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8

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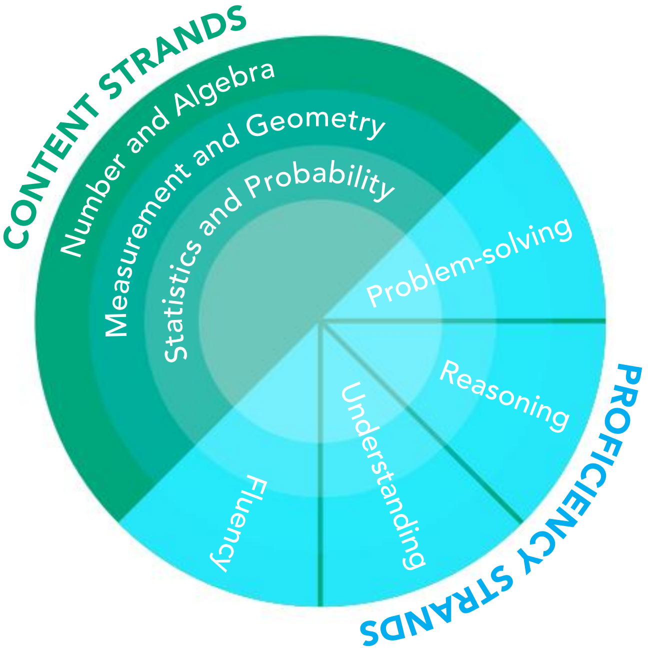
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

Pearson Mathematics shares these aims.

Pearson Mathematics Second Edition is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills.



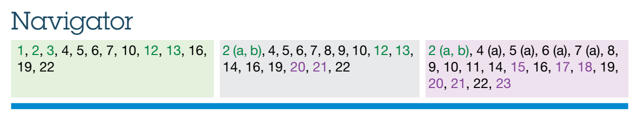
Supporting all learners

Differentiation

Pearson Mathematics Second Edition has been written and designed for the needs of the full ability spectrum of students in Australian classrooms.

Recall—Each chapter begins with a review of assumed and necessary knowledge for the chapter. For students needing extra revision, Recall Worksheets are available for each Recall question, with explanations to refresh understanding and exercises to practise skills.

Exercises—All Exercises include questions according to the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has ‘Open-ended’ questions to encourage students’ creative thinking and ability to communicate mathematics effectively.



Navigator

1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 16, 19, 22	2 (a, b), 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 19, 20, 21, 22	2 (a, b), 4 (a), 5 (a), 6 (a), 7 (a), 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

Navigator—Three optional graded pathways through *every* Exercise, with every question rated as Foundation, Standard or Advanced level.

Challenge—Every chapter includes a full page Challenge section for early finishers and advanced students.

Regular revision and reinforcement

Pearson Mathematics Second Edition has a broad range of cumulative and chapter-based revision.

Half-time—A mid-chapter review of chapter content so far.

Chapter review—An end-of-chapter thorough review of chapter content.

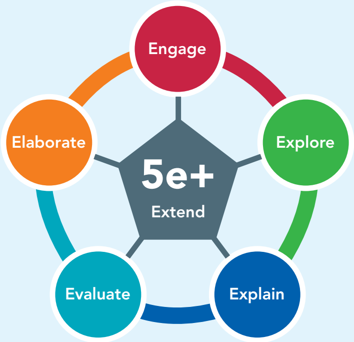
Mixed review—Cumulative revision that mixes content from previous chapters.

Encouraging inquiry and problem solving

Investigation

These are scaffolded to be accessible to all learners and structured following a 5e+ format.

Investigation



Exploration Spreadsheet and CAS

Pearson Mathematics Second Edition understands the importance placed on students developing technological literacy. Explorations with CAS technology and spreadsheet software allow students to learn and practice their technical proficiency, while also helping to develop students' deeper understanding of the mathematical concepts covered.

Exploration Spreadsheet

Exploration CAS

Problem solving, Puzzles and Games

These are located throughout every chapter and give students fun opportunities to develop their problem-solving skills and logical reasoning.

Problem solving

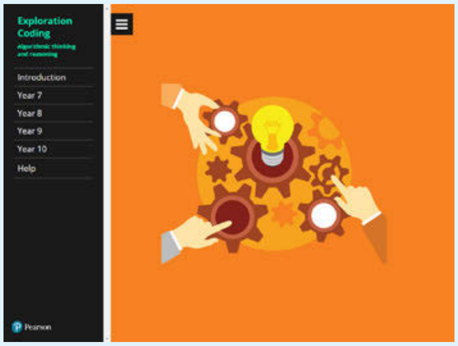
Puzzle

Game

Exploration Coding

Algorithmic thinking is an increasingly relevant component of mathematics education for the 21st century. Algorithmic thinking and coding tasks encourage learners to understand and develop reasoning skills with computational procedures, algorithms and logical problem-solving, exploring a coding environment within a mathematical context.

Exploration Coding



Real-life contexts and engagement

Exploration Task and STEM

Practical contexts provide useful learning opportunities and natural pathways into interesting, relevant mathematics.

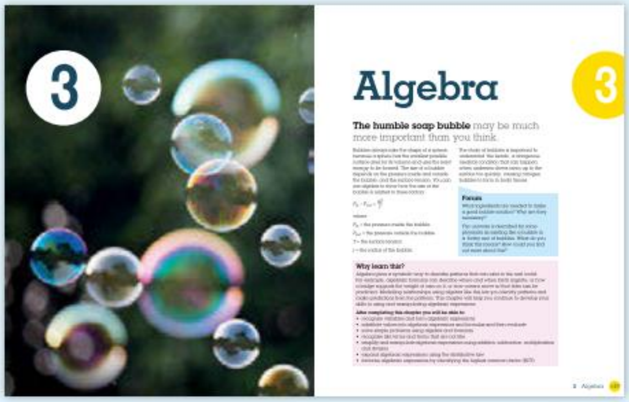
Exploration tasks are rich tasks that encourage student engagement, questioning and creative thinking. STEM activities allow students to explore topics related to Science, Technology, Engineering and Design with a Mathematics perspective.

Exploration Task

Exploration STEM

Home Page

Anticipate the 'Why learn this?' question and provide a motivating entry into every new chapter topic, including discussion-provoking Forum questions.



3

Algebra

The humble soap bubble may be much more important than you think.

Why learn this?

Pearson Mathematics Second Edition has been designed to capture students' interest, with the incorporation of Maths 4 Real, Gamespace and more.

Maths 4 Real

Scenarios that help students make connections with mathematics in the real world.

Maths 4 Real

Relative Wealth

Are you wealthy?

Gamespace

Skill consolidation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games and solve-the-riddle, find-the-clue tasks and games.

Gamespace

MAGIC ALGEBRA

MAGIC TRICK 6: MIND READER

MAGIC TRICK 4: WHEN IS MY BIRTHDAY?

MAGIC TRICK 5: CREATE YOUR OWN

MAGIC TRICK 1

MAGIC TRICK 2

1

Integers and indices

1

How low can you go? Humans have explored much of the land areas of our planet, but have seen very little of the vast depths of Earth's oceans.

The highest point on the Earth's surface is the summit of Mt Everest, a height of 8848 m above sea level (+8848 m). The lowest point on Earth is the bottom of the Mariana Trench, in the Pacific Ocean. It is believed to reach at least 10 994 m below sea level (-10 994 m). In 1960, naval lieutenant Don Walsh and engineer Jacques Piccard made the first expedition to the bottom of the trench. They reached a depth of -10 916 m. When they resurfaced, the windows of their submersible were cracked from the pressure of the water. Because of the dangers of high pressures, much of the Earth's oceans have only been explored to a depth of -300 m. Only a few submersibles can go beyond -3000 m, and even military submarines usually travel no deeper than -500 m. Below these depths, an amazing underwater world is waiting to be explored.

Forum

Have you ever been in negative temperatures? If you have, where were you?

If you were able to pick up Mt Everest and place it in the Mariana Trench, it would be completely covered by water. How much water would there be between the top of Mt Everest and sea level?

Why learn this?

Positive numbers are only half the story! You need to be able to work with numbers that are less than zero—the negative side of the number line. Negative numbers are used to show actions that are opposite to positives, such as a move backwards instead of forwards, or a fall in price instead of a rise.

Indices (also called powers or exponents) provide a convenient way to write and work with very large numbers. Having rules for working with indices helps make calculations more efficient.

After completing this chapter you will be able to:

- use directed numbers (integers) in everyday situations
- do the four operations (+, −, ×, ÷) on directed numbers
- use a number line to help with directed number calculations
- apply the order of operations rules to directed number calculations
- understand and use index notation
- work efficiently with index notation by applying the appropriate rules.

Recall 1

Prepare for this chapter by trying the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 To answer the following, it may help to draw a diagram.

- **(a)** At 7 am on a winter morning, the temperature was 4 °C. At midday, the temperature was 13 °C. By midnight it had dropped to -2 °C.
  - **(i)** By how many degrees did the temperature increase, from 7 am to midday?
  - **(ii)** By how many degrees did the temperature decrease, from midday to midnight?
  - **(iii)** What was the difference between the temperature at 7 am and the temperature at midnight?
- **(b)** A city building has 5 storeys above the ground floor and 2 basement levels below the ground floor.
  - **(i)** Jade parked her car in the 2nd basement level, got in the lift and moved up 5 floors. At which floor did she get out?
  - **(ii)** Erin got in the lift on the 4th floor. She went down 5 floors, then up 2 floors. At which floor did she get out?

2 Write the value of the following. **(a)** 2^3 **(b)** 3^4 **(c)** 5^2
(d) 10^6 **(e)** $\sqrt{36}$ **(f)** $\sqrt{49}$ **(g)** $\sqrt[3]{64}$ **(h)** $\sqrt[3]{8}$

3 Imagine that you are standing at -4 on a large number line. Which number would you land on if you walked:

(a) 7 places to the right **(b)** 5 places to the left?

4 Calculate: **(a)** 2×8 **(b)** 7×9 **(c)** 38×6 **(d)** 15×19
(e) $18 \div 3$ **(f)** $54 \div 9$ **(g)** $324 \div 3$ **(h)** $616 \div 4$

5 Calculate: **(a)** $(8 \div 4) + (35 \div 5)$ **(b)** $6 \times 8 + 2 \times 3$ **(c)** $50 - 5 \times 2 + 3$

6 Use a factor tree to find the prime factors of the following, then write each number as a product of its prime factors in index form.

(a) 24 **(b)** 36 **(c)** 120

7 Find the value of the following. **(a)** $4^2 + 3^2$ **(b)** $2^2 + 5^2$ **(c)** $8^2 - 6^2$
(d) $9^2 - 3^2$ **(e)** $5^2 \times 2^4$ **(f)** $3^3 \times 5 \times 7$ **(g)** $9^2 \div 3^2$ **(h)** $2^3 \times 7 \div 2^2$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Comparing powers

In this activity, you will explore what can make numbers raised to powers greater or smaller.

Integers review

1.1

A **positive** (+) or a **negative** (-) sign is used to show the direction of a number. Some words associated with negative numbers are: down, loss, below, decrease, lose, withdrawal. Some words associated with positive numbers are: up, profit, above, increase, gain, deposit.

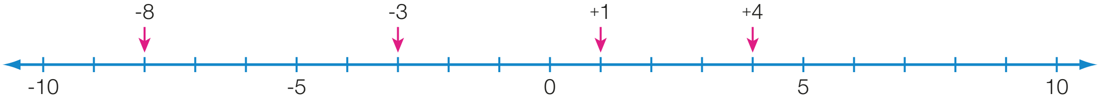
For example, a deposit of $50 into your bank account could be written as +50, and a withdrawal of $180 could be written as -180.

Date	Transaction	Balance
		$500
17 July	ATM Withdrawal -$180	$320
20 July	Birthday money from Mum +$50	$370

What is an integer?

The **integers** are all of the positive whole numbers 1, 2, 3, … , the negative whole numbers -1, -2, -3, … , and zero. (The use of '…' in mathematics shows that the sequence of numbers continues forever.) Positive and negative integers are also called *directed numbers*.

Integers can be represented on a number line:



The numbers on a number line get larger in value as you move from left to right.

If a number has no sign in front of it, assume it is positive. 4 = +4

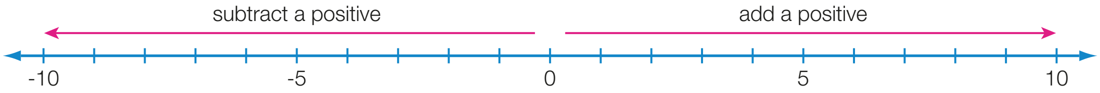
From the number line above, you can write the statement $+4 > -8$. You could put this in words as 'positive 4 is greater than negative 8'. You could also write the statement $-3 < +1$, which is the same as saying 'negative three is less than positive one'. Number lines can also be written vertically, with the positive numbers above the negative ones. A thermometer is an example of a vertical number line.

Adding and subtracting integers

Adding and subtracting positive integers is the straightforward addition you have been doing for years.

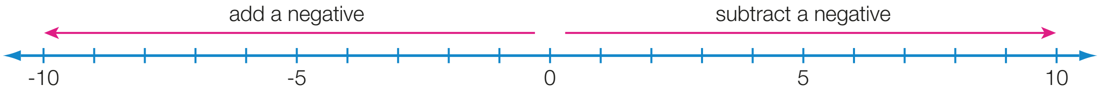
To *add a positive integer*, imagine moving that many places to the *right* on the number line (or upwards on a vertical number line), in the positive direction.

To *subtract a positive integer*, imagine moving that many places to the *left* on the number line (or downwards on a vertical number line), in the negative direction.



Adding a negative integer is the opposite of adding a positive one. It means you move in the negative direction, to the *left* on the number line (or downwards on a vertical number line). You can see that this is the same movement as subtracting a positive integer.

Subtracting a negative integer is the opposite of subtracting a positive one. It means you move in the positive direction, to the *right* on the number line (or upwards on a vertical number line).



Simplifying addition

When adding and subtracting integers, brackets are often placed around the second number and its sign, to separate it from the addition or subtraction symbol; for example, $+5 + (+9)$ or $+7 - (+3)$. However, as you can write a positive integer without the + sign in front, you can drop the positive signs in front of the numbers, remove the brackets, and simply write $5 + 9$ or $7 - 3$.

You can see from the previous number lines that subtracting a negative integer is the same as adding the positive integer. This means that you could write $8 - (-2)$ as $8 + 2$.

You can also see that adding a negative integer is the same as subtracting the positive integer. This means you could write $4 + (-10)$ as $4 - 10$.

$+ (+)$ and $- (-)$ can be replaced with $+$

$- (+)$ and $+ (-)$ can be replaced with $-$

You could also say:

When the two signs are the *same*, add.

When the two signs are *different*, subtract.

Worked example 1

W.E. 1

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) $-4 + (+11)$ **(b)** $-9 + (+3)$ **(c)** $+2 - (+8)$ **(d)** $-5 - (+11)$

	Thinking	Working
(a) 1	Simplify the addition by writing positive integers without their signs, and removing the brackets.	(a) $-4 + (+11)$ $= -4 + 11$
2	Imagine (or draw) a number line. Start at the first number (-4), then move the number of places indicated by the second number (11) in the positive direction (to the right). Write your answer.	$= 7$
(b) 1	Simplify the addition by writing positive integers without their signs, and removing the brackets.	(b) $-9 + (+3)$ $= -9 + 3$
2	Imagine (or draw) a number line. Start at the first number (-9), then move the number of places indicated by the second number (3) in the positive direction (to the right). Write your answer.	$= -6$

(c)	**1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	(c) +2 – (+8) = 2 – 8
	2	Imagine (or draw) a number line. Start at the first number (2), then move the number of places indicated by the second number (8) in the negative direction (to the left). Write your answer.	= -6
(d)	**1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	(d) -5 – (+11) = -5 – 11
	2	Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (11) in the negative direction (to the left). Write your answer.	= -16

Worked example 2

W.E. 2

Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +9 + (-7) **(b)** -12 + (-1) **(c)** +4 – (-6) **(d)** -5 – (-7)

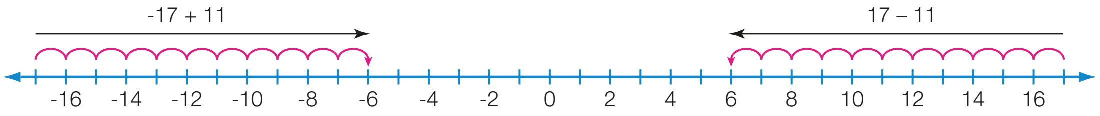
Thinking			**Working**
(a)	**1**	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	(a) +9 + (-7) = 9 – 7
	2	Complete this straightforward subtraction. Write your answer.	= 2
(b)	**1**	Remove the brackets and write a single subtraction symbol between the numbers. Write positive integers without their signs.	(b) -12 + (-1) = -12 – 1
	2	Imagine (or draw) a number line. Start at the first number (-12), then move the number of places indicated by the second number (1) in the negative direction (to the left). Write your answer.	= -13
(c)	**1**	Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.	(c) +4 – (-6) = 4 + 6
	2	Complete this straightforward addition.	= 10

(d) **1**	Remove the brackets and write a single addition symbol between the numbers. Write positive integers without their signs.	(d) -5 – (-7) = -5 + 7
2	Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (7) in the positive direction (to the right). Write your answer.	= 2

Using number line symmetry

The number line is symmetrical about zero. A positive number is the same distance from zero as its negative opposite; for example, negative 6 is the same distance from zero as positive 6. This can be useful when adding integers, especially large ones.

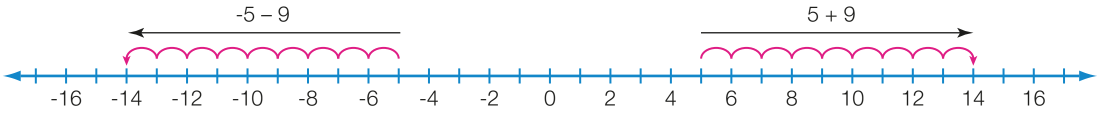
For example, if you show -17 + 11 on a number line, it is the mirror image of showing 17 – 11.



So, to calculate -97 + 64, you could simply do 97 – 64, then change the sign of the answer.

97 – 64 = 33 -97 + 64 = -33

Similarly, to calculate -5 – 9, you could consider the mirror image situation: 5 + 9.



So, to calculate -53 – 49, you could simply do 53 + 49, and change the sign of the answer.

53 + 49 = 102 -53 – 49 = -102

1.1 Integers review

Navigator

Answers p. 616

1 (columns 1–3), 2, 3 (columns 1–3), 4, 5, 6, 8, 9, 10, 13, 15, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 14, 15, 16, 17	1 (columns 3–4), 3 (columns 3–4), 4, 5 (row 2), 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19

Fluency

W.E. 1

1 Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) +4 + (+9)
(b) -7 + (+5)
(c) +5 + (+3)
(d) +2 – (+6)
(e) +10 – (+13)
(f) -3 + (+8)
(g) +12 + (+6)
(h) -1 – (+9)
(i) -15 + (+8)
(j) +8 – (+9)
(k) -8 – (+13)
(l) -19 + (+11)
(m) +16 – (+9)
(n) -14 + (+7)
(o) -4 – (+22)
(p) -17 + (+23)

2 Use your calculator to evaluate the following.

(a) $-3 + -5$
(b) $22 - 12$
(c) $13 + -29$
(d) $84 + 12$
(e) $39 + +6$
(f) $-46 - -12$
(g) $-52 + -16 + 12$
(h) $22 - 13 + 6$
(i) $46 - +24 + -13 - -20$

3 Simplify the following by writing a single symbol between the two numbers, then calculate the answer. **W.E. 2**

(a) $+5 + (-4)$
(b) $+7 + (-9)$
(c) $+5 - (-11)$
(d) $-7 - (-6)$
(e) $-4 - (-9)$
(f) $-3 - (+8)$
(g) $-14 - (+3)$
(h) $+12 - (-8)$
(i) $+19 - (+13)$
(j) $0 - (-3)$
(k) $-8 - (-16)$
(l) $+15 + (-22)$
(m) $-11 - (-7)$
(n) $-13 + (-9)$
(o) $+25 + (-31)$
(p) $-27 - (-16)$

4 Write a negative or a positive integer to describe the following situations.

(a) 350 m above sea level
(b) a loss of $4800
(c) rewinding 6 seconds of an audio recording
(d) depositing $73 into your bank account
(e) 2 levels below the ground floor of a building
(f) 19 metres under water
(g) a company profit of $10 750
(h) skipping 34 minutes of a TV episode
(i) withdrawing $200 from your bank account
(j) a plane flying at an altitude of 8100 metres

5 Use the symbols greater than (>) or less than (<) between the following pairs of integers to show their relationship. A number line may be useful.

(a) +2 ___ +6
(b) -3 ___ +1
(c) +5 ___ -4
(d) -1 ___ -3
(e) +5 ___ -10
(f) -6 ___ -8
(g) +16 ___ -16
(h) -33 ___ -12

6 Write the following in descending order (largest to smallest).

(a) +4, 0, -7, +11, -2
(b) -23, 1, 0, -9, +7
(c) -3, 4, 0, 11, -15, 1
(d) -5, 8, 19, -43, -2, 6
(e) 14, -72, 5, 26, -1, -38
(f) 32, -19, 0, 17, -56, 4

7 Calculate:

(a) $2 + 7 - 5$
(b) $-3 + 10 - 5$
(c) $-6 + 4 - 8$
(d) $-15 + 9 + 8$
(e) $11 + 14 - 23$
(f) $-7 - 8 - (-9)$
(g) $4 + 5 - (-5)$
(h) $-6 + (-9) - (+9)$

Understanding

8 **(a)** Copy the diagram's vertical line and label it according to the following information.

A bird is 24 metres above the water.

A scuba diver is 17 metres below the surface.

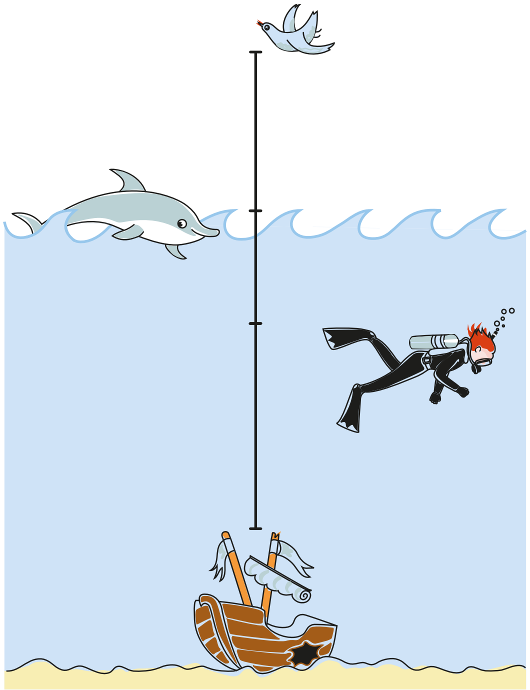
The wreck of a ship is 31 metres below the diver.

A dolphin is just on the surface of the water.

(b) Use your diagram to find out:

(i) the distance between the bird and the diver

(ii) the distance between the wreck and the dolphin.



9 Ahmed has saved $110 to buy a new bicycle, but the price is $200. Ahmed's parents lend him the rest of the money, to be repaid later at $15 per month.

(a) Write an integer expression to show how much Ahmed needs to borrow from his parents.

(b) Complete and extend the following table to find how long it would take Ahmed to repay his loan.

Month	Initially	After 1 month	After 2 months	After 3 months
Money owing				

10 The maximum temperatures and minimum temperatures recorded during a week in June on Mount Kosciuszko are shown in this table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (°C)	11	13	8	6	9	12	13
Minimum (°C)	-3	-1	-7	-5	-4	-1	0

(a) On which day was the lowest minimum recorded?

(b) On which day was the highest minimum recorded?

(c) What was the difference between the maximum and the minimum temperatures on:

(i) Thursday **(ii)** Friday?

(d) On which day was the difference between the maximum and the minimum temperatures the greatest?

11 On Monday, Rachel withdrew $120 from her bank account at an ATM. On Tuesday, she used her bankcard to pay $87 for her shopping from the same account. On Thursday, her employer deposited her salary of $243 into the account, and her mum also deposited $50 for her birthday. On Friday, Rachel used the account to pay a $109 bill online.

(a) Write one long integer calculation that shows each of the events mentioned above as an addition or a subtraction. (Hint: Begin by writing the $120 withdrawal as -120.)

(b) Complete your calculation to find the following.

(i) Did Rachel have more or less money in her account by the end of the week?

(ii) How much more or less?

12 Puerto Rico Trench is the deepest trench on the floor of the Atlantic Ocean. The bottom of the trench is 8605 metres below sea level. Mount McKinley is the highest mountain in North America. It is 6194 metres above sea level. If Mount McKinley could be picked up and dropped into the Puerto Rico Trench, what depth of water would be between the top of the mountain and sea level?

Drawing a diagram could help with Question **12**.

Reasoning

13 Using the number lines on page 5, find the number you would have to add to each of the following to get an answer of +5.

(a) 0 **(b)** -3 **(c)** -12 **(d)** -10 **(e)** +6 **(f)** +16 **(g)** -9

14 A magic square is one for which the numbers in each row, column and diagonal add up to the same 'magic' total.

Complete the following magic squares, by first working out the magic total.

(a)

-6		-2
	-3	
		0

(b)

6			-18
	2		8
	0	-8	
12		-2	-12

15 Calculate the following by considering the 'mirror image' of each on the opposite side of the number line.

(a) -27 + 14 **(b)** -59 + 36 **(c)** -87 + 62

(d) -31 – 29 **(e)** -68 – 43 **(f)** -75 – 58

(g) -47 + (-62) **(h)** -71 – (-26) **(i)** -96 – (+31)

16 **(a)** Complete the following calculations.

(i) -2 + 3 **(ii)** -5 + 11 **(iii)** -8 + 17 **(iv)** -21 + 34

(b) Now, reverse the order of the two numbers in the above additions, and calculate the resulting subtractions (e.g. -4 + 6 becomes 6 + -4, or 6 – 4).

(c) What do you notice? Comment on your observation.

(d) What is the name for the property of numbers that makes it possible to add them or multiply them in any order?

(e) Use your observation from part **(c)** to calculate the following.

(i) -34 + 45 **(ii)** -53 + 69 **(iii)** -72 + 99 **(iv)** -98 + 113

Open-ended

17 The thermometer at the weather station on Mount Wellington reads -4 °C at 5 am one day in July. By 2 pm, the temperature has reached the day's maximum of 9 °C. Suggest what the thermometer reading might have been at:

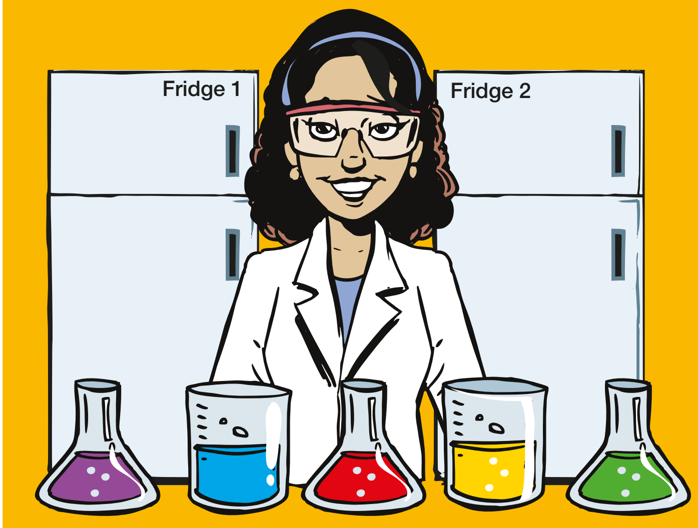
(a) 8:30 am **(b)** midday.

18 List three integers that give a negative answer when added to -17.

19 List three integers that give a positive answer when subtracted from -17.

Problem solving

Lab maths



A	B	C	D	E
Max: 2 °C	Max: 0 °C	Max: -4 °C	Max: -3 °C	Max: 13 °C
Min: -3 °C	Min: -4 °C	Min: -9 °C	Min: -7 °C	Min: 2 °C

Professor Berner has two fridges in her laboratory in which she needs to safely store five different chemicals, A, B, C, D and E.

Each chemical has a maximum and a minimum safe temperature at which it can be stored—anything outside this range and it will explode!

Each fridge can be set to any temperature Professor Berner chooses.

Use the information on the chemical labels to decide Professor Berner's temperature settings and which chemicals are stored in each fridge.

Strategy options

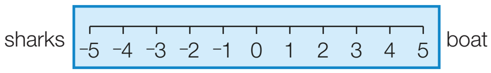
- Guess and check.
- Work backwards.

Investigation

Walking the plank

Equipment required: 1 large number line from -5 to +5 (optional), 1 die marked with +3, +2, +1, -1, -2, -3 and 1 die marked with +, +, +, –, –, –

You have been captured by a gang of pirates who plan to make you walk the plank. However, the pirate captain decides to give you a chance to save yourself. The plank is marked as shown below.



The pirate captain has two dice: one with the sides marked +, +, +, –, –, – (called the ‘operations die’), and one marked -3, -2, -1, +1, +2, +3 (called the ‘number die’). The captain places you at point 0 and rolls the dice.

Depending on the roll of the dice, you’ll have to either move forwards or backwards, according to the following instructions.

- If the operations die shows a ‘+’, you turn to face the boat (the positive end of the number line).
- If it shows a ‘–’, you face the sharks (the negative end).
- If the number die shows a positive number, you walk forwards that number of steps in the direction you are facing.
- If it shows a negative number, you walk backwards that number of steps.

For example, if the captain rolls (–) (+1), you face the sharks and walk forward one step to -1. If the captain rolls (–) (-1), you face the sharks and walk backwards one step to +1.

The captain keeps rolling and you keep moving until you move past +5 or -5. If you go past +5, you are safely back on the boat, but if you go past -5, you join the sharks.

The Big Question

Is this game fair? How many moves will it take to save you?

Engage

1 Copy the diagram of the plank. Starting at 0, use it to find where you would be if the pirate captain rolled (+) (-3), followed by (–) (-2).

2 Return to 0. Roll the two dice again, and move as they tell you. Keep rolling and moving until you are either with the sharks or back on the boat. Keep track of how many moves it takes.

Explore

3 (a) A ‘fair game’ means that both you and the captain have an equal chance of winning. To decide if the game is fair, make a list of all the possible outcomes of rolling the dice.

(b) To find out a reasonable number of moves in which you could expect to get a result (sharks or boat), you will need to play the game several times. (Take turns being the pirate captain and the ‘victim’!)

Strategy options

- Make a table.
- Act it out.

Explain

4 (a) What is the minimum number of moves possible that would bring you to safety?

(b) What was the largest number of moves you needed to complete a game?

Elaborate

5 Complete the following to answer the Big Question.

(a) State whether you and the captain have an equal chance of winning, and explain how you decided.

(b) What is a ‘reasonable’ number of moves needed to complete the game? How did you decide this?

(c) How often would you expect to get a result in the ‘reasonable’ number of moves you have chosen?

6 Imagine you are teetering on the edge of the plank at -5. Show how it is possible to still be on -5 after four different rolls of the dice.

Evaluate

7 How confident are you in the answer you gave for a ‘reasonable number of moves’? What could you do to become more confident?

8 Did playing this game help you get better at adding and subtracting integers? Could it be useful for someone new to the topic? Explain your answer.

Extend

9 How would you expect your ‘reasonable number of moves’ to change if the plank extended from -8 to +8? Make a prediction and then test it by playing the game.

1.2

Integer multiplication

You are probably familiar with multiplying positive numbers. For example:

$^{+}4 \times {}^{+}5 = {}^{+}20$ can be written simply as $4 \times 5 = 20$

What happens when one number is negative? To help you understand this situation, imagine that you owe a friend \$5. You could write this as -5. If you owe 4 friends \$5 each, you owe 4 lots of -5, which you can write as $-5 + -5 + -5 + -5 = -20$. If you paid them back, you would have \$20 less than you did before.

You can write this as:

$4 \times -5 = -20$, or
$-5 \times 4 = -20$

Multiplying a positive number by a negative number gives a negative result.

What happens when both of the numbers being multiplied are negative? Consider the following number pattern.

$3 \times -5 = -15$
$2 \times -5 = -10$
$1 \times -5 = -5$
$0 \times -5 = 0$
$-1 \times -5 = ?$

The numbers on the right-hand side of the equals symbol are increasing by 5. You need to replace the ? with $^{+}5$ to continue the pattern.

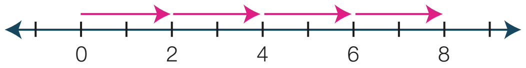
$-1 \times -5 = {}^{+}5$
$-2 \times -5 = {}^{+}10$
$-3 \times -5 = {}^{+}15$ and so on.

Multiplying a negative number by another negative number gives a positive result.

Using a number line

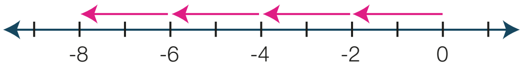
You can use a number line to demonstrate integer multiplication, if you think of multiplication as repeated addition.

$4 \times 2 = 8$



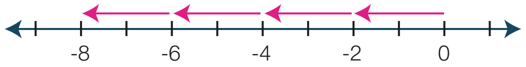
adding 4 lots of 2

$4 \times -2 = -8$



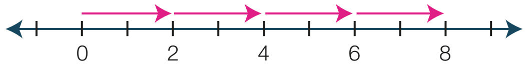
adding 4 lots of -2

$-4 \times 2 = -8$



subtracting 4 lots of 2

$-4 \times -2 = 8$



subtracting 4 lots of -2
(Subtracting a negative is the same as adding a positive.)

Multiplying two numbers with like signs gives a positive answer.
Multiplying two numbers with non-like signs gives a negative answer.
This can be summarised as:

$(+) \times (+) = (+)$ $(+) \times (-) = (-)$
$(-) \times (-) = (+)$ $(-) \times (+) = (-)$

Worked example 3

W.E. 3

Calculate:

(a) -5 × -7

(b) -6 × +9

Thinking	Working
(a) Do the multiplication as though both numbers are positive (5 × 7 = 35), then write the sign of the answer (– × – = +). If the final answer is positive you can leave out the + sign.	(a) -5 × -7 = 35
(b) Do the multiplication as though both numbers are positive (6 × 9 = 54), then write the sign of the answer (– × + = –).	(b) -6 × +9 = -54

When finding the product of more than two numbers, just repeat this same process, as shown below.

Worked example 4

W.E. 4

Calculate: +4 × -3 × -1

Thinking	Working
1 Multiply the first two numbers, placing the correct sign in front of the answer. (Here, + × – = –)	+4 × -3 × -1 = -12 × -1
2 Multiply the product of the first two numbers by the third number, again ensuring the correct sign is in front of the answer. (Here, – × – = +, so you can leave the sign off the final answer.)	= 12

Multiplying a number by 1 does not change the size of a number. In a similar way, multiplying by -1 also does not change the size of a number. However, multiplying by -1 does change the sign of the number, as shown above.

Squaring integers—the difference between $(-3)^2$ and -3^2

To 'square' a number, you multiply it by itself; for example, $5^2 = 5 \times 5 = 25$.

When you square a negative number, you need to place brackets around the number and the sign to avoid confusion.

$(-3)^2$ is read as 'the square of -3'.
$(-3)^2 = -3 \times -3 = +9$

-3^2 is different from $(-3)^2$. Because it doesn't have brackets, -3^2 is read as 'the negative of 3 squared'. $-3^2 = -(3 \times 3) = -9$

1.2 Integer multiplication

Navigator

Answers p. 617

1, 2, 3, 4, 5 (a–c), 6 (a–d), 7 (a–c), 8, 9 (a–d), 10, 13	1 (columns 2–3), 2 (columns 2–3), 3, 4, 5 (a–f), 6, 7, 8, 9, 10, 11, 12, 13	1 (columns 3–4), 2 (columns 2–3), 4, 5 (d–h), 6, 7, 8, 9, 10, 11, 12, 14

Fluency

W.E. 3

1 Calculate:

(a) $+6 \times +5$ **(b)** $+7 \times +3$ **(c)** $+11 \times +7$ **(d)** $+6 \times +9$

(e) $+8 \times -5$ **(f)** $+12 \times -2$ **(g)** $+9 \times -3$ **(h)** $+5 \times -7$

(i) $-2 \times +4$ **(j)** $-2 \times +8$ **(k)** $-5 \times +5$ **(l)** $-4 \times +4$

(m) -6×-5 **(n)** -2×-4 **(o)** -3×-2 **(p)** -5×-3

(q) -12×5 **(r)** 7×-20 **(s)** 9×-10 **(t)** -11×-12

W.E. 4

2 Calculate:

(a) $-2 \times -4 \times 3$ **(b)** $-1 \times 3 \times -4$ **(c)** $3 \times -5 \times -1$

(d) $4 \times 5 \times -2$ **(e)** $-9 \times 1 \times 2$ **(f)** $8 \times -1 \times 3$

(g) $-2 \times -5 \times -5$ **(h)** $-5 \times -2 \times -2$ **(i)** $-8 \times -1 \times -4$

(j) $-3 \times -3 \times -3$ **(k)** $-2 \times -2 \times -2$ **(l)** $-10 \times -10 \times -10$

3 Copy and complete by following the pattern.

(a)
$3 \times 2 = 6$
$3 \times 1 = 3$
$3 \times 0 = 0$
$3 \times -1 =$ ______
$3 \times -2 =$ ______
$3 \times -3 =$ ______

(b)
$-2 \times 3 = -6$
$-2 \times 2 = -4$
$-2 \times 1 = -2$
$-2 \times 0 =$ ______
$-2 \times -1 =$ ______
$-2 \times -2 =$ ______

(c)
$-2 \times -2 = 4$
$-2 \times -1 = 2$
$-2 \times 0 = 0$
$-2 \times 1 =$ ______
$-2 \times 2 =$ ______
$-2 \times 3 =$ ______

4 **(a)** $2 \times -2 \times 2 \times -2$ equals:

A -16 **B** 0 **C** 8 **D** 16

(b) $4 \times -3 \times -3 \times 2$ equals:

A -72 **B** -12 **C** 12 **D** 72

5 Find the value of the unknown number, using your knowledge of multiplication.

(a) $+4 \times ? = +12$ **(b)** $+2 \times ? = 8$ **(c)** $-3 \times ? = +15$

(d) $-(10 \times ?) = +30$ **(e)** $-4 \times ? = (+24)$ **(f)** $-(6 \times ?) = -36$

(g) $? \times +7 = -(-56)$ **(h)** $-(? \times +8) = -(-72)$

6 Calculate:

(a) $(-6)^2$ **(b)** -6^2 **(c)** $(-10)^2$ **(d)** -10^2

(e) $2 \times (-5)^2$ **(f)** $-5^2 \times -3$ **(g)** 4×-10^2 **(h)** $(-10)^2 \times -7$

(i) $(-2)^2 \times (-2)^2$ **(j)** $-2^2 \times (-5)^2$ **(k)** $-3^2 \times -3^2$ **(l)** $(-8)^2 \times -2^2$

Understanding

7 Evaluate the following expressions using the values $x = -2$, $y = +3$, $a = +1$ and $b = 0$.

(a) xy **(b)** ab **(c)** $5ay$ **(d)** $x - ab$ **(e)** axy **(f)** $ax + by$

8 An electronic goods store is trying to sell old DVD and CD players. To clear this stock, they are selling them below 'cost price' (the price the store bought them for).

DVD player: cost price \$26, sale price \$19
CD player: cost price \$15, sale price \$9

For the following questions, write your answers as negative integers.

(a) Find the difference between the cost price and sale price, to calculate the loss the store will make on:

(i) a DVD player **(ii)** a CD player.

(b) The store sold 23 DVD players. Calculate the loss made.

(c) The store sold 19 CD players. Calculate the loss made.

(d) Calculate the total loss made by the store on these goods.

9 In the card game 'Krummy', each player's final score is found by counting the number of cards of each suit, and giving points as shown in the table.

Card	Score
♥ Hearts	-5 points each
♦ Diamonds	-3 points each
♣ Clubs	+2 points each
♠ Spades	+4 points each

Find the score of a player who finishes with these cards.

(a) 3 diamonds **(b)** 7 hearts

(c) 5 hearts, 6 clubs **(d)** 6 diamonds, 9 spades

(e) 4 hearts, 5 clubs, 3 spades **(f)** 3 hearts, 3 diamonds, 8 clubs

(g) 6 hearts, 2 diamonds, 4 spades **(h)** 4 hearts, 4 diamonds, 5 clubs, 3 spades

Reasoning

10 **(a)** Calculate the following.

(i) -1×-1 **(ii)** $-1 \times -1 \times -1$

(iii) $-1 \times -1 \times -1 \times -1$ **(iv)** $-1 \times -1 \times -1 \times -1 \times -1$

(v) $-1 \times -1 \times -1 \times -1 \times -1 \times -1$ **(vi)** $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

(b) Copy and complete.

In a multiplication that contains directed numbers, the answer is positive if there is an ______ number of negative values. The answer is negative if there is an ______ number of negative values.

(c) Use the above result to find the value of the following.

(i) eighty-six -1s multiplied together **(ii)** ninety-nine -1s multiplied together

(iii) $(-1)^{37}$ **(iv)** $(-1)^{50}$

11 Write the following in expanded form, then use your result from Question **10(b)** to find the value of them. (Hint: The expanded form of $(-4)^3$ is $-4 \times -4 \times -4$.)

(a) $(-4)^3$ **(b)** $(-3)^4$ **(c)** $(-2)^5$ **(d)** $-(-6)^2$

12 Hong pays her energy bills by 'direct debit'. An amount is automatically taken out of her bank account each month by the energy suppliers. Hong believes this method of payment is better than receiving a large bill every 3 months.

Each month, $25 for gas and $35 for electricity is 'debited' from Hong's account.

(a) Calculate the amount that will be debited after 3 months for **(i)** gas and **(ii)** electricity. Write your answers as negative integers.

(b) Hong receives a statement from her gas and electricity companies after 3 months, telling her the cost of the energy she has actually used.

Her statements are as follows: Gas $93, Electricity $102.

(i) Calculate the difference between the amounts on Hong's statements and your answers to part **(a)**.

(ii) Should Hong adjust her monthly direct debit amounts for her gas or electricity bills? Give reasons for your answer.

Open-ended

13 Copy and complete. Give at least three different answers.

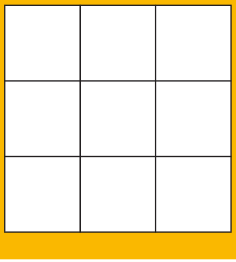
$\_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ = -24$

14 Kemahl is playing the card game Krummy (see Question **9**). At the end of a round, Kemahl has -22 points and cards from three of the four suits in his hand. What could the cards be? Give two possible combinations, stating the number of cards of each suit.

Puzzle

The 1 dilemma

Draw a 3 × 3 grid like the grid shown. Then, arrange five 1s and four -1s in the grid so that the product of every row, column and diagonal is 1.



Integer division

1.3

Division is the inverse operation to multiplication.

If you know that 4 × 3 = 12, you also know that 12 ÷ 3 = 4, and 12 ÷ 4 = 3.

Similarly, if you know that -2 × 3 = -6, you also know that -6 ÷ -2 = 3, and -6 ÷ 3 = -2.

The rules that apply to multiplying negative numbers also apply to dividing negative numbers.

Dividing two numbers with 'like' (same) signs gives a positive answer.

(+) ÷ (+) = (+)   (–) ÷ (–) = (+)

Dividing two numbers with 'non-like' (different) signs gives a negative answer.

(+) ÷ (–) = (–)   (–) ÷ (+) = (–)

Worked example 5

W.E. 5

Calculate:

(a) -56 ÷ -7   **(b)** 20 ÷ -4

Thinking	Working
(a) Do the division as though both numbers are positive (56 ÷ 7 = 8), then put the sign of the answer (– ÷ – = +). If an answer is positive, leave out the '+' sign.	(a) -56 ÷ -7 = 8
(b) Do the division as though both numbers are positive (20 ÷ 4 = 5), then put the sign of the answer (+ ÷ – = –).	(b) 20 ÷ -4 = -5

Division calculations can also be written as fractions. For example, -60 ÷ 4 can be written as $\frac{-60}{4}$. In this case, as the result of the division will be negative, you can write the sign in front of the fraction: $-\frac{60}{4}$. This is also the case if the denominator is the negative number: $\frac{60}{-4} = -\frac{60}{4}$.

1.3 Integer division

Navigator

Answers p. 617

1 (columns 1–2), 2 (columns 1–2), 3, 4 (a–d), 5, 6 (a–c), 8, 10, 12, 15	1 (columns 2–3), 2 (columns 2–3), 3, 4 (a–d), 5, 6 (a–c), 8, 9, 10, 11, 12, 13, 15, 16	1 (columns 3–4), 2 (columns 3–4), 4 (e–f), 6 (d–f), 7, 9, 11, 13, 14, 15, 16

Fluency

W.E. 5

1 Calculate:

(a) $9 \div -3$ **(b)** $6 \div -2$ **(c)** $20 \div -5$ **(d)** $21 \div -7$

(e) $-8 \div 4$ **(f)** $-30 \div 5$ **(g)** $-54 \div 6$ **(h)** $-32 \div 4$

(i) $-12 \div -3$ **(j)** $-18 \div -2$ **(k)** $-49 \div -7$ **(l)** $-48 \div -8$

(m) $-63 \div -9$ **(n)** $45 \div -5$ **(o)** $-16 \div 4$ **(p)** $70 \div -7$

(q) $-240 \div 8$ **(r)** $-600 \div 6$ **(s)** $240 \div -12$ **(t)** $-2300 \div -10$

2 Calculate the following.

(a) $\frac{48}{-3}$ **(b)** $\frac{-77}{11}$ **(c)** $\frac{-65}{5}$ **(d)** $\frac{120}{-8}$

(e) $\frac{52}{-2}$ **(f)** $\frac{-93}{-3}$ **(g)** $\frac{-84}{-4}$ **(h)** $\frac{-96}{4}$

(i) $\frac{-90}{-6}$ **(j)** $\frac{100}{-5}$ **(k)** $\frac{-360}{40}$ **(l)** $\frac{-240}{-80}$

3 **(a)** $\frac{-28}{-4}$ equals:

A -32 **B** -24 **C** -7 **D** 7

(b) $162 \div -3$ equals:

A -159 **B** -54 **C** 54 **D** 159

4 Evaluate the following.

(a) $\frac{-25}{5}$ **(b)** $\frac{56}{8}$ **(c)** $\frac{-22}{2}$ **(d)** $\frac{-22}{-2-9}$

(e) $\frac{46 + -2}{11}$ **(f)** $\frac{99 \div -11}{15 \div 5}$

Understanding

5 Without evaluating, decide whether the answers to the following will be positive or negative.

(a) $\frac{9 \times -11 \times 3}{-15 \times -5}$ **(b)** $\frac{5 \times -10 \times 2 \times -4}{-10 \times -2 \times -3}$ **(c)** $\frac{-6 \times 12 \times -2 \times -4}{8 \times -2 \times -7}$

6 Evaluate the following expressions, given that $a = 12$, $b = 15$ and $c = -3$.

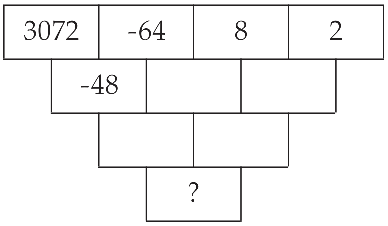
(a) $a \div c$ **(b)** $2a \div c$ **(c)** $b \div c$ **(d)** $4c \div a$

(e) $2b \div 2c$ **(f)** $2a - b \div c$

7 Evaluate the following expressions, given that $x = -8$ and $y = 4$.

(a) $\frac{x - y}{3}$ (b) $\frac{-3x - y}{5}$ (c) $\frac{-5xy}{5}$

8 Use your calculator to find the unknown value '?'. Each number in the pyramid is the division of the two numbers above it. The first division has been done already: $3072 \div -64 = -48$.



9 A department store has 4 floors below the ground floor and 11 floors above it. A shopper on the top level takes a lift to the bottom level, stopping every 3 floors to look around.

(a) Draw a diagram to represent the situation, clearly showing the lift and floor numbers.

(b) How many times will the shopper stop on the way to the lowest floor?

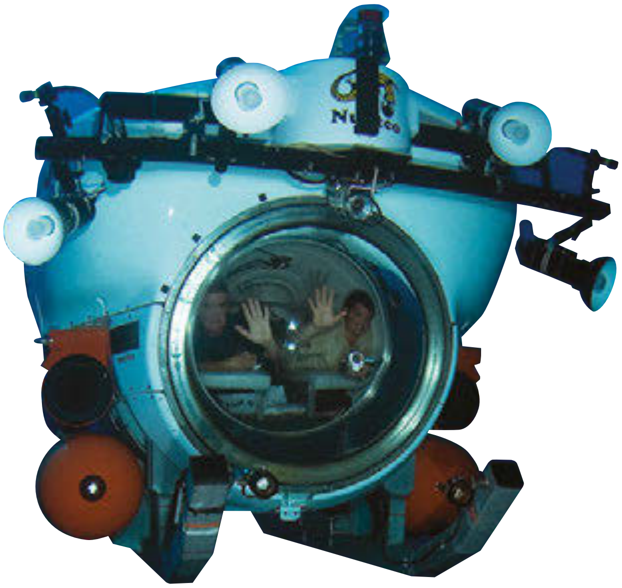
(c) List the floor numbers the shopper stops at.

10 A group of 6 people loses \$240 trying to win a lottery. Write each person's share of the loss as a negative integer.

11 A submersible begins at the ocean surface and is lowered 6 metres every minute. If the ocean is 96 metres deep in this area, calculate:

(a) the time taken for the submersible to reach the ocean floor

(b) the time taken to rise back to the surface if the submersible rises at a rate of 2 metres per minute.



12 The temperature falls from 24 °C to -4 °C between the hours of 7 pm and 2 am one night in the desert. Find the average hourly temperature change by dividing the overall change by the number of hours. Write your answer as a negative integer.

Reasoning

13 Beau has bought a home theatre system and some new furniture from a store offering a 'buy now, pay later' scheme. The total cost of the home theatre system and furniture was \$8600. Beau plans to pay \$200 every month.

(a) How many months will it take Beau to pay off his debt?

(b) How many years and months is this?

(c) After 1 year, the store sends Beau a statement showing how much he still owes. Write a negative integer to show this amount.

14 You are playing a game that involves rolling two dice with unusual numbers on each face.

The six faces of the green die show:	+24	-36	+12	-48	-60	+72
The six faces of the blue die show:	-1	+2	-3	+4	-6	+12

To find out the score for your dice roll, you need to divide the value showing on the green die by the value showing on the blue die.

Make a list, in order from smallest to largest, of all the possible scores you can get.

Open-ended

15 Write three numbers that give a negative integer when divided by 8.

16 ☐ ÷ ☐ > -1

Write different integers from the following list in each box to make the above statement true. Make at least three different true statements. (Remember, '>' means 'is greater than'.)

-2, 3, -4, 5, 6, -7, -8

Game

3 in a row

Equipment required:
1 standard pack of 52 playing cards with the picture cards (Jacks, Queens, Kings) removed; about 10 coloured counters each (or 10 small pieces of paper with the player's initials on them)

How to win:
The first player to get three of their counters in a line (horizontally, vertically, or diagonally) is the winner.

How to play:
In this game, red (♥ and ♦) card values are negative, while black (♠ and ♣) card values are positive.

Shuffle the cards and place them face down on the table. Each player draws one card from the top of the pack. The player with the higher value card goes first. (Aces are worth 1 point.)

On your turn, take the top two cards from the deck and place them face up in front of you. You can either add, subtract, multiply or divide the two values on the cards. Say the calculation and the answer out loud. Then, only if you stated it correctly, place a counter on the square that shows the answer to the calculation. Then it is the next player's turn.

If you use all the cards, reshuffle the pack and continue.

-2	-7	-16	-2	6	-1	12	3
8	10	1	-12	-4	18	5	-7
36	2	-24	6	0	-8	-3	14
-4	30	7	3	16	-9	2	-5
-6	-16	-10	5	-1	-24	4	8
24	4	0	-18	-36	7	-12	1
15	9	-3	10	-2	-30	11	16
12	-5	-8	13	24	-6	-10	-4

Gamespace

CRYOGENIC CRISIS

While cruising the outer galaxy, you are captured by three scary looking aliens who bring you to their home planet, Zush.

They place you in a vat of freezing liquid, which is at -20 °C. If it gets to -40 °C you will be frozen alive.

The aliens challenge you to an ‘integers duel’ to decide your fate. You have no choice but to accept.

EQUIPMENT REQUIRED:

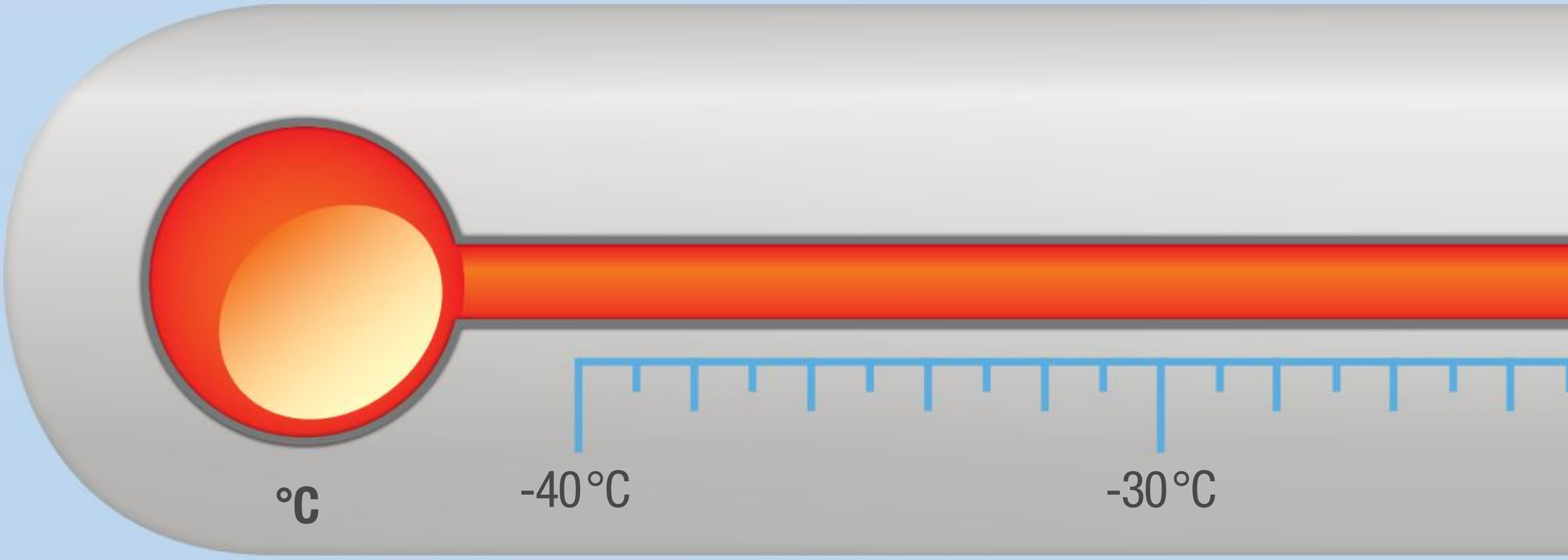
2 players, 1 counter (or something to use as a marker), 1 die, 1 coin

HOW TO PLAY:

- Place the counter on the thermometer at -20 °C.
- Decide who will be the victim (the person in the vat) and who will be the alien.
- Roll the die to decide who goes first (higher number first).

On your turn:

- Toss the coin to find the operation.
    - If the coin shows heads, add (+).
    - If the coin shows tails, subtract (—).
- To find the number that you will add or subtract, roll the die.
- Find the number you have rolled in the following table.



Die roll	Choose from these numbers	Die roll	Choose from these numbers
⚀	-1, -2, 5	⚃	0, 6, -2
⚁	2, 1, -6	⚄	1, -3, -6
⚂	3, 0, -4	⚅	5, -1, 4

- Choose any two numbers from the list next to your roll and find their product (multiply them together).
- If your coin shows heads, add this amount (the product) to the number your marker is on. If your coin shows tails, subtract it from the number your marker is on.
- When you have calculated the answer, move the marker up or down the temperature scale to this number.

SaÜle move:

'Alien' player goes first and the coin toss shows heads, so they will add. Alien rolls a ⚂, and chooses -4 and 3 from the list in the table.

-4 × 3 = -12

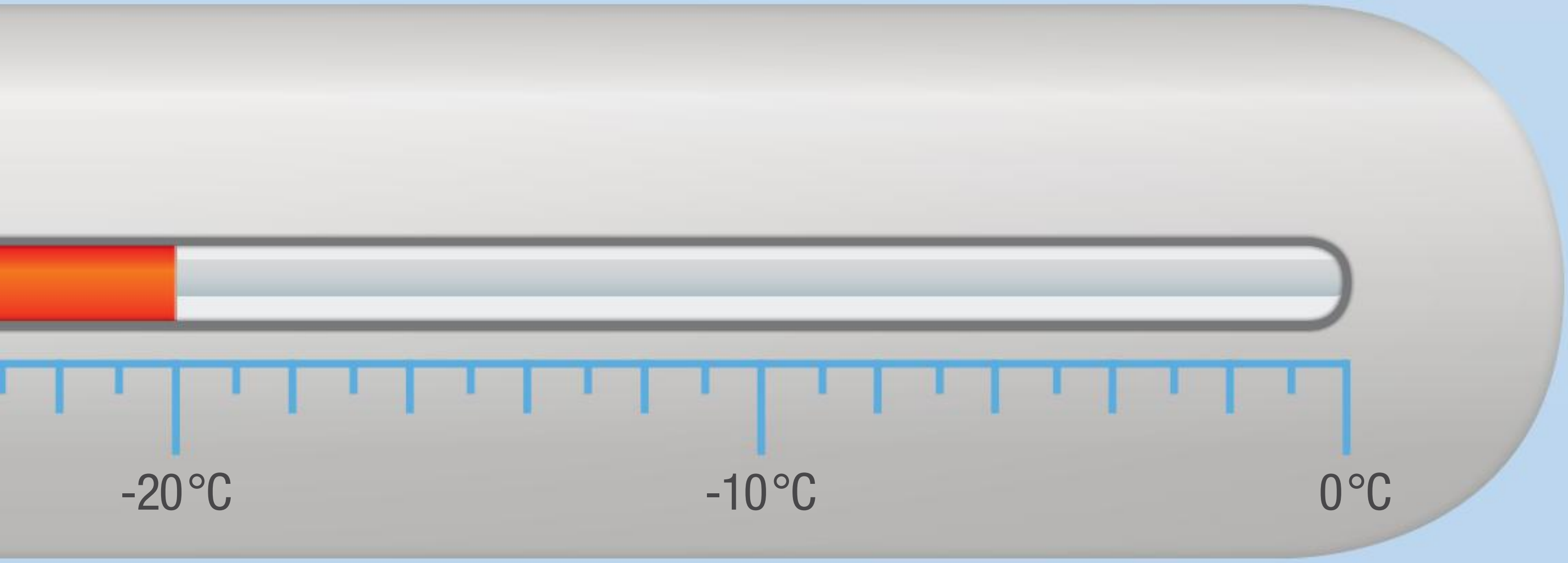
-20 + (-12) = -32

They move the counter from -20 °C to -32 °C.

HOW TO WIN:

The victim wins if they move the counter up to or beyond 0 °C.

The alien wins if they move the counter to or beyond -40 °C.



Half-time 1

1.1 1 Simplify the following by writing a single symbol between the two numbers, then calculate the answer.

(a) $9 - (+6)$ (b) $14 - (+21)$ (c) $-9 - (+8)$ (d) $-31 - (+27)$

(e) $-8 + (-8)$ (f) $-14 + (-20)$ (g) $7 - (-10)$ (h) $-35 - (-21)$

1.2 2 Calculate:

(a) -4×6 (b) -7×-8 (c) 10×-14 (d) -2×-12

(e) $(-3)^2$ (f) -5^2 (g) -2^3 (h) $(-4)^3$

1.1 3 The temperature at 6 am on a cold winter's morning in Moscow was -11 °C. By midday, the temperature had risen to 3 °C. By 6 pm, the temperature had dropped to -5 °C.

(a) Write the increase in temperature from 6 am to midday as a positive integer.

(b) Write the decrease in temperature from midday to 6 pm as a negative integer.

1.3 4 Calculate:

(a) $-40 \div -8$ (b) $-63 \div 7$ (c) $55 \div -11$ (d) $-7 \div -1$

(e) $\frac{-32}{4}$ (f) $\frac{-72}{9}$ (g) $\frac{-36}{-12}$ (h) $\frac{-60}{-15}$

1.1 5 Write the following sets of integers in ascending order (smallest to largest).

(a) -37, 7, -30, 0, -3, 3 (b) 54, -20, -1, -5, -40

1.1 6 Calculate the following.

(a) $-3 + 4$ (b) $-9 + 7$ (c) $4 - 5$ (d) $-8 - 6$

(e) $+4 - (+2) + (-5)$ (f) $-7 + (-8) - (-10)$ (g) $-9 + 5 - 7$ (h) $18 - 25 + 11$

1.2 7 Maya spent $5 every week buying a lottery ticket. After 16 weeks, she won a $50 prize.

(a) How much had Maya spent on tickets at the end of the 16-week period?

(b) Write the overall amount Maya had gained or lost at the end of the same period as a positive or a negative integer.

1.1 8 Write an integer to represent each of the following situations.

(a) a withdrawal of $120 from a bank account

(b) an increase of 7 cm in the height of a child

1.3 9 Four friends each invested $30 000 to start up a business. At the end of 1 year, the business made a profit of $80 000.

(a) What was the share of the profit for each friend?

(b) Considering their initial investment, write the overall profit or loss made by each friend as a positive or a negative number.

1.1 10 Calculate the following by considering the 'mirror image' of the calculation on the positive side of the number line.

(a) $-67 + 45$ (b) $-83 + 57$ (c) $-39 - (+24)$ (d) $-19 - (-31)$

Combined operations with integers

1.4

The order of operations for working with negative integers is the same as the order you use when working with positive integers.

The order of operations rules are:

- do any calculation inside brackets first
- then find the value of any numbers in index form (powers and roots)
- then do division and multiplication in the order they appear, moving from left to right
- then do addition and subtraction in the order they appear, moving from left to right.

Worked example 6

W.E. 6

Find the value of:

(a) $-7 \times 4 \div -2 \times -1$

(b) $-3 \times (-2 + -6) - 4$

Thinking	Working
(a) As there is only multiplication and division involved, work from left to right.	(a) $-7 \times 4 \div -2 \times -1$ $= -28 \div -2 \times -1$ $= 14 \times -1$ $= -14$
(b) 1 Find the value of what's inside the brackets first. If the answer is negative, keep it inside the brackets to avoid confusion.	(b) $-3 \times (-2 + -6) - 4$ $= -3 \times (-8) - 4$
2 Do multiplication and division as you move from left to right.	$= 24 - 4$
3 Do subtraction and addition as you move from left to right.	$= 20$

Using the negative sign

The negative sign (–) can be used to indicate subtraction and also to indicate a negative number. To avoid any confusion, make sure you indicate a negative number by placing brackets around the number and the sign wherever necessary. This is very important when applying the order of operations, as any powers must be calculated before subtracting or adding, and squaring a negative number will change its sign.

For example: $5 + (-4)^2 = 5 + 16$
$= 21$

$5 - 4^2 = 5 - 16$
$= -11$

Worked example 7

W.E. 7

Find the value of:

(a) $6 - 3^2 + 7 \times -2$

(b) $9 + (-4)^2 \times (7 - 2)$

Thinking	Working
(a) **1** As there are no calculations in brackets, calculate indices first.	(a) $6 - 3^2 + 7 \times -2$ $= 6 - 9 + 7 \times -2$
2 Next, multiply and divide from left to right.	$= 6 - 9 + -14$
3 Simplify any addition or subtraction of directed numbers. Do them from left to right.	$= 6 - 9 - 14$ $= -3 - 14$ $= -17$
(b) **1** Do the calculation in brackets first.	(b) $9 + (-4)^2 \times (7 - 2)$ $= 9 + (-4)^2 \times 5$
2 Find the value of numbers in index form.	$= 9 + 16 \times 5$
3 Next, multiply and divide from left to right.	$= 9 + 80$
4 Do addition and subtraction from left to right.	$= 89$

Calculators and integers

To show a negative number on the calculator, press the (–) key before entering the number. On some calculators, the key looks like this: +/–. Pressing this key repeatedly changes the sign of an entered number from positive to negative and back again.

You should also locate and know how to use the 'open bracket' and 'close bracket' keys, (and), the 'square root' key, √, and the 'square' key, x^2. If your calculator does not have a 'square' key like this, press ^ 2.

Almost all calculators have the order of operations built into them, so you can enter the numbers and operations as they are written on the page. Make sure you enter brackets in the correct positions.

1.4 Combined operations with integers

Navigator

1, 2 (rows 1–2), 3 (a–d), 6, 9 (a–b), 12, 13	2 (rows 1–2), 3 (a–d), 4 (a–d), 5 (a), 6, 7, 8, 9, 10, 12, 13	2 (rows 3–4), 3 (e–f), 4, 5 (b), 8, 9, 10, 11, 12, 13

Answers p. 618

Equipment required: calculator for Questions **3**, **4**, **12**

Fluency

1 By completing the steps, find the value of:

(a)

$$\underbrace{(16-8)}\underbrace{+\,\text{-}5\times 2}\underbrace{-\,18\div 2}$$

$$=\underbrace{\boxed{}\quad\boxed{}\quad\boxed{}}$$

$$=\boxed{}$$

(b)

$$\text{-}6\underbrace{-\,4\times 2}\underbrace{+\,3^2}\times\underbrace{(4+1)}$$

$$=\text{-}6\ \boxed{}\ \underbrace{\boxed{}\times\boxed{}}$$

$$=\underbrace{\text{-}6\ \boxed{}\quad\boxed{}}$$

$$=\boxed{}$$

2 Find the value of: **W.E. 6**

(a) 45 ÷ -9 × -2
(b) -32 ÷ -8 × 5
(c) 56 ÷ -7 ÷ -2
(d) 4 + (6 × -3) – 2
(e) -2 + (-64 ÷ -8)
(f) -5 × (-7 – 4)
(g) 7 + (14 ÷ -2) – 3
(h) 44 ÷ (-12 + 1)
(i) (-6 × 5) + (-24 ÷ -6)
(j) (-8 + 2) × (4 – 10)
(k) (12 × -3) ÷ (-2 – 2)
(l) (-5 × -9) – (7 – 10)

3 Find the value of the following. You may like to use a calculator to check your answers. **W.E. 7**

(a) $20 - 5^2 + 10 \div \text{-}2$
(b) $7 + (\text{-}4)^2 \times (3 - 5)$
(c) $30 \div \text{-}6 \times 5 + 5^2$
(d) $5 \times 8 \div \text{-}2 + (\text{-}8)^2$
(e) $(7 - 10) \times (20 \div \sqrt{4})$
(f) $(18 - \sqrt{16}) \times \text{-}2 + 4$
(g) $\sqrt{100} - 5^2 \times (1 - 3)$
(h) $(\sqrt{81} - \sqrt{36}) \times (\text{-}10)^2$
(i) $\dfrac{\text{-}4^2}{2} - 3 \times \text{-}6$
(j) $\dfrac{(17-5)}{\text{-}2} \times \text{-}3 + 4$

4 Find the value of the following. You may like to use a calculator to check your answers.

(a) $81 \div (9 - 18) + (16 - 25)$
(b) $\text{-}27 - (42 - 91) + 17$
(c) $40 \div (3 - 8) + 7^2$
(d) $\text{-}19 + 10 \times \text{-}3 + (\text{-}8)^2$
(e) $(\text{-}37 + 21)^2 - \sqrt{169}$
(f) $\text{-}14 \times (\text{-}6)^2 \div \text{-}2$
(g) $\text{-}(27 - 42)^2 \div \text{-}5$
(h) $(57 - 4^2) \times \text{-}3 - 21$
(i) $\text{-}18 \times \text{-}5 \div 3^2 - 54$
(j) $\text{-}18 \times \text{-}5 \div (3^2 - 54)$

5 **(a)** The expression -4 – (2 × -3) + 6 equals:

A -2 **B** -12 **C** 8 **D** 24

(b) $5 + 6 \times (\text{-}3) + (\text{-}4)^2$ is equal to:

A 3 **B** -17 **C** -29 **D** -49

Understanding

6 Chen has $250 in his bank account at the start of the month. Over the month he makes the following transactions as shown.

Answer the questions below using numbers, brackets (if necessary), and operation symbols (×, ÷, +, –) to write expressions that show how to calculate the amount.

Chen Bank Statement

STATEMENT PERIOD: **1/10/16–30/10/16**

Account Transactions

Transaction Details	**Withdrawal**	**Deposit**	**Balance**
Opening balance			**$250.00**
Week 1	$50.00		
	$50.00		
	$50.00		
Week 2	$80.00		
	$80.00		
Week 3		$100.00	
Week 4	$80.00	$100.00	
Closing Balance			

(a) How much did Chen have in his account at the end of Week 1?

(b) How much did Chen have in his account at the end of Week 3?

(c) During the month, Chen got a letter from his bank saying he had 'overdrawn' on his account—he now owed money to the bank! After which withdrawal did Chen receive this letter? How much did he owe the bank?

(d) What was Chen's final closing balance at the end of Week 4?

7 A company made a profit of $3 million per month for 7 months, then lost $9 million per month for 5 months. What was their result for the year?

8 Taylor places some water at 24 °C in the freezer. The freezer is able to lower the water's temperature by 4 °C per hour. What will be the temperature of the water after:

(a) 3 hours **(b)** 6 hours **(c)** 7 hours **(d)** 10 hours?

9 To play one round of golf you play the 18 holes of a course. For each hole, it is expected a player will take a certain number of shots to get the ball in the hole. This is called 'par' for the hole and it is given a score of zero. If you take one, two or three shots less than par, or 'under par', you score -1, -2 or -3, respectively. If you take one, two or three shots more than par, or 'over par', you score +1, +2 or +3, respectively.

Polly plays a round of golf at a course where 'par' for the whole course is 72 shots. Polly has the following results on her round.

- 4 holes at 1 under par
- 1 hole at 2 under par
- 5 holes at par
- 7 holes at 1 over par
- 1 hole at 10 over par.

(a) Write a single calculation that Polly can use to find her finishing score.

(b) **(i)** What was Polly's finishing score?

(ii) How many shots over or under par for the course is this?

(c) **(i)** Polly has a handicap of 14, which means that 14 is subtracted from her finishing score to find her official score. What was Polly's official score, taking into account her handicap?

(ii) How many shots over or under par for the course is this?

Reasoning

10 **(a)** Find the value of the following.

(i) $19 - 7 - 15 + 14$ **(ii)** $19 - (7 - 15) + 14$

(b) Explain why having the brackets in the second part gives you a very different answer from the first part.

11 Two of the following contain a pair of brackets that is unnecessary. Find them and rewrite the calculation without them, checking that you still get the same answer.

(a) $8 + (16 - 23) - 9 = -8$ **(b)** $14 - (8 - 19) - 10 = 15$

(c) $(11 \times -8) + (24 \div -2) = -100$ **(d)** $(5 - 2) \times 4 - (3 - 7) = 16$

Do each calculation with and without a pair of brackets and compare your answers.

Open-ended

12 **(a)** Use your calculator to find the value of $(-15 + 11) \times -2 + (-4)^2$, and write your answer.

(b) Re-enter the expression into your calculator, leaving out all of the brackets.

(c) Explain how your calculator gave you two different answers, given that it follows the rules for the order of operations.

13 Copy and complete. (Different answers are possible.)

$-3 - (\_\_\_\_\_ \div \_\_\_\_\_) = 5$

Puzzle

Animal speed challenge

Equipment required: 1 stopwatch/timer

Time yourself to see how quickly you can complete each of the following chains of calculations.

Check your answers with some classmates.

Look at the scoring scale below to see which animal you are!

1 Start with 4 → Multiply by 7 → Double it → Divide by -8 → Subtract 10 → Add 3 → Multiply by 2

2 Start with 3 → Cube it → Divide by 9 → Add -6 → Multiply by 4 → Add 20 → Add 10

3 Start with 10 → Add 17 → Multiply by 2 → Subtract -10 → Divide by -8 → Square it → Add 6

How did you go?

Less than 30 seconds: Champion Cheetah
30 seconds to 1 minute: Galloping Giraffe
1 minute to 2 minutes: Zippy Zebra
More than 2 minutes: Leisurely Lion

Now, write one of your own for a friend to complete. Make sure it has at least seven steps. Also, make sure that you are able to complete it yourself first!

Maths 4 Real

The ultimate cool

How cold can things get? The lowest temperature ever recorded on Earth was -89 °C at the Russian research station Vostok in Antarctica. The lowest temperature in the northern hemisphere, recorded in Oimekon, Siberia, was a mere -78 °C.

The average Antarctic temperatures for different seasons are shown in the following table.

Season	Inland temperature (°C)	Coastal temperature (°C)
Autumn	-58	-9
Winter	-60	-12
Spring	-48	-8
Summer	-32	-2

At these sorts of temperatures, it becomes painful to breathe because the moisture on the tiny hairs inside your nose freezes instantly. However, other parts of the universe can get even colder. The coldest places in the Solar System are the bottom of the craters at the south pole of the Moon. Here, the temperature has been measured at -240 °C. Parts of deep space, at -270 °C, have the lowest temperature of anything in nature.

1 How much warmer is:

(a) the lowest temperature ever recorded in the northern hemisphere, compared to the lowest temperature recorded in Antarctica?

(b) the average winter inland temperature in Antarctica, compared to the temperature at the bottom of the craters at the south pole of the Moon?

2 What is the difference between the average coastal and inland temperatures in Antarctica in:

(a) autumn **(b)** winter **(c)** spring **(d)** summer?

What is the 'ultimate' cool? For centuries, scientists have been interested in finding the lowest temperature possible. In early 2001, scientists at the Australian National University produced the coldest known substance for the first time in Australia. It was a super-cold cloud of atoms reaching very close to -273 °C. To create the cloud, the temperature of one million atoms was lowered using laser beams.

Theoretically, the coldest temperature possible is -273.15 °C, which is also called 'absolute zero' on the Kelvin temperature scale. This is where atoms reach their lowest possible level of movement. The Australian scientists didn't quite get there: they were about 100 billionths of a degree above absolute zero.

3 A United Nations report predicted that rising levels of greenhouse gases in the atmosphere will cause the average global surface temperature to rise between 1 °C and 6 °C by the end of the century. How would this affect Antarctic temperatures? For each season, show what would happen to the average inland temperatures if the temperature rose by:

(a) 1 °C **(b)** 6 °C

4 Look at the following table of temperatures.

Temperature (°C)	Temperature fact
+5600	Temperature at the Sun's surface
+1064	Gold melts
+100	Boiling point of water (at sea level)
+57	Highest recorded temperature on Earth
+37	Human body temperature
0	Water freezes
-37	Car antifreeze freezes
-89	Lowest recorded temperature on Earth
-273.15	Absolute zero

Find the difference between:

(a) human body temperature and the temperature at which car antifreeze freezes

(b) the temperatures of the Sun's surface and absolute zero

(c) the temperatures at which gold melts and water freezes

(d) the highest and lowest temperatures recorded on Earth.

Research

Present a report on other temperature scales, such as the Fahrenheit and Kelvin scales. Describe how to convert these to degrees Celsius. Explain why -273.15 °C is called absolute zero.

1.5 Multiplying and dividing numbers in index form

Index form

To **square** a number, you multiply it by itself.

To **cube** a number, you multiply it by itself, then multiply it by itself again.

For example: '4 squared' $= 4 \times 4$ $= 4^2$ '4 cubed' $= 4 \times 4 \times 4$ $= 4^3$

There is a shorthand way of writing the multiplication. It can be used for any number of times that the number is multiplied by itself. A smaller number, called the **index** (or **power**, or **exponent**), is written to the top right of the number. The index tells you how many times the number (called the **base**) appears in the multiplication; for example, $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$.

Numbers written in this way are said to be written in **index form**. The plural of index is indices.

index or power or exponent

base

$$\underbrace{4^3}_{\text{index form}} = \underbrace{4 \times 4 \times 4}_{\text{expanded form}} = 64$$

Two or more different base numbers must be written individually as numbers in index form, as shown in part **(b)** of the following example.

Worked example 8

W.E. 8

Write the following in index form.

(a) $7 \times 7 \times 7 \times 7$ **(b)** $2 \times 2 \times 2 \times 5 \times 5$

Thinking	Working
(a) Count the number of times the base appears in the string of multiplications and use this number as the index. (Here, there are four 7s multiplied together, so 4 is the index.)	(a) $7 \times 7 \times 7 \times 7$ $= 7^4$
(b) Count the number of times each base appears in the string of multiplications and use these numbers as the index of each base. (Here, you have the product of three 2s and two 5s.)	(b) $2 \times 2 \times 2 \times 5 \times 5$ $= 2^3 \times 5^2$

Expanded form

Numbers in index form can be written in **expanded form** by writing a string of multiplications equal to the number of the index. Index numbers with different bases must be treated separately.

Worked example 9

W.E. 9

Write the following in expanded form.

(a) 8^5 **(b)** $3^2 \times 4^3$

Thinking	Working
(a) Use the index to find the number of times the base will appear in the string of multiplications. (Here, an index of 5 means there are five 8s multiplied together.)	(a) 8^5 $= 8 \times 8 \times 8 \times 8 \times 8$
(b) Use the index of each base to find the number of times each base will appear in the string of multiplications. (Here, the index of 3 is 2 and the index of 4 is 3, so you have two 3s and three 4s multiplied together.)	(b) $3^2 \times 4^3$ $= 3 \times 3 \times 4 \times 4 \times 4$

Indices on the calculator

You can use your calculator to find the value of numbers expressed in index form. Your calculator should have a key that looks like ^ or x^y. These keys both work in the same way. For example, to find 6^4 you would enter 6 ^ 4 =, or 6 x^y 4 =, to get 1296.

Multiplying index numbers

How can you calculate $2^3 \times 2^5$?

Method 1

Evaluate each part separately and find the product. Remember that to evaluate a number or an expression means to calculate its value.

$2^3 = 8$ and $2^5 = 32$

$$\begin{aligned} 2^3 \times 2^5 &= 8 \times 32 \\ &= 256 \end{aligned}$$

Method 2

Write each index number in expanded form, then find the product.

$$\begin{aligned} 2^3 \times 2^5 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 256 \end{aligned}$$

Developing a rule for multiplying index numbers

Complete the following table by writing the product of the numbers in the first column, first in expanded form, then as a single number in index form.

Product	In expanded form	As an index number
$3^2 \times 3^4$		
$4^3 \times 4^4$		
$7^2 \times 7$		
$11^2 \times 11^2$		
$20^3 \times 20^5$		
$6^5 \times 6$		

For each row of your completed table, look closely at the index numbers in the first and third columns.

Can you see a pattern, or connection, between the numbers? Describe it. (Remember that $7 = 7^1$ and $6 = 6^1$.)

Did you notice that the index of the final product is equal to the sum of the indices of the numbers being multiplied?

$$\begin{aligned} &3^2 \times 3^4 \\ &= 3^{(2+4)} \\ &= 3^6 \end{aligned} \qquad \begin{aligned} &4^3 \times 4^4 \\ &= 4^{(3+4)} \\ &= 4^7 \end{aligned}$$

This pattern can be stated as a rule:

> When multiplying numbers in index form with the same base,
> keep the base and add the indices.

If the numbers do not have the same base, the indices cannot be added. For example, $2^4 \times 3^4$ cannot be written more simply in index form.

Worked example 10

W.E. 10

Simplify the following.

(a) $3^7 \times 3^5$

(b) $2^3 \times 3^5 \times 2^4 \times 3^6$

Thinking	Working
(a) Are the bases the same? If yes, keep the base and add the indices.	(a) $3^7 \times 3^5$ $= 3^{7+5}$ $= 3^{12}$
(b) Are the bases the same? If not, group the numbers that have the same base and add their indices. Write the answer as a product of the index numbers with different bases.	(b) $2^3 \times 3^5 \times 2^4 \times 3^6$ $= 2^3 \times 2^4 \times 3^5 \times 3^6$ $= 2^{3+4} \times 3^{5+6}$ $= 2^7 \times 3^{11}$

Odd and even powers

Recall that multiplying two negative numbers gives a positive product, and multiplying a positive and a negative number gives a negative product. Consider these powers of -2, written in expanded form and multiplied together:

$(-2)^2 = -2 \times -2 = 4$
$(-2)^3 = -2 \times -2 \times -2 = -8$
$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$
$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$

Can you see a pattern here? Will $(-2)^6$ be a positive or a negative number? What about $(-2)^7$?

Negative numbers raised to an even power give a positive result.

Negative numbers raised to an odd power give a negative result.

Worked example 11

W.E. 11

Simplify, if possible, then evaluate the following.

(a) $(-3)^2 \times (-3)^3$ **(b)** $-7^2 \times (-4)^4$ **(c)** $-2^2 \times -2^5$

	Thinking	Working
(a) 1	Are the bases the same? (Yes.)	(a) $(-3)^2 \times (-3)^3$
2	If yes, simplify by keeping the base (-3) and adding the indices.	$= (-3)^5$
3	Find the value of the product in index form, noting that the power is an odd number, so the answer will be negative.	$= -243$
(b) 1	Are the bases the same? (No.)	(b) $-7^2 \times (-4)^4$
2	If not, evaluate each index number separately. (Note here that the value of one will be negative, -7^2, and one will be positive, $(-4)^4$.)	$= -49 \times 256$
3	Find the value of the final product.	$= -12\,544$
(c) 1	Are the bases the same? (Yes.)	(c) $-2^2 \times -2^5$
2	If yes, simplify by keeping the base (2) and adding the indices. Note that multiplying the two negative signs in front of the bases will give a positive answer.	$= +2^7$
3	Find the value of the index number.	$= 128$

Dividing index numbers

How can you calculate $5^8 \div 5^5$? As you saw for multiplying index numbers, there are two methods you can use.

Method 1

Evaluate each number separately and find the **quotient** using a calculator:

$$
\begin{aligned}
5^8 \div 5^5 &= 390\,625 \div 3125 \\
&= 125
\end{aligned}
$$

Method 2

Write each number in expanded form, then divide by cancelling common factors before simplifying the indices:

$$
\begin{aligned}
5^8 \div 5^5 &= \frac{{}^1\cancel{5} \times {}^1\cancel{5} \times {}^1\cancel{5} \times {}^1\cancel{5} \times {}^1\cancel{5} \times 5 \times 5 \times 5}{{}_1\cancel{5} \times {}_1\cancel{5} \times {}_1\cancel{5} \times {}_1\cancel{5} \times {}_1\cancel{5}} \\
&= \frac{5 \times 5 \times 5}{1} \\
&= 5^3 \\
&= 125
\end{aligned}
$$

Notice that the division of numbers with the same base in index form also has a pattern:

$$
\begin{aligned}
5^8 \div 5^5 &= 5^{8-5} \\
&= 5^3
\end{aligned}
$$

> When dividing numbers with the same base in index form,
> keep the base and subtract the indices.

Worked example 12

W.E. 12

Simplify the following.

(a) $3^9 \div 3^4$

(b) $(2^{11} \times 5^4) \div (2^9 \times 5^3)$

Thinking	Working
(a) Are the bases the same? (Yes.) If yes, write the base (3) and subtract the indices.	(a) $3^9 \div 3^4$ $= 3^{9-4}$ $= 3^5$
(b) **1** Are the bases the same? (No.) If not, rewrite the division as a fraction.	(b) $(2^{11} \times 5^4) \div (2^9 \times 5^3)$ $= \frac{2^{11} \times 5^4}{2^9 \times 5^3}$
Subtract the indices of numbers with the same base.	$= 2^{11-9} \times 5^{4-3}$
2 Write the answer as a product of index numbers with different bases.	$= 2^2 \times 5$

The difference between 'simplify' and 'evaluate'

To simplify an expression involving index numbers, apply the rules for multiplying and dividing them, writing it in a simpler form. Note that examples 10 and 12 asked for expressions to be simplified. The answer was given in index form—its actual value was not calculated.

To evaluate an expression involving index numbers, find the value of the answer as an actual number. Example 11 asked for expressions to be simplified first, then evaluated. The answer was given as a number.

1.5 Multiplying and dividing numbers in index form

Navigator

1, 2 (rows 1–3), 3 (rows 1–2), 4 (row 1), 5 (rows 1–3), 6, 7, 8, 10, 14, 15 (a–b), 17	1 (column 2), 2 (rows 2–4), 3 (rows 3–4), 4, 5 (rows 2–4), 6, 7, 8 (a–c), 9, 10, 11, 12 (row 1), 13, 14, 15, 18, 20	1 (column 2), 2 (rows 3–4), 3 (rows 3–4), 4, 5 (rows 3–5), 6, 8, 11, 12, 13, 14, 15, 16, 18, 19, 20

Answers p. 618

Equipment required: calculator may be used for Questions **4**, **9**, **10**, **13**, **15 (c)**, **19**

Fluency

W.E. 8

1 Write the following in index form.

(a) $2 \times 2 \times 2 \times 2$
(b) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
(c) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
(d) $11 \times 11 \times 11 \times 11 \times 11 \times 11$
(e) $13 \times 13 \times 13 \times 13 \times 13 \times 13$
(f) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
(g) $5 \times 5 \times 5 \times 5 \times 6 \times 6 \times 6$
(h) $2 \times 2 \times 2 \times 3 \times 3$
(i) $4 \times 4 \times 4 \times 4 \times 10 \times 10 \times 10 \times 10$
(j) $7 \times 7 \times 7 \times 7 \times 7 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
(k) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
(l) $11 \times 11 \times 11 \times 13 \times 13 \times 13 \times 13 \times 13 \times 13$
(m) $6 \times 6 \times 6 \times 6 \times 7 \times 7 \times 7 \times 9 \times 9$
(n) $12 \times 14 \times 14 \times 14$

W.E. 9

2 Write the following in expanded form.

(a) 5^7
(b) 4^6
(c) 7^3
(d) 9^6
(e) 10^4
(f) 3^7
(g) $2^3 \times 5^4$
(h) $3^7 \times 11^5$
(i) $9^5 \times 7^5$
(j) $2 \times 4^6 \times 6^4$
(k) $3^4 \times 7^2 \times 12$
(l) $9^3 \times 11 \times 17^2$

Remember that 3 is the same as 3^1.

W.E. 10

3 Simplify the following.

(a) $5^2 \times 5^3$
(b) $3^7 \times 3$
(c) $6^4 \times 6^2$
(d) $4^5 \times 4^2$
(e) $9^2 \times 9^7$
(f) $17^5 \times 17^6$
(g) $63^4 \times 63^{12}$
(h) $4^3 \times 5^2 \times 4^6 \times 5^3$
(i) $2^7 \times 6^3 \times 2^3 \times 6^4$
(j) $7^3 \times 8^4 \times 7 \times 8^8$
(k) $11^2 \times 9^2 \times 11^5 \times 9^5$
(l) $10^{10} \times 3^8 \times 10 \times 3^7$

W.E. 11

4 Simplify, if possible, then evaluate the following.

(a) $(-2)^2 \times (-2)^5$
(b) $(-6)^7 \times (-6)^2$
(c) $(-3)^4 \times (-3)^5$
(d) $(-7)^2 \times (-7)^2$
(e) $-5^2 \times 3^3$
(f) $(-2)^2 \times -4^2$
(g) $-6^2 \times (-8)^4$
(h) $-7^2 \times -9^2$

W.E. 12

5 Simplify the following.

(a) $7^4 \div 7^2$
(b) $3^9 \div 3^4$
(c) $2^8 \div 2^5$
(d) $13^4 \div 13^2$
(e) $17^6 \div 17^3$
(f) $25^8 \div 25^5$
(g) $(-2)^5 \div (-2)^3$
(h) $(-4)^4 \times (-4)^3$
(i) $(-6)^3 \div (-6)^2$
(j) $(3^4 \times 2^6) \div (3^2 \times 2^3)$
(k) $(6^5 \times 4^3) \div (6^3 \times 4^2)$
(l) $(9^{11} \times 13^5) \div (9^9 \times 13^4)$
(m) $(5^{12} \times 7^6) \div 7^5$
(n) $(11^2 \times 8^3) \div (11 \times 8)$
(o) $(2^7 \times 3^5 \times 4^2) \div (2^5 \times 4)$

Understanding

6 **(a)** In index form $4 \times 4 \times 4 \times 4 \times 4 \times 4$ is:

A 24 **B** 6^4 **C** 4^6 **D** 1024

(b) In expanded form 3^5 is:

A 3×5 **B** $3 \times 3 \times 3 \times 3$ **C** $5 \times 5 \times 5$ **D** $3 \times 3 \times 3 \times 3 \times 3$

(c) The value of 7^5 is:

A 35 **B** 2401 **C** 16 807 **D** 78 125

(d) The value of -5^4 is:

A -625 **B** -20 **C** 20 **D** 625

7 Copy and complete the following table.

Words	Index form	Expanded form	Value
			49
		$6 \times 6 \times 6 \times 6 \times 6$	
	8^3		
Eleven to the power of six			

8 Evaluate the following expressions, given that $x = 2$ and $y = -4$.

(a) $x^2 \times y$ **(b)** $x^2 \times y^2$ **(c)** $x^2 \times x^2 \times y^2$

(d) $y^3 \times y^2$ **(e)** $6 \times x^4 \div 4$ **(f)** $(6^2 \div 12) \times y^2 \div -2$

9 A scientist studying a population of insects finds that the population doubles every week.

If the scientist begins with 50 insects, at the end of 1 week there will be $50 \times 2 = 100$ insects. At the end of 2 weeks, the population will be:

Week 1 population × 2
$= 100 \times 2$
$= (50 \times 2) \times 2$
$= 50 \times 2^2$ insects.

(a) Use index notation to write how many insects there will be after:

(i) 3 weeks **(ii)** 5 weeks **(iii)** 8 weeks.

(b) Evaluate each of your answers in the previous part to find the actual number of insects after each period.

10 **(a)** Complete the table of powers of 5.

$(-5)^1$	$(-5)^2$	$(-5)^3$	$(-5)^4$	$(-5)^5$	$(-5)^6$	$(-5)^7$	$(-5)^8$	$(-5)^9$	$(-5)^{10}$	$(-5)^{11}$	$(-5)^{12}$
-5	25	-125									

(b) What is the pattern?

(c) How can you predict whether the answer is positive or negative?

11 The Wimbledon tennis championship consists of 7 rounds of matches: Rounds 1, 2, 3 and 4, the quarterfinals, the semifinals and the final. Half the players (the losers of their matches) leave the competition after each round, so that there are only 2 players remaining in the final.

(a) Starting at the final and working back through each round, write the number of players playing in each round as a number in index form.

(b) How many players are there at the start of the championship?

12 Simplify the multiplications first, then the divisions, then evaluate the following.

(a) $\frac{8^2 \times 8^3}{8^4}$ **(b)** $\frac{5^5 \times 5^4}{5^6}$ **(c)** $\frac{1^{13} \times 1^5}{1^9}$

(d) $\frac{(-4)^2 \times (-4)^5}{(-4)^4}$ **(e)** $\frac{(-3)^3 \times (-3)^2}{-3}$ **(f)** $\frac{(-2)^5 \times (-2)^3}{-2^2}$

13 (a) Evaluate each index number separately, then find the product or quotient.

(i) $4^5 \times 4^3$ **(ii)** $7^5 \div 7^2$

(iii) $(9^2 \times 9^3) \div 9^4$ **(iv)** $(5^3 \times 5^4) \div (5^4 \times 5^3)$

(b) Repeat the first part, this time using the rules for multiplying and/or dividing index numbers to simplify, then evaluate the result.

(c) Which process is more efficient—the first method or the second?

Reasoning

14 (a) Write the value of each of the following.

(i) 1^5 **(ii)** 1^7 **(iii)** 1^{15}

(iv) 0^4 **(v)** 0^8 **(vi)** 0^{11}

(b) What conclusions can you draw from this about:

(i) powers of 1 **(ii)** powers of 0?

15 (a) State whether the value of the following will be positive or negative.

(i) $(-4)^3$ **(ii)** -4^3 **(iii)** $(-6)^6$ **(iv)** -6^6

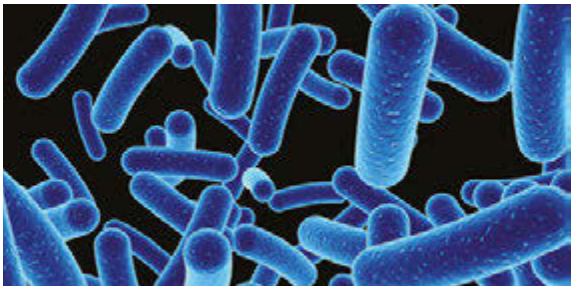
(b) Explain how brackets may affect the signs of the answers for part **(a)**.

(c) Find the value of the following.

(i) $-3^3 \times 4^2$ **(ii)** $(-2)^4 \times (-3)^2$ **(iii)** $-2^5 \times -5^2$

(iv) $(-4)^3 \times (-5)^2$ **(v)** $-2^3 \times (2^4)$ **(vi)** $-2^3 \times (-2)^4$

16 Bacteria A splits into two after every 20 minutes and Bacteria B splits into two after every 40 minutes.



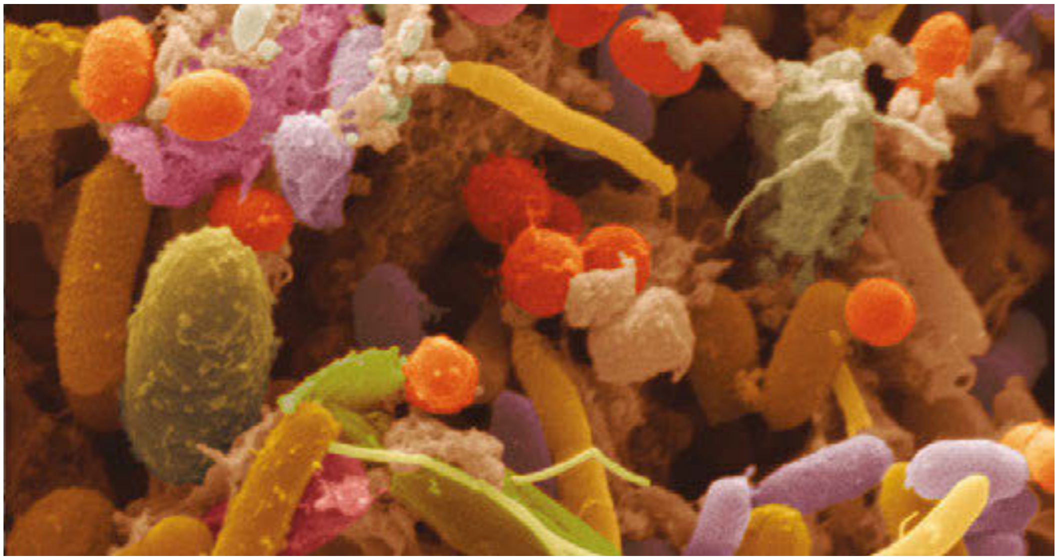
(a) Complete the following bacteria population table.

