileptons have a baryon number of 0.
- Quarks have $B = +\frac{1}{3}$ and antiquarks have $B = -\frac{1}{3}$.
- Leptons have a lepton number (L) of + 1, and antileptons have a lepton number of –1. Baryons and antibaryons have a lepton number of 0.
- Baryon number and lepton number are always conserved in a reaction.
- 14.2 Feynman diagrams are a graphical representation of particle interactions showing time along the horizontal axis and space along the vertical axis.
- Examples of particle interactions that can be represented with a Feynman diagram include electron–electron scattering, electron–positron (Bhabha) scattering and annihilation, and neutron decay.
- Feynman diagrams can show baryon decay in terms of baryons or as quarks.
- 14.3 Symmetry reversal includes charge reversal, time reversal and crossing reversal.
- Symmetry is important as it is not always conserved, and this opens up new fields to explore.
- Conservation of energy and momentum applies to all symmetry operations.
- Symmetry is important as it allows physics to predict possible interactions that may or may not be successful.
- 14.4 Particle accelerators were developed in the 1930s to study the structure of matter by using collisions of high energy particles.
- The Australian Synchrotron uses high energy electrons in circular paths to produce synchrotron radiation for analytical and imaging purposes.

Key terms

- annihilation
- baryon number
- charge-reversal symmetry
- conservation of baryon number
- conservation of lepton number
- crossing symmetry
- Feynman diagram
- lepton number
- symmetry
- time-reversal symmetry
- vertex

Key formulas

Baryon number	$B = \frac{1}{3}(n_q - n_{\bar{q}})$
Lepton number	$L = (n_l - n_{\bar{l}})$

Revision questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: ★ = low; ★★ = medium; ★★★ = high.

Multiple choice

1 A particle is made up of the following combination: $\overline{s}\overline{s}\overline{s}$. Which one of the following contains terms that all apply to the particle?

A meson, antiquarks, gluons

B baryon, antiquarks, gluons

C antibaryon, antiquarks, antigluons

D antibaryon, antiquarks, gluons

2 Which one of the following correctly lists the baryon number and lepton number respectively for the charmed bottom xi prime particle $\Xi_{cb}^{\prime 0}$ given that it is a quark composite (dcb)?

A –1, –1

B –1, 0

C +1, 0

D +1, +1

3 For the decay reaction $\Xi^0 \rightarrow p + \pi^-$, the identity of the particles and quark composition are Ξ^0, charmed Xi baryon (usc); p, proton (uud); π^-, pion ($d\overline{u}$).

Which one of the following is a true characterisation in terms of baryon and lepton conservation of the reaction?

A allowed

B forbidden; conservation of baryon number is violated

C forbidden; conservation of lepton number is violated

D forbidden; conservation of both baryon and lepton number is violated

4 Which one of the following best describes the mediating particle and the fundamental force, respectively?

A boson, weak force

B gluon, strong nuclear force

C meson, weak force

D quark, strong nuclear force

A pion (π^+) is a quark composite ($u\overline{d}$). Its decay can be represented in a Feynman diagram as shown in Figure 1. Use this diagram to answer Questions 5 and 6.

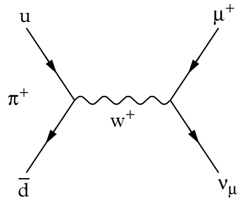


FIGURE 1

5 Which one of the following best states the baryon number before and after the interaction?

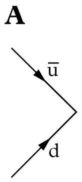
A 0, 0

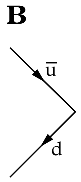
B 1, 1

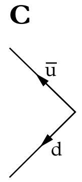
C 2, 2

D $+\frac{1}{3}$, 0

6 If the meson in Figure 1 was made up of $d\overline{u}$ quark combination instead of a $u\overline{d}$ combination, which one of the diagrams in Figure 2 would best represent its initial interaction?







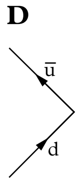


FIGURE 2

7 Which one of the following correctly lists the baryon number and lepton number respectively for the double charmed anti-omega particle $\overline{\Omega}_{\overset{*}{c}\bar{c}}$, which is a quark composite ($\bar{s}\bar{c}\bar{c}$)?

A –1, –1

B –1, 0

C +1, 0

D +1, +1

8 A particle has a baryon number of 0, and a lepton number of −1. Which one of the following could it be?

A strange D meson, D_s^-, ($s\bar{c}$)

B double charmed bottom anti-Omega, $\overline{\Omega}_{\overset{*}{c}\bar{c}\,b}$, ($\bar{c}\bar{c}\bar{b}$)

C tau, τ

D muon antineutrino, $\bar{\nu}_\mu$

9 For the decay reaction $\Sigma^- \rightarrow \Lambda^0 + e^+ + \bar{\nu}_e$, the identity of the unfamiliar particles and quark composition are Σ^-, sigma, (dds) and Λ^0, lambda, (uds).

Which one of the following is a true characterisation in terms of baryon and lepton conservation of the reaction?

A allowed
B forbidden; conservation of baryon number is violated
C forbidden; conservation of lepton number is violated
D forbidden; conservation of both baryon and lepton number is violated

10 For the decay reaction $\pi^+ \rightarrow \mu^+ + \gamma$, the identity of the particles and quark composition are π^+, pion ($u\bar{d}$); μ^+, antimuon (lepton) and γ, gamma (electromagnetic radiation).

Which one of the following is a true characterisation in terms of baryon and lepton conservation of the reaction?

A allowed
B forbidden; conservation of baryon number is violated
C forbidden; conservation of lepton number is violated
D forbidden; conservation of both baryon and lepton number is violated

Short answer

Describe and explain

★ **11** **Define** the concept of lepton number and baryon number, and explain their difference.

★ **12** **Explain** the purpose of a Feynman diagram as outlined in this section.

★ **13** **Recall** the conventions used in constructing Feynman diagrams.

★ **14** **Sketch** the Feynman symbol for a Z^0 boson.

★★ **15** **Describe** the significance of symmetry in particle interactions.

★★ **16** Consider the β decay of a neutron.

a **Sketch** a Feynman diagram that indicates what happens at the quark level.
b **Recall** whether baryon number and lepton number are conserved.

★★★ **17** **Explain** the following interactions using Feynman diagrams:

a electron and positron annihilating into a photon, only to then 'pair produce' another electron and positron
b neutron decaying into a proton in terms of the particles at a nucleon level, and showing the mediating boson
c neutron decaying into a proton in terms of the quarks, showing the role of the W^- boson

★★★ **18** **Summarise** your understanding of Feynman diagrams by creating a sketch for the following interaction. A tau lepton decays into a tau neutrino mediated by a W^- boson. The W^- boson immediately decays into an electron and an electron antineutrino.

Apply, analyse and interpret

★★ **19** **Determine** the baryon number for these quark composites:

a uuc **b** $\overline{ss}\bar{b}$ **c** $\bar{u}b$

★★ **20** Consider this decay reaction:
$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$.

a **Determine** the lepton number for the reactants and the products.
b **Compare** the *L* values to determine whether lepton number is conserved.

★★ **21** A suspected reaction is between the meson π^- and a proton p, to form a kappa zero particle K^0 made of the quarks $d\bar{s}$.

$$\pi^- + p \rightarrow K^0$$

Determine whether the reaction violates the lepton or baryon conservation laws and hence whether it is likely to be observed.

★★★ **22** The following reactions depict the decay of a muon before and after symmetry reversal.

i $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ **ii** $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

a **Deduce**, with stated reasons, the type of symmetry reversal that is occurring.
b **Comment** on whether lepton number has been conserved in each case.

★★★ **23** Figure 3 depicts the interaction of some fundamental particles.

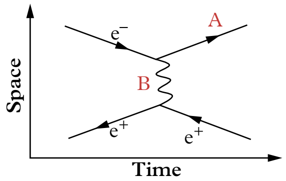


FIGURE 3

- **a** **Deduce** the charge on particle A and explain your reasoning.
- **b** **Recall**, with reasons, whether this is a scattering or an annihilation interaction.
- **c** **Deduce** the identity of boson B and explain how you distinguished it from the other bosons.

Investigate, evaluate and communicate

★★★ **24** The Feynman diagram in Figure 4 represents the interaction of some fundamental particles.

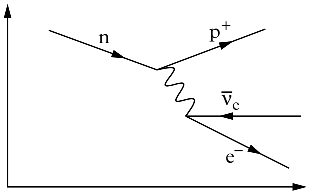


FIGURE 4

- **a** **Recall** the labels given to the vertical and horizontal axes respectively.
- **b** **Symbolise** the Feynman diagram in the form of a conventional equation.
- **c** **Sketch** a Feynman diagram after charge-reversal symmetry is applied.
- **d** **Sketch** a Feynman diagram after time-reversal symmetry is applied.
- **e** **Comment** on which reaction has the greater probability of occurring: the original interaction or the time-reversed interaction.

★★★ **25** **Assess** this beta negative decay of a neutron for conservation of baryon number:

$n \rightarrow p + e^- + \bar{\nu}_e$

★★★ **26** Figure 5a shows an interaction in which two particles annihilate. It shows these two particles approaching and exchanging an electron and then emitting two different particles instead. The diagram on the left has then undergone a crossing symmetry operation to produce the diagram on the right. The interaction in Figure 5b is called Compton scattering.

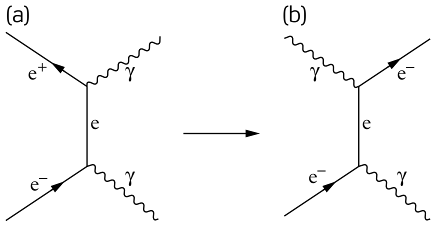


FIGURE 5

- **a** **Describe** the annihilation reaction in Figure 5a.
- **b** **Explain** why you can say a crossing operation has been applied.
- **c** **Explain** why the diagram in Figure 5b represents Compton scattering.

★★★ **27** Physicists claim that once a Feynman diagram has been proposed for a particular particle interaction, they can then predict other interactions by applying a crossing symmetry operation to the diagram.

- **a** Apply a further crossing operation to the Compton scattering in Figure 5b to show the interaction in Figure 6 and identify the particle indicated by the question mark.

