PEARSON mathematics

NEW SOUTH WALES

S.B.

STAGES 5.1, 5.2 AND 5.3



AUSTRALIAN CURRICULUM



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We would like to thank our authors for their extraordinary dedication and their contribution to the development of this project.



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Fibonacci poem

The poem on the inside front cover is called a Fib. The sequence of syllables in the lines follows a Fibonacci sequence: 1, 1, 2, 3, 5, 8.

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PEARSON mathematics
NEW SOUTH WALES



Pearson Mathematics New South Wales provides you with a 'nuts and bolts', no-nonsense Mathematics series that saves you time and delivers differentiated pathways for students of all abilities, at an affordable price. We've built this series from the ground up specifically for the Australian Curriculum, developing it for the NSW syllabus in conjunction with the national edition. We've found it important to create our resources for the Australian Curriculum from scratch to ensure we can incorporate the latest research about how students learn Mathematics best.

Student Book

The *Pearson Mathematics New South Wales 9 Stages 5.1, 5.2 and 5.3* Student Book delivers a differentiated pathway for every student. We've built Foundation, Standard and Advanced pathways into every exercise so you can truly differentiate Mathematics for all students.

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- worked solutions to questions
- over 150 Bridging Worksheets to support and scaffold students across Years 7 and 8 who struggle with secondary mathematics concepts, so that they are able to complete exercises
- over 250 Recall Worksheets enabling students to master the specific prerequisite skills required for each chapter.



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ProductLink

Digital teacher resources including editable teaching programs, syllabus correlations, Recall worksheets and chapter tests are available for free when you purchase **Pearson Mathematics New South Wales**. To access, visit **www.pearsonplaces.com.au** and log in. Click on 'Toolkit', then select 'ProductLink' and browse to your title.

USING PEARSON mathematics NEW SOUTH WALES

Pearson Mathematics New South Wales provides a differentiated pathway for every student, enabling you to support the full range of student abilities.

Pearson Mathematics New South Wales differentiated pathway structure

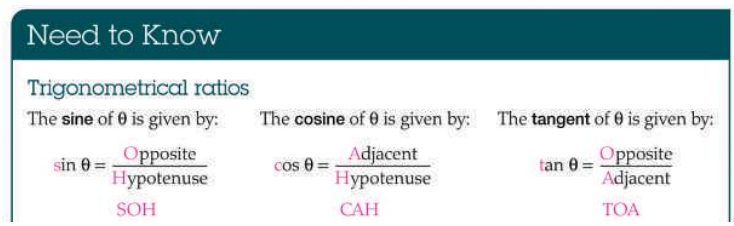
Recall	Each chapter has a **Recall** section that tests whether students have the prerequisite skills and understanding to begin a chapter. Every Recall question has a matching **Recall Worksheet** available free on ProductLink that teaches the relevant skill and allows students to practise it.	**Below Stage Level**
Bridging	Over 150 **Bridging Worksheets** are included with Pearson eBook 3.0 to support and scaffold students across Years 7 and 8 who struggle with secondary mathematics concepts, so that they are able to complete specific Student Book exercises.	
Foundation	A **Foundation** pathway through each exercise is shown in the **Navigator** feature to ensure students at this level are given sufficient skills practice as well as some exposure to the higher-order aspects of Working Mathematically.	**3 Navigator Pathways At Stage Level**
Standard	A **Standard** pathway through each exercise is shown in the **Navigator** feature to ensure students at this level are given a balance of skills practice and the higher-order aspects of Working Mathematically.	
Advanced	An **Advanced** pathway through each exercise is shown in the **Navigator** feature to ensure students at this level are given skills practice as well as an emphasis on the higher-order aspects of Working Mathematically.	
Challenge	Each chapter includes a **Challenge** page of high-level extension work for the most able students.	**Above Stage Level**

Pearson Mathematics New South Wales provides students with just the right amount of information in one no-frills book at each year level.

In Years 9 and 10, **Pearson Mathematics New South Wales** covers the full range of **Stages 5.1**, **5.2** and **5.3** (and **Stage 4** revision where relevant), with content from all Stages explicitly labelled for maximum flexibility. To assist planning for Stage 6, the symbol ◊ indicates material for students intending to study Preliminary Mathematics General/ HSC Mathematics General 2; § is for Mathematics ('2 Unit'); and # is for Mathematics Extension 1.

Stage 4 Stage 5.1 Stage 5.2 Stage 5.3
Stage 5.2◊ Stage 5.3§ Stage 5.3#

Need to Know sections provide concise, accurate theory that promotes understanding.


Need to Know

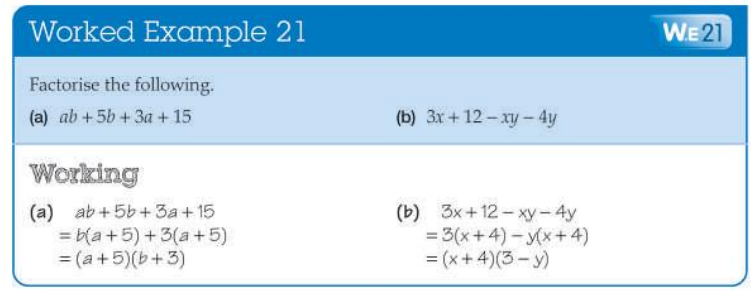
Trigonometrical ratios

The **sine** of θ is given by: $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ SOH

The **cosine** of θ is given by: $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ CAH

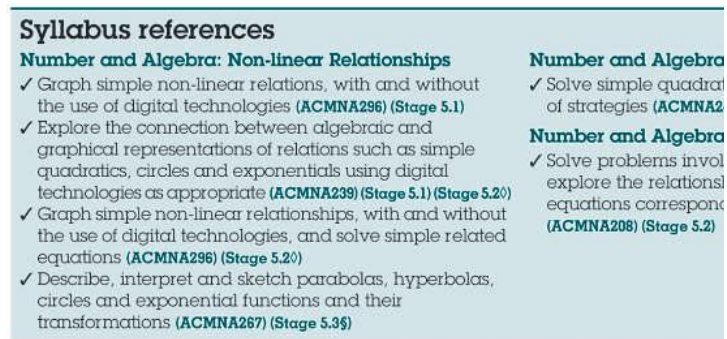
The **tangent** of θ is given by: $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$ TOA

Worked Examples clearly demonstrate the working that students will need to show to achieve full marks in a test.

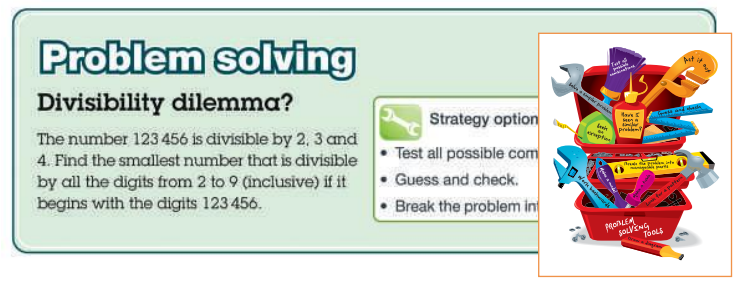


Pearson Mathematics New South Wales is a perfect match to the NSW Syllabus for the Australian Curriculum, including all the higher-order aspects of Working Mathematically and the technology requirements.

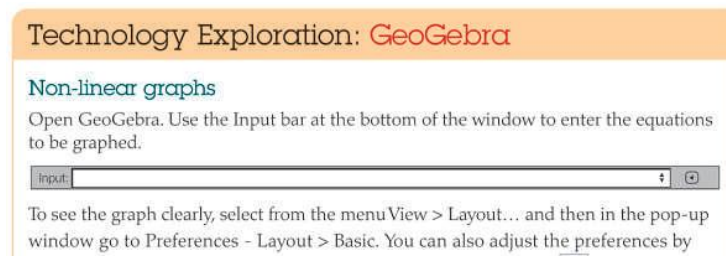
Syllabus references are provided at the start of every chapter, while a detailed **Syllabus Correlation** document (available free on ProductLink) shows where every element of syllabus content is covered.



Questions in every exercise are categorised into 'Fluency and Understanding' and 'Reasoning, Open-ended and Communicating'. High quality **Problem solving** tasks also appear regularly.



Technology Explorations appear regularly throughout the text, providing students with a deeper understanding of concepts covered and making it easy to integrate ICT into the classroom.



Pearson Mathematics New South Wales provides regular graded revision and reinforcement to help your students consolidate their learning.

Half-time—mid-chapter review



Chapter review—end-of-chapter review



Mixed review—cumulative multi-chapter review



NAPLAN practice—end-of-chapter section to help students prepare for NAPLAN tests



PEARSON mathematics

NEW SOUTH WALES

Stage 5 Curriculum Grid for Year 9

This maps the NSW Mathematics Syllabus (Stage 5) for the Australian Curriculum content to the *Pearson Mathematics New South Wales 9 Stages 5.1, 5.2 and 5.3* Student Book.

Detailed syllabus references are given at the start of each chapter. A full Stage 5 curriculum audit, including syllabus content dot points, is available on ProductLink.

Stage 5.1	Year 9
Number and Algebra	
Financial Mathematics	**Ch. 1**
Solve problems involving earning money	1.3, 1.4
Solve problems involving simple interest **(ACMNA211)**	1.5, 1.6
Indices	Ch. 2
Extend and apply the index laws to variables, using positive-integer indices and the zero index **(ACMNA212)**	2.1, 2.2
Simplify algebraic products and quotients using index laws **(ACMNA231)**	2.1, 2.2
Apply index laws to numerical expressions with integer indices **(ACMNA209)**	2.2
Linear Relationships	Ch. 4
Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software **(ACMNA294)**	4.1, 4.6
Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software **(ACMNA214)**	4.1
Sketch linear graphs using the coordinates of two points **(ACMNA215)**	4.5
Solve problems involving parallel lines **(ACMNA238)**	4.6
Non-Linear Relationships	Ch. 8
Graph simple non-linear relations, with and without the use of digital technologies **(ACMNA296)**	8.1
Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technologies as appropriate **(ACMNA239)**	8.1
Measurement and Geometry	
Area and Surface Area	Ch. 3
Calculate the areas of composite shapes **(ACMMG216)**	3.2
Solve problems involving the surface areas of right prisms **(ACMMG218)**	3.3, 3.4
Numbers of Any Magnitude	Ch. 2
Investigate very small and very large time scales and intervals **(ACMMG219)**	2.4
Express numbers in scientific notation **(ACMNA210)**	2.4
Right-Angled Triangles (Trigonometry)	Ch. 6
Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles **(ACMMG223)**	6.1
Apply trigonometry to solve right-angled triangle problems **(ACMMG224)**	6.3, 6.4
Solve right-angled triangle problems, including those involving angles of elevation and depression **(ACMMG245)**	6.5
Properties of Geometrical Figures	Ch. 5
Use the enlargement transformation to explain similarity **(ACMMG220)**	5.2
Solve problems using ratio and scale factors in similar figures **(ACMMG221)**	5.2
Statistics and Probability	
Single Variable Data Analysis	Ch. 9
Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources **(ACMSP228)**	7.2
Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal' **(ACMSP282)**	7.3, 7.4
Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread **(ACMSP283)**	7.4
Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data **(ACMSP253)**	7.2
Probability	Ch. 7
Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' **(ACMSP226)**	7.6

Stage 5.2	Year 9
Number and Algebra	
Ratios and Rates	Ch. 8
Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems **(ACMNA208)**	8.8
Algebraic Techniques	Ch. 2
Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate **(ACMNA213)**	2.6
Factorise algebraic expressions by taking out a common algebraic factor **(ACMNA230)**	2.7
Expand binomial products and factorise monic quadratic expressions using a variety of strategies **(ACMNA233)**	2.6, 2.8
Indices	Ch. 2
Apply index laws to algebraic expressions involving integer indices	2.3
Equations	Ch. 4
Solve linear equations **(ACMNA215)**	4.3
Solve linear equations involving simple algebraic fractions **(ACMNA240)**	4.3
Substitute values into formulas to determine an unknown **(ACMNA234)**	4.2, 4.6
Solve problems involving linear equations, including those derived from formulas **(ACMNA235)**	4.4
Linear Relationships	Ch. 4
Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line	4.8
Solve problems involving parallel and perpendicular lines **(ACMNA238)**	4.8
Non-Linear Relationships	Ch. 8
Graph simple non-linear relationships, with and without the use of digital technologies, and solve simple related equations **(ACMNA296)**	8.2, 8.3, 8.5, 8.7
Explore the connection between algebraic and graphical representations of relationships such as simple quadratics, circles and exponentials using digital technologies as appropriate **(ACMNA239)**	8.2, 8.3, 8.5, 8.7
Measurement and Geometry	
Area and Surface Area	Ch. 3
Calculate the surface areas of cylinders and solve related problems **(ACMMG217)**	3.5
Solve problems involving surface area for a range of prisms, cylinders and composite solids **(ACMMG242)**	3.5
Volume	Ch. 3
Solve problems involving the volumes of right prisms **(ACMMG218)**	3.6
Solve problems involving volume for a range of prisms, cylinders and composite solids **(ACMMG242)**	3.6
Right-Angled Triangles (Trigonometry)	Ch. 6
Solve right-angled triangle problems, including those involving direction and angles of elevation and depression **(ACMMG245)**	6.6
Properties of Geometrical Figures	Ch. 5
Formulate proofs involving congruent triangles and angle properties **(ACMMG243)**	5.5, 5.6
Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar **(ACMMG220)**	5.3
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes **(ACMMG244)**	5.4, 5.5, 5.6
Statistics and Probability	
Probability	Ch. 7
List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events **(ACMSP225)**	7.7

Stage 5.3	Year 9
Number and Algebra	
Algebraic Techniques	Ch. 2
Expand binomial products using a variety of strategies **(ACMNA233)**	2.9
Factorise monic and non-monic quadratic expressions **(ACMNA269)**	2.9
Equations	Ch. 2
Rearrange literal equations	2.5
Linear Relationships	Ch. 4
Find the midpoint and gradient of a line segment (interval) on the Cartesian plane **(ACMNA294)**	4.2
Find the distance between two points located on the Cartesian plane **(ACMNA214)**	4.2
Sketch linear graphs using the coordinates of two points **(ACMNA215)**	4.9

1 Financial mathematics

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Convert the following fractions to decimals.

(a) $\frac{17}{100}$ **(b)** $\frac{57}{1000}$ **(c)** $\frac{321}{100\,000}$

2 Convert the following fractions to decimals. If the decimal is a recurring decimal, use the appropriate notation.

(a) $\frac{16}{25}$ **(b)** $\frac{9}{8}$ **(c)** $\frac{23}{40}$ **(d)** $\frac{1}{6}$ **(e)** $\frac{10}{11}$ **(f)** $\frac{5}{7}$

3 Convert the following decimals to fractions in simplest form.

(a) 0.79 **(b)** 0.003 **(c)** 0.014 **(d)** 1.065 **(e)** 10.140 **(f)** 0.0902

4 Evaluate the following.

(a) $5.8 + 2.9$ **(b)** $12.7 - 3.6$ **(c)** 5.2×1.5 **(d)** $6.2 \div 0.03$

5 Convert the following fractions and decimals to percentages. (Round your answers to two decimal places, if necessary.)

(a) $\frac{14}{20}$ **(b)** 0.91 **(c)** 0.032 **(d)** $\frac{23}{28}$

6 Write the following percentages as fractions in simplest form.

(a) 85% **(b)** 22% **(c)** 6.7% **(d)** $\frac{1}{2}$%

7 Write the following percentages in decimal form.

(a) 49% **(b)** 30% **(c)** 2.5% **(d)** 370%

8 Calculate the following:

(a) $\frac{5}{6}$ of 90 m **(b)** $\frac{3}{4}$ of 105 kg **(c)** $\frac{11}{10}$ of \$25 600

9 Find:

(a) 5% of 20 m **(b)** 10% of 63.5 km **(c)** 22% of 18.1 cm

Syllabus references

Number and Algebra: Fractions, Decimals and Percentages

- ✓ Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies **(ACMNA158) (Stage 4)**
- ✓ Solve problems involving the use of percentages, including percentage increases and decreases, with and without the use of digital technologies **(ACMNA187) (Stage 4)**

Number and Algebra: Financial Mathematics

- ✓ Investigate and calculate 'best buys', with and without the use of digital technologies **(ACMNA174) (Stage 4)**
- ✓ Solve problems involving profit and loss, with and without the use of digital technologies **(ACMNA189) (Stage 4)**
- ✓ Solve problems involving earning money **(Stage 5.1)**
- ✓ Solve problems involving simple interest **(ACMNA211) (Stage 5.1)**

Percentages review

1.1

Stage 4

Need to Know

Finding a percentage of an amount

'Per cent' means 'out of 100' or 'for every hundred', so change the percentage to a fraction with a denominator of 100, simplifying if necessary, and multiply by the amount.

For example:

$$40\% \text{ of } 35 = \frac{\cancel{40}^{2}}{\cancel{100}_{5}} \times 35$$
$$= 14$$

Some common fraction and percentage equivalents

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	$0.\dot{3}$	$0.\dot{6}$	0.1	0.2	0.05	0.01
Percentage	50%	25%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%	5%	1%

Increasing or decreasing by a given percentage

A percentage can represent a change in a quantity as a percentage increase or decrease of the original amount. The original amount can be increased by a percentage by adding the given percentage to 100%, then multiplying this new percentage by the original amount. The original amount can be decreased by subtracting the given percentage from 100% before multiplying it by the original amount.

For example:

100 increased by 10% = 110% of 100
= 110

100 decreased by 10% = 90% of 100
= 90

Writing one amount as a percentage of another

1. Make sure both amounts are the same type or measured in the same units. Convert units if necessary.
2. Write one amount as a fraction of the other.
3. Convert this fraction into a percentage by multiplying by 100%.

Rounding errors

Most financial applications of percentages use decimals instead of fractions. For practical purposes, the rounded approximation of a recurring decimal is used in calculations. For example, $\frac{5}{6}$ expressed as a decimal and rounded to one decimal place is 0.8. If we rounded it to two decimal places, we would use 0.83.

An error is introduced as soon as a rounded value is used. To minimise this error, round off as late as possible in a calculation or use fractions for as long as possible before converting to a decimal and rounding off. When a rounded answer is given, state its accuracy (e.g. when rounding to two decimal places, write 'correct to 2 d.p.').

Technology Exploration: Calculator

Fractions and decimals on the calculator

If you are working with decimals on your calculator, keep all the decimal places in the calculator display and only round off your final answer. Scientific calculators have a key that enables you to convert between fractions, decimals and mixed numbers. It may look like this S⇔D or this $a\frac{b}{c}$.

To enter the fraction $\frac{2}{9}$, for example, you would press $\frac{\square}{\square}$, then enter 2 and 9, using the cursor ◁▷ to move between numerator and denominator. To convert to a decimal, press S⇔D. To convert back to a fraction, press S⇔D again. To enter a mixed number such as $3\frac{2}{9}$, press SHIFT $\frac{\square}{\square}$ and use the cursor to enter the numbers. Pressing S⇔D will switch between the mixed number and decimal form. Pressing S⇔D will convert the improper fraction back to a mixed number.

Worked Example 1

WE1

Calculate the percentages of the given amounts:

(a) 35% of $62

(b) 27.5% of $300

Working

Method 1: *Use basic percentages*

(a) $35\% = 30\% + 5\%$
$= 3 \times 10\% + 5\%$

10% of 62
$= 62 \div 10$
$= 6.2$

5% of 62
$= (10\% \text{ of } 62) \div 2$
$= 6.2 \div 2$
$= 3.1$

$3 \times 10\% + 5\%$
$= (3 \times 6.2) + 3.1$
$= 18.6 + 3.1$
$= 21.7$

35% of $62 = $21.70

(b) $27.5\% = 25\% + 2.5\%$

25% of 300
$= 300 \div 4$
$= 300 \div 2 \div 2$
$= 150 \div 2$
$= 75$

$2.5\% = 25\% \div 10$
$= 75 \div 10$
$= 7.5$

$25\% + 2.5\%$
$= 75 + 7.5$
$= 82.5$

27.5% of $300 = $82.50

Working

Method 2: *Fraction or decimal multiplication*

(a) 35% of 62
$= 35\% \times 62$
$= \frac{35}{100} \times 62$ or 0.35×62
$= \frac{{}^{7}\cancel{35}}{{}_{20}\cancel{100}} \times \frac{62}{1}$
$= \frac{7}{{}_{10}\cancel{20}} \times \frac{{}^{31}\cancel{62}}{1}$
$= \frac{217}{10}$
$= 21.7$

35% of $62 = $21.70

(b) 27.5% of 300
$= 27.5\% \times 300$
$= \frac{27.5}{100} \times 300$ or 0.275×300
$= 27.5 \times 3$
$= 82.5$

27.5% of $300 = $82.50

Worked Example 2 WE2

Increase or decrease the following amounts by the given percentages. If necessary, round decimal answers to two decimal places.

(a) Increase $420 by 65%

(b) Decrease 87 kg by 13.6%

Working

(a) $(100 + 65)\% = 165\%$
$= 1.65$

$1.65 \times 420 = 693$

$420 increased by 65% is $693.

(b) $(100 - 13.6)\% = 86.4\%$
$= 0.864$

$0.864 \times 87 = 75.168$

87 kg decreased by 13.6% is 75.17 kg.

Worked Example 3 WE3

Round your answers to the following questions to one decimal place.

(a) Tyler kicked 13 goals from 18 shots at goal. Calculate his kicking accuracy as a percentage.

(b) A packet of 'Chocky' biscuits is on sale for $2.35, a saving of 57c. Write the saving as a percentage of the original price.

Working

Method 1: Fraction multiplication

(a) $\frac{13}{18} \times \frac{100}{1}\%$

$= \frac{13}{\cancel{18}_{9}} \times \frac{\overset{50}{\cancel{100}}}{1}$

$= \frac{650}{9}$

$= 72\frac{2}{9}\%$ or $72.\dot{2}\%$

Tyler's kicking accuracy is 72.2% (1 d.p.).

(b) $2.35 + $0.57 = $2.92

$\frac{0.57}{2.92} \times \frac{100}{1}\%$

$= \frac{57}{2.92}\%$

$= 19.520\,54\ldots\%$

The saving is 19.5% of the original price (1 d.p.).

Working

Method 2: Convert to a decimal, then multiply

(a) $\frac{13}{18} = 13 \div 18 = 0.7222\ldots$

$0.7222\ldots \times 100\% = 72.2222\ldots\%$

Tyler's kicking accuracy is 72.2% (1 d.p.).

(b) $2.35 + $0.57 = $2.92

$\frac{0.57}{2.92} = 0.57 \div 2.92 = 0.1952\ldots$

$0.1952\ldots \times 100\% = 19.52\ldots\%$

The saving is 19.5% of the original price (1 d.p.).

Answers page 420

1.1 Percentages review

Stage 4

Navigator

Q1 Columns 1 & 2, Q2 (a) & (b), Q3, Q4 (a), Q5, Q6, Q7, Q9, Q10, Q11, Q15, Q16, Q17	Q1 Columns 2 & 3, Q2 (b) & (c), Q3 (a)–(f), Q4, Q5, Q7, Q8, Q10, Q11, Q12, Q14, Q15, Q16, Q17	Q1 Column 3, Q2 (c) & (d), Q3 (c)–(h), Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17

Fluency and understanding

WE1

1 Calculate the percentages of the given amounts. If necessary, round decimal answers to two decimal places.

(a) 25% of $40 **(b)** 45% of $50 **(c)** 30% of $150

(d) 11% of 60 L **(e)** 66% of 400 m **(f)** 77% of 35 kg

(g) 9% of 20 kg **(h)** 59% of 74 kg **(i)** 88% of 60 L

(j) 0.5% of $3500 **(k)** 12.5% of 1500 m **(l)** 7.2% of 15 L

(m) $5\frac{3}{4}$% of $17 000 **(n)** $6\frac{1}{5}$% of $90 **(o)** $\frac{2}{3}$% of $200 000

WE2

2 Increase or decrease the following amounts by the given percentages. If necessary, round decimal answers to two decimal places.

(a) Increase 240 by:

(i) 50% **(ii)** 35% **(iii)** 8% **(iv)** 140%

(b) Decrease 5700 by:

(i) 20% **(ii)** 75% **(iii)** 46% **(iv)** 98%

(c) Increase $64.15 by:

(i) 10% **(ii)** 260% **(iii)** 7.5% **(iv)** $6\frac{3}{4}$ %

(d) Decrease 9.8 seconds by:

(i) 50% **(ii)** 13% **(iii)** 6.3% **(iv)** $3\frac{1}{3}$ %

WE3

3 Round your answers to the following questions to one decimal place.

(a) Ash kicked 9 goals from 13 shots at goal. Calculate Ash's percentage kicking accuracy.

(b) A jar of 'Nuttie's' peanut butter is on sale for $3.18, a saving of 64 cents. Write the saving as a percentage of the original price.

(c) Maryam had 22 shots at the netball goals and got 14 in. Calculate Maryam's percentage shooting accuracy.

(d) A box of 'Crunchy-Bix' cereal has been reduced by $1.02 to $4.37. Calculate the percentage saving based on the original cost price.

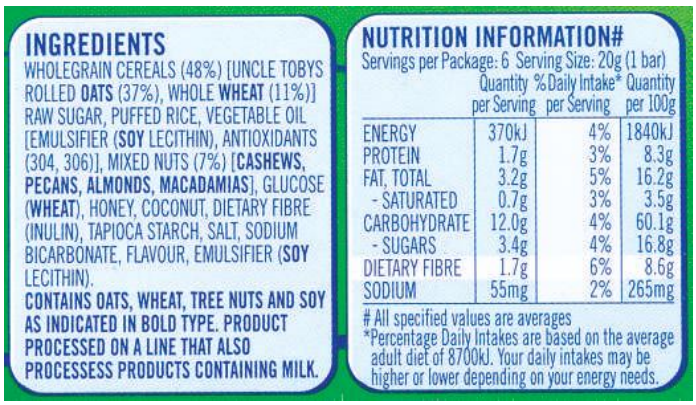
(e) Raf got 76 of 114 serves into the tennis court. Calculate the percentage accuracy of Raf's serve.

(f) A tin of 'Beanz' baked beans is on sale for $1.17, a saving of 32 cents. Write the saving as a percentage of the original price.

(g) Brett bowled a cricket ball at a set of stumps 54 times. He hit the stumps 41 times. Calculate Brett's stump-hitting accuracy as a percentage of the number of balls bowled.

(h) A tub of 'Yo-Cal' yoghurt has been reduced by 74 cents to $2.11. Calculate the percentage saving based on the original cost price.

4 This is the nutritional label from a packet of muesli bars. The 'serving size' is one bar, which has a mass of 20 g.



INGREDIENTS

WHOLEGRAIN CEREALS (48%) [UNCLE TOBYS ROLLED **OATS** (37%), WHOLE **WHEAT** (11%)] RAW SUGAR, PUFFED RICE, VEGETABLE OIL [EMULSIFIER (**SOY** LECITHIN), ANTIOXIDANTS (304, 306)], MIXED NUTS (7%) [**CASHEWS, PECANS, ALMONDS, MACADAMIAS**], GLUCOSE (**WHEAT**), HONEY, COCONUT, DIETARY FIBRE (INULIN), TAPIOCA STARCH, SALT, SODIUM BICARBONATE, FLAVOUR, EMULSIFIER (**SOY** LECITHIN).

CONTAINS OATS, WHEAT, TREE NUTS AND SOY AS INDICATED IN BOLD TYPE. PRODUCT PROCESSED ON A LINE THAT ALSO PROCESSESS PRODUCTS CONTAINING MILK.

NUTRITION INFORMATION#

Servings per Package: 6 Serving Size: 20g (1 bar)

	Quantity per Serving	% Daily Intake* per Serving	Quantity per 100g
ENERGY	370kJ	4%	1840kJ
PROTEIN	1.7g	3%	8.3g
FAT, TOTAL	3.2g	5%	16.2g
- SATURATED	0.7g	3%	3.5g
CARBOHYDRATE	12.0g	4%	60.1g
- SUGARS	3.4g	4%	16.8g
DIETARY FIBRE	1.7g	6%	8.6g
SODIUM	55mg	2%	265mg

All specified values are averages
*Percentage Daily Intakes are based on the average adult diet of 8700kJ. Your daily intakes may be higher or lower depending on your energy needs.

(a) Use the information in the 'Quantity per serving' column to calculate the percentage mass of the muesli bar that is:

(i) carbohydrate

(ii) protein

(iii) fat (total).

(b) Sugar is a type of carbohydrate.

(i) What percentage of the mass of a muesli bar is sugar?

(ii) What percentage of the carbohydrate mass is sugar?

Answer to one demical place.

5 The label on a bottle of tropical fruit juice says:

Pineapple juice:	78%
Orange juice:	15.5%
Passionfruit juice:	4.5%
Guava juice:	2%

There is 2.5 L of juice in the bottle. Calculate the volume of each type of juice, rounding answers to the nearest mL if necessary.

6 Approximately 65% of the mass of an adult human is water. Akira weighs 58 kg. How much of Akira's mass is water, to the nearest 100 g?

7 Stuart earns $1023.87 a week working as a plumber. He receives a weekly pay increase of 3.5%.

(a) Calculate Stuart's new weekly wage.

(b) How much more is Stuart now earning per week?

8 Charlotte is a runner whose personal best time for her favourite race is 3 min 34 seconds.

(a) Charlotte wants to decrease her time by 4%. How many seconds is this?

(b) At the state titles, Charlotte lowered her time by 6.9 seconds. What percentage improvement is this, correct to two decimal places?

9 There are two male and seven female gorillas in a zoo enclosure.

(a) What fraction of the gorillas are female?

(b) What percentage of the gorillas are male?

(c) One extra female is added to the enclosure. Recalculate your answers to **(a)** and **(b)**.

10 Copy and complete the following table.

	Common fraction	Decimal fraction	Percentage
(a)	$\frac{1}{2}$		
(b)		0.45	
(c)			72%
(d)	$\frac{5}{4}$		
(e)		0.56	
(f)			12.5%

11 Approximately $\frac{7}{8}$ of an iceberg lies under water. The mass of the iceberg is 150 000 tonnes.

(a) What mass of the iceberg is under water?

(b) What percentage of the whole mass is this?

12 At the end of the first week of trading on the share market, shares in Jaydeep's company had increased by 17.5%. The shares were worth $3.40 each at the beginning of the week.

(a) What is the value of a share now?

(b) At the end of the second week, the share price had decreased by 9% compared with its price at the beginning of that week. Calculate the value of one share at the end of the second week.

(c) Calculate the overall increase or decrease at the end of the 2 weeks as a percentage of the original price of $3.40, correct to two decimal places.

Reasoning, communicating and open-ended

13 In her Drama class, $\frac{3}{5}$ of Anita's overall mark is allocated to the practical exam and the remaining $\frac{2}{5}$ is allocated to the written exam. Anita scored 82% in her practical exam and 76% in the written exam. Calculate Anita's overall percentage for the subject, correct to 1 decimal place.

14 The local newspaper had the following headline: 'Prime Minister's approval rating 67%'. The article stated that the newspaper had surveyed all 180 000 of its readers, of which 14% had responded. Of those who responded, 67% said that they thought the Prime Minister was doing a good job. How many people actually said this?

15 **(a)** **(i)** Increase 100 by 20%.

(ii) Now decrease the answer obtained by 20%.

(b) **(i)** Increase 300 by 30%.

(ii) Now decrease the answer obtained by 30%.

(c) Explain why you do not arrive back at the starting number after an increase and decrease by the same percentage.

16 Write three whole numbers that when increased by 140% give an answer between 100 and 200.

17 Trent is looking to buy a new iPod. In one store, the iPod was on sale for $258, an advertised saving of $31. In another store, the same model iPod had a marked price of $299, but a sale sticker next to it said 'take 25% off the marked price'. Trent decided that $31 sounded like a better saving than 25% and bought the iPod from the first store.

(a) Was this the right choice in terms of saving money? Explain why or why not.

(b) Describe a simple calculation Trent could have done while standing in the shop that could have helped him make a more effective decision.

Problem solving

Age-old dilemmas

1 My mother is three times my age.

My sister is 75% of my age and 15% of my grandmother's age.

My father is 40, 4 years older than my mother.

Find my age, my grandmother's age and my sister's age.

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

2 Harry, Larry and Garry are three brothers who all go to secondary school.

Harry and Larry are an exact number of years apart. They have their birthdays on the same day. Garry's birthday is on a different day.

Harry is 80% of Larry's age.

Garry is 110% of Larry's age.

How old is each brother?

1.2 Buying and selling

Stage 4

Need to Know

Profit and loss

The cost of buying or manufacturing goods is called the cost price. The price that a retailer sells the product to the consumer for is the selling price. If the selling price is higher than the cost price, the retailer makes a **profit**. If the selling price is less than the cost price, then a **loss** is made.

Profit or loss can be expressed as a percentage of the cost price or selling price.

$$\%\ \text{Profit} = \frac{\text{profit in \$}}{CP} \times 100\% \qquad \text{or} \qquad \frac{\text{profit in \$}}{SP} \times 100\%$$

where CP = cost price, SP = selling price

To find the % Loss, substitute 'Loss in $' into the numerators of the above formulas.

Markup

Markup is the amount added to the cost price of an article to produce a particular percentage profit. This percentage is the markup percentage.

Calculating the cost price

If the selling price and markup of an article are known, the cost price of the article can be calculated by dividing the selling price by 100% plus the markup.

$$\text{Cost Price } (CP) \underset{\div\,(100\% + \text{markup}\%)}{\overset{\times\,(100\% + \text{markup}\%)}{\rightleftarrows}} \text{Selling Price } (SP)$$

Discounts

A **discount** is a reduction in the marked price of an item.

If the selling price and discount are known, the marked price of the article can be calculated by dividing the selling price by 100% minus the discount.

For discounts:

$$\text{Marked Price } (MP) \underset{\div\,(100\% - \text{discount}\%)}{\overset{\times\,(100\% - \text{discount}\%)}{\rightleftarrows}} \text{Sale Price } (SP)$$

Worked Example 4 WE4

Calculate the sale price of a $46 shirt discounted by 25%.

Working

Method 1: Find and subtract the discount amount

$25\% \text{ of } \$46 = 46 \div 4$
$= \$11.50$

$\text{Sale price} = 46 - 11.50$
$= \$34.50$

Working

Method 2: Find the remaining percentage

$100\% - 25\% = 75\%$

$\text{Sale price} = 75\% \text{ of } \$46$
$= 0.75 \times 46$
$= \$34.50$

Worked Example 5 WE5

A home theatre system costs an electrical store $510. It sells the system for $965. Calculate, correct to one decimal place, the profit made as a percentage of:

(a) the cost price **(b)** the selling price.

Working

(a) $\text{Profit} = SP - CP$
$= \$(965 - 510)$
$= \$455$

$$\%\ \text{Profit} = \frac{\text{profit}}{CP} \times 100\%$$
$$= \frac{455}{510} \times 100\%$$
$= 89.2\%$ (1 d.p.)

(b) $\text{Profit} = \$455$

$$\%\ \text{Profit} = \frac{\text{profit}}{SP} \times 100\%$$
$$= \frac{455}{965} \times 100\%$$
$= 47.2\%$ (1 d.p.)

Worked Example 6 WE6

A furniture store has a policy of selling their chairs and sofas with a markup on the cost price of 65%. What would a sofa that cost the store $300 be sold for?

Working

$65\% + 100\% = 165\%$
$= 1.65$

$\text{Selling price} = 1.65 \times \$300$
$= \$495$

Worked Example 7 WE7

A jewellery store has a policy of marking up the cost price of their goods by 75%. Calculate the cost price of a necklace that was sold for $56.

Working

$100\% + 75\% = 175\%$
$= 1.75$

175% of $CP = \$56$

$1.75 \times CP = \$56$

$CP = \$56 \div 1.75$
$= \$32$

Worked Example 8 WE8

A smartphone has a sale price of $539 after a discount of 22.5% is applied. What was the original marked price of the smartphone? Answer to the nearest dollar.

Working

$100\% - 22.5\% = 77.5\%$
$= 0.775$

77.5% of $MP = \$539$

$0.775 \times MP = \$539$

$MP = \$539 \div 0.775$
$= \$695$ (nearest $)

Answers page 420

1.2 Buying and selling

Stage 4

Navigator

Q1, Q2, Q3, Q4, Q5, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q19, Q22	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14, Q15, Q16, Q18, Q19, Q20, Q22	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23

Fluency and understanding

WE4

1 Calculate the sale price of the following items:

(a) a $24 pair of shorts discounted by 15%

(b) a $95 pair of shoes discounted by 25%

(c) a $63.99 skirt discounted by 40%

(d) a $419.95 suit discounted by 37.5%.

2 **(a)** A camera costs a camera shop $360. The shop sells the camera for $504. Calculate, correct to one decimal place, the profit as a percentage of: WE5

(i) the cost price **(ii)** the selling price.

(b) An old car costs a used car saleman $4200. He sells the car for $4050. Calculate, correct to one decimal place, the loss as a percentage of:

(i) the cost price **(ii)** the selling price.

3 A clothing store has a policy of selling jeans and T-shirts with a markup% on the cost price of 92%. Calculate the selling price of the following items, rounding your answer to the nearest 5 cents: WE6

(a) a pair of jeans, cost price $39.30 **(b)** a T-shirt, cost price $8.75.

4 A hardware store has a policy of marking up the cost price of their goods by 85%. Calculate the cost price of the following items, rounding your answer to the nearest cent: WE7

(a) a packet of nails, selling price $3.70 **(b)** a cordless drill, selling price $51.00.

5 **(a)** A tennis racquet has a sale price of $149 after a discount of 15% has been applied. What was the original marked price of the racquet? Answer to the nearest dollar. WE8

(b) A plasma TV has a sale price of $2497 after a discount of 7.5% has been applied. What was the original marked price of the TV? Answer to the nearest dollar.

6 If a table cost $486 and was subsequently sold for $350, the percentage profit or loss, based on the cost price, was:

A $\frac{(486 - 350)}{486} \times 100\%$ loss **B** $\frac{(486 - 350)}{350} \times 100\%$ loss

C $\frac{(486 - 350)}{350} \times 100\%$ profit **D** $\frac{(486 - 350)}{486} \times 100\%$ profit

7 If a laptop is sold for $960 at a 114% profit, the cost of the laptop was:

A $\$960 \times 2.14 \times 960$ **B** $\$960 \times 2.14$

C $\$960 \times 1.14$ **D** $\frac{\$960}{2.14}$

8 If a desk is for sale for $140 after a 25% discount, it means that it was originally marked at:

A $105 **B** $175 **C** $186.67 **D** $245

9 Calculate the selling price for the following cost prices and percentage profits (on cost price).

(a) $CP = \$48$, percentage profit = 45% **(b)** $CP = \$60$, percentage profit = 60%

10 A builder buys $840 worth of timber to build an extension. If he is allowed a 12.5% discount, how much does he pay?

11 A hairdresser gets a discount of 24% on hair products bought in bulk. How much is saved on $370 worth of products?

12 A store purchases a bulk order of 120 watches for $960.

(a) Calculate the profit made if the watches are sold individually for $19.95 each.

(b) Calculate the total profit as a percentage of the cost price.

13 A store purchased a bulk order of pyjamas cheaply for a sale. If a customer paid $11 for them at the sale, what were they marked at prior to the sale if they were reduced by 30%?

14 In Australia, a Goods and Services Tax (GST) of 10% is added to the price of all products and services purchased by consumers (with the exception of some basic grocery items). GST is added after the product has been marked up by the retailer. Mei is a clothing store owner who has just purchased a large number of T-shirts for $6.49 each. She adds a markup of 180%, then the 10% GST.

(a) What is the final selling price of the T-shirts? (Answer to the nearest cent.)

(b) What amount of the final price is GST?

15 Most of the time the marked prices we see on goods in shops have the 10% GST already added to the price. Find the original, pre-GST, price of:

(a) a DVD with a marked price of $29.90

(b) a vacuum cleaner with a marked price of $349.

16 A mechanic buys a car at an auction for $3120. The car is then repaired at a cost of $430. It is resprayed and has its panels straightened for $840. The car is then advertised and sold for $9800. Calculate the percentage profit on the selling price if the cost of advertising was $80.

17 Louise bought shares in the following companies through a share broker (a person who buys and sells shares on behalf of others). The share broker charges a fee of 2% of the value of the shares bought and sold.

150 shares in Oro Mining, bought for $39.05 each
200 shares in Beacon Bank, bought for $19.86 each
250 shares in JeansEast, bought for $6.14 each

(a) Calculate the share broker's fee.

(b) Including the fee, how much did Louise pay for her shares in total?

Louise instructs her share broker to sell her shares once they reach the following prices:

Oro Mining	$42
Beacon Bank	$21.50
JeansEast	$8

Her share broker does so and subtracts his fee.

(c) Calculate Louise's profit from the sale of the shares.

(d) Calculate Louise's profit as a percentage of the cost of her shares, including her share broker's fee. State your answer correct to two decimal places.

Reasoning, communicating and open-ended

18 Evie's favourite shoe shop has a 'buy one pair, get the second pair half-price' offer. Another shoe shop nearby is offering 20% off everything. If the regular retail price of the shoes Evie wants in both stores is $49.95, which store is the cheaper place for Evie to buy two pairs? How much would she save, compared with the other store?

19 A furniture store purchases a dining table and chairs for $750. It marks up this price by 45% to obtain its selling price.

(a) What is the selling price of the dining set? State your answer correct to the nearest dollar.

(b) The furniture is not selling, so the selling price is discounted by 45%. Calculate the sale price to the nearest dollar.

(c) The markup on the cost price was 45%. Why doesn't a 45% discount on the selling price return the price to the original cost price?

(d) What percentage saving will they be able to advertise if they take the price back to the cost price? State your answer correct to the nearest whole number.

20 A stallholder at a market buys 20 kg of peanuts for $65, 5 kg of hazelnuts for $30 and 1 kg of cashews for $13. He mixes them together and puts them into plastic bags each containing 200 g of mixed nuts and sells each bag for $1.00. If the plastic bags cost 2 cents each, what is the percentage profit on the selling price of the nuts?

21 If you wanted to be able to advertise a discount of 30% on goods that cost you $5000 and still make a profit of 10%, at what price would you initially advertise them? Give your answer to the nearest dollar.

22 **(a)** An outdoor furniture setting has a marked price of $999. State three different percentage discounts, correct to one decimal place, that would make the selling price between $750 and $800. Show your calculations.

(b) A lounge suite is sold for $2450. State three different percentage discounts, correct to one decimal place, that could have been applied to the lounge suite if you know the marked price was somewhere between $3000 and $3200. Also state the marked price for your discount. Show the calculations that allow you to state your answer.

23 Using an example, explain why taking an additional 20% off an item already discounted by 30% is not the same as one single overall discount of 50%.

Problem solving

Patrick's prized painting

Patrick is an art lover who bought a painting for $6000. A year later, Patrick has some financial difficulties and has to sell his painting for $7000. As soon as he is able to, he buys the painting back again. Unfortunately he has to pay $8000 to get it back. A year later Patrick moves house. He no longer has room for the painting, so he sells it for $9000.

How much profit or loss does Patrick make?

Strategy options

- Act it out.
- Draw a diagram.
- Break the problem into manageable parts.

1.3 Earning an income

Stage 5.1

Need to Know

An **award** is a document that sets out the minimum conditions of employment for employees in a certain occupation or industry. It covers rates of pay, standard hours of work, penalty rates, leave entitlements and other working conditions.

Wage requirements for employers

Employees must be paid at least the minimum rate in the award for their industry—it can be more, but it can't be less. Many awards have special conditions for junior employees (those under 21 years of age).

Types of income

Wages: Based on an hourly rate of pay. Extra work may be paid as **overtime** at penalty rates (time-and-a-half, double time).

Salary: Based on weekly, monthly or annual rates of pay. Overtime is not usually paid; time off in lieu may be given instead.

Commission: A percentage of the value of sales made, or a certain amount for every sale that is added to a **retainer** or flat fee.

Piece work: Payment made per item produced.

Leave loading

Leave loading is calculated at 17.5% of four weeks' wages. It was introduced to compensate workers for overtime opportunities missed while on holiday.

To calculate leave loading, divide the yearly wage by 13, then find 17.5% of this amount. This extra money is added to the normal wage for four weeks.

Worked Example 9

WE 9

Sim is employed at a photographic studio. Sim's job is covered by an award that states that he must be paid a standard hourly rate of $15.80 for a standard 38-hour Monday to Friday week. On Saturday he is paid time-and-a-half (1.5 times the standard hourly rate) for the first 3 hours and double time (twice the standard rate) for any additional hours thereafter. One particular week, Sim worked 4.5 hours on Saturday in addition to his standard hours. How much money did he earn?

Working

Standard hours: 38
Time-and-a-half: 3 hours
Double time: 1.5 hours

Standard hours: $38 \times 15.8 = \$600.40$
Time-and-a-half: $3 \times 15.8 \times 1.5 = \$71.10$
Double time: $1.5 \times 15.8 \times 2 = \$47.40$

Total = $718.90

Worked Example 10 WE10

Lina works as a real estate agent. She receives a retainer of $450 a fortnight, plus a commission of 2.3% of the value of the property she sells. How much does Lina earn in a fortnight in which she sells a house for $340 000?

Working

$C = 2.3\%$ of $\$340\,000$
$= 0.023 \times 340\,000$
$= \$7820$

Total income $= 7820 + 450$
$= \$8270$

Worked Example 11 WE11

Emma is employed sewing dresses. She is paid at the rate of $7.50 per dress and she can complete 42 dresses each week. Calculate her weekly income.

Working

Income $= 42 \times 7.5$
$= \$315$

1.3 Earning an income

Answers page 421

Stage 5.1

Navigator

Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q16, Q20	Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q12, Q14, Q16, Q18, Q19, Q20	Q1, Q2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q19, Q20

Fluency and understanding

1 Jake works in a clothing store. Under his award he is paid $14.70 per hour for a standard 38-hour week, time-and-a-half for the first 2 hours on Saturday and double time for any hours after that. How much will he receive if he works a standard week, and then 5 hours on Saturday? WE9

2 Brian works as a secondhand car salesperson. He is paid a retainer of $1800 per fortnight plus a commission of 1.5% of the value of his sales. In a fortnight in which he sells a Commodore for $28 000, a Captiva for $32 200 and a Cruze for $17 600, how much will Brian be paid? WE10

3 A semitrailer driver is paid $12.75 per tonne of timber delivered to a wood-chipping mill. If he delivers 120 tonnes to the mill, how much will he be paid? WE11

4 A casual worker earns $10.75 per hour stacking shelves in a supermarket. If she works 18.5 hours in 5 days, she will be paid:

A 18.5×10.75 **B** $18.5 \times 10.75 \times 5$ **C** $18.5 \times \frac{10.75}{5}$ **D** 18.5×5

5 If a retail assistant, paid at the rate of $9.80 per hour, works 7 hours standard plus 3 hours at time-and-a-half and 1 hour double time, the wages for that week will be:

A $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 19.6)$ **B** $\$(7 \times 9.8 + 3 \times 14.7 + 1 \times 19.6)$

C $\$(7 \times 9.8 + 3 \times 9.8 + 1 \times 9.8)$ **D** $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 4.9)$

6 A salesperson is paid a $130 retainer per week plus 2.5% of sales. If $20 000 worth of goods are sold during one week, the income will be:

A $\$2.5 \times 20\,000$ **B** $\$(130 + 2.5 \times 20\,000)$

C $\$(130 + 0.025 \times 20\,000)$ **D** $\$0.025 \times 20\,000$

7 The following time sheet summarises the hours worked by four employees. Calculate the total income of each employee for the:

(a) week

(b) fortnight

(c) month

(d) year

for each of them, if the standard rate is $16.40 per hour. (Assume 52 weeks, 12 months and 26 fortnights in a year.)

	Standard hours	Time-and-a-half	Double time
Adam	35	2	0
Britta	25	4	4
Con	30	0	5
Deng	20	2	2

8 A real estate agent is paid commission on the sale of houses at the following rate:

2% on the first $150 000
1.5% on the next $100 000
1% on the rest.

How much would the seller have to pay in commission if a house was sold for:

(a) $280 000 **(b)** $340 000 **(c)** $495 000 **(d)** $600 000?

9 Mike is a shearer who earns up to $450 a day. If he is paid $3.90 for every sheep he shears, what is the maximum number of sheep he is likely to shear in one day?

10 Aleesha is 18 and works part time at a fast food outlet. The adult weekly wage for her position is $656 for a standard week consisting of 38 hours. Aleesha receives 80% of the adult hourly rate.

(a) Calculate Aleesha's hourly rate.

Aleesha works from 4 p.m. to 8 p.m., Wednesday to Sunday. Her award states that after 6 p.m. on weekdays, she must be paid an extra 10% of her hourly rate. An extra 25% is paid for all hours worked on Saturday, and an extra 75% for all hours worked on a Sunday.

(b) Calculate Aleesha's weekly wage.

11 Lottie has an annual income of $53 508.

(a) Calculate her weekly wage.

(b) Calculate the leave loading on her four weeks' annual leave.

12 Marco works as a part-time hairdresser. His award states that a full-time person in his position must be paid a minimum wage of $650 for an ordinary 38-hour week, plus an extra 33% of the hourly rate for any hours worked on a Saturday. Calculate Marco's weekly wage if he works 8 hours a day for 4 weekdays and 5 hours on Saturday.

13 Hayley is a telemarketer with an insurance company. She is paid a retainer of $275 a week, plus $50 commission for every customer she signs up on a new insurance policy. One week, Hayley sells five $149 policies and three $179 policies.

(a) Calculate Hayley's earnings for the week.

(b) Calculate Hayley's commission as a percentage of the value of the products she sold. Give your answer to one decimal place.

14 Erica sells cars and receives a weekly retainer of $150 plus 0.75% of the value of the cars she sells. In a particular week Erica earns $1275. What was the value of the cars Erica sold that week?

15 Patrick is paid $28 per hour for the first 38 hours worked in a week. He is then paid time-and-a-half for the next 6 hours worked and double time for anything beyond that. How many hours did Patrick work in a week in which he earned:

(a) $1190 **(b)** $1372 **(c)** $1652?

(d) How many hours overtime would need to be worked so that his total overtime payments were at least as much as his standard hours payment?

Reasoning, communicating and open-ended

16 Another difference between earning wages and earning a salary is that some types of payments are additional to wage earnings, but are already incorporated into a salary.

(a) Hong gets paid a basic weekly wage of $450. Her leave loading, added onto her pay for holidays, is 17.5% of her basic weekly wage.

(i) Calculate her leave loading per holiday week.

(ii) What pay will she receive during a 2-week holiday period?

(b) Barry's income is $1250 per fortnight. If he receives leave loading of 17.5%, what pay will he receive for a 4-week holiday period?

17 When employed as a salesman, Nabil was given a choice as to how he would be paid.

Option A: A retainer of $250 per week plus 2.5% of his sales.
Option B: No retainer but 3.5% of his sales.

(a) Nabil needs to earn at least $1250 per week. What value of sales does he need to make under each payment option to meet this requirement? Give answers correct to the nearest dollar.

(b) What value of sales will give Nabil the same pay regardless of the option chosen?

(c) So, when would it be better for Nabil to choose Option A?

18 Lucy is the manager of a toy store. She earns a salary of $48 700, which is paid fortnightly.

(a) If there are 26 fortnights in a year, calculate Lucy's fortnightly pay to the nearest cent.

(b) If Lucy works a 40-hour week, what is her equivalent hourly rate?

(c) During the mid-year stocktake, Lucy worked an extra 2 hours each weekday. She was not paid overtime. What was Lucy's equivalent hourly rate for that week?

Hint

Calculate the hourly rate of a full-time person first.

19 Malik is an 18-year-old first-year apprentice electrician. His industry award states that first-year apprentices should be paid 40% of the adult hourly rate of $16.78, rising to 52% in the second year, 70% in the third year and 82% in the fourth and final year. Malik works 8 hours a day, 4 days a week. He goes to TAFE on the fifth day (he isn't paid for this). He also gets a tool allowance in his pay of $17 a week. Malik is wondering if his apprenticeship is worth it. He thinks that he could earn more working the same number of hours in a fast food outlet. The award for that industry states that 18-year-old employees earn 70% of the minimum weekly adult wage of $600 for a 38-hour week, rising by 10% every year until they turn 21 and are on the full adult rate.

(a) Do some calculations to decide whether Malik would earn more at a fast food outlet than on his apprenticeship, either now or in the future.

(b) What would you advise Malik to do? (Some additional information to consider: fully qualified electricians can earn between $40 000 and $80 000 a year.)

20 Tegan has a casual position in a local supermarket. As a casual, Tegan is not entitled to leave and other benefits, but she is paid an extra 25% of the regular hourly rate of $15.80 an hour. She gets a further 10% per hour on Saturdays and Sundays.

Tegan has weekly living expenses (food, rent, transport, etc.) of $300.

Give two or three different combinations of weekday and weekend hours that Tegan could work in order to earn $300 a week.

Problem solving

Careless Carly

Carly is given a savings account by her parents with some money already deposited into it. Carly is trying to avoid withdrawing money from the account but is not very good at sticking to her plan, withdrawing $\$m$ every week. Soon she realises she has already withdrawn half the available money from the account. The next week Carly withdraws another $\$m$ and realises the ratio of withdrawn money to money remaining is now 3 : 2.

What is the ratio of withdrawn money to money remaining when Carly withdraws another $\$m$?



Strategy options

- Guess and check.
- Test all possible combinations.

Half-time 1



Stage 4 Q1–5 **Stage 5.1 Q6–8**

1 **(a)** Giaan had 15 shots at the basketball ring from the free throw line. She got 11 in. Calculate Giaan's percentage shooting accuracy, correct to one decimal place. Ex 1.1

(b) A box of 100 'T42' tea bags is on sale for $5.38, a saving of 91 cents. Calculate the value of the saving as a percentage of the original price, correct to one decimal place.

2 Mo bought an old desk for $155. He bought some special polish for $47.50, polished it up and then sold the desk for $260. Ex 1.2

(a) Calculate Mo's profit as a percentage of the cost price (taking into account all costs), correct to one decimal place.

(b) Calculate Mo's profit as a percentage of the selling price, correct to one decimal place.

3 Find the selling price of the following items. Ex 1.2

(a) coffee table, cost price $48, markup 65%

(b) recliner chair, cost price $197, markup 40%

(c) bookcase, marked price $67, discount 17.5%

(d) chest of drawers, marked price $299, discount $33\frac{1}{3}$%

4 Lucas wants to buy a new skateboard. In shop A it has a marked price of $170 and a sticker that says 'take $40 off'. Shop B has the same skateboard with a marked price of $165 and a sticker that says 'take 25% off'. Decide which skateboard is cheaper. Ex 1.1

5 Naomi has been out shopping for bargains. She has bought a pair of jeans for $65 that had been reduced by 10%, a jacket for $47 that had been reduced by 35% and a scarf for $11 that had been reduced by 20%. Calculate the original price on each of the items and find out how much Naomi saved in total. Ex 1.2

6 Ying is working as a grape picker in a vineyard. He is paid $3.85 for each bucket of grapes picked. How much does Ying earn if he picks 17 buckets in one day? Ex 1.3

7 Sheri is a salesperson for a publishing company. She is paid a $365 weekly retainer as well as 1.7% of the value of the sales she makes. One week she made the following sales: $10 980, $4372, $23 065, $17 834 and $31 178. Calculate Sheri's earnings for the week. Ex 1.3

8 Franco works as a waiter in a restaurant. He is paid an hourly rate of $18.95, with time-and-a-half for any hours worked on Saturday or after 10 p.m. on weekdays and double time for any hours worked on Sunday. Ex 1.3

Here is Franco's time sheet for the week:

Tuesday	5 p.m.–10 p.m.
Wednesday	5 p.m.–10 p.m.
Thursday	5 p.m.–midnight
Friday	5 p.m.–midnight
Saturday	noon–4 p.m.
Sunday	noon–4 p.m.

Calculate Franco's wage for the week.

1.4 Tax and other pay deductions

Stage 5.1

Need to Know

Tax is money collected by the government from Australian residents. It is used to fund public services, such as hospitals, schools, the police and defence forces and transport (road and train networks). The Australian Taxation Office (ATO) collects the tax.

Income tax

The Federal Government collects **income tax** by having employers withhold a certain amount of income from each employee every time they are paid (usually fortnightly or monthly). This is known as the Pay As You Go (**PAYG**) system. The amount of tax withheld depends on your income.

The tax table for the financial year 2012–2013

The incomes given are annual incomes and a financial year goes from 1 July in one year to 30 June in the next. Each row in the table shows the tax payable on incomes within a certain range. The ranges shown on each row of the table are often called 'tax brackets'.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3 572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

We can see from the table that no tax is paid if the annual income is $18 200 or less. This is known as the tax-free threshold.

Your income before tax is referred to as **gross income**. The amount remaining after tax and other deductions are paid is known as **net income** (sometimes called 'take home pay').

Tax deductions

If you spend money on something that helps you earn your income, you may be entitled to claim that cost as a **tax deduction**, which reduces the amount of income on which you pay tax. For example, uniforms, safety equipment, education expenses for courses related to job.

Tax deductions lower the taxable income, so the ATO calculates a tax refund based on the new taxable income after deductions have been made.

Taxable income = gross income − tax deductions

Medicare levy

From its introduction, the Medicare levy has been 1.5% of taxable income. However, in 2013, the levy was increased to 2% to pay for the National Disability Insurance Scheme. It is deducted from any tax refund owed.

Other pay deductions

Apart from income tax, other deductions from an employee's pay must be agreed to in writing by the employee. Unless it forms part of the employee's award or contract of employment, it is illegal for an employer to deduct money for things such as uniforms or tools without the employee's consent.

Superannuation

Superannuation (also known as 'super') is a way of saving money for retirement. Employers are required to regularly pay a certain percentage of their employee's **earnings** into an account or fund. This rate has been increasing from 9% in 2012 to reach 12% by 2019.

This payment is made in addition to the employee's salary. The money can then be accessed when a person retires (provided they are over a certain age) or in an extreme situation such as death, illness or disability. Employees can also make voluntary contributions to their super in addition to those made by their employer. These can be claimed as a tax deduction or a 'salary sacrifice' pay deduction.

Net income = gross income – (tax + voluntary super contributions + any authorised deductions)

Worked Example 12

WE12

Calculate **(a)** the annual income tax and **(b)** the annual net income for an annual gross income of $68 000. Assume no other pay deductions are made.

Working

(a) Tax = $3572 + 32.5c for each $1 over $37 000

Amount over $37 000 = 68 000 – 37 000
= $31 000

Tax on $31 000 = 0.325 × 31 000
= $10 075

Annual tax payable = 3 572 + 10 075
= $13 647

(b) Annual net income = 68 000 – 13 647
= $54 353

Worked Example 13

WE13

Calculate **(a)** the tax that is deducted each pay and **(b)** the net income each pay for a gross income of $980 paid fortnightly. Assume no other pay deductions are made. (Assume there are 26 fortnights in a year.)

Working

(a) Annual gross income = 980 × 26
= $25 480

Tax = 19c for each $1 over $18 200

Amount over $18 200 = 25 480 – 18 200
= $7280

Tax on $31 000 = 0.19 × 7280
= $1383.20

Fortnightly tax payable = $1383.20 ÷ 26
= $53.20

(b) Fortnightly net income
= 980 – 53.20
= $926.80

Worked Example 14 WE14

Carrie is an engineer whose annual gross salary is $95 000. During the year she had work-related expenses totalling $1200, paid membership fees to Engineers Australia of $255 and donated $650 to the Red Cross.

(a) Calculate Carrie's taxable income.

(b) Calculate how much income tax Carrie should have paid using the table on page 22.

Working

(a) Taxable income
$= 95\,000 - (1200 + 255 + 650)$
$= 95\,000 - 2105$
$= \$92\,895$

(b) Tax payable is $17 547 + 37 cents for each $1 over $80 000.

Amount over $80 000
$= 92\,895 - 80\,000$
$= \$12\,895$

Tax on $12 895 = 37% of $12 895
$= 0.37 \times 12\,895$
$= \$4771.15$

Tax payable $= 4771.15 + 17\,547$
$= \$22\,318.15$

Answers page 421

1.4 Tax and other pay deductions

Stage 5.1

Navigator

Q1, Q2 (a)–(d), Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q13	Q1 (a)–(f), Q2 (c)–(f), Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q13	Q1 (e)–(h), Q2 (c)–(f), Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13

Fluency and understanding

WE12

1 Calculate **(i)** the annual income tax and **(ii)** the annual net income for the following annual gross incomes. Assume no other pay deductions are made.

(a) $45 674 **(b)** $77 365 **(c)** $125 000 **(d)** $29 576

(e) $14 033 **(f)** $200 000 **(g)** $59 960 **(h)** $5976

WE13

2 Calculate **(i)** the tax deducted each pay and **(ii)** the net income each pay for the following gross incomes. Assume no other pay deductions are made and that there are 12 months, 26 fortnights and 52 weeks in a year.

(a) $725, paid fortnightly

(b) $4630, paid monthly

(c) $1262.39, paid fortnightly

(d) $176, paid weekly

(e) $8635.24, paid monthly

(f) $537.04, paid weekly

WE14

3 Rosanna is a nurse who receives an annual gross income of $58 700.

(a) What is her taxable income if she donates $200 a year to Amnesty International, has work expenses totalling $575 and pays $160 for her Nurses Federation membership.

(b) Calculate how much income tax Rosanna should pay annually using the tax table on page 22.

4 The tax payable on $58 538 is closest to:

A $5922 **B** $10 572 **C** $10 600 **D** $17 022

5 Karl's fortnightly gross income is $1856. There are 26 fortnights in a year.

(a) Calculate Karl's annual gross income.

(b) Use your answer to **(a)** to calculate Karl's fortnightly PAYG tax, using the tax table on page 22.

(c) Find Karl's fortnightly 'net' pay (take-home pay).

(d) On top of his pay, Karl's employer must pay superannuation at a rate of 9% of his gross income. Calculate the employer's fortnightly super contribution.

SAARBORDE Pty Ltd	PAY PERIOD 06/09/2011–20/09/2011		
Employee Name		Employee Number	
Karl		000758	
Description	**Hours**	**Rate**	**Amount**
Standard	38	Basic	$1856·00
TOTAL GROSS PAY			$1856·00

Description	**Amount**
PAYG Tax	
NET PAY	

Description	**Amount**
Employer super contribution	

6 Leanne earns a gross annual income of $67 200 and is paid monthly. Every month her employer also calculates 9% of her gross income for the month and pays it into her superannuation fund.

(a) How much is paid into Leanne's fund each month?

Leanne decides to make a voluntary super contribution of $80 each pay. She asks that this payment be deducted from her gross income.

(b) Including Leanne's voluntary contribution, how much is paid into her super account by the end of the year?

(c) Calculate Leanne's monthly net income by deducting her tax and her voluntary super contribution.

7 Adila has two permanent part-time jobs. In her first job, she works 15 hours a week at an hourly rate of $18.29 per hour. In her second job, she works 12 hours a week at an hourly rate of $16.43 an hour. For her second job, Adila bought some protective clothing that cost $125. Her union fees cost her $97 a year. Assume there are 52 weeks in a year.

(a) Calculate Adila's annual gross income.

(b) Calculate Adila's annual taxable income.

(c) Calculate the tax payable on Adila's taxable income (exclude the Medicare levy).

8 At the beginning of the financial year Justin started in a new position with an annual gross salary of $54 500. There are 52 weeks in a year.

(a) How much should Justin pay per year in tax?

(b) How much is Justin's weekly gross income?

(c) How much per week does Justin get withheld as PAYG tax? You need to include the 2% Medicare levy.

Justin decides to leave his job after 34 weeks in the position, and travel around Australia without working for the rest of the financial year. Before leaving, Justin had earned $35 635 and paid $6767 in PAYG tax including the 2% Medicare levy.

(d) Use the table on page 22 to calculate how much tax Justin should pay on the amount he earned in the financial year.

(e) How much tax should Justin be refunded? Round your answer to the nearest dollar.

9 Julian has a taxable income of $74 000.

(a) Calculate the Medicare levy of 2% he pays on this income.

(b) How much of this Medicare levy will go towards the National Disability Insurance Scheme since the levy was increased from 1.5% of taxable income?

Reasoning, communicating and open-ended

10 Brad and Jennifer are married. At the moment Brad has a taxable income of $80 000 and Jennifer does not do paid work.

(a) How much tax does Brad pay?

(b) What percentage (correct to the nearest whole number) of his taxable income does this represent?

Brad and Jennifer decide they would be better off if they both worked part-time. They are able to obtain positions that give each of them a taxable income of $40 000.

(c) What is their combined tax contribution?

(d) What percentage, correct to the nearest whole number, of their combined taxable income does this represent?

(e) How much better off are Brad and Jennifer under this scheme?

(f) Why is this different to Brad's contribution when he was the sole earner?

11 The five rows in the tax table on page 22 are sometimes called tax brackets. If you change to a higher-paid job, you may move from one tax bracket to the next. The following pairs of incomes represent the 'border' between two tax brackets. Calculate the difference in the tax paid on them.

(a) $37 000 and $37 001 **(b)** $80 000 and $80 001 **(c)** $180 000 and $180 001

(d) Use your results to explain why the amounts added to each of the tax rates given in the table are $4650, $17 550 and $54 550.

12 Do you think the tax table is fairly designed? Give a reason for your answer.

13 Why do you think the government has made it compulsory for employers to pay superannuation for each of their employees?

Simple interest

1.5

Stage 5.1

Need to Know

Interest

Whenever money is borrowed, the borrower pays the lender for the use of that money. This payment is known as **interest**. Interest is paid on top of the repayment of the **principal**, the amount borrowed.

It is charged as a 'percentage rate'—a percentage of the amount borrowed, calculated at regular time periods.

Money that is invested in a bank or other financial institution can earn interest also because it uses the investor's money to conduct business.

Simple interest

Simple interest is calculated on the initial amount borrowed or invested, regardless of how that amount changes over the time period of the loan or investment.

The amount of interest paid depends on:

- the initial amount of money borrowed or invested, known as the principal, P
- the percentage **rate of interest** per time period that is charged or paid, R
- the number of time periods for which the money is borrowed or invested, N.

The interest rate, R, can be expressed as a decimal by dividing the percentage rate by 100.

The percentage rate, R, is often the annual percentage rate and referred to as the rate 'per annum' abbreviated to p.a.

Interest can be calculated quarterly, monthly or even daily. In these cases we need to express T as a fraction of a year by dividing the annual rate by 4, 12 or 365.

Simple interest formula

$$I = PRN$$

Where I = simple interest (in dollars)
P = principal (in dollars)
R = rate of interest per time period as a percentage
N = number of time periods

When calculating daily interest, it is important to carefully count the number of days the balance remains at a certain amount. The balance changes on the day a withdrawal or deposit is made. Make sure the number of days used adds up to the total number of days in the month.

Worked Example 15

WE15

Calculate the simple interest paid on a loan of \$5000 at 16.4% p.a. interest over 4 years.

Working

$P = \$5000$
$R = 16.4\% = 0.164$
$N = 4$
$I = PRN$
$I = 5000 \times 0.164 \times 4$
$= \$3280$

Worked Example 16

WE16

Barbara borrows \$24 000 to set up a business. She repaid the money at the end of 5 years and 3 months, paying 18.5% p.a. in simple interest, payable quarterly.

(a) How much interest did she pay quarterly?

(b) What was the total amount of money repaid?

Working

(a) $P = \$24\,000$
$R = 18.5\%$
$= 0.185$
$N = \frac{3}{12} = \frac{1}{4}$
$I = PRN$
$I = \$24\,000 \times 0.185 \times \frac{1}{4}$
$= \$1110$ per quarter

(b) $N = 5$ years 3 months
$= 5\frac{1}{4}$ years
$= \frac{21}{4}$ years
$= 21$ quarters
$I = \$1110 \times 21$
$= \$23\,310$
Total repaid $= P + I$
$= \$24\,000 + \$23\,310$
$= \$47\,310$

Worked Example 17

WE17

Craig's bank statement for September is shown.

Date	Transaction		Balance
Sept 1			$490.15
Sept 6	Deposit	$180	$670.15
Sept 19	Withdrawal	$400	$270.15

If the account pays 6% interest, calculate the interest accrued in September if interest is calculated:

(a) daily, to four decimal places

(b) on the minimum monthly balance.

Working

(a) $P = \$490.15, \$670.15, \$270.15$

$R = 6\% = 0.06$

$N = \frac{5}{365}, \frac{13}{365}, \frac{12}{365}$

$I = PRN$

$I = \$490.15 \times 0.06 \times \frac{5}{365}$

$= \$0.4029$

$I = \$670.15 \times 0.06 \times \frac{13}{365}$

$= \$1.4321$

$I = \$270.15 \times 0.06 \times \frac{12}{365}$

$= \$0.5329$

$I = \$0.4029 + \$1.4321 + \$0.5329$

$= \$2.3679$

$= \$2.37$

(b) minimum monthly balance $= \$270.15$

$P = \$270.15$

$R = 6\% = 0.06$

$N = \frac{30}{365}$

$I = PRN$

$I = \$270.15 \times 0.06 \times \frac{30}{365}$

$= \$1.33$

Worked Example 18

WE18

Allan put $12 500 into a cash management account. He received $13 740.15 when he withdrew all of the money 9 months later. What was the average rate of return, p.a., on his capital (correct to one decimal place)?

Working

$I = \$13\,740.15 - \$12\,500$

$= \$1240.15$

$P = 12\,500$

$R = ?$

$N = \frac{9}{12} = \frac{3}{4}$

$I = PRN$

$1240.15 = 12\,500 \times R \times \frac{3}{4}$

$1240.15 = 9375R$

$\frac{1240.15}{9375} = R$

$R = 0.13228$

$= 13.2\%$

Technology Exploration: Spreadsheet

Using iteration to investigate savings plans

Equipment required: 1 brain, 1 computer with Excel or any other spreadsheet program

In mathematics, iteration occurs when the output from a mathematical rule becomes the input to the rule the next time the rule is processed.

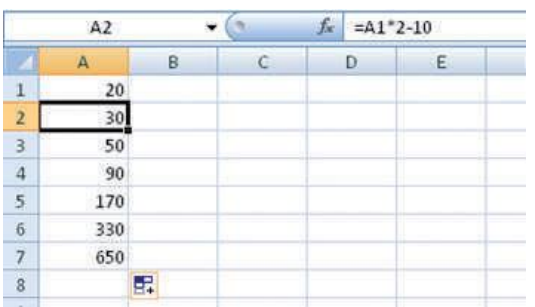
Consider the rule of doubling a number and taking away 10.

If the first input is 20, then the rule becomes $2 \times 20 - 10$ and the output is 30. The output now becomes the next input. That is, the rule is applied to 30. The second output is 50, as $2 \times 30 - 10 = 50$. Each time the process is applied, an iteration has been performed. The first iteration gave a result of 30 and the second iteration gave a result of 50.

Excel allows us to use iteration in a very simple manner. Simply enter the start value in cell A1 and the rule into A2. (Cell A2 is highlighted so you can see the formula.)

1 Copy the formula down as far as you like. The spreadsheet below shows the first six iterations of the rule. 30 is the result of the first iteration and is shown in cell A2.

Find the value of the fourth, tenth and twentieth iterations. Extend the table beyond the size shown below.



Saving options

Suppose you are able to save $1200 each year towards a major holiday in 10 years' time. You have to choose the best plan from the following options.

	Payment*	Interest rate
Plan A: Regular saver	$100 per month	9% p.a. paid monthly
Plan B: Periodical saver	$600 per 6 months	10% p.a. paid 6-monthly
Plan C: Yearly lump sum	$1200 per year	9% p.a. paid yearly

* All payments commence at the beginning of the payment period.

Investigating Plan A: Regular saver

Calculate the monthly rate of interest. Dividing 9% p.a. by 12 gives a rate of 0.75% per month or 0.0075 per month as a decimal rate.

The amount saved at the beginning of the second month is calculated as follows:

Initial deposit = $100

Interest on deposit = $0.0075 \times 100 = \$0.75$

The second deposit = $100

Total = $\$(100 + 0.75 + 100)$

= $200.75

The amount saved at the beginning of the third month is calculated as follows:

Previous balance = $200.75

Interest on balance = $0.0075 \times 200.75 = \$1.51$

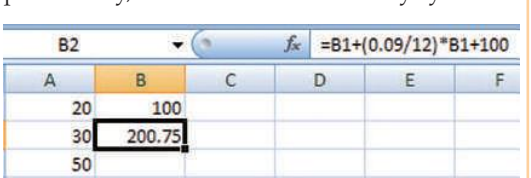
The third deposit = $100

Total = $\$(200.75 + 1.51 + 100)$

= $302.26

To perform these calculations using Excel, enter an initial amount of $100 into B1 and the formula =B1+(0.09/12)*B1+100 into B2.

If you have entered this correctly, you should see a result of 200.75 as the amount in the account at the beginning of the second month. Fix this cell to show two decimal places only, so it matches our money system.



2 Explain what each part of the formula =B1+(0.09/12)*B1+100 does.

3 Copy the formula into cell B3. This shows the amount in the account at the beginning of the third month. Does it match the value calculated earlier? What has happened to the formula?

4 Extend the table to find the amount saved at the beginning of each month for months 3 to 13 inclusive of the savings plan. (Hint: The end of the 12th month occurs at the beginning of the 13th month.)

5 How much is saved at the end of one year (not including payment of $100 at the beginning of month 13)?

Investigating Plan B: Periodical saver

To calculate the 6-monthly rate of interest, divide 10% p.a. by 2. This gives a rate of 5% per 6 months or 0.05 per month as a decimal rate.

At the beginning of the second 6-month period, the amount saved is calculated as:

The initial deposit = $600

Interest on deposit = $0.05 \times 600 = \$30$

The second deposit = $600

$$\text{Total} = \$(600 + 30 + 600)$$
$$= \$1230$$

Copy and complete the calculation below for the beginning of the third 6-month period:

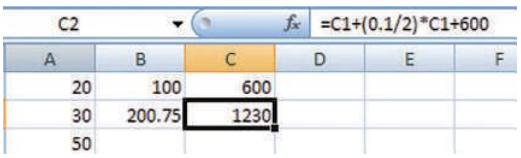
Previous balance = $

Interest on balance = $0.0 \times 1230 = \$0.50$

The third deposit = $600

$$\text{Total} = \$(1230 + 0.50 + 600)$$
$$= \$0.50$$

We will enter this plan in column C of our spreadsheet. Cell C2 has been highlighted to show you the formula to use.



6 Explain what each part of the formula =C1+(0.1/2)*C1+600 does.

7 Copy the formula to cell C3. Does this give you the same value you calculated earlier for the amount in the account at the beginning of the third 6-month period?

8 How much is saved at the end of one year if the payment of $600 (at the beginning of the second year) is not included?

Investigating Plan C: Yearly lump sum

9 Find the amount saved after one year. Do not include the payment made at the start of the second year.

Comparing the plans

10 Write a brief report that compares the plans and the amount saved in each plan after one year. Which plan would you recommend? How practical are the plans?

1.5 Simple interest

Answers page 422

Stage 5.1

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q11, Q12, Q13, Q17, Q19, Q21, Q23	Q1, Q2, Q3, Q4, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q16, Q20, Q21, Q22, Q23	Q1, Q2, Q3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q20, Q21, Q22, Q23

Fluency and understanding

WE15

1 (a) Calculate the simple interest payable on a loan of $45 000 at a rate of 12% p.a. for 3 years.

(b) Calculate the simple interest payable on a loan of $3500 at an annual rate of 7% for 4 years.

(c) Calculate the simple interest payable on a loan of $11 450 at 8.2% p.a. for 5 years.

(d) Calculate the simple interest payable on a loan of $8500 at 5.5% p.a. for 6 months.

WE16

2 Chloe borrowed $15 000 to buy a new car. She repaid the money over $3\frac{1}{2}$ years, at a simple interest rate of 12.5% p.a., payable every 6 months.

(a) How much interest did Chloe pay every 6 months?

(b) What was the total amount of money repaid?

WE17

3 (a) A statement for an account offering 6.9% p.a. simple interest is shown for the month of July.

Date	Transaction	Balance
July 1		$350
July 5	Withdrawal $170	$180
July 22	Deposit $450	$630

How much interest is payable for July if interest is calculated:

(i) daily, to four decimal places

(ii) on the minimum monthly balance?

(b) A statement for an account offering 7.8% p.a. is shown for the month of May.

Date	Transaction	Balance
May 1		$65.20
May 16	Deposit $530	$595.20
May 22	Withdrawal $260	$335.20
May 27	Withdrawal $300	$35.20

How much interest is payable for May if interest is calculated:

(i) daily, to four decimal places

(ii) on the minimum monthly balance?

WE18

4 After investing $5000 into a fixed deposit account for 2 years for a trip to London, Inika received $5890. What percentage return p.a. did she get?

5 Calculate the simple interest in the following cases:

(a) $P = \$440$, $R = 0.08$, $N = 5$ years

(b) $P = \$600$, $R = 0.09$, $N = 7$ years

(c) $P = \$369$, $R = 0.12$, $N = 3$ months

(d) $P = \$3800$, $r = 16.8\%$, $N = 20$ days

6 The simple interest on $870 for 4 years at 15% p.a. is:

A $870 \times 15 \times 4$

B $870 \times 0.15 \times 4$

C $870 \times 1.15 \times 4$

D $870 \times 0.15 \times 4 + 870$

7 What is the total amount that is paid back on a loan of $8150 over 3 years 6 months at 18.5% p.a.?

8 Calculate R, then write as a percentage (p.a.) correct to one decimal place when:

(a) $P = \$245$, $N = 5$ years, $I = \$110$

(b) $P = \$420$, $N = 2.25$ years, $I = \$132$

9 Calculate N (in years) correct to one decimal place when:

(a) $P = \$245$, $R = 0.06$, $I = \$116$

(b) $P = \$120$, $R = 0.07$, $I = \$32$

10 Calculate P (in dollars) when:

(a) $N = 3.5$ years, $R = 0.05$, $I = \$35$

(b) $N = 2$ years, $R = 0.08$, $I = \$216$

11 To save for a trip to Thailand, Claudia puts her tax refund cheque of $950 into an account paying 8% p.a. simple interest. How long (in years, correct to one decimal place) will her money take to accumulate to $1300?

12 How much will be paid out to Sol, who puts $1400 into a bank account paying 6% simple interest p.a., if he withdraws the total amount after 240 days?

13 Victor agrees to lend Esme $1500 so that she can go on a holiday to Fiji. They agree that Esme will make quarterly payments, with simple interest to be paid at a rate of 12% p.a. As well as the principal, Esme will pay the debt in full in 5 years. Calculate Esme's quarterly payments.

14 Manny borrows $6100 from his mother to buy a car. He agrees to pay 11% simple interest each year without paying off the principal. After 2 years, in addition to the interest, he pays $2500 off his principal, reducing the principal for the rest of the loan. How much in total did he pay his mother if he pays off the loan in another 1.5 years?

15 **(a)** Calculate the interest payable on an account in the month of April if it pays 5.8% interest p.a., and it has a balance of $155 for 3 days, $820 for 19 days, and $580 for the rest of the month, if the interest is paid on:

(i) minimum monthly balance

(ii) daily balance.

(b) How much more interest is paid under the conditions in part **(ii)** than in part **(i)**?

16 After investing $6000 into a savings account paying 7% interest p.a., Ian received $6238. For how many days (correct to the nearest day) was the money invested?

17 What amount of money must be invested at 9% p.a. in order to earn $540 in interest in 3 months?

18 Tula repays her father a total of $350 in interest to cover a debt for a motorbike which she incurred 15 months ago, agreeing to pay 4% p.a. simple interest. How much did she borrow?

Reasoning, communicating and open-ended

19 Consider the statement shown at right for a savings account.

(a) Copy and complete the statement for the 4-month period shown.

(b) State the minimum balance for each month.

(c) Calculate the interest owed by the bank if the interest rate is 2.75% p.a. and it is paid on the minimum monthly balance.

Late in each month there has been a withdrawal. Assume that these withdrawals were delayed until the first day of the next month.

Date	Transaction		Balance
Jul 1			$975.00
17	Deposit	$197	
28	Withdrawal	$641	
Aug 4	Deposit	$299	
19	Deposit	$862	
29	Withdrawal	$176	
Sep 6	Deposit	$194	
28	Withdrawal	$800	
Oct 16	Deposit	$426	
27	Withdrawal	$500	
31	Balance		

(d) What are the minimum monthly balances now?

(e) How much interest is now earned?

(f) What percentage change has there been in the amount earned?

(g) In light of the calculations you have done, what advice would you offer people or businesses who want to earn as much interest as possible on their savings?

20 Phillip borrows $15 000 from his mother to help pay for his new car. He needs to pay back the money within 3 years and pay 5% simple interest for the period of the loan. He intends to make monthly repayments to his mother.

(a) How much interest will Phillip need to pay?

(b) How much in total does Phillip owe his mother?

(c) How much is each monthly repayment?

(d) Assuming $\frac{1}{3}$ of the interest is repaid each year and $\frac{1}{3}$ of the principal is repaid each year, what percentage of each payment is interest?

(e) How much interest does Phillip still need to repay after the first 12 months?

(f) If the interest calculation was done again based on the principal still owing after 12 months, how much interest would be owing?

(g) Why is there a difference in the answers to parts **(e)** and **(f)**?

21 Hung earns $640 on an investment of $8000. Give three possible simple interest rates as percentages per annum, and their corresponding time periods, that give interest of $640 on an investment of $8000.

22 Michael the miser is sure that the bank has made a mistake on the interest it added to his account for the month.

Interest is calculated daily at a rate of 3.5% p.a. Here is Michael's statement.

Date	Transaction	Balance
1 June	Opening balance	$578.12
9 June	$250.03 deposit	$828.15
21 June	$185 withdrawal	$643.15
30 June	Closing balance	$643.15

Here are Michael's calculations.

$P = \$578.12, \$828.15, \$643.15$

$R = 3.5\% = 0.035$

$N = \frac{9}{365}, \frac{13}{365}, \frac{10}{365}$

$I = (\$578.12 \times 0.035 \times \frac{9}{365}) + (\$828.15 \times 0.035 \times \frac{13}{365}) + (\$643.15 \times 0.035 \times \frac{10}{365})$

$= \$0.4989 + 1.0324 + \$0.6167$

$= \$2.15$

The amount of interest given on his bank statement is $2.01.

Has the bank made a mistake? Or has Michael? Explain the difference in the two amounts of interest.

23 'Neither a borrower nor a lender be' is a line from Shakespeare's famous play *Hamlet*. Do you think this is good advice? Give some reasons for your answer.

Payment methods

1.6

Stage 5.1

Need to Know

Payments without cash

- **Debit cards** allow money for purchases to be transferred electronically from the customer's bank account to another account. This is done through the Electronic Funds Transfer at Point of Sale (**EFTPOS**) system. The money must be present in the bank account in order for it to be transferred and the bank may charge a small fee for each EFTPOS transaction. Credit card companies such as VISA and MasterCard provide debit cards that, in addition to being used for EFTPOS, can also be used overseas and for online purchases.
- **Credit cards** look like debit cards and are used in a similar way, but there is one important difference. When a credit card is used to purchase goods, the bank that issued the card pays the cost then sends the purchaser a bill, called a statement, for that amount (plus any other credit purchases) at the end of the month. Most cards have an 'interest free' period of up to 55 days. This period begins on the first day of the statement period and ends on the date when payment is due. If the full amount of the bill is paid by the due date, no interest will be charged. However, if the bill is not paid in full by the due date, interest will be charged even if the minimum repayment amount stated on the bill has been paid. The interest rate is stated as an Annual Percentage Rate (APR). This varies from card to card but is usually somewhere between 15% and 20%. These cards usually have an annual fee of $25 to $40. Some credit cards do not have an interest-free period and calculate interest from the day the purchase is made.

  Credit card interest is calculated using the method outlined below.

  * The balance (the amount owing) at the end of each day of the statement period is determined. These daily balances are then added up and divided by the number of days in the statement period to get an 'Average Daily Balance' or ADB.
  * A daily interest rate is calculated by dividing the annual rate by 365.
  * The ADB is multiplied by the daily interest rate, then by the number of days in the statement period, to give the amount of interest for the period.
  * This interest will then be charged on the next statement if the amount is not paid in full by the due date.
- **Cheques** have been used as a substitute for cash for a long time, but are being used less and less as electronic banking has become more common. To write a cheque you enter the name of the person or company being paid, the amount in words and numbers, and your signature. The cheque must then be deposited at a bank. Many banks charge a cheque handling fee, and the money takes several days to become available.
- **Lay-by** means the store holds the item until the item has been completely paid for. No interest has to be paid for this form of payment.
- **Deferred payment** is usually used to describe the payment method for repaying university or college fees that have been accumulated by using HELP—the Higher Education Loan Program. Payment begins once the borrower (the student) begins earning above a certain amount set by the government. Repayments are calculated as a percentage of the borrower's income.

- **Buying on terms** means that large purchases, such as furniture, computers or cars, can be bought. Customers take possession of the goods then pay them off. 'Interest-free' terms allow customers a period (usually 6–12 months) during which no interest is charged. During this period, a small monthly repayment plus a monthly account fee is paid. An 'establishment fee' is paid at the beginning of the period when the account is set up. The remaining balance of the account must be paid at the end of the interest-free period to avoid interest being charged from that point on.
- **'Buy now, pay later'** terms are similar except that no monthly repayments are required during the interest-free period, although you may choose to make them to avoid a big payment at the end of the period. The interest on any outstanding amount at the end of the interest-free period is calculated in the same way as credit card interest. However, if you have not fully paid for the item by the end of this period, a very high interest rate is charged (often between 20% and 30%).

Fees and charges

In addition to charging for EFTPOS transactions, banks may also charge monthly account-keeping fees and/or fees for withdrawing cash from ATMs (Automatic Teller Machines), particularly if the ATM belongs to another bank, or for paying bills online.

Worked Example 19

WE19

Jemima's bank charges her a $2 account-keeping fee every month, plus the following charges.

- 50c for a withdrawal from the bank's ATMs
- $2.00 for a withdrawal from another bank's ATMs
- 75c for EFTPOS transactions
- 50c for transferring money online

In one month, Jemima makes two withdrawals from her own bank's ATM, three withdrawals from another bank's ATM, seven EFTPOS transactions and two online transfers.

Calculate the total amount of Jemima's bank charges for the month.

Working

Bank ATM: $2 \times \$0.50 = \$1.00$
Other bank ATM: $3 \times \$2.00 = \$6.00$
EFTPOS: $7 \times \$0.75 = \$5.25$
Online transfers: $2 \times \$0.50 = \$1.00$
Account keeping fee: $\$2.00$

Total fees = $\$15.25$

Jemima's bank will charge her $15.25 for the month.

Worked Example 20

WE20

Ali has a credit card with an Annual Percentage Rate (APR) of 18.4% interest and an interest-free period of 55 days. He has just received a statement covering the period from 24 May to 23 June. If Ali's average daily balance (ADB) for this period is $2184.36, calculate the interest that will be payable if he does not pay the account in full by the due date.

Working

$R = 18.4\%$ APR $\div 365$
$= 0.0504\%$
$= 0.000\,504$

$N =$ 24 May to 23 June
$= 31$ days

$P = \$2184.36$
$I = PRN$
$= \$2184.36 \times 0.000\,504 \times 31$
$= \$34.13$

Worked Example 21

WE21

Ravi wants to purchase a new lounge suite. He has seen one advertised for $2799, but he does not have the money. The store offers Ravi a 12-month interest-free payment plan where he must pay a minimum monthly repayment of 3% of the cost, a $25 up-front application fee and a $3.95 monthly account-keeping fee.

(a) How much will Ravi pay in fees over the 12 months?

(b) If he only makes the minimum monthly repayments, how much will Ravi have paid off the cost of the lounge suite after 12 months?

(c) If Ravi does not pay the remainder of the cost at the end of the 12 months, he will be charged 29% p.a. interest until it is all paid off. Calculate the interest that will be charged after the 13th month. (Assume there are 30 days in the month.)

Working

(a) $(\$3.95 \times 12) + \$25 = \$47.40 + \$25$
$= \$72.40$

(b) 3% of $\$2799 = 0.03 \times 2799$
$= \$83.97$ per month

$\$83.97 \times 12 = \$1007.64$ after 1 year

(c) $P = \$2799 - \$1007.64$
$= \$1791.36$

$R = 29\% \div 365$
$= 0.079\,452\%$
$= 0.000\,795$

$T = 30$ days

$I = PRT$
$= \$1791.36 \times 0.000\,795 \times 30$
$= \$42.70$

Answers page 422

1.6 Payment methods

Stage 5.1

Navigator

Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q10, Q12	Q1 (a)–(b), Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q12	Q1 (c)–(d), Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

WE19

1 Federation Bank charges customers the following fees on their 'Easy Saver' account.

- a $3 monthly account-keeping fee
- 70c for withdrawals from the bank's ATMs
- $1.50 for withdrawals from other banks' ATMs
- 70c for EFTPOS transactions
- 50c for online transfers

Calculate how much the bank will charge the following customers in a month if the following transactions are made:

(a) five ATM withdrawals (one from a different bank), four EFTPOS payments, three online transfers

(b) six EFTPOS payments, four ATM withdrawals (two from the bank's ATMs), five bills paid online

(c) two withdrawals from the bank's ATMs, eight EFTPOS payments

(d) four withdrawals from other banks' machines, six online transfers.

WE20

2 Shannon has a credit card with an annual interest rate of 16.99% and an interest-free period of 55 days. She has just received a statement covering the period from 15 September to 14 October. If Shannon's average daily balance (ADB) for this period is $3205.48, calculate the interest that will be payable if she does not pay the account in full by the due date.

WE21

3 John wants to buy a home entertainment system priced at $3299 but doesn't have the money. The store offers him a 6-month interest-free plan with a minimum monthly repayment of 5% of the cost, a $20 up-front application fee and a monthly account-keeping fee of $3.50.

(a) How much will John pay in fees over 6 months?

(b) How much will John have paid off the cost of the TV after 6 months if he only makes the minimum monthly repayments?

(c) If John does not pay the remainder of the cost at the end of 6 months, he will be charged 24% p.a. interest until it is paid off. Calculate the interest that will be charged after the seventh month. (Assume there are 30 days in the month.)

4 Jackson wanted to buy a new telescope for $109.95, but did not have enough money, so he placed it on lay-by. He paid a 10% deposit and needs to make payments of $15 a week.

(a) How long will it take Jackson to pay off his telescope?

(b) How much will his last repayment be?

5 Farouk is 18 years old and has just obtained a credit card that has an interest rate of 17.35% and an interest-free period of 55 days. His statement after his first month is shown.

St Geoff Bank

Account Number: 8675 0032 6701 2130

Statement Period: 03/07/11 to 02/08/11

Date	Purchase	Amount	Balance
	Opening balance		$0.00
5 July	Shoes	$59.00	$59.00
11 July	Petrol	$72.30	$131.30
17 July	Concert ticket	$124.80	$256.10
22 July	Phone and internet bill	$89.95	$346.05
29 July	Groceries	$147.32	$493.37
1 August	Ski jacket and goggles	$239.86	$ 733.23
	Closing balance		$733.23

(a) How many days are in the statement period?

(b) Calculate the average daily balance by finding the balance at the end of each day, adding them up and dividing by the number of days in the statement period. Remember to include the days at the beginning and end of the statement period (shown on the bill).

(c) Use your answer to **(b)** to calculate the interest that Farouk will be charged if he does not pay the balance in full by the due date.

Hint

If no purchases have been made on a particular day, the closing balance is the same as the previous day's balance.

6 Yolanda has bought a new spa bath worth $4300 using a 'buy now, pay later' 24-month payment plan. The plan requires her to pay an up-front establishment fee of $25 and a monthly account-keeping fee of $2.95. Yolanda does not have to make any monthly repayments, although she can do so if she wants to. Yolanda must pay the full amount by the end of the 24-month period or she will pay 30% p.a. interest every month on whatever is owing until it is all paid off.

(a) Calculate Yolanda's fees for the 24 months.

(b) If Yolanda does not pay anything off the spa in the 24-month period, how much interest will be added to her account by the end of the 25th month? (Assume there are 30 days in the month.)

(c) Aside from the account-keeping fees, what voluntary monthly repayment amount should Yolanda make in order to have the spa paid off by the end of the 24 months?

7 Poh and Dzung are trying to decide which method they should use to pay their rent of $950 each month. They can withdraw cash from their account using an ATM, then go into a bank to deposit it into their landlord's account. Alternatively, they can post a cheque directly to their landlord's workplace. Poh's bank charges $6 to process a cheque.

(a) The ATM will only allow withdrawals of up to $400 each time, at a cost of 70c per withdrawal. How much will it cost to withdraw the $950 from the ATM?

(b) Which method of payment would you recommend to Poh and Dzung?

Reasoning, communicating and open-ended

8 Edwin is leasing a work car using a hire purchase agreement. He pays a monthly amount of $810 to the finance company for the use of the car over a 5-year period. This monthly amount includes the cost of the car and interest. At the end of the 5 years, Edwin is the owner of the car.

(a) Calculate the total amount Edwin pays for the car over 5 years.

(b) If the initial value of the car is $39 995, how much has Edwin paid in interest?

(c) Using the value of the car as the principal, calculate the equivalent rate p.a. of simple interest that Edwin has paid, correct to one decimal place.

9 Shelby has used her credit card to buy furniture for her new house. This month, her credit card statement is for the period from 1 March to 31 March. The total owing on the credit card is $8691.57. Shelby is unable to pay her bill, so interest and a $9 late payment fee are added to the balance. The interest rate of the card is 20.99% p.a.

(a) Calculate the interest that is added if the ADB over the statement period was $6982.45.

(b) Calculate the opening balance on Shelby's account at the beginning of April.

(c) During April, Shelby makes two purchases: a plasma screen TV for $1299 on the 14th and a new lounge suite for $2499 on the 21st. Calculate the balance on her account on 30 April.

(d) If Shelby again doesn't pay her bill, how much interest will be calculated on April's balance? (You will need to calculate the ADB for April.)

(e) How much in interest and fees will have been charged to Shelby's account after 2 months of no payments?

(f) Use your answers from **(a)**–**(e)** to explain why credit card debt can become a problem for some people.

10 Jack and Jill would like to purchase some furniture for $1999. They have two payment options. Option 1 is lay-by the furniture by placing a 10% deposit up front and make payments of $160 a month until it is paid off. Option 2 is to use a 'buy now, pay later' arrangement of an upfront fee of $25, a monthly account fee of $3, and no other compulsory repayments for 12 months, after which an interest rate of 29% p.a. is applied.

(a) If they choose to lay-by, calculate how long it will be before Jack and Jill will be able to take home their furniture.

(b) How much extra will they pay if they choose to 'buy now, pay later', even if no interest is applied?

(c) Which payment plan would you choose? Give at least one reason for your answer.

(d) **(i)** Why might people choose 'buy now, pay later' over lay-by, even though it may mean paying extra money?

(ii) Why might people opt to lay-by instead?

11 Kezia's bank charges her 50c for a withdrawal from the bank's ATMs, $2.00 for withdrawing from another bank's ATMs, 75c every time she uses EFTPOS and 50c every time she uses the internet to transfer money or pay bills online.

Kezia's bank statements show that she is losing between $10 and $20 a month in fees and charges.

(a) Give some combinations of fees that have a sum between $10 and $20.

(b) Give Kezia a couple of pieces of advice to help her lower this amount.

12 List some of the advantages and disadvantages of using a debit card instead of a credit card.

Challenge 1



1 Find five consecutive numbers such that the sum of the first and the fifth is 1708.

2 If 7^{2048} is divided by 100, what is the remainder?

3 Which numbers less than 100 have 5 as their smallest prime factor?

4 If m and n are different whole numbers, what is the smallest value of m for which $m^3 = n^4$ is possible?

5 A palindromic number reads the same backwards as it does forwards. There was one palindromic date in 2002, 20/02/2002 (20 February 2002). In the USA, dates are written with the month number first, then the day number, then the year. For example, 15 April 2003 is written as 04/15/2003.

(a) Write three palindromic dates in Australia for the 21st century.

(b) Write three palindromic dates in the USA for the 21st century.

6 Copy and complete the pattern below.

0, 3, 9, 18, 30, ____, ____, ____

7 Copy and fill in the missing numbers if the difference between each term in the sequence is the same.

$\frac{1}{12}$, ____, ____, $\frac{1}{7}$

8 If it takes 12 students 30 minutes to clean up 100 dishes after lunch on a school camp, how long will it take eight students to clean up 90 dishes?

9 What percentage is m of n?

A $\frac{n}{m}$ **B** $\frac{100n}{m}$ **C** $\frac{m}{n}$ **D** $\frac{100m}{n}$

10 When 1000^{2010} is written as a numeral, the number of digits written is:

A 2010 **B** 2011 **C** 6030 **D** 6031

11 Successive discounts of 10%, 20% and $33\frac{1}{3}\%$ are equivalent to a single discount of:

A 48% **B** 50% **C** 52% **D** $63\frac{1}{3}\%$

12 Which of the following will simplify to 160?

A $\frac{40}{\frac{1}{4}}$ **B** $1000 \div 160\,000$ **C** 80×20 **D** $3.2 \div 0.2$

Question **13** may have more than one answer. List all the letters corresponding to correct answers.

13 Which of the following will simplify to 5?

A $\sqrt{13^2 - 12^2}$ **B** 2.5^2 **C** $29 - 4 \times 6$ **D** $40 \div 4 \times 2$

Chapter review 1

D.I.Y. Summary

Key Words

Stage 4	discount	loss	rate of interest
Stage 5.1	award	income	retainer
	buying on terms	income tax	salary
	cheque	interest	simple interest
	commission	lay-by	superannuation
	credit card	net income	tax
	debit card	overtime	tax deduction
	deferred payment	PAYG	taxable income
	earnings	piece work	wages
	EFTPOS	principal	
	gross income	profit	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

Stage 4 Q1, 2 **Stage 5.1 Q3–11**

1 When a business sells a product for more than it was bought or made for, a _____ has been made.

2 _____ is a payment based on a percentage of sales made.

3 Simple interest is calculated by multiplying the _____, the _____ (in decimal form) and the time period.

4 Before tax and other deductions are made, a person's earnings are referred to as _____.

5 Work-related expenses and donations to recognised charities are both examples of _____s.

6 When a person works _____ they should be paid at a rate higher than their normal rate of pay.

7 What does EFTPOS stand for?

8 Explain the difference between salary and wages.

9 What are some of the extra costs associated with buying on terms?

10 The set of rules outlining working conditions for employees, including wages, overtime and leave, is called an _____.

11 _____ is an amount of money paid into an account or fund to be accessed at retirement.

Stage 4 Q1–5, 13, 20 Stage 5.1 Q6–10, 11, 12, 14, 15, 16, 17, 18, 19

Fluency and understanding

1 Calculate the following. Ex 1.1

(a) 130% of $47.85 **(b)** 0.5% of $52 000

2 **(a)** Ami is a hockey goalkeeper. In one game she stopped 13 of 16 shots on goal. Calculate Ami's 'stopping' percentage. Ex 1.1

(b) A box of soft-centred chocolates is on sale for $7.34, a saving of $2.45. Calculate the dollar saving as a percentage saving, correct to one decimal place.

3 Calculate the selling price on the following items. Give answers correct to the nearest cent where necessary. Ex 1.2

(a) A $75 shirt discounted by 30% **(b)** A $29.99 DVD discounted by $12\frac{1}{2}$ %

4 A timber dining table cost $800 to construct and was sold for $1250. The percentage profit, based on cost price, is closest to: Ex 1.2

A 36% **B** 43.75% **C** 56.25% **D** 64%

5 The Goods and Services Tax (GST) is a 10% tax added to the price of most retail items. Ex 1.2

(a) Reprice the following so that the prices include GST:

(i) a $5.60 bottle of shampoo **(ii)** a $37.35 hairdryer.

(b) The retail prices of the following items have GST included. Calculate the 'pre-GST' price of:

(i) a $129 calculator **(ii)** a $38.95 computer game.

6 Andrea works as a casual in a supermarket. She earns $9.45 per hour for hours worked between 7 a.m. and 7 p.m., time-and-a-half for any hours worked after 7 p.m. or on Saturday and double time for any hours on Sunday. Ex 1.3

Calculate Andrea's wage for the following week.

Monday	Wednesday	Friday	Saturday	Sunday
8 a.m.–4 p.m.	8 a.m.–4 p.m.	1 p.m.–10 p.m.	10 a.m.–3 p.m.	10 a.m.–3 p.m.

7 Nazim is picking grapes in a vineyard. He is paid $4.75 for every bucket of grapes picked. How much can Nazim earn in a 5-day week if he picks an average of 35 buckets in a day? Ex 1.3

8 How much income tax is payable on the following taxable incomes? (Use the tax table on p. 22.) Ex 1.4

(a) $5900 **(b)** $47 890 **(c)** $82 780

9 Evan borrows $7000 to purchase a car. He will pay 8.5% simple interest over a period of 4 years. Ex 1.5

(a) How much interest is payable?

(b) How much will Evan pay back altogether?

(c) If Evan makes monthly repayments, how much will these repayments need to be to ensure he pays off both the principal and the interest in the 4 years?

10 Olivia has a credit card with an annual interest rate of 17.3%. The card has an interest-free period of 55 days. She has just received a statement covering the period from 12 June to 11 July. The average daily balance owing on the card over this period was $4509.82. Calculate the interest that will be payable if Olivia does not pay the full amount owing by the end of the interest free period. Ex 1.6

11 Peter is a salesperson for an audiovisual company. He is paid a weekly retainer of $250 plus 3.5% of the value of his sales. One week, Peter sold products worth $4500, $7890, $2630 and $6100. Calculate his income for the week.

12 Calculate the percentage rate of simple interest paid if a $5000 investment over $6\frac{1}{2}$ years gave a return of $7730.

13 A shop purchases crystal vases for $75 each.

(a) At what price should they be sold if there is an 85% markup?

(b) The vases are not selling well so they are discounted by 10%. What is their new selling price?

(c) What percentage markup does the discounted price represent on the cost price?

14 Effie works as a cleaner and is paid $19.50 per hour for a standard 38-hour week. If she works overtime, she is paid time-and-a-half for the first 2 hours of overtime each day and double time for any hours after that. In one week Effie worked 3 hours overtime on Tuesday, 1 hour overtime on Wednesday and 4 hours overtime on Friday. Find her wage for the week.

15 Jenny has a gross weekly income of $860. She donates $750 per year to registered charities, and has work-related expenses totalling $690 annually.

(a) What is Jenny's annual gross income, assuming there are 52 weeks in a year?

(b) What is Jenny's taxable income?

(c) How much should Jenny pay in tax per year?

16 Natalie purchased an $80 necklace with lay-by. She had to pay a 20% deposit to begin the lay-by.

(a) How much was the initial payment?

(b) If Natalie paid $15 per week, how many weeks did she take to pay off the necklace?

17 A monthly statement for a bank account offering 7% p.a. has the entries shown for the month of June below.

(a) How much interest is payable for June if interest is calculated:

(i) daily

(ii) on the minimum monthly balance?

(b) By how much is **(i)** greater than **(ii)**?

Date	Transaction	Balance
June 1	Opening balance	$100
June 14	Deposit $285	$385
June 21	Withdrawal $207	$178

18 Andrew has purchased a $7000 boat on terms, with an interest-free period of 24 months. He must pay a $25 establishment fee, a monthly account fee of $3.50 and a minimum monthly repayment of $60.

(a) **(i)** How much will Andrew have paid off the cost of the boat after 24 months if he only makes the minimum repayments?

(ii) How much will still be owing?

(iii) How much will Andrew have paid in fees?

(b) If Andrew wants to have the boat paid in full by the end of the interest-free period, how much should his monthly repayments be?

(c) If Andrew has not paid the amount owing in full at the end of 24 months, an interest rate of 29% p.a. is charged. How much interest will be charged on the amount calculated in part **(a) (ii)** after the 25th month? (Assume 30 days in a month.)

Reasoning and communicating

19 Freya is 19 and has a part-time position at a supermarket. Her hourly rate for weekdays is $15.43, with time-and-a-half paid on Saturdays, and double time on Sundays.

Tanya is also 19 and works in the same supermarket. Because she is a casual employee, her conditions are different to Freya's. Tanya gets an extra 25% on the hourly rate on weekdays. On weekends she gets a further 10% on top of her casual rate.

Both Freya and Tanya work for 18 hours during the week, 4 hours on Saturday and 4 hours on Sunday.

(a) Who earned more money?

(b) Why do casual workers receive an extra 25% on top of their hourly rate?

20 Annika works in a jewellery store. Her manager has asked her to place a markup% of 78% on the cost price of all pairs of earrings. These earrings were later discounted by 20% during a mid-year sale. Find the overall profit or loss made by the store on a pair of earrings sold in the sale if the initial cost price of the earrings was $46.60.

NAPLAN practice 1

Numeracy: Non-calculator

1 Julie bought 36 cans of soft drink for a party. 75% of the cans are empty at the end of the party. How many are still full?

A 4 **B** 9 **C** 16 **D** 27

2 Ali has 240 DVDs in his collection. Two-thirds of the DVDs are animated movies. Of the rest, 20% are action movies. How many action movies does Ali have in his collection?

A 16 **B** 48 **C** 80 **D** 160

3 The GST is a 10% tax added to the price of many retail items. A toaster costs $33.50 before the addition of GST. Which calculation will give the new price of the toaster that includes the GST?

A $33.50 + 10 **B** $33.50 × 1.1 **C** $33.50 + 0.1 **D** $33.50 + 1.1

4 Emilio works part time in a takeaway food shop. He gets $15.80 an hour, except on Saturdays, when he gets one-and-a-half times his hourly rate for the first 3 hours and double his hourly rate for any hours worked after that.

One week, Emilio works 15 hours between Monday and Friday and 5 hours on Saturday. Which calculation will give Emilio's total wage for the week?

A 15.8×20 **B** $(15.8 \times 15) + (15.8 \times 1.5 \times 5)$

C $(15.8 \times 15) + (15.8 \times 1.5 \times 3) + (15.8 \times 2 \times 2)$ **D** $(15.8 \times 15) + (17.3 \times 3) + (17.8 \times 2)$

5 Raelene is going to invest $5000 at 6% simple interest for 2 years. If she invested her money for twice as long, the amount of interest she would earn on her investment would:

A decrease slightly **B** double **C** halve **D** increase slightly.

Numeracy: Calculator allowed

6 Rohan bought a new phone at a 35% discount. If the phone was originally priced at $99, how much did Rohan pay for the phone?

A $64 **B** $34.65 **C** $64.35 **D** $134

7 Of the 2153 books stored on the shelves of the local library; 847 are out on loan. What percentage of the library books are out on loan?

A 28.23% **B** 39.34% **C** 71.76% **D** 254.54%

8 Tyson purchases a $1579 home theatre system on the following terms: 20% deposit up front, with monthly repayments of $85 for the next 2 years. How much more money is paid using this method of payment than paying the full price up front?

9 Yang earns a yearly salary of $57 300. The income tax rules that apply to Yang state that she must pay $4650 plus 30c for every dollar over $37 000. Calculate Yang's annual income tax bill.

10 A large chain of clothing stores has a policy of making 85% profit on all jeans and trousers sold. How much would a pair of jeans with a marked retail price of $79.95 have cost the store? (Answer to the nearest cent.)

Algebra

2

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Simplify the following expressions.

(a) $3x + 4x + 7x$ **(b)** $5y + 7y - 4y$ **(c)** $3k^2 + 5k^2 - 7k^2$

2 Simplify the following expressions.

(a) $5y + 6h - 2h - 3y$ **(b)** $6x^2 + 5y^3 - 2x + 3y^3$

3 Simplify the following expressions.

(a) $3x \times 4y$ **(b)** $5k \times 4k$ **(c)** $-5g \times 6g \times 2g$

4 Simplify, writing your answer in index form.

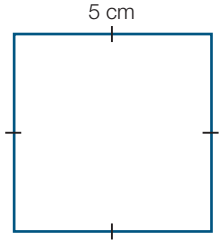
(a) $2^4 \times 2^9$ **(b)** $6^8 \div 6^2$ **(c)** $(5^2)^2$ **(d)** $(8.6325 \times 5.15)^0$

5 Expand:

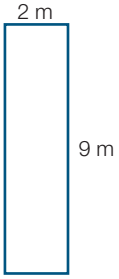
(a) $\left(\frac{4^2}{5^3}\right)^3$ **(b)** $(2^4 \times 5^2)^4$ **(c)** $\left(\frac{7^2 \times 3^4}{4^5}\right)^2$

6 Find **(i)** the perimeter and **(ii)** the area of each of the following.

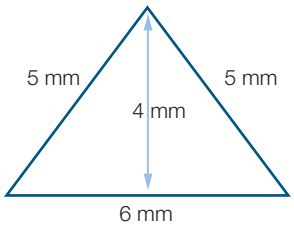
(a)



(b)



(c)



7 Expand the following expressions.

(a) $4(x + 2)$ **(b)** $6(x - 3)$ **(c)** $-2(x + 4)$

8 Factorise the following expressions.

(a) $6x - 12$ **(b)** $10x - 25$ **(c)** $-3x + 15$

Syllabus references

Number and Algebra: Indices

- ✓ Extend and apply the index laws to variables, using positive-integer indices and the zero index **(ACMNA212) (Stage 5.1)**
- ✓ Simplify algebraic products and quotients using index laws **(ACMNA231) (Stage 5.1)**
- ✓ Apply index laws to numerical expressions with integer indices **(ACMNA209) (Stage 5.1)**
- ✓ Apply index laws to algebraic expressions involving integer indices **(ACMNA209) (Stage 5.2)**

Number and Algebra: Algebraic Techniques

- ✓ Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate **(ACMNA213) (Stage 5.2)**
- ✓ Factorise algebraic expressions by taking out a common algebraic factor **(ACMNA230) (Stage 5.2)**
- ✓ Expand binomial products and factorise monic quadratic expressions using a variety of strategies **(ACMNA233) (Stage 5.2)**
- ✓ Expand binomial products using a variety of strategies **(ACMNA233) (Stage 5.3$)**
- ✓ Factorise monic and non-monic quadratic expressions **(ACMNA269) (Stage 5.3$)**

Number and Algebra: Equations

- ✓ Rearrange literal equations **(Stage 5.3$)**

Measurement and Geometry: Numbers of any Magnitude

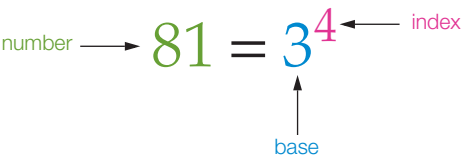
- ✓ Investigate very small and very large time scales and intervals **(ACMMG219) (Stage 5.1)**
- ✓ Express numbers in scientific notation **(ACMNA210) (Stage 5.1)**

2.1 Introducing index laws using variables

Stage 5.1

Need to Know

Index form



$81 = 3 \times 3 \times 3 \times 3 = 3^4$

Expanded form | Index form

Index laws for integers:

$2^4 \times 2^5 = 2^{4+5} = 2^9$	When multiplying numbers in index form that have the same **base**, keep the base and add the indices.
$\frac{3^6}{3^4} = 3^{6-4} = 3^2$	When dividing numbers in index form that have the same base, keep the base and subtract the indices.
$(4^2)^5 = 4^{2 \times 5} = 4^{10}$	When raising a number in index form to a power, keep the base and multiply the indices.
$(2 \times 5)^3 = 2^3 \times 5^3$	When the product of factors in brackets is raised to a power, each factor in the brackets is raised to the same power.
$\left(\frac{7}{3}\right)^4 = \frac{7^4}{3^4}$	When the quotient of factors is raised to a power, each factor in the brackets is raised to the same power.
$2^0 = 1$, $4975^0 = 1$ $6.347^0 = 1$, $\left(\frac{5}{7}\right)^0 = 1$	Any number, except 0, raised to the zero index is equal to 1 (0^0 is undefined).

Multiplication with indices

To calculate $a^2 \times a^3$, write each power in expanded form and count the number of factors in the multiplication string.

$a^2 = a \times a$	There are two factors of base a in the multiplication string.
$a^3 = a \times a \times a$	There are three factors of base a in the multiplication string.
$a^2 \times a^3 = a \times a \times a \times a \times a = a^5$	There are five factors of base a in the multiplication string.

Notice that $2 + 3 = 5$, so $a^2 \times a^3 = a^{2+3} = a^5$

$a^m = a \times a \times \ldots \times a \times a$	There are m factors of base a in the multiplication string.
$a^n = a \times a \times \ldots \times a \times a$	There are n factors of base a in the multiplication string.

To multiply a^m by a^n,

$a^m \times a^n = a^{m+n}$	There are $m + n$ factors of base a in the multiplication string.

Division with indices

As for multiplication, a pattern for the division of numbers in index form can be identified.

To calculate $a^5 \div a^3$, write the division as a fraction, write each power in expanded form and cancel the common factor of a^3.

$$\frac{a^5}{a^3} = \frac{a \times a \times a \times a \times a}{a \times a \times a}$$
$$= a^2$$

Notice that $5 - 3 = 2$ so $a^5 \div a^3 = a^{5-3}$
$= a^2$

To divide a^m by a^n,

$a^m \div a^n = a^{m-n}$ There are $m - n$ factors of base a in the multiplication string.

Power of a power

To calculate $(a^5)^4$, rewrite as a product and then use the index law of multiplication.

$(a^5)^4 = a^5 \times a^5 \times a^5 \times a^5 = a^{5+5+5+5}$
$= a^{4 \times 5}$ or $a^{5 \times 4}$
$= a^{20}$

To find the power of a power,

$(a^m)^n = a^{m \times n}$

Index laws for variables

$a^m \times a^n = a^{m+n}$ When multiplying **variables** in index form that have the same base, keep the base and add the indices.

$a^m \div a^n = \frac{a^m}{a^n}$
$= a^{m-n}$ When dividing variables in index form that have the same base, keep the base and subtract the indices.

$(a^m)^n = a^{m \times n}$ When raising a variable in index form to a power, keep the base and multiply the indices. This rule is known as the 'power of a power' law.

Remember that $a = a^1$.

Worked Example 1

WE1

Simplify each of the following, leaving your answers in index form.

(a) $7^3 \times 7^8$ **(b)** $c^3 \times c^6$ **(c)** $4h^5 \times 3h^9$

Working

(a) $7^3 \times 7^8$
$= 7^{3+8}$
$= 7^{11}$

(b) $c^3 \times c^6$
$= c^{3+6}$
$= c^9$

(c) $4h^5 \times 3h^9$
$= 4 \times 3 \times h^5 \times h^9$
$= 12h^{5+9}$
$= 12h^{14}$

Worked Example 2 WE2

Simplify each of the following, leaving your answers in index form.

(a) $\frac{11^9}{11^5}$ (b) $\frac{d^7}{d^3}$ (c) $20c^8e^7 \div 4c^3e^2$

Working

(a) $\frac{11^9}{11^5}$
$= 11^{9-5}$
$= 11^4$

(b) $\frac{d^7}{d^3}$
$= d^{7-3}$
$= d^4$

(c) $\frac{20c^8e^7}{4c^3e^2}$
$= \frac{{}^{5}\cancel{20}c^{8-3}e^{7-2}}{{}_{1}\cancel{4}}$
$= 5c^{8-3}e^{7-2}$
$= 5c^5e^5$

Worked Example 3 WE3

Simplify each of the following, leaving your answers in index form.

(a) $(6^3)^4$ (b) $(2^3)^2 \times (2^4)^3$ (c) $(5^3)^4 \div (5^5)^2$

(d) $(r^3)^5$ (e) $(p^3)^4 \times (p^5)^2$ (f) $(m^5)^3 \div (m^2)^6$

Working

(a) $(6^3)^4$
$= 6^{3\times4}$
$= 6^{12}$

(b) $(2^3)^2 \times (2^4)^3$
$= 2^{3\times2} \times 2^{4\times3}$
$= 2^6 \times 2^{12}$
$= 2^{6+12}$
$= 2^{18}$

(c) $(5^3)^4 \div (5^5)^2$
$= 5^{3\times4} \div 5^{5\times2}$
$= 5^{12} \div 5^{10}$
$= 5^{12-10}$
$= 5^2$

(d) $(r^3)^5$
$= r^{3\times5}$
$= r^{15}$

(e) $(p^3)^4 \times (p^5)^2$
$= p^{3\times4} \times p^{5\times2}$
$= p^{12} \times p^{10}$
$= p^{12+10}$
$= p^{22}$

(f) $\frac{(m^5)^3}{(m^2)^6}$
$= \frac{m^{5\times3}}{m^{2\times6}}$
$= \frac{m^{15}}{m^{12}}$
$= m^{15-12}$
$= m^3$

2.1 Introducing index laws using variables

Answers page 424

Navigator

Stage 5.1

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Fluency and understanding

1 Simplify each of the following, leaving your answers in index form. WE1

(a) $2^4 \times 2^5$ **(b)** $3^7 \times 3^4$ **(c)** $5^{11} \times 5^7$

(d) $a^3 \times a^4$ **(e)** $b^6 \times b^5$ **(f)** $5c^3 \times c^2$

(g) $3d^3 \times 2d^2$ **(h)** $4b^4 \times 10b^4$ **(i)** $7b^2 \times 4b^2$

(j) $4a^5 \times 3a$ **(k)** $7a^3 \times 2a$ **(l)** $20a^3 \times 10a^4$

(m) $6a^4b^2 \times 4a^5$ **(n)** $9a^3b^3 \times 3ab^2$ **(o)** $5b^4c^5 \times 6b^3c^2$

(p) $-5a^4b^6 \times 7b^2$ **(q)** $-9e^5f^5 \times -4ef$ **(r)** $3a^5b^8 \times -8ab^2$

2 Simplify each of the following, leaving your answers in index form. WE2

(a) $3^8 \div 3^5$ **(b)** $4^7 \div 4^2$ **(c)** $9^5 \div 9^3$

(d) $m^5 \div m^2$ **(e)** $4b^5 \div b^3$ **(f)** $5g^7 \div g^3$

(g) $\frac{n^6}{n^4}$ **(h)** $\frac{14b^7}{7b^5}$ **(i)** $\frac{22c^6}{11c^2}$

(j) $\frac{6a^5}{3a^4}$ **(k)** $\frac{12a^7}{8a^3}$ **(l)** $\frac{14e^4f^5}{10ef^2}$

(m) $10a^7b^7 \div 5a^4b^3$ **(n)** $-24a^5b^3 \div 21a^2b$ **(o)** $144a^6b^8 \div -12a^5b^7$

3 Simplify each of the following, leaving your answers in index form. WE3

(a) $(2^3)^4$ **(b)** $(3^2)^5$ **(c)** $(5^4)^2$

(d) $(3^2)^5 \times (3^4)^2$ **(e)** $(4^3)^2 \times (4^5)^3$ **(f)** $(7^2)^5 \times (7^4)^2$

(g) $(5^2)^8 \div (5^3)^2$ **(h)** $(8^5)^8 \div (8^2)^3$ **(i)** $(11^4)^8 \div (11^3)^9$

(j) $(b^7)^3$ **(k)** $(k^8)^4$ **(l)** $(a^6)^2$

(m) $y^5 \times (y^3)^4$ **(n)** $(k^3)^8 \times k^3$ **(o)** $(m^6)^3 \times m^2$

(p) $(g^2)^5 \times (g^4)^2$ **(q)** $(y^2)^4 \times (y^5)^3$ **(r)** $(t^3)^3 \times (t^2)^6$

(s) $(p^3)^4 \div (p^5)^2$ **(t)** $(n^7)^2 \div (n^2)^4$ **(u)** $(b^8)^3 \div (b^3)^7$

4 **(a)** $-4r^3 \times -r^2$ can be simplified to give:

A $4r^1$ **B** $-4r^5$ **C** $4r^5$ **D** $4r^6$

(b) The product of $-abc$, $-5a$ and $-2a$ is:

A $-10a^3bc$ **B** $-7a^2bc$ **C** $10abc$ **D** $10a^2bc$

5 Copy and complete each of the following by writing the correct index number in the box.

(a) $\dfrac{x^7 \times x^9}{x^{12}}$

$= \dfrac{x^{\square}}{x^{12}}$ (Add indices)

$= x^{\square}$ (Subtract indices)

(b) $\dfrac{6y^4 \times y^6}{y^2 \times y^5}$

$= \dfrac{6y^{\square}}{y^{\square}}$ (Add indices)

$= 6y^{\square}$ (Subtract indices)

6 Simplify each of the following by completing any multiplication of variables first.

(a) $\dfrac{x^8 \times x^7}{x^6}$ **(b)** $\dfrac{t^7}{t^2 \times 3t^4}$ **(c)** $\dfrac{c^6 \times c^9}{c^5 \times c^3}$

(d) $\dfrac{j^4 \times j^8}{j^2 \times j^5}$ **(e)** $\dfrac{4a^7 \times 3a^4}{a^3}$ **(f)** $\dfrac{8q \times 3q^6}{12q^4}$

7 (a) $\dfrac{f^8 \times f^6}{f^{10}}$ simplifies to:

A f^4 **B** f^6 **C** f^{48} **D** f^{12}

(b) $\dfrac{15x^9 \times 3x^6}{9x^{10} \times x^4}$ simplifies to:

A $5x^9$ **B** $9x$ **C** $5x$ **D** $9x^9$

(c) $-36k^4n^3 \div 9n^2k^3$ can be simplified to:

A $4k^4n$ **B** $-4k^7n^6$ **C** $-4kn$ **D** $45k^{12}n^6$

(d) $\dfrac{42x^5y^7z^6}{14x^4y^2z^3}$ simplifies to:

A $3xy^5z^2$ **B** $28xy^5z^3$ **C** $3xy^5z^3$ **D** $3x^9y^9z^9$

8 (a) How many groups of $3a^2$ will divide into $15a^4$?

(b) Use your answer to **(a)** to rewrite this as a multiplication that has $15a^4$ as the product.

9 (a) $\dfrac{a^8 \times b^5}{a^2 \times a^6}$ simplifies to:

A $a^{16}b^5$ **B** $a^{10}b^{11}$ **C** b^5 **D** b^{11}

(b) $\dfrac{3c^7 \times 8d^4}{12c^3 \times 4c^4}$ simplifies to:

A $2d^4$ **B** $\dfrac{d^4}{2}$ **C** $\dfrac{c^4d^4}{24}$ **D** $\dfrac{d^4}{24}$

10 Simplify each of the following.

(a) $(m^5)^2 \div m^7$ **(b)** $(p^6)^4 \div p^{10}$ **(c)** $\dfrac{(k^4)^3}{k^9}$ **(d)** $\dfrac{(n^8)^2}{n^6}$

11 (a) $(k^2)^3 \times (k^5)^2$ simplifies to:

A k^{16} **B** k^{12} **C** k^{60} **D** k^{15}

(b) $(b^5)^4 \div (b^3)^5$ simplifies to:

A b^4 **B** b **C** b^5 **D** b^{12}

Reasoning, communicating and open-ended

12 Solve for x in the following equations.

(a) $5^{x+2} = 5^{2x-1}$ **(b)** $3^{x+1} = 3$ **(c)** $2^{2x+4} = 1$ **(d)** $4(7^x + 7^x) = 56$

13 When Josh started his part-time job, he decided to save some of his wages on a weekly basis. He was paid $50.00 a week, so he decided to start by saving 5c the first week, 10c the next week, 20c the third week, continuing to double the amount he saved each week. He was shocked to find that, by the 11th week, he could not afford to keep up his plan.

(a) Copy the following table and complete the second column to show how much Josh saved each week for the first 5 weeks.

Week	Amount saved each week (c)	Amount saved each week (c) in index form
1		
2		
3		
4		
5		

(b) Complete the third column in this table by writing these amounts in index form.

(c) Write a rule in index form for finding the amount Josh saved in the nth week.

(d) Use your rule to calculate how much Josh was supposed to save in the 11th week.

(e) How much would Josh need to earn each week if he was to keep up his saving plan for 6 months (26 weeks)?

14 Find three different expressions for ■ and ▲ in terms of a and b to make this a true statement:

■ × ▲ = $-3a^2b^3$

15 Write three possible divisions involving indices that will give an answer of $5a^2b^3$.

16 Create three index number division expressions and simplify them using any of $24a^2b^3$, $3ab$, $48a^5b^6$, $22a^2b^2$ or 2×3^2a. Check your answers by giving the divisions to a friend to simplify.

17 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

18 The index laws are sometimes used incorrectly. Below is a list of equations that 'look' reasonable, but are actually incorrect.

In each case, explain the mistake that has been made and give the correct answer in its place. Explain why your answer is correct.

(a) $x^2x^4 = x^8$ **(b)** $\frac{x^9}{x^3} = x^3$ **(c)** $(x^2)^4 = x^{2^4} = x^{16}$

Hint

If the bases on both sides of the equation are the same, the expression for the powers must be equal.

Hint

Look for a pattern. What values do you get if 1 is subtracted from a power of 2?

2.2 More index laws

Stage 5.1

Need to Know

To **expand** an **expression** is to remove the brackets in the expression.

Expanding products

Expand $(2 \times a)^3$.

$$\begin{aligned}(2 \times a)^3 &= (2 \times a) \times (2 \times a) \times (2 \times a) \\ &= 2 \times 2 \times 2 \times a \times a \times a \\ &= 2^3 \times a^3\end{aligned}$$

So $(2 \times a)^3 = 2^3 \times a^3$.

Each factor in the brackets has been raised to the power of 3.

Expand $(kp)^4$.

$$\begin{aligned}(kp)^4 &= kp \times kp \times kp \times kp \\ &= k \times k \times k \times k \times p \times p \times p \times p \\ &= k^4 \times p^4 \\ &= k^4 p^4\end{aligned}$$

Each factor in the brackets has been raised to the power of 4.

So $(ab)^m = a^m b^m$.

Expanding quotients

Expand $\left(\frac{m}{n}\right)^5$.

$$\left(\frac{m}{n}\right)^5 = \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n}$$

$$= \frac{m^5}{n^5}$$

Each variable in the brackets has been raised to the power of 5.

So $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

The zero index

If $2^0 = 1$, $45\,678^0 = 1$ and $0.000\,123^0 = 1$, is $a^0 = 1$ also?

To simplify $\frac{a^m}{a^m}$, subtract indices (second index law).

$$\begin{aligned}\frac{a^m}{a^m} &= a^{m-m} \\ &= a^0\end{aligned}$$

Alternatively, treating it as a fraction division,

$$\frac{a^m}{a^m} = 1$$

Therefore $a^0 = 1$.

Negative indices

Evaluating 2^{-6}.

Consider $2^6 = 64$, $2^5 = 32$, $2^4 = 16$, $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ and $2^0 = 1$.

The index on the left-hand side of each equation is one less than the previous one and the number on the right-hand side of each equation is half the previous one. Continuing this pattern:

$2^{-1} = \frac{1}{2}$ $\quad 2^{-2} = \frac{1}{2^2}$ $\quad 2^{-3} = \frac{1}{2^3}$

So, $2^{-6} = \frac{1}{2^6}$

In general, $2^{-m} = \frac{1}{2^m}$.

Note that any base number can be used, e.g. $3^{-4} = \frac{1}{3^4}$, $81^{-6} = \frac{1}{81^6}$.

Index laws for powers with the same base

The following laws are true if $a, b \neq 0$ (a and b cannot be zero). Only integer values for m and n are considered here.

Multiplication of powers	$a^m \times a^n = a^{m+n}$	When multiplying numbers with the same base, keep the base and add the indices.
Division of powers	$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	When dividing numbers with the same base, keep the base and subtract the indices.
Power of a power	$(a^m)^n = a^{mn}$	When raising a number in index form to a power, keep the base and multiply the indices.
Expanding products	$(ab)^m = a^m b^m$	When removing brackets around a product raised to a power, raise each number to that power.
Expanding quotients	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	When removing brackets around a quotient raised to a power, raise each number to that power.
The zero index	$a^0 = 1$ $(a \neq 0)$	Any variable or product of variables raised to the power of 0 is equal to 1. Note that 0^0 is undefined.
Negative indices with a numerical base	Example $2^{-m} = \frac{1}{2^m}$	A number raised to a negative power is equal to 1 divided by the same number raised to the positive power.

Worked Example 4

WE4

Use an index law to expand each of the following, leaving your answers in index form.

(a) $(3 \times 5)^4$ **(b)** $(ab)^5$ **(c)** $(5p^4q)^3$

Working

(a) $(3 \times 5)^4$
$= 3^4 \times 5^4$

(b) $(ab)^5$
$= a^5 \times b^5$
$= a^5 b^5$

(c) $(5p^4q)^3$
$= 5^3 \times (p^4)^3 \times q^3$
$= 5^3 \times p^{4\times 3} \times q^3$
$= 5^3 \times p^{12} \times q^3$
$= 5^3 p^{12} q^3$

Worked Example 5 WE5

Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{4}{3}\right)^5$ (b) $\left(\frac{p}{q}\right)^7$ (c) $\left(\frac{2a}{c^4}\right)^3$

Working

(a) $\left(\frac{4}{3}\right)^5$

$= \frac{4^5}{3^5}$

(b) $\left(\frac{p}{q}\right)^7$

$= \frac{p^7}{q^7}$

(c) $\left(\frac{2a}{c^4}\right)^3$

$= \frac{(2a)^3}{(c^4)^3}$

Worked Example 6 WE6

Simplify each of the following, leaving your answers in index form.

(a) $2^3 \times 6^3$ (b) $5^4 \times h^4$ (c) $m^2 \times n^2$

Working

(a) $2^3 \times 6^3$

$= (2 \times 6)^3$

$= 12^3$

(b) $5^4 \times h^4$

$= (5h)^4$

(c) $m^2 \times n^2$

$= (mn)^2$

Worked Example 7 WE7

Simplify each of the following, leaving your answers in index form.

(a) $\frac{16^3}{4^3}$ (b) $\frac{t^4}{2^4}$ (c) $\frac{c^8}{d^8}$

Working

(a) $\frac{16^3}{4^3}$

$= \left(\frac{16}{4}\right)^3$

$= 4^3$

(b) $\frac{t^4}{2^4}$

$= \left(\frac{t}{2}\right)^4$

(c) $\frac{c^8}{d^8}$

$= \left(\frac{c}{d}\right)^8$

Worked Example 8 WE8

Simplify each of the following.

(a) $4p^0$ (b) $\dfrac{2y^4 \times y^3}{y^5 \times y^2}$ (c) $\dfrac{m^3n^6}{m^3n}$

Working

(a) $4p^0$
$= 4 \times p^0$
$= 4 \times 1 = 4$

(b) $\dfrac{2y^4 \times y^3}{y^5 \times y^2}$
$= \dfrac{2y^{4+3}}{y^{5+2}}$
$= \dfrac{2y^7}{y^7}$
$= 2y^{7-7}$
$= 2y^0$
$= 2 \times 1$
$= 2$

(c) $\dfrac{m^3n^6}{m^3n}$
$= m^{3-3}n^{6-1}$
$= m^0n^5$
$= 1 \times n^5$
$= n^5$

Worked Example 9 WE9

Simplify the following, writing your answer with positive indices.

(a) 3^{-2} (b) 5^{-3} (c) 8^{-5}

Working

(a) $3^{-2} = \dfrac{1}{3^2}$ (b) $5^{-3} = \dfrac{1}{5^3}$ (c) $8^{-5} = \dfrac{1}{8^5}$

2.2 More index laws

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Stage 5.1

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Fluency and understanding

1 Use an index law to expand each of the following, leaving your answers in index form. WE4

(a) $(6 \times 5)^2$ (b) $(11 \times 3)^4$ (c) $(12 \times 9)^5$
(d) $(4x)^2$ (e) $(p^3q)^2$ (f) $(a^4f)^3$
(g) $(2p)^5$ (h) $(p^3q^5)^7$ (i) $(c^2d^7)^4$
(j) $(bc)^3$ (k) $(6a^4)^3$ (l) $(9p^5q^4)^2$
(m) $(7m^3n^6)^2$ (n) $(2ab^2)^3$ (o) $(3ef^4)^4$

WE5

2 Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{12}{5}\right)^2$ (b) $\left(\frac{7}{3}\right)^4$ (c) $\left(\frac{9}{2}\right)^8$

(d) $\left(\frac{y}{x}\right)^3$ (e) $\left(\frac{d}{e}\right)^5$ (f) $\left(\frac{m^3}{n}\right)^4$

(g) $\left(\frac{w^4}{z}\right)^2$ (h) $\left(\frac{r^5}{s}\right)^4$ (i) $\left(\frac{d^3}{g}\right)^2$

(j) $\left(\frac{3}{w^6}\right)^4$ (k) $\left(\frac{8}{m^2}\right)^3$ (l) $\left(\frac{2}{r^4}\right)^3$

(m) $\left(\frac{h^3}{g^2}\right)^5$ (n) $\left(\frac{a^3}{c^5}\right)^4$ (o) $\left(\frac{t^4}{v^3}\right)^3$

(p) $\left(\frac{5a}{b}\right)^3$ (q) $\left(\frac{c^4}{2x}\right)^6$ (r) $\left(\frac{f^6}{6g}\right)^3$

WE6

3 Simplify each of the following, leaving your answers in index form.

(a) $3^2 \times 4^2$ (b) $4^6 \times 5^6$ (c) $8^4 \times 7^4$

(d) $4^8 \times h^8$ (e) $3^4 \times p^4$ (f) $7^9 \times r^9$

(g) $a^5 \times m^5$ (h) $k^7 \times t^7$ (i) $p^8 \times n^8$

WE7

4 Simplify each of the following, leaving your answers in index form.

(a) $\frac{15^4}{3^4}$ (b) $\frac{24^3}{6^3}$ (c) $\frac{132^5}{11^5}$

(d) $\frac{r^5}{3^5}$ (e) $\frac{n^7}{6^7}$ (f) $\frac{g^9}{4^9}$

(g) $\frac{x^6}{y^6}$ (h) $\frac{k^{11}}{n^{11}}$ (i) $\frac{g^{10}}{h^{10}}$

WE8

5 Simplify each of the following.

(a) k^0 (b) $7g^0$ (c) $7 + g^0$

(d) a^0b^2 (e) $3m^0n^0$ (f) $c^3d^0e^0$

(g) $-3a^4b^0$ (h) $-8g^{11}h^0$ (i) $-(4pr)^0$

(j) $y^4 \div y^4$ (k) $3x^7 \div x^7$ (l) $24m^6 \div 8m^6$

(m) $\frac{3x^4z}{z}$ (n) $\frac{h^5k^8}{h^5}$ (o) $\frac{8m^3n^4}{16m^3}$

(p) $\frac{a^5b^4}{a^2b^4}$ (q) $\frac{6h^4r^3}{2h^4r^3}$ (r) $\frac{9b^4d^3}{6b^4d^3}$

(s) $\frac{12e^3f^2}{16e^3}$ (t) $\frac{15r^2s^9}{27r^2}$ (u) $\frac{25k^4p^2}{15k^4}$

Hint

Remember that any number (except 0) divided by itself equals one.

WE9

6 Simplify the following, writing your answers with positive indices.

(a) 4^{-2} (b) 2^{-3} (c) 5^{-4}

(d) 3^{-3} (e) 5^{-2} (f) 4^{-3}

7 **(a)** $(2k)^6$ simplifies to:

A $2k^6$ **B** $12k$ **C** $64k^6$ **D** $12k^6$

(b) $(3a^4)^3$ simplifies to:

A $27a^{12}$ **B** $9a^{12}$ **C** $9a^7$ **D** $27a^7$

(c) $\left(\frac{3}{g}\right)^4$ simplifies to:

A $\frac{12}{g^4}$ **B** $\frac{81}{g^4}$ **C** $\frac{12}{g}$ **D** $\frac{81}{g}$

(d) $\left(\frac{k^8}{7}\right)^2$ simplifies to:

A $\frac{k^{10}}{7}$ **B** $\frac{k^{16}}{49}$ **C** $\frac{k^{16}}{7}$ **D** $\frac{k^{10}}{49}$

8 Simplify each of the following by first applying the index law for expanding products, then completing the multiplication, applying other index laws if necessary.

(a) $(xy)^2 \times (xy)^4$ **(b)** $(gh)^7 \times (gh)^2$ **(c)** $(3x)^3 \times (4x)^2$

(d) $(4y)^2 \times (5xy)^2$ **(e)** $(cd)^3 \times (c^3d^2)^3$ **(f)** $(x^2y^3)^2 \times (xy)^2$

9 Simplify each of the following by first applying the index law for expanding quotients, completing the multiplication, then simplifying if possible.

(a) $\left(\frac{f}{g}\right)^3 \times \left(\frac{f}{g}\right)^2$ **(b)** $\left(\frac{p}{q}\right)^6 \times \left(\frac{p}{q}\right)^4$ **(c)** $\left(\frac{d^6}{q^4}\right)^2 \times \left(\frac{d^5}{q^2}\right)^3$

(d) $\left(\frac{a^4}{b^2}\right)^2 \times \left(\frac{b}{a}\right)^6$ **(e)** $\left(\frac{2a}{b}\right)^2 \times \left(\frac{a}{b}\right)^3$ **(f)** $\left(\frac{3x^2}{y}\right)^3 \times \left(\frac{2x}{y}\right)^2$

10 Simplify each of the following by applying the expanding products and the powers of powers index laws, then the index law for division.

(a) $\frac{(w^4z^3)^3}{(w^5z^2)^2}$ **(b)** $\frac{(c^7e^5)^2}{(c^4e^2)^3}$ **(c)** $\frac{(pq)^{12}}{(p^2q^3)^3}$ **(d)** $\frac{(vw)^9}{(v^3w^2)^2}$

11 Simplify each of the following.

(a) $(x^3)^0$ **(b)** $(d^5)^0$ **(c)** $(k^0)^4$ **(d)** $(p^0)^2$

(e) $(m^3)^4 \times (m^2)^0$ **(f)** $(f^5)^4 \times (f^3)^0$ **(g)** $(t^0)^3 \times t^4$ **(h)** $((q^4)^5 \times (q^5)^4)^0$

(i) $\frac{x^6 \times x^5}{x^{11}}$ **(j)** $\frac{3k^2}{k \times 5k}$ **(k)** $\frac{4a^3}{a^2 \times 7a}$ **(l)** $\frac{n^4 \times n^3}{n^5 \times n^2}$

(m) $\frac{2b^6 \times 5c^8}{c^7 \times 4c}$ **(n)** $\frac{d^6 \times e^2}{3e \times 2e}$ **(o)** $\frac{c^4 \times d}{2c^2 \times 5c^2}$ **(p)** $\frac{p^7 \times q^5}{15p^2 \times p^5}$

12 Use the negative index law to write the following with positive indices, then evaluate.

(a) 4^{-2} **(b)** 3^{-3} **(c)** 5^{-4}

(d) $\frac{1}{2^{-1}}$ **(e)** $\frac{1}{4^{-3}}$ **(f)** $\frac{1}{3^{-2}}$

(g) 4×3^{-2} **(h)** 3×4^{-3} **(i)** 2×5^{-2}

(j) $-\frac{7 \times 3^{-2}}{3}$ **(k)** $-\frac{6 \times 2^{-4}}{2^{-3}}$ **(l)** $-\frac{3 \times 3^{-4}}{3^{-6}}$

13 **(a)** $\frac{1}{2^{-4}}$ is the same as:

A 16 **B** 8 **C** $\frac{1}{16}$ **D** -16

(b) $\frac{1}{8}$ is the same as:

A 2^3 **B** 2^{-3} **C** 3^{-2} **D** $\frac{1}{2^{-3}}$

Reasoning, communicating and open-ended

14 Kellie and Eva are given the following question as part of their indices homework.

Simplify $\left(\frac{4x^2}{2x}\right)^3$.

Both girls had the correct answer, but Eva finished much faster. 'How did you do it so fast?' Kellie asked Eva.

Explain why Eva's method took fewer steps.

Here is Kellie's working:

$\left(\frac{4x^2}{2x}\right)^3$

$= \frac{4^3 \times x^{2\times 3}}{2^3 \times x^3}$

$= \frac{64x^6}{8x^3}$

$= 8x^3$

Here is Eva's working:

$\left(\frac{4x^2}{2x}\right)^3$

$= (2x)^3$

$= 8x^3$

15 By rewriting the expression if required and then using index laws, simplify each of the following using the smallest possible base.

(a) $\frac{9^{2n}}{9^n}$ **(b)** $2^n \times 4^{n+1}$ **(c)** $\frac{125}{5^n} \times \frac{5^{2n}}{25}$

16 **(a)** Using index notation, write the numbers 125, 25, 5 and 1, using base 5.

(b) Using index notation, write the next five numbers using the pattern in part **(a)**.

(c) Write the whole sequence of nine numbers in this pattern without using index notation.

17 A model for the number of planets, N, created in a section of a galaxy is given by the formula $N = \left(\frac{n^2T^3p}{b}\right)^2$. In the formula:

n is the number of suns
T is the temperature of the surrounding space
p is the number of planets already present
b is the number of black holes.

(a) Write the formula without brackets.

(b) **(i)** How many new planets does the model predict if $T = 1$, $n = 1000$ and $b = p$?

(ii) If the temperature of the surrounding space is a negative number, will we obtain a positive or a negative answer? Explain your reasoning.

18 The intensity of sound is measured in watts per square metre. The level of sound (loudness) is measured in decibels. The threshold of human hearing is 10^{-12} watts/m^2 and is defined as a level of sound of 0 decibels (dB). All other decibel levels are calculated by comparing the sound intensity with the threshold of human hearing. To do this we follow a simple rule:

1 Divide the intensity measured in watts/m^2 by the sound intensity of the threshold of human hearing.

For example, normal conversation is measured at 10^{-6} watts/m^2.

$$\frac{10^{-6}}{10^{-12}} = 10^{-6-(-12)}$$
$$= 10^{6}$$

This means that normal conversation is 1 million times as loud as the threshold of human hearing.

2 Multiply the index by 10. This is the intensity level in decibels.

Intensity level = $10 \times 6 = 60$ dB

To convert from decibels to find the sound intensity, we reverse the process.

For example, the sound level for a quiet library = 40 dB.

To find the sound intensity:

1 Divide the number of decibels by 10.

$$\frac{40}{10} = 4$$

2 Use this number as an index for 10 and multiply by the threshold for human hearing.

The sound intensity of a quiet library $= 10^{4} \times 10^{-12}$
$= 10^{-8}$ watts/m^2

(a) Copy and complete the following table.

Source	Sound level (loudness; dB)	Sound intensity (watts/m^2)
Jet aircraft, 50 m away		100
Threshold of pain		10
Chainsaw, 1 m distance	110	
Nightclub, 1 m from speaker		0.01
Busy traffic	80	
Vacuum cleaner, 1 m distance	70	
Average home		0.000 000 1
Rustling of leaves in the distance	10	

(b) If a whisper is 20 dB, how many times as loud is normal conversation?

(c) If the intensity of a rock concert is 10^{0} watts/m^2 and the level of intensity of the threshold of discomfort is 120 dB, what comment could you make comparing these two measures?

(d) Which is louder, thunder at a sound level of 110 dB or a loud TV at a sound intensity of 10^{-4} watts/m^2?

19 **(a)** Write at least two expressions involving indices and multiplication that, when simplified, equal $36m^4n^8$.

(b) Write at least two expressions involving indices and division that can be simplified to $\frac{4m^4}{5n^6}$.

20 Write three different terms that $27x^3y^7$ could be divided by so that no x appears in the answer.

21 Write a question that could be simplified to $\frac{3x^2}{y^3}$ by using:

(a) any two index laws with an expression that has only positive indices

(b) any three index laws with an expression that has only positive indices.

Show, step by step, how your question can be simplified to obtain the final answer.

Problem solving

Square cubes

Equipment required: 1 brain, calculator

'I have been studying the cube numbers. You know, some of them are also square numbers!' It was smart Alec, bragging again.

'You're kidding!' said Sue. 'A number can't be a square and a cube.'

'Yes it can,' said Alec. 'Like, 64 is both 8 squared and 4 cubed.'

If Alec is right, there may be more.

1 Can you find any other two-digit numbers that are a square and a cube?

2 With the help of your calculator, how many can you find under 1000?



Strategy options

- Test all possible combinations.
- Look for a pattern.
- Break the problem into manageable parts.

Negative indices with a variable base

2.3

Stage 5.2

Need to Know

Negative indices with a variable as the base

Consider two ways to simplify $a^2 \div a^6$.

To simplify $a^2 \div a^6$, subtract indices.

$a^2 \div a^6 = a^{2-6}$

$= a^{-4}$

Alternatively, expand the numerator and denominator and cancel the common factors.

$$a^2 \div a^6 = \frac{a \times a}{a \times a \times a \times a \times a \times a}, a \neq 0$$

$$= \frac{1}{a^4}, a \neq 0$$

The expressions $a^{-4} = \frac{1}{a^4}, a \neq 0$

a^{-4} is the **reciprocal** of a^4. Reciprocals always multiply to give a product of 1.

$a^4 \times a^{-4} = a^0$ or $a^4 \times \frac{1}{a^4} = \frac{a^4}{a^4}$

$= 1$ $= 1$

So, $a^{-m} = \frac{1}{a^m}, a \neq 0$

Negative indices	$a^{-m} = \frac{1}{a^m} (a \neq 0)$	A number raised to a negative power is equal to 1 divided by the same number raised to the positive power.

Worked Example 10 WE10

Simplify the following, writing your answer with positive indices.

(a) m^{-5} (b) $4p^{-3}$ (c) $6x^{-5}y^2$

Working

(a) $m^{-5} = \frac{1}{m^5}$

(b) $4p^{-3} = 4 \times p^{-3}$
$= 4 \times \frac{1}{p^3}$
$= \frac{4}{p^3}$

(c) $6x^{-5}y^2 = \frac{6y^2}{x^5}$

Answers page 425

2.3 Negative indices with a variable base

Stage 5.2

Navigator

Q1, Q2, Q3, Q4, Q5 Columns 1 & 2, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9

Fluency and understanding

WE10

1 Simplify the following, writing your answer with positive indices.

(a) x^{-5} (b) y^{-3} (c) b^{-7}
(d) $3e^{-3}$ (e) $7x^{-5}$ (f) $3g^{-4}$
(g) $8p^{-5}q^{-2}$ (h) $2x^{-3}y^{-5}$ (i) $12a^{-3}b^{-9}$
(j) $3p^3q^{-1}$ (k) $5x^2y^{-7}$ (l) $7a^{-5}b^2$

2 Express each of the following with positive indices.

(a) $\frac{1}{d^{-2}}$ (b) $\frac{1}{k^{-5}}$ (c) $\frac{3}{a^{-1}}$
(d) $\frac{x^3}{y^{-6}}$ (e) $\frac{p^7}{q^{-2}}$ (f) $\frac{m^{-4}}{n^{-4}}$
(g) $\left(\frac{a^2}{b^3}\right)^{-4}$ (h) $\left(\frac{m^4}{n^5}\right)^{-2}$ (i) $\left(\frac{x^{-5}}{y^2}\right)^{-3}$

3 Rewrite with positive indices.

(a) $(x+4)^{-1}$ (b) $(x-y)^{-2}$ (c) $5(c-3)^{-1}$

4 **(a)** a^{-4} is the same as:

A $-a^4$ **B** $-4a$ **C** $\frac{1}{a^4}$ **D** $\frac{1}{a^{-4}}$

(b) $\frac{1}{y^2}$ is the same as:

A $-y^2$ **B** y^{-2} **C** $-y^{-2}$ **D** $2y$

5 Simplify, writing your answer with positive indices.

(a) $x^2 \times x^{-5}$ **(b)** $p^{-3} \times p$ **(c)** $2c^{-4} \times 6c^{-1}$

(d) $a^{-3} \div a^4$ **(e)** $k^{-2} \div k^{-4}$ **(f)** $3d^{-3} \div 4d^{-2}$

(g) $(m^{-3})^{-3}$ **(h)** $(xy^{-2})^{-4}$ **(i)** $\left(\frac{c^{-3}d}{2c^2d^{-2}}\right)^{-1}$

Reasoning, communicating and open-ended

6 Evaluate the expression $y^{-2} \div x^{-1}$ when $x = \frac{1}{2}$ and $y = \frac{2}{3}$.

7 **(a)** If I divide a number by its square, what is the result?

(b) If a rectangular room has one side length of $12a$ m and an area of 84 m^2, what is the length of the other side of the room? ($b = \frac{A}{w}$ where b is the breadth, A the area and w the length.)

8 Write a question that could be simplified to $\frac{3x^2}{y^3}$ by using:

(a) two index laws with an expression that has at least one negative index

(b) three index laws with an expression that has at least one negative index.

Show, step by step, how your question can be simplified to obtain the final answer.

9 Jodie made four mistakes when simplifying the following index question. Find them, explain to Jodie what she did incorrectly and redo the question to get the correct answer. Jodie's working is shown below:

$$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{6x^4y^6}{2x^{-2}y^5}$$
$$= 4x^2y$$

2.4 Scientific notation and significant figures

Stage 5.1

Need to Know

Powers of 10

When multiplying by a power of 10 with a positive index, move the decimal point to the right by the same number of places as the value of the index. This makes the number bigger. Zeroes are inserted as the values for any places not filled by other digits.

When multiplying by a power of 10 with a negative index, move the decimal point to the left by the same number of places as the value of the index after the negative sign. This makes the number smaller. Zeroes are inserted as the values for any places not filled by other digits.

This is the same as dividing by the reciprocal power.

$$\begin{aligned} 6.5 \times 10^{-4} &= \frac{6.5}{10^4} \\ &= \frac{6.5}{10\,000} \\ &= 0.000\,65 \end{aligned}$$

Scientific notation

When dealing with very large or very small numbers, **scientific notation** is often used. A number is in scientific notation (also known as 'standard form') if it is written as the product of a number, a, and a power of 10, where 'a' is any number greater than or equal to 1 but less than 10:

$a \times 10^b$ where $1 \leq a < 10$ and b is an integer.

Scientific notation on a calculator

When the result of an arithmetic calculation is larger or smaller than the display on a calculator allows, it is written in scientific notation. However, it looks a little different from the way it is written.

For example:

$4\,589\,765\,125 \times 849\,235\,713 = 3.897\,792\,458 \times 10^{18}$

On a calculator, this result may be written 3.897792458E18 where E18 represents $\times 10^{18}$.

Similarly, 3.7792311E-5 would represent $3.779\,231\,1 \times 10^{-5}$.

Significant figures

Significant figures are used in measurement because all measurements are approximate as they are only as accurate as measuring devices will allow. Using significant figures is just another way of rounding. The number of significant figures indicates how accurate the measurements are.

Rounding using significant figures

1 Write the number in scientific notation: $23\,467.8 = 2.346\,78 \times 10^4$.

2 Decide how many significant figures you require.

$2.346\,78 \times 10^4 \approx 2.3 \times 10^4$ to two significant figures

$\approx 2.35 \times 10^4$ to three significant figures

$\approx 2.347 \times 10^4$ to four significant figures.

Remember to check the next digit after the one being rounded to decide whether to round up or down.

Counting significant figures

- All non-zero digits are significant. The first significant figure in a number is the first non-zero digit:

  2.346 78 has six significant figures.

- Zeroes between non-zero digits or at the end of a decimal number are significant:

  1**0** 2**0**3 has five significant figures.

  0.03**0** has two significant figures.

- Zeroes at the end of an integer or at the beginning of a decimal number are not significant.

  1 350 000 has three significant figures.

  0.000 013 5 also has three significant figures.

Writing significant figures

- Zeroes appearing at the end of a decimal number after rounding are significant figures and must be retained to give the required number of significant figures.

  7.999 is written as 8.00 to three significant figures.

- Zeroes sometimes need to be added.

  7.2 is written as 7.20 to three significant figures.

Accuracy of measurement

All measurements are only as accurate as the measuring equipment used can measure. Very large and very small measurements are usually approximated further by rounding to a certain number of significant figures. However, all calculations should be made using the maximum number of digits possible on a calculator until a final answer is achieved, before any rounding is carried out.

Using units for very large and very small measurements

For very small measurements, 'milli', 'micro' and 'nano' are used, where 'milli' means $\times 10^{-3}$, 'micro' means $\times 10^{-6}$ and 'nano' means $\times 10^{-9}$.

For very large measurements, 'mega', 'giga' and 'tera' are used, where 'mega' means $\times 10^3$, 'giga' means $\times 10^6$ and 'tera' means $\times 10^9$.

It is always important to use the correct abbreviation for units, although some units can correctly be abbreviated in different ways. For example, kilobytes can be abbreviated to kB, KB, k or K.

Worked Example 11 WE11

Find the value of each of the following.

(a) 4.35×10^6 **(b)** 1.78×10^{-3}

Working

(a) $4.35 \times 10^6 = 4\,350\,000$

(Move the decimal six places to the right and insert four zeroes.)

(b) $1.78 \times 10^{-3} = 0.001\,78$

(Move the decimal point three places to the left and insert two zeroes.)

Worked Example 12 WE12

Express each of the following in scientific notation.

(a) 5800 **(b)** 0.000 603 4

Working

(a) $5800 = 5.8 \times 10^3$

(b) $0.000\,603\,4 = 6.034 \times 10^{-4}$

Worked Example 13 WE13

Round each of the following to the number of significant figures specified.

(a) 50 842 to three significant figures **(b)** 0.007 497 to three significant figures

Working

(a) $50\,842 = 5.084\,2 \times 10^4$
$\approx 5.08 \times 10^4$ or 50 800

(b) $0.007\,497 = 7.497 \times 10^{-3}$
$\approx 7.50 \times 10^{-3}$ or 0.007 50

2.4 Scientific notation and significant figures

Answers page 425

Navigator

Stage 5.1

Q1 Columns 1 & 2, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9 Column 1, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q19, Q23, Q24	Q1 Columns 2 & 3, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9 Column 2, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q19, Q20, Q23, Q24	Q1 Columns 3 & 4, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9 Column 3, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q20, Q21, Q22, Q24

Equipment required: calculator for Question **9**

Fluency and understanding

1 Find the value of each of the following. WE11

(a) 9.8×10^3 **(b)** 6.5×10^7 **(c)** 3.25×10^6 **(d)** 4.2071×10^5

(e) 2×10^{-3} **(f)** 5×10^{-5} **(g)** 7×10^{-8} **(h)** 5.032×10^{-2}

(i) 8.26×10^{-3} **(j)** 4.18×10^{-5} **(k)** 7.14×10^{-4} **(l)** 3.55×10^{-6}

2 Express each of the following in scientific notation. WE12

(a) 2560 **(b)** 90 020 **(c)** 614 000

(d) 190 000 **(e)** 8 050 000 **(f)** 56 000 000

(g) 0.305 **(h)** 0.03 **(i)** 0.0004

(j) 0.002 01 **(k)** 0.000 072 466 **(l)** 0.000 000 532 18

3 Round each of the following to the number of significant figures indicated in the brackets. WE13

(a) 394 024 (2) **(b)** 0.054 02 (3) **(c)** 674.5 (2)

(d) 0.003 (3) **(e)** 1 679 000 (2) **(f)** 0.899 5 (3)

4 To how many significant figures have each of the following been written?

(a) 825 **(b)** 63.73 **(c)** 145 000

(d) 0.0034 **(e)** 3.008 **(f)** 0.0040

5 **(a)** A fly's wing flap is 3 milliseconds. Express this time in seconds using scientific notation.

(b) The time for light to travel once around the Equator is 134 milliseconds. Express this time in seconds using scientific notation.

(c) The average length of a blink is 300–400 milliseconds. Express this range of times in seconds using scientific notation.

6 Express each quantity described below using scientific notation.

(a) The distance around the Earth's equator is 40 075 km.

(b) The thickness of a soap bubble is about 0.000 000 1 m.

(c) A 10-cent coin is about 0.0015 m thick.

(d) Pluto is approximately 5900 million km from the Sun.

(e) The poliomyelitis virus has a diameter of 0.000 000 3 m.

(f) The melting point of gold is 1063°C.

7 Express each quantity described below as a number.

(a) Radio waves travel at about 3×10^8 m/s.

(b) In a hydrogen atom, the average distance between the proton and the electron is 1.5×10^{-15} m.

(c) The thickness of fine paper is about 1.2×10^{-4} m.

(d) The aluminium in a soft-drink can contains around 4×10^{24} atoms.

(e) This distance from the Sun to Jupiter is approximately 7.78×10^8 km.

(f) The temperature at the centre of the Sun is about 1.5×10^7°C.

8 Find the largest number of the following:

A 1.325×10^5 **B** 2.853×10^2 **C** 9.851×10^{-2} **D** 6.312×10^4

9 Use a calculator to evaluate each expression. Express your answers in scientific notation.

(a) $2.4 \times 10^6 \times 5.9 \times 10^4$

(b) $7.34 \times 10^3 \times 2.9 \times 10^{-6}$

(c) $7.86 \times 10^3 \div (2.62 \times 10^{-8})$

(d) $4.38 \times 10^9 \div (2.19 \times 10^{-3})$

(e) $6.2 \times 10^{-4} - 1.34 \times 10^{-5}$

(f) $4.6 \times 10^{-10} - 3.01 \times 10^{-11}$

(g) $2(4.3 \times 10^3 - 6.9 \times 10^7)$

(h) $5(9.1 \times 10^5 - 2.7 \times 10^9)$

(i) $\dfrac{3.21 \times 10^3 \times 4.19 \times 10^4}{5 \times 10^6}$

(j) $\dfrac{5.92 \times 10^4 \times 8.01 \times 10^{12}}{3 \times 10^5}$

(k) $\dfrac{3(2.5 \times 10^6 + 4.9 \times 10^7)}{6.5 \times 10^{-4}}$

(l) $\dfrac{4(7.3 \times 10^9 + 2.1 \times 10^4)}{8.6 \times 10^{-5}}$

Hint

A negative power in the denominator can be written in the numerator with a positive power.

10 Convert the following to the units given in brackets, writing your answer in scientific notation.

(a) 63 nanoseconds (seconds)

(b) 857 megahertz (hertz)

(c) 2562 gigalitres (litres)

(d) 59 micrometres (metres)

11 Given that $2^{10} = 1024$ and 1 byte = 8 bits, 1 kilobyte = 1024 bytes, 1 megabyte = 1024 kilobytes, 1 gigabyte = 1024 megabytes and 1 terabyte = 1024 gigabytes, find the number of bits in 1 terabyte giving your answer:

(a) in index form

(b) in scientific notation.

12 Convert the following to the units given in brackets, writing your answer in scientific notation.

(a) 6 megabytes (kilobytes)

(b) 1.5 gigabytes (megabytes)

(c) 64 kilobytes (megabytes)

13 Estimate the answer to the following by first writing the numbers in scientific notation.

(a) $8\,975\,328 \times 4129$

(b) $0.023\,75 \div 178.4$

14 Express each quantity described below using scientific notation.

(a) The diameter of the nucleus of an atom is about 0.000 000 000 000 01 m.

(b) The highest Sherpa village on the slopes of Mount Everest is about 4000 m above sea level.

(c) Fine capillaries, which carry blood to the body's tissues, are about 0.000 02 m in diameter.

(d) The filament of a light bulb reaches a temperature of 2730°C.

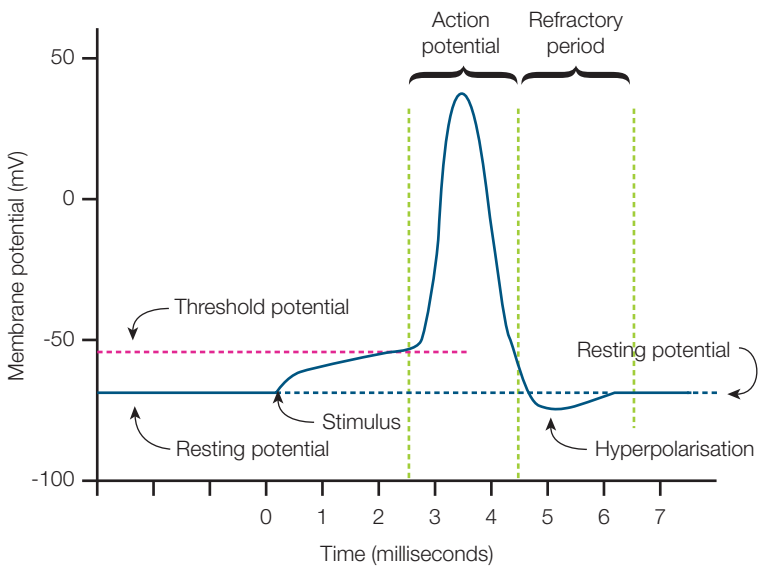
(e) The number of athletes who attended the Sydney 2000 Olympic Games was 10 651.

(f) About 100 million million atoms could fit on the point of a household pin.

(g) The Sun is approximately 150 000 000 km from Earth. The temperature at the surface of the Sun is about 6000°C.

(h) Australia has the highest rate of skin cancer in the world. More than 140 000 cases of skin cancer are reported each year in Australia.

15 The nervous system controls movement in the body. Electrical impulses travel through nerves to the muscles causing them to contract. Here is a graph showing the time it takes for a nerve to transmit an impulse. Use the graph to answer the following questions, expressing all times in seconds using scientific notation.



Events that characterise the transmission of a nerve impulse.

(a) How long does it takes for a nerve to return to resting potential after it has been stimulated?

(b) How long does it takes for a nerve to reach hyperpolarisation after it has reached the threshold potential?

(c) How long does the action potential last?

(d) How long does the refractory period last?

16 **(a)** A personal computer in 1982 required approximately 1000 nanoseconds (1 microsecond) to perform an operation. Modern computers require between 0.3 and 1 nanosecond to perform an operation. How much faster are modern computers than the computers used in 1982? Express this range in seconds using scientific notation.

(b) The RAM of a personal computer in 1982 had a response time of 200 nanoseconds. The RAM of modern computers has a response time of between 5 and 10 nanoseconds. How much faster is the response time of the RAM in modern computers compared with computers in 1982? Express this range in seconds using scientific notation.

Hint

Express numbers in scientific notation before you compare them.

17

Timescale	Event
4570 mya	Planet Earth first formed
4440 mya	Earth cooled enough to form oceans and the atmosphere
3800 mya	First life on Earth: single-celled bacteria (microbial mat)
2400 mya	First photosynthesis by plants to produce oxygen
1700–1200 mya	Multicellular plants and fungi (sea dwellers)
850–630 mya	Ice age 'snowball Earth'—extinction event
580 mya	Multicellular animals: *Charnia* (filter feeders such as sponges; sea dwellers)
550 mya	First creatures capable of movement: trilobites (first arthropod)
530 mya	First predators with mouth, teeth and stomach: echinoderms (*Arkarua*)
490–476 mya	Plants and invertebrates on land: arthropods (insects, giant dragonfly)
370 mya	Vertebrate animals on land
251 mya	Extinction event
230 mya	Dinosaurs
150 mya	First birds (type of dinosaur)
130–90 mya	Flowering plants and social insects (bees, termites, ants)
65 mya	Extinction of the dinosaurs (except birds)
6 mya	Humans (upright apes)
200 000	Modern humans

Note: mya means millions of years ago.

Above is a time line of the development of life on Earth. Use this time line to answer the following questions. Write all answers in scientific form.

(a) What is the age of the Earth?

(b) How long after the Earth cooled and the atmosphere developed did the first life on Earth appear?

(c) The ability of single-celled bacteria to create oxygen changed the Earth's atmosphere and allowed multicellular plants to develop. When did photosynthesis begin?

(d) For how long was the Earth completely covered in snow and ice (snowball Earth)?

(e) What was the time span over which multicellular plants and fungi developed?

(f) Remains of the first predator, an echinoderm, have been found in South Australia and named *Arkarua*. When did this animal first evolve?

(g) How long after invertebrates first moved out of the water and onto land did land vertebrates evolve?

(h) For how long was the Earth inhabited by dinosaurs?

(i) How many years before the first humans appeared on Earth did the first birds evolve?

(j) Extinction events are events that have extinguished almost all life on Earth. Recent extinction events are thought to have been due to volcanic eruptions or collisions with meteorites producing dust storms that blocked out the Sun, causing plants to die and disrupting the entire food chain. When were the last two extinction events?

(k) What is the time span from the formation of the Earth to the beginning of modern humans?

18 Light travels at a speed of about 3×10^8 m/s.

(a) Given that speed $= \frac{\text{distance}}{\text{time}}$, how far does light travel in:

(i) 2 minutes **(ii)** 6 weeks **(iii)** 1 year (365.25 days)? (This is called a *light year*.)

Express your answers in scientific form and in metres.

(b) If the mean distance of the Earth from the Sun is 1.496×10^8 km, approximately how long does light from the Sun take to reach Earth? Express your answer in minutes, correct to one decimal place.

19 Neil Armstrong sent messages back from the Moon on 20 July 1969. Radio waves travel at about 3.0×10^8 m/s. If the Moon is 384 000 km away from Earth, how long did it take, in seconds, for a message to travel from the Moon to Earth?

$\left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$

20 The thickness of a sheet of paper is approximately 2×10^{-4} m and the thickness of a sheet of cellophane is approximately 2×10^{-3} m.

(a) Which is thinner?

(b) Find the difference in thickness between the two sheets.

Reasoning, communicating and open-ended

21 Two bacteria cultures, one containing 6.2×10^{11} cells and the other 3.5×10^9 cells, are combined in the one flask.

(a) How many cells are there in the flask?

(b) If the bacteria population doubles in size every 12 hours, how many cells will there be after 3 days?

(c) Find the value of your answer to **(b)**.

22 The planet Venus is approximately 108 million km from the Sun.

(a) Express this quantity in scientific notation.

(b) Write this distance in metres using scientific rotation.

(c) If the orbit of Venus around the Sun is assumed to be circular, find the distance, in kilometres, covered by Venus in one orbit. Answer in scientific notation.

(d) Earth is about 1.5×10^{11} m from the Sun. Find the closest distance, in metres, between Venus and Earth. Answer in scientific notation.

23 Estimate the diameter of a hair from your head and write your answer in scientific notation.

24 Josh wrote:

$3 \times 10 = 30^2$

$= 900$

Explain to Josh why he is incorrect.

25 Estimate the number of red blood cells in your body and write your answer in scientific notation. (Children typically have 3.8–5.5 million red blood cells per microlitre of blood. Children weighing 40–60 kg would have approximately 2.7–4.5 L of blood.)

Problem solving

Stairway to the Moon

A standard piece of paper is approximately 0.1 mm thick.

If you cut it in half and place the two halves on top of each other, you now have a 'stack' of two pieces 0.2 mm thick. If you now cut the stack in half and place one half on top of the other, you have four pieces 0.4 mm thick.

The Moon is 363 100 km from the Earth at its closest point. How many 'cut 'n' stacks' will you need to do to have a pile of paper that reaches all the way to the Moon? (Assuming, of course, you are able to cut the stack as many times as you like.)

Strategy options

- Act it out.
- Make a table.
- Look for a pattern.

2.5 Rearranging formulas

Stage 5.3

Need to Know

A **formula** is an **equation** showing the relationship between two or more variables or unknowns. It is written with one variable on the left-hand side of the equation and all other variables on the right-hand side. The variable on the left-hand side is called the subject of the formula.

When the variable required is not the subject of the formula, we need to rearrange the equation. The formula for finding the velocity, v, of an object after accelerating over a distance d is $v^2 = u^2 + 2ad$. We can rearrange this to make u, a or d the subject; that is, we can rearrange the formula to the form $u = \ldots$, $a = \ldots$ or $d = \ldots$.

We need to use the correct order of operations to rearrange the formula correctly.

Alternatively, the values could be substituted into the original formula and the equation solved in the usual way.

Sometimes before the required variable can become the subject of the formula, the equations need to be undone with operations such as squaring both sides or taking the square root or reciprocal of both sides.

Worked Example 14

WE14

The formula for finding the velocity, v, of an object after accelerating over a distance, d, is $v^2 = u^2 + 2ad$.

(a) Rearrange the formula to make a the subject.

(b) Find a when $v = 5$, $u = 1$ and $d = 2$.

Working

(a)

$$v^2 = u^2 + 2ad$$
$$v^2 - u^2 = u^2 + 2ad - u^2$$
$$v^2 - u^2 = 2ad$$
$$\frac{v^2 - u^2}{2d} = \frac{2ad}{2d}$$
$$\frac{v^2 - u^2}{2d} = a$$
$$a = \frac{v^2 - u^2}{2d}$$

(b)

$$a = \frac{v^2 - u^2}{2d}$$
$$a = \frac{5^2 - 1^2}{2 \times 2}$$
$$a = \frac{25 - 1}{4}$$
$$a = \frac{24}{4}$$
$$a = 6$$

Worked Example 15 WE15

Rearrange the formula in each of the following to make the variable in brackets the subject of the formula.