Time (min)	Number of Bacteria A	Number of Bacteria B
0	1	1
20		
40		
60		
80		
100		
120		
140		

(b) What will be the population of each after 3 hours?

(c) How many more of Bacteria A than Bacteria B will there be after 3 hours?

(d) If Bacteria A was competing for food and space with Bacteria B, which bacteria would be the winner? Give a reason for your answer.



Open-ended

17 Write one example of each of the following that has a value between 20 and 100.

(a) a number that is a power of 2

(b) a number that is a power of 3

(c) a number that is a power of 4 and a power of 2

18 Here is some of Joel's homework on index numbers. He only has one question correct.

Write the value of the following index numbers.

1	$4^6 = 24$	**4**	$7^2 = 14$
2	$3^3 = 9$	**5**	$5^4 = 20$
3	$2^5 = 10$	**6**	$9^1 = 9$

(a) How do you think Joel obtained his answers?

(b) Which question did he 'accidentally' get correct? Why was this?

(c) What would you tell Joel to help him have a better understanding of index numbers?

19 Hayden and Tao were arguing about the solution to $3^3 \times 3^3 + 3^3 \times 3^3$. Hayden said it was 2×3^6 and Tao said it was 18^6. Who was correct, and why?

20 **(a)** Write a division between any two of the eight expressions below, so that the answer is 2.

$10^2, 2^3, 4^2, 2^5, 4^3, 2 \times 5^2, 2 \times 3^2, 6^2$

(b) Write two more divisions that have an answer of 2, using two different pairs of the above expressions.

Game

Closest to 500

Equipment required: 2 dice, calculator

How to win:
The winner of the game is the player who gets closest to 500 after 10 rolls.

How to play:
Copy a blank version of the table shown for each person.

Roll both dice. Choose one number as the base, the other as the exponent (power).

Six is a 'wild card'. If a player rolls a six, they must choose any whole number between 1 and 5 to take its place.

Find the resulting value (using the calculator if necessary) and then add or subtract your new number to your current total. It is possible to go into the negative numbers if you want.

Do this for 10 rolls and then compare your final total with that of your playing partner(s).

An example of a player's table after three rolls is shown below.

Roll	Base	Exponent	Value	Add/Subtract
1	2	3	$2^3 = 8$	8
2	6	2	$6^2 = 36$	8 + 36 = 44
3	4	5	$4^5 = 1024$	44 – 1024 = -980
4				
5				
6				
7				
8				
9				
10				

Investigation

Cyclic powers

Equipment required: calculator

Do you recognise the sequence of numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, …? They are the multiples of 2 (the even numbers). If you look at the last digit of each number, you can see a repeating, or cyclic, pattern: 2, 4, 6, 8, 0, 2, 4, 6, 8, 0, …

The multiples of 5 form this sequence: 5, 10, 15, 20, 25, 30, … Here, the last digits form a shorter repeating pattern: 5, 0, 5, 0, 5, 0, …

In this investigation you will look for patterns in the sequences of powers.

The Big Question

Do sequences of powers contain patterns? Can you use these patterns to make predictions?

Engage

Here are the powers of 2.

$2^1 = 2$ (2)
$2^2 = 4$ (2×2)
$2^3 = 8$ $(2 \times 2 \times 2)$
$2^4 = 16$ $(2 \times 2 \times 2 \times 2)$
$2^5 = 32$ $(2 \times 2 \times 2 \times 2 \times 2)$

You can write the powers as a sequence: 2, 4, 8, 16, 32, …

The next number in the sequence is found by multiplying the previous number by 2. Notice that the numbers in this sequence get very big very quickly.

1 **(a)** Write the first 5 powers of 2 listed above, then calculate the next 5 numbers in the sequence. (Your calculator should have a key with a symbol such as ^ that calculates powers.)

(b) Write the last digit in each of the 10 numbers. Can you see a pattern? If not, calculate more powers until the pattern becomes clear. Predict what the last digit of 2^{16} will be, then calculate it to check your prediction.

2 How many numbers are there in one 'cycle' of the pattern? (That is, how many numbers does the pattern go through before beginning again?) What power of 2 will begin the fifth cycle of the pattern?

Explore

3 Are there patterns in the sequences of other powers? Write the sequences of the powers of 3, 4, 5, 6, 7, 8 and 9. Write any patterns formed by the last digits of the numbers in the sequence.

Strategy options

- Guess and check.
- Make a table.
- Look for a pattern.

Explain

4 **(a)** For each sequence of powers, describe any patterns you found in the last digits.

(b) The patterns vary in the number of digits that appear before the pattern begins again, or 'cycles'. How many digits in:

(i) the shortest cycle

(ii) the longest cycle?

5 Can you explain why some patterns have short cycles and some have long cycles? Think about the multiplications done to generate each number in the sequence of powers.

Elaborate

6 Use your pattern to predict the last digits of the 23rd and 39th powers of each of the numbers from 2 to 9. Explain how you arrived at your prediction. (You can try to check your predictions, but your calculator may struggle with some of the bigger numbers!)

7 What is unusual about the sequences of powers for 1 and 10?

Evaluate

8 Approximately how many numbers did you need to write before you saw a pattern in the sequence? Did you get better at seeing patterns as you went along?

Extend

9 Some numbers, such as 16, 81, 256 and 729, appear in more than one sequence of powers. What are the connections between these numbers and the sequences that they appear in?

Powers of powers, products and quotients

1.6

Raising a number in index form to a power

Consider $(7^2)^3$. This is 7^2 multiplied by itself three times (that is, multiplied so that it appears in the multiplication three times). You can write this as follows.

$$\begin{aligned}(7^2)^3 &= (7 \times 7) \times (7 \times 7) \times (7 \times 7)\\ &= 7^6\end{aligned}$$

or:

$$\begin{aligned}(7^2)^3 &= 7^2 \times 7^2 \times 7^2\\ &= 7^{(2+2+2)}\\ &= 7^6\end{aligned}$$

Notice that 2×3 equals 6. So, you can simplify by multiplying the indices together:

$$\begin{aligned}(7^2)^3 &= 7^{(2 \times 3)}\\ &= 7^6\end{aligned}$$

When raising a number in index form to a power, keep the base and multiply the indices; e.g. $(3^4)^3 = 3^{12}$.

Worked example 13

W.E. 13

Simplify $(2^3)^5$

Thinking	Working
Keep the base and multiply the indices.	$(2^3)^5$ $= 2^{3 \times 5}$ $= 2^{15}$

Raising a product to a power

How could you calculate $(4 \times 7)^3$?

You could find the product in brackets first, then raise it to the power:

$$\begin{aligned}(4 \times 7)^3 &= (28)^3\\ &= 21\,952\end{aligned}$$

Or, you could raise each factor in the product to the power. You can show this by writing the expression in expanded form first:

$$\begin{aligned}(4 \times 7)^3 &= (4 \times 7) \times (4 \times 7) \times (4 \times 7)\\ &= 4 \times 4 \times 4 \times 7 \times 7 \times 7\\ &= 4^3 \times 7^3\\ &= 64 \times 343\\ &= 21\,952\end{aligned}$$

If a product of factors in brackets has been raised to a power, then each factor in the brackets is raised to that power.

e.g. $(3 \times 5)^4 = 3^4 \times 5^4$

Raising a quotient to a power

What does $\left(\frac{2}{5}\right)^3$ simplify to? It has a base of $\frac{2}{5}$ and an index of 3.

$$\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$

$$\left(\frac{2}{5}\right)^3 = \frac{2 \times 2 \times 2}{5 \times 5 \times 5}$$

$$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$$

Each number in the brackets has been raised to the power of 3.

> If a quotient in brackets has been raised to a power, then each number in the brackets is raised to that power.
>
> e.g. $\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$

Worked example 14

W.E. 14

Expand the brackets in the following.

(a) $(2 \times 3)^4$

(b) $\left(\frac{5}{7}\right)^2$

Thinking	Working
(a) Raise every factor in the brackets to the power outside the brackets.	(a) $(2 \times 3)^4$ $= 2^4 \times 3^4$
(b) Raise every number in the brackets to the power outside the brackets.	(b) $\left(\frac{5}{7}\right)^2$ $= \frac{5^2}{7^2}$

The zero power

To simplify $5^3 \div 5^3$, you can use two methods.

Method 1

Use the rule for dividing index numbers, which is to keep the base and subtract the indices.

$$\begin{aligned} 5^3 \div 5^3 &= 5^{(3-3)} \\ &= 5^0 \end{aligned}$$

Method 2

Write in expanded form and cancel common factors.

$$5^3 \div 5^3 = \frac{5^3}{5^3} = \frac{\cancel{5}\times\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times\cancel{5}} = 1$$

(Don't forget that any number divided by itself is 1.)

From Method 1, you can see that $5^3 \div 5^3 = 5^0$

From Method 2, you can see that $5^3 \div 5^3 = 1$

Therefore, $5^0 = 1$.

Try this with $\frac{8^4}{8^4}$. Do you get $8^0 = 1$?

Any number raised to the power of zero equals one.
(The exception is zero itself: 0^0 is undefined.)

1.6 Powers of powers, products and quotients

Navigator

1, 2 (columns 1–2), 3 (row 1), 4, 5, 7 (column 1), 8, 11, 13 (a–b), 14, 19	1, 2 (columns 2–3), 3 (row 2), 4, 5, 6, 7 (columns 2–3), 8, 9, 11, 12 (row 1), 13 (a–d), 14, 16, 19	1, 2 (columns 3–4), 3 (row 2), 4, 5, 6, 7 (columns 3–4), 8, 9, 10, 12, 13 (c–f), 15, 16, 17, 18

Answers p. 620

Equipment required: calculator for Questions **11**, **17**

Fluency

1 Simplify the following.

W.E. 13

(a) $(9^6)^2$ **(b)** $(14^7)^3$ **(c)** $(6^8)^8$ **(d)** $(10^5)^5$

(e) $(4^2)^3$ **(f)** $(5^2)^0$ **(g)** $(2^4)^5$ **(h)** $(7^0)^2$

2 Expand the brackets in the following.

W.E. 14

(a) $(4 \times 3)^2$ **(b)** $(2 \times 5)^3$ **(c)** $(3 \times 2)^5$ **(d)** $(4 \times 7)^4$

(e) $(7 \times 10)^3$ **(f)** $(8 \times 9)^1$ **(g)** $(2 \times 3 \times 1)^6$ **(h)** $(4 \times 5 \times 6)^2$

(i) $\left(\frac{3}{5}\right)^3$ **(j)** $\left(\frac{2}{9}\right)^2$ **(k)** $\left(\frac{10}{13}\right)^3$ **(l)** $\left(\frac{12}{13}\right)^2$

(m) $\left(\frac{1}{2}\right)^5$ **(n)** $\left(\frac{1}{10}\right)^3$ **(o)** $\left(\frac{1}{11}\right)^6$ **(p)** $\left(\frac{1}{12}\right)^2$

3 Evaluate the following.

(a) 7^0 **(b)** 20^0 **(c)** $\left(\frac{5}{6}\right)^0$ **(d)** $\left(-3\frac{1}{2}\right)^0$ **(e)** $(-0.5234)^0$

(f) 3×5^0 **(g)** $2^2 \times 6^0$ **(h)** -3^0 **(i)** -6×5^0 **(j)** $3 \times (-6)^0$

4 **(a)** $(5^6)^2$ simplifies to:

A 5^8 **B** 5^4 **C** 5^3 **D** 5^{12}

(b) $(4 \times 3)^0$ simplifies to:

A 1 **B** 2 **C** 7 **D** 12

5 What is $(2^3)^2 \times 2^3$ equal to?

A 4^9 **B** 2^8 **C** 2^9 **D** 2^{35}

6 State whether the following equations are true (T) or false (F). If false, where possible state the correct answer in index form.

(a) $(3 + 4)^2 = 3^2 + 4^2$ **(b)** $(12 - 8)^2 = 4^2$ **(c)** $(-3)^2 + (6)^3 = (-18)^2$

(d) $(-4)^3 \times (-2)^3 = 8^3$ **(e)** $(5)^2 \times (2)^3 = 10^3$ **(f)** $8^7 \div 8^0 = 8^7$

Understanding

Simplify the numerators and denominators first, then look to cancel common factors.

7 Use a combination of the rules for working with index numbers to simplify the following. Leave your answers in index form (do not evaluate).

(a) $(8^2 \times 7^3)^2$ **(b)** $\left(\frac{6^2}{7^2}\right)^3$ **(c)** $\frac{(2 \times 4)^5}{2}$ **(d)** $\frac{(5^3)^2}{5^2}$

(e) $\frac{3^4 \times 4^7}{9^2 \times 3^4}$ **(f)** $\frac{3^7 \times 4^0}{3^5 \times 4^5}$ **(g)** $\frac{(7 \times 8)^4}{7^2}$ **(h)** $\frac{(8 \times 9 \times 10)^3}{(9 \times 8)^3}$

(i) $\frac{(4^3 \times 5)^4}{(5^2 \times 4)^2}$ **(j)** $(2 \times 3)^4 \times \left(\frac{1}{3}\right)^2$ **(k)** $\left(\frac{3}{2}\right)^4 \times \left(\frac{2}{3}\right)^3$ **(l)** $\left(\frac{1}{3^5}\right)^2 \times \left(\frac{2}{3}\right)^2$

8 Write the following as the product of two or three prime factors, raised to a single power; e.g. $15^2 = (5 \times 3)^2$.

(a) 10^6 **(b)** 21^3 **(c)** 35^2 **(d)** 45^4 **(e)** 70^5 **(f)** 77^7

9 Write the following as the product of two or more prime factors in index form. The index of each prime factor may be different; e.g. $24 = 2^3 \times 3$.

(a) 36 **(b)** 48 **(c)** 63 **(d)** 72 **(e)** 75 **(f)** 84

10 Evaluate the following without a calculator by writing the numerators and denominators as products of prime factors, then simplifying.

(a) $\frac{9 \times 14}{15 \times 21}$ **(b)** $\frac{12 \times 18}{27 \times 24}$ **(c)** $\frac{45 \times 21}{35 \times 36}$ **(d)** $\frac{54 \times 60}{48 \times 33}$

11 **(a)** By trial and error, find the largest number that is a power of 2 and that will fit on a 10-digit calculator display.

(b) Find the largest power of 9 that will fit on a 10-digit calculator display.

12 Pronumerals are the symbols used to represent unknown numbers. You can apply the rules for working with index numbers to pronumerals. Use one of the index number rules you have learnt in this section to simplify the following.

(a) $(a^3)^2$ **(b)** $\left(\frac{c}{d}\right)^4$ **(c)** $(xy)^5$ **(d)** m^0

(e) $(2x)^2$ **(f)** $5p^0$ **(g)** $\left(\frac{b^2}{c}\right)^5$ **(h)** $\left(\frac{3a}{b}\right)^3$

Reasoning

13 Use the appropriate index law to find the value of x in each of the following.

(a) $10^{12} \div 10^5 = 10^x$

(b) $(5^6)^3 = 5^x$

(c) $(6^x)^2 = 6^{20}$

(d) $\frac{7^4 \times 7^x}{7^{11}} = 1$

(e) $\frac{(3^4)^x}{3^{12}} = 1$

(f) $\frac{(5^6)^x \times (5^x)^2}{5^4} = 5^4$

14 Kelvin and Terry were given the following question as part of their indices homework.

Evaluate: $\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

Here is Kelvin's working:

$$\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$$
$$= \left(\frac{27 \times 32}{9 \times 8}\right)^2$$
$$= \frac{729 \times 1024}{81 \times 64}$$
$$= \frac{746\,496}{5184}$$
$$= 144$$

Here is Terry's working:

$$\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$$
$$= (3 \times 2^2)^2$$
$$= (3 \times 4)^2$$
$$= 12^2$$
$$= 144$$

'Wow', said Kelvin. 'We both got the same answer but your working doesn't look as complicated as mine'. 'No', said Terry. 'And I didn't have to use my calculator either!'

(a) How is Terry's method different from Kelvin's?

(b) Both answered the question correctly, but who did it more efficiently? Give a reason for your choice.

15 Write the following numbers as index numbers raised to a second index; for example, $64 = (2^2)^3$.

(a) 16

(b) 81

(c) 625

(d) 729

(e) 256

(f) 1 000 000

Open-ended

16 Find two or three ways of writing the following as a product of two or three factors, raised to a single power.

(a) 24^3

(b) 36^5

(c) 48^2

(d) 80^4

17 1000 can be written as 10^3, or as $5^3 \times 2^3$. Find at least two different ways of writing 625 000 using indices.

18 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

19 The index laws are sometimes used incorrectly. Below is a set of calculations that look reasonable, but are actually incorrect.

In each case, explain the mistake and give the correct answer. Explain why your answer is correct.

(a) $3^2 \times 3^4 = 3^8$

(b) $\frac{7^9}{7^3} = 7^3$

(c) $(5^2)^4 = 5^{24} = 5^{16}$

(d) $13 \times 4^0 = 1$

Challenge 1

1 The mean of -5, -3, 0, 4 and 9 is:

A -1 **B** 1 **C** $3\frac{4}{5}$ **D** $4\frac{3}{4}$

2 Reece bought a new table for \$100. He then changed his mind and sold it for \$110. He changed his mind again and bought it back for \$130, then sold it again for \$150. What overall profit or loss did Reece make?

A \$30 loss **B** \$10 loss **C** \$20 profit **D** \$30 profit

3 Three consecutive numbers are such that twice the greatest added to three times the least is -31. Find the numbers.

4 My father is 30 years older than I am, and my mother is 24 years older than I am. How old was I when my father's age was double my mother's? Explain your answer.

5 If $x^y = 64$, find all the possible pairs of values for x and y.

6 **(a)** By writing 2^{15} as $(2^3)^5$ and 3^{10} as $(3^2)^5$, find out whether $2^{15} > 3^{10}$ is a true or a false statement. Do not use a calculator.

(b) Use the same method to decide which is larger, 2^{27} or 3^{18}.

7 If the numbers $a = 2^{80}$, $b = 3^{60}$ and $c = 5^{40}$ are written in ascending order, then what order do they appear in?

A a, b, c **B** a, c, b **C** b, c, a **D** c, a, b

8 What is the value of $(-1)^5 - (-1)^4$?

A 1 **B** 0 **C** -1 **D** -2

9 $2^3 \times 2^2 \times 3^3 \times 3^2$ is equal to:

A 6^5 **B** 6^6 **C** 36^5 **D** 36^{10}

10 If $10^x - 10 = 99\,990$, then x is equal to:

A 4 **B** 5 **C** 6 **D** 9

11 The value of $\frac{8^2}{2^8}$ is:

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 2 **D** 4

12 If $y^x = 256$, find all the possible whole number pairs of values for x and y.

13 If $x = -3$, which of the following expressions has the largest value?

A $x^2 - 3$ **B** $(x - 3)^2$ **C** $(x + 3)^3$ **D** $x^2 + 3$

14 What is the last digit in the number represented by 4^{3827}?

15 What is the last digit in the number represented by 3^{2004}?

Chapter review 1

Maths literacy

- base
- cube
- evaluate
- expanded form
- exponent
- index
- index form
- indices
- integers
- negative
- positive
- power
- simplify
- square

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The __________ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

2 Two negative numbers multiply to give a __________ result.

3 A __________ number divided by a positive number gives a negative result.

4 5^3 is written in __________, while $5 \times 5 \times 5$ is written in ____________.

5 6^7 is read as 'six to the __________ of 7'.

6 The number that is raised to a power is called the __________.

7 Other names for 'power' are __________ and __________.

8 The addition of two negative numbers will always give a __________ answer.

9 To __________ means to find a value by doing a calculation.

10 Any number (except zero) raised to the __________ of zero is equal to 1.

11 When you __________ an expression, you do not need to calculate an actual value.

Fluency

1 Write a positive or a negative integer to represent each of the following. **1.1**

- **(a)** The bottom of a lake is 23 metres below sea level.
- **(b)** A business made a profit of $840 000.
- **(c)** You deposit $350 into your bank account.
- **(d)** The value of a share in a mining company rose by $4.

2 Place the following in ascending order. **1.1**

(a) -6, 9, 14, -23, 0 **(b)** 8, -15, 5, -7, -2 **(c)** 34, -11, 0, 6, 12

3 Calculate: **1.1**

(a) $-3 + (+10)$ **(b)** $7 - (+9)$ **(c)** $-5 - (+6)$ **(d)** $-11 + (+4)$

(e) $8 + (-5)$ **(f)** $-13 + (-11)$ **(g)** $2 - (-18)$ **(h)** $-12 - (-4)$

4 Calculate: **1.1**

(a) $4 + 7 - 9$ **(b)** $-2 + 5 - 1$ **(c)** $-15 + 23 - 8$

(d) $6 + (-3) - (+10)$ **(e)** $-7 - (-8) + 3$ **(f)** $5 - (-14) - 20$

5 Find the following products. **1.2**

(a) -14×2 **(b)** 4×-5 **(c)** -6×-6 **(d)** -7×-12

(e) 15×-9 **(f)** -22×8 **(g)** -60×30 **(h)** -28×-200

(i) $(-6)^2$ **(j)** -7^2 **(k)** $-2^2 \times 3^2$ **(l)** $(-4)^2 \times (-1)^2$

6 Find the following quotients. **1.3**

(a) $-36 \div 3$ **(b)** $55 \div -11$ **(c)** $\dfrac{-28}{-4}$ **(d)** $-27 \div 3$

(e) $66 \div -11$ **(f)** $\dfrac{-91}{-7}$ **(g)** $72 \div -9$ **(h)** $-45 \div -5$

(i) $\dfrac{-80}{4}$ **(j)** $440 \div -10$ **(k)** $-320 \div 8$ **(l)** $\dfrac{-78}{6}$

7 Evaluate the following expressions. **1.4**

(a) $45 \div 9 \times -2 - 4$ **(b)** $46 + (-6 \times 7) + 20 \div -5$

(c) $-9 \times -5 - 3 \times 4 + 2$ **(d)** $-8 + (-18) \div -3 - 4 \times -4$

(e) $-4 + (-6)^2 \div 9$ **(f)** $-7 \times -8 - 5^2 + (-10)$

8 Write the following in index form and find the value of each. **1.5**

(a) $9 \times 9 \times 9 \times 9 \times 9$ **(b)** $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

(c) $3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$ **(d)** $8 \times 8 \times 10 \times 10 \times 10$

9 Write the following in expanded form. **1.5**

(a) $2^4 \times 5^3$ **(b)** $13^3 \times 8^6$ **(c)** $4^2 \times 7^2 \times 9^5$ **(d)** $6^6 \times 10^2 \times 17$

10 Simplify the following. **1.5**

(a) $7^3 \times 7^2$ **(b)** $3^6 \times 3^2$ **(c)** $5^2 \times 5^2$ **(d)** $2^2 \times 2^6$

(e) $3^6 \times 2^6 \times 3^3 \times 2^4$ **(f)** $4^2 \times 7^2 \times 4^3 \times 7$ **(g)** $5^2 \times 2^3 \times 2^3 \times 5^4$ **(h)** $7^5 \times 11^5 \times 7^2 \times 11$

11 Simplify the following. **1.5**

(a) $5^6 \div 5^2$ **(b)** $\dfrac{7^4}{7}$ **(c)** $11^5 \div 11^3$ **(d)** $\dfrac{2^6}{2^3}$

(e) $(3^6 \times 4^3) \div (3^5 \times 4^2)$ **(f)** $\dfrac{7^3 \times 10^5}{7 \times 10^3}$ **(g)** $(8^2 \times 5^7) \div (8 \times 5^4)$ **(h)** $\dfrac{2 \times 9^3 \times 13^3}{9^2 \times 13}$

12 Evaluate the following. **1.5**

(a) $(-3)^4$ **(b)** -3^4 **(c)** $(-5)^3$

(d) -1^7 **(e)** $-4^2 \times (-3)^2$ **(f)** $(-2)^4 \times (-3)^4$

13 Simplify, then evaluate the following. **1.6**

(a) $(3^5)^2$ **(b)** $(7^2)^3$ **(c)** $(3^5)^3$ **(d)** $(2^4)^4$

(e) 2^0 **(f)** 6×3^0 **(g)** $\dfrac{5^6}{5^6}$ **(h)** $\dfrac{2^{10}}{2^{10}}$

14 Expand the brackets. **1.6**

(a) $(4 \times 11)^3$ **(b)** $(8 \times 9)^5$ **(c)** $(3 \times 5)^7$ **(d)** $(7 \times 10)^4$

(e) $\left(\dfrac{1}{2}\right)^3$ **(f)** $\left(\dfrac{3}{4}\right)^2$ **(g)** $\left(\dfrac{5}{6}\right)^4$ **(h)** $\left(\dfrac{8}{9}\right)^7$

Understanding

1.1

15 The minimum overnight temperatures for 1 week at Mt Hotham were -2 °C, -3 °C, 1 °C, 2 °C, -3 °C, -2 °C, 0 °C. Find the mean minimum overnight temperature for the week. (The mean is found by adding all values, then dividing the result by the number of values.)

1.2

16 In the game called Count'Em Up, red tokens are worth 5 points, black tokens are worth -3 points and white tokens are worth -1 point. Calculate the total point score at the end of a round for each of the following players.

Ava: 2 red, 3 black and 1 white
Georgia: 3 red, 4 black and 2 white
Rose: 3 red, 5 black and 4 white
Wei: 2 red, 5 black and 2 white

1.5

17 A scientist is observing the behaviour of bacterial cells. She finds that each cell divides in two every 24 hours.

(a) If the scientist isolates a single cell in a dish, complete the following table to show how many cells there will be after a certain number of days.

Time (number of days)	0	1	2	3	4
Number of cells	1				

(b) Extend the pattern in your table to find how many cells there will be after 1 week.

(c) Write the 'Number of cells' row of the table as a series of index numbers.

(d) Use the pattern to find how many cells there will be after 2 weeks.

1.3

18 Three friends invest a total of $270 in a lottery (they put in $90 each). If they collect prizes worth a total of:

(a) $60, find the loss for each friend

(b) $300, find the profit for each friend

(c) $28 500, find the profit for each friend.

1.4

19 An ice-cube tray filled with water at a temperature of 21 °C is put in the freezer, where it takes 3 hours to freeze solid at 0 °C. What is the average hourly drop in temperature?

1.5

20 Choose the best answer.

(a) $5^3 \times 2^3$ is the same as:

A 10^3 **B** $(5 \times 2)^6$ **C** 10^6 **D** both B and and C

(b) $7^6 \times 7^6$ is the same as:

A 49^6 **B** 7^{12} **C** both A and and B **D** 49^{12}

21 Simplify if possible, then evaluate:

1.5, 1.6

(a) $\left(\frac{3}{7}\right)^0$ **(b)** $(-8)^0$ **(c)** $\frac{(-6)^5}{(-6)^5}$ **(d)** $\left(-\frac{2}{5}\right)^0$

(e) $\frac{(-3)^6}{(-3)^5}$ **(f)** $\frac{(-5)^8}{(-5)^6}$ **(g)** $\frac{(-2)^{10}}{(-2)^5}$ **(h)** $\frac{(-3)^{20}}{(-3)^{17}}$

(i) $\frac{2^3 \times 2^4}{2^5}$ **(j)** $\frac{10^5 \times 10}{-10^2}$ **(k)** $\frac{3^4 \times 4^2 \times 3^2 \times 4}{(3 \times 4)^3}$ **(l)** $\left(\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2$

22 Evaluate the following without using a calculator by writing the numbers as products of prime factors and simplifying the calculation.

1.6

(a) $\frac{25 \times 12}{15 \times 18}$ **(b)** $\frac{10^2 \times 45}{21^2 \times 25}$ **(c)** $\frac{9 \times 7}{10^4 \times 21}$

Reasoning

23 Calculate the value of x in the following equations.

1.5

(a) $15^7 \div 15^4 = 15^x$ **(b)** $12^{12} \div 12^x = 12^4$ **(c)** $34^x \div 34^{10} = 1$

24 16 can be written as $(2^2)^2$. Rewrite the following as an index number raised to a second index, using the base given in brackets.

1.6

(a) 36^2 (6) **(b)** 128^4 (2) **(c)** $(1000^3)^2$ (10) **(d)** 4900^7 (70)

25 **(a)** Find the magic sum for the following 4×4 magic square.

1.1, 1.2, 1.3

(b) Complete the magic square.

(c) Make a new 4×4 magic square by dividing each number in the completed square by -2. What is the magic sum of this magic square?

(d) Make another new 4×4 magic square by multiplying each number in the original magic square by -2. What is the magic sum of this magic square?

		2	-20
-18		-8	
	-2		8
10	-12	-4	

26 Van is organising a table tennis tournament for the local clubs in his area. The tournament will have 3 rounds, then the semifinals, then the final. The losers in each round leave the competition. How many players will Van need to invite to compete in round 1 in order to end up with 2 players in the final?

1.5

27 Insert brackets where necessary to make the following statements true.

1.4

(a) $-3 - 4 \times -2 = 14$ **(b)** $2 + -3 \times 4 + -2 = -6$

(c) $16 \div -4 + 3 \times -2 + 1 = -7$ **(d)** $4 + -3 \times -2 \div -6 \times -1 + 5 = 0$

(e) $2 - 3 + 6 \times 3 + 2 \times -1 - 6 = -37$ **(f)** $24 \div -2 \div -3 + 4 \times -2 - 5 = -24$

28 State whether the following are true (T) or false (F).

1.5, 1.6

(a) $5^3 \times 5^2 = 5^6$ **(b)** $8^3 \times 8^4 \times 8^6 > 8^9 \times 8^4$ **(c)** $17 + 17 + 17 + 17 = 17^4$

(d) $(3^6)^3 = (3^3)^6$ **(e)** $(4 + 6)^4 = (10^2)^2$ **(f)** $(4 + 3)^2 = 4^2 + 3^2$

Numeracy practice 1

Non-calculator

1 Two numbers added together equal -5. The same two numbers multiply together to give -24. What are the two numbers?

2 You have \$5 in your pocket. You owe a friend \$8, and another friend owes you \$12. If everyone pays what they owe, how much money do you have now?

3 Which of the following is equal to $9^2 \times 3^3$?

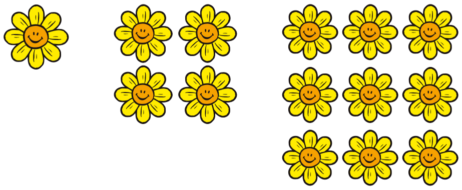
A $9 \times 2 \times 3 \times 3$

B $3 \times 3 \times 3 \times 3 \times 3$

C $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

D 27^5

4 Dora is making this flower pattern, made of an increasing number of shapes.



(a) Complete the table to show the number of flowers she needs for each shape in her pattern.

Shape	1	2	3	4	5
Number of flowers					

(b) What sequence of numbers does the second row of the table form?

(c) How many flowers would she need for Shape 10?

Calculator allowed

5 $(-2)^2 - 9$ is equal to:

A -13 **B** -5 **C** 5 **D** 13

6 A microwave oven can heat a frozen meal by 10 °C every minute. A meal taken out of a freezer is at -20 °C. How many minutes should the meal be heated to be at 50 °C when served?

7 If $A^3 = B^2 = C^6 = 64$, then what are the values of A, B and C?

8 Which of the following is not equal to $\sqrt{4096}$?

A $\frac{2^8}{2^2}$ **B** $(2^3)^2$ **C** 2048 **D** 4^3

2

Fractions, decimals and percentages

2

Diamonds lose their glitter.

The 2008–09 global financial crisis badly affected Botswana's diamond industry.

Botswana is a small country in Africa that produces a large number of diamonds. Diamonds are the main source of income for the country, making up 70% of the total value of Botswana's exports. In the global financial crisis of 2008–09, a decrease in demand caused Botswana's diamond exports to drop by 90%. This, along with a drop in other mineral exports, caused a loss of 50% of the country's income. Botswana's government was forced to ask for a $1.5 billion loan from the African Development Bank to pay for imported food, fuel and other basic needs.

By the second half of 2009, the demand for diamonds was starting to rise again. Some mines that had been closed were opened again and production gradually returned to previous levels.

Forum

Why were the fall in exports and loss of income reported using percentages instead of dollar amounts?

How can a drop in the export of just one product (diamonds) have such a large effect on the economy of the whole country?

How might the drop in diamond exports affect the people and communities in Botswana?

Why learn this?

Imagine you need to work out which discounted jacket is the best deal, or how much profit you made by selling your skateboard on the internet. You need good skills in fractions, decimals and percentages. These skills are not just important in the world of business and money. You can use fractions, decimals and percentages to analyse and present all kinds of information. This can help you decide how useful or meaningful the information is.

After completing this chapter you will be able to:

- convert between fractions, decimals and percentages
- identify different types of decimal numbers and use the correct notation for writing them
- calculate with negative fractions and decimals
- estimate percentages
- write one amount as a percentage of another
- find percentages of amounts
- use percentages to calculate increases, decreases, discounts, mark-ups, profit and loss
- solve problems involving fractions, decimals and percentages.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Evaluate the following, writing your answers in simplest form.

(a) $\frac{2}{3}+\frac{3}{5}$ **(b)** $\frac{5}{6}-\frac{1}{4}$ **(c)** $\frac{4}{9}\times\frac{3}{8}$ **(d)** $\frac{4}{15}\div\frac{4}{5}$

2 Evaluate the following.

(a) $2.6+3.04+5.3+2.101$ **(b)** $5.29-4.831$ **(c)** 12.7×5

(d) 4.2×3.1 **(e)** $3.72\div 8$ **(f)** $12.64\div 0.04$

3 Round the following decimals to the number of decimal places shown in brackets.

(a) 2.0335 (2) **(b)** 3.1884 (2) **(c)** 2.9096 (3) **(d)** 4.086 19 (3)

4 Simplify the following fractions. **(a)** $\frac{27}{45}$ **(b)** $\frac{16}{92}$ **(c)** $\frac{56}{21}$

5 Write the following mixed numbers as improper fractions.

(a) $2\frac{1}{2}$ **(b)** $4\frac{2}{9}$ **(c)** $3\frac{7}{20}$

6 Write the following improper fractions as mixed numbers in simplest form.

(a) $\frac{17}{7}$ **(b)** $\frac{40}{9}$ **(c)** $\frac{95}{10}$

7 Evaluate:

(a) 7.08×10 **(b)** 0.49×1000 **(c)** 0.8×100

(d) $5.21\div 100$ **(e)** $34.65\div 10$ **(f)** $4.037\div 1000$

8 Calculate the following. **(a)** $\frac{5}{6}$ of \$90 **(b)** $\frac{7}{12}$ of 180 kg **(c)** $\frac{9}{8}$ of 200 m

9 Write the following fractions as percentages.

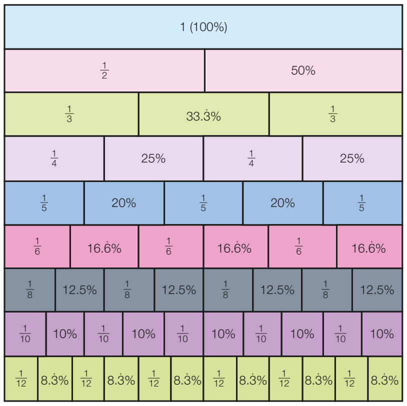
(a) $\frac{1}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{2}{5}$ **(d)** $\frac{7}{10}$

10 Write the following decimals as percentages.

(a) 0.35 **(b)** 0.9 **(c)** 0.04 **(d)** 1.27

Exploration Task

You can download this activity from the eBook or the Pearson Places website.



Is this right?

In this activity, you will explore ways to identify mistakes made when trying to convert a ratio into a decimal. What is the best way to discover mistakes?

2.1

Working with fractions and decimals

Fractions and decimals are used to represent parts of wholes. They are two different ways of writing a number. To be able to work with decimals and fractions is a useful everyday skill.

Converting decimals to fractions

Decimal numbers use place value to represent the fractional parts of a number. You can use place value to convert decimals to fractions.

For example, write the number 13.2452 in a place value table, like this.

tens	ones	•	tenths	hundredths	thousandths	ten-thousandths
10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$
1	3	.	2	4	5	2

From this, 13.2452 can be written in expanded fractional form as

$13 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{2}{10\,000}$ or $13\frac{2452}{10\,000}$ (which simplifies to $13\frac{613}{2500}$ when you cancel a common factor of 4).

To convert a decimal to a fraction:

- use the place value column of the last digit to get the denominator of the fraction
- write all the digits of the decimal part in the numerator
- simplify the fraction if possible.

Worked example 1

W.E. 1

Write 0.384 as a fraction in simplest form.

Thinking	Working
1 The last digit in the decimal is in the thousandths column, so 1000 is the denominator. Write the other digits as the numerator.	$0.384 = \frac{384}{1000}$
2 Simplify the fraction. (Here, cancel common factors of 4 and 2.)	$= \frac{96}{250}$ $= \frac{48}{125}$

The division symbol itself looks like a fraction.

Converting fractions to decimals

A fraction has a line or bar, which separates the top number (the numerator) from the bottom number (the denominator). This line can be thought of as division.

So, $\frac{1}{2} = 1 \div 2$, or 1 shared between 2. If you perform this division, you get the decimal equivalent for $\frac{1}{2}$, which is 0.5.

You can also use equivalent fractions to convert a fraction to a decimal. Write the fraction as an equivalent fraction with a denominator that is a multiple of 10, such as 10 or 100 or 1000. You can then write a decimal number using the digits of the numerator, with as many decimal places as there are 0s in the denominator.

So, $\frac{1}{2} = \frac{5 \times 1}{5 \times 2} = \frac{5}{10}$, which can be written as 0.5.

To convert a fraction to a decimal, perform the division, e.g. $\frac{3}{4} = 3 \div 4 = 0.75$.
Or, convert to an equivalent fraction with a denominator that is a multiple of 10, e.g. $\frac{3}{4} = \frac{75}{100} = 0.75$.

Some fractions are used so often that it is useful to know the decimal equivalent. If you don't already know them, try to learn the following.

Common fractions and their decimal equivalents

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.5	$0.\dot{3}$	$0.\dot{6}$	0.25	0.75	0.2	0.125	0.1	0.01	0.001

Notice that the decimal equivalents for $\frac{1}{3}$ and $\frac{2}{3}$ have a dot above the decimal digit. This is to show that the digit is repeated forever; that is, $\frac{1}{3} = 0.333\ldots$ These types of decimals are called **recurring decimals**. You will learn more about these in the next section.

To compare quantities expressed as fractions and decimals, you need to change all values to the same format. It is often easier to convert all values to decimal form.

The easiest way to compare fractions and decimals is to convert the fractions to decimals.

Fractions on a calculator

The key for entering fractions into your calculator will usually look like ⊟. To enter a fraction, you enter ⊟, then enter the numerator and the denominator, using the arrow keys to move between them. To enter a mixed number, you would usually enter SHIFT ⊟.

On some calculators, the key may look like $a\frac{b}{c}$ or $\frac{a}{b}$. To enter a fraction, such as $\frac{5}{6}$, you would enter 5 $a\frac{b}{c}$ 6. You may need to press = for the fraction to be displayed on the screen. To enter a mixed number, such as $4\frac{3}{8}$, you would enter 4 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 8. The calculator may use symbols such as ⌐, ⌞ or _ to separate the whole numbers, numerator or denominator.

Check that you can work with fractions correctly on your calculator by entering some additions and subtractions for which you already know the answers. For example, if you enter $\frac{3}{8} + 1\frac{2}{3}$ correctly, the answer of $2\frac{1}{24}$ should appear on the screen.

To convert a mixed number to an improper fraction, you can usually enter the mixed number followed by **SHIFT** and $a\,{}^{b}\!/_{c}$ or **SHIFT** and **S⇔D**. (You might need to press **=** to see the result.)

You may also notice that pressing the **S⇔D** key or pressing the $a\,{}^{b}\!/_{c}$ key twice will convert a fraction to its decimal form.

Worked example 2

W.E. 2

Arrange the following list in ascending order (smallest to largest) by first converting the fraction values to decimals.

$\frac{5}{8}, \frac{3}{5}, \frac{3}{4}$, 0.69, 0.686

Thinking	Working
1 Convert each of the fractions into decimals. (You could use your calculator for this step, or you could convert them to equivalent fractions with denominators of 10 or 100.)	$\frac{5}{8} = 5 \div 8 = 0.625$ $\frac{3}{5} = 3 \div 5 = 0.6$ $\frac{3}{4} = 3 \div 4 = 0.75$
2 List the numbers in order from smallest to largest, by comparing the decimal digits in each place value column.	0.6, 0.625, 0.686, 0.69, 0.75
3 Substitute the fraction values back into the list.	$\frac{3}{5}, \frac{5}{8}$, 0.686, 0.69, $\frac{3}{4}$

Rounding decimals

Some decimal numbers have a lot of decimal places. Often you don't need all of the decimal places, or it doesn't make sense to use them. For example, you might calculate that you need 1.697 214 metres of wood for a project. The third decimal digit represents millimetres (thousandths of a metre). The fourth, fifth and sixth decimal places represent such a tiny amount that they are impossible to measure accurately and are not necessary. In these cases, it is better to 'round' the number to a certain number of decimal places.

It is important to state how many decimal places a number has been rounded to. You can do this by using the abbreviation 'd.p.' or 'dec. pl.'. You can also say that a rounded number has been written 'correct to' a certain number of decimal places.

Remember, if the digit to the right of the digit being rounded is 5 or more, round up. If it is less than 5, round down.

To round a number to a given number of decimal places, look at the digit to the right of the digit being rounded. If it is:

- 0, 1, 2, 3 or 4, round down. Keep the digit being rounded the same, and delete the digits following it;

  for example, 1.697 214 rounded to 3 d.p. is 1.697
- 5, 6, 7, 8 or 9, round up. Increase the digit being rounded by 1, and delete the digits following it. If the digit being rounded is 9, then write the zero of 10, and carry the one across to the next number to the left;

  for example, 1.789 24 rounded to 2 d.p. is 1.79
  24.5999 rounded to 3 d.p. is 24.600

Working with remainders

Division calculations do not always give whole number answers. On a calculator, remainders are usually given in decimal form. It is important to understand what the remainder means for that particular question.

Worked example 3

W.E. 3

A length of wood is $2\frac{1}{2}$ m long. How many pieces, each 600 mm long, can be cut from this length, and how much will be left over? Ignore any wastage that might occur in cutting. You may wish to use a calculator.

Thinking	Working
1 Write the amounts using the same unit. (In most cases, the smaller unit is more convenient.)	$2\frac{1}{2}$ m = 2500 mm
2 Decide the operation required. 'How many' tells you this is a division question. Do the division.	$25\cancel{0}\cancel{0} \div 6\cancel{0}\cancel{0}$ $= 25 \div 6$ $= 4.166\,666\,667$
3 Write the whole number of pieces.	4
4 Multiply the whole number of pieces (4) by the size of each piece (600 mm) and use this to calculate the number of mm left over.	$4 \times 600 = 2400$ $2500 - 2400 = 100$
5 Write the answer.	You will get 4 pieces 600 mm long and will have 100 mm of wood left over.

In the example above, the decimal remainder after the division represents the length of wood left over after all the pieces of the required length (four 600 mm pieces) have been cut off. 0.166 666 667 of 2500 m is 100 mm.

2.1 Working with fractions and decimals

Navigator

Answers p. 622

1 (columns 1–3), 2 (column 1), 3, 4, 5, 6, 7, 9 (columns 1–2), 10, 12, 14, 21, 23	1 (columns 3–4), 2 (column 2), 3, 4, 5, 6, 7, 8, 9 (columns 1–2), 10, 11, 12, 13, 14, 15, 19, 20, 21, 23	1 (columns 3–4), 2 (column 3), 3, 4, 5, 6, 8, 9 (column 3), 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 22, 24

Equipment required: calculator may be used for Questions **1–4**, **6**, **9**, **12–17**, **20**

Fluency

1 Write the following decimals as fractions in simplest form using equivalent fractions or your calculator. **W.E. 1**

(a) 0.8
(b) 0.05
(c) 0.002
(d) 0.0009
(e) 0.14
(f) 0.62
(g) 0.31
(h) 0.85
(i) 0.711
(j) 0.684
(k) 0.625
(l) 0.128
(m) 0.203
(n) 0.094
(o) 0.1560
(p) 0.7009

2 Arrange the following lists into ascending order (smallest to largest) by converting the fraction values to decimals. **W.E. 2**

(a) $\frac{2}{5}$, 0.399, $\frac{4}{5}$, 0.382, $\frac{3}{4}$
(b) $\frac{9}{10}$, $\frac{9}{8}$, 0.88, 0.89, 0.899
(c) $\frac{1}{8}$, 0.112, 0.099, $\frac{1}{4}$, 0.07
(d) $\frac{2}{5}$, $\frac{1}{3}$, 0.3, $\frac{3}{8}$, 0.2
(e) $\frac{1}{2}$, 0.555, 0.58, $\frac{3}{5}$, 0.55
(f) 0.291, 0.302, $\frac{3}{10}$, $\frac{2}{9}$, $\frac{2}{3}$
(g) $2\frac{4}{5}$, $2\frac{3}{4}$, 2.278, 2.932, $2\frac{9}{10}$
(h) $1\frac{1}{5}$, $1\frac{2}{3}$, $1\frac{3}{8}$, 1.029, 1.243
(i) $4\frac{1}{5}$, $4\frac{1}{4}$, 4.295, 4.199, 4.201
(j) $3\frac{1}{5}$, 3.45, 3.439, $3\frac{2}{3}$, 3.482

3 Use a calculator, if necessary, to answer the following questions. **W.E. 3**

(a) A length of wood is 2.6 m long. How many pieces, each 40 cm long, can be cut from this length, and how much wood will be left over?

(b) A bag holds 750 g of flour. How many cups, each containing 120 g of flour, can be filled from the bag, and how much will be left over?

(c) A container holds $3\frac{1}{4}$ litres of juice. How many 200 mL cups can be filled from the container, and how much juice will be left over? (1 litre = 1000 mL)

(d) A bus can carry 52 passengers. If every bus except the last one is filled to capacity, how many buses will be needed to transport 650 people, and how many people will be on the last bus?

Understanding

4 **(a)** $1\frac{5}{8}$ written as a decimal is:

A 0.16
B 1.58
C 1.625
D 1.63

(b) $2\frac{1}{3}$ written as a decimal, correct to 2 decimal places, is:

A 0.34
B 2.13
C 2.33
D 2.34

5 Write the following decimals as mixed numbers in simplest form.

(a) 7.5
(b) 3.4
(c) 1.25
(d) 5.64
(e) 13.02
(f) 27.96
(g) 9.045
(h) 124.706

6 Write the following mixed numbers as decimals, using a calculator where necessary.

(a) $51\frac{7}{10}$ **(b)** $28\frac{24}{200}$ **(c)** $7\frac{1}{8}$ **(d)** $67\frac{7}{25}$

(e) $39\frac{1}{16}$ **(f)** $5\frac{3}{20}$ **(g)** $14\frac{13}{80}$ **(h)** $24\frac{31}{400}$

7 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (Hint: First, find the value of the smallest interval on the number line.)

(a) 0.5, 1.3, $1\frac{1}{2}$, $\frac{1}{5}$ **(b)** $\frac{2}{2}$, 0.8, 1.6, $1\frac{4}{5}$

8 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{3}{4}$, $\frac{7}{10}$, 0.34, 0.47, $1\frac{1}{3}$ **(b)** 1.05, $\frac{7}{8}$, 0.58, $\frac{8}{5}$, $\frac{6}{4}$

9 Use a calculator to convert the fractions to decimal form, then place a > (greater than), < (less than) or = (equal to) symbol between the pairs of numbers to make the following statements correct.

(a) 0.84 ______ $\frac{18}{21}$ **(b)** 2.29 ______ $2\frac{12}{39}$ **(c)** 0.912 ______ $\frac{114}{125}$

(d) 0.64 ______ $\frac{16}{25}$ **(e)** 1.83 ______ $1\frac{18}{23}$ **(f)** 0.97 ______ $\frac{98}{99}$

(g) $\frac{1}{3}$ ______ 0.3 **(h)** $\frac{2}{9}$ ______ 0.23 **(i)** $\frac{2}{3}$ ______ 0.67

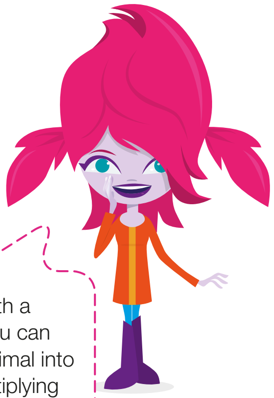
10 Which of the following fractions is greater than $\frac{3}{4}$ and less than $\frac{7}{8}$?

A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{9}{10}$ **D** $\frac{1}{2}$

11 **(a)** Express each first quantity as a fraction of the second quantity. Write the fraction in simplest terms.

- **(i)** first quantity 1.5 hours; second quantity 2.4 hours
- **(ii)** first quantity 200 m; second quantity 322.5 m
- **(iii)** first quantity 80.6 mL; second quantity 200 mL
- **(iv)** first quantity 20.6 kg; second quantity 12.5 kg
- **(v)** first quantity 18.6 m^2; second quantity 12 m^2
- **(vi)** first quantity 14.4 g; second quantity 1.5 g

(b) Express each fraction from part **(a)** as a decimal, rounding your answers to 3 decimal places where necessary.



If you have a fraction with a decimal number in it, you can sometimes turn the decimal into a whole number by multiplying it by 10. But remember, you must multiply the numerator and the denominator by the same number.

12 Mr Scully is out buying materials to build the set for the school play. Calculate how much he will pay for the following materials. Round your answers to the nearest 5 cents.

- **(a)** 6 lengths of timber, at \$8.99 per length
- **(b)** 9.4 metres of canvas, at \$11.20 per metre
- **(c)** 5.6 metres of ribbon, at \$0.95 per metre

13 The exchange rate between Australian dollars (A\$) and United States dollars (US\$) varies every day. On a particular day, A\$1 is worth US\$0.72. How much are the following amounts worth in US\$?

(a) A\$10 **(b)** A\$50 **(c)** A\$100 **(d)** A\$267 **(e)** A\$1845

14 George buys 6 bottles of milk and 3 packets of biscuits from a supermarket. The price of 1 bottle of milk is \$2.90 and price of 1 packet of biscuits is \$2.45. How much did he pay in total?

15 The most overdue library book ever was a copy of *Febrile Diseases* by Dr J. Currie. The book was borrowed from the University of Cincinnati Medical Library in 1823 by Mr M. Dodd, and was returned by his great-grandson in 1968. If the library fine for an overdue book was \$18.30 per year, then how much would his great-grandson have had to pay? (Luckily, in this case the library did not make Mr Dodd pay the fine!)

16 The largest crab in the world is the giant spider crab, which is found off the south-eastern coast of Japan. It has a claw span of 2.74 m. If an average person has a width of 36 cm across the waist, how many people could fit in the claw span of the giant spider crab?

17 The highest wave ever recorded in Australia was 24.9 m, off Macquarie Island. If the average surfer is 180 cm tall, how many surfers standing on top of one another would it take to reach the top of the wave?

Convert measurements to centimetres first.

Reasoning

18 An insect was climbing a wall 2.7 m high. In the first 20 minutes the insect started at the bottom and climbed $\frac{1}{3}$ of the height of the wall. In the second 20 minutes it climbed $\frac{1}{4}$ of the remaining height and in the third 20 minutes it climbed $\frac{1}{5}$ of the remaining height.

- **(a)** Calculate the distances climbed in each 20-minute period.
- **(b)** Calculate how far the insect still had to climb to reach the top of the wall.
- **(c)** Express the distance remaining as a fraction of the height of the wall.

19 Angela has to fill 50 gift bags by putting an equal amount of lollies in each. She has 4 packets of lollies that weigh 375 g each. Angela knows that the average mass of one lolly is 2.8 g.

- **(a)** Explain how Angela can use this information to find how many lollies she can put in each gift bag.
- **(b)** Use the method you described to find the answer.
- **(c)** About how many lollies will Angela have left over? Explain why this answer may not be exact.

20 The force of gravity varies from planet to planet. This means that the weight of an object will also vary, depending on which planet it is on. Objects on Jupiter weigh 2.6 times their weight on Earth. Objects on Mars weigh 0.38 times their 'Earth weight'. Calculate the weight of a 5 kg bag of potatoes on **(a)** Jupiter and **(b)** Mars.

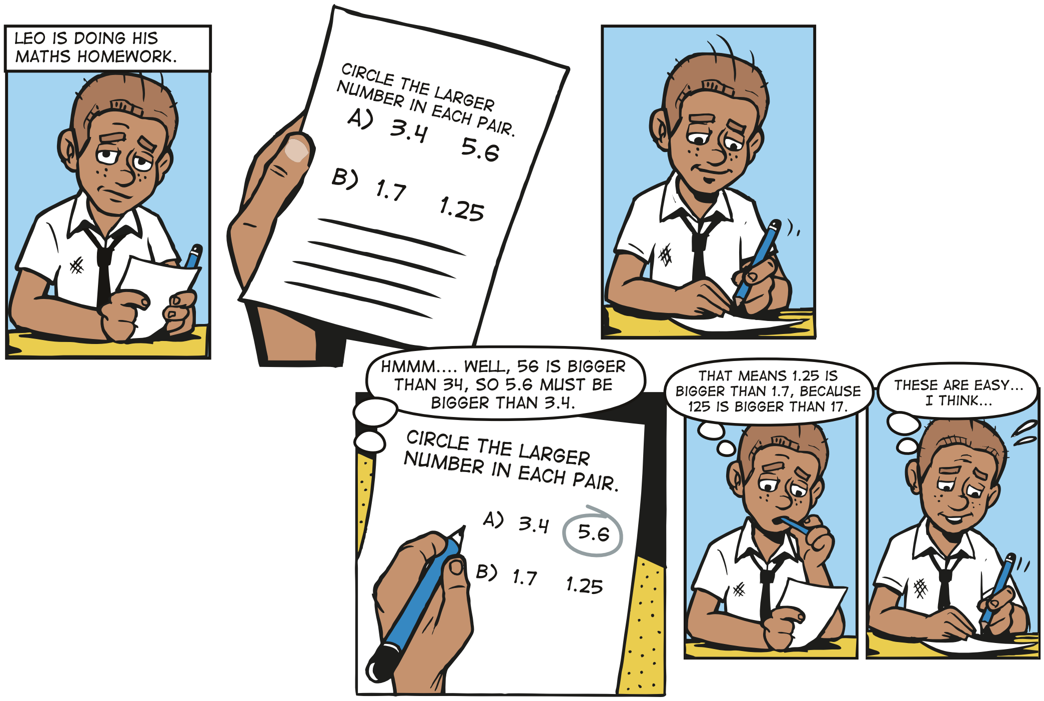
Open-ended

21 Write three fractions with three different denominators that have decimal values between 0.2 and 0.4.

22 Write three decimals, each with a different number of decimal places, that have fraction values between $\frac{9}{10}$ and $\frac{9}{8}$.

23 Write two fractions equivalent to $\frac{4}{5}$.

24



(a) Explain to Leo why his reasoning is incorrect.

(b) Describe to Leo how to compare two decimal numbers.

Puzzle

Sudoku

Equipment required: grid paper

To solve a Sudoku you need to use the digits 1–9 to fill in the blank squares so that each row, column and small 3 × 3 box contains each of the digits 1–9 only once.

Copy the grid into your book and complete it.

5		8	2				6	
	7	2			6			5
6			1			4		
	8					2		4
			8		1			
	6	4	3			5	8	
			7		8	1		3
2						6		
		1	6		3		7	2

Types of decimals

2.2

Terminating decimals

Dividing two whole numbers can give another whole number; for example, $\frac{12}{3} = 4$.

Recall that you can write a division as a fraction. Sometimes, the result of a division is a decimal number that has a certain number of digits. These are called **terminating decimals**. ('Terminating' means 'stopping'.)

Examples of terminating decimals: $\frac{18}{5} = 18 \div 5 = 3.6$

$$\frac{9}{8} = 9 \div 8 = 1.125$$

$$\frac{88}{200} = 88 \div 200 = 0.44$$

'Terminating' is like the word 'terminal' which describes an end point. For example, aircraft depart from and arrive at 'terminals'. Can you see the similarity here?

Non-terminating decimals

Sometimes, the division of one number by another gives a decimal number that has an infinite number of digits (so the digits continue forever). These are called **non-terminating decimals**. If some or all of the decimal digits follow a repeating pattern, then it is a recurring decimal. ('Recurring' means 'repeating'.)

Examples of recurring decimals: $\frac{1}{3} = 1 \div 3 = 0.333\,333\,3\ldots$

$$\frac{4}{15} = 4 \div 15 = 0.266\,666\ldots$$

$$\frac{11}{7} = 11 \div 7 = 1.571\,428\,571\ldots$$

Writing recurring decimals

To write a recurring decimal:

- write any non-recurring digits, then the digits that form the repeating pattern
- place a dot on top of the first and last digits of the repeating pattern, or draw a dash across the whole of the repeating pattern. For example, $0.357\,357\,357\ldots$ can be written as $0.\dot{3}5\dot{7}$ or $0.\overline{357}$.

  Using a dot or dash notation is called writing the number in **exact decimal form**.

Some examples of recurring decimals written in exact decimal form are:

$\frac{1}{3} = 1 \div 3 = 0.333\,333\ldots$ and is written as $0.\dot{3}$ or $0.\overline{3}$

$\frac{4}{15} = 4 \div 15 = 0.266\,666\,666\ldots$ and is written as $0.2\dot{6}$ or $0.2\overline{6}$

$\frac{1547}{9999} = 1547 \div 9999 = 0.154\,715\,471\,547\ldots$ and is written as $0.\dot{1}54\dot{7}$ or $0.\overline{1547}$.

Worked example 4 W.E. 4

Use a calculator to write each of the following fractions in exact decimal form.

(a) $\frac{1}{7}$ (b) $\frac{1}{13}$ (c) $\frac{1}{12}$

Thinking	Working
(a) 1 Use a calculator to do the division.	(a) $\frac{1}{7} = 1 \div 7 = 0.142\,857\,142$
2 Look for the repeating pattern.	The repeating pattern is 142 857.
3 Write the answer in exact decimal form by placing a dot above the first and last digits in the repeating pattern.	$\frac{1}{7} = 0.\dot{1}42\,85\dot{7}$
(b) 1 Use a calculator to do the division.	(b) $\frac{1}{13} = 1 \div 13 = 0.076\,923\,076$
2 Look for the repeating pattern.	The repeating pattern is 076 923.
3 Write the answer in exact decimal form by placing a dot above the first and last digits in the repeating pattern.	$\frac{1}{13} = 0.\dot{0}76\,92\dot{3}$
(c) 1 Use a calculator to do the division.	(c) $\frac{1}{12} = 1 \div 12 = 0.083\,333\,333$
2 Look for the repeating pattern.	The repeating pattern is just 3.
3 Write the answer in exact decimal form by placing a dot above the repeating digit.	$\frac{1}{12} = 0.08\dot{3}$

Writing recurring decimals as fractions

Previously, you have converted decimals to fractions by using the place value of the last digit. Given that a recurring decimal does not have a last digit, a new method is needed for converting recurring decimals to fractions.

You can write a recurring decimal as a fraction by first creating two expressions with the same decimal part. The decimal part is eliminated when the two expressions are subtracted.
For example: $2.4444\ldots - 0.4444\ldots = 2$

Worked example 5

W.E. 5

Write the following recurring decimals as fractions.

(a) $0.\dot{1}\dot{5}$ **(b)** $1.2\dot{4}$

Thinking	Working
(a) 1 Let x = the recurring decimal.	(a) $x = 0.151515\ldots$ [1]
2 Create a second expression by multiplying by the power of 10 that has the same number of zeros as the number of digits in the repeating pattern. (Here, multiply by 100.)	$100x = 15.151515\ldots$ [2]
3 Subtract the expressions created in the previous steps.	$[2] - [1]$: $100x = 15.151515\ldots$ $-\quad x = 0.151515\ldots$ $99x = 15$
4 Solve the equation that results and simplify your answer if necessary.	$x = \frac{15}{99}$ $= \frac{5}{33}$
(b) 1 Let x = the recurring decimal.	(b) $x = 1.24444\ldots$ [1]
2 Create a second expression by multiplying by the power of 10 that has the same number of zeros as the number of digits in the repeating pattern.	$10x = 12.4444\ldots$ [2]
3 Subtract the expressions created in the previous steps.	$[2] - [1]$: $10x = 12.4444\ldots$ $-\quad x = 1.2444\ldots$ $9x = 11.2$
4 Solve the equation that results and simplify your answer if necessary. (Here, multiply the numerator and denominator by 10 to eliminate the decimal place.)	$x = \frac{11.2}{9}$ $= \frac{112}{90}$ $= \frac{56}{45}$ or $1\frac{11}{45}$

Irrational numbers

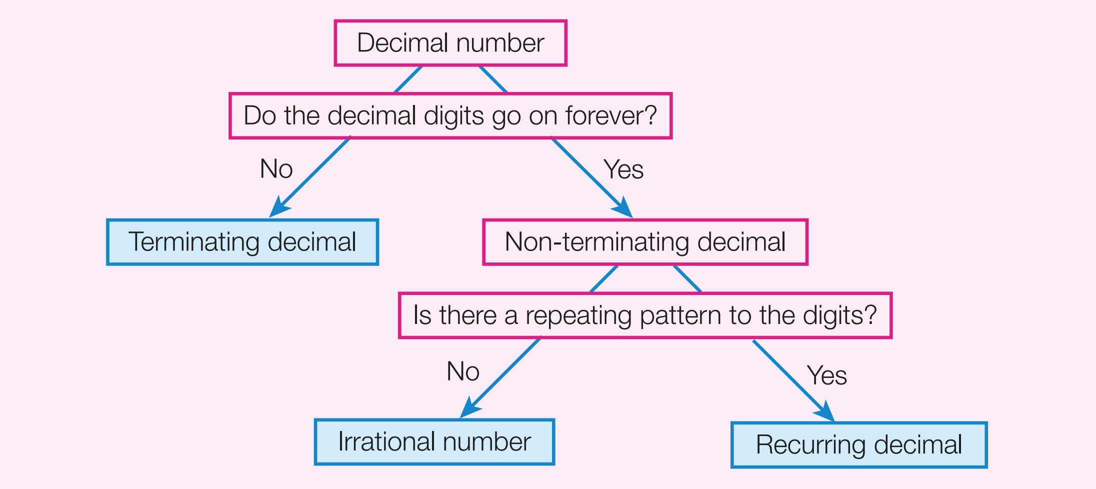
Some decimals are both non-terminating and non-recurring. They have an infinite number of digits with no pattern. These decimals cannot be written as fractions, and are called **irrational numbers**.

Irrational numbers include special numbers such as π (which you will learn about when you study circle measurement). Irrational numbers also include the square roots of some numbers such as 2, 3 and 5 ($\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$). Square roots that are irrational numbers are called **surds**. Because irrational numbers are non-terminating decimals, you can only write an approximate value for them in decimal form, correct to a certain number of decimal places. The only way to write the exact value of a surd is to write it in surd form (for example, $\sqrt{2}$, $\sqrt{3}$ or $\sqrt{5}$).

For example:

$$\begin{aligned} \sqrt{2} &= 1.414\,213\,562\ldots \\ &\approx 1.4 && \text{(1 d.p.)} \\ &\approx 1.41 && \text{(2 d.p.)} \\ &\approx 1.414\,214 && \text{(6 d.p.)} \end{aligned}$$

(Note that '≈' means 'is approximately equal to'.)



Worked example 6

W.E. 6

Use a calculator to express the following as decimals. Classify them as either terminating, recurring, or irrational.

(a) $12 \div 7$ **(b)** $\frac{5}{16}$ **(c)** $\sqrt{50}$ **(d)** $\sqrt{156.25}$

Thinking	Working
(a) 1 Use the calculator to perform the division.	(a) $12 \div 7 = 1.714\,285\,714\ldots$
2 Do the decimal digits seem to go on forever? (Yes.) Is there a repeating pattern to the digits? (Yes, the repeating pattern is 714 285.) If both answers are yes, the number is a recurring decimal.	Recurring

(b)	**1**	Use the calculator to perform the division.	(b) $\frac{5}{16} = 0.3125$
	2	Do the decimal digits seem to go on forever? (No.) If there is a definite number of decimal places, the number is a terminating decimal.	Terminating
(c)	**1**	Use the calculator to find the value of the square root.	(c) $\sqrt{50} = 7.071\,067\,812\ldots$
	2	Do the decimal digits seem to go on forever? (Yes.) Is there a repeating pattern to the digits? (No.) If there is no repeating pattern, the number is irrational.	Irrational
(d)	**1**	Use the calculator to find the value of the square root.	(d) $\sqrt{156.25} = 12.5$
	2	Do the decimal digits seem to go on forever? (No.) If there is a definite number of decimal places, the number is a terminating decimal.	Terminating

2.2 Types of decimals

Navigator

1 (columns 1–3), 2 (columns 1–3), 3 (columns 1–2), 4, 5, 7 (row 1), 8, 9, 10, 13, 15, 16, 19, 20	1 (columns 3–4), 2 (columns 3–4), 3 (columns 3–4), 5, 6, 7 (row 2), 8, 10, 12, 13, 14, 15, 16, 17, 19, 20	1 (columns 3–4), 2 (columns 3–4), 3 (columns 3–4), 5, 6, 7 (row 2), 8, 11, 12, 13, 14, 15, 17, 18, 19, 20

Answers p. 623

Equipment required: calculator

Fluency

1 Use a calculator to write each of the following fractions in exact decimal form.

W.E. 4

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{9}$ (d) $\frac{5}{9}$

(e) $\frac{1}{11}$ (f) $\frac{2}{11}$ (g) $\frac{3}{11}$ (h) $\frac{2}{13}$

(i) $\frac{1}{15}$ (j) $\frac{2}{15}$ (k) $\frac{4}{15}$ (l) $\frac{7}{15}$

(m) $\frac{1}{18}$ (n) $\frac{5}{18}$ (o) $\frac{7}{18}$ (p) $\frac{11}{18}$

The repeating pattern does not always begin at the start of the decimal.

W.E. 5

2 Write the following recurring decimals as fractions.

(a) $0.\dot{4}$ (b) $0.\dot{1}$ (c) $0.\dot{2}$ (d) $0.\dot{7}$

(e) $0.\dot{4}\dot{2}$ (f) $0.\dot{3}\dot{2}$ (g) $0.\dot{1}\dot{7}$ (h) $0.\dot{2}\dot{4}$

(i) $0.\dot{0}1\dot{2}$ (j) $0.\dot{2}1\dot{3}$ (k) $0.\dot{3}3\dot{2}$ (l) $0.\dot{2}2\dot{4}$

(m) $0.12\dot{5}$ (n) $0.1\dot{2}\dot{6}$ (o) $0.3\dot{2}1\dot{4}$ (p) $4.1\dot{0}2\dot{6}$

W.E. 6

3 Use a calculator to express the following as decimals and classify them as either terminating, recurring or irrational.

(a) $9 \div 8$ (b) $\frac{26}{4}$ (c) $\sqrt{449}$ (d) $\sqrt{84}$

(e) $\frac{5}{9}$ (f) $\sqrt{58.4}$ (g) $\frac{2}{11}$ (h) $22 \div 3$

(i) $\sqrt{216}$ (j) $15 \div 6$ (k) $\sqrt{533.61}$ (l) $\frac{11}{12}$

(m) $\sqrt{342.25}$ (n) $\sqrt{360}$ (o) $35 \div 9$ (p) $\sqrt{96.4}$

4 (a) The recurring decimal 4.563 213 213 21… written in exact decimal form is:

A $4.\overline{563\,21}$ B $4.563\,2\dot{1}$ C $4.563\,\overline{21}$ D $4.56\overline{3\,21}$

(b) The fraction $\frac{27}{55}$ written in exact decimal form is:

A $0.\overline{490}$ B $0.\dot{4}\dot{9}\dot{0}$ C $0.4\overline{90}$ D $0.\dot{4}9\dot{0}$

5 Choose the exact decimal form equivalent to the fraction $\frac{4}{90}$.

A 0.4 B 0.04 C $0.\dot{4}$ D $0.0\dot{4}$

6 Which of these is $0.\dot{7}\dot{3}$ converted into a fraction?

A $\frac{73}{100}$ B $\frac{8}{9}$ C $\frac{73}{99}$ D $\frac{73}{9}$

Understanding

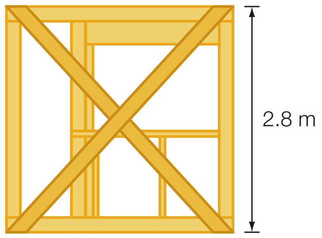
7 If you evaluate $\sqrt{2}$ on a calculator, you get a non-terminating decimal value of 1.414 213…. Although you cannot write an exact decimal value, you can state that the value of $\sqrt{2}$ lies in between the values of 1.4 and 1.5, and is closer to 1.4.

State the two decimal values that the following lie between, correct to 1 decimal place. Also state which of the two values is closer to the exact value.

(a) $\sqrt{3}$ (b) $\sqrt{5}$ (c) $\sqrt{12}$ (d) $\sqrt{20}$

(e) $\sqrt{8}$ (f) $\sqrt{7}$ (g) $\sqrt{47}$ (h) $\sqrt{68}$

8 The diagonal of a square is equal to its side length multiplied by $\sqrt{2}$. Luca is strengthening a large house frame by placing two diagonals across it, as shown. Using a calculator, find the length of wood Luca needs for the diagonals, correct to 3 decimal places.



9 Lily is carrying a full bucket of water. $\frac{1}{8}$ of the water leaks out. Convert the amount of water that leaks out into exact decimal form.

10 A shop sells a packet of 11 lollipops for $7.

(a) How much does each lollipop cost? Write your answer in exact decimal form, then as a price (rounded to the nearest cent).

(b) If you can buy just one lollipop using the coins in your pocket, how much will you pay?

11 Find the fraction equivalent of $0.\dot{9}$.

12 Arrange the following in ascending order.

(a) $\frac{1}{3}, \sqrt{9}, \sqrt{3}, 1.3, 0.9$

(b) $0.5, \sqrt{5}, \frac{5}{9}, \sqrt{59}, \frac{9}{5}$

(c) $\frac{4}{11}, 0.4, \sqrt{4}, 0.36, \sqrt{11}$

(d) $1.6, \frac{1}{6}, \sqrt{6}, \frac{1}{16}, \sqrt{16}$

The result to Question **11** might surprise you!

Reasoning

13 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{4}{3}, 0.\dot{6}, \frac{11}{6}, 0.3, \frac{3}{5}$

(b) $1.04, 0.4, \frac{4}{7}, 0.\dot{4}, \frac{7}{4}$

14 For each set of numbers, draw the number line below, and indicate the approximate position of each number with a labelled arrow.

(a) $\sqrt{3}, \frac{13}{7}, \frac{7}{6}, \sqrt{6}, \sqrt{7}$

(b) $\sqrt{5}, \frac{15}{14}, \sqrt{11}, \frac{40}{11}, \sqrt{15}$

15 **(a)** The value of $\frac{11}{19}$ is closest to:

A 0.5 **B** 0.579 **C** 0.58 **D** 0.6

(b) The value of $\sqrt{75}$ is closest to:

A 8 **B** 8.66 **C** 8.7 **D** 9

16 **(a)** What fraction of an hour is a minute?

(b) Convert your answer into exact decimal form.

17 Mac works in a factory 8 hours each day.

(a) What fraction of a day does he work? Write the answer in simplest form.

(b) Convert your answer into exact decimal form.

(c) Is the answer in the previous part a recurring decimal?

18 **(a)** Use a calculator to write $\frac{1}{9}$ and $\frac{1}{11}$ as decimals with eight digits after the decimal point.

(b) Add together the two decimal values.

(c) Add the two fraction values together.

(d) Do you get the same answer?

(e) Repeat the first three parts with $\frac{2}{3}$ and $\frac{5}{9}$.

(f) Explain the difference between the fraction and decimal sums.

Open-ended

19 Which is bigger, 0.3 or $0.\dot{3}$? Explain your answer.

20 Write five proper fractions, with varying numbers of digits in both numerator and denominator, that have only 9s in the denominator. Can you predict the recurring digits in the decimal equivalent? Check your prediction with your calculator.

Puzzle

Hitori

Hitori is a Japanese logic puzzle.

Copy the following grid and numbers into your book.

4	4	1	1	3
3	5	3	1	2
2	4	5	4	2
1	2	3	3	4
2	1	2	3	5

How to play:

The aim is to colour in squares according to the following rules. You must colour in squares so that:

- each number can be seen only once in each row and column; where a number is repeated, the extras must be coloured in
- you do not colour two squares that are next to each other, except diagonally
- the squares that are not coloured in are all joined. You should be able to trace an unbroken path, moving horizontally, vertically or diagonally between all of them.

Hint: Try circling the numbers that have to be left uncoloured (for example, numbers directly next to coloured-in cells).

Investigation

Terminating and recurring decimals

Equipment required: scientific calculator

This investigation focuses on unit fractions. Unit fractions are fractions with a numerator of 1.

The Big Question

Which decimal equivalents of the unit fractions are terminating and which are recurring?

Can you predict whether a fraction will produce a terminating or a recurring decimal?

Engage

1 **(a)** **(i)** Convert $\frac{1}{2}$ to its decimal equivalent by calculating $1 \div 2$. Is it a terminating or a recurring decimal?

(ii) Convert $\frac{1}{3}$ to its decimal equivalent by calculating $1 \div 3$. Is it a terminating or a recurring decimal?

(b) $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$. Find the decimal equivalent of $\frac{1}{4}$ by calculating $1 \div 4$. Now, find it another way by taking the decimal equivalent of $\frac{1}{2}$ and halving it (by dividing by 2). How do the decimal digits show that $\frac{1}{4}$ is $\frac{1}{2} \div 2$?

Explore

2 Investigate the unit fractions with denominators from 2 to 50. Which ones produce terminating decimals? Which ones produce recurring decimals?

Strategy options

- Guess and check.
- Make a table.
- Look for a pattern.

Explain

3 **(a)** Collect together all the fractions that produced terminating decimals, together with their decimal equivalents. Describe any patterns you can see:

(i) in the fraction denominators

(ii) in the decimal digits.

Look carefully at the factors of the denominators. Try breaking them down into prime factors.

(b) Collect together some of the fractions that produced recurring decimals, together with their decimal equivalents.

Describe any patterns you can see:

(i) in the fraction denominators

(ii) in the decimal digits.

Elaborate

4 **(a)** Use these patterns to predict a fraction with a denominator greater than 50 that will produce a terminating decimal. Write a predicted value for this decimal. Test your prediction. Were you correct?

(b) Answer the Big Question by writing a general rule that enables you to tell whether a fraction will produce a terminating or a recurring decimal, just by looking at the denominator.

(c) Using examples, describe how:

(i) you can operate on (add, subtract, multiply or divide) a recurring decimal to turn it into another recurring decimal

(ii) you can operate on (add, subtract, multiply or divide) a terminating decimal to turn it into a recurring decimal.

(d) Is it possible to turn a recurring decimal into a terminating decimal?

Evaluate

5 **(a)** Have you found all of the terminating decimals for the fractions between $\frac{1}{2}$ and $\frac{1}{50}$? How confident are you of this?

(b) How far into the process of changing fractions into decimals were you when you began to see patterns? Were you confident enough to rely on these patterns to predict the answers or did you continue to test individual fractions?

Extend

6 Investigate the fraction family of 'sevenths' ($\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \ldots \frac{6}{7}$), 'ninths' ($\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \ldots \frac{8}{9}$), or 'elevenths' ($\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \ldots \frac{10}{11}$) and their decimal equivalents.

(a) Describe any number patterns you find as you go from one fraction to the next.

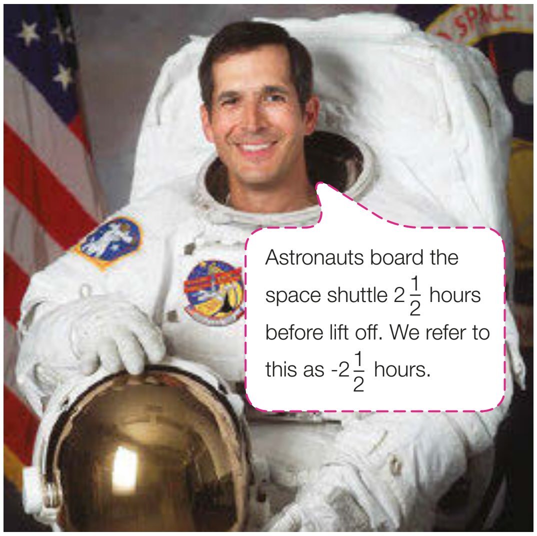
(b) Write the decimal equivalents of fraction pairs that add to 1; for example, $\frac{2}{7}$ and $\frac{5}{7}$. Describe any patterns or connections you can see between the two.

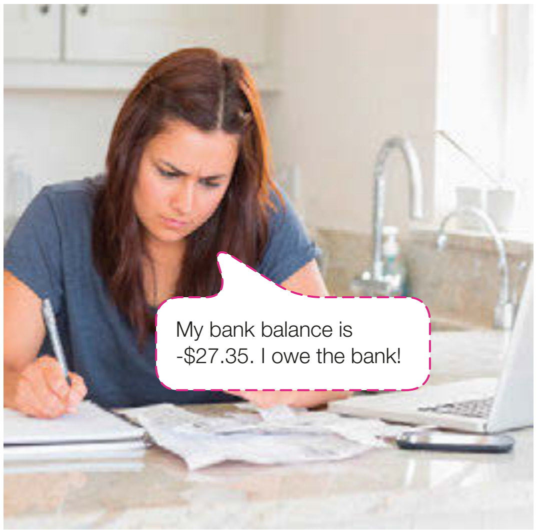
2.3

Negative fractions and decimals

Like integers, fractions and decimals can be positive or negative.

Here are some examples of negative fractions or decimals:





Negative fractions and decimals can be located on a number line in much the same way that you locate positive fractions and decimals.

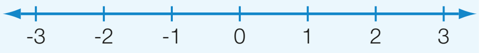
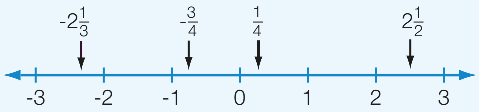
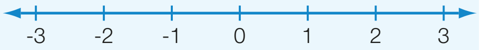
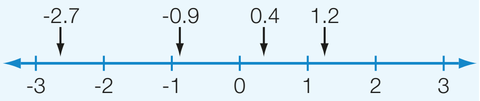
Worked example 7

W.E. 7

Place the following sets of numbers on separate number lines.

(a) $2\frac{1}{2}, -\frac{3}{4}, \frac{1}{4}, -2\frac{1}{3}$

(b) -2.7, 0.4, 1.2, -0.9

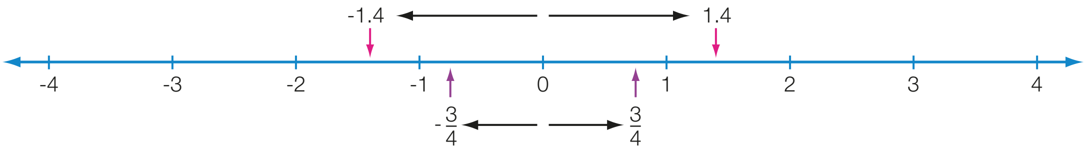
Thinking	Working
(a) 1 Draw a number line and place zero in the middle. Mark the positive integers at equal intervals, starting at zero and moving to the right. Mark the negative integers at equal intervals, starting at zero and moving to the left.	(a) 
2 Locate each of the numbers by first determining whether to move left or right from zero. Use the marked integers and the intervals between them as a guide to where to locate the numbers.	
(b) 1 Draw a number line, place zero in the middle and mark the positive and negative integers at equal intervals, as before.	(b) 
2 Locate each of the numbers by first determining whether to move left or right from zero. Use the marked whole numbers and the intervals between them as a guide to where to locate the numbers.	

Number line symmetry

The number line is symmetrical about zero, which means that a negative number is always the same distance from zero as the positive number is.

For example: -1.4 is the same distance to the left of zero as 1.4 is to the right of zero.

$-\frac{3}{4}$ is the same distance to the left of zero as $\frac{3}{4}$ is to the right of zero.



Adding and subtracting negative decimals

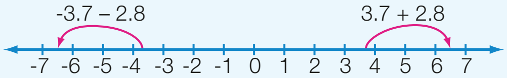
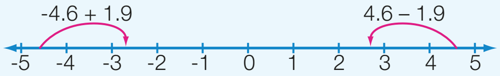
The symmetry of the number line can also help you add and subtract negative decimal numbers. Any calculation on one side of the number line has an equivalent 'mirror image' calculation on the opposite side.

Worked example 8

W.E. 8

Calculate the following. You might find it helpful to imagine the equivalent 'mirror image' calculation on the number line.

(a) $-3.7 + (-2.8)$ **(b)** $-4.6 - (-1.9)$

Thinking	Working
(a) 1 Simplify the addition by writing a single symbol between the two numbers.	(a) $-3.7 + (-2.8)$ $= -3.7 - 2.8$
2 Imagine the calculation on the number line. Imagine the 'mirror image' of this calculation on the opposite side of the number line. Notice that the original calculation moves to the left on the number line (subtraction), while the mirror image calculation moves to the right on the number line (addition).	
3 Complete the 'mirror image' calculation.	$3.7 + 2.8 = 6.5$
4 The answer to the original calculation has the opposite sign to the 'mirror image' calculation.	$-3.7 - 2.8 = -6.5$
(b) 1 Simplify the addition by writing a single symbol between the two numbers.	(b) $-4.6 - (-1.9)$ $= -4.6 + 1.9$
2 Imagine the calculation on the number line. Imagine the 'mirror image' of this calculation on the opposite side of the number line. Notice that the original calculation moves up the number line (addition), while the mirror image calculation moves down the number line (subtraction).	
3 Complete the 'mirror image' calculation.	$4.6 - 1.9 = 2.7$
4 The answer to the original calculation has the opposite sign to the 'mirror image' calculation.	$-4.6 + 1.9 = -2.7$

Adding and subtracting negative fractions

To add and subtract negative fractions, you need to apply the methods you have previously learnt for adding and subtracting fractions, along with the rules for adding and subtracting negative numbers.

Worked example 9

W.E. 9

Calculate the following, writing your answers in simplest form.

(a) $-\frac{4}{5}+\frac{1}{2}$

(b) $-1\frac{3}{4}+\left(-\frac{5}{6}\right)$

Thinking	Working
(a) 1 Write the fractions as equivalent fractions with a common denominator.	(a) $-\frac{4}{5}+\frac{1}{2}$ $=-\frac{8}{10}+\frac{5}{10}$
2 Add the numerators and simplify the answer if possible.	$=-\frac{3}{10}$
(b) 1 Simplify the addition by writing a single symbol between the two fractions.	(b) $-1\frac{3}{4}+\left(-\frac{5}{6}\right)$ $=-1\frac{3}{4}-\frac{5}{6}$
2 Write any mixed numbers as improper fractions.	$=-\frac{7}{4}-\frac{5}{6}$
3 Write the fractions as equivalent fractions with a common denominator.	$=-\frac{21}{12}-\frac{10}{12}$
4 Subtract the numerators and simplify the answer if possible.	$=-\frac{31}{12}$ or $-2\frac{7}{12}$

Multiplying and dividing negative fractions and decimals

To multiply or divide negative fractions and decimals:

1. Ignore any positive or negative signs, and apply the usual methods for multiplication and division.
2. Find the sign of the answer by applying the rules for multiplying and dividing integers.

   Two ‘like’ signs give a positive answer: (+) × (+) or (–) × (–) = (+)

   Two different signs give a negative answer: (+) × (–) or (–) × (+) = (–)

Negative numbers on the calculator

To enter a negative fraction or decimal on your calculator you can usually press the (–) or +/– key, then enter the fraction or decimal in the usual way.

2.3 Negative fractions and decimals

Navigator

Answers p. 624

1 (columns 1–2), 2 (column 1), 3 (row 1), 4 (columns 1–2), 5 (row 1), 7, 8, 9, 16 (column 1), 17, 20	1 (columns 2–3), 2 (columns 2–3), 3 (rows 2–3), 4 (columns 3–4), 5 (columns 1–3), 6, 7, 9, 10, 11, 12, 14, 15, 16 (column 2), 17, 18, 19, 21	1 (columns 2–3), 2 (columns 3–4), 3 (rows 3–4), 4 (columns 3–4), 5 (columns 2–4), 6, 7, 10, 11, 12, 13, 14, 15, 16 (column 2), 18, 19, 21, 22

Equipment required: calculator for Question **16**

Fluency

W.E. 7

1 Show each of the following sets of numbers on a number line.

(a) $-3\frac{1}{2}, -2\frac{3}{4}, -\frac{1}{2}, 1\frac{1}{4}$ **(b)** -1.4, -1.9, 0.4, 2.2 **(c)** $-\frac{4}{5}, \frac{1}{5}, -2\frac{1}{4}, 1\frac{2}{5}$

(d) -3.6, -2.6, -6.1, 1.6 **(e)** $-\frac{6}{5}, -\frac{5}{6}, -2\frac{1}{6}, \frac{1}{5}$ **(f)** -0.5, 0.05, -5.2, 0.75

W.E. 8

2 Calculate the following. It may be useful to imagine the equivalent 'mirror image' calculation on the number line.

(a) $-0.5 + 0.4$ **(b)** $-0.3 - 0.7$ **(c)** $-1.5 + (-1.6)$ **(d)** $-2.4 - (-1.3)$

(e) $-7.2 + 6.8$ **(f)** $-5.9 - 8.3$ **(g)** $-4.6 + (-3.9)$ **(h)** $5.2 - (-4.3)$

(i) $7.61 + (-5.92)$ **(j)** $3.47 - (-10.98)$ **(k)** $0.8 - 2.34$ **(l)** $-9.87 + 11.3$

(m) $19.3 - 21.27$ **(n)** $-31.4 + 39.2$ **(o)** $-26.04 - (-15.97)$ **(p)** $13.92 - (-17.64)$

W.E. 9

3 Calculate the following, writing your answers in simplest form.

(a) $-\frac{3}{4} - \frac{3}{4}$ **(b)** $-\frac{1}{5} + \left(-\frac{3}{5}\right)$ **(c)** $\frac{2}{7} - \frac{5}{7}$ **(d)** $-\frac{8}{9} - \left(-\frac{1}{9}\right)$

(e) $-\frac{3}{5} + \left(-\frac{1}{4}\right)$ **(f)** $\frac{7}{8} - \frac{9}{10}$ **(g)** $-\frac{1}{6} - \left(-\frac{1}{5}\right)$ **(h)** $-\frac{2}{3} + \frac{5}{6}$

(i) $-\frac{2}{5} + 1$ **(j)** $-\frac{1}{2} + 3$ **(k)** $-1\frac{1}{3} - (-2)$ **(l)** $1 + \left(-1\frac{1}{4}\right)$

(m) $-1\frac{1}{2} + 2\frac{1}{4}$ **(n)** $-3\frac{1}{3} - 2\frac{1}{2}$ **(o)** $4\frac{1}{5} - \left(-\frac{7}{10}\right)$ **(p)** $1\frac{3}{4} + \left(-\frac{5}{6}\right)$

4 Calculate the following, writing your answers in simplest form.

(a) $-\frac{3}{7} \times \frac{1}{2}$ **(b)** $-\frac{1}{2} \div \frac{1}{4}$ **(c)** $\frac{5}{2} \times -\frac{1}{3}$ **(d)** $\frac{3}{5} \div -\frac{1}{3}$

(e) $-4 \div \frac{1}{8}$ **(f)** $-\frac{1}{6} \times -\frac{3}{4}$ **(g)** $6 \div -\frac{2}{3}$ **(h)** $-\frac{5}{8} \times -\frac{4}{5}$

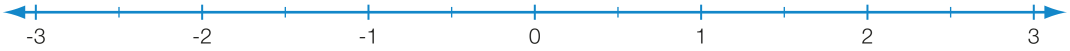
(i) $-2\frac{1}{2} \times -1\frac{3}{5}$ **(j)** $3\frac{2}{3} \div -\frac{1}{2}$ **(k)** $-4\frac{1}{6} \times 1\frac{3}{5}$ **(l)** $-6\frac{3}{4} \div \frac{9}{10}$

5 Calculate the following, rounding answers to 2 decimal places where necessary.

(a) -0.4 × 0.8	**(b)** -0.6 ÷ 2	**(c)** -3 × 1.5	**(d)** 2.7 ÷ -3
(e) -0.72 ÷ -0.3	**(f)** -12.5 × -7.2	**(g)** -1.2 ÷ 0.5	**(h)** 0.05 × -7.4
(i) 14.25 × 6.3	**(j)** 24 ÷ -3.4	**(k)** 29.5 × -11.23	**(l)** -56.2 ÷ 10.1

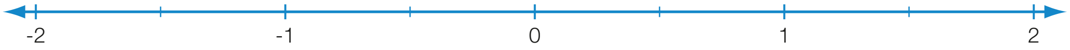
6 Copy the following number line. Use arrows to show the locations of the following fractions. (You may need to estimate the position of some numbers.)

$-\frac{2}{3}, \frac{4}{3}, -1\frac{1}{2}, \frac{11}{6}, -\frac{7}{3}$



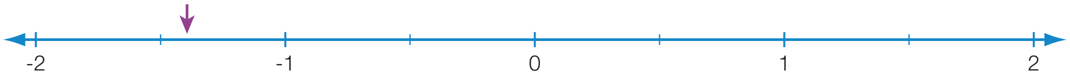
7 Copy the following number line. Use arrows to show the locations of the following decimals. (You may need to estimate the position of some numbers.)

-0.7, 1.3, -1.1, 0.65, -0.05



8 The number closest to the position indicated by the arrow on the number line is:

A -2.6	**B** -1.4	**C** -0.3	**D** 1.4



Understanding

9 The following table shows the maximum and minimum temperatures recorded at Thredbo during 5 days in winter.

Temperature (°C)	Mon	Tues	Wed	Thur	Fri
Maximum	5.2	-0.8	6.1	3.5	-1.2
Minimum	-1.1	-4.3	-1.7	-2.9	-5.6

(a) On which day was the coolest maximum recorded?

(b) On which day was the warmest minimum recorded?

(c) On which day was the difference between the maximum and minimum temperatures the greatest?

(d) Calculate **(i)** the mean maximum temperature and **(ii)** the mean minimum temperature, to 1 decimal place.

10 In the morning, the UV level was 1.17. In the afternoon, the UV level was 7.5. How much has the UV level increased from morning to afternoon?

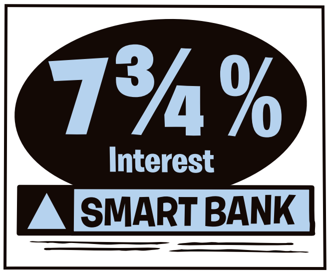
11 A drone is flying 28.5 m above the ground. If it rises 11.7 m higher and then descends 18.6 m, how high above the ground is the drone now?

12 A scuba diver swims to a depth of -35.5 m before rising 13.8 m. What is the diver's depth now?

To find the mean, add up all the temperatures and divide by how many temperatures there are.

13 The executives of a bank meet every month to decide whether to raise or lower interest rates. Over an 8-month period, they decided to make these changes to interest rates.

Increase by 1.2%, no change, increase by $\frac{3}{4}$%, decrease by 0.8%, decrease by $1\frac{1}{4}$%, no change, decrease by $\frac{1}{2}$%, decrease by 0.6%.



(a) Write each of the percentage changes with a positive or a negative sign to show if it is an increase or a decrease.

(b) Work out the overall increase or decrease in interest rates at the end of the 8-month period. Write your answer in **(i)** fraction form and **(ii)** decimal form, using a negative sign if necessary.

14 Write the following in order from smallest to largest: -3, 0, $\frac{3}{10}$, 1.7, -3.4, -$\frac{3}{4}$, -0.9, 1. Use a number line, if necessary, to help you.

15 Write the following in order from smallest to largest. Use a number line, if necessary, to help you. -$2\frac{1}{5}$, 0, -1.8, -$\frac{5}{4}$, 1.1, $\frac{4}{5}$, -2, $\frac{3}{4}$

16 Use your calculator to evaluate the following. Write answers correct to 3 decimal places.

(a) $(78.9 \times 3.4) - (54.8 \times 7.77)$

(b) $(54.9 \times 5.6) - (6.2 \times 103.7)$

(c) $(-14.7 + 5.8) \times (4.3 - 8.25)$

(d) $(56 - 72.8) \times (-12.3 + 17.1)$

(e) $\sqrt{7} - \sqrt{3} - \sqrt{6}$

(f) $\sqrt{3} - \sqrt{2} - (\sqrt{5})^2$

(g) $(2.3)^4 - (5.9)^5 + (1.5)^2$

(h) $(\sqrt{6})^2 - (10.5)^4 - (1.005)^2$

(i) $(3.8 - 5.7)^2 - (\sqrt{8})$

(j) $(2\sqrt{10})^2 - (53.66)^2$

Reasoning

17 Chloe receives a statement from her bank every month. The statement is a list of all the transactions (deposits and withdrawals) made on the account, and the account balance (how much money is in the account after each transaction). Here is part of Chloe's statement:

Date	Transaction	Balance
1 May		$257.89
7 May	ATM Withdrawal $180.00	$77.89
12 May	Cheque deposit $65.00	$142.89
16 May	Shopping paid for by EFTPOS $97.63	$45.26
23 May	Insurance, paid by direct debit $58.17	-$12.91
30 May	Salary deposit ________	$460.94

(a) Locate the negative number on Chloe's statement. What is the '-' symbol used to indicate here?