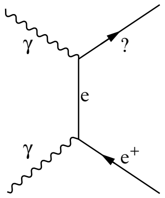


FIGURE 6

- **b** **Explain** how you applied this operation without ending up with the original diagram.
- **c** **Decide**, with reasons, whether the new interaction would be called electron–positron annihilation or electron–positron pair production.

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UNIT 4

Practice exam questions

Revolutions in modern physics

Multiple choice

1 A very fast spaceship is travelling at a relativistic speed near a planet. Deduce which one of the following observers is measuring its rest length.

- **A** observer on the spaceship measuring the time taken for light to travel between two points on the planet
- **B** observer on the planet measuring the time taken for light to travel from the front to the back of the spaceship
- **C** observer on the spaceship measuring the time taken for light to travel from the front to the back of the spaceship
- **D** observer on the planet measuring the difference in the arrival time of light from the front and the back of the spaceship

2 Select the graph in Figure 1 that represents the ratio of relativistic momentum to Newtonian momentum as a function of speed.

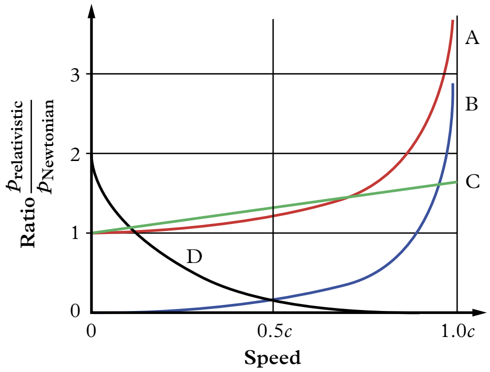


FIGURE 1

3 Calculate the energy equivalent to 1 g of uranium.

- **A** 3×10^5 J
- **B** 9×10^{13} J
- **C** 3×10^{16} J
- **D** 9×10^{16} J

4 Ultraviolet photons of 200 nm wavelength are incident on platinum, which has a work function of 5.6 eV. Determine the maximum kinetic energy of the emitted photoelectrons.

- **A** 0.6 eV
- **B** 1.2 eV
- **C** 5.6 eV
- **D** 6.2 eV

5 The four lowest energy levels of a hydrogen atom are shown in Figure 2. Four different photons are incident on a hydrogen atom in its ground state. Their energies are given: W = 8.6 eV, X = 10.2 eV, Y = 12.1 eV, Z = 12.6 eV. Determine which photon/s could be absorbed by the atom.

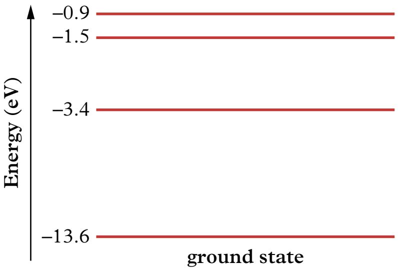


FIGURE 2

- **A** W only
- **B** X and Y only
- **C** Y and Z only
- **D** X, Y and Z only

6 Select the statement that best describes an antiproton.

- **A** the mass of a proton and the charge of an electron
- **B** the mass of an electron and the charge of a proton
- **C** the mass of a neutron and the charge of a proton
- **D** the mass of a proton and the charge of a neutron

7 Select the sequence that correctly lists the four fundamental forces in nature in order of decreasing strength.

- **A** strong nuclear, electromagnetic, weak nuclear, gravitational

B electromagnetic, strong nuclear, weak nuclear, gravitational

C strong nuclear, gravitational, weak nuclear, electromagnetic

D strong nuclear, electromagnetic, gravitational, weak nuclear

8 Select one of the following that represents a meson.

A udc

B $\overline{u}\overline{d}$

C cc

D $c\overline{c}$

9 Select one of the following that correctly lists the baryon number and lepton number respectively for the charmed-lambda particle Λ^{+}_{c} given that it is a quark composite (udc).

A –1, –1

B –1, 0

C +1, 0

D +1, +1

10 Select the statement that is a correct description of the Feynman diagram in Figure 3.

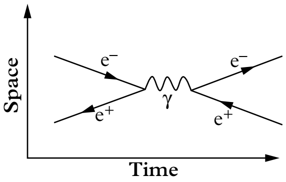


FIGURE 3

A An electron and a positron collide and form a photon, which forms an electron and positron.

B An electron turns into a positron and a gamma particle.

C An electron and a positron bounce off a gamma particle.

D An electron and a positron bounce off a gamma particle and turn into their opposites.

Short answer

11 Pions are particles that are present in cosmic rays striking Earth. Pions decay with an average life of 26 ns. A pion is approaching Earth at a speed of 0.98*c*. Determine the life of the pion as measured by Earth observers.

12 Alpha Centauri, our nearest star, lies 4.37 light-years from the Sun. Determine the fraction of the speed of light at which a ship would have to travel for the crew to experience only 1 year in the transit.

13 A spacecraft of rest mass 100 kg is travelling through space at 0.5*c*. Determine the ratio of the relativistic to the Newtonian calculations of its momentum.

14 A demonstration of the photoelectric effect is set up in which light with a wavelength of 310 nm shines upon a metal. As a result, electrons are ejected from the metal at a non-relativistic speed of 0.0024*c*.

a Determine the work function of the metal.

b The wavelength of the light is doubled to 620 nm. Predict whether any electrons are produced from the metal, and if so, at what speed.

15 A hydrogen atom absorbs a photon of UV light and its electron jumps from level 1 to level 4.

a Determine the change in energy of the atom in joules.

b Determine the wavelength of the photon in nanometres.

16 The decay reaction $\Sigma^{-} \rightarrow \Lambda^{0} + e^{+} + \overline{\nu}_{e}$ involves two unfamiliar particles with quark compositions of Σ^{-}, sigma, (dds) and Λ^{0}, lambda, (uds). They have the same baryon number of +1 and the same lepton number of 0. Determine whether the reaction is allowable or forbidden, based on conservation of baryon and lepton number.

17 Carbon-14 is a radioactive nuclide that decays according to this reaction:

$$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e + \overline{\nu}_{e}.$$

It sheds some of its excess energy by turning one of its neutrons into a proton. It uses some of its excess energy to form a temporary (virtual) intermediate particle called the W^{-} boson, which disappears by forming an electron and an electron antineutrino.

$$n \rightarrow p + e^{-} + \overline{\nu}_{e}$$

The interaction can be represented by a Feynman diagram, the start of which is shown in Figure 4.

Construct a labelled Feynman diagram to represent this interaction.

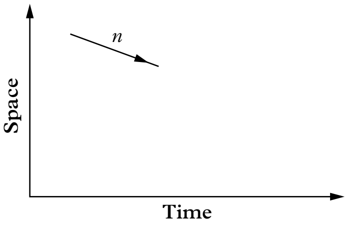


FIGURE 4

CHAPTER 15

Practical manual

This chapter is a guide to all the mandatory practicals included in the QCAA *Physics General Senior Syllabus*. Please refer to your obook assess for access to the suggested practicals from the syllabus. These practicals are not prescriptive and schools may complete the practicals to their resources.

The practicals in this chapter have been trialled, and safety instructions are provided; however, it is the legal obligation of the teacher to perform their own risk assessments prior to participating in any practical activity.

While completing the practicals, specific safety hazards will be highlighted at the top of the practical. This page provides general safety information that should always be followed when in a laboratory.

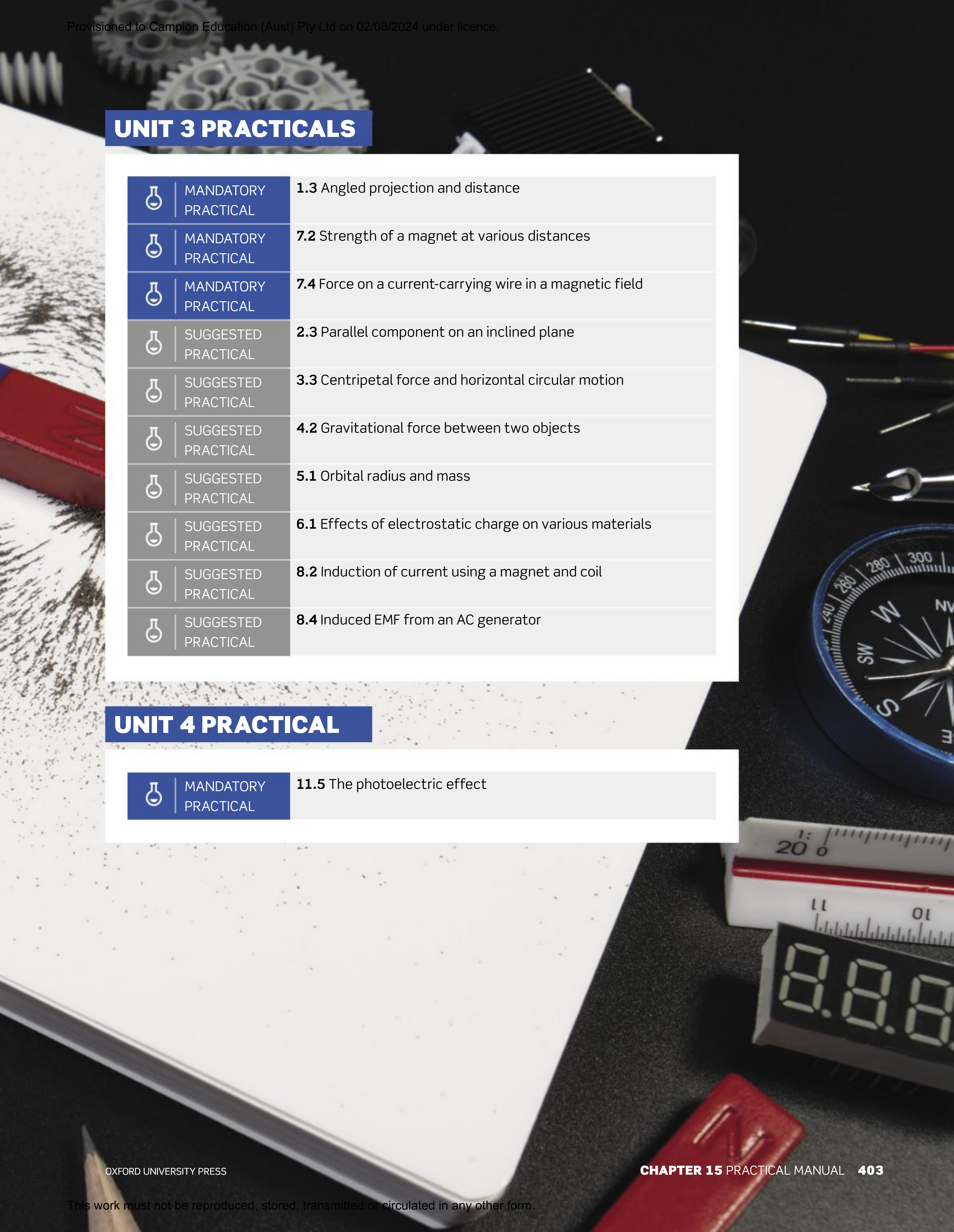
SAFETY

This chapter will highlight key safety concerns for each practical on the page; however, there are some general safety concerns to be considered in all practicals.

