

Solomon Islands MATHEMATICS Year 8 Learner's Book

Book **1**



Solomon Islands
MATHEMATICS
Year 8 Learner's Book

Book 1

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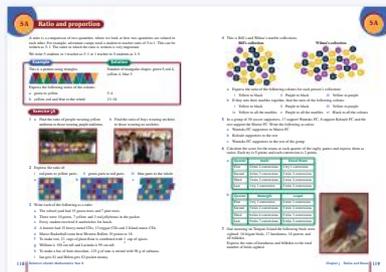
HOW TO USE THIS BOOK

The **Solomon Islands Mathematics** series has been written to cover the General Learning Outcomes of the Solomon Islands Secondary Mathematics Syllabus Years 7 to 9.



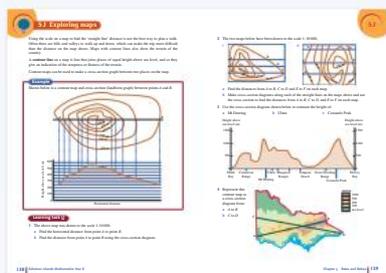
Chapter opening pages

Chapter opening pages include a contemporary or historical context for the content and provide learners with a list of the skills that are covered in the chapter.



Theory and exercise sections

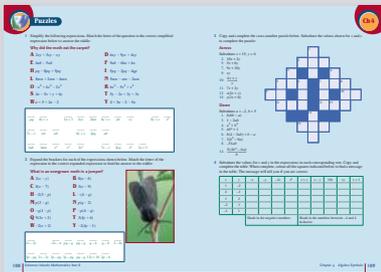
Theory and exercise sections contain explanations, examples and exercises designed to develop understanding of concepts and provide opportunities for students to practise new skills.



Explorations

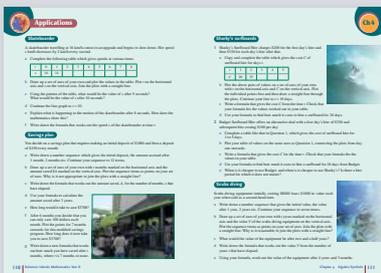
Explorations are scattered throughout the chapters, allowing students to work independently on non-standard problems and construct their own understandings.

These features are found at the end of each chapter



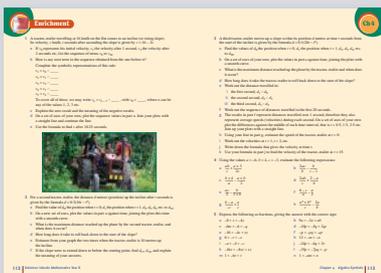
Puzzles

Puzzles are included for extra skills practice.



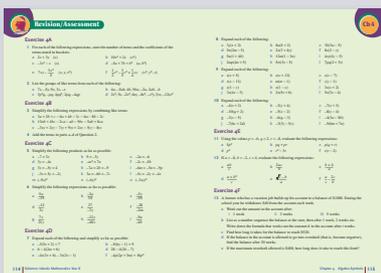
Application

Application sections investigate and apply mathematical ideas in a creative way and provide activities for a range of learner abilities.



Enrichment

Enrichment sections contain challenging tasks for learners to apply and extend their understanding of concepts.



Revision questions

Revision questions provide opportunities for learners to consolidate understanding of concepts.

Solomon Islands Mathematics Year 8 Learner's Book

Introduction

This book is written to help you learn Mathematics by actively participating in a variety of activities. The book has a total of 14 chapters. Each chapter focuses on a particular topic from one of the strands in the Solomon Islands Junior Secondary Mathematics Syllabus. The strands are *Number, Measurement, Algebra, Geometry, Statistics and Probability*. We hope that the activities in this book will encourage you to learn Mathematics effectively, and to gain enjoyment and enrichment from the topics and contexts involved.

Chapter organisation

The chapter order provides opportunities to revise topics studied in earlier years, to learn new knowledge and skills, and to review and develop your understanding throughout the school year.

The Number strand

The chapters that will further develop your number skills include *Percentages, Ratios and Rates* and *Indices*. You will continue your understanding of numbers, the ways they are represented and the quantities for which they stand. You will develop accuracy, efficiency and confidence in calculating, both mentally and on paper. You will refine your ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements. It is important to maintain your competency in the four basic operations, and to apply them confidently with whole numbers, directed numbers, decimals and fractions. By extending these skills to percentages, ratios and rates, and indices, you will be able to apply Mathematics to solving problems in real-life contexts.

The Measurement strand

The chapters that will further develop your measuring skills include *Length and Perimeter* and *Area and Volume*. You will need to apply the standard metric units to a range of practical situations, and be able to use a range of tools to measure accurately and with precision. You will need to understand and use appropriate formulae to calculate areas and volumes and apply them to real-life problems. You will also learn to estimate measurements so that you will know whether you receive approximately the right amount when buying goods in a shop.

The Algebra strand

The chapters that will develop your algebra skills in Year 8 are *Algebra Symbols, Equations and Inequations* and *The Coordinate Plane*. You will learn to recognise patterns and relationships in Mathematics and the real world, and be able to generalise from them. You will develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams, to represent and communicate mathematical relationships, concepts and generalisations. You will also have opportunities to use algebraic expressions confidently to solve practical problems.

The Geometry strand

The chapters that will develop your geometric skills include *Polygons and Parallel Lines*, *Transformations*, *Pythagoras* and *Polyhedra and Networks*. Geometry is concerned with size, shape, position and the properties of space. This year you will gain knowledge of geometrical relations in both two and three dimensions, and recognise and appreciate their occurrence in the environment. You will develop spatial awareness, and the ability to recognise and make use of the geometrical properties and the symmetries of everyday objects. You will also appreciate how to use geometric models as aids to solve practical problems in time and space.

The Statistics and Probability Strand

There is one chapter for your study of *Statistics* and another for *Probability*. Statistics deals with the collecting, organising, presenting and interpreting of numerical information (data). Our newspapers are full of statistical information, and it is important that you can understand statements and graphs to check the accuracy of the conclusions. Your study of Probability will help you describe the chance of various events occurring. Games of chance are a fun way of learning about probability but the concepts can be applied to many real-life situations when outcomes are uncertain, and so the study of Probability develops our decision-making skills.

How to learn Mathematics

As you work through the chapters you will be asked to work on your own, work with a partner or in a group, and sometimes with the whole class. Therefore, you must be willing to participate actively in all the tasks and not rely on the teacher or friends for answers. When you actively participate you will learn a great deal as well.

Making mistakes

Learning Mathematics is a skill, like riding a bicycle. You cannot learn to ride a bicycle by just listening to the teacher telling you how to ride, you can only learn by doing it. Nobody has learnt to ride a bicycle without falling off many times. Making mistakes is part of the learning process and this is also true for Mathematics. The more familiar you are with the topic, the fewer mistakes you are likely to make. Like bicycle riding, Mathematics learning needs lots of practice and the exercises in this book are designed to help you practice until you become confident with each new skill. Homework is a chance to further practice the skills learnt in class, and what you can't do on your own, you can ask your teacher or a friend the next day.

Developing skills

Mathematics is more than a series of facts and rules. It involves understandings and skills that can be applied to new situations. After each lesson it is useful to reflect on your learning and in particular about the problem-solving strategies that you used that day. Those same strategies may be useful for other problems in the future. And if you discover a new skill, show it to a friend. Not only will your friend benefit, but it will help you remember it too!

Suggested teaching plan for the Year 8 Learner's Book

Semester 1

Weeks	Sub-strands	Allocated Times
	Number	
1	Chapter 1: Percentages	3 Weeks
2		
3		
	Measurement	
4	Chapter 2: Length and Perimeter	3 Weeks
5		
6		
	Geometry	
7	Chapter 3: Polygons and Parallel Lines	2 Weeks
8		
	Algebra	
9	Chapter 4: Algebra Symbols	2 Weeks
10		
	Number	
11	Chapter 5: Ratios and Rates	3 Weeks
12		
13		
	Probability and Statistics	
14	Chapter 6: Statistics	3 Weeks
15		
16		
	Geometry	
17	Chapter 7: Transformations	3 Weeks
18		
19		
20	<i>Mid-Year Examinations</i>	1 Week
Mid-year Holidays		

Semester 2

Weeks	Sub-strands	Allocated Times
	Algebra	
21	Chapter 8: Equations and Inequations	3 Weeks
22		
23		
	Number	
24	Chapter 9: Indices	2 Weeks
25		
	Algebra	
26	Chapter 10: The Coordinate Plane	3 Weeks
27		
28		
	Geometry	
29	Chapter 11: Pythagoras	3 Weeks
30		
31		
	Probability and Statistics	
32	Chapter 12: Probability	3 Weeks
33		
34		
	Measurement	
35	Chapter 13: Area and Volume	3 Weeks
36		
37		
	Geometry	
38	Chapter 14: Polyhedra and Networks	2 Weeks
39		
40	<i>Final Examinations</i>	1 Week
End-of-year Holidays		

CHAPTER

1

Percentages



Percentages

Percentages are often used to compare quantities and to describe changes in numbers. They are normally given as a ratio or a fraction that has a denominator of 100. For example, it is estimated that in 2012, the population of the Solomon Islands was growing at approximately 2.17% per year, that 37.3% of the population were between 0 and 14 years of age, and 48.6% of those children were girls.

At the end of Years 6 and 9 in Solomon Islands schools learners sit examinations, and graduation ceremonies are held to tell parents and learners of the outcome of the exams, who will continue at school and who will leave school. The results of examinations are often given as percentages. It has been recorded that high percentages of learners who leave during Years 6 and 9 are those that come from rural areas.

This chapter covers the following skills:

- Learning the definition of a percentage
Per cent means 'out of 100'
- Converting percentages to fractions and decimals
- Converting fractions and decimals to percentages
- Finding percentages of quantities
- Expressing a quantity as a percentage
- Applying percentages to situations of discount and interest
- Calculating percentage increase and decrease
- Percentage change =
$$\frac{\text{Increase or decrease in quantity}}{\text{original quantity}} \times 100$$

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.1.1.1** Define percentage and identify the symbol %.
- 8.1.2.1** Use diagrams to represent percentages.
- 8.1.2.2** Express quantities as percentages and fractions.
- 8.1.2.3** Place percentages in ascending or descending order.
- 8.1.3.1** Change percentages into fractions in their simplest form.
- 8.1.4.1** Convert fractions to percentages by multiplying by 100.
- 8.1.4.2** Change mixed numbers to percentages by first changing the mixed numbers to improper fractions and multiplying by 100.
- 8.1.5.1** Convert percentages to decimals by dividing the numerator by 100.
- 8.1.6.1** Change decimals to percentages by multiplying the decimal number by 100.

8.1.7.1 Calculate percentages by changing the 'of' sign to 'x' then evaluate by multiplying.

8.1.8.1 Express one quantity as the percentage of another by writing it as a fraction and then multiplying it by 100.

8.1.9.1 Calculate the percentage increase and decrease, using the formula:
Percentage change =
$$\frac{\text{Increase or decrease in quantity}}{\text{original quantity}} \times 100$$

8.1.10.1 Calculate percentages for practical applications of percentages and compare the changes in percentages and numbers.

8.1.11.1 Define discount.

8.1.12.1 Calculate the discounted price for various items.

8.1.13.1 Calculate costs, selling prices, discounts, increases and decreases using percentages. Compare discounts offered in shops using percentages.

1A

Introducing percentages

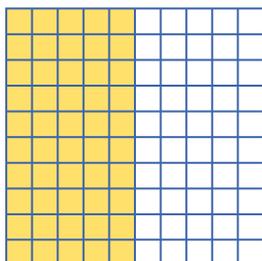
Per cent means ‘out of 100’. The symbol % is used to indicate a percentage.

10% of a dollar is 10 cents.

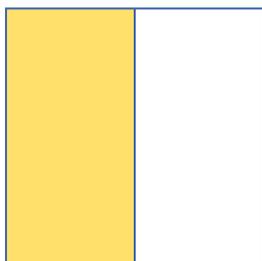
20% of a dollar is 20 cents.

50% of a dollar is 50 cents.

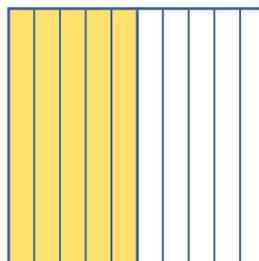
The grid below contains 100 boxes. It can be used to represent percentages.



50% is shaded = $\frac{50}{100}$



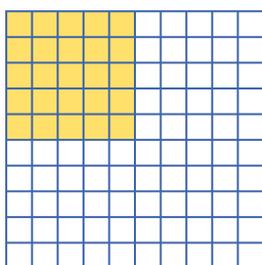
$\frac{1}{2}$



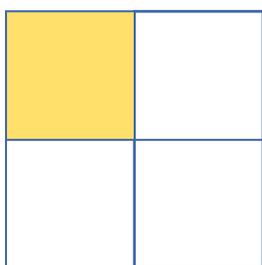
$\frac{1}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10}$
 $= 0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 0.5$

50% is shaded = $\frac{50}{100} = \frac{1}{2} = 0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 0.5$

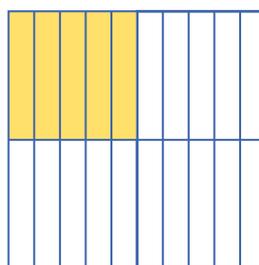
If 25 boxes out of 100 are shaded, 25% is shaded = $\frac{25}{100} = \frac{1}{4} = 0.25$



25% is shaded = $\frac{25}{100}$



$\frac{1}{4}$



0.25

Exercise 1A

1 Write the following amounts as percentages of 1 dollar:

a 16 cents

b 27 cents

c 3 cents

d 87 cents

e 41 cents

f 77 cents

g 94 cents

h 35 cents

i 19 cents

j 0 cents

k 55 cents

l 100 cents

m 24 cents

n 58 cents

o 60 cents

p 49 cents

2 Place the following percentages in order from smallest to largest:

a 14%, 28%, 9%, 93%, 41%

b 49%, 91%, 67%, 3%, 24%

c 98%, 13%, 27%, 32%, 69%

d 28%, 63%, 25%, 12%, 99%

e 94.6%, 23.8%, 9.5%, 9.56%

f 5.67%, 56.7%, 84.9%, 8.49%

g $57\frac{1}{2}\%$, $7\frac{2}{5}\%$, $27\frac{2}{3}\%$, $57\frac{1}{7}\%$

h $30\frac{1}{3}\%$, $13\frac{1}{4}\%$, $3\frac{4}{9}\%$, $93\frac{1}{4}\%$

i $6\frac{1}{2}\%$, $2\frac{1}{6}\%$, $26\frac{1}{4}\%$, $26\frac{1}{6}\%$

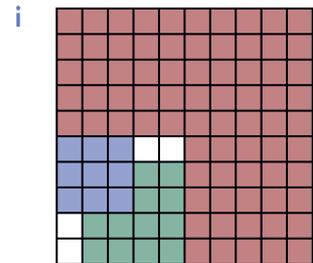
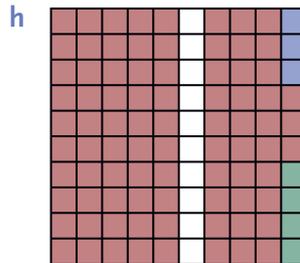
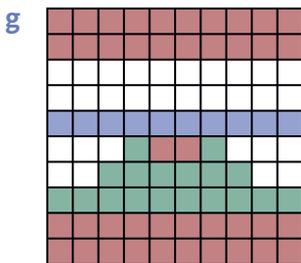
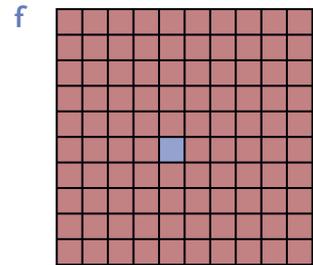
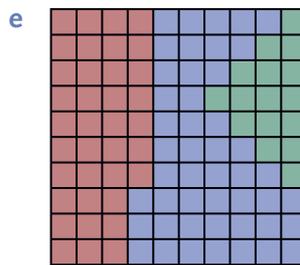
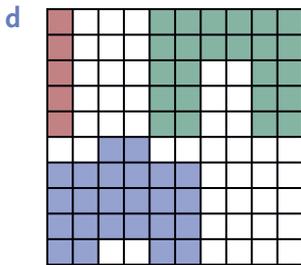
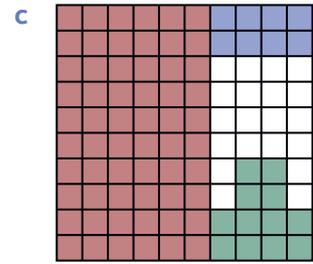
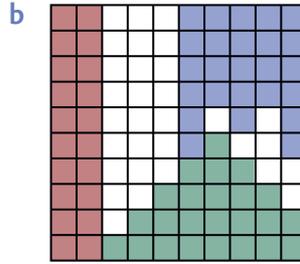
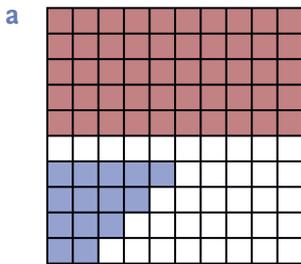
j $44\frac{1}{4}\%$, $42\frac{1}{2}\%$, $40\frac{1}{10}\%$, $40\frac{1}{3}\%$

3 Count the number of shaded squares in each grid below.

i What percentage of each grid is red?

ii What percentage of each grid is blue?

iii What percentage of each grid is green?



4 Copy and complete the table below:

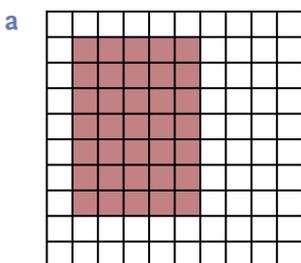
Fraction	Decimal	Percentage
		40%
	0.8	
$\frac{3}{10}$		
		70%
	0.5	
$\frac{3}{5}$		
		10%

5 Copy and complete the table below:

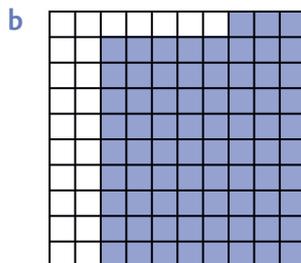
Fraction	Decimal	Percentage
$\frac{1}{3}$		
		$66\frac{2}{3}\%$
	0.75	
		5%
	0.2	
	0.125	
$\frac{9}{10}$		

6 Express the coloured region in each diagram as:

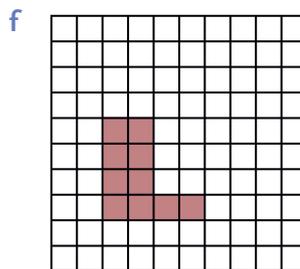
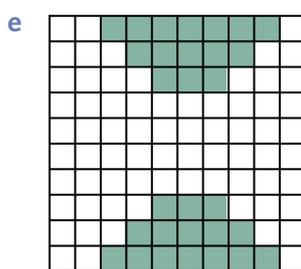
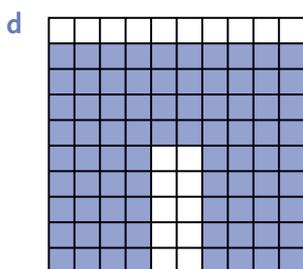
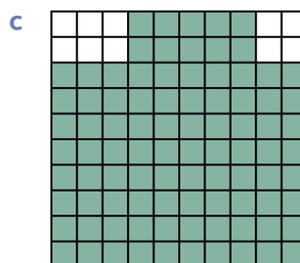
i a percentage



ii a fraction



iii a decimal



7 Kelly has 64 cents, Michael has 38 cents, Julian has 49 cents and Maddie has 83 cents.

- Write these amounts as percentages of a dollar.
- Who has the highest percentage?
- Who has the lowest percentage?
- Who has more than 75%?

8 If 12% of learners were away from school with the flu, what percentage of learners was at school?

9 All Year 9 learners at Biula Provincial Secondary School choose one of the three optional subjects for Term 1: Industrial Arts, Home Economics and Agriculture. If 26% of learners choose Industrial Arts and 38% choose Agriculture, what percentage of Year 9 learners choose Home Economics?

10 If 47% of the learners at a school are girls, what percentage of the learners must be boys?



- At a school, 3% of Year 8 learners have red hair, 5% have black hair and 26% have blond hair. The other learners have brown hair. What percentage of Year 8 learners have brown hair?
- At the same school, 7% of learners play hockey, 29% play tennis, 15% play squash and the others play soccer.
 - What percentage of learners play soccer?
 - List the sports in order from most popular to least popular.

50% can be 50 cents out of a dollar or 50 cents out of 100 cents.

$50\% = \frac{50}{100} = \frac{1}{2}$ so 50 cents is half a dollar.

Also, since 100% of a dollar is one dollar, 200% of a dollar must be two dollars.

Example

Convert the percentages to fractions:

a 75%

$$75\% = \frac{75}{100}$$

$$= \frac{3}{4}$$



b 150%

$$150\% = \frac{150}{100}$$

$$= 1 \frac{50}{100}$$

$$= 1 \frac{1}{2}$$

c $14\frac{1}{2}\%$

$$14\frac{1}{2} = \frac{29}{2}$$

$$14\frac{1}{2}\% = \frac{29}{2 \times 100}$$

$$= \frac{29}{200}$$

Exercise 1B

1 Write the following percentages as fractions in their simplest form:

- | | | | | | |
|--------|-------|--------|--------|--------|--------|
| a 23% | b 51% | c 9% | d 1% | e 86% | f 100% |
| g 14% | h 32% | i 160% | j 400% | k 20% | l 95% |
| m 360% | n 66% | o 750% | p 12% | q 125% | r 2% |

2 Write the following percentages as fractions in simplest form:

- | | | | | | |
|---------------------|---------------------|----------------------|----------------------|----------------------|---------------------|
| a $15\frac{1}{2}\%$ | b $42\frac{1}{2}\%$ | c $120\frac{1}{2}\%$ | d $300\frac{1}{2}\%$ | e $10\frac{1}{4}\%$ | f $16\frac{3}{4}\%$ |
| g $66\frac{2}{3}\%$ | h $3\frac{4}{5}\%$ | i $\frac{1}{4}\%$ | j $\frac{1}{2}\%$ | k $\frac{3}{4}\%$ | l $\frac{1}{3}\%$ |
| m $4\frac{1}{2}\%$ | n $20\frac{3}{4}\%$ | o $10\frac{2}{5}\%$ | p $\frac{4}{5}\%$ | q $212\frac{5}{8}\%$ | r $84\frac{2}{7}\%$ |

3 A bottle of soft drink is 75% full.

- | | |
|---------------------------------|-----------------------------------|
| a What fraction has been drunk? | b What percentage has been drunk? |
| c What fraction is left? | d What percentage is left? |

4 Milly planted some corn seeds and had 84% of the seeds germinate.

- | | |
|-----------------------------|--|
| a What fraction germinated? | b What percentage failed to germinate? |
|-----------------------------|--|

5 Mrs Teng divides a block of chocolate equally between her five children. What percentage does each child receive?

6 Kelvin uses $12\frac{1}{2}\%$ of a 200-gram packet of sugar in a cake recipe. What fraction of the packet does he use?

To change a fraction to a percentage, it is necessary to find an equivalent fraction with 100 as the denominator.

For example, to convert $\frac{3}{4}$ to a percentage the denominator, 4, would have to be multiplied by 25. So the fraction would need to be multiplied by $\frac{25}{25}$ to make a suitable equivalent fraction. That is:

$$\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

If you have a mixed number, convert it to an improper fraction first.

Example

Write the fractions as percentages:

a $\frac{3}{5}$

$$\begin{aligned}\frac{3}{5} &= \frac{3}{5} \times \frac{20}{20} = \frac{60}{100} \\ &= 60\%\end{aligned}$$

b $2\frac{1}{10}$

$$\begin{aligned}2\frac{1}{10} &= \frac{21}{10} \\ &= \frac{21}{10} \times \frac{10}{10} = \frac{210}{100} \\ &= 210\%\end{aligned}$$

Solution

Exercise 1C

1 Write the following fractions as percentages:

a $\frac{1}{4}$

b $\frac{2}{5}$

c $\frac{7}{10}$

d $\frac{3}{4}$

e $\frac{1}{2}$

f $\frac{2}{3}$

g $\frac{1}{8}$

h $\frac{5}{6}$

i $\frac{1}{3}$

j $\frac{3}{10}$

2 Write the following mixed numbers as percentages:

a $2\frac{1}{5}$

b $4\frac{1}{2}$

c $4\frac{3}{4}$

d $1\frac{3}{20}$

e $5\frac{1}{6}$

f $2\frac{4}{5}$

g $7\frac{1}{7}$

h $3\frac{1}{3}$

i $2\frac{3}{8}$

j $1\frac{2}{9}$

3 If $\frac{3}{5}$ of a cake has been eaten:

a what percentage has been eaten?

b what percentage is left?

4 A melon is cut into eight equal pieces; what percentage of the melon does each slice represent?

5 Hong has five trays of seedlings to plant. Each tray contains 12 plants. If Hong has planted three trays, what percentage of seedlings does he still have to plant?

6 In the Solomon Islands, a survey indicates that 29% of males and 24% of females smoke. Assume the Solomon Islands population is 50% male and 50% female.

a What fraction of males smoke?

b What fraction of females smoke?

c What percentage of the Solomon Islands population smoke?

d What fraction of the Solomon Islands population does not smoke?

To change a percentage to a decimal, simply divide the percentage number by 100. This can be done quickly by moving the decimal point two places to the left.

Example

Convert the percentages to decimals:

a 13%

$$\begin{aligned} 13\% &= \frac{13}{100} \\ &= 13 \div 100 \\ &= 0.13 \end{aligned}$$

b 270%

$$\begin{aligned} 270\% &= \frac{270}{100} \\ &= 270 \div 100 \\ &= 2.7 \end{aligned}$$

c 8.5%

$$\begin{aligned} 8.5\% &= \frac{8.5}{100} \\ &= 8.5 \div 100 \\ &= 0.085 \end{aligned}$$

Solution

Exercise 1D

1 Write the following percentages as decimals:

- | | | | | | |
|-------|-------|-------|-------|-------|-------|
| a 40% | b 32% | c 17% | d 24% | e 55% | f 80% |
| g 29% | h 67% | i 64% | j 86% | k 3% | l 78% |

2 Write the following percentages as decimals:

- | | | | | | |
|--------|--------|--------|--------|--------|--------|
| a 45% | b 31% | c 5% | d 1% | e 97% | f 100% |
| g 16% | h 50% | i 130% | j 420% | k 170% | l 340% |
| m 426% | n 225% | o 850% | p 542% | q 306% | r 127% |

3 Write the following percentages as decimals:

- | | | | |
|----------|---------|----------|----------|
| a 12.5% | b 6.5% | c 302.5% | d 140.5% |
| e 13.25% | f 5.75% | g 16.2% | h 8.8% |
| i 0.5% | j 0.25% | k 0.4% | l 0.75% |

4 Meika drinks 62% of a 1-litre bottle of soft drink.

- Write this percentage as a decimal.
- How many litres of soft drink did Meika drink?

5 Kieran's engine only has 23% of its oil. If the engine's oil capacity is 1 litre, how much oil is in the engine?

6 Home loan interest rates are 6.75%. Write this percentage as a decimal.

7 a The interest rate on credit cards is 16.65%. Write this percentage as a decimal.

b The daily interest rate is 0.045 61%. Express this percentage as a decimal.

c A business has a mark-up on its prices of 200%. Express this percentage as a decimal.

To convert a decimal to a percentage express it as a fraction out of 100. A short cut is to multiply the decimal by 100. This can be done quickly by moving the decimal point two places to the right.

Example

Convert the decimals to percentages:

a 0.35

$$0.35 = \frac{35}{100} \\ = 35\%$$

b 2.6

$$2.6 = 2.6 \times 100\% \\ = 260\%$$

c $0.\overline{56}$

$$0.\overline{56} = 56.\overline{56} \times 100\% \\ = 56.\overline{56}\%$$

Solution**Exercise 1E**

1 Change the following decimals into percentages:

a 0.75

b 0.5

c 0.3

d 0.55

e 0.92

f 0.63

g 0.41

h 0.7

i 0.66

j 0.02

k 0.62

l 0.96

m 0.212

n 0.345

o 0.457

p 0.240

q 0.001

r 0.0062

s 0.887

t 0.016

2 Change the following decimals into percentages:

a 2.88

b 3.09

c 5.19

d 1.45

e 10.65

f 9.19

g 2.81

h 4.2

i 5.987

j 5.099

k 3.005

l 7.002

m 2.4555

n 90.876

o 567.04

p 21.33333

3 Complete the following table for recurring decimals and percentages:

Decimal	Percentage
$0.\dot{3}$	
$0.\overline{12}$	
0.0 $\dot{1}$	
$2.\dot{4}$	

Decimal	Percentage
	$22.\dot{2}\%$
	$12.\overline{06}\%$
	$66.\dot{6}\%$
	$93.\overline{361}\%$

4 Daniel has hiked 0.87 kilometres of his 1-kilometre trip. What is this as a percentage?

5 Ratah purchases 1.5 metres of material which is cut from a 30-metre roll of fabric. What percentage of the roll of fabric did Ratah buy?

6 Solo drinks 0.125 litres of a 0.375-litre bottle of drink. What percentage of the drink is this?

7 A shift worker has worked 5.5 hours of an 8-hour shift. What percentage of his shift has he worked?

Finding percentages of quantities

1F

To calculate a percentage of a quantity, write the percentage as fraction, change the 'of' to '×', then multiply as usual. Some calculators have a percentage button.

Example

Find the following:

a 10% of \$380

$$\begin{aligned} 10\% \text{ of } \$380 &= \frac{10}{100} \text{ of } 380 \\ &= \frac{10}{100} \times \frac{380}{1} \\ &= \$38 \end{aligned}$$

b $12\frac{1}{2}\%$ of \$90

$$\begin{aligned} 12\frac{1}{2}\% \text{ of } \$90 &= \frac{25}{2}\% \text{ of } \$90 \\ &= \frac{25}{200} \times \frac{90}{1} \\ &= \frac{1}{8} \times \frac{90}{1} \\ &= \frac{90}{8} \\ &= \$11.25 \end{aligned}$$

Solution

Exercise 1F

1 Find:

a 20% of 70

b 25% of 98

c 10% of 420

d 30% of 860

e 50% of 46

f 25% of 240

g 35% of 140

h 12% of 480

i 75% of 480

j 30% of 170

k 15% of 300

l 2% of 400

2 Find:

a $12\frac{1}{2}\%$ of 80

b $3\frac{1}{2}\%$ of 1400

c $66\frac{2}{3}\%$ of 90

d $\frac{1}{2}\%$ of 600

e $3\frac{4}{5}\%$ of 750

f $16\frac{3}{4}\%$ of 1200

g $5\frac{1}{2}\%$ of 128

h $\frac{3}{4}\%$ of 900

i 6.5% of 200

j 10.2% of 500

k 3.3% of 11 000

l 40.5% of 400

3 a Copy and complete the table below, to find the total number of wickets for each player.

b Which player took the most wickets?

c Which two players took the same number of wickets?

Player	Percentage of wickets per over	Total number of overs bowled	Total number of wickets
Katy	10%	120	
Lencia	15%	560	
Wasi	25%	344	
Kimo	3.75%	320	

To express one quantity as a percentage of another, write the numbers as a fraction. Multiply the fraction by 100 to make it a percentage. Always ensure that the quantities are in the same units.

Example

- 1 What percentage is 75 of 300?

$$\begin{aligned} 75 \text{ out of } 300 &= \frac{75}{300} \times \frac{100}{1} \% \\ &= \frac{75}{3} \% \\ &= 25\% \end{aligned}$$

- 2 What percentage is 20c of \$5.00?

$$\begin{aligned} 20\text{c out of } \$5.00 &= 20\text{c as a percentage of } 500\text{c} \\ &= \frac{20}{500} \times \frac{100}{1} \% \\ &= \frac{20}{5} \% \\ &= 4\% \end{aligned}$$

Solution

Exercise 1G

- 1 Find the first number as a percentage of the second number:

- | | | | |
|--------------|---------------|---------------|-------------|
| a 16 of 32 | b 12 of 60 | c 19 of 76 | d 10 of 100 |
| e 6 of 72 | f 12 of 96 | g 24 of 120 | h 16 of 64 |
| i 40 of 4000 | j 130 of 2600 | k 55 of 137.5 | l 11 of 220 |

- 2 Find the first number as a percentage of the second number:

- | | | |
|-----------------------|---------------------|------------------------|
| a 50 cm of 1000 cm | b 15 min of 60 min | c 25 cents of 55 cents |
| d 12 min of 120 min | e 18 min of 45 min | f 48° of 360° |
| g 100° of 360° | h \$5 of \$500 | i 13 mm of 104 mm |
| j 5 min of 3 hours | k 12 cm of 3 metres | l 45 m of 500 km |
| m 5 seconds of 1 hour | n 32 m of 4 km | o 90 min of 12 hours |

- 3 Yu scored 45 of a possible 50 marks on a test. What is her score as a percentage?

- 4 a Catherine obtained 64 marks out of a possible 82 marks for her Maths test. What is this score as a percentage?

- b Catherine got 45 marks out of possible 60 marks for her Science test. What is the score as a percentage?

- c In which test did Catherine do better?



- 5 Mia has \$25 to spend at the trade show in China Town. She spent \$5 on ice cream and \$2 on peanuts. What percentage of her money did she spend?

- 6 Rory has 16 techno CDs. If Rory has a total of 80 CDs, what percentage of the CD collection is techno?

- 7 Lulu spends 15 minutes each night on English homework. If she spends a total of 2 hours on homework each night, what percentage of homework time does Lulu spend on English?

Percentage increase and decrease

1H

Percentages are used when people discuss issues involving changes in number. For example, rainforest destruction and increases in pollution levels are usually given as a percentage change. Retailers often use percentages when discounting the price of goods for a sale.

$$\text{Percentage change} = \frac{\text{increase or decrease in quantity}}{\text{original quantity}} \times 100$$

Example

- 1 A book was bought for \$5 and sold for \$9.50. Calculate the percentage increase.

$$\begin{aligned}\text{The increase in price} &= \$9.50 - \$5 \\ &= \$4.50\end{aligned}$$

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{increase in quantity}}{\text{original quantity}} \times 100 \\ &= \frac{4.50}{5} \times \frac{100}{1} \\ &= 90\%\end{aligned}$$

- 2 A school canteen sells 90 coconuts per day during sunny days but only 30 coconuts per day during rainy days. Calculate the percentage decrease.

$$\begin{aligned}\text{Decrease} &= 90 - 30 \\ &= 60\end{aligned}$$

$$\begin{aligned}\text{Percentage decrease} &= \frac{60}{90} \times \frac{100}{1} \\ &= 66\frac{2}{3}\%\end{aligned}$$

Exercise 1H

- Find the percentage increase from 50 to:
a 75 b 100 c 80 d 65 e 95 f 118
- Find the percentage decrease from 60 to:
a 30 b 15 c 42 d 24 e 8 f 1
- The number of players at a sports club decreases from 90 to 75. What is the percentage decrease?
- Before the Goods and Services Tax (GST) came into effect, a medium tin of Solomon Blue Taiyo cost \$4.00. After the GST the Solomon Blue Taiyo cost \$6.50 with the GST and the effect of rounding. What is the actual percentage increase in the price?
- The number of Year 8 learners this year is 88, which is 8 more than last year. What is the percentage increase?
- The temperature drops from 36°C to 24°C when the cool change arrives. What is the percentage decrease in temperature?
- As the low depression arrives, the wind speed increases from 20 km/h up to gusts of 80 km/h. What is the percentage increase?
- A 2-litre container of ice cream normally costs \$50.00 but was reduced to \$48.00 one week. What was the percentage decrease?



11 Exploring percentages

Percentages are used to compare quantities and express changes in numbers.

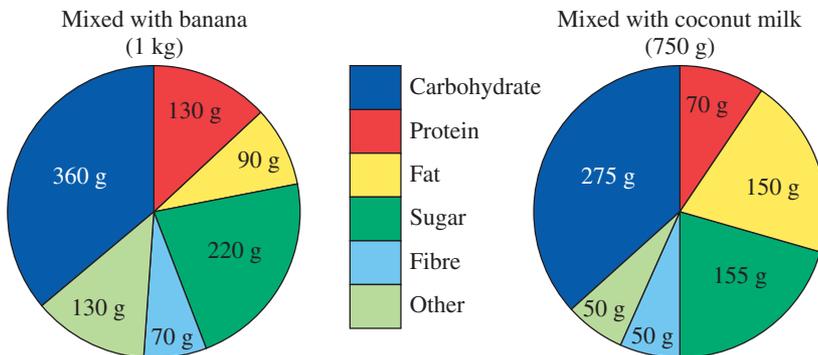
Learning task 11

- 1 The table below shows the breakdown of nutrients in a popular breakfast cereal.



Nutrient	Mass per 100 g serve	Percentage
Protein	21.9 g	
Fat	0.6 g	
Carbohydrate	71.7 g	
Fibre	2.7 g	
Sodium	0.6 g	
Potassium	0.15 g	
Vitamins and minerals	2.35 g	
Total	100 g	

- Copy and complete the table to show the percentages of each nutrient.
 - Add a new column to the table and add in the mass per 200 g serve for each nutrient.
 - Which nutrient has the highest percentage?
 - Which nutrients have the same percentage?
- 2 The pie charts below compare cassava puddings that are mixed with either banana or coconut milk.



- Construct a table for each type of cassava pudding and show the amount of each nutrient in grams.
- Convert the amount of each nutrient to a percentage of the total amount for each type of pudding.
- Which type of pudding has the higher percentage of sugar, fat and fibre?
- Which type of pudding do you think is the healthier?
- If the cassava pudding that is mixed with banana costs \$45.50, find the price per gram.
- If the cassava pudding that is mixed with coconut milk costs \$36.50, find the price per gram.
- Which cassava pudding is cheaper per gram?

- 3** Mix Fruits is a new type of smoothie sold at the Panatina Plaza Cafeteria. The amount of each ingredient is shown in the table below.

Mix Fruits 600 mL	
Banana	135 mL
Pawpaw	250 mL
Ice cubes	80 mL
Apple	120 mL
Lemon juice	15 mL

- a** Copy the table and add a percentage column.
b Calculate the total percentage of pawpaw in the drink.
 A 1-litre jumbo-sized drink is also sold that contains exactly the same percentages of each ingredient.
c Calculate the volume of each ingredient in the jumbo-sized drink.

- 4** The table below shows the prices for a range of food products in 1987 and 2002.

- a** Copy the table and use a calculator to find the percentage increases for each product over the 15 years.

For example, the cost of a 1.25 L bottle of Coca-Cola increased from \$9.50 to \$16.50.

$$\begin{aligned}\text{Increase} &= \$16.50 - \$9.50 \\ &= 7.00\end{aligned}$$

$$\begin{aligned}\% \text{ increase} &= \frac{\text{increase}}{\text{original cost}} \times 100 \\ &= \frac{7.00}{9.50} \times 100 \\ &= 73.68\%\end{aligned}$$

The price of Coca-Cola has increased by about 74% over the 15 years.

- b** Assuming that the rate of increase remains constant over the next 15 years from 2002, predict the price of each product in the year 2017.

For example, the price of Coca-Cola was \$16.50 in 2002.

$$\begin{aligned}\text{Prediction for 2017} &= 2002 \text{ cost} + \% \text{ increase} \times 2002 \text{ cost} \\ &= \$16.50 + 74\% \times \$16.50 \\ &= \$28.70\end{aligned}$$

- c** Compare the percentage increases for the various products. Which products increased the most? Can you find reasons to explain the differences?

Product	1987	2002	Increase	% Increase	Prediction for 2017
Coca-Cola 1.25 L	\$9.50	\$16.50	\$7.00	74%	\$28.70
Coffee 150 g	\$43.90	\$64.10			
Tomato sauce 600 mL	\$9.90	\$18.90			
Tea bags 100	\$22.90	\$43.80			
Cigarettes 200 carton	\$159.90	\$697.00			

Sometimes the prices on items are reduced or discounted by a certain percentage. To calculate the discount price, subtract the discounted amount from the original price.

Example

- 1 A dress priced at \$900 is reduced by 12%. Calculate the discount price.

Solution

$$12\% \text{ of } \$900 = \frac{12}{100} \times \frac{900}{1} \\ = \$108$$

So \$108 is the discount amount.

$$\text{Sale price} = \text{original price} - \text{discount} \\ = 900 - 108 \\ = \$792.00$$

The discount price is \$792.00.

An alternative method can be used to find a discounted price. The percentage paid can be calculated using the rule:

$$\bullet \text{ Percentage paid} = 100\% - \% \text{ discount}$$

Example

- 2 A jacket priced at \$780 is reduced by 15%. Calculate the discount price.

Solution

$$\text{Percentage paid} = 100\% - 15\% \\ = 85\%$$

$$85\% \text{ of } \$780 = \frac{85}{100} \times \frac{780}{1} \\ = \$663.00$$

So the discount price is \$663.00.

Exercise 1J

- Calculate the amount of the discount on a \$2400.00 CD stacker if it is offered at the following percentage discounts:

a 10%	b 12.5%	c 15%	d 25%
e 50%	f 62.5%	g 75%	h 90%
- Calculate the percentage paid if an item is offered for sale with the following percentage discounts:

a 8%	b 17.5%	c 20%	d 28%
e 55%	f 69%	g 78%	h 80%
- A jeanery is having a stocktake sale with 20% off the price of all items. How much would you pay for the following items?

a Jeans for \$800	b A windcheater for \$350	c A shirt for \$680
d A T-shirt for \$230	e A jacket for \$1360	f A jumper for \$950
- Tom has a choice of a \$650 scooter with a 10% discount or a \$750 scooter with a 15% discount. Which is the better buy?

- 5** A music store has a sale with 10% off all stock. How much will you pay for the following items?
- A top-10 CD priced at \$350
 - A jazz CD priced at \$280
 - A classical CD priced at \$560
 - A children's CD priced at \$240
- 6** A department store has a sale on. What price will you pay for each item?
- A hair dryer for \$680 which is discounted by 5%
 - A CD player with a retail price of \$1100 which is discounted by 8%
 - A DVD player for \$13 950 which is discounted by 12%
 - A television for \$8900 which is discounted by 7%
- 7** A business offers its customers a discount of 5% for accounts that are paid within 14 days. How much will each of the following customers save if they pay their accounts within 14 days?
- The Norris family has an account for \$7280.
 - The Lowe family has an account for \$1050.
 - The Conway partnership has an account for \$720.
 - Manderville has an account for \$12 960.
- 8** You buy a new tennis racquet and are offered a 15% discount on your next purchase. A tennis dress is \$1050. How much would it cost with the 15% discount?
- 9** A family purchases a cot and pram for their new baby, and is offered a discount of 10% off the purchase of either a high chair or a car seat. The high chair costs \$2290 and the car seat costs \$2590.
- How much would the high chair cost with the discount?
 - How much would they pay for the car seat with the discount?
 - Assuming the family needs both items, which item should they buy with the discount offer?
- 10** A family purchases a new computer and is offered a discount of 12.5% if they pay cash. The computer costs \$36 080 and the accessories including a printer, scanner and CD burner are an extra \$6640.
- How much is the computer with the discount?
 - How much are the accessories with the discount?
 - Find the total amount of the package with the discount.





1K Exploring shopping

Learning task 1K

1 Calculate the discount price of the items below if they are sold for 25% discount:

- a Sweet potatoes at \$10.00 per heap b Bush limes at \$5.00 per heap c Pineapples with prices of \$8.00 and \$15.00



2 Calculate the discount price of the items below if they are sold for 20% discount.

- a Melons at \$40 and \$50 b Yam heaps at \$8, \$10, \$15 and \$20



3 Calculate the cost of the following items given below after the GST of 15% has been added.

- a BMX bicycle at \$1220 b Acoustic guitar at \$2980



4 The Solomon Islands government imposes a 15% tax on some of the goods that are sold in the country. To calculate the price of an item before the Goods Tax (GT) is added, simply divide the sales price by 1.15.

For example, a guitar was sold for \$2875. To calculate the price of the guitar before GT, divide the sales price by 1.15, that is $\$2875 \div 1.15 = \2500 . Calculate the price of these items before GT was added.

- a A set of drums at \$6980 b An amp speaker at \$4580



- 5 The Rav 4 vehicle shown below was bought at a price of \$80 000. Calculate the cost before the GT of 15% was added.



- 6 Not all products increased in price when the GT was introduced into the Solomon Islands. Most electrical products already included 12% sales tax on the wholesale price. Find the cost today, assuming they have decreased by 12%:

- a Laptop \$15 000



- b TV \$2500



- c Camera \$2000





Puzzles

- 1 Convert the following percentages to fractions and simplify. Match the percentage to the correct fraction shown below to solve the riddle:

Why did the dinosaur cross the road?

A 20%

B 25%

C 45%

D 30%

E 10%

H 42%

I 90%

K 15%

L $66\frac{6}{7}\%$

N 35%

O 75%

S 12%

T 85%

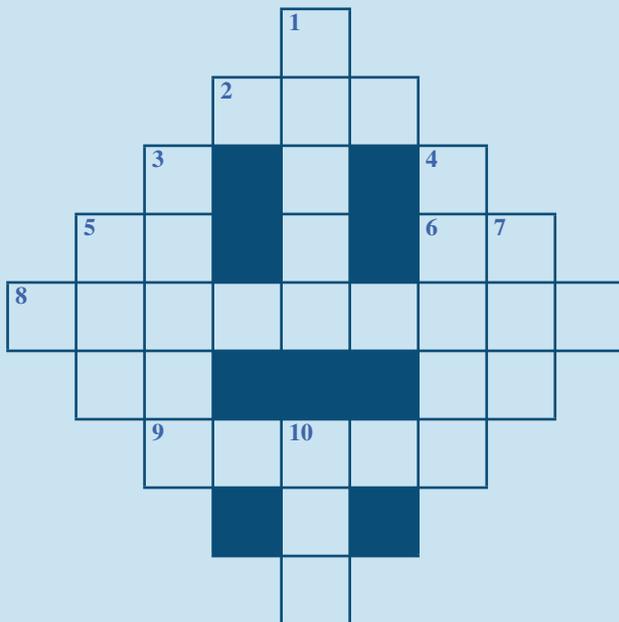
U $12\frac{5}{8}\%$

V $62\frac{5}{8}\%$

Y $37\frac{5}{8}\%$

$\frac{1}{4}$	$\frac{1}{10}$	$\frac{9}{20}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{3}{25}$	$\frac{1}{10}$							
$\frac{9}{20}$	$\frac{21}{50}$	$\frac{9}{10}$	$\frac{9}{20}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{7}{20}$	$\frac{3}{25}$		$\frac{21}{50}$	$\frac{1}{5}$	$\frac{3}{10}$		
$\frac{7}{20}$	$\frac{3}{4}$	$\frac{17}{20}$	$\frac{1}{10}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{1}{10}$	$\frac{3}{10}$		$\frac{3}{8}$	$\frac{1}{10}$	$\frac{17}{20}$

- 2 Calculate the percentages to complete the cross-number puzzle below:



Across

- 30% of 410
- 25% of 240
- 10% of 220
- 10 000% of 1 345 247
- 50% of 40 610

Down

- 175% of 24 104
- 200% of 10 241
- 125% of 10 164
- 75% of 844
- $12\frac{1}{2}\%$ of 1608
- $37\frac{1}{2}\%$ of 888

3 Calculate the percentage the first number is of the second number for each problem shown below. Match the letter to the correct percentage to solve the riddle:

Why did the turtle cross the road?

A 48 of 64

E 24 of 200

G 11 of 88

H 15 of 150

I 16 of 800

L 12 of 32

N 24 of 120

O 16 of 64

S 40 of 4000

T 130 of 260

U 55 of 137.5

V 11 of 220

50%	25%	12.5%	12%	50%	50%	25%	50%	10%	12%		
1%	10%	12%	37.5%	37.5%	1%	50%	75%	50%	2%	25%	20%



4 Find the percentage increase when 50 increases to each of the following values, then match the letter to the correct answer below to solve the riddle:

Why did the chewing gum cross the road?

A 55

B 80

C 65

E 75

H 70

I 150

K 125

L 51

O 60

R 52.5

S 62.5

T 50.5

U 52

W 51.5

Y 102.5

Z 100

60%	50%	30%	10%	4%	25%	50%	200%	1%	3%	10%	25%
25%	1%	4%	30%	150%	1%	20%	1%	40%	50%		
1%	4%	5%	1%	2%	50%						



Applications

Sporting percentages

$$\text{Sports percentages} = \frac{\text{points scored by the team}}{\text{points scored against the team}} \times \frac{100}{1} \%$$

Bob and John's tennis team is on top of the ladder but the newspaper does not print the percentage. Copy the table below into your workbooks and calculate the percentage for each week and for the season.

Points for	Points against	Percentage	Cumulative points for	Cumulative points against	Cumulative percentage
63	49		63	49	
68	50		63 + 68 = 131	49 + 50 = 99	
63	40				
72	0				
54	62				
57	44				
72	37				
67	51				

Keep the scores for your sports team, a football team or one of your school teams and calculate the weekly and cumulative percentages.

Simple interest

Simple interest or flat-rate interest is interest charged on the original amount of money borrowed, called the **principal**. The interest also depends on the length of time the money is borrowed, called the **term** and the interest rate charged per year, called the **rate**.

- Simple interest = principal \times rate \times term

$$SI = \frac{P \times R \times T}{100}$$

For example, to find the interest paid on \$5000 borrowed for 3 years at 6% per annum:

$$\begin{aligned} SI &= \frac{5000}{1} \times \frac{6}{100} \times \frac{3}{1} \\ &= \frac{5000 \times 6 \times 3}{100} \\ &= \$900 \end{aligned}$$

The interest is \$900, so \$5900 is paid back altogether.



- a The Carsons purchase a new car through a finance company. After they trade in their old car they borrow \$130 000 over 4 years at 7% per annum.
- How much interest do they pay?
 - What is the total cost of the car after the trade-in?
 - How much do they have to repay each month (48 months in total)?
- b Matthew buys a new tractor for \$645 700. He uses the manufacturer's finance company and borrows the entire value of the tractor at $3\frac{1}{4}\%$ simple interest over 5 years.
- How much interest does Matthew pay?
 - What is the total cost of the tractor?
 - How much does he have to repay each month?

How big is a bottle?

Champagne bottles come in a variety of sizes. A standard bottle holds 750 mL. Why 750 mL? One suggestion is that 750 mL is about one lungful of air and glass blowers used to expand the bottles by blowing into the hot glass. Some of the names of the bottles come from the Bible. Jeroboam and Rehoboam were both Kings of Israel, and Nebuchadnezzar was the most powerful King of Babylon. The names may have been chosen so that the bigger bottles were named after more rich and powerful leaders.



- a Copy and complete the table below:

Name	Volume	Number of standard bottles	Increase in volume from a standard bottle	Percentage increase from standard bottle
Standard	750 mL	1		
Magnum	1.5 litres	2	750 mL	100%
Marie-Jeanne	2.25 litres	3	1500 mL	200%
Jeroboam	3 litres	4		
Rehoboam	4.5 litres			
Methuselah	6 litres			
Salmanazar	9 litres			
Balthazar	12 litres			
Nebuchadnezzar	15 litres			

- b Think up your own names for the bottles, using a hierarchy of people or place names for the increasing size of the bottles. You could use Solomon Islands sporting heroes, politicians, cities or cars.



Enrichment

- 1 When I am 40% older than I am now, I will be 28 years old. How old am I now?
- 2 When I am 15% older than I am now, I will be 21 years old. How old am I now?
- 3 An investment portfolio has a value now of \$148 500. This portfolio has increased by 10% from its original value. What was the original value of the portfolio?
- 4 A house at Ngossi is now valued at \$200 080, which is an increase of 22% on the original price paid. Find the purchase price.
- 5 Commission is a form of payment for work, which is based on a percentage of the value of items or goods sold. A car salesperson receives 5% commission on all car sales. If she sells cars to the value of \$875 000, her wage will be 5% of sales:

$$\frac{5}{100} \times \frac{875000}{1} = \$43750$$

- a Ellen sells perfume to earn extra money. She earns 7% commission on her sales. How much does she earn each month if she sells perfume to the value of:
 - i \$1400?
 - ii \$3200?
 - iii \$2650?
 - iv \$6005?
 - b A car salesperson is paid a base wage of \$2000 per week and receives 3% commission on all car sales. If he sells cars to the value of \$270 000, \$165 000, \$332 800 and \$245 000 over a 4-week period, what will his wage be each week?
- 6 Simple interest on borrowed money can be calculated using the formula:

$$\text{Simple interest} = \text{principal} \times \text{rate} \times \text{time}$$
$$SI = \frac{P \times R \times T}{100}$$

If the principal is paid back during the loan period, then the amount of interest paid is affected.

- a Calculate the simple interest charged on \$100 000 at 12% for 5 years, if the principal is paid back at the end of the 5 years and the interest is paid annually. Compare this to the interest charged over the term of the loan if the interest is paid annually along with an equal portion of the principal ($\$100\,000 \div 5 = \$20\,000$).
- b Calculate the interest at the end of the first year.
- c If the borrower pays back the interest calculated in part b and \$20 000 of the principal, how much principal is still owing?
- d Calculate the interest charged in the second year.
- e Calculate the interest charged in the third, fourth and fifth years, remembering to reduce the principal owing each time.
- f Calculate the total interest paid on this loan.
- g What would happen if you paid your principal every 6 months instead of 12-monthly?

Loan A: A relative has some surplus money to invest for 3 years and offers your brother \$68 000 to buy a car. Their terms are 12% per annum simple interest, paid annually and the full \$68 000 is to be repaid at the end of the 3 years.

Loan B: A bank will loan your brother the \$68 000 for 4 years at 18% per annum simple interest (or 9% per 6 months), with half-yearly repayments of interest and principal.

- h Calculate which loan option charges the least interest.
- i Which loan do you recommend your brother choose and why?

- 7** Compound interest is the interest added to the principal before the next calculation of interest. For example, \$20 000 is invested at 12% per annum compound interest for 3 years, calculate the interest each year.

Principal = \$20 000, rate = 12% = 0.12, term = 1 year

$$\begin{aligned}\text{First year} \quad I &= PRT \\ &= 20\,000 \times 0.12 \times 1 \\ &= 2400\end{aligned}$$

$$\begin{aligned}\text{Second year} \quad I &= (20\,000 + 2400) \times 0.12 \times 1 \\ &= 2688.00\end{aligned}$$

$$\begin{aligned}\text{Third year} \quad I &= 25\,088.00 \times 0.12 \times 1 \\ &= 3010.56\end{aligned}$$

$$\text{Total interest} \quad I = 2400 + 2688 + 3010.56 = 8098.56$$

$$\text{Amount of investment } A = 20\,000 + 8098.56 = \$28\,098.56$$

- a** Calculate the amount of compound interest on an investment of \$8000 at 10% for 5 years.
b Use the calculations above to show that the compound interest formula is:

$$A = P \times (1 + r)^t \quad \text{where } A = \text{amount of investment, } P = \text{principal,} \\ r = \text{rate of interest, and } t = \text{number of years.}$$

Check the formula using the interest in part a.

- c** You invest \$100 000 at 9% per annum.
- Calculate your investment at the end of 5 years.
 - Calculate your investment at the end of 10 years.
 - Calculate your investment at the end of 20 years.

Inflation is a measure of the change in the purchasing power of your money. Annual inflation is approximately 1% currently and can be calculated using the compound interest formula above.

- d** A car costs \$240 000 today. How much will it cost in 5 years' time if inflation is:
- 1%?
 - 2%?
 - 8%?

- 8** Businesses calculate the depreciation on an item, which is an estimate of how much value the item has lost. Usually businesses calculate the depreciation as a percentage of the value of the goods at the start of the financial year. If goods are purchased during the year, then depreciation is calculated as a proportion of the year.

Item	Purchase date	Purchase price	Depreciation percentage
Tractor	1/7/96	\$36 000	30%
Cattle yards	1/7/96	\$102 000	20%
Dam	1/8/96	\$25 000	33%
Cattle crush	1/10/97	\$275 000	20%
Lawn mower	10/11/97	\$32 250	20%
Ute	14/9/98	\$70 000	30%

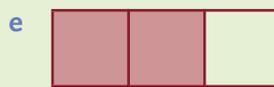
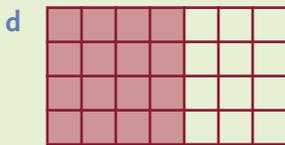
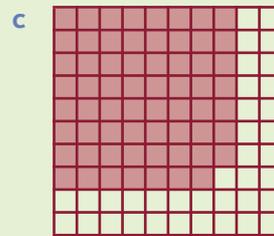
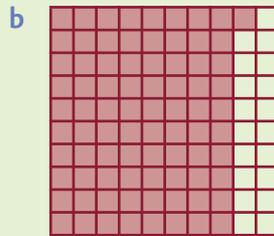
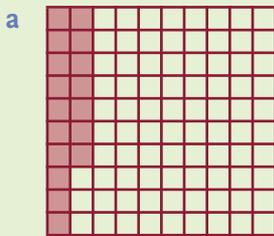
For each item, calculate its value at the end of the financial year (30 June) in which it was purchased.



Revision/Assessment

Exercise 1A

1 Express the following diagrams as percentages:



2 Complete the table:

	Fraction	Decimal	Percentage
a			90%
b	$\frac{1}{3}$		
c		0.48	
d	$\frac{1}{6}$		
e			$66\frac{2}{3}\%$
f		$0.\dot{2}$	

- A school offers three summer sports for boys. If 46% of boys play basketball, 20% play cricket and 16% play tennis during summer, what percentage of boys do not play sport during summer?
- Eighty-eight per cent of the cakes at a cake stall were sold.
 - What fraction of cakes were sold?
 - Express the percentage as a decimal.
- A cake is cut into 20 equal slices, what percentage of the cake does each slice represent?
- Ingrid has completed 0.65 of her Science assignment. What percentage is still to be done?

Exercise 1B

7 Change these percentages to fractions:

- | | | | |
|---------------------|---------------------|-------------------|--------------------|
| a 44% | b 300% | c 25% | d 15% |
| e $12\frac{1}{2}\%$ | f $33\frac{1}{3}\%$ | g $\frac{2}{5}\%$ | h $5\frac{1}{2}\%$ |

Exercise 1C

8 Write the following fractions and mixed numbers as percentages:

a $\frac{9}{10}$

b $\frac{11}{20}$

c $\frac{1}{50}$

d $\frac{5}{9}$

e $1\frac{2}{25}$

f $2\frac{1}{2}$

g $5\frac{1}{4}$

h $1\frac{1}{5}$

Exercise 1D

9 Write the following percentages as decimals:

a 30%

b 44%

c 81%

d 7%

e 205%

f 160%

g 0.9%

h 12.5%

Exercise 1E

10 Write the following decimals as percentages:

a 0.77

b 0.2

c 0.89

d 0.06

e 2.1

f 1.55

g 3.008

h 4.9

Exercise 1F

11 Find:

a 30% of 160

b 45% of 20

c 12% of 400

d 7% of 1000

e $2\frac{1}{2}\%$ of 200

f $\frac{3}{4}\%$ of 400

g 6.1% of 500

h 15.5% of 600

12 Stefan obtained 95% for his English test, which totalled 140 marks. How many marks did Stefan score?

13 Marsha scored 112 marks out of 120 for her History test. What percentage did she get?

Exercise 1G

14 Find the first quantity as a percentage of the second:

a 5 of 50

b 12 of 120

c 4 of 20

d 3 of 36

e 12 min of 60 min

f 10 m of 400 m

g 2 cm of 1 m

h 24 cents of \$2

i 50 mg of 4 g

Exercise 1H

15 A shopkeeper purchased a lounge chair for \$2500 and sold it for \$5250. What was the percentage increase?

16 A supermarket reduces ripe mangoes from \$39 per kilogram to \$25 per kilogram. What is the percentage decrease?

Exercise 1J

17 A camera valued at \$6950 was on sale for 15% off the retail price. What was its new price?

18 A shirt valued at \$699.50 is on sale for 25% off the retail price. What is its new price?

CHAPTER

2

Length and
Perimeter



Length and Perimeter

Solomon Islands is made up of hundreds of islands that spread out over a large area of the South Pacific Ocean. People have sailed from island to island using dugout canoes for centuries. Canoes are used to sail short distances between villages and longer distances between islands. Dugout canoes are also used for fishing and for travelling to gardens. They continue to be the main form of transport that islanders in the Solomon Islands use today.

This chapter covers the following skills:

- Estimating the length of lines and objects by using known lengths
- Using measuring instruments accurately
- Using common prefixes of units and converting between units
- Units of length
 $1 \text{ km} = 1000 \text{ m}$
 $1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$
 $1 \text{ cm} = 10 \text{ mm}$
- Calculating the perimeters of shapes including polygons and circles
- Perimeter of shape is the distance around the outside of the shape
- Perimeter of square = $4a$
Perimeter of rectangle = $2(a + b)$
Perimeter of rhombus = $4a$
Perimeter of parallelogram = $2(a + b)$
- Circumference of circle:
 $C = \pi D$ or $C = 2\pi r$
- Estimating uncertainty in measurements

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.2.1.1** Estimate the lengths of objects by comparing them to known lengths.
- 8.2.2.1** Use rulers and tapes to accurately measure the lengths of objects and shapes.
- 8.2.3.1** Identify standard units for lengths and their corresponding values in metric units:
 $1 \text{ kilometre (km)} = 1000 \text{ metres (m)}$
 $1 \text{ metre (m)} = 100 \text{ centimetre (cm)}$
 $1 \text{ centimetre} = 10 \text{ millimetres (mm)}$
- 8.2.4.1** Change metric units of length from one to another using the appropriate conversion factors.
- 8.2.5.1** Identify different units that are used to measure objects.
- 8.2.6.1** Add and subtract lengths with same and or different units by first converting them to the same unit.

8.2.7.1

Define the terms 'perimeter' and 'polygon'.

8.2.7.2

Calculate the perimeter of polygon shapes.

8.2.8.1

Calculate the perimeter of quadrilateral shapes with different units.

8.2.9.1

Define the circumference of a circle.

8.2.10.1

Measure the circumference of circular objects by using ropes and tapes.

8.2.10.2

Write a general statement in words connecting the diameter of a circle to the circumference.

8.2.11.1

Define diameter of a circle.

8.2.11.2

Divide the circumference by the diameter of regular objects to identify the relationship between them.

8.2.12.1

Define radius and arc length.

8.2.12.2

Calculate the perimeter of any circles using: $C = \pi D$ or $C = 2\pi r$

8.2.12.3

Find different arc lengths for circles.

8.2.13.1

Explain the term 'composite shape'.

8.2.13.2

Calculate the perimeter of composite shapes.

8.2.13.3

Calculate the error that may arise when measurements are given as percentages or other units.

2A

Estimating lengths

In order to guess or estimate the size of an object we need to compare it to a known length. The objects must be close to each other so that perspective doesn't fool us.

Example

Estimate the height of the mother if her daughter, Lily (wearing the blue T-shirt), is 0.9 metres tall.



Solution

The mother is about two times taller than the height of her daughter, Lily.

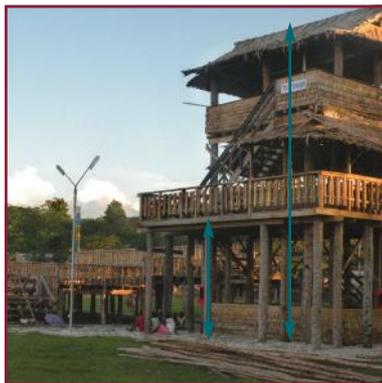
$$\begin{aligned} \text{Height} &\approx 2 \times 0.9 \\ &= 1.8 \end{aligned}$$

So the height of the mother is approximately 1.8 metres.

Exercise 2A

1 Estimate the following lengths:

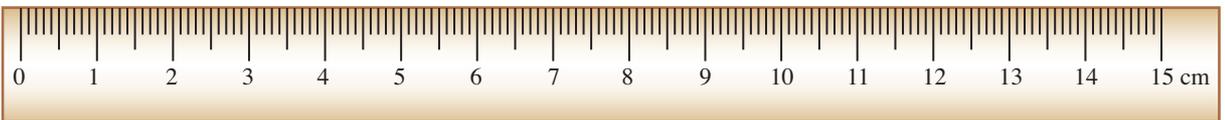
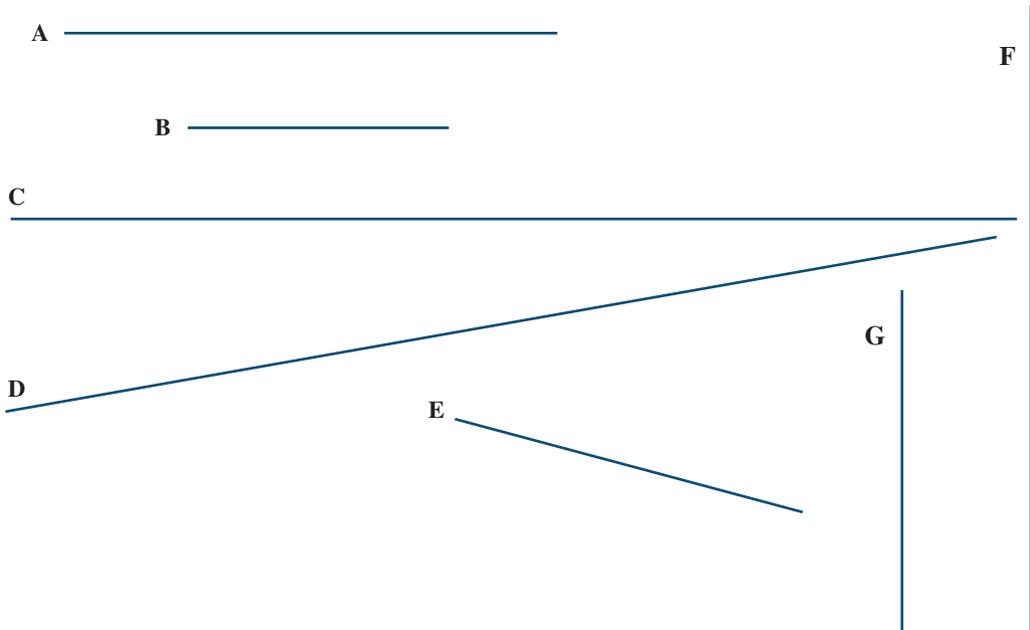
- a The height of the Arts Festival TV tower if the height of the ground floor posts is 112 centimetres.
- b The height of the statue if the height of the young boy is 4.5 metres.



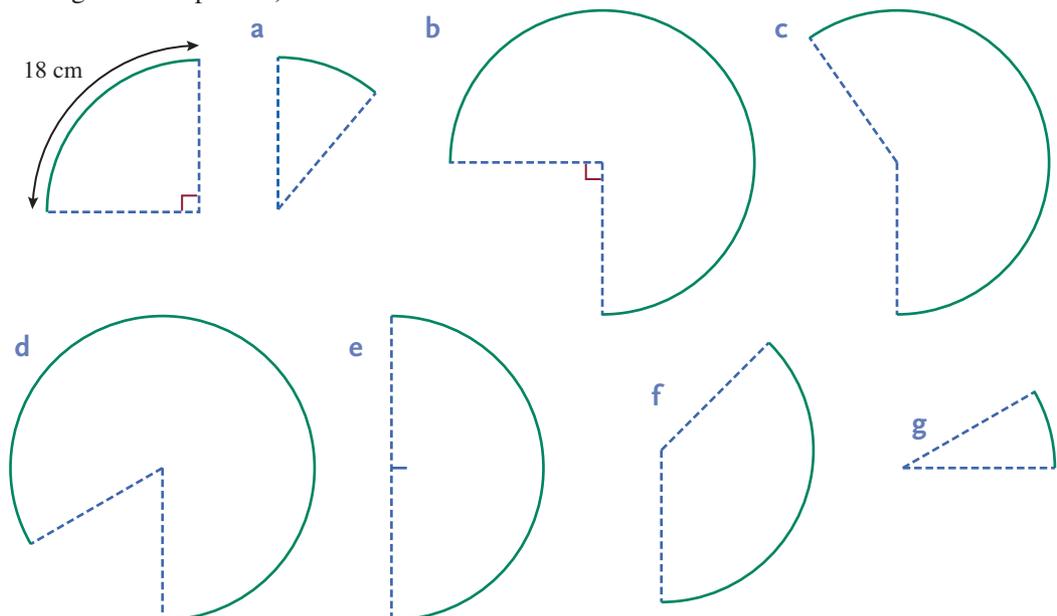
- c The height of Radike Samo (wearing the cap) if Patrick's height is 1.6 metres.
- d The height of the roof of the Arts Festival Pasifika Stage if the height of the stage is 1.2 metres.



- 2 Estimate the lengths of the following lines by using the scale given below, then measure them to check your answers. List the lines in order of easiest to hardest to estimate. Explain the order.



- 3 Estimate the lengths of each circular section (arc) by using the first one as a guide (the angles are important):



It is important to measure lengths as accurately as possible, especially when making things. Material needs to be cut properly when making clothes to avoid waste. Pieces of wood need to be cut to the correct length when making a door that will shut properly. Carpenters use the following ‘rule of thumb’: measure twice, cut once.

We usually use rulers or tapes to measure length. The length must start on the zero of the ruler and the scale be read off as accurately as possible.

Example

Use the ruler below to measure the height of the cup.



Solution

Place the ruler in the same direction as the object.

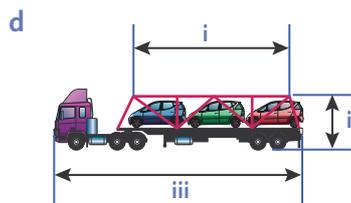
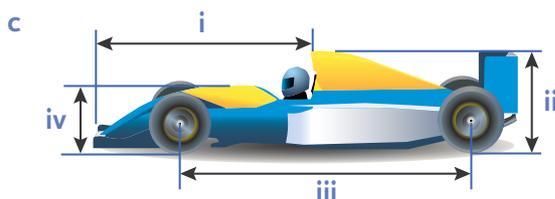
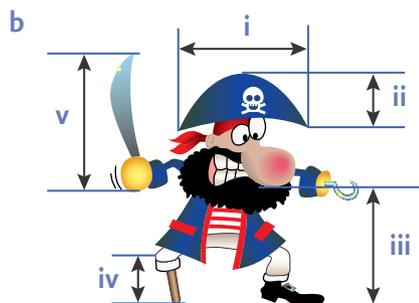
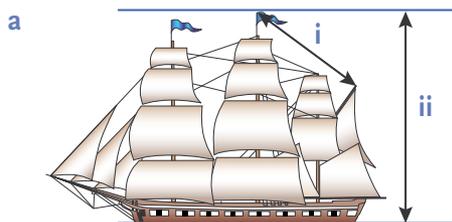
Make sure that the bottom of the object is lined up with the zero.

Look closely at the scale at the top of the cup and read it as accurately as possible.

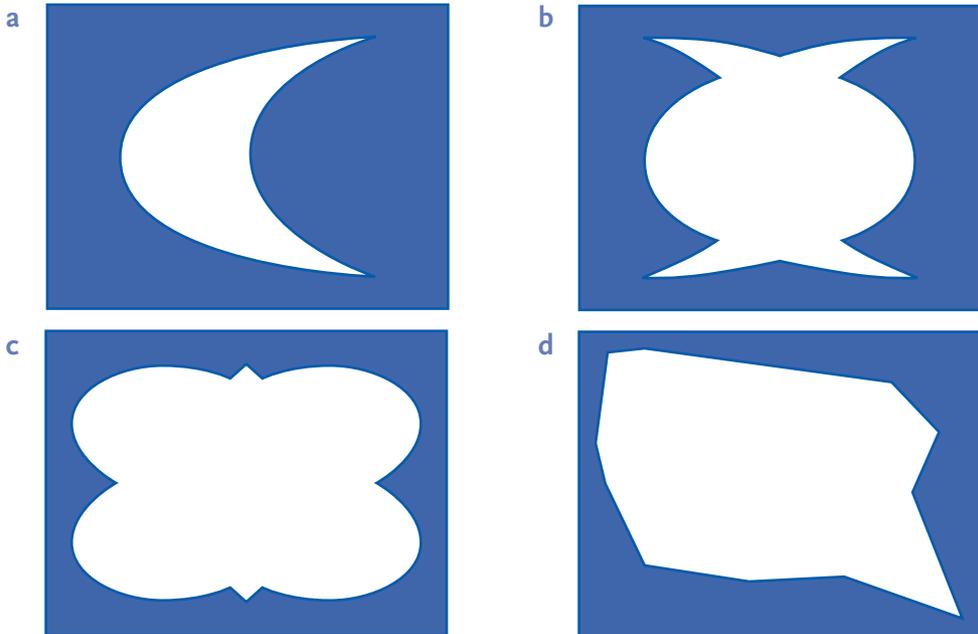
The height of the cup is 13.5 cm.