(a) $A = \pi r^2$ (r)

(b) $T = 2\pi\sqrt{\frac{l}{g}}$ (l)

Working

(a) $A = \pi r^2$

$\frac{A}{\pi} = r^2$

$r = \sqrt{\frac{A}{\pi}}$

(b) $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$

$\frac{T^2}{(2\pi)^2} = \frac{l}{g}$

$l = \frac{T^2 g}{4\pi^2}$

2.5 Rearranging formulas

Answers page 426

Stage 5.2

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q10, Q13	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13	Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: calculator for Questions **5**, **6** and **7**

Fluency and understanding

WE14

1 (a) (i) Rearrange the formula to make k the subject if $P = mk$.

(ii) Find k when $P = 32$ and $m = -4$.

(b) (i) Rearrange the formula to make b the subject if $y = mx + b$.

(ii) Find b when $y = 9$, $m = 5$ and $x = 2$.

(c) (i) Rearrange the formula to make h the subject if $C = ah - g$.

(ii) Find h when $C = 19$, $a = -24$ and $g = -7$.

(d) (i) Rearrange the formula to make x the subject if $a = mx + c$.

(ii) Find x when $a = -21$, $m = 4$ and $c = 3$.

(e) (i) Rearrange the formula to make p the subject if $k = \frac{mp - 2}{n}$.

(ii) Find p when $k = 42$, $m = -4$ and $n = 5$.

(f) (i) Rearrange the formula to make w the subject if $u = \frac{w}{v} - xy$.

(ii) Find w when $u = 6$, $x = -3$, $y = 7$ and $v = -2$.

WE15

2 Rearrange the formula in each of the following to make the variable in brackets the subject of the formula. (Assume all variables are positive.)

(a) $V = \pi r^2 h$ (r) **(b)** $E = \frac{1}{2}mv^2$ (v) **(c)** $F = \frac{dh^2}{k}$ (h)

(d) $k = 7\sqrt{\frac{m}{n}}$ (m) **(e)** $d = 2\sqrt{\frac{b}{c}}$ (b) **(f)** $m = x\sqrt{\frac{y}{z}}$ (y)

3 Rearrange each of the following to make x the subject.

(a) $x + y = z$ **(b)** $x - 3a = 2b$ **(c)** $kx - m = n$

(d) $dx + c = f$ **(e)** $rt - x = p$ **(f)** $vy - x = -w$

(g) $\frac{2x + y}{p} = k$ **(h)** $\frac{t + rx}{w} = m$ **(i)** $\frac{mp + px}{m} = n$

(j) $cdx + e = gh$ **(k)** $axp - k = mn$ **(l)** $s - tux = vw$

(m) $b + \frac{xd}{y} = f$ **(n)** $\frac{mx}{n} + kp = t$ **(o)** $ab - \frac{dx}{c} - f = o$

4 The formula for finding the velocity, v, of a moving object is $v^2 = u^2 + 2as$. A correct rearrangement of the formula would be:

A $u^2 = v^2 + \frac{2as}{2}$ **B** $s = \frac{v^2 - u^2}{2a}$ **C** $a = \frac{2s}{v^2 - u^2}$ **D** $a = \frac{u^2 - v^2}{2s}$

5 The formula for finding the velocity, v m/s, of an object starting with a velocity of u m/s and undergoing a constant acceleration of a m/s^2 over time t s is $v = u + at$.

(a) Rearrange the formula to make t the subject.

(b) Find the value of t when:

(i) $a = 14$ m/s^2, $u = 20$ m/s and $v = 76$ m/s

(ii) $a = 2.4$ m/s^2, $u = 5.7$ m/s and $v = 12.9$ m/s.

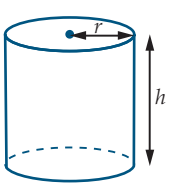
6 The circumference (C) of a circle is related to the radius (r) by the formula $C = 2\pi r$. Use the calculator value for π and give your answer correct to two decimal places.

(a) Rearrange the formula to make r the subject.

(b) Find the value of r when:

(i) $C = 628$ **(ii)** $C = 16.4$

7 The formula for the volume (V) of a cylinder is $V = \pi r^2 h$. Rearrange the formula to find h when $r = 5$ and $V = 157$. Use the calculator value for π and give your answer correct to two decimal places.



8 The perimeter of a rectangle is given by $P = 2(l + w)$, where l and w represent the length and width of the rectangle, respectively.

Rearrange the formula to make w the subject. Find the width of the rectangle if the length is 23 m and the perimeter is 84 m.

Reasoning, communicating and open-ended

9 The kinetic energy (E) of an object, in joules, is found using the formula $E = \frac{1}{2}mv^2$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Determine E when $m = 2$ kg and $v = 1.2$ m/s.

(b) Determine m when $E = 162$ joules and $v = 9$ m/s.

(c) Determine v when $E = 1.96 \times 10^6$ joules and $m = 8$ kg.

(d) Explain why there are two answers for part **(c)** and what each answer means.

10 The density (D) of a substance is related to its mass (M) and its volume (V) by the formula $D = \frac{M}{V}$.

(a) Find the difference in the mass of two objects made of the same material with a density of 7 g/cm^3 if one has a volume of 12 cm^3 and the other has a volume of 15 cm^3.

(b) A flotation device is to be made of an outer material with a density of 1.8 g/cm^3 and a mass of 2.4 kg. It is filled with an inner material of density 0.9 g/cm^3 and a mass of 7.8 kg. Will the device float in sea water of density 1.025 g/cm^3?

Hint

Find the total mass and the total volume.

11 The area of a trapezium is given by the formula $A = \frac{(a+b)h}{2}$.

(a) Rearrange the formula to make b the subject.

(b) Use this rearranged formula to find the value of b if $a = 9$, $h = 6$ and $A = 66$.

(c) We could substitute these values for a, h and A into the original formula and solve the equation $66 = \frac{(9+b) \times 6}{2}$ to find the value of b. Explain why we would rearrange the formula to find the value of b.

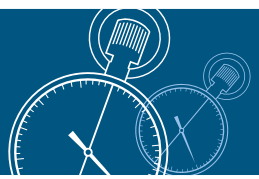
12 Consider this formula: $S = \frac{n}{2}[2a + (n-1)d]$

Expand all the brackets in this expression and use this form to explain why it is difficult to make n the subject.

13 Ruth has been rearranging the formula for the area of a trapezium, $A = \frac{(a+b)h}{2}$. She has produced the following working while trying to make a the subject. Where has she made a mistake? Now rearrange the formula to make b the subject.

$$A = \frac{h(a+b)}{2}$$
$$2A = h(a+b)$$
$$\frac{2A}{h} = a + b$$
$$a = b - \frac{2A}{h}$$

Half-time 2



Stage 5.1 Q1–7 **Stage 5.2 Q8–10**

Ex 2.1

1 Simplify each of the following, expressing your answer with positive indices.

(a) $3x^3y \times 4x^4y$ **(b)** $\dfrac{8x^3y^5}{2xy^2}$ **(c)** $\dfrac{3m^4n^2 \times 2m^5n^3}{8m^3n^4}$

Ex 2.4

2 Write the following numbers correct to three significant figures.

(a) 0.2999 **(b)** 80 **(c)** 62 078

Ex 2.2

3 Simplify each of the following, expressing your answer with positive indices.

(a) $(3x^4y^5)^2$ **(b)** $\left(\dfrac{a^4b^3}{6ab^4}\right)^2$ **(c)** $\dfrac{5^3x^4y^6}{3x^3y^2} \times \dfrac{x^6y}{x^7y^5}$

Ex 2.4

4 Express each quantity described below in scientific notation.

(a) The Moon is 384 000 km away.

(b) 500 mL of water is lost through the skin daily.

Ex 2.4

5 To how many significant figures have each of the following been written?

(a) 36 050 000 **(b)** 0.000 000 852 **(c)** 0.0830

Ex 2.1

6 Simplify the following, leaving your answer in index form.

(a) $\dfrac{(2^3)^4 \times (5^6)^2}{(2^5) \times (5^2)^4}$ **(b)** $\dfrac{(3^2 \times 5^4)^2}{25 \times 27}$ **(c)** $\dfrac{8^3 \times 9^2}{6^4}$

Ex 2.4

7 Express each quantity described below as a number.

(a) The number of bacteria that will fit on a pin head is approximately 1.1×10^{14}.

(b) The nucleus of a leaf cell is about 4.4×10^{-6} m in diameter.

Ex 2.3

8 Simplify each of the following, expressing your answer with positive indices:

(a) $\dfrac{(p^2q^3)^{-1} \times p^5q^{-2}}{p^3q^2}$ **(b)** $\dfrac{(c^{-3}d^{-4})^2 \times c^2d^6}{c^5d^3}$

(c) $\dfrac{4(a^2b)^3 \times 3a^{-5}b^{-2}}{6ab}$ **(d)** $\dfrac{6(mn^3)^2 \times 4m^{-4}n^{-1}}{3m^2n}$

Ex 2.3

9 Express each of the following with positive indices.

(a) x^{-5} **(b)** $3p^{-4}$ **(c)** $5m^{-3}n$ **(d)** $6p^{-4}q^{-1}r^2$

Ex 2.5

10 Rearrange the formula $z = 2xy - 3y$ to make x the subject and then find the value of x when $z = 7$ and $y = 5$.

Expanding the brackets

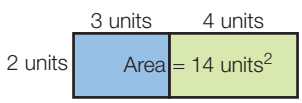
2.6

Stage 5.2

Need to Know

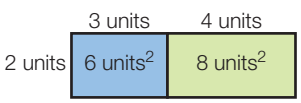
The distributive law

Brackets are used to change the order of operations, allowing addition or subtraction to be done before multiplication or division. The expression is written without brackets by multiplying every term inside the brackets by the number outside the brackets. This is called expanding the brackets.



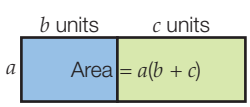
$2(3+4) = 2 \times 7$
$= 14$

and

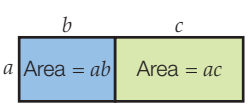


$2(3+4) = 2 \times 3 + 2 \times 4$
$= 6 + 8$
$= 14$

If variables replace numbers, the expression is rewritten without the brackets the same way.



$a(b+c)$ =



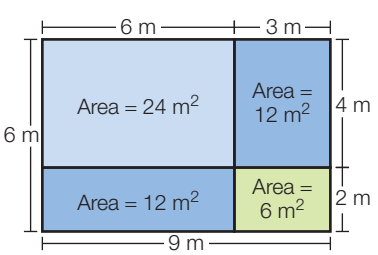
$ab + ac$

This is known as the **distributive law**. Every term inside the brackets is multiplied by the term outside the brackets.

$a(b+c) = ab + ac$

Binomial products

Consider a room 6 metres long and 4 metres wide. If it is extended by 3 metres in length and 2 metres in width, as in the diagram, find the new area.



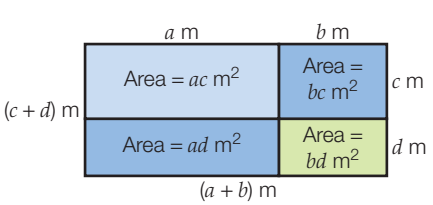
The total area can be found in two ways:

$(6+3)(4+2) = 9 \times 6$
$= 54 \text{ m}^2$

or

$6 \times 4 + 6 \times 2 + 3 \times 4 + 3 \times 2$
$= 24 + 12 + 12 + 6$
$= 54 \text{ m}^2$

If a room a metres long and c metres wide has been extended by b metres in length and d metres in width, we can find the new area using the diagram.



Area $= (a+b)(c+d)$
$= a \times c + a \times d + b \times c + b \times d$
$= ac + ad + bc + bd \text{ m}^2$

A **binomial expansion** is the result of multiplying two terms in one set of brackets by two terms in another set of brackets. Every term in the second bracket is multiplied by every term in the first bracket.

$$(a + b)(c + d) = ac + ad + bc + bd$$

Binomial expansion can be completed using FOIL.

First: Multiply the first terms in each bracket.

Outside: Multiply the outside terms in each bracket.

Inside: Multiply the inside terms in each bracket.

Last: Multiply the last terms in each bracket.

Often after expansion, like terms can be collected and the expression simplified, leaving three terms called a quadratic trinomial.

Worked Example 16

WE16

Expand:

(a) $5(p + 4)$

(b) $2m(4m - 3n)$

Working

(a) $5(p + 4) = 5 \times p + 5 \times 4$
$= 5p + 20$

(b) $2m(4m - 3n) = 2m \times 4m + 2m \times -3n$
$= 8m^2 - 6mn$

Worked Example 17

WE17

Expand and simplify:

(a) $4a(a - 3) - 2(a + 5)$

(b) $3(2 - b) - (4 - b)$

Working

(a) $4a(a - 3) - 2(a + 5)$
$= 4a \times a + 4a \times (-3) + (-2) \times a + (-2) \times 5$
$= 4a^2 - 12a - 2a - 10$
$= 4a^2 - 14a - 10$

(b) $3(2 - b) - (4 - b)$
$= 3(2 - b) - 1(4 - b)$
$= 3 \times 2 + 3 \times (-b) + (-1) \times 4 + (-1) \times (-b)$
$= 6 - 3b - 4 + b$
$= 2 - 2b$

Worked Example 18 WE18

Expand each of the following expressions.

(a) $(c+4)(d+2)$ **(b)** $(2m-4)(3n-5)$ **(c)** $(2p-3r)(3t+4)$

Working

(a) $(c+4)(d+2)$
$= c \times (d+2) + 4 \times (d+2)$
$= c \times d + c \times 2 + 4 \times d + 4 \times 2$
$= cd + 2c + 4d + 8$

(b) $(2m-4)(3n-5)$
$= 2m \times (3n-5) + (-4) \times (3n-5)$
$= 2m \times 3n + 2m \times (-5) + (-4) \times 3n + (-4) \times (-5)$
$= 6mn - 10m - 12n + 20$

(c) $(2p-3r)(3t+4)$
$= 2p \times (3t+4) + (-3r) \times (3t+4)$
$= 2p \times 3t + 2p \times 4 + (-3r) \times 3t + (-3r)$
$= 6pt + 8p - 9rt - 12r$

Worked Example 19 WE19

Expand each of the following expressions. Simplify where possible.

(a) $(x-2)(x+5)$ **(b)** $(t-3)(t-7)$ **(c)** $(4x+3)(5x-2)$

Working

(a) $(x-2)(x+5)$
$= x \times x + x \times 5 + (-2) \times x + (-2) \times 5$
$= x^2 + 5x - 2x - 10$
$= x^2 + 3x - 10$

(b) $(t-3)(t-7)$
$= t \times t + t \times (-7) + (-3) \times t + (-3) \times (-7)$
$= t^2 - 7t - 3t + 21$
$= t^2 - 10t + 21$

(c) $(4x+3)(5x-2)$
$= 4x \times 5x + 4x \times (-2) + 3 \times 5x + 3 \times (-2)$
$= 20x^2 - 8x + 15x - 6$
$= 20x^2 + 7x - 6$

Answers page 427

2.6 Expanding the brackets

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q6 Column 1, Q7, Q8, Q9, Q10, Q14, Q15	Q1 Column 2, Q2 Column 1, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6 Column 2, Q7, Q8, Q9, Q10, Q14, Q16	Q1 Column 3, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16

Fluency and understanding

WE16

1 Expand each of the following expressions.

(a) $2(x+5)$ **(b)** $4(a+9)$ **(c)** $8(b+3)$

(d) $8(x-2)$ **(e)** $9(x-2)$ **(f)** $3(k-4)$

(g) $-7(x+1)$ **(h)** $-4(d+8)$ **(i)** $-6(x+3)$

(j) $-9(c-6)$ **(k)** $-2(x-5)$ **(l)** $-5(x-6)$

(m) $x(x+3)$ **(n)** $m(m+5)$ **(o)** $x(x+9)$

(p) $-x(x-8)$ **(q)** $-x(x-4)$ **(r)** $-n(n-9)$

(s) $3x(x+2)$ **(t)** $2u(u-6)$ **(u)** $5x(x-1)$

(v) $-2x(x+7)$ **(w)** $-7q(q+1)$ **(x)** $-9b(b-6)$

Hint

Remember $-(x+2)$ really means $-1(x+2)$.

WE17

2 Expand and simplify each expression.

(a) $3(x-2)+2(x+4)$ **(b)** $2(y+1)+3(y-2)$

(c) $4(2x-3)-(6x-1)$ **(d)** $3(4a+5)-(7a+2)$

(e) $a(2a+3)-2(3a-5)$ **(f)** $3x(x+2)-4(x-1)$

(g) $7y(y-3)-5(y+2)$ **(h)** $6k(2k+3)-3k(k-3)$

WE18

3 Expand each of the following.

(a) $(x+6)(y+5)$ **(b)** $(x+4)(y+7)$ **(c)** $(c+d)(e+3)$

(d) $(r+5)(t-8)$ **(e)** $(a+2)(b-1)$ **(f)** $(p+3)(q-2)$

(g) $(a-3)(y-4)$ **(h)** $(m-2)(n-6)$ **(i)** $(k-5)(m-4)$

(j) $(4p+q)(m+n)$ **(k)** $(7x+y)(a+k)$ **(l)** $(3p+2r)(4s+t)$

(m) $(2a+1)(b-3)$ **(n)** $(3x-1)(x+2)$ **(o)** $(3m+2)(n-1)$

(p) $(2a-3b)(5m-3n)$ **(q)** $(5c-2d)(3x-4y)$ **(r)** $(7p-2q)(2r-3t)$

WE19

4 Expand each of the following and simplify.

(a) $(a+3)(a+2)$ **(b)** $(c+5)(c+3)$ **(c)** $(y+7)(y+6)$

(d) $(x-3)(x+1)$ **(e)** $(y-2)(y+9)$ **(f)** $(a-7)(a+12)$

(g) $(b-2)(b-1)$ **(h)** $(x-4)(x-3)$ **(i)** $(p-8)(p-7)$

(j) $(3m+1)(2m+3)$ **(k)** $(4d+5)(2d+3)$ **(l)** $(3k+5)(7k+4)$

(m) $(4y-3)(2y+1)$ **(n)** $(5k+3)(2k-7)$ **(o)** $(2a+5)(4a-3)$

(p) $(6a-5)(2a-1)$ **(q)** $(4p-3)(3p-5)$ **(r)** $(3r-2)(8r-7)$

5 Expand each of the following and simplify.

(a) $(x + y)(x + y)$ **(b)** $(m + n)(m + n)$ **(c)** $(a + b)(a + b)$

(d) $(p - r)^2$ **(e)** $(c - d)^2$ **(f)** $(g - h)^2$

(g) $(2c + 3d)(2c + 3d)$ **(h)** $(3m + 2n)(3m + 2n)$ **(i)** $(5a + 3b)(5a + 3b)$

(j) $(5x - 4y)(5x - 4y)$ **(k)** $(2j - 3k)(2j - 3k)$ **(l)** $(7c - 5d)^2$

6 Expand each of the following and simplify.

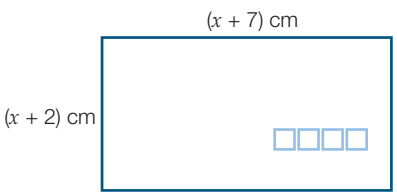
(a) $(m + n)(m - n)$ **(b)** $(p + q)(p - q)$ **(c)** $(a - b)(a + b)$

(d) $(3a + 2b)(3a - 2b)$ **(e)** $(4x - 3y)(4x + 3y)$ **(f)** $(7m - 3n)(7m + 3n)$

7 When simplified, $5m(-3 + 2n) - n(10m - 1)$ is:

A $n - 15m$ **B** $10n - 15m - 1$ **C** $-15m - 20mn + n$ **D** $15m + n$

8 Consider this envelope.



(a) Write the expression (in factorised form) for the area of this envelope.

(b) Use your answer to find the area when $x = 10$.

(c) What value would x represent if the area of the envelope is 104 cm^2? Try substituting different values for x until you find the appropriate one.

(d) Expand the expression you obtained for the area in part **(a)**.

(e) Check that the expanded expression is correct by calculating the area when $x = 10$ and comparing this with the answer obtained in part **(b)**.

9 $(3x - 2)(3x + 2)$ is the same as:

A $6x^2 + 4$ **B** $9x^2 - 12x - 4$ **C** $9x^2 + 6x - 4$ **D** $9x^2 - 4$

10 A vegetable bed, 15 m long and 3 m wide, is to be extended by x metres in both length and width.

(a) Write expressions for each of the following.

(i) The new length of the garden bed.

(ii) The new width of the garden bed.

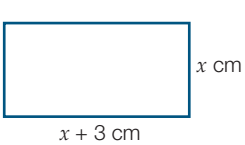
(iii) The new area of the garden bed as the product of factors.

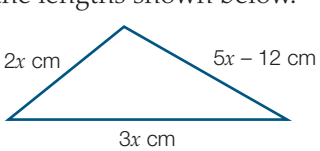
(iv) The new area of the garden bed as the expanded product of factors.

(b) Compare the area of the new garden bed with the area of the original garden bed. By how much has the area increased?

(c) Use your answer from part **(b)** to calculate how much extra area of garden bed is available if both the length and width are increased by 2 m.

11 The sides of a rectangle and a triangle have the lengths shown below.





(a) Write the expression for the area of the rectangle using the given lengths.

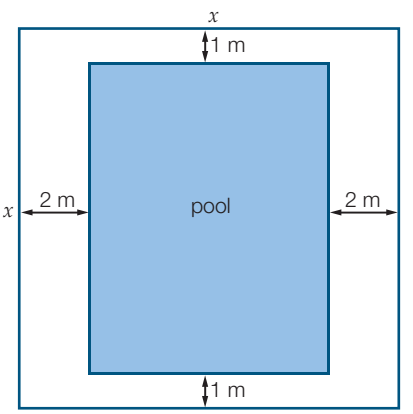
(b) Write the expression for the perimeter of the triangle using the given lengths.

(c) Find two values for x so that the area of the rectangle is the same as the perimeter of the triangle.

Reasoning, communicating and open-ended

12 There are a number of ways to check that $x(x+2) = x^2 + 2x$ is a true statement. Choose two of these methods and show clearly how you are able to check that $x(x+2) = x^2 + 2x$ is true. Can you use these methods to show that $a(b+c) = ab + ac$? If so, show how.

13 A backyard, square in shape, is to have an in-ground rectangular swimming pool constructed in it. A path around the pool is to be laid as shown in the diagram.



(a) Write expressions for each of the following:

(i) the total area of the backyard

(ii) the width of the swimming pool

(iii) the length of the swimming pool

(iv) the area of the swimming pool.

(b) Using your answers to parts **(i)** and **(iv)**, write an expression for the area of the path. State your answer in simplest expanded form.

14 Choose two variables to represent the length and width of a rectangle. Increase each dimension by a fixed amount (e.g. increase the length by 2 units and the width by 3 units). Find the area of the new rectangle.

15 Three students gave the following incorrect answers when expanding $-3(5x - 2)$.

(a) $-15x - 2$ **(b)** $5x + 6$ **(c)** $-8x + 5$

Explain the errors made by each student. What is the correct answer?

16 Three students obtained the following incorrect answers when expanding $(x - 3)(x + 4)$.

(a) $x^2 + 7x + 12$ **(b)** $2x - 12$ **(c)** $x^2 + x - 7$

Explain what errors were made by each student. What is the correct answer?

Factorising using common factors

2.7

Stage 5.2

Need to Know

To **factorise** an expression is to write it as a product of its factors.

The opposite or inverse of expanding is called factorising.

Expanding	Factorising
$a(b + c) = ab + ac$	$ab + ac = a(b + c)$

Factorising using the highest common factor (HCF)

- Place the HCF outside brackets.
- Divide each term by the **HCF**; this quotient goes inside the brackets.
- If the HCF is not taken out as a factor, the expression is not fully factorised (e.g. $4x + 8 = 2(2x + 4)$ is not fully factorised).
- A negative sign may be taken out as well as the HCF; be careful to change signs inside the brackets (e.g. $-5x + 10 = -5(x - 2)$).

This procedure is called 'taking out the highest common factor'. The HCF may be any positive number, a variable or the product of a number and one or more variables. Once an expression is factorised, no matter what method has been used, it will have brackets in it.

Worked Example 20

WE20

Factorise each of the following.

(a) $5x + 20$

(b) $y^2 - y$

(c) $-9y - 6z + 3$

(d) $3a^2bc - acd$

(e) $-8mn^2 - 12m^2n$

(f) $a(b + 3) + 2(b + 3)$

Working

(a) $5x + 20 = 5(x + 4)$

(b) $y^2 - y = y(y - 1)$

(c) $-9y - 6z + 3 = -3(3y + 2z - 1)$

(d) $3a^2bc - acd = ac(3ab - d)$

(e) $-8mn^2 - 12m^2n = -4mn(2n + 3m)$

(f) $a(b + 3) + 2(b + 3) = (b + 3)(a + 2)$

Answers page 428

2.7 Factorising using common factors

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q9, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 1, Q5, Q6, Q8, Q9, Q10, Q12, Q13	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13

Fluency and understanding

WE20

1 Factorise each of the following.

(a) $3x + 9$ **(b)** $12y + 24$ **(c)** $6 + 42a$

(d) $2k - 6$ **(e)** $4m - 20$ **(f)** $8b - 4$

(g) $7x - xy$ **(h)** $4s - 9st$ **(i)** $3a - ab$

(j) $15y + 5w + 10x$ **(k)** $2a + 4b - 8c$ **(l)** $3x - 15y + 9$

(m) $3ac + 2abc$ **(n)** $5xy^2 + 2x^2y$ **(o)** $9f^2gh - 12fg^2$

(p) $6gh^2 + 18g^2h$ **(q)** $16x^2y - 40xy^2$ **(r)** $18ab - 42a^2b$

(s) $-4m - 20$ **(t)** $-7d - 49$ **(u)** $-12r - 60$

(v) $x(a + 3) + 5(a + 3)$ **(w)** $m(n - 2) + 9(n - 2)$ **(x)** $3(x + 5) + y(x + 5)$

2 Factorise each of the following.

(a) $10x + 25$ **(b)** $16y + 36$ **(c)** $14a - 35$

(d) $12d - 18$ **(e)** $18 - 27m$ **(f)** $36 - 15k$

(g) $24abc - 10b + 7bc$ **(h)** $5km - 3kn + 2ghk$ **(i)** $c + 4bc + 2ac$

(j) $x^2 + 3x$ **(k)** $y^2 + 6y$ **(l)** $5k + k^2$

(m) $8m - 3m^2$ **(n)** $4p^2 - 7p$ **(o)** $9a^2 - 5a$

(p) $6d - 9d^2$ **(q)** $22g - 14g^2$ **(r)** $24a - 32a^2$

3 Factorise each of the following by taking out the highest negative common factor.

(a) $-2m - 12$ **(b)** $-4k - 24$ **(c)** $-5h - 25$

(d) $-x^2 - x$ **(e)** $-y^2 - 3y$ **(f)** $-p^2 - 4p$

(g) $-16a + 4$ **(h)** $-21b + 7$ **(i)** $-30r + 6$

(j) $-24p + 40$ **(k)** $-18w + 16$ **(l)** $-45k - 63$

(m) $-15x^2 - 36x$ **(n)** $-14y^2 - 49y$ **(o)** $-20r^2 - 44r$

(p) $-8d - 4f - 6g$ **(q)** $-15a - 10b - 5c$ **(r)** $-14c - 28d - 7f$

(s) $-12x^2 - 16x + 20xy + 4xz$ **(t)** $-2km + 4km^2 - 6k - 8k^2$ **(u)** $-10b^2 + 4bc - 8bc^2 + 16ab$

4 Factorise each of the following.

(a) $4(y - 1) + w(y - 1)$ **(b)** $5(m - 3) + n(m - 3)$ **(c)** $5(p - 2) + r(p - 2)$

(d) $p(q + 5) - 2(q + 5)$ **(e)** $y(2x + 3) - (2x + 3)$ **(f)** $a(3p + 5) - (3p + 5)$

(g) $d(f - 1) - 6(f - 1)$ **(h)** $5d(d - 2) - 4(d - 2)$ **(i)** $3m(m - 6) - 7(m - 6)$

5 When factorised, the expression $3ab^2c - 6abc + 3bc$ is:

A $3bc(ab - 2a)$ **B** $3abc(b - 2 + 3a)$

C $3bc(ab - 2a + 1)$ **D** $6bc(ab - a + 2bc)$

6 When fully factorised, the expression $-2x(3m - 4) + 5y(3m - 4)$ is:

A $(3m - 4)(5y - 2x)$ **B** $(5y - 4)(3m - 2x)$

C $(-3m + 4)(-2x + 5y)$ **D** $(-2x - 4)(3m + 5y)$

7 **(a)** Write the expression for 'Eight times the product of two numbers added to twelve times one of the numbers'.

(b) Factorise the expression in **(a)**.

8 A rectangular pizza is of length d cm and width $(c + 2)$ cm. Another rectangular pizza of the same width has a length of 5 cm. The two pizzas are joined to make a longer pizza.

(a) Write the area of the new shape in factorised form.

(b) What is the relationship between c and d if the pizza is square?

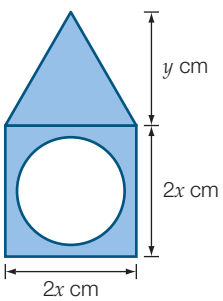
9 A floor-to-ceiling window of width x m is to be installed in a wall 5 m long and $(y + 3)$ m high. The remaining part of the wall is to be plastered.

(a) Find the length of the wall to be plastered after the window is installed.

(b) Find the area of the wall to be plastered after the window is installed.

Reasoning, communicating and open-ended

10 A wood template is to be made for the front of a letter box with the dimensions shown.



(a) Write an expression that would show the area of timber used in the construction before the circular opening is cut out.

(b) Express this answer in a factorised form.

(c) If the diameter of the circular cut out needs to be 2 cm less than the width, find the area of the timber cut out for the opening.

(d) Write this in expanded form.

(e) Show that the total area of timber that will be used is $(4 - \pi)x^2 + (y + 2\pi)x - \pi$ cm^2.

(f) What area of timber would be wasted if the template was cut from a rectangular piece of timber with the smallest possible dimensions?

11 A lasagne is baked in a tray $(3p + 2)$ cm long and $(2p - 1)$ cm wide. A slice, the full width of the tray and $(r + 2)$ long is taken from the lasagne.

(a) Find the area of the lasagne, before the slice was removed, in factorised form.

(b) Find the area of the slice taken from the lasagne in factorised form.

(c) Use your answers to parts **(a)** and **(b)** to find the area remaining after the slice has been removed. Write your answer in simplest factorised form.

(d) How could you find the area in part **(c)** more simply? Show your working.

12 Make up three different expressions whose highest common factor is $3x$.

13 In a test, students were asked 'Factorise the expression $-5x + 10$'. Three students wrote these answers; one answer was correct, the other two answers were incorrect. Identify the correct answer and explain the mistakes the other two students made.

A $-5(x + 2)$ **B** $-5(x + 10)$ **C** $-5(x - 2)$

Problem solving

Divisibility dilemma?

The number 123 456 is divisible by 2, 3 and 4. Find the smallest number that is divisible by all the digits from 2 to 9 (inclusive) if it begins with the digits 123 456.

Strategy options

- Test all possible combinations.
- Guess and check.
- Break the problem into manageable parts.

2.8 Further factorisation

Stage 5.2

Need to Know

Factorising by grouping in pairs

If there are four terms in an expression and there is no common factor for all four terms, we can sometimes group the expression in pairs of terms so that there is a common factor in each pair. If this gives us another common factor in brackets, we can factorise further. This technique is called **grouping in pairs**. Sometimes you may have to consider which two terms to pair together.

Factorising quadratic trinomials

Expanding some products gives quadratic expressions when simplified. Because expanding and factorising are opposite operations, the expanding process can be reversed to convert these trinomials back into factor form. The middle term in the trinomial can be split into two terms and then factorised by grouping in pairs as long as the product of the coefficients of the two middle terms equals the last term.

For example:

$x^2 + 9x + 14$ is converted to $x^2 + 2x + 7x + 14$ because $2 + 7 = 9$ and $2 \times 7 = 14$.

Worked Example 21 WE21

Factorise the following.

(a) $ab + 5b + 3a + 15$

(b) $3x + 12 - xy - 4y$

Working

(a)
$$\begin{aligned} & ab + 5b + 3a + 15 \\ &= b(a + 5) + 3(a + 5) \\ &= (a + 5)(b + 3) \end{aligned}$$

(b)
$$\begin{aligned} & 3x + 12 - xy - 4y \\ &= 3(x + 4) - y(x + 4) \\ &= (x + 4)(3 - y) \end{aligned}$$

Worked Example 22 WE22

Factorise $df + 6g - 6f - dg$.

Working

$$\begin{aligned} df + 6g - 6f - dg &= df - 6f - dg + 6g \\ &= f(d - 6) - g(d - 6) \\ &= (d - 6)(f - g) \end{aligned}$$

Worked Example 23

WE23

For each of the following expressions, **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 5x + 6x + 30$ **(b)** $p^2 - 4p - 4p + 16$ **(c)** $a^2 - 12a + 12a - 144$

Working

(a) **(i)** $x^2 + 5x + 6x + 30 = x(x + 5) + 6(x + 5)$
$= (x + 5)(x + 6)$

(ii) $x^2 + 5x + 6x + 30 = x^2 + 11x + 30$

(iii) $x^2 + 11x + 30 = (x + 5)(x + 6)$

(iv) $5 \times 6 = 30$

The product of the coefficients of the two middle terms gives the last term.

(b) **(i)** $p^2 - 4p - 4p + 16 = p(p - 4) - 4(p - 4)$
$= (p - 4)(p - 4)$
$= (p - 4)^2$

(ii) $p^2 - 4p - 4p + 16 = p^2 - 8p + 16$

(iii) $(p - 4)^2 = p^2 - 8p + 16$

(iv) $-4 \times -4 = 16$

The product of the coefficients of the two middle terms gives the last term.

(c) **(i)** $a^2 - 12a + 12a - 144 = a(a - 12) + 12(a - 12)$
$= (a - 12)(a + 12)$

(ii) $a^2 - 12a + 12a - 144 = a^2 - 144$

(iii) $a^2 - 144 = (a - 12)(a + 12)$

(iv) $-12 \times +12 = -144$

The product of the coefficients of the two middle terms gives the last term.

Worked Example 24

WE24

Factorise the following trinomials by splitting the middle term.

(a) $x^2 + 9x + 14$ **(b)** $x^2 - 7x + 12$

Working

(a) $x^2 + 9x + 14$
$= x^2 + 2x + 7x + 14$
$= x(x + 2) + 7(x + 2)$
$= (x + 2)(x + 3)$

(b) $x^2 - 7x + 12$
$= x^2 - 3x - 4x + 12$
$= x(x - 3) - 4(x - 3)$
$= (x - 3)(x - 4)$

Answers page 428

2.8 Further factorisation

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q12, Q13, Q14	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

WE21

1 Factorise the following.

(a) $xy + 4y + 3x + 12$ **(b)** $pq + q + 5p + 5$ **(c)** $mn + 2n + 9m + 18$

(d) $ab + bf + ad + df$ **(e)** $pq + qt + pr + rt$ **(f)** $km + 3nk + 6m + 18n$

(g) $np - 7n + p - 7$ **(h)** $ab - 5b + a - 5$ **(i)** $mn - 2n + 7m - 14$

(j) $ab - 6b - 4a + 24$ **(k)** $x^2 + 2x - 8x - 16$ **(l)** $y^2 + 4y - 6y - 24$

WE22

2 Factorise the following.

(a) $2p + 3k + kp + 6$ **(b)** $mn + pq + np + mq$ **(c)** $cd + 12 + 6c + 2d$

(d) $ad - hk - dk + ah$ **(e)** $xy - 8 - 4y + 2x$ **(f)** $eg - 3 - g + 3e$

(g) $bc - 1 + c - b$ **(h)** $xy - 3 + 3x - y$ **(i)** $mk - 3n + 3m - nk$

WE23

3 For each of the following expressions, **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 7x + 2x + 14$ **(b)** $j^2 + 3j + 5j + 15$ **(c)** $z^2 + z + 8z + 8$

(d) $x^2 + 2x - 8x - 16$ **(e)** $y^2 + 4y - 6y - 24$ **(f)** $x^2 + 3x - 4x - 12$

(g) $a^2 - 4a + 5a - 20$ **(h)** $k^2 - 2k + 5k - 10$ **(i)** $m^2 - 7m + 4m - 28$

(j) $y^2 - 3y - 7y + 21$ **(k)** $c^2 - 9c - 5c + 45$ **(l)** $d^2 - 8d - 12d + 96$

(m) $m^2 + 3m + 3m + 9$ **(n)** $r^2 + 6r + 6r + 36$ **(o)** $k^2 + 11k + 11k + 121$

(p) $p^2 - 5p - 5p + 25$ **(q)** $d^2 - 7d - 7d + 49$ **(r)** $n^2 - 8n - 8n + 64$

(s) $c^2 - 3c + 3c - 9$ **(t)** $t^2 - 2t + 2t - 4$ **(u)** $b^2 - 9b + 9b - 81$

WE24

4 Factorise the following trinomials by splitting the middle term.

(a) $x^2 + 14x + 24$ **(b)** $x^2 + 11x + 24$ **(c)** $x^2 - 10x + 24$

(d) $x^2 + 8x + 12$ **(e)** $x^2 - 7x + 12$ **(f)** $x^2 - 13x + 12$

(g) $x^2 + 8x + 15$ **(h)** $x^2 + 11x + 18$ **(i)** $x^2 + 10x + 16$

(j) $x^2 - 2x - 24$ **(k)** $x^2 + 5x - 24$ **(i)** $x^2 - 10x - 24$

5 By taking out the common factor from $k(j - 1) + (j - 1)$ we obtain:

A $(j - 1)(k + 1)$ **B** $(j + 1)(k - 1)$ **C** $(j - 1)(k)$ **D** $(j + 1)(k + 1)$

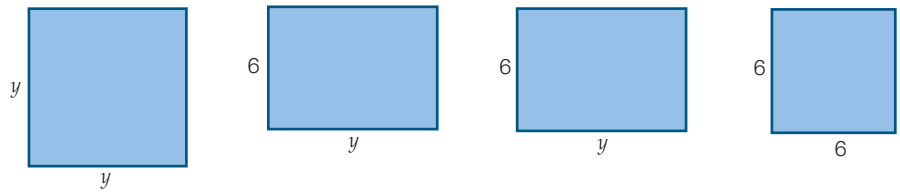
6 Square a number, x, subtract four times x, add 12 and subtract three times x.

(a) Write the expression for the information given.

(b) Simply the expression in **(a)**.

(c) Factorise the expression in **(a)**.

7



(a) Write the expression for the total area of the four shapes above.

(b) Factorise the expression in **(a)**.

(c) Draw a diagram to show how the four shapes can fit together to make a square (include dimensions on your diagram).

Reasoning, communicating and open-ended

8 **(a)** Factorise $ab + ac + ad + 7b + 7c + 7d$ by grouping in threes. That is, take out a common factor from three terms at a time.

(b) Show that, if $b = a$ and $c + d = 7$, the expression is a perfect square.

9 **(a)** Add five different positive numbers to $x^2 + 10x$ so that it factorises differently each time.

(b) Factorise each of the new expressions you have created in part (a).

(c) Explain why there are only five possible positive numbers that can be added in part (a).

(d) Explain why there are an infinite number of negative numbers that could be added to the expression so creating an infinite number of quadratic trinomials that have $x^2 + 10x$ as the first two terms.

10 **(a)** Factorise $35y^2 - 21y$.

(b) If this represents the area of a rectangle in cm^2 and the factors represent the width, write the dimensions of the rectangle.

(c) If an area of $15y - 9$ cm^2 is added to the area in part **(a)**, find the combined area in factorised form.

(d) Write the dimensions of the new rectangle if the factors in part **(c)** represent the length and width.

11 A trapezium has an area of $\frac{3x^2 + 2xy - 6x - 4y}{2}$ m^2 where x and y are both positive numbers and $x > 2$.

(a) Write the area in factorised form.

(b) If the smaller factor represents the height of the trapezium, what is the sum of the lengths of the two parallel non-equal sides?

12 Without expanding the factorised expression in the following, add a term to:

(a) $x^2 - 5x$ so that it factorises as $(x + 4)(x - 9)$

(b) $x^2 - 5x$ so that it factorises as $(x - 2)(x - 3)$

(c) $x^2 + 6$ so that it factorises as $(x + 1)(x + 6)$.

13 **(a)** Find two terms that, when added to $x^2 + ax$, can be factorised by grouping in pairs.

(b) Factorise your expression in **(a)**.

14 **(a)** Draw a rectangle of length l and width w. Extend the length by a certain amount and the width by a different amount. Use your diagram to write the area of the rectangle in two different ways.

(b) Draw a square of side s. Extend one pair of opposite sides by a certain amount and the pair of opposite sides by a different amount. Use your diagram to write the area of the new rectangle in two different ways.

(c) Draw a square of side s and extend the sides by the same amount to form a new square. Use your diagram to find the area of the new square in two different ways.

2.9 Expanding and factorising special products

Stage 5.3§

Need to Know

There are two special products formed from the expansion of $(a + b)(c + d)$.

Expanding perfect squares

The square of a sum or a difference of two terms is called a **perfect square**. Perfect squares are expanded in exactly the same way as other binomial products.

Square of a sum

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

or

Square of a difference

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Perfect square rules:

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Note that the only difference in these two rules is the sign of the middle term and that this sign is the same as the sign inside the brackets.

Expanding the difference of two squares

The **difference of two squares** is another useful rule found when the sum of two terms is multiplied by the difference of those terms.

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ab - b^2\\ &= a^2 - b^2\end{aligned}$$

Difference of two squares rule:

$(a + b)(a - b) = a^2 - b^2$

As factorising is the opposite of expansion, the special product rules can be used in reverse to factorise special products.

Factorising perfect squares

$a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$

Factorising the difference of perfect squares

$a^2 - b^2 = (a + b)(a - b)$

Worked Example 25 WE25

Expand each of the following expressions using a perfect square rule.

(a) $(p+4)^2$ **(b)** $(d-3)^2$ **(c)** $(2f+5)^2$ **(d)** $(3m-2n)^2$

Working

(a) $(p+4)^2 = p^2 + 2 \times p \times 4 + 16$
$= p^2 + 8p + 16$

(b) $(d-3)^2 = d^2 - 2 \times d \times 3 + 9$
$= d^2 - 6d + 9$

(c) $(2f+5)^2 = (2f)^2 + 2 \times 2f \times 5 + 25$
$= 4f^2 + 20f + 25$

(d) $(3m-2n)^2$
$= (3m)^2 - 2 \times 3m \times 2n + 2n \times 2n$
$= 9m^2 - 12mn + 4n^2$

Worked Example 26 WE26

Use the difference of two squares rule to expand the following.

(a) $(x+3)(x-3)$ **(b)** $(5-k)(5+k)$

(c) $(3x+2)(3x-2)$ **(d)** $(4m-5n)(4m+5n)$

Working

(a) $(x+3)(x-3) = x^2 - 3^2$
$= x^2 - 9$

(b) $(5-k)(5+k) = 5^2 - k^2$
$= 25 - k^2$

(c) $(3x+2)(3x-2) = (3x)^2 - (2)^2$
$= 9x^2 - 4$

(d) $(4m-5n)(4m+5n) = (4m)^2 - (5n)^2$
$= 16m^2 - 25n^2$

Worked Example 27 WE27

Factorise the following using a perfect square rule.

(a) $x^2 + 6x + 9$ **(b)** $x^2 - 10x + 25$ **(c)** $4x^2 - 12x + 9$

Working

(a) $x^2 + 6x + 9$
$= x^2 + 2 \times 3x + 3^2$
$= (x+3)^2$

(b) $x^2 - 10x + 25$
$= x^2 - 2 \times 5x + 5^2$
$= (x-5)^2$

(c) $4x^2 - 12x + 9$
$= (2x)^2 - 2 \times 2x \times 3 + 3^2$
$= (2x-3)^2$

Worked Example 28 WE28

Factorise the following using the difference of two squares rule.

(a) $x^2 - 4$ (b) $x^2 - 25$ (c) $16x^2 - 9$

Working

(a) $x^2 - 4$
$= x^2 - 2^2$
$= (x + 2)(x - 2)$

(b) $x^2 - 25$
$= x^2 - 5^2$
$= (x + 5)(x - 5)$

(c) $16x^2 - 9$
$= (4x)^2 - 3^2$
$= (4x + 3)(4x - 3)$

Answers page 430

2.9 Expanding and factorising special products

Navigator

Stage 5.3§

Q1 Column 1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11 Column 1, Q13 (a), Q14, Q16	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Column 1 & 2, Q13, Q14, Q16	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16

Fluency and understanding

WE25

1 Expand each of the following expressions using a perfect square rule.

(a) $(d - 1)^2$ (b) $(m + 5)^2$ (c) $(k + 2)^2$
(d) $(p - 3)^2$ (e) $(x - 8)^2$ (f) $(y - 12)^2$
(g) $(m + n)^2$ (h) $(w - k)^2$ (i) $(6 - c)^2$
(j) $(y + 6)(y + 6)$ (k) $(x + 9)(x + 9)$ (l) $(k - 7)(k - 7)$
(m) $(4y + 3)^2$ (n) $(2w + 9)^2$ (o) $(3x + 5)^2$
(p) $(6k - 1)^2$ (q) $(7 - 2a)^2$ (r) $(3 - 8d)^2$
(s) $(1 - 4c)(1 - 4c)$ (t) $(6 + 7y)(6 + 7y)$ (u) $(4g - h)(4g - h)$
(v) $(3a + d)^2$ (w) $(2a - 3b)^2$ (x) $(5m + 8n)^2$

WE26

2 Use the difference of two squares rule to expand the following.

(a) $(c - 5)(c + 5)$ (b) $(a + 4)(a - 4)$ (c) $(k + 8)(k - 8)$
(d) $(9 + x)(9 - x)$ (e) $(3 - y)(3 + y)$ (f) $(1 - k)(1 + k)$
(g) $(m - n)(m + n)$ (h) $(x - y)(x + y)$ (i) $(c + d)(c - d)$
(j) $(v + w)(v - w)$ (k) $(p + q)(p - q)$ (l) $(h - g)(h + g)$
(m) $(2m - 1)(2m + 1)$ (n) $(3x + 4)(3x - 4)$ (o) $(6b - 7)(6b + 7)$
(p) $(1 + 5y)(1 - 5y)$ (q) $(8 - 5a)(8 + 5a)$ (r) $(3 - 2d)(3 + 2d)$
(s) $(4a + 5c)(4a - 5c)$ (t) $(3m - 2n)(3m + 2n)$ (u) $(2x - 7y)(2x + 7y)$
(v) $(4 + ab)(4 - ab)$ (w) $(3 - cd)(3 + cd)$ (x) $(5 + jk)(5 - jk)$

3 Factorise the following using a perfect square rule. WE27

(a) $x^2 + 4x + 4$ **(b)** $y^2 - 16y + 64$ **(c)** $9b^2 - 30b + 25$

(d) $t^2 - 24t + 144$ **(e)** $x^2 + 12x + 36$ **(f)** $4p^2 + 36p + 81$

(g) $m^2 + 20m + 100$ **(h)** $9s^2 + 24s + 16$ **(i)** $25h^2 - 30h + 9$

(j) $4f^2 - 20f + 25$ **(k)** $c^2 - 4cd + 4d^2$ **(l)** $49n^2 - 28kn + 4k^2$

4 Factorise the following using a perfect square rule. WE28

(a) $c^2 - 25$ **(b)** $k^2 - 100$ **(c)** $36m^2 - 49$

(d) $h^2 - 36$ **(e)** $m^2 - 81$ **(f)** $9y^2 - 121$

(g) $n^2 - 16$ **(h)** $v^2 - 144$ **(i)** $16c^2 - 25d^2$

(j) $4p^2 - 25$ **(k)** $36h^2 - k^2$ **(l)** $25m^2 - 9n^2$

5 $(a + 5)^2$ is the same as:

A $a^2 + 5a + 10$ **B** $a^2 + 10a + 25$ **C** $a^2 + 10a + 10$ **D** $a^2 + 25$

6 $(2k - 7)^2$ is the same as:

A $2k^2 - 28k + 49$ **B** $4k^2 + 28k + 49$ **C** $4k^2 - 49$ **D** $4k^2 - 28k + 49$

7 When expanded, $(2 - 4x)(2 + 4x)$ is:

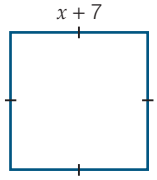
A $4 - 16x^2$ **B** $4 + 8x$ **C** $4 - 8x^2$ **D** $4 - 8x$

8 When expanded, $(ab + 3cd)(ab - 3cd)$ is:

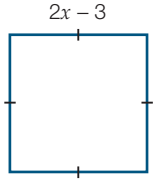
A $ab - 9cd$ **B** $a^2b^2 + 9c^2d^2$ **C** $a^2b^2 - 9c^2d^2$ **D** $a^2b^2 + 6c^2d^2$

9 **(a)** Find the area, in terms of x, of each of these squares. Expand your expressions.

(i)



(ii)



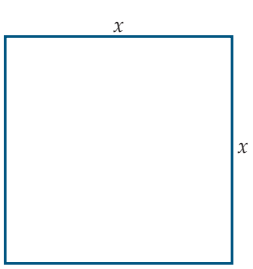
(b) Calculate the area of each when $x = 4$ cm.

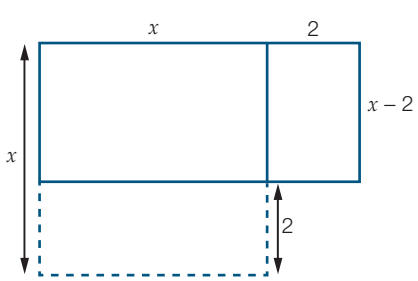
10 A square playing field is to be changed to a rectangular shape by adding 2 m to the length and subtracting 2 m from the width.

(a) Write an expression for the area of the square playing field.

(b) Write an expression for the area of the new rectangular playing field.

(c) Which playing field has the larger area? How much more area is available?





Reasoning, communicating and open-ended

11 Because 16 can be written as $(10 + 6)$ or $(20 - 4)$, 16^2 can be written as $(10 + 6)^2$ or $(20 - 4)^2$. Evaluate the following, without a calculator, using your knowledge of special products.

(a) 23^2 **(b)** 49^2 **(c)** 112^2

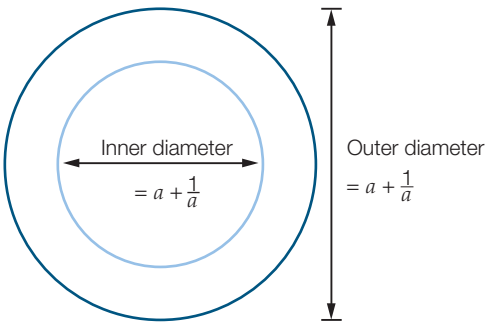
(d) 3.8^2 **(e)** 6.01^2 **(f)** 11.96^2

(g) 102×98 **(h)** 2.3×1.7 **(i)** 6.04×5.96

12 A steel washer has the dimensions shown.

(a) Find the area of one side of the washer by using your knowledge of the expansion of special products.

(b) What can you say about the area of the washer for any value of a where $a > 0$?



13 **(a)** Find the shaded area using:

Area of a trapezium $= \frac{1}{2}(a + b)h$

(b) Find the shaded area using:

Area of a triangle $= \frac{1}{2}bh$

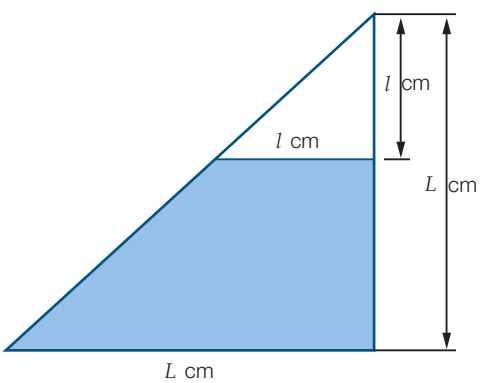
(c) Write an equation using part **(a)** and part **(b)**.

(d) If $L = 5l$, find the value of the shaded area:

(i) using your answer to part **(a)**

(ii) using your answer to part **(b)**.

(e) Show that the shaded area is 96% of the area of the large triangle when $L = 5l$.



14 Find three values for a and b so that $a^2 - b^2$ produces a square number.

15 Add a middle term to $x^2 + 16$ so that it will factorise as a perfect square.

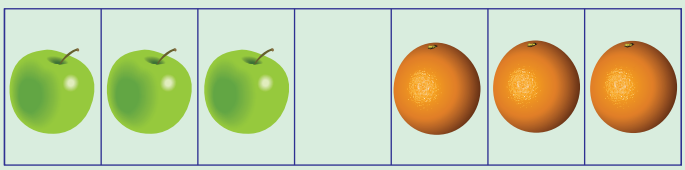
16 Explain why an expression that contains two terms where both are squares cannot be factorised unless the sign between the two squares is a subtraction sign.

Problem solving

Oranges and apples

The aim is to end up with the oranges and apples in opposite positions. You can make two types of moves:

- You can slide a piece of fruit one square into an empty space.
- You can hop a piece of fruit over a different type of fruit.



1 What is the minimum number of moves needed so that the oranges are on the left and the apples are on the right?

2 If there were four or five pieces of each fruit, how many moves would it take?

Strategy options

- Solve a simpler problem.
- Act it out.
- Look for a pattern.

Challenge 2

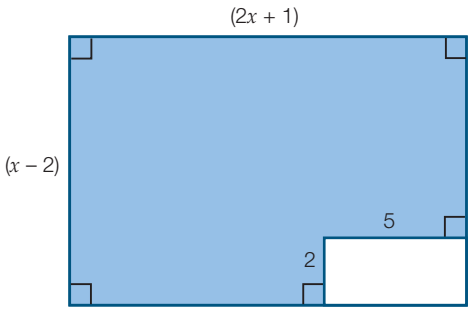


1 Write two numbers that have a sum of 27 and a product of 72.

2 Bridget once said, 'The day before yesterday I was 15 and next year I will be 18'. When is Bridget's birthday and on what day did she say this?

3 How many perfect cubes are there between 2000 and 10 000?

4 Part of a rectangle is shaded as shown in the diagram.



(*Note*: Diagram not to scale.)

Which expression gives the area of the shaded part?

A $2x^2 - x - 8$ **B** $2x^2 - x - 12$ **C** $2x^2 - 3x - 8$ **D** $2x^2 - 3x - 12$

5 If ♦ represents a digit, find the value of ♦ if $[3(170 + \blacklozenge)]^2 = 272\,48\blacklozenge$.

6 a, b and c are positive whole numbers. Show that $(a - b)(b - c)(c - a)$ is divisible by 2.

7 Is the number $3^{404} + 4^{495}$ divisible by 5? Show why or why not.

8 Steven thinks of four different numbers. The sums Steven gets by pairing the numbers are 6, 7, 9, 9, 11 and 12. What are the four numbers?

9 A formula used in the science of optics is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

(a) Make f the subject of the formula.

(b) Make v the subject of the formula.

10 **(a)** Show that the difference between any two-digit number and the number formed by reversing the digits is divisible by 9. (Hint: let the two-digit number be $10a + b$.)

(b) The difference between any three-digit number and the number formed by reversing its digits is obtained. By what number is it divisible?

(c) The difference between any four-digit number and the number formed by reversing its digits is obtained. By what number is it divisible?

Chapter review 2

D.I.Y. Summary

Key Words

Stage 5.1	expanded form	index form	scientific notation
	index	significant figures	variable
Stage 5.2	base	expand	highest common factor
	binomial expansion	expression	quadratic trinomial
	distributive law	factorise	reciprocal
Stage 5.3	difference of two squares	grouping in pairs	perfect square

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

Stage 5.1 Q1 **Stage 5.2 Q2, 3** **Stage 5.3 Q4**

1 If we write a number in _____, then it has a number between 1 and 10 multiplied by a positive or a negative power of 10.

2 If an expression is factorised to obtain the product of two brackets that are exactly the same, it is called a _____.

3 To _____ an expression is to remove brackets. The reverse operation is to _____ the expression and put it into brackets.

4 An expression of the form $x^2 - y^2$ is called the _____.

Stage 5.1 Q1, 2, 4, 5, 14, 15, 20 **Stage 5.2 Q3, 6–11, 17, 18, 19** **Stage 5.3 Q12, 13, 16, 21**

Fluency and understanding

1 Simplify each of the following. Ex 2.1

(a) $a^3b^5 \times a^2b^2$ (b) $m^4n^7 \times m^3n$ (c) $\dfrac{p^4q^6}{p^3q^9}$

(d) $\dfrac{(x^5)^2 \times x^8}{(x^4)^4}$ (e) $\dfrac{(x^2)^3 \times (y^4)^2}{xy^3 \times x^5}$ (f) $\dfrac{c^2d^4}{c^4d^2} \times \dfrac{c^6d}{(cd)^2}$

2 $(2x^3y^2)^3$ is the same as: Ex 2.2

A $2x^3y^6$ B $8x^3y^6$ C $2x^9y^6$ D $8x^9y^6$

3 Express each of the following with positive integers. Ex 2.3

(a) $-5m^{-1}$ (b) a^2b^{-6} (c) $x^{-2}y^{-3}$ (d) $a^{-4}b^{-9}$

4 Write each of the following in scientific notation. Ex 2.4

(a) 0.000 000 000 000 002 mm (b) 0.000 15 kg

(c) 320 000 000 L (d) 480 000 000 000 000 km

5 Write each of the following as a basic numeral. Ex 2.4

(a) 5.4×10^7 (b) 8.273×10^{-4}

(c) 1.4×10^{-5} (d) 7.162×10^3

6 Rearrange each of the following to make x the subject.

Ex 2.5

(a) $z = \dfrac{3 - xy}{2}$ **(b)** $x + 2a = 4x$

(c) $s = xt + 8t^2$ **(d)** $\dfrac{2 + 4z}{3x} = 10$

7 Expanded and simplified, $2c(3b - 4) - 3b(-5 + 2c)$ is:

Ex 2.6

A $15b - 8c$ **B** $21b + 8c$ **C** $12bc - 12c$ **D** $7b + 14c$

8 Expand:

Ex 2.6

(a) $4(x + 7)$ **(b)** $x(x - 3)$ **(c)** $-5x(2x - 1)$

9 **(a)** Expand and simplify:

Ex 2.6

(i) $4(2x - 3) + 5(x + 1)$ **(ii)** $3x(6x + 5) - 2(3 - 2x)$

(b) Find the value of each expression when x has the value 4.

10 Expand and simplify:

Ex 2.6

(a) $(x + 8)(x + 2)$ **(b)** $(2a + 5)(3a - 4)$

11 Factorise:

Ex 2.7

(a) $8a + 12$ **(b)** $-15k - 20k^2$

(c) $5p^2q - 3pq + 2pq^2 - pqr$ **(d)** $-24a - 6ab - 72$

12 Expand and simplify:

Ex 2.9

(a) $(x + 12)^2$ **(b)** $(5c - 2d)^2$ **(c)** $(4x - 3)(4x + 3)$

13 Factorise by grouping in pairs.

Ex 2.8

(a) $ab + 4b + 3a + 12$ **(b)** $ab - 2b + a - 2$

(c) $ab - 6a - 5b + 30$ **(d)** $ab - 7 + 7b - a$

14 Express the following in scientific form.

(a) A 10-cent coin is about 0.0015 m thick.

(b) Pluto is approximately 5900 million km from the Sun.

15 Express each of the following as a basic numeral.

(a) The distance from the Sun to Jupiter is approximately 7.78×10^8 km.

(b) The temperature at the centre of the Sun is believed to be about 1.5×10^7 °C.

16 The length of a greeting card is 20 cm and the width is $(5 + x)$ cm.

(a) Draw a diagram of the card.

(b) Write an expression for the perimeter.

(c) Factorise the expression obtained in part **(b)**.

(d) Write an expression for the area (in factorised form).

(e) Expand the expression obtained in part **(d)**.

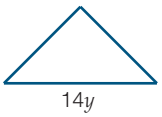
(f) Check that the two expressions obtained for the area are equivalent by substituting a value for x.

17 If the area of a rectangular postage stamp is $4x^2 + 12x$, what is the width of the stamp if the height is $2x$?

18 The mass of an object can be determined by finding the product of its density and its volume. If the density can be represented by $(m - 2)$ and the volume by $(k + 8)$, write an expression for the mass.

Suggest possible values that m and k could take to give a mass of 48.

19 Find the height of this triangular support structure if the base is $14y$ and the area enclosed by the beams is $35y^2 - 21y$.



Reasoning and communicating

20 The heart pumps 7000 L of blood daily.

(a) Express this quantity in scientific notation.

(b) How many litres of blood are pumped by the heart:

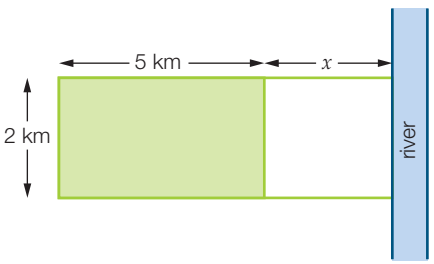
(i) each week

(ii) each second, to one decimal place

(iii) in 3 years (assuming no leap years)?

(c) Would this measurement be exact and the same for each person?

21 The Greenwood family had a rectangular block of land 2 km by 5 km for grazing sheep, as seen in the diagram. They then bought the land adjoining this leading down to the river. Depending on the time of year and the weather conditions, the width of the river can vary, so the land available for grazing sheep changes.



(a) Write an expression for the total area available for grazing sheep at any particular time. (Assume the land remains rectangular.)

(b) Suppose there are $\frac{1}{2}x$ sheep per km^2 of land. How many sheep are there altogether? Write your answer in expanded form.

(c) Fourteen sheep are moved to another location.

(i) Write the expression for the number of sheep remaining.

(ii) Expand $(x + 7)(x - 2)$. How is this related to the expression in **(i)**?

(iii) Explain what happens when $x = 2$ km.

NAPLAN practice 2

Numeracy: Calculator allowed

1 Which of the following numbers is the largest?

A 3.2×10^{18} **B** 9.6×10^{3} **C** 3.2×10^{-18} **D** 9.6×10^{-3}

2 What is the value of $7(x - 9)$, when $x = 15$?

3 Factorise the expression $14x + 21$.

Numeracy: Non-calculator

4 Which one of the following expressions is equivalent to $3(5x - 2)$?

A $15x - 1$ **B** $15x - 6$ **C** $15x + 1$ **D** $15x + 6$

5 A rule for y in terms of x is $y = 12 - 3x$. Write an equivalent rule for x in terms of y.

6 Which one of the following expressions is a fully factorised form of $9xy + 12y$?

A $3(3xy + 4y)$ **B** $9y(x + 2)$ **C** $y(9x + 12)$ **D** $3y(3x + 4)$

7 When the expression $-3x^2y + 6xy$ is fully factorised, the result is:

A $3xy(2 - x)$ **B** $-3xy(2 - x)$ **C** $-3xy(x - y)$ **D** $-3xy(x + y)$

3 Measurement

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Calculate the following.

(a) 7.21×10 **(b)** 18.21×1000

(c) $1571.23 \div 100$ **(d)** $0.74 \div 10\,000$

2 Copy and complete the following conversions.

(a) 4.2 cm = _____ mm **(b)** 3.9 m = _____ cm

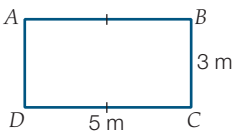
(c) 7.1 km = _____ m **(d)** 450 mm = _____ m

(e) 18.6 cm = _____ m **(f)** 5060 m = _____ km

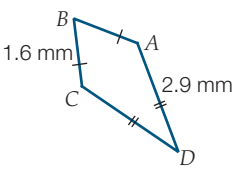
(g) 1100 cm = _____ km **(h)** 3720 mm = _____ km

3 What is the length of AB in each of the following diagrams? Where necessary, give your answers to one decimal place.

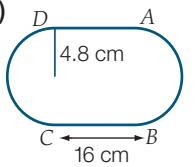
(a)



(b)

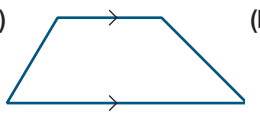


(c)

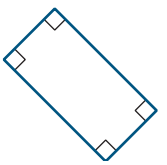


4 Match the shape to its name.

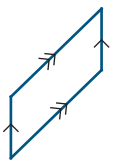
(a)



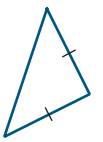
(b)



(c)



(d)

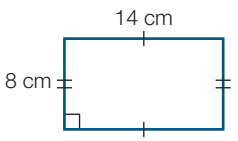


A rectangle **B** isosceles triangle

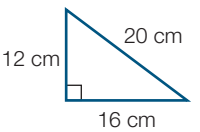
C parallelogram **D** trapezium

5 Calculate the perimeter and area of each of these shapes to the nearest whole number:

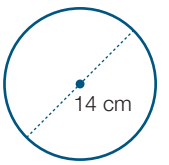
(a)



(b)



(c)



Syllabus references

Measurement and Geometry: Length

- ✓ Investigate the relationship between features of circles, such as the circumference, radius and diameter; use formulas to solve problems involving circumference **(ACMMG197) (Stage 4)**
- ✓ Find perimeters of parallelograms, trapeziums, rhombuses and kites **(ACMMG196) (Stage 4)**

Measurement and Geometry: Area and Surface Area

- ✓ Calculate the areas of composite shapes **(ACMMG216) (Stage 5.1)**
- ✓ Solve problems involving the surface areas of right prisms **(ACMMG218) (Stage 5.1)**
- ✓ Calculate the surface areas of cylinders and solve related problems **(ACMMG217) (Stage 5.2)**
- ✓ Solve problems involving surface area for a range of prisms, cylinders and composite solids **(ACMMG242) (Stage 5.2)**

Measurement and Geometry: Volume

- ✓ Solve problems involving the volumes of right prisms **(ACMMG218) (Stage 5.2)**
- ✓ Solve problems involving volume for a range of prisms, cylinders and composite solids **(ACMMG242) (Stage 5.2)**

Perimeter

3.1

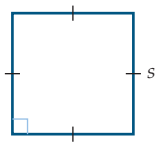
Stage 4

Need to Know

The **perimeter** of a shape is the total length of its boundary, measured in units such as cm or km.

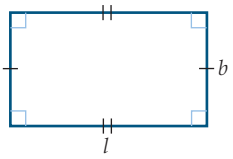
Perimeter formulas

Square



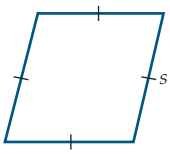
$P = 4s$

Rectangle



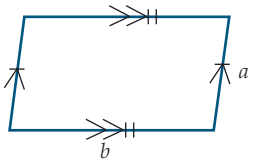
$P = 2l + 2b$ or $P = 2(l + b)$

Rhombus



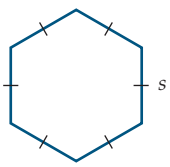
$P = 4s$

Parallelogram



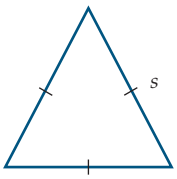
$P = 2a + 2b$ or $P = 2(a + b)$

Regular hexagon



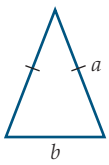
$P = 6s$

Equilateral triangle



$P = 3s$

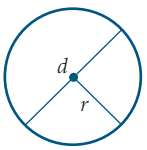
Isosceles triangle



$P = 2a + b$

Circle formulas

The perimeter of a circle is called the **circumference**.



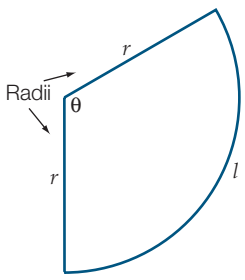
Circumference of a circle

$C = \pi d$ or $C = 2\pi r$ where d = diameter and r = radius.

A **sector** of a circle is the area contained within two radii and the arc that joins them. An **arc** is part of the circumference of a circle. The distance along the arc of a sector (l) is in proportion to the angle **subtended** by (standing on) the arc at the centre of the circle.

Arc length of a sector

$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle measured in degrees subtended at the centre.



Perimeter of a sector

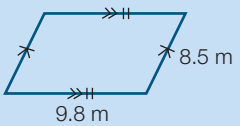
$P = l + 2r$

Worked Example 1

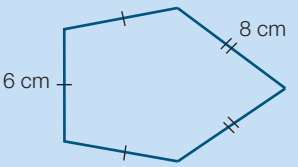
WE1

Find the perimeter of these shapes.

(a)



(b)



Working

(a) $P = 2(a + b)$
$P = 2(8.5 + 9.8)$
$= 2 \times 18.3$
$P = 36.6$ m

(b) $P = 8 + 8 + 6 + 6 + 6$
$= 2 \times 8 + 3 \times 6$
$= 34$ cm

Worked Example 2

WE2

(a) Find the diameter of a circle with a circumference of 8.5 cm, correct to one decimal place.

(b) Find the radius of a circle with a circumference of 40 mm, correct to one decimal place.

Working

(a) $C = \pi d$

$\frac{C}{\pi} = d$

$\frac{8.5}{\pi} = d$

$d = 2.705\ldots$
$d = 2.7$ cm (1 d.p.)

(b) $C = 2\pi r$

$\frac{C}{2\pi} = r$

$\frac{40}{2\pi} = r$

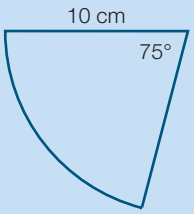
$r = 6.366\ldots$
$r = 6.4$ mm (1 d.p.)

Worked Example 3

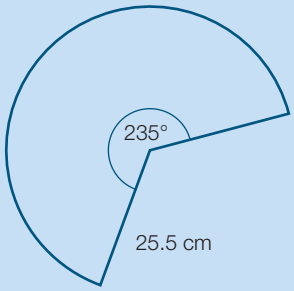
WE3

Calculate **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

(a)



(b)



Working

(a) (i) $l = \frac{\theta}{360} \times 2\pi r$

$r = 10$ cm, $\theta = 75°$

$l = \frac{75}{360} \times 2 \times \pi \times 10$

$= \frac{75}{\cancel{360}_{18}} \times \cancel{20}^{1} \times \pi$

$= \frac{\cancel{75}^{25}\pi}{\cancel{18}_{6}}$

$= \frac{25\pi}{6}$

$= 13$ cm (nearest whole number)

(ii) $P = l + 2r$

$r = 10$ cm, $l = \frac{25\pi}{6}$ cm

$P = 2 \times 10 + \frac{25\pi}{6}$

$= 33$ cm (nearest whole number)

(b) (i) $l = \frac{\theta}{360} \times 2\pi r$

$r = 25.5$ cm, $\theta = 235°$

$l = \frac{235}{360} \times 2 \times \pi \times 25.5$

$= \frac{235}{\cancel{360}_{180}} \times \cancel{2}^{1} \times \pi \times 25.5$

$= \frac{5992.5\pi}{180}$

$= 105$ cm (nearest whole number)

(ii) $P = l + 2r$

$r = 25.5$ cm, $l = \frac{5992.5\pi}{180}$ cm

$P = 2 \times 25.5 + \frac{5992.5\pi}{180}$

$= 156$ cm (nearest whole number)

Answers page 432

3.1 Perimeter

Stage 4

Navigator

Q1, Q2, Q3, Q4, Q5 (a)–(e), Q6, Q7, Q10, Q12	Q1, Q2, Q3 (a)–(d), Q5, Q6, Q7, Q8, Q10, Q12, Q13	Q1 Column 2, Q3 (d)–(f), Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14

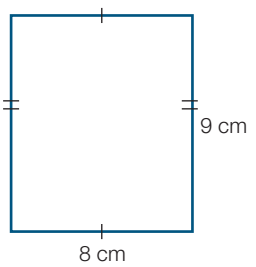
Equipment required: tape measure or ruler for Question **12**

Fluency and understanding

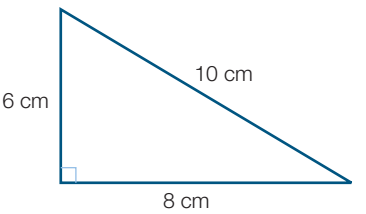
WE1

1 Find the perimeters of the following shapes using the measurements given.

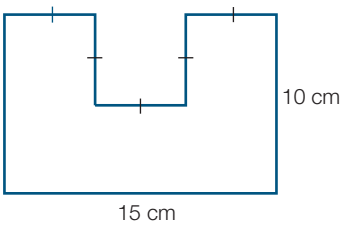
(a)



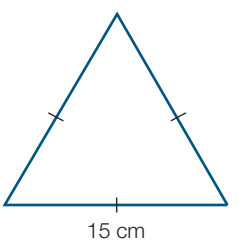
(b)



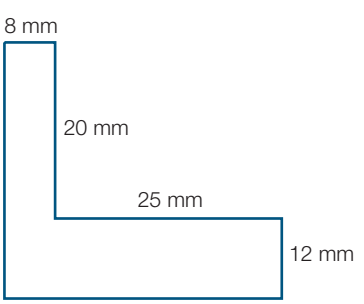
(c)



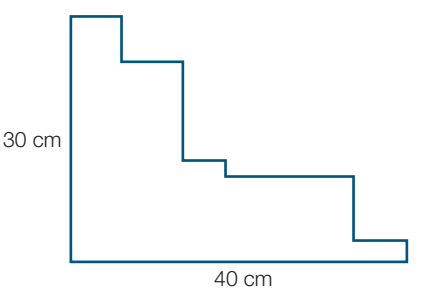
(d)



(e)



(f)



WE2

2 **(a)** Find, correct to one decimal place, the diameter of a circle with a circumference of:

(i) 15 m **(ii)** 32 m **(iii)** 141.39 mm **(iv)** 188.52 mm

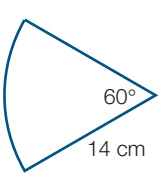
(b) Find, correct to one decimal place, the radius of a circle with a circumference of:

(i) 93 km **(ii)** 71 km **(iii)** 804.248 cm **(iv)** 1.2566 cm

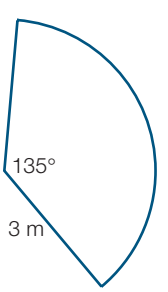
WE3

3 Find **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

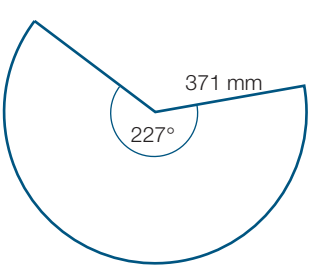
(a)



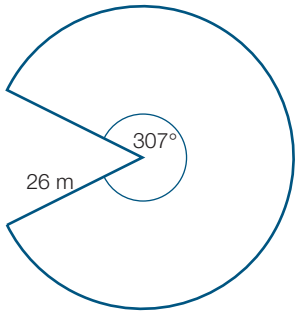
(b)



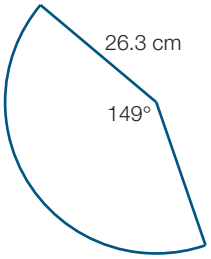
(c)



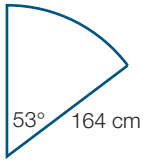
(d)



(e)

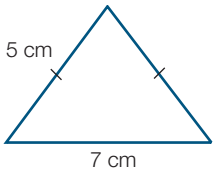


(f)



4 Choose the correct answer.

The perimeter of the triangle at right is:

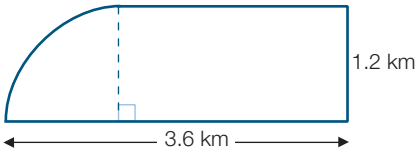


A 12 cm **B** 17 cm

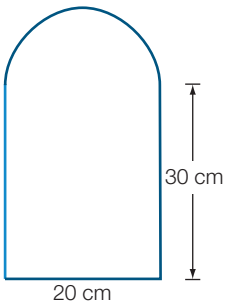
C 19 cm **D** 35 cm

5 Find the perimeters of these shapes, giving your answers to one decimal place where necessary.

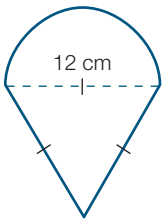
(a)



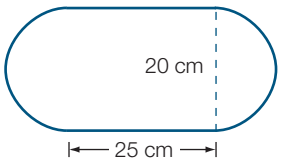
(b)



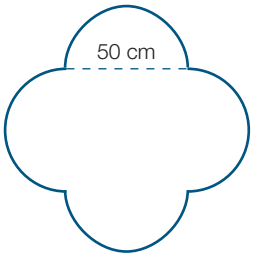
(c)



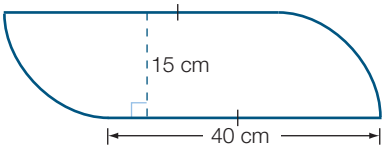
(d)



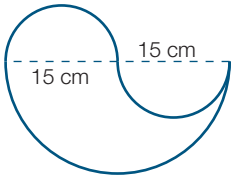
(e)



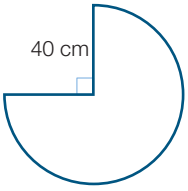
(f)



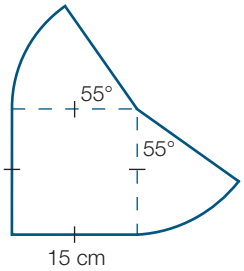
(g)



(h)



(i)



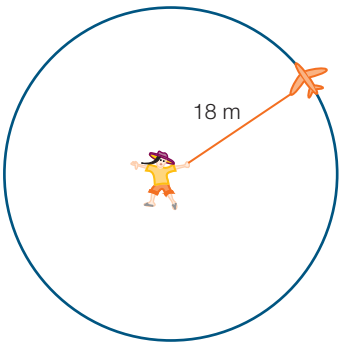
Warning

Dotted or dashed lines are only to indicate measurements. Make sure you don't include them in your perimeter calculation.

6 Find the diameter of a DVD if its circumference is 37.7 cm. Write your answer correct to one decimal place.

7 Find the circumference of Earth if the diameter, to the nearest 100 km, is 12 800 km. Give your answer correct to the nearest 100 km.

8 Andrea is flying her model aeroplane on a wire 18 m long, as shown.



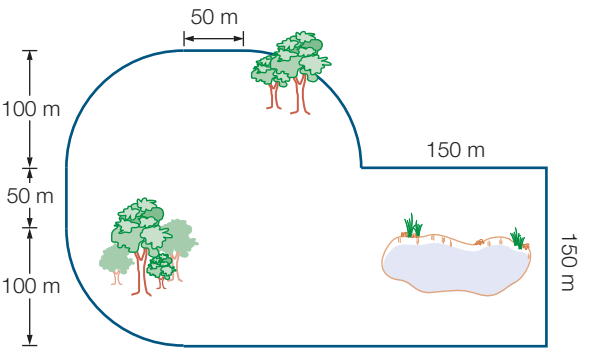
(a) How far does it fly in one circuit? (Answer to the nearest metre.)

(b) How long will it take to complete one circuit if it is flying at a speed of 5 metres per second? (Answer to the nearest tenth of a second.)

(c) How long would the wire need to be if the distance for one circuit was 200 m? (Answer to the nearest centimetre.)

9 Every night Nicole walks her dog twice around the park. The plan of the park is shown.

How far does she walk in km? (Answer to the nearest metre.)



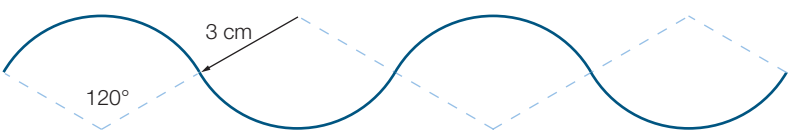
Hint

Drawing a diagram can help you 'picture' a problem.

Reasoning, communicating and open-ended

10 If a plane travels halfway around the world at an altitude of 11 300 m, what extra distance, in km, will it fly compared with the distance measured at the surface of the Earth? (The radius of the Earth is approximately 6400 km.) Round your answer to one decimal place.

11 A sheet of corrugated iron has corrugations that are based on the arc of a circle of radius 3 cm and a central angle of 120°. Four corrugations are shown below.



How many corrugations are formed when a flat sheet of steel 1 m wide is bent to form a sheet of corrugated iron?

12 Estimate first, then use a tape measure or ruler to measure the perimeter of:

(a) the front cover of this book

(b) the top of your desk

(c) the whiteboard in your room

(d) the floor of the classroom.

13 Find whole number diameters of three different circles whose circumferences are between 50 cm and 90 cm.

14 Sketch and label a shape with a boundary containing straight as well as curved sections whose perimeter is between 30 and 40 cm.

Area of composite shapes

3.2

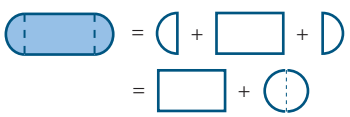
Need to Know

Stage 5.1

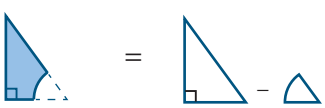
Area is a measure of the amount of surface within a boundary.

Composite shapes

Composite shapes are made from different combinations of basic shapes. The area of composite shapes is found by breaking them up into simpler shapes, calculating the individual areas of the simpler shapes and adding the areas together.



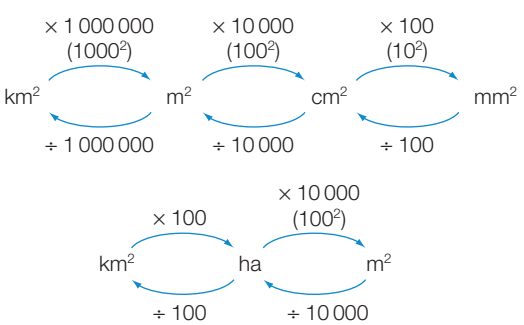
For some shapes, the area remaining is found by subtracting the area of a piece that is not included from the area of the overall shape.



Exact answers, rounding and accuracy

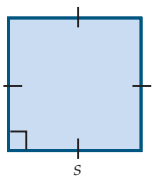
The exact areas of circular shapes involve π, which is an irrational number. Approximate areas can be calculated by substituting the calculator value for π and rounding the answer to the required number of decimal places. Calculations should be simplified, if possible, before evaluation. Rounding should only be performed once at the very end of the calculation. This ensures that the most accurate final answer is obtained. Using rounded answers through a problem means errors are accumulated with each calculation, which results in an inaccurate final answer.

Converting units of area



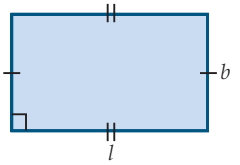
Area formulas

Square



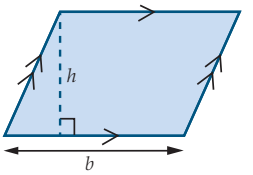
$A = s^2$

Rectangle



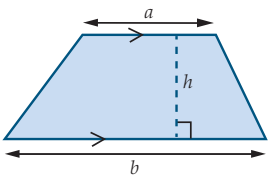
$A = lb$

Parallelogram



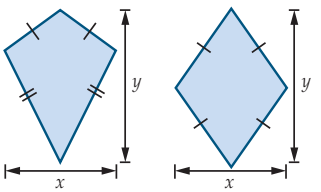
$A = bh$

Trapezium



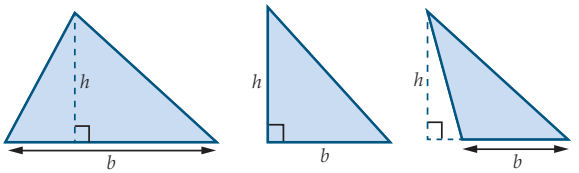
$A = \frac{(a+b)h}{2}$

Kite and rhombus



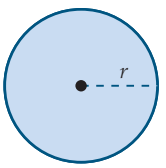
$A = \frac{xy}{2}$

Triangle



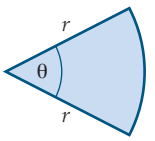
$A = \frac{bh}{2}$

Circle



$A = \pi r$

Sector of a circle



$A = \frac{\theta}{360^\circ} \times \pi r^2$

where θ is the angle measured in degrees subtended at the centre of the circle.

Worked Example 4

WE4

Copy and complete the following conversions.

(a) 17.4 ha = _____ m^2

(b) 20 000 m^2 = _____ km^2

Working

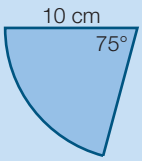
(a) 17.4×100^2
$= 174\,000\ m^2$

(b) $20\,000 \div 1000^2$
$= 0.02\ km^2$

Worked Example 5

WE5

Find the exact area of this sector.



Working

$$A = \frac{\theta}{360} \times \pi r^2$$

$r = 10$ cm, $\theta = 75°$

$$A = \frac{75}{360} \times \pi \times 10^2$$

$$= \frac{\overset{25}{\cancel{75}}}{\underset{12}{\cancel{360}}} \times \pi \times 10 \times \overset{1}{\cancel{10}}$$

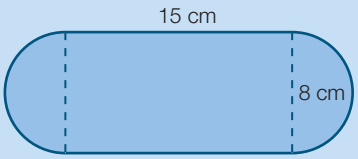
$$= \frac{250\pi}{12}$$

Worked Example 6

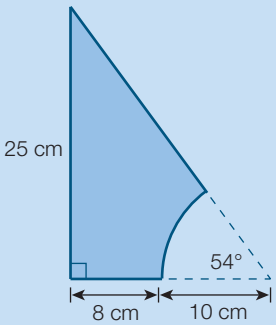
WE6

Find the shaded area of each shape. Write your answers to the nearest whole number.

(a)



(b)



Working

(a) Area = Area of rectangle + Area of circle

$$A = lb + \pi r^2$$

$$= 15 \times 8 + \pi \times 4^2;\ l = 15 \text{ cm},\ b = 8 \text{ cm and } r = 4 \text{ cm}$$

$$= 120 + 16\pi$$

$$= 170 \text{ cm}^2 \text{ (nearest whole number)}$$

(b) Area = Area of triangle − Area of sector

$$A = \frac{bh}{2} - \left(\frac{\theta}{360} \times \pi r^2\right)$$

$$= \frac{\overset{9}{\cancel{18}} \times 25}{\underset{1}{\cancel{2}}} - \left(\frac{\overset{27}{\cancel{54}}}{\underset{18}{\cancel{360}}} \times \pi \times 10^{\overset{1}{\cancel{2}}}\right);$$

$b = 18$ cm, $r = 10$ cm, $\theta = 54°$ and $h = 25$ cm

$$= 225 - \left(\frac{270}{18}\pi\right)$$

$$= 178 \text{ cm}^2 \text{ (nearest whole number)}$$

Answers page 432

3.2 Area of composite shapes

Stage 5.1

Navigator

Q1 Column 1, Q2, Q3 (a)–(f), Q4, Q5, Q6, Q7, Q8, Q10, Q13, Q15, Q16, Q19, Q22	Q1 Column 2, Q2, Q3 (d)–(i), Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q21, Q22	Q1 Column 2, Q2 (d)–(f), Q3 (g)–(i), Q5, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21

Fluency and understanding

WE4

1 Copy and complete the following conversions.

(a) 3 m^2 = ____ cm^2

(b) 0.172 m^2 = ____ cm^2

(c) 72.5 cm^2 = ____ mm^2

(d) 0.6 cm^2 = ____ mm^2

(e) 9 ha = ____ m^2

(f) 4.07 ha = ____ m^2

(g) 35 m^2 = ____ ha

(h) 695 mm^2 = ____ cm^2

(i) 127 cm^2 = ____ m^2

(j) 835 cm^2 = ____ m^2

(k) 43.8 ha = ____ m^2

(l) 6850 m^2 = ____ ha

(m) 290 cm^2 = ____ m^2

(n) 39.6 cm^2 = ____ mm^2

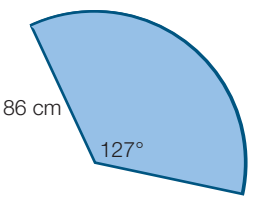
(o) 15 000 000 m^2 = ____ km^2

(p) 10 080 ha = ____ km^2

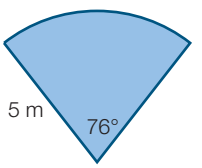
WE5

2 Calculate the exact area of each of these sectors.

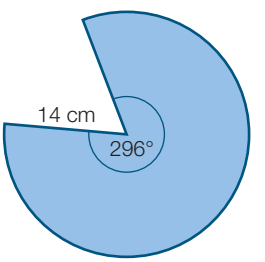
(a)



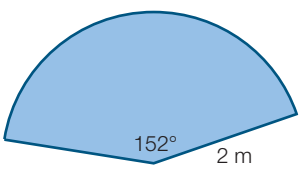
(b)



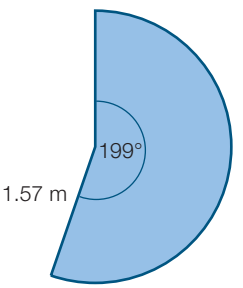
(c)



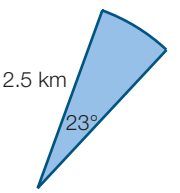
(d)



(e)



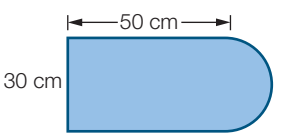
(f)



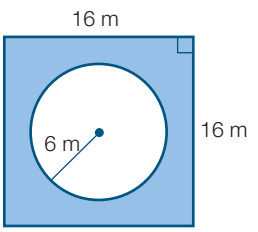
WE6

3 Find the shaded area of each of the following shapes. Write your answers correct to the nearest whole number.

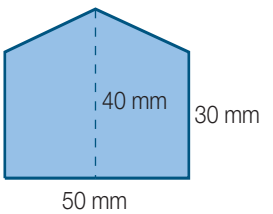
(a)



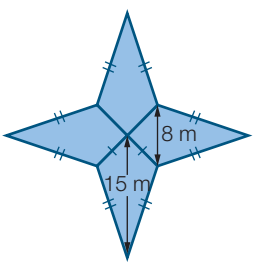
(b)



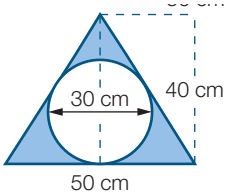
(c)



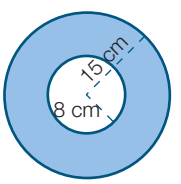
(d)



(e)



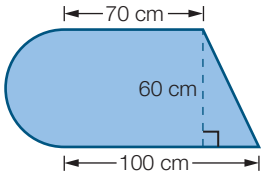
(f)



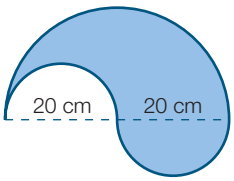
Warning

Do not round any answers until *after* you have added or subtracted the areas. Otherwise your result may be less accurate.

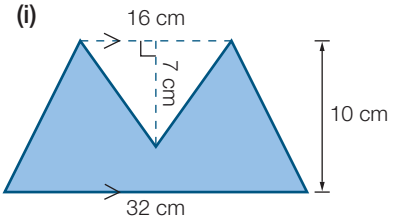
(g)



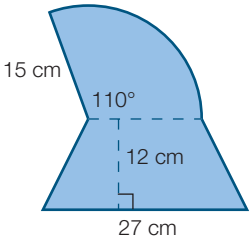
(h)



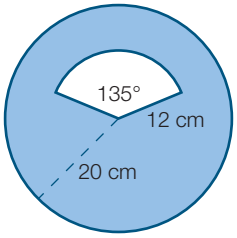
(i)



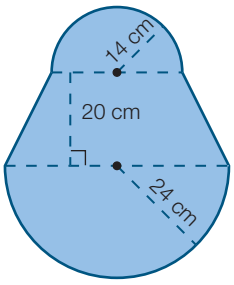
(j)



(k)

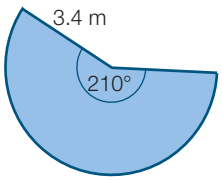


(l)



4 Choose the correct answer.

The area of the sector shown on the right is closest to:



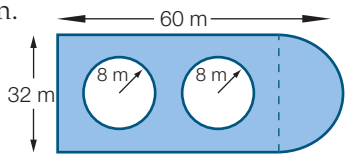
A 12.5 m^2 **B** 21.1 m^2

C 21.2 m^2 **D** 36.3 m^2

5 A garden lawn has two ponds within its boundary, as shown.

(a) Find the area of the lawn.

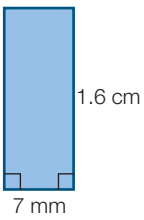
(b) How much seed is needed to grass the lawn if 1 kg of seed covers 8 m^2?



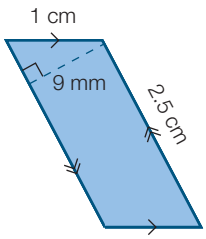
6 Find the area of each shape, correct to two decimal places where necessary, in:

(i) square millimetres **(ii)** square centimetres.

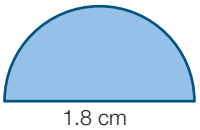
(a)



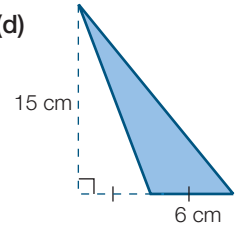
(b)



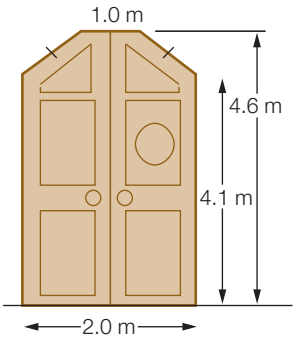
(c)



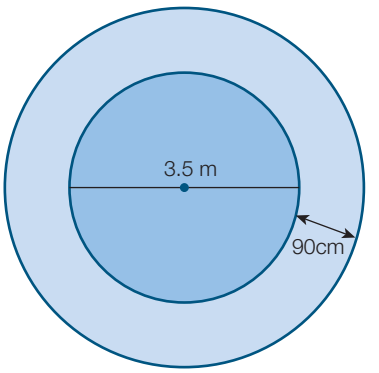
(d)



7 The doors at a local church are shaped as shown. What is the exact area of each pair of doors?



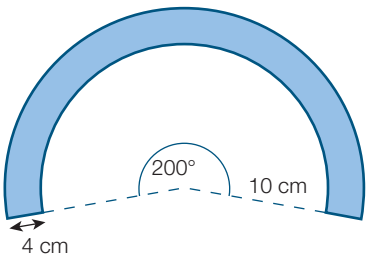
8 An **annulus** is the area between two circles that have the same centre but different radii. A circular pond 3.5 m in diameter is surrounded by a path 90 cm wide. The shape of the path is an annulus. The path is to be paved with small rectangular bricks that are 20 cm long and 10 cm wide.



(a) Calculate the area of the path.

(b) Calculate the number of bricks required to pave the path.

9 Nicola has made a collar from a sector of an annulus, as shown in the diagram. What area of fabric has been used for the collar? Give your answer correct to one decimal place.



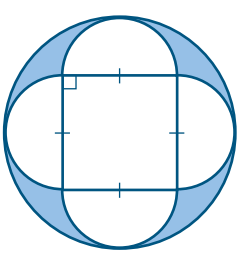
10 Circular plastic drink coasters 8 cm wide are manufactured by feeding a rectangular strip of plastic 9 cm wide into a machine that cuts out the coasters. The machine starts 1 cm from one end of the strip and leaves 1 cm between coasters and 1 cm on the other end of the strip. For this machine, answer the following questions.

(a) How long would a strip of plastic have to be to cut out 50 coasters?

(b) What would be the total area of wasted plastic (to the nearest cm^2)?

(c) What percentage of the strip is used for making coasters? (Give your answer to the nearest whole number.)

11 An architect is designing a decorative circular courtyard 22 m in diameter. The shaded area is to be a garden and the rest will be pavement.



(a) Find the area of the pavement, correct to one decimal place.

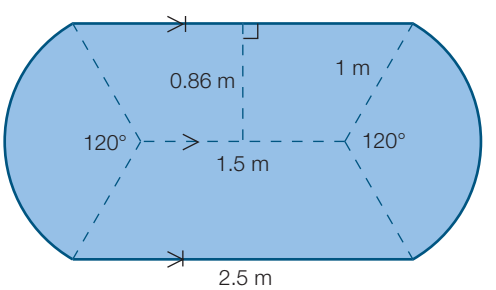
(b) Find the area of the garden, correct to one decimal place.

(c) What percentage of the courtyard will be pavement? (Give your answer to the nearest whole number.)

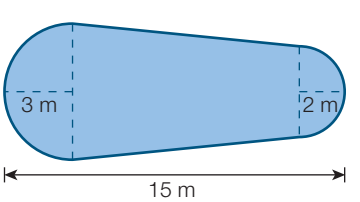
Hint

How many of the radii of the smaller circle make up the side length of the square?

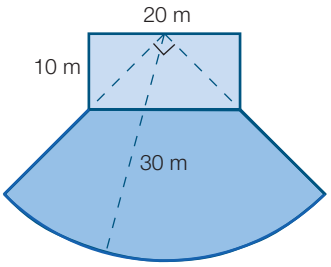
12 Norbert's dining table is the shape shown. Calculate its area, giving your answer correct to two decimal places.



13 Jennifer has a rectangular lawn at the back of her house that measures 20 m by 18 m. She is putting in a pond with the shape shown in the diagram. What area of lawn will she have left if the grass is to go all the way to the edge of the pond? (Give your answer correct to one decimal place.)



14 A new theatre is to be built. The design of the theatre includes a rectangular stage area with a portion of a quadrant of a circle for the audience to be seated in, as shown in the diagram. Calculate the total area of the theatre to the nearest whole number.



Reasoning, communicating and open-ended

15 How many rectangular house-blocks 16 m × 20 m would fit in a 15 ha subdivision for a housing estate, assuming 20% of the area of the subdivision is used for roads?

16 A square has diagonals that are 20 cm long.

(a) Remembering that the diagonals of a square bisect each other at right angles, find the area of the square.

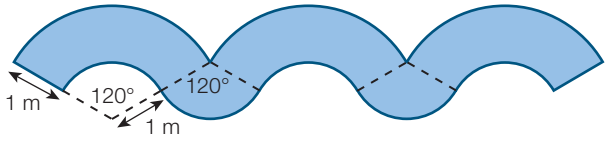
(b) Calculate the length of the sides of the square from the area you found, correct to two decimal places.

Hint

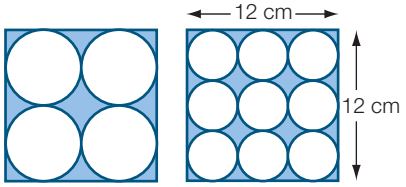
Use Pythagoras' Theorem.

17 The area of a sector of a circle is 1000 cm^2. The radius of the sector is 30 cm. Calculate the size of the angle at the centre of the sector, correct to the nearest degree.

18 Part of the front of a shop awning consists of three sectors of an annulus and two sectors of a circle, as shown in the diagram. Calculate the total area of this section. (Give your answer correct to one decimal place.)



19 Touching circles are drawn inside two squares with side length 12 cm. Four identical circles (called a 4-Pak) just fit into the first square and nine (called a 9-Pak) fit into the second.



(a) What is the radius of each circle in:

(i) the 4-Pak **(ii)** the 9-Pak?

(b) Which of the shaded background areas is bigger?

20 A trapezium with an area of 900 cm^2 has parallel sides 40 cm apart. If one of these sides is twice as long as the other, the lengths of the parallel sides would be:

A 20 cm, 10 cm **B** 24 cm, 12 cm **C** 30 cm, 15 cm **D** 40 cm, 20 cm

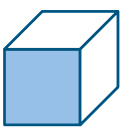
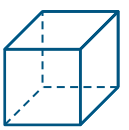
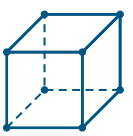
21 Steve's Pizza Shop sells pizzas that are 14 cm in radius. He decides to stop making them circular and instead make them rectangular. He uses approximately the same amount of ingredients and makes them the same thickness. Assuming that the length and width of the rectangular pizzas are in whole centimetres, write at least three pairs of dimensions that are practical.

22 The perimeter of a rectangle is 60 cm. Calculate two possible areas for the rectangle.

3.3 Solids and nets

Stage 5.1

Need to Know

Face	Edge	Vertex
The flat part of a **solid**.  A cube has 6 faces.	The line segment where two faces meet.  A cube has 12 edges.	A corner where several edges and faces meet.  A cube has 8 vertices.

A solid where all faces are polygons is known as a **polyhedron**.

A polyhedron where all faces are identical regular polygons is a **regular polyhedron**.

A **net** is the 2D outline of a solid that shows all the faces. A net can be folded to make the 3D solid.

Prisms have a polygon base and a regular cross-section (any cut parallel to the base gives an identical shape to the base).

Answers page 432

3.3 Solids and nets

Stage 5.1

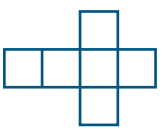
Navigator

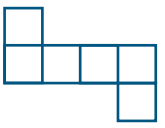
Q1, Q2 (a), Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q14	Q1, Q2 (a), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14	Q1, Q2 (b), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14

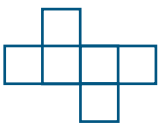
Equipment required: scissors, nets, thread, glue and a ruler for Questions **7** and **8**

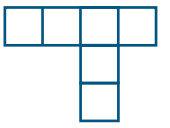
Fluency and understanding

1 Which of the following could not be a net for a cube?

A 

B 

C 

D 

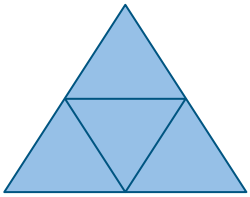
2 Draw a possible net for each of the following solids.

(a) a rectangular prism with dimensions of length 5 cm, width 2 cm and height 3 cm

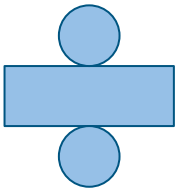
(b) a pentagonal prism of side length 2 cm and height 5 cm

3 Name the solids that the nets below would fold into.

(a)



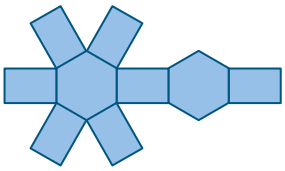
(b)



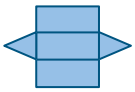
(c)

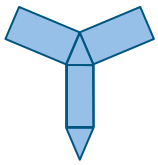


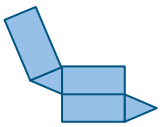
(d)

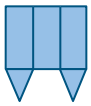


4 Which of the following nets *cannot* be folded to make a triangular prism?

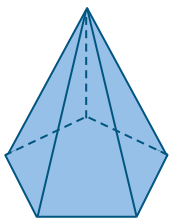
A 

B 

C 

D 

5 Which description matches the shape shown?



A 5 faces, 10 edges, 5 vertices

B 6 faces, 10 edges, 6 vertices

C 10 faces, 10 edges, 6 vertices

D 6 faces, 9 edges, 6 vertices

6 Draw the net of the external packaging for the following objects (e.g. no folding flaps).

(a)



(b)



(c)



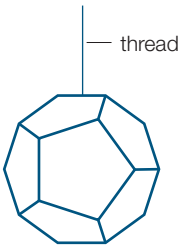
7 Draw a net of a cube with side lengths of 3 cm on cardboard, cut it out and construct a solid cube.

8 Follow these steps to construct solids from the nets shown.

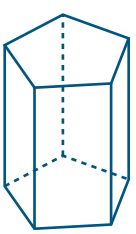
(i) Copy and cut out each net, including the tabs, and then crease along the fold lines so that a sharp line is formed.

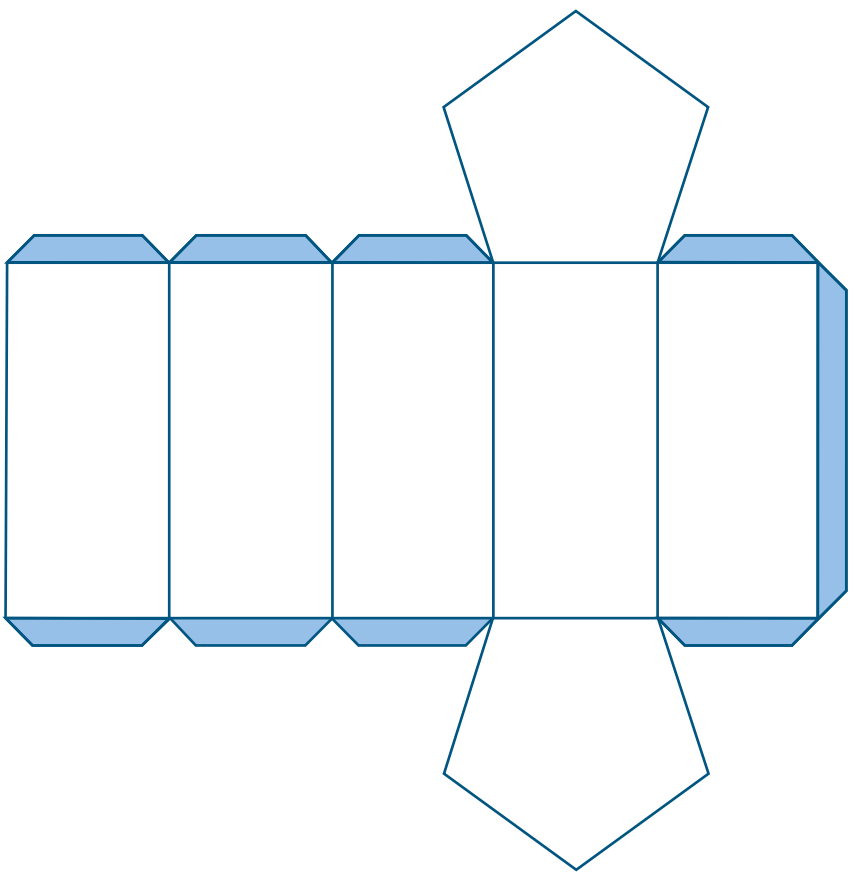
(ii) Fold the nets so the marked fold lines are on the inside of the model.

(iii) Insert a length of thread in your models before glueing the last tab. The thread may be used for hanging the models in a display later on.

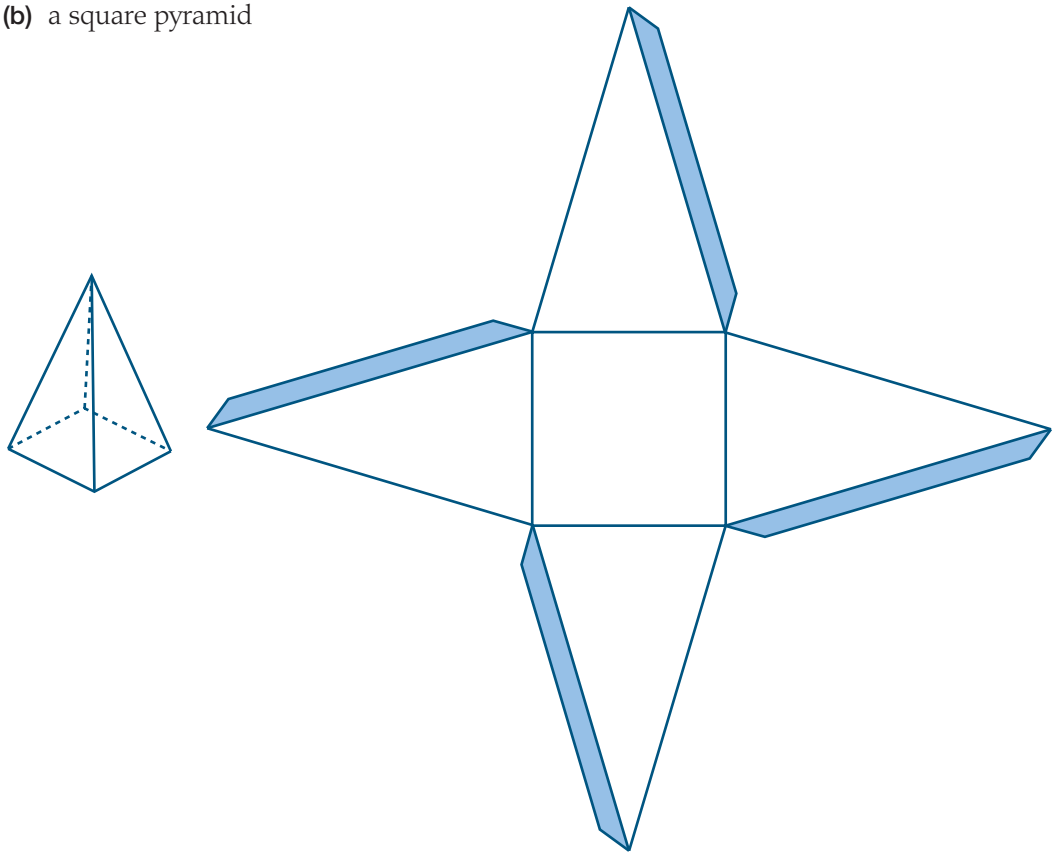


(a) a pentagonal prism



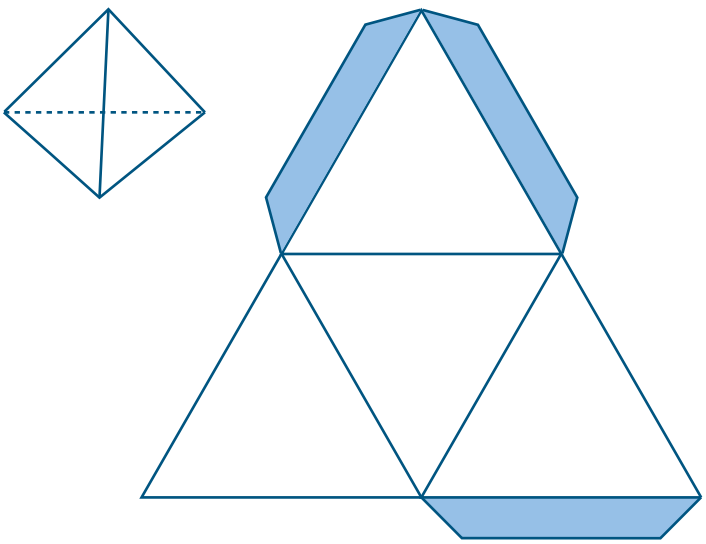


(b) a square pyramid



(c) a tetrahedron:

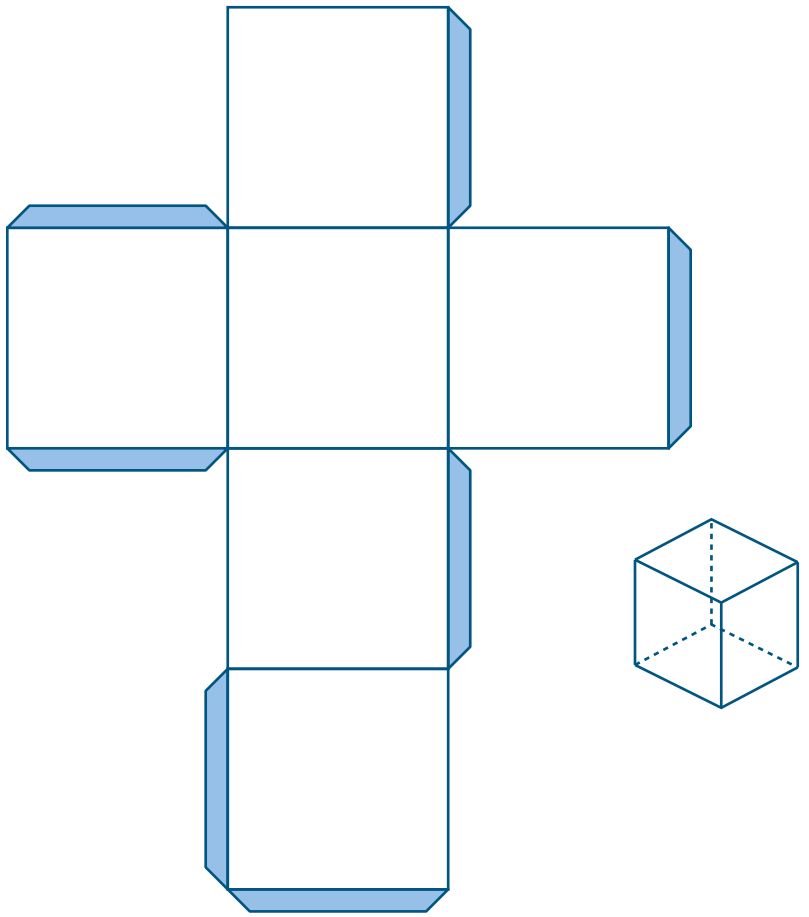
- has four identical triangular faces
- is a rectangular polyhedron, as each face is an equilateral triangle
- is a triangular pyramid.



Hint

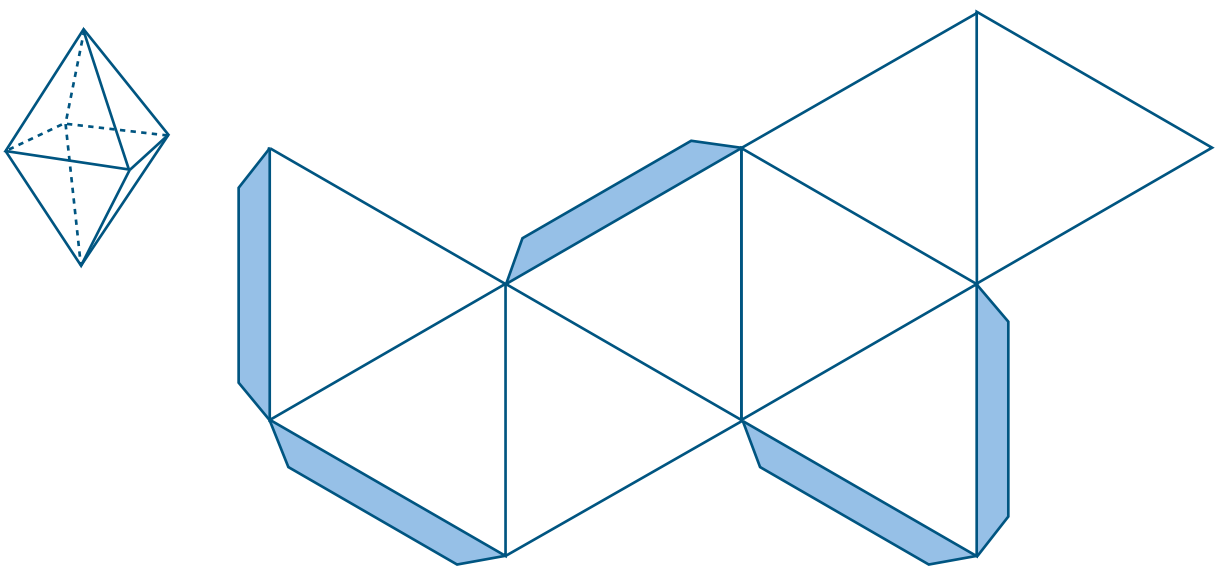
A tetrahedron is very strong. The atoms in diamonds are arranged in a lattice of tetrahedrons. Diamond is one of the hardest substances known.

(d) a cube



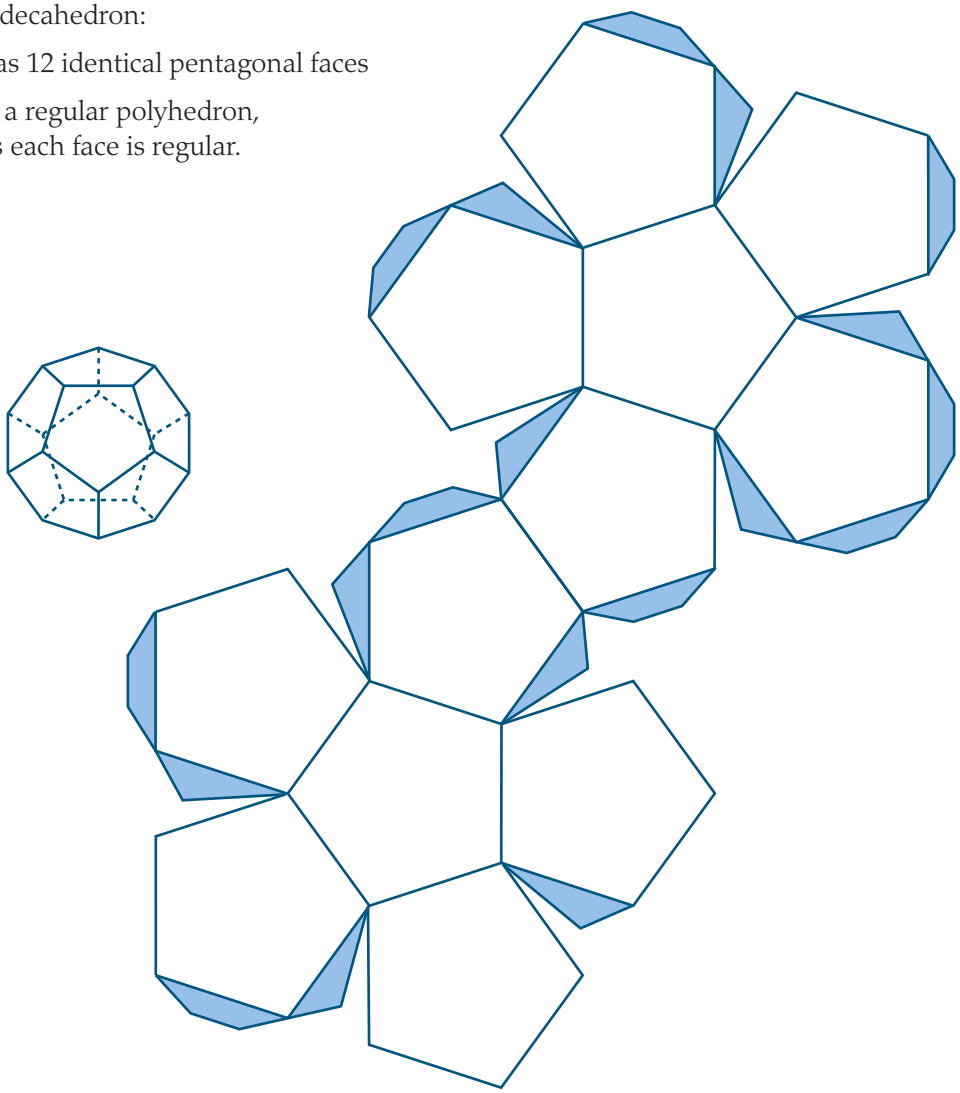
(e) an octahedron:

- has eight identical triangular faces
- is a regular polyhedron, as each face is an equilateral triangle
- can be thought of as two square-based pyramids joined together at the bases.



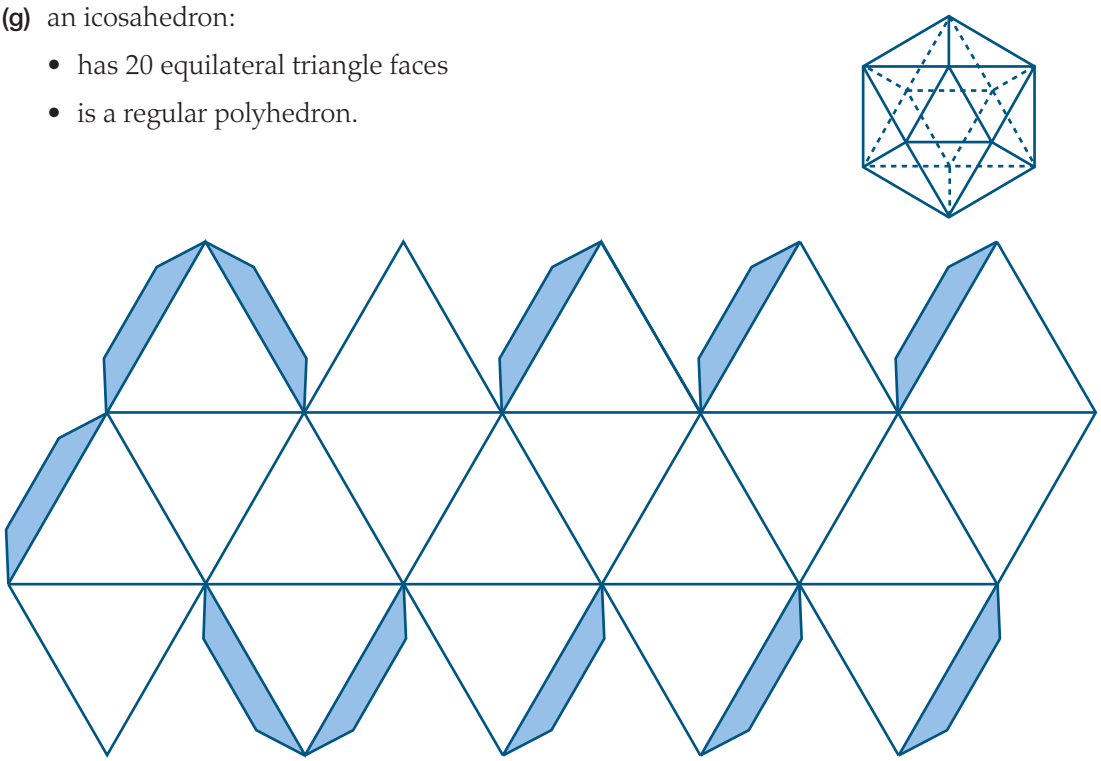
(f) a dodecahedron:

- has 12 identical pentagonal faces
- is a regular polyhedron, as each face is regular.



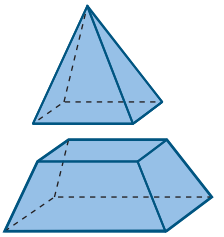
(g) an icosahedron:

- has 20 equilateral triangle faces
- is a regular polyhedron.

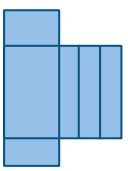


Reasoning, communicating and open-ended

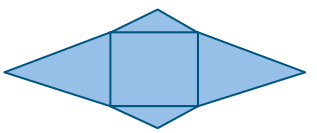
9 A square pyramid has been cut into two, as shown. How many faces, vertices and edges does each part have?



10 Roland drew nets of two solids shown below.



Net of a rectangular prism



Net of a rectangular pyramid

(a) Explain why the two nets above will not work.

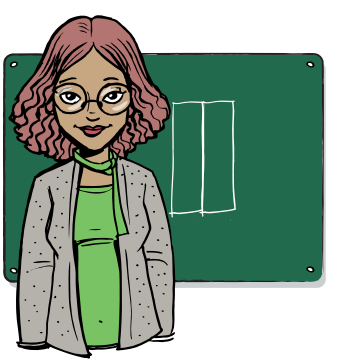
(b) Correct Roland's drawings.

11 Draw a net of a solid that has nine faces.

12 State three solids whose nets have no rectangles.

13 Can a solid have only three faces? Can a polyhedron have only three faces? Explain your answers.

14 Saxon can only see part of the net the teacher is talking about. He can see two quadrilaterals. Draw a possible net and the solid it can fold into.



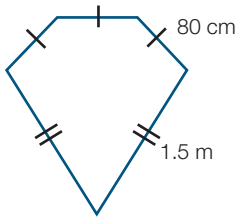
Half-time 3



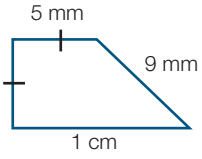
Stage 4 Q1 Stage 5.1 Q2–7

1 Find the perimeter of each shape, in the smaller units if two are given, rounding your answer to the nearest whole number. Ex.3.1

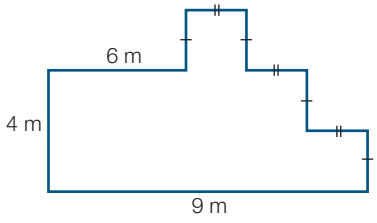
(a)



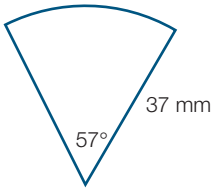
(b)



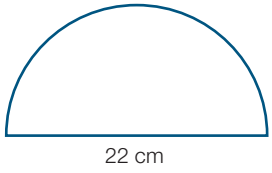
(c)



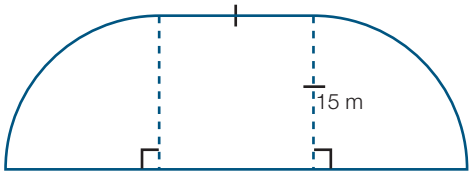
(d)



(e)

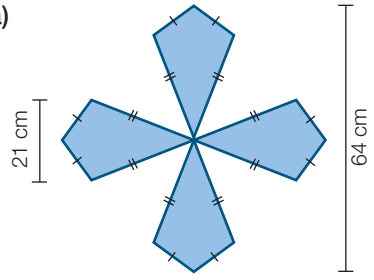


(f)

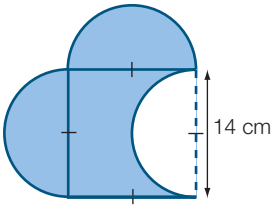


2 Find the shaded area, correct to the nearest whole number. Ex.3.2

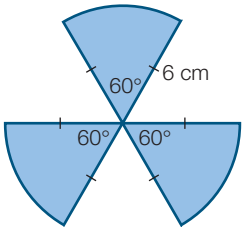
(a)



(b)



(c)



3 Calculate the distance along the surface of Earth (to the nearest 100 km) from the Equator to the South Pole given that the diameter of Earth is approximately 12 800 km. Ex.3.1

4 A trapezium has an area of 600 cm^2. Its parallel sides are 15 and 25 cm long. Find the distance between the parallel sides. Ex.3.2

5 Copy and complete the following conversions. Ex.3.2

(a) 4 cm^2 = _____ mm^2

(b) 7.5 m^2 = _____ cm^2

(c) 0.26 km^2 = _____ m^2

(d) 19.8 ha = _____ m^2

(e) 3.05 km^2 = _____ ha

(f) 274 cm^2 = _____ m^2

6 The perimeter of a right-angled triangle is 90 cm. Two of the sides are 15 and 39 cm long. Find the area of the triangle. Ex.3.2

7 Draw the net for a triangular prism. Ex.3.3

3.4 Surface area of prisms

Stage 5.1

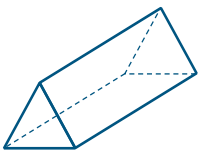
Need to Know

The **surface area** of a polyhedron is the total area of its faces.

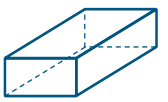
To find the surface area of a polyhedron, find the area of each face, then add all the areas together.

Prisms

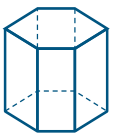
A prism is a three-dimensional object that has a uniform **cross-section** in the shape of a polygon.



Triangular prism



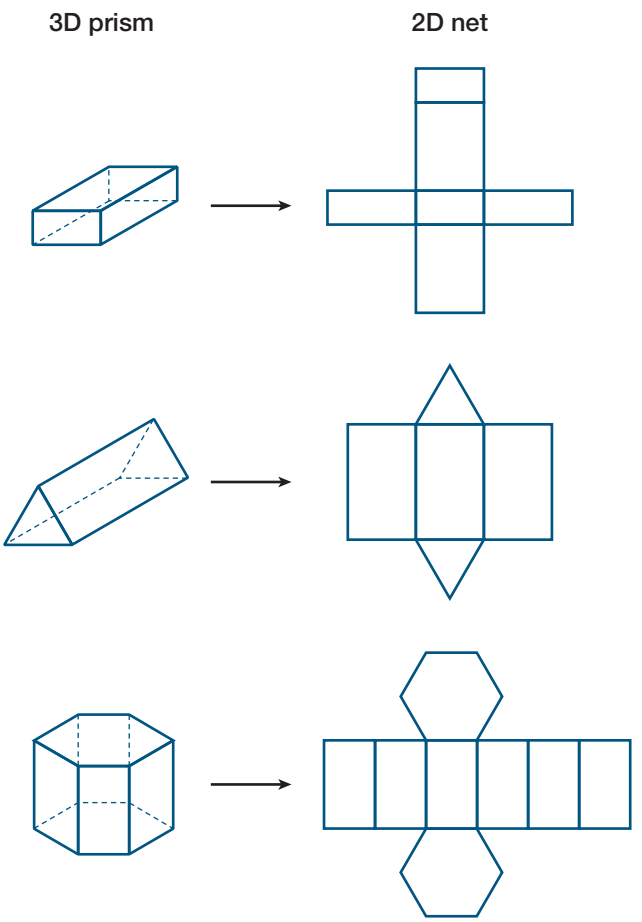
Rectangular prism



Hexagonal prism

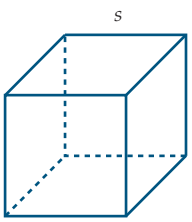
Nets of prisms

To calculate surface area, it is helpful to consider the net of the object.

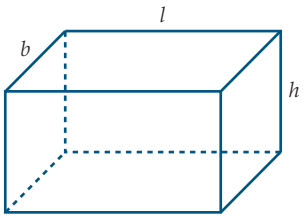


Formulas for surface areas

The surface area of a cube = $6s^2$, where s is the side length.



The surface area of a rectangular prism $= 2lb + 2lh + 2bh$
$= 2(lb + lh + bh)$,
where l, b and h are the three side lengths.

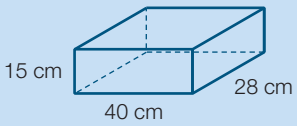


Worked Example 7

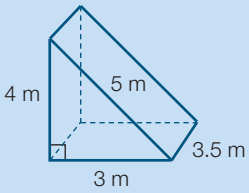
WE7

Draw the nets of the following polyhedra and use them to calculate their surface area.

(a)

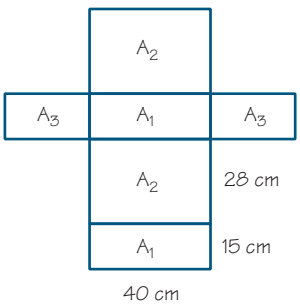


(b)



Working

(a)



$SA = 2A_1 + 2A_2 + 2A_3$

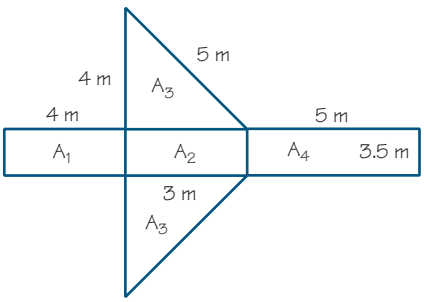
$SA = 2l_1b_1 + 2l_2b_2 + 2l_3b_3$

$= 2 \times (40 \times 15) + 2 \times (40 \times 28) + 2 \times (28 \times 15)$

$= 1200 + 2240 + 840$

$= 4280 \text{ cm}^2$

(b)



$SA = A_1 + A_2 + 2A_3 + A_4$

$= l_1b_1 + l_2b_2 + 2\frac{bh}{2} + l_3b_3$

$= (3.5 \times 4) + (3 \times 3.5) + 2 \times \left(\frac{3 \times 4}{2}\right) + (5 \times 3.5)$

$= 14 + 10.5 + 12 + 17.5$

$= 54 \text{ cm}^2$

Answers page 433

3.4 Surface area of prisms

Stage 5.1

Navigator

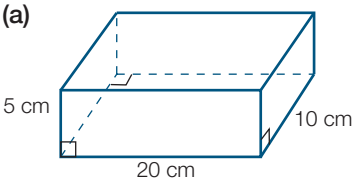
Q1, Q2, Q3, Q5, Q8 | Q1, Q2, Q3, Q5, Q6, Q8 | Q1, Q2, Q3, Q4, Q5, Q6, Q7

Fluency and understanding

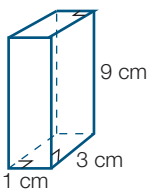
WE7

1 Draw nets of the following polyhedra and use them to calculate their surface area.

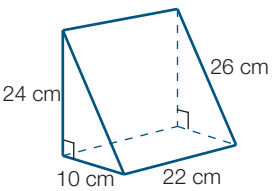
(a)



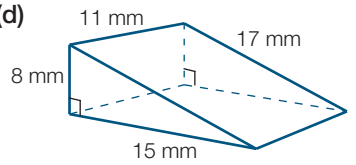
(b)



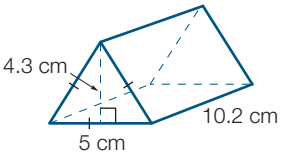
(c)



(d)



(e)



2 **(a)** The surface area of a cube with edge length 10 cm is:

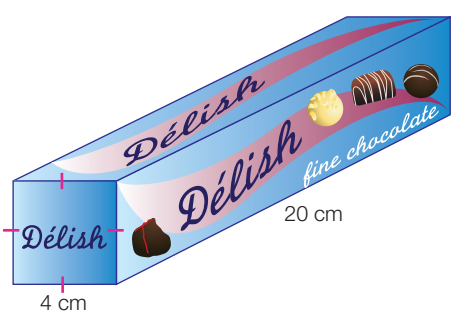
A 200 cm^2 **B** 400 cm^2 **C** 600 cm^2 **D** 800 cm^2

(b) The surface area of a rectangular box with dimensions 40 cm, 30 cm and 20 cm is:

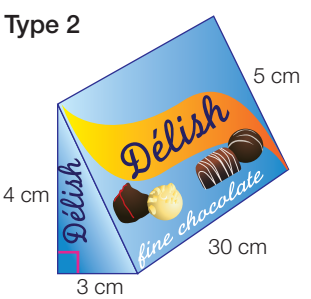
A 1800 cm^2 **B** 2400 cm^2 **C** 2600 cm^2 **D** 5200 cm^2

3 Joshua has to design a box for a new brand of chocolate. Assuming there is no overlap, which of these designs uses less cardboard?

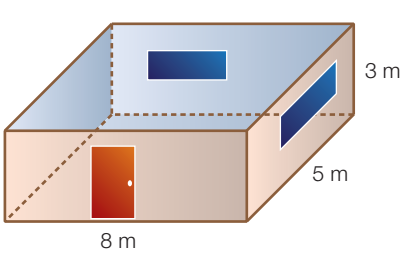
Type 1



Type 2

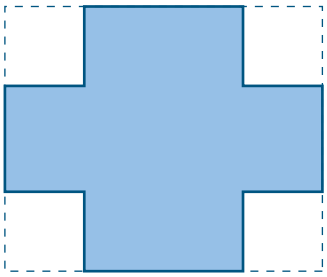


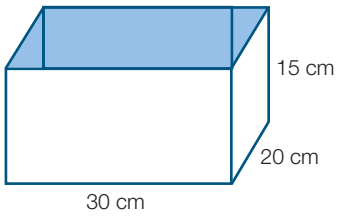
4 Leonardo has been hired to paint the walls and ceiling of a room, as shown in the diagram. The room has dimensions 8 m × 5 m × 3 m and it has two windows, each 2 m × 1.5 m, and a door 0.9 m × 2.3 m. Find the area to be painted (excluding the windows and door).



Reasoning, communicating and open-ended

5 The rectangular piece of cardboard on the left has four squares cut from the corners to make an open box whose dimensions are 30 cm × 20 cm × 15 cm.





(a) What was the length and width of the original piece of cardboard?

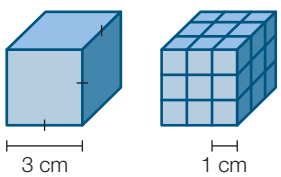
(b) What percentage of the original piece of cardboard is wasted in making the box?

6 Chewing your food is an important step in the digestive process. Chewing breaks down a large piece of food, making it easier to digest. Demonstrate your calculations using a cube to model a piece of food.

(a) Calculate the surface area of this 3 cm cube.

(b) Imagine the cube is chewed into smaller cubes, each with a side length of 1 cm. Calculate the surface area of these smaller cubes.

(c) By how many times has the surface area of the original cube increased?



7 What is the effect on the surface area of a cube if you double the length of its edges? Demonstrate your answer with two suitable cubes, one with an edge length that is double that of the other cube.

8 A rectangular box has a surface area of 960 cm^2. It has a square end, as shown. Each side length is a whole number of centimetres. Find two different sets of dimensions that satisfy these conditions.



Problem solving

Gold, set, match

The following statement has appeared on several 'weird facts' websites:

'A lump of pure gold that is only as big as a matchbox can be flattened into a sheet about the size of a tennis court.'

A matchbox is 5.5 cm long, 3.5 cm wide and 1.5 cm high. A tennis court is 23.77 m long and 10.97 m wide.

1 If the 'fact' is true, and a matchbox-sized lump of gold was flattened out into a sheet to cover a tennis court, how thick would the sheet of gold be? Give your answer in mm.

2 Considering the size of your answer, do you think the 'fact' could actually be true?

Strategy options

- Draw a diagram.
- Make a model.
- Break the problem into manageable parts.

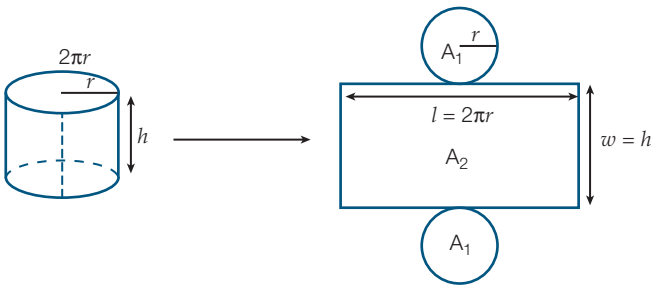
3.5 Surface area of cylinders and composite shapes

Stage 5.2

Need to Know

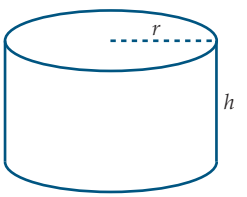
A net of a closed cylinder

To find the surface area of a cylinder, draw the net and calculate the area of each surface.



The surface area of a cylinder

The surface area of a closed cylinder = $2\pi r^2 + 2\pi rh$, where r is the radius of the circular face and h is the height of the cylinder.



If the cylinder is open at the top, the formula becomes

$$SA = \pi r^2 + 2\pi rh.$$

If the cylinder is open at both ends, the formula becomes

$$SA = 2\pi rh.$$

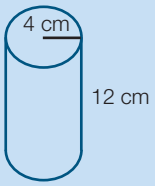
Composite solids

If the surface area of a composite solid is to be found, calculate the surface area of each solid separately but subtract the areas where the solids are joined.

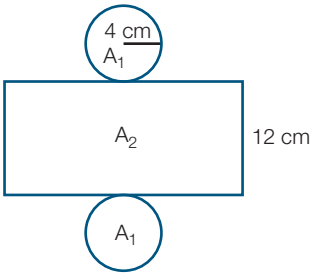
Worked Example 8

WE8

Find the surface area of this cylinder, correct to one decimal place.



Working

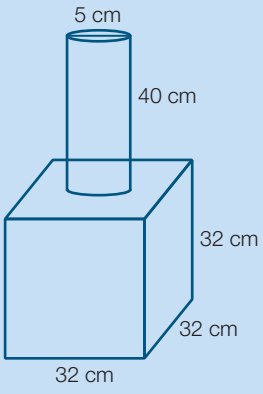


$$\begin{aligned} SA &= 2A_1 + A_2 \\ &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 12 \\ &= 32\pi + 96\pi \\ &= 128\pi \\ &= 402.1 \text{ cm}^2 \text{ (1 d.p.)} \end{aligned}$$

Worked Example 9

WE9

Find the surface area of the composite shape given below, correct to the nearest square centimetre.



Working

$$\begin{aligned} SA &= \text{SA of cube} + \text{SA of cylinder} - 2 \times \text{SA of overlap} \\ &= 6l^2 + 2\pi r^2 + 2\pi rh - 2\pi r^2 \\ &= 6l^2 + 2\pi rh \\ &= 6 \times 32^2 + \pi \times 2.5 \times 40 \\ &= 6458.159265 \\ &= 6458 \text{ cm}^2 \end{aligned}$$

Answers page 434

3.5 Surface area of cylinders and composite shapes

Stage 5.2

Navigator

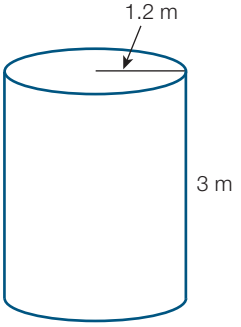
Q1, Q2, Q3, Q4, Q5, Q7, Q9 | Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9 | Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9

Fluency and understanding

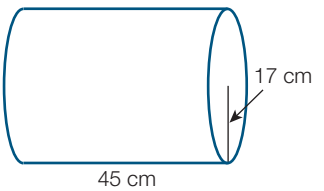
WE8

1 Find the surface area of the following cylinders, correct to one decimal place.

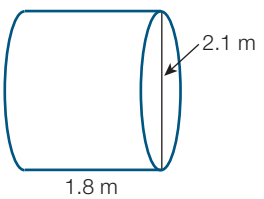
(a)



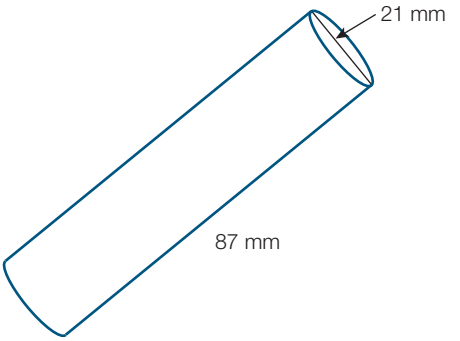
(b)



(c)

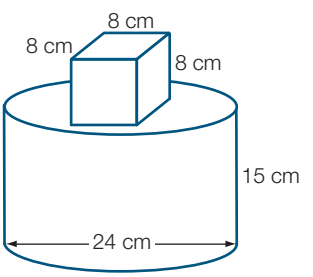


(d)



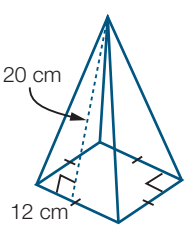
WE9

2 Find the surface area of the composite shape given, correct to the nearest square centimetre.

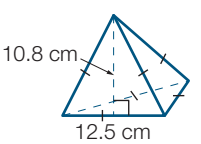


3 Find the surface area of the following figures, correct to one decimal place.

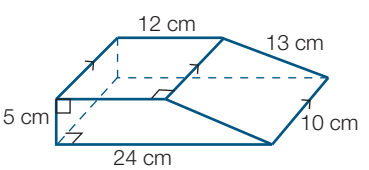
(a)



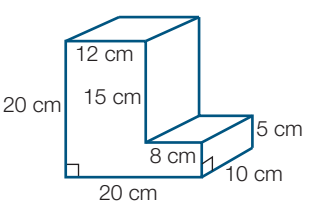
(b)



(c)



(d)



4 Paper labels are made to wrap around the curved surface of cans of food. A 1 cm overlap is allowed at the end so the label can be glued down. Calculate the area of paper required to label each of the following cans, to the nearest square centimetre.

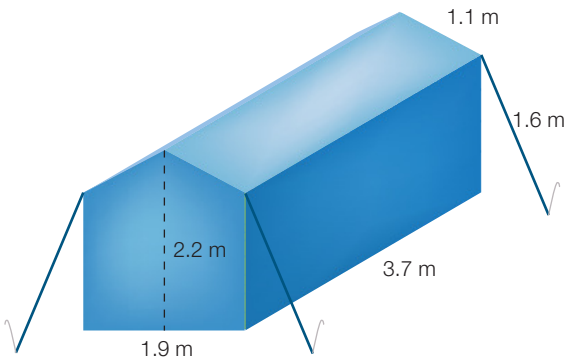
(a)



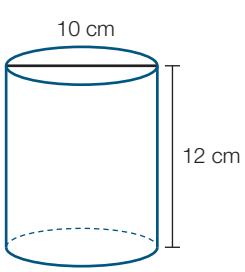
(b)



5 Calculate, to the nearest whole number, the area of the material used to make this tent, including the floor. (Assume that there is no overlap at the edges.)



6 **(a)** Draw the net of this cylinder, labelling the measurements to the nearest whole number.



(b) The area of each circle on the ends of the cylinder can be found by the calculation:

A $\pi \times 10^2$ **B** $\pi \times 5^2$

C $2 \times \pi \times 10$ **D** $\pi \times 10$

(c) The width (shorter side) of the rectangular part of the net is:

A 5 cm **B** 10 cm **C** 12 cm **D** 24 cm

(d) The length of the rectangle can be found by the calculation:

A $\pi \times 10^2$ **B** $\pi \times 5^2$ **C** $\pi \times 10$ **D** $2 \times \pi \times 10$

Reasoning, communicating and open-ended

7 **(a)** Find the surface area, correct to the nearest mm^2, of a rectangular prism of length 72 mm, width 30 mm and height 45 mm.

(b) Find the surface area, correct to the nearest mm^2, if a cylindrical hole is drilled vertically from top to bottom through the centre of the prism in part **(a)** if the diameter of the hole is 6 mm.

(c) Find the surface area, correct to the nearest mm^2, if a cylindrical hole is drilled horizontally from front to back through the centre of the prism in part **(a)** if the diameter of the hole remains 6 mm.

(d) Find the surface area, correct to the nearest mm^2, if a cylindrical hole is drilled horizontally from side to side through the centre of the prism in part **(a)** if the diameter of the hole remains 6 mm.

8 The inside of a cylindrical can (excluding the lid) 12 cm high and of diameter 7 cm is to be coated to prevent corrosion. What is the total surface area to be covered, correct to the nearest cm^2?

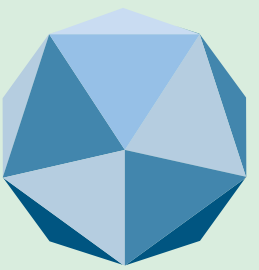
9 **(a)** A length of hose 50 m long and 2 cm in diameter is to be coated on the outside to increase the life of the hose. What is the total surface area to be covered? (Give your answer in m^2 correct to 2 decimal places.)

(b) If the hose is now attached to a cylindrical tank 1.8 m high and 1.5 m in diameter, and the outside of this tank is also to be coated, what is the total surface area to be covered? (Give your answer in m^2 correct to 2 decimal places.)

10 Find the dimensions of three cylinders that would have a total surface area between 500 and 600 cm^2.

Problem solving

Icosahedron



The chicken pox virus has the shape of an icosahedron, a solid with 20 triangular faces.

An icosahedron is one of the five Platonic solids. Each face of a Platonic solid is an identical regular polygon.

A particular icosahedron has a surface of 20 equilateral triangles of edge length 20 cm.

Find the surface area of the icosahedron.
Round your answer to one decimal place.

Strategy options

- Draw a diagram.
- Have I seen a similar problem?
- Break the problem into manageable parts.

Volume and capacity

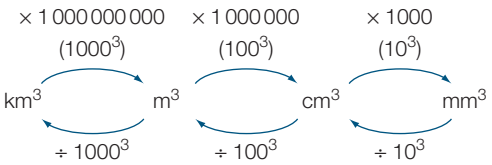
3.6

Stage 5.2

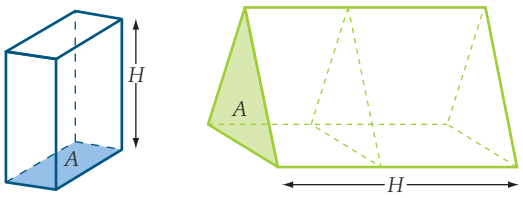
Need to Know

The **volume** of a solid is the total amount of space that the solid occupies.

Converting units of volume



Volume formula



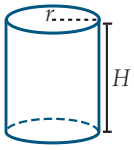
Volume of prism

$V = AH$ where
A = area of the base
H = perpendicular height of the prism

The base is the face that has a 'uniform cross-section'.

Volume of cylinder

$V = \pi r^2 H$



Capacity

Capacity refers to the maximum amount of fluid that a container can hold, usually measured in millilitres (mL), litres (L), kilolitres (kL) or megalitres (ML). A 10 cm × 10 cm × 10 cm cube has a capacity of 1 L.

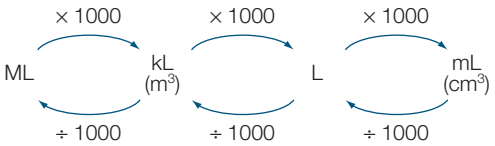
Converting units of capacity

1 mL = 1 cm^3

1 L = 1000 mL
= 1000 cm^3

1 kL = 1000 L
= 1 m^3

1 ML = 1000 kL

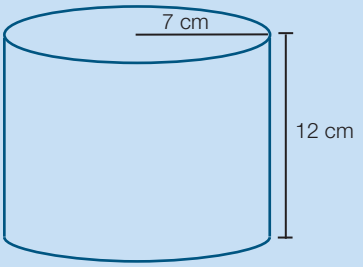


Worked Example 10

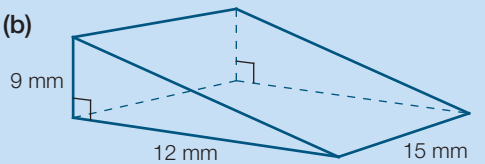
WE10

Find the volume of these solids, to the nearest whole number.

(a)



(b)



Working

(a) $V = AH$
$= \pi r^2 \times H$
$= \pi \times 7^2 \times 12$
$= 1847 \text{ cm}^3$ (nearest whole number)

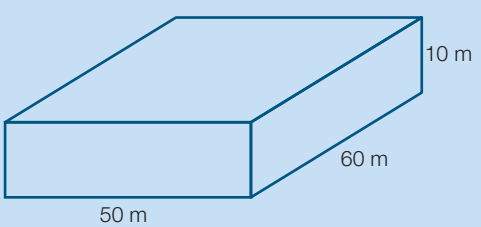
(b) $V = AH$
$= \frac{bh}{2} \times H$
$= \frac{12 \times 9}{2} \times 15$
$= 810 \text{ mm}^3$

Worked Example 11

WE11

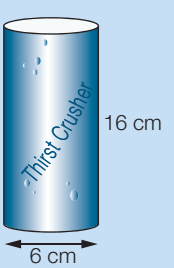
Find the capacity of each of the following in the units specified, rounded to the nearest whole number.

(a)



Water tank (in ML)

(b)



Can of soft drink (in mL)

Working

(a) $V = A \times H$
$= 60 \times 50 \times 10$
$= 30\,000 \text{ m}^3$

$\therefore$ Capacity $= 30\,000$ kL
$= 30$ ML

(b) $V = \pi r^2 \times H$, where $r = \frac{6}{2} = 3$ cm
$= \pi \times 3^2 \times 16$
$= 452 \text{ cm}^3$ (nearest whole number)

$\therefore$ Capacity $= 452$ mL

3.6 Volume and capacity

Answers page 435

Stage 5.2

Navigator

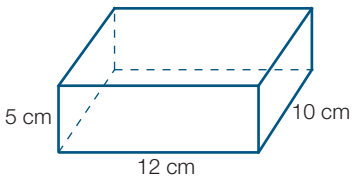
Q1, Q2, Q3, Q4, Q5 (a), Q6, Q7, Q11, Q12, Q13	Q1, Q2, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13	Q1 (a), (c), (e), Q2, Q5 (b)–(d), Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

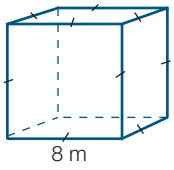
1 Find the volume of these solids, to the nearest whole number.

WE10

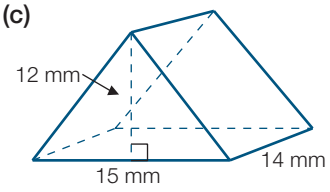
(a)



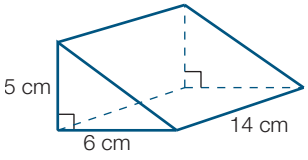
(b)



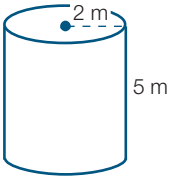
(c)



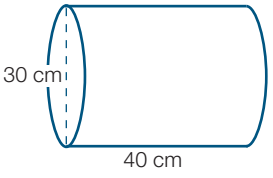
(d)



(e)



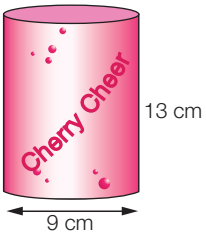
(f)



2 Find the capacity of each of the following in the units specified.

WE11

(a)



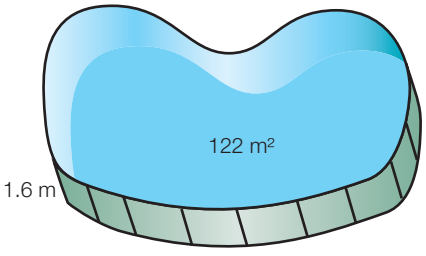
Cylindrical can of soft drink (in mL, to the nearest whole number)

(b)



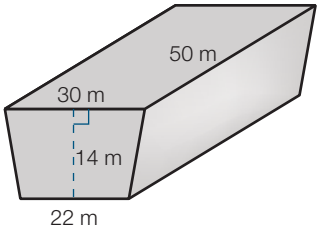
Square-based milk carton (in L, to one decimal place)

(c)



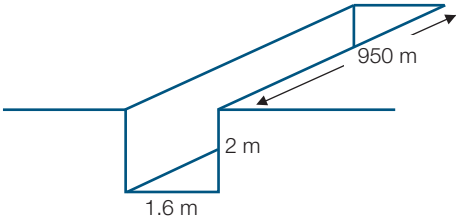
A kidney-shaped swimming pool (in kL, to the nearest whole number)

(d)



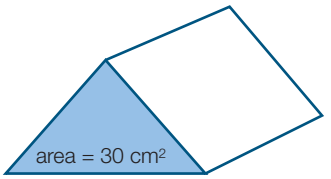
A storage tank on a ship (in ML, to one decimal place)

3 The council is digging a trench to lay some new pipe. Find the volume of soil that will have to be removed.

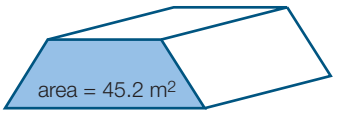


4 Find the height of these prisms.

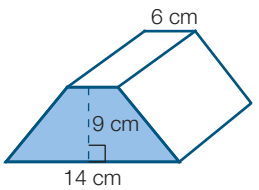
(a) volume = 600 cm^3



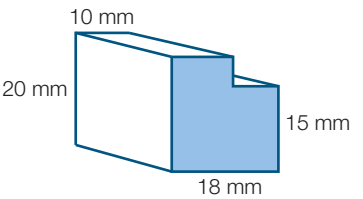
(b) volume = 560.48 cm^3



(c) volume = 1080 m^3

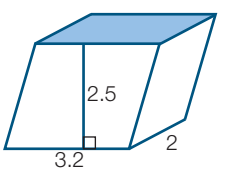


(d) volume = 9600 mm^3



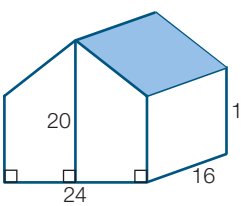
5 All the lengths in the diagrams below are in metres. The volume of each solid is:

(a)



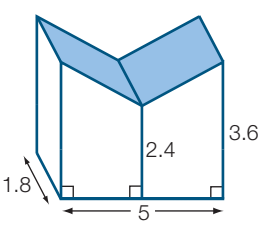
A 8 m^3
B 11.4 m^3
C 15.4 m^3
D 16 m^3

(b)



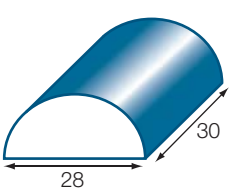
A 1320 m^3
B 1408 m^3
C 6528 m^3
D 7680 m^3

(c)



A 16 m^3
B 27 m^3
C 54 m^3
D 86.4 m^3

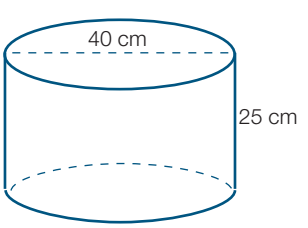
(d)



A 9236 m^3
B 11 760 m^3
C 36 945 m^3
D 79 168 m^3

Reasoning, communicating and open-ended

6 Jenny is making a fruit punch for a party. She is using the cylindrical bowl shown and will fill it to 5 cm from the top. If each of the party cups holds 250 mL, find the number of drinks that can be served from the bowl.



7 A cylindrical water tank has a capacity of 8 kL and a height of 2 m.

(a) Find its radius to the nearest cm.

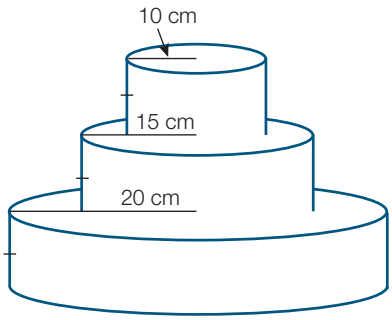
(b) Water starts leaking out of a full tank at the rate of 25 L/day. How many days will it be before the tank is half empty?

8 Find the volume left after a wedge of 52° has been cut from a cheese wheel of diameter 30 cm and height 10 cm. Write your answer to the nearest cm^3.



9 **(a)** Find the volume of a three-tier cake if each tier is 7 cm high and the radii are 20 cm, 15 cm and 10 cm. Give your answer correct to the nearest cm^3.

(b) The baker decides to use the same volume of ingredients to make a cake with a square base of side length 40 cm. How high will it be, correct to the nearest cm?



10 A steel pipe has a diameter of 24 cm and is 1 cm thick. It is made in lengths of 2 m.

(a) What is the capacity of the pipe (to the nearest mL)?

(b) How many cubic centimetres of steel are in the length of pipe (to the nearest cm^3)?

(c) How many kilolitres of water, correct to one decimal place, would pass through one length of pipe in an hour if the water is flowing at a rate of 2 metres per second?



11 Give an example of a substance that we might measure in:

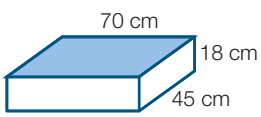
(a) mL **(b)** L **(c)** kL

12 Write the dimensions of any rectangular prism that has a capacity of each of the following.

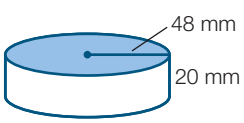
(a) 1 mL **(b)** 1 L **(c)** 1 kL **(d)** 1 ML

13 Sabanth was set the following two questions for homework.

(i) Calculate the volume of this prism, in m^3.



(ii) Calculate the volume of this cylinder, in cm^3.



Here is his workng:

$V = lwh$
$= 70 \times 45 \times 18$
$= 56\,700\ cm^3$
$= 567\ m^3$

$V = \pi r^2 h$
$= \pi \times 48^2 \times 20$
$= 144\,764.6\ mm^3$ (1 d.p.)
$= 14\,476.5\ cm^3$ (1 d.p.)

(a) Identify the error that Sabanth has made.

(b) Give Sabanth a tip to remember, so he can avoid similar errors in the future.

Challenge 3



1 The volume of a cube is 19 683 cm^3. What is its surface area?

2 How many seconds will you have lived by the time you turn 15? (Assume three leap years and the same time of day as your birth.)

3 In a game of soccer, 11 players are on the field and two reserves are interchanged as needed. The game lasts 40 minutes. If each of the 13 members of the team had equal time on the field, how long was each of them off the field (to the nearest second)?

4 Bernard clicked his fingers once. One minute later he clicked them again. Two minutes later he clicked them again, then again 4 minutes later and 8 minutes later. If he continues this pattern, how many times will Bernard have clicked his fingers in one day?

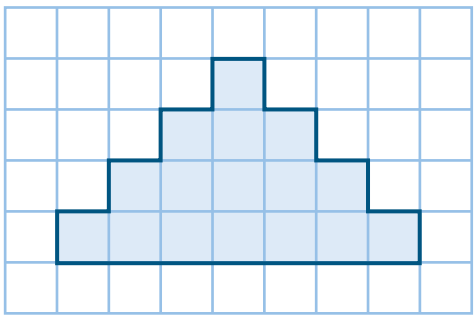
5 How can the contents of a 720-mL jug of cordial be divided evenly between two containers using three unmarked jugs that hold 150 mL, 330 mL and 390 mL?

6 A cube has its side lengths doubled. What happens to its volume? What happens to its surface area?

7 These diagrams are made up of semicircles, with the largest semicircle in each diagram having a diameter of 24 cm. Express the ratio 'perimeter of first figure : perimeter of second figure', in simplest form.

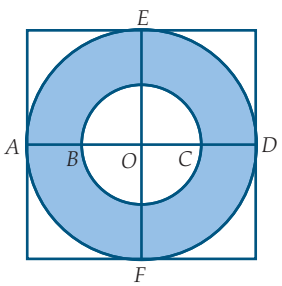


8 The area of the figure on the right is 64 cm^2. What is its perimeter?

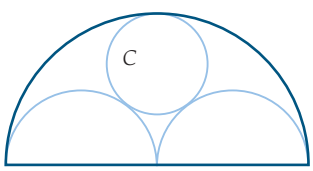


9 *A*, *E*, *D* and *F* are the midpoints of the sides of the square. $AD \perp EF$, *AD* and *EF* intersect at *O* and *B* is the midpoint of *AO*. The circles have radii *OB* and *OA*, respectively. The fraction of the square covered by the shaded area between the circles is:

A $\frac{3\pi}{16}$ **B** $\frac{\pi}{4}$ **C** $\frac{\pi}{16}$ **D** $\frac{1}{2}$



10 Two semicircles of radius 1 cm are drawn on the diameter of a semicircle of radius 2 cm. A circle *C* touches all three semicircles, as shown in the diagram. Calculate the radius of the circle *C*.



Chapter review 3

D.I.Y. Summary

Key Words

Stage 5.1	arc	net	sector
	area	perimeter	solid
	circumference	polyhedron	subtended
	cross-section	prism	surface area
Stage 5.2	capacity	volume	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

Stage 5.1 Q1–6 **Stage 5.2 Q7–10**

1 The word _____ refers to the total distance along the boundary of a figure.

2 The perimeter of a circle is called the _____.

3 _____ is the amount of surface within a boundary. To convert between units, square the conversion factor for length.

4 An _____ is part of the circumference of a circle.

5 A _____ of a circle is the area between two radii and the arc that joins them.

6 A _____ is a solid in which every face is a polygon.

7 To find the _____ of a regular polyhedron, multiply the area of identical faces by the number of times each face appears.

8 The volume of a _____ or cylinder is found by multiplying the area of the base by the height.

9 The _____ of a container is usually measured in millilitres or litres, where $1\text{ L} = 1000\text{ cm}^3$.

10 _____ is the amount of space occupied by a solid. To convert between units, cube the conversion factor for length.

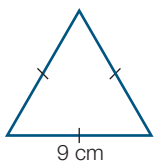
Stage 4 Q1 **Stage 5.1 Q2–7, 10, 11, 12, 15, 16, 18** **Stage 5.2 Q8, 9, 13, 14, 17, 19–22**

Fluency and understanding

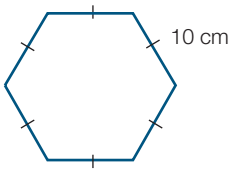
1 Find the perimeters of these shapes.

Ex 3.1

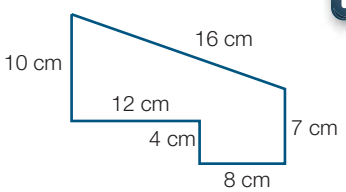
(a)



(b)



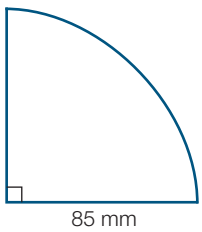
(c)



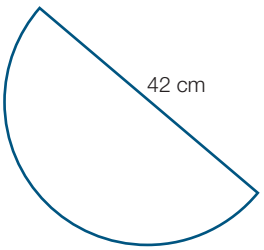
2 Calculate **(i)** the arc length and **(ii)** the perimeter of each sector to the same number of decimal places as the length in each question.

Ex 3.1

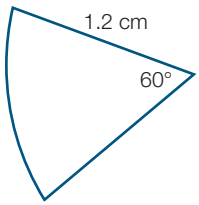
(a)



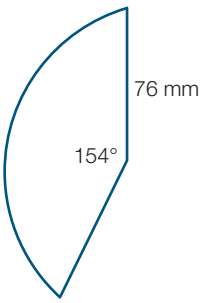
(b)



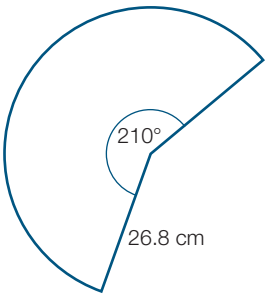
(c)



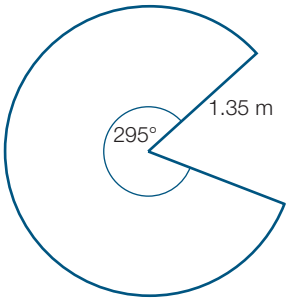
(d)



(e)



(f)



3 Copy and complete the following area conversions.

Ex 3.2

(a) $4.2\text{ cm}^2 =$ _____ mm^2

(b) $20\,000\text{ m}^2 =$ _____ ha

(c) $682\,500\text{ cm}^2 =$ _____ m^2

(d) $528\text{ ha} =$ _____ km^2

(e) $80.7\text{ ha} =$ _____ m^2

(f) $0.009\text{ m}^2 =$ _____ cm^2

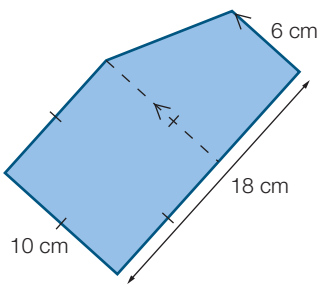
4 Calculate the area of each of the sectors in Question **2**, giving your answers to the same number of decimal places as given for the length.

Ex 3.2

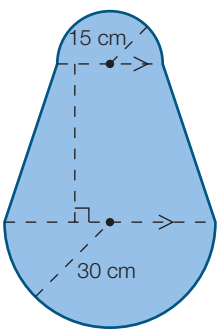
5 Find the shaded areas of the following composite shapes.

Ex 3.2

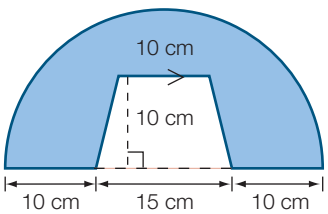
(a)



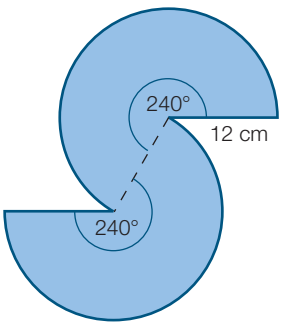
(b)



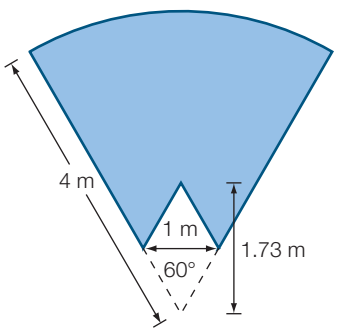
(c)



(d)



(e)

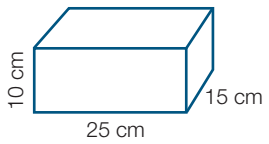


6 Draw the net for a regular tetrahedron.

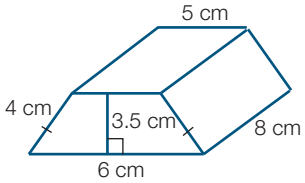
Ex 3.3

7 Find the surface area of each of these solids, correct to two decimal places where necessary. Ex.3.4

(a)

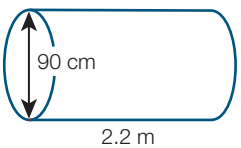


(b)



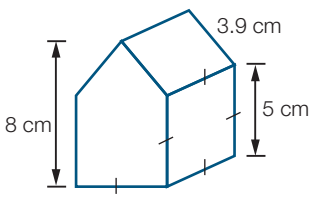
8 Find the surface area of each of these solids, correct to two decimal places where necessary. Ex.3.5

(a)



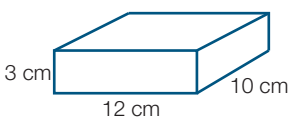
(Answer in m^2.)