→ Tie back long hair.
→ Wear a lab coat, safety goggles and enclosed shoes at all times.
→ Check electrical cables before use.
→ Familiarise yourself with your school's safety procedures and the locations of safety kits.
→ If ever in doubt, ask your teacher before proceeding.
→ Always be aware of your peers in the lab and act sensibly.

It is each teacher and school's responsibility to conduct a risk assessment prior to any practical covered in this book (either online or printed).

FIGURE 1 The magnetic field around a bar magnet being mapped using iron filings.



UNIT 3 PRACTICALS

MANDATORY PRACTICAL	**1.3** Angled projection and distance
MANDATORY PRACTICAL	**7.2** Strength of a magnet at various distances
MANDATORY PRACTICAL	**7.4** Force on a current-carrying wire in a magnetic field
SUGGESTED PRACTICAL	**2.3** Parallel component on an inclined plane
SUGGESTED PRACTICAL	**3.3** Centripetal force and horizontal circular motion
SUGGESTED PRACTICAL	**4.2** Gravitational force between two objects
SUGGESTED PRACTICAL	**5.1** Orbital radius and mass
SUGGESTED PRACTICAL	**6.1** Effects of electrostatic charge on various materials
SUGGESTED PRACTICAL	**8.2** Induction of current using a magnet and coil
SUGGESTED PRACTICAL	**8.4** Induced EMF from an AC generator

UNIT 4 PRACTICAL

MANDATORY PRACTICAL	**11.5** The photoelectric effect

1.3 MANDATORY PRACTICAL

Angled projection and distance

CAUTION: BE CAREFUL LAUNCHING PROJECTILES. DO NOT STAND IN THE PROJECTILE'S PATH.

Unit 3, Topic 1: Conduct an experiment to determine the horizontal distance travelled by an object projected at various angles from the horizontal.

Source: *Physics 2019 v1.2 General Senior Syllabus*
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Context

Projecting an object at various angles makes analysing the trajectory of the projectile more complex. The vertical force on an object projected upwards means it will reach a maximum height before falling to the ground. This acceleration is directed downwards even as the projectile moves upwards. However, the horizontal motion is one of constant speed as ideally there are no net forces acting horizontally. The maximum range should be for an angle of elevation of 45° for which the projectile falls to the same starting height, but there are several complications in examining this. This investigation allows examination of competing forces.

Aim

To determine the horizontal distance travelled by an object projected at various angles from the horizontal.

Materials

- Projectile launcher and projectile (Figure 1)
- Data-logger (optional)
- Light gate for data-logger (optional)
- Metre ruler or 3 m tape measure
- Sheet of butcher's paper or white A3 paper, or as needed
- Sheet of carbon paper A4 size
- Masking tape

Method

Set up the equipment as shown in Figure 2.

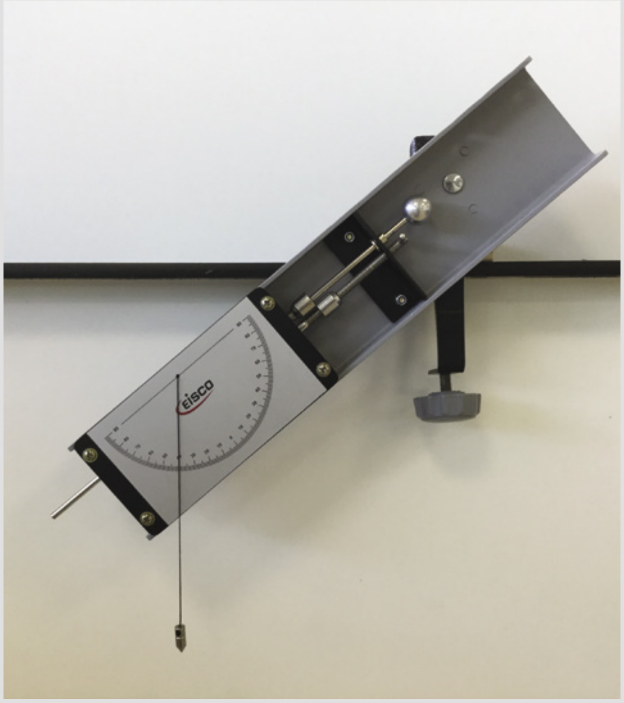

FIGURE 1 Typical projectile launcher used in schools. The launch velocity can be adjusted by changing the compression of the spring.

Part 1: Measuring launch velocity (optional)

1 Use a light gate attached to the launcher to measure the time for the ball's diameter to pass through the light gate (see the instruction manual for the launcher).

Part 2: Measuring horizontal range

2 Set the launcher to the first angle of elevation (suggest 20°) and fire the projectile to establish an approximate landing point (1–2 m is suitable). Adjust the landing platform so it is under this point. Tape a sheet of white paper in place and place the carbon paper on top.

3 Fire the launcher for Trial 1, Test 1 and mark the impact point and label. Do two more repetitions (tests) for this trial angle. Mark points as Trial 1, Tests 2 and Test 3.

4 Optional: Measure the velocity of the projectile leaving the photogate.

If the settings on the launcher are not altered, this velocity should not change as the ball is fairly light.

5 Change the angle for the next trial (suggest 30°) and repeat the three tests. Label these points accordingly. Record times or velocity as necessary.

6 Continue to change the angle and mark impact points. Suggest 40°, 45°, and 50° but try to include 60°, and 70°.

Results

1 Copy and complete Table 1 below with the results of your experiment.

2 Create a graph and place the horizontal range (s_x) on the vertical axis, and the launch angle (θ°) on the horizontal axis. Place error bars on the vertical axis.

3 Sketch a curved line of best fit.

4 Interpret the graph to deduce the angle/s of maximum range.

Discussion

1 Assess how maximum horizontal range and launch angle are related.

2 Discuss the uncertainty in determining the maximum range.

3 Decide if it is a single angle or a range of angles that can be considered to give maximum range.

4 Resolve the claim that complementary launch angles (e.g. 40° and 50°, or 30° and 60°) give the same range.

5 Discuss whether the launch velocity was constant for all trials (optional as it requires photogate).

6 Evaluate the experiment and suggest improvements.

7 Propose, with reasons, a follow-on investigation.

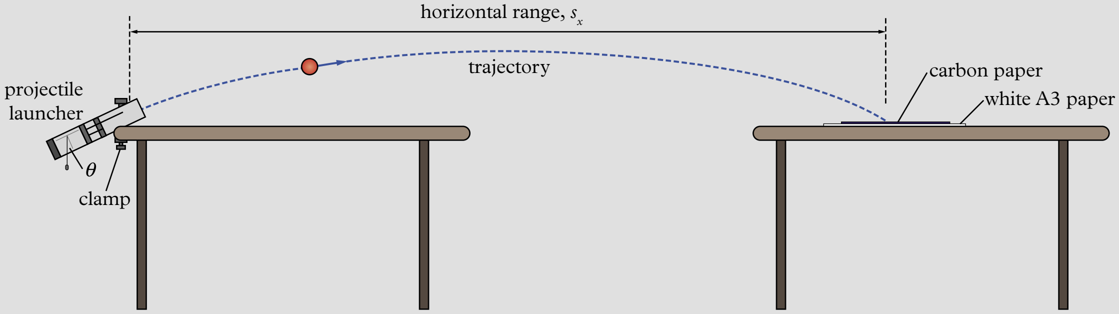

FIGURE 2 Arrangement of desks to ensure the impact is at the same vertical displacement as the launch

TABLE 1 Results of projection

Angle, θ	Dependent variables	Replicates				Uncertainty, δ (± m) for range
		Test 1	Test 2	Test 3	Average	Launch velocity, u (m s^{-1})
20°	Horizontal range, s_x (m)					δ = _____ ± _____ m
	Time or speed at photogates					u = _____ m s^{-1}
30°	Horizontal range, s_x (m)					δ = _____ ± _____ m
	Time or speed at photogates					u = _____ m s^{-1}
40°	Horizontal range, s_x (m)					δ = _____ ± _____ m
	Time or speed at photogates					u = _____ m s^{-1}
45°	Horizontal range, s_x (m)					δ = _____ ± _____ m
	Time or speed at photogates					u = _____ m s^{-1}
50°	Horizontal range, s_x (m)					δ = _____ ± _____ m
	Time or speed at photogates					u = _____ m s^{-1}

7.2
MANDATORY PRACTICAL

Strength of a magnet at various distances

Unit 3, Topic 2: Conduct an experiment to investigate the strength of a magnet at various distances.

Source: *Physics 2019 v1.2 General Senior Syllabus*
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Context

Magnetic force, if it is at all like electrostatic force or gravitational force, obeys an inverse square relationship with distance. However, unlike masses and charges which can exist as single entities, magnets come as north–south dipoles, which may affect the relationship. The longer the magnets, the closer the relationship should come to being an inverse-squared relationship.

Aim

To investigate the strength of a magnet at various distances.

Materials

- 2 × bar magnets
- Electronic balance
- Ruler
- Retort stand, boss head and clamp

Method

1. Place a bar magnet vertically upright on the pan of an electronic balance (Figure 1). Zero the balance.
2. Place another magnet in a clamp directly above the first magnet so that the unlike poles face each other. There will be an attractive force, so the scale reading on the balance should be a negative value. If like poles are facing each other, the magnet on the pan will fall over (unless held firmly in a wooden support or similar).
3. Start with the end of the clamped magnet 30 cm from the magnet on the balance and take a scale reading. If it is not zero, start with a 1 m separation (hold it in your hand).
4. Reduce the separation distance (r) by 5 cm at a time until r = 10 cm, and then in 2 cm increments, and take scale readings of the balance in grams. There is no need to reduce the separation less than 4 cm. Ensure that the two magnets are in a line.