(b) On 27 May, Chloe went to an ATM and tried to withdraw $50, but the machine would not give her any money! Why not?

(c) What was the value of Chloe's salary that was deposited on 30 May?

18 Peter must practise guitar for at least $\frac{3}{4}$ of an hour each day. Today he practised for $1\frac{1}{3}$ hours.

(a) How much extra practice has he done?

(b) Write the answer in minutes.

19 Use a number line to explain why $\frac{1}{2} > \frac{1}{4}$ but $-\frac{1}{2} < -\frac{1}{4}$.

Open-ended

20 Write three decimal numbers that are greater than -2.1 but less than -1.2.

21 Write three numbers in fraction form that are greater than $-\frac{7}{8}$ but less than 0.

22 Dan is having trouble with negative decimals. He can add and subtract positive numbers successfully, but often gets questions with negatives wrong. Here are two questions that Dan has done incorrectly:

$-8.3 + (-4.5) = -3.8$

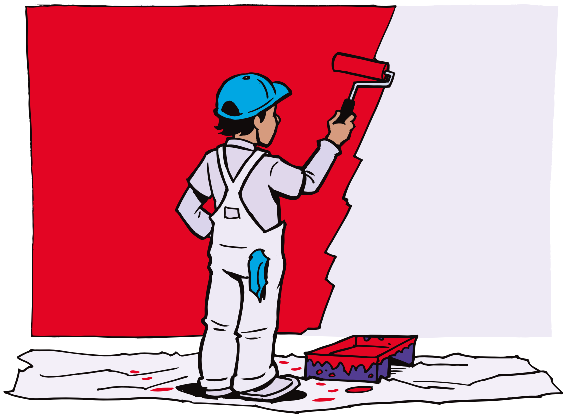
$-6.7 - (-2.9) = -9.6$

(a) Explain the errors that Dan has made.

(b) Give him some tips as to how he can do these types of questions successfully.

2.4

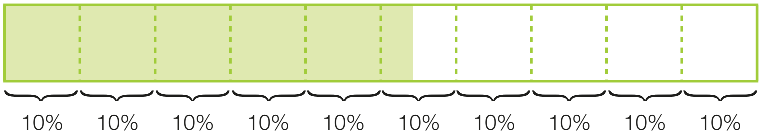
Estimating percentages



Per cent means 'for every hundred' or 'out of a hundred'. The symbol % is used for per cent. (Notice that this symbol % even looks a little like a rearranged 100.) A **percentage** is a value out of 100.

It can be useful to imagine what a particular percentage looks like. For example, you might describe the painting of this wall as being about 60% complete.

To estimate a percentage of an amount you can mentally split the object into 10 parts of 10% each. (Recall that 100% always represents the whole.) If you divide up the rectangle below like this, you can see that the shaded area is somewhere between 50% and 60%.



The shading is a little short of half-way between 50% and 60%. A better estimate of the shading in the rectangle above would be 53%.

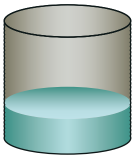
Worked example 10

W.E. 10

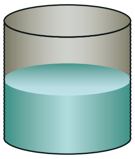
Which of the following water tanks is approximately:

(a) 50% full **(b)** 15% full **(c)** 80% full?

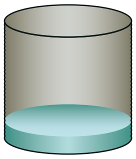
(i)



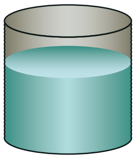
(ii)



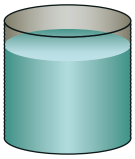
(iii)



(iv)



(v)



Thinking	Working
(a) **1** Is the percentage required bigger, smaller or equal to a half?	(a) 50% = half
2 50% is half full. Which tank looks half full?	Tank (ii) is 50% full.
(b) **1** Is the percentage required bigger, smaller or equal to a half?	(b) 15% < 50%

	2	15% is less than one half, so there are two possibilities—tanks **(i)** and **(iii)**. Compare them by mentally dividing them into 10% pieces and counting how many are full.	Tank (iii) is 15% full.
(c)	1	Is the percentage required bigger, smaller or equal to a half?	(c) 80% > 50%
	2	80% is greater than one-half, so there are two possibilities—tanks **(iv)** and **(v)**. Compare them by mentally dividing them into 10% pieces and counting how many are full.	Tank (v) is 80% full.

2.4 Estimating percentages

Navigator

1, 2, 3 (a–b), 4, 6, 10	1, 2, 3 (c), 4, 5, 7, 8, 9, 10, 11	3 (c), 5, 7, 8, 9, 10, 11

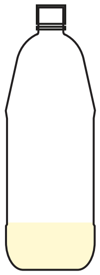
Answers p. 625

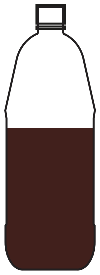
Fluency

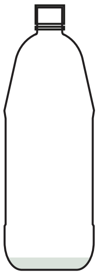
W.E. 10

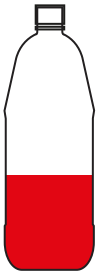
1 Which of the following bottles of soft drink is approximately:

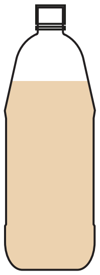
(a) 5% full **(b)** 90% full **(c)** 80% full **(d)** 20% full?

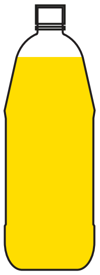

Lemonade


Cola


Soda water


Raspberry

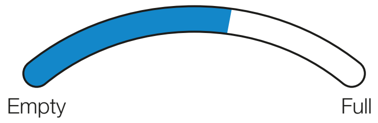

Creamy soda


Lemon squash

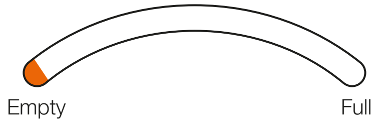
2 Which of the following fuel gauges shows a petrol tank that is approximately:

(a) 40% full **(b)** 15% full **(c)** 60% full **(d)** 75% full?

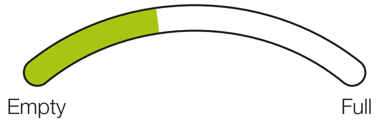
Gauge 1



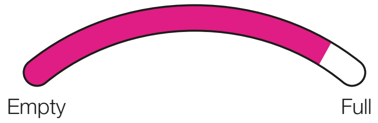
Gauge 2



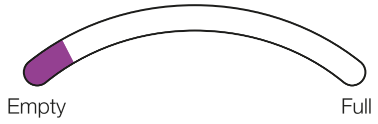
Gauge 3



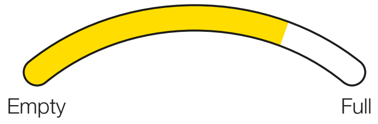
Gauge 4



Gauge 5



Gauge 6



3 **(a)** Approximately what percentage of this rectangle is shaded?

A 10% **B** 30% **C** 60% **D** 80%

(b) Approximately what percentage of the square is shaded?

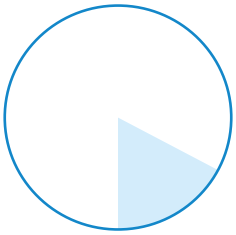
A 15% **B** 30%

C 45% **D** 75%

(c) Approximately what percentage of the circle is shaded?

A 16% **B** 27%

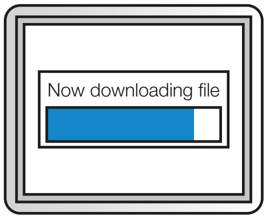
C 55% **D** 78%



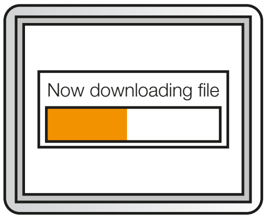
4 A screen displays the amount of a file being downloaded by gradually filling a rectangular bar until the download is complete. Which display shows approximately:

(a) 33% downloaded **(b)** 70% downloaded

(c) 82% downloaded **(d)** 22% downloaded?

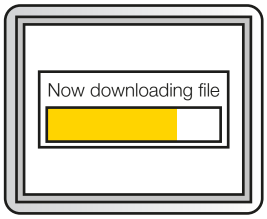


Display A



Display B

Display C



Display D

Display E

Display F

5 A smartphone has following battery level indicators. What percentage does each image appear to indicate?

(a)

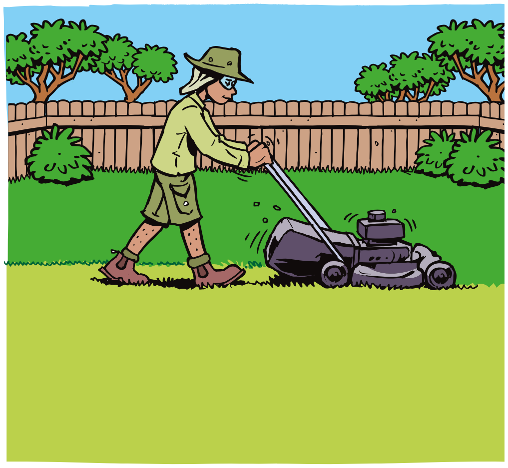
(b)

(c)

(d)

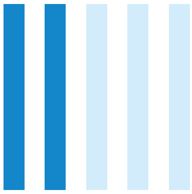
Understanding

6 What percentage of lawn does John still need to mow? (Assume that the cartoon shows the complete lawn, and consider the rectangular area shown.)



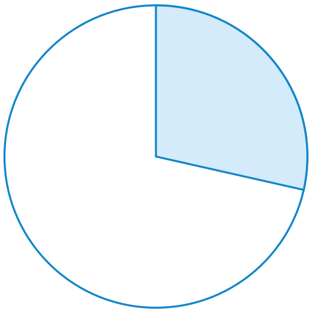
7 Describe how full this water dispenser is, using a percentage value.

8 A mobile phone displays its signal strength as 2 bars out of 5. What percentage is the signal strength?



Reasoning

9 28% of this circle is shaded.



(a) Draw three similar sized circles, then shade in your estimate of:

(i) 40% of the circle

(ii) $\frac{5}{6}$ of the circle

(iii) 0.7 of the circle.

(b) Did you find it easier to estimate the percentage, the decimal or the fraction amount? Why do you think this was the case?

Open-ended

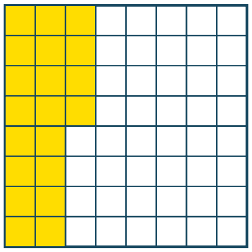
10 **(a)** Draw at least three different shapes and shade 25% of each of them.

(b) Redraw the shapes and shade a different 25% of each in a totally different way.

(c) Redraw the shapes again and this time, shade 65% of each.

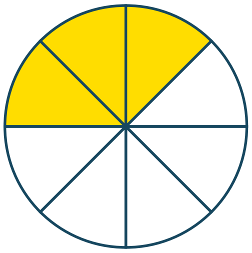
11 Helga has worked out the percentage of the areas shaded as follows.

(a)



24% shaded

(b)



30% shaded

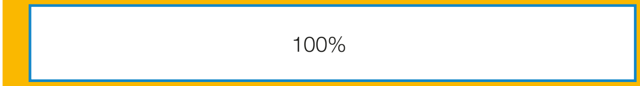
Explain Helga's mistake in her calculations for each shape.

Game

Best estimate

How to win:
The aim is to get the most points in five rounds.

How to play:
On a piece of paper, draw a bar 10 cm long and 1 cm wide. This bar represents 100%.



Using a ruler, the first player draws another bar on a separate piece of paper. Their bar must be shorter than the 100% bar. They must know the exact percentage it represents.

The second player has to say what percentage the second bar represents. (No rulers are allowed, but you can move the paper about.)

Once the second player has guessed, the first player states the actual answer and the error is written down.

For example: If you guess 80% when the actual answer is 83%, then the error is 3.

The players then swap roles. When both players have had a turn, one round is complete.

Each round, the player with the lowest error earns a point.

Play best of five rounds.

Alternatives:

- Change the length of the 100% bar to 20 cm.
- Try guessing percentages that are greater than 100%.

Writing fractions and decimals as percentages

2.5

Percentages are easy to compare with each other. Also, percentages are often easier than fractions to imagine. Because of this, it is useful to be able to convert fractions and decimals into percentages. Some percentages are used so often that it is useful to know their fraction or decimal equivalent. If you don't already know them, try to learn the following.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	$0.\dot{3}$	$0.\dot{6}$	0.2	0.1	0.05	0.02	0.01
Percentage	50%	25%	75%	$33.\dot{3}\%$	$66.\dot{6}\%$	20%	10%	5%	2%	1%

When a fraction has a denominator that is a factor of 100, then you can easily convert it to a fraction out of 100. This is the same as the percentage.

For example:

$$\frac{3}{4} \underset{\times 25}{\overset{\times 25}{=}} \frac{75}{100} = 75\%$$

$$\frac{1}{5} \underset{\times 20}{\overset{\times 20}{=}} \frac{20}{100} = 20\%$$

However, for fractions that have denominators which cannot be easily converted to 100, you will need to use another method.

To write a fraction or decimal as a percentage, multiply the fraction or decimal by 100%.

There are two methods you can use to multiply a fraction by 100%. You can multiply two fractions together, as shown in Method 1 of the following example. Or you can convert the fraction to a decimal, then multiply by 100%, as shown in Method 2 of the example. Method 1 is a more convenient 'pen and paper' method, while Method 2 is useful if you have a calculator.

Multiplying by 100% is equivalent to multiplying by 1, because 100% is the same as one whole. If you multiply a fraction or a decimal by 100%, the value of the fraction or decimal is not being changed, it is just being represented in a different way.

Worked example 11

W.E. 11

Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

(a) $\frac{7}{8}$

(b) $\frac{11}{15}$

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 4.)	(a) $\frac{7}{8} \times 100\%$ $= \frac{7}{{}_{2}\cancel{8}} \times \frac{{}^{25}\cancel{100}}{1}\%$ $= \frac{175}{2}\%$
2 Simplify the answer.	$= 87\frac{1}{2}\%$ or 87.5%
(b) 1 Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 5.)	(b) $\frac{11}{15} \times 100\%$ $= \frac{11}{{}_{3}\cancel{15}} \times \frac{{}^{20}\cancel{100}}{1}\%$ $= \frac{220}{3}$
2 Simplify the answer.	$= 73\frac{1}{3}\%$ or $73.\dot{3}\%$

Method 2: Convert to a decimal, then multiply

Thinking	Working
(a) Convert the fraction to decimal form first, then multiply the decimal by 100%.	(a) $\frac{7}{8} \times 100\%$ $= 7 \div 8 \times 100\%$ $= 0.875 \times 100\%$ $= 87.5\%$ or $87\frac{1}{2}\%$
(b) Convert the fraction to decimal form first, then multiply the decimal by 100%. (When working with recurring decimals, write out a couple of decimal places so you can place the decimal point in the correct position.)	(b) $\frac{11}{15} \times 100\%$ $= 11 \div 15 \times 100\%$ $= 0.7333\ldots \times 100\%$ $= 73.\dot{3}\%$ or $73\frac{1}{3}\%$

Worked example 12

W.E. 12

Write $3\frac{1}{2}$ as a percentage.

Method 1: Fraction multiplication

Thinking	Working
1 Write the mixed number as an improper fraction. Write 100% as an improper fraction, then multiply the two fractions. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 2.)	$3\frac{1}{2} \times 100\%$ $= \frac{7}{{}_{1}\cancel{2}} \times \frac{{}^{50}\cancel{100}}{1}\%$ $= \frac{350}{1}\%$
2 Simplify the answer.	$= 350\%$

Method 2: Convert to a decimal, then multiply

Thinking	Working
Convert the mixed number or decimal form first, then multiply the decimal by 100%.	$3\frac{1}{2} \times 100\%$ $= 3.5 \times 100\%$ $= 350\%$

Worked example 13

W.E. 13

Find the percentage equivalent of each of the following.

(a) 0.8 **(b)** 1.65 **(c)** 0.032

Thinking	Working
(a) Multiply by 100%. Show this by moving the decimal point two places to the right. Fill in empty place value columns with zeros.	(a) $0.8 = 0.8 \times 100\%$ $= 80\%$
(b) Multiply by 100%. Show this by moving the decimal point two places to the right.	(b) $1.65 = 1.65 \times 100\%$ $= 165\%$
(c) Multiply by 100%. Show this by moving the decimal point two places to the right.	(c) $0.032 = 0.032 \times 100\%$ $= 3.2\%$

Percentage error

When you compare an exact (or true) value with an approximate value that has been rounded, estimated or measured, the difference between them is often called the 'error'. It is useful to find this error as a **percentage error** of the exact value, so that you can easily compare the errors of different values.

For example, the difference between an actual value of 2.0 and an approximate value of 1.5 is 0.5, which is 25% of the actual value. The difference between an actual value of 122.0 and an approximate value of 121.5 is also 0.5, but in this example it is only 0.4% of the actual value.

The smaller the percentage error, the less significant it is when compared to the actual value.

Use these steps to write a percentage error.

- Compare the actual value and the approximate value. Find the error by subtracting one value from the other, to find the difference between the values.
- Divide the error by the actual value.
- Multiply this by 100 to calculate the percentage error and write a % symbol.

Worked example 14

W.E. 14

Calculate the percentage error in the following.

(a) actual value = 2.3, approximate value = 2

(b) actual value = 99.65, approximate value = 100

Thinking	Working
(a) **1** Find the error by calculating the difference between the two values.	(a) $2.3 - 2 = 0.3$ Error $= 0.3$
2 Divide this error by the actual value.	$\frac{0.3}{2.3} = 0.1304\ldots$
3 Multiply this by 100 to calculate the percentage error and write a % symbol.	$0.1304 \times 100\% = 13.04\%$ (2 d.p.) Percentage error $= 13.04\%$
(b) **1** Find the error by calculating the difference between the two values.	(b) $100 - 99.65 = 0.35$ Error $= 0.35$
2 Divide this error by the actual value.	$\frac{0.35}{99.65} = 0.0035\ldots$
3 Multiply this by 100 to calculate the percentage error and write a % symbol.	$0.0035 \times 100\% = 0.35\%$ Percentage error $= 0.35\%$ (2 d.p.)

2.5 Writing fractions and decimals as percentages

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 6, 7, 8, 9, 11, 13, 14, 17 (a), 22, 24	1 (columns 3–4), 2 (columns 3–4), 3 (columns 3–4), 4, 5, 6, 7, 10, 11, 12, 14, 15, 16, 17, 20, 22, 23, 24, 25	1 (columns 3–4), 2 (columns 3–4), 3 (column 4), 5, 6, 7, 10, 11, 12, 15, 16, 18, 19, 20, 21, 23, 25

Answers p. 625

Equipment required: calculator

Fluency

1 Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

W.E. 11

(a) $\frac{9}{100}$ **(b)** $\frac{1}{10}$ **(c)** $\frac{7}{50}$ **(d)** $\frac{19}{20}$

(e) $\frac{3}{5}$ **(f)** $\frac{1}{4}$ **(g)** $\frac{3}{2}$ **(h)** $\frac{16}{8}$

(i) $\frac{1}{3}$ **(j)** $\frac{2}{3}$ **(k)** $\frac{1}{6}$ **(l)** $\frac{2}{9}$

(m) $\frac{32}{40}$ **(n)** $\frac{9}{16}$ **(o)** $\frac{56}{80}$ **(p)** $\frac{14}{32}$

(q) $\frac{73}{80}$ **(r)** $\frac{13}{60}$ **(s)** $\frac{55}{66}$ **(t)** $\frac{23}{90}$

2 Write the following as percentages. Write any answers that are not whole numbers in exact decimal form.

W.E. 12

(a) $1\frac{1}{4}$ **(b)** $1\frac{2}{5}$ **(c)** $3\frac{1}{5}$ **(d)** $5\frac{1}{2}$

(e) $2\frac{3}{8}$ **(f)** $2\frac{3}{4}$ **(g)** $5\frac{7}{10}$ **(h)** $4\frac{5}{8}$

(i) $4\frac{2}{3}$ **(j)** $5\frac{1}{3}$ **(k)** $1\frac{7}{9}$ **(l)** $7\frac{5}{6}$

3 Find the percentage equivalent of each of the following.

W.E. 13

(a) 0.9 **(b)** 0.4 **(c)** 0.8 **(d)** 0.6

(e) 0.17 **(f)** 0.47 **(g)** 0.82 **(h)** 0.53

(i) 0.051 **(j)** 0.438 **(k)** 0.007 **(l)** 0.342

(m) 9.2 **(n)** 5.1 **(o)** 2.02 **(p)** 9.01

4 **(a)** The percentage equivalent of 0.7 is:

A 0.07% **B** 0.70% **C** 7% **D** 70%

(b) The percentage equivalent of 3.45 is:

A 0.0345% **B** 0.345% **C** 34.5% **D** 345%

5 **(a)** Which of the following is equivalent to $\frac{9}{50}$?

A 9% **B** 18% **C** 45% **D** 90%

(b) Which of the following is equivalent to $\frac{3}{20}$?

A 3% **B** 12% **C** 15% **D** 30%

6 Copy and complete the table of commonly used fractions, decimals and percentages.

Percentage	5%			20%	25%	$33\frac{1}{3}\%$	40%		60%	$66\frac{2}{3}\%$		80%	100%
Fraction			$\frac{1}{8}$								$\frac{3}{4}$		
Decimal		0.1						0.5					1

W.E. 14

7 Calculate the percentage error if the actual value is 10 and the approximate value is 11.

Understanding

8 Lou estimated that $\frac{7}{10}$ of the crowd at a rugby match were Broncos supporters. What percentage is this?

9 After a cyclone destroyed most of the banana crop, the price of bananas rose to $3\frac{3}{4}$ times the original price. What percentage of the old price was this?

10 At the end of the year, Ron was paid a bonus of $\frac{3}{4}$ of his salary. Andrew was paid a bonus of $\frac{2}{3}$ of his salary. What percentage of their salary did Ron and Andrew each get?

11 In one term, Patrick achieved the following test results in maths.

Measurement $\frac{33}{40}$ Algebra $\frac{26}{30}$ Geometry $\frac{12}{15}$

(a) Convert Patrick's test results to percentages. Round your answers to the nearest whole percentage.

(b) In which topic did Patrick achieve his best result?

12 In a Maths class with 24 students, 17 students got a grade of B and 7 got a grade of B+.

(a) What percentage of students got a grade of B?

(b) What percentage of students got a grade of B+?

13 A teacher calculates her students' scores for a test by dividing the marks obtained by the total marks to get a decimal score. Part of her mark book appears as follows.

Student	Result
Abby	0.675
Brayden	0.95
Ciara	0.9
Dakota	0.55
Elmer	0.425
Felix	0.825

Find the percentage mark obtained by:

(a) Abby **(b)** Brayden **(c)** Ciara

(d) Dakota **(e)** Elmer **(f)** Felix

14 Rebecca needs to score at least 80% on the theory test for her learner's permit. She knows that the test contains 40 questions. What is the minimum number of questions Rebecca must get correct in order to receive her learner's permit?

15 A chemist testing a rock sample found that it contained 0.246 parts of calcium, 0.034 parts of magnesium and 0.0012 parts of potassium. Find the percentage equivalent of each of these three decimals.

16 Sam estimates his height to be 164 cm, but his actual height is 162 cm.

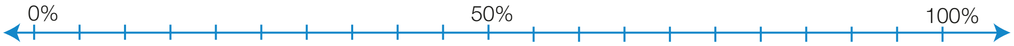
(a) Calculate the error.

(b) Calculate the percentage error.

17 For each of these sets of numbers, draw the percentage number line shown and indicate the position of each number on the line. (You may need to estimate the position of some numbers.)

(a) 25%, $\frac{2}{5}$, 0.04, $\frac{13}{20}$

(b) 3%, 0.3, $\frac{3}{4}$, 34%



18 For each of these sets of numbers, draw the number line shown and indicate the position of each number. (You may need to estimate the position of some numbers.)

(a) 120%, 0.8, 52.5%, 1.95, $\frac{11}{10}$

(b) 115%, 1.01, $\frac{5}{3}$, 270%, $\frac{14}{25}$

19 An advertisement for a brand of bread claims that it now has '20% more fibre!' The nutritional information table on the packaging shows that a 40 g serve of bread contains 3.4 g of fibre. Before the improvement was made, the same 40 g serving contained 2.8 g of fibre.

(a) Write the increase in the amount of fibre as a fraction of the original amount, in simplest form.

(b) Convert this fraction to a percentage, rounding your answer to 2 decimal places.

(c) Is the bread manufacturer's claim of '20% more fibre' justified? Explain your answer.

In Question **19**, you can turn decimal numbers into whole numbers by multiplying by 10.

Reasoning

20 Imogen compared two toy stores to find out the number of customers who bought goods as a fraction of the total number of customers who entered the store. The results for the first store in 3 days were $\frac{122}{200}$, $\frac{143}{220}$ and $\frac{275}{330}$. The results for the second store were $\frac{192}{400}$, $\frac{168}{320}$ and $\frac{336}{480}$.

(a) Calculate the percentage of paying customers for each day for both stores.

(b) Which store had the greater percentage of paying customers each day?

(c) Which store had the greater percentage over the 3 days?

21 A nutritionist analysing a sample of savoury biscuits obtained the following results.

Nutrient	Proportion
Protein	0.133 parts
Dietary fibre	0.03 parts
Sodium	0.0073 parts
Carbohydrate	0.594 parts (of which 0.005 is sugar)
Fat	0.172 parts (of which 0.084 is saturated)

(a) Calculate the percentage equivalent of protein, dietary fibre, sodium, carbohydrate and fat in the sample.

(b) Calculate the percentage equivalent of the fat content in the sample that is not made up of saturated fats.

(c) Calculate the percentage equivalent of the carbohydrate content in the sample that is not sugar.

(d) A batch is rejected if it is found to contain greater than 1% sodium. The next batch analysed was found to have an extra 0.0052 parts of sodium compared with the batch above. Calculate the percentage equivalent of sodium of this batch to find if this batch will be accepted.

22 Seventy-five Year 8 students went on an excursion. Emily and Emma each had to count the total number of students. Emily counted 76 students but Emma counted 73 students.

(a) Calculate the percentage error made by Emily.

(b) Calculate the percentage error made by Emma.

23 In a cross-country race, a 0.25 portion of the track is flat and straight; the next 0.275 portion of the track is curved uphill; the next 0.225 portion of the track is slightly downhill; the next 0.15 portion of the track is straight grassland; and the last 0.1 portion of the track is flat and straight. What percentage equivalent of the track is:

(a) curved uphill **(b)** slightly downhill **(c)** straight grassland?

Open-ended

24 Fill in each box below with one of the digits 0–9 to make the equation correct. Try to find three different combinations.

$\square.3\square = \square\square\square\,\%$

25 Write at least three different fractions that have equivalent percentage values between 68% and 75%. Only one of them can have a denominator of 100.

Puzzle

KenKen

Equipment required: grid paper

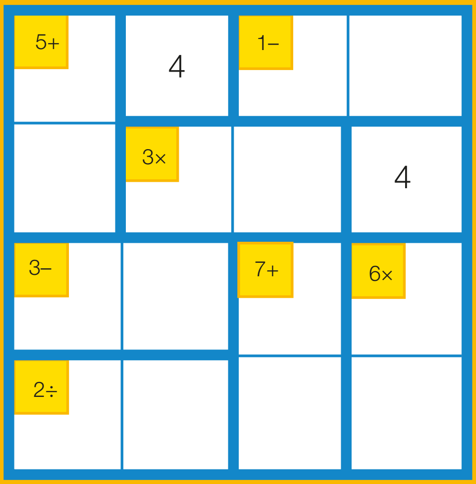
This grid is made of 16 squares, in 4 rows and 4 columns. Each square needs a number in it. (Some numbers are already given.)

The grid squares are grouped together in areas with thick outlines. For each outlined area, the number in the yellow top left corner is the 'answer' for that area. You must find the numbers that can be calculated together to match that answer, using the operator given (+ – × ÷).

For example, the area of two squares with the given answer '3×' must contain two numbers that can be multiplied together to make 3. The numbers must be 1 and 3, because the calculation can only be 1 × 3 = 3 or 3 × 1 = 3.

- Each number from 1 to 4 must be used once in each row and once in each column.

Copy this puzzle onto grid paper and try to solve it.



Half-time 2

2.4 1 Estimate the percentage of the jug that contains cordial.

2.2 2 Express the following in decimal form. State whether they are terminating decimals, recurring decimals, or irrational numbers.

(a) $15 \div 6$ **(b)** $\frac{7}{11}$

(c) $\sqrt{54}$ **(d)** $\sqrt{174.24}$

(e) $83 \div 12$ **(f)** $\sqrt{75}$

2.1 3 Hasharan is cutting ribbon to wrap around gift boxes. She has a 4 m roll of ribbon, and needs 35 cm of ribbon per box.

(a) How many gift boxes will Hasharan be able to wrap?

(b) What length of ribbon will she have left over?

2.5 4 Write each of the following fractions and decimals as percentages.

(a) 4.56 **(b)** $\frac{3}{4}$ **(c)** $\frac{71}{80}$ **(d)** 0.432 **(e)** $1\frac{2}{5}$

2.2 5 Write the following recurring decimals in fraction form.

(a) $1.\dot{3}$ **(b)** $3.2\dot{2}\dot{6}$ **(c)** $5.1\overline{902}$

2.1, 2.2, 2.3 6 Write the following in order from smallest to largest.

(a) $-\frac{5}{9}, \sqrt{5}, \frac{3}{8}, -0.7, \frac{1}{2}, 1.099$ **(b)** $-1.23, 0.874, -\frac{2}{3}, \frac{17}{8}, \sqrt{13}$

2.2 7 Write each of the following in exact decimal form.

(a) $\frac{2}{3}$ **(b)** $\frac{5}{7}$ **(c)** $\frac{1}{9}$ **(d)** $\frac{5}{11}$

2.1 8 Write the following decimals as fractions or as mixed numbers in simplest form.

(a) 0.438 **(b)** 1.072 **(c)** 2.0506 **(d)** 13.7005

2.3 9 Calculate the following.

(a) $-\frac{1}{5} + \frac{1}{2}$ **(b)** $-\frac{1}{4} \times \left(-\frac{6}{7}\right)$ **(c)** $-\frac{3}{10} - \left(-\frac{7}{12}\right)$ **(d)** $-\frac{5}{4} \div \frac{2}{3}$

(e) $-7.8 + (-5.49)$ **(f)** $0.3 - (-11.46)$ **(g)** -2.6×3.05 **(h)** $18 \div -1.2$

2.5 10 The fraction of the human body that is water varies between $\frac{11}{20}$ and $\frac{13}{20}$. Write this fraction range as a percentage range.

2.6

Writing percentages as fractions and decimals

When shopping, a '50% off' sale is the same as a half-price sale.

You can write 50% as $\frac{50}{100}$ or 50 ÷ 100, which is equivalent to the decimal 0.5 or the fraction $\frac{1}{2}$.

To write a percentage as a fraction or a decimal:

- divide the value of the percentage by 100
- for a fraction, write the division by 100 in fraction form, then simplify if possible
- for a decimal, divide by 100. Show this by moving the decimal point two places to the left.

Worked example 15

W.E. 15

Write the following percentages as fractions in simplest form.

(a) 16% **(b)** 120%

Thinking	Working
(a) 1 Divide the value of the percentage by 100. **2** Write the division as a fraction and simplify if possible.	(a) 16% $= 16 \div 100$ $= \frac{16}{100}$ $= \frac{4}{25}$
(b) 1 Divide the value of the percentage by 100. **2** Write the division as a fraction and simplify if possible.	(b) 120% $= 120 \div 100$ $= \frac{120}{100}$ $= \frac{6}{5}$ or $1\frac{1}{5}$

Worked example 16

W.E. 16

Write the following 'fractional percentages' as fractions in simplest form.

(a) $\frac{1}{4}\%$ **(b)** $6\frac{2}{3}\%$

Thinking	Working
(a) **1** Divide the value of the percentage by 100.	(a) $\frac{1}{4}\% = \frac{1}{4} \div 100$
2 Perform the division (recall that to divide by a fraction, you must multiply by its inverse).	$= \frac{1}{4} \div \frac{100}{1}$ $= \frac{1}{4} \times \frac{1}{100}$
3 Write the answer, simplifying if possible.	$= \frac{1}{400}$
(b) **1** Divide the value of the percentage by 100.	(b) $6\frac{2}{3}\% = 6\frac{2}{3} \div 100$
2 Write both values as improper fractions so that the division can be performed.	$= \frac{20}{3} \div \frac{100}{1}$
3 Perform the division (recall that to divide by a fraction, you must multiply by its inverse).	$= \frac{20}{3} \times \frac{1}{100}$ $= \frac{20}{300}$
4 Write the answer, simplifying if possible.	$= \frac{2}{30}$ $= \frac{1}{15}$

Worked example 17

W.E. 17

Write the following percentages as fractions in simplest form.

(a) 48.2% **(b)** 3.75%

Thinking	Working
(a) **1** Divide the value of the percentage by 100.	(a) $48.2\% = 48.2 \div 100$
2 Write the division as a fraction, then multiply the numerator and denominator by a power of 10 to remove the decimal point. (Here, multiply by 10.)	$= \frac{48.2}{100}$ $= \frac{482}{1000}$
3 Simplify the answer if possible. (Here, divide by a common factor of 2.)	$= \frac{241}{500}$

(b) **1** Divide the value of the percentage by 100.	(b) $3.75\% = 3.75 \div 100$
2 Write the division as a fraction, then multiply the numerator and denominator by a power of 10 to remove the decimal point. (Here, multiply by 100.)	$= \frac{3.75}{100}$ $= \frac{375}{10000}$
3 Simplify the answer if possible. (Here, divide by a common factor of 25, then 5.)	$= \frac{15}{400}$ $= \frac{3}{80}$

Worked example 18

W.E. 18

Write the following percentages as decimals.

(a) 23% **(b)** 171% **(c)** 3.49%

Thinking	**Working**
(a) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left.	(a) $23\% = 23 \div 100$ $= 0.23$
(b) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left.	(b) $171\% = 171 \div 100$ $= 1.71$
(c) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left. Fill in any empty place value columns with zeros.	(c) $3.49\% = 3.49 \div 100$ $= 0.0349$

Worked example 19

W.E. 19

Write the following ‘fractional percentages’ as decimals.

(a) $\frac{1}{2}\%$ **(b)** $3\frac{1}{5}\%$

Thinking	**Working**
(a) **1** Write the fraction value as a decimal value.	(a) $\frac{1}{2}\% = 0.5\%$
2 Divide the decimal value by 100.	$0.5 \div 100$ $= 0.005$
(b) **1** Write the mixed number value as a decimal value.	(b) $3\frac{1}{5}\% = 3.2\%$
2 Divide the decimal value by 100.	$3.2 \div 100$ $= 0.032$

It is important to remember the difference between $\frac{1}{2}$% and $\frac{1}{2}$.

- $\frac{1}{2}$% means 'one half of 1%'. This is one half of 0.01, or 0.005.
- $\frac{1}{2}$ means 'one half of the whole', which is 50%.

2.6 Writing percentages as fractions and decimals

Navigator

Answers p. 626

1 (columns 1–3), 2 (columns 1–2), 3 (columns 1–2), 4, 5 (row 1), 6, 7, 9, 10, 12, 14, 17, 19, 22 (a)	1 (columns 3–4), 2 (columns 3–4), 3 (columns 3–4), 4, 5 (rows 2–3), 6, 7, 8, 11, 13, 14, 15, 16, 17, 18, 20, 22, 23	1 (column 4), 2 (columns 3–4), 3 (columns 3–4), 4 (rows 3–4), 5 (rows 2–3), 6, 7, 8, 11, 13, 14, 15, 16, 18, 20, 21, 22, 23

Equipment required: calculator may be used for Questions **5**, **22(a)**

Fluency

W.E. 15

1 Write the following percentages as fractions in simplest form.

(a) 17% **(b)** 48% **(c)** 9% **(d)** 65%
(e) 58% **(f)** 76% **(g)** 117% **(h)** 129%
(i) 240% **(j)** 315% **(k)** 138% **(l)** 360%

W.E. 16

2 Write the following 'fractional percentages' as fractions in simplest form.

(a) $\frac{1}{2}\%$ **(b)** $\frac{1}{5}\%$ **(c)** $\frac{3}{7}\%$ **(d)** $\frac{3}{8}\%$
(e) $\frac{1}{10}\%$ **(f)** $\frac{2}{3}\%$ **(g)** $\frac{5}{6}\%$ **(h)** $\frac{2}{9}\%$
(i) $2\frac{1}{4}\%$ **(j)** $6\frac{1}{2}\%$ **(k)** $5\frac{3}{5}\%$ **(l)** $8\frac{3}{4}\%$

W.E. 17

3 Write the following percentages as fractions in simplest form.

(a) 8.3% **(b)** 12.29% **(c)** 50.5% **(d)** 62.5%
(e) 2.1% **(f)** 3.4% **(g)** 0.55% **(h)** 0.4%
(i) 10.25% **(j)** 87.5% **(k)** 150.2% **(l)** 420.5%

W.E. 18

4 Write the following percentages as decimals.

(a) 80% **(b)** 27% **(c)** 40% **(d)** 15%
(e) 66% **(f)** 92% **(g)** 3% **(h)** 5%
(i) 110% **(j)** 265% **(k)** 700% **(l)** 304%
(m) 0.5% **(n)** 7.2% **(o)** 40.5% **(p)** 6.25%

5 Write the following 'fractional percentages' as decimals.

W.E. 19

(a) $\frac{1}{4}\%$
(b) $\frac{1}{8}\%$
(c) $\frac{7}{10}\%$
(d) $\frac{2}{5}\%$
(e) $2\frac{1}{2}\%$
(f) $10\frac{3}{4}\%$
(g) $37\frac{1}{5}\%$
(h) $18\frac{4}{5}\%$
(i) $25\frac{1}{3}\%$
(j) $5\frac{2}{3}\%$
(k) $2\frac{5}{9}\%$
(l) $23\frac{5}{6}\%$

6 2.8% in its simplest fraction form is:

A $\frac{14}{500}$
B $\frac{28}{1000}$
C $\frac{7}{250}$
D $2\frac{4}{5}$

7 $\frac{3}{4}\%$ expressed as a fraction is:

A $\frac{3}{400}$
B $\frac{34}{100}$
C $\frac{3}{4}$
D $3\frac{4}{100}$

8 What are the equivalent decimal and fraction for $13\frac{1}{3}\%$?

A $0.1\dot{3}$, $13\frac{1}{3}$
B $0.1\dot{3}$, $\frac{2}{15}$
C $0.01\dot{3}$, $\frac{1}{13}$
D $0.001\dot{3}$, $\frac{1}{300}$

9 The decimal equivalent of 500% is:

A 50 000
B 5
C 0.5
D 0.05

Understanding

10 An advertisement states that 86% of dentists recommend using 'Squeeky Clean' toothpaste. What fraction of dentists is this?

11 Gina has a $9\frac{1}{2}\%$ pay rise. What fraction of her salary does this represent?

12 Madison travelled 45% of the journey across Europe from Paris to Istanbul by train. What is the decimal equivalent of the percentage travelled by train?

13 Jacob received 92.5% on his Australian History test. What is the decimal equivalent of Jacob's result?

14 A bouquet of roses contains 40% red roses, 32% pink roses and 28% yellow roses. What are the fractions of red, pink and yellow roses?

15 A car company introduced a new car which is $7\frac{1}{2}\%$ more fuel efficient than the previous model. What fraction of fuel efficiency does this represent?

16 An economist predicts that inflation will increase by 0.8% this month. What is the decimal equivalent of this increase?

17 In rugby league, bruisings and strains are 30% of injuries, while more serious injuries such as fractures are 16% and concussions are 9%.

(a) Write each of these percentages as fractions.

(b) What fraction of the total number of injuries do bruisings, strains, fractures and concussions make up altogether?

(c) What fraction of the total number of injuries are due to other causes?

18 Place the following in ascending order.

(a) $20\%, \frac{1}{4}, 0.02, \frac{2}{5}$ **(b)** $0.45, \frac{4}{5}, 4.5\%, \frac{5}{4}$

(c) $\frac{3}{10}, 0.03, \frac{1}{3}, 33\%$ **(d)** $0.72, 7\frac{1}{2}\%, 7.2\%, \frac{7}{2}$

19 Employment figures show that 28% of employed people in a suburb are tradespeople. What fraction of the employed workforce are tradespeople?

Reasoning

20 One quarter of the 200 Year 8 students surveyed at St Gabriel's College said that they were driven to school and 30% said that they arrived by bus. What fraction of students are not driven to school and do not take the bus?

21 The Burandoc Water Authority is trying to estimate the volume of water available in the reservoir. At present, it is 85% full. It is estimated that if there is no rain over the next 6 months the volume will be 48% of capacity. However, if the area receives normal rainfall during this period, the volume will only decrease by 17% of capacity.

(a) What fraction of the reservoir currently contains water?

(b) What fraction of the reservoir will contain water if there is no rain during the next 6 months?

(c) By what fraction would the water volume be reduced if there is no rain during the next 6 months?

(d) What fraction of the reservoir would remain unfilled if there is a normal amount of rain during these 6 months?

Open-ended

22 **(a)** Write at least three fractions with denominators less than 100 that have a percentage equivalent between 50% and 60%.

(b) Write at least three decimals that have a percentage equivalent between 25% and 26%. Each of the decimals given should have a different number of decimal places.

23 Mason writes: $3\frac{1}{4} = 3.25\%$

(a) Explain to Mason the difference between $3\frac{1}{4}$ and 3.25%.

(b) Correctly convert $3\frac{1}{4}$ into its percentage form.

Game

First to change

Equipment required:
calculator, 1 die

How to win:
You win a point if you are first to convert a percentage to a fraction in simplest form.

The first person to get 5 points wins.

How to play:
Roll the die two times and make the numbers rolled into a percentage, in the order in which they are rolled. For example, if a 3 is rolled and then a 6, the percentage is 36%.

You and your opponent must then race each other to convert the percentage to a fraction in simplest form. You can work it out in your head or with pen and paper. The first with an answer taps the table and immediately gives their answer.

Check the answer on the calculator using the fraction key.

If the answer is correct the player gets a point. If it is incorrect, the opponent gets a point.

For the example of 36%, if you give $\frac{36}{100}$ or $\frac{18}{50}$ as your answer, your opponent gets a point, because the answer must be $\frac{9}{25}$.

2.7 Writing one amount as a percentage of another

In a netball match, Kate scored 17 goals from 23 shots at goal, while her teammate Mel scored 13 goals from 19 shots at goal. Who was the more accurate goal shooter? Kate scored more goals, but she had more shots. Both girls missed 6 of their shots at goal. What do you think?

Percentages are very useful in these situations. Because percentage values are always 'out of 100', you can easily compare them.

To write one amount as a percentage of another:

1. make sure both amounts are the same type, or measured in the same units; convert units if necessary
2. write a fraction with the 'part amount' as the numerator and the 'whole amount' as the denominator
3. convert this fraction into a percentage.

Not all calculations of percentage give whole number answers. Remainders can be written in fraction form, exact decimal form or as rounded decimals, depending on the situation.

The **S⇔D** or **$a^b/_c$** key on your calculator may be useful for converting between answers in decimal form and fraction form.

Worked example 20

W.E. 20

Express the first amount as a percentage of the second.

Give your answers in **(i)** fractional and **(ii)** exact decimal form.

(a) 23, 40

(b) 35, 30

Method 1: Multiply fraction by 100%

Thinking	Working
(a) **1** Write a fraction with the first amount as the numerator and the second amount as the denominator.	(a) $\frac{23}{40}$

2 Multiply by $\frac{100}{1}\%$, simplifying the multiplication by cancelling common factors. (Here, the common factor is 20.)	$\frac{23}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$
3 Perform the simplified multiplication.	$= \frac{115}{2}\%$
4 Write your answer as a mixed number (fractional form), and as the equivalent decimal.	$= \text{(i) } 57\frac{1}{2}\%$ or (ii) 57.5%
(b) 1 Write a fraction with the first amount as the numerator and the second amount as the denominator.	(b) $\frac{35}{30}$
2 Multiply by $\frac{100}{1}\%$, simplifying the multiplication by cancelling common factors. (Here, common factors of 5, then 2, are cancelled.)	$\frac{{}^{7}\cancel{35}}{{}_{6}\cancel{30}} \times \frac{100}{1}\%$ $= \frac{7}{{}_{3}\cancel{6}} \times \frac{{}^{50}\cancel{100}}{1}\%$
3 Perform the simplified multiplication.	$= \frac{350}{3}\%$
4 Write your answer as a mixed number (fractional form), and as the equivalent decimal.	$= \text{(i) } 116\frac{2}{3}\%$ or (ii) $116.\dot{6}\%$

Method 2: Convert to decimal, then multiply by 100%

Thinking	**Working**
(a) 1 Write a fraction with the first amount as the numerator and the second amount as the denominator.	(a) $\frac{23}{40}$
2 Convert the fraction to a decimal by performing the division. (A calculator could be used for this step.) Keep all of the decimal places (do not round off).	$= 0.575$
3 Multiply the decimal value by 100%.	$0.575 \times 100\%$
4 Write your answer in decimal form. Convert any decimal remainder to a fraction, and write in fractional form.	$= \text{(i) } 57\frac{1}{2}\%$ or (ii) 57.5%

(b) **1** Write a fraction with the first amount as the numerator and the second amount as the denominator.	(b) $\frac{35}{30}$
2 Convert the fraction to a decimal by performing the division. (A calculator could be used for this step.) Keep all of the decimal places (do not round off).	$= 1.1666\ldots$
3 Multiply the decimal value by 100%.	$1.1666\ldots \times 100\%$
4 Write your answer in exact decimal form. Convert any decimal remainder to a fraction, and write in fractional form.	$= \text{(i) } 116\frac{2}{3}\% \text{ or (ii) } 116.\dot{6}\%$

Worked example 21

W.E. 21

Express the first amount as a percentage of the second: 40 cents, \$5.

Method 1: Multiply fraction

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator, multiplied by $\frac{100}{1}\%$.	$\frac{40}{5\cancel{00}} \times \frac{1\cancel{00}}{1}\%$
3 Simplify by cancelling common factors. (Here, divide by 100, then by 5.)	$= \frac{\overset{8}{\cancel{40}}}{\underset{1}{\cancel{5}}} \times \frac{1}{1}\%$ $= \frac{8}{1} \times \frac{1}{1}\%$ $= 8\%$
4 Write the answer.	40 cents is 8% of \$5.

Method 2: Convert to decimal, then multiply

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator.	$\frac{40}{500}$
3 Convert this fraction to a decimal by performing the division.	$= 0.08$
4 Multiply by 100%.	$= 0.08 \times 100\%$ $= 8\%$
5 Write the answer.	40 cents is 8% of \$5.

2.7 Writing one amount as a percentage of another

Navigator

1 (columns 1–3), 2, 3 (a), 4, 6, 8, 10, 11, 15, 20 (a–b), 21, 22, 23	1 (columns 3–4), 2 (column 1), 3 (b), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23	1 (columns 3–4), 2 (column 2), 3 (b), 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

Answers p. 627

Equipment required: calculator

Fluency

W.E. 20

1 Express the first amount as a percentage of the second. Give your answers in **(i)** fractional and **(ii)** exact decimal form.

(a) 13, 50 **(b)** 16, 20 **(c)** 27, 45 **(d)** 42, 48

(e) 35, 80 **(f)** 15, 18 **(g)** 54, 96 **(h)** 15, 27

(i) 86, 54 **(j)** 90, 70 **(k)** 72, 66 **(l)** 26, 22

W.E. 21

2 Express the first amount as a percentage of the second. Round your answer to 2 decimal places where necessary.

(a) 25 cents, $4 **(b)** 40 m, 2 km

(c) 7 mm, 2 cm **(d)** 30 seconds, 4 minutes

(e) 750 g, 2.5 kg **(f)** 5 days, 4 weeks

(g) 80 kg, 1 tonne **(h)** 85 mL, 2 L

(i) 600 mm, 7 m **(j)** 50 minutes, 40 hours

Remember that units have to be the same, so convert both values to the smaller unit first.

3 **(a)** Which of the following shows how to calculate 17 as a percentage of 30?

A $\frac{17}{30} \times \frac{100}{1}\%$ **B** $\frac{30}{17} \times \frac{100}{1}\%$ **C** $\frac{17}{100} \times \frac{30}{1}\%$ **D** $\frac{30}{100} \times \frac{17}{1}\%$

(b) Which of the following is 40 cm as a percentage of 2 m?

A 2% **B** 5% **C** 20% **D** 500%

Understanding

4 Cory received 38 marks out of 40 for a maths test. Find his result as a percentage.

5 In an election for house captain, Jessica received 324 votes. If 540 students voted, what percentage of them voted for Jessica?

6 Keira planted 30 small trees in her new garden, but only 19 survived after a year. What percentage of the new trees survived? (Answer in exact decimal form.)

7 Sam improved his best time for a 200 m sprint by 2 seconds during a training session. His previous best time was 25 seconds. Write Sam's improvement as a percentage of his previous best time.

8 Hamish improved his shot-put throw from 20 m to 23 m. By what percentage of his 20 m throw did he increase his distance?

9 Lara has a part-time job that pays $160 per week. She receives a pay rise of $24 per week. Express Lara's pay rise amount as a percentage of her original pay.

10 Smith has read 185 pages of a book of 490 pages. What percentage of the pages has he read?

11 Phil saved $250 from his $3500 salary each month to pay for a holiday. What percentage of his salary has he saved?

12 During one particular season, Cuong records 18 wins from 27 sets of tennis played. What percentage of sets played did Cuong win that season? (Answer in fractional percentage form.)

13 Harry scores 5 of his team's 16 goals during a football match. What percentage of his team's goals does he score? (Answer in fractional percentage form.)

14 Adila spends $90 each week on food. If her total income is $400 per week, what percentage of it is spent on food? (Answer in fractional percentage form.)

15 According to *Guinness World Records*, Antonio Gomesdos Santos stood motionless (apart from involuntary blinking) for approximately 15 hours on 30 July 1988. What percentage of a day was this? (Answer in decimal form.)

16 According to *Guinness World Records*, the tallest man in reliably recorded history was Robert Wadlow of Illinois, USA, whose height was measured to be 272 cm. If the average male height today is about 1.8 m, express Robert Wadlow's measured height as a percentage of the average male height. (Answer in exact decimal form.)

17 A 30 gram serving of a breakfast cereal contains 2.5 grams of dietary fibre. What percentage of the serving of cereal is this? (Answer in fractional percentage form.)

18 A sofa is advertised in a furniture sale catalogue at the discounted price of $699, a saving of $350. Find the saving as a percentage of the original price of the sofa. Round your answer to the nearest whole number.

Reasoning

19 In their basketball game on Friday night, Nathan scored 12 goals from 16 shots, and his teammate Dayo scored 14 goals from 18 shots.

(a) Estimate who is the more accurate scorer, then use percentages to check your estimate. (The more accurate scorer will have made a greater percentage of their shots as goals.) Round your answers to 1 decimal place if necessary.

(b) In their game the following week, Nathan scored 13 goals from 18 shots, and Dayo scored 11 goals from 14 shots. Add the number of goals and the number of shots to those of the previous week. Use your total to calculate a new, two-game percentage for Nathan and for Dayo. (Again, round your answers to 1 decimal place if necessary.)

(c) Based on your answer to the previous part, who improved their scoring percentage from one week to the next?

20 Round your answers to the following to 1 decimal place.

(a) Jaime estimates that she spends 2 hours a day online. What percentage of a day is this?

(b) Jaime also estimates that she spends $\frac{1}{3}$ of a day sleeping. What percentage of her waking hours does she spend online?

(c) Jaime does not include the 7 hours that she spends at school as part of her 'online time'. Recalculate her 'online' time as a percentage of her free time (excluding school time and sleeping time).

(d) Describe what happens to the value of the percentage as you move from **(a)** to **(b)** to **(c)**. Explain why this happens.

21 Lin won a $500 gift card. She used the gift card to spend $110 on games, $90 on jewellery, $79 on a pair of sunglasses and $49 on a T-shirt.

(a) What percentage of money did she spend on games and jewellery together?

(b) What percentage of money did she spend on the sunglasses and the T-shirt together?

(c) What percentage of the money remains on the gift card?

Open-ended

22 Write three different amounts of money that when expressed as a percentage of $4 are greater than 32% and less than 39%.

23 Bella has calculated that she is 89% of her dad's height (to the nearest whole percentage). Give three possible pairs of heights for Bella and her dad, if they are both between 1 m and 2 m tall.

Problem solving

League tables

In many sports, percentage is used to rank teams that have the same number of wins and losses. It is calculated by writing the points the team has scored as a fraction of the points scored against them, then multiplying by 100.

Team percentage $\frac{\text{points for}}{\text{points against}} \times \frac{100}{1}$

Half-way through round 4 of the competition, the Coburg Comets are third on the ladder, while the Mitcham Mozzies are fourth.

Team	Matches played	Points for	Points against	%
Comets	4	192	147	130.6
Mozzies	3	104	81	128.4

The Comets have already played their match for the round, but the Mozzies have not. What would the final scores of the Mozzies match need to be to overtake the Comets' percentage?

Strategy options

- Guess and check.
- Solve a simpler problem.

2.8

Finding a percentage of an amount

Being able to find a percentage of an amount is very useful. Percentages can be written as fractions out of 100, so you can use fraction multiplication to find a percentage of an amount (Method 1 of the example below). Alternatively, you can multiply the amount by the decimal equivalent of the percentage (Method 2 of example).

The key thing to remember is that, in maths, the word 'of' can mean multiplication.

> To find a percentage of an amount:
>
> **1** replace the 'of' in the expression with '×'
> **2** convert the percentage to a fraction or a decimal
> **3** perform the multiplication and simplify your answer.

Fractions are often easier to work with when performing calculations by hand. Decimals are much quicker if you have a calculator available.

Worked example 22

W.E. 22

Find the following, rounding answers to 2 decimal places where necessary.

(a) 12% of \$350 **(b)** 5.2% of 30 kg

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Substitute '×' for 'of' and omit any units.	(a) 12% of \$350 $= 12\% \times 350$
2 Write the percentage as a fraction, and the amount as an improper fraction with a denominator of 1. Simplify the multiplication by cancelling common factors. (Here, divide by 10 and 5, then by 2.)	$= \frac{12}{{}_{2}\cancel{100}} \times \frac{{}^{7}\cancel{350}}{1}$ $= \frac{{}^{6}\cancel{12}}{{}_{1}\cancel{2}} \times \frac{7}{1}$
3 Simplify, then perform the multiplication.	$= 6 \times 7$
4 State your answer, including any units.	= \$42

(b) **1** Substitute '×' for 'of'.	(b) 5.2% of 30 kg $= 5.2\% \times 30$
2 Write the percentage as a fraction, and the amount as an improper fraction with a denominator of 1. Simplify by cancelling common factors. (Here, divide by 10, then by 4.)	$= \frac{\overset{13}{\cancel{52}}}{{}_{25}\cancel{1000}} \times \frac{3\cancel{0}}{1}$
3 Simplify, then perform the multiplication and write the answer.	$= \frac{13}{25} \times 3$ $= \frac{39}{25}$
4 Write your answer in the required form with the correct units.	$= 1.56$ kg

Method 2: Decimal multiplication

Thinking	**Working**
(a) **1** Substitute '×' for 'of'.	(a) 12% of \$350 $= 12\% \times 350$
2 Write the percentage as a decimal.	$= 0.12 \times 350$
3 Perform the multiplication and write the answer.	$= \$42$
(b) **1** Substitute '×' for 'of'.	(b) 5.2% of 30 kg $= 5.2\% \times 30$
2 Write the percentage as a decimal.	$= 0.052 \times 30$
3 Perform the multiplication and write the answer with the correct units.	$= 1.56$ kg

If the percentage you are finding is less than 100%, then your answer will always be smaller than the original amount. It is important to check if your answer is reasonable.

2.8 Finding a percentage of an amount

Navigator

Answers p. 627

1 (columns 1–2), 2, 3, 4, 5, 6, 7, 8, 12, 14, 22	1 (columns 2–3), 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 22	1 (columns 2–3), 2 (b), 3 (a), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Equipment required: calculator may be used for Questions **1**, **7**, **8**, **10–21**

Fluency

W.E. 22

1 Find the following, rounding answers to 2 decimal places where necessary.

(a) 6% of \$200 **(b)** 20% of \$150 **(c)** 64% of \$50

(d) 80% of \$25.50 **(e)** 75% of \$279.95 **(f)** 120% of \$35.74

(g) 150% of 30 kg **(h)** 160% of 25 kg **(i)** 90% of 4 kg

(j) 23.2% of 40 min **(k)** 42.5% of 70 min **(l)** 15.7% of 60 min

(m) 3.4% of 50 L **(n)** 9.3% of 500 L **(o)** 0.6% of 3000 L

2 **(a)** Which of the following shows how to calculate 33% of 70?

A $\frac{33}{1} \times \frac{100}{1} \times \frac{70}{1}$ **B** $\frac{33}{100} \times \frac{70}{1}$ **C** $\frac{33}{70} \times \frac{100}{1}$ **D** $\frac{70}{30} \times \frac{100}{1}$

(b) Which of the following shows how to find $7\frac{1}{2}\%$ of 600?

A $\frac{7}{200} \times \frac{600}{1}$ **B** $\frac{3}{40} \times \frac{600}{1}$ **C** $\frac{15}{2} \times \frac{600}{1}$ **D** 7.5×600

3 **(a)** Which of the following shows how to calculate 3.5% of 15?

A 3.5×15 **B** 35×15 **C** 0.035×15 **D** 0.035×0.15

(b) Which of the following shows the calculation for finding 19% of 6000?

A 19×6000 **B** 19×60 **C** 0.19×6000 **D** 0.19×6

Understanding

4 Nick got 85% of the questions right in a test containing 80 questions. How many questions did Nick answer correctly?

5 A 750 gram packet of breakfast cereal is only 20% full. How many grams of cereal are in the packet?

6 About 65% of the mass of an adult human is water. Ollie weighs 64 kg. How much of Ollie's mass is water, to the nearest kilogram?

7 In a 45-minute workout, Jim spends 20% of the time warming up. How much time does he spend warming up?

8 The revised edition of a book has 12% more pages than the previous edition. If the previous edition had 350 pages, how many pages are in the new edition?

9 A survey of 5500 households reveals that 4 in every 5 regularly sort their rubbish for recycling.

(a) Write this result as a percentage.

(b) Calculate the number of households that sort their rubbish.

10 The Cancer Council estimates that 2 in 7 cancer deaths are related to tobacco (smoking).

(a) Write this figure as a percentage, rounded to 1 decimal place.

(b) Of 412 000 cancer deaths, how many will be related to tobacco?

11 A geologist finds a rock of mass 600 g containing a substantial amount of gold. She has the rock analysed and finds that the rock is 19.4% gold.

(a) How many grams of gold are in the rock?

(b) If the current price of gold is \$45 per gram, how much is this gold worth?

12 Forty-five per cent of the residents in a particular suburb use internet shopping. If the suburb contains 23 000 people:

(a) how many people use internet shopping

(b) how many people don't use internet shopping?

13 **(a)** A furniture store advertises a 40% reduction on the price of its beds.

(i) Calculate the dollar value of the percentage reduction on a bed priced at $1800.

(ii) Calculate the new, reduced price of the bed.

(b) The same store advertises a $33\frac{1}{3}$ % reduction on the price of sofas.

(i) Calculate the dollar value of the percentage reduction on a sofa priced at $450.

(ii) Calculate the new, reduced price of the sofa.

14 The population of a small town increases by 6.5% during the summer holidays. If the town's population is normally 1600, by how many does it increase during the summer holidays?

15 The Republic of Banano increases its banana crop by 4.4% one year. If the previous year's crop was 4200 tonnes, how many tonnes were produced in the new crop? Answer to the nearest tonne.

16 A meeting of 700 residents votes on a proposed road closure. If 2.1% of the residents voted for the closure:

(a) how many residents voted for the closure

(b) what percentage did not vote for the closure

(c) how many residents did not vote for the closure?

17 A 250 mL bottle of fruit juice drink contains 25% real fruit juice. The rest of the drink is water and sugar.

(a) What is the volume of real fruit juice in the bottle?

(b) What is the volume made up by water and sugar?

Reasoning

18 Aaron is studying dance. 60% of his overall marks for the subject are allocated to the practical exam, and the remaining 40% is allocated to the theory exam. Aaron scored 84% in his practical exam and 77% in the theory exam. Calculate Aaron's overall percentage for the subject, correct to 1 decimal place.

19 Kim's monthly salary is $4500. She spends 25% on house rent, 5% on travel and 12% on groceries.

(a) How much does she spend on house rent?

(b) How much does she spend on travel?

(c) How much money is left with Kim after house rent, travel and grocery expenses?

20 A chemical company is planning to increase its production of fertiliser by 20%. At present, it produces 11 200 litres of fertiliser per week. The Environmental Protection Agency is concerned that this will increase the company's waste water output to greater than the $1\frac{1}{2}$% of the volume of the total production that is allowed. At present, waste water is only $\frac{3}{5}$% of the volume of the total production. The company claims that with new production procedures, the total waste water produced will only be 168 litres per week.

- **(a)** **(i)** Calculate the amount of fertiliser represented by the 20% increase in production.
  - **(ii)** Calculate the total amount of fertiliser that will be produced after the increase.
- **(b)** Calculate the amount of waste water that is produced at present.
- **(c)** Calculate the maximum amount of waste water that is allowed.
- **(d)** What percentage of the total production does the company claim the waste water will be?
- **(e)** Is the chemical company's planned increase in its waste water production more than is allowed?

Open-ended

21 There are 60 books on Naomi's bookshelf.

- **(a)** Naomi says that 21% of her books are recipe books. Explain why this is not an accurate statement.
- **(b)** Find at least three percentage values that are possible for the percentage of recipe books.

22 Two Year 8 classes each have 25 students. In one class, 20% of the students play basketball. In the other class, 20 of the students play basketball.

- **(a)** Which of the two classes has more basketball players?
- **(b)** Explain how '20% of the students' is different from '20 students'.

Problem solving

Cordial contents

Three jugs of cordial have volumes of 500 mL, 1250 mL and 1800 mL and they are each filled to capacity. The contents of the three jugs are combined together into another large jug. After this is completed the percentage of cordial in the large jug is 9.507%. Tilly knows that one of the original jugs contained 5% cordial, the second contained 7% cordial and the third contained 12.5% cordial. Work out which jug contained which percentage of cordial prior to mixing.

Strategy options

- Solve a simpler problem.
- Break problem into manageable parts.

Gamespace

Alice in Numberland

Alice was doing her homework while checking Facebook, messaging her friends and Googling for an English assignment. She fell asleep while waiting for one of her friends to reply …

She awoke in a strange land full of question marks, numbers and equations floating in the air.

I'm late!

A well-dressed rabbit went bounding by, muttering, 'I'm late, I'm late! How late you ask? I'm 20% of a minute late for the dentist!'

A kangaroo in a bow tie bounced past, crying, 'I'm later, I'm later, 15% of an hour late for my barbecue!'

'How strange', thought Alice.

Then she noticed a snail sliding along, whispering, 'I've just travelled 6 mm. That's 5% of my journey.'

(a) How late was the rabbit for the dentist (in seconds)?

(b) How late was the kangaroo for the barbecue (in minutes)?

(c) How much further does the snail have to travel?

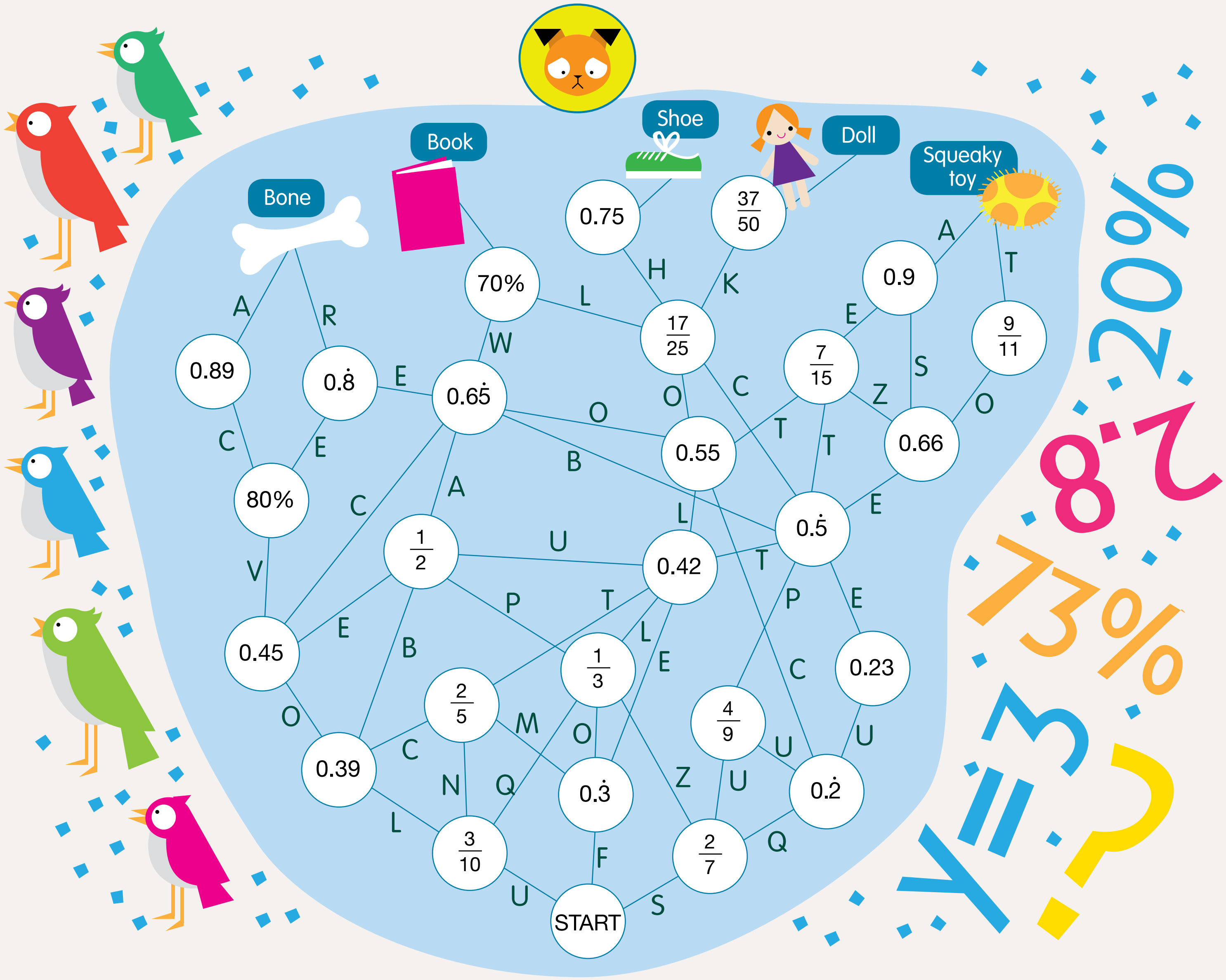
The pool of tears

Alice wandered until she saw an old dog sobbing in the far distance. He had forgotten where he had buried a favourite object and his tears had formed a large lake.

To find the lost object and the magic word to return it, Alice has to step on the stones in the correct order and collect the letters on each path she takes. At each stone, Alice must follow the path to the stone that has the largest number. Begin at START and find the path Alice must take.

(a) What object has the dog lost?

(b) What is the magic word that will return the object?



A fraction of her size

Alice stumbled upon some tiny muffins with 'eat me' written on them, and some little glass bottles with the words 'drink me' inscribed on them. 'Hmmm,' she thought, 'I am a little hungry,' and nibbled one of the muffins. She started shrinking …

(a) If Alice was originally 150 cm tall and shrank to 120 cm tall, by what percentage of her original size did the muffin make her shrink?

Alice didn't like her new, smaller height, so she sipped one of the drinks …

(b) She grew from 120 cm to 300 cm! By what percentage did she increase in size?

(c) By what percentage of her current enlarged size does Alice need to decrease to return to her original size?

'Convince us, or else!'

Alice found herself in a mathematical court room, with a jury of birds and a walrus as the judge. Little fractions kept nipping Alice's head.

The judge declared, 'Unless you can convince the jury of the following mathematical statements, we will simply have to keep you in Numberland forever'.

Alice must convince the jury that the following statements are true.

(a) 0.2% = 0.002

(b) If I have 50 tarts and I gain another 100 tarts, then I have 200% extra.

How could Alice explain these statements to the jury?

2.9

Increasing or decreasing by a given percentage

Calculating a percentage increase or decrease

A percentage can be used to represent a change in a quantity. You can calculate the result of a percentage increase or decrease by using the fact that the original amount represents 100%, which has a decimal equivalent of 1.0.

> To increase or decrease an amount by a given percentage:
>
> **1** add the percentage to, or subtract it from, 100%, then write it as a decimal scale factor
>
> **2** multiply this decimal by the amount to be increased or decreased.

Worked example 23

W.E. 23

Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to 2 decimal places where necessary.

(a) Increase $2700 by 22%.

(b) Decrease 163 kg by 45%.

Thinking	Working
(a) 1 Add the percentage increase to 100%. Write this new percentage as a decimal scale factor.	(a) 100% + 22% = 122% = 1.22
2 Multiply the scale factor by the original amount to find the new amount.	1.22 × 2700
3 Write the answer.	= $3294
(b) 1 Subtract the percentage decrease from 100%. Write this new percentage as a decimal scale factor.	(b) 100% − 45% = 55% = 0.55
2 Multiply the scale factor by the original amount to find the new amount.	0.55 × 163
3 Write the answer.	= 89.65 kg

Expressing an increase or a decrease as a percentage

Often, an increase or a decrease is described as the percentage change of an original amount. To calculate this, express the amount of increase or decrease as a fraction of the original amount, and convert it to a percentage.

Worked example 24

W.E. 24

Tori earns \$950 per week as an electrician. She receives a pay increase of \$32 per week. Write Tori's pay increase as a percentage of her original pay, correct to 1 decimal place.

Thinking	Working
1 Write the amount of the increase as a fraction of the original amount.	$\frac{32}{950} \times 100\%$
2 Convert the fraction to a percentage, rounding to the specified number of decimal places.	$= 0.033\,68\ldots \times 100\%$ $= 3.4\%$ (1 d.p.)

2.9 Increasing or decreasing by a given percentage

Navigator

1, 2, 3, 4, 5, 7 (a), 9 (a), 14 (a), 16	1 (a–b), 2, 3, 4, 5, 6, 7, 8, 9 (a–b), 10, 12 (a), 13, 14, 15, 16	1 (c–d), 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Answers p. 628

Equipment required: calculator

Fluency

W.E. 23

1 Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to 2 decimal places where necessary.

(a) Increase 1500 by: **(i)** 15% **(ii)** 25% **(iii)** 45% **(iv)** 120%

(b) Decrease 2200 by: **(i)** 10% **(ii)** 40% **(iii)** 66% **(iv)** 95%

(c) Increase \$83.75 by: **(i)** 23% **(ii)** 340% **(iii)** 3.5% **(iv)** $3\frac{2}{3}\%$

(d) Decrease 5.2 km by: **(i)** 6% **(ii)** 74% **(iii)** 2.8% **(iv)** $1\frac{3}{4}\%$

W.E. 24

2 (a) Georgia earns \$1253 per week working as a designer. She receives a pay increase of \$49 per week. Write Georgia's pay increase as a percentage of her original pay, correct to 1 decimal place.

(b) Jayden earns \$67 982 per year working as a legal assistant. He receives a pay increase of \$2850 per year. Write Jayden's pay increase as a percentage of his original pay, correct to 1 decimal place.

(c) This month, the Ling family reduced their electricity bill by \$15.76 compared to last month. If last month's bill was \$109.43, calculate the reduction in this month's bill as a percentage of the previous bill, correct to 1 decimal place.

(d) This month, the Edwards family used 13 kL (kilolitres) less water than last month. If they used 190 kL last month, calculate the reduction in this month's water usage as a percentage of last month's usage, correct to 1 decimal place.

3 Which of the following calculations decreases \$129.50 by 40%?

A 129.5×0.4 **B** 1.295×40 **C** 0.6×129.5 **D** 1.4×129.5

Understanding

4 Summer's weekly rent is set to increase by 15%. If she currently pays $750 a week, what will she pay after the increase?

5 Last year, 87 453 people attended the Royal Show. This year, only 79 014 people attended. Write the drop in attendance as a percentage decrease.

6 Yang earns $834.72 a week working on a building site. His pay increases to $886.70 a week. Write the amount of Yang's pay rise as a percentage increase of his original pay, correct to 1 decimal place.

7 Sam buys a new sports car for $45 000. As soon as he drives it out of the showroom its value decreases by 12%.

(a) Find the value of the car after it leaves the showroom.

After just 3 years, the value of the car will have decreased by 35% of its original value.

(b) What will the car be worth in 3 years' time?

(c) Write the 3-year decrease in value as an average yearly loss, in dollars per year.

8 Lizzie is a swimmer whose personal best time for her favourite race is 1 min 8.23 s.

(a) Write Lizzie's time in seconds.

(b) Lizzie is aiming to decrease her time by 5%. Use your answer to the previous question to calculate this decrease as a number of seconds, correct to the nearest hundredth of a second.

(c) At the state titles, Lizzie lowered her time by 2.65 seconds. Write this improvement as a percentage of her previous best time, to the nearest whole number.

9 **(a)** Margaret purchased a block of land for $385 000. A year later land prices had increased by 9%. What is Margaret's block of land worth now?

(b) The following year, land prices had risen by 7.5% compared to the previous year. What is Margaret's block of land worth now?

(c) Margaret later sold her block of land for $469 000. What percentage increase on the original price is this, to the nearest whole number?

Reasoning

10 At the end of the first week of trading on the sharemarket, shares in Deepak's company had increased by 12%. The shares were worth $2.50 each at the beginning of the week.

(a) What was the value of a share at the end of the first week?

(b) By the end of the second week, the share price had decreased by 7% compared to the start of the week. Calculate the value of one share at the end of the second week.

(c) Calculate the overall increase or decrease at the end of the 2 weeks as a percentage of the original price of $2.50 per share.

11 The population of the town of Greenhills decreased in 1 year by 4%. The population at the beginning of the year was 17 000.

(a) Calculate the population at the end of the year.

(b) If the population continues to drop by 4% for the next 2 years, calculate the population:

(i) at the end of the second year **(ii)** at the end of the third year.

(c) Do the three 4% decreases in population give the same result as one 12% decrease? Explain why or why not.

12 Pheap improves his maths test result by 20%.

(a) If his first result was 27 out of 60 marks, what was his improved result, if the second test was also out of 60?

(b) If Pheap continues to improve by this percentage with each test and all tests are out of 60 marks, how many more tests must he take to achieve over 90%?

13 Melissa is 165 cm tall, and her younger sister Laura is 150 cm tall.

(a) Calculate Melissa's height as a percentage increase of Laura's height, to the nearest per cent.

(b) Calculate Laura's height as a percentage decrease of Melissa's height, to the nearest per cent.

(c) Explain how the same height difference of 15 cm can produce these two different answers.

14 The amount of metal ore produced at a mine is 6900 tonnes per year.

(a) Calculate the reduced amount of ore mined if production drops by 15%.

(b) If production continues to drop by 15% each year, how many more years will it be before the amount mined falls below 4000 tonnes?

(c) At this rate, will the mine ever stop producing ore? Explain.

Open-ended

15 **(a)** Calculate the number of people with brown eyes in your class as a percentage of the whole class.

(b) Calculate the increase in this percentage if one extra brown-eyed person joins the class.

(c) Repeat the first part for the number of people with non-brown eyes in the class.

(d) Repeat the second part for the people with eyes that aren't brown in the class.

(e) Were the percentage increases you calculated for the two groups the same? Explain why or why not.

16 How many centimetres taller would you be if your height increased by 8%? (Answer to the nearest cm.)

Puzzle

Break the code

CABLE is the word needed to crack the safe. The letters C, A, B, L, E each represent a digit from 1 to 5.

Using the following equations, find the values of C, A, B, L and E.

$L - 2 = B$
$E + B = L$
$C + E = B$

Investigation

Supermarket specials

Equipment required: calculator (optional), shopping catalogues (optional)

Many shops, particularly supermarkets, put items 'on special' by reducing their price. These 'specials' are advertised in different ways, including percentage discounts, dollar savings and 'multi-buy' offers, such as 'buy two, get the third free'.

The Big Question

Is a 'multi-buy' deal better than a discounted price?

Engage

1 Imagine you are in the supermarket. Without doing any calculations, which of the following specials looks better value to you?

Deal A Tuna: 6 tins for the price of 5

Deal B Tuna: $1.08. Save 22c a tin

2 A supermarket is offering a 'buy two, get the third free', multi-buy deal on packets of biscuits. The usual price of the biscuits is $3 per packet.

- **(a)** How much would you pay for the multi-buy deal?
- **(b)** How much do you save in this deal?
- **(c)** Calculate this saving as a percentage of the full price of the three packets.
- **(d)** If you buy the multi-buy deal, what is the cost of each packet?
- **(e)** How much do you save, per packet?
- **(f)** Calculate the saving on one packet as a percentage of the original price of the packet.
- **(g)** What do you notice?

Explore

3 To investigate the Big Question, choose a product and its price, and create several different 'multi-buy' deals, such as '2 for the price of 1' and '3 for the price of 2'. For each deal, calculate the amount saved as both a dollar saving and a percentage saving. You might like to use a shopping catalogue to get ideas, or to check the price of an item.

Strategy options

- Look for a pattern.
- Test all possible combinations.

Explain

4 For each of the 'multi-buy' deals you investigated:

- **(a)** give a percentage reduction that the store could use to advertise an equivalent discount off the price of a single product
- **(b)** calculate how much the individual price of the item would need to be reduced by to make it better value than the multi-buy.

Elaborate

5 Most multi-buy deals give you just 1 product for free, such as 3 for 2, 4 for 3, 5 for 4 and so on.

- **(a)** As the number of items in a multi-buy deal increases, what happens to:
  - **(i)** the dollar saving
  - **(ii)** the overall percentage discount?
- **(b)** Explain the difference between your two answers to the first part.

6 Go back to the tuna deals you considered in the first question. Which deal is cheaper for buying 6 tins of tuna?

7 Answer the Big Question by summarising your results from **3–6**.

8 Imagine that you are in the supermarket. You see one brand of corn chips with a discounted price of $1.30 per packet. Another brand of the same sized packet of corn chips has a regular price of $1.79, but has a '3 for 2' offer. Which special represents better value for money? Show how you arrived at your answer.

Evaluate

9 Give some reasons as to why stores might offer 'multi-buy' deals for certain products, instead of advertising a percentage or a dollar discount on individual items.

10 Imagine you are doing the grocery shopping for yourself or your family. List some pros and cons of purchasing 'multi-buy' items. For which types of item would it be good to buy 'multi-buy' specials?

Extend

11 Another common promotion is the 'buy one and get the second one half-price'.

- **(a)** When would store managers use this type of promotion?
- **(b)** When might this type of promotion be good value to you?
- **(c)** Assuming both items are the same price, what percentage of the original price are you paying for each of them? Use an example to demonstrate how you arrived at your answer.

2.10

Financial applications of percentages

Businesses that sell products to people are called retailers. People buying these products are called consumers. Retailers either make their own products or purchase them from suppliers. The cost of buying or manufacturing the products (including things such as the cost of materials and transport) is called the **cost price (CP)**.

To make a **profit**, the retailer needs to sell their product at a higher price than the cost price. This higher price is called the **selling price (SP)**. The amount added onto the cost price to make the selling price is called the **mark-up**. It is often calculated as a percentage of the cost price.

If the selling price is less than the cost of the item plus any associated costs (such as transport), then a **loss** is made.

As well as adding their mark-up to the cost of a product, a business must also add a government tax called the **Goods and Services Tax (GST)**, which is an extra 10% of the selling price.

A business will sometimes offer products at a lower price, or **sale price**, to make them more attractive to consumers. This reduction in price is called a **discount**.

- Cost price (CP): the total cost of making or buying the goods
- Selling price (SP): cost price + mark-up
- Mark-up: amount added to cost price in order to make a profit
- Discount: decrease in the selling price to attract more buyers
- Sale price = (selling price) – (discount amount)
- GST: Goods and Services Tax of 10% added to selling price
- Profit: difference between SP and CP, where SP > CP
- Loss: difference between SP and CP where SP < CP

Calculating discounts and mark-ups

Worked example 25

W.E. 25

Calculate the sale price of a $350 stereo discounted by 25%, rounding your answer to the nearest cent.

Method 1: Find and subtract the discount amount

Thinking	Working
1 Calculate the dollar amount of the current price represented by the discount.	25% of $350 = 0.25 × 350 = $87.50
2 Subtract the discount amount from the original price to find the sale price.	Sale price = $350 – $87.50 = $262.50

Method 2: Calculate the remaining percentage

Thinking	Working
1 Subtract the percentage discount from 100%.	$100\% - 25\%$ $= 75\%$
2 Find the remaining percentage value of the price.	$\text{Sale price} = 75\% \text{ of } \$350$ $= 0.75 \times 350$ $= \$262.50$

Worked example 26

W.E. 26

An electrical goods store determines the selling price of appliances by applying a 68% mark-up. Calculate the selling price of a toaster bought at a cost price of $23.20.

Method 1: Find and add on the mark-up amount

Thinking	Working
1 Calculate the dollar amount of the cost price represented by the mark-up (to the nearest cent).	$68\% \text{ of } \$23.20$ $= 0.68 \times 23.20$ $= \$15.78$
2 Add the mark-up amount onto the cost price to find the selling price.	$SP = CP + \text{mark-up}$ $= \$23.20 + \$15.78$ $= \$38.98$

Method 2: Calculate the percentage increase

Thinking	Working
1 Add the percentage mark-up onto 100%.	$100\% + 68\% = 168\%$
2 Calculate this new, increased percentage of the cost price, by multiplying by the percentage in decimal form.	$SP = 168\% \times CP$ $= 1.68 \times 23.20$ $= \$38.98$

Note from the previous two examples that Method 1 is useful if you need to know the actual dollar amount of the discount or mark-up, while Method 2 is a more direct way of finding the reduced or increased price.

Make sure you don't confuse 'selling price' (SP) with 'sale price'. Selling price is the price of the goods after a mark-up has been added. Sale price is the price of goods on sale after they have been discounted.

Calculating the original price

Consumers only see the final price of products after the mark-up has been added or discounts subtracted. If you know the value of the mark-up or discount, you can calculate the price of the products before the discount or mark-up was applied. Just reverse the process used to find the selling price.

In Method 2 of the example above, you multiply the cost price of a toaster by the mark-up to find the selling price. If you divide the selling price by the mark-up, you arrive back at the cost price.

$$\begin{tikzcd}
\$23.20 \arrow[r, "\times 1.68"] & \$38.98 \\
\text{Cost price} & \text{Selling price} \arrow[l, "\div 1.68"']
\end{tikzcd}$$

In Method 2 of worked example 25, you calculated the sale price by multiplying the original selling price (often called the 'marked price') by the reduced percentage value. If you divide the sale price by this percentage value, you arrive back at the original price.

$$\begin{tikzcd} \$350 \arrow[r, "\times 0.75"] & \$262.50 \\ \text{Marked price} & \text{Sale price} \arrow[l, "\div 0.75"] \end{tikzcd}$$

For mark-ups:

$$\begin{tikzcd} \text{Cost price (CP)} \arrow[r, shift left, "\times (100\% + \text{mark-up }\%)"] & \text{Selling price (SP)} \arrow[l, shift left, "\div (100\% + \text{mark-up }\%)"] \end{tikzcd}$$

For discounts:

$$\begin{tikzcd} \text{Marked price} \arrow[r, shift left, "\times (100\% - \text{discount }\%)"] & \text{Sale price} \arrow[l, shift left, "\div (100\% - \text{discount }\%)"] \end{tikzcd}$$

Convert percentages to their decimal forms before multiplying or dividing.

Worked example 27

W.E. 27

(a) 'Great Groceries' supermarket marks up all of its bakery products by 75%. If the selling price of a loaf of bread is \$3.49, calculate the cost price.

(b) 'Discount Computers' store is having a 20% sale on all items. Calculate the original marked price of a hard drive that has been discounted to \$135.

Thinking	Working
(a) **1** Summarise the question by writing an equation.	(a) 175% of cost price = \$3.49
2 Rewrite the equation, replacing the percentage with its decimal value, and 'of' with a multiplication symbol.	1.75 × cost price = \$3.49
3 Solve the equation by dividing both sides by the decimal value of the percentage.	Cost price = 3.49 ÷ 1.75 = \$1.99
(b) **1** Summarise the question by writing an equation.	(b) 80% of marked price = \$135
2 Rewrite the equation, replacing the percentage with its decimal value, and 'of' with a multiplication symbol.	0.8 × marked price = \$135
3 Solve the equation by dividing both sides by the decimal value of the percentage.	Marked price = 135 ÷ 0.8 = \$168.75

Profit and loss

The profit or loss made on a sale is the difference between the cost price and the selling price. Profit or loss is usually expressed as a percentage of the cost price or the selling price.

You make a profit when you make money on the price you paid. You make a loss when you lose money on the price you paid.

$$\% \text{ Profit} = \frac{\text{profit in \$}}{\text{CP}} \times 100\% \text{ or } \frac{\text{profit in \$}}{\text{SP}} \times 100\%$$

where CP = cost price, SP = selling price.

Replace 'profit' with 'loss' in these formulas to calculate the percentage loss.

Worked example 28

W.E. 28

(a) Laura's wedding dress cost $1450. After her wedding, she sold it for $900.

- **(i)** Calculate the loss Laura made on the dress.
- **(ii)** Express the loss as a percentage of the cost price, rounded to the nearest per cent.

(b) Ajay bought a tent for $125 and then resold it online for $139.

- **(i)** How much profit did Ajay make on the tent?
- **(ii)** Express Ajay's profit as a percentage of the selling price, rounded to the nearest per cent.

Thinking	Working
(a) (i) Find the loss by finding the difference between the cost price (CP) and the selling price (SP).	(a) $\text{Loss} = CP - SP$ $= \$1450 - \$900$ $= \$550$
(ii) Write the loss as a fraction of the cost price, then convert it to a percentage. Round your answer to the nearest whole number.	$\frac{550}{1450} \times 100\%$ $= 38\%$
(b) (i) Find the profit by finding the difference between the cost price (CP) and the selling price (SP).	(b) $\text{Profit} = SP - CP$ $= 139 - 125$ $= \$14$
(ii) Write the profit as a fraction of the selling price, then convert it to a percentage. Round your answer to the nearest whole number.	$\frac{14}{139} \times 100\%$ $= 10\%$

2.10 Financial applications of percentages

Navigator

Answers p. 628

1, 2, 3 (a–b), 4, 5, 6, 9, 10, 11, 13, 17, 21, 29	1 (c–d), 2 (c–d), 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19 (a–b), 21, 22, 26, 28, 29, 30	1 (c–d), 2 (c–d), 3, 4, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 30, 31

Equipment required: calculator

Fluency

W.E. 25

1 Calculate the sale price of the following items, rounding your answers to the nearest cent.

(a) A $520 surfboard discounted by 20%.

(b) A shirt originally priced at $18 discounted by 10%.

(c) A $34.50 book discounted by 12.5%.

(d) A phone with a marked price of $399.95 discounted by 35%.


When you're working with amounts of money, remember to include $ signs as well as 2 decimal places (cents) in your answer.

W.E. 26

2 (a) A clothing store determines the selling price of T-shirts by applying a 53% mark-up. Calculate the selling price of a T-shirt bought at a cost price of $15.

(b) A mark-up of 74% applies to all goods at Shelley's Shoe Shop. Find the selling price of a pair of shoes with a cost price of $25.

(c) Glen adds a mark-up of 60% to the cost price of the wooden toys he makes before selling them at his market stall. Find the selling price of a toy that cost $12.50 to make.

(d) A computer games store adds a mark-up of 78.5% to the cost price of all of its console games. Find the selling price of a game that cost the store $30.25.

W.E. 27

3 (a) Deb's Designer Dresses has a policy of marking up the cost price of all of its dresses by 82%. If the selling price of a dress is $200, calculate the cost price.

(b) Amanda's music store offers a storewide discount of 15% on all second-hand vinyl records. Calculate the original marked price of a second-hand vinyl record that has been discounted to $49.30.

(c) Calculate the cost price of a pair of sunglasses that has been marked up by 76% to a selling price of $69.95.

(d) Calculate the original marked price of a cricket bat discounted by 30% and on sale for $169.

W.E. 28

4 (a) Rosa buys a ticket to a concert for $65, but is unable to go and sells it to a friend for $39.

(i) Calculate the loss Rosa made on the ticket.

(ii) Express the loss as a percentage of the cost price, rounded to the nearest per cent.

(b) Travis bought an old car for $1500 and then resold it for $1950 without spending any money on it.

(i) How much profit did Travis make on the car?

(ii) Express Travis' profit as a percentage of the selling price, rounded to the nearest per cent.

5 For each of the following, express the profit or loss as a percentage of the cost price, rounded to the nearest whole number.

(a) A painting purchased for $750 and sold for $2000.

(b) A book purchased for $45 and sold for $20.

(c) Sporting goods purchased for $320 and sold for $520.

(d) Jewellery purchased for $980 and sold for $1750.

6 An electrical goods retailer buys a washing machine for $580 and applies a standard mark-up of 70% to all of its goods. The selling price for one of its washing machines is:

A $174 **B** $406 **C** $754 **D** $986

7 A magazine is advertised for 20% off the marked price and you are charged $7.60. The original marked price of the magazine is closest to:

A $6.10 **B** $7.80 **C** $9.10 **D** $9.50

8 A bookstore applies a mark-up of 92% to the cost price of all of its books. The cost price of a book with a selling price of $35 is closest to:

A $2.80 **B** $32.20 **C** $18.23 **D** $67.20

9 Calculate **(i)** the dollar value of the discount and **(ii)** the discount sale price of the following items. (Round your answers to the nearest cent.)

(a) A games console originally priced at $379 on sale at a 10% discount.

(b) A suitcase with an original price of $199 discounted by 40%.

(c) A $20 T-shirt discounted by 30%.

(d) A discount of 25% on a $99.95 pair of sunglasses.

10 Sanjay invests $30 000 in the sharemarket only to see his shares drop in value to $22 000.

(a) Find the amount of Sanjay's loss.

(b) Find his percentage loss by expressing the loss as a percentage of the original amount invested.

Understanding

11 In Australia, a Goods and Services Tax (GST) of 10% is added to the price of all products and services purchased by consumers (except for some basic grocery items).

Calculate **(i)** the amount of GST and **(ii)** the new, GST-inclusive price of the following items.

(a) a $560 bike **(b)** a $64 pair of jeans

(c) a $19 calculator **(d)** a $2640 television

12 A professional designer charges $90 per hour plus 10% GST. She makes an invoice for 8 hours of work.

(a) What is the total amount charged excluding GST?

(b) What is the total GST charged?

(c) What is the total amount charged including GST?

13 A builder calculates a job will cost $2450 before adding GST. Calculate:

(a) the 10% GST to be added

(b) the final amount (including GST).

14 Mitchell buys an antique desk for $150. He spends $45 to repair it and sells it for $320.

(a) How much profit did Mitchell make, taking the cost of the repair into account?

(b) Calculate the value of the profit as a percentage of the total cost. (Write your answer rounded to 1 decimal place.)

15 It costs a furniture company $180 to make a sofa. The cost of transport to the company's stores is $7.50 per sofa. The company then sells the sofas for $350.

(a) How much profit does the company make on the sofas, taking transport and production costs into account?

(b) Express the profit as a percentage of the selling price to 2 decimal places.

16 Lee bought an old motorbike for $2200 and spent $250 repairing it. He then sold it for $3000.

(a) Calculate how much profit Lee earned, considering the money spent repairing it.

(b) What was the profit as a percentage of the total cost?

17 A designer store applies a 200% mark-up to all of the clothing it sells. What would be the selling price of a gown that has a cost price of $150?

18 A store purchases a bulk order of 72 photo frames for $324.

(a) Calculate the profit made if the photo frames are sold individually for $7 each.

(b) Calculate the profit as a percentage of the cost price to 2 decimal places.

19 Most of the time, the marked prices you see on goods in shops have the GST already added. Find the original, pre-GST price of:

(a) a book with a price tag of $32.95

(b) a microwave oven with a price tag of $279

(c) a boat priced at $29999

(d) a T-shirt priced at $39.95.

20 A store takes delivery of a large container with 1000 packets of soap which cost the store $2750. The store policy is to add a 70% mark-up and then add the GST. Find the selling price of each packet of soap.

Reasoning

21 When out shopping, you may have noticed that some percentages are used more commonly than others, such as 10%, 20%, 25% and 50%. Explain why these percentage values are used, rather than, for example, 17%, 36% or 42%.

22 Julia makes and sells jars of jam to a local cafe, which then sells the jam to customers. Julia adds a 70% mark-up to the cost of making the jam before selling it to the cafe. The cafe adds its own mark-up of 85% to set the price at which the jam is sold to cafe customers. Express your answers to the following to the nearest 5 cents.

(a) If a jar of jam costs Julia $1.35 to make, at what price does she sell each jar to the cafe?

(b) At what price does the cafe sell the jam to its customers?

(c) Express the cafe's selling price of the jam as a percentage of Julia's initial cost price, correct to the nearest whole number.

23 **(a)** For the following situations, calculate the profit made as a percentage of the cost price, rounding your answers to 2 decimal places where necessary.

(i) CP = \$45, SP = \$54

(ii) CP = \$360, SP = \$504

(iii) CP = \$4200, SP = \$4350

(iv) CP = \$480, SP = \$600

(b) Repeat the above question, this time calculating the profit as a percentage of the selling price.

(c) For each of the situations, compare your answers to the first two parts. Which gives a higher percentage—calculating profit using the cost price or using the selling price? Explain your answer.

24 Billy Bob's Bargain Basement discounts the selling price of its toys by 15%. If the store's mark-up on toys is 50% of the cost price, find:

(a) the discounted sale price of a toy that has a cost price of \$45

(b) the percentage profit made by the store on the sale of the discounted toy

(c) the decrease in percentage profit made by discounting the sale price of the toy.

25 Tariq sees an item in a bargain store initially marked down by 50% with a sign that says 'take an extra 20% off'. If, after these two discounts, the price Tariq paid was \$11.20, what was the original price of the item?