Exercise 2B

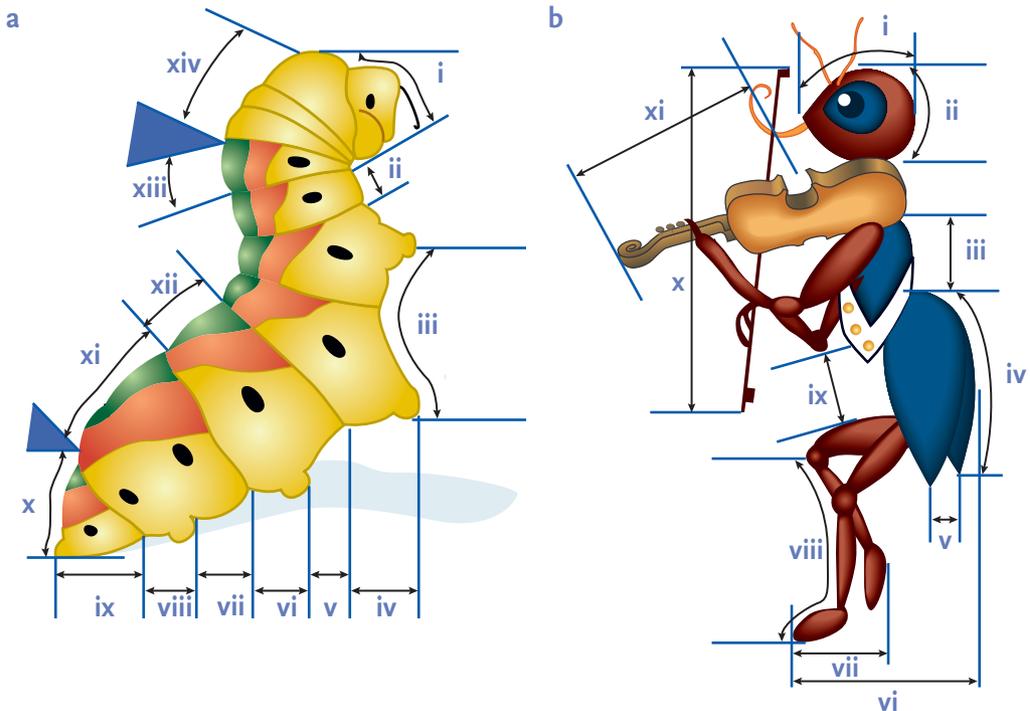
- I Accurately measure the lengths of the following black-arrowed lines to the nearest millimetre:



- 2 Find the distance around these white silhouettes by using a piece of string. Give your answer to the nearest millimetre:



- 3 Measure these lengths to the nearest millimetre:



2C

Converting units of length

The standard unit of length in the metric system is the metre. The smaller and larger length units are related to it via a series of conversion factors based on the number 10.

Milli means ‘a thousandth’ and so a millimetre (mm) is a thousandth of a metre (m), or there are 1000 millimetres in 1 metre.

$$1000 \text{ mm} = 1 \text{ m}$$

Centi means ‘a hundredth’ and so a centimetre (cm) is a hundredth of a metre (m), or there are 100 centimetres in 1 metre.

$$100 \text{ cm} = 1 \text{ m}$$

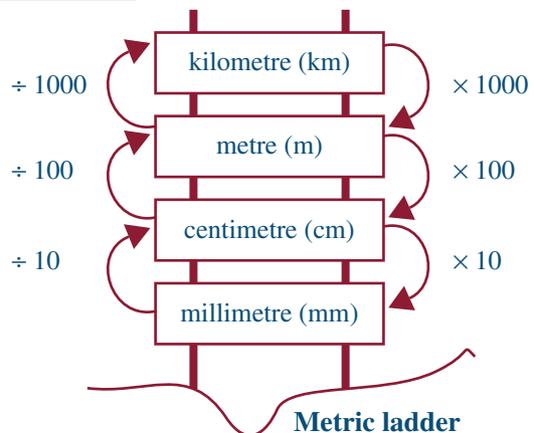
Kilo means ‘thousands of’ and so a kilometre (km) is a thousand metres (m), or there are 1000 metres in 1 kilometre.

$$1000 \text{ m} = 1 \text{ km}$$

The units can be thought of as being connected by either multiplication or division by using powers of 10. Use the metric ladder as a visual guide.

When climbing down the ladder, multiply.

When climbing up the ladder, divide.



Example

1 Find the factor that can be applied to convert:

a centimetres to millimetres $\times 10$

b millimetres to metres $\div 1000$

2 Use a conversion factor to fill in the blanks:

a $20 \text{ m} = \underline{\quad} \text{ cm}$ $20 \times 100 = 2000 \text{ cm}$

b $80 \text{ mm} = \underline{\quad} \text{ cm}$ $80 \div 10 = 8 \text{ cm}$

c $6\frac{1}{2} \text{ cm} = \underline{\quad} \text{ mm}$ $6\frac{1}{2} \times 10 = 65 \text{ mm}$

Solution

Exercise 2C

1 State the factor that would have to be applied to convert the following quantities:

a Metres to centimetres b Centimetres to millimetres c Kilometres to metres

d Millimetres to centimetres e Centimetres to metres f Metres to kilometres

2 Fill in the spaces:

- a $310 \text{ m} = \underline{\hspace{2cm}} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$
 b $\underline{\hspace{2cm}} \text{ km} = 0.063 \text{ m} = \underline{\hspace{2cm}} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$
 c $\underline{\hspace{2cm}} \text{ km} = \underline{\hspace{2cm}} \text{ m} = 1490 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$
 d $\underline{\hspace{2cm}} \text{ km} = \underline{\hspace{2cm}} \text{ m} = \underline{\hspace{2cm}} \text{ cm} = 8890 \text{ mm}$
 e $\underline{\hspace{2cm}} \text{ km} = 2560 \text{ m} = \underline{\hspace{2cm}} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$
 f $9.709 \text{ km} = \underline{\hspace{2cm}} \text{ m} = \underline{\hspace{2cm}} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

3 Convert the following into the units indicated:

- a $260 \text{ m} = \underline{\hspace{2cm}} \text{ cm}$ b $2900 \text{ km} = \underline{\hspace{2cm}} \text{ m}$ c $0.9 \text{ m} = \underline{\hspace{2cm}} \text{ cm}$
 d $580 \text{ m} = \underline{\hspace{2cm}} \text{ km}$ e $960 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$ f $0.07 \text{ m} = \underline{\hspace{2cm}} \text{ cm}$
 g $90 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$ h $720 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$ i $90 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$
 j $3\frac{3}{4} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$ k $9\frac{1}{5} \text{ m} = \underline{\hspace{2cm}} \text{ cm}$ l $5\frac{1}{4} \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

4 List the following lengths in descending order:

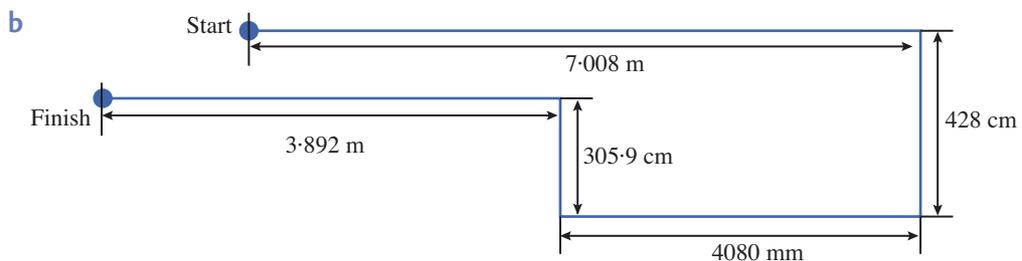
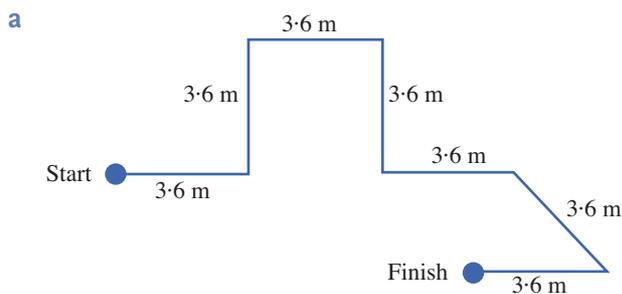
- a 129 mm , 13 cm , 0.224 m , 0.0009 km b 34 m , 560 cm , 0.0004 km , 2418 mm
 c 423 cm , 4501 mm , 67 m , 0.003 km d 42.1 m , 390 cm , 2490 mm , 0.0078 km

5 Each day Lucinda runs 230 metres more than her brother Christopher. If Christopher runs 1.4 km , 1.8 km , 700 m , 2.8 km and 4200 m each day for a week, find:

- a the total distance run by Christopher expressed in metres
 b the total distance run by Lucinda expressed in kilometres
 c the extra distance run by Lucinda compared to Christopher expressed in centimetres and millimetres.

6 Express the total length of these paths in:

- i metres ii centimetres iii millimetres



When working with situations where things are measured in different units, it is important to express each measurement in the same unit before they are combined. Usually the smaller unit is chosen to avoid the use of decimals.

When lengths are compared they must be expressed as the same unit. The length with the biggest value is the longest.

Example

1 Find the answer to each of the following:

a $5 \text{ km} + 2300 \text{ m}$

b $5.8 \text{ cm} - 26 \text{ mm}$

2 Which is longer, a pencil that is 25 cm long or one that is 224 mm long?

Solution

The units used are kilometres and metres. Work in the smaller unit, which is metres:

$$5 \text{ km} \times 1000 = 5000 \text{ m}$$

(express 5 km in metres)

$$5000 + 2300 = 7300 \text{ m}$$

The answer is 7300 m or 7.3 km.

$$5.8 \text{ cm} \times 10 = 58 \text{ mm}$$

(express 5.8 cm in mm)

$$58 - 26 = 32 \text{ mm}$$

The answer is 32 mm or 3.2 cm.

$$25 \text{ cm} \times 10 = 250 \text{ mm}$$

$$250 \text{ mm or } 224 \text{ mm}$$

250 mm is longer, i.e. the first pencil.

Exercise 2D

1 Find the answers to the following:

a $56 \text{ mm} + 25 \text{ mm}$

b $26 \text{ m} + 5 \text{ m}$

c $32 \text{ mm} + 48 \text{ mm}$

d $598 \text{ km} + 12 \text{ km}$

e $365 \text{ mm} - 25 \text{ mm}$

f $29 \text{ cm} - 14 \text{ cm}$

g $125 \text{ m} - 12 \text{ m}$

h $12 \text{ mm} - 9 \text{ mm}$

i $32 \text{ m} + 25 \text{ m} + 236 \text{ m}$

j $2 \text{ mm} + 3 \text{ mm} + 23 \text{ mm}$

k $14 \text{ km} + 2 \text{ km} + 56 \text{ km}$

l $69 \text{ cm} + 23 \text{ cm} + 95 \text{ cm}$

2 Find the answers to the following, expressing your answers in the smaller unit:

a $7.9 \text{ m} + 68 \text{ cm}$

b $7.9 \text{ km} + 390 \text{ m}$

c $58 \text{ cm} + 3.4 \text{ m}$

d $59 \text{ mm} + 4.2 \text{ cm}$

e $54 \text{ cm} + 130 \text{ mm}$

f $73 \text{ mm} + 2.9 \text{ cm}$

g $436 \text{ m} + 0.3 \text{ km}$

h $93 \text{ mm} + 5.3 \text{ cm}$

i $56 \text{ km} + 789 \text{ m}$

j $23 \text{ m} + 980 \text{ cm}$

k $45 \text{ cm} + 251 \text{ mm}$

l $569 \text{ cm} + 5.63 \text{ m} + 231 \text{ cm}$

3 Find the answers to the following, expressing your answers in the larger unit:

a $7.09 \text{ km} + 34 \text{ m}$

b $57 \text{ cm} + 4.3 \text{ m}$

c $87 \text{ mm} + 8.3 \text{ cm}$

d $86 \text{ m} + 5.9 \text{ km}$

e $78 \text{ m} - 270 \text{ cm}$

f $7.3 \text{ cm} - 59 \text{ mm}$

g $736 \text{ m} - 0.234 \text{ km}$

h $93 \text{ m} - 783 \text{ cm}$

i $600 \text{ m} + 2.6 \text{ km}$

j $256 \text{ mm} + 56.1 \text{ cm}$

k $78.9 \text{ km} + 9800 \text{ m}$

l $5.6 \text{ m} - 450 \text{ cm}$

- 4 A team of javelin athletes in the local athletics club had their throws measured.
- For each group state the longest and shortest throw.
 - Find the difference, in centimetres, between the longest and shortest throws.

Group 1: 3.8 m, 310 cm, 4.9 m, 280 cm

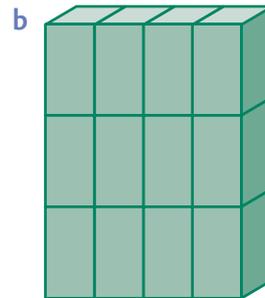
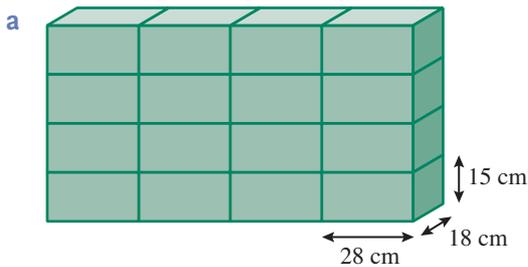
Group 2: 4250 mm, 5.9 m, 432 cm, 570 mm

Group 3: 470 cm, 4.75 m, 5280 mm, 0.0045 km

- 5 A group of boys had a football-kicking competition. Each boy kicked with his right and left foot, and the results are shown in this table.
- Find the total distance recorded by the boys when they kicked with their right foot.
 - Find the total distance recorded by the boys when they kicked with their left foot.
 - Find the difference between each boy's right and left kick.
 - List the boys in order from longest to shortest kick with their:
 - right foot
 - left foot

	Right foot	Left foot
William	17.4 m	586 cm
Matt	2140 cm	704 cm
Chao	1960 cm	6.4 m
Ling	20.4 m	789 cm
Rian	15.9 m	1600 cm
Martin	25.36 m	1987 cm

- 6 Boxes that are 28 cm long, 18 cm wide and 15 cm tall are stacked on top of each other in various ways. Find the length, width and height, expressed in metres, of each of the following stacks:



- 7 Semaika, Lui and Katalulu were the youngest children who joined the 2012 Easter Road Race held in Honiara. The distances each covered were:

Semaika: 373.1 m Lui: 1.0426 km Katalulu: 243 121 cm

- Which child walked the longest distance?
 - What distance, in centimetres, did he walk further than the others?
- 8 Chris, Kim, Alice and Emma are children in Lilisiana village that walk different distances to school each day. If the distances these children walk are given in the ratio of 1:2:3:6, then find the actual distance each child walks to school if the total distance walked is 12 km.

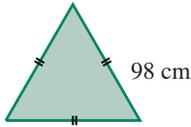


Polygons are shapes with straight sides. The **perimeter** of a shape is the distance around the outside. When finding the perimeter of a shape, make sure that the lengths are given in the same units before adding them up.

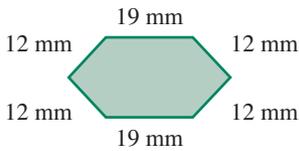
Example

Find the perimeter of the following:

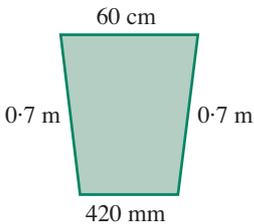
a



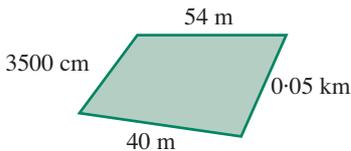
b



c



d



Solution

All the sides are the same length.

$$\begin{aligned} \text{Perimeter} &= 3 \times 98 \\ &= 294 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 12 + 12 + 12 + 12 + 19 + 19 \\ &= 4 \times 12 + 2 \times 19 \\ &= 48 + 38 \\ &= 86 \text{ mm} \end{aligned}$$

Change the measurements to the same unit:

$$0.7 \text{ m} = 70 \text{ cm}$$

$$420 \text{ mm} = 42 \text{ cm}$$

$$\begin{aligned} \text{Perimeter} &= 60 + 42 + 70 + 70 \\ &= 242 \text{ cm} \end{aligned}$$

Change the measurements to the same unit:

$$0.05 \text{ km} = 50 \text{ m}$$

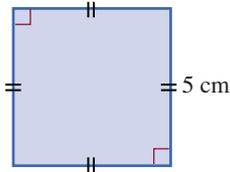
$$3500 \text{ cm} = 35 \text{ m}$$

$$\begin{aligned} \text{Perimeter} &= 50 + 35 + 54 + 40 \\ &= 179 \text{ m} \end{aligned}$$

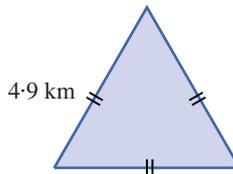
Exercise 2E

I Find the perimeter of the following:

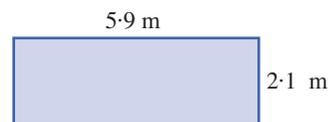
a



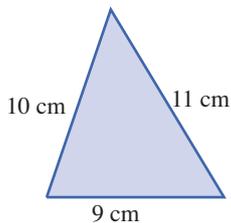
b



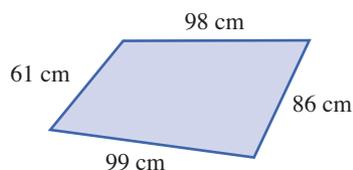
c



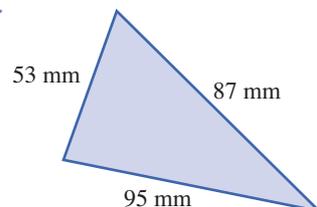
d



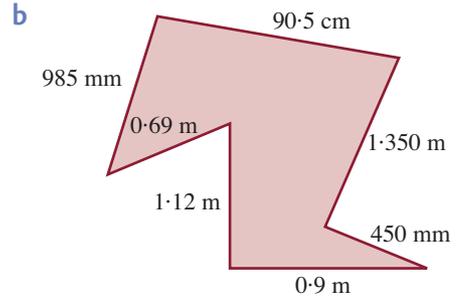
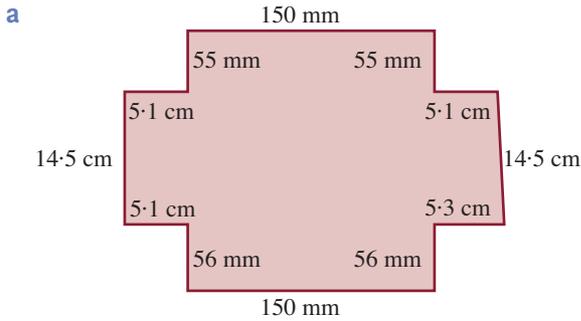
e



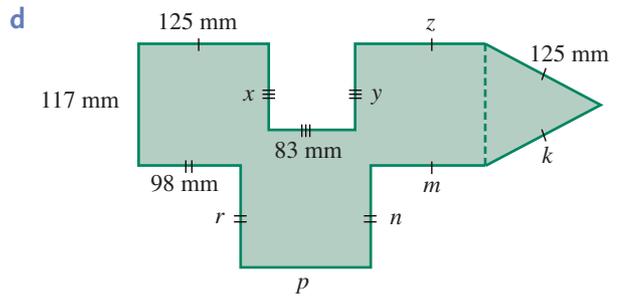
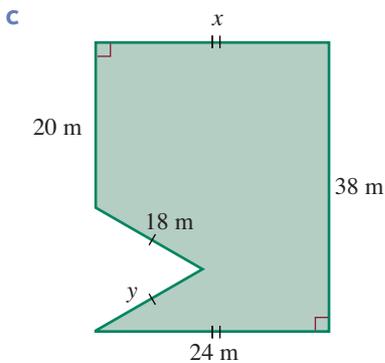
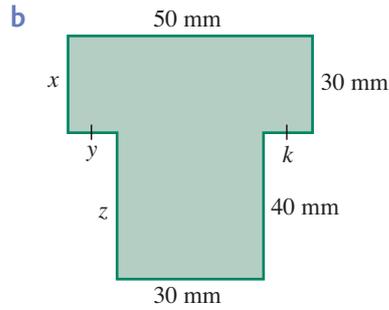
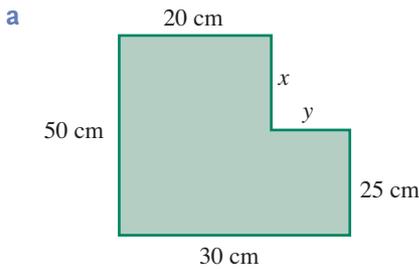
f



2 Find the perimeter of the following:



3 Mark the missing lengths on the diagram and find the perimeter of each shape:



4 Find the perimeter of the window frames shown below:

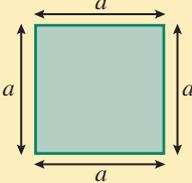
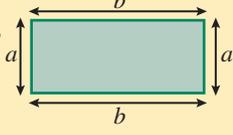
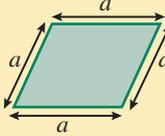
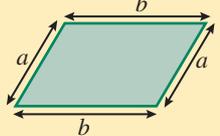
a Length 1.5 m
Width 1.2 m

b Side lengths 32 cm

c Length 1.6 m
Width 75 cm



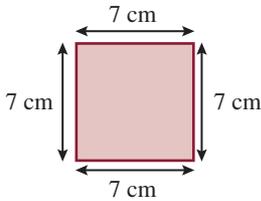
The perimeter of a shape is the distance around the outside. When finding the perimeter of a shape make sure that the lengths are given in the same units and then add them up. Rectangles, squares, rhombuses and parallelograms are special four-sided polygons whose opposite sides are equal. This makes it easy to find their perimeters using a formula.

<p>Square</p> <p>Perimeter = $a + a + a + a$ = $4a$</p> 	<p>Rectangle</p> <p>Perimeter = $a + a + b + b$ = $2a + 2b$ = $2(a + b)$</p> 
<p>Rhombus</p> <p>Perimeter = $a + a + a + a$ = $4a$</p> 	<p>Parallelogram</p> <p>Perimeter = $a + a + b + b$ = $2a + 2b$ = $2(a + b)$</p> 

Example

Find the perimeter of the following quadrilaterals:

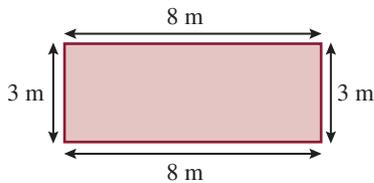
a



Quadrilateral is a square

$$\begin{aligned} \text{Perimeter} &= 4a \\ &= 4 \times 7 \\ &= 28 \text{ cm} \end{aligned}$$

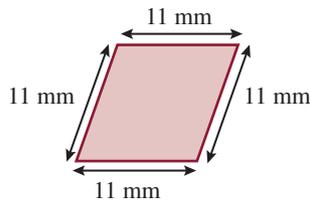
b



Quadrilateral is a rectangle

$$\begin{aligned} \text{Perimeter} &= 2(a + b) \\ &= 2(8 + 3) \\ &= 2 \times 11 \\ &= 22 \text{ m} \end{aligned}$$

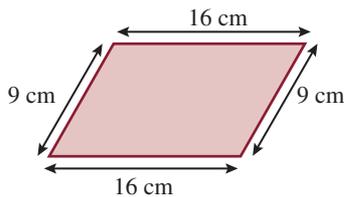
c



Quadrilateral is a rhombus

$$\begin{aligned} \text{Perimeter} &= 4a \\ &= 4 \times 11 \\ &= 44 \text{ mm} \end{aligned}$$

d

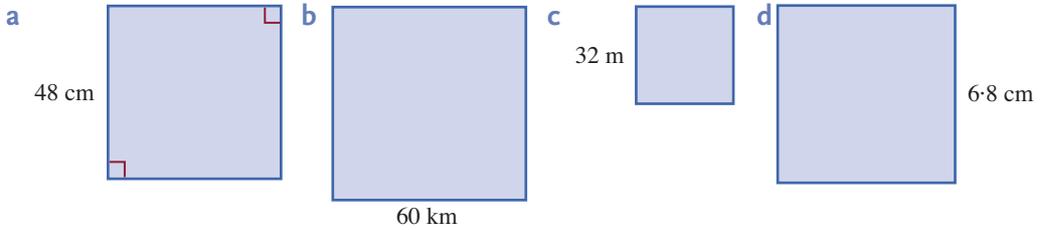


Quadrilateral is a parallelogram

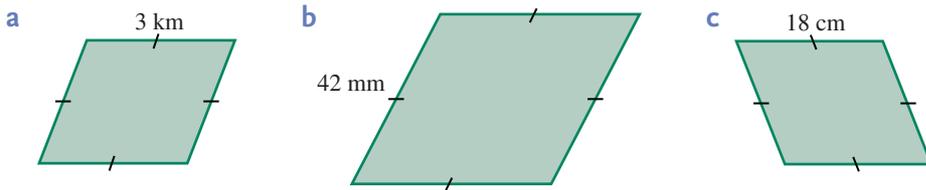
$$\begin{aligned} \text{Perimeter} &= 2(a + b) \\ &= 2(16 + 9) \\ &= 2 \times 25 \\ &= 50 \text{ cm} \end{aligned}$$

Exercise 2F

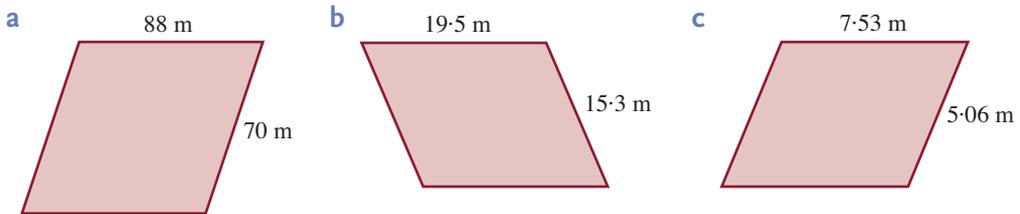
- 1 Use a formula to find the perimeter of these squares:



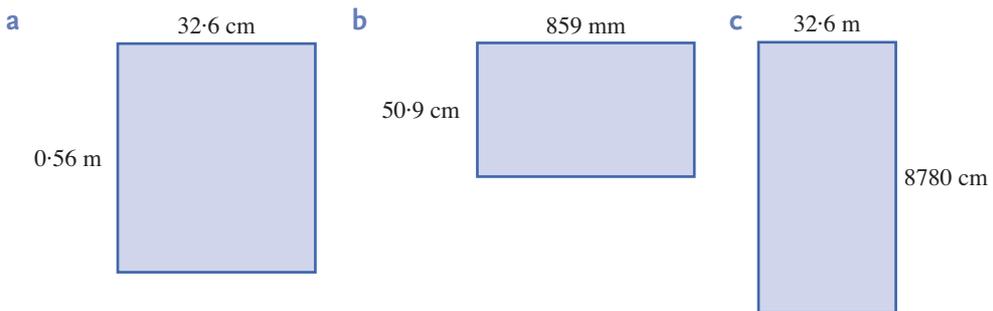
- 2 Use a formula to find the perimeter of these rhombuses:



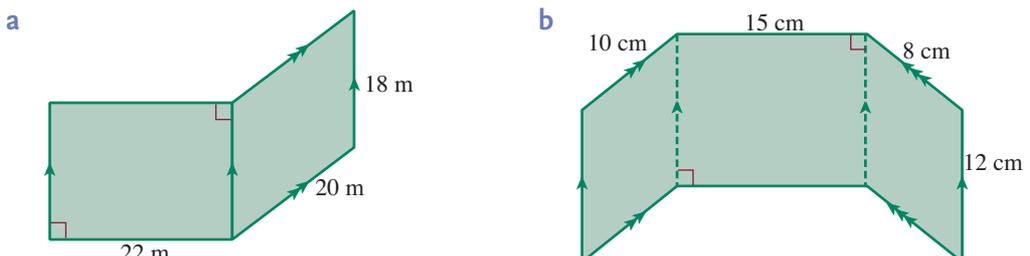
- 3 Use a formula to find the perimeter of the following parallelograms:



- 4 Use the formula to find the perimeter of the following rectangles, expressing the answers in centimetres and metres:



- 5 Use a formula to find the perimeter of these shapes:



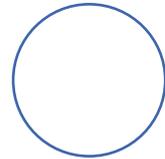


2G Exploring the circumference of a circle

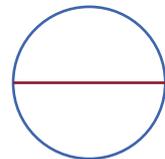
So far, we have studied the perimeter of shapes that generally have straight edges. The perimeter or the distance around a circle is called the **circumference**. The following activity is designed to help you develop a rule for finding the circumference of a circle.

Learning task 2G

- 1 Find five objects that are circular, for example a glue stick, a flower pot, the classroom clock, a rubbish bin, a bangle, a roll of sticky tape, a CD, a measuring cylinder.
- 2 Measure the circumference of each object, using a tape measure or a length of string.
- 3 Now measure the diameter. This is the distance from one side of the circle to the other and passing through the centre.
- 4 Copy and complete the table below:



Circumference



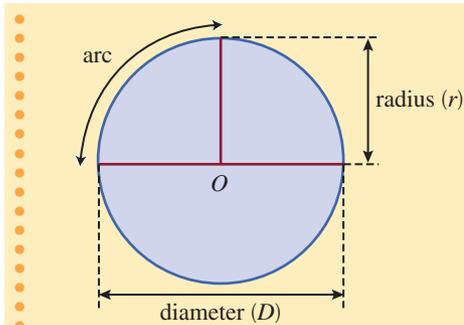
Diameter

Object	Circumference	Diameter	Circumference ÷ Diameter
a			
b			
c			
d			
e			

- 5 Divide the circumference by the diameter to complete the last column in the table. (A calculator may be useful here.)
- 6 For all the circular objects you measured, about how many times bigger is the circumference than the diameter?
- 7 Write a general statement in words connecting the diameter to the circumference of the circle.
- 8 Given that the radius of a circle is half the diameter, write a general statement in words connecting the radius to the circumference of a circle.
- 9 Estimate the diameter of the front wheel of a bicycle. Using the results found above, estimate the circumference of the bicycle wheel shown in the photo.
- 10 Estimate the diameter and circumference of the spare tyre shown in the photo.



The perimeter of a circle is called its **circumference** and a part of a circle is called an **arc**. One way to find the circumference of a circle is to place some string around it and then measure the length of the string. This is not accurate. There is a formula or rule that we can use to find an accurate answer for the circumference. To use it we need to know either the radius or the diameter of the circle.



Formula for circumference

$$C = \pi D$$

$$\text{or } C = 2\pi r$$

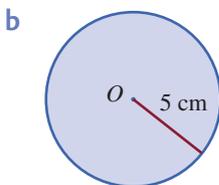
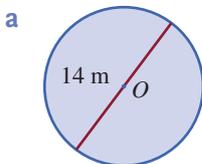
where π is approximately 3.14.

For accuracy use a calculator, which gives a better approximation for the number π as 3.141 592 7...

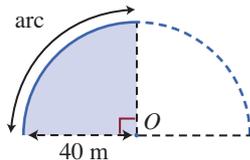
Answers in exact form are given in terms of π .

Example

- 1 Find the circumference of these circles, expressing your answer to 2 decimal places:



- 2 Find the length of the arc, giving your answer in exact form and correct to 2 decimal places:



- 3 Find the diameter of a circle whose circumference is 31.42 m. Give your answer correct to the nearest metre.

Solution

$$C = \pi D$$

$$C = \pi \times 14 = 14\pi$$

$$= 14 \times 3.14$$

$$= 43.98 \text{ cm}$$

exact form
2 decimal places

$$C = 2\pi r$$

$$= 2 \times \pi \times 5 = 10\pi$$

$$= 10 \times 3.14$$

$$= 31.42 \text{ cm}$$

exact form
2 decimal places

Arc is a quarter circle with radius 40 cm

$$C = \frac{1}{4}(2\pi r)$$

$$= \frac{1}{4} \times 2 \times \pi \times 40$$

$$= 20 \times \pi$$

$$= 20\pi \text{ m}$$

$$= 62.83 \text{ m}$$

exact form
2 decimal places

$$C = \pi D$$

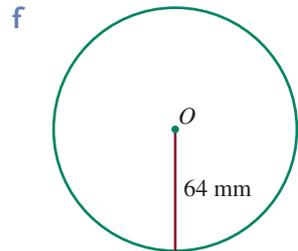
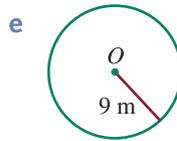
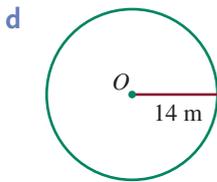
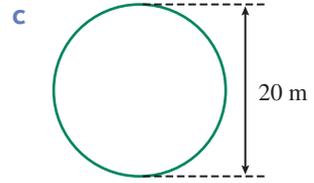
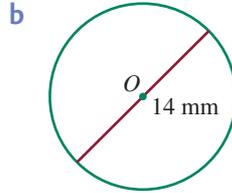
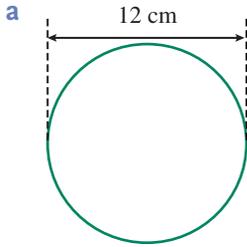
Using the formula:

$$31.42 = \pi D$$

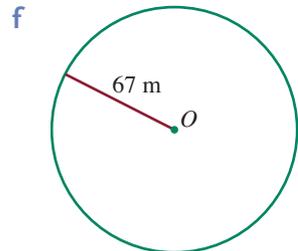
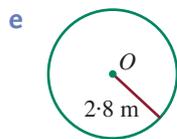
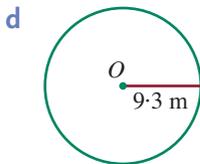
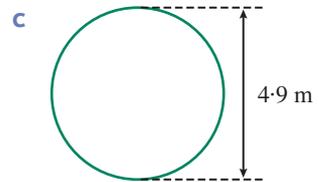
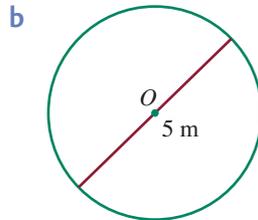
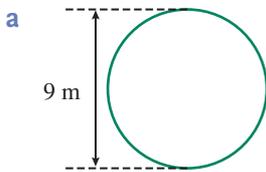
$$D = \frac{31.42}{\pi} = 10 \text{ m}$$

Exercise 2H

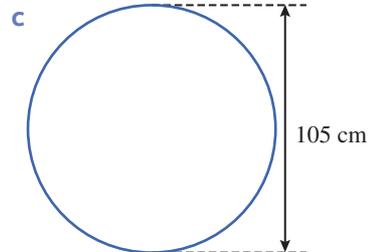
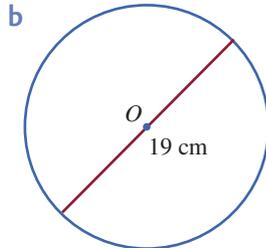
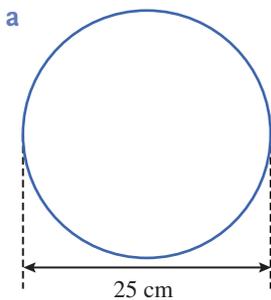
1 Find the circumference of each of the following circles, expressed to 2 decimal places:

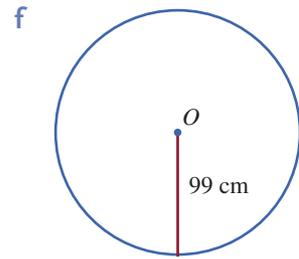
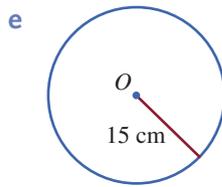
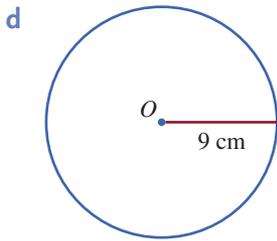


2 Find the circumference of the following circles, expressed to the nearest metre:

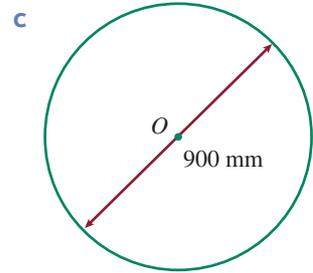
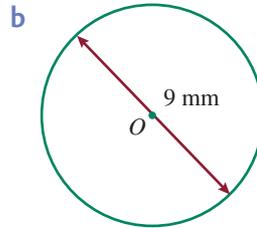
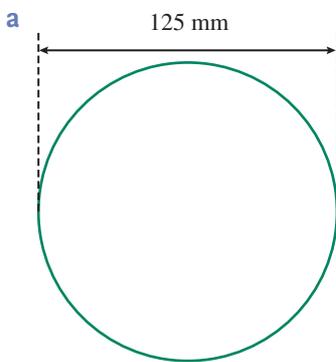


3 Find the circumference of the following circles correct to the nearest centimetre:

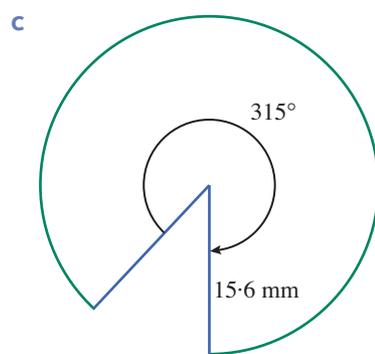
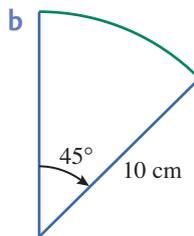
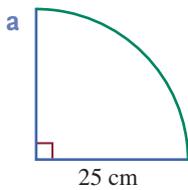




4 Find the circumference of the following circles to the nearest millimetre:



5 Find the length of these arcs to the nearest centimetre:



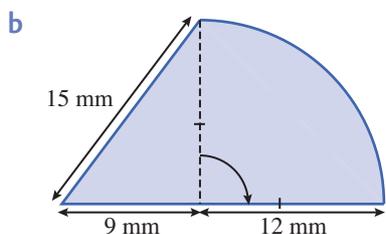
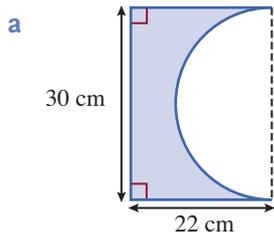
6 Estimate the circumference of the netball hoop if the circumference of the netball is 680 mm.



Shapes are often made up of curved and straight sides. When finding the perimeter of these, find the length of each part then add them up. Make sure that the lengths are converted into the same units where necessary.

Example

Find the perimeter of the shaded region:



Solution

Arc is half a circle with diameter 30 cm

$$C = \frac{1}{2} \times \pi \times D$$

$$= \frac{1}{2} \times \pi \times 30 \approx 47.12 \text{ cm}$$

$$\text{Perimeter} = 22 + 22 + 30 + 47.12$$

$$= 121.12 \text{ cm}$$

Arc is quarter of a circle with radius 12 mm

$$C = \frac{1}{4} \times 2\pi r$$

$$= \frac{1}{4} \times 2\pi \times 12$$

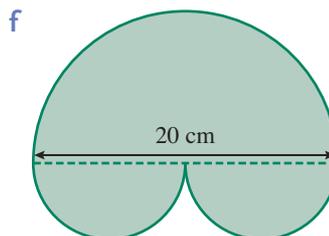
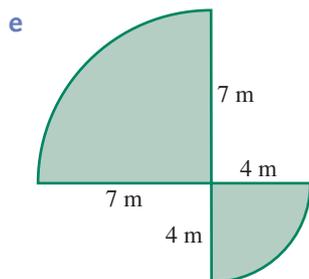
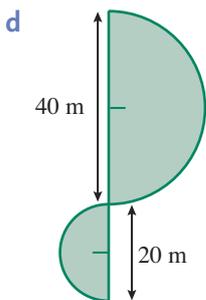
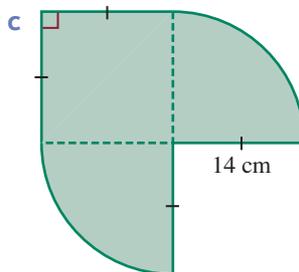
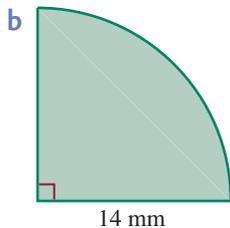
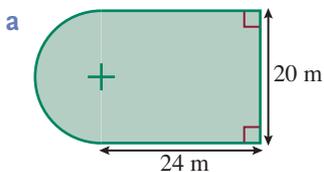
$$= 6 \times \pi = 18.85 \text{ mm}$$

$$\text{Perimeter} = 9 + 12 + 15 + 18.85 \text{ mm}$$

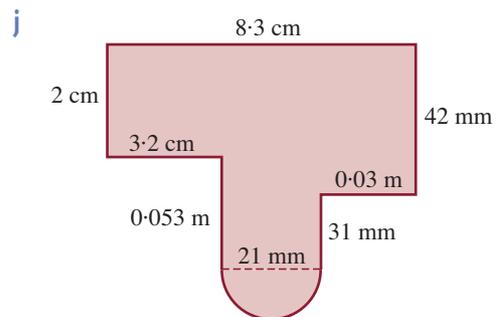
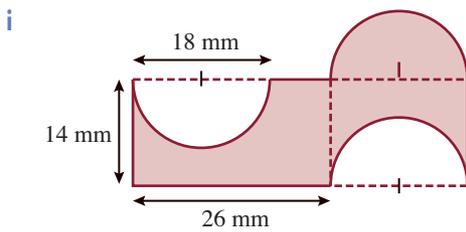
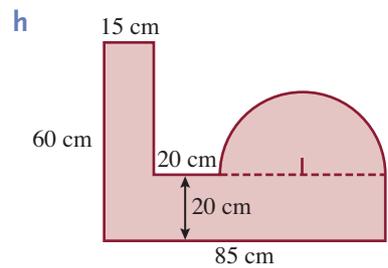
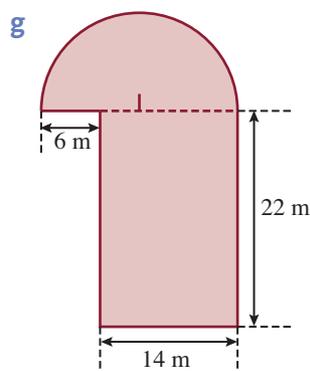
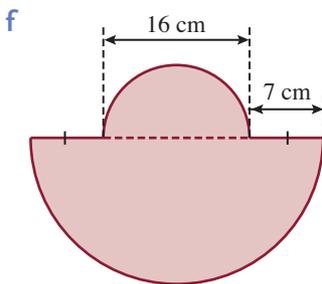
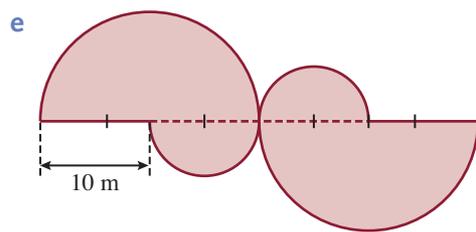
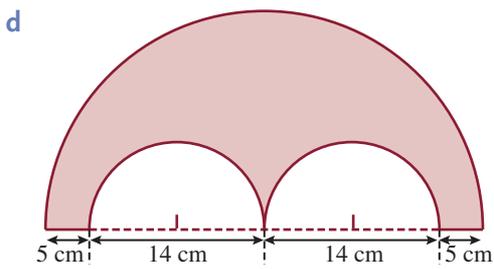
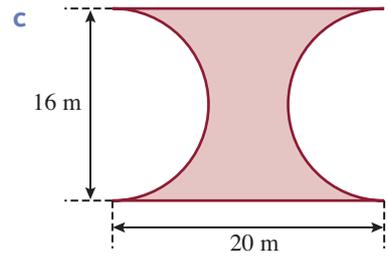
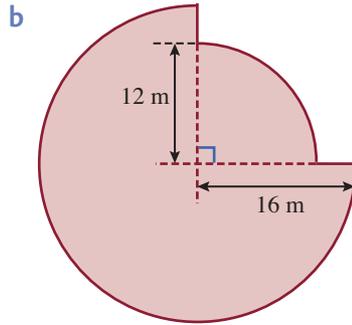
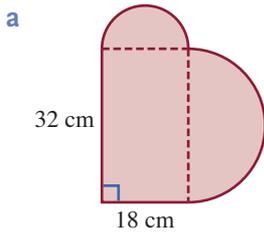
$$= 54.85 \text{ mm}$$

Exercise 21

1 Find the perimeter of the following shapes expressed to 2 decimal places:



2 Find the perimeter of these shapes expressed to 2 decimal places:



When we make measurements, it is not possible to be completely accurate. There is always uncertainty. Human error is involved in the way we read the scales and this produces errors in the measurements we record.

Example

- 1 William is to have his height measured by Lucinda. As Lucinda is shorter than William, she is not able to read his height very well.

Lucinda reads the scale as 153 cm. If William is only 150 cm tall:

- a find the error in her measurement
b express this as a percentage of his true height.

Solution

$$153 - 150 = 3 \text{ cm}$$

$$\frac{3}{150} = 2\%$$

- 2 Sammi uses a stopwatch to time the winner of the Under-11 long-distance race. The watch does not work very well and the correct time is somewhere between 1 second above or below the time showing.

If the watch shows a time of 15.8 seconds find:

- a the slowest time possible
b the fastest time possible
c the difference between the fastest and slowest times, which is the tolerance of the watch.

$$15.8 + 1 = 16.8 \text{ seconds}$$

$$15.8 - 1 = 14.8 \text{ seconds}$$

$$16.8 - 14.8 = 2 \text{ seconds}$$

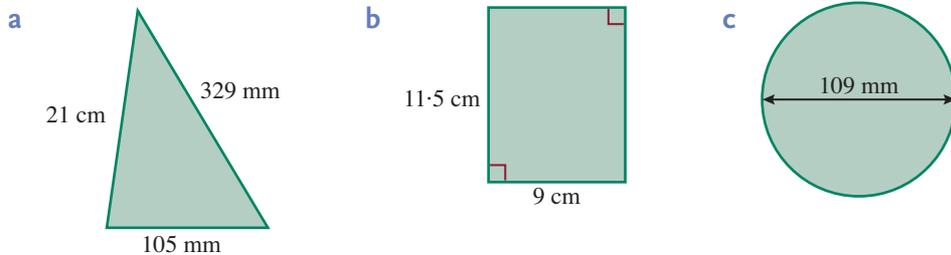


Exercise 2]

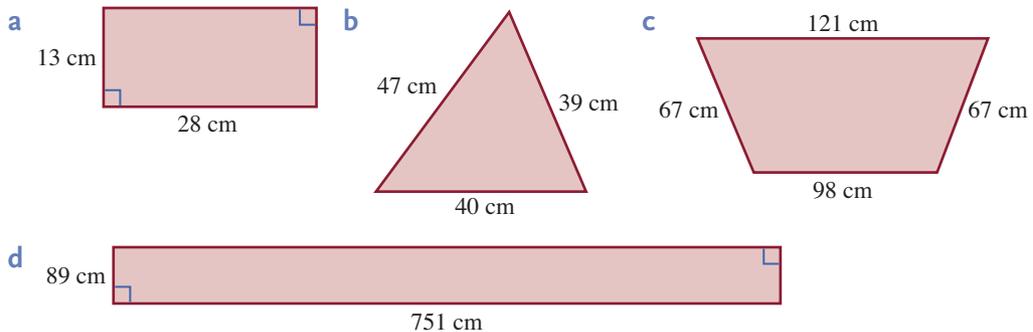
- 1 Jeremy has a 1-metre tape measure that was cut and glued back together. One centimetre is missing between the 20 and 21 cm mark. State the true measurement for these objects, which he measured using the tape measure:
- a Length of stapler measured as 12.5 cm b Length of desk measured as 96 cm
c Length of chopstick measured as 19 cm d Width of book measured as 24.5 cm
e Height of seedling measured as 6.8 cm f Height of child measured as 99.9 cm
- 2 Aba measures the height of his sister Lydia as 1.76 m. In fact, she is 1.65 m tall.
- a Find the error of the measurement.
b Express this error as a percentage of her true height.
- 3 Lilly's watch loses 4 seconds every hour. If the time was set correctly at midnight, state the true time and so the error of the watch when it shows:
- a 1 am b 2 am c 5:30 am d 9:15 am e 4:45 pm

- 4 Luan uses a faulty stopwatch to record the finishing times of the first six competitors in the Under-9, 100-metre race. The watch is 1.4 seconds fast. Find the true times if the first six times are recorded as 15.6, 16.1, 16.4, 16.9, 17 and 21.5 seconds.
- 5 Mandy has a set of scales that she uses to weigh snails in her garden, which is part of a science experiment. The scales are faulty and the true weight is 9 grams above the number showing. Find the true weight of the following snails if these are the weights showing on the scales:
- a Goldy: 49 g b Slimy: 78 g c Sleepy: 56 g d Tiger: 98 g
e Crunchy: 79 g f Crazy: 91 g g Huge One: 290 g h Baby: 23 g
- 6 James has a ruler that is 3 millimetres short, which means that when he measures a length to be 29 mm the true length is really 3 millimetres longer—32 mm.

James has recorded his measurements on these diagrams. State the true length of the perimeter of these shapes. Give your answers correct to the nearest millimetre.



- 7 Bali measured the following shapes and wrote the lengths on the diagram. He is unable to measure lengths very well. The lengths could be somewhere between 2 cm less or 2 cm more than the length shown on the diagrams. Find the maximum and minimum perimeter of each shape:



- 8 Lengths of pipe made in a factory are, on average, 120 cm long, but can vary between 140 and 100 cm. A number of lengths are welded together and used to make lamp posts. Find the shortest and longest lamp posts that can be made when the following number of lengths of pipe are welded together:
- a 4 b 5 c 6 d 7 e 8 f 9
- 9 The tolerance of a set of weighing scale is 5 g. State the heaviest and lightest weights that the scales would show when mangoes of known weights of 230 g, 182 g and 93 g are placed on them.



Puzzles

1 Convert the units in the left-hand column as indicated, and then draw a line to the correct answer in the right-hand column. Write the letter through which each line passes above the related question number in the code boxes below to answer to the riddle:

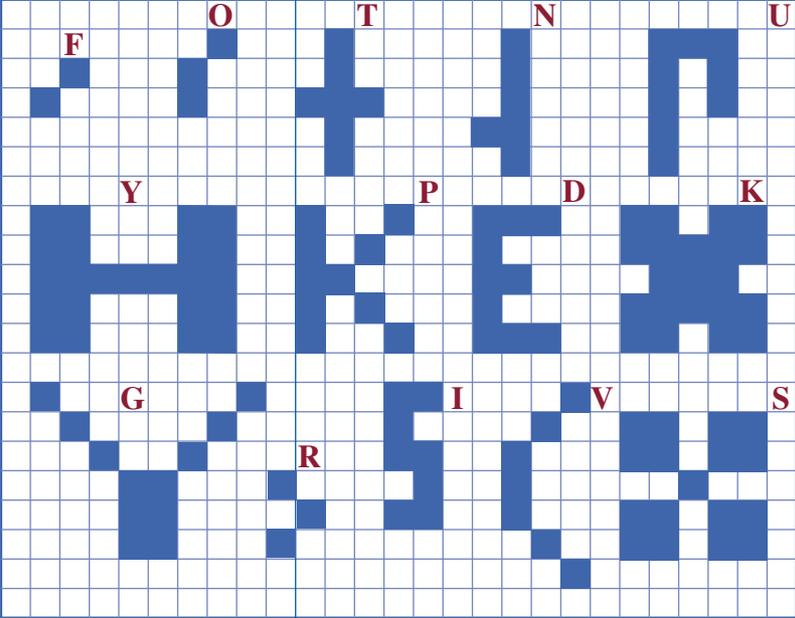
What should a successful movie producer always do?

1	3 km to m	<input type="checkbox"/>				<input type="checkbox"/>	2.1
2	74 m to cm	<input type="checkbox"/>				<input type="checkbox"/>	210
3	21 mm to cm	<input type="checkbox"/>				<input type="checkbox"/>	21
4	3 m to cm	<input type="checkbox"/>				<input type="checkbox"/>	3000
5	0.74 km to cm	<input type="checkbox"/>				<input type="checkbox"/>	0.21
6	21 cm to mm	<input type="checkbox"/>				<input type="checkbox"/>	30
7	333 m to km	<input type="checkbox"/>				<input type="checkbox"/>	3
8	300 cm to m	<input type="checkbox"/>				<input type="checkbox"/>	3300
9	7.4 m to cm	<input type="checkbox"/>				<input type="checkbox"/>	300
10	21 cm to m	<input type="checkbox"/>				<input type="checkbox"/>	330
11	0.333 m to mm	<input type="checkbox"/>				<input type="checkbox"/>	74 000
12	0.21 m to cm	<input type="checkbox"/>				<input type="checkbox"/>	740
13	300 mm to cm	<input type="checkbox"/>				<input type="checkbox"/>	74
14	3.3 km to m	<input type="checkbox"/>				<input type="checkbox"/>	7400
15	30 cm to m	<input type="checkbox"/>				<input type="checkbox"/>	0.33
16	7.4 cm to mm	<input type="checkbox"/>				<input type="checkbox"/>	0.3
17	3.3 m to cm	<input type="checkbox"/>				<input type="checkbox"/>	0.333
18	330 m to km	<input type="checkbox"/>				<input type="checkbox"/>	333

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18								

2 Find the perimeter of each figure, then write the corresponding letter above each answer below to solve the riddle:

What did the bird say to the parachutist?



$\frac{36}{22}$	$\frac{28}{18}$	$\frac{32}{24}$		$\frac{18}{8}$	$\frac{10}{10}$	$\frac{12}{20}$		
$\frac{18}{22}$	$\frac{36}{12}$		$\frac{30}{10}$	$\frac{10}{10}$	$\frac{20}{20}$	$\frac{16}{16}$	$\frac{36}{36}$	



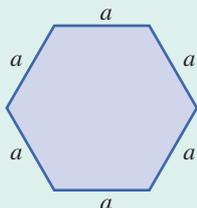
Perimeter

You are set the task of raising money for the school's building fund or a charity. The challenge is to raise enough money to arrange \$1 coins around the boundary of the school fence.

- Working in groups, estimate the length of the fence line around the school and use it to find the total amount that could be raised.
- Now measure the distance accurately and find the amount of money that could be raised.
- How much would be raised if 10 cent, 20 cent or 50 cent coins were used instead?

Regular polygons

A regular hexagon of side a is shown.



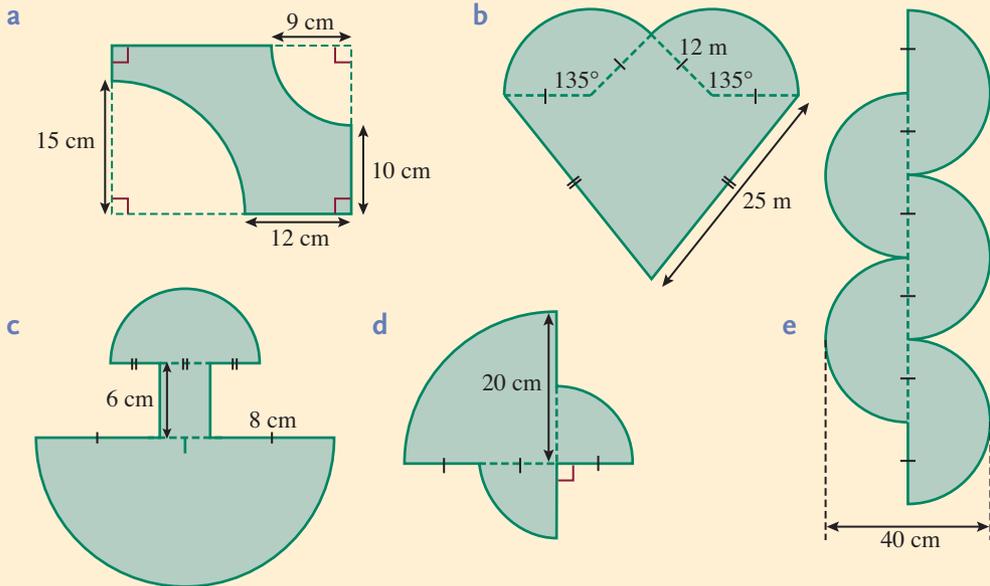
Copy the table below and draw in each of the regular polygons of side a . Write a formula for the perimeter of each shape in terms of a .

Measure the actual length and perimeter of your polygon and check that your formula works.

Regular polygon	Shape	Number of sides	Formula for perimeter	Measured side length	Measured perimeter
Equilateral triangle					
Square					
Pentagon					
Hexagon					
Septagon					
Octagon					
Nonagon					
Decagon					

Enrichment

1 Find the perimeter of these shapes correct to 1 decimal place:



2 Bill walks once around a large circle whose radius is 100 metres. His sister Voula walks once around a circle whose diameter is 100 metres. Which person walks furthest? Find the extra distance that is walked by that person to the nearest metre.

3 a Wire in the shape of circles is bent to form squares. Find the edge length of the squares that are formed from circles with the following radii:

- i 12 mm ii 39 cm iii 4 m iv 59 cm v 34 m

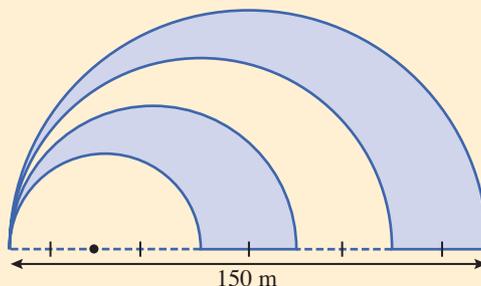
b Find the edge length of the squares that are formed from circles with the following diameters:

- i 6 cm ii 35 cm iii 2 m iv 4.4 m v 1 km

4 Regular shapes made of wire are bent into perfect circles. Find the diameter of the circles that are formed from:

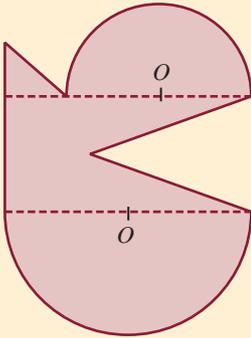
- a an equilateral triangle edge length 15 cm b a hexagon with side length 9 m
- c a nonagon with side length 10 mm d a heptagon with side length 70 cm
- e a dodecagon with side length 3 m f a pentagon with side length 12.5 mm

5 Find the perimeter of this figure expressed to 2 decimal places:

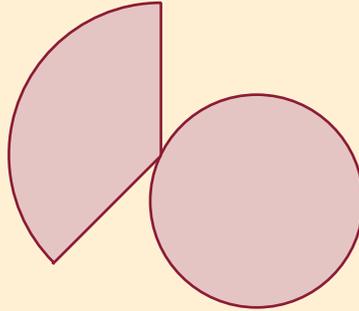


- 6 Find the perimeter of the following shapes to the nearest centimetre by measuring appropriate lengths and angles:

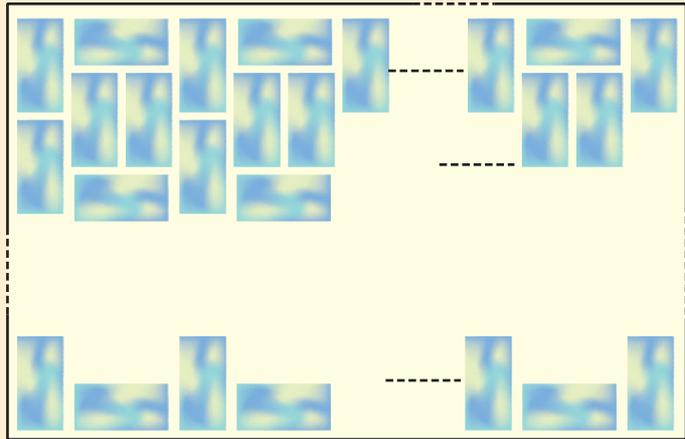
a



b



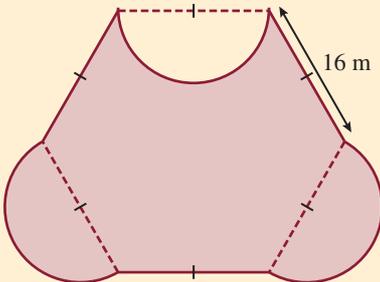
- 7 Tiles that are 100 mm wide and 204 mm long are placed in this pattern along a bathroom wall. The grout distance between the tiles is 4 mm. If the length of the wall is 3.748 m and its height is 3.332 m, find the number of tiles needed for the job.



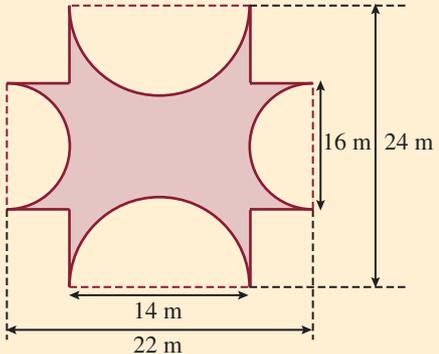
- 8 Wire is sold in 20-metre rolls. Three people need the following lengths of wire to complete jobs: 1600 cm, 2560 cm and 9800 cm. How should the rolls of wire best be used to minimise wastage if parts of rolls can be joined together? How much wire is left on any partly used rolls?

- 9 Find the perimeter of these symmetrical shapes expressed to 2 decimal places:

a



b



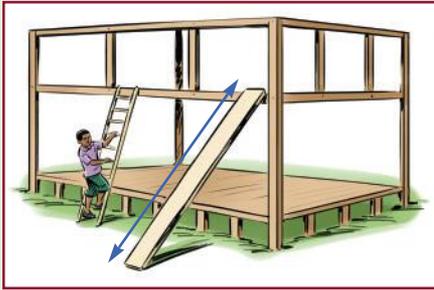
- 10 Boxes that are 28 cm long, 18 cm wide and 15 cm tall are stacked on top of each other in a room that measures 3.4 m long by 3 m wide by 2.4 m tall. Find the largest number of boxes that can be stacked in the room and the distance that is left between the top of the boxes and the ceiling.



Revision/Assessment

Exercise 2A

- 1 a Estimate the length of the plank of wood if the boy, on the far left is 1.4 m tall.



- b Estimate the height of the tree, assuming the cyclist is 1.4 m tall.



- 2 Estimate the lengths of these lines:

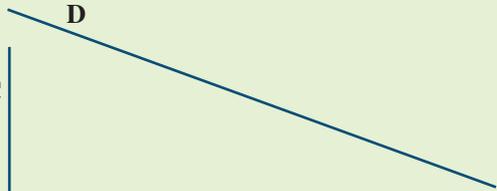
A



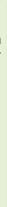
B



D



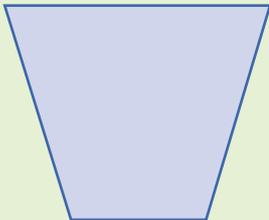
C



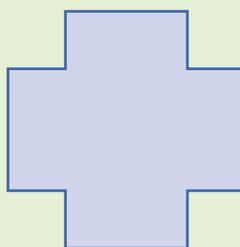
Exercise 2B

- 3 Accurately measure the perimeters of the following shapes to the nearest millimetre:

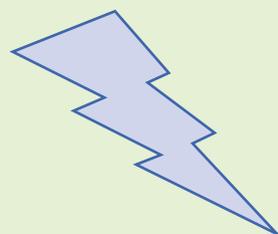
a



b

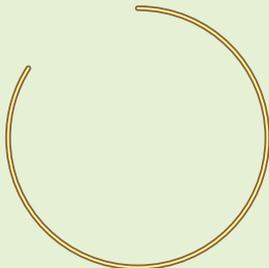


c

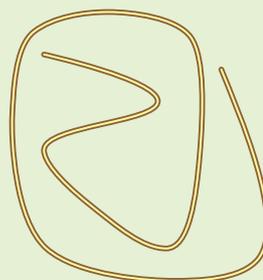


- 4 Using a piece of string, measure these lengths as accurately as possible:

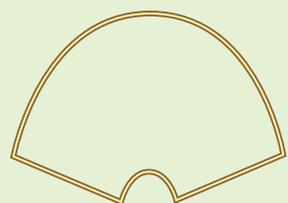
a



b



c



Exercise 2C

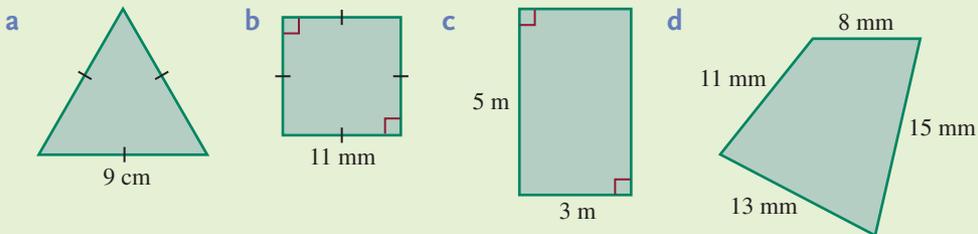
- 5 What factor is needed to convert:
- a metres to centimetres? b millimetres to metres?
- c metres to kilometres? d centimetres to millimetres?
- 6 Convert the following:
- a $560 \text{ m} = \underline{\hspace{2cm}} \text{ cm}$ b $340 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$
- c $45 \text{ km} = \underline{\hspace{2cm}} \text{ m}$ d $80 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$
- e $1400 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$ f $560 \text{ m} = \underline{\hspace{2cm}} \text{ km}$
- 7 List the following lengths in order from shortest to longest:
23 cm, 98 mm, 1.09 m, 234 mm, 102 cm
- 8 Rahul's pet snake grows 15 mm each month. If it is 14 cm this month, state its length in centimetres each month for a year.

Exercise 2D

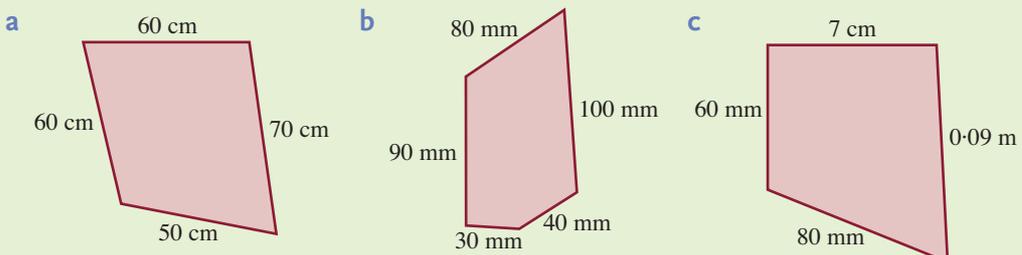
- 9 Find the answers to the following, expressed in the smaller unit:
- a $45 \text{ cm} + 590 \text{ mm}$ b $18 \text{ m} + 790 \text{ cm}$
- c $43 \text{ mm} + 8 \text{ cm} + 1 \text{ m}$ d $0.78 \text{ km} + 67 \text{ m}$
- e $8.9 \text{ m} + 67 \text{ cm}$ f $5670 \text{ mm} + 45 \text{ cm} + 1.2 \text{ m}$
- 10 Arrange these lengths from shortest to longest: 1.2 m, 900 mm, 45.8 cm, 0.0897 km
- 11 Eighteen lengths of string are tied together to make one long length. Find the length of the string in metres if each piece of string was 45 mm long.
- 12 A 12-metre long piece of liquorice needs to be shared equally between six children. What length in millimetres will each child get?

Exercise 2E

- 13 Find the perimeter of the following shapes:



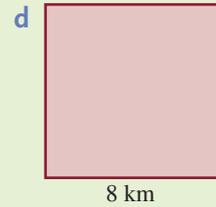
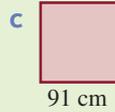
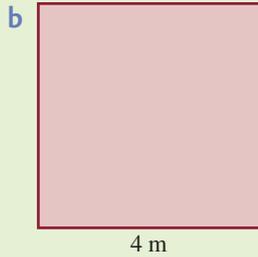
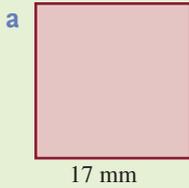
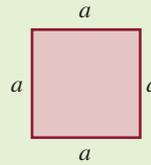
- 14 Find the perimeter of the following shapes. Express your answer in centimetres:



Exercise 2F

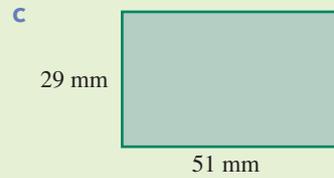
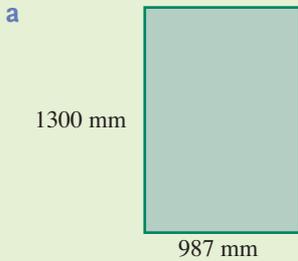
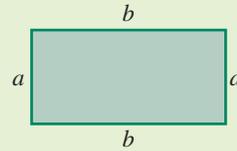
15 Find the perimeter of these squares using the formula:

Perimeter = $4 \times$ edge length



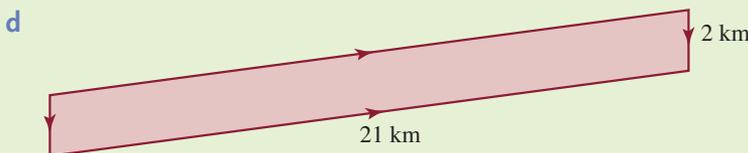
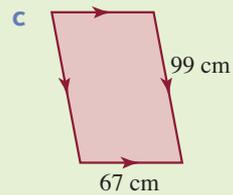
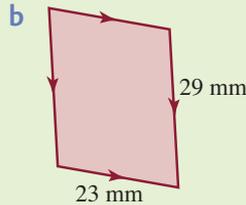
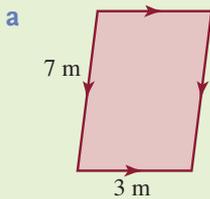
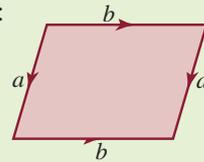
16 Find the perimeter of these rectangles using the formula:

Perimeter = $2(a + b)$



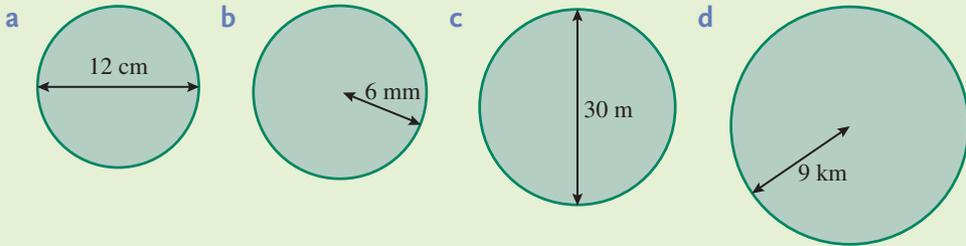
17 Find the perimeter of these parallelograms using the formula:

Perimeter = $2(a + b)$

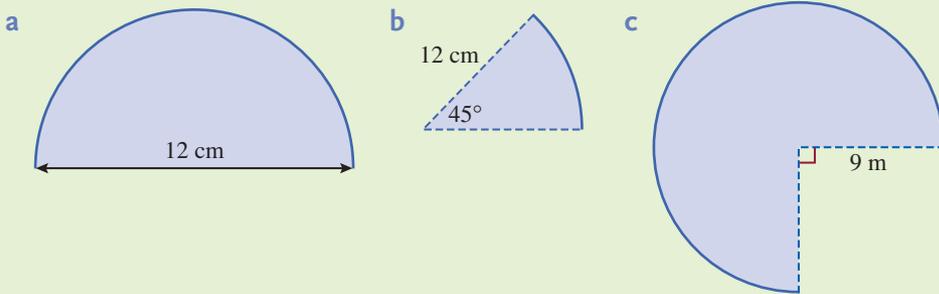


Exercise 2H

18 Find the perimeter of these circles expressed to 2 decimal places:

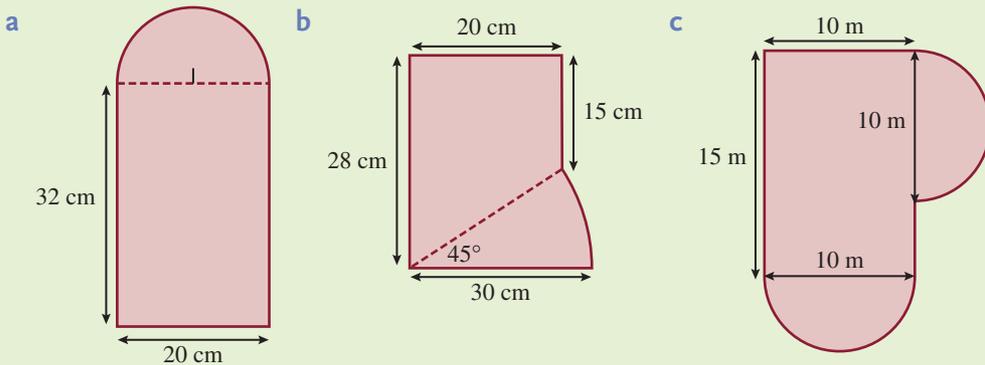


19 Find the length of these arcs to the nearest centimetre:



Exercise 2I

20 Find the perimeter of these shapes correct to 2 decimal places:



Exercise 2J

21 Seng's height is measured as 132 cm but he is actually 128 cm tall. Find the error and express this error as a percentage of his true height.