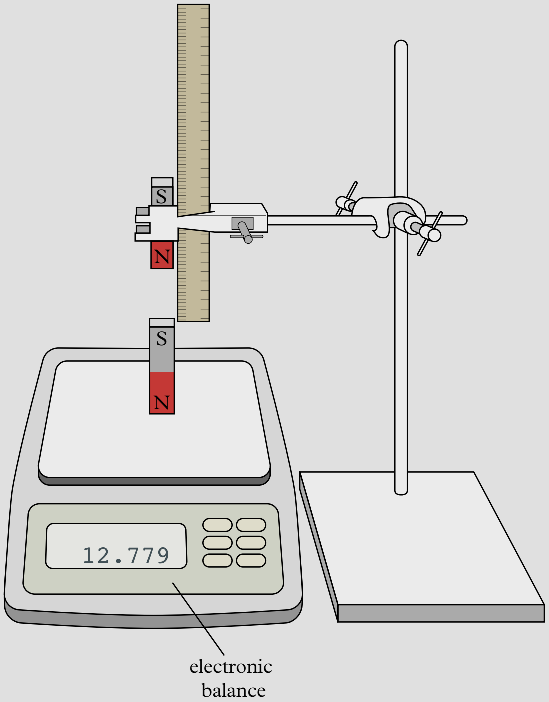

FIGURE 1 Balance and magnet assembly

Results

1 Copy and complete Table 1 below with the results of your experiment.

2 Calculate the force in newton (N) from the scale readings in grams.

3 Calculate the mean and determine the uncertainty.

4 Using the results you recorded in Table 1, construct a graph of the data with separation distance (m) on the *x*-axis, and force (N) on the *y*-axis. Add custom error bars.

5 Deduce the relationship between force and distance. It will be inverse, but does it appear to be inverse or inverse squared?

6 Construct a linearised graph. Note: if the graph looks like an inverse square relationship ($y \propto \frac{1}{x^2}$) then linearise it. See if your prediction is confirmed.

7 If the relationship does not linearise, the relationship is likely to be a power relationship in the form $y = kx^a$. The power (a) will be −2 for an inverse squared relationship, or −3 for a inverse cubed relationship. It could be in between these extremes; it may even be less than −2.0. Calculate the logarithm to base 10 for the force and distance data and use a log–log graph to determine the power (k).

$y = kx^a$ (power relationship)

$\log_{10} y = a \log_{10} x + \log_{10} k$ (logs taken of both sides)

$y = mx + c$ (equation appears in this form)

$\frac{\log_{10} y}{\log_{10} x}$ = gradient = a

8 Construct a graph with $\log_{10} F$ (force) on the vertical axis and $\log_{10} r$ on the horizontal axis.

9 Interpret the relationship, with justification from the data. (Note: gradient is the power, a.)

Discussion

1 Deduce the relationship between F and r.

2 Propose a modification that would reduce random error and propose another that would reduce systematic error.

3 (Optional) This practical offers many opportunities for modification such as those listed below. Choose one and:

- **a** categorise it as a refinement, extension or a redirection
- **b** identify a dependent and an independent variable to be investigated
- **c** identify several controlled variables
- **d** propose a correctly worded research question

Some ideas:

- How does the relationship change if you used one magnet and a piece of steel instead of two magnets?
- What would the relationship between force and distance be like for two electromagnets?
- When does an intervening medium affect the relationship? Would you get the same results if the magnets were in water?
- Where does the relationship between force and distance break down? It seems that other effects come into play when the magnets are really close.
- Why would temperature affect the relationship? Would a heating coil around each magnet be appropriate for keeping the magnets hot?

TABLE 1 Results

Distance	Scale reading (g)		Force between magnets (N)			
r (m)	Test 1	Test 2	Test 1	Test 2	Mean	δ

7.4 MANDATORY PRACTICAL

Force on a current-carrying wire in a magnetic field

Unit 3, Topic 2: Conduct an experiment to investigate the force acting on a conductor in a magnetic field.

Source: *Physics 2019 v1.2 General Senior Syllabus*
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Context

The force on a current-carrying wire has been derived as $F = BIL \sin\theta$, and the direction of the force has been established by the right-hand rule (Figure 1). In this experiment the wire will be at 90° to the magnetic field, and the length will be measured and kept constant. Hence, the relationship is expected to be linear with $F \propto I$, providing B, L and θ are kept constant. A graph of F versus I will produce a gradient of $B \times L$. Substitute for L to determine the magnetic field strength (assumed uniform) between the poles of the permanent magnets. Analysis of uncertainty will allow you to estimate the magnetic field strength and its percentage uncertainty.

Aim

To investigate the relationship between force and current for a conductor in a magnetic field.

Materials

- Laboratory power supply
- Variable resistor (rheostat)
- Connecting wires
- Ammeter
- Electronic balance
- 2 × rare earth magnets
- Stiff copper wire (e.g. 0.5–1 mm diameter, 25 cm length)
- Cork
- Retort stand, boss head and clamp
- Aluminium channel (1 cm × 1 cm × 4 cm long)

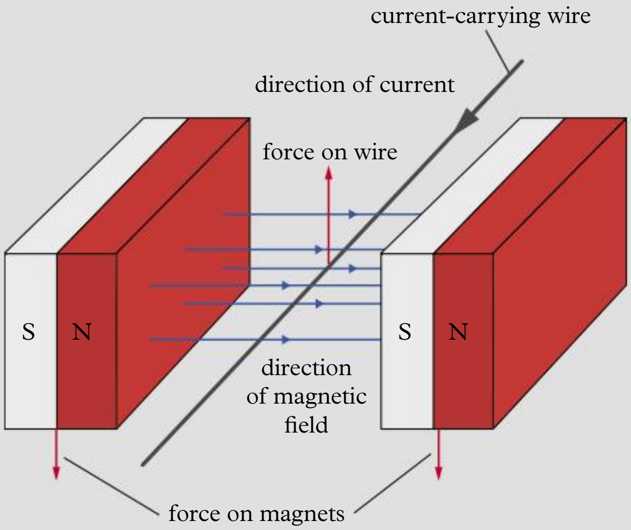

FIGURE 1 Forces on the wire and magnets

Method

The magnets should be placed on the balance with unlike poles facing each other. They need to be kept apart so that the copper wire can pass between them. A small length of aluminium channel 1 cm high × 1 cm wide × 4 cm long to form a 'yoke' is appropriate. The magnets will not need fixing in place as their attraction will be sufficient to stop them sliding off.

1 The copper wire needs to be bent into a square loop so that one side can pass freely between the magnets, and the rest supported by a fixed clamp in a boss head on a retort stand.

2 Set up the equipment as shown in the Figure 2. Ensure that the current will flow in the correct direction. Note the direction of the magnetic field in the diagram. Remember that it is the inside poles of the magnets that produce the magnetic field. Position the parts as shown in the diagram.

3 With the magnets in place on the yoke (the aluminium channel), zero the balance.

4 Set the power supply to 2 V DC and adjust the resistor so that no current flows.

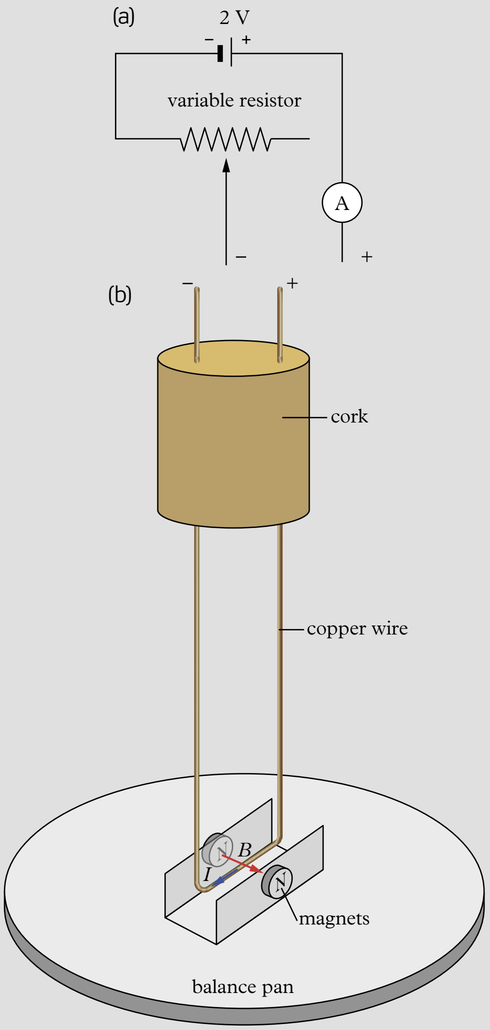

FIGURE 2 (a) Electric circuit and (b) yoke assembly for experiment set-up

5 Increase the current in 0.1 A increments and take a scale reading in grams. The current should be in a direction to make the yoke get pushed down by the magnetic field (see Figure 1); that is, the force on the wire should be up. Use a hand rule or, if uncertain, check by trial and error. There may be no response until about 0.4 A. Stop when 7 trials have been made. Repeat once more to have duplicates of each trial.

Results

Length of wire in the field (i.e. diameter of magnet) = ______ mm = ______ m

1. Copy and complete Table 1 below with the results of your experiment.
2. Construct a graph with current on the horizontal axis, and force on the vertical axis.
3. Construct custom error bars for the vertical axis.
4. Create a linear trendline, and calculate the R^2 value.
5. Construct maximum and minimum lines of best fit within the error bars, and determine the maximum and minimum gradients.

Discussion

1. Deduce the relationship between F and I.
2. Determine the average magnetic field strength of the permanent magnets, using the gradient value.
3. Determine the uncertainty in the magnetic field strength.
4. Calculate the maximum and minimum magnetic field strengths.
5. Explain why the balance pan moved down when a current passed through the wire.

TABLE 1 Results of scale readings

Current (A)	Scale reading (g)		Force (N)			Uncertainty in force
	Test 1	Test 2	Test 1	Test 2	Average	δ (N)

11.5

MANDATORY PRACTICAL

The photoelectric effect

Unit 4, Topic 2: Conduct an experiment (or use a simulation) to investigate the photoelectric effect. Data such as the photoelectron energy or velocity, or electrical potential difference across the cathode and anode, can be compared with the wavelength or frequency of incident light. Calculation of work functions and Planck's constant using the data would also be appropriate.