26 Martha's favourite clothing shop is offering a 'buy two, get the third half price' offer on T-shirts. The shop next door is offering 20% off the same brand of T-shirts. If the regular retail price of the T-shirts in both stores is \$19.95, which store is the cheaper place for Martha to buy three T-shirts? How much would she save, compared to the other store?

27 A number of people together own an earth-moving business. Each person owns a certain percentage of the business. Profits are divided according to the percentage of the money they invested. For example, someone investing 7% of the operating capital (the amount of money needed to run the business) receives 7% of the profits. The operating capital is $675 000.

(a) Calculate the amount contributed by a person who has invested 7% of the operating capital.

(b) If the company makes $150 000 yearly profit, how much does the person who invested 7% receive?

(c) If the company continued to make this profit each year, how many years would it be before the person has received back their investment?

28 It is very useful to be able to calculate some common percentages in your head. Knowing their fraction equivalents is very helpful.

(a) Write the fraction equivalents of:

(i) 25%　**(ii)** 10%　**(iii)** 20%　**(iv)** $33\frac{1}{3}\%$

(b) Chantal knows that $50\% = \frac{1}{2}$. When she sees a '50% off' sale sign, she halves the original price to find the new price. Use your answers to the first part to help Chantal extend her method so she can calculate 25%, 10%, 20% and $33\frac{1}{3}\%$ off.

Open-ended

29 A sports store is having a sale on all sporting equipment. If a tennis racquet is normally priced at $299, write three different, common discount percentages and find the sale price for each.

30 Many stores, such as department stores, advertise sale items using percentage discounts, as seen in this catalogue:

40% off

DOWNIA 'White Collection' 85/15 white duck down and feather quilts

e.g. quilt QS*
was $349
save $140
now
$209

40% off

TONTINE LUXE 'All Seasons' 80/20 white duck down and feather quilts

e.g. quilt QS*
was $279.95
save $112
now
$167.95

exclusive to Myer

At other times, stores advertise sale items using dollar discounts as seen in this catalogue:

now
$45

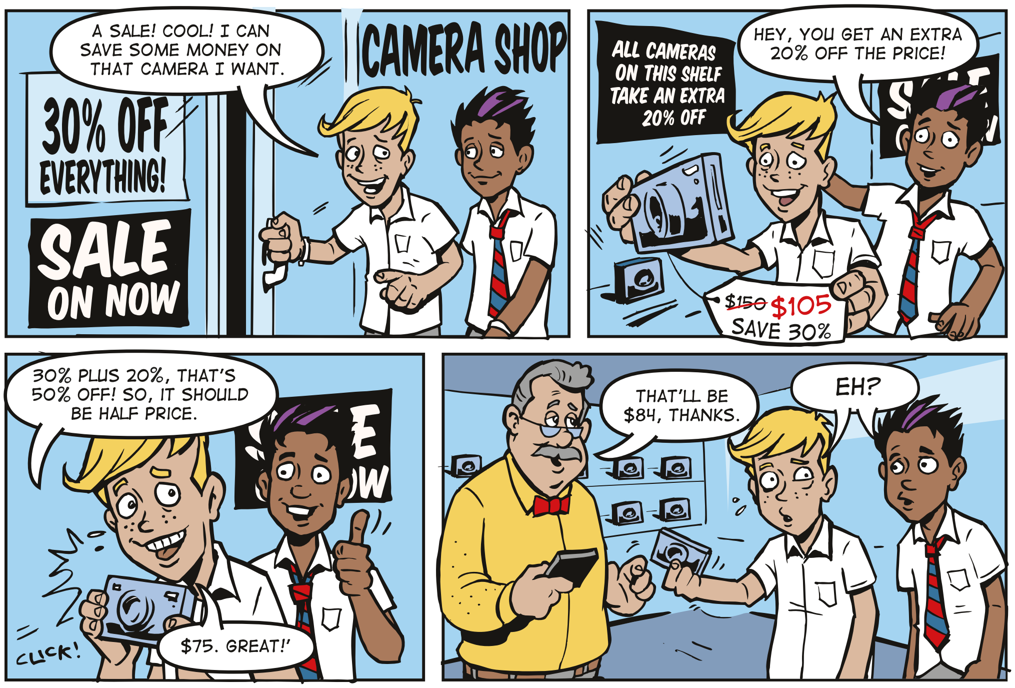
DOWNIA 'Double Down Surround' pillow
was $69.95 save $24.95

now
$54.95

DENTONS 'Medi Rest' foam pillow
was $94.95 save $40

Why do you think this is? Try to give a couple of reasons for your answer.

31



The boys think the camera should only cost $75. The shop assistant says the camera now costs $84. What mistake have the boys made? Explain how the shop assistant calculated the new price of $84.

Problem solving

The plummeting price

Frida puts a price tag of $45 on a superhero costume for sale in her fancy-dress store. Unfortunately, there is little interest in the costume, so she reduces the price to $27. As the costume still does not sell at the reduced price, Frida reduces the price again, by the same percentage as the first reduction.

1 What is the new reduced price of the costume?

2 How many reductions at the same rate would reduce the price to less than one-quarter of the original price?

3 If the initial price was different, would it take the same number of reductions to do this?

Strategy options

- Break problem into manageable parts.
- Test all possible combinations.

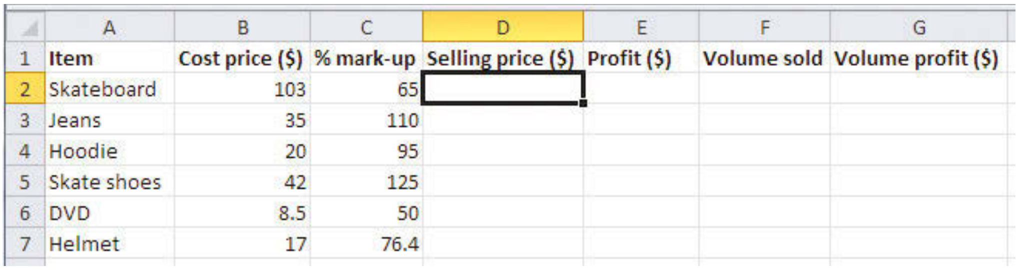
Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Sam's Skate Shop

A spreadsheet can help businesses keep track of their sales and profits. In this task you will set up a spreadsheet for Sam, who has just opened a shop selling skateboards, protective gear, skate clothing and skating DVDs.

1 Open a new spreadsheet and enter the following details.



	A	B	C	D	E	F	G
1	Item	Cost price ($)	% mark-up	Selling price ($)	Profit ($)	Volume sold	Volume profit ($)
2	Skateboard	103	65				
3	Jeans	35	110				
4	Hoodie	20	95				
5	Skate shoes	42	125				
6	DVD	8.5	50				
7	Helmet	17	76.4				

2 To calculate the selling price, enter the formula **=B2*(1+(C2/100))** into cell **D2**. Then select the cells from **D2** down to **D7** and **Fill Down** to copy this formula down the column. Explain how this formula calculates the selling price. You should refer to the part of the formula that is in brackets.

3 As you are working with money, format the cells in column **D** to show 2 decimal places. Highlight the column by selecting **D**. Then select **Format Cells…**, select the **Number** tab and in the **Number** category enter the **Decimal places** as 2.

4 Enter a formula in cell **E2** to calculate the dollar profit. Make sure your formula begins with the **=** symbol. Copy this formula down the column, and format the cells as in step **3**. If you have entered the formula correctly, your table should look like this.

	A	B	C	D	E	F	G
1	Item	Cost price ($)	% mark-up	Selling price ($)	Profit ($)	Volume sold	Volume profit ($)
2	Skateboard	103	65	169.95	66.95		
3	Jeans	35	110	73.50	38.50		
4	Hoodie	20	95	39.00	19.00		
5	Skate shoes	42	125	94.50	52.50		
6	DVD	8.5	50	12.75	4.25		
7	Helmet	17	76.4	29.99	12.99		

Do not work out the profits and enter them manually, as you will be relying on the use of formulas later on.

5 In one typical week, Sam sold 9 skateboards, 27 pairs of jeans, 42 hoodies, 38 pairs of skate shoes, 86 DVDs and 15 helmets. Enter this information into your spreadsheet under the **Volume sold** heading. To calculate the profit made on the volume of each item sold, enter a formula into cell **G2** to multiply the profit made on one item by the volume of that item sold. Format the cells as in step **3**. If you have entered the formula correctly, your table should look like this.

	A	B	C	D	E	F	G
1	Item	Cost price ($)	% mark-up	Selling price ($)	Profit ($)	Volume sold	Volume profit ($)
2	Skateboard	103	65	169.95	66.95	9	602.55
3	Jeans	35	110	73.50	38.50	27	1039.50
4	Hoodie	20	95	39.00	19.00	42	798.00
5	Skate shoes	42	125	94.50	52.50	38	1995.00
6	DVD	8.5	50	12.75	4.25	86	365.50
7	Helmet	17	76.4	29.99	12.99	15	194.82

6 Find Sam's total profit on all the items sold for the week. Select all of the cells in column **G** of the table, plus the empty cell below them. Find the $\sum$ symbol (on the far right of the toolbar under the **Home** tab) and from the down arrow next to it select **Sum**. The total profit will appear at the bottom of the column.

Answer the following questions by changing the numbers on your spreadsheet and seeing what happens.

7 Sam must make a profit of at least $4300 a week to pay the rent on his shop, to pay his electricity, water and phone bills, and to pay Will, his casual employee who works on weekends. By changing the numbers in the **Volume Sold** column (but keeping them in roughly the same proportion as in **5**), find a minimum number of each item that Sam must sell to make the minimum profit.

8 Sam sells out of DVDs and will not receive more of them to sell until next week. How many extra skateboards would he need to sell to make up for the loss of DVD profits, assuming he sells the same number of the other items as in **5**?

9 Sam believes that the mark-up on his jeans and skate shoes is too high, and that he would sell more at 30% less mark-up. Write the selling price, the profit, the volume sold and the volume profit for the jeans and skate shoes from the original spreadsheet in **5** before using your spreadsheet to answer the following questions.

(a) Find the new selling prices of the jeans and skate shoes after lowering their mark-up by 30%.

(b) How many extra pairs of jeans and shoes will Sam have to sell at this price to make the same volume profit as before?

Taking it further

10 Insert some extra columns and a formula into your spreadsheet so that you can enter the number of items in stock at the beginning of the week, and calculate the number left after the 'Volume Sold' data is entered.

11 Insert a **Discount %** column and a **Sale Price** column after the **Selling Price** column. Enter a number of common discounts (such as 10% or 25%) into the **Discount %** column. Enter formulas in these columns and calculate the sale price.

(a) If Sam has a '20% off everything' sale, calculate a minimum combination of items that need to be sold to get the minimum profit of $4300.

(b) Explain why a 20% discount on the selling price of an item is not the same as 20% less mark-up.

Maths 4 Real

Relative Wealth

Are you wealthy?

Many people aspire to be wealthy. Many of their choices, such as which career path to follow, are based on whether they think it will help them to become wealthier. You might not think of yourself or your family as particularly wealthy. Here are some facts to consider.

In 2016, there were 7.42 billion people living in the world. Of these:

- 1 in 5 lived on less than US$1.25 a day.
- nearly 1 in 2 lived on less than US$2.50 a day.
- nearly 4 in 5 lived on less than US$10 a day.

1 (a) Write 1 in 5 as:
   (i) a fraction   (ii) a percentage.
   (b) Repeat the above question for 1 in 2 and 4 in 5.
   (c) Why do you think these statistics are reported using the phrase '1 in …' instead of using a fraction or a percentage?

2 In 2014 the median income of an Australian household was $998 per week. How many dollars a day is this?

3 In 2016 the minimum wage (required by law) that an Australian adult should be paid was $656.90 a week. How many dollars a day is this?

4 In 2016 a single person on the Commonwealth government's unemployment allowance was entitled to $263.80 per week. How many dollars a day is this?

5 Look again at the facts about how much money people live on, then at your answers to the previous three questions. If the countries of the world were listed from richest to poorest, where do you think Australia might be positioned? Give some reasons for your answer.

6 If Australia is one of the world's wealthier countries, how is it that there are people in Australian society who are poor, or living in poverty?

7 In 2007, the poorest 40% of the world's population received 5% of global income. The richest 20% received 75% of global income. (Global income can be thought of as the money received by all of the people working around the world.)
   (a) Rewrite the above statement using fractions.
   (b) Which statement do you think has more impact—the one with fractions or percentages?

The Millennium Development Goals

In 2000, the world's leaders came together for a meeting of the United Nations (UN).They agreed to work towards a set of goals that would make the world a better and fairer place, known as the Millennium Development Goals. The first Goal was to get rid of extreme poverty and hunger, or, more specifically, to take the number of people living on less than US$1 per day in 1990, and halve it by 2015. Other Goals included stopping the spread of diseases such as malaria and AIDS, and ensuring that all children could complete a primary education.

To achieve the goal of reducing poverty, the UN suggested that the wealthier countries should give 0.7% of their national income in aid to developing countries. However, the average amount given by countries in 2006 was only 0.33%.

8 Write: (a) 0.7% and (b) 0.33% as fractions.

Despite this, the UN reported that in 2005 the number of people in extreme poverty was 1.4 billion, down from 1.8 billion in 1990. By 2010, the number of people living in extreme poverty had fallen to half the number in 1990, five years ahead of the scheduled goal.

9 (a) Write the reduction in the number of people in extreme poverty as a percentage of the 1990 figure.
   (b) How many more millions of people were moved out of extreme poverty by 2010 to achieve the Millennium Development Goal?

It's not just about money

Poverty is not just about a lack of money or being unable to buy things such as food, fuel, clothes or tools. People in poverty often do not get access to healthcare (such as immunisations) or medical treatment, so their years of a healthy life are reduced. Children in poverty are often sent out to work instead of school. They miss out on an education that would provide them with the literacy and numeracy skills they need to get a better paying job.

10 Suggest some ways in which aid money from richer nations could be used to help move people out of extreme poverty.

How can I help?

There are many, many organisations devoted to eliminating the poverty and inequality that exists in the world, including: the Red Cross, World Vision, UNICEF and Oxfam. As well as directly helping those in need, these organisations work to raise other people's awareness of the problems that poverty causes. They suggest ways people can help, either by donating money, sponsoring a child or buying products made or grown by people in poverty. Other organisations, such as Doctors Without Borders, and Engineers Without Borders, are groups of professional people who work as volunteers in poorer countries, in their areas of expertise.

Research

Find out about the other Millennium Development Goals, and the progress that has been made towards achieving them.

Find out more about the organisations devoted to helping people in poverty, and make a list of practical ways in which you can support them. Share this list with your class.

Find out some other facts about poverty and inequality, either within Australia or within other countries. Create an online advertisement or campaign to raise awareness of the issues, and to tell people what they can do to help.

Challenge 2

1 What is the average (the mean) of $\frac{2}{3}$, 0.7 and 55%?

A $\frac{23}{36}$ **B** 175% **C** $1\frac{11}{12}$ **D** 5.75

2 Jasmine bought a large tub of yoghurt. On the first day she ate $\frac{1}{4}$ of the yoghurt. The next day she ate $\frac{1}{3}$ of what was left, and on the third day she ate $\frac{2}{3}$ of what was left. What fraction of the yoghurt remained after the third day?

3 What fraction is half-way between $\frac{1}{4}$ and $\frac{1}{2}$?

4 A piece of fabric cost Michael $25. He purchases another piece of similar fabric that is twice as long and twice as wide as the first piece. How much should this second piece cost him?

5 Which of the following is the best value deal for a box of chocolates that normally sells for $5.04?

A save $1.70 **B** cost price of $3.20, +5%

C discount of $33\frac{1}{3}$% **D** buy 2, get 1 free

6 Write an expression using four 8s and any of the operation symbols (+, –, ×, ÷) that equals 65.

7 Find the sum of $\frac{1}{2} + \frac{0.1}{2} + \frac{1}{0.2}$, expressing your answer in decimal form.

8 Michael picks three *different* digits from the set {1, 2, 3, 4, 5} and forms a mixed number by placing the digits in the spaces $\square\frac{\square}{\square}$. The fractional part of the mixed number must be less than 1. What is the difference between the largest and the smallest mixed numbers that can be formed?

A $4\frac{4}{15}$ **B** $4\frac{3}{10}$ **C** $4\frac{7}{20}$ **D** $4\frac{3}{5}$

9 Suppose that x^* means $\frac{1}{x}$ so that for example, $5^* = \frac{1}{5}$. Which of the following statements are true?

A $2^* + 4^* = 6^*$ **B** $3^* \times 5^* = 15^*$ **C** $7^* - 3^* = 4^*$ **D** $12^* \div 3^* = 4^*$

10 The sum of seven consecutive integers is always:

A odd **B** a multiple of 7 **C** a multiple of 3 **D** even

11 Find the following sum without using a calculator.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512} + \frac{1}{1024} + \frac{1}{2048} + \frac{1}{4096} + \frac{1}{8192}$$

12 One liquid contains 22.5% water, another liquid contains 27% water. A glass is filled with 5 parts of the first liquid and 7 parts of the second liquid. What percentage of the liquid in the glass is water?

Chapter review 2

Maths literacy

- cost price (CP)
- discount
- exact decimal form
- Goods and Services Tax (GST)
- irrational number
- loss
- mark-up
- non-terminating decimal
- per cent
- percentage
- profit
- recurring decimal
- sale price
- selling price (SP)
- surd
- terminating decimal

Copy and complete the following using the words and phrases from this list where appropriate. A word or phrase may be used more than once.

1 The word ___________ literally means per hundred or 'out of a hundred'.

2 A decimal such as $2.6\dot{3}$ is written in ___________.

3 A decimal that has an infinite number of digits after the decimal point is called a ___________.

4 To convert a decimal to a ___________, you multiply by 100 and add the percentage symbol.

5 The amount added to the cost price of goods before selling them is known as the ___________.

6 A number with a defined number of digits after the decimal point is called a ___________.

7 An ___________ is a non-terminating decimal that cannot be written as a fraction.

8 A ___________ is applied to goods by decreasing the marked price by a given percentage.

9 An irrational number written in the form of a square root is called a ___________.

10 To make a profit, the ___________ must be higher than the ___________.

Fluency

Equipment required: calculator

1 Write the following decimals as fractions or mixed numbers in simplest form. **2.1**

(a) 0.58 (b) 0.0124 (c) 2.605

2 Write the following in ascending order. **2.3**

(a) $\frac{2}{9}$, $-\frac{3}{10}$, -0.283, 0.26, $\frac{3}{4}$, $\sqrt{7}$ (b) $-\frac{1}{3}$, $\frac{7}{10}$, $\sqrt{2}$, 0.22, -0.26, 0.253

3 Write the following mixed numbers as decimals. **2.1**

(a) $28\frac{5}{8}$ (b) $15\frac{3}{40}$ (c) $53\frac{261}{320}$

4 How many 60 cm lengths of wood can be cut from a 2.5 m long plank? What length of wood is left over? **2.1**

5 Use a calculator to express the following numbers in decimal form, then classify them as terminating decimals, recurring decimals, or irrational numbers. **2.2**

(a) $53 \div 4$ (b) $\sqrt{85.2}$ (c) $\frac{11}{9}$ (d) $\sqrt{125}$ (e) $\sqrt{30.25}$

6 The fraction $\frac{19}{11}$, written in exact decimal form, is: **2.2**

A $\overline{1.72}$　　**B** $1.\overline{72}$　　**C** $1.\dot{7}2$　　**D** $1.7\dot{2}$

7 Write the following recurring decimals in fraction form. **2.2**

(a) $0.\dot{2}$　　**(b)** $0.3\dot{2}$　　**(c)** $0.\dot{3}\dot{2}$

8 Calculate the following, giving your answers as mixed numbers in simplest form. **2.3**

(a) $-\frac{1}{2} + \frac{3}{4}$　　**(b)** $\frac{8}{9} - 1\frac{1}{6}$　　**(c)** $-\frac{3}{10} \times -\frac{5}{6}$　　**(d)** $-5 \div \frac{1}{3}$

9 Calculate the following. **2.3**

(a) $-3.4 + 2.8$　　**(b)** $8.6 - 10.2$　　**(c)** 4.2×-6.1　　**(d)** $20.4 \div -5$

10 Approximately what percentage of the shape is shaded? **2.4**

A 20%　　**B** 34%　　**C** 67%　　**D** 75%

11 Convert the following to percentages. Express any remainders as fractions. **2.5**

(a) $\frac{23}{50}$　　**(b)** $\frac{6}{5}$　　**(c)** $2\frac{4}{5}$　　**(d)** $\frac{25}{40}$

12 Convert the following to percentages. **2.5**

(a) 0.6　　**(b)** 0.35　　**(c)** 1.12　　**(d)** 0.03

13 Convert the following to fractions in simplest form. **2.6**

(a) 65%　　**(b)** 154%　　**(c)** $5\frac{5}{6}\%$　　**(d)** 4.7%

14 Convert the following to decimals. **2.6**

(a) 80%　　**(b)** 7%　　**(c)** 2.09%　　**(d)** $6\frac{1}{4}\%$

15 Express the first amount as a percentage of the second, giving answers rounded to 2 decimal places if necessary. **2.7**

(a) 27, 30　　**(b)** 35, 16　　**(c)** 4 cm, 2 m　　**(d)** 30c, $4.50

16 Find the following, giving answers rounded to 2 decimal places if necessary. **2.8**

(a) 35% of 84 m　　**(b)** $4\frac{2}{5}\%$ of 2000 kg　　**(c)** $7\frac{1}{3}\%$ of 4500 L

(d) 14% of $48　　**(e)** 0.59% of $4000　　**(f)** 9.5% of $90

17 **(a)** Increase $450 by **(i)** 35% and **(ii)** 67.2%. **2.9**

(b) Decrease $19.95 by **(i)** 20% and **(ii)** 7.5%.

Round answers to 2 decimal places where necessary.

18 Calculate: **2.10**

(a) the sale price of a $90 tennis racquet discounted by 15%

(b) the selling price of a $950 ring that will be marked up by 80%

(c) the original marked price of a pair of sunglasses discounted by 30% to $62.30

(d) the cost price of a dress that has been marked up by 85% to $130.

19 Kayla bought a concert ticket for \$145, but was unable to go, so she sold it for \$120. Calculate Kayla's profit or loss as a percentage of what she paid for the ticket, correct to 2 decimal places. **2.10**

Understanding

20 The number of pre-booked tickets for a 1-day cricket match is 62% of the 75 242 tickets available. How many tickets were pre-booked? **2.8**

A 4665 **B** 28 592 **C** 46 650 **D** 121 358

21 Michelle made deposits of \$210, \$25, \$45.50 and \$66.75 into her bank account during one month and withdrawals of \$35.75, \$56.90, \$214 and \$102.50 during the same period. Use a negative number to show how much her balance decreased during this period. **2.3**

22 Franco has a 5 kg bag of nuts that he needs to use to fill smaller 375 g bags. **2.3**

(a) How many bags can he fill?

(b) What mass of nuts will he have left over?

23 The value of a property increases by 17.5% one year. By what fraction did the property's value increase? **2.6**

24 Approximately 85% of an iceberg lies under water. The average mass of an iceberg is 150 000 tonnes. What mass of the iceberg sits above water? **2.7**

25 A geography book states that 0.7092 of the Earth's surface is covered by ocean. Write this value as a percentage. **2.5**

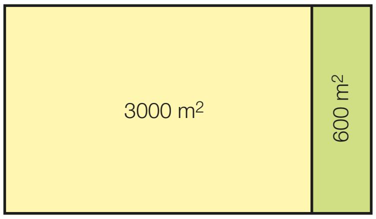
26 The total area of Australian desert is 42.556% as large as the Sahara Desert in Africa, which has an area of 9 000 000 square kilometres. Find the area of Australian desert to the nearest square kilometre. **2.8**

27 Tom bought an old car for \$3500. He spent \$2700 to repair and restore it, then sold it for \$9400. **2.10**

(a) Calculate the profit Tom made, taking all costs into account.

(b) Express the profit as a percentage of the selling price, rounding your answer to the nearest per cent.

28 Tye buys a neighbouring 600 m^2 property to add to his 3000 m^2 farm. By what percentage has the area of his farm increased? **2.7**



29 Gao currently earns \$600 a week. If he receives a pay rise of $2\frac{1}{3}\%$, what will his new weekly pay be? **2.8**

30 Arala runs a newsagency. She buys boxes of pens, then sells the pens individually. A box of 60 pens costs her \$17. **2.9**

(a) To work out what she will sell them for individually, Arala needs to know what she paid per pen. Calculate the cost of each pen, to the nearest cent.

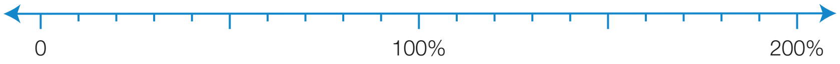
(b) Arala wants to sell each pen for 25% more than she paid for them. Calculate the price Arala should sell the pens for.

31 Nazim had 62 friends on his favourite new social media site. One month later, he had 97 friends. Write this increase as a percentage of his initial number of friends. Round your answer to 1 decimal place. **2.7**

32 The minimum overnight temperatures for one week at Mt Hotham were -2 °C, -3 °C, -1 °C, 2 °C, -3 °C, -2 °C, 0 °C. Find the mean minimum overnight temperature for the week, rounding your answer to 1 decimal place. (The mean is found by adding all values, then dividing the result by the number of values.) **2.1**

33 Copy the number line below and place the following percentages on it: **2.5**

14%, 140%, 4%, 104%, $\frac{1}{4}$%



Reasoning

34 A certain cheese contains $7\frac{1}{2}$% fat. How much fat is in a 600 gram piece? **2.8**

35 **(a)** The decimal value of $\frac{4}{7}$ is closest to: **2.1**

A 0.57 **B** 0.571 **C** 0.58 **D** 0.6

(b) The decimal approximation of $\sqrt{23}$ is closest to:

A 4.7 **B** 4.796 **C** 4.8 **D** 5

36 The combined capacity of all of the dams that supply water to Melbourne is 1 810 500 ML (where 1 ML = 1 000 000 L, called 'one megalitre'). In May 2010, the dams were 33.3% full of water, higher than the 27.2% recorded on the same day the previous year. **2.8**

(a) How many ML of water was in the dams in May 2010?

(b) How many more ML was present in May 2010 than in May 2009?

(c) How many more ML would need to flow into the dams to achieve a capacity of 35% full?

37 A games store applies a mark-up of 80% to the cost price of all of its products, then adds 10% GST. **2.10**

(a) Find the final price of a game that had a cost price of $33.25, to the nearest 5 cents.

(b) The GST charged by the store must be given to the government. What is the amount of GST in the final price of the game?

(c) What profit does the store make on the game, allowing for GST?

(d) The store now discounts the price of the game by 25%. What is the new price of the game?

(e) What is the amount of GST in this new price?

(f) What profit does the store make on the game now, allowing for GST?

38 A packet of 12 chocolate frogs is on sale for $4.75. **2.10**

(a) How much does each frog in the packet cost? Write your answer in exact decimal form.

(b) Write the cost of each frog rounded to the nearest cent.

(c) How much extra will you pay if you buy 12 frogs individually at this price?

(d) Where does this 'extra' amount come from?

Numeracy practice 2

Non-calculator

1 Jye cuts a pizza into 12 equal slices. He eats 75% of the slices.

How many slices of pizza are left? ___________

2 A ticket costs $75. A fee of 10% is added to the price. Which calculation will give the new price?

A 75 + 10 **B** 75 + 0.1 **C** 75 × 0.1 **D** 75 × 1.1

3 Vicky bought a house for $400 000. At the end of 1 year, its value had increased by 7%. At the end of the next year, the value had increased by another 5%. What was Vicky's house worth after 2 years?

4 A skateboard is on sale for 20% off the marked price. If the marked price is $140, how much is the sale price?

A $102 **B** $112 **C** $120 **D** $160

Calculator allowed

5 At midday during winter in Canberra the temperature was 11.7 °C. At 6 pm, it had dropped by 5.4 °C. By midnight, the temperature was 15.2 °C cooler than at midday. What was the temperature at midnight?

A 6.3 °C **B** -3.5 °C **C** -35 °C **D** -9.8 °C

6 There were only 13 students in Jo's class on Wednesday. The other 7 were absent.

What percentage of Jo's class was absent?

A 7% **B** 35% **C** 54% **D** 65%

7 Justin bought a multi-bag of chocolate bars. The bag contained 15 chocolate bars and it cost $3.70. What is the price of each chocolate bar, correct to the nearest cent?

8 Maryam scored 28 goals from 40 shots during netball training. What percentage is this?

Mixed review A

Equipment required: calculator

Fluency

1 Calculate the following. **1.1–1.3**

(a) $7 - (-9)$ **(b)** $-3 + (-10)$ **(c)** $-14 - (+8)$ **(d)** $-21 + 9$

(e) -4×-9 **(f)** 7×-3 **(g)** $-60 \div 15$ **(h)** $-\frac{48}{8}$

2 Write the following decimals as fractions in simplest form. **2.1**

(a) 0.36 **(b)** 0.05 **(c)** 0.75 **(d)** 0.002

3 A TV has been bought for a cost price of $560 by a retailer, who will add a mark-up of 78% before selling it in the store. Calculate: **2.10**

(a) the value of the mark-up

(b) the selling price after the mark-up has been applied.

4 Evaluate the following by simplifying first where possible. **1.5**

(a) $2^3 \times 5^2$ **(b)** $3^4 \times 10^3$ **(c)** $-4^2 \times 5$

(d) $-3 \times (-2)^4$ **(e)** $(5^2 \times 3^3) \div (5 \times 3)$ **(f)** $(7^4 \times 8^4) \div (7^2 \times 8^2)$

(g) $\frac{4^2 \times 6}{2^3}$ **(h)** $\frac{8^2 \times 2^3}{4^2}$ **(i)** $\frac{3^3 \times 4^4}{2^6}$

5 Convert the following fractions to percentages. (Round answers to 2 decimal places if necessary.) **2.5**

(a) $\frac{17}{20}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{5}{8}$ **(d)** $\frac{20}{35}$

6 Write the following fractions as recurring decimals using the correct notation. **2.2**

(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{5}{9}$

7 Write the following percentages **(i)** as decimals and **(ii)** as fractions in simplest form. **2.6**

(a) 70% **(b)** 45% **(c)** 120% **(d)** 8%

8 Find the following percentages. (Round answers to 2 decimal places if necessary.) **2.8**

(a) 20% of $240 **(b)** 65% of 700 kg **(c)** 42% of 8.5 m

(d) 3% of 16 seconds **(e)** 78% of $19.40 **(f)** 110% of 400 m

9 For each of the following, draw a number line from -3 to 3 and indicate the positions of the numbers (you may need to estimate the positions of some). **2.3**

(a) $-\frac{1}{2}, -\frac{4}{3}, 1\frac{1}{2}, \frac{2}{3}$ **(b)** 0.7, -1.2, -0.5, 2.1, 0.02

10 Which of the following is equivalent to 10^4? **1.5**

A 40 **B** 100 000 **C** 10 000 **D** 400

11 Evaluate: **1.4**

(a) $-9 + 2 \times -5 + 7$ **(b)** $-4 + (-3)^2 + 10 \div 5$

12 A pair of sunglasses that has a marked price of \$74 will be discounted by 40%. Calculate: **2.10**

(a) the value of the discount

(b) the sale price after the discount has been applied.

13 Simplify the following, then evaluate. **1.5, 1.6**

(a) $6^2 \times 5^2 \times 5^4 \times 6^1 \times 5^3 \times 6^2$ **(b)** $(7^2 \times 10^1 \times 10^3 \times 7^6) \div (7^2 \times 5^2 \times 5 \times 2^2)$

(c) $(10^2 \times 10^1 \times 10^0) \div (2^0 \times 5^0 \times 5 \times 2)$ **(d)** $\dfrac{3^0 \times 4 \times 5^2}{2^0} \times \dfrac{(2^3 \times 5^2)^0}{(12 \times 10 \times 5)^0}$

Understanding

14 Dhanya is studying a group of 30 insects in her science lab. After 1 month, the number of insects had doubled, giving 30×2 (60) insects. After 2 months, the number had doubled again, giving $30 \times 2 \times 2$, or 30×2^2 insects. After 3 months, there were $30 \times 2 \times 2 \times 2$, or 30×2^3 insects. The group continues to double every month. **1.5**

(a) Show, using a number in index form, how many insects there will be after **(i)** 6 months and **(ii)** 12 months.

(b) Evaluate each of your answers to give an actual number of insects after 6 and 12 months.

15 An ice cube tray filled with water at room temperature (20 °C) is placed in a freezer, where it takes 3 hours to freeze solid (0 °C). What is the average hourly change in temperature? Give your answer as a negative fraction in simplest form. **2.1**

16 Show, by evaluating, that $(2 \times 3)^2 = 2^2 \times 3^2$. **1.5, 1.6**

17 Ella works in a clothing shop. Her manager has asked her to place the following items on sale at a 20% discount. Find the sale price for the following. **2.10**

(a) Jeans, \$65 **(b)** Jacket, \$89 **(c)** T-shirts, \$24 **(d)** Socks, \$8.50

Reasoning

18 Casey is keeping track of how many goals she scores in her netball matches. She wants to steadily improve her accuracy. **2.8**

Match 1: 10 goals out of 14 shots
Match 2: 15 goals out of 20 shots
Match 3: 13 goals out of 18 shots
Match 4: 12 goals out of 16 shots

Convert each of Casey's results to a percentage, then use them to rank the matches from her best performance to her worst. Round the answers to 1 decimal place where necessary. Would you say that Casey is steadily improving her accuracy?

19 **(a)** As a decimal, the value of $\frac{8}{17}$ is closest to: **2.2, 2.4**

A 0.4 **B** 0.47 **C** 0.48 **D** 0.5

(b) As a decimal, the value of $\sqrt{78}$ is closest to:

A 8 **B** 8.8 **C** 8.9 **D** 9

20 Ali has performed the following calculation in a maths assignment. **1.5, 1.6**

$2^4 + 3^3 + 5^2$
$= 8 + 9 + 10$
$= 27$

His teacher has marked this as incorrect. Explain to Ali what he has done wrong, and what he needs to remember when working with numbers in index form.

3

Algebra

3

The humble soap bubble may be much more important than you think.

Bubbles always take the shape of a sphere, because a sphere has the smallest possible surface area for its volume and uses the least energy to be formed. The size of a bubble depends on the pressure inside and outside the bubble, and the surface tension. You can use algebra to show how the size of the bubble is related to these factors:

$$P_{\text{in}} - P_{\text{out}} = \frac{4T}{r}$$

where

P_{in} = the pressure inside the bubble

P_{out} = the pressure outside the bubble

T = the surface tension

r = the radius of the bubble.

The study of bubbles is important to understand 'the bends', a dangerous medical condition that can happen when undersea divers swim up to the surface too quickly, causing nitrogen bubbles to form in body tissues.

Forum

What ingredients are needed to make a good bubble solution? Why are they necessary?

The universe is described by some physicists as existing like a bubble in a 'frothy sea' of bubbles. What do you think this means? How could you find out more about this?

Why learn this?

Algebra gives a symbolic way to describe patterns that can exist in the real world. For example, algebraic formulas can describe where and when birds migrate, or how a bridge supports the weight of cars on it, or how oceans move so that tides can be predicted. Modelling relationships using algebra like this lets you identify patterns and make predictions from the patterns. This chapter will help you continue to develop your skills in using and manipulating algebraic expressions.

After completing this chapter you will be able to:

- recognise variables and form algebraic expressions
- substitute values into algebraic expressions and formulas and then evaluate
- solve simple problems using algebra and formulas
- recognise like terms and terms that are not like
- simplify and manipulate algebraic expressions using addition, subtraction, multiplication and division
- expand algebraic expressions using the distributive law
- factorise algebraic expressions by identifying the highest common factor (HCF).

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Write algebraic expressions to represent each of the following.

(a) ten more than x **(b)** m divided by three
(c) two lots of l **(d)** y less than twenty

2 Calculate:

(a) $6 + 3 \times 4$ **(b)** $2 \times -3 - 6 \times 5$
(c) $5 \times (14 - 10) \div 10$ **(d)** $5 - 11 + (16 - 15) \times 8 + 2$

3 Substitute each of the values of m into the formula $l = 2m + 1$ to find l.

(a) $m = 4$ **(b)** $m = -2$

4 Answer true (T) or false (F) for each of the following statements.

(a) $y = 3x - 4$ is an expression **(b)** $3m$ is a term
(c) $2x + 3 - 4a$ is an equation **(d)** $7y - 10$ is an expression

5 Express each number as a product of its prime factors.

(a) 18 **(b)** 28 **(c)** 42 **(d)** 660

6 Write each of the following in index form.

(a) $3 \times 3 \times 3 \times 3$ **(b)** six cubed
(c) fourteen squared **(d)** five to the power of seven

7 Write these numbers in expanded form, then evaluate.

(a) 9^2 **(b)** $(-3)^4$ **(c)** $10^4 - 2^5$

8 Write the highest common factor (HCF) of each of the following.

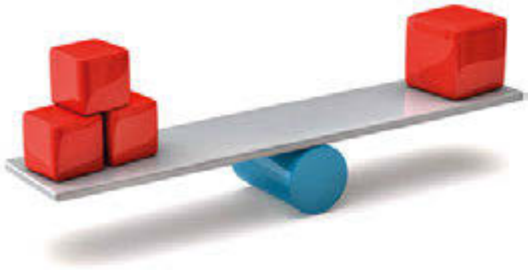
(a) 6 and 10 **(b)** 15 and 55 **(c)** 10, 30 and 42

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Which is larger?

In this activity, you will consider which might be larger: $4m$, or $m + 4$? Is your answer true for all values of m?



Variables and expressions

3.1

The language of algebra

To use algebra to solve problems, you need to understand the language of algebra. In Year 7 you were introduced to many new words and conventions for this.

A **variable** is an unknown amount in an algebraic expression or equation. Variables are represented by a letter or a symbol, which is called a **pronumeral**. For example, the amount of water used by a Year 8 student in a day is a variable, and it can be represented by a symbol such as a, x, θ or β.

A **term** is one 'item' in an equation or expression. It can be:

- a number or a variable by itself: -7, x
- a number multiplied by a variable: $12c$, $5k$
- different variables multiplied together: mn, $-abc$
- a variable multiplied by itself a number of times: a^3, $-b^2$
- the product of a number and several variables: $-3xy$, $4a^2b$.

An **expression** is formed when terms are added or subtracted: $3x + 4y$.

An **equation** joins two expressions with an equals symbol: $3x + 6 = 2x - 4$.

A **coefficient** of a variable is a number that multiplies the variable. The coefficient is written in front of the variable. It may include a negative sign. For example, 8 is the coefficient of $8x$, while -6 is the coefficient of $-6y$.

Note that the coefficient of a single variable is 1. For example, the coefficient of x is 1.

A **constant** is a term with no variable. It is a number written by itself. 6 is the constant in the expression $3x + 6$.

Worked example 1

W.E. 1

For $6a + 3b - 4 = 2ab - c + 7$

(a) identify whether this is an equation or an expression

(b) write the coefficient of b

(c) write all the terms in the equation or expression

(d) list the variables

(e) write any constants.

Thinking	Working
(a) Look for an equals symbol. If there is one, it is an equation. If there is not, it is an expression.	(a) It is an equation because it has an equals symbol.
(b) Look for the number in front of the b. This is the coefficient of b.	(b) 3 is the coefficient of b.

(c) Look for parts of the equation separated by addition, subtraction or equals. Include the sign that comes before the term.	(c) $6a$, $3b$, -4, $2ab$, $-c$ and 7
(d) Look for the letters or symbols in the equation.	(d) a, b and c are variables.
(e) Look for numbers that are by themselves.	(e) -4 and 7 are constants.

Algebraic conventions

A convention is a rule for how maths is written, so that everyone understands what is being stated.

- A multiplication symbol is not needed between variables and numbers or other variables ($5 \times b = 5b$, $a \times d \times f = adf$).
- Division can be shown as a fraction ($3a \div b = \frac{3a}{b}$).
- Indices show variables being multiplied by themselves ($c \times c = c^2$).
- Numbers are written before a variable ($a \times 4 = 4a$).
- Variables are written in alphabetical order ($y \times z \times x = xyz$).

Worked example 2

W.E. 2

Rewrite the following using algebraic conventions.

$t \times m \times 6 \times m \div 5$

Thinking	Working
1 Replace the division symbol with a fraction bar.	$\frac{t \times m \times 6 \times m}{5}$
2 Rewrite with the numbers in the multiplication at the start.	$= \frac{6 \times t \times m \times m}{5}$
3 Remove multiplication symbols.	$= \frac{6tmm}{5}$
4 Write in alphabetical order.	$= \frac{6mmt}{5}$
5 Use indices to show variables being multiplied by themselves.	$= \frac{6m^2t}{5}$
Note that these steps can be done in any order.	

Worked example 3

W.E. 3

Pete has b blue pens and r red pens. Sue has three times as many blue pens as Pete. She has two fewer red ones than Pete.

(a) Write an expression for the number of pens Pete has altogether.

(b) If Pete has 16 pens to start with, write an equation to show this information.

(c) Write an expression for the number of blue pens Sue has.

(d) Write an expression for the number of red pens Sue has.

(e) If Sue has 28 pens altogether, write an equation to show this information.

(f) If Pete loses four of his pens, write an expression in terms of b and r to show how many pens Pete has now.

Thinking	Working
(a) Identify the variables (b and r) that need to be used and the operation needed to be done on them (+).	(a) $b + r$
(b) Equate the expression in the first part to the information given.	(b) $b + r = 16$
(c) Decide what needs to be done to the first variable ($3 \times b$).	(c) $3b$
(d) Decide what needs to be done to the second variable ($r - 2$).	(d) $r - 2$
(e) Use the information from the previous two parts to form an expression ($3b + r - 2$) and equate to the information given.	(e) $3b + r - 2 = 28$
(f) Identify what operation needs to be done on the expression in the first part and write a new expression.	(f) $b + r - 4$

3.1 Variables and expressions

Navigator

Answers p. 631

1, 2, 3, 4, 5, 6 (column 1), 7, 8 (a–b), 9 (a–b), 12, 18	1, 2, 3, 6 (column 2), 7, 8 (c–d), 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	1, 2, 3, 6 (column 2), 7, 8 (c–d), 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Fluency

W.E. 1

1 For each of the following:

(i) identify whether it is an expression or an equation

(ii) write the coefficient of b

(iii) write all the terms in the expression or equation

(iv) list the variables

(v) write any constants.

(a) $x + 3b - 4$ **(b)** $-2a + 4b - c = 5$ **(c)** $7a - b = 3c + a$

(d) $3d + 4f - 3bd - 3 + 7b$ **(e)** $5g - 3r + 5rf = 7 - 3b$ **(f)** $b + f - 3bf + 8$

W.E. 2

2 Rewrite the following using algebraic conventions.

(a) $4 \times d \times c$ **(b)** $2 \times t \div 7$ **(c)** $k \times g \times 4 \times k$ **(d)** $6 \div (f \times g)$

W.E. 3

3 Nerida has w watches and r rings. Donna has two times as many watches and two fewer rings than Nerida.

(a) Write an expression for the number of watches and rings Nerida has altogether.

(b) If Nerida has eight watches and rings to start with, write an equation to show this information.

(c) Write an expression for the number of watches Donna has.

(d) Write an expression for the number of rings Donna has.

(e) If Donna has 14 watches and rings altogether, write an equation to show this information.

(f) If Nerida loses two of her rings, write an expression in terms of w and r to show how many watches and rings Nerida has now.

4 **(a)** The coefficient of x in $6y + 7xy + 5x$ is:

A 5 **B** 7 **C** $5x$ **D** $6y$

(b) The coefficient of xy in $5 + 8xy - 2y + 4x$ is:

A 2 **B** 4 **C** 8 **D** $8xy$

(c) The coefficient of a in $5b + 2ab + 6$ is:

A 0 **B** 2 **C** 5 **D** 6

(d) The coefficient of x in $4xy - 6y + x + 8$ is:

A 0 **B** 1 **C** 4 **D** 6

5 **(a)** The constant in the expression $\frac{1}{2}x^2y - 3 + 5xy$ is:

A -3 **B** $\frac{1}{2}$ **C** 3 **D** 5

(b) The constant in $3ef + 7efg + 12 + 11ef + 4e$ is:

A 2 **B** 7 **C** 11 **D** 12

6 Write the following without division and multiplication symbols or brackets. Do not simplify your expressions.

(a) $x \div 6$

(b) $h \div 9$

(c) $6 \times a \div 11$

(d) $15 \div (3 \times r)$

(e) $21 \div (12 \times v)$

(f) $4 \times s \div 19$

(g) $8 \div x - u \div 6$

(h) $h \div 5 + 4 \div i$

(i) $c \times u \div 5 + 9 \times y$

(j) $q \div (7 \times c) - g \times h \div 4$

(k) $v \times z \div 6 - 8 \div (f \times s)$

(l) $3 \div (t \times r) + 6 \times w \div (y \times z)$

(m) $4 \times h \times b \div (2 \times r)$

(n) $6 \times c \times a \div (5 \times e \times u)$

7 There are a apples and p pears in a fruit dish. There are 5 apples and 4 pears in a second dish. The total number of pieces of fruit is:

A $a + p$ **B** $5a + 4p$ **C** $a + 5a + p + 4p$ **D** $a + p + 9$

8 Kim is x years old. Write an algebraic expression for each of the following.

(a) Jane's age, which is four times Kim's age

(b) Irene's age, which is 5 more than Kim's age

(c) Hayley's age, which is 10 less than twice Kim's age

(d) the present age of Kim's aunt, if her aunt is four times as old as Kim will be 2 years from now.

9 Andrew has y number of pencils in his pencil case.

(a) Kayla has $y + 7$ pencils in her pencil case. What does this mean?

(b) Simon has $2y$ pencils in his pencil case. What does this mean?

(c) Suppose Emily has $2y - 2$ pencils in her pencil case. Does she have more or fewer pencils than Simon?

(d) If Andrew took out 3 pencils from his pencil case, how many would he have left?

(e) If Andrew gave the 3 pencils to Kayla, how many would she now have?

(f) If Simon took 3 pencils out of his pencil case, how many would he have left?

(g) If Simon gave the 3 pencils to Emily, how many would she now have?

10 Suppose the number of cups you have is m and the number of glasses you have is n.

(a) How many cups and glasses do you have altogether?

(b) If you broke 2 glasses and threw them away, how many glasses would you now have?

(c) How many cups and glasses would you now have altogether?

(d) If you then bought another set of 8 cups, how many cups would you have?

(e) How many cups and glasses would you now have altogether?

11 Suppose x is the number of people in Sweats Sports Store at 3:00 pm on a certain Friday afternoon.

(a) If there are twice as many people at 3:30 pm, write this in terms of x.

(b) If the number of people at 4:00 pm is $3x$, what does this mean?

(c) If five people leave the store between 4:00 pm and 4:15 pm, and no one comes in, write an expression for the number of people in the store at 4:15 pm.

(d) If nine people come into the store and two leave between 4:15 pm and 4:30 pm, write an expression for the number of people in the store at 4:30 pm.

Understanding

12 In a glass jar there are some different coloured jelly beans, including g green jelly beans.

(a) There are twice as many red jelly beans as green ones. Write an expression for the number of red jelly beans in terms of g.

(b) The number of blue jelly beans is five less than the number of green ones. Write an expression for the number of blue jelly beans in terms of g.

(c) The number of black jelly beans is four more than the number of green ones. Write an expression for the number of black jelly beans in terms of g.

13 On a supermarket shelf there are different types of chocolate. There are m types of milk chocolate.

(a) The number of soft-centred types is one more than twice the number of milk chocolate types. Write an expression for the number of soft-centred types in terms of m.

(b) There are three more types of chocolate-nut than soft-centred types. Write an expression for the number of chocolate-nut types in terms of m.

(c) The number of types of white chocolate is two more than one-third of the amount of types of milk chocolate. Write an expression for the number of white chocolate types in terms of m.

14 Lauren takes h hours and m minutes to finish her mathematics project. Tim takes twice as long as Lauren to complete his project. The time Tim takes in minutes is:

A $2h + 2m$ **B** $2h + 120m$ **C** $60h + 2m$ **D** $120h + 2m$

Reasoning

15 Ryan is playing Crazy Maze on his smartphone. He starts with p points. Ryan plays the game for 45 minutes and every 5 minutes he notes how many points he has. The number of points he has is shown in the table below.

(a) At what time did Ryan have the most points?

(b) If Ryan stopped playing when he had the most points, then how many more points is this than when he started?

(c) At what time did Ryan have the fewest points?

(d) If Ryan stopped playing when he had the fewest points, then how many points would he have lost compared to when he started?

(e) At what times should Ryan have stopped playing if he wanted to 'break even' (not gain and not lose any points)?

Time	Number of points
10:00 pm	p
10:05 pm	$p + 2$
10:10 pm	$p + 7$
10:15 pm	$p + 1$
10:20 pm	$p + 4$
10:25 pm	p
10:30 pm	$p - 4$
10:35 pm	p
10:40 pm	$p - 10$
10:45 pm	$p - 2$

16 Suppose e stands for the number of people expected to be at a party. The table on the right shows how many people were at the party during the evening.

Write a paragraph describing what happened during the evening.

Time	Number of people
6:00 pm	$e - 20$
6:10 pm	$e - 18$
6:15 pm	$e - 15$
6:20 pm	$e - 11$
6:25 pm	$e - 12$
6:30 pm	$e - 6$
7:00 pm	$e - 3$
8:00 pm	$e - 13$
9:30 pm	$e - 18$
11:00 pm	$e - 20$

Open-ended

17 Ella has been answering some questions involving algebra, with k representing the variable in the expression.

(a) What could k represent?

(b) In this case, what would $2k$ represent?

(c) What would $k + 4$ represent?

(d) What would $\frac{k}{2} - 1$ represent?

18 **(a)** Write a question of your own that is similar to Question **3**.

(b) Answer the question you have written.

Problem solving

Tricky algebra

Instructions

1 Pick a number (keep it secret).

2 Multiply your number by 3.

3 Add 6.

4 Take away the number you started with.

5 Divide by 2.

6 Again, take away the number you started with.

You have finished with 3.

Use algebra to show why this always works.

Strategy options

- Work backwards.
- Make a model.

Hint: Let your number be 'x'.

Create your own algebra trick and give it to a friend for them to work out.

3.2

Substitution for variables

Variables in an expression have unknown values. If you want to know the value for an expression when the variables have certain values, you can replace the variables in the expression with those values. This process is called **substitution**.

In the expression $3x + 2y$, x and y are variables. Where $x = 5$ and $y = 1$, the expression can be evaluated by substituting 5 for x and 1 for y:

$$\begin{aligned} 3x + 2y &= 3 \times (5) + 2 \times (1) \\ &= 17 \end{aligned}$$

Because the variables can vary, different values for the variable will give different values for the expression. Where $x = -3$ and $y = 7$, the expression can be evaluated by substituting -3 for x and 7 for y:

$$\begin{aligned} 3x + 2y &= 3 \times (-3) + 2 \times (7) \\ &= 5 \end{aligned}$$

Worked example 4

W.E. 4

Evaluate the expression $2b - a$ for $a = 3$ and $b = 10$.

Thinking	Working
1 Substitute the values for the variables, taking care to insert multiplication symbols where necessary.	$a = 3$ and $b = 10$ $2b - a = 2 \times 10 - 3$
2 Evaluate.	$= 20 - 3$ $= 17$

When substituting negative numbers, it is useful to place brackets around the number and its sign.

Worked example 5

W.E. 5

Evaluate the expression $2x^2 - 3y$ for $x = -3$ and $y = -1$.

Thinking	Working
1 Substitute the values for the variables, taking care to place brackets around the negative numbers and to insert multiplication symbols where necessary.	$x = -3$ and $y = -1$ $2x^2 - 3y = 2 \times (-3)^2 - 3 \times (-1)$
2 Evaluate.	$= 2 \times 9 + 3$ $= 18 + 3$ $= 21$

You can use these skills to complete a table of values.

Worked example 6

W.E. 6

Use the rule $y = 3x$ to complete the following table.

x	-1	0	2
y			

Thinking	Working
1 Substitute each of the x-values into the rule to find the y-value.	$x = -1,\ y = 3x$ $= 3 \times (-1)$ $= -3$ $x = 0,\ y = 3x$ $= 3 \times (0)$ $= 0$ $x = 2,\ y = 3x$ $= 3 \times (2)$ $= 6$
2 Complete the table using the y-values calculated.	

x	-1	0	2
y	-3	0	6

3.2 Substitution for variables

Navigator

1 (a–o), 2 (column 1), 3 (a–i), 5 (column 1), 8, 10, 14 (a), 15	1 (columns 1–2), 2 (columns 2–3), 3 (columns 1–2), 4 (columns 1–2), 5 (columns 2–3), 6, 7, 8, 10, 11, 12, 14, 15	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4 (columns 2–3), 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14

Answers p. 632

Fluency

W.E. 4

1 Evaluate the following expressions for $a = 3$ and $b = 6$.

(a) $a + 2b$ **(b)** $3a + 2b$ **(c)** $2a + 5b$

(d) ab **(e)** $3ab$ **(f)** $10ab - a$

(g) $8b - 16a$ **(h)** $10ab - b$ **(i)** $6b - ab$

(j) $12a^2$ **(k)** $2b^3$ **(l)** $4a^3$

(m) $a^2 - b^3$ **(n)** $3a^3 - 2b^2$ **(o)** $2a^5 - 2b^3$

(p) $\frac{2a^2}{b}$ **(q)** $\frac{a^3}{b^2}$ **(r)** $\frac{144a^2}{3b^3}$

(s) $a^2 - b - \frac{a}{3}$ **(t)** $a^3 + 2b^2 - \frac{b}{2}$ **(u)** $3b^2 + \frac{a^5}{2} - 5$

W.E. 5

2 Evaluate the following expressions for $x = -2$ and $y = -5$.

(a) $x + y$ **(b)** $2x + 12y$ **(c)** $10x + 5y$

(d) $y - 4x$ **(e)** $2y - 3x$ **(f)** $2x - 3y$

(g) $-5y - 3 + x$ **(h)** $8 - 6y + 2x$ **(i)** $7y - 4 - 4x$

(j) $y^2 - x^2$ **(k)** $x^3 + y^3$ **(l)** $-5y^2$

W.E. 6

3 Use the rules given to complete the following tables.

(a) $y = x - 2$

x	-2	0	-4	-8
y				

(b) $y = x - 4$

x	-5	-3	-2	-7
y				

(c) $y = x - 9$

x	-7	-1	-5	-9
y				

(d) $y = 4x - 1$

x	1	2	3	4
y				

(e) $y = 2x - 4$

x	-2	0	1	2
y				

(f) $y = 3x + 14$

x	-4	-7	-20	11
y				

(g) $y = -6x$

x	-3	-2	-1	-5
y				

(h) $y = -5x - 3$

x	-1	0	3	5
y				

(i) $y = 3 - 4x$

x	-3	-1	2	4
y				

(j) $y = \frac{x}{2}$

x	8	6	12	20
y				

(k) $y = \frac{x}{3} - 2$

x	15	9	12	30
y				

(l) $y = \frac{x}{4} - 1$

x	12	-4	8	-24
y				

(m) $y = x^2$

x	0	1	2	3
y				

(n) $y = 2x^3 - 1$

x	-2	0	2	4
y				

(o) $y = x^2 + 3x + 1$

x	0	2	4	8
y				

4 Evaluate these expressions for $d = -2$ and $e = 5$.

(a) $-3de - 6$ **(b)** $-5de + 100$ **(c)** $-8de + 15$

(d) $\frac{10}{e}$ **(e)** $\frac{40}{d}$ **(f)** $\frac{e}{d}$

(g) $\frac{2}{d} - \frac{5}{e}$ **(h)** $\frac{12}{d} - \frac{30}{e}$ **(i)** $\frac{15}{e} - \frac{18}{d}$

(j) $e^3 - 3d^2$ **(k)** $e^2 - 3d^2$ **(l)** $2e^3 - 3d^2$

5 Evaluate the following expressions for $m = 3$ and $n = -2$.

(a) $2(3m - n)$ **(b)** $4n(m - 3)$ **(c)** $m(6 + 3n)$

(d) $6m(8 - n)$ **(e)** $3m(n - 2)$ **(f)** $6n(3m - 4)$

(g) $\frac{m+n}{3}$ **(h)** $\frac{2m+n}{5}$ **(i)** $\frac{6m-n}{4}$

(j) $\frac{2m+n}{n}$ **(k)** $\frac{5(m+2n)}{5m}$ **(l)** $\frac{3(m-4n)}{4n}$

6 If $u = 6$ and $v = 8$ are substituted into the expression $\frac{v(3u+v-2)}{u}$, then the result is:

A 16 **B** 20 **C** 25 **D** 32

7 If the values $a = -1$, $b = 2$ and $c = -3$ are substituted into the expression $ab + bc$, then the result is:

A -8 **B** -4 **C** -2 **D** 4

8 If $d = 3$ and $e = -4$, then $de(d + e)$ equals:

A -12 **B** 84 **C** 12 **D** -84

Understanding

9 Boon buys d books at \$$y$ each.

(a) Write an expression to show how much Boon spent.

(b) Write an expression to show how much Boon received if he sold k books for \$$m$ each.

(c) Write an expression to calculate the profit Boon made.

(d) If $d = 7$, $y = 12$, $k = 5$ and $m = 21$, find the profit Boon made.

(e) One book is returned to Boon and the money refunded, because the book was missing the last page. Does Boon still make a profit? Explain why or why not.

10 Asha has a dollars, Mayoarini has $2b$ dollars and Danni has $3c$ dollars.

(a) What is the difference between Danni's amount of money and Asha's amount of money if Danni has more money than Asha?

(b) Mayoarini gives her money to Asha. What is the difference now between Danni's amount of money and Asha's amount of money if Asha now has more money than Danni?

(c) How much money is there altogether if $a = 3$, $b = 5$ and $c = 4$?

11 Michael has scored r runs in a cricket game. His friend Ravi has scored $2r - 4$ runs.

(a) What is the smallest value that r could be?

(b) Show that Ravi and Michael score the same number of runs if $r = 4$. Explain.

(c) If $r > 4$, did Ravi or Michael score more runs? Explain.

Reasoning

12 It takes $\frac{3N}{P+1}$ minutes to package biscuits, where N is the number of biscuits and P is the number of packing machines.

(a) How much time does it take to package 1500 biscuits if 99 machines are used?

(b) Forty-nine machines are to be used to package 2200 biscuits in 2 hours. Show why this is not possible.

(c) Five faulty machines have been repaired so that now there are 54 machines available. Show whether it is now possible to package the 2200 biscuits in 2 hours.

Open-ended

13 Find two different sets of values for the pronumerals M and N so that $\frac{3M+N}{2M-N}$ gives a whole number when M and N are substituted and the expression evaluated.

14 Lisa, Anna and Saji each choose an integer between -10 and +10.

Let a represent Lisa's number, b represent Anna's number and c represent Saji's number.

(a) Find one set of possible values for a, b and c if the sum of the three numbers is 0.

(b) Find three different sets of possible values for each girl's number if the product of the three numbers is -100 and a, b and c take different values.

15 Find three sets of whole number values for a and b so that $a^3 - 3b^2$ has a result that is less than 20.

Game

4 in a row

Equipment required: 1 die

Each player needs to make a copy of the game board below.

Game board

-4	-3	-2	-1	0
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Expressions		
$x^2 + 2$	x^3	$4x - 1$
$3x - 4$	$x + 3$	$5x - 3$
$5x + 1$	$2x - 1$	$2 - x$
$\frac{x^2}{2}$	$x + 10$	

How to win:
The winner is the first player to get 4 numbers in a row on their game board.

How to play:
Player 1 rolls the die and then has 30 seconds to choose an expression from the Expressions table. The player must substitute the number rolled into the expression, and circle the answer on their game board.

The other players check that Player 1 has correctly substituted the number. If the player has incorrectly circled a number, then the player to spot the mistake gets to circle the correct number.

It is then the next player's turn.

If the answer is a decimal, round to the nearest whole number.

Using formulas

3.3

Formulas (or ‘formulae’) are algebraic equations that are used in practical situations. Formulas can be used to calculate an unknown quantity from other known quantities.

You have already used formulas to find the perimeter and area of simple shapes. For example, a rectangle with length l and width w has perimeter P, given by the formula $P = 2(l + w)$ or $P = 2l + 2w$. The rectangle also has area A given by the formula $A = lw$. By substituting known values for l and w into these formulas, you can find the unknown values for the perimeter and area.

Worked example 7

W.E. 7

The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F). Use this formula to convert 29 degrees Celsius into degrees Fahrenheit.

Thinking	Working
1 Write the formula.	$F = \frac{9C}{5} + 32$
2 Substitute the value or values given ($C = 29$).	$= \frac{9(29)}{5} + 32$
3 Evaluate.	$= 84.2$ degrees Fahrenheit

3.3 Using formulas

Navigator

1, 2, 3, 4, 5, 6, 8, 10, 14 (a–b), 16	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17

Answers p. 632

Fluency

W.E. 7

1 The perimeter of a square can be found using the formula $P = 4s$, where P is the perimeter and s is the side length of the square. Use this formula to find the perimeter of a square with the following side lengths.

(a) 10 metres **(b)** 5.5 metres **(c)** $8\frac{1}{3}$ metres **(d)** 205 centimetres

2 A science centre sells tickets that cost \$4 for children and \$9 for adults. The total ticket money received by the centre is given by the formula $N = 4c + 9a$, where c is the number of child tickets sold and a is the number of adult tickets sold. Find N for:

(a) $c = 8$ and $a = 10$ **(b)** $c = 12$ and $a = 17$ **(c)** $c = 31$ and $a = 83$

3 The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius (C) to degrees Fahrenheit (F). Use this formula to convert the following values from degrees Celsius (°C) to degrees Fahrenheit (°F).

(a) 10 °C **(b)** 5 °C **(c)** 35 °C

4 The formula $C = \frac{5F - 160}{9}$ converts degrees Fahrenheit (F) to degrees Celsius (C). Find the value of C when F is:

(a) 32 °F **(b)** 41 °F **(c)** 50 °F

5 The distance travelled by a moving object travelling at a constant speed can be calculated using the formula $d = st$, where

d = distance travelled in metres
s = speed of object in metres per second
t = time in seconds that the object is in motion.

Calculate the value of d when:

(a) $s = 8, t = 4$ **(b)** $s = 4\frac{1}{2}, t = 1\frac{1}{2}$

6 An elevator of power w kilowatts can lift m kilograms up h metres in a time of t seconds, as given by the formula $t = \frac{mh}{25w}$. If $w = 20$, the time in seconds to lift a group of people of total mass 400 kilograms up to a height of 50 metres is:

A 20 seconds **B** 25 seconds **C** 35 seconds **D** 40 seconds

7 The amount C dollars to be paid for some hardware is found from the formula:
$C = H + 0.5L + 0.2S$

where H is the cost of a hammer
L is the number of long 50 mm nails bought
S is the number of short 10 mm nails bought.

When the hammer is $32 and there are 110 long nails and 25 short nails, the amount to be paid is:

A $55.40 **B** $92.00 **C** $66.50 **D** $42.50

Understanding

8 The cost of printing photographs is found by the formula $C = 2 + 0.2n$, where C is the cost of printing and n is the number of photos printed.

(a) Calculate how much Leila has to pay if she wants to print:

(i) 10 photos **(ii)** 30 photos **(iii)** 78 photos.

(b) Leila does not want to spend more than $10 printing photos. She calculates that to do this, she can only print a maximum of 52 photos. Is this correct?

9 The salespeople who work at Joe's Motors are paid according to the formula $P = 250 + 100c$, where P is the total pay for one week (in dollars) and c is the number of cars that they sell that week.

(a) Warren sells no cars one week. How much money does he earn?

(b) Caitlin sells 5 cars in the same week. How much does she earn?

(c) Claudio wants to earn more than $1000 one week. What is the minimum number of cars he has to sell?

10 The formula $d = 4.9t^2$ tells you how far a falling object travels, where d is the distance the object falls in metres and t is the time it falls in seconds.

(a) Suppose a football is dropped from Sydney Tower. What distance does it fall in 2 seconds?

(b) Divers on the high cliffs in Acapulco, Mexico, usually take 4 seconds to hit the water. Approximately how high are the cliffs?

(c) Wayne the skydiver always counts out 10 seconds just after he jumps out of the plane before he opens his parachute. How far does he fall in this time?

11 Temperatures can be converted from degrees Celsius (C) to degrees Fahrenheit (F) using a less accurate but simpler formula than the formula used in Question **3**. The formula is $F = 2C + 30$.

(a) Use this formula to convert 10 °C to degrees Fahrenheit.

(b) Compare your answer to the previous part with your answer to Question **3 (a)**.

(c) Use this formula to convert 5 °C to degrees Fahrenheit.

(d) Compare your answer to the previous part with your answer to Question **3 (b)**.

(e) Use this formula to convert 35 °C to degrees Fahrenheit.

(f) Compare your answer to the previous part with your answer to Question **3 (c)**.

(g) In most parts of Australia, daily minimum temperatures are rarely below 0 °C and daily maximum temperatures are rarely above 40 °C. For the temperature range from 0 °C to 40 °C, how close are the values from the approximate formula $F = 2C + 30$ to the values from the exact formula $F = \frac{9}{5}C + 32$?

12 The formula for finding how much water goes into a rectangular fish tank is $V = ldw$,

where V = the volume of water (in cubic centimetres)

l = the length (in cm) of the tank

d = the depth (in cm) of the water in the tank

w = the width (in cm) of the tank.

(a) How much water does a tank hold if its length is 80 cm, its width is 30 cm and its depth is 20 cm?

(b) How much water would you put in the tank if you want the water level to be 5 cm from the top?

(c) Chun is pumping the water from her smaller fish tank into her larger one. The smaller tank is 50 cm long, 20 cm wide and 20 cm deep. The larger tank is 100 cm long, 50 cm wide and 40 cm deep. After she's finished, how much *more* water will she need to add to the large tank to fill it completely?

Reasoning

13 Some scientists are trying find out if the fish population in a lake is decreasing. To estimate the number of fish in the lake, they catch some fish, count them, tag them and then release them back into the lake. On later visits, they catch more fish and count how many have tags and how many do not have tags. The total number of fish F in the lake can be estimated by $F = 2nt - 5$, where n is the number of fish caught in one visit and t is the number of caught fish that have tags.

- **(a)** **(i)** On one visit, the scientists catch 12 fish and 7 of these have tags. From this, what is the estimated total number of fish in the lake?
    - **(ii)** On the next visit, a month later, the scientists catch 11 fish and 8 of these are tagged. From this, what is the estimated total number of fish in the lake now?
- **(b)** Based on these two results, is the population of fish in the lake decreasing? Why?
- **(c)** On a later visit, the scientists catch 15 fish and estimate the total in the lake to be 145. How many of the caught fish had tags?

14 Banks use the formula $I = PRT$ to calculate simple interest, which is the amount of money earned by a simple interest investment.

I stands for the *simple interest* (the amount earned)

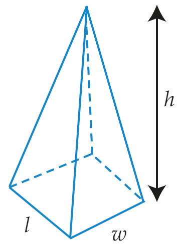
P stands for the *principal*, which is the amount of money invested at the start

R stands for the *rate* of interest, as a decimal value. Interest rates are usually given as a percentage per year ('p.a.').

T stands for the *time* that the money is invested, in years.

- **(a)** How much interest would you earn on \$100 invested for 1 year with a simple interest rate of 7% p.a.?
- **(b)** How much interest would you earn on \$1000 invested for 5 years with a simple interest rate of 12% p.a.?
- **(c)** Suppose you have \$1000 to invest for 5 years, and you have a choice between two banks: Aussie Bank and Matilda Bank. Aussie Bank offers a simple interest rate of 11% p.a., but charges a once-only fee of \$100. Matilda Bank's simple interest rate is 12% p.a., but it charges a once-only fee of \$160.
    - **(i)** Which bank should you invest your money in?
    - **(ii)** How much more would you earn with this bank?
    - **(iii)** Suppose you have \$2000 to invest for 5 years. Which bank should you invest your money in? How much more would you earn with this bank?

15 The volume V cubic metres of a pyramid of height h metres that has a rectangular base is given by the formula $V = \frac{lwh}{3}$, where l is the length and w is the width of the base.



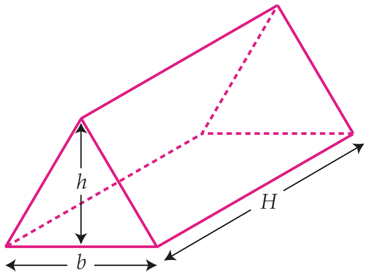
- **(a)** A pyramid is 30 metres long, 25 metres wide, and 60 metres high. What is its volume?
- **(b)** The pyramid is made from stone blocks with a volume of five cubic metres each. If the pyramid is completely solid, how many blocks were used to build the pyramid?

Open-ended

16 The formula $A = lw$ is used to find the area of a rectangle. Given that the area is 20 cm^2 and the length l is longer than the width w, find the possible dimensions of two different rectangles.

17 The volume of a triangular prism is given by $V = \frac{bhH}{2}$, where b is the base length, h is the perpendicular height of the triangle and H is the height of the prism.

Find two different sets of whole-number values for b, h and H if V is a whole number between 12 000 and 25 000, and each of b, h and H is at least 20.



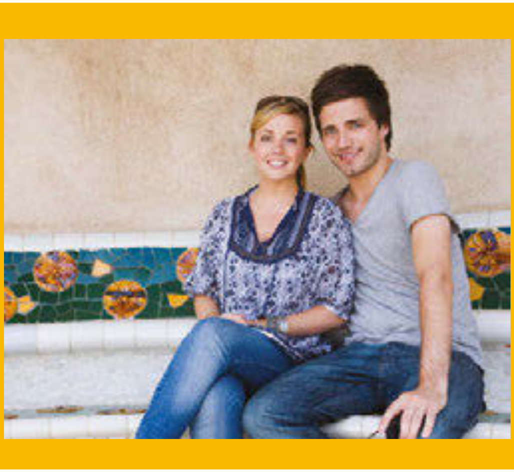
Puzzle

How old am I?

- Three years ago, my age was a prime number.
- This year, my age is a multiple of 4.
- I have a brother whose age twenty years ago was half my current age.
- The sum of the digits of my age is equal to one more than the product of the digits.

What is my age?

What is my brother's age?



Investigation

Take your medicine

Equipment required: calculator

It is very important that the amount of medicine needed by a sick person is calculated correctly. The correct amount is called the dose or dosage.

Medicine doses for children are usually smaller than those for adults. There are some formulas that help doctors to calculate the dose to give children. Here are two formulas that use a child's age to decide the dose they need:

Formula 1

$$X = \frac{A}{A + 12} \times N$$

where

X = the child's dosage in mL

A = the child's age

N = the usual adult's dosage in mL

Formula 2

$$Y = \frac{4 \times A + 20}{100} \times N$$

where

Y = the child's dosage in mL

A = the child's age

N = the usual adult's dosage in mL

A more accurate way of calculating a child's dosage uses the height and weight of the child to compare them to an average adult. This formula is more commonly used by health professionals.

Formula 3

$$Z = \frac{w^{0.425} \times h^{0.725}}{240} \times N$$

where

Z = the child's dosage in mL

w = the child's weight in kilograms

h = the child's height in centimetres

N = the usual adult's dosage in mL

The Big Question

According to these three dosage formulas, when should a child be given an adult dose of medicine?

Engage

1 Use Formula 1 to find what fraction of the usual adult's dose is needed by someone who is:

(a) 1 year old

(b) 7 years old

(c) 12 years old

(d) 18 years old.

Write your answers as decimal values (round to 2 decimal places).

2 Repeat the first question using Formula 2.

Explore

3 Use Formula 3 to find what fraction of the usual adult's dose is needed by someone who has:

- **(a)** weight 4 kg and height 50 cm
- **(b)** weight 10 kg and height 75 cm
- **(c)** weight 25 kg and height 115 cm.

Write your answers as decimal values (round to 2 decimal places).

You will need to use the ^ key on your calculator for this question.

4 Substitute the different ages, heights and weights given in the table below into the three formulas to find the child dosage amounts for each, with an adult dose of 50 mL each time.

	Age	Weight	Height
(a)	2	11 kg	88 cm
(b)	6	22 kg	106 cm
(c)	10	35 kg	132 cm
(d)	14	50 kg	156 cm
(e)	18	64 kg	170 cm

Strategy options

- Make a table.
- Look for a pattern.

Explain

5 Consider the dosages you calculated using Formula 1. Describe how the dosage changes as the child gets older. Is it a steady increase?

6 Repeat the previous question, this time considering the dosages you calculated using Formula 2.

7 Repeat Question **5**, this time considering the dosages you calculated using Formula 3.

8 Use your answers to the previous three questions to answer the Big Question.

9 Compare the differences in the dosages calculated by the three different formulas. Comment on any pattern or trend you can see.

Elaborate

10 Children are usually considered to be adults at age 18. Which formula best agrees with this?

11 Both Formulas 1 and 2 give answers that are not sensible when some values are used, so doctors and nurses should be aware of this.

The formulas have limitations: you need to limit the values to certain ages.

- **(a)** Describe the limitations of Formula 1. (If you are unsure, try calculating the dosage for 'children' aged 20, 30 and 40 years.)
- **(b)** Describe the limitations of Formula 2 (again, use dosages for a wide range of ages).
- **(c)** Why do you think Formula 3 is a more commonly used formula?

Evaluate

12 Do you think you need to know how to substitute correctly into these formulas? Why or why not?

13 Do you think it is important for doctors and nurses to be able to use formulas correctly? Why or why not?

14 Which formula would be best for you if you needed to take some medicine? Explain why.

Extend

15 Research some other formulas that are used to find a child's dose based on an adult's dose.

16 Based on your results above, create your own formula to quickly estimate a child's recommended dose based on an adult's recommended dose.

Maths 4 Real

Astronomical algebra

Throughout history, mathematicians and astronomers have tried to predict the alignment of objects in the night sky. The most spectacular of these alignments is a solar eclipse, and algebra is crucial for making accurate predictions of when these occur.

A solar eclipse is when the Moon crosses between the Sun and the Earth, and the Moon's shadow crosses the surface of the Earth. There will be 224 solar eclipses this century. The longest, at 6 minutes 39 seconds, occurred on 22 July 2009.

Around the year 300 IJÁ, the Greek astronomer Aristarchus of Samos used eclipses to calculate the sizes of the Sun and the Moon (relative to the size of the Earth), and their distances from the Earth. He was also one of the first people to come up with the theory that the planets move around the Sun, not around the Earth. In the year 62, another Greek astronomer named Heron accurately described the causes of a lunar eclipse.

In 1619, the German astronomer Johannes Kepler published an important formula that connected the average distance of the planets from the Sun (the radius of the orbit, R) to the amount of time for the planet to make one complete orbit (the period, T).

The formula stated:

$\frac{R^3}{T^2} = K$, where R = average radius
T = period
K = a constant number

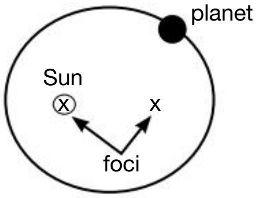
1 Earth's orbit can be said to have an average radius of 100 astronomical units and a period of 365.25 days. Using these values, find the value of K to 2 decimal places.

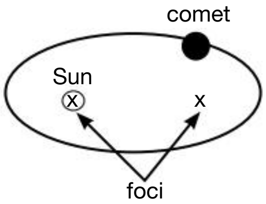
2 The orbit of Mars has an average radius of 152.4 astronomical units and a period of 686.98 days. Find the value of K to 2 decimal places.

3 What do you notice about your answers to the first two questions?

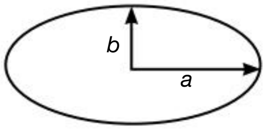
4 Using your value of K, calculate the period of Saturn using Kepler's modified formula $T^2 = \frac{R^3}{K}$ if the average radius of Saturn's orbit is 951 astronomical units.

An important discovery is that the paths of the planets around the Sun are shaped like an ellipse (a 'flattened' circle) rather than a perfect circle. The Sun is not at the centre of the ellipse. It is at a point called a focus of the ellipse (every ellipse has two foci). Comets also travel in elliptical paths around the Sun, but their ellipse is usually flatter than for a planet.





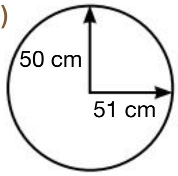
The *eccentricity* of an ellipse is a number between 0 and 1 that describes how flat the ellipse is. The flatter an ellipse, the greater its eccentricity.



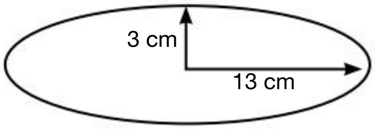
The formula for the eccentricity of an ellipse is $e = \frac{\sqrt{a^2 - b^2}}{a}$, where a and b are the minimum and maximum distances from the centre to the ellipse, as shown above. a is called the semi-major axis and b is the semi-minor axis.

5 Find the eccentricity of the following ellipses, correct to 1 decimal place.

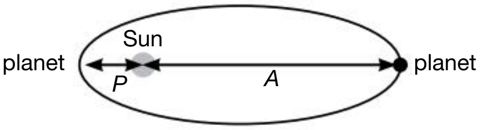
(a)



(b)



If you know the planet's closest distance to the Sun P and its furthest distance from the Sun A you can calculate the eccentricity of its orbit using the formula $e = \frac{A - P}{A + P}$.



6 Below are the closest and furthest distances from the Sun of four bodies in our solar system. Calculate the eccentricity of the orbit for each one, correct to 1 decimal place.

(a) Earth: $P \approx 147.1$ million km, $A \approx 152.1$ million km

(b) Mars: $P \approx 206.6$ million km, $A \approx 249.2$ million km

(c) Pluto: $P \approx 4435.0$ million km, $A \approx 7304.3$ million km

(d) Halley's Comet: $P \approx 89$ million km, $A \approx 5250$ million km

Research

Find out more about an astronomer such as Aristarchus of Samos, Heron, Kepler, Galileo or Sir Isaac Newton, and how they worked with algebra. Present your findings to the class.

3.4 Simplifying expressions

Like terms

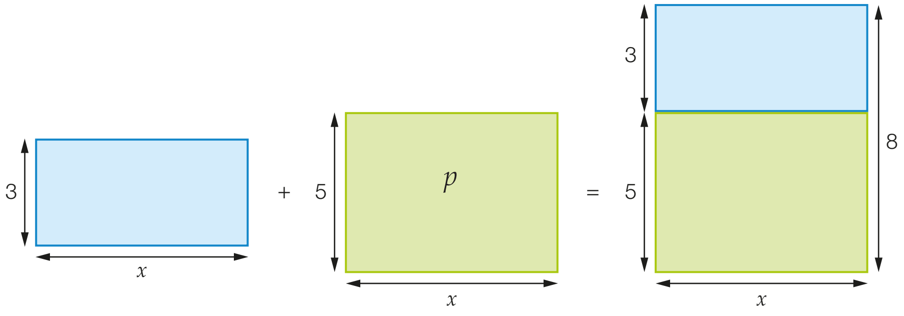
Algebraic terms that have exactly the same variables are called **like terms**.

You can simplify algebraic expressions by collecting like terms. To collect like terms, add their coefficients.

$3x$ and $5x$ are like terms and can be simplified by adding the coefficients of x.

$3x + 5x = 8x$

For example, see what happens when you add rectangles of the same length, x m, together. One has a width of 3 m and the other has a width of 5 m.



(Area = $3x$ m^2) + (Area = $5x$ m^2) = (Area = $8x$ m^2)

If $x = 4$:

$$\begin{aligned} \text{LHS} &= 3x + 5x \\ &= 3 \times 4 + 5 \times 4 \\ &= 12 + 20 \\ &= 32 \end{aligned} \qquad \begin{aligned} \text{RHS} &= 8 \times x \\ &= 8 \times 4 \\ &= 32 \end{aligned}$$

$3x$ and $4y$ are **not like terms** (often called 'non-like' terms or 'unlike' terms) and cannot be simplified.

Like terms can be simplified by adding their coefficients.

Non-like terms cannot be simplified.

Be careful when there are several variables. To help you identify like terms, always write terms with more than one variable in alphabetical order.

- $5ab$ and $2ac$ are not like terms, as $ab \neq ac$. ($\neq$ means 'is not equal to'.)
- $3xy$ and $5xyz$ are not like terms, as $xy \neq xyz$.
- $6x$ and $4x^2$ are not like terms, as $x \neq x^2$.

However:

- $3xy$ and $5yx$ are like terms, as $5yx = 5xy$.
- $11x^2$ and $5x^2$ are like terms.
- $3p^3q^4$ and $-5p^3q^4$ are like terms.

Worked example 8

W.E. 8

In each of the following, write whether the pair of terms is like or not like.

(a) $5t$ and $3t$ **(b)** $2xy$ and $2x$ **(c)** $6m^2n$ and $4mn^2$ **(d)** $3xy^2$ and $7y^2x$

Thinking	Working
(a) Do the terms have exactly the same variables? (Both terms have t as their only variable.)	(a) 5t and 3t are like terms.
(b) Do the terms have exactly the same variables? (The first term has $x \times y$, but the second term only has x.)	(b) 2xy and 2x are not like terms.
(c) Do the terms have exactly the same variables? (The first term has $m \times m \times n$, while the second term has $m \times n \times n$.)	(c) $6m^2n$ and $4mn^2$ are not like terms.
(d) Do the terms have exactly the same variables? (The first term has $x \times y \times y$ and the second term has $y \times y \times x$. So the terms have exactly the same variables, just in a different order.)	(d) $3xy^2$ and $7y^2x$ are like terms.

An expression can be simplified by collecting like terms.

Worked example 9

W.E. 9

Simplify the following expressions, if possible, then check your answers by substituting the value for the variables given in brackets.

(a) $5x + 3y - x + 7y$ ($x = 1$, $y = 2$) **(b)** $7x^2 + 4xy + 8yx - 2x$ ($x = 3$, $y = 2$)

Thinking	Working
(a) 1 Rearrange the expression so that all the like terms are grouped together. Remember to keep the sign in front of each term with that term.	(a) $5x + 3y - x + 7y$ $= 5x - x + 3y + 7y$
2 Collect like terms.	$= 4x + 10y$

	3	Substitute the given values into both expressions.	$5x + 3y - x + 7y$ $= 5 \times 1 + 3 \times 2 - 1 + 7 \times 2$ $= 5 + 6 - 1 + 14$ $= 24$ $4x + 10y$ $= 4 \times 1 + 10 \times 2$ $= 24$
	4	Check that both expressions give the same value.	As both expressions equal 24, the simplification is correct.
(b)	1	Rearrange the expression so that all the like terms are grouped together. Reorder variables in the terms in alphabetical order. Remember to keep the sign in front of each term with that term. Note that $7x^2$ and $-2x$ are not like terms.	(b) $7x^2 + 4xy + 8yx - 2x$ $= 7x^2 - 2x + 4xy + 8yx$ $= 7x^2 - 2x + 4xy + 8xy$
	2	Collect like terms.	$7x^2 - 2x + 12xy$
	3	Substitute the given values into both expressions.	$7x^2 + 4xy + 8yx - 2x$ $= 7 \times 3^2 + 4 \times 3 \times 2 + 8 \times 2 \times 3 - 2 \times 3$ $= 63 + 24 + 48 - 6$ $= 129$ $7x^2 - 2x + 12xy$ $= 7 \times 3^2 - 2 \times 3 + 12 \times 3 \times 2$ $= 63 - 6 + 72$ $= 129$
	4	Check that both expressions give the same value.	As both expressions equal 129, the simplification is correct.

3.4 Simplifying expressions

Navigator

Answers p. 633

1 (column 1), 2 (columns 1–2), 3 (column 1), 4, 5, 6, 7, 8 (a), 11, 12, 17	1 (column 2), 2 (columns 2–3), 3 (column 2), 5, 6, 7, 8, 9, 11, 12, 13, 15, 16, 17	1 (column 2), 3 (columns 2–3), 3 (column 2), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Fluency

W.E. 8

1 In each of the following, write whether the pair of terms is like or not like.

(a) $4w$ and $6w$

(b) $7u$ and $7w$

(c) $7t$ and $6i$

(d) $3e$ and $5e$

(e) $6d$ and $3de$

(f) yx and $33xy$

(g) $9xyz$ and $5xyz$

(h) $6x$ and $52xyz$

(i) $6xy$ and $8yx$

(j) $2xyz$ and $4zyx$

(k) $4x^2y$ and $7x^2y$

(l) $8xy^2$ and $12yx^2$

2 Simplify the following expressions, if possible.

(a) $6a + 11a$
(b) $15f + 6f$
(c) $7v - 2v$
(d) $5v - 8v$
(e) $-7w + 17w$
(f) $-12d + 15d$
(g) $7d + 8d + 9 - 3d$
(h) $9f + 5 - 2f + 5f$
(i) $-3r + 6u + 10r - 9u$
(j) $6m - 9n - 13m - 2n$
(k) $7f + 8fg - 4f - 6fg$
(l) $8ij + 14j - 2ij + 6j$

3 Simplify these expressions:

(a) $3hdw + 6hd + 7d - 4hd + 8 - 2d$
(b) $4x + 15 + 6x + 7xyz - 7 - 3xyz$
(c) $5x^2 - 6y - 13x^2 - 4y + 2$
(d) $7x^3 - y - 12x^3 - 5y + xy$
(e) $28ab + 40 + 30a - 10ab - 6a + 15b$
(f) $9s + 40st + 5 - 100st - 5s + 15$
(g) $-9b - 4b^3 + ab - b^3 + 8b - 6ab$
(h) $6a^3 - 8a^2 - 4a - 5a - 4a^3 - 61$

4 Which of these is a like term for $6y$?

A $6x$
B $14y$
C $6 + y$
D 6

5 Which of these is a like term for mn?

A $9n$
B mnp
C amn
D $8nm$

6 Which of these is a like term for $5xyz$?

A $22xy$
B $2xzj$
C $xy + z$
D $6zxy$

Understanding

W.E. 9

7 Simplify the following expressions, if possible, then check your answers by substituting the value for the variables given in brackets.

(a) $8t + 7t + 2d + 5d$ $(d = 1, t = 3)$
(b) $40w + 50w + 100v + 34v$ $(v = 2, w = 1)$
(c) $6a + 5b + 4a - 3b$ $(a = 2, b = 1)$
(d) $18f - 5f + 6g - 3g$ $(f = 1, g = 4)$
(e) $-6b - 8b + 10b$ $(b = 5)$
(f) $-9b - 4b + 18b$ $(b = 4)$

8 Answer true (T) or false (F) for each of the following.

(a) These two rules are really the same:

Rule 1: 'Take a number and multiply it by three. Take the same number and multiply it by seven. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

(b) These two rules are really the same:

Rule 1: 'Take a number and multiply it by ten. Take another number and multiply it by five. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by fifteen.'

(c) These two rules are really the same:

Rule 1: 'Take a number and multiply it by fourteen. Take the same number and multiply it by ten. Then subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by four.'

9 A store is selling all its books for c dollars and all its vinyl records for d dollars each. Tom bought eight books and five records. His friend Ayman bought seven books and six records.

(a) Write an expression for the amount Tom paid for his collection.

(b) Write an expression for the amount Ayman paid for his collection.

(c) Write an expression for the amount they paid altogether. Simplify this expression.

10 On each birthday, Alysia is given three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write the expression for the amount of money Alysia received on her fourth birthday.

(b) Write an expression for the total amount of money Alysia received for her first four birthdays.

(c) How much money did Alysia receive altogether for her first four birthdays if $d = 5$?

11 **(a)** By drawing three rectangles each with a width of 2 cm, show that $2 \times 3 + 2 \times 4 = 2 \times 7$.

(b) Substitute 'y' for '2' and rewrite the statement above using algebra.

Reasoning

12 **(a)** Which of the following expressions cannot be simplified by collecting like terms?

A $5x^2 + 4x$ **B** $3x^2 + 2x^2$ **C** $2xy + 6yx$

D $4xy^2 + 5x^2y$ **E** $5xy + xy$ **F** $4xy - xy$

(b) Simplify the remaining expressions. Explain why these expressions can be simplified.

13 Jess has x ten-cent coins and $10x$ fifty-cent coins.

(a) Write an expression for the total number of coins Jess has.

(b) Ryan has $44x$ coins. Explain what this means, in terms of the coins that Jess has.

(c) Write an expression for the value of Jess' coins in dollars.

(d) Can you find the value of Ryan's coins? Why or why not?

14 A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.

(a) If x is the number of coins on the first square, how many are stacked on the sixth square?

(b) How many coins in total are there on the board in these six squares?

(c) If each coin's value is $\$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins between \$300 and \$400.

Open-ended

15 Consider the algebraic expressions $a + b$, $2a + b$, $3a + b$.

Find two sets of values for a and b so that the sum of the three expressions is a multiple of ten.

16 The sequence of terms: $x + y$, $3x + 2y$, $5x + 3y$, $7x + 4y$, $9x + 5y$, … follows a pattern.

Find two sets of values for x and y so that the sum of the first five terms in this sequence is a multiple of ten.

17 Lindy was simplifying $7xy - xy$ and wrote: $7xy - xy = 7$

(a) What mistake did Lindy make?

(b) Give Lindy a suggestion to help her correct this mistake.

Half-time 3

1 If Michael has n \$2 coins and m \$1 coins, write an expression for how much money he has altogether. **3.1**

2 Simplify: **3.4**

(a) $7x + 10x$ **(b)** $14y - 9y$ **(c)** $3x + 2y - x + 4y$

(d) $4x^2 + 3xy - 2x^2 + 4x^2$ **(e)** $10a + 13b - 3b - 26a$ **(f)** $4x + 2a - 3x + 2a^2$

3 If $x = 3$, $y = -4$ and $z = 5$, find: **3.2**

(a) $3x + 2y$ **(b)** $2z - 3y$ **(c)** $7x + 2y - z$

(d) $6x^2 - y$ **(e)** $10xy^2 + z$ **(f)** $2xyz + 3z^3$

4 Use the rules to complete the following tables. **3.2**

(a) $y = 3x - 4$

x	-2	0	1	2
y				

(b) $y = 3 + 2x$

x	-3	-1	2	4
y				

(c) $y = 2x^2 + 3$

x	-1	0	3	5
y				

5 The following formulas are often used in mathematics and science. **3.3**

(a) For $V = IR$, find V where $I = 1.8$ and $R = 20$.

(b) For $v = u + at$, find v where $u = 20$, $a = 1.2$ and $t = 15$.

(c) For $t = a + (n - 1)d$, find t where $a = 2.6$, $n = 10$ and $d = -0.4$.

(d) For $f = \dfrac{vu}{v + u}$, find f where $v = 25$ and $u = 20$.

(e) For $s = ut + \frac{1}{2}at^2$, find s where $u = 3$, $t = 10$ and $a = 9.8$.

6 Susan gets a ticket to be served at the deli section of her local supermarket. The number on her ticket is t. The tickets are given out in consecutive order (1, 2, 3, …). **3.1**

(a) John gets the ticket after Susan. Write an expression for the number on John's ticket.

(b) Jill is the person who is served just before Susan. Write an expression for the number on Jill's ticket.

7 Simplify the following expressions. **3.4**

(a) $4xy + 3x - 2y + 5xy$ **(b)** $2x^2 - 4x - 3xy + 5x^2$

(c) $8m^2 - 12m + m^2 + 8m + 5$ **(d)** $9pq - qp + 4p^2q - 3p^2q^2 + 4pq^2$

8 For each expression state: **3.1**

(i) the number of terms it contains

(ii) the coefficient of x

(iii) the constant.

(a) $2x + 3$ **(b)** $a - 5x - 4$ **(c)** $x + 7 + 4x^2$

(d) $-2x - 14xy$ **(e)** $6ax - 5 - \frac{1}{2}x$ **(f)** $3x + 4y + 6$

(g) $4x^2y - 2 + x$ **(h)** $-10 - 12x$ **(i)** $7x$

Gamespace

MAGIC ALGEBRA

MAGIC TRICK 1

Think of a number.
Double it.
Add the original number.
Divide by 3

Abracadabra, your answer is the number you started with.

Magician's secret: How does this trick work? Let the original number you thought of be X. Now do each of the steps on X. Write each step as you go.

MAGIC TRICK 2

Think of a number.
Double it.
Add 8.
Multiply by 3.
Divide by 6.
Subtract your original number.

Alakazoom, your answer is 4.

Magician's secret: Discover how this works by letting the number you think of be X and doing each of the steps on it.

MAGIC TRICK 3: MIND READER

Choose any two-digit number and add together both the digits. Subtract the total from your original number. Look up your answer on the chart to find its magic symbol. Now stare at the symbol and think about it really hard. Can you find your symbol near the page number on this page? Try it again with a different starting number.

0 ♍	1 ◆	2 ●	3 ♋	4 △	5 ●	6 ◆	7 ♐
8 ♋	9 ✋	10 ♐	11 ♍	12 ☐	13 ⍓	14 ●	15 △
16 ♐	17 △	18 ✋	19 ◆	20 ♐	21 ♍	22 ♋	23 ♍
24 ☐	25 ♍	26 ⍓	27 ✋	28 ♋	29 ♐	30 ⍓	31 ◆
32 ●	33 ♐	34 ♋	35 ♐	36 ✋	37 ☐	38 ☐	39 ●
40 ♋	41 ☐	42 ◆	43 ●	44 ♍	45 ✋	46 ♐	47 ♋
48 ◆	49 ✋	50 ♍	51 △	52 ◆	53 △	54 ✋	55 ☐
56 ♍	57 ♋	58 ♐	59 ✋	60 ♐	61 ♋	62 △	63 ✋
64 △	65 △	66 ◆	67 ♋	68 ●	69 ◆	70 ♍	71 △
72 ✋	73 ♋	74 ●	75 ☐	76 △	77 ♋	78 ✋	79 ♐
80 ♋	81 ✋	82 ♍	83 ♍	84 ⍓	85 ♐	86 ◆	87 ♋
88 ☐	89 ☐	90 ✋	91 ⍓	92 ☐	93 ●	94 ⍓	95 ✋

Magician's secret: Can you work out the secret?

Use algebra to show how the trick works.

(Hint: Any two-digit number can be broken down into tens and units: 42 = 40 + 2. So, if a is the tens digit and b is the units digit, then the two-digit number can be written as $10a + b$.)

MAGIC TRICK 4: WHEN IS MY BIRTHDAY?

Take the month number of your birthday (i.e. Jan. = 1, Feb. = 2 and so on).