22 Maria's stopwatch loses 0.9 seconds every 10 seconds. What are the true times of the following?

- a 12.9 seconds b 31.7 seconds c 61.3 seconds d 1 minute

23 Willie's tape measure got broken and when it was stuck back together it was missing the first 12.5 cm. Find the true lengths in centimetres, if the tape measure was used to record these lengths:

- a 2489 cm b 26.9 m c 38 790 mm d 19 m e 0.01 km

CHAPTER

3

Polygons and Parallel Lines



Polygons and Parallel Lines

Polygons are closed, flat geometric shapes with straight sides. In architecture today, geometric shapes form the basis of many designs. Triangles and quadrilaterals are used in construction to provide strength. Regular polygons provide symmetry. Tessellating polygons create interesting patterns and designs.

This chapter covers the following skills:

- Naming angles
- Classifying angles
 - Acute
 - Right
 - Obtuse
 - Straight
 - Reflex
 - Perigon
- Identifying complementary angles
Add to 90°
- Identifying supplementary angles
Add to 180°
- Calculating angles and parallel lines
 - Vertically opposite
 - Corresponding
 - Alternate
 - Co-interior
- Classifying triangles
 - Side properties
 - Scalene
 - Isosceles
 - Equilateral
 - Angle properties
 - Acute
 - Right
 - Obtuse
 - Angle sum is 180°
- Classifying exterior angles of quadrilaterals
Angle sum is 360°
- Classifying polygons
Angle sum = $(n - 2) \times 180$
- Applying geometric bisections

Specific Learning Outcome (SLO)

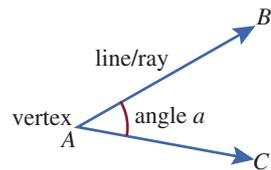
Learners should be able to:

- 8.3.1.1** Name angles using letters as labels for lines and vertices.
- 8.3.1.2** Draw angles and name them.
- 8.3.2.1** Name and classify angles according to size.

- 8.3.3.1** Calculate missing angles using the properties complementary, supplementary etc.
- 8.3.4.1** Define and identify transversals and parallel lines.
- 8.3.5.1** Name different types of angles: *vertically opposite; corresponding; alternate; co-interior or allied angles.*
- 8.3.6.1** Find the size of missing angles using the relationship between the angles and parallel lines.
- 8.3.7.1** Identify the 'side' and 'angle' properties for different triangles.
- 8.3.7.2** Name triangles according to their 'side' and 'angle' properties.
- 8.3.7.3** Measure angles using protractors.
- 8.3.8.1** Identify the *exterior angle properties* for any triangle.
- 8.3.8.2** Find the sizes of exterior angles using the *exterior angle properties.*
- 8.3.9.1** Define a quadrilateral.
- 8.3.10.1** Name six quadrilaterals: *rectangle; square; kite; parallelogram; rhombus; trapezium.*
- 8.3.10.2** Identify the properties of particular quadrilaterals.
- 8.3.11.1** Find the size of the missing angles in a quadrilateral using the angle sum of a quadrilateral
- 8.3.12.1** Define the term 'polygon' and identify particular polygon shapes.
- 8.3.12.2** Identify and name polygon shapes according to the number of sides.
- 8.3.13.1** Calculate the total angles in a quadrilateral using the angle-sum of a triangle.
- 8.3.14.1** Calculate the total angles in any polygon shapes using the formula:
Total angle – sum = $(n - 2) \times 180^\circ$
- 8.3.15.1** Use a compass to:
 - bisect an angle
 - bisect a line segment
 - identify the centre of a circle
 - draw the circumference of a circle.

Naming angles

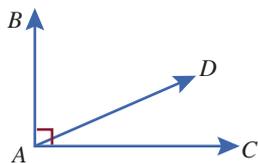
When naming angles we use the three letters positioned at the endpoints of the lines, making sure that the letter at the vertex is placed in the middle. The angle BAC shown could be named $\angle BAC$, or $\angle CAB$, or $\angle A$, or a° , or $\hat{B}AC$ or $C\hat{A}B$.



Types of angles

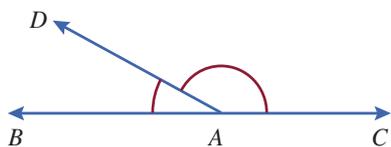
Angles can be described according to their size. They may be classified as one of the following types:

Type of angle	Diagram	Description
Acute angle		An angle between 0° and 90°
Right angle		90° or one quarter turn indicated by the little square
Obtuse angle		An angle between 90° and 180°
Straight angle		180° or one half turn or two quarter turns
Reflex angle		An angle between 180° and 360°
Perigon or full circle		360° or one complete turn



Complementary angles

The angle sum of $\angle BAD$ and $\angle DAC$ is 90° , therefore $\angle BAD$ and $\angle DAC$ are complementary angles. Any two angles which add together to give 90° are called **complementary** angles.



Supplementary angles

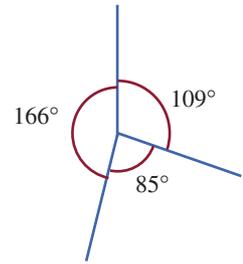
The angle sum of $\angle BAD$ and $\angle DAC$ is 180° , therefore $\angle BAD$ and $\angle DAC$ are supplementary. Any two angles which add together to give 180° are called **supplementary** angles.

Angles in a circle

Several smaller angles can be added together to form 360° , or one complete turn or a revolution. This is also known as a perigon.

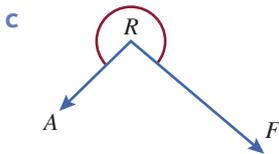
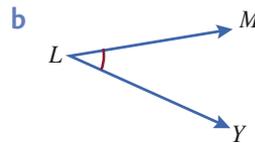
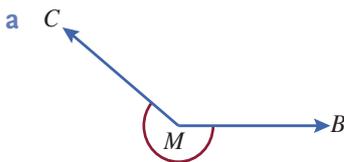
$$\text{Angles } 166^\circ + 109^\circ + 85^\circ = 360^\circ$$

Therefore they add to form a revolution.



Exercise 3A

1 Give two names for the angles shown below:



2 Draw a diagram and clearly label it to represent the angles known as:

a $\angle AWC$

b $\angle BLT$

c $\angle MUD$

d $\angle HGJ$

e a°

f k°

g m°

h p°

i $\hat{S}ML$

j $\hat{Q}UZ$

k $\hat{A}MP$

l $\hat{D}FS$

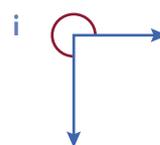
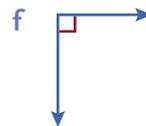
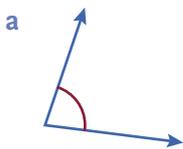
m $\angle B$

n $\angle C$

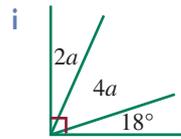
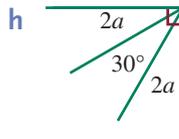
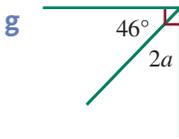
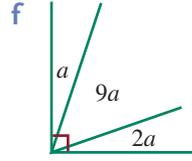
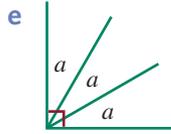
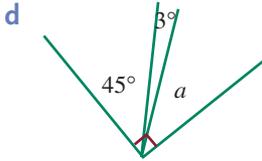
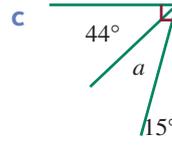
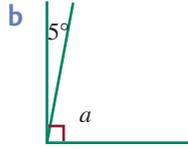
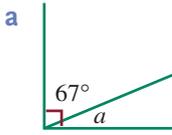
o $\angle F$

p $\angle W$

3 For each of the following angles state its type:



4 Find the size of a in each of the following:



5 Which of the following sets of angles are complementary?

a $23^\circ, 77^\circ$

b $36^\circ, 64^\circ$

c $47^\circ, 43^\circ$

d $58^\circ, 22^\circ$

e $63^\circ, 27^\circ$

f $10^\circ, 21^\circ, 69^\circ$

6 Find the complement of:

a 25°

b 46°

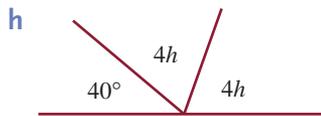
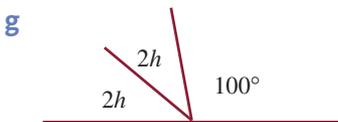
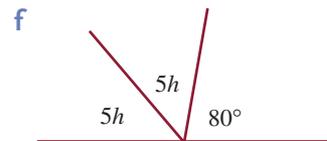
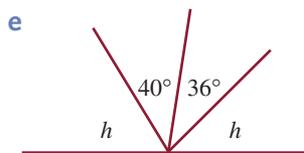
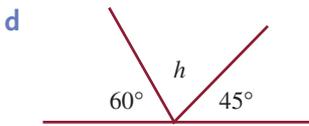
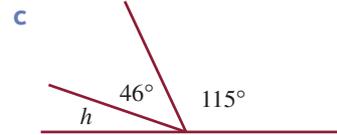
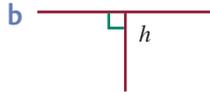
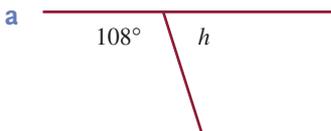
c 68°

d 51°

e 79°

f 81°

7 Find the value of h in each of the following:



8 Which of the following sets of angles are supplementary?

a $30^\circ, 150^\circ$

b $47^\circ, 133^\circ$

c $112^\circ, 78^\circ$

d $12^\circ, 56^\circ, 112^\circ$

e $12^\circ, 57^\circ, 111^\circ$

f $90^\circ, 2^\circ, 41^\circ, 116^\circ$

9 Find the supplement of:

a 35°

b 52°

c 96°

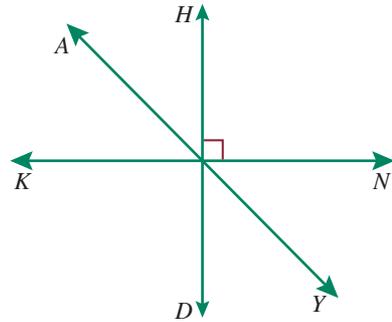
d 107°

e 161°

f 186°

10 Copy the diagram shown and colour or outline:

- a an acute angle red
- b a right angle yellow
- c an obtuse angle orange
- d a straight angle black
- e a reflex angle blue

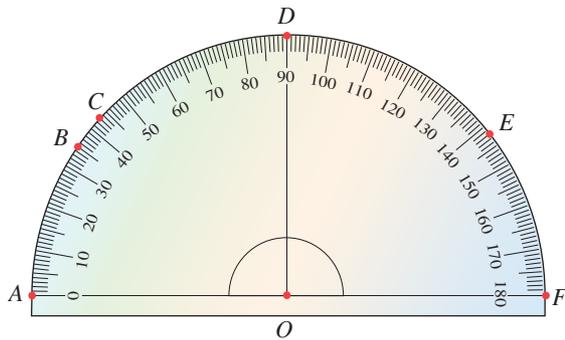


11 Use your protractor to draw the following angles. Ensure that they are correctly labelled:

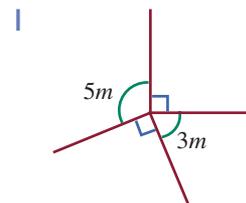
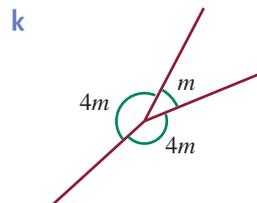
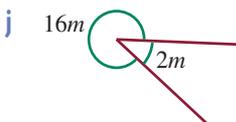
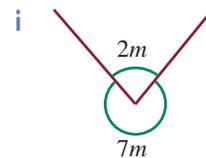
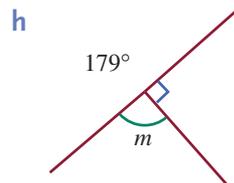
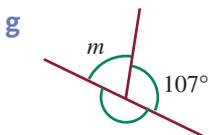
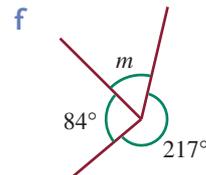
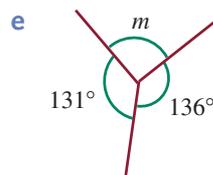
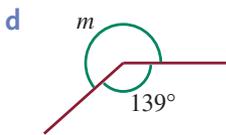
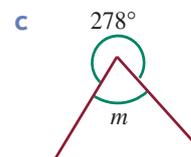
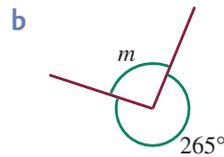
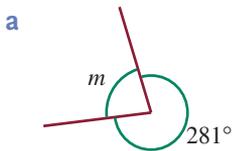
- a $\angle T = 65^\circ$
- b $f^\circ = 85^\circ$
- c $\angle RAT = 105^\circ$
- d $\angle K = 184^\circ$
- e $\angle S = 256^\circ$
- f $\angle AMY = 303^\circ$

12 Use a protractor or another method to accurately measure each of the following angles:

- a $\angle AOB$
- b $\angle AOC$
- c $\angle COD$
- d $\angle DOE$
- e $\angle AOF$
- f $\angle EOF$
- g $\angle DOF$
- h $\angle BOC$



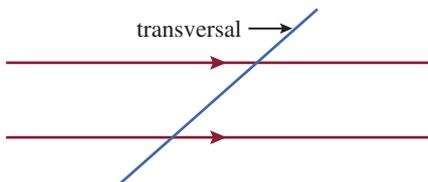
13 Find the value of m in each of the following:



3B

Angles and parallel lines

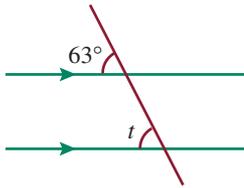
A **transversal** is the name given to the line that crosses a pair of parallel lines. The angles created when a transversal crosses a pair of parallel lines can be related in a variety of ways.



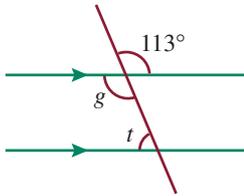
Type of angle	Diagram	Description
Vertically opposite (X)		Vertically opposite angles are equal: $\therefore \angle c = \angle d$ Remember the X shape.
Corresponding (F)		Corresponding angles are equal: $\therefore \angle e = \angle f$ Remember the F shape.
Alternate (Z)		Alternate angles are equal: $\therefore \angle p = \angle q$ Remember the Z shape.
Co-interior (C) or allied angles		Co-interior angles are supplementary, i.e. add up to 180° : $\therefore \angle t + \angle m = 180^\circ$ Remember the C shape.

Example

- 1 Find the value of t :



- 2 Find the value of t :



Solution

First, establish the relationship between the angles: t and 63° are corresponding angles
 $\therefore t = 63^\circ$.

Corresponding angles are equal.

g is the angle vertically opposite to 113°
 $\therefore g = 113^\circ$.

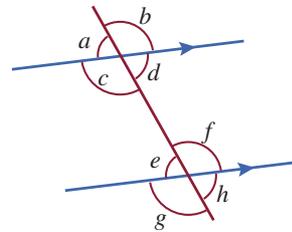
Angles g and t are co-interior angles and therefore add up to 180° .

$$\begin{aligned} t + 113^\circ &= 180^\circ \\ \therefore t &= 180^\circ - 113^\circ \\ &= 67^\circ \end{aligned}$$

Exercise 3B

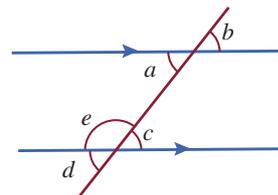
- 1 List the pairs of angles in the diagram below that are:

- vertically opposite
- co-interior
- alternate
- corresponding
- equal
- supplementary
- complementary

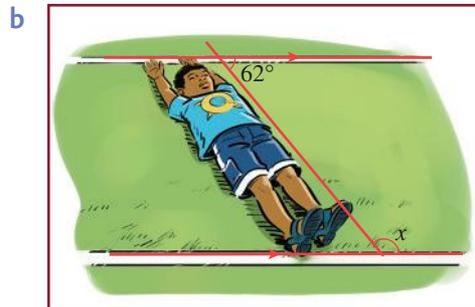
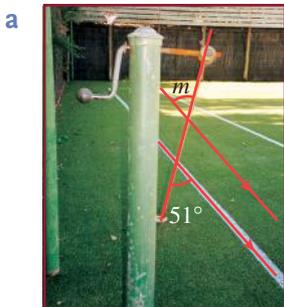


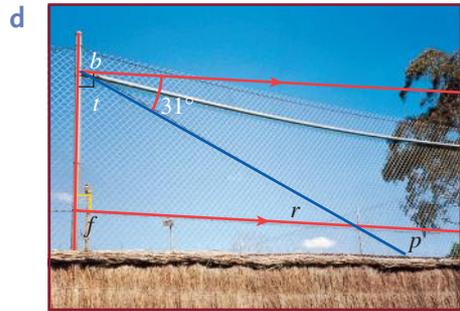
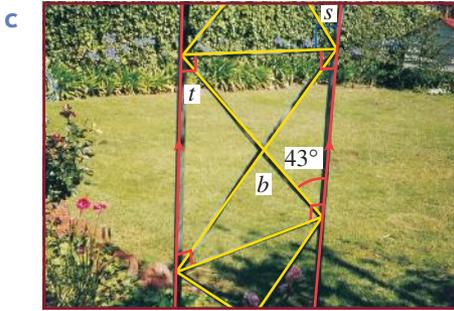
- 2 Use the information in the diagram to fill in the blanks below:

- The corresponding angle to a is _____.
- The vertically opposite angle to a is _____.
- The alternate angle to a is _____.
- The co-interior angle to a is _____.

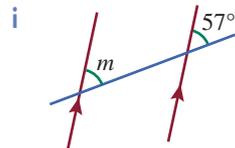
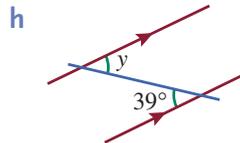
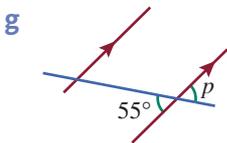
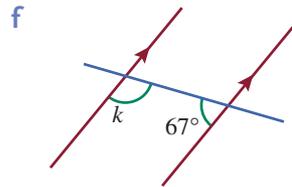
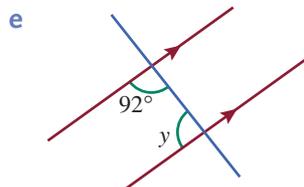
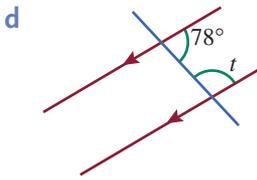
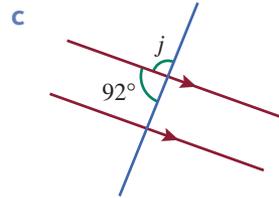
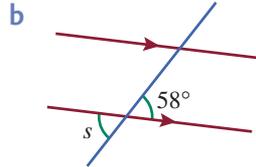
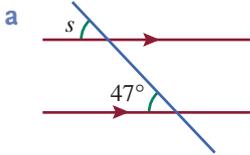


- 3 Find the value of the pronumeral in each of the following:

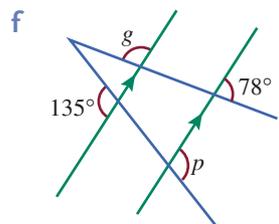
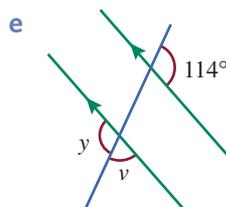
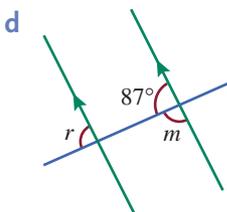
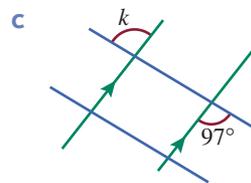
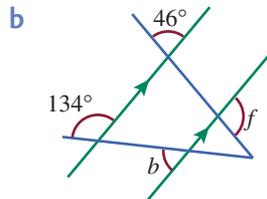
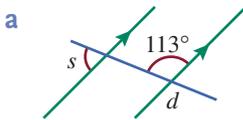




- 4 For each of the following, state:
- the relationship between the angles shown
 - the value of the pronumeral

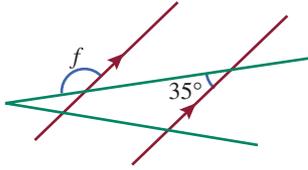


- 5 Find the value of the pronumeral in each of the following:

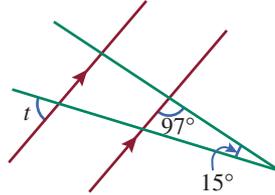


6 Find the value of the pronumeral in each of the following

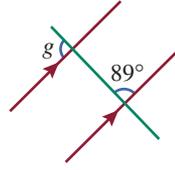
a



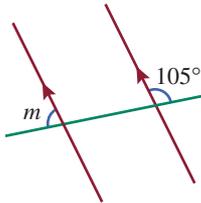
b



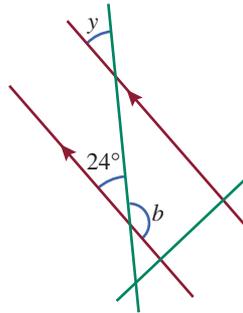
c



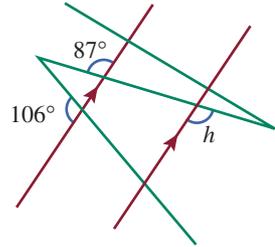
d



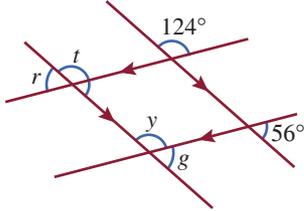
e



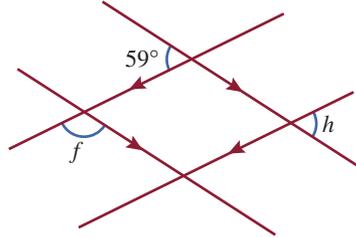
f



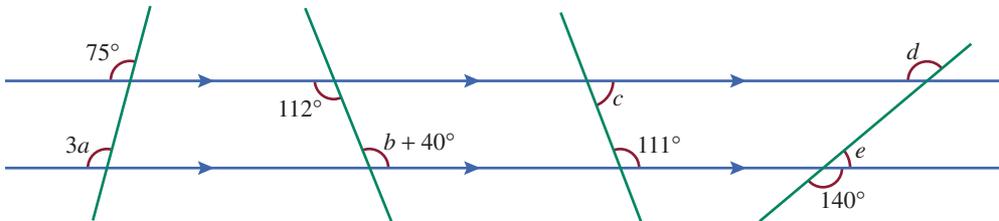
g



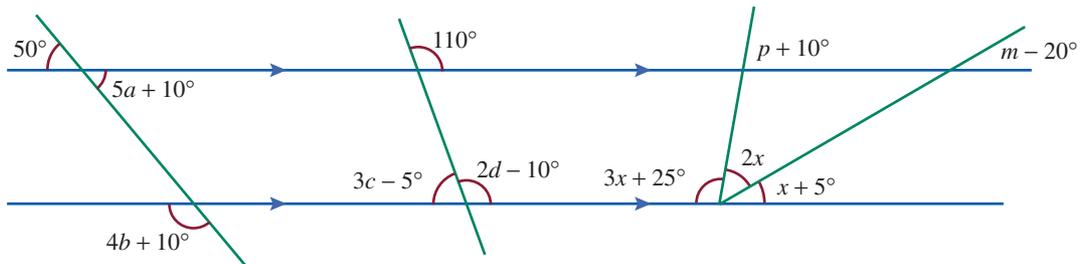
h



7 Find the value of the pronumeral in each of the following:

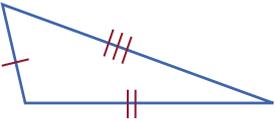
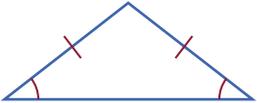
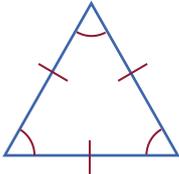


8 Find the value of the pronumeral in each of the following:

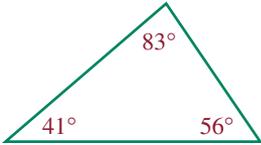
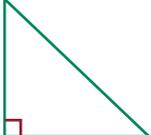
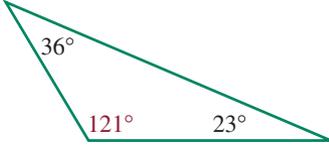


There are different types of triangles. We can name them according to their side properties or their angle properties. Triangles named according to their side properties are scalene, isosceles or equilateral, and triangles named according to their angle properties are acute, obtuse and right-angled.

Side properties

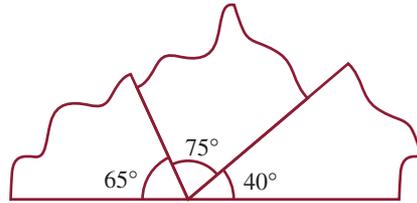
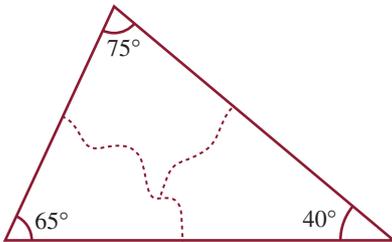
Type of triangle	Diagram	Description
Scalene		All the side lengths are different in a scalene triangle.
Isosceles		In an isosceles triangle two of the sides are equal in length and two angles are equal in size.
Equilateral		An equilateral triangle has three sides that are equal in length and three equal angles. Each of the angles in an equilateral triangle is 60° .

Angle properties

Type of triangle	Diagram	Description
Acute		In an acute-angled triangle none of the angles is equal but they are all acute, that is, all the internal angles are between 0° and 90° .
Right		In a right-angled triangle, one of the internal angles is 90° .
Obtuse		In an obtuse-angled triangle, one of the internal angles is obtuse, that is, between 90° and 180° .

Interior angle sum of a triangle

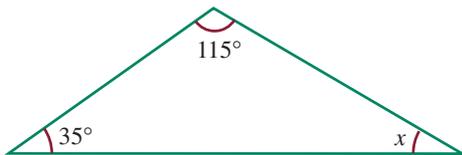
The angles in a triangle can be cut off and placed together to form a straight angle:



• The angle sum of a triangle is 180° .

Example

- 1 Find the unknown angle x :



Solution

In the triangle, the known internal angles add to 150° :

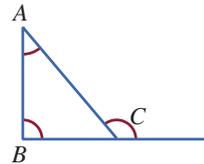
$$\begin{aligned} x + 150^\circ &= 180^\circ \\ x &= 180^\circ - 150^\circ \\ &= 30^\circ \end{aligned}$$

Exterior angle properties of a triangle

Angle C is known as an exterior angle in the triangle shown.

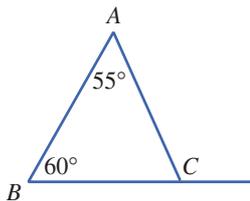
The exterior angle is equal to the sum of the two interior opposite angles:

$$\angle A + \angle B = \angle C$$



Example

- 2 Find the size of the exterior angle $\angle C$:



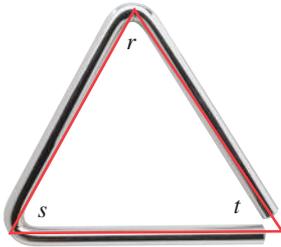
Solution

$$\begin{aligned} \angle A &= 55^\circ, \angle B = 60^\circ \\ \angle C &= \angle A + \angle B \\ &= 55^\circ + 60^\circ \\ &= 115^\circ \end{aligned}$$

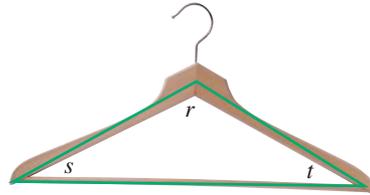
Exercise 3C

- 1 Look at the triangles in the following pictures. Give each triangle two names, one according to its side properties and the other according to its angle properties.

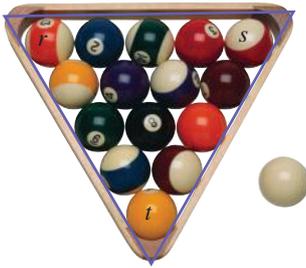
a



b



c



d



e



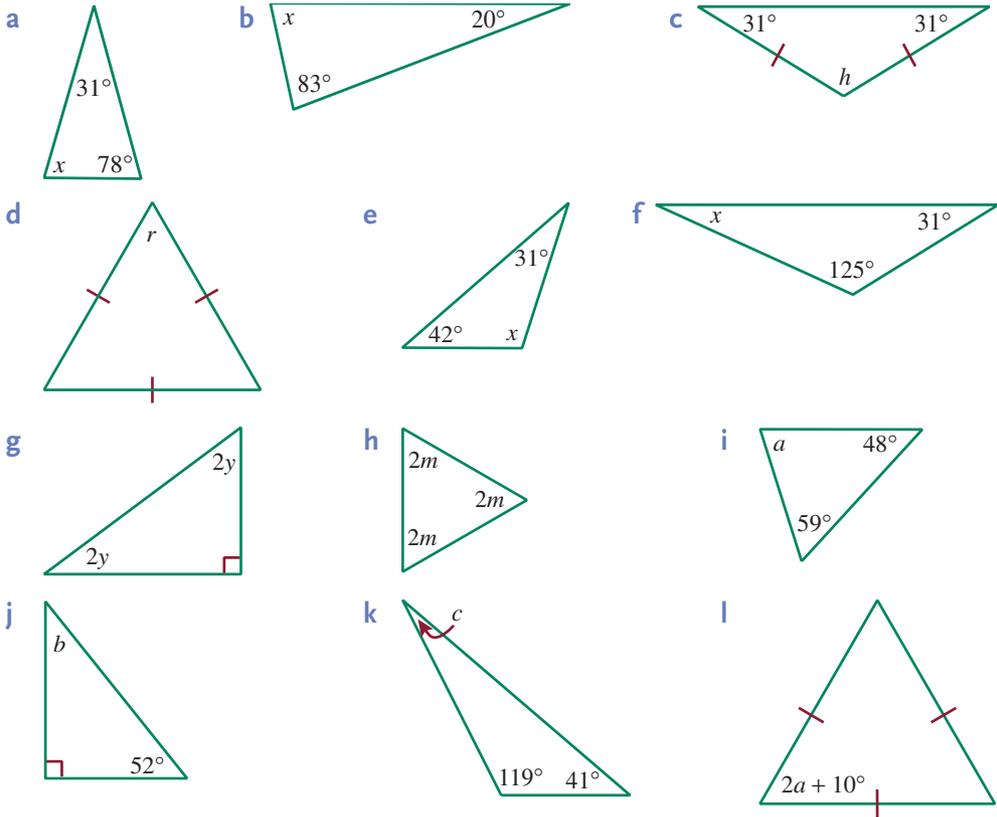
f



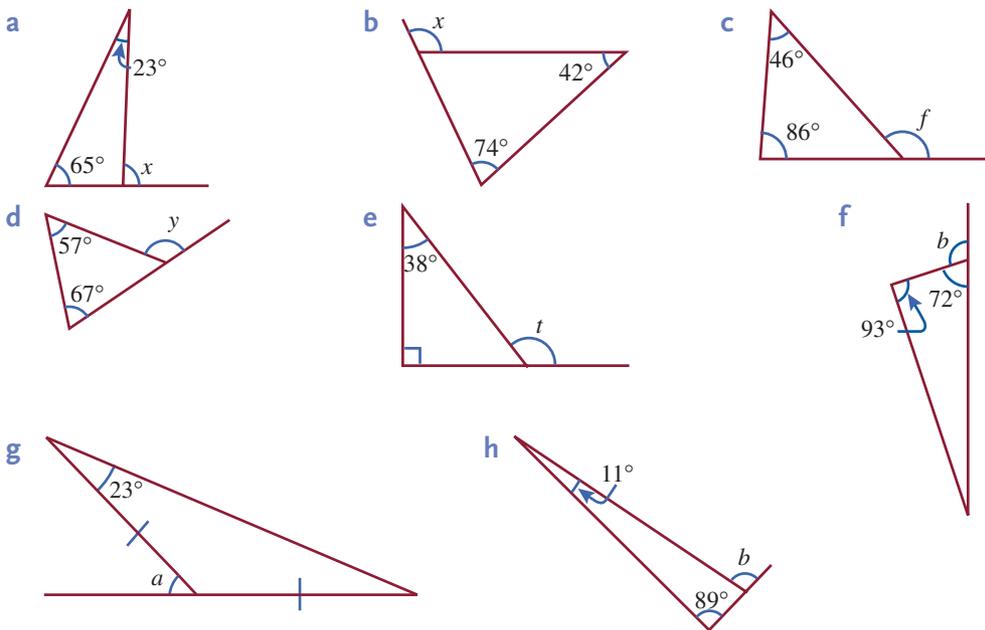
- 2 Measure the angles in each of the triangles above using a protractor. Copy and complete the table below for each of the triangles. Include the size of each angle, then add the angles to find the angle sum for each triangle.

Triangle	Side and angle name	$\angle r$	$\angle s$	$\angle t$	Angle sum $= r + s + t$
a					
b					
c					
d					
e					
f					

3 Calculate the value of the pronumeral in each of the following triangles:



4 Using the exterior angle properties of a triangle to find the size of the unknown angle in the following triangles:



3D

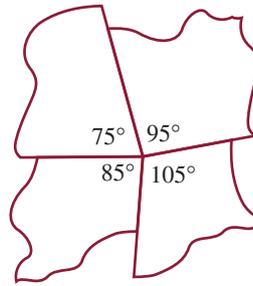
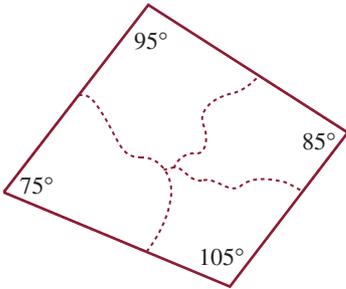
Properties of quadrilaterals

Quadrilaterals are figures with four corners or vertices and four sides. One corner would be called a vertex. There are six main quadrilaterals that have special properties.

Type of quadrilateral	Diagram	Description
Rectangle		In a rectangle , opposite sides are equal and all angles are equal and measure 90° .
Square		In a square , all sides are equal and all angles are equal and measure 90° .
Parallelogram		A parallelogram has two pairs of opposite sides that are equal and parallel. The opposite angles in a parallelogram are also equal.
Rhombus		A rhombus is a parallelogram with all equal sides. The opposite angles are also equal.
Kite		A kite has two long sides and two short sides. It has two pairs of adjacent sides equal and one pair of opposite angles equal.
Trapezium		A trapezium has one pair of opposite sides that are parallel.

Interior angle sum of a quadrilateral

A quadrilateral with angles measuring 65° , 95° , 85° and 115° is shown below.

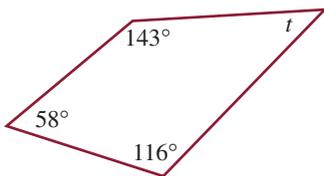


The corners of the quadrilateral can be folded back to form a complete revolution.

• The angle sum of a quadrilateral is 360° .

Example

Find the missing angle:



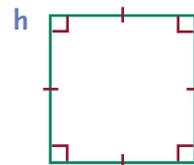
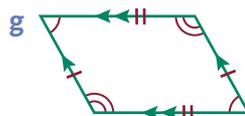
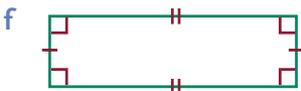
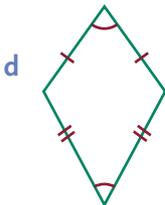
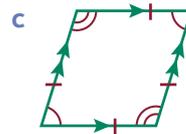
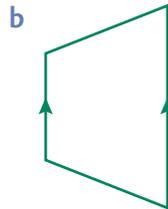
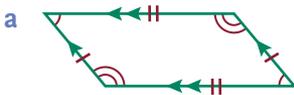
Solution

The shape is a quadrilateral.
There are 360° in a quadrilateral.

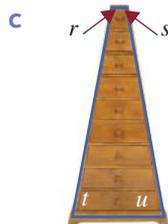
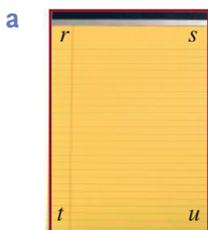
$$\begin{aligned} t &= 360^\circ - (116^\circ + 58^\circ + 143^\circ) \\ &= 360^\circ - 317^\circ \\ &= 43^\circ \end{aligned}$$

Exercise 3D

1 Name each of the following quadrilaterals:



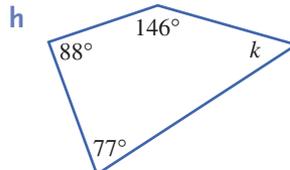
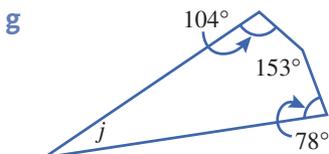
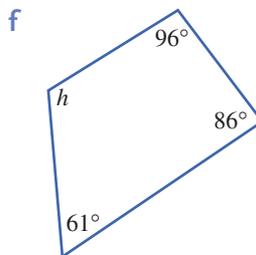
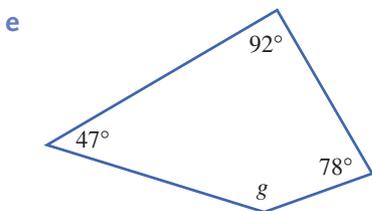
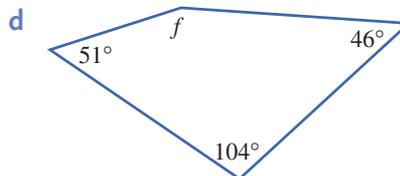
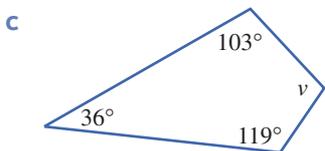
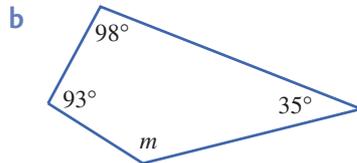
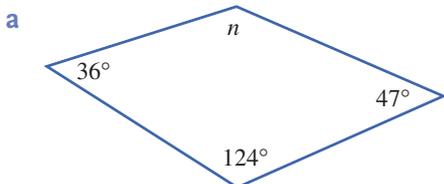
2 Use the following pictures to identify as many quadrilaterals as you can:



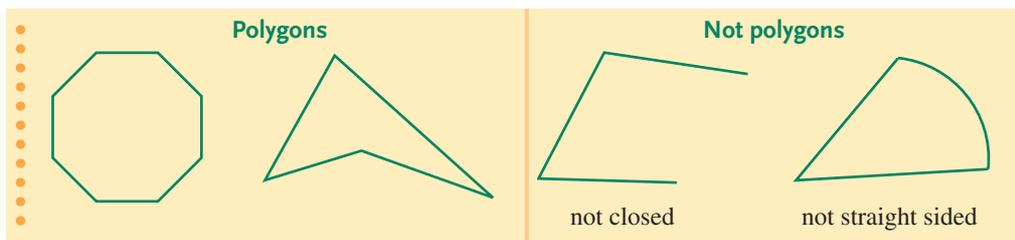
3 Measure the angles in each of the quadrilaterals above using a protractor. Copy and complete the table below for each of the quadrilaterals. Include the size of each angle, then add to find the angle sum for each quadrilateral.

Quadrilateral	Name	$\angle r$	$\angle s$	$\angle t$	$\angle u$	Angle sum $= r + s + t + u$
a						
b						
c						

4 Find the size of the unknown angle in each of the following quadrilaterals:



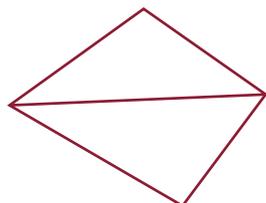
Polygon is a Greek word meaning ‘many angles’. A **polygon** is a plane-enclosed figure with straight sides.



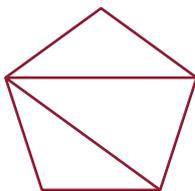
In a regular polygon the sides are of equal length and the angles are equal in size. Some polygons are given a special name. Polygons with more than ten sides do not always have a special name. For example, a fourteen-sided polygon is called a 14-gon. In general, a polygon with n sides is called an n -gon.

Angle sum of a polygon

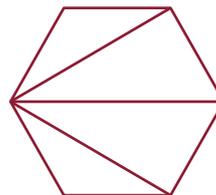
We can split any polygon into triangles to assist us when finding their angle sum.



$$\begin{aligned} &\text{Two triangles} \\ &= 2 \times 180^\circ \\ &= 360^\circ \end{aligned}$$



$$\begin{aligned} &\text{Three triangles} \\ &= 3 \times 180^\circ \\ &= 540^\circ \end{aligned}$$



$$\begin{aligned} &\text{Four triangles} \\ &= 4 \times 180^\circ \\ &= 720^\circ \end{aligned}$$

We can use this approach to find the angle sum of any polygon.

- In an n -sided polygon there are $n - 2$ triangles.
- So the angle sum in any polygon is $(n - 2) \times 180^\circ$.

Example

- Find the sum of the angles in an octagon.



Solution

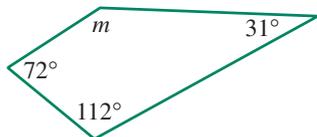
There are 6 triangles in an octagon (8 sides).
Sum of the angles = $6 \times 180^\circ$
= 1080°

When we have polygons with many sides it is not always practical to draw the shape and divide it into triangles. It can be useful to know a relationship between the number of sides and the number of triangles.

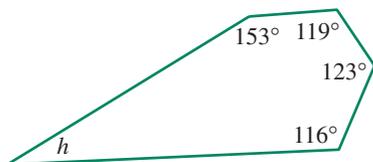
Example

- 2 Find the sum of the angles in an octagon (8 sides).

- 3 Find the missing angle:



- 4 Find the missing angle:



Solution

There are 8 sides and $8 - 2 = 6$ triangles
 Sum of the angles = $(8 - 2) \times 180^\circ$
 $= 1080^\circ$

The shape is a quadrilateral.
 Angle sum of all quadrilaterals
 $= 360^\circ$
 $m = 360^\circ - (72^\circ + 112^\circ + 31^\circ)$
 $= 360^\circ - 215^\circ$
 $= 145^\circ$

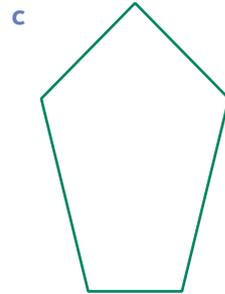
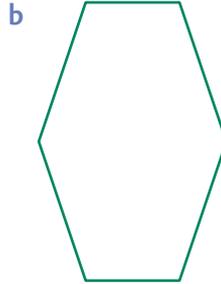
The shape is a pentagon.
 Angle sum of all pentagons
 $= 540^\circ$
 $h = 540^\circ - (153^\circ + 119^\circ + 123^\circ + 116^\circ)$
 $= 540^\circ - 511^\circ$
 $= 29^\circ$

Exercise 3E

- 1 Copy and complete the following table of regular shapes into your workbook.
 Choose names from this list:
 nonagon, hexagon, decagon, pentagon, rhombus, heptagon, square, octagon.

Number of sides	Name of polygon	Regular shape
3	Equilateral triangle	
4		
5		
6		
7		
8		
9		
10		

2 Name the following polygons and find their angle sum by first splitting them into triangles:



3 Find the total number of degrees, i.e. the angle sum, in the following polygons without drawing them first:

a decagon

b 54-agon

c 17-agon

d 350-agon

e 15-agon

f dodecagon

g heptagon

h 1000-agon

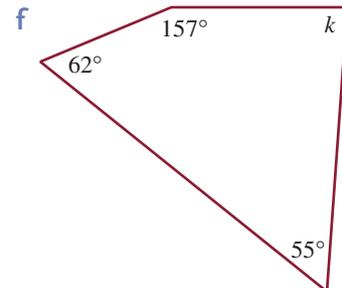
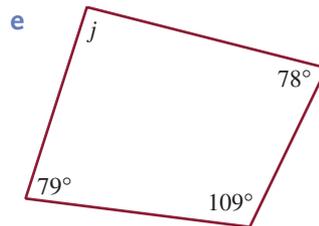
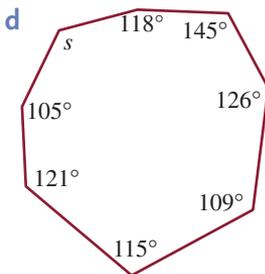
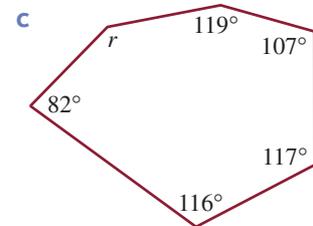
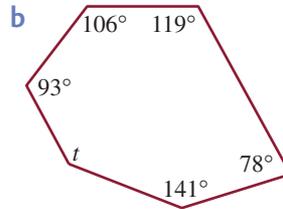
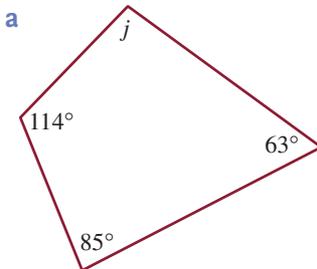
i 25-agon

j 18-agon

k 21-agon

l 45-agon

4 Find the value of the missing angle in each of the following polygons:



5 List five man-made items you can see in your classroom that are in the shape of a polygon. Indicate both the item and the polygon it replicates.

6 There are many naturally (i.e. in nature) occurring polygons. Find three different examples of naturally occurring polygons and make an accurate sketch in your workbook. Correctly label each polygon.



3F Exploring geometric bisections

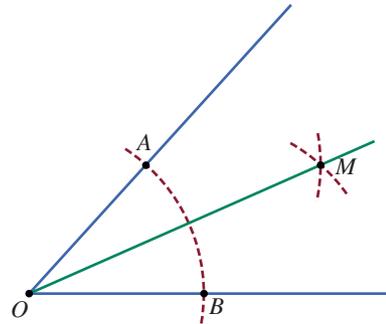
An angle can be bisected (cut in half) by using only a compass and straight edge.

Bisecting an angle

Place the point of the compass on the vertex of the angle, O , and draw an arc that cuts both arms of the angle. Label the points where it cuts A and B . Place the point of the compass on both A and B and draw two arcs inside the arms of the angle.

Label the point where they cross M .

Join points O and M . The line OM bisects the angle.

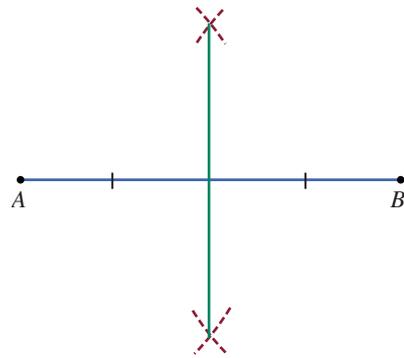


Bisecting a line

Place the point of the compass on the ends of the line at A and B .

Draw arcs from each point to intersect above and below the line AB .

Join the points of intersection of the arcs. This line will bisect AB .



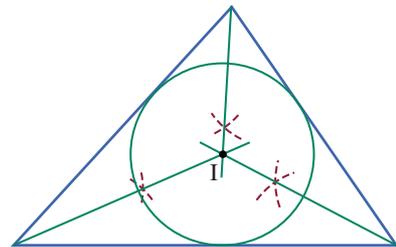
Finding the incentre

Bisect each angle using the method shown above.

Extend each of the lines to intersect inside the triangle.

Label this point I for the incentre of the triangle.

Place the point of the compass on I and open it to reach the closest point on the triangle. Draw an incircle.

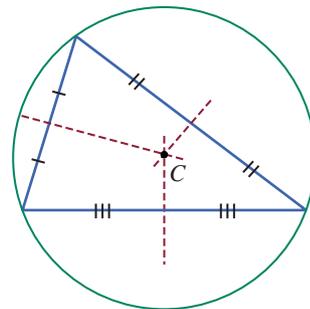


Finding the circumcentre

Bisect each side of the triangle using the method shown above.

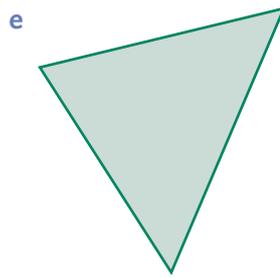
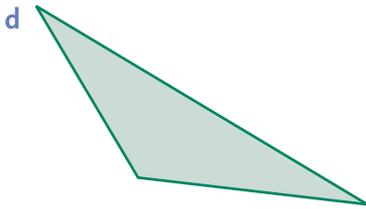
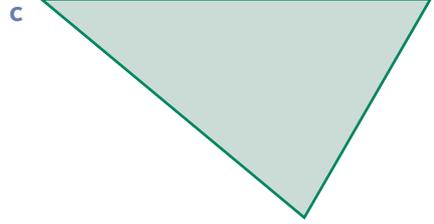
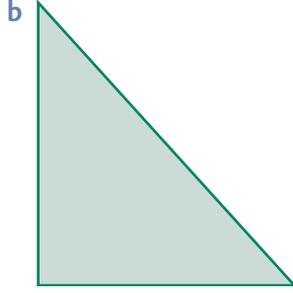
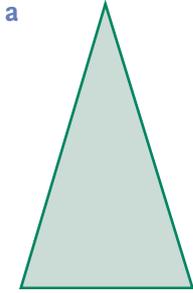
Extend each of the lines to intersect inside the triangle.

Label this point C for the circumcentre of the triangle. Place the point of the compass on C and open it to reach a vertex of the triangle. Draw a circumcircle.

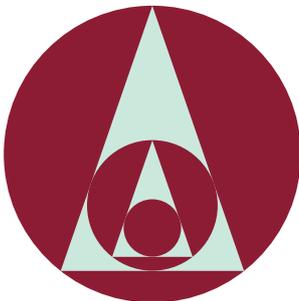
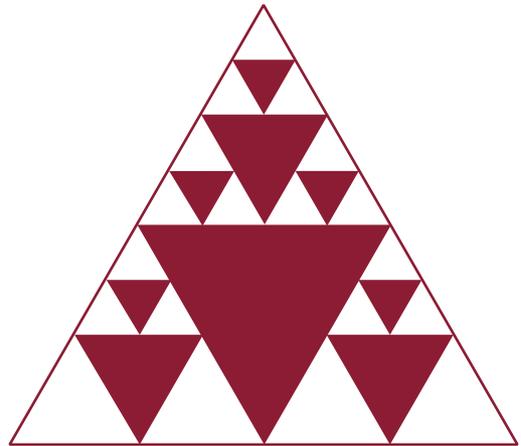


Learning task 3F

1 Copy the triangles and use your protractor to find each incentre and to draw the incircles.



- 2 a Copy the triangle design. Using the method shown, draw incircles in each of the triangles to create a new design.
- b Create your own design using triangles and circles.



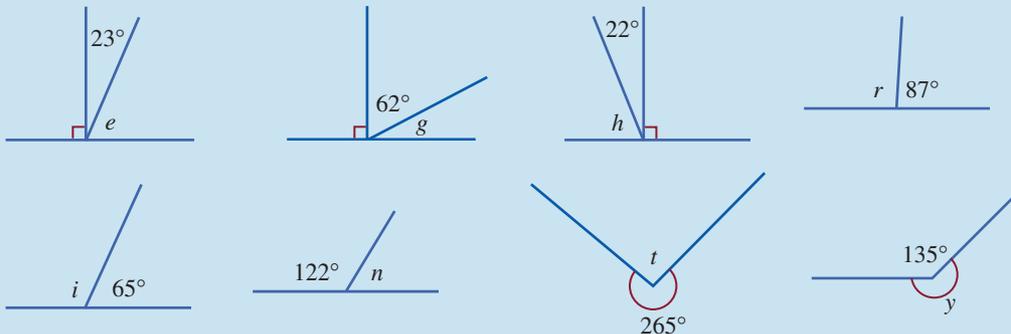
- 3 Draw a similar design of concentric incentre triangles and circumcircles. Start with:
- an equilateral triangle
 - a right-angled isosceles triangle
 - a reflex-angled scalene triangle



Puzzles

- 1 Find the unknown angles in the following diagrams. Match the letter of the angle to the correct angle below, to solve the riddle:

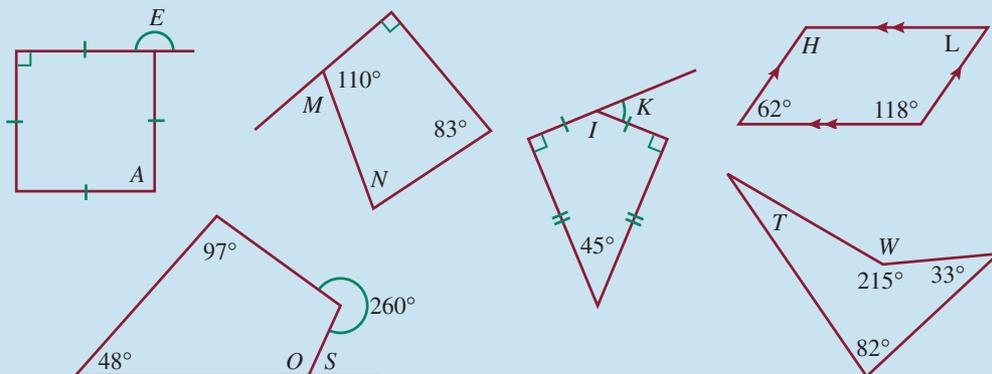
Who invented fractions?



- | | | | | | | | | | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\overline{68^\circ}$ | $\overline{67^\circ}$ | $\overline{58^\circ}$ | $\overline{93^\circ}$ | $\overline{225^\circ}$ | $\overline{95^\circ}$ | $\overline{68^\circ}$ | $\overline{67^\circ}$ | $\overline{67^\circ}$ | $\overline{225^\circ}$ | $\overline{28^\circ}$ | $\overline{68^\circ}$ | $\overline{95^\circ}$ | $\overline{68^\circ}$ |
|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|

- 2 Find the unknown angles in the following quadrilaterals. Match the letter of the angle to the correct angle below, to solve the riddle:

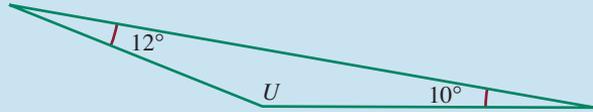
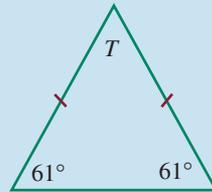
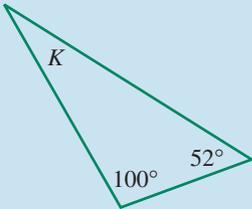
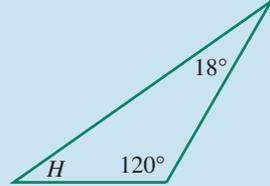
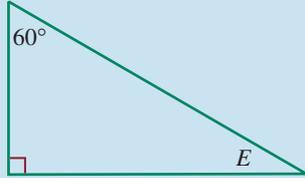
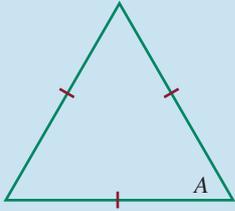
What is an IG?



- | | | | | | | | | | | | | |
|------------------------|------------------------|------------------------|------------------------|-----------------------|------------------------|-----------------------|------------------------|------------------------|------------------------|------------------------|-----------------------|------------------------|
| $\overline{90^\circ}$ | $\overline{77^\circ}$ | $\overline{180^\circ}$ | $\overline{65^\circ}$ | $\overline{45^\circ}$ | $\overline{135^\circ}$ | $\overline{70^\circ}$ | $\overline{115^\circ}$ | $\overline{65^\circ}$ | $\overline{118^\circ}$ | $\overline{115^\circ}$ | $\overline{70^\circ}$ | $\overline{180^\circ}$ |
| $\overline{145^\circ}$ | $\overline{135^\circ}$ | $\overline{30^\circ}$ | $\overline{118^\circ}$ | $\overline{77^\circ}$ | $\overline{115^\circ}$ | $\overline{30^\circ}$ | $\overline{115^\circ}$ | $\overline{135^\circ}$ | $\overline{62^\circ}$ | $\overline{180^\circ}$ | $\overline{30^\circ}$ | |

- 3 Find the unknown angles in the following triangles. Match the letter of the angle to the correct angle below, to solve the riddle:

How do you make varnish disappear?



<u> </u>											
58°	60°	28°	30°	35°	158°	58°	58°	42°	30°	45°	

- 4 Unscramble the letters below to form a word. Write down the first letter in each of the words to find the answer to the riddle:

What do you call an escaped parrot?

ROLLGAMARELAP

LOVA

GNHTEL

TAHYC

MYTREGGE

TAGOCON

AGONNNO





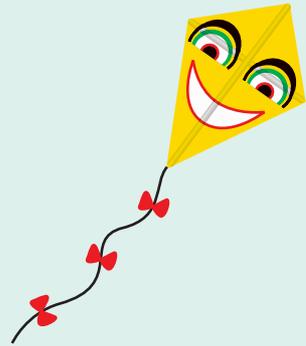
Applications

Making a kite

Using thin strips of bamboo or similar material, a thin plastic sheet, string and streamers, construct a kite with short sides of length 50 cm and long sides that measure 90 cm in length.

Angles in the sun

Move outside, stand with the sun behind you. Using your arms and the shadows they create, try to make an acute, an obtuse and a reflex angle. Get your partner to trace over your shadow using chalk and accurately label your angles. Swap over.

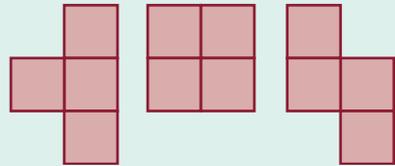


Polygon path!

Plan a 'polygon path' around your classroom, neighbourhood or schoolyard. You must include ten different stations and each station must be accompanied by a question based on the geometry in this chapter, to be answered by your classmates. Present your 'polygon path' on a piece of A4 paper. Ask a classmate, or the class, to complete your trail.

Tetrominoes

A tetromino is a shape formed by joining four squares together edge to edge. Three tetrominoes are shown. There are five different tetrominoes possible. Can you draw them all?

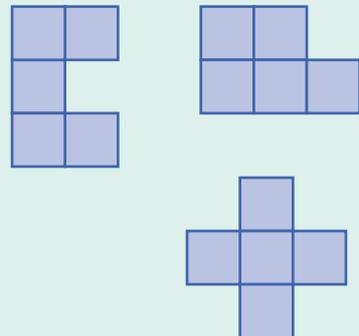


Note that a rotated or reflected shape is not classed as a different shape.

Make a set of pentominoes and see if you can fit them together to make a 6 by 10 rectangle.

Pentominoes

A pentomino is a shape formed by joining five squares together edge to edge. Three pentominoes are shown. How many different pentominoes are possible?



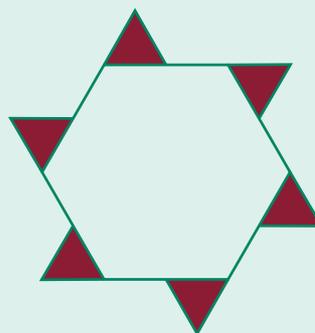
Exterior angles of polygons

Draw a hexagon and extend each one of the sides past the vertex to create an external angle as shown.

Colour in the regions inside the exterior angles.

Cut out the coloured regions and place together with no overlapping edges.

What is the size of the angle formed by adding together the regions? Draw a series of polygons that include regular and irregular shapes and repeat the above task. Copy and complete the table below to record your results:



Polygon	Diagram	Number of exterior angles	Size of each exterior angle	Sum of exterior angles
Triangle				
Quadrilateral				
Pentagon				
Hexagon		6	60	360°
Heptagon				
Octagon				

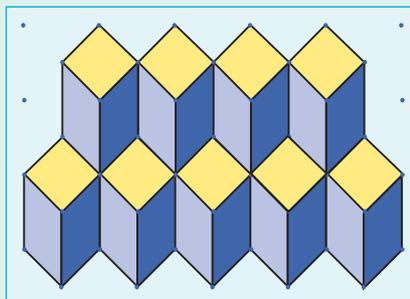
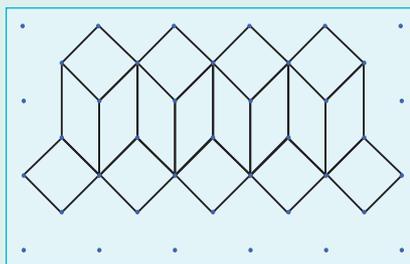
Copy and complete this statement:

The sum of the exterior angles of a polygon is equal to _____.

Patchwork

A tessellation is produced by repeating shapes to cover a page, leaving no spaces. Tessellations are often used in patchwork quilts. One popular design is the three-dimensional cube.

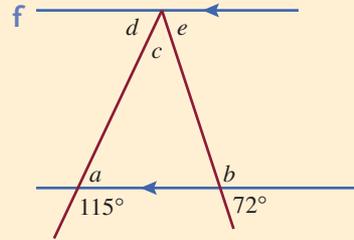
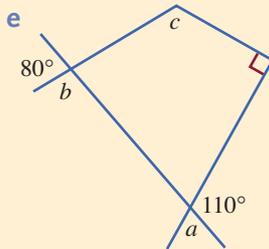
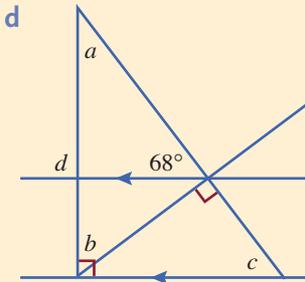
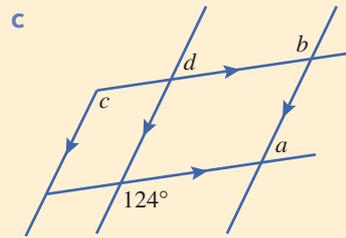
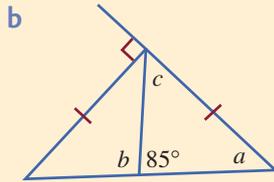
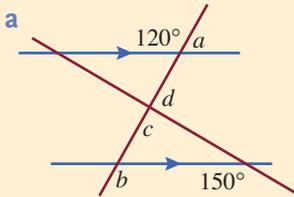
- Using grid paper as shown, draw a series of tessellating parallelograms and squares.
- Colour in the shapes to give a three-dimensional effect.
- Experiment with different ways to colour in the shapes to give different effects.
- Cut out the parallelograms and squares from material and sew them together to form a section of a quilt, or use brightly coloured paper to make a poster.



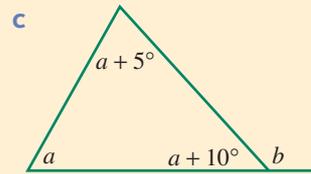
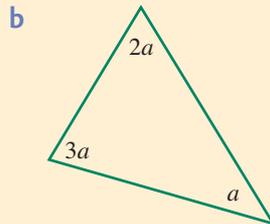
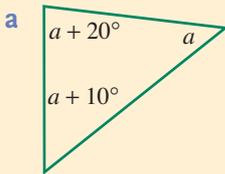


Enrichment

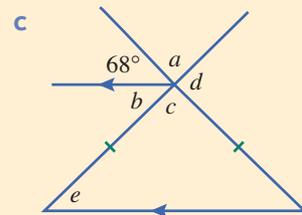
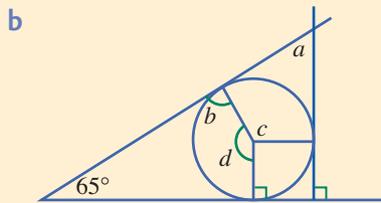
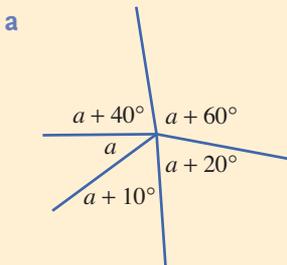
1 Without using a protractor find the unknown angles:



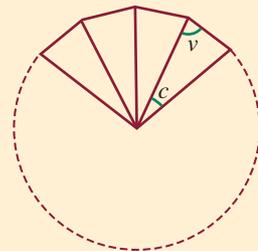
2 Find the pronumeral in each of the following:



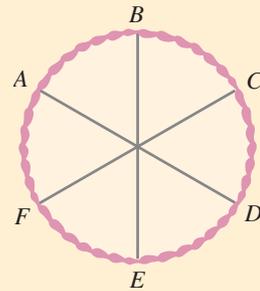
3 Find the value of the pronumerals:



4 The sum of the vertex angles (v) in a certain regular polygon is three times the sum of the angles at its centre (c). Calculate the number of sides in this regular polygon.



- 5 Niamh was cutting up her birthday cake when she noticed that she had formed several adjacent angles.
- Measure the acute angles Niamh has created.
 - What is their sum?
 - What do you notice about the opposite angles?



- 6 Naomi ordered some pizzas for a party of 11 people. She believed she could cut one pizza into 11 pieces with only four cuts.
- Draw a circle and make one cut.
What is the maximum number of pieces?
 - Draw another circle and make two cuts.
What is the maximum number of pieces?
 - Draw another circle and make three cuts.
What is the maximum number of pieces?
 - Copy and complete the table below, and use the working column to generate a pattern.



Number of cuts	Diagram	Maximum number of pieces	Working out
1		2	$\frac{1 \times 2}{2} + 1$
2		4	$\frac{2 \times 3}{2} + 1$
3		7	
4			
5			
6			

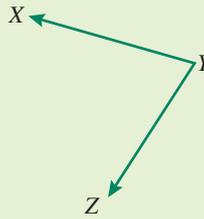
- If n = the number of cuts, find a rule for P , the maximum number of pizza pieces.
- Use the rule to find the maximum number of pieces possible from the following number of cuts:
 - 8
 - 15
 - 18
 - 20
- Use the rule to find the minimum number of cuts for the following number of pieces:
 - 29
 - 79
 - 46
 - 326
- Is Naomi right? Could cut one pizza be cut into 11 pieces with only four cuts?



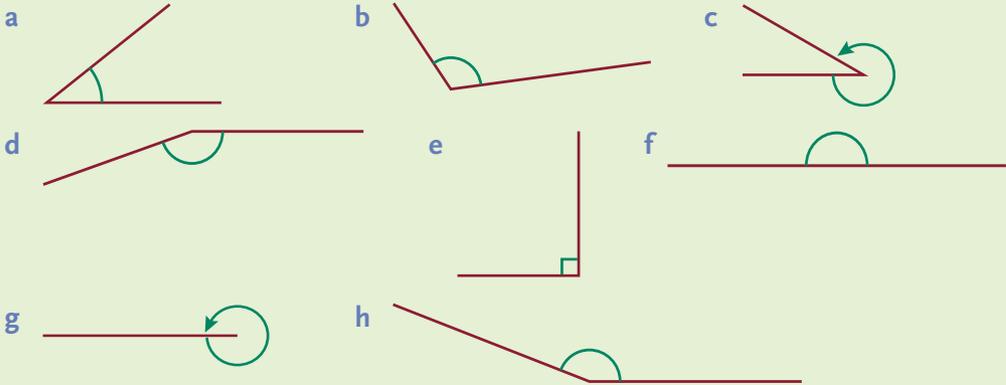
Revision/Assessment

Exercise 3A

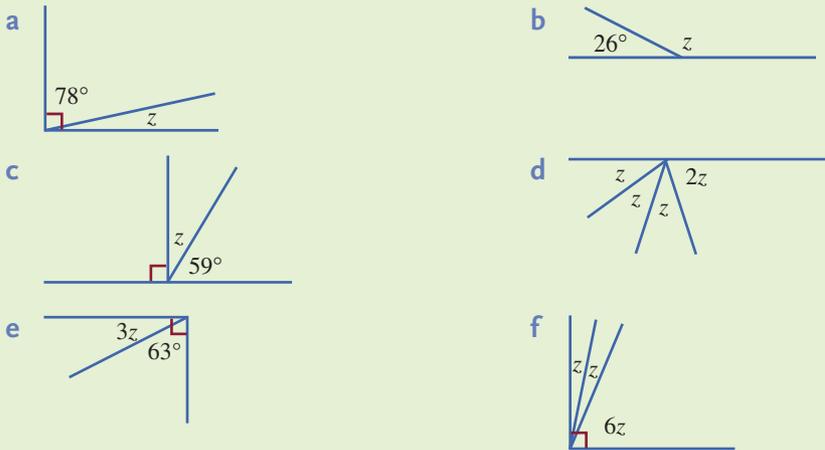
1 Give two names for the angle shown:



2 For each of the following angles state its type:

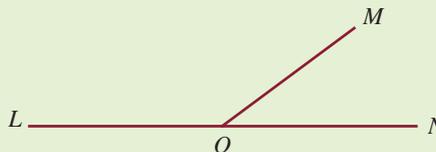


3 Find the value of z in each of the following:



4 Using your protractor, measure the size of the following angles:

- a acute angle $\angle MON$
- b obtuse angle $\angle LOM$
- c reflex angle $\angle MON$



5 Use your protractor to draw the following angles.

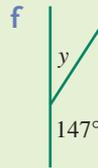
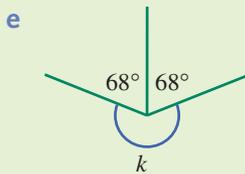
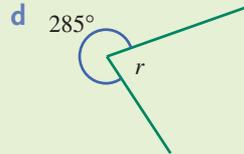
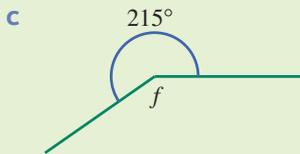
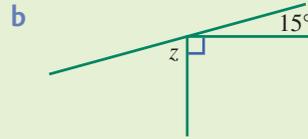
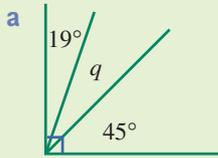
a $\angle K = 73^\circ$

b $j = 225^\circ$

c $\angle UVW = 315^\circ$

d $\angle t = 92^\circ$

6 Find the value of the pronumerals in each of the following:



7 Which angle is complementary to 39° ?

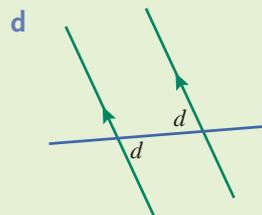
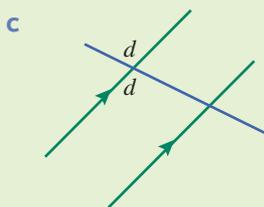
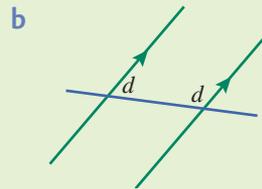
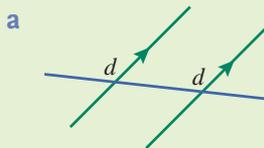
8 Write down the angle complementary to $(5 - n)^\circ$.