Source: *Physics 2019 v1.2 General Senior Syllabus*
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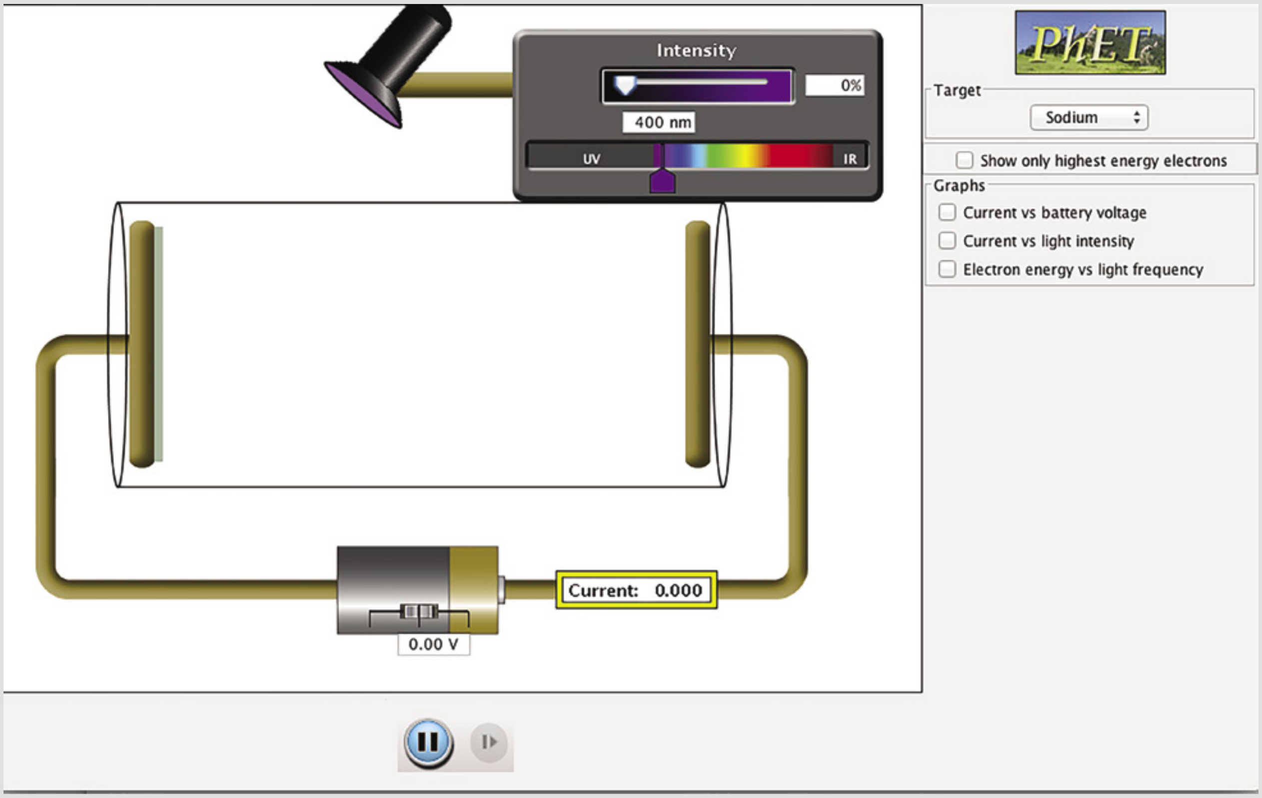
Context

In the early years of the 20th century, physicists had unsuccessfully attempted to use the wave model for light to explain the emission of electromagnetic radiation from a black body. In 1905, Max Planck proposed a solution that showed the energy of light came in discrete packets called 'quanta'. The energy of a photon was given by $E = hf$ where h is Planck's constant and f is the frequency of the light. Albert Einstein then used Planck's equation to describe the minimal element of electromagnetic radiation called a 'photon'. Einstein used the concept of a photon to explain the photoelectric effect.

Aim

To investigate the photoelectric effect, by:

1. investigating the electric current across the cathode and anode as a function of the intensity of incident light
2. investigating the electric current across the cathode and anode as a function of wavelength
3. determining the work functions of various metals by measuring their stopping voltage.



Intensity 0%

400 nm

Target: Sodium

☐ Show only highest energy electrons

☐ Current vs battery voltage

☐ Current vs light intensity

☐ Electron energy vs light frequency

Current: 0.000

0.00 V

FIGURE 1 PhET Photoelectric effect lab

Materials

- PhET Interactive Simulations, University of Colorado Boulder, https://phet.colorado.edu/en/simulation/photoelectric

Part 1: Current as a function of intensity of light

Aim

To investigate the electric current from the cathode and anode as a function of the intensity of incident light.

Background

The intensity of light is proportional to the number of photons striking the metal. As more photons strike the metal, more electrons are ejected from the metal surface. As a result, there are more electric charges passing through the external circuit (the ammeter) in a given time period and, hence, more electric current.

Variable	Parameters
Controlled variables	Target: sodium
	Battery voltage, $\Delta V = 0$ V
	Wavelength, $\lambda = 100$ nm
Independent variable	Light intensity = 0% to 100%
Dependent variable	Electric current, I (A)

Method

1 Set the wavelength to 100 nm (default) so photoemission can occur.

2 Set the intensity slider to 0% and record the current. Increase the intensity in 20% increments and record the current at each value. Enter the data in Table 1.

3 Increase wavelength in 100 nm increments (up to 600 nm) and measure the current for changing intensities as in step 2 above. Enter the data in Table 1. Note: if you are limited for time, just test two wavelengths.

Results

1 Copy and complete Table 1 below with the results of your experiment

TABLE 1 Current (A) at various intensities and wavelengths

Wavelength, λ	Intensity (%)					
	0	20	40	60	80	100
100 nm						
200 nm						
300 nm						
400 nm						
500 nm						
600 nm						

2 Analyse your data by constructing a graph that depicts the relationship.

Discussion

1 Justify whether or not the hypothesis was confirmed and propose a conclusion.

2 Evaluate the design of the investigation and comment on the limitations of the data.

Part 2: Threshold frequency for various metals

Aim

To investigate the electric current from the cathode and anode as a function of the frequency of incident light.

Background

Metals have a specific threshold wavelength (and threshold frequency) for producing a photoelectric current. This frequency can be used to determine the experimental value for the work function of the metal, and allow a comparison with the accepted value to be made.

Variable	Parameters
Controlled variables	Target: sodium, zinc, copper, platinum, calcium trialled individually
	Battery voltage, $\Delta V = 0$ V
	Light intensity = 100%
Independent variable	Wavelength, $\lambda = 100$ nm to 600 nm
Dependent variable	Electric current, as shown by electron movement between cathode and anode

Method

1 Select sodium as the first target metal, and set the light intensity at 100%. Begin with the light at a low frequency (long wavelength) and slowly move the slider until you see electrons move across the screen from left to right (cathode to anode).

2 Move the slider back and forth (or type the exact values into the wavelength display if the slider can't be adjusted finely enough) to locate the minimum wavelength for which no photons are emitted. This is the threshold wavelength. Enter the data in Table 2.

Results

1 Copy and complete Table 2 below with the results of your experiment.

2 Calculate the threshold frequency using the wave equation, $f_0 = \frac{c}{\lambda}$ ($c = 3 \times 10^8$ m s^{-1}).

3 Determine the work function of the metal by multiplying the threshold frequency by Planck's constant ($h = 6.626 \times 10^{-34}$ J s). An example has been done for you.

4 Determine the work function W (in J) by applying the formula $W = hf_0$.

5 Calculate the work function (in eV) by using the conversion 1 eV = 1.60×10^{-19} J.

Discussion

1 Propose reasons why different metals have different threshold frequencies.

2 Assess the accuracy of the experimental results for work function by comparing your values with the accepted values and determining a percentage error.

3 Propose why some electrons can be moving across the screen when near the threshold wavelength but no current shows on the meter. Hint: consider the scale reading uncertainty of the ammeter.

TABLE 2 Threshold wavelength and frequency

Metal target	Threshold wavelength, λ (nm)	Threshold frequency, f (Hz)	Work function, W (J) $W = hf_0$	Work function, W (eV) (1 eV = 1.60×10^{-19} J)
Sodium	540	5.56×10^{14}	3.68×10^{-19}	2.30
Zinc				
Copper				
Platinum				
Calcium				

Part 3: Photoelectron energy as a function of frequency

Aim

To determine the work function of various metals by measuring their stopping voltage.

Background

You have seen that light of a suitable frequency is able to eject electrons from the metal and cause a current to flow in the circuit (shown by the ammeter) and hence a potential difference between cathode and anode. If a battery is placed in the circuit with opposite polarity, it can be adjusted to 'stop' the current and cause it to fall to zero. This means the electrons will be stopped from moving across the screen. The potential difference applied to stop the current is equivalent to the kinetic energy of the electron that was ejected by the light shining on the metal. Use $E_{k\ (max)}$ to indicate the maximum kinetic energy of the electrons.

Variable	Parameters
Controlled variables	Target: sodium
	Light intensity = 100%
Independent variable	Wavelength, λ = 125 nm to 600 nm
Dependent variable	Battery voltage, $\Delta V = 0$ V

Method

1 Set the wavelength to 125 nm, the intensity to 100% and the target as sodium. Note: in a real experiment (and not a simulation), you might like to start the wavelength at 100 nm; however, the voltage cannot be adjusted below the −0.8 V required to stop photoelectrons being emitted from sodium with incident light at 100 nm.

2 The battery slider by default is at 0.00 V. Move the slider on the battery towards the negative potential so that the electrons no longer reach the opposite plate. The stopping voltage can be changed to specific values by entering the values into the box on the slider. Be careful to determine the stopping voltage to the nearest 0.01 V. Adjust the voltage such that the ejected electrons stop just short of the negative plate. If the electrons hit the negative plate, the stopping voltage must be increased. Try 0.01 V increments when getting close. Record this as the stopping voltage ΔV (V).