(b)

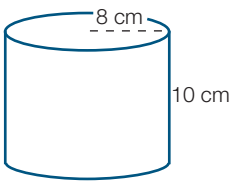


9 Find the volume of the following figures, to the nearest whole number. Ex.3.6

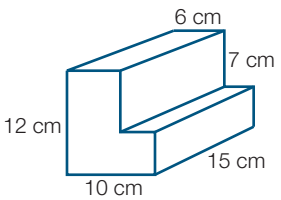
(a)



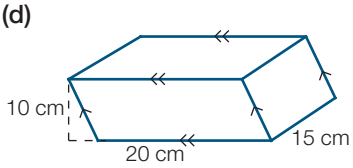
(b)



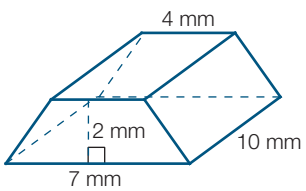
(c)



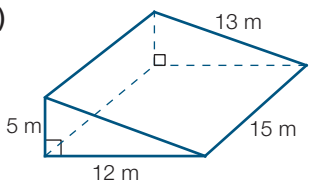
(d)



(e)

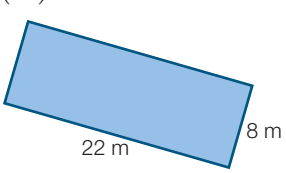


(f)

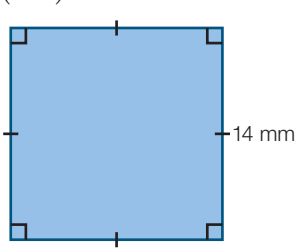


10 Find the area of each figure below in the units indicated in brackets.

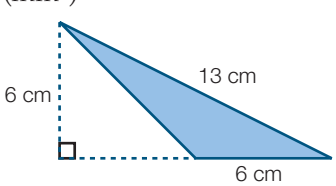
(a) (ha)



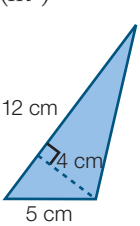
(b) (cm^2)



(c) (mm^2)

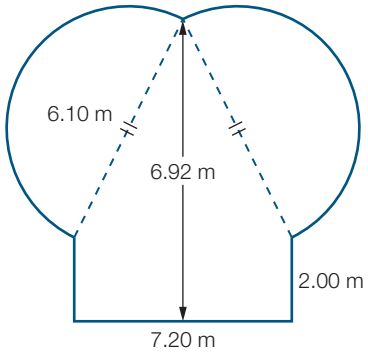


(d) (m^2)

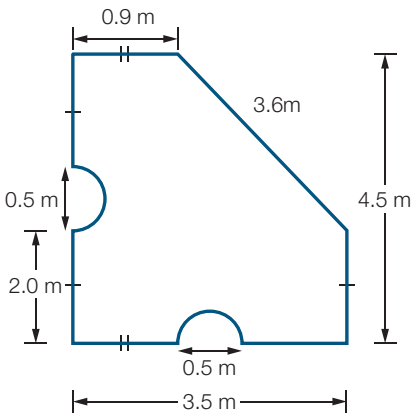


11 Find the area of the following shapes. (Give your answers correct to two decimal places.)

(a)



(b)



12 A farmer planted a crop in a field 300 m long and 500 m wide. One month later, seven 'crop circles', each with a diameter of 21 m, were found one morning, flattening the crop in the circle. What area of crop remained? (Give your answer to the nearest m^2.)

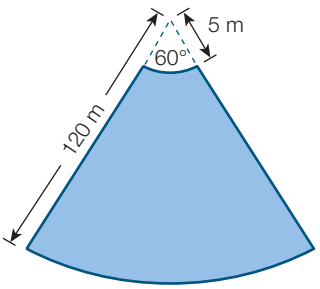
13 Find the capacity of a cylindrical soft drink can that is 5.5 cm wide and 13 cm high. (Give your answer in mL, correct to two decimal places.)

14 A cube has a volume of 512 cm^3. Find its surface area.

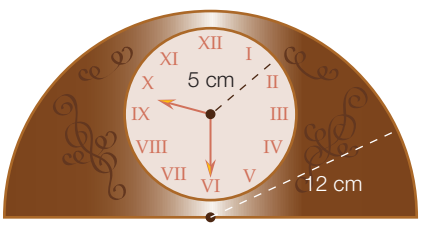
15 A snowball-throwing court has the measurements shown.

(a) Calculate the area that the thrower has to land the snowball in, to the nearest square metre.

(b) Find the perimeter of the shaded area, to the nearest metre.



16 An antique clock has a circular face of radius 5 cm on a semicircular background of radius 12 cm. The area around the clock face is decorated. Find the size of the decorated area to the nearest cm^2.



17 A cylinder is to be made from a piece of sheet metal that is 30 cm square. If the height and diameter of the cylinder are to be 8 cm each, what area of metal is wasted? Write your answer to the nearest whole cm^2.

Reasoning and communicating

18 **(a)** The inside surface of a rectangular swimming pool is to be tiled. If the pool is 12 m long, 5.5 m wide and 2.2 m deep, what is the tiled area?

(b) If the tiles are 12 cm long and 8 cm wide, how many will be needed (to the nearest 100)?

(c) How many litres will the pool hold when it is full?

19 A cylindrical paint tin has a radius of 11 cm and a capacity of 12.16 L.

(a) What is the volume of paint in cm^3?

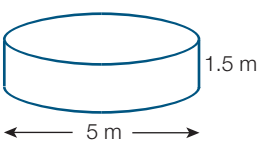
(b) What is the height of the tin to the nearest cm?

(c) If the paint just fills a rectangular paint tray that is 33 cm long and 22 cm wide, how deep is the tray? (Round your answer to two decimal places.)

20 A cylindrical container has a radius of 7 cm and a height of 10 cm. What must be the height of a rectangular container 7 cm long and 10 cm wide if the two containers are to have the same volume? (Answer to the nearest centimetre.)

21 **(a)** How many metric cups (250 mL) of water would it take to fill a water tank of diameter 5 m and depth 1.5 m?

(b) How long will the tank take to fill if a hose delivers 2 cups per second?



22 The hour hand of a clock is 4.5 cm long and the minute hand is 6 cm.

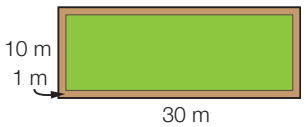
(a) To the nearest cm, how much further does the tip of the minute hand travel in the course of a day (24 hours) than the hour hand?

(b) What is the speed of each tip in centimetres per minute (cm/min)? Write your answer correct to two decimal places.

NAPLAN practice 3

Numeracy: Non-calculator

1 A 30 m × 10 m garden is bordered on the inside by a 1-m wide path. The area of the path in m^2 is:

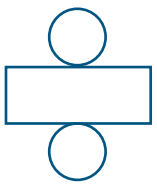


A 39 **B** 72 **C** 76 **D** 80

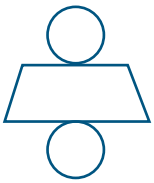
2 A regular tetrahedron is a solid with four equilateral triangles as faces. Find the surface area of a tetrahedron where the base of the triangular face is 6 cm and the height is 5.2 cm.

3 The net that could NOT form a cylinder is:

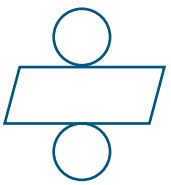
A



B



C



D

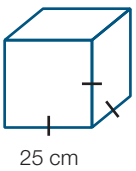


4 A cube has a surface area of 294 cm^2. How long is an edge of the cube (in cm)?

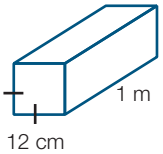
Numeracy: Calculator allowed

5 The shape with the biggest volume is:

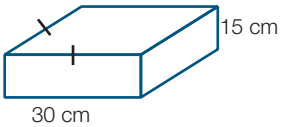
A



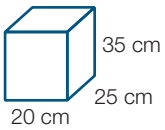
B



C



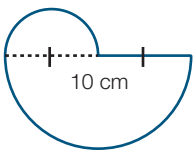
D



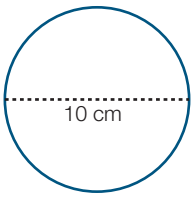
6 The perimeters of the shapes below, in order from smallest to largest, are:

A I, II, III **B** II, III, I **C** I, III, II **D** III, II, I

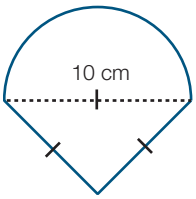
I



II



III



7 One full turn of the wheels on this car moves the car forward 195 cm. What is the diameter of the wheels? (Answer to the nearest cm.)



8 A cylindrical can full of water has a diameter of 50 cm and a height of 50 cm. How many smaller cylinders with a diameter and height of 5 cm can it fill?

A 10 **B** 100 **C** 1000 **D** 10 000

Mixed review A

Stage 4 Q1, 2, 19 Stage 5.1 Q3–14, 20, 21, 23, 25, 26, 27 Stage 5.2 Q15–18, 22, 24

Fluency and understanding

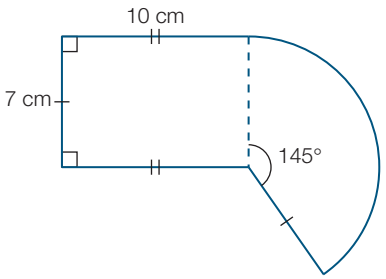
1 Calculate: Ex 1.1

(a) 85% of 250 **(b)** 20% of $0.40 **(c)** 90% of 80

2 Find the percentage that: Ex 1.1

(a) 25 is of 175 **(b)** 90 is of 120 **(c)** 45 is of 360 **(d)** 100 is of 40

3 Find the area of this composite shape. Write your answer to the nearest whole number. Ex 3.2



4 Complete these conversions. Ex 3.2

(a) 20 mm^2 = _____ cm^2 **(b)** 23.67 km^2 = _____ m^2

(c) 0.007 m^2 = _____ cm^2 **(d)** 14.5 ha^2 = _____ m^2

5 Simplify the following expressions. Ex 2.1

(a) $x^3 \times x^5$ **(b)** $x^8 \div x^4$ **(c)** $\frac{x^3 \times x^6}{x^5}$

6 Simplify: Ex 2.1

(a) $10g \times 4h \times 2gh^2$ **(b)** $-12k \times 3k^3$ **(c)** $9ef \times 6e^2 \times 2g$

7 A casual worker earns $9.70 per hour for a standard 8-hour day. He is paid time-and-a-half for the first two hours of overtime on any day and double time for any hours after that. How much will he earn if on one day he works: Ex 1.3

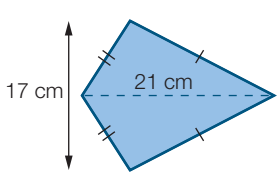
(a) 5 hours **(b)** 8 hours **(c)** 9 hours **(d)** 12 hours?

8 Simplify each of the following. Ex 2.2

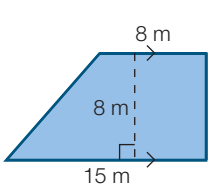
(a) z^0 **(b)** $5z^0$ **(c)** $(5z^0)$ **(d)** $3z^0 + 2z^0$

9 Find the shaded area of each of these shapes, correct to one decimal place where necessary. Ex 3.2

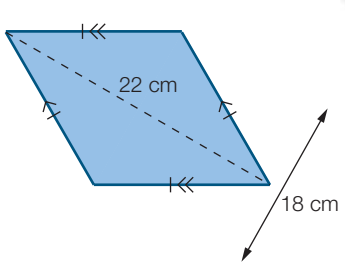
(a)



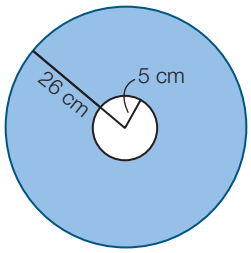
(b)



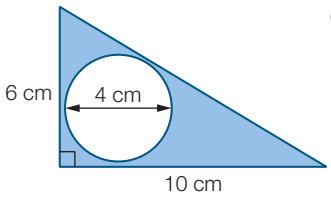
(c)



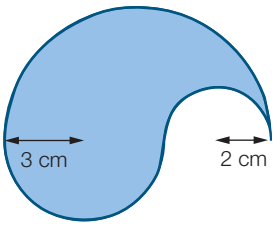
(d)



(e)



(f)



10 Use the following income tax table to find out how much income tax should be paid on the taxable incomes below. Ex.1.4

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3 572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

(a) $6300 **(b)** $18 600 **(c)** $47 650 **(d)** $82 700

11 Write the following in scientific notation. Ex.2.4

(a) 45 000 000 **(b)** 5 270 000 000 **(c)** 0.0003 **(d)** 0.000 000 298

12 **(a)** Find the volume of a cylinder with a base diameter of 24 cm and a height of 16 cm, correct to one decimal place. Ex.3.6

(b) What is the capacity of this cylinder, correct to one decimal place?

13 A department store has a policy that a profit of 87% must be achieved on all furniture sold.

(a) What must a lounge suite, bought by the store for $300, sell for? (Answer to the nearest $10.)

(b) A dining table has a marked price of $899. How much did the store purchase it for? (Answer to the nearest $1.)

14 Simplify: Ex.2.1

(a) $4gh^4 \times 3g^3$ **(b)** $12a^2b^6 \times 2ab^3 \times 5a^4b^2$ **(c)** $\frac{5x^5y^6}{15x^4y}$

15 After investing $8000 in a savings account paying 6.8% interest p.a., Justine received $8321.93. For how many days, correct to the nearest day, was her money invested? Ex.1.5

16 Expand these expressions and simplify where possible. Ex.2.6

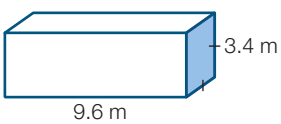
(a) $-8(x + 2)$ **(b)** $5y(6 - 11y)$ **(c)** $9(p + 7) + (3p - 5)$

17 Factorise: Ex.2.7

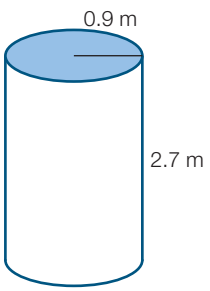
(a) $4m - 28$ **(b)** $7ed^2 + 35d$ **(c)** $8xy^2 - 2y$

18 Calculate the **(i)** surface area and **(ii)** volume of the following solids. State answers correct to two decimal places. Ex.3.5, 3.6

(a)



(b)



Ex 2.9

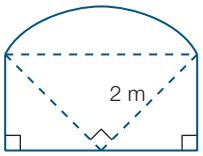
19 Expand these expressions and simplify where possible.

(a) $(x-3)(x+3)$ **(b)** $(x+6)^2$ **(c)** $(2x-5)^2$ **(d)** $(x+8)(x-8)$

20 Simplify:

(a) $\dfrac{3m^5n^6 \times 4mn^2}{2m^2n^5 \times 3m^4n^3}$ **(b)** $\dfrac{5p^7q^4}{3p^3q} \div \dfrac{10p^5q^2}{9p^6q^6}$ **(c)** $\dfrac{5(x^3)^4y^6}{(x^2y)^2} \times \left(\dfrac{xy^3}{x^2y^2}\right)^3$

21 Find the area of the following shape.
Write your answer correct to one decimal place.



22 Kinetic energy is the energy (E) of a moving object, measured in joules (J). It is calculated by the formula $E = \dfrac{mv^2}{2}$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Rearrange the formula for kinetic energy to make mass the subject.

(b) Use your rearranged formula to find the mass of a bowling ball that has a kinetic energy of 32.4 J and a velocity of 3.5 m/s. Round your answer to one decimal place.

23 Mario is paid a retainer of $250 per week plus a commission of 9.5% on the value of sales. Calculate the income he would earn in one week if he sold goods to the value of:

(a) $1200 **(b)** $2500 **(c)** $3750 **(d)** $4210

24 **(a)** A cylindrical water tank has a capacity of 5000 L. If it has a height of 2.5 m, find its base radius.

(b) If it was emptied at a rate of 225 litres per day, how many days would it take for the tank to be less than half full?

Reasoning and communicating

25 What happens to the total surface area of a rectangular prism if you multiply each dimension by two?

26 When employed as a sales assistant, Bettina was given a choice as to how she would be paid.

Option A: A retainer of $150 per week plus 5% of sales.

Option B: No retainer but 7.5% of sales.

(a) How much would Bettina need to sell to earn at least $950 per week under each of the options?

(b) What value of sales gives the same result regardless of the option chosen?

(c) When would it be better for Bettina to choose Option A?

27 Six circular discs of radius 5 cm fit snugly into the bottom of a rectangular box. Find the percentage of empty space at the bottom of the box, correct to two decimal places, if the discs are arranged in:

(a) one row of six **(b)** two rows of three.

(c) Which box is the more efficient container?

4 Linear relationships

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 **(a)** If $P = 2k + 5$, find P when $k = -9$. **(b)** If $C = 9 - 4d$, find C when $d = 4$.

2 Plot these coordinate points on a Cartesian plane and label them.

(a) $A(1, 2)$ **(b)** $B(0, 3)$ **(c)** $C(-2, 0)$ **(d)** $D(-4, 2)$

3 Complete a table of values for each of the following equations then plot the graph of each.

x	-2	-1	0	1	2
y					

(a) $y = x + 4$ **(b)** $y = x - 2$ **(c)** $y = 2x$

4 Solve the following equations to find the value of the variable.

(a) $x + 1 = 7$ **(b)** $3 + y = 18$ **(c)** $a + 4 = -6$ **(d)** $b - 5 = 16$

5 Solve the following equations to find the value of the variable.

(a) $2g = 38$ **(b)** $0.5x = 21$ **(c)** $-9y = -54$ **(d)** $\frac{x}{7} = 5$

6 Expand the brackets and simplify.

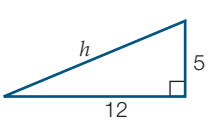
(a) $-3(5x + 2)$ **(b)** $2(7 - 3x) + 6$ **(c)** $3x + 1 + 6(2x + 7)$

7 Find the rule connecting x and y in the given table of values.

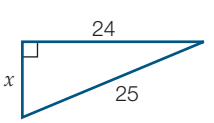
x	0	1	2	3	4	5
y	6	4	2	0	-2	-4

8 Use Pythagoras' Theorem to find the unknown sides in the following triangles.

(a)



(b)



Syllabus references

Number and Algebra: Linear Relationships

- ✓ Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software **(ACMNA294) (Stage 5.1)**
- ✓ Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software **(ACMNA214) (Stage 5.1)**
- ✓ Solve problems involving parallel lines **(ACMNA238) (Stage 5.1)**
- ✓ Sketch linear graphs using the coordinates of two points **(ACMNA215) (Stage 5.1)**
- ✓ Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line **(Stage 5.2)**
- ✓ Solve problems involving parallel and perpendicular lines **(ACMNA238) (Stage 5.2)**
- ✓ Find the midpoint and gradient of a line segment (interval) on the Cartesian plane **(ACMNA294) (Stage 5.3§)**
- ✓ Find the distance between two points located on the Cartesian plane **(ACMNA214) (Stage 5.3§)**
- ✓ Sketch linear graphs using the coordinates of two points **(ACMNA215) (Stage 5.3§)**
- ✓ Solve problems using various standard forms of the equation of a straight line **(Stage 5.3§)**

Number and Algebra: Equations

- ✓ Solve linear equations **(ACMNA215) (Stage 5.2)**
- ✓ Solve linear equations involving simple algebraic fractions **(ACMNA240) (Stage 5.2)**
- ✓ Solve problems involving linear equations, including those derived from formulas **(ACMNA235) (Stage 5.2)**
- ✓ Substitute values into formulas to determine an unknown **(ACMNA234) (Stage 5.2)**

Introducing coordinate geometry

4.1

Stage 5.1

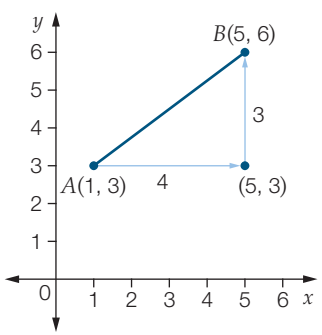
Need to Know

- The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.
- The point of intersection of these axes is called the **origin**, which has the **coordinates** (0, 0).
- All points can be described by an ordered pair known as coordinates (x, y) as measured from the origin.
- An **interval** is the line segment formed when two points are joined.

The distance between two points

Pythagoras' Theorem can be used to find the distance between two points on a number plane.

For example, to find the distance between (1, 3) and (5, 6), plot the points on a Cartesian plane, join the points with a straight line and then draw a **right-angled triangle** by moving 4 units right and 3 units up from the point (1, 3) to reach (5, 6). Pythagoras' Theorem can be used to find the distance between (1, 3) and (5, 6).

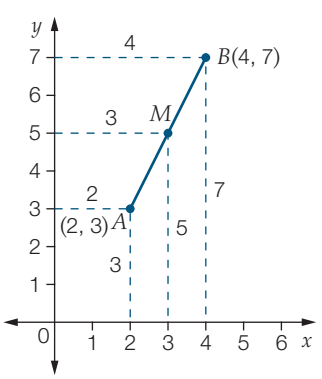


$$\text{Distance} = \sqrt{4^2 + 3^2}$$
$$= 5$$

The midpoint of an interval

The **midpoint** of an interval is halfway between the two points that form the interval. The average of the x-coordinates of the two points gives the x-coordinate of the midpoint and the average of the y-coordinates of the two points gives the y-coordinate of the midpoint.

For example, for points (2, 3) and (4, 7), plot the points on a Cartesian plane, join the points with a straight line then draw dotted lines parallel to the axes from the points as shown on the diagram. The coordinates of the midpoint lie halfway between the coordinates of the two points.



The x-coordinate of the midpoint is 3, the average of the x-coordinates 2 and 4. The y-coordinate of the midpoint is 5, the average of the y-coordinates 3 and 7. So the coordinates of the midpoint of the interval joining (2, 3) and (4, 7) are (3, 5).

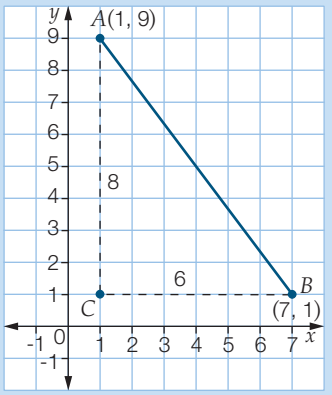
$$x\text{-coordinate of midpoint} = \frac{2 + 4}{2}$$
$$= 3$$
$$y\text{-coordinate of midpoint} = \frac{3 + 7}{2}$$
$$= 5$$

Worked Example 1

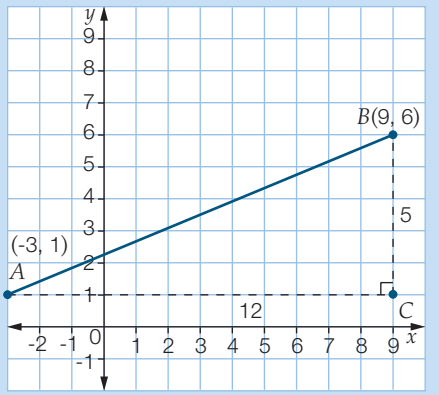
WE1

Use Pythagoras' Theorem to find the distance between points A and B for each of the following.

(a)



(b)



Working

(a) $AB^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100$

$AB = \sqrt{100}$
$= 10$

(b) $AB^2 = 12^2 + 5^2$
$= 144 + 25$
$= 169$

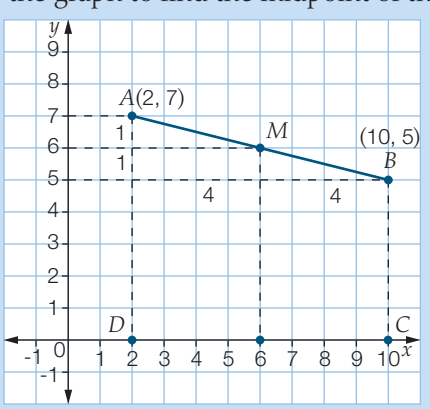
$AB = \sqrt{169}$
$= 13$

Worked Example 2

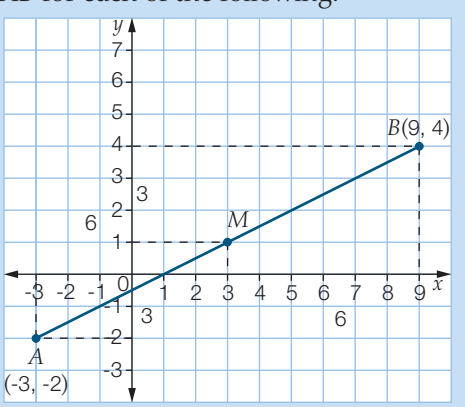
WE2

Use the graph to find the midpoint of the interval AB for each of the following.

(a)



(b)



Working

(a) The average of 2 and 10 $= \frac{2+10}{2}$
$= 6$

The average of 7 and 5 $= \frac{7+5}{2}$
$= 6$
$M = (6, 6)$

(b) The average of -3 and 9 $= \frac{-3+9}{2}$
$= 3$

The average of -2 and 4 $= \frac{-2+4}{2}$
$= 1$
$M = (3, 1)$

Technology Exploration: GeoGebra

Finding the distance between two points and the midpoint of the line joining them

(a) Open GeoGebra and use the Interval between Two Points tool to draw an interval between the points $A(2, 1)$ and $B(6, 4)$. Use the Distance or Length tool. The length of the interval will appear on the screen. Select the Move tool to move either of the points around. The distance will change accordingly.

(b) Place a point C anywhere on the interval joining points A and B. Use the Distance or Length tool to measure the distances AC and BC. Use the Move tool to move the point along the line until the two distance measurements are equal. The coordinates of the point will be shown in the Algebra view. This will be the midpoint between the two points. Use the Move tool to move A and/or B around. The Point C will remain at the midpoint and its new coordinates will be shown in the Algebra view.

4.1 Introducing coordinate geometry

Answers page 437

Navigator

Stage 5.1

Q1, Q2, Q3, Q4, Q5, Q8, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9

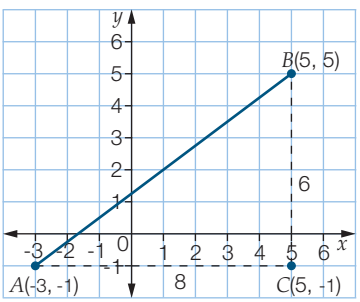
Equipment needed: Graph or grid paper

Fluency and understanding

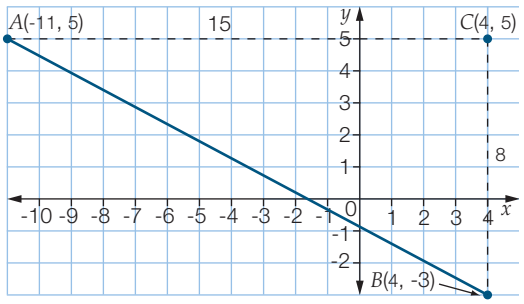
1 Use Pythagoras' Theorem to find the distance between points A and B for each of the following.

WE1

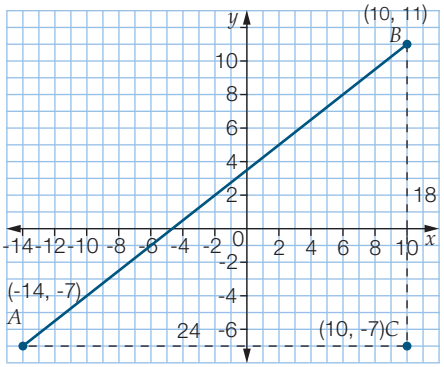
(a)



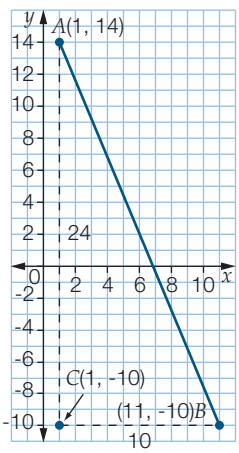
(b)



(c)



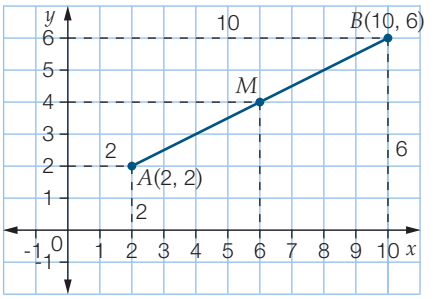
(d)



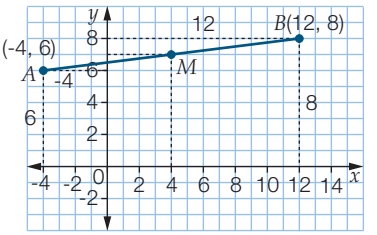
WE2

2 Use the graph to find the midpoint of the interval AB for each of the following.

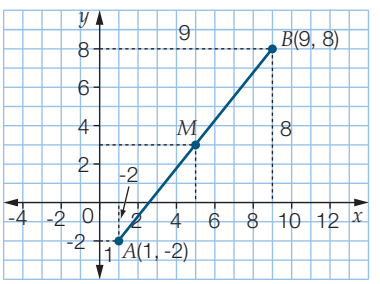
(a)



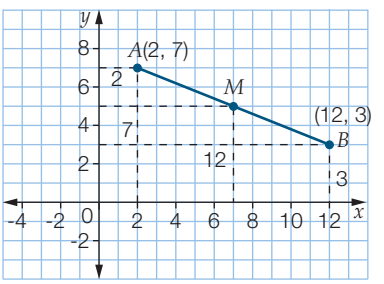
(b)



(c)



(d)



3 The distance between (1, -5) and (6, 7) is:

A 4 **B** 17 **C** 13 **D** 7

4 A line AB is drawn from Point A at (3, 14) to Point B (7, 10). Point C lies on the line AB halfway between points A and B. Find the coordinates of Point C.

5 Find the coordinates of the midpoint of the line joining (-4, 6) and (2, -2).

6 Use a diagram and Pythagoras' Theorem to show that the points $A(1, 2)$, $B(4, 6)$ and $C(8, 9)$ form an isosceles triangle.

Reasoning, communicating and open-ended

7 A quadrilateral has vertices $A(0, 2)$, $B(3, 6)$, $C(15, 1)$ and $D(12, -3)$.

(a) Find the length of each side.

(b) Find the midpoints of the diagonals.

(c) What type of quadrilateral is quadrilateral $ABCD$?

8 Prove and explain why the points $A(1, 4)$, $B(-4, -8)$, $C(9, -8)$ and $D(14, 4)$ are the vertices of a rhombus.

9 Choose three points that do not lie on the x- or y-axes that would form an isosceles triangle when joined in order and justify your choice.

Using coordinate geometry formulas

4.2

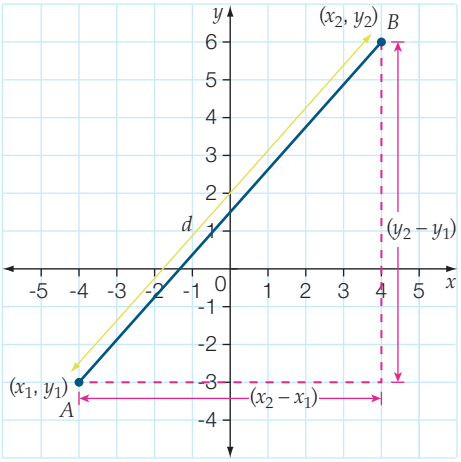
Stage 5.3 §

Need to Know

The distance between two points

- The difference between the x-coordinates $(x_2 - x_1)$ gives the length of one side of a right-angled triangle.
- The difference between the y-coordinates $(y_2 - y_1)$ gives the length of another side.
- These two lines form two of the sides of a right-angled triangle, with the distance between the two points, d, the length of the hypotenuse.
- We use Pythagoras' Theorem to find the distance between the two points.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



The distance, d, between any two Cartesian coordinates (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Exact and approximate answers

The exact distance between two points is often a surd, an irrational number.

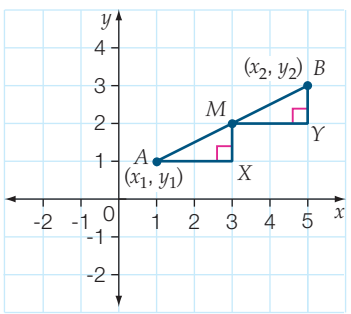
For example, $\sqrt{35}$.

Unless an approximate answer is required, answers should always be left in exact form.

The midpoint of an interval

The midpoint of an interval is halfway between the two points that form the interval. The average of the x-coordinates of the two points gives the x-coordinate of the midpoint and the average of the y-coordinates of the two points gives the y-coordinate of the midpoint.

For points (x_1, y_1) and (x_2, y_2), the average of x_1 and x_2 is $\frac{x_1 + x_2}{2}$ and the average of y_1 and y_2 is $\frac{y_1 + y_2}{2}$.



The midpoint $M(x, y)$ between two points (x_1, y_1) and (x_2, y_2) is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Worked Example 3

WE3

Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form.

(a) (-4, 6) and (2, -2) **(b)** (-2, 1) and (2, -4) **(c)** (3, 5) and (5, 2)

Working

(a) Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - -4)^2 + (-2 - 6)^2}$$
$$= \sqrt{6^2 + (-8)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

(b) Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (2, -4)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - -2)^2 + (-4 - 1)^2}$$
$$= \sqrt{4^2 + (-5)^2}$$
$$= \sqrt{41} \text{ units}$$

(c) Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (5, 2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - 3)^2 + (2 - 5)^2}$$
$$= \sqrt{(2)^2 + (-3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13} \text{ units}$$

Worked Example 4

WE4

Find the coordinates of the midpoint between (-4, 6) and (2, -2).

Working

Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$M(x, y) = \left(\frac{-4 + 2}{2}, \frac{6 + -2}{2}\right)$$
$$M(x, y) = \left(\frac{-2}{2}, \frac{4}{2}\right)$$
$$M(x, y) = (-1, 2)$$

4.2 Using coordinate geometry formulas

Answers page 437

Stage 5.3 §

Navigator

Q1 Column 1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q14	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q14	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Fluency and understanding

1 Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form. WE3

(a) (2, 5) and (5, 9) **(b)** (-4, 2) and (8, -3) **(c)** (-4, 13) and (4, -2)

(d) (-6, 34) and (4, 10) **(e)** (-4, 5) and (4, -1) **(f)** (-11, -4) and (13, 3)

(g) (2, -5) and (3, -1) **(h)** (-1, 3) and (5, 2) **(i)** (4, -3) and (-2, 2)

(j) (-3, -5) and (-1, -7) **(k)** (3, -4) and (-1, 2) **(l)** (2, -7) and (6, 1)

2 Find the coordinates of the midpoints between the following Cartesian points. WE4

(a) (-3, 4) and (0, 0) **(b)** (-3, 5) and (1, 3) **(c)** (-9, -11) and (7, -3)

(d) (-1, 1) and (2, 3) **(e)** (-5, 8) and (3, 7) **(f)** (-2, -4) and (5, 3)

(g) (1, 2) and (6, 7) **(h)** (-2, -5) and (7, 0) **(i)** (0, 0) and (9, 11)

3 The distance between (-4, -5) and (4, 1) is:

A 4 **B** 5 **C** 10 **D** 9

4 A line AB is drawn from Point A at (-2, 10) to Point B (4, 1). Point C lies on the line AB halfway between Points A and B. Find the coordinates of Point C.

5 **(a)** The midpoint of CD is (6, 5). If the coordinates of Point C are (3, 8), find the coordinates of Point D.

(b) The midpoint of EF is (-1, -2). If the coordinates of Point E are (-7, 6), find the coordinates of Point F.

(c) The midpoint of AB is (2, -4). If the coordinates of Point A are (-5, -8), what are the coordinates of Point B?

6 Use the distance formula and Pythagoras' Theorem to show that the points A(-3, -2), B(1, 1) and C(-2, 5) form an isosceles right-angled triangle.

7 A triangle has the vertices (0, 5), (3, 3) and (1, 0).

(a) Find the length of each side, correct to two decimal places.

(b) What type of triangle is it?

Reasoning, communicating and open-ended

Hint

Plot the known points on a Cartesian plane.

8 The proposed course for a yacht race has been mapped out with buoys positioned at $A(-3, -4)$, $B(-1, 3)$ and $C(9, 8)$, but the position of the last buoy $D(x, y)$ is yet to be finalised. The race will begin at position $O(0, 0)$, proceed through buoys A, B, C and D and finish back at the starting point O.

(a) If the coordinates indicate distances measured in kilometres, what is the distance from the starting position O to the first buoy A?

(b) The judge's boat must be placed in the middle of buoys A and C. What are the coordinates of the judge's boat?

(c) If the judge's boat is also in the middle of buoys B and D, as with A and C, what are the coordinates (x, y) of buoy D?

(d) What is the greatest distance that the yachts must travel between any two successive buoys?

(e) What geometric shape is formed by the buoys $ABCD$?

9 A quadrilateral has the vertices $A(0, -2)$, $B(-2, -6)$, $C(-6, -8)$ and $D(-4, -4)$.

(a) Find the midpoints of the diagonals.

(b) Find the length of each side.

(c) Find the length of each diagonal.

(d) What type of quadrilateral is it?

10 Prove and explain how the points $A(3, 5)$, $B(5, 1)$, $C(-1, -2)$ and $D(-3, 2)$ are the vertices of a rectangle.

11 If $E(1, 6)$, $F(-4, 3)$ and $G(-1, -2)$ are the vertices of a square, find the missing vertex.

12 Points $A(-6, 3)$, $B(2, 5)$, $C(0, -1)$ and $D(-2, 1)$ are joined to form a quadrilateral. The midpoints of AB, BC, CD, and DA are joined to form another quadrilateral $PQRS$. Show that $PQRS$ is a parallelogram.

13 Choose three points that would form an equilateral triangle when joined in order and justify your choice.

14 Choose four points that would form a kite when joined in order and justify your choice.

Solving linear equations

4.3

Stage 5.2

Need to Know

Linear relationships

A **linear relationship** between two variables exists if a uniform change in one variable, the **independent variable**, causes a uniform change in the other, the **dependent variable**.

A linear relationship produces a **linear graph** (or **straight-line graph**) and is represented by the equation $y = mx + b$.

Linear equations

A **linear equation** is a linear relationship where the dependent variable has a specific value.

We **solve** linear equations to find the value of the independent variable. A linear equation has only one solution. Linear graphs can be used to solve linear equations.

Solving linear equations

- Identify the operations used to construct the equation and use **inverse operations** to deconstruct the equation in the opposite order to the way it was constructed.
- If there is more than one term containing the unknown, collect those terms together on one side of the equation. Collect all numerical values on the other side.
- If fractions are involved, express each term with a common denominator and then eliminate the fractions by multiplying each term on both sides by the common denominator.
- If brackets are involved, expand the brackets.

Worked Example 5

WE 5

Solve the following linear equations. Check your solutions by substitution.

(a) $3x - 5 = -11$ **(b)** $7 - 2x = 19$

Working

(a)
$$3x - 5 = -11$$
$$3x - 5 + 5 = -11 + 5$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$

Check: LHS $= 3 \times -2 - 5$
$= -6 - 5$
$= -11$
$=$ RHS

(b)
$$7 - 2x = 19$$
$$-2x + 7 = 19$$
$$-2x + 7 - 7 = 19 - 7$$
$$-2x = 12$$
$$\frac{-2x}{-2} = \frac{12}{-2}$$
$$x = -6$$

Check: LHS $= 7 - 2 \times -6$
$= 7 + 12$
$= 19$
$=$ RHS

Worked Example 6

WE6

Solve the following linear equations. Check your solutions by substitution.

(a) $\frac{2x}{5} - 1 = 7$

(b) $\frac{2x+4}{3} = 4$

Working

(a)
$$\frac{2x}{5} - 1 = 7$$
$$\frac{2x}{5} - 1 + 1 = 7 + 1$$
$$\frac{2x}{5} = 8$$
$$\frac{2x}{5} \times 5 = 8 \times 5$$
$$2x = 40$$
$$\frac{2x}{2} = \frac{40}{2}$$
$$x = 20$$

Check: LHS $= \frac{2 \times 20}{5} - 1$
$= \frac{40}{5} - 1$
$= 8 - 1$
$= 7$
$=$ RHS

(b)
$$\frac{2x+4}{3} = 4$$
$$\frac{3 \times (2x+4)}{3} = 4 \times 3$$
$$2x + 4 = 12$$
$$2x + 4 - 4 = 12 - 4$$
$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

Check: LHS $= \frac{2 \times 4 + 4}{3}$
$= \frac{8+4}{3}$
$= \frac{12}{3}$
$= 4$
$=$ RHS

Worked Example 7

WE7

Solve the linear equation $5x - 2 = 3x + 8$. Check your solution by substitution.

Working

$$5x - 2 = 3x + 8$$
$$5x - 3x - 2 = 3x - 3x + 8$$
$$2x - 2 = 8$$
$$2x - 2 + 2 = 8 + 2$$
$$2x = 10$$
$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

Check: LHS $= 5 \times 5 - 2$
$= 25 - 2$
$= 23$

RHS $= 3 \times 5 + 8$
$= 15 + 8$
$= 23$

LHS $=$ RHS

Worked Example 8

WE8

Solve the linear equation $3(x + 4) = 2(1 - x)$. Check your solution by substitution.

Working

$$3(x + 4) = 2(1 - x)$$
$$3x + 12 = 2 - 2x$$
$$3x + 2x + 12 = 2 - 2x + 2x$$
$$5x + 12 = 2$$
$$5x + 12 - 12 = 2 - 12$$
$$5x = -10$$
$$\frac{5x}{5} = \frac{-10}{5}$$
$$x = -2$$

Check: LHS $= 3(-2 + 4)$
$= 6$

RHS $= 2(1 - -2)$
$= 6$

LHS = RHS

Worked Example 9

WE9

Solve the linear equation $\frac{2a - 1}{5} = \frac{a + 7}{4}$. Check your solution by substitution.

Working

$$\frac{2a - 1}{5} = \frac{a + 7}{4}$$
$$\frac{4(2a - 1)}{20} = \frac{5(a + 7)}{20}$$
$$\frac{4(2a - 1)}{20} \times 20 = \frac{5(a + 7)}{20} \times 20$$
$$4(2a - 1) = 5(a + 7)$$
$$8a - 4 = 5a + 35$$
$$8a - 5a - 4 = 5a - 5a + 35$$
$$3a - 4 = 35$$
$$3a - 4 + 4 = 35 + 4$$
$$3a = 39$$
$$\frac{3a}{3} = \frac{39}{3}$$
$$a = 13$$

Check: LHS $= \frac{2 \times 13 - 1}{5}$
$= \frac{26 - 1}{5}$
$= \frac{25}{5}$
$= 5$

RHS $= \frac{13 + 7}{4}$
$= \frac{20}{4}$
$= 5$

LHS = RHS

Answers page 438

4.3 Solving linear equations

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q6 Column 1, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q13 Column 1, Q14, Q17 (a), Q18, Q20	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12 Column 2, Q13 Column 2, Q14, Q15, Q17 (a) & (b), Q18, Q20, Q21	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5 Column 2, Q6 Column 3, Q7, Q8, Q9, Q10, Q12 Column 3, Q13 Column 3, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21

Fluency and understanding

WE5

1 Solve the following linear equations. Check your solutions by substitution.

(a) $2h + 9 = 5$ **(b)** $5b + 3 = 28$ **(c)** $6x + 7 = 1$

(d) $6r - 3 = 2$ **(e)** $3f - 2 = -14$ **(f)** $5k - 3 = -1$

(g) $12 = 7x + 6$ **(h)** $-5 = 2x + 13$ **(i)** $-2 = 9y - 2$

(j) $-3b + 14 = -4$ **(k)** $-2a - 4 = 0$ **(l)** $-5b - 6 = 9$

(m) $-7 - p = 8$ **(n)** $15 - 4e = -1$ **(o)** $9 - 4g = -23$

(p) $4 = 17 - 5y$ **(q)** $-3 = 8 - 4p$ **(r)** $-5 = 11 - 8y$

WE6

2 Solve these linear equations. Check your solutions by substitution.

(a) $\frac{2x}{3} + 9 = 13$ **(b)** $\frac{3x}{7} + 5 = 8$ **(c)** $\frac{5x}{3} - 8 = 7$

(d) $4 - \frac{2x}{3} = 10$ **(e)** $8 - \frac{3x}{4} = -1$ **(f)** $12 - \frac{3x}{2} = 3$

(g) $\frac{2x + 3}{11} = 1$ **(h)** $\frac{3x + 1}{5} = -1$ **(i)** $\frac{7x + 4}{6} = 3$

(j) $\frac{1 - 7x}{4} = -5$ **(k)** $\frac{2 - 3x}{5} = 1$ **(l)** $\frac{9 - 4x}{3} = -5$

(m) $\frac{4x - 6}{7} = -2$ **(n)** $\frac{7x - 4}{5} = -12$ **(o)** $\frac{3x - 1}{8} = -2$

WE7

3 Solve the following linear equations. Check your solutions by substitution.

(a) $7c + 2 = 6c - 5$ **(b)** $4b + 1 = 2b - 15$ **(c)** $9f + 5 = 5f - 3$

(d) $5d - 4 = 2d + 17$ **(e)** $5e - 3 = 2e + 9$ **(f)** $3a - 5 = a + 7$

(g) $3g + 12 = 5 + 2g$ **(h)** $2h - 7 = 13 + h$ **(i)** $4x - 3 = 6 + x$

(j) $k + 6 = 9 - k$ **(k)** $10j + 1 = 25 - 2j$ **(l)** $3y - 22 = -4 - 3y$

(m) $2m + 5 = 4m - 3$ **(n)** $5 + 4n = 7n - 1$ **(o)** $7x - 4 = 9x - 6$

(p) $2 - y = 3 - 2y$ **(q)** $19 - p = 16 - 2p$ **(r)** $6 + 3x = 21 - 2x$

(s) $15a - 17 = 16a - 19$ **(t)** $14 - 3x = 5 - 2x$ **(u)** $30 - p = 5p + 6$

WE8

4 Solve the following linear equations. Check your solutions by substitution.

(a) $3(x - 2) = 2(x + 1)$ **(b)** $7(x + 4) = 3(x - 4)$ **(c)** $4(x - 5) = 5(3x + 7)$

(d) $5(2x + 3) = 9(x + 1)$ **(e)** $5(2x + 3) = 2(3x + 1)$ **(f)** $4(3x - 2) = 3(2x + 9)$

(g) $2(4x + 7) = 3(5 - x)$ **(h)** $3(5x + 4) = 4(2 - 3x)$ **(i)** $4(3x - 11) = 5(29 - 3x)$

5 Solve the following linear equations. Check your solutions by substitution.

WE9

(a) $\frac{b+4}{3} = \frac{b-8}{5}$

(b) $\frac{a-3}{2} = \frac{a+6}{3}$

(c) $\frac{c+7}{4} = \frac{c+2}{2}$

(d) $\frac{d+5}{3} = \frac{d-1}{9}$

(e) $\frac{2x-3}{4} = \frac{x+6}{3}$

(f) $\frac{3y-2}{7} = \frac{y+6}{4}$

(g) $\frac{5g-1}{4} = \frac{7-2g}{5}$

(h) $\frac{7k-3}{4} = \frac{13-5k}{8}$

(i) $\frac{4-3m}{2} = \frac{1-5m}{3}$

(j) $\frac{2-3j}{5} = \frac{3-4j}{6}$

(k) $\frac{8x+3}{2} = 3x-4$

(l) $\frac{6p+4}{11} = p+9$

(m) $\frac{2(3x-5)}{3} = \frac{3(5x+1)}{4}$

(n) $\frac{4(2y+3)}{5} = \frac{5(2-y)}{2}$

6 Solve these equations. Check your solutions by substitution.

(a) $\frac{x}{2} - 3 = 0$

(b) $\frac{x}{8} - 6 = -5$

(c) $\frac{x}{9} + 3 = 3$

(d) $3 - \frac{x}{4} = 12$

(e) $5 - \frac{x}{7} = 11$

(f) $-4 - \frac{x}{3} = -8$

(g) $\frac{4x}{5} = 3$

(h) $\frac{7x}{13} = 0$

(i) $\frac{2x}{7} = -4$

(j) $\frac{x+3}{2} = 9$

(k) $\frac{x+7}{3} = 5$

(l) $\frac{x+5}{9} = 3$

(m) $\frac{x-6}{2} = 8$

(n) $\frac{x-9}{8} = 6$

(o) $\frac{x-4}{3} = 17$

7 (a) To obtain a from $2a + 7$ you would need to:

A add 7 then divide by 2 **B** subtract 7 then divide by 2

C divide by 2 then subtract 7 **D** multiply by 2 then add 7

(b) To obtain p from $5p - 4$ you would need to:

A add 4 then multiply by 5 **B** add 4 then divide by 5

C divide by 5 then add 4 **D** multiply by 5 then add 4

(c) To obtain x from $-6x - 11$ you would need to:

A add 11 then multiply by -6 **B** multiply by -6 then add 11

C add 11 then divide by -6 **D** subtract 11 then divide by -6

(d) To obtain n from $1 - 4n$ you would need to:

A add 4 then multiply by 1 **B** add 1 then divide by -4

C subtract 1 then divide by -4 **D** subtract 1 then multiply by -4

8 **(a)** The equation $2z + 1 = 11$ has the solution:

A $z = 4$ **B** $z = 5$ **C** $z = 6$ **D** $z = -5$

(b) The equation $4x - 5 = 3$ has the solution:

A $x = 3$ **B** $x = -1$ **C** $x = 2$ **D** $x = 0$

(c) The equation $7 - 3q = 22$ has the solution:

A $q = 4$ **B** $q = -3$ **C** $q = 5$ **D** $q = -5$

(d) The equation $5 - 8n = -11$ has the solution:

A $n = 2$ **B** $n = -3$ **C** $n = 0$ **D** $n = -2$

(e) The equation $\frac{r}{3} - 4 = 2$ has the solution:

A $r = 10$ **B** $r = -6$ **C** $r = 9$ **D** $r = 18$

(f) The equation $\frac{a + 3}{4} = 6$ has the solution:

A $a = 5$ **B** $a = 7$ **C** $a = 21$ **D** $a = 27$

9 **(a)** The linear equation $\frac{2a}{5} - 3 = 1$ has the solution:

A $a = -10$ **B** $a = -5$ **C** $a = 4$ **D** $a = 10$

(b) The linear equation $\frac{4y}{3} + 7 = -1$ has the solution:

A $y = -9$ **B** $y = -6$ **C** $y = 6$ **D** $y = 9$

10 **(a)** $9x + 4 = 2x - 3$ has the solution:

A $x = 1$ **B** $x = -1$ **C** $x = 2$ **D** $x = 4$

(b) $2y + 7 = -8 - 3y$ has the solution:

A $y = 2$ **B** $y = 1$ **C** $y = -2$ **D** $y = -3$

(c) $4(3k - 2) = 5(k + 4)$ has the solution:

A $k = 3$ **B** $k = -2$ **C** $k = 1$ **D** $k = 4$

(d) $\frac{p - 2}{2} = \frac{p + 1}{3}$ has the solution:

A $p = 8$ **B** $p = 5$ **C** $p = -4$ **D** $p = 0$

11 Write an equation for each of the following and then solve the equation.

(a) 7 is added to x to give a result of 19.

(b) 5 is subtracted from y to give a result of 3.

(c) Three times a certain number is 14.

(d) One-quarter of a certain number is -5.

(e) Three more than two times a number is 13.

(f) 7 less than four times a number is -12.

12 Solve these equations. (First simplify each equation.)

(a) $3x + 2x + 7 = 32$ **(b)** $2x + x - 4 = -10$ **(c)** $6x + 3x - 4 = 14$

(d) $3(x + 5) + 8 = -4$ **(e)** $2(x + 3) + 1 = -5$ **(f)** $5(x - 2) - 3 = 7$

(g) $3(7 - x) - 6 = 0$ **(h)** $8(2 - x) + 1 = 9$ **(i)** $-4(3 - x) + 8 = -12$

(j) $-2(1 - 4x) - 3x = -2$ **(k)** $-7(5 - 2x) - 6x = 13$ **(l)** $-(8 - 5x) + 4x = -26$

13 Solve these linear equations.

(a) $\frac{4(2x+3)}{7} = 1$ **(b)** $\frac{3(4x+1)}{5} = -4$ **(c)** $\frac{5(3x+2)}{2} = 1$

(d) $\frac{2(x-1)}{3} = 5$ **(e)** $\frac{4(3x-2)}{5} = -1$ **(f)** $\frac{7(2x+1)}{2} = -2$

(g) $\frac{6(1-4x)}{15} = \frac{2}{3}$ **(h)** $\frac{3(2-5x)}{8} = -\frac{1}{2}$ **(i)** $\frac{3(7-2x)}{16} = \frac{1}{4}$

14 Subtracting 11 from four times a certain number gives the same result as subtracting two times the number from 7. Write an equation and solve it to find the unknown number.

15 A certain number when added to 36 gives the same result as adding 6 to this number and doubling the result. Write an equation and solve it to find the unknown number.

16 Half of the result of three times a certain number added to one is the same as one-fifth of the result when 4 is subtracted from this certain number. Write an equation and solve it to find the unknown number.

Reasoning, communicating and open-ended

17 Consecutive numbers, such as 6, 7 and 8, can be represented algebraically by n, $n + 1$ and $n + 2$, because to get from one number to the next consecutive number we simply add 1. Similarly, three consecutive odd numbers can be represented algebraically by n, $n + 2$ and $n + 4$ where n is an odd number. Using this information, write each of the following as an equation and solve to find the unknown numbers.

(a) Find the three consecutive odd numbers that sum to 195.

(b) Find the four consecutive even numbers that sum to 204.

(c) Find the six consecutive multiples of seven that sum to 609.

Hint

n does not have to be the first number. Three consecutive numbers could be $n - 1$, n and $n + 1$.

18 The ancient Egyptians solved equations using a method of guessing and checking. We call this method the 'rule of false position'.

For example, consider the equation $x + \frac{x}{9} = 80$, which the ancient Egyptians would have said as 'a number added to one-ninth of itself is 80'.

First, choose a possible value for x that is divisible by 9; for example, 9. Substitute this into the equation.

$$9 + \frac{9}{9} = 9 + 1 = 10$$

Because the answer we want is eight times the answer we found, try a number eight times the value of the number we tried.

$$9 \times 8 = 72$$

$$72 + \frac{72}{9} = 72 + 8 = 80$$

So, the solution to the equation 'a number added to one-ninth of itself is 80' is 72.

For the following questions, form an equation, then use the rule of false position to solve it.

(a) A number added to one-sixth of itself is 28.

(b) A number added to one-quarter of itself is 45.

(c) A number added to one-eighth of itself is 99.

19 On a photocopied test some numbers are not clear. One equation reads $2x + 7 =$ but the number on the right-hand side is unreadable. If we know that x is a whole negative number, give three examples of what the number on the right-hand side could be.

20 Use at least three mathematical operations to make up three different equations that have a solution of 4.

21 Make up three equations similar to those in Question **3**, where the solution is a positive whole number.

Problem solving

Burning down

Two different-sized candles are lit. Each candle burns at a different rate and one is 3 cm longer (or taller) than the other.

The longer candle was lit at 3.30 p.m. and the shorter one at 5 p.m.

At 7.30 p.m. they were both the same length.

The longer candle burned out at 9.30 p.m., whereas the shorter candle burned out at 9 p.m.

How long was each candle originally?

Strategy options

- Make a model.
- Draw a diagram.
- Guess and check.
- Break the problem into manageable parts.

Solving problems using linear equations

4.4

Stage 5.2

Need to Know

Problem-solving strategies

- Determine the unknown(s) you need to find and represent each with a variable.
- Form an equation, using the key words in the statement of the problem.
- Solve the equation using inverse operations.
- Answer the question in words.
- Check that your answers make sense and satisfy all conditions of the problem.

Worked Example 10

WE10

Tim is twice Sally's age. How old are Tim and Sally now, if in 10 years' time the sum of their ages will be 89? Use s to represent Sally's age.

Working

Sally's present age = s

Tim's present age = $2s$

In 10 years' time:
Sally's age = $s + 10$
Tim's age = $2s + 10$

$$\begin{aligned} 3s + 20 &= 89 \\ 3s &= 69 \\ s &= 23 \end{aligned}$$

$$\begin{aligned} 2s &= 2 \times 23 \\ &= 46 \end{aligned}$$

Sally is 23 years old and Tim is 46 years old.

Checking the solution

In 10 years' time, they will be 33 and 56 years old respectively, $33 + 56 = 89$. Our answers solve the problem.

Answers page 438

4.4 Solving problems using linear equations

Stage 5.2

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q14, Q15	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency and understanding

WE10

1 Kate's father is three times her age. How old are Kate and her father now if, in 4 years time, the sum of their ages is 68? Use K to represent Kate's age.

2 Andrew has x coins in his pocket. He tells Lori that she has five fewer coins than he has. If they have 17 coins between them, how many coins does each have?

3 Phil has $\$p$ in his piggy bank and Bill has $\$b$ in his. If Phil has three times the amount of money Bill has in his piggy bank and they save \$5 each per week for the next 10 weeks, the total amount of money in both piggy banks will be \$200. Which of the following equations represents this situation?

A $4p + 100 = 200$ **B** $4b + 100 = 200$

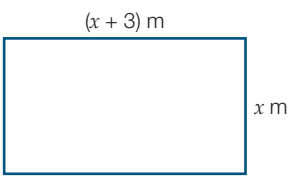
C $3b + 100 = 200$ **D** $3b + 1p + 100 = 150$

4 Choose the correct answer.

If $a = b + 2cx$, then:

A $x = \frac{a+b}{2c}$ **B** $x = \frac{a-b}{2c}$ **C** $x = \frac{a}{2c} - b$ **D** $x = a - \frac{2c}{b}$

5 If the perimeter of this rectangular block of land is 46 m, find the length and width of the block.



6 Three students take different routes home from school. Mark needs to walk twice as far as Annette, whereas Sue needs to walk four times as far as Annette. If the three cover a total distance of 2.1 km (or 2100 m), how far does each need to travel to get home from school?

7 Ben bought five ice-cream cones and received \$1.75 change from his \$10 note. How much did each ice-cream cone cost?

8 Peter is 7 years older than Sharon and John is twice as old as Peter. Find how old Peter is if the mean (arithmetic average) of their ages is 19.

9 Joseph has 24 cousins. He knows that he has six more female cousins than male cousins. How many female cousins does he have?

10 In 3 years' time Joanne will be twice as old as her daughter. If Joanne is 45 years old now, how old is her daughter?

Reasoning, communicating and open-ended

11 A triangle has angles of $(2x + 11)°$, $(3x - 2)°$ and $(4x)°$. Find each angle.

12 An isosceles triangle has a perimeter of 45 cm. Its equal sides each measure 6 cm longer than the base. How long is the base of the triangle?

13 Gabby is twice as old as Kate and Kate is 3 years older than Joe. Brett is four times as old as Joe. If the sum of the ages of Kate and Brett is 63, find how old each person is.

14 A stock market analyst was investigating the trends of two independent stocks and was surprised to see that both stocks were at the same unit value after a 12-month period. He knew that both stocks had started at the same value but had undergone different trends over the past year. One stock had risen to three times its original unit value then lost \$8 a unit before finally finishing at $\frac{3}{4}$ of this reduced unit value. The other stock added \$3 a unit to its original unit value before crashing to $\frac{1}{5}$ of its improved unit value. It then showed a late improvement by increasing the unit value to seven times the reduced unit value. Express this information as an equation and solve it to find the original unit value of the stocks.

15 The sum of three consecutive whole numbers is a number between 70 and 100. Give three possible starting numbers for the group of three consecutive whole numbers.

Hint

Let one of the ages be represented by a variable and express the other ages in terms of this variable.

Problem solving

Fractions in fractions in fractions

The fraction $\frac{2008}{1998}$ may be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \frac{1}{d}}}$ where a, b, c and d are all positive integers.

Can you find the value of d?

Hint

- Write the fraction $\frac{2008}{1998}$ as a mixed number.

  This should help you find the value of a.

- $\frac{x}{y} = \cfrac{1}{\frac{y}{x}}$ may be useful.

Strategy options

- Break the problem into manageable parts.
- Solve a simpler problem.
- Work backwards.

4.5 Plotting linear graphs

Stage 5.1

Need to Know

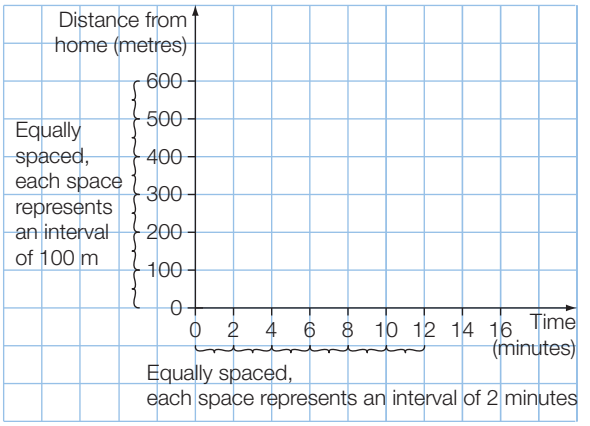
Linear relationships can be represented both graphically (using graphs) and algebraically (using equations).

Each axis on a graph should have a clearly marked scale and a label.

A linear relationship is a relationship between two variables that, when graphed, produces a straight line.

Plotting graphs accurately

Use graph paper. Each axis should have a clearly marked scale and a label. The scale may or may not be the same for each axis. On an axis, the scale divisions should be equally spaced with intervals representing equal amounts. An example is shown opposite.



Worked Example 11

WE 11

For each of the following table of values:

(i) plot a graph to represent the information

(ii) write an equation to represent the linear relationship.

(a)

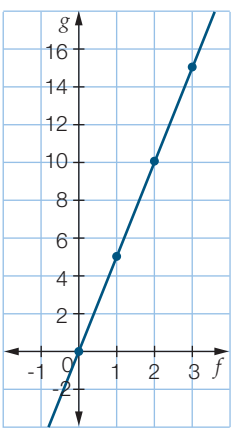
f	0	1	2	3
g	0	5	10	15

(b)

v	0	1	2	3
w	7	5	3	1

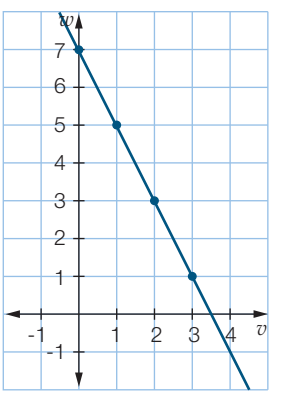
Working

(a) (i)



(ii) $g = 5f$

(b) (i)



(ii) -2v is in our equation
7 needs to be added.
$w = -2v + 7$

Worked Example 12

WE12

Use the equation given for each of the following to:

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $M = 9n$

n	-3	0	3	6
M				

(b) $L = 5k - 8$

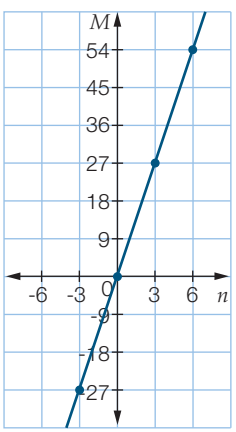
k	-4	0	4	8
L				

Working

(a) **(i)**

n	-3	0	3	6
M	-27	0	27	54

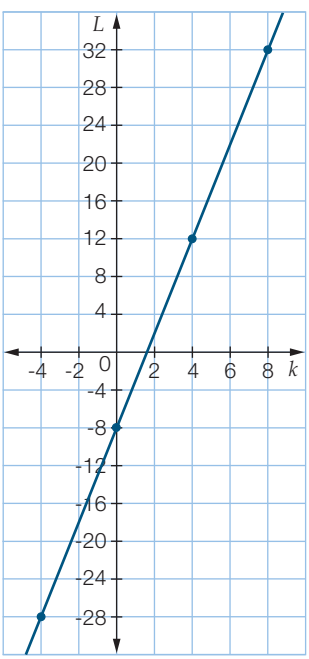
(ii)



(b) **(i)**

k	-4	0	4	8
L	-28	-8	12	32

(ii)



Worked Example 13

WE13

Lisa, a travel agent working for an airline, receives a base wage of $160 each week and a bonus of $40 for each flight ticket sold.

No. of tickets sold in a week (n)	0	1	2	3	4	5
Weekly wage (W)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table.

(c) Write an equation to represent the information.

(d) If Lisa sold 12 tickets one week, how much did she earn that week?

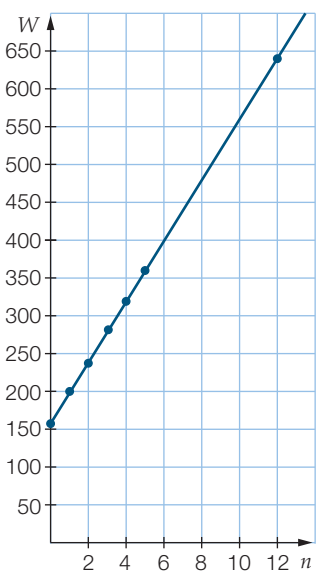
(e) Show, by substitution, that the point (8, 480) lies on the line.

Working

(a)

No. of tickets sold per week (n)	0	1	2	3	4	5
Weekly wage (W)	160	200	240	280	320	360

(b)



(c) $m = \frac{\text{rise}}{\text{run}}$ $b = 160$

$= 40$

$W = 40n + 160$

(d) $W = 40n + 160$

$W = 40 \times 12 + 160$

$= 480 + 160$

$= 640$

Lisa would earn $640.

(e) $W = 40n + 160$

$= 40 \times 8 + 160$

$= 320 + 160$

$= 480$

(8, 480) lies on the line.

4.5 Plotting linear graphs

Answers page 438

Stage 5.1

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q7, Q9, Q10	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: graph or grid paper

Fluency and understanding

1 For each of the following tables of values:

WE11

(i) plot a graph to represent the information

(ii) write an equation to represent the linear relationship.

(a)

a	0	1	2	3
b	0	2	4	6

(b)

c	0	3	6	9
d	5	2	-1	-4

2 Use the equation given for each of the following to:

WE12

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $A = 4L$

L	0	10	20	30
A				

(b) $Q = 5M$

M	0	2	4	6
Q				

(c) $J = 2G - 12$

G	0	4	8	12
J				

(d) $d = -6e + 5$

e	0	3	6	9
d				

(e) $B = \frac{3R}{5}$

R	-20	0	20	40
B				

(f) $C = \frac{5F}{6} - 4$

F	-6	0	6	12
C				

3 Eddy, a television salesperson, receives a base wage of $80 each week and a bonus of $100 for each television he sells.

WE13

Number of televisions sold	0	1	2	3	4	5
Weekly wage (dollars)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table. Use the horizontal axis for the number of televisions sold and the vertical axis for weekly wage (dollars).

(c) Write an equation to repesent the information.

(d) If Eddy sold eight televisions in one week, how much did he earn that week?

(e) Show by substitution, that the point (7, 780) lies on the graph.

4 **(a)** In the linear relationship $V = 3W - 14$, when $W = 7$ the value of V will be:

A -14 **B** -8 **C** -2 **D** 7

(b) In the linear relationship $P = 5R$, P has a value of -30 when R is equal to:

A -10 **B** -6 **C** 6 **D** 10

(c) In the linear relationship $C = \frac{5}{9}F - 32$, the value of C when F is equal to 63 is:

A -32 **B** -3 **C** 3 **D** 18

5 A no-name brand detergent can be bought wholesale for 95c a litre. To supply information to customers quickly, the wholesaler has prepared a graph representing this information.

(a) Copy and complete this table.

Number of litres	0	1	2	3	4	5	6	7
Cost (dollars)								

(b) Plot the cost of the detergent against the number of litres of detergent bought. This is the wholesaler's graph. Use the horizontal axis for the number of litres and the vertical axis for the cost (dollars).

(c) Is the relationship linear? How can you tell?

(d) From your graph, how much would it cost to buy:

(i) 3 L of detergent **(ii)** 2.4 L of detergent **(iii)** 5.7 L of detergent?

(e) From your graph, how many litres of detergent (to the nearest tenth of a litre) could be bought for:

(i) \$3.80 **(ii)** \$2.50 **(iii)** \$5.20?

(f) Write a relationship connecting the cost in dollars, C, to the number of litres of detergent, d, bought.

6 Steve is cycling at a constant speed of 185 metres per minute.

(a) Copy and complete the following table.

Time (minutes)	0	1	2	3	4	5	6	7
Distance cycled (metres)								

(b) Plot the distance against the time. Use the horizontal axis for time (minutes) and the vertical axis for distance cycled (metres).

(c) From the graph, how far does Steve cycle in 2.7 minutes?

(d) How long does it take Steve to cycle 700 metres?

(e) Extend your graph to find how long it takes Steve to cycle 2 km.

(f) Write the rule (or equation) connecting the distance cycled to the time taken.

(g) If Steve continued cycling, what distance would he have covered in 40 minutes?

Reasoning, communicating and open-ended

7 Phuong works at the stock exchange buying and selling shares for different international and local companies on behalf of her client. She needs to be able to convert quickly from one currency to another, and uses either a calculator or a linear graph to obtain her information.

(a) If \$20 in Australian currency converts to \$21 in American (US) currency, draw up a graph showing the relationship between the two currencies. Plot Australian currency on the horizontal axis and American currency on the vertical axis.

Hint

What is \$A1.00 equivalent to in \$US?

Use the graph to convert from one currency to the other for each of these amounts (round off to the nearest 10c).

(b) A$7.00 to US$

(c) A$17.60 to US$

(d) US$5.50 to A$

(e) US$9.60 to A$

8 Wally, Sarah and Cleo are collecting money for an upcoming charity walk. Wally is asking for $1 for each kilometre he walks. Sarah, on the other hand, is asking for a $4 donation plus $0.50 per kilometre walked. Finally, Cleo is asking for a donation of $8 regardless of how many kilometres she walks.

(a) Draw up and complete a table of values to show the amount donated for each kilometre walked for Wally, Sarah and Cleo for distances from 0 to 10 kilometres.

(b) Use your table of values to plot graphs showing the amount donated for each kilometre walked for Wally, Sarah and Cleo. You should plot each graph on the same set of axes. Plot distance on the horizontal axis and the amount donated on the vertical axis.

(c) From the graph and table of values, identify the distance at which all three collect the same amount of money.

(d) Identify the distances for which:

(i) Wally raises the most money

(ii) Sarah raises the most money

(iii) Cleo raises the most money.

9 Frank and Bruno are two emerging football players about to sign contracts with FC Mathematica. In the first year, Frank earns $120 000, which will increase by $60 000 for each successive year that he stays with the club. Bruno earns $80 000 in his first year, which increases by $80 000 for each successive year that he stays with the club. Explain in which years Frank will earn more than Bruno and in which years Bruno will earn more than Frank, assuming each stays with the club.

10 Draw three different linear relationships that show the same amount of distance travelled over different periods of time. Explain what happens to the graph as the time increases.

Problem solving

How many edges?

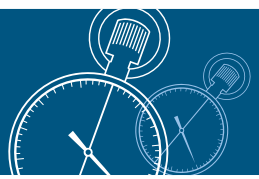
Equipment required: 1–2 brains, scissors, ruler and several pieces of A4 paper

Instructions: Divide an A4 rectangular sheet of paper into two pieces using a single straight cut. Then divide one of those pieces into two, also using a single straight cut. What is the maximum number of edges that the resulting three pieces can have in total? What would the minimum number of edges of the three pieces be?

Strategy options

- Act it out.
- Look for a pattern.
- Test all possible combinations.

Half-time 4



Stage 5.1 Q1, 2, 3 **Stage 5.2 Q4–7** **Stage 5.3 Q8**

Ex.4.5

1 Decide which, if any, of the following tables represent linear relationships.

(a)

x	-1	0	1	2
y	-5	-3	-1	1

(b)

x	-2	0	2	4
y	5	2	-1	-3

(c)

x	0	1	2	3
y	0	1	1	2

(d)

x	-4	0	4	8
y	0	1	2	3

Ex.4.1

2 Find the distance between the points (-1, 7) and (4, -5) by plotting the points on a Cartesian plane and using Pythagoras' Theorem.

Ex.4.5

3 Use the equation $P = 3 - 4R$ to:

(a) copy and complete the following table of values

R	0	2	4	6
P				

(b) plot the graph of the relationship.

Ex.4.3

4 Solve the following linear equations.

(a) $2x - 5 = 13$ **(b)** $8 - 3y = -7$ **(c)** $\frac{3x}{4} - 2 = 10$

(d) $\frac{4x + 1}{5} = -7$ **(e)** $3x - 2 = 5x - 8$ **(f)** $5(x - 1) = 2(8 - x)$

(g) $4(3p - 5) = 5(7 - 2p)$ **(h)** $\frac{a + 11}{3} = \frac{3 - a}{4}$ **(i)** $\frac{3m - 5}{2} = \frac{2m + 25}{3}$

Ex.4.4

5 Kelvin and Rob are training to be weightlifters. Rob weighs 6 kg more than Kelvin. Kelvin can lift one-and-a-half times his weight less 6 kg. Rob can lift a quarter more than his weight. They found they can both lift the same weight. How much do Kelvin and Rob each weigh?

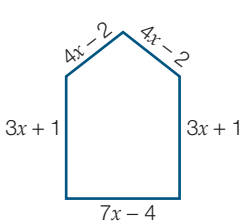
Ex.4.3

6 The equation $3x - 4 = 2(x - 5)$ has the solution:

A $x = -2$ **B** $x = -1$ **C** $x = -6$ **D** $x = 6$

Ex.4.4

7 The total perimeter of this shape is 36 m. Find x.



Ex.4.2

8 Find the midpoint of the line joining (6, 2) and (3, 8) using the midpoint formula.

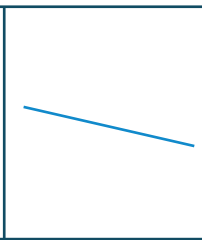
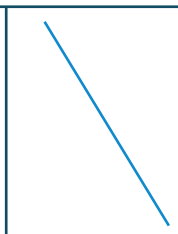
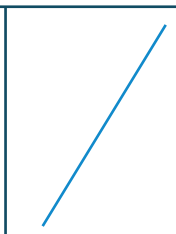
Gradient

4.6

Stage 5.1

Need to Know

The measure of a line's steepness is called the **gradient** or **slope** of the line. The gradient of a line describes the change in the y values as the x value increases by 1.

Line				
Magnitude of gradient	Small	Large	Large	Small
Sign of gradient	Negative	Negative	Positive	Positive

The gradient of a line can be found by considering the ratio of the vertical **rise** to the horizontal **run** between any two points on the line.

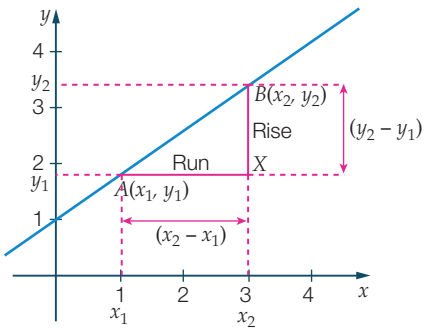
$$\text{Gradient } AB = \frac{\text{vertical rise}}{\text{horizontal run}}$$

We use the letter m to represent the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$



Special cases for the gradient

The gradient of any horizontal line is zero.

The gradient of any vertical line is **undefined**.

Technology Exploration: GeoGebra

Finding the gradients of lines and exploring the gradients of parallel lines

Part A

1. Open GeoGebra and use the Line through Two Points tool to draw a line through the points $A(2, 1)$ and $B(6, 3)$. Plot the point $C(6, 1)$ and use the Interval between Two Points tool to join AC and AB. This will form a right-angled triangle ACB with the right angle at C.
2. Use the Distance or Length tool to measure the lengths of AC (the run) and CB (the rise). Calculate $\frac{rise}{run}$ to get 0.5.
3. Now select the Slope tool. Click on the line AB and a small shaded triangle will appear with $m = 0.5$. This is the gradient of the line.
4. Use the Move tool to move point A or B around. Move point C to form a right-angled triangle ACB with the right angle at C. Recalculate $\frac{rise}{run}$. The value obtained should be the same as the value of m on the screen. The direction of the interval must be used in the calculations, so note that movements to the right and up are positive while movements to the left and down are negative.
5. Investigate what happens to m if the line AB is a horizontal line.
6. Investigate what happens to m if the line AB is a vertical line. (See the Algebra view.)
7. Copy and complete the following sentences.
   - **(a)** Lines that slope upwards from left to right have a ..... gradient.
   - **(b)** Lines that slope downwards from left to right have a ..... gradient.
   - **(c)** Lines that are parallel to the x-axis have a ..... gradient.
   - **(d)** Lines that are parallel to the y-axis have a ..... gradient.

Part B

1. Plot a point anywhere in the Graphics view. Select the Parallel Line tool and click on the line AB. A line parallel to AB will be drawn through the new point. Use the Slope tool to find the gradient of the new line.
2. Move the line AB around, finding a new gradient each time. Find the gradient of the parallel line. What do you notice about the gradients of the parallel lines?
3. Copy and complete the following sentence:

   Lines that are parallel have the ..... gradient.

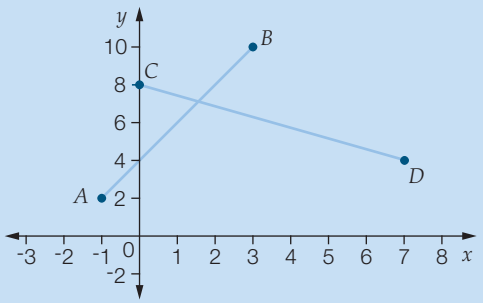
Worked Example 14

WE14

Find the gradient of:

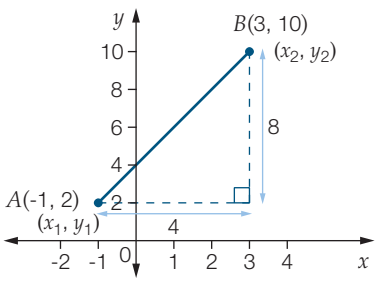
(a) the line *AB*

(b) the line *CD*.



Working

(a)



$$m = \frac{\text{rise}}{\text{run}}$$

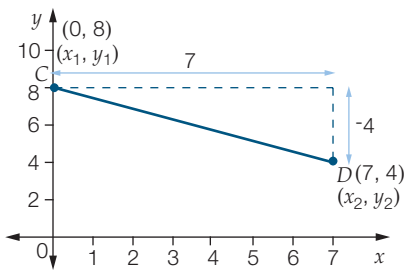
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{10 - 2}{3 - -1}$$

$$= \frac{8}{4}$$

$$= 2$$

(b)



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 8}{7 - 0}$$

$$= \frac{-4}{7}$$

$$= -\frac{4}{7}$$

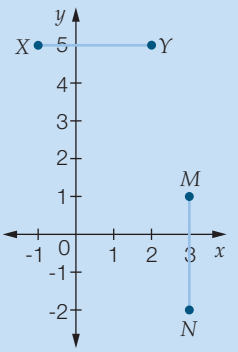
Worked Example 15

WE15

Find the gradients of:

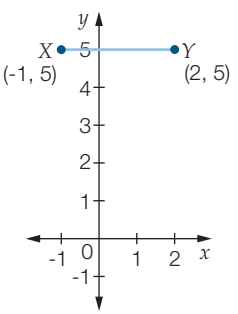
(a) the line segment XY

(b) the line segment MN.



Working

(a)



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

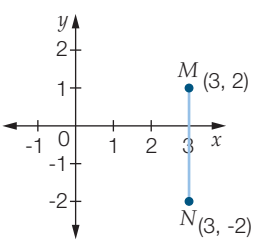
$$= \frac{5 - 5}{2 - 1}$$

$$= \frac{0}{1}$$

$$= 0$$

The gradient is equal to zero.

(b)



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 1}{3 - 3}$$

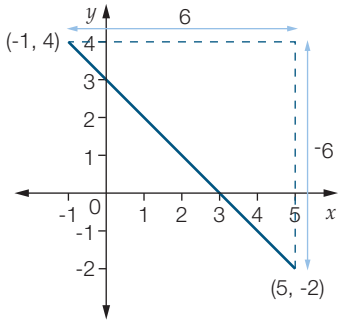
$$= \frac{-3}{0}$$

This cannot be evaluated, so the gradient is undefined.

Worked Example 16 WE16

Find the gradient of the line joining (5, -2) and (-1, 4).

Working



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 - 4}{5 - -1}$$

$$= \frac{-6}{6}$$

$$= -1$$

4.6 Gradient

Answers page 440

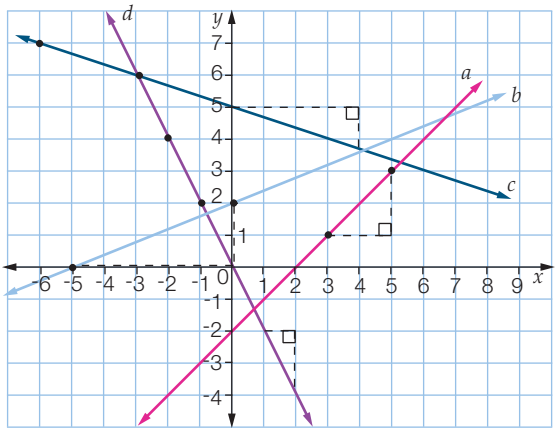
Navigator

Stage 5.1

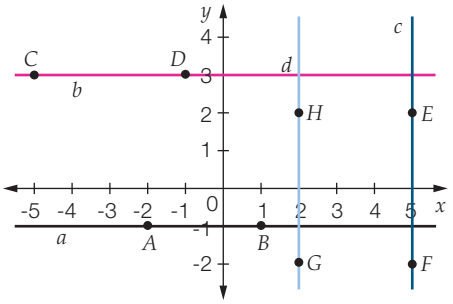
Q1, Q2, Q3 Column 1, Q4, Q5, Q6, Q7, Q10, Q11, Q15	Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q15	Q1, Q2, Q3 Column 3, Q4, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15

Fluency and understanding

1 Find the gradient of each of the lines *a*–*d* using the points and triangles indicated. WE14



2 Find the gradient of each of the lines *a*–*d* using the points on the lines as indicated. WE15



WE16

3 Find the gradient of the line joining each of the following pairs of points.

(a) (2, 3) and (4, 7) **(b)** (1, 5) and (3, 11) **(c)** (5, 8) and (6, 4)

(d) (7, 0) and (-1, 8) **(e)** (0, -2) and (3, 4) **(f)** (1, -4) and (3, -2)

(g) (-7, 3) and (5, -4) **(h)** (-6, 2) and (-2, -2) **(i)** (-1, -3) and (-4, -2)

(j) (2, -1) and (-7, -1) **(k)** (4, 2) and (-3, 2) **(l)** (-3, -5) and (1, -5)

(m) (-1, -5) and (-1, 5) **(n)** (3, -4) and (3, 8) **(o)** (-7, 2) and (-7, 5)

4 Use your answers to Questions **1** and **2** to answer the following.

(a) If a line is sloping up to the right, does it have a positive or a negative gradient?

(b) If a line is sloping down to the right, does it have a positive or a negative gradient?

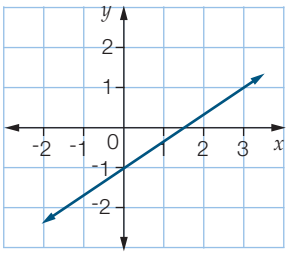
(c) What is the gradient of a horizontal line?

(d) What is the gradient of a vertical line?

5 (a) The gradient of this line is:

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$

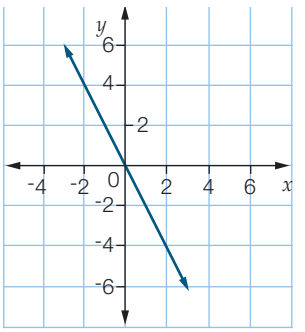
C $\frac{2}{3}$ **D** $\frac{3}{2}$



(b) The gradient of this line is:

A -2 **B** $-\frac{1}{2}$

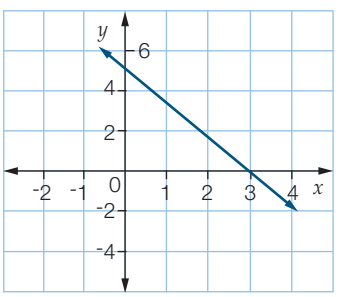
C $\frac{1}{2}$ **D** 2



(c) The gradient of this line is:

A $-\frac{5}{3}$ **B** $-\frac{3}{5}$

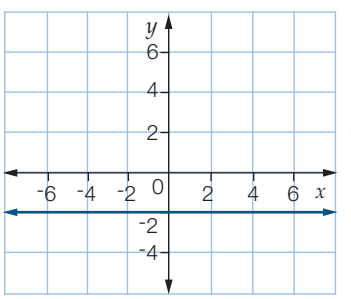
C $\frac{3}{5}$ **D** $\frac{5}{3}$



(d) The gradient of this line is:

A -2 **B** -1

C 0 **D** 2



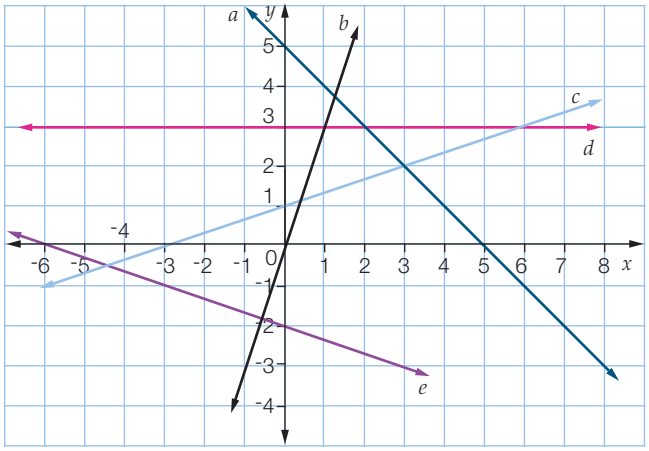
6 For each line *a*–*e* decide whether the gradient is:

A positive

B negative

C zero

D undefined.

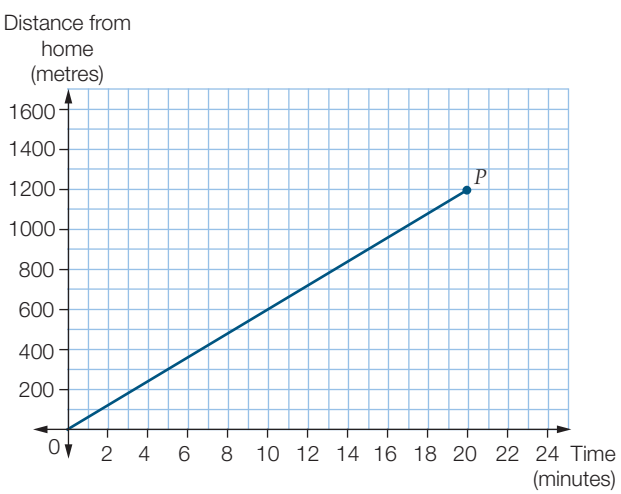


7 **(a)** Find the gradient of each of the lines in Question **6**.

(b) Which line has the steepest positive gradient?

(c) Which line has the steepest negative gradient?

8 Thanh decided to walk to school rather than take the bus. This graph shows his distance from home at any particular time on the walk. The school is located at point *P*.



(a) How far is the school from Thanh's home?

(b) How long did it take Thanh to walk to school?

(c) How far did Thanh walk every minute?

(d) What is the gradient of this graph?

(e) What was Thanh's speed in m/min on the walk?

(f) How far from home was Thanh after walking for:

(i) 6 minutes **(ii)** 12 minutes **(iii)** 18 minutes **(iv)** *t* minutes?

(g) Write the equation to find the distance from home, *D* (in metres), after Thanh has walked *t* minutes.

9 Anna drove her car at a constant speed for 30 seconds, covering a distance of 450 m.

(a) Represent this information on a distance–time graph. (The distance travelled should be shown on the vertical axis and the time taken on the horizontal axis.)

(b) At what speed was Anna travelling?

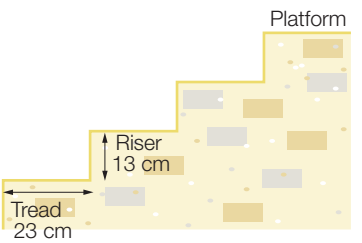
(c) What is the gradient of your graph?

(d) Write an equation connecting the distance travelled to the time taken.

(e) Using your equation, calculate how far Anna would have travelled after 53 seconds if she maintained the same speed.

Reasoning, communicating and open-ended

10 A standard step is 13 cm high (also called the riser) and has a width of 23 cm (also called the tread). A wheelchair ramp is to be constructed to reach a platform that is four steps above ground level. If wheelchair access ramps have a gradient of 1:12, (this means a rise of 1 for a run of 12) find how far from the bottom of the stairs the ramp would need to start if it is to reach the top of the platform.



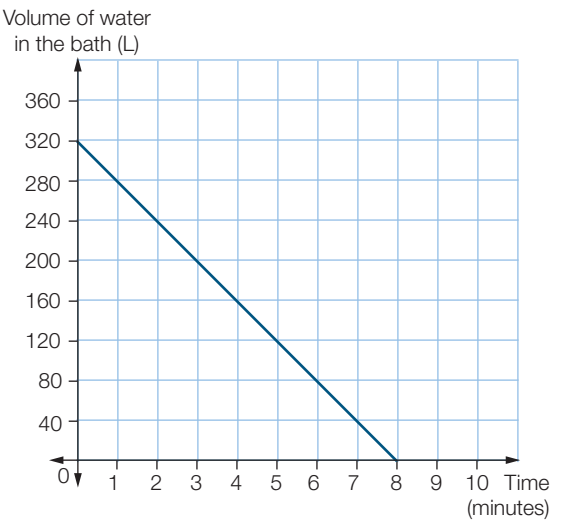
11 Using the 5 × 5 grid labelled with the numbers 1–25, show how many lines with different gradients can be drawn between any two of these points.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Hint

The gradients can be whole numbers, fractions, decimals positive and negative.

12 Sylvia is draining a bathtub of water. The graph shows the volume of water (in litres) in the bath after a number of minutes.



(a) How much water did the bath contain originally?

(b) How long did it take for the water to drain from the bath completely?

(c) What is the gradient of this graph?

(d) At what rate was the water being drained away? (That is, how many litres of water drained away each minute?)

(e) What does the negative represent in your answer to part **(c)**?

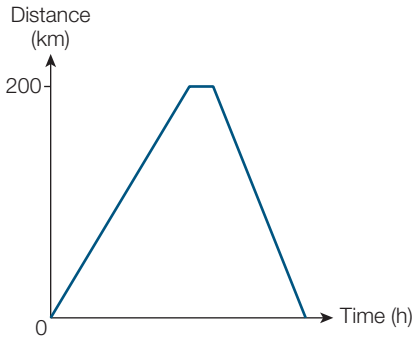
(f) How much water was left in the bath after:

(i) 4 minutes **(ii)** 6 minutes **(iii)** 2 minutes **(iv)** t minutes?

(g) Write the equation for the volume of water left in the bath, V (in litres), after t minutes.

(h) Why is this graph only drawn in the first quadrant?

13 Monique works for a courier company and had to make a long-distance delivery one very rainy day. This graph represents her journey. The rain had cleared for Monique's return journey, so she could travel twice as fast as she did on her way to make the delivery. It took her 4 hours and 10 minutes to make the delivery.



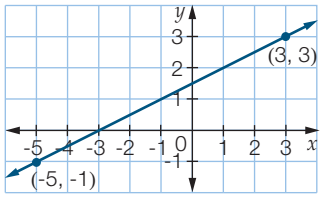
(a) At what speed was Monique travelling on her way to make the delivery?

(b) At what speed was Monique travelling on her return journey?

(c) How long did the entire trip take if Monique stopped for 45 minutes for lunch?

14 The gradient of the line joining two points A and B is $-\frac{2}{3}$. If A is the point (1, 6), write three possible coordinates for B.

15 Kate and Sally were looking at a diagram the teacher had drawn on the whiteboard and the question written underneath, 'What is the gradient of this line?'



Kate wrote $m = \frac{\text{rise}}{\text{run}}$

$= \frac{3 - -1}{3 - -5}$

$= \frac{4}{8}$

$= \frac{1}{2}$

Sally wrote $m = \frac{\text{rise}}{\text{run}}$

$= \frac{3 - 1}{3 - 5}$

$= \frac{2}{-2}$

$= -1$

The teacher tells Kate she is correct. Explain Sally's mistake.

Problem solving

Lost in the desert

An aeroplane has crashed in the desert at a point that could be labelled (-15, 12) on a Cartesian plane. A search party sets out by 4WD from the point (18, 3) and travels in a straight line passing through the point (3, 9). A light plane sets out from point (3, -15) and travels in a straight line passing over the point (-3, -6).

1 Will either of the search parties pass the point where the aeroplane lies if they continue on these paths? Explain how you obtained your answer.

2 If either (or both) of the original routes does not pass the point where the aeroplane lies, give a different point (but with the same x-coordinate) that the searchers should pass through (or over) to be on course to locate the aeroplane.

Strategy options

- Draw a diagram.
- Break the problem into manageable parts.

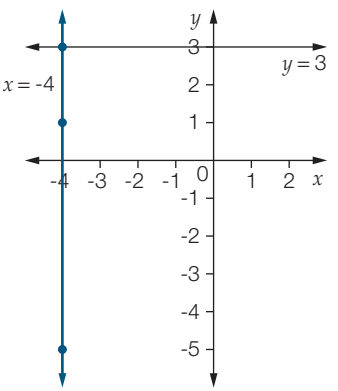
4.7 Vertical and horizontal graphs

Stage 5.1

Need to Know

Linear graphs of the form:

- $y = b$ are straight lines parallel to the x-axis (i.e. horizontal lines).
  For example, the graph of $y = 3$ is a line parallel to the x-axis through the point (0, 3).
- $x = a$ are straight lines parallel to the y-axis (i.e. vertical lines).
  For example, the graph of $x = -4$ is a line parallel to the y-axis through the point (-4, 0).



Worked Example 17

WE 17

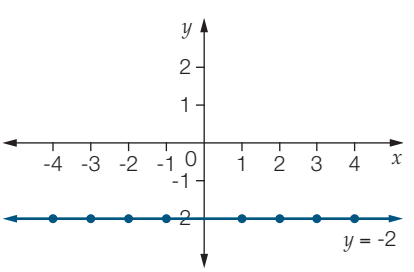
Plot the following graphs, showing any intercepts on the axes.

(a) $y = -2$ **(b)** $x = 5$

Working

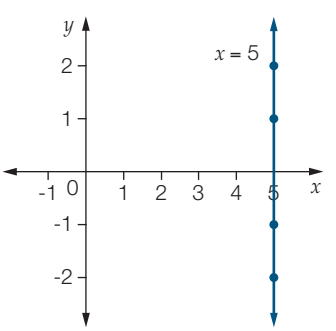
(a)

x	-4	-3	-2	-1	0	1	2	3	4
y	-2	-2	-2	-2	-2	-2	-2	-2	-2



(b)

x	5	5	5	5	5
y	-2	-1	0	1	2



Hint

The scales on the x- and y-axes do not need to be the same.

4.7 Vertical and horizontal graphs

Answers page 441

Navigator

Stage 5.1

Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 Column 1, Q6, Q7, Q8	Q1 Column 2, Q2, Q3, Q4, Q5 Column 2, Q6, Q7, Q8	Q1 Column 3, Q2, Q3, Q4 Column 3, Q5 Column 3, Q6, Q7, Q8, Q9

Fluency and understanding

WE17

1 Plot the graphs of the following, showing any intercepts on the axes.

(a) $y = 2$ **(b)** $y = 4$ **(c)** $x = -3$

(d) $x = -1$ **(e)** $x = 6$ **(f)** $x = \frac{5}{2}$

(g) $y = -3$ **(h)** $x = -2$ **(i)** $y = -\frac{3}{4}$

(j) $x = 4$ **(k)** $y = -\frac{7}{2}$ **(l)** $y = 5$

2 Write the equation of the straight line given.

(a) $m = 0$ and $b = -6$ **(b)** $m = 0$ and $b = 2$

(c) gradient 0 and y-intercept $\frac{2}{3}$ **(d)** gradient 0 and y-intercept $\frac{3}{4}$

3 (a) A coordinate point that *does not* lie on the line $y = 5$ is:

A (0, 5) **B** (-1, -5) **C** (-5, 5) **D** (5, 5)

(b) A coordinate point that *does not* lie on the line $x = 1$ is:

A (1, 0) **B** (-1, 1) **C** (1, -1) **D** (1, 11)

(c) A coordinate point that *does not* lie on the line $y = -8$ is:

A (8, -8) **B** (-8, -8) **C** (-8, 0) **D** (0, -8)

4 Sketch each pair of linear graphs on the same number plane. What coordinate point is common to both graphs in each pair?

(a) $x = 2$ and $y = 8$ **(b)** $y = 4$ and $x = -3$ **(c)** $y = 0$ and $x = -5$

(d) $y = 2$ and $x = 3$ **(e)** $y = -5$ and $x = 10$ **(f)** $x = 0$ and $y = 7$

5 Sketch the graphs of the following linear relationships using the most appropriate method, showing any relevant points.

(a) $2y = -6$ **(b)** $y = -6$ **(c)** $2x = 8$

(d) $x + 1 = 0$ **(e)** $x + 4 = 0$ **(f)** $y - 3 = 0$

(g) $5x = 4$ **(h)** $3y = 2$ **(i)** $2y = 5$

(j) $2y = 7$ **(k)** $3x = -5$ **(l)** $3y = -4$

(m) $5x = 0$ **(n)** $6x = 0$ **(o)** $7y = 0$

(p) $3y - 6 = 0$ **(q)** $2y - 8 = 0$ **(r)** $4x + 12 = 0$

(s) $4y - 3 = 0$ **(t)** $3x - 8 = 0$ **(u)** $2x + 7 = 0$

Reasoning, communicating and open-ended

6 Five streets form the boundary of an undeveloped outer suburban block of land. Taking the southernmost point of the block as the coordinate point (0, 0), each of the roads can be plotted on a number plane. The graphs that represent the roads are $y = 2x$, $x = 3$, $y = 10$, $x + 2 = 0$ and $2y + 3x = 0$.

The local council plans to subdivide this land by putting in new roads along the lines $x = 0$, $y = -5x$ and $3y - 10x = 0$.

(a) Sketch each of the linear graphs that represent the boundary roads on the same number plane.

(b) From your sketch, determine the coordinate points that are common to each adjacent pair of graphs.

(c) Sketch each of the linear graphs that represent the new roads.

(d) Will an old shed located at the coordinate point (-1, 5) be affected by the new road?

7 Write three equations whose graphs are parallel to the x-axis. Sketch the graphs for each.

8 Write three equations whose graphs are parallel to the y-axis. Sketch the graphs for each.

9 Draw three different pairs of linear graphs on the same number plane. Each pair of graphs must have a graph parallel to the x-axis and the other parallel to the y-axis. From these graphs, determine the coordinate point which is common to each pair.

Sketching linear graphs using the gradient and *y*-intercept

4.8

Stage 5.2

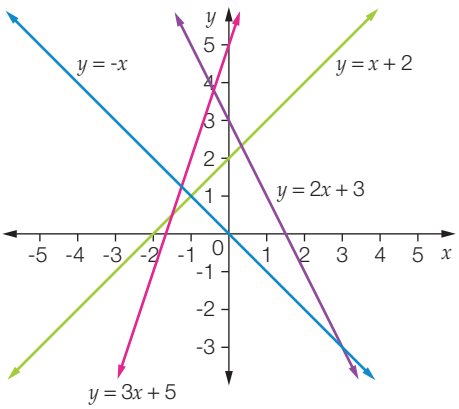
Need to Know

The gradient–intercept form

$y = mx + b$, where m is the gradient and b is the y coordinate of the y-**intercept**.

Graphs of equations in the form of $y = mx + b$ are linear graphs.

For example: $y = 2x - 6$ is a linear graph.



The effect of *m*

As m increases and b stays constant, the gradient of the graph increases. The coefficient m gives the value of the gradient, $m = \frac{rise}{run}$.

For example: $y = 3x + 1$ has a larger gradient than $y = 2x + 1$.

The effect of *b*

As b increases and m stays constant, the y-intercept increases. The value of the constant term b is the value of the y coordinate of the y-intercept.

For example: $y = 3x + 5$ has a y-intercept of (0, 5).

Sketching linear graphs that pass through the origin

Graphs of equations in the form of $y = mx$ are graphs through the origin. The origin is the y-intercept. This is one point on the graph. Use the gradient to find a second point. The line through the origin and this second point is the graph of the equation.

For example: the graphs of $y = 2x$, $y = 6x$, $y = -9x$ all pass through the origin.

Sketching other linear graphs using gradient and *y*-intercept

Identify the y-intercept using the value of b in the equation $y = mx + b$. This is one point on the graph. Using $m = \frac{rise}{run}$, identify a second point on the graph. The line through these two points is the graph of the equation.

The general form

$ax + by + c = 0$, where a, b and c are constants and a and $b \neq 0$.

In this equation, the gradient $m = \frac{-a}{b}$ and the y coordinate of the y-intercept $= \frac{-c}{b}$ can be found by rearranging the equation to make y the subject.

For example: $3x + 2y - 8 = 0$.

$m = -\frac{3}{2}$

y coordinate of the y-intercept $= \frac{8}{2}$

$= 4$

If $3x + 2y - 8 = 0$ is rearranged to give $y = -\frac{3}{2}x + 4$, the answers are the same.

Parallel and perpendicular lines

- Two distinct lines in the same plane with the same gradient are **parallel** lines. The **converse** of this statement is also true: Parallel lines have the same gradient, $m_1 = m_2$.

  For example: the graphs of $y = 3x - 5$ and $y = 3x + 1$ are parallel lines.
- The product of the gradients of two **perpendicular** lines is equal to -1, $m_1 m_2 = -1$. Conversely, it can be shown that if the product of the gradients of two lines $m_1 m_2 = -1$, then the lines are perpendicular.

  For example: the graphs of $y = 2x + 1$ and $y = 1 - \frac{x}{2}$ are perpendicular lines.

Worked Example 18

WE18

State the gradient and coordinates of the y-intercept of the graphs of each of the following linear equations.

(a) $y = -3x + 2$

(b) $y = x - 1$

(c) $2y = 5x - 4$

(d) $y + 2x = 6$

Working

(a) $y = -3x + 2$

gradient $= -3$

y-intercept: $(0, 2)$

(b) $y = x - 1$

$= 1x + (-1)$

gradient $= 1$

y-intercept: $(0, -1)$

(c) $2y = 5x - 4$

$y = \frac{5}{2}x - 2$

gradient $= \frac{5}{2}$

y-intercept: $(0, -2)$

(d) $y + 2x = 6$

$y = 6 - 2x$

$= -2x + 6$

gradient $= -2$

y-intercept: $(0, 6)$

Worked Example 19

WE19

Sketch the graph of the following relationships.

(a) $y = 4x$ **(b)** $5x - 2y = 0$

Working

(a) y-intercept:

When $x = 0$, $y = 4 \times 0$

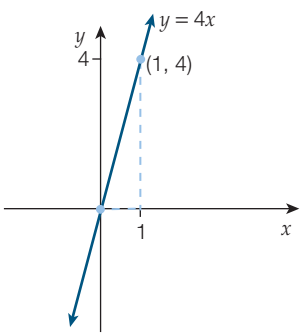
$y = 0$

y-intercept is $(0, 0)$

$m = 4$

$= \frac{4}{1}$

rise = 4; run = 1



(b) y-intercept: When $x = 0$, $-2y = 0$

$y = 0$

y-intercept is $(0, 0)$

$5x - 2y = 0$

$5x - 5x - 2y = 0 - 5x$

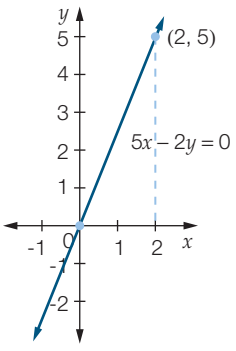
$-2y = -5x$

$\frac{-2y}{-2} = \frac{-5x}{-2}$

$y = \frac{5}{2}x$

$m = \frac{5}{2}$

rise = 5; run = 2



Worked Example 20

WE20

Use the y-intercept and the gradient to sketch the graph of $y = 2x + 1$.

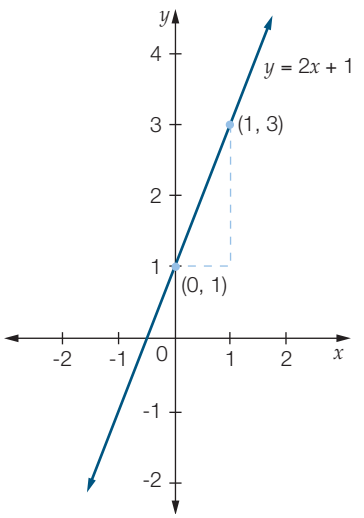
Working

y-intercept: $(0, 1)$

Gradient $= 2$

$$= \frac{2}{1}$$

$$= \frac{\text{rise}}{\text{run}}$$



Worked Example 21

WE21

Use the y-intercept and the gradient to sketch the graph of $2x + 3y = 6$.

Working

$$2x + 3y = 6$$

$$3y = 6 - 2x$$

$$y = \frac{6 - 2x}{3}$$

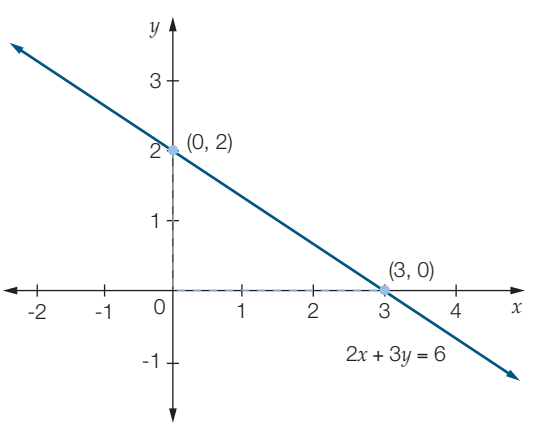
$$= 2 - \frac{2x}{3}$$

$$= -\frac{2x}{3} + 2$$

y-intercept: $(0, 2)$

Gradient $= -\frac{2}{3}$

$$= \frac{\text{rise}}{\text{run}}$$



Worked Example 22

WE22

Find the gradient of a line parallel to each of the following.

(a) $y = 2x - 5$ **(b)** $2x + 3y = 6$

Working

(a) $y = 2x - 5$

$m = 2$

Any parallel line will have a gradient of 2.

(b) $2x + 3y = 6$

$2x - 2x + 3y = 6 - 2x$

$\frac{3y}{3} = -\frac{2}{3}x + \frac{6}{3}$

$y = -\frac{2}{3}x + 2$

$m_1 = -\frac{2}{3}$

Any parallel line will have a gradient of $-\frac{2}{3}$.

Worked Example 23

WE23

Find the gradient of a line perpendicular to each of the following.

(a) $y = 4x + 3$ **(b)** $3x - 5y = 10$

Working

(a) $y = 4x + 3$

$m_1 = 4$

$m_2 = -\frac{1}{m_1}$

$= -\frac{1}{4}$

(b) $3x - 5y = 10$

$3x - 3x - 5y = 10 - 3x$

$-5y = -3x + 10$

$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{10}{-5}$

$y = \frac{3}{5}x - 2$

$m_1 = \frac{3}{5}$

$m_2 = -\frac{1}{m_1}$

$= -\frac{5}{3}$

Answers page 445

4.8 Sketching linear graphs using the gradient and y-intercept

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 2, Q3 Column 3, Q4 Column 1, Q5, Q6 Column 1, Q7 Column 1, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q18, Q20	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6 Column 2, Q7 Column 2, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q17, Q18, Q20	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6 Column 3, Q7 Column 3, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

Fluency and understanding

WE18

1 State the gradient and the coordinates of the y-intercept of the graphs of each of the following linear equations.

(a) $y = x + 3$ **(b)** $y = 2x + 1$ **(c)** $y = 7x + 3$

(d) $y = 3x - 2$ **(e)** $y = x - 4$ **(f)** $y = -4x + 3$

(g) $y = -x + 4$ **(h)** $y = 6 - 2x$ **(i)** $y = -5 + 2x$

WE19

2 Sketch the graphs of the following relationships.

(a) $y = 2x$ **(b)** $y = 3x$ **(c)** $y = -x$

(d) $y = -2x$ **(e)** $y = \frac{2}{3}x$ **(f)** $y = \frac{4}{5}x$

(g) $4x + y = 0$ **(h)** $6x + y = 0$ **(i)** $4x + 3y = 0$

WE20

3 Use the y-intercept and the gradient to sketch the graph of each of the equations in Question **1**.

WE21

4 Use the y-intercept and the gradient to sketch the graph of each of the following.

(a) $2x + y = 4$ **(b)** $-x + y = -3$ **(c)** $-3x + y = -2$

(d) $4x - y = 1$ **(e)** $2x - y = -3$ **(f)** $3x + 2y = 4$

(g) $2x + 3y = 3$ **(h)** $2x - 5y = 10$ **(i)** $7x + 3y - 6 = 0$

WE22

5 Find the gradient of lines parallel to each of the following.

(a) $y = 3x + 5$ **(b)** $y = 7 - 4x$ **(c)** $2y = 6x - 5$

(d) $6x - 2y = 3$ **(e)** $2x + 5y = 10$ **(f)** $4x - 3y = 8$

WE23

6 Find the gradient of lines perpendicular to each of the equations given in Question **5**.

7 Write the equation of the straight line given:

(a) $m = 3$ and $b = 4$ **(b)** $m = 1$ and $b = 5$ **(c)** $m = 2$ and $b = 0$

(d) gradient is 4 and y-intercept is 0 **(e)** gradient is -5 and y-intercept is 1 **(f)** gradient is 3 and y-intercept is -4

8 **(a)** The equation of a linear graph with y-intercept 3 and gradient of -2 is:

A $y = 2x + 3$ **B** $2x - y = 3$ **C** $3x + 6 = 2$ **D** $2x + y = 3$

(b) The equation of a linear graph with gradient $\frac{1}{2}$ and y-intercept 1 is:

A $y = 2x + 1$ **B** $y = x + 2$ **C** $x - 2y = -2$ **D** $x + 2y = -2$

9 When using the y-intercept and gradient method of graphing, we first mark in the y-intercept and then use the gradient to locate a second coordinate point.

(a) A line has a gradient of $\frac{3}{4}$. From the y-intercept, we move:

A 3 units right and 4 units up **B** 4 units right and 3 units up

C 3 units right and 4 units down **D** 5 units right and 3 units down

(b) A line has a gradient of $-\frac{2}{5}$. From the y-intercept, we move:

A 2 units right and 5 units down **B** 5 units right and 2 units down

C 2 units left and 5 units up **D** 5 units left and 2 units down

(c) A line has a gradient of 5. From the y-intercept, we move:

A 1 unit right and 5 units up **B** 5 units right and 5 units up

C 1 unit right and 5 units down **D** 5 units right and 5 units down

(d) A line has a gradient of -3. From the y-intercept, we move:

A 3 units right and 3 units up **B** 1 unit right and 3 units down

C 0 units right and 3 units down **D** 1 unit right and 3 units up

10 (a) A coordinate point that lies on the line $y = x + 7$ is:

A (2, 10) **B** (-3, -4) **C** (0, -7) **D** (-4, 3)

(b) A coordinate point that lies on the line $3x + 2y = 11$ is:

A (5, -2) **B** (2, 3) **C** (-3, -1) **D** (4, 0)

(c) A coordinate point that lies on the line $3y - x + 6 = 0$ is:

A (0, 2) **B** (2, 0) **C** (3, -1) **D** (-3, 1)

11 Find the gradient of the line perpendicular to:

(a) a line with gradient 2

(b) a line with gradient 0

(c) the line passing through (4, 5) and (3, -2)

(d) the line passing through (-2, 6) and (3, 8).

12 Match the following.

(a) A line parallel to the line joining the points (2, 3) and (4, 7)

(b) A line perpendicular to the line joining the points (2, 3) and (4, 7)

(c) A line parallel to the line joining the points (7, 0) and (-1, 8)

(d) A line perpendicular to the line joining the points (7, 0) and (-1, 8)

(e) A line perpendicular to the line joining the points (-1, -3) and (-4, -2)

(f) A line parallel to the line joining the points (4, 2) and (-3, 2)

(g) A line parallel to the line joining the points (3, -4) and (3, 8)

A A line joining (-6, 2) and (-2, -2)

B A line joining (0, -2) and (3, 4)

C A line joining (-3, -5) and (1, -5)

D A line joining (7, 3) and (5, 4)

E A line joining (-1, -5) and (-1, 5)

F A line joining (1, -4) and (3, -2)

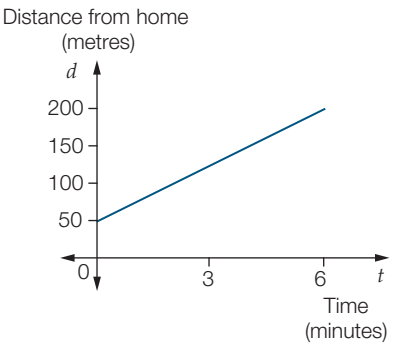
G A line joining (1, 5) and (3, 11)

13 What is special about the following lines?

(a) Lines parallel to the line joining (4, 2) and (-3, 2).

(b) Lines parallel to the line joining (3, -4) and (3, 8).

14 Consider this distance–time graph for Cassia as she walks to the shops.



(a) Write the intercept with the vertical axis.

(b) Find the gradient of this graph.

(c) Use this information to write an equation linking the distance from home (d) to the time (t).

(d) At what speed was she walking? (Refer to the equation.)

(e) If Cassia continued walking at the same speed, how far from home would she be after 20 minutes?

15 A distance–time graph is represented by the equation $d = 30t + 20$, where d is the distance from Newcastle in kilometres and t is the time in hours.

(a) Write the gradient of the line.

(b) What is the speed of travel?

(c) How far from Newcastle did the journey start?

(d) If the journey had started 90 km from Newcastle and a constant speed of 100 km/h was maintained, what equation could now be written to represent the relationship between d and t?

Reasoning, communicating and open-ended

16 An oil-tanker is being filled with crude oil. The oil-tanker initially contained 10 000 tonnes of oil and, over an 8-hour period, it received a further 96 000 tonnes of crude oil. Three full storage tanks, each of 48 000 tonnes capacity, were being used to transfer the crude oil to the tanker. Each tank was being emptied simultaneously at the same rate of 4000 tonnes per hour.

(a) Sketch a graph showing the amount of crude oil (in tonnes) in the oil-tanker during the 8-hour period.

(b) Find the gradient of this graph.

(c) Use this information to write an equation relating the amount of crude oil (m) in the tanker to the time (t).

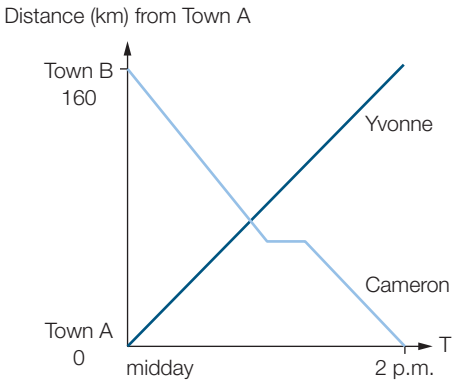
(d) Sketch a graph showing the amount of crude oil (m) left in one of the storage tanks over the 8-hour period as oil is pumped out of it.

(e) Write an equation relating the amount of crude oil (m) left in one of the storage tanks to the time (t).

(f) How much crude oil was left in each of the storage tanks after the 8-hour period?

(g) If the pumps from one of the storage tanks were under repair, how long would it take to transfer the same amount of crude oil from the remaining storage tanks?

17 This graph shows the journeys two friends made between Towns A and B, leaving at midday and arriving at their respective destinations 2 hours later.



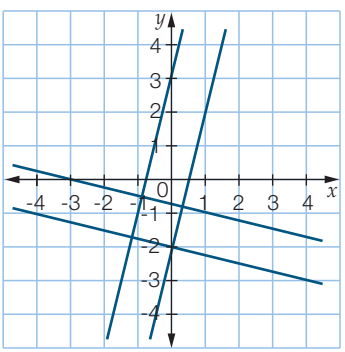
Yvonne travels at a constant speed straight from Town A to Town B. Cameron travels from Town B towards Town A at a constant speed for the first hour. He rests for 15 minutes, then continues to Town A at the same speed that Yvonne is travelling.

Find how far each of them was from Town A after travelling for half an hour.

18 State at least three different coordinate points that lie on each of the following lines.

(a) $y = x - 4$ **(b)** $2x - 5y = -10$ **(c)** $4y - x + 2 = 0$

19 When the equations $y = 4x + 3$, $y = 4x - 2$, $y = -\frac{1}{4}x - 2$ and $y = -\frac{1}{4}x - \frac{3}{4}$ are plotted on a Cartesian plane, the area enclosed by the points of their intersection is a square.



Can you find another set of sloping lines that enclose a square?

20 **(a)** Write an equation for a line parallel to $y = 4x - 3$.

(b) Write an equation for a line perpendicular to $y = 4x - 3$.

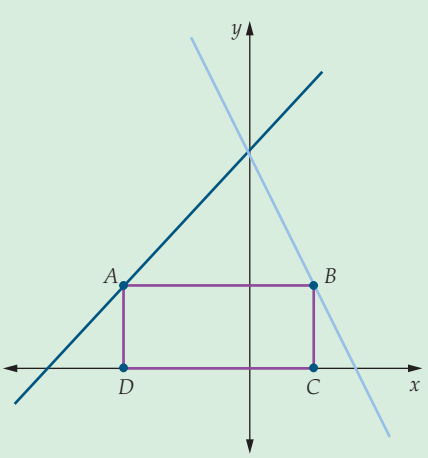
Hint

Remember the properties of parallel and perpendicular lines.

Problem solving

Area of a rectangle

In the diagram, $ABCD$ is a rectangle with A on the line $y = x + 10$, B on the line $y = -2x + 10$ and C and D on the x-axis. If $AD = 4$, what is the area of rectangle $ABCD$?



Strategy options

- Solve a simpler problem.
- Break the problem into manageable parts.

4.9 Sketching linear graphs using the *x*- and *y*-intercepts

Stage 5.3 §

Need to Know

A linear equation can be sketched by using the x- and y-intercepts as two points on the line. This method is not suitable for graphs in the form of $y = mx$.

To find the x-intercept, substitute $y = 0$ into the equation.

To find the y-intercept, substitute $x = 0$ into the equation.

Worked Example 24

WE 24

Use the x- and y-intercepts to sketch the graph of $2x + y = 4$.

Working

$2x + y = 4$

When $y = 0$:

$2x + 0 = 4$

$2x = 4$

$x = 2$

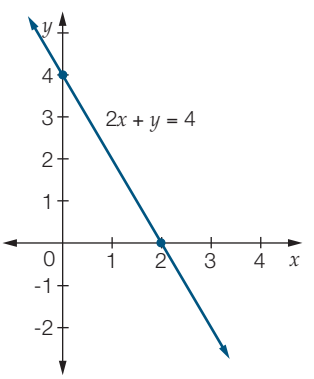
$\therefore$ x-intercept $= (2, 0)$

When $x = 0$:

$2 \times 0 + y = 4$

$y = 4$

$\therefore$ y-intercept $= (0, 4)$



Worked Example 25 WE25

Use the x- and y-intercepts to sketch the graph of $y = 6x - 2$.

Working

$y = 6x - 2$

When $y = 0$:

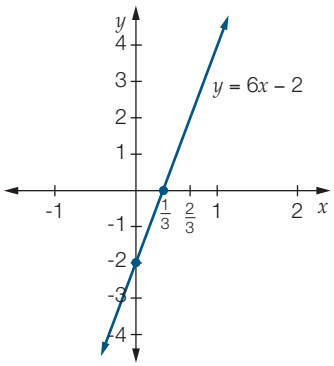
$0 = 6x - 2$

$6x = 2$

$x = \frac{2}{6}$

$= \frac{1}{3}$

$\therefore$ x-intercept $= \left(\frac{1}{3}, 0\right)$



When $x = 0$:

$y = 6 \times 0 - 2$

$= -2$

$\therefore$ y-intercept $= (0, -2)$

4.9 Sketching linear graphs using the x- and y-intercepts

Answers page 449

Navigator

Stage 5.3 §

Q1 Column 1, Q2 Column 1, Q3, Q4, Q6, Q7	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q6, Q7

Fluency and understanding

1 Use the x- and y-intercepts to sketch a graph for each of the following. WE24

(a) $x + 2y = 4$ **(b)** $3x + y = 6$ **(c)** $3x - y = 6$

(d) $x - y = 3$ **(e)** $4x - 2y = 12$ **(f)** $2x - 5y = 20$

(g) $2x + 5y = 10$ **(h)** $4x + 3y = 12$ **(i)** $3x + 4y = 2$

2 Use the x- and y-intercepts to sketch a graph for each of the following. WE25

(a) $y = 2x - 8$ **(b)** $y = 3x - 9$ **(c)** $y = x + 5$

(d) $y = x + 2$ **(e)** $y = 6 - 3x$ **(f)** $y = 2 - 4x$

(g) $2y = 3x + 4$ **(h)** $5y = 2x - 5$ **(i)** $3y = 6 - 2x$

3 **(a)** The coordinates of two points that lie on the line $2x + 3y = 6$ are:

A (2, 0) and (0, 3) **B** (-2, 0) and (0, -3)

C (3, 0) and (0, 2) **D** (-3, 0) and (0, -2)

(b) The coordinates of two points that lie on the line $3y = 2x - 6$ are:

A (-3, 0) and (0, 2) **B** (3, 0) and (0, -2)

C (-3, 0) and (0, -2) **D** (2, 0) and (0, -3)

4 Todd has seen a Blu-ray player for $175 that he wishes to buy.
The store will allow him to pay it off in weekly instalments of $25.

(a) How much would Todd still owe the store:

(i) after 1 week **(ii)** after 3 weeks **(iii)** after n weeks?

(b) Write an equation showing the relationship between the amount (A) that Todd owes the store and the number of weeks (n) he has been paying off the Blu-ray player.

(c) **(i)** Find A when $n = 0$.

(ii) What does this mean?

(d) **(i)** Find n when $A = 0$.

(ii) What does this mean?

(e) Sketch the graph of this relationship. Write A on the vertical axis and n on the horizontal axis.

5 A milk vat is being drained so that regular cleaning and maintenance can take place. The volume, V, of milk remaining in the vat after a given time, t, is given by the relationship $V = 630 - 15t$, where V is in litres and t is in minutes.

(a) How much milk is left in the vat after 5 minutes?

(b) How much milk did the vat originally contain?

(c) How long will it take for the vat to be completely empty?

(d) Use this information to graph this relationship on a number plane.

Reasoning, communicating and open-ended

6 Chantal has borrowed $7200 from her parents to buy a car. She agrees to repay the amount, plus an extra $300 in interest, to her parents at the rate of $300 per month.

(a) What is the total amount she owes to her parents?

(b) How long will it take to pay off the loan?

(c) Sketch a graph of this relationship. (Put amount owing A, in $, on the vertical axis and time, t, in months on the horizontal axis.)

(d) Write an equation describing the amount, A, Chantal owes her parents after t months.

(e) How much money does Chantal owe after 6 months?

(f) How much money has Chantal repaid after 14 months?

7 Find three different pairs of x- and y-intercepts when the gradient of the graph is equal to 3.

Challenge 4



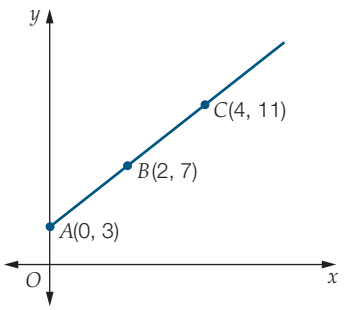
1 It takes Vinny 30 minutes to walk around the block, but Jesse can ride around on his bike in 15 minutes. If Vinny starts walking at 5:35 p.m. and Jesse follows at 5:48 p.m., when will Jesse catch up to Vinny?

2 Jakki is travelling through country Victoria and is trying to decide on her mode of travel for the next 5 days. If she hires a bicycle for 2 days and a car for the other 3 days, she will pay $150. If she hires a bicycle for 4 days and a car for the remaining 1 day, she will pay $90. What is the cost of hiring a bicycle for one day?

3 Nina uses the following table to produce a straight line graph.

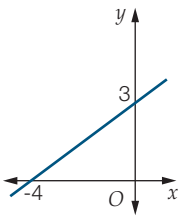
x	0	2	4
y	3	7	11

The point $D(7, w)$ also lies on the line. Find the value of w.

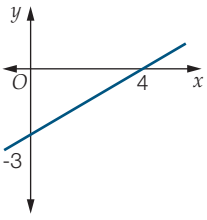


4 Which line has a gradient of $-\frac{3}{4}$? Diagrams are not to scale.

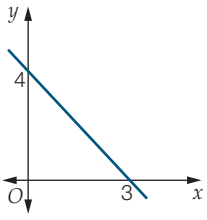
A



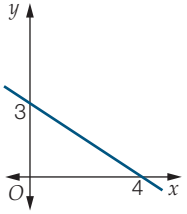
B



C



D



5 **(a)** The square $ABCD$ is to be plotted on a number plane. Given $A(1, 1)$, $B(3, 2)$ and $C(2, 4)$, find the coordinates of D.

(b) Write the gradients of AB and BC. What do you notice about your answers?

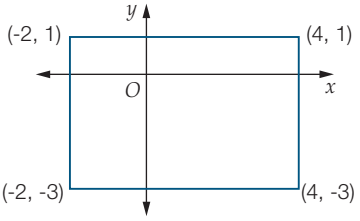
(c) Write the gradients of AD and DC. What do you notice about your answers?

(d) What can you say about the gradients of the parallel sides of the square?

(e) What can you say about the gradients of the perpendicular sides of the square?

6 The square $KLMN$ is to be constructed on the side KL. Given $K(-2, -1)$ and $L(1, 3)$, find two pairs of values for the coordinates of M and N.

7 The diagonals of this rectangle intersect at a point. What are the coordinates of this point?



8 Solve each equation for x.

(a) $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}$; $x \neq b$

(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2$; $a \neq 0, b \neq 0$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}$; $x \neq \pm c$

Chapter review 4

D.I.Y. Summary

Key Words

Stage 5.1	coordinates	midpoint	run
	gradient	origin	slope
	interval	rise	undefined
Stage 5.2	dependent variable	inverse operation	perpendicular
	general form	linear equation	solve
	gradient–intercept form	linear graph	straight line
	independent variable	linear relationship	
	intercept	parallel	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

Stage 5.1 Q1, 2 **Stage 5.2 Q3, 4, 5, 6**

1 The _____ or _____ is a measure of a line's steepness.

2 The _____ can be found by dividing the vertical rise by the horizontal _____ between two points.

3 The graph of a linear relationship is called a _____.

4 To solve an equation, we perform the _____ in the reverse order.

5 A _____ has only one solution.

6 The point where a linear graph cuts the y-axis is called the y-_____.

Stage 5.1 Q1, 9–12, 18, 21, 22, 24 **Stage 5.2 Q4–8, 13, 14** **Stage 5.3 Q2, 3, 15–17, 19–20, 23**

Fluency and understanding

1 **(a)** Plot the points $P(-3, 5)$ and $Q(6, 11)$ on a Cartesian plane and join the points PQ. Ex 4.1

(b) Find the exact length PQ using Pythagoras' Theorem.

(c) Find the midpoint of PQ.

2 Use the distance formula to find the length of each side of a triangle formed by the points $X(1, -9)$, $Y(4, -5)$ and $Z(6, 3)$. Ex 4.2

3 **(a)** Use the midpoint formula to find the midpoints of each side of the polygon formed by the points $A(1, -2)$, $B(6, -6)$, $C(4, 3)$ and $D(-4, 2)$. Ex 4.2

(b) What shape is this polygon?

4 Solve the following equations. Ex 4.3

(a) $3x - 7 = -22$ **(b)** $7x + 4 = -3$ **(c)** $\frac{x}{5} + 9 = 0$

(d) $5 - \frac{x}{6} = -7$ **(e)** $6(x - 2) + 7 = -5$ **(f)** $4(2x - 5) = 0$

5 Solve these equations. Ex 4.3

(a) $\frac{4x}{3} - 20 = -8$ **(b)** $\frac{2x-3}{5} = 7$

(c) $\frac{3-9x}{2} = -4$ **(d)** $\frac{4(7x+1)}{11} = 8$

6 Solve these equations. Ex 4.3

(a) $4x - 5 = 8 - 2x$ **(b)** $8x + 5 = 3(2x - 11)$ **(c)** $5x + 18 = 3x$

(d) $2x = 9 - 7x$ **(e)** $\frac{1-4x}{8} = \frac{3x-7}{4}$ **(f)** $\frac{2x-5}{3} = \frac{x+7}{4}$

7 **(a)** The linear equation $\frac{x}{8} - 5 = 4$ has the solution: Ex 4.3

A $x = 8$ **B** $x = 72$ **C** $x = 37$ **D** $x = 13$

(b) The linear equation $6x + 21 = 4x + 1$ has the solution:

A $x = 2$ **B** $x = 10$ **C** $x = -10$ **D** $x = -2$

8 Pete keeps pet mice. He has quite a lot of mice. He has just been given four more. His friend Mal also keeps pet mice, but he has 15 fewer than Pete has now. Ex 4.4

(a) Write an expression to show how many pet mice Pete has now.

(b) Write an expression to show how many pet mice Mal has now.

(c) If Mal has 10 mice now, use an equation to find out how many mice Pete had originally.

9 A 68 cm screen television set is on sale for $1850. Terry arranges to pay $500 as a deposit and then weekly payments of $90. Ex 4.5

(a) Copy and complete this table.

Number of weeks	0	1	2	3	4	5	6
Amount paid (dollars)							

(b) Plot the amount paid for the television set against the number of weeks. Extend your graph to show the amount paid up to 20 weeks. (Plot number of weeks along the horizontal axis.)

(c) From your graph, how much would Terry have paid after 11 weeks?

(d) How long would it take Terry to pay for the television set in full?

(e) Write a relationship connecting the amount paid in dollars, A, to the number of weeks, n.

(f) Use this relationship to calculate the amount paid after 9 weeks. Check to see whether you obtained the same result from your graph.

10 Find the gradient of the line joining (3, 2) and (4, 7). Ex 4.6

11 **(a)** The gradient of the line joining the points (-5, 3) and (2, -4) is: Ex 4.6

A -1 **B** $\frac{1}{7}$ **C** 1 **D** $\frac{7}{3}$

(b) The gradient of the line joining the points (-2, 3) and (3, 3) is:

A 0 **B** $\frac{1}{5}$ **C** 1 **D** 5

12 Sketch these graphs on the one number plane: Ex 4.7

(a) $y = -5$ **(b)** $y = 3$ **(c)** $x = -2$

13 Plot the graph of $y = 2x + 4$ by first completing a table of values for x and y. Find the gradient, x-intercept and y-intercept for this line. Ex 4.8

14 Write four points that lie on each of the following lines and draw the graph. Ex.4.8

(a) $y = 0$ **(b)** $x = -2$ **(c)** $y = 4$ **(d)** $x = 0$

15 **(a)** The y-intercept of the graph of the linear equation $y + 2x = 3$ is: Ex.4.9

A -3 **B** -2 **C** 2 **D** 3

(b) The x-intercept of the graph of the linear equation $3x + 5y = 45$ is:

A 3 **B** 5 **C** 15 **D** 45

(c) The gradient of the graph of the linear equation $7x - 4y = 28$ is:

A $-\frac{7}{4}$ **B** $-\frac{4}{7}$ **C** $\frac{4}{7}$ **D** $\frac{7}{4}$

16 Find the y-intercept and gradient for each of the following, and sketch the graph of each. Ex.4.9

(a) $y = 2x - 4$ **(b)** $2y = 3x - 4$ **(c)** $x + y = 5$

(d) $3x + 2y = 4$ **(e)** $y = 3x$ **(f)** $y = -2x$

17 Find the x- and y-intercepts for each of the following and hence sketch graphs of each, showing the intercepts on the axes. Ex.4.9

(a) $4x - y = 4$ **(b)** $2x + 5y = 20$ **(c)** $y - 3x = 6$ **(d)** $2x + y = 5$

18 Polly walked at a constant speed for 5 minutes, covering 350 metres.

(a) To construct a distance–time graph, what two points can you use? (Do not draw the graph.)

(b) What is the gradient between these two points?

(c) What is Polly's average speed during these 5 minutes?

(d) Write an equation relating distance covered, d metres, to time taken, t minutes.

(e) Use your equation to find the distance Polly would travel in 8 minutes if she maintained her walking speed.

19 **(a)** The sum of three consecutive whole numbers is 66. What are the numbers?

(b) The sum of four consecutive even whole numbers is 100. What are the numbers?

20 The length of a rectangular paddock is 7 m less than twice the width. If the perimeter is 58 m, write an equation and solve it to find the length and width of the paddock.

Reasoning and communicating

21 Two lines are given by $kx + 6y - 5 = 0$ and $3x + 2y + 4 = 0$. Find the value of k such that the lines are parallel.

22 A bicycle manufacturing company needs to make a certain component to increase its range of products. $3000 needs to be spent on set-up costs and each unit that is produced costs an additional $5.75.

(a) Construct a cost function that describes the total cost of producing x components.

(b) Find the cost of producing 500 components.

(c) How many components could be produced on a budget of $9000?

(d) Draw a graph of the cost function to find the cost of 1000 components.

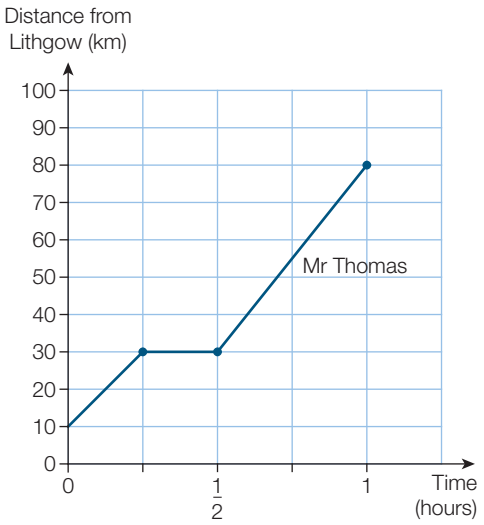
23 An electrician charges a fixed fee plus an hourly rate. A job that takes 2 hours to complete costs $280 and a job that takes 3.5 hours to complete costs $437.50.

(a) Find the linear model to express the charge $\$C$ for t hours of work.

(b) Find the amount she earns if she spends 8 hours on one job.

(c) Find the time taken to complete the job if the charge was $306.25.

24 Mrs Thomas, who works in Lithgow, has arranged to meet her husband at home in exactly 1 hour. Mr Thomas works closer to home but on the same route. He leaves work at the same time as Mrs Thomas but needs to make a stop along the way, as seen in the graph below; however, he still arrives home on time.



(a) Copy this graph onto graph paper.

(b) How far from Lithgow do they live?

(c) How fast was Mr Thomas travelling in the first 15 minutes?

(d) How long did he stop on his trip home?

(e) How fast did he travel in the last leg of his trip?

(f) How far from Lithgow does Mr Thomas work?

(g) If Mrs Thomas drives at a constant speed from Lithgow, rule a line on your graph that represents her travel.

(h) At what speed does Mrs Thomas need to travel to arrive home 1 hour later?

(i) Write a linear equation that represents her travel, giving the distance from Lithgow at any particular time.

(j) At what time does Mrs Thomas pass Mr Thomas on the way home? (Use your graph.)

(k) At what distance from home does she pass him?

NAPLAN practice 4

Numeracy: Non-calculator

1 Which expression is equivalent to $5 - 7x$?

A $7x - 5$ **B** $-7x - 5$ **C** $-7x + 5$ **D** $7x + 5$

2 What is the gradient of the straight line connecting the points (-3, -4) and (-1, 8)?

A 8 **B** 4 **C** 2 **D** 6

3 What is the value of b in the equation $5b - 4 = 2b + 17$?

4 If $y = -x + 4$ and $y = 2x - 8$, which point lies on both lines?

A (-3, 7) **B** (4, 0) **C** (1, -6) **D** (3, 1)

5 Which of these points lies on the straight line joining (4, 4) and (20, 12)?

A (5, 5) **B** (6, 6) **C** (10, 7) **D** (14, 8)

Numeracy: Calculator allowed

6 What number comes next in the following sequence?

25, 50, 75, 100, …

7 $y = 2x - 1$
$y = 3x + 2$

Which value of x satisfies both these equations?

A $x = -3$ **B** $x = -1$ **C** $x = 1$ **D** $x = 3$

8 Find the value of x that makes the equation $\frac{3x - 5}{2} = 5$ true.

9 The sum of two consecutive numbers is 55. What are the numbers?

A 33 and 22 **B** 22 and 23 **C** 27 and 28 **D** 30 and 25

10 An isosceles triangle has two equal sides and a third side that is 15 cm longer. Find the length of the longest side if the perimeter is 84 cm.

5 Geometry

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

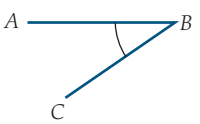
1 Draw angles of the following sizes, using a protractor and a ruler.

(a) 48° **(b)** 127° **(c)** 235° **(d)** 315°

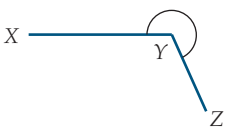
2 Name the type of each angle drawn in Question **1** using a word from this list: acute; right; obtuse; straight; reflex.

3 Use a protractor to measure the following angles.

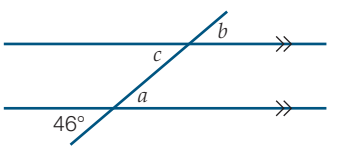
(a)



(b)



4 Find the value of the variables in this diagram.



5 Solve for x.

(a) $x + 7 = 180$ **(b)** $2x + 40 + 3x = 360$ **(c)** $x + 20 + 5x - 80 = 360$

6 Use the scale and measure this picture to determine the actual measurements, in metres, of:

(a) the length of the car **(b)** the height of the car.



Scale 100 : 1

7 Write the following as unit ratios. Round answers to two decimal places if necessary.

(a) 300:50 **(b)** 14:4 **(c)** 1400:64 **(d)** 258 000:376

8 **(a)** The population of Adelaide is approximately 9.4 times the population of Darwin. If the population of Darwin is 125 000, what is the population of Adelaide?

(b) A baker's basic biscuit recipe requires 2.5 times as much flour as sugar. If 10 cups of flour are used in one batch of biscuits, how much sugar should be used?

Syllabus references

Number and Algebra: Equations

✓ Solve simple linear equations **(ACMNA179) (Stage 4)**

Measurement and Geometry: Angle Relationships

✓ Recognise the geometrical properties of angles at a point **(Stage 4)**

✓ Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal **(ACMMG163) (Stage 4)**

✓ Solve simple numerical problems using reasoning **(ACMMG164) (Stage 4)**

Measurement and Geometry: Properties of Geometrical Figures

✓ Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral **(ACMMG166) (Stage 4)**

✓ Use the enlargement transformation to explain similarity **(ACMMG220) (Stage 5.1)**

✓ Solve problems using ratio and scale factors in similar figures **(ACMMG221) (Stage 5.1)**

✓ Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar **(ACMMG220) (Stage 5.2)**

✓ Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes **(ACMMG244) (Stage 5.2)**

✓ Formulate proofs involving congruent triangles and angle properties **(ACMMG243) (Stage 5.2)**

Angle review

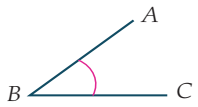
5.1

Stage 4

Need to Know

Angles at a point

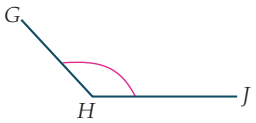
Acute angle:



Less than 90°

$0 < \angle ABC < 90°$

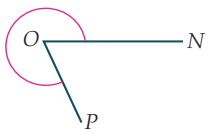
Obtuse angle:



Greater than 90° but less than 180°

$90° < \angle GHJ < 180°$

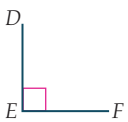
Reflex angle:



Greater than 180° but less than 360°

$180° < \angle NOP < 360°$

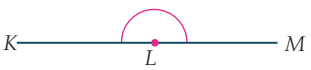
Right angle:



Exactly 90°

$\angle DEF = 90°$

Straight angle:



Exactly 180°

$\angle KLM = 180°$

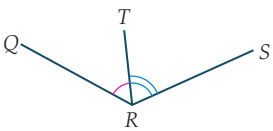
Revolution or angle of revolution:



Exactly 360°

Pairs of angles

Adjacent angles:



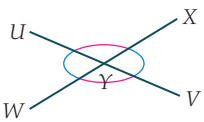
Angles that have a common vertex and a common arm.

R is a common **vertex**.

TR is a side common to both $\angle QRT$ and $\angle TRS$.

$\angle QRT$ and $\angle TRS$ are a pair of adjacent angles.

Vertically opposite angles:



A pair of angles formed when two straight lines intersect. They are on opposite sides of the vertex.

Vertically opposite angles are equal.

The line segments UV and WX intersect at Y.

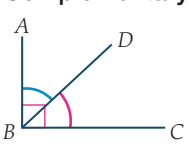
$\angle UYW$ and $\angle XYV$ are a pair of vertically opposite angles.

$\angle UYW = \angle XYV$

$\angle UYX$ and $\angle WYV$ are another pair of vertically opposite angles.

$\angle UYX = \angle WYV$

Complementary angles:

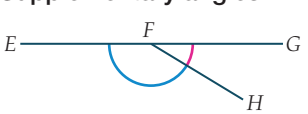


A pair of angles whose sum is 90°.

$\angle ABD$ and $\angle DBC$ are a pair of complementary angles.

$\angle ABD + \angle DBC = 90°$

Supplementary angles:



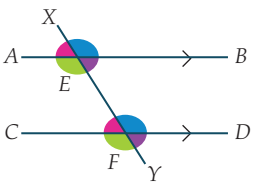
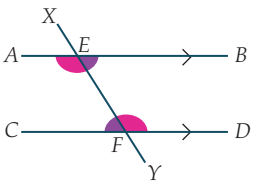
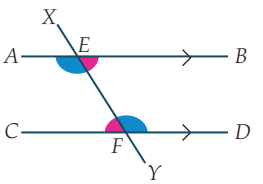
A pair of angles whose sum is 180°.

$\angle EFH$ and $\angle GFH$ are a pair of supplementary angles.

$\angle EFH + \angle GFH = 180°$

Angles on parallel lines

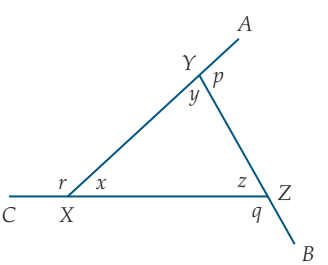
$AB \parallel CD$, the transversal XY cuts AB at E and CD at F.

Corresponding angles:	**Alternate angles:**	**Co-interior (allied) angles:**
		
When a transversal cuts a pair of parallel lines, four pairs of equal corresponding angles are formed. The pairs of equal corresponding angles are: $\angle AEX = \angle CFE$ $\angle BEX = \angle DFE$ $\angle AEF = \angle CFY$ $\angle BEF = \angle DFY$	When a transversal cuts a pair of parallel lines, two pairs of equal alternate angles are formed. The pairs of equal alternate angles are: $\angle AEF = \angle DFE$ $\angle BEF = \angle CFE$	When a transversal cuts a pair of parallel lines, two pairs of supplementary co-interior angles are formed. The pairs of supplementary co-interior angles are: $\angle AEF + \angle CFE = 180°$ $\angle BEF + \angle DFE = 180°$

Exterior angle of a triangle

The **exterior angle** of a triangle is the angle formed when any side is extended. The exterior angle of a triangle is equal to the sum of the opposite **interior angles**.

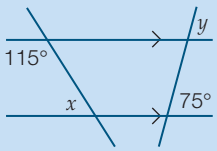
$p = x + z$, $q = x + y$ and $r = y + z$

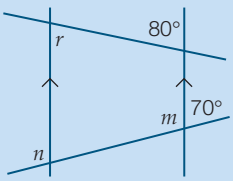


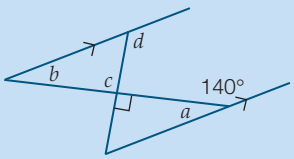
Worked Example 1

WE1

Find the value of the variables in each diagram. Give reasons for your answer.

(a) 

(b) 

(c) 

Working

(a) $x = 180° - 115°$
$= 65°$ (co-interior angles between parallel lines)

$y = 75°$ (corresponding angles on parallel lines)

(b) $m = 180° - 70°$ (straight angle)
$= 110°$

$n = m$
$= 110°$ (corresponding angles on parallel lines)

$r = 80°$ (alternate angles on parallel lines)

(c) $a = 180° - 140°$ (straight angle)
$= 40°$

$b = a$
$= 40°$ (alternate angles on parallel lines)

$c = 90°$ (vertically opposite)

$d = b + c$
$d = 40° + 90°$
$= 130°$ (exterior angle of a triangle)

5.1 Angle review

Answers page 454

Stage 4

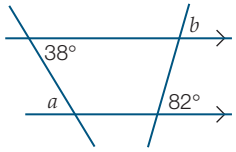
Navigator

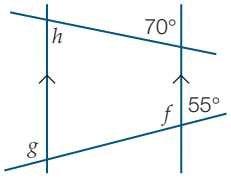
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

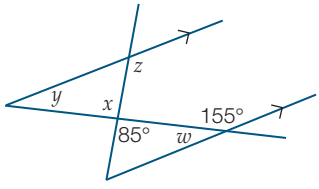
Fluency and understanding

1 Find the value of the variables in each diagram. Give reasons for your answer.

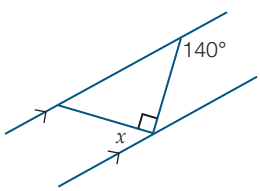
WE1

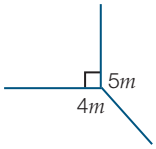
(a) 

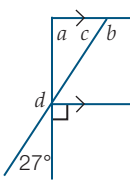
(b) 

(c) 

2 Find the value of the variables in each diagram. Give reasons for your answer.

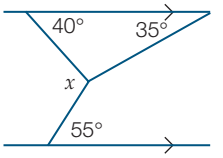
(a) 

(b) 

(c) 

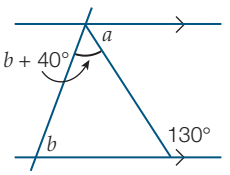
3 In the diagram, the value of x is:

A 75 **B** 95 **C** 125 **D** 140

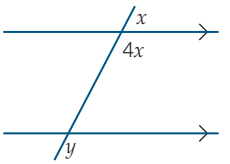


4 Find the value of the variables in each diagram. Give reasons for your answer.

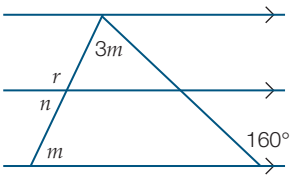
(a)



(b)



(c)



5 Write True or False for each of the following statements.

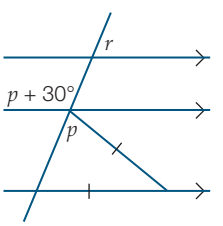
(a) Alternate angles are always equal.

(b) Co-interior angles are always supplementary.

(c) If the sum of a pair of adjacent angles is greater than 180°, then one of the angles must be an obtuse angle.

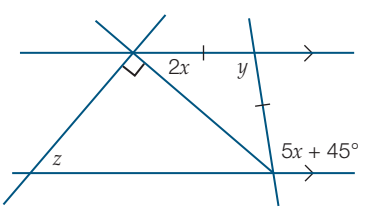
(d) The exterior angle of a triangle is always an obtuse angle.

6 Find the value of the variables in the diagram. Give reasons for your answer.



Reasoning, communicating and open-ended

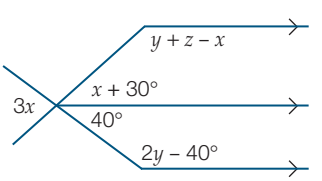
7 Find the values of x, y and z, giving reasons for your answer.



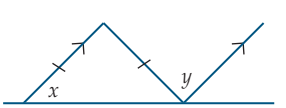
8 **(a)** Refer to the diagram on the right. Form an equation and solve it to find x. Give reasons for your answer.

(b) Form an equation and solve it to find y. Give reasons for your answer.

(c) Hence, form an equation and solve it to find z.



9 Choose three possible values for x. Show that $2x + y = 180°$ and use this to find y for each of your x-values.



10 **(a)** Write a possible pair of values for x and y.

(b) Write a possible pair of values for m and n.

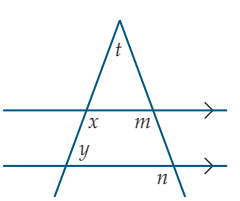
(c) Write an equation to find t in terms of n and y.

(d) Rewrite this equation to find t in terms of m and x.

(e) Use the equation in part **(c)** to find the value of t for your values of n and y.

(f) Use the equation from part **(d)** to find the value of t for your values of x and m.

(g) Compare your answers for parts **(e)** and **(f)**. Were they the same?



Enlarging and reducing

5.2

Stage 5.1

Need to Know

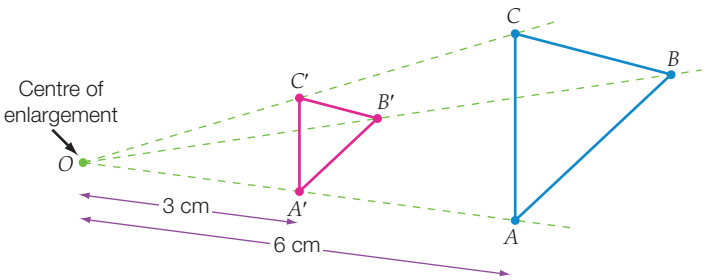
Scale factor

The **scale factor** is the proportion by which a copy of a figure is larger or smaller than the original.

If the scale factor is greater than 1, the resulting figure is an **enlargement**.

If the scale factor is less than 1, the resulting figure is a **reduction**.

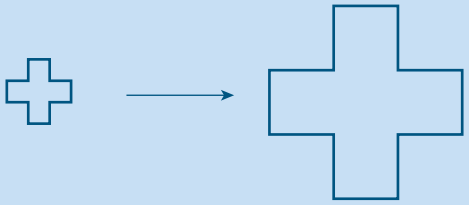
To enlarge or reduce a shape, we first need to choose a point of reference, known as the **centre of enlargement** (called O). Measure the distance from O to each of the points on the original shape and multiply that distance by the scale factor to determine the distance from O to the matching corners of the enlarged or reduced shape.



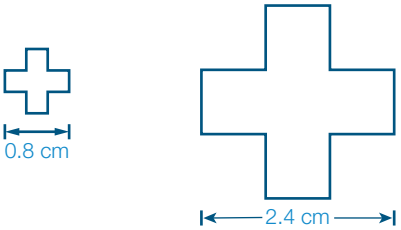
Worked Example 2

WE2

What scale factor has been used in this enlargement?



Working



Enlarged length : original length

$2.4 : 0.8$

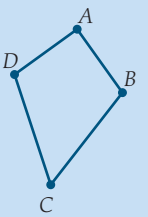
$= 3 : 1$

Scale factor $= 3$

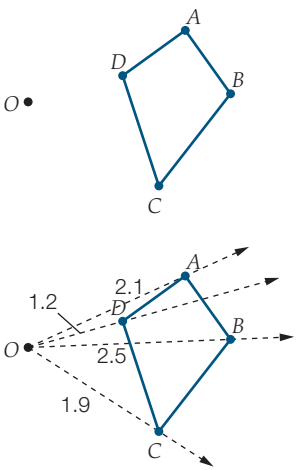
Worked Example 3

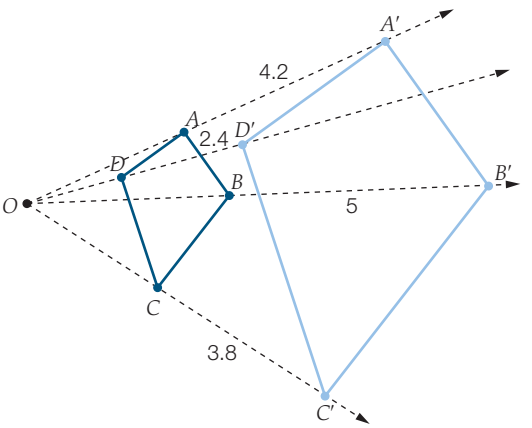
WE3

Enlarge the following shape by a scale factor of 2.



Working

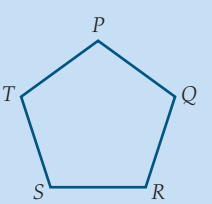




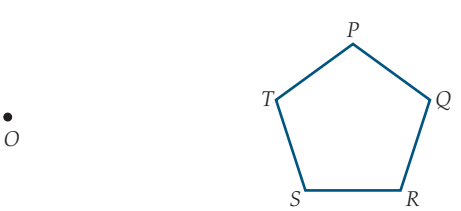
Worked Example 4

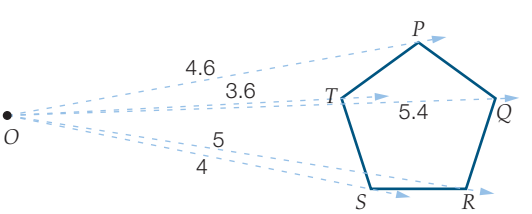
WE4

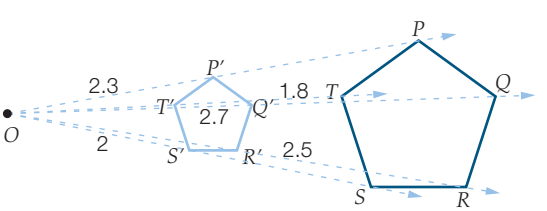
Reduce the following shape by a scale factor of $\frac{1}{2}$.



Working







Technology Exploration: GeoGebra

Enlarging or reducing a figure

Open the GeoGebra program.

1 From the menu, select Options > Point Capturing > Snap to Grid.

2 Right-click in the Graphics view to hide the Axes.

3 If a larger font is required, then from the menu select Options > Font Size.

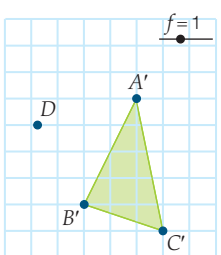
Changing the size of a figure by a given scale factor

4 Select the Slider tool. Click on the Graphics view where you would like the slider to appear. In the box that appears, name the slider '*f*' and set Min to 0, Max to 3 and Increment to 0.25.

5 Select the Polygon tool. Construct a triangle by clicking on any three points in the Graphics view and then clicking back on the starting point.

6 Select the Enlarge Object from Point by Factor tool. Click inside the triangle and then click to the side of the triangle where you would like the point to appear. When the pop-up box appears, enter *f* for the number value.

7 Your screen should look similar to this.



8 Use the Move tool to move your points to adjust your picture.

9 With the Move tool still selected, drag the point on the slider.

(a) What happens when you make the value of *f* greater than 1?

(b) What happens when you make the value of *f* less than 1?

10 Select the Ray Through Two Points tool. By clicking *D* first, draw rays through *A*, *B* and *C*. What do you notice about the rays? (What other points do they pass through?)

11 Move any of the points *A*, *B*, *C* or *D* about the Graphics view using the Move tool. What do you notice about the points *A*′, *B*′ and *C*′?

Investigating the lengths associated with enlargements

The lengths of the sides of the triangle are displayed in the Algebra view, with *c* representing the length of side *AB* in the original figure and *c*′ representing the length of the matching side *A*′*B*′ in the enlargement.

12 In the Input bar, enter: cxfactor=f*c. This will create a new result in the Algebra view called cxfactor. Press Enter. This is the length of *c* multiplied by the scale factor shown on the slider.

13 Repeat Step **12** to create: bxfactor=f*b.

14 Repeat Step **12** to create: axfactor=f*a. What do you notice about the values for:

(a) axfactor and length *a*′

(b) bxfactor and length *b*′

(c) cxfactor and length *c*′?

15 Using the Move tool, drag any of the points *A*, *B*, *C* or *D* around the screen. What do you notice about the values for:

(a) axfactor and length *a*′

(b) bxfactor and length *b*′

(c) cxfactor and length *c*′?

16 Change the value of the factor by moving the slider. What do you notice about the values for:

(a) axfactor and length *a*′

(b) bxfactor and length *b*′

(c) cxfactor and length *c*′?

17 Select the Distance or Length tool . Click on Point *D* and then on Point *A* to measure the interval *AD*. Repeat the process for intervals *DB*, *DC*, *DA'*, *DB'* and *DC'*. Use the Move tool to drag measurement labels away from overlapping points or sides.

18 In the Input bar, enter: distance DAxfactor = distance DA*f.

19 Repeat Step **18** for distance *DB* and distance *DC*. What do you notice about the values of:

(a) distance DAxfactor and the distance *DA'*

(b) distance DBxfactor and the distance *DB'*

(c) distance DCxfactor and the distance *DC'*?

20 Use the Move tool to drag any of the points *A*, *B*, *C* or *D* about. What do you notice about the values of:

(a) distance DAxfactor and the distance *DA'*

(b) distance DBxfactor and the distance *DB'*

(c) distance DCxfactor and the distance *DC'*?

21 Change the value of the scale factor by moving the slider. What do you notice about the values of:

(a) distance DAxfactor and the distance *DA'*

(b) distance DBxfactor and the distance *DB'*

(c) distance DCxfactor and the distance *DC'*?

Investigating the areas of similar figures

The areas of the two triangles should be given in the Algebra view as poly1 and poly1'.

22 In the Input bar, enter: poly1xfactor=poly1*f. What do you notice about the values of poly1xfactor and poly1'?

23 In the Input bar, enter: poly1xfactorsquared=poly1*f*f. What do you notice about the values of poly1xfactorsquared and poly1'?

Answers page 455

5.2 Enlarging and reducing

Stage 5.1

Navigator

Q1, Q2 (a), Q3 (a), Q4, Q5, Q7, Q9, Q10, Q12, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14	Q1, Q2 (a), Q3 (a), Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

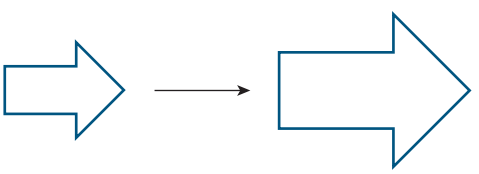
Equipment required: protractor for Question **8**, ruler for all

Fluency and understanding

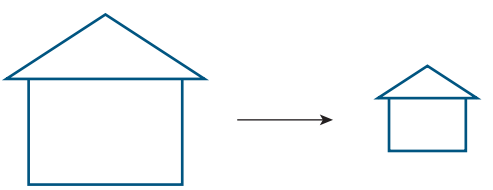
WE2

1 What scale factors have been used in each of the following enlargements or reductions?

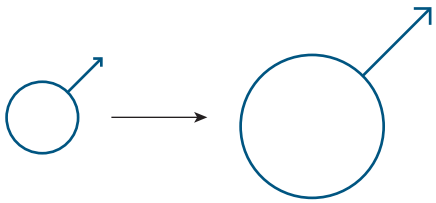
(a)



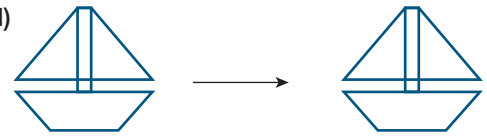
(b)



(c)



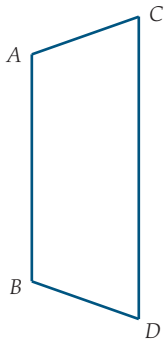
(d)



2 (a) Enlarge this shape by a scale factor of:

(i) 3

(ii) 1.5

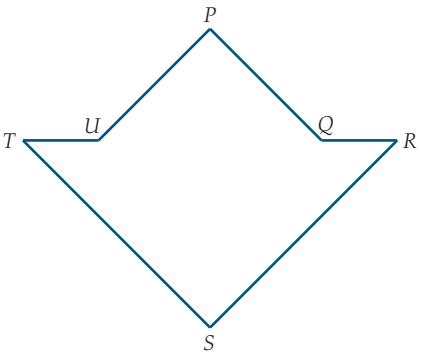


WE3

(b) Enlarge this shape by a scale factor of:

(i) 4

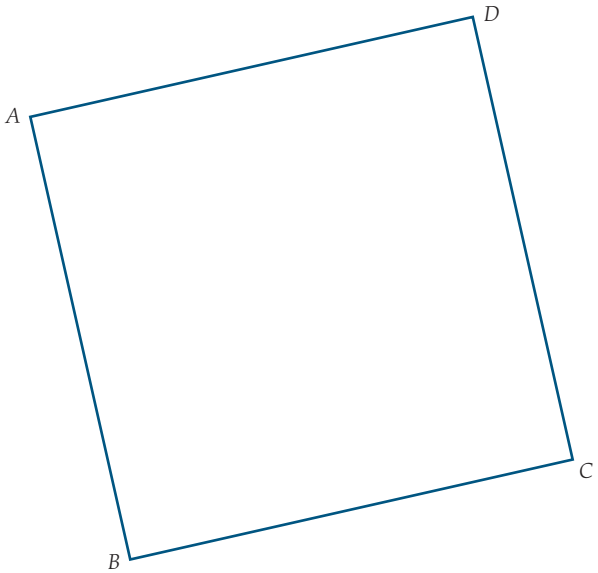
(ii) $2\frac{1}{4}$



3 (a) Reduce this shape by a scale factor of:

(i) $\frac{1}{2}$

(ii) $\frac{1}{3}$

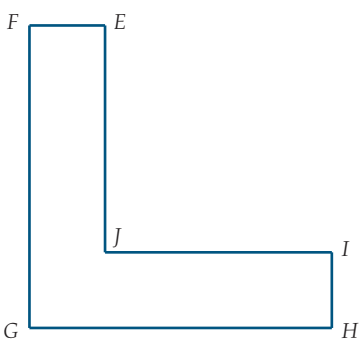


WE4

(b) Reduce this shape by a scale factor of:

(i) $\frac{3}{4}$

(ii) 0.6



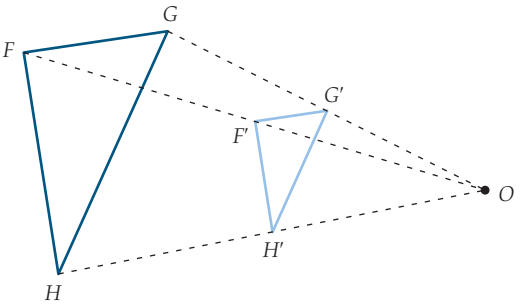
4 **(a)** ΔFGH has been reduced to $\Delta F'G'H'$. Measure each side of both triangles and find the following ratios.

(i) $F'G' : FG$

(ii) $F'H' : FH$

(iii) $G'H' : GH$

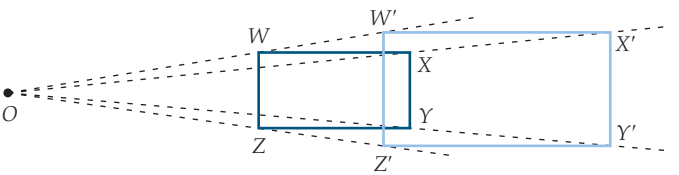
(b) What is the scale factor?



5 **(a)** Rectangle $WXYZ$ has been enlarged to rectangle $W'X'Y'Z'$. Measure each side of both rectangles and find the following ratios.

(i) $W'X' : WX$ **(ii)** $X'Y' : XY$ **(iii)** $Y'Z' : YZ$ **(iv)** $Z'W' : ZW$

(b) What is the scale factor?



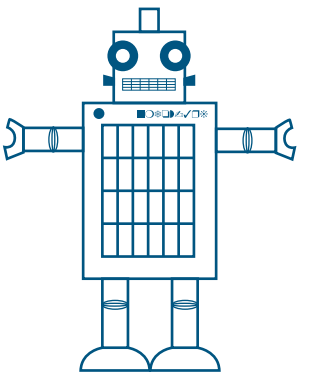
6 A scale factor of $\frac{1}{4}$ has been used to draw this toy robot. The actual height of the robot is:

A 1.2 cm

B 18 cm

C 19.2 cm

D 20 cm



7 Rectangle $ABCD$ has dimensions of 2 cm × 3 cm and is to be enlarged by a scale factor of 3. Draw this rectangle and use a centre of enlargement to produce the enlarged shape.

8 A regular pentagon $JKLMN$ has a side length of 8 cm. Accurately construct this pentagon using a ruler and a protractor and enlarge it by a scale factor of $1\frac{1}{4}$ using a centre of enlargement, O. (Choose any reasonable point as point O.)

Hint

The internal angle of a regular pentagon is 108°.

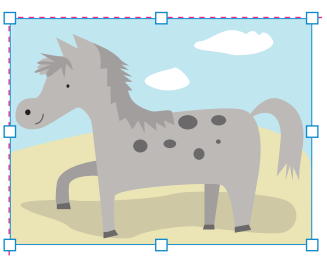
9 Kylee is using her laptop to collect some images for a project. She has one rectangular image that measures 4.8 cm by 6.2 cm. By clicking and dragging her mouse on a corner of the image, Kylee enlarges it so that it now measures 16.8 cm by 21.7 cm.

(a) What is the scale factor of this enlargement?

Later, Kylee finds that the image is now too big. She clicks and drags her mouse to reduce the image to 7.2 cm by 9.3 cm.

(b) What is the scale factor of this reduction of the enlarged image used in part **(a)**?

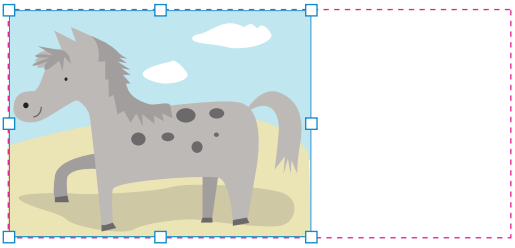
(c) Overall, has the original image been enlarged or reduced? Find the scale factor for this overall change.



(d) Kylee takes a copy of her original image and clicks and drags the middle of one of the sides instead of the corner.

She finds that the length of the image changes from 4.8 cm to 10.2 cm, but the width remains the same as before.

Is this image a true enlargement of the original image? Explain why or why not.



Reasoning, communicating and open-ended

10 The area of a square is 36 cm^2. What is the area of the square after the dimensions of the square have been reduced by a scale factor of $\frac{1}{3}$?

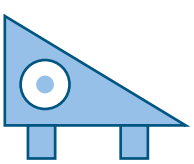
11 The picture on the right shows a scale drawing of a paperclip after it has been enlarged by a scale factor of 2. To make a scale drawing of the paperclip that is 4.9 cm long, what scale factor would need to be applied to the real paperclip?



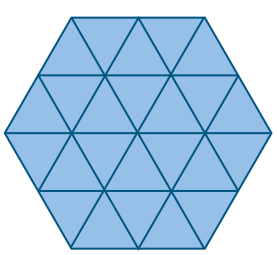
12 Draw a polygon with more than three sides. Reduce the shape by a scale factor of $\frac{1}{2}$. Enlarge the original shape by a scale factor of 3.

13 The area of a rectangle is 72 cm^2 after its length and width are enlarged by a scale factor of 3. What could the dimensions of the original rectangle be?

14 For the picture on the right, draw three scale drawings, stating the scale factor used each time.



15 Gina and Conan were playing a game using this hexagonal board. Gina noticed that the board consisted of three different-sized equilateral triangles. She then commented, 'There are four small triangles in the medium-sized triangle, so the scale factor must be 4, and there are nine small triangles in the large triangle so the scale factor must be 9'. Conan didn't agree.



(a) Draw the three different-sized triangles.

(b) Complete scale drawings of the small triangle with a scale factor of 4 and a scale factor of 9.

(c) What might Conan's answer be? How could he explain this answer to Gina?

5.3 Similarity and similar triangles

Stage 5.2

Need to Know

Similar figures

In **similar** figures:

- all pairs of matching sides are in the same ratio, *and*
- all pairs of matching angles are equal.

To show that two shapes are similar, we use the ~ symbol or the ||| symbol.

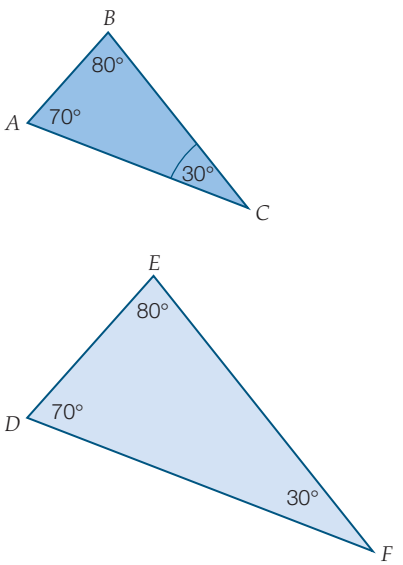
For example, 'triangle ABC is similar to triangle PQR' is written as:

$\Delta ABC \sim \Delta PQR$ or $\Delta ABC \,|||\, \Delta PQR$

It is not necessary to know about all sides and angles to determine that two triangles are **similar triangles**.