9 Which angle is supplementary to 39° ?

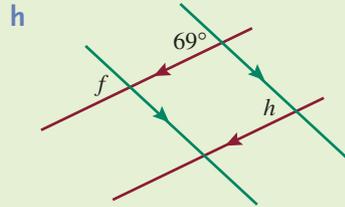
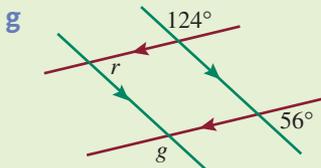
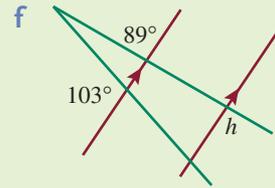
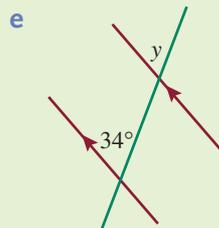
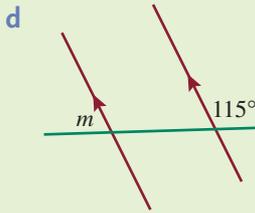
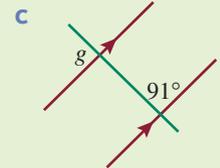
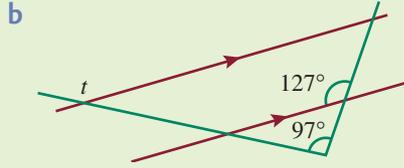
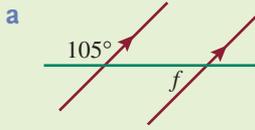
10 Write down the angle supplementary to $(12 + 3n)^\circ$.

Exercise 3B

11 Name the pair of angles shown in each diagram:

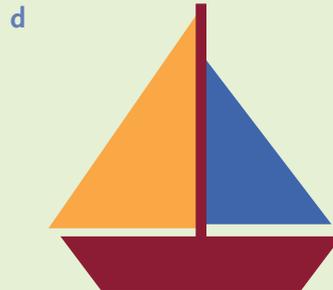
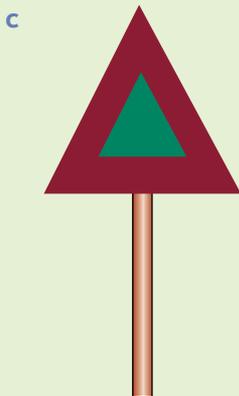
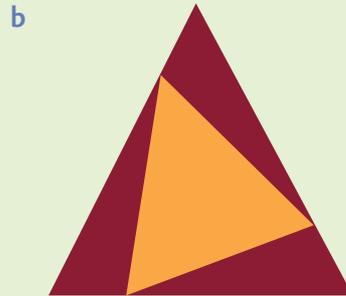
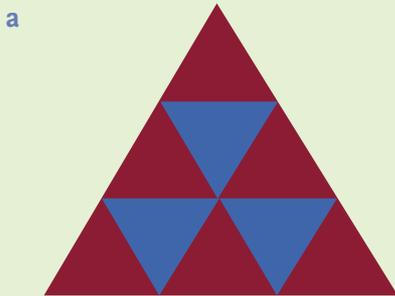


12 Find the value of the pronumerals in each of the following:



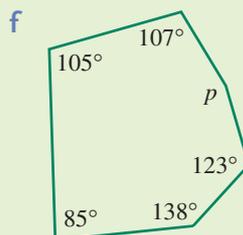
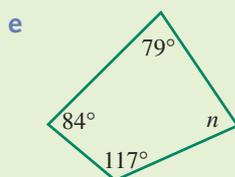
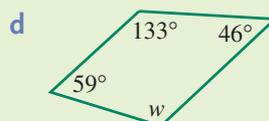
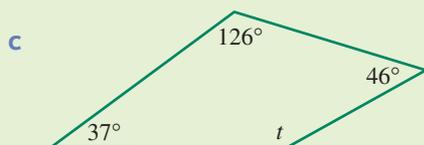
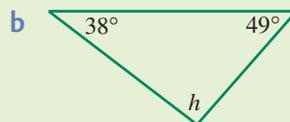
Exercise 3C

13 Look at the triangles in the following pictures. Give each triangle two names, one according to its side properties and the other according to its angle properties.



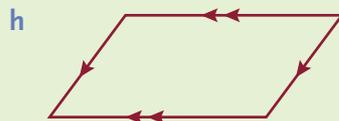
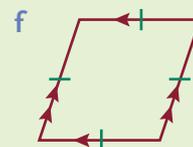
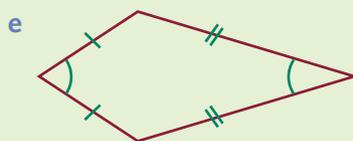
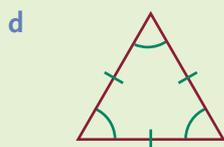
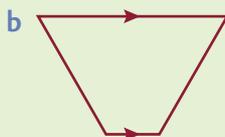
Exercises 3C and 3D

14 Find the value of the unknown angle in each of the following shapes:



Exercise 3E

15 Write down the correct mathematical name for each of the shapes shown below:



16 Find the angle sum of a:

- a hexagon b pentagon c nonagon d 13-agon e 100-agon

17 What is the size of each angle in the regular polygons below?

- a octagon b 21-agon c 17-agon

CHAPTER

4

Algebra
Symbols



Algebra Symbols

Solomon Islands became an independent nation from Great Britain on the 7th July 1978. Since then, each year the government and people of the Solomon Islands celebrate the anniversary of their nationhood.

The 30th anniversary was celebrated on the 7th of July 2007. It was a big occasion throughout the country. During the celebration people wore T-shirts in the colours of the national flag: blue, yellow and green. Blue represents the ocean, yellow represents the sky and green represents the forests of the Solomon Islands.

This chapter covers the following skills:

- Working with mathematical expressions
- Finding equivalent forms for various mathematical expressions
- Using the distributive law
 $a(b + c) = ab + ac$
- Evaluating mathematical expressions
- Deducing a formula which models a particular situation
- Verifying a deduced formula
- Listing values derived from a formula in sequence form
- Exploring expressions on a CAS calculator

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.4.1.1** Define the term 'algebraic expression'.
Expression—a mathematical statement that is made up of terms, coefficients, constants separated by addition and subtraction signs.
- 8.4.1.2** Explain the words: terms; pronumerals; coefficients; like terms; unlike terms; constants.
- 8.4.2.1** Evaluate pronumerals in mathematical statements.
- 8.4.3.1** Identify terms, coefficients and constants in expressions.
- 8.4.4.1** Write algebraic expressions for statements that represent quantities.
- 8.4.5.1** Group like terms in expressions.
- 8.4.6.1** Simplify expressions by adding and subtracting like terms.
Like terms: same bases and same powers

8.4.7.1

Simplify terms and expressions when multiplying and dividing using the following rules:

Same sign = Positive:

$$p \times t = pt \quad -p \times -t = pt$$

$$p \times t = pt, \quad -p \times -t = pt$$

$$p \div t = pt \quad -p \div -t = pt$$

Different signs = Negative:

$$-a \times -b = ab \quad a \times -b = -ab$$

$$-a \times b = -ab, \quad a \times -b = -ab - \frac{a}{b}$$

$$a \div b = \frac{a}{b}$$

8.4.8.1

Expand brackets by multiplying terms outside the bracket with those inside the brackets—i.e. distributive law.

8.4.8.2

Write algebraic expressions using brackets.

8.4.9.1

Explain the term 'evaluate'.

8.4.9.2

Substitute values into expressions

8.4.10.1

Explain the term 'formula'

8.4.11.1

Use formulas to solve practical problems.

A **pronumeral** is a letter that represents an unknown number.
For example, x might represent the number of school days in a year.

A **term** usually contains products or quotients of pronumerals and numbers.
The term $3x^2$ means $3 \times x \times x$ or 3 lots of x^2 .

A **coefficient** is the number multiplying a pronumeral or product of pronumerals.
3 is the coefficient of x^2 in the term $3x^2$.

Like terms have exactly the same letter make up, other than order.
 $6x^2y$ and $14yx^2$ are like terms.

The value of the product stays the same, regardless of the order.
For example, $2 \times 3 = 3 \times 2$, so x^2y and yx^2 is the same term.
 $4a^2bc$ and $3bc^2a$ are unlike terms. Only like terms may be added or subtracted.

Example

1 Write down the value of the pronumeral used in each of the following:

a n = the number of times Australia has hosted the Olympic Games

b p = the number of hours in a day

2 For the expression $5x^2 - 2x - 6$, find:

a the number of terms

b the coefficient of x

c the coefficient of x^2

d the constant term

3 Write an algebraic expression to represent the following:
A certain number n is doubled and the result is subtracted from 10.

Solution

Australia has hosted the Olympic Games twice: $n = 2$.

There are 24 hours in a day: $p = 24$.

There are 3 terms: $5x^2$, $2x$ and 6.

The coefficient of x is -2 .

The coefficient of x^2 is 5.

The constant term is -6 . It is the term without a pronumeral.

Doubling the number n gives $2n$.
Subtracting this result from 10 then gives $10 - 2n$.

The expression is $10 - 2n$.

Exercise 4A

1 Write down the value of the pronumeral used in each of the following statements:

a n = the number of days in a non-leap year

b g = the number of girls in the classroom

c b = the number of boys in the classroom

d t = the number of teachers in the classroom

e T = the total number of people in the classroom

f s = the speed at which I need to ride my bike to travel 50 kilometres in 2 hours

g L = the number of lessons in the school day

h S = the number of sandwiches you eat for lunch

2 Write down the value of the pronumeral for each of the following photographs:

b = the number of golf balls

t = the number of tees

$n = b + t$

a



b



3 Write down the coefficient of x in each of the following:

a $7x$

b $-8x$

c $2x - 3y$

d $9y + x$

e $4x^2 - 8x$

f $\frac{2}{3}x$

g x

h $-\frac{4}{3}x$

i $\frac{3x}{4}$

j $\frac{5x}{6}$

k $\frac{7x}{3}$

l $2 - \frac{3x}{4}$

4 For each of the following expressions, state the number of terms and the coefficient of the term noted in brackets:

a $3x + 4y$ (y)

b $7x^2 + 3x$ (x^2)

c $7x^2 + x$ (x)

d $-a + 3ab + b^2$ (b^2)

e $-a + 3ab + b^2$ (a)

f $-a + 3ab + b^2$ (ab)

g $4xy - \frac{2x^2}{3}$ (x^2)

h $4xy - \frac{2x^2}{3}$ (x)

i $4xy - \frac{2x^2}{3}$ (xy)

5 Write an algebraic expression for each of the following statements:

a Eight is added to the number n .

b Six is added to three times the number n .

c Four is added to five times a certain number x .

d Thirty is subtracted from six times a certain number y .

e A certain number m is multiplied by negative five and then eight is added.

f A certain number q is divided by four and then six is added.

g Twelve is divided by negative x .

h The number x is multiplied by negative three and the result is divided by five.

i Four times the number z is subtracted from negative three.

j Three times the number y is divided into negative twelve.

6 List the groups of like terms from each of the following:

a $4x, -5y, 2x, 7y, -x$

b $2a, -3ab, 6b, 7ba, -4a, 4ab, -5b$

c $6p^2q, -pq, 12pq^2, 9pq, qp$

d $3x^2, 2x, -x^2, 3xy, 5x^2, -x^2y, 6yx, -3yx^2$

e $5c^2d^2, c^2d, -d^2c, 4c^2d^2, -dc$

f $x^2, y^2, 2x, 2x^2, 2y, 2y^2, y, -xy, x$

g $10cd^2, 4c^3d, 10d^2c, 6d^2, dc$

h $3a^2, b^2, 5a, -b^2, b, 2b^2, b, 6ab, 9ab$

i $2x^2y, 3xy^2, 5xy, 8yx, y^2x$

j $mnp, mpn^2, np^2m, pnm, p^2mn$

4B

Adding and subtracting like terms

Only like terms can be added or subtracted. Remember that the terms ab and ba are like terms.

Example

Simplify the following expressions:

a $x + x + x + y + y$

b $2xyz + 5xyz - 6yzx$

c $2abc + 3bac - 7cba$

d $3a + 9 + 6b - 4 - 2a - 8b$

Solution

$$\begin{aligned} x + x + x + y + y \\ = 3x + 2y \end{aligned}$$

$$\begin{aligned} 2xyz + 5xyz - 6yzx \\ = 2xyz + 5xyz - 6xyz \\ = 7xyz - 6xyz \\ = 1xyz \\ = xyz \end{aligned}$$

$$\begin{aligned} 2abc + 3bac - 7cba \\ = 2abc + 3abc - 7abc \\ = 5abc - 7abc \\ = -2abc \end{aligned}$$

$$\begin{aligned} 3a + 9 + 6b - 4 - 2a - 8b \\ = 3a - 2a + 6b - 8b + 9 - 4 \\ = a - 2b + 5 \end{aligned}$$

Exercise 4B

1 Simplify the following expressions:

a $m + m + m + n + n$

c $x + x + x - y + y$

e $m + m + n + n + n$

g $g + g + h - h - h$

b $x + y + x + y + x + x$

d $a - b + a - a - b + a$

f $s + s + t + t - s - s + t$

h $a + a + a - b - b - b$

2 Simplify the following expressions:

a $10x - 5x$

d $7x - 12x$

g $6b - 5b$

j $8c - 16c$

b $5x - 10x$

e $15a - 7a$

h $5b - 6b$

k $25p - 9p$

c $12x - 7x$

f $7a - 15a$

i $16c - 8c$

l $9p - 25p$

3 Simplify:

a $-2d - 3d$

d $-5q - 10q$

g $-y - 3y$

b $-3x - 4x$

e $-x - x$

h $-m - 10m$

c $-8m - 12m$

f $-2y - y$

i $-3q - 7q$

4 Simplify the following expressions:

a $4y + 8y$

d $4y - 8y$

g $-4y - 8y$

j $-8y + 2y$

b $8y + 4y$

e $-8y + 4y$

h $-8y - 4y$

k $2y - 8y$

c $8y - 4y$

f $-4y + 8y$

i $8y - 8y$

l $6y - 8y$

5 Simplify the following expressions:

a $10y + 3y$

b $3y + 10y$

c $10y - 3y$

d $3y - 10y$

e $-10y + 3y$

f $-3y + 10y$

g $-3y - 10y$

h $-10y - 3y$

i $10y - 10y$

6 Simplify the following like terms:

a $2xy - 4xy$

b $6xy - 9yx$

c $6ab - 5ab$

d $9ab - 6ba$

e $4pq - 8pq + 9pq$

f $7pq - 2pq - 4qp$

g $8mn + 2mn - 16mn$

h $15mn - 17nm - 5nm$

i $19xy - 15xy - 6xy$

j $7xy - 3yx - xy$

k $7wx - 12wx + 5wx$

l $-6rs + 2rs + 4rs$

m $-x^2 + 4x^2 - 2x^2$

n $-y^2 - 5y^2 + 2y^2$

7 Simplify the following expressions by combining like terms:

a $4x + 5x + y + 6y$

b $5m - 2m + 3n - 6n$

c $6xy - 2xy + zy - 2zy$

d $6pq - 8q - 2qp + 5q$

e $2x - 3y - 4x + 5y$

f $3x + 2 - 4x - 5$

g $a - 5 - 2a + 2$

h $4 + 3x - 2 - 5x$

i $6x + 3y - 2 - 4x - 5y + 1$

j $3d - 2e + 3e - 4d$

k $8x - 2y + 7 - 9x + y - 6$

l $3g + 6h - 4g - 6h$

m $4x^2 + 5x - 2x^2 - 8x$

n $2x^2 + x - 5x^2 - 7x$

o $3x^2 + 2x - 4 + x^2 + x + 5$

p $4x^2 - 7x + 1 - 6x^2 + 5x - 3$

8 Simplify the following expressions by combining like terms:

a $2a + 3b + c - 3a + 2b - 2c - 5a - 6b - 4c$

b $6ab + 3bc - ca - 2ab - 4bc - 7ab + 2ca$

c $-7xy + 5yz - 2zy + 3zx + 4xy - 6yz - 6zx$

d $2x^2 + 5x + 6 - 2x^2 - 8x - 9$

e $x^2 - 3x + 9 - 4x^2 - 2x - 19$

f $2x^2 - 5x + 1 + y^2 + 3y - 4x^2 - 2x - 19 - 5y^2 - 8y$

g $x^2 - 2x + 1 + y^2 - 2y - 2x^2 - x - 6 - 2y^2 - 3y + 4$

9 Copy and complete the following tables by adding the terms across the top of the table to each of the terms down the side of the table.

a

+	3	2y	5x	4xy
2				
3y				
4x				
2xy				

b

+	5	y	2x	5xy
1				
y				
3x				
xy				

c

+	2	-y	4x	2xy
4				
-y				
-x				
-xy				

To simplify algebraic expressions, use the rules for multiplying or dividing directed numbers.

When multiplying different pronumerals, simply leave out the multiplication sign.

$x \times y = xy$	$-x \times -y = xy$	$-x \times y = -xy$	$x \times -y = -xy$
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When dividing different pronumerals, write the expression as a fraction:

$x \div y = \frac{x}{y}$	$-x \div -y = \frac{x}{y}$	$x \div -y = -\frac{x}{y}$	$-x \div y = -\frac{x}{y}$
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In summary, when the signs of two numbers are the same, the answer is positive, and when the signs are different, the answer is negative.

Example

1 Simplify these expressions:

a $-5 \times x$

b $-4x \times 2y$

2 Simplify these expressions:

a $-7x \div y$

b $20a \div -4$

c $-b \div 6 \times -a$

d $-2xy \times 3x \div -4y$

Solution

$$-5 \times x = -5x$$

$$\begin{aligned} -4x \times 2y &= -4 \times x \times 2 \times y \\ &= -4 \times 2 \times x \times y \\ &= -8xy \end{aligned}$$

$$-7x \div y = -\frac{7x}{y}$$

$$\begin{aligned} 20a \div -4 &= \frac{20a}{-4} \\ &= -5a \end{aligned}$$

$$\begin{aligned} -b \div 6 \times -a &= -\frac{b}{6} \times -\frac{a}{1} \\ &= \frac{ab}{6} \end{aligned}$$

$$\begin{aligned} -2xy \times 3x \div -4y &= -2 \times 3 \times x \times x \times y \div -4y \\ &= -6x^2y \div -4y \\ &= \frac{-6x^2y}{-4y} \\ &= \frac{6x^2y}{4y} \\ &= \frac{3x^2}{2} \end{aligned}$$

Exercise 4C

1 Simplify the following products as far as possible:

a $-3 \times x$

b $7 \times -y$

c $-a \times -11$

d $b \times -9$

e $-a \times b$

f $-m \times -n$

g $-14 \times x \times y$

h $-m \times n \times -10$

i $-a \times -b \times -c$

j $-x \times 6 \times 7$

k $-y \times -8 \times 6$

l $-4 \times -7 \times -p$

m $-3 \times a \times -9 \times b$

n $12 \times -x \times -8 \times y$

o $-a \times -b \times -b$

p $-8 \times -x \times 9 \times x$

q $4 \times -x^2 \times y \times -11$

r $-8 \times m \times -n \times -m$

s $-a \times b \times -4 \times a \times -7 \times 5 \times -b$

2 Simplify the following products as far as possible:

a $-8 \times 2x$

b $6 \times -9y$

c $-3a \times -7$

d $y \times -5x$

e $-m^2 \times 5n$

f $-7c \times -9b$

g $-5x \times 6y$

h $7p \times -9q$

i $-5x \times -5x$

j $3y \times -5y$

k $-2a \times 4b \times -7$

l $-6m \times -5n \times -9p$

m $-3x \times 5y \times -2z$

n $3a \times -6b \times -7c$

o $-4x \times -3y \times -5x$

p $(-10x)^2$

q $(-13y)^2$

r $(-9xy)^2$

s $(-12ab)^2$

t $(-11mn)^2$

u $(-15pqr)^2$

3 Copy and complete the following tables by multiplying the terms across the top of the table by each of the terms down the side of the table:

a

\times	3	y	$-2x$	xy
-1				
-y				
2x				
3xy				

b

\times	-2	7y	$-3x$	$-xy$
2				
$-3y$				
$-4x$				
$-2xy$				

4 Express the following quotients in fraction form, giving the correct sign with the answer:

a $-x \div 4$

b $m \div -9$

c $-p \div -10$

d $8 \div -x$

e $-7 \div 5x$

f $-3x \div -7y$

g $-8a \div 9b$

h $-1 \div 6x^2$

i $-3x^2 \div -8y^2$

5 Simplify the following algebraic fractions by cancelling:

a $\frac{3a}{-15}$

b $\frac{-7a}{14}$

c $\frac{-2y}{-10}$

d $\frac{-12}{4x}$

e $\frac{27}{-9y}$

f $\frac{-28}{-7m}$

g $\frac{-8}{24p}$

h $\frac{5}{-20q}$

i $\frac{-16}{-48xy}$

j $\frac{10x}{-8}$

k $\frac{-12x}{8}$

l $\frac{-18y}{-12}$

m $\frac{-40}{12x}$

n $\frac{12}{-30ab}$

o $\frac{-25}{-35w}$

p $\frac{-18}{16m}$

6 Express the following as fractions, giving the answer with the correct sign:

a $-7 \times x \div -y$

b $9 \times -a \div -b$

c $-m \times -6 \div -q$

d $-p \times -q \div r$

e $-6 \times -x \div y$

f $-p \times -q \div -r$

g $6 \times -r \div -s$

h $12 \times -m \div -n$

i $-a \times -b \div -c$

7 Express the following as fractions and cancel where possible:

a $-24 \times x \div -48$

b $-26 \times -x \div 13$

c $x \div -16 \times -24$

d $-10x \div -5$

e $-27y \div 18$

f $-12 \div -8m$

g $32 \div -12y$

h $-ab \div 4a$

i $-55xy \div -5y$

j $-2x \times 3xy \div -y$

k $-5xy \times -2x \div -5y$

l $-12xy \times 6xy \div 24x$

8 Copy and complete the following tables, by dividing the numbers across the top of the table by each of the terms down the side of the table.

a

\div	-24	12y	-x	36xy
1				
-3y				
2x				
-4xy				

b

\div	-10	20y	-5x	30xy
2				
-5y				
x				
-2xy				

9 My bank account is currently overdrawn by \$150. The balance is $-\$150$.

a If I withdraw another \$25, what would the balance be then?

b Write an expression for the balance if I withdraw a further $\$d$ from the balance in part a.

b How much do I need to deposit on pay day so that my balance is brought back to $\$2d$?

10 A BMX bicycle that I bought at Solo Shop, Chinatown, for \$940, loses \$70 in value after each year of use.

a What would its value be after 1 year?

b What would its value be after 2 years?

c What would its value be after 3 years?

d What would its value be after y years? (Write an expression involving y .)

e After how many years is it worth half of its original value?

f After how many years is it worth nothing?



11 The sunglasses I bought for $\$D$ lose $\$L$ in value after each year of use. What will its value be after T years of use?

In algebra the term outside the bracket ‘distributes’ onto the terms inside and multiplies each.

$$a(b + c) = ab + ac \text{ and } a(b - c) = ab - ac$$

The term a can also come after the bracket, and we would have:

$$(b + c)a = ba + ca \text{ and } (b - c)a = ba - ca$$

The sign immediately in front of a term belongs to that term:

$$-a(b + c) = -ab - ac \text{ and } -a(b - c) = -ab + ac$$

Example

Expand the following expressions:

a $5(x - 6)$

$$\begin{aligned} &5(x - 6) \\ &= 5 \times x - 5 \times 6 \\ &= 5x - 30 \end{aligned}$$

b $-6x(2x - 11)$

$$\begin{aligned} &-6x(2x - 11) \\ &= -6x \times 2x + 6x \times 11 \\ &= -12x^2 + 66x \end{aligned}$$

c $-9x + 5(x + 2)$

$$\begin{aligned} &-9x + 5(x + 2) \\ &= -9x + 5x + 10 \\ &= -4x + 10 \end{aligned}$$

d $-8(3x - 2) - 10$

$$\begin{aligned} &-8(3x - 2) - 10 \\ &= -24x + 16 - 10 \\ &= -24x + 6 \end{aligned}$$

Solution

Exercise 4D

1 Copy and complete the following. Compare your answers:

a $-4 \times (7 - 3)$
 $= -4 \times \underline{\quad}$
 $= \underline{\quad}$

b $-4 \times 7 + 4 \times 3$
 $= \underline{\quad} + \underline{\quad}$
 $= \underline{\quad}$

2 Copy and complete the following. Compare your answers:

a $-8 \times (12 + 4)$
 $= -8 \times \underline{\quad}$
 $= \underline{\quad}$

b $-8 \times 12 - 8 \times 4$
 $= \underline{\quad} - \underline{\quad}$
 $= \underline{\quad}$

3 Expand (remove brackets from) each of the following:

a $-(x + y)$

b $-(a + b)$

c $-(m + n)$

d $-(x + 4)$

e $-5(y + 7)$

f $-10(p + 8)$

g $(a + 4) \times -8$

h $(c + 5) \times -9$

i $(b + 9) \times -5$

j $(6 + p) \times -5$

k $(12 + q) \times -3$

l $(5 + n) \times -10$

m $-4(x + 2)$

n $-4(x - 2)$

o $-4(2 - x)$

4 Expand each of the following:

- | | | | | | | | |
|---|-------------|---|---------------------|---|----------------------|---|----------------------|
| a | $12(x - y)$ | b | $8(a - b)$ | c | $9(m - n)$ | d | $8(x - 7)$ |
| e | $3(x - 9)$ | f | $2(x - 14)$ | g | $-2(13 - m)$ | h | $-9(7 - q)$ |
| i | $-6(9 - p)$ | j | $(7 - x) \times -8$ | k | $(18 - b) \times -3$ | l | $(15 - m) \times -9$ |

5 Expand each of the following:

- | | | | | | | | |
|---|-------------|---|-------------|---|--------------|---|-------------|
| a | $x(y + z)$ | b | $m(p + q)$ | c | $a(b + c)$ | d | $r(s - t)$ |
| e | $p(q - r)$ | f | $l(m - n)$ | g | $m(n - 8)$ | h | $a(b - 12)$ |
| i | $-z(y - 8)$ | j | $-c(9 - b)$ | k | $-p(14 - q)$ | l | $-m(9 - n)$ |

6 Expand each of the following:

- | | | | | | | | |
|---|----------------|---|-----------------------|---|------------------------|---|---------------|
| a | $9(3x + 2)$ | b | $3(4y + 6)$ | c | $2(9b + 4)$ | d | $5(7q - 6)$ |
| e | $4(6p - 12)$ | f | $4(3s - 7)$ | g | $-5(2x + 5y)$ | h | $-8(6m + 9n)$ |
| i | $-2(8a + 12b)$ | j | $(3a - 7b) \times -6$ | k | $(8m - 6n) \times -11$ | l | $-(2m - 3n)$ |

7 Expand each of the following and simplify as far as possible:

- | | | | | | |
|---|--------------------------|---|------------------------|---|----------------------------|
| a | $3(2x + 6) + 9$ | b | $7(7y - 1) + 10$ | c | $4 - 5(3m + 6)$ |
| d | $30 - 9(b - 2)$ | e | $4(2a + 3b - 4c) + 5b$ | f | $4x(3x + 2)$ |
| g | $2x(3x + 2) - 3(3x + 2)$ | h | $-6y(5y - 8)$ | i | $-3x(2x + 5) - 4x(3x - 2)$ |

8 Write an algebraic expression using brackets for the following and then remove the brackets:

- I think of a number n , add 3 and then multiply the total by -7 .
- I think of a number n , subtract 4, then multiply the result by -5 .
- I think of a number n , double it, subtract 3, then multiply the result by -4 .
- I think of a number n , multiply it by 4, subtract 6, then multiply the result by -3 .
- I think of a number n , halve it, subtract 3, then multiply the result by -6 .

9 The price of a lollipop at the school canteen is 2 dollars.

- Write an expression for the price of a lollipop if it is discounted by d dollars.
- Using brackets, write down an expression for the cost of five discounted lollipops.
- Keeping the brackets in place, write an expression for the change I would get from 10 dollars, when I buy the five discounted lollipops.
- Remove the brackets to your answer in part c and simplify the answer as far as possible.



10 At the school canteen a bag of jelly beans costs x cents.



- Write an expression for the price of a bag of jelly beans if it is discounted by 50 cents.
- Using brackets, write down an expression for cost of six discounted bags of jelly beans.
- Keeping the brackets in place, write an expression for the change I would get from five dollars, when I buy six discounted bags of jelly beans.
- Expand the brackets and simplify the expression.

Pronumerals are letters that represent unknown numbers. When values for pronumerals are substituted into an expression and a result is worked out, it is called **evaluating** the expression.

When you substitute values and evaluate expressions, remember the rules for directed numbers and order of operations (BODMAS).

Example

Find the values of the following expressions where $x = 3$ and $y = -4$.

a $8xy$

$$\begin{aligned} 8xy &= 8 \times 3 \times -4 \\ &= -96 \end{aligned}$$

b $4(2x - 8) - y$

$$\begin{aligned} 4(2x - 8) - y &= 4 \times (2 \times 3 - 8) - -4 \\ &= 6 \times (6 - 8) + 4 \\ &= 6 \times -2 + 4 \\ &= -12 + 4 \\ &= -8 \end{aligned}$$

c $2y^2 + 5y - 3x$

$$\begin{aligned} 2y^2 + 5y - 3x &= 2 \times (-4)^2 + 5 \times -4 - 3 \times 3 \\ &= 2 \times 16 - 20 - 9 \\ &= 32 - 20 - 9 \\ &= 3 \end{aligned}$$

d $\frac{2x + y}{5}$

$$\begin{aligned} \frac{2x + y}{5} &= \frac{2 \times 3 + -4}{5} \\ &= \frac{6 - 4}{5} \\ &= \frac{2}{5} \end{aligned}$$

Solution

Exercise 4E

1 Find the values of the following expressions where $x = -4$:

a $3x$

b $7x$

c $\frac{1}{2}x$

d $x + 9$

e x^2

f $4x + 16$

g $5 + 3x$

h $5 - 3x$

i $-7 + 2x$

j $-7 - 2x$

k $x^2 + x$

l $x^2 - x$

2 Find the values of the following where $x = -5$ and $y = 4$:

a $2xy$

b $3xy - 1$

c $\frac{1}{4}xy + 1$

d $2x + 6y$

e $9x - y$

f $4y - 3x$

g $4(2x + 1) + y$

h $2(3x - 1) + 2y$

i $4(2x - 4) + y$

j $7 - y^2$

k $3x^2 + 2y + x$

l $x^2 \div 2y$

m $x^2 + 2x$

n $3x^2 + x$

o $2x^2 - x + 1$

p $x^2 + y^2$

q $(x + 2)^2$

r $y^2 + 4y + 4$

s $y^2 - x^2$

t $(y - x)(y + x)$

3 Using the values $p = -5$, $q = 3$, $r = -4$, evaluate the following expressions:

- a qr b pq c prq d $5p^2$
 e $pq + pr$ f $p(q + r)$ g $q^2 + 4q$ h $q(q + 4)$
 i $q^2 + r^2$ j p^2 k $r^2 - 3r$ l $r(r - 2)$

4 If $a = 5$, $b = -6$, $c = 3$, evaluate the following expressions:

- a $\frac{ab}{c}$ b $\frac{2ac}{b}$ c $\frac{b+4}{a}$ d $\frac{a+c}{b}$ e $\frac{a-b}{c}$ f $\frac{2a+b}{2c}$

5 Complete the following tables by substituting each value of x into the given expression:

- a

x	-3	-2	-1	0	1
$x+1$					

 b

x	-2	-1	0	1	2
$x-2$					
- c

x	-2	-1	0	1	2
$6-x$					

 d

x	-2	-1	0	1	2
$4x-1$					
- e

x	-2	-1	0	1	2
x^2					

 f

x	-3	-2	-1	0	1
x^2+4					

6 Copy and complete the following tables by substituting each value of x and y into the expressions:

Values		Expressions					
x	y	xy	$x+y$	$x-y$	$y-x$	$4x-4y$	$4(x-y)$
2	5						
-2	5						
2	-5						
-2	-5						

7 Copy and complete the following tables by substituting each value of x and y into the expressions:

Values		Expressions					
x	y	xy	$x+y$	$x-y$	$y-x$	$\frac{x}{y}$	$\frac{2x}{y-1}$
10	5						
8	2						
-3	3						
-15	-5						

A **formula** is a rule that allows us to make calculations which depend on variable quantities.

Example

- 1** A skier pushes off at a speed of 2 km/h and travels for t seconds down a slope. The speed s (km/h) reached after t (seconds) is given by the formula $s = 2 + 3t$.

- a** Find the speed reached by the skier after:
- i** 1 second
 - ii** 2 seconds
 - iii** 5 seconds
- b** Find the time taken to reach a speed of 26 km/h.

- 2** A learner opens a bank account with \$200 and adds \$15 to the account each week. Letting A be the amount in the account after t weeks, write a formula which relates A to t .

- a** Find the amount in the account after:
- i** 1 week
 - ii** 2 weeks
 - iii** 10 weeks
- b** List a number sequence for the amount at the start, then after 1, 2, 3 weeks etc.
- c** Which term in the formula gives the weekly increase in the account?
- d** How long does it take to save \$500?

Solution

Substituting the various times into the formula:

$$s = 2 + 3 \times 1, \text{ so } s = 5 \text{ km/h}$$

$$s = 2 + 3 \times 2, \text{ so } s = 8 \text{ km/h}$$

$$s = 2 + 3 \times 5, \text{ so } s = 17 \text{ km/h}$$

$$\text{Solve } 26 = 2 + 3t$$

$$24 = 3t$$

$$t = 8 \text{ seconds}$$

The required formula is $A = 200 + 15t$.

$$A = 200 + 15 \times 1, \text{ so } A = 215$$

$$A = 200 + 15 \times 2, \text{ so } A = 230$$

$$A = 200 + 15 \times 10, \text{ so } A = 350$$

The sequence is 200, 215, 230, 245, 260, ...

15 is the weekly increase.

$$\text{Solve } 500 = 200 + 15t$$

$$300 = 15t$$

$$t = 300 \div 15$$

$$= 20 \text{ weeks}$$

Exercise 4F

- 1** Lisa opens an account with \$150 and adds \$10 to the account each week. The amount A (in dollars) in the account after t weeks, is given by the formula $A = 150 + 10t$.
- a** Find the amount in the account after:
- i** 1 week
 - ii** 2 weeks
 - iii** 10 weeks

- b** List the number sequence for the amount at the start, then after 1, 2 weeks etc.
- c** Which number in the formula tells you the increase in the deposit each week?
- d** How long does it take to save \$500?
- 2** A learner who has a holiday job builds up his account to a balance of \$800. During the school year he withdraws \$50 from the account each week. The balance B (dollars) in the account after t weeks, is given by the formula $B = 800 - 50t$.
- a** Find the amount in the account after:
- i** 1 week **ii** 2 weeks **iii** 5 weeks
- b** List as a number sequence the balance at the start, then after 1 week, 2 weeks etc. How is any next term obtained from the one before it? Which number in the formula tells you the decrease in the balance each week?
- c** Find how long it takes for the balance to reach \$300.
- d** If the balance in the account is allowed to go into overdraft (that is become negative), find the balance after twenty weeks.
- e** If the maximum overdraft allowed is $-\$300$, how long does it take to reach this limit?
- 3** Another learner starts the year with a balance of \$1000. During the school year he withdraws \$85 from the account each week. Write down a formula which gives the balance B (dollars) in the account after t weeks.
- a** Find the amount in the account after:
- i** 1 week **ii** 3 weeks **iii** 6 weeks
- b** List as a number sequence the balance at the start, then after 1 week, 2 weeks etc.
- c** How is any next term obtained from the one before it?
- d** Which number in the formula tells you the decrease in the balance each week?
- e** How long does it take for the balance to reach \$235?
- f** If the balance in the account is allowed to go into overdraft (that is become negative), find the balance after 15 weeks.
- g** If the maximum overdraft allowed is $-\$700$, how long does it take to reach this limit?
- 4** A third learner commences a savings plan for which she opens an account with \$140 and adds \$12 to the account each week. Letting A be the amount (in dollars) in the account after t weeks, write a formula which relates A to t .



- a** Find the amount in the account after:
- i** 1 week
- ii** 3 weeks
- iii** 7 weeks
- b** List as a number sequence the amount at the start, then after 1 week, 2 weeks etc.
- c** How is any next term obtained from the one before it?
- d** Which number in the formula tells you the increase in the deposit each week?
- e** How long does it take to save \$380?

- 5 The speed s , in kilometres per hour, reached after t seconds by a motor bike moving down a slope, is given by the formula $s = 4 + 2t$.
- Find the speed reached after:
 - 0 seconds (push-off speed)
 - 1 second
 - 5 seconds
 - 10 seconds
 - List as a sequence the speed attained after 0, 1, 2, 3 ... seconds.
 - How is any next term in the sequence obtained from the one before it?
 - Which number in the formula tells you the increase in the speed each second?
 - How long does it take the motor bike to reach 30 km/h?

- 6 Another motor bike rider pushes off down a slope at 5 km/h and increases her speed by 1.5 km/h every second. Write down a formula that gives her speed s (km/h) after t seconds.

- Find the speed reached after:
 - 0 seconds (push-off speed)
 - 1 second
 - 4 seconds
 - 6 seconds
- List as a sequence the speeds attained after 0, 1, 2, 3 ... seconds.
- How is any next term in the sequence obtained from the one before it?
- Which number in the formula tells you the increase in the speed each second?
- How long does it take the motor bike to reach 17 km/h?



- 7 In each table, a number sequence has been listed giving the values of the variables A and t . Write down the formula in each case that relates A to t .

a

t	0	1	2	3	4
A	10	13	16	19	22

b

t	0	1	2	3	4
A	10	2	-6	-14	-22

c

t	0	1	2	3	4
A	30	24	18	12	6

d

t	0	1	2	3	4
A	15	23	31	39	47



Puzzles

- 1 Simplify the following expressions. Match the letter of the question to the correct simplified expression below to answer the riddle:

Why did the moth eat the carpet?

A $2xy + 3xy - xy$

E $6ab - 5ab$

H $pq - 8pq + 9pq$

L $8mn + 2mn - 6mn$

O $-x^2 + 4x^2 - 2x^2$

S $4x - 5x + y + 6y$

W $a + 5 + 2a - 2$

D $6xy - 9yx + 4xy$

F $9ab - 6ba + ba$

I $5pq - 2pq - 4qp$

N $5mn - nm - 2nm$

R $6x^2 - 5x^2 + x^2$

T $5y - 2x + 3y + 3x$

Y $4 + 3a - 2 - 5a$

$-pq$	$8y + x$	$3a + 3$	$4xy$	$2mn$	$8y + x$	ab	xy	$8y + x$	x^2
$7y - x$	ab	ab	$8y + x$	$2pq$	ab				
$4ab$	$4mn$	x^2	x^2	$2x^2$	$7y - x$	$2pq$	x^2	$3a + 3$	

- 2 Expand the brackets for each of the expressions shown below. Match the letter of the expression to the correct expanded expression to find the answer to the riddle:

What is an overgrown moth in a jumper?

A $2(x - y)$

B $8(a - b)$

C $8(x - 7)$

D $3(x - 9)$

H $-2(3 - p)$

L $-(4 - q)$

M $p(3 - q)$

N $p(q - 2)$

O $-q(1 - p)$

P $-p(4 - q)$

Q $9(3x + 2)$

T $3(4y + 6)$

W $-2(x + 2)$

Y $-2(4y - 1)$



$2x - 2y$	$-2x - 4$	$pq - q$	$pq - q$	$q - 4$	$q - 4$	$2 - 8y$
$3p - pq$	$2x - 2y$	$3p - pq$	$3p - pq$	$pq - q$	$12y + 18$	$2p - 6$

- 3 Copy and complete the cross-number puzzle below. Substitute the values shown for x and y to complete the puzzle:

Across

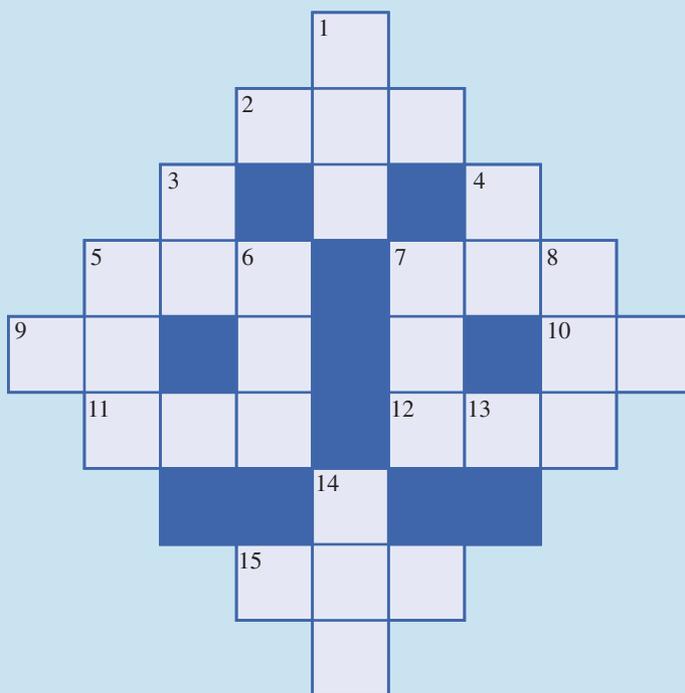
Substitute $x = 15$, $y = 6$

2. $10x + 2y$
5. $5x + 6y$
7. $9x + 10y$
9. xy
10. $\frac{4x + y}{y}$
11. $7x + 2y$
12. $x(4x + y)$
15. $y(2x + 6)$

Down

Substitute $a = -2$, $b = 5$

1. $b(6b - a)$
3. $1 - 2ab$
4. $a^2 + b^2$
5. $4b^2 + 1$
6. $b(4 - 3ab) + b - a$
7. $3(b^2 - 9a)$
8. $-51ab$
14. $\frac{5(4b^2 - 6a)}{b}$



- 4 Substitute the values for x and y in the expressions in each corresponding row. Copy and complete the table. When complete, colour all the squares indicated below to find a message in the table. The message will tell you if you are correct.

x	y	$-x$	$-y$	$-2x$	x^2	$x + y$	$x - y$	$10x$	xy	$x + 1$
1	-2									
4	-1									
1	2									
-2	3									
-1	1									
Shade in the negative numbers.						Shade in the numbers between -4 and 4 inclusive.				



Applications

Skateboarder

A skateboarder travelling at 16 km/h comes to an upgrade and begins to slow down. Her speed s km/h decreases by 2 km/h every second.

- a Complete the following table which gives speeds at various times.

t	0	1	2	3	4	5	6	7	8
s	16	14							

- b Draw up a set of axes of your own and plot the values in the table. Plot t on the horizontal axis and s on the vertical axis. Join the plots with a straight line.
- c Using the pattern of the table, what would be the value of s after 9 seconds? What would be the value of s after 10 seconds?
- d Continue the line graph to $t = 10$.
- e Explain what is happening to the motion of the skateboarder after 8 seconds. How does the mathematics show this?
- f Write down the formula that works out the speed s of the skateboarder at time t .

Savings plan

You decide on a savings plan that requires making an initial deposit of \$1000 and then a deposit of \$150 every month.

- a Write down a number sequence which gives the initial deposit, the amount accrued after 1 month, 2 months etc. Continue your sequence to 12 terms.
- b Draw up a set of axes of your own with t months marked on the horizontal axis and the amount saved \$ A marked on the vertical axis. Plot the sequence terms as points on your set of axes. Why is it not appropriate to join the plots with a straight line?
- c Write down the formula that works out the amount saved, A , for the number of months, t , that have elapsed.
- d Use your formula to calculate the amount saved after 3 years.
- e How long would it take to save \$3700?
- f After 6 months you decide that you can only save 100 dollars each month. Plot the points for 7 months onwards for this modified savings program. How long does it now take you to save \$3700?
- g Write down a new formula that works out how much you have saved after t months, where t is 7 months or more.



Sharky's surfboards

- 1 Sharky's Surfboard Hire charges \$200 for the first day's hire and then \$150 for each day's hire after that.

- a Copy and complete the table which gives the cost C of surfboard hire for days t .

t	1	2	3	4	5
C	20	35			

- b Plot the above pairs of values on a set of axes of your own with t on the horizontal axis and C on the vertical axis. Plot the individual points first and then draw a straight line through the plots. Continue your line to $t = 10$ days.
- c Write a formula that gives the cost C from the time t . Check that your formula fits the values worked out in your table.
- d Use your formula to find how much it costs to hire a surfboard for 28 days.
- 2 Budget Surfboard Hire offers an alternative deal with a first day's hire of \$350 and subsequent hire costing \$100 per day.
- a Complete a table like that in Question 1, which gives the cost of surfboard hire for 1 to 5 days.
- b Plot your table of values on the same axes as Question 1, connecting the plots from day one onwards.
- c Write a formula that gives the cost C for the time t . Check that your formula fits the values in your table.
- d Use your formula to find how much it costs to hire a surfboard for 28 days from Budget.
- e When is it cheaper to use Budget, and when is it cheaper to use Sharky's? Is there a hire period for which it does not matter?



Scuba diving

Scuba diving equipment initially costing \$8000 loses \$1000 in value each year when sold as a second-hand item.

- a Write down a number sequence that gives the initial value, the value after 1 year, 2 years etc. Continue your sequence to seven terms.
- b Draw up a set of axes of your own with t years marked on the horizontal axis and the value V of the scuba diving equipment on the vertical axis. Plot the sequence terms as points on your set of axes. Join the plots with a straight line. Why is it reasonable to join the plots with a straight line?
- c What would the value of the equipment be after two and a half years?
- d Write down the formula that works out the value V from the number of years t that have elapsed.
- e Using your formula, work out the value of the equipment after 4 years and 3 months.





Enrichment

- 1 A tractor-trailer travelling at 16 km/h on the flat comes to an incline (or rising slope). Its velocity, v km/h, t seconds after ascending the slope is given by $v = 16 - 2t$.
- If v_0 represents his initial velocity, v_1 the velocity after 1 second, v_2 the velocity after 2 seconds etc, list the sequence of terms v_0 to v_{16} .
 - How is any next term in the sequence obtained from the one before it?
Complete the symbolic representations of this rule:
 $v_1 = v_0 - \underline{\hspace{2cm}}$
 $v_2 = v_1 - \underline{\hspace{2cm}}$
 $v_3 = v_2 - \underline{\hspace{2cm}}$
 $v_4 = v_3 - \underline{\hspace{2cm}}$
 $v_5 = v_4 - \underline{\hspace{2cm}}$
To cover all of these, we may write $v_n = v_{n-1} - \underline{\hspace{2cm}}$, with $v_0 = \underline{\hspace{2cm}}$, where n can be any of the values 1, 2, 3 etc.
 - Explain the zero result and the meaning of the negative results.
 - On a set of axes of your own, plot the sequence values in part a. Join your plots with a straight line and continue the line.
 - Use the formula to find v after 10.25 seconds.



- 2 For a second tractor-trailer, the distance d metres (position) up the incline after t seconds is given by the formula $d = 0.3(16t - t^2)$.
- Find the value of d_0 , the position when $t = 0$, d_1 the position when $t = 1$, d_2, d_3, d_4 , etc. to d_{16} .
 - On a new set of axes, plot the values in part a against time, joining the plots this time with a smooth curve.
 - What is the maximum distance reached up the plane by the second tractor-trailer, and when does it occur?
 - How long does it take to roll back down to the start of the slope?
 - Estimate from your graph the two times when the tractor-trailer is 10 metres up the incline.
 - If the slope were to extend down to before the starting point, find d_{17}, d_{18} , and explain the meaning of your answers.

- 3** A third tractor–trailer moves up a slope so that its position d metres at time t seconds from the start of the incline is given by the formula $d = 0.1(20t - t^2)$.
- Find the values of d_0 the position when $t = 0$, d_1 the position when $t = 1$, d_2 , d_3 , d_4 , etc. to d_{20} .
 - On a set of axes of your own, plot the values in part a against time, joining the plots with a smooth curve.
 - What is the maximum distance reached up the plane by the tractor–trailer and when does it occur?
 - How long does it take the tractor–trailer to roll back down to the start of the slope?
 - Work out the distance travelled in:
 - the first second, $d_1 - d_0$
 - the second second, $d_2 - d_1$
 - the third second, $d_3 - d_2$
 - Work out the sequence of distances travelled in the first 20 seconds.
 - The results in part f represent distances travelled over 1 second, therefore they also represent average speeds (velocities) during each second. On a set of axes of your own plot the differences against the middle of each time interval, that is $t = 0.5$, 1.5 , 2.5 etc. Join up your plots with a straight line.
 - Using your line in part g, estimate the speed of the tractor–trailer at $t = 0$.
 - Work out the velocities at $t = 1$, $t = 2$, etc.
 - Write down the formula that gives the velocity at time t .
 - Use your formula in part j to find the velocity of the tractor–trailer at $t = 15$.
- 4** Using the values $a = -6$, $b = 4$, $c = -3$, evaluate the following expressions:
- | | |
|--|---|
| a $\frac{ab}{c} + \frac{a+3}{2c}$ | b $\frac{2ac}{b} - \frac{b}{c-1}$ |
| c $\frac{b+4}{a} - \frac{a+b}{2}$ | d $\frac{2ab}{c} + \frac{2-a}{b}$ |
| e $\frac{ac}{2} - \frac{b}{3+b}$ | f $\frac{6-c}{a} - \frac{a}{2}$ |
| g $\frac{b-a}{-c} + \frac{a}{c}$ | h $\frac{a^2+b^2}{c} - \frac{2a}{b}$ |
- 5** Express the following as fractions, giving the answer with the correct sign:
- | | |
|-----------------------------------|-----------------------------------|
| a $-8 \div x \div -4y$ | b $9a \div -3a \div ab$ |
| c $-6m \div -6 \div -q$ | d $-10p \div -5q \div 2qr$ |
| e $-16 \div -4x \div xy$ | f $-p \div -pq \div -qr$ |
| g $6 \div -r \div -s$ | h $12 \div -m \div -n$ |
| i $-a \div -b \div -c$ | j $-24p \div -6q \div 2r$ |
| k $-16x \div -8xy \div xy$ | l $-35p \div -7pq \div -p$ |
| m $1 \div -6r \div t$ | n $1 \div -mn \div n$ |



Revision/Assessment

Exercise 4A

1 For each of the following expressions, state the number of terms and the coefficients of the terms noted in brackets:

a $2x + 3y$ (x)

b $10x^2 + 2x$ (x^2)

c $-3x^2 - x$ (x)

d $-5a + 7b + b^2$ (a, b^2)

e $7xy - \frac{3x^2}{5}$ (x, y, x^2)

f $\frac{1}{2}x^2 - \frac{3}{5}y^2 + \frac{1}{4}xy$ (x^2, y^2, x)

2 List the groups of like terms from each of the following:

a $7x, -5y, 9x, 3y, -x$

b $4a, -5ab, 4b, 9ba, -2a, 2ab, -b$

c $3p^2q, -pq, 4pq^2, 2pq, -4qp$

d $2x^2, 5x, -2x^2, 6xy, -8x^2, -x^2y, 3yx, -12yx^2$

Exercise 4B

3 Simplify the following expressions by combining like terms:

a $3a + 2b + c - 6a + 4b - 3c - 6a - 8b - 2c$

b $13ab + 4bc - 2ca - ab - 9bc - 5ab + 8ca$

c $-3xy + 2yz - 7zy + 9zx + 2xy - 5yz - 8zx$

4 Add the terms in parts a–d of Question 2.

Exercise 4C

5 Simplify the following products as far as possible:

a $-7 \times 3x$

b $9 \times -3y$

c $-2a \times -6$

d $3y \times -4x$

e $-m^2 \times 7n$

f $-2c \times -5b$

g $3y \times -5y \times 4$

h $-7a \times 2b \times -9$

i $-6m \times -5n \times -9p$

j $-3x \times 5y \times -2z$

k $3a \times -6b \times -7c$

l $-5x \times -2y \times -4x$

m $(-9x)^2$

n $(-6y)^2$

o $(-3xy)^2$

6 Simplify the following expressions as far as possible:

a $\frac{5a}{-15}$

b $\frac{-2a}{14}$

c $\frac{-5y}{-10}$

d $\frac{-12}{3x}$

e $\frac{27}{-3y}$

f $\frac{-28}{-4m}$

g $\frac{7x}{21y}$

h $\frac{-12y}{-48x}$

i $\frac{-9a}{-6b}$

Exercise 4D

7 Expand each of the following and simplify as far as possible:

a $-3(5x + 2) + 7$

b $-5(6y - 1) + 9$

c $6 - 4(2m + 6)$

d $28 - 4(2b - 7)$

e $-4x(3x + 6) - 3x(2x - 1)$

f $-4p(2p + 3m) + 16p^2$

8 Expand each of the following:

a $3y(x + 2)$

b $8a(b + 2)$

c $3b(3a - 5)$

d $5n(2m - 5)$

e $2x(3 + 4y)$

f $8x(2 - y)$

g $5a(1 + 4b)$

h $12m(1 - 3n)$

i $4xy(4z - 5)$

j $2mp(4n + 5)$

k $5rt(3s - 5)$

l $7pq(2 + 3r)$

9 Expand each of the following:

a $x(x + 5)$

b $x(x + 12)$

c $x(x - 7)$

d $x(x - 11)$

e $m(m - 1)$

f $z(z - 1)$

g $y(1 - y)$

h $x(1 - x)$

i $3x(x + 2)$

j $3x(4x - 3)$

k $2x(5x + 6)$

l $5x(3x - 4)$

10 Expand each of the following:

a $-4(x + 2)$

b $-3(y + 4)$

c $-7(x + 3)$

d $-10(q + 2)$

e $-5(x - 2)$

f $-8(y - 4)$

g $-3(x - 5)$

h $-6(q - 3)$

i $-4(3a - 8b)$

j $-7(6c + 2d)$

k $-3(3z - 5y)$

l $-5(6m + 7n)$

Exercise 4E

11 Using the values $p = -6$, $q = 2$, $r = -8$, evaluate the following expressions:

a $5p^2$

b $pq + pr$

c $p(q + r)$

d p^2

e $r^2 - 3r$

f $r(r - 2)$

12 If $a = -8$, $b = -2$, $c = 4$, evaluate the following expressions:

a $\frac{ab}{c}$

b $\frac{2ac}{b}$

c $\frac{b + 4}{a}$

d $\frac{a + b^2}{c}$

e $\frac{\sqrt{c} - b}{a}$

f $\frac{a}{c} - \frac{2c}{b}$

Exercise 4F

13 A learner who has a vacation job builds up his account to a balance of \$1000. During the school year he withdraws \$40 from the account each week.

a Work out the amount in the account after:

i 1 week

ii 2 weeks

iii 5 weeks

b List as a number sequence the balance at the start, then after 1 week, 2 weeks etc.

Write down the formula that works out the amount A in the account after t weeks.

c Find how long it takes for the balance to reach \$520.

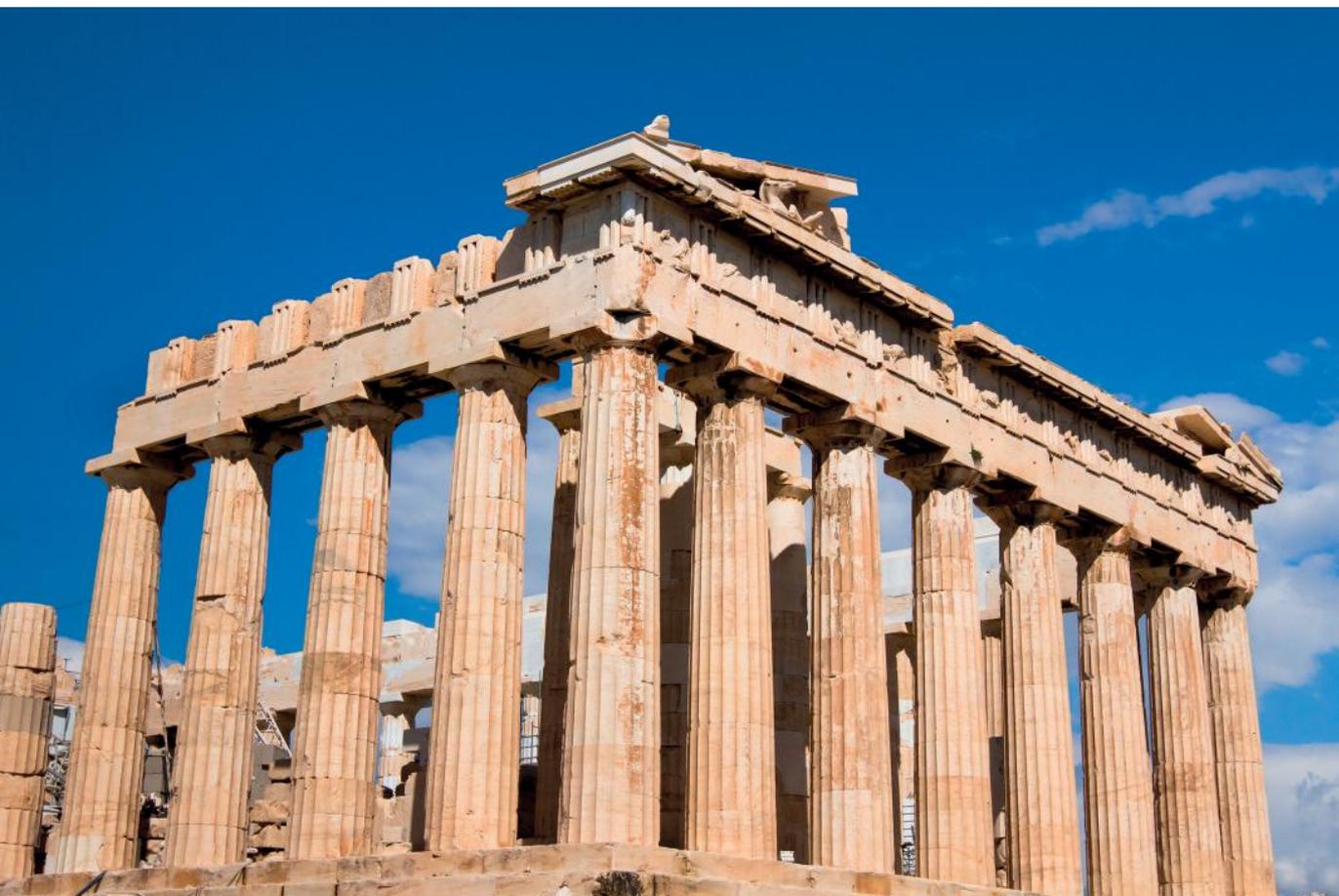
d If the balance in the account is allowed to go into overdraft (that is, become negative), find the balance after 30 weeks.

e If the maximum overdraft allowed is \$400, how long does it take to reach this limit?

CHAPTER

5

Ratios and Rates



Ratios and Rates

Special ratios and proportions have been used in architecture since ancient times. The golden rectangle can be divided into a square and a rectangle that is similar to the original rectangle. Golden rectangles have a ratio of length to width of approximately 1:1.62, and are considered very pleasing to look at. The Parthenon (built 447–432 BC), the temple on the Acropolis in Athens, is built entirely of marble from the quarries on Mount Pentelikon. Measuring about 70 m by 31 m, it is one of the largest, as well as one of the finest, Doric temples. It is fronted by 8 columns and flanked by 17 columns. The front portico has the length and width of a golden rectangle. The Ancient Greek civilisation held that geometry and proportion represented perfection.

This chapter covers the following skills:

- Recognising and writing ratios in their simplest form
- Using ratios to determine the quantities or amounts required
- Sharing amounts according to ratios
- Identifying the quantities connected in a rate
- Interpreting everyday rates
- Reading information from rate graphs
- Comparing rates
- Working with speed, distance and time

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.5.1.1** Define ratio and proportion.
- 8.5.1.2** Correctly interpret the symbol that is used to represent ratio: ': '.
- 8.5.1.3** Write or express quantities as ratios.
- 8.5.2.1** Expressing ratios of quantities as fractions in their simplest form.
- 8.5.3.1** Simplify ratios of quantities that are given in different units.
- 8.5.4.1** Simplify fractional ratios by using a common number to remove all the denominators.
- 8.5.5.1** Find missing numbers and quantities for given ratios.
- 8.5.6.1** Divide lines into different parts using given ratios.
- 8.5.7.1** Identify special ratios for the lengths of lines constructed inside triangles.
- 8.5.8.1** Share quantities according to given ratios.
- 8.5.8.2** Solve practical problems using given ratios.
- 8.5.9.1** Define rate.
Rate—measure of how one quantity changes with respect to another.
- 8.5.10.1** Calculate rates of given quantities.
- 8.5.10.2** Use given rates to find quantities.

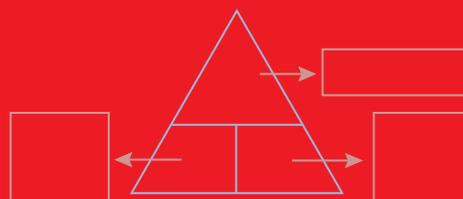
8.5.11.1

Define speed.

Speed—the rate at which distance changes with respect to time. It is measured in units of distance divided by time.

8.5.12.1

Use the given diagram to calculate the rates of speed, distances and time.



8.5.13.1

Interpret time graphs.

8.5.14.1

Use scale ratios to calculate actual measurements for maps and plans.

8.5.15.1

Explain the term 'contour line'.

8.5.15.2

Draw cross-section diagrams from contours on a map.

8.5.15.3

Estimate heights of key features of a map using cross-section diagrams

A ratio is a comparison of two quantities where we look at how two quantities are related to each other. For example, adventure camps need a student to teacher ratio of 5 to 1. This can be written as 5:1. The order in which the ratio is written is very important.

We write 5 students to 1 teacher as 5:1 or 1 teacher to 5 students as 1:5.

Example

This is a pattern using triangles.



Express the following ratios of the colours:

a green to yellow

5:4

b yellow, red and blue to the whole

13:18

Solution

Number of triangular shapes: green 5, red 4, yellow 4, blue 5

Exercise 5A

1 a Find the ratio of people wearing yellow uniforms to those wearing purple uniforms.



b Find the ratio of boys wearing neckties to those wearing no neckties.

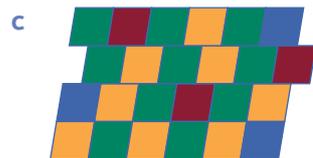
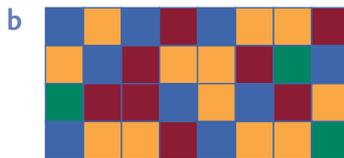
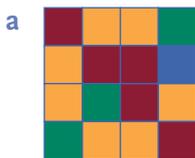


2 Express the ratio of:

i red parts to yellow parts

ii green parts to red parts

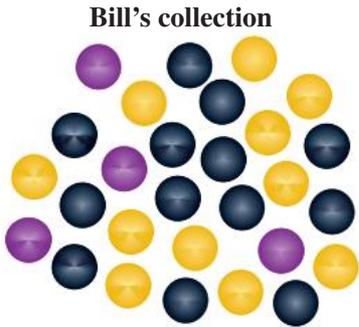
iii blue parts to the whole



3 Write each of the following as a ratio:

- The school yard had 15 guava trees and 7 pine trees.
- There were 10 green, 7 yellow and 3 red jellybeans in the packet.
- Every student received 8 sandwiches for lunch.
- A learner had 15 heavy-metal CDs, 13 reggae CDs and 2 Island music CDs.
- Marist Basketball team beat Western Bullets 19 points to 18.
- To make roti, $2\frac{1}{2}$ cups of plain flour is combined with $\frac{1}{2}$ cup of spices.
- William is 150 cm tall and Lucinda is 99 cm tall.
- To make a bar of fruit chocolate, 125 g of nuts is mixed with 98 g of sultanas.
- Ian gets \$1 and Helen gets \$3 pocket money.

- 4 This is Bill's and Wilma's marble collections.



- a Express the ratio of the following colours for each person's collection:
- Yellow to black
 - Purple to black
 - Yellow to purple
- b If they mix their marbles together, find the ratio of the following colours:
- Yellow to black
 - Purple to black
 - Yellow to purple
 - Yellow to all the marbles
 - Purple to all the marbles
 - Black to all the colours
- 5 In a group of 30 soccer supporters, 17 support Wantoks FC, 8 support Koloale FC and the rest support the Marist FC. Write the following as ratios:
- Wantoks FC supporters to Marist FC
 - Koloale supporters to the rest
 - Wantoks FC supporters to the rest of the group
- 6 Calculate the score for the teams at each quarter of the rugby games and express them as ratios. Each try is 5 points and each conversion is 2 points.

a

Quarter	Avaiki	Diesel Power
First	2 tries 2 conversions	1 try 1 conversion
Second	4 tries 3 conversions	2 tries 2 conversions
Third	3 tries 3 conversions	2 tries 2 conversions
Last	1 try 1 conversion	2 tries 2 conversions

b

Quarter	Matangiki	Luapel
First	1 try 1 conversion	2 tries 1 conversion
Second	3 tries 2 conversions	3 tries 2 conversions
Third	6 tries 6 conversions	5 tries 3 conversions
Last	9 tries 7 conversions	8 tries 5 conversions

- 7 One morning on Tetepari Island the following birds were sighted: 16 frigate birds, 17 kurukurus, 18 parrots and 48 bilikikis.
Express the ratio of kurukurus and bilikikis to the total number of birds sighted.



Ratios like fractions can be simplified using multiplication or division.

The numbers in a ratio can be increased by multiplying, and reduced by dividing.

Simplifying ratios is similar to finding the simplest equivalent fraction.

The simplest form of a ratio is one that has the smallest pair of whole numbers.

Example

- 1 The ratio of students to teachers when going on a dangerous activity such as caving is 5 students to 1 teacher (5:1). Express the ratio when two of these groups are combined.

- 2 A group of 18 students has 2 teachers (18:2). Express the ratio 18:2 in simplest form.

Solution

Ratios can be written in fraction form such as:

$$5:1 = \frac{5}{1}$$

For 2 groups, multiply top and bottom by 2.

$$\frac{5}{1} \times \frac{2}{2} = \frac{10}{2} = 10:2$$

which means 10 students to 2 teachers.

or

$$\times 2 \left(\begin{array}{c} 5:1 \\ \\ 10:2 \end{array} \right) \times 2$$

Ratios can be written in fraction form such as:

$$18:2 = \frac{18}{2}$$

To simplify, divide top and bottom by 2.

$$\frac{18}{2} \div \frac{2}{2} = \frac{9}{1} = 9:1$$

or

$$\div 2 \left(\begin{array}{c} 18:2 \\ \\ 9:1 \end{array} \right) \div 2$$

The simplest ratio is 9 students to 1 teacher.

When finding ratios the units of each quantity must be the same.

Don't forget conversions such as: 1 kg = 1000 g 1 L = 1000 mL

Example

- 3 35 mL of food dye is added to 2 litres of water. Find the ratio of food dye to water.

Solution

$$\div 5 \left(\begin{array}{c} 35 \text{ mL} : 2 \text{ L} \\ 35 \text{ mL} : 2000 \text{ mL} \\ \\ 7 : 400 \end{array} \right) \div 5$$

Example

4 Simplify the ratio $\frac{1}{2} : \frac{2}{3}$.

Solution

$$\begin{array}{c} \frac{1}{2} : \frac{2}{3} \\ \times 6 \quad \swarrow \quad \searrow \quad \times 6 \\ 3 : 4 \end{array}$$

Exercise 5B

1 Simplify the following ratios by cancelling:

a 12:3

b 25:5

c 45:25

d 39:13

e 24:14

f 42:14

g 3:5:1:5

h 1:2:4

i 1:8:30

j 0:9:3:6

k 10:8:14:4

l 3:8:1:8

2 Fill in the missing numbers:

a $1:12 = 2:\underline{\quad}$

b $2:3 = 6:\underline{\quad}$

c $7:3 = 14:\underline{\quad}$

d $5:11 = 10:\underline{\quad}$

e $1:15 = \underline{\quad}:60$

f $4:11 = \underline{\quad}:33$

g $13:2 = \underline{\quad}:6$

h $7:8 = \underline{\quad}:24$

3 One cup of self-raising flour is added to one and a half cups of plain flour, what is the ratio of plain to self-raising flour?

4 If $12\frac{1}{2}$ kilograms of salt is added to 120 litres of water in a massage bath, what is the ratio of salt in kilograms to the volume of water in the bath in litres?

5 If $7\frac{1}{2}$ cups of flour is combined with $10\frac{1}{2}$ cups of egg mixture to make pancake dough, what is the ratio of flour to egg mixture?

6 To make fertiliser for roses, $3\frac{1}{2}$ grams of fertiliser is added to 100 litres of water. State the ratio of fertiliser to water.

7 If 13.5 mL of red food dye is added to 2.5 mL of yellow dye, state the ratio of red to yellow dye in the mixture.

8 Simplify the following ratios:

a 12 m:250 cm

b 600 mL:1 L

c 600 g:2.5 kg

d 3500 g:1.5 kg

e 35 min:1 h

f 2400 s:30 min

g 1.5 t:1800 kg

h 1800 m:2.5 km

i 32 mm:18 cm

j 15 s:1 min

k 420 mm:36 cm

l 250 g:0.4 kg

9 Simplify the following ratios:

a $\frac{1}{2} : \frac{1}{5}$

b $\frac{2}{3} : 1$

c $1 : \frac{3}{4}$

d $\frac{2}{5} : \frac{3}{5}$

e $\frac{5}{8} : \frac{1}{4}$

f $\frac{2}{3} : \frac{3}{8}$

g $1\frac{1}{4} : 2\frac{1}{2}$

h $5\frac{1}{4} : 2\frac{1}{5}$

i $1:6\frac{1}{4}$

j $1\frac{1}{16} : 3\frac{1}{4}$

10 Write fractions that are less than 1 and which simplify to the following ratios:

a 2:1

b 3:2

c 6:1

d 5:4

e 3:11

Ratios can be used to determine unknown quantities.

Example

On a canoeing camp the ratio of learners to teachers needs to be 3:1. Find the number of teachers required if there are:

a 27 students

Ratio of students to teachers is 3:1

$$\times 9 \left(\begin{array}{c} 3:1 \\ \curvearrowright \\ 27:9 \end{array} \right) \times 9$$

so 9 teachers are needed.

b 33 students

Ratio of students to teachers is 3:1

$$\times 11 \left(\begin{array}{c} 3:1 \\ \curvearrowright \\ 33:11 \end{array} \right) \times 11$$

so 11 teachers are needed.