Multiply your number by 4.

Add 14.

Multiply the result by 25.

Subtract 230.

Add the day number of your birthday, (e.g. if born on 19 May, add 19).

Subtract 120 from your answer.

Hey presto, the first digit is the month you were born in and the next two digits are the day you were born on.

Magician's secret: How does this work? Choose m as the month number and d as the day number to help you find out.

MAGIC TRICK 5: CREATE YOUR OWN

Create your own "When is my birthday?" or "Guess the number" magic algebra trick.

3.5

Multiplying and dividing algebraic terms

You already know that $5 \times 6 \times 2 = 5 \times 2 \times 6$
$= 60$

You can multiply numbers in any order and the answer is always the same. This is because when you multiply numbers, the grouping or order of multiplication does not change the result. This is the **associative law**.

This also works in algebra. You can multiply numbers and variables (algebraic pronumerals) in any order and the answer is still the same. This means that you can rearrange multiplication to group numbers and pronumerals separately. Remember, pronumerals are written in alphabetical order.

$$\begin{aligned} 5a \times 2 &= 5 \times a \times 2 \\ &= 5 \times 2 \times a \\ &= 10a \end{aligned}$$

$$\begin{aligned} 2d \times 3c &= 2 \times d \times 3 \times c \\ &= 2 \times 3 \times c \times d \\ &= 6cd \end{aligned}$$

When multiplying algebraic terms, you should simplify them by:

- multiplying the numbers
- then multiplying the variables (algebraic pronumerals)
- writing variables in alphabetical order, after the numbers.

Worked example 10

W.E. 10

Simplify:

(a) $6a \times 3$

(b) $5a \times -2c \times b \times 3$

Thinking	Working
(a) Multiply the numbers ($6 \times 3 = 18$) and multiply this product by the variable (a).	(a) $6a \times 3 = 18a$
(b) Multiply the numbers ($5 \times -2 \times 3 = -30$) and multiply this product by the product of the variables ($a \times c \times b = abc$), writing them in alphabetical order.	(b) $5a \times -2c \times b \times 3 = -30abc$

When dividing algebraic terms:

- first, write the division as a fraction
- then simplify the fraction by dividing both the numerator and the denominator by the **highest common factor (HCF)** of the numerator and the denominator
- remember that common factors can be numbers and/or variables.

Worked example 11

W.E. 11

Simplify: $\frac{-15ef}{10f}$

Thinking	**Working**
1 Identify the HCF of the numerator and the denominator (HCF = $5f$). Then divide the HCF into both the numerator and the denominator.	$\frac{{}^{3}\cancel{-15}}{{}_{2}\cancel{10}} \times \frac{e\,{}^{1}\cancel{f}}{{}_{1}\cancel{f}}$
2 Write the simplified expression.	$= -\frac{3e}{2}$

Worked example 12

W.E. 12

Simplify: $14ab \div 7bc$

Thinking	**Working**
1 Write as a fraction.	$14ab \div 7bc = \frac{14ab}{7bc}$
2 Identify the HCF of the numerator and the denominator ($7b$). Then divide it into both the numerator and the denominator.	$= \frac{{}^{2}\cancel{14}a\,{}^{1}\cancel{b}}{{}_{1}\cancel{7}\,{}_{1}\cancel{b}c}$
3 Write the simplified expression.	$= \frac{2a}{c}$

3.5 Multiplying and dividing algebraic terms

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 8, 9 (a–c), 10 (a–c), 12	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4, 5, 6, 8, 9, 10, 11, 12, 13	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 5, 6, 7, 8, 9, 10, 11, 12, 13

Answers p. 634

Fluency

W.E. 10

1 Simplify:

(a) $5 \times 3a$ **(b)** $6 \times 2a$ **(c)** $9g \times 2$

(d) $7 \times 8z$ **(e)** $3z \times 12$ **(f)** $9p \times 6$

(g) $11e \times 6f$ **(h)** $9m \times 8n$ **(i)** $12j \times 11k$

(j) $6gh \times k \times 3$ **(k)** $8gh \times 3 \times 2k$ **(l)** $6b \times 5eh \times 3$

(m) $4 \times -5y$ **(n)** $-3 \times -7y$ **(o)** $-2a \times -8$

(p) $11a \times -4b$ **(q)** $-3r \times -6uq$ **(r)** $-7u \times 8rq$

(s) $-4j \times -2 \times 10k$ **(t)** $10j \times -2k \times 3$ **(u)** $8 \times -u \times 3v \times 2$

W.E. 11

2 Simplify:

(a) $\frac{14a}{7}$ (b) $\frac{12b}{3}$ (c) $\frac{63e}{9}$

(d) $\frac{15f}{5f}$ (e) $\frac{10x}{12x}$ (f) $\frac{x}{15x}$

(g) $\frac{15ab}{5}$ (h) $\frac{24ef}{8e}$ (i) $\frac{26ab}{13a}$

(j) $\frac{ef}{8e}$ (k) $\frac{7mn}{28m}$ (l) $\frac{11ef}{77f}$

(m) $\frac{-22ab}{11ab}$ (n) $\frac{-6ab}{3ba}$ (o) $\frac{-16ef}{-4ef}$

(p) $\frac{-9h}{-45gh}$ (q) $\frac{-7g}{-77gh}$ (r) $\frac{-9i}{-36ij}$

W.E. 12

3 Simplify:

(a) $20c \div 5$ (b) $40d \div 4$ (c) $12g \div 6$

(d) $60h \div 10h$ (e) $18x \div 27x$ (f) $20x \div 22x$

(g) $18cd \div 9$ (h) $56cd \div 8$ (i) $30gh \div 6$

(j) $gh \div 3g$ (k) $45gh \div 18g$ (l) $50gh \div 16g$

(m) $8cd \div -4c$ (n) $-50ab \div 30a$ (o) $60cd \div -12d$

(p) $12ab \div -8ba$ (q) $-ij \div -6ij$ (r) $-11ij \div -ji$

4 The expression $4 \times 6x \times -2y$ can be simplified to:

A $8xy$ B $10xy$ C $48xy$ D $-48xy$

5 $-2 \times 5g \times -a$ simplifies to:

A $-7ga$ B $-10ag$ C $7ga$ D $10ag$

6 $\frac{-12mad}{-9md}$ simplifies to:

A $\frac{4a}{3}$ B $-\frac{4m}{3}$ C $\frac{4m}{3}$ D $-\frac{4a}{3}$

7 $-a \times -2b \times -3c \div -6bad$ simplifies to:

A $-12a$ B $\frac{c}{d}$ C $6cd$ D $-\frac{6d}{c}$

Understanding

8 Simplify the product of the terms $3abc$, $-5d$ and -4.

9 Two toolboxes are shown. The volume of each is given by the product of the length, width and height.

(a) What is the volume of toolbox 1?

(b) What is the volume of toolbox 2?

(c) Find the volume of each toolbox if $w = 4$ and $j = 12$.

(d) How many of the smaller toolbox will fit inside the larger toolbox?

Toolbox 1

Toolbox 2

Reasoning

10 A necklace is made from 3 strands of gold wire. The shortest strand is x mm long. Each strand is 1.2 times longer than the previous one.

(a) Write expressions to show the lengths (in mm) of each strand.

(b) Write an expression in terms of x to show the total length (in mm) of gold wire.

(c) If gold wire costs $\$d$ per millimetre, write an expression to show the total cost of the gold used.

(d) $6y$ small diamonds are added to the necklace and each strand has the same number of diamonds. Write an expression to show how many diamonds are inserted in each strand.

(e) If each diamond costs $\$c$, write an expression to show the total cost of all the diamonds.

(f) Write an expression to show the total cost of materials needed to make the necklace if it also needs a fastener, which costs $\$f$.

(g) If $x = 30$, $d = 5$, $y = 4$, $c = 40$ and $f = 34$, find the total cost of materials needed to make the necklace.

Open-ended

11 Joshua simplified an expression. His work is shown below. Explain the mistake he has made and write the correct answer.

$3k \times -2k = k$

12 Choose two terms, each containing the variables p and r, so that the quotient of the two terms gives a result of $-\frac{3p}{7r}$. (Remember, the quotient is the result after one term has been divided by the other.)

13 Choose three terms, each containing the variables m and n, so that the product of two terms divided by the third gives a result of $6mn$.

Problem solving

Make a profit

A T-shirt company pays fixed expenses of \$400 per week to run their T-shirt factory. It also costs another \$12 to make each T-shirt. The information in the table shows how many T-shirts the company sells each week at certain prices.

Profit = (income from selling T-shirts) – (cost of making T-shirts)

1 What number of T-shirts must be made, to make a profit?

2 What number of T-shirts must be made, to make the largest profit?

Sale price per T-shirt	Number of T-shirts sold
\$10	45
\$15	40
\$20	35
\$25	30
\$30	25
\$35	20
\$40	15
\$45	10

Strategy options

- Guess and check.
- Make a table.
- Look for a pattern.

3.6

Expanding brackets

Expanding is the process of removing brackets, using the **distributive law.** You have seen this before with numbers.

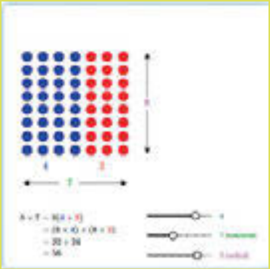
For example:
$$\begin{aligned} 3 \times 102 &= 3 \times (100 + 2) \\ &= 3 \times 100 + 3 \times 2 \\ &= 300 + 6 \\ &= 306 \end{aligned}$$

Interactive

The distributive law

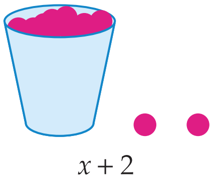
Explore visually how the distributive law works with changing numbers.

Go to the eBook or the Pearson Places website to access this interactive.

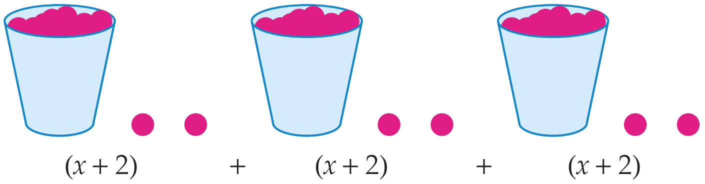


What is meant by $3(x + 2)$?

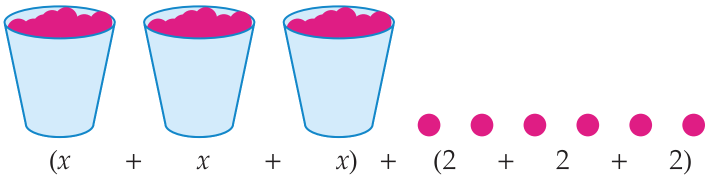
You can see what this expression means by using x to represent the number of counters in a cup. This means that $x + 2$ is the number of counters in the cup plus 2 more.



If you have $3(x + 2)$, this represents 3 lots of $(x + 2)$ or $3 \times (x + 2)$. It can be shown by 3 cups with x counters in each and 2 more counters with each cup.



You can rearrange this as 3 cups, each with x counters, and 6 more counters, which will gives a total of $3x + 6$ counters.



Therefore, you can write $3(x + 2) = 3x + 6$.

The variables in algebra simply represent unknown numbers, so the distributive law must also work in algebra.

For example:

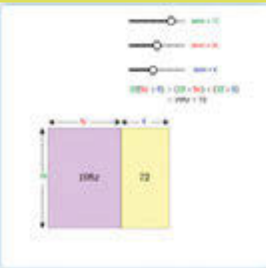
$$\begin{aligned}3(x+2) &= 3\times x+3\times 2\\ &= 3x+6\end{aligned}$$

Interactive

Expanding brackets

Explore the distributive law and expanding brackets with algebra, by showing multiplications as changing areas.

Go to the eBook or the Pearson Places website to access this interactive.



Expanding using the distributive law:

$$a(b+c)=ab+ac \quad \text{and} \quad a(b-c)=ab-ac$$

Worked example 13

W.E. 13

Use the distributive law to find 24 × 101.

Thinking	Working
1 Write one number in the product as the sum or difference of two numbers that are easy to multiply by (101 = 100 + 1).	24×101 $=24\times(100+1)$
2 Expand the brackets.	$=24\times 100+24\times 1$
3 Simplify.	$=2400+24$ $=2424$

Worked example 14

W.E. 14

Expand each of the following expressions.

(a) $5(a-2)$ **(b)** $2a(a+3b)$ **(c)** $-3(a-b)$

Thinking	Working
(a) 1 Multiply each term in the bracket by the term in front of the bracket.	(a) $5(a-2)$ $=5\times a-5\times 2$
2 Simplify.	$=5a-10$
(b) 1 Multiply each term in the bracket by the term in front of the bracket.	(b) $2a(a+3b)$ $=2a\times a+2a\times 3b$
2 Simplify.	$=2a^2+6ab$

(c) 1 Multiply each term in the bracket by the term in front of the bracket, making sure to change signs if necessary.	(c) $-3(a-b)$ $= -3 \times a - 3 \times -b$
2 Simplify.	$= -3a + 3b$

Expanding and collecting like terms

After an expression is expanded using the distributive law, it may contain like terms. You can collect like terms and simplify the expression.

Worked example 15

W.E. 15

Simplify the following expressions by expanding the brackets and collecting like terms.

(a) $3(r-4)-8r$ **(b)** $5(a+6b)+a(9-10a)$

Thinking	Working
(a) 1 Expand all brackets.	(a) $3(r-4)-8r$ $= 3r - 12 - 8r$
2 Simplify by collecting like terms.	$= -5r - 12$
(b) 1 Expand all brackets.	(b) $5(a+6b)+a(9-10a)$ $= 5a + 30b + 9a - 10a^2$
2 Simplify by collecting like terms.	$= 14a + 30b - 10a^2$

Worked example 16

W.E. 16

Simplify the following expressions by expanding the brackets and collecting like terms.

(a) $8d-(2d+3)$ **(b)** $7(a-3b)-5(a+3b)$ **(c)** $3m(3n-4t)-4n(2m-t)$

Thinking	Working
(a) 1 Rewrite the expression, inserting a 1 and a multiplication symbol in front of the brackets.	(a) $8d-(2d+3)$ $= 8d - 1 \times (2d+3)$
2 Expand the brackets, taking care to change signs where needed.	$= 8d - 2d - 3$
3 Simplify by collecting like terms.	$= 6d - 3$
(b) 1 Expand all brackets, taking care to change signs where needed.	(b) $7(a-3b)-5(a+3b)$ $= 7a - 21b - 5a - 15b$
2 Simplify by collecting like terms.	$= 2a - 36b$
(c) 1 Expand all brackets, taking care to change signs where needed.	(c) $3m(3n-4t)-4n(2m-t)$ $= 9mn - 12mt - 8mn + 4nt$
2 Simplify by collecting like terms.	$= mn - 12mt + 4nt$

3.6 Expanding brackets

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4 (column 1), 5 (a–f), 6, 10 (a–b), 13, 15	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4 (column 2), 5 (column 1), 6, 7, 8, 9, 10, 11 (a–b), 13, 14, 15	1 (columns 2–3), 2 (column 3), 3 (columns 2–3), 4 (column 2), 5 (column 2), 7, 8, 9, 10, 11, 12, 13, 14, 15

Answers p. 634

Fluency

W.E. 13

1 Use the distributive law to find the following.

(a) 21×101 (b) 17×102 (c) 16×101

(d) 23×98 (e) 41×99 (f) 37×97

(g) 54×11 (h) 38×1010 (i) 64×1100

W.E. 14

2 Expand the following expressions.

(a) $5(x + 12)$ (b) $2(s - 5)$ (c) $12(a + 5)$

(d) $s(4 + t)$ (e) $l(6 - r)$ (f) $a(2 + h)$

(g) $3(4x + 5)$ (h) $9(6x - 1)$ (i) $2(3h + 4)$

(j) $2m(n + r)$ (k) $3p(r - z)$ (l) $6p(r + t)$

(m) $5t(t + 2)$ (n) $2p(5p - 7)$ (o) $4m(4 + 2m)$

(p) $3m(5m + 7n)$ (q) $4k(p - 5k)$ (r) $10x(2x - 5y)$

3 Expand the following expressions.

(a) $-2(a + 4)$ (b) $-3(b - 9)$ (c) $-(c + d)$

(d) $-(d - x)$ (e) $-e(x + 7)$ (f) $-f(f - y)$

(g) $-5(3g + 10)$ (h) $-8(3 - 7h)$ (i) $-11(2i + 4l)$

(j) $-2j(8x + 12y)$ (k) $-3kl(4x - 15y)$ (l) $-12mn(3p - 5np)$

W.E. 15

4 Simplify the following expressions by expanding the brackets and collecting like terms.

(a) $5(7 - d) + 11d$ (b) $8(3 - e) - 12e$

(c) $6n(m - 4) - 12nm$ (d) $4x(y - 2) - 7xy$

(e) $5t(7 - 4k) - 40tk$ (f) $9v(5 - 10w) - 50v$

(g) $6m(7n - 4) + 4(8mn - 4m)$ (h) $2a(5b - 8) + 2(6a - 12ab)$

(i) $9x(5 - y) + 3(10x - 4xy)$ (j) $3t(4 - 8z) + 6(7tz - 20t)$

W.E. 16

5 Simplify the following expressions by expanding the brackets and collecting like terms.

(a) $3x - 4(x + 2)$ (b) $5x - (2x + 2)$

(c) $8 - 3(x + 2)$ (d) $14 - (3x - 7)$

(e) $5x - 3(5x - 2)$ (f) $7x - 3(x - 2)$

(g) $5(x + 2) - 3(2x + 1)$ (h) $4(3x - 2) - 12(x + 5)$

(i) $3(3x + 4) - 5(7x - 3)$ (j) $5(x - 3) - (5x - 1)$

(k) $5(2x - 3y) - 3(3x + 4y)$ (l) $4(3x - 5y) - 2(5x + 2y)$

(m) $c(3d + 4e) - 2c(d + 5e)$ (n) $d(5e + 2f) - 4d(e + 4b)$

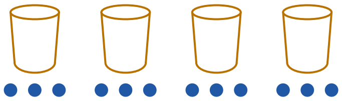
(o) $m(2n - 3t) - t(2m - n)$ (p) $a(2p - q) - q(2p - 3a)$

If there's a minus in front of a bracket such as $(x + 5) - (3x + 1)$, then you should put 1 × between the minus and bracket: $(x + 5) - 1 \times (3x + 1)$.

Understanding

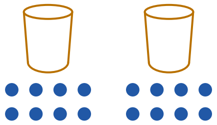
6 Which expression is modelled by the cups and counters in each diagram?

(a)



A $4x + 3$ **B** $4(x + 12)$ **C** $4(x + 3)$ **D** $2x + 12$

(b)

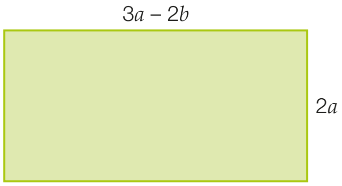


A $x + 16$ **B** $2x + 8$ **C** $2x + 16$ **D** $2(x + 16)$

7 Expanding and simplifying $3y(2y + a) - a(3y - a)$ gives:

A $6y^2 - ay$ **B** $6y^2 - 3ay + a^2$ **C** $3ay - 6y^2$ **D** $6y^2 + a^2$

8 A rectangle has width $2a$ metres and length $(3a - 2b)$ metres. Write the expression for the area and expand the brackets.



9 **(a)** **(i)** Expand $33(x + 5)$. **(ii)** Substitute $x = 100$ and evaluate.

(b) **(i)** Expand $105(y + 3)$. **(ii)** Substitute $y = 30$ and evaluate.

(c) Explain why your answers to parts **(a) (ii)** and **(b) (ii)** are the same.

Reasoning

10 A shopkeeper buys a bookcase for $\$u$ and then sells it for $\$v$.

(a) The profit is the selling price minus the cost price. Write an expression for the shopkeeper's profit.

(b) The shopkeeper decides to buy and sell 6 bookcases. Write an expression for the total profit.

(c) The shopkeeper increases the selling price of the bookcases by 50%. If she sells 6 bookcases at this increased price, how much profit does she make?

11 A rectangle has length $(5 + x)$ m and width 5 m.

(a) Draw a diagram to represent this.

(b) Find the area of the rectangle.

(c) Split your diagram into two parts, with one part being a square of side length 5 m.

(d) Find the area of each part of your diagram.

(e) Use your diagram and the answers to parts **(b)** and **(d)** to explain why $5 \times (5 + x) = 25 + 5x$.

12 When Lucy was asked to multiply 8 by 8.125, she immediately answered 65. She said it was easy to work out, using $20 \times 20.05 = 401$ and $4 \times 4.25 = 17$. She also said that she used the distributive law. How did she find the answer so quickly?

Open-ended

13 Quentin was working on algebraic expressions, expanding brackets and then collecting like terms. His answer was: $20x - xy$

Write a question that Quentin's answer is correct for.

14 Michael was simplifying by expanding and collecting like terms. His working is shown.

$$\begin{aligned} 3(2x - 1) - (5x - 2) &= 6x - 3 - 5x - 2 \\ &= x - 5 \end{aligned}$$

(a) Identify the mistake that Michael made.

(b) Correct his mistake by completing the question.

(c) Suggest how Michael can avoid making this mistake in the future.

15 Write three expressions containing brackets that can be expanded to give the first term as $12ef$.

Puzzle

The rat race

It was the end of the National Rodent Racing season, but the main race results have been destroyed.

Luckily, people can remember the following facts about the race results.

There were eight rats in the race.

- Lin finished after Bo and Cam.
- Cam finished before Jay, Lin, Femi, Tigger and Bo.
- Zuri finished before Dale and Cam.
- Tigger finished before Dale, Femi and Lin.
- Femi finished after Lin, Cam, Tigger, Bo and Dale.
- Jay finished after Bo and Cam.
- Jay finished before Lin and Femi.
- Dale finished after Cam, Tigger and Lin.
- Tigger finished after Jay and Cam.

Using these facts, can you find each rat's position in the race results and see which rat won the race?

Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Farmer Jones' dilemma

Farmer Jones has a problem to solve. She wants to build a rectangular pig pen for her pigs so that they will have the largest possible area to move around in.

But first, Farmer Jones needs to fence a rectangular paddock for her bull. She has 400 m of fencing wire and she also wants the bull to have the largest possible area. Her son Joe offered to help work out the dimensions. He first wrote the formula for the perimeter of a rectangle:

$P = 2L + 2W$

Next, he rearranged the formula to find the width (W):

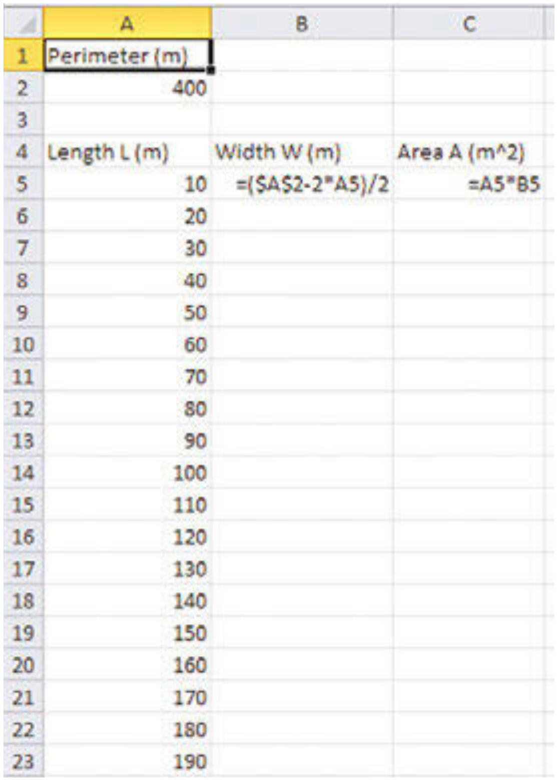
$$W = \frac{P - 2L}{2}$$

He knows $P = 400$ m. He used a spreadsheet to calculate W for different lengths, starting with $L = 10$ m and increasing the length by 10 m each time. He then found the area for each length and width:

$A = LW$

The formulas Joe used are shown in the spreadsheet below.

(Hint: Remember that in Excel, the **$** symbol means that the cell reference will not change when the formula is copied using **Fill Down**. The symbol ***** means multiplication.



	A	B	C
1	Perimeter (m)		
2	400		
3			
4	Length L (m)	Width W (m)	Area A (m^2)
5	10	=(A2-2*A5)/2	=A5*B5
6	20		
7	30		
8	40		
9	50		
10	60		
11	70		
12	80		
13	90		
14	100		
15	110		
16	120		
17	130		
18	140		
19	150		
20	160		
21	170		
22	180		
23	190		

By following the same steps as Joe, find the dimensions of the rectangle that would enclose the largest area.

1 **(a)** In a new spreadsheet, enter the 'Perimeter' heading in cell **A1** and the perimeter length (400) in cell **A2**. (Do not include units with the numbers.) Enter the column headings as shown above, starting with 10 as the first length. Increase the length by 10 each row, up to 190.

Enter the formulas to find the width and the area as shown. Use **Fill Down** to copy the formulas down the columns.

(b) What was special about the rectangle that Joe found?

2 Joe wonders whether this shape would always give the largest area. He wants to build a chicken coop with 50 m of chicken wire for his pet chickens.

(a) By using a spreadsheet, find the dimensions of the chicken coop that would enclose the largest area.

(Hint:You can use a similar spreadsheet as before, but you will need to change the perimeter and use smaller amounts for the length. Don't forget that length does not need to be a whole number.)

(b) What do you notice about this rectangle? Is it the same type of rectangle as Joe found in **1(b)**?

(c) What area does this rectangle now enclose?

3 Farmer Jones is impressed with her son's findings, but wants to check them. She has found the right fencing wire to build the pig pen and still wants to maximise the area enclosed. She sets up a spreadsheet similar to Joe's.

(a) Choose a suitable length of fencing wire (100 m to 200 m) for Farmer Jones. Use this value in the spreadsheet to show that the type of rectangle with the biggest area is always the same.

(b) Using your chosen length of fencing wire, what are the dimensions of Farmer Jones' pig pen?

Taking it further

4 Farmer Jones is so happy with her new spreadsheet skills that she wants to try something new. She now wants to enclose a rectangular paddock with 800 m of fencing wire for her sheep. However, one side of the paddock will be the straight river that runs through her property, so this will not need to be fenced. She wants to make sure that the sheep have the largest area possible.

(a) What is the new formula:

- **(i)** for the perimeter
- **(ii)** for the width
- **(iii)** for the area?

(b) Use a spreadsheet to find the dimensions of this new paddock.

(c) What do you notice about this rectangle?

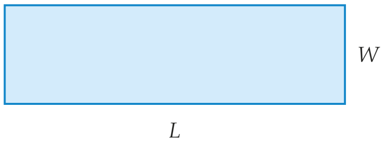
3.7 Factorising

Factorising means writing an expression as the product of factors.

$10 = 2 \times 5$ $\quad$ $2a = 2 \times a$

$2(a + b)$ is a factorised expression, as it is the product of 2 and $(a + b)$.

Factorised expressions can be useful in many situations. For example, how do you find the perimeter of a rectangle?



To find a rectangle perimeter, you can double the length and double the width and add these two amounts. Writing this as a formula:

$P = 2L + 2W$, where L = length and W = width. This is the expanded form.

However, you could add the length and width together first and then double that amount. Writing this as a formula:

$P = 2(L + W)$. This is the **factorised form**.

This means that $2L + 2W = 2(L + W)$.

The expanded form = the factorised form.

You also know that $2(L + W) = 2L + 2W$ by expanding the brackets.

The factorised form = the expanded form.

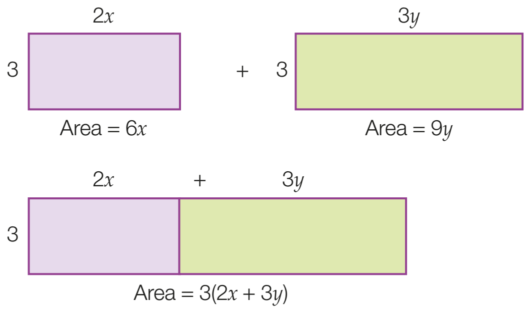
This shows that factorising is the reverse of expanding.

What is the sum of the areas of the two rectangles on the right?

The total area can be written in two equally correct forms.

Expanded form: $6x + 9y$

Factorised form: $3(2x + 3y)$



Factorising an expression

- Write each term in the expression as a product of prime factors.
- Circle all common factors to find the highest common factor (HCF) of all the terms in the expression.
- Write the HCF in front of the brackets.
- Write the sum or difference of the other factors inside the brackets.
- Write the expression in expanded form.
- Check your answer by expanding the brackets.

For example: $6x + 9y = \textcircled{3} \times 2 \times x + \textcircled{3} \times 3 \times y$ $\quad$ (3 is the HCF of $6x$ and $9y$)

$= 3(2x + 3y)$

Check: $3(2x + 3y) = 6x + 9y$

Worked example 17

W.E. 17

Find the highest common factor (HCF) of the following pairs of algebraic terms.

(a) $10x$ and 15 **(b)** $12a$ and $18b$ **(c)** $6c^2d$ and $8cd^2$

		Thinking	Working
(a)	**1**	Write each term as the product of prime factors and circle the common factors.	(a) $10x = 2 \times \enclose{circle}{5} \times x$ $15 = 3 \times \enclose{circle}{5}$
	2	Write the HCF.	$\text{HCF} = 5$
(b)	**1**	Write each term as the product of prime factors and circle the common factors.	(b) $12a = \enclose{circle}{2} \times 2 \times \enclose{circle}{3} \times a$ $18b = \enclose{circle}{2} \times 3 \times \enclose{circle}{3} \times b$
	2	Multiply the common factors to find the HCF.	$\text{HCF} = 2 \times 3$ $= 6$
(c)	**1**	Write each term as the product of prime factors and circle the common factors.	(c) $6c^2d = \enclose{circle}{2} \times 3 \times \enclose{circle}{c} \times c \times \enclose{circle}{d}$ $8cd^2 = \enclose{circle}{2} \times 2 \times 2 \times \enclose{circle}{c} \times \enclose{circle}{d} \times d$
	2	The HCF is the product of the common factors.	$\text{HCF} = 2 \times c \times d$ $= 2cd$

It is sometimes convenient to factorise with a negative number as a common factor. Be careful to use the correct negative or positive sign inside the brackets. Check the multiplication and remember that multiplying the same sign gives a positive, while multiplying different signs gives a negative:

$(-) \times (-) = (+)$ $(-) \times (+) = (-)$
$(+) \times (+) = (+)$ $(+) \times (-) = (-)$

For example, factorising an algebraic expression:

$$\begin{aligned} -2x^2 - 4xy &= \enclose{circle}{-2} \times \enclose{circle}{x} \times x + \enclose{circle}{-2} \times 2 \times \enclose{circle}{x} \times y \\ &= -2x(x + 2y) \end{aligned}$$

Worked example 18

W.E. 18

Factorise the following expressions.

(a) $3y + 3$ **(b)** $10a - 12$ **(c)** $-15b + 10c$

		Thinking	Working
(a)	**1**	Write each term as the product of its prime factors and circle the common factors. If all the factors in a term are circled, write 1 as another factor.	(a) $3y + 3$ $= \enclose{circle}{3} \times y + \enclose{circle}{3} \times 1$
	2	Write the HCF outside the brackets and the remaining terms inside the brackets.	$= 3 \times (y + 1)$
	3	Leave out the multiplication symbols.	$= 3(y + 1)$

(b) **1**	Write each term as the product of its prime factors, and circle the common factors.	(b) $10a - 12$ $= \textcircled{2} \times 5 \times a - \textcircled{2} \times 2 \times 3$
2	Write the HCF outside the brackets and the remaining terms inside the brackets.	$= 2 \times (5 \times a - 2 \times 3)$
3	Simplify and leave out the multiplication symbols.	$= 2(5a - 6)$
(c) **1**	Write each term as the product of its prime factors. Change the + to – - so that a negative sign can be included in the common factor. Circle the common factors.	(c) $-15b + 10c$ $= \textcircled{-}3 \times \textcircled{5} \times b - \textcircled{-}2 \times \textcircled{5} \times c$
2	Write the HCF, including the negative sign, outside the brackets and the remaining terms inside the brackets.	$= -5 \times (3 \times b - 2 \times c)$
3	Leave out the multiplication symbols.	$= -5(3b - 2c)$

3.7 Factorising

Navigator

Answers p. 635

1, 2 (column 1), 3 (column 1), 4 (column 1), 5, 6, 7, 8, 9, 13, 14, 16	2 (column 2), 3 (column 2), 4 (column 2), 5, 6, 7, 8, 12, 13, 14, 15, 16	2 (column 3), 3 (column 3), 4 (column 3), 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16

Fluency

1 Find the highest common factor of the following sets of numbers.

(a) 24 and 96 **(b)** 150 and 900 **(c)** 126 and 630

(d) 32, 48 and 128 **(e)** 315, 520 and 1500 **(f)** 63, 243, 405 and 729

W.E. 17

2 Find the highest common factor (HCF) of the following pairs of algebraic terms.

(a) $6m$ and 9 **(b)** $14p$ and 21 **(c)** $22d$ and 55

(d) $8x$ and $36y$ **(e)** $30d$ and $45c$ **(f)** $24a$ and $54b$

(g) $2p^2$ and $8p^4$ **(h)** $9t^5$ and $12t$ **(i)** $15d^5$ and $10d^3z^3$

(j) fm^4 and f^4m^2 **(k)** a^2b and a^3b^2 **(l)** x^4y^2 and x^2y^3

(m) $20mn^4$ and $25m^3n^2$ **(n)** $18s^3t^6$ and $15st$ **(o)** $6g^2h$ and $15gh^3i$

W.E. 18

3 Factorise the following expressions.

(a) $3t + 15$ **(b)** $2h + 14$ **(c)** $2a - 8$

(d) $9p + 3$ **(e)** $20p - 5$ **(f)** $15z + 3$

(g) $12 - 20b$ **(h)** $24 + 10y$ **(i)** $18 - 14q$

(j) $3px + 12$ **(k)** $2kt - 20$ **(l)** $4hs + 14$

(m) $-8ab - 16$ **(n)** $-5mn - 15$ **(o)** $-12kr - 132$

4 Factorise the following expressions.

(a) $3lx + 15x$
(b) $10pm - 20m$
(c) $14pq + 28q$
(d) $6vx - 3x$
(e) $30ab + 20a$
(f) $18xy - 40y$
(g) $16p + 14pq$
(h) $15l - 20lm$
(i) $22j - 55jl$
(j) $30mz - 20m$
(k) $32hk + 36k$
(l) $40h + 28hi$
(m) $-12ab + 16b$
(n) $-14xy + 21x$
(o) $-72m - 56mn$

You can check your answers by expanding the brackets. This should give the original expression again.

5 Factorise the following expressions.

(a) $a^2 + a$
(b) $b^2 + 4b$
(c) $c^2 - c$
(d) $d - d^2$

6 Factorise the following expressions.

(a) $3a^2 + 3a$
(b) $2b^2 + 4b$
(c) $5c^2 + 5c^3$
(d) $6d^2 - 6d$

7 The highest common factor of $20x$ and $14xy$ is:

A $140xy$
B $7y$
C $2y$
D $2x$

8 The HCF of $2mn$ and $2m^6$ is:

A m^6n
B $2m$
C $2m^6n$
D $2mn$

Understanding

9 Marge has \$$5ab$ and Lisa has \$$25c$. Write the total amount of money they have together, in factorised form.

10 The kinetic energy of a moving object with a mass of m kilograms and a speed of v metres per second is $\frac{mv^2}{2}$. The object's gravitational potential energy when it is h metres above the ground is mgh, where g is the acceleration due to gravity.

(a) Write the total energy of the object (kinetic energy plus gravitational potential energy).
(b) Write the total energy in factorised form.

11 Factorise:

(a) $3(x + 1) - y(x + 1)$
(b) $7(3x - 4) + y(3x - 4)$
(c) $5(x + 7) + 2y(x + 7)$

Reasoning

12 The expression $ax + ay + bx + by$ contains four terms.

(a) Factorise $ax + ay$.
(b) Factorise $bx + by$.
(c) Write the expression for the sum of your factorised answers for the first two parts.
(d) Factorise the expression in the previous part by taking the expression in brackets to be the highest common factor.
(e) Use the method above to factorise $3x + 3b + xy + by$.

13 **(a)** What is the highest common factor of the expression $4(a + 1) + b(a + 1)$?
(b) Factorise $4(a + 1) + b(a + 1)$.

Open-ended

14 Write three pairs of terms whose highest common factor is 8.

15 A rectangle has a width of x cm and an area given by the formula $A = 10x - x^2$.

(a) Factorise the expression for the area to find an expression for the length of the rectangle.

(b) If the length is greater than the width, find a possible value for x.

16 Stephen has factorised $9x + 3$ as shown below.

$9x + 3 = 3(3x + 3)$

His friend Mohammed tells him that he is wrong.

Mohammed has written $9x + 3 = 3(3x + 1)$.

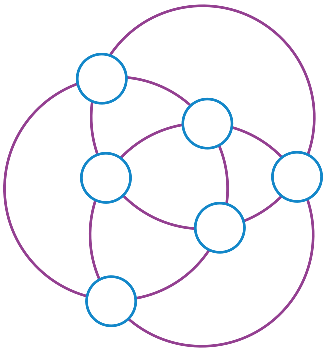
State who is correct. Explain where the other person has made a mistake and what can be done to correct this mistake.

Challenge 3

1 When four consecutive even numbers are added, the total is 140. Use algebra to find the numbers.

2 In how many different ways can you add four positive odd numbers to total 16? You can use any positive odd number as many times as you like.

3 Three large circles overlap. Six smaller circles are located on the points where the circles cross, as shown.



(a) Copy this diagram and then write the digits 0, 1, 2, 3, 4 and 5 in the blue circles so that the totals of the numbers in the blue circles around each large circle are the same.

(b) Copy this diagram again and then write the expressions x, $x + 1$, $x + 2$, $x + 3$, $x + 4$ and $x + 5$ in the blue circles so that the totals of the numbers in the blue circles around each large circle are the same.

(c) Find six other integers or expressions for which this will work. (Integers include positive and negative whole numbers and zero; e.g. -7, 0, 4.)

4 Given the factorisations $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ and $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$, answer the following questions, without using a calculator.

(a) Write 3^6 and 4^6 as perfect cubes.

(b) Show that 25 is a factor of $3^6 + 4^6$.

(c) Show that 4 is a factor of $3^6 - 5^3$.

5 Write an algebraic expression in each empty cell to make a magic square for which the sum of each row, column and diagonal is $(4x + 2)$.

$x + 8$	$x - 6$		$x + 5$
		$x + 2$	x
$x - 4$	$x + 6$	$x + 7$	

6 $\dfrac{a + \frac{1}{b}}{b + \frac{1}{a}}$ is equal to:

A $\frac{a}{b}$ **B** $\frac{b}{a}$ **C** ab **D** 1

Chapter review 3

Maths literacy

- associative law
- coefficient
- constant
- distributive law
- equation
- expanding
- expression
- factorised form
- factorising
- formulas
- highest common factor (HCF)
- like terms
- not like terms
- substitution
- term
- variable

Copy and complete the following using the terms from this list, where appropriate. A term may be used more than once.

1 $4b + 5$ is called an __________. The letter b in $4b + 5$ is called a __________ because it can take the place of any number. The number 5 is called a __________ and the number 4 is the __________ of b.

2 When you replace x with a certain number, this is called __________.

3 To simplify $4e + 3f - 2e + 6f$, collect __________. The expression then simplifies to $2e + 9f$.

4 When you obtain the expression $4c + 12$ from $4(c + 3)$, you are __________ the bracket using the __________.

5 When you write $6a + 18$ as $6(a + 3)$ you are __________ the expression.

6 Algebraic equations used in practical situations are called __________.

Fluency

1 You have x power points in your house. **3.1**

(a) A house next door has $x + 3$ power points. How does this compare to the number in your house?

(b) The other house next door has $(x - 2)$ power points. How does this compare to the number in your house?

(c) The block of flats across the road has $6x$ power points. How does this compare to the number in your house?

2 Evaluate the following expressions for $a = 4$ and $b = 5$. **3.2**

(a) $3a + 2b$ **(b)** $2ab - 12$ **(c)** $\frac{8b}{a} + 2b$ **(d)** $\frac{10}{b} + \frac{ab}{10}$

3 Evaluate the following expressions for $a = 2$ and $b = 3$. **3.2**

(a) $7(2a - b)$ **(b)** $3a(b + 17)$ **(c)** $2a(3 - 5b)$

4 Evaluate the following expressions for $a = -2$ and $b = -3$. **3.2**

(a) $-2a + b - 5$ **(b)** $ab - 4 - \frac{6b}{2}$

5 Evaluate the following expressions for $x = 2$ and $y = 12$. **3.2**

(a) $\frac{x + y}{2}$ **(b)** $\frac{2x + 3y}{5}$ **(c)** $\frac{12(x - y)}{y}$

6 **(a)** Using the formula $F = ma$, find F where $m = 100$ and $a = 6.3$. **3.3**

(b) Using the formula $v^2 = u^2 + 2as$, find v where $u = 1$, $a = 10$ and $s = 4$.

7 Simplify the following, if possible, by collecting like terms. **3.4**

(a) $-j - 9j$ **(b)** $5ty - 2ty$ **(c)** $7ghi - 4ghi$

(d) $24ab - 5ab + 7$ **(e)** $6a + 12b - 7a + 11b$ **(f)** $2a^2 - 4a$

8 Simplify: **3.5**

(a) $15a \times 2b$ **(b)** $6ab \times c \times 11$ **(c)** $-2a \times -7 \times b$

9 Simplify: **3.5**

(a) $24a \div 20a$ **(b)** $\frac{25ab}{5a}$ **(c)** $-4ab \div -44a$

10 Expand: **3.6**

(a) $7(2 + a)$ **(b)** $a(2b + 6)$ **(c)** $4a(3a + 5)$

11 Simplify by expanding and collecting like terms: **3.6**

(a) $2(4 + a) + 7a$

(b) $10a(6 + 2b) + 7(2a - ab)$

(c) $2a^2(3a^4 + 11a) + 3a(5a^5 - 2a^2)$

12 Factorise: **3.7**

(a) $3a - 15$ **(b)** $20ab + 16$ **(c)** $-15p + 20q$

Understanding

13 Suppose there are m jelly beans in a packet. Melissa, Roderigo and Huong have bought some packets of jelly beans. The following tells you how many jelly beans each has at certain times. **3.1**

Time	Melissa	Roderigo	Huong
12:30	m	0	0
12:31	$m - 2$	$2m$	0
12:32	$m - 4$	$2m - 10$	$3m$
12:33	$m - 6$	m	$3m - 5$
12:34	$m - 8$	$m - 6$	$2m$
12:35	$m - 14$	0	$2m - 8$

Write a paragraph telling a story about what happened during the 5 minutes.

14 In a school hall there are t teachers, g female students and b male students. **3.1**

(a) How many people are there in total?

(b) Eight teachers, six female students and half the boys exit the hall. How many people remain inside?

(c) Half the teachers and female students who had left re-enter the hall. How many people are now in the hall?

15 The heights (in centimetres) of three Year 8 students, Mei, Sienna and Liam, are $3a + 2$, $4b - 5$, $6c + 1$ (in order). **3.2**

If $a = 52$, $b = 41$, and $c = 26$, what is each person's height?

16 The heights (in cm) of Year 8 students William, Noah and Yumi are $4d^2 - 8$, $d^3 - 45$, $2d^2 + 18d$ (in order). If $d = 6$, who is the tallest and what is their height? **3.2**

17 If $a = 1$, $b = 2$, $c = 3$ and so on, with the number increasing by 1 for each letter, find the value of $ab - bc + cd - de + ef - fg$. **3.2**

18 Fatima draws a design made of straight lines. She first draws a line of length u cm straight up, then another line of length r cm to the right and a line of length d cm straight down. **3.2**

(a) Write an expression for the total length of the three lines.

(b) If Fatima continues this design 10 times, what is the total length of the lines?

(c) If $r = d$, $r = \frac{1}{2}u$ and $u = 7$, what is your answer to the previous part?

19 The speed v metres per second at which a wheel of radius r metres turns is given by the formula $v = \frac{6.28r}{t}$, where t seconds is the time the wheel takes to make one turn. Find the speed of the wheel if $r = 0.35$ metres and $t = 1.57$ seconds. **3.2**

20 Apples are sold at a market. A standard bag contains x apples. A bonus bag of apples has twice the number of apples as a standard bag plus an extra 4 apples. A case of apples contains 2 fewer apples than 6 standard bags of apples. Three customers each buy a bonus bag of apples and five customers each buy a case of apples. **3.2**

(a) Write an expression to show the number of apples in a bonus bag.

(b) Write an expression to show the number of apples in a case of apples.

(c) Write an expression to show the number of apples sold.

(d) Expand and simplify your answer.

(e) How many apples were sold if a standard bag contains a dozen apples ($x = 12$)?

21 A tint is used to protect glass in a rectangular window of length L in metres and width W in metres. The cost in dollars of applying the tint is $7 \times$ area of glass. **3.2**

(a) Write an expression for the cost of tinting the window in terms of L and W.

(b) Find the cost of tinting a window 2.5 m long and 0.8 m wide.

22 The area of a triangle is half the product of its base length and its height: $A = \frac{bh}{2}$. **3.3, 3.6**

(a) Write the expression for the area of a triangle whose base is $4x$ cm and whose height is $(3x + 2)$ cm.

(b) Expand and simplify the expression found in the first part.

(c) What is the area of the triangle if the height is 14 cm?

Reasoning

23 A stencil is made by cutting a triangle out of a rectangle of length l cm and width w cm. **3.6, 3.7**

(a) The base of the triangle is half the length of the rectangle and the height of the triangle is 6 cm less than the width of the rectangle. Find the area of the triangle in terms of l and w in: **(i)** factorised form and **(ii)** expanded form.

(b) Show that the area of the rectangle that is left once the triangle is cut out is $\frac{3lw}{4} + \frac{3l}{2}$.

(c) Factorise this answer.

(d) Find the area of the rectangle that is left once the triangle is cut out if $l = 20$ cm and $w = 16$ cm by:

- **(i)** subtracting the area of the triangle from the area of the rectangle
- **(ii)** substituting the values for l and w into your factorised answer in the previous part.

24 a, b, c represent three numbers.

- **(a)** Write an expression that represents b being increased by 1.
- **(b)** Write an expression for the product of a and your answer to part **(a)**.
- **(c)** Write an expression for the product of c and your answer to part **(a)**.
- **(d)** Write an expression for the sum of the answers to parts **(b)** and **(c)**.
- **(e)** Factorise the answer found in part **(b)**.
- **(f)**
  - **(i)** Write an expression for the answer to part **(e)** divided by $a + c$.
  - **(ii)** Simplify your answer to part **(i)**.
  - **(iii)** Subtract 1 from your answer to part **(ii)**.
  - **(iv)** What do you notice about your answer to part **(iii)**?
- **(g)** Repeat the question using the values $a = 3$, $b = 7$, $c = 8$.

Numeracy practice 3

Non-calculator

1 What is the value of $4a^2b$ when $a = -3$ and $b = 2$?

A -72 **B** -48 **C** 48 **D** 72

2 Which expression is equivalent to $5(2x - 1)$?

A $10x - 1$ **B** $10x - 5$ **C** $10x + 1$ **D** $10x + 5$

3 Which expression is equivalent to $3xy - 8x^2 + 2yx - 4x^2$?

A $3xy - 12x^2 + 2yx$ **B** $5xy - 12x^2$ **C** $-7x^2y$ **D** $5xy - 4x^2$

4 Which expression is equivalent to $4 - 8k$?

A $8k - 4$ **B** $8k + 4$ **C** $-8k + 4$ **D** $-8k - 4$

Calculator allowed

5 Which expression is equivalent to $12x - 9y$?

A $9(3x - y)$ **B** $12(x - 9y)$ **C** $3(4x - 3)$ **D** $3(4x - 3y)$

6 If $a = 2$, what is the value of $\frac{3a - 4}{2a}$?

A 2 **B** $\frac{1}{2}$ **C** -2 **D** $-\frac{1}{2}$

7 A rule for y in terms of x is $y = 12 - 3x$. If $x = 2.3$, then the value of y is:

A 5.1 **B** 16.6 **C** 20.7 **D** 34.5

8 Dylan is x years old. His brother, Liam, is 8 years old. John's age is three times Dylan's age, plus Liam's age. Find an expression in terms of x for John's age.

A $x + 11$ **B** $8x + 3$ **C** $3x + 8$ **D** $3x - 8$

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Planes, trains, boats and automobiles

What are the best ways to transport goods from one country to another? You have a budget of $5 million to purchase goods from Asian capital cities and transport them to different cities in Australia. Should you transport the goods to each different city directly, or move everything to a central city first? What methods of transport are the cheapest? Your task is to use maps and calculations to find the most efficient routes and the best way to make a profit.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

The scale of the universe

You live in a universe full of strange, interesting creatures and objects of vastly different sizes. How large and small can these different things be, relative to each other? Your task is to explore the scale of the universe, using appropriate mathematics to compare the sizes of different kinds of creatures and objects.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Fishing, angling, netting

What is the real difference between fishing with a rod and fishing with a net? In 2015, the Victorian Government decided to phase out the use of commercial fishing nets in Port Phillip Bay. Your task is to model the effects of different fishing methods on fish populations and fishing behaviours.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

Quick division

Explore how computers work with numbers by learning how an algorithm can test whether numbers will divide into each other or not.

4

Ratio and rate

4

What a dummy! Would you create a fake passenger just so you could travel faster on the freeway?

In peak-hour traffic on freeways, there may be up to 20 'driver-only' vehicles for every 1 vehicle with passengers. To encourage people to share their vehicles and take more passengers, many places now have transit lanes. Only vehicles with passengers are allowed to travel in transit lanes. Because these lanes have fewer vehicles, travelling in them is faster. By sharing vehicles, people also save time, fuel, money and the environment.

One driver put a dummy in the passenger seat to make it look like there was a passenger, so he could drive in the transit lane. He was caught and paid a large fine. Many people around the world from places including New Zealand, Australia and the USA have all had this same idea—and have been caught.

Forum

How do you think the comparison of the number of driver-only vehicles to the number of passenger-carrying vehicles might change outside of peak hour?

Can you think of a time or a place where the situation might be the opposite: there is 1 'driver-only' vehicle for every 20 vehicles carrying passengers?

Why learn this?

An understanding of ratios is important for completing many tasks. A baker making a cake, a builder mixing concrete, a photographer enlarging an image or a cartographer drawing a map all need skills in working with ratios and scale factors. Rates similarly enable you to compare quantities of different types, or to see how an amount, such as a population, is changing over time.

After completing this chapter you will be able to:

- write and simplify ratios
- understand the relationship between ratios, fractions and percentages
- use scale factors and unit ratios to find unknown quantities
- use ratios to calculate one quantity given the other
- use scale ratios to interpret scale drawings, diagrams and maps
- share a quantity in a given ratio
- calculate with rates such as speed
- use the unitary method to compare different rates
- calculate population growth rates.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Copy and complete the following conversions.

(a) 2000 mm = ______ cm **(b)** 3.6 km = ______ m

(c) 4 t = ______ kg **(d)** 350 mL = ______ L

2 Copy and complete the following pairs of equivalent fractions.

(a) $\frac{2}{3} = \frac{8}{\square}$ **(b)** $\frac{\square}{2} = \frac{21}{6}$

3 Simplify each of the following by cancelling common factors and converting improper fractions to mixed numbers.

(a) $\frac{36}{48}$ **(b)** $\frac{25}{15}$ **(c)** $\frac{35}{49}$ **(d)** $\frac{100}{76}$

4 Convert to improper fractions. **(a)** $2\frac{1}{7}$ **(b)** $2\frac{3}{8}$

5 Write the following:

(a) as mixed numbers in simplest form

(b) as decimals.

(i) $\frac{12}{5}$ **(ii)** $\frac{9}{2}$ **(iii)** $\frac{25}{8}$ **(iv)** $\frac{52}{10}$

6 Round the following to 2 decimal places.

(a) 0.525 17 **(b)** 3.899

7 Find:

(a) 30% of 150 **(b)** 8% of 40 **(c)** 102% of 76

8 **(a)** 5 out of 20 students buy their lunch at the school canteen. What percentage is this?

(b) 9 out of 16 pet owners own dogs. What percentage is this?

9 Solve the following equations. **(a)** $\frac{x}{3} = 7$ **(b)** $\frac{d}{8} = \frac{3}{4}$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Ratios

In this activity, you will explore how to calculate a ratio made of other ratios.

Writing ratios

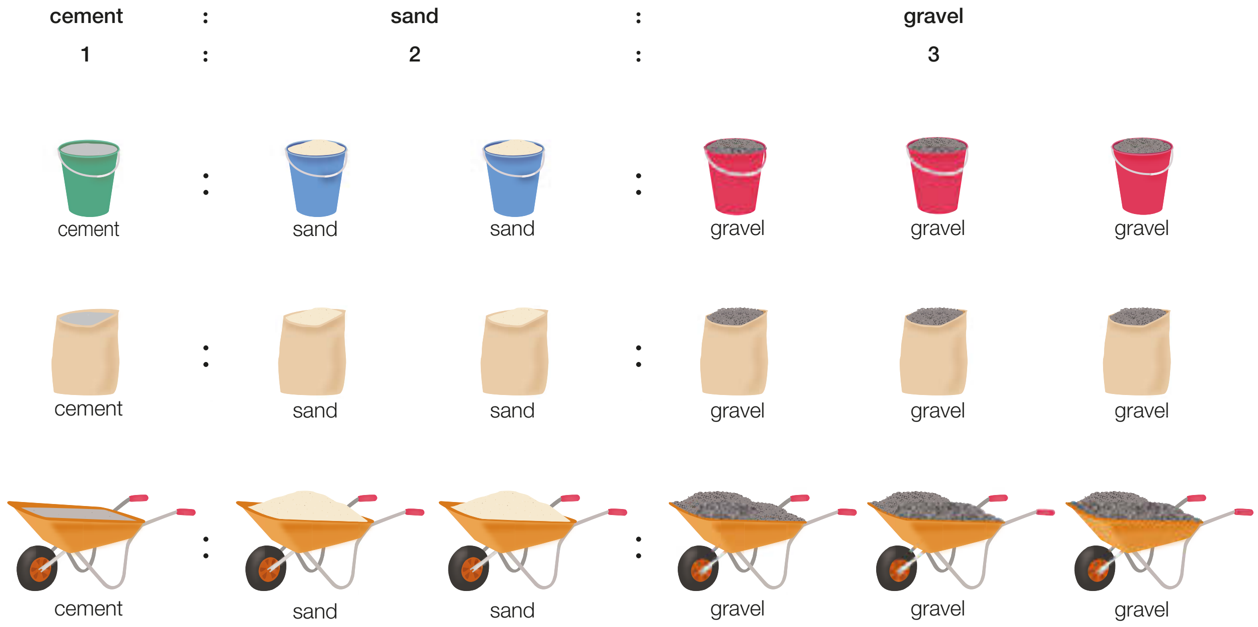
4.1

A **ratio** is a comparison of two or more amounts often written using a : symbol.

The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

A common ratio used for making concrete is to mix 1 part cement with 2 parts sand and 3 parts gravel. You can write this ratio as cement : sand : gravel = 1 : 2 : 3.

The actual measured amounts of these ingredients don't matter. It is the ratio, the proportion of each ingredient compared with the others, that is important. You could measure them out using buckets, bags or wheelbarrows and still get the correct mixture (if the buckets, bags or wheelbarrows are each the same size).



Part : part ratios and part : whole ratios

Suppose that an animal shelter has 13 dogs, 11 cats and 6 birds. You could write a ratio that compares the number of cats to the number of dogs as 11 : 13. This type of ratio compares two separate parts of a whole and is called a **part : part ratio**. You could write another part : part ratio that compared the number of cats, birds and dogs as 11 : 6 : 13. (Note that the order of the numbers in the ratio is the same as the written order of the animals.)

You can also write a ratio that compares the number of cats to the total number of animals as 11 : 30. This type of ratio compares one part to the whole, so it is called a **part : whole ratio**.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

The estimated ratio of the number of insects in the world to the number of humans is 200 million to 1!

Ratios, fractions and percentages

It is often useful to express part : whole ratios as fractions or percentages of the whole.

For example, the ratio cats : total animals of 11 : 30 means that $\frac{11}{30}$ of the animals are cats, which can be converted to $36.\dot{6}\%$. The number of dogs could be written as $\frac{13}{30}$, or $43.\dot{3}\%$.

Part : part ratios can also be expressed as fractions or percentages. If the ratio of cats to dogs is 11 : 13, then you can say that the number of cats is $\frac{11}{13}$, or 84.6% of the number of dogs. The number of dogs is $\frac{13}{11}$, or 118.2% of the number of cats. The important thing to understand is that you are writing one part as a fraction of the other part, not as a fraction of the overall whole.

Simplifying ratios

When the paint in the groups of tins below is mixed, each group gives the same shade of purple. The ratio of blue paint tins to red paint tins is 1 : 3 and 2 : 6 and 4 : 12 in each group. These ratios are **equivalent ratios**, because they represent the same proportions.

When the paint in each group of tins is mixed, the fraction of red paint in each mixture will be $\frac{3}{4}, \frac{6}{8}, \frac{12}{16}$, respectively. These three fractions are equivalent and all simplify to $\frac{3}{4}$.

Ratios can be simplified in a similar way to fractions, by dividing each number in the ratio by a common factor. If you divide by the highest common factor (HCF), you find the ratio in **simplest form**. For example, the HCF of the numbers in the ratio 4 : 12 is 4.

$$\begin{array}{rcccl} & 4 & : & 12 & \\ \div 4 & \downarrow & & \downarrow & \div 4 \\ = & 1 & : & 3 & \end{array}$$

1 : 3

2 : 6

4 : 12

To write a ratio in simplest form, divide each number in the ratio by the highest common factor (HCF).

Worked example 1

W.E. 1

A box of chocolates has 7 plain, 5 nut and 3 soft-centred chocolates.

(a) Write a ratio that compares the number of nut chocolates to the number of plain chocolates, in simplest form.

(b) Write a ratio that compares the number of soft-centred chocolates to the total number in the box, in simplest form.

(c) Write the number of soft-centred chocolates as a percentage of the total number in the box.

(d) Write the number of soft-centred chocolates as a fraction of the number of plain chocolates.

Thinking	Working
(a) Write the ratio in words, in the correct order, with the numbers underneath. Simplify by dividing by a common factor, if possible. (Here, there are no common factors, because the ratio is already in simplest form.)	(a) Nut : plain = 5 : 7
(b) **1** Find the number of parts in the whole (the total number of chocolates).	(b) 7 + 5 + 3 = 15
2 Write the ratio in words, in the correct order, then write the numbers underneath. Simplify by dividing by the HCF. (Here, the HCF is 3.)	Soft-centred : total = 3 : 15 = 1 : 5 (÷ 3)
(c) Write the part : whole ratio as a fraction, then convert the fraction to a percentage.	(c) $\frac{1}{5}$ $= \frac{20}{100}$ = 20% 20% of the chocolates in the box are soft-centred.
(d) Write the part : part ratio, then use the numbers in the ratio to write the fraction. Simplify the fraction if possible.	(d) Soft-centred : plain = 3 : 7 $= \frac{3}{7}$

4.1 Writing ratios

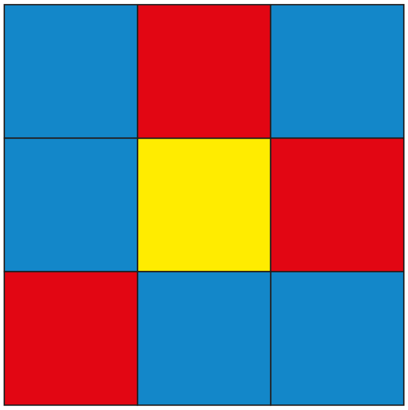
Navigator

1, 2, 3 (column 1), 4, 5, 6, 7, 8 (a), 9, 12, 14, 15	2, 3 (column 2), 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15	2, 3 (column 3), 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15

Answers p. 636

Fluency

1 Look at the following coloured squares that appeared on one side of a Rubik's cube and then write the following ratios.



- **(a)** blue squares : red squares
- **(b)** red squares : blue squares
- **(c)** yellow squares : blue squares
- **(d)** yellow squares : red squares
- **(e)** blue squares : total squares
- **(f)** yellow squares : total squares
- **(g)** red squares : total squares in simplest form
- **(h)** blue squares : red and yellow squares

W.E. 1

2 A greengrocer has 9 boxes of apples, 7 boxes of bananas and 4 boxes of pears.

- **(a)** Write a ratio that compares the number of boxes of pears to the number of boxes of apples, in simplest form.
- **(b)** Write a ratio that compares the number of boxes of pears to the total number of boxes of fruit, in simplest form.
- **(c)** Write the number of boxes of pears as a percentage of the total number of boxes of fruit.
- **(d)** Write the number of boxes of bananas as a fraction of the number of boxes of apples.

3 Write each of the following ratios in simplest form.

(a) 2 : 8	**(b)** 3 : 15	**(c)** 4 : 16
(d) 4 : 10	**(e)** 12 : 34	**(f)** 180 : 150
(g) 20 : 300	**(h)** 500 : 4000	**(i)** 225 : 325
(j) 25 : 10 : 5	**(k)** 24 : 4 : 8	**(l)** 33 : 15 : 27
(m) 3 : 12 : 6	**(n)** 36 : 4 : 16	**(o)** 670 : 500 : 215

4 There are 26 students in a Year 8 maths class. 15 of them live in town.

(a) The ratio of students living out of town to students living in town is:

A 15 : 26 **B** 26 : 15 **C** 15 : 11 **D** 11 : 15

(b) The percentage of the class that live out of town (to 1 decimal place) is:

A 42.3% **B** 57.7% **C** 73.3% **D** 136.4%

(c) The number of students living out of town as a fraction of the number of students living in town is:

A $\frac{11}{26}$ **B** $\frac{15}{26}$ **C** $\frac{11}{15}$ **D** $\frac{15}{11}$

5 Lara mixes 8 litres of white paint with 3 litres of yellow paint and 2 litres of red paint to get the colour she wants for her living room. Find, in simplest form:

- **(a)** the ratio of yellow paint to white paint to red paint
- **(b)** the ratio of white paint to the total amount of paint in the mixture
- **(c)** the ratio of yellow paint to the total amount of paint in the mixture
- **(d)** the fraction of red paint in the mixture.

6 **(a)** Aimi is at an AFL match and estimates that for every 2 Sydney supporters there are 3 Brisbane supporters. Write her estimate as a ratio.

- **(b)** Write the number of Brisbane supporters as a fraction of the number of Sydney supporters.
- **(c)** Write the number of Sydney supporters as a fraction of the number of Brisbane supporters.

Understanding

7 A plank of wood 24 m long has an 8 m long piece cut off it. Express, in simplest form:

- **(a)** the ratio of the length remaining to the length cut off
- **(b)** the ratio of the length cut off to the length of the original piece
- **(c)** the length of the shorter piece as a fraction of the longer piece
- **(d)** the length of the longer piece as a percentage of the original whole.

8 A lawnmower's fuel mixture is 4% oil. The rest of the mixture is petrol.

(a) Write the ratio of petrol to oil in simplest form.

(b) Oliver has 200 mL of oil. How much petrol should he add to make the correct fuel mixture for his lawnmower?

9 Gek is a travel agent selling return airfares to London for $1950 and to Bangkok for $1150.

(a) Write a ratio comparing the London fare to the Bangkok fare, in simplest form.

(b) Write the London fare as a percentage of the Bangkok fare to the nearest whole number.

10 Mira mixes 24 buckets of sand with 6 buckets of cement to prepare a mortar mix.

(a) Write the ratio of sand to cement in simplest form.

(b) What fraction of her mortar mixture is cement?

11 Ryan uses a mixture of petrol and oil to power his go-kart. 2% of the mixture is oil, the rest is petrol.

(a) Write the ratio of oil to petrol in simplest form.

(b) What fraction of the mixture is oil?

Reasoning

12 In his art lesson, Milan made pink paint by mixing 4 cups of red paint with 4 cups of white paint. His friend Ali mixed 6 cups of red with 3 cups of white.

(a) Write the ratio, in simplest form, of red paint to white paint for:

(i) Milan (ii) Ali

(b) Whose mixture would have produced a whiter shade of pink? Explain your answer.

13 Jo is making cordial. She follows the instructions and mixes 100 mL of cordial concentrate with 400 mL of water.

(a) What percentage of the total mixture is concentrate?

(b) Jo decides the cordial is too weak. She adds another 50 mL of concentrate. What fraction of the resulting mixture is concentrate?

Open-ended

14 **(a)** Write three ratios that are equivalent to 5 : 2. Check your answers by simplifying them.

(b) Write three ratios that are equivalent to 50 : 75. Check your answers by simplifying them.

15 A group of primary school students were asked to name their favourite Australian animal. The results were:

Kangaroo	3
Koala	5
Platypus	7
Quokka	8

Use these results to write two fractions and two ratios that give some information about the group of students.

Game

The game of Nim

Equipment required: a heap of counters or stones

There are several different versions of this game. It is based on an ancient Chinese game called *jian shizi*, which means 'choosing stones'.

How to play:
Make two randomly sized piles of stones or counters.

When it is your turn, you can either take the same number of stones from *both* heaps, or a number of stones from *one* heap.

How to win:
The player who takes the last stone is the winner.

Alternative rules:
Each turn, you must take one or more stones from only *one* heap (not from both). You can take from different heaps on different turns.

Simplifying ratios

4.2

Ratios and units

Usually ratios are used to compare quantities of the same type, such as lengths, masses or numbers of students or animals.

When writing ratios, you do not write the measurement units of the quantities being compared, because it is assumed that the units are the same. To write a ratio comparing quantities with different units, you must first convert the amounts to the same unit (the smaller unit is usually more convenient). When the numbers are converted so that the units are the same, then you don't need to write the units.

> Ratios should be written in simplest form, using whole numbers only.
>
> If a ratio contains fractions or decimals, multiply all parts in the ratio by the same number to convert them to an equivalent whole number ratio.
>
> Each quantity in the ratio must be expressed in the same units, which should not be written.

Worked example 2

W.E. 2

Write the ratio 25 m : 1500 cm in simplest form.

Thinking	Working
1 Are the units the same? If not, write both quantities in the smaller unit.	$25\text{ m} : 1500\text{ cm}$ $= 2500\text{ cm} : 1500\text{ cm}$
2 Remove the units.	$= 2500 : 1500$
3 Divide both numbers by the HCF (500).	$= 5 : 3$

Worked example 3

W.E. 3

Write the ratio $2\frac{1}{2} : 1 : 3$ in simplest form.

Thinking	Working
1 Write any mixed numbers as improper fractions.	$2\frac{1}{2} : 1 : 3$ $= \frac{5}{2} : 1 : 3$
2 Multiply by the denominator to turn the fraction into a whole number. Multiply the other numbers in the ratio by the denominator as well to maintain the ratio.	$= \frac{5}{2} \times 2 : 1 \times 2 : 3 \times 2$ $= 5 : 2 : 6$

Worked example 4

W.E. 4

Write the ratio 3.5 : 4.25 in simplest form.

Thinking	Working
1 Multiply both sides by 10. Keep multiplying by 10 until all parts in the ratio are whole numbers.	3.5 : 4.25 = 35 : 42.5 = 350 : 425
2 Simplify if possible by cancelling common factors. (In this case, both numbers have been divided by 5, then by 5 again.)	= 70 : 85 = 14 : 17

4.2 Simplifying ratios

Navigator

Answers p. 637

1, 2 (column 1), 3 (row 1), 4 (a–h), 5, 6, 8, 14 (a–b), 17	1 (i–m), 2 (column 2), 3 (rows 1–2), 4 (a–h), 6, 7, 8, 10, 11, 12, 14, 15, 17	2 (column 2), 3 (rows 2–3), 4 (e–l), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18

Equipment required: ruler for Question **17**

Fluency

1 Write each of the following ratios in simplest form.

(a) 2 : 4 **(b)** 96 : 48 **(c)** 10 : 15 **(d)** 75 : 60 **(e)** 50 : 125
(f) 750 : 75 **(g)** 49 : 56 **(h)** 60 : 65 **(i)** 100 : 104 **(j)** 3500 : 2100
(k) 7 : 14 : 21 **(l)** 60 : 100 : 40 **(m)** 35 : 25 : 20

W.E. 2

2 Write each of the following ratios in simplest form.

(a) 5 cm : 3 mm **(b)** 75 cm : 1.5 m
(c) 5 km : 600 m : 300 m **(d)** 8000 m : 5 km : 4.5 km
(e) 60 cents : $2 **(f)** $3.50 : 45 cents
(g) 650 g : 3.5 kg **(h)** 6 L : 500 mL
(i) 45 seconds : 2 minutes **(j)** 480 kg : 3 tonnes
(k) 8 months : 1 year **(l)** 2.5 L : 2000 mL
(m) 8 hours : 180 minutes **(n)** 10 minutes : 20 seconds
(o) 5400 g : 6 kg : 1.2 kg **(p)** 4 days : 4 weeks : 1 fortnight
(q) 80 cm : 2.5 m : 20 cm **(r)** $19 : $20 : 60 cents

Make sure quantities are expressed in the same units.

W.E. 3

3 Write each of the following ratios in simplest form.

(a) $3\frac{1}{2}:1$ **(b)** $4:3\frac{2}{3}$ **(c)** $4:2\frac{1}{3}$ **(d)** $5\frac{1}{8}:2$

(e) $3\frac{1}{3}:2$ **(f)** $4:3\frac{1}{5}$ **(g)** $5:2\frac{1}{4}$ **(h)** $7\frac{3}{4}:6$

(i) $5\frac{1}{7}:3$ **(j)** $3\frac{5}{6}:9$ **(k)** $6\frac{1}{8}:9$ **(l)** $6:6\frac{2}{3}$

W.E. 4

4 Write each of the following ratios in simplest form.

(a) $4.7:3.1$ **(b)** $1.7:5.2$ **(c)** $2.9:3.4$ **(d)** $4.5:7.2$

(e) $0.75:1.55$ **(f)** $2.04:2.52$ **(g)** $2.82:3.09$ **(h)** $3.96:2.61$

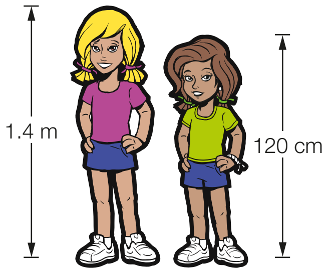
(i) $5.6:3.52$ **(j)** $8.1:2.75$ **(k)** $2.2:6.64$ **(l)** $2.84:6.8$

5 The simplest form of the ratio $6:33$ is:

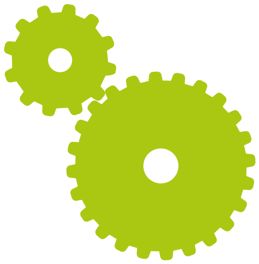
A $0.18:1$ **B** $2:11$ **C** $11:2$ **D** $1:5\frac{1}{2}$

6 Express each of the stated ratios in simplest form.

(a) The ratio of the taller girl's height to the shorter girl's height.

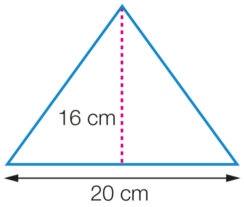


(b) The ratio of the number of teeth on the small gear to the big gear.

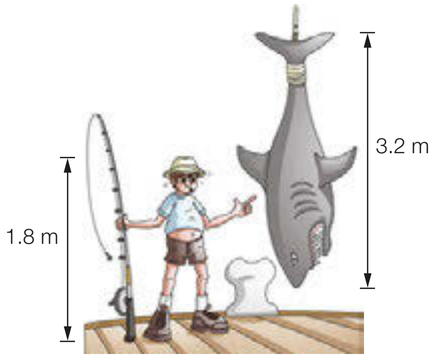


(c) The ratio of the larger can's mass to the smaller can's mass.

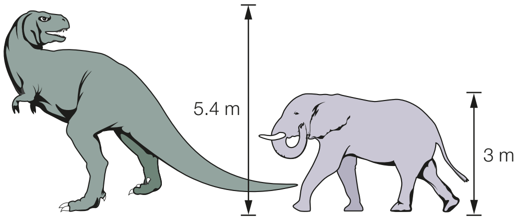
(d) The ratio of the triangle's height to its base length.



(e) The ratio of the shark's length to the person's height.



(f) The ratio of the dinosaur's height to the elephant's height.



Understanding

7 An unopened box of Megafibre breakfast cereal contains 1.3 kg of cereal. Another box that has been opened and partly eaten now contains 450 grams.

(a) What is the ratio of the contents of the opened box to the contents of the unopened box? Write your answer in simplest form.

(b) Write the mass of the opened box as a fraction of the mass of the unopened box.

8 **(a)** A carpenter makes shelves in three different lengths: 45 cm, 75 cm and 1.05 m. Write a ratio comparing the lengths, from shortest to longest, in simplest form.

(b) Orange juice comes in three different-sized bottles of 250 mL, 600 mL and 1 L. Write the ratio comparing the bottle sizes, from smallest to largest, in simplest form.

9 Nancy is 57 years old. Her grandson Will is 11 months old. Nancy is 1.7 m tall and Will is 89 cm tall.

(a) Write the ratio of Nancy's age to Will's age.

(b) Write Will's age as a fraction of his grandma's age.

(c) Write the ratio of Nancy's height to Will's height.

(d) Write Nancy's height as a percentage of Will's height.

10 Last year, Gloria grew 2 tonnes of peaches in her orchard, but her crop this year is down by 800 kg.

(a) Write the ratio of last year's crop to this year's crop, in simplest form.

(b) Write this year's crop as a fraction of last year's crop.

(c) Write the decrease in the mass of the crop this year as a percentage of last year's crop.

11 Tasso has three part-time jobs which he works at for 20 hours, 12.5 hours and 4.5 hours per week, respectively.

(a) Write a ratio to compare the amount of time he spends at his three jobs.

(b) What is the ratio of total hours worked to the hours at his first job?

(c) What percentage of his total hours does he spend at his second job? Write your answer to the nearest whole per cent.

12 Find the simplest ratio of two children's ages if they are:

(a) 14 and $10\frac{1}{2}$ years old

(b) 12 years 3 months and 7 years old.

13 Silvio's fruit salad contains $6\frac{3}{4}$ apples and $4\frac{1}{2}$ kiwifruit. (He ate the missing fractions while he was making the fruit salad.) Silvio is delighted with the taste of the dessert, and wants to find the ratio of the fruits in terms of whole pieces of fruit. What is the ratio of apples to kiwifruit in simplest form?

Reasoning

14 A fruit juice mixture contains 1.2 L of orange juice and 0.8 L of pineapple juice.

(a) What is the simplest ratio of orange to pineapple juice in the mixture?

(b) If 1 L of water is added to the mixture, what is the new ratio of orange juice to pineapple juice to water?

(c) What fraction of the final mixture is orange juice?

15 Pardeep is making a fruit punch for a party. She mixes 2.5 L juice with 1.25 L soda water and 750 mL ginger beer.

(a) Write the ratio for the 3 parts of the mixture, in simplest form.

(b) What fraction of the mixture is ginger beer? Write your answer in simplest form.

(c) Pardeep mixes the punch thoroughly, then divides it equally into 3 smaller containers. How much ginger beer is there in each container?

16 Troy completes a triathlon in 2 hours and 10 minutes. The triathlon consists of 3 'legs' (parts): a 1500 m swim, a 40 km ride, and a 10 km run. Troy's swim and run take 20 minutes and 40 minutes, respectively.

(a) Write the ratio of (swim distance) : (ride distance) : (run distance) in simplest form.

(b) Write the ratio of (swim time) : (ride time) : (run time) in simplest form.

(c) Which 'leg' of the triathlon takes the longest time? Write the time taken for this leg as a percentage of the total time to the nearest whole number.

(d) Consider the two ratios you wrote for parts **(a)** and **(b)**. Explain why the ratios are not equivalent. (Why are the times to complete each leg not in the same proportion as the distances of the legs?)

Open-ended

17 Use a ruler to draw a rectangle that has a length 1.5 times as long as its width. Write the ratio length : width in simplest form. Compare your rectangle and ratio with that of a classmate. Is their rectangle the same size as yours? Is their ratio the same?

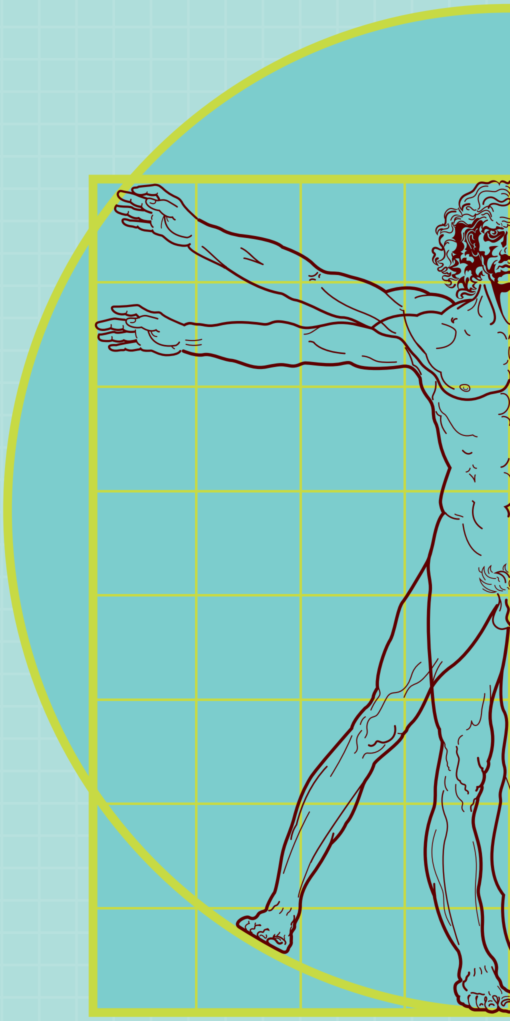
18 Troy wants to improve his time in the triathlon (see Question **16**). Suggest two ways he could allocate his time between the three sports when devising a monthly training program.

Gamespace

Human ratios

Equipment required:
ruler and string or tape measure

Throughout history, humans have had different ideas about what makes a correctly proportioned, 'beautiful' face or body. Check out these strange ideas for yourself.



Gulliver's Travels

In Jonathan Swift's classic story *Gulliver's Travels*, Lemuel Gulliver is shipwrecked and swims to the island of Lilliput, where the people are an average height of 14 cm. As Gulliver's only clothes are those he is wearing, the Lilliputians make new clothing for him. They take only two measurements—the distance from his neck to his knee and then the distance around his thumb. From this they calculate his body shape using these ratios:

- twice around the thumb is once around the wrist
- twice around the wrist is once around the neck
- twice around the neck is once around the waist.

1 Write the ratios from the sentence above.

2 Check yourself and students in your class to see if these ratios are true.

Vitruvian Man

Many people believe Leonardo Da Vinci used a number of specific ratios in his famous drawing *Vitruvian Man*, shown on this page.

- The maximum width of a person's shoulders is always a quarter of their height.
- The length of a person's outstretched arms is equal to their height.
- The length of a person's hand, from the wrist to the end of their longest finger, is one-tenth of their height.

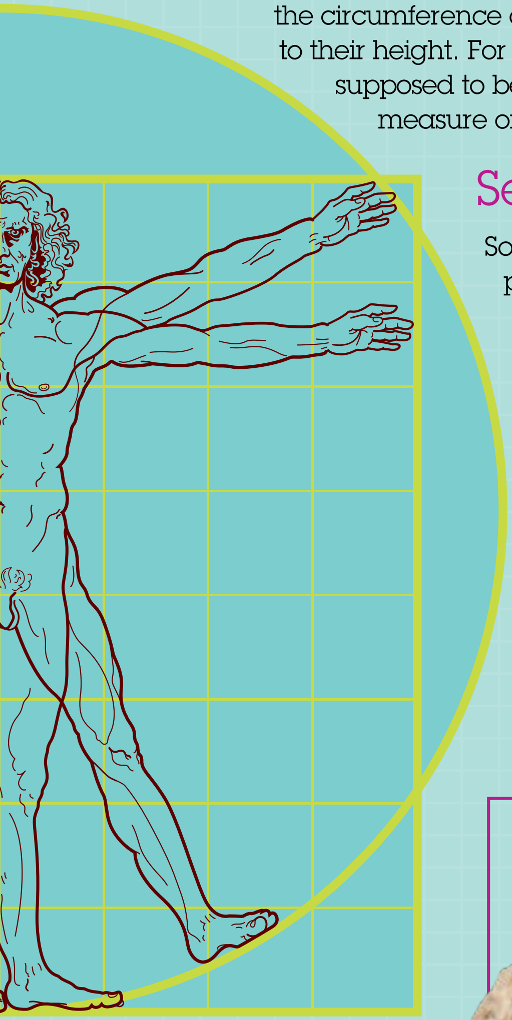
3 Write these sentences as ratios.

4 Which of these ratios do you think is likely to be true?

5 Test these ratios on yourself and students in your class.

Heads up

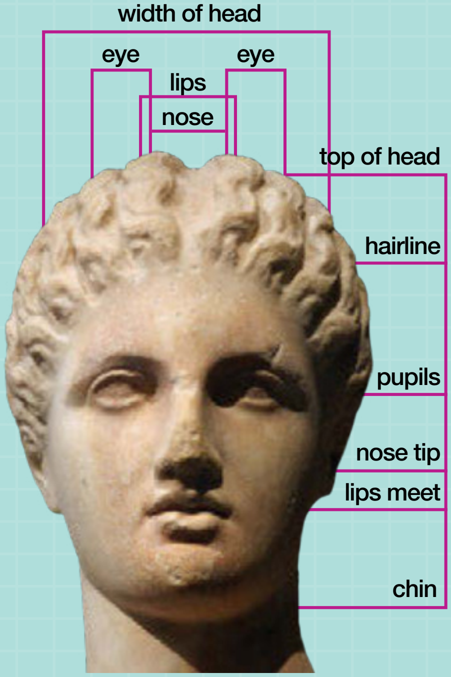
Another interesting body proportion is that of the circumference of a person's head compared to their height. For most people, this ratio is supposed to be 1 : 3. Test this out using a tape measure or some string and a ruler.



Seven up

Some ancient Greeks had particular ideas of what the 'perfect' human was like.

They believed that the 'perfect' human body was 7 heads tall. Find out if this ratio applies to you by measuring from your chin to the top of your head, and comparing to your height.



The perfect face?

Some ancient Greeks believed that a perfect face was in proportion to the 'golden ratio' of 1.618. (See the Maths 4 Real on page 238 to investigate this further.)

6 Using your ruler and the statue face pictured, find each measurement listed below to the nearest millimetre. Remember, you are measuring the distance between the two locations. Write your measurements next to the given letters *a*–*l*.

a = top-of-head to chin
b = top-of-head to pupil
c = pupil to nose tip
d = pupil to lip
e = width of nose
f = outside distance between eyes
g = width of head
h = hairline to pupil
i = nose tip to chin
j = lips to chin
k = length of lips
l = nose tip to lips

7 Now, use the measurements calculated previously to complete the following table. Remember: $\frac{a}{g}$ means measurement a divided by measurement g (round to 3 d.p.).

Ratio	Value
$\frac{a}{g}$	
$\frac{b}{g}$	
$\frac{i}{j}$	
$\frac{i}{c}$	
$\frac{e}{l}$	
$\frac{f}{h}$	
$\frac{k}{e}$	

Are your ratios in the table the same as the golden ratio of 1.618?

What can you conclude about facial proportions? Is this ancient Greek idea of the 'perfect face' realistic?

4.3

Unit ratios and scale factors

Ratios that contain large numbers or numbers that cannot be simplified can be more challenging to understand and interpret. For example, consider the two buildings shown.

The Empire State Building is 443 m tall.

The Sydney Tower is 305 m tall.

You can write the ratio of their heights in simplest form as 443 : 305. This is correct, but it does not really help you to *understand* their relative sizes. The Empire State Building is taller, but how many times taller?

Writing the ratio of their heights as a **unit ratio** makes the comparison more meaningful.

To write a unit ratio, divide both numbers in the ratio by the second number.

Empire State Building : Sydney Tower
$443 : 305$
$\div 305 \quad \div 305$
$1.45 : 1$ (rounded to 2 decimal places)

The unit ratio tells you that the Empire State Building is 1.45, or close to $1\frac{1}{2}$ times, the height of the Sydney Tower. Because $1.0 = 100\%$, you can also say that the Empire State Building's height is 145% of the height of the Sydney Tower or that its height is 45% taller than the height of the Sydney Tower.

1.45 is the **scale factor** you multiply the height of the Sydney Tower by to get the height of the Empire State Building:

$305 \text{ m} \times 1.45 \approx 442 \text{ m}$ (allowing a small error due to rounding)

Dividing the height of the Empire State Building by 1.45 gives you the height of the Sydney Tower:

$443 \div 1.45 \approx 306 \text{ m}$ (with a small rounding error)

- To write the ratio $a : b$ as a unit ratio, calculate $a \div b$ and express the ratio as $\frac{a}{b} : 1$ (divide both of the numbers in the ratio by b).
- The quotient $\frac{a}{b}$ is the scale factor. It tells you the size of a in terms of how many times the size of b it is. It is the number you must multiply b by to get a.
- If a is greater than b ($a > b$), then $\frac{a}{b}$ is greater than 1, and more than 100%.
- If a is smaller than b ($a < b$), then $\frac{a}{b}$ is less than 1, and less than 100%.

Worked example 5

W.E. 5

Write 34 : 19 as a unit ratio, rounded to 2 decimal places.

Thinking	Working
1 Divide both numbers in the ratio by the second number (19 in this case).	$34 : 19$ $= \frac{34}{19} : \frac{19}{19}$
2 Write your answer, rounding to the specified number of decimal places.	$= 1.79 : 1$ (2 d.p.)

Using the scale factor to find an unknown quantity

If you have a unit ratio or a scale factor that compares two quantities, you can use it to find one of the quantities if you know the other one. The following examples show different types of scale factor questions and different methods of using the scale factor to find an unknown quantity.

Worked example 6

W.E. 6

The ratio of the height of an elephant to the height of a lion is 3.07 : 1. If the elephant's height is 4 m, what is the height of the lion? Write your answer correct to 1 decimal place.

Method 1: Use the scale factor

Thinking	Working
1 Write the unit ratio, first in words, then in numbers underneath.	height of elephant : height of lion $= 3.07 : 1$
2 You know the quantity on the left side of the ratio (the height of the elephant). To find the quantity on the right side of the ratio, divide by the scale factor. Round to the number of decimal places the question asks for.	$4 \div 3.07$ $= 1.3$ m (1 d.p.)
3 Write your answer in words.	The lion is 1.3 m tall.

Method 2: Write and solve an equation

Thinking	Working
1 Write the ratio in words with the unit ratio underneath. Write a ratio that compares the known quantity (the height of the elephant) with the unknown. Use a pronumeral to represent the unknown. (Here, L represents the height of the lion.)	elephant's height : lion's height $= 3.07:1$ $= 4:L$
2 Write the ratio in fraction form with the unknown, the height of the lion (L), as the numerator.	$\frac{\text{lion's height}}{\text{elephant's height}} = \frac{L}{4}$
3 Form an equation by writing the unit ratio as an equivalent fraction on the other side of an equals symbol.	$\frac{L}{4} = \frac{1}{3.07}$
4 Solve the equation. (Here, multiply both sides by 4 to solve for L.) Round to the number of decimal places the question asks for.	$L = \frac{1}{3.07} \times 4$ $= 1.3$ (1 d.p.)
5 Write your answer in words.	The lion is 1.3 m tall.

Worked example 7

W.E. 7

In a recipe for pancakes there is four times as much flour as sugar and 1.6 times as much flour as milk.

(a) If you use 3 cups of sugar how much flour is required?

(b) If you use 2 cups of flour how much milk is required?

Method 1: Use a unit ratio

Thinking	Working
(a) 1 Write a unit ratio, first in words, then in numbers underneath.	(a) flour : sugar $= 4:1$
2 You know the quantity on the right side of the ratio (the cups of sugar). To find the quantity on the left side of the ratio, multiply by the scale factor.	3 cups of sugar × 4 = 12 cups of flour
(b) 1 Write a unit ratio, first in words, then in numbers underneath.	(b) flour : milk $= 1.6:1$
2 You know the quantity on the left side of the ratio (the cups of flour). To find the quantity on the right side of the ratio, divide by the scale factor.	2 cups of flour ÷ 1.6 = 1.25 cups of milk

Method 2: Write and solve an equation

Thinking	Working
(a) 1 Give pronumerals to the two quantities.	(a) Let f = number of cups of flour Let s = number of cups of sugar
2 Use the given scale factor to write a formula that shows the relationship between these two quantities.	$f = 4s$
3 Substitute the known value (the amount of sugar) into the formula and evaluate.	$f = 4 \times 3$ $f = 12$
4 Write the answer in words.	12 cups of flour are needed.
(b) 1 Give pronumerals to the two quantities.	(b) Let f = number of cups of flour Let m = number of cups of milk
2 Write a formula that shows the relationship between these two quantities.	$f = 1.6m$
3 Substitute the known value (the amount of flour) into the formula.	$2 = 1.6 \times m$
4 Solve the equation. (Here, solve for *m* by dividing both sides by 1.6.)	$m = \frac{2}{1.6}$ $m = 1.25$
5 Write the answer in words.	1.25 cups of milk are needed.

Worked example 8

W.E. 8

A stegosaurus is believed to have weighed about 5 tonnes, but with a brain that weighed only 50 g. How many times heavier is the stegosaurus than its brain?

Thinking	Working
1 Write the ratio and express both quantities in the same unit (grams).	5 tonnes : 50 g = 5 000 000 g : 50 g
2 Convert to a unit ratio by dividing both numbers in the ratio by the second number (50) and leaving out the units.	$= \frac{5\,000\,000}{50} : \frac{50}{50}$ $= 100\,000 : 1$
3 Write your answer in words.	The stegosaurus is 100 000 times heavier than its brain.

4.3 Unit ratios and scale factors

Navigator

Answers p. 637

1 (columns 1–2), 2, 3, 4, 5, 8, 11, 16	1 (columns 2–3), 2, 3, 4, 5, 6, 8, 9, 11, 12, 13, 15 (a–b), 16, 17	1 (columns 2–3), 2, 3, 4 (a–b), 6, 7, 8, 9, 10, 12, 13, 14, 15, 17

Equipment required: calculator

Fluency

W.E. 5

1 Write each of the following as a unit ratio, rounded to 2 decimal places.

(a) 17 : 16 **(b)** 733 : 525 **(c)** 766 : 329

(d) 29 : 37 **(e)** 1087 : 5230 **(f)** 3078 : 2100

(g) 65.5 : 13.4 **(h)** 4.25 : 0.8 **(i)** 3.56 : 5

(j) 6.25 : 8.33 **(k)** 20.375 : 18.116 **(l)** 651.9 : 300.7

W.E. 6

2 **(a)** The ratio of Thanh's height to Hamish's height is 1.16 : 1. If Hamish has a height of 147 cm, what is Thanh's height? Write your answer correct to 1 decimal place.

(b) The ratio of the value of the Australian dollar to the British pound sterling is 2.38 : 1. How many Australian dollars will Patrick get for 250 pounds?

(c) The times taken for the planets Venus and Mercury to orbit around the Sun are in the ratio 2.55 : 1. If Venus takes 224.65 days to orbit the Sun, then how many days does Mercury take?

(d) At Callum's Pizza Shop the unit ratio of ham to pineapple on a Hawaiian pizza is 3.5 : 1. If 200 g of ham is used on a large pizza, how much pineapple is used?

W.E. 7

3 **(a)** Megan's recipe for concrete requires 3.7 kg of cement for every litre of water.

(i) If Megan uses 10 kg of cement how much water is required?

(ii) If Megan uses 6 litres of water how much cement is required?

(b) There are roughly 6.3 sheep for every person in New Zealand. If the population of New Zealand is 4.7 million, how many sheep are there?

(c) Miriam saves three times as much money as Shannon does. If Miriam saves \$150 how much does Shannon save?

(d) The circumference (perimeter) of a circle is approximately 3.14 times the length of the diameter (the distance across the circle through the centre). What is the circumference of a circle whose diameter is 14.5 cm?

4 Round your answers to the following questions to 2 decimal places.

W.E. 8

(a) An adult female elephant weighs 3400 kg. A baby elephant weighs 160 kg. How many times heavier is the adult elephant than the baby?

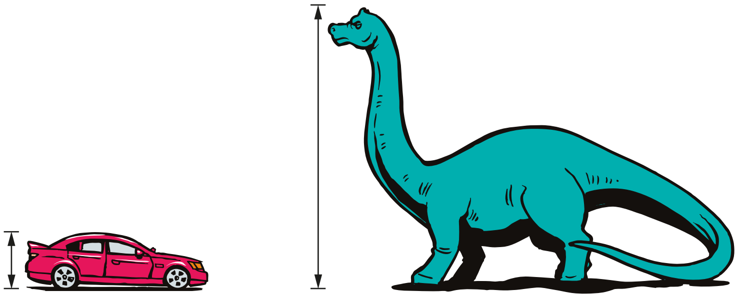
(b) The populations of Australia and Indonesia in 2016 are shown below.

Australia: 24 million people Indonesia: 260 million people

How many times greater than the population of Australia is the population of Indonesia?

(c) A Formula One car has a top speed of 415 kilometres per hour (km/h). A family car has a top speed of 230 km/h. How many times faster is the top speed of the Formula One car than the family car?

(d) Look at the heights indicated on the picture below. How many times taller than the car is the dinosaur?



Understanding

5 The two longest rivers in the world are the Nile and the Amazon. The Nile is 1.034 times as long as the Amazon. If the Amazon is 6448 km long, how long is the Nile? Give your answer to the nearest km.

6 The diameters of the planets Jupiter and Earth are in the ratio 20.93 : 1.

(a) If Earth has a diameter of 12 760 km, then what is the diameter of Jupiter, to the nearest km?

(b) If a model of the Earth is tennis-ball sized (6.36 cm diameter), then what should be the diameter size of a model of Jupiter, for the models to be in correct proportion?

7 Lawnmower fuel is a mixture of petrol and oil. 25 litres of petrol are needed for every litre of oil in the mixture. Dion has 8 litres of petrol. How much oil does he need to add?