Tests for similar triangles

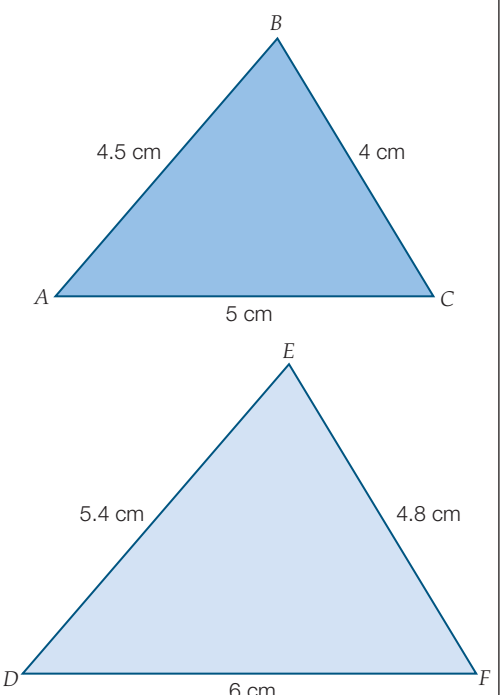
1 Angle, Angle, Angle (AAA)



$\Delta ABC \sim \Delta DEF$ (AAA)

Three pairs of angles are of the same magnitude. (It is only necessary to know the values of two of the pairs because the angle sum of a triangle is 180° and so we can calculate the value of the third pair.)

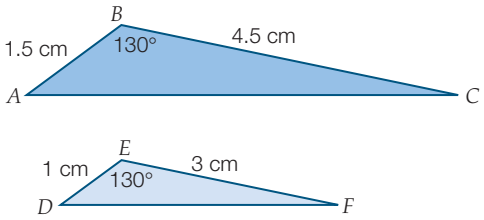
2 Side, Side, Side (SSS)



$\Delta ABC \sim \Delta DEF$ (SSS)

Three pairs of matching sides are in the same ratio.

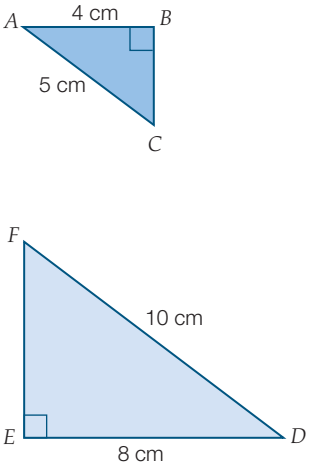
3 Side, Angle, Side (SAS)



$\Delta ABC \sim \Delta DEF$ (SAS)

Two pairs of matching sides have lengths in the same ratio and the included angles are equal.

4 Right-angle, Hypotenuse, Side (RHS)



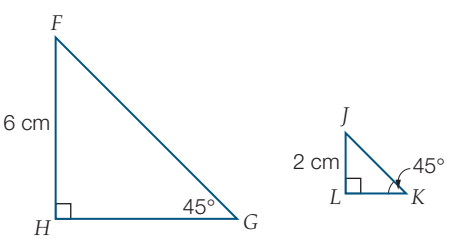
$\Delta ABC \sim \Delta DEF$ (RHS)

The hypotenuse and one side of a right-angled triangle are in the same ratio as the hypotenuse and one side of another right-angled triangle.

For two similar figures, the scale factor is the ratio of matching sides.

$$\frac{JL}{FH} = \frac{2}{6} = \frac{1}{3}$$

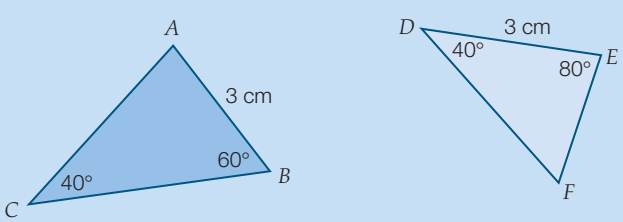
$\therefore \Delta JKL$ is one-third the size of ΔFGH or ΔFGH is three times the size of ΔJKL.



Worked Example 5

WE5

Use a similarity test to show that triangles *ABC* and *DEF* are similar.



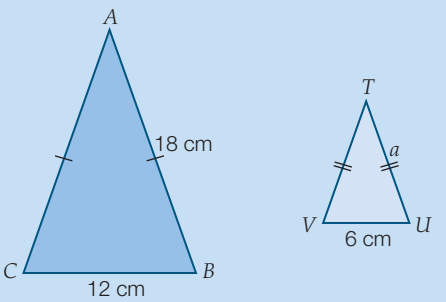
Working

$\angle ABC = \angle EFD = 60°$
$\angle ABC = \angle EFD = 60°$
$\angle CAB = \angle DEF = 80°$

$\therefore \Delta ABC \sim \Delta EFD$ (AAA)

Worked Example 6 WE6

Given that the following triangles are similar, find the value of a.



Working

Method 1: Use the scale factor

$$\text{Scale factor} = \frac{UV}{BC}$$
$$= \frac{6}{12}$$
$$= \frac{1}{2}$$
$$a = 18 \times \frac{1}{2}$$
$$= \frac{18}{1} \times \frac{1}{2}$$
$$= 9 \text{ cm}$$
$$a = 9\text{cm}$$

Working

Method 2: Set up an equation

$$\frac{TU}{AB} = \frac{UV}{BC}$$
$$\frac{a}{18} = \frac{6}{12}$$
$$a = \frac{6}{12} \times 18$$
$$= \frac{1}{2} \times \frac{18}{1}$$
$$= \frac{18}{2}$$
$$a = 9\text{cm}$$

Answers page 455

5.3 Similarity and similar triangles

Stage 5.1

Navigator

Q1 (a), (b), Q2 (a), (c), Q3, Q4, Q5, Q6 (a), (b), Q7, Q8, Q11, Q12, Q13	Q1 (b), (c), Q2 (a), (b), Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13	Q1 (c), (d), Q2 (b), (d), Q4, Q5, Q6 (c), (d), Q7, Q8, Q9, Q10, Q11, Q12, Q13

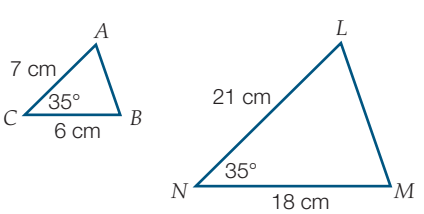
Equipment required: ruler and protractor for Question **9**

WE5

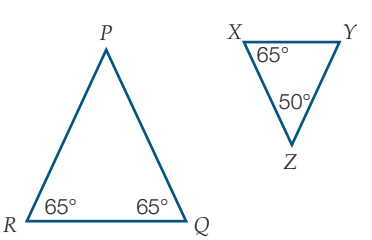
Fluency and understanding

1 Use a similarity test to show that the following pairs of triangles are similar.

(a)



(b)



Hint

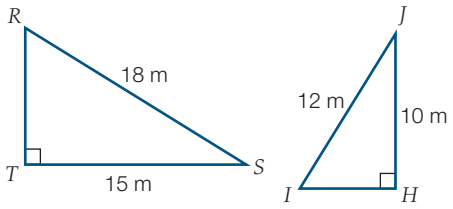
Remember to use the correct mathematical symbols:

∠ means 'angle'

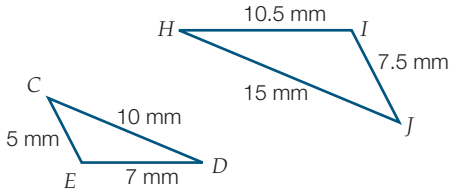
Δ means 'triangle'

~ means 'is similar to'.

(c)



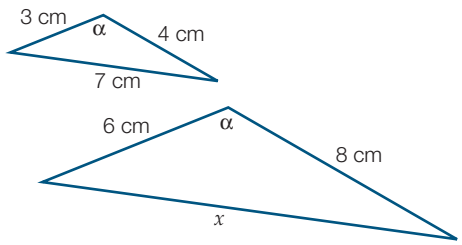
(d)



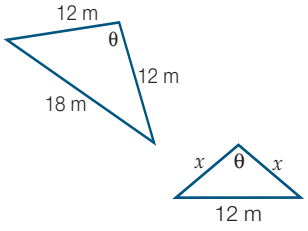
2 Find the value of x in each of the following pairs of similar triangles.

WE6

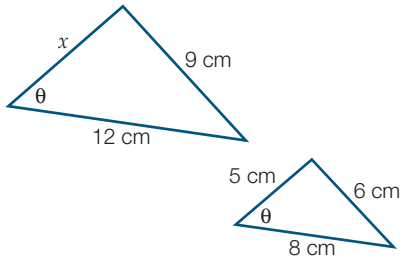
(a)



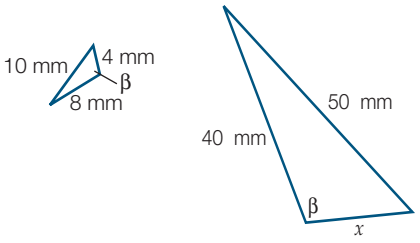
(b)



(c)



(d)



3 Given triangle ABC is similar to triangle EFG, then:

(a) the matching side to AB is:

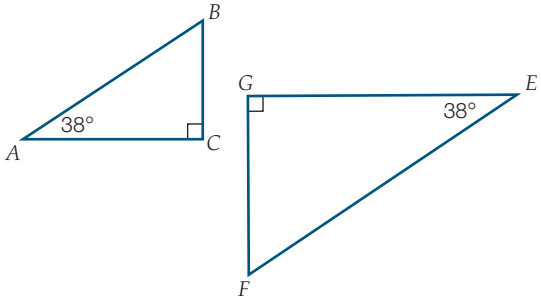
A FG **B** EF

C BC **D** EG

(b) the matching angle to $\angle EFG$ is:

A $\angle CAB$ **B** $\angle ACB$

C $\angle ABC$ **D** $\angle BAC$



4 Given triangle HJK is similar to triangle LMN, then:

(a) the matching side to LM is:

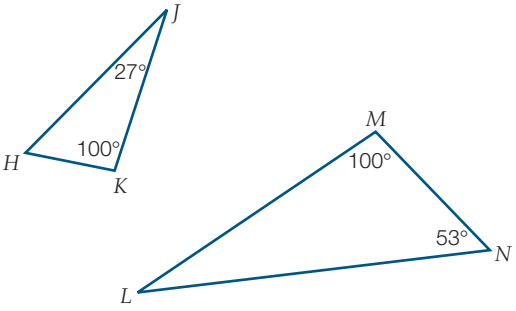
A HJ **B** MN

C JK **D** LN

(b) the matching angle to $\angle JHK$ is:

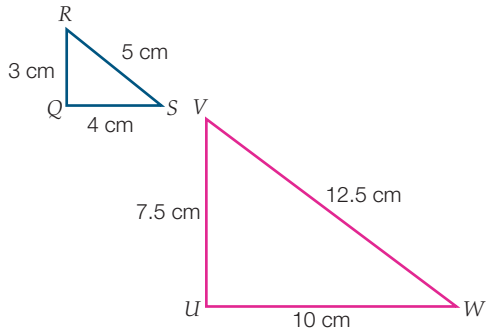
A $\angle LMN$ **B** $\angle NLM$

C $\angle LNM$ **D** $\angle MLN$

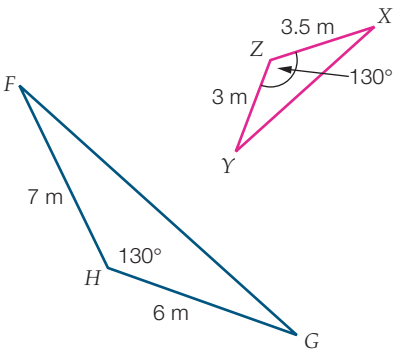


5 Find the scale factor for the following pairs of similar triangles. (The blue triangles are the originals.)

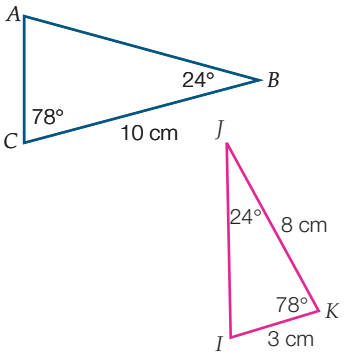
(a)



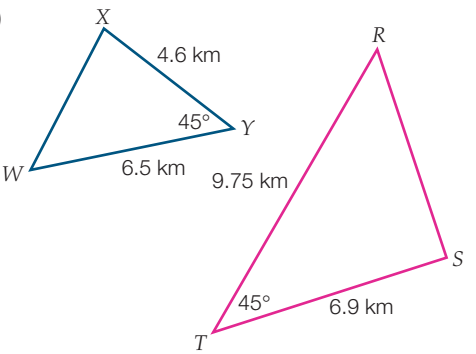
(b)



(c)

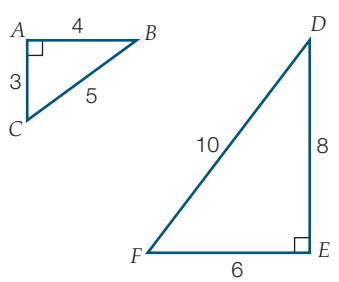


(d)

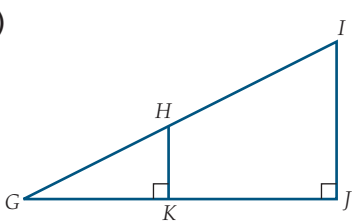


6 Show that these pairs of triangles are similar.

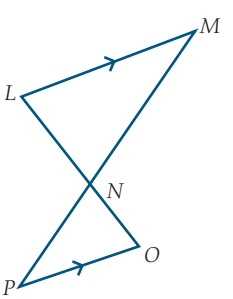
(a)



(b)



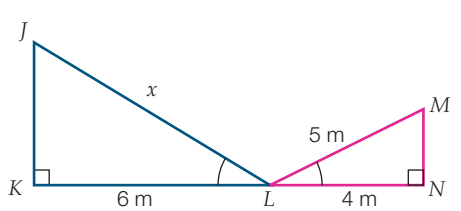
(c)



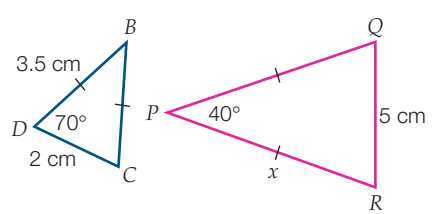
7 The following pairs of triangles are similar. (The blue triangles are the originals.)
For each pair:

(i) state the scale factor **(ii)** find the value of the variable.

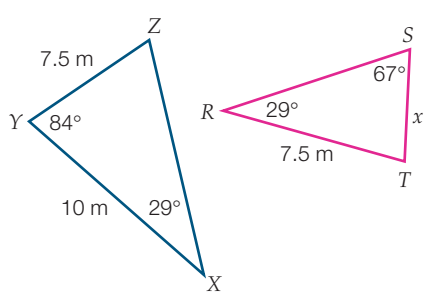
(a)



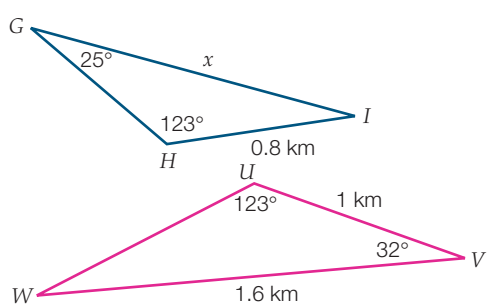
(b)



(c)



(d)



8 A 4-m ladder is leaning against a wall. The base of the ladder is 2 m from the wall. A second 5-m ladder is leaning against a wall. The base of this ladder is 3 m from the wall.

(a) Draw a diagram showing the triangle formed by the wall, the first ladder and the ground.

(b) Draw a diagram showing the triangle formed by the wall, the second ladder and the ground.

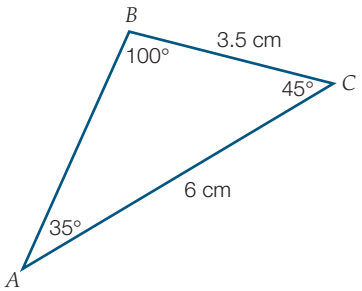
(c) Are the two triangles drawn in **(a)** and **(b)** similar? Justify your answer.

Reasoning, communicating and open-ended

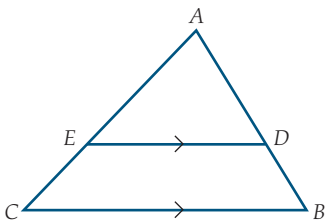
9 ΔABC and ΔPQR are similar triangles. ΔABC is shown here. $\angle PRQ = 45°$, $\angle RQP = 100°$ and $PR = 9$ cm. Use this information to:

(a) determine the scale factor

(b) construct ΔPQR using a protractor and ruler using the same scale.



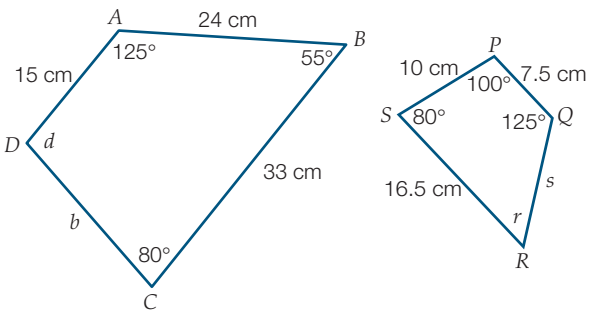
10 ΔABC and ΔADE are similar. Explain why any triangle divided with a line parallel to any side will produce two similar triangles.



11 Other two-dimensional shapes (and three-dimensional shapes) can be classed as similar. The quadrilaterals shown are similar.

(a) How can you prove this?

(b) Find the size of any unknown sides or angles in either quadrilateral.



12 Draw a pair of similar triangles that have angles of 30°, 60° and 90°. State the scale factor.

13 On a maths test, one of Geordie's justifications for the similarity of two triangles was:

$$\angle GHI = \angle UVT = 38°$$

(a) Draw two similar triangles that match this justification.

(b) On the triangles drawn in **(a)**, state the minimum amount of information necessary (angle magnitudes and/or side lengths) to be able to justify that the two triangles are similar.

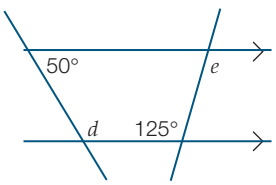
Half-time 5

Stage 4 Q1, 2, 3 **Stage 5.1 Q4, 5, 6, 7, 8**

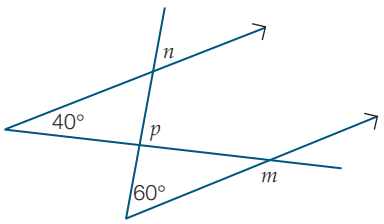
Ex.5.1

1 Find the value of the variables in each figure.

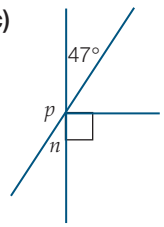
(a)



(b)



(c)



Ex.5.1

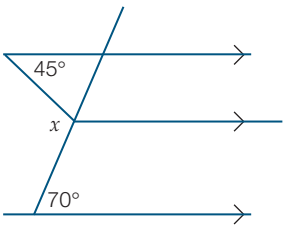
2 The value of x in the diagram is:

A 45°

B 65°

C 70°

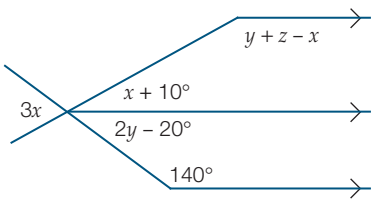
D 115°



Ex.5.1

3 Find the value of the variables in the diagram.

Give reasons for your answers.

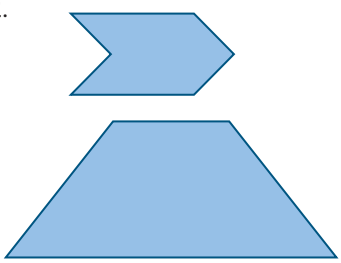


Ex.5.2

4 Enlarge this shape by a scale factor of 2.

Ex.5.2

5 Reduce this shape by a factor of $\frac{1}{2}$.



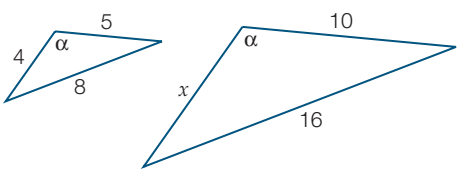
Ex.5.2

6 The area of a square is 49 m^2. Find the area of the shape after its sides are enlarged by a scale factor of 3.

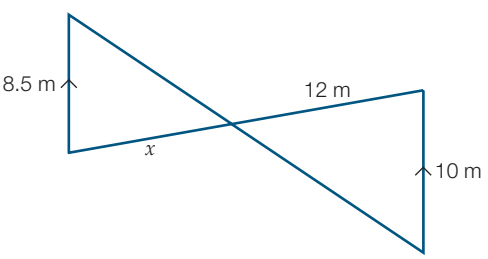
Ex.5.2

7 Find the scale factor for the following pair of similar triangles, then find the value of x.

(a)



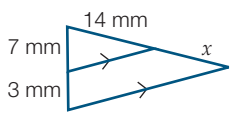
(b)



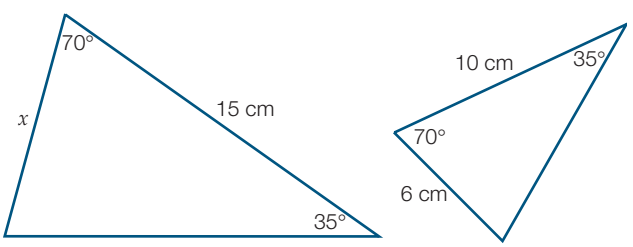
Ex.5.3

8 In each of the following, show that two triangles are similar and then find the value of x.

(a)



(b)



Using similar triangles

5.4

Stage 5.2

Need to Know

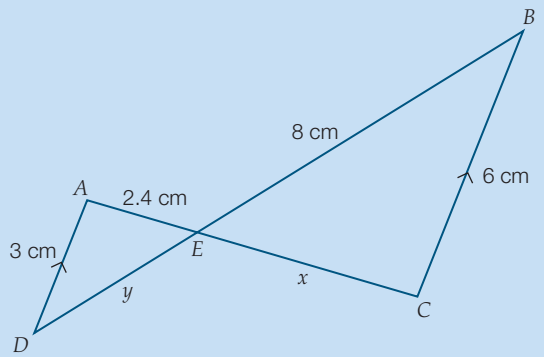
Solving problems using similar triangles

- Many questions and problems can be solved by first identifying that two triangles are similar and then using the scale factor to find the length of any unknown sides.
- Unknown angles can be found by identifying matching angles.
- Recall angle facts relating to parallel lines being crossed by a transversal.
- If two triangles are connected or overlapping, it is often useful to draw the two triangles separately and change the orientation of one so that they both have the same orientation.

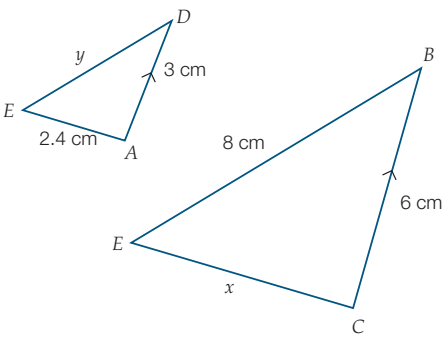
Worked Example 7

WE7

Show that the two triangles are similar and then find the values of x and y.



Working



$\angle AED = \angle BEC$ (vertically opposite angles)

$\angle DAE = \angle BCE$ (alternate angles)

$\angle ADE = \angle CBE$ (alternate angles)

$\therefore \Delta ADE \sim \Delta BCE$ (AAA)

$$\text{Scale factor} = \frac{BC}{DA}$$
$$= \frac{6}{3}$$
$$= 2$$

$$\frac{x}{EA} = 2$$
$$x = EA \times 2$$
$$= 2.4 \times 2$$
$$= 4.8 \text{ cm}$$

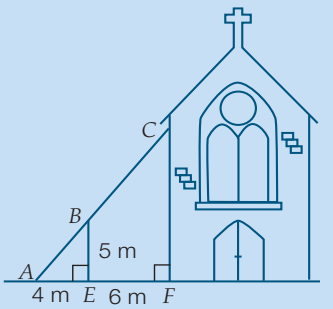
$$\frac{EB}{y} = 2$$
$$y = \frac{EB}{2}$$
$$= \frac{8}{2}$$
$$= 4 \text{ cm}$$

$x = 4.8$ cm and $y = 4$ cm

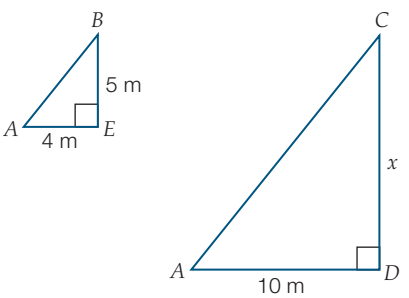
Worked Example 8

WE8

The wall at a local church needs a support to keep it in place (as shown). The support is positioned on the ground 10 m from the base of the wall. There is a 5-m vertical prop for this support placed 6 m from the wall. How far up the wall does the support reach?



Working



$\angle BAE = \angle CAD$ (common angle)
$\angle BEA = \angle CDA = 90°$ (given in diagram)
$\angle ABE = \angle ACD$ (angles in triangles = 180°)
$\therefore \Delta ABE \sim \Delta ACD$ (AAA)

$$\frac{CD}{BE} = \frac{AD}{AE}$$
$$\frac{x}{5} = \frac{10}{4}$$
$$x = \frac{10}{4} \times 5$$
$$= \frac{5}{2} \times \frac{5}{1}$$
$$= \frac{25}{2}$$
$$= 12.5 \text{ m}$$

$\therefore$ The support reaches 12.5 m up the wall.

Answers page 456

5.4 Using similar triangles

Stage 5.2

Navigator

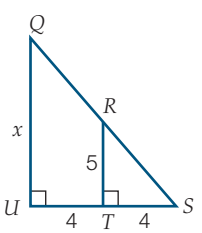
Q1 (a), (c), Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1 (b), (c), Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1 (b), (d), Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10

Fluency and understanding

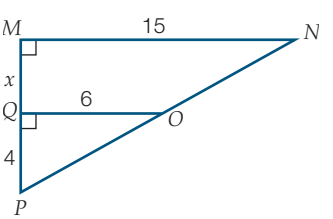
WE7

1 In each of the following diagrams, show that the two triangles are similar and then find the value of x.

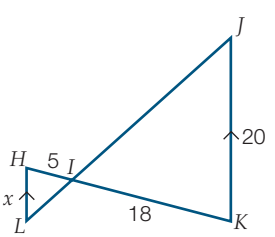
(a)



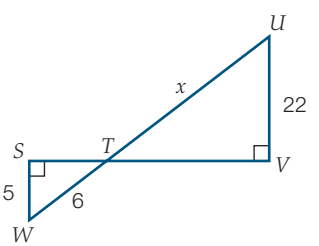
(b)



(c)

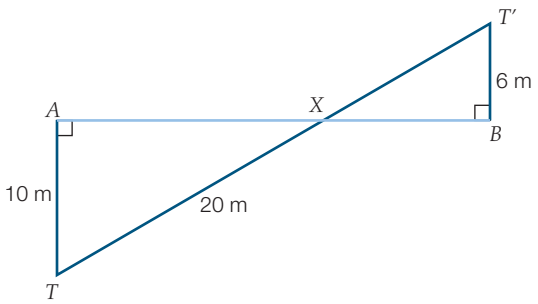


(d)



WE8

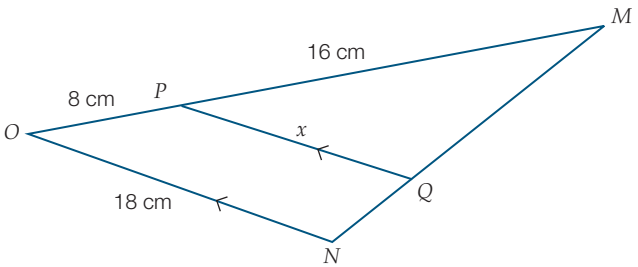
2 A tap T is placed on one end of a 10 m wall. A 20 m water pipe runs from the tap to a Point X as shown on the diagram. It is to be extended to another tap at T' on another wall BT' 6 m long.



(a) Show that the triangles ATX and $BT'X$ in the diagram are similar.

(b) Calculate the total length of new pipe XT' required.

3 The diagram below shows two similar triangles.



(a) Which of the following equations could be used to determine the length of PQ?

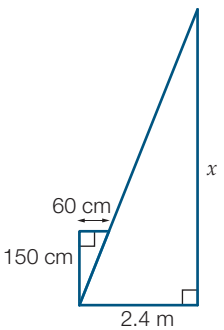
A $\frac{8}{18} = \frac{x}{16}$ **B** $\frac{x}{18} = \frac{8}{16}$ **C** $\frac{x}{18} = \frac{16}{24}$ **D** $\frac{x}{16} = \frac{8}{24}$

(b) The scale factor that produces the larger triangle from the smaller is:

A 0.5 **B** $0.\dot{6}$ **C** 1.5 **D** 2

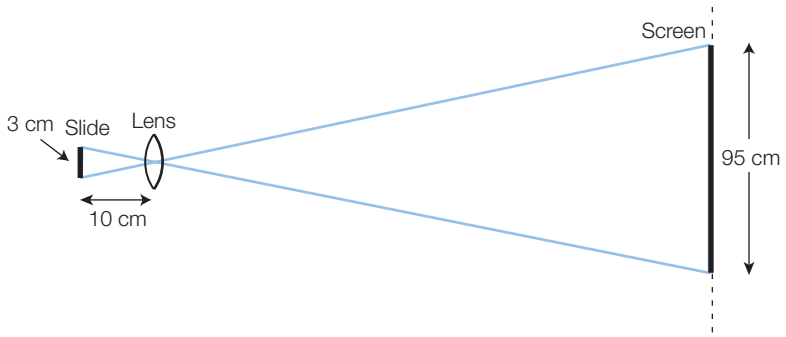
4 Noah is standing on a long ladder leaning against a wall. The base of the ladder is 2.4 m from the base of the wall. His friend Sean is holding the ladder to prevent it wobbling. Sean is standing with his shoulders 150 cm directly above the base of the ladder. His arms, 60 cm in length, are stretched horizontally, parallel with the ground, to hold the ladder.

A diagram of the information is shown.



How far up the wall does the ladder reach?

5 A light source projects the image of a photographic slide onto a screen, as shown in the diagram.

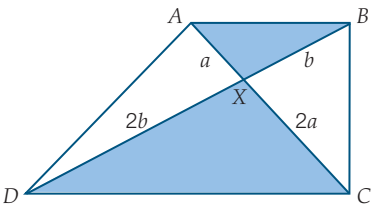


The slide is 3 cm high and positioned 10 cm behind a lens. Calculate the distance between the lens and the screen if the image on the screen is 95 cm high.

6 Martin plays tennis at night under floodlights. When he stands 4.5 m from the base of a floodlight, he casts a shadow 1.4 m long. If Martin is 1.7 m tall, how high is the floodlight?

Reasoning, communicating and open-ended

7 A quadrilateral *ABCD* is drawn as shown with diagonals intersecting at Point *X*.



(a) Show that the shaded triangles are similar.

(b) Hence, show that *ABCD* is a trapezium.

(c) Will the non-shaded triangles also be similar? Explain why or why not.

8 The leader of the local Scout troop was explaining to the Scouts how to use a stick to estimate the height of a tree.

Step 1: Have someone hold the stick vertically on the ground.

Step 2: Lie on the ground in a position so that, when looking at the tree on a line of sight that goes through the stick, the top of the tree and the top of the stick are in line.

Step 3: Unfortunately, the leader had forgotten Step 3.

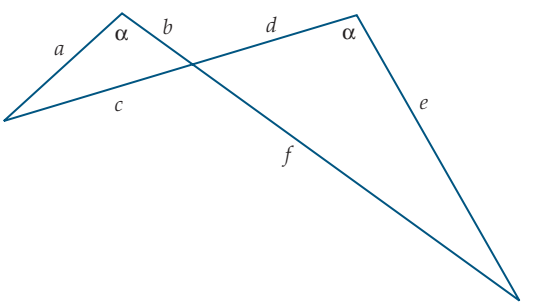
(a) Draw a diagram of the situation and use your knowledge of similar triangles to complete the instructions.

(b) List any practical problems you can see with this method.

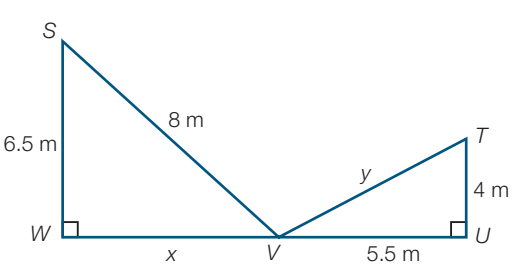
9 State a value for each of the variables in the diagram so that the two triangles are similar.

Hint

Be sure that $a + b > c$.



10 The teacher drew the following diagram on the board and asked the students to calculate the lengths of *x* and *y*. Santino's answers are shown.



Santino's answers:

To calculate x:

$$\frac{x}{5.5} = \frac{6.5}{4}$$

$$x = \frac{6.5}{4} \times 5.5$$

$$x = 8.94 \text{ m}$$

To calculate y:

$$\frac{y}{8} = \frac{4}{6.5}$$

$$y = \frac{4}{6.5} \times 8$$

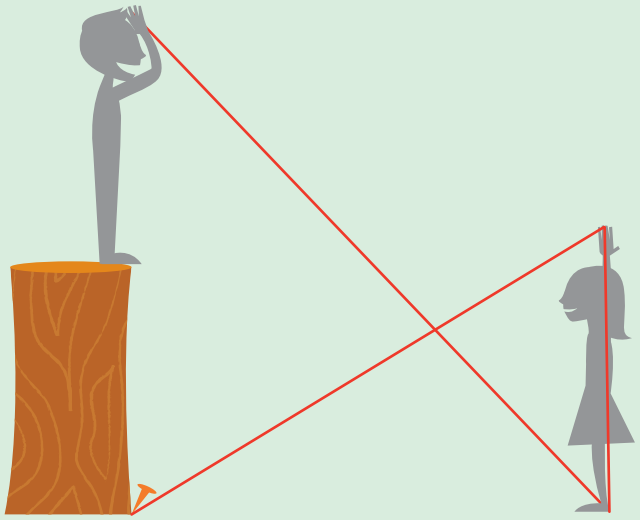
$$y = 4.92 \text{ m}$$

(a) Explain why the values Santino calculated for *x* and *y* are not correct.

(b) What method could Santino use to calculate the values of *x* and *y*?

Problem solving

Curiously crossed



Gina and Gerard are twins who are both exactly 1.5 metres tall. Gina has a very long rope. Gerard attaches one end of it to the bottom of a 1.5-metre tree stump with a tent peg. Gina picks up the rope and holds it above her head as high and as tightly as she can so it is 2 m above the ground. She lets the rest of the rope fall down behind her back and stands on it. Gerard then picks up the other end of the rope and stands on the tree stump so the rope crosses itself. This end of the rope just reaches the top of his head. How high off the ground do the ropes cross?

Strategy options

- Draw a diagram.
- Test all possible combinations.
- Break the problem into manageable parts.
- Look for a pattern.

5.5 Triangles and congruency

Stage 5.2

Need to Know

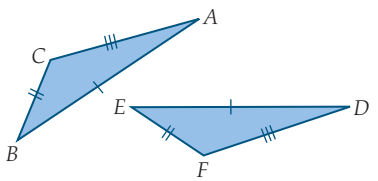
Two triangles are **congruent** if the matching sides are the same length and the matching angles are the same size. They are identical in all respects.

If ΔABC is congruent to ΔDEF, it is written $\Delta ABC \equiv \Delta DEF$. $\equiv$ means 'is congruent to'.

There are four tests for **congruent triangles**.

Tests for congruent triangles

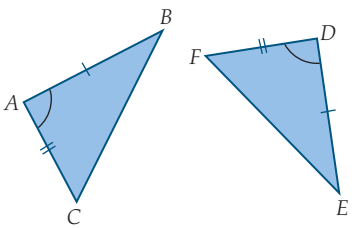
1 Side Side Side (SSS)



Two triangles are congruent if the matching sides have equal length.

$AB = DE$ (side)
$BC = EF$ (side)
$CA = FD$ (side)
$\Delta ABC \equiv \Delta DEF$ (SSS)

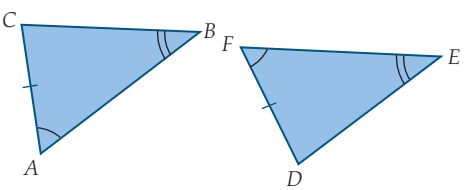
2 Side Angle Side (SAS)



Two triangles are congruent if two matching sides have equal lengths and the angles included by these sides are equal.

$AB = DE$ (side)
$\angle CAB = \angle FDE$ (angle)
$AC = DF$ (side)
$\Delta ABC \equiv \Delta DEF$ (SAS)

3 Angle Angle Side (AAS)

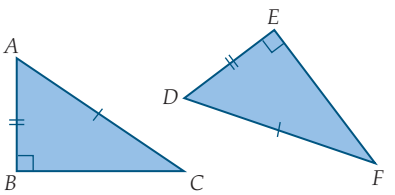


Two triangles are congruent if two matching angles are equal and a matching side is equal in length.

$\angle CAB = \angle DFE$ (angle)
$\angle CBA = \angle DEF$ (angle)
$AC = DF$ (side)
$\Delta ABC \equiv \Delta FED$ (AAS)

Note that the matching side does not need to be included between the two equal angles.

4 Right-angle Hypotenuse Side (RHS)



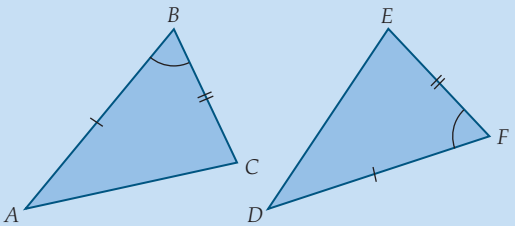
Two right-angled triangles are congruent if the hypotenuse and one pair of matching sides are equal in length.

$\angle ABC = \angle DEF = 90°$ (right angle)
$AC = DF$ (hypotenuse)
$AB = DE$ (side)
$\Delta ABC \equiv \Delta DEF$ (RHS)

Worked Example 9

WE9

Show that the triangles ABC and DEF are congruent, stating the congruency test used.



Working

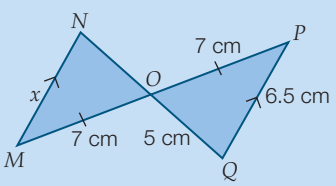
$AB = DF$ (given)
$\angle ABC = \angle DFE$ (given)
$BC = FE$ (given)

$\Delta ABC \equiv \Delta DFE$ (SAS)

Worked Example 10

WE10

Prove that $\Delta MNO \equiv \Delta PQO$, and hence find the value of x.



Working

ΔMNO and ΔPQO

$\angle MNO = \angle PQO$ (alternate angles, $MN \parallel QP$)

$\angle MON = \angle POQ$ (vertically opposite angles)

$MO = PO = 7$ cm (given)

$\Delta MNO \equiv \Delta PQO$ (AAS)

$MN = PQ$ and $NO = OQ$ (matching sides in congruent triangles)

$x = 6.5$ cm

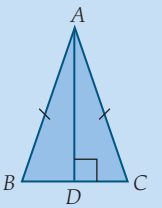
Worked Example 11 WE11

(a) ΔABC is a triangle with $AB = AC$. AD is perpendicular to BC. Prove that $\angle ABC = \angle ACB$.

(b) What type of triangle is ΔABC? Explain your answer.

(c) What triangle property have you just proved?

(d) Show that $BD = DC$.



Working

(a) ΔABD and ΔACD.

$\angle ADB = \angle ADC = 90°$ (given)

$AB = AC$ (given)

AD is a common side

$\Delta ABD \equiv \Delta ACD$ (RHS)

$\angle ABC = \angle ACB$ (matching angles in congruent triangles)

(b) ΔABC is an isosceles triangle because $AB = AC$.

(c) The base angles (angles opposite equal sides) of an isosceles triangle are equal.

(d) Matching sides of the congruent triangles ABD and ACD are equal.
$\therefore BD = CD$

Answers page 456

5.5 Triangles and congruency

Stage 5.2

Navigator

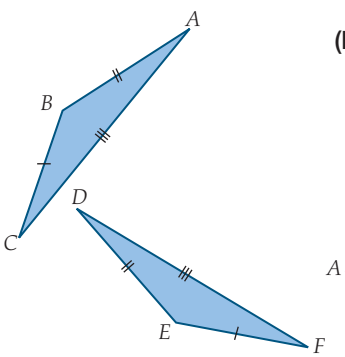
Q1 Columns 1 & 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 1, Q9, Q11 (a), (b), Q16	Q1 Columns 1 & 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q8 Column 2, Q9, Q11 (a)–(d), Q12, Q14, Q16	Q1 Columns 2 & 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 2, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency and understanding

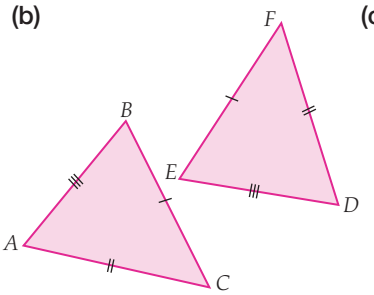
WE9

1 Show that each of the following pairs of triangles are congruent, stating the congruency test used in each.

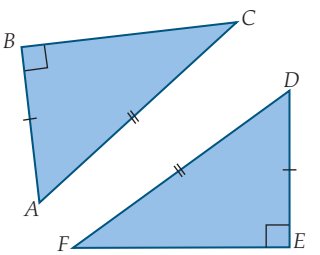
(a)



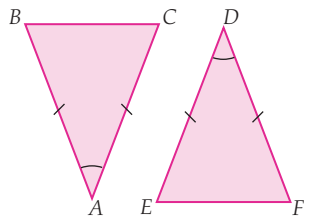
(b)



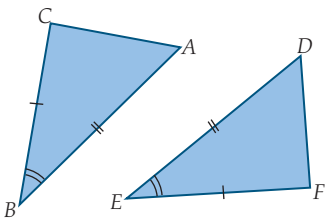
(c)



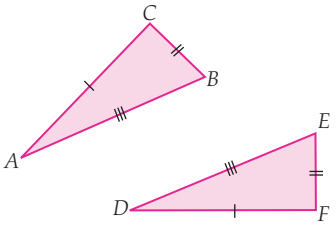
(d)



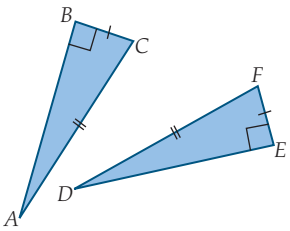
(e)



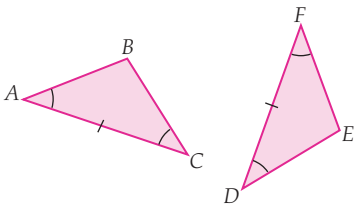
(f)



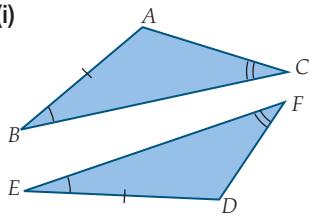
(g)



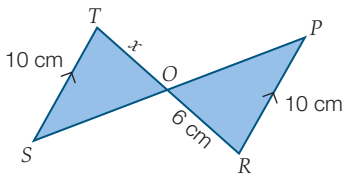
(h)



(i)



2 Prove that $\Delta STO \equiv \Delta POR$, and hence find the value of x.



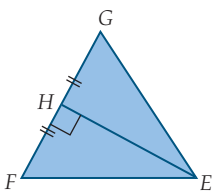
WE10

3 **(a)** In ΔEFG, $FH = HG$ and EH is perpendicular to FG. Prove that $\angle EFH = \angle EGH$.

(b) What type of triangle is ΔEFG? Explain your answer.

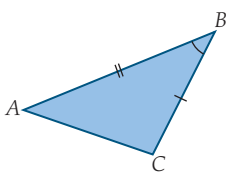
(c) What triangle property have you just proved?

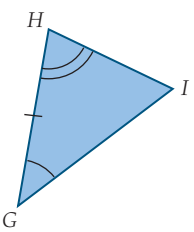
(d) Show that $EF = EG$.

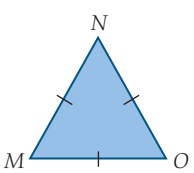


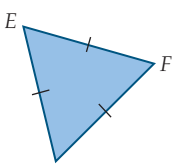
WE11

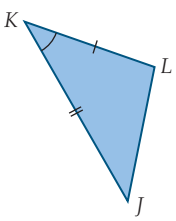
4 Find pairs of congruent triangles among the following diagrams. State the test used to show that they are congruent. Letter names must be in the correct order.

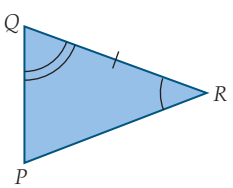








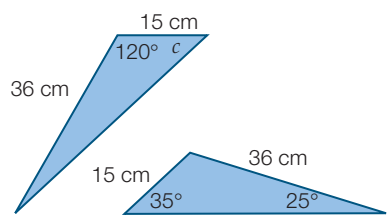




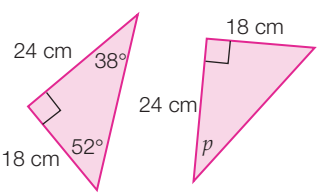
5 **(a)** State the test used to prove the following pairs of triangles are congruent.

(b) Use congruency to find the value of the unknown in each pair of triangles.

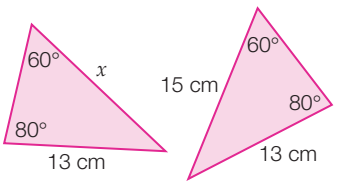
(i)



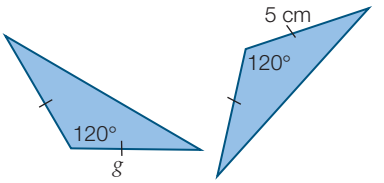
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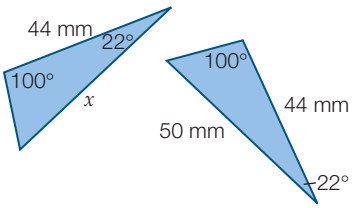
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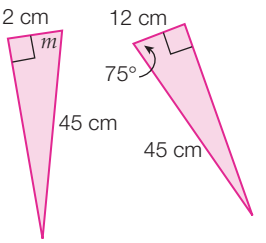
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(v)

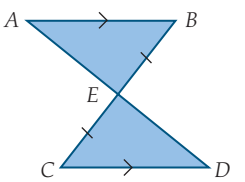


(vi)



6 Explain why two triangles with three equal angles may not necessarily be congruent. Show this by drawing diagrams.

7 Complete the following proof by copying it and writing in the missing steps.



$AB \parallel CD$. AD and BC intersect at E so that $BE = EC$.
Prove that $AB = CD$.

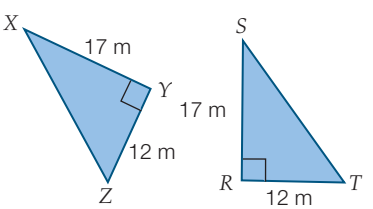
Proof:

In Δs ABE and DCE,

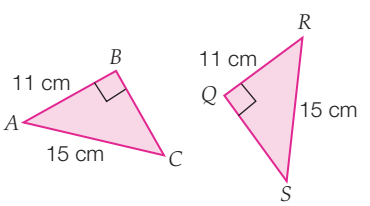
$\angle$_____ = $\angle$_____ (alternate angles, $AB \parallel CD$)
_____ = _____ (given)
$\angle$_____ = $\angle$_____ (vertically opposite angles)
$\therefore \Delta$_____ $\equiv \Delta$_____ (AAS congruency)
$\therefore$ _____ = _____ (matching sides of congruent triangles)

8 Prove that each of the following pairs of triangles is congruent, giving reasons for your answer.

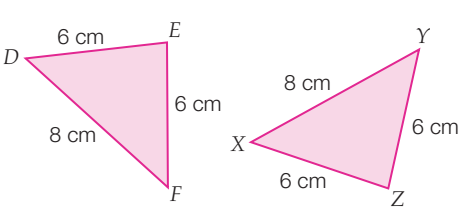
(a)



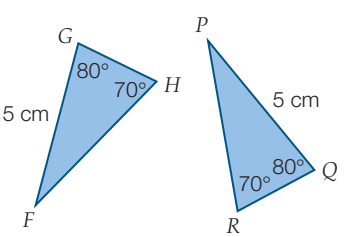
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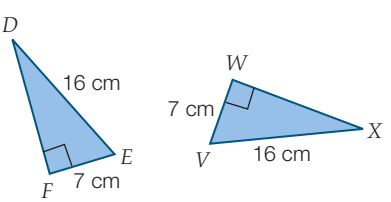
(c)



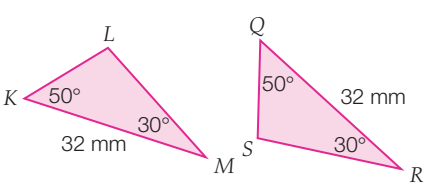
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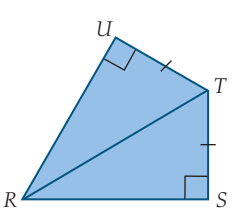
(e)



(f)

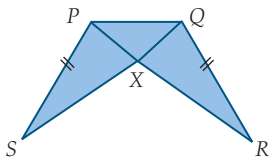


9 Prove that $\Delta RUT \equiv \Delta RST$.



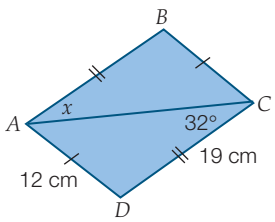
Reasoning, communicating and open-ended

10 If $SP = RQ$ and $\angle SPQ = \angle RQP$, prove that $\Delta SPQ \equiv \Delta RQP$.

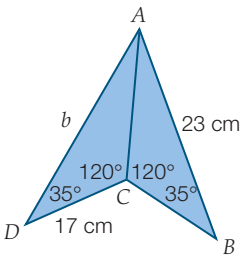


11 Prove that the following pairs of triangles are congruent and hence find the value of the variables.

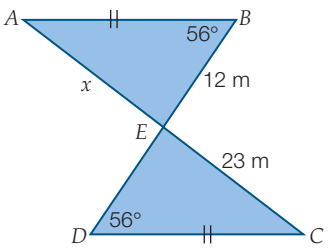
(a)



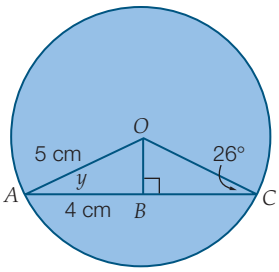
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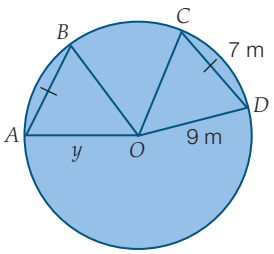
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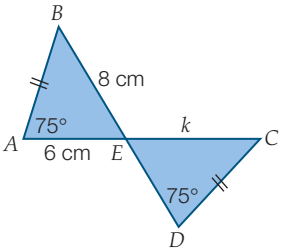
(d)



(e)



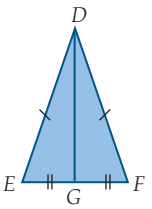
(f)



12 ΔDEF is an isosceles triangle with $DE = DF$. G is the midpoint of EF.

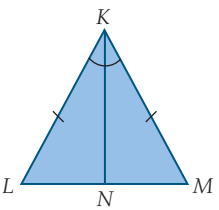
(a) Show that $DG \perp EF$.

(b) If ΔDEF is equilateral, prove that $\angle EDG = \angle FDG = 30°$.

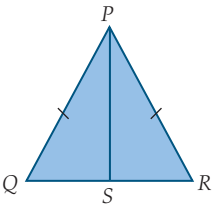


13 ΔKLM is an equilateral triangle with $KL = LM = MK$. N is a point on LM such that $\angle LKN = \angle MKN$.

Show that KN is the perpendicular bisector of LN (i.e. $LN = NM$ and $KN \perp LM$).

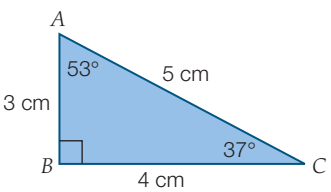


14 ΔPQR is an equilateral triangle with $PQ = QR = RP$. S is a point on QR that is joined to P. Explain why this is not enough information to prove that $QS = SR$.



15 Draw four pairs of congruent triangles and put just enough information on each diagram to show congruence. Each pair should represent a different test. Write the name of the test that applies to each pair of triangles.

16 ΔABC is right angled at B. The lengths of its sides and the size of its angles are shown on the figure. Draw four triangles with enough information marked on them so that each one is congruent with ΔABC using each of the congruence tests.



5.6 Special quadrilaterals and their properties

Stage 5.2

Need to Know

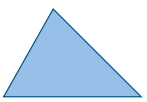
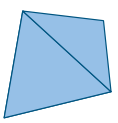
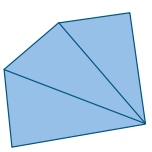
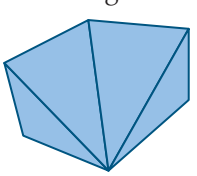
Polygons

A **polygon** is a plane figure bounded by straight sides. A **regular polygon** has all its sides and angles equal.

Angle sum of a polygon

If a polygon is divided into triangles, there are always two fewer triangles than sides.

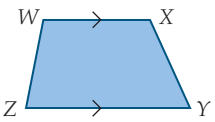
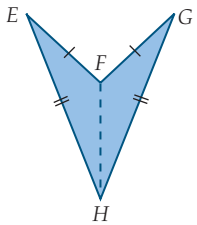
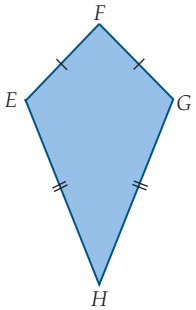
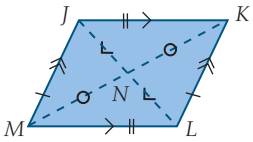
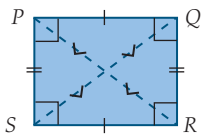
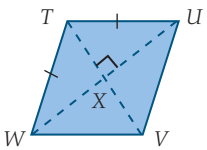
The sum of the interior angles of a polygon with n sides = $(n - 2) \times 180°$.

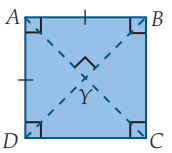
Shape	n	Angle sum using triangles	Angle sum using $(n - 2) \times 180°$
Triangle 	3	$1 \times 180° = 180°$	$(n - 2) \times 180°$ $= (3 - 2) \times 180°$ $= 1 \times 180°$ $= 180°$
Quadrilateral 	4	$2 \times 180° = 360°$	$(n - 2) \times 180°$ $= (4 - 2) \times 180°$ $= 2 \times 180°$ $= 360°$
Pentagon 	5	$3 \times 180° = 540°$	$(n - 2) \times 180°$ $= (5 - 2) \times 180°$ $= 3 \times 180°$ $= 540°$
Hexagon 	6	$4 \times 180° = 720°$	$(n - 2) \times 180°$ $= (6 - 2) \times 180°$ $= 4 \times 180°$ $= 720°$

Definitions and properties

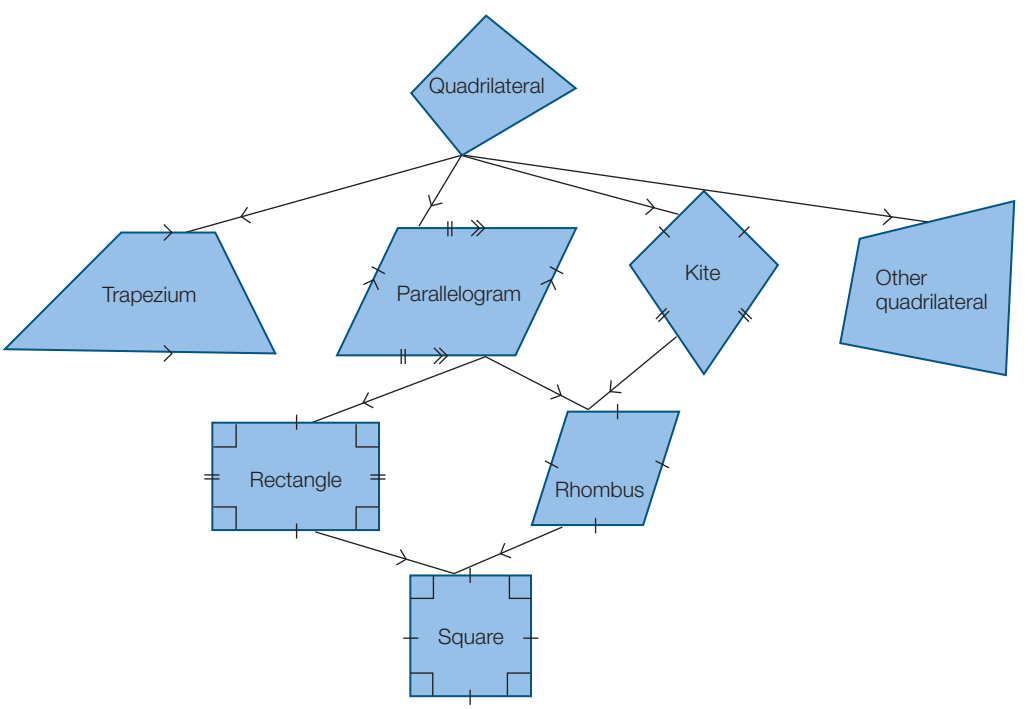
A definition is the minimum amount of information needed to describe the figure accurately.

A property is an attribute or characteristic of a figure that can be proved to be always true once the figure has been defined. Congruence can be used to prove properties of special quadrilaterals.

Shapes and definitions	Properties
Trapezium A trapezium is a quadrilateral with only one pair of opposite sides parallel.  $WXYZ$ is a trapezium in which $WX \parallel ZY$.	A trapezium has no special properties.
Kite A kite is a quadrilateral with two pairs of adjacent sides equal. $EFGH$ is a kite in which $EF = FG$, $EH = HG$.  Concave kite  Convex kite	• The pair of angles between the unequal sides are equal: $\angle FEH = \angle FGH$ • The angles between the equal sides are bisected by the diagonal through them. • The diagonal through the equal angles is an axis of symmetry. • The diagonals are perpendicular: $FH \perp EG$ • The diagonal that passes through the equal angles is bisected by the axis of symmetry.
Parallelogram A parallelogram is a quadrilateral with both pairs of opposite sides parallel.  $JKLM$ is a parallelogram in which $JK \parallel ML$, $JM \parallel KL$.	• Both pairs of opposite sides are equal: $JK = ML$, $JM = KL$ • Both pairs of opposite angles are equal: $\angle MJK = \angle MLK$, $\angle JML = \angle JKL$ • The diagonals bisect each other $JN = NL$, $MN = NK$
Rectangle A rectangle is a parallelogram with a right angle.  $PQRS$ is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus A rhombus is a parallelogram with two adjacent sides that are equal.  $TUVW$ is a rhombus in which $TU = TW$ (or a kite with all sides equal).	A rhombus has all the properties of a parallelogram, plus: • all the sides are equal: $TU = TW = WV = UV$ • the diagonals bisect the angles of the rhombus: $\angle WTV = \angle UTV$, $\angle WVT = \angle UVT$, $\angle TWU = \angle VWU$, $\angle TUW = \angle VUW$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$, $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$

Shapes and definitions	Properties
Square A square is a rectangle with one pair of adjacent sides equal.  *ABCD* is a square in which $AB = AD$. or A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90°$.	A square has all the properties of a rectangle, and all the properties of a rhombus: • all the sides are equal: $AB = AD = DC = BC$ • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90°$ • the two diagonals and the two lines joining the midpoints are axes of symmetry • the diagonals are equal: $AC = BD$

Pictorial summary of the quadrilateral family

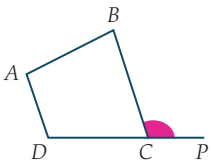


Use the definitions of each type of quadrilateral as your proof.

Technology Exploration: GeoGebra

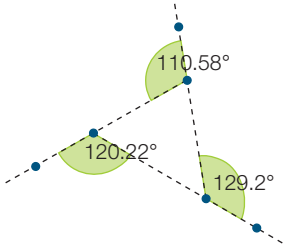
Exterior angle sum of polygons

An exterior angle is formed if one side of a polygon is extended from a vertex (corner). An exterior angle will always be less than 180°. In the diagram, side *DC* has been extended from *C* to *P*. The angle *PCB* is one of the exterior angles of the polygon.



Is the sum of the exterior angles of a polygon always the same for all polygons? If so, why?

1 Open GeoGebra. Right-click in the Graphics view to hide the Axes and Grid. Use the Ray Through Two Points tool to construct a triangle with each side extended in a 'Catherine wheel' arrangement, as shown at right. Use the Move tool to place a point on each extended arm, then use the Angle tool (anticlockwise) to measure each of the exterior angles. Add the three angles by entering Sum[{α, β, γ}] in the Input bar.



2 Draw a different triangle. Make an exterior angle on each vertex, and measure and sum the angles as in **1**. What is the total sum of the exterior angles?

3 Use GeoGebra to find the sum of the exterior angles of a parallelogram, kite, pentagon and hexagon by drawing the shape and extending each side past the vertex, as shown above. Measure the exterior angles formed in each shape and find the sum of the exterior angles.

4 Use GeoGebra to draw any convex polygon (i.e. a polygon with all interior angles less than 180°) with more than six sides. Mark the exterior angles of this polygon, measure the exterior angles formed in each shape and find their sum.

5 Write a sentence describing what you have found regarding the sum of exterior angles of convex polygons.

6 Explain why this property does not apply to concave polygons (polygons with one or more interior angles greater than 180°).
(Hint: Can you form all the exterior angles for concave polygons?)

7 By exploring the sum of the exterior angles of triangles, quadrilaterals, pentagons and hexagons, find a rule for calculating the sum of the exterior angles of convex polygons.

Sum of the exterior angles of a convex polygon = …

8 Use your rule to find the sum of the exterior angles of a regular icosagon (20-sided polygon). (A regular polygon has all sides of equal length.)

9 Do you believe your findings are correct for polygons of any number of sides? Justify your answer.

10 Do you believe that the number and variety of shapes you studied is enough to give a good indication that the rule you discovered is correct?

11 How could you make your investigation of the sum of exterior angles in polygons more thorough?

12 **(a)** Find a rule to calculate the size of each exterior angle of any regular polygon where n is the number of sides of the polygon.

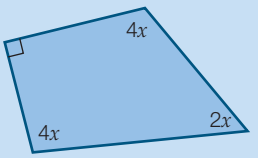
(b) Use this rule to show that the size of each interior angle, in degrees, of any regular polygon is $\frac{180°(n-2)}{n}$, where n is the number of sides of the polygon.

Worked Example 12

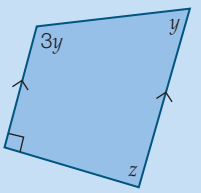
WE12

Find the value of the variables in each figure.

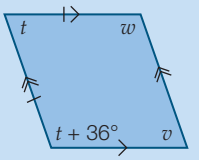
(a)



(b)



(c)



Working

(a) This is a quadrilateral.

$2x + 4x + 4x + 90° = 360°$

$10x + 90° = 360°$

$10x = 270°$

$x = 27°$

(b) This is a trapezium.

$y + 3y = 180°$ (co-interior angles between parallel lines)

$4y = 180°$

$y = 45°$

$z + 90° = 180°$ (co-interior angles between parallel lines)

$z = 90°$

(c) The figure is a parallelogram with one pair of adjacent sides equal; hence, it is a rhombus.

$t + t + 36° = 180°$ (co-interior angles add to 180°)

$2t + 36° = 180°$

$2t = 144°$

$t = 72°$

$v = 72°$ (opposite angles of a rhombus are equal)

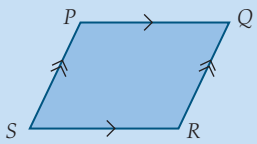
$w = 108°$ (interior angles between parallel lines)

$t = 72°$, $v = 72°$, $w = 108°$

Worked Example 13

WE13

$PQRS$ is a parallelogram is which $PQ \parallel SR$ and $PS \parallel QR$.



(a) Use congruence to prove that $\angle SPQ = \angle QRS$.

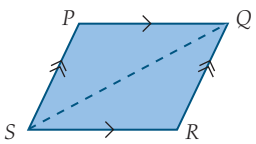
(b) Prove that $PQ = SR$ and $PS = QR$.

(c) Prove that $\angle PSR = \angle RQP$.

(d) What properties of a parallelogram have you proved?

Working

(a)



In ΔSPQ and ΔQRS,

$\angle PQS = \angle QSR$ (alternate angles, $PQ \parallel SR$)

$\angle PSQ = \angle SQR$ (alternate angles, $PS \parallel QR$)

QS is a common side

$\Delta SPQ \equiv \Delta QRS$ (AAS)

$\angle SPQ = \angle QRS$ (matching angles in congruent triangles)

(b) $PQ = SR$ (matching sides of congruent triangles)

$PS = QR$ (matching sides of congruent triangles)

(c) If the diagonal PR is drawn, then the same steps as in (a) can be used to prove that $\Delta PSR \equiv \Delta RQP$.

$\angle PSR = \angle RQP$ (matching angles in congruent triangles)

(d) The opposite angles of a parallelogram are equal.

The opposite sides of a parallelogram are equal.

Worked Example 14

WE14

$ABCD$ is a rhombus in which $AB \parallel DC$ and $AD \parallel BC$. The diagonals AC and BD intersect at E.

(a) Prove that $\angle ABD = \angle CBD$.

(b) Prove that $\Delta ABE \equiv \Delta CBE$.

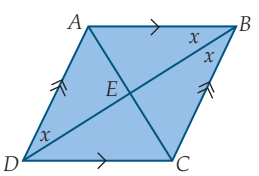
(c) Prove that $\angle AEB = \angle CEB = 90°$.

(d) Prove that $AE = CE$.

(e) What properties of a rhombus have you proved?

Working

(a)



$\angle CDB = \angle ABD = x$ (alternate angles, $AB \parallel DC$)

$BC = CD$ (adjacent sides of a rhombus)

ΔBCD is isosceles

$\angle CBD = \angle CDB = x$ (base angles of an isosceles triangle)

$\angle ABD = \angle CBD$ (both equal x)

(b) Consider ΔABE and ΔCBE.

$AB = BC$ (adjacent sides of a rhombus)

$\angle ABE = \angle CBE$ (proved in **(a)**)

BE is a common side

$\Delta ABE \equiv \Delta CBE$ (SAS)

(c) $\angle AEB = \angle CEB$ (matching angles in congruent triangles)

$\angle AEB + \angle CEB = 180°$ ($\angle AEC = 180°$)

$\angle AEB = 180°$

$\angle AEB = 90°$

$\angle AEB = \angle CEB = 90°$

(d) $AE = CE$ (matching sides in congruent triangles)

(e) The diagonals bisect the angles of a rhombus.

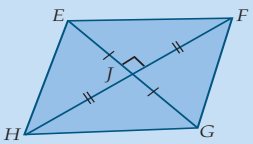
The diagonals of a rhombus intersect at right angles.

One diagonal bisects the other diagonal.

Worked Example 15

WE15

The diagonals of the quadrilateral *EFGH* bisect each other at right angles at *J*. Prove that *EFGH* is a rhombus.



Working

$EJ = GJ$,
$HJ = FJ$,
$\angle FJG = \angle HJG = \angle HJE = \angle FJE = 90°$

In the quadrilateral, the pairs of opposite sides must be parallel and a pair of adjacent sides must be equal.

Consider ΔEJF and ΔGJH.

$EJ = GJ$ (given)
$\angle EJF = \angle GJH$ (both 90°)
$FJ = HJ$ (given)
$\Delta EJF \equiv \Delta GJH$ (SAS)

$\angle EFJ = \angle GHJ$ (matching angles in congruent triangles)

$\angle EFH$ and $\angle GHF$ are a pair of equal alternate angles contained between EF and HG.

$\therefore EF \parallel HG$

Similarly, $\Delta EJH \equiv \Delta GJF$
$\angle EHF = \angle HFG$.

$\angle EHF$ and $\angle HFG$ are a pair of equal alternate angles contained between EF and HG.

$\therefore EH \parallel FG$

Consider ΔEJF and ΔGJF.

$EJ = GJ$ (given)
$\angle EJF = \angle GJF$ (both 90°)
JF is a common side

$\Delta EJF \equiv \Delta GJF$ (SAS)

$EF = GF$ (matching sides in congruent triangles)

$\Delta EJF \equiv \Delta GJH$ (SAS)

$EF = GF$ are a pair of equal adjacent sides of $EFGH$ and $EF \parallel HG$, $EH \parallel FG$ gives both pairs of opposite sides parallel. Hence, $EFGH$ is a parallelogram with a pair of adjacent sides equal, so $EFGH$ is a rhombus.

Answers page 458

5.6 Special quadrilaterals and their properties

Stage 5.2

Navigator

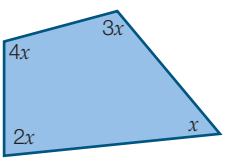
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q16, Q17, Q18, Q25	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q18, Q19, Q21, Q23, Q25	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19, Q20, Q21, Q22, Q23, Q24

Fluency and understanding

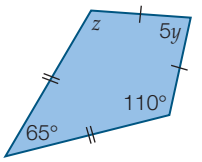
WE12

1 Find the value of the variables in each figure.

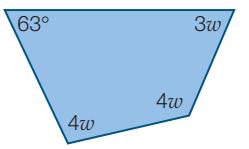
(a)



(b)

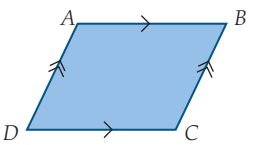


(c)



WE13

2 $ABCD$ is a parallelogram is which $AB \parallel DC$ and $AD \parallel BC$.



(a) Use congruence to prove that $\angle DAB = \angle BCD$.

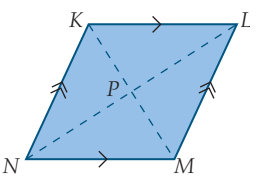
(b) Prove that $AB = DC$ and that $AD = BC$.

(c) Prove that $\angle ADC = \angle CBA$.

(d) What properties of a parallelogram have you proved?

WE14

3 $KLMN$ is a rhombus in which $KL \parallel NM$ and $KN \parallel LM$. The diagonals KM and LN intersect at P.



(a) Prove that $\angle KLN = \angle MLN$.

(b) Prove that $\Delta KLP \equiv \Delta MLP$.

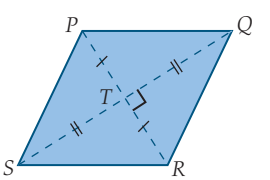
(c) Prove that $\angle KPL = \angle MPL = 90°$.

(d) Prove that $KP = MP$.

(e) What properties of a rhombus have you proved?

WE15

4 The diagonals of the quadrilateral $PQRS$ bisect each other at right angles at T. Prove that $PQRS$ is a rhombus.

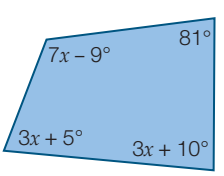


5 The size of each of the angles in a regular pentagon is:

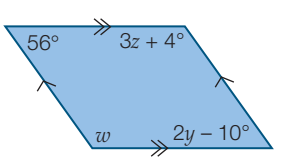
A 60° **B** 90° **C** 108° **D** 120°

6 Find the value of the variables in each figure.

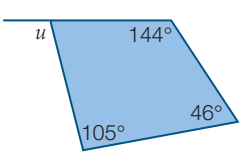
(a)



(b)



(c)



7 Write True or False for each statement.

(a) A trapezium has one pair of opposite sides equal and parallel.

(b) A square is a special rectangle.

(c) The diagonals of a kite are equal in length.

(d) The diagonals of a rhombus bisect each other at right angles.

8 In which regular polygon is each interior angle equal to 120°?

9 Which statement is True?

A A kite has one pair of opposite angles equal.

B A rectangle is a special kite.

C A rhombus is a rectangle.

D A rhombus is a special trapezium.

10 Which statement is False?

A All parallelograms have two pairs of parallel sides.

B All trapeziums have one pair of parallel sides of equal length.

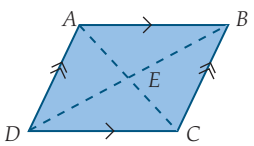
C All kites have a pair of equal angles.

D All rectangles have a right angle.

11 **(a)** What is the sum of the angles of an octagon?

(b) What is the size of each interior angle of a regular octagon?

12 $ABCD$ is a parallelogram whose diagonals intersect at E.
Using triangles ADC and CBA, prove that $\angle ADC = \angle CBA$.



(a) Copy and complete this proof by filling in the missing steps.

Proof: In Δs ADC and CBA,
$\angle ACD = \angle CAB$ (_____ $AB \parallel DC$)
$\angle DAC = \angle BCA$ (_____ AD___BC)
AC is a _____
$\therefore \Delta ADC \equiv \Delta CBA$ (_____)
$\therefore \angle ADC = \angle CBA$ (_____)

(b) Show that $\angle DAB = \angle BCD$.

13 In Question **12** you have proved that $\Delta ADC \equiv \Delta CBA$. Using this result:

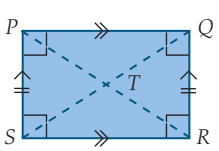
(a) show that the opposite angles of a parallelogram are equal

(b) show that the opposite sides of a parallelogram are equal

(c) if the diagonals intersect at point E, prove that $\Delta AEB \equiv \Delta CED$

(d) show that the diagonals of a parallelogram bisect each other.

14 $PQRS$ is a rectangle in which $PS = QR$ and PR and QS intersect at T.
Prove that $\Delta PRS \equiv \Delta QSR$.



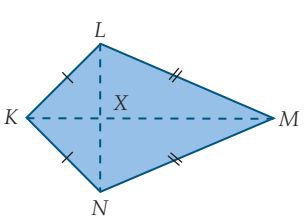
(a) Copy and complete this proof by filling in the missing steps.

Proof: In Δs PRS and QSR:
$\angle$_____ = $\angle$_____ = 90° (given)
_____ is a common side
_____ = _____ (given)
$\therefore \Delta$_____ $\equiv \Delta$_____ (SAS)

(b) Hence, prove that $PR = QS$.

(c) What special property of a rectangle did you prove in **(b)**?

15 $KLMN$ is a kite. KM and LN intersect at X.



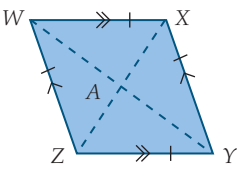
(a) Use triangles KLM and KNM to prove that $\angle KLM = \angle KNM$.

(b) Prove that $\Delta LKX \equiv \Delta NKX$.

(c) Show that $\angle LXK = \angle NXK = 90°$.

(d) What properties of a kite have you proved?

16 $WXYZ$ is a rhombus as shown in the diagram. WY and XZ intersect at A.

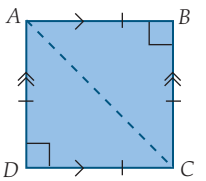


(a) Prove that $\angle WYX = \angle WYZ$.

(b) Explain why $\angle WZX = \angle YZX$.

(c) What property of a rhombus have you proved?

17 $ABCD$ is a square as shown in the diagram. AC is a diagonal.

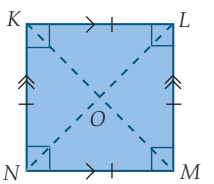


(a) Prove that $\angle ACD = \angle ACB$.

(b) Prove that $\angle ACD = 45°$.

(c) What properties of a square have you proved?

18 $KLMN$ is a square as shown. KM and LN intersect at O.



(a) Prove that $KM = NL$.

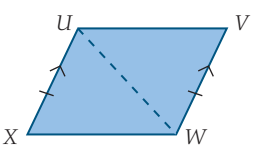
(b) Prove that $KO = OM$ and that $NO = OL$.

(c) Prove that $\angle KOL = 90°$.

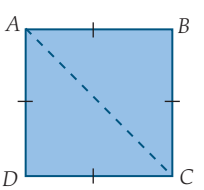
(d) What properties of a square have you proved?

Reasoning, communicating and open-ended

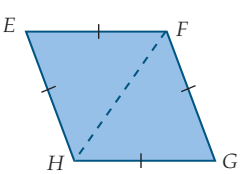
19 $UVWX$ is a quadrilateral in which $UX \parallel VW$ and $UX = VW$. Prove that $UVWX$ is a parallelogram.



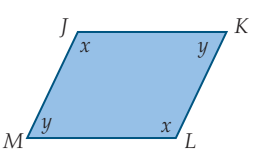
20 $ABCD$ is a quadrilateral in which all the sides are equal and $\angle ABC = 90°$. Prove that $ABCD$ is a square.



21 $EFGH$ is a quadrilateral in which all the sides are equal. Prove that $EFGH$ is a rhombus.



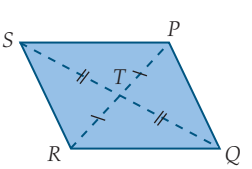
22 $JKLM$ is a quadrilateral in which both pairs of opposite angles are equal; that is, $\angle MJK = \angle MLK$ and $\angle JML = \angle JKL$.



(a) Prove that $\angle MJK + \angle JML = 180°$.

(b) Hence, prove that $JKLM$ is a parallelogram.

23 $SPQR$ is a quadrilateral in which the diagonals bisect each other at T, as shown. Prove that $SPQR$ is a parallelogram.



24 **(a)** Simon knows that the statement 'All squares have equal sides' is true. He wonders whether the converse (opposite) statement 'All shapes with four equal sides are squares' is true. Explain to him whether or not it is true.

(b) Select another true statement about a special quadrilateral. Write the converse statement and explain whether or not it is true.

25 Explain why a square is a rectangle but a rectangle is not necessarily a square.

Challenge 5

Note: In this section the diagrams are not drawn to scale.

1 In ΔABC, sides AC and BC are equal and side AB is shorter than AC.

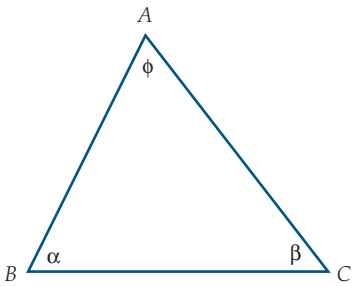
Which statement is true?

A α, β and ϕ are all equal

B $\alpha = \beta$

C $\beta = \phi$

D $\alpha = \phi$



2 How can you use six matches to form four equilateral triangles, each with a side length of one match?

3 In the diagram, JLM and KLN are straight lines. $JK = KL = LJ$.

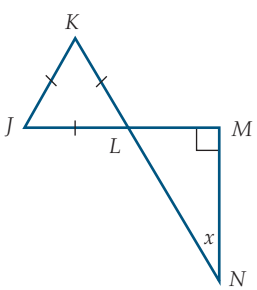
$\angle LMN = 90°$. What is the size of x?

A 15°

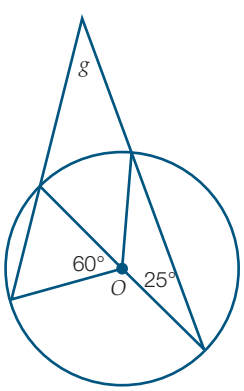
B 30°

C 45°

D 60°



4 Find the value of g in this diagram.



5 In the quadrilateral $PQRS$, $PS = PR = PQ$.

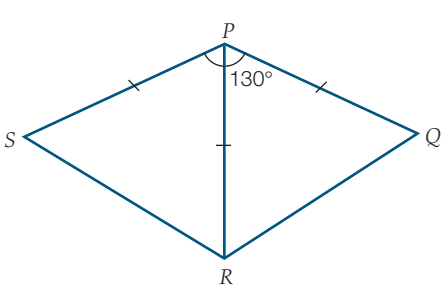
If $\angle SPQ = 130°$, then the size of $\angle SRQ$ is:

A 25°

B 115°

C 125°

D 135°



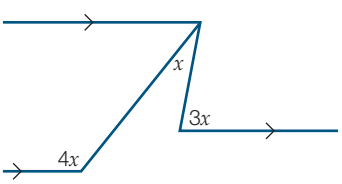
6 $\angle ABC$, $\angle CBD$ and $\angle DBA$ together make up an angle of revolution. If $\angle ABC$ is acute and $\angle CBD$ is obtuse, which of the following are possible for $\angle DBA$? There may be more than one answer.

A acute **B** obtuse **C** reflex **D** straight

7 What is the value of x?

A 22.5 **B** 30

C 36 **D** 45



Chapter review 5

D.I.Y. Summary

Key Words

Stage 4	alternate angles	corresponding angles	interior angle
	co-interior angles	exterior angle	vertex
Stage 5.1	centre of enlargement	reduction	
	enlargement	scale factor	
Stage 5.2	congruent	polygon	similar
	congruent triangles	regular polygon	similar triangles

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

Stage 4 Q1, 2 **Stage 5.1 Q3, 4** **Stage 5.2 Q5–8**

1 When a transversal cuts a pair of parallel lines, the pairs of _____ are equal, the pairs of _____ are equal, and pairs of _____ are supplementary.

2 The _____ of a triangle is equal to the sum of the interior opposite angles.

3 Multiplying the sides of a shape by a _____ greater than one will produce an _____ of the shape.

4 _____ have the same shape, but not necessarily the same size.

5 Which of the tests for similar triangles states that pairs of matching sides must be in the same ratio?

6 Which of the tests for similar triangles states that the matching hypotenuses and one other pair of sides of right-angled triangles must be in the same ratio?

7 Which of the tests for similar triangles states that one pair of matching angles must be equal and the pairs of matching sides on either side of the angle must be in the same ratio?

8 _____ triangles are exactly the same shape and size.

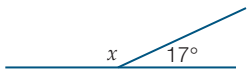
Stage 4 Q1, 2, 19 **Stage 5.1 Q3, 4, 20** **Stage 5.2 Q5–18, 21**

Fluency and understanding

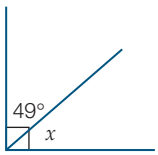
1 Find the value of x in the following diagrams.

Ex 5.1

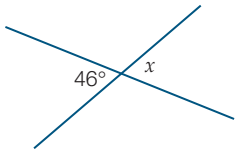
(a)



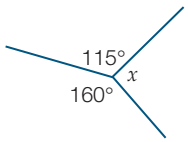
(b)



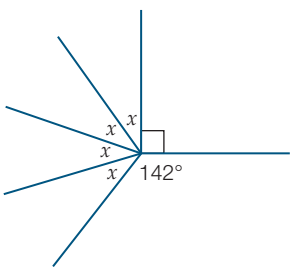
(c)



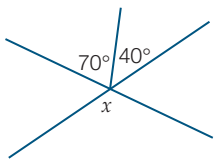
(d)



(e)



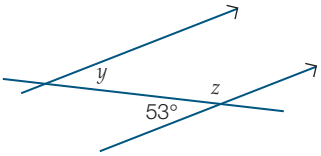
(f)



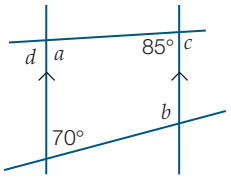
2 Find the value of the variables in the following diagrams.

Ex.5.1

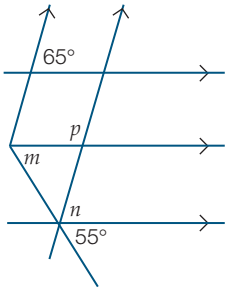
(a)



(b)



(c)

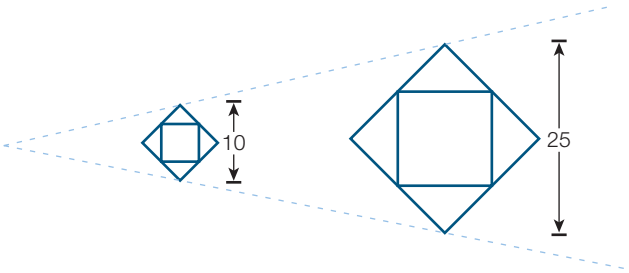


3 In the figure at right, what is the scale factor for:

(a) the reduction

(b) the enlargement?

Ex.5.2



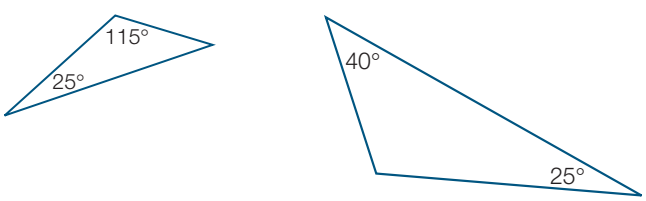
4 Enlarge the following shape by a factor of 2.4.



Ex.5.2

5 Show that the following pair of triangles are similar.

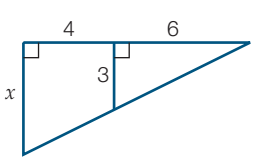
Ex.5.3



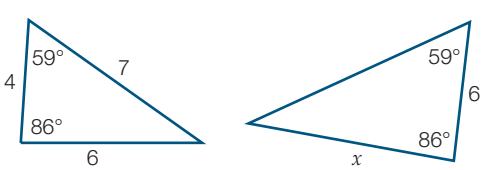
6 Find the value of x in the following diagrams.

Ex.5.3

(a)

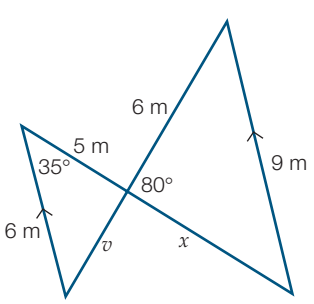


(b)



7 Show that the following pair of triangles are similar, then find the lengths of the unknown sides.

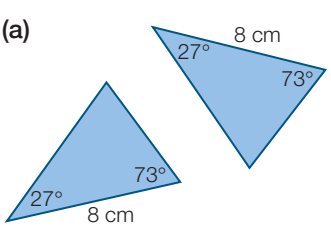
Ex.5.4



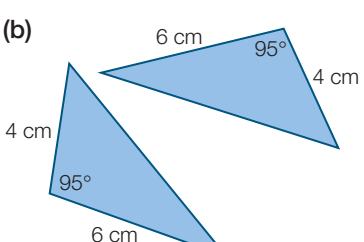
8 Explain why each of the following pairs of triangles are congruent.

Ex.5.5

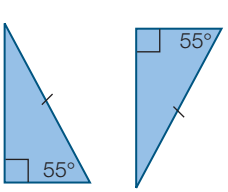
(a)



(b)



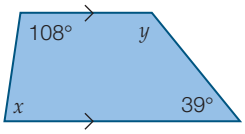
(c)



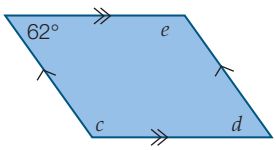
9 Find the value of the variables in the following diagrams.

Ex 5.6

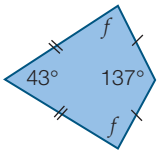
(a)



(b)



(c)



10 Match the name of the shape with its correct description.

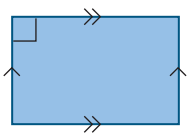
Ex 5.6

(a) Adjacent sides of equal length, no parallel sides	**A** Rectangle
(b) One pair of opposite sides parallel	**B** Kite
(c) Four right angles, opposite sides equal in length	**C** Isosceles triangle
(d) Two sides equal in length	**D** Trapezium

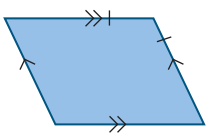
11 Identify each of the following shapes using the name that has all the properties shown.

Ex 5.6

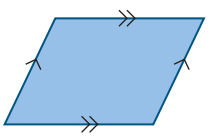
(a)



(b)

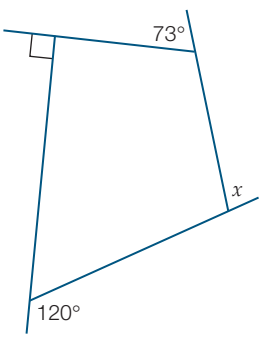


(c)

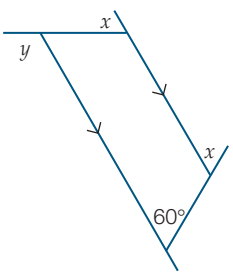


12 Find the value of the variables in each of these quadrilaterals.

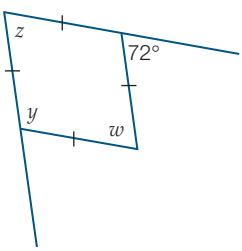
(a)



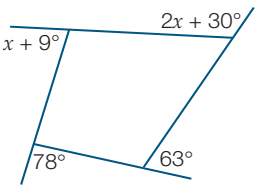
(b)



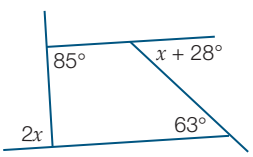
(c)



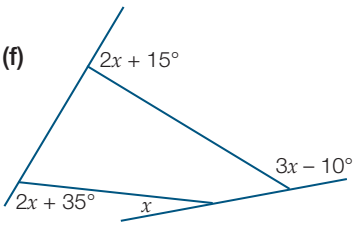
(d)



(e)

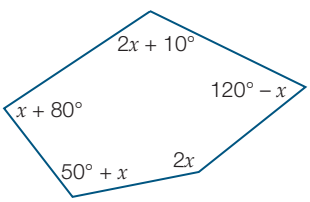


(f)

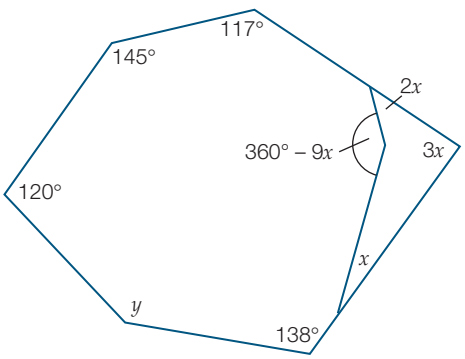


13 Find the values of the variables in each of these shapes.

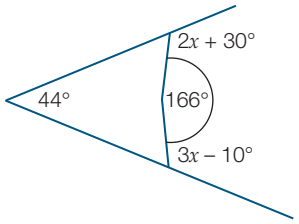
(a)



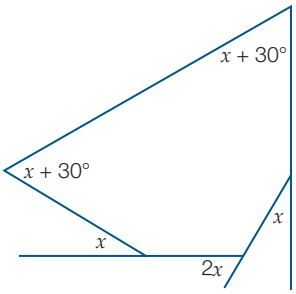
(b)



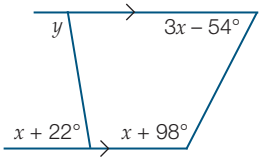
(c)



(d)



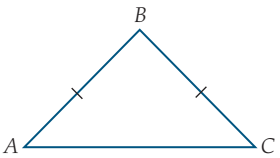
(e)



14 In ΔABC, $BA = BC$.

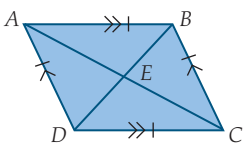
(a) Prove that $\angle BAC = \angle BCA$ by drawing the line BD perpendicular to AC, meeting it at D.

(b) What property of isosceles triangles have you just proved?



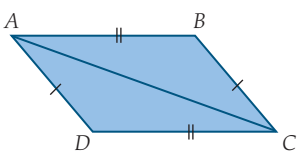
15 **(a)** For the diagram shown, prove AC ⊥ BD.

(b) What special property of a rhombus have you just proved?

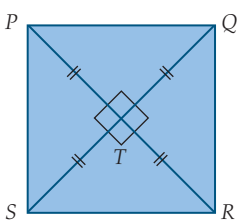


16 Show the following.

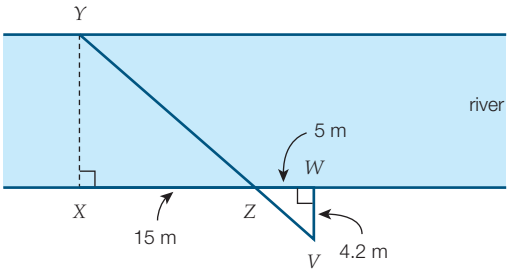
(a) If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.



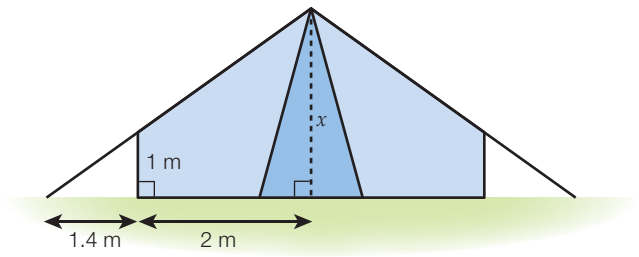
(b) If the diagonals of a quadrilateral are equal and bisect each other at right angles, then the quadrilateral is a square.



17 Angela is on a walk through the forest when she comes to a river. She wants to know the distance across the river. She measures the distances shown and draws a diagram to help her. Find the distance across the river.

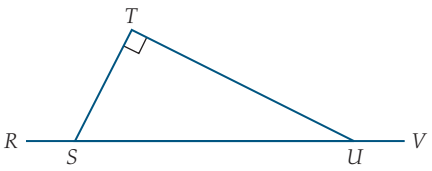


18 Use your knowledge of similar triangles to calculate the height of the tent shown.



Reasoning and communicating

19 Prove that $\angle TSR + \angle TUV = 270°$



20 A postcard measuring 11 cm × 8.5 cm is enlarged so that its new length and width are 120% the size of the original dimensions.

(a) What is the scale factor of the enlargement?

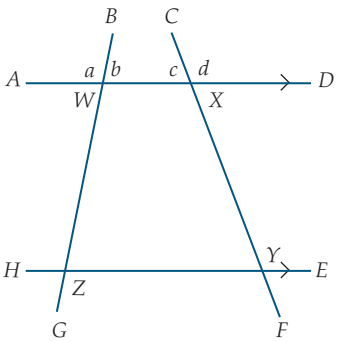
(b) What are the dimensions of the enlarged postcard?

(c) Find the area of **(i)** the original postcard and **(ii)** the enlarged image.

(d) Find the ratio $\frac{\text{area of photocopied image}}{\text{area of original postcard}}$. Explain why this ratio is not the same as the scale factor you found in **(a)**.

(e) Photocopiers enlarge images according to area, not length. What percentage enlargement would you need to enter into the photocopier to achieve the same enlarged postcard?

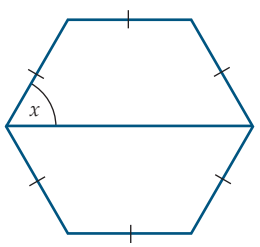
21 Use the diagram to prove that the sum of the interior angles of a trapezium is 360°.



NAPLAN practice 5

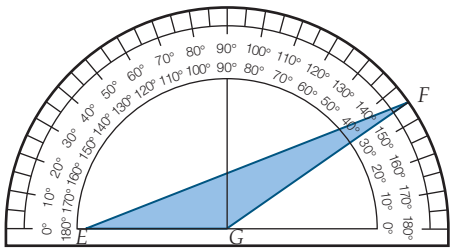
Numeracy: Non-calculator

1 Two isosceles trapeziums fit together to make the regular hexagon shown.



What is the value of x?

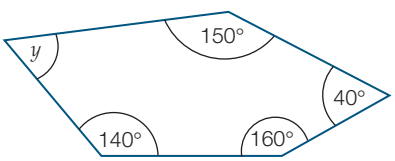
2 ΔEFG is shaded below. What is the value of $\angle FGE$?



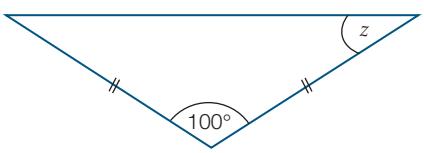
A 35° **B** 45° **C** 145° **D** 155°

Numeracy: Calculator allowed

3 What is the value of y?

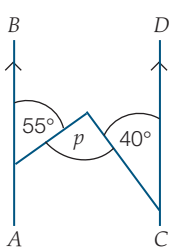


4 What is the value of z?



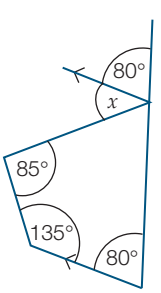
5 In the diagram, AB and CD are parallel.

What is the value of p?



6 In the diagram, a pair of parallel lines are shown.

What is the value of x?



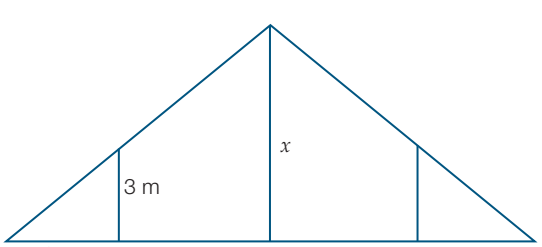
7 Liam is 1.5 m tall. His shadow is 2 m long. A tree nearby has an 8 m shadow. How tall is the tree?

A 7.5 m **B** 7 m **C** 6 m **D** 4 m

8 The support beams for a triangular barn are being put in place. How long will the support beam in the centre of the barn need to be?

A 6 m **B** 9 m

C 12 m **D** 15 m



9 A poster measuring 1.5 m × 0.9 m is enlarged by a scale factor of 3. What is the area of the enlarged poster?

Mixed review B

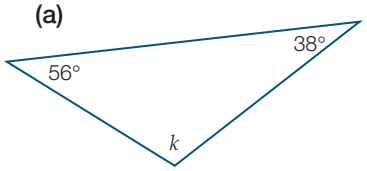
Stage 4 Q1, 2, 13 Stage 5.1 Q3–7, Q14–19 Stage 5.2 Q8–11 Stage 5.3 Q12, 20

Fluency and understanding

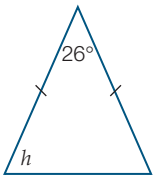
1 Find the perimeter of a semicircle of radius 12 m. Write your answer correct to two decimal places. Ex 3.1

2 Find the value of the variables in each of the following. Ex 5.1

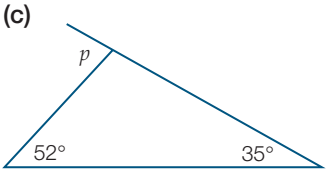
(a)



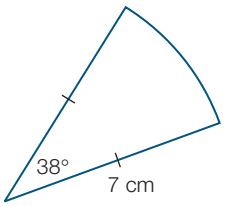
(b)



(c)



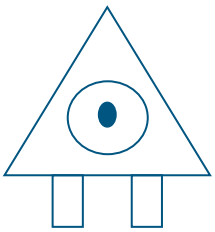
3 Find the area of the sector shown in the diagram. Give your answer correct to two decimal places. Ex 3.2



4 For the illustration shown: Ex 5.2

(a) enlarge by a scale factor of 3

(b) reduce by a scale factor of $\frac{1}{2}$.



5 Find the gradient of the line joining each of the following pairs of points. Ex 4.6

(a) (0, 4) and (5, 14) (b) (12, 8) and (5, 15) (c) (-8, -11) and (6, -4)

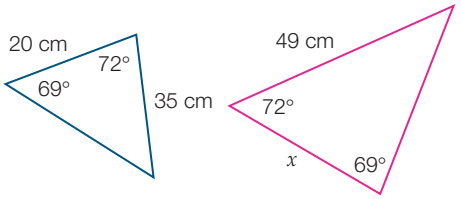
6 Calculate the simple interest in the following cases. Ex 1.5

(a) $P = \$5000$, $R = 4.7\%$ p.a., $T = 3$ years

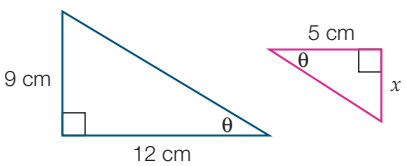
(b) $P = \$690$, $R = 8.7\%$ p.a., $T = 1.5$ years

7 Given that the following triangles are similar, state the scale factor and find the value of the variable. (The blue triangles are the originals.) Ex 5.2

(a)



(b)



8 Write the rules for the area of a: Ex 3.2

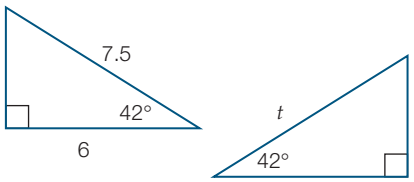
(a) kite (b) rhombus (c) trapezium.

9 Find the surface area of a cylinder of height 10 cm and a base radius of 4 cm. Give your answer correct to two decimal places. Ex 3.5

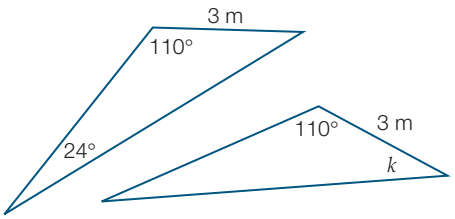
10 Find the values of the variables in the following congruent triangles.

Ex 5.5

(a)



(b)



11 Solve the following equations.

Ex 4.3

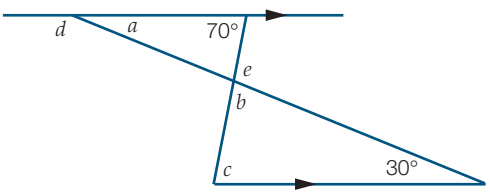
(a) $\frac{3x}{5} - 7 = 4$ **(b)** $\frac{4x+7}{4} = 5$ **(c)** $\frac{2(3x+8)}{5} = -2$

12 Factorise the following expressions.

Ex 2.8

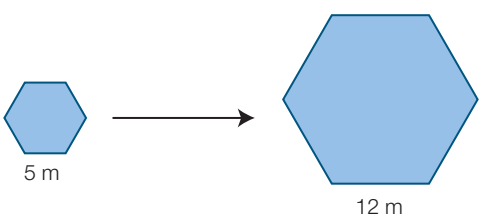
(a) $y^2 + 7y + 3y + 21$ **(b)** $cd + 3c + 4d + 12$ **(c)** $t^2 - 4t - 5t + 20$

13 Find the values of the variables in the diagram below.

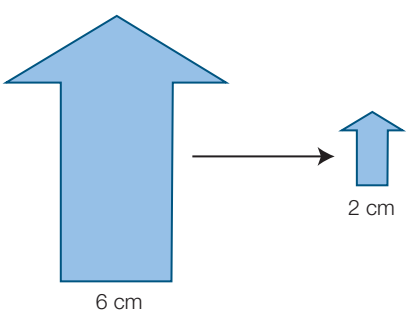


14 What scale factors have been used in the following?

(a)

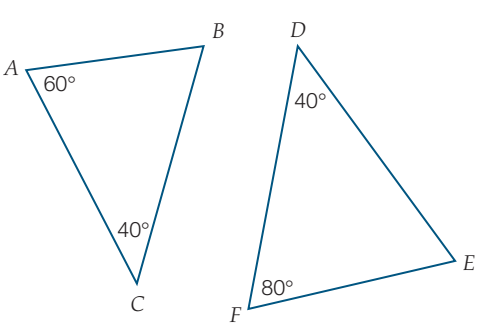


(b)

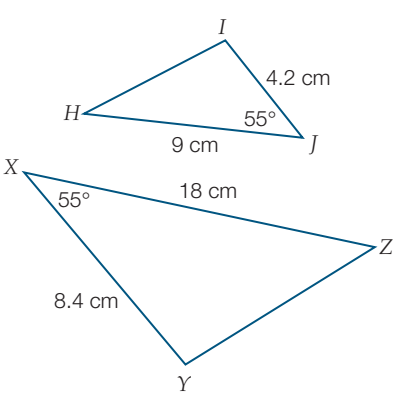


15 Show that the following pairs of triangles are similar.

(a)

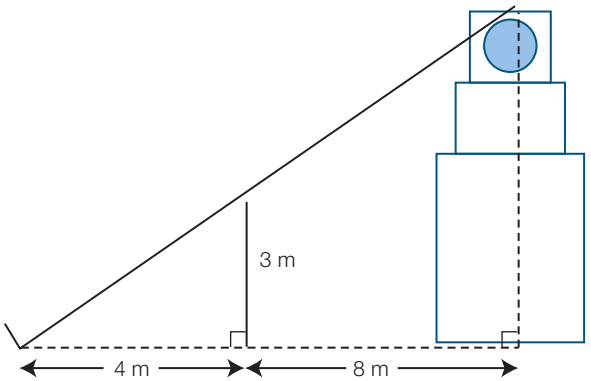


(b)



Reasoning and communicating

16 The clock on the Town Hall is being repaired. A safety wire has been attached to the top of the Town Hall, as shown. How high is the Town Hall?



17 A standard size photo is 10 cm × 15 cm.

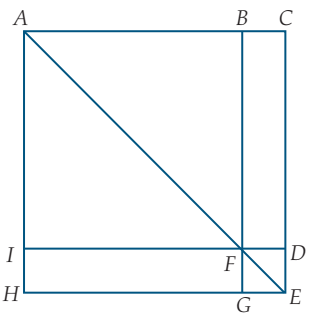
(a) If the photo is enlarged to 30 cm × 45 cm, what is the scale factor of the enlargement?

(b) Calculate the area of **(i)** the standard size photo and **(ii)** the enlargement.

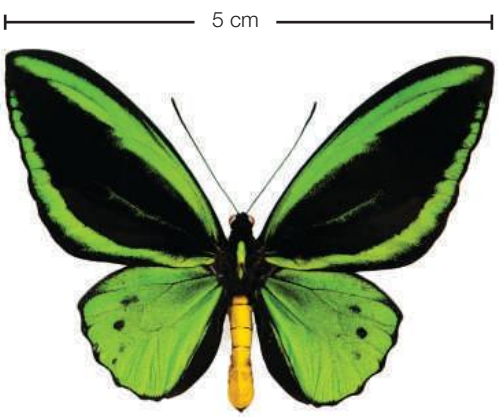
(c) Calculate the ratio $\frac{\text{area of enlargement}}{\text{area of standard size}}$.

(d) Compare your answers to **(a)** and **(c)**. What do you notice?

18 The diagram below has three different-sized triangles. Name one example of each triangle and show that all three are similar.



19 What scale factor is needed to enlarge this image of a butterfly to a width of 27 cm?



20 **(a)** Expand the brackets in the expression $x(x + 4) - 21$.

(b) Factorise this new expression as the product of two brackets.

(c) What value of x in this new expression gives 110×100?

(d) Use the last answer to show that $103 \times 107 - 21 = 110 \times 100$.

Trigonometry

6

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

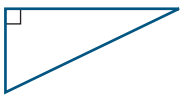
1 Solve the following equations. Where necessary, express your answer as a decimal correct to four decimal places.

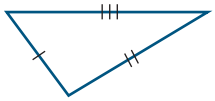
(a) $2x + 16 = 41$ **(b)** $38 - x = 25$ **(c)** $x^2 + 10 = 74$

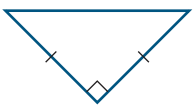
(d) $2x^2 = 98$ **(e)** $\frac{x}{15} = 0.356$ **(f)** $\frac{30}{x} = 0.4783$

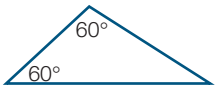
2 State whether the following triangles are: **(i)** equilateral, scalene or isosceles and **(ii)** acute angled, obtuse angled or right angled.

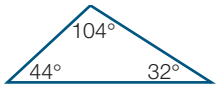
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

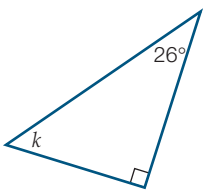
3 **(a)** For the formula $F = ma$, find F if $m = 6$ and $a = 7$.

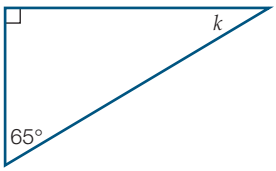
(b) Find S if $a = 21$, $b = 24$ and $c = 35$ in the formula $S = \frac{a + b + c}{2}$.

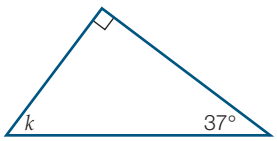
(c) For the formula $E = \frac{1}{2}mv^2$, find E when $m = 3$ and $v = 6$.

(d) For the formula $E = mx^2$, find m when $E = 100$ and $x = 5$.

4 Find the value of k in each of these triangles.

(a) 

(b) 

(c) 

5 Simplify the following ratios and then express the answer as a fraction.

(a) 8 : 10 **(b)** 36 : 24 **(c)** $\frac{5}{6} : \frac{2}{3}$

(d) $3\frac{1}{3} : 2\frac{1}{2}$ **(e)** 54 cm : 72 cm **(f)** 3.2 m : 400 cm

Syllabus references

Measurement and Geometry: Right-angled Triangles (Trigonometry)

- ✓ Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles **(ACMMG223) (Stage 5.1)**
- ✓ Apply trigonometry to solve right-angled triangle problems **(ACMMG224) (Stage 5.1)**
- ✓ Solve right-angled triangle problems, including those involving angles of elevation and depression **(ACMMG245) (Stage 5.1)**
- ✓ Solve right-angled triangle problems, including those involving direction and angles of elevation and depression **(ACMMG245) (Stage 5.2◊)**

6.1 Introduction to trigonometry

Stage 5.1

Need to Know

Trigonometry

Trigonometry is the study of the relationships between side lengths and angles in right-angled triangles.

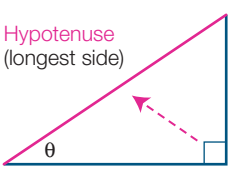
There are three ratios, sine θ, cosine θ and tangent θ, that compare the different lengths of the sides of a right-angled triangle. These ratios are called **trigonometric ratios**. To define these trigonometric ratios we need to name the sides of the right-angled triangle.

Naming the sides of a right-angled triangle

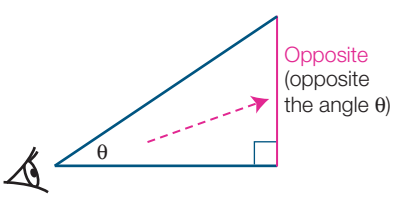
When naming the sides we need to specify which angle we are referring to. This angle is called the **reference angle**. At this level, angles are measured in **degrees**.

Greek letters such as θ, α, β, γ and ϕ are often used to identify the reference angle.

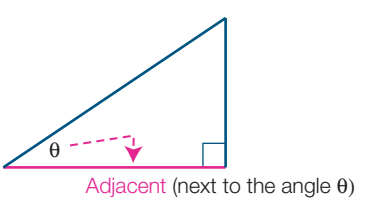
- The **hypotenuse** is always the longest side. It is always opposite the right angle.



- The **opposite side** is the side opposite the reference angle θ. To help you identify it, imagine that you are standing looking at the angle θ. From this view, the opposite side is behind θ.



- The **adjacent side** is the side next to the angle θ. It is between the right angle and θ.



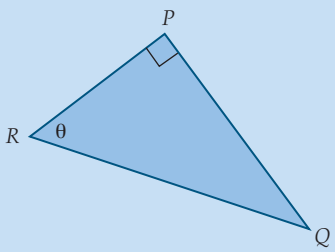
The side names can be abbreviated to H, O and A.

Sides can also be identified in relation to a given angle, i.e. side '*a*' is opposite angle *A*.

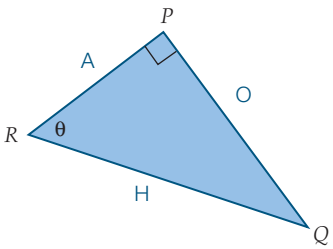
Worked Example 1

WE1

In the following triangle, label the hypotenuse (H), the opposite (O) and the adjacent (A) sides.



Working



Worked Example 2

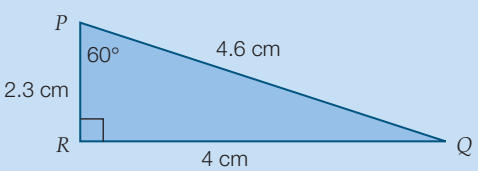
WE2

For the triangle pictured, with a reference angle of 60°, what is the length of:

(a) the hypotenuse

(b) the opposite side

(c) the adjacent side?



Working

(a) Length of hypotenuse = 4.6 cm.

(b) Length of opposite side = 4 cm.

(c) Length of adjacent side = 2.3 cm.

Answers page 462

6.1 Introduction to trigonometry

Stage 5.1

Navigator

Q1 (a)–(c), Q2 (a)–(c), Q3 (a), (b), Q4, Q5, Q6, Q7, Q8, Q9, Q10	Q1 (a), (c), (e), Q2 (a), (c), (e), Q3 (a), (b), Q4, Q6, Q7, Q8, Q9, Q10	Q1 (d)–(f), Q2 (d)–(f), Q3 (b), (c), Q4, Q6, Q7, Q8, Q9, Q10

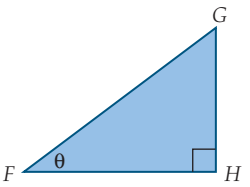
Equipment required: ruler and protractor for Questions **7, 8**

Fluency and understanding

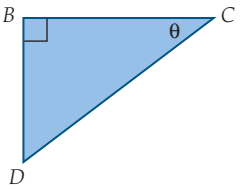
WE1

1 Copy each of the following triangles and use the reference angle to identify the hypotenuse, the opposite side and the adjacent side. Label these sides with H, O and A, respectively.

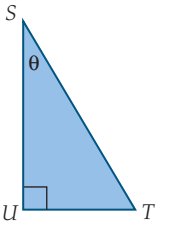
(a)



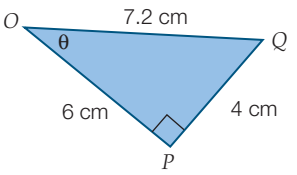
(b)



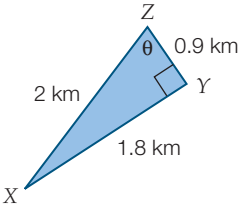
(c)



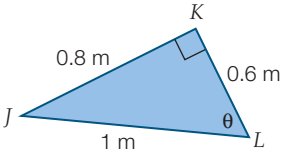
(d)



(e)



(f)



WE2

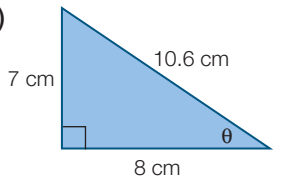
2 For each of the following triangles, state the length of the:

(i) hypotenuse

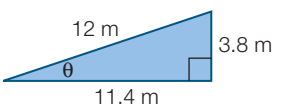
(ii) opposite side

(iii) adjacent side.

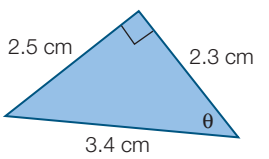
(a)



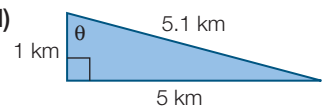
(b)



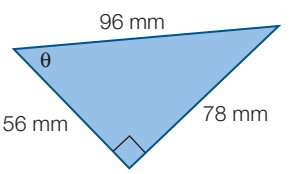
(c)



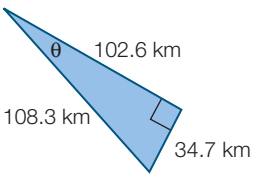
(d)



(e)



(f)



3 (a) Which of the following statements is incorrect?

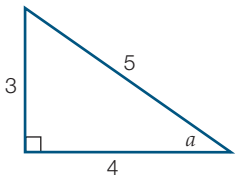
A The hypotenuse is the longest side in a right-angled triangle.

B The hypotenuse is opposite the right angle.

C The shorter side next to the reference angle is called the adjacent side.

D A right-angled triangle always has two sides equal.

(b) From this triangle determine which of the following represents the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for the angle a.



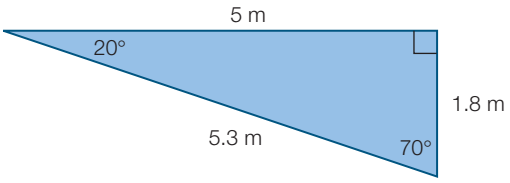
A $\frac{3}{5}$ **B** $\frac{3}{4}$

C $\frac{4}{5}$ **D** $\frac{4}{3}$

(c) Using the diagram from part **(b)**, what would be the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for the other acute angle in the triangle?

A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{4}{3}$ **D** $\frac{3}{4}$

4 Consider the following triangle.



(a) State the length of the hypotenuse.

(b) If 20° is the reference angle, state the length of the opposite side.

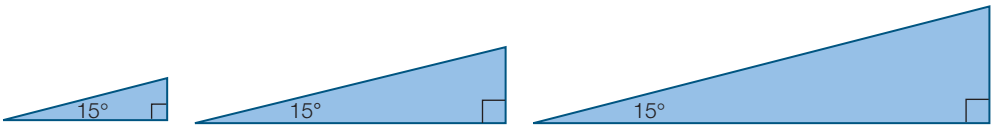
(c) If the adjacent side is 1.8 m long, what is the size of the reference angle?

5 A right-angled triangle has three sides of 5, 12 and 13.

(a) Which of these sides would be the hypotenuse?

(b) Draw a diagram of the triangle with the vertices labelled A, B and C, with the right angle at B. Mark the lengths on the sides.

6 (a) In the following right-angled triangles, measure the length of each side.

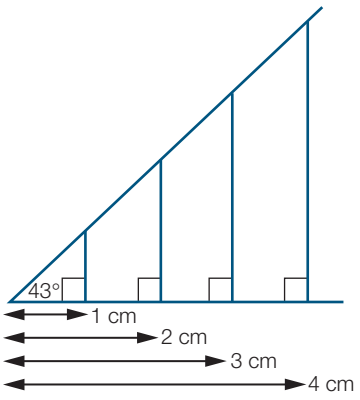


(b) For each triangle in **(a)**, use 15° as the reference angle to find the value of $\frac{\text{opposite}}{\text{hypotenuse}}$.

(c) For each triangle in **(a)**, use 15° as the reference angle to find the value of $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Reasoning, communicating and open-ended

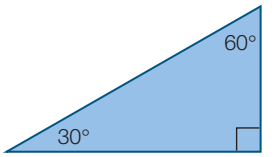
7 (a) Use a protractor to draw an angle of 43°. Mark off distances of 1 cm, 2 cm, 3 cm and 4 cm and rule a perpendicular line at each point. A diagram is shown here.



(b) Measure the heights of the four triangles created and, using 43° as the reference angle, calculate the ratio $\frac{\text{opposite}}{\text{adjacent}}$ in each triangle. Write your ratios in decimal form, rounded to one decimal place.

(c) The four triangles that you have drawn are similar triangles because they all have equal angles (the AAA similarity test). How does this help you explain your answer to **(b)**?

8 **(a)** Use a ruler and protractor to accurately draw three differently sized triangles with angles of 30°, 60° and 90°. One triangle is shown here.



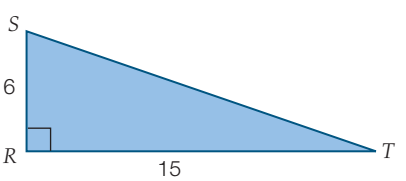
(b) Use your triangles from **(a)** to copy and complete the table below. Write measurements to the nearest mm.

Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°						
2	30°						
3	30°						
1	60°						
2	60°						
3	60°						

(c) What do you notice about the values calculated in the last three columns?

(d) Because the three triangles drawn in **(a)** are of different sizes but have the same angles, they are similar triangles. What do you know about the side lengths in similar triangles?

9 The teacher drew the diagram on the right on the board. He asked the students to complete the triangle and label the hypotenuse, opposite and adjacent sides.

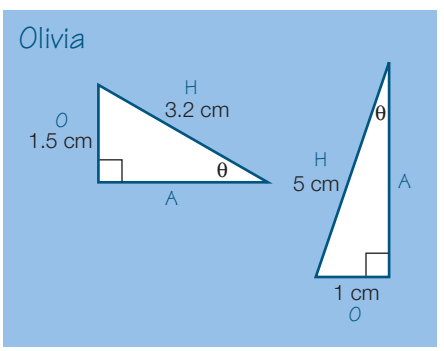


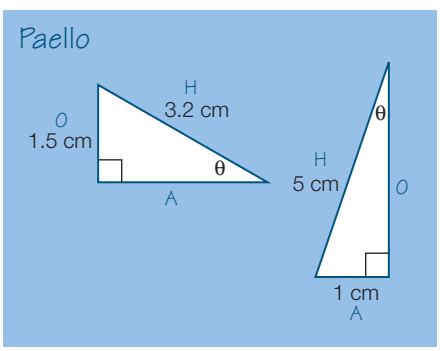
Nash said, 'The opposite side is 15'. Aisha said, 'The opposite side is 6'. They are both right.

(a) What piece of information did the teacher forget to add to the triangle that makes it possible for both students to be correct?

(b) Draw the completed triangles of Nash and Aisha.

10 A class was given two right-angled triangles and asked to label the hypotenuse, opposite and adjacent sides, relative to the reference angle θ. The answers of two students, Olivia and Paello, are shown below.





Paello says to Olivia: 'Your answer is different to mine. I think it's wrong, because the adjacent side is always the one on the bottom of a right-angled triangle.'

Write what you would say to Paello to explain and correct his misunderstanding.

Trigonometric ratios

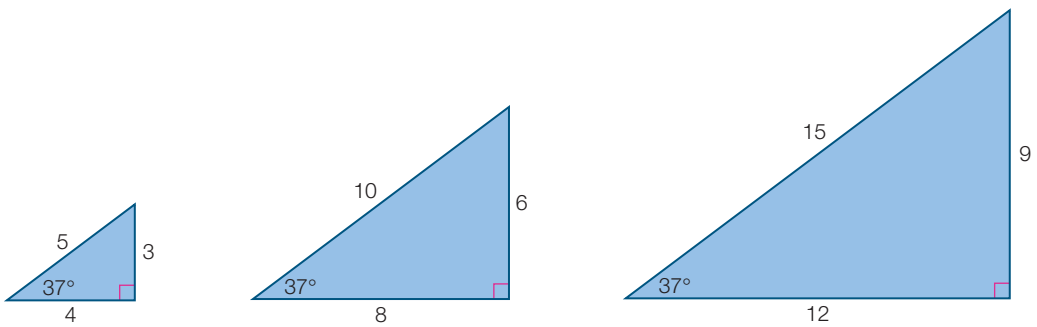
6.2

Stage 5.1

Need to Know

Similarity and trigonometrical ratios

All triangles with three angles the same are similar because all the angles are equal and the sides are in the same proportion. This means that the ratios between matching side lengths are constant, regardless of the size of the triangle.



Using the reference angle of 37°, the ratio of $\frac{\text{opposite}}{\text{adjacent}}$ is the same for each of the above triangles: $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$. This is known as the **tangent** (tan) ratio.

Likewise, the ratios of $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$, using the reference angle of 37°, are the same for all of the above triangles.

The $\frac{\text{opposite}}{\text{hypotenuse}}$ ratio is called the **sine** (sin) ratio and the $\frac{\text{adjacent}}{\text{hypotenuse}}$ ratio is called the **cosine** (cos) ratio.

Each ratio uses two of the three sides of a right-angled triangle. For a specific reference angle, the value of a particular ratio is always the same regardless of the size of the triangle.

For example:

The sine of 37° (sin 37°) = 0.60 (correct to two decimal places).

The cosine of 37° (cos 37°) = 0.80 (correct to two decimal places).

The tangent of 37° (tan 37°) = 0.75 (correct to two decimal places).

Trigonometric definitions

The **sine** of θ is given by: $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

The **cosine** of θ is given by: $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

The **tangent** of θ is given by: $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

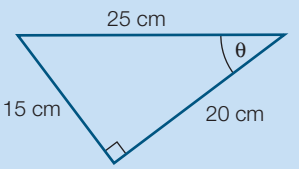
A common acronym to help with remembering the trigonometric ratios is:

SOH CAH TOA

Worked Example 3

WE3

Find the exact value of tan θ.



Working

$$\tan\theta = \frac{O}{A}$$

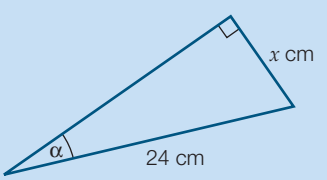
$$\tan\theta = \frac{15}{20}$$

$$= \frac{3}{4}$$

Worked Example 4

WE4

If $\sin\alpha = \frac{1}{3}$, use the sine ratio to find the value of x in the following triangle.



Working

$$\sin\alpha = \frac{O}{H}$$

$$\frac{1}{3} = \frac{x}{24}$$

$$\frac{1}{3} \times 24 = x$$

$$x = 8 \text{ cm}$$

Worked Example 5

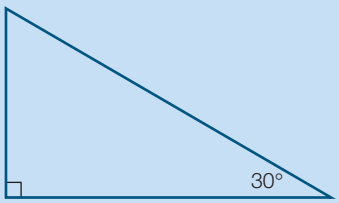
WE5

Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

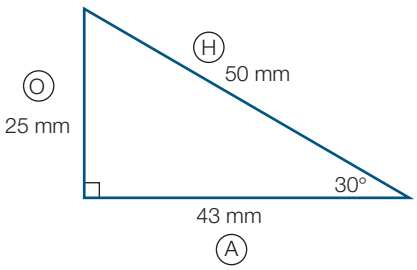
(a) sin 30°

(b) cos 30°

(c) tan 30°



Working



(a) $\sin 30° = \frac{O}{H}$

$= \frac{25}{50}$

$= \frac{1}{2}$

$= 0.50$

(b) $\cos 30° = \frac{A}{H}$

$= \frac{43}{50}$

$= 0.86$

(c) $\tan 30° = \frac{O}{A}$

$= \frac{25}{43}$

$= 0.58$

Warning

The measuring technique in Worked Example 5 is provided to demonstrate how the trigonometric ratios are found and requires an accurately drawn scale diagram. In the next section you will learn how to find these values accurately by using your calculator. You should never assume that a diagram in a textbook or on a test is drawn to scale.

6.2 Trigonometric ratios

Answers page 463

Stage 5.1

Navigator

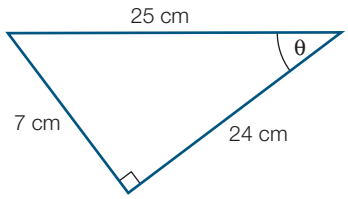
Q1 (a) (i), (b) (i), (c) (i), Q2 (a) (i), (b) (i), (c) (i), Q3, Q4, Q6 (a), (b), Q7, Q8, Q10, Q11, Q12, Q15	Q1 (a) (ii), (b) (ii), (c) (ii), Q2 (a) (ii), (b) (ii), (c) (ii), Q3, Q4, Q6 (b), (c), Q7, Q9, Q10, Q11, Q12, Q14, Q15	Q1 (a) (iii), (b) (iii), (c) (iii), Q2 (a) (iii), (b) (iii), (c) (iii), Q3, Q4, Q5, Q6 (c), (d), Q8, Q9, Q11, Q12, Q13, Q15

Fluency and understanding

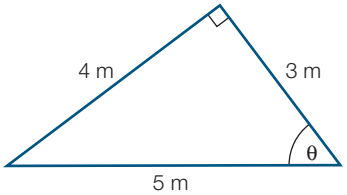
1 (a) Find the exact value of tan θ in each triangle.

WE3

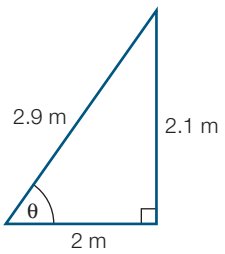
(i)



(ii)

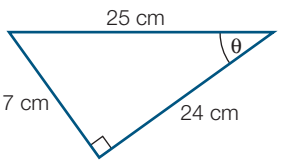


(iii)

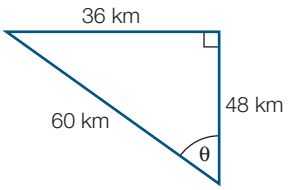


(b) Find the exact value of sin θ in each triangle.

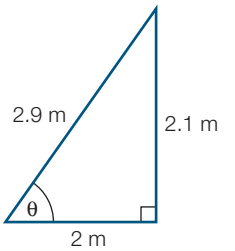
(i)



(ii)

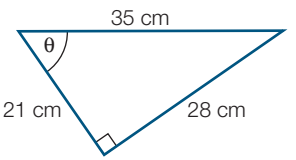


(iii)

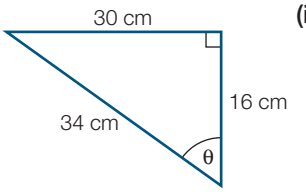


(c) Find the exact value of cos θ in each triangle.

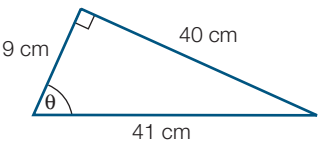
(i)



(ii)



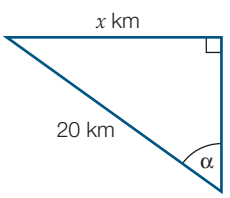
(iii)



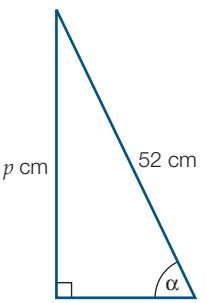
WE4

2 **(a)** Find the value of the variable (correct to two decimal places) in the following triangles, where sin $\alpha =$

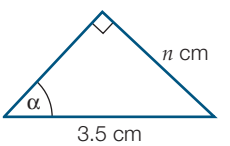
(i) $\frac{1}{3}$



(ii) $\frac{3}{4}$

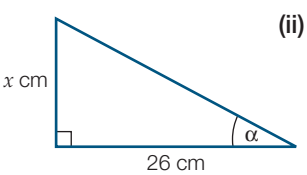


(iii) $\frac{1}{5}$

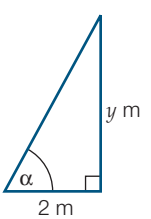


(b) Find the value of the variable (correct to two decimal places) in the following triangles, where tan $\alpha =$

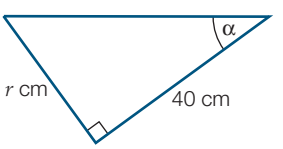
(i) $\frac{1}{2}$



(ii) 3

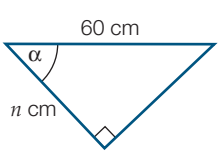


(iii) 0.52

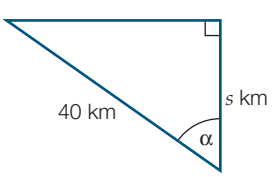


(c) Find the value of the variable (correct to two decimal places) in the following triangles, where cos $\alpha =$

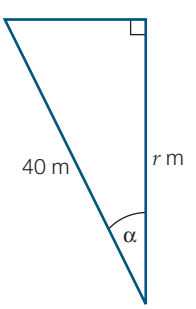
(i) $\frac{4}{5}$



(ii) 0.3



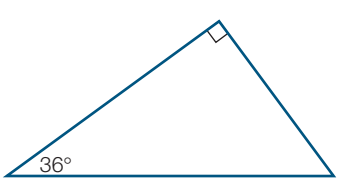
(iii) 0.56



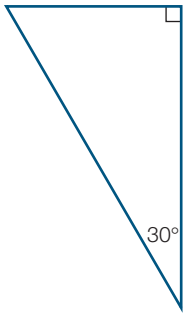
WE5

3 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

(a) sin 36° **(b)** cos 36° **(c)** tan 36°



4 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:



(i) tan 30° **(ii)** cos 30°

(iii) sin 30° **(iv)** sin 60°

(v) tan 60° **(vi)** cos 60°

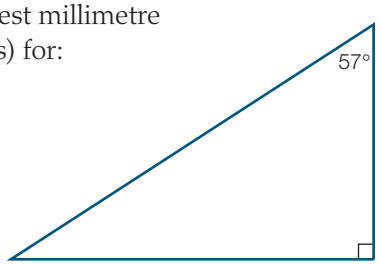
(b) What do you notice about the values of cos 30° and sin 60°?

(c) What do you notice about the values of cos 60° and sin 30°?

Hint

What is the value of the unlabelled angle in the diagram?

5 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:



(i) cos 57° **(ii)** tan 57°

(iii) sin 57° **(iv)** sin 33°

(v) cos 33° **(vi)** tan 33°

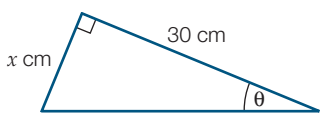
(b) What do you notice about the values for sin 57° and cos 33°?

(c) What do you notice about the values for sin 33° and cos 57°?

Hint

What is the value of the third angle in this triangle?

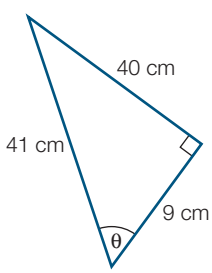
6 **(a)** If $\tan\theta = \frac{2}{3}$, then the value of x in this triangle is:



A 15 **B** 20

C 40 **D** 45

(b) In this diagram, cos θ is:



A $\frac{9}{41}$ **B** $\frac{40}{41}$

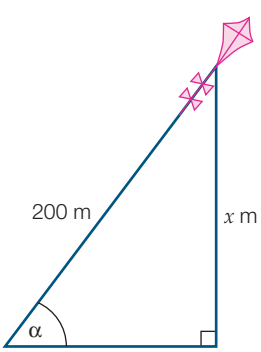
C $\frac{40}{9}$ **D** $\frac{41}{9}$

(c) The value for sin 71° would be the same as:

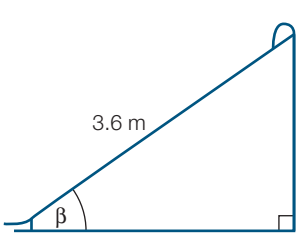
A cos 71° **B** cos 19°

C sin 19° **D** cos 29°

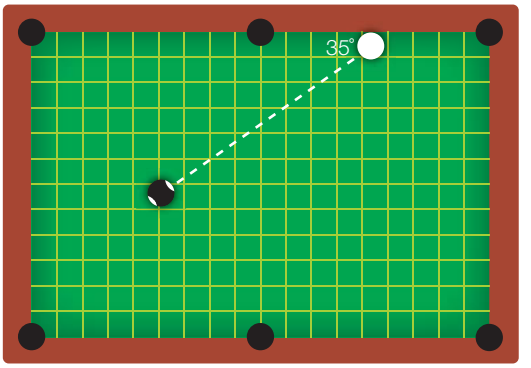
7 One windy day, Nam was flying a kite. Nam's kite string was 200 m long. If $\sin\alpha = \frac{1}{2}$, how high above the ground was the kite?



8 A slippery dip is 3.6 m long. If $\cos\beta = \frac{7}{9}$, find the distance between the foot of the slippery dip and the bottom of the supporting tower.



9 In a game of pool, the cue ball is at the edge of the table and is hit 152 cm, at an angle of 35° with the side of the table. It hits the black ball, which is 125 cm further down the table. Calculate the value of cos 35°, correct to two decimal places.



Reasoning, communicating and open-ended

10 A 7.5-m ladder is leaning against a wall at an angle of 45°. The bottom of the ladder is 5.3 m from the base of the wall.

(a) Draw a diagram to scale (1 cm = 1 m) showing the information provided.

(b) What type of triangle is formed by the ladder, the wall and the ground?

(c) How far up the wall does the ladder reach?

(d) Explain how you can find the answer to **(c)** without doing any calculations.

11 A slide that is 5 m long is to be built in a playground. The recommended angle of the slide with the ground is 37°. Parents are concerned that the slide may be too high for children.

(a) If $\sin 37° = \frac{3}{5}$, what is the height of the slide with this recommended slide angle?

(b) The parents are only happy if the height at the top of the slide is no greater than 2.5 m. What changes could be made so that this can happen?

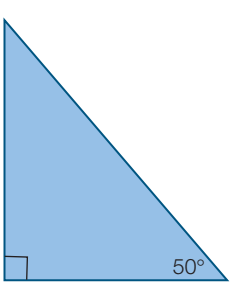
12 The sine of an angle is $\frac{12}{13}$.

(a) Draw the triangle and label the sides.

(b) Calculate the length of the other side.

(c) Determine the cosine ratio.

13 A right-angled triangle with a reference angle of 50° is shown.



(a) Measure the side lengths (in mm) and then calculate the values of sin 50°, cos 50° and tan 50°, correct to three decimal places.

(b) In what way is the value of tan 50° different to sin 50° and cos 50°? Explain why this happens.

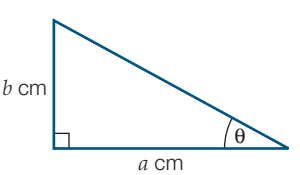
(c) Draw a right-angled triangle that has:

(i) a tan θ value of less than 1

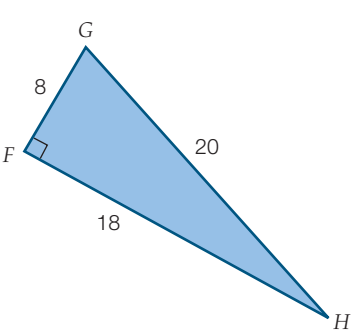
(ii) a tan θ value of more than 1.

(d) Can the values of sin θ and cos θ be greater than 1? Why/why not?

14 In the triangle opposite, $\tan \theta = \frac{7}{10}$. Write down at least three possible pairs of values for the lengths of sides a and b.



15 The teacher of a Year 9 class drew the following triangle on the board and asked the students to find the sin and cos ratios for the angles at G and H.



After completing the task, Rashira looked at her answers and noticed that she had written $\sin H = \frac{8}{20}$ and $\cos G = \frac{8}{20}$. She said to her friend, 'They're the same!' That can't be right, can it? How would you explain to Rashira that she has the correct answers?

Using trigonometry to find side lengths

6.3

Stage 5.1

Need to Know

Trigonometrical ratios

The **sine** of θ is given by:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

SOH

The **cosine** of θ is given by:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

CAH

The **tangent** of θ is given by:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

TOA

To use trigonometry to find side lengths in right-angled triangles, you first need to identify which trigonometric ratio to use. To do this, identify which side lengths are involved in the problem.

For example, if the adjacent side and the hypotenuse are the given lengths, use the cosine ratio.

Calculating an unknown side of a right-angled triangle

To calculate an unknown side length of a right-angled triangle using trigonometry:

1. Draw the right-angled triangle, showing all known information.
2. Label the sides with O, A and H in relation to the given angle.
3. Use SOH–CAH–TOA to decide which trigonometric ratio to use.
4. Form an equation and solve for the unknown.

Technology Exploration: Calculator

Using a calculator to find a trigonometric ratio

A calculator can be used to find the value of a ratio by pressing one of the following keys:
sin **cos** **tan**.

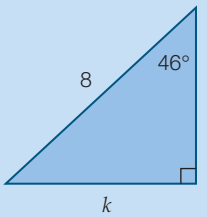
For example, to find the sine of 67°, press **sin** **6** **7** **=**.

Many trigonometric ratio values have many decimal places. It is more accurate to use the full value in your calculations and round your final answer than to use a rounded ratio in your calculations. Make sure your calculator is set to 'degrees' mode to calculate trigonometric values using degrees.

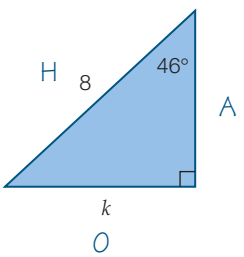
Worked Example 6

WE6

Find the value of k, correct to two decimal places.



Working



$\theta = 46°$
$H = 8$
$O = k$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 46° = \frac{k}{8}$$

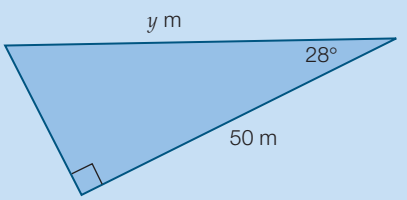
$$k = 8 \times \sin 46°$$

$$\approx 5.75 \text{ (2 d.p.)}$$

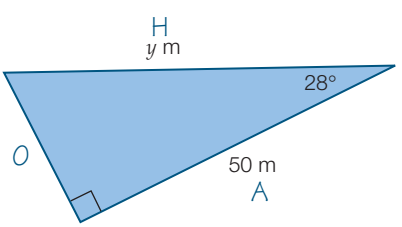
Worked Example 7

WE7

Find the value of y, correct to two decimal places.



Working



$\theta = 28°$
$A = 50$ m
$H = y$ m

$$\cos\theta = \frac{A}{H}$$

$$\cos 28° = \frac{50}{y}$$

$$y \times \cos 28° = 50$$

$$y = \frac{50}{\cos 28°}$$

$$\approx 56.63 \text{ m (2 d.p.)}$$

6.3 Using trigonometry to find side lengths

Answers page 463

Stage 5.1

Navigator

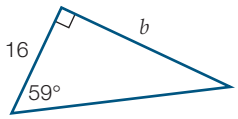
Q1, (a)–(c), Q2 (a)–(c), Q3, Q5, Q6, Q7, Q8, Q9, Q11, Q12	Q1 (a), (c), (e), Q2 (a), (c), (e), Q3, Q5, Q7, Q9, Q10, Q11, Q12, Q13	Q1 (d)–(f), Q2 (d)–(f), Q4, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

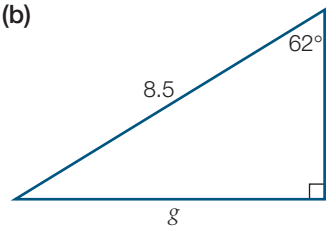
1 Find the value of each variable correct to two decimal places.

WE6

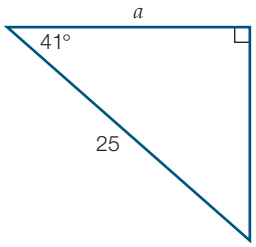
(a)



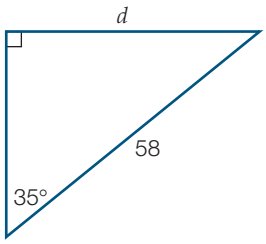
(b)



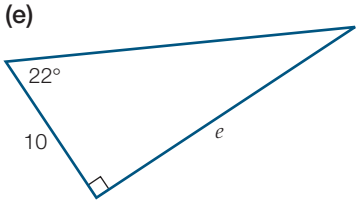
(c)



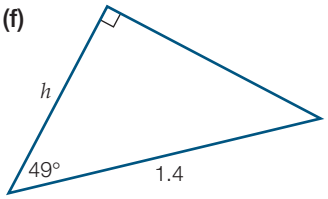
(d)



(e)



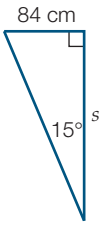
(f)



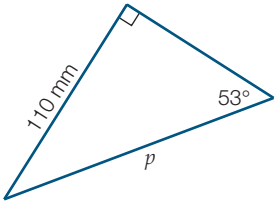
2 Find the value of the variable (correct to two decimal places) in each of the following triangles.

WE7

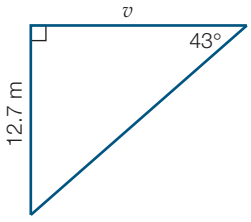
(a)



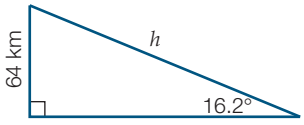
(b)



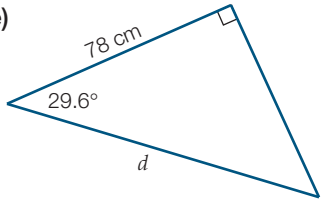
(c)



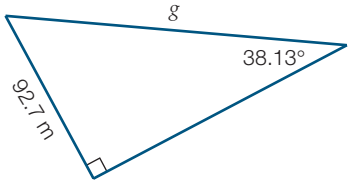
(d)



(e)

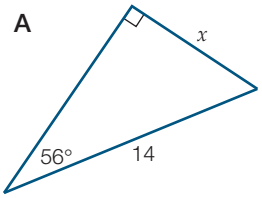


(f)

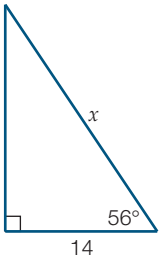


3 (a) For which of the following triangles can $\sin 56° = \frac{14}{x}$ be solved to give x?

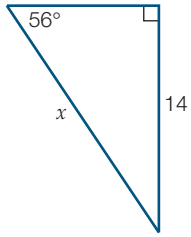
A



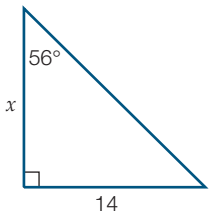
B



C

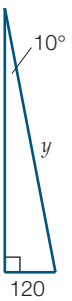


D

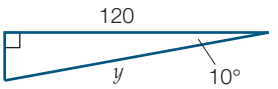


(b) For which of the following triangles can $\cos 10° = \frac{y}{120}$ be solved to give y?

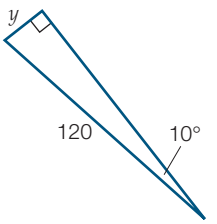
A



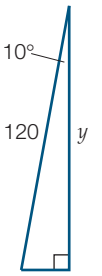
B



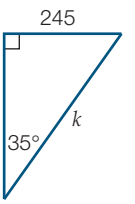
C



D



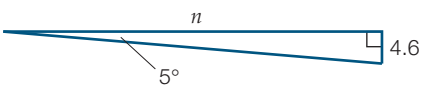
4 (a) The value of k could be found by using the equation:



A $\sin 35° = \frac{k}{245}$ **B** $\tan 35° = \frac{245}{k}$

C $\sin 35° = \frac{245}{k}$ **D** $\cos 35° = \frac{245}{k}$

(b) The value of n could be found by using the equation:

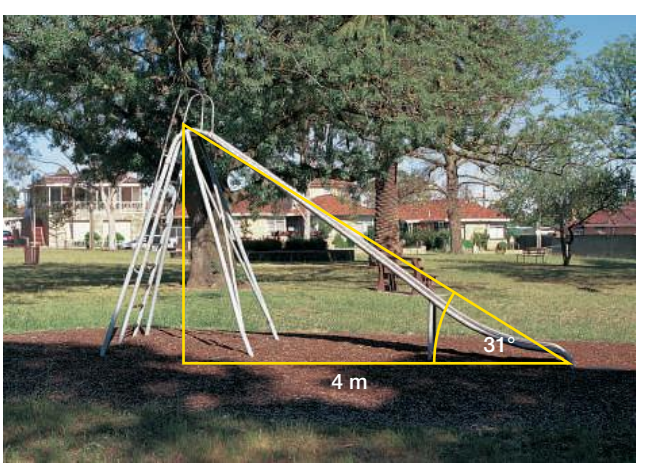


A $\sin 5° = \frac{4.6}{n}$ **B** $\cos 5° = \frac{4.6}{n}$

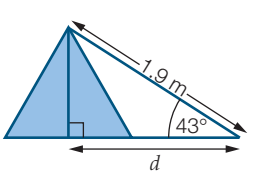
C $\tan 5° = \frac{n}{4.6}$ **D** $\tan 5° = \frac{4.6}{n}$

For the following questions, express all answers correct to two decimal places.

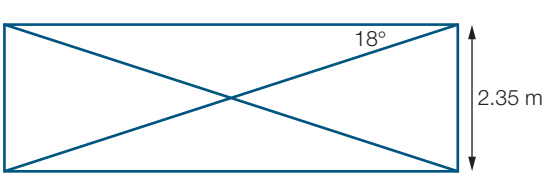
5 Use the information given in the photograph to find the height of the slide.



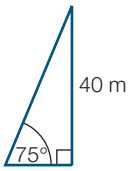
6 Calculate how far away a tent peg is from a tent pole if the 1.9-m support wire is pegged so that it makes an angle of 43° with the ground.



7 A rectangular gate has diagonal wooden supports that make an angle of 18° to the horizontal sections of the frame. If the gate is 2.35 m high, how long would one of the diagonal supports be?

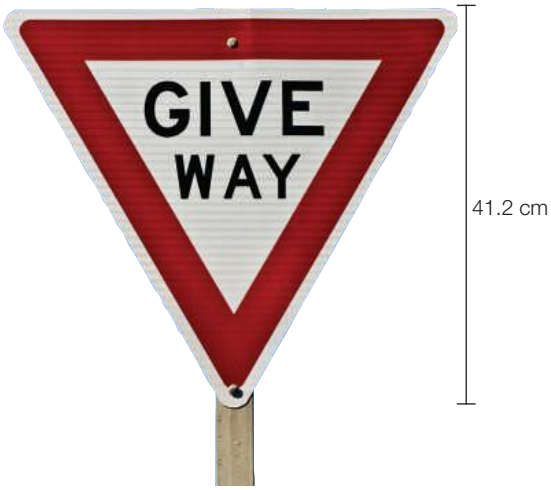


8 Members of a stunt team wish to be catapulted into the air in a capsule at an angle of 75° to the horizontal. Assuming they travel along a straight path, what horizontal distance will they have covered when they have reached a height of 40 m?

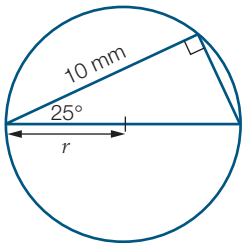


Reasoning, communicating and open-ended

9 A road sign is in the shape of an equilateral triangle with a vertical measurement of 41.2 cm. How long is each side of the road sign?



10 Find the radius of this circle.



11 An angle in a right-angled triangle measures 63°. Give two possible pairs of lengths of the adjacent side and the hypotenuse.

12 Shown is Caillan's working for a trigonometry question.

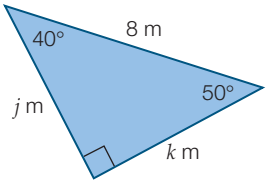
(a) What might the question have been? (Draw the triangle.)

(b) Calculate the length of the third side of the triangle.

$$\cos 15° = \frac{20}{y}$$
$$y \times \cos 15° = 20$$
$$y = \frac{20}{\cos 15°}$$
$$y = 20.7 \text{ cm}$$

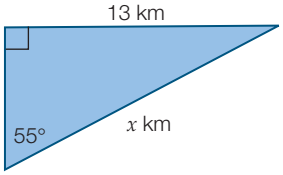
13 The following answers were given by a student on a trigonometry test.

(i) Find the value of k.



$$\cos\theta = \frac{A}{H}$$
$$\cos 40° = \frac{k}{8}$$
$$8 \times \cos 40° = k$$
$$k = 6.13 \text{ m}$$

(ii) Find the value of x.



$$\sin\theta = \frac{O}{H}$$
$$\sin 55° = \frac{13}{x}$$
$$13 \times \sin 55° = x$$
$$x = 10.65 \text{ km}$$

(a) Explain the mistake the student has made in each question.

(b) Show the correct calculations and answers.

14 Explain the differences between using Pythagoras' Theorem to find an unknown side of a right-angled triangle and using trigonometric ratios. For each method, state the minimum information that must be known in order to apply the method successfully.

Problem solving

Finding the height of a tree

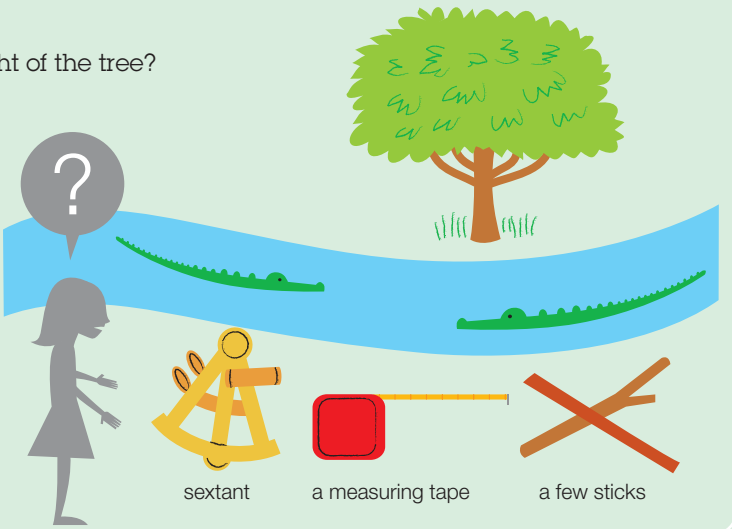
Your task is to find the height of a tree that is on the other side of a river that cannot be crossed. You have been given a 100-m tape measure and a sextant.

A sextant is an instrument for measuring the angle from your horizontal line of sight to the top of another object. This object may be higher or lower than your line of horizontal sight. You do not have to reach the object to be able to find the angle, but it must be within sight.

How can you find the height of the tree?

Strategy options

- Draw a diagram.
- Break the problem into manageable parts.

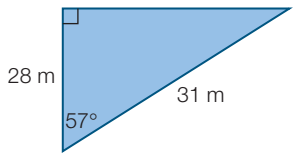


Half-time 6

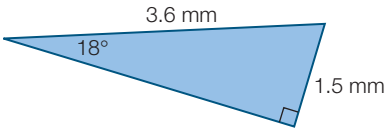
Stage 5.1 Q1–6

1 Write the letter names (O, A or H) of the two sides that would be used in any calculations. Ex.6.2

(a)

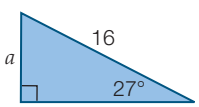


(b)

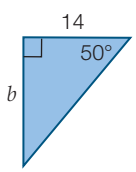


2 Find the value of each variable, correct to two decimal places. Ex.6.3

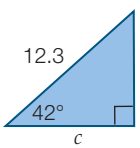
(a)



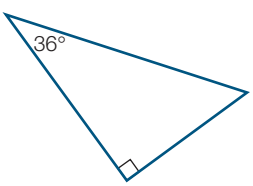
(b)



(c)



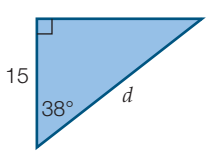
3 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for: Ex.6.2



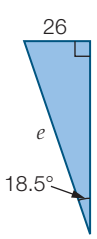
(a) sin 36° **(b)** cos 36° **(c)** tan 36°

4 Find the value of each variable, correct to two decimal places. Ex.6.3

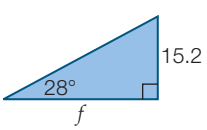
(a)



(b)

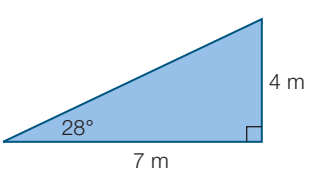


(c)

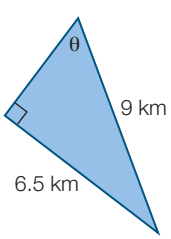


5 Label the sides of the following right-angled triangles as hypotenuse (H), opposite (O) and adjacent (A). Ex.6.1

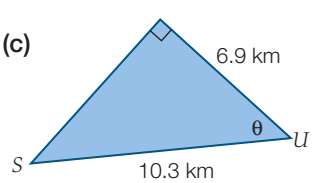
(a)



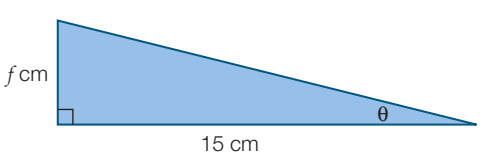
(b)



(c)



6 If $\tan\theta = \frac{2}{5}$, what is the value of f? Ex.6.2



6.4 Using trigonometry to find angles

Stage 5.1

Need to Know

Finding angles in right-angled triangles

The trigonometric ratios sine, cosine and tangent are the ratios of two specific side lengths of a right-angled triangle. If two side lengths are known, they will form one of the trigonometric ratios. The angle that corresponds to this ratio can be found by rearranging the trigonometric equation to make the angle the subject. This requires the use of the trigonometric inverse.

Trigonometric inverse notation

Inverse trigonometric ratios are written as $\sin^{-1}\left(\frac{O}{H}\right)$, $\cos^{-1}\left(\frac{A}{H}\right)$ and $\tan^{-1}\left(\frac{O}{A}\right)$.

Trigonometric ratio	Inverse
sine of θ $\sin\theta = \frac{O}{H}$	$\theta = \sin^{-1}\left(\frac{O}{H}\right)$
cosine of θ $\cos\theta = \frac{A}{H}$	$\theta = \cos^{-1}\left(\frac{A}{H}\right)$
tangent of θ $\tan\theta = \frac{O}{A}$	$\theta = \tan^{-1}\left(\frac{O}{A}\right)$

Note that the notation used here can appear to be misleading. The $^{-1}$ does not mean 'to the power of -1' as it does in algebra. When talking about $\sin^{-1}\theta$ you should always say 'inverse sin of θ', not 'sine to the power of negative one'.

Technology Exploration: Calculator

Using a calculator to find inverse ratios

Decide which inverse ratio applies to the situation.

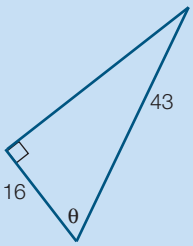
On most calculators, the inverse trigonometric functions are found by pressing **2nd F**, **SHIFT** or **INV** before pressing **sin**, **cos** or **tan**.

Perform the appropriate division: $\frac{O}{H}$, $\frac{A}{H}$ or $\frac{O}{A}$.

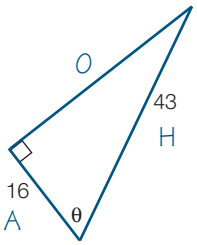
Worked Example 8

WE8

Find the value of θ correct to the nearest degree.



Working



$$\cos\theta = \frac{A}{H}$$

$$\cos\theta = \frac{16}{43}$$

$$\theta = \cos^{-1}\left(\frac{16}{43}\right)$$

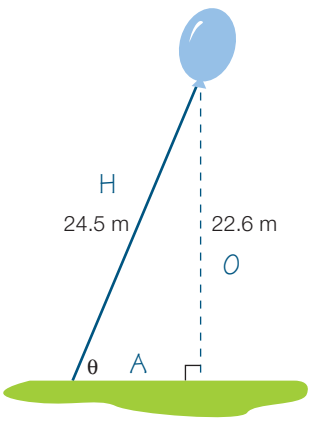
$$\approx 68°$$

Worked Example 9

WE9

A helium-filled balloon is attached to the ground by a 24.5-m string. A light breeze is blowing so that the base of the balloon is at a height of 22.6 m above the ground. What is the smaller of the two angles, to the nearest degree, that the string makes with the ground?

Working



$$\sin\theta = \frac{O}{H}$$

$$\sin\theta = \frac{22.6}{24.5}$$

$$\theta = \sin^{-1}\left(\frac{22.6}{24.5}\right)$$

$$\approx 67.287°$$

The string makes an angle of 67° with the ground.

Answers page 463

6.4 Using trigonometry to find angles

Stage 5.1

Navigator

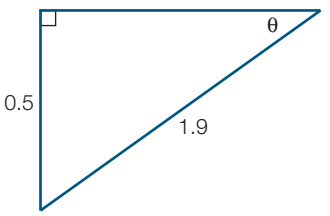
Q1, (a)–(c), Q2, Q3, Q4, Q5, Q6, Q8, Q9	Q1 (c)–(e), Q2, Q3, Q4, Q5, Q7, Q8, Q9	Q1 (f)–(h), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Fluency and understanding

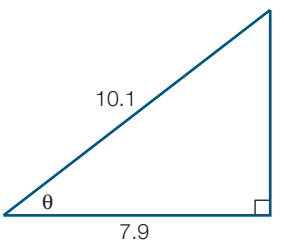
WE8

1 Find the value of θ, correct to the nearest degree, in each of the following.

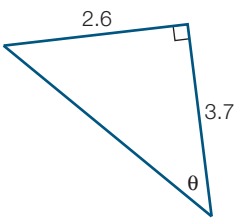
(a)



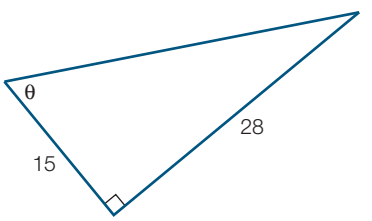
(b)



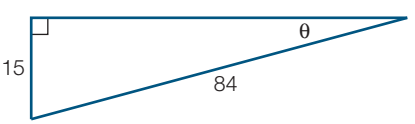
(c)



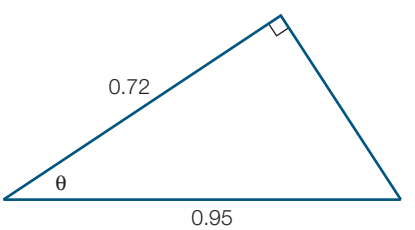
(d)



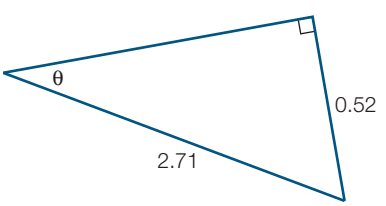
(e)



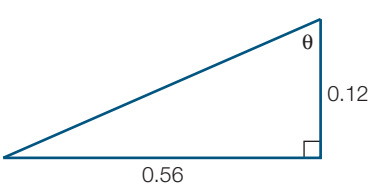
(f)



(g)

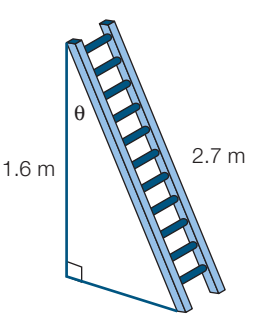


(h)

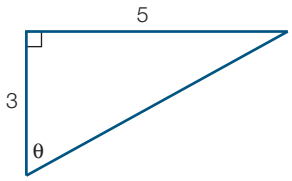


WE9

2 The top of a 2.7-m ladder, leaning against a wall, reaches a height of 1.6 m. What angle, to the nearest degree, does the ladder make with the wall?



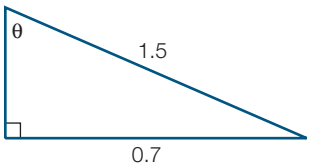
3 **(a)** The value of θ can be found by using the equation:



A $\cos\theta = \frac{3}{5}$ **B** $\tan\theta = \frac{5}{3}$

C $\sin\theta = \frac{3}{5}$ **D** $\tan\theta = \frac{3}{5}$

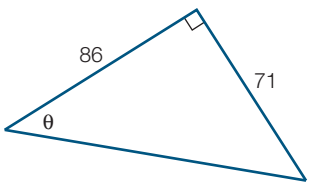
(b) The value of θ in this triangle would be equal to:



A $\tan^{-1}\left(\frac{0.7}{1.5}\right)$ **B** $\cos^{-1}\left(\frac{1.5}{0.7}\right)$

C $\sin^{-1}\left(\frac{0.7}{1.5}\right)$ **D** $\cos^{-1}\left(\frac{0.7}{1.5}\right)$

(c) The value of θ in this triangle would be equal to:



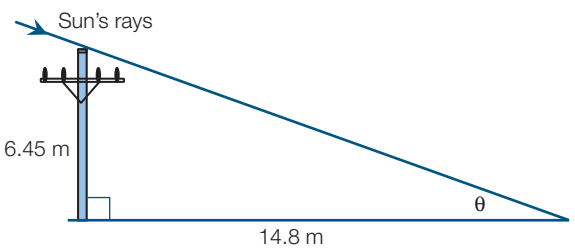
A $\cos^{-1}\left(\frac{71}{86}\right)$ **B** $\tan^{-1}\left(\frac{86}{71}\right)$

C $\sin^{-1}\left(\frac{71}{86}\right)$ **D** $\tan^{-1}\left(\frac{71}{86}\right)$

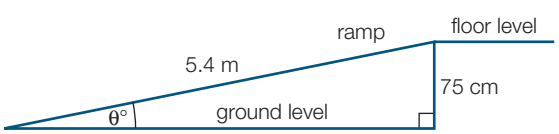
4 A children's seesaw is made from a plank of wood 3.4 m long supported in the middle, as shown in the photograph. If the greatest height reached by one end of the seesaw is 0.9 m, what angle does the seesaw make with the ground? Round your answer to the nearest degree.



5 Find (to the nearest degree) the angle of the Sun's rays, θ, when a 6.45 m high power pole casts a shadow 14.8 m long.



6 A builder uses a ramp to bring in sand in a wheelbarrow from ground level to floor level inside a building under construction. Floor level is 75 cm above ground level. If the ramp is 5.4 m long, what angle, to the nearest degree, does the ramp make with the ground?



Reasoning, communicating and open-ended

7 **(a)** When the opposite side and hypotenuse are similar in length, $\sin\theta$ is large. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of $\sin\theta$.

(b) When the opposite side is small and the hypotenuse is large, $\sin\theta$ is very small. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of $\sin\theta$.

(c) Complete the following sentences:

(i) When the adjacent side and hypotenuse are similar in length, $\cos\theta$ is _____.

(ii) When the adjacent side is small and the hypotenuse is large, $\cos\theta$ is _____.

8 One angle between 0° and 90° gives a trigonometric ratio of 1.

(a) What does a ratio of 1 tell you about the lengths of the two sides involved?

(b) Which ratio is it? Explain how you know.

(c) What type of right-angled triangle gives this ratio a value of 1?

(d) What is the value of the angle involved?

9 Two of the side lengths of a right-angled triangle are 9 and 15. What could the reference angle be?

10 An access ramp for a motorised wheelchair must make an angle with the horizontal that is between 6° and 9°. Find three different ramp lengths and the angles they must be placed at to give smooth access to a platform that is 65 cm high.

Applications of trigonometry

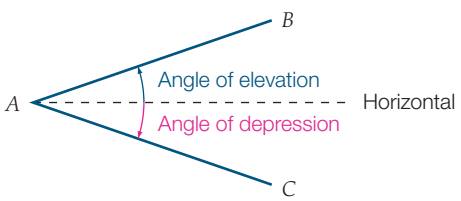
6.5

Stage 5.1

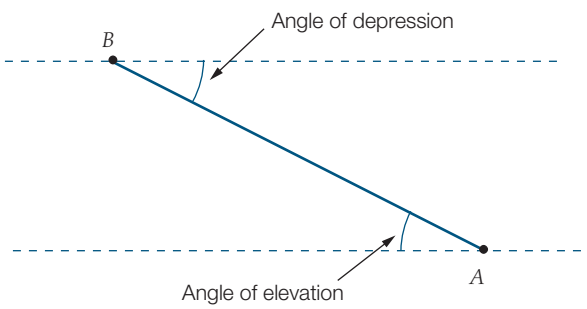
Need to Know

Angles of elevation and depression

The **angle of elevation** from A to B is the angle the upward-sloping line AB makes with the horizontal.



The **angle of depression** from A to C is the angle the downward-sloping line AC makes with the horizontal.



Angles of elevation and depression are always measured from the horizontal.

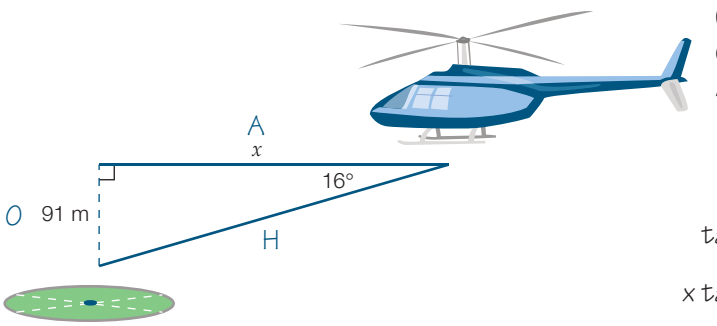
The angle of elevation from Object A to Object B is the same as the angle of depression from Object B to Object A. (As the line AB crosses between two parallel lines, the angle of elevation and the angle of depression are alternate angles. Hence, they are equal.)

Worked Example 10

WE10

A helicopter, flying at a constant altitude of 91 m, sights the middle of a landing pad at an angle of depression of 16°. How far, in metres, must the helicopter travel to be directly over the landing spot at an altitude of 91 m?

Working



$\theta = 16°$
$O = 91$ m
$A = x$ m

$$\tan\theta = \frac{O}{A}$$

$$\tan 16° = \frac{91}{x}$$

$$x \tan 16° = 91$$

$$x = \frac{91}{\tan 16°}$$

$$\approx 317.35 \text{ (2 d.p.)}$$

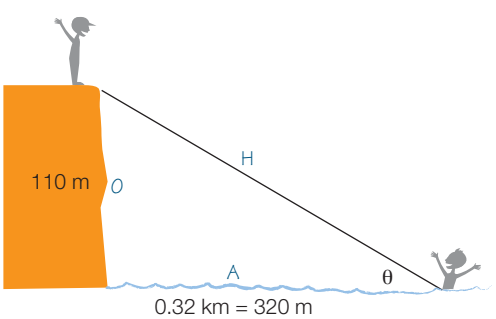
The helicopter must travel 317 m before it is directly over the landing pad.

Worked Example 11

WE11

A swimmer, 0.32 km from the base of a cliff, looks up and waves to her friend who is at the top of the 110-m cliff. Find the angle of elevation from the swimmer to the top of the cliff. Give your answer to the nearest degree.

Working



$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{110}{320}$$

$$\theta = \tan^{-1}\left(\frac{110}{320}\right)$$

$$\approx 18.97 \text{ (2 d.p.)}$$

The angle of elevation from the swimmer to the top of the cliff is 19° (to the nearest degree).