Exercise 5C

1 Find the missing number:

a $2:3 = 4:\underline{\quad}$

b $3:5 = 9:\underline{\quad}$

c $4:7 = \underline{\quad}:28$

d $2:7 = \underline{\quad}:14$

e $15:\underline{\quad} = 45:12$

f $\underline{\quad}:11 = 18:22$

g $1\frac{1}{2}:3 = 3:\underline{\quad}$

h $3:5:1 = 7:\underline{\quad}$

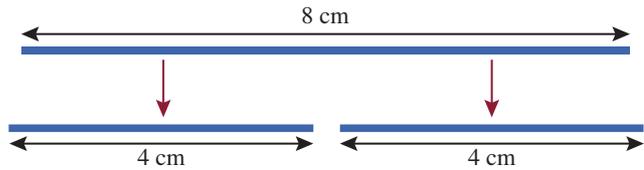
i $2\frac{1}{4}:3\frac{1}{2} = 6\frac{3}{4}:\underline{\quad}$

- 2 The ratio of adults to children on a bushwalk is 4:5. If there are 24 adults, how many children are there on the bushwalk?
- 3 Bill and Bob divide their paper round money in the ratio 2:5, so that Bill gets \$8:20.
- How much will Bob get?
 - What was the total amount of the paper round?
- 4 A concrete slab mixture requires sand, cement and screenings to be mixed in the ratio 5:4:1. If 12 kg of sand is placed into the mixer, what weight of cement and screenings should be added?
- 5 A two-stroke petrol mixture is made by mixing petrol and oil in the ratio 25:1.
- How many litres of petrol need to be added to 250 mL of oil to make this mixture?
 - How much oil needs to be added to 20 litres of petrol?
- 6 An alloy is made by combining copper, tin and zinc in the ratio 10:12:8.
- Express this ratio in simplest form.
 - If 44 kg of zinc is used in a smelter, what weights of the other metals are required?

Exploring ratios in triangles 5D



When a line is divided into two parts in the ratio 1:1 it is cut into two equal parts.



Learning task 5D

1 Draw a 12 cm line and divide it into two parts which divide in the ratio:

a 1:2

b 1:3

c 1:5

2 a Which point divides the line interval SZ in the following ratios?

i 1:6

ii 3:4

iii 4:3

iv 2:5

v 6:1

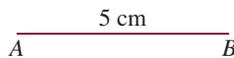
vi 5:2



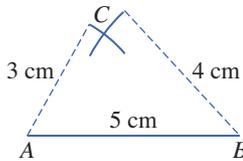
b Draw a line 18 centimetres long and divide it in the ratio 1:2:3. What are the lengths of the parts?

3 The lengths of lines inside triangles form special ratios. Construct the following triangles:

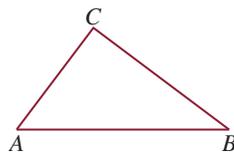
a Draw the line AB length 5 cm.



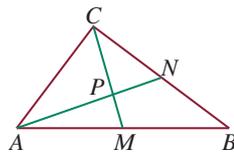
Draw two arcs above the line and 3 cm from A , 4 cm from B . Mark in the point C where the arcs cross.



Draw in the triangle ABC and rub out the construction lines.



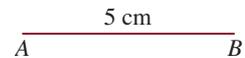
Locate the point M , which is halfway between A and B , and point N , which is the midpoint of the line BC . Draw the lines CM and AN . Mark the point where they cross as P .



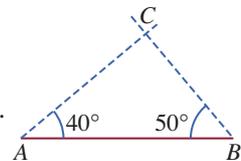
Measure the lines and find the ratio of:

i $PM:CP$ ii $AP:PN$

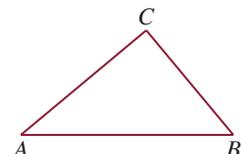
b Draw the line AB length 5 cm.



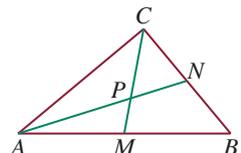
Draw the angles as shown and extend the lines to cross at point C .



Draw in the triangle ABC and rub out the construction lines.



Locate the point M that is halfway between A and B , and point N , which is the midpoint of the line BC . Draw the lines CM and AN . Mark the point where they cross as P .



Measure the lines and find the ratio of:

i $PM:CP$ ii $AP:PN$

5E

Sharing using ratios

Often quantities are not shared equally but in a given ratio. The ratio indicates the way in which the amount is to be shared. There are two ways to find answers to these questions.

Example

A lottery win of \$900 is to be shared between Peppi and Jesse in the ratio 4:5. Find the amount that each person will receive.

Solution

Method 1 Share method:

Peppi + Jesse have $4 + 5 = 9$ shares.

$$\$900 \div 9 = \$100$$

Each share is worth \$100.

Peppi will get $4 \times 100 = \$400$.

Jesse will get $5 \times 100 = \$500$.

Method 2 Fraction method:

Between them Peppi + Jesse have

$4 + 5 = 9$ shares.

Peppi will get $\frac{4}{9}$ of \$900 = $\frac{4}{9} \times \$900 = \400 .

Jesse will get $\frac{5}{9}$ of \$900 = $\frac{5}{9} \times \$900 = \500 .

Exercise 5E

- 1 a Share 420 in the ratio 3:1. b Share 3450 in the ratio 1:4.
 c Share 4536 in the ratio 2:1. d Share 6270 in the ratio 7:3.
- 2 Share \$36 in the following ratios:
 a 1:2 b 1:5 c 3:7 d 3:5 e 4:5 f 5:7
 g 1:2:3 h 1:3:4 i 2:3:4 j 1:1:2 k 1:2:3:4 l 1:2:2:3
- 3 At White River Community High School, classes were analyzed for the number of girls and boys. Copy and complete this:

Class and number	Ratio of boys to girls	Number of boys	Number of girls
8A, 24 students	2:1		
8B, 25 students	2:3		
8C, 21 students	3:4		
8D, 24 students	1:3		
8E, 22 students	5:6		
8F, 24 students	3:1		

- 4 Abe, Beryl and Chris have won a lottery prize of \$96 000. It is to be divided in the ratio 2:3:1, respectively, between them.
- a How much will each person receive?
 b What is the ratio of Abe's and Beryl's winnings to the total prize?

- 5 The table below shows the number of Coconut and Navy biscuits that are produced at the Delight Factory in Honiara each day of the week. Copy the table into your workbook and complete it.

Day	Total production (kg)	Coconut Biscuit : Navy Biscuit	Coconut Biscuits (kg)	Navy Biscuits (kg)
Monday	260	2:3		
Tuesday	1704	1:2		
Wednesday	567	2:5		
Thursday	810	2:1		
Friday	1265	4:1		
Total				

- 6 A total of 1200 learners at the Solomon Islands College of Higher Education (SICHE) were surveyed about their favourite food. The ratio for cassava with slippery cabbage, rice with taiyo and potato with fish was 1:2:3. Find the number of students who chose each type of local food.
- 7 Langalanga shell money necklaces are made out of shell beads that are strung together. The pattern that is used to produce these necklaces is: two red beads for every black bead. A necklace is to have 72 beads. Find the number of each colour bead that is to be used on the necklace.
- 8 Tamana's recipe for a fruit drink says mix 1 cup of bush lime with 3 cups of sugar and 6 cups of water. Write her ingredients as a ratio and calculate how many litres of each ingredient she will need to make 1.5 litres of fruit drink.
- 9 Concrete is made by mixing screenings, sand and cement in the ratio 3:5:1. Find the weight of each that is to be used to make 117 tonnes of concrete.
- 10 The lengths of the sides of a triangle are in the ratio 1:2:4. If the perimeter is 39.2 cm find the length of each side.
- 11 Liquid adhesive is made by mixing components A and B in the ratio 1:20. If 12.6 mL of adhesive is required, find the amount of each component that needs to be used.
- 12 Red, white, yellow and black jelly beans are made at a factory. Use the following production details to find the number of each colour made each day for a week:

Day	Total production	Ratio of red : white : yellow : black
Monday	29 872 700	2:3:4:1
Tuesday	31 846 221	3:5:1:2
Wednesday	63 159 018	1:2:3:7
Thursday	18 949 284	4:1:3:1
Friday	36 171 660	2:1:3:8



5F Rates

A rate is a measure of how one quantity changes with respect to another. Familiar rates might be the speed of a car measured in kilometres per hour, or the exchange rate of the Solomon Islands dollar as $\text{US}\$1 = \text{SBD}\7.40 or $\frac{\text{SBD}\$7.40}{\text{US}\$1}$.

Example

- 1 If rope costs \$2.20 per metre, find the cost of 12 m of rope.
- 2 If 15 metres of chain costs \$975.00, find the cost of 20 metres of chain.
- 3 The exchange rate between the Solomon Islands dollar and the Australian dollar on a particular day is $\text{SBD}\$1 = \text{A}\0.80 .
 - a Find the amount of Solomon Islands currency that is the same as:
 - i A\$10
 - ii A\$50
 - b Find the amount of Australian currency that is the same as:
 - i SBD\$10
 - ii SBD\$50

Solution

$$\begin{aligned} \text{Length} \times \text{rate} &= 12 \text{ m} \times \frac{\$2.20}{1 \text{ m}} \\ &= 12 \times 2.2 \\ &= \$26.40 \end{aligned}$$

$$\text{Rate of dollar per metre} = \frac{975.00}{15} = \$65$$

$$\begin{aligned} \text{Cost of 20 m} &= \text{length} \times \text{rate} \\ &= 20 \text{ m} \times \$65/\text{m} = \$1300 \end{aligned}$$

$$10 \div 0.8 = \$12.50$$

$$50 \div 0.8 = \$62.50$$

$$10 \times 0.8 = \$8$$

$$50 \times 0.8 = \$40$$

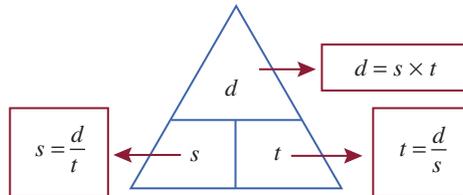
Exercise 5F

- 1 What units would you use to measure the following rates?
 - a A fast-growing tree growing taller (____/day)
 - b Jack Iroga running in a 200-metre race (____/s)
 - c The price of petrol sold from a petrol station (cents/____)
 - d A tortoise moving across a field
 - e An aircraft flying from Honiara to Gizo
 - f Children at a party eating small cakes
 - g A car consuming petrol
- 2 Give a real-life example for the following rates:

a Tries per match	b Goals per quarter	c Cents per litre
d Metres per second	e People per day	f Litres per kilometre
g Dozen per hour	h People per car	i Goats per hectare

- 3** Express each of the following situations using a rate in simplest form:
- The cost for tiles was \$3480 for 12 square metres.
 - The cost of filling a 9-kilogram gas cylinder was \$360.
 - A 20-litre can of paint cost \$900.
 - The cost for a 15-minute mobile telephone call was \$73.50.
- 4** Laus Lawn Mowing Service states that they can mow 2700 m² of lawn in 90 minutes. At this rate, how long will it take to mow a lawn of the following area?
- 540 m²
 - 6300 m²
 - 45 000 m²
- 5** If the cost of lawn-mower fuel is \$12.50 per litre, what is the value of fuel carried by tankers with the following capacity?
- 3000 litres
 - 45 000 litres
 - 130 000 litres
- 6** If 15 metres of fencing wire costs \$90, find the costs of these lengths:
- 45 metres
 - 90 metres
 - 37.5 metres
- 7** An O League soccer match at Lawson Tama, Honiara, was attended by 10 000 people. If the gate takings were \$1 450 000, then at this rate what would be the total gate takings for crowds of:
- 1000?
 - 5000?
 - 6500?
 - 11 420?
- 8** The cost of advertising in the local paper is \$8.00 per word, with a minimum charge of \$196.00. Find the cost of advertisements with the following number of words:
- 20
 - 29
 - 55
 - 127
- 9** When a bar of metal is heated in a furnace, its temperature rises at the rate of 20 degrees Celsius per minute. If it is at 15°C when placed into the furnace, find the temperature of the metal after:
- 15 minutes
 - 25 minutes
 - 1020 seconds
 - an hour
- 10** A 1-kilogram packet of lawn seed is able to cover 60 m².
- At this rate, what weight of seed would be needed to cover the following areas?
 - 45 m²
 - 90 m²
 - 210 m²
 - 250 m²
 - Find the area that the following weights of seeds will cover:
 - 2.5 kg
 - 3.6 kg
 - 18 kg
 - 200 grams
- 11** A telephone company charges a customer \$27.00 for 6 minutes.
- What will be the charge for calls lasting:
 - 2 minutes?
 - 9 minutes?
 - half a minute?
 - 90 seconds?
 - How long were the calls that cost:
 - \$459.00?
 - \$540?
 - \$97.20?
 - \$129.60?
- 12** A dock of length 64 metres can hold 16 boats.
- Find the length of dock that is needed for the following number of boats:
 - 32
 - 48
 - 36
 - 140
 - How many boats can be most efficiently moored in docks of length:
 - 192 m?
 - 44 m?
 - 57 m?
 - 1728 m?

When travelling across the Solomon Islands in regions where the islands are far apart, it is important to estimate the time a journey will take by knowing the boat's speed and the distance that needs to be travelled. **Speed** is the rate at which distance changes with respect to time. It is measured in units of distance divided by time, such as metres per second, kilometres per hour. The diagram can be used to express the speed s , distance d and time t in terms of the other two measures.



Example

- Find the speed of a cyclone that travels 72 km in 9 hours.
- A car travels with an average speed of 45 km/h, how far does travel in 3 hours?
- How long does it take for a cyclone to travel 200 km while travelling at 12 km per hour?
- Find the speed in km/h of a car that travels 5 km in 10 minutes.

Solution

$$s = \frac{d}{t} = \frac{72 \text{ km}}{9 \text{ h}} = 8 \text{ km/h}$$

$$d = s \times t = 45 \text{ km/h} \times 3 \text{ h} = 135 \text{ km}$$

$$t = \frac{d}{s} = \frac{200 \text{ km}}{12 \text{ km/h}} = 16.6 \text{ h}$$

$$s = \frac{d}{t} = \frac{5 \text{ km}}{10 \text{ min}} = \frac{5 \text{ km}}{\frac{10}{60} \text{ h}} = 5 \times \frac{60}{10} = 30 \text{ km/h}$$

Exercise 5G

- Select the unit of speed from km/h or m/s that would be best used to state the speed of the following:

a A racing car	b An aeroplane
c A beetle in the garden	d A snail moving across a lawn at night
e A sprinter in the Olympic Games	f A bus
- Calculate the speed in kilometres per hour (km/h) for the following:
 - A car travelling 510 km in 6 h
 - A plane travelling 1400 km in 5 h
 - A skier skiing 30 km in 2 h
 - Joe taking 3 hours to run 24 km on a fun run
 - A skateboarder travelling 16 km in 3 h
 - Mabel walking 1800 m in half an hour
 - A racing car travelling 665 km in 3.5 h
 - A bus travelling 1748 km in 19 h
 - A car travelling 20 km in $1\frac{1}{2}$ h
 - A camel travelling 74.25 km in $8\frac{1}{4}$ h



3 Find the time for the following journeys:

- a A beetle moving at 18 cm/s over a distance of 1.8 m
- b A ball rolling down a 45-cm slope with a speed of 10 cm/s
- c A marble rolling down a 45-cm slope with a speed of 9 cm/s
- d A snail crossing a 2-m wide garden bed at a speed of 20 mm/s
- e A turtle crawling 10.5 m at a speed of 25 cm/s
- f A car travelling 50 km at an average speed of 9 km/h



4 A motor mower travels at 4 m/s. At this rate, find the distance it could travel in:

- a 20 seconds
- b 45 seconds
- c 2 minutes
- d 7 minutes
- e half an hour
- f $3\frac{1}{2}$ hours

5 Calculate the following speeds in kilometres per hour (km/h):

- a A horse runs 4 km in 10 minutes.
- b A fish swims 0.5 km in 5 minutes.
- c A car travels 80 km in 45 minutes.
- d A submarine cruises 135 km in 3 h 30 minutes.
- e A motorbike completes a 5 km lap in 2 minutes.
- f In 3 h 10 minutes a jet travels 1995 km.
- g A homing pigeon flies 308 km in 4 h 20 minutes.
- h An iron woman runs 48 km in 2 h 40 minutes.



6 In the ocean, the bottlenose dolphin travels at 64 km/h, the killer whale 48 km/h, the mako shark 32 km/h and the penguin at 24 km/h. The fastest marine animal is the bluefin tuna, which travels at 88 km/h. Find the distances that each of these animals would swim in:

- a 15 minutes
- b half an hour
- c three-quarters of an hour
- d two and a half hours

7 A motorbike travels at a speed of 80 km/hr. Find the distance that it travels in:

- a 2 hours
- b 5 hours
- c $3\frac{1}{2}$ hours
- d $5\frac{1}{4}$ hours
- e $6\frac{2}{3}$ hours
- f $10\frac{1}{2}$ hours

8 A downhill skier's best speed is recorded as 15 m/s. Find the distance that she can travel in:

- a 10 s
- b 15 s
- c 30 s
- d 1 minute
- e 4 minutes
- f 15 minutes

9 Find the distance travelled by each of the following:

- a A bicycle rider travelling for 4 h at a speed of 20 km/h
- b A motorbike rider riding for 15 minutes at 5 m/s
- c A boat travelling at 90 km/h for 6 h 15 min
- d A submarine travelling at 45 km/h for 2 h



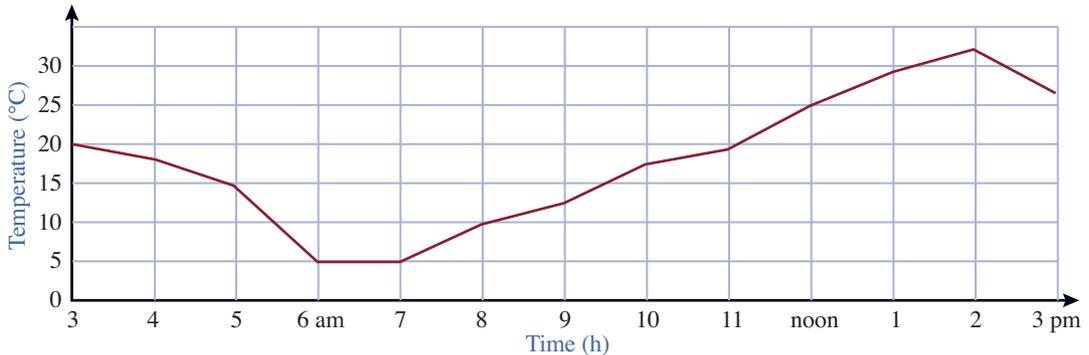
5H

Graphs and time

Graphs can be drawn to show lots of information such as the way temperature changes throughout the day or the progress of a learner on a fun run. In both these cases time is the important quantity against which the other quantities are measured.

Example

Lillian recorded the temperature every hour for 12 hours and used the information to draw the following graph:

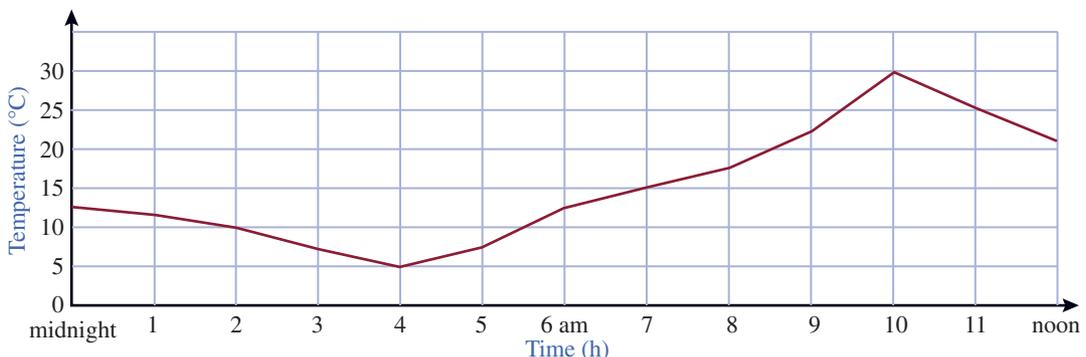


Solution

- a** When was the temperature below 15°C ? The temperature was below 15°C between 5 am and 9:30 am.
- b** What was the change in temperature between the following times?
- i** 3 am to 5 am There was a drop of 5°C in 2 hours.
 - ii** 6 am to 7 am There was no change.
 - iii** 7 am to noon There was a rise of 20°C in 5 hours.
- c** Use your answers above to state the change in temperature per hour between:
- i** 3 am and 5 am There was a decrease of 2.5°C per hour.
 - ii** 6 am and 7 am There was no change.
 - iii** 7 am and noon There was an increase of 4°C per hour.

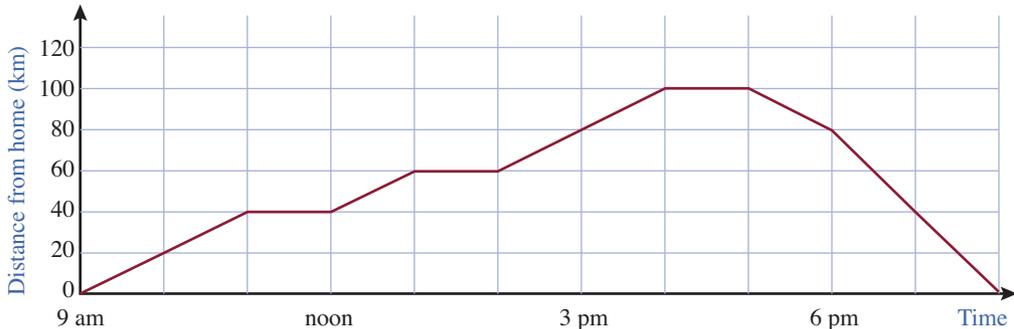
Exercise 5H

- I** The graph below shows the temperature recorded each hour over a 12-hour period.



- a Using the graph, find the time of day when the temperature reached:
- i 5°C ii 10°C iii 15°C iv 25°C v 30°C
- b What is the change in temperature between the following times?
- i 1 am and 4 am ii 7 am and 10 am iii 4 am and noon
- c Use your answers above to state the change in temperature per hour between:
- i 1 am and 4 am ii 7 am and 10 am iii 4 am and noon

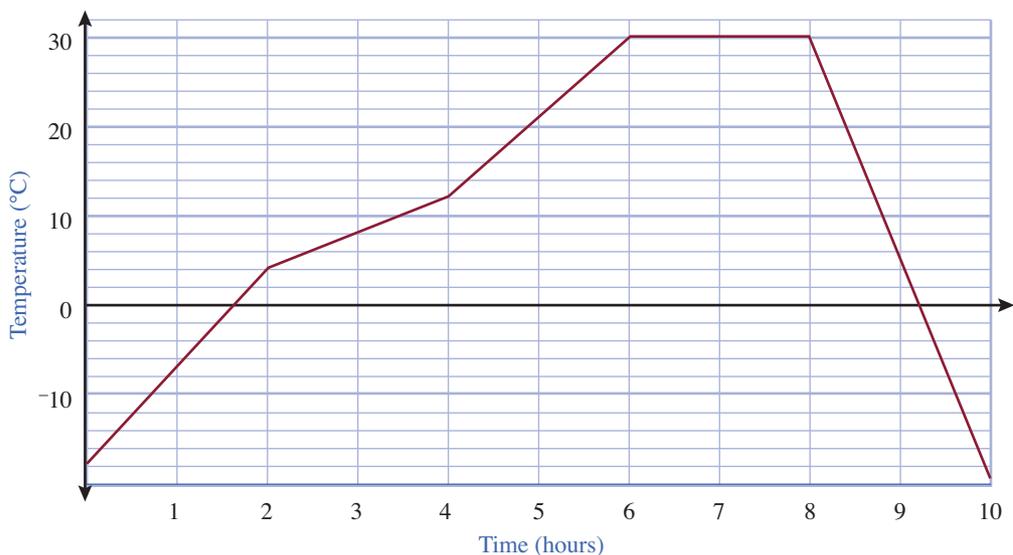
- 2 This graph shows how far a family is from home while on a Sunday drive.



- a
- i Where is the car at 9 am?
- ii Where is the car at 11 am?
- iii How far did the car travel between 9 am and 11 am?
- iv What was the speed of the car in this part of the journey?
- b Between what times did the car not move?
- c
- i Where was the car at noon?
- ii Where was the car at 1 pm?
- iii How far did the car travel between noon and 1 pm?
- iv What was the speed of the car in this part of the journey?
- d
- i Where was the car at 2 pm?
- ii Where was the car at 4 pm?
- iii How far did the car travel between 2 pm and 4 pm?
- iv What was the speed of the car in this part of the journey?
- e At what time did the family begin the return journey?
- f At what speed did the car travel between the times of:
- i 5 pm and 6 pm?
- ii 6 pm and 8 pm?



- 3 This graph shows the temperature of a tub of ice cream which is in a freezer at time zero hours.



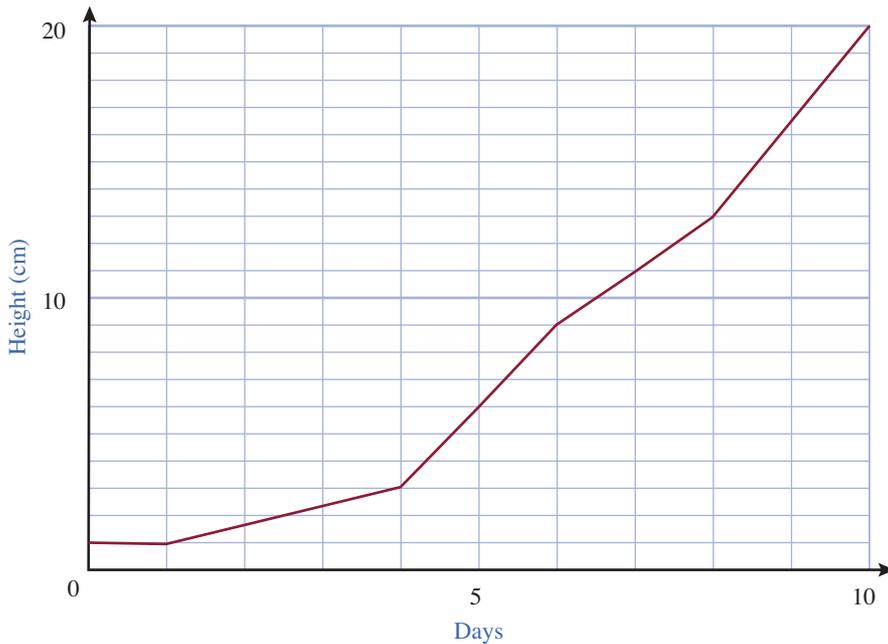
- a Fill in the temperature of the ice cream in the table.

Time (h)	0	1	2	3	4	5	6	7	8	9	10
Temp. (°C)											

- b At what rate did the ice cream heat up between the times of:
- i zero and 2 hours?
 - ii 2 hours and 4 hours?
 - iii 4 hours and 6 hours?
 - iv 6 hours and 8 hours?
- c What was the temperature of the room in which the experiment was conducted?
- d When was the ice cream placed back into the freezer?
- e Explain what might have happened in the experiment to produce the five straight-line sections of the graph.



- 4 This graph shows the growth of a tomato seedling in a science experiment at school.



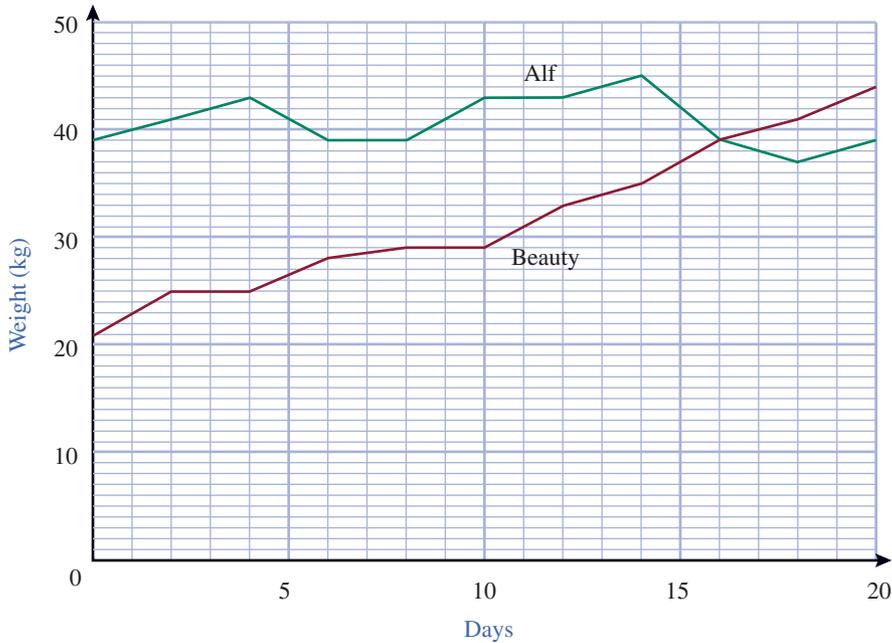
- a Copy and complete table below:

Day	0	1	2	3	4	5	6	7	8	9	10
Height											

- b Find the rate of growth of the tomato plant from:
- the start until day 1
 - day 1 to day 4
 - day 4 to day 6
 - day 8 to day 10
- c Find the average rate of growth of the tomato plant throughout the experiment.



- 5 The graph below shows the weight of two pet dogs owned by Mr Pedro. Each dog was weighed every 2 days for 20 days.



- a Copy and complete the table below, showing the weight of each dog.

Day	0	2	4	6	8	10	12	14	16	18	20
Alf											
Beauty											

- b Find the rate at which each dog changes in weight for each of the two-day intervals.
 c On which day was the weight of the dogs the same?
 d Make a story to match the graph of Alf's weight.
 e Make a story to match the graph of Beauty's weight.
 f Write the ratio of the weight of the two dogs on each of the days they were weighed.



Ratios can be used to reduce the size of plans or maps so that they can be drawn on paper. The ratio connects the length on the map to the actual distance.

Example

- 1 The length on a building plan is 2 cm. What is the actual length if the scale of the map is 1:20?
- 2 The distance between two points is 2 km. What is the distance on a map if the scale is 1:10 000?

Solution

$$2 \times \frac{20}{1} = 40 \text{ cm} = 0.4 \text{ m}$$

$$2 \text{ km} = 2 \times 1000 \times 100 = 200\,000 \text{ cm}$$

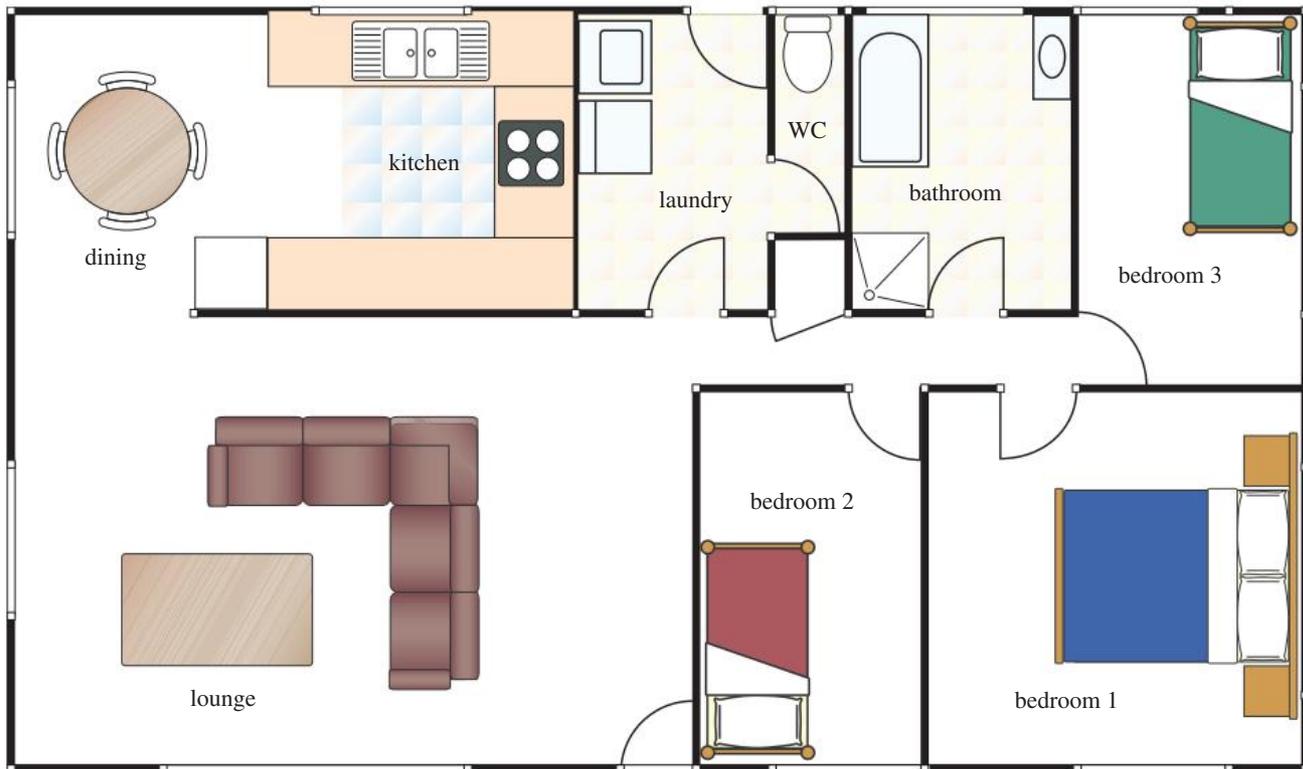
$$200\,000 \times \frac{1}{10\,000} = 20 \text{ cm}$$

So the line on the map will be 20 cm long.

Exercise 51

- 1 The scale on a building plan is 1:20. Find the actual length if on the plan a line is:
- | | | | | |
|----------------------|---------------------|---------------------|---------------------|----------------------|
| a 3 cm | b 5 cm | c 11 cm | d 12 cm | e 18 cm |
| f 2.5 cm | g 4.8 cm | h 5.2 cm | i 6.75 cm | j 8.9 cm |
| k $10\frac{1}{4}$ cm | l $5\frac{1}{4}$ cm | m $3\frac{1}{8}$ cm | n $6\frac{2}{3}$ cm | o $12\frac{7}{8}$ cm |
- 2 The scale on a map is 1:10 000. Find the length of a line on the map that joins positions which are the following distances apart:
- | | | | | |
|---------------------|---------------------|---------------------|---------------------|----------------------|
| a 1 km | b 3 km | c 1.5 km | d 5.8 km | e 7.8 km |
| f 580 m | g 780 m | h 1200 m | i 1800 m | j 2500 m |
| k $6\frac{1}{2}$ km | l $2\frac{1}{4}$ km | m $7\frac{3}{4}$ km | n $1\frac{3}{8}$ km | o $12\frac{5}{8}$ km |
- 3 Find the actual lengths of the following lines on a building plan for which the scale is given:
- | | |
|--------------------------|-------------------------------------|
| a 2 cm, scale is 1:300 | b 5 cm, scale is 1:400 |
| c 8 cm, scale is 1:500 | d 1.8 cm, scale is 1:200 |
| e 3.6 cm, scale is 1:800 | f $5\frac{3}{4}$ cm, scale is 1:250 |
| g 120 mm, scale is 1:100 | h 160 mm, scale is 1:200 |
| i 510 mm, scale is 1:500 | j 240 mm, scale is 1:200 |
- 4 Find the lengths on a map of the following lines for which the actual length and scale is given:
- | | |
|-----------------------------|-------------------------------------|
| a 4 km, scale is 1:10 000 | b 8 km, scale is 1:40 000 |
| c 9 km, scale is 1:50 000 | d 2.6 km, scale is 1:20 000 |
| e 2.8 km, scale is 1:80 000 | f $18\frac{1}{4}$ m, scale is 1:250 |
| g 240 m, scale is 1:10 000 | h 650 m, scale is 1:20 000 |
| i 960 m, scale is 1:500 000 | j 2530 m, scale is 1:50 000 |

5 A house plan drawn to a scale of 1:100 is shown below:



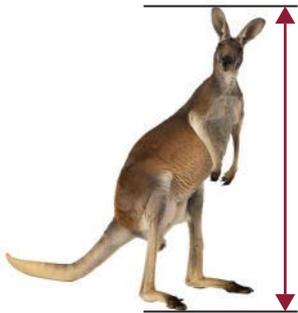
- a What distance in the house is represented by 1 cm on the plan?
 b Use the house plan to complete the table. Measure distances to the nearest 0.5 cm.

Room	Length (m)	Width (m)	Area (m ²)
Dining/kitchen			
Lounge			
Bedroom 1			
Bedroom 2			
Bedroom 3			
Bathroom			
Laundry			
Toilet			

- c If carpet costs \$400 per square metre, find the cost to the nearest dollar to carpet:
 i bedroom 1 ii bedroom 2 iii bedroom 3 iv the lounge room
- d Find the area of tiles required for:
 i the kitchen ii the laundry and toilet iii the bathroom
- e If it costs \$600 per square metre to tile a floor, including tiles and labour, calculate the cost of tiling each room.

6 Use the scale to find the indicated lengths of the following animals:

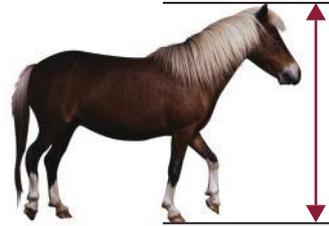
a 1:50



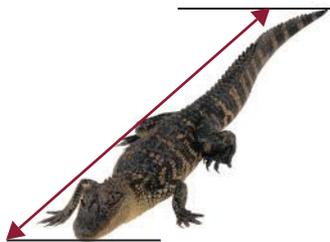
b 1:60



c 1:45



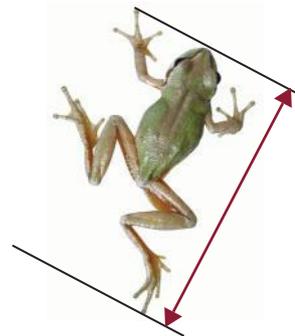
d 1:100



e 1:10



f 1:2





5J Exploring maps

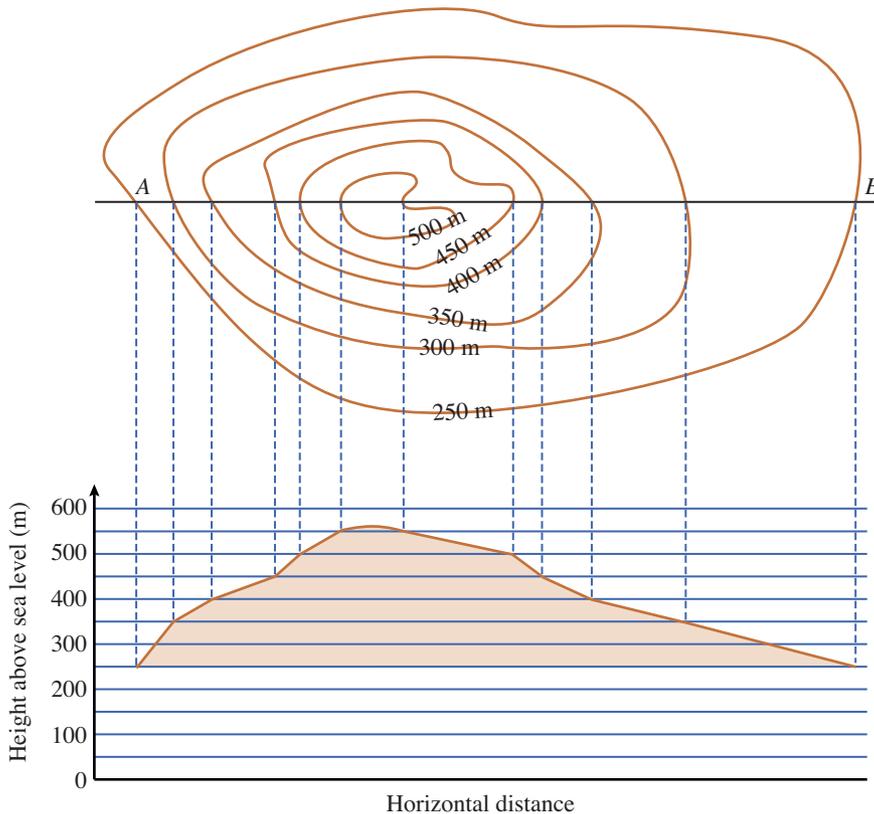
Using the scale on a map to find the ‘straight line’ distance is not the best way to plan a walk. Often there are hills and valleys to walk up and down, which can make the trip more difficult than the distance on the map shows. Maps with contour lines also show the terrain of the country.

A **contour line** on a map is line that joins places of equal height above sea level, and so they give an indication of the steepness or flatness of the terrain.

Contour maps can be used to make a cross-section graph between two places on the map.

Example

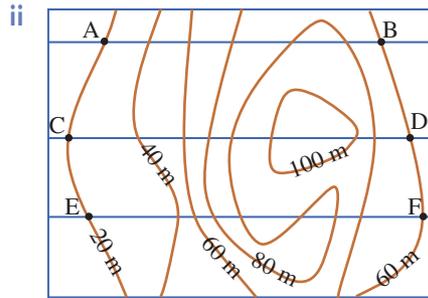
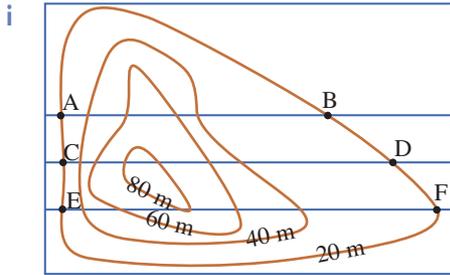
Shown below is a contour map and cross-section (landform graph) between points *A* and *B*.



Learning task 5J

- I The above map was drawn to the scale 1:10 000.
 - a Find the horizontal distance from point *A* to point *B*.
 - b Find the distance from point *A* to point *B* using the cross-section diagram.

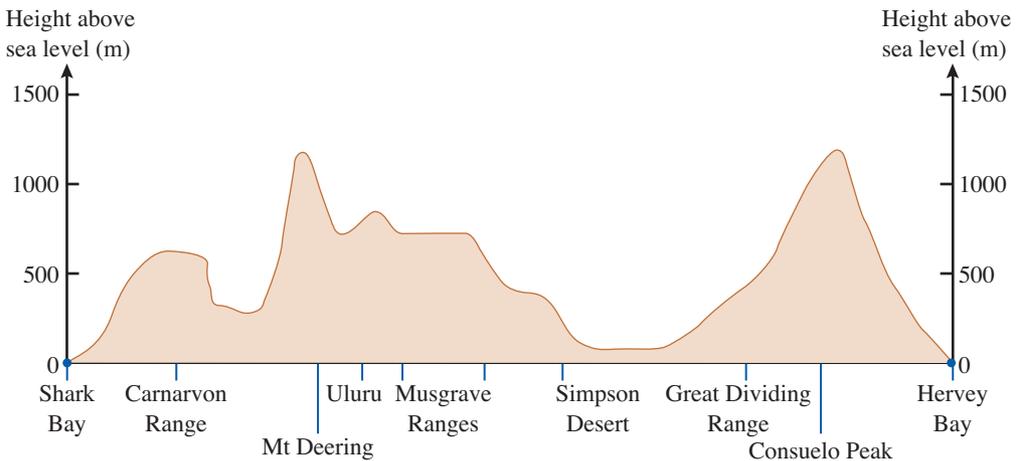
2 The two maps below have been drawn to the scale 1 : 10 000.



- a Find the distances from *A* to *B*, *C* to *D* and *E* to *F* on each map.
- b Make cross-section diagrams along each of the straight lines on the maps above and use the cross-section to find the distances from *A* to *B*, *C* to *D*, and *E* to *F* on each map.

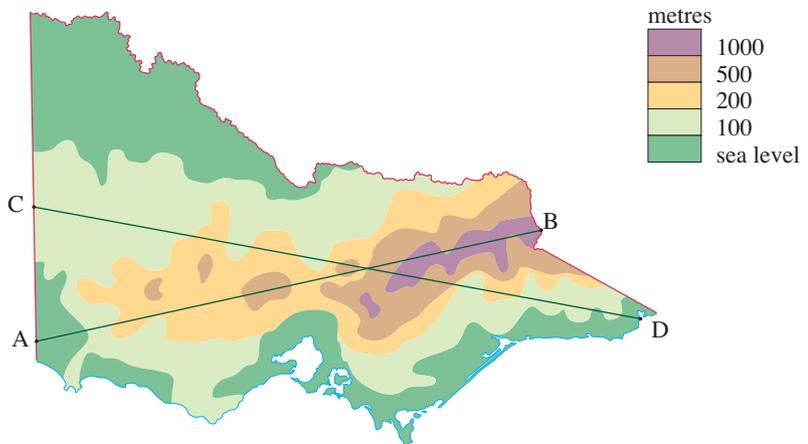
3 Use the cross-section diagram shown below to estimate the height of:

- a Mt Deering
- b Uluru
- c Consuelo Peak



4 Represent this contour map as a cross-section diagram from:

- a *A* to *B*
- b *C* to *D*





Puzzles

- 1 Find the value of the letters below by using equivalent ratios. Match the letter to the correct number to solve the riddle:

When is the best time to buy bonito fish at the Honiara Central Market?

$1:12 = 3:A$

$2:3 = 8:C$

$7:3 = 14:E$

$2:G = 6:12$

$4:7 = 16:H$

$1:5 = 6:I$

$N:5 = 8:20$

$1:5 = 3:O$

$4:11 = P:22$

$2:R = 14:21$

$7:T = 28:36$

$V:2 = 3:6$

$W:9 = 35:45$

$Y:6 = 10:12$

$2:5 = 4:Z$

<u>7</u>	<u>28</u>	<u>6</u>	<u>2</u>	<u>9</u>	<u>28</u>	<u>6</u>	<u>5</u>	<u>36</u>	<u>3</u>	<u>6</u>
<u>4</u>	<u>15</u>	<u>30</u>	<u>2</u>	<u>4</u>	<u>12</u>	<u>28</u>	<u>6</u>	<u>38</u>	<u>8</u>	



- 2 Calculate the actual length of a 5 cm line on a map by using the scales below. Match the letter to the correct measurement to solve the riddle:

What was the snail doing on the highway?

$A 1:100$

$B 1:50$

$D 1:10$

$E 1:1000$

$M 1:2000$

$N 1:5000$

$O 2:15$

$R 2:45$

$T 2:75$

$U 2:50$

$Y 5:6$

$Z 3:500$

<u>5 m</u>	<u>2.5 m</u>	<u>37.5 cm</u>	<u>1.25 m</u>	<u>1.875 m</u>	<u>37.5 cm</u>	<u>250 m</u>	<u>50 m</u>			
<u>100 m</u>	<u>50 m</u>	<u>1.875 m</u>	<u>1.125 m</u>	<u>50 m</u>	<u>5 m</u>	<u>50 cm</u>	<u>5 m</u>	<u>6 cm</u>		

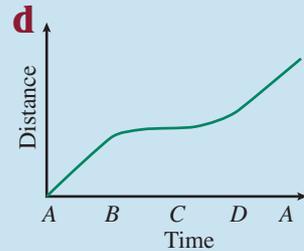
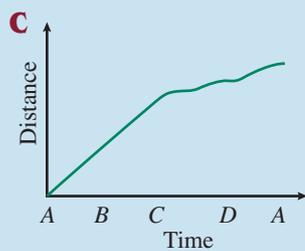
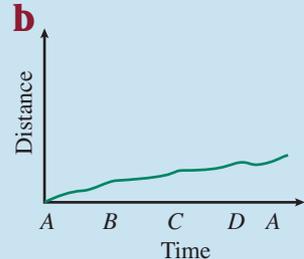
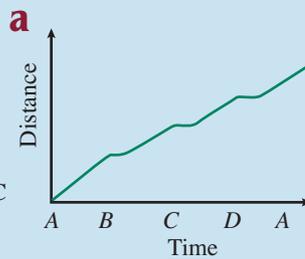
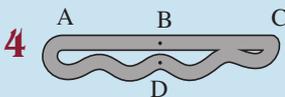
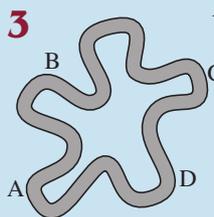
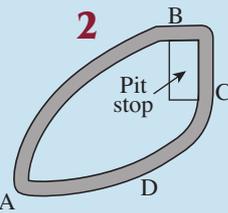
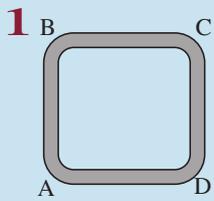
3 Find the speed of the car for each of the following journeys. Match the letter to the correct speed to solve the riddle:

What do you call a racing car driver at the North Pole?

Speed	Distance	Time
A	200 km	4 h
C	30 m	6 min
F	120 m	24 s
K	45 km	30 min
O	24 m	15 s
R	520 km	8 h
T	1320 km	12 h
Y	75 m	5 s
W	125 m	5 s

_____ _____ _____ _____ _____ _____ _____ _____ _____
 25 m/s 50 km/h 15 m/s 96 m/min 5 m/s 5 m/s 110 km/h 65 km/h 50 km/h 5 m/min 90 km/h

4 Match the travel graphs to the racing tracks shown below:





Applications

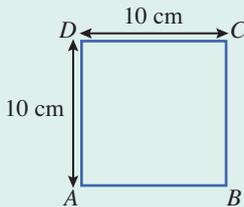
The Parthenon



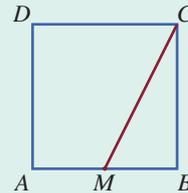
The Parthenon and other Greek temples were constructed using the most pleasing rectangle (the golden rectangle), which is in the ratio 1:1.62.

This is called the golden ratio. The exact value is $\frac{\sqrt{5}-1}{2}$.

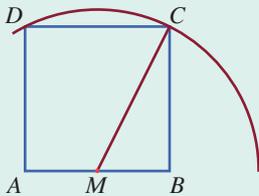
- a Copy the following construction to make a golden rectangle.



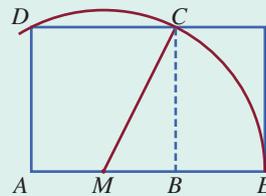
Step 1: Draw a square with 10 cm sides.



Step 2: Mark the midpoint of the base (AB) with the letter M and draw in the line MC .



Step 3: Use the line MC as a radius and draw part the circle using a compass as shown.



Step 4: Extend the base of the square so that it meets the arc. Label this point as E . Using the line AE as the base complete the rectangle.

- b Use squares of different sizes and draw different golden rectangles. Measure each one and check that the sides are in the ratio 1:1.62.

School crossing safety

The speed limits around school crossings are 40 km/h.

- Measure the width of your school crossing to the nearest metre.
- Find how long it takes (to the nearest minute) for students to walk this distance.
- Convert 40 km/h to metres per second and use this to find how far from the crossing a car needs to be for a student to cross the road safely (assuming that the car fails to slow down). This is known as the safe distance.
- Find the safe distances as a car's speed increases and present your findings on a graph.

Designing houses

Prefabricated units are used to construct simple and cheap housing. The units are 10-metre cubes which can be placed together in different configurations so that they join along their edges, either all on one level or as a multistorey building. Four units are used in total.

- Use a set of small cubes to find all the possible ways in which four units can be arranged. Draw all the possible designs on isometric paper.
- On centimetre dot paper, draw a floor plan of one of your house designs, stating the scale you have used.
- Construction costs are \$100 000 for each unit on ground level, \$250 000 for one or more storeys high. State the construction cost for your design.
- On isometric paper, make a realistic drawing of your design. Add other features such as water features, basketball rings, playground equipment and so on.
- Present your work in an advertising folder.

Favourite rectangles

Survey your class on which of the rectangles shown is the most appealing to the eye. Using a ruler, measure the sides of each rectangle and find the ratio of the length to the width.

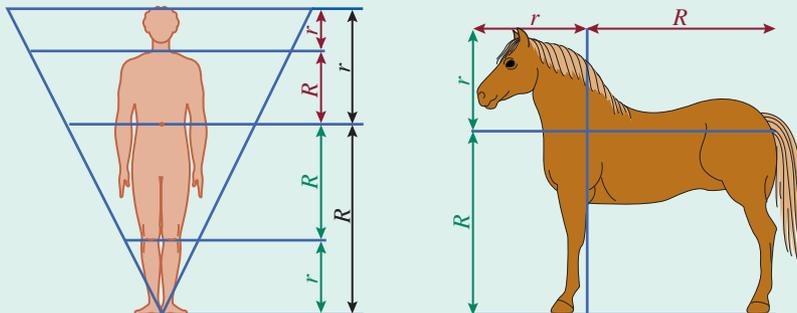
- Which rectangle is most popular?
- Which rectangle forms a golden ratio?



Living golden ratios

The bodies of humans and some animals have proportions that form golden ratios.

- Measure the pairs of lengths marked R and r and show that the ratios are approximately 1:1.6.



- Find photographs of other animals and find golden ratios.
- Find more examples of golden ratios in the human face.
- Investigate the use of golden ratios in the work of artists, for example Leonardo da Vinci's *Mona Lisa*.



Enrichment

1 Paper is sold in different grades. The papers have different weights and the weight is described in terms of the number of grams it weighs for the area of a square metre (gsm).

- White newsprint (50 gsm)
- Cartridge (110 gsm)
- Bank paper (40 gsm)
- Photocopy paper (70 gsm)

For each paper type above state its weight in:

- a g per square cm
- b g per square mm
- c kg per square m
- d tonne per square km

2 Audio speaker cable costs \$45.00 for 1.2 metres. A 10% discount is given for orders of continuous lengths of 10 metres or more.

Find the cost of purchasing these lengths of cable:

- a 3 metres
- b 9800 cm
- c 12.25 metres
- d 6500 cm

3 At the Under-12 football Grand Final, it is planned to use balloons in the opening event. Burnscreek Primary School has the colours of red, black and yellow and St John Primary School has the colours of red, blue and green. If the balloons are to be purchased in the ratio of the club colours, then how many of each colour should be bought if 5400 balloons are to be bought in total?

4 Concrete for a slab is made by mixing screenings, sand and cement in the ratio 3:5:1. A slab that is 12 metres long, 9 metres wide and 1 metre deep is to be poured.

- a Find the volume of each material that is to be used.
- b Using the following densities, find the weight of each material that is needed to make the concrete for the slab:

Sand: 1500 kg/m^3 Cement: 1900 kg/m^3 Screenings: 2000 kg/m^3

5 Draw three different-sized triangles with side lengths in the ratios:

- a 1:2:2
- b 2:3:4
- c 3:4:5

6 Draw polygons with a perimeter of 72 cm whose lengths are in the following ratios:

- a 1:1:1
- b 1:2:3:4
- c 1:2:3:4:5
- d 1:2:3:3:4:5



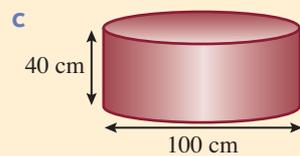
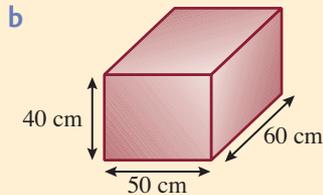
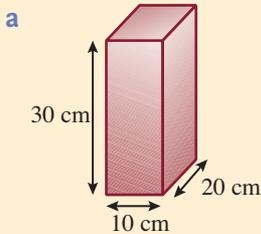
7 Find the distance travelled by the following:

- a An eagle diving at 120 km/h for 25 seconds
- b A bullet travelling at 125m/s for a minute
- c A helicopter travelling at 50 km/h for $4\frac{1}{2}$ hours

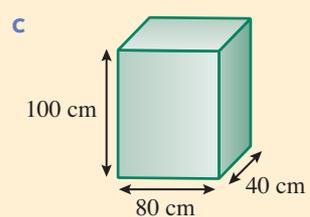
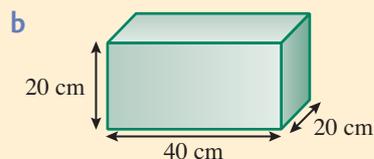
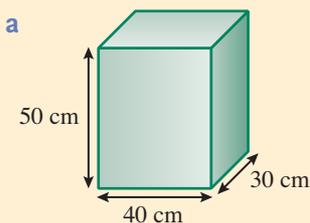
8 The fastest animal is the Arctic falcon, which has been observed to dive at speeds of 208 km/h. Find the distance that the falcon would travel in a dive that lasts:

- a 20 s
- b 5 s
- c 30 s
- d 1 minute

- 9 The density of plastic A is 1.5 g/cm^3 and the density of plastic B is 0.9 g/cm^3 . A mixture of plastics A and B in the ratio 1:2 is required to produce electrical resistors. Find the average density of this plastic and use it to find the weight of the following resistors:



- 10 A ream of paper has 500 A4 sheets. Each sheet is $210 \text{ mm} \times 297 \text{ mm}$ and its weight is 80 gsm. What is the weight of the ream to the nearest tenth of a kilogram?
- 11 Fill in five values of a and b for each of the following. State the relationship between a and b :
- a $a:b = 4:12$ b $18:b = 2a:12$ c $a:b = b:12$
- 12 Bill, Fiona and Guido have the following pulse rates when exercising:
 Bill: 120 beats/min Fiona: 210 beats/min Guido: 180 beats/min
 Express each person's heartbeat in:
- a beats per hour b beats per second c beats per day
- 13 A pumpkin was weighed at regular intervals and found to grow at the constant rate of 190 g/day over a four-week period.
- a If its weight was 1.2 kg at the start of the trial, find its weight at the end of each week of the trial.
- b Express the rate of the increase in weight as:
- i g/h ii g/min iii g/s
- 14 A petrol pump is able to deliver petrol at the rate of 20 L/min .
- a Express this rate in:
- i L/h ii mL/min iii mL/h
- b How long will it take to fill cars with petrol tanks of the following capacities?
- i 60 litres ii 45 litres iii 25 litres iv 120 litres
- 15 Find the weights of the following samples of iron if the density of iron is 7.9 g/cm^3 :
- a 14 cm^3 b 98 cm^3 c 39 cm^3 d 120 cm^3 e 3000 cm^3
- 16 The density of water at 4°C is 1 g/cm^3 . Find the weight of water in these fish tanks if they were filled to the top:



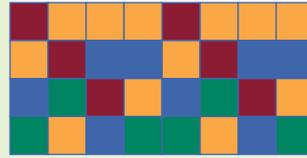


Revision/Assessment

Exercise 5A

1 Express the following as ratios in simplest form:

- a Red squares to yellow squares
- b Blue squares to green squares
- c Red and yellow squares to the total



2 The following plastic models are used to make a hanging transport mobile:

5 trains, 4 cars, 6 trucks, 2 planes and 3 buses.

Express in simplest form the ratio of each type of vehicle to the total number of vehicles.

Exercise 5B

3 Simplify these ratios:

- a 24:36
- b 12:2
- c 16:4
- d 18:3
- e 3:36

4 Find the missing numbers:

- a $7:8 = \underline{\quad}:40$
- b $3:2 = 12:\underline{\quad}$
- c $2:\underline{\quad} = 12:24$
- d $\underline{\quad}:4 = 1:12$

5 Simplify the following ratios:

- a 15 m:250 cm
- b 900 g:1.2 kg
- c 25 min:2 hours

6 To make paste, $1\frac{1}{2}$ cups of flour are added to 3 cups of water. Express this as a ratio and simplify.

Exercise 5C

7 The ratio of red balloons to green balloons in a display is 2:3.

a Find the number of green balloons if there are:

- i 40 red balloons
- ii 100 red balloons
- iii 250 balloons in total

b Find the number of balloons in total if there are:

- i 15 green balloons
- ii 351 green balloons

8 Yellow and red cordials are combined in the ratio 1:2.

a How much red cordial needs to be added to 30 mL of yellow cordial?

b If this cordial mixture is added to water in the ratio 1:200, then how much water needs to be added to the above volume?

Exercise 5E

9 A lottery win of \$96 000 is shared in the ratio 3:2:1. Find the amount that each should receive.

10 Pewter is made by mixing lead and tin in the ratio 1:4. How much lead and tin is there in a pewter mug of 150 g?

11 Sally's Superfertiliser has lime, potash and nitrates mixed in the ratio 1:4:3. How much potash is there in a 20 kg bag?

Exercise 5F

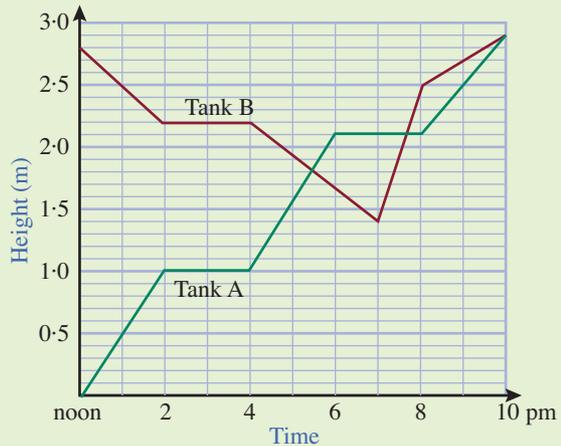
- 12** What quantity is changing in the following rates? Give an example of each:
 a mm/day b \$/m c \$/kg d L/h
- 13** Express each of the following situations as a simple ratio:
 a It costs \$480 to fill a 96-litre gas cylinder.
 b The netball team scored 52 goals in a game (four quarters).

Exercise 5G

- 14** Find the speed of the following in the units indicated:
 a A snake slithers 20 metres in 30 seconds (m/min).
 b An ant travels 20 metres in 3 minutes (cm/min).
- 15** Find the distance that a jet travelling at 800 km/h will cover in:
 a 3 hours b 30 minutes c 15 seconds

Exercise 5H

- 16** This graph shows the height of water that is held in two water tanks in Gizo between noon and 10 pm of the same day.



- a Record the water height of each tank at hourly intervals in this table:

Time	Noon	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm	7 pm	8 pm	9 pm	10 pm
Tank A											
Tank B											

- b Between which times does the water level in tank A increase?
 c Between which times does the water level in tank B decrease?
 d What is the average rate at which the water level increases in each tank?
 e Find the ratio of the water height in tank A to that in tank B at the following times:
 i 2 pm ii 7 pm iii 10 pm

Exercise 5I

- 17** Find the actual length of the following lines that are drawn on a map with scale 1 : 100 000:
 a 120 mm b 48 mm c 9.5 cm d 12.8 cm

CHAPTER

6

Statistics



Statistics

Statistics can be used to record interesting facts about a country, such as population growth rates, death rates, birth rates and fertility rates. Fertility rates are recorded as the number of children born per woman. According to the World Factbook, in 2012 the Solomon Islands had one of the highest fertility rates in the world, with approximately 3.51 children born per woman. Countries such as Australia have much lower figures, with a fertility rate of 1.77. Governments use statistics such as these when planning for issues such as population change.

This chapter covers the following skills:

- Collecting numerical data, discrete and continuous, by using class intervals
- Drawing histograms and column graphs and finding the mode
- Finding the median, range and interquartile range of a set of data
 - The median is the middle value when the data is written as an ordered list.
 - The range is the difference between the highest and lowest values.
 - The lower quartile or Q_1 is the middle value of the lower half.
 - The upper quartile or Q_3 is the middle value of the upper half.
 - The interquartile range (IQR) = $Q_3 - Q_1$.
- Drawing stem-and-leaf plots
- Finding the mean or average of a set of data
 - Let n stand for the number of values.
 - The notation Σx means the sum of the values.
 - To calculate the mean (or average) we use $\bar{x} = \frac{\Sigma x}{n}$.
- Comparing the mean and median
- Finding the mean absolute difference
The mean absolute difference is $\frac{\Sigma |x - \bar{x}|}{n}$.

8.6.5.1

Explain the terms: median, range and inter-quartile range (IQR).

8.6.5.2

Calculate the median, range and IQR of a set of scores.

8.6.1.1

Define and identify the properties of a stem-and-leaf plot.

8.6.1.2

Order data using a stem-and-leaf plot.

8.6.7.1

Define the term 'mean'.

8.6.7.2

Explain the formula for calculating the mean of a set of scores.

8.6.8.1

Calculate the mean or average of a set of scores.

8.6.9.1

Explain the term 'mean absolute difference'.

8.6.9.2

Explain the symbol that is use for absolute values: $|-a| = a$

8.6.10.1

Calculate the mean and the mean absolute difference for given set of scores.

8.6.11.1

Explain the term 'absolute value'.

8.6.11.2

Explain which of mean or median provides the best measure for the centre of a set of scores.

8.6.11.3

Find the mean and median of a set of given scores.

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.6.1.1 Define and identify discrete and continuous numerical data.
- 8.6.2.1 Record data in frequency tables.
- 8.6.3.1 Identify the properties of column graphs and histograms.
- 8.6.4.1 Appropriately display data using column graphs or histograms.
- 8.6.4.2 Define and identify the modal score in a set of scores.

Numerical data, as the name suggests, is data that involves counting or measurement. There are two different types of numerical data: discrete and continuous.

Discrete numerical data involves distinct values. These are often whole numbers, but not always. If collecting data involves counting then it will be discrete numerical data.

Examples of discrete numerical data are the number of children in a family, 1, 2, 3, 4, 5, and shoe sizes, $7, 7\frac{1}{2}, 8, 8\frac{1}{2}, 9, 9\frac{1}{2}, 10, 10\frac{1}{2}$.

Example

- 1 The number of children in each of the families of a class of Year 8 learners is given below:

2, 5, 3, 1, 4, 4, 3, 3, 2, 2, 5, 1, 2,
3, 2, 2, 1, 3, 2, 2, 2, 4, 3, 1, 2

The numbers can be displayed as a frequency table as shown.

Solution

No. of children	Frequency
1	4
2	10
3	6
4	3
5	2
Total	25

We may need to group discrete numerical data when there is a large variation in the numbers.

Example

- 2 The number of DVDs hired over the course of a year by a sample 30 customers was recorded:

53, 74, 36, 55, 62, 39, 54, 42, 63, 61,
29, 34, 37, 41, 47, 58, 60, 72, 38, 45,
48, 52, 67, 76, 44, 52, 71, 53, 64, 21

This data can be grouped into classes 21–30, 31–40 and so on.

Solution

No. of DVDs	Tally	Frequency
21–30		2
31–40		5
41–50		6
51–60		7
61–70		6
71–80		4
	Total	30

Continuous numerical data is numerical data where every number on a scale has meaning. If collecting data involves measuring, then it is probably continuous numerical data. This type of data will need to be grouped so that it can be analysed. Ideally the groups should be the same size.

An example of continuous numerical data is grouped height in cm: 150–, 160–, 170–, 180– etc.

Example

- 3 The heights, in cm, of a group of 25

Year 8 learners are:

144, 153, 167, 178, 171, 149, 153,
161, 167, 166, 156, 154, 162, 156,
166, 162, 160, 164, 148, 157, 170,
165, 163, 174, 168

This is continuous data and will need to be grouped into classes such as 140–, 150–, 160– and so on.

Solution

Height	Tally	Frequency
140–		3
150–		6
160–		12
170–		4
	Total	25

Exercise 6A

- 1 The number of registered voters in each home for the 2010 National General Election for Naha 4 was recorded:
1, 2, 2, 4, 3, 3, 2, 5, 3, 2, 3, 2, 1, 1, 1, 2, 1, 2, 4, 2, 3, 4, 3, 2, 1, 4, 2, 2, 1, 2
- a Display this information in a frequency table.
b What was the highest number of voters in a home?
c What is the most common number of voters in a home?
- 2 A group of Year 8 learners conducted a survey at school to find the number of pets learners have. The results are:
2, 3, 6, 1, 4, 2, 3, 2, 1, 5, 3, 3, 2, 1, 0, 1, 1, 2, 3, 2, 1, 2, 2, 0, 4, 3, 2, 2, 1, 4, 2, 3
- a Display this information in a frequency table.
b How many learners had more than three pets?
- 3 The heights, to the nearest centimetre, of 30 soccer players are:
185, 192, 175, 189, 178, 185, 193, 196, 189, 194, 197, 203, 201, 184, 198,
201, 178, 189, 186, 193, 187, 183, 192, 196, 185, 184, 189, 194, 194, 189

- a Copy and complete the table below:

Heights	Tally	Frequency
175–		
180–		
185–		
190–		
195–		
200–		
	Total	30

- b Repeat this for the different classes:

Heights	Tally	Frequency
170–		
180–		
190–		
200–		
	Total	30

- c Which way of grouping the data do you prefer? Why?

- 4** The times taken by a group of runners during the Annual Easter road race in Honiara in 2012 are recorded below.
 23, 27, 18, 19, 42, 26, 34, 38, 36, 32, 40, 41, 29, 33, 31, 30, 27, 25, 24, 23
 22, 26, 21, 22, 37, 28, 38, 33, 28, 22, 24, 26, 28, 27, 30, 25, 26, 23, 28, 31
 Display this in a frequency table, using the classes 15–, 20–, 25–, 30– and so on.
- 5** The ages of patients who went to Rove Clinic one weekend are recorded below:
 12, 26, 35, 43, 65, 98, 45, 67, 3, 57, 89, 75, 64, 58, 59, 73, 65, 72, 56, 42,
 1, 22, 35, 16, 42, 56, 52, 67, 2, 53, 61, 67, 71, 28, 51, 67, 49, 69, 34, 70
- What was the age of the youngest person to go to Rove Clinic during the weekend?
 - What was the age of the oldest person to visit the clinic?
 - Display this information in a frequency table, using classes 1–10, 11–20 and so on.
- 6** The weights, in kilograms, of 25 full-term newborn babies born in Kilufi Hospital in one month are given below:
 4.12, 2.76, 3.67, 4.03, 3.17, 3.85, 3.52, 2.97, 4.25, 2.98, 3.93, 3.69, 2.86,
 3.41, 3.78, 3.84, 2.97, 3.26, 3.81, 2.93, 4.18, 3.70, 3.48, 3.25, 3.77
- What was the lightest weight recorded?
 - What was the heaviest weight recorded?
 - Display this information in a frequency table, with a class size of 1 kg.
 - How many babies weighed over 4 kilograms?
 - How many babies weighed under 3 kilograms?
- 7** In an attempt to budget, a family recorded its weekly shopping bill at the Honiara Central Market, to the nearest dollar, for 6 months:
 181, 219, 194, 197, 185, 254, 236, 267, 185, 143, 178, 132, 188,
 205, 224, 187, 253, 221, 237, 245, 259, 193, 224, 218, 162, 177
- Display this information in a frequency table, with a class size of \$10.
 - If the weekly budget is \$200, how often do they go over budget?
- 8** An environmental group recorded the number of plastic bags people used when they went to the supermarket. The results of their survey are given below:
 0, 1, 3, 7, 12, 4, 0, 8, 5, 6, 9, 5, 0, 1, 1, 8, 10, 12, 10, 11, 2, 7, 8, 5, 6, 8, 7, 5, 6, 8, 4,
 11, 10, 8, 9, 12, 10, 8, 9, 0, 2, 12, 7, 6, 8, 10, 0, 1, 9, 0, 11, 9, 7, 8, 0, 1, 1, 8, 7, 5
- Display this information in a frequency table with classes 0–1, 2–3, 4–5, 6–7, 8–9, 10–11 and 12–13.
 - Which interval had the highest frequency?
 - Describe what the frequency table shows. Is this what you expected? Explain your answer.
- 9** Collect some data on a topic of your own choice.
- Display your data and organise it into a frequency table.
 - Describe what the frequency shows. Justify the statements you make about your data.

Column graphs and histograms can be used to display numerical data.

The **mode** is the most frequent score. For grouped or continuous data, the modal class is the most frequent interval.

Column graphs for discrete data have the following features:

- All the bars are the same width.
- The axes should be labelled. (The vertical axis is often labelled ‘Frequency’.)
- The horizontal axis should be evenly consecutively numbered.

Example

1 Discrete data

The frequency table shows the number of children in the families of a group of Year 8 learners.

No. of children	Frequency
1	4
2	10
3	6
4	3
5	2
Total	25

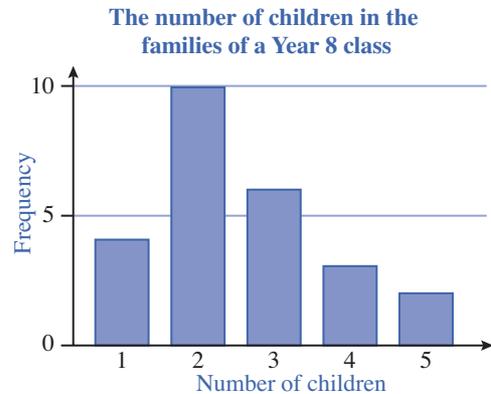
2 Grouped discrete data

The frequency table shows the number of DVDs hired over the course of a year by a sample of 30 customers.