8 **(a)** Express the ratio of the heights of Mt Kosciuszko (2228 m) to Mt Everest (8848 m) as a unit ratio. Round your answer to 2 decimal places.

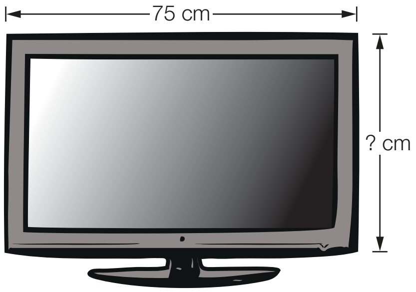
(b) How many times taller than Mt Kosciuszko is Mt Everest?

9 The ratio of the top speeds of the gazelle and the cheetah is 0.724 : 1. If a gazelle can run at a top speed of 76 kilometres per hour, how fast can a cheetah run? Give your answer to the nearest km/h.

10 A flat screen TV has an 'aspect ratio' of 16 : 9. The aspect ratio is the ratio of the screen's width to its height.

(a) Write the aspect ratio as a unit ratio.

(b) How tall is this TV screen, if it is 75 cm wide?



11 Copy and complete the following table.

Quantities	Ratio	Unit ratio	Scale factor	Explanation
$a : b$	4 : 5	0.8 : 1	0.8	a is 0.8 times the size of b
$x : y$	7 : 1	7 : 1		x is seven times as large as y
$m : p$	12 : 4			
	240 : 12			h is one-twentieth the size of g
	3.8 : 15.2			r is one-quarter the size of j
$d : e$	4000 : 20			

Reasoning

12 The system of measurement used in Australia before the metric system was called the imperial system. The scale factors for converting some of the imperial units of length are as follows.

Conversion	Scale factor
feet to inches	$\times 12$
feet to yards	$\times \frac{1}{3}$
miles to yards	$\times 1760$

(a) How many inches are in 3 feet?

(b) How many feet are in 10 yards?

(c) How many inches are in 6 feet?

(d) How many yards are in 3 miles?

(e) How many miles are in 10 560 feet?

(f) What would I do to convert from yards to inches?

(g) What would I do to convert from miles to feet?

13 A travel agent adds 3.5% to the cash price of a plane ticket if it is paid for by credit card.

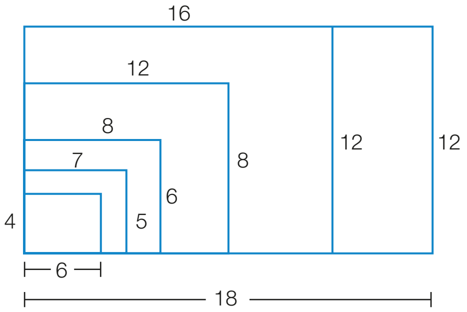
(a) Write the unit ratio for the ratio of the credit card price to the cash price.

(b) Meena paid $361.20 for a ticket with her credit card. What would she have paid if she had paid in cash? (Answer to the nearest cent.)

14 The scale factor between the height of a 1.75 m tall father and the height of his 12-year-old son is 1.08. Find the two possible heights for the son and explain how you arrived at each. Which do you think is more likely to be the correct height?

15 The aspect ratio of an image produced by a camera is the ratio of the image's length to its width. Images taken with many digital cameras have an aspect ratio of 3 : 2. The paper used to print photos comes in the following range of sizes (traditionally measured in inches). The relative sizes of the paper are shown in the diagram below.

Length (inches)	Width (inches)
6	4
7	5
8	6
12	8
16	12
18	12



(a) Copy the table and add two extra columns, one for the ratio $L:W$ and one for the unit ratio $\frac{L}{W}:1$. Complete both columns for each paper size.

(b) Use your table to decide which paper sizes provide the best 'fit' for an image that has an aspect ratio of 3 : 2. Explain your choice.

(c) Imagine that you work in a photographic printing business. A customer has some images with an aspect ratio of 3 : 2 and wants them printed on '7 by 5' paper. You sit down at your computer and start with an image in the smallest standard size of 6 by 4. To increase the width of the print from 4 to 5, you find that you must multiply by a scale factor of 1.25 ($4 \times 1.25 = 5$).

- **(i)** If you increase the length of the '6 by 4'–sized image by the same scale factor, then what is the new length of the image?
- **(ii)** What is the problem with this new image length?

(d) Describe a potential solution to this problem.

Open-ended

16 The following table gives the approximate unit ratio of (human life expectancy) : (animal's life expectancy) for some common animal pets.

Animal	Unit ratio
Blue-tongue lizard	3.6 : 1
Horse	3.8 : 1
Dog	5.7 : 1
Cat	6.3 : 1
Budgie	7.6 : 1
Rabbit	11.7 : 1
Guinea pig	16.9 : 1
Mouse	25 : 1

If you don't have any pets, ask a friend how old their pets are and use this information for Question **16**.

These ratios can be used to calculate how old an animal is in 'human years'.

(a) Calculate the ages in human years of any pets you have (if you know how old they are).

(b) Use the average human life expectancy of 76 years to calculate the life expectancies of the animals in the table, to the nearest year.

(c) What does it mean if a particular animal has a high value for this unit ratio?

17 Brett has a space on his lounge room wall 1.7 m long and 1.2 m wide to hang a new flat screen TV.

The 'aspect ratio' of a TV screen is the ratio of its length to its width. The two most common screen aspect ratios are 16 : 9 and 4 : 3.

(a) Find three sets of screen dimensions (length and width) in the ratio 4 : 3 that would fit in the wall space.

(b) Find three sets of screen dimensions (length and width) in the ratio 16 : 9 that would fit in the wall space.

(c) Which aspect ratio would be best for Brett's space? Explain your reasoning.

(d) Use your answer to the previous part to find the dimensions of the screen that takes up the greatest amount of wall space.

Problem solving

Cartoon capers

Mario has an art panel from a comic that he wants to enlarge by a scale factor of 10 to print on a poster. The comic art panel is a rectangle 10 cm long and 7.5 cm high.

The poster will also be rectangular, 1.2 m long and 84 cm high.

1 What area of the poster paper will be left blank after the art is enlarged by the scale factor?

2 (a) What scale factor should Mario use so that he uses the biggest possible area of poster paper (without losing any of the image)?

(b) What area of poster paper will be left blank if Mario uses this new scale factor to enlarge the image?

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

Using ratios to find amounts

4.4

A painter mixes two different coloured paints to get a desired shade. If the shade needs red paint mixed with yellow paint in the ratio 3 : 2, and the painter uses 21 litres of red paint in the mixture, then how would you calculate the volume of yellow paint? If you know only the ratio and one of the amounts, then you can calculate the other amount.

For example, one method is simply to list multiples of the given ratio (equivalent ratios) until 21 appears in the 'red paint' position in the ratio:

red : yellow
3 : 2
6 : 4
9 : 6
12 : 8
15 : 10
18 : 12
21 : 14

So, 14 litres of yellow paint are needed.

A more efficient method is to use a pronumeral, such as '*n*', to represent the unknown amount. You can then use the other numbers in the ratios to find the value of *n*. This is shown in the following example.

Worked example 9

W.E. 9

Find the value of *n* for the ratio *n* : 24 if *n* : 24 = 9 : 4.

Thinking	Working
1 Write the ratio with the unknown value underneath the ratio with the two known values.	$9:4$ $= n:24$
2 Consider the side of the ratios in which both values are known (the right-hand side). Use these values to find the multiplier that turns the first ratio into the second. (Here, multiply 4 by 6 to get 24.)	$$\begin{tikzcd} 9:4 \arrow[d, bend right, "\times 6"'] \arrow[d, bend left, "\times 6"] \\ n:24 \end{tikzcd}$$
3 Because the ratios are equivalent, multiplying the known value on the other side of the ratio (9) by the same multiplier (6) gives the value of the unknown (*n*).	$n = 9 \times 6$
4 Write the answer.	$n = 54$

Worked example 10

W.E. 10

A chemist makes a batch of medicine by mixing a chemical with water in the ratio 3 : 7. If the chemist begins with 600 mL of the chemical, how much water must be added?

Thinking	Working
1 Write the ratio in words, then the two numerical ratios underneath each other. Use a pronumeral to represent the unknown value.	chemical : water $= 3 : 7$ $= 600 : w$ (where w = volume of water)
2 Consider the side of the ratios in which both values are known (the left-hand side). Use these values to find the multiplier that turns the first ratio into the second. (Here, multiply 3 by 200 to get 600.)	$$\begin{tikzcd} 3 : 7 \arrow[d, bend right, "\times 200"'] \arrow[d, bend left, "\times 200"] \\ 600 : w \end{tikzcd}$$
3 Because the ratios are equivalent, multiplying the known value on the other side of the ratio (7) by the same multiplier (200) gives the value of the unknown (*w*).	$w = 7 \times 200$ $w = 1400$
4 Write the answer in words.	1400 mL of water must be added.

In the previous two examples, the number that you multiplied the first ratio by to get the second (the 'multiplier') was an easily identified whole number. In some ratios, this number is less easy to find. Methods for finding the unknown quantity in these situations are shown in the following examples.

Worked example 11

W.E. 11

Find the value of b for the ratio $3 : b$ if $3 : b = 5 : 7$.

Method 1: Use an intermediate unit ratio

Thinking	Working
1 Write the ratio containing the unknown value underneath the ratio with the two known values.	$5 : 7$ $= 3 : b$
2 Divide both numbers in the top ratio by the number that is above the known value in the second ratio (in this case 5, which is above 3). This gives a unit ratio. Show the division using arrows. **3** Now, multiply the unit ratio to find the known amount in the second ratio. (Here, multiply by 3.)	$$\begin{tikzcd} 5 : 7 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 1 : \frac{7}{5} \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 3 : \frac{7}{5} \times 3 \\ 3 : b \end{tikzcd}$$

4 Use the sequence of operations used to turn the first ratio into the second (shown here in pink) to find *b*.	$b = 7 \div 5 \times 3$ $b = 4.2$

Method 2: Use fractions to solve an equation

Thinking	**Working**
1 Write each ratio as a fraction, ensuring that the pronumeral is a numerator. (This means that the second number in the other ratio, 7, is also a numerator.) Because the ratios are equivalent, the fractions are also equivalent.	$5:7$ $= 3:b$ $\frac{b}{3} = \frac{7}{5}$
2 Solve the equation for *b*. (Here, multiply both sides by 3.)	$b = \frac{7}{5} \times 3$
3 Write the answer.	$b = 4.2$

Worked example 12

W.E. 12

The ratio of Uppma's height to Brett's height is 9 : 10. If Brett is 152 cm tall, use a calculator to find Uppma's height. Write your answer to the nearest centimetre.

Method 1: Use an intermediate unit ratio

Thinking	**Working**
1 Write the ratio in words, then the numerical ratios underneath each other. Use a pronumeral to represent the unknown value. (Here, use *h* to represent Uppma's height.)	Uppma : Brett $= 9:10$ $= h:152$ (where h = Uppma's height)
2 Divide both numbers in the top ratio by the number that is sitting above the known number in the second ratio (in this case 10, which is above 152). This gives a unit ratio. Show the division using arrows.	$\begin{tikzcd} 9 \arrow[d, bend right, "\div 10"'] & : & 10 \arrow[d, bend left, "\div 10"] \\ \frac{9}{10} & : & 1 \end{tikzcd}$
3 Now, multiply the unit ratio to find the known amount in the second ratio. (Here, multiply by 152.)	$\begin{tikzcd} \frac{9}{10} \arrow[d, bend right, "\times 152"'] & : 1 \arrow[d, bend left, "\times 152"] \\ \frac{9}{10} & \times 152 \end{tikzcd}$
4 Use the sequence of operations shown to turn the first ratio into the second (shown here in pink) to find *h*.	$h = 9 \div 10 \times 152$ $h = 137$ cm
5 Write your answer in words.	Uppma is 137 cm tall.

Method 2: Use fractions to solve an equation

Thinking	Working
1 Write the ratio in words, then the numerical ratios underneath each other. Use a pronumeral to represent the unknown value. (Here, use *h* to represent Uppma's height.)	Uppma : Brett $= 9:10$ $= h:152$ (where $h =$ Uppma's height)
2 Write each ratio as a fraction, ensuring that the pronumeral and the corresponding number in the other ratio (*h* and 9 in this case) are both numerators. Because the ratios are equivalent, the fractions are also equivalent.	$\frac{h}{152} = \frac{9}{10}$
3 Solve the equation for *h*. (Here, multiply both sides of the equation by 152.)	$h = \frac{9}{10} \times 152$ $= 137$
4 Write your answer in words.	Uppma is 137 cm tall.

4.4 Using ratios to find amounts

Navigator

Answers p. 638

1 (columns 1–2), 2, 5, 6 (column 1), 7 (a–b), 8, 10, 13, 15 (a), 18	1 (columns 2–3), 2, 3, 4 (a), 5, 6 (columns 1–2), 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19	1 (columns 2–3), 3, 4, 5, 6 (columns 2–3), 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19

Equipment required: calculator for Questions **6**, **7**, **9–19**

Fluency

W.E. 9

1 Find the value of each unknown.

(a) $21:g = 7:3$
(b) $8:h = 1:7$
(c) $i:72 = 5:8$
(d) $a:15 = 2:5$
(e) $b:6 = 5:3$
(f) $c:18 = 5:3$
(g) $13:2 = p:18$
(h) $5:3 = u:300$
(i) $4:7 = v:700$
(j) $2:9 = 24:r$
(k) $18:5 = 180:t$
(l) $11:6 = 330:w$

2 The following directions are given on a bottle of concentrated household cleaner: 'Dilute 2 parts of concentrated cleaner with 5 parts water'.

(a) How much water should be added to 300 mL of the concentrated cleaner?

(b) How much of the diluted solution mixture is there?

3 Jenny needs to add glitter to her container of motion sand in the ratio 3 : 10. She has 540 g of motion sand. How much glitter should she add?

4 Ann is mixing a drink that needs cola cordial mixed with water in the ratio of 1 : 4.

(a) She has poured 200 mL of water into her glass. How much cola cordial should she add?

(b) If Ann wants to make 1 litre of cola drink, then how much cola cordial and water should she mix?

W.E. 10

5 (a) Tacky-bond glue comes in two tubes containing Part A and Part B pastes. These have to be mixed in the ratio 1 : 4 for maximum strength. If Elio measures out 5 mL of Part A, how much Part B should he mix with it for maximum strength?

(b) A training college handbook describes a course as having 3 practical lessons for every 8 theory lessons. If the course has 24 theory lessons, then how many lessons are practicals?

(c) A particular outdoor activity can only be done when there is a teacher–student ratio of 1 : 6. If 96 students participate, then how many teachers must be there?

(d) A thick-shake is made of milk and ice-cream in the ratio 5 : 2, and contains 400 mL of milk. Find the amount of ice-cream in the shake.

W.E. 11

6 Find the value of each unknown.

(a) $a : 44 = 3 : 8$

(b) $c : 32 = 2 : 5$

(c) $b : 20 = 2 : 9$

(d) $i : 10 = 11 : 4$

(e) $h : 12 = 9 : 8$

(f) $g : 30 = 7 : 4$

(g) $6 : x = 10 : 1$

(h) $2 : x = 5 : 11$

(i) $3 : x = 2 : 7$

(j) $2 : x = 8 : 13$

(k) $4 : x = 8 : 15$

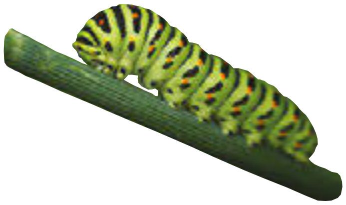
(l) $4 : x = 5 : 9$

W.E. 12

7 (a) The ratio of an average African elephant's height to a prehistoric mammoth's height is 5 : 7. If the elephant's average height is 321 cm, find the height of the mammoth. Write your answer to the nearest centimetre.

(b) When correctly proportioned, the Australian flag has a width to length ratio of 10 : 19. For a project, Harley wants to make an Australian flag that is 80 cm long. How wide should Harley's flag be, to the nearest centimetre?

(c) An entomologist finds that the ratio of caterpillars to moths on a particular tree is 6 : 5. If there are 35 moths, how many caterpillars are there?



(d) Sunbeam Island Resort advertises that the daylight 'sun to cloud' ratio for the island is 20 : 3. If the average number of hours of sun is 12 during summer, then what is the average number of hours of cloudy conditions each day? Give your answer **(i)** as a decimal and **(ii)** in hours and minutes.

8 **(a)** If $a:16=5:4$, then a equals:

A 1	**B** 4	**C** 16	**D** 20

(b) If $40:9=800:b$, then b equals:

A 900	**B** 450	**C** 180	**D** 40

9 **(a)** If $x:4=9:7$, then x equals:

A $\frac{9}{7}\times 4$	**B** $\frac{9}{7}\times\frac{1}{4}$	**C** $\frac{7}{9}\times 4$	**D** $\frac{7}{9}\times\frac{1}{4}$

(b) If $19:y=2:5$, then y equals:

A $\frac{2}{5}\times 19$	**B** $\frac{2}{5}\times\frac{1}{19}$	**C** $\frac{5}{2}\times 19$	**D** $\frac{5}{2}\times\frac{1}{19}$

Understanding

10 The ratio of the dimensions of the rug shown is $9:5$. Find the length of the rug.

11 Amber and Yusuf put their money together to buy a bag of lollies. Amber puts in 90c, and Yusuf puts in 50c. They weigh their shares of lollies, which are in the same ratio as the amount of money each contributed.

(a) If Yusuf gets 75 grams of lollies, then what mass of lollies does Amber get?

(b) What was the total mass of the lolly bag?

12 Linda and Jean own 35% and 65% of a company, respectively. When yearly profits are divided in proportion to each owner's investment in the company, Linda receives \$450 000.

(a) How much does Jean receive (to the nearest dollar)?

(b) What was the company's total profit?

13 Bob's recipe for concrete is to mix cement, sand and gravel in the ratio $1:2:4$. He has one 10 kg bag of gravel left, and wants to use it all to make concrete. How much cement and sand should he add?

14 A jam manufacturer sells jam in 250 g and 450 g jars. The prices for each jar are to be in the same ratio as the jar sizes. If the price of the larger jar is \$3.69, what should be the price of the smaller jar?

Reasoning

15 Voters in the electorate of Wallyville could vote for the Action Party, the Barry Party, the Save the Whale Party or the Democratic Party. Election results showed the ratio of votes received was $5:1:2:6$, respectively.

(a) If the Barry Party received 850 votes, how many votes did the Democratic Party receive?

(b) The Save the Whale Party received 1700 votes. Find the number of votes for the Action Party.

(c) What fraction of voters voted for the Barry Party?

(d) How many voters were there in the total?

16 Jesse's local petrol station sells three types of petrol: e10 petrol (a mixture of petrol and ethanol), 91-octane petrol and 95-octane petrol. The prices were \$1.25, \$1.32 and \$1.38 per litre, respectively.

(a) Write the ratio of the three prices in simplest form.

(b) Assuming the ratio between the prices stays constant and e10 petrol goes up in price to \$1.30 per litre, find the new prices of the other two types of petrol to the nearest cent per litre.

(c) Assuming the ratio between the prices remains constant and 91-octane petrol drops in price to \$1.25 per litre, find the cost of the other two types of petrol to the nearest cent per litre.

Open-ended

17 The maximum allowed ratio of students to skiing instructors is 9 : 1. Groups with fewer than 7 students cannot ski. If there are 10 instructors available, suggest different group sizes so that students can take a skiing lesson with no students missing out.

18 The ratio of Mary's height to her daughter Alexi's height is 3 : 2. List three pairs of reasonable heights that fit this ratio.

19 On an exam, the ratio of the marks allocated for multiple-choice questions to short-answer questions to long-answer questions is 5 : 6 : 4.

(a) Give two different whole number examples of how many marks could be allocated to each section of the test.

(b) To make it easier to record the results as percentages, the teacher wants the total marks to add up to 100 exactly. Is this possible? Explain your reasoning.

Puzzle

Sudoku

Equipment required: grid paper

Copy the grid and numbers.

Fill in the spaces with the digits 1 to 9, so that each row, column, and bold 3×3 square contains each of the numbers 1–9 only once.

	6		7		2		3	
	7	2				9		
	8		9	3		1		7
5		8	1	9				
1			6		5			3
				4	7	5		2
7		9		8	4		5	
		6				2	4	
	3		5		1		8	

Investigation

Bicycle gears

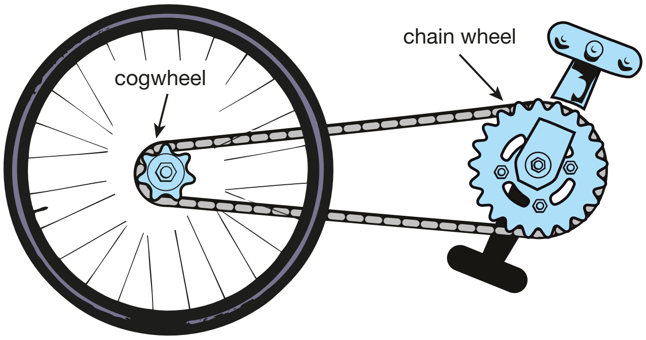
Equipment required: 1 bike with gears (optional)

The Big Question

How many bike gears are necessary to compete in the Tour de France?

Engage

Single-speed or fixed-gear bicycles have a front chain wheel (sometimes called the front sprocket) attached to the pedals. A rear cogwheel (sometimes called the rear sprocket) is attached to the rear bike wheel. The chain wheel is connected to the cogwheel by a chain.



Multi-speed bicycles often have two or more chain wheels that can combine with any of several differently sized rear cogwheels. These different combinations are called gears. A bicycle with three front chain wheels and six rear cogwheels would have 18 different gear combinations or speeds.

1 If a bicycle has two front chain wheels and five rear cogwheels, then how many gears does it have?

2 If a 24-gear bicycle has three chain wheels, then how many rear cogwheels does it have?

3 If a 21-gear bicycle has seven rear cogwheels, then how many chain wheels does it have?

The 'gear ratio' is found by counting the number of teeth on both the chain wheel and the cogwheel and calculating the unit ratio. For example, the chain wheel in the diagram on this page has 21 teeth and the rear cogwheel has 7 teeth.

So, the ratio (chain wheel) : (cogwheel)
= 21 : 7
Gear ratio = 3 : 1

This ratio means that when the pedals are turned, the smaller cogwheel must rotate faster than the chain wheel. For every rotation of the chain wheel, the rear wheel must rotate three times.

Although the back wheel turns three times faster, the effort to make it turn is also three times greater. Lower gear ratios (which happen when a smaller chain wheel is chosen) need less effort to turn the pedals, but the pedals must be turned more times to travel the same distance.

Some gear specifications for a mountain bike are shown in the following table.

Front chain wheel (Number of teeth)
48
38
28

Rear cogwheel (Number of teeth)
28
24
21
18
16
14

4 **(a)** How many gear combinations are possible for the mountain bike?

(b) Find the gear ratios for each combination and order them from largest to smallest. Are any ratio values close enough that they might be considered the same?

Explore

The Tour de France is a cycling endurance race held over a 3-week period around France and other parts of Europe. A typical bike for this competition might have the following gear arrangement: two front chain wheels with 39 and 53 teeth, and 10 rear cogwheels with 11, 12, 13, 14, 15, 16, 17, 18, 19 and 21 teeth.

5 **(a)** Calculate the gear ratio for every gear combination.

(b) How many different gears can the cyclist choose?

(c) Does your answer to the previous part depend on your definition of 'different'?

Strategy options

- Make a table.
- Test all possible combinations.
- Break problem into manageable parts.

Explain

6 Explain how you decided whether two gear ratios were different or the same.

7 Which gears would a cyclist use when going up a hill? Why? Which gear would a cyclist choose when travelling at top speed downhill? Why?

Elaborate

8 Why would a professional cyclist ride a bike with some gears that have almost the same ratio? (Hint: When changing gears, separate changes must be made for the front chain wheel and the rear cogwheel, because they cannot be changed at the same time.)

9 Use your answers from **5**, **6** and **8** to answer the Big Question.

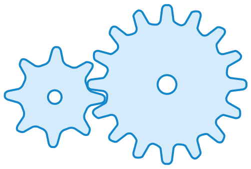
Evaluate

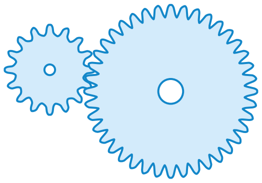
10 Consider how you went about answering the Big Question. Which strategies did you find useful?

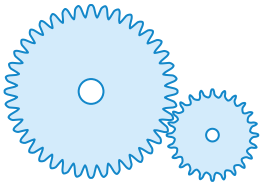
11 'When solving problems, mathematicians not only need to define the problem, but also need to define the solution.' How does this statement apply to this investigation?

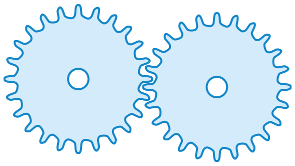
Extend

12 The letters 'rpm' stand for 'revolutions per minute', For each of the gears shown below, use gear ratios to find the speed of the second cog, given that the first cog is turning at 12 rpm.









13 If you wanted to create a gear ratio of exactly 3.5 : 1, list three possibilities for the number of teeth on each cog. Which combination would you choose to put on a bicycle? Why?

14 Research the gears used by competition road cyclists and track riders. Can you explain why they use different gear ratios?

15 Research how the gears in a car work. Why do some cars have 3, 4 or 5 forward gears? How does reverse gear work? Why is there only one reverse gear?

Maths 4 Real

The golden ratio

– fact or fiction?

Throughout history, people have been interested in mysteries and patterns in nature. The number 1.618, known as the 'golden ratio' or the 'divine proportion', has fascinated humans for centuries. It is represented by the symbol ϕ, the Greek letter 'phi'. (1.618 is actually a rounded approximation of ϕ. The actual value is $\frac{1+\sqrt{5}}{2}$, which is an irrational number with an infinite number of decimal places.)

In Dan Brown's top-selling novel *The Da Vinci Code* he describes the following situations in which the golden ratio appears.

Statement	Fact or fiction?
Human proportions 'Da Vinci actually exhumed corpses to measure the exact proportions of human bone structure. He was the first to show that the human body is literally made of building blocks whose proportional ratios always equal phi.' For example, the ratio of a person's height to the distance from their navel to the ground is 1.618.	✓ Fact: Da Vinci did exhume bodies and measure bones as part of his studies of human anatomy. He was taught maths by Pacioli, a mathematician who was fascinated by the golden ratio. Da Vinci illustrated some of Pacioli's work. ✗ Fiction: There is no evidence that Leonardo believed that human proportions always equalled phi. The ratio of height to the distance between the navel and the ground varies from person to person.
The nautilus – A spiral-shelled sea creature 'Can you guess what the ratio is of each spiral's diameter to the next?' 'Phi, the divine proportion.'	✓ Fact: The growth of the nautilus shell is governed by the golden ratio. The shape of the spiral is the Fibonacci spiral, shown elsewhere on this page. ✗ Fiction: The diameter ratios are not in this ratio and vary from spiral to spiral.
Honey bees 'Did you know that if you divide the number of female bees by the number of male bees in any beehive in the world, you always get the same number?' 'Phi.'	✗ Fiction: There is nothing true about this statement. The ratio of male to female bees is estimated to vary from 5 to 1 to 50 to 1. The numbers vary from hive to hive, region to region and season to season.

Dan Brown, *The Da Vinci Code*, Random House Australia, 2004

Here are some other commonly held beliefs about the golden ratio:

Belief	Fact or fiction?
Beauty The golden proportion is what is most pleasing to the eye. 'Beautiful' objects, people, plants and animals are in the proportions of the golden ratio.	✓ Fact: Some cultures have used the golden ratio to define perfection. Ancient Greeks believed that the 'perfect face' was in the golden ratio. ✗ Fiction: Recent research has shown a wide range of ratios that are considered beautiful by people. The concept of beauty varies dramatically from culture to culture and over time.
Architecture Ancient Greek architects used the golden ratio in their building designs. The Egyptian pyramids were built using the golden ratio.	✓ Fact: Euclid, an ancient Greek mathematician, was one of the first to discover and investigate the golden ratio. ✗ Fiction: There is no evidence that the ratio was deliberately used in Ancient Greek architecture or by the Egyptians.
Art Artists have used the golden ratio in their paintings.	✓ Fact: Artists including Sérusier, Gris, Severini and Salvador Dali deliberately used the golden ratio in their art work. ✗ Fiction: There is no evidence that earlier artists such as Michelangelo and Dürer used the golden ratio in their art.

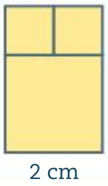
Although there is little evidence to support many of the claims made about the golden ratio, there is a real and interesting link between the golden ratio, a shape called the golden rectangle and a famous group of numbers called the Fibonacci sequence.

The golden rectangle

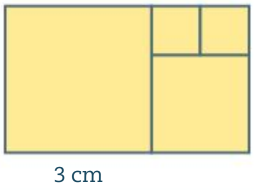
1 Draw a square of side length 1 cm. Calculate length ÷ width for this square.

2 Attach another square to form a rectangle. Calculate length ÷ width for this rectangle.

3 Now, attach a square to the larger side of the rectangle to form a new rectangle as shown. Calculate length ÷ width for this new rectangle.



4 Continue attaching larger squares to the longer side of the rectangles formed at the previous stage until you run out of space. Calculate length ÷ width for the new rectangle formed at each stage.



5 What do you notice about your answers to the length ÷ width calculation at each stage? If you can't draw a conclusion yet, continue creating larger rectangles and calculating length ÷ width.

The Fibonacci sequence

If you write the side lengths of each of the squares drawn above, you should get the following sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, …

This is called the Fibonacci sequence, named after the Italian mathematician who was the first to study it in detail. The numbers in the sequence are known as Fibonacci numbers.

6 Can you see how each number in the sequence is generated? (Hint: Look at the previous two numbers.)

7 What are the 10th and 11th numbers in the Fibonacci sequence?

8 Calculate the 11th number ÷ 10th number, and round your answer to 3 decimal places. What do you notice?

Fibonacci numbers in nature

Fibonacci numbers (and the golden ratio) appear in nature, most clearly, in the spiral patterns seen in plants such as sunflowers, pine cones and pineapples. The pine cone at the bottom of the previous page shows two sets of spirals, one set curving anticlockwise, the other clockwise. You can observe that the number of spirals in each set are two consecutive Fibonacci numbers. ('Consecutive' means that one number follows the other in the sequence.)

9 Count the number of clockwise and anticlockwise spirals on the pine cone (some have been marked in for you). Do you find two consecutive Fibonacci numbers?

10 Draw a 'Fibonacci spiral' by drawing quarter circles inside each of the squares used to make a golden rectangle, as shown.

Research

11 Research the work of Fibonacci and other occurrences of Fibonacci numbers in nature.

12 It is commonly believed that the golden ratio has been used in art and architecture for centuries.

- **(a)** Do some research to see if you can find any evidence that ancient Greek buildings (such as the Parthenon) or Leonardo da Vinci's paintings (such as the Mona Lisa) contain golden rectangles.
- **(b)** Can you find evidence that the golden ratio is used today? Take some measurements of doors, windows, TV screens, picture frames, paper sizes, vases and so on.

Half-time 4

4.1

1 For the following illustration, write the ratio of:

(a) dogs to people (b) legs to heads (c) leads to dogs.

4.4

2 Roberto has planted potatoes and carrots. The area they take up is in the ratio of 2 : 5. If the potatoes take up 50 m^2, what area do the carrots take up?

4.2

3 Write the following ratios in simplest form.

(a) 28 : 32 (b) 180 mm : 120 mm (c) 10 kg : 350 g

(d) $2\frac{1}{2} : \frac{1}{3}$ (e) 2.5 : 0.45 (f) $3\frac{1}{4}$ years : 9 months

4.1

4 In a small business, Lucas owns 5 shares, Jenny owns 2 shares and Mia owns 3 shares. If they are the only shareholders, then find:

(a) the ratio of shares owned by Lucas to those owned by Jenny to those owned by Mia

(b) the fraction of all the shares owned by Jenny, in simplest form

(c) the combined percentage of the business owned by Jenny and Mia.

4.4

5 Find the value of each unknown.

(a) $d : 28 = 6 : 7$ (b) $27 : f = 9 : 2$ (c) $1 : 5 = 12 : q$ (d) $4 : 13 = 88 : t$

4.3

6 The scale factor of the life expectancy of Australian females to males is approximately 1.06. If the life expectancy of the average male is 79 years, then what is the life expectancy of the average female? Give your answer to the nearest year.

4.3

7 The tallest building in the world is currently the Burj Khalifa in Dubai. It is 828 m high, which is 1.83 times as tall as a former world's tallest building, the Petronas Towers in Malaysia. How tall is the Petronas Towers? (Answer to the nearest metre.)

4.1, 4.2

8 Pedro has had to raise the price of bananas in his fruit shop from $2.75 per kg to $3.50 per kg.

(a) Write the ratio of the old price to the new price, in simplest form.

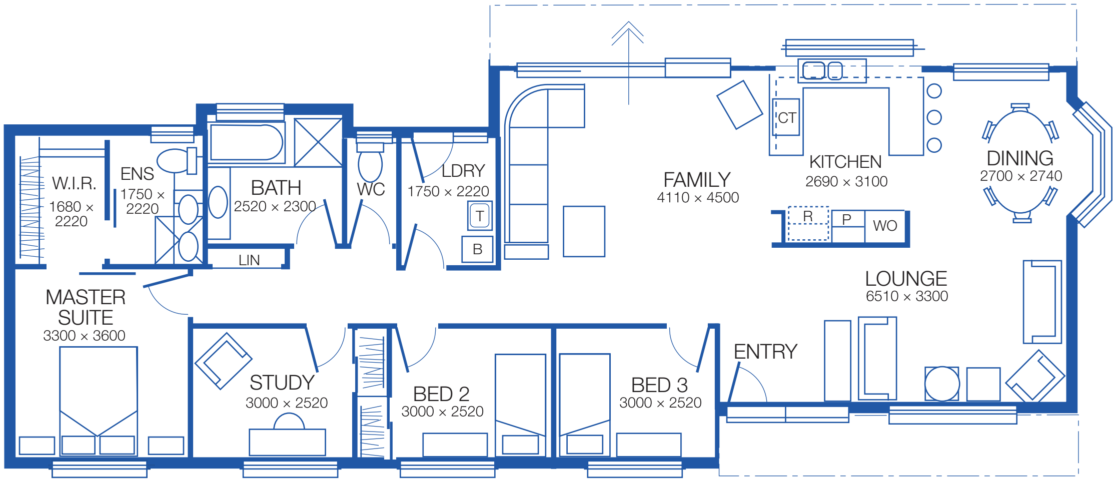
(b) Write the new price as a percentage of the old price, to the nearest percentage.

(c) What is the percentage increase of the old price?

4.5

Scale drawings

Scale drawings use ratios to present accurate information about distances in a diagram of a convenient size. Maps and building plans are two common examples of scale drawings.



Scale 1 : 10 000

A scale drawing should always include its ratio. (For example, 1 : 1 000 000).

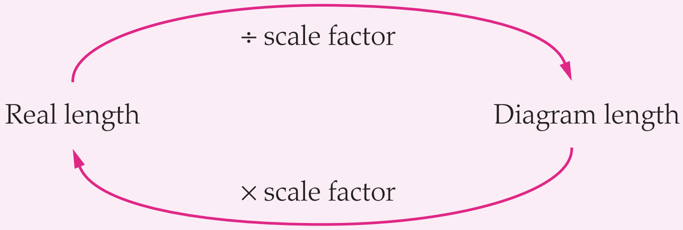
The first number in the ratio represents a distance on the diagram. The second number in the ratio is how many times bigger that distance is in real life.

As with other ratios, **scale ratios** should be written without units and in simplest form. For example, a plan of a house may use a scale of 5 millimetres to represent a metre, or 5 mm : 1 m. To convert this to a scale ratio, express both values in the same units and simplify:

$$\begin{aligned} & 5\text{ mm} : 1\text{ m} \\ = \; & 5\text{ mm} : 1000\text{ mm} \\ = \; & 5 : 1000 \\ = \; & 1 : 200 \end{aligned}$$

If the scale ratio is a unit ratio, then the larger value in the scale ratio is the scale factor. A scale factor of 200 means that in real life the object is 200 times as big as in the diagram. Another way to think of this is that the object or length was shrunk by a factor of 200 to make it fit on the page.

To write a ratio as a scale ratio, convert to appropriate smaller units and simplify by cancelling common factors.



When very small objects are enlarged by a scale factor, then the operations shown above are reversed.

Worked example 13

W.E. 13

Convert 2 mm : 1 m to a scale ratio and underline the scale factor.

Thinking	Working
1 Write both quantities in the smaller units (mm).	2 mm : 1 m = 2 mm : 1000 mm
2 Simplify by cancelling any common factor and leaving out the units. Underline the scale factor.	= 1 : $\underline{500}$

Worked example 14

W.E. 14

The diagram at right uses the scale of 1 : 80. Find the real length of the car in metres.

Thinking	Working
1 Measure the length of the car (in mm) on the diagram (65 mm).	Diagram length = 65 mm
2 Multiply the diagram length by the scale factor to get the real length.	Real length = 65 mm × 80 = 5200 mm
3 Convert your answer to metres.	= 5.2 m

Worked example 15

W.E. 15

The height of a tower is 70 m. How high should it be drawn on a diagram with a scale of 1 : 200? Answer in millimetres.

Thinking	Working
1 Divide the real length by the scale factor to find the diagram height.	70 m ÷ 200 = 0.35 m
2 Convert the diagram height to mm.	= 350 mm

4.5 Scale drawings

Navigator

1, 2, 3, 4, 5 (columns 1–2), 6 (columns 1–2), 7 (columns 1–2), 8, 9, 10 (columns 1–2), 11 (column 1), 12, 15, 17, 22	1, 3, 4, 5 (columns 2–3), 6 (columns 2–3), 7 (columns 2–3), 8, 9, 10 (columns 2–3), 11 (columns 1–2), 12, 13, 14, 15, 16, 17, 18, 20, 22	1, 4, 5 (columns 2–3), 6 (columns 2–3), 7 (columns 2–3), 8, 9, 10 (columns 2–3), 11 (columns 2–3), 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Answers p. 638

Equipment required: ruler, calculator may be used

Fluency

1 Complete the following sentences.

(a) A scale of 1 : 20 on a drawing means that 1 cm on the drawing represents _____ cm of the actual object.

(b) A scale on a map of 1 : 900 means that 1 cm on the map represents an actual distance of _____ m in real life.

(c) A scale of 1 : 450 000 on a plan means 1 cm on the plan represents _________ km in real life.

(d) A scale of 1 : 25 on a plan means 2 cm on the plan represents _________ m in real life.

2 The scale of a map is 1 : 200 000.

(a) This means that every 1 cm on the map represents _________ cm in real life.

(b) In other words, 1 cm on the map represents _________ km in real life.

(c) On the map, the distance from Eastland Shopping Centre to Knox Shopping Centre is shown as 4 cm. What is this distance in real life, in km?

(d) If Brighton Beach and Frankston Beach are 30 km apart, then how far apart will they be on this map?

3 The scale of a drawing is 1 : 500.

(a) This means that a building in real life is _____ times as big as in the drawing.

(b) If the Eureka Tower is 59.4 cm in the drawing, what is its actual height in metres?

(c) A male giraffe can grow to 6 m in height. What would this be in the drawing?

4 Convert the following to scale ratios and underline the scale factor.

W.E. 13

(a) 1 cm : 6 m
(b) 1 mm : 40 cm
(c) 2 mm : 50 cm
(d) 5 cm : 3 m
(e) 5 mm : 4 m
(f) 2 mm : 1 m
(g) 4 cm : 1 m
(h) 4 mm : 50 cm

5 A council plan is drawn to a scale of 1 : 100. Find the real length in metres for each of the following diagram lengths.

W.E. 14

(a) 2 cm
(b) 3 cm
(c) 5 cm
(d) 8 mm
(e) 23 mm
(f) 19 mm
(g) 40.3 cm
(h) 92.5 cm
(i) 1.2 m

W.E. 15

6 A house is drawn to scale. Calculate what the following lengths should be in the diagram using a scale of 1 : 400. Answer in millimetres.

(a) 2 m **(b)** 4 m **(c)** 8 m
(d) 40 cm **(e)** 40 m **(f)** 80 m
(g) 320 cm **(h)** 1200 mm **(i)** 5200 mm
(j) 480 cm **(k)** 0.8 m **(l)** 0.24 m

Remember that in most cases, the real length will be bigger than the diagram length. So, to get the real length you multiply by the scale factor.

7 A map is drawn to the scale of 1 : 2000. Find the real distances in metres for each of the following lengths measured on the map.

(a) 3 cm **(b)** 5 cm **(c)** 10 cm
(d) 2.6 cm **(e)** 38.4 cm **(f)** 43 mm
(g) 9 mm **(h)** 17.9 mm **(i)** 51 mm

8 Are the following scales also unit ratios?

(a) 4 : 7000 **(b)** 6 : 120 000 **(c)** 1 : 400
(d) 1 : 500 000 **(e)** 65 : 500 **(f)** 1 cm : 17 mm

9 In the ratio 1 : 3500, the scale factor is:

A $\frac{1}{3500}$ **B** 1 **C** 35 **D** 3500

10 Find the real length in metres for each of the following diagram lengths. The scale ratio is given in brackets in each case.

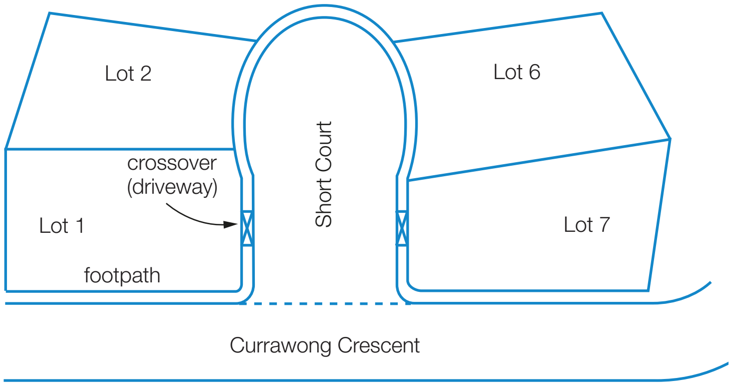
(a) 7 cm (1 : 100) **(b)** 5 mm (1 : 1000) **(c)** 4 cm (1 : 2000)
(d) 8 cm (1 : 500) **(e)** 6 mm (1 : 50) **(f)** 40 mm (1 : 3000)
(g) 21 mm (1 : 30 000) **(h)** 13 cm (1 : 40 000) **(i)** 8.8 cm (1 : 2000)

11 Find the diagram length in millimetres for each of the following real lengths. The scale ratio is given in brackets in each case.

(a) 6 m (1 : 300) **(b)** 16 m (1 : 2000) **(c)** 700 m (1 : 200)
(d) 16 km (1 : 4000) **(e)** 12 km (1 : 50 000) **(f)** 180 m (1 : 60 000)
(g) 50 cm (1 : 20) **(h)** 400 cm (1 : 500) **(i)** 36 m (1 : 400)

Understanding

12 Part of a map of a new subdivision is drawn to a scale of 1 : 1000 as shown. Find the real lengths corresponding to each of the following.

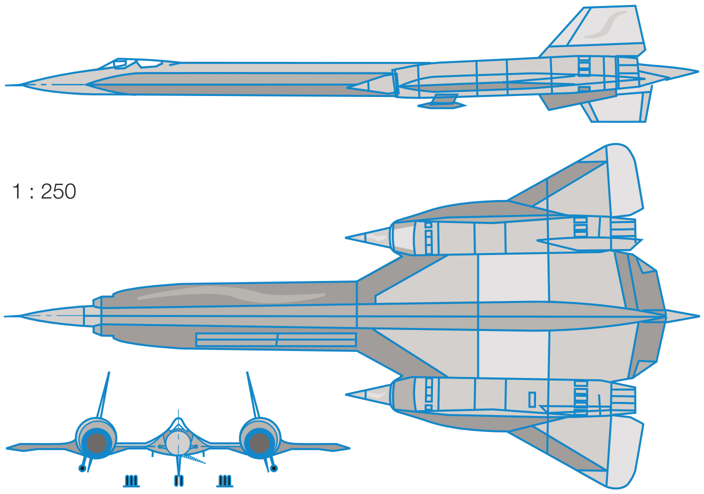


(a) the frontage (length of the block on Short Court) of Lot 1
(b) the length of the longest boundary of Lot 7
(c) the width of Currawong Crescent
(d) the width of the footpath

13 An engineer's drawing of an aircraft is shown below. Find, to the nearest metre:

(a) the length of the aircraft

(b) the width of the aircraft (wingspan).



14 Use the scale of the map below and a ruler to estimate the distance (to the nearest km) between:

(a) Melbourne and Sydney

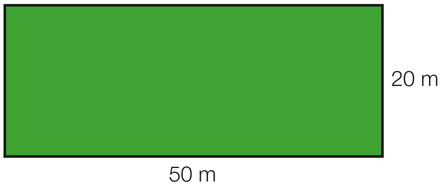
(b) Brisbane and Townsville

(c) Adelaide and Darwin

(d) Hobart and Perth.

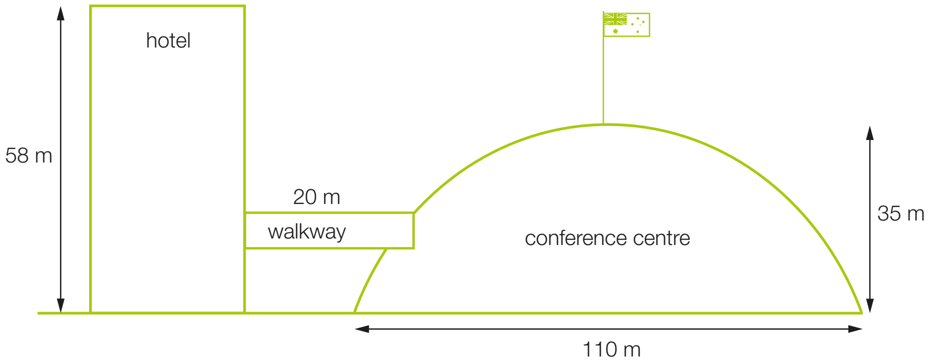
Reasoning

15 Franjo is about to draw an accurate scale diagram of his block of land, which has the measurements shown (not to scale).



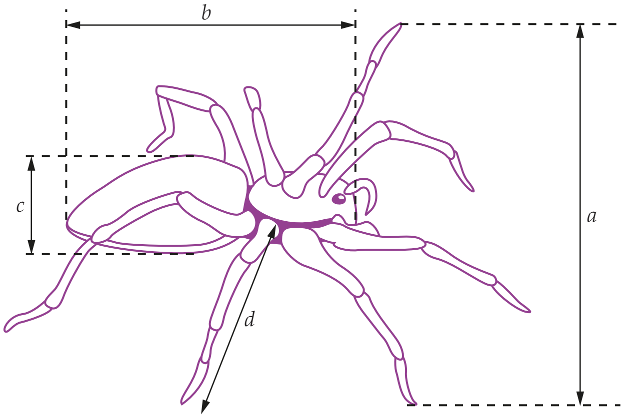
- **(a)** Find the dimensions of Franjo's diagram if he uses a scale of:
  - **(i)** 1 cm : 5 m
  - **(ii)** 4 mm : 1 m
  - **(iii)** 1 : 200
  - **(iv)** 1 : 400
- **(b)** Which of the above scales would best cover an A4 sheet of paper?

16 A developer has a proposal for a city project. A section of her rough plans for the project appears below. If a scale model is made of the proposed development with a scale of 1 : 2000, then what are the lengths on the model of the following? Answer in millimetres.

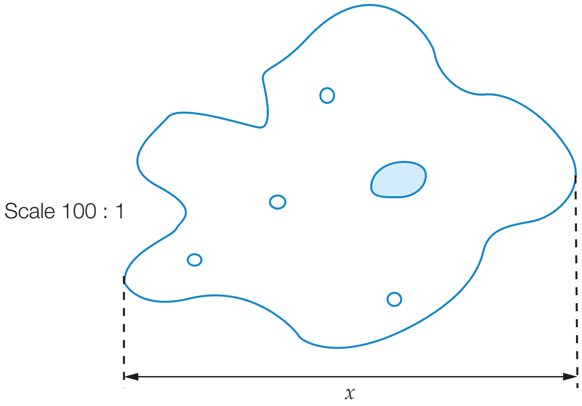


- **(a)** the height of the hotel
- **(b)** the length of the conference centre
- **(c)** the height of the conference centre
- **(d)** the length of the walkway

17 Scale drawings aren't always smaller than the real thing. The diagram opposite is of a spider, drawn to a scale of 2 : 1 (i.e. the real size of the spider is 0.5 or $\frac{1}{2}$ the diagram size). Find the real length of each distance marked with a variable on the diagram. (Answer in mm.)



18 Find the real length of the amoeba (a microscopic single-celled organism) using the scale shown.



19 The scale ratio for a housing estate plan is 1 : 900. A rectangular block measures 3 cm by 2.4 cm on the plan.

(a) Find the area of the block on the plan in cm^2.

(b) Find the actual dimensions of the block in cm.

(c) Find the actual area of the block in cm^2.

(d) Multiply the area of the block on the plan by the scale factor. Is the answer the same as the actual area? Explain why or why not.

(e) Find the actual dimensions of the block in metres.

(f) Find the actual area of the block in m^2.

(g) What is the relationship between the actual area in cm^2 and the area in m^2?

Open-ended

20 An architect needs a scale model of a 140 m high building. The finished model must be between 20 cm and 50 cm tall. List three whole-number scale factors that could be used.

21 A cartographer (map maker) needs to show an entire suburb on a single A4 piece of paper. Allowing for borders, the maximum map size is 18 cm by 25 cm. The dimensions of the suburb are roughly 5 km by 7 km. List two proper scale ratios that the cartographer could use. Which would you recommend and why?

22 Sometimes, a scale drawing uses a scale that increases the size of the drawn object compared with the actual object. List three examples of where this might be the case.

4.6

Sharing an amount in a given ratio

To share an amount in a given ratio:

1. add the values in the ratio to find the total number of parts
2. divide the amount by the total number of parts to find the size of one part
3. multiply the values in the ratio by the size of one part.

Worked example 16

W.E. 16

Share 45 in the ratio stated in the brackets. (2 : 7)

Method 1: Find the size of one part

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 2 + 7$ $= 9$
2 Divide the full amount by the number of parts to find the size of one part.	Size of one part $= \frac{45}{9}$ $= 5$
3 Multiply the ratio by the size of one part.	$2 : 7$ $= 5 \times 2 : 5 \times 7$ $= 10 : 35$
4 Check that the new amounts add up to the original amount.	$10 + 35 = 45$ ✓

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 2 + 7$ $= 9$
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{2}{9}$ $\quad$ $\frac{7}{9}$
3 To find each part, multiply the full amount by the fraction.	$\frac{2}{9} \times 45 = 10$ $\quad$ $\frac{7}{9} \times 45 = 35$
4 Check that the new amounts add up to the original total.	$10 + 35 = 45$ ✓

Worked example 17

W.E. 17

Jake, Abdul and Pendo share a box containing 24 chocolates in the ratio 4 : 3 : 1. How many chocolates does each receive?

Method 1: Find the size of one part

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 4 + 3 + 1$ $= 8$
2 Divide the amount by the number of parts to find the size of one part.	Size of one part $= \frac{24}{8}$ $= 3$
3 Multiply the ratio by the size of one part.	$4 : 3 : 1$ $= 4 \times 3 : 3 \times 3 : 1 \times 3$ $= 12 : 9 : 3$
4 Check that the new amounts add up to the original total.	$12 + 9 + 3 = 24$ ✓
5 Explain the answer.	Jake receives 12 chocolates. Abdul receives 9 chocolates. Pendo receives 3 chocolates.

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 4 + 3 + 1$ $= 8$
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{4}{8} \qquad \frac{3}{8} \qquad \frac{1}{8}$
3 To find each part, multiply the amount by the fraction.	$\frac{4}{8} \times 24 = 12 \qquad \frac{3}{8} \times 24 = 9 \qquad \frac{1}{8} \times 24 = 3$
4 Check that the new amounts add up to the original total.	$12 + 9 + 3 = 24$ ✓
5 Explain the answer.	Jake receives 12 chocolates. Abdul receives 9 chocolates. Pendo receives 3 chocolates.

4.6 Sharing an amount in a given ratio

Navigator

Answers p. 639

1, 2 (columns 2–3), 3, 4, 7, 11, 12	2 (columns 2–3), 3, 4, 5, 6, 7, 9, 10, 11, 12	2 (columns 2–3), 3, 4, 5, 6, 7, 8, 9, 10, 12, 13

Equipment required: calculator for Questions **8**, **9**

Fluency

1 In the following ratios, how many parts are there in total?

(a) 5 : 15 **(b)** 9 : 3 : 15 **(c)** 24 : 5 : 12 : 7

W.E. 16

2 Share the following amounts in the ratios stated in the brackets.

(a) 30 (2 : 3) **(b)** 20 (3 : 1) **(c)** 42 (5 : 1)

(d) 39 (2 : 11) **(e)** 84 (16 : 5) **(f)** 38 (12 : 7)

(g) 56 (4 : 3 : 1) **(h)** 42 (2 : 4 : 1) **(i)** 54 (2 : 7 : 9)

(j) 33 (2 : 5 : 4) **(k)** 50 (2 : 3 : 5) **(l)** 96 (1 : 7 : 4)

W.E. 17

3 **(a)** Kayla has a packet of 45 jelly beans that she must share with her little brother in the ratio 3 : 2. How many jelly beans do Kayla and her brother receive?

(b) Ms Everage makes a batch of 84 lamingtons, which she wants to divide in the ratio 2 : 7 : 3 and put into separate containers for afternoon tea on three days. How many lamingtons should she put into each container?

Understanding

4 After 21 cricket matches, the win : loss ratio of Adrian's cricket team is 5 : 2. How many matches has the team won, and how many have they lost?

5 The instructions on a container of orange juice concentrate say to mix with water in the ratio 1 : 4. If Boutros wants to make 2 L of juice according to the instructions, then how many mL of concentrate and water should he mix?

6 Two-stroke motor fuel is made by mixing 1 part oil with 24 parts petrol. Anastasia wants to mix 5 litres of two-stroke fuel. How much oil and petrol does she need?

7 The ratio of boys to girls in Year 8 at MacGregor High is 7 : 8. If there are 300 students in Year 8, how many of each gender are there?

8 A lottery win is to be divided among the following people in the same ratio as the amounts they contributed to the tickets, which are as follows:

Karine	$5
André	$3
Vivian	$2
Julian	$2

If the lottery win is $57 000, then how much should each person receive?

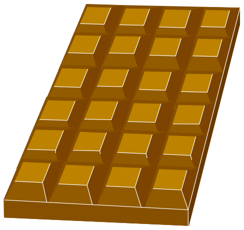
Reasoning

9 A nursery sells a fertiliser with an N : P : K (nitrogen : phosphorus : potassium) ratio of 3 : 2 : 4. (The capital letters are the chemical symbols for these elements.)

(a) How many kilograms of each element are in a 2.7 tonne load of the fertiliser?

(b) If 300 kg of phosphorus (P) was added to the 2.7 tonne load, what would be the new ratio of elements?

10 The block of chocolate shown is divided between two groups of students according to the number of students in each group.



(a) If there are 4 students in the first group and 2 in the second group, how many pieces of chocolate does each group get?

(b) How many pieces does each student get?

(c) One member of each group decides not to have any chocolate. If the chocolate is redistributed according to the new groups, how many more pieces of chocolate does each student get now?

11 A 6.4 m length of timber is to be cut so that the two pieces are in the ratio 2 : 3. How far along should the cut be made? (Answer in centimetres.)

Open-ended

12 Kelly, Nadine and Petra have lunch together. When the bill comes it does not show individual prices for each thing they ordered. The total bill is $115. All three had a main course and a drink but Nadine also had a dessert. Suggest two ways the bill could be divided and explain your reasoning.

13 Ms Footsie and Ms Nikkei form a partnership and invest $5 million and $9 million, respectively, to buy a television station. They sell the station for $42 million some time later.

(a) How should the $42 million be divided between the two partners?

(b) Ms Footsie is claiming she should receive $19 million from the sale, only $4 million less than her partner. Can you suggest a reason why?

Problem solving

Catch 22

1 Write any three different single-digit numbers and add them together. Keep this number for later.

2 Use the three single-digit numbers to write six possible two-digit numbers. Do not repeat any digits.

3 Add together all six two-digit numbers.

4 Divide the number you have in step **3** by the total from step **1** and record your answer.

5 Try this procedure again, starting with a different set of three numbers. What do you notice? Why do you think this works?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

4.7 Rates

A **rate** is a way of comparing two amounts of different quantities. Rates usually include the word *per* which means 'for each' and which often uses the symbol / or the letter 'p'. A rate is similar to a unit ratio, because it is the amount of the first quantity for every 1 unit of the second quantity. Some examples of rates are:

If the price of apples is \$4.96/kg, you will pay \$4.96 for 1 kg of apples.

Hair grows at an average rate of 1.3 cm per month.

Speed

Speed is a rate that compares distance and time. It is used to describe how fast something is travelling. For example, a car could be travelling at 60 kilometres per hour (km/h or kph), or a cheetah could be running at 25 metres per second (m/s or $\mathrm{m\,s}^{-1}$).

A car speed of 60 km/h means that in 1 hour, a car will travel a distance of 60 km. This could be a constant speed, but it is more likely that a car will speed up and slow down at various times during the hour, in which case you would be calculating an average speed.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

This can also be written as the formula: $s = \frac{d}{t}$

Worked example 18

W.E. 18

Calculate the average speed for the following. Express your answers correct to 2 decimal places where necessary.

(a) a car that travels 360 km in $4\frac{1}{2}$ hours

(b) an athlete who sprints 100 metres in 10.49 seconds

Thinking	Working
(a) 1 Write the formula for calculating average speed.	(a) $s = \frac{d}{t}$ $d = 360$ km $t = 4.5$ h
2 Substitute the values for distance and time.	$s = \frac{360}{4.5}$
3 Evaluate. Note that as the original units are km and hours, the speed unit is km/h.	average speed = 80 km/h
(b) 1 Write the formula for calculating average speed.	(b) $s = \frac{d}{t}$ $d = 100$ m $t = 10.49$ s
2 Substitute the values for distance and time.	$s = \frac{100}{10.49}$
3 Evaluate, rounding your answer to the stated number of decimal places. Note that as the original units are metres and seconds, the speed unit is m/s.	average speed = 9.53 m/s

The unitary method

A rate represents a certain number of a first quantity for every 1 of a second quantity. Calculating a number per 1 'unit' (where a 'unit' might be for example litres, kilograms, boxes or people) is a useful method for solving problems. This is called the **unitary method**. The first part of this method is similar to calculating a unit ratio, which was discussed in Section 4.3.

Worked example 19

W.E. 19

(a) A recipe for blueberry muffins needs 150 g of blueberries. The recipe makes 12 muffins. What mass of blueberries is required to make 40 muffins?

(b) 5 kg of potatoes costs $12.99. How much would 16 kg cost?

Thinking	Working
(a) 1 Calculate the amount needed for one 'unit' (one muffin). 2 Multiply the amount per unit by the number of units (40 muffins).	(a) 150 g for 12 muffins ÷ 12 ÷ 12 = 12.5 g for 1 muffin × 40 × 40 = 500 g for 40 muffins
(b) 1 Calculate the price per unit (per kilo). 2 Multiply the price per unit by the number of units (16 kg).	(b) $12.99 for 5 kg ÷ 5 ÷ 5 = $2.598 for 1 kg × 16 × 16 = $41.57 for 16 kg

You can modify the unitary method slightly if you are working with numbers that have common factors. Instead of finding the cost or amount per one unit, you can find it for 10 or 20 units or a number of units that is a common factor.

For example, if 30 L of petrol costs \$45, then you can calculate the cost of 50 L by first finding the cost of 10 L.

\$45 for 30 L
= \$15 for 10 L (÷ 3)
= \$75 for 50 L (× 5)

Similarly, if one number is a multiple of another, your task is simpler. For example, if 100 g of mixed nuts costs \$2.50, then you can simply multiply by 3 to find the cost of 300 g.

\$2.50 for 100 g
= \$7.50 for 300 g (× 3)

Percentage rates of change

A rate is often used to describe how quickly or slowly a quantity is changing over time. Speed is an example of distance changing over time. To track change over times such as months, years or decades, the rate is often calculated as a percentage increase in the previous quantity. Percentage rates are commonly used to describe the yearly growth in a population (of a country, city or school), or financial growth (such as the value of shares in a business or the price of oil).

A percentage rate is calculated by writing the change (the amount of increase or decrease) as a fraction of the previous amount, then converting to a percentage.

For example, if the population of a suburb increases in one year from 250 to 270 people, the percentage rate of change is: $\frac{20}{250} \times 100\%$, or 8% per year.

If the population continues to grow at this rate in the following year, the population at the end of that year would be 108% of 270: $1.08 \times 270 = 292$ (rounded to a whole number).

4.7 Rates

Navigator

Answers p. 639

1, 2, 3, 4, 6 (a), 7, 8, 9, 11, 13, 16 (a–c), 18, 19, 25	2, 3, 4, 6, 7, 8, 10, 11, 12, 13, 16, 17, 18, 19, 25	3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 17, 20, 21, 22, 23, 24

Equipment required: calculator

Fluency

1 On her way to the city, Jackie took a picture of the following road sign.
   - **(a)** What does this road sign mean?
   - **(b)** If Jackie travels at 100 km/h, what does this mean in terms of distance travelled and time taken?
   - **(c)** If Jackie travels at a constant speed of 100 km/h, how far does she drive in 2 hours?

2 Explain what the following rates mean in your own words:
   - **(a)** Robert reads at a rate of 294 words/minute.
   - **(b)** Jack earns \$21/h mowing his neighbour's lawn.
   - **(c)** Flour costs \$0.55 per 100 g.
   - **(d)** Fuel is used by a sportscar at a rate of 11.8 L per 100 km.

3 Calculate the average speed for the following. Express your answers correct to 2 decimal places where necessary.

W.E. 18

(a) 80 kilometres travelled in 2 hours

(b) 200 kilometres travelled in 4 hours

(c) 160 kilometres travelled in 2.4 hours

(d) 150 kilometres travelled in 3.9 hours

(e) 1120 kilometres travelled in 12 hours

(f) running 400 m in 48.80 seconds

(g) swimming 100 m freestyle in 49.2 seconds

(h) floating 3.5 km in 20 minutes

4 (a) A recipe for Anzac biscuits requires 125 g of butter. The recipe makes 24 biscuits. What mass of butter is required to make 100 biscuits? (Answer to the nearest gram.)

W.E. 19

(b) 4 kg of tomatoes costs \$19.75. How much would 10 kg cost?

(c) A box of 24 mangoes costs \$65. How much would 20 mangoes cost?

(d) A recipe for vegetable soup requires three stalks of celery. The recipe feeds four people. How much celery is required if soup is to be made for 11 people?

5 The following table shows the change in population of Australia’s three largest cities, as well as Australia’s total population, over the year from 2013 to 2014.

City	Population 2013	Population 2014	Change
Sydney	4 372 802	4 451 841	+ 79 039
Melbourne	4 177 864	4 269 138	+ 91 274
Brisbane	2 140 701	2 176 799	+ 36 098
Australia – Total	23 285 739	23 610 906	+ 325 167

(a) Answer the following questions for each of the cities, and also for the whole of Australia.

(i) Calculate the increase in population as a number of people per week.

(ii) Express the change in population as a percentage increase in the 2013 population. (Round answers to 2 decimal places.)

(b) If the population continued to grow by the rate calculated in the previous part, then what was the population at June 2015?

6 (a) Mal wants to get fit. He jogs 5 km a day for a week. If 5 km takes him 45 minutes to complete, his average speed is closest to:

A 0.15 km/h   B 3.75 km/h   C 5.45 km/h   D 6.7 km/h

(b) Angela walks 4 km each day. Today, it takes her 40 minutes. Angela’s average speed, in metres per second, is:

A 100 m/s   B 1.67 m/s   C 1.5 m/s   D 0.0017 m/s

7 Find the cost of petrol, in cents per litre, for each of the following situations. Give your answers correct to the nearest tenth of a cent.

(a) It costs \$37 for 30 litres.

(b) It costs \$31.40 for 25 litres.

(c) It costs \$22.84 for 15.7 litres.

(d) It costs \$66.52 for 46.8 litres.

8 A typical healthy resting heart rate for an adult is in the range 60–80 beats per minute (bpm). Find the number of beats per day if the rate is:

(a) 60 bpm **(b)** 80 bpm

Understanding

9 **(a)** Emily walks 22 kilometres at a constant speed of 4 km/h. How much time does it take?

(b) Luan averaged 80 km/h in her car for 3 hours. How far did she travel?

10 Here are Martha's ingredients for scones. The quantities shown make 12 scones. Rewrite the quantities of ingredients to make 40 scones.

4 cups self-raising flour

2 tablespoons icing sugar mixture

$\frac{3}{4}$ cup milk

$\frac{3}{4}$ cup thickened cream

1 egg, lightly beaten

11 In one-day cricket, the 'run rate' is the number of runs that must be scored each over by the batting team to win the match. In one match, Australia need to score 215 runs to win a 50-over cricket match against England.

(a) Find the run rate required at the start of the innings.

(b) If Australia scored 42 runs in the first 10 overs, find the run rate required for the next 40 overs for Australia to win the match.

(c) The opening batsman scored 20 runs from 16 balls. Express his run rate as the number of runs per ball.

(d) The next batsman scored 68 runs from 89 balls. Express his run rate as the number of runs per ball.

12 Calculate the following. Use mental strategies wherever possible.

(a) Ham costs \$13/kg. How much does it cost for:

(i) 500 g **(ii)** 250 g?

(b) Dried apricots cost \$9/kg. How much does it cost for:

(i) 100 g **(ii)** 300 g?

(c) 30 L of petrol costs \$42. How much does it cost for:

(i) 20 L **(ii)** 50 L?

(d) Chocolate-coated nuts cost \$3/100 g. How much does it cost for:

(i) 250 g **(ii)** 350 g?

13 Water costs \$1.7924 per kilolitre (kL). For each of the following charges for water supply, find the amount of water used, correct to 4 decimal places.

(a) \$52.40 **(b)** \$16.55 **(c)** \$22.40 **(d)** \$18.90

14 The following currency conversion table gives some conversion rates for the Australian dollar to other currencies on a particular day. The first column tells you how much of the other currency you will get if you exchange one Australian dollar (A$1). The second column tells you how many Australian dollars you will get if you exchange one unit (dollar, real, pound or yen) of the other currencies.

	A$1	**Worth in A$**
American dollar	0.989 139	1.010 98
Brazilian real	1.652 45	0.605 163
British pound	0.618 406	1.617 06
Japanese yen	80.770 5	0.012 229 4
New Zealand dollar	1.305 34	0.766 085

(a) Calculate how much A$500 would be worth in:

- **(i)** American dollars
- **(ii)** Brazilian reals
- **(iii)** British pounds sterling.

(b) You have 250 000 Japanese yen. How much would you get in Australian dollars for this amount?

(c) You have 750 New Zealand dollars. How much would you get in Australian dollars for this amount?

15 The following table gives the estimated population and the percentage growth rate of that population in June 2016 for the world and a number of countries in the Asia–Pacific region.

Region	**Population (million)**	**Growth rate (%)**
World	7432.7	1.13
Australia	24.3	1.42
China	1382.3	0.46
India	1326.8	1.20
Indonesia	260.6	1.17

(a) Assuming the population growth rate remains constant for each country, find an estimate for the population of the world and each of the countries listed above for:

(i) 2017 **(ii)** 2018.

Round your answers to 1 decimal place (0.1 million).

(b) Which country has the:

(i) fastest growth rate **(ii)** slowest growth rate?

(c) What would a growth rate of zero tell you about the population of a country?

(d) What would a negative growth rate tell you about the population of a country?

16 Calculate the following as rates:

(a) Jin walked 348 m in 4 minutes. What is her walking speed in metres per minute?

(b) Martin paid $7 for a 2 kg bag of potatoes. What is the cost per kilogram?

(c) Lily bought 5 m of wood that cost $45. What is the cost per metre?

(d) A pack of 4 black pens costs $5. What is the cost for 1 pen?

(e) It took 36 hours to refill Jo's 34 920 L pool. At what rate did her pool fill?

(f) It took 1 hour to iron 20 shirts. How many shirts are ironed in a minute?

17 The nutritional information table on a packet of cream biscuits looks like this.

	Per serving: 17 g (one biscuit)	Per 100 g
Protein		5.2 g
Fat – total		26.7 g
Fat – saturated		13.1 g
Carbohydrate – total		66.1 g
Carbohydrate – sugars		27.6 g

Use the unitary method to complete the 'per serving' column.

18 Emily and Josh went bushwalking and travelled approximately 6.5 km in the first 3 hours. Answer to 2 decimal places.

(a) What was their average speed?

(b) If they maintained this speed for the next $5\frac{1}{2}$ hours, how far would they travel in total over the day?

Reasoning

19 Kieran and Jess walk to the bus stop each day. The stop is approximately 1.8 km away. If it takes them 20 minutes, calculate their average speed in:

(a) metres/minute **(b)** km/h

20 In April 2016 the following world records stood. All events are athletics (running), except for the 100 m swimming.

	100 m	100 m swimming	400 m	5000 m	Marathon (42 km)
Men	9.58 s	46.91 s	43.18 s	12 min 37.35 s	2 h 2 min 57 s
Women	10.49 s	52.07 s	47.60 s	14 min 11.15 s	2 h 15 min 25 s

(a) What is the average speed of the men's 100 m running record in m/s, to 1 decimal place?

(b) If the men could maintain the speed calculated, how much time would it take to cover the 42 km of the full marathon? (Answer to the nearest minute.)

(c) What is the average speed in m/s for the women's 100 m and 5000 m running events, to 2 decimal places?

(d) In which event is the ratio of the men's time to the women's time closest to 1:1?

(e) Complete the following sentence. (Write the answer correct to 2 decimal places.)

Men can run 100 m ____________ times faster than they can swim 100 m.

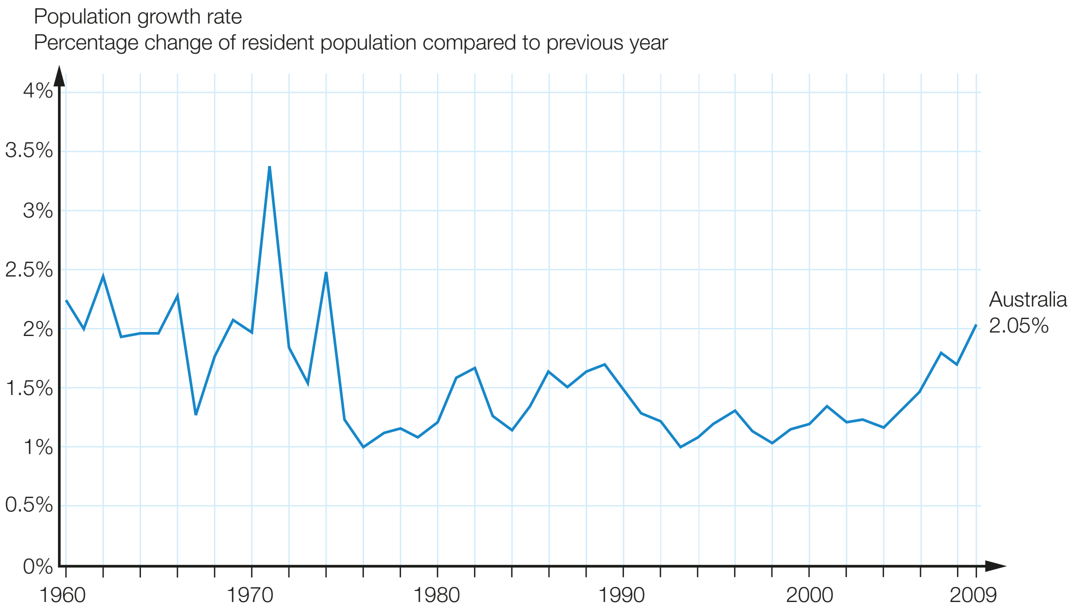
21 **(a)** As the use of electricity increases, so does its cost. For the first 1000 kilowatt hours (kWh) the cost is 19.05 cents per kWh. Any extra energy consumed is charged at 19.51 cents per kWh. Find the cost of:

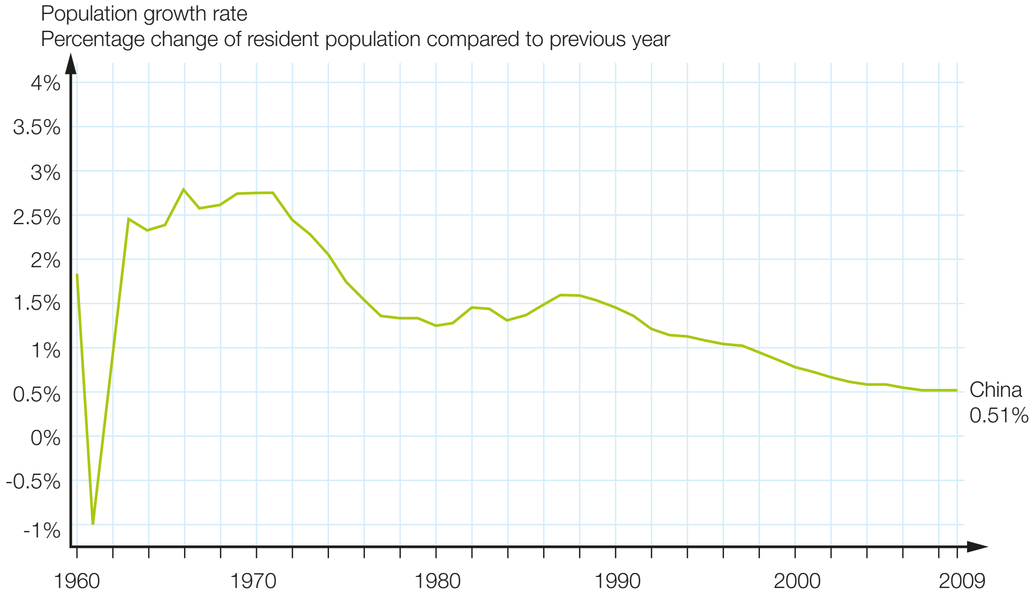
(i) 1203 kWh **(ii)** 1424 kWh **(iii)** 1046 kWh **(iv)** 1822 kWh

(b) Now find, to 2 decimal places, the number of kWh used if the total cost is:

(i) \$172.50 **(ii)** \$199.85 **(iii)** \$203.45 **(iv)** \$220.90

22 The graphs below show the population growth rates for Australia (blue) and China (green) over a period of 50 years.





(a) What does a growth rate of 0% indicate?

(b) What unusual thing happened to the Chinese population in the early 1960s?

(c) Summarise the population growth rate of Australia over the past 50 years.

(d) Summarise the population growth rate of China over the past 50 years.

(e) List some possible reasons for the differences between the two countries.

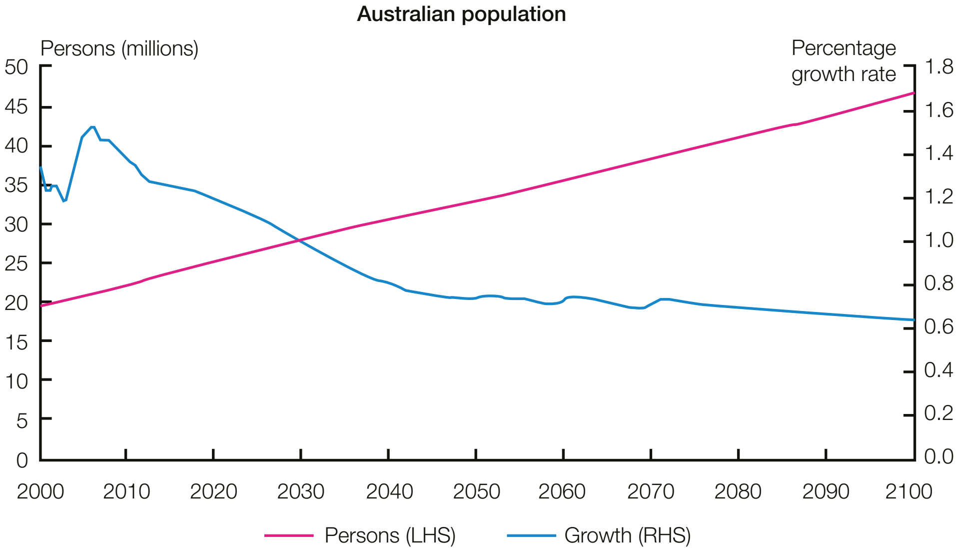
(f) What further information do you need to be able to calculate how many people were added to each population in 2009?

Look at the difference in the numbers on the *y*-axis for the Australian and Chinese graphs.

23 The pink line on the graph below shows the population of Australia steadily increasing. It represents the number of people predicted for the period 2000–2100. The blue line on the graph shows the predicted percentage growth rate of the population. After a sharp increase near the beginning, it decreases for most of the time shown.

(a) What are the two main ways that the population of a country can grow?

(b) Explain how the population can keep increasing even though the growth rate is decreasing.



Open-ended

24 Make a list of factors that influence the rate at which a population of a city (or a country) can grow.

25 By estimating the time between haircuts and the length of hair that is cut each time, calculate the growth rate of your hair in mm/week.

Problem solving

Numberrrrrrrrs

1 Write your favourite single-digit number.

2 Multiply that number by 9.

3 Multiply this new number by 12 345 679. (Unless your calculator has more than 8 digits in its display, you will need to do this by hand to see the full effect.)

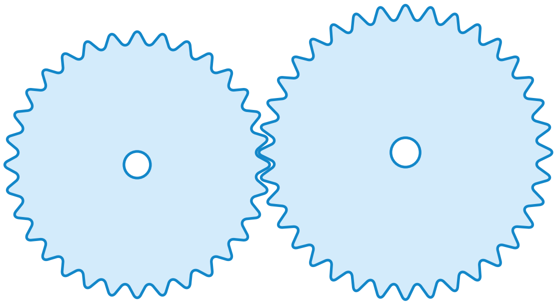
4 If you did choose your favourite number, then you should like the answer you see. Does this result work for other single-digit numbers? Can you explain how this works?

Strategy options

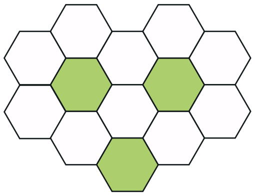
- Guess and check.
- Work backwards.
- Test all possible combinations.

Challenge 4

1. A train is travelling from Melbourne to Sydney at 80 km/h. Another train is travelling from Sydney to Melbourne at 90 km/h. How far apart are they, half an hour before they pass each other?
2. Kevin and Sanna take 6 hours to clean their house. If Sanna does twice as much work as Kevin, then how much time would it take Kevin to clean the house on his own?
3. If six cats eat a total of nine tins of cat food in three days, how many tins of cat food should two cats eat in six days?
4. One liquid contains 22.5% water, another liquid contains 27% water. A glass is filled with 5 parts of the first liquid and 7 parts of the second liquid. What percentage of the liquid in the glass is water?
5. A train travels from A to B at a speed of 40 km/h, and from B to A at a speed of 60 km/h. What was its average speed for the whole journey?
6. A tank can be filled by one pipe in 3 hours, by a second pipe in 5 hours, or by a third pipe in 10 hours. If all three pipes are used together, then how much time will it take to fill the tank?
7. Two gears work together. There are 32 teeth on one gear and 36 teeth on the other gear. The first gear makes 128 revolutions per second. How often, during a 24 hour period, will the same two teeth from each gear be together?



8. A square floor is tiled with a large number of regular hexagonal tiles. Each green tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 green tiles.



   (a) Explain why the ratio of the number of green tiles to white tiles is 1 : 2.

   (b) Justify your answer by completing the pattern in two ways, starting with a green tile in the top left-hand corner, then starting with a white tile in the top left-hand corner.

9. It takes 10 employees 25 days to drain a lake and remove the weeds.

   (a) Assuming a constant ratio between the number of employees and the time taken, how much time will it take:

      (i) five employees

      (ii) three employees

      (iii) 30 employees

      (iv) 100 employees

      (v) 300 employees?

   (b) What might be wrong with assuming that the ratio is constant between employees and time?

Chapter review 4

Maths literacy

- equivalent ratios
- part:part ratio
- part:whole ratio
- rate
- ratio
- scale factor
- scale ratio
- simplest form
- speed
- unit ratio
- unitary method

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 A __________ is a comparison of the relative amounts of two quantities that are measured in the same units.

2 The __________ helps you solve problems by calculating a number per unit.

3 A __________ compares two parts of a whole.

4 __________ are created if you multiply or divide every quantity in a ratio by the same number.

5 A __________ is found by dividing the first quantity in a ratio, a, by the second quantity, b, and then expressing the ratio as $\frac{a}{b}:1$.

6 In a unit ratio, the number that is not 1 is called the __________.

7 The comparison of two quantities that use different units is called a __________.

8 An example of a rate is average __________ which is calculated using $\frac{\text{distance travelled}}{\text{time taken}}$.

9 A __________ is used on diagrams that are drawn smaller or larger than their actual size.

10 A __________ compares a part to the whole.

Fluency

1 A vet's waiting room contains 4 dogs, 5 cats and 3 budgies. **4.1**

(a) Find the ratio of:

  (i) the number of dogs to the number of cats

  (ii) the number of cats to the number of dogs to the number of budgies

  (iii) the number of budgies to the number of dogs

  (iv) the number of cats to the total number of animals.

(b) What percentage of the animals are cats? (Round your answer to the nearest whole number.)

(c) Write the number of budgies as a fraction of the number of dogs.

2 Simplify the following ratios. **4.1, 4.2**

(a) 14 : 24

(b) 80 : 16 : 32

(c) 5 cm : 20 mm

(d) 3 hours : 40 minutes

(e) 4 days : 1 week

(f) 3 L : 1500 mL

3 The ratio $2\frac{1}{4}:3\frac{1}{2}$ simplifies to: **4.2**

A 2 : 1 **B** 3 : 2 **C** 9 : 7 **D** 9 : 14

4 Express the following as unit ratios to 2 decimal places. **4.3**

(a) 220 : 75 **(b)** 15 : 29 **(c)** 12 : 20

5 The Yangtze River in China is 6300 km long, while the Colorado River in the USA is 2333 km long. How many times longer is the Yangtze than the Colorado? (Answer correct to 1 decimal place.) **4.3**

6 Anna is using a recipe from a very old recipe book. The recipe asks for 2.5 pints of water, but Anna's measuring jug is in litres. How many litres of water should Anna use, if 1 pint = 0.568 L? **4.3**

7 Find the value of the pronumeral in each case. **4.4**

(a) $a : 42 = 11 : 6$ **(b)** $5 : 7 = 100 : b$ **(c)** $7 : c = 49 : 77$

(d) $12 : 10 = a : 15$ **(e)** $9 : 21 = 24 : b$ **(f)** $c : 3 = 9 : 2$

8 **(a)** Find a if $a : b = 1.6$ and $b = 5$. **(b)** Find q if $p : q = 5.2$ and $p = 26$. **4.4**

9 Instructions for making lawnmower fuel mixture say that 200 mL of oil should be added to 5 L of petrol. **4.2, 4.4**

(a) Write the ratio oil : petrol in simplest form.

(b) Robert has 30 L of petrol. What volume of oil should he add to get the correct fuel mixture?

10 Convert the following to proper scales, and underline the scale factor. **4.5**

(a) 1 cm : 2 m **(b)** 5 mm : 12 km **(c)** 3 m : 15 km

11 Find the real length in metres for each of the following diagram lengths. The scale ratio is given in brackets in each case. **4.5**

(a) 4 cm (1 : 100) **(b)** 14 mm (1 : 20 000) **(c)** 5.2 cm (1 : 5000)

12 Find the diagram length in millimetres for each of the following real lengths. The scale ratio is given in brackets in each case. **4.5**

(a) 5 m (17 : 7500) **(b)** 38 km (1 : 200 000) **(c)** 245 km (1 : 500 000)

13 Share each amount given below in the ratio stated in brackets. **4.6**

(a) 50 (2 : 3) **(b)** 72 (7 : 5) **(c)** 32 (3 : 5)

14 **(a)** Find Jake's average speed if he travels 6 km in 1.5 hours. **4.7**

(b) Find the average speed of a car that has travelled 450 km in 6 hours.

(c) Gazza earns $600 000 a year for 25 games of football. How much does he earn per game?

(d) If it costs $75.45 to put 45.3 litres of petrol in a tank, what was the price per litre?

(e) 3 kg of onions costs $4.50. How much would it cost for:

(i) 6 kg **(ii)** 5 kg?

(f) A recipe for minestrone soup requires three potatoes and 200 g of pumpkin. The recipe feeds six people. What amounts of potato and pumpkin are needed to make soup for 21 people?

Understanding

15 Maxine runs 12 km at an average speed of 8 km per hour. If she begins at 9 am, then at what time will she finish? **4.1**

16 Isobel spends 12 minutes of her half-hour recess in the queue at the canteen. **4.2**

(a) What is the ratio of time spent in the queue to the time of recess, in simplest form?

(b) What percentage of her recess time did Isobel spend in the queue?

17 Francoise's fruit punch is made from chopped fruit and juice in the ratio 2 : 5. If she uses 24 cups of juice, then how many cups of fruit should she chop? **4.3**

18 Akio and Tsutomu are sumo wrestlers whose masses are in the ratio 8 : 5. If Akio has a mass of 200 kg, what is Tsutomu's mass? **4.3**

19 Water flows through a water-efficient shower at a rate of 7.5 L/min. How much water is used for a 6-minute shower? **4.4**

20 An analysis of a fertiliser reveals a ratio of carbon to nitrogen of 2.9 : 1. If the fertiliser contained 335 g of nitrogen, how much carbon did it contain? **4.4**

21 A hockey team scores a total of 137 goals one season, compared to 88 goals scored against them. For every goal scored against them, how many goals does the team score? (Answer to 1 decimal place.) **4.6**

22 Adrian, Gavin and Kevin invest funds in the ratio 3 : 4 : 6, respectively, to finance a search for lost treasure. They share what they find according to this ratio. If they are successful and discover a chest containing 520 identical gold coins, how should their find be shared? **4.6**

23 The floor plan of a house has a scale of 1 : 125. **4.7**

(a) What are the real dimensions in metres of a bedroom that measures 2.7 cm by 2.6 cm on the plan?

(b) What is the area of the bedroom on the plan, in square centimetres?

(c) What is the real area of the room, in square centimetres?

(d) How many times larger is the real area of the room than the area of the room on the plan?

Reasoning

24 The small town of Splitsville has a population of 14 500. One year later, 178 babies had been born, 69 people had died, 103 people had moved away and 152 people had moved into the town. **4.4**

(a) Calculate the overall change in population as a percentage change in the population at the start. Round your answer to 2 decimal places.

(b) If the population continues to grow at the rate you calculated, find the population in 2 years' time.

25 Concentrated disinfectant needs to be diluted so that the final solution contains 10% disinfectant. How much water should be added to 35 mL of disinfectant? **4.6**

26 In a recent Twenty20 cricket match, Australia made a score of 185 from its 20 overs. **4.6**

(a) When they begin batting, at what rate must the Pakistani team score per over to win the match?

(b) After 8 overs, Pakistan has scored 63 runs. At what rate must they now score (runs per over) to win the match?

(c) After 18 overs, the Pakistani team has averaged 8.5 runs per over. To win, how many runs do they need from the last two overs?

Numeracy practice 4

Non-calculator

1 The ratio $3\frac{1}{3}:2$, expressed in whole numbers in simplest form, is:

A 5:3
B 10:2
C 10:3
D 10:6

2 A soccer team has three defenders, five midfielders and three strikers. What fraction of the team are midfielders?

A $\frac{3}{11}$
B $\frac{5}{11}$
C $\frac{3}{11}$
D $\frac{5}{6}$

3 A map is drawn using the scale ratio 1:5000. If a distance on the map is 20 cm, how long is the actual distance?

A 20 m
B 10 000 cm
C 250 m
D 1 km

4 The exchange rate between Australian and New Zealand dollars is 1 : 1.15. If $200 AUD is exchanged, how many New Zealand dollars would this be?

A $174
B $200
C $200.15
D $230

5 Lawnmower fuel consists of petrol and oil, mixed in the ratio 5:2. Luke has 3.5 L of oil. How much petrol does he need to make a fuel mixture?

Calculator allowed

6 Angelo got 60% for his first maths test. After studying hard for the next two tests, he increased his mark by 25% both times. His mark on the third test is closest to:

A 48%
B 75%
C 82%
D 94%

7 A recipe for choc chip muffins requires 180 g of choc chips. The recipe makes 12 muffins. What mass of choc chips is required to make 20 muffins?

8 In a class of 22 students, the ratio of girls to boys is 6 : 5. Three extra girls and two extra boys join the class. What is the new ratio of girls to boys?

A 3 : 2
B 5 : 4
C 6 : 5
D 9 : 7

9 Lachlan, Hannah and Ella own 40%, 35% and 25% of a company, respectively. The company profits are divided according to their share of ownership. If the company profits are $240 000, then the amount Ella is paid to the nearest dollar is:

A $60 000
B $84 000
C $156 000
D $171 429

10 Jacob wants to take three hours for his 6 km bushwalk. He walks only 1 km in the first hour. What speed will he need to average in the final two hours to make the 6 km as planned?

A 2.5 m/s
B 2.5 km/h
C 4 km/h
D 5 km/h

Mixed review B

Fluency

1 Convert each of the following percentages to fractions in simplest form. **2.6**

(a) 23% **(b)** $\frac{2}{5}$ % **(c)** 12.5% **(d)** 0.7%

2 Convert each of the following decimals to percentages. **2.5**

(a) 0.14 **(b)** 0.03 **(c)** 0.286 **(d)** 1.9

3 Calculate: **2.8**

(a) 30% of 24 **(b)** 4.8% of 65 **(c)** 86% of 330

4 Calculate the sale price of each of the following items if they are discounted by 15%. **2.10**

(a) a pair of jeans marked at $80 **(b)** a stereo marked at $650

5 Simplify, then evaluate: **1.5, 1.6**

(a) $(2^3)^2$ **(b)** $\frac{5^8}{5^6}$ **(c)** $15^9 \div 15^9$

6 Simplify each of the following expressions. **3.4**

(a) $7a^2 - 5a^2$ **(b)** $5t^2 + 7t - t + 2t^2$ **(c)** $12q + 7q^2 - 3p^2 + q^2$

7 Evaluate each of the following expressions when $g = 2$ and $h = 5$. **3.2**

(a) $gh - \frac{16}{g}$ **(b)** $7h - 8g$ **(c)** $30gh + g(h + 4)$

8 Simplify: **3.5**

(a) $5j \times 7k$ **(b)** $14pq \div 2q$ **(c)** $2a^2 \times 3ab$

9 Factorise the following expressions. **3.7**

(a) $5h - 45$ **(b)** $y^2 + 7y$ **(c)** $12mn + 8m$

10 Convert the following to decimals. **2.6**

(a) 17% **(b)** 6.5% **(c)** 400% **(d)** 0.006%

11 Calculate, giving answers in simplest form, where necessary. **1.1–1.3, 2.3**

(a) $-7 + (-8)$ **(b)** $-3 - (-12)$ **(c)** -5×6 **(d)** $-\frac{28}{7}$

(e) $-\frac{3}{8} + \left(-\frac{7}{10}\right)$ **(f)** $-1\frac{1}{8} - \left(-\frac{5}{6}\right)$ **(g)** $-12.83 - (-7.5)$ **(h)** $0.46 + (-1.9)$

12 Find the value of the unknown in each of the following. **4.4**

(a) $a : 16 = 3 : 4$ **(b)** $8 : h = 88 : 121$ **(c)** $120 : 42 = 20 : n$

(d) $c : 6 = 20 : 8$ **(e)** $p : 12 = 3 : 5$ **(f)** $k : 14 = 27 : 35$

13 Share each amount given according to the ratio stated in brackets. **4.6**

(a) 48 (5 : 3) **(b)** 630 (2 : 7) **(c)** 88 (2 : 3 : 6)

14 An orange juice drink has 16% actual orange juice, with the rest made up of sugar and water. What percentage of the drink is not actual juice? **2.8**

15 Convert the following fractions to percentages. (Express your answer as fractional percentages where appropriate.) **2.5**

(a) $\frac{9}{100}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{3}{16}$ **(d)** $2\frac{1}{4}$

16 Increase the following amounts by the percentages given. **2.9**

(a) 600 by 9% **(b)** 54 by 8% **(c)** 150 by 80%

17 Simplify each of the following ratios. **4.1, 4.2**

(a) 15 : 33 **(b)** 14 weeks : 6 weeks **(c)** 50 seconds : 2 minutes

(d) $3\frac{1}{2} : 5$ **(e)** 6 : 4.4 **(f)** 0.2 : 6.32

Understanding

18 Kadim drove 175 km in 2 hours, then drove the next 120 km in 1.5 hours. Calculate his overall average speed, to the nearest whole number. **4.7**

19 Order the following values from smallest to largest.

(a) 76%, 0.7, $\frac{7}{9}$ **(b)** 15.2%, $\frac{3}{20}$, 0.105 **(c)** $5\frac{1}{5}$, 5.06, 500% **2.6**

20 A car valued at $6000 decreases in value by 6.5% in the first year, then by 5% in the second year. **2.9**

(a) What is the value of the car at the end of the first year?

(b) What is the value of the car after 2 years?

21 Sophie's and Dean's heights are in the ratio 8 : 9. If Sophie is 166 cm tall, how tall is Dean to the nearest centimetre? **4.4**

22 Jenny buys a new fridge for $529. She then moves into a share house where there is already a fridge, so she sells her new fridge for $425. **2.10**

(a) What loss has Jenny made on the sale of her fridge?

(b) Find Jenny's percentage loss by expressing the loss as a percentage of the cost price.

23 Dzung slept for $7\frac{1}{2}$ hours last night. What percentage of a day is this? **2.5**

24 Michelle's pay is increased by $7\frac{1}{2}$%. If her fortnightly pay was originally $1250, what is her income now? **2.9, 2.10**

25 The formula for converting degrees Celsius into degrees Fahrenheit is $F = \frac{9C}{5} + 32$. **3.3**

Use this formula to convert the following Celsius temperatures into degrees Fahrenheit.