6.5 Applications of trigonometry

Answers page 464

Stage 5.1

Navigator

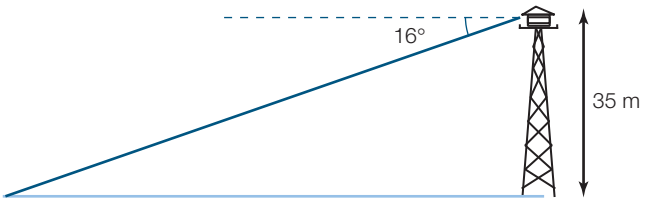
Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12

Fluency and understanding

Find all angles to the nearest degree and all lengths to two decimal places, unless told otherwise.

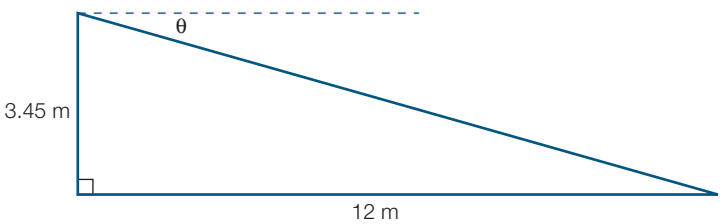
WE10

1 Michelle, a forestry officer on watch from a lookout 35 m above the ground, notices a small brush fire at an angle of depression of 16°. How far, in metres, would a fire-fighting aircraft, currently directly over the lookout, need to fly to drop its load on the fire?

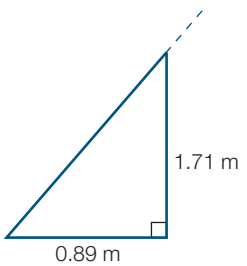


WE11

2 The crew on a fishing boat spot a shark near the surface of the water nearby. They estimate that the horizontal distance between themselves and the shark is 12 m. If the crew are 3.45 m above sea level, find the angle of depression to the shark.



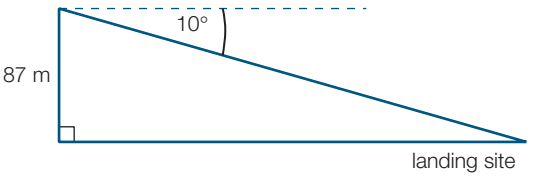
3 Find the angle of elevation of the Sun (to two decimal places) if Phillip is 1.71 m tall and he casts a 0.89-m shadow.



4 A 5.8-m tree casts a shadow across the ground. If the angle of elevation of the Sun is 55°, the length of the shadow is:

A 4.1 m **B** 4.8 m **C** 7.1 m **D** 10.1 m

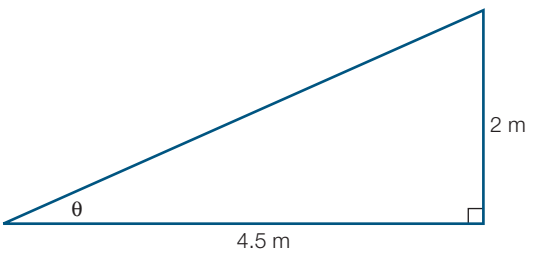
5 From a hot air balloon, the angle of depression to a marked landing site is 10°. If the observation deck in the balloon is 87 m above the ground, the horizontal distance the balloon needs to travel to reach the landing site is:



A 86 m **B** 88 m **C** 493 m **D** 501 m

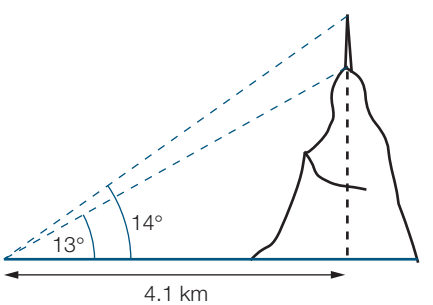
6 A lifeguard spots a swimmer in distress from a lifeguard station 2.8 m high. If the angle of depression from the lifeguard to the swimmer is 17°, find the distance to the swimmer along the lifeguard's line of sight.

7 Martin has stopped his car at an intersection. He is 4.5 m from the base of a traffic signal, which is red, and judges the position of the green light to be 2 m higher than his horizontal line of sight. Through what angle of elevation must he raise his eyes to view the green light signal?



Reasoning, communicating and open-ended

8 A communications tower is on top of a nearby mountain. From a position 4.1 km away from the base of the mountain, the angle of elevation to the bottom of the tower is 13°, whereas the angle of elevation to the top of the tower is 14°. Find the height of the tower to the nearest metre.



9 From her vantage point on a cliff, Maria sights two swimmers in a direct line in front of her at angles of depression of 38.6° and 53.9°. If Maria is 50 m above the water level, find the distance between the two swimmers.

10 The angle of elevation from Ned's car to the top of a nearby tree is 27°. Give two pairs of possible values for the height of the tree and the horizontal distance of the tree from the car.

11 Draw a diagram that shows the angle of elevation and the angle of depression between two objects. Explain why these two angles are equal.

12 Germaine's maths test contained the following question:

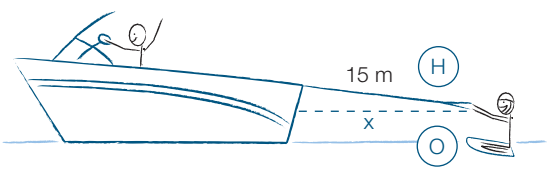
'Bianca was kneeboarding behind a boat on a 15-m rope. If the rope has an angle of depression of 4°, how far is Bianca from the back of the boat?'

Here is Germaine's answer. The teacher had marked it as incorrect.

(a) Identify the mistake and calculate the correct answer.

(b) How would you explain to Germaine the mistake she made?

Germaine's answer:



$\sin 4° = \frac{x}{15 \text{ m}}$

$x = \sin 4° \times 15$

$x = 1.05 \text{ m}$

Bearings

6.6

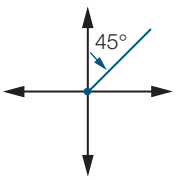
Stage 5.2◊

Need to Know

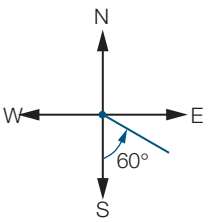
Bearings are used to indicate the direction of one object from another object or point and are used extensively in navigation, orienteering and map reading.

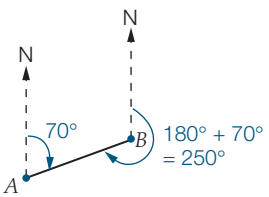
There are two types of bearings:

True bearings: A true bearing describes a direction measured as an angle clockwise from north. True bearings are always written using three figures. For example, an angle of 45° measured from north would be written as 045°T.



Compass bearings: A compass bearing describes a direction measured as an angle either side of north or south. For example, an angle of S60°E is found by facing south and then turning 60° towards the east.





Note that the bearing from one point A to another point B is not the same as the bearing from B to A.

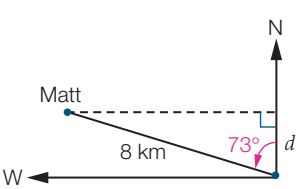
Bearing of B from A = 070°T or N70°E
Bearing of A from B = 250°T or S70°W

Worked Example 12

WE12

Matt hikes for 8 km from his base camp on a bearing of N73°W. How far north is Matt from his base camp? Give your answer correct to one decimal place.

Working



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 73° = \frac{d}{8}$$

$$d = 8 \times \cos 73°$$

$$\approx 2.339 \text{ km}$$

Matt is 2.3 km north of the base camp.

Answers page 464

6.6 Bearings

Stage 5.2

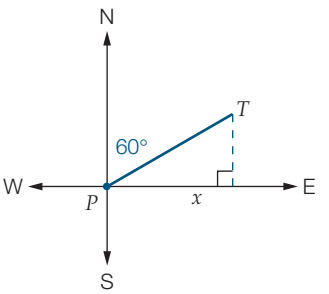
Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9

Fluency and understanding

WE12

1 A train leaves Paddistone station and travels 40 km on a bearing of 060°T to Tarrigal station. How far east of Paddistone station is Tarrigal station?



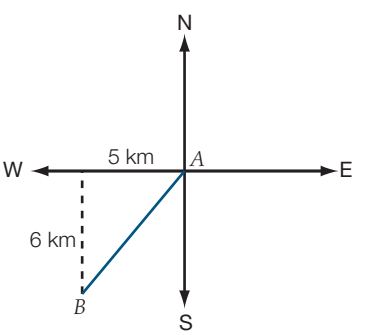
2 To reach a cattle station from the nearest town, a traveller must drive 73 km north then 52 km east. If the traveller flew directly to the cattle station from the town:

(a) find the distance flown, correct to the nearest kilometre

(b) find the bearing for the flight, correct to the nearest degree.

3 For the given diagram, the bearing of B from A, to the nearest degree, would be:

A S50°W **B** S56°W **C** S34°W **D** S40°W



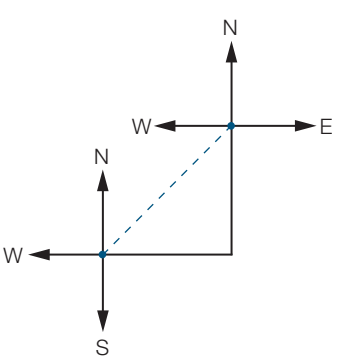
4 A man on a horse rode 4 km due south before turning and travelling 6.5 km due west. The bearing the horse and man must travel on to return to their starting point is:

A N31.6°E

B N58.4°E

C S31.6°W

D S58.4°W



5 A ship sails 134 km north then 81 km west from Port A to Port B to avoid a cyclone.

(a) How far would it have travelled if it had sailed directly from Port A to Port B? Give your answer in kilometres correct to 1 d.p.

(b) What bearing would it have taken for the direct route?

Reasoning, communicating and open-ended

6 A ship travels 45 km on a bearing of N42°W.

(a) How far north of its starting point is the ship?

(b) How far west of its starting point is the ship?

7 A hiker walks 2.4 km east, then walks S75°W until she is due south of her starting position.

(a) Draw a diagram of the scenario.

(b) How far is she from her starting point?

(c) How far did she walk on the bearing of S75°W before she was due south of her starting position?

8 Sarah is standing 100 m due south of a tower. Dougal is standing 140 m due west of the same tower. Using both compass bearings and true bearings, find the bearing of:

(a) Dougal from Sarah

(b) Sarah from Dougal.

9 A hiker has wandered away from his campsite and is lost. He has been located by a search-and-rescue team x km south and 2 km to the east of his camp.

(a) Draw a diagram to represent this situation and find a possible value for x if he was more than 2 km but no more than 10 km from his camp.

(b) Find the distance, correct to 2 decimal places, he was from his campsite using this value for x and the bearing the rescue party needs to take from the campsite to locate him.

Challenge 6



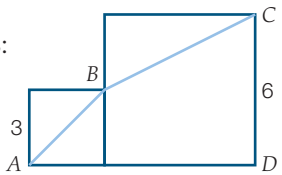
1 If m, n are integers, where $m > n$ and $n \geq 1$, then $m^2 - n^2$, $2mn$, $m^2 + n^2$ are Pythagorean triples (i.e. a set of three numbers that obey Pythagoras' Theorem).

(a) Show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$.

(b) Find the Pythagorean triples with no common factors, whose smallest number is:

(i) 3 **(ii)** 5 **(iii)** 7 **(iv)** 8 **(v)** 9

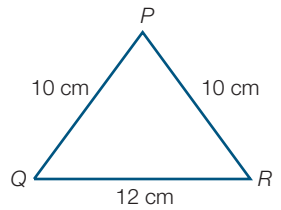
2 A square of side length 3 cm is placed next to a square of side length 6 cm as shown. The perimeter of the quadrilateral $ABCD$ is:



A $3\sqrt{2}$ cm **B** $3\sqrt{5}$ cm

C $6\sqrt{2}$ cm **D** $3(5 + \sqrt{2} + \sqrt{5})$ cm

3 In ΔPQR, $PQ = PR = 10$ cm, $QR = 12$ cm.



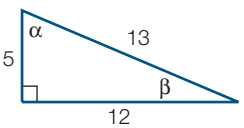
(a) What type of triangle is PQR?

(b) The perpendicular from P to QR meets QR at S. Calculate the length of PS.

(c) The value of $\sin R$ is:

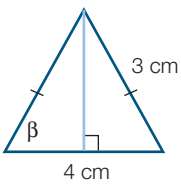
A $\frac{4}{5}$ **B** $\frac{2}{3}$ **C** $\frac{3}{5}$ **D** $\frac{3}{10}$

4 Which of the following is equivalent to $\cos\alpha°$?



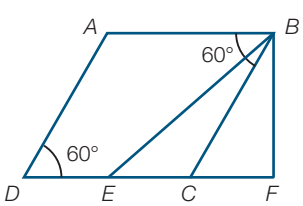
A $\sin\alpha°$ **B** $\cos\beta°$ **C** $\tan\beta°$ **D** $\sin\beta°$

5 For the triangle shown, which statement is correct?



A $\sin\beta = \frac{3}{4}$ **B** $\cos\beta = \frac{4}{3}$ **C** $\cos\beta = \frac{2}{3}$ **D** $\tan\beta = \frac{5}{2}$

6 $ABCD$ is a rhombus in which the opposite angles B and D are each 60°. CD is bisected at E and BE is drawn. BF is perpendicular to DC. Let the sides of the rhombus be $2x$ units.



(a) Write down the length of EC.

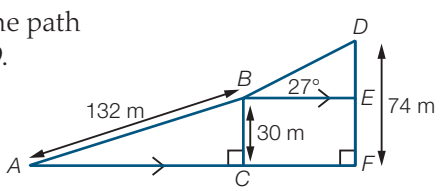
(b) Use trigonometry to find the length of CF.

(c) Calculate the length of BF as a surd.

(d) Calculate the size of $\angle EBF$.

(e) Hence, write down the size of $\angle EBC$, correct to the nearest minute.

7 A mountain bike rider rides up a hill. The slope of the path he follows is shown in the diagram from A to B to D.



(a) The first stage of the track from A to B is 132 m long. The track rises 30 m over this distance (i.e. $BC = 30$ m).

Find the angle of elevation (θ), to the nearest degree, of the track AB.

(b) If the second stage of the track is inclined at 27° to the horizontal, find the length of the track BD to the nearest metre.

(c) The winning time for a rider to climb the hill from A to D was 1 min and 30 s. Calculate the average speed of the bike in metres per second (m/s). Give your answer correct to one decimal place.

Chapter review 6

D.I.Y. Summary

Key Words

Stage 5.1	adjacent side	degree	sine
	angle of depression	hypotenuse	tangent
	angle of elevation	opposite side	trigonometric ratios
	cosine	reference angle	trigonometry
Stage 5.2	bearing	compass bearing	true bearing

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

Stage 5.1 Q1–4 **Stage 5.2 Q5, 6**

1 When using the trigonometric ratios we must know which angle is to be used. We call this angle the _____.

2 The _____ of an angle is found by dividing the opposite side length by the length of the adjacent side.

3 The only trigonometric ratio that can have a value greater than 1 is _____.

4 The _____ is measured from the horizontal up to the line of sight, whereas the _____ is measured from the horizontal down to the line of sight.

5 A _____ is measured from the north or from the south.

6 A _____ is measured clockwise from north.

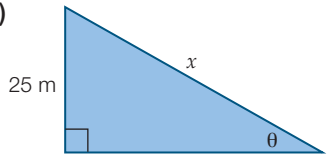
Stage 5.1 Q1–6, 8–12, 15–17 **Stage 5.2 Q7, 13, 14**

Fluency and understanding

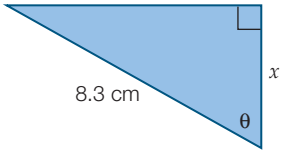
1 Name the two labelled sides of the following triangles as opposite, adjacent or hypotenuse and give the trigonometric ratio that would be used to calculate the unknown side.

Ex 6.1

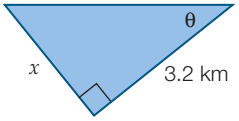
(a)



(b)



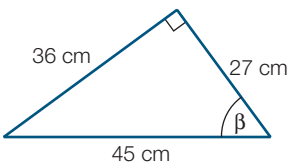
(c)



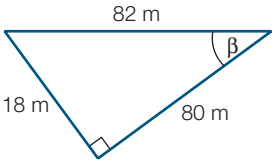
2 Find the value of tan β° for each triangle, correct to four decimal places where appropriate.

Ex 6.2

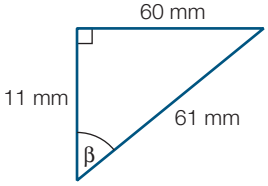
(a)



(b)



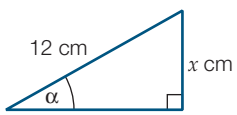
(c)



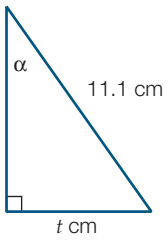
3 If $\sin\alpha = \frac{1}{3}$, find the value of the variable for each triangle, correct to two decimal places.

Ex 6.2

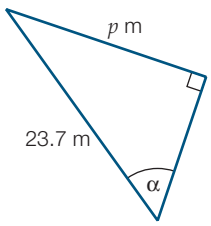
(a)



(b)



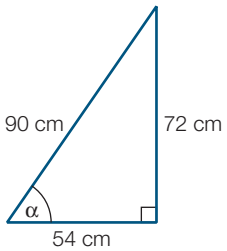
(c)



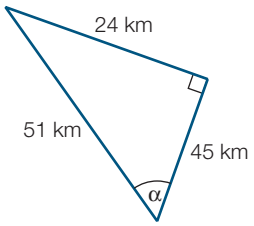
4 Find the exact value of $\cos\alpha°$ for each triangle.

Ex 6.2

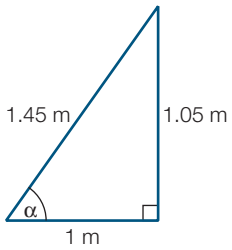
(a)



(b)



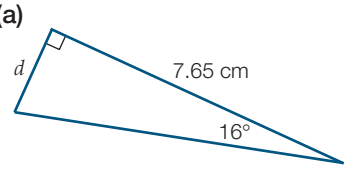
(c)



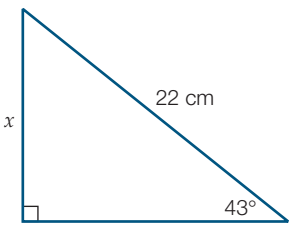
5 Find the value of the unknowns in the following triangles. Give your answers correct to two decimal places for side lengths and to the nearest degree for angles.

Ex 6.3, 6.4, 6.5

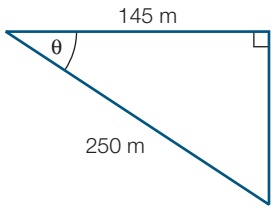
(a)



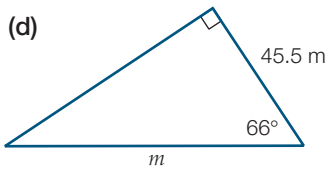
(b)



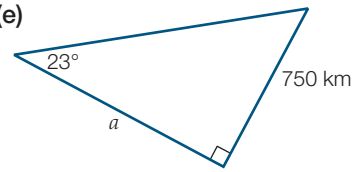
(c)



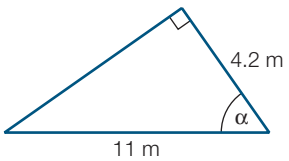
(d)



(e)



(f)



6 A kite is attached to a string that is 15 m long. If the string makes an angle of 26° to the horizontal, the height of the kite is:

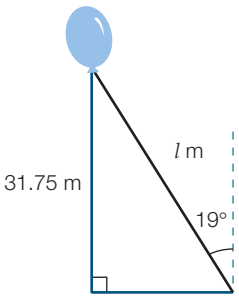
Ex 6.5

A 6.6 m **B** 7.4 m **C** 13.5 m **D** 34.2 m

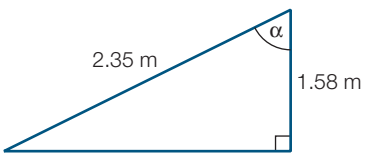
7 A hiker travels 13.2 km on a bearing of 125°T. How far south and east is the hiker from the starting position? Express your answer correct to two decimal places.

Ex 6.6

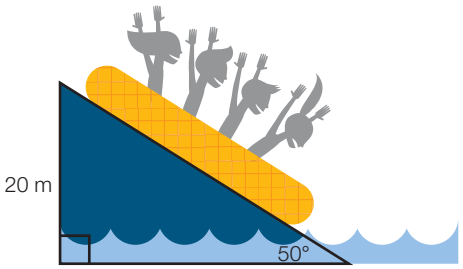
8 A balloon on the end of a string is being blown by the wind. If the balloon is 31.75 m above the ground and the string makes an angle of 19° to the vertical, find the length of the string.



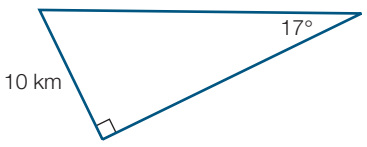
9 A children's slide has a vertical ladder 1.58 m high. Find the angle the slide makes with this ladder if it is 2.35 m long.



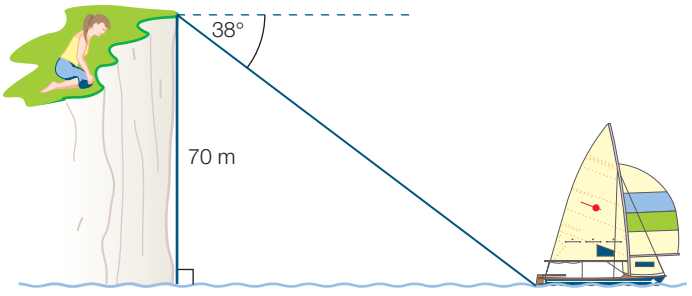
10 At an amusement park, a scary water raft ride descends a slide at an angle of 50° to the horizontal. If the slide starts 20 m above the ground, how long is the slide?



11 An orienteering trail follows a triangular course as shown in the diagram. Find the total length of the course, correct to one decimal place.



12 The angle of depression from the top of a 70 m cliff to a boat out at sea is 38°. How far away is the boat from the base of the cliff?



13 A submarine travelled 56 km on a bearing of 025°T from Point *A* to Point *B*. How far to the north has the submarine travelled from its starting point?

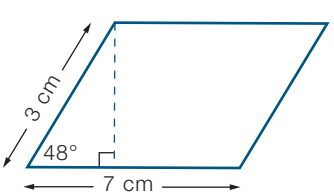
14 A four-wheel drive solar car was being tested in the Simpson Desert. It travelled north for 25 km, then turned and travelled 14 km due west. The driver would like to travel straight back to the starting point. What bearing must the car travel to return to its starting point, to the nearest degree?

Reasoning and communicating

15 A 365-cm ladder leaning against a wall makes an angle of 68.2° with the ground. If the ladder slips 76 cm down the wall, what angle (correct to one decimal place) does it now make with the ground?

16 **(a)** Find the height of this parallelogram. Give your answer correct to two decimal places.

(b) Hence, find the area of this parallelogram.

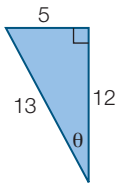


17 When Colin is 85 m away from a building he notices a Spiderman at an angle of elevation of 62° climbing up the side of the building. What will be the angle of elevation when the Spiderman has climbed up a further 10 m? Give your answer correct to one decimal place.

NAPLAN practice 6

Numeracy: Non-calculator

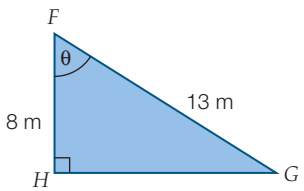
1 In the diagram on the right, the exact value of $\sin\theta$ is:



A $\frac{5}{12}$ **B** $\frac{5}{13}$

C $\frac{12}{13}$ **D** $\frac{13}{12}$

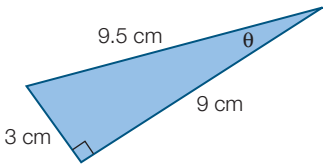
2 Which trigonometric ratio would be used to calculate angle θ?



A sin **B** cos

C tan **D** Pythagoras' Theorem

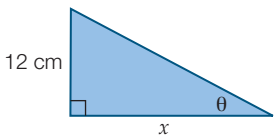
3 What is the tan ratio for θ in the triangle shown?



A $\frac{1}{3}$ **B** $\frac{9}{9.5}$

C $\frac{3}{9.5}$ **D** $\frac{9.5}{9}$

4 If $\tan\theta = \frac{3}{4}$, then the value of the variable in the diagram on the right is:

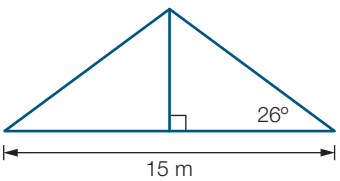


A 3 cm **B** 4 cm

C 9 cm **D** 16 cm

Numeracy: Calculator allowed

5 Part of a roof frame is shown. The frame has two symmetrical sides. What is the minimum length of timber needed for the diagonal parts of the frame?



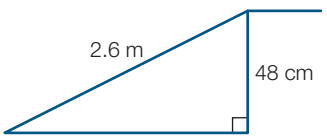
A 8 m **B** 9 m

C 16 m **D** 17 m

6 An aeroplane is a horizontal distance of 4000 m from the airport, flying at 2500 m, when it is instructed to descend for landing. To the nearest degree, at what angle will the aeroplane need to descend?

A 32° **B** 39° **C** 51° **D** 58°

7 A wheelchair ramp stretches from the ground to the top of a step 48 cm high. The ramp is 2.6 m long. To the nearest degree, what angle does the ramp make with the ground?



A 10° **B** 11°

C 32° **D** 33°

8 A rescue helicopter hovering 95 m above the ocean spots a swimmer in distress at an angle of depression of 32°. What horizontal distance will the helicopter need to travel to be directly above the swimmer?

A 95 m **B** 112 m **C** 152 m **D** 179 m

Statistics and probability

7

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 To one decimal place, calculate the mean, median and range of the following data.

20, 17, 24, 28, 32, 14, 21, 18, 14, 22, 16, 25

2 A normal die is rolled once. Calculate the following probabilities, expressing your answers in fraction form.

(a) P(rolling a 2) **(b)** P(not rolling a multiple of 3) **(c)** P(rolling 4 or 5)

3 Find the mean, correct to two decimal places, and the median of the following data set.

Number of brothers and sisters	0	1	2	3	4	5	6
Frequency	12	17	23	20	18	4	2

4 List the sample space for drawing a card of a particular suit from a normal pack of playing cards.

Syllabus references

Statistics and Probability: Data Collection and Representation

- ✓ Investigate techniques for collecting data, including census, sampling and observation **(ACMSP284) (Stage 4)**
- ✓ Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes **(ACMSP206) (Stage 4)**
- ✓ Identify and investigate issues involving numerical data collected from primary and secondary sources **(ACMSP169) (Stage 4)**

Statistics and Probability: Single Variable Data Analysis

- ✓ Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data **(ACMSP171) (Stage 4)**
- ✓ Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources **(ACMSP228) (Stage 5.1)**
- ✓ Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal' **(ACMSP282) (Stage 5.1)**
- ✓ Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread **(ACMSP283) (Stage 5.1)**
- ✓ Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data **(ACMSP253) (Stage 5.1)**

Statistics and Probability: Probability

- ✓ Identify complementary events and use the sum of probabilities to solve problems **(ACMSP204) (Stage 4)**
- ✓ Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' **(ACMSP226) (Stage 5.1)**
- ✓ List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events **(ACMSP225) (Stage 5.2)**

7.1 Investigating data

Stage 4

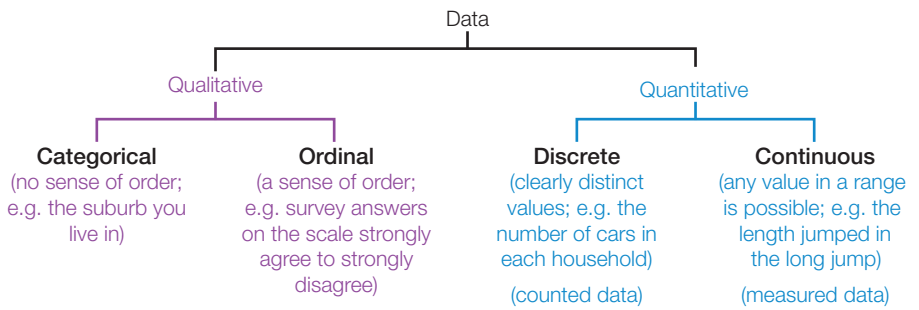
Need to Know

Data types

Data can be divided into two broad groups:

- **quantitative data** sets, where the data consist of numerical values
- **qualitative data** sets, where the data consist of categories or descriptive terms.

The different types of data can be summarised in the following diagram:



Sources of data

Primary data is data collected directly by an observer.

Observational data is primary data obtained by direct observation.

Secondary data is data that has not been collected directly and has come from another source.

A **random sample** is one in which every member of the population has the same chance of being selected. A random number generator may be used to identify the population members to be selected.

One of the ways to describe data sets is by using a single value to represent the whole data set. This value is called a **measure of location**. Three measures of location are commonly used: mean, median and mode. It should be noted that the mode is not strictly a measure of location.

Worked Example 1

WE1

Decide whether the following data sets are discrete or continuous.

(a) The number of goals scored by each of the teams in each of their games in the 2010 FIFA World Cup.

(b) The crowd size at each of the games in the 2010 FIFA World Cup.

(c) The height of each of the players at the 2010 FIFA World Cup.

Working

(a) *Discrete data (This is a counted value.)*

(b) *Discrete data (This is a counted value.)*

(c) *Continuous data (This is a measured value.)*

Worked Example 2

WE2

The following data were taken from the Australian Bureau of Statistics Average Weekly Earnings bulletin (ABS: 6302.0 August 2010) released on 18 November 2010.

Males	Full-time adult ordinary time earnings	Full-time adult total earnings	All employees total earnings
2009			
May	1268.70	1335.70	1120.50
August	1289.30	1355.80	1138.70
November	1310.50	1377.00	1160.10
2010			
February	1326.30	1394.30	1175.30
May	1336.90	1407.40	1183.60
August	1343.90	1417.20	1188.10

According to the Explanatory Notes, estimates of average weekly earnings are calculated by dividing estimates of weekly total earnings by estimates of the number of employees.

(a) Which measure of location is being used here?

(b) Can you see any potential problems with using this as the measure of location?

(c) By what percentage did the full-time ordinary male earnings increase from May 2009 to August 2010? Express your answer correct to two decimal places.

(d) Why are the figures in the third column lower than the equivalent figures in the other two columns?

Working

(a) This is the mean.

(b) The mean can be affected by extreme values, whether they are very low or very high.

(c) $1343.90 - 1268.70 = 75.20$

Convert to percentage change, based on original value:

$$\frac{75.20}{1268.70} \times 100\% = 5.93\%$$

(d) This column includes males who are only employed part-time. These males would naturally earn less per week than full-time employees.

Answers page 465

7.1 Investigating data

Stage 4

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17

Equipment needed: scientific calculator or CAS for Questions **6**, **9** and **15**

Fluency and understanding

WE1

1 For each of these data sets, state whether it is discrete or continuous.

(a) The number of apples in each 2-kg bag at the supermarket.

(b) The weight of each bag of apples individually selected by customers in a fruit shop.

(c) The number of matches in each of 30 boxes of matches checked during quality control.

(d) The time taken by each competitor to complete the 100-m freestyle at the most recent Olympic Games.

WE2

2 The following data were taken from the Australian Bureau of Statistics Average Weekly Earnings bulletin (ABS: 6302.0 August 2010) released on 18 November 2010.

Females	Full-time adult ordinary time earnings	Full-time adult total earnings	All employees total earnings
2009			
May	1054.60	1069.10	729.90
August	1066.20	1080.20	737.50
November	1079.40	1093.80	747.30
2010			
February	1093.10	1108.10	757.40
May	1105.70	1121.20	766.40
August	1116.70	1132.40	774.20

According to the Explanatory Notes, estimates of average weekly earnings are calculated by dividing estimates of weekly total earnings by estimates of the number of employees.

(a) By what percentage did the full-time ordinary female earnings increase from May 2009 to August 2010? Express your answer correct to two decimal places.

(b) In August 2010, for females, what percentage of *Full-time adult ordinary time earnings* was the *All employees total earnings*? Express your answer correct to two decimal places.

(c) Calculate this same percentage for males. You will need to look back to the figures in Worked Example 2, page 293.

(d) Make some comment about the difference between the two percentages obtained in **(b)** and **(c)**. Include at least one reason why this is the case.

3 For each of these data sets, state whether it is quantitative or qualitative.

(a) The colour of each car in the car park at the supermarket.

(b) The amount of pocket money received each week by each member of your class.

(c) The number of grains of sand in each of 10 separate matchboxes filled at the beach.

(d) The response of each person surveyed to a question with possible answers: agree, unsure, disagree.

4 **(i)** For each of the data sets in Question **3** that you said was quantitative, state whether it is discrete or continuous.

(ii) For each of the data sets in Question **3** that you said was qualitative, state whether it is categorical or ordinal.

5 Some road death data related to motorcyclists are included in the following table.

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Number of deaths	55	43	72	17	36	8	10	4
Deaths per 10 000 registered motorcyclists	3.71	3.16	5.17	4.30	4.67	6.51	20.28	3.94

Make some comments about the data, with reference to safety and dangerous places for motorcyclists.

6 **(a)** Select a random sample of 10 out of a population of whole numbers from 1 to 100 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

(b) Select a random sample of 5 out of a population of 25 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

7 Choose the most precise data type for each of the following.

(a) Height of players in the Australian netball team

A discrete **B** continuous **C** categorical **D** qualitative

(b) Shoe size of players in the National Basketball League

A discrete **B** continuous **C** categorical **D** ordinal

(c) Eyesight test results of umpires on a cricket umpires panel

A discrete **B** categorical **C** qualitative **D** ordinal

8 In a sample of 50 seedlings, nine had white flowers, 12 had mauve flowers, 18 had crimson flowers, two had pink flowers, eight had magenta flowers, and one had purple flowers. If this sample was representative of the general population, what mean number of seedlings would be needed if a garden designer required:

(a) 32 magenta

(b) 10 pink

(c) 120 mauve?

9 A random number generator can be used to simulate the rolling of a die. The following are the results for a group of 30 rolls:

5, 5, 5, 3, 5, 2, 5, 6, 3, 1, 3, 3, 2, 2, 5, 6, 1, 1, 6, 1, 4, 6, 3, 3, 5, 5, 4, 1, 1, 5

(a) Which number appeared the most often?

(b) Do you think every group of 30 rolls would produce the same results?

(c) Use a random number generator to produce three lots of 30 rolls and comment on your results.

10 Chad is conducting a survey related to the school canteen. The first few questions from Chad's survey are shown below.

Question 1: How often do you buy something from the school canteen?

Question 2: What is the best thing about the school canteen?

Question 3: What items should be available at the school canteen that are not presently available?

His friend Narelle tells Chad that his questions are too open and he would be better off including some suggested answers to his questions that can be chosen by the people surveyed.

Chad agrees with this suggestion. Give some options that Chad may include for each of these questions.

11 The Bureau of Meteorology records the following data for the North Adelaide weather station with regards to rainfall, measured in mm. These data are based on records kept since 1883.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	21.5	20.2	25.0	44.1	64.6	73.4	68.3	64.4	55.1	45.7	31.0	27.6
Median	16.1	9.9	18.4	38.0	57.8	67.6	63.9	63.5	49.7	44.6	25.8	22.3

(a) Why are the mean figures consistently higher than the median figures?

(b) Based on this, for any given month, is the actual rainfall recorded likely to be less than the mean value or the median value?

(c) If you were lobbying for the building a new dam, which figures would you quote in your media releases?

(d) The biggest difference between the two figures is for February. Why do think this is the case?

Reasoning, communicating and open-ended

12 Sophie records the number of days each week the maximum temperature is greater than 22°C. Explain why Sophie's set of data is discrete and not continuous.

13 The Student Representative Council (SRC) at your school has been informed by the people who work at the school canteen that the manners displayed by students have been falling away. The SRC is concerned by this and decides to do some research.

(a) The SRC decides to conduct a survey with 50 students. The school population is 740. How would you recommend the SRC should go about selecting the 50 students to survey?

Concern is then raised that the students may not give absolutely truthful answers on the survey, so it is decided to observe interactions in the canteen.

(b) Can you see any problems with this approach?

(c) How would you suggest the SRC investigate this issue?

14 A large jar contains 25 black marbles, 34 blue marbles and 11 white marbles.

(a) A sample of 10 marbles is drawn from the jar. What would you expect the sample to contain if it truly represented the contents of the jar?

(b) If you drew five such samples from the jar, would you expect them all to be the same?

15 **(a)** Use a random number generator of some kind to produce 20 numbers from 1 to 20. Record your results for each number individually in a frequency table. (Repeated numbers are counted.)

(b) Use the same method to produce 60 numbers from 1 to 20. Again, record your answers in a frequency table. (Repeated numbers are counted.)

(c) Collate the results from the rest of the class, recording your results in a frequency table.

(d) What conclusion can you draw from this work?

16 Your teacher has directed you to construct a recording sheet related to the types of vehicle that pass your school in a certain period of time.

(a) How many different types of vehicle do you think should be included on your sheet?

(b) If left to design the recording sheet individually, do you think other students will come up with the same list as you?

(c) Would it be a problem if different students had different sheets and were recording data on different streets that bordered your school?

17 The Office for National Statistics is the United Kingdom's equivalent of our Bureau of Statistics. In November 2009 they released a statistical bulletin titled *2009 Annual Survey of Hours and Earnings*. Some of the statistics published are reproduced below.

Median gross weekly earnings

		Employee jobs paid at adult rates and not affected by absence		
		Full-time	**Part-time**	**All**
£ per week April 2009	Men	531.1	143.6	491.0
	Women	426.4	155.6	309.8
	All	488.7	152.9	397.3

(a) What percentage of the male income for full-time employees does the female income for full-time employees represent?

(b) What do you notice about the part-time figure for women?

(c) What additional information would you like before making a definitive statement about the reason for the result found in **(b)**?

The following additional information was also published.

Median gross hourly earnings, excluding overtime

		Employee on adult rates and not affected by absence		
		Full-time	**Part-time**	**All**
£ per hour April 2009	Men	12.97	7.71	12.42
	Women	11.39	7.86	9.68
	All	12.34	7.83	10.99

(d) How does this additional information affect your answer to **(c)**? Is this now sufficient information to state why females working part-time are paid more, on average, than men?

Further to the information already provided, the bulletin also tells us that in April 2009 the mean hourly earnings for men employed full-time was £16.07 compared with £13.43 for women, whereas for part-time employees the figures were £11.98 for men with £10.40 for women.

(e) What is different about these mean figures compared with the median figures? Which figures would you be more inclined to use if you were writing a newspaper article about them?

7.2 Statistics in the media

Stage 5.1

Need to Know

Statistics are frequently printed in newspapers and magazines and discussed and reported on television in news bulletins, consumer programs and advertisements. However, careful thought about what is read and heard is needed. Some of the issues that may arise include:

- insufficient evidence being provided to support the conclusions made
- percentages used without an indication of the actual size of the survey
- raw figures used that are based on very different populations
- oversimplification of the results.

Not everything read or heard is incorrect, but it is a good idea to think critically about the key statistical issues involved.

Graphs can become misleading if care is not taken. Line graphs and bar charts need to have a zero on their scale. Graphs involving area are frequently inaccurate as only one measurement matches the correct scale ratio.

To analyse data in the media:

- read or listen to the information closely, ascertain the facts
- properly constructed graphs can assist with correct interpretation of the information
- understand that it is not always possible to be exact with data interpretation but it is possible to be reasonably confident in interpreting data correctly.

7.2 Statistics in the media

Answers page 466

Stage 5.1

Navigator

Q1, Q2 (a) & (b), Q3, Q4, Q9, Q10	Q1, Q2 (a), (b) & (c), Q3, Q4, Q5, Q6, Q8, Q9, Q10	Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Fluency and understanding

This article appeared in *The Age* on 21 January 2010.

NBA hotshot Bogut closes in on Shark

HAVING made his presence felt in the National Basketball Association, Andrew Bogut is starting to cast his considerable shadow over the rest of Australia's sporting stars in money matters.

The Milwaukee Bucks centre has shot up from fifth to second in *BRW*'s latest list of the nation's top 50 sports earners, behind occasional golfer Greg Norman.

The 25-year-old's five-year, US$60 million (A$65 million) contract extension has kicked in for this NBA season and Bogut has responded with his most productive season, averaging 15.1 points and 10.4 rebounds a game.

BRW listed Bogut as earning $14 million a year, hot on Norman's ($15 million) tail, even if the two-time British Open champion's wealth has long been coming from off the golf course. Harry Kewell remained our best-paid football player despite his earnings dipping from $10.5 million to $9.5 million at Turkish club Galatasaray.

Formula One ace Mark Webber ($9 million) was next best, one of four involved in motor sports that made the top 10, ahead of motocross star Chad Reed ($8.8 million), NASCAR driver Marcos Ambrose ($5.44 million) and 2007 MotoGP world champion Casey Stoner ($5 million).

Geoff Ogilvy ($6.34 million) is the highest full-time golfer on this list, while Lleyton Hewitt has dropped out of the top 10, his earning power shrinking from $6.6 million to $2.88 million.

Australia's cricketers remain the best-paid locally based sportsmen. Skipper Ricky Ponting ($4.26 million) is the sport's top earner, despite never having really cashed in on the lucrative Indian Premier League.

Fast bowler Brett Lee ($3.52 million) has shown that playing power does not always equate to earning power, having been rated as the nation's second-best-paid cricketer despite missing the Ashes and home summer. Vice-captain Michael Clarke was ranked 21st with $2.54 million and Michael Hussey ($1.30 million) was at 41. Wild child Andrew Symonds made the list at 50 with $1.22 million.

TOP 5 MONEY-EARNERS IN AUSTRALIAN SPORT	
1 Greg Norman (golf)	**$15m**
2 Andrew Bogut (basketball)	**$14m**
3 Harry Kewell (soccer)	**$9.5m**
4 Mark Webber (Formula One)	**$9m**
5 Chad Reed (motocross)	**$8.8m**

Source: *The Age*, 21 January 2010, Sport 9

1 **(a)** Draw up a table and include all of the named Australian sporting stars and estimate their positions in the Top 50 Earners list. Why is it difficult to be precise about some of the positions?

(b) What percentage drop in earnings did Harry Kewell suffer when moving clubs?

(c) What percentage slide did Lleyton Hewitt suffer in his earnings?

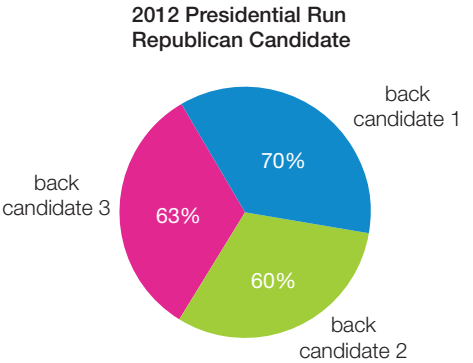
(d) How do you think the figure of $14m for Andrew Bogut was arrived at?

The *Herald Sun*, reporting on the same issue, noted that 'Samantha Stosur was the second highest earning tennis player, with $1.53 million'.

(e) Where would this place Samantha Stosur in your Top 50?

2 (a) In the early stages of the nomination process for the Republican Party leading up to the 2012 election for the US President, an American news network produced a graph similar to the one shown here.

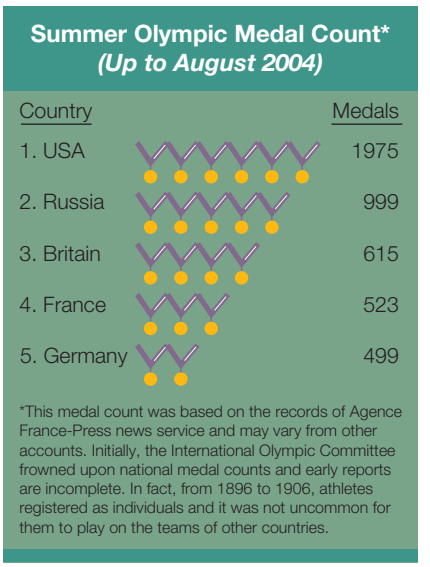
What problems can you see with this graph? Why do you think the graph has been drawn this way? Could you draw an accurate graph from the information provided?



(b) The graph on the right appeared in a magazine.

(i) What problems can you see with the graph?

(ii) Redraw the graph in a more accurate fashion.

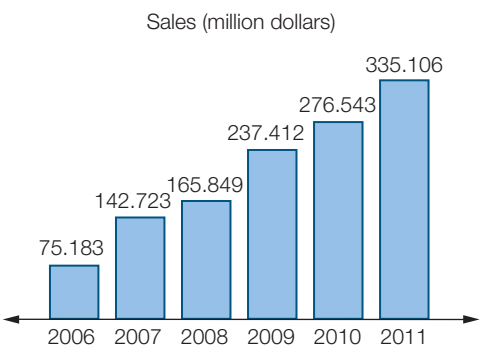


Source: http://www.forbes.com/sites/naomirobbins/2012/02/16/misleading-graphs-figures-not-drawn-to-scale/

(c) **(i)** What is the key reason the graph shown at right is misleading?

(ii) Why would you draw a graph like this?

(iii) Redraw it in a more accurate fashion.

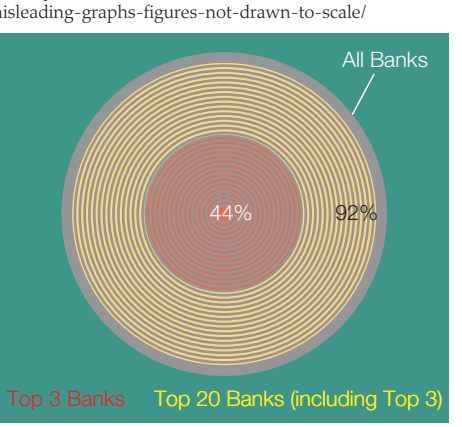


Source: http://www.forbes.com/sites/naomirobbins/2012/02/16/misleading-graphs-figures-not-drawn-to-scale/

(d) Once area is involved graphs can become misleading very quickly. Measure the radius of the red section and use this to calculate its area. Find the area of the yellow section. It includes the red section so is just an ordinary circle calculation. Find the area of the grey section. Again, this is just an ordinary circle that includes the other sections.

(i) What percentage of the full circle is represented by the red section?

(ii) What percentage of the full circle is represented by the yellow section?



Market share among US banks

Source: http://evalblog.com/2012/01/06/tragic-graphic-the-new-york-times-checks-facts-not-math/

(iii) Redraw this as a divided bar chart.

(iv) Why is it easier to draw this as a divided bar chart than in the form that was chosen?

3 The following article by John Rolfe appeared in the *Daily Telegraph* on 19 January 2010.

Why car insurers are ripping us off?

INSURERS have slugged Sydney drivers an extra $100 to make up for poor returns on investments—even though crash and theft rates have fallen dramatically.

Premiums have increased at up to 10 times the rate of inflation, figures obtained by the *Daily Telegraph* show. The price of the average greenslip rose 14% last year and comprehensive cover increased 5%. Yet Sydney's overall price inflation was just 1.3% during that time.

When premium increases are compared to crash and theft rates the differences are even more stark. The most recent accident statistics show a 6% annual fall, while car robberies declined 9%.

The *Daily Telegraph's* investigation exposes the cost of cover is no longer driven by risk, but by financial markets. Insurers make money by investing customers' premiums before that money is paid out in claims.

KPMG insurance group head Brian Greig said: "The returns they were able to generate pre-global financial crisis were substantially higher than they are now. When that (investment) lever is a little flat they look at other levers to generate acceptable returns, because sharemarket analysts are always on to them to achieve that."

The main "lever" is premium increases.

"Insurers are using premiums to compensate for lower investment returns," said Damian Smith, CEO of insurance comparison service RateCity.

Until the GFC, "premium growth wasn't that important", Mr Smith said. "But now insurers are using premiums to try to make up for the investments."

Data from the Motor Accidents Authority of NSW shows the average greenslip purchased during the three months to September 2009 cost $390, up $49 on a year earlier. Before the GFC took effect in 2007, compulsory third party insurance had fallen to $319, lower than 2004 levels.

"The increase seen over the last two reporting periods is directly related to the impact of the GFC," MAA spokesman Christian Fanker said.

Figures from Insurance Statistics Australia—the source the insurance industry uses—reveal comprehensive cover increased 4.9% last financial year. For a Sydney household driving a late-model family car, this would add more than $50 to premiums.

This means the total increase imposed on most drivers exceeds $100.

The number of crashes on NSW roads fell to 9692 per million cars in 2008—down 9.2% from 10 531 in 2007, RTA statistics show. There were 42 833 crashes in 2008. Official crash data for 2009 has not been made public.

(a) According to the article, how much did the average 'greenslip' cost in the 3-month period to September 2009?

(b) How much did the same cover cost a year earlier?

(c) What percentage change is this? Does this agree with what the article claims is the increase?

(d) If the cost had increased at the stated inflation rate, what should the premium have been?

(e) The article claims comprehensive premiums have increased by more than $50. Assuming it is exactly $50, what does this make the old and new costs for comprehensive cover?

(f) What is the average annual increase in greenslip cost from 2007 to 2009?

(g) Why does the article mention the number of crashes?

4 The following article by Beth Rivers appeared in *PerthNow* on 10 January 2010.

Perth's most prized suburbs

CANNING Vale was in demand in 2009, with more houses sold in that suburb than any other, but Guildford delivered the biggest median house price rise.

Landgate Valuation Services' figures show Canning Vale was way out in front with 506 houses sold, ahead of Thornlie with 340 and Ellenbrook with 330.

Real Estate Institute of WA president Alan Bourke said the result was to be expected because of the first homebuyer's boost stimulating outer-metropolitan suburbs. The best performing suburbs in terms of median price rises last year were Guildford, Mt Richon and Victoria Park.

From 2008 to 2009, Guildford saw a rise in its median house price of 17.26 per cent, from $565 000 to $662 500. The median in Mt Richon increased 16.86 per cent from $422 500 to $493 750, and Victoria Park's median increased 15.65 per cent, from $575 000 to $665 000.

The suburbs whose median prices were hit the hardest were North Beach, with its median price falling 35.38 per cent ($1.22 million to $790 000), Ascot, which fell by 32.14 per cent ($766 250 to $520 000), and Mt Lawley's median price fell by 28.28 per cent ($1.22 million to $875 000). Mr Bourke said North Beach was just one example of the many top-end suburbs "hurt" during 2009.

"There was more activity in the bottom end of the market so the median moved down (in the top end)", he said. "There was also a lot of land released across the road from the Ascot Racecourse, which would have pushed the median price down there."

Peppermint Grove retained its crown as being Perth's most expensive suburb, with a median house price of $4.2 million, while Medina, near Kwinana, was the least expensive suburb with a median of $230 000. Most recent REIWA data showed more than 31 500 properties were sold last year.

About 80% were houses and 20 per cent multi-residential, including apartments and units.

Best performing suburbs			
Median sale price		2008	2009
1	Guildford	$565 000	$662 500
2	Mount Richon	$422 500	$493 750
3	Victoria Park	$575 000	$665 000
4	Darlington	$515 000	$590 000
5	Orelia	$297 500	$339 000
6	Maylands	$670 000	$754 000
7	South Fremantle	$825 000	$925 000
8	Stoneville	$457 500	$511 250
9	Fremantle	$795 000	$880 000
10	Joondanna	$633 000	$697 500

Worst performing suburbs			
Median sale price		2008	2009
1	North Beach	$1 222 500	$790 000
2	Ascot	$766 250	$520 000
3	Mount Lawley	$1 220 000	$875 000
4	Mosman Park	$1 522 500	$1 150 000
5	Forrestdale	$520 000	$395 000
6	Applecross	$2 075 000	$1 602 500
7	Perth	$875 000	$690 000
8	Marmion	$885 000	$698 750
9	Kallaroo	$660 000	$545 000
10	Swanbourne	$1 600 000	$1 322 500

Suburbs by volume sold		2008
1	Canning Vale	506
2	Thornlie	340
3	Ellenbrook	330
4	Clarkson	277
5	Gosnells	275
6	Ballajura	256
7	Baldivis	238
8	Port Kennedy	235
9	Butler	224
10	Tapping	218

(a) Calculate the percentage increase for each of the ten best performing suburbs from 2008 to 2009.

(b) Calculate the percentage decrease for each of the ten worst performing suburbs from 2008 to 2009.

(c) Make some comment about the Perth housing market, based on these figures.

(d) What further information would make the 'Suburbs by volume sold' table more useful to us?

(e) According to Mr Bourke, 'There was more activity in the bottom end of the market so the median moved down (in the top end).' What do you think this means? Why does one affect the other?

(f) Why do these figures refer to the median house price? What would be the problems with using either the mean or the mode to represent house prices?

5 The following article by Michelle Griffin appeared in *The Sydney Morning Herald* on 17 May 2011.

Peer guide for parents

WHEN Australian teenagers tell their parents that everyone else is allowed to go to the shopping centre alone or stay up late, their parents will finally have a guide to what other families are really saying.

The average age at which teenagers are allowed to make their own decisions is tabled in a survey released today by the federally funded parenting advice website Raising Children. Most children can choose their own clothes by the time they are $12\frac{1}{2}$, but the average age for lifting a teenager's curfew is $17\frac{1}{2}$—six months off legal adulthood.

While it's not possible to tell parents exactly what age to allow teenagers to make decisions, the website's content director, parenting expert Warren Cann, hopes the survey gives parents a point of comparison. "If you find yourself well below or above the norms, you might want to think why," he says.

The survey asked parents what they believed was the right age for children to be allowed to make decisions. Mr Cann believes parents may yield to child pressure earlier. "Kids tend to think they're ready two years before their parents want them to be," he says. The survey suggests the average age for getting a mobile phone is 14, but telecommunications research says teenagers typically get their first mobile at 13. The survey of 360 parents found a strong resistance to allowing younger teenagers open access to websites and social media. The average age of allowing them open access is $15\frac{1}{2}$. On average, parents don't want their children on Facebook or other social media before $14\frac{1}{2}$, even though Facebook allows them to register from 13, and many children as young as eight have active pages.

Primary school teacher and mother of three Simone Traynor allowed her oldest son, Jacob, now 17, to use Facebook from 14. He is allowed to use it even when doing his homework, as long as he maintains good marks in VCE. Ms Traynor's 15-year-old son is now allowed to log on but she will not allow her 12-year-old daughter Sonja to register for an account. Ms Traynor said: "Brandon is not that interested. Sonja's got plenty of real-life social activity, and it seems more vulnerable [for her]: girls can be nasty with these things."

Mother of two Kristin has allowed her daughter, now 16, on Facebook since she was 12, but polices her children's internet time through a net nanny, by reading their internet histories and by setting time limits. "She has to spend an hour off all screens before bed—no phone, iPod, iPad, laptop, nothing. She can read a book or come and have a conversation."

The survey was commissioned to mark the launch of the Raising Children's teenage section, which was built to respond to a growing demand for advice on coping with adolescents.

(a) How many people took part in this survey?

(b) Do you think this is enough people to justify the media interest in the results?

(c) To participate in this survey you needed to visit the website and access the survey online. What type of survey is this?

(d) One of the questions in the survey was: 'At what age do you think adolescent boys/girls should be allowed to make their own decisions about the following issues?' A list of issues then followed. Do you think the results for the survey might have been different if the question was: 'At what age did you allow your adolescent boys/girls to make their own decisions about the following issues?'

(e) One of the findings of the survey was reported as follows.

'The average age at which teenagers were permitted to wield a mobile phone was 14. This is 12 months older than 13, at which the telcos say is the average age children start using a mobile phone.'

Why do you think there is such a difference in these reported ages?

(f) How much confidence would you have, in general, with the findings and conclusions of this survey?

Reasoning, communicating and open-ended

6 The following article by Anna Caldwell appeared in the *Courier Mail* on 2 January 2010.

Coast residents betting on Mermaid strip for lotto win

HOPEFUL Queenslanders hankering for a new-year windfall—or perhaps an upgrade to the homes and lifestyles of the rich and famous—have swarmed newsagencies ahead of tonight's $30 million Gold Lotto megadraw.

And at no place in the state is there a firmer belief that the next big win is just a ticket away than at Golden Sands News at Mermaid Beach on the Gold Coast.

The multi-purpose shop is wedged between two very lucky newsagents. About 1 km in one direction is Nobby Beach News, where a punter took out the $20 million Oz Lotto in August and about 1 km in the other direction is Lucky Mermaid News where a couple won $53 million on Oz Lotto in June.

Figures released by Golden Casket have crowned the glitter strip Queensland's number one lotto hot spot for first division prize wins for the second year running.

Sixteen lucky first division winners on the Gold Coast shared in a mammoth $106.37 million in 2009.

Darrell Hiscock, who owns Golden Sands News, said that everyone had been tipping his shop to offer up the next golden ticket.

Mathew Graham, from Nobby News, said people stopped by the suburb just because they believed that visiting the Mermaid strip enhanced their chances of winning millions.

(a) Why are people tipping Golden Sands News as the seller of the winning Gold Lotto ticket?

(b) In your opinion, is there a statistical basis for their confidence?

(c) What other information would you like to have access to before you made a statement like this about where a winning ticket is likely to be sold?

Other people look at lists of previously drawn numbers and use these as the basis for the numbers they select. Some people look at the numbers most frequently drawn, whereas others look at those that have not been drawn for the longest time.

(d) Try to explain the reasoning both of these groups use to justify their thinking. Do you support either argument yourself?

7 The following fictional article is of a type often seen around election time. Read it and answer the questions that follow.

Poll: Voters back Smith

Mr John Smith's return to the Federal Liberal leadership appears to have buoyed the Coalition's [Liberal and National parties] electoral chances, with one in four voters saying they are at least 'a bit more likely' to vote Liberal at the next federal election. Mr Smith was clearly preferred by female voters (47% support) over the leader of the Australian Labor Party, Mr Roger Casey (41%).

Men preferred Mr Casey (54%) to Mr Smith (45%). Support for the ALP among men was 47%, compared to the Coalition's (45%), while from women the Coalition attracted 47% support compared to the ALP's 44%. The survey also asked voters to identify those characteristics they believed Mr Casey or Mr Smith possessed. The results of the survey are summarised opposite.

SUNDAY BUGLE POLL

Voting intentions	%	Preferred PM	%
ALP	46	Casey	48
Coalition	46	Smith	46
Other/Don't know	8	Neither/Don't know	6

SUNDAY BUGLE POLL: RATING THE LEADERS

	Casey (%)	Smith (%)		Casey (%)	Smith (%)
Decisive	68	53	Sincere	22	55
Forward thinking	68	48	Too arrogant	75	15
Good for Australia	43	55	Honest	20	62
Weak	10	30	Capable	72	72
A good leader	66	43	None of the above	1	2

Source: Corio Market Research, 1030 voters from Vic., NSW. Poll done by telephone.

(a) Do the survey results agree with the heading 'Voters back Smith'? Discuss.

(b) Comment on the sample size—is it appropriate for a conclusion?

(c) The survey was done by telephone—is this the best way to do such a survey? Discuss.

(d) Why have the results for choice of preferred PM been separated into male and female opinions?

(e) Why don't the totals of male and female voters' opinions add up to 100%?

(f) Discuss the categories that describe Roger Casey and John Smith, such as decisive and forward thinking. Comment on the appropriateness of these categories. Can you add some of your own?

(g) Is this survey just about the preferred Prime Minister or is it about something else?

8 The Australian Bureau of Statistics releases more than just statistics. They also conduct some analysis of the data and produce reports such as the one below.

DEMOGRAPHIC, SOCIAL AND ECONOMIC CHARACTERISTICS: LIFE EXPECTANCY

This article is part of a comprehensive series released as 'The Health and Welfare of Australia's Aboriginal and Torres Strait Islander Peoples'.

KEY MESSAGES

At the national level for 2005-2007, the gap between Aboriginal and Torres Strait Islander and non-Indigenous life expectancy was 11.5 years for males and 9.7 years for females.

Life expectancy at birth for Aboriginal and Torres Strait Islander males is estimated to be 67.2 years, and 72.9 years for females.

This topic provides an overview of estimates of Aboriginal and Torres Strait Islander life expectancy. Data presented are from ABS experimental life tables for Aboriginal and Torres Strait Islander Australians and provide context for the detailed health and welfare information presented in other topics in this release.

LIFE EXPECTANCY

Life expectancy is a summary measure of mortality. It represents the estimated number of years a newborn baby would live if current mortality patterns remained constant throughout the baby's lifetime (see Box 1.6).

At the national level for 2005–2007:

- Life expectancy at birth for Aboriginal and Torres Strait Islander males is estimated to be 67.2 years, 11.5 years less than life expectancy at birth for non-Indigenous males (78.7 years).
- Life expectancy at birth for Aboriginal and Torres Strait Islander females is estimated to be 72.9 years, 9.7 years less than life expectancy at birth for non-Indigenous females (82.6 years).
- Life expectancy at birth differs across the states and territories, with the highest life expectancy estimates in New South Wales (69.9 years for males and 75.0 years for females) and lowest in the Northern Territory (61.5 years for males and 69.2 years for females). As shown in table 1.6, differences in life expectancy at birth estimates between non-Indigenous and Aboriginal and Torres Strait Islander Australians are greatest in the Northern Territory (14.2 years for males and 11.9 years for females) and Western Australia (14.0 years for males and 12.5 years for females), and lowest in New South Wales (8.8 years for males and 7.5 years for females).

1.6 LIFE EXPECTANCY AT BIRTH (a), by Indigenous status 2005–2007

	Life Expectancy at Birth			
	Indigenous years	Non-Indigenous years	Total (b) years	Difference Between Non-Indigenous and Indigenous Life Expectancy at Birth (c) years
		Males		
NSW	69.9	78.7	78.5	8.8
Qld	68.3	78.6	78.4	10.4
WA	65.0	79.0	78.7	14.0
NT	61.5	75.7	72.0	14.2
Aust (d)	**67.2**	**78.7**	**78.5**	**11.5**
		Females		
NSW	75	82.5	82.4	7.5
Qld	73.6	82.5	82.3	8.9
WA	70.4	82.9	82.5	12.5
NT	69.2	81.2	77.6	11.9
Aust (d)	**72.9**	**82.6**	**82.4**	**9.7**

(a) Due to significant changes in methodology, estimates of life expectancy at birth for 2005–2007 are not comparable to previously published estimates.

(b) Estimates of life expectancy at birth for the total population presented in this publication differ from estimates in ABS 2007, Deaths, Australia, 2006, cat. no. 3302.0.

(c) Differences are based on unrounded estimates.

(d) Includes all states and territories.

Source: ABS 2009, Experimental Life Tables for Aboriginal and Torres Strait Islander Australians, 2005–2007, cat. no. 3302.0.55.003, ABS, Canberra

(a) Why do you think only three states and the Northern Territory are recorded separately in the article above?

(b) According to the information in the article, what was the life expectancy for non-Indigenous males and females at this time?

(c) Why do you think there are such differences in life expectancy for Aboriginal and Torres Strait Islanders compared to non-Indigenous Australians?

Another government body, the Australian Institute of Health and Welfare, contributes the following additional information.

Australia appears to compare poorly with other countries for Indigenous life expectancy at birth. For example, Registered Indians (Aboriginal people) in Canada have an estimated life expectancy of about 6-8 years below that of the non-Aboriginal population (Tjemka, M. 2002. Statistics Canada, cat. no. 82-003). In New Zealand, Maori life expectancy is 8-9 years below that of the non-Maori population (www.stats.govt.nz). Although comparing these very different populations is perhaps over-simplifying the situation, it does illustrate that the Australian Indigenous life expectancy gap between non-Indigenous persons is approximately twice as large as in comparable developed nations.

Unfortunately, it is difficult to accurately assess the life expectancy gap between Indigenous and non-Indigenous persons. This is due to the fact that the population counts of Indigenous persons in Australia both over time and between jurisdictions is inaccurate, and that Indigenous persons are often undercounted when recording deaths.

(d) This extract mentions a couple of problems that arise when dealing with the issue of the life expectancy of the Australian Indigenous population. What are these problems?

(e) Why would you want to attempt to make comparisons with Canada, New Zealand and the United States?

The Australian Bureau of Statistics can provide further data to assist in our analysis.

In 2009, the median age at death of Aboriginal and Torres Strait Islander Australian males varied across the states and territories, from 48 years in South Australia to 57 years in New South Wales, compared with 67 years in the Northern Territory to 79 years in South Australia for non-Indigenous males. Similarly, the median age at death of Aboriginal and Torres Strait Islander Australian females was lower than for non-Indigenous females, 53 years in South Australia to 66 years in New South Wales compared with 72 years in Northern Territory to 85 years in South Australia.

3.6 MEDIAN AGE AT DEATH, Indigenous status (a) 2004–2009

	NSW	Qld	SA	WA	NT	Total (b)
	Males					
Indigenous						
2004	55.8	53.7	49.5	50.0	43.8	51.2
2005	54.3	51.1	42.4	52.8	45.8	50.4
2006	59.3	55.6	50.4	47.9	45.4	52.4
2007	58.1	54.7	50.5	53.3	45.9	53.1
2008	59.9	53.2	49.0	51.2	52.1	53.8
2009	57.2	53.2	48.0	50.6	48.3	52.5
Non-Indigenous						
2004	77.0	76.2	77.6	76.3	63.0	76.8
2005	77.2	76.4	77.9	76.6	63.7	76.9
2006	77.8	76.7	78.3	76.9	64.7	77.4
2007	78.1	77.1	78.7	76.9	64.6	77.7
2008	78.5	77.3	79.2	77.1	66.3	78.0
2009	78.4	77.2	79.3	77.3	66.6	78.0

	NSW	Qld	SA	WA	NT	Total (b)
	Females					
Indigenous						
2004	62.7	57.9	53.5	63.6	54.0	60.1
2005	65.8	59.5	47.5	57.8	50.4	57.9
2006	64.8	57.0	59.3	57.0	55.3	59.0
2007	63.0	59.5	58.3	59.3	55.7	59.5
2008	63.8	62.3	53.5	64.0	56.0	60.5
2009	65.9	62.6	53.0	58.7	55.4	61.3
Non-Indigenous						
2004	82.8	82.5	83.3	82.3	71.3	82.7
2005	83.1	82.6	83.7	83.2	70.5	83.1
2006	83.5	83.1	84.1	83.1	75.0	83.4
2007	83.7	83.3	84.3	83.4	69.3	83.6
2008	84.2	83.7	84.6	84.1	75.7	84.1
2009	84.1	83.4	84.6	83.6	71.8	83.9

(a) Care should be exercised when comparing median age at death of Indigenous and non-Indigenous Australians. See Commentary.

(b) Data are for NSW, Qld, WA, SA and NT combined, based on state or territory of usual residence. Victoria, Tasmania and the Australian Capital Territory are excluded due to small numbers of registered Indigenous deaths.

(f) Write a couple of sentences describing the conclusion you would draw from this additional information.

9 Write your own media release that includes statistics in the body of the article as well as in a table.

10 The following advertisement appeared in the Victorian press.



The Victorian Government developed this advertisement in 2010 with funds from the Community Support Fund.

Write a paragraph or two about this advertisement. You might like to include comment on the following questions or take a different angle altogether.

What do you think about this advertisement? What do you think about the statistics that are quoted in the advertisement? How would they have been calculated? Do you think this would be effective in reducing problem gambling?

Statistics from grouped data

7.3

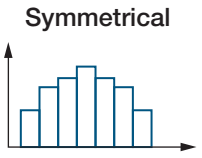
Stage 5.1

Need to Know

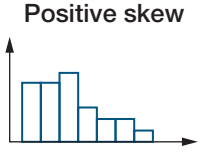
Symmetry and skew

A **symmetrical graph** has frequencies that are evenly distributed about the median or middle value, whereas a **skewed graph** has higher frequencies on one side of the median.

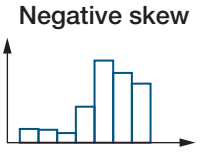
- If a graph is symmetrical, or close to symmetrical, then the mean, median and mode will be very close to each other.



- If most of the results fall on the left of the graph, we say it has **positive skew**.

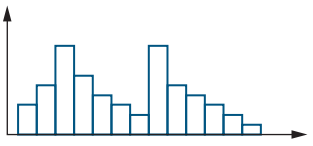


- If most results fall on the right of the graph, we say it has **negative skew**.

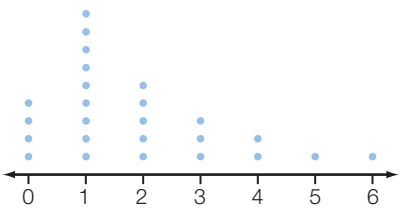


Data displays

If a data distribution has two modes it is said to be **bi-modal**. (The two values do not have to be exactly the same.)



A **dot plot** is a statistical display that uses a number line and a dot to represent each data value. If there are multiple occurrences of the same value, they are stacked on top of each other. The dot plot on the right represents the number of brothers and sisters for the students in a Year 9 class.



Stem plots (stem-and-leaf plots) are useful to determine whether data are symmetrical, skewed or bi-modal because they look like frequency graphs turned on their side.

Positive skew

STEM	LEAF
0	1 3 4 4 5
1	2 5 8 3 5 6
2	3 3 4 5 6 8
3	0 2 4 4
4	4 5
5	2
6	8

Symmetrical

STEM	LEAF
0	1
1	2 5 8
2	3 3 4 5 6 8
3	0 2 4 4 6 8 9
4	4 5 7 7 8 9
5	2 3 6
6	8

Negative skew

STEM	LEAF
0	1 2
1	4 8 9
2	1 1 4 6
3	0 2 6 7 7
4	4 4 5 6 7 8 9
5	0 1 3 4 5
6	4 5 5 5 6 7 8

Mean and median class interval

To find the mean of grouped data, carry out the following steps.

Step 1 Find the class centre or midpoint by adding the end points of the class interval and dividing by 2. We enter these values in a column labelled x.

Step 2 Multiply the midpoint value, x, by the frequency, f, and enter this product in a new column labelled fx.

Step 3 Total all the values in the fx column and write the sum as Σfx (sigma fx).

Step 4 We then divide Σfx by the sum of all the frequencies Σf. This will give us the mean.

$$\text{mean} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\Sigma fx}{\Sigma f}$$

The **median class interval** is the group, or bin, that contains the median value.

Grouping data means that individual values are lost but the interval that contains the median can still be identified.

Worked Example 3

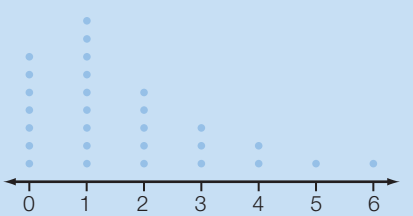
WE3

Use the following data displays to decide for each data set whether it is symmetrical, positively skewed, negatively skewed or bi-modal.

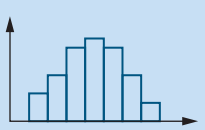
(a)

STEM	LEAF
2	1 1 3 4 6 9
3	3 5 7
4	2 3
5	0 3 6 7
6	3 4 6 8 9 9
7	1 2
8	4 5

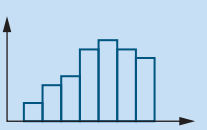
(b)



(c)



(d)



Working

(a) There are two peaks (or modes) for the data: bi-modal.

(b) There are more values pushed to the left: positively skewed.

(c) The graph is symmetrical.

(d) There are more values pushed to the right: negatively skewed.

Worked Example 4

WE4

A manufacturer has orders from 50 different retailers for a particular product each week. One week the number of orders were summarised as follows.

Number of orders	Frequency (f)
100–149	5
150–199	9
200–249	7
250–299	8
300–349	11
350–399	6
400–449	0
450–499	4

Use the grouped data to:

(a) calculate the mean number of orders. Give your answer correct to two decimal places.

(b) calculate the median class interval for the number of orders

(c) draw a bar chart of the orders

(d) describe the spread of the data.

Working

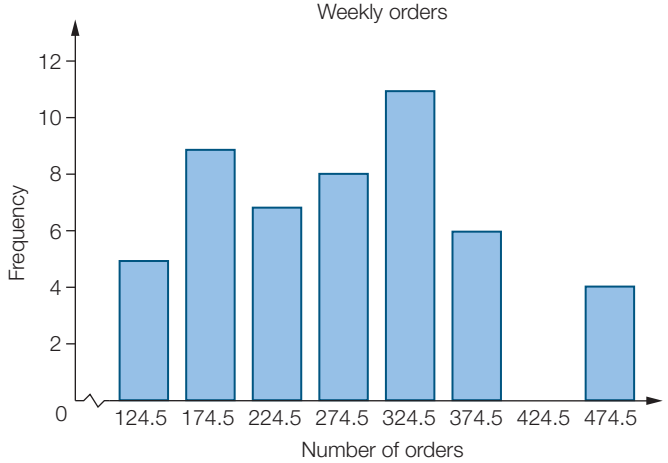
(a)

Number of orders	x	Frequency	fx
100–149	124.5	5	622.5
150–199	174.5	9	1570.5
200–249	224.5	7	1571.5
250–299	274.5	8	2196.0
300–349	324.5	11	3569.5
350–399	374.5	6	2247.0
400–449	424.5	0	0.0
450–499	474.5	4	1898
		$\Sigma f = 50$	$\Sigma fx = 13\,675$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{13\,675}{50}$$
$$= 273.50$$

(b) Both the 25th and 26th values are in the 250–299 class, so the median class interval is somewhere between 250 and 299 orders.

(c)



(d) The graph is not symmetrical. It is slightly negatively skewed. There do not appear to be any outliers.

Worked Example 5

WE5

A wool producer wants to keep some of his lambs for breeding. To do this, the lambs must reach a certain breeding weight. He decides to use statistics to help him plan a breeding program. The data in the frequency table gives the weight, in kg, of the lambs in his flock.

Weight (kg)	Frequency
25–<30	5
30–<35	12
35–<40	14
40–<45	16
45–<50	13
50–<55	10
55–<60	2

(a) Calculate the mean weight of the lambs. Give your answer correct to two decimal places.

(b) Calculate the median class interval for the weights of the lambs.

(c) Draw a histogram of the data and draw a frequency polygon over it.

(d) Describe the spread of the data in terms of its skewness.

Working

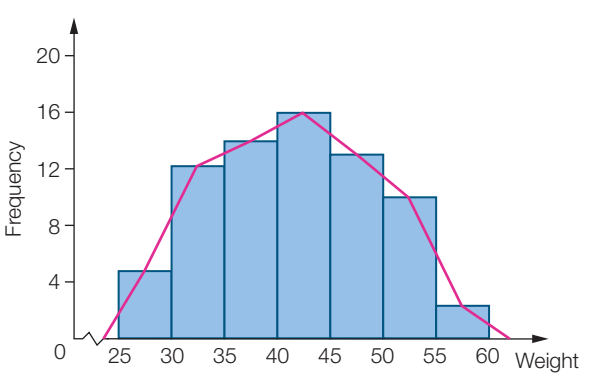
(a)

Weight (kg)	x	Frequency	fx
25–<30	27.5	5	137.5
30–<35	32.5	12	390
35–<40	37.5	14	525
40–<45	42.5	16	680
45–<50	47.5	13	617.5
50–<55	52.5	10	525
55–<60	57.5	2	115
		$\Sigma f = 72$	$\Sigma fx = 2990$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{2990}{72}$$
$$= 41.53 \text{ kg (correct to 2 d.p.)}$$

(b) Both the 36th and 37th values are in the 40–<45 class, so the median class interval is between 40 and 45 kg.

(c)



(d) The graph has a symmetrical shape.

7.3 Statistics from grouped data

Answers page 468

Navigator

Stage 5.1

Q1, Q2 (a), Q3 (a), Q4, Q5, Q6, Q7, Q8, Q13, Q14, Q15	Q1, Q2 (a), Q3 (a), Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q14, Q15	Q1, Q2 (b), Q3 (b), Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q14, Q15

Equipment required: CAS calculator or Excel spreadsheet may be used for Question **5**

Fluency and understanding

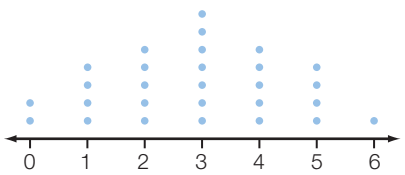
1 Use the following data displays to decide for each data set whether it is symmetrical, positively skewed, negatively skewed or bi-modal.

WE3

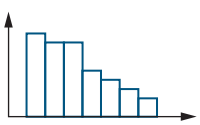
(a)

STEM	LEAF
2	1 4 9
3	3 5 7
4	2 3
5	0 3 6 7 7 9
6	3 4 6 8 9 9
7	1 2 3 5 6
8	4 5 5 6

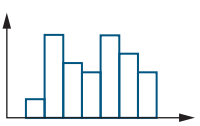
(b)



(c)



(d)



2 **(a)** A bakery records the number of sales of a new type of specialty bread over a 60-day period. The number of sales of the bread are summarised in the table.

WE4

Number of loaves	Frequency (f)
60–69	2
70–79	2
80–89	3
90–99	6
100–109	12
110–119	13
120–129	15
130–139	7

Use the grouped data to:

(i) calculate the mean number of sales (give your answer correct to two decimal places)

(ii) calculate the median class interval for the number of sales

(iii) draw a bar chart of the sales

(iv) describe the spread of the data.

(b) The number of cars in a council car park was recorded every hour over a 60-hour period starting from midnight. The data are summarised below.

Number of cars	Frequency (f)
0–49	9
50–99	8
100–149	17
150–199	5
200–249	13
250–299	8

Use the grouped data to:

(i) calculate the mean number of cars (give your answer correct to two decimal places)

(ii) calculate the median class interval for the number of cars

(iii) draw a bar chart of the cars

(iv) describe the spread of the data.

WE5

3 (a) A horse trainer has a large stable of horses. He needs to keep records of each horse's weight. The weights are entered into a frequency table so that the trainer can gather some statistics on the horses in his stable. The data in the frequency table give the weights, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	3
300–<350	13
350–<400	17
400–<450	20
450–<500	16
500–<550	14
550–<600	4

(i) Calculate the mean weight of the horses. Give your answer correct to the nearest kilogram.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Draw a histogram of the data and draw a frequency polygon over it.

(iv) Describe the spread of the data in terms of its skewness.

(b) Another horse trainer has a stable of horses. She also keeps records of each horse's weight. The weights are entered into a frequency table so the trainer can gather some statistics on the horses in her stable. The data in the frequency table give the weights, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	5
300–<350	11
350–<400	21
400–<450	27
450–<500	18
500–<550	11
550–<600	7

(i) Calculate the mean weight of the horses. Give your answer correct to the nearest kilogram.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Draw a histogram of the data and draw a frequency polygon over it.

(iv) Describe the spread of the data in terms of its skewness.

4 For each of the following stem plots, calculate the **(i)** mean, **(ii)** median and **(iii)** comment on the skewness of the data. Give your answers correct to two decimal places.

(a)

STEM	LEAF
4	8
5	2 3 4 5 6 7 7
6	4 5 5 6 7
7	3 4 5
8	2
9	1
10	9

(b)

STEM	LEAF
11	5 7
12	4 5
13	2 3 4 9
14	0 0 2 3 4
15	6 7 8 9 9 9
16	1 3 3 4 5 6
17	0 5 6 7 8

(c)

STEM	LEAF
21	0 2
22	1 8 8
23	0 1 2 3
24	1 2 5 6 7 9
25	1 2 3
26	1 1 3
27	1 2

5 If you have access to a CAS calculator or an Excel spreadsheet, use it to help you with this question; otherwise complete it using the normal methods.

The numbers that follow are the career points scored by the players on the list of the Brisbane Broncos at the start of the 2010 NRL season.

0	0	0	32	4	0	238	8	28	0
16	302	4	0	130	8	1163	114	16	8
88	8	0	0	582	0	16	44	0	0
60	62	82	0	0	56	36			

(a) Using grouped data, draw a histogram and use it to comment on the symmetry or skewness of the data.

(b) Do you think any of the values might be a possible outlier? If so, which ones?

(c) Using grouped data, calculate the mean and median class of the data.

Hint

To find the class centre, add the end points and divide by 2.

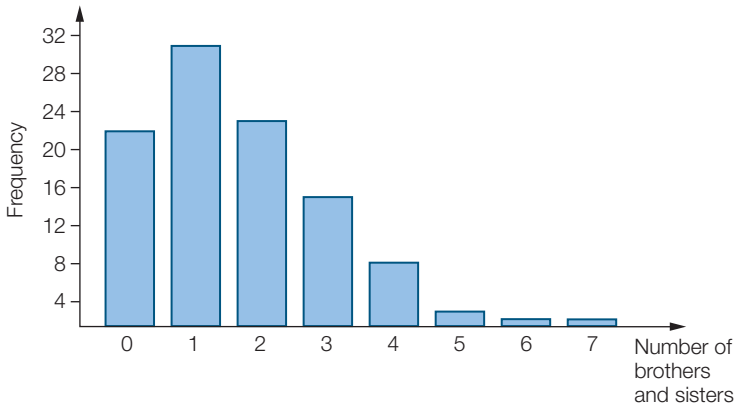
6 **(a)** The class centre for a class interval of 0–19 is:

A 9 **B** 9.5 **C** 10 **D** 10.5

(b) The class centre for a class interval of 10–20 is:

A 14.5 **B** 15 **C** 15.25 **D** 15.5

7 The graph on the right shows the number of brothers and sisters for each of the students in Year 9.



(a) Describe the data in terms of its skewness.

(b) Estimate the mean number of brothers and sisters.

(c) Now, calculate the mean number of brothers and sisters. How good was your estimate?

(d) Do you think this is a realistic spread of brothers and sisters? What could you do to check?

8 Find the class intervals for each of the following lists of class centres for discrete data.

(a) 5.5, 15.5, 25.5, 35.5, 45.5

(b) 2, 7, 12, 17, 22

(c) 24.5, 74.5, 124.5, 174.5, 224.5

(d) 25.5, 75.5, 125.5, 175.5, 225.5

(e) 11, 14, 17, 20, 23

(f) 27, 32, 37, 42, 47

9 This frequency table shows the time taken, in minutes, by the students at a particular school to complete a cross-country race.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

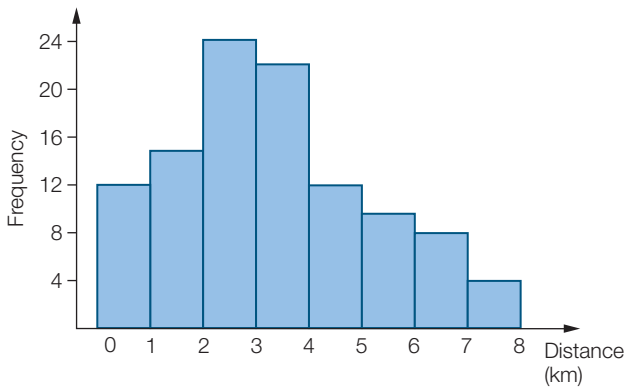
(a) Calculate the mean time taken by the students to complete the cross-country race.

(b) Draw a histogram and use it to describe the data set.

(c) Five students did not complete the race within the time limit. How would their results affect the mean time taken?

(d) How might you record their times in order to include them in the histogram?

10 The histogram on the right shows the distance travelled to school, measured in km, by each of the students in Year 9 at Westbow High School.



(a) Describe the graph in terms of symmetry or skewness.

(b) Make an estimate of the mean distance travelled to school.

(c) What factors would make the data distribution have this shape?

Reasoning, communicating and open-ended

11 This table shows the number of AFL games played by each of the 45 players on the list of the Sydney Swans at the beginning of the 2010 season.

Number of games	Frequency
0–39	22
40–79	7
80–119	7
120–159	3
160–199	2
200–239	3
240–279	1

(a) Calculate the mean number of games played.

(b) State the game interval that represents the median class.

(c) Which of these measures of location do you think does the better job of describing the number of games played?

(d) In fact, 13 of the players have yet to play a game. Recalculate the two measures of location, leaving out these players.

(e) Does this give a better representation of the measures of location for the number of games? Explain your reasoning.

12 **(a)** For discrete data, what can you tell about a class interval if the class centre is a whole number?

(b) For discrete data, what can you tell about a class interval if the class centre is a decimal number with a decimal part of .5?

(c) For discrete data, can a class centre ever be other than a whole number or a '.5' number? Explain your reasoning.

13 **(a)** Write two different lists of class intervals that have class centres 5 apart.

(b) Write two different lists of class intervals that have class centres 10 apart.

14 **(a)** Construct a stem plot with at least 20 values such that the median is bigger than the mean.

(b) Construct a dot plot with at least 20 values such that the median is smaller than the mean.

15 Draw a symmetrical frequency graph that has at least 20 values, using class intervals of 0–4, 5–9, etc. and then use it to construct a frequency table.

(a) Estimate the mean from the frequency table.

(b) Now make up the actual data values and calculate the mean.

(c) Comment on what you find.

Comparing data sets

7.4

Stage 5.1

Need to Know

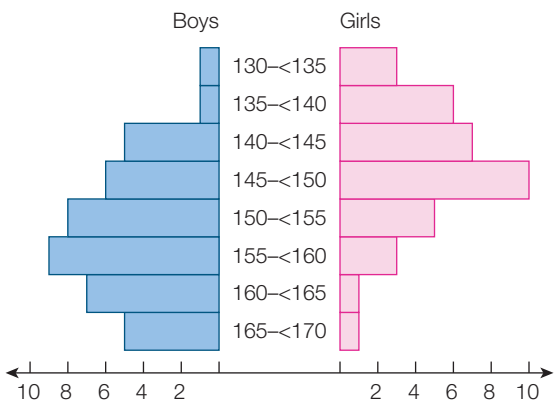
Back-to-back data displays

Data sets can be compared by drawing a **back-to-back stem plot** or a **back-to-back histogram**.

Back-to-back stem plot

Boys	Stem	Girls
4	13_L	4 4
9 8	13_H	6 7 9
3 3 2	14_L	0 1 1 3 4
9 8 6 6 5 5	14_H	5 5 5 7
3 2 2 0 0	15_L	0 1 4
8	15_H	5 6
4	16_L	3
6	16_H	

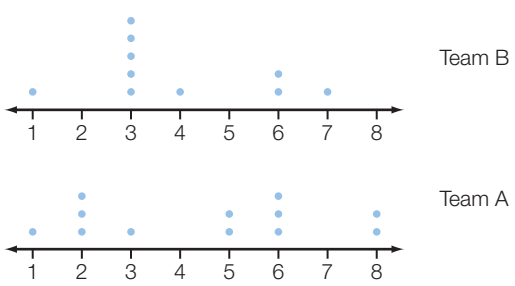
Back-to-back histogram



Other data displays

We can also compare data sets by looking at side-by-side column or bar graphs. This usually involves some interpretation of what is shown in the graph.

A **parallel dot plot** can also be used to compare data. It suffers in comparison to a back-to-back stem plot as it plots the individual data values and could, therefore, consist of many data values each represented only once or twice. A parallel dot plot is best employed when the range is small.



Worked Example 6

WE6

The weights (in kg) of the players on the 2010 playing lists for each of the two AFL teams based in South Australia are as follows.

Adelaide:	99	85	100	85	99	84	82	81	80	80	93	97	75
	82	90	86	84	102	84	87	82	83	99	76	87	74
	98	92	79	89	79	82	98	86	89	75	88	82	77
	92	86	87	80	87	84	81	80					
Port Adelaide:	84	82	85	90	87	80	80	80	83	83	80	98	79
	91	85	99	84	83	96	100	88	85	98	93	86	98
	95	78	99	80	89	87	98	92	90	84	95	82	94
	85	100	80	80	70	85	101	75					

(a) Find the range for the complete data set.

(b) Use the range to decide on a suitable class interval for a stem plot.

(c) Draw a back-to-back stem plot to display the data and make a statement about which team seems to be heavier.

(d) Calculate the mean (correct to 2 d.p.) and median weights for each team.

(e) Use these statistics to modify the statement you made in **(c)**.

Working

(a) $102 - 70 = 32$

(b) We will have a class interval of 5, with the first interval being 7_L; that is, 70–74.

(c)

Adelaide	Stem	Port Adelaide
4	7_L	0
9 9 7 6 5 5	7_H	5 8 9
4 4 4 4 3 2 2 2 2 2 1 1 0 0 0 0	8_L	0 0 0 0 0 0 0 2 2 3 3 3 4 4 4
9 9 8 7 7 7 7 6 6 6 5 5	8_H	5 5 5 5 5 6 7 7 8 9
3 2 2 0	9_L	0 0 1 2 3 4
9 9 9 8 8 7	9_H	5 5 6 8 8 8 8 9
2 0	10_L	0 0 1

It would seem that Port Adelaide is the heavier team because it has more players weighing 90 kg and over. It also has fewer players weighing less than 80 kg.

(d) Adelaide: Mean $= \frac{4047}{47} = 86.11$ kg Median = 85 kg

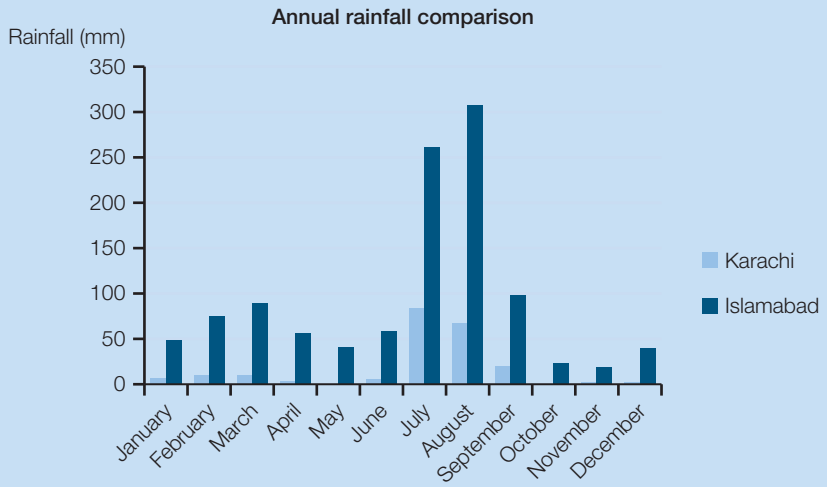
Port Adelaide: Mean $= \frac{4116}{47} = 87.57$ kg Median = 85 kg

(e) These statistics confirm the conclusion made in **(c)** that Port Adelaide is slightly heavier. However the difference is marginal.

Worked Example 7

WE7

Pakistan is quite diverse climatically. The average monthly rainfall, in mm, for two of its largest cities is shown below.



(a) Describe the average monthly rainfall for the two cities using just what you can see in the graph.

(b) What makes this a difficult task?

(c) What additional information would you like to have to make a more detailed analysis of the average monthly rainfall?

Working

(a) Islamabad is much wetter than Karachi, but both follow the same general pattern of having the most rain in July, August and September.

(b) The rainfall figures for Karachi are so low that it is difficult to see them sometimes.

(c) If we had the actual numbers, rather than just the graphical view, we would have a better chance of conducting a sensible statistical analysis.

Answers page 469

7.4 Comparing data sets

Stage 5.1

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q10	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11

Fluency and understanding

WE6

1 The heights (in cm) of the players on the 2010 playing lists for each of the two AFL teams based in West Australia are as follows.

Fremantle:	174	191	176	182	188	185	189	180	190	182	186
	181	186	189	190	192	194	193	186	193	194	188
	191	191	199	184	193	192	203	211	184	188	181
	180	194	177	187	182	184	191	191	192	189	194
	184	186									
West Coast Eagles:	195	183	178	189	188	179	188	201	186	185	186
	183	187	189	196	194	184	189	203	192	169	192
	191	186	187	185	186	198	195	196	189	189	195
	183	189	190	194	182	191	167	202	183	175	186
	189	181									

(a) Find the range for each team.

(b) Use the range to decide on a suitable class interval for a stem plot.

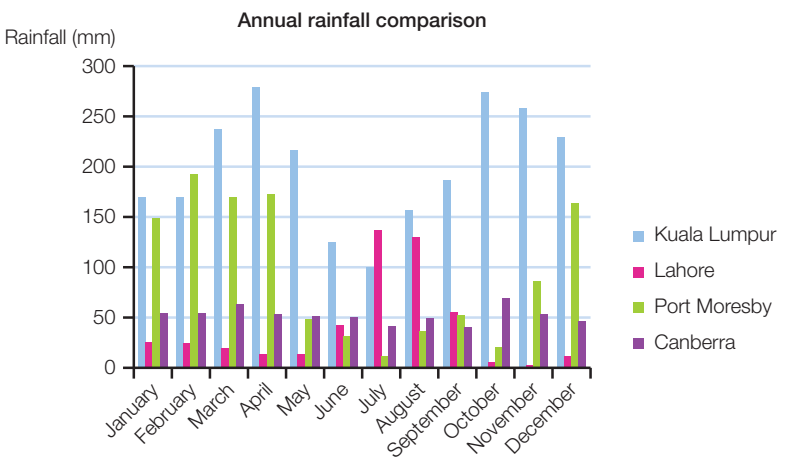
(c) Draw a back-to-back stem plot to display the data and make a statement about which team seems to be taller.

(d) Calculate the mean (correct to 2 d.p.) and median heights for each team.

(e) Use these statistics to modify the statement you made in **(c)**.

WE7

2 The following graph shows the annual monthly rainfall, in mm, for Kuala Lumpur (Malaysia), Lahore (Pakistan), Port Moresby (Papua New Guinea) and Canberra (Australia).



Although it is interesting to compare data such as these for different places around the world, this graph does not quite work.

(a) What would you consider to be the biggest problem associated with this graph?

(b) How could you overcome this problem?

(c) Estimate the monthly rainfall figures for Kuala Lumpur and Port Moresby and use these figures to draw a new comparative graph.

(d) Use your estimated figures to calculate the mean, median and range of the rainfall for each of these two cities. Then, use these figures and your graph to compare the rainfall in each of these two cities.

3 The US PGA is one of the major events in the world of golf. The table below shows a summary of the scoring in the first two rounds of the 2010 event.

Score	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
Frequency Day 1	0	1	4	6	10	22	17	21	25	13	10	6	8	4	3	1	2	1
Frequency Day 2	4	2	11	8	14	23	13	13	18	19	6	10	3	2	4	2	1	1

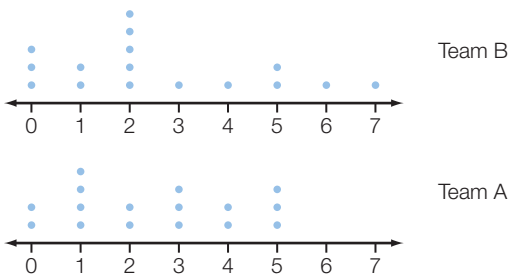
(a) Group the data in 2s, with the first group being 66–67, and draw a back-to-back histogram of the data.

(b) Why was it appropriate to group the data in the way done in **(a)**?

(c) Use the histogram to help you decide on which day scoring was easier. In golf, the lower the score the better it is.

(d) Use the raw data in the frequency table to calculate the mean and median scores for each day. Do these values confirm what the histogram shows?

4 The following parallel dot plot shows the number of tries scored in each game of the season by two rugby teams.

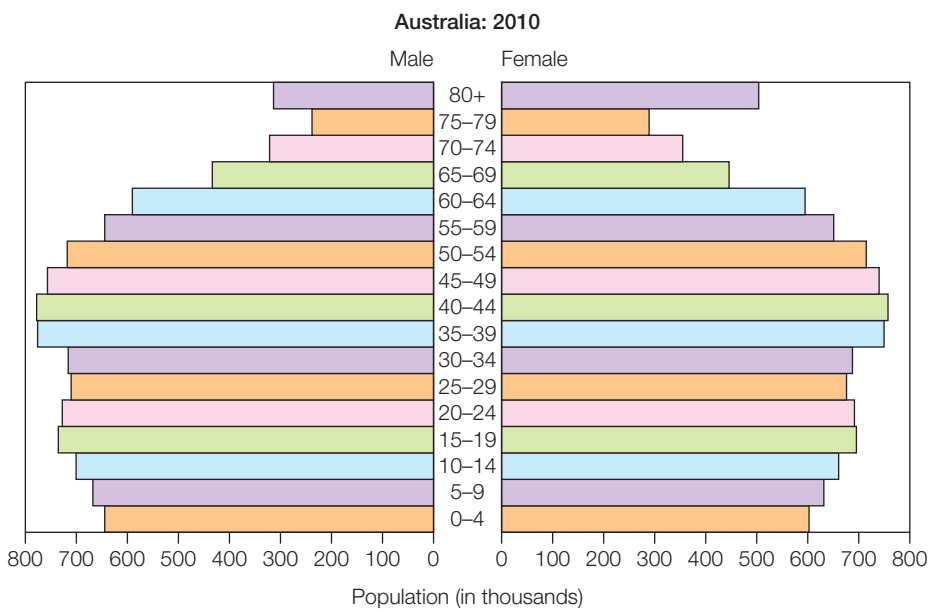


(a) Make a comment about what this graphs tells us about the two teams.

(b) Calculate the mean and the median for each team.

(c) Now make another comment comparing the two teams.

5



(a) For which age groups does it appear there are more males than females?

(b) Why are the 80+ bars longer than the 75–79 bars?

(c) How useful do you think this graph is in helping you compare male to female age distribution in Australia?

6 Are Year 9 girls taller or shorter than Year 9 boys? One way to investigate this is to sample some Year 9 students. However, the Australian Bureau of Statistics has organised this for you already through the Census at Schools program. One of the features of the program is that you can sample Australian students based on many different criteria.

From that website, the following data about a sample of 50 Year 9 NSW students in 2010 were obtained.

Male heights (cm):	176	181	154	177	191	190	180	175	164	166
	147	188	175	159	157	183	173	174	177	185
Female heights (cm):	145	162	160	167	153	155	162	178	163	167
	164	161	174	165	171	164	160	170	169	162
	174	167	154	153	168	144	165	160	168	172

(a) Draw a back-to-back stem plot of the data. Use a class interval of 5 (i.e. 16_L and 16_H).

(b) What conclusion would you draw from the stem plot drawn?

(c) Is it a concern that there are more girls than boys in the sample? Explain your reasoning here.

(d) Do you think another sample would produce the same results? Give some detail in your answer.

(e) Do you think a sample from another state or territory would give different results? What do you think would be reasonable ranges in which you would find the heights of Year 9 Australian students for males and females?

7 The average monthly rainfall, in mm, for four places in Australia is shown below.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	276.5	307.6	179.0	66.3	32.5	20.5	13.8	16.4	10.9	24.9	60.2	131.7
B	17.5	11.6	21.6	18.3	41.7	61.4	56.1	49.6	40.6	27.4	18.4	18.4
C	24.5	24.5	20.3	17.4	22.5	22.0	18.9	18.5	20.4	24.5	20.7	21.7
D	87.3	86.6	111.4	154.6	197.1	211.2	225.6	234.9	200.1	173.7	139.3	113.9

(a) Calculate the mean, median and range for each of these places.

(b) Describe the rainfall pattern for each place, including some mention of the seasons of the year.

(c) The four places, in no particular order, are Broken Hill (in outback NSW), Port Lincoln (a coastal town in SA), Weeaproinah (in southern Vic.) and Townsville (in tropical north Qld). You are now to decide which place is which. Include some justification for your choices.

Reasoning, communicating and open-ended

8 In Question **3** you looked at some data from the 2010 US PGA event. In golf events such as this, which are played over four rounds, approximately half the field is eliminated after the first two rounds and does not compete in the last two rounds. The following table shows the total score for each player for the first two rounds and the third and fourth rounds, if they were still competing.

Rounds 1 and 2:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	1	1	2	10	5	7	13	9	9	14	15	11	10	10
Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	7	5	5	4	2	4	5	0	1	2	0	1	1	

Rounds 3 and 4:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	0	2	3	4	6	2	5	5	4	9	8	9	2	4
Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	2	3	2	1	0	0	0	0	0	0	0	0	0	

(a) Given we prefer between 5 and 10 groups, explain why grouping in 5s, with a first group of, say, 136–140, would be the best way to group the data if we were to draw a back-to-back histogram of the data.

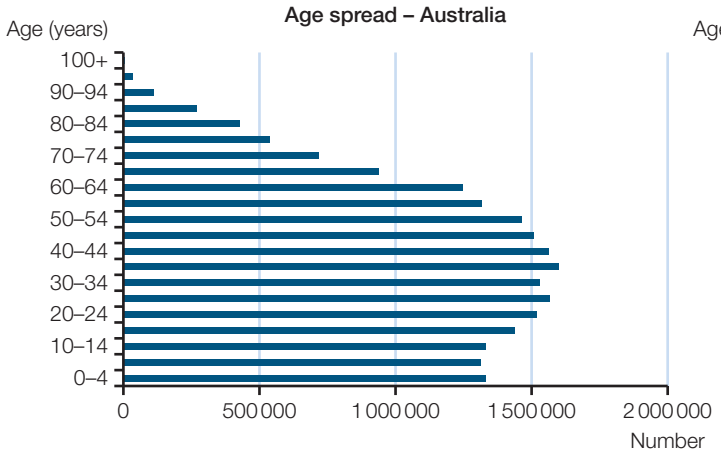
(b) What is the concern with this grouping for Rounds 3 and 4 data?

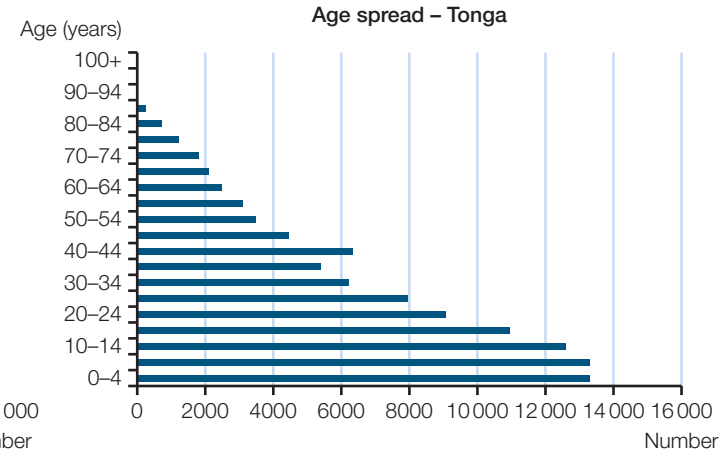
(c) Go ahead and draw the histogram using this grouping.

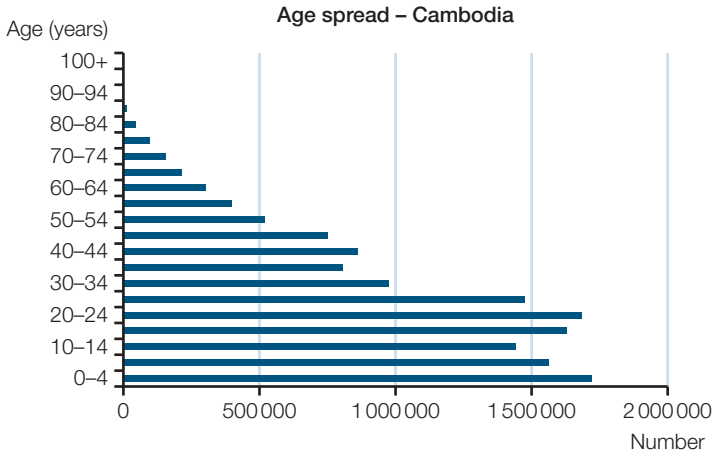
(d) Why do you think the two sides of the histogram seem so different?

(e) Find the mean and median for each data set and use these to add some further detail to your comments.

9 The following graphs show the age spread for people in three countries.







Compare the ages of people in the three countries using as much detail as you can.

10 Construct a back-to-back stem plot where the mean of the right-hand side is greater than the median and, on the left-hand side, the mean is less than the median. State what the data may represent.

11 The website of the Australian Bureau of Statistics has some great animated population pyramids of the Australian population that can be broken down into states or viewed in various ways to provide more information. Go to this website and investigate what you can do. Write a short report about what you discovered.

Half-time 7

Stage 5.1 Q1–5

Ex.7.3

1 For each of the following stem plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to two decimal places.

(a)

STEM	LEAF
3	0 1 9 9
4	4 5 6 7 9
5	3 8 9 9
6	4 5 6
7	7
8	0 1

(b)

STEM	LEAF
12	1 4 7
13	2 5 6 9
14	3 4 7 8 9
15	0 1 2 3 4
16	4 6 7 7
17	1 3 4

Ex.7.1

2 Choose the most precise data type for each of the following.

(a) The rating, on a scale of 1–5, given by members of the audience at a movie premiere.

A discrete **B** ordinal **C** categorical **D** qualitative

(b) The time taken by each of the competitors to complete the marathon at the 2012 Olympic Games.

A discrete **B** continuous **C** categorical **D** qualitative

(c) The number of buckets of apples picked by each of the pickers at an orchard during one day.

A discrete **B** continuous **C** categorical **D** ordinal

Ex.7.1

3 A local company thinks it needs to change its corporate image and so has hired a graphic artist to produce a new company logo. The graphic artist has come up with three possible logos. You are responsible for finding out which logo should be adopted by the company. Explain how you would go about reaching a decision.

Ex.7.4

4 The following table shows the heights of 40 students sampled from the Year 9 students at a particular school. The measurement has been taken to the nearest cm. The teacher knows that one group is boys and the other is girls, but has forgotten which is which. Draw a back-to-back stem plot and use this to help you decide which is which.

Group 1:	164	153	149	167	181	175	154	167	177	189
	155	166	180	170	173	155	156	183	177	160
Group 2:	172	166	155	161	172	155	151	158	167	171
	166	171	173	155	160	160	179	166	154	153

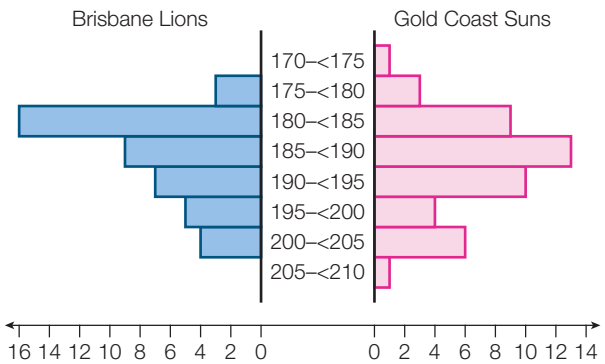
Ex.7.4

5 The following back-to-back histogram shows the heights of the players on the 2011 playing lists of the two Queensland-based AFL teams.

(a) Which team appears to be taller?

(b) What other information would you like to have to more fully answer this question?

(c) The mean and median heights for the Brisbane Lions are 188 cm and 186 cm, respectively, whereas for the Gold Coast Suns they are 189 cm and 188 cm, respectively. Does this information make it easier for you to compare the sides?



Understanding probability

7.5

Stage 5.1

Need to Know

Probability is a measure of the **chance** of an event occurring and ranges from **impossible** to **certain**. If the event is impossible, the probability is 0. If the event is certain, the probability is 1. Therefore, probability values range between 0 and 1.

$$0 \leq P(\text{event}) \leq 1 \quad \text{or} \quad 0\% \leq P(\text{event}) \leq 100\%$$

Complementary events

A probability event can consist of many successful outcomes. A list of the total possible outcomes, successful and unsuccessful, is called the **sample space**.

For example: Rolling a die has a sample space of 1, 2, 3, 4, 5, 6. The event of rolling an even number has three successful outcomes: 2, 4 and 6.

Complementary events are all of the successful and all of the unsuccessful events that comprise the sample space, so the probabilities of complementary events add to 1.

For example: Rolling an even number on a die and rolling an odd number are complementary events as together they comprise the entire sample space.

In general, $P(\text{A}) + P(\text{not A}) = 1$, where A and not A are complementary events.

This can also be expressed as $P(\text{not A}) = 1 - P(\text{A})$.

Probabilities can be determined in various ways.

Theoretical probability

The **theoretical probability** of an event is a calculation based on the number of successful outcomes that would occur under ideal circumstances.

For example: When tossing a fair coin, the theoretical probability is 0.5 as there is one successful outcome out of two possible outcomes.

Empirical probability

Empirical probabilities are probabilities determined from experiments or estimated from collected data.

$$P(\text{successful event}) = \frac{\text{number of successful equally likely outcomes}}{\text{total number of possible equally likely outcomes}}$$

Relative frequency can be used to estimate the probability of an event occurring. These estimates become more stable as the number of trials increases.

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Experimental probabilities are not expected to be exactly the same as theoretical probabilities, although they approach theoretical probability values as the number of trials increases.

For example: The theoretical probability for tossing a head on a fair coin is 0.5, but it is quite possible to get 8 heads in 10 tosses giving an experimental probability of 0.8.

Worked Example 8

WE 8

Narelle has been having some trouble with her car. Over the course of the past 14 days she has failed to get it to start on 8 days. What is the estimated probability that Narelle will be unable to start her car on the 15th day?

Working

Narelle has failed to start the car on 8 of 14 days.

$$P(\text{Narelle is unable to start the car}) = \frac{8}{14}$$
$$= \frac{4}{7}$$

Worked Example 9

WE 9

In a box containing 24 chocolates, there are 10 cream-centred and 8 hard-centred, and the rest are filled with nuts. One chocolate is chosen at random. Find the probability of choosing:

(a) a cream-centred chocolate

(b) a hard-centred chocolate

(c) a chocolate filled with nuts

(d) a chocolate that is not cream-centred

(e) a chocolate that is not filled with nuts.

Working

Let
C = choosing a cream-centred chocolate
H = choosing a hard-centred chocolate
N = choosing a chocolate filled with nuts

(a) $P(C) = \frac{10}{24}$
$= \frac{5}{12}$

(b) $P(H) = \frac{8}{24}$
$= \frac{1}{3}$

(c) $P(N) = \frac{6}{24}$
$= \frac{1}{4}$

(d) $P(\text{not } C) = 1 - P(C)$
$= 1 - \frac{5}{12}$
$= \frac{7}{12}$

or $P(\text{not } C) = \frac{8+6}{24}$
$= \frac{14}{24}$
$= \frac{7}{12}$

(e) $P(\text{not } N) = 1 - P(N)$
$= 1 - \frac{1}{4}$
$= \frac{3}{4}$

Worked Example 10 WE10

Five coins were flipped a different number of times each and the results recorded in the table below.

	Coin 1	Coin 2	Coin 3	Coin 4	Coin 5
Heads	21	14	16	54	109
Tails	17	19	20	49	117

(a) What was the relative frequency of heads for Coin 3?

(b) For which coin was the relative frequency of tails greatest?

Working

(a) Relative frequency of heads $= \frac{16}{36}$

$= \frac{4}{9}$

$= 0.\overline{4}$

(b)

	Coin 1	Coin 2	Coin 3	Coin 4	Coin 5
Tails	0.447	0.576	0.556	0.476	0.518

Coin 2 has the highest relative frequency of tails.

7.5 Understanding probability

Answers page 471

Navigator

Stage 5.1

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q11, Q12, Q15, Q16, Q18, Q22	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q20, Q22, Q23	Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23

Equipment required: coins, drawing pins, calculator, paperback novels for Question **17**

Fluency and understanding

1 Madelene and Pieter enjoy playing against each other in ten-pin bowling. Over the past few years Madelene has won 26 times and Pieter has won 28 times. Estimate the probability that Madelene will win their next game. WE8

2 To decide which lane he would swim in for the butterfly swimming event, Rohan drew a numbered token by random selection from a bag. The bag contained eight different tokens numbered from 1 to 8 to represent the eight lanes of the pool. Find the probability that Rohan: WE9

(a) swims in lane 1

(b) swims in lane 8

(c) does not swim in lane 8

(d) swims in an even-numbered lane

(e) swims in one of lanes 2, 3 or 4.

(f) swims in lane 2 or 5.

WE10

3 A fruit bowl contains one each of the following fruits: apple, banana, orange, apricot, pear, nectarine and peach. The following table gives the results for five separate experiments in which fruits were chosen by random selection from the bowl (and replaced after selection).

Fruit	Experiment 1	Experiment 2	Experiment 3	Experiment 4	Experiment 5
apple	5	4	3	5	4
banana	2	3	3	6	2
orange	6	6	3	2	7
apricot	7	8	4	7	8
pear	3	9	4	9	4
nectarine	2	10	4	2	6
peach	4	2	5	5	2

Answer the following, expressing all relative frequencies to two decimal places.

(a) For which experiment was the relative frequency of bananas greatest, and what was it?

(b) For which experiment was the relative frequency of bananas least, and what was it?

(c) For which experiment was the relative frequency of apples greatest, and what was it?

(d) For which experiment was the relative frequency of apples least, and what was it?

(e) Which fruit, in which experiment, had the lowest relative frequency, and what was it?

(f) Which fruit, in which experiment, had the highest relative frequency?

4 Your class has collected the following data from observations of the cars passing your school.

Make	Holden	Ford	Toyota	Mazda	Honda	Hyundai	Other
Frequency	35	27	31	15	7	10	42

Assuming that these figures are a reliable estimate of all cars on the road, find the probability, written as a decimal correct to two decimal places, that the next car observed is:

(a) a Ford **(b)** a Holden or a Mazda **(c)** not a Toyota.

5 Elliot, Kris and Austin are UNO fanatics. So far this year Elliot has won 45 games, Kris has won 38 games and Austin has won 67 games. Estimate an approximate value for the probability for each to win the next game.

6 (a) The probability of choosing by random selection a black jelly bean from the selection of black and pink jelly beans shown is:



A $\frac{1}{4}$ **B** $\frac{2}{5}$ **C** $\frac{3}{5}$ **D** $\frac{2}{3}$

(b) The probability of choosing by random selection a blue ball from a bag containing only six blue balls is:

A 0 **B** $\frac{1}{5}$ **C** $\frac{1}{6}$ **D** 1

(c) Aaron has passed eight out of the last 10 maths tests. Based on these data, the probability that he will *not* pass his next maths test is:

A 0 **B** $\frac{1}{5}$ **C** $\frac{4}{5}$ **D** 1

7 Five dice were rolled a different number of times each and the results recorded in the table below.

	Die 1	Die 2	Die 3	Die 4	Die 5
1	10	14	21	31	18
2	12	19	28	28	22
3	13	21	23	30	21
4	11	13	25	26	17
5	14	17	22	19	16
6	19	15	26	24	19

(a) For which die was the relative frequency of 1 the highest?

A die 1 **B** die 2 **C** die 4 **D** die 5

(b) For which die was the relative frequency of 6 the lowest?

A die 1 **B** die 2 **C** die 4 **D** die 5

(c) Put in order, from smallest to largest, the relative frequency for a 2 appearing by random selection.

(d) Put in order, from smallest to largest, the relative frequency for a 5 appearing by random selection.

8 Bill has beaten Dario in 15 of their last 25 badminton matches. The estimated probability that Dario will win the next match is:

A $\frac{1}{4}$ **B** $\frac{3}{8}$ **C** $\frac{2}{5}$ **D** $\frac{3}{5}$

9 A basketballer has scored the following points in the last 20 games.

22 15 19 27 16 31 40 29 19 26 14 22 33 41 44 32 31 20 24 30

Using these past results as a guide, estimate the following probabilities.

(a) The probability that the basketballer scores less than 20 points in the next game is:

A $\frac{1}{5}$ **B** $\frac{1}{4}$ **C** $\frac{3}{10}$ **D** $\frac{1}{3}$

(b) The probability that the basketballer scores more than 35 points in the next game is:

A $\frac{1}{10}$ **B** $\frac{3}{20}$ **C** $\frac{1}{5}$ **D** $\frac{1}{4}$

(c) The probability that the basketballer scores between 21 and 34 points in the next game is:

A $\frac{9}{20}$ **B** $\frac{11}{20}$ **C** $\frac{2}{3}$ **D** $\frac{9}{11}$

10 The maximum daily temperatures were recorded for the previous fortnight. They were:

23°C, 25°C, 19°C, 18°C, 20°C, 21°C, 21°C, 24°C, 29°C, 28°C, 24°C, 25°C, 20°C, 22°C

Using these as a guide, estimate the probability that the next maximum daily temperature is:

(a) 20°C **(b)** below 20°C **(c)** above 20°C

(d) below 30°C **(e)** above 30°C **(f)** above 25°C.

(g) Find the sum of your answers to **(a)**, **(b)** and **(c)**. Do you obtain the result you expect?

11 A die numbered 1–6 is biased so that $P(1) = P(2) = P(3) = \frac{1}{4}$ and $P(4) = P(5) = P(6)$.

(a) What is the fractional value for $P(4)$?

(b) Find P(even number).

(c) Find P(not a multiple of 3).

12 If the probability of choosing by random selection a red jelly bean from a packet is $\frac{4}{9}$, what is the probability of not selecting a red jelly bean?

13 A teacher compiled this table of absences from his class for first term.

No. of students absent	No. of days
0	10
1	15
2	12
3	9
4	3
5	1

(a) How many school days were there in Term 1? Assume for the remainder of this question that the figures in the table are an indication of what could be expected in Term 2.

(b) Based on the data collected, what is the estimated probability that on any particular day in Term 2 the number of absentees will be:

(i) zero students **(ii)** two students **(iii)** five students

(iv) less than four students **(v)** more than one student **(vi)** at least one student?

(c) Express each of the fractional answers from **(b)** as a decimal, correct to two decimal places if necessary.

(d) Express each of the fractional answers from **(b)** as a percentage, correct to two decimal places if necessary.

14 The table below gives the life expectancy for men at various ages.

Current age (years)	20	25	30	35	40	45	50
Further life expectancy (years)	52.1	47.4	42.8	38.2	33.8	29.6	25.5

Current age (years)	55	60	65	70	75	80	85
Further life expectancy (years)	21.7	18.2	15.0	12.1	9.6	7.5	5.7

(a) What is the expected age of death for a man currently:

(i) 20 years of age **(ii)** 30 years of age **(iii)** 40 years of age

(iv) 55 years of age **(v)** 65 years of age **(vi)** 75 years of age?

(b) What do you notice about the figures? Is the figure constant?

Half of all men of a particular age are expected to live to at least their current life expectancy figure. So, half of all men currently 25 years of age are expected to be alive at the age of 72.4. The further life expectancy for a 72-year-old is 11.0 years.

(c) What fraction of men currently 25 years of age would you expect to live to the age of 83 years?

(d) Who would be interested in these figures, and why?

15 Find:

(a) P(being born on 17 June). Ignore leap years.

(b) P(being born in June).

(c) P(being born on the 17th of any month).

(d) P(being born in a month containing exactly 30 days).

(e) P(not being born in February).

16 A fair six-sided die is rolled. Find the probability of obtaining:

(a) a 2

(b) a 7

(c) either a 1 or a 2

(d) an odd number

(e) a number larger than 2

(f) a number between 0 and 7.

17 For each of the following experiments, complete the following steps.

(i) Predict the number of times each of the outcomes will be obtained.

(ii) Conduct the experiment and record the outcomes.

(iii) Draw up a table of results.

(iv) Use the table to calculate the probabilities for each of the outcomes achieved in the experiment.

(v) Compare your predictions with the actual results.

(a) Toss two coins together 60 times. Record how many times the outcomes are two heads, or two tails, or one head one tail.

(b) Drop a drawing pin onto your page 100 times.

The two outcomes are point up:  and point down: 

(c) Use the random number key on your calculator. Push the button 100 times and note how many times each digit appears in the first place after the decimal point.

(d) Open to any page in a paperback novel. Take the first paragraph and record how often each letter occurs.

18 Chandra opens the batting for his local cricket club in the Under 14s. Over the season so far he has scored the following runs.

34, 21, 1, 14, 45, 23, 35, 0, 10, 56, 36, 12

Use this as a guide to estimate the following for his next score.

(a) P(score from 0 to 9)

(b) P(score from 10 to 19)

(c) P(score from 20 to 29)

(d) P(score from 30 to 39)

(e) P(score from 40 to 49)

(f) P(score from 50 to 59)

(g) P(score greater than or equal to 60)

(h) Add the probabilities in **(a)** to **(g)** and explain the meaning of your answer.

(i) Chandra is asked to play in the seniors, instead of the U14s, in his next match. Could we use these same figures to predict the likelihood of his score? Explain your answer.

Reasoning, communicating and open-ended

19 Over the whole basketball season Eloise scored with two-thirds of the free throws she had. In the Grand Final she is lining up for her third free throw. The opposition coach believes Eloise is sure to miss because she has scored with her first two. Explain why you think the coach is right or wrong about this.

20 To help choose his lotto numbers Sylvester records the frequency with which numbered balls are drawn out over a 2-year period.

(a) Offer two reasons why Sylvester might think this information is useful.

(b) In your opinion, is this information useful? Explain your opinion in a sentence or two.

21 A biased die, marked 1 to 6, is such that:

$P(6) = P(2) = P(1) = \frac{1}{12}$ $P(\text{not } 6) = 5 \times P(5) + P(1)$ $P(3) = P(4) + P(2)$

Find the probability of rolling each number.

22 You have an unbiased eight-sided die, with faces numbered 1 to 8. Write an event for each of the following probabilities.

(a) $P(A) = \frac{1}{4}$ **(b)** $P(B) = \frac{1}{2}$ **(c)** $P(C) = \frac{5}{8}$

23 In some board games, such as Monopoly, you need to throw a double 6 on a normal six-sided die before you can begin.

(a) Is there something special about double 6? Is it harder, or easier, to throw than, say, double 3?

(b) **(i)** Roll two dice and record the number of trials needed to get to each of the double results. Complete the table at right to help you.

You do not have to get 1, 1 before 2, 2. Just keep counting. For example, 2, 2 may occur as trial 12 and 5, 5 as trial 30, etc.

Event	Trial no.
1, 1	
2, 2	
3, 3	
4, 4	
5, 5	
6, 6	

(ii) Repeat this experiment 10 times and find the mean number of trials for each of the six doubles to occur.

(iii) Answer part **(a)** of this question again.

Probability events

7.6

Stage 5.1

Need to Know

Probability of events involving 'and', 'or' and 'not'

Theoretical probabilities can be obtained by counting the number of possible successful outcomes and dividing by the number of possible equally likely outcomes. When probability events involve 'and', both events must occur to be counted. When probability events involve 'or', either event can occur to be counted. When probability events involve 'not', neither event can occur to be counted.

Probability of events involving two attributes

A collection may have two or more different attributes. Collections with two attributes can be represented by two-way tables and Venn diagrams.

Two-way tables

A **two-way table** is an efficient way of representing two attributes.

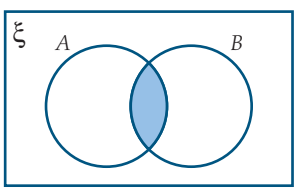
For example: Gender and handedness can be represented in a two-way table.

The table can be completed from information collected and then probabilities for 'and', 'or' and 'not' can be determined.

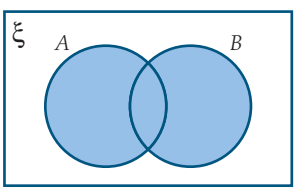
Venn diagrams

Venn diagrams can also be used to represent two attributes and shaded to show the three key situations of 'and', 'or' and 'not'.

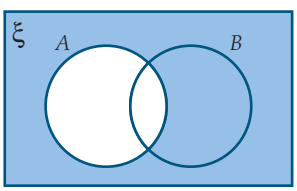
A and *B*



A or *B*

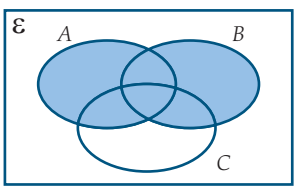


A' (not *A*)

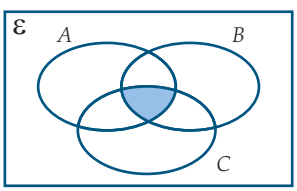


Venn diagrams can also be used to represent three attributes. When dealing with three attributes, there are too many different possible situations to show a diagram for each. A selection is shown below.

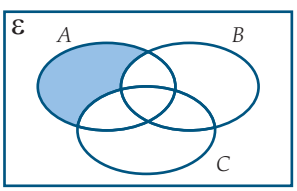
A or *B*



A and *B* and *C*



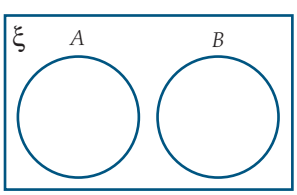
A and not (*B* or *C*)



Mutually exclusive events

Mutually exclusive events are not able to occur at the same time; if one event occurs, then the other cannot.

Mutually exclusive events can be represented by the Venn diagram shown. Note that there is no overlap.



Technology Exploration: Spreadsheet

A probability experiment

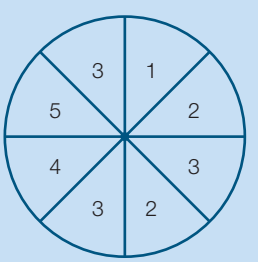
A spreadsheet offers an easily accessible way to see how the relative frequencies of outcomes stabilise as the number of trials rises.

1. Open a new spreadsheet and enter =RANDBETWEEN(1,6) in cell A1. Then click on cell A1 and put the cursor in the bottom right-hand corner so the cursor changes to a cross. Click and drag the corner down to, say, cell A24. You will now have 24 numbers in the range 1 to 6 displayed in your spreadsheet.
2. Enter the heading 'Number' in C1, 'Frequency' in D1 and 'Percentage' in E1. Enter the numbers 1 to 6 in cells C2 to C7. Then, in cell D2 enter =COUNTIF(A1:A360,1). Next, copy this formula into cells D3 to D7 but change the last number to 2, etc. to match the number in Column C. Then, in cell E2 enter =D2/D7*100 and copy it into cells E3 to E7.
3. You can now explore how the percentage frequencies change as you increase the number of trials. To do this, go back into column A and click and drag the last entry down another 50 or so cells each time.

Worked Example 11

WE11

The spinner shown in the diagram at right is spun once. Find the following probabilities.



(a) $P(2)$

(b) $P(4)$

(c) $P(2 \text{ or } 4)$

(d) $P(\text{even number})$

(e) $P(\text{not } 2)$

Working

(a) $P(2) = \frac{2}{8} = \frac{1}{4}$

(b) $P(4) = \frac{1}{8}$

(c) $P(2 \text{ or } 4) = \frac{3}{8}$

(d) $P(\text{even number}) = \frac{3}{8}$

(e) $P(\text{not } 2) = \frac{6}{8} = \frac{3}{4}$

Worked Example 12

WE12

A survey of some Year 9 students produced the following results.

	Only child	Not only child	
Left handed	10	21	31
Right handed	31	65	96
	41	86	127

Calculate the following:

(a) the number of left-handed students surveyed

(b) the number of only children surveyed

(c) P(left-handed only child)

(d) P(right-handed student who is not an only child).

Working

(a) 31

(b) 41

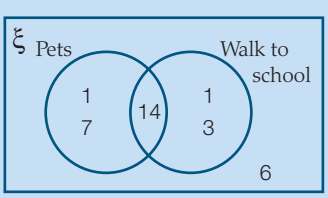
(c) $\frac{10}{127}$

(d) $\frac{65}{127}$

Worked Example 13

WE13

A survey of 50 Year 9 students was conducted and the number of responses was recorded in the Venn diagram shown.



Two questions were asked:

- Do you have a pet?
- Do you walk to school regularly (at least 4 times a week)?

Use the Venn diagram to anwer the following questions.

What is the probability that a student chosen at random:

(a) has a pet

(b) does not regularly walk to school

(c) has a pet and regularly walks to school

(d) has a pet or regularly walks to school?

Working

(a) $\frac{31}{50}$ or 62%

(b) $\frac{23}{50}$ or 46%

(c) $\frac{14}{50}$ or 28%

(d) $\frac{44}{50}$ or 88%

Answers page 472

7.6 Probability events

Stage 5.1

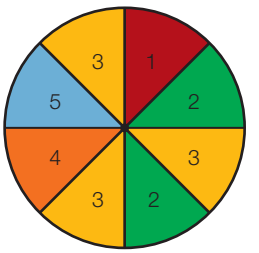
Navigator

Q1, Q2, Q3, Q6, Q7, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

WE11

1 The spinner shown at right is spun once.
Find the following probabilities.



(a) $P(3)$ **(b)** $P(5)$

(c) $P(3 \text{ or } 5)$ **(d)** $P(\text{odd number})$

(e) $P(5 \text{ or even number})$ **(f)** $P(5 \text{ or odd number})$

(g) $P(\text{not } 3 \text{ or } 4)$ **(h)** $P(\text{not even})$

WE12

2 A survey of some Year 9 students produced the following results.

	Only child	Not only child	
Left handed	9	12	21
Right handed	33	59	92
	42	71	113

Calculate the following:

(a) the number of left-handed students surveyed

(b) the number of only children surveyed

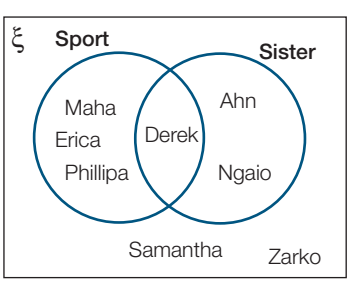
(c) $P(\text{left-handed only child})$

(d) $P(\text{right-handed student who is not an only child})$.

WE13

3 A survey of 50 Year 9 students was conducted and the number of responses was recorded.

- Who has a sister?
- Who regularly plays sport at the weekend?



Use the Venn diagram to answer the following questions.

What is the probability that a student chosen at random:

(a) has a sister

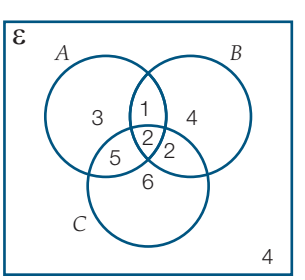
(b) does not regularly play sport at the weekend

(c) has a sister and regularly plays sport at the weekend

(d) has a sister or regularly plays sport at the weekend?

4 This Venn diagram shows the number of students who match combinations of three attributes. The attributes are:

A = 'travels to school by bus'
B = 'is an only child'
C = 'has a part-time job'



Use the diagram to calculate the probability that a student chosen at random:

(a) is represented in the diagram

(b) is the only child in their family

(c) travels to school by bus and has a part-time job

(d) travels to school by bus or has a part-time job

(e) is an only child and has a part-time job

(f) displays none of these three attributes.

5 A bowl contains 30 balls, numbered from 1 to 30. A ball is drawn from the bowl. Find:

(a) P(2 or 10)

(b) P(even or 7)

(c) P(odd or multiple of 8)

(d) P(multiple of 7 or multiple of 8)

(e) P(multiple of 4 or multiple of 9)

(f) P(4 or multiple of 5).

6 Which of the following pairs of events are not mutually exclusive?

A obtaining a head on the toss of a coin—obtaining a tail on the toss of a coin

B choosing an odd-numbered horse in a race—choosing horse number 7 in a race

C winning first prize in a raffle with a blue ticket—winning first prize in a raffle with a yellow ticket

D playing for England in the World Cup—playing for Brazil in the World Cup

7 A bowl contains 20 identical counters with one different number from 1 to 20 written on each. The following sets have been defined.

A = 'multiples of 4' B = 'multiples of 3' C = '1, 3, 5, 7, 9' D = '1, 2, 3, 5, 8, 13'

(a) State whether or not each of the following pairs of sets are mutually exclusive.

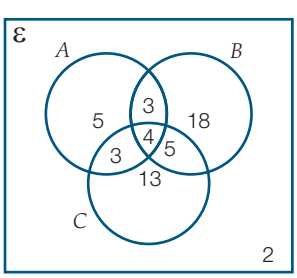
(i) A and B **(ii)** A and C **(iii)** A and D

(iv) B and C **(v)** B and D **(vi)** C and D

(b) Find the probability of each of A, B, C and D if a single counter is drawn from the bowl.

Reasoning, communicating and open-ended

8 This Venn diagram shows the number of students displaying combinations of three attributes. The attributes are:



A = 'left handed' B = 'play basketball' C = 'play guitar'

A student is chosen at random from those surveyed for this Venn diagram. Find the probability the student:

(a) is a left-handed guitarist

(b) is either left handed or a basketballer, but not both

(c) is not a guitarist

(d) is neither a guitarist nor left handed

(e) displays none of the three attributes

(f) displays at least one of the three attributes

(g) displays at most one of the three attributes

(h) is a right-handed guitarist who plays basketball.

9 Consider the letters of the word *left* and the letters of the word *right*. A bag contains 26 pieces of card, each with one different letter of the alphabet written on it. A letter is drawn at random from the bag. Find the following probabilities.

(a) The letter is in the word *left*.

(b) The letter is in the word *right*.

(c) The letter is in either word.

(d) Explain in two different ways how you can get the answer to part **(c)**.

(e) The letter is in both words.

(f) The letter is not in *left*.

(g) The letter is not in *right*.

(h) The letter is in neither word.

10 A survey of some members of the local sporting club produced the following results. However, some of the values are missing.

	Married	Not married	
Left handed	21	6	
Right handed	83		135
		58	162

Calculate the following:

(a) the number of left-handed members surveyed

(b) the number of right-handed members surveyed who are not married

(c) the number of married members surveyed

(d) P(right handed and married)

(e) P(left handed and not married)

(f) P(married).

11 Draw a Venn diagram with three overlapping circles and write some questions like those in Question **8**.

12 Draw a spinner such that $P(\text{blue}) > P(\text{green})$, $P(\text{green}) > P(\text{pink})$ and $P(\text{pink}) > P(\text{yellow})$.

Representing probability

7.7

Stage 5.2

Need to Know

Multiple events

Multiple events are events that can occur simultaneously, such as tossing a coin and rolling a die, or events that occur one after another, such as tossing a coin three times. Outcomes of multiple events can be shown in arrays or tree diagrams.

Arrays

An array can be used to list all the possible equally likely outcomes in the sample space. Probabilities can then be calculated.

For example: An array can be used to list the sample space for tossing two dice.

Tree diagrams

Another method of representing the sample space is to construct a **tree diagram**. This tree diagram can then be used to calculate probabilities for multiple events.

Using a tree diagram

- Draw the first part of the tree with the number of branches to match the number of outcomes for the first event.
- To each of the branches, add the number of branches to match the number of outcomes for the second event.
- List the sample space.
- Count the number of equally likely successful outcomes and the total number of outcomes.
- Find the probability using the formula:

$$P(\text{success}) = \frac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely possible outcomes}}.$$

- Simplify, if possible.

Sampling with and without replacement

Tree diagrams can be used for **sampling with replacement** and **sampling without replacement**. When replacement occurs, for subsequent events the probabilities remain the same.

For example, the probability of selecting an Ace from a pack of cards is $\frac{1}{13}$. If the card is replaced, the probability of selecting a second Ace remains $\frac{1}{13}$. However, if the Ace is not replaced, the probability of selecting a second Ace is now $\frac{3}{51}$, simplified to $\frac{1}{17}$.

Worked Example 14 WE14

Ann-Marie has rolled two dice and is interested in the sum of the two numbers showing uppermost.

(a) Draw an array to show all outcomes.

(b) List the sample space showing all possible outcomes.

(c) Using the sample space, find:

(i) P(total of 7) **(ii)** P(total is less than 6)

Working

(a)

Die No. 1

Die No. 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(b) There are 36 possible outcomes, giving the sample space: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

(c) **(i)** For a total of 7, there are 6 outcomes, therefore:

$$P(\text{total of 7}) = \frac{6}{36} = \frac{1}{6}$$

(ii) For a total less than 6, there are 10 outcomes, therefore:

$$P(\text{less than 6}) = \frac{10}{36} = \frac{5}{18}$$

Worked Example 15 WE15

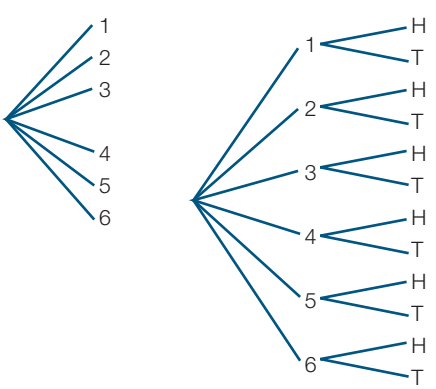
A die is thrown and then a coin is flipped.

(a) Use a tree diagram to list the sample space.

(b) Find P(an even number followed by a head).

Working

(a)



(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)

(b) 3 and 12

$$P(\text{even number and head}) = \frac{3}{12}$$

$$= \frac{1}{4}$$

Worked Example 16 WE16

A bowl contains three apples and two pears. Joshua wants two pieces of fruit and doesn't care what he has, so he randomly chooses two pieces of fruit from the bowl.

(a) Draw a tree diagram to show all the possible outcomes.

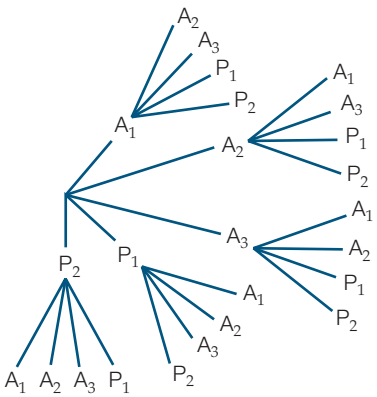
(b) Write out the sample space, making sure you distinguish between different pieces of the same fruit.

(c) Find the probability that Joshua chooses:

(i) two apples **(ii)** two pears **(iii)** two different pieces of fruit.

Working

(a)



(b) A1, A2, A3, P1 and P2 represent the fruit.

A1A2, A1A3, A1P1, A1P2, A2A1, A2A3, A2P1, A2P2, A3A1, A3A2, A3P1, A3P2, P1A1, P1A2, P1A3, P1P2, P2A1, P2A2, P2A3, P2P1

(c) (i) $\frac{6}{20} = \frac{3}{10}$

(ii) $\frac{2}{20} = \frac{1}{10}$

(iii) $\frac{12}{20} = \frac{6}{10} = \frac{3}{5}$

Worked Example 17 WE17

A card is selected at random from a shuffled pack of cards. It is then replaced and another card is selected.

(a) **(i)** Find the probability that the first card chosen is a King.

(ii) Find the probability that the second card chosen is also a King.

(b) Repeat part **(a)** without replacement to:

(i) find the probability that the first card chosen is a King

(ii) find the probability that the second card chosen is also a King.

Working

(a) (i) $\frac{4}{52} = \frac{1}{13}$

(ii) $\frac{4}{52} = \frac{1}{13}$

(b) (i) $\frac{4}{52} = \frac{1}{13}$

(ii) $\frac{3}{51} = \frac{1}{17}$

Answers page 472

7.7 Representing probability

Stage 5.2

Navigator

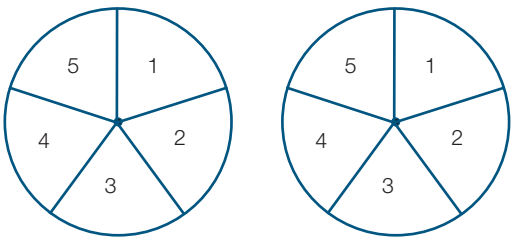
Q1, Q2, Q3, Q4, Q5, Q6, Q10, Q15, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q16

Fluency and understanding

For each of the following questions, it may be a good idea to first draw up a sample space using one of the techniques covered.

WE14

1 Con has two spinners as shown here. Draw a sample space to help you answer the questions. If Con spins them both, find the probability of getting:



(a) a total of 6

(b) a total of 10

(c) a total of less than 7

(d) a total not equal to 5.

WE15

2 A coin is flipped and then a die is rolled.

(a) Use a tree diagram to list the sample space.

(b) Find P(a tail followed by a multiple of 3).

WE16

3 A bowl contains four apples and two bananas. Joshua wants two pieces of fruit and doesn't care what he has, so he randomly chooses two pieces of fruit from the bowl.

(a) Write out the sample space, making sure you distinguish between different pieces of the same fruit.

(b) Find the probability that Joshua chooses:

(i) two apples **(ii)** two bananas **(iii)** two different pieces of fruit.

WE17

4 A card is selected at random from a shuffled pack of cards. It is then replaced and another card is selected.

(a) **(i)** Find the probability that the first card chosen is a Heart.

(ii) Find the probability that the second card chosen is also a Heart.

(b) Repeat part **(a)** without replacement to:

(i) find the probability that the first card chosen is a Heart

(ii) find the probability that the second card chosen is also a Heart.

5 A coin is flipped and then a six-sided die is rolled. By first drawing a tree diagram, find the probability of:

(a) getting a head followed by a 6

(b) getting a head followed by an even number

(c) getting a head followed by a number less than 5

(d) getting a tail followed by a multiple of 3

(e) not getting a head or a 5.

6 The menu at Shirley's local restaurant has four entrées: prawn cocktail, oysters, pâté and satay; and five main courses: lamb, fish, beef, pork and poultry.

(a) List all the possible different meals.

(b) Find, assuming random selection:

(i) P(pâté and pork)

(ii) P(oysters or fish).

(c) Shirley does not like satay or lamb. If she chooses a dish at random, what are the chances she has a meal she will enjoy?

7 Heath has four suits (black, blue, grey and brown) and five shirts (white, blue, yellow, pink and beige). Heath randomly chooses his clothes each day.

(a) The probability that Heath wears the grey suit is:

A $\frac{1}{4}$ **B** $\frac{1}{9}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(b) The probability that Heath wears the blue suit and the blue shirt is:

A $\frac{1}{20}$ **B** $\frac{7}{24}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(c) The probability that Heath wears the black suit or the yellow shirt is:

A $\frac{1}{60}$ **B** $\frac{1}{9}$ **C** $\frac{2}{9}$ **D** $\frac{2}{5}$

8 A coin is tossed three times.

(a) Draw a tree diagram to show all possible outcomes.

(b) Find the probability that there will be only one tail recorded.

(c) Find the probability that the second toss will record a head.

(d) Find the probability that the same outcome is recorded for all three tosses.

(e) Find the probability that at least one head is recorded.

9 Aldo is making his choice for elective classes for Year 10. He must choose one subject from each of these subject blocks.

Block 1	Block 2
French	German
Art	Metalwork
ICT	Bonus Maths
	Drama

(a) List all the possible subject combinations that Aldo can choose.

(b) Find the following, assuming the subjects are chosen randomly:

(i) P(Art and Drama)

(ii) P(chooses exactly one language)

(iii) P(not ICT)

(iv) P(at least one language)

(v) P(neither Art nor German).

(c) If he *must* do Bonus Maths, what is the probability that Aldo also does French?

10 Angela is on holiday at the Solaris Beach Resort. She needs to choose her activities for the next day. Her choices are shown in the following table. Assume she makes one choice from each session by random selection.

Session 1	Session 2
Wind surfing	Water skiing
Scuba diving	Wave boarding
Body boarding	Wave pool
	Surfing

(a) How many different combinations of two activities can Angela select?

(b) Find the following:

(i) P(wind surfing and wave boarding)

(ii) P(scuba diving and any of the Session 2 activities)

(iii) P(body boarding and not surfing).

(c) Angela decides that she will finish the day in the wave pool. In how many ways can she now plan her day?

Reasoning, communicating and open-ended

11 We know that if two events, A and B, are mutually exclusive, then $P(A) + P(B) = P(A \text{ or } B)$. The opposite of this is also true:
If $P(A) + P(B) \neq P(A \text{ or } B)$, then the events are not mutually exclusive.

Consider the following situation where a red die and a blue die are rolled at the same time.

Let A = the red die shows 3
B = the blue die shows 4
C = the sum of the two numbers showing is 6
D = the difference between the two numbers showing is 1

(a) Copy and complete the following diagram to show the sample space.

		Red die					
		1	2	3	4	5	6
Blue die	1	1, 1	2, 1				
	2	1, 2					
	3						
	4						
	5						
	6						

(b) Calculate:

(i) $P(A)$ **(ii)** $P(B)$ **(iii)** $P(C)$ **(iv)** $P(D)$

(c) Calculate:

(i) $P(A \text{ or } B)$ **(ii)** $P(A \text{ or } C)$ **(iii)** $P(A \text{ or } D)$

(iv) $P(B \text{ or } C)$ **(v)** $P(B \text{ or } D)$ **(vi)** $P(C \text{ or } D)$

(d) Are any of the pairs of events mutually exclusive?

12 A bowl contains two blue marbles, two green marbles and two yellow marbles. Two marbles are drawn from the bowl.

(a) If the marbles are not replaced after being drawn, find the probability that:

(i) each marble is blue

(ii) one marble is blue and the other yellow

(iii) each marble is a different colour

(iv) neither marble is green.

(b) Recalculate the same probabilities if the marbles are now replaced after being drawn.

13 **(a)** A coin is tossed. List the elements in the sample space.

(b) How many outcomes are possible?

(c) A 10c coin and a 20c coin are tossed. List the elements in the sample space.

(d) How many outcomes are possible?

(e) A 10c coin, a 20c coin and a 50c coin are tossed. Write out the sample space.

(f) How many events are possible?

(g) Is there a connection between the number of outcomes possible for each coin and the total number of outcomes possible for the combined events? Explain your answer.

14 A particular cube has two green faces, two blue faces and two red faces. It is rolled eight times and the face uppermost is recorded.

(a) How many elements are there in the sample space? (Hint: Don't try to write them out—apply the rule from Question **13**.)

(b) What is the probability that eight green faces in a row will occur?

(c) Why is **(b)** one of the easiest questions that could be asked about this event?

(d) What are the other equivalent questions with the same answer?

(e) Consider the question *'What is the probability that none of the faces will be green?'* This seems like an application of complementary probabilities to the event stated in **(b)**, but it is not quite as easy as that. Why not?

15 Construct a situation that has a sample space of at least 15 elements. Make up at least four questions to go with your sample space. Your teacher may get you to share your questions with other members of the class.

16 Make up some questions associated with a bowl of marbles. Make sure your questions include some with 'and', 'or' and 'not'.

Problem solving

Birthday cards

Two of your friends happen to have their birthdays close together. You decide to post them each a birthday card. You purchase the cards and write a nice message inside each card. You then address the two envelopes but get distracted when you are putting the cards in the envelopes. What is the probability that you put the cards in the correct envelopes?

Now consider the same question if there were three friends, so three cards and three envelopes. What is the probability that you get all the cards correct? What about getting just one of them correct? What about getting none of them correct? Can you get exactly two of them correct? Explain your answers.

Strategy options

- Make a table.
- Test all possible combinations.

Challenge 7



Stage 5.1 Q1–7

1 This is a set of scores: 18, 20, 20, 25, 37.

A whole-number score between 18 and 37 is added to the set of scores.

For the new set of scores, indicate whether each of the following statements is correct or incorrect.

(a) The mean of the scores could stay the same.

(b) The mean of the scores could decrease.

(c) The median of the scores could stay the same.

(d) The median of the scores could decrease.

2 **(a)** A set of five numbers has a range of 5 and a mean of 5. None of the numbers is 5. Write a set of possible numbers.

(b) Add a sixth number to your set in order to increase the mean to 6. (The range will change.)

3 A factor of the number 48 is chosen at random. What is the probability that the chosen factor does not itself have a factor of 3?

4 A prime number die has the numbers 2, 3, 5, 7, 11 and 13 on it. Two such dice are thrown. What is the probability that:

(a) the sum shown is even

(b) the number on one die is larger than the number on the other?

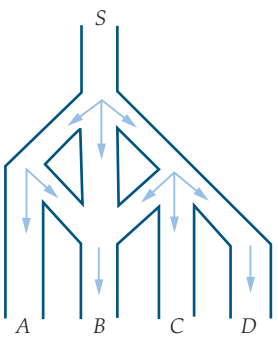
5 A barrel contains 18 marbles that are coloured white, black or green. If you draw one marble at random, then you are most likely to draw a black, and you are twice as likely to draw a white compared with a green. What could the contents of the barrel be? (There are three possible answers.)

6 It is estimated that 85% of students in Australia have internet access at home.

(a) Two students are selected at random. What is the probability that neither of them has internet access at home?

(b) Based on a recent survey, 60% of students who have internet access at home have also joined Facebook. A student is selected at random. What is the probability that the student has internet access at home but has not joined Facebook?

7 Rémy the mouse is put in a maze. He starts at Point S and can only move in the direction of the arrows; he cannot double back. At any junction he is equally likely to choose any of the forward paths.



(a) Show that the probability that Rémy ends up at Point A is $\frac{1}{6}$.

(b) Find the probability that Rémy ends up at Point B.

(c) Find the probability that Rémy ends up at Point C.

(d) Find the probability that Rémy ends up at Point D.

Chapter review 7

D.I.Y. Summary

Key Words

back-to-back histogram	impossible	qualitative data
back-to-back stem plot	measure of location	quantitative data
bi-modal	median class interval	random sample
categorical data	mutually exclusive	relative frequency
certain	negative skew	sample space
chance	observational data	secondary data
complementary events	ordinal data	skewed graph
continuous data	parallel dot plot	stem plot
discrete data	positive skew	symmetrical graph
dot plot	primary data	theoretical probability
empirical probabilities	probability	two-way table

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The _____ or _____ of an event lies between 0 and 1 inclusive.

2 If we collect data, such as recording the types of vehicles that pass the school, we are dealing with _____.

3 Data collected by another person or agency that you have accessed are called _____.

4 Non-numerical data that are simply divided into categories are called _____ .

5 The general name for any type of numerical data is _____.

6 Exact data values that are countable are called _____.

7 The mean and the median are the two most commonly used _____ .

8 If a data set has its values bunched to the right of the histogram, it is said to display _____.

9 An event with a probability of 1 is said to be _____.

10 We can use a _____ to assist us in finding the sample space.

11 Any data value that arises from measurement is going to be _____.

12 Events that cannot occur at the same time are said to be _____.

Fluency and understanding

1 State whether the following data sets are discrete or continuous. Ex 7.1

(a) the times taken by competitors to complete the Sydney–Hobart yacht race in a particular year

(b) the number of occupants in cars that go through a toll booth

2 A survey was conducted at the local rugby club concerning a possible change to the club jumper. The following table summarises the percentage support for each of the three new designs, as well as the current design. Ex 7.1

	Existing design	New design #1	New design #2	New design #3
Junior players	36	25	19	20
Junior parents	78	12	8	2
Senior players	46	31	18	5
Supporters	42	25	12	21
Sponsors	19	34	21	26

As the member of the club committee in charge of the survey, what recommendation would you make to the club? Give as much detail as you think is appropriate.

3 The following fictional article contains information about the opinions of voters on the suggested introduction of tolls on certain roads in the Greater Sydney area. Ex 7.2

Toll roads – poll reaction

The State Government is facing a big electoral backlash over its proposal for a system of linked toll roads in the Greater Sydney area, according to a *Sunday Bugle* poll conducted by BM Market Research Centre in marginal seats along the suggested route.

The poll showed at least half the voters surveyed would actively seek to avoid paying tolls by finding alternative routes.

Residents of the affected areas have increased worries about traffic on their local suburban roads growing massively if the plan is implemented. The poll shows that the Government could lose three of the four seats in which the survey was conducted.

SUNDAY BUGLE POLL: TOLL ROADS

How marginal electorates would vote . . .

	Government %	Opposition %
Smithfield	33	47
Fairfield	43	43
Auburn	38	38
Strathfield	46	32

. . . and who avoid tolls

Smithfield	59%
Fairfield	50%
Auburn	52%
Strathfield	49%

Source: Telephone poll of 1200 voters conducted by BM Market Research Centre

(a) Explain what is meant by the heading. Do the survey results support this 'big electoral backlash'?

(b) Add up the percentages for State Government and Opposition voters in the Smithfield electorate. Why don't they add up to 100%? Why is this important to the judgment of the poll?

(c) The article suggests the Government could lose three of the four electorates surveyed. Which of the four mentioned do you think is the one they would not lose?

(d) The poll was conducted by telephone. Do you think this would be the best way to conduct such a poll? Write a sentence or two outlining your views on this matter.

4 For each of the following stem plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to two decimal places.

Ex.7.3

(a)

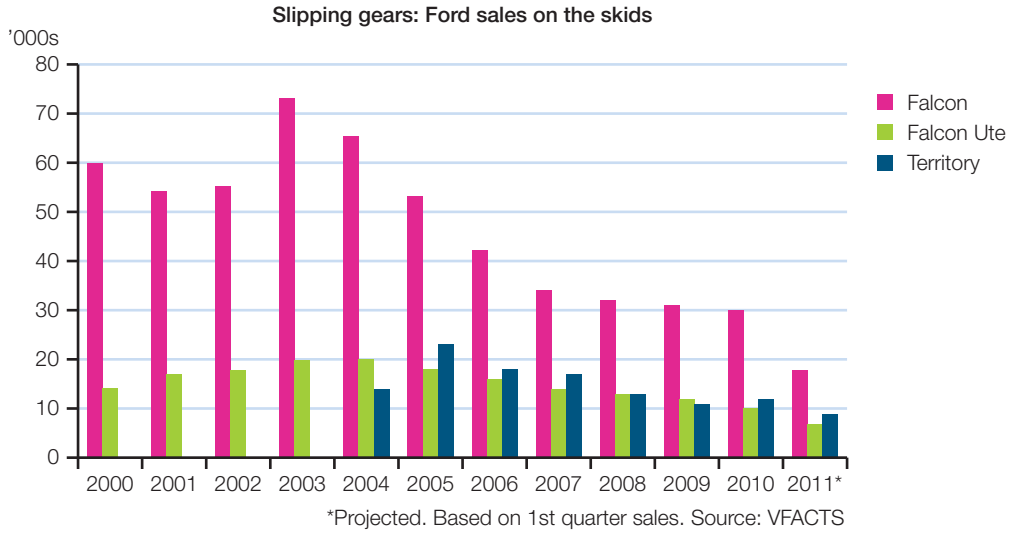
STEM	LEAF
4	0 1
5	4 5
6	2 4 5
7	4 5 6 7 8
8	1 3 5 5 6 6
9	0 0 2 4 5

(b)

STEM	LEAF
23	0 3 7 8 8
24	2 5 6 9
25	3 4 7 8 9
26	3
27	5 6
28	0

5 The following graph appeared in *The Age* (15 April 2011, p. 2).

Ex.7.4



(a) What has happened to the annual sales of the Falcon since the introduction of the Territory?

(b) Compare the trend in the sales of the Falcon with the sales of the Falcon Ute across the years shown in the graph.

(c) For 2004, estimate the sales of the three vehicle types shown in the graph.

(d) The number of Falcon Utes sold in 2004 was how many per cent of the number of Falcons sold in 2004?

(e) The number of Territorys sold in 2004 was how many per cent of Falcons sold in 2004?

(f) Repeat **(c)**–**(e)** for 2010.

(g) Make a comment about the percentages you have just calculated.

(h) What could you say about the total sales of these three Ford vehicles?

6 Last year I bought a packet of a particular brand of biscuits every week. Of these, 40 packets contained broken biscuits. Based on these previous results, what is the estimated probability that the next packet I buy will contain broken biscuits?

Ex.7.5

7 A jar contains five coloured blocks, one of each of the colours red, white, blue, green and gold. In an experiment in which 20 blocks were drawn from the jar, one at a time with replacement, the following results were obtained.

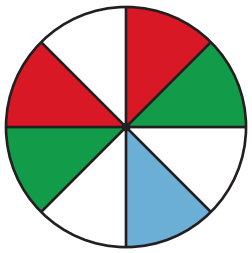
Ex.7.5

Colour	red	white	blue	green	gold
Frequency	4	3	1	7	5

(a) What is the relative frequency of drawing out a white block?

(b) What is the relative frequency of drawing out a gold block?

8 A spinner, coloured as shown on the right, is spun once. Find:



(a) P(white) **(b)** P(green or red)

(c) P(not white) **(d)** P(neither green nor red)

(e) P(white or green or red).

9 Two normal dice are rolled at the same time.

(a) How many elements are there in the sample space?

(b) Find the probability of the following events.

(i) P(total of 5) **(ii)** P(total less than 11)

(iii) P(at least one even number in the pair) **(iv)** P(no prime numbers in the pair)

10 An area 20 m × 20 m is pegged out in a nature reserve and the number of various species within the area is counted. The count includes three Coast Wattle, 12 Bidgee Widgee, seven Karkalla, four White Correa, six Running Postman and two Moonah. The nature reserve has a total area of 7.5 hectares. (Remember, 1 ha = 10 000 m^2.)

(a) If the control area is representative of the entire reserve, state the number of each species you would expect to find in the reserve.

(b) Do you think it is reasonable to make this assumption about the relationship between the control area and the entire reserve? What would you do to improve the reliability of your estimate?

11 Do Year 9 girls have longer or shorter arm spans than Year 9 boys?

The following data about a sample of 50 Year 9 NSW students in 2010 were obtained from the Census at Schools website.

Male arm span (cm):	166	174	163	178	198	145	180	175	176	171
	133	178	165	152	163	191	169	173	178	182
Female arm span (cm):	158	165	166	170	153	157	151	180	168	160
	170	165	155	153	171	164	159	170	169	164
	173	163	156	149	171	129	154	150	166	172

(a) Draw a back-to-back stem plot of the data. Use a class interval of 10.

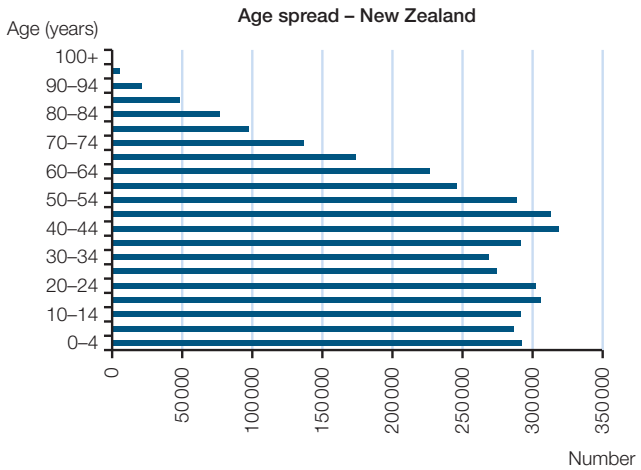
(b) What conclusion would you draw from the stem plot drawn?

(c) Is it a concern that there are more girls than boys in the sample?

(d) Does this mean there are more Year 9 girls than boys in NSW schools? Explain your reasoning here.

(e) How different would the results be if they were for Year 7 students or Year 11 students? Give some detail in your answer.

12 The following graph shows the age spread of the population of New Zealand.



Describe the age spread of the population of New Zealand using as much detail as you can.

Ex 7.6

Ex 7.7

13 The coach of the local junior soccer team has recorded the number of goals his team scores each week in the competition over a number of years.

Assume that the figures in the table are an indication of what can be expected from his team in the future.

Number of goals	Number of games
0	5
1	9
2	7
3	4
4	3
5	1

(a) What is the estimated probability that in the next game the number of goals scored will be:

(i) five goals

(ii) less than two goals

(iii) no more than four goals

(iv) either two or four goals?

(Give your answers in fractional form.)

(b) State each of the probabilities from **(a)** as a decimal, correct to two decimal places, and as a whole-number percentage.

14 A die is tossed twice. What is the probability of tossing:

(a) a pair of sixes

(b) any pair of identical numbers

(c) two even numbers?

Reasoning and communicating

15 Some newspapers pose a daily question and ask readers to respond either by telephone or by logging onto a website. The *Herald Sun*, published in Melbourne, recently posed the question 'Should youth detainees be barred from watching real life stories of crime?' The results were 87.8% Yes and 12.2% No. In smaller type, the actual numbers were displayed: 922 Yes and 128 No.

(a) Do you think you could have predicted the result for this question?

(b) Rephrase the question so it is not so pointed. Do you think the results would be the same if you asked this question?

(c) Do you think this sample size is big enough to draw any conclusions about the broader population?

(d) What would that population actually be—the readers of the *Herald Sun* or the residents of Melbourne, or Victoria, or Australia? Write a sentence or two giving your thoughts about this.

(e) What would you need to do if you wanted to improve the reliability of these results?

16 Two five-sided dice, one green and one blue, are rolled at the same time.

(a) Draw up a table to show the sample space possible.

(b) Find the following probabilities.

(i) P(total of 9)

(ii) P(total less than 8)

(iii) P(total no more than 5)

(iv) P(at least one prime number)

(v) P(difference between numbers less than 3)

(vi) P(at least one odd number)

(vii) P(both numbers even)

(viii) P(not a double)

(c) Are the events 'getting a 5 on the green die' and 'getting a 3 on the blue die' mutually exclusive? Explain your answer.

17 Every year *The Economist* publishes what it calls the Big Mac Index. This compares the cost of a Big Mac in various countries around the world using US$ as the comparative currency. This index can give the reader a simple view of the relative cost of (some) goods in these various countries. For 2010 the following figures were published.

Country	Big Mac price (US$)	Country	Big Mac price (US$)	Country	Big Mac price (US$)
United States	3.73	Hong Kong	1.90	Russia	2.33
Argentina	3.56	Hungary	3.33	Saudi Arabia	2.67
Australia	3.84	Indonesia	2.51	Singapore	3.08
Brazil	4.91	Israel	3.86	South Africa	2.45
Britain	3.48	Japan	3.67	Sri Lanka	2.82
Canada	4.00	Latvia	2.80	Sweden	1.86
Chile	3.34	Lithuania	2.71	Switzerland	6.56
China	1.95	Malaysia	2.19	South Korea	6.19
Colombia	4.39	Mexico	2.50	Taiwan	2.34
Costa Rica	3.83	New Zealand	3.59	Thailand	2.17
Czech Republic	3.43	Norway	7.20	Turkey	3.89
Denmark	4.90	Pakistan	2.46	UAE	2.99
Egypt	2.28	Peru	3.54	Ukraine	1.84
Estonia	2.62	Philippines	2.19	Uruguay	1.74
Euro area	4.33	Poland	2.60		

(a) Which country has the lowest price and what is that price?

(b) Which country has the highest price and what is it?

(c) What is the range of prices?

(d) You want to draw a histogram showing this data. Why is a price range of 50 cents appropriate in this case?

(e) Draw the histogram, using \$1.50–< \$2.00 as the first interval.

The mean of the data is 3.336 and the median is 3.205.

(f) Take a random sample of five prices and find the mean and median of those five prices.

(g) Take another random sample of five prices and find the mean and median of those five prices.

(h) Take two different random samples of 10 prices and find the mean and median for each group of 10.

(i) Which size sample would you expect to do a better job of representing the whole list? Did your sampling process confirm this?

NAPLAN practice 7

Numeracy: Non-calculator

1 The probability of rolling a 3 on a standard six-sided die is:

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{6}$ **D** $\frac{3}{4}$

2 For each of the following data sets, state whether it is discrete or continuous.

(a) the number of potatoes in each 5-kg bag at the supermarket

(b) the wrist circumference of each player on the list of the AFL team Greater Western Sydney

(c) the time taken by each competitor to complete a particular stage of the Tour de France

(d) the number of brothers and sisters for each of the students in your class

Numeracy: Calculator allowed

Questions **3** and **4** refer to the following frequency table.

Value	2	3	4	5	6	7	8	9
Frequency	4	5	2	1	8	5	7	2

3 The median of the data is:

A 4 **B** 5.5 **C** 6 **D** 8

4 The interquartile range of the data is:

A 3 **B** 5 **C** 6 **D** 8

5 The spellcheck on Bob's computer goes crazy and rearranges the letters in a word in a random order. If the letters in the word FRIEND are rearranged randomly, what is the probability that the new arrangement ends with either an i or an e?

A $\frac{1}{3}$ **B** $\frac{2}{5}$ **C** $\frac{2}{4}$ **D** $\frac{2}{3}$

6 Ros has a 70% chance of passing her English exam and an 80% chance of passing mathematics. The probability that Ros will pass both exams is:

A 1.5 **B** 0.56 **C** 0.44 **D** 0.06

7 This list shows the number of films that nine members of a film club watched in April.

Number of films watched	0, 1, 2, 2, 3, 4, 5, 5, 5

Which of the following is true for these data?

A mean > median = mode **B** mean < median < mode

C mean = median = mode **D** mean = median < mode

8 Non-linear relationships and proportion

Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

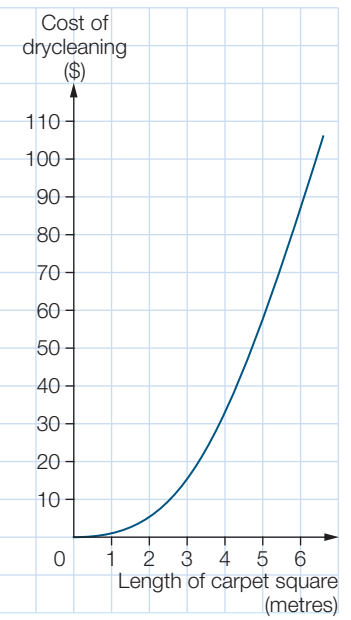
1 **(a)** If $p = 2m^2 - 3m + 5$, find the value of p when:

(i) $m = 1$ **(ii)** $m = 0$ **(iii)** $m = -1$

(b) Find the value of $x(x - 3)$ when:

(i) $x = 5$ **(ii)** $x = -3$ **(iii)** $x = 0$ **(iv)** $x = 3$

2 Consider this graph showing the cost charged to dryclean carpet squares of different lengths.



(a) Find the approximate cost to dryclean a carpet square with a length of:

(i) 3 m **(ii)** 5.5 m

(b) Find the approximate length of the carpet square if the drycleaning cost is:

(i) \$48.00 **(ii)** \$18.75

3 Expand the following.

(a) $3x(x + 4)$ **(b)** $(x + 2)(x + 5)$

(c) $(x - 7)(x + 3)$ **(d)** $(x - 5)(x + 5)$

4 Factorise the following.

(a) $x^2 + 2x$ **(b)** $x^2 - 16$ **(c)** $x^2 + 12x + 36$

5 Factorise the following.

(a) $x^2 + 2x + 5x + 10$ **(b)** $x^2 - 2x - 3x + 6$

(c) $x^2 - 14x + x - 14$ **(d)** $x^2 - 7x - 4x + 28$

6 Solve for x.

(a) $3x + 5 = 7$ **(b)** $5x - 2 = 11$ **(c)** $6x - 5 = -11$

Syllabus references

Number and Algebra: Non-linear Relationships

- ✓ Graph simple non-linear relations, with and without the use of digital technologies **(ACMNA296) (Stage 5.1)**
- ✓ Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technologies as appropriate **(ACMNA239) (Stage 5.1) (Stage 5.2◊)**
- ✓ Graph simple non-linear relationships, with and without the use of digital technologies, and solve simple related equations **(ACMNA296) (Stage 5.2◊)**
- ✓ Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations **(ACMNA267) (Stage 5.3§)**

Number and Algebra: Equations

- ✓ Solve simple quadratic equations using a range of strategies **(ACMNA241) (Stage 5.2)**

Number and Algebra: Ratios and Rates

- ✓ Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems **(ACMNA208) (Stage 5.2)**

Non-linear graphs

8.1

Stage 5.1

Need to Know

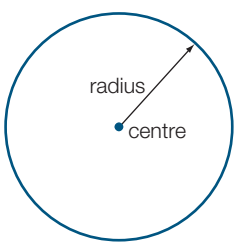
A non-linear graph is any graph connecting variables that do not graph as a straight line. Common non-linear graphs are **parabolas**, **circles** and **exponential graphs**.

Parabolas

A parabola is a U-shaped graph and can be drawn as ∪ or ∩. Quadratic equations graph as parabolas.

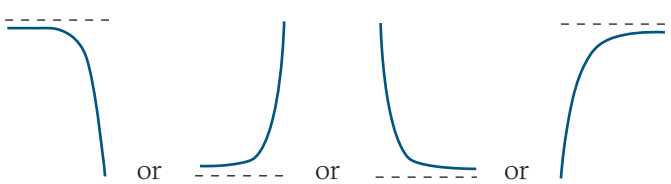
Circles

Circles map points that remain a fixed distance, called the radius, from a given point, the centre of the circle.



Exponential graphs

Exponential graphs are used to show rapid growth and decay and are drawn as shown below.



All exponential graphs approach lines that they do not touch or cross. These lines are called **asymptotes**.

Technology Exploration: GeoGebra

Non-linear graphs

Open GeoGebra. Use the Input bar at the bottom of the window to enter the equations to be graphed.



To see the graph clearly, select from the menu View > Layout… and then in the pop-up window go to Preferences - Layout > Basic. You can also adjust the preferences by right-clicking in the Graphics view. The tools Move Graphics View  , Zoom Out and Zoom In may also be used.

In the Input bar, use ^ to enter powers and * to enter multiplication.