3 Repeat for the next wavelength until reaching the stopping voltage of 0.00 V.

Results

1 Copy and complete Table 3 below with the results of your experiment.

TABLE 3 Kinetic energy of electrons from sodium

Wavelength, λ (nm)	Frequency, f (Hz)	Stopping voltage, ΔV (V)	Kinetic energy, $E_{k\,(max)}$ (J)
125	2.4×10^{15}	−7.60	1.20×10^{-18}
200			
300			
400			
450			
500			
538			
540			
600			

2 Determine the frequency of the light using the wave equation $c = f\lambda$, with $c = 3 \times 10^8$ m s^{-1}, and add it to the table.

3 Determine the maximum kinetic energy using the formula $E_{k\,(max)} = \Delta Vq$ ($q = 1.60 \times 10^{-19}$ J). The first row has been done as an example.

4 Construct a graph of the maximum kinetic energy (vertical axis) of the electron versus the frequency of the light.

5 Determine a linear trendline and the equation for the line.

Discussion

The ejection of an electron (photoemission) occurs when an electron absorbs the energy of a photon. The minimum amount of energy needed to release an electron from the metal is called the work function (W). Any extra energy beyond the work function for the metal is given to the electron as kinetic energy. Thus, the excess energy is the difference between the incident photon energy ($E = hf$) and the work function needed to just remove the electron; that is, $E_k = hf - W$.

This is in the form of the linear equation $y = mx + c$, so if frequency (f) is plotted on the horizontal axis and kinetic energy is plotted n the vertical axis, the gradient will be h (Planck's constant) and the intercept (c) on the vertical axis will the work function (W).

1 Determine the value for Planck's constant using the gradient of the graph of E_k versus f.

2 Assess the accuracy of the experimental value by comparing it with the accepted value and determining the percentage error.

3 Determine the work function for sodium by considering the vertical intercept of the graph (which will be in joule). Convert this value to electron-volts (eV) by using the conversion 1 eV = 1.60×10^{-19} J.

4 Assess the accuracy of the experimental value of the work function for sodium by comparing it with the accepted value and determining the percentage error.

5 Evaluate the design of the investigation and comment on the limitations of the data.

6 Synthesise the data from Parts 2 and 3 to make a judgement about the accuracy of the two methods for determining the work function of sodium.

7 Determine and assess the experimental work function values of other metals such as zinc, copper, platinum and calcium (if you have time) by repeating the steps above.

GLOSSARY

A

accuracy
the difference between the measured value and the true or accepted value of the observed quantity

albedo
the fraction of light that is reflected by a body or surface; it is a measure of the relative brightness of the surface

angular momentum
for circular motion, the momentum of a particle in which the velocity vector points along the radius of the circular path and is equal to *mvr* (symbol: *L*; unit: kg m^2 s^{-1})

annihilation
the process that occurs when a subatomic particle collides with its respective antiparticle to produce other particles such as photons

anode
the positive terminal, which attracts electrons

antibaryon
a composite subatomic particle made up of three antiquarks held together by the strong nuclear force

antimatter
matter that is composed of the antiparticles of those particles that constitute ordinary matter

antiparticles of matter
a particle that has the same mass and opposite charge and/or spin as a corresponding particle; for example, positron and electron

antiproton
the antiparticle of the proton, with an electric charge of –1e. It is relatively stable but it is typically short-lived because any collision with a proton causes both particles to be annihilated in a burst of energy

antiquark
a particle with the same mass and opposite charge to a corresponding quark; for example, a strange quark and an antistrange quark are said to be antiparticles

applied forces
forces applied to an object by a person or another object

average speed
the rate of change of distance calculated by the formula: average speed $= \frac{\text{distance}}{\text{time}}$; a scalar quantity (symbol: *v*; SI unit: metres per second; unit symbol: m s^{-1})

B

baryon
composite subatomic particle made up of three quarks (or three antiquarks) held together by the strong nuclear force

baryon number
a quantum number of a system defined by $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks; it is strictly conserved and additive

black body
an object that absorbs all radiation falling on it, at all wavelengths; it is a perfect absorber or emitter of radiation

black-body radiation
the radiation emitted by a black body from the conversion of thermal energy, and which has a characteristic frequency distribution that depends on the temperature

black hole
a region of space where the gravitational field is so intense that no matter or radiation can escape

Bohr model
electrons orbit the nucleus in particular circular orbits called stationary states with fixed angular momentum and energy, their distance from the nucleus (their radius) being proportional to their energy. When an electron moves between these stationary states, it is accompanied by the emission or absorption of a photon

C

cathode
the negative electrode from which electrons are emitted

centripetal acceleration
the acceleration experienced by any object moving in a circular path directed towards the centre of motion (symbol: a_c; SI unit: metres per second squared; unit symbol: m s^{-2})

centripetal force
the force acting on an object travelling in a circle that constantly either pulls or pushes the object towards the centre of motion (symbol: F_c ; SI unit: newton; unit symbol: N)

charge-reversal symmetry
that interactions are not affected if all charges are swapped (i.e. positive for negative and vice versa)

cognitive verb
task word that provides information about what an answer requires

coherent
waves with the same frequency and amplitude, and a constant phase relationship. Conventional light sources such as candles and incandescent bulbs are incoherent sources; laser beams are coherent

combine (vectors)
add two or more vectors to determine the resultant vector

complementary angles
angles that add to 90°; for example, 60° is the complementary angle (complement) of 30° and 75° is the complement of 15°

components
(of a vector) a depiction of the influence of a vector in a given direction

conservation of baryon number
in any interaction baryon number is conserved

conservation of lepton number
in any interaction lepton number is conserved

conservation of momentum
for a collision occurring between object 1 and object 2 in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision

cosmological red shift
shift in the wavelengths of sufficiently distant light sources (galaxies, quasars) due to the expansion of the universe

Coulomb's law
states that like electric charges repel and opposite electric charges attract, with a force proportional to the product of the electric charges and inversely proportional to the square of the distance between them, expressed by the formula $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$

Coulomb's law constant, *k*
a constant of proportionality relating the force between charged objects to the magnitude of their charge and separation distance; $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

crossing symmetry
if a particle interaction is observed to occur, any of the particles can be replaced by its antiparticle on the other side of the interaction

D

dependent variable
the variable (often denoted by *y*) that responds to the independent variable; it 'depends' on the independent variable

discrete
individually separate and distinct

E

electric charge
a physical property of an object that causes it to experience a force when placed in an electromagnetic field

electric field
a region of space near an electrically charged particle or object within which a force would be exerted on other electrically charged particles or objects

electric field strength
the intensity of an electric field at a particular location (symbol: *E*; SI unit: newton per coulomb; unit symbol: N C^{-1})

electric potential
the electrostatic potential energy stored per unit charge at any given point (symbol: *V*; SI unit: volt; unit symbol: V)

electrical potential difference
the work done in moving a unit charge between the final and the initial positions in an electric field (symbol: ΔV; SI unit: volt; unit symbol: V)

electrical potential energy
the capacity of electric charge carriers to do work due to their position in an electric field (symbol: W; SI unit: joule; unit symbol: J)

electromagnetic force
the second strongest of the four fundamental forces; the electromagnetic force is mediated by photons

electromagnetic induction
the production of an electromotive force (EMF) or voltage across an electrical conductor due to its dynamic interaction with a magnetic field

electromagnetic radiation
energy radiating out as synchronised oscillations of electric and magnetic fields, or electromagnetic waves, propagated at the speed of light in a vacuum

electromagnetic waves
waves produced by an oscillating electric charge that radiate out at the speed of light as mutually perpendicular electric and magnetic fields

electromotive force
(EMF) a difference in potential that tends to give rise to an electric current, also written as emf; it is measured in volts (V)

electron volt (eV)
a unit of energy equal to the work done on an electron in accelerating it through an electrical potential difference of 1 volt (unit: eV); equivalent to 1.60×10^{-19} J

elementary particle
a particle with no substructure, and thus not composed of other particles

elevation angle
the angle at which a projectile is launched with respect to the horizontal

energy
the capacity to do work in which it is transformed or transferred

error analysis
a calculation of the precision and accuracy of experimental results

event
an act or action with a distinct beginning and end, e.g. the swing of a pendulum, ticking of a watch, motion of an object (including a light pulse) from one place to another, decay of a nucleus, propagation of an energy wave (light, sound, water)

F

Faraday's law
a law stating that when the magnetic flux linking a circuit changes, an electromotive force (EMF) is induced in the circuit proportional to the rate of change of the flux linkage

Feynman diagrams
the graphical representation of particle interactions with time along the horizontal axis and space along the vertical axis

first law of planetary motion (law of orbits)
all planets move about the Sun in elliptical orbits, having the Sun as one of their foci

force
a push or pull between objects, which may cause one or both objects to change speed and/or the direction of their motion (i.e. accelerate) or change their shape

frame of reference
an arbitrary set of axes with reference to which the position or motion of something is described or physical laws are formulated

free-fall acceleration
the acceleration of a body falling freely in a vacuum near the surface of an astronomical body in the local gravitational field

frequency
the number of waves that move past a given point in one second (symbol: f; SI unit: hertz; unit: Hz)

friction
the resistance to motion of a surface moving relative to another

fundamental forces
those that act between bodies of matter and are mediated by one or more particles. In order from strongest to weakest: the strong nuclear force, electromagnetic force, the weak force, the gravitational force

G

gauge bosons
mediating particles that govern particle interaction and the mediation of the four fundamental forces. There are four gauge bosons in the Standard Model

generation
a division of the elementary particles according to the Standard Model. There are three generations, or families, of elementary particles grouped according to mass and charge

gluon
the fundamental exchange particle that operates between quarks and hence underlies the strong nuclear force between nucleons (protons and neutrons) in a nucleus

gravitational field
the region of space surrounding a body in which another body experiences a force of gravitational attraction

gravitational field direction
towards the direction of the net gravitational force

gravitational field strength
(g) is the net force (F) per unit mass (m) at a particular point in the gravitational field

gravity
the force of attraction between objects with mass

greenhouse effect
the process by which certain gases in the atmosphere, known as greenhouse gases, warm Earth by absorbing the long wavelength radiation from Earth's surface

H

hadron
particle composed of quarks and gluons (optional term)

Higgs boson
an elementary particle in the Standard Model that acts as a 'force carrier' for the Higgs field – a field that pervades the universe and is responsible for giving certain elementary particles mass. It is similar to the way a photon is a force carrier for the electromagnetic field

horizontal component
the resolution of a projectile's velocity in the horizontal direction

Hubble's law
there is a direct proportion between the distance to a galaxy and its recessional velocity as determined by the red shift