(a) 0 °C **(b)** 20 °C **(c)** 35 °C **(d)** 150 °C

Reasoning

26 Gerard and Hao invest $12 000 and $18 000, respectively, to start their small business. After the first year, the profits come to $12 500. How should they split the profits? **2.10, 4.6**

27 An amount of money increases by 10% and becomes $121. It is then increased by a further 10%. **2.9**

(a) What was the final amount of money?

(b) What was the original amount of money?

(c) What single percentage increase would take you from the original amount to the final amount in one step?

5


NORTHWEST MONSOONS
BARRA'MIRRI MAYALTHA
STRONG WINDS HEAVY RAIN LIGHT WIND
HIGH TIDES
GROWTH PERIOD
MAGPIE GEESE
SHELLFISH
CYCAS MEDIA
SEASON
MAIN FEATURES
EAST SOUTH EAST
MIDAWARR
FLAT WATER & CLOUDS
BUSH FRUITS
ROOT CROPS
BARRAMUNDI
FOOD SOURCES
NORTH & NORTHWEST
DHULUDUR'
CLOUDY
HUMID
THUNDER AND LIGHTING
THREADFIN SALMON
TURTLES
STINGRAYS
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
WIND
SHARKS AND STINGRAYS
SMALL SHARKS MANGO FRUIT
HONEY
MUD CRABS
TURTLE EGGS
COOL NIGHTS HEAVY DEWS
BURNING OFF, FOGS
DHARRATHARRA
SOUTH SOUTHEAST
CALENDAR MONTHS
GROUND HOT TO WALK ON
NO WATER
RARRANHDHARR
SOUTHEAST & NORTHEAST

Measurement

5

Timelines and timecircles. If you can measure something, does that mean you know what it is?

Time can be measured in many ways. Atomic clocks use the regular vibrations of atoms to measure time with a high level of precision, losing less than 1 second every 1 million years. But what exactly *is* time? Great minds throughout history have tried to answer that question.

Western cultures have traditionally seen time as a straight line, but other cultures have traditionally seen time as a circle.

While people often talk of time*lines*, you can also see time as the cycles and rhythms of nature: the changing seasons that bring different types of food, the rise and fall of the tides, and the waxing and waning of the Moon. Opposite is a calendar that shows how the Yolngu people of north-east Arnhem Land traditionally measure a year of time compared with the European months.

In Australian Aboriginal culture, the past is not gone: the Dreaming is both part of the past and part of the present. The events themselves and what they mean for people now are important, not the precise time the events occurred.

Forum

Do you think time is like a straight line or like a circle? Or is it like something else? Why do you think this?

Why learn this?

All of us use measurement every day; for example, you measure time for school timetables or when to have a meal. Many jobs require good measurement skills. Builders, electricians, chefs, farmers, town planners, fashion designers and environmental scientists all need to be able to calculate length, area, volume and capacity efficiently. A landscape gardener, for example, may need to calculate the length of wood needed for the border of a garden bed and the volume of soil required to fill it. She might need to calculate the area she can cover with lawn or seed, and what capacity of rainwater tank to use for the garden. Surveyors and cartographers use many measurements of the landscape to create accurate maps for people to use. Precise measurement of time allows you to find which athlete won the race and whether they broke a record. Accurate measurement is important!

After completing this chapter you will be able to:

- calculate the perimeter and area of triangles and quadrilaterals such as parallelograms, rectangles, trapeziums and kites
- investigate the relationships between the radius, diameter and circumference of a circle
- understand and apply the formulas for calculating circumference and area of circles
- calculate the perimeter and area of composite shapes
- calculate the volume and capacity of prisms and cylinders
- convert between units of length, area, volume and capacity
- write and calculate elapsed time using both 24-hour and 12-hour (am/pm) notation
- understand how time zones work.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website

1 Round each value correct to 2 decimal places.

(a) 7.823 **(b)** 15.0047 **(c)** 0.986 **(d)** 106.4971

2 Calculate each of the following, correct to 2 decimal places if appropriate.

(a) 4.13×6 **(b)** 5.25^2 **(c)** 3.14×12

(d) 7.9×8.52 **(e)** $6.3 \times (5.1 + 4.4)$ **(f)** 13.9×7.8^2

3 Copy and complete each of the following conversions.

(a) 4.5 km = _____ m **(b)** 69 cm = _____ m **(c)** 4.8 cm = _____ mm

(d) 46 mm = _____ cm

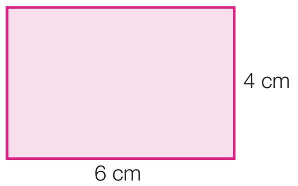
(e) 6780 mm = _____ m **(f)** 90 000 mm = _____ km

4 Calculate each of the following, expressing your answers in metres.

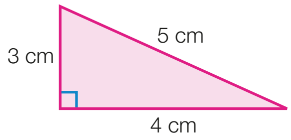
(a) 1.2 m + 40 cm + 5 cm + 3 m **(b)** 3.245 m + 73 cm + 108.9 cm

5 Find **(i)** the perimeter and **(ii)** the area of the following shapes.

(a)

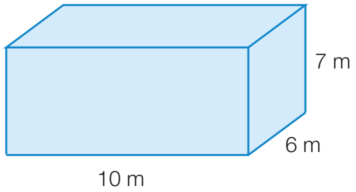


(b)

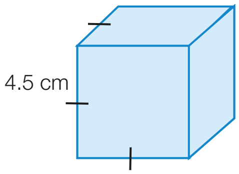


6 Find the volume of the following solids.

(a)



(b)



Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Can squares and rectangles be equal?

In this activity, you will investigate how a square and rectangle with the same perimeter can have different areas. Which will have the larger area? Is this always the case?

Perimeter

5.1

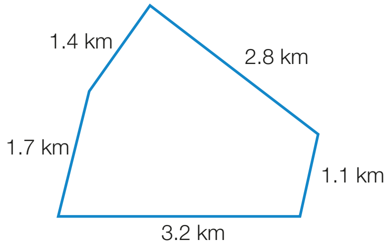
Perimeter is the total length of the boundary of a two-dimensional shape. It is measured using units of length such as kilometres (km), metres (m), centimetres (cm) or millimetres (mm). You will often see markings such as | and || on the sides of shapes. These show that the marked sides are the same length. For example, two edges marked with | (that is, marked with a short line across the edge) are the same length.

Worked example 1

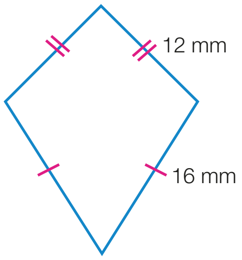
W.E. 1

Find the perimeter of each of the following shapes.

(a)



(b)

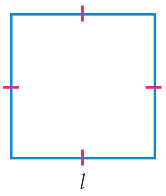
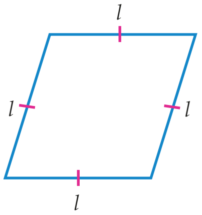
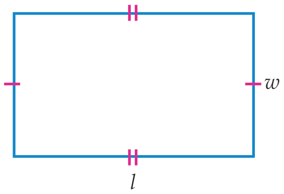
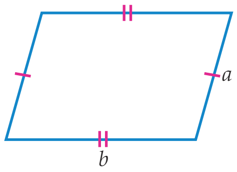


Thinking	Working
(a) **1** Add the side lengths.	(a) $P = 1.7 + 1.4 + 2.8 + 1.1 + 3.2$
2 Write the total length, including the unit.	$= 10.2$ km
(b) **1** The sides marked with the same symbol have equal lengths. Multiply the lengths by the number of times each appears.	(b) $P = 2 \times 12 + 2 \times 16$
2 Add the products.	$= 24 + 32$
3 Write the total length, including the unit.	$= 56$ mm

Perimeter formulas

You can use formulas to find perimeters. Usually, the letter *P* is used as a pronumeral to represent the perimeter. If there are sides of equal length, then they are represented by the same pronumeral. Any pronumeral can be used, but usually the letters *l* (length) and *w* (width) are used for rectangles. The letter *b* (base) is used for distances along the bottom of shapes such as triangles and parallelograms.

If there is no convenient formula for the perimeter, then you can simply add all the side lengths. When equal lengths are added, you can use multiplication to simplify the calculation.

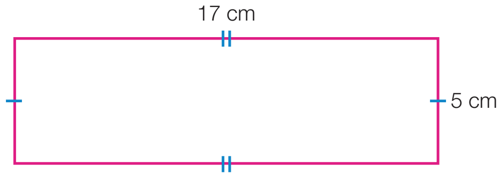
Square	Rhombus	Rectangle	Parallelogram
 $P = 4l$	 $P = 4l$	 $P = 2l + 2w$ or $P = 2(l + w)$	 $P = 2a + 2b$ or $P = 2(a + b)$

Worked example 2

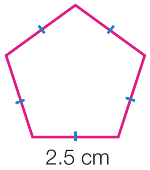
W.E. 2

Use a formula to calculate the perimeter of each shape.

(a)



(b)



Thinking	Working
(a) **1** Write the formula for the perimeter of a rectangle.	(a) $P = 2l + 2w$
2 Substitute values for length and width.	$= 2 \times 17 + 2 \times 5$
3 Add the products.	$= 34 + 10$
4 Write the total length, including the unit.	$= 44$ cm
(b) **1** Use *s* to represent the length of the side marked ǀ. Write a formula that multiplies *s* by the number of equal sides.	(b) $P = 5s$
2 Substitute the side length.	$= 5 \times 2.5$
3 Write the product with units of length.	$= 12.5$ cm

5.1 Perimeter

Navigator

1 (columns 1–2), 2 (columns 1–2), 3, 4, 5, 7, 9, 11

1 (columns 2–3), 2 (columns 2–3), 3, 5, 6, 7, 8, 9, 11

1 (columns 2–3), 2 (columns 2–3), 3, 5, 6, 7, 8, 9, 10, 12

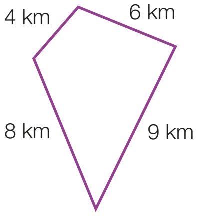
Answers p. 642

Fluency

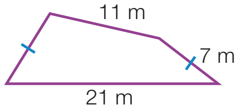
W.E. 1

1 Find the perimeter of each shape.

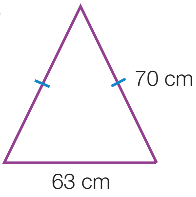
(a)



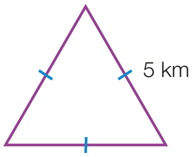
(b)



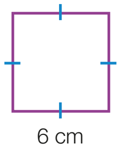
(c)



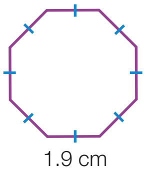
(d)



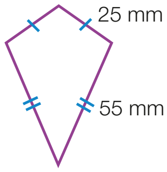
(e)



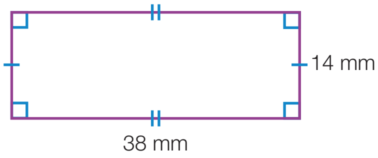
(f)



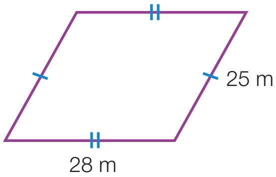
(g)



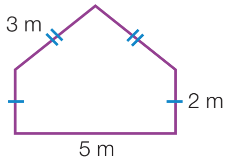
(h)



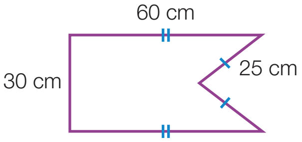
(i)



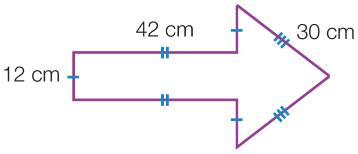
(j)



(k)



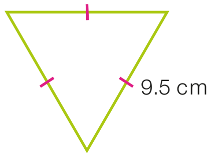
(l)



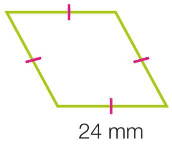
W.E. 2

2 Use a formula to calculate the perimeter of each shape.

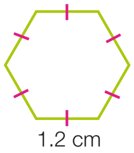
(a)



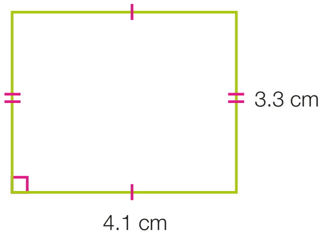
(b)



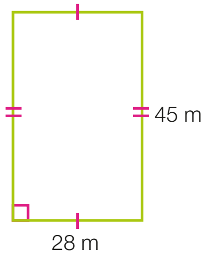
(c)



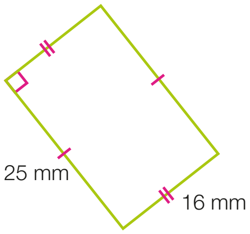
(d)



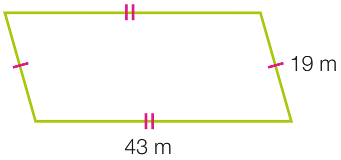
(e)



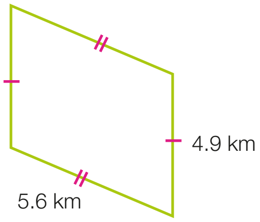
(f)



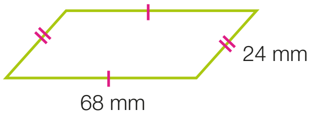
(g)



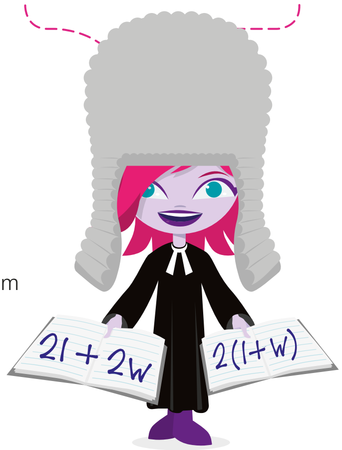
(h)



(i)



Both of these are correct. The expression $2l + 2w$ is equivalent to $2(l + w)$.



3 Find the perimeter of the following (drawing a diagram may help).

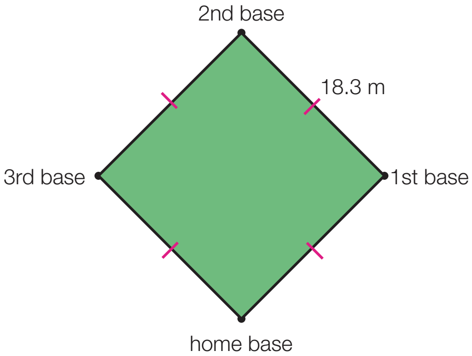
(a) a square of side length 57 mm

(b) a rectangle of width 5.8 m and length 10.7 m

(c) a parallelogram of base 66 cm and a sloping side length of 32 cm

(d) a rhombus of side length 4.25 mm.

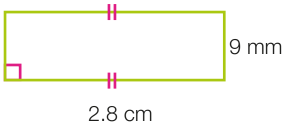
4 How far is the distance around the softball diamond shown?



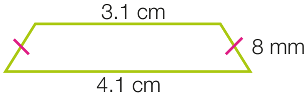
Understanding

5 Find the perimeter of each shape below.

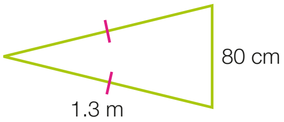
(a)



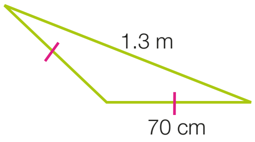
(b)



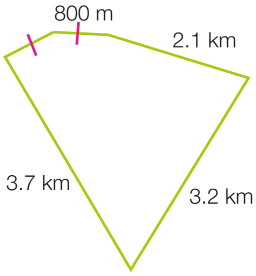
(c)



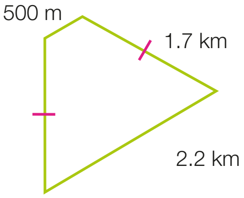
(d)



(e)

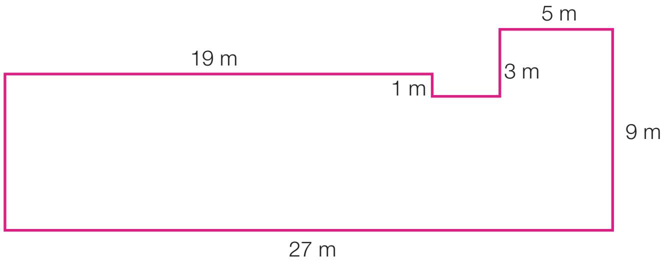


(f)



All lengths must have the same units before you can add them up.

6 The roof plan for a new house is shown below. Use it to calculate the length of guttering attached all the way around each edge of the roof.



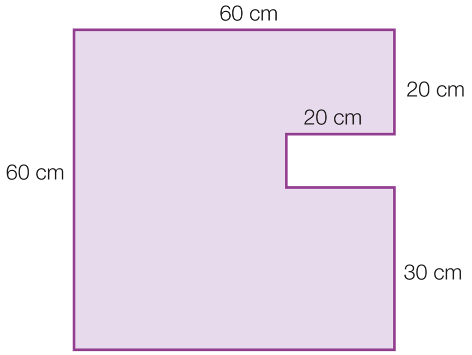
7 **(a)** The length of tape needed to go along the perimeter of the figure shown in the diagram is:

A 190 cm

B 200 cm

C 220 cm

D 280 cm



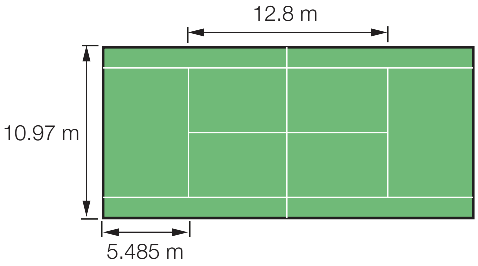
(b) The perimeter of the tennis court shown here is:

A 23.77 m

B 29.255 m

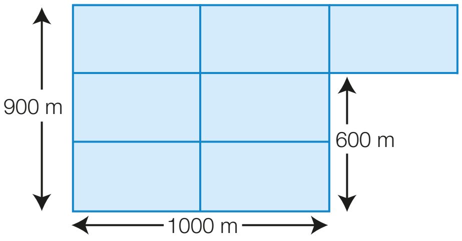
C 58.51 m

D 69.48 m



Reasoning

8 A farmer needs to fence a new property. The fence needs to go along the boundary of the property, and also needs to divide it up into rectangles, as shown.



(a) What is the total length of fence required?

(b) If the type of fence used by the farmer costs $5.75 per metre, what will it cost to fence the property?

(c) If the fencing contractor can build the fence at a rate of 20 metres per hour, how much time will the job take?

9 **(a)** Find the side length of a square if it has the same perimeter as a rectangle of length 19 cm and width 11 cm.

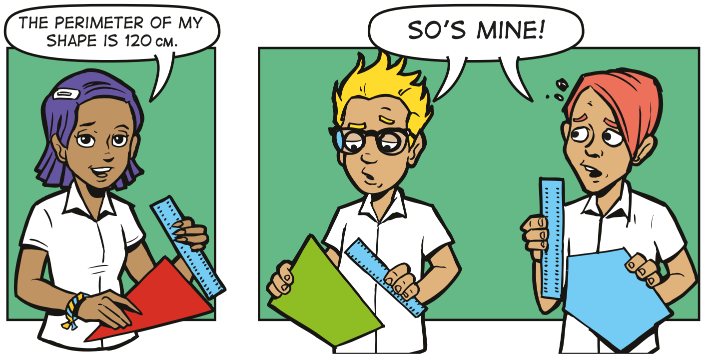
(b) Find the width of a rectangle if its length is 28 cm and it has the same perimeter as a square of side 20 cm.

10 **(a)** Find the perimeter of a square tiled area containing 25 square tiles if each tile is 40 cm wide.

(b) If the number of tiles is doubled, find the perimeter of all the different rectangles that can be formed, using every tile.

Open-ended

11 Write possible side lengths for the triangle, quadrilateral and pentagon that the students are holding below.



12 Brett is designing a new vegetable garden for his backyard.

(a) Brett's friend Emma is helping him and needs to know the dimensions of the garden. Brett tells Emma that the garden will be in the shape of an isosceles triangle (a triangle with two sides the same length), and will have a perimeter of 56 m. Find three possible sets of measurements for Brett's vegetable garden if the side lengths have to be whole numbers of metres.

(b) If Brett tells Emma that the perimeter can be any length from 56 m to 100 m, find two new sets of dimensions for Brett's vegetable garden if the two equal sides of the isosceles triangle are to be twice as long as the third side. All side lengths need to be whole metres.

Game

That formula is mine

How to win:
The winner is the first to 100 points.

List of formulas

$P = 2a + 2b + c$

$P = 4a + b$

$P = 4a + 2b$

$P = 2a + b$

$P = 3a + 3b$

$P = 6a + 2b + c$

$P = 4a + b + c$

$P = 3a + 2b + c$

Equipment required: 1 die, 2 counters

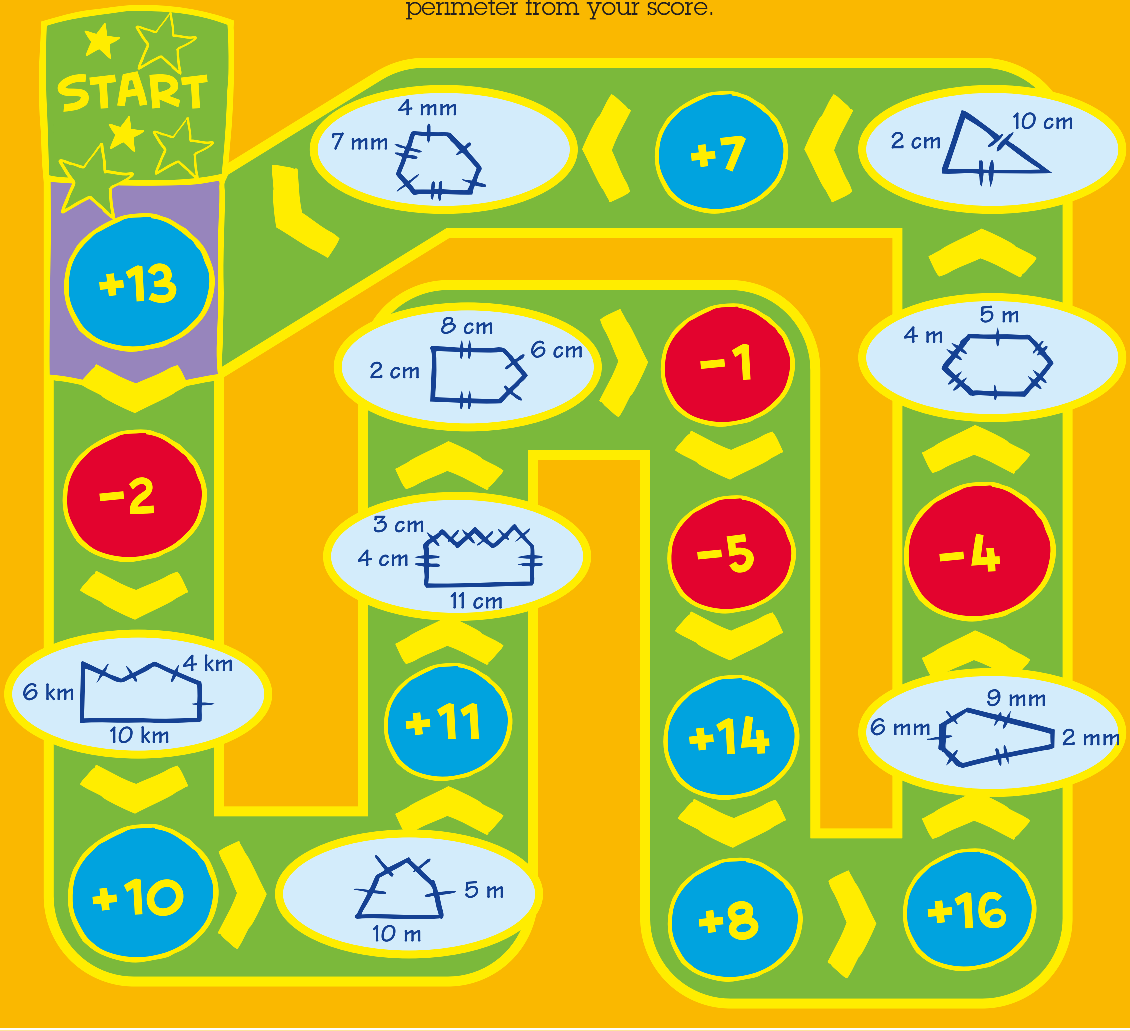
How to play:

1 Take turns choosing a formula from the list until all formulas are taken.

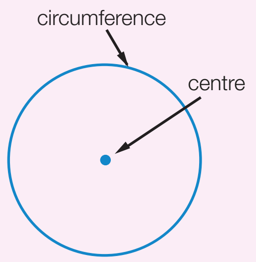
2 Each person starts with 30 points.

3 On your turn, roll the die and move your counter that many steps along the path.

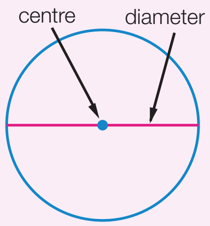
- If you land on a blue circle, add the points to your score as shown.
- If you land on a red circle, subtract the points from your score as shown.
- If you land on a shape and you chose the formula for its perimeter, score points equal to the length of the perimeter.
- If you land on a shape and you didn't choose the formula for its perimeter, then subtract points equal to the length of the perimeter from your score.



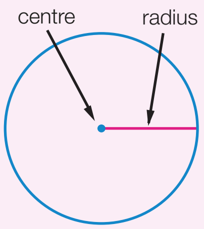
5.2 Circle relationships



The perimeter of a circle is called the **circumference**.



The **diameter** of a circle is any straight line from the circumference through the centre to the circumference at the other side.



The **radius** of a circle is any straight line from the centre to the circumference.

To calculate the perimeters and areas of circles or of shapes with circular parts, you need to understand how the circumference, diameter and radius of a circle are related to each other. In this exercise, you will investigate and establish these relationships.

5.2 Circle relationships

Navigator

Answers p. 642

1, 2, 3, 4, 6 (a), 7, 9	1, 2, 3, 4, 5, 6 (a), 7, 8, 9	1, 2, 3, 4, 5, 6, 7, 8, 9

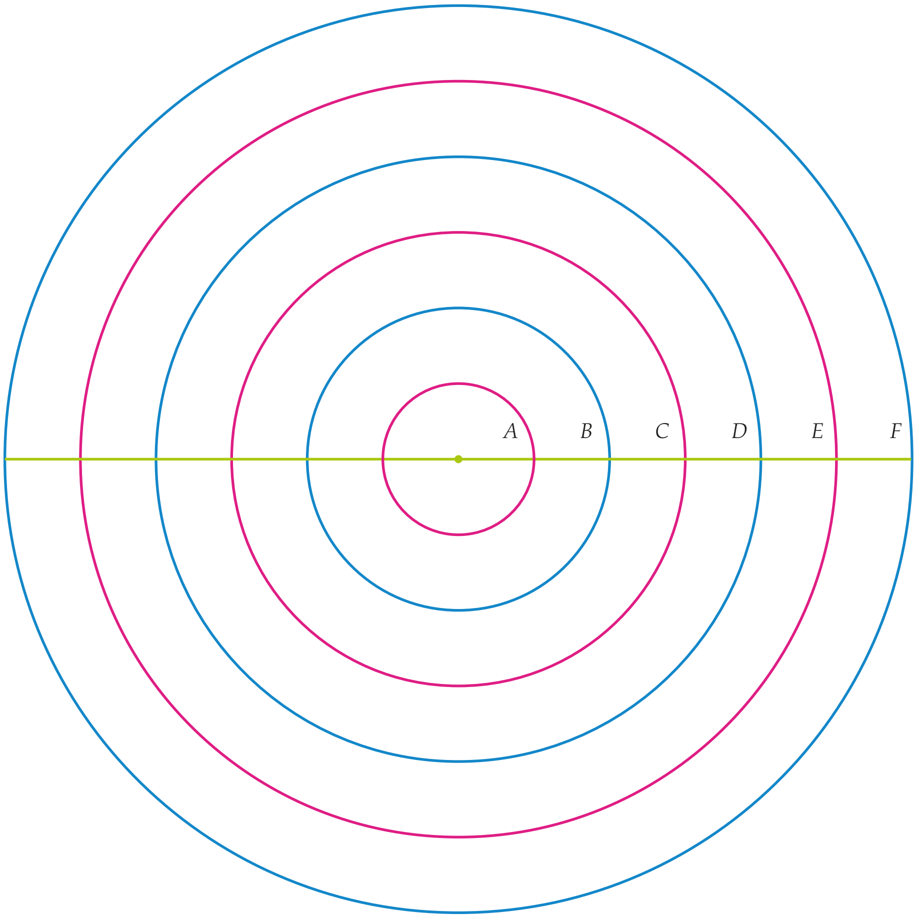
Equipment required: ruler, string, compass (optional), circular objects (optional—e.g. cups, plates, lids, drink bottles, wheels) for Question **1**; scientific calculator for Question **7(b)**

Fluency

1 Use your ruler to measure the diameter and radius of circles *A*–*F* on the following page. Record your measurements in a table. Measure and record the circumference of each circle by carefully laying a piece of string (or something similar) along it. Mark the beginning and end of the circumference on the string, then hold the string straight against a ruler to measure the length.

Alternatively, choose six circular objects, or use your compass to draw six circles of different sizes, then measure and record the diameter, radius and circumference using your ruler and string. (If you are using a compass, remember that the opening width of your compass will be the same as the radius of the circle.)

The diameter is the widest distance across the circle.



Circle	Radius *r* (cm)	Diameter *d* (cm)	Circumference *C* (cm)
A			
B			
C			
D			
E			
F			

Understanding

2 **(a)** Use your completed table from the previous question to describe any patterns or connections you can see between the numbers in the 'radius', 'diameter' and 'circumference' columns.

(b) To see the connections more clearly, add two columns to the end of your table, with the headings $\frac{C}{d}$ and $\frac{C}{r}$. Calculate these two unit ratios for each circle that you measured, round to 2 decimal places, and record your results in the new columns. These unit ratios give a scale factor for the length of the circumference compared to the diameter or radius.

Do you remember what 'unit ratio' means? Turn to Section 4.3.

3 Use your completed table from Question **1** and the unit ratios you calculated in Question **2** to copy and complete the following sentences.

(a) The circumference of a circle is approximately ___________ times the size of its diameter.

(b) The circumference of a circle is approximately ___________ times the size of its radius.

(c) The length of the diameter of a circle is ___________ the length of its radius.

Reasoning

4 Based on your answers to Questions **2** and **3**, if a circle has a diameter of 4 m, predict the lengths of the:

(a) radius **(b)** circumference.

5 Based on your answers to Questions **2** and **3**, if the radius of a circle is doubled, predict what will happen to:

(a) the diameter **(b)** the circumference.

6 **(a)** Explain whether or not the measurements of circumference that you made with string and ruler are accurate.

(b) How would your calculations of $\frac{C}{d}$ and $\frac{C}{r}$ be affected if you measured the circumference as:

(i) smaller than actual size **(ii)** bigger than actual size?

7 **(a)** The exact value of the ratio $\frac{\text{circumference}}{\text{diameter}}$ is represented by the symbol π (the Greek letter pi).

π is an irrational number, which means that if written as a decimal, it is a non-terminating, non-recurring decimal. Describe what the digits in a 'non-terminating, non-recurring' decimal look like.

(b) Find the 'π' key on your calculator. Write the value of π as a decimal approximation, rounded to:

(i) 2 decimal places **(ii)** 3 decimal places **(iii)** 4 decimal places.

Open-ended

8 **(a)** Choose a length for the diameter of a circle that is between the lengths of two diameters listed in the table you drew in Question **1**. (For example, if your table contains diameters of 4 cm and 6 cm, you could choose 5 cm or 5.5 cm.) Use your circumference measurements for the two known diameters to estimate the circumference of a circle with your chosen diameter.

(b) Repeat the question for a different value that is between two different diameters.

9 Cricket grounds vary in diameter and circumference. For example, the Sydney Cricket Ground is much smaller than the Melbourne Cricket Ground. Estimate the diameter of a cricket ground, such as a school oval. Find the approximate distance that a cricket team will run if they do a lap of honour after a win on that ground. Assume the cricket ground is circular.

Exploration CAS

Equipment required: TI-Nspire CAS or Casio ClassPad CAS

The relationship between the circumference and diameter of a circle

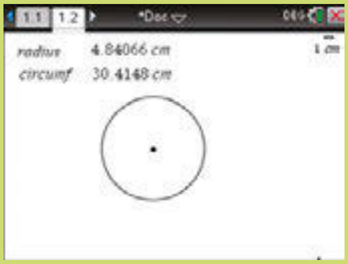
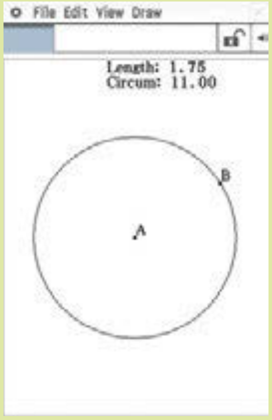
You can use CAS technology to construct different circles and to explore their properties. By doing this, you will become more familiar with how to use your CAS to help you study and understand circle geometry.

Investigating the relationship between the diameter and circumference of a circle

The perimeter of a circle is called the *circumference*.

The *radius* of a circle is the distance from the centre of the circle to the circumference.

The *diameter* of a circle is any straight line from the circumference through the centre to the circumference at the other side. It is double the size of the radius.

Using TI-Nspire CAS	**Using Casio ClassPad CAS**
Add **Geometry** to your document. To make sure that the values will be displayed with enough decimal places, select **menu** > **Graphs & Geometry Settings…** and set the **Display Digits** to **Float 6**.  Display Digits: Float 6 Graphing Angle: Radian Geometry Angle: Degree ☐ Automatically hide plot labels ☑ Show axis end values ☐ Show tool tips for function manipulation	From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen.
Select **menu** > **Shapes** > **Circle**, to select the centre point and radius of a circle to be drawn. To measure the length of the circle radius, select **menu** > **Measurement** > **Length**. Select the circle centre point and then select a point on the circle. You will then be able to place a caption showing this length. To show that this length is the radius, you can create a text label to be placed next to this value. Select **menu** > **Actions** > **Text**, then enter 'radius' to place this as a caption, which can be moved around the screen. To measure the length of the circumference, select **menu** > **Measurement** > **Length** and then select a point on the circle. You will then be able to place a caption showing this circumference. You can similarly create another text caption for this.  radius 4.84066 cm circumf 30.4148 cm	You can now draw a circle as one of the built-in default shapes. Select **Draw** > **Basic Object** > **Circle** to select the centre point and radius of a circle to be drawn. To measure the length of the circle radius, select the circle centre point and then select a point on the circle. Then select **Draw** > **Measurement** > **Length**. The length of the radius will appear.  Length: 1.75 Circum: 11.00 To measure the length of the circumference, select the circle and then select **Draw** > **Measurement** > **Circumference/Perimeter**. The length of the circumference will appear.

You can select the circle and drag it to change its size. This will change the radius and circumference length values. Change the size of your circle several times and write your different radius and circumference values in a table such as the one below.

Radius (cm)	4.562 83							
Circumference (cm)	28.6691							

To analyse these radius and circumference values, you can enter them all into a spreadsheet.

Using TI-Nspire CAS

Add **Lists & Spreadsheet** to your document.

Enter your circle data into the spreadsheet, with column **A** 'radius' and column **B** 'circumference'.

To explore the relationship between these values, you can calculate their ratio. To do this, in the **=** row under column **C**, enter the formula **= circumference/radius** to calculate the circumference values divided by their radius values.

Alternatively, you can calculate the circumference/radius ratio in the **Geometry** screen.

Select **menu** > **Actions** > **text** and enter the text **ratio = circumference/radius**. Then select **menu** > **Actions** > **Calculate** and select the words in the ratio equation text, followed by the number values on the screen. The calculated ratio will appear.

radius 5.04752 cm

circumf 31.7145 cm

$ratio = \frac{circumf}{radius}$ 6.28319

Using Casio ClassPad CAS

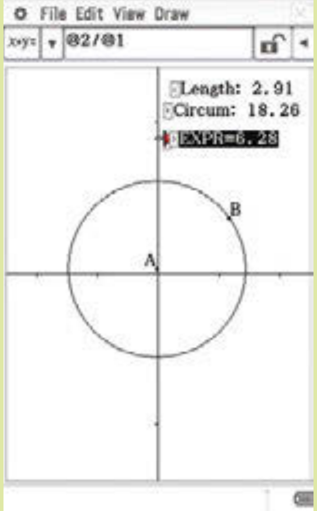
From the menu select **Spreadsheet**.

Enter your circle data into the spreadsheet, with radius values in column **A** and circumference values in column **B**.

To explore the relationship between these values, you can calculate their ratio. To do this, in cell **C1** enter the formula **=B1/A1** to calculate the circumference value divided by the radius value. Then select cell **C1** and select **Edit** > **Fill** > **Fill Range**, and set the **Range** to **C1:C8** (or to the **C** cell row where your data ends).

Alternatively, you can calculate the circumference/radius ratio in the **Geometry** screen.

Select **Draw** > **Expression**, then select the right arrow in the top right corner of the screen. This will display the box where you can enter a calculation.



@2/@1

Length: 2.91

Circum: 18.26

EXPR=6.28

Your circumference and radius measurements should now have a number label attached. To enter a measurement into the calculation circumference/radius, select the measurement's attached number label. This will appear in the calculation as an **@** symbol with the number label.

The calculated ratio will appear.

Now change the size of the circle and observe what happens to the calculated value of the ratio.

1 **(a)** What do you notice about the values in column **C**, the circle circumference divided by the radius?

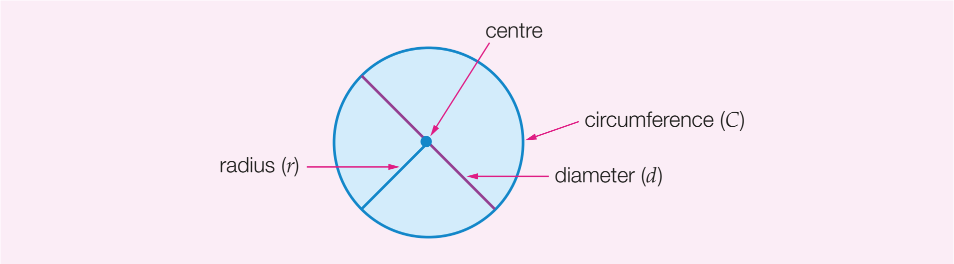
(b) Write a sentence about the relationship between the circumference and the radius of a circle.

(c) Write a formula to link the circumference (C) and radius (r) of a circle.

(d) Remembering the circle diameter $d = 2 \times r$, rewrite your formula to now link the circumference (C) and diameter (d) of the circle.

Circumference

5.3



For any circle:

- the length of the diameter d is twice the length of the radius r, so: $d = 2r$
- the length of the circumference C is a bit more than three times the diameter: $C \approx 3d$ (Remember that the ≈ symbol means 'approximately equal to'.)

The exact formula for circumference is given by $C = \pi d$, where π (the Greek letter *pi*) is an **irrational number** (a non-terminating, non-recurring decimal).

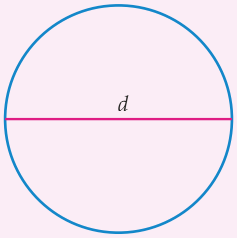
The value of π is approximately 3.141 592 654. This is just slightly more than $\pi \approx 3\frac{1}{7}$ or $\frac{22}{7}$ or $\pi \approx 3.14$ (correct to 2 decimal places), so people sometimes use these approximations for π when calculating by hand.

You should use the π key on your calculator, unless you are estimating circle measurements or your calculator does not have a π key.

Formulas for calculating the circumference of a circle

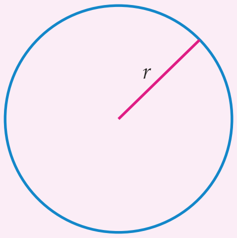
$C = \pi d$

where d is the length of the diameter



$C = 2\pi r$

where r is the length of the radius



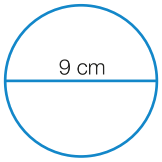
The value of π cannot be written as an exact decimal, but computers have been able to approximate it correct to more than a trillion digits!

Worked example 3

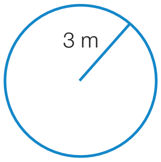
W.E. 3

Calculate the circumference of the following circles correct to 2 decimal places.

(a)



(b)



Thinking	Working
(a) **1** Choose the formula that involves diameter.	(a) $C = \pi d$
2 Substitute the value for d into the formula.	$C = \pi \times 9$
3 Use the π key on your calculator to evaluate.	$C \approx 28.274\,333\,88$
4 Round the answer to 2 decimal places and include the units.	$C = 28.27$ cm (correct to 2 d.p.)
(b) **1** Choose the formula that involves radius.	(b) $C = 2\pi r$
2 Substitute the value for r into the formula.	$C = 2 \times \pi \times 3$
3 Use the π key on your calculator to evaluate.	$C \approx 18.849\,555\,92$
4 Round the answer to 2 decimal places and include the units.	$C = 18.85$ m (correct to 2 d.p.)

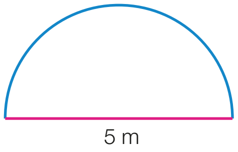
The perimeters of shapes can have both curved sections and straight sections. If the curved sections are parts of circles, you can calculate their length as a fraction of the circumference and then find the total length of the perimeter.

Worked example 4

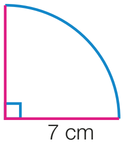
W.E. 4

Find the perimeter of each of the following shapes correct to 2 decimal places.

(a)



(b)



Thinking	Working
(a) **1** Identify the parts that make up the perimeter (half of a circumference and a straight line equal to the diameter). Use the diameter formula for circumference to write a formula for perimeter.	(a) $P = \frac{C}{2} + d$ where $C = \pi d$ $P = \frac{\pi d}{2} + d$

2 Substitute for *d*. $= \frac{\pi \times 5}{2} + 5$

3 Write the answer rounded to the correct number of decimal places and include the units. $= 12.853\,98\ldots$

$= 12.85$ m (correct to 2 d.p.)

(b) 1 Identify the parts that make up the perimeter (a quarter of a circumference and two straight lines equal to the radius). Use the radius formula for circumference to write a formula for perimeter.

(b) $P = \frac{C}{4} + 2r$ where $C = 2\pi r$

$P = \frac{2\pi r}{4} + 2r$

2 Substitute for *r*. $= \frac{2 \times \pi \times 7}{4} + 2 \times 7$

3 Write the answer rounded to the correct number of decimal places and include the units. $= 24.995\,57\ldots$

$= 25.00$ cm (correct to 2 d.p.)

It is most accurate to keep all of the decimal places on your calculator until you reach your final answer, then round the values only at the end.

5.3 Circumference

Navigator

1 (columns 1–2), 2, 3, 5, 8, 9	1 (columns 2–3), 2, 3, 4, 5, 6, 8, 9, 10, 11	1 (columns 2–3), 2, 3, 4, 5, 6, 7, 8, 10, 11

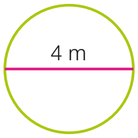
Answers p. 643

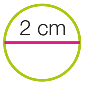
Equipment required: scientific calculator

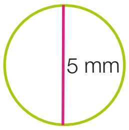
Fluency

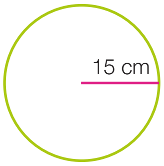
W.E. 3

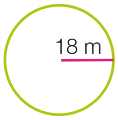
1 Calculate the circumference of the following circles correct to 2 decimal places.

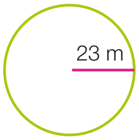
(a)


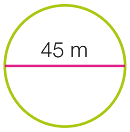
(b)


(c)


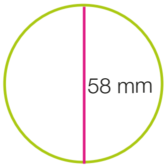
(d)


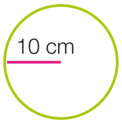
(e)


(f)


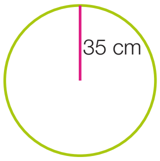
(g)


(h)


(i)


(j)


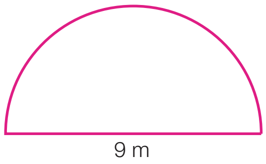
(k)


(l)


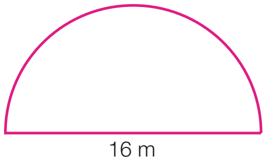
W.E. 4

2 Find the perimeter of each of the following shapes correct to 2 decimal places.

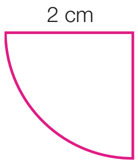
(a)



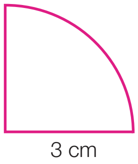
(b)



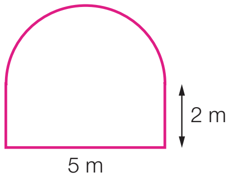
(c)



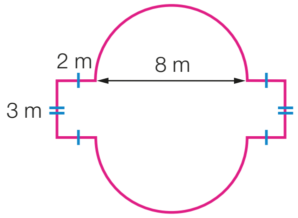
(d)



(e)



(f)



Understanding

3 **(a)** The distance around the outside of a round biscuit tin lid, if the lid has a radius of 21 cm, is closest to:

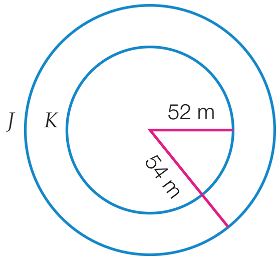
A 66 cm **B** 131 cm **C** 132 cm **D** 133 cm

(b) A bike tyre has a diameter of 70 cm. The circumference of the tyre is closest to:

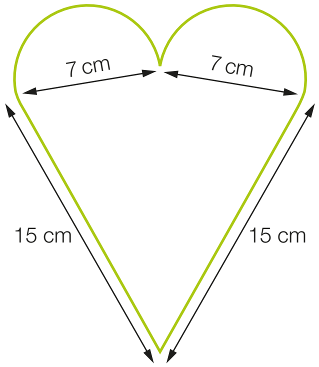
A 110 cm **B** 220 cm **C** 290 cm **D** 440 cm

4 A bicycle wheel has a diameter of 60 cm. How far would it move if it is rolled through four revolutions (complete turns)? Give your answer in metres, correct to 2 decimal places.

5 Jane and Kate run once around the circular track shown. Jane runs along path *J*, while Kate takes path *K*. How much further than Kate does Jane run? Give your answer correct to 2 decimal places.



6 A box of chocolates is shaped like a heart as shown. Find the perimeter of the box correct to the nearest mm.

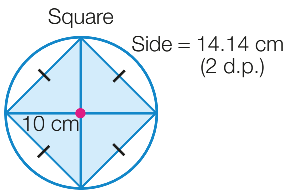


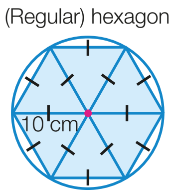
Reasoning

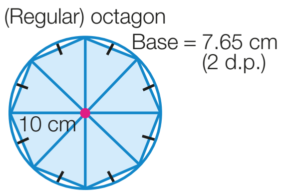
7 **(a)** Copy the table. Calculate the perimeter of each shape to fill in the second column.

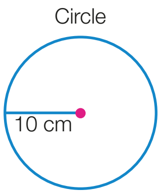
Then calculate the ratio $\frac{P}{D}$ for each shape and write these values in the last column.

Round values to 2 decimal places.









	Perimeter or circumference, *P* (cm)	**Diagonal length or diameter, *D* (cm)**	**Ratio $\frac{P}{D}$**
Square		20	
Hexagon		20	
Octagon		20	
Circle		20	

(b) For a circle, $\frac{P}{D}$ or $\frac{\text{circumference}}{\text{diameter}} = \pi$, which is slightly more than 3. For which shape is the ratio $\frac{P}{D}$ exactly 3?

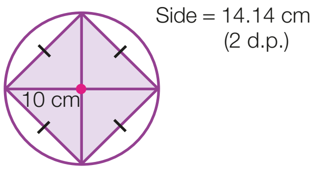
(c) Which of the three polygons above has a ratio of $\frac{P}{D}$ closest to π?

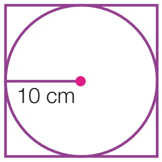
(d) Name another polygon for which the ratio $\frac{P}{D}$ would be even closer to π.

8 **(a)** Find the approximate perimeter of the internal square.

(b) Find the perimeter of the external square.

(c) Calculate C using $C = 2\pi r$. Comment on this value compared to the values you found for the perimeters of the squares in the first two parts.



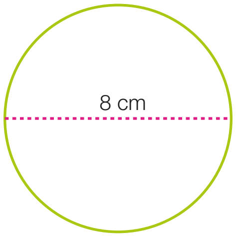


Open-ended

9 A semicircle has a perimeter between 50 cm and 60 cm. Write a possible value for the radius of the semicircle.

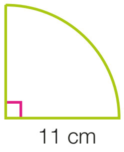
10 Riya found the circumference of a circle, but lost her work and could only remember part of the answer. Her answer began with the digits 314 followed by some zeros. Riya's friend said she could tell that the diameter must have been approximately a power of 10, such as 100 or 1000. Explain how her friend knew this.

11 Yogi was asked to find the perimeter of the following two shapes. Here is his working:



$C = 2 \times \pi \times 8$

$= 50.27$ cm (2 d.p.)



$P = \dfrac{2 \times \pi \times 11}{4}$

$= 17.28$ cm (2 d.p.)

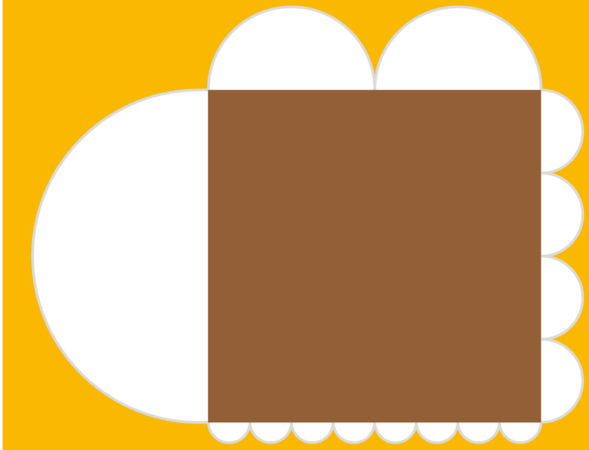
(a) Explain how you can tell, without using a calculator, that each answer is incorrect.

(b) For each one, explain the mistake that Yogi has made.

(c) Calculate the correct answers.

Problem solving

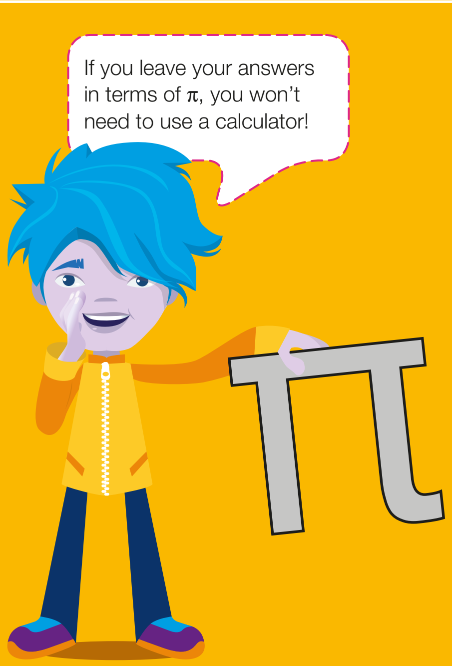
The icing on the cake



A large square cake is being decorated with a series of curved pieces of icing, each in the shape of a semicircle. The semicircles along any edge are the same size as each other.

1 If the biggest curve of icing is 80 cm long, what are the lengths of each of the other curved pieces?

2 What is the total length of icing around the cake?



Strategy options

- Make a table.
- Break problem into manageable parts.

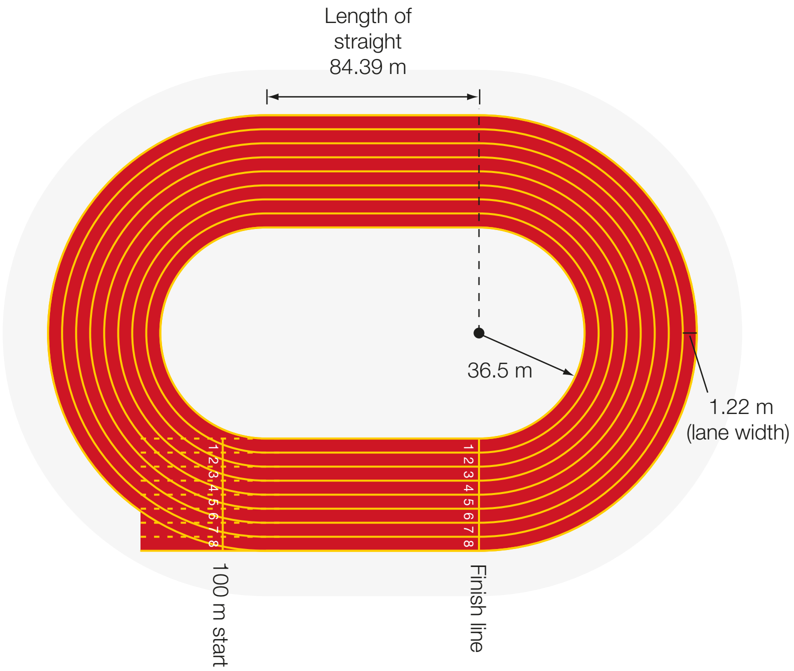
Investigation

Staggered starts

Equipment required:
scientific calculator

An Olympic-standard running track must be built according to very strict guidelines. The International Association of Athletics Federations (IAAF) says that it must be made of two straight sections that are 84.39 m in length, and two curved sections that have an internal radius of 36.5 m. The running lanes must be 1.22 m wide.

Athletes in the 200 m and 400 m races line up in a ‘staggered start’ formation, so the competitor in lane 2 starts in front of lane 1, lane 3 starts in front of lane 2, and so on, to lane 8. Can you see why this is necessary?



The Big Question

To ensure a 'fair' race, how far in front of the lane 1 runner should the lane 8 runner start?

Engage

1 Athletes must stay within their lane when running or they will be disqualified. Top athletes train to run as close as possible to the inside line of their lane (the one closer to the inside of the track). Why is this?

Explore

2 Use the measurements provided by the IAAF to calculate and compare the distances run by athletes in lane 1 and lane 8 if they run one complete lap of the track.

(You may have to make some assumptions about how far the athlete runs from the lane lines. If you do, make sure you state any assumptions clearly.)

Strategy options

- Draw a diagram.
- Have I seen a similar problem?
- Break problem into manageable parts.

Explain

3 **(a)** If the perimeter of the track is calculated using the measurements for the internal radius and straight sections given on the diagram, it is less than 400 m. Why is this?

(b) By how much should you adjust the radius so that one lap of the track is exactly 400 m?

(c) How do your answers to this question apply to an athlete running in that lane?

4 Describe how to find the radius of the curve run by the athlete in lane 8.

Elaborate

5 Use your answers to **2**, **3** and **4** to answer the Big Question. Explain any assumptions you made.

6 The distance that the athlete in lane 8 stands in front of the athlete in lane 1 at the start of the race is called the 'stagger distance'. Use the 'stagger distance' you have found for lane 8 to find the stagger distances for each of lanes 2 to 7.

Evaluate

7 Describe how you approached this problem. List the main steps or tasks you undertook to solve it.

8 Did you need to make any assumptions in order to solve the problem? What were they? Do you think they were reasonable?

9 In races such as the 200 m and the 400 m, the middle two lanes (Lane 4 and Lane 5) are seen as being the best lanes to run in, and are usually given to the fastest qualifiers for the race. Suggest some possible reasons for this.

10 For an athletics track to be rated as 'Olympic standard', it must meet very strict and specific measurement standards. Why do you think this is?

11 If you had to mark the starting positions for each lane on the track, how would you go about it?

Extend

12 Imagine that the workers making the track decided to round up the measurements for the radius and the straights to 37 m and 85 m, respectively.

(a) How would this change the total length of the track?

(b) The formula $t = \frac{d}{s}$ can be used to calculate the time taken to complete a race, where d = distance and s = average speed. Olympic-standard 400 m runners can run at an average speed of 8.9 m/s. Use the formula to calculate the time they would take to complete 400 m, running at that speed.

(c) Now, calculate how much time it would take the same runner to complete a race on the track that is the length you calculated in **(a)**.

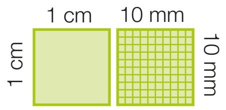
(d) Find the difference in your answers to **(b)** and **(c)**. Is this a significant difference? How would these race results be different compared to races on a standard track?

5.4

Area

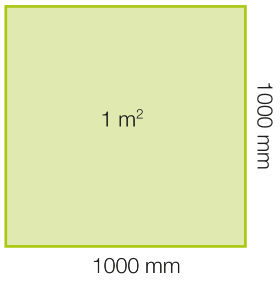
Converting units of area

Area is the amount of surface within the boundary of a shape. Area is measured in 'square' units, such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) or square kilometres (km^2). When you find the area of a shape, you are counting the number of whole squares and parts of whole squares that would cover the surface exactly.



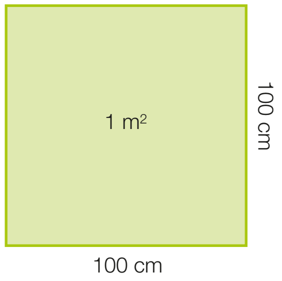
$$1 \text{ cm} = 10 \text{ mm}$$

$$\begin{aligned} 1 \text{ cm}^2 &= 10 \text{ mm} \times 10 \text{ mm} \\ &= 100 \text{ mm}^2 \end{aligned}$$



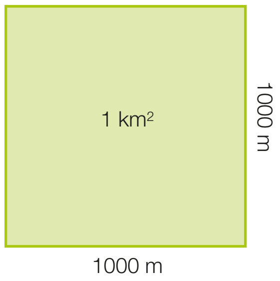
$$1 \text{ m} = 1000 \text{ mm}$$

$$\begin{aligned} 1 \text{ m}^2 &= 1000 \text{ mm} \times 1000 \text{ mm} \\ &= 1\,000\,000 \text{ mm}^2 \end{aligned}$$



$$1 \text{ m} = 100 \text{ cm}$$

$$\begin{aligned} 1 \text{ m}^2 &= 100 \text{ cm} \times 100 \text{ cm} \\ &= 10\,000 \text{ cm}^2 \end{aligned}$$



$$1 \text{ km} = 1000 \text{ m}$$

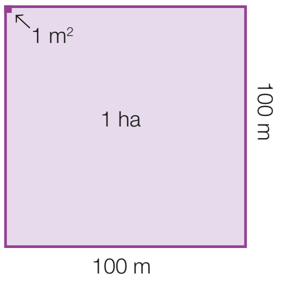
$$\begin{aligned} 1 \text{ km}^2 &= 1000 \text{ m} \times 1000 \text{ m} \\ &= 1\,000\,000 \text{ m}^2 \end{aligned}$$

Area can also be measured in units called **hectares**. A hectare (ha) is an area of 10 000 m^2. It is the area of a square that is 100 m long and 100 m wide.

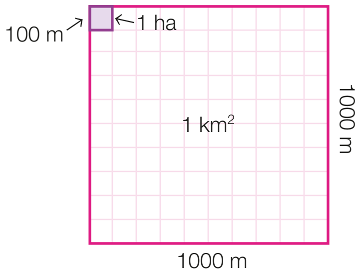
Hectares are often used to describe the size of a property, such as a farm. A hectare is smaller than a square kilometre.

- A hectare is approximately the size of two hockey or soccer fields placed side by side.
- A square kilometre is the approximate size of a small suburb.

$$1 \text{ ha} = 10\,000 \text{ m}^2$$



$$1 \text{ km}^2 = 100 \text{ ha}$$



Remember when you convert between units of area, this is *not* the same as converting between units of length. Although 1 m is equal to 100 cm, 1 m^2 is *not* equal to 100 cm^2. Because the units are square units, the conversion factor (the number you multiply or divide by) is the *square* of the conversion factor for length.

Units of length:

$$\begin{tikzcd}
\text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

Units of area:

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"]
\end{tikzcd}$$

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 10^2"] & \text{ha} \arrow[l, bend left, "\div 10^2"] \arrow[r, bend left, "\times 100^2"] & \text{m}^2 \arrow[l, bend left, "\div 100^2"]
\end{tikzcd}$$

Worked example 5

W.E. 5

Copy and complete the following conversions.

(a) 3.5 m^2 = _____ mm^2

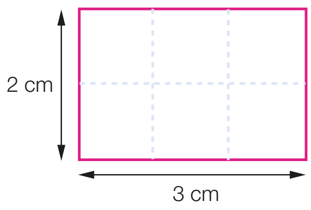
(b) 27 000 m^2 = _____ ha

Thinking	Working
(a) You are converting from a larger unit to a smaller unit, so need to multiply. There are 1000^2 mm^2 in every m^2, so multiply by 1000^2 to convert m^2 to mm^2.	(a) 3.5×1000^2 $= 3.5 \times 1\,000\,000$ $= 3\,500\,000$ mm^2
(b) You are converting from a smaller unit to a larger unit, so need to divide. There are 100^2 m^2 in every ha, so divide by 100^2.	(b) $27\,000 \div 100^2$ $= 27\,000 \div 10\,000$ $= 2.7$ ha

If you need to calculate the area of a shape in units different from the ones given, it may be easier to convert the dimensions of the shape into the new units of length first, then calculate the area.

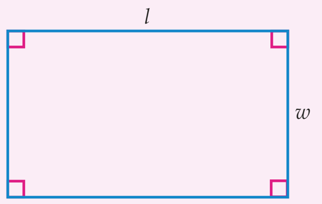
Area of rectangles and squares

The area of a **rectangle** is found by multiplying the length by the width. There are two rows of three 1-centimetre squares inside this rectangle, so $2 \times 3 = 6$ cm^2.



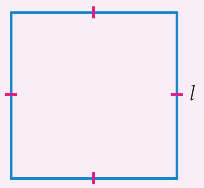
The area of a **square** is found by multiplying the side length by itself (because length = width).

Area of a rectangle



$A = lw$

Area of a square



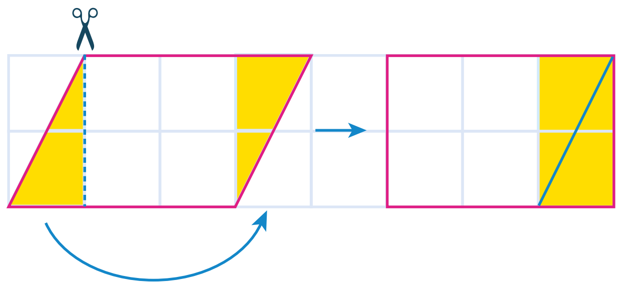
$A = l^2$

where l = length of one side, w = width (length of other side)

Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

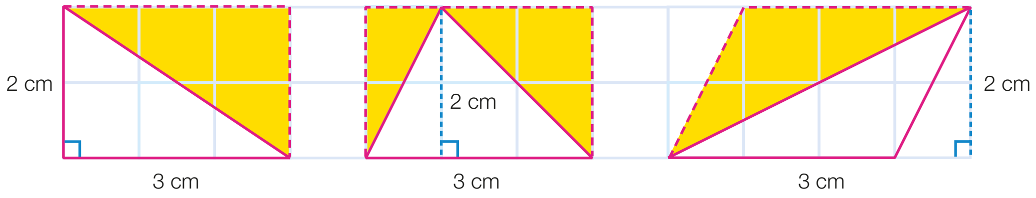
You can cut and rearrange a parallelogram to form a rectangle as shown.



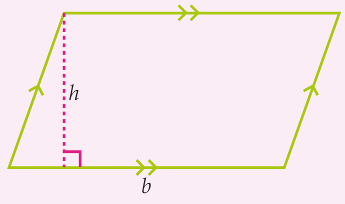
You can see from the diagram that a parallelogram with a **base** length of 3 cm and a **height** of 2 cm has the same area as a 3 cm × 2 cm rectangle. The height is **perpendicular** to the base. ('Perpendicular' means 'at right angles to'.)

Triangles

Triangles have an area equivalent to either half of a rectangle or half of a parallelogram. The rectangles and parallelogram below each have an area of 6 cm^2 (3 cm × 2 cm), so each of the triangles below has an area of 3 cm^2; which is half of 6 cm^2.

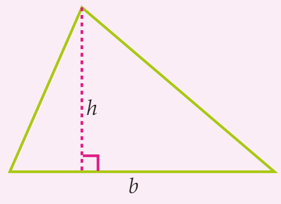


Area of a parallelogram



$A = bh$

Area of a triangle



$A = \frac{1}{2}bh$ or $\frac{bh}{2}$

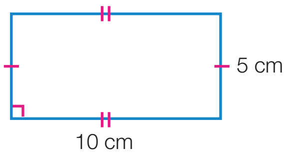
where b = base length, h = height perpendicular to base

Worked example 6

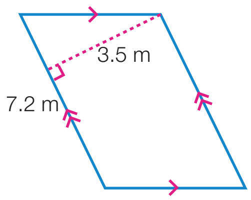
W.E. 6

Find the area of each of the following shapes.

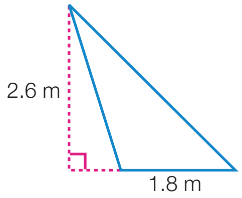
(a)



(b)



(c)

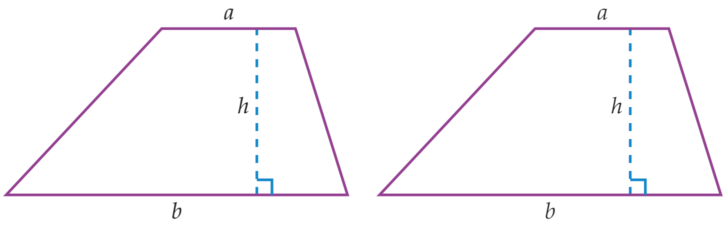


Thinking	Working
(a) **1** Write the formula for the area of a rectangle.	(a) $A = lw$
2 Substitute the values for *l* and *w*.	$= 10 \times 5$
3 Evaluate and include the units.	$= 50\text{ cm}^2$
(b) **1** Write the formula for the area of a parallelogram.	(b) $A = bh$
2 Substitute the values for *b* and *h*.	$= 7.2 \times 3.5$
3 Evaluate and include the units.	$= 25.2\text{ m}^2$
(c) **1** Write the formula for the area of a triangle.	(c) $A = \frac{1}{2}bh$
2 Substitute the values for *b* and *h*.	$= \frac{1}{2} \times 1.8 \times 2.6$
3 Evaluate and include the units.	$= 2.34\text{ m}^2$

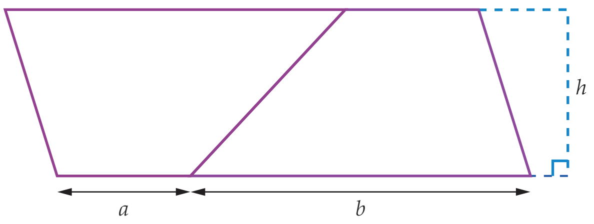
Trapeziums

A **trapezium** is a quadrilateral with only one pair of opposite sides parallel. It is sometimes also called a trapezoid.

The two trapeziums below are identical.



If the first shape is rotated around 180° and joined to the other, then they form a parallelogram, as shown below.



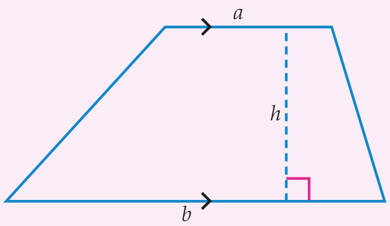
The area of this parallelogram is (base length) × height

$= (a + b) \times h$

$= (a + b)h$

However, this parallelogram contains two of the same trapezium, so the area of one trapezium is half of this amount, $\frac{(a+b)\times h}{2}$ or $\frac{1}{2}(a+b)\times h$.

Area of a trapezium



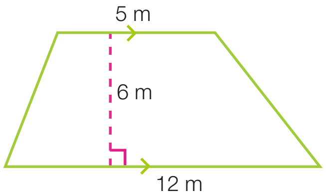
$$A = \tfrac{1}{2}(a+b)h \quad \text{or} \quad \frac{(a+b)h}{2}$$

where a and b = the lengths of the parallel sides, h = height perpendicular to a and b

Worked example 7

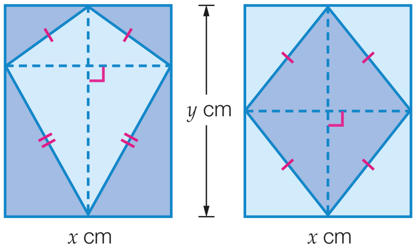
W.E. 7

Find the area of the following trapezium.



Thinking	Working
1 Write the formula for the area of a trapezium.	$A = \frac{1}{2}(a+b)h$
2 Substitute the values for a, b and h.	$= \frac{1}{2} \times (5 + 12) \times 6$
3 Evaluate and include the units.	$= 8.5 \times 6$ $= 51\ \text{m}^2$

Kites



A **kite** is a quadrilateral with two pairs of equal adjacent sides. 'Adjacent sides' means sides that are next to each other.

A **rhombus** is a special kind of kite that has four equal sides. Because both pairs of its sides are parallel, a rhombus is also a special kind of parallelogram, with four equal sides.

The diagonals of a kite or rhombus intersect at right angles.

(*Note:* In a quadrilateral, a 'diagonal' is a line between opposite corners. The diagonals in these diagrams are shown as dashed lines.)

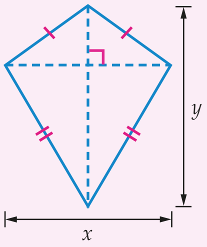
The area of a kite or rhombus is equal to half the area of a rectangle that encloses it, as shown here.

The length and width of the enclosing rectangle are equivalent to the diagonals of the rhombus or kite (shown in the following diagrams as the lengths x and y).

If the diagonals are $x = 2$ cm and $y = 4$ cm, as shown, then the area of the shape can be calculated as two identical (mirror-image) triangles, defined by the edges and diagonals:

$$
\begin{aligned}
A &= \frac{1}{2} \times \left(\frac{x}{2}\right) \times y + \frac{1}{2} \times \left(\frac{x}{2}\right) \times y \\
&= \frac{1}{2}xy \\
&= \frac{1}{2} \times 2 \times 4 \\
&= 4 \text{ cm}^2 \text{ for both shapes}
\end{aligned}
$$

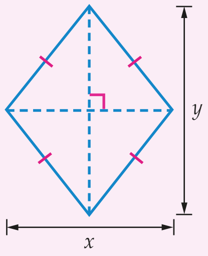
Area of a kite



Kite

$$A = \frac{1}{2}xy$$

Area of a rhombus



Rhombus

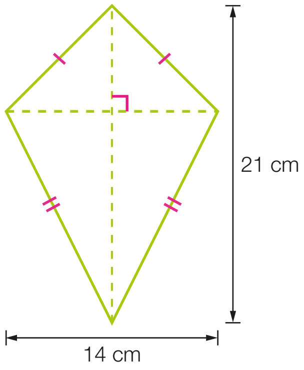
$$A = \frac{1}{2}xy$$

where x and y are the lengths of the diagonals

Worked example 8

W.E. 8

Find the area of the following kite.



Thinking	Working
1 Write the formula for the area of a kite.	$A = \frac{1}{2}xy$
2 Substitute the values for x and y.	$= \frac{1}{2} \times 14 \times 21$
3 Evaluate and include the units.	$= 147 \text{ cm}^2$

5.4 Area

Navigator

1, 2 (column 1), 3, 4 (column 1), 5 (column 1), 6 (column 1), 8, 9, 11, 12, 19, 22, 24	1, 2 (column 2), 3, 4 (column 2), 5 (column 2), 6 (column 2), 7, 8, 9, 10, 11, 12, 13, 16, 17, 19, 22, 23, 24	1 (c–d), 2 (column 2), 3, 4 (column 2), 5 (column 2), 6 (column 2), 7, 8, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 23

Answers p. 643

Equipment required: calculator for Questions **4–24**

Fluency

1 Convert the following measurements.

(a) **(i)** 1 km = ______ m **(ii)** 1 km^2 = ______ m^2

(b) **(i)** 1 cm = ______ m **(ii)** 1 cm^2 = ______ m^2

(c) **(i)** 28 m = ______ cm **(ii)** 28 m^2 = ______ cm^2

(d) **(i)** 5400 m = ______ km **(ii)** 5400 m^2 = ______ km^2

2 Copy and complete the following conversions.

W.E. 5

(a) 5 m^2 = ____ cm^2 **(b)** 0.0065 m^2 = ____ cm^2

(c) 94 cm^2 = ____ mm^2 **(d)** 9.76 cm^2 = ____ mm^2

(e) 12 ha = ____ m^2 **(f)** 1.003 ha = ____ m^2

(g) 50 000 m^2 = ____ ha **(h)** 9800 mm^2 = ____ cm^2

(i) 67 000 cm^2 = ____ m^2 **(j)** 950 cm^2 = ____ m^2

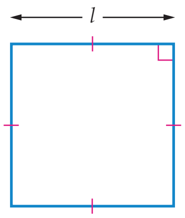
(k) 23.6 ha = ____ m^2 **(l)** 3400 m^2 = ____ ha

(m) 460 cm^2 = ____ m^2 **(n)** 345 cm^2 = ____ mm^2

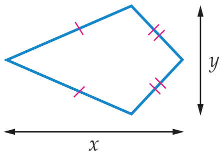
(o) 9 000 000 m^2 = ____ km^2 **(p)** 5570 ha = ____ km^2

3 For each of the following, **(i)** identify the shape and **(ii)** write the formula for the area.

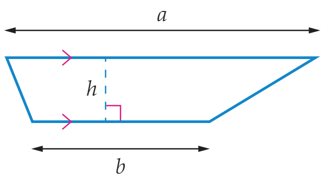
(a)



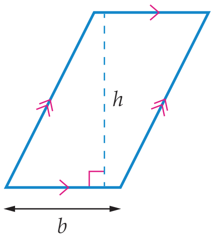
(b)



(c)



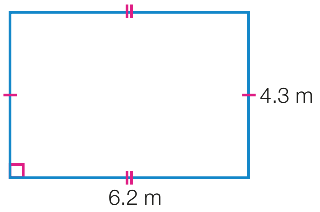
(d)



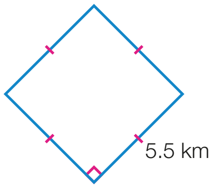
W.E. 6

4 Find the area of each of the following shapes.

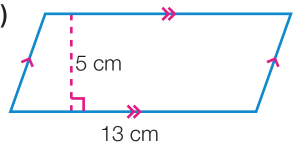
(a)



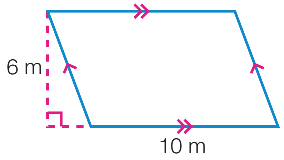
(b)



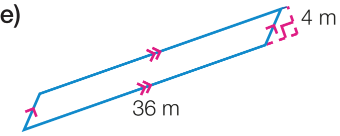
(c)



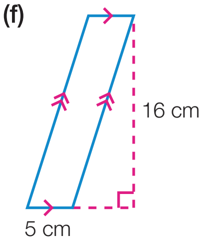
(d)



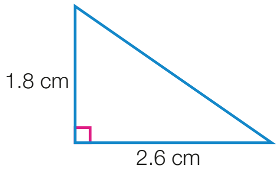
(e)



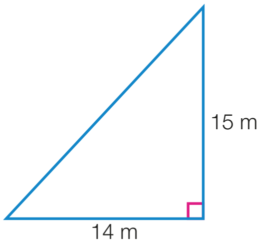
(f)



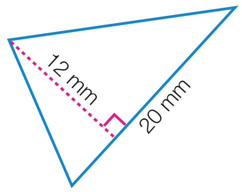
(g)



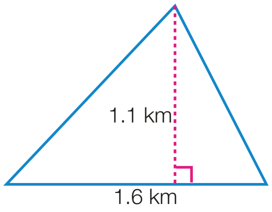
(h)



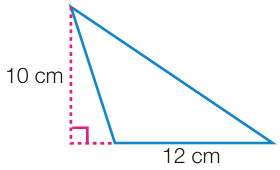
(i)



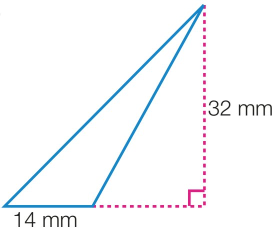
(j)



(k)



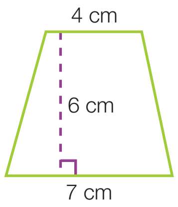
(l)



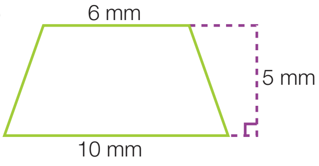
W.E. 7

5 Find the area of each of the following trapeziums.

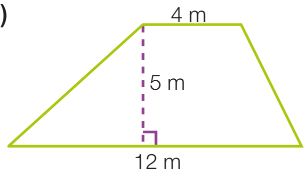
(a)



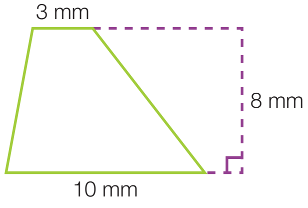
(b)



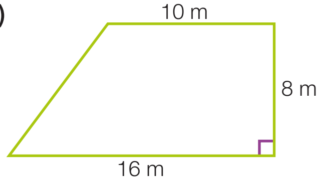
(c)



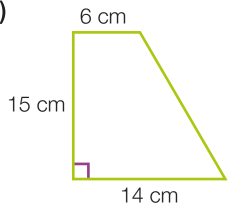
(d)



(e)



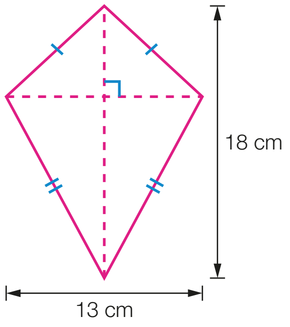
(f)



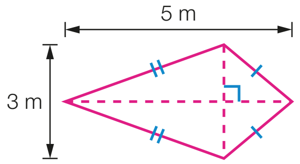
W.E. 8

6 Find the area of each of the following kites.

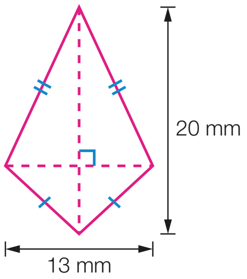
(a)



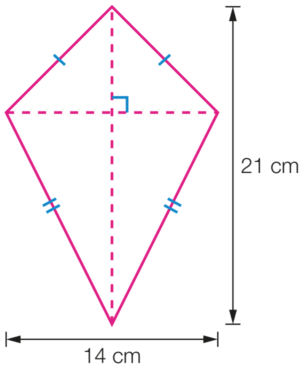
(b)



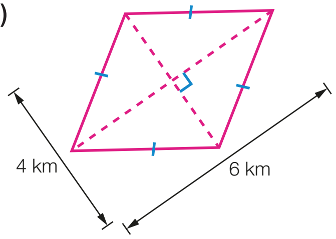
(c)



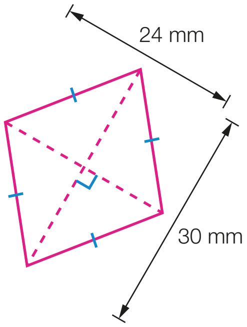
(d)



(e)

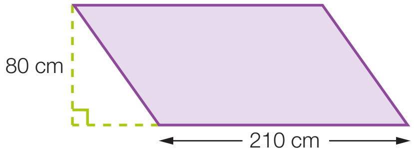


(f)

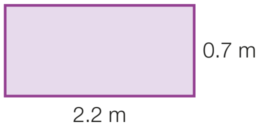


7 Find the area of the following figures in: **(i)** m^2 **(ii)** cm^2.

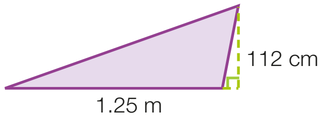
(a)



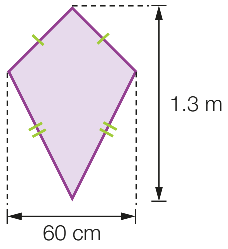
(b)



(c)



(d)



8 **(a)** A trapezium with perpendicular height 8 cm and parallel sides measuring 10 cm and 18 cm has an area of:

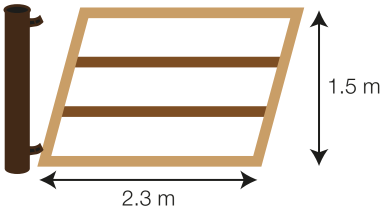
A 224 cm^2 **B** 180 cm^2 **C** 162 cm^2 **D** 112 cm^2

(b) A rhombus with side length 20 cm has diagonals 24 cm and 32 cm long. Its area is:

A 384 cm^2 **B** 400 cm^2 **C** 560 cm^2 **D** 768 cm^2

Understanding

9 A very old gate has changed shape to appear as shown. What area does the gate's frame now enclose?



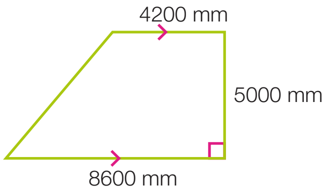
10 A parallelogram with base length 25 cm has an area of 375 cm^2. The height of the parallelogram is:

A 30 cm **B** 20 cm **C** 15 cm **D** 7.5 cm

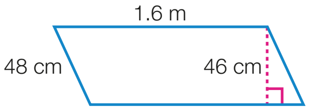
11 Sam wants to pave an area using slate costing $32 per square metre. The area he wishes to pave is shaped as shown.

(a) Find the area to be paved in m^2.

(b) What will be the total cost of the slate?



12 Which of the following calculations shows how to calculate the area of the parallelogram, in cm^2?



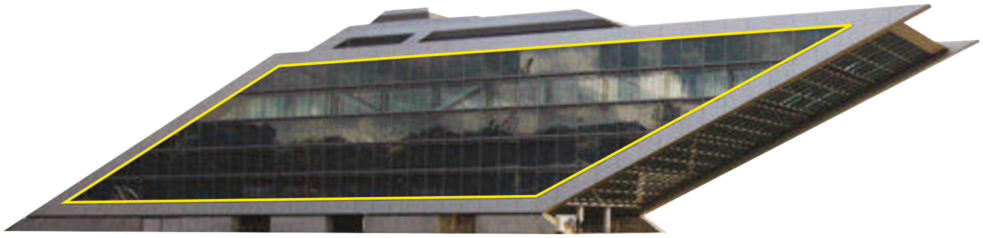
A $\frac{46 \times 48}{2}$ **B** 1.6×0.48

C 160×48 **D** 160×46

13 A beachside town has houses along 5 km of beachfront, occupying land that stretches up to 150 m inland from the beach. Find the area occupied by the town in:

(a) square metres **(b)** hectares **(c)** square kilometres.

14 This parallelogram-shaped building is in Hamburg, Germany. If the base length of the parallelogram shape enclosing the windows is 85 m, and each of the five floors is 3.5 m high, what is the area of glass on the front face of the building shown?



15 This mosaic was found on the floor of an ancient Roman building. It is made of tiny square tiles, which are each approximately 2 cm by 2 cm. At the centre of the floor is a pattern of eight white rhombuses with a side length of 40 cm and diagonal lengths of 36 cm and 72 cm. In the middle of each big rhombus is a smaller coloured rhombus whose dimensions are one-third the size.

Calculate:

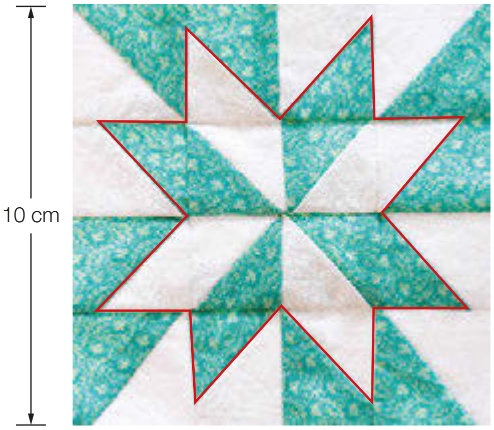
(a) the total area of the eight-rhombus pattern

(b) the perimeter of the eight-rhombus pattern

(c) the number of white tiles needed to make the pattern.

Reasoning

16 In this patchwork quilt design, a square 10 cm long is divided into 32 identical green and white isosceles triangles. Two triangles make up a parallelogram. Find the area of the star figure made up by eight parallelograms.



17 Kwan has cut out a square of side length 8 cm from a sheet of metal.

(a) Hugh has cut out a parallelogram of the same area but with a base length of 10 cm. What will be the height of Hugh's parallelogram?

(b) Daniella wants to cut out a triangle with the same area but with a base length of 8 cm. What will be the height of her triangle?

18 **(a)** A rectangle of width 6 cm and length 8 cm has the same area as a trapezium of height 4 cm. What do the lengths of the parallel sides of the trapezium add to?

(b) The areas of a rectangle, a triangle and a trapezium are all 128 cm^2. If the heights of all three shapes are 8 cm, what are the lengths of each shape?

19 **(a)** **(i)** Find the side length of a square that has the same area as a rectangle which is 18 cm by 50 cm.

(ii) Find the difference in the perimeters of the two shapes.

(b) **(i)** Find the width of a rectangle that is 24 cm long, if its area is the same as a square of side 18 cm.

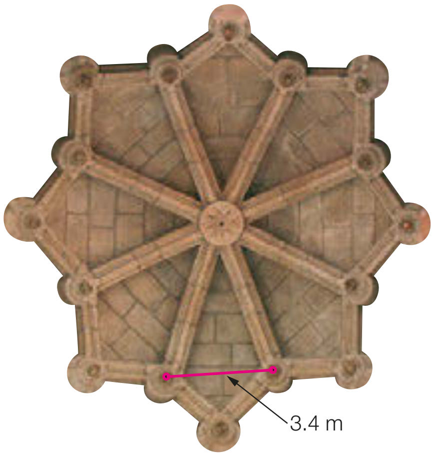
(ii) Find the difference in the perimeters of the two shapes.

20 Omar has to design a kite with an area of 2 m^2. For the kite to fly well, the length of one diagonal has to be 2.5 times the length of the other diagonal. How long does he need to make each of the diagonals of his kite? Write answers in metres correct to 1 decimal place.

21 This pattern is found on the ceiling of a cathedral in Spain. It has eight identical kites arranged in a circle. The long diagonal of each kite is 1.5 times as long as the short diagonal, which is 3.4 m long.

(a) Calculate the total area of the kites, in m^2.

(b) If a circle is drawn so that the outermost vertex (corner) of each kite lies on the circumference of the circle, then what is the circumference of the circle, correct to 2 decimal places?



Open-ended

22 Amanda's restaurant has a floor plan in the shape of a parallelogram. The floor has an area of 48 m^2. Give three possible combinations for the base length and height (in whole numbers of metres) for the floor of Amanda's restaurant.

23 **(a)** Ramzi has built a new deck at the rear of his house. It is in the shape of a trapezium with an area of 80 m^2. He needs to buy some railing for the parallel sides of the deck but has forgotten the lengths of these two sides. He knows that these lengths are whole numbers of metres and that the distance between the parallel sides is 10 metres. Give three possible combinations for the lengths of the parallel sides.

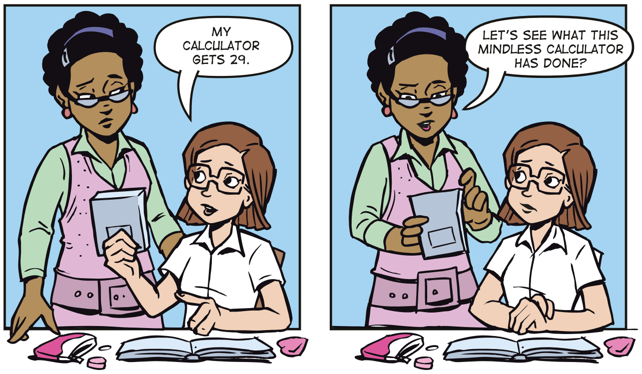
(b) Calculate the length of railing he will need to buy for one of your combinations of lengths.

24 Antoinette is trying to find the area in cm^2 for a trapezium with dimensions $a = 10$ cm, $b = 4$ cm and $h = 6$ cm.

Antoinette's calculator working shows: $\frac{1}{2} \times 10 + 4 \times 6$.

(a) State the error that Antoinette has made.

(b) Find the actual area of the trapezium.



Problem solving

Tiling a floor

A rectangular floor area needs to be tiled. The floor has a perimeter of 280 cm. Its length is 2.5 times its width. 160 square tiles must be used.

What is the area of each square tile, to the nearest cm^2?

Strategy options

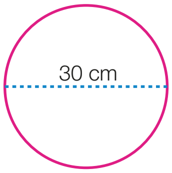
- Guess and check.
- Work backwards.
- Test all possible combinations.

Half-time 5

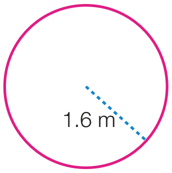
1 Find the circumference of the following circles.

5.3

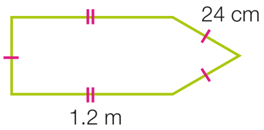
(a)



(b)

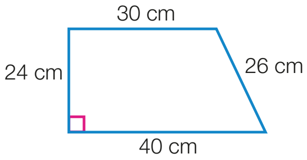


2 Find the perimeter of the shape in centimetres.



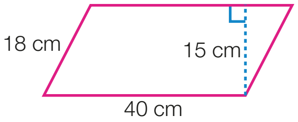
5.1

3 Find the area and perimeter of the trapezium.



5.1, 5.4

4 Draw a rectangle that has the same base length and area as the parallelogram.



5.4

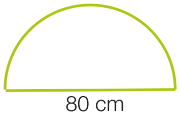
5 Find the perimeter of a regular octagon of side length 16 mm.

5.1

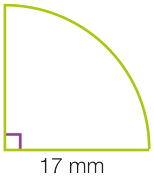
6 Find the perimeter of the following shapes, correct to 2 decimal places.

5.3

(a)

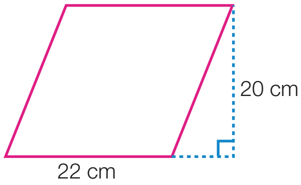


(b)



7 (a) Find the area of the parallelogram.

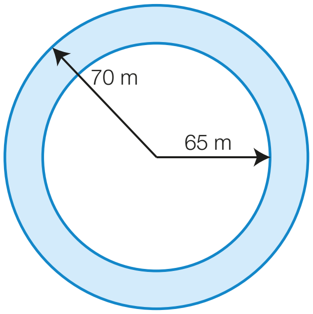
(b) Draw two triangles that have the same base length and same area as the parallelogram.



5.4

8 A running track has an inside radius of 65 m and an outside radius of 70 m. How much further would you run if you ran on the outside of the track instead of on the inside of the track? (Give your answer correct to the nearest metre.)

5.3



9 A circle has a radius of 11 cm. Find the side length of a square with the same perimeter as the circumference of the circle. Round your answer to 2 decimal places.

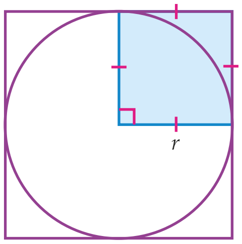
5.3

10 Estimate (do not calculate) the circumference of a circle with a diameter of 25 cm.

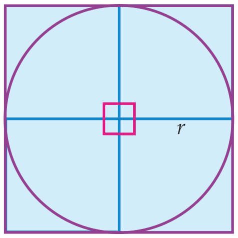
5.2

5.5 Area of a circle

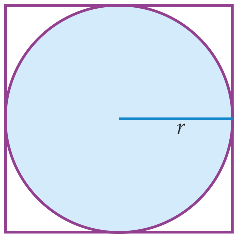
To find an approximation for the area of a circle, consider a circle enclosed by a square with a side length the same as the diameter of the circle. (The sides of the square just touch the circle at one point on each side.) This square can be divided into four identical smaller squares.



The area of the shaded square is r^2.

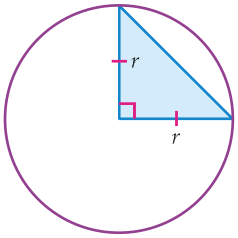


The area of the square that encloses the circle is $4r^2$ (that is, $r^2 \times 4$).

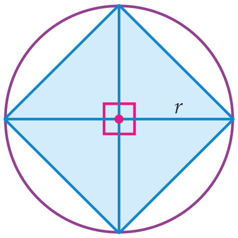


The area of the circle must be less than $4r^2$.

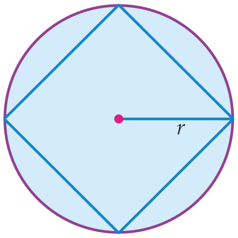
Now consider a square drawn inside the circle, with vertices (corners) that touch the circumference. This square can be divided into four identical smaller triangles.



The area of the shaded triangle is $\frac{r^2}{2}$.



The area of the square inside the circle is $4 \times \frac{r^2}{2} = 2r^2$.

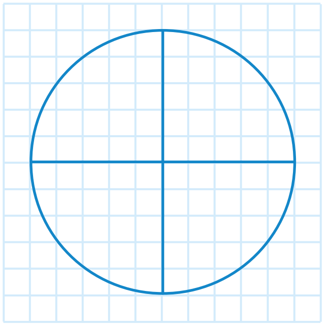


The area of the circle must be more than $2r^2$.

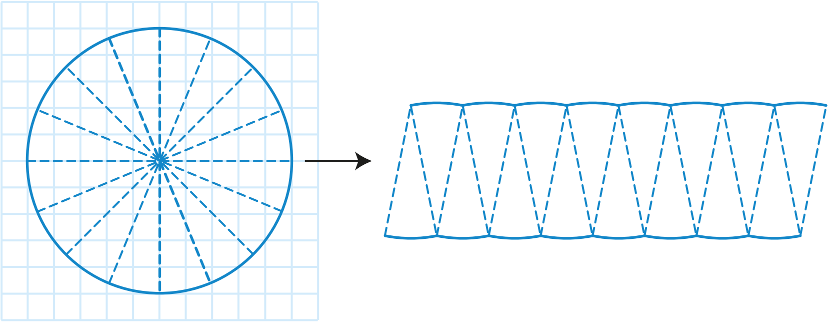
So, the area of the circle is greater than $2r^2$, but less than $4r^2$. You can write this mathematically as $2r^2 <$ (area of circle) $< 4r^2$.