1 Draw the graph of $y = x^2 + 1$.

2 Compare this graph to the one plotted in Worked Example 1.

3 Draw the graphs of:

(a) $y = 2x^2$ **(b)** $y = x^2 - 5$ **(c)** $y = -x^2$

4 Identify the type of non-linear relationship given by the equations in **1** and **3** above.

5 Draw the graph of $x^2 + y^2 = 25$.

6 Compare this graph to the one plotted in Worked Example 2.

7 Draw the graphs of:

(a) $x^2 + y^2 = 64$ **(b)** $x^2 + y^2 = 100$ **(c)** $x^2 + y^2 = 12$

8 Identify the type of non-linear relationship given by the equations in **5** and **7** above.

9 Draw the graph of $y = 2^x - 3$.

10 Compare this graph to the one plotted in Worked Example 3.

11 Draw the graphs of:

(a) $y = 2^x + 1$ **(b)** $y = 3^x - 4$ **(c)** $y = 1 - 2^x$

12 Identify the type of non-linear relationship given by the equations in **9** and **11** above.

Worked Example 1

WE1

(a) Copy and complete the following table of values for the equation $y = x^2 + 1$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y											

(b) Plot the graph of the equation given above using the table of values from part **(a)**.

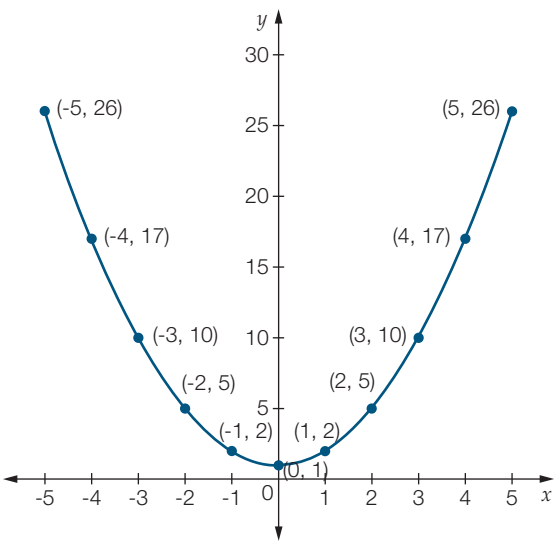
(c) Use the graph to identify the type of non-linear relationship given by the equation.

Working

(a)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	26	17	10	5	2	1	2	5	10	17	26

(b)



(c) Quadratic equation.

Worked Example 2

WE2

(a) Copy and complete the following table of values for the equation $x^2 + y^2 = 25$.

x	-5	0	5
y			

(b) Plot the graph of the equation given above using the table of values from part **(a)**.

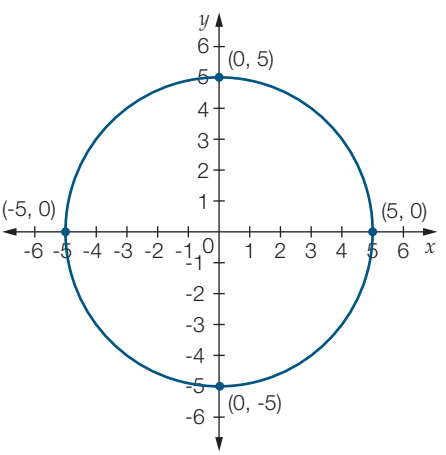
(c) Use the graph to identify the type of non-linear relationship given by the equation.

Working

(a)

x	-5	0	5
y	0	±5	0

(b)



(c) Circle equation.

Worked Example 3

WE3

(a) Copy and complete the following table of values for the equation $y = 2^x - 3$.

x	-2	-1	0	1	2	3	4	5
y								

(b) Plot the graph of the equation given above using the table of values in part **(a)**.

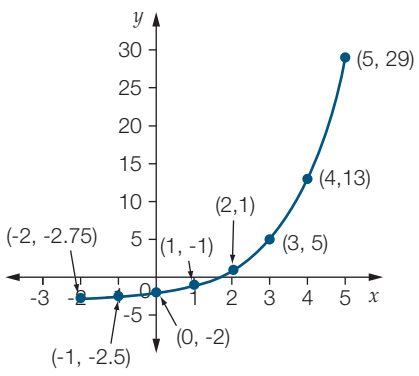
(c) Use the graph to identify the type of non-linear relationship given by the equation.

Working

(a)

x	-2	-1	0	1	2	3	4	5
y	-2.75	-2.5	-2	-1	1	5	13	29

(b)



(c) Exponential equation.

8.1 Non-linear graphs

Answers page 475

Navigator

Stage 5.1

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Fluency and understanding

1 **(a)** Copy and complete the table of values for each of the following equations.

WE1

(i) $y = 3x^2$ **(ii)** $y = x^2 - 4x + 3$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y											

(b) Plot the graph of each of the equations using the table of values in part **(a)**.

(c) Use the graph to identify the type of non-linear relationship given by the equation.

WE2

2 **(a)** Copy and complete the table of values for each of the following equations.

(i) $x^2 + y^2 = 36$

x	-6	0	6
y			

(ii) $x^2 + y^2 = 4$

x	-2	0	2
y			

(b) Plot the graph of each of the equations given above using the tables of values in part **(a)**.

(c) Use the graph to identify the type of non-linear relationship given by the equation.

WE3

3 **(a)** Copy and complete the table of values for each of the following equations.

(i) $y = 2^x$ **(ii)** $y = 3^x$

x	-2	-1	0	1	2	3	4	5
y								

(b) Plot the graph of each of the equations given above using the table of values in part **(a)**.

(c) Use the graph to identify the type of non-linear relationship given by the equation.

4 Plot the graph of the following circle relationships using an appropriate table of values.

(a) $x^2 + y^2 = 16$ **(b)** $x^2 + y^2 = 9$ **(c)** $x^2 + y^2 = 1$

5 Plot the graph of the following parabolas using an appropriate table of values.

(a) $y = x^2 + 6$ **(b)** $y = 3 - x^2$ **(c)** $y = x^2 - 9$

6 Plot the graph of the following exponential relationships using an appropriate table of values.

(a) $y = -2^x$ **(b)** $y = 3^x + 1$ **(c)** $y = 2^x - 1$

7 Which of the following equations would graph as a circle?

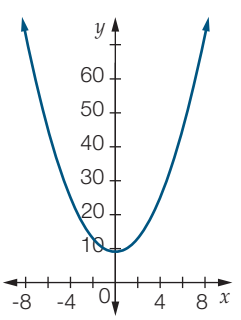
A $y - x^2 = 4$ **B** $y = 5^x - 9$ **C** $x^2 + y^2 = 81$ **D** $y = 5x^2 - 3$

Reasoning, communicating and open-ended

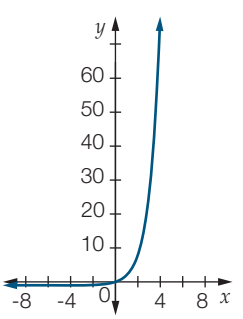
8 Match the following graphs with the equations.

(a) $x^2 + y^2 = 49$ **(b)** $y = 9 + x^2$ **(c)** $y = 3^x - 1$

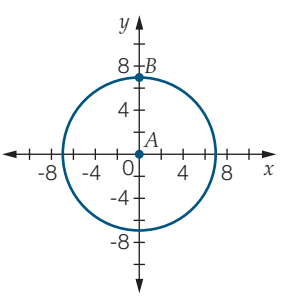
A



B



C



9 **(a)** Explain how you would recognise the equation of a circle relationship.

(b) Explain how you would recognise the equation of an exponential relationship.

(c) Explain how you would recognise the equation of a parabola.

10 Write an equation that would graph as a:

(a) circle **(b)** exponential **(c)** parabola

Quadratic relationships

8.2

Stage 5.2◊

Need to Know

Quadratic relationships

A **quadratic relationship** is a very common type of non-linear relationship. The graph of a quadratic relationship is a special shape called a parabola.

The general **quadratic equation** is given by $y = ax^2 + bx + c$, where $a \neq 0$.

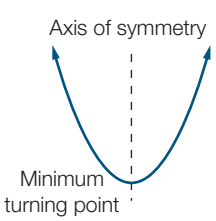
All quadratic equations have a term where the highest power of the independent variable is 2, the dependent variable has a highest power of 1, and all powers are integers.

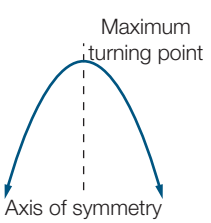
$y = x^2 + 3x - 4$ and $y = -2x^2 + 9$ are examples of quadratic equations where x is the independent variable and y is the dependent variable.

Axis of symmetry and turning point

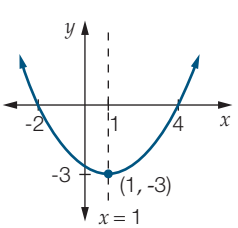
A parabola is the graph of a quadratic relationship. Each parabola has an **axis of symmetry** and a **turning point**. An axis of symmetry is an imaginary vertical line that divides the parabola into two mirror-image or symmetrical halves passing through the turning point. The turning point or **vertex** is where the graph changes direction.

A minimum turning point occurs where the graph changes direction at the lowest point (the lowest y-value) of the graph. A maximum turning point occurs where the graph changes direction at the highest point (the highest y-value) of the graph.





The parabola shown on the right has a minimum turning point at (1, -3) and an axis of symmetry at $x = 1$.



Graphing quadratic equations

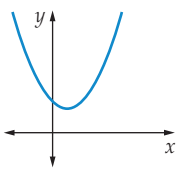
1. Complete a table of values.
2. Draw a set of appropriately sized and scaled axes.
3. Plot the points from the table.
4. Join the points with a smooth curve.
5. Label the graph, the turning point and the axis of symmetry.

Features of the parabola:

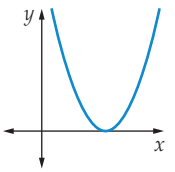
- Every parabola is symmetrical about the axis of symmetry.
- Every parabola has a turning point.
- A parabola can have a minimum turning point or a maximum turning point.

x- and *y*-intercepts

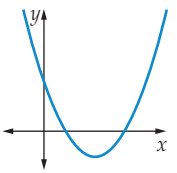
- All parabolas have a y-intercept.
- A parabola can have zero, one or two x-intercepts.



No x-intercepts



1 x-intercept



2 x-intercepts

Worked Example 4

WE4

State which of the following are quadratic relationships.

(a) $g = 8f + 6f^2$ **(b)** $p^2 = r - 3r^2$ **(c)** $d = 4b^3 - b^2 + 5$

Working

(a) The highest power of f is 2 and the highest power of g is 1. This is a quadratic relationship.

(b) The highest power of r is 2 and the highest power of p is 2. This is not a quadratic relationship.

(c) The highest power of b is 3 and the highest power of d is 1. This is not a quadratic relationship.

Worked Example 5

WE5

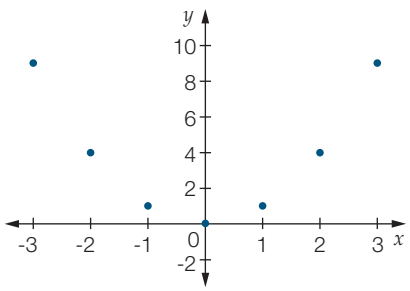
Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

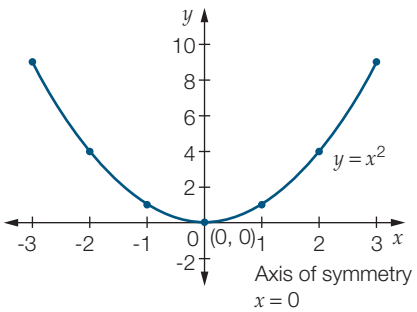
(a) $y = x^2$ **(b)** $y = 2x^2$ **(c)** $y = -x^2$

Working

(a) $y = x^2$

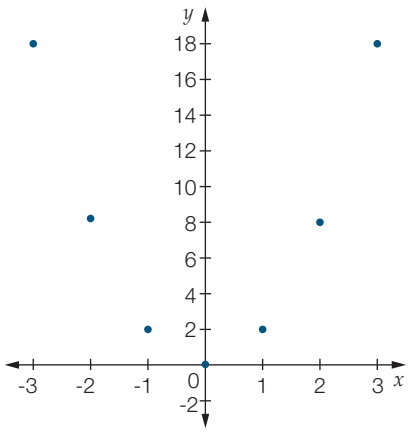
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

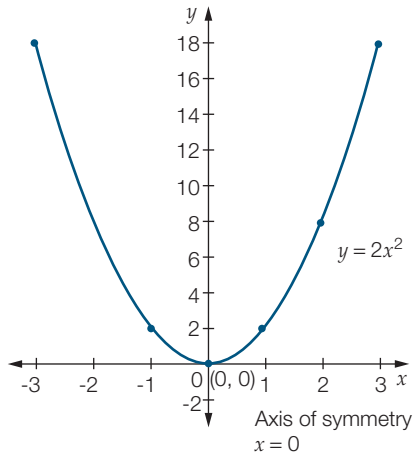




(b) $y = 2x^2$

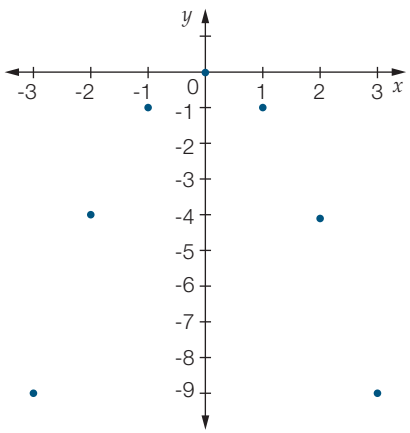
x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

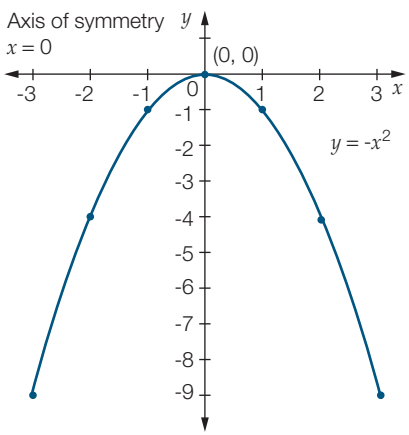




(c) $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9





Worked Example 6

WE6

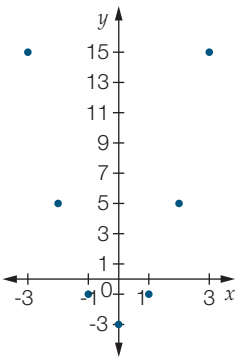
Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

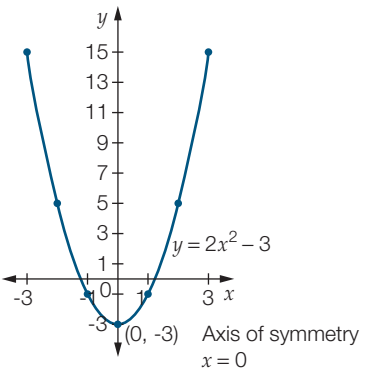
(a) $y = 2x^2 - 3$ **(b)** $y = 2 - x^2$ **(c)** $y = x^2 - 2x - 3$

Working

(a) $y = 2x^2 - 3$

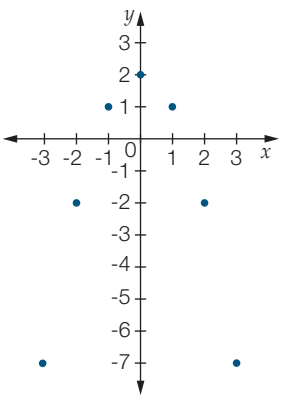
x	-3	-2	-1	0	1	2	3
y	15	5	-1	-3	-1	5	15

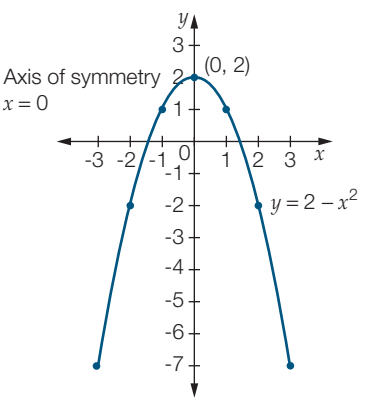




(b) $y = 2 - x^2$

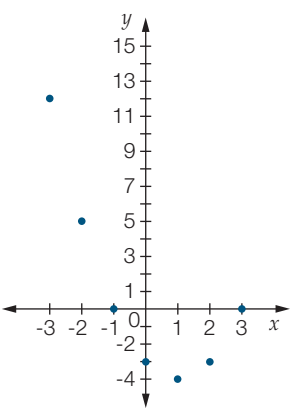
x	-3	-2	-1	0	1	2	3
y	-7	-2	1	2	1	-2	-7

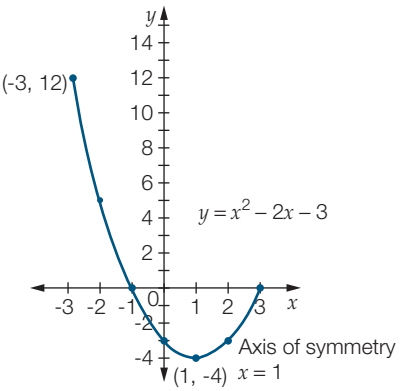




(c) $y = x^2 - 2x - 3$

x	-3	-2	-1	0	1	2	3
y	12	5	0	-3	-4	-3	0





Worked Example 7

WE7

For the graph shown, write down:

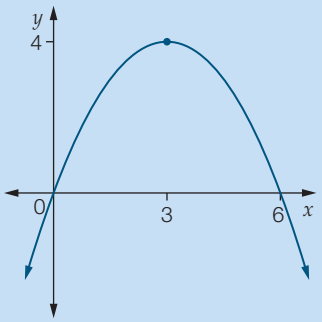
(a) the coordinates of the turning point

(b) the type of turning point

(c) the equation of the axis of symmetry

(d) the y-intercept

(e) the x-intercepts.



Working

(a) There is a turning point at (3, 4).

(b) Because $y = 4$ is the maximum value of y, this is a maximum turning point.

(c) The axis of symmetry is the line $x = 3$.

(d) The y-intercept is (0, 0).

(e) The x-intercepts are (0, 0) and (6, 0).

Worked Example 8

WE8

For each of the following equations:

(a) $y = x^2 - 4x + 5$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = x^2 - 4x + 3$

(i) Complete the table of values.

x	-2	-1	0	1	2	3	4	5
y								
(x, y)								

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write down the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write down the equation of the axis of symmetry of the parabola.

(vi) Write down the coordinates of the y-intercept.

(vii) Write down the coordinates of the x-intercepts.

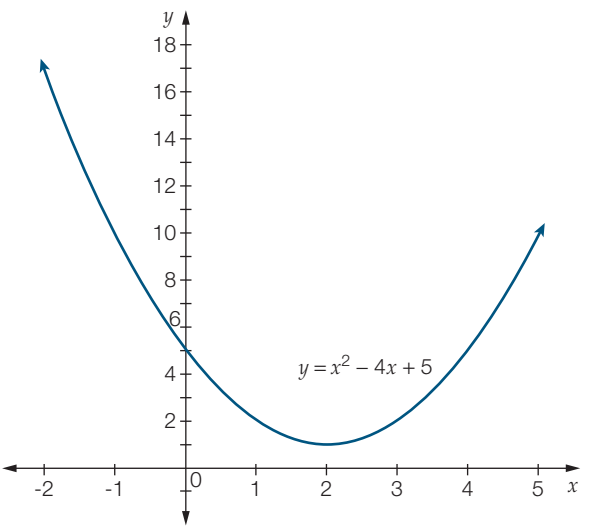
(viii) From the graph, find the values of x when $y = 8$. Hence, state the values of x when the given equation is equal to 8. (Answer correct to one decimal place.)

Working

(a) (i) $y = x^2 - 4x + 5$

x	-2	-1	0	1	2	3	4	5
y	17	10	5	2	1	2	5	10
(x, y)	(-2, 17)	(-1, 10)	(0, 5)	(1, 2)	(2, 1)	(3, 2)	(4, 5)	(5, 10)

(ii)



(iii) The turning point is (2, 1).

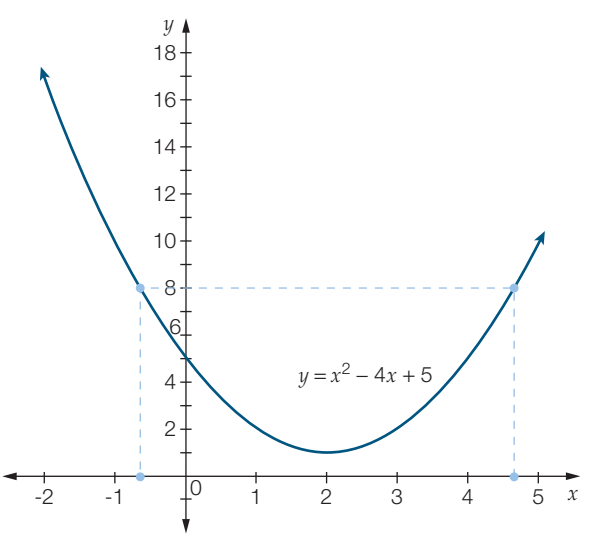
(iv) $y = 1$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is $x = 2$.

(vi) The y-intercept is (0, 5).

(vii) There are no x-intercepts.

(viii)



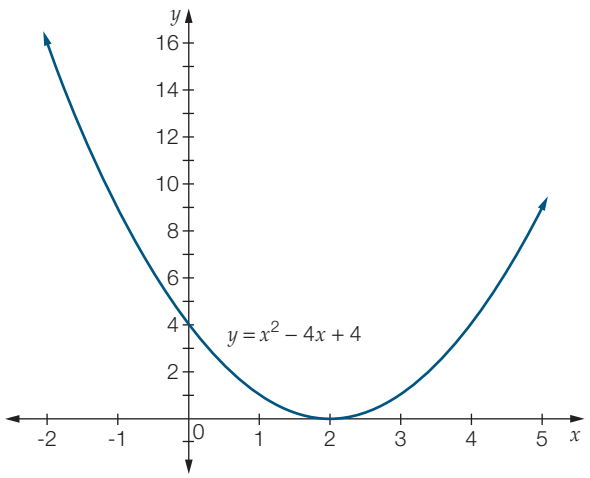
The graph shows $x \approx 4.6$ and -0.6 when $y = 8$.

So, $x^2 - 4x + 5 = 8$ when $x \approx 4.6$ and -0.6.

(b) **(i)** $y = x^2 - 4x + 4$

x	-2	-1	0	1	2	3	4	5
y	16	9	4	2	01	1	4	9
(x, y)	(-2, 16)	(-1, 9)	(0, 4)	(1, 1)	(2, 0)	(3, 1)	(4, 4)	(5, 9)

(ii)



(iii) The turning point is (2, 0).

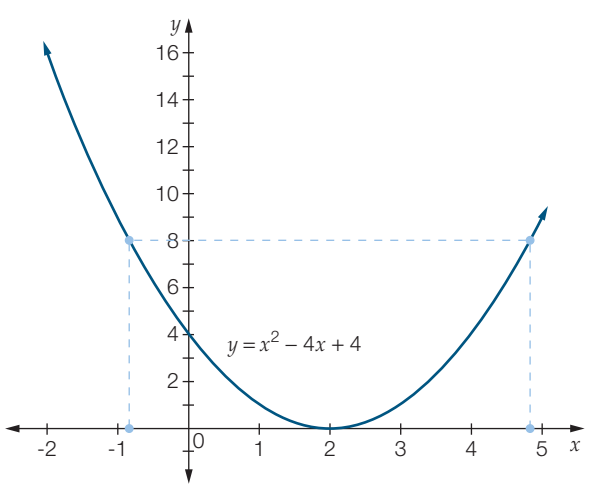
(iv) $y = 0$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is $x = 2$.

(vi) The y-intercept is (0, 4).

(vii) There is one x-intercept at (2, 0).

(viii)



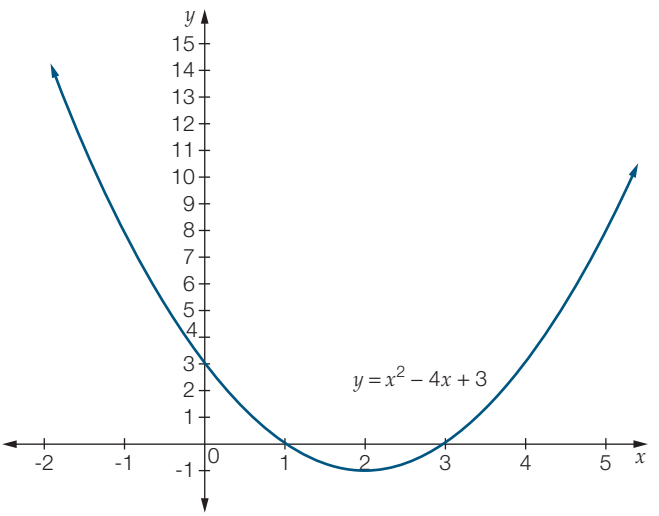
The graph shows $x \approx 4.8$ and -0.8 when $y = 8$.

So, $x^2 - 4x + 4 = 8$ when $x \approx 4.8$ and -0.8.

(c) **(i)** $y = x^2 - 4x + 3$

x	-2	-1	0	1	2	3	4	5
y	15	8	3	0	-1	0	3	8
(x, y)	(-2, 15)	(-1, 8)	(0, 3)	(1, 0)	(2, -1)	(3, 0)	(4, 3)	(5, 8)

(ii)



(iii) The turning point is (2, -1).

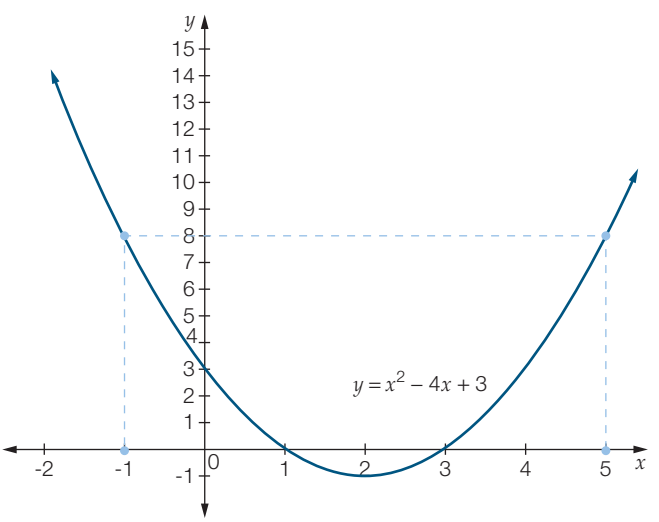
(iv) $y = -1$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is $x = 2$.

(vi) The y-intercept is (0, 3).

(vii) There are two x-intercepts, one at (1, 0) and one at (3, 0).

(viii)



The graph shows $x = -1$ and 5 when $y = 8$.

So, $x^2 - 4x + 3 = 8$ when $x = -1$ and 5.

8.2 Quadratic relationships

Answers page 478

Stage 5.2◊

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q21, Q22, Q24	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q19, Q20, Q22, Q23	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q17, Q18, Q19, Q20, Q21, Q22, Q23

Equipment required: graph paper for most questions

Fluency and understanding

1 State which of the following are quadratic relationships. WE4

(a) $m = 4k^2 - 6$ **(b)** $a = -b^2$ **(c)** $s = 4t^4 - 5t^2 + 7$

(d) $r = 4w - 3$ **(e)** $l = 2 - 3n^2$ **(f)** $a^2 = 9 - x^2$

2 Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive. WE5

(a) $y = 3x^2$ **(b)** $y = -2x^2$ **(c)** $y = \frac{x^2}{2}$

3 Plot the graph of the following quadratic equations for integer values of x between -4 and 4 inclusive. WE6

(a) $y = 3x^2 - 3$ **(b)** $y = -2x^2 + 3$ **(c)** $y = \frac{x^2}{2} - 2$

(d) $y = 3x^2 - 6x$ **(e)** $y = -2x^2 + 5x + 3$ **(f)** $y = \frac{x^2}{2} - \frac{3x}{2} - 2$

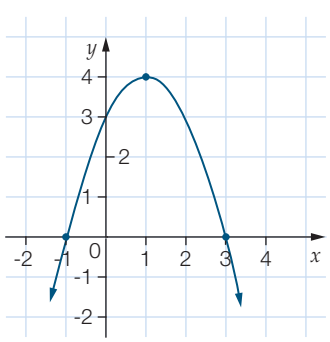
4 For each of the following graphs, write down: WE7

(i) the coordinates of the turning point **(ii)** the type of turning point

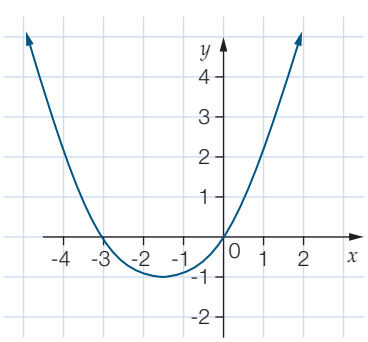
(iii) the equation of the axis of symmetry **(iv)** the y-intercept

(v) the x-intercepts.

(a)



(b)



5 For each of the following equations: WE8

(a) $y = x^2 - 2x + 2$ **(b)** $y = x^2 - 2x - 3$ **(c)** $y = x^2 - 2x + 1$

x	-2	-1	0	1	2	3	4	5
y								

(i) copy and complete the table of values.

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write down the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write down the equation of the axis of symmetry of the parabola.

(vi) Write down the coordinates of the y-intercept.

(vii) Write down the coordinates of the x-intercepts.

(viii) From the graph, find the values of x when $y = 5$. Hence, state the values of x when the given equation is equal to 5.

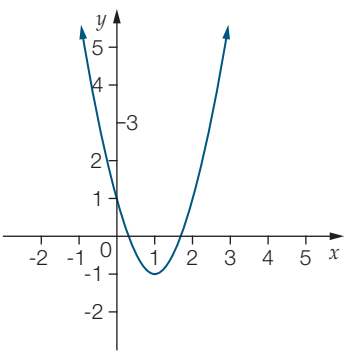
6 (a) Which of the following does not show a quadratic relationship?

A $v = w - 4w^2$ **B** $f^2 = 3d^2 - 1$ **C** $p = 6r^2 - 7$ **D** $k^2 = t$

(b) If $p = 3m^2 + 4$, then if m equals 5, p will be equal to:

A 19 **B** 34 **C** 75 **D** 79

7 The graph of $y = 2x^2 - 4x + 1$ is shown.



(a) For this graph the turning point would be:

A (1, 0) **B** (1, 1)

C (1, -1) **D** (-1, 1)

(b) The value(s) of x for $2x^2 - 4x + 1 = 0$ would be approximately:

A -0.8 and 1.8 **B** 1 and 0

C 0.3 and 1.7 **D** 0 and 2

(c) The value(s) of x for $2x^2 - 4x + 1 = 1$ would be approximately:

A 0 and 2 **B** 0.3 and 1.7 **C** 0 and 1 **D** -0.8 and 1.8

8 Compare your graphs in Question **2 (a)–(c)** to your graphs in Question **3 (a)–(c)**.

Comment on any similarities and differences.

9 Compare your graphs in Question **2 (a)–(c)** to your graphs in Question **3 (d)–(f)**.

Comment on any similarities and differences.

10 (a) Complete the following table of values for $y = x^2 - 4$. (The first column has been completed for you.)

x	-4	-3	-2	-1	0	1	2	3	4
y	12								
(x, y)	(-4, 12)								

(b) Plot the points onto a number plane. Join them with a smooth curve.

(c) What are the coordinates of the turning point?

(d) Does the parabola have a maximum or a minimum turning point?

(e) Write down the equation of the axis of symmetry of the parabola.

(f) Write down the coordinates of the y-intercept.

(g) Write down the coordinates of the x-intercepts.

(h) From the graph, find the value of y when $x = 3.5$ (correct to one decimal place).

(i) From the graph, find the values of x when $y = -2$ (correct to one decimal place).

(j) State the values of x when $x^2 - 4 = 5$. Hence, find the values of x when $x^2 - 9 = 0$.

11 (a) Complete the following table of values for $y = -2x^2 - 6x$.

x	-5	-4	-3	-2	-1	0	1	2
x^2								

(b) Plot the graph of $y = -2x^2 - 6x$ using graph paper.

(c) Write down the coordinates of the turning point. State whether the parabola has a maximum or a minimum turning point.

(d) Write down the coordinates of any x- and y-intercepts.

(e) What are the x-values when $y = 4$?

(f) If $y = 4$, what does $-2x^2 - 6x$ equal? Hence, state the x-values when $-2x^2 - 6x = 4$. These x-values are the solution to the equation $-2x^2 - 6x = 4$.

(g) Use the graph to find the x-values that solve the equation $-2x^2 - 6x = -8$.

(h) Find the x-values that solve $-2x^2 - 6x = 0$.

12 (a) Complete a table of values for $y = x^2 - x - 2$ for integer values of x between -3 and 5.

(b) Plot the graph of $y = x^2 - x - 2$ for the points in your table.

(c) Write down the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(d) Write down the coordinates of any x- and y-intercepts.

(e) Use the parabola to solve x in each of the following equations.

(i) $x^2 - x - 2 = 0$ **(ii)** $x^2 - x - 2 = 4$ **(iii)** $x^2 - x - 2 = 2$

(iv) $x^2 - x - 2 = -2.25$ **(v)** $x^2 - x - 2 = -4$ **(vi)** $x^2 - x - 2 = 1\frac{1}{2}$

13 (a) Plot the graph of $y = x^2 + 6x + 10$. Use integer values of x between -7 and 1.

(b) What are the coordinates of the turning point?

(c) Write down the coordinates of any x- and y-intercepts.

(d) Use the graph to solve each of the following equations.

(i) $x^2 + 6x + 10 = 1$ **(ii)** $x^2 + 6x + 10 = 10$

(iii) $x^2 + 6x + 10 = 0$ **(iv)** $x^2 + 6x + 10 = 15$

14 (a) Plot the graphs of the following parabolas on separate axes on graph paper. (Remember to complete a table of values first.)

(i) $y = -x^2 + 4x$ for $-1 \leq x \leq 5$

(ii) $y = x^2 + 3x - 10$ for $-6 \leq x \leq 3$

(iii) $y = -x^2 + 2x + 3$ for $-2 \leq x \leq 4$

(b) Use your graphs to solve the following equations to find x.

(i) $-x^2 + 4x = 0$ **(ii)** $x^2 + 3x - 10 = 0$ **(iii)** $-x^2 + 2x + 3 = 4$

(iv) $-x^2 + 4x = 4$ **(v)** $x^2 + 3x - 10 = 8$ **(vi)** $-x^2 + 2x + 3 = 0$

(vii) $-x^2 + 4x = 2$ **(viii)** $x^2 + 3x - 10 = 4$ **(ix)** $-x^2 + 2x + 3 = -2$

Reasoning, communicating and open-ended

Hint

Use only realistic values of x for your table of values.

15 A postage stamp has a length of $(x + 2)$ cm and a width of $(x + 1)$ cm.



(a) Write an equation for the area, h, of the postage stamp in expanded form.

(b) Complete a table of values for x and A.

(c) Plot a graph of your equation showing x on the horizontal axis (use 1 cm = 1 cm) and A on the vertical axis (use 1 cm = 1 cm^2).

(d) Use the graph to find the value x represents if the area of the stamp is 8 cm^2. Did you obtain one or two values for x in this problem? Explain.

(e) What is the length and width of the stamp if its area is 8 cm^2? (Give your answers correct to one decimal place.)

16 Karrie, a keen golfer, hits a golf ball along the fairway. The height of the golf ball, h, in metres, at any time, t, in seconds, during its flight is given by the equation $h = -5t^2 + 20t$.

(a) Complete a table of values for t and h.

(b) Plot the graph of $h = -5t^2 + 20t$. Show the values of t on the horizontal axis (use 1 cm = 0.25 s) and values for h on the vertical axis (use 1 cm = 2 m).

Use the graph to answer the following questions.

(c) How high does the ball get above the ground?

(d) How long does it take to reach this maximum height?

(e) When does the ball first reach a height of 12 m?

(f) How long does it take before the ball hits the ground?

(g) For how long was the ball at least 8 m above the ground?

17 A worker's vacuum flask was accidentally dropped from the top of a multistorey building. The relationship between the height, h, above the ground of the flask (in metres) and the time, t, for it to fall to that height (in seconds) was found to be:

$h = 125 - 5t^2$

(a) Complete a table of values for t and h.

(b) Plot the graph of $h = 125 - 5t^2$. Show values of t on the horizontal axis (use 1 cm = 1 s) and values for the height h on the vertical axis (use 1 cm = 20 m).

Hint

The first second occurs between $t = 0$ and $t = 1$.

(c) What is the height of the building?

(d) How long did it take for the flask to hit the ground?

(e) How high above the ground was the flask after 2.5 s? (Answer correct to the nearest metre.)

(f) What distance, to the nearest metre, had the flask fallen after 2.5 s?

(g) How long did it take for the flask to fall 50 m? (Answer correct to one decimal place.)

(h) How long did it take for the flask to fall 100 m? (Answer correct to one decimal place.)

(i) What distance did the flask fall during the fourth second?

(j) During which second did the flask fall the greatest distance?

18 The speed of a car was tracked by radar in a police helicopter over a period of 10 minutes. The police found that the car's speed could be represented by the equation, $s = 3t^2 - 30t + 135$, where s is the speed of the car in km/h after t minutes have passed.

(a) Plot a graph representing this information, showing values of t on the horizontal axis. (Use 1 cm = 2 minutes) and values of s on the vertical axis (use 1 cm = 20 km/h).

Use the graph to answer the following questions.

(b) What was the maximum speed recorded for the car?

(c) What was the minimum speed recorded for the car?

(d) What was the speed of the car after 3 minutes?

(e) What was the speed of the car after 7 minutes?

(f) When was the car travelling at 80 km/h?

(g) When was the car travelling at 120 km/h?

(h) The speed limit along this section of road is 100 km/h. In which time intervals was the car driven above the speed limit?

19 The entrance to a one-way railway tunnel forms an arch. At any particular distance, d, across the base of the tunnel, starting from the left, the height, h, of the arch can be found using the equation $h = -d^2 + 6d$. Both h and d are in metres.

(a) Plot the graph of $h = -d^2 + 6d$. (Show d on the horizontal axis.)

Use the graph to answer the following questions.

(b) How wide is the base of the tunnel?

(c) What is the maximum height of the tunnel?

(d) How high is the tunnel 1 m along the base from either side of the tunnel?

For a train to pass safely through the tunnel there must be a minimum vertical clearance of 1 m between the train and the arch. Assume the train has a rectangular cross-section. Use this information to answer the following questions.

(e) If a particular train happened to be 4 m wide, what would be its maximum allowable height?

(f) What is the maximum allowable height of a train that is 3 m wide?

(g) Find the maximum width of a train if it just fits through the tunnel and is 4.8 m high.

(h) Decide whether trains of the following dimensions would fit through the tunnel.

(i) width 2.6 m and height 6 m **(ii)** width 3.8 m and height 4.5 m

20 The volume of liquid in a container can be found using the equation $V = 3t^2 + 4t$, where V is in cm^3 and t is the number of seconds after pouring commences.

(a) Plot the graph of $V = 3t^2 + 4t$. (Use values of t from 0 to 7 and show t on the horizontal axis.)

Use your graph to answer the following questions.

(b) Find the amount of liquid in the container at the start.

(c) Find the amount of liquid in the container after 4 minutes.

(d) The container can hold only 132 cm^3. How long, in seconds, does it take to fill the container?

(e) How much liquid is added in the first 2 seconds?

(f) What are the feasible values for time and for volume?

21 **(a)** Write down at least three expressions that are quadratic expressions.

(b) Write down at least three expressions that are not quadratic expressions.

22 Make up at least two quadratic equations. For each equation:

(a) draw up a table of values

(b) plot the graph

(c) find the turning point

(d) find the value of any x- and y-intercepts.

23 **(a)** If the length of a rectangle is 3 cm longer than its width, write down the area of the rectangle.

(b) Draw a graph of this relationship.

(c) Choose a value for the area of the rectangle and use your graph to find the dimensions of your rectangle, correct to two decimal places.

24 Josh is having some trouble filling in a table of values for a quadratic equation. Explain what he is doing wrong.

(a) $y = -x^2 + 2x$

x	-3	-2	-1	0
y	3	0	-1	0

(b) $y = 2x^2 + 1$

x	-2	-1	0	1	2
y	17	5	1	5	17

Solving quadratic equations

8.3

Stage 5.2

Need to Know

The Null Factor Law

If the product of two or more numbers is zero, then at least one of the numbers must be zero.

If $ab = 0$, then $a = 0$, $b = 0$ or both a and $b = 0$.

To use the **Null Factor Law** to solve quadratic equations, two requirements need to be met.

1. One side of the equation must equal zero.
2. The other side must be the product of factors. (Factorisation may be necessary to achieve this.)

If the quadratic expression is not in a factorised form, we must try to factorise it first. Not all quadratics will factorise to give real number solutions.

Solving non-factorised quadratics

For equations in the form of $ax^2 + bx = 0$:

- identify the highest common factor (HCF) and factorise
- use the Null Factor Law to find solutions.

For equations in the form of $x^2 - a^2 = 0$:

- insert $+ ax - ax$ to make four terms
- factorise by grouping in pairs to get $(x + a)(x - a) = 0$
- use the Null Factor Law to find solutions.

For equations in the form of $x^2 + 2ax + a^2 = 0$ or $x^2 - 2ax + a^2 = 0$:

- split $2ax$ into ax and ax to make four terms
- factorise by grouping in pairs to get $(x + a) = 0$ or $(x - a) = 0$
- use the Null Factor Law to find solutions.

Notice that there is only one solution for quadratics in this form. Remember that some parabolas only touch the x-axis at one point. The equation to find the x-intercept of these parabolas will have only one solution.

Worked Example 9 WE9

Solve the following equations using the Null Factor Law.

(a) $x(x+3)=0$ **(b)** $(x+3)(x-2)=0$ **(c)** $(x-1)^2=0$

Working

(a) $x(x+3)=0$
$x=0$ or $x+3=0$
$x=0$ or $x=-3$

(b) $(x+3)(x-2)=0$
$x+3=0$ or $x-2=0$
$x=-3$ or $x=2$

(c) $(x-1)^2=0$
$(x-1)(x-1)=0$
$x-1=0$
$x=1$

Worked Example 10 WE10

Solve the following equations using the Null Factor Law.

(a) $2x^2+4x=0$ **(b)** $7x^2=x$

Working

(a) $2x^2+4x=0$
$2x(x+2)=0$ ($2x$ is the HCF)
$2x=0$ or $x+2=0$
$x=0$ or $x=-2$

(b) $7x^2=x$
$7x^2-x=0$
$x(7x-1)=0$ (x is the HCF)
$x=0$ or $7x-1=0$
$x=0$ or $7x=1$
$x=\frac{1}{7}$

Worked Example 11 WE11

Solve the following equations using the Null Factor Law.

(a) $x^2+10x+21=0$ **(b)** $x^2-12x+27=0$

Working

(a) $x^2+10x+21=0$
$x^2+3x+7x+21=0$
$x(x+3)+7(x+3)=0$
$(x+3)(x+7)=0$
$x+3=0$ or $x+7=0$
$x=-3$ or $x=-7$

(b) $x^2-12x+27=0$
$x^2-9x-3x+27=0$
$x(x-9)-3(x-9)=0$
$(x-9)(x-3)=0$
$x-9=0$ or $x-3=0$
$x=9$ or $x=3$

Worked Example 12

WE12

Solve the following equations using the Null Factor Law.

(a) $x^2 - 49 = 0$ **(b)** $x^2 - 7 = 0$

Working

(a)

$$\begin{aligned} x^2 - 49 &= 0 \\ x^2 - 7x + 7x - 49 &= 0 \\ x(x-7) + 7(x-7) &= 0 \\ (x-7)(x+7) &= 0 \\ x - 7 = 0 \quad &\text{or} \quad x + 7 = 0 \\ x = 7 \quad &\text{or} \quad x = -7 \end{aligned}$$

(b)

$$\begin{aligned} x^2 - 7 &= 0 \\ x^2 - \sqrt{7}x + \sqrt{7}x - 7 &= 0 \\ x(x-\sqrt{7}) + \sqrt{7}(x-\sqrt{7}) &= 0 \\ (x-\sqrt{7})(x+\sqrt{7}) &= 0 \\ (x-\sqrt{7}) = 0 \quad &\text{or} \quad (x+\sqrt{7}) = 0 \\ x = \sqrt{7} \quad &\text{or} \quad x = -\sqrt{7} \end{aligned}$$

Worked Example 13

WE13

Solve the following equations using the Null Factor Law.

(a) $x^2 - 6x + 9 = 0$ **(b)** $2x^2 + 48x = -288$

Working

(a)

$$\begin{aligned} x^2 - 6x + 9 &= 0 \\ x^2 - 3x - 3x + 9 &= 0 \\ x(x-3) - 3(x-3) &= 0 \\ (x-3)(x-3) &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

(b)

$$\begin{aligned} 2x^2 + 48x &= -288 \\ 2x^2 + 48x + 288 &= 0 \\ 2(x^2 + 24x + 144) &= 0 \\ 2(x^2 + 12x + 12x + 144) &= 0 \\ 2(x(x+12) + 12(x+12)) &= 0 \\ 2(x+12)(x+12) &= 0 \\ (x+12)^2 &= 0 \\ (x+12)(x+12) &= 0 \\ x + 12 &= 0 \\ x &= -12 \end{aligned}$$

Answers page 481

8.3 Solving quadratic equations

Stage 5.2

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q6, Q7, Q8, Q9, Q15	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6, Q7, Q8, Q9, Q10, Q11	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

WE9

1 Solve the following equations using the Null Factor Law.

(a) $x(x-1)=0$ **(b)** $x(x+8)=0$ **(c)** $x(x+5)=0$

(d) $(x-4)(x-1)=0$ **(e)** $(x-2)(x-6)=0$ **(f)** $(x-3)(x-9)=0$

(g) $(x-4)(x+1)=0$ **(h)** $(x-3)(x+7)=0$ **(i)** $(x-5)(x+2)=0$

(j) $(x+4)(x+6)=0$ **(k)** $(x+7)(x+2)=0$ **(l)** $(x+3)(x+10)=0$

(m) $(x-4)^2=0$ **(n)** $(x+7)^2=0$ **(o)** $(x-9)^2=0$

WE10

2 Solve the following equations using the Null Factor Law.

(a) $x^2-6x=0$ **(b)** $x^2-x=0$ **(c)** $x^2-8x=0$

(d) $x^2+7x=0$ **(e)** $x^2+5x=0$ **(f)** $x^2+x=0$

(g) $2x^2-8x=0$ **(h)** $3x^2-9x=0$ **(i)** $7x^2-28x=0$

(j) $3x^2+27x=0$ **(k)** $5x^2+45x=0$ **(l)** $12x^2+4x=0$

WE11

3 Solve the following equations using the Null Factor Law.

(a) $x^2+6x+8=0$ **(b)** $x^2+9x+20=0$ **(c)** $x^2+12x+32=0$

(d) $x^2-11x+28=0$ **(e)** $x^2-12x+35=0$ **(f)** $x^2-15x+56=0$

(g) $x^2+2x-8=0$ **(h)** $x^2+2x-15=0$ **(i)** $x^2-9x-22=0$

WE12

4 Solve the following equations using the Null Factor Law.

(a) $x^2-9=0$ **(b)** $x^2-1=0$ **(c)** $x^2-16=0$

(d) $2x^2-98=0$ **(e)** $3x^2-27=0$ **(f)** $7x^2-28=0$

(g) $x^2-11=0$ **(h)** $x^2-29=0$ **(i)** $x^2-47=0$

WE13

5 Solve the following equations using the Null Factor Law.

(a) $x^2-6x+9=0$ **(b)** $x^2-2x+1=0$ **(c)** $x^2-8x+16=0$

(d) $x^2+10x+25=0$ **(e)** $x^2+12x+36=0$ **(f)** $x^2+18x+81=0$

(g) $2x^2-28x+98=0$ **(h)** $3x^2+18x+27=0$ **(i)** $7x^2-28x+28=0$

6 **(a)** The solution(s) to $x(x+1)=0$ is/are:

A $x=-1$ **B** $x=1$ **C** $x=0, x=-1$ **D** $x=0, x=1$

(b) The solution(s) to $(x+2)(x-3)=0$ is/are:

A $x=2, x=3$ **B** $x=-2, x=3$ **C** $x=2, x=-3$ **D** $x=-2, x=-3$

7 When three times a number is added to the square of that number, the result is seven times the number. Use an equation to find the number if it is greater than zero.

8 The product of $(2x + 1)$ and $(2x - 3)$ is equal to the product of $(x + 2)$ and $(5x - 6)$. Find x.

9 The height of a golf ball can be represented by the equation $h = 5t - t^2$, where t represents the time in seconds.

The golf ball will be 4 m above the ground when t equals:

A 0 seconds **B** 4 seconds **C** 1 second **D** Never

Reasoning, communicating and open-ended

10 A square vegetable garden has one side extended by 4 m and the other side reduced by 4 m. The area is now 65 m^2. What was the side length of the original vegetable garden?

Hint

Write an equation and solve it to find the original length of the vegetable garden.

11 A graph has equation $y = (x + 1)(x - 4)$. Find the x-intercepts of this graph.

12 Determine the x-intercepts of the graphs of the following quadratic relationships.

(a) $y = x^2 - 144$ **(b)** $y = 6x^2 + 24x$ **(c)** $y = x^2 - 12x + 36$

13 **(a)** Expand $(x - 6)(x + 2)$.

(b) Use your answer in part **(a)** to find the solution to $x^2 - 4x - 12 = 0$.

(c) How are your solutions in part **(b)** related to the constant term in $y = x^2 - 4x - 12$?

(d) Expand $(x - 5)(x + 1)$.

(e) Use your answer in part **(d)** to find the solution to $x^2 - 4x - 5 = 0$.

(f) How are your solutions for part **(e)** related to the constant term in $y = x^2 - 4x - 5$?

(g) Solve the quadratic equation $x^2 - 4x = 0$.

(h) What is the value of the constant term in $x^2 - 4x$?

(i) How are your solutions for part **(g)** related to the constant term in $y = x^2 - 4x$?

(j) Expand $(x - 1)(x - 3)$.

(k) Use your answer in part **(j)** to find the solution to $x^2 - 4x + 3 = 0$.

(l) How are your solutions for part **(k)** related to the constant term in $y = x^2 - 4x + 3$?

(m) Solve the quadratic equation $x^2 - 4x + 4 = 0$.

(n) How are your solutions in part **(m)** related to the constant term in $y = x^2 - 4x + 4$?

(o) What does the constant term in the equations in parts **(c)**, **(f)**, **(i)** and **(n)** represent graphically?

(p) What do the equations in parts **(b)**, **(e)**, **(g)**, **(k)** and **(m)** all have in common?

(q) What information does the solution to these five equations give you about the graphs of each of these equations?

(r) What happens to the x-intercepts of the graphs of these equations as the constant term increases?

(s) Write a statement about the solutions to $x^2 - 4x + c = 0$ where $c > 4$. Hence, make a statement about the x-intercepts of graphs in the form of $y = x^2 - 4x + c$ where $c > 4$.

14 Decide on two values of x that will be the x-intercepts of a quadratic graph. Write down at least two possible equations for the graph.

15 Louise is solving the equation $8x - x^2 = 16$. She writes:

$8x - x^2 = 16$
$x(8 - x) = 16$
$x = 16$ or $8 - x = 16$
$x = 16$ or $x = -8$

Explain to Louise what she has done incorrectly and write the correct solution.

Problem solving

Stopping distances

The distance it takes for a car to stop could be described by the equation $y = 0.005x^2 + 0.12x$, where y is the distance (m) it takes to stop and x is the speed (km/h) of the car.

(a) What extra distance will it take a car to stop if it is travelling at 60 km/h instead of 40 km/h through a school zone?

(b) What is the extra distance required for a car to stop if it is exceeding a 100 km/h speed limit by 10 km/h?

(c) Show that the speed of the car must be less than 12 km/h to ensure it does not hit a stationary car 2 m in front of it.

(d) What is the effect on the stopping distance of wiping 5 km/h off a 100 km/h speed?



Strategy options

- Have I seen a similar problem?
- Break the problem into manageable parts.

Sketching parabolas using transformations

8.4

Stage 5.3 §

Need to Know

A **transformation** is any change that can be made to a graph or shape using a **dilation**, **reflection**, **translation** or rotation.

Dilations and reflections

A dilation is when the graph of x^2 is multiplied by a constant, a, to form graphs in the form of $y = ax^2$. The **coefficient** of x^2, a, is referred to as the **dilation factor**.

For equations of the form $y = ax^2$:

- When the coefficient of x^2 is negative ($a < 0$) the parabola is inverted. These graphs are reflected in the x-axis.
- When the coefficient of x^2 is greater than 1 ($a > 1$), the parabola is narrower than $y = x^2$.
- When the coefficient of x^2 is greater than zero but less than 1 ($0 < a < 1$), the parabola is wider than $y = x^2$.
- When the coefficient of x^2 is greater than -1 but less than zero ($-1 < a < 0$) the parabola is also wider than $y = x^2$.

Translations

Horizontal translations:

The graph of $y = (x - h)^2$ is the graph of $y = x^2$ translated h units to the right.
If the value of h is negative, this will mean a translation to the left.

Vertical translations:

The graph of $y = x^2 + k$ is the graph of $y = x^2$ translated k units upwards. If the value of k is negative, this will mean a translation downwards.

Combined horizontal and vertical translations:

The graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated h units to the right and k units upwards. If the value of h is negative, this will mean a translation to the left. If the value of k is negative, this will mean a translation downwards.

Locating *x*- and *y*-intercepts

The *y*-intercept occurs where the parabola intersects the *y*-axis (i.e. when $x = 0$). The *y*-value of the *y*-intercept can therefore be found by substituting $x = 0$ into the quadratic equation.

If $y = x^2 - 5x + 6$,

$y = 0 - 5 \times 0 + 6$ (substituting $x = 0$)

$= 6$

The coordinates of the *y*-intercept would be (0, 6).

The *x*-intercept occurs where the parabola intersects with the *x*-axis (when $y = 0$). The *x*-value of the *x*-intercept can therefore be found by substituting $y = 0$ into the quadratic equation.

If $y = (x - 2)(x - 3)$, (this quadratic, when expanded, gives $y = x^2 - 5x + 6$)

$0 = (x - 2)(x - 3)$ (substituting $y = 0$)

$x - 2 = 0$ or $x - 3 = 0$ (using the Null Factor Law)

$x = 2$ or $x = 3$

The coordinates of the *x*-intercepts would be (2, 0) or (3, 0).

Many quadratics will have two *x*-intercepts.

Parabolas with equations in the form of $y = (x - h)^2$ will have only one *x*-intercept.

Some parabolas will have no *x*-intercepts.

Intercepts and the turning point

All quadratic graphs have a vertex or turning point. This can be a minimum value or a maximum value. When a graph has two *x*-intercepts, the *x*-coordinate of the turning point is halfway between the two *x*-intercepts.

For the quadratic equation $y = (x - 3)(x + 5)$, the *x*-intercepts are (3, 0) and (-5, 0) and $x = -1$ is the *x*-coordinate of the turning point.

The corresponding *y*-coordinate can be found by substituting the *x*-value into the quadratic equation.

$y = (-1 - 3)(-1 + 5)$

$y = -4 \times 4$

$= -16$

For the quadratic equation $y = (x - h)^2$, the *x*-intercept and turning point is $(h, 0)$.

When a graph has only one *x*-intercept, the turning point is the *x*-intercept, i.e. for $y = (x - 4)^2$, the *x*-intercept and turning point is (4, 0).

For the quadratic equation $y = x^2 + k$, the turning point is $(0, k)$. The turning point is also the *y*-intercept.

If *k* is a positive integer there will be no *x*-intercepts, i.e. for $y = x^2 + 3$, the coordinates of the turning point are (0, 3) and there are no *x*-intercepts.

For the quadratic equation $y = (x - h)^2 + k$, the turning point is (h, k), i.e. for $y = (x - 2)^2 + 7$, the coordinates of the turning point are (2, 7).

In a quadratic equation in the form of $y = ax^2 + bx + c$, the *x*-value of the turning point can be found using $x = -\frac{b}{2a}$.

Technology Exploration: GeoGebra

Investigating non-linear equations

1 Open GeoGebra. From the menu, select Options > Advanced… and in the pop-up window go to/Preferences-Graphics > Grid and select Show Grid. In the same window, go to the Basic tab and in Dimensions set the x-axis values from -10 to 10 and set the y-axis values from -8 to 8. In the same window, go to the xAxis tab and set Label to x, then go to the yAxis tab and set Label to y. In both these tabs, select 'Distance' and set it to 1. Finally, from the main menu select Options > Point Capturing > Automatic and then select Options > Labelling > No New Objects.

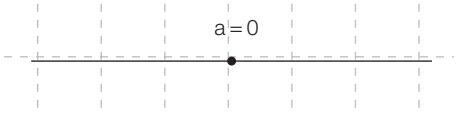
2 If a larger font is required, select from the menu Options > Font Size. Choose an appropriate size from the list provided.

Investigating parabolas

All parabolas can be represented by the equation $y = ax^2 + bx + c$, where a represents the dilation and c represents the y-coordinate of the y-intercept (where the line cuts the y-axis). We will now create parabolas using different values of a, b and c to investigate how the graphs change when we change these values.

3 Select the Slider 

tool. Click in the right-hand side of the Graphics view to create a slider there. In the pop-up window that appears, in Name, type a. Change Increment to 0.2. Click on the Slider tab and change Width to 200. Click Apply. A line like this will appear on the screen.



4 Construct a second slider by repeating Step **3**, naming it b.

5 Construct a third slider by repeating Step **3**, naming it c.

6 Set the slider values for a, b and c to 0.

7 In the Input bar at the bottom of the window, type $y=x^2$ (the ^ is used to show indices) and press enter. A parabola should appear on the screen.

8 In the Input bar, enter $y=a*x^2+b*x+c$ (the * is used to show multiplication). Increase the slider value for a from -5 to 5. A parabola should appear on the screen, and it should change as the slider value changes.

9 Write what you see as the slider values for a increase from -5 to 5. What happens when $a = 0$? Explain. What happens when $a = 1$? Explain. What happens when $a = -1$? Explain.

10 Set the value of a to 1 and increase the value of c from -5 to 5. Describe what happens to the parabola as the c value changes. What do you notice about the shape of each graph? What do you notice about the y-intercept for each graph? What do you notice about the x-intercepts? (Hint: c = -4 and -1 should help you.)

11 Leave the a value at 1 and set the c value to 0. Change the b value from -5 to 5. Write about any connection between the graphs and the b value. Use algebra to explain this connection. You should also notice that an equation appears in the Algebra view (on the left-hand side of the window).

12 Find a value for each of b and c so that the parabola moves to the right. Explain why you chose these values.

13 Find a value for each of b and c so that the parabola moves to the left. Explain why you chose these values.

Worked Example 14

WE14

(a) Plot the graph of $y = x^2$ for $-2 \leq x \leq 2$.

(b) Plot the graphs of **(i)** $y = -x^2$, **(ii)** $y = 2x^2$ and **(iii)** $y = \frac{1}{2}x^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

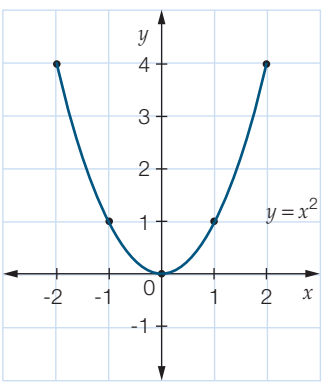
(c) Compare each of the graphs in part **(b)** to $y = x^2$ using the words narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

Working

(a)

x	-2	-1	0	1	2
y	4	-1	0	1	4



(b) **(i)** $y = -x^2$

x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

(ii) $y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

(iii) $y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	2	0.5	0	0.5	2

(c) **(i)** $y = -x^2$ is inverted, but the same width as $y = x^2$.

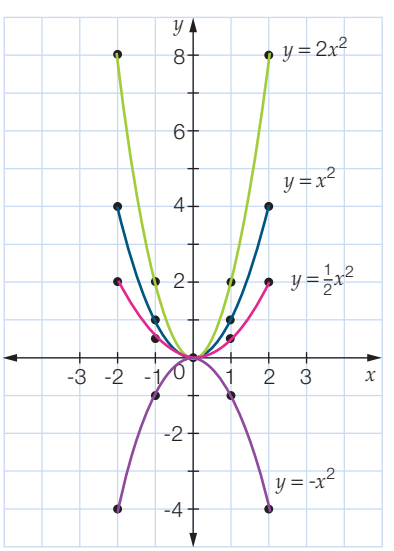
(ii) $y = 2x^2$ is narrower than $y = x^2$.

(iii) $y = \frac{1}{2}x^2$ is wider than $y = x^2$.

(d) **(i)** The dilation factor, a, is -1.

(ii) The dilation factor, a, is 2.

(iii) The dilation factor, a, is $\frac{1}{2}$.



Worked Example 15

WE15

(a) Plot the graph of $y = x^2$ for $-4 \leq x \leq 4$.

(b) Plot the graphs of **(i)** $y = (x - 1)^2$ and **(ii)** $y = (x + 2)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

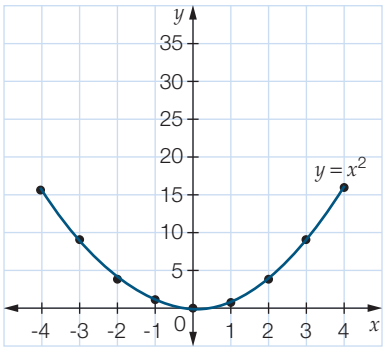
(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

Working

(a) $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

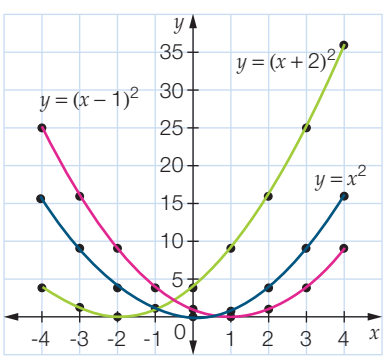


(b) **(i)** $y = (x - 1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	25	16	9	4	1	0	1	4	9

(ii) $y = (x + 2)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	4	1	0	1	4	9	16	25	36



(c) **(i)** The graph of $y = (x - 1)^2$ has been translated 1 unit to the right.

(ii) The graph of $y = (x + 2)^2$ has been translated 2 units to the left.

(d) **(i)** The x-intercept of $y = (x - 1)^2$ is the point $(1, 0)$. The x-value of the intercept is $x = 1$. This is the number subtracted from x before squaring in the equation.

(ii) The x-intercept of $y = (x + 2)^2$ is the point $(-2, 0)$. The x-value of the intercept is $x = -2$. This is the number subtracted from x before squaring in the equation.

Worked Example 16

WE16

(a) Plot the graph of $y = x^2$ for $-4 \leq x \leq 4$.

(b) Plot the graphs of **(i)** $y = x^2 + 3$ and **(ii)** $y = x^2 - 4$ on the same set of axes as your graph in part **(a)** using the same values for x.

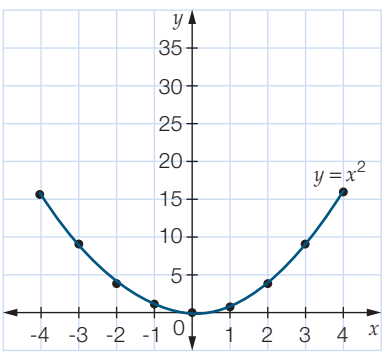
(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the y-intercept for each graph in part **(b)** and identify the y-intercept from the equation.

Working

(a)

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

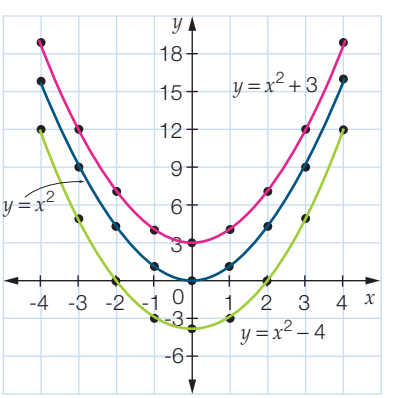


(b) **(i)** $y = x^2 + 3$

x	-4	-3	-2	-1	0	1	2	3	4
y	19	12	7	4	3	4	7	12	19

(ii) $y = x^2 - 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12



(c) **(i)** The graph of $y = x^2$ has been translated 3 units upwards.

(ii) The graph of $y = x^2$ has been translated 4 units downwards.

(d) **(i)** The y-intercept of $y = x^2 + 3$ is the point (0, 3). The y-value of the intercept is $y = 3$. This is the constant in the equation.

(ii) The y-intercept of $y = x^2 - 4$ is the point (0, -4). The y-value of the intercept is $y = -4$. This is the constant in the equation.

Worked Example 17

WE17

(a) Plot the graph of $y = x^2$ for $-5 \leq x \leq 5$.

(b) Plot the graphs of **(i)** $y = (x - 3)^2 + 1$ and **(ii)** $y = (x + 1)^2 - 5$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

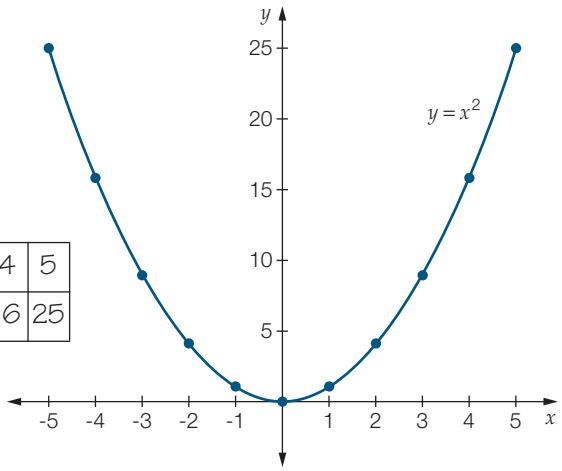
(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

Working

(a) $y = x^2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

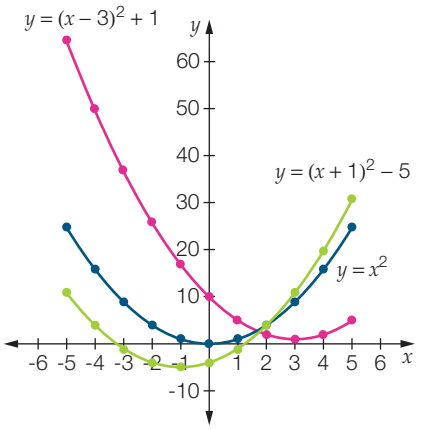


(b) **(i)** $y = (x - 3)^2 + 1$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	65	50	37	26	17	10	5	2	1	2	5

(ii) $y = (x + 1)^2 - 5$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	11	4	-1	-4	-5	-4	-1	4	11	20	31



(c) **(i)** The graph of $y = x^2$ has been translated 3 units to the right and 1 unit upwards.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left and 5 units down.

(d) **(i)** The y-intercept is $(0, 10)$.

(ii) The y-intercept is $(0, -4)$.

(e) **(i)** The graph has no x-intercepts.

(ii) The graph has two x-intercepts.

Worked Example 18

WE18

(a) Plot the graph of $y = (x - 1)^2 - 9$ for $-5 \leq x \leq 5$.

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x - h)^2 + k$, write down the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x - 4)(x + 2)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

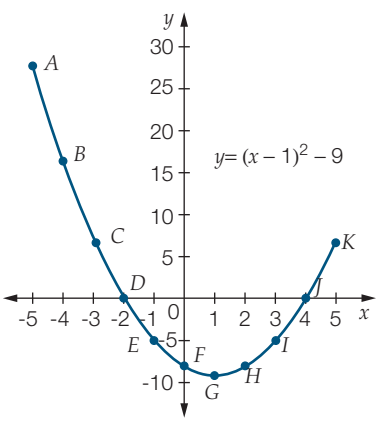
(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write down the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

Working

(a) $y = (x - 1)^2 - 9$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	27	16	7	0	-5	-8	-9	-8	-5	0	7



(b) The turning point is (1, -9).

(c) $h = 1$ and $k = -9$

$(h, k) = (1, -9)$

Yes, it is the same point.

(d) $y = (x - 1)^2 - 9$

$= x^2 - 2x + 1 - 9$

$= x^2 - 2x - 8$

$(x - 4)(x + 2)$

$= x^2 - 4x + 2x - 8$

$= x^2 - 2x - 8$

$(x - 1)^2 - 9 = (x - 4)(x + 2)$

$(x - 4)(x + 2) = 0$

$x = 4$ and $x = -2$

Average of 4 and -2 $= \frac{4 + -2}{2} = 1$

Yes, this is the same value.

(e) $y = (x - 1)^2 - 9 = (1 - 1)^2 - 9 = -9$

Yes, this is the same value.

(f) $a = 1$, $b = -2$, $c = -8$

$-\frac{b}{2a} = -\frac{-2}{2} = 1$

Yes, this is the same value.

8.4 Sketching parabolas using transformations

Answers page 482

Stage 5.3 §

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 1 & 2, Q8 Columns 1 & 2, Q9, Q10, Q11, Q12, Q13 (a)–(h), Q14, Q17, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 Columns 2 & 3, Q9, Q10, Q11, Q12, Q13 (a)–(h), Q14, Q17, Q18, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 3 & 4, Q8 Columns 3 & 4, Q9, Q10, Q11, Q12, Q13 (i)–(p), Q14, Q15, Q16, Q17, Q18, Q20

Fluency and understanding

1 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$. WE14

(b) Plot the graphs of **(i)** $y = -2x^2$, **(ii)** $y = 3x^2$ and **(iii)** $y = \frac{x^2}{4}$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$ using the words narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

2 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$. WE15

(b) Plot the graphs of **(i)** $y = (x-2)^2$ and **(ii)** $y = (x+1)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

3 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$. WE16

(b) Plot the graphs of **(i)** $y = x^2 + 4$ and **(ii)** $y = x^2 - 2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the y-intercept for each graph in part **(b)** and identify the y-intercept from the equation.

4 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$. WE17

(b) Plot the graphs of **(i)** $y = (x-1)^2 - 2$ and **(ii)** $y = (x+2)^2 + 1$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

5 **(a)** Plot the graph of $y = (x-2)^2 - 1$ for $-5 \leq x \leq 5$. WE18

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x-h)^2 + k$, write down the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x-1)(x-3)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write down the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

6 **(a)** State whether the graph of each of these equations will be narrower or wider than the parabola $y = x^2$.

(i) $y = 5x^2$ **(ii)** $y = \frac{7}{8}x^2$ **(iii)** $y = -\frac{1}{6}x^2$ **(iv)** $y = -14x^2$

(b) Which of the graphs of the equations in part **(a)** would be upside down (inverted) compared with $y = x^2$?

(c) State the dilation factor of each of the following.

(i) $y = 4x^2$ **(ii)** $y = \frac{1}{5}x^2$ **(iii)** $y = 7x^2 + 2x$ **(iv)** $y = \frac{2}{3}x^2 - 3x + 1$

(d) Will a parabola be narrower or wider than $y = x^2$ if it has a dilation factor of:

(i) 3 **(ii)** $\frac{4}{5}$ **(iii)** 16 **(iv)** $\frac{1}{7}$?

7 How is the graph of $y = x^2$ translated (moved) to obtain each of the following parabolas?

(a) $y = x^2 + 4$ **(b)** $y = x^2 - 7$ **(c)** $y = x^2 - 1$ **(d)** $y = x^2 + 2$

(e) $y = x^2 - 8$ **(f)** $y = x^2 + 5$ **(g)** $y = x^2 + \frac{1}{2}$ **(h)** $y = x^2 - \frac{3}{4}$

8 How is the graph of $y = x^2$ translated (moved) to obtain each of the following parabolas?

(a) $y = (x - 5)^2$ **(b)** $y = (x + 4)^2$ **(c)** $y = (x + 1)^2$ **(d)** $y = (x - 7)^2$

(e) $y = (x + 6)^2$ **(f)** $y = (x - 3)^2$ **(g)** $y = (x - \frac{2}{3})^2$ **(h)** $y = (x + \frac{3}{2})$

9 For each of the following quadratic relationships:

(i) find the transformations of the graph $y = x^2$ that have taken place; and

(ii) find the turning point.

(a) $y = 2(x - 1)^2$ **(b)** $y = -(x + 2)^2$ **(c)** $y = (x + 3)^2 - 2$

(d) $y = (x + 5)^2 + 3$ **(e)** $y = (x + 4)^2 - 3$ **(f)** $y = -2(x + 3)^2$

10 **(a)** To obtain the graph of $y = x^2 + 6$ we need to move the graph of $y = x^2$:

A 6 units to the right **B** 6 units to the left

C 6 units up **D** 6 units down

(b) To obtain the graph of $y = (x + 6)^2$ we need to move the graph of $y = x^2$:

A 6 units to the right **B** 6 units to the left

C 6 units up **D** 6 units down

(c) To obtain the graph of $y = (x - 2)^2$ we need to move the graph of $y = x^2$:

A 2 units to the right **B** 2 units to the left

C 2 units up **D** 2 units down

(d) To obtain the graph of $y = x^2 - 4$ we need to move the graph of $y = x^2$:

A 4 units to the right **B** 4 units to the left

C 4 units up **D** 4 units down

11 To obtain the graph of $y = (x - 1)^2 - 3$, we need to move the graph of $y = x^2$:

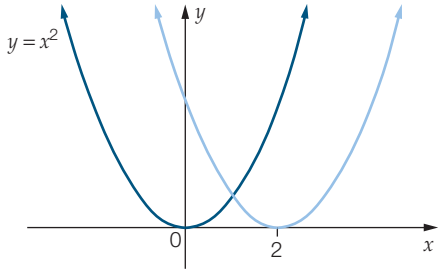
A 1 unit to the right and 3 units down

B 3 units to the right and 1 unit down

C 1 unit to the left and 3 units up

D 3 units to the right and 1 unit down

12 (a) The equation of the unlabelled parabola shown could be:



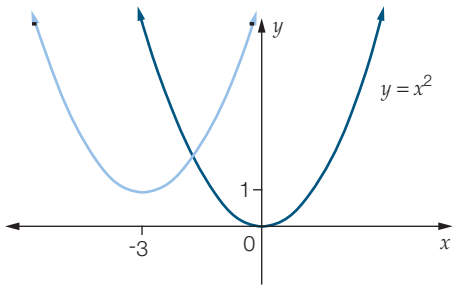
A $y = x^2 + 2$

B $y = x^2 - 2$

C $y = (x - 2)^2$

D $y = (x + 2)^2$

(b) The equation of the unlabelled parabola shown could be:



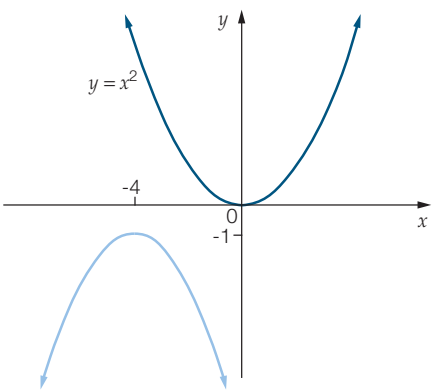
A $y = x^2 + 1$

B $y = (x + 3)^2 + 1$

C $y = 3x^2 + 1$

D $y = (x - 3)^2 + 1$

(c) The equation of the unlabelled parabola shown could be:



A $y = (x + 4)^2 - 1$

B $y = -(x + 4)^2 - 1$

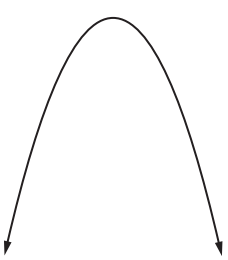
C $y = (x - 4)^2 - 1$

D $y = 4(x - 4)^2 - 1$

13 State the changes that should be made to the graph of $y = x^2$ to obtain the graph of each of the following.

		Dilation	Reflection in *x*-axis	Translation in *x* direction	Translation in *y* direction
(a)	$y = (x + 1)^2 + 2$				
(b)	$y = (x - 3)^2 + 1$				
(c)	$y = 5x^2 + 4$				
(d)	$y = -x^2 - 6$				
(e)	$y = -(x - 2)^2 + 3$				
(f)	$y = (x - 4)^2 - 5$				
(g)	$y = (x + 2)^2 - 7$				
(h)	$y = 3x^2 - 5$				
(i)	$y = -x^2 - 2$				
(j)	$y = -(x - 5)^2 + 2$				

14 When graphed, how many of the following equations would have a shape like that of the parabola shown here?



Identify them.

(a) $y = x^2 + 2x - 5$

(b) $y = -x^2 - 2x + 4$

(c) $y = 1 - x^2$

(d) $y = 3 - 4x + x^2$

(e) $y = 6 - 2x - x^2$

15 The turning point of a parabola with the equation $y = (x - 2)^2 + 4$ is:

A (-2, -4) **B** (-2, 4) **C** (2, 4) **D** (2, -4)

Reasoning, communicating and open-ended

16 **(a)** A quadratic graph has a turning point at (-2, 5) and a dilation factor of 1. Find the equation of the graph.

(b) Find the y-intercept (where $x = 0$).

(c) Draw the graph, clearly labelling the turning point and the y-intercept.

17 Find the new equation when the following transformations are completed on the graph $y = x^2$.

(a) The graph is translated 3 units to the right and 2 units up.

(b) The graph is translated down 2 units and to the right 4 units.

18 **(a)** The sum of two numbers, x and y, is 20. Express y in terms of x.

(b) Write the product of x and y, P, in terms of x and sketch a graph of P.

(c) Use the graph to find the values of x and y that will make the product as large as possible.

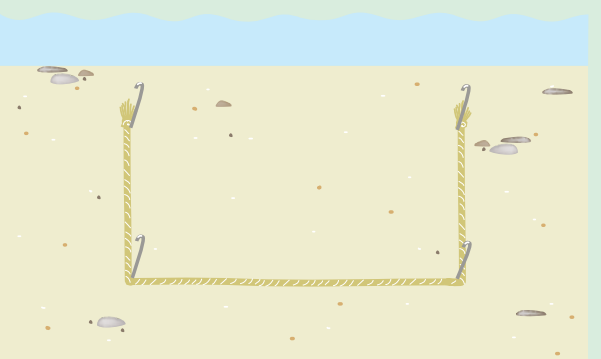
19 Write down at least two equations that show a quadratic relationship in the form $y = (x - h)^2 + k$. For each one, write down the transformations that the graph $y = x^2$ must undergo to be the graph of the relationship you have written.

20 Write equations for three parabolas that have no x-intercepts.

Problem solving

Let's go camping

You decide to go camping at a camping ground by a lake. The manager of the camping ground has designed a novel way of determining individual campsites. She gives you a 60-m long piece of rope and four pegs. The only condition is that your campsite must be rectangular. You decide to pitch your tent by the lake and only need to use the rope on three sides. Find the largest campsite that you could make.



Strategy options

- Test all possible combinations.
- Make a table.
- Make a model.
- Draw a diagram.

Half-time 8



Stage 5.1 Q1 **Stage 5.2 Q2–7** **Stage 5.3 Q8–10**

1 Plot the graphs of each of the following equations using an appropriate table of values. Ex 8.1

(a) $x^2 + y^2 = 144$ **(b)** $y = x^2 - 7$ **(c)** $y = 2x + 3$

2 Which of the following is not a quadratic expression? Ex 8.2

A $2x^2 + 3x + 5$ **B** $-x + 3x^2$ **C** $3x + 2$ **D** $4x^2 - 3x - 1$

3 Solve each of the following. Ex 8.3

(a) $x(x + 4) = 0$ **(b)** $2x(x - 3) = 0$ **(c)** $(x + 2)(x - 3) = 0$

4 Lisa throws a ball upwards. The equation of the flight of the ball is $h = -5t^2 + 8t + 1.3$, where h is the height of the ball in metres and t is the time in seconds. Ex 8.2

(a) Complete the table of values below.

t	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
h									

(b) Draw a graph showing this relationship.

(c) From the graph, find when the ball first reaches a height of 3 m.

(d) How high is the ball after 1.2 seconds?

5 Solve each of the following. Ex 8.3

(a) $x^2 + 4x + 4 = 0$ **(b)** $x^2 + 12x + 36 = 0$ **(c)** $x^2 - 18x + 81 = 0$

6 Solve each of the following. Ex 8.3

(a) $x^2 - 6x = 0$ **(b)** $2x^2 + 20x + 50 = 0$ **(c)** $3x^2 = 27x$

7 Solve each of the following. Ex 8.3

(a) $x^2 - 64 = 0$ **(b)** $(x + 1)^2 = 0$ **(c)** $(x - 2)^2 = 4$

8 **(a)** Complete a table of values for $y = 2x^2 + 4x$ from $x = -3$ to $x = 3$ inclusive. Ex 8.4

(b) Plot the points on a number plane and join them using a smooth curve.

(c) Write down the coordinates of the turning point.

(d) State whether the parabola has a maximum or a minimum turning point.

(e) Write down the coordinates of the x- and y-intercepts.

9 **(a)** Plot the graph of $y = x^2 - 2x + 3$. Use integer values of x between -3 and 5 inclusive. Ex 8.4

(b) What are the coordinates of the turning point?

(c) Write down the coordinates of any x- and y-intercepts.

(d) Use the graph to solve the following equations.

(i) $x^2 - 2x + 3 = 3$ **(ii)** $x^2 - 2x + 3 = 2$

(iii) $x^2 - 2x + 3 = 6$ **(iv)** $x^2 - 2x + 3 = 0$

10 Consider the equation $y = -x^2 + 1$. Ex 8.4

(a) Set up a table of values using integer values of x between -3 and 3 inclusive.

(b) Plot the points on a number plane and join them with a smooth curve.

(c) Find the coordinates of the turning point.

(d) Find the coordinates of the y-intercept.

(e) Find the coordinates of the x-intercepts.

8.5 Circle relationships

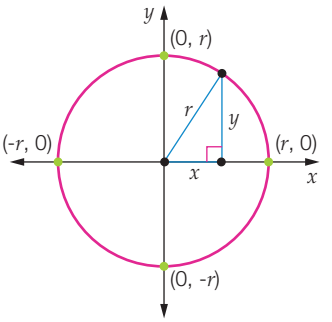
Stage 5.2◊

Need to Know

Circle equations are important non-linear relationships.

Circles with centre at the origin

All points on the circle are the same distance from the centre of the circle, the origin, so the graph of a circle and Pythagoras' Theorem can be used to derive the equation of a circle with the centre at the origin.



The equation of a circle with centre (0, 0) and radius r is:

$$x^2 + y^2 = r^2$$

In circle equations, both the x and y variables are squared.

Graphing a circle from its equation

Find the radius of the circle, then find four points on the circle circumference to give maximum and minimum values by adding and subtracting the radius from the centre coordinates (0, 0).

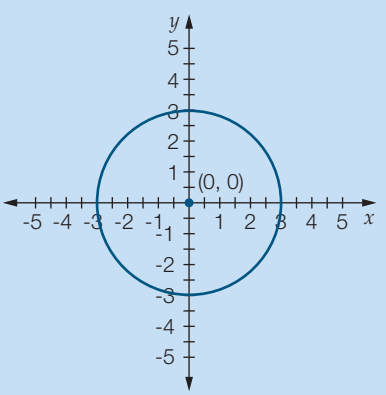
For a circle with centre (0, 0) and radius r, the four points are $(r, 0)$, $(0, r)$ $(-r, 0)$ and $(0, -r)$.

Worked Example 19

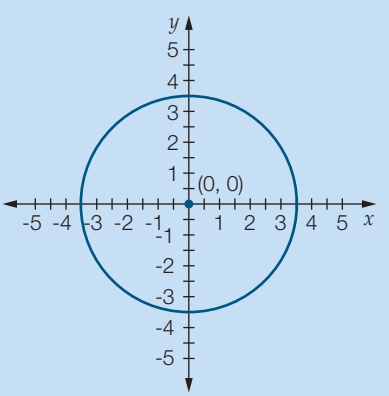
WE19

Determine the centre and radius of each of the following, and hence the equation of the circle.

(a)



(b)



Working

(a) Centre = (0, 0), Radius = 3

Equation

$x^2 + y^2 = r^2$

$x^2 + y^2 = 3^2$

$x^2 + y^2 = 9$

(b) Centre = (0, 0), Radius = 3.5

Equation

$x^2 + y^2 = r^2$

$x^2 + y^2 = 3.5^2$

$x^2 + y^2 = 12.25$

8.5 Circle relationships

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Stage 5.2◊

Navigator

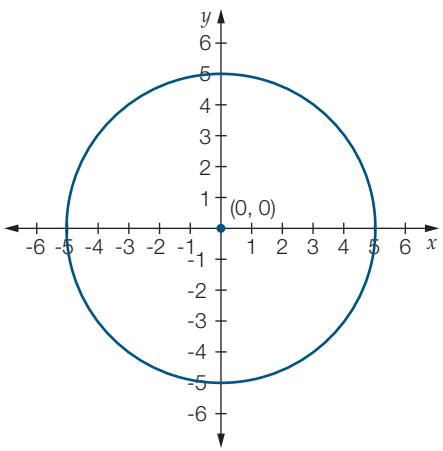
Q1, Q2, Q3, Q4 (a) & (b), Q5, Q6, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9

Fluency and understanding

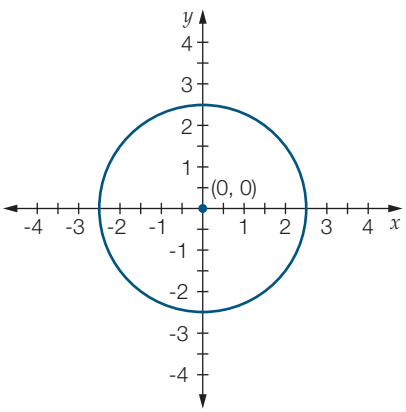
WE19

1 Determine the centre and radius of each of the following, and hence the equation of the circle.

(a)



(b)



2 Find the radius of the following circles.

(a) $x^2 + y^2 = 4$ **(b)** $x^2 + y^2 = 25$

(c) $x^2 + y^2 = 49$ **(d)** $x^2 + y^2 = 81$

(e) $x^2 + y^2 = 6.25$ **(f)** $x^2 + y^2 = 1.96$

3 Graph each of the circles in question **2** on separate number planes.

4 Find the equation of a circle with its centre at the origin if its radius is:

(a) 2 units **(b)** 8 units **(c)** 0.6 units

(d) 1.2 units **(e)** $\frac{1}{2}$ unit **(f)** $1\frac{3}{5}$ units

5 Which point lies on the circle with equation $x^2 + y^2 = 25$?

A (2, 3) **B** (3, -4) **C** (2, -3) **D** (1, 4)

6 Decide whether the equation would graph as a circle in each of the following. Answer Yes or No.

(a) $y - x^2 = 4$ **(b)** $y^2 = x^2 - 9$ **(c)** $x^2 = 1 - y^2$

(d) $y = 25 - x^2$ **(e)** $x^2 + y^2 = 7$ **(f)** $y^2 = x + 4$

Reasoning, communicating and open-ended

7 **(a)** Explain how to determine whether a point lies inside, on or outside a circle.

(b) Use this reasoning to decide whether the following points lie inside, on or outside the circle with equation $x^2 + y^2 = 13^2$.

(i) (3, 10) **(ii)** (-2, 3) **(iii)** (5, -12) **(iv)** (15, -2)

8 Find the equation of a circle with its centre the origin that passes through the point (-8, -15).

9 List two points that lie on the circle $x^2 + y^2 = 100$ but do not lie on the x- or y-axes.

8.6 Circle transformations

Stage 5.3 §

Need to Know

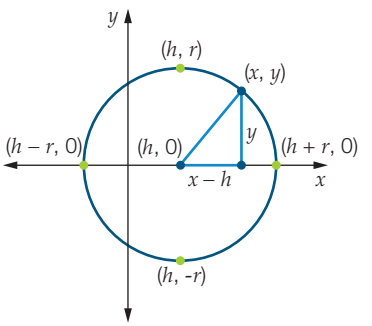
Translated circles

A circle equation can be translated horizontally.

If the circle centre is moved h units to the right, it will have the coordinates $(h, 0)$, and all points on the circle will move h units to the right.

The equation becomes $(x - h)^2 + y^2 = r^2$.

The four points on the circumference would be $(h + r, 0)$, (h, r), $(h - r, 0)$ and $(h, -r)$.

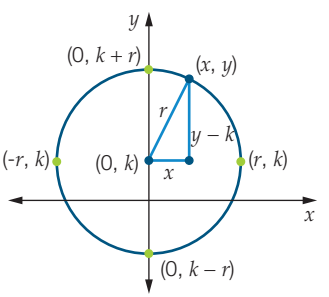


A circle equation can be translated vertically.

If the circle centre is moved k units up, it will have the coordinates $(0, k)$ and all points on the circle will move k units up.

The equation becomes $x^2 + (y - k)^2 = r^2$.

The four points on the circumference would be (r, k), $(0, k + r)$, $(-r, k)$ and $(0, k - r)$.

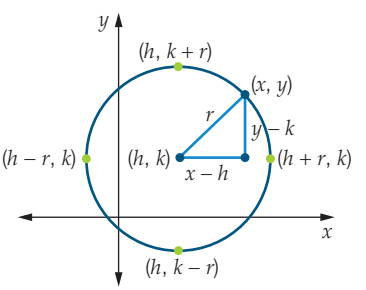


A circle can be translated both horizontally and vertically.

If the circle centre is moved h units to the right and k units up, it will have the coordinates (h, k) and all points on the circle will move h units to the right and k units up.

The equation becomes $(x - h)^2 + (y - k)^2 = r^2$.

The four points on the circumference would be $(h + r, k)$, $(h, k + r)$, $(h - r, k)$ and $(h, k - r)$.



The important features of a circle are:

- the centre with coordinates (h, k)
- the radius, r.

Circles with centre the origin can be translated by h units to the right and k units up. The new centre has coordinates (h, k) but the radius remains the same.

The general equation of a circle is given by $(x - h)^2 + (y - k)^2 = r^2$.

Technology Exploration: GeoGebra

Investigating circles

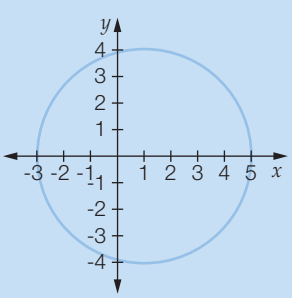
1. Open a new GeoGebra window and set it up as for the Technology Exploration in section 8.4. Select the Circle with Centre and Radius tool, and use this tool to draw a circle with radius 4 and centre at the origin. Use the New Point tool to insert a point on the circle in each quadrant. Use the coordinates of each point, as given in the Algebra view, to show that $x^2 + y^2 = 16$. If you do not get 16 exactly, explain why.
2. Use the Input bar to enter the equation $(x–h)^2 + (y–k)^2 = r^2$. Set up three new sliders and call them h, k and r, and set the r values from 0 to 5. Set both h and k values to zero and r to 4. What do you see? Explain.
3. Leave the k value on 0 and the r value on 4. Change the slider value of h from -5 to 5. Write what you see.
4. Now set the value of h to 0 and change the slider value of k from -5 to 5. Write what you see.
5. What happens when neither h nor k is zero? Choose integer values for each of h and k. Write these values down and write the centre of the circle that is drawn. What do you see?
6. Use the New Point tool to insert a point on this circle in each quadrant. Use the coordinates of each point (given in the Algebra view) to show that $(x-h)^2 + (y-k)^2 = 16$. If you do not get 16 exactly, explain why.
7. Set the values of h and k to zero and change the value of r from 0 to 5. Describe what happens.

Worked Example 20

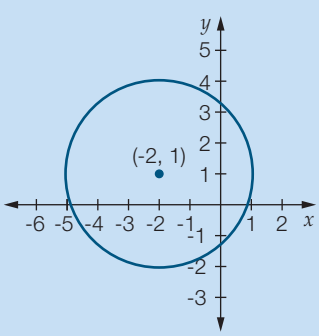
WE20

Determine the centre and radius of each of the following, and hence the equation of the circle.

(a)



(b)



Working

(a) Centre = (h, k)
Centre = (1, 0)
Radius = 5 – 1
= 4
$(x-h)^2 + (y-k)^2 = r^2$
$(x-1)^2 + y^2 = 16$

(b) Centre = (h, k)
Centre = (-2, 1)
Radius = (1 – -2)
= 3
$(x-h)^2 + (y-k)^2 = r^2$
$(x+2)^2 + (y-1)^2 = 9$

Worked Example 21

WE21

Sketch each of the following graphs of circles.

(a) $(x+1)^2+(y+2)^2=9$ **(b)** $x^2+(y-2)^2=6$

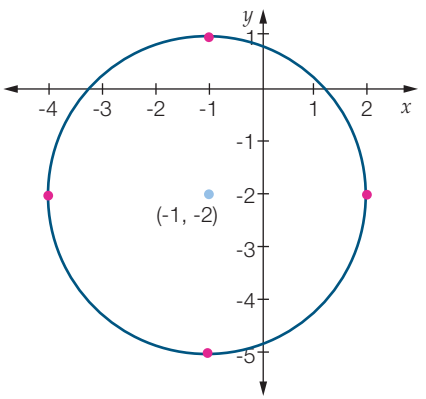
Working

(a) $(x+1)^2+(y+2)^2=9$

Centre: (-1, -2)

Radius: $\sqrt{9}=3$

Points (2, -2) (-1 ,1) (-4, -2) (-1, -5)



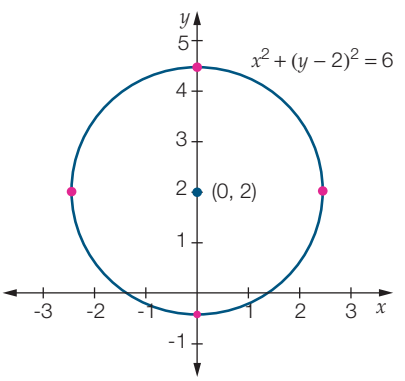
(b) $x^2+(y-2)^2=6$

Centre: (0, 2)

Radius = $\sqrt{6}$

Points: $(\sqrt{6}, 2)$ $(0, \sqrt{6}+2)$ $(-\sqrt{6}, 2)$, $(0, \sqrt{6}-2)$

$\approx$ (2.45, 2), (0, 4.45), (-2.45, 2), (0, -0.45)



Worked Example 22

WE22

A path around an area on a coordinate map is described by the equation $(x-3)^2+(y-2)^2=9$.

(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

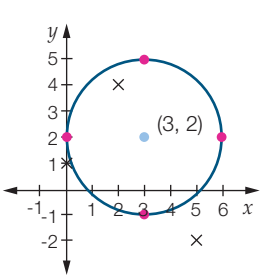
(i) (0, 1) **(ii)** (2, 4) **(iii)** (5, -2)

Working

(a) Centre (3, 2) and radius = 3

(b) (0, 2), (6, 2) (3, -1) (3, 5)

(c)



(d) **(i)** (0, 1) is outside the circle.
(ii) (2, 4) is inside the circle.
(iii) (5, -2) is outside the circle.

8.6 Circle transformations

Answers page 487

Stage 5.3 §

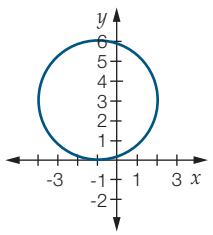
Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11

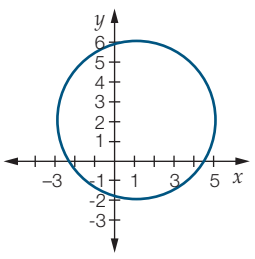
Fluency and understanding

1 Determine the centre and radius of each of the following, and hence the equation of the circle. WE 20

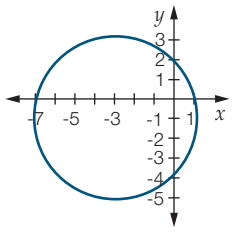
(a)



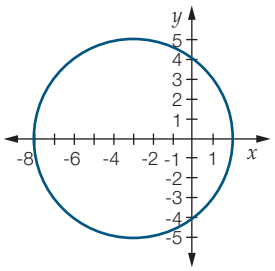
(b)



(c)



(d)



2 Sketch each of the following graphs of circles. WE 21

(a) $(x+2)^2+(y+3)^2=25$ **(b)** $(x-2)^2+(y-1)^2=16$

(c) $(x+1)^2+(y-1)^2=4$ **(d)** $x^2+(y-1)^2=9$

(e) $(x+2)^2+y^2=7$ **(f)** $(x-3)^2+(y+1)^2=1$

3 A path around an area on a coordinate map is described by the equation $(x+1)^2+(y-3)^2=16$. WE 22

(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

(i) (1, 2) **(ii)** (3, 6) **(iii)** (-2, 0)

4 Which of the following equations would graph as a circle?

A $x^2+y=4$ **B** $x^2+(y-3)^2=5$ **C** $y^2=x^2+9$ **D** $y^2=x-3$

5 Write down the equation if the circle with equation $x^2+y^2=36$ undergoes the following transformations.

(a) a move of 4 units to the right and 3 units up

(b) a move of 6 units to the left and 4 units up

(c) a move of 7 units to the right

(d) a move of 5 units down

(e) a dilation to a radius of 7

6 What are the centre coordinates and radius of a circle with the equation $(x-5)^2+(y+3)^2=81$?

A (5, 3), 9 **B** (-5, 3), 9 **C** (5, -3), 81 **D** (5, -3), 9

Reasoning, communicating and open-ended

7 A circle has an equation $(x+4)^2+(y+2)^2=25$. What would be the equation of this circle if it is translated 2 units left and 6 units up and the radius is decreased by 1 unit?

8 **(a)** Show, by substitution, that the equations $(x-8)^2+(y+8)^2=169$, $(x+9)^2+(y-9)^2=169$ and $x^2+y^2=25$ are three circles that intersect at (3, 4) and (-4, -3).

(b) Draw a graph of each of these three circles on the same set of axes.

9 A circular path with a radius of 5 metres has its centre 2 metres east and 3 metres north of a sculpture. It has been decided to move the path 9 metres further east and 1 metre further north and that its radius is to be increased by 8 metres.

(a) Write an equation to represent the original circle if the sculpture is positioned at the origin.

(b) Write an equation to represent the circle after it has been moved.

(c) Show that a point that is 1 metre west and 1 metre south of the sculpture lies on both circles.

10 **(a)** Write down two equations that would graph as circles with a centre at the origin.

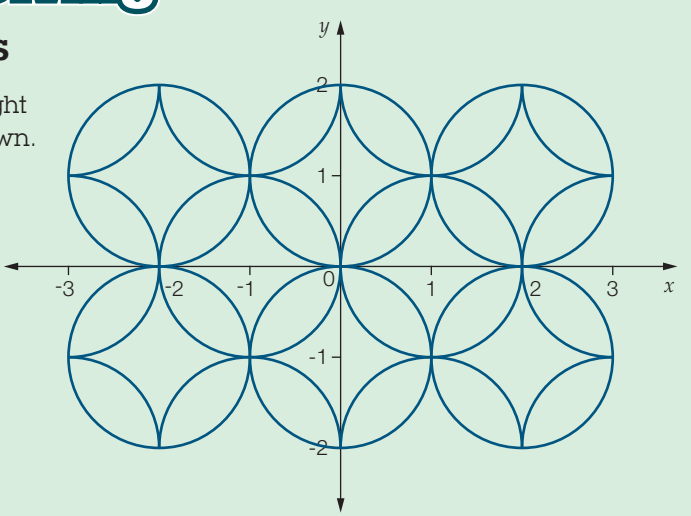
(b) Write down two equations that would graph as circles with the same radius but with centres in different quadrants.

11 Make a design with three overlapping circles and write down the equations for the circles you used in your design.

Problem solving

Circles and circles

Find the equations for the eight full circles in the pattern shown.



Strategy options

- Draw a diagram.
- Solve a simpler problem.
- Look for a pattern.
- Guess and check.

Exponential relationships

8.7

Stage 5.2◊

Need to Know

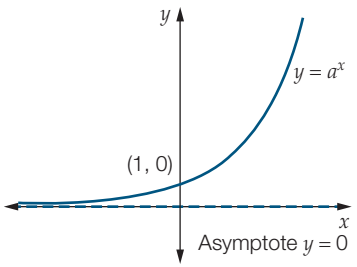
Exponential relationships

An exponential relationship is a relationship between two quantities that involves an **exponent** (or **index**), where the exponent is a variable. Exponential graphs always go in the same direction, either continually getting bigger or continually getting smaller, but, unlike straight lines, there is a limiting feature to this. Exponential graphs will always have a value that they will get closer and closer to but never reach. This value is called the **asymptote** of the graph.

The simplest exponential equations are in the form $y = a^x$ where a is the base and is any non-zero number and x is the index or exponent.

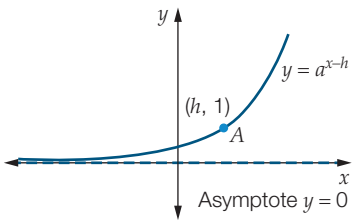
Because $a^0 = 1$ for all values of a except 0, all these exponential graphs will have a y-intercept of (0, 1).

As x gets smaller (i.e. negatively large), the value of y gets closer and closer to 0, but it never reaches it. The line with the equation $y = 0$ (the x-axis) is the asymptote.

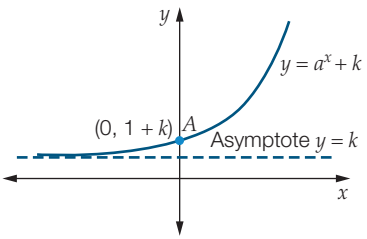


Translations of exponential graphs

An exponential graph with equation $y = a^x$ can be translated k units to the right to give the equation $y = a^{x-h}$. The asymptote remains $y = 0$.

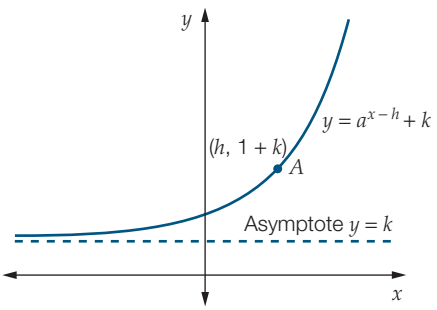


An exponential graph with an equation $y = a^x$ can be translated k units up to give the equation $y = a^x + k$. Here the asymptote becomes $y = k$.



The general equation of an exponential relationship is $y = a^{x-h} + k$, where

- a is the base
- x is the exponent
- h indicates the translation to the right
- k indicates the translation upwards
- $y = k$ is the asymptote.



Worked Example 23

WE 23

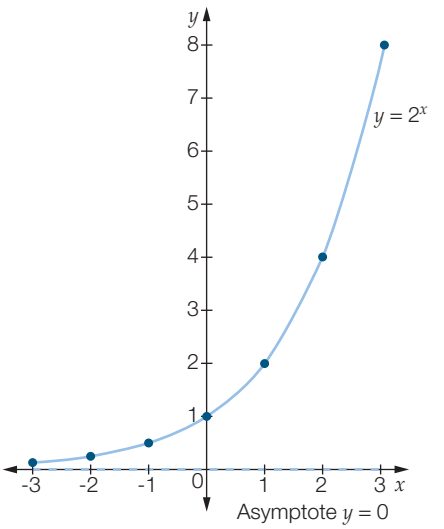
Plot the following graphs by setting up tables of values from -3 to 3 inclusive.

(a) $y = 2^x$ **(b)** $y = 2^{x+1}$ **(c)** $y = 2^x + 3$

Working

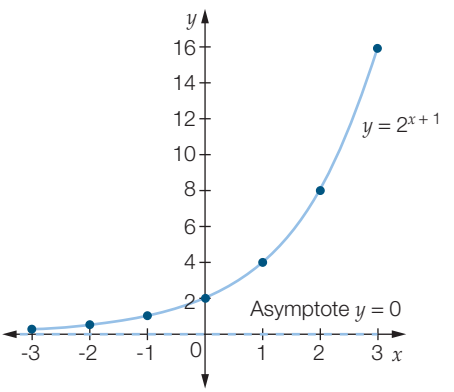
(a)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



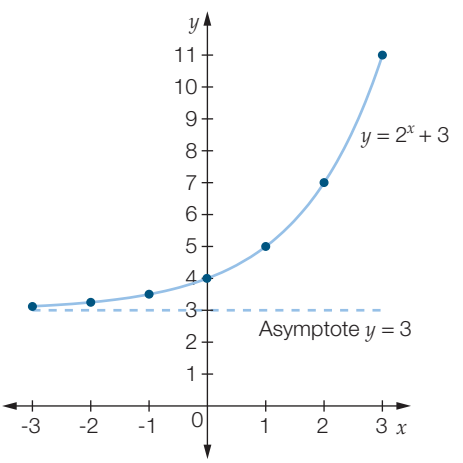
(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



(c)

x	-3	-2	-1	0	1	2	3
y	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	5	7	11



Worked Example 24

WE24

Bacteria in a culture are multiplying rapidly. Each bacterial cell divides into two, then each of these cells divides into two and so on. This can be expressed in the form $N = 2^t$ where N is the number of bacteria and t is the time in hours.

(a) For the relationship $N = 2^t$, copy and complete the table of values for t and N.

t	0	1	2	3	4	5	6
N							

(b) Use this table of values to plot a graph of $N = 2^t$. Join the points with a smooth curve.

(c) Use your graph to determine how long it takes for the number of bacteria to exceed 40.

(d) Evaluate N for $t = 5.3$ (give your answer correct to the nearest integer) and explain why this value is meaningless in this situation.

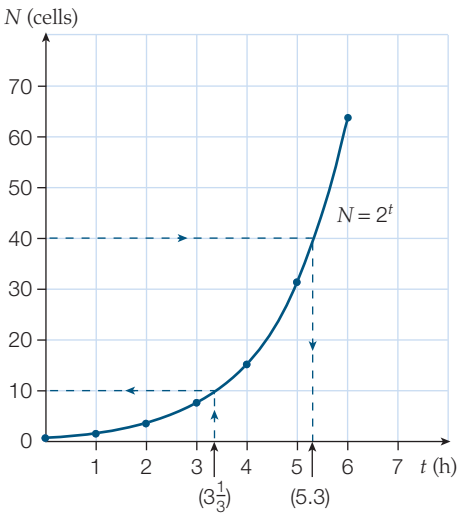
(e) Determine how many bacteria there are after 3 hours 20 minutes.

Working

(a)

t	0	1	2	3	4	5	6
N	1	2	4	8	16	32	64

(b)



(c) From the graph, when $N = 40$, $t = 5.3$ hours. However, it takes a full hour for the 32 cells present at $t = 5$ to become 64 cells, so $t = 6$.

(d) The answer using a calculator is ≈ 39.4, but cell division only takes place at the end of each hour, so this value is meaningless in this situation.

(e) At 3 hours 20 minutes there will still only be 8 cells, as the next cell division does not occur until $t = 4$.

Technology Exploration: GeoGebra

Investigating exponentials

1 Open a new GeoGebra window and set it up as for the GeoGebra activity in section 8.4. Use the Input bar to enter the equation $y = a\text{^}(h) + k$. Set up three new sliders as you did in previous GeoGebra activities and call them h, k and a, and set the a values from 0 to 5. Set both h and k values to 0 and set a to 1. What do you see?

2 Leave the k value on 0 and the a value on 1. Change the slider value of h from -5 to 5. Write what you see.

3 Now set the value of h to 0 and change the slider value of k from -5 to 5. Write what you see.

4 What happens when neither h nor k is zero? Choose integer values for each of h and k. Write these values down and write the equation of the asymptote formed. What do you see?

5 Set the values of h and k to 0 and change the value of a from 0 to 5. Describe what happens.

Answers page 488

8.7 Exponential relationships

Stage 5.2

Navigator

Q1, Q2, Q3, Q4, Q6	Q1, Q2, Q3, Q4, Q6	Q1, Q2, Q3, Q4, Q5, Q6

Equipment required: graph paper for Question **6**

Fluency and understanding

WE23

1 Plot the following graphs by setting up tables of values from -3 to 3 inclusive.

(a) $y = 3^x$ **(b)** $y = 3^{x+1}$ **(c)** $y = 3^x - 2$

WE24

2 A gardener decided to propagate his orchid (make new plants). He did this by dividing the bulb on his plant after it finished flowering into three. He then planted these smaller bulbs giving him three orchids in total. He continued to do this at the same time every year with every orchid he had grown from the original orchid. This situation can be modelled by the equation $P = 3^t$ where P is the number of orchids he has and t is the time in years.

(Assume that a healthy orchid grows from every divided bulb and all orchids survive for at least 6 years.)

(a) For the relationship of orchids to the time in years where $P = 3^t$, copy and complete the table of values below.

t (years)	0	1	2	3	4	5	6
P (orchids)							

(b) Use this table of values to plot a graph of $P = 3^t$. Join the points with a smooth curve.

(c) Use your graph to determine how many years it takes for the number of orchids to exceed 125.

(d) Evaluate P for $t = 3.8$ (give your answer correct to the nearest integer) and explain why this value is meaningless in this situation.

(e) Determine how many orchids there are after $5\frac{1}{2}$ years.

3 **(a)** Plot the graphs of $y = 2^x$ and $y = 3^x$ on the same set of axes.

(b) By comparing asymptotes, y-intercepts and shape, describe the similarities and differences between the two graphs.

4 An exponential graph of $y = 5^x$ is shifted 4 units left and 3 units down. Which of the following is the equation of the new graph?

A $y = 5^{(x-4)} + 3$ **B** $y = 5^{(x-4)} - 3$ **C** $y = 5^{(x+4)} + 3$ **D** $y = 5^{(x+4)} - 3$

Reasoning, communicating and open-ended

5 The population of a town has been increasing at a rate of 10% each year since the beginning of 2005. This can be modelled by the equation $P = 2000 \times 1.1^t$ where P is the number of people in the town t years after the beginning of 2005. Initially, when $t = 0$, the population numbered 2000.

(a) Set up a table to show the population of the town for the first 10 years after 2005.

(b) Draw a graph showing this information.

(c) From the graph, approximately how long did it take for the town to double in size?

(d) Jason said it would take 10 years for the population to double. Explain why this isn't the case.

6 Substitute your own values for h and k into the equation of an exponential in the form of $y = 2^{(x-h)} + k$ and draw a graph of this relationship.

Problem solving

How many triangles?

△

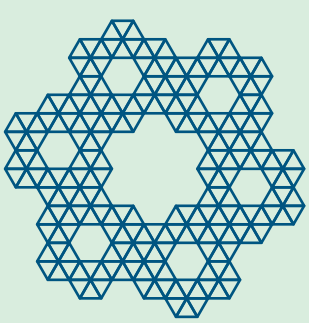
Six equilateral triangles are joined to make a hexagon.



Six hexagons are joined to make the hexagon pattern as shown.



Six of these hexagon patterns are joined to make a larger pattern.



Six of these larger hexagon patterns are joined in the same way to make an even larger pattern. How many triangles would there be in this pattern?

Strategy options

- Look for a pattern.
- Make a table.

8.8 Direct proportion

Stage 5.2

Need to Know

The simplest type of relationship between two continuous quantities is one in which any change in one quantity results in an identical change to the other quantity. For example, if we double one quantity, the other will double also; halve one quantity and the other will halve also. In this case we say that one quantity is in direct proportion to the other quantity.

The relationship between two quantities, say a and b, is one of **direct proportion** when a increases as b increases and the graph of the relationship passes through the origin.

$a \propto b$ means 'a is directly proportional to b'.

$a \propto b$ is not the same as $a = b$.

In general, there is direct proportion between two variables, x and y, when

$y \propto x^n$ where n is any positive number;

or, as an equation, when

$y = kx^n$ where k is the **constant of proportionality**

Two variables are in direct proportion if the graph of y plotted against x^n is a linear graph passing through the origin, (0, 0).

As x increases, y increases.

Worked Example 25

WE 25

Which of the following could be examples of direct proportion?

(a)

x	1	3	6	8
y	2	6	12	18

(b)

s	1	2	3	5
t	2	8	18	50

(c)

v	1	2	3	4
w	4	2	1	0.5

(d)

x	1	2	3	4
y	5	7	7	3

Working

(a) Could be direct proportion. (y is increasing as x is increasing.)

(b) Could be direct proportion. (t is increasing as s is increasing.)

(c) Not direct proportion. (w is decreasing as v is increasing.)

(d) Not direct proportion. (q is fluctuating as p is increasing.)

Worked Example 26 WE26

Find the equation for each of the following tables of values.

(a)

x	1	3	6	8
y	2	6	12	16

(b)

s	0	1	2	3	5
t	0	2	8	18	50

(c)

a	0	1	4	9
b	0	3	6	9

Working

(a) $\frac{y}{x} = \frac{2}{1} = \frac{6}{3} = \frac{12}{6} = \frac{16}{8} = 2$

$\therefore k = 2$

$y \propto x$

$y = kx$

$y = 2x$

(b) $\frac{t}{s} = \frac{2}{1} = 2, \quad \frac{t}{s} = \frac{8}{2} = 4, \quad \frac{t}{s} = \frac{18}{3} = 6,$

$\frac{t}{s} = \frac{50}{5} = 10$

s^2	0	1	4	9	25
t	0	2	8	18	50

$\frac{t}{s^2} = \frac{2}{1} = \frac{8}{4} = \frac{18}{9} = \frac{50}{25} = 2$

$\therefore k = 2$

$t \propto s^2$

$t = ks^2$

$t = 2s^2$

(c) $\frac{b}{a} = \frac{3}{1} = 3, \quad \frac{b}{a} = \frac{6}{4} = \frac{3}{2}, \quad \frac{b}{a} = \frac{9}{9} = 1$

$\sqrt{a}$	0	1	2	3
b	0	3	6	9

$\frac{b}{\sqrt{a}} = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = 3$

$\therefore k = 3$

$b \propto \sqrt{a}$

$b = k\sqrt{a}$

$b = 3\sqrt{a}$

Answers page 489

8.8 Direct proportion

Stage 5.2

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Fluency and understanding

WE25

1 Which of the following could be examples of direct proportion?

(a)

v	1	2	4	5
w	7	3	2	1

(b)

p	1	2	3	4
q	3	4	5	0

(c)

x	1	2	3	4
w	0	1	2	4

(d)

s	1	2	6	5
t	5	10	30	25

WE26

2 Find the equation for each of the following tables of values.

(a)

a	0	1	2	3
b	0	4	8	12

(b)

w	0	1	2	3
v	0	5	10	15

(c)

c	0	1	4	6
d	0	0.5	2	3

(d)

g	0	4	6	8
h	0	1	1.5	2

(e)

e	0	1	2	3
h	0	3	6	9

(f)

p	0	2	4	6
q	0	3	6	9

(g)

i	1	2	3	0
j	2	8	18	0

(h)

a	0	1	2	3
d	0	3	12	27

(i)

m	0	1	2	3
n	0	4	32	108

(j)

g	1	2	3	0
h	2	16	54	0

(k)

p	0	1	2	3
q	0	0.5	2	4.5

(l)

b	0	1	2	3
f	0	0.2	0.8	1.8

(m)

r	0	1	2	3
s	0	3	24	81

(n)

p	0	1	2	3
m	0	0.25	2	6.75

(o)

t	0	1	2	3
u	0	0.5	4	13.5

(p)

v	0	1	4	9
w	0	2	4	6

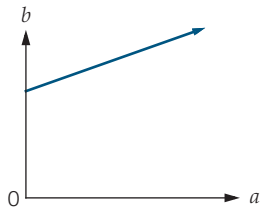
(q)

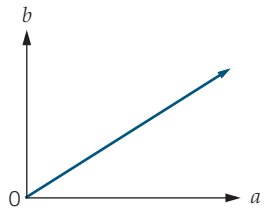
u	0	1	4	9
t	6	0.1	0.2	0.3

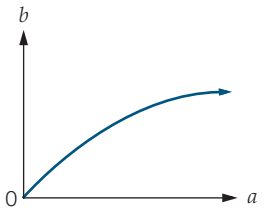
(r)

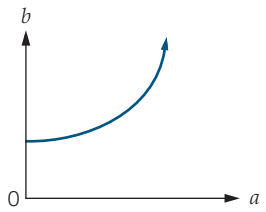
d	0	1	4	9
l	0	$\frac{1}{3}$	$\frac{2}{3}$	1

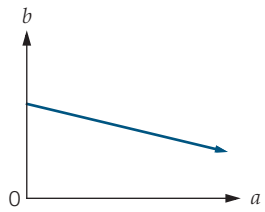
3 State whether or not each graph could be an example of direct proportion.

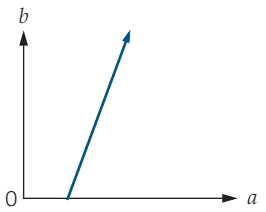
(a)


(b)


(c)


(d)


(e)


(f)


4 Express each of these relationships involving direct proportion as an equation, using k as the constant.

(a) $m \propto n$

(b) $a \propto b^2$

(c) $g \propto h^3$

(d) t is proportional to w.

(e) N is proportional to p^2.

(f) y is proportional to the cube of x.

(g) V is proportional to the square of r.

(h) L is proportional to the square root of d.

5 (a) The volume of a sphere (V) is directly proportional to the cube of its radius (r). This may be written as:

A $V \propto r^2$ **B** $V \propto r^3$ **C** $V \propto r$ **D** $V \propto \sqrt{r}$

(b) The radius of a cylinder (r) is directly proportional to the square root of the volume (V). This may be written as:

A $r \propto \sqrt{V}$ **B** $r \propto V^2$ **C** $r \propto V^3$ **D** $r \propto V$

6 State the value of the constant of proportionality in each of the following.

(a) $p = 3q$ **(b)** $a = 0.2b^2$ **(c)** $M = \frac{1}{4}n$ **(d)** $y = 200\sqrt{x}$

(e) $c = n^5$ **(f)** $F = \frac{a}{6}$ **(g)** $x = \frac{w^3}{20}$ **(h)** $P = \frac{r^4}{35}$

7 In each of the following table of values find the constant of proportionality and hence find the missing values in the table.

(a) $p \propto q$

q	5	7	9	
p	15	21		33

(b) $m \propto n^2$

n	1	3	5	
m	7		175	252

(c) $d \propto c^3$

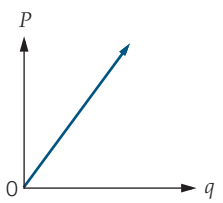
c	1		4	5
d	1	8		125

(d) $w \propto \sqrt{x}$

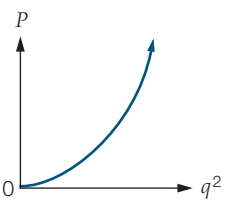
x	1		16	49
w	3	6	12	

8 Which graph shows direct proportion of the form $p \propto q^2$?

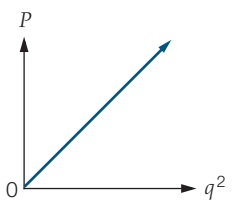
A



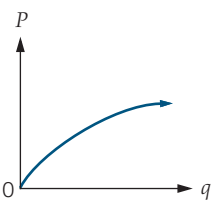
B



C



D



9 A student measures the current and voltage of a simple electrical circuit to determine the value of a resistor within the circuit. The results obtained are shown in the table.

Current, *I* (amps)	0.001	0.002	0.003	0.004
Voltage, *V* (volts)	3	6	9	12

(a) Graph the relationship between I amps and V volts. Show I on the horizontal axis and V on the vertical axis.

(b) Calculate the constant of proportionality, in this case called the resistance R (ohms).

(c) Calculate the voltage if 0.01 amps of current is flowing through the circuit.

(d) What is the current flowing through the circuit when a voltage of 45 volts is measured in the circuit?

10 The following table shows the relationship between the surface area of a sphere and its radius.

Radius, *r* (cm)	2	7	15	20	50
Surface area, *A* (cm^2)	50.24	615.44	2 826	5 024	31 400

(a) Determine the constant of variation for this relationship.

(b) Express this relationship as an equation using $\pi = 3.14$.

Reasoning, communicating and open-ended

11 The distance to the horizon is proportional to the square root of the height above sea level. A man standing on a cliff 32 m above sea level can see a distance of 18 km.

(a) How far could he see if he was standing on the viewing platform of a lighthouse 18 m above the cliff top?

(b) What height above sea level would he need to be to see a distance of 100 km?

12 The following table shows the circumference, surface area and volume of spheres of various radii, in similar units.

Radius, *r*	**Circumference, *C***	**Surface area, *A***	**Volume, *V***
2	12.56	50.24	33.49
4	25.12	200.96	267.95
6	37.68	452.16	904.32

8	50.24	803.84	2143.57
10	62.80	1256.00	4186.67
12	75.36	1808.64	7234.56

(a) Find the ratio $\frac{C}{r}$, $\frac{A}{r}$ and $\frac{V}{r}$ for each pair of values.

(b) Which of the ratios is constant? Describe your results.

(c) Plot graphs of C against r, A against r and V against r.

(d) Plot a graph of A against r^2. Describe the graph.

(e) Plot a graph of V against r^3. Describe the graph.

(f) Which of the graphs show direct proportion between the quantities? Write a relationship for each of these.

(g) Calculate the constant of proportionality, k, for each of these relationships. Express each as an equation in terms of π (where $\pi = 3.14$).

13 **(a)** Write two different relationships of the type $y = kx^n$, where $n > 0$ and $0 < k < 1$. Use the same value of n for both of your relationships.

(b) Sketch the graph of $y = x^n$ for your chosen value of n and, on the same set of axes, sketch the two relationships you have written. Describe what happens as k decreases from 1 to 0.

Problem solving

Relationships between variables

Determine the relationship between x and y for each of these tables.

1

x	0	1	2	3	4
y	0	3	12	27	48

2

x	0	1	2	3	4
y	1	5	25	125	625

Strategy options

- Look for a pattern.
- Guess and check.

Challenge 8



1 The solutions of the equation $x^2 - 13x - 48 = 0$ are:

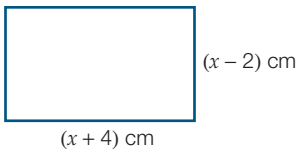
A -3, 16 **B** 3, -16 **C** 4, -12 **D** -6, 8

2 How many positive integers less than or equal to 3000 can be written without using the digits 4, 5 or 6?

3 If $(x - 2)(2x + 3) = 0$, then possible values for $2x + 3$ are:

A 0 only **B** 0 and 3 **C** 0 and 7 **D** $-\frac{3}{2}$ and $-\frac{7}{2}$

4 The perimeter of the rectangle in the diagram is 112 cm. Its total area is:



A 44 cm^2 **B** 187 cm^2

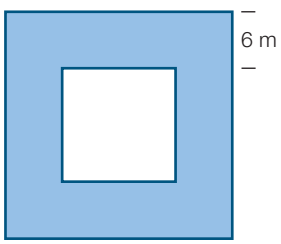
C 775 cm^2 **D** 3127 cm^2

5 If $u \# v$ means $2u - v$, what is the value of $6 \# (2 \# 5)$?

6 The mean of the solutions of the quadratic equation $x^2 + 10x - 11 = 0$ is:

A -10 **B** -5 **C** 5 **D** 10

7 A 6-m wide strip is mown around the inside edges of a square lawn. If eight-ninths of the lawn has been mowed (shaded area), calculate the area, in square metres, that still has to be mown (unshaded area).



8 Two three-digit numbers, *abc* and *def*, have the following property:

$$\begin{array}{r} a\ b\ c \\ +\ d\ e\ f \\ \hline 1\ 0\ 0\ 0 \end{array}$$

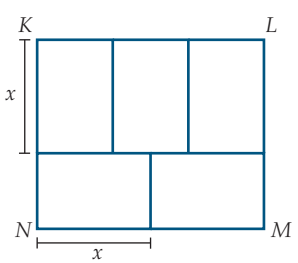
None of *a*, *b*, *c*, *d*, *e* and *f* is zero. Therefore, the value of $a + b + c + d + e + f$ is:

A 10 **B** 19 **C** 21 **D** 28

9 If $x + \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ equals:

A 25 **B** 23 **C** 27 **D** 3

10 Five identical rectangles are arranged to form a larger rectangle *KLMN*, as shown. The area of *KLMN* is 4000 cm^2. The length, *x* cm, of each of the identical rectangles is closest to:



A 33 cm **B** 35 cm

C 37 cm **D** 39 cm

Chapter review 8

D.I.Y. Summary

Key Words

Stage 5.1	circle	exponential	parabola
Stage 5.2	asymptote	direct proportion	quadratic relationship
	axis of symmetry	Null Factor Law	turning point
	constant of proportionality	quadratic equation	vertex
Stage 5.3	coefficient	dilation factor	transformation
	dilation	reflection	translation

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once

Stage 5.1 Q1, 2 **Stage 5.2 Q3–6** **Stage 5.3 Q7–9**

1 A _____ is an expression where the highest power of x is 2 and the highest power of y is 1.

2 A _____ is the type of graph produced by the equation $y = x^2$.

3 The _____ passes through the _____ of a quadratic graph.

4 Another name for the turning point is the _____.

5 Using the _____ it is possible to find the solutions for x when an equation equals 0.

6 The equation $x^2 + y^2 = 9$ graphs as a _____ with a radius of 3.

7 The graph $y = x^2$ is a _____ of the graph $y = -x^2$.

8 The graph $y = x^2$ can be changed into $y = 2(x - 1)^2 + 3$ by undergoing a number of _____s.

9 The main features of a sketch curve of a quadratic relationship are: y-intercept, x-intercepts and the _____.

Equipment required: graph paper for Question **14**.

Stage 5.1 Q1 **Stage 5.2 Q2–4, 9, 11–15, 17–21, 24** **Stage 5.3 Q5–8, 10, 16, 22, 23**

Fluency and understanding

1 Which of the following equations would NOT graph as a parabola.

A $y = 4 - x^2$ **B** $y = 4x^2$ **C** $y^2 = 4 - x^2$ **D** $y = \frac{x^2}{4}$

Ex 8.1

2 Plot the graph of $y = x^2 - 2x - 8$.

Ex 8.2

(a) Write down the coordinates of the x- and y-intercepts.

(b) Write down the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(c) Write down the equation of the axis of symmetry.

(d) From the graph, find the value of y when $x = 3$.

(e) What are the values of x when $y = 7$?

(f) Hence state the values of x when $x^2 - 2x - 8 = 7$.

(g) Write down the values of x when $x^2 - 2x - 8 = 0$.

3 Solve the following using the Null Factor Law. Ex 8.3

(a) $2x(x+4)=0$ **(b)** $(x-7)(x+1)=0$ **(c)** $x^2-16=0$

(d) $x^2+6x+9=0$ **(e)** $x^2=8x$ **(f)** $x^2+36=12x$

4 Solve these quadratic equations. Ex 8.3

(a) $x^2-4x=0$ **(b)** $x^2-6=0$ **(c)** $x^2+12=0$

(d) $3x^2=27$ **(e)** $(x-3)(x+7)=11$ **(f)** $(x-3)(x+2)=6$

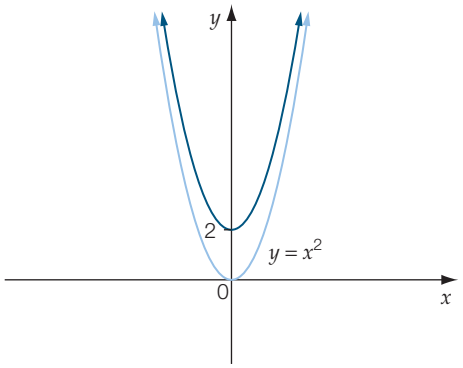
5 **(a)** The equation of the unlabelled parabola shown could be: Ex 8.4

A $y=x^2+2$

B $y=x^2-2$

C $y=(x-2)^2$

D $y=(x+2)^2$



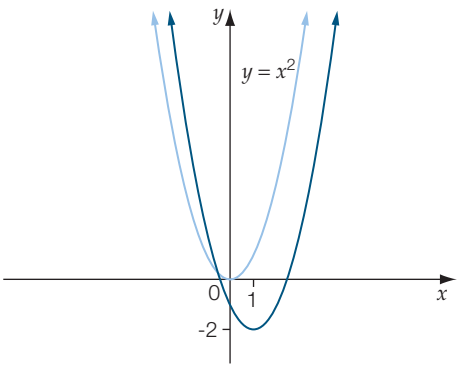
(b) The equation of the unlabelled parabola shown could be:

A $y=(x-1)^2-2$

B $y=(x-1)^2+2$

C $y=(x+1)^2+2$

D $y=(x+1)^2-2$



6 State the changes that should be made to the graph of $y=x^2$ to obtain the graph of each of the following. Ex 8.4

		Dilation	Reflection in *x*-axis	Translation in *x* direction	Translation in *y* direction
(a)	$y=(x-1)^2+3$				
(b)	$y=-x^2+4$				
(c)	$y=-(x+2)^2-1$				

7 Sketch the graphs of: Ex 8.4

(a) $y=x^2+8x+16$ **(b)** $y=-x^2+2x-1$

(c) $y=3x^2+9x$ **(d)** $y=x^2-4x+4$

8 **(a)** The graph of the quadratic relationship $y=x^2-64$ crosses the x-axis at: Ex 8.4

A $x=-8$ and $x=8$ **B** $x=8$ **C** $x=-8$ **D** $x=64$

(b) The graph of the quadratic relationship $y=x^2-8x+16$ crosses the x-axis at:

A $x=2$ and $x=8$ **B** $x=-4$ and $x=4$ **C** $x=4$ **D** $x=-4$

9 Identify the centre and radius for each of the following circle equations, then sketch the graphs. Ex 8.5

(a) $x^2+y^2=9$ **(b)** $x^2+y^2=5$

10 Sketch graphs for each of the following circle equations.

Ex.8.6

(a) $x^2 + (y-2)^2 = 25$ **(b)** $(x+3)^2 + (y-1)^2 = 16$

11 Sketch graphs for each of the following equations, showing the asymptotes.

Ex.8.7

(a) $y = 2^{-x}$ **(b)** $y = 2^{x-1} + 3$

12 Find the equation for each of the following tables of values.

Ex.8.8

(a)

a	1	3	4
b	3	9	12

(b)

g	0	4	9
h	0	6	9

13 **(a)** The volume of a cube is directly proportional to the cube of its height. This may be written as:

Ex.8.8

A $V \propto H^3$ **B** $V = H^3 + 1$ **C** $V \propto H^{1/3}$ **D** $V = H^{1/3}$

(b) The surface area of a cylinder is directly proportional to the square of its radius. This may be written as:

A $A = r^2 + 3r$ **B** $A \propto r^2$ **C** $A \propto \sqrt{r}$ **D** $A \propto \dfrac{1}{r^2}$

(c) The profit of a company is directly proportional to the square root of its number of employees. This may be written as:

A $p \propto n^2$ **B** $p = n^2$ **C** $p = \sqrt{n^2}$ **D** $p \propto \sqrt{n}$

14 A softball thrown by an outfielder to the pitcher follows a parabolic path where its height, h metres, at any time, t seconds, is given by the relationship:

$$h = -5t^2 + 10t + 1$$

(a) Complete a table of values for this relationship.

(b) Plot the graph of this relationship on graph paper.

(c) What was the height of the softball after 0.5 seconds?

(d) When was the ball first at a height of 5 m above the ground?

(e) During which time interval was the ball above a height of 2 m?

(f) What was the maximum height of the softball during its flight?

(g) How long was the ball in flight if it was caught at a height of 1.6 m above the ground on its downward path?

15 A jet of water from an outlet level with the surface of a pond follows a parabolic path. The height of the water at any point can be described by the equation:

$$h = -d^2 + 6d$$

where h is the height of the water a horizontal distance d from the water outlet. Both h and d are in metres.

(a) Sketch the graph of $h = -d^2 + 6d$.

(b) From your sketch, state the maximum height of the water jet above the surface of the pond.

(c) What horizontal distance does the jet of water cover from its outlet back to the surface of the pond?

16 Describe the transformation that has been performed when $(x+2)^2 + (y-1)^2 = 25$ has been changed to $(x-4)^2 + (y-3)^2 = 16$.

17 **(a)** Show that the graphs of $y = 2^{x+1} + 5$ and $y = 3^{x+2} - 2$ have the same y-intercept.

(b) Write down the equation of the asymptotes for each graph.

Reasoning and communicating

18 Di and Sam found the following table of values, but some values were missing. Di could not see a pattern, but Sam thought she could see one.

x	1	2	3	4	5	
y	0.1		2.7		12.5	21.6

(a) Is there a relationship between the two values? If so, write the relationship.

(b) Use the relationship to fill in the missing values.

19 Leonard and Faye want to develop a vegetable garden in their backyard. They have 20 m of chicken wire and enough posts to erect a fence around the four sides of the rectangular garden.

(a) If the length of the rectangular garden is x m, find an expression (in terms of x) for the width of the garden.

(b) Show that the area of the garden, A m^2, is given by the quadratic expression $A = -x^2 + 10x$.

(c) Sketch the graph of the area of the vegetable garden, A m^2, against the length of the garden, x m. (Show length on the horizontal axis.)

(d) What should the dimensions of the garden be if the area is to be as large as possible?

(e) What is the area of the largest vegetable garden that Leonard and Faye can make?

20 A book cover has an area of 64 cm^2. If the length is x cm and the length and width add to 16 cm, find the dimensions of the book cover.

21 The sum of two numbers is 50. What are the numbers if their product is to be as large as possible?

22 A parabola has a minimum turning point at (3, 5) and a dilation factor of 1. What is the equation of the curve?

23 A circle has an equation $(x + 1)^2 + (y - 3)^2 = 121$. What would be the equation of this circle if it is translated 4 units right and 3 units down and the radius is decreased by 2 units?

24 $v^2 = u^2 + 2as$ is a formula used in motion. If $u = 0$, the formula becomes $v^2 = 2as$. If a is a constant:

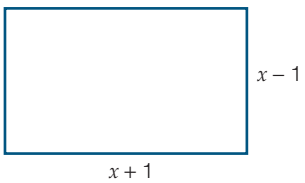
(a) what happens to v when s is doubled?

(b) what happens to s when v is reduced to $\frac{3}{4}$ of its original value?

NAPLAN practice 8

Numeracy: Non-calculator

1 Find the value of x in the following rectangle given that the area is 8 m^2.



2 Here is a table of values for x and y.

x	-3	-2	-1	0	1
y	18	8	2	0	2

Which of the following is a correct rule for y in terms of x?

A $y = 6x$ **B** $y = 4x$ **C** $y = x^2 + 3$ **D** $y = 2x^2$

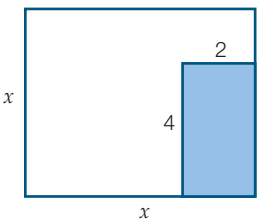
3 The solution/s to $x^2 - 4 = 0$ is/are:

A $x = -2$ **B** $x = 0$ **C** $x = 2$ and -2 **D** $x = 2$

Numeracy: Calculator allowed

4 Jane drew this plan for her square block of land.
(All measurements are in metres.)

The shaded area shows the garage. Apart from the garage, what is the area of the land?



A $x^2 + 4x$ **B** $(x + 4)(x - 2)$

C $(x - 4)(x - 2)$ **D** $x^2 - 8$

5 Complete the table of values below that would be used to plot points for the relationship $y = x^2 - 3x + 2$.

x	-3	-2	-1	0	1	2	3
y							

6 If $x = 2$, then which of these expressions will be equal to 6?

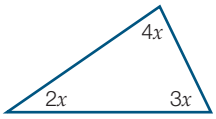
A $x^2 + 5x - 3$ **B** $(x + 1)(x - 4)$ **C** $4x^2 - 2x + 3$ **D** $-x^2 + 4x + 2$

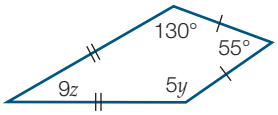
Mixed review C

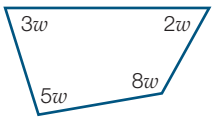
Stage 4 Q1 Stage 5.1 Q2–5, 12, 14 Stage 5.2 Q6–10, 13, 15 Stage 5.3 Q11, 16

Fluency and understanding

1 Find the value of the pronumeral in each figure. Ex 5.1

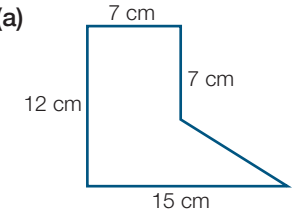
(a) 

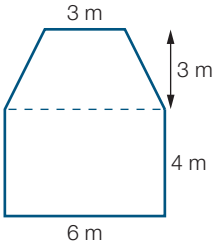
(b) 

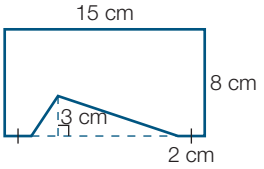
(c) 

2 An investment of $5000 is made at 3.5% p.a. for 6 years. Find the difference in interest earned if interest is compounded monthly compared with annually. Ex 1.5

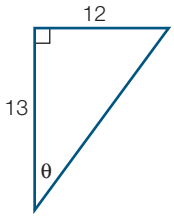
3 Find the area of each of the following shapes. Ex 3.2

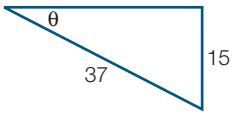
(a) 

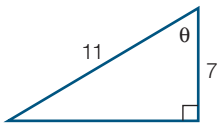
(b) 

(c) 

4 Find the value of θ in degrees, correct to one decimal place in each of the following. Ex 6.4

(a) 

(b) 

(c) 

5 Find the mean for each of the following grouped data frequency tables. Give your answers correct to two decimal places. Ex 7.3

(a)

Class interval	Frequency
20–<25	8
25–<30	6
30–<35	3
35–<40	2
40–<45	5
45–<50	5

(b)

Class interval	Frequency
111–<115	12
116–<120	15
121–<125	17
126–<130	19
131–<135	13
136–<140	11

6 Solve the following equations. Ex 4.3

(a) $\frac{3a+4}{7} = -2$ (b) $\frac{4a+3}{2} = \frac{2a}{5}$ (c) $\frac{3a-2}{3} = \frac{4a-5}{2}$

7 Solve each of the following for x. Ex 8.3

(a) $2x^2 - 50 = 0$ (b) $4x^2 + 2x = 0$ (c) $x^2 - 6x + 9 = 0$

8 Sketch the graphs of each of the following, clearly showing the turning point.

Ex.8.2

(a) $y = (x + 3)(x - 2)$ **(b)** $y = x^2 + 2x + 5$ **(c)** $y = 2x^2 - 4x$

9 Sketch the graph of each of the following quadratics, showing all important features.

Ex.8.2

(a) $y = x^2 + 6x - 4$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = -2x^2 + 3x$

10 Consider the equation $y = 2x^2 + x$.

Ex.8.2

(a) Draw up and fill in a table of values with x-values from -3 to 3.

(b) Use the table of values to draw the graph of the equation.

(c) Find the turning point of the graph.

(d) Determine the x- and y-intercepts for the graph.

11 Factorise the following quadratic expressions.

Ex.2.8

(a) $6a^2 - 24$ **(b)** $3a^2 + 6a + 3$ **(c)** $x^2 + 5x - 7x - 35$

12 The 26 letters of the alphabet are written on 26 identical cards and placed in a bag. A card is chosen at random. What is the probability that the card chosen is:

(a) a vowel

(b) one of the letters in the word 'card'

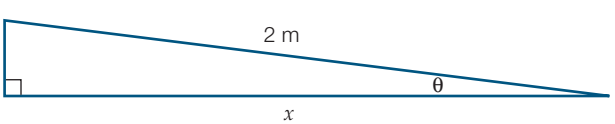
(c) not a letter in the word 'letter'?

13 Jodie is standing on the top of a sand dune and spots a dolphin in the water. If the distance from Jodie to the dolphin along her line of sight is 100 m, find the angle of depression from Jodie to the dolphin if she is 4.3 m above sea level. Answer to one decimal place.

Reasoning and communicating

14 To meet Australian standards, ramps for wheelchair access that are longer than 152 cm must only rise 1 cm vertically for every 14 cm horizontal distance.

(a) What is the largest angle, θ, between the ramp and the horizontal that can be used to build a wheelchair ramp 2 m long? Answer to the nearest degree.



(b) Use your answer to **(a)** to find:

(i) the horizontal length, x

(ii) the height of the top of the ramp. Answer to the nearest mm.

15 A book cover has an area of 60 cm^2. If the width is 7 cm less than the length, find the dimensions of the book cover.

16 A parabola has x-intercepts at $x = 4$ and $x = 6$.

(a) Determine the x-coordinate of the turning point.

(b) If the coefficient of x^2 is 1, determine the equation of the graph.

Answers

Worked solutions and answers to ***all*** activities appear on the eBook.

Chapter 1

Recall 1 (p. 2)

1 (a) 0.17 (b) 0.057 (c) 0.00321

2 (a) 0.64 (b) 1.125 (c) 0.575
(d) $0.1\dot{6}$ (e) $0.\overline{90}$ (f) $0.\overline{714285}$

3 (a) $\frac{79}{100}$ (b) $\frac{3}{1000}$ (c) $\frac{7}{500}$
(d) $\frac{213}{200}$ (e) $\frac{507}{50}$ (f) $\frac{451}{5000}$

4 (a) 8.7 (b) 9.1 (c) 7.8 (d) $206\frac{2}{3}$

5 (a) 70% (b) 91% (c) 3.2% (d) 82.14%

6 (a) $\frac{17}{20}$ (b) $\frac{11}{50}$ (c) $\frac{67}{1000}$ (d) $\frac{1}{200}$

7 (a) 0.49 (b) 0.3 (c) 0.025 (d) 3.7

8 (a) 75 m (b) 78.75 kg (c) $28 160

9 (a) 1 m (b) 6.35 km (c) 3.982 cm

Exercise 1.1 (p. 6)

1 (a) $10 (b) $22.50 (c) $45
(d) 6.6 L (e) 264 m (f) 26.95 kg
(g) 1.8 kg (h) 43.66 kg (i) 52.8 L
(j) $17.50 (k) 187.5 m (l) 1.08 L
(m) $977.50 (n) $5.58 (o) $1333.33

2 (a) (i) 360 (ii) 324 (iii) 259.2 (iv) 576
(b) (i) 4560 (ii) 1425 (iii) 3078 (iv) 114
(c) (i) $70.57 (ii) $230.94 (iii) $68.96 (iv) $68.48
(d) (i) 4.9 s (ii) 8.53 s (iii) 9.18 s (iv) 9.47 s

3 (a) 69.2% (b) 16.8% (c) 63.6% (d) 18.9%
(e) 66.7% (f) 21.5% (g) 75.9% (h) 26.0%

4 (a) (i) 60% (ii) 8.5% (iii) 16%
(b) (i) 17% (ii) 28.3%

5 Pineapple: 1.95 L Orange: 0.388 L
Passionfruit: 0.113 L Guava: 0.05 L

6 37.7 kg

7 (a) $1059.71 (b) $35.84

8 (a) 8.56 s (b) 3.22%

9 (a) $\frac{7}{9}$ (b) 22.22% (c) $\frac{8}{10}$ or $\frac{4}{5}$, 20%

10 (a) 0.5, 50% (b) $\frac{9}{20}$, 45% (c) $\frac{18}{25}$, 0.72
(d) 1.25, 125% (e) $\frac{14}{25}$, 56% (f) $\frac{1}{8}$, 0.125

11 (a) 131 250 t (b) 87.5%

12 (a) $4.00 (b) $3.64 (c) 7.06%

13 79.6% **14** 16 884

15 (a) (i) 120 (ii) 96
(b) (i) 390 (ii) 273
(c) When you are increasing and decreasing by the same percentage, you are not multiplying and dividing by the same number.

16 45, 59 and 80

17 (a) No, 25% of $299 is $74.75, a greater saving than $31.
(b) 25% is equivalent to $\frac{1}{4}$, which Trent could have found by dividing $300 (rounding up slightly) by 4 to get $75. Because we rounded up the actual answer will be slightly less, but still greater than $31.

Exercise 1.2 (p. 12)

1 (a) $20.40 (b) $71.25 (c) $38.39 (d) $262.47

2 (a) (i) 40% (ii) 28.6% (b) (i) 3.6% (ii) 3.7%

3 (a) $75.45 (b) $16.80

4 (a) $2 (b) $27.57

5 (a) $175 (b) $2699

6 A **7** D **8** C

9 (a) $69.60 (b) $96

10 $735 **11** $88.80 **12** (a) $1434 (b) 149.4%

13 $15.71

14 (a) $19.99 (b) $1.82

15 (a) $27.18 (b) $317.27

16 54.4%

17 (a) $227.29 (b) $11 591.79 (c) $756.21 (d) 6.52%

18 She would save $4.99 by going to her favourite shoe shop.

19 (a) $1088 (b) $598
(c) 45% is applied to two different prices. First, 45% of the cost price of $750 is found and added on. Second, 45% of the selling price of $1088 is found and subtracted. This is a larger amount than the 45% mark up.
(d) 31%

20 14.9% **21** $7857.14

22 **(a)** 24.2%: $\$999 \times (1 - 0.242) = \$999 \times 0.758 = \$757.24$
23%: $\$999 \times (1 - 0.23) = \$999 \times 0.77 = \$769.23$
20.8%: $\$999 \times (1 - 0.208) = \$999 \times 0.792 = \$791.21$

(b) Example:
Discount 20%, marked price $3062.50;
$\$3062.50 \times 0.8 = \$2450$
Discount 21.5%, marked price $3121.02;
$\$3121.02 \times 0.785 = \$2450$
Discount 23%, marked price $3181.82;
$\$3181.82 \times 0.77 = \$2450$

23 Example: A $100 pair of shoes.

A 30% discount gives a sale price of $70. An additional 20% discount means that 20% of $70 (not the original $100) is subtracted. After a 20% discount, sale price = $56.

One 50% discount off the original $100 gives a sale price of $50.

Exercise 1.3 (p. 17)

1 $690.90 **2** $2967 **3** $1530

4 A **5** B **6** C

7

	(a) Week	(b) Fortnight	(c) Month	(d) Year
Adam	$623.20	$1246.40	$2700.53	$32 486.40
Britta	$639.60	$1279.20	$2771.60	$33 259.20
Con	$656.00	$1312	$2846.67	$34 112
Deng	$442.80	$885.60	$1918.80	$23 025.60

8 **(a)** $4800 **(b)** $5400 **(c)** $6950 **(d)** $8000

9 115 sheep

10 **(a)** $\$656 \div 38 \times 0.8 = \$13.81$ per hour **(b)** $339.73

11 **(a)** $1029.00 **(b)** $720.30

12 $661.30 **13** **(a)** $675 **(b)** 31.2%

14 $150 000

15 **(a)** 41 hours **(b)** 45 hours **(c)** 50 hours

(d) 20.5 hours overtime is exactly the same

16 **(a)** **(i)** $78.75 **(ii)** $1057.50

(b) $2937.50

17 **(a)** Option A: $40 000; Option B: $35714

(b) $25 000

(c) If Nabil expects sales less than $25 000 per week, he should choose Option A

18 **(a)** $1873.08 **(b)** $23.41 **(c)** $18.73

19 **(a)** Currently Malik is earning $\$16.78 \times 0.4 \times 32 + 17 = \$231.78$ a week.

Working the same number of hours in a fast food outlet, he would earn $\$600 \div 38 \times 0.7 \times 32 = \$353.68$.

Next year (assuming the same number of hours), his wages would be:

Apprentice: $\$16.78 \times 0.52 \times 32 + 17 = \$296.22$ a week

Fast food: $\$600 \div 38 \times 0.8 \times 32 = \$404.21$ a week

In the third year, his wages would be:

Apprentice: $\$16.78 \times 0.7 \times 32 + 17 = \$392.87$ a week

Fast food: $\$600 \div 38 \times 0.9 \times 32 = \$454.74$ a week

In the fourth year, his wages would be:

Apprentice: $\$16.78 \times 0.82 \times 32 + 17 = \$457.31$ a week

Fast food: $\$600 \div 38 \times 32 = \$505.26$

(b) Malik does not earn more than a casual fast food employee during his apprenticeship. However, he is training to become a fully qualified tradesperson capable of earning far more than a casual fast food worker. This calculation assumes 32 hours a week as a casual worker, but for casuals there is no guaranteed number of hours, and no leave. Malik's current position is more stable and secure.

20 16 hours during the week: $\$15.80 \times 1.25 \times 16 = \$316$

14 hours on a weekend = $(\$15.80 \times 1.25 \times 1.1 \times 14) = \$304.15$

10 weekday hours + 5 weekend hours:
$(10 \times \$15.80 \times 1.25) + (5 \times \$15.80 \times 1.25 \times 1.1) = \$306.13$

Half-time 1 (p. 21)

1 **(a)** 73.3% **(b)** 14.5%

2 **(a)** 28.4% **(b)** 22.1%

3 **(a)** $79.20 **(b)** $275.80 **(c)** $55.28 **(d)** $199.33

4 Lucas should buy the $165 skateboard with a 25% discount in shop B (it costs $123.75 compared with the other one costing $130).

5 Jeans: $72.22
Jacket: $72.31
Scarf: $13.75
In total Naomi saved $35.28

6 $65.45

7 $1851.29

8 $758

Exercise 1.4 (p. 24)

1 **(a)** **(i)** $6391.05 **(ii)** $39 282.95

(b) **(i)** $16 690.63 **(ii)** $60 674.37

(c) **(i)** $34 197 **(ii)** $90 803

(d) **(i)** $2161.44 **(ii)** $27 414.56

(e) **(i)** $0 **(ii)** $14 033

(f) **(i)** $63 547 **(ii)** $136 453

(g) **(i)** $11 034 **(ii)** $48 926

(h) **(i)** $0 **(ii)** $5976

2 **(a)** **(i)** $4.75 **(ii)** $720.25

(b) **(i)** $800.33 **(ii)** $3829.67

(c) **(i)** $106.85 **(ii)** $1155.54

(d) **(i)** $0 **(ii)** $176

(e) **(i)** $2190.62 **(ii)** $6444.62

(f) **(i)** $129.48 **(ii)** $407.56

3 **(a)** $57 765 **(b)** $10 320.63

4 B

5 **(a)** $48 256 **(b)** $278.08 **(c)** $1577.92 **(d)** $167.04

6 (a) \$504 (b) \$7008 (c) \$4404.42

7 (a) \$24 518.52 (b) \$24 296.52 (c) \$1158.34

8 (a) \$9259.50 (b) \$1048.08 (c) \$199.03

(d) \$4025.35 (e) \$2742

9 (a) \$1480 (b) \$370

10 (a) \$17 547 (b) 22% (c) \$9094

(d) 11% (e) \$8453

(f) If the income is split then each individual gets the benefit of the tax-free threshold and a lower percentage of income tax.

11 (a) \$0.33 (b) \$0.37 (c) \$0.45

(d) These figures are the amount of tax paid up to the beginning of that tax bracket.

12 The tax table is designed so that those who earn more, pay more tax. This could be argued as being unfair because those who work hard and earn more are penalised, or fair because those who can afford more can contribute more.

13 Compulsory superannuation means that Australian workers should all have an income after they retire, and are therefore less likely to fall into poverty after retirement.

Exercise 1.5 (p. 31)

1 (a) \$16 200 (b) \$980 (c) \$4694.50 (d) \$233.75

2 (a) \$937.50 (b) \$21 562.50

3 (a) (i) \$2.03 (ii) \$1.05

(b) (i) \$1.37 (ii) \$0.23

4 8.9%

5 (a) \$176 (b) \$378 (c) \$11.07 (d) \$34.98

6 B **7** \$13 427.13

8 (a) $R = 0.090$, 9.0% (b) $R = 0.140$, 14.0%

9 (a) 7.9 years (b) 3.8 years

10 (a) \$200 (b) \$1350

11 4.6 years **12** \$1455.23 **13** \$120 **14** \$8036

15 (a) (i) \$0.75 (ii) \$3.29

(b) \$2.54

16 207 days **17** \$24 000 **18** \$7000

19 (a)

Date		Transaction		Balance
Jul	1			\$975.00
Jul	17	Deposit	\$197	\$1172.00
Jul	28	Withdrawal	\$641	\$531.00
Aug	4	Deposit	\$299	\$830.00
Aug	19	Deposit	\$862	\$1692.00
Aug	29	Withdrawal	\$176	\$1516.00
Sep	6	Deposit	\$194	\$1710.00
Sep	28	Withdrawal	\$800	\$910.00
Oct	16	Deposit	\$426	\$1336.00
Oct	27	Withdrawal	\$500	\$836.00
Oct	31	Balance		\$836.00

(b) July: \$531; August: \$531; September: \$910; October: \$836

(c) \$6.49

(d) July: \$975; August: \$531; September: \$1516; October: \$910

(e) \$9.07

(f) 40%

(g) Try to make withdrawals at the start of each month.

20 (a) \$2250 (b) \$17 250 (c) 479.17

(d) 13% (e) \$1500 (f) \$1000

(g) Part (e) assumes the \$15 000 is owed for the whole 3 years, whereas part (f) doesn't.

21 4% for 2 years, 8% for 1 year, 16% for 6 months

22 Michael has overcalculated the amount of days the balance was in his account. The opening balance was in his account for 8 days instead of 9, and the second balance was in his account for 12 days instead of 13.

23 As a borrower you are in debt and must repay the debt out of your own income, which gives you less money to spend. If you pay interest you will end up paying back far more than you borrowed. As a lender, you are relying on people to pay back the money you loaned. If they don't or can't repay, you have lost money.

Exercise 1.6 (p. 38)

1 (a) \$11.60 (b) \$14.10 (c) \$10 (d) \$12

2 \$44.76

3 (a) \$41 (b) \$989.70 (c) \$45.55

4 (a) 7 weeks (b) \$8.95

5 (a) 31 days (b) \$251.33 (c) \$3.70

6 (a) \$95.80 (b) \$106.03 (c) \$179.17/month

7 (a) \$2.10

(b) Using an ATM then depositing into their landlord's account, because this saves \$3.90 each month

8 (a) \$48 600 (b) \$8605 (c) 4.3%

9 (a) \$124.48 (b) \$8825.05 (c) \$12 623.05

(d) \$179.32 (e) \$321.80

(f) People often use a credit card because they want the goods but do not have the cash to pay for them at the time. If they cannot pay the bill at the end of the month, interest and late fees are added to the balance. If several payments are missed, then interest is charged on the interest, and paying off the debt becomes very difficult.

10 (a) 12 months (b) \$61

(c) Students' own answers.

(d) (i) Goods are able to be taken home straight away.

(ii) People do not want to pay fees, and are not sure they will have the full sum of money to avoid paying high interest at the end of the period.

11 (a) Four ATM withdrawals, two other bank ATM withdrawals, three EFTPOS transactions and five online internet transfers = \$10.75

(b) Kezia could try to stop using other banks ATMs altogether, and withdraw more money from her own bank's ATM each time, so she is only being charged an ATM fee a couple of times each month.

12 Debit card: advantage—use money that you actually have

disadvantage—if you do not have the money at the time, you cannot purchase

Credit card: advantage—can purchase even if you do not actually have the cash

disadvantage—interest charged if bill not paid on time, tempted to overspend.

Challenge 1 (p. 41)

1 Numbers are x, $x+1$, $x+2$, $x+3$, $x+4$, $x+x+4=1708$, $2x=1704$, $x=852$

Numbers are 852, 853, 854, 855, 856

2 $7^1=7$, $7^2=49$, $7^3=343$, $7^4=2401$, $7^5=16\,807$, $7^6=117\,649$, $7^7=823\,543$, $7^8=5\,764\,801$, …

The powers of 7 end in 07, 49, 43 or 01 in that order and then the pattern is repeated.

2048 is divisible by 4, so the last two digits of 7^{2048} will be the same as the last two digits of 7^4, that is 01.

Hence, $7^{2048} \div 100$ has a remainder of 1.

3 The numbers will end in 5 because numbers ending in 0 have 2 as their smallest prime factor. Will be ≥ 25.

$5=1\times5$ (1 is not prime), $25=5\times5$, $35=5\times7$, $55=5\times11$, $65=5\times13$, $85=5\times17$, $95=5\times19$.

Numbers are 5, 25, 35, 55, 65, 85, 95

4 If $m=n=1$ then $m^3=n^4$. But m and n are different.

$(2^4)^3=(2^3)^4$, $m=16$, $n=8$.

$(3^4)^3=(3^3)^4$, $m=81$, $n=27$ etc. Smallest value is $m=16$.

5 (a) 21/02/2012, 22/02/2022, 23/02/2032, 11/02/2011, 10/02/2001 etc.

(b) 10/02/2001, 2 October, 2001; 11/02/2011, 2 November, 2011; 09/02/2090, 2 September, 2090; 01/02/2010, 2 January, 2010

6 The pattern is that the numbers increase by the next multiple of 3. The missing terms are 45, 63 and 84.

7 This is really dividing the interval between $\frac{1}{12}$ and $\frac{1}{7}$ into three equal parts.

$\left(\frac{1}{7}-\frac{1}{12}\right)\div3=\frac{5}{84}\times\frac{1}{3}=\frac{5}{252}$; the numbers are

$\frac{1}{12}+\frac{5}{252}=\frac{26}{252}=\frac{13}{126}$ and $\frac{26}{252}+\frac{5}{252}=\frac{31}{252}$

8 12 students, 30 min, 100 dishes; four students, 90 min, 100 dishes; eight students, 45 min, 100 dishes; eight students, 4.5 min, 10 dishes; 8 students, 40.5 min, 90 dishes. It will take 40.5 min.

9 D

10 D 1000^1 has four digits $=3\times1+1$; $1000^2=1\,000\,000$ has seven digits $=3\times2+1$;
so, 1000^{2010} has $3\times2010+1=6031$ digits

11 C Let cost be \$$x$ so $D_1=0.1\times x$, $C_1=x-0.1x=0.9x$

$D_2=0.2\times0.9x=0.18x$, $C_2=0.9x-0.18x=0.72x$

$D_3=\frac{1}{3}\times0.72x=0.24x$, $C_3=0.72x-0.24x=0.48x$

So total discount $=0.52x$, i.e. 52%.

12 A **13** A, C

Chapter review 1 (p. 42)

1 (a) \$62.21 **(b)** \$260

2 (a) \$81.25% **(b)** 25%

3 (a) 52.50 **(b)** 26.24

4 C

5 (a) (i) \$6.16 **(ii)** \$41.09

(b) (i) \$117.27 **(ii)** \$35.41

6 \$415.80 **7** \$831.25

8 (a) \$0 **(b)** \$7111.25 **(c)** \$18 575.60

9 (a) \$2380 **(b)** \$9380 **(c)** 195.42/month

10 \$64.13 **11** \$989.20 **12** 8.4%

13 (a) \$138.75 **(b)** \$124.88 **(c)** 66.5%

14 \$1004.25

15 (a) \$44 720 **(b)** \$43 280 **(c)** \$6534

16 (a) \$16 **(b)** 5 weeks

17 (a) (i) \$1.11 **(ii)** \$0.58

(b) \$0.53

18 (a) (i) \$1440 **(ii)** \$5560 **(iii)** \$109

(b) \$291.67/month **(c)** \$132.53

19 (a) Tanya earned more money (\$516.91 compared with Freya's \$493.76).

(b) Casual workers receive more because they don't receive work benefits, including sick pay and holiday leave, and their job isn't guaranteed to be ongoing.

20 \$19.76 profit

NAPLAN practice 1

1 B **2** A **3** B **4** C

5 B **6** C **7** B **8** \$776.80

9 \$10 740 **10** \$43.22

Chapter 2

Recall 2 (p. 47)

1 (a) $14x$ **(b)** $8y$ **(c)** k^2

2 (a) $4h+2y$ **(b)** $8y^3+6x^2-2x$

3 (a) $12xy$ **(b)** $20k^2$ **(c)** $-60g^3$

4 (a) 2^{13} **(b)** 6^6 **(c)** 5^4 **(d)** 1

5 (a) $\frac{4^6}{5^9}$ **(b)** $2^{16}\times5^8$ **(c)** $\frac{7^4\times3^8}{4^{10}}$

6 (a) (i) 20 cm (ii) 25 cm^2

(b) (i) 22 m (ii) 18 m^2

(c) (i) 16 mm (ii) 12 mm^2

7 (a) $4x + 8$ (b) $6x - 18$ (c) $-2x - 8$

8 (a) $6(x - 2)$ (b) $5(2x - 5)$ (c) $-3(x - 5)$

Exercise 2.1 (p. 51)

1 (a) 2^9 (b) 3^{11} (c) 5^{18} (d) a^7 (e) b^{11}

(f) $5c^5$ (g) $6d^5$ (h) $40b^8$ (i) $28b^4$ (j) $12a^6$

(k) $14a^4$ (l) $200a^7$ (m) $24a^9b^2$ (n) $27a^4b^5$ (o) $30b^7c^7$

(p) $-35a^4b^8$ (q) $36e^6f^6$ (r) $-24a^6b^{10}$

2 (a) 3^3 (b) 4^5 (c) 9^2 (d) m^3 (e) $4b^2$

(f) $5g^4$ (g) n^2 (h) $2b^2$ (i) $2c^4$ (j) $2a$

(k) $\frac{3}{2}a^4$ (l) $\frac{7}{5}e^3f^3$ (m) $2a^3b^4$ (n) $-\frac{8}{7}a^3b^2$ (o) $-12ab$

3 (a) 2^{12} (b) 3^{10} (c) 5^8 (d) 3^{18} (e) 4^{21}

(f) 7^{18} (g) 5^{10} (h) 8^{34} (i) 11^5 (j) b^{21}

(k) k^{32} (l) a^{12} (m) y^{17} (n) k^{27} (o) m^{20}

(p) g^{18} (q) y^{23} (r) t^{21} (s) p^2 (t) n^6

(u) b^3

4 (a) C (b) A

5 (a) $\frac{x^7 \times x^{12}}{x^{12}}$

$= \frac{x^{16}}{x^{12}}$

$= x^4$

(b) $\frac{6y^4 \times y^6}{y^2 \times y^5}$

$= \frac{6y^{10}}{y^7}$

$= 6y^3$

6 (a) x^9 (b) $\frac{t}{3}$ (c) c^7 (d) j^5 (e) $12a^8$ (f) $2q^3$

7 (a) A (b) C (c) C (d) C

8 (a) $5a^2$ (b) $3a^2 \times 5a^2$

9 (a) C (b) B

10 (a) m^3 (b) p^{14} (c) k^3 (d) n^{10}

11 (a) A (b) C

12 (a) 3 (b) 0 (c) -2 (d) 1

13 (a), (b)

Week	Amount saved each week (c)	Amount saved each week in index form (c)
1	5	5×2^0
2	10	5×2^1
3	20	5×2^2
4	40	5×2^3
5	80	5×2^4

(c) $a = 5 \times 2^{n-1}$ (d) \$51.20

(e) \$1 677 721.60

14 $-3a$ and ab^3, a^2 and $-3b^3$, $3b$ and $-a^2b^2$

15 $10a^5b^4 \div 2a^3b$, $5a^2b^5 \div b^2$, $15a^4b^3 \div 3a^2$

16 $24a^2b^3 \div 22a^2b^2 = \frac{12b}{11}$

$2 \times 3^2a \div 3ab = \frac{6}{b}$

$48a^5b^6 \div 24a^2b^3 = 2a^3b^3$

17 $m = 3$, $n = 8$; $m = 4$, $n = 6$; $m = 12$, $n = 2$

18 (a) The indices have been multiplied instead of added. The correct answer is x^6.

(b) The indices have been divided instead of subtracted. The correct answer is x^6.

(c) The index inside the brackets has been raised to the power of the index outside the brackets, when instead they should have been multiplied together. The correct answer is x^8.

Exercise 2.2 (p. 57)

1 (a) $6^2 \times 5^2$ (b) $11^4 \times 3^4$ (c) $12^5 \times 9^5$ (d) $16x^2$

(e) p^6q^2 (f) $a^{12}f^3$ (g) $32p^5$ (h) $p^{21}q^{35}$

(i) c^8d^{28} (j) b^3c^3 (k) $216a^{12}$ (l) $81p^{10}q^8$

(m) $49m^6n^{12}$ (n) $8a^3b^6$ (o) $81e^4f^{16}$

2 (a) $\frac{12^2}{5^2}$ (b) $\frac{7^4}{3^4}$ (c) $\frac{9^8}{2^8}$ (d) $\frac{y^3}{x^3}$

(e) $\frac{d^5}{e^5}$ (f) $\frac{m^{12}}{n^4}$ (g) $\frac{w^8}{z^2}$ (h) $\frac{r^{20}}{s^4}$

(i) $\frac{d^6}{g^2}$ (j) $\frac{81}{w^{24}}$ (k) $\frac{512}{m^6}$ (l) $\frac{8}{r^{12}}$

(m) $\frac{h^{15}}{g^{10}}$ (n) $\frac{a^{12}}{c^{20}}$ (o) $\frac{t^{12}}{v^9}$ (p) $\frac{125a^3}{b^3}$

(q) $\frac{c^{24}}{64x^6}$ (r) $\frac{f^{18}}{216g^3}$

3 (a) 12^2 (b) 20^6 (c) 56^4 (d) $(4h)^8$ (e) $(3p)^4$

(f) $(7r)^9$ (g) $(am)^5$ (h) $(ht)^7$ (i) $(pn)^8$

4 (a) 5^4 (b) 4^3 (c) 12^5 (d) $\left(\frac{r}{3}\right)^5$ (e) $\left(\frac{n}{6}\right)^7$

(f) $\left(\frac{g}{4}\right)^9$ (g) $\left(\frac{x}{y}\right)^6$ (h) $\left(\frac{k}{n}\right)^{11}$ (i) $\left(\frac{g}{h}\right)^{10}$

5 (a) 1 (b) 7 (c) 8 (d) b^2

(e) 3 (f) c^3 (g) $-3a^4$ (h) $-8g^{11}$

(i) -1 (j) 1 (k) 3 (l) 3

(m) $3x^4$ (n) k^8 (o) $\frac{n^4}{2}$ (p) a^3

(q) 3 (r) $\frac{3}{2}$ (s) $\frac{3f^2}{4}$ (t) $\frac{5s^9}{9}$

(u) $\frac{5p^2}{3}$

6 (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{625}$ (d) $\frac{1}{27}$

(e) $\frac{1}{25}$ (f) $\frac{1}{64}$

7 (a) C (b) A (c) B (d) B

8 (a) x^6y^6 (b) g^9h^9 (c) $432x^5$ (d) $400x^2y^4$

(e) $c^{12}d^9$ (f) x^6y^8

9 (a) $\frac{f^5}{g^5}$ (b) $\frac{p^{10}}{q^{10}}$ (c) $\frac{d^{27}}{q^{14}}$ (d) a^2b^2

(e) $\frac{4a^5}{b^5}$ (f) $\frac{108x^8}{y^5}$

10 (a) w^2z^5 (b) c^2e^4 (c) p^6q^3 (d) v^3w^5

11 (a) 1 (b) 1 (c) 1 (d) 1

(e) m^{12} (f) f^{20} (g) t^4 (h) 1

(i) 1 (j) $\frac{3}{5}$ (k) $\frac{4}{7}$ (l) 1

(m) $\frac{5b^6}{2}$ (n) $\frac{d^6}{6}$ (o) $\frac{d}{10}$ (p) $\frac{q^5}{15}$

12 (a) $\frac{1}{16}$ (b) $\frac{1}{27}$ (c) $\frac{1}{625}$ (d) 2 (e) 64 (f) 9

(g) $\frac{4}{9}$ (h) $\frac{3}{64}$ (i) $\frac{2}{25}$ (j) $-\frac{7}{27}$ (k) -3 (l) -27

13 (a) A (b) B

14 Eva's method is faster because simplifying the expression in the brackets first before raising it to the index makes the mathematics simpler.

15 (a) 3^{2n} (b) 2^{3n+2} (c) 5^{n+1}

16 (a) $5^3, 5^2, 5^1, 5^0$ (b) $5^{-1}, 5^{-2}, 5^{-3}, 5^{-4}, 5^{-5}$

(c) 125, 25, 5, 1, $\frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \frac{1}{3125}$

17 (a) $\frac{n^4T^6p^2}{b^2}$

(b) (i) 10^{12}

(ii) We will obtain a positive answer, because the number inside the brackets will be squared, and a negative or a positive number squared gives a positive answer.

18 (a)

Source	Sound level (loudness, dbs)	Sound intensity, watts/m^2
Jet aircraft, 50 m away	140	100
Threshold of pain	130	10
Chainsaw, 1 m distance	110	0.1
Nightclub, 1 m from speaker	100	0.01
Busy traffic	80	0.0001
Vacuum cleaner, 1 m distance	70	0.000 01
Average home	50	0.000 000 1
Rustling of leaves in the distance	10	0.000 000 000 001

(b) 3

(c) The intensity of a rock concert is at the same level as the threshold of discomfort.

(d) Thunder at a sound level of 110 db.

19 (a) $(6m^2)^2 \times (n^2)^4, (mn^2)^4 \times 6^2$

(b) $(2m^2)^2 \div 5(n^3)^2, \frac{(4m^2n)^2}{20n^8}$

20 $(2x)^3, x^3y^2, (3xy)^3$ **21** Students' own answers.