I

impact speed
the speed at which an object impacts a surface

impact velocity
the velocity of a projectile immediately before striking the ground; the magnitude of impact velocity is impact speed

inclined plane
a flat surface raised at one end, used as an aid for raising or lowering a load

independent variable
a variable (often denoted by x) whose variation does not depend on that of another

inertial frame of reference
a non-accelerating frame of reference in which Newton's laws of motion hold

L

launch velocity
the velocity at which a projectile is launched; is a vector quantity that has both magnitude and direction, usually specified as an angle of elevation to the horizontal

law of conservation of energy
energy cannot be created nor destroyed, but only changed from one form to another or transferred from one object to another

length contraction
the shorter measurement made by an observer moving relative to the object in the direction of the length being measured

Lenz's law
states that the direction of an induced electric current is such that it produces a current whose magnetic field opposes the change in the circuit or the magnetic field that produces it

lepton
a class of elementary particles that respond only to the weak force and the gravitational force. They can carry one unit of electric charge or are neutral, and those that are charged experience the electromagnetic force. There are six leptons in the Standard Model

lepton number
a conserved quantum number representing the difference between the number of leptons and the number of antileptons in an elementary particle reaction:

$$L = (n_{l} - n_{\bar{l}})$$

light-year (ly)
the distance travelled by light in one year (9.5×10^{15} m)

linearising
a process of transforming non-linear data by applying a mathematical function to one of the variables so that the relationship between the variables becomes closer to a straight line

M

magnetic field
a region of space where a magnetic force is experienced

magnetic field line
the direction an isolated north pole would move in the field

magnetic flux
a measurement of the total magnetic field that passes through a given area; a measure of the number of magnetic field lines passing through the given area (symbol: ϕ; SI unit: weber; unit symbol: Wb)

magnetic flux density
the strength of a magnetic field or the number of magnetic field lines per unit area (symbol: B; SI unit: weber per square metre; unit symbol: Wb m^{-2} or T)

magnetism
a phenomenon associated with magnetic fields, which arise from the motion of electric charges

mass
an object's resistance to motion; also commonly stated as the amount of matter in an object (symbol: m; SI unit: kilogram; unit symbol: kg)

mass–energy equivalence relationship
relates change in mass to change in energy, given by $\Delta E = \Delta mc^2$

matter
a physical substance that has mass and takes up space by having volume, especially as distinct from energy

mean lifetime
the average time before decay of an elementary particle as measured by an observer at rest to the particle; also known as rest life

mediating particles
a descriptive name for the gauge bosons, which govern the interaction of the four fundamental forces; also known as carrier particles or exchange particles

meson
subatomic particle composed of one quark and one antiquark, held together by the strong nuclear force

momentum
the product of an object's mass and its velocity (Newtonian); it is a vector quantity and is conserved in interactions (symbol: p; unit symbol: kg m s^{-1})

muon
an elementary particle similar to the electron but with a greater mass; it is the product of the decay of pions

muon neutrino
an almost massless and neutral elementary particle produced in radioactive decay of pions

mutual induction
the production of an EMF in a circuit by a change in the current in an adjacent circuit that is linked to the first by the flux lines of a magnetic field

N

Newton's law of universal gravitation
states that the force of attraction between each pair of point particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

normal force
the force acting along an imaginary line drawn perpendicular to the surface

O

orbitals
regions of space around the nucleus of an atom where an electron is likely to be found

P

parabola
a graph of a quadratic function for which the power a is 2, e.g. $y = x^2$

paradox
a self-contradictory conclusion from true premises

particle zoo
a term used in particle physics to describe the relatively extensive list of particles by comparison to the variety of species in a zoo

period
the time taken to complete one revolution calculated by the formula:
$\text{period} = \dfrac{\text{time}}{\text{no. of revolutions}}$ (symbol: T; SI unit: second; unit symbol: s)

photoelectric effect
the emission of electrons (or other free carriers) when light shines on a material

photon
a quantum of all forms of electromagnetic radiation

pion
a subatomic particle produced in the atmosphere as a result of the collision of cosmic ray protons with nitrogen and oxygen atoms

Planck's constant
a fundamental constant used in quantum mechanics that relates frequency to energy (symbol: h; SI unit: joule second; unit symbol: J s), equal to 6.626×10^{-34} J s. Also known as the Planck constant

Planck equation
a relationship that relates frequency to energy: $E = hf$. It accounts for the quantised nature of light and plays a key role in understanding phenomena such as the photoelectric effect and Planck's law of black-body radiation

precision
the uncertainty of the measurement

principal quantum number
n, is a discrete variable assigned to each electron in an atom to describe the energy level of the electron, with higher numbers representing higher potential energy (further from the nucleus)

proper length
the length as measured by an observer at rest with respect to the object

proper time interval
the time between two events measured by an observer at rest to the events

Q

quanta
the smallest discrete packets of energy of electromagnetic waves, also later known as photons

quantised
form into quanta with certain discrete energies

quark
subatomic particles governed by the strong nuclear force that constitute hadrons; there are six quarks in the Standard Model

R

random error
error due to the uncertainty of the measurement equipment and the uncontrollable effects of procedure and environment on a measurement result

range
the horizontal displacement of a projectile upon impact

relative motion
the motion of an object with regard to some other moving object; the motion is not calculated with reference to Earth, but is the velocity of the object in reference to the other moving object as if it were in a static state

relativistic length
the length as measured by an observer moving with respect to the object in the direction of motion

relativistic momentum
the momentum of an object as measured by an observer moving relative to the object

relativistic time interval
the time between two events measured by an observer moving with respect to the events; also known as dilated time (interval)

relativity of simultaneity
events that are simultaneous in one frame of reference are not necessarily simultaneous in another frame of reference, even if both frames are inertial

reliable
constant and dependable, or consistent and repeatable

resolution
determining the components of a vector, usually at right angles to each other

rest mass
the mass of an object when measured in the same reference frame as the observer

resultant vector
a single vector that is a combination of two or more other vectors

rotational speed
the number of revolutions an object does per second, as distinguished from the term 'average speed', which is the linear speed

Rutherford's model
a small, central positively charged nucleus with negatively charged electrons orbiting around it

S

scale reading limitation
the inability of an instrument to resolve small measurement differences

Schwarzschild radius
a point near a black hole where the gravity is so powerful that nothing, not even light, can escape

scientific notation
a shorthand way of expressing very large or very small numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10

second law of planetary motion (law of areas)
a radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time

secondary evidence
data that has been compiled from records of primary sources by someone not directly involved in the primary event

sidereal period
the time it takes for a planet to complete one orbit of another body relative to the stars

simultaneity
the relation between two events assumed to be happening at the same time in a frame of reference

solenoid
a long straight coil of wire used to generate a controlled and almost uniform magnetic field

Standard Model
a theory describing three of the four known fundamental forces in the universe, as well as classifying all known elementary particles

Stefan-Boltzmann law
states that the energy radiated by an ideal black body is proportional to the fourth power of the temperature in kelvin

stopping potential
the negative potential on the collector at which the photoelectric current becomes zero

strong nuclear force
the strongest of the four fundamental forces; binds quarks together to make subatomic particles such as protons and neutrons and underlies interactions between all particles containing quarks; also called the strong force

symmetry
when a particle interaction is subjected to a certain operation and it appears exactly the same after the operation

synodic period
time taken for a planet to appear in front of the same constellation of stars as seen from Earth

systematic error
an error that is due to the accuracy of a measurement process that causes readings to deviate from the accepted value by a consistent amount each time a measurement is made

T

tangential velocity
the linear velocity of an object undergoing circular motion, where the magnitude is the speed of the object, and the direction is a tangent to the circular path at that moment (directed towards the centre); for circular motion, the term is usually abbreviated to 'velocity'

tension
the pulling force transmitted along a rope, string, cable or chain on an object

tesla
the SI unit of magnetic field strength; $1\ \mathrm{T} = 1\ \mathrm{N}\ \mathrm{C}^{-1}\ \mathrm{m}^{-1}\ \mathrm{s}$

theory of special relativity
a theory that all motion must be defined relative to a frame of reference; it consists of two principal postulates. It explains how space and time are linked for objects moving at constant speed in a straight line, and forms part of the basis of modern physics

the two postulates of special relativity
the two assumptions of the theory of special relativity: 1, that the laws of physics are the same in all inertial frames of reference; 2, that the speed of light in a vacuum has the same value c in all inertial frames of reference

third law of planetary motion (law of periods)
the square of the sidereal period of a planet is directly proportional to the cube of its mean distance from the Sun: $T^2 \propto r^3$

threshold frequency
the minimum frequency of a photon that can eject an electron from a surface

time dilation
the difference in the time interval between two events as measured by observers moving with respect to each other

time-reversal symmetry
an interaction looks the same if the flow of time is reversed (products become reactants, and reactants become products)

trajectory
the path taken by a projectile in flight

transverse waves
a wave where the direction of oscillation of particles is perpendicular to the direction of energy transfer

U

uniform circular motion
the motion of an object travelling at a constant speed in a circle

uniform electric field
a field that has constant field strength, as found between charged parallel plates

uniform motion
motion of an object that is not undergoing acceleration

V

vector
a variable quantity, such as force, that has magnitude and direction

vertex
the point where particles interact (plural, *vertices*). At the vertex they will emit or absorb new particles, deflect one another, or change type

vertical component
the resolution of a projectile's velocity in the vertical direction

W

wave model of light
uses wave characteristics such as wavelength, frequency and speed to describe the behaviour of light such as polarisation, interference and refraction

wavelength
the distance between corresponding points on successive waves with the same velocity (symbol: λ; SI unit: metre; unit symbol: m)

wave–particle duality
every particle or quantum entity may be partly described in terms not only of particles, but also of waves

weak nuclear force
the third strongest force of the four fundamental forces; it is responsible for radioactive decay and is mediated by W (W^+, W^-) and Z^0 bosons

weight
a measure of the force of gravity acting on an object (symbol: F_g; SI unit: newton; unit symbol: N)

Wien's displacement law
states that the black-body radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature

work function
the minimum energy required to remove an electron from a solid (symbol: W; SI unit: joule; unit symbol: J). Formula: $W = hf_0$.