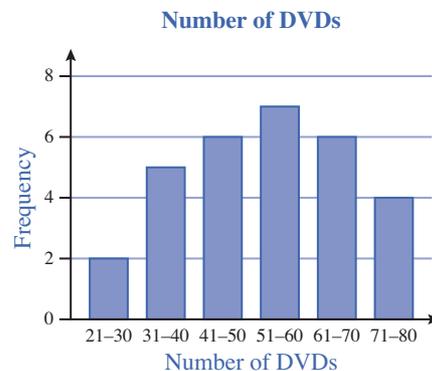
No. of DVDs	Frequency
21–30	2
31–40	5
41–50	6
51–60	7
61–70	6
71–80	4
Total	30

Solution

A column graph displaying the data is shown below.



A column graph of the data is shown below.



Histograms for continuous data have the following features:

- All bars are the same width.
- There are no gaps between bars.
- Axes should be labelled.
- The horizontal axis should have regular intervals.

Example

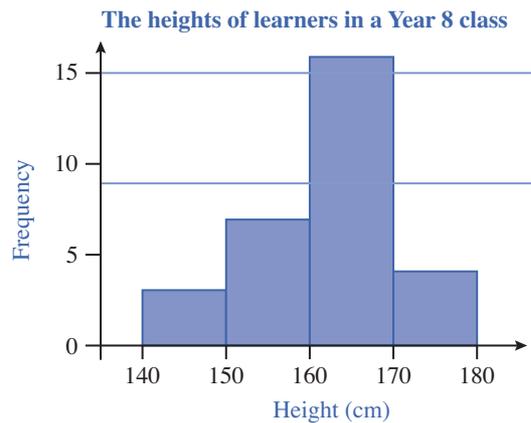
3 Continuous data

The frequency table shows the heights of a group of Year 8 learners.

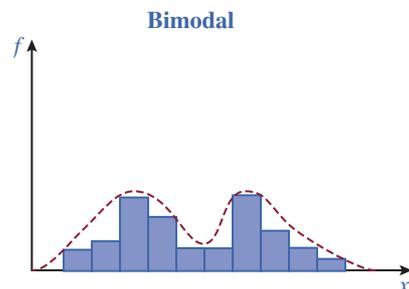
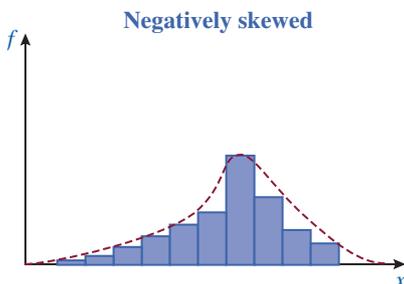
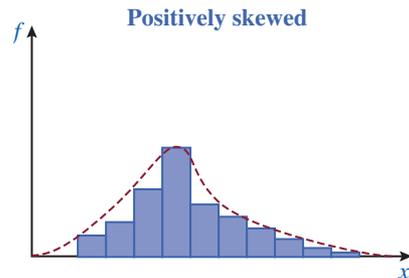
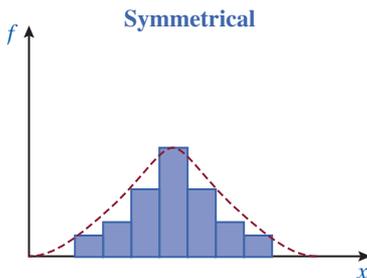
Height (cm)	Frequency
140–	3
150–	7
160–	16
170–	4
Total	30

Solution

A histogram displaying the data is shown below.



We draw histograms so that we can see the spread of the data. Usually the data is symmetrical with a peak in the middle. Sometimes the data is skewed or tails off to one side. Occasionally the graph has two peaks and we call these graphs bimodal.



Exercise 6B

- 1 The number of pets a group of Year 8 learners have is recorded in the frequency table below:

Number of pets	Frequency
0	2
1	7
2	11
3	7
4	3
5	1
6	1



Display this information as a labelled column graph.

- 2 A certain style of shoe was very popular but the shop owner still had a lot of shoes left over for the sales. The number of shoes and shoe sizes still available are given in the table below.

Shoe size	Frequency
$5\frac{1}{2}$	10
6	7
$6\frac{1}{2}$	2
7	2
$7\frac{1}{2}$	0
8	0
$8\frac{1}{2}$	3
9	4
$9\frac{1}{2}$	4
10	8



Display this information as a labelled column graph and comment on what the graph shows.

- 3 The frequency table for the height, to the nearest centimetre, of 30 soccer players is given:

Height	Frequency
175–	3
180–	1
185–	12
190–	7
195–	4
200–	3

Display this information as a carefully labelled histogram.

- 4 The weights, to the nearest kilogram, of the same 30 soccer players are given below:

86, 72, 95, 99, 108, 103, 93, 96, 79, 89, 97, 74, 101, 94, 88,
104, 108, 89, 96, 71, 89, 95, 92, 73, 97, 74, 95, 97, 93, 96

- Record this data in a frequency table with classes 70–, 75– and so on.
- Display this information as carefully labelled histogram.
- Comment on how this histogram compares with the histogram in Question 3.

- 5 The ages given below are those of patients that visited the Honiara National Referral Hospital during one weekend.

12, 26, 35, 43, 65, 98, 45, 67, 3, 57, 89, 75, 64, 58, 59, 73, 65, 72, 56, 42,
1, 22, 35, 16, 42, 56, 52, 67, 2, 53, 61, 67, 71, 28, 51, 67, 49, 69, 34, 70

Display this information in a column graph using the classes 1–10, 11–20 and so on. Comment on what the graph shows.

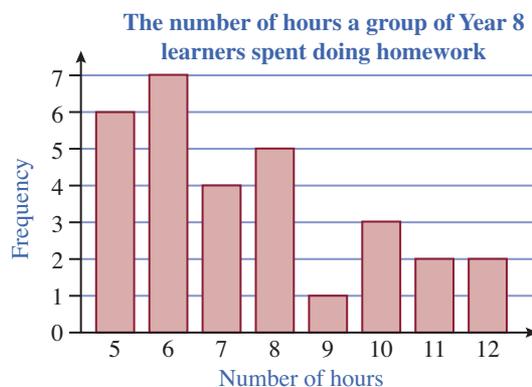
- 6 These are the weekly Solomon Islands Electricity Authority (SIEA) bills, to the nearest dollar, for a period of six months for a family:

193, 169, 187, 194, 221, 236, 197, 205, 177, 185, 195, 219, 224,
207, 193, 181, 223, 229, 202, 192, 204, 218, 214, 178, 188, 247

Display this data as a column graph. Describe the shape of the graph.

- 7 The column graph shows the number of hours spent on homework each week by a group of Year 8 learners.

- How many learners spent 6 hours doing homework?
- What was the greatest number of hours a learner spent doing homework?
- How many learners spent less than 7 hours doing homework?
- How many learners spent more than 10 hours doing homework?
- How many learners were surveyed?



When analysing data it is often useful to know a measure of the centre. Consider the heights of the students in your class. How do you compare with the others? Are you shorter or taller than most of the class? Maybe your height is somewhere in the middle.

The **median** is the middle value when the heights are written in order. Half the scores are above the median, half the scores are below the median. If there is an even number of results, the median is the average of the middle pair.

- The median is the middle score.

The **range** is a measure of the spread of the data. In Year 8 classes there is often a big difference between the tallest and shortest person. The range measures the spread of heights in the class.

- The range is the difference between the highest and the lowest values.

The **interquartile range** or IQR is a measure of the spread of the middle half of the data. To find the IQR we need to find the upper and lower quartiles. The **quartiles** are points that divide the data into quarters.

The **median** is the middle quartile sometimes called Q_2 .

The **lower quartile** or Q_1 is the $\frac{1}{4}$ point in the middle of the lower half of the data.

The **upper quartile** or Q_3 is the $\frac{3}{4}$ point in the middle of the upper half of the data.

- The interquartile range (IQR) = $Q_3 - Q_1$



Example

- 1 Find the median height (in cm) and the range for a group of 25 Year 8 learners whose heights are:

144, 153, 167, 178, 171, 149, 153,
161, 167, 166, 156, 154, 162, 156,
166, 162, 160, 164, 148, 157, 170,
165, 163, 174, 168

Solution

First, we need to rewrite the heights in order:

144, 148, 149, 153, 153, 154, 156,
156, 157, 160, 161, 162, **162**, 163,
164, 165, 166, 166, 167, 167, 168,
170, 171, 174, 178

There are 25 learners in the group, so the median height is the 13th score. The 13th height is 162.

The median height is 162.

The range = $178 - 144 = 34$.

Example

- 2 Find the interquartile range for a group of 25 Year 8 learners whose heights are:

144, 148, 149, 153, 153, 154, 156,
156, 157, 160, 161, 162, **162**, 163,
164, 165, 166, 166, 167, 167, 168,
170, 171, 174, 178

- 3 A new learner who is 185 cm tall joins the class.

a How does this affect the median and the range?

b How does this affect the interquartile range?

Solution

Do not include the median.

The lower quartile is halfway between the middle pair of the bottom half of the data:

144, 148, 149, 153, 153, **154**,
156, 156, 157, 160, 161, 162

$$Q_1 = \frac{154 + 156}{2}$$

$$Q_1 = 155 \text{ cm}$$

The upper quartile is halfway between the middle pair of the top half of the data:

163, 164, 165, 166, 166, **167**,
167, 168, 170, 171, 174, 178

$$Q_3 = 167 \text{ cm}$$

$$\text{IQR} = 167 - 155 = 12$$

The heights in order are now:

144, 148, 149, 153, 153, 154, 156,
156, 157, 160, 161, 162, **162**, **163**,
164, 165, 166, 166, 167, 167, 168,
170, 171, 174, 178, 185

The new median is halfway between the middle pair:

$$\text{New median} = \frac{162 + 163}{2}$$

New median is 162.5 cm.

The new range = 185 - 144 = 41.

The lower quartile is the middle value of the bottom half of the data:

144, 148, 149, 153, 153, 154, **156**,
156, 157, 160, 161, 162, 162

$$Q_1 = 156 \text{ cm}$$

The upper quartile is the middle value of the top half of the data:

163, 164, 165, 166, 166, 167, **167**,
168, 170, 171, 174, 178, 185

$$Q_3 = 167 \text{ cm}$$

$$\text{IQR} = 167 - 156 = 11$$

The addition of the new learner had little effect on the median and IQR, but there is a larger range.

Exercise 6C

- 1 Find the median, the range and the IQR for the heights of 30 children who played soccer in the park.
125, 132, 175, 129, 178, 125, 133, 136, 129, 134, 137, 143, 201, 184, 138, 141, 118, 129, 126, 193, 127, 123, 132, 136, 185, 124, 129, 134, 134, 129
- 2 Find the median, the range and the IQR for the weights of the 30 soccer players whose weights, in kg, are:
86, 72, 95, 99, 108, 103, 93, 96, 79, 89, 97, 74, 101, 94, 88, 104, 108, 89, 96, 71, 89, 95, 92, 73, 97, 74, 95, 97, 93, 96
- 3 Find the median, the range and the IQR for the times, in minutes, in the cross-country running event:
23, 27, 18, 19, 42, 26, 34, 38, 36, 32, 40, 41, 29, 33, 31, 30, 27, 25, 24, 23, 22, 26, 21, 22, 37, 28, 38, 33, 28, 22, 24, 26, 28, 27, 30, 25, 26, 23, 28, 31
- 4 Find the median, the range and the IQR for the recorded ages of death:
12, 26, 35, 43, 65, 98, 45, 67, 3, 57, 89, 75, 64, 58, 59, 73, 65, 72, 56, 42, 1, 22, 35, 16, 42, 56, 52, 67, 2, 53, 61, 67, 71, 28, 51, 67, 49, 69, 34, 70
- 5 The heights, in cm, of the players in two Year 8 basketball teams were:
Team A: 152, 156, 163, 167, 172, 174, 176, 200
Team B: 154, 170, 171, 171, 172, 173, 174, 175
Find the median height and the range of the heights for each team. Comment on the results.
- 6 The netball coach is trying to choose between two shooters for the finals competition. The number of goals each shooter scored in the 12-week season are recorded below:
Shooter A: 36, 2, 34, 2, 16, 4, 42, 28, 6, 4, 38, 20
Shooter B: 23, 24, 16, 18, 12, 16, 24, 26, 14, 10, 28, 18
 - a Find the median, range and IQR for each player.
 - b On the basis of these results, which shooter should the coach choose to play in the finals?
- 7 The intelligence quotient or IQ scores of a group of 15 learners were recorded in their first year at primary school and their first year of secondary school for comparison.
Primary school: 108, 89, 95, 107, 105, 128, 94, 102, 108, 111, 104, 93, 94, 85, 63
Secondary school: 109, 91, 93, 109, 103, 140, 92, 104, 106, 115, 100, 92, 95, 83, 56
 - a Find the median, range and IQR of these scores.
 - b What does this tell you about the change in the intelligence quotient of learners as they progress through primary school?
- 8 Conduct a class survey to measure the length of each person's pencil in millimetres. Record the results for girls and boys separately.
 - a Calculate the median, range and IQR for all the girls' pencil lengths and the boys' pencil lengths.
 - b What conclusions can you make about the lengths of girls' pencils and the lengths of boys' pencils?



Stem-and-leaf plots are also known as stemplots. They are a clever way of displaying numerical data. The stemplot must always have a key to show what each stem and leaf represents. The leaves must be evenly spread and when this is done carefully the stemplot looks a little like a histogram on its side. In an **ordered** stem-and-leaf plot the leaves are written in numerical order and this can be very useful for finding the median.

For two-digit numbers we divide the numbers so that the tens are the stem and the units are the leaves. For example, 49 is split as 4|9.

For three-digit numbers we could divide the numbers in two ways; 149 could be split as 14|9 or 1|49.

Example

- 1 The football scores for The Lakers football teams last season's games were:

67, 85, 56, 69, 99, 97, 59, 65, 84,
97, 49, 72, 89, 78, 66, 81, 92, 88,
87, 73, 79, 85, 82, 53, 61

Display the scores as a stemplot.

- 2 The heights, in cm, of a group of 26 learners competing in a race were:

144, 153, 167, 178, 171, 149, 153,
161, 167, 166, 156, 154, 162, 156,
166, 162, 160, 164, 148, 157, 170,
165, 163, 174, 168, 170

Display the heights as an ordered stemplot.

Solution

Football scores

4	9
5	3 6 9
6	1 5 6 7 9
7	2 3 8 9
8	1 2 4 5 5 7 8 9
9	2 7 7 9

Key: 4|9 represents 49

To display these heights as an ordered stem-and-leaf plot, we could divide the numbers so that the tens are the stem.

Heights

14	4 8 9
15	3 3 4 6 6 7
16	0 1 2 2 3 4 5 6 6 7 7 8
17	0 0 1 4 8

Key: 14|9 represents 149



Example

- I The number of dogs registered as pets in 20 electorates were recorded as:
576, 435, 695, 673, 490, 533, 674,
587, 512, 454, 537, 555, 423, 656,
612, 507, 382, 754, 626, 572

What is the median number of dogs in these electorates?

**Solution**

To display the numbers as an ordered stem-and-leaf plot, we could divide the numbers so that the hundreds are the stem.

Number of dogs

3		82
4		23 35 54 90
5		07 12 33 37 55 72 76 87
6		12 26 56 73 74 95
7		54

Key: 4|23 represents 423

To find the median number of dogs we need the average of the 10th and 11th numbers in the ordered stem-and-leaf plot.

Number of dogs

3		82
4		23 35 54 90
5		07 12 33 37 55 72 76 87
6		12 26 56 73 74 95
7		54

Key: 4|23 represents 423

The 10th number is 555 and the 11th number is 572.

$$\begin{aligned}\text{Median} &= \frac{555 + 572}{2} \\ &= 563.5 \text{ (say 564)}\end{aligned}$$

The median number of dogs is 564.

Exercise 6D

- I These are the final scores for a football team's last 30 home and away games:
123, 87, 69, 74, 106, 78, 125, 112, 105, 78, 86, 89, 93, 76, 68,
126, 92, 78, 100, 72, 81, 102, 110, 94, 101, 78, 92, 78, 61, 103
- Display these scores as an ordered stem-and-leaf plot.
 - What was the median, the range and the IQR for these scores?
 - How many scores are above 100?
 - How many scores are below 80?

- 2** The ages of the people who attended a village meeting organised by the Live and Learn NGO group are given below:

22, 26, 35, 43, 65, 38, 45, 47, 3, 57, 39, 55, 64, 58, 59, 43, 65, 32, 56, 42, 21, 22, 35, 16, 42, 56, 52, 47, 2, 53, 61, 67, 51, 28, 51, 57, 49, 39, 34, 30

- a** Display these ages as a stem-and-leaf plot.
b What was the median age of those who attended the workshop?



- 3** The weights, in kilograms, of 25 full-term newborn babies were:

4.12, 2.76, 3.67, 4.03, 3.17, 3.85, 3.52, 2.97, 4.25, 2.98, 3.93, 3.69, 2.86, 3.41, 3.78, 3.84, 2.97, 3.26, 3.81, 2.93, 4.18, 3.70, 3.48, 3.25, 3.77

- a** Display these weights in a stem-and-leaf plot with units as the stem.
b What was the median weight recorded?

- 4** These are the weekly shopping bills, to the nearest dollar, for a period of 6 months for a family:

193, 169, 187, 194, 221, 236, 197, 205, 177, 185, 195, 219, 224, 207, 193, 181, 223, 229, 202, 192, 204, 218, 214, 178, 188, 247

- a** Display this information as an ordered stem-and-leaf plot.
b Find the median shopping bill.
c Do you think \$200 was a reasonable amount to put in the family budget for shopping?

- 5** The average daily temperatures ($^{\circ}\text{C}$) recorded for a month were:

28.1, 27.6, 25.6, 26.7, 28.9, 25.4, 29.3, 26.5, 27.8, 29.1, 24.7, 26.8, 27.2, 28.3, 27.6, 26.6, 25.4, 26.3, 28.2, 26.4, 28.3, 29.7, 28.4, 27.3, 26.5, 27.5, 28.2, 27.6, 26.4, 27.0, 27.4

- a** Display this information as a stem-and-leaf plot.
b Find the median and range for these temperatures.
c How many days had temperatures above 27°C ?

A more commonly used measure of the centre is called the **mean** or the **average**. To find the mean we add up all the values and then divide the total by the number of values. We use the notation \bar{x} to represent the mean. The answer is often not one of the values originally given.

Statisticians are often interested to know the average number of children in a family. This will show whether the population is expected to increase or decrease, and whether we should build more schools, hospitals, sports facilities and so on. Recently, in some countries, there has been a decrease in the average number of children in a family and some economists feel that this is a cause of concern. What is the average number of children in the families at your school?

Example

- 1 Find the average number of children in the families of Year 8 learners if the number of children in each family is:

5, 3, 2, 4, 5, 3, 2, 7, 4, 1, 5, 1, 6,
2, 3, 4, 2, 4, 3, 1, 3, 6, 5, 3, 4.

- 2 Use the data in the frequency table to find the mean number of children.

x	f	$x \times f$
1	3	3
2	4	8
3	6	18
4	5	20
5	4	20
6	2	12
7	1	7
Total	25	88

Solution

There are 25 children in the class.

n is the number of values.

$$n = 25$$

Σx is the sum of the values.

$$\Sigma x = 88$$

\bar{x} is the mean.

$$\begin{aligned}\bar{x} &= \frac{\Sigma x}{n} \\ &= \frac{88}{25} \\ &= 3.52\end{aligned}$$

There is an average of 3.52 children in the families of the Year 8 class.

\bar{x} is the mean.

$$\begin{aligned}\bar{x} &= \frac{\Sigma x}{n} \\ &= \frac{88}{25} \\ &= 3.52\end{aligned}$$

The mean number of children is 3.52.

Exercise 6E

- 1 The results of the survey of the number of plastic bags people used when they went to the supermarket are given below:
0, 1, 3, 7, 12, 4, 0, 8, 5, 6, 9, 5, 0, 1, 1, 8, 10, 12, 10, 11, 2, 7, 8, 5, 6, 8, 7, 5, 6, 8, 4, 11, 10, 8, 9, 12, 10, 8, 9, 0, 2, 12, 7, 6, 8, 10, 0, 1, 9, 0, 11, 9, 7, 8, 0, 1, 1, 8, 7, 5
 - a What was the average number of plastic bags used?
 - b How many people surveyed used more than the average number of bags?
- 2 What was the average height (in cm) of these soccer players?
185, 192, 175, 189, 178, 185, 193, 196, 189, 194, 197, 203, 201, 184, 198, 201, 178, 189, 186, 193, 187, 183, 192, 196, 185, 184, 189, 194, 194, 189
- 3 What was the average weight (in kg) of these soccer players?
86, 72, 95, 99, 108, 103, 93, 96, 79, 89, 97, 74, 101, 94, 88, 104, 108, 89, 96, 71, 89, 95, 92, 73, 97, 74, 95, 97, 93, 96
- 4 The weights, in kilograms, of some newborn babies were:
4.12, 2.76, 3.67, 4.03, 3.17, 3.85, 3.52, 2.97, 4.25, 2.98, 3.93, 3.69, 2.86, 3.41, 3.78, 3.84, 2.97, 3.26, 3.81, 2.93, 4.18, 3.70, 3.48, 3.25, 3.77
 - a What was the mean weight recorded?
 - b If a new baby is born weighing 2.51 kg, would this result in an increase or decrease in the average?
 - c Calculate the new average.
- 5 The annual salaries, in dollars, of some first-year graduates were recorded as:
35 000, 23 000, 15 000, 24 000, 16 000, 26 000, 31 000, 28 000, 23 000, 25 000, 31 000, 41 000, 29 000, 18 000, 32 000, 34 000, 42 000, 9000, 24 000, 22 000
 - a What are the median and the mean of these salaries?
 - b The \$9000 salary was recorded incorrectly and it should be \$69 000. What effect did this error have on the mean and the median?
 - c How many graduates had salaries above the mean?
 - d How many graduates had salaries above the median?
- 6 The average daily temperatures ($^{\circ}\text{C}$) for the month of July in the highlands of Guadalcanal were recorded as:
28.1, 27.6, 25.6, 26.7, 28.9, 25.4, 29.3, 26.5, 27.8, 29.1, 24.7, 26.8, 27.2, 28.3, 27.6, 26.6, 25.4, 26.3, 28.2, 26.4, 28.3, 29.7, 28.4, 27.3, 26.5, 27.5, 28.2, 27.6, 26.4, 27.0, 27.4
 - a Find the mean for these temperatures.
 - b How many days had temperatures above the mean?
- 7
 - a If 7 values have a mean of 4.5, what is their sum?
 - b If 10 values have a mean of 27.6, what is their sum?
- 8
 - a The mean of 3 values is 2.8. Write down 3 possible values.
 - b The mean of 5 values is 7.4. Write down 5 possible values.

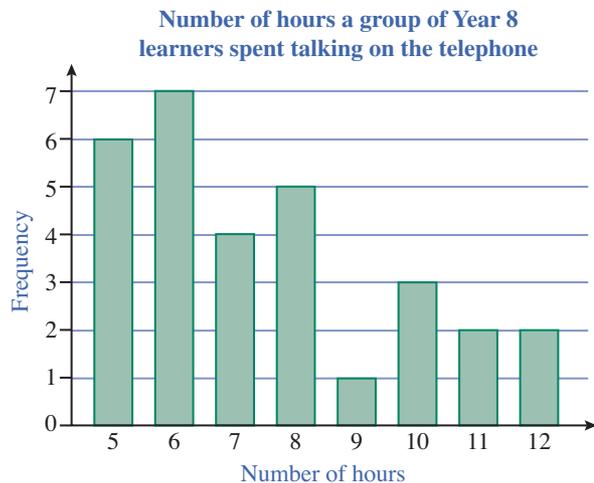
- 9 The number of pets a group of Year 8 learners have is recorded in the frequency table below:

Number of pets	Frequency
0	2
1	7
2	11
3	7
4	3
5	1
6	1



What is the average number of pets for these learners?

- 10 The column graph shows the number of hours that a group of Year 8 learners spent talking on the telephone each week. What is the mean number of hours spent on the phone?
- 11 The average exam result for a Year 8 class of 18 learners was 76%. The average for another Year 8 class of 27 learners was 54%. What was the overall average for the two classes?



- 12 These are the weekly shopping bills, to the nearest dollar, for a period of 6 months for a family:
- 193, 169, 187, 194, 221, 236, 197, 205, 177, 185, 195, 219, 224, 207, 193, 181, 223, 229, 202, 192, 204, 218, 214, 178, 188, 247
- Find the average weekly shopping bill.
 - Do you still think \$200 was a reasonable amount to put in the family budget for shopping?
 - The last bill was \$247. What would the last bill need to have been so that the mean was \$200?
- 13 The average weekly earnings of the employees in a shop was \$425. When a new manager was appointed the average increased to \$430. If there were originally 7 employees, what is the new manager's weekly earnings?
- 14 Use string to measure the neck circumference to the nearest centimetre of all the girls and all the boys in your class.
- Which have the thickest necks in your class, the girls or the boys?
 - Explain your answer with reference to suitable histograms and averages.

With the mean as a measure of the centre, an appropriate measure of spread could be the **mean absolute difference**. The term ‘absolute value’ is the numerical value ignoring whether it is positive or negative. The symbol $|x|$ is used to represent the absolute value of x . For example $|2| = 2$ and $|-3| = 3$. The mean absolute difference is the average of the difference between the mean and each of the other values when we ignore the sign.

Example

Find the mean and mean absolute difference for these test results:

11, 12, 12, 12, 13, 13, 13, 13, 14, 15

Solution

There are 10 results in this sample.

$$\bar{x} = \frac{\sum x}{10} = \frac{128}{10} = 12.8$$

Mean absolute difference

$$= \frac{\sum |x - \bar{x}|}{10} = \frac{8.4}{10} = 0.84$$

This is a small value as these results are very close.

x	$x - \bar{x}$	$ x - \bar{x} $
11	-1.8	1.8
12	-0.8	0.8
12	-0.8	0.8
12	-0.8	0.8
13	0.2	0.2
13	0.2	0.2
13	0.2	0.2
13	0.2	0.2
14	1.2	1.2
15	2.2	2.2
Total	0	8.4

Exercise 6F

- 1 The number of babies born each year in Kilufi hospital, Malaita Province, is shown in the table below:

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
No. of babies	75	87	92	76	84	78	62	59	43	36

- a Find the mean number of babies born per year in the hospital during this period.
 b Find the mean absolute difference for the number of babies born during this period.
- 2 The average minimum night-time temperatures for Adaua Secondary School, Malaita Province, are given in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maximum temperature ($^{\circ}\text{C}$)	26	26	24	20	17	14	13	15	17	20	22	24

- a Find the mean of the average minimum night-time temperatures for the year.
 b Find the mean absolute difference for the average minimum night-time temperatures for this period.

Most of the time the mean or the median are both appropriate measures of the centre of a sample as they both give roughly the same answer. However, the median is a better measure to use when dealing with skewed data. This is data that has some unusually high or unusually low values. For example, the median is often preferred when looking at house prices because it is common to find some very expensive houses in some areas.

Example

The price of houses listed for sale in a newspaper are given below:

\$1 250 000, \$1 000 000, \$1 950 000,
\$950 000, \$1 350 000, \$1 050 000,
\$1 500 000, \$1 100 000

- a Find the median and the mean house prices.

Solution

Median house price:

850 000, 950 000, 1 000 000, 1 050 000,
1 100 000, 1 250 000, 1 350 000, 1 500 000,
1 950 000, 2 600 000

$$\begin{aligned} \text{Median} &= \frac{1\,100\,000 + 1\,250\,000}{2} \\ &= 1\,175\,000 \end{aligned}$$

The median price is \$1 175 000.

Mean house price:

$$\begin{aligned} \bar{x} &= \frac{1\,360\,000}{10} \\ &= 1\,360\,000 \end{aligned}$$

The mean price is \$1 360 000.

- b Which is the better measure of the centre?

The median is a much better measure of the centre, because the mean is inflated by the high price of \$2 600 000.

Exercise 6G

The life expectancy, in hours, of each of two different brands of light bulbs was tested and the results were recorded in the table below:

Brand A	235, 412, 628, 534, 578, 602, 398, 437, 525, 298, 384, 484, 533, 431, 572, 389, 471, 519, 483, 358, 422, 585, 461, 438, 523
Brand B	542, 193, 712, 643, 278, 373, 181, 789, 852, 235, 531, 644, 587, 201, 289, 887, 356, 277, 482, 276, 367, 795, 378, 357, 476

- Display the data for Brand A in a histogram using class sizes of 100.
- Display the data for Brand B in a histogram using class sizes of 100.
- Describe the differences between these two types of light bulbs.
- Find the mean and the median life expectancy for each brand of light bulb.
- Find the mean absolute difference.
- Comment on what these values tell us.

Statistical methods are useful for answering questions.

For example: Farmer Joe wishes to know which brand of string beans will produce the best crop of string beans. The brands available are Super Beans and Healthy Beans.

To answer his question the farmer decides to grow five plants of each brand, and to compare the results by collecting and analysing the data. He plans to report his results and conclusions to his farmer friends.

Farmer Joe could use some discrete numerical methods covered earlier in Chapter 6.

He would need to count the number of string beans produced by each plant on a particular day. He might record his data on a table as a tally chart and calculate the totals for each plant and brand.

To show his results to his farmer friends, he might choose to draw some column graphs.

Farmer Joe could also do some calculations to compare central values and the spread (or range) for each brand. He chooses the means and the inter-quartile ranges (IQR) for his statistical analysis.

Finally he presents his findings, his graphs and calculations to his farmer friends on a chart with his reasons for which brand, Super Beans or Healthy Beans, produce the greatest number of string beans.

However, Farmer Joe could have used some continuous numerical methods, also covered earlier in Chapter 6, to investigate his two brands of beans.

He would need to measure the lengths of each string bean produced by each plant on a particular day. He might record his data on a table like the one for Example 2 in Exercise 6A.

To show his results to his farmer friends he might choose to draw some histograms.

Farmer Joe could calculate the median bean length and the range of the bean lengths for each brand as his statistical analysis.

Finally, he presents his findings, his graphs and calculations to his farmer friends on a chart with his reasons for which brand, Super Beans or Healthy Beans, produce the longest string beans.



Exercise 6H

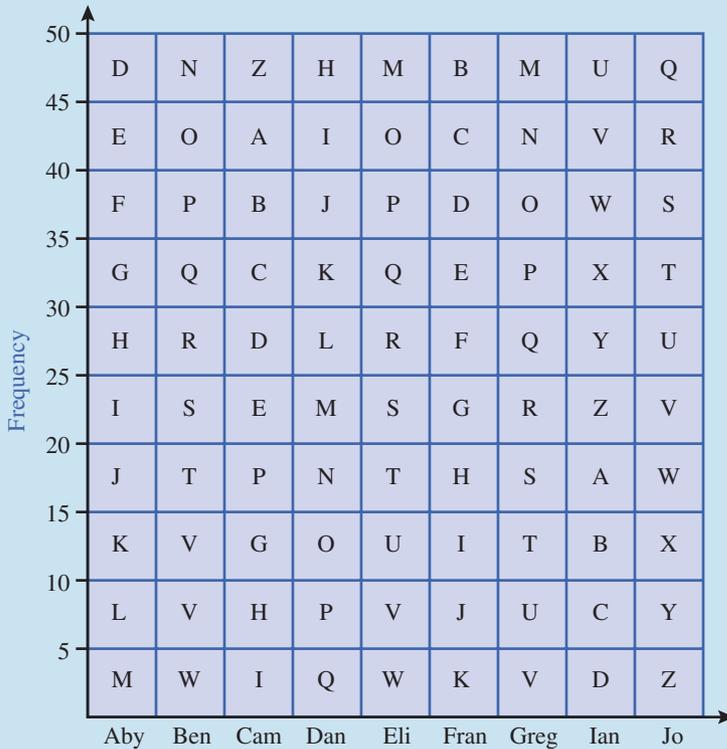
- 1 With a partner, choose a question that you can investigate that requires collecting and analysing your own discrete numerical data. You could modify Farmer Joe's investigation for counting string beans or choose your own topic. You might choose to count numbers of bananas, piglets, dogs, fish, or any other item that is readily available in your village. Prepare a chart of your data, graphs calculations and conclusions, and present them to your class. Be prepared to justify your conclusions with facts from your statistical analysis.
- 2 With a partner, choose a question that you can investigate that requires collecting and analysing your own continuous numerical data. You could modify Farmer Joe's investigation for measuring the lengths of string beans or choose your own topic. You might choose to measure the lengths of fish in a catch, the lengths of canoes owned by fishermen in your village, the weight of each fish in a catch, the volume of coconut cream from several coconuts, or any other items that are readily available to measure in your village. Prepare a chart of your data, graphs calculations and conclusions, and present them to your class. Be prepared to justify your conclusions with facts from your statistical analysis.





Puzzles

- 1 The scores for a group of friends in a game are shown in the frequency table below. Colour in the columns on the frequency graph to represent the scores for each person. Write down the letter in the square at the top of each column to find a term used in statistics.



	Score
Aby	40
Ben	30
Cam	25
Dan	5
Eli	15
Fran	35
Greg	45
Ian	10
Jo	10

- 2 Calculate the summary statistics for the data represented in the following frequency tables, then match the letter to the correct value below to solve the riddle:

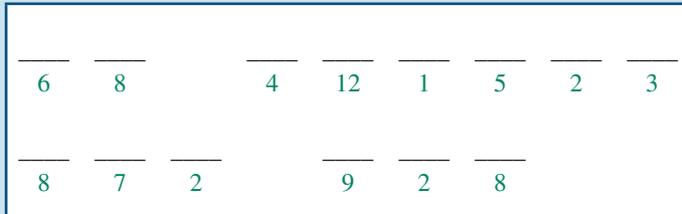
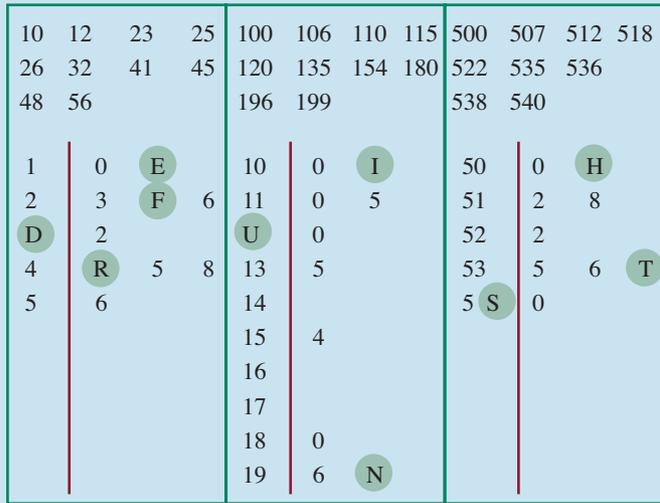
What is it called when a person is eating tuna fish while surfing?

<table border="1"> <thead> <tr> <th>x</th> <th>f</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>4</td> </tr> <tr> <td>4</td> <td>5</td> </tr> <tr> <td>5</td> <td>2</td> </tr> <tr> <td>6</td> <td>5</td> </tr> </tbody> </table> <p>A mean C highest score E median</p>	x	f	3	4	4	5	5	2	6	5	<table border="1"> <thead> <tr> <th>x</th> <th>f</th> </tr> </thead> <tbody> <tr> <td>5</td> <td>2</td> </tr> <tr> <td>6</td> <td>2</td> </tr> <tr> <td>7</td> <td>4</td> </tr> <tr> <td>10</td> <td>7</td> </tr> </tbody> </table> <p>I mean M lowest score N highest score</p>	x	f	5	2	6	2	7	4	10	7	<table border="1"> <thead> <tr> <th>x</th> <th>f</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>6</td> </tr> <tr> <td>5</td> <td>4</td> </tr> <tr> <td>8</td> <td>2</td> </tr> <tr> <td>10</td> <td>3</td> </tr> </tbody> </table> <p>V mean W lowest score X number of scores</p>	x	f	2	6	5	4	8	2	10	3
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4.5	5	4	15	8	6	4.5	10	2	4.5	5.2	4
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3 Match the letters in the stemplots with their correct values below to find the answer to the riddle:

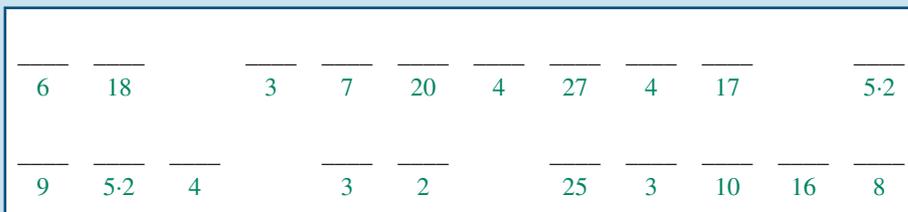
What did the computer do at Bells Beach?



4 For each set of data, calculate the summary statistics. Match the correct letters to the values below to solve the riddle:

What is the best way to stir up trouble when fishing?

<p>1 3 6 6 10</p> <p>A Mean</p> <p>B Median</p> <p>C Range</p>	<p>2 6 17 19 20</p> <p>E Highest score</p> <p>F Lowest score</p> <p>G Median</p>	<p>3 4 8 15 18 20 30</p> <p>H Median</p> <p>I Range</p> <p>M Interquartile range</p>
<p>1 1 1 2 3 7 7 7 7</p> <p>N Mean</p> <p>O Median</p> <p>P Mode</p>	<p>6 10 1 3 14 26</p> <p>R Mean</p> <p>S Median</p> <p>W Range</p>	<p>2 2 6 12 12 13 20</p> <p>X Median</p> <p>Y Range</p> <p>Z Interquartile range</p>





Applications

Time series data



Record some data over time. These could include:

- the temperature each day
- the number of absences at your school each day
- the heart rate of a student during exercise
- the length of a shadow.

Display this information as a line graph and comment on the results.

Random numbers using π

π is used to find the area and circumference of a circle. π is just a number, but it has decimal places that go forever without repeating in a pattern. The decimal places of π can be used to generate random numbers in probability. The first 15 digits of π are 3.141 592 653 598 79.

- a Which digit does not appear in this list?
- b How often does the digit 3 appear in the first 15 digits of π ?
- c How often does the digit 8 appear in the first 15 digits of π ?
- d Which digit has the highest frequency in the first 15 digits of π ?
- e Find the first 500 decimal places of π by searching the Internet.
- f Find the frequency of each of the digits 0, 1, 2, ... 9 in the first 500 decimal places of π .
- g Use the first 500 decimal places of π to find one-digit random numbers.
- h Use the first 500 decimal places of π to find two-digit random numbers.
- i Using a calculator or computer, generate random numbers from 0 to 9.
- j Using a calculator or computer, generate random numbers from 0 to 99.
- k Compare results.

Sampling

When we are doing a statistical survey, it is often impossible to survey everybody involved. It is usually better to survey a sample or smaller group of people. Imagine you are a school principal and are considering changing the rules on uniform in your school, which has more than 1000 students. You don't really need to survey the whole school and may only have time to question 50 families. How do you decide which 50 families to survey so that other members of the school community accept the results as fair?

One way is to randomly select the families by using random numbers. Most modern calculators have a random number function. However, random numbers can also be generated with dice or spinners.

Example

A teacher wants to randomly select a group of five learners from a class of 24.

First she gives each learner a number from 1 to 24. To get random numbers from 1 to 24 she creates a 6×6 table and writes in the numbers as below:

		Score on first die					
		1	2	3	4	5	6
Score on second die	1	1	2	3	4	5	6
	2	7	8	9	10	11	12
	3	13	14	15	16	17	18
	4	19	20	21	22	23	24
	5	25	26	27	28	29	30
	6	31	32	33	34	35	36

The dice are rolled in turn and the resulting numbers identify a random number in the table. For example, if the first die is 2 and the second die is 3, then the random number will be 14. Any numbers greater than 24 or any repeated numbers are ignored. Continue rolling the dice until five different random numbers are generated that correspond to the learners.

Instead of dice, hexagonal spinners can be used to generate the random numbers in the same way. By varying the number of dice or spinners, or the number of edges to a spinner, rules for generating other random numbers can be invented. However, the result of each roll or spin must be equally likely to fairly generate random numbers.

- Use the calculator to get a random number.
- Multiply the number by 24.
- Add one to the number.
- Ignore the decimal places.

The calculator gives the numbers:

0.1059	2.5416	3.5416	3
0.1850	4.44	5.44	5
0.5921	14.2104	15.2104	15
0.2825	6.78	7.78	7
0.6396	15.3504	16.3504	16

She selects the 3rd, 5th, 15th, 7th and 16th student.

Explain how you could use random numbers to select:

- a a group of 3 learners from your class
- b a number between
 - i 1 and 100
 - ii 1 and 20
 - iii 1 and 50
 - iv 100 and 200
- c a month
- d a date this year
- e a date in the 21st century



Enrichment

- 1 Two lots of information can be displayed on the same set of axes.

Month	Maximum temp. (°C)	Minimum temp. (°C)
January	26	14
February	26	14
March	24	13
April	20	11
May	17	8
June	14	7
July	13	6
August	15	6
September	17	8
October	20	9
November	22	11
December	24	13

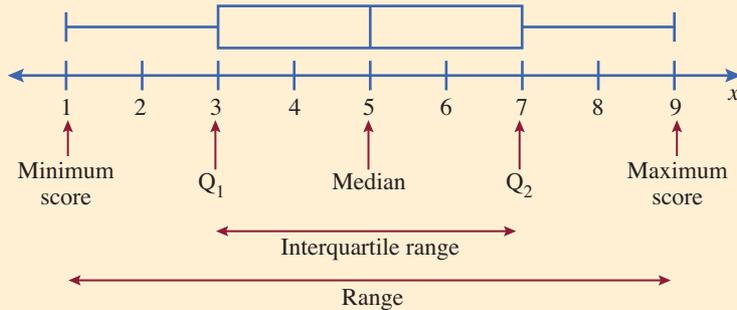
- Display the maximum and minimum temperatures for Melbourne on the same set of axes
 - When was the difference between maximum and minimum temperatures greatest?
 - When was the difference between the temperatures smallest?
 - Calculate the difference in temperature for each month and display as a time series graph. Compare this graph with the graph in part a.
- 2 The frequency table shows the heights, in centimetres, of the soccer players:

Height	Frequency
175–	3
180–	1
185–	12
190–	7
195–	4
200–	3

If you were only given the frequency table and asked to estimate the mean:

- What would be the highest possible mean?
 - What would be the lowest possible mean?
 - What would be the best possible estimate?
- 3 The average height of a basketball squad is 178 cm. Another person joins the squad and the average height increases to 180 cm. If the new player is 194 cm tall, how many people were originally in the squad?

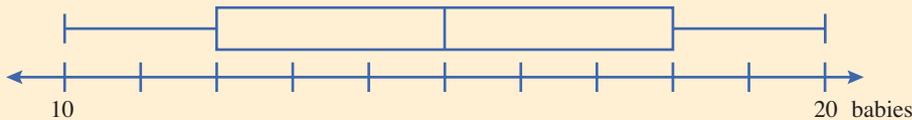
Boxplots can be drawn to represent the spread of data between each quartile.



4 Draw a boxplot to represent the following data:

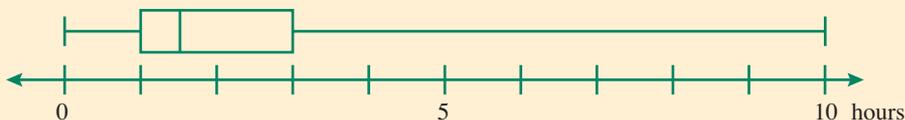
- Lowest score 30, $Q_1 = 42$, median score = 50, $Q_3 = 72$, highest score = 80
- Lowest score 10, $Q_1 = 12$, median score = 30, $Q_3 = 32$, highest score = 40
- Lowest score 5, $Q_1 = 8$, median score = 12, $Q_3 = 22$, highest score = 28
- Lowest score 12, $Q_1 = 15$, median score = 23, $Q_3 = 32$, highest score = 46
- 2, 3, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9, 10, 10
- 144, 153, 167, 178, 171, 149, 153, 161, 167, 166, 156, 154, 162, 156, 166, 162, 160, 164, 148, 157, 170, 165, 163, 174, 168
- 235, 412, 628, 534, 578, 602, 398, 437, 525, 298, 384, 484, 533, 431, 572, 389, 471, 519, 483, 358, 422, 585, 461, 438, 523

5 The boxplot below represents the number of babies born in a hospital in each month of the year.



- Write down the lowest score, Q_1 , the median score, Q_2 , the highest score, and the IQR.
- Comment on the results.

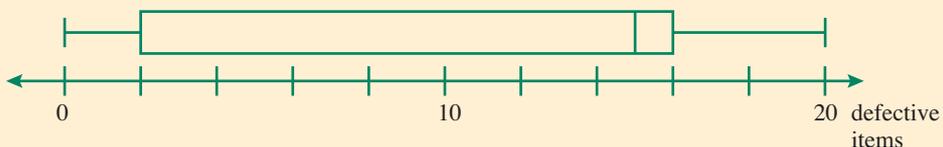
6 The boxplot below represents the numbers of hours students spent playing computer games per day on the weekend.



- Write down the lowest score, Q_1 , the median score, Q_2 , the highest score and the IQR.
- Comment on the results

7 The boxplot below represents the number of defective items made in a factory each day.

- Write down the lowest score, Q_1 , the median score, Q_2 , the highest score and the IQR.
- Comment on the results.





Revision/Assessment

Exercise 6A

- 1 The times in seconds that 50 people waited in a queue at a fast-food store were recorded and the results were:
- 125, 146, 98, 104, 122, 86, 74, 111, 96, 102, 89, 94, 105, 128, 143, 75,
125, 144, 131, 98, 86, 92, 71, 87, 96, 132, 117, 104, 93, 94, 116, 132, 76,
81, 78, 72, 112, 98, 86, 74, 106, 114, 88, 93, 97, 101, 126, 114, 123, 136
- a Display this information in a frequency table with the classes 70–, 80–, 90–, ...
- b The company claims it serves its customers within 2 minutes. How many customers waited for more than 2 minutes?

Exercise 6B

- 2 The number of days sick leave that 100 teachers took in 12 months are listed in the table below:

Days	Frequency
0	22
1	26
2	15
3	11
4	8
5	6
6	5
7	5
8	3

- a Display the data as a column graph.
- b Describe the shape of the column graph.

- 3 The frequency table is grouped into the classes 0–, 2–, 4– and so on.

Scores	Frequency
0–	13
2–	3
4–	7
6–	10
8–	15
10–	8
12–	4

- a Display the data as a histogram.
- b Which is the most frequent group?

Exercise 6C

- 4 The costs of a cup of coffee in a survey of 16 cafes were:
 \$2.40, \$2.50, \$2.90, \$3.50, \$2.00, \$2.60, \$2.60, \$2.70,
 \$2.20, \$2.40, \$2.40, \$2.50, \$2.20, \$2.70, \$2.80, \$2.30
- What was the median price for a cup of coffee?
 - What was the range for the price of a cup of coffee?

Exercise 6D

- 5 The ages of a group of women when they had their first child were:
 28, 27, 25, 24, 17, 16, 29, 26, 23, 24, 24, 29, 32, 36, 31, 30, 25, 24, 22, 27, 26,
 29, 28, 31, 34, 36, 42, 40, 35, 36, 38, 28, 25, 24, 31, 18, 32, 16, 19, 41, 24
- Display this as an ordered stem-and-leaf plot.
- 6 Find the median, the range and the interquartile range for the ages in Question 5.

Exercise 6E

- 7 Find the mean price for a cup of coffee using the data in Question 4.
- 8 Find the mean number of days sick leave for the teachers in Question 2.
- 9 A data processor will lose his job if his average typing speed in four tests is less than 50 words per minute. The results of his first three tests were 52, 46 and 48. What score does he need in the last test to keep his job?

Exercise 6F

- 10 The number of thefts from parked cars in a large town was recorded for 18 months:

Month	Number of thefts
January	54
February	63
March	78
April	82
May	56
June	34
July	45
August	61
September	65

Month	Number of thefts
October	74
November	78
December	82
January	90
February	81
March	72
April	53
May	42
June	31

- Find the mean of the data, correct to 2 decimal places.
- Find the mean absolute difference for the data, correct to 2 decimal places.

CHAPTER

7

Transformations



Transformations

Translations, reflections, rotations and dilations are called transformations. Transformations appear around us in our everyday life. Every time you look in a mirror you see a reflection of your face. Did you know that most faces are not symmetrical—there are minor differences between the left and right sides of your face? The hands on a clockface continually rotate. When you play board games you translate the counters around the board.

This chapter covers the following skills:

- Identifying transformations
 - Translation
 - Reflection
 - Rotation
 - Enlargement
- Finding congruent shapes
- Exploring tessellations
- Reducing and enlarging shapes using a grid
- Flipping, sliding and turning in 3D
- Using mathematical language for transformations
- Producing patterns using transformations
- Looking for line and rotational symmetry

Specific Learning Outcome (SLO)

Learners should be able to:

- 8.7.1.1** Explain the term ‘transformation’.
- 8.7.2.1** Identify four types of transformation of an object or shape: translation, reflection, rotation and enlargement.
- 8.7.3.1** Define translation.
- 8.7.4.1** Describe the translations of objects using ‘up’, ‘down’, ‘left’ and ‘right’.
- 8.7.4.2** Use mathematical language to describe a translation of an object.
- 8.7.5.1** Define reflection.
- 8.7.5.2** Identify the properties of reflections.
- 8.7.6.1** Reflect objects and shapes in a mirror line and identify their properties.
- 8.7.7.1** Identify any lines of symmetry in objects and shapes.
- 8.7.8.1** Define rotation.
- 8.7.8.2** Identify the properties of rotations: centre (point) of rotation; directions: clockwise or anticlockwise; and size of angle in a turn.
- 8.7.8.3** Rotate an object given its centre of rotation, and the directions and angle size of its turn.

- 8.7.9.1** Define enlargement and reduction: Enlargement—Increase the size of an object. Reduction—Decrease the size of an object.
- 8.7.10.1** Enlarge shapes or objects on a variety of grids.
- 8.7.11.1** Find the scale factors for the enlargement or reduction of objects.
- 8.7.11.2** Enlarge a shape or object by a given scale factor.
- 8.7.12.1** Calculate the areas and volumes of shapes.
- 8.7.12.2** Use scale factors to calculate areas and volumes of shapes enlarged by given scale factors.
- 8.7.13.1** Identify the relationship between the scale factors for length and volume.
- 8.7.14.1** Define the term ‘congruent’.
- 8.7.14.2** Compare the properties of shapes and objects to identify whether they are similar.
- 8.7.14.3** Identify the effects of different combinations of transformations.

A transformation gives an object a new shape, position or size.

Isometric transformations do not change the shape of the object. The most common isometric transformations are translation, reflection and rotation.

A **translation** shifts the shape to a new position.

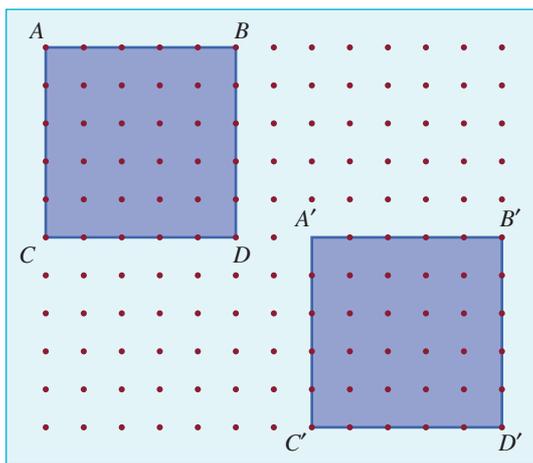
A **reflection** flips the shape or produces a mirror image.

A **rotation** turns the shape about a fixed point.

An **enlargement** is a non-isometric transformation, in which an object is made bigger. A **reduction** is a non-isometric transformation in which an object is made smaller.

After a transformation an object is called the image.

The square $ABCD$ is known as the 'object'.



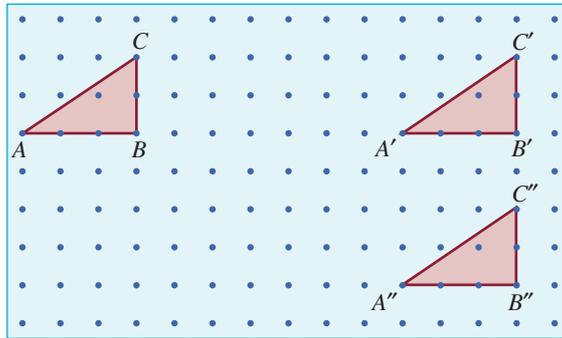
The square $A'B'C'D'$ is known as the 'image'.

Exercise 7A

Decide whether the photographs below represent translation, reflection, rotation, enlargement or reduction:



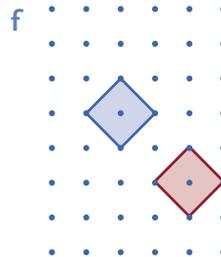
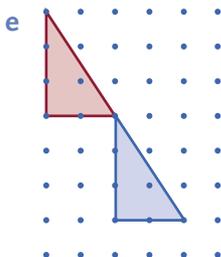
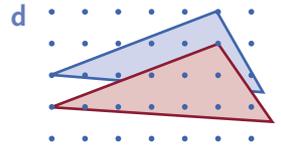
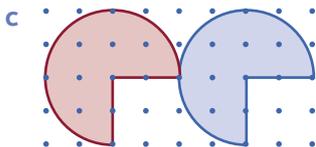
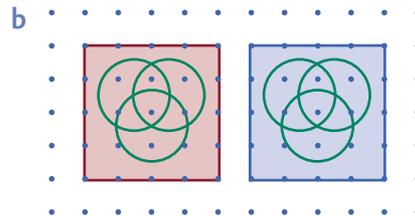
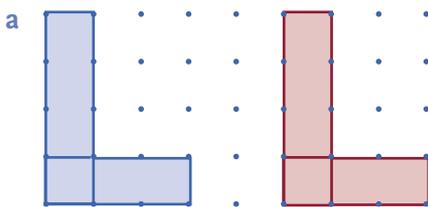
'Translation' is a term used to describe the movement of an object across a surface in one direction to another position. The size and shape of the object is unchanged.



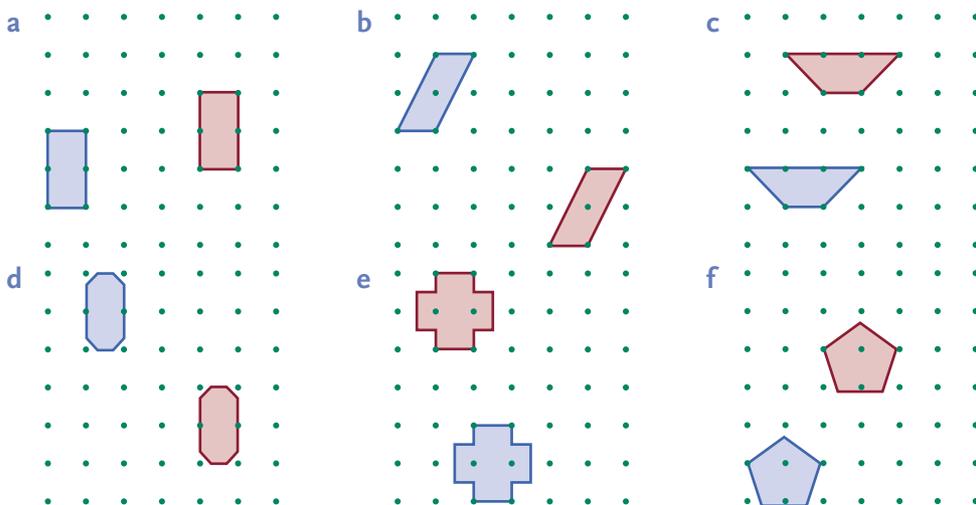
The triangle ABC has been translated 10 units to the right to $A'B'C'$ and then translated four units down to $A''B''C''$. This can be written as 10R 4D. Count the number of units across from B to B' . It is the same number of units across as A is to A' . Translations in two dimensions involve movement right and left and also movement up and down.

Exercise 7B

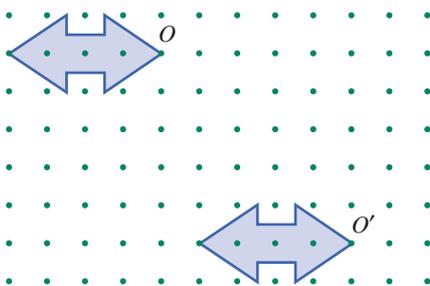
- I Describe the translation of the blue to the red shape. Use language such as up, down, right and left. Each translation has direction and distance. Remember to write the distance and the direction in which the object has been translated:



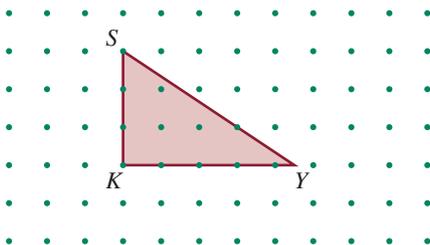
- 2 The blue objects have been translated to the red images. What is the translation that has taken place in each figure below?



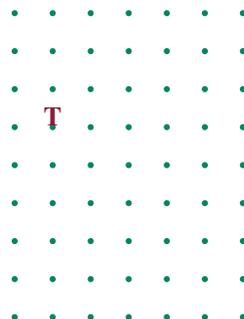
- 3 Describe the translation of shape O , using direction and distance.



- 4 Triangle SKY is translated left 3 units and down 2 units. Copy the grid below and draw the image and label it $S'K'Y'$.



- 5 Copy the grid and translate the letter T:
- 3 units right and 4 units down. Label this point T' .
 - 2 units right and 1 unit up. Label this point T'' .
 - 1 unit left and 3 units up. Label this point T''' .
 - 2 units right and 2 units down. Label this point T'''' .



6 Copy the grid shown and translate the B:

- a 1 unit left and 4 units up. Label this point B'.
- b 5 units right and 1 unit down. Label this point B''.
- c 1 unit left and 2 units down. Label this point B'''.
- d 1 unit right and 5 units up. Label this point B''''.

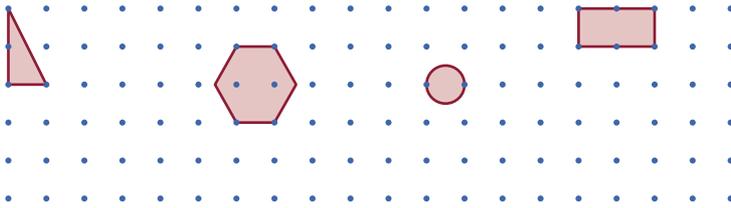


7 State the final position of the image relative to the object after each of the following translations (R, right; L, left; U, up; D, down):

- a 4R 2D 3L 4D
- b 6L 3U 4R 5U
- c 8R 2D 4L 3U 6L 4D
- d 9R 3U 4L 5U 6L 8U
- e 8R 2U 4L 9D 6R 2U 4L 3D 2R 9U
- f 6L 2D 3R 5D 3L 4U 6R 2U 4R 6D

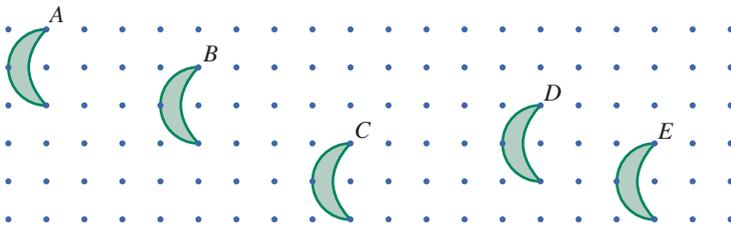
8 Copy the grid and draw the images of the following shapes after translation:

- a Triangle 5R 2D
- b Hexagon 2L 2D
- c Circle 1L 2U
- d Rectangle 2R 3D



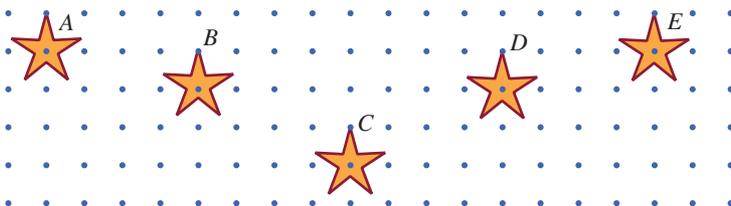
9 Use the diagram below to determine the translations required to move from:

- a A to B
- b A to C
- c B to E
- d C to B
- e E to D
- f D to B
- g E to A
- h E to C



10 Using the diagram shown, state the translations required to move from:

- a A to B
- b A to C
- c B to E
- d C to B
- e E to D
- f D to B
- g E to A
- h E to C

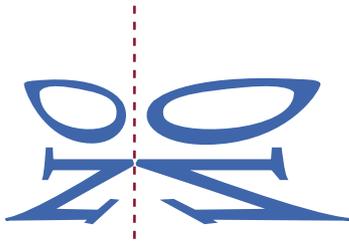


A 'flip' is a term used to describe a shape which is turned over. The mathematical name for a flip is a **reflection**. Reflection produces a mirror image of the shape. The line halfway between a shape and its reflection is called the **line of reflection** or **mirror line**. A shape that maps exactly onto itself about a fixed line has line symmetry. Some regular shapes have more than one line of symmetry. Note that the shape and size of the object is unchanged.

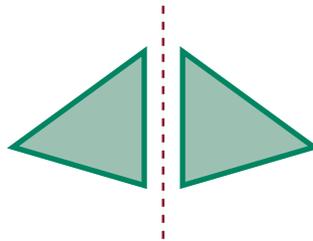
Exercise 7C

- 1 Look carefully at these pairs of shapes. For each pair that is not a reflection, write the reason. Redraw the shapes, using tracing paper, to show a reflection:

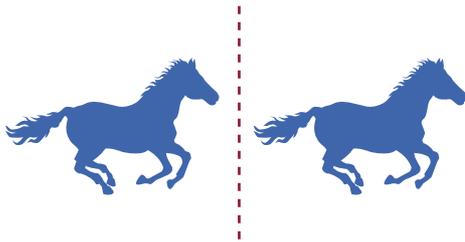
a



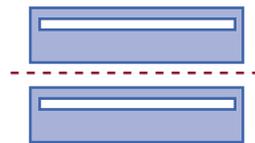
b



c



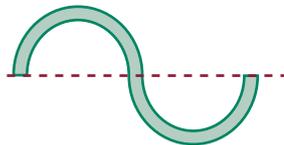
d



e



f

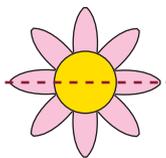


- 2 For each pair of shapes, decide if the transformation is a reflection in the given line. If not, give a reason:

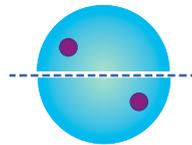
a



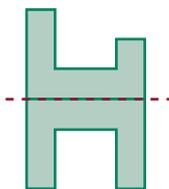
b



c



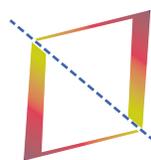
d



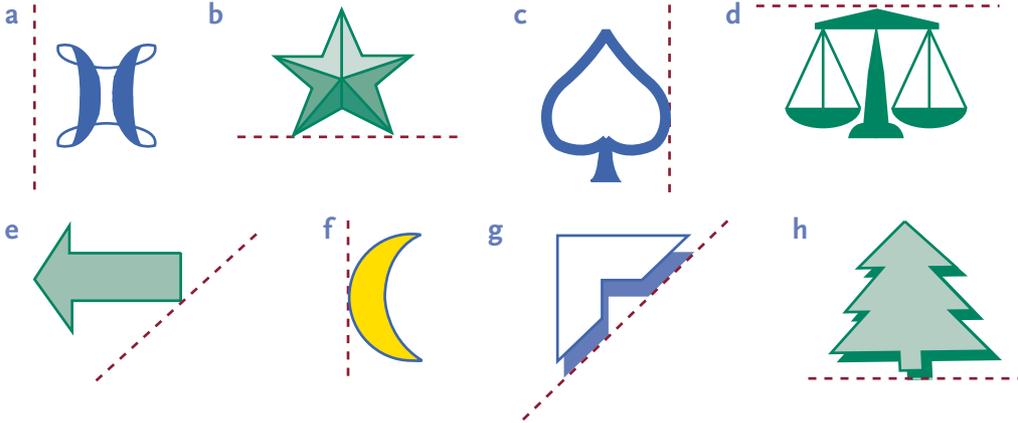
e



f



3 Reflect each shape in the line of reflection or mirror line. You may use tracing paper:



4 Continue this pattern by reflecting it in the line of reflection shown.



5 a For each of the shapes below decide if there is line of symmetry, i.e. is the shape symmetrical? Can the shape be folded to map exactly onto itself?

b Draw in any lines of symmetry.



6 a Redraw the letters in the word 'Monday' into your workbook.

b Which letters have line symmetry?

c Draw in any lines of symmetry.

d Reflect the letters in the word MONDAY in the two mirror lines labelled i and ii.



7 Draw each letter of the alphabet as capital letters.

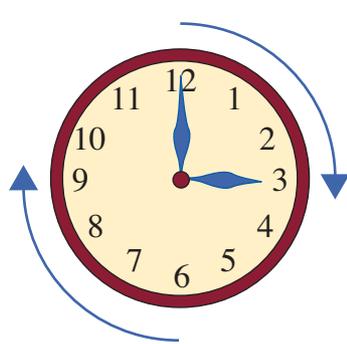
a For each letter, sketch any line of symmetry.

b List all the capital letters according to their order of symmetry, i.e. group those with no lines of symmetry, 1 line, 2 lines ... and so on.

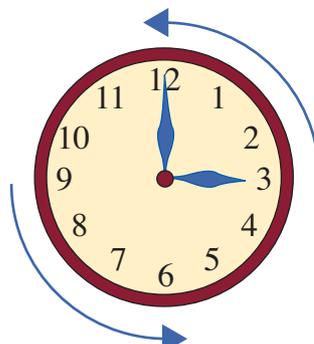
7D

Rotation

Shapes can also be transformed by **rotation** or a 'turn'. Each rotation has a direction and an angle or amount of turn. A rotation is about a fixed point. Direction is either clockwise or anticlockwise. The angle of rotation is measured in degrees. Remember to write the angle and the direction of the rotation. The shape and size of the object is unchanged.



Clockwise



Anticlockwise

Exercise 7D

1 Copy the symbols below, then rotate in the direction given about the fixed point shown:

- a 180° clockwise b 90° clockwise c 90° anticlockwise d 180° clockwise

A

B

C

D

- e 90° clockwise f 180° clockwise g 90° clockwise h 180° anticlockwise

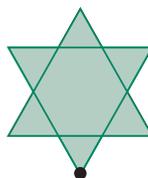


2 Copy the shapes and then rotate them in the direction indicated about the fixed point:

a

 90° clockwise

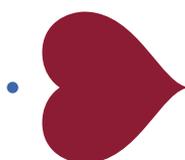
b

 90° anticlockwise

c

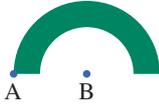
 180° clockwise

d

 180° anticlockwise

3 We can create some interesting shapes using rotations. Rotate each of the following shapes as instructed. Draw both the original object and its image to see the pattern created:

a



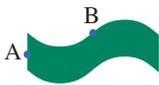
Rotate 180° about A and B

b



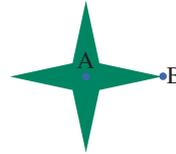
Rotate at 90° intervals clockwise about A for a full circle

c



Rotate 180° about A and B.

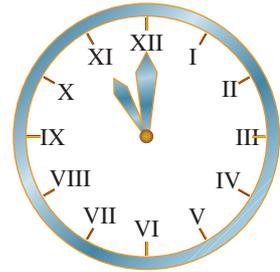
d



Rotate at 45° intervals clockwise about A for a full circle then rotate at 90° intervals about B for a full circle

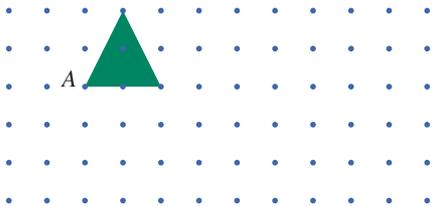
4 The face of a clock is shown below. Each hand undergoes rotation about the centre of the clockface. State the rotation required when moving the minute hand clockwise from:

- | | | |
|--------------|--------------|-------------|
| a XII to I | b XII to II | c I to III |
| d II to I | e III to VII | f VI to I |
| g VII to XII | h III to I | i XII to IX |
| j II to IX | k I to VI | l X to V |



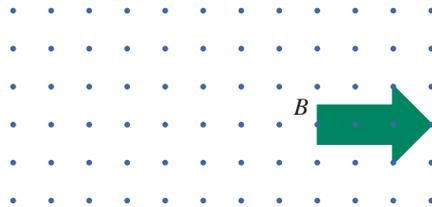
5 Copy the grids below then draw the final image after the transformations:

a



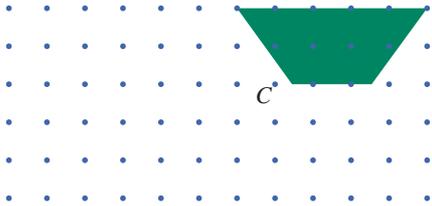
Rotate 90° anticlockwise about A, translate 3R then 2D

b



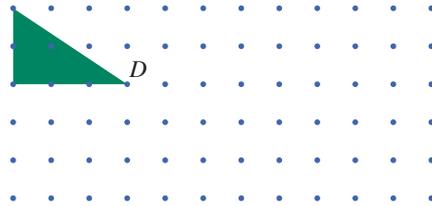
Rotate 180° clockwise about B, translate 1L then 3U

c



Rotate 180° anticlockwise about C then translate 2U and 3L, and then reflect in the longest side

d

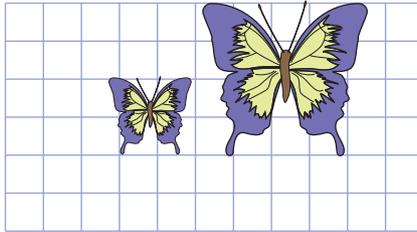


Rotate 180° anticlockwise about D and reflect in the hypotenuse, then translate 5R then 3U

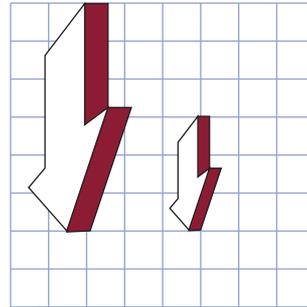
When we increase the size of an object it is called an **enlargement**. When we decrease the size of an object it is called a **reduction**. When all dimensions are changed by the same amount, the shape of the object remains the same. Two objects with the same shape, but of different size are said to be **similar**.

Similar shapes

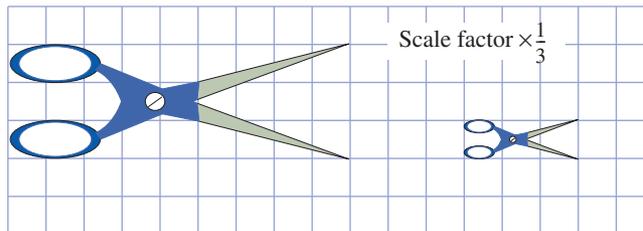
These pairs of shapes are similar. All corresponding side lengths have been enlarged or reduced by the same amount. This is called the scale factor.