Exercise 2.3 (p. 64)

1 (a) $\frac{1}{x^5}$ (b) $\frac{1}{y^3}$ (c) $\frac{1}{b^7}$ (d) $\frac{3}{e^3}$

(e) $\frac{7}{x^5}$ (f) $\frac{3}{g^4}$ (g) $\frac{8}{p^5q^2}$ (h) $\frac{2}{x^3y^5}$

(i) $\frac{12}{a^3b^9}$ (j) $\frac{3p^3}{q}$ (k) $\frac{5x^2}{y^7}$ (l) $\frac{7b^2}{a^5}$

2 (a) d^2 (b) k^5 (c) $3a$

(d) x^3y^6 (e) p^7q^2 (f) $\frac{n^4}{m^4}$

(g) $\frac{b^{12}}{a^8}$ (h) $\frac{n^{10}}{m^8}$ (i) $x^{15}y^6$

3 (a) $\frac{1}{x+4}$ (b) $\frac{1}{(x-y)^2}$ (c) $\frac{5}{c-3}$

4 (a) C (b) B

5 (a) $\frac{1}{x^3}$ (b) $\frac{1}{p^2}$ (c) $\frac{12}{c^5}$

(d) $\frac{1}{a^7}$ (e) k^2 (f) $\frac{3}{4d}$

(g) $\frac{1}{m^9}$ (h) $\frac{y^8}{x^4}$ (i) $\frac{2c^6}{d^3}$

6 $\frac{9}{8}$

7 (a) 1 divided by the original number

(b) $\frac{7}{a}$ m

8 (a) Students' own answers.

(b) Students' own answers.

9 $3^2 = 9$ not 6. $(y^4)^2 = y^8$ not y^6. $6 \div 2 = 3$ not 4. $x^4 \div x^{-2} = x^6$ not x^2.

$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{9x^4y^8}{2x^{-2}y^5} = \frac{9x^6y^3}{2}$

Exercise 2.4 (p. 69)

1 (a) 9800 (b) 65 000 000 (c) 3 250 000

(d) 420 710 (e) 0.002 (f) 0.000 05

(g) 0.000 000 07 (h) 0.050 32 (i) 0.008 26

(j) 0.000 041 8 (k) 0.000 714 (l) 0.000 003 55

2 (a) 2.56×10^3 (b) 9.002×10^4 (c) 6.14×10^5
(d) 1.9×10^5 (e) 8.05×10^6 (f) 5.6×10^7
(g) 3.05×10^{-1} (h) 3×10^{-2} (i) 4×10^{-4}
(j) 2.01×10^{-3} (k) 7.2466×10^{-5} (l) 5.3218×10^{-7}

3 (a) 390 000 (b) 0.054 0 (c) 670
(d) 0.003 00 (e) 1700 000 (f) 0.900

4 (a) 3 (b) 4 (c) 3 (d) 2 (e) 4 (f) 2

5 (a) 3×10^{-3} seconds (b) 1.34×10^{-1} seconds
(c) $3\text{-}4 \times 10^{-1}$ seconds

6 (a) 4.0075×10^4 km (b) 1×10^{-7} m
(c) 1.5×10^{-3} m (d) 5.9×10^9 km
(e) 3×10^{-8} m (f) 1.063×10^3 °C

7 (a) 300 000 000 m/s
(b) 0.000 000 000 000 001 5 m
(c) 0.000 12 m
(d) 4 000 000 000 000 000 000 000 000 atoms
(e) 778 000 000 km
(f) 15 000 000 °C

8 A

9 (a) 1.416×10^{11} (b) 2.1286×10^{-2}
(c) 3.0×10^{11} (d) 2.0×10^{12}
(e) 6.066×10^{-4} (f) 4.299×10^{-10}
(g) $-1.379\,914 \times 10^8$ (h) $1.349\,545 \times 10^{10}$
(i) $2.689\,98 \times 10$ (j) $1.580\,64 \times 10^{12}$
(k) $2.376\,923\,077 \times 10^{11}$ (l) $3.395\,358\,605 \times 10^{14}$

10 (a) 6.3×10^{-8} s (b) 8.53×10^8 Hz
(c) 2.562×10^{12} L (d) 5.9×10^{-5} m

11 (a) 2^{43} (b) 8.80×10^{12}

12 (a) 6×10^3 kB (b) 1.5×10^2 MB (c) 6.4×10^{-2} MB

13 (a) 3.75×10^{10} (b) 1.33×10^{-4}

14 (a) 1×10^{-14} m (b) 4×10^3 m (c) 2×10^{-5} m
(d) 2.73×10^3°C (e) 1.0651×10^4 athletes
(f) 1×10^{14} atoms (g) 1.5×10^8 km, 6×10^3°C
(h) 1.4×10^5 cases

15 (a) 6×10^{-3} seconds (b) 2.5×10^{-3} seconds
(c) 2×10^{-3} seconds (d) 2×10^{-3} seconds

16 (a) $9.99\text{-}9.997 \times 10^{-4}$ seconds (b) $1.9\text{-}1.95 \times 10^{-4}$ seconds

17 (a) 4.57×10^9 years (b) 6.4×10^8 years
(c) 2.4×10^9 years ago (d) 2.2×10^8 years
(e) 5×10^8 years (f) 5.3×10^8 years
(g) 1.2×10^8 years (h) 1.65×10^8 years
(i) 1.44×10^8 years
(j) 6.5×10^7 years ago and 2.51×10^8 years ago
(k) 4.5698×10^9 years

18 (a) (i) 3.6×10^{10} m (ii) $1.088\,64 \times 10^{15}$ m
(iii) $9.467\,28 \times 10^{15}$ m
(b) 8.3 minutes

19 1.28 seconds

20 (a) paper (b) 1.8×10^{-3} m

21 (a) 6.235×10^{11} cells (b) 3.9904×10^{13} cells
(c) 39 904 000 000 000 cells

22 (a) 1.08×10^8 km (b) 1.08×10^{11} m
(c) 6.79×10^8 km or 6.79×10^{11} m
(d) 4.2×10^7 km or 4.2×10^{10} m

23 5×10^{-3} cm

24 3×10^2
$= 3 \times 100$
$= 300$
Indices are evaluated before multiplication. Brackets would be needed to change the order of operations to get Josh's answer.
$(3 \times 10)^2 = 30^2$

25 2×10^{13}
$= 900$

Exercise 2.5 (p. 75)

1 (a) (i) $k = \frac{P}{m}$ (ii) -8
(b) (i) $b = y - mx$ (ii) -1
(c) (i) $h = \frac{C + g}{a}$ (ii) $-\frac{1}{2}$
(d) (i) $x = \frac{a - c}{m}$ (ii) -6
(e) (i) $p = \frac{kn + 2}{m}$ (ii) -53
(f) (i) $w = v(u + xy)$ (ii) 30

2 (a) $r = \sqrt{\frac{V}{\pi h}}$ (b) $v = \sqrt{\frac{2E}{m}}$ (c) $h = \sqrt{\frac{Fk}{d}}$
(d) $m = \frac{k^2 n}{49}$ (e) $b = \frac{d^2 c}{4}$ (f) $y = \frac{m^2 z}{x^2}$

3 (a) $x = z - y$ (b) $x = 3a + 2b$ (c) $x = \frac{m + n}{k}$
(d) $x = \frac{f - c}{d}$ (e) $x = rt - p$ (f) $x = vy + w$
(g) $x = \frac{kp - y}{2}$ (h) $x = \frac{mw - t}{r}$ (i) $x = \frac{mn - mp}{p}$
(j) $x = \frac{gh - e}{cd}$ (k) $x = \frac{mn + k}{ap}$ (l) $s = \frac{x - vw}{tu}$
(m) $x = \frac{y(f - b)}{d}$ (n) $x = \frac{n(t - kp)}{m}$ (o) $x = \frac{c(ab - f)}{d}$

4 B

5 (a) $t = \frac{v-u}{a}$

(b) (i) $t = 4$ s (ii) $t = 3$ s

6 (a) $r = \frac{C}{2\pi}$

(b) (i) $r = 99.95$ (ii) $r = 2.61$

7 $h = \frac{V}{\pi r^2}$; $h = 2.00$

8 $w = \frac{P}{2} - l$; $w = 19$ m

9 (a) 1.44 (b) 4 (c) ±700

(d) When calculating a square root of a number there is a negative and a positive solution. The negative solution means that the object is travelling in the opposite direction to the positive solution.

10 (a) 21 g (b) Yes (the overall density is 1.02 g/cm^3)

11 (a) $\frac{2A}{h} - a$ (b) $b = 13$

(c) If we had to recalculate b for different values of A, a and h, it would be easier to have b as the subject.

12 $s = an + \frac{dn^2}{2} - \frac{dn}{2}$

It's difficult to make n the subject as there are terms with n and n^2 in this expression.

13 She has made a mistake in the final line. She should have taken b away from $\frac{2A}{h}$. $a = \frac{2A}{h} - b$; $b = \frac{2A}{h} - a$

Half-time 2 (p. 78)

1 (a) $12x^7y^2$ (b) $4x^2y^3$ (c) $\frac{3m^6n}{4}$

2 (a) 0.300 (b) 80.0 (c) 62 100

3 (a) $9x^8y^{10}$ (b) $\frac{a^6}{36b^2}$ (c) $\frac{125}{3}$

4 (a) 3.84×10^5 km (b) 5×10^2 mL

5 (a) 4 (b) 3 (c) 3

6 (a) $2^7 \times 5^4$ (b) 3×5^6 (c) 2^5

7 (a) 110 000 000 000 000 bacteria (b) 0.000 004 4 m

8 (a) $\frac{1}{q^7}$ (b) $\frac{1}{c^9d^5}$ (c) 2 (d) $\frac{8n^4}{m^4}$

9 (a) $\frac{1}{x^5}$ (b) $\frac{3}{p^4}$ (c) $\frac{5n}{m^3}$ (d) $\frac{6r^2}{p^4q}$

10 $x = \frac{z+3y}{2y}$, $x = 2.2$

Exercise 2.6 (p. 82)

1 (a) $2x + 10$ (b) $4a + 36$ (c) $8b + 24$

(d) $8x - 16$ (e) $9x - 18$ (f) $3k - 12$

(g) $-7x - 7$ (h) $-4d - 32$ (i) $-6x - 18$

(j) $-9c + 54$ (k) $-2x + 10$ (l) $-5x + 30$

(m) $x^2 + 3x$ (n) $m^2 + 5m$ (o) $x^2 + 9x$

(p) $-x^2 + 8x$ (q) $-x^2 + 4x$ (r) $-n^2 + 9n$

(s) $3x^2 + 6x$ (t) $2u^2 - 12u$ (u) $5x^2 - 5x$

(v) $-2x^2 - 14x$ (w) $-7q^2 - 7q$ (x) $-9b^2 + 54b$

2 (a) $5x + 2$ (b) $5y - 4$ (c) $2x - 11$

(d) $5a + 13$ (e) $2a^2 - 3a + 10$ (f) $3x^2 + 2x + 4$

(g) $7y^2 - 26y - 10$ (h) $9k^2 + 27k$

3 (a) $xy + 5x + 6y + 30$ (b) $xy + 7x + 4y + 28$

(c) $ce + 3c + de + 3d$ (d) $rt - 8r + 5t - 40$

(e) $ab - a + 2b - 2$ (f) $pq - 2p + 3q - 6$

(g) $ay - 4a - 3y + 12$ (h) $mn - 6m - 2n + 12$

(i) $km - 4k - 5m + 20$ (j) $4mp + 4np + mq + nq$

(k) $7ax + 7kx + ay + ky$ (l) $12ps + 3pt + 8rs + 2rt$

(m) $2ab - 6a + b - 3$ (n) $3x^2 + 5x - 2$

(o) $3mn - 3m + 2n - 2$ (p) $10am - 6an - 15bm + 9bn$

(q) $15cx - 20cy - 6dx + 8dy$ (r) $14pr - 21pt - 4qr + 6qt$

4 (a) $a^2 + 5a + 6$ (b) $c^2 + 8c + 15$

(c) $y^2 + 13y + 42$ (d) $x^2 - 2x - 3$

(e) $y^2 + 7y - 18$ (f) $a^2 + 5a - 84$

(g) $b^2 - 3b + 2$ (h) $x^2 - 7x + 12$

(i) $p^2 - 15p + 56$ (j) $6m^2 + 11m + 3$

(k) $8d^2 + 22d + 15$ (l) $21k^2 + 47k + 20$

(m) $8y^2 - 2y - 3$ (n) $10k^2 - 29k - 21$

(o) $8a^2 + 14a - 15$ (p) $12a^2 - 16a + 5$

(q) $12p^2 - 29p + 15$ (r) $24r^2 - 37r + 14$

5 (a) $x^2 + 2xy + y^2$ (b) $m^2 + 2mn + n^2$

(c) $a^2 + 2ab + b^2$ (d) $p^2 - 2pr + r^2$

(e) $c^2 - 2cd + d^2$ (f) $g^2 - 2gh + h^2$

(g) $4c^2 + 12cd + 9d^2$ (h) $9m^2 + 12mn + 4n^2$

(i) $25a^2 + 30ab + 9b^2$ (j) $25x^2 - 40xy + 16y^2$

(k) $4j^2 - 12jk + 9k^2$ (l) $49c^2 - 70cd + 25d^2$

6 (a) $m^2 - n^2$ (b) $p^2 - q^2$

(c) $a^2 - b^2$ (d) $9a^2 - 4b^2$

(e) $16x^2 - 9y^2$ (f) $49m^2 - 9n^2$

7 A

8 (a) $(x + 7)(x + 2)$ cm^2 (b) 204 cm^2

(c) 6 (d) $x^2 + 9x + 14$

(e) $10^2 + 9 \times 10 + 14 = 204$

9 D

10 (a) (i) $(15 + x)$ m (ii) $(3 + x)$ m

(iii) $(15 + x)(3 + x)$ m^2 (iv) $(45 + 18x + x^2)$ m^2

(b) New garden bed has $(18x + x^2)$ m^2 more area

(c) 40 m^2 extra area

11 (a) $x(x + 3)$ (b) $10x - 12$ (c) 3 cm, 4 cm

12 Two possible methods are to substitute values for x and to use the areas of rectangles.

13 **(a)** **(i)** x^2 m^2 **(ii)** $(x - 4)$ m

(iii) $(x - 2)$ m **(iv)** $(x - 2)(x - 4)$ m^2

(b) $(6x - 8)$ m^2

14 length = l, width = w; increase length by 5 units, increase width by 7 units; area of new rectangle = $(l + 5)(w + 7)$

15 $-3(5x - 2) = -3 \times 5x - (-3) \times 2 = -15x + 6$

(a) This student forgot to multiply the -2 by -3.

(b) This student forgot to multiply the $5x$ by -3.

(c) This student said that $-3 \times 5 = -8$ and $-3 \times -2 = 5$.

16 $(x - 3)(x + 4) = x^2 + 4x - 3x - 12 = x^2 + x - 12$

(a) This student expanded $(x + 3)(x + 4)$.

(b) This student might have said that $x^2 + x = 2x$.

(c) This student said that $-3 \times 4 = -7$.

Exercise 2.7 (p. 86)

1 **(a)** $3(x + 3)$ **(b)** $12(y + 2)$ **(c)** $6(1 + 7a)$

(d) $2(k - 3)$ **(e)** $4(m - 5)$ **(f)** $4(2b - 1)$

(g) $x(7 - y)$ **(h)** $s(4 - 9t)$ **(i)** $a(3 - b)$

(j) $5(3y + w + 2x)$ **(k)** $2(a + 2b - 4c)$ **(l)** $3(x - 5y + 3)$

(m) $ac(3 + 2b)$ **(n)** $xy(5y + 2x)$ **(o)** $3fg(3fh - 4g)$

(p) $6gh(h + 3g)$ **(q)** $8xy(2x - 5y)$ **(r)** $6ab(3 - 7a)$

(s) $-4(m + 5)$ **(t)** $-7(d + 7)$ **(u)** $-12(r + 5)$

(v) $(a + 3)(x + 5)$ **(w)** $(n - 2)(m + 9)$ **(x)** $(x + 5)(3 + y)$

2 **(a)** $5(2x + 5)$ **(b)** $4(4y + 9)$ **(c)** $7(2a - 5)$

(d) $6(2d - 3)$ **(e)** $9(2 - 3m)$ **(f)** $3(12 - 5k)$

(g) $b(24ac - 10 + 7c)$ **(h)** $k(5m - 3n + 2gh)$ **(i)** $c(1 + 4b + 2a)$

(j) $x(x + 3)$ **(k)** $y(y + 6)$ **(l)** $k(5 + k)$

(m) $m(8 - 3m)$ **(n)** $p(4p - 7)$ **(o)** $a(9a - 5)$

(p) $3d(2 - 3d)$ **(q)** $2g(11 - 7g)$ **(r)** $8a(3 - 4a)$

3 **(a)** $-2(m + 6)$ **(b)** $-4(k + 6)$ **(c)** $-5(h + 5)$

(d) $-x(x + 1)$ **(e)** $-y(y + 3)$ **(f)** $-p(p + 4)$

(g) $-4(4a - 1)$ **(h)** $-7(3b - 1)$ **(i)** $-6(5r - 1)$

(j) $-8(3p - 5)$ **(k)** $-2(9w - 8)$ **(l)** $-9(5k + 7)$

(m) $-3x(5x + 12)$ **(n)** $-7y(2y + 7)$ **(o)** $-4r(5r + 11)$

(p) $-2(4d + 2f + 3g)$ **(q)** $-5(3a + 2b + c)$ **(r)** $-7(2c + 4d + f)$

(s) $-4x(3x + 4 - 5y - z)$

(t) $-2k(m - 2m^2 + 3 + 4k)$

(u) $-2b(5b - 2c + 4c^2 - 8a)$

4 **(a)** $(y - 1)(4 + w)$ **(b)** $(m - 3)(5 + n)$

(c) $(p - 2)(5 + r)$ **(d)** $(q + 5)(p - 2)$

(e) $(2x + 3)(y - 1)$ **(f)** $(3p + 5)(a - 1)$

(g) $(f - 1)(d - 6)$ **(h)** $(d - 2)(5d - 4)$

(i) $(m - 6)(3m - 7)$

5 C **6** A

7 **(a)** $8ab + 12a$ **(b)** $4a(2b + 3)$

8 **(a)** $(c + 2)(d + 5)$ **(b)** $c = d + 3$

9 **(a)** $(5 - x)$ m **(b)** $(5 - x)(y + 3)$ m^2

10 **(a)** $4x^2 + xy$ cm^2 **(b)** $x(4x + y)$ cm^2

(c) $\pi(x - 1)^2$ cm^2 **(d)** $\pi x^2 - 2\pi x + \pi$ cm^2

(e) $4x^2 + xy - (\pi x^2 - 2\pi x + \pi)$
$= (4 - \pi)x^2 + (y + 2\pi)x - \pi$ cm^2

(f) $\pi x^2 + xy - 2\pi x + \pi$ cm^2

11 **(a)** $(3p + 2)(2p - 1)$ cm^2 **(b)** $(r + 2)(2p - 1)$ cm^2

(c) $(2p - 1)(3p - r)$

(d) If you calculate the length of the lasagne *after* the slice has been removed, it is then easier to calculate the required area:

New length = $3p + 2 - (r + 2) = 3p + 2 - r - 2 = 3p - r$

New area = $(2p - 1)(3p - r)$ cm^2

12 Sample answers: $3x + 12xy$; $6x + 9xb$; $12abx + 15cx$

13 The correct answer is C, $-5(x - 2)$. The first answer had the wrong sign in front of the constant in the brackets, and the second answer had the wrong sign and value of the constant in the brackets.

Exercise 2.8 (p. 90)

1 **(a)** $(x + 4)(y + 3)$ **(b)** $(p + 1)(q + 5)$

(c) $(m + 2)(n + 9)$ **(d)** $(a + f)(b + d)$

(e) $(p + t)(q + r)$ **(f)** $(k + 6)(m + 3n)$

(g) $(p - 7)(n + 1)$ **(h)** $(a - 5)(b + 1)$

(i) $(m - 2)(n + 7)$ **(j)** $(a - 6)(b - 4)$

(k) $(x + 2)(x - 8)$ **(l)** $(y + 4)(y - 6)$

2 **(a)** $(k + 2)(p + 3)$ **(b)** $(m + p)(n + q)$

(c) $(d + 6)(c + 2)$ **(d)** $(a - k)(d + h)$

(e) $(x - 4)(y + 2)$ **(f)** $(e - 1)(g + 3)$

(g) $(b + 1)(c - 1)$ **(h)** $(y + 3)(x - 1)$

(i) $(k + 3)(m - n)$

3 **(a)** **(i)** $(x + 2)(x + 7)$

(ii) $x^2 + 9x + 14$

(iii) $x^2 + 9x + 14 = (x + 2)(x + 7)$

(iv) $2 \times 7 = 14$

(b) **(i)** $(j + 5)(j + 3)$

(ii) $j^2 + 8j + 15$

(iii) $j^2 + 8j + 15 = (j + 5)(j + 3)$

(iv) $5 \times 3 = 15$

(c) **(i)** $(z + 8)(z + 1)$

(ii) $z^2 + 9z + 8$

(iii) $z^2 + 9z + 8 = (z + 8)(z + 1)$

(iv) $8 \times 1 = 8$

(d) **(i)** $(x - 8)(x + 2)$

(ii) $x^2 - 6x - 16$

(iii) $x^2 - 6x - 16 = (x - 8)(x + 2)$

(iv) $-8 \times 2 = -16$

(e) (i) $(y-6)(y+4)$

(ii) $y^2-2x-24$

(iii) $y^2-2x-24=(y-6)(y+4)$

(iv) $-6\times 4=-24$

(f) (i) $(x-4)(x+3)$

(ii) x^2-x-12

(iii) $x^2-x-12=(x-4)(x+3)$

(iv) $-4\times 3=-12$

(g) (i) $(a+5)(a-4)$

(ii) a^2+a-20

(iii) $a^2+a-20=(a+5)(a-4)$

(iv) $5\times -4=-20$

(h) (i) $(k+5)(k-2)$

(ii) $k^2+3k-10$

(iii) $k^2+3k-10=(k+5)(k-2)$

(iv) $5\times -2=-10$

(i) (i) $(m+4)(m-7)$

(ii) $m^2-3m-28$

(iii) $m^2-3m-28=(m+4)(m-7)$

(iv) $4\times -7=-28$

(j) (i) $(y-7)(y-3)$

(ii) $y^2-10y+21$

(iii) $y^2-10y+21=(y-7)(y-3)$

(iv) $-7\times -3=21$

(k) (i) $(c-5)(c-9)$

(ii) $c^2-14c+45$

(iii) $c^2-14c+45=(c-5)(c-9)$

(iv) $-5\times -9=45$

(l) (i) $(d-12)(d-8)$

(ii) $d^2-20d+96$

(iii) $d^2-20d+96=(d-12)(d-8)$

(iv) $-12\times -8=96$

(m) (i) $(m+3)^2$

(ii) m^2+6m+9

(iii) $m^2+6m+9=(m+3)^2$

(iv) $3\times 3=9$

(n) (i) $(r+6)^2$

(ii) $r^2+12r+36$

(iii) $r^2+12r+36=(r+6)^2$

(iv) $6\times 6=36$

(o) (i) $(k+11)^2$

(ii) $k^2+22k+121$

(iii) $k^2+22k+121=(k+11)^2$

(iv) $11\times 11=121$

(p) (i) $(p-5)^2$

(ii) $p^2-10p+25$

(iii) $p^2-10p+25=(p-5)^2$

(iv) $-5\times -5=25$

(q) (i) $(d-7)^2$

(ii) $d^2-14d+49$

(iii) $d^2-14d+49=(d-7)^2$

(iv) $-7\times -7=49$

(r) (i) $(n-8)^2$

(ii) $n^2-16n+64$

(iii) $n^2-16n+64=(n-8)^2$

(iv) $-8\times -8=64$

(s) (i) $(c-3)(c+3)$

(ii) c^2-9

(iii) $c^2-9=(c-3)(c+3)$

(iv) $-3\times 3=-9$

(t) (i) $(t-2)(t+2)$

(ii) t^2-4

(iii) $t^2-4=(t-2)(t+2)$

(iv) $-2\times 2=-4$

(u) (i) $(b-9)(b+9)$

(ii) b^2-81

(iii) $b^2-81=(b-9)(b+9)$

(iv) $-9\times 9=-81$

4 **(a)** $(x+2)(x+12)$ **(b)** $(x+3)(x+8)$ **(c)** $(x-4)(x-6)$

(d) $(x+2)(x+6)$ **(e)** $(x-3)(x-4)$ **(f)** $(x-1)(x-12)$

(g) $(x+3)(x+5)$ **(h)** $(x+2)(x+9)$ **(i)** $(x+2)(x+8)$

(j) $(x-6)(x+4)$ **(k)** $(x+8)(x-3)$ **(l)** $(x+2)(x-12)$

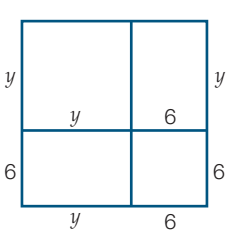
5 A

6 **(a)** $x^2-4x+12-3x$ **(b)** $x^2-7x+12$

(c) $(x-4)(x-3)$

7 **(a)** $y^2+12y+36$ **(b)** $(y+6)^2$

(c)



8 **(a)** $(a+7)(b+c+d)$

(b) The expression would be equal to $(a+7)(a+7)$, which is a perfect square: $(a+7)^2$.

9 **(a)** $x^2+10x+9$, $x^2+10x+16$, $x^2+10x+21$, $x^2+10x+24$, $x^2+10x+25$

(b) $(x+1)(x+9)$, $(x+2)(x+8)$, $(x+3)(x+7)$, $(x+4)(x+6)$, $(x+5)(x+5)$

(c) There are only five pairs of positive numbers that will add to 10: 1 & 9, 2 & 8, 3 & 7, 4 & 6, 5 & 5.

(d) If the smaller of any two positive numbers that differ by 10 is made negative and the numbers are added, then the sum will be 10, e.g. 15 – 5, 32 – 22.There is an infinite number of possibilities.

10 (a) $7y(5y - 3)$ (b) $7y$ cm by $(5y - 3)$ cm

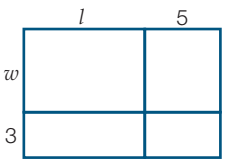
(c) $(7y + 3)(5y - 3)$ (d) 3 cm by $(5y - 3)$ cm

11 (a) $\frac{1}{2}(3x + 2y)(x - 2)$ (b) $3x + 2y$

12 (a) -36 (b) $+6$ (c) $7x$

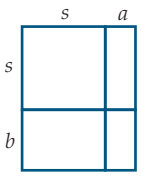
13 (a) xy and ay (b) $(x + a)(x + y)$

14 (a)



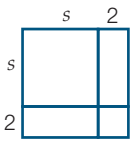
Area = $lw + 3l + 5w + 15$
or
Area = $(l + 5)(w + 3)$

(b)



Area = $s^2 + as + bs + ab$
or
Area = $(s + a)(s + b)$

(c)



Area = $s^2 + 2s + 2s + 4$
or
Area = $(s + 2)^2$

Exercise 2.9 (p. 94)

1 (a) $d^2 - 2d + 1$ (b) $m^2 + 10m + 25$

(c) $k^2 + 4k + 4$ (d) $p^2 - 6p + 9$

(e) $x^2 - 16x + 64$ (f) $y^2 - 24y + 144$

(g) $m^2 + 2mn + n^2$ (h) $w^2 - 2wk + k^2$

(i) $c^2 - 12c + 36$ (j) $y^2 + 12y + 36$

(k) $x^2 + 18x + 81$ (l) $k^2 - 14k + 49$

(m) $16y^2 + 24y + 9$ (n) $4w^2 + 36w + 81$

(o) $9x^2 + 30x + 25$ (p) $36k^2 - 12k + 1$

(q) $49 - 28a + 4a^2$ (r) $9 - 48d + 64d^2$

(s) $1 - 8c + 16c^2$ (t) $36 + 84y + 49y^2$

(u) $16g^2 - 8gh + h^2$ (v) $9a^2 + 6ad + d^2$

(w) $4a^2 - 12ab + 9b^2$ (x) $25m^2 + 80mn + 64n^2$

2 (a) $c^2 - 25$ (b) $a^2 - 16$ (c) $k^2 - 64$

(d) $81 - x^2$ (e) $9 - y^2$ (f) $1 - k^2$

(g) $m^2 - n^2$ (h) $x^2 - y^2$ (i) $c^2 - d^2$

(j) $v^2 - w^2$ (k) $p^2 - q^2$ (l) $h^2 - g^2$

(m) $4m^2 - 1$ (n) $9x^2 - 16$ (o) $36b^2 - 49$

(p) $1 - 25y^2$ (q) $64 - 25a^2$ (r) $9 - 4d^2$

(s) $16a^2 - 25c^2$ (t) $9m^2 - 4n^2$ (u) $4x^2 - 49y^2$

(v) $16 - a^2b^2$ (w) $9 - c^2d^2$ (x) $25 - j^2k^2$

3 (a) $(x + 2)^2$ (b) $(y - 8)^2$ (c) $(3b - 5)^2$

(d) $(t - 12)^2$ (e) $(x + 6)^2$ (f) $(2p + 9)^2$

(g) $(m + 10)^2$ (h) $(3s + 4)^2$ (i) $(5h - 3)^2$

(j) $(2f - 5)^2$ (k) $(c - 2d)^2$ (l) $(7n - 2k)^2$

4 (a) $(c + 5)(c - 5)$ (b) $(k + 10)(k - 10)$

(c) $(6m + 7)(6m - 7)$ (d) $(h + 6)(h - 6)$

(e) $(m + 9)(m - 9)$ (f) $(3y + 11)(3y - 11)$

(g) $(n + 2)(n - 2)$ (h) $(v + 12)(v - 12)$

(i) $(4c + 5d)(4c - 5d)$ (j) $(2p + 5)(2p - 5)$

k) $(6h + k)(6h - k)$ (l) $(5m + 3n)(5m - 3n)$

5 B **6** D **7** A **8** C

9 (a) (i) $x^2 + 14x + 49$ (ii) $4x^2 - 12x + 9$

(b) (i) 121 cm^2 (ii) 25 cm^2

10 (a) x^2 m^2 (b) $(x^2 - 4)$ m^2

(c) Square playing field is larger by 4 m^2.

11 (a) 529 (b) 2401 (c) 12 544

(d) 14.44 (e) 36.120 1 (f) 143.041 6

(g) 9996 (h) 3.91 (i) 35.998 4

12 (a) π (b) The area remains constant (π).

13 (a) $\frac{1}{2}(L + l)(L - l)$ (b) $\frac{1}{2}L^2 - \frac{1}{2}l^2$

(c) $(L + l)(L - l) = L^2 - l^2$

(d) (i) $12l^2$ cm^2 (ii) $12l^2$ cm^2

(e) Area of large triangle = $12.5l^2$

$$\% \text{ of area shaded} = \frac{12l^2}{12.5l^2} \times 100\% = 96\%$$

14 Sample answers: $a = 5$ $b = 4$; $a = 13$ $b = 5$; $a = 10$ $b = 6$; any Pythagorean triple where a = length of hypotenuse and b = either of the other side lengths.

15 $\pm 8x$

16 As there is no term in x, the coefficient of x must be zero. If the expression was expanded to 4 terms, the two middle terms would have to be equal but opposite in sign. Therefore the product of these coefficients must be negative. As all squares are positive, the sign connecting them must be a subtraction sign.

Challenge 2 (p. 98)

1 $a + b = 27$, $a \times b = 72 = 6 \times 12 = 8 \times 9 = 4 \times 18 = 3 \times 24$. Numbers are 3 and 24.

2 Let today be x. The day before yesterday is $x - 2$. These two days need to be in different years.
If today is 1 January, year y, then on 30 December, year $(y - 1)$, she is 15. Her birthday is on 31 December. So on 31 December, year $(y - 1)$, she turns 16. On 31 December, year y, she is 17 and on 31 December, year $(y + 1)$, next year, she is 18. She said this on 1 January in the year of her 17th birthday.

3 $12^3 = 1728$, $13^3 = 2197$, $20^3 = 8000$, $21^3 = 9261$, $22^3 = 10\,648$. There are nine perfect cubes between 2000 and 10 000.

4 D: shaded area $= (2x + 1)(x - 2) - 10 = 2x^2 - 3x - 12$.

5 $(3 \times 174)^2 = 272\,484$, so ♦ represents 4.

6 If a, b and c are positive whole numbers then at least two of them are either both odd or both even. The difference between two odd numbers will be even and the difference between two even numbers will be even; hence, at least one of $(a - b)$, $(b - c)$ or $(c - a)$ will be even and so the product will have a factor of 2.

7 Powers of 3 end in 3, 9, 7 or 1, and the pattern is repeated. $404 \div 4 = 101$, so 3^{404} ends in 1.

Powers of 4 end in 4 or 6, and the pattern is repeated. Hence, odd powers of 4 end in 4.

$3^{404} + 4^{495}$ ends in $(1 + 4 = 5)$, so it is divisible by 5.

8 $x + y = 6$; $1 + 5$, $2 + 4$
$x + z = 7$; $1 + 6$, $2 + 5$, $3 + 4$
$x + w = 9$; $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$
$y + z = 9$; $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$
$y + w = 11$; $1 + 10$, $2 + 9$, $3 + 8$, $4 + 7$, $5 + 6$
$z + w = 12$; $1 + 11$, $2 + 10$, $3 + 9$, $4 + 8$, $5 + 7$
By trial and error: if $x = 1$, $y = 5$, $z = 6$, $w = 8$, $z = 4$, so not the answer. Same result if $x = 5$. If $s = 2$, $y = 4$, $z = 5$, $w = 7$, and these results satisfy the other equations.
Numbers are 2, 4, 5 and 7.
Or, using pairs of equations you can get: $z - y = 1$, $w - y = 3$, $y - x = 2$, $w - x = 5$, So $y + x = 9$ and $z - y = 1$, so $2z = 10$, $z = 5$
$y + w = 11$ and $w - y = 3$, so $2w = 14$, $w = 7$
$x + y = 6$ and $y - x = 2$, so $2y = 8$, $y = 4$
$x + w = 9$ and $w - x = 5$, so $2x = 4$, $x = 2$

9 (a) $f = \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, $\frac{v + u}{uv} = \frac{1}{f}$, $f = \frac{uv}{u + v}$

(b) $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, $= \frac{1}{v} = \frac{u - f}{fu}$, $v = \frac{uf}{u - f}$

10 (a) Let the two-digit number be $10a + b$. The number formed by reversing the digits is $10b + a$. The difference $= 10a + b - (10b + a) = 9a - 9b = 9(a - b)$, which is divisible by 9.

(b) The three digit number is $100a + 10b + c$. The number formed by reversing the digits is $100c + 10b + a$. The difference $= 100a + 10b + c - (100c + 10b + a) = 99a - 99c = 99(a - c)$, which is divisible by 99 (and hence by the factors of 99).

(c) The four digit number is $1000a + 100b + 10c + d$. The number formed by reversing the digits is $1000d + 100c + 10b + a$. The difference $= 1000a + 100b + 10c + d - (1000d + 100c + 10b + a) = 999a + 99b - 99c - 999d = 9(111a + 11b - 11c - 111d)$, which is divisible by 9 (and hence by 3).

Chapter review 2 (p. 99)

1 (a) a^5b^7 (b) m^7n^8 (c) $\frac{P}{q^3}$
(d) x^2 (e) y^5 (f) c^2d

2 D

3 (a) $\frac{-5}{m}$ (b) $\frac{a^2}{b^6}$ (c) $\frac{1}{x^2y^3}$ (d) $\frac{1}{a^4b^9}$

4 (a) 2×10^{-15} mm (b) 1.5×10^{-4} kg
(c) 3.2×10^8 L (d) 4.8×10^{14}

5 (a) 54 000 000 (b) 0.000 827 3
(c) 0.000 014 (d) 7162

6 (a) $x = \frac{3 - 2z}{y}$ (b) $x = \frac{2a}{3}$
(c) $x = \frac{s - 8t^2}{t}$ (d) $x = \frac{1 + 2z}{15}$

7 A

8 (a) $4x + 28$ (b) $x^2 - 3x$ (c) $-10x^2 + 5x$

9 (a) (i) $13x - 7$ (ii) $18x^2 + 19x - 6$
(b) (i) 45 (ii) 358

10 (a) $x^2 + 10x + 16$ (b) $6a^2 + 7a - 20$

11 (a) $4(2a + 3)$ (b) $-5k(3 + 4k)$
(c) $pq(5p - 3 + 2q - r)$ (d) $-6(4a + ab + 12)$

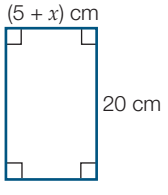
12 (a) $x^2 + 24x + 144$ (b) $25c^2 - 20cd + 4d^2$
(c) $16x^2 - 9$

13 (a) $(a + 4)(b + 3)$ (b) $(b + 1)(a - 2)$
(c) $(b - 6)(a - 5)$ (d) $(a + 7)(b - 1)$

14 (a) 1.5×10^{-3} (b) 5.9×10^9 km

15 (a) 778 000 000 km (b) 15 000 000°C

16 (a) $(5 + x)$ cm



(b) $(50 + 2x)$ cm (c) $2(25 + x)$
(d) $20(5 + x)$ cm^2 (e) $100 + 20x$
(f) Sample answer: $x = 10$; $20 \times 15 = 100 + 20 \times 10$

17 $2x + 6$

18 $(m - 2)(k + 8)$; $m = 5$, $k = 8$

19 $5y - 3$

20 (a) 7×10^3 L
(b) (i) 4.9×10^4 L (i) 0.1 L (iii) 7.665×10^6 L
(c) The amount of blood each person has will vary between people, as well as at different times for the same person. However, this provides a suitable approximation.

21 (a) $10 + 2x$ (b) $x^2 + 5x$
(c) (i) $x^2 + 5x - 14$
(ii) $x^2 + 7x - 2x - 14$, they are the same expression
(iii) There are no sheep.

NAPLAN practice 2

1 A **2** 42 **3** $7(2x + 3)$

4 B **5** $x = \frac{12 - y}{3}$ **6** D **7** A

Chapter 3

Recall 3 (p. 102)

1 **(a)** 72.1 **(b)** 18 210 **(c)** 15.7123 **(d)** 0.000074

2 **(a)** 42 mm **(b)** 390 cm **(c)** 7100 m
(d) 0.45 m **(e)** 0.186 m **(f)** 5.06 km
(g) 0.011 km **(h)** 0.003 72 km

3 **(a)** 5 m **(b)** 1.6 mm **(c)** 15.1 cm

4 **(a)** D **(b)** A **(c)** C **(d)** B

5 **(a)** 44 cm, 112 cm^2 **(b)** 48 cm, 96 cm^2
(c) 44 cm, 154 cm^2

Exercise 3.1 (p. 106)

1 **(a)** 34 cm **(b)** 24 cm **(c)** 60 cm
(d) 45 cm **(c)** 130 mm **(f)** 140 cm

2 **(a)** **(i)** 4.8 m **(ii)** 10.2 m **(iii)** 45.0 mm **(iv)** 60.0 mm
(b) **(i)** 14.8 km **(ii)** 11.3 km **(iii)** 128.0 cm **(iv)** 0.2 cm

3 **(a)** **(i)** 15 cm **(ii)** 43 cm
(b) **(i)** 7 m **(ii)** 13 m
(c) **(i)** 1470 mm **(ii)** 2212 mm
(d) **(i)** 139 m **(ii)** 191 m
(e) **(i)** 68 cm **(ii)** 121 cm
(f) **(i)** 152 cm **(ii)** 480 cm

4 B

5 **(a)** 9.1 km **(b)** 111.4 cm **(c)** 42.8 cm
(d) 112.8 cm **(e)** 314.2 cm **(f)** 127.1 cm
(g) 94.2 cm **(h)** 268.5 cm **(i)** 88.8 cm

6 12.0 cm **7** 40 200 km

8 **(a)** 113 m **(b)** 22.6 seconds **(c)** 31.83 m

9 2.342 km

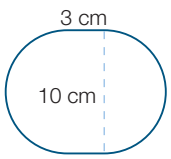
10 35.5 km

11 Just under 16 corrugations (15.9 on the calculator)

12 Students' own answers.

13 Any whole number value from 16 to 28 cm (e.g. a diameter of 20 cm gives a circumference of ≈ 62.83 cm; a diameter of 25 cm gives a circumference of ≈ 78.54 cm; a diameter of 28 cm gives a circumference of ≈ 87.96 cm).

14 Possible shape with $P \approx 37.4$ cm:



Exercise 3.2 (p. 112)

1 **(a)** 30 000 cm^2 **(b)** 1720 cm^2 **(c)** 7250 mm^2
(d) 60 mm^2 **(e)** 90 000 m^2 **(f)** 40 700 m^2
(g) 0.0035 ha **(h)** 6.95 cm^2 **(i)** 0.0127 m^2
(j) 0.0835 m^2 **(k)** 438 000 m^2 **(l)** 0.685 ha
(m) 0.029 m^2 **(n)** 3960 mm^2 **(o)** 15 km^2
(p) 100.8 km^2

2 **(a)** 8196.9 cm^2 **(b)** 16.6 m^2 **(c)** 506.3 cm^2
(d) 5.3 m^2 **(e)** 4.3 m^2 **(f)** 1.3 km^2

3 **(a)** 1853 cm^2 **(b)** 143 cm^2 **(c)** 1750 mm^2
(d) 240 m^2 **(e)** 293 cm^2 **(f)** 506 cm^2
(g) 6514 cm^2 **(h)** 628 cm^2 **(i)** 184 cm^2
(j) 468 cm^2 **(k)** 1087 cm^2 **(l)** 1973 cm^2

4 C

5 **(a)** 1408 m^2 **(b)** 176 kg

6 **(a)** **(i)** 112 mm^2 **(ii)** 1.12 cm^2
(b) **(i)** 225 mm^2 **(ii)** 2.25 cm^2
(c) **(i)** 127.23 mm^2 **(ii)** 1.27 cm^2
(d) **(i)** 4500 mm^2 **(ii)** 45 cm^2

7 8.95 m^2

8 **(a)** 12.44 m^2
(b) At least 623 bricks would be required to tile the path, although taking into account wastage, say 630 bricks.

9 167.6 cm^2

10 **(a)** 451 cm **(b)** 1546 cm^2 **(c)** 62%

11 **(a)** 311.1 m^2 **(b)** 69.1 m^2 **(c)** 82%

12 5.53 m^2 **13** 289.6 m^2 **14** 807 m^2 **15** 375 blocks

16 **(a)** 200 cm^2 **(b)** 14.14 cm

17 127° **18** 11.5 m^2

19 **(a)** **(i)** 3 cm **(ii)** 2 cm
(b) They are the same.

20 C

21 The area of the top of the pizza is 616 cm^2, so any rectangle that gives this area with integer values: approximately 25×25; 22×28; 31×20 etc.

22 Area = 200 cm^2 (length 20 cm, width 10 cm); Area = 216 cm^2 (length 18 cm, width 12 cm).

Exercise 3.3 (p. 116)

1 D

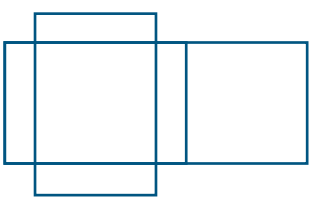
2 Several possibilities for each—teacher to check.

3 **(a)** tetrahedron **(b)** cylinder
(c) cone **(d)** hexagonal prism

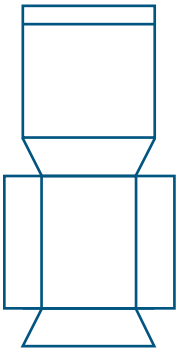
4 D **5** B

6 Sample answers—teacher to check.

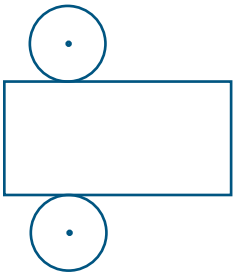
(a)



(b)



(c)



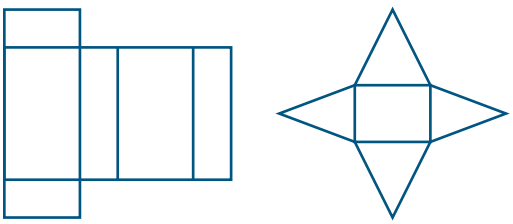
7 Students' own answers—check construction measurements carefully.

8 Students' own constructions.

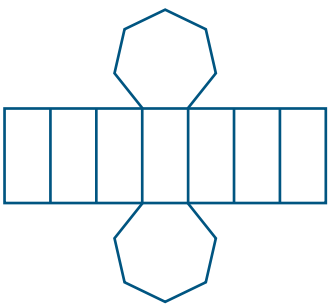
9 Top part has five faces, five vertices and eight edges. Bottom part has six faces, eight vertices and twelve edges.

10 (a) Rectangular prism: edges that are meant to meet are of different lengths, no top to match the base. Rectangular pyramid: the triangles all need to have the same sloping side lengths to meet at the apex of the pyramid.

(b)



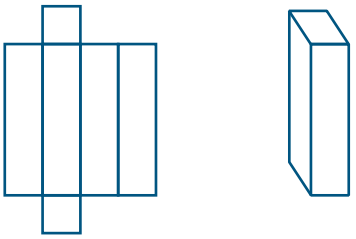
11



12 tetrahedron, octahedron, dodecahedron

13 No, a face is a flat surface; therefore, a cylinder is not suitable. A polyhedron always has more than three faces. The simplest polygon used to construct a polyhedron is a triangle and it requires a minimum of four faces to create a three-dimensional shape.

14 a rectangular prism



Half-time 3 (p. 123)

1 (a) 540 cm (b) 29 mm (c) 30 m
(d) 111 mm (e) 57 cm (f) 107 m

2 (a) 1344 cm^2 (b) 273 cm^2 (c) 57 cm^2

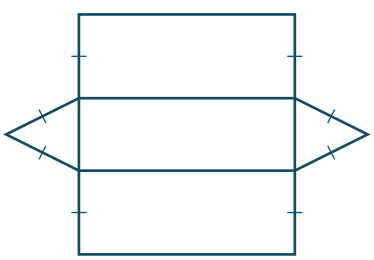
3 10 100 km

4 30 cm

5 (a) 400 mm^2 (b) 75 000 cm^2 (c) 260 000 m^2
(d) 198 000 m^2 (e) 305 ha (f) 0.0274 m^2

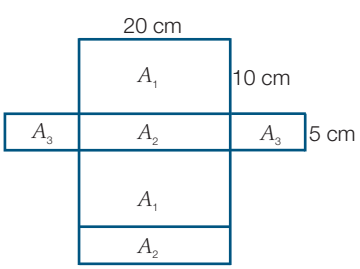
6 270 cm^2

7

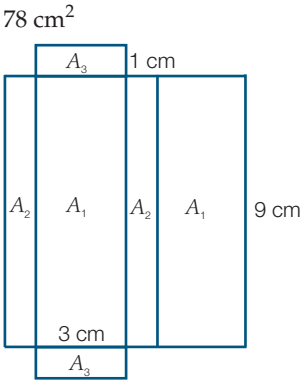


Exercise 3.4 (p. 126)

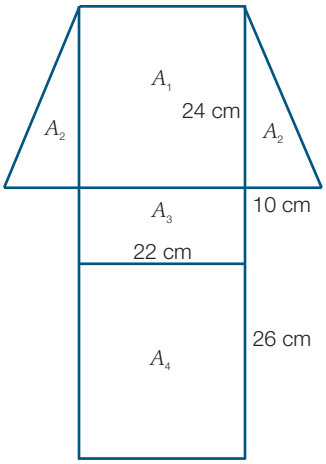
1 (a) 700 cm^2



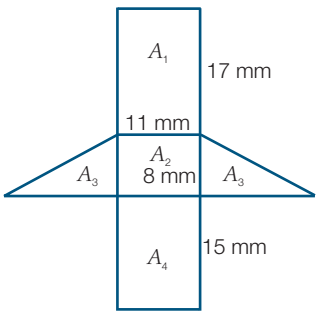
(b) 78 cm^2



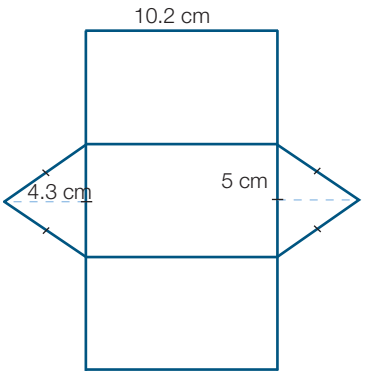
(c) 1560 cm^2



(d) 560 mm^2



(e) 174.5 cm^2



2 (a) C (b) D

3 Type 1 uses 352 cm^2; Type 2 uses 372 cm^2, so Type 1 uses less.

4 109.93 m^2

5 (a) 60 cm long, 50 cm wide (b) 30%

6 (a) 54 cm^2 (b) 162 cm^2 (c) 3.06 times

7 Four times larger; possible choice, l = 10 cm, 20 cm, SA = 600 cm^2, 2400 cm^2

8 Any of these combinations (length, width, height) in centimetres: (2, 2, 119), (4, 4, 58), (6, 6, 37), (8, 8, 26), (10, 10, 19), (12, 12, 14), (16, 16, 7), (20, 20, 2)

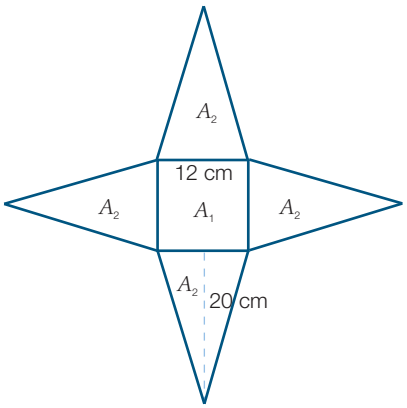
Exercise 3.5 (p. 130)

1 (a) 31.7 m^2 (b) 6622.5 cm^2

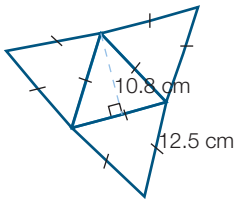
(c) 18.8 m^2 (d) 6432.4 mm^2

2 2292 cm^2

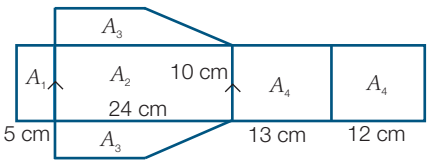
3 (a) 624 cm^2



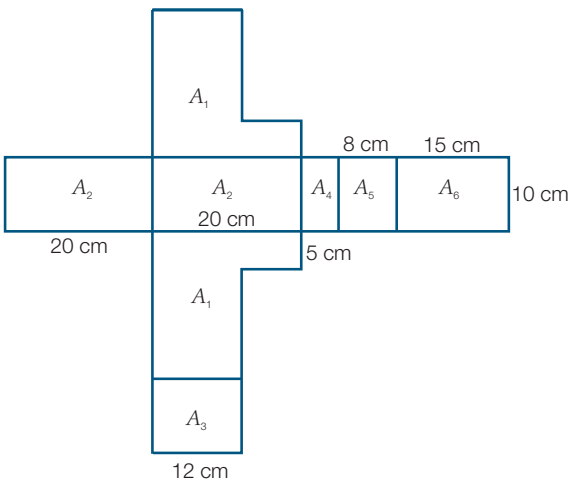
(b) 270 cm^2



(c) 720 cm^2



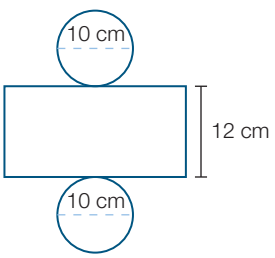
(d) 1360 cm^2



4 (a) 250 cm^2 (b) 85 cm^2

5 34 m^2

6 (a)



(b) B (c) C (d) C

7 (a) 13 500 mm^2 (b) 14 292 mm^2
(c) 14 009 mm^2 (d) 14 801 mm^2

8 3.2 cm^2

9 (a) 6.2 m^2 (b) 18.17 m^2

10 Sample answers: $r = 7$ cm, $h = 5$ cm; $r = 8$ cm, $h = 3$ cm; $r = 6$ cm, $h = 8$ cm

Exercise 3.6 (p. 135)

1 (a) 600 cm^3 (b) 512 m^3 (c) 1260 mm^3
(d) 210 cm^3 (e) 63 m^3 (f) 28 274 cm^3

2 (a) 827 mL (b) 3.2 L (c) 195 kL (d) 18.2 ML

3 3040 m^3

4 (a) 20 cm (b) 12.4 m
(c) 12 cm (d) 30 mm

5 (a) D (b) C (c) B (d) A

6 100 cups

7 (a) 113 cm (b) 160 days

8 6048 cm^3

9 (a) 15 944 cm^3 (b) 10 cm

10 (a) 76 027 mL (b) 14 451 cm^3 (c) 273.7 kL

11 (a) Examples are: Medicine, cans of drink, bottles of juice.
(b) The amount of petrol in the tank of a car, daily water usage in a household.
(c) Capacity of a swimming pool, water tank capacity.

12 Possible dimensions:
(a) 2 cm, 5 mm, 1 cm (b) 20 cm, 5 cm, 10 cm
(c) 2 m, 50 cm, 1 m (d) 20 m, 5 m, 10 m

13 (a) (i) 56 700 $cm^3 = 0.0567$ m^3. Need to move two decimal places for each length conversion, hence six places overall.
(ii) 144 765 $mm^3 \approx 145$ cm^3. Need to move one decimal place for each length conversion, hence three places overall.
(b) Work in the changed units from the start; that is:
(i) 0.45 m, 0.18 m, 0.7 m (ii) 4.8 cm, 2 cm.

Challenge 3 (p. 138)

1 $A = 6s^2$. $v = s^3 = 19\,683$, $s = 27$. $A = 6 \times 27^2 = 4374$ cm^2.

2 3 years of 366 days and 12 years of 365 days = 5478 days.
Number of seconds = $5478 \times 24 \times 60 \times 60 = 473\,299\,200$.

3 Time off field = $\frac{2}{13} \times 40$ min = 6.15 min = 6 min 9 s.

4 Time: $1 + 2 + 4 + 8 + \ldots = 24 \times 60 = 1440$ min.

Number of clicks = 1 more than number of terms in above time sum.

$1 + 2 = 3$, $1 + 2 + 4 = 7$, $1 + 2 + 4 + 8 = 15 = 2^4 - 1$

Find which power of 2 is ≤ 1440. $2^{10} = 1024$. He clicks his fingers $10 + 1 = 11$ times.

5 Require 360 mL in each of the two containers.

Fill the 330-mL jug and use it to fill the 150-mL jug. Pour the remaining 180 mL into one container. Pour the 150 mL back into the 720-mL jug.

Repeat, giving 360 mL in one container. Pour the remaining 360 mL from the 720-mL jug into the second container.

6 Volume increases by a factor of 8, the surface area by a factor of 4.

7 Perimeter of first figure = $12\pi + 12\pi = 24\pi$.
Perimeter of second figure = $3 \times 4\pi + 12\pi = 24\pi$.

Ratio = 1:1.

8 Each small square has an area of 4 cm^2.
Length of side = 2 cm.

Perimeter = $14 \times 2 + 8 \times 2 = 44$ cm

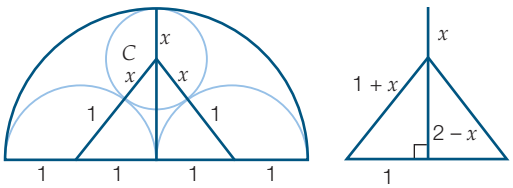
9 Answer: A

Let $OB = x$ cm, $OA = 2x$ cm, $AD = 4x$ cm.

Area of square = $AD^2 = 16x^2$ cm^2.
Area of annulus = $\pi(2x + x)(2x - x) = 3\pi x^2$ cm^2.

Fraction covered $\frac{3\pi x^2}{16x^2} = \frac{3\pi}{16}$.

10



Let the radius of the circle, centre C, be x cm.

In the right-angled triangle,

$$(1 + x)^2 = 1^2 + (2 - x)^2$$
$$1 + 2x + x^2 = 1 + 4 - 4x + x^2$$
$$6x = 4$$
$$x = \frac{2}{3} \text{ cm.}$$

The radius of a circle centre C is $\frac{2}{3}$ cm.

Chapter review 3 (p. 139)

1 (a) 27 cm (b) 60 cm (c) 57 cm

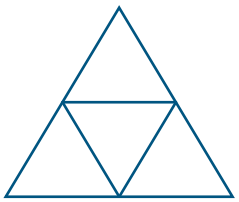
2 (a) (i) 134 mm (ii) 304 mm
(b) (i) 66 cm (ii) 108 cm
(c) (i) 1.3 cm (ii) 3.7 cm
(d) (i) 204 mm (ii) 356 mm
(e) (i) 98.2 cm (ii) 151.8 cm
(f) (i) 6.95 m (ii) 9.65 m

3 (a) 420 mm^2 (b) 2 ha (c) 68.25 m^2
(d) 5.28 km^2 (e) 807 000 m^2 (f) 90 cm^2

4 (a) 5675 mm^2 (b) 693 cm^2 (c) 0.75 cm^2
(d) 7762 mm^2 (e) 1316.2 cm^2 (f) 4.69 m^2

5 (a) 164 cm^2 (b) 3792 cm^2 (c) 356 cm^2
(d) 603 cm^2 (e) 7.51 m^2

6



7 (a) 1550 cm^2 (b) 190.5 cm^2

8 (a) 7.49 m^2 (b) 179 cm^2

9 (a) 360 cm^3 (b) 2011 cm^3 (c) 1380 cm^3
(d) 3000 cm^3 (e) 110 mm^3 (f) 450 m^3

10 (a) 0.017 6 ha (b) 1.96 cm^2
(c) 1800 mm^2 (d) 0.002 4 m^2

11 (a) 61.34 m^2 (b) 12.30 m^2

12 147 575 m^2 13 308.86 mL 14 384 cm^2

15 (a) 7527 m^2 (b) 361 m

16 148 cm^2 17 598 cm^2

18 (a) 143 m^2 (b) 14 900 (c) 145 200 L

19 (a) 12 160 cm^3 (b) 32 cm (c) 16.75 cm

20 22 cm

21 (a) 117 810 (b) 16 h 21 min 45 sec

22 (a) 8.48 m
(b) hour hand: 0.04 cm/min,
minute hand: 0.63 cm/min

NAPLAN practice 3

1 C 2 62.4 cm^2 3 B 4 7 cm

5 D 6 A 7 62 cm 8 C

Mixed review A (p. 145)

1 (a) 212.5 (b) \$0.08 (c) 72

2 (a) 14.29% (b) 75% (c) 12.5% (d) 250%

3 132 cm^2

4 (a) 20 $mm^2 = 0.2\ cm^2$
(b) 23.67 $km^2 = 23\,670\,000\ m^2$
(c) 0.007 $m^2 = 70\ cm^2$ (d) 14.5 ha = 145 000 m^2

5 (a) x^8 (b) x^4 (c) x^4

6 (a) $80g^2h^3$ (b) $-36k^4$ (c) $108e^3fg$

7 (a) \$48.50 (b) \$77.60 (c) \$92.15 (d) \$145.50

8 (a) 1 (b) 5 (c) 1 (d) 5

9 (a) 178.5 cm^2 (b) 92 m^2 (c) 198 cm^2
(d) 2045.2 cm^2 (e) 17.4 cm^2 (f) 47.1 cm^2

10 (a) \$45 (b) \$1890 (c) \$7845 (d) \$18 549

11 (a) 4.5×10^7 (b) 5.27×10^9
(c) 3×10^{-4} (d) 2.98×10^{-7}

12 (a) 7238.2 cm^3 (b) 7238.2 mL

13 (a) \$560 (b) \$481

14 (a) $12g^4h^4$ (b) $120a^7b^{11}$ (c) $\frac{xy^5}{3}$

15 216 days

16 (a) $-8x - 16$ (b) $30y - 55y^2$ (c) $12p + 58$

17 (a) $4(m - 7)$ (b) $7d(ed + 5)$ (c) $2y(4xy - 1)$

18 (a) (i) 153.68 m^2 (ii) 110.98 m^3
(b) (i) 20.36 m^2 (ii) 6.87 m^3

19 (a) $x^2 + 4x - 21$ (b) $x^2 + 12x + 36$
(c) $4x^2 - 20x + 25$ (d) $x^2 - 64$

20 (a) 2 (b) $\frac{3p^5q^7}{2}$ (c) $5x^5y^7$

21 5.1 m^2

22 (a) $m = \frac{2E}{v^2}$ (b) 5.3 kg

23 (a) \$364 (b) \$487.50 (c) \$606.25 (d) \$649.95

24 (a) 0.8 m (approx.) (b) 12 days

25 Surface area is multiplied by four.

26 (a) Option A: at least \$16 000; Option B: at least \$12 667.
(b) \$6000
(c) Option A is better if Bettina expects to sell less than \$6000 worth of goods per week.

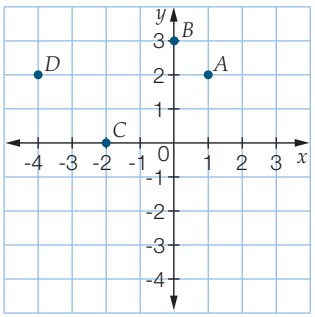
27 (a) 21.46%
(b) 21.46%
(c) They are equally efficient.

Chapter 4

Recall 4 (p. 148)

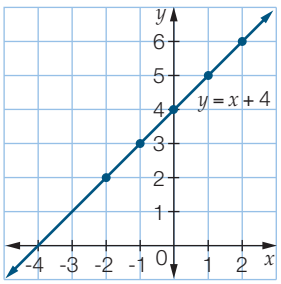
1 (a) $P = -13$ (b) $C = -7$

2



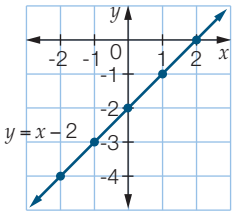
3 (a)

x	-2	-1	0	1	2
y	2	3	4	5	6



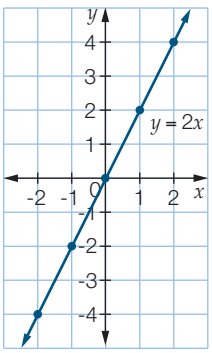
(b)

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



(c)

x	-2	-1	0	1	2
y	-4	-2	0	2	4



4 (a) $x = 6$ (b) $y = 15$ (c) $a = -10$ (d) $b = 21$

5 (a) $g = 19$ (b) $x = 42$ (c) $y = 6$ (d) $x = 35$

6 (a) $-15x - 6$ (b) $20 - 6x$ (c) $15x + 43$

7 $y = 6 - 2x$

8 (a) $h = 13$ (b) $x = 7$

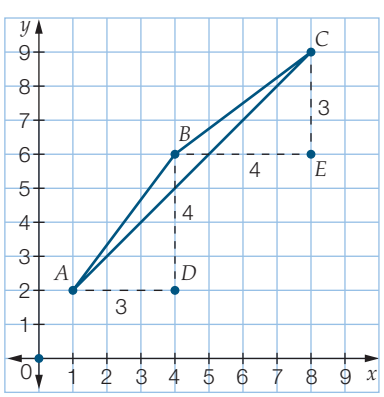
Exercise 4.1 (p. 151)

1 (a) 10 (b) 17 (c) 30 (d) 26

2 (a) (6, 4) (b) (4, 7) (c) (5, 3) (d) (7, 5)

3 C **4** (5, 12) **5** (-1, 2)

6



$AB = 5$; $BC = 5$

$\therefore \Delta ABC$ is isosceles

7 (a) $AB = 5$, $BC = 13$, $CD = 5$, $DA = 13$

(b) midpoint AC = (7.5, 1.5), midpoint BD = (7.5, 1.5)

(c) $ABCD$ is a rectangle (diagonals bisect each other).

8 $AB = 13$, $BC = 13$, $CD = 13$, $DA = 13$. $\therefore$ $ABCD$ is a rhombus as all sides are equal.

9 Sample answer

A (1, 1), B (4, 1), C (1, 4)

$AB = 3$, $AC = 3$

$\therefore \Delta ABC$ is isosceles

Exercise 4.2 (p. 155)

1 (a) 5 (b) 13 (c) 17 (d) 26

(e) 10 (f) 25 (g) $\sqrt{17}$ (h) $\sqrt{37}$

(i) $\sqrt{61}$ (j) $2\sqrt{2}$ (k) $2\sqrt{13}$ (l) $4\sqrt{5}$

2 (a) (-1.5, 2) (b) (-1, 4) (c) (-1, -7) (d) (0.5, 2)

(e) (-1, 7.5) (f) (1.5, -0.5) (g) (3.5, 4.5) (h) (2.5, -2.5)

(i) (4.5, 5.5)

3 C **4** (1, 5.5)

5 (a) (9, 2) (b) (5, -10) (c) (9, 0)

6 $AB = \sqrt{(1 - -3)^2 + (1 - -2)^2} = 5.$

$BC = \sqrt{(-2 - 1)^2 + (5 - 1)^2} = 5.$

$AC = \sqrt{(-2 - -3)^2 + (5 - -2)^2} = \sqrt{50}.$

Two sides are of equal length, therefore ΔABC is isosceles.

$\sqrt{AB^2 + BC^2} = \sqrt{5^2 + 5^2} = \sqrt{50}$. Pythagoras' Theorem holds and therefore ΔABC is a right-angled triangle.

7 (a) Side lengths are 3.61, 3.61 and 5.10 (2 d.p.).

(b) Isosceles

8 (a) 5 (b) (3, 2) (c) (7, 1)

(d) From B to C, 11.18 (2 d.p.)

(e) Parallelogram

9 (a) Midpoint of AC = (-3, -5), midpoint of BD = (-3, -5)

(b) All sides are 4.47 (2 d.p.) (c) $AC = 8.49$, $BD = 2.83$

(d) Rhombus

10 $AB = \sqrt{20}$, $BC = \sqrt{45}$, $CD = \sqrt{20}$, $DA = \sqrt{45}$. Therefore opposite sides are of equal length. $AC = \sqrt{65}$, $BD = \sqrt{65}$. Therefore diagonals are also of equal length and hence $ABCD$ is a rectangle.

11 (4, 1)

12 Coordinates of $PQRS$ are $P(-2, 4)$, $Q(1, 2)$, $R(-1, 0)$, $S(-4, 2)$. PQ has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, QR has length $\sqrt{8}$ and gradient 1, RS has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, SP has length $\sqrt{8}$ and gradient 1. Therefore opposite sides have equal length and gradient and hence $PQRS$ is a parallelogram.

13 $A(-1, 0)$, $B(1, 0)$ and $C(0, \sqrt{3})$. $AB = 2$, $BC = 2$ and $CA = 2$. Therefore ΔABC is an equilateral triangle.

14 $A(0, 8)$, $B(3, 12)$, $C(6, 8)$ and $D(3, 0)$. $AB = 5$, $BC = 5$, $CD = \sqrt{73}$ and $DA = \sqrt{73}$. $ABCD$ has two pairs of adjacent, equal sides and is therefore a kite.

Exercise 4.3 (p. 160)

1 (a) $h = -2$ (b) $b = 5$ (c) $x = -1$ (d) $r = \frac{5}{6}$ (e) $f = -4$
(f) $k = \frac{2}{5}$ (g) $x = \frac{6}{7}$ (h) $x = -9$ (i) $y = 0$ (j) $b = 6$
(k) $a = -2$ (l) $b = -3$ (m) $p = -15$ (n) $e = 4$ (o) $g = 8$
(p) $y = 2\frac{3}{5}$ (q) $p = 2\frac{3}{4}$ (r) $y = 2$

2 (a) $x = 6$ (b) $x = 7$ (c) $x = 9$ (d) $x = -9$ (e) $x = 12$
(f) $x = 6$ (g) $x = 4$ (h) $x = -2$ (i) $x = 2$ (j) $x = 3$
(k) $x = -1$ (l) $x = 6$ (m) $x = -2$ (n) $x = -8$ (o) $x = -5$

3 (a) $c = -7$ (b) $b = -8$ (c) $f = -2$ (d) $d = 7$ (e) $e = 4$
(f) $a = 6$ (g) $g = -7$ (h) $h = 20$ (i) $x = 3$ (j) $k = \frac{3}{2}$
(k) $j = 2$ (l) $y = 3$ (m) $m = 4$ (n) $n = 2$ (o) $x = 1$
(p) $y = 1$ (q) $p = -3$ (r) $x = 3$ (s) $a = 2$ (t) $x = 9$
(u) $p = 4$

4 (a) $x = 8$ (b) $x = -10$ (c) $x = -5$ (d) $x = -6$
(e) $x = -3\frac{1}{4}$ (f) $x = 5\frac{5}{6}$ (g) $x = \frac{1}{11}$ (h) $x = -\frac{4}{27}$
(i) $x = 7$

5 (a) $b = -22$ (b) $a = 21$ (c) $c = 3$ (d) $d = -8$
(e) $x = 16\frac{1}{2}$ (f) $y = 10$ (g) $g = 1$ (h) $k = 1$
(i) $m = -10$ (j) $j = \frac{3}{2}$ (k) $x = -5\frac{1}{2}$ (l) $p = -19$
(m) $x = -2\frac{1}{3}$ (n) $y = \frac{26}{41}$

6 (a) $x = 6$ (b) $x = 8$ (c) $x = 0$ (d) $x = -36$ (e) $x = -42$
(f) $x = 12$ (g) $x = 3\frac{3}{4}$ (h) $x = 0$ (i) $x = -14$ (j) $x = 15$
(k) $x = 8$ (l) $x = 22$ (m) $x = 22$ (n) $x = 57$ (o) $x = 55$

7 (a) B (b) B (c) C (d) C

8 (a) B (b) C (c) D (d) A (e) D (f) C

9 (a) D (b) B

10 (a) B (b) D (c) D (d) A

11 (a) $x + 7 = 19, x = 12$ (b) $y - 5 = 3, y = 8$
(c) $3x = 14, x = 4\frac{2}{3}$ (d) $\frac{x}{4} = -5, x = -20$
(e) $2x + 3 = 13, x = 5$ (f) $4x - 7 = -12, x = -1\frac{1}{4}$

12 (a) $x = 5$ (b) $x = -2$ (c) $x = 2$ (d) $x = -9$ (e) $x = -6$
(f) $x = 4$ (g) $x = 5$ (h) $x = 1$ (i) $x = -2$ (j) $x = 0$
(k) $x = 6$ (l) $x = -2$

13 (a) $x = -\frac{5}{8}$ (b) $x = -\frac{23}{12}$ (c) $x = -\frac{8}{15}$ (d) $x = 8\frac{1}{2}$
(e) $x = \frac{1}{4}$ (f) $x = -\frac{11}{14}$ (g) $x = -\frac{1}{6}$ (h) $x = \frac{2}{3}$
(i) $x = 2\frac{5}{6}$

14 $4x - 11 = 7 - 2x, x = 3$

15 $x + 36 = 2(x + 6), x = 24$

16 $\frac{3x + 1}{2} = \frac{x - 4}{5}, x = -1$

17 (a) $n + n + 2 + n + 4 = 195, n = 63$. The three numbers are 63, 65 and 67.
(b) $n + n + 2 + n + 4 + n + 6 = 204, n = 48$. The four numbers are 48, 50, 52 and 54.
(c) $n + n + 7 + n + 14 + n + 21 + n + 28 + n + 35 = 609, n = 84$. The six numbers are 84, 91, 98, 105, 112 and 119.

18 (a) $x + \frac{x}{6} = 28, x = 24$ (b) $x + \frac{x}{4} = 45, x = 36$
(c) $x + \frac{x}{8} = 99, x = 88$

19 If $x = -1, 2x + 7 = 5$. If $x = -2, 2x + 7 = 3$. If $x = -10, 2x + 7 = -13$.

20 $\frac{x}{2} + 1 = x - 1, \frac{3x + 4}{2} = 8, 5(x - 1) = 3(x + 1)$

21 $x + 2 = 2x + 1, 2x - 7 = x - 3, 20 - x = x + 18$

Exercise 4.4 (p. 166)

1 Kate is 15 years old and her father is 45 years old.

2 Andrew has 11 coins and Lori has six coins.

3 B **4** B

5 length = 13 m, width = 10 m

6 Annette: 300 m; Mark: 600 m; Sue: 1.2 km

7 \$3.65 **8** Peter is 16 years old. **9** 15

10 21 **11** 49°, 55°, 76° **12** 11 cm

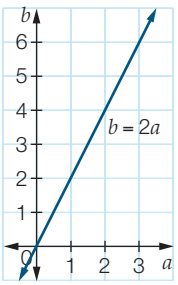
13 Joe is 12, Kate is 15, Brett is 48 and Gabby is 30.

14 $\frac{3(3a - 8)}{4} = \frac{7(a + 3)}{5}, a = \$12$

15 23, 26, 30

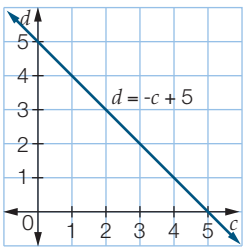
Exercise 4.5 (p. 171)

1 (a) (i)



(ii) $b = 2a$

(b) (i)

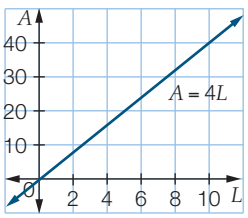


(ii) $d = -c + 5$

2 (a) (i)

L	0	10	20	30
A	0	40	80	120

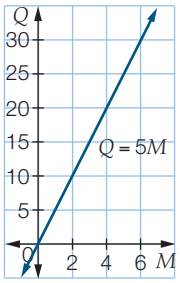
(ii)



(b) (i)

M	0	2	4	6
Q	0	10	20	30

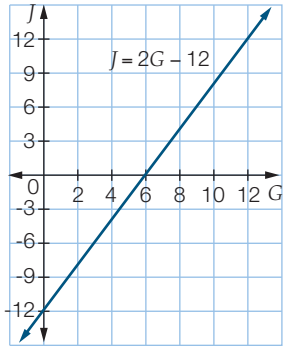
(ii)



(c) (i)

G	0	4	8	12
J	-12	-4	4	12

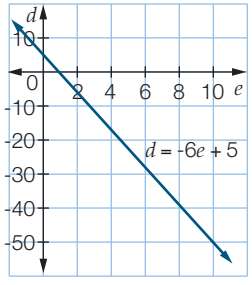
(ii)



(d) (i)

e	0	3	6	9
d	5	-13	-31	-49

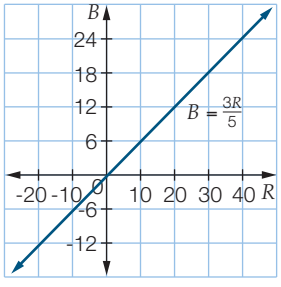
(ii)



(e) (i)

R	-20	0	20	40
B	-12	0	12	24

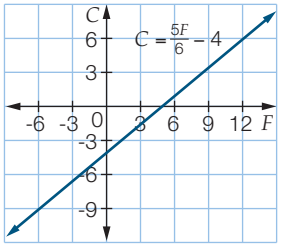
(ii)



(f) (i)

F	-6	0	6	12
C	-9	-4	1	6

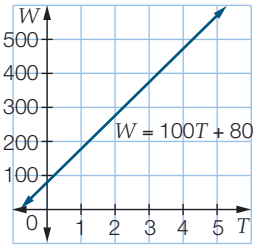
(ii)



3 (a)

No. televisions sold	0	1	2	3	4	5
Weekly wage ($)	80	180	280	380	480	580

(b)



(c) $W = 100T + 80$ (d) \$880

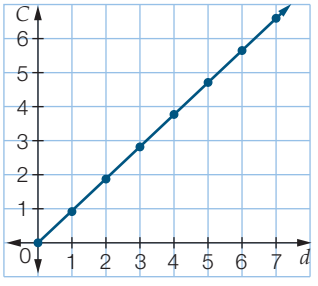
(e) $W = 100T + 80$, $W = 100 \times 7 + 80 = 780$

4 (a) D (b) B (c) C

5 (a)

No. litres	0	1	2	3	4	5	6	7
Cost ($)	0	0.95	1.90	2.85	3.80	4.75	5.70	6.65

(b)



(c) Yes, the graph is in a straight line.

(d) (i) \$2.85 (ii) \$2.28 (iii) \$5.42

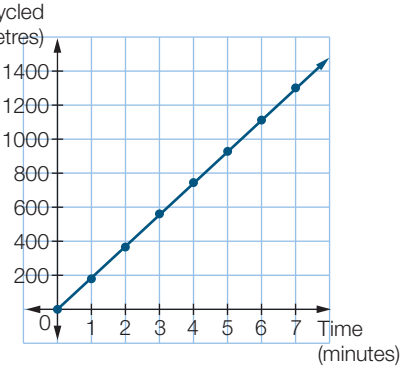
(e) (i) 4 L (ii) 2.6 L (iii) 5.5 L

(f) $C = 0.95d$

6 (a)

Time (min)	0	1	2	3	4	5	6	7
Distance cycled (m)	0	185	370	555	740	925	1110	1295

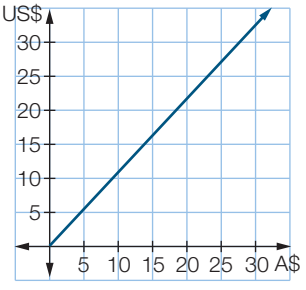
(b)



(c) 499.5 m (d) 3.8 min (e) 10.8 min

(f) $D = 185t$ (g) 7400 m (7.4 km)

7 (a)

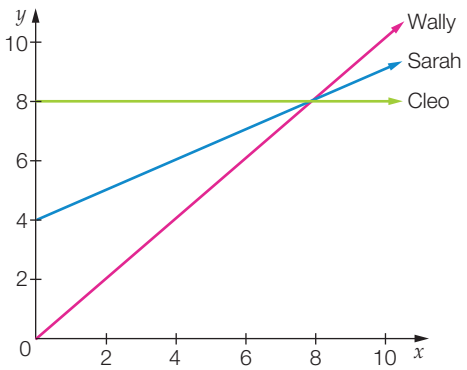


(b) US$7.40 (c) US$18.50 (d) A$5.20 (e) A$9.10

8 (a)

D	Wally	Sarah	Cleo
0	0	4	8
1	1	4.5	8
2	2	5	8
3	3	5.5	8
4	4	6	8
5	5	6.5	8
6	6	7	8
7	7	7.5	8
8	8	8	8
9	9	8.5	8
10	10	9	8

(b)



(c) 8 km

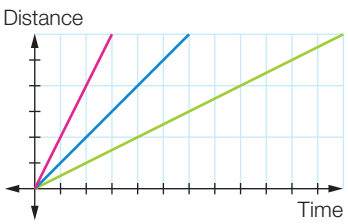
(d) (i) Wally collects the most money above 8 km.

(ii) Sarah never collects the highest amount of money for any distance.

(iii) Cleo collects the most money between 0 and 8 km.

9 In the first 2 years Frank earns more than Bruno, they earn the same in the third year, and after that Bruno earns more.

10



As the time taken to travel the same distance increases, the line becomes less steep (has a smaller gradient).

Half-time 4 (p. 174)

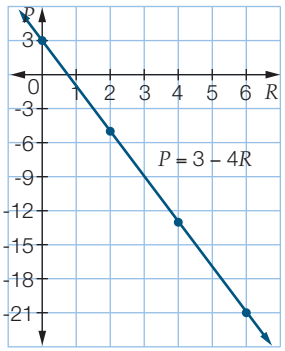
1 (a) Yes (b) No (c) No (d) Yes

2 13

3 (a)

R	0	2	4	6
P	3	-5	-13	-21

(b)



4 (a) $x = 9$ (b) $y = 5$ (c) $x = 16$ (d) $x = -9$ (e) $x = 3$

(f) $x = 3$ (g) $p = \frac{5}{2}$ (h) $a = -5$ (i) $m = 13$

5 Kelvin weighs 54 kg, Rob weighs 60 kg

6 C 7 $x = 2$ m 8 (4.5, 5)

Exercise 4.6 (p. 179)

1 a: 1; b: $\frac{2}{5}$; c: $-\frac{1}{3}$; d: -2

2 a: 0; b: 0; c: undefined; d: undefined

3 (a) 2 (b) 3 (c) -4 (d) -1 (e) 2

(f) 1 (g) $-\frac{7}{12}$ (h) -1 (i) $-\frac{1}{3}$ (j) 0

(k) 0 (l) 0 (m) undefined

(n) undefined (o) undefined

4 (a) positive (b) negative

(c) zero (d) undefined

5 (a) C (b) A (c) A (d) C

6 a: B; b: A; c: A; d: C; e: B

7 (a) a: -1; b: 3; c: $\frac{1}{3}$; d: 0; e: $-\frac{1}{3}$ (b) b (c) a

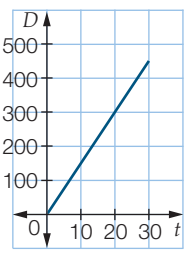
8 (a) 1200 m (b) 20 min (c) 60 m (d) 60

(e) 60 m/min

(f) (i) 360 m (ii) 720 m (iii) 1080 m (iv) $60t$ m

(g) $D = 60t$

9 (a)



(b) 15 m/s or 54 km/h (c) 15

(d) $D = 15t$ (e) 795 m

10 The ramp should be 555 cm from the first step: $4 \times 13 = 52$ cm in vertical rise. Because the gradient is 1:12, the horizontal run is $12 \times 52 = 624 - 3 \times 23$ (tread of three stairs) = 555 cm.

11 24 gradients; there are 22 (11 positive and 11 negative due to a mirror image) that are either whole numbers or fractions. The 23rd is $m = 0$ and the 24th is undefined.

12 **(a)** 320 L **(b)** 8 min **(c)** -40

(d) 40 L/min **(e)** The volume of water is decreasing.

(f) **(i)** 160 L **(ii)** 80 L **(iii)** 240 L **(iv)** $(320 - 40t)$ L

(g) $V = 320 - 40t$

(h) Neither time nor the volume of water can be a negative value.

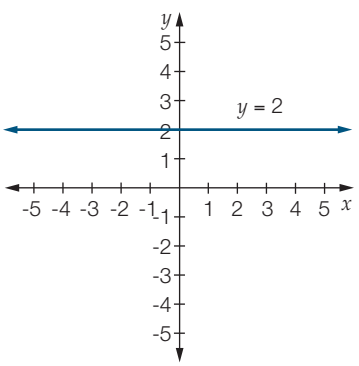
13 **(a)** 48 km/h **(b)** 96 km/h **(c)** 7 h

14 (-2, 8), (4, 4), (7, 2)

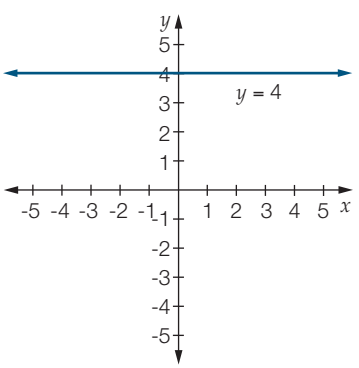
15 Sally has made a mistake by adding the negative values of x and y to the positive values instead of subtracting them from the positive values.

Exercise 4.7 (p. 185)

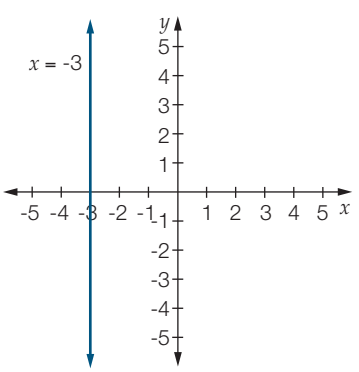
1 **(a)**



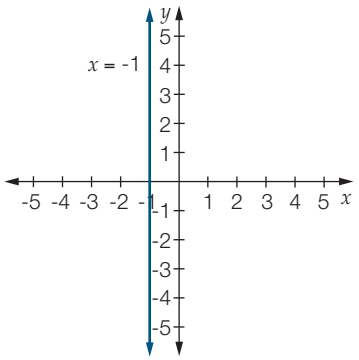
(b)



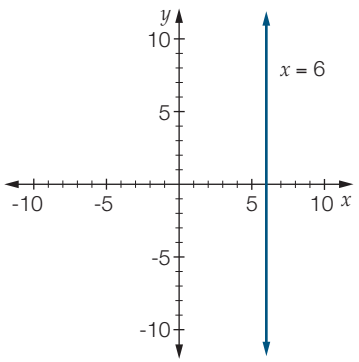
(c)



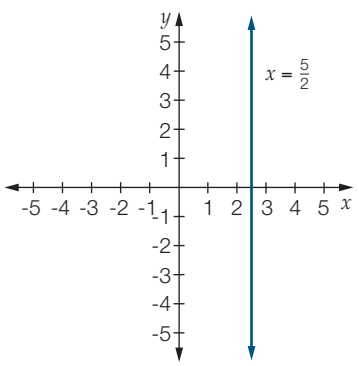
(d)



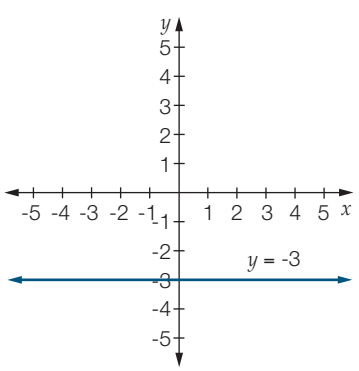
(e)



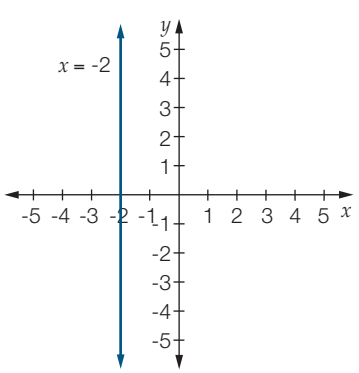
(f)



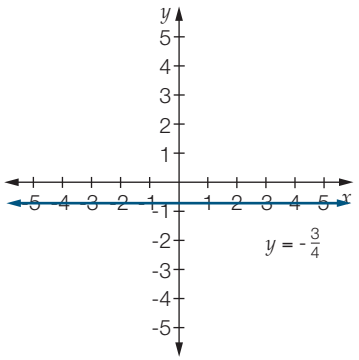
(g)



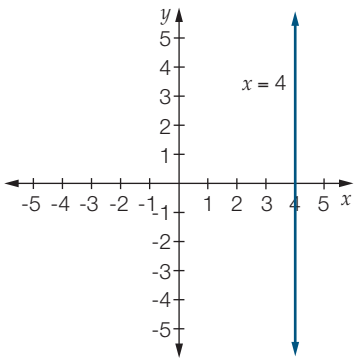
(h)



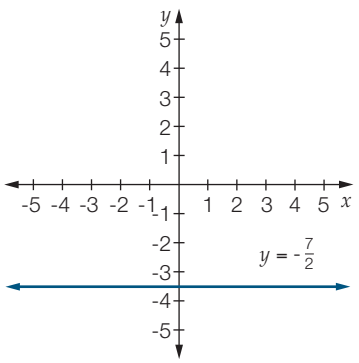
(i)



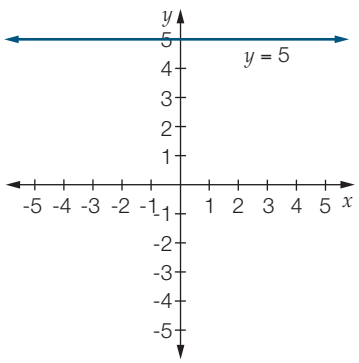
(j)



(k)



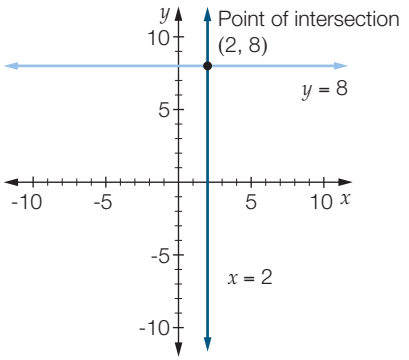
(l)



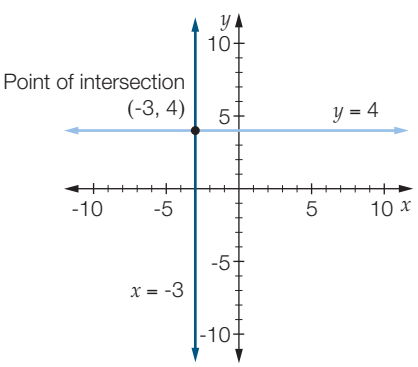
2 (a) $y = -6$ (b) $y = 2$ (c) $y = \frac{2}{3}$ (d) $y = \frac{3}{4}$

3 (a) B (b) B (c) C

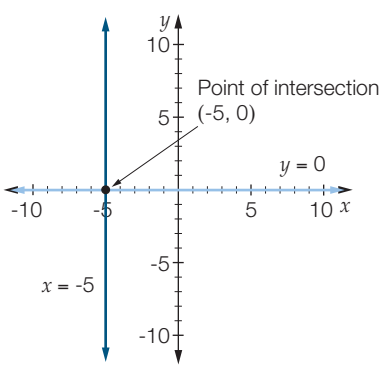
4 (a)



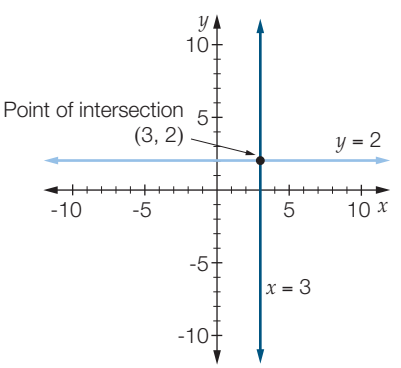
(b)



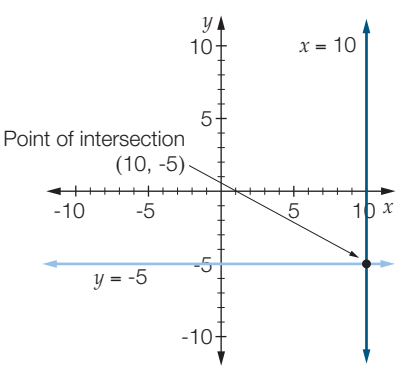
(c)



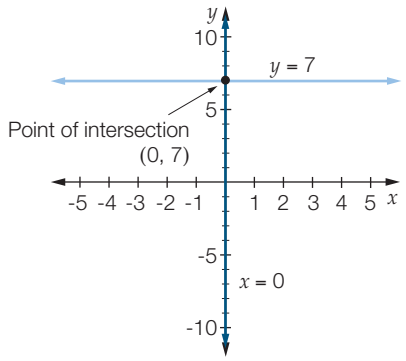
(d)



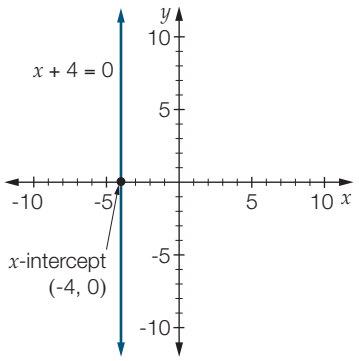
(e)



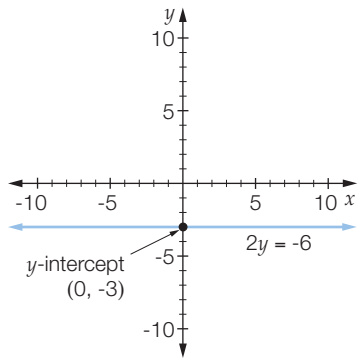
(f)



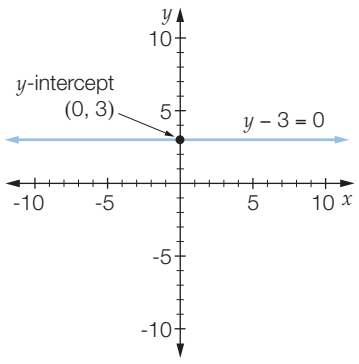
(e)



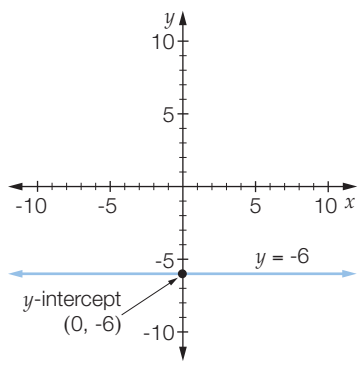
5 (a)



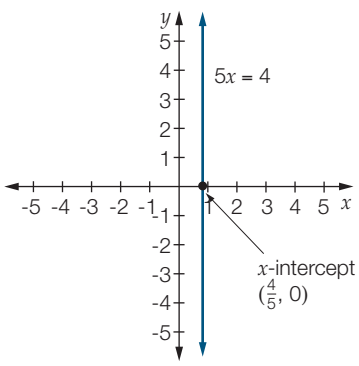
(f)



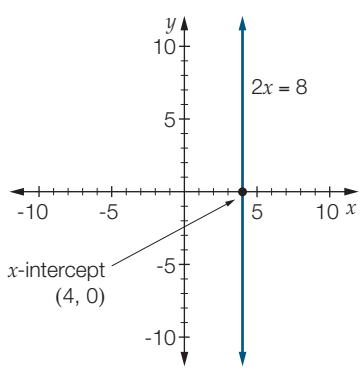
(b)



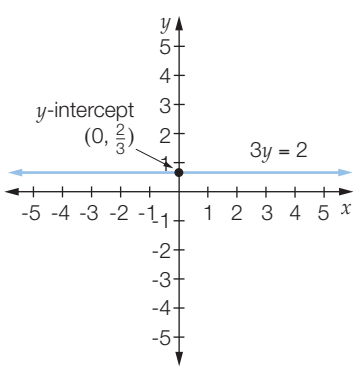
(g)



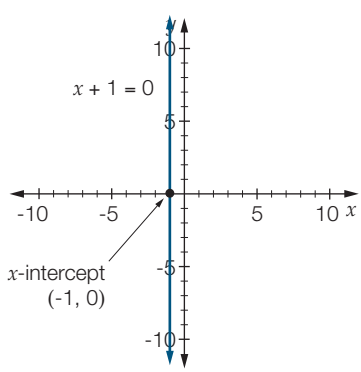
(c)



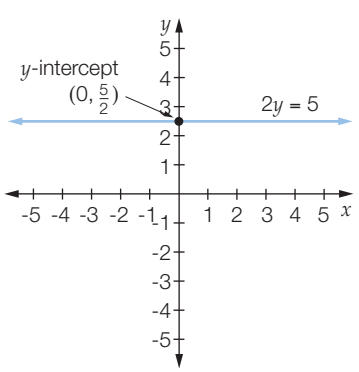
(h)



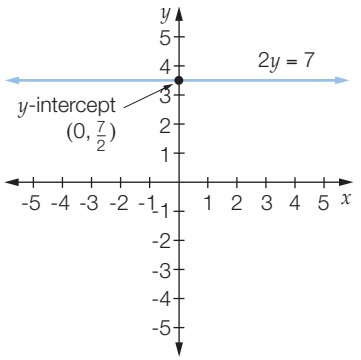
(d)



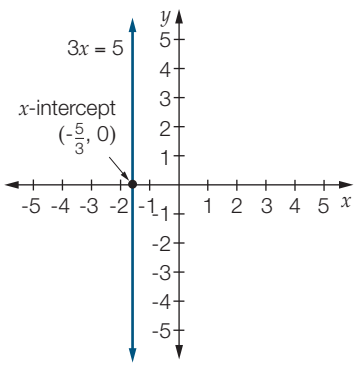
(i)



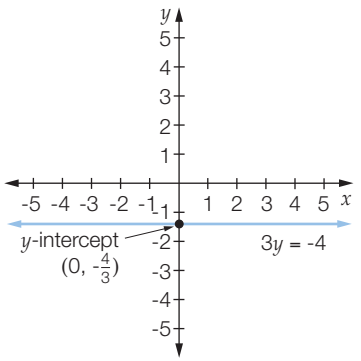
(j)


y
x
2y = 7
y-intercept
(0, 7/2)

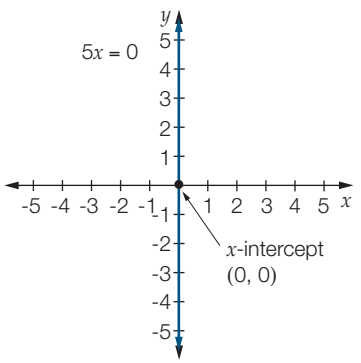
(k)


y
x
3x = 5
x-intercept
(-5/3, 0)

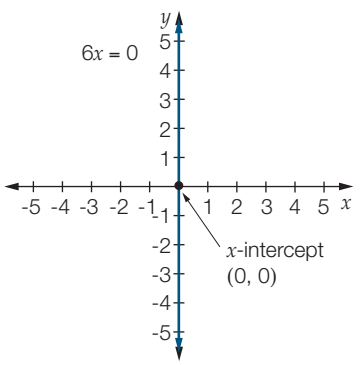
(l)


y
x
y-intercept
(0, -4/3)
3y = -4

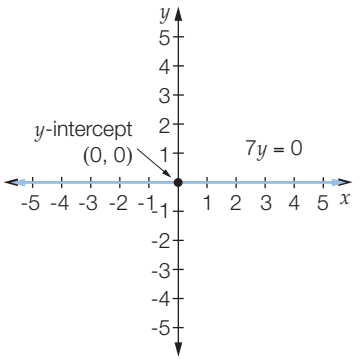
(m)


y
x
5x = 0
x-intercept
(0, 0)

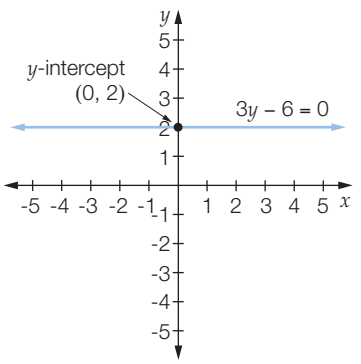
(n)


y
x
6x = 0
x-intercept
(0, 0)

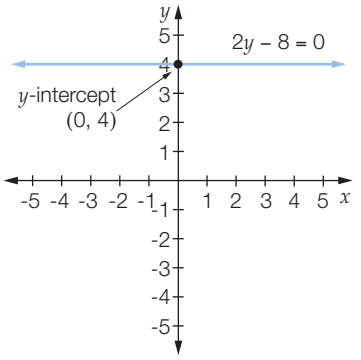
(o)


y
x
y-intercept
(0, 0)
7y = 0

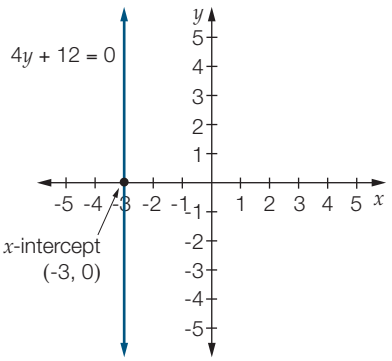
(p)


y
x
y-intercept
(0, 2)
3y – 6 = 0

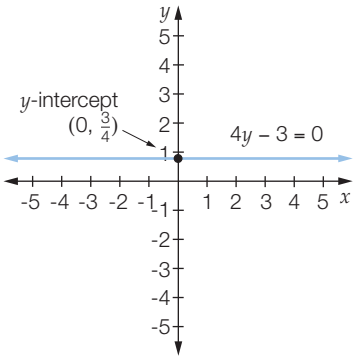
(q)


y
x
2y – 8 = 0
y-intercept
(0, 4)

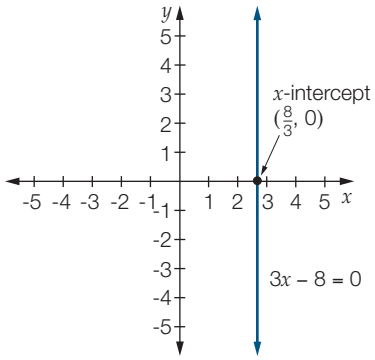
(r)


y
x
4y + 12 = 0
x-intercept
(-3, 0)

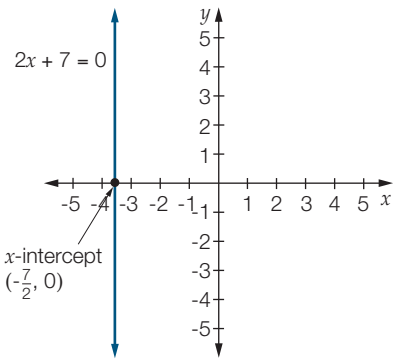
(s)


y
x
y-intercept
(0, 3/4)
4y – 3 = 0

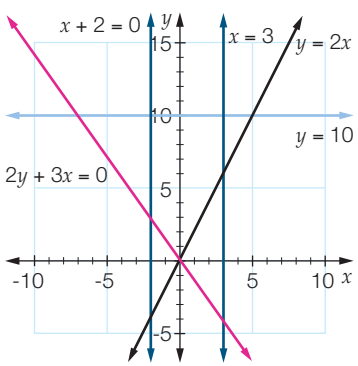
(t)



(u)

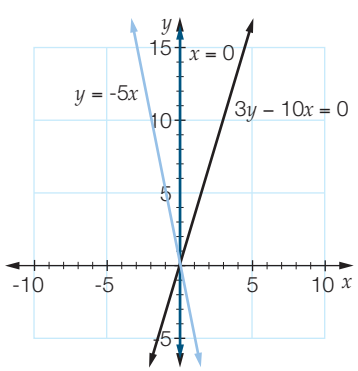


6 (a)



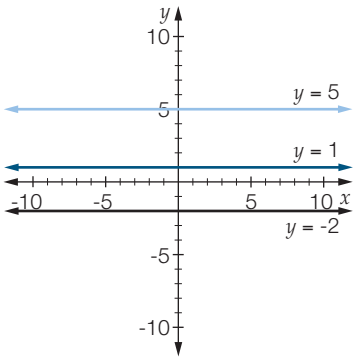
(b) (0, 0), (3, 6), (3, 10), (-2, 3), (-2, 10), $(-6\frac{2}{3}, 10)$, (5, 10)

(c)

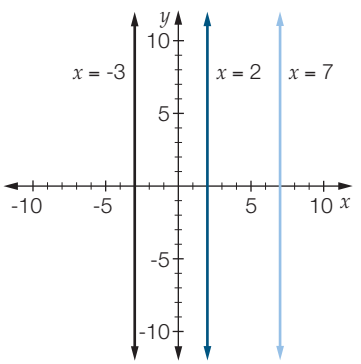


(d) Yes. It is located in the path of the road.

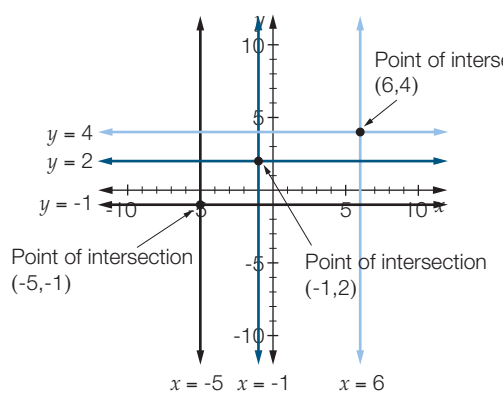
7 $y = -2, y = 1, y = 5$



8 $x = -3, x = 2, x = 7$



9

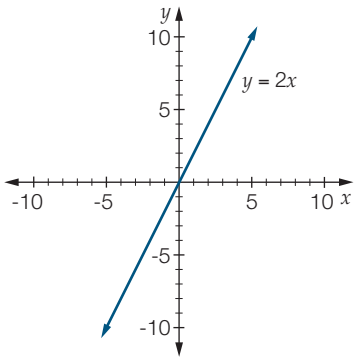


Exercise 4.8 (p. 192)

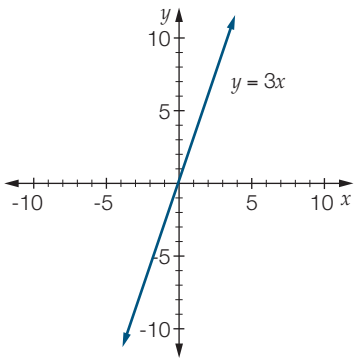
1

	Gradient	*y*-intercept
(a)	1	(0, 3)
(b)	2	(0, 1)
(c)	7	(0, 3)
(d)	3	(0, -2)
(e)	1	(0, -4)
(f)	-4	(0, 3)
(g)	-1	(0, 4)
(h)	-2	(0, 6)
(i)	2	(0, -5)

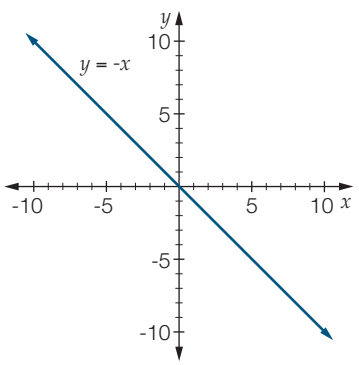
2 (a)



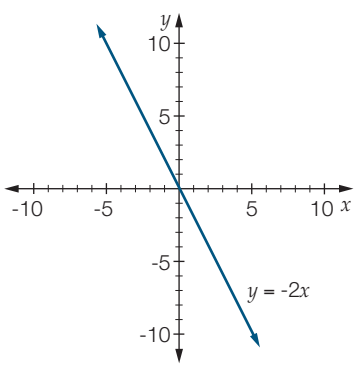
(b)



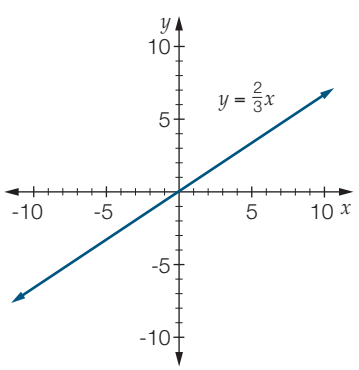
(c)



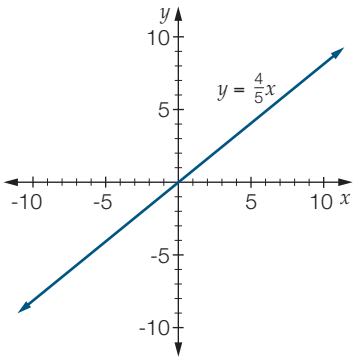
(d)



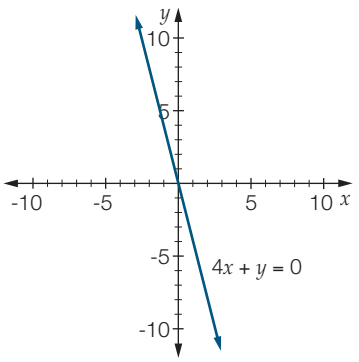
(e)



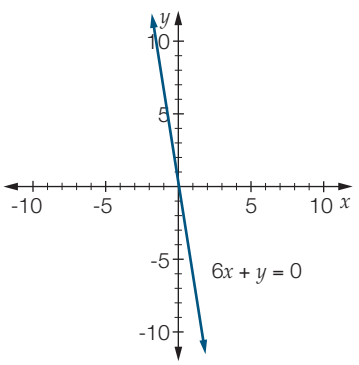
(f)



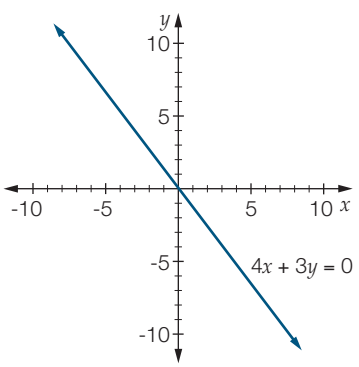
(g)



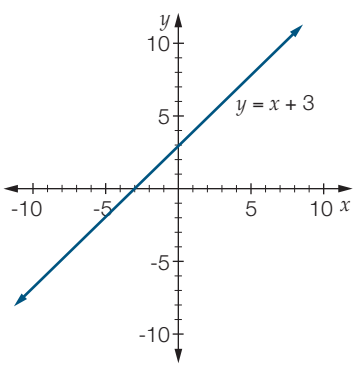
(h)



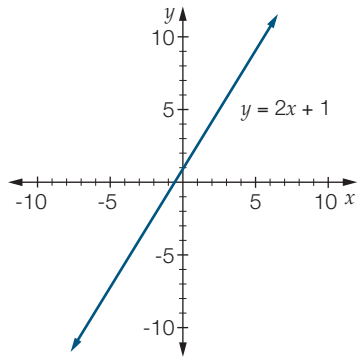
(i)



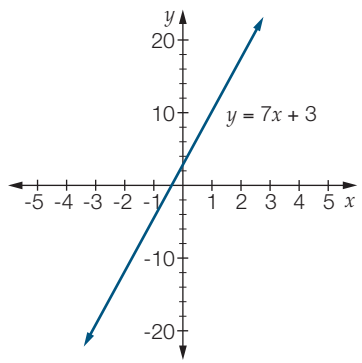
3 (a)



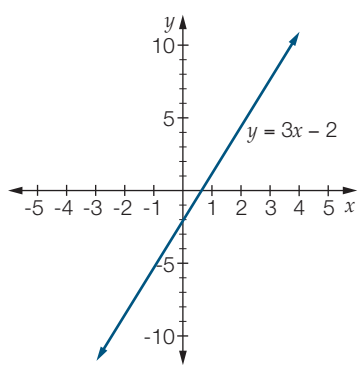
(b)



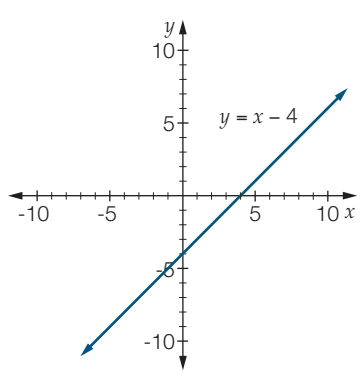
(c)



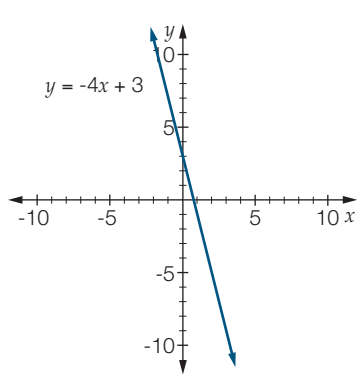
(d)



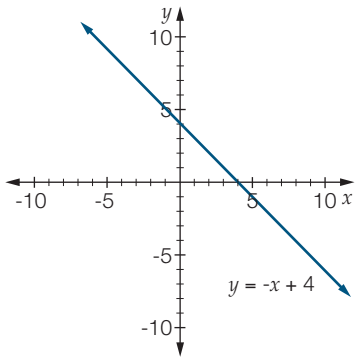
(e)



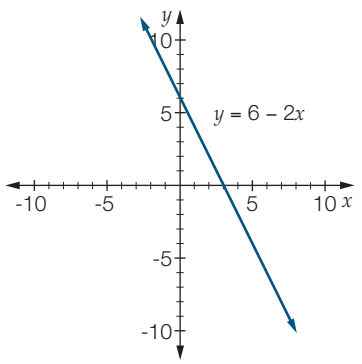
(f)



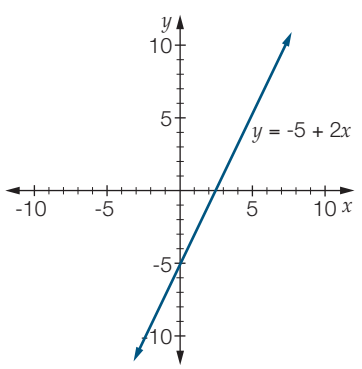
(g)



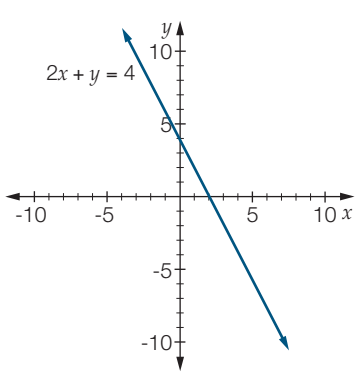
(h)



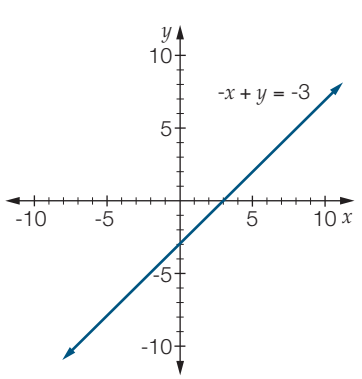
(i)



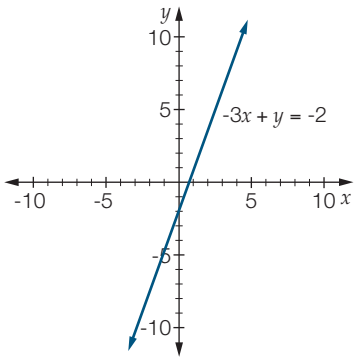
4 (a)



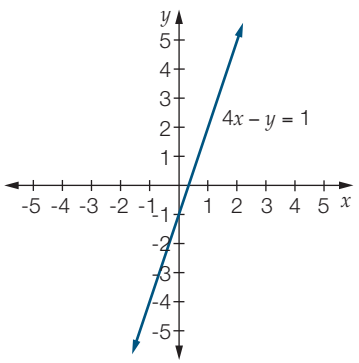
(b)



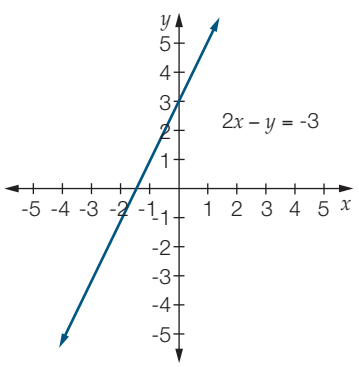
(c)



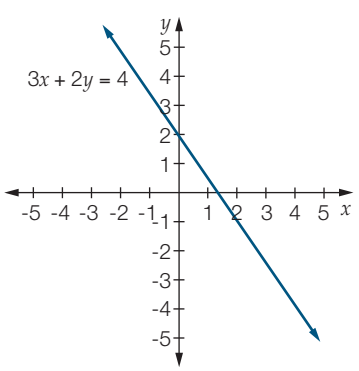
(d)



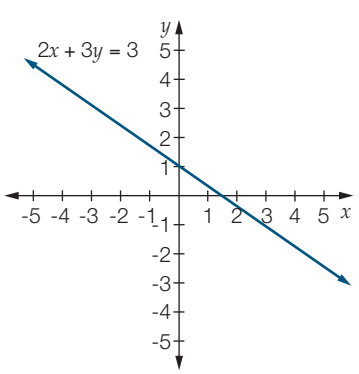
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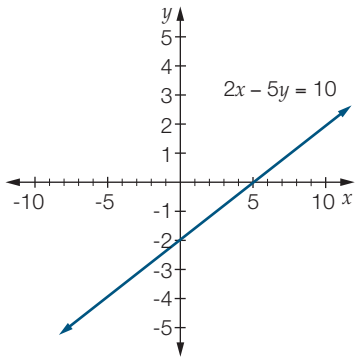
(f)



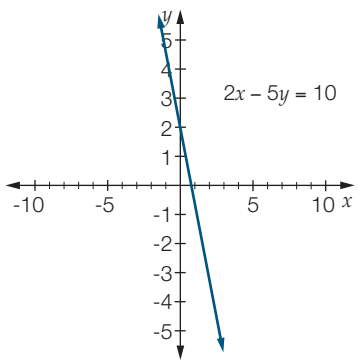
(g)



(h)



(i)



5 (a) $m = 3$ (b) $m = -4$ (c) $m = 3$
(d) $m = 3$ (e) $m = -\frac{2}{5}$ (f) $m = \frac{4}{3}$

6 (a) $-\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{3}$
(d) $-\frac{1}{3}$ (e) $\frac{5}{2}$ (f) $-\frac{3}{4}$

7 (a) $y = 3x + 4$ (b) $y = x + 5$ (c) $y = 2x$
(d) $y = 4x$ (e) $y = -5x + 1$ (f) $y = 3x - 4$

8 (a) D (b) C

9 (a) B (b) B (c) A (d) B

10 (a) D (b) A (c) C

11 (a) $-\frac{1}{2}$ (b) undefined (c) $-\frac{1}{7}$ (d) $-\frac{2}{5}$

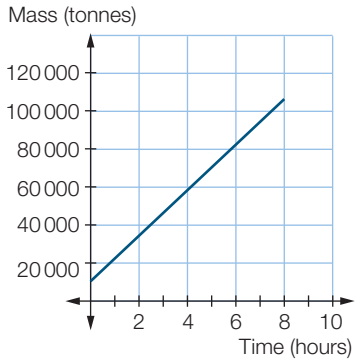
12 (a) B (b) D (c) A (d) F
(e) G (f) C (g) E

13 (a) The lines are horizontal. (b) The lines are vertical.

14 (a) (0, 50) (b) 25 (c) $d = 25t + 50$
(d) 50 m/min (e) 550 m

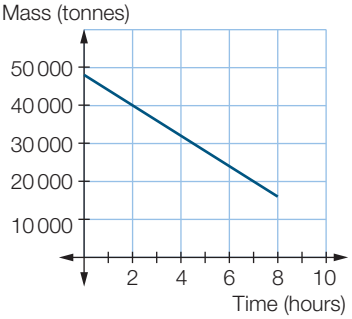
15 (a) 30 (b) 30 km/h (c) 20 km
(d) $d = 100t + 90$

16 (a)



(b) 12 000 (c) $m = 12\,000t + 10\,000$

(d) Mass (tonnes)



(e) $m = 48\,000 - 4000t$

(f) 16 000 tonnes

(g) 12 hours

17 Cameron is 110 km away from town A, Yvonne is 40 km away from town A.

18 **(a)** (0, -4), (10, 6), (15, 11) **(b)** (0, 2), (-5, 0), (10, 6)

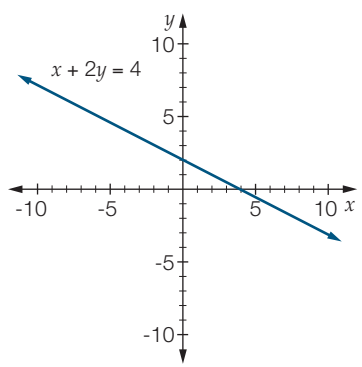
(c) (0, -0.5), (4, 0.5), (2, 0)

19 $y = x$, $y = x + 1$, $y = -x + 2$, $y = -x + 3$

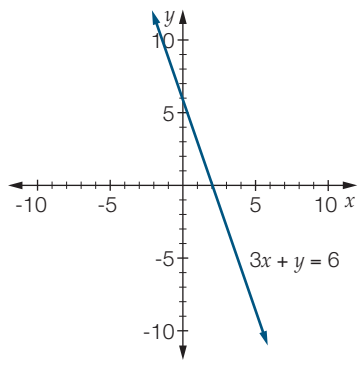
20 **(a)** $y = 4x - 9$ **(b)** $y = -\frac{1}{4}x + 2$

Exercise 4.9 (p. 197)

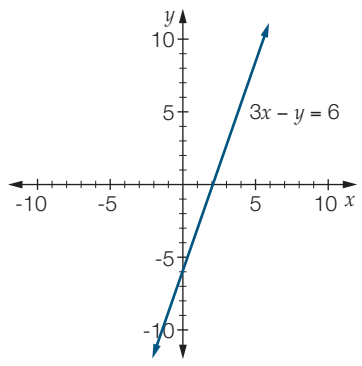
1 **(a)**



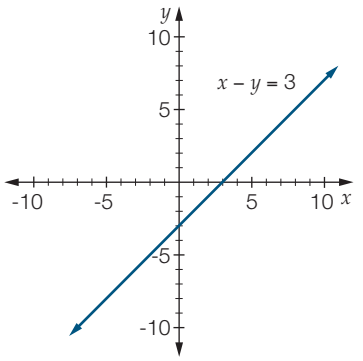
(b)



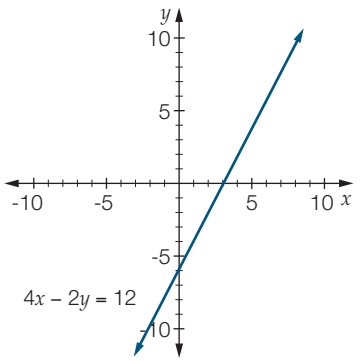
(c)



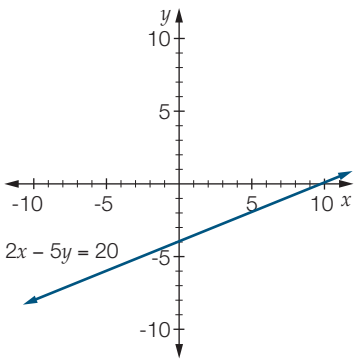
(d)



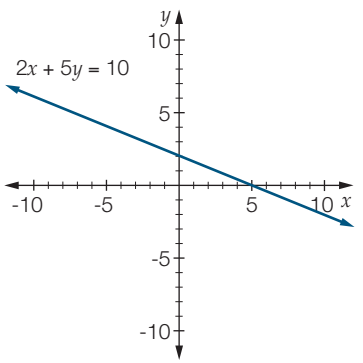
(e)



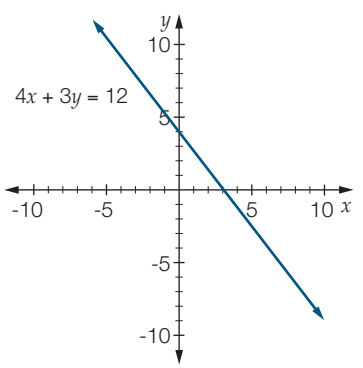
(f)



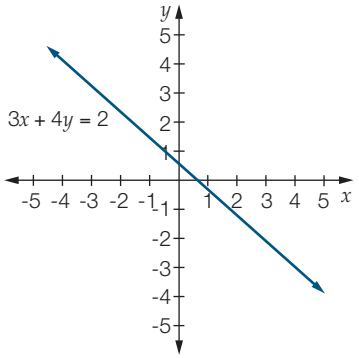
(g)



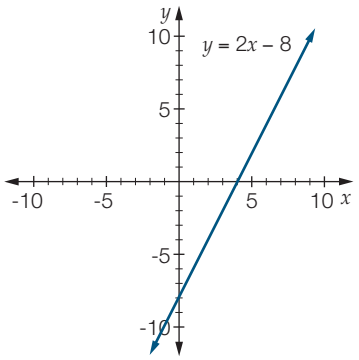
(h)



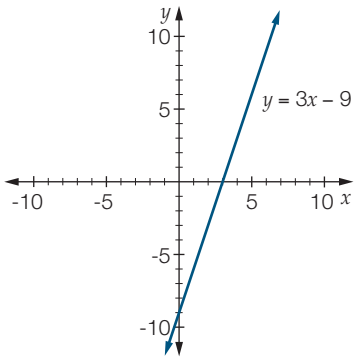
(i)



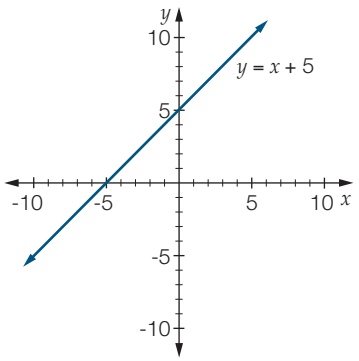
2 (a)



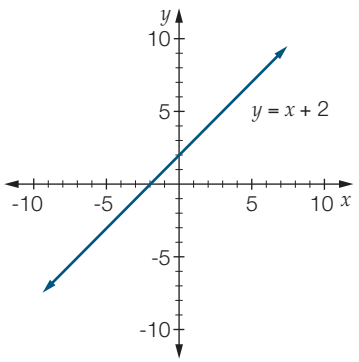
(b)



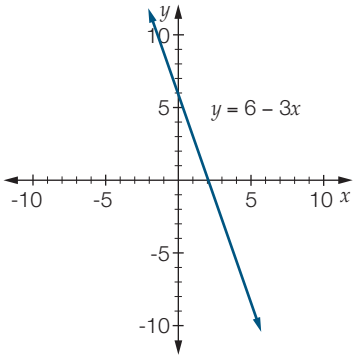
(c)



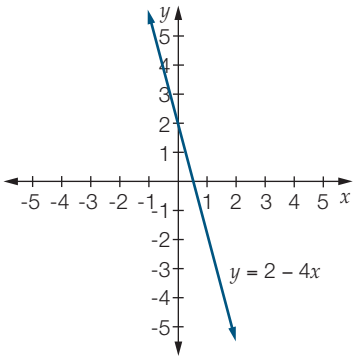
(d)



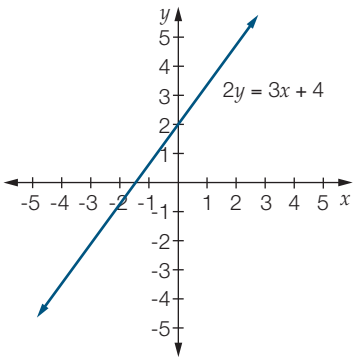
(e)



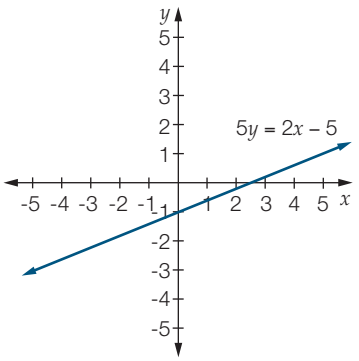
(f)



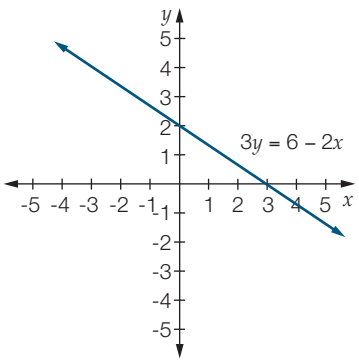
(g)



(h)



(i)



3 (a) C (b) B

4 **(a)** **(i)** \$150 **(ii)** \$100 **(iii)** $175 - 25n$

(b) $A = 175 - 25n$

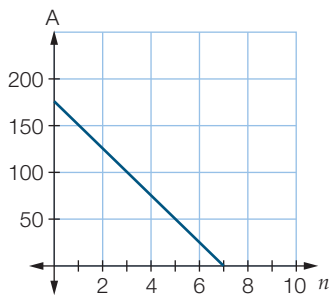
(c) **(i)** $A = 175$

(ii) This means that Todd owes \$175 at the time of purchase.

(d) **(i)** $n = 7$

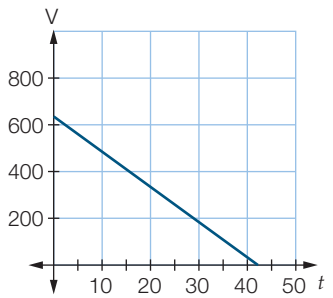
(ii) This means that Todd will have paid off the full amount in 7 weeks.

(e)



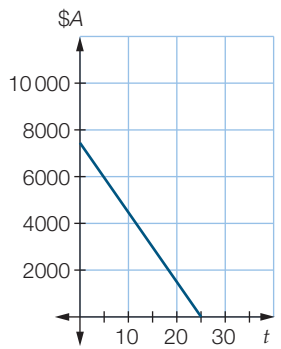
5 **(a)** 555 L **(b)** 630 L **(c)** 42 min

(d)



6 **(a)** \$7500 **(b)** 2 years and 1 month

(c)



(d) $\$A = 7500 - 300t$ **(e)** \$5700 **(f)** \$4200

7 (-1, 0) and (0, 3); (1, 0) and (0, -3); (2, 0) and (0, -6)

Challenge 4 (p. 199)

1 Jesse travels twice as fast as Vinny. He leaves 13 minutes after Vinny, so in the next 13 minutes he will travel twice as far as Vinny has travelled in the 26 minutes since he started. Thus, Jesse will catch up with Vinny at this time:
5.48 p.m. + 13 min = 6.01 p.m.

2 Let the bicycle cost \$$x$ per day and the car cost \$$y$ per day.

$2x + 3y = 150$ and $4x + y = 90$
$12x + 3y = 270$
$10x = 120$
$x = 12$ and $y = 42$

The bike costs \$12 per day and the car costs \$42 per day.

3 The equation of the line is $y = 2x + 3$; $w = 17$.

4 D

5 **(a)** From A to B is across 2 and up 1. From B to A it is back 2 and down 1. Hence, from C to D is back 2 and down 1, so D is (0, 3).

(b) The gradient of $AB = \frac{2-1}{3-1} = \frac{1}{2}$; the gradient of $BC = \frac{4-2}{2-3} = \frac{2}{-1} = -2$. The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

(c) The gradient of $AD = -2$, the gradient of $DC = \frac{1}{2}$. The gradients have a product of -1 and are the same as the gradients of their opposite sides.

(d) The parallel sides of the square have the same gradient.

(e) The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

6 Either plot the points, or by analysis see that from K to L is 3 across and 4 up. From L, go back 4 and up 3 to give M (-3, 6). From K, go back 4 and up 3 to give N (-6, 2). For the second set of values, from L go across 4 and down 3 to give M (5, 0). From K, go across 4 and down 3 to give N (2, -4). This process is based on the fact that the gradient of the perpendicular sides is the negative reciprocal of the gradient of KL. The hypotenuse of our triangle is still 5, the length of the side.

Answers: M (-3, 6) and N (-6, 2) or M (5, 0) and N (2, -4).

7 (1, -1)

8 **(a)** $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}$, $\frac{x-a}{x-b} = \frac{x-b-x}{x-b}$, $x - a = -b$, $x = a - b$

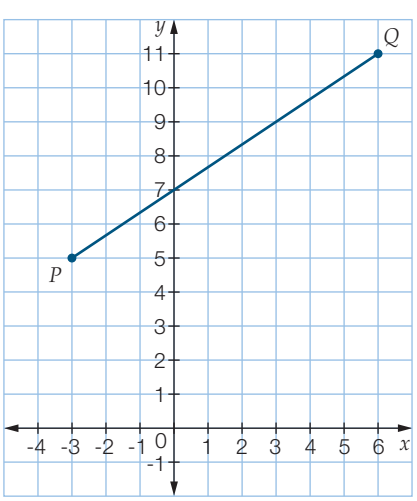
(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2$, $\frac{a^2-b^2}{abx} = a^2 - b^2$, $abx = 1$, $x = \frac{1}{ab}$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}$, $(a-b)(x+c) = (a+b)(x-c)$, $ax + ac - bx - bc$

$= ax - ac + bx - bc$, $2ac = 2bx$, $x = \frac{ac}{b}$.

Chapter review 4 (p. 200)

1 **(a)**



(b) 10.82 (2 d.p.) **(c)** (1.5, 8)

2 $x = \sqrt{68}$, $y = 13$, $z = 5$

3 $M(AB) = \left(\frac{7}{2}, -4\right)$, $M(BC) = \left(5, -\frac{3}{2}\right)$,

(a) $M(CD) = \left(0, \frac{5}{2}\right)$, $M(DA) = (1, -2)$ **(b)** Triangle

4 **(a)** $x = -5$ **(b)** $x = -1$ **(c)** $x = -45$

(d) $x = 72$ **(e)** $x = 0$ **(f)** $x = \frac{5}{2}$

5 **(a)** $x = 9$ **(b)** $x = 19$ **(c)** $x = \frac{11}{9}$ **(d)** $x = 3$

6 **(a)** $x = \frac{13}{6}$ **(b)** $x = -19$ **(c)** $x = -9$ **(d)** $x = 1$

(e) $x = \frac{3}{2}$ **(f)** $x = \frac{41}{5}$

7 **(a)** B **(b)** C

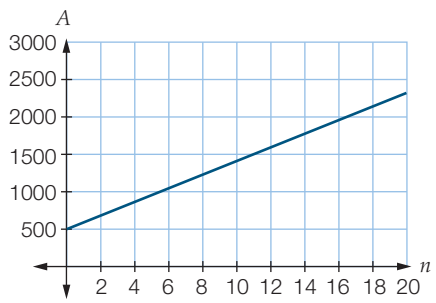
8 **(a)** $p + 4$ **(b)** $p - 11$

(c) $10 = p - 11$, $p = 21$. Pete had 21 mice originally.

9 **(a)**

No. weeks	0	1	2	3	4	5	6
Amount paid ($)	500	590	680	770	860	950	1040

(b)

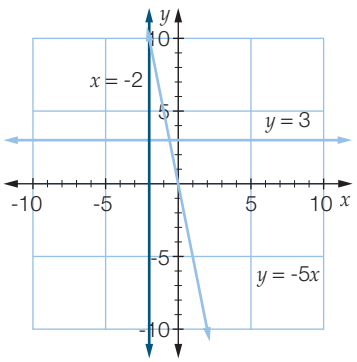


(c) $1490 **(d)** 15 weeks

(e) $A = 500 + 90n$ **(f)** $1310

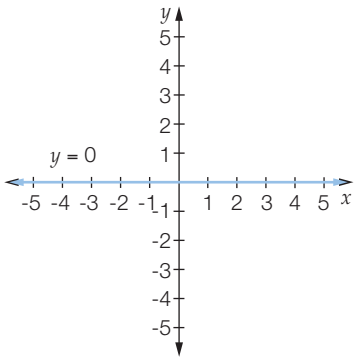
10 5 **11** **(a)** A **(b)** A

12

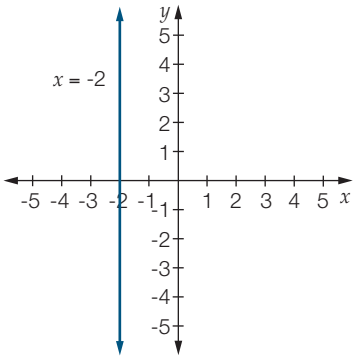


13 gradient = 2, x-intercept = (-2, 0), y-intercept = (0, 4)

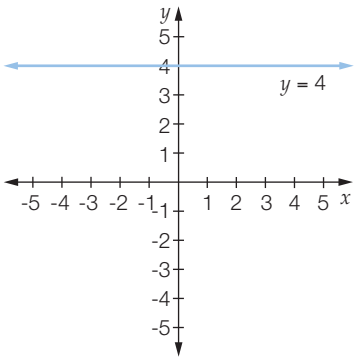
14 **(a)** (-2, 0), (0, 0), (1, 0), (3, 0) or any points that have a y value of zero.



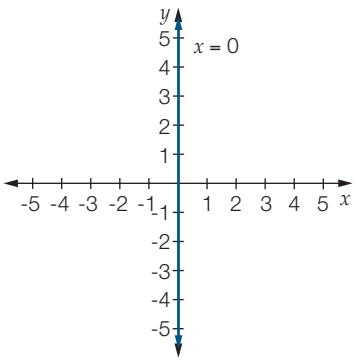
(b) (-2, -5), (-2, -1), (-2, 1), (-2, 4) or any points that have an x value of -2.



(c) (-2, 4), (0, 4), (1, 4), (3, 4) or any points that have a y value of 4.

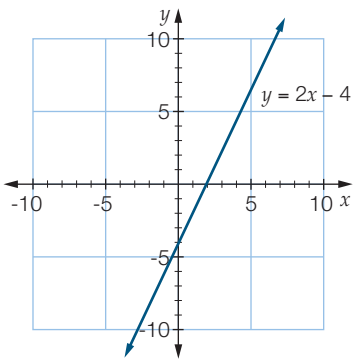


(d) (0, -5), (0, -1), (0, 1), (0, 4) or any points that have an x value of -2.

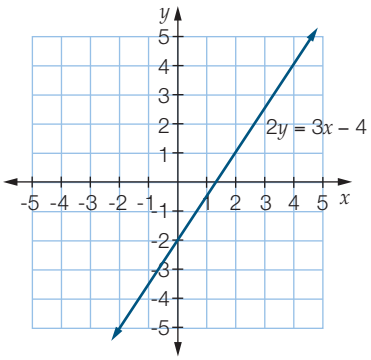


15 **(a)** D **(b)** C **(c)** D

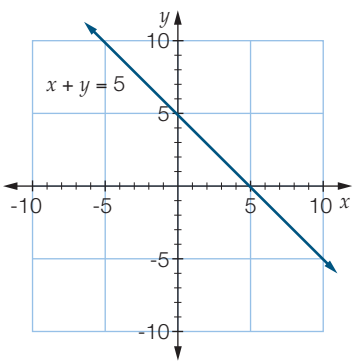
16 **(a)** y-intercept: (0, -4), gradient = 2



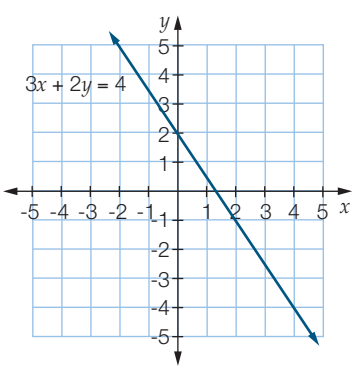
(b) y-intercept: (0, -2), gradient = $\frac{3}{2}$



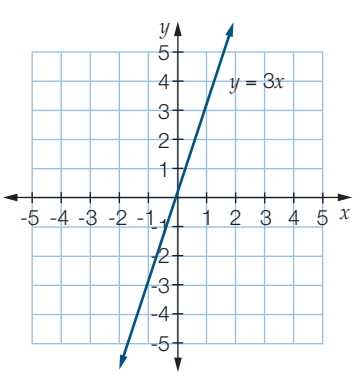
(c) y-intercept: (0, 5), gradient = -1



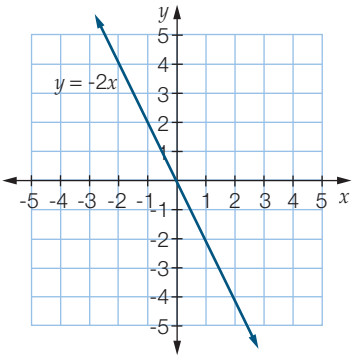
(d) y-intercept: (0, 2), gradient = $-\frac{3}{2}$



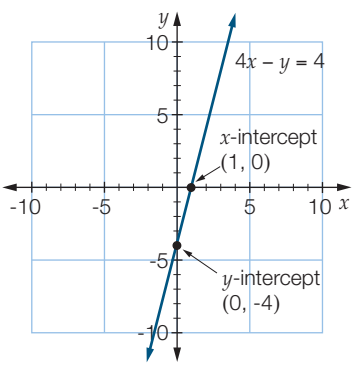
(e) y-intercept: (0, 0), gradient = 3



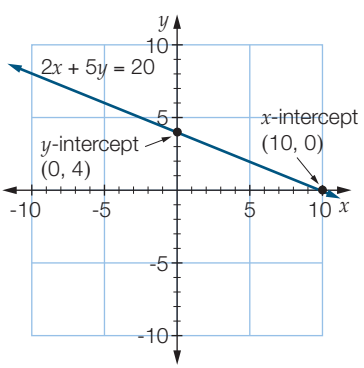
(f) y-intercept: (0, 0), gradient = -2



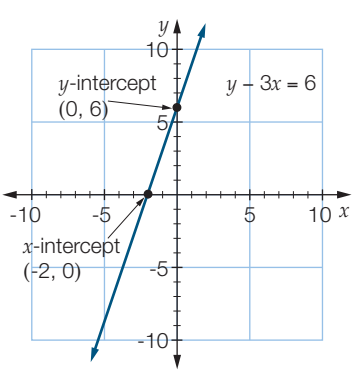
17 (a)



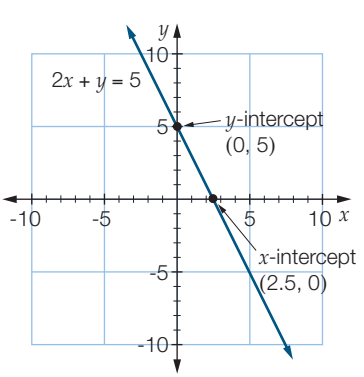
(b)



(c)



(d)



18 **(a)** (0, 0) and (5, 350) **(b)** 70

(c) 70 m/min **(d)** $d = 70t$ **(e)** 560 m

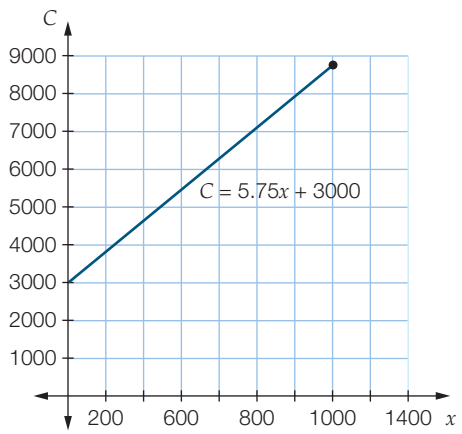
19 **(a)** 21, 22, 23 **(b)** 22, 24, 26, 28

20 $58 = 2(2x - 7) + 2x$, $x = 12$. Therefore the length is 17 m and the width is 12 m.

21 $k = 9$

22 **(a)** $C = 5.75x + 3000$ **(b)** \$5875 **(c)** 1043

(d)



The cost of 1000 components is \$8750.

23 **(a)** $C = 70 + 105t$ **(b)** \$910

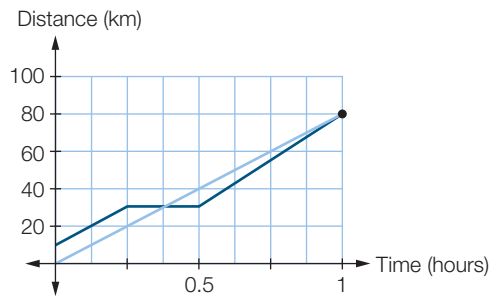
(c) 2 hours 15 minutes

24 **(a)** Students to check own graphs carefully.

(b) 80 km **(c)** 80 km/h **(d)** 15 min

(e) 100 km/h **(f)** 70 km

(g)



(h) 80 km/h **(i)** $d = 80t$

(j) 22.5 min after she left **(k)** 50 km from home

NAPLAN practice 4

1 C **2** D **3** $b = 7$ **4** B

5 C **6** 125 **7** A **8** 5

9 C **10** 38 cm

Chapter 5

Recall 5 (p. 204)

1 Students to draw own angles.

2 **(a)** acute **(b)** obtuse **(c)** reflex **(d)** reflex

3 **(a)** 35° **(b)** 245°

4 $a = 46°$, $b = 46°$, $c = 46°$

5 **(a)** 173 **(b)** 64 **(c)** 70

6 **(a)** 4.8 m **(b)** 1.5 m

7 **(a)** 6 : 1 **(b)** 3.5 : 1 **(c)** 21.88 : 1 **(d)** 686.17 : 1

8 **(a)** 1 175 000 **(b)** 4 cups

Exercise 5.1 (p. 207)

1 **(a)** $a = 38°$, alternate angles on parallel lines; $b = 82°$, corresponding angles on parallel lines

(b) $f = 125°$, straight angle; $g = 125°$, corresponding angles on parallel lines; $h = 70°$, alternate angles on parallel lines

(c) $w = 25°$, straight angle; $x = 85°$, vertically opposite angle; $y = 25°$, alternate angles on parallel lines; $z = 110°$, exterior angle equals sum of two interior opposite angles

2 **(a)** $x = 50°$, supplementary angles

(b) $m = 30°$, angles in a revolution

(c) $a = 90°$, corresponding angles on parallel lines; $b = 117°$, corresponding angles on parallel lines, $c = 63°$, straight angle; $d = 153°$, straight angle

3 B

4 **(a)** $a = 50°$, cointerior angles on parallel lines; $2b + 40 = 130$, $b = 45°$, two opposite interior angles equal to exterior angle of triangle

(b) $x = 36°$, straight angle; $y = 144°$, corresponding angles on parallel lines

(c) $m = 40°$, two opposite interior angles equal to exterior angle of triangle; $n = 40°$, alternate angles on parallel lines; $r = 140°$, straight angle.

5 **(a)** False **(b)** False **(c)** True **(d)** False

6 $p = 75°$; equal angles of an isosceles triangle, $r = 75°$; corresponding angles on parallel lines

7 $y = 5x + 45$, alternate angles on parallel lines, hence $9x + 45 = 180°$, $x = 15°$, sum of angles in a triangle; $y = 5 \times 15 + 45 = 120°$; $z = 60°$, sum of angles in a triangle.

8 **(a)** $3x = 40 + (x + 30)$, vertically opposite angles; $2x = 70$, $x = 35°$

(b) $(2y - 40) + 40 = 180$, co-interior angles supplementary on parallel lines; $2y = 180$, $y = 90°$

(c) $65 + (90 + z - 35) = 180$, co-interior angles supplementary on parallel lines; $120 + z = 180$. $z = 60°$

9 y creates a straight angle with 2 angles. These both have the same value as x; one because it is an equal angle to x in an isosceles triangle, the other because it is an alternate angle on parallel lines to x. Therefore, $2x + y = 180$. If $x = 40°$, $y = 100°$. If $x = 50°$, $y = 80°$. If $x = 70°$, $y = 40°$.

10 **(a)** x and y must sum to 180°. For example, $x = 100°$ and $y = 80°$.

(b) m and n must be equal. For example, $m = n = 120°$.

(c) $t = 180 - y - (180 - n)$, $t = -y + n$

(d) $t = -(180 - x) + m$ **(e)** $t = -80 + 120 = 40°$

(f) $t = -(180 - 100) + 120 = 40°$ **(g)** Yes

Exercise 5.2 (p. 212)

1 **(a)** 1.5 **(b)** 0.5 **(c)** 2 **(d)** 1

2 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

3 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

4 **(a)** **(i)** 1 cm : 2 cm **(ii)** 1.5 cm : 3 cm
(iii) 1.75 cm : 3.5 cm
(b) 0.5

5 **(a)** **(i)** 3 cm : 2 cm **(ii)** 3.3 cm : 2.2 cm
(iii) 3 cm : 2 cm **(iv)** 1.5 cm : 1 cm
(b) 1.5

6 C

7 Students' own answers—the dimensions of $A'B'C'D'$ should be 6 cm × 9 cm.

8 Students' own answers—side lengths of $J'K'L'M'N$ should be 10 cm.

9 **(a)** 3.5 **(b)** $\frac{3}{7}$
(c) Enlarged by a scale factor of 1.5
(d) It is not a true enlargement because only the length has been stretched, altering the actual shape of the original image.

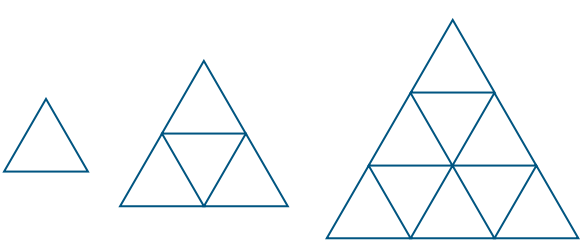
10 4 cm^2 **11** 1.75

12 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

13 The area of an enlarged rectangle is the area of the original rectangle multiplied by the square of the scale factor. Area of original rectangle = $72 \div 3^2 = 8$. The dimensions of the original rectangle are either 1×8 or 2×4.

14 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

15 **(a)**



(b) Scale factor of 4 has a side length of 4 cm (or area of 16 small triangles); scale factor of 9 has a side length of 9 cm (or an area of 81 small triangles).

(c) Because the side length of the medium triangle has length of two small triangles, the scale factor is 2; the small triangle has been enlarged by a scale factor of 3 to obtain the large triangle because it has a side length of three triangles.

Exercise 5.3 (p. 218)

1 **(a)** SAS **(b)** AAA **(c)** RHS **(d)** SSS

2 **(a)** $x = 14$ cm **(b)** $x = 8$ m **(c)** $x = 7.5$ cm **(d)** $x = 20$ mm

3 **(a)** B **(b)** C

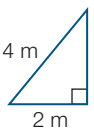
4 **(a)** C **(b)** C

5 **(a)** 2.5 **(b)** 0.5 **(c)** 0.8 **(d)** 1.5

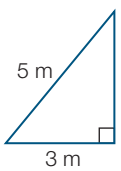
6 **(a)** $\Delta ABC \sim \Delta EDF$ (SSS) **(b)** $\Delta GHK \sim \Delta GIJ$ (AAA)
(c) $\Delta LMN \sim \Delta OPN$ (AAA)

7 **(a)** **(i)** $\frac{2}{3}$ **(ii)** 7.5 m
(b) **(i)** 2.5 **(ii)** 8.75 cm
(c) **(i)** $\frac{3}{4}$ **(ii)** 5.625 m
(d) **(i)** 1.25 **(ii)** 1.28 km

8 **(a)**



(b)



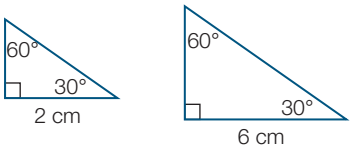
(c) Not similar; matching side lengths do not have the same ratio.

9 **(a)** 1.5
(b) Students' own answers—check carefully that $QR = 5.25$ cm and $QPR = 35°$.

10 All angles remain the same (AAA).

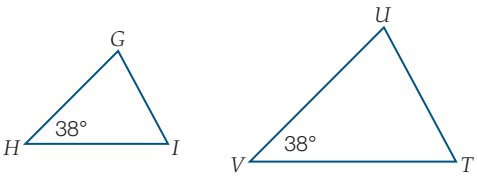
11 **(a)** Matching angles are the same and matching side lengths are of the same ratio.
(b) $b = 20$ cm, $d = 100°$, $r = 55°$, $s = 12$ cm

12



Scale factor = 3.

13 **(a)**



(b) Two side lengths (which enclose the given angle) or one more angle must be stated to justify similarity.

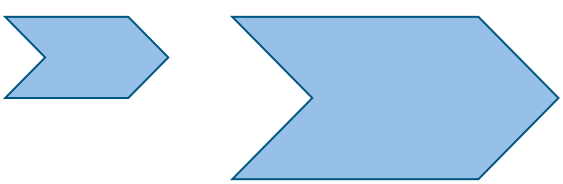
Half-time 5 (p. 222)

1 **(a)** $d = 130°$, $e = 125°$ **(b)** $n = 60°$, $m = 140°$, $p = 100°$
(c) $n = 47°$, $p = 133°$

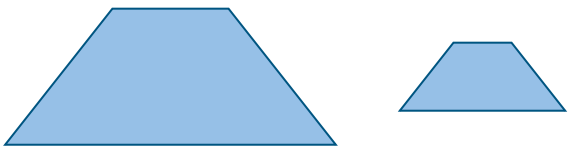
2 D

3 $(2y - 20) + 140 = 180$ (allied angles on parallel lines), $y = 30°$
$3x = x + 10 + 40$ (vertically opposite angles), $x = 25°$
$30 + z - 25 + 35 = 180$ (allied angles on parallel lines), $z = 140°$

4



5



6 441 m^2

7 **(a)** $x = 8$ **(b)** 10.2 m

8 **(a)** AAA, $x = 6$ mm **(b)** AAS, $x = 9$ cm

Exercise 5.4 (p. 224)

1 **(a)** AAA; $x = 10$ **(b)** AAA; $x = 6$

(c) AAA; $x = 5.56$ **(d)** AAA; $x = 26.4$

2 **(a)** AAA means that the triangles are similar

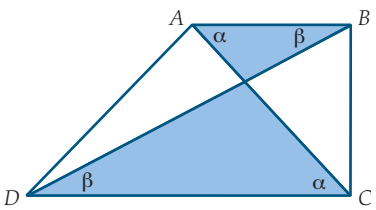
(b) 12 m

3 **(a)** C **(b)** C

4 6 m **5** 316.67 cm **6** 7.2 m

7 **(a)** Equally opposite angles at x mean that SAS shows the shaded triangles are similar.

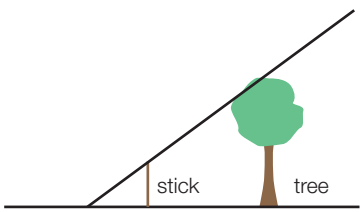
(b)



As the triangles are similar and the alternate angles are equal, AB and CD must be parallel. Hence $ABCD$ is a trapezium.

(c) No, the remaining two triangles only have one pair of vertically opposite angles, which is not enough to justify similarity.

8 **(a)**



Measure these three distances: from the stick to the tree; from the stick to the place on the ground you look from; and the length of the stick. Use similar triangles to find the height of the tree.

(b) Students' own answers.

9 Ratio of corresponding sides must be equal $\frac{a}{e} = \frac{b}{d} = \frac{c}{f}$.

One possible solution is: $a = 3, b = 2, c = 4, e = 6, d = 4, f = 8$

$\therefore \frac{3}{6} = \frac{2}{4} = \frac{4}{8}$

10 **(a)** The triangles might not be similar, thus cannot be calculated in this way.

(b) Pythagoras' Theorem.

Exercise 5.5 (p. 230)

1 **(a)** SSS **(b)** SSS **(c)** RHS

(d) SAS **(e)** SAS **(f)** SSS

(g) RHS **(h)** AAS **(i)** AAS

2 $\angle STO = \angle PRO$ (alternate angles, $ST \parallel RP$)
$\angle SOT = \angle POR$ (vertically opposite angles)
$ST = PR = 10$ cm (given)
$\Delta STO \equiv \Delta PRO$ (AAS)
$TO = RO$ (matching sides in congruent triangles)
$x = 6$ cm

3 **(a)** In Δs EFH and EGH,

$FH = GH$ (given)

$\angle FHE = \angle GHE$ (both 90°)

EH is a common side

$\therefore \Delta EFH \equiv \Delta EGH$ (SAS)

$\therefore \angle EFH = \angle EGH$ (matching angles in congruent triangles)

(b) ΔEFG is isosceles because it has two equal angles; i.e. $\angle EFH = \angle EGH$.

(c) If the perpendicular bisector of a side of a triangle passes through the opposite vertex, then the triangle is isosceles with the other two sides being equal in length.

(d) $EF = EG$ because they are matching sides in the congruent triangles EFH and EGH.

4 $\Delta MNO \equiv \Delta DFE$ (SSS), $\Delta ABC \equiv \Delta JKL$ (SAS), $\Delta GHI \equiv \Delta RQP$ (AAS)

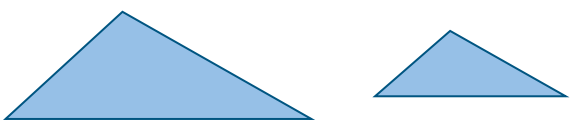
5 **(a)** **(i)** SAS **(ii)** SAS **(iii)** AAS

(iv) SAS **(v)** AAS **(vi)** RHS

(b) **(i)** $c = 35°$ **(ii)** $p = 38°$ **(iii)** $x = 15$ cm

(iv) $g = 5$ cm **(v)** $x = 50$ mm **(vi)** $m = 75°$

6 Two triangles with three equal angles are equiangular and so are similar. To be congruent they must have at least one pair of matching sides equal. The following triangles are equiangular but not congruent.



7 Proof: In Δs ABE and DCE,
$\angle ABE = \angle DCE$ (alternate angles, $AB \parallel CD$)

$BE = CE$ (given)

$\angle AEB = \angle DEC$ (vertically opposite angles)

$\therefore \Delta ABE \equiv \Delta DCE$ (AAS congruency)

$\therefore AB = CD$ (matching sides of congruent triangles)

8 **(a)** In Δs XYZ and SRT, $XY = SR$ (both 17 m)

$\angle XYZ = \angle SRT$ (both 90°)

$YZ = RT$ (both 12 m)

$\therefore \Delta XYZ \equiv \Delta SRT$ (SAS)

(b) In Δs ABC and RQS, $\angle ABC = \angle RQS$ (both 90°)

$AB = RQ$ (both 11 cm)

$AC = RS$ (both 15 cm)

$\therefore \Delta ABC \equiv \Delta RQS$ (RHS)

(c) In Δs DEF and XZY, $DE = XZ$ (both 6 cm)

$EF = ZY$ (both 6 cm)

$DF = XY$ (both 8 cm)

$\therefore \Delta DEF \equiv \Delta XZY$ (SSS)

(d) In Δs FGH and PQR,

$\angle FGH = \angle PQR$ (both 80°)

$\angle FHG = \angle PRQ$ (both 70°)

$GF = QP$ (both 5 cm)

$\therefore \Delta FGH \equiv \Delta PQR$ (AAS)

(e) In Δs DEF and XVW, $\angle DFE = \angle XWV$ (both 90°)

$DE =$ XV (both 16 cm)

$FE = WV$ (both 7 cm)

$\therefore \Delta DEF \equiv \Delta XVW$ (RHS)

(f) In Δs KLM and QSR,

$\angle LKM = \angle SQR$ (both 50°)

$\angle LMK = \angle SRQ$ (both 30°)

$KM = QR$ (both 32 cm)

$\therefore \Delta KLM \equiv \Delta QSR$ (AAS)

9 In Δs TUR and TSR, $\angle TUR = \angle TSR$ (both 90°)

TR is a common side

$TU = TS$ (given)

$\therefore \Delta TUR \equiv \Delta TSR$ (RHS)

10 In Δs SQP and RPQ,

$SP = RQ$ (given)

$\angle SPQ = \angle RQP$ (given)

PQ is a common side

$\therefore \Delta SQP \equiv \Delta RPQ$ (SAS)

11 (a) In Δs ABC and CDA, $AB = CD$ (given)

$BC = DA$ (given)

AC is a common side

$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)

$x = 32°$ (matching angles in congruent triangles)

(b) In Δs ADC and ABC, $\angle ADC = \angle ABC = 35°$ (given)

$\angle ACD = \angle ACB = 120°$ (given)

AC is a common side

$\therefore \Delta ADC \equiv \Delta ABC$ (AAS)

$b = 23$ cm (matching sides in congruent triangles)

(c) In Δs ABE and CDE, $\angle ABE = \angle CDE = 56°$ (given)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = CD$ (given)

$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)

$x = 23$ cm (matching sides in congruent triangles)

(d) In Δs OAB and OCB, $\angle OBA = \angle OBC = 90°$

$OA = OC$ (radii)

OB is a common side

$\therefore \Delta OAB \equiv \Delta OCB$ (RHS)

$y = 26°$ (matching angles in congruent triangles)

(e) In Δs ABO and CDO, $AB = CD$ (given)

$AO = CO$ (radii)

$BO = DO$ (radii)

$\therefore \Delta ABO \equiv \Delta CDO$ (SSS)

$y = 9$ m (matching sides in congruent triangles)

(f) In Δs ABE and DCE,

$\angle BAE = \angle CDE = 75°$(given)

$AB = DC$ (given)

$\angle AEB = \angle DEC$ (vertically opposite angles)

$\therefore \Delta ABE \equiv \Delta DCE$ (AAS)

$k = 8$ cm (matching sides in congruent triangles)

12 (a) In Δs DEG and DFG,

$DE = DF$ (given)

$EG = FG$ (G is the midpoint of EF)

GD is a common side

$\therefore \Delta DEG \equiv \Delta DFG$ (SSS)

$\therefore \angle DGE = \angle DGF$ (matching angles of congruent triangles)

But $\angle DGE + \angle DGF = 180°$, therefore $\angle DGE = 90°$ and hence $DG \perp EF$.

(b) If ΔDEF is equilateral, then all interior angles are 60°.

Because $\Delta DEG \equiv \Delta DFG$ (SSS)

then $\angle EDG = \angle FDG = 60 \div 2 = 30°$

13 In Δs KLN and KMN,

$KL = KM$ (given)

$\angle LKN = \angle MKN$ (given)

KN is a common side

$\therefore \Delta KLN \equiv \Delta KMN$ (SAS)

$\therefore LN = NM$ (matching sides of congruent triangles)

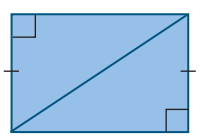
$\angle KNL = \angle KNM$ (matching angles of congruent triangles)

But $\angle KNL + \angle KNM = 180°$, hence $KN \perp LM$.

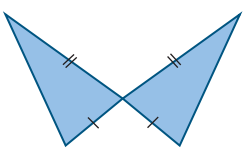
Therefore KN is the perpendicular bisector of LM.

14 In Δs QPS and RPS we only have two pairs of equal sides. Either another pair of sides needs to be equal, the angles between the equal sides need to be equal, or PS perpendicular to QR, to be able to prove that the triangles are congruent. The equal angles that are given are not included by the sides.

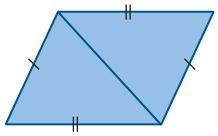
15



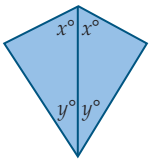
RHS



SAS

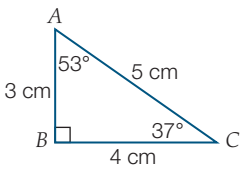


SSS

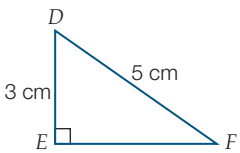


AAS

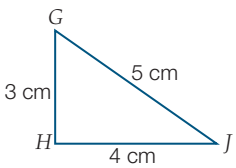
16



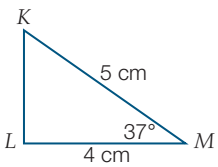
Original ΔABC



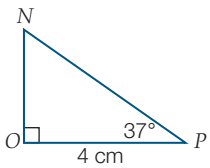
RHS



SSS



SAS



AAS

Exercise 5.6 (p. 242)

1 **(a)** $x = 36°$ **(b)** $y = 15°, z = 110°$ **(c)** $w = 27°$

2 **(a)** Join DB. In Δs DAB and ΔBCD,

$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel DC$)

DB is a common side

$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel BC$)

$\Delta DAB \equiv \Delta DCB$ (AAS)

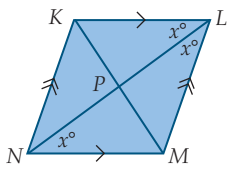
$\angle DAB = \angle BCD$ (matching angles in congruent triangles)

(b) $AB = DC$ (matching sides in congruent triangles);
$AD = BC$ (matching sides in congruent triangles)

(c) If the diagonal AC is drawn, then the same steps as in **(a)** can be used to prove that $\Delta ADC \equiv \Delta CBD$. $\angle ADC = \angle CBA$ (matching angles in congruent triangles).

(d) The opposite angles of a parallelogram are equal. The opposite sides of a parallelogram are equal.

3 **(a)** Let $\angle KLN = x°$ and mark other angles that are equal to $x°$ on the diagram.



$\angle LNM = \angle KLN = x°$ (alternate angles, $KL \parallel NM$)

$LM = MN$ (adjacent sides of a rhombus)

ΔLMN is isosceles

$\angle MLN = \angle LNM = x°$ (equal angles of an isosceles triangle)

$\angle KLN = \angle MLN$ (both equal $x°$)

(b) In Δs KLP and MLP

$KL = LM$ (adjacent sides of a rhombus)

$\angle KLP = \angle MLP$ (proved in **(a)**)

LP is a common side

$\Delta KLP \equiv \Delta MLP$ (SAS)

(c) $\angle KPL = \angle MPL$ (matching angles in congruent triangles)

$\angle KPL + \angle MPL = 180°$ ($\angle KPM = 180°$)

$\angle KPL = \angle MPL = 90°$

(d) $KP = MP$ (matching sides in congruent triangles)

(e) The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

4 $PT = TR$, $QT = TS$, $\angle PTQ = \angle QTR = \angle RTS = \angle STP = 90°$

In Δs PTQ and QTR, $PT = TR$ (given)

$\angle PTQ = \angle QTR = 90°$ (given)

QT is a common side

$\Delta PTQ \equiv \Delta QTR$ (SAS)

$PQ = QR$ (matching sides in congruent triangles)

In Δs PTQ and RTS, $PT = TR$ (given)

$\angle PTQ = \angle RTS = 90°$ (vertically opposite angles)

$QT = TS$ (given)

$\Delta PTQ \equiv \Delta RTS$ (SAS)

$\angle PQT = \angle RST$ (matching angles in congruent triangles)

$\angle PQS$ and $\angle RSQ$ are a pair of equal alternate angles contained between PQ and SR.

$PQ \parallel SR$

Repeating these steps in ΔPTS and ΔRTQ allows you to prove that $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so $PQRS$ is a rhombus.

5 C

6 **(a)** $x = 21°$ **(b)** $w = 124°, y = 33°, z = 40°$ **(c)** $u = 115°$

7 **(a)** False **(b)** True **(c)** False **(d)** True

8 Hexagon **9** A **10** B

11 **(a)** 1080° **(b)** 135°

12 (a) Proof: In Δs ADC and CBA,
$\angle ACD = \angle CAB$ (alternate angles, $AB \parallel DC$)
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
AC is a common side

$\therefore \Delta ADC \equiv \Delta CBA$ (AAS)

$\therefore \angle ADC = \angle CBA$ (matching angles in congruent triangles)

(b) Repeat **(a)** in Δs ABD and CDB to prove that they are congruent, Hence, $\angle DAB = \angle BCD$ (matching angles in congruent triangles).

13 (a) $\angle ADC = \angle CBA$ and $\angle DAB = \angle BCD$, hence the opposite angles of a parallelogram are equal.

(b) Because $\Delta ADC \equiv \Delta CBA$, then $AB = DC$ (matching sides) and $AD = BC$ (matching sides). Hence the opposite sides of a parallelogram are equal.

(c) In Δs AEB and CED,

$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel DC$)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = DC$ (opposite sides of a parallelogram are equal, proved above)

$\Delta AEB \equiv \Delta CED$ (AAS)

(d) $AE =$ EC (matching sides of congruent triangles);
$BE = ED$ (matching sides of congruent triangles).
Hence, AC and BD bisect each other at E.

14 (a) Proof: In Δs PRS and QSR:
$\angle PSR = \angle QRS = 90°$ (given)

PS is a common side

$PS = QR$ (given)

$\therefore \Delta PRS \equiv \Delta QSR$ (SAS)

(b) $PR = QS$ (matching sides of congruent triangles)

(c) The diagonals of a rectangle are equal.

15 (a) In Δs KLM and KNM, $KL = KN$ (given)

$LM = NM$ (given)

KM is a common side

$\Delta KLM \equiv \Delta KNM$ (SSS)

Hence, $\angle KLM = \angle KNM$ (matching angles of congruent triangles)

(b) In Δs LKX and NKX, $KL = KN$ (given)

$\angle LKX = \angle NKX$ (matching angles of congruent triangles KLM and KNM)

KX is a common side

$\Delta LKX \equiv \Delta NKX$ (SAS)

(c) $\angle LXK = \angle NXK$ (matching angles of congruent triangles), but, $\angle LXK + \angle NXK = 180°$ (LXN is a straight angle). Hence, $\angle LXK = \angle NXK = 90°$.

(d) One pair of opposite angles of a kite are equal; these are the angles between the unequal sides. The diagonals of a kite intersect at right angles.

16 (a) In Δs WYX and WYZ, $WX = WZ$ (given)

$XY = ZY$ (given)

WY is a common side

$\Delta WYX \equiv \Delta WYZ$ (SSS)

$\angle WYX = \angle WYZ$ (matching angles of congruent triangles)

(b) Repeat **(a)** for Δs WZX and YXZ, then $\angle WZX = \angle YZX$ (matching angles of congruent triangles).

(c) The diagonals of a rhombus bisect the angles of the rhombus.

17 (a) In Δs ACD and ACB, $AD = BC$ (given)

$DC = AB$ (given)

AC is a common side

$\Delta ACD \equiv \Delta ACB$ (SSS)

$\angle ACD = \angle ACB$ (matching angles of congruent triangles)

(b) Because $\angle DCB = 90°$, $\angle ACD = \angle ACB = 45°$.

(c) The diagonals of a square bisect the angles of the square.

18 (a) In Δs KLM and LKN, $LM = KN$ (given)

$\angle KLM = \angle LKN = 90°$ (given)

KL is a common side

$\Delta KLM \equiv \Delta LKN$ (SAS)

$KM = NL$ (matching sides of congruent triangles)

(b) ΔLKN is a right-angled isosceles triangle so $\angle KLN = \angle KNL = 45°$

ΔLMN is a right-angled isosceles triangle so $\angle MLN = \angle MNL = 45°$

Hence $\angle KLO = \angle LKO = 45°$

ΔKLO is isosceles so $KO = OL$.

Similarly, you can show that ΔMLO is isosceles and $OL = OM$.

Hence $KO = OM$ (both equal OL)

Since ΔKNO is isosceles, $KO = NO$ so $LO = ON$ (both equal KO)

(c) In ΔKLO since $\angle KLO = \angle LKO = 45°$ then $\angle KOL = 90°$ (angle sum of ΔKLO is 180°)

(d) The diagonals of a square bisect each other at right angles and bisect the angles of the square.

19 In Δs UXW and WVU,

$UX = WV$ (given)

$\angle XUW = \angle VWU$ (alternate angles, $UX \parallel VW$)

UW is a common side

$\Delta UXW \equiv \Delta WVU$ (SAS)

Hence $\angle XWU = \angle VUW$ (matching angles of congruent triangles)

But these are a pair of equal alternate angles between XW and UV, so $XW \parallel UV$.