Y

Young's double slit experiment
demonstrates the wave nature of light by allowing two coherent beams of light to overlap on a screen to form an interference pattern

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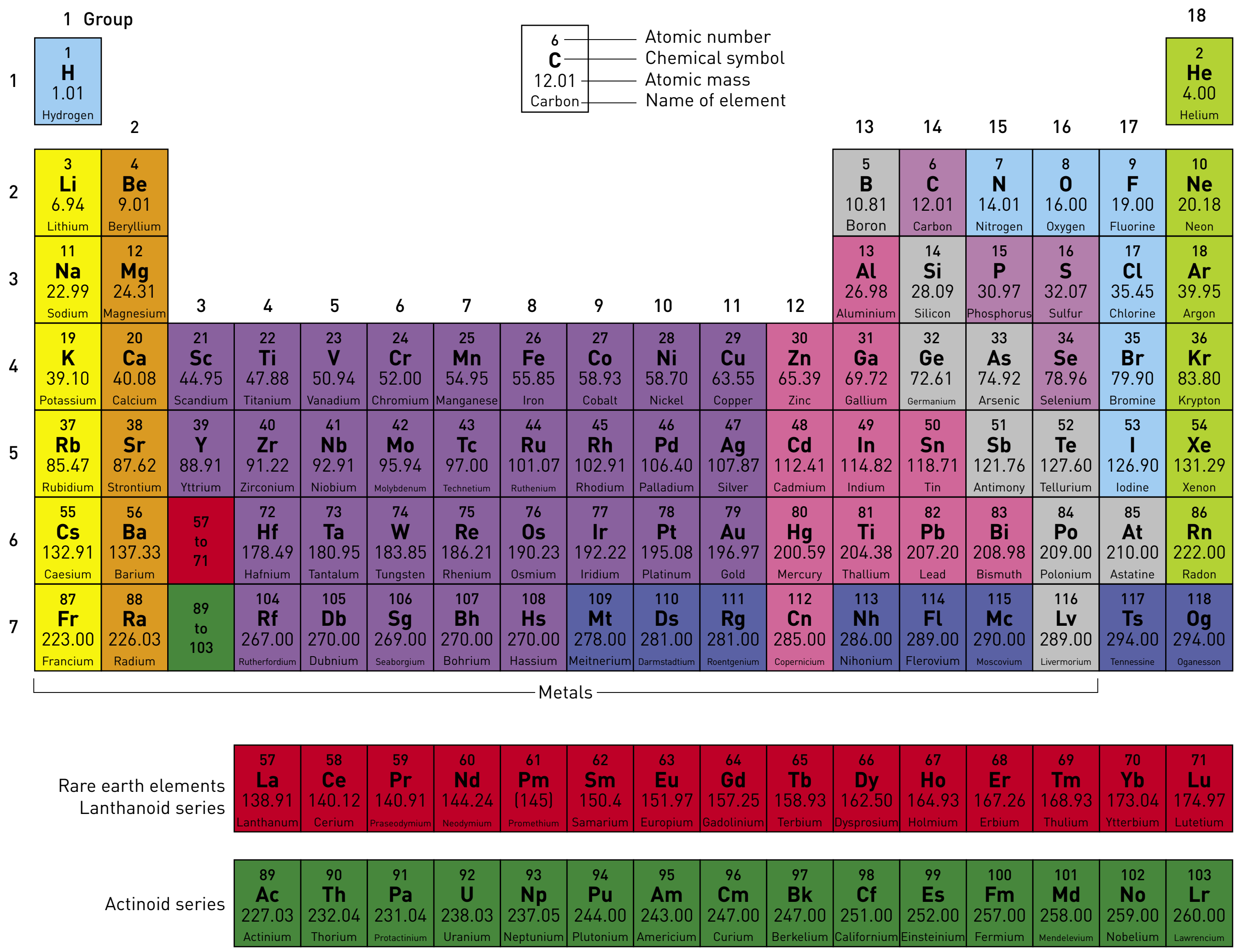
APPENDIX

Periodic table



6 — Atomic number
C — Chemical symbol
12.01 — Atomic mass
Carbon — Name of element

	1 Group	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 **H** 1.01 Hydrogen																	2 **He** 4.00 Helium
2	3 **Li** 6.94 Lithium	4 **Be** 9.01 Beryllium											5 **B** 10.81 Boron	6 **C** 12.01 Carbon	7 **N** 14.01 Nitrogen	8 **O** 16.00 Oxygen	9 **F** 19.00 Fluorine	10 **Ne** 20.18 Neon
3	11 **Na** 22.99 Sodium	12 **Mg** 24.31 Magnesium											13 **Al** 26.98 Aluminium	14 **Si** 28.09 Silicon	15 **P** 30.97 Phosphorus	16 **S** 32.07 Sulfur	17 **Cl** 35.45 Chlorine	18 **Ar** 39.95 Argon
4	19 **K** 39.10 Potassium	20 **Ca** 40.08 Calcium	21 **Sc** 44.95 Scandium	22 **Ti** 47.88 Titanium	23 **V** 50.94 Vanadium	24 **Cr** 52.00 Chromium	25 **Mn** 54.95 Manganese	26 **Fe** 55.85 Iron	27 **Co** 58.93 Cobalt	28 **Ni** 58.70 Nickel	29 **Cu** 63.55 Copper	30 **Zn** 65.39 Zinc	31 **Ga** 69.72 Gallium	32 **Ge** 72.61 Germanium	33 **As** 74.92 Arsenic	34 **Se** 78.96 Selenium	35 **Br** 79.90 Bromine	36 **Kr** 83.80 Krypton
5	37 **Rb** 85.47 Rubidium	38 **Sr** 87.62 Strontium	39 **Y** 88.91 Yttrium	40 **Zr** 91.22 Zirconium	41 **Nb** 92.91 Niobium	42 **Mo** 95.94 Molybdenum	43 **Tc** 97.00 Technetium	44 **Ru** 101.07 Ruthenium	45 **Rh** 102.91 Rhodium	46 **Pd** 106.40 Palladium	47 **Ag** 107.87 Silver	48 **Cd** 112.41 Cadmium	49 **In** 114.82 Indium	50 **Sn** 118.71 Tin	51 **Sb** 121.76 Antimony	52 **Te** 127.60 Tellurium	53 **I** 126.90 Iodine	54 **Xe** 131.29 Xenon
6	55 **Cs** 132.91 Caesium	56 **Ba** 137.33 Barium	57 to 71	72 **Hf** 178.49 Hafnium	73 **Ta** 180.95 Tantalum	74 **W** 183.85 Tungsten	75 **Re** 186.21 Rhenium	76 **Os** 190.23 Osmium	77 **Ir** 192.22 Iridium	78 **Pt** 195.08 Platinum	79 **Au** 196.97 Gold	80 **Hg** 200.59 Mercury	81 **Ti** 204.38 Thallium	82 **Pb** 207.20 Lead	83 **Bi** 208.98 Bismuth	84 **Po** 209.00 Polonium	85 **At** 210.00 Astatine	86 **Rn** 222.00 Radon
7	87 **Fr** 223.00 Francium	88 **Ra** 226.03 Radium	89 to 103	104 **Rf** 267.00 Rutherfordium	105 **Db** 270.00 Dubnium	106 **Sg** 269.00 Seaborgium	107 **Bh** 270.00 Bohrium	108 **Hs** 270.00 Hassium	109 **Mt** 278.00 Meitnerium	110 **Ds** 281.00 Darmstadtium	111 **Rg** 281.00 Roentgenium	112 **Cn** 285.00 Copernicium	113 **Nh** 286.00 Nihonium	114 **Fl** 289.00 Flerovium	115 **Mc** 290.00 Moscovium	116 **Lv** 289.00 Livermorium	117 **Ts** 294.00 Tennessine	118 **Og** 294.00 Oganesson

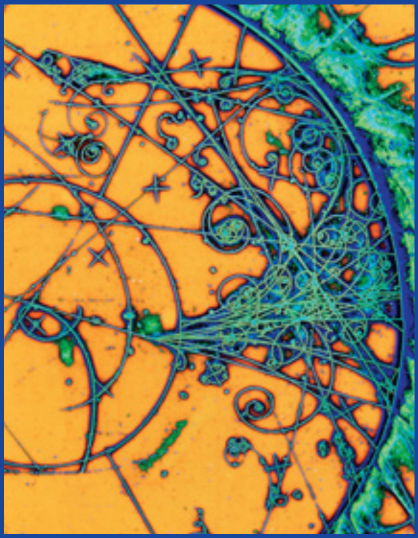
Metals

Rare earth elements Lanthanoid series	57 **La** 138.91 Lanthanum	58 **Ce** 140.12 Cerium	59 **Pr** 140.91 Praseodymium	60 **Nd** 144.24 Neodymium	61 **Pm** (145) Promethium	62 **Sm** 150.4 Samarium	63 **Eu** 151.97 Europium	64 **Gd** 157.25 Gadolinium	65 **Tb** 158.93 Terbium	66 **Dy** 162.50 Dysprosium	67 **Ho** 164.93 Holmium	68 **Er** 167.26 Erbium	69 **Tm** 168.93 Thulium	70 **Yb** 173.04 Ytterbium	71 **Lu** 174.97 Lutetium
Actinoid series	89 **Ac** 227.03 Actinium	90 **Th** 232.04 Thorium	91 **Pa** 231.04 Protactinium	92 **U** 238.03 Uranium	93 **Np** 237.05 Neptunium	94 **Pu** 244.00 Plutonium	95 **Am** 243.00 Americium	96 **Cm** 247.00 Curium	97 **Bk** 247.00 Berkelium	98 **Cf** 251.00 Californium	99 **Es** 252.00 Einsteinium	100 **Fm** 257.00 Fermium	101 **Md** 258.00 Mendelevium	102 **No** 259.00 Nobelium	103 **Lr** 260.00 Lawrencium

METALS
- 🟨 alkali metal
- 🟧 alkaline earth metal
- 🟥 lanthanide
- 🟩 actinide
- 🟪 transition metals
- ■ post-transition metals

NON-METALS
- 🟦 diatomic non-metals
- ■ polyatomic non-metals
- 🟩 noble gases

OTHER
- ⬜ metalloids
- 🟦 unknown chemical properties



The front cover shows subatomic particles captured in a bubble chamber. The spiral tracks are due to electrons and positrons moving through the bubble chamber. This method allows physicists to see the interactions between particles and use theories such as the Standard Model to explain the interactions of particles.

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