Scale factor $\times 2$



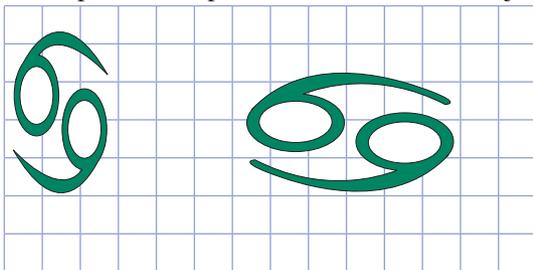
Scale factor $\times \frac{1}{2}$



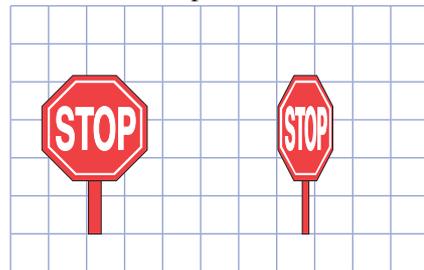
Scale factor $\times \frac{1}{3}$

Not similar shapes

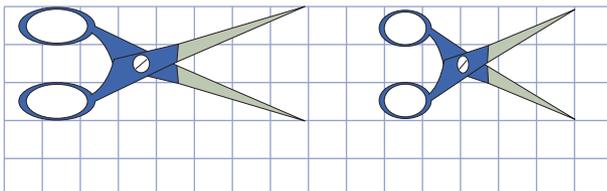
These pairs of shapes are not similar. The objects are not the same shape.



The length has been increased more than the height. The object has been stretched more in one dimension.



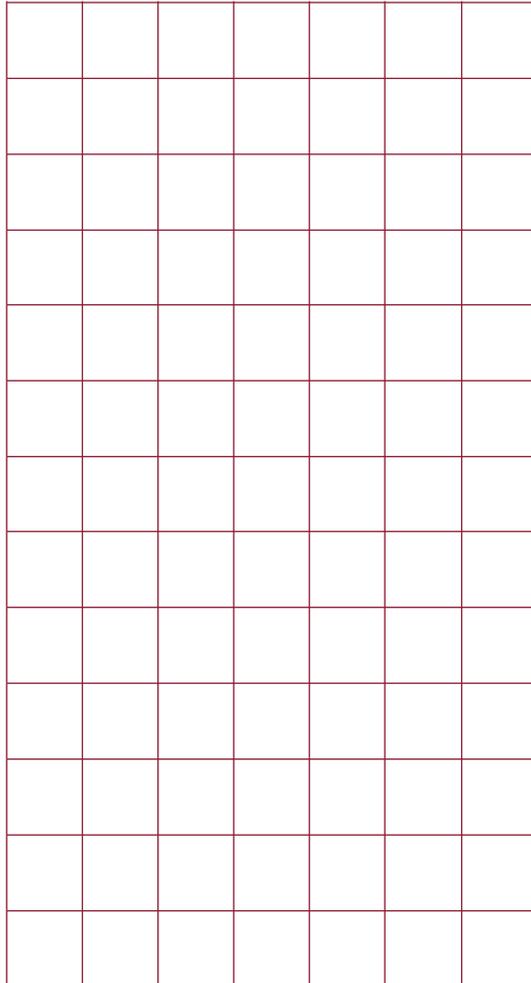
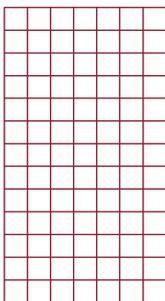
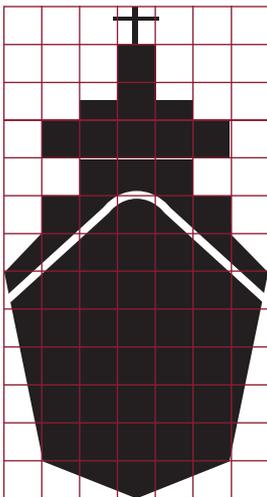
The height of the object remains unchanged.



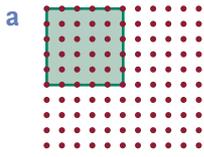
The length has been decreased while the height has not changed.

Exercise 7E

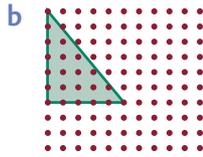
- a** Enlarge the picture onto the larger grid. Carefully mark on the larger grid where each line crosses a grid line. Your final picture should be similar, i.e. the same shape as, but larger than the original picture.
- b** How many times bigger is the enlargement? Hint: Measure the length of the squares in the grids.
- c** Reduce the picture onto the small grid. Your final picture should be similar, i.e. the same shape as, but smaller than the original picture.
- d** What is the scale factor for the reduced picture compared to the original?
- e** Enlarge or reduce a picture of your own. What is its scale factor?



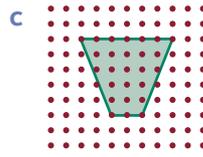
2 Enlarge each of the following shapes by the scale factor given:



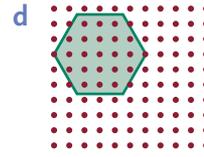
Scale factor of 2



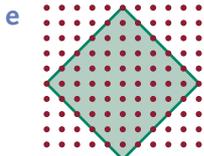
Scale factor of 2



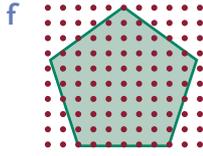
Scale factor of 3



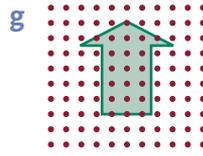
Scale factor of 3



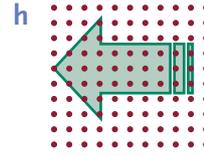
Scale factor of 4



Scale factor of 4

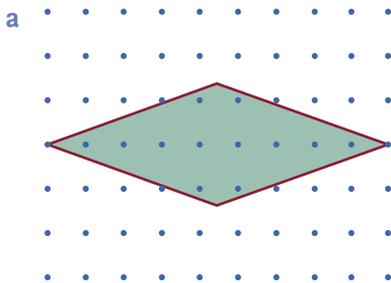


Scale factor of 2

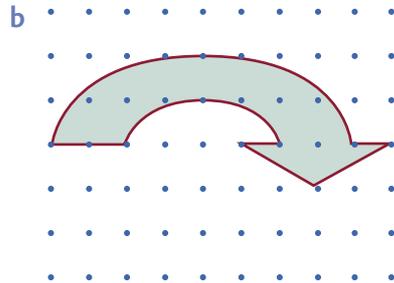


Scale factor of 2

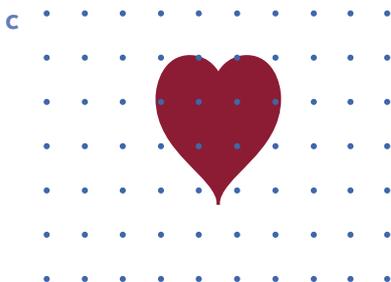
3 Reduce each of the following shapes by the scale factor given:



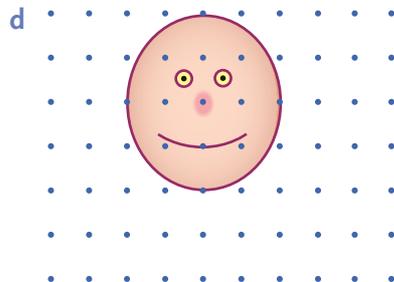
Scale factor of $\frac{1}{2}$



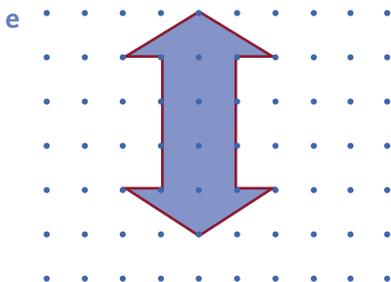
Scale factor of $\frac{1}{2}$



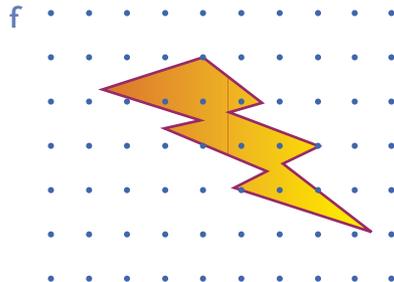
Scale factor of $\frac{1}{3}$



Scale factor of $\frac{1}{3}$



Scale factor of $\frac{1}{4}$

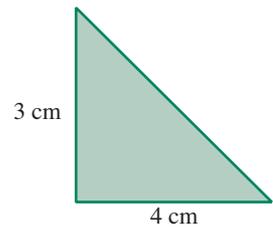


Scale factor of $\frac{1}{4}$

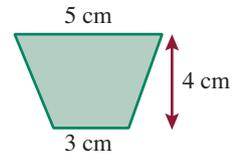


Learning task 7F

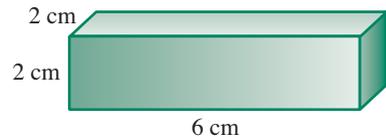
- 1 a Draw a triangle with dimensions as shown.
- b Find its area.
- c Enlarge it by a scale factor of 3 and find the area of the image.
- d How much bigger is the area of the image than that of the object?
- e By what factor has the area of the object changed?



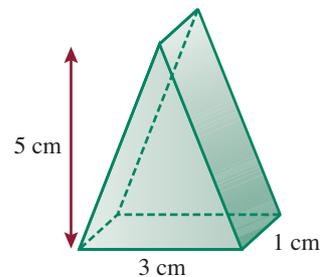
- 2 a Draw a trapezium with the dimensions as shown.
- b Find its area.
- c Enlarge it by a scale factor of 3 and find the area of the image.
- d How much bigger is the area of the image than that of the object?
- e By what factor has the area of the object changed?
- f What is the relationship between the scale factors for length and area?



- 3 a Draw a rectangular prism with the dimensions as shown.
- b Find the volume of the prism.
- c Enlarge it by a scale factor of 2 and find the volume of the image.
- d How much bigger is the volume of the image than that of the object?
- e By what factor has the volume of the object changed?



- 4 a Draw the triangular prism using the dimensions shown.
- b Find its volume.
- c Enlarge it by a scale factor of 2 and find the volume of the image.
- d How much bigger, in volume, is the image than the object?
- e By what factor has the volume of the object changed?
- f Describe the relationship between the scale factor for length and volume.

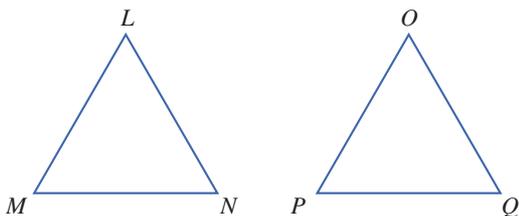


- 5 If a rectangle that has an area of 3 cm^2 is enlarged by a factor of 2, what is the area of the image in cm^2 ?
- 6 If a circle that has an area of 4 cm^2 is enlarged by a factor of 3, what is the area of the image in cm^2 ?
- 7 If a prism that has a volume of 4 cm^3 is enlarged by a factor of 2, what is the volume of the image in cm^3 ?
- 8 If a sphere that has a volume of 24 cm^3 is reduced by a factor of 2, what is the volume of the image in cm^3 ?
- 9 If a shape is enlarged by a factor of k , by how much is the area increased, in terms of k ?
- 10 If a solid is enlarged by a factor of k , by how much is the volume increased, in terms of k ?

Two shapes are **congruent** if they have the same length sides and the same size angles, that is, they are identical. If you can use a series of transformations to map one object onto the other, then the two shapes are congruent.

Example

Is triangle LMN congruent with triangle OPQ ?



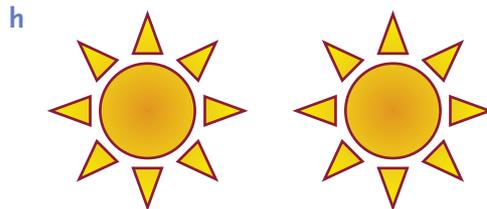
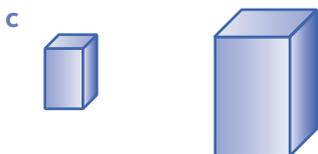
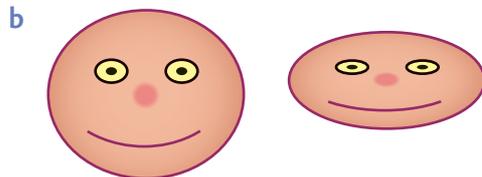
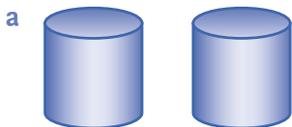
Solution

Triangle LMN can be transformed so that it is the same as triangle OPQ .

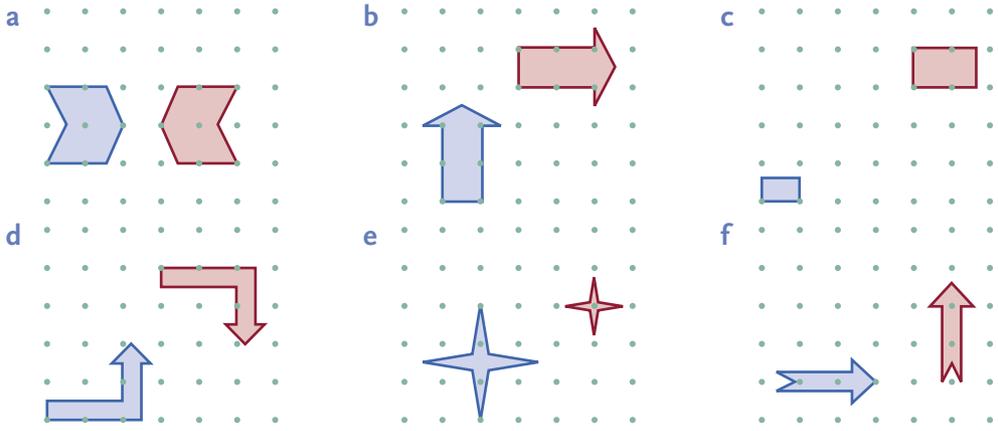
We can say that the two triangles are congruent and we write $\triangle LMN \cong \triangle OPQ$.

Exercise 7G

- 1 Which of the following pairs of shapes are congruent; that is, which one can be mapped exactly onto the other?



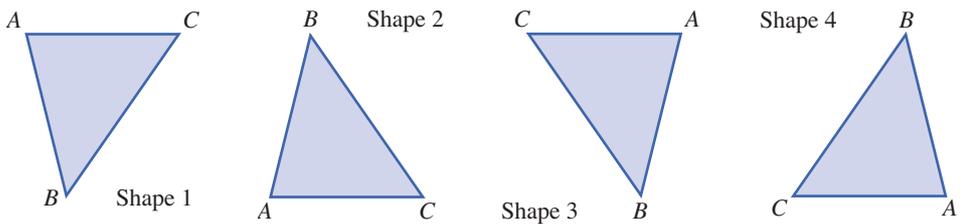
2 Describe how the blue shape has been translated, reflected, rotated, enlarged or reduced to obtain its red image.



3 a Copy the table below into your workbook.

	Shape 1	Shape 2	Shape 3	Shape 4
$\angle A$				
$\angle B$				
$\angle C$				
\overline{AC}				
\overline{BC}				
\overline{AB}				

b For the triangles shown below, measure the size of each angle (in degrees) and the length of each side (in mm). Use your results to help you complete the table.



- Write a sentence describing your observations about the corresponding lengths in each shape.
- Write a sentence describing your observations about the corresponding angles in each shape.
- Use your table to make a list of the triangles that are congruent.

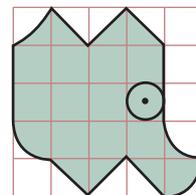


7H Exploring tessellations

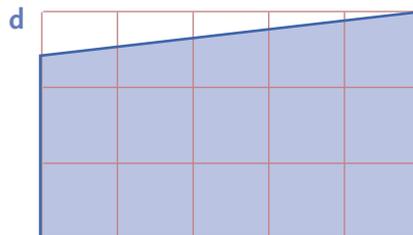
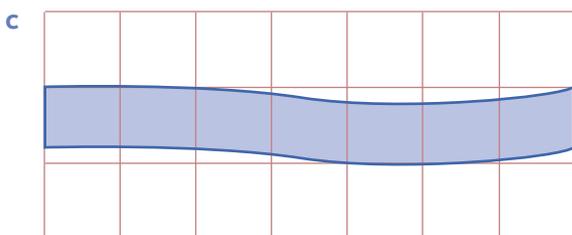
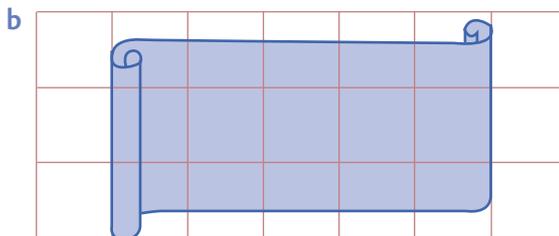
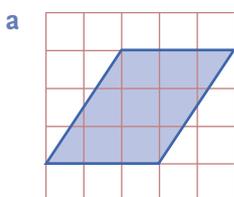
- 1 On grid paper draw a shape using squares, which has an area of five squares. Label your shape N .
 - a Translate your shape once. Label the image N' .
 - b Translate the image N' . Label the image N'' .
 - c Write a set of instructions for drawing your shape and its images after translation. Make sure you include the direction and the number of units.
 - d Give your instructions to your partner so that your partner can draw your shape and its images. Compare your partner's shapes with your drawings.
 - e Discuss any differences with your partner to try to find a reason for them.

- 2 Copy this shape onto grid paper to produce a tessellation.

- a Translate the shape four units right. Repeat until you have completed the row.
- b Translate the shape four units down. Repeat until you have finished the column.
- c Keep translating across, down and up until the page is filled.
- d Explain why translating the shape filled the whole page.



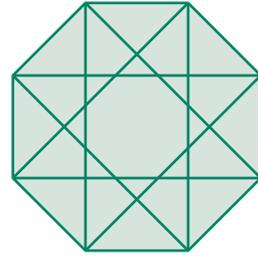
- 3 For each of these shapes, find how many units across it would have to be translated so that it tessellates. How many units down? Do all of the shapes tessellate? Explain your answer.



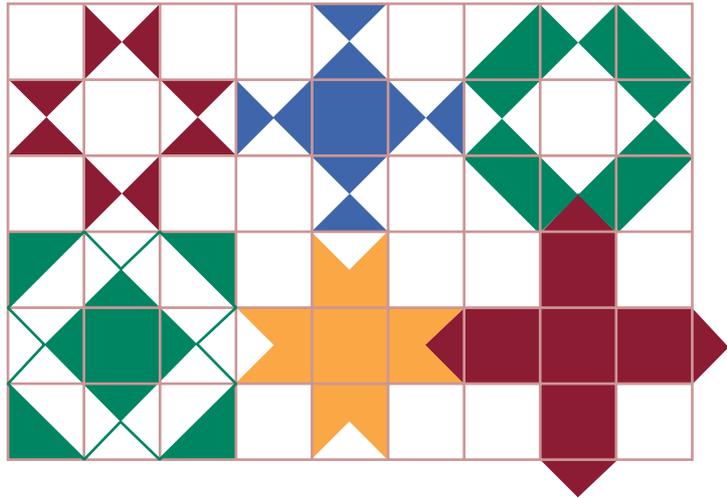
- 4 Copy each of the shapes above onto 1 cm grid paper. Show that the shapes tessellate by drawing at least three rows of pattern. Colour your shapes to show an interesting pattern.
- 5 After a translation, is the shape congruent to its image? Explain your answer.
- 6 After a reflection, is the shape congruent to its image? Explain your answer.

7 A tessellation can be produced by translating an octagon.

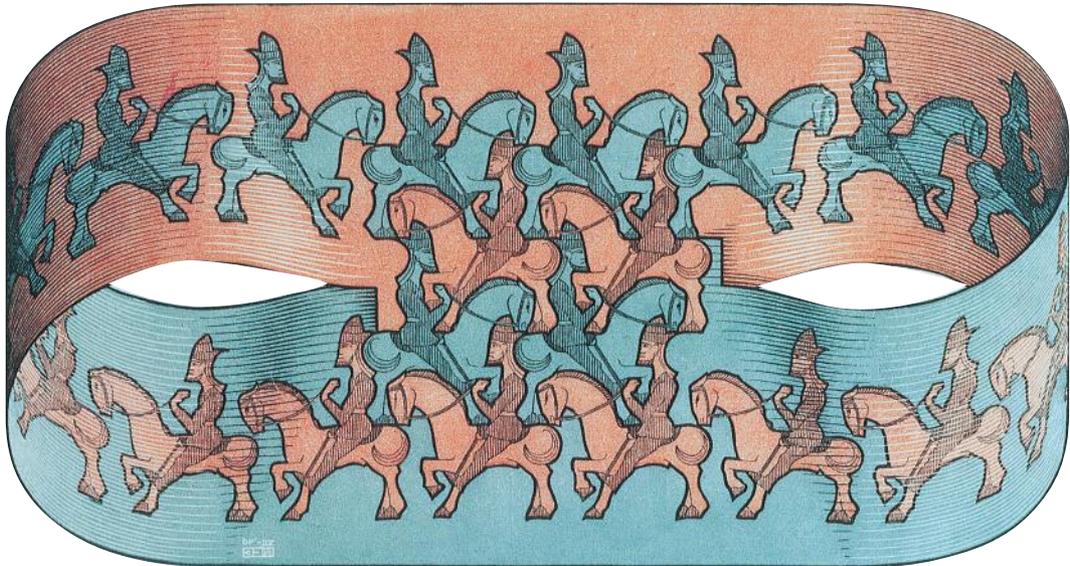
- a** Using isometric grid paper, draw a series of octagons as shown. Translate the shapes to produce a tessellation of 3 across and 3 down.



- b** Colour in the shapes to create a design.
c Experiment with different ways to colour in the shapes to give different effects.



- 8 a** Study the Escher artwork below.
b Describe the translations used to produce the tessellations.
c Design your own tessellation using translations.



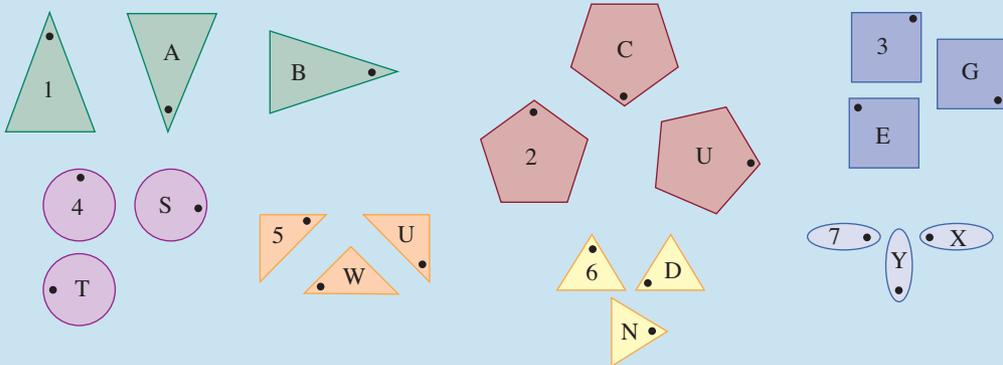
M. C. Escher's *Horsemen* © 2002 Cordon Art B.V. – Baarn – Holland. All rights reserved.



Puzzles

- 1 Match the number of each shape to the letter of the shape that has been rotated by 90° clockwise. Use the letters to solve the riddle:

What do you call a rabbit that is owned by a beetle?



1	2	3	4	5	6	6	7		

- 2 Match the letter and the translated number to find the names of the explorers who travelled to Cape York in Australia, in the 1800s. Each letter has been translated 2 units to the right and 3 units up from its matching number.

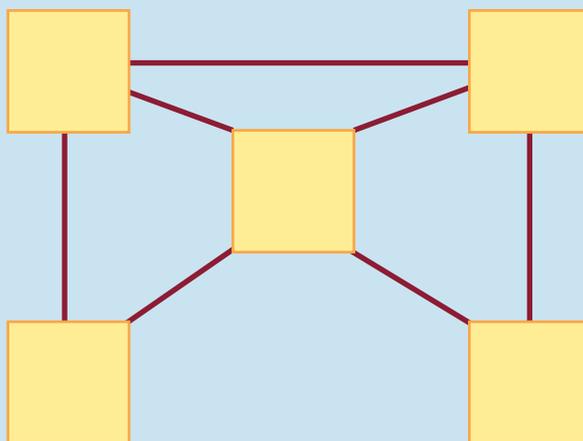
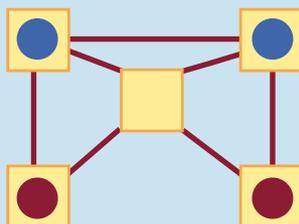
			O					S	
						E			
		D							B
	9					12			
				7				N	
4			A		I		8		T
		J		H			R		
						6			
	2		5				10		
1		11			3				

1	2	3	4	5	6	7	8	3	9	10	11	7	3	12						

3 To play this strategy game you will need two blue and two red counters.

The counters are moved by sliding along the lines from one box to another. Move one space at a time. Take turns to move. You win if you block your opponent's counters.

Start like this.



Play so that you can work out a strategy to win.

Can the first player always win?

You have finished when you have worked out how one colour can always win.

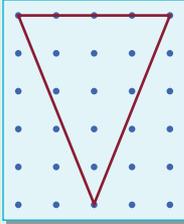




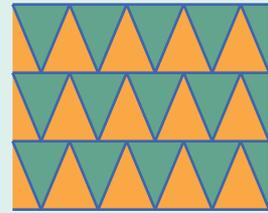
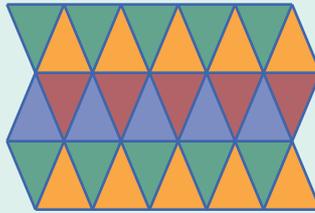
Applications

Tessellations

- 1 M. C. Escher based many of his drawings on tessellations of a triangle.

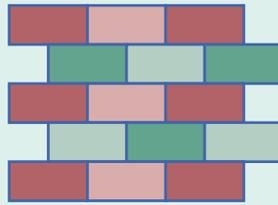
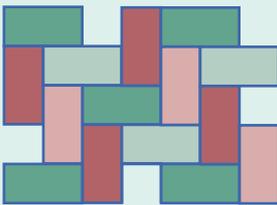


On 1 cm grid paper join five dots and construct an isosceles triangle. Cut out your triangle. Trace around your triangle on dot paper to make a pattern that covers half the page leaving no spaces. An example is shown below. This is called **tessellation**. We say that this triangle tessellates. Can your triangle tessellate in another pattern?



Repeat using a scalene triangle of your choice. Remember that scalene triangles have three different side lengths. Colour your tessellation to show the pattern. Do all triangles tessellate? Discuss this with your partner and write a short explanation.

- 2 We can tessellate a rectangle because the angles fit together well in a number of different patterns.



On 1 cm dot paper draw a quadrilateral that has four different side lengths.

Investigate until you find a way of tessellating your quadrilateral. You are allowed to flip or turn your shape as well as slide it. Can you find another way to tessellate your quadrilateral? Parallelograms and rhombi tessellate in several ways. Quadrilaterals with four different side lengths are not as easy to tessellate. Do all quadrilaterals tessellate? Discuss this with your partner and write a short explanation.

- 3 Use the library to research the topic of symmetry. Write a report on your findings and use diagrams to illustrate your ideas.
- 4 a Use the library or the Internet to research the topic of tessellations in art. Include tessellations of a variety of shapes. Make a note of the artist and the year the artwork was completed. Display this artwork on A3 paper and give to your teacher to display in your classroom.
- b Investigate a range of shapes to find whether they tessellate. Do hexagons or octagons tessellate?

Border patterns

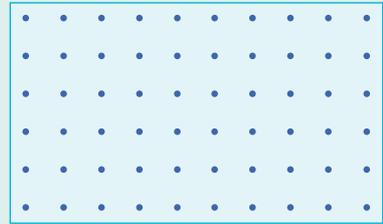
Copy and continue the border pattern.



From the instructions given, draw the border pattern.

Start at a dot at the top left corner. Follow these directions in order:

- Down 4, right 4, up 3, left 2, down 1, right 1, down 1, left 2, up 3, right 4, repeat.
- Continue the pattern across the page.
Colour the pattern to show its repeated nature.



- a Design a border pattern of your own on 5 mm dot paper or grid paper.
- b Write instructions for your pattern using direction language such as up, down, left and right.
- c Give your instructions to your partner and ask your partner to draw your pattern. Compare your partner's pattern to your design.

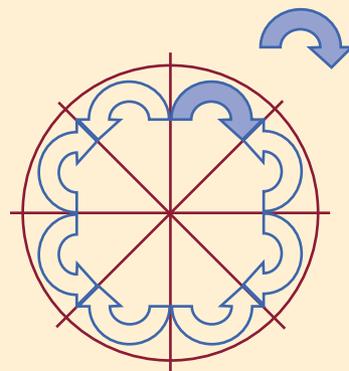
Cartoons

- a Cut out one of your favourite cartoons.
- b Draw a suitable grid over the top of the cartoon.
- c On a large sheet of poster paper, draw a grid using a scale factor of 5.
- d Copy the cartoon onto the enlarged grid on the poster paper.
- e Colour in the cartoon and decorate your classroom.



Enrichment

- 1 Kaleidoscopes can be made by reflecting a picture or shape about a number of axes in a circle.
 - a Draw a large circle and divide into 8 equal sectors.
 - b Choose a simple design and draw it in one of the sectors.
 - c Reflect the shape in each of the lines.
 - d Choose a more complex design and try dividing your circle into more than 8 sectors.



- 2 If we enlarge a picture using different scale factors for the horizontal and vertical directions, the picture becomes distorted. This is called a dilation.

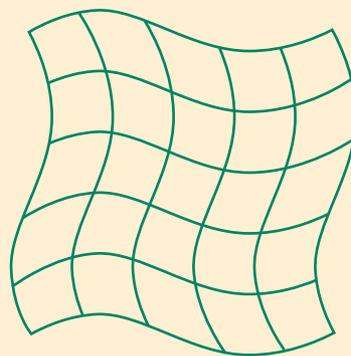
The picture below has been dilated by a scale factor of 2 in the horizontal direction.



Now the picture has been dilated by a factor of one-half in the vertical direction.



- a Collect some of your favourite cartoons and use different scale factors to distort the pictures.
- b What happens if you use grids with wavy lines?



- 3 Copy the shapes below and dilate them by the scale factor shown:

a



Scale factor of 5 in the vertical direction

b



Scale factor of 5 in the horizontal direction

c



Scale factor of 4 in the vertical direction

d



Scale factor of 4 in the horizontal direction

e



Scale factor of $\frac{1}{2}$ in the vertical direction

f



Scale factor of $\frac{1}{3}$ in the horizontal direction

- 4 MAM has a line of symmetry. Make a list of other words that have line symmetry:

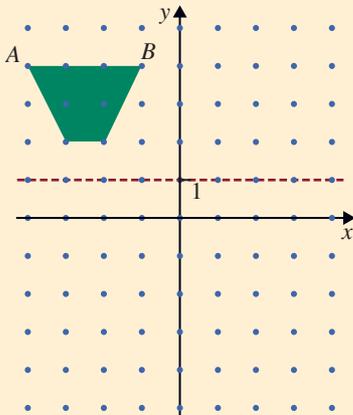


- 5 MOW has rotational symmetry. Make a list of other words that have rotation symmetry:



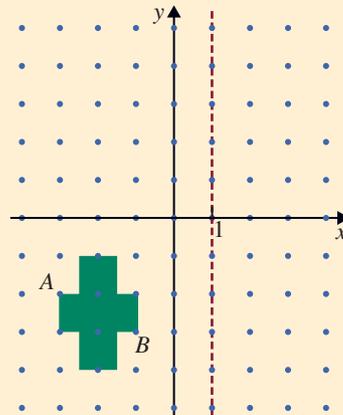
- 6 i Transform each of the following objects by using the directions below.
 ii Write down the coordinates of points A and B.
 iii Write down the coordinates of the images of points A and B after the specified transformation.

a



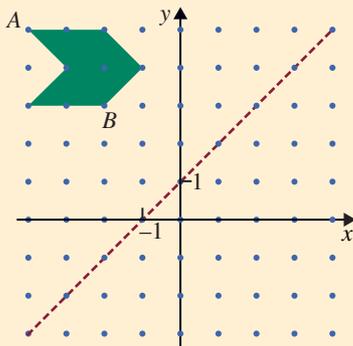
Translate 3R then 2D then reflect in the dotted line

b



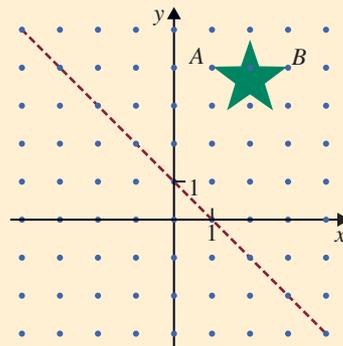
Translate 4R then 6U then reflect in the dotted line

c



Reflect in the dotted line then translate 2L then 3U

d



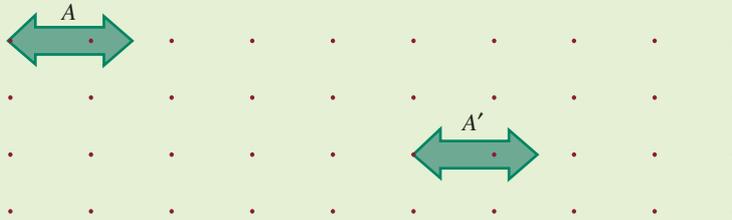
Reflect in the dotted line then translate 3R then 2U



Revision/Assessment

Exercise 7B

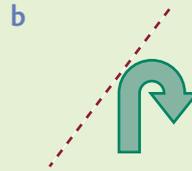
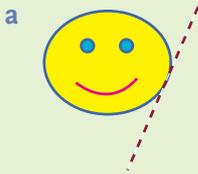
- 1 Describe the translation of A using direction and distance.



- 2 Triangle NMO is translated left 3 units and down 2 units. Draw a diagram to show this. Label as shape and image.

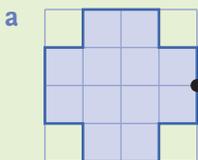
Exercise 7C

- 3 When a shape has been reflected, the image is the same size as the original shape. True or false? Explain.
- 4 Copy the shapes into your workbook then reflect in the mirror line shown. You may use tracing paper.

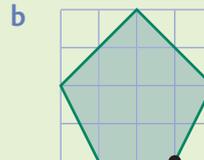


Exercise 7D

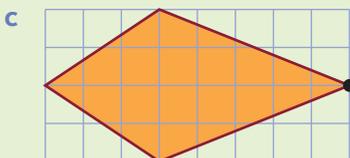
- 5 If you have transformed a shape 60° clockwise, what transformation have you used? Explain with a diagram.
- 6 Rotate these shapes about the point of rotation and the angle indicated.



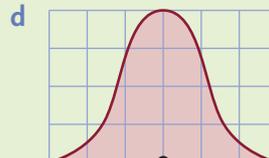
180° clockwise



180° anticlockwise



90° clockwise

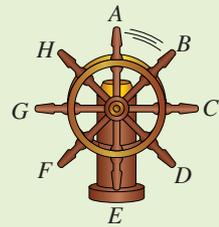


180° clockwise

- 7 The wheel at the helm of a boat is shown here. Each spoke undergoes rotation about the centre of the wheel.

State the rotation required to move from:

- a A to B b A to C c B to E d C to B
 e E to F f G to B g H to D h E to C



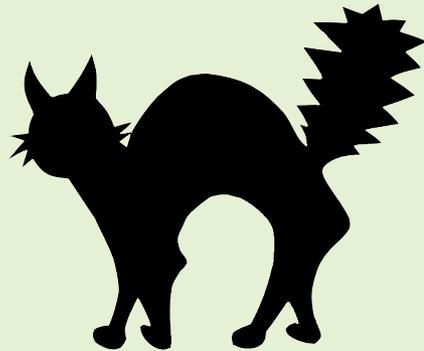
Exercise 7E

- 8 Copy the shapes into your workbook on a 5 mm grid. Redraw the shape onto a 1 cm grid to enlarge by a factor of 2.

a



b



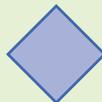
Exercise 7G

- 9 Explain the meaning of the term 'congruence'.
 10 Draw two congruent triangles.
 11 Draw two quadrilaterals that are congruent.
 12 Draw two quadrilaterals that are not congruent and explain why.
 13 Pick the pair of congruent quadrilaterals from the ones shown below:

a



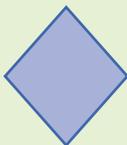
b



c



d



e



f



- 14 If all the angles in one quadrilateral are the same size and in the same order as those in another quadrilateral, does that mean that the two quadrilaterals are congruent? Explain your answer.

Answers

These are selected answers only. A set of Fully Worked Solutions can be found in the Teacher's Resource.

Chapter 1

Exercise 1A

- 1** a 16% b 27% c 3% d 87%
 e 41% f 77% g 94% h 35%
 i 19% j 0% k 55% l 100%
 m 24% n 58% o 60% p 49%

- 2** a 9%, 14%, 28%, 41%, 93%
 b 3%, 24%, 49%, 67%, 91%
 c 13%, 27%, 32%, 69%, 98%
 d 12%, 25%, 28%, 63%, 99%
 e 9.5%, 9.56%, 23.8%, 94.6%
 f 5.67%, 8.49%, 56.7%, 84.9%
 g $7\frac{2}{5}\%$, $27\frac{2}{3}\%$, $57\frac{1}{7}\%$, $57\frac{1}{2}\%$
 h $3\frac{4}{9}\%$, $13\frac{1}{4}\%$, $30\frac{1}{3}\%$, $93\frac{1}{4}\%$
 i $2\frac{1}{6}\%$, $6\frac{1}{2}\%$, $26\frac{1}{6}\%$, $26\frac{1}{4}\%$
 j $40\frac{1}{10}\%$, $40\frac{1}{3}\%$, $42\frac{1}{2}\%$, $44\frac{1}{4}\%$

- 3** i a 50% b 20% c 60%
 d 5% e 37% f 99%
 g 42% h 83% i 75%
 ii a 14% b 25% c 8%
 d 24% e 47% f 1%
 g 10% h 3% i 9%
 iii a 0% b 24% c 12%
 d 24% e 16% f 0%
 g 18% h 4% i 12%

4	Fraction	Decimal	Percentage
	$\frac{4}{10}$	0.4	40%
	$\frac{8}{10}$	0.8	80%
	$\frac{3}{10}$	0.3	30%
	$\frac{7}{10}$	0.7	70%
	$\frac{5}{10}$	0.5	50%
	$\frac{3}{5}$	0.6	60%
	$\frac{1}{10}$	0.1	10%

5	Fraction	Decimal	Percentage
	$\frac{1}{3}$	0. $\dot{3}$	$33\frac{1}{3}\%$
	$\frac{2}{3}$	0. $\dot{6}$	$66\frac{2}{3}\%$
	$\frac{3}{4}$	0.75	75%
	$\frac{5}{100}$	0.05	5%
	$\frac{2}{10}$	0.2	20%
	$\frac{1}{8}$	0.125	12.5%
	$\frac{9}{10}$	0.9	90%

- 6** a i 35% ii $\frac{7}{20}$ iii 0.35

- b i 75% ii $\frac{3}{4}$ iii 0.75

- c i 90% ii $\frac{9}{10}$ iii 0.9

- d i 80% ii $\frac{8}{10}$ iii 0.8

- e i 30% ii $\frac{3}{10}$ iii 0.3

- f i 10% ii $\frac{1}{10}$ iii 0.1

- 7** a 64%, 38%, 49%, 83% b Maddie
 c Michael d Maddie

- 8** 88% **9** 36% **10** 53% **11** 66%

- 12** a 49% b Soccer, tennis, squash, hockey

Exercise 1B

- 1** a $\frac{23}{100}$ b $\frac{51}{100}$ c $\frac{9}{100}$ d $\frac{1}{100}$

- e $\frac{43}{50}$ f 1 g $\frac{7}{50}$ h $\frac{8}{25}$

- i $1\frac{3}{5}$ j 4 k $\frac{1}{5}$ l $\frac{19}{20}$

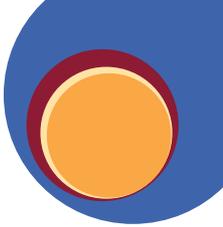
- m $3\frac{3}{5}$ n $\frac{33}{50}$ o $7\frac{1}{2}$ p $\frac{3}{25}$

- q $\frac{5}{4}$ r $\frac{1}{50}$

- 2** a $\frac{31}{200}$ b $\frac{17}{40}$ c $\frac{241}{200}$

- d $\frac{601}{200}$ e $\frac{41}{400}$ f $\frac{67}{400}$

- g $\frac{2}{3}$ h $\frac{19}{500}$ i $\frac{1}{400}$



- j $\frac{1}{200}$ k $\frac{3}{400}$ l $\frac{1}{300}$
 m $\frac{9}{200}$ n $\frac{83}{400}$ o $\frac{13}{125}$
 p $\frac{1}{125}$ q $\frac{1701}{800}$ r $\frac{59}{70}$

3 a $\frac{1}{4}$ b 25% c $\frac{3}{4}$ d 75%

4 a $\frac{21}{25}$ b 16%

5 20%

6 $0.125 \times 200 \text{ g} = 25 \text{ g}$ $\frac{25 \text{ g}}{200 \text{ g}} = \frac{1}{8}$

Exercise 1C

- 1 a 25% b 40% c 70%
 d 75% e 50% f 66.6%
 g 12.5% h 83.3% i 33.3%
 j 30%
 2 a 220% b 450% c 475%
 d 115% e 516.6% f 280%
 g 714.29% h 333.3% i 237.5%
 j 122.2%
 3 a 60% b 40%

4 $\frac{1}{8} \times 100\% = 12.5\%$

5 Seedlings left to plant = $60 - 36 = 24$ or 2 trays to plant
 $\frac{24}{60} \times 100\% = 40\%$ $\frac{2}{5} \times 100\% = 40\%$

6 a $\frac{29}{100}$ b $\frac{6}{25}$ c 26.5% d $\frac{147}{200}$

Exercise 1D

- 1 a 0.4 b 0.32 c 0.17 d 0.24
 e 0.55 f 0.80 g 0.29 h 0.67
 i 0.64 j 0.86 k 0.03 l 0.78
 2 a 0.45 b 0.31 c 0.05 d 0.01
 e 0.97 f 1.00 g 0.16 h 0.50
 i 1.30 j 4.20 k 1.70 l 3.40
 m 4.26 n 2.25 o 8.50 p 5.42
 q 3.06 r 1.27
 3 a 0.125 b 0.065 c 3.025
 d 1.405 e 0.1325 f 0.0575
 g 0.162 h 0.088 i 0.005
 j 0.0025 k 0.004 l 0.0075
 4 a 0.62 b 0.62 litres

- 5 0.23 litres 6 0.0675
 7 a 0.1665 b 0.0004561 c 2

Exercise 1E

- 1 a 75% b 50% c 30% d 55%
 e 92% f 63% g 41% h 70%
 i 66% j 2% k 62% l 96%
 m 21.2% n 34.5% o 45.7% p 24%
 q 0.1% r 0.62% s 88.7% t 1.6%
 2 a 288% b 309% c 519%
 d 145% e 1065% f 919%
 g 281% h 420% i 598.7%
 j 509.9% k 300.5% l 700.2%
 m 245.55% n 9087.6% o 56704%
 p 2133.333%

Decimal	Percentage	Decimal	Percentage
0.3	33.3%	0.2	22.2%
0.12	12.12%	0.1206	12.06%
0.01	1.1%	0.6	66.6%
2.4	244.4%	0.93361	93.361%

- 4 87% 5 5% 6 33.3% 7 68.75%

Exercise 1F

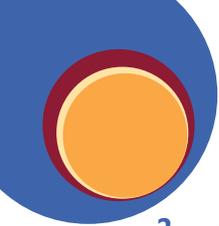
- 1 a 14 b 24.5 c 42 d 258
 e 23 f 60 g 49 h 57.6
 i 360 j 51 k 45 l 8
 2 a 10 b 49 c 60 d 3
 e 28.5 f 201 g 7.04 h 6.75
 i 13 j 51 k 363 l 162

Player	Percentage of wickets per over	Total number of overs bowled	Total number of wickets
Katy	10%	120	10% of 120 = 12
Lencia	15%	560	84
Wasi	25%	344	86
Kimo	3.75%	320	12

- b Wasi c Katy and Kimo

Exercise 1G

- 1 a 50% b 20% c 25% d 10%
 e 8.33% f 12.5% g 20% h 25%
 i 1% j 5% k 40% l 5%



- 2 a 5% b 25% c 45-45%
 d 10% e 40% f 13-33%
 g 27-78% h 1% i 12-5%
 j 2-78% k 4% l 0-009%
 m 0-14% n 0-8% o 12-5%
- 3 90%
- 4 a 78% b 75% c Maths
- 5 28% 6 20% 7 12-5%

Exercise 1H

- 1 a 50% b 100% c 60%
 d 30% e 90% f 136%
- 2 a 50% b 75% c 30%
 d 60% e 86-67% f 98-33%
- 3 16-67% 4 62-5% 5 10%
- 6 33-33% 7 300% 8 4%

Learning task 1I

Nutrient	Mass per 100 g serve	Percentage	Mass per 200 g serve
Protein	21.9 g	21.9%	43.8 g
Fat	0.6 g	0.6%	1.2 g
Carbohydrate	71.7 g	71.7%	143.4 g
Fibre	2.7 g	2.7%	5.4 g
Sodium	0.6 g	0.6%	1.2 g
Potassium	0.15 g	0.15%	0.3 g
Vitamins and minerals	2.35 g	2.35%	4.7 g
Total	100 g	100%	200 g

- c Carbohydrates d Fat and sodium

2 a & b

Nutrient	Un-toasted (g per 1 kg)	%	Toasted (g per 750 g)	%
Carbohydrate	360 g	36%	275 g	36.67%
Protein	130 g	13%	70 g	9.33%
Fat	90 g	9%	150 g	20%
Sugar	220 g	22%	155 g	20.67%
Fibre	70 g	7%	50 g	6.67%
Other	130 g	13%	50 g	6.67%

- c Untoasted, toasted, untoasted
 d Untoasted: less fat and more protein
 e \$0-004 75/g
 f \$0-004 87/g
 j Untoasted muesli

3

Ingredient	Volume (mL per 600 mL)	%	Volume (mL of jumbo size)
Banana	135	22.5%	225
Pawpaw	250	41.67%	416.7
Ice cubes	80	13.33%	133.3
Apple	120	20%	200
Lemon juice	15	2.5%	25
Total	600	100%	1000

4 a & b

Product	Increase	% Increase	Prediction for 2017
Coca-Cola 1.25 L	\$7.00	74%	\$28.70
Coffee 150 g	\$20.20	46.01%	\$93.60
Tomato sauce 600 mL	\$9.00	90.91%	\$36.10
Tea bags 100	\$20.90	91.27%	\$83.80
Cigarettes 200 carton	\$537.10	335.90%	\$3038.20

- c Cigarettes (anti-smoking campaign), then sauce and tea bags (common-use products)

Exercise 1J

- 1 a \$240 b \$300 c \$360 d \$600
 e \$1200 f \$1500 g \$1800 h \$2160
- 2 a 92% b 82.5% c 80% d 72%
 e 45% f 31% g 22% h 20%
- 3 a \$640 b \$280 c \$544.00
 d \$184.00 e \$1088.00 f \$760
- 4 \$650 scooter
- 5 a \$315 b \$252 c \$504 d \$216
- 6 a \$646 b \$1012
 c \$12 276 d \$8277
- 7 a \$364 b \$52.50 c \$36 d \$648
- 8 \$892.50
- 9 a \$2061 b \$2331 c The car seat
- 10 a \$31 570 b \$5810 c \$37 380

Learning task 1K

- 1 a \$7.70 b \$3.75
 c \$6.00 + \$11.25
- 2 a \$32 + \$40 b \$6, \$8, \$12 + \$16
- 3 a \$1403 b \$3427



2

	Fraction	Decimal	Percentage
a	$\frac{9}{10}$	0.9	90%
b	$\frac{1}{3}$	0. $\dot{3}$	$33\frac{1}{3}\%$
c	$\frac{12}{25}$	0.48	48%
d	$\frac{1}{6}$	0.1 $\dot{6}$	16. $\dot{6}\%$
e	$\frac{2}{3}$	0. $\dot{6}$	$66\frac{2}{3}\%$
f	$\frac{2}{9}$	0.2 $\dot{2}$	22.2%

3 18%

4 a $\frac{22}{25}$ **b** 0.88

5 5% **6** 35%

7 a $\frac{11}{25}$ **b** 3 **c** $\frac{1}{4}$ **d** $\frac{3}{20}$

e $\frac{1}{8}$ **f** $\frac{1}{3}$ **g** $\frac{1}{250}$ **h** $\frac{11}{200}$

8 a 90% **b** 55% **c** 2% **d** 55.56%

e 108% **f** 250% **g** 525% **h** 120%

9 a 0.30 **b** 0.44 **c** 0.81 **d** 0.07

e 2.05 **f** 1.60 **g** 0.009 **h** 0.125

10 a 77% **b** 20% **c** 89% **d** 6%

e 210% **f** 155% **g** 300.8% **h** 499.9%

11 a 48 **b** 9 **c** 48 **d** 70

e 5 **f** 3 **g** 30.5 **h** 93

12 133 **13** 93.3%

14 a 10% **b** 10% **c** 20% **d** 8.33%

e 20% **f** 2.5% **g** 2% **h** 12%

i 1.25%

15 110% **16** 35.90% **17** \$590.75 **18** \$524.63

Chapter 2

Exercise 2A

1 a 10 m **b** 250 cm **c** 2 m **d** 7.2 m

2 A 6.5 cm, B 3.4 cm, C 13.2 cm, D 13.2 cm, E 4.7 cm, F 8.2 cm, G 4.4 cm

3 a 8 cm **b** 54 cm **c** 45 cm **d** 60 cm

e 36 cm **f** 27 cm **g** 6 cm

Exercise 2B

1 a **i** 16 mm **ii** 28 mm

b **i** 17 mm **ii** 7 mm **iii** 15 mm

iv 6 mm **v** 18 mm

c **i** 28 mm **ii** 13 mm **iii** 38 mm

iv 9 mm

d **i** 21 mm **ii** 7 mm **iii** 33 mms

2 a 123 mm **b** 170 mm **c** 150 mm **d** 128 mm

3 a **i** 16 mm **ii** 5 mm **iii** 26 mm

iv 9 mm **v** 5 mm **vi** 7 mm

vii 7 mm **viii** 7 mm **ix** 12 mm

x 15 mm **xi** 17 mm **xii** 10 mm

xiii 8 mm **xiv** 16 mm

b **i** 18 mm **ii** 15 mm **iii** 10 mm

iv 26 mm **v** 4 mm **vi** 25 mm

vii 13 mm **viii** 28 mm **ix** 10 mm

x 46 mm **xi** 27 mm

Exercise 2C

1 a $\times 100$ **b** $\times 10$ **c** $\times 1000$

d $\div 10$ **e** $\div 100$ **f** $\div 1000$

2 a 310 m = 31 000 cm = 310 000 mm

b 0.000 063 km = 0.063 m = 6.3 cm = 63 mm

c 0.0149 km = 14.9 m = 1490 cm = 14 900 mm

d 0.008 89 km = 8.89 m = 889 cm = 8890 mm

e 2.56 km = 2560 m = 256 000 cm = 2 560 000 mm

f 9.709 km = 9709 m = 970 900 cm = 9 709 000 mm

3 a 26 000 cm **b** 2 900 000 m

c 90 cm **d** 0.58 km

e 96 cm **f** 7 cm

g 900 mm **h** 7.2 m

i 0.9 m **j** 37.5 mm

k 920 cm **l** 52.5 mm

4 a 0.0009 km, 0.224 m, 13 cm, 129 mm

b 34 m, 560 cm, 2418 mm, 0.0004 km

c 67 m, 4501 mm, 423 cm, 0.003 km

d 42.1 m, 0.0078 km, 390 cm, 2490 mm

5 a 10.9 km = 10 900 m

Christopher runs 10 900 m

b Lucinda runs an extra $230 \times 5 = 1150$ m

\therefore She runs $10.9 \text{ km} + 1.15 \text{ km} = 12.05 \text{ km}$

c $1.15 \text{ km} = 115 000 \text{ cm} = 1 150 000 \text{ mm}$

6 a **i** 25.2 m **ii** 2520 cm **iii** 25 200 mm

b **i** 22.319 m **ii** 2231.9 cm **iii** 22 319 mm

Exercise 2D

1 a 81 mm **b** 31 m **c** 80 mm

d 610 km **e** 340 mm **f** 15 cm

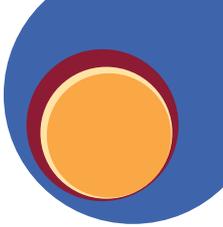
g 113 m **h** 3 mm **i** 293 m

j 28 mm **k** 72 km **l** 187 cm

2 a 858 cm **b** 8290 m **c** 398 cm

d 101 mm **e** 670 mm **f** 102 mm

g 736 m **h** 146 mm **i** 56 789 m



- j 3280 cm k 701 mm l 1363 cm
- 3 a** 7.124 km b 4.87 m c 17 cm
d 5.986 km e 75.3 m f 1.4 cm
g 0.502 km h 85.17 m i 3.2 km
j 81.7 cm k 88.7 km l 1.1 m
- 4 a** Group 1 longest = 4.9 m, shortest = 280 cm
Group 2 longest = 5.9 m, shortest = 570 mm
Group 3 longest = 5280 mm, shortest = 0.0045 km
- b** Group 1 = 210 cm
Group 2 = 533 cm
Group 3 = 78 cm
- 5 a** 120.06 m b 63.06 m
- c** William = 11.54 m
Matt = 14.36 m
Chao = 13.20 m
Ling = 12.51 m
Rian = 10 cm (left foot was further)
Martin = 5.49 cm
- d** i Martin, Matt, Ling, Chao, William, Rian
ii Martin, Rian, Ling, Matt, Chao, William
- 6 a** Length = 1.12 m b Length = 0.6 m
Width = 0.18 m Width = 0.18 m
Height = 0.6 m Height = 0.84 m
- 7 a** Katalulu
b 138 861 cm more than Lui and 205 811 cm more than Semaika.
- 8** Chris: 1 km Kim: 2 km
Alice: 3 km Emma: 6 km

Exercise 2E

- 1 a** 20 cm b 14.7 km c 16 m
d 30 cm e 344 cm f 235 mm
- 2 a** 1018 mm or 101.8 cm
b 6400 mm
- 3 a** $P = 160$ cm
missing lengths are $y = 10$ cm, $x = 25$ cm
b $P = 240$ mm
missing lengths are $y = k = 10$ mm, $z = 40$ mm, $x = 30$ mm
c $P = 142$ m
missing lengths are $x = 24$ m, $y = 18$ m
d $P = 1395$ mm
missing lengths are $x = y = 83$ mm,
 $z = m = k = 125$ mm, $r = n = 98$ mm, $p = 110$ mm
- 4 a** 5.4 m b 256 cm c 4.7 m

Exercise 2F

- 1 a** 192 cm b 240 km c 128 m
d 27.2 cm
- 2 a** 12 km b 168 mm c 72 cm
- 3 a** 316 m b 69.6 m c 25.18 cm
- 4 a** 177.2 cm b 2.736 m c 240.8 m

- 5 a** 120 m b 90 cm

Exercise 2H

- 1 a** 37.70 cm b 43.98 mm c 62.83 m
d 87.96 m e 56.55 m f 402.12 mm
- 2 a** 28 m b 16 m c 15 m
d 58 m e 18 m f 421 m
- 3 a** 79 cm b 60 cm c 330 cm
d 57 cm e 94 cm f 622 cm
- 4 a** 393 mm b 28 mm c 2827 mm
- 5 a** 39 cm b 8 cm c 8.6 cm
- 6** Approximately 900 mm

Exercise 2I

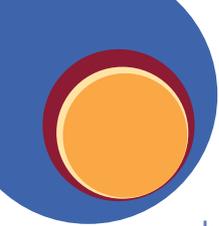
- 1 a** 99.40 m b 49.98 mm c 99.96 cm
d 154.20 m e 39.27 m f 62.80 cm
- 2 a** 128.54 cm b 102.25 m c 90.27 m
d 113.67 cm e 114.25 m f 86.26 cm
g 95.42 m h 318.54 cm i 146.82 mm
j 32.4 cm

Exercise 2J

- 1 a** 12.5 cm b 95 cm c 19 cm
d 23.5 cm e 6.8 cm f 98.9 cm
- 2 a** 11 cm b $6\frac{2}{3}\%$
- 3 a** 12:59-56, 4 s b 1:59-52, 8 s
c 5:29-38, 22 s d 9:14-23, 37 s
e 4:43-53, 67 s
- 4** 14.2, 14.7, 15, 15.5, 15.6 and 20.1 s
- 5 a** 58 g b 87 g c 65 g d 107 g
e 88 g f 100 g g 299 g h 32 g
- 6 a** 653 mm b 422 mm c 352 mm
- 7 a** 74 and 90 cm b 120 and 132 cm
c 345 and 361 cm d 1672 and 1688 cm
- 8 a** 400 and 560 cm b 500 and 700 cm
c 600 and 840 cm d 700 and 980 cm
e 800 and 1120 cm f 900 and 1260 cm
- 9** 227.5 g, 232.5 g; 179.5 g, 184.5 g; 90.5 g, 95.5 g

Enrichment

- 1 a** 81.7 cm b 106.5 m c 86.26 cm
d 102.8 cm e 354.2 cm
- 2** Bill: 628.32 m Voula: 314.16
Bill walks an extra 314 m correct to the nearest metre.
- 3 a** i 18.85 mm ii 61.26 cm
iii 6.28 m iv 78.54 cm
v 53.41 m



- b i 4.71 cm ii 27.49 cm
 iii 1.57 m iv 3.46 m
 v 785.40 m

- 4 a $P = 45$ cm $D = 14.32$ cm
 b $P = 54$ cm $D = 17.19$ m
 c $P = 90$ cm $D = 28.65$ cm
 d $P = 490$ cm $D = 155.97$ cm
 e $P = 36$ cm $D = 11.46$ m
 f $P = 62.5$ mm $D = 19.89$ mm

5 $P = 719.7$ m

6 a 165 mm b 263 mm

7 576 tiles

8 Person A should buy one roll, use 1600 cm and have 400 cm spare.

Person B should buy one roll, and use 560 cm from the 600 cm spare.

Person C should buy 5 rolls, use 9800 cm and have 200 cm spare.

There would therefore be 40 cm of wire left over.

9 a $P = 123.40$ m b $P = 126.25$ m

10 Stacking with base on floor:

12 boxes will fit along the length

16 boxes will fit along the width

16 boxes exactly will fit along the height.

Total number of boxes = $12 \times 16 \times 16 = 3072$
 if stacked this way.

Stacking with base on side:

18 boxes will fit along the length

20 boxes will fit along the width

8 boxes will fit along the height.

Total number of boxes = $18 \times 20 \times 8 = 2880$
 if stacked this way.

Stacking with base on the front:

22 boxes will fit along the length

16 boxes will fit along the width

8 boxes will fit along the height.

Total number of boxes = $22 \times 16 \times 8 = 2816$
 if stacked this way.

The largest number is 3072 boxes would fit in the room with no gap between the top of the boxes and the ceiling.

Revision/Assessment

- 1 a 4 m b 5 m
- 2 Exact measurements are shown.
 a 6 cm b 3 cm c 2.7 cm d 6.8 cm
- 3 a 111 mm b 120 mm c 113 mm
- 4 a 90 mm b 195 mm c 98 mm
- 5 a $\times 100$ b $\div 1000$ c $\div 1000$ d $\times 10$
- 6 a 56 000 cm b 3400 mm c 45 000 m
 d 0.8 m e 140 cm f 0.56 km

7 98 mm, 23 cm, 234 mm, 102 cm, 1.09 m

8 15.5 cm, 17 cm, 18.5 cm, 20 cm, 21.5 cm, 23 cm, 24.5 cm, 26 cm, 27.5 cm, 29 cm, 30.5 cm, 32 cm

9 a 1040 mm b 2590 cm c 1123 mm
 d 847 m e 957 cm f 7320 mm

10 45.8 cm, 900 mm, 1.2 m, 0.0897 m

11 The string would be 0.81 m.

12 Each child would receive 2000 mm.

13 a 27 cm b 44 mm c 16m d 47 mm

14 a 240 cm b 34 cm c 30 cm

15 a 68 mm b 16 m c 364 cm d 32 km

16 a 4574 mm b 27 m c 160 mm

17 a 20 m b 104 mm c 332 cm d 46 km

18 a 37.70 cm b 37.70 mm
 c 94.25 m d 56.55 km

19 a 19 cm b 9 cm c 42 m

20 a 115.42 cm b 116.56 cm c 61.42 m

21 The error in measurement was 4 cm, which is 3.125% of Seng's true height.

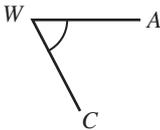
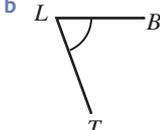
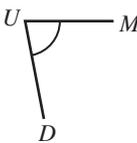
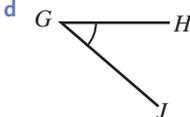
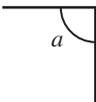
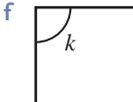
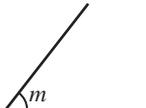
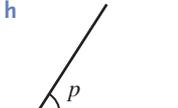
22 a 14.1 s b 34.6 s c 66.8 s d 65.4 s

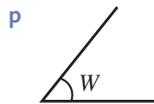
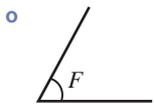
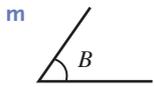
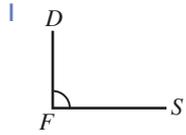
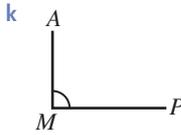
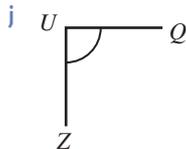
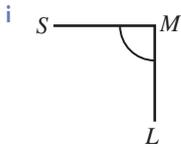
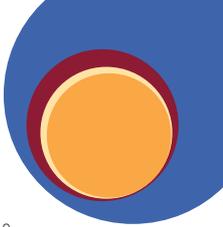
23 a 2476.5 cm b 2677.5 cm c 3866.5 cm
 d 1887.5 cm e 987.5 cm

Chapter 3

Exercise 3A

- 1 a M° , $C\hat{M}B$, $B\hat{M}C$, $\angle CMB$, $\angle BMC$
 b L° , $M\hat{L}Y$, $Y\hat{L}M$, $\angle MLY$, $\angle YLM$
 c R° , $A\hat{R}F$, $F\hat{R}A$, $\angle ARF$, $\angle FRA$
 d P° , $M\hat{P}X$, $X\hat{P}M$, $\angle MPX$, $\angle XPM$

- 2 a  b 
- c  d 
- e  f 
- g  h 



- 3** a Acute angle
 c Acute angle
 e Perigon
 g Obtuse angle
 i Reflex angle
 b Reflex angle
 d Reflex angle
 f Right angle
 h Straight angle

- 4** a 23° b 85° c 31°
 d 42° e 30° f 7.5°
 g 22° h 15° i 12°

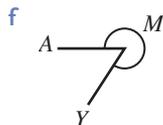
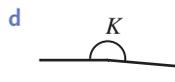
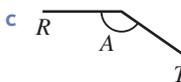
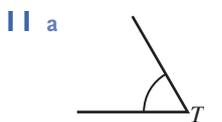
5 c and e

- 6** a 65° b 44° c 22°
 d 39° e 11° f 9°

- 7** a 72° b 90° c 19°
 d 75° e 52° f 10°
 g 20° h 17.5°

8 a, b, d, and e

- 9** a 145° b 128° c 84° d 73° e 19°
 f There is no supplementary angle for this.



- 12** a 35° b 43° c 47° d 52°

- e 180° f 38° g 90° h 8°

- 13** a 79° b 95° c 82° d 221°

- e 93° f 59° g 73° h 91°
 i 40° j 20° k 40° l 22.5°

Exercise 3B

- 1** a $a d, c b, e h, f g$
 b $c e, d f$
 c $c f, d e$
 d $a e, c g, b f, d h$
 e $a d, c b, e h, f g, a e, c g, b f, d h, c f, d e$
 f $a c, a b, b d, c d, f e, e g, g h, h f, c e, d f, b h, a g$
 g Complementary angles add to 90° ; no right angles are marked on diagram.

- 2** a d b b c c d e

- 3** a $m = 51^\circ$
 b $x = 180 - 62 = 118^\circ$
 c $t = 43^\circ, b = 90 - 43 = 47^\circ$
 d $t = 90 - 31 = 59^\circ, b = 90^\circ, f = 90^\circ, r = p = 31^\circ$

- 4** a $s = 47^\circ$, corresponding angles
 b $s = 58^\circ$, vertically opposite angles
 c $j = 88^\circ$, supplementary angles
 d $t = 102^\circ$, co-interior angles
 e $y = 88^\circ$, co-interior angles
 f $k = 113^\circ$, co-interior angles
 g $p = 55^\circ$, vertically opposite angles
 h $y = 39^\circ$, alternate angles
 i $m = 57^\circ$, corresponding angles

- 5** a $d = 113^\circ, s = 67^\circ$ b $b = 46^\circ, f = 134^\circ$
 c $k = 97^\circ$ d $r = 87^\circ, m = 93^\circ$
 e $y = 114^\circ, v = 66^\circ$ f $g = 102^\circ, p = 135^\circ$

- 6** a $f = 145^\circ$ b $t = 68^\circ$
 c $g = 91^\circ$ d $m = 75^\circ$
 e $y = 24^\circ, b = 156^\circ$ f $h = 87^\circ$
 g $t = y = 124^\circ, g = r = 56^\circ$
 h $h = 59^\circ, f = 121^\circ$

- 7** $a = 25^\circ, b = 72^\circ, c = 69^\circ, d = 140^\circ, e = 40^\circ$

- 8** $a = 8^\circ, b = 30^\circ, c = 25^\circ, d = 60^\circ, m = 50^\circ, p = 70^\circ, x = 25^\circ$

Exercise 3C

- 1** a Acute-angled equilateral triangle
 b Obtuse-angled isosceles triangle
 c Acute-angled equilateral triangle
 d Acute-angled equilateral triangle
 e Acute-angled isosceles triangle
 f Obtuse-angled scalene triangle



2

	Side and angle name	$\angle r$	$\angle s$	$\angle t$	Angle sum $= r + s + t$
a	acute-angled scalene triangle	63°	60°	57°	180°
b	obtuse-angled isosceles triangle	120°	30°	30°	180°
c	acute-angled equilateral triangle	60°	60°	60°	180°
d	acute-angled equilateral triangle	60°	60°	60°	180°
e	acute-angled isosceles triangle	65°	65°	50°	180°
f	obtuse-angled scalene triangle	30°	50°	100°	180°

- 3 a 71° b 77° c 118° d 60°
 e 107° f 24° g 22.5° h 30°
 i 73° j 38° k 20° l 25°
- 4 a 88° b 116° c 132° d 124°
 e 128° f 108° g 46° h 100°

Exercise 3D

- 1 a Parallelogram b Trapezium
 c Rhombus d Kite
 e Trapezium f Rectangle
 g Parallelogram h Square
- 2 a Rectangle b Rectangle c Trapezium

3

Name	$\angle r$	$\angle s$	$\angle t$	$\angle u$	Angle sum $= r + s + t + u$
a rectangle	90°	90°	90°	90°	360°
b rectangle	90°	90°	90°	90°	360°
c trapezium	100°	100°	80°	80°	360°

- 4 a 153° b 134° c 102° d 159°
 e 143° f 117° g 25° h 49°

Exercise 3E

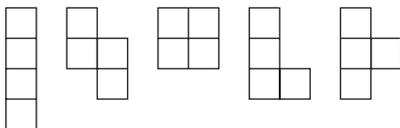
Number of sides	Name of polygon	Regular shape
3	Equilateral triangle	

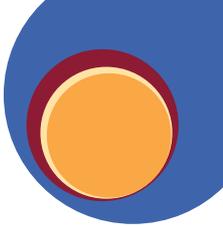
Number of sides	Name of polygon	Regular shape
4	Square	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	

- 2 a Rectangle, 360° b Hexagon, 720°
 c Pentagon, 540° d Octagon, 1080°
- 3 a 1440° b 9360° c 2700°
 d $62\ 640^\circ$ e 2340° f 1800°
 g 900° h $179\ 640^\circ$ i 4140°
 j 2880° k 3420° l 7740°
- 4 a 98° b 183° c 179°
 d 241° e 94° f 86°

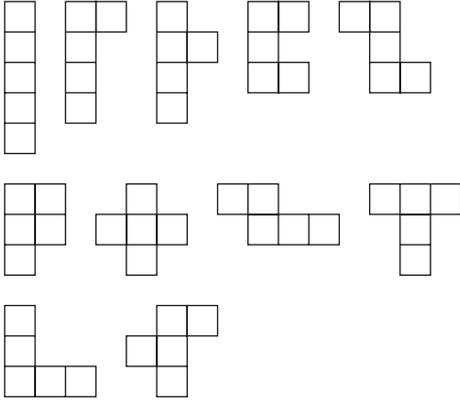
Applications

Tetrominoes





Pentominoes



Exterior angles of polygons

Polygon	Diagram	No. of exterior angles	Size of each exterior angle	Sum of exterior angles
Triangle		3	120°	360°
Quadrilateral		4	90°	360°
Pentagon		5	72°	360°
Hexagon		6	60°	360°
Heptagon		7	51 $\frac{3}{7}$ °	360°
Octagon		8	45°	360°

The sum of the exterior angles of a polygon is 360°.