Hence $UX \parallel VW$ and $XW \parallel UV$, so $UVWX$ is a parallelogram.

20 In Δs ABC and ADC, $AB = AD$ (given)

$BC = DC$ (given)

AC is a common side

$\Delta ABC \equiv \Delta ADC$ (SSS)

Hence, $\angle ADC = \angle ABC = 90°$ (matching angles of congruent triangles). Because ΔABC is right angled and isosceles, then $\angle BAC = \angle BCA = 45°$. Similarly, $\angle DAC = \angle DCA = 45°$. Hence, $\angle DAB = \angle DCB = 90°$. Now $\angle DAB$ and $\angle ADC$ are a pair of supplementary co-interior angles between AB and DC so $AB \parallel DC$, and $\angle DAB$ and $\angle ABC$ are a pair of supplementary co-interior angles between AD and BC. Thus, $AD \parallel BC$. Hence, $ABCD$ is a parallelogram with a pair of adjacent sides equal, so $ABCD$ is a rhombus. But $ABCD$ is a rhombus with one angle a right angle, so $ABCD$ is a square.

21 In Δs EFH and GFH, $EF = HG$ (given)

$EH = GF$ (given)

FH is a common side

$\Delta EFH \equiv \Delta GFH$ (SSS)

Hence $\angle EHF = \angle GFH$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EH and FG, so $EH \parallel FG$. Similarly, $\angle DFH = \angle GHF$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EF and HG, so $EF \parallel HG$. Hence, $EFGH$ is a parallelogram. But $EF = FG$, so EFGH is a parallelogram with a pair of adjacent sides equal. Thus, $EFGH$ is a rhombus.

22 **(a)** $2x + 2y = 360°$ (angle sum of a quadrilateral); $x + y = 180°$. Hence, $\angle MJK + \angle JML = 180°$.

(b) $\angle MJK$ and $\angle JML$ are a pair of supplementary co-interior angles between JK and ML. Hence $JK \parallel ML$. But $\angle MJK + \angle JKL = 180°$, so $\angle MJK$ and $\angle JKL$ are a pair of supplementary co-interior angles between JM and KL. Hence, $JM \parallel KL$. $JKLM$ has both pairs of opposite sides parallel, so it is a parallelogram.

23 In Δs SPT and QRT, $ST = QT$ (given)

$\angle STP = \angle QTR$ (vertically opposite angles)

$TP = TR$ (given)

$\Delta SPT \equiv \Delta QRT$ (SAS)

Hence $\angle PST = \angle TQR$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SP and RQ, so $SP \parallel RQ$. Similarly, you can prove that $\Delta SRT \equiv \Delta QPT$ (SAS) so that $\angle RST = \angle TQP$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SR and PQ, so $SR \parallel PQ$. Hence, $SP \parallel RQ$ and $SR \parallel PQ$, so $SPQR$ is a parallelogram.

24 **(a)** Not true. A rhombus has four equal sides but no right angle, so it is not a square.

(b) All rectangles have their opposite sides parallel. The converse is: 'All quadrilaterals with their opposite sides parallel are rectangles'. This is false, as parallelograms have opposite sides parallel but are not always rectangles.

25 A square is a rectangle because it has two pairs of parallel sides all meeting at 90° angles, the defining features of a rectangle. However, for a rectangle to be a square its four sides must also be of equal length, which is not necessarily the case.

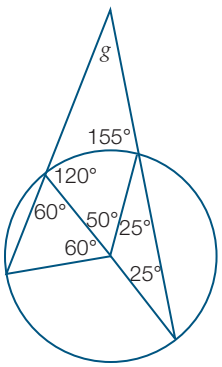
Challenge 5 (p. 245)

1 D

2 Arrange the matches as a three-dimensional tetrahedron. The four faces are equilateral triangles.

3 B

4 $g = 360 - (120 + 50 + 155) = 35°$



5 B

6 B, C, D

7 B

Chapter review 5 (p. 246)

1 **(a)** $x = 163°$ **(b)** $x = 41°$ **(c)** $x = 46°$

(d) $x = 85°$ **(e)** $x = 32°$ **(f)** $x = 110°$

2 **(a)** $y = 53°, z = 127°$ **(b)** $a = 95°, b = 110°, c = 95°, d = 85°$

(c) $m = 55°, n = 65°, p = 115°$

3 **(a)** 0.4 **(b)** 2.5

4 Students' own diagrams.

5 The missing angles in the triangles are 40° and 115°, so the triangles are similar (AAA).

6 **(a)** 5 **(b)** 9

7 A vertically opposite angle and an alternate angle on parallel lines means the triangles are similar by AAA.

$v = 4$ m, $x = 7.5$ m

8 **(a)** AAS **(b)** SAS **(c)** AAS

9 **(a)** $x = 72°, y = 141°$ **(b)** $c = 118°, d = 62°, e = 118°$

(c) $f = 90°$

10 **(a)** B **(b)** D **(c)** A **(d)** C

11 **(a)** rectangle **(b)** rhombus **(c)** parallelogram

12 **(a)** $x = 77$ **(b)** $x = 60, y = 120$

(c) $w = 72\ y = 108, z = 72$ **(d)** $x = 60$

(e) $x = 40$ **(f)** $x = 40$

13 **(a)** $x = 56$ **(b)** $x = 24, y = 128$ **(c)** $x = 43.6$

(d) $x = 30$ **(e)** $x = 34, y = 124$

14 (a) $BD \perp AC$

In Δs ABD and CBD, $\angle BDA = \angle BDC = 90°$ ($BD \perp AC$)

$AB = CB$ (given)

BD is a common side

$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)

$\therefore \angle BAD = \angle BCD$ (matching angles of congruent triangles)

$\therefore \angle BAC = \angle BCA$

(b) The angles opposite equal sides in an isosceles triangle are equal.

15 (a) $AE = CE$

$DE = BE$

$\Delta ABE \equiv \Delta BEC \equiv \Delta CED \equiv \Delta DEA$ (SSS)

$\therefore \angle AEB = \angle BEC = \angle CED = \angle DEA = 90°$

$\therefore AC \perp BD$

(b) The diagonals of a rhombus bisect each other at right angles.

16 (a) In Δs ABC and ADC, $AB = CD$ (given)

$BC = DA$ (given)

AC is a common side

$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)

$\therefore \angle BAC = \angle DCA$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between AB and DC, so $AB \parallel DC$.

Similarly, $\angle BCA = \angle DAC$ and hence $BC \parallel AD$.

Hence $ABCD$ is a parallelogram because it is a quadrilateral with both pairs of opposite sides parallel.

(b) In Δs PQT and RQT, $PT = TR$ (given)

$\angle PTQ = \angle RTQ$ (both 90°, given)

QT is a common side

$\therefore \Delta PQT \equiv RQT$ (SAS)

$\therefore PQ = QR$ (matching sides of congruent triangles)

Similarly, you can prove that $\Delta PQT \equiv RST$ (SAS) so that $PQ = SR$.

$\angle QPT = \angle SRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PQ and SR, so $PQ \parallel SR$.

Similarly, you can prove that $\Delta PST \cong \Delta RQT$ (SAS).

$\angle SPT = \angle QRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PS and QR, so $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so it is a rhombus.

But Δs PQT and RQT are right-angled isosceles triangles, so $\angle PQT = \angle RQT = 45°$.

Hence $\angle PQR = 90°$.

$\therefore PQRS$ is a square.

17 12.6 m

18 2.43 m

19 Let $\angle TSU = x°$.

$\angle TSR = (180 - x)°$ ($RSUV$ a straight line)

$\angle TUS = (90 - x)°$ (angle sum of ΔTSU)

$\angle TUV = (90 + x)°$ ($RSUV$ a straight line)

$\therefore \angle TSR + \angle TUV = (180 - x)° + (90 + x)° = 270°$

20 (a) 1.2

(b) 13.2 cm by 10.2 cm

(c) **(i)** 93.5 cm^2 **(ii)** 134.64 cm^2

(d) 1.44—this ratio is different from the scale factor as each side has been increased by the scale factor, so the area has increased by the square of the scale factor.

(e) 144%

21 $a + b = 180°$ (AD a straight line)

Similarly $c + d = 180°$

Now $\angle ZWX = a$ (vertically opposite angles)

and $\angle YXW = d$ (vertically opposite angles)

$\angle YZW = b$ (corresponding angles, $AD \parallel HE$)

$\angle ZYX = c$ (corresponding angles, $AD \parallel HE$)

$\angle ZWX + \angle YXW + \angle ZYX + \angle YZW = a + d + c + b$
$= 180° + 180° = 360°$

NAPLAN practice 5

1 60° **2** C **3** 50° **4** 40°

5 95° **6** 40° **7** C **8** B

9 12.15 m^2

Mixed review B (p. 252)

1 61.70m

2 (a) 86° **(b)** 77° **(c)** 87°

3 16.25 cm^2

4 Students' own answers.

5 (a) 2 **(b)** -1 **(c)** $\frac{1}{2}$

6 (a) \$705 **(b)** \$90.05

7 (a) Scale factor = 1.4, $x = 28$ cm

(b) Scale factor = $\frac{5}{12}$, $x = 3.75$ cm

8 (a) $A = \frac{xy}{2}$ **(b)** $A = \frac{xy}{2}$ **(c)** $A = \frac{1}{2}h(a + b)$

9 351.86 cm^2

10 (a) 7.5 **(b)** $k = 46°$

11 (a) $x = \frac{55}{3}$ **(b)** $x = \frac{13}{4}$ **(c)** $x = -\frac{13}{3}$

12 (a) $(y + 7)(y + 3)$ **(b)** $(d + 3)(c + 4)$ **(c)** $(t - 4)(t - 5)$

13 $a = 30°$, $b = 80°$, $c = 70°$, $d = 150°$, $e = 100°$

14 (a) 2.4 **(b)** $\frac{1}{3}$

15 (a) AAA **(b)** SAS, scale factor 2

16 9 m

17 **(a)** 3

(b) **(i)** 150 cm^2 **(ii)** 1350 cm^2

(c) 9

(d) $9 = 3^2$, the scale factor of the area enlargement is the square of the scale factor of the enlarged lengths.

18 Any of ΔAEH, ΔAFI, ΔFEG, ΔACE, ΔABF, ΔFDE. All triangles have one right angle and two 45° angles
$\therefore$ they are similar triangles (AAA).

19 5.4

20 **(a)** $x^2 + 4x - 21$ **(b)** $(x + 7)(x - 3)$ **(c)** $x = 103$

(d) Students to show the equality of the expressions.

Chapter 6

Recall 6 (p. 255)

1 **(a)** 12.5 **(b)** 13 **(c)** ± 8 **(d)** ± 7

(e) 5.34 **(f)** 62.7221

2 **(a)** equilateral and acute angled

(b) scalene and right angled

(c) scalene and acute angled

(d) isosceles and right angled

(e) equilateral and acute angled

(f) scalene and obtuse angled

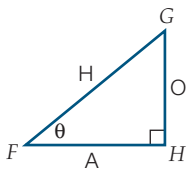
3 **(a)** 42 **(b)** 40 **(c)** 54 **(d)** 4

4 **(a)** 64° **(b)** 25° **(c)** 53°

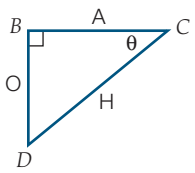
5 **(a)** $\frac{4}{5}$ **(b)** $\frac{3}{2}$ **(c)** $\frac{5}{4}$ **(d)** $\frac{4}{3}$ **(e)** $\frac{3}{4}$ **(f)** $\frac{4}{5}$

Exercise 6.1 (p. 258)

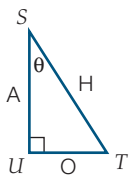
1 **(a)**



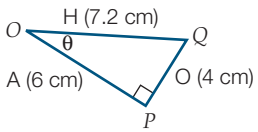
(b)



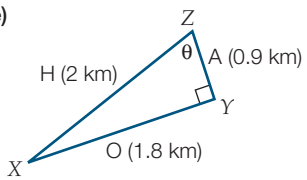
(c)



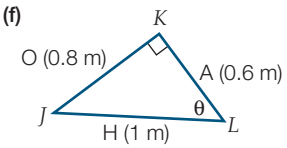
(d)



(e)



(f)



2 **(a)** **(i)** 10.6 cm **(ii)** 7 cm **(iii)** 8 cm

(b) **(i)** 12 m **(ii)** 3.8 m **(iii)** 11.4 m

(c) **(i)** 3.4 cm **(ii)** 2.5 cm **(iii)** 2.3 cm

(d) **(i)** 5.1 km **(ii)** 5 km **(iii)** 1 km

(e) **(i)** 96 mm **(ii)** 78 mm **(iii)** 56 mm

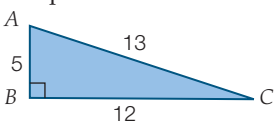
(f) **(i)** 108.3 km **(ii)** 34.7 km **(iii)** 102.6 km

3 **(a)** D **(b)** B **(c)** C

4 **(a)** 5.3 m **(b)** 1.8 m **(c)** 70°

5 **(a)** 13

(b) Sample answer:



6 **(a)** Triangle 1: H: 2.25 cm; O: 0.55 cm; A: 2.2 cm

Triangle 2: H: 4.25 cm; O: 1 cm; A: 4.1 cm

Triangle 3: H: 6.25 cm; O: 1.55 cm; A: 6.15 cm

(b) $\frac{0.55}{2.55}; \frac{1}{4.25}; \frac{1.55}{6.25}$

(c) $\frac{2.2}{2.25}; \frac{4.1}{4.25}; \frac{6.15}{6.25}$

7 **(a)** Students' own diagram.

(b) $\frac{0.9}{1} = 0.9; \frac{1.8}{2} = 0.9; \frac{2.7}{3} = 0.9; \frac{3.6}{4} = 0.9$

(c) Corresponding sides have an equal ratio in similar triangles.

8 **(a)** Students' own diagrams.

(b) Length measurements will depend on students' diagrams.

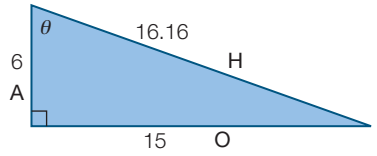
Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°				≈0.577	≈0.5	≈0.866
2	30°				≈0.577	≈0.5	≈0.866
3	30°				≈0.577	≈0.5	≈0.866
1	60°				≈1.732	≈0.866	≈0.5
2	60°				≈1.732	≈0.866	≈0.5
3	60°				≈1.732	≈0.866	≈0.5

(c) The ratios of corresponding sides are the same for each triangle.

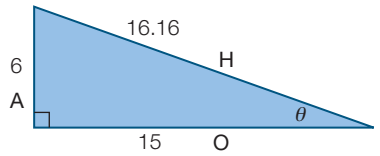
(d) Students' own response.

9 **(a)** The teacher has not marked in the angle.

(b) Nash



Aisha



10 The adjacent side is between the right angle and the reference angle. It is not always on the bottom of a triangle.

Exercise 6.2 (p. 263)

1 **(a)** **(i)** $\frac{7}{24}$ **(ii)** $\frac{4}{3}$ or $1\frac{1}{3}$ **(iii)** $\frac{21}{20}$ or $1\frac{1}{20}$ or 1.05

(b) **(i)** $\frac{7}{25}$ **(ii)** $\frac{3}{5}$ **(iii)** $\frac{21}{29}$

(c) **(i)** $\frac{3}{5}$ or 0.6 **(ii)** $\frac{8}{17}$ **(iii)** $\frac{9}{41}$

2 **(a)** **(i)** $x = 6.\dot{6}$ **(ii)** $p = 39$ **(iii)** $n = 0.7$

(b) **(i)** $x = 13$ **(ii)** $y = 6$ **(iii)** $r = 20.8$

(c) **(i)** $n = 48$ **(ii)** $s = 12$ **(iii)** $r = 22.4$

3 **(a)** 0.59 **(b)** 0.81 **(c)** 0.73

4 **(a)** **(i)** 0.58 **(ii)** 0.87 **(iii)** 0.50

(iv) 0.87 **(v)** 1.73 **(vi)** 0.50

(b) same **(c)** same

5 **(a)** **(i)** 0.54 **(ii)** 1.54 **(iii)** 0.84

(iv) 0.54 **(v)** 0.84 **(vi)** 0.65

(b) same **(c)** same

6 **(a)** B **(b)** A **(c)** B

7 100 m **8** 2.8 m **9** 0.82

10 **(a)** Students to draw a scale diagram.

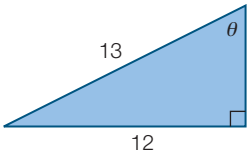
(b) right-angled isosceles triangle

(c) 5.3 m

(d) The two shorter sides of a right-angled isosceles triangle are equal.

11 **(a)** 3 m **(b)** Angle from the ground could be lowered to 30° or less.

12 **(a)**



(b) 5 **(c)** $\frac{5}{13}$

13 **(a)** sin 50° = 0.766; cos 50° = 0.643; tan 50 = 1.192

(b) tan 50° has a value greater than 1. The opposite side is longer than the adjacent side.

(c) **(i)** Any right-angled trianlge with a reference angle less than 45°.

(ii) Any right-angled triangle with a reference angle greater than 45°.

(d) sin θ and cos θ can not be greater than 1 because the hypotenuse (the bottom number on the ratio) will always be longer than the other two sides.

14 Any, as long as the ratio between b and a is 7:10. Sample answers: $a = 10$, $b = 7$; $a = 20$, $b = 14$; $a = 100$, $b = 70$.

15 The opposite and adjacent side change position with the two reference angles, so sin H is the same as cos G.

Exercise 6.3 (p. 269)

1 **(a)** 26.63 **(b)** 7.51 **(c)** 18.87 **(d)** 33.27

(e) 4.04 **(f)** 0.92

2 **(a)** 313.49 cm **(b)** 137.73 mm **(c)** 13.62 m

(d) 229.40km **(e)** 89.71 cm **(f)** 150.13 m

3 **(a)** C **(b)** D

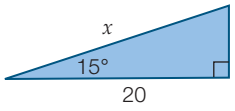
4 **(a)** C **(b)** D

5 2.40 m **6** 1.39 m **7** 7.60 m

8 10.72 m **9** 47.57 cm **10** 5.52 mm

11 The adjacent side divided by the hypotenuse must be approximately equal to 0.45.

12 **(a)** A question might have been to find the hypotenuse, as shown in the triangle below.



(b) 5.36

13 **(a)** **(i)** Use of the wrong reference angle in the equation. Student has calculated the length of j.

(ii) Incorrect method of rearranging the equation. The unknown is the denominator in the equation, so the known side should be divided by the trigonometric ratio.

(b) **(i)** $\cos\theta = \frac{A}{H}$

$\cos 50° = \frac{k}{8}$

$\cos 50° \times 8 = k$

$k = 5.14$ m

(ii) $\sin\theta = \frac{O}{H}$

$\sin 55° = \frac{13}{x}$

$x = \frac{13}{\sin 55°}$

$x = 15.87$ km

14 To find a side length with Pythagoras' Theorem, the side lengths of two sides must be known. Trigonometry can calculate side lengths with one side length and one angle known.

Half-time 6 (p. 273)

1 **(a)** A, H **(b)** O, H

2 **(a)** 7.26 **(b)** 16.68 **(c)** 9.14

3 **(a)** 0.59 **(b)** 0.81 **(c)** 0.73

4 **(a)** 19.04 **(b)** 81.94 **(c)** 28.59

5 **(a)** 4 m = opposite, 7 m = adjacent, unknown = hypotenuse.

(b) 9 km = hypotenuse, 6.5 km = opposite, unknown = adjacent

(c) 10.3 km = hypotenuse, 6.9 km = adjacent, unknown = opposite

6 $f = 6$

Exercise 6.4 (p. 276)

1 **(a)** 15° **(b)** 39° **(c)** 35° **(d)** 62°

(e) 10° **(f)** 41° **(g)** 11° **(h)** 78°

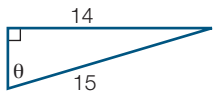
2 54°

3 **(a)** B **(b)** C **(c)** D

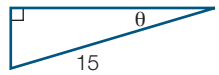
4 15°

5 24°

6 8°

7 (a) 

$\sin\theta = 0.933$

(b) 

$\sin\theta = 0.067$

(c) (i) close to 1 (ii) close to 0

8 (a) A ratio of 1 means that the two sides are equal in length.

(b) Tangent ratio—the other two ratios involve the hypotenuse, which is always longer than the other two sides.

(c) isosceles triangle

(d) 45°

9 There are three possible triangles, with reference angles:
$\sin^{-1}\left(\frac{9}{15}\right) = 36.87°$, $\cos^{-1}\left(\frac{9}{15}\right) = 53.13°$, $\tan^{-1}\left(\frac{9}{15}\right) = 30.96°$,
$\tan^{-1}\left(\frac{15}{9}\right) = 59.04°$.

10 ramp length = 450 cm, $\theta = 8.3°$
ramp length = 500 cm, $\theta = 7.5°$
ramp length = 550 cm, $\theta = 6.8°$

Exercise 6.5 (p. 281)

1 122.06 m

2 16°

3 63°

4 A

5 C

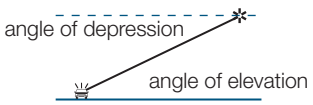
6 9.58 m

7 24°

8 76 m

9 26.17 m

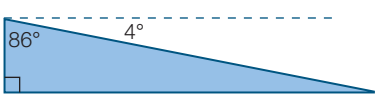
10 Height of tree ÷ distance from car ≈ 0.51

11 

The two horizontal lines are parallel. The angle of elevation and the angle of depression are alternate angles, which we know are equal.

12 (a) Germaine has based her calculations on the reference angle of 4° between the rope and the boat; however, 4° is the angle of depression. The correct answer is 14.96 m.

(b) The angle of depression must go down from the horizontal, which means in this case it is not inside the triangle. The reference angle inside the triangle (to use the sine ratio) is 86°.



Exercise 6.6 (p. 284)

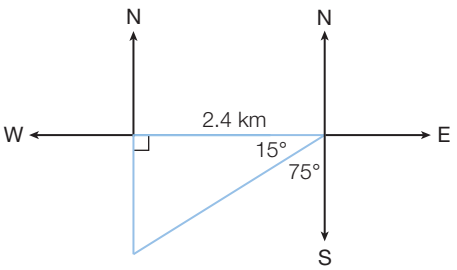
1 34.64 km

2 (a) 90 km (b) N35°E or 35°T

3 D 4 B

5 (a) 156.6 km (b) N31°W or 329°T

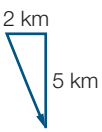
6 (a) 33.44 km (b) 30.11 km

7 (a) 

(b) 0.64 km (c) 2.48 km

8 (a) N54°W or 306°T (b) S54°E or 126°T

9 Sample answer

(a) 

(b) 5.39 km or 11.2°T

Challenge 6 (p. 286)

1 (a) $(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2$
$= m^4 + 2m^2n^2 + n^4$
$= (m^2 + n^2)^2$

(b) (i) $m = 2$, $n = 1$:3, 4, 5

(ii) $m = 3$, $n = 1$:8, 6, 10 → 3, 4, 5
$m = 3$, $n = 2$:5, 12, 13

(iii) $m = 4$, $n = 2$:12, 16, 20 → 3, 4, 5
$m = 4$, $n = 3$:7, 24, 25

(iv) $m = 4$, $n = 1$:15, 8, 17 → 8, 15, 17

(v) $m = 5$, $n = 1$:24, 10, 26 → 5, 12, 13
$m = 5$, $n = 2$:21, 20, 29 → 20, 21, 29
$m = 5$, $n = 3$:16, 30, 34 → 8, 15, 17
$m = 5$, $n = 4$:9, 40, 41

2 $AB = 3\sqrt{2}$, $BC = 3\sqrt{5}$,
Perimeter $= 3\sqrt{2} + 3\sqrt{5} + 6 + 9 = 3(5 + 2\sqrt{5})$ cm
Answer: D

3 (a) Isosceles (b) $PS^2 = 100 - 36$; $PS = 8$ cm (c) A

4 D

5 C

6 (a) $EC = DE = x$

(b) $\frac{CF}{CB} = \cos 60°$, $CF = 2x \times 0.5 = x$

(c) $BF^2 = CB^2 - CF^2 = 4x^2 - x^2 = 3x^2$. $BF = \sqrt{3}x$

(d) $\tan\angle EBF = \frac{EF}{BF} = \frac{2x}{\sqrt{3}x} = \frac{2}{\sqrt{3}}$. $\angle EBF = 49°6'$. (49.1°)

(e) $\angle CBF = 30°$. $\angle EBC = 49°6' - 30° = 19°6'$

7 (a) $\sin\theta = \frac{30}{132}$, $\theta = 13.13° \approx 13°$

(b) $\sin 27 = \frac{44}{BD} = BD = \frac{44}{\sin 27} = 96.9 \approx 97$ m.

(c) Total length of track = 132 + 97 = 229 m.

Average speed = $\frac{229}{90} = 2.54 \approx 2.5$ m/s.

Chapter review 6 (p. 287)

1 (a) 25m = opposite, x = hypotenuse, sin

(b) 8.3 cm = hypotenuse, x = adjacent, cos

(c) 3.2 km = adjacent, x = opposite, tan

2 (a) 1.3333 **(b)** 0.225 **(c)** 5.4545

3 (a) $x = 4$ cm **(b)** $t = 3.7$ cm **(c)** $p = 7.9$ m

4 (a) $\frac{3}{5}$ **(b)** $\frac{15}{17}$ **(c)** $\frac{20}{29}$

5 (a) 2.19 cm **(b)** 15.00 cm **(c)** 55° **(d)** 111.87 m **(e)** 1766.89 m **(f)** 68°

6 A **7** 7.57 km south and 10.81 km east

8 33.58 m **9** 48°

10 26.11 m

11 76.9 km

12 89.6 m

13 50.75 km

14 151°T or S29°E

15 46.1°

16 (a) 2.23 cm **(b)** 15.61 cm^2

17 63.4°

NAPLAN practice 6

1 B **2** B **3** A **4** D

5 D **6** A **7** B **8** C

Chapter 7

Recall 7 (p. 291)

1 mean = 20.9, median = 20.5, range = 18

2 (a) $\frac{1}{6}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{3}$

3 mean 2.36; median 2

4 spade, club, diamond, heart

Exercise 7.1 (p. 294)

1 (a) discrete **(b)** continuous

(c) discrete **(d)** continuous

2 (a) 5.89% **(b)** 69.33% **(c)** 88.41%

(d) The full-time figures are not far apart, but these figures are quite different. This would seem to indicate that women work fewer hours than men when part-time and that more women work part-time.

3 (a) qualitative **(b)** quantitative

(c) quantitative **(d)** qualitative

4 (i) **(b)** discrete **(c)** discrete

(ii) **(a)** categorical **(d)** ordinal

5 The safest state, with the lowest registered deaths per 10 000 registered motorcyclists, is Victoria. Although the most motorcyclists are killed in Queensland, the rate is much worse in the Northern Territory. The deaths per 10 000 motorcyclists gives a more accurate picture than the raw figures.

6 Students will produce their own answers.

7 (a) B **(b)** A **(c)** D

8 (a) 200 seedlings **(b)** 250 seedlings 500 seedlings

9 (a) 5

(b) No, every sample will have its own unique composition.

(c) Students will produce their own answers and comment on the similarities and differences in their samples.

10 Sample answers:
Question 1: Every day; once a week; at least once a month etc.
Question 2: Range of products: prices; quality of food; service etc.
Question 3: Students to provide own answer based on their own school canteen.

11 (a) The mean will be affected by the years when there is quite a lot of rainfall, increasing the mean value. The median is consistently lower, as Adelaide is normally relatively dry apart from in these wet years.

(b) It is more likely to have less rain than the mean value.

(c) You would use the median figures because it is more likely that these figures will make it seem that Adelaide is drier than it may actually be.

(d) February is the driest month on whichever measure you use. So, the exceptionally wet Februarys will have more effect on the mean value.

12 The results are discrete because there is a fixed, countable number of values possible.

13 (a) Assign each student a number and then use random numbers to select the sample.

(b) Student behaviour may be different if they see they are being observed.

(c) Students to provide their own answer, but something like the following may be appropriate. A combination of the two strategies given may be useful. It would also be useful to speak with the canteen workers to get more detail regarding their concerns.

14 (a) Expect 4 black, 5 blue and 2 white marbles. Note that due to rounding this adds to 11.

(b) You should not expect every sample to be the same. There are variations in every experimental situation.

15 Students will provide their own answers.

16 **(a)** Students will come up with their own list of vehicle types.

(b) It is unlikely every student would arrive at exactly the same list.

(c) If the intent was to collate class results, then it would be a problem if the lists were not the same.

17 **(a)** 80.29%

(b) It is higher than the equivalent figure for men.

(c) You might like to see the number of hours worked part-time and/or the pay per hour for part-time work.

(d) It shows that women seem to be paid more for part-time work than men. It helps, but is probably not all you would like to see.

(e) The means indicate that men earn more than women in part-time work, which is the opposite of what the medians tell us. If you were trying to promote the notion that women are paid less than men, then you would probably use the mean values.

Exercise 7.2 (p. 299)

1 **(a)** The shaded numbers are approximations, the others are certain.

Position	Sports identity
1	Greg Norman
2	Andrew Bogut
3	Harry Kewell
4	Mark Webber
5	Chad Reed
8	Geoff Ogilvy
9	Marcos Ambrose
10	Casey Stoner
13	Ricky Ponting
17	Brett Lee
19	Lleyton Hewitt
21	Michael Clarke
41	Michael Hussey
50	Andrew Symonds

It is difficult to be precise about some of these positions because there are large gaps between the positions mentioned in the article.

(b) approximately 9.5% drop

(c) approximately 56.4% drop

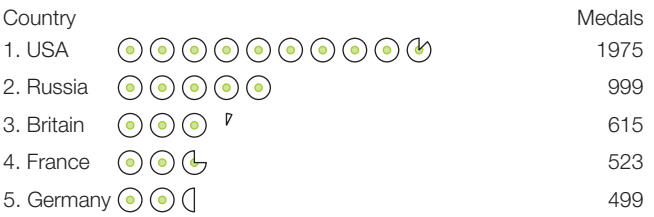
(d) $\frac{65}{5} \neq 13$ so it is not just a straight division. The contract must have some higher up–front payments.

(e) $1.53m is better than 41st but not as good as 21st ($2.54m). Somewhere around the low 30s would seem appropriate.

2 **(a)** What a mess! A pie graph assigns percentages of a whole and these percentages clearly do not add up to 100. The explanation given is that it was possible to offer support to more than one candidate. That is fine, but a pie chart should never have been used to represent it. You would not be able to draw a correct graph from this information as you would need to know which one candidate each person supported.

(b) **(i)** There is not a consistent value for each medal. Germany has two medals for 499, so each medal seems to be worth 250 medals. At this rate the USA should have $\frac{1975}{250} = 7.9$, say 8 medals.

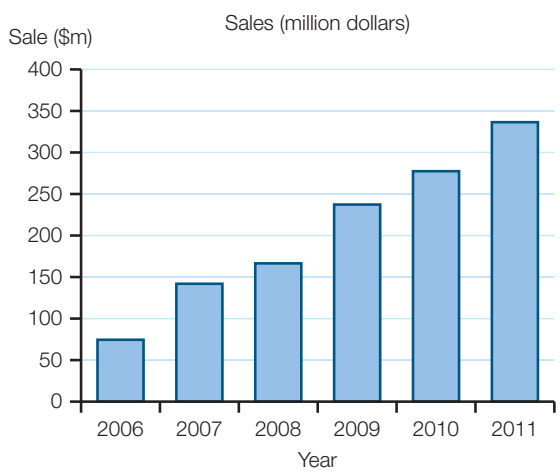
(ii) For the following graph we will let each medal represent 200 medals.



(c) **(i)** The vertical scale does not start at 0.

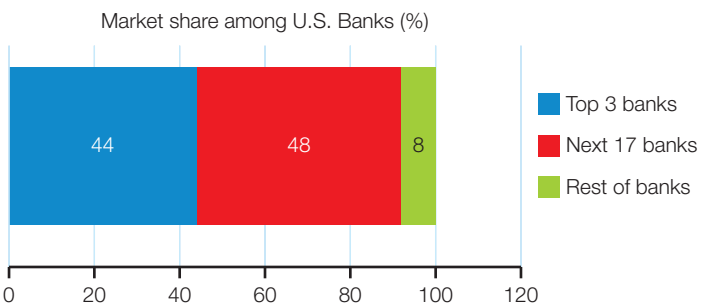
(ii) This makes the increase seem greater than it really is.

(iii)



(d) **(i)** approx 19% **(ii)** approx 82%

(iii)



(iv) The width of the chart is constant so the length is all that determines the portion size.

3 (a) $390 (b) $341

(c) 14.4%, so the article has not exaggerated the rise in its claim of 14%

(d) $345.43

(e) old: $1020.41; new: $1070.41

(f) 11.1% p.a. increase

(g) Crash repairs are a big part of the cost of insurance. If car crashes are reduced the overall cost of providing insurance cover should also reduce.

4 (a) Rounded to one decimal place:

Guildford 17.3%; Mount Richon 16.9%; Victoria Park 15.7%; Darlington 14.6%; Orelia 13.9%; Maylands 12.5%; South Fremantle 12.1%; Stoneville 11.7%; Fremantle 10.7%; Joondanna 10.2%

(b) Rounded to one decimal place:

North Beach -35.4%; Ascot -32.1%; Mount Lawley -28.3%; Mosman Park -24.5%; Forrestdale -24.0%; Applecross -22.8%; Perth -21.1%; Marmion -21.0%; Kallaroo -17.4%; Swanbourne -17.3%

(c) There is wide variability in prices both in absolute terms and in percentage change terms. It might even be described as volatile. The big losses seem to be associated with some of the more expensive suburbs.

(d) The number of houses in each of these suburbs. This would enable a percentage turnover to be calculated.

(e) House buyers concentrated on lower priced houses so the prices of houses in the more expensive suburbs fell as there was little competition for these properties.

(f) Mode is almost useless as there is such a range of prices possible. It might be that the only two houses sold for the same price were the least expensive houses of all; this is hardly representative. Mean is affected by very low and very high prices.

5 (a) 360

(b) Students to provide their own answer, but it is likely they will say it is not a very big sample.

(c) self-selecting

(d) Yes, the questions are quite different. It is quite possible for a person to hold a view about what should happen in general but have a different view about the same issue as it directly affects them.

(e) The telcos may be talking themselves up. It would be interesting to see some data to support this view. However, it is quite possible, as the survey methodology seems to be quite flawed.

(f) These findings are probably not very reliable. The sample size is relatively small and the selection process would not lead to robust results.

6 (a) It is between two outlets that have both recently sold tickets that won multimillion dollar prizes, so it could be next.

(b) No, this is nonsense.

(c) At the very least you would want to know how many tickets were sold. What percentage of tickets sold are winners? But even this is ridiculous as each ticket, no matter where sold, has the same chance of winning.

(d) If the numbers have been drawn often then they are lucky numbers that will continue to be drawn. If they haven't been drawn for a long time then it is their turn to be drawn. Both arguments are bereft of statistical merit.

7 (a) Mr Casey is the preferred Prime Minister (48%) compared with Mr Smith (46%). Only female voters preferred Mr Smith (47%) compared with Mr Casey (41%). The heading is misleading.

(b) Sample size 1030, $n = 1030^2 = 1\,060\,900$. This is a reasonable sample size, given that the voting population may be less than 10 million.

(c) Telephone survey is the most convenient sampling method.

(d) The figures for female opinions (as in part (a)) give some justification for the heading 'Voters back Smith'—but this is not true for male voters.

(e) When you add the votes for either preferred Prime Minister, the total male votes equal 99% and the total female votes equal 88%. The missing 1% and 12% of votes respectively are due to the 'neither' and 'don't know' categories chosen by some persons surveyed.

(f) These categories attempt to identify the grounds for the voters' opinions. We do not know whether these categories are important to the voters.

(g) This poll does indicate that the preferred Prime Minister is Mr Casey (in the general population), but only slightly. The survey also covers the personal qualities of the candidates.

8 (a) The other States and the ACT probably don't have a high enough Indigenous population to make the statistics accurate.

(b) Males 78.7 years and females 82.6 years.

(c) Students to provide their own answer, but would probably include thoughts such as lack of medical facilities in many traditional communities and the impact of certain illnesses and diseases, such as diabetes, on the Indigenous communities.

(d) It is difficult because the ATSI population count is probably not accurate and deaths are not always reported.

(e) Canada, New Zealand and the United States also have significant Indigenous populations and the countries themselves are quite similar to Australia.

(f) Students to provide their own answer, but some of the following may be included. The percentage increase for Indigenous males is 2.54% compared to 1.56% for non-Indigenous males and for females it is 2.00% for the Indigenous population compared to 1.09% for the non-Indigenous population. So the rate of increase is higher but this is due to the lower starting figure. A fair society would want to see these differences become much smaller.

9 Students will provide their own answers.

10 The most interesting question is where the figure for the probability of finding buried treasure came from. The other probability can probably be calculated although it is open to debate what the expression 'top prize at the pokies' means. Each machine would have a different 'top prize' so this makes things more difficult. However, the thrust of the advertisement about the unlikelihood of winning a huge prize on the pokies is correct.

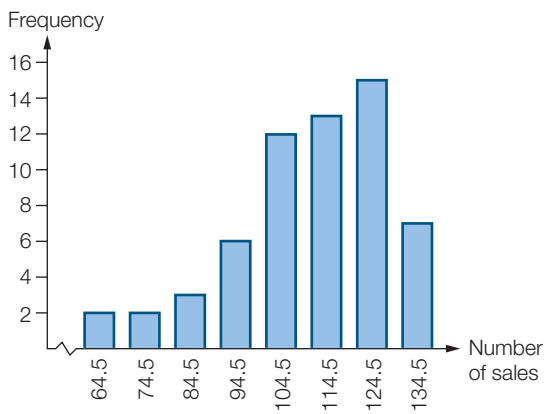
Exercise 7.3 (p. 313)

1 **(a)** negatively skewed **(b)** symmetrical

(c) positively skewed **(d)** bi-modal

2 **(a)** **(i)** 110.83 **(ii)** 110–119

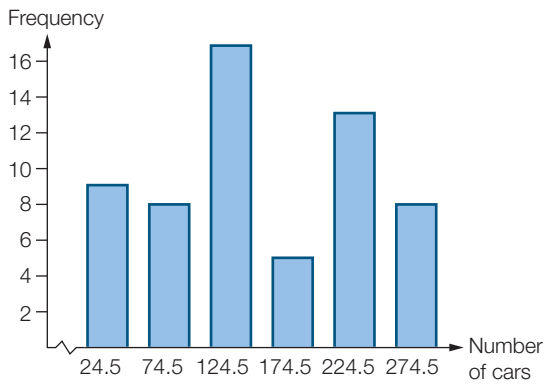
(iii)



(iv) The graph is not symmetrical. It is negatively skewed and there are no outliers.

(b) **(i)** 148.67 **(ii)** 100–149

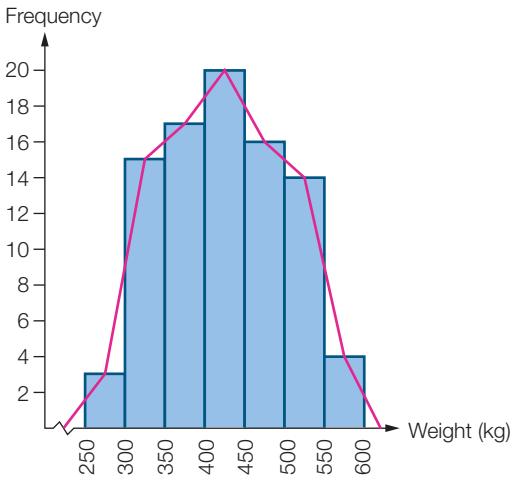
(iii)



(iv) The graph is relatively symmetrical, although the 150–199 bin is a little low for true symmetry.

3 **(a)** **(i)** 427 kg **(ii)** 400 – < 450 kg

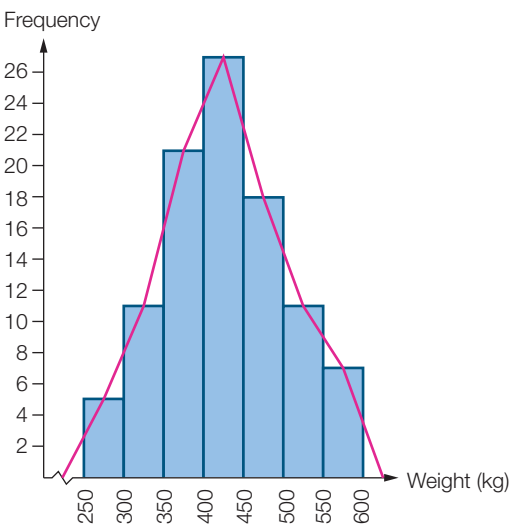
(iii)



(iv) The data look symmetrical.

(b) **(i)** 427 kg **(ii)** 400 – < 450 kg

(iii)



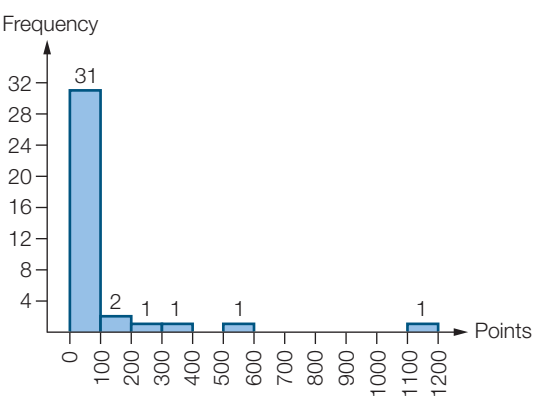
(iv) The data look symmetrical.

4 **(a)** **(i)** 66.47 **(ii)** 65 **(iii)** positively skewed

(b) **(i)** 151.13 **(ii)** 157.5 **(iii)** negatively skewed

(c) **(i)** 242.57 **(ii)** 245 **(iii)** symmetrical

5 **(a)**



The graph is positively skewed and it is not symmetrical.

(b) Certainly 1163 is an outlier, and an argument could be made for 582 as well.

(c) mean is 112.16, median class is 0–99

6 **(a)** B **(b)** B

7 **(a)** The data set is positively skewed.

(b) Students will provide their own answer, but a value about 2 would seem reasonable.

(c) 1.72; students will make their own comment about the reasonableness of their estimate.

(d) Students will make their own comment, but may suggest conducting a survey within their own school.

8 **(a)** 1–10; 11–20; 21–30; 31–40; 41–50

(b) 0–4; 5–9; 10–14; 15–19; 20–24

(c) 0–49; 50–99; 100–149; 150–199; 200–249

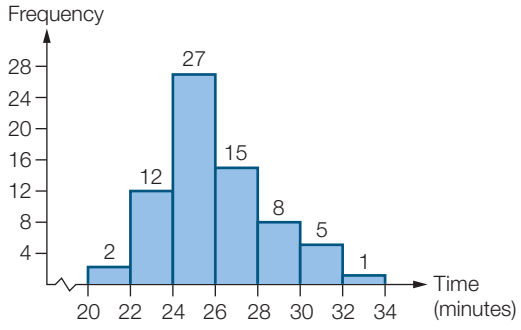
(d) 1–50; 51–100; 101–150; 151–200; 201–250

(e) 10–12; 13–15; 16–18; 19–21; 22–24

(f) 25–29; 30–34; 35–39; 40–44; 45–49

9 **(a)** 25.97 minutes

(b)



The data set is moderately positively skewed.

(c) Their times would increase the mean.

(d) You might just assign them a time, say 34 minutes.

10 **(a)** The data set is slightly positively skewed.

(b) Somewhere around 3 km would seem reasonable.

(c) Not too many students would be seen in the early bars as they would be living almost next to the school. The highest bars are a reflection of the reducing distance from school.

11 **(a)** 71.94 **(b)** 40–79 games

(c) In this case the mean probably does the better job

(d) Mean: 93.25; median: between 40–79 and 80–119 games, so let's say about 80 games.

(e) Both these values are probably better measures of location because they represent the players who have actually played. There is no guarantee any of those on 0 will ever play AFL.

12 **(a)** The two end numbers are both odd or both even.

(b) One of the end numbers is odd and the other is even.

(c) No, because there are no possibilities left after those described in **(a)** and **(b)**.

13 **(a)** 1–5, 6–10, 11–15, etc. and 100–104, 105–109, 110–114, etc.

(b) 0–9, 10–19, 20–29 etc. and 1–10, 11–20, 21–30 etc.

14 **(a)** Students are to provide their own answers.

(b) Students are to provide their own answers.

15 **(a)** Students are to provide their own answers.

(b) Students are to provide their own answers.

(c) Students are to provide their own answers.

Exercise 7.4 (p. 320)

1 **(a)** 37 for Fremantle and 36 for West Coast Eagles

(b) We will have a class interval of 5, with the first being 16_H (165 – 169).

(c)

Fremantle	Stem	West Coast Eagles
	16_H	7 9
4	17_L	
7 6	17_H	5 8 9
4 4 4 4 2 2 2 1 1 0 0	18_L	1 2 3 3 3 3 4
9 9 9 8 8 8 7 6 6 6 6 5	18_H	5 5 6 6 6 6 6 7 7 8 8 9 9 9 9 9 9
4 4 4 4 3 3 3 2 2 2 1 1 1 1 1 0 0	19_L	0 1 1 2 2 4 4
9	19_H	5 5 5 6 8
3	20_L	1 2 3
	20_H	
1	21_L	

The two teams seem to be very close as far as height is concerned. Some more work would need to be done to make a more definitive statement. This may include calculating the mean and median heights for the two teams.

(d) Fremantle: Mean = 188.20 cm, Median = 188.5 cm; West Coast Eagles: Mean = 187.93 cm, Median = 188.5 cm

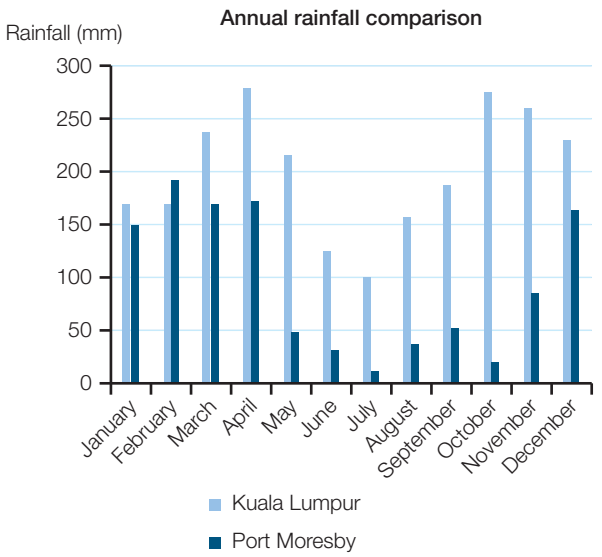
(e) The statistical calculations confirm that the heights are very close for the two teams. The median height is exactly the same and Fremantle's mean height is less than half a centimetre bigger than WCE.

2 **(a)** The figures for Lahore, and to a lesser extent Canberra, are lost on the graph owing to the much higher rainfall in the other two places.

(b) Restrict graphs such as this to comparing things that are closer in value.

(c) The actual figures are as shown here. (Students have estimated these figures so there is no expectation that they will have exactly the same answers.)

	Kuala Lumpur	Port Moresby
January	171	150
February	169	194
March	237	170
April	279	173
May	216	49
June	126	31
July	102	12
August	157	37
September	188	53
October	275	20
November	259	87
December	230	164

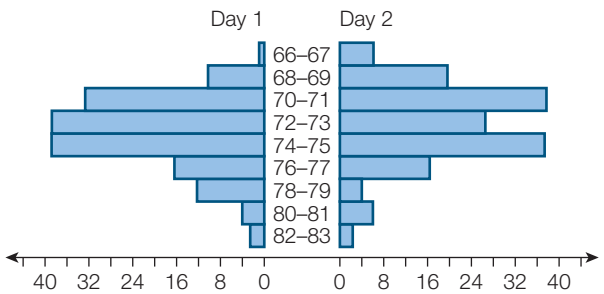


(d) Kuala Lumpur: mean: 200.75; median: 202; range: 177
Port Moresby: mean: 95; median: 70; range: 182

(These figures are again from the estimated values, so students will almost certainly have different answers.)
Although both places have some months with high rainfall, Kuala Lumpur is much more consistently wet, as indicated by its much higher mean and median values. The larger range for Port Moresby is brought about by its very dry season in the middle of the year. Kuala Lumpur is wetter in every month except February and even then they are quite close.

3 (a)

Score range	Frequency Day 1	Frequency Day 2
66–67	1	6
68–69	10	19
70–71	32	37
72–73	38	26
74–75	38	37
76–77	16	16
78–79	12	5
80–81	4	6
82–83	3	2



(b) Grouping this way gave nine columns for each part of the histogram. This fits well into our preferred number of 5 to 10.

(c) Day 2 appears to have had marginally lower scores.

(d) Day 1: Mean = 73.55 and Median = 73
Day 2: Mean = 72.84 and Median = 73
This confirms the answer given in (c).

4 (a) The scores for Team A are more concentrated. The two highest scores were achieved by Team B.

(b) Team A: Mean = 2.5, Median = 2.5;
Team B: Mean = 2.625, Median = 2

(c) The concentration of values makes the median for Team A higher but the two highest scores for Team B help make its mean value slightly higher.

5 (a) From birth to about 49 years of age.

(b) These bars include everybody older than 79. Each of the other bars covers a 5-year range, but these cover up to 20 or 30, depending on how long people live.

(c) It gives a good picture of the differences between the ages but is not as useful comparing males to females because it is difficult to see the values exactly.

6 (a)

Male	Stem	Female
	14_L	4
7	14_H	5
4	15_L	3 3 4
9 7	15_H	5
4	16_L	0 0 0 1 2 2 2 3 4 4
6	16_H	5 5 7 7 7 8 8 9
4 3	17_L	0 1 2 4 4
7 7 6 5 5	17_H	8
3 1 0	18_L	
8 5	18_H	
1 0	19_L	

(b) Year 9 NSW boys are clearly taller than the equivalent girls.

(c) This would normally be a concern, but even with fewer boys the seven tallest students are boys and the two shorest are girls.

(d) Another sample may, but is unlikely to, give the same results. We would, however, expect that all samples would show that the boys are taller than the girls.

(e) The same basic structure of results would probably apply for the whole of Australia. It would seem fair to say that boys would be in the range 145–195 cm and girls 140–185 cm.

7 (a) The following table shows the results.

	Mean	Median	Range
A	95.03	46.35	296.7
B	31.88	24.5	49.8
C	21.33	21.2	7.1
D	161.31	164.15	148.3

(b) A has a wet season (basically summer) and a dry season (winter and spring). This is best reflected in its very high range. It is likely to be in the tropics.
B has low annual rainfall with most rain occuring in the winter.
C is quite a dry place with a consistent rainfall pattern. This is reflected in the closeness of the mean and median values and the very low range.

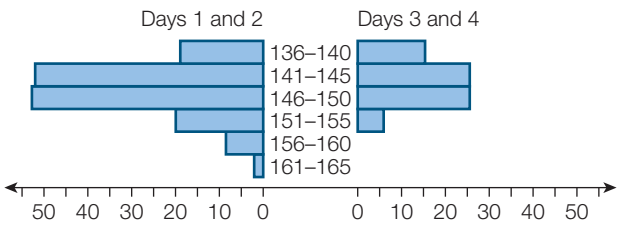
D is the wettest of the four places with most of its rain coming in the winter months. However, its high range value indicated it has some relatively dry months. These occur in the summer.

(c) Tropical places have high rainfall in the summer months, so A must be Townsville. Broken Hill and Port Lincoln are both dry places but, as Port Lincoln is on the coast it will likely have more rain. So B is Port Lincoln and C is Broken Hill. This leaves D to be Weeaproinah, which is the wettest place in Victoria.

8 (a) This grouping would give six groups for the first pair of days, so it fits neatly into our preferred range of 5 to 10.

(b) There are only four groups for the school pair of days because no one scored over 153.

(c)



(d) There are only half as many results on the right-hand side and none at all in the highest two categories. However, it appears that the scores for the second pair of days are lower because there are almost as many scores in the lowest score category as on the first pair of days. As approximately half the players are eliminated after the first two rounds, it is safe to say that the better performing players will proceed to Rounds 3 and 4.

(e) For Days 1 and 2: Mean = 146.4 and Median = 146;
For Days 3 and 4: Mean = 144.6 and Median = 145.
This confirms that scoring was lower on Days 3 and 4.

9 It is important to note the different scale used in the Tonga graph compared with Australia and Cambodia. Australia has a bigger spread of population (range) compared with both Cambodia and Tonga. Cambodia has a higher birth weight than Australia. Overall, Tonga has a smaller population compared with the other two countries.

10 Students will provide their own answers.

11 Students will provide their own answers.

Half-time 7 (p. 324)

1 (a) (i) 54.32 (ii) 53 (iii) positive skew

(b) (i) 149.46 (ii) 149.5 (iii) symmetrical

2 (a) B (b) B (c) A

3 Students will provide their own answer, but it should contain at least some of the following elements:

- Construct a survey instrument that shows the three new logos as well as the current one.
- Decide on important stakeholders to be surveyed—employees, customers, general public etc.
- How many of each will be surveyed?
- Do their opinions count equally?
- What will represent success for your survey?

4

Group 1	Stem	Group 2
9	14_H	
4 3	15_L	1 3 4
6 5 5	15_H	5 5 5 8
4 0	16_L	0 0 1
7 7 6	16_H	6 6 6 7
3 0	17_L	1 1 2 2 3
7 7 5	17_H	9
3 1 0	18_L	
9	18_H	

It is likely that Group 1 is the boys.

5 (a) Gold Coast Suns appear to be slightly taller. They have the tallest player and more players in the second tallest band. The Brisbane Lions have a lot of players in the 180–<185 band, which will bring down their average height.

(b) The mean and median would be the least you would require. Having the actual heights would be most useful.

(c) This extra information confirms that the Gold Coast Suns are slightly taller than the Brisbane Lions. In fact the statistics are very close.

Exercise 7.5 (p. 327)

1 $\frac{13}{27}$

2 (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$ (f) $\frac{1}{4}$

3 (a) experiment 4; 0.17 (b) experiment 5; 0.06

(c) experiment 1; 0.17 (d) experiment 2; 0.10

(e) peach; experiment 2; 0.05 (f) pear; experiment 4; 0.25

4 (a) 0.16 (b) 0.3 (c) 0.81

5 Elliot: $\frac{3}{10}$; Kris: $\frac{19}{75}$; Austin: $\frac{67}{150}$

6 (a) B (b) D (c) B

7 (a) C (b) B

(c) Die 1, Die 4, Die 2, Die 3, Die 5

(d) Die 4, Die 5, Die 3, Die 2, Die 1

8 C

9 (a) B (b) B (c) B

10 (a) $\frac{1}{7}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) 1 (e) 0 (f) $\frac{1}{7}$

(g) 1, as expected, as these three outcomes cover all possibilities.

11 (a) $\frac{1}{12}$ (b) $\frac{5}{12}$ (c) $\frac{2}{3}$

12 $\frac{5}{9}$

13 (a) 50

(b) (i) $\frac{1}{5}$ (ii) $\frac{6}{25}$ (iii) $\frac{1}{50}$

(iv) $\frac{23}{25}$ (v) $\frac{1}{2}$ (vi) $\frac{4}{5}$

(c) (i) 0.20 (ii) 0.24 (iii) 0.02

(iv) 0.92 (v) 0.50 (vi) 0.80

(d) (i) 20% (ii) 24% (iii) 2%

(iv) 92% (v) 50% (vi) 80%

14 (a) (i) 72.1 years (ii) 72.8 years (iii) 73.8 years
(iv) 76.7 years (v) 80 years (vi) 84.6 years

(b) As you get older your expected age at death increases.

(c) $\frac{1}{4}$ (half of a half)

(d) Insurance companies; it helps them set the premiums (prices) for policies.

15 (a) $\frac{1}{365}$ (b) $\frac{6}{73}$ (c) $\frac{12}{365}$ (d) $\frac{24}{73}$ (e) $\frac{337}{365}$

16 (a) $\frac{1}{6}$ (b) 0 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$ (f) 1

17 Students' own answers.

18 (a) $\frac{2}{12} = \frac{1}{6}$ (b) $\frac{3}{12} = \frac{1}{4}$ (c) $\frac{2}{12} = \frac{1}{6}$

(d) $\frac{3}{12} = \frac{1}{4}$ (e) $\frac{1}{12}$ (f) $\frac{1}{12}$ (g) 0

(h) Add to 1, and this is because all possibilities shown so far have been covered.

(i) The seniors are likely to be a different standard and these figures may not apply.

19 The coach is wrong. $P(\text{success}) = \frac{2}{3}$ for every throw.

20 (a) Sylvester may believe that balls that have appeared frequently will continue to appear frequently. Alternatively, he may think that the balls that haven't been drawn often will be 'due' their turn.

(b) No it is not useful. The balls do not have memories and will come out randomly next time, regardless of what has happened in the past.

21 $P(1) = P(2) = P(6) = \frac{1}{12}$, $P(3) = \frac{1}{3}$, $P(4) = \frac{1}{4}$, $P(5) = \frac{1}{6}$

22 Examples of solutions:

(a) A = getting a multiple of 4

(b) B = getting an odd number

(c) C = not getting a composite number

23 (a) No, the probability of rolling any double number, whether it is a double 6, double 3 or double 1, is the same, $\frac{1}{36}$.

(b) Students' own answers.

Exercise 7.6 (p. 336)

1 (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$

(e) $\frac{1}{2}$ (f) $\frac{5}{8}$ (g) $\frac{1}{2}$ (h) $\frac{5}{8}$

2 (a) 21 (b) 42 (c) $\frac{9}{113}$ (d) $\frac{59}{113}$

3 (a) Derek, Ahn and Ngaio

(b) Ahn, Ngaio, Samantha and Zarko

(c) Derek

(d) Maha, Erica, Phillipa, Derek, Ahn and Ngaio

4 (a) 27 (b) 9 (c) 7 (d) 19 (e) 4 (f) 4

5 (a) $\frac{1}{15}$ (b) $\frac{8}{15}$ (c) $\frac{3}{5}$ (d) $\frac{7}{30}$ (e) $\frac{1}{3}$ (f) $\frac{7}{30}$

6 B

7 (a) (i) No (ii) Yes (iii) No
(iv) No (v) No (vi) No

(b) $P(A) = \frac{1}{4}$ $P(B) = \frac{3}{10}$ $P(C) = \frac{1}{4}$ $P(D) = \frac{3}{10}$

8 (a) $\frac{7}{53}$ (b) $\frac{31}{53}$ (c) $\frac{28}{53}$ (d) $\frac{20}{53}$ (e) $\frac{2}{53}$ (f) $\frac{51}{53}$

(g) $\frac{38}{53}$ (h) $\frac{5}{53}$

9 (a) $\frac{2}{13}$ (b) $\frac{5}{26}$ (c) $\frac{4}{13}$

(d) You can write out the sample space or use $P(\text{either}) = P(\text{left}) + P(\text{right}) - P(\text{both})$

(e) $\frac{1}{26}$ (f) $\frac{11}{13}$ (g) $\frac{21}{26}$ (h) $\frac{9}{13}$

10 (a) 27 (b) 52 (c) 104

(d) $\frac{83}{162}$ (e) $\frac{6}{162} = \frac{3}{81}$ (f) $\frac{104}{162} = \frac{52}{81}$

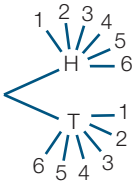
11 Students' own answers.

12 Students will provide their own answers.

Exercise 7.7 (p. 342)

1 (a) $\frac{1}{5}$ (b) $\frac{1}{25}$ (c) $\frac{3}{5}$ (d) $\frac{21}{25}$

2 (a)



(b) $\frac{1}{6}$

3 (a)

A1A2	A1A3	A1A4	A1B1	A1B2
A2A1	A2A3	A2A4	A2B1	A2B2
A3A1	A3A2	A3A4	A3B1	A3B2
A4A1	A4A2	A4A3	A4B1	A4B2
B1A1	B1A2	B1A3	B1A4	B1B2
B2A1	B2A2	B2A3	B2A4	B2B1

(b) (i) $\frac{2}{5}$ (ii) $\frac{1}{15}$ (iii) $\frac{8}{15}$

4 (a) (i) $\frac{1}{4}$ (ii) $\frac{1}{4}$

(b) (i) $\frac{1}{4}$ (ii) $\frac{4}{17}$

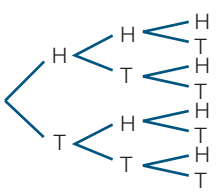
5 (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{5}{12}$

6 (a) Prawn, Lamb; Prawn, Fish; Prawn, Beef; Prawn, Pork; Prawn, Poultry; Oysters, Lamb; Oysters, Fish; Oysters, Beef; Oysters, Pork; Oysters, Poultry;
Pâté, Lamb; Pâté, Fish; Pâté, Beef; Pâté, Pork, Pâté, Poultry; Satay, Lamb; Satay, Fish; Satay, Beef; Satay, Pork; Satay, Poultry

(b) (i) $\frac{1}{20}$ (ii) $\frac{2}{5}$ (c) $\frac{3}{5}$

7 (a) A (b) A (c) D

8 (a)



(b) $\frac{3}{8}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) $\frac{7}{8}$

9 (a) FG, FM, FB, FD, AG, AM, AB, AD, IG, IM, IB, ID

(b) (i) $\frac{1}{12}$ (ii) $\frac{5}{12}$ (iii) $\frac{2}{3}$ (iv) $\frac{1}{2}$ (v) $\frac{1}{2}$

(c) $\frac{1}{3}$

10 (a) 12

(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$

(c) 3

11 (a)

Blue die \ Red die	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4,4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

(b) (i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{5}{36}$ (iv) $\frac{5}{18}$

(c) (i) $\frac{11}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{7}{18}$ (iv) $\frac{5}{18}$

(v) $\frac{7}{18}$ (vi) $\frac{5}{12}$

(d) Yes, but C and D are the only mutually exclusive pair.

12 (a) (i) $\frac{1}{15}$ (ii) $\frac{2}{15}$ (iii) $\frac{4}{5}$ (iv) $\frac{2}{5}$

(b) (i) $\frac{1}{9}$ (ii) $\frac{1}{9}$ (iii) $\frac{2}{3}$ (iv) $\frac{4}{9}$

13 (a) H, T

(b) 2

(c) HH, HT, TH, TT

(d) 4

(e) HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(f) 8

(g) Yes. To find the number of events for a combination you multiply together the number possible for each event within the combination.

14 (a) $3^8 = 6561$ (b) $\frac{1}{6561}$

(c) There is only one way it can happen: GGGGGGGG.

(d) 'What is the probability that all the faces will be blue?' and 'What is the probability that all the faces will be red?'

(e) The complementary question to the statement in (b) would actually be: 'What is the probability that not all of the faces are green?'

15 Students' own answers.

16 Students will provide their own answers.

Challenge 7 (p. 346)

1 (a) correct (b) correct (c) correct (d) incorrect

2 (a) 2, 4, 6, 6, 7 (other answers may be possible)

(b) 11

3 The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

3, 6, 12, 24, 48 all have factors of 3; the other five factors do not have a factor of 3. Therefore, the probability of not having factor of 3 is $\frac{1}{2}$.

4

+	2	3	5	7	11	13
2	4	5	7	9	13	15
3	5	6	8	10	14	16
5	7	8	10	12	16	18
7	9	10	12	14	18	20
11	13	14	16	18	22	24
13	15	16	18	20	24	26

(a) $\frac{13}{18}$

(b) $\frac{5}{6}$, except when the numbers are the same, one number is always larger.

5 9 black, 6 white, 3 green; or 12 black, 4 white, 2 green; or 15 black, 2 white, 1 green

6 (a) $\frac{3}{20} \times \frac{3}{20} = \frac{9}{400} = 0.022\,5$

(b) $\frac{17}{20} \times \frac{2}{5} = \frac{17}{50} = 0.34$

7 (a) At first junction he has three choices, one of which leads to A. At the second junction he has two choices, one of which leads to A. The probability that he ends up at $A = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

(b) Probability that he ends up at $B = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{11}{18}$.

(c) Probability that he ends up at $C = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

(d) Probability that he ends up at $D = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Chapter review 7 (p. 347)

1 (a) continuous (b) discrete

2 For every group except sponsors the existing jumper has the greatest level of support. If the sponsors are more important than the other groups, then a decision to go with new design #1 may be justified, but the sponsors are not really overwhelming in their support for this. In all likelihood the current design should be retained.

3 (a) While there may be good reason to think that the introduction of toll roads would be unpopular, the survey does not really provide any evidence to suggest that this one issue would be enough for voters to vote against the Government. In addition to this, the percentage results are quite close making it difficult to predict the actual outcome.

(b) Government + Opposition voters equal 80% (33 + 47%) for Smithfield. 20% of people surveyed from this electorate chose to support neither the Government nor the Opposition. This is important for the overall judgment of the poll because, if this 20% of voters then decided to vote for either of the two major parties, the result could change dramatically.

(c) Strathfield is the most likely candidate for this.

(d) A telephone survey is certainly convenient. However, whatever method is used it would be important to establish in which State electorate the respondents were registered to vote.

4 (a) (i) 75.13 (ii) 78 (iii) negative skew

(b) (i) 251.83 (ii) 251 (iii) positive skew

5 (a) The sales of the Falcon have fallen considerably. In 2004 there were approximately 64 000 sold, and by 2010 this had fallen to approximately 30 000. So, the number sold had more than halved. In this same time the sales of the Ford Territory have fluctuated but have usually been in the 10 000–15 000 range.

(b) The sales of the Falcon fell initially before seeing a resurgence in 2003, but have then fallen steadily, although up to the end of 2010 the number seeemed to have become relatively stable. The estimate for 2011 sees another fall. In the same time the sales of the Falcon Ute rose steadily to a peak in 2004 and then fell at about the same slight rate as they rose.

(c) Falcon: 65 000; Falcon Ute: 20 000; Territory: 13 000

(d) 31% (will depend on estimates)

(e) 20% (will depend on estimates)

(f) Falcon: 30 000; Falcon Ute: 10 000; Territory: 11 000; 33%; 37%

(g) The percentage comparisions for both the Falcon Ute and the Territory have increased. The Territory has almost doubled its comparative value.

(h) The total number has fallen enormously. The peak year was 2004 with approximately 98 000 vehicles and the worst year was either 2009 or 2010—approximately 51 000 vehicles each. The predictions for 2011 are worse. The total number of vehicles sold in these categories has roughly halved.

6 $\frac{10}{13}$

7 (a) $\frac{3}{20}$ (b) $\frac{1}{4}$

8 (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$ (e) $\frac{7}{8}$

9 (a) 36

(b) (i) $\frac{1}{9}$ (ii) $\frac{11}{12}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{4}$

10 (a) Coast Wattle: 562; Bidgee Widgee: 2250; Karkalla: 1312; White Correa: 750; Running Postman: 1125; Moonah: 375

(b) It is probably not reasonable to apply this across the entire reserve because different areas could support different vegetation types. To improve reliability you should take a few more samples from widely spaced areas of the reserve.

11 (a)

Male	Stem	Female
	12	9
3	13	
5	14	9
2	15	0 1 3 3 4 5 6 7 8 9
9 6 5 3 3	16	0 3 4 4 5 5 6 6 8 9
8 8 8 6 5 4 3 1	17	0 0 0 1 1 2 3
2 0	18	0
8 1	19	

(b) The arm span for boys is longer than for girls.

(c) Not in this case, because the longest three arm spans are all boys.

(d) No, this does not mean there are more girls than boys; it is just the way this sample was constructed.

(e) You would expect the values to be lower for Year 7 students and higher for Year 11 students. For Year 11 we would probably expect that the boys would have larger arm spans than the girls. Things are not so clear for year 7 students because it will depend on when the students undergo their growth spurt.

12 The spread of the population of New Zealand shows a bump around the ages 35–54. This is where the so-called 'baby boomers' can be found in the graph. The rest of the graph shows that the population is relatively stable until about the age of 65, when things start to decline fairly rapidly. This is the age when the number of deaths starts to increase. By the time we get to the late 70s or 80s there are only a few people left.

13 (a) (i) $\frac{1}{29}$ (ii) $\frac{14}{29}$ (iii) $\frac{28}{29}$ (iv) $\frac{10}{29}$

(b) (i) 0.03, 3% (ii) 0.48, 48%

(iii) 0.97, 97% (iv) 0.34, 34%

14 (a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

15 (a) Yes, the question is fairly transparent in what is sought for the answer.

(b) What sort of TV shows should youth detainees be allowed to watch? The problem with this question is that it is not just a Yes/No answer. The results would probably not be the same.

(c) The sample size is probably OK, but the method by which it has been obtained is not.

(d) The broader population could only be the readers of the *Herald Sun*.

(e) The sample would need to be more representative of the population, whatever that was determined to be.

16 (a)

		Green die				
		1	2	3	4	5
Blue die	1	1, 1	2, 1	3, 1	4, 1	5, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5

(b) (i) $\frac{2}{25}$ (ii) $\frac{19}{25}$ (iii) $\frac{2}{5}$ (iv) $\frac{21}{25}$

(v) $\frac{19}{25}$ (vi) $\frac{21}{25}$ (vii) $\frac{4}{25}$ (viii) $\frac{4}{5}$

(c) No—$P(5 \text{ on green}) + P(3 \text{ on blue}) \neq P(5 \text{ on green or } 3 \text{ on blue})$.

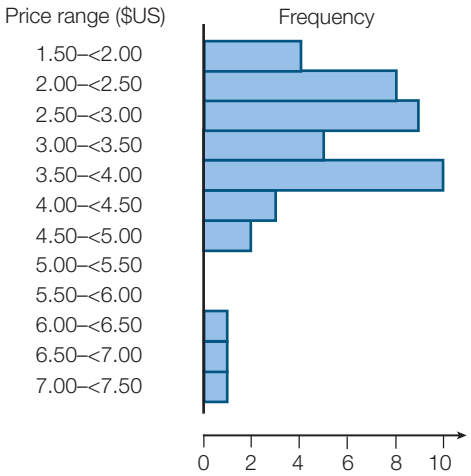
17 (a) Uruguay, \$1.74

(b) Norway, \$7.20

(c) \$5.46

(d) Only 50 cents or \$1 would be sensible intervals, and 50 cents gives 12 intervals, which is just a bit bigger than we normally like. However, there are lots of prices in the \$2–\$4 range, so this helps separate those prices.

(e)



(f) Students to provide their own answers.

(g) Students to provide their own answers.

(h) Students to provide their own answers.

(i) A sample of 10 would be expected to do a better job, but this may not always be the case with individuals results.

NAPLAN practice 7

1 C

2 (a) discrete (b) continuous

(c) continuous (d) discrete

3 C 4 B 5 A

6 B 7 D

Chapter 8

Recall 8 (p. 354)

1 (a) (i) 4 (ii) 5 (iii) 10

(b) (i) 10 (ii) 18 (iii) 0 (iv) 0

2 (a) (i) \$16 (ii) \$75

(b) (i) 4.6 m (ii) 3.2 m

3 (a) $3x^2 + 12x$ (b) $x^2 + 7x + 10$

(c) $x^2 - 4x - 21$ (d) $x^2 - 25$

4 (a) $x(x + 2)$ (b) $(x - 4)(x + 4)$

(c) $(x + 6)^2$

5 (a) $(x + 2)(x + 5)$ (b) $(x - 2)(x - 3)$

(c) $(x - 14)(x + 1)$ (d) $(x - 7)(x - 4)$

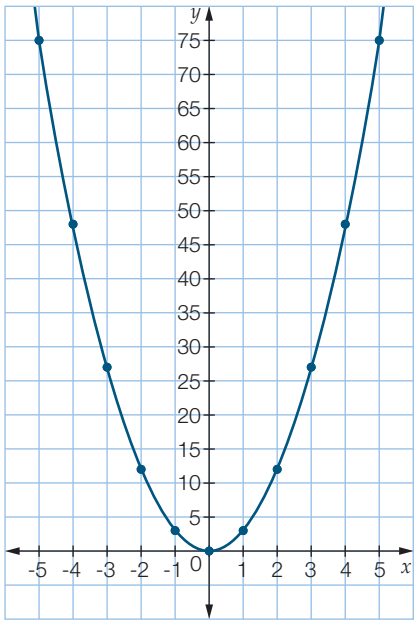
6 (a) $x = \frac{2}{3}$ (b) $x = \frac{13}{5}$ (c) $x = -1$

Exercise 8.1 (p. 359)

1 (a) (i)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	75	48	27	12	3	0	3	12	27	48	75

(b) (i)

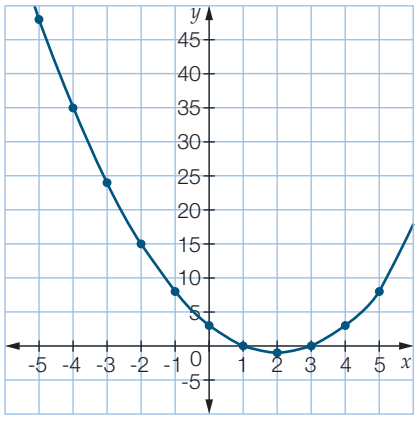


(c) (i) quadratic

1 (a) (ii)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	48	35	24	15	8	3	0	-1	0	3	8

(b) (ii)

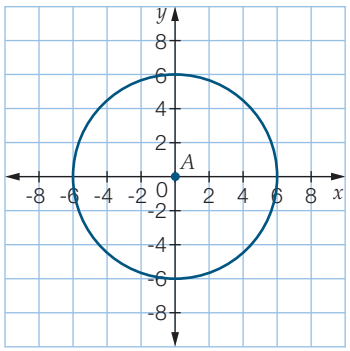


(c) (ii) quadratic

2 (a) (i)

x	-6	0	6
y	0	±6	0

(b) (i)

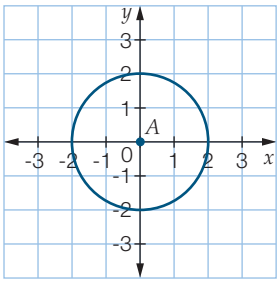


(c) (i) circle

2 (a) (ii)

x	-2	0	2
y	0	±2	0

(b) (ii)

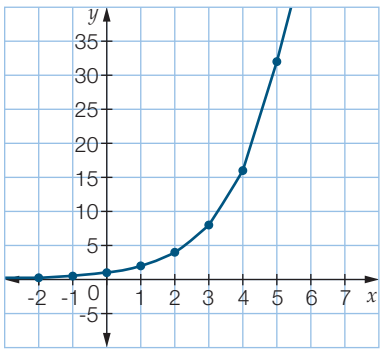


(c) (ii) circle

3 (a) (i)

x	-2	-1	0	1	2	3	4	5
y	0.25	0.5	1	2	4	8	16	32

(b) (i)

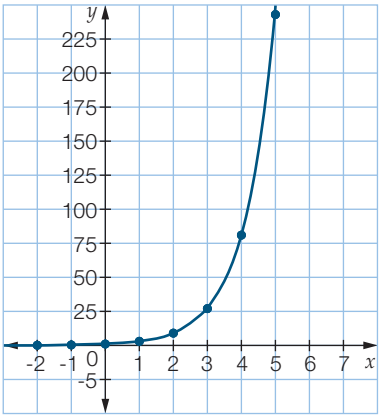


(c) (i) exponential

3 (a) (ii)

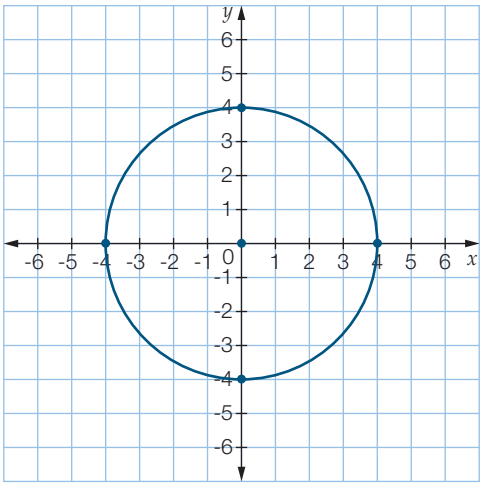
x	-2	-1	0	1	2	3	4	5
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

(b) (i)

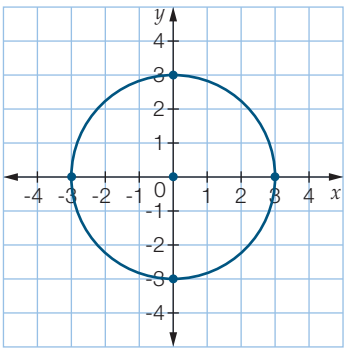


(c) (ii) exponential

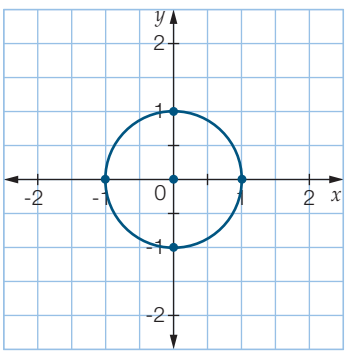
4 (a)



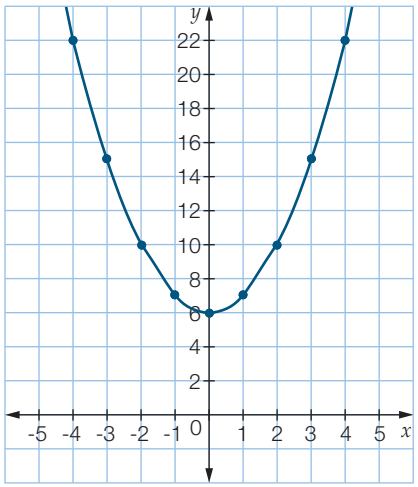
(b)



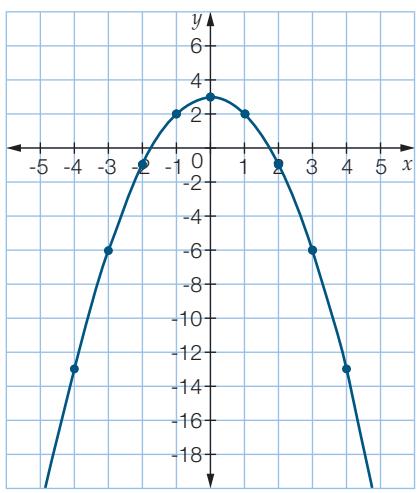
(c)



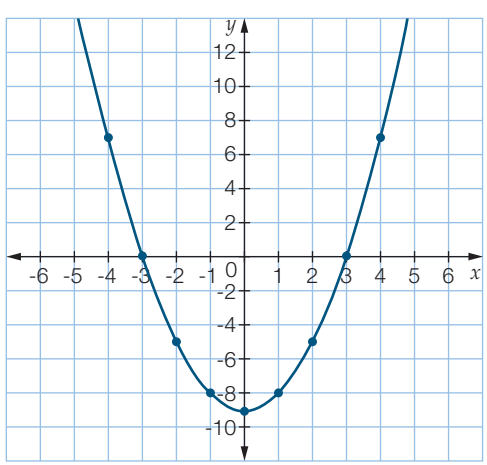
5 (a)



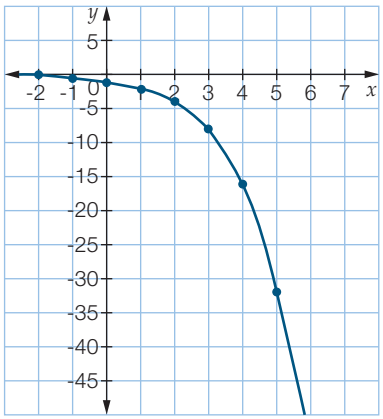
(b)



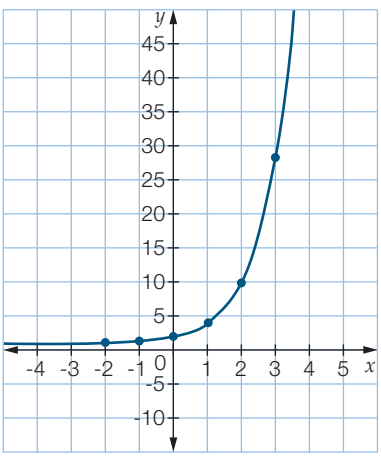
(c)



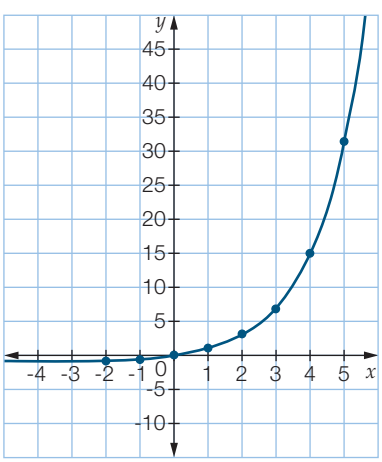
6 (a)



(b)



(c)



7 C

8 (a) ↔C; (b) ↔A; (c) ↔B

9 (a) Both the x and y variables are squared and they have the same coefficient.

(b) The x variable is the index of a number (the base).

(c) The y variable has an index of 1 but the x variable has an index of 2.

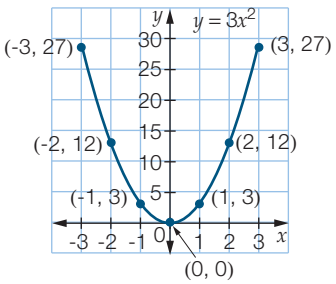
10 Sample answers

(a) $x^2 + y^2 = 144$ (b) $y = 4^x$ (c) $y = x^2 - 9$

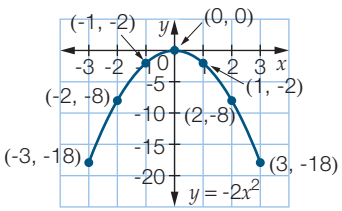
Exercise 8.2 (p. 369)

1 **(a)**, **(b)** and **(e)** are quadratic relationships.

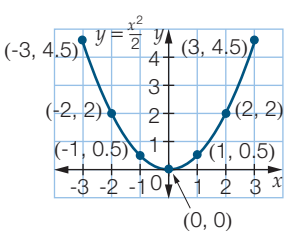
2 **(a)**



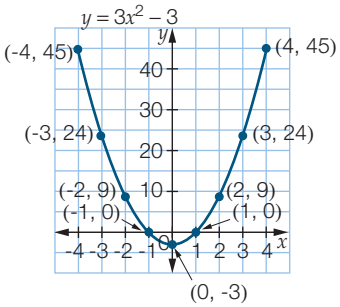
(b)



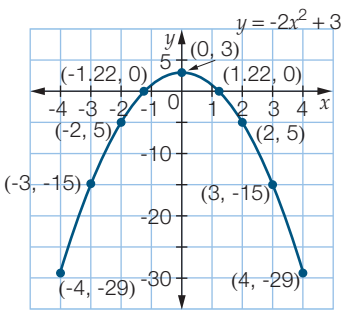
(c)



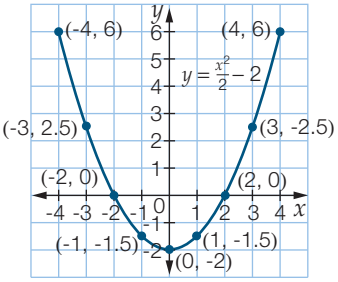
3 **(a)**



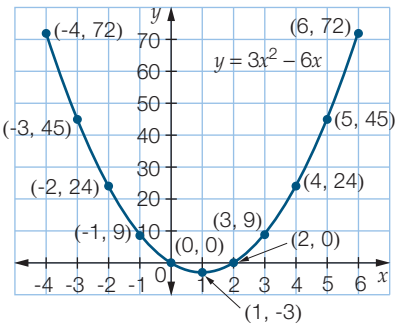
(b)



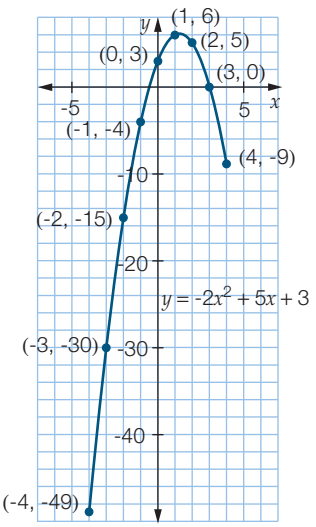
(c)



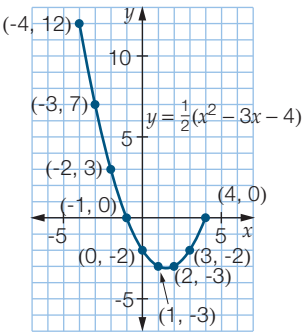
(d)



(e)



(f)



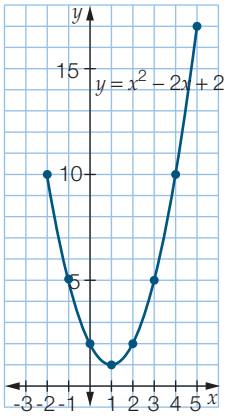
4 **(a)** **(i)** (1, 4) **(ii)** maximum
(iii) $x = 1$ **(iv)** (0, 3)
(v) (-1, 0) and (3, 0)

(b) **(i)** $(-\frac{3}{2}, -1)$ **(ii)** minimum
(iii) $x = -\frac{3}{2}$ **(iv)** (0, 0)
(v) (-3, 0) and (0, 0)

5 **(a)** **(i)**

x	-2	-1	0	1	2	3	4	5
y	10	5	2	1	2	5	10	17

(ii)



(iii) (1, 1) (iv) minimum turning point

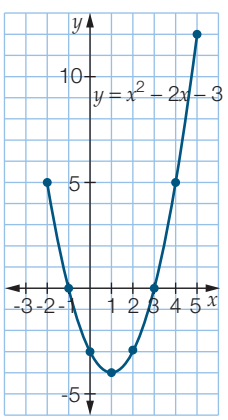
(v) $x = 1$ (vi) (0, 2)

(vii) no x-intercepts (viii) $x = -1$ or $x = 3$

(b) (i)

x	-2	-1	0	1	2	3	4	5
y	5	0	-3	-4	-3	0	5	12

(ii)



(iii) (1,-4) (iv) minimum turning point

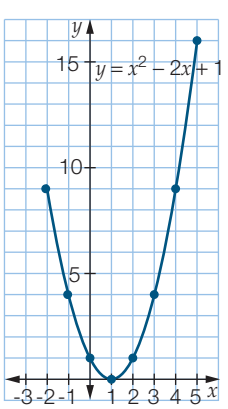
(v) $x = 1$ (vi) (0, -3)

(vii) (-1, 0) and (3, 0) (viii) $x = -2$ or $x = 4$

(c) (i)

x	-2	-1	0	1	2	3	4	5
y	9	4	1	0	1	4	9	16

(ii)



(iii) (1, 0) (iv) minimum turning point

(v) $x = 1$ (vi) (0, 1)

(vii) (1, 0) (viii) $x = -1.2$ or $x = 3.2$

6 (a) B (b) D

7 (a) C (b) C (c) A

8 (a) Both graphs concave up. If you move the graph of $y = 3x^2$ down 3 units, you get the graph of $y = 3x^2 - 3$.

(b) Both graphs concave down. If you move the graph of $y = -2x^2$ up 3 units, you get the graph of $y = -2x^2 + 3$.

(c) Both graphs concave up. If you move the graph of $y = \frac{x^2}{2}$ down 2 units you get the graph of $y = \frac{x^2}{2} - 2$.

9 (a) Both graphs concave up. The graph of $y = 3x^2 - 6x$ has the same dilation as $y = 3x^2$, but is moved to the right and down compared to $y = 3x^2$.

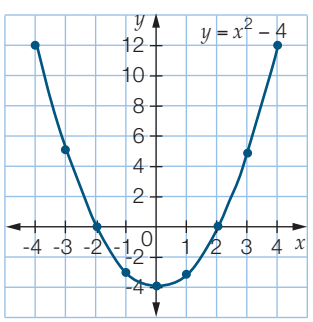
(b) Both graphs concave down. The graph of $y = -2x^2 + 5x + 3$ has the same dilation as $y = -2x^2$, but is moved to the right and upwards compared to $y = -2x^2$.

(c) Both graphs concave up. The graph of $y = \frac{x^2}{2} - \frac{3x}{2} - 2$ has the same dilation as $y = \frac{x^2}{2}$, but is moved to the right and down compared to $y = \frac{x^2}{2}$.

10 (a)

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12
(x, y)	(-4, 12)	(-3, 5)	(2, 0)	(-1, -3)	(0, -4)	(1, -3)	(2, 0)	(3, 5)	(4, 12)

(b)



(c) (0, -4) (d) minimum (e) $x = 0$

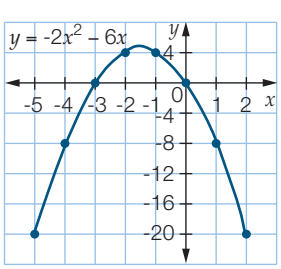
(f) (0, -4) (g) (-2, 0) and (2, 0) (h) 8.3

(i) -1.4; 1.4 (j) $x = -3$ or 3

11 (a)

x	-5	-4	-3	-2	-1	0	1	2
y	-20	-8	0	4	4	0	-8	-20

(b)



(c) $(-1\frac{1}{2}, 4\frac{1}{2})$ maximum turning point

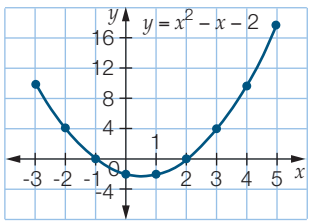
(d) (-3, 0); (0, 0) (e) -2; -1 (f) 4; $x = -2, -1$

(g) $x = -4, 1$ (h) $x = -3, 0$

12 (a)

x	-3	-2	-1	0	1	2	3	4	5
y	10	4	0	-2	-2	0	4	10	18

(b)



(c) $(\frac{1}{2}, -2\frac{1}{4})$; minimum turning point

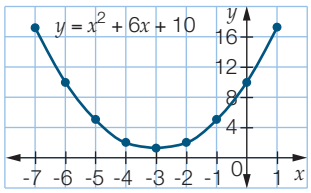
(d) (-1, 0); (2, 0); (0, -2)

(e) **(i)** $x = -1, 2$ **(ii)** $x = -2, 3$ **(iii)** $x \approx -1.6, 2.6$

(iv) $x = 0.5$ **(v)** no real x-values

(vi) $x \approx -1.4, 2.4$

13 (a)

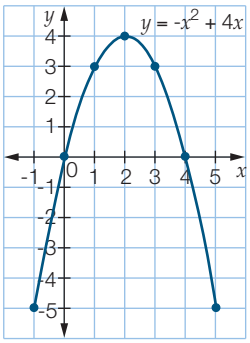


(b) (-3, 1) **(c)** no x-intercepts, (0, 10)

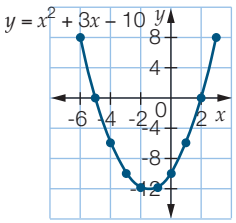
(d) **(i)** $x = -3$ **(ii)** $x = -6, 0$

(iii) no real x-values **(iv)** $x \approx -6.7, 0.7$

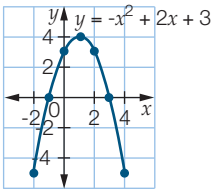
14 (a) **(i)**



(ii)



(iii)



(b) **(i)** $x = 0, 4$ **(ii)** $x = -5, 2$ **(iii)** $x = 1$

(iv) $x = 2$ **(v)** $x = -6, 3$ **(vi)** $x = -1, 3$

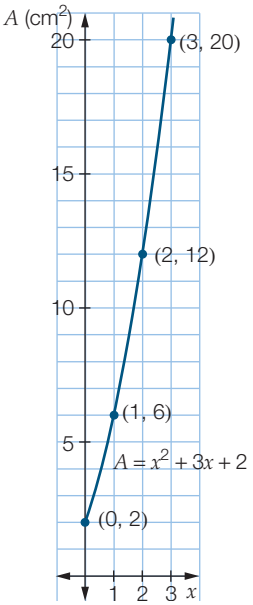
(vii) $x \approx 0.6, 3.4$ **(viii)** $x \approx -5.5, 2.5$ **(ix)** $x \approx -1.4, 3.4$

15 (a) $A = x^2 + 3x + 2$

(b)

x	0	1	2	3
A	2	6	12	20

(c)



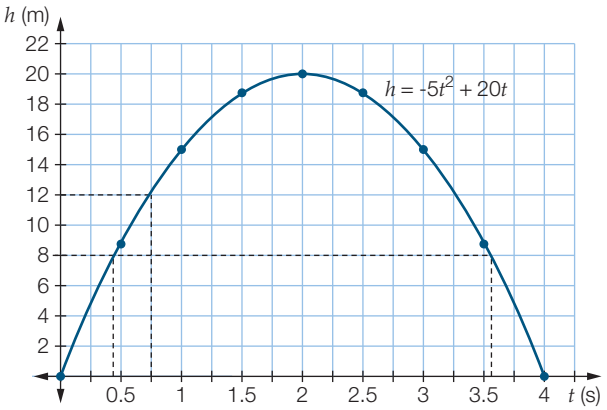
(d) $x = 1.4$. This should be the only value you obtain, because it is unrealistic for x to be negative.

(e) length = 3.4 cm, width = 2.4 cm

16 (a)

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	0	8.75	15	18.75	20	18.75	15	8.75	0

(b)

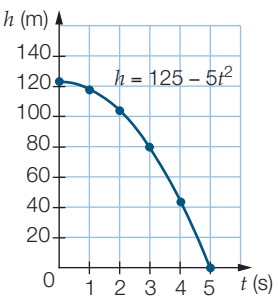


(c) 20 m **(d)** 2 s **(e)** ≈ 0.75 s **(f)** 4 s **(g)** ≈ 3.2 s

17 (a)

t	0	1	2	3	4	5
h	125	120	105	80	45	0

(b)

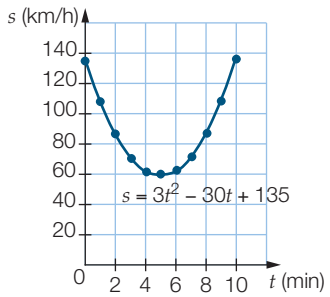


(c) 125 m **(d)** 5 s **(e)** 94 m **(f)** 31 m

(g) 3.2 s **(h)** 4.5 s **(i)** 35 m

(j) fifth second; i.e. between $t = 4$ and $t = 5$ s

18 **(a)**

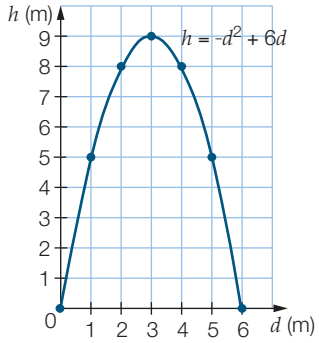


(b) 135 km/h **(c)** 60 km/h **(d)** 72 km/h

(e) 72 km/h **(f)** 2.4 min; 7.6 min

(g) 0.5 min; 9.5 min **(h)** $0 \le t < 1.35$ and $8.65 < t \le 10$

19 **(a)**

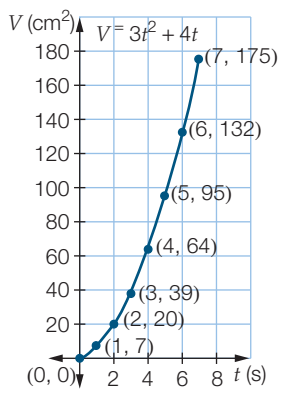


(b) 6 m **(c)** 9 m **(d)** 5 m

(e) 4 m **(f)** 5.75 m **(g)** 3.6 m

(h) **(i)** Yes **(ii)** No

20 **(a)**



(b) 0 **(c)** 64 cm^3 **(d)** 6 s **(e)** 20 cm^3

(f) $0 \le t \le 6$, $0 \le V \le 132$

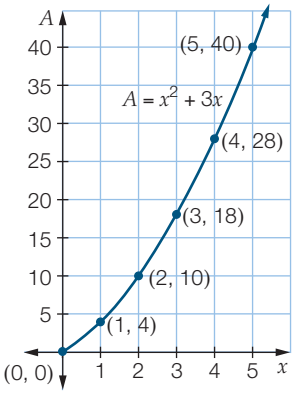
21 **(a)** Student answers will vary; any expressions such as $x^2 + 2x$, $3x^2 - 6x$, $\frac{1}{2}(x^2 - 3x - 4)$.

(b) Student answers will vary; any expressions such as $y + 4$, $x^2 + y^2$, $4x^3 + 3x^2 + 2x$.

22 Student answers will vary.

23 **(a)** $A = x^2 + 3x$, where x is the width of the rectangle.

(b)



(c) If $A = 10$, width = 2 and length = 5.

24 **(a)** Josh thought that $-x^2 = (-x)^2$.

(b) Josh thought that $2x^2 = (2x)^2$.

Exercise 8.3 (p. 378)

1 **(a)** $x = 0$ or $x = 1$ **(b)** $x = 0$ or $x = -8$ **(c)** $x = 0$ or $x = -5$

(d) $x = 1$ or $x = 4$ **(e)** $x = 2$ or $x = 6$ **(f)** $x = 3$ or $x = 9$

(g) $x = -1$ or $x = 4$ **(h)** $x = -7$ or $x = 3$ **(i)** $x = -2$ or $x = 5$

(j) $x = -6$ or $x = -4$ **(k)** $x = -7$ or $x = -2$

(l) $x = -10$ or $x = -3$ **(m)** $x = 4$

(n) $x = -7$ **(o)** $x = 9$

2 **(a)** $x = 0$ or $x = 6$ **(b)** $x = 0$ or $x = 1$ **(c)** $x = 0$ or $x = 8$

(d) $x = -7$ or $x = 0$ **(e)** $x = -5$ or $x = 0$ **(f)** $x = -1$ or $x = 0$

(g) $x = 0$ or $x = 4$ **(h)** $x = 0$ or $x = 3$ **(i)** $x = 0$ or $x = 4$

(j) $x = -9$ or $x = 0$ **(k)** $x = -9$ or $x = 0$ **(l)** $x = -\frac{1}{3}$ or $x = 0$

3 **(a)** $x = -2, -4$ **(b)** $x = -4, -5$ **(c)** $x = -8, -4$

(d) $x = 4, 7$ **(e)** $x = 5, 7$ **(f)** $x = 7, 8$

(g) $x = -4, 2$ **(h)** $x = -5, 3$ **(i)** $x = -2, 11$

4 **(a)** $x = -3$ or $x = 3$ **(b)** $x = -1$ or $x = 1$ **(c)** $x = -4$ or $x = 4$

(d) $x = -7$ or $x = 7$ **(e)** $x = -3$ or $x = 3$ **(f)** $x = -2$ or $x = 2$

(g) $x = -\sqrt{11}$ or $x = \sqrt{11}$ **(h)** $x = -\sqrt{29}$ or $x = \sqrt{29}$

(i) $x = -\sqrt{47}$ or $x = \sqrt{47}$

5 **(a)** $x = 3$ **(b)** $x = 1$ **(c)** $x = 4$

(d) $x = -5$ **(e)** $x = -6$ **(f)** $x = -9$

(g) $x = 7$ **(h)** $x = -3$ **(i)** $x = 2$

6 **(a)** C **(b)** B

7 Let n be the number, $n^2 + 3n = 7n$; $n^2 - 4n = 0$; $n = 4$

8 $x = -9$ or $x = 1$

9 C **10** 9 m **11** (-1, 0), (4, 0)

12 **(a)** (12, 0), (-12, 0) **(b)** (0, 0), (-4, 0) **(c)** (6, 0)

13 **(a)** $x^2 - 4x - 12$ **(b)** $x = -2$ or $x = 6$

(c) The product of the solutions equals the constant term.

(d) $x^2 - 4x - 5$ **(e)** $x = -1$ or $x = 5$

(f) The product of the solutions equals the constant term.

(g) $x = 0$ or $x = 4$ **(h)** 0

(i) The product of the solutions equals the constant term.

(j) $x^2 - 4x + 3$ (k) $x = 1$ or $x = 3$

(l) The product of the solutions equals the constant term.

(m) $x = 2$

(n) The product of the repeated solutions equals the constant term.

(o) The constant term represents the y-intercept.

(p) All equations have the first two terms, x^2 and $-4x$, in common.

(q) The solution tells you where the graphs intersect the x-axis.

(r) As the constant term increases, the distance between the x-intercepts decreases until the case where there is only one x-intercept.

(s) When $c > 4$, the equation $x^2 - 4x + c = 0$ has no solution. Hence, a graph of $y = x^2 - 4x + c$, $c > 4$ has no x-intercepts.

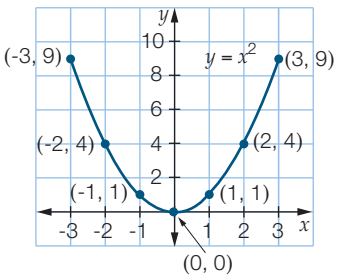
14 Let $x = 0$ or $x = 2$; two possible equations are $y = x^2 - 2x$ and $y = 2x^2 - 4x$.

15 Louise needed to use the Null Factor Law (i.e. she had to first set the equation to equal 0).

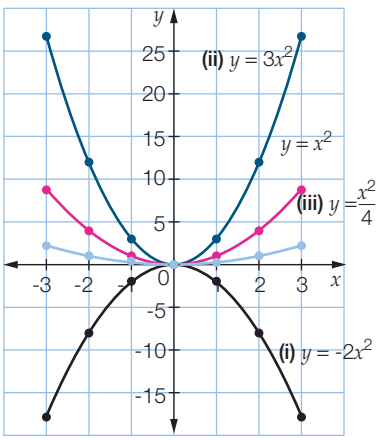
$0 = x^2 - 8x + 16$
$\therefore 0 = (x - 4)^2$
$\therefore x - 4 = 0$
$\therefore x = 4$

Exercise 8.4 (p. 389)

1 (a)



(b)



from left to right

• (-3, 27)	• (-3, 9)	• (-3, 2.25)	• (-3, -18)
(-2, 12)	(-2, 4)	(-2, 1)	(-2, -8)
(-1, 3)	(-1, 1)	(-1, 0.25)	(-1, -2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)
(1, 3)	(1, 1)	(1, 0.25)	(1, -2)
(2, 12)	(2, 4)	(2, 1)	(2, -8)
(3, 27)	(3, 9)	(3, 2.25)	(3, -18)

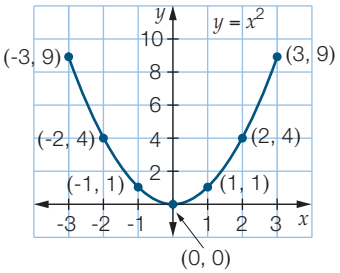
(c) (i) $y = -2x^2$ is reflected in the x-axis and narrower than $y = x^2$

(ii) $y = 3x^2$ is narrower than $y = x^2$

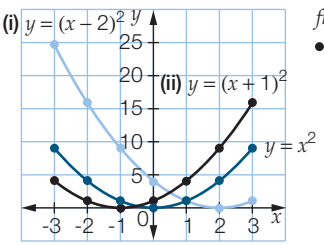
(iii) $y = \frac{1}{4}x^2$ is wider than $y = x^2$

(d) (i) -2 (ii) 3 (iii) $\frac{1}{4}$

2 (a)



(b)



from left to right

• (-3, 4)	• (-3, 9)	• (-3, 25)
(-2, 1)	(-2, 4)	(-2, 16)
(-1, 0)	(-1, 1)	(-1, 9)
(0, 1)	(0, 0)	(0, 4)
(1, 4)	(1, 1)	(1, 1)
(2, 9)	(2, 4)	(2, 0)
(3, 16)	(3, 9)	(3, 1)

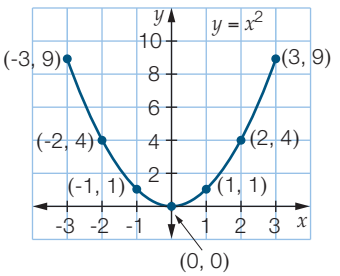
(c) (i) The graph of $y = x^2$ has been translated 2 units to the right to give the graph of $y = (x - 2)^2$.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left to give the graph of $y = (x + 1)^2$.

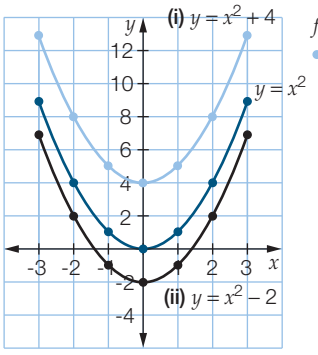
(d) (i) (2, 0). The x-value of the intercept, 2, is the number subtracted from x before squaring in the equation.

(ii) (-1, 0). The x-value of the intercept, -1, is the number subtracted from x before squaring in the equation.

3 (a)



(b)



from left to right

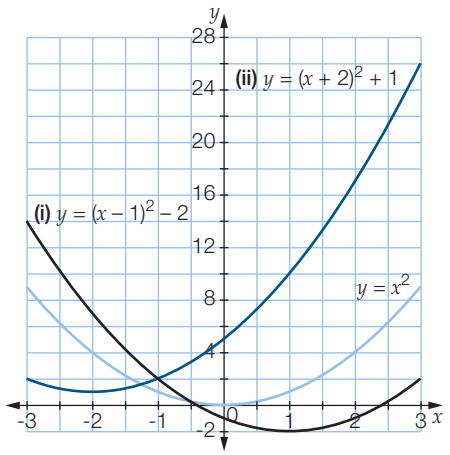
• (-3, 13)	• (-3, 9)	• (-3, 7)
(-2, 8)	(-2, 4)	(-2, 2)
(-1, 5)	(-1, 1)	(-1, -1)
(0, 4)	(0, 0)	(0, -2)
(1, 5)	(1, 1)	(1, -1)
(2, 8)	(2, 4)	(2, 2)
(3, 13)	(3, 9)	(3, 7)

(c) (i) The graph of $y = x^2$ has been translated 4 units upwards to give the graph of $y = x^2 + 4$.

(ii) The graph of $y = x^2$ has been translated 2 units downwards to give the graph of $y = x^2 - 2$.

(d) (i) (0, 4) (ii) (0, -2)

4 (a), (b)



(c) (i) The graph of $y = (x - 1)^2 - 2$ is the graph of $y = x^2$ translated 1 unit right and 2 units down.

(ii) The graph of $y = (x + 2)^2 + 1$ is the graph of $y = x^2$ translated 2 units left and 1 unit up.

(d) (i) $y = (x - 1)^2 - 2$ has a y-intercept of -1.

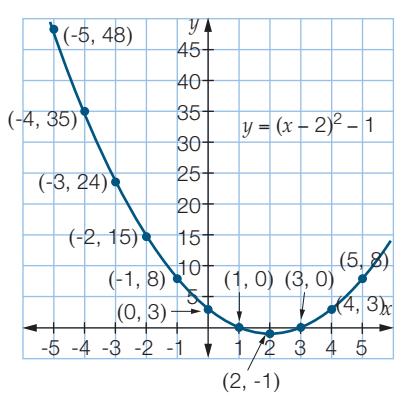
(ii) $y = (x + 2)^2 + 1$ has a y-intercept of 5.

(e) $y = x^2$ has one x-intercept.

(i) $y = (x - 1)^2 - 2$ has two x-intercepts.

(ii) $y = (x + 2)^2 + 1$ has zero x-intercepts.

5 (a)



(b) (2,-1) (c) $h = 2$, $k = -1$. Yes

(d) Both equations expand to $x^2 - 4x + 3$. The x-coordinates of the x-intercepts are 1 and 3, so the value of the x-coordinate of the turning point is the same as the average of these two values.

(e) $y = -1$, which is the same as the y-coordinate of the turning point.

(f) $a = 1$, $b = -4$, $c = 3$. The value of $-\frac{b}{2a}$ is 2, which is the same as the value of the x-coordinate of the turning point.

6 (a) (i) narrower (ii) wider (iii) wider (iv) narrower

(b) (iii) and (iv)

(c) (i) 4 (ii) $\frac{1}{5}$ (iii) 7 (iv) $\frac{2}{3}$

(d) (i) narrower

(ii) wider

(iii) narrower

(iv) wider

7 (a) up 4 units (b) down 7 units

(c) down one unit (d) up 2 units

(e) down 8 units (f) up 5 units

(g) up $\frac{1}{2}$ unit (h) down $\frac{3}{4}$ unit

8 (a) 5 units to the right (b) 4 units to the left

(c) 1 unit to the left (d) 7 units to the right

(e) 6 units to the left (f) 3 units to the right

(g) $\frac{2}{3}$ unit to the right (h) $\frac{3}{2}$ units to the left

9 (a) (i) dilation factor 2, moved right 1

(ii) (1, 0)

(b) (i) reflected in x-axis, moved left 2

(ii) (-2, 0)

(c) (i) moved left 3 down 2

(ii) (-3, -2)

(d) (i) moved left 5 up 3

(ii) (-5, 3)

(e) (i) moved left 4, down 3

(ii) (-4, -3)

(f) (i) reflected in the x-axis, dilation factor 2, moved left 3

(ii) (-3, 0)

10 (a) C (b) B (c) A (d) D

11 A

12 (a) C (b) B (c) B

13

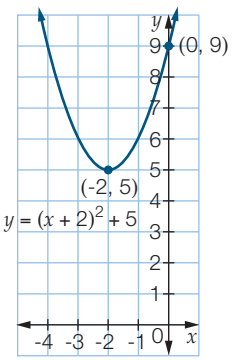
		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x+1)^2 + 2$	–	–	1 unit left	2 units up
(b)	$y = (x-3)^2 + 1$	–	–	3 units right	1 unit up
(c)	$y = 5x^2 + 4$	by factor of 5 (narrower)	–	–	4 units up
(d)	$y = -x^2 - 6$	–	invert	–	6 units down
(e)	$y = -(x-2)^2 + 3$	–	invert	2 units right	3 units up
(f)	$y = (x-4)^2 - 5$	–	–	4 units right	5 units down
(g)	$y = (x+2)^2 - 7$	–	–	2 units left	7 units down
(h)	$y = 3x^2 - 5$	by factor of 3 (narrower)	–	–	5 units down
(i)	$y = -x^2 - 2$	–	invert	–	2 units down
(j)	$y = -(x-5)^2 + 2$	–	invert	5 units right	2 units up

14 (b), (c) and (e)

15 C

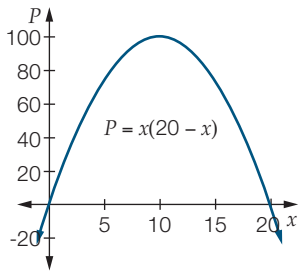
16 (a) $y = (x+2)^2 + 5$ (b) $y = 9$

(c)



17 (a) $y = (x-3)^2 + 2$ (b) $y = (x-4)^2 - 2$

18 (a) $y = 20 - x$ (b) $P = x(20 - x)$



(c) $x = 10$ and $y = 10$, $P = 100$

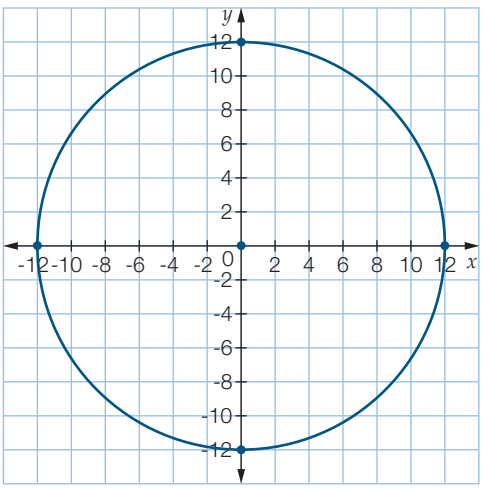
19 $y = (x-2)^2 + 3$.
Translaed 2 units to the right and 3 units up.

$y = (x+3)^2 - 4$.
Translated 3 units to the left and 4 units down.

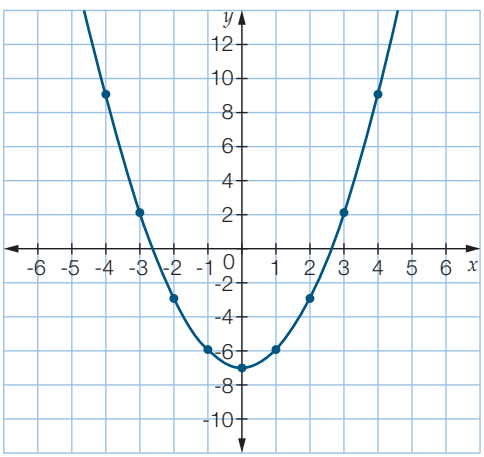
20 $y = (x-5)^2 + 2$; $y = (x+4)^2 + 7$; $y = -(x-5)^2 - 3$

Half-time 8 (p. 393)

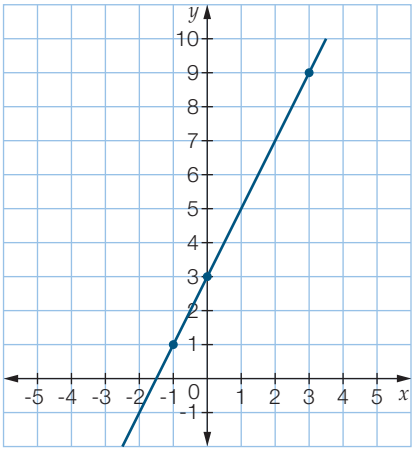
1 (a) $x^2 + y^2 = 144$



(b) $y = x^2 - 7$



(c) $y = 2x + 3$



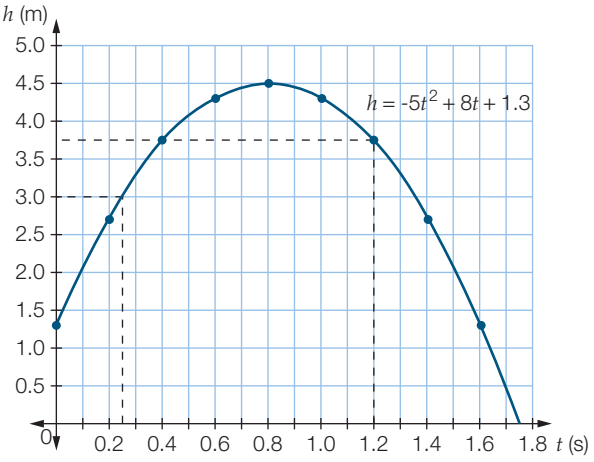
2 C

3 (a) $x = -4$ or $x = 0$ (b) $x = 0$ or $x = 3$ (c) $x = -2$ or $x = 3$

4 (a)

t	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
h	1.3	2.7	3.7	4.3	4.5	4.3	3.7	2.7	1.3

(b)



(c) 0.25 s (d) 3.7 m

5 (a) $x = -2$ (b) $x = -6$ (c) $x = 9$

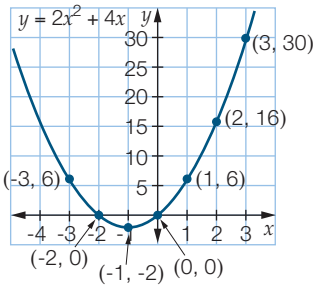
6 (a) $x = 0$ or $x = 6$ (b) $x = -5$ (c) $x = 0$ or $x = 9$

7 (a) $x = -8$ or $x = 8$ (b) $x = -1$ (c) $x = 0$ or $x = 4$

8 (a) $y = 2x^2 + 4x$

x	-3	-2	-1	0	1	2	3
y	6	0	-2	0	6	16	30

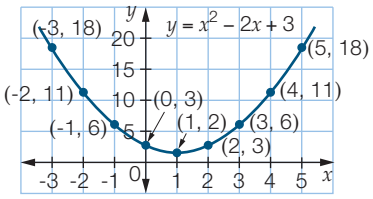
(b)



(c) (-1, -2) (d) minimum turning point

(e) x-intercepts (-2, 0) and (0, 0); y-intercept (0, 0)

9 (a)



(b) (1, 2)

(c) y-intercept is (0, 3); there are no x-intercepts

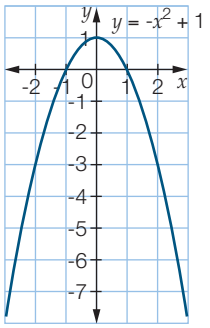
(d) (i) $x = 0$ or $x = 2$ (ii) $x = 1$

(iii) $x = -1$ or $x = 3$ (iv) No solutions

10 (a) $y = -x^2 + 1$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

(b)



(c) (0, 1) (d) (0, 1) (e) (-1, 0) and (1, 0)

Exercise 8.5 (p. 395)

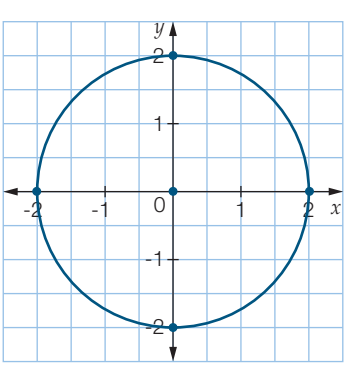
1 (a) centre: (0, 0); radius = 5
Equation: $x^2 + y^2 = 25$

(b) centre: (0, 0); radius = 2.5
Equation: $x^2 + y^2 = 6.25$

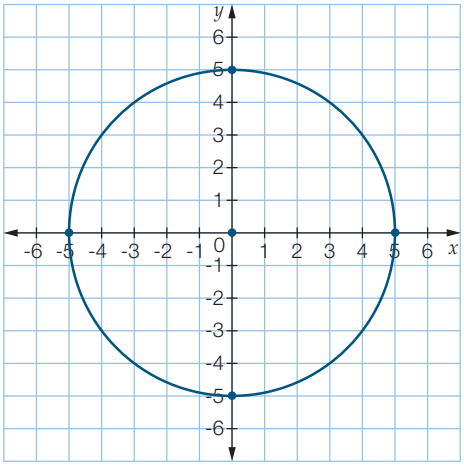
2 (a) radius = 2 (b) radius = 5 (c) radius = 7

(d) radius = 9 (e) radius = 2.5 (f) radius = 1.4

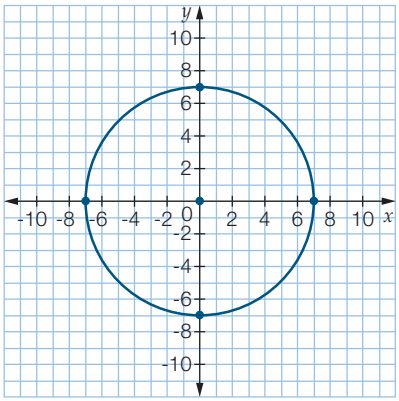
3 (a) $x^2 + y^2 = 4$



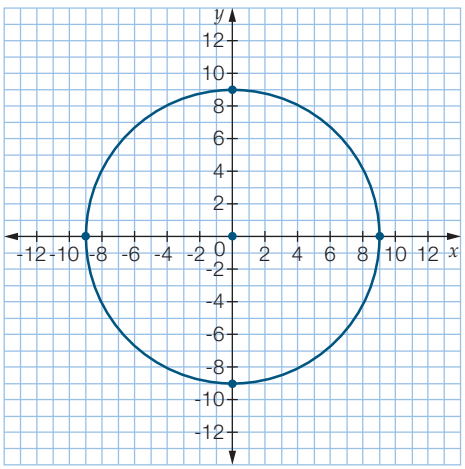
(b) $x^2 + y^2 = 25$



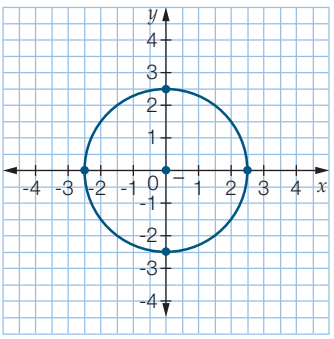
(c) $x^2 + y^2 = 49$



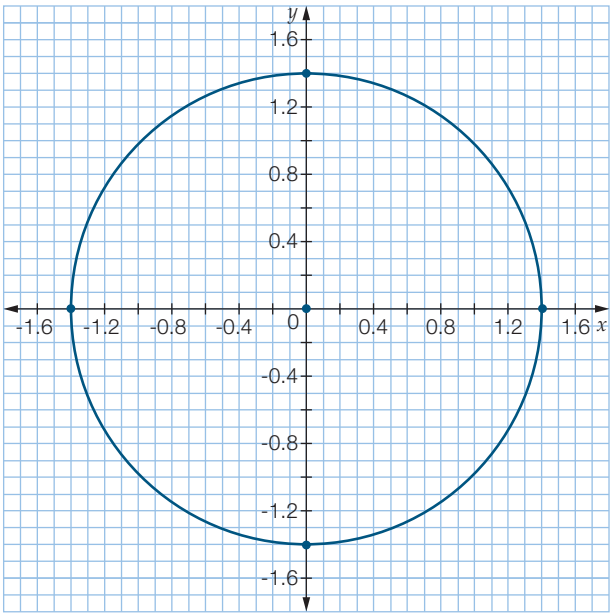
(d) $x^2 + y^2 = 81$



(e) $x^2 + y^2 = 6.25$



(f) $x^2 + y^2 = 1.96$



4 (a) $x^2 + y^2 = 4$ (b) $x^2 + y^2 = 64$
(c) $x^2 + y^2 = 0.36$ (d) $x^2 + y^2 = 1.44$
(e) $x^2 + y^2 = 0.25$ (f) $x^2 + y^2 = 2.56$

5 B

6 (a) No (b) No (c) Yes
(d) Yes (e) Yes (f) No

7 (a) Substitute the coordinates into the equation. If the value of $x^2 + y^2 > r^2$, the point lies outside the circle. If the value of $x^2 + y^2 = r^2$, the point lies on the circle. If the value of $x^2 + y^2 < r^2$, the point lies inside the circle.

(b) (i) $x^2 + y^2 = 3^2 + 10^2$
$= 109$
$109 < 169$
(3, 10) lies inside the circle

(ii) $x^2 + y^2 = (-2)^2 + 3^2$
$= 13$
$13 < 169$
(-2, 3) lies inside the circle

(iii) $x^2 + y^2 = (5)^2 + (-12)^2$
$= 169$
$169 = 169$
(5, -12) lies on the circle

(iv) $x^2 + y^2 = (15)^2 + (-2)^2$
$= 229$
$229 > 169$
(15, -2) lies outside the circle

8 $x^2 + y^2 = 289$

9 Sample answer
(6, 8), (8, 6)

Exercise 8.6 (p. 399)

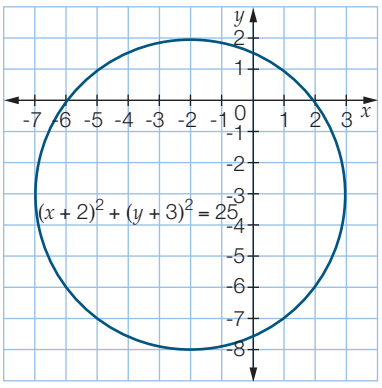
1 **(a)** centre: (-1, 3); radius = 3; $(x + 1)^2 + (y - 3)^2 = 9$

(b) centre: (1, 2); radius = 4; $(x - 1)^2 + (y - 2)^2 = 16$

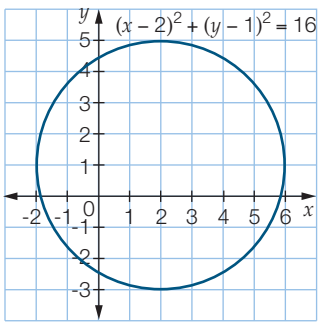
(c) centre (-3, -1); radius = 4; $(x + 3)^2 + (y + 1)^2 = 16$

(d) centre (-3, 0); radius = 5; $(x + 3)^2 + y^2 = 25$

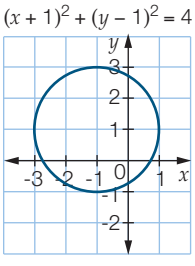
2 **(a)**



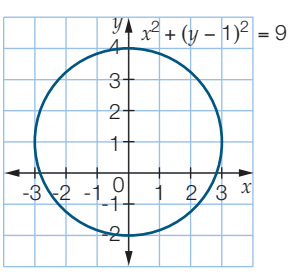
(b)



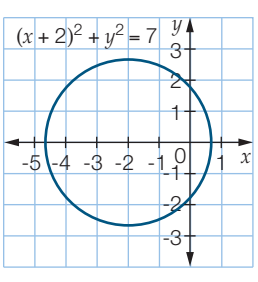
(c) $(x + 1)^2 + (y - 1)^2 = 4$



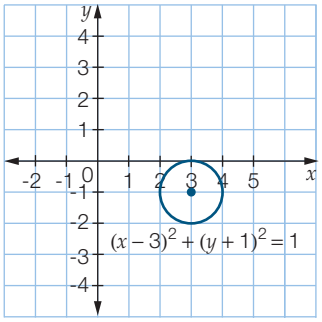
(d)



(e)



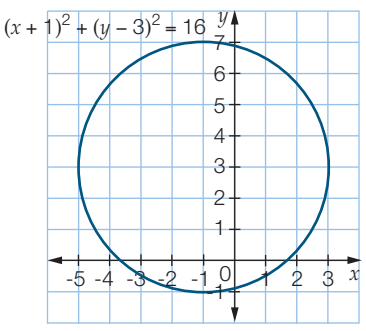
(f)



3 **(a)** centre = (-1, 3), radius = 4

(b) Points on circle are (3, 3), (-1, 7), (-5, 3), (-1, -1).

(c)



(d) **(i)** inside **(ii)** outside **(iii)** inside

4 B

5 **(a)** $(x - 4)^2 + (y - 3)^2 = 36$

(b) $(x + 6)^2 + (y - 4)^2 = 36$ **(c)** $(x - 7)^2 + y^2 = 36$

(d) $x^2 + (y + 5)^2 = 36$ **(e)** $x^2 + y^2 = 49$

6 D **7** $(x + 6)^2 + (y - 4)^2 = 16$

8 **(a)** point (3, 4)

$(x - 8)^2 + (y + 8)^2 = 169$
$(3 - 8)^2 + (4 + 8)^2 = 169$
$(-5)^2 + 12^2 = 169$
$25 + 144 = 169$—true

$(x + 9)^2 + (y - 9)^2 = 169$
$(3 + 9)^2 + (4 - 9)^2 = 169$
$12^2 + (-5)^2 = 169$
$25 + 144 = 169$—true

$x^2 + y^2 = 25$
$3^2 + 4^2 = 25$
$9 + 16 = 25$—true

Point (-4, -3)

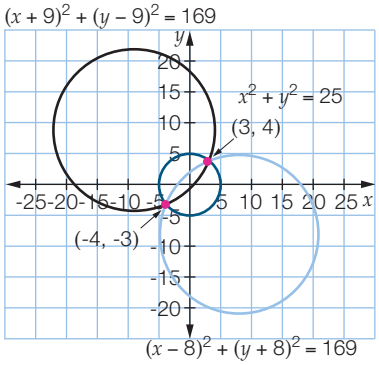
$(x - 8)^2 + (y + 8)^2 = 169$
$(-4 - 8)^2 + (-3 + 8)^2 = 169$
$(-12)^2 + 5^2 = 169$
$144 + 25 = 169$—true

$(x + 9)^2 + (y - 9)^2 = 169$
$(-4 + 9)^2 + (-3 - 9)^2 = 169$
$5^2 + (-12)^2 = 169$
$25 + 144 = 169$—true

$x^2 + y^2 = 25$
$(-4)^2 + (-3)^2 = 25$
$16 + 9 = 25$—true

The three circles intersect at the points (3, 4) and (-4, -3).

(b) $(x + 9)^2 + (y - 9)^2 = 169$



9 (a) $(x - 2)^2 + (y - 3)^2 = 25$ **(b)** $(x - 11)^2 + (y - 4)^2 = 169$

(c) Testing point (-1, -1) in part **(a)** equation:

$(-1 - 2)^2 + (-1 - 3)^2 = 25$—true

Testing point (-1, -1) in part **(b)** equation:

$(-1 - 11)^2 + (-1 - 4)^2 = 169$—true

10 (a) Student answers will vary. Examples include $x^2 + y^2 = 9$ and $x^2 + y^2 = 100$.

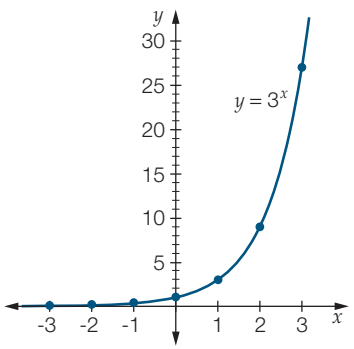
(b) Student answers will vary. Examples include $(x - 5)^2 + (y - 6)^2 = 9$ and $(x + 14)^2 + (y + 8)^2 = 9$.

11 Student answers will vary.

Exercise 8.7 (p. 404)

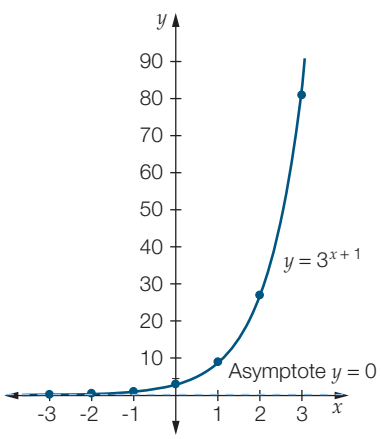
1 (a)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



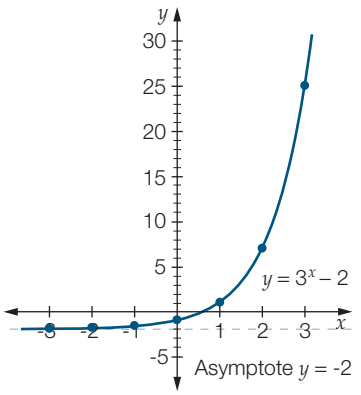
(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81



(c)

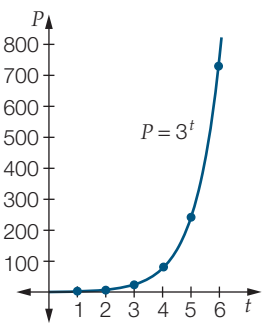
x	-3	-2	-1	0	1	2	3
y	$-\frac{53}{27}$	$-\frac{17}{9}$	$-\frac{5}{3}$	-1	1	7	25



2 (a)

t	0	1	2	3	4	5	6
P	1	3	9	27	81	243	729

(b)

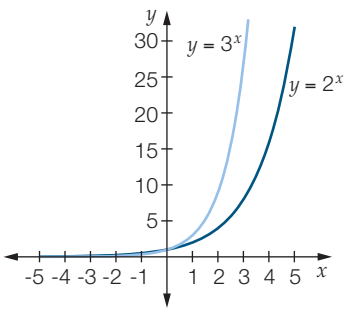


(c) 5 years

(d) 65. This value is meaningless as he never has 65 orchids. The values are not continuous in this situation.

(e) 243

3 (a)



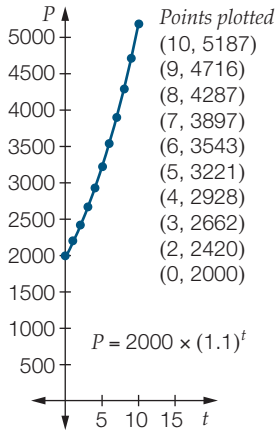
(b) Both graphs have a y-intercept of 1 and $y = 0$ is the equation of the asymptote for both graphs. $y = 3^x$ is steeper than $y = 2^x$.

4 D

5 (a)

	Beginning of year					
t	2005	2006	2007	2008	2009	2010
Population, P	2000	2200	2420	2662	2928	3221
	Beginning of year					
t	2011	2012	2013	2014	2015	
Population, P	3543	3897	4287	4716	5187	

(b)

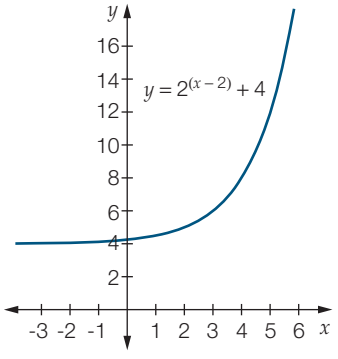


Points plotted
(10, 5187)
(9, 4716)
(8, 4287)
(7, 3897)
(6, 3543)
(5, 3221)
(4, 2928)
(3, 2662)
(2, 2420)
(0, 2000)

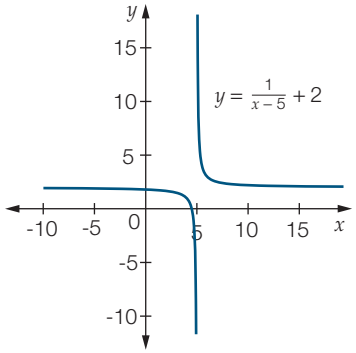
(c) 7.3 years

(d) The increase is happening on the previous year's population; the 10% is compounded annually and it would therefore take less than 10 years for the population to double.

6 (a) $h = 2, k = 4$
$y = 2^{(x-2)} + 4$



(b) $h = 5, k = 2$
$y = \dfrac{1}{x-5} + 2$



Exercise 8.8 (p. 408)

1 (a) not direct proportion (decreasing)

(b) not direct proportion (fluctuating)

(c) not direct proportion (not through origin)

(d) could be direct proportion

2 (a) $b = 4a$ **(b)** $v = 5w$ **(c)** $d = \frac{1}{2}c$

(d) $h = \frac{1}{4}g$ **(e)** $h = 3e$ **(f)** $q = 1.5p$

(g) $j = 2i^2$ **(h)** $d = 3a^2$ **(i)** $n = 4m^3$

(j) $h = 2g^3$ **(k)** $q = \frac{1}{2}p^2$ **(l)** $f = \frac{1}{5}b^2$

(m) $s = 3r^3$ **(n)** $m = \frac{1}{4}p^3$ **(o)** $u = \frac{1}{2}t^3$

(p) $w = 2\sqrt{v}$ **(q)** $t = \frac{1}{10}\sqrt{u}$ **(r)** $l = \frac{1}{3}\sqrt{d}$

3 **(a), (c), (d), (e)** and **(f)** not direct proportion

(b) is in direct proportion (linear and through origin)

4 (a) $m = kn$ **(b)** $a = kb^2$ **(c)** $g = kh^3$

(d) $t = kw$ **(e)** $N = kp^2$ **(f)** $y = kx^3$

(g) $V = kr^2$ **(h)** $L = k\sqrt{d}$

5 (a) B **(b)** A

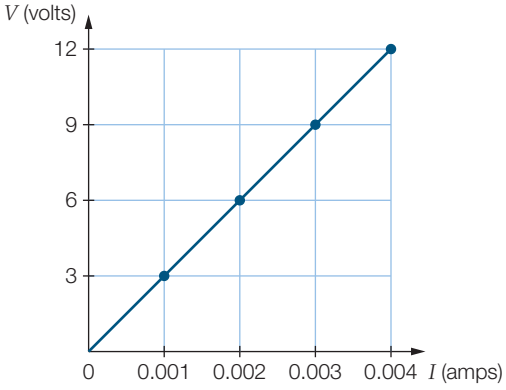
6 (a) 3 **(b)** 0.2 **(c)** $\frac{1}{4}$ **(d)** 200

(e) 1 **(f)** $\frac{1}{6}$ **(g)** $\frac{1}{20}$ **(h)** $\frac{1}{35}$

7 (a) $k = 3, q = 11, p = 27$ **(b)** $k = 7, m = 63, n = 6$
(c) $k = 1, c = 2, d = 64$ **(d)** $k = 3, x = 4, w = 21$

8 C

9 (a)



(b) 3000 ohms **(c)** 30 volts **(d)** 0.015 amps

10 (a) $k = 12.56$ **(b)** $A = 4\pi r^2$

11 (a) 22.5 km **(b)** 988 m

12 (a) $\dfrac{C}{r} = 6.28$

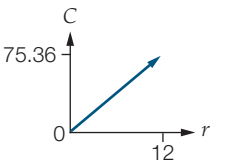
$\dfrac{A}{r} = 25.12, 50.24, 75.36, 100.48, 125.60, 150.72$

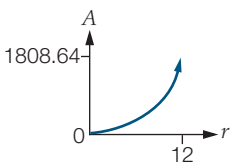
$\frac{V}{r}$ = 16.745, 66.9875, 150.72, 267.95, 418.67, 602.88

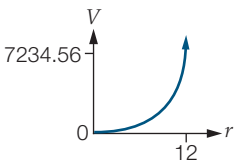
(b) $\frac{C}{r} = 6.28$; $\frac{A}{r}$ is increasing by a constant value

with each pair of values; $\frac{V}{r}$ seems to be no relationship

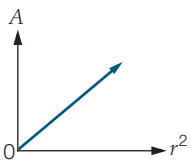
(c)



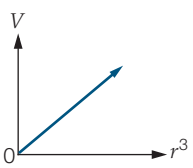




(d) linear relationship



(e) linear relationship



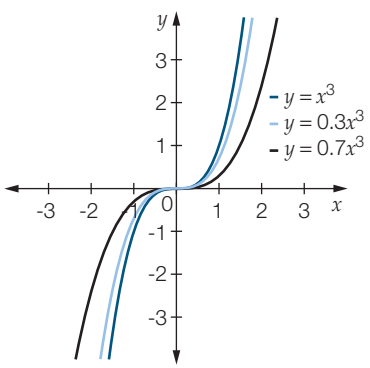
(f) $C \propto r$; $A \propto r^2$; $V \propto r^3$

(g) $k = 6.28$, $C = 2\pi r$; $k = 12.56$, $A = 4\pi r^2$;

$k = 4.19$, $V = \frac{4}{3}\pi r^3$

13 (a) $y = 0.3x^3$, $y = 0.7x^3$

(b)



As k decreases from 1 to 0, the graph is dilated horizontally.

Challenge 8 (p. 412)

1 A **2** 1028 **3** C **4** C

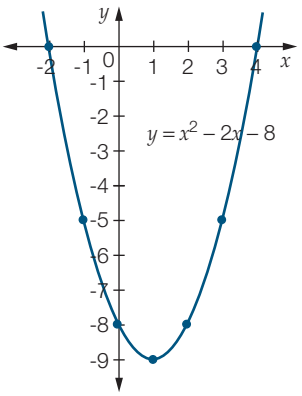
5 13 **6** B **7** 36 m^2 **8** D

9 B **10** B

Chapter review 8 (p. 413)

1 C

2



(a) x-intercepts (-2, 0) and (4, 0); y-intercept (0, -8)

(b) (1, -9) minimum turning point

(c) $x = 1$ **(d)** $y = -5$ **(e)** -3 and 5

(f) $x = -3$ or $x = 5$ **(g)** $x = -2$ or $x = 4$

3 (a) $x = -4$ or $x = 0$ **(b)** $x = -1$ or $x = 7$ **(c)** $x = -4$ or $x = 4$

(d) $x = -3$ **(e)** $x = 0$ or $x = 8$ **(f)** $x = 6$

4 (a) $x = 0$ or $x = 4$ **(b)** $x = -\sqrt{6}$ or $x = \sqrt{6}$

(c) No solution **(d)** $x = -3$ or $x = 3$ **(e)** $x = -8$ or $x = 4$

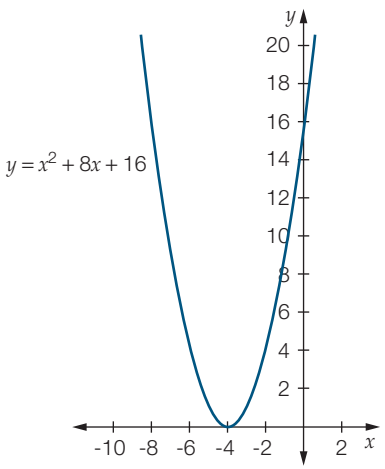
(f) $x = -3$ or $x = 4$

5 (a) A **(b)** A

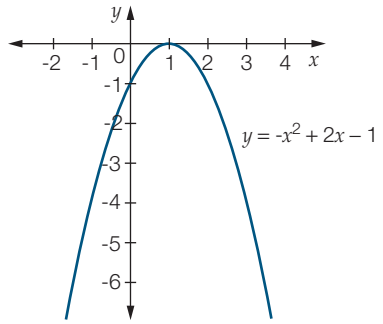
6

		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x-1)^2 + 3$	No dilation	No reflection	1 unit right	3 units up
(b)	$y = -x^2 + 4$	No dilation	Invert	No translation	4 unit up
(c)	$y = -(x+2)^2 - 1$	No dilation	Invert	2 units left	1 unit down

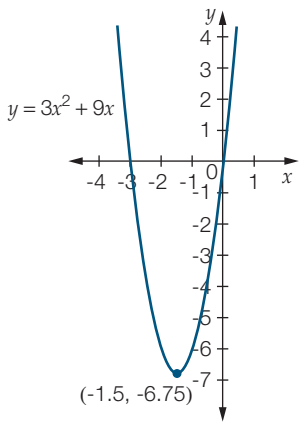
7 (a)



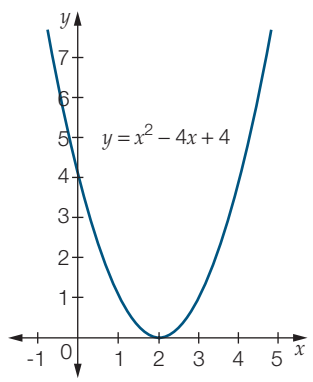
(b)



(c)



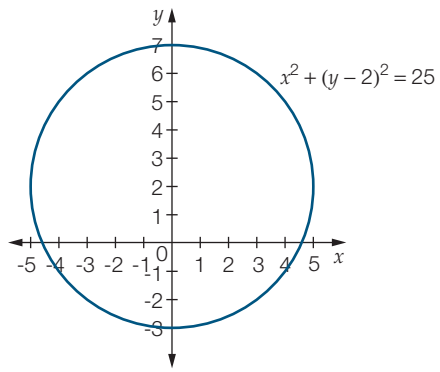
(d)



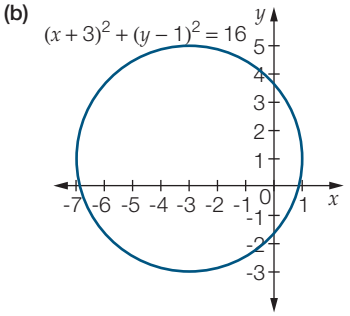
8 **(a)** A **(b)** C

9 **(a)** $(0, 0), 3$ **(b)** $(0, 0), \sqrt{5}$

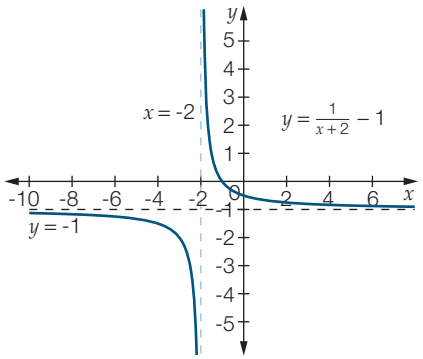
10 **(a)**



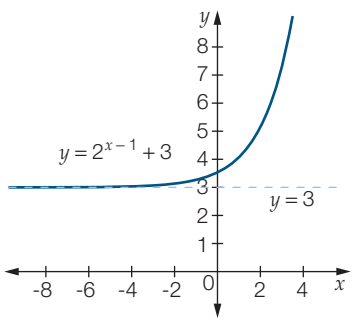
(b)



11 **(a)**



(b)



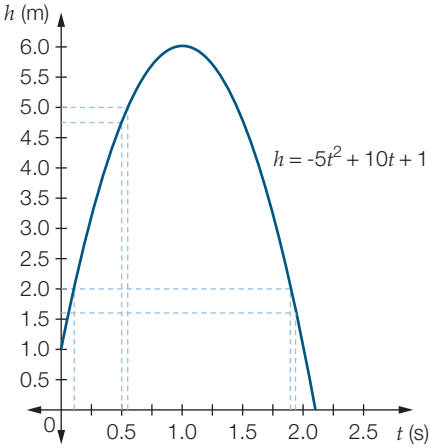
12 **(a)** $b = 3a$ **(b)** $h = 3\sqrt{g}$

13 **(a)** A **(b)** B **(c)** D

14 **(a)**

t	0	0.5	1	1.5	2
h	1	4.75	6	4.75	1

(b)

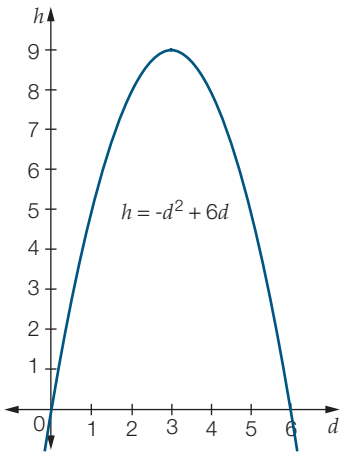


(c) 4.75 m **(d)** ≈ 0.55 s

(e) between 0.1 and 1.9 s

(f) 6 m **(g)** ≈ 1.95 s

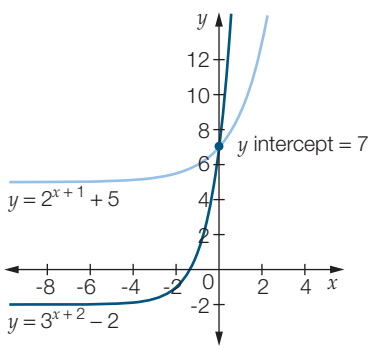
15 (a)



(b) 9 m **(c)** 6 m

16 The circle $(x-4)^2 + (y-3)^2 = 16$ has been translated 6 units to the right, 2 units upwards and the radius decreased by 1 unit compared with the circle $(x+2)^2 + (y-1)^2 = 25$.

17 (a)



(b) $y = 2^{x+1} + 5$ horizontal asymptote $y = 5$; $y = 3^{x+2} - 2$ horizontal asymptote $y = -2$

18 (a) Yes, $y = \frac{1}{10}x^3$

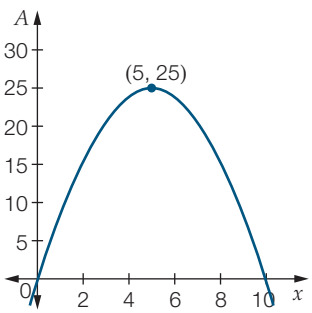
(b)

x	1	2	3	4	5	6
y	0.1	0.8	2.7	6.4	12.5	21.6

19 (a) Width $= 10 - x$

(b) A = length × width $= x(10-x) = 10x - x^2 = -x^2 + 10x$

(c)



(d) 5 m × 5 m **(e)** 25 m²

20 length = 8 cm, width = 8 cm

21 25 and 25

22 $y = (x-3)^2 + 5$

23 $(x-3)^2 + y^2 = 81$

24 (a) When s is doubled, v increases by $\sqrt{2}$.

(b) When v is reduced by $\frac{1}{4}$, s is reduced by $\frac{7}{16}$.

NAPLAN practice 8

1 $x = 3$ **2** D **3** C **4** D

5 $y = x^2 - 3x + 2$

x	-3	-2	-1	0	1	2	3
y	20	12	6	2	0	0	2

6 D

Mixed review C (p. 418)

1 (a) $x = 20°$ **(b)** $y = 26°$, $z = 5°$ **(c)** $w = 20°$

2 6166.51 − 6146.28 = $20.23

3 (a) 104 cm² **(b)** 37.5 m² **(c)** 103.5 cm²

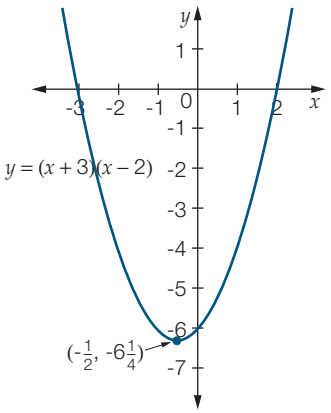
4 (a) 42.7° **(b)** 23.9° **(c)** 50.5°

5 (a) 33.36 **(b)** 125.24

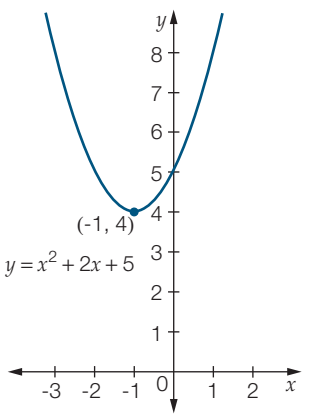
6 (a) $a = -6$ **(b)** $a = -\frac{15}{16}$ **(c)** $a = \frac{11}{6}$

7 (a) $x = -5$ or $x = 5$ **(b)** $x = -\frac{1}{2}$ or $x = 0$ **(c)** $x = 3$

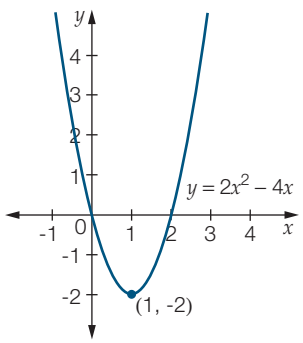
8 (a)



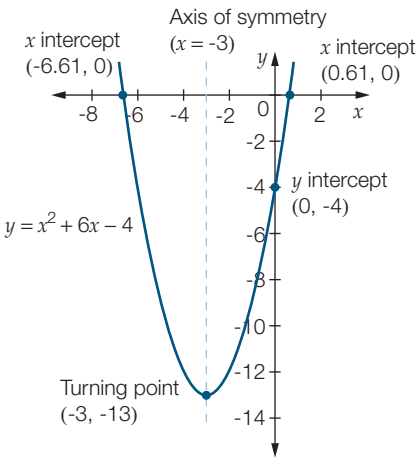
(b)



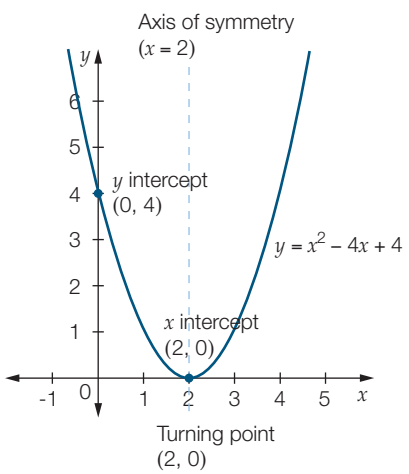
(c)



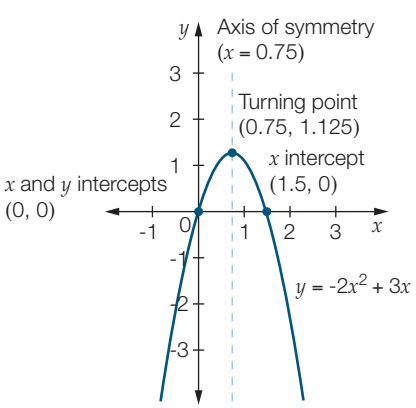
9 (a)



(b)



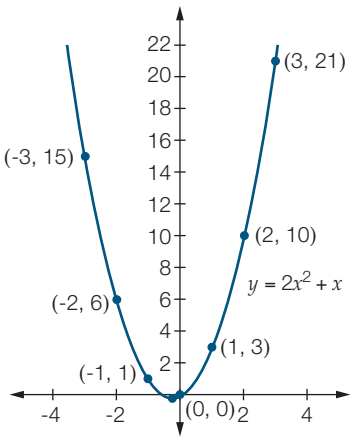
(c)



10 (a) $y = 2x^2 + x$

x	-3	-2	-1	0	1	2	3
y	15	6	1	0	3	10	21

(b)



(c) $(-\frac{1}{4}, -\frac{1}{8})$

(d) x-intercepts $(-\frac{1}{2}, 0)$ and $(0, 0)$; y-intercept $(0, 0)$

11 (a) $6(a + 2)(a - 2)$ **(b)** $3(a + 1)^2$ **(c)** $(x + 5)(x - 7)$

12 (a) $\frac{5}{26}$ **(b)** $\frac{2}{13}$ **(c)** $\frac{11}{13}$

13 2.5°

14 (a) 4°

(b) (i) 1.995 m **(ii)** 140 mm

15 length = 12 cm, width = 5 cm

16 (a) $x = 5$ **(b)** $y = (x - 4)(x - 6)$ or $y = x^2 - 10x + 24$

Glossary and index

adjacent side the side of a right-angled triangle beside a given or required angle (p. 256)

alternate angles angles found between two lines but on opposite sides of a transversal. Alternate angles between parallel lines are equal. (p. 206)

angle of depression the angle between the horizontal and the line of sight to a lower object (p. 279)

angle of elevation the angle between the horizontal and the line of sight to a higher (elevated) object (p. 279)

annulus the area between two circles that have the same centre but different radii (p. 114)

arc a section of the circumference of a circle (p. 103)

area the amount of surface within a boundary (p. 109)

asymptote a line that a graph approaches but does not cross or touch for particular values of the independent variable (pp. 355, 401)

award a document that sets out the minimum working conditions of employment for employees in a certain occupation or industry (p. 16)

axis of symmetry an imaginary line along which a mirror could be placed and the reflection would be indistinguishable from the original (p. 361)

back-to-back histogram two horizontal histograms of similar data, with the same y-axis but plotted in opposite directions on the x-axis (p. 317)

back-to-back stem plot two stem plots, one with its leaves going to the left from the stem and the other with the leaves going to the right, used to compare two sets of (related) data (p. 317)

base the number that is raised to an index or power (pp. 48, 63)

bearing indicates the direction of one object from another object or point (p. 283)

bi-modal a data distribution that has two modes (p. 309)

binomial an expression with two terms (p. 80)

buy now, pay later a method of purchase where goods can be taken home without repayments having to be made during the interest-free period. Interest is paid on any outstanding amount at the end of the interest-free period (p. 36)

buying on terms a method of purchase where goods can be taken home before payment, under certain conditions; also known as hire purchase (p. 36)

capacity the amount that a container can hold; usually refers to gases and liquids. Measured in units based on the litre (p. 133)

categorical data non-numerical data that is simply divided into categories (p. 292)

centre of enlargement a point of reference, (called O) from which the distance to each of the points on the original shape is measured and multiplied by the scale factor to determine the distance from O to the matching corners of the enlarged or reduced shape (p. 209)

certain associated with a probability of 1 (p. 325)

chance the likelihood of an event (p. 325)

cheque a form on which an order is written to a bank to pay the stated sum from the drawer's account (p. 35)

circle a plane curve with the equation $x^2 + y^2 = r^2$ (pp. 355, 394, 396)

circumference the perimeter of a circle (p. 103)

coefficient the number factor in a term (p. 381)

co-interior angles angles found between two lines and on the same side of a transversal. Co-interior angles between parallel lines are supplementary. (p. 206)

commission payment for work done based on a percentage of sales made or business gained (p. 16)

compass bearing a direction measured as an angle either side of north or south (p. 283)

complementary events are all of the successful and all of the unsuccessful events that comprise the sample space, where the sum of the probabilities of each outcome add to 1. (p. 325)

composite shape a shape made up of more than one basic shape (pp. 109, 128)

congruent exactly the same in size and shape (p. 228)

congruent triangles triangles that are exactly the same shape and size. There are four tests that can be used to determine if two triangles are congruent: AAA (Angle, Angle, Angle), SAS (Side, Angle, Side), AAS (Angle, Angle, Side) and RHS (Right angle, Hypotenuse, Side). (p. 228)

constant of proportionality the constant value in a relationship that is a direct proportion (p. 406)

continuous data numerical data for which the possible values are all of the real numbers within a particular range; hence, there are an infinite number of possible outcomes that cannot be counted (p. 292)

converse a statement that inverts a statement that has been made (p. 188)

coordinates numbers used to describe a position on a Cartesian plane (p. 149)

corresponding angles angles found either both above or both below two lines and on the same side of the transversal. Corresponding angles on parallel lines are equal. (p. 206)

cosine (cos) the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for a particular angle in a right-angled triangle (p. 261)

credit card a card issued by a bank or financial institution that obtains the user credit from the institution in order to pay a business (p. 35)

cross-section a slice through a solid so that the shape is similar to that of the base. In a prism, the cross-section is uniform (identical to the base). (p. 124)

debit card similar to a credit card, but the money is deducted from the account within a few days and no interest is charged (p. 35)

deferred payment a payment option for HECS-HELP, where payment can be put off until the borrower is earning over a certain amount (p. 35)

degree a unit used to measure the size of angles; a full turn is equal to 360° (p. 256)

dependent variable a variable whose value is dependent on the value of another variable to which it is connected by a rule (p. 157)

difference of two squares a technique for factorising quadratics: $a^2 - b^2 = (a - b)(a + b)$ (p. 92)

dilation a change in the size of a graph or shape (p. 381)

dilation factor the number that can be used to compare an original figure to its dilated size (p. 381)

direct proportion a relationship between two variables such that one variable increases as the other increases, and when one variable is 0, the other variable is also 0 (p. 406)

discount a reduction in the marked price of goods for sale (p. 10)

discrete data exact values that are also countable (e.g. number of students present) (p. 292)

distributive law the way to multiply out the brackets, e.g. $a(b + c) = ab + ac$ (p. 79)

dot plot a statistical display that uses a number line and a dot to represent each data value. If there are multiple occurrences of the same value, they are stacked on top of each other. (p. 309)

earnings income received for performing work (p. 23)

EFTPOS Electronic Funds Transfer at Point Of Sale, where a card is used at the time of purchase and the funds are withdrawn directly from the purchaser's account (p. 35)

empirical probabilities probabilities determined from experiments or estimated from collected data (p. 325)

enlargement a figure that has matching sides in exactly the same ratio as another smaller figure. The figures are said to be similar. (p. 209)

equation two expressions, one on either side of an equals sign (p. 74)

equivalence the left-hand side of the equation is equal to the right-hand side of the equation

expand multiply; used usually when multiplying brackets (p. 54)

expanded form the form of an expression in which all factors have been multiplied out fully (p. 48)

exponent *see* index

exponential describes a relationship between two quantities that involves an exponent (or index), where the exponent is the variable (pp. 355, 401)

expression a collection of terms added or subtracted together (p. 54)

exterior angle an angle formed when the side of a polygon is extended outside the shape; the angle is between the extended side and the adjacent side (pp. 206, 237)

factorise to write an expression as a product of its factors (p. 85)

formula a rule that is expressed using pronumerals and an equals sign (p. 74)

general form a form in which no actual values are used and represents any possible version of that form (p. 188)

gradient the measure of a line's steepness, also called slope (p. 175)

gradient–intercept form an equation in the form $y = mx + b$, where the gradient is m and the y-value of the intercept is b (p. 187)

gross income earnings before deductions have been made (p. 22)

grouping in pairs a technique for factorising a four-term polynomial where terms are put in pairs, a common factor is taken out and the remaining factor in each pair is common (p. 88)

highest common factor (HCF) the largest number that occurs in each of the sets of factors of a given group of numbers (p. 85)

hire purchase *see* buying on terms

hypotenuse the longest side in a right-angled triangle (pp. 256, 261)

impossible associated with a probability of 0 (p. 325)

income money received for work and from business activities and investments, such as shares, property or savings (pp. 16, 22)

income tax a direct tax levied on earnings (income) (p. 22)

independent variable a variable in a rule that determines the value of another variable (the dependent variable) in the rule (p. 157)

index (*plural* **indices**) a number, written smaller, to the right and above another number (the base), which shows the number of times the base is multiplied by itself; also called a power or exponent (pp. 48, 54, 63, 66, 401)

index form short way of writing a number multiplied by itself using a base raised to an index (p. 48)

intercept the value at which a line cuts the x- or y-axis (p. 187)

interest when money is borrowed or invested, interest is paid (either by the individual or the bank) for the use of that money (p. 27)

interior angle an angle inside a shape (p. 206)

interval the line segment formed when two points are joined (pp. 149, 153)

inverse operation an 'opposite' operation; for example, + and − or × and ÷ (p. 157)

inverse proportion a relationship between two variables in which one increases by a factor when the other decreases by the same factor

irrational number a number that cannot be expressed in the form $\frac{a}{b}$, where $b \neq 0$ and a and b are whole numbers

lay-by a system of payment where an item is paid for at intervals before being taken home after payment in full (p. 35)

linear equation an equation in which the power of the pronumeral(s) is 1 (p. 157)

linear graph a straight-line graph (pp. 157, 184)

linear relationship a relationship between two variables that, when graphed, gives a straight line (pp. 157, 168)

loss when goods are sold for less than what they were bought or made for, a loss is made (p. 10)

markup the amount added onto the cost price of an item sold (p. 10)

measure of location measures of data that are representative of the data set. The median and mean are measures of location. (p. 292)

median class interval a class interval in grouped data that contains the median (middle) value (p. 309)

midpoint halfway between two given coordinates that define an interval (pp. 149, 153)

mutually exclusive not able to occur at the same time; if one event occurs, then the other cannot (p. 333)

negative skew a statistical graph that is not symmetrical, with the mean of the data on the right side of the median. It has a long tail on the left of the median, so is also called a skew to the left. (p. 309)

net a diagram consisting of each face of a solid, used to help construct these shapes (pp. 116, 124, 128)

net income earnings after deductions have been made, also known as take-home pay (p. 22)

Null Factor Law states that if the product of factors is zero, then one or more of the factors must equal zero (p. 375)

observational data primary data obtained by direct observation (p. 292)

opposite side the side of a right-angled triangle opposite a given or required angle (p. 256)

ordinal data non-numerical data that has some sort of order associated with it (p. 292)

origin the point of intersection of the vertical and horizontal axes (pp. 149, 187)

overtime hours worked in excess of minimum hours set for a time period (p. 16)

parabola a curve that describes a quadratic relationship (pp. 355, 361)

parallel a reference to lines that lie in the same plane that are always the same distance apart and will never intersect (pp. 188, 206)

parallel dot plot two dot plots drawn with their own scale lines but parallel to each other to enable comparisons (p. 317)

PAYG Pay As You Go, which describes the system by which income tax is paid to the government (p. 22)

percentage out of one hundred (p. 3)

perfect square a quadratic trinomial expression that can be factorised to give $(a \pm b)^2$ (p. 92)

perimeter the distance around the outside of a shape (p. 103)

perpendicular a description of lines that meet/intersect at right angles (p. 188)

piece work work for a payment per item produced (p. 16)

polygon a many-sided (or many-angled) plane shape (p. 234)

polyhedron (*plural* **polyhedra**) a solid where all faces are polygons, shapes with straight edges (p. 116)

positive skew a statistical graph that is not symmetrical, with the mean of the data on the left side of the median. It has a long tail on the right of the median, so is also called a skew to the right. (p. 309)

power *see* index

primary data data collected directly by an observer (p. 292)

principal the initial amount of money borrowed or invested (p. 27)

prism a three-dimensional shape with a uniform cross-section where the cross-section is a polygon and the sides are straight (pp. 116, 124, 133)

probability the study of chance in which the likelihood of the occurrence of a specific event is expressed as a number between, or equal to, 0 and 1 (pp. 325, 333)

profit when goods are sold for more than what they were bought or made for, a profit is made (p. 10)

proof a step-by-step logical connection of statements known to be true to verify that another statement is always true

Pythagoras' Theorem a famous theorem in mathematics that states that, in right-angled triangles, 'the square of the hypotenuse is equal to the sum of the squares on the other two sides'. Often expressed as $c^2 = a^2 + b^2$. (pp. 153, 394)

quadratic equation an equation in which the highest power of the pronumeral(s) is 2 (p. 361)

quadratic relationship a relationship between two variables where the rule for connection is a polynomial containing terms where the highest power of the pronumeral is 2 (p. 361)

qualitative data data that are not numerical. Qualitative data can be categorical or ordinal. (p. 292)

quantitative data any type of numerical data (p. 292)

random sample sampling in which there is no pattern involved in the sampling process (p. 292)

rate of interest the percentage per year (per annum) applied to the amount borrowed or invested to calculate interest (p. 27)

rational approximation an approximate value for an irrational number

reciprocal the result of dividing an expression, term or number into 1. The product of an expression, term or number and its reciprocal is 1. (p. 63)

reduction a figure resulting from the application of a scale factor that is less than 1 (p. 209)

reference angle the angle in a right-angled triangle that determines the opposite and adjacent sides used in trigonometric ratios (p. 256)

reflection the transformation of a shape that would result from looking at the reflection of the shape in a mirror held along the line of reflection (p. 381)

regular polygon a polygon where all the sides and angles are equal (p. 234)

regular polyhedron a polyhedron where all faces are identical regular polygons (p. 116)

relative frequency a proportion found by dividing the number of outcomes of a particular result by the total number of trials (p. 325)

retainer a fixed amount paid to an employee (usually each week) as a base payment to which commission earned is added (p. 16)

right-angled triangle a triangle that contains a right angle (p. 149)

rise the vertical distance between two points on the Cartesian plane, used in the equation

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$ (p. 175)

run the horizontal distance between two points on the Cartesian plane, used in the equation

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$ (p. 175)

salary a fixed annual income paid at regular intervals (p. 16)

sample space a listing of all the outcomes possible for a particular experiment (p. 325)

sampling with replacement random sampling where the sample taken from any population is returned, without damage, to the population before a subsequent sample is taken. The size of the population remains constant. (p. 339)

sampling without replacement random sampling where the sample taken from any population is not returned to the population before a subsequent sample is taken. The size of the population decreases with each sample taken. (p. 339)

scale factor the number that relates the size of a map or drawing to the original it represents (p. 209)

scientific notation (standard form) a number is in scientific notation if it is written as $a \times 10^b$ where $1 \leq a \leq 10$ and b is an integer (p. 66)

secondary data data that have come from another source, such as the internet, and have not been collected directly (p. 292)

sector the area contained within two radii of a circle and the arc that joins them (pp. 103, 110)

significant figures the number of digits in a number that are counted or written after the number has been approximated (p. 66)

similar having the same shape but not necessarily the same size (p. 216)

similar triangles triangles where matching sides are in the same proportion and where matching angles are equal (p. 216)

simple interest the interest calculated on the initial principal for the entire term of a loan/investment (p. 27)

sine (sin) the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ for a particular angle in a right-angled triangle (p. 261)

skewed graph a statistical graph that is not symmetrical in shape (p. 309)

slope *see* gradient

solid a three-dimensional figure that has length, width and depth (p. 116)

solve find the solution (p. 157)

stem plot (stem-and-leaf plot) a statistical graph in which the leaf value represents the units digit for each value and the stem represents the other digit(s). The leaf values are ordered from smallest to largest, usually on the right-hand side of the corresponding stem values. (p. 309)

straight line the shortest distance between two points. When referring to graphs, an equation of the form $y = mx + b$. (p. 157)

subtended refers to the angle formed by an interval at a point that does not lie on that line (p. 103)

superannuation regular payments made to a fund to be accessed upon retirement (p. 23)

surd a number (irrational) that can only be expressed exactly using the radical ($\sqrt{\ }$) sign

surface area the total area of all the surfaces of a solid (pp. 124, 128)

symmetrical graph a graph that has an axis of symmetry that divides the graph into mirror image halves (p. 309)

tangent (tan) the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for a particular angle in a right-angled triangle (p. 261)

tax money collected by the government from Australian residents to pay for public services such as hospitals, schools, roads and security (p. 22)

tax deduction work-related expense incurred by the taxpayer in the course of earning an income (p. 22)

taxable income gross income minus allowable tax deductions (p. 22)

theorem a rule that has been proved to be always true

theoretical probability is based on what would occur under ideal circumstances. We use these theoretical probabilities as the basis for the probabilities we assign to events. (p. 325)

transformation any change that can be made to a graph or shape using rotation, dilation, reflection or translation (p. 381)

translation the transformation of a graph that keeps the shape, size and orientation of the graph exactly the same, but changes its location; usually involving a vertical or horizontal movement or a combination of the two (p. 381)

tree diagram a diagram consisting of lines branching out from other lines used to represent a sample space. Probabilities can be assigned to each branch and can then be used to calculate the probability of a particular outcome. (p. 339)

trigonometric ratio the ratio of sides of a right-angled triangle for particular angles. There are three trigonometric ratios: sine θ, cosine θ and tangent θ. (pp. 256, 261, 274)

trigonometry the part of mathematics that studies the relationship between the measures of sides and angles in right-angled triangles (p. 256)

true bearing a bearing that has been measured in a clockwise direction from north (p. 283)

turning point the point at which the gradient of a curve changes from positive to negative or negative to positive (p. 361)

two-way table information presented in rows and columns (p. 333)

undefined a value that cannot be calculated (p. 175)

variable in algebra, an unknown amount that can vary (pp. 49, 74)

vertex the corner of a shape (pp. 116, 205, 361); *see also* turning point

volume the amount of space inside a three-dimensional shape (p. 133)

wages income usually based on an hourly rate of pay. The rate may be higher for extra hours worked above the standard number or for work done on weekends or public holidays. (p. 16)