Enrichment

- 1 a $a = 60^\circ, b = 120^\circ, c = 90^\circ, d = 90^\circ$
 b $a = 45^\circ, b = 95^\circ, c = 50^\circ$
 c $a = 56^\circ, b = 124^\circ, c = 124^\circ, d = 56^\circ$
 d $a = 22^\circ, b = 68^\circ, c = 68^\circ, d = 90^\circ$
 e $a = 70^\circ, b = 100^\circ, c = 120^\circ$
 f $a = 65^\circ, b = 108^\circ, c = 43^\circ, d = 65^\circ, e = 72^\circ$
- 2 a $a = 50^\circ$ b $a = 30^\circ$ c $a = 55^\circ, b = 115^\circ$
- 3 a $a = 46^\circ$
 b $a = 25^\circ, b = 90^\circ, c = 155^\circ, d = 115^\circ$
 c $a = 44^\circ, b = 68^\circ, c = 44^\circ, d = 136^\circ, e = 68^\circ$
- 4 8 sides
- 5 a $\angle AOB = 55^\circ$ $\angle BOC = 68^\circ$
 $\angle COD = 57^\circ$ $\angle DOE = 55^\circ$
 $\angle EOF = 68^\circ$ $\angle FOA = 57^\circ$

- b 360° c They are equal.
- 6 a 2 pieces b 4 pieces c 7 pieces

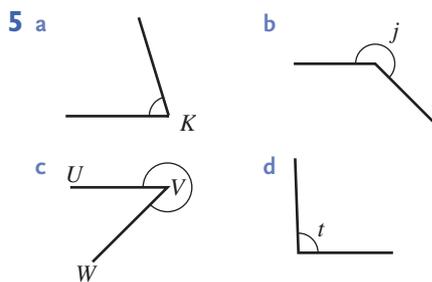
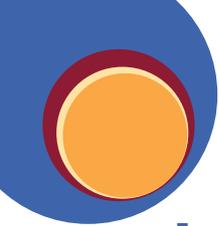
Number of cuts	Maximum number of pieces	Working out
1	2	$\frac{1 \times 2}{2} + 1$
2	4	$\frac{2 \times 3}{2} + 1$
3	7	$\frac{3 \times 4}{2} + 1$
4	11	$\frac{4 \times 5}{2} + 1$
5	16	$\frac{5 \times 6}{2} + 1$
6	22	$\frac{6 \times 7}{2} + 1$

e $P = \frac{n(n+1)}{2} + 1$

- f i 37 ii 121 iii 172 iv 211
 g i 7 cuts ii 12 cuts iii 9 cuts iv 25 cuts
 h Yes

Revision/Assessment

- 1 $\hat{X}\hat{Y}Z, \hat{Z}\hat{Y}X$ or $\angle XYZ, \angle ZYX, \angle Y$
- 2 a Acute angle b Obtuse angle
 c Reflex angle d Obtuse angle
 e Right angle f Straight angle
 g Perigon h Obtuse angle
- 3 a 12° b 154° c 31°
 d 36° e 9° f 11-25°
- 4 a 35° b 145° c 325°



- 6 a 26° b 75° c 145°
 d 75° e 224° f 33°

7 51° because $51^\circ + 39^\circ = 90^\circ$

8 $(85 + n)^\circ$

9 141° because $141^\circ + 39^\circ = 180^\circ$

10 $(168 - 3n)^\circ$

- 11 a Corresponding angles
 b Co-interior angles
 c Vertically opposite angles
 d Alternate angles

- 12 a 75° b 150° c 89°
 d 65° e 34° f 89°
 g $r = 56^\circ, g = 124^\circ$ h $h = 69^\circ, f = 69^\circ$

- 13 a Equilateral, acute-angled triangle
 b Isosceles, acute-angled triangle
 c Isosceles, acute-angled triangle
 d Scalene, right-angled triangle

- 14 a 23° b 93° c 151°
 d 122° e 80° f 162°

- 15 a Octagon b Trapezium
 c Hexagon d Equilateral triangle
 e Kite f Rhombus
 g Pentagon h Parallelogram

- 16 a 720° b 540° c 1260°
 d 1980° e $17\,640^\circ$

- 17 a 135° b 162.86° c 158.82°

Chapter 4

Exercise 4A

- 1 a $n = 365$ f $s = 25 \text{ km/h}$
- 2 a $b = 3, t = 2, n = 3 + 2 = 5$
 b $b = 1, t = 4, n = 1 + 4 = 5$
- 3 a 7 b -8 c 2 d 1
 e -8 f $\frac{2}{3}$ g 1 h $-\frac{4}{3}$
 i $\frac{3}{4}$ j $\frac{5}{6}$ k $\frac{7}{3}$ l $-\frac{3}{4}$

- 4 a 2, 4 b 2, 7 c 2, 1
 d 3, 1 e 3, -1 f 3, 3
 g $2, -\frac{2}{3}$ h 2, 0 i 2, 4

- 5 a $n + 8$ b $3n + 6$
 c $4 + 5x$ or $5x + 4$ d $6y - 30$
 e $-5m + 8$ f $\frac{q}{4} + 6$

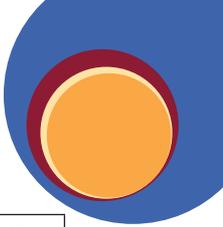
g $-\frac{12}{x}$ h $-\frac{3x}{5}$

i $-3 - 4z$ j $-\frac{3y}{12}$

- 6 a $4x, -x, 2x, -5y, 7y$
 b $2a, -4a; 6b, -5b; -3ab, 7ba, 4ab$
 c $-pq, qp, 9pq; 6p^2q; 12pq^2$
 d $2x; 3x^2; -x^2 5x^2; 3xy, 6yx; -x^2y, -3yx^2$
 e $5c^2d^2, 4c^2d^2; c^2d; -d^2c; -dc$
 f $x^2, 2x^2; 2x, x; y^2, 2y^2; 2y, y; -xy$
 g $10cd^2, 10d^2c; 4c^3d; 6d^2; dc$
 h $3a^2; b^2, -b^2, 2b^2; 5a; b, b; 6ab, 9ab$
 i $5xy, 8yx; 2x^2y; 3xy^2, y^2x$
 j $mnp, pnm; mnp^2; np^2m, p^2mn$

Exercise 4B

- 1 a $3m + 2n$ b $4x + 2y$ c $3x$
 d $2a - 2b$ e $2m + 3n$ f $3t$
 g $2g - h$ h $3a - 3b$
- 2 a $5x$ b $-5x$ c $5x$
 d $-5x$ e $8a$ f $-8a$
 g b h $-b$ i $8c$
 j $-8c$ k $16p$ l $-16p$
- 3 a $-5d$ b $-7x$ c $-20m$
 d $-15q$ e $-2x$ f $-3y$
 g $-4y$ h $-11m$ i $-10q$
- 4 a $12y$ b $12y$ c $4y$
 d $-4y$ e $-4y$ f $4y$
 g $-12y$ h $-12y$ i 0
 j $-6y$ k $-6y$ l $-2y$
- 5 a $13y$ b $13y$ c $7y$
 d $-7y$ e $-7y$ f $7y$
 g $-13y$ h $-13y$ i 0
- 6 a $-2xy$ b $-3xy$ c ab
 d $3ab$ e $5pq$ f pq
 g $-6mn$ h $-7mn$ i $-2xy$
 j $3xy$ k 0 l 0
 m x^2 n $-4y^2$



- 7 a $9x + 7y$ b $3m - 3n$ c $4xy - zy$
 d $4pq - 3q$ e $-2x + 2y$ f $-x - 3$
 g $-a - 3$ h $-2x + 2$ i $2x - 2y - 1$
 j $-d + e$ k $-x - y + 1$ l $-g$
 m $2x^2 - 3x$ n $-3x^2 - 6x$ o $4x^2 + 3x + 1$
 p $-2x^2 - 2x - 2$

- 8 a $-6a - b - 5c$ b $-3ab - bc + ca$
 c $-3xy - 3yz - 3zx$ d $-3x - 3$
 e $-3x^2 - 5x - 10$
 f $-2x^2 - 7x - 4y^2 - 5y - 18$
 g $-x^2 - 3x - y^2 - 5y - 1$

9 a

+	3	2y	5x	4xy
2	5	2y + 2	5x + 2	4xy + 2
3y	3 + 3y	5y	5x + 3y	4xy + 3y
4x	3 + 4x	4x + 2y	9x	4xy + 4x
2xy	3 + 2xy	2xy + 2y	5x + 2xy	6xy

b

+	5	y	2x	5xy
1	6	y + 1	2x + 1	5xy + 1
y	y + 5	2y	2x + y	5xy + y
3x	3x + 5	3x + y	5x	5xy + 3x
xy	xy + 5	y + xy	2x + xy	6xy

c

+	2	-y	4x	2xy
4	6	4 - y	4x + 4	2xy + 4
-y	2 - y	-2y	4x - y	2xy - y
-x	2 - x	-x - y	3x	2xy - x
-xy	2 - xy	-y - xy	4x - xy	xy

Exercise 4C

- 1 a $-3x$ b $-7y$ c $11a$
 d $-9b$ e $-ab$ f mn
 g $-14xy$ h $10mn$ i $-abc$
 j $-42x$ k $48y$ l $-28p$
 m $27ab$ n $96xy$ o $-ab^2$
 p $72x^2$ q $44x^2y$ r $-8m^2n$
 s $140a^2b^2$
- 2 a $-16x$ b $-54y$ c $21a$
 d $-5xy$ e $-5m^2n$ f $63bc$
 g $-30xy$ h $-63pq$ i $25x^2$
 j $-15y^2$ k $56ab$ l $-270mnp$
 m $30xyz$ n $126abc$ o $-60x^2y$
 p $100x^2$ q $169y^2$ r $81x^2y^2$
 s $144a^2b^2$ t $121m^2n^2$ u $225p^2q^2r^2$

3 a

x	3	y	-2x	xy
-1	-3	-y	2x	-xy
-y	-3y	-y^2	2xy	-xy^2
2x	6x	2xy	-4x^2	2x^2y
3xy	9xy	3xy^2	-6x^2y	3x^2y^2

b

x	-2	7y	-3x	-xy
2	-4	14y	-6x	-2xy
-3y	6y	-21y^2	9xy	3xy^2
-4x	8x	-28xy	12x^2	4x^2y
-2xy	4xy	-14xy^2	6x^2y	2x^2y^2

- 4 a $-\frac{x}{4}$ b $\frac{m}{9}$ c $\frac{p}{10}$
 d $-\frac{8}{x}$ e $-\frac{7}{5x}$ f $\frac{3x}{7y}$
 g $\frac{8a}{9b}$ h $-\frac{1}{6x^2}$ i $\frac{3x^2}{8y^2}$

- 5 a $-\frac{a}{5}$ b $-\frac{a}{2}$ c $\frac{y}{5}$ d $-\frac{3}{x}$
 e $-\frac{3}{y}$ f $\frac{4}{m}$ g $-\frac{1}{3p}$ h $-\frac{1}{4q}$
 i $\frac{1}{3xy}$ j $-\frac{5x}{4}$ k $-\frac{3x}{2}$ l $\frac{3y}{2}$
 m $\frac{10}{3x}$ n $-\frac{2}{5ab}$ o $\frac{5}{7w}$ p $-\frac{9}{8m}$

- 6 a $\frac{7x}{y}$ b $\frac{9a}{b}$ c $-\frac{6m}{q}$
 d $\frac{pq}{r}$ e $\frac{6x}{y}$ f $-\frac{pq}{r}$
 g $\frac{6r}{s}$ h $\frac{12m}{n}$ i $-\frac{ab}{c}$
- 7 a $\frac{x}{2}$ b $2x$ c $\frac{3x}{2}$
 d $2x$ e $-\frac{3y}{2}$ f $\frac{3}{2m}$
 g $-\frac{8}{3y}$ h $-\frac{b}{4}$ i $11x$
 j $6x^2$ k $-2x^2$ l $-3xy^2$



8 a

÷	-24	12y	-x	36xy
1	-24	12y	-x	36xy
-3y	$\frac{8}{y}$	-4	$\frac{x}{3y}$	-12x
2x	$-\frac{12}{x}$	$\frac{6y}{x}$	$-\frac{1}{2}$	18y
-4xy	$\frac{6}{xy}$	$-\frac{3}{x}$	$\frac{1}{4y}$	-9

b

÷	-10	20y	-5x	30xy
2	-5	10y	$-\frac{5x}{2}$	15xy
-5y	$\frac{2}{y}$	-4	$\frac{x}{y}$	-6x
x	$-\frac{10}{x}$	$\frac{20y}{x}$	-5	30y
-2xy	$\frac{5}{xy}$	$-\frac{10}{x}$	$\frac{5}{2y}$	-15

9 a $-\$175$ **b** $-\$175 - d$ **c** $3d + \$175$

10 a $\$870$ **b** $\$800$

c $\$730$ **d** $\$870 - 20y$

e $435 = 870 - 70y$ **f** $0 = 870 - 70y$
 $-435 = -70y$ $-870 = -70y$
 $y = 6.2$ years $y = 12.4$ years

11 Value = $D - L \times T$

Exercise 4D

1 a $-4 \times 4 = -16$ **b** $-28 + 12 = -16$

2 a $-8 \times 16 = -128$ **b** $-96 - 32 = -128$

3 a $-x - y$ **b** $-a - b$ **c** $-m - n$

d $-x - 4$ **e** $-5y - 35$ **f** $-10p - 80$

g $-8a - 32$ **h** $-9c - 45$ **i** $-5b - 45$

j $-30 - 5p$ **k** $-36 - 3q$ **l** $-50 - 10n$

m $-4x - 8$ **n** $-4x + 8$ **o** $-8 + 4x$

4 a $12x - 12y$ **b** $8a - 8b$ **c** $9m - 9n$

d $8x - 56$ **e** $3x - 27$ **f** $2x - 28$

g $-26 + 2m$ **h** $-63 + 9q$ **i** $-54 + 6p$

j $-56 + 8x$ **k** $-54 + 3b$ **l** $-135 + 9m$

5 a $xy + xz$ **b** $mp + mq$ **c** $ab + ac$

d $rs - rt$ **e** $pq - pr$ **f** $lm - ln$

g $mn - 8m$ **h** $ab - 12a$ **i** $-yz + 8z$

j $-9c + bc$ **k** $-14p + pq$ **l** $-9m + mn$

6 a $27x + 18$ **b** $12y + 18$ **c** $18b + 8$

d $35q - 30$ **e** $24p - 48$ **f** $12s - 28$

g $-10x - 25y$ **h** $-48m - 72n$ **i** $-16a - 24b$

j $-18a + 42b$ **k** $-88m + 66n$ **l** $-2m + 3n$

7 a $6x + 27$ **b** $49y + 3$

c $-15m - 26$

d $48 - 9b$

e $8a + 17b - 16c$

f $12x^2 + 8x$

g $6x^2 - 5x - 6$

h $-30y^2 + 48y$

i $-18x^2 - 7x$

8 a $-7(n+3) = -7n - 21$

b $-5(n-4) = -5n + 20$

c $-4(2n-3) = -8n + 12$

d $-3(4n-6) = -12n + 18$

e $-6\left(\frac{n}{2} - 3\right) = -3n + 18$

9 a $2 - d$

b $5(2 - d)$

c $10 - 5(2 - d)$

d $5d$

10 a $x - 50$

b $6(x - 50)$

c $500 - 6(x - 50)$

d $800 - 6x$

Exercise 4E

1 a -12 **b** -28 **c** -2 **d** 5 **e** 16 **f** 0

g -7 **h** 17 **i** -15 **j** 1 **k** 12 **l** 20

2 a -40 **b** -61 **c** -4 **d** 14

e -49 **f** 31 **g** -32 **h** -24

i -52 **j** -9 **k** 78 **l** $\frac{25}{8}$

m 15 **n** 70 **o** 56 **p** 41

q 9 **r** 36 **s** -9 **t** -9

3 a -12 **b** -15 **c** 60 **d** 125

e 5 **f** 5 **g** 21 **h** 21

i 25 **j** 25 **k** 28 **l** 24

4 a -10 **b** -5 **c** $-\frac{2}{5}$ **d** $-\frac{4}{3}$ **e** $\frac{11}{3}$ **f** $\frac{2}{3}$

5 a **b** **c** **d** **e** **f**

x	-3	-2	-1	0	1
x+1	-2	-1	0	1	2

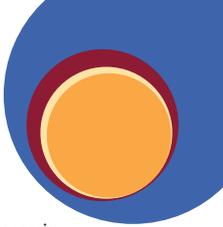
x	-2	-1	0	1	2
x-2	-4	-3	-2	-1	0

x	-2	-1	0	1	2
6-x	8	7	6	5	4

x	-2	-1	0	1	2
4x-1	-9	-5	-1	3	7

x	-2	-1	0	1	2
x ²	4	1	0	1	4

x	-3	-2	-1	0	1
x ² +4	13	8	5	4	5



Values		Expressions					
x	y	xy	$x+y$	$x-y$	$y-x$	$4x-4y$	$4(x-y)$
2	5	10	7	-3	3	-12	-12
-2	5	-10	3	-7	7	-28	-28
2	-5	-10	-3	7	-7	28	28
-2	-5	10	-7	3	-3	12	12

Values		Expressions					
x	y	xy	$x+y$	$x-y$	$y-x$	$\frac{x}{y}$	$\frac{2x}{y-1}$
10	5	50	15	5	-5	2	5
8	2	16	10	6	-6	4	16
-3	3	-9	0	-6	6	-1	-3
-15	-5	75	-20	-10	10	3	5

Exercise 4F

- 1 a i \$160 ii \$170 iii \$250
 b {150, 160, 170 ...} c 10 d 35 weeks
- 2 a i \$750 ii \$700 iii \$550
 b {800, 750, 700 ...}
 To get the next term, subtract 50 from the term before it.
 50 gives the decrease
 c 10 weeks d -\$200 e 22 weeks
- 3 $B = 1000 - 85t$
 a i \$915 ii \$745 iii \$490
 b {1000, 915, 830 ...}
 c To get the next term, subtract 85 from the term before it.
 d 85 e 9 weeks
 f -\$275 g 20 weeks
- 4 $A = 140 + 12t$
 a i \$152 ii \$176 iii \$224
 b {140, 152, 164 ...}
 c The next term is obtained by adding the weekly increase of \$12.
 d 12 e 20 weeks
- 5 a i 4 km/h ii 6 km/h
 iii 14 km/h iv 24 km/h
 b {4, 6, 8, 10, 12 ...}
 c The next term is obtained by adding the increase of 2 km/h to the previous speed.
 d 2 e 13 s
- 6 $s = 5 + 1.5t$
 a i 5 km/h ii 6.5 km/h
 iii 11 km/h iv 14 km/h
 b {5, 6.5, 8, 9.5 ...}

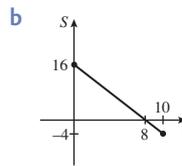
c The next term is obtained by adding the increase in speed of 1.5 km/h to the previous speed.

- d 1.5 e 8 s
 7 a $A = 10 + 3t$ b $A = 10 - 8t$
 c $A = 30 - 6t$ d $A = 15 + 8t$

Applications

Skateboarder

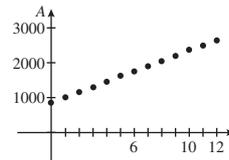
t	0	1	2	3	4	5	6	7	8
s	16	14	12	10	8	6	4	2	0



- c $t = 9, s = -2; t = 10, s = -4$
 e The skateboarder starts going back down the hill. This is shown by the negative speed.
 f $s = 16 - 2t$

Savings plan

- a 1000, 1150, 1300, 1450, 1600, 1750, 1900, 2050, 2200, 2350, 2500, 2650
 b It is not appropriate to join plots because the value only increases at each monthly interval.

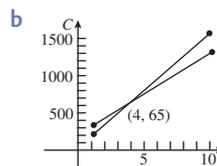


- c $A = 1000 + 150t$ d \$6400
 e 18 months f 24 months
 g $A = 1900 + 1000(t-6)$ or $A = 1300t + 100t$

Sharky's surfboards

1 a

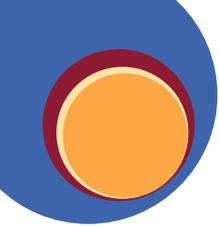
t	1	2	3	4	5
C	200	350	500	650	800



- c $C = 200 + 150(t-1)$ d \$4250
 $= 150t + 50$

2 a

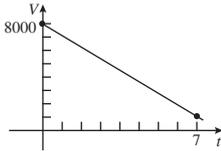
t	1	2	3	4	5
C	350	450	550	650	750



- c $C = 350 + 100(t - 1)$ d \$3050
 $= 100t + 250$
 e $t < 4$ days it is cheaper to use Sharky's, $t > 4$ days it is cheaper to use Budget, $t = 4$ it is the same for both.

Scuba diving

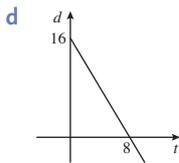
- a 8000, 7000, 6000, 5000, 4000, 3000, 2000
 b It is appropriate to join plots because the equipment is constantly decreasing in value.



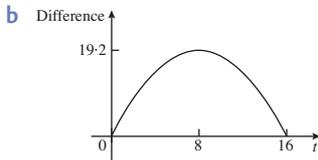
- c \$5500 d $V = 8000 - 1000t$
 e $V = 8000 - 1000(4 \cdot 25) = \3750

Enrichment

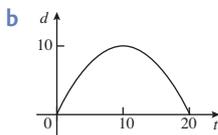
- 1 a 16, 14, 12, 10, 8, 6, 2, 0, -2, -4, -6, -8, -10, -12, -14, -16, -18
 b By subtracting 2 km/h from the previous velocity:
 $v_1 = v_0 - 2$, $v_2 = v_1 - 2$, $v_3 = v_2 - 2$, $v_4 = v_3 - 2$,
 $v_5 = v_4 - 2$, $v_n = v_{n-1} - 2$
 c The zero velocity means the tractor-trailer has stopped, the negative results means that the tractor-trailer is rolling backwards down the hill.



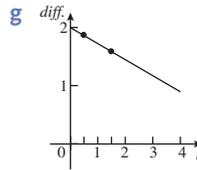
- e -4.5 km/h or 4.5 km/h back down the slope
 2 a 0, 4.5, 8.4, 11.7, 14.4, 16.5, 18, 18.9, 19.2, 18.9, 18, 16.5, 14.4, 11.7, 8.4, 4.5, 0



- c $d = 19.2$ m, at $t = 8$ s
 d 8 s
 e Approximately 2.5 s, 13.5 s
 f $d_{17} = -5.1$, $d_{18} = -10.8$, the position is now behind the starting point.
 3 a 0, 1.9, 3.6, 5.1, 6.4, 7.5, 8.4, 9.1, 9.6, 9.9, 10, 9.9, 9.6, 9.1, 8.4, 7.5, 6.4, 5.1, 3.6, 1.9, 0



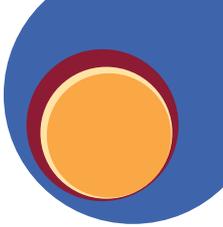
- c $d = 10$ m, $t = 10$ s d 10 s
 f 1.9, 1.7, 1.5, 1.3, 1.1, 0.9, 0.7, 0.5, 0.3, 0.1, -0.1, -0.3, -0.5, -0.7, -0.9, -1.1, -1.3, -1.5, -1.7, -1.9



- h $t = 0$, $v = 2$ i $t = 1$, $v = 1.8$, $t = 2$, $v = 1.6$
 j $v = 2 - 0.2t$ k -1 m/s
 4 a $\frac{17}{2}$ b 10 c $-\frac{1}{3}$ d 18
 e $\frac{59}{7}$ f $\frac{3}{2}$ g $5\frac{1}{3}$ h $-14\frac{1}{3}$
 5 a $\frac{2}{xy}$ b $\frac{3}{ab}$ c $\frac{m}{q}$ d $\frac{p}{q^2r}$
 e $\frac{4}{x^2y}$ f $\frac{-1}{q^2r}$ g $\frac{6}{rs}$ h $\frac{12}{mn}$
 i $-\frac{a}{bc}$ j $\frac{2p}{qr}$ k $\frac{2}{xy^2}$ l $\frac{-5}{qp}$
 m $\frac{-1}{6rt}$ n $\frac{-1}{mn^2}$

Revision/Assessment

- 1 a 2, 2 b 2, 10 c 2, -1
 d 3, -5, 1 e 2, 7y, 7x, $-\frac{3}{5}$ f 3, $\frac{1}{2}$, $-\frac{3}{5}$, $\frac{1}{4}y$
 2 a $7x, -x, 9x; -5y, 3y$
 b $4a, -2a; -5ab, 9ba, 2ab; 4b, -b$
 c $3p^2q; -pq, -4qp, 2pq; 4pq^2$
 d $2x^2, -2x^2, -8x^2; 5x; 6xy, 3yx; -x^2y, -12x^2$
 3 a $-9a - 2b - 4c$ b $7ab - 5bc + 6ca$
 c $-xy - 10yz + zx$
 4 a $15x - 2y$
 b $2a + 6ab + 3b$
 c $3p^2q - 3qp + 4pq^2$
 d $-8x^2 + 5x + 9xy - 13yx^2$
 5 a -21x b -27y c 12a
 d -12xy e $-7m^2n$ f 10bc
 g $-60y^2$ h 126ab i -270mnp
 j 30xyz k 126abc l $-40x^2y$
 m $81x^2$ n $36y^2$ o $9x^2y^2$
 6 a $-\frac{a}{3}$ b $-\frac{a}{7}$ c $\frac{y}{2}$ d $\frac{4}{x}$ e $-\frac{9}{y}$
 f $\frac{7}{m}$ g $\frac{x}{3y}$ h $\frac{y}{4x}$ i $\frac{3a}{2b}$



- 7** a $-15x + 1$ b $-30y + 14$
 c $-8m - 18$ d $-8b + 56$
 e $-18x^2 - 21x$ f $8p^2 - 12pm$
- 8** a $3xy + 6y$ b $8ab + 16a$
 c $9ab - 15b$ d $10mn - 25n$
 e $6x + 8xy$ f $16x - 8xy$
 g $5a + 20b$ h $12m - 36mn$
 i $16xyz - 20xy$ j $8mp + 10mp$
 k $15rst - 25tr$ l $14pq + 21pqr$

- 9** a $x^2 + 5x$ b $x^2 + 12x$
 c $x^2 - 7x$ d $x^2 - 11x$
 e $m^2 - m$ f $z^2 - z$
 g $y - y^2$ h $x - x^2$
 i $3x^2 + 6x$ j $12x^2 - 9x$
 k $10x^2 + 12x$ l $15x^2 - 20x$

- 10** a $-4x - 8$ b $-3y - 12$
 c $-7x - 21$ d $-10q - 20$
 e $-5x + 10$ f $-8y + 32$
 g $-3x + 15$ h $-6q + 18$
 i $-12a + 32b$ j $-42c - 14d$
 k $-9z + 15y$ l $-30m - 35n$

- 11** a 180 b 36 c 36
 d 36 e 88 f 80

- 12** a 4 b 32 c $-\frac{1}{4}$ d -1 e $-\frac{1}{2}$ f 2

- 13** a i \$960 ii \$920 iii \$800
 b {1000, 960, 920 ...}
 $A = 1000 - 40r$
 c 12 weeks d -\$200 e 35 weeks

Chapter 5

Exercise 5A

- 1** a 12:9 or 4:3 b 3:5
- 2** a i 5:7 ii 3:5 iii 1:16
 b i 8:12 ii 3:8 iii 9:32
 c i 3:8 ii 10:3 iii 3:24
- 3** a 15:7 b 10:7:3 c 1:8
 d 15:13:2 e 19:18 f $2\frac{1}{2} : \frac{1}{2}$
 g 150:99 h 125:98 i 1:3

4 a

	Bill	Wilma
i	11:13	14:9
ii	4:13	9:9
iii	11:4	14:9

- b i 25:22 ii 13:22 iii 25:13
 iv 25:60 v 13:60 vi 22:60

- 5** a 17:5 b 8:22 c 5:25

6 a

	Bendigo Bullants	Benalla Bulldozers	Ratio
First quarter	15 points	14 points	15:14
Second quarter	30 points	27 points	30:27
Third quarter	57 points	64 points	57:64
Last quarter	77 points	78 points	77:78

b

	Beaufort Bullies	Bully Baddies	Ratio
First quarter	7 points	15 points	7:15
Second quarter	22 points	23 points	22:23
Third quarter	42 points	42 points	42:42
Last quarter	63 points	64 points	63:64

- 7** Zebras 16:120
 Lions 36:120
 Elephants 2:120
 Tigers 18:120
 Monkeys 48:120

Exercise 5B

- 1** a 4:1 b 5:1 c 9:5 d 3:1
 e 12:7 f 3:1 g 7:3 h 3:10
 i 3:50 j 1:4 k 3:4 l 19:9
- 2** a 24 b 9 c 6 d 22
 e 4 f 12 g 39 h 21
- 3** 1:5:1 **4** 12:5:120 **5** 7:5:10:5
 = 3:2 = 5:48 = 5:7
- 6** $3\frac{1}{2}:100$ **7** $13\frac{1}{2}:2\frac{1}{2}$
 = 7:200 = 27:5
- 8** a 24:5 b 3:5 c 6:25 d 7:3
 e 7:12 f 4:3 g 5:6 h 18:25
 i 8:45 j 1:4 k 7:6 l 5:8
- 9** a 5:2 b 2:3 c 4:3 d 2:3
 e 5:2 f 16:9 g 1:2 h 105:44
 i 4:25 j 17:52

- 10** a $\frac{2}{1}$ b $\frac{3}{2}$ c $\frac{6}{1}$ d $\frac{5}{4}$ e $\frac{3}{11}$

Exercise 5C

- 1** a 6 b 15 c 16 d 4 e 4
 f 9 g 6 h 2 i 10:5



2 30

3 a \$20.50

b \$28.70

4 9.6 kg of cement and 2.4 kg of screenings

5 a 6.25 litres of petrol b 0.8 litres of oil

6 a 5:6:4 b 55 kg of copper and 66 kg of tin

Learning task 5D

1 a 4 cm and 8 cm b 3 cm and 9 cm

c 2 cm and 10 cm

2 a i T ii V iii W

iv U v Y vi X

b 3 cm, 6 cm and 9 cm

3 a i 1:2 ii 2:1

b i 1:2 ii 2:1

Exercise 5E

1 a 315:105

b 690:2760

c 3024:1512

d 4389:1881

2 a \$12:\$24

b \$6:\$30

c \$10.80:\$25.20

d \$13.50:\$22.50

e \$16:\$20

f \$15:\$21

g \$6:\$12:\$18

h \$4.50:\$13.50:\$18

i \$8:\$12:\$16

j \$9:\$9:\$18

k \$3.60:\$7.20:\$10.80:\$14.40

l \$4.50:\$9:\$9:\$13.50

3

Class and number	Ratio of boys to girls	No. of boys	No. of girls
8A, 24 students	2:1	16	8
8B, 25 students	2:3	10	15
8C, 21 students	3:4	9	12
8D, 24 students	1:3	6	18
8E, 22 students	5:6	10	12
8F, 24 students	3:1	18	6

4 a Abe will receive \$32 000, Beryl will receive \$48 000 and Chris will receive \$16 000.

b $80\,000:96\,000 = 5:6$

5

Day	Total production (kg)	Red:green	No. of red	No. of green
Monday	260	2:3	104	156
Tuesday	1704	1:2	568	1136
Wednesday	567	2:5	162	405
Thursday	810	2:1	540	270
Friday	1265	4:1	1012	253
Total	4606	1193:1110	2386	2220

6 200 for cassava, 400 for rice and 600 for potato

7 24 black beads and 48 red beads

8 1:3:6, 0.15 L bush lime, 0.45 L sugar, 0.9 L water

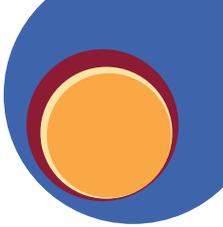
9 39 t of screenings, 65 t of sand and 13 t of cement

10 5.6 cm, 11.2 cm and 22.4 cm

11 0.6 mL of component A and 12 mL of component B

12

Day	Total production	Ratio of red:white:yellow:black	Red	White	Yellow	Black
Monday	29 872 700	2:3:4:1	5 974 540	8 961 810	11 949 080	2 987 270
Tuesday	31 846 221	3:5:1:2	8 685 333	14 475 555	2 895 111	5 790 222
Wednesday	63 159 018	1:2:3:7	4 858 386	9 716 772	14 575 158	34 008 702
Thursday	18 949 284	4:1:3:1	8 421 904	2 105 476	6 316 428	2 105 476
Friday	36 171 660	2:1:3:8	5 167 380	2 583 690	7 751 070	20 669 520



Exercise 5F

- 1** a mm/day b m/s c cents/litre
d cm/min e km/h f cakes/child
g km/L
- 2** a Scoring rate of a rugby team
b Scoring rate of a football team
c Cost of petrol
d Speed of a falcon
e Attendance records at the soccer
f Petrol consumed by a racing car
g Number of rolls made in a bakery
h Number of passengers in cars travelling on a freeway
i Carrying capacity on a large goat farm
- 3** a \$2900/m² b \$40/kilogram of gas
c \$45/litre of paint d \$4.90/min
- 4** a 18 min b 210 min
c 1500 min or 25 h
- 5** a \$37 500 b \$562 500 c \$1 625 000
- 6** a \$27 b \$54 c \$22.50
- 7** a \$145 000 b \$725 000
c \$942 500 d \$1 655 900
- 8** a \$196 b \$232
c \$440 d \$1016
- 9** a 315 degrees b 515 degrees
c 355 degrees d 1215 degrees
- 10** a i 0.75 kg ii 1.5 kg
iii 3.5 kg iv 4.17 kg
b i 150 m² ii 216 m²
iii 1080 m² iv 12 m²
- 11** a i \$9.00 ii \$40.50
iii \$2.25 iv \$6.75
b i 102 min ii 120 min
iii 21.6 min iv 28.8 min
- 12** a i 128 m ii 192 m
iii 144 m iv 560 m
b i 48 boats ii 11 boats
iii 14 boats iv 432 boats

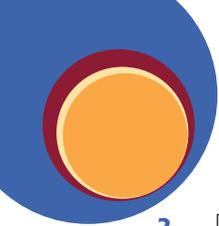
Exercise 5G

- 1** a km/h b km/h c m/s
d m/s e m/s f km/h
- 2** a 85 km/h b 280 km/h c 15 km/h
d 8 km/h e 5.33 km/h f 3.6 km/h
g 190 km/h h 92 km/h i 13.33 km/h
j 9 km/h

- 3** a 100 s b 4.5 s c 5 s
d 100 s e 42 s f $\frac{5}{9}$ h
- 4** a 80 m b 180 m c 480 m
d 1680 m e 7200 m f 50 400 m
- 5** a 24 km/h b 6 km/h
c 106.7 km/h d 38.57 km/h
e 150 km/h f 630 km/h
g 71.08 km/h h 18 km/h
- 6** a 16 km b 32 km c 48 km d 160 km
12 km 24 km 36 km 120 km
8 km 16 km 24 km 80 km
6 km 12 km 18 km 60 km
22 km 44 km 66 km 220 km
- 7** a 160 km b 400 km c 280 km
d 420 km e 533.3 km f 840 km
- 8** a 150 m b 225 m c 450 m
d 900 m e 3600 m f 13 500 m
- 9** a 80 km b 4500 m c 562.5 km
d 90 km

Exercise 5H

- 1** a i 4:00 am ii 2:00 am and 5:30 am
iii 7:00 am iv 9:20 am and 11:00 am
v 10:00 am
b i 7 degrees ii 15 degrees iii 16 degrees
c i $2\frac{1}{3}$ degrees/h ii 5 degrees/h
iii 2 degrees/h
- 2** a i at home ii 40 km from home
iii 40 km iv 20 km/h
b The car was stationary between 11:00 am and noon, between 1:00 and 2:00 pm and between 4:00 and 5:00 pm.
c i 40 km from home ii 60 km from home
iii 20 km iv 20 km/h
d i 60 km from home ii 100 km from home
iii 40 km iv 20 km/h
e 5:00 pm
f i 20 km/h ii 40 km/h



3 a

Time (h)	Temp. (°C)
0	-18
1	-7
2	4
3	8
4	12
5	21
6	30
7	30
8	30
9	5
10	-19

- b** i 11°C/h ii 4°C/h
 iii 9°C/h iv 0°C/h
- c** 30°C d at 8 hours

e The ice cream was removed from freezer, placed in a room with temperature of 30 degrees. After 2 hours a fan was turned on for a period of 2 hours and the room cooled. The fan was turned off and the temperature reached 30° again. After another 2 hours the ice cream was returned to the freezer.

4 a

Day	Height
1	1
2	1.6
3	2.2
4	3
5	6
6	9
7	11
8	13
9	16.5
10	20

- b** i 0 cm/day ii $\frac{2}{3}$ cm/day
 iii 3 cm/day iv $3\frac{1}{2}$ cm/day
- c** 2 cm/day

5 a

Day	Alf	Beauty
0	39	21
2	41	25
4	43	25
6	39	28
8	39	29
10	43	29
12	43	33
14	45	35
16	39	39
18	37	41
20	39	44

- b** Alf: 1 kg/day, 1 kg/day, -2 kg/day, 0 kg/day, 2 kg/day, 0 kg/day, 1 kg/day, -3 kg/day, -1 kg/day, 1 kg/day

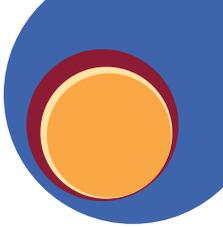
Beauty: 2 kg/day, 0 kg/day, 1.5 kg/day, 0.5 kg/day, 0 kg/day, 2 kg/day, 1 kg/day, 2 kg/day, 1 kg/day, 1.5 kg/day

- c** Day 16
- e** Beauty was unhappy when he moved in to the local zoo and was not eating properly. He made friends with Alf and as he started to settle in his eating improved. His weight steadily gained until he outweighed Alf at the end of the 20-day observation period.

- f** Ratio of Alf's weight to Beauty's weight:
 13:7
 41:25
 43:25
 39:28
 39:29
 43:29
 43:33
 9:7
 1:1
 37:41
 39:44

Exercise 5I

- 1 a** 60 cm **b** 100 cm **c** 220 cm
d 240 cm **e** 360 cm **f** 50 cm
g 96 cm **h** 104 cm **i** 135 cm
j 178 cm **k** 205 cm **l** 105 cm
m 62.5 cm **n** 133.33 cm **o** 257.5 cm
- 2 a** 10 cm **b** 30 cm **c** 15 cm
d 58 cm **e** 78 cm **f** 5.8 cm
g 7.8 cm **h** 12 cm **i** 18 cm
j 25 cm **k** 65 cm **l** 22.5 cm
m 77.5 cm **n** 13.75 cm **o** 126.25 cm



- 3** a 6 m b 20 m c 40 m
 d 3.6 m e 28.8 m f 14.375 m
 g 12 m h 32 m i 255 m
 j 48 m
- 4** a 40 cm b 20 cm c 18 cm
 d 13 cm e 3.5 cm f 7.3 cm
 g 2.4 cm h 3.25 cm i 0.192 cm
 j 5.06 cm
- 5** a 1 m

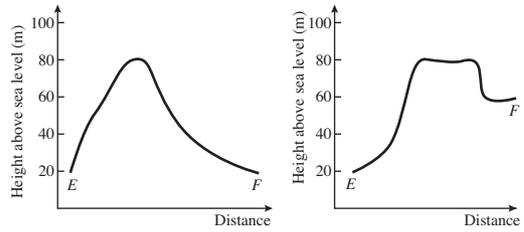
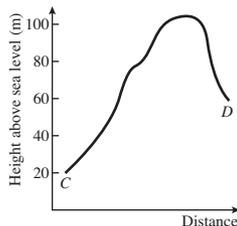
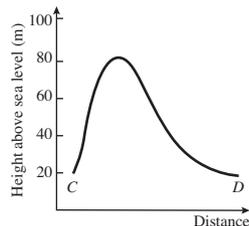
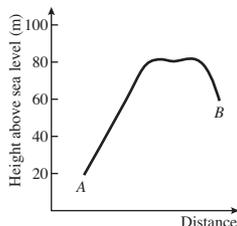
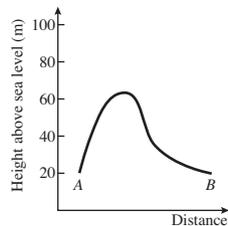
b

Room	Length (m)	Width (m)	Area (m ²)
Dining/kitchen	7.5	4	30
Lounge	9	6	54
Bedroom 1	5	5	25
Bedroom 2	3	5	15
Bedroom 3	3	5	15
Bathroom	3	4	12
Laundry	4	2.5	10
Toilet	3	1	3

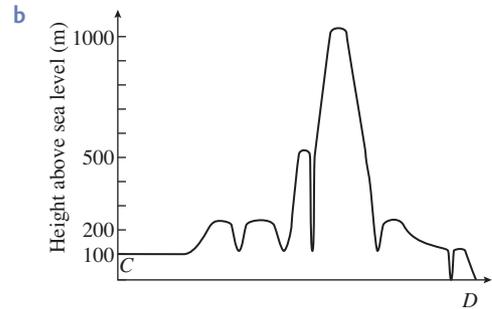
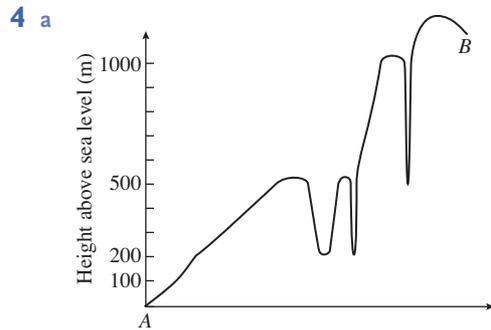
- c i \$10 000 ii \$6000 iii \$6000 iv \$21 600
 d i 4 m² ii 13 m² iii 12 m²
 e i \$2400 ii \$7800 iii \$7200
- 6** a 2 m b 2.4 m c 1.305 m
 d 4.5 m e 39 cm f 7 cm

Learning task 5j

- 1** a 950 m b 1000 m
- 2** a i \overline{AB} 350 m ii \overline{AB} 600 m
 \overline{CD} 430 m \overline{CD} 850 m
 \overline{EF} 500 m \overline{EF} 850 m
- b i \overline{AB} 360 m ii \overline{AB} 630 m
 \overline{CD} 450 m \overline{CD} 820 m
 \overline{EF} 440 m \overline{EF} 710 m



- 3** Mt Deering 1219 m
 Uluru 868 m
 Consuelo Peak 1174 m



Enrichment

- 1** a White: 0.005 g b White: 0.000 05 g
 Bank: 0.004 g Bank: 0.000 04 g
 Cartridge: 0.011 g Cartridge: 0.000 11 g
 Photocopy: 0.007 g Photocopy: 0.000 07 g
- c White: 0.05 kg d White: 50 t
 Bank: 0.04 kg Bank: 40 t
 Cartridge: 0.11 kg Cartridge: 110 t
 Photocopy: 0.07 kg Photocopy: 70 t
- 2** a \$112.50 b \$367.50
 c \$413.40 d \$243.75
- 3** 1800 red balloons and 900 of each of the other colours
- 4** a 12 m³ of cement, 36 m³ of screenings and 60 m³ of sand
 b Sand: 90 000 kg
 Cement: 22 800 kg
 Screenings: 72 000 kg



- 5 a** sides 1 cm, 2 cm, 2 cm
sides 2 cm, 4 cm, 4 cm
sides 3 cm, 6 cm, 6 cm
- b** sides 2 cm, 3 cm, 4 cm
sides 4 cm, 6 cm, 8 cm
sides 6 cm, 9 cm, 12 cm
- c** sides 3 cm, 4 cm, 5 cm
sides 6 cm, 8 cm, 10 cm
sides 9 cm, 12 cm, 15 cm
- 6 a** sides 24 cm, 24 cm, 24 cm
- b** sides of 7.2 cm, 14.4 cm, 21.6 cm, 28.8 cm
- c** 4.8 cm, 9.6 cm, 14.4 cm, 19.2 cm, 24 cm
- d** 4 cm, 8 cm, 12 cm, 12 cm, 16 cm, 20 cm
- 7 a** 0.83 km **b** 7.5 km **c** 225 km
- 8 a** 1.16 km **b** 0.289 km
- c** 1.73 km **d** 3.47 km
- 9 a** 6.6 kg **b** 132 kg **c** 345.58 kg
- 10** 2.5 kg

11 a

<i>a</i>	1	2	3	4	5
<i>b</i>	3	6	9	12	15

$$a = \frac{b}{3}$$

b

<i>a</i>	1	2	3	4	5
<i>b</i>	108	54	36	27	21.6

$$a = \frac{108}{b}$$

c

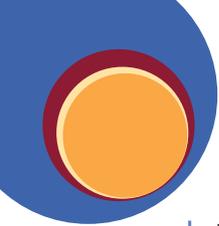
<i>a</i>	3	12	27	48	75
<i>b</i>	6	12	18	24	30

$$b = \sqrt{12a}$$

- 12 a** Bill: 7200 beats/h
Fiona: 12 600 beats/h
Guido: 10 800 beats/h
- b** Bill: 2 beats/s
Fiona: 3.5 beats/s
Guido: 3 beats/s
- c** Bill: 172 800 beats/day
Fiona: 302 400 beats/day
Guido: 259 200 beats/day
- 13 a** 1.2 kg, 2.53 kg, 3.86 kg, 5.19 kg and 6.52 kg
- b** i 7.92 g/h ii 0.13 g/mins iii 0.002 g/s
- 14 a** i 1200 L/h ii 20 000 mL/min
iii 1 200 000 mL/h
- b** i 3 min ii 2.25 min
iii 1.25 min iv 6 min
- 15 a** 110.6 g **b** 774.2 g **c** 308.1 g
d 948 g **e** 23.7 kg
- 16 a** **b** 16 kg **c** 320 kg

Revision/Assessment

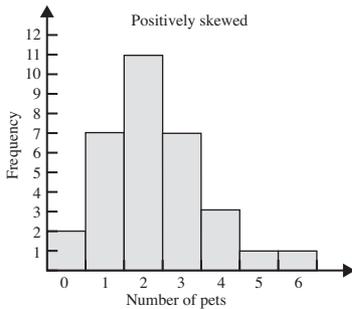
- 1 a** 1:2 **b** 4:3 **c** 9:16
- 2** Trains 1:4 Cars 1:5 Trucks 3:10
Planes 1:10 Buses 3:20
- 3 a** 2:3 **b** 6:1 **c** 4:1 **d** 6:1 **e** 1:12
- 4 a** 35 **b** 8 **c** 4 **d** $\frac{1}{3}$
- 5 a** 6:1 **b** 3:4 **c** 5:24
- 6** 1:2
- 7 a** i 60 ii 150 iii 150
b i 25 ii 585
- 8 a** 60 mL **b** 18 litres
- 9** \$48 000, \$32 000 and \$16 000
- 10** 30 g of lead and 120 g of tin
- 11** 10 kg
- 12 a** Height and time, e.g. rainfall
b Dollars and length, e.g. cost of buying carpet
c Dollars and weight, e.g. price per weight of potatoes
d Litres and time, e.g. litres of petrol consumed in an hour of a journey
- 13 a** \$5/litre **b** 13 goals/quarter
- 14 a** 40 m/min **b** 666 cm/min
- 15 a** 2400 km **b** 400 km **c** $3\frac{1}{3}$ km
- 16 a**
- | Time | Tank A | Tank B |
|-------|--------|--------|
| Noon | 0 | 2.8 |
| 1 pm | 0.5 | 2.5 |
| 2 pm | 1 | 2.2 |
| 3 pm | 1 | 2.2 |
| 4 pm | 1 | 2.2 |
| 5 pm | 1.5 | 1.9 |
| 6 pm | 2.1 | 1.7 |
| 7 pm | 2.1 | 1.4 |
| 8 pm | 2.1 | 2.5 |
| 9 pm | 2.5 | 2.7 |
| 10 pm | 2.9 | 2.9 |
- b** Between 12:00 noon and 2:00 pm, between 4:00 and 6:00 pm and between 8:00 and 10:00 pm
- c** Between 12:00 noon and 2:00 pm and between 4:00 and 7:00 pm
- d** Tank A: 29 cm/h, Tank B: 1 cm/h
- e** i 5:11 ii 3:2 iii 1:1
- 17 a** 12 km **b** 4.8 km
c 9.5 km **d** 12.8 km



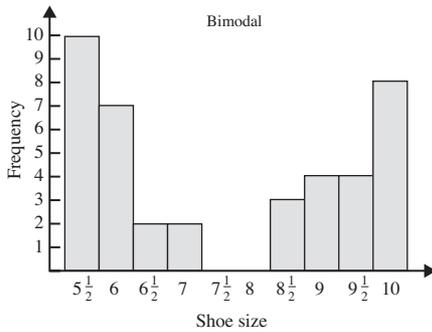
- b The interval from 8 to 9 has the highest frequency.
- c The table shows two peaks, one for 0 or 1 bag, the other for 6–9 bags. This shows that two groups of people are using the supermarket: shoppers who pop in for a few items (1 or no bags) and those who do a large weekly stock up (6–9 bags).

Exercise 6B

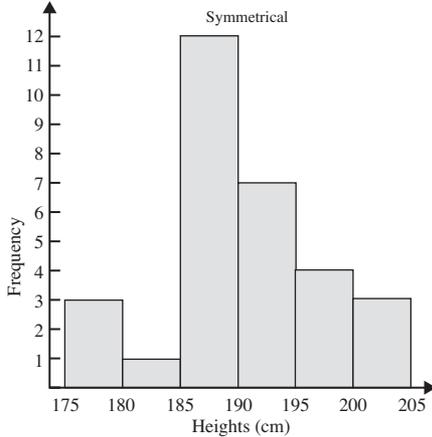
1



2



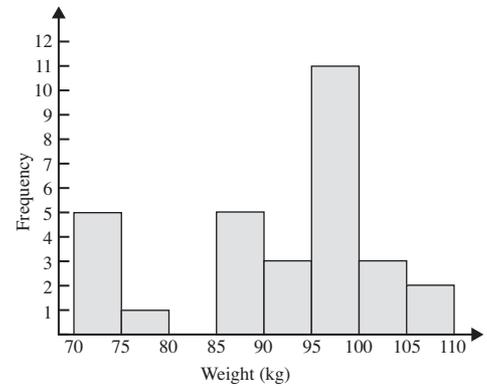
3



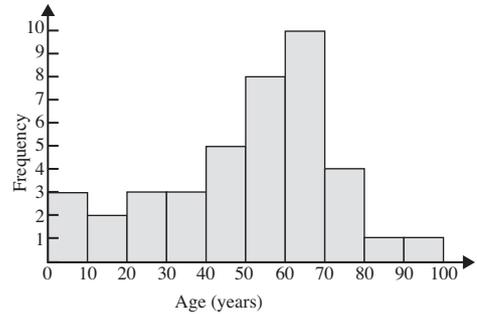
4 a

No.	Frequency
70–	5
75–	1
80–	0
85–	5
90–	3
95–	11
100–	3
105–	2

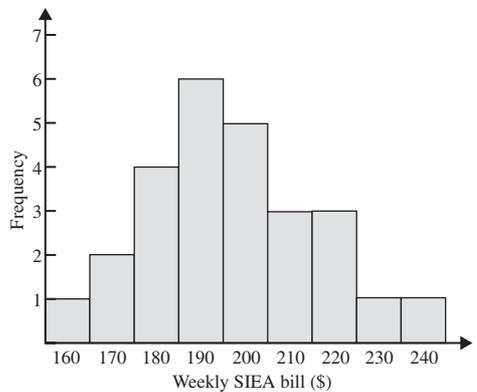
b



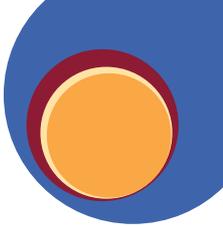
5



6



7 a 7 b 12 c 13 d 4 e 30



Exercise 6C

- 1** Median: 133.5 Range: $201 - 118 = 83$
 IQR: $Q_1 = 129$ $Q_3 = 141$
 IQR = $Q_3 - Q_1 = 141 - 129 = 12$
- 2** Median: 94.5 Range: $108 - 71 = 37$
 IQR: $Q_1 = 88$ $Q_3 = 97$
 IQR = $Q_3 - Q_1 = 97 - 88 = 9$
- 3** Median: 27.5 Range: $42 - 18 = 24$
 IQR: $Q_1 = 24$ $Q_3 = 32.5$
 IQR = $Q_3 - Q_1 = 32.5 - 24 = 8.5$
- 4** Median: 56 Range: $98 - 1 = 97$
 IQR: $Q_1 = 35$ $Q_3 = 67$
 IQR = $Q_3 - Q_1 = 67 - 35 = 32$
- 5** Team A median: 169.5 range: $200 - 152 = 48$
 Team B median: 171.5 range: $175 - 154 = 21$
 The medians are similar but there is more variation in height for team A.
- 6 a** Shooter A
 median: 18 range: $42 - 2 = 40$
 IQR: $Q_1 = 4$ $Q_3 = 35$
 IQR = $Q_3 - Q_1 = 31$
 Shooter B
 median: 18 range: $28 - 10 = 18$
 IQR: $Q_1 = 15$ $Q_3 = 24$
 IQR = $Q_3 - Q_1 = 9$
- 7 a** Primary
 median: 102 range: $128 - 63 = 65$
 IQR: $Q_1 = 93$ $Q_3 = 108$
 IQR = $Q_3 - Q_1 = 15$
 Secondary
 median: 100 range: $140 - 56 = 84$
 IQR: $Q_1 = 92$ $Q_3 = 109$
 IQR = $Q_3 - Q_1 = 17$
- b** The median appears to be slightly lower in secondary school learners than in primary school learners. It has fallen from 102 to 100. The range has increased from 65 to 84 and IQR from 15 to 17.5. This indicates that the spread of IQ has increased. There is more variation in the IQs of secondary school learners than in primary school learners.

Exercise 6D

- 1 a**

6	1 8 9
7	2 4 6 8 8 8 8 8
8	1 6 7 9
9	2 2 4 3
10	0 1 2 3 5 6
11	0 2
12	3 5 6
- b** Median: 90.5 Range: $126 - 61 = 65$
 IQR: $Q_1 = 78$ $Q_3 = 103$
 IQR = $Q_3 - Q_1 = 25$
- c** 10 **d** 11

- 2 a**

0	2 3
1	6
2	1 2 2 6 8
3	0 2 4 5 5 8 9 9
4	2 2 3 3 5 7 7 9
5	1 1 2 3 5 6 6 7 7 8 9
6	1 4 5 5 7

b 44

- 3 a**

2	76 86 93 97 97 98
3	17 25 26 41 48 52 67 69 70 77 78 81 84
	85 93
4	03 12 18 25

c 3.67 kg

- 4 a**

16	9
17	7 8
18	1 5 7 8
19	2 3 3 4 5 7
20	2 4 5 7
21	4 8 9
22	1 3 4 9
23	6
24	7

b \$199.50 **c** Yes

- 5 a**

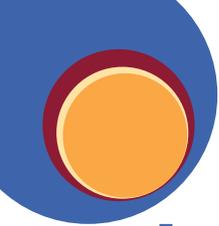
24	7
25	4 4 6
26	3 4 4 5 5 6 7 8
27	0 2 3 4 5 6 6 6 8
28	1 2 2 3 3 4 9
29	1 3 7

b Median: 27.4 Range: $29.7 - 24.7 = 5$

c 18 days

Exercise 6E

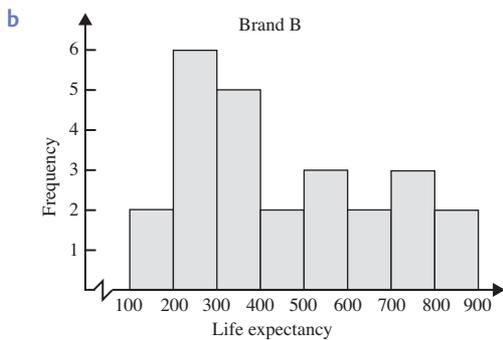
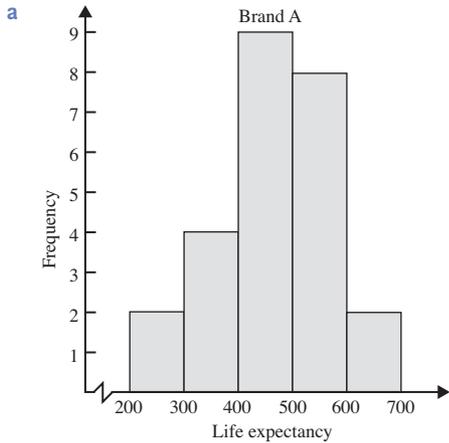
- 1 a** 6.133 bags **b** 33
- 2** 189.97 cm **3** 91.33 kg
- 4 a** 3.53 kg **b** A decrease **c** 3.49 kg
- 5 a** \$26 400; median: \$25 500
b \$29 400; median: \$27 000
c 8 **d** 10
- 6 a** 27.32°C **b** 16 days
- 7 a** $\Sigma x = 31.5$ **b** $\Sigma x = 276$
- 8 a** Various answers are possible, e.g. 2.7, 2.8, 2.9
b Various answers are possible, e.g. 7.2, 7.3, 7.4, 7.5, 7.6
- 9** $\bar{x} = 2.28$
- 10** $\bar{x} = 7.5$ hours
- 11** $\bar{x} = 62.8$, $Av = 62.8\%$
- 12 a** $\bar{x} = \$203$ **b** Yes **c** $x = \$169$
- 13** $x = \$465$



Exercise 6F

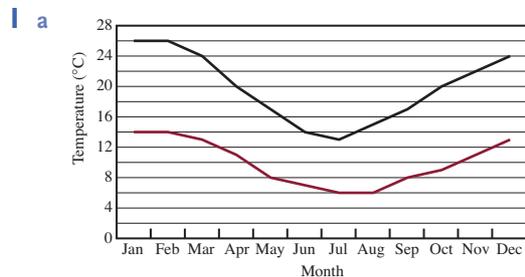
- 1 a 69.2 b 15.36
 2 a 19.83°C b 3.86°C

Exercise 6G

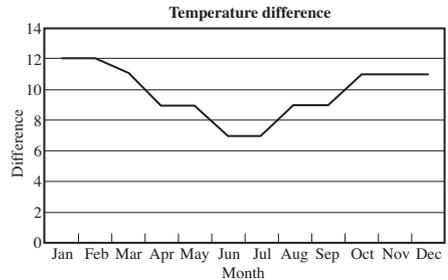


- c More variation in Brand B. Brand A is more consistent.
 d Mean: A 468 hours B 468 hours
 Median: A 471 hours B 378 hours
 e A 76.24, B 185.88
 Brand A has a lower absolute mean difference as the spread of data is smaller.
 f The mean for both brands is the same. Brand A has a higher median because Brand B is positively skewed.

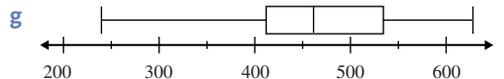
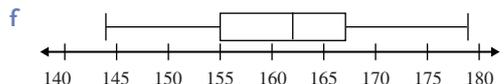
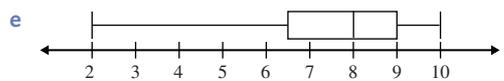
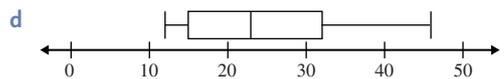
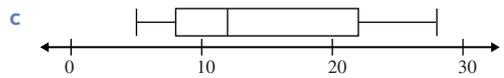
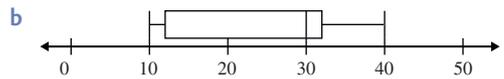
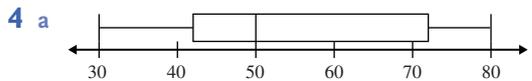
Enrichment



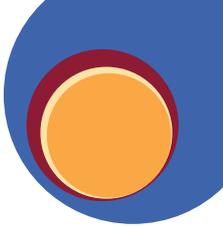
- b The temperature difference was greatest between January and February.
 c The difference was smallest for June and July.
 d



- 2 a \bar{x} max = 191.83 cm
 b \bar{x} min = 187.83 cm
 c $\bar{x} = \frac{\bar{x} \text{ max} + \bar{x} \text{ min}}{2} = 189.83 \text{ cm}$
 3 $178 \times n + 194 = 180 \times (n + 1)$
 $194 - 180 = 180n - 178n$
 $14 = 2n$
 $n = 7$



- 5 a Lowest: 10 Q_1 : 12
 Median: 15 Q_3 : 18
 Highest: 20 IQR: 6
 6 a Lowest: 0 Q_1 : 1
 Median: 1.5 Q_3 : 3
 Highest: 10 IQR: 2
 7 a Lowest: 0 Q_1 : 2
 Median: 15 Q_3 : 16
 Highest: 20 IQR: 14



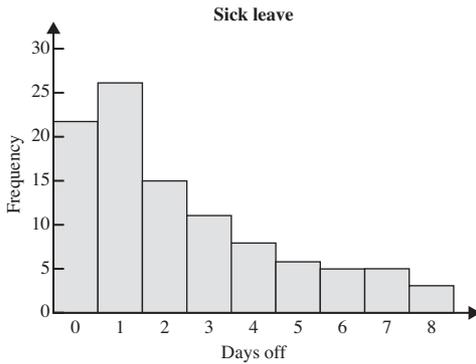
Revision/Assessment

1 a

Time	Frequency
70–	7
80–	7
90–	11
100–	6
110–	6
120–	6
130–	8
140–	4

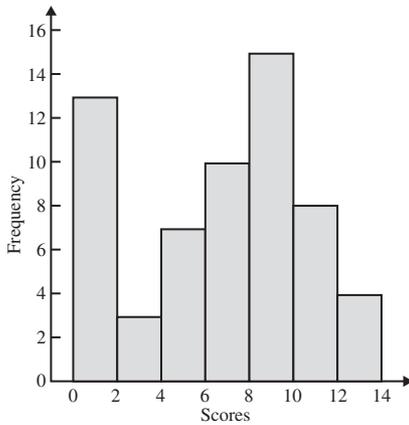
b 13

2



b The graph is positively skewed.

3 a



c The most frequent interval is from 8 up to but not including 10.

4 a \$2.50

b \$1.50

5

```

1 | 6 6 7 8 9
2 | 2 3 4 4 4 4 4 5 5 5 6 6 7 7 8 8 8 9 9 9
3 | 0 1 1 1 2 2 2 4 5 6 6 6 6 8
4 | 0 1 2
  
```

6 Median: 28 Range: 26 IQR: 8

7 $\bar{x} = \$2.54$

8 $\bar{x} = 2.40$

9 $x = 54$ words per min

10 a 63.39 thefts

b 14.61

Chapter 7

Exercise 7A

a Reflection

b Rotation

c Reflection

d Translation

e Reflection

f Reduction and translation

Exercise 7B

1 a R5

b L5

c L4

d D1

e L2 U3

f R2 D2

2 a R4 U1

b R4 D3

c R1 U3

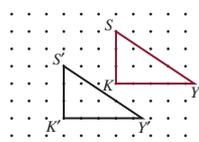
d R3 D3

e L1 U4

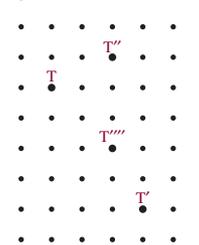
f R2 U3

3 Right 5 Down 5

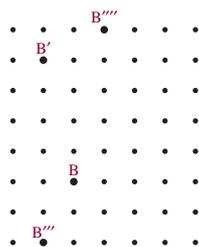
4



5 T'''



6



7 a 1R 6D

b 2L 8U

c 2L 3D

d 1L 16U

e 8R 1U

f 4R 7D

9 a 4R 1D

b 8R 3D

c 12R 2D

d 4L 2U

e 3L 1U

f 9L 1U

g 16L 3U

h 8L

10 a 4R 1D

b 8R 3D

c 12R 1U

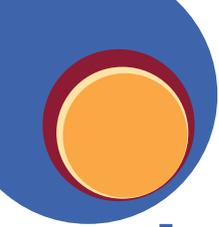
d 2U 4L

e 4L 1D

f 8L

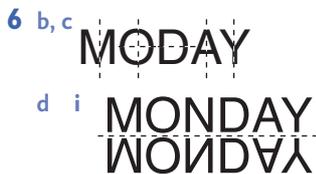
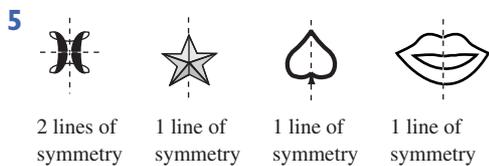
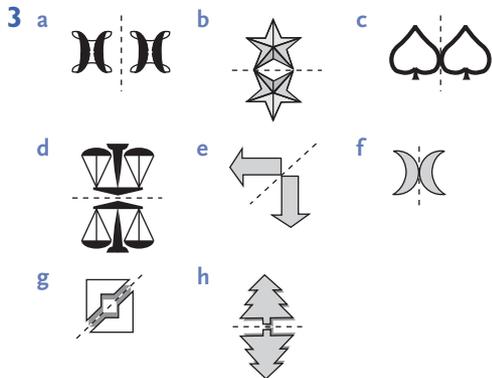
g 16L

h 8L 3D



Exercise 7C

- 1 a The shapes are not congruent.
 b They have been reflected.
 c The images are not mirror images. This is a translation.
 d The images are not mirror images. This is a translation.
 e This is reflection.
 f The shape has been reflected then translated.
- 2 a Reflection
 b Reflection
 c This is not a reflection as the shapes are not congruent.
 d This is not a reflection as the shapes are not congruent.
 e Reflection
 f This is not a reflection as the shapes are not congruent.



Exercise 7D

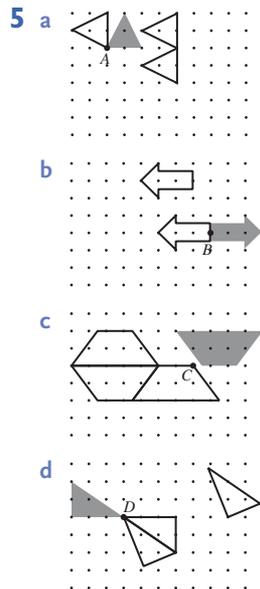
- 1 a 180° clockwise b 90° clockwise

 c 90° anticlockwise d 180° clockwise

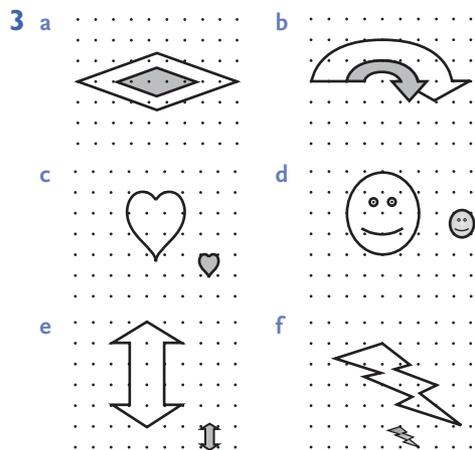
- e 90° clockwise f 180° clockwise

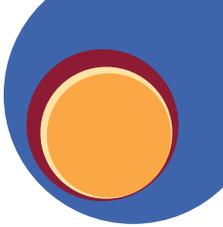
 g 90° clockwise h 180° anticlockwise

- 4 a 30° clockwise b 60° clockwise
 c 60° clockwise d 330° clockwise
 e 120° clockwise f 210° clockwise
 g 150° clockwise h 300° clockwise
 i 270° clockwise j 210° clockwise
 k 150° clockwise l 210° clockwise



Exercise 7E

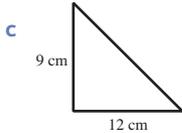




Learning task 7F



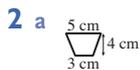
b $A = \frac{1}{2}bh$
 $= \frac{1}{2} \times 3 \times 4$
 $= 6 \text{ cm}^2$



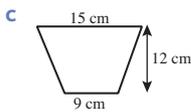
$A = \frac{1}{2}bh$
 $= \frac{1}{2} \times 9 \times 12$
 $= 54 \text{ cm}^2$

d 9 times

e $3^2 = 9$ times
 since $6 \times 3^2 = 54 \text{ cm}^2$



b $A = \frac{1}{2}(a+b)h$
 $= \frac{1}{2}(3+5) \times 4$
 $= 16 \text{ cm}^2$

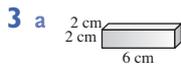


$A = \frac{1}{2}(a+b)h$
 $= \frac{1}{2}(9+15)12$
 $= 144 \text{ cm}^2$

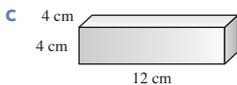
d 9 times

e $3^2 = 9$ times, since $16 \times 3^2 = 144 \text{ cm}^2$

f The square of the linear change gives the change in area.



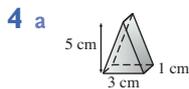
b $V = 24 \text{ cm}^3$



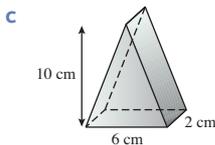
$V = 192 \text{ cm}^3$

d 1 time $\frac{192}{24} = 8$ times

e $2^3 = 8$ times since $24 \times 8 = 192 \text{ cm}^3$



b $V = 7\frac{1}{2} \text{ cm}^3$



$V = 60 \text{ cm}^3$

d 8 times

e $2^3 = 8$ times, since $7\frac{1}{2} \times 8 = 60 \text{ cm}^3$

f The cube of the linear change gives the change in volume.

5 $3 \times 2^2 = 12 \text{ cm}^2$

6 $4 \times 3^2 = 36 \text{ cm}^2$

7 $4 \times 2^3 = 32 \text{ cm}^3$

8 $24 \times \left(\frac{1}{2}\right)^3 = 3 \text{ cm}^3$

9 k^2

10 k^3

Exercise 7G

1 a, d, e, h

2 There are various answers to this question. One possible set of transformations is given:

a Rotated 180° clockwise about the vertex to the right of the original shape, then translated 1 unit right.

b Rotated 90° clockwise about the left point of the base, then translated 2 units right, 4 units up.

c Enlarged by scale factor $\times 1\frac{1}{2}$, then translated 4 units right, 3 units up.

d Reflected in the vertex of the arrowhead, then translated 3 units right.

e Reduced by scale factor $\times \frac{1}{2}$, then translated 3 units right, 1 unit up.

f Rotated 90° anticlockwise about the middle vertex of the base of the arrow, then translated 4 units right, $\frac{1}{2}$ unit up.

3 a, b

	Shape 1	Shape 2	Shape 3	Shape 4
$\angle A$	75°	75°	75°	75°
$\angle B$	50°	50°	50°	50°
$\angle C$	55°	55°	55°	55°
\overline{AC}	20 mm	20 mm	20 mm	20 mm
\overline{BC}	26 mm	26 mm	26 mm	26 mm
\overline{AB}	22 mm	22 mm	22 mm	22 mm

c The corresponding lengths are the same in each shape.

d The corresponding angles in each shape are the same.

e Shape 1 \equiv Shape 2 \equiv Shape 3 \equiv Shape 4.

Applications

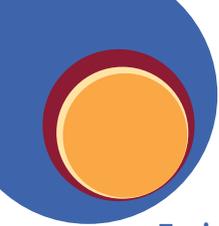
Tessellations

1 All triangles will tessellate.

2 All quadrilaterals will tessellate and special quadrilaterals usually tessellate in more than one pattern.

Border patterns





Enrichment

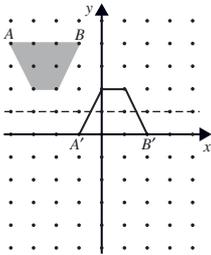
4 Line symmetry

MAM
WOW
MUM

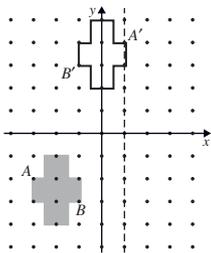
5 Rotational symmetry

MOW
SOS
SIS

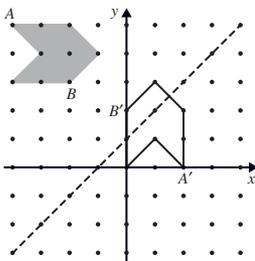
6 i a



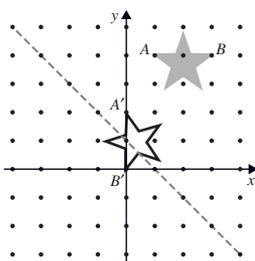
b



c



d



- ii a $A(-4, 4)$ $B(-1, 4)$ iii a $A(-1, 0)$ $B(2, 0)$
 b $A(-3, -2)$ $B(-1, -3)$ b $A(1, 4)$ $B(-1, 3)$
 c $A(-4, 5)$ $B(-2, 3)$ c $A(2, 0)$ $B(0, 2)$
 d $A(1, 4)$ $B(3, 4)$ d $A(0, 2)$ $B(0, 0)$

Revision/Assessment

1 The arrow has been translated 5 units to the right and 2 units down.

3 True. After reflection, a shape is congruent to its image; the size and shape does not change.

4 a

b

5 The arrow has been rotated 60° clockwise.



6 a

b

c

d

7 a 45° clockwise

b 90° clockwise

c 135° clockwise

d 45° anticlockwise or 315° clockwise

e 45° clockwise

f 135° clockwise

g 180° clockwise

h 90° anticlockwise or 270° clockwise

9 Congruent shapes have the same shape and are the same size. All corresponding angles and sides are equal.



13 b and f are congruent.

14 If all corresponding angles are equal then the quadrilaterals must have the same shape but the side lengths may be different. These shapes are similar but not congruent. For example, the rectangles below each have four right angles, but the corresponding side lengths are not equal.



Solomon Islands MATHEMATICS Year 8 Learner's Book

Book 1

Mathematical knowledge is essential for full participation in Solomon Islands life, both at school as learners and in the future as adults.

Mathematics is the exploration and use of patterns, relationships and variations in quantity, space, and time, as well as the interpretation of statistical data.

Solomon Islands Mathematics Year 8 Learner's Book integrates these aspects of mathematics into a wide range of social, cultural, scientific, technological, environmental, health and economic contexts, representing both real-life and hypothetical situations.

Learners in secondary schools will build on their existing knowledge and skills in five main strands:

- Number
- Measurement
- Algebra
- Geometry
- Probability and Statistics.

Learners will acquire effective strategies for investigating, interpreting, explaining and making sense of the world, using numbers, symbols and graphs. They will develop the ability to think creatively, critically, strategically and logically. These skills and approaches will have long-term applications throughout the learners' lives.