To find a more accurate rule for the area of a circle, you can cut the circle into pieces and rearrange them to form a shape that you know how to find the area of.

Step 1 Use a compass to draw three circles of the same size on centimetre grid paper. A radius of 4 or 5 cm is convenient. Divide the circle into four quarters. Estimate the area by counting the squares in a quarter and multiplying by 4. Write your estimate for the area of your circle.



Step 2 Divide the circle into sixteenths by dividing the quarters in half, then in half again. Cut them out and stick them onto plain paper or into your workbook as shown below, making sure that you alternate the curved edge between the top and bottom to make the new shape as 'straight' as possible.



Step 3 You should be able to see that the shape formed by the sectors of the circle looks like a rectangle. Write the rule for finding the area of a rectangle.

Step 4 The circumference of the original circle has been cut up into the small curved edges along the top and bottom sides of the rectangle, so the length of the rectangle, l, should be approximately equal to half the circumference, $2\pi r$. So you can write:

$$l = \frac{\cancel{2}\pi r}{\cancel{2}} = \pi r$$

Step 5 The two shorter sides of the rectangle should be the length of the radius in the original circle. So you can write w (width) $= r$.

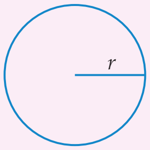
Step 6 Write the rule for the area of the rectangle again. Now, replace the two dimensions in the formula with $l = \pi r$ and $w = r$.

$$\begin{aligned} A &= l \times w \\ &= \pi r \times r \\ &= \pi r^2 \end{aligned}$$

Step 7 Use the formula $A = \pi r^2$ to calculate the area of the circle. Is the calculated area greater or less than the area you found by counting squares?

It was stated earlier that the area of a circle would be between $2r^2$ and $4r^2$ (the 'inside' and 'outside' squares at the beginning of this section). The value of π is approximately 3.141 592 654, so $A = \pi r^2$ agrees with this condition.

Area of a circle



$$A = \pi r^2$$

where r = radius

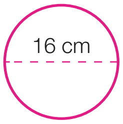
Worked example 9

W.E. 9

Find the area of the following circles, correct to 2 decimal places.

(a)

(b)



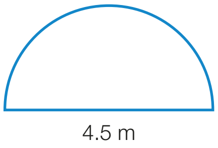
Thinking	Working
(a) **1** Write the formula.	(a) $A = \pi r^2$
2 Substitute for *r*.	$= \pi \times 77^2$
3 Evaluate and round the answer to the correct number of decimal places. Include the units.	$= 18\,626.50 \text{ mm}^2$
(b) **1** Write the formula.	(b) $A = \pi r^2$
2 Halve the diameter to get the radius.	$d = 16$ cm, so $r = 8$ cm
3 Substitute for *r*.	$A = \pi \times 8^2$
4 Evaluate and round the answer to the correct number of decimal places. Include the units.	$= 201.06 \text{ cm}^2$

Worked example 10

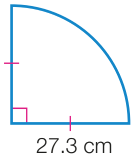
W.E. 10

Find the area of the following shapes, correct to 2 decimal places.

(a)



(b)



Thinking	Working
(a) **1** Modify the area formula to show the fraction of the whole circle being calculated (in this case, one half).	(a) $A = \frac{\pi r^2}{2}$
2 Find the radius by halving the diameter.	$d = 4.5$ m, so $r = 2.25$ m
3 Substitute into the formula.	$A = \frac{\pi \times 2.25^2}{2}$
4 Evaluate and round the answer to the correct number of decimal places. Include the units.	$= 7.95 \text{ m}^2$

(b) 1 Modify the area formula to show the fraction of the whole circle being calculated (in this case, one quarter).

(b) $A = \dfrac{\pi r^2}{4}$

2 Substitute into the formula.

$A = \dfrac{\pi \times 27.3^2}{4}$

3 Evaluate and round the answer to the correct number of decimal places. Include the units.

$= 585.35 \text{ cm}^2$

5.5 Area of a circle

Navigator

1 (columns 1–2), 2 (columns 1–2), 3, 4, 5, 6, 8, 9, 11 (a–b), 16	1 (columns 2–3), 2 (columns 2–3), 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16	1 (columns 2–3), 2 (columns 2–3), 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Answers p. 644

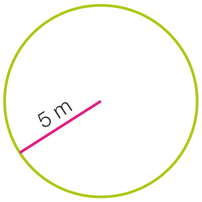
Equipment required: scientific calculator

Fluency

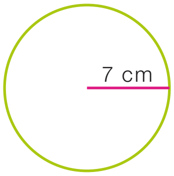
W.E. 9

1 Find the area of the following circles, correct to 2 decimal places.

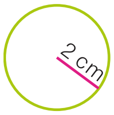
(a)



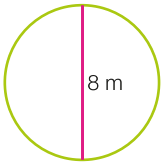
(b)



(c)

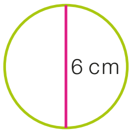


(d)

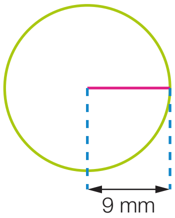


(e)

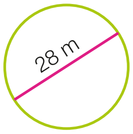
(f)



(g)



(h)



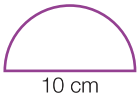
(i)

Always use the radius in the circle area formula. If the diameter is given, divide it by 2 to find the radius.

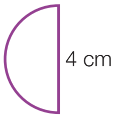
W.E. 10

2 Find the area of the following shapes, correct to 2 decimal places.

(a)

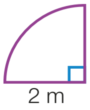


(b)

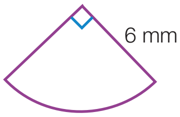


(c)

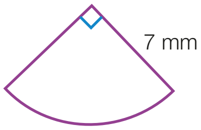
(d)



(e)



(f)



3 Calculate the area of each of the following circles, correct to the nearest cm^2.

(a) a circle of radius 35 cm **(b)** a circle of diameter 68 cm

4 **(a)** The area of a circle with a diameter of 25 m is closest to:

A 157 m^2 **B** 490 m^2 **C** 491 m^2 **D** 1963 m^2

(b) If a circle has a radius of 75 mm, then its area, correct to 2 decimal places, is:

A 471.24 mm^2 **B** 4417.86 mm^2 **C** 17 671.46 mm^2 **D** 70 685.83 mm^2

Understanding

5 Find the area, correct to 2 decimal places, of the glass in a pair of spectacles if the radius of each circular lens is 4.2 cm.

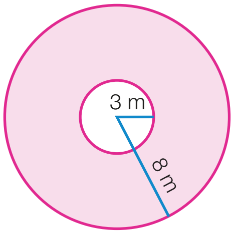
6 The playing field at Eastern Districts Cricket Club is a circle with a diameter of 106 m. Adam wants to lay new turf on the entire playing surface.

(a) What area of turf should Adam order? (Answer to the nearest square metre.)

(b) If the turf costs $7.50 per square metre, how much will this cost?

7 Charlotte has designed a new logo for her business using seven circles. If the central circle has a diameter of 4 cm and the smaller circles each have a diameter of 2 cm, calculate the total area of the circles used in the logo, to 2 decimal places.

8 An 'annulus' is the ring shape formed when a larger circle is drawn around a smaller circle. It is the shaded area between the circumferences of the two circles, as shown. For the questions below, give answers to 2 decimal places.

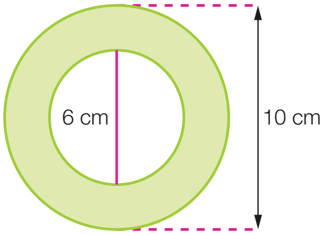


(a) Calculate the area of the larger circle.

(b) Calculate the area of the smaller circle.

(c) Subtract the smaller area from the larger area to find the area of the annulus.

9 What is the area of the annulus below, correct to the nearest whole number?



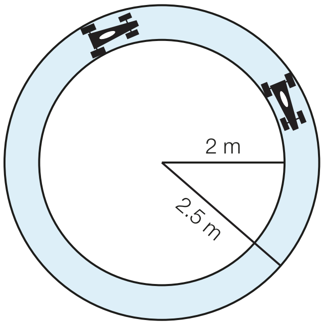
A 13 cm^2

B 50 cm^2

C 101 cm^2

D 201 cm^2

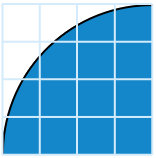
10 Find the area of a circular model racetrack of outer radius 2.5 m and inner radius 2 m. Give your answer to 2 decimal places.



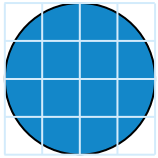
Reasoning

11 For each of the following shapes, estimate the area by considering the grid squares that are covered or partially covered. Write your answer to the nearest whole square unit. Check your answers by calculating the area of each shape.

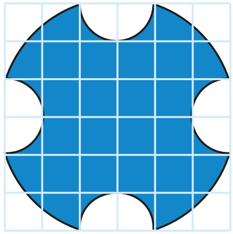
(a)



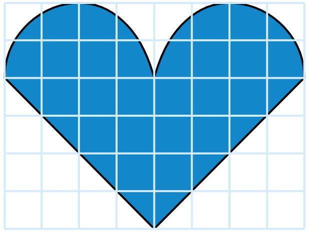
(b)



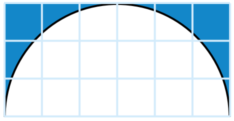
(c)



(d)



12 The fraction of the rectangle that is shaded is closest to:



A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

13 A factory produces soft drink cans. Factory machines cut 150 circles out of a rectangular sheet of metal that is 1.68 m long and 1.13 m wide. The circles are 10 cm in diameter.

(a) Find the area of one circle, correct to 2 decimal places.

(b) Find the total area of all the circles in m^2, correct to 2 decimal places.

(c) Find the area of metal that remains after the circles have been removed, in m^2, correct to 2 decimal places.

14 **(a)** The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle. The formula for the area of a circle is $A = \pi r^2$.

Copy and complete the following table.

Radius r (cm)	Circumference C (cm)	C^2 (cm^2)	Area A (cm^2)	$\frac{C}{A}$	$\frac{C^2}{A}$
1					
2					
5					
10					

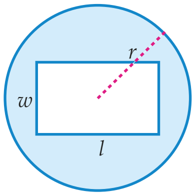
(b) What do you notice about the ratios $\frac{C}{A}$ and $\frac{C^2}{A}$? Which ratio seems to give a constant result?

(c) Divide the constant ratio by π. What number do you get?

(d) Write a formula connecting the circumference and area of a circle.

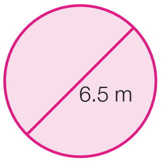
Open-ended

15 Write some dimensions for the rectangle and circle so that the shaded area is somewhere between 400 cm^2 and 500 cm^2.



16 Ajay and Simon were working together on their maths homework when they came across the following question.

Find the area of the following circle, correct to 2 decimal places.



Here is Simon's working:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 6.5^2 \\ &= 132.73\ \text{m}^2 \end{aligned}$$

Here is Ajay's working:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 3.25 \times 2 \\ &= 20.42\ \text{m}^2 \end{aligned}$$

(a) Is either student correct? What is the correct answer?

(b) Explain the mistakes that have been made, and give each student some advice to help answer these types of problems correctly in the future.

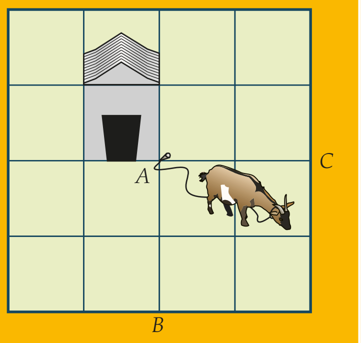
Problem solving

Victa the goat

Victa, the grass-eating goat, lives in a square paddock that is 8 m long. He is tied to the corner of a square shelter at point *A*. The dimensions of the shelter are a quarter of the dimensions of the paddock. Victa is just able to reach the grass at the four edges of the paddock, for example at points *B* and *C*. What percentage (to the nearest whole per cent) of the grass will still need to be mowed?

Strategy options

- Draw a diagram.
- Solve a simpler problem.
- Break problem into manageable parts.



Exploration CAS

Equipment required: TI-Nspire CAS or Casio ClassPad CAS

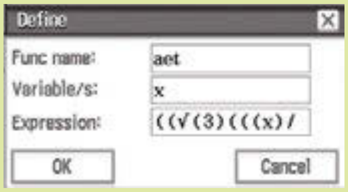
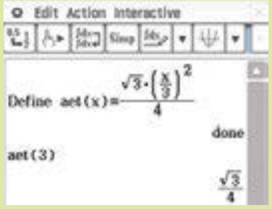
Area of equilateral triangles

Sir Cumference, a mathematics professor, claims to have found a new rule to find the area of an equilateral triangle using just the perimeter of the triangle. (Remember that an equilateral triangle has all sides the same length and all angles the same size.) The professor says the rule is:

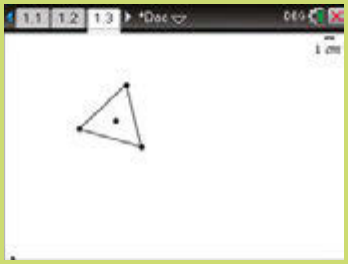
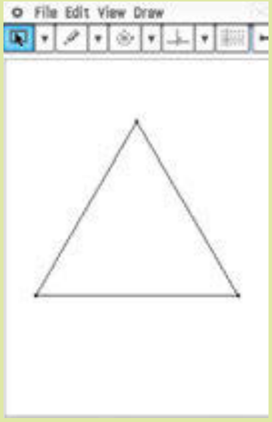
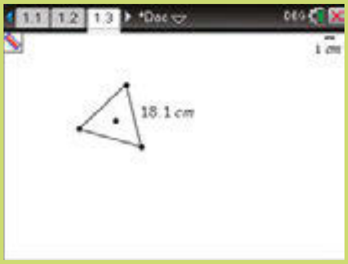
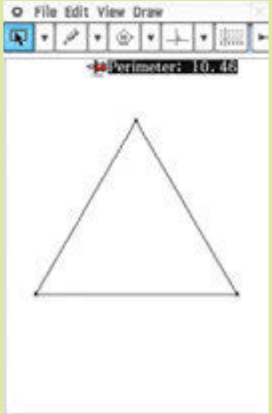
$$\text{Area of equilateral triangle} = \frac{\sqrt{3}\left(\frac{x}{3}\right)^2}{4}$$

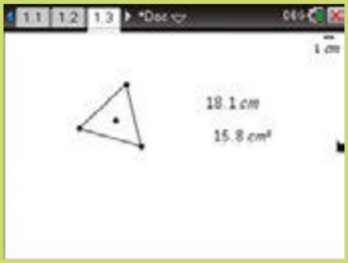
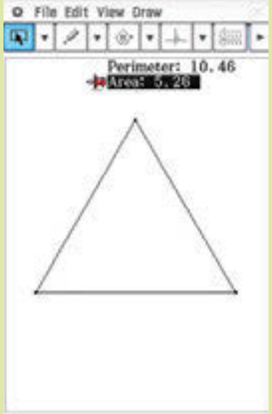
where x is the perimeter of the equilateral triangle.

The first step in the process is to define the rule and store it on your CAS so you can use it whenever you like. Use the name **aet**(x) to stand for **a**rea of **e**quilateral **t**riangle. It is a good idea to name functions so you can identify them again later.

Using TI-Nspire CAS	Using Casio ClassPad CAS
From the **Home** screen, select **Scratchpad** > **Calculate**. Select **menu** > **Actions** > **Define** and then enter: **aet(x)** = $\frac{\sqrt{3}\left(\frac{x}{3}\right)^2}{4}$ To use the square root template, enter **ctrl** > **x²**. To use the fraction template, enter **ctrl** > ÷.  Define aet(x)= $\frac{\sqrt{3}\cdot\left(\frac{x}{3}\right)^2}{4}$ Done	From the menu select **Main**. Open the **Keyboard**, go to the **Math1** tab and use the fraction template ▭, the square root template √▪ and the squared template x^2 to enter the expression: $\frac{\sqrt{3}\left(\frac{x}{3}\right)^2}{4}$ Then highlight the expression, select **Interactive** > **Define** and fill in the settings as shown below. Note that the function name **aet** must be entered using the **abc** tab from the **Keyboard**, but the ***x*** should be done as usual.  Func name: aet Variable/s: x Expression: ((√(3)(((x)/
To check you have entered the rule correctly, enter **aet(3)**. The answer should appear as $\frac{\sqrt{3}}{4}$.  aet(3) $\frac{\sqrt{3}}{4}$	To check you have entered the rule correctly, enter **aet(3)**. The answer should appear as $\frac{\sqrt{3}}{4}$.  Define aet(x)= $\frac{\sqrt{3}\cdot\left(\frac{x}{3}\right)^2}{4}$ done aet(3) $\frac{\sqrt{3}}{4}$ (If a decimal answer appears, then you need to make sure you are in **Standard** mode. If the word at the bottom of the screen shows **Decimal**, then select the word to change it to **Standard**.)

You will now use your CAS to explore whether or not Sir Cumference is right. You are going to draw an equilateral triangle, find its perimeter and area, and decide whether this gives the same answer as using the formula.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Now add Geometry to your document. Select **menu** > **Shapes** > **Regular Polygon**, then select the centre point and a vertex of the polygon to be drawn. Rotate until the polygon becomes a triangle, then press **enter** to draw the shape as an equilateral triangle.  	From the menu select **Geometry**. Select the polygon tool shown opposite. Enter the **Number of Sides** as 3 and you can then select a point on the screen to draw an equilateral triangle.  Hide the vertex labels by selecting the three vertices and then selecting **Edit** > **Properties** > **Hide Name**. 
To measure the perimeter of your triangle, select **menu** > **Measurement** > **Length** and select the triangle. You will then be able to place a caption showing this perimeter.  	To measure the perimeter of this triangle, select the three sides and then select **Draw** > **Measurement** > **Circumference/Perimeter**. The length of the perimeter will appear. 

Using TI-Nspire CAS	**Using Casio ClassPad CAS**
To measure the area of your triangle, select **menu** > **Measurement** > **Area** and select the triangle. You will then be able to place a caption showing this area.  18.1 cm 15.8 cm² Record the perimeter and area values.	To measure the area of your triangle, select the three sides and then select **Draw** > **Measurement** > **Area**. The area will appear. Record the perimeter and area values.  Perimeter: 10.46 Area: 5.28
Press **esc** and then select one of the vertices to drag it to a new location, changing the shape of the triangle. The values for the perimeter and area will change. Change the size of the triangle to record at least six different pairs of values for the triangle perimeter and area.	Now select one of the vertices and drag it to a new location, changing the shape of the triangle. The values for the perimeter and the area will change. Change the size of the triangle to record at least six different pairs of values for the triangle perimeter and area.  Perimeter: 9.23 Area: 4.10
From the **Home** screen, again select **Scratchpad** > **Calculate**. To see whether Sir Cumference's formula works, you can enter **aet** followed by your measured perimeter value in brackets, to see whether this gives the measured area value.	From the menu, again select **Main**. To see whether Sir Cumference's formula works, you can enter **aet** followed by your measured perimeter value in brackets, to see whether this gives the measured area value.  Define aet(x)=$\frac{\sqrt{3}\cdot\left(\frac{x}{3}\right)^2}{4}$ done aet(3) $\frac{\sqrt{3}}{4}$ aet(9.23) 4.098845312

Check your pairs of values and decide whether Sir Cumference has a correct formula.

5.6

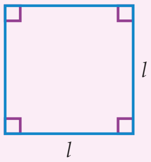
Finding the area of composite shapes

A **composite shape** is a shape that is made up of simpler shapes.

If you can find the areas of the simpler shapes, then these can be added to calculate the total area.

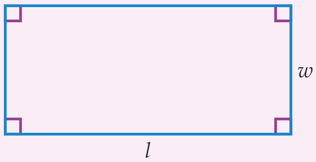
There can be more than one way of dividing a composite shape into simpler shapes.

Area formula summary



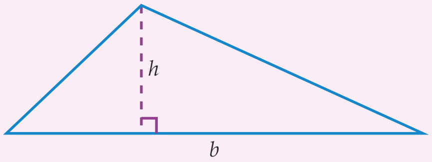
Square

$A = l^2$



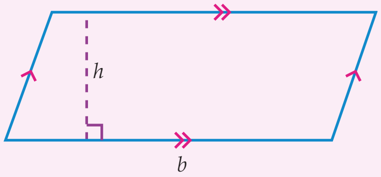
Rectangle

$A = lw$



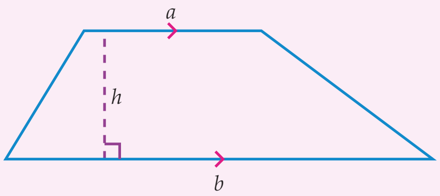
Triangle

$A = \frac{1}{2}bh$



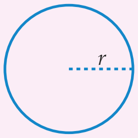
Parallelogram

$A = bh$



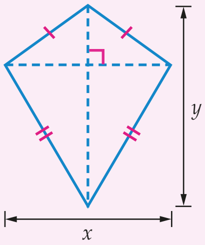
Trapezium

$A = \frac{1}{2}(a + b)h$



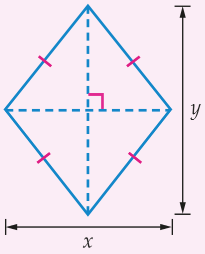
Circle

$A = \pi r^2$



Kite

$A = \frac{1}{2}xy$



Rhombus

$A = \frac{1}{2}xy$

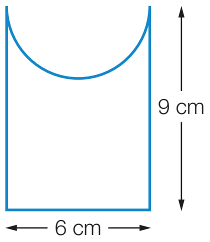
Worked example 11

W.E. 11

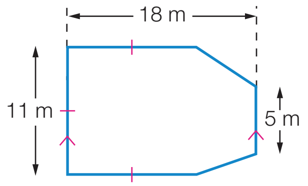
For each of the following composite shapes:

- **(i)** draw dashed lines to divide the shape into simpler shapes
- **(ii)** redraw each shape separately.

(a)



(b)

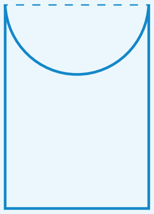


Thinking

Working

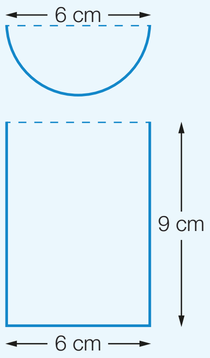
(a) **(i)** Identify the shapes within the composite shape (here, one half of a circle and a rectangle) and draw a dashed line to indicate the different shapes.

(a) (i)



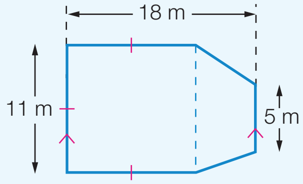
(ii) Redraw each shape separately.

(ii)



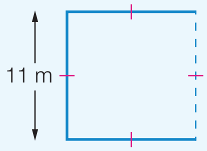
(b) **(i)** Identify the shapes within the composite shape (here, a square and a trapezium) and draw a dashed line to indicate the different shapes.

(b) (i)



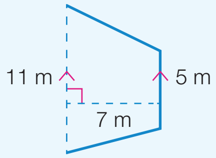
(ii) Redraw each shape separately. Work out any missing lengths that would be needed for the area formula and label them.

(ii)



Find the height of the trapezium.

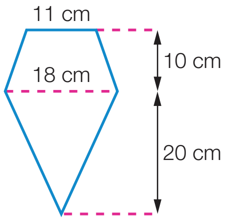
18 – 11 = 7 m



Worked example 12

W.E. 12

Find the area of this composite shape.



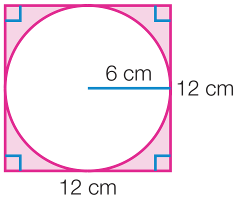
Thinking	Working
1 Identify the individual shapes that make up the total area.	$A_{total} = A_{trapezium} + A_{triangle}$
2 Write the formulas for each of the individual shapes. Use different pronumerals for the different heights (*h* for the trapezium and *H* for the triangle).	$= \frac{1}{2}(a+b)h + \frac{1}{2}bH$
3 Substitute the known lengths into the formulas. (Remember that multiplying by $\frac{1}{2}$ is the same as dividing by 2.)	$= \frac{1}{2}(11+18) \times 10 + \frac{1}{2} \times 18 \times 20$
4 Evaluate the total area, writing your answer with the correct units.	$= 325 \text{ cm}^2$

Areas of parts of shapes can be found by subtracting the areas that are not included from the overall area.

Worked example 13

W.E. 13

Find the shaded area in the following shape, correct to 2 decimal places.



Thinking	Working
1 Write the shaded area as the subtraction of the area of one shape from the area of the other.	$A_{shaded} = A_{square} - A_{circle}$
2 Write the formulas for each of the individual shapes.	$= l^2 - \pi r^2$
3 Substitute the known lengths into the formula.	$= 12^2 - \pi \times 6^2$ $= 144 - 36\pi$
4 Evaluate the difference, writing your answer with the correct units.	$= 30.90 \text{ cm}^2$

By not using rounded values for the areas of individual shapes (such as the circle in the previous example) and instead finding the final answer as one complete calculation, you can avoid creating any rounding errors. Make sure you know how to enter these calculations into your calculator in one step, using the bracket keys as necessary to group calculations.

5.6 Finding the area of composite shapes

Navigator

1, 2 (a, d), 3 (a–d), 6, 8, 10, 13	1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14

Answers p. 644

Equipment required: scientific calculator

Fluency

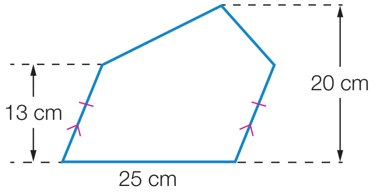
W.E. 11

1 For each of the following:

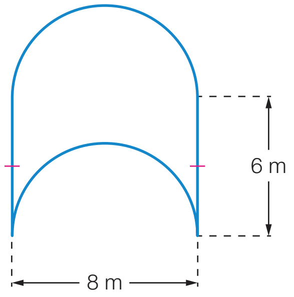
(i) draw dashed lines to divide the shape into simpler shapes

(ii) redraw each shape separately.

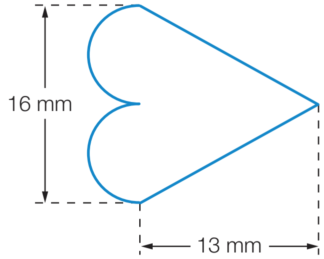
(a)



(b)



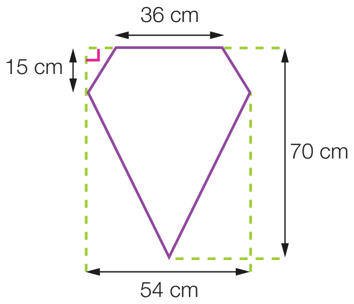
(c)



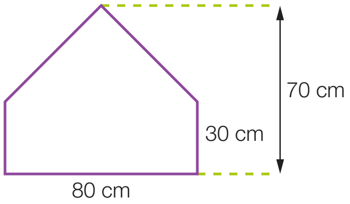
W.E. 12

2 Find the area of these composite shapes, correct to 2 decimal places where appropriate.

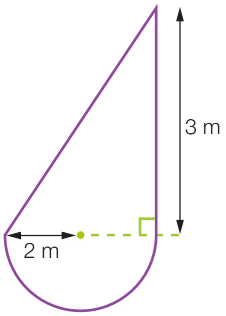
(a)



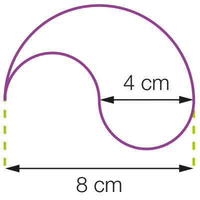
(b)



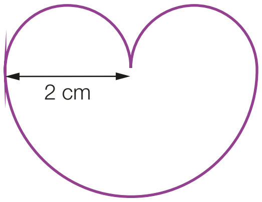
(c)



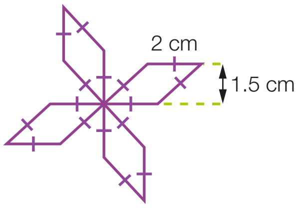
(d)



(e)



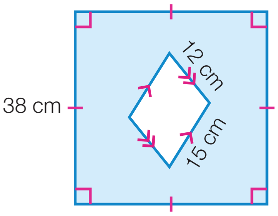
(f)



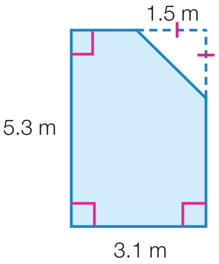
W.E. 13

3 Find the shaded area in each of the following shapes, correct to 2 decimal places where appropriate.

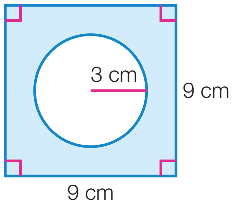
(a)



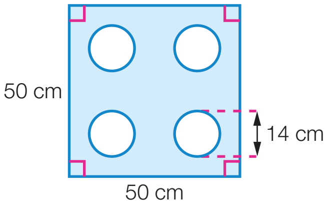
(b)



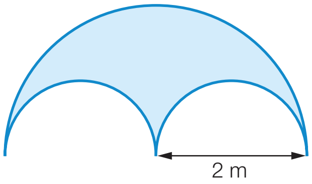
(c)



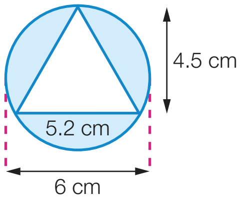
(d)



(e)



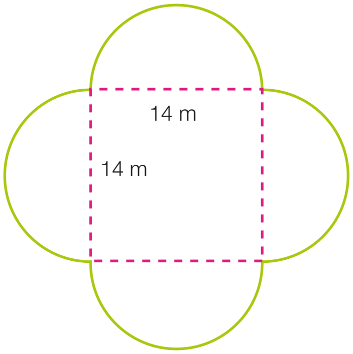
(f)



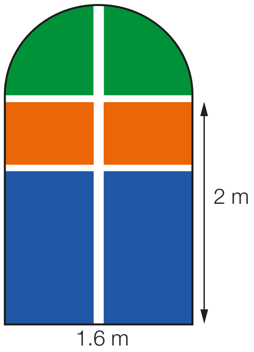
4 What is the area of the following shape, correct to the nearest m^2?

A 196 m^2 **B** 308 m^2

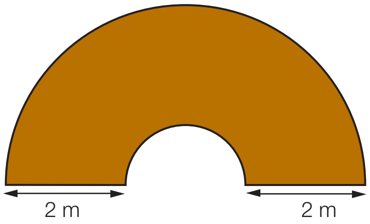
C 504 m^2 **D** 1232 m^2



5 A stained-glass window is shaped as shown. What area of coloured glass, correct to 2 decimal places, does it contain? (Ignore the parts of the frame between the glass areas.)

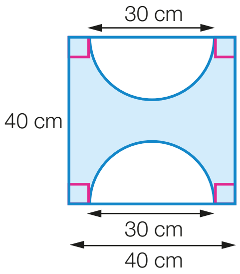


6 Michael needs to buy some pine bark for a garden bed that is shaped as shown. What area does this need to cover? Give your answer correct to 2 decimal places.

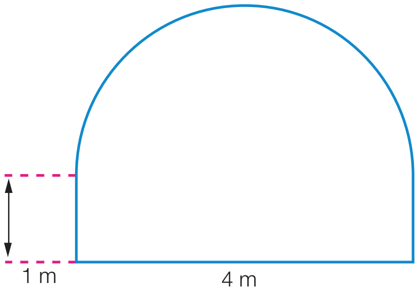


7 Find the perimeter and area of each shape, correct to 2 decimal places.

(a)



(b)

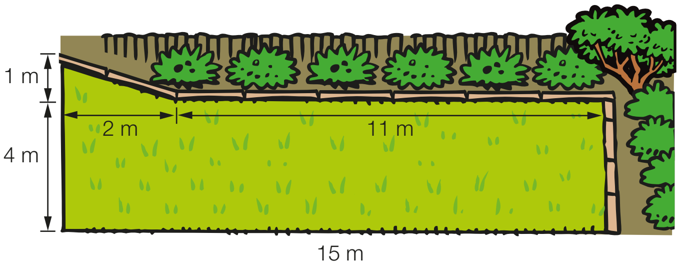


Understanding

8 A new home owner wants to lay 'instant lawn' over her front yard, which has the shape below.

(a) What is the area of the yard?

(b) If instant lawn costs $6.50 per square metre, then how much will this cost?



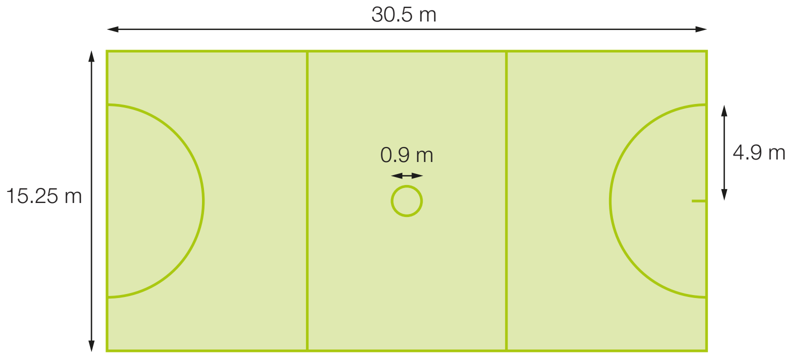
9 Consider the diagram of a netball court (see over page).

- The court is the shape of a rectangle 30.5 m in length and 15.25 m wide.
- The court is divided into thirds across its length.
- In the very centre of the middle third is a circle 0.9 m in diameter.
- At the ends of the court, in the middle of each of the shorter sides, are two semicircles, with a radius of 4.9 m. These are called the 'goal circles'.

(a) Find the area of the court.

(b) The person playing goal attack can only move across two of the thirds of the court. What area of the court is this? (Answer correct to 2 decimal places.)

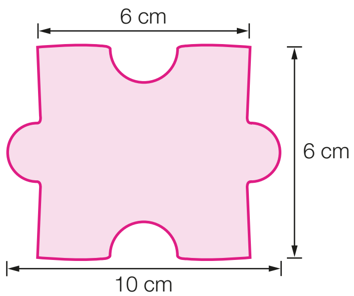
(c) The centre player can move all over the court, except in the two goal circles. What area of the court is this? Answer correct to 2 decimal places.



10 A puzzle contains the piece shown. Its area is closest to:

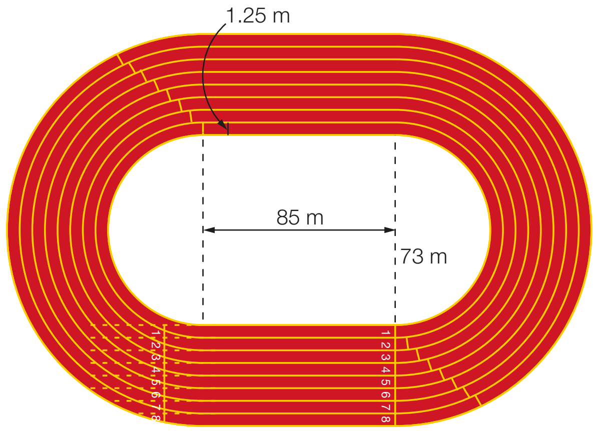
A 28 cm^2 **B** 36 cm^2

C 48 cm^2 **D** 60 cm^2



Reasoning

11 Athletics running tracks are placed around the perimeter of a shape that is a rectangle with a semicircle at each end, as shown. The rectangle is 85 m × 73 m. There are 8 lanes each 1.25 m wide around the perimeter of the shape. Answer to 2 decimal places.



(a) Find the distance you would run doing one lap of:

  (i) the inside lane, running along the inside line

  (ii) the outside lane, running along the outside line.

(b) Find the area of:

  (i) the field inside the track

  (ii) the area of the track and field

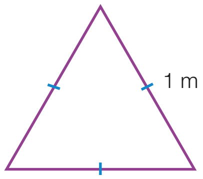
  (iii) the area of the track.

12 When the perpendicular height of a triangle is not known, the area can be calculated using the three side lengths by a formula known as Heron's formula:

$A = \sqrt{s(s - a)(s - b)(s - c)}$, where a, b and c are the side lengths and

$s = \dfrac{a + b + c}{2}$. The value s is half the perimeter, called the semi-perimeter.

(a) Use Heron's formula to find the area of the equilateral triangle shown, correct to 2 decimal places.



(b) A regular hexagon is drawn inside a circle of radius 1 m. The hexagon is made up of six of the equilateral triangles from part **(a)**.

(i) Find the area of the hexagon, correct to 2 decimal places.

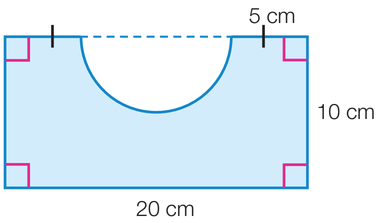
(ii) Find the area of the circle, correct to 2 decimal places.

(iii) Use your answers so far to find the area of the shaded region, correct to 2 decimal places.

Open-ended

13 Draw a composite shape, made of a rectangle and a trapezium, that has an area of 60 cm^2. Show the appropriate dimensions.

14 Lucy was asked to find the perimeter and area of this shape, rounding her answer to the nearest whole number.



(a) Find all the errors Lucy has made in her working below.

(b) Calculate the correct answers.

Perimeter

Semicircle:

$\pi d + d$
$= \pi \times 10 + 10$
$\approx 31 + 10$
$= 41$ cm

$P = 5 + 10 \times 2 + 20 + 41$
$= 5 + 20 + 20 + 41$
$= 86$ cm

Area

Rectangle:

$A = 2l + 2w$
$= 2 \times 20 + 2 \times 10$
$= 40 + 20$
$= 60\ cm^2$

Semicircle:

$A = \frac{\pi r^2}{2}$
$= \frac{\pi \times 10^2}{2}$
$\approx 157\ cm^2$

Required area: $157 - 60 = 97\ cm^2$

Problem solving

Doubling a square

Farmer George owns one hectare of land in the shape of a square. One day, the king said to him "Farmer George, I will let you double the size of your land, but your farm must still be in the shape of a square".

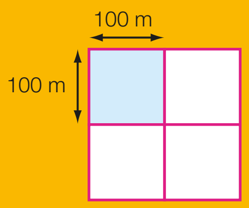
Farmer George jumped up and down in excitement, and then got busy with some maths. If he simply doubled the length and width of his farm, it would then be four times as big, not double.

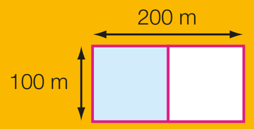
If he put two squares next to each other, the farm would be double in size, but it would not be a square.

How can Farmer George double the size of the land while keeping it shaped as a square?

Strategy options

- Guess and check.
- Make a model.





Gamespace

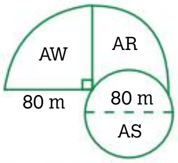
Are we there yet?

You are going on a trip around Australia. Your maths teacher has given you some maths tasks to answer from each of the tourist attractions you plan to visit.

Start at the airport nearest your home, then follow the flight paths in an anticlockwise direction, completing the tasks as you go.

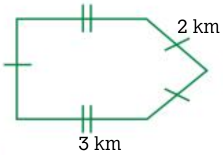
Perth Zoo – areas include Australian Walkabout (AW), Asian Rainforest (AR) and African Savannah (AS).

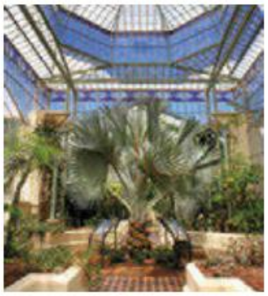
Which has more area, the Australian Walkabout or the African Savannah?



Adelaide Botanical Gardens has bicycles for hire.

How far do you ride in completing one circuit of the trail shown?

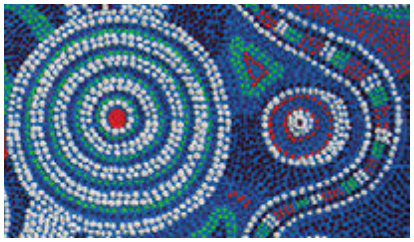




Melbourne Observation Deck

Instead of taking the elevator to the top of the 253 m tower, you walk up the 1254 stairs. How high is each stair in cm, correct to 1 decimal place?

Darwin – Mindil Beach Sunset Market



The largest circle on the painting has a diameter of 13 cm and the smaller circle has a diameter of 5 cm. What is the difference between the circumference of the two circles (to the nearest cm)?

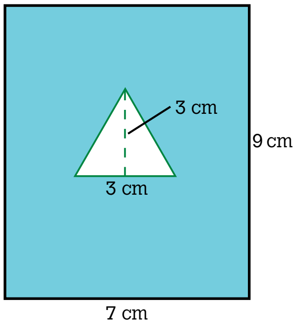
The Wheel of Brisbane

The wheel is 60 m high, so the diameter of the wheel would be about 58 m. How many revolutions (to the nearest half revolution) would it take to travel a whole kilometre?

Sydney Harbour Bridge

The middle of the top of the arch is 134 m above the water and there is 49 m clearance below the bridge. The span (length across) is 503 m. If the arch was a semicircle over the same span, how much higher would the bridge be?

Canberra – National Gallery of Australia



The centre of one of the pieces of art is as shown in the diagram. What is the area of the shaded section?

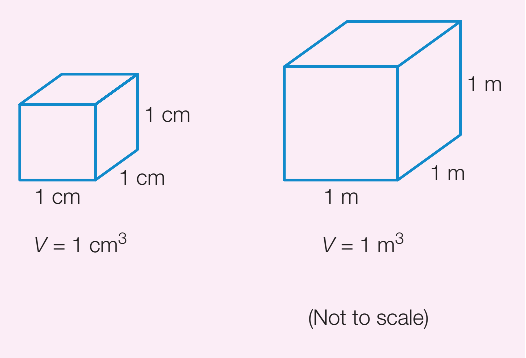
Hobart – Seven Mile Beach

What is the area of the beach (in m^2) if it is 11 kilometres long and the average width is 20 metres?

5.7 Volume and capacity

Units of volume

Volume is the amount of space taken up by three-dimensional objects, and is measured in units such as cubic centimetres (cm^3) and cubic metres (m^3).



When you find the volume of a **solid**, you are counting the number of whole cubes and parts of whole cubes that would fill the solid exactly.

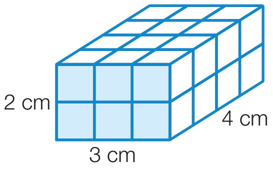
Prisms

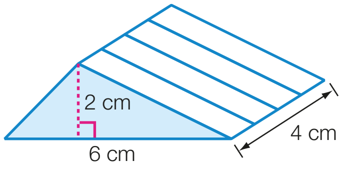
A **prism** is a solid that has a **uniform cross-section**. This means that the shape on one of the ends of the prism continues unchanged, throughout the height of the prism. This shape is called the base of the prism.

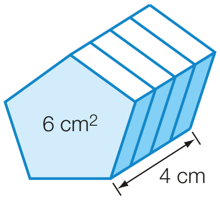
The height of the prism is the perpendicular distance from the base to the top.

The base of a prism is a polygon. It is not always at the bottom of the shape.

Below are three prisms: a rectangular-based prism, a triangular-based prism and a pentagonal-based prism.

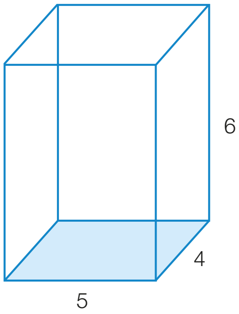




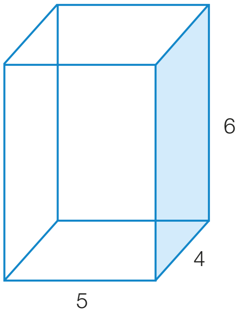


Finding the volume of a rectangular prism

Recall that to find the volume of a rectangular prism, you must multiply together the length (l), width (w) and height (h). Using the definition of a prism, you can see that any one of the faces of the rectangular prism can act as the base.



$V = lwh$
$V = 5 \times 4 \times 6$
$= 120 \text{ cm}^3$



$V = lwh$
$V = 6 \times 4 \times 5$
$= 120 \text{ cm}^3$

If you look at the formula $V = lwh$, you can see that multiplying the values of l and w together gives the area of the rectangular base: $A = lw$. So, you can write the formula

$V = lwh$
as
$V = Ah$

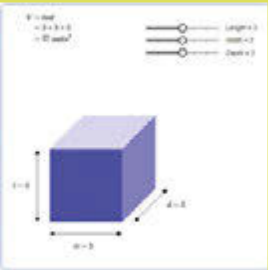
where A = the area of the base.

Interactive

Volume

Explore how the volume of a prism changes by varying the dimensions.

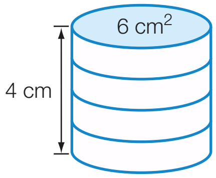
Go to the eBook or the Pearson Places website to access this interactive.



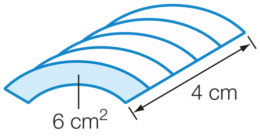
Because all prisms have a uniform cross-section, the volume of all prisms can be calculated in the same way: find the area of the base and multiply it by the height.

Volume of solids with a uniform cross-section

There are other solids that are not prisms but that also have a uniform cross-section. A solid with a circular uniform cross-section is called a **cylinder**.



The solid shown here with the odd-shaped curvy base has no special name, but it also has a uniform cross-section.

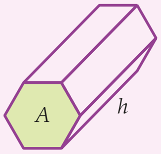


The rectangular, triangular and pentagonal prisms shown in blue on the previous page and the cylinder and the odd-shaped solid shown here all have the same volume, because they all have the same cross-sectional area (6 cm^2) and the same height (4 cm).

Volume of solids with a uniform cross-section:

$V = Ah$

where A = area of the cross-section, h = height (and h is perpendicular to A)

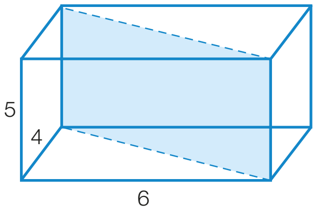


Triangular prisms

If you make a diagonal cut from corner to corner across the base and up through the height of the rectangular prism, you create a triangular prism that has half the volume of the rectangular prism.

You can write the volume of this triangular prism as:

$$V = \frac{1}{2} \times 6 \times 4 \times 5 = 60 \text{ cm}^3 \qquad \text{or} \qquad V = \frac{6 \times 4 \times 5}{2} = 60 \text{ cm}^3$$



You could also use the formula $V = Ah$ to find the volume. But because the shape of the base of the prism is a triangle, its area is given by $A = \frac{1}{2}bw$, where b = length of base and w = the width, which is also the triangle's perpendicular height to its base.

Note that although you would usually use h to represent the height of a triangular base (4 cm in the above diagram with a base of 6 cm), here we use w because this is also the width of the prism. This avoids confusion with the use of h for the height of the prism (5 cm in the above diagram).

Substituting in the formula for A gives $V = \frac{1}{2}bwh$.

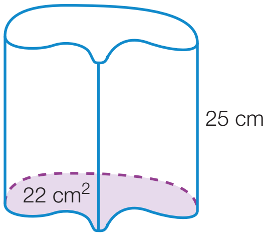
This is demonstrated in the second part of the example below.

Worked example 14

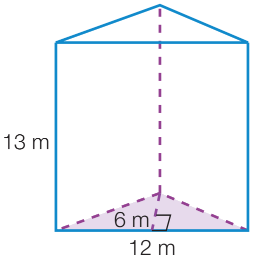
W.E. 14

Find the volume of each solid. Give answers correct to 2 decimal places, where appropriate.

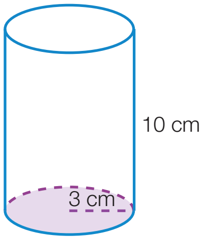
(a)



(b)



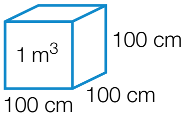
(c)



Thinking			Working
(a)	1	Write the formula.	(a) $V = Ah$
	2	Substitute the values for A and h into the formula. Note that the base area A is already given.	$= 22 \times 25$
	3	Evaluate, and write the answer with the appropriate units of volume.	$= 550 \text{ cm}^3$

(b)	**1**	Write the general formula.	(b) $V = Ah$
	2	Substitute the area formula for the triangular base.	$= \frac{1}{2}bw \times h$
	3	Substitute the values for *b*, *w* and *h*.	$A = \frac{1}{2} \times 12 \times 6 \times 13$
	4	Evaluate the volume and write the answer with the appropriate units.	$= 468 \text{ m}^3$
(c)	**1**	Write the general formula.	(c) $V = Ah$
	2	Substitute the area formula for the circular base.	$= \pi r^2 \times h$
	3	Substitute the values for *r* and *h*.	$= \pi \times 3^2 \times 10$
	4	Evaluate the volume. Round the answer correct to 2 decimal places and write it with the correct units.	$= 282.74 \text{ cm}^3$

Converting units of volume



One 'linear' metre (the unit of length) is equal to 100 cm. However, one *cubic* metre is equal to $100 \times 100 \times 100 \text{ cm}^3$.

$$\begin{aligned} 1 \text{ m}^3 &= (100 \text{ cm})^3 \\ &= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1\,000\,000 \text{ cm}^3 \end{aligned}$$

Units of volume are cubic units. Converting between units of volume is not the same as converting units of length. The conversion factor (the number you multiply or divide by) is the cube of the conversion factor for length.

Units of length:

$$\begin{tikzcd} \text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"] \end{tikzcd}$$

Units of volume:

$$\begin{tikzcd} \text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

Worked example 15

W.E. 15

Copy and complete the following conversions.

(a) $14.5\ \text{cm}^3 = \_\_\_\_\ \text{mm}^3$ **(b)** $37\,000\ \text{m}^3 = \_\_\_\_\ \text{km}^3$

Thinking	Working
(a) Converting from a larger unit to a smaller unit, so multiply. There are 10^3 mm^3 in every cm^3, so multiply by 10^3.	(a) 14.5×10^3 $= 14.5 \times 1000$ $= 14\,500\ \text{mm}^3$
(b) Converting from a smaller unit to a larger unit, so divide. There are 1000^3 m^3 in every km^3, so divide by 1000^3.	(b) $37\,000 \div (1000)^3$ $= 37\,000 \div 1\,000\,000\,000$ $= 0.000\,037\ \text{km}^3$

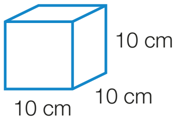
Units of capacity

Capacity is a term usually used for the volume of space inside a container.

One litre (L) is the space inside a 10 cm × 10 cm × 10 cm cube.

One millilitre (mL) is the space inside one cubic centimetre, which is one thousandth of one litre.

One kilolitre (kL) is the space inside one cubic metre, which is 1000 litres.



$$\begin{aligned} 1\ \text{kL} &= 1000\ \text{L} \\ &= 1\ \text{m}^3 \\ &= 1\,000\,000\ \text{cm}^3 \end{aligned}$$

$$\begin{aligned} 1\ \text{L} &= 1000\ \text{mL} \\ &= 1000\ \text{cm}^3 \end{aligned}$$

$$1\ \text{cm}^3 = 1\ \text{mL}$$

$$\begin{tikzcd} \begin{matrix}\text{kL}\\(\text{m}^3)\end{matrix} \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \begin{matrix}\text{mL}\\(\text{cm}^3)\end{matrix} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

Worked example 16

W.E. 16

Copy and complete the following conversions.

(a) 2.25 L to ____ mL **(b)** $600\ \text{cm}^3$ to ____ L **(c)** $40\,000\ \text{cm}^3$ to ____ m^3

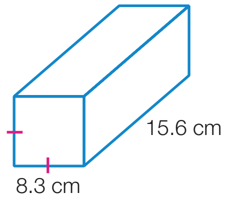
Thinking	Working
(a) Converting from a larger unit to a smaller unit, so multiply by 1000.	(a) $2.25\ \text{L} = 2.25 \times 1000\ \text{mL}$ $= 2250\ \text{mL}$
(b) **1** Change the volume to the equivalent capacity.	(b) $600\ \text{cm}^3 = 600\ \text{mL}$
2 Converting from a larger unit to a smaller unit, so divide by 1000.	$= 600 \div 1000\ \text{L}$ $= 0.6\ \text{L}$
(c) There are $1\,000\,000\ \text{cm}^3$ in $1\ \text{m}^3$, so divide by 1 000 000 (smaller unit to larger unit).	(c) $40\,000\ \text{cm}^3 \div 1\,000\,000 = 0.04\ \text{m}^3$

To find the capacity of a container or hollow solid, find the volume of the interior, then convert to units of capacity (mL, L, kL).

Worked example 17

W.E. 17

Find the capacity of the container, in litres. Round your answer correct to one decimal place.



Thinking	**Working**
1 Write the formula for volume.	$V = Ah$
2 Substitute the formula for A, the area of the square base.	$= l^2 \times h$
3 Substitute the values.	$= 8.3^2 \times 15.6$
4 Evaluate the volume, including the units.	$= 1074.684\ \text{cm}^3$
5 Convert the answer to mL. (1 mL = 1 cm^3)	Capacity = 1074.684 mL
6 Convert mL to L by dividing by 1000.	= 1.074 684 L
7 Round the answer to 1 decimal place.	= 1.1 L

5.7 Volume and capacity

Navigator

1 (columns 1–2), 2 (column 1), 3 (column 1), 4, 5 (columns 1–2), 6, 7, 9, 13, 14, 15, 20	1 (columns 2–3), 2 (column 2), 3 (column 2), 4, 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20	1 (columns 2–3), 2 (column 2), 3 (column 2), 4, 5 (columns 2–3), 6, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 21

Answers p. 645

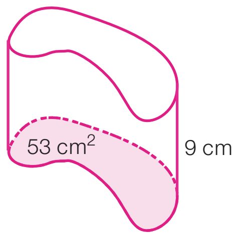
Equipment required: scientific calculator

Fluency

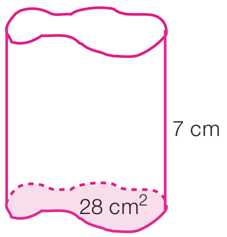
W.E. 14

1 Find the volume of each solid. Give answers correct to 2 decimal places, where appropriate.

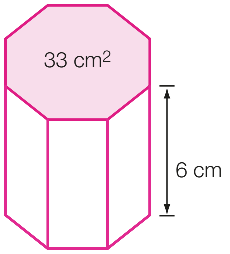
(a)



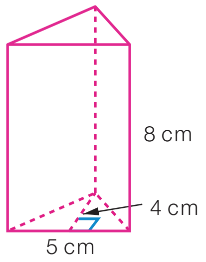
(b)



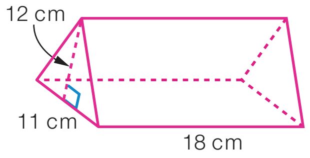
(c)



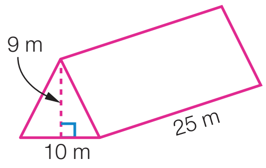
(d)



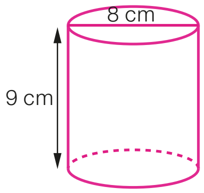
(e)



(f)

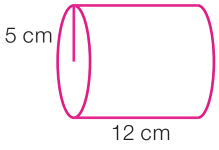


(g)



(h)

(i)



W.E. 15

2 Copy and complete the following conversions.

(a) 20 cm^3 = _____ mm^3
(b) 195 cm^3 = _____ mm^3
(c) 8.3 m^3 = _____ cm^3
(d) 25.46 m^3 = _____ cm^3
(e) 0.75 km^3 = _____ m^3
(f) 57 km^3 = _____ m^3
(g) 150 mm^3 = _____ cm^3
(h) 3830 mm^3 = _____ cm^3
(i) 47 900 cm^3 = _____ m^3
(j) 6070 cm^3 = _____ m^3
(k) 0.48 m^3 = _____ cm^3
(l) 97 500 m^3 = _____ km^3
(m) 360 000 m^3 = _____ km^3
(n) 0.0025 km^3 = _____ m^3
(o) 0.04 cm^3 = _____ mm^3
(p) 0.0356 m^3 = _____ cm^3

W.E. 16

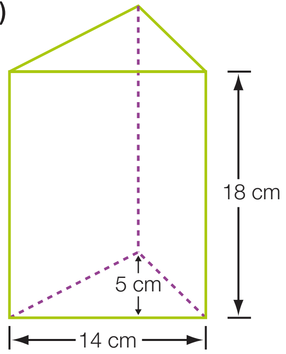
3 Copy and complete the following conversions.

(a) 2000 mL = _____ L
(b) 55 000 mL = _____ L
(c) 800 mL = _____ L
(d) 40 mL = _____ L
(e) 5 L = _____ mL
(f) 95 L = _____ mL
(g) 0.03 L = _____ mL
(h) 0.5 L = _____ mL
(i) 25 cm^3 = _____ mL
(j) 48 cm^3 = _____ mL
(k) 33 mL = _____ cm^3
(l) 140 mL = _____ cm^3
(m) 5.8 m^3 = _____ kL
(n) 1.7 m^3 = _____ kL
(o) 0.13 kL = _____ m^3
(p) 0.05 kL = _____ m^3

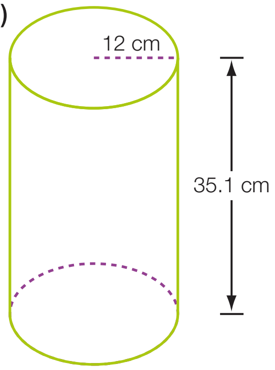
W.E. 17

4 Find the capacity of the following containers, in millilitres. Round your answers correct to 1 decimal place where necessary.

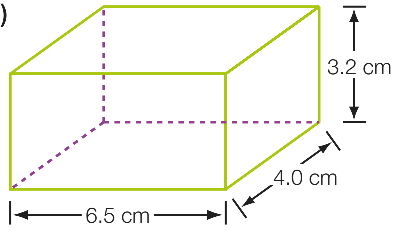
(a)



(b)

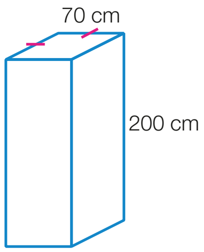


(c)

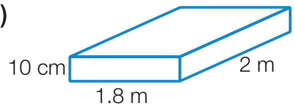


5 Find the volume of each of the following shapes **(i)** in cm^3 and **(ii)** in m^3. For the cylinders, round your answers to 2 decimal places.

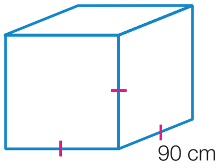
(a)



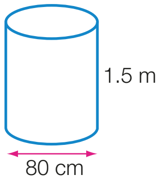
(b)



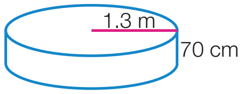
(c)



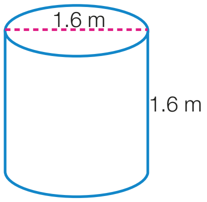
(d)



(e)



(f)



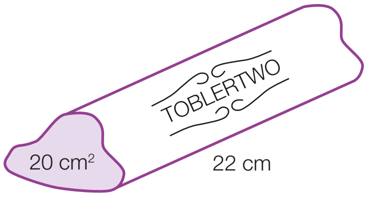
6 **(a)** 16 L is equal to:

A 0.016 kL **B** 160 mL **C** 0.16 kL **D** 1.6 kL

(b) 3450 cm^3 is equal to:

A 3.45 mm^3 **B** 34 500 mm^3 **C** 0.003 45 m^3 **D** 34.5 m^3

7 Find the volume of a chocolate bar shaped as shown.



Understanding

8 The shape of one of the fermenters shown can be modelled as a cylinder of height 1.5 m and a radius of 60 cm.

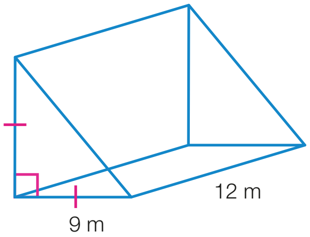
(a) Find the volume of a fermenter in m^3, correct to 3 decimal places.

(b) What is the capacity of the fermenter to the nearest litre?

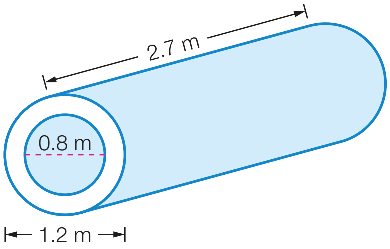
9 Calculate the volume of this triangular prism, by:

(a) calculating the volume of a rectangular prism, then halving it

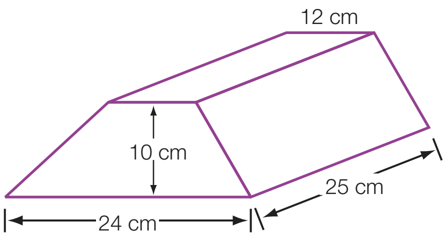
(b) finding the area of the triangular base and multiplying it by the height.



10 Find the volume of the hollowed-out cylinder, correct to 2 decimal places.



11 Find the capacity of this prism in litres.



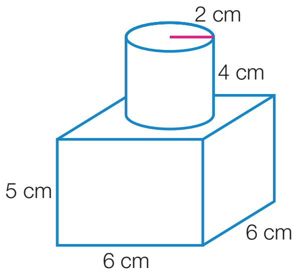
12 The capacity of the model aircraft fuel tank shown is closest to:

A 67.24 mL

B 205.12 mL

C 230.27 mL

D 1440 mL



13 During a science experiment, Alia pours 45 mL of acid into a flask already containing 0.75 L of acid. How much acid is now in the flask:

(a) in mL

(b) in L?

14 What is the capacity of a cylindrical soft drink can that has a diameter of 6 cm and a height of 13 cm? Answer to the nearest mL.

Reasoning

15 Larissa and Dave are installing a pool in their backyard. It is rectangular and will be 4 m wide, 12 m long, and have a depth of 1.5 m.

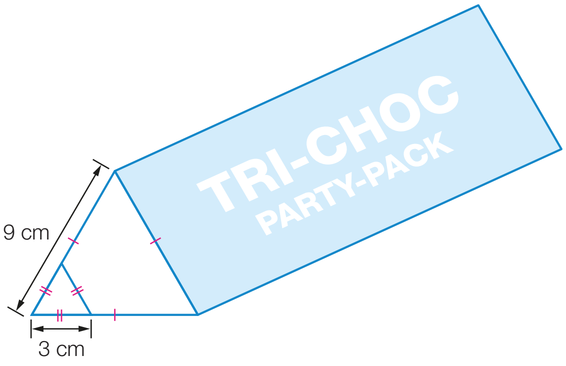
(a) Draw a diagram that shows this information.

(b) If it costs $1.68 for every 1000 litres of water, what will it cost to fill the pool?

16 A steel pinhead has a volume of 1 mm^3. If pinheads are melted down, how many would it take to make a millilitre of steel?

17 A drink jug holds 2 litres of juice. How many glasses can be filled by the jug, if the glasses are cylindrical, their internal diameter is 8 cm and they are filled to a depth of 12 cm?

18 A confectioner makes chocolate bars shaped like a triangular prism. The base of the prism is an equilateral triangle with a side length of 3 cm. The prism is 12 cm long. The company also produces a giant 'party-pack' version, where a larger triangular prism-shaped box has the regular size bars packed tightly inside. All the dimensions of this giant box are three times the size of the regular box. How many regular boxes will fit inside the giant box?



19 Rainfall is measured in millimetres. If 5 mm of rain is recorded in a particular place, any bucket or other container with straight sides will fill to a depth of 5 mm.

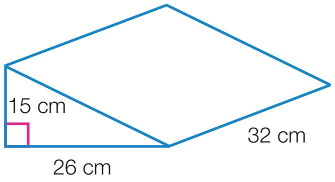
(a) When 5 mm of rain falls, how many litres of rain will be collected in a tank from a roof of equivalent horizontal area 200 m^2?

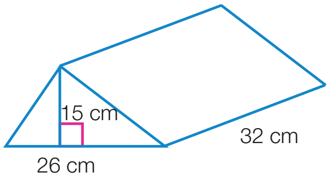
(b) What roof area is needed if 25 mm of rain must fill a 10 000 L tank?

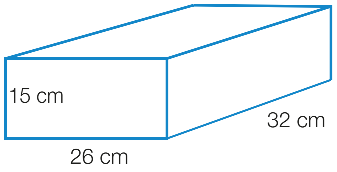
Open-ended

20 Sally is designing a cylindrical container for a new sports drink, PowerPlus. The container is to have a capacity of 500 mL. Help Sally design three different containers for PowerPlus. How might she decide which is the best-shaped container? (You should consider both the height and the diameter of the container.)

21 The two triangular prisms below would each fit inside the rectangular prism.







(a) Explain how each of the triangular prisms could be produced from the rectangular prism, then compare the volume of the three solids.

(b) Draw another triangular prism of height (*h*) 26 cm that could be produced from the rectangular prism and that has the same volume as the triangular prisms above.

Problem solving

Baffling box

Chiara is making a wooden gift box and has gone to a timber supplier to buy the wood. The trouble is, she can remember the area of each face, but not the dimensions.

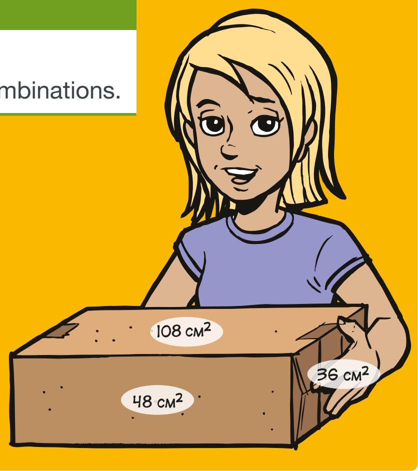
1 Work out the dimensions from the information she remembers (see the illustration).

2 Find the volume of the proposed gift box.

3 What is the total area of wood needed for all six of the box's faces?

Strategy options

- Guess and check.
- Test all possible combinations.



5.8

Time

One day is the time taken for the Earth to complete one full rotation on its axis facing the Sun, and it is divided into 24 hours. One year is the time taken for the Earth to complete one orbit all the way around the Sun.

Unlike other units of measurement, many units for time are not based on the number 10. Instead, many time units are based on the number 60.

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days

2 weeks = 1 fortnight
1 month = 28–31 days
1 year = 12 months
1 year ≈ 52 weeks

1 year = 365 days*
1 decade = 10 years
1 century = 100 years
1 millennium = 1000 years

* or 366 days in a 'leap year' (usually every 4 years)

24-hour time

12-hour time divides the day into two 12-hour blocks, 'am' and 'pm', to label whether the hours are before noon ('ante meridiem') or after noon ('post meridiem').

24-hour time instead numbers the 24 hours of the day starting at midnight, using a four-digit system:

12 am (midnight)	0000	12 pm (midday)	1200
1 am	0100	1 pm	1300
9:30 am	0930	9:30 pm	2130
		11:59 pm	2359

Note that with 24-hour time, there is no need to use 'am' or 'pm' after the time. Many countries use 24-hour time in train and bus timetables, cinema times and shop opening hours.

Worked example 18

W.E. 18

Express the following in 24-hour time.

(a) 6:30 am **(b)** 7:45 pm

Thinking	Working
(a) For times between 1 am and 1 pm, remove the colon, and write it as a four-digit number (add a zero in front if you have only three digits).	(a) 0630
(b) For times from 1 pm onwards, remove the colon, write it as a four-digit number, and add 1200.	(b) 0745 + 1200 = 1945

Worked example 19

W.E. 19

Write the following 24-hour times as 12-hour (am or pm) times.

(a) 0852 **(b)** 1914

Thinking	Working
(a) 1 Do the first two digits make a number greater than 12? (No.)	(a) 0852
2 Rewrite the time and put a separating colon between the hour and the minute number. If the first digit is a zero, it can be dropped.	8:52
3 As the hour number is less than 12, it is before midday, so write 'am' after the time.	8:52 am
(b) 1 Do the first two digits make a number greater than 12? (Yes.)	(b) 1914
2 Subtract 1200 from the number.	1914 – 1200 = 0714
3 Rewrite the time and put a separating colon between the hour and the minute number.	7:14
4 As the original hour number is greater than 12, it is after midday, so write 'pm' after the time.	7:14 pm

Calculating elapsed time

A period of time that has passed is called **elapsed time**.

Worked example 20

W.E. 20

Calculate how much time has passed between the times given.

10:56 am to 3:33 pm

Thinking	Working
1 Look at the starting time and work out how many minutes to the next hour.	10:56 am to 11:00 am = 4 min
2 Work out how many whole hours occur before the finishing time.	11:00 am to 3:00 pm = 4 h
3 Work out any remaining minutes to the finishing time.	3:00 pm to 3:33 pm = 33 min
4 Add up the hours and minutes separately.	4 min + 4 h + 33 min = 4h 37 min

Worked example 21

W.E. 21

What is the time:

(a) $4\frac{1}{2}$ hours after 11:47 am

(b) 9 hours and 25 minutes before 1315?

Thinking	Working
(a) 1 Ignore the fraction and add the whole number hours onto the time.	(a) 11:47 am + 4 h = 3:47 pm
2 Work out how many minutes the fraction part is.	$\frac{1}{2}$ h = 30 min
3 Add the minutes onto the time. Make sure you go up to the next hour when you reach 60 minutes.	3:47 pm + 30 min = 3:47 pm + 13 min + 17 min = 4:00 pm + 17 min
4 Write the answer.	= 4:17 pm
(b) 1 Subtract the number of hours from the time.	(b) 1315 – 9 h = 0415
2 Now, subtract the number of minutes, remembering to move into the previous hour if necessary.	0415 – 25 min = 0415 – 15 min – 10 min = 0400 – 10 min = 0350

Time zones

The world is divided into **time zones** corresponding to the 24 hours in one day. These are shown by columns at the top of the map of the world shown opposite. Places that are in the same time zone set their clocks to the same time. You can see from the map that some large countries, such as Australia and China, are spread across several time zones. Australia has three time zones, meaning that clocks in different cities are set to different times. China has chosen to use one time zone across the whole country, meaning that all clocks are set to the same time.

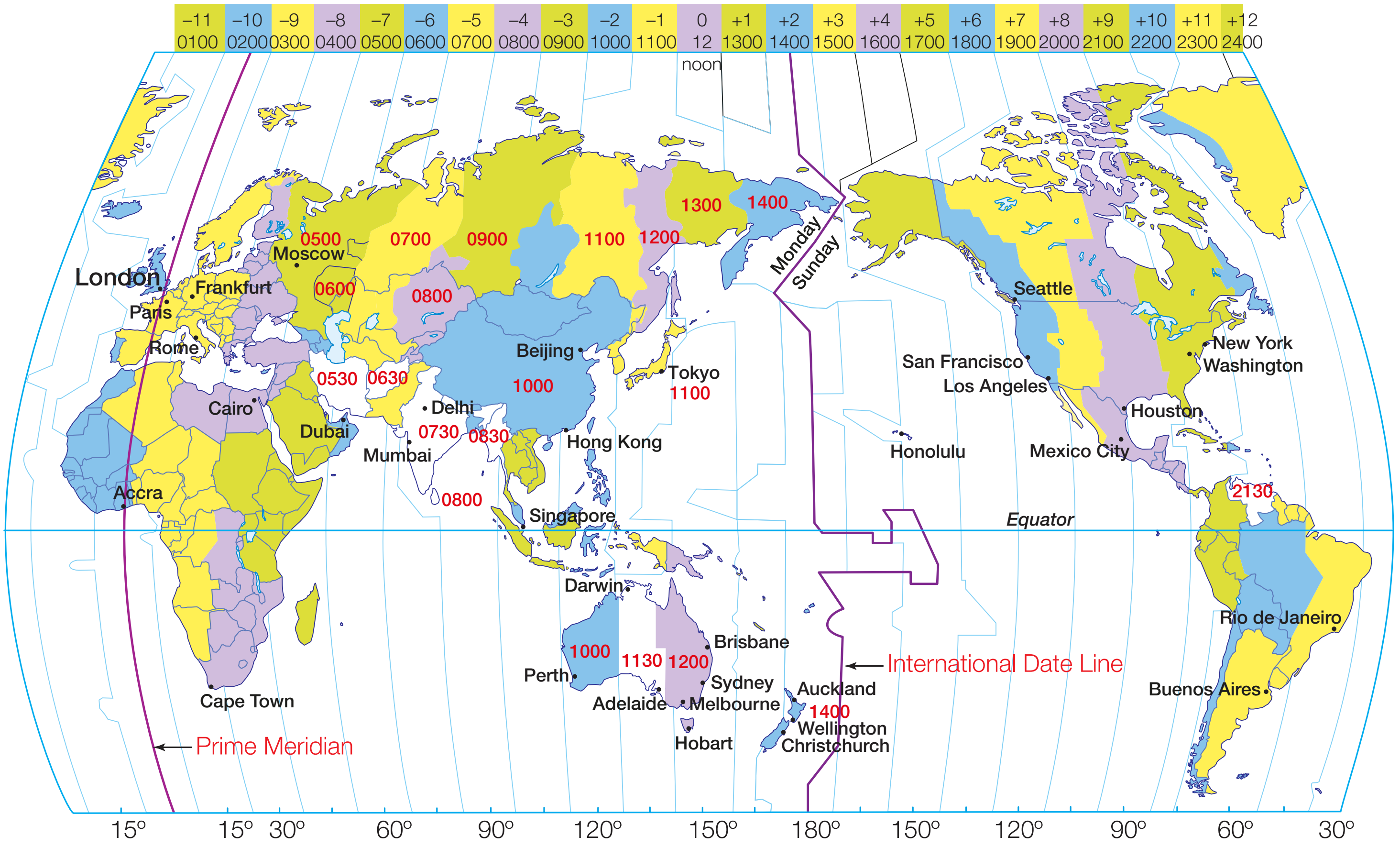
Reading a time zone map

The first line of numbers across the top of the time zone map shows how many hours ahead (+) or behind (–) each time zone is, compared to Australian Eastern Standard Time (AEST). AEST covers the eastern states of Australia (not including daylight saving time). The second line of numbers shows what time it would be in each time zone if it was 1200 hours (noon) in this time zone, for example in the cities of Sydney, Melbourne or Brisbane.

Greenwich Mean Time

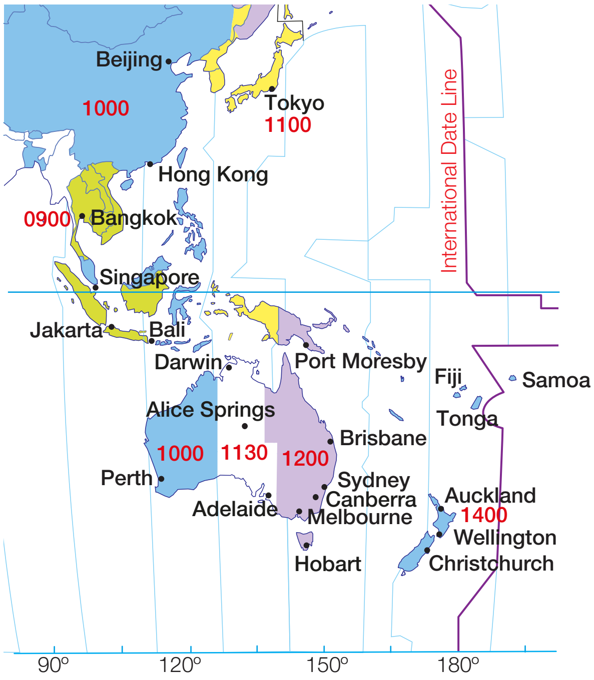
The origin point for setting the official time around the world was originally in a place called Greenwich (pronounced 'gren-itch') in London. The time zone for Greenwich is known as Greenwich Mean Time or GMT. An imaginary vertical line running through Greenwich (called the prime meridian) divides the world into the eastern and western hemispheres. The time around the world was then set according to how many time zones east or west of Greenwich each place was. For example, Western Australia is 8 time zones east of Greenwich, so the time in WA is 8 hours in front of GMT. Brazil is 3 time zones to the west of Greenwich, so the time in Brazil is 3 hours behind GMT.

International time today is based on a scientific standard that is a more precise version of GMT, called Coordinated Universal Time (UTC). However, the system of time zones is still used to set time around the world.



Working with different time zones

It sometimes helps to think of whether a person in a different time zone would be ahead of you or behind you, time wise. People in time zones to the east of you are ahead, so they will be having breakfast while you are sleeping. People in time zones to the west of you are behind you, so they may still be asleep when you have finished breakfast.



The International Date Line

There is an important line shown on the map 12 hours later or 12 hours earlier than GMT (depending on whether you move east or west). This line is known as the International Date Line (or IDL). The local times in the places either side of the IDL are the same, but they are on different days.

For example, when it is 7 am on Monday in Tonga, just to the west of the IDL, it is 7 am on Sunday in Samoa, to the east of the IDL.

Australian and Asian time zones

Time zone	Time	States
Australian Western Standard Time (AWST)	GMT plus 8 hours	WA
Australian Central Standard Time (ACST)	GMT plus 9.5 hours	SA, NT
Australian Eastern Standard Time (AEST)	GMT plus 10 hours	Qld, NSW, Vic, Tas, ACT

Some countries to the north of Australia, such as Papua New Guinea, Japan and China, are in these Australian time zones, so the time in these countries will be the same.

Daylight saving

Some states in Australia apply daylight saving time (DST) to have more hours of daylight in the evening (and fewer hours of daylight in the early morning) during summer. This is done by turning the clock forward by one hour near the start of spring (October) and then turning the clock back one hour in autumn (April). A handy way to remember this is the phrase ‘spring forward, fall back’ (remember that ‘fall’ is another name for autumn).

5.8 Time

Navigator

1, 2, 3 (column 1), 4 (column 1), 5, 6, 7, 8, 9, 10, 12, 13, 17, 20, 22, 23, 24	1, 2, 3 (column 2), 4 (column 2), 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24	1, 2, 3 (column 2), 4 (column 2), 5, 6, 8, 9, 11, 12, 13, 14, 15, 16, 18, 19, 21, 23, 24

Answers p. 646

Fluency

W.E. 18

1 Express the following in 24-hour time.

(a) (i) 2:30 am (ii) 2:30 pm
(b) (i) 6:15 am (ii) 6:15 pm
(c) (i) 11:43 am (ii) 11:43 pm
(d) (i) midday (ii) midnight

W.E. 19

2 Write the following 24-hour times as 12-hour (am or pm) times.

(a) 1354
(b) 0833
(c) 0539
(d) 1634
(e) 1830
(f) 1902
(g) 0147
(h) 0320

W.E. 20

3 Work out how much time has passed between the times given.

(a) 2:55 am to 5:41 am
(b) 1849 to 2304
(c) 1934 to 2220
(d) 1:22 am to 6:13 am
(e) 10:52 pm to 1:09 am
(f) 2340 to 0243
(g) 0143 to 1152
(h) 2:03 am to 12:44 am
(i) 1027 to 0100
(j) 9:49 pm to 3:01 pm

W.E. 21

4 What is the time:

(a) 3 hours after 4:15 am
(b) 5 hours after 0123
(c) 6 hours and 10 min before 2012
(d) 4 hours and 40 min before 11:59 pm
(e) 5 hours and 23 min after 10:55 am
(f) 7 hours and 13 min before 1443
(g) $3\frac{1}{2}$ hours before 1621
(h) $4\frac{1}{2}$ hours before 1:08 am
(i) $7\frac{1}{4}$ hours after 9:49 am
(j) $9\frac{1}{4}$ hours after 1158?

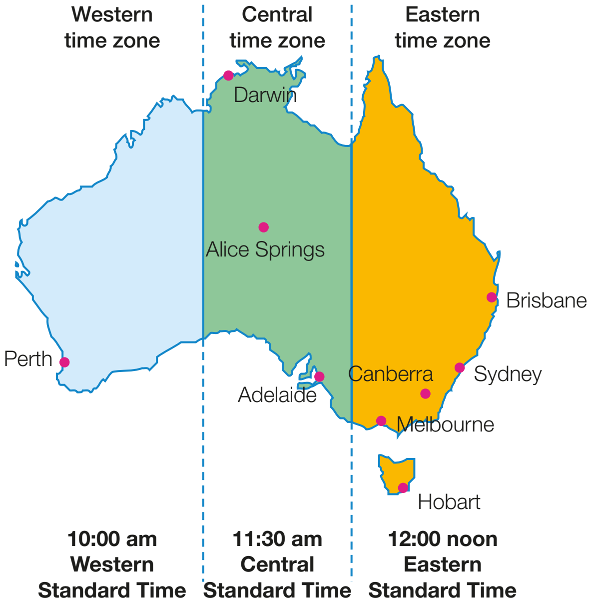
5 Look at this map showing the three Australian time zones. Central Standard Time (CST) is half an hour behind Eastern Standard Time (EST). Western Standard Time (WST) is 2 hours behind Eastern Standard Time.

(a) If it is 3:00 pm in Melbourne, what time is it in Perth?

(b) If it is 4:15 pm in Brisbane, what time is it in Adelaide?

(c) If it is 11:45 pm in Adelaide, what time is it in Canberra?

(d) If it is 7:10 am in Perth, what time is it in Darwin?



6 The AFL game between the West Coast Eagles and Collingwood is being televised live. The broadcast starts at 1930 on a Friday night in June in Perth. Use the map of Australian time zones to find:

(a) the time the broadcast starts in Melbourne

(b) the time the broadcast starts in Adelaide.

Understanding

7 Estimate how much time it takes you to do the following.

(a) run 100 metres

A 15 seconds **B** 15 minutes **C** 15 hours **D** 15 days

(b) boil an egg

A 4 seconds **B** 4 minutes **C** 4 hours **D** 4 days

(c) drive from Melbourne to Sydney

A 10 seconds **B** 10 minutes **C** 10 hours **D** 10 days

8 State whether each of the following is true (T) or false (F).

(a) It takes about $365\frac{1}{4}$ days for the Earth to travel around the Sun.

(b) There are 2 weeks in a fortnight.

(c) A day is the time taken for the Earth to rotate once about its axis.

(d) There are exactly 52 weeks in a year.

(e) There are exactly 4 weeks in a month.

(f) There are exactly 10 years in a decade.

9 Jodie got to school at 8:40 am If her journey to school took 63 minutes, at what time did she leave home?

10 Dirk left home at five past eight in the morning and caught the 8:13 am train, which was 1 minute late. The train trip took 12 minutes, and it took him 6 minutes to walk from the station to the school.

(a) At what time did he get to school?

(b) How many minutes did it take him to get from his home to school?

11 Mark started a marathon at 10:49 am and he finished at 2:36 pm. The time that he took to run the course was:

A 3 h 47 min

B 4 h

C 4 h 13 min

D 13 h 25 min

12 Aaron's flight from Brisbane to Cairns arrived at 0915. If the flight took 2 hours and 20 minutes, at what 24-hour time did he leave Brisbane?

13 The late flight from Perth to Melbourne leaves Perth at 12:30 am and travels through the night. The flying time is 4 hours 20 minutes. When the plane lands, what will be the time in:

(a) Perth **(b)** Melbourne?

14 The following table shows when high and low tides were predicted at North Bondi Beach in Sydney over a period of 5 days. Over this 5 days there will be two high tides and two low tides each day.

June		Low tide		High tide	
8	Mon	0246	1400	0837	2040
9	Tue	0324	1438	0916	2116
10	Wed	0401	1517	0956	2153
11	Thur	0440	1557	1035	2230
12	Fri	0519	1639	1116	2309

Give your answers to these questions in am or pm times.

(a) When was high tide at North Bondi Beach on 11 June?

(b) When was low tide at North Bondi Beach on 10 June?

(c) How much earlier was the am high tide on 10 June than on 11 June?

(d) What was the elapsed time between the first low tide and the first high tide on 12 June?

(e) On which dates did low tide occur between 2 am and 4 am?

(f) How much earlier was the pm low tide on 8 June than on 12 June?

15 Grace works in Canberra. Her manager has asked her to attend a video conference between her office and the company's office in Beijing, China. If the phone meeting is scheduled for 10:00 am Thursday in Canberra, for what time will the meeting be scheduled in Beijing? Use the time zone map on page 337 to help find the answer.

16 Western Australia, Queensland and the Northern Territory do not apply daylight saving time. In all other states, clocks are moved forward one hour during spring (usually early October) until autumn (usually early April).

(a) Copy and complete the table to show the time during the daylight saving months. Use the time zone map on page 337.

State	Standard time	Daylight saving time
WA		
NT		
SA		
Qld		
NSW	12 noon	
Vic		
Tas		

(b) At 3:30 pm in Sydney in January, what time is it in Darwin?

(c) At noon on Christmas day in Hobart, Dinara telephoned her Mum, who lives in Adelaide. What time was it in Adelaide?

(d) At the Perth Airport during summer, a row of clocks display the times in different cities in Australia. Copy and complete the table below.

Perth	Darwin	Adelaide	Sydney	Melbourne	Brisbane
 09:45					

Reasoning

17 Jamie is backpacking around Europe. He gets on a train from Geneva (Switzerland) to Milan (Italy), a trip that will take 3 hours and 50 minutes. When he arrives in Milan, he needs to change trains to go to Rome. If the train left Geneva at 1415, will Jamie make the 1820 train to Rome? Explain your answer.

18 On 1 December 2010, what was the exact age in years, months and days of a woman who was born on 20 January 1985?

19 Corporal Green is writing a timetable for a training day for his army recruits. The day must start with a half-hour parade ground drill at 0800. This is to be followed by a one-hour lesson on navigation theory, a one-hour lesson on military history, then a one and a half hour fitness session. Recruits then get 35 minutes to shower and have lunch, before a three-quarters of an hour lesson on weapons handling and a two-and-a-quarter hour practice session on the firing range. Finally, recruits have a one-hour session on setting up a bush camp. Complete Corporal Green's timetable, writing all times as 24-hour times.

Recruit training day	
Activity	**Starting time**
Parade ground drill	0800

20 **(a)** If a plane leaves Brisbane at 0930 local time and arrives in Alice Springs at 1300 local time, how much time has the trip taken?

(b) If a plane leaves Canberra at 1422 local time and arrives in Perth at 1715 local time, how much time has the trip taken?

21 The following table shows the flying times from Sydney to various cities and the time differences between these cities and AEST (Australian Eastern Standard Time). If Yvonne leaves Sydney at 8:00 am, calculate her local arrival time at each city.

City	Time zone	Flight time
Hong Kong	AEST less 2 h	7 h 50 min
Tokyo	AEST less 1 h	9 h 30 min
Auckland	AEST plus 2 h	2 h 45 min
Bangkok	AEST less 3 h	9 h 30 min

Open-ended

22 Elizabeth recorded the amount of time she spent on homework in a week. The table shows her record.

Day	Amount of time	
Monday	1 h 30 min	1.3 h
Tuesday	2 h	2.0 h
Wednesday	1 h	1.0 h
Thursday	1 h	1.0 h
Friday	45 min	0.45 h

Elizabeth added her numbers in the last column and got a total time of 5.75 hours, which she converted to $5\frac{3}{4}$ hours. 'That's funny!' she thought. 'I'm sure I spent more time than that!'

What is wrong with Elizabeth's adding? Give her some advice so that she can add time correctly in future.

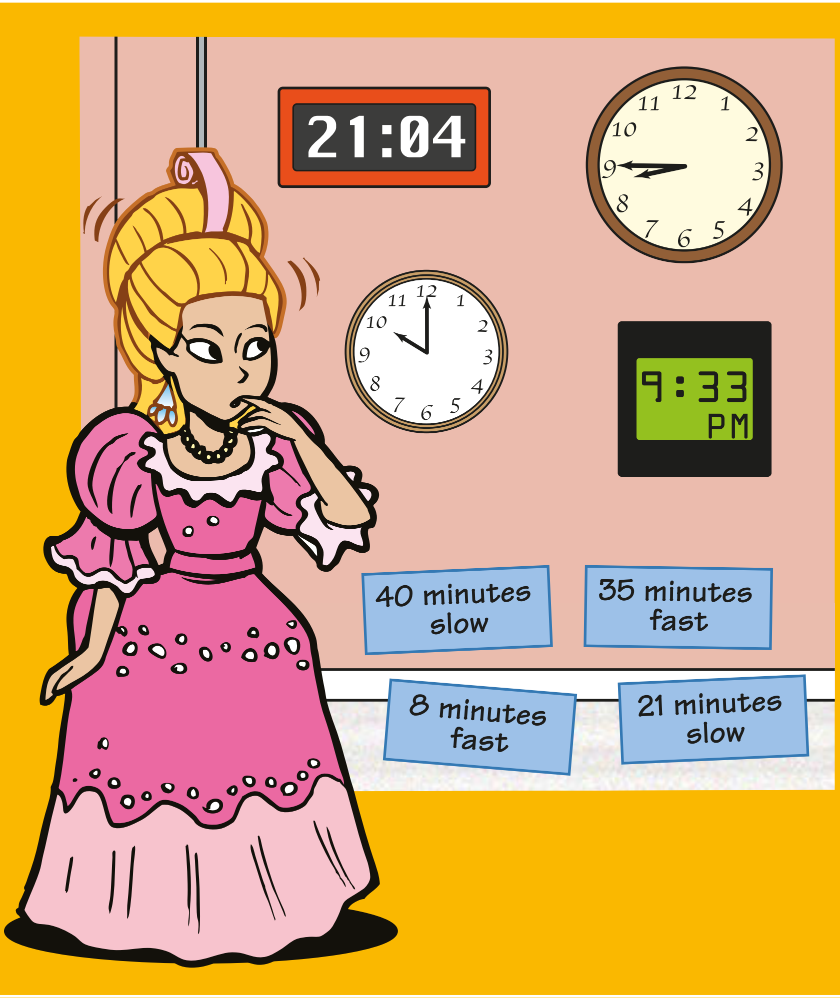
23 A TV show has a total running time of 1 hour. It must include 22 minutes of advertisements. How many ad breaks would you have and how much time would each ad break last?

24 In 2009, Western Australia voted against applying daylight saving time. Make a list of the advantages and disadvantages of turning the clocks forward by one hour over the hotter months.

Puzzle

Time gone cuckoo

As Cinderella arrived at the ball and stepped down from her carriage, she wondered what the time was. Cinderella ran over to the clock shop and looked in the window. In the window display of the shop there were four clocks and four signs that had been taken down and placed under the clocks. The signs read '40 minutes slow', '8 minutes fast', '35 minutes fast' and '21 minutes slow'. Cinderella has no idea which sign belongs to which clock. How many minutes does Cinderella have until midnight?



Challenge 5

1 A growing substance doubles its volume every minute. At 9:20 am a small amount is placed in a container and at 10:00 am the container just fills. The time at which the container was one-quarter full was:

A 9:30 am **B** 9:40 am **C** 9:58 am **D** 9:59 am

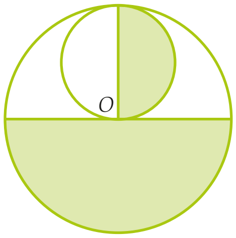
2 A piece of string is used to form a square of area 196 cm^2. Rounded to the nearest whole number, the area of the largest circle that can be formed from this piece of string is:

A 16 cm^2 **B** 196 cm^2 **C** 250 cm^2 **D** 3057 cm^2

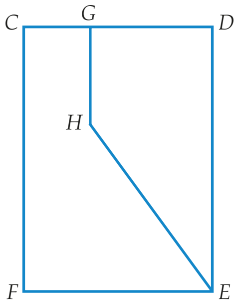
3 In the diagram, each circle is divided into two equal areas. *O* is the centre of the larger circle. The area of the larger circle is 144π cm^2. The total area of the shaded regions is:

A 54π cm^2 **B** 90π cm^2

C 108π cm^2 **D** 144π cm^2



4 In the diagram, the rectangle *CDEF* is divided into two regions *CGHEF* and *GDEH* of equal area. If $GD = 60$ cm, $CF = 90$ cm and $GH = 30$ cm, what is the length of *CG*?



5 Chen had a 10 am appointment 60 km from his home. He drove at an average speed of 80 km/h for the trip and arrived 20 minutes late for the appointment. At what time did he leave his home?

A 9:35 am **B** 9:20 am

C 9:15 am **D** 9 am

6 If 1 August is a Thursday, what is the date of the second Saturday in September?

7 Petra's doctor has instructed her to take a course of 20 tablets. She has to take two tablets every 8 hours. How many days will it take Petra to complete the course?

8 A clock is set to the correct time at 9 am, but loses 3 minutes every hour. What is the correct time when the clock next shows 6:30?

9 If it takes 40 seconds to saw a log into five pieces, how much time will it take to saw it into eight pieces?

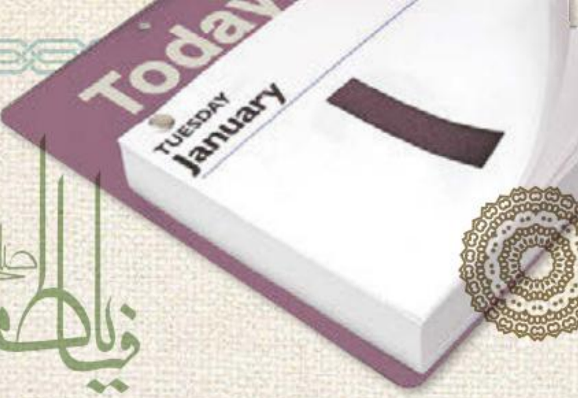
10 Brent needs to cook rice for exactly 15 minutes, but he has only a 7-minute egg timer and an 11-minute egg timer. Explain how he can measure 15 minutes exactly.

11 How much time will it take a 1 kilometre-long train going at 20 kilometres per hour to get completely through a 2 kilometre-long tunnel?

12 Kaley's daughter Eleanor was born on Friday, 1 May 2009. What is the first year in which her birthday again falls on a Friday?

13 A cricket club always holds its board meetings on the second Tuesday of the month. What are the only possible dates for the meeting each month?

Maths 4 Real



Today's date is...

Calendars are designed so they fit either the solar cycle or the lunar cycle. The solar cycle is the time it takes the Earth to complete one orbit of the Sun, which can be measured by observing the seasons of a year. The lunar cycle is the time it takes the Moon to complete one orbit of the Earth, which can be measured by observing the phases of the Moon.

The standard calendar used in Australia (and most other countries) is known as the Gregorian calendar (named after Pope Gregory XIII, who introduced it). The Gregorian calendar is a solar calendar of 365 days, with an extra day usually added every fourth year to make a 'leap year'. This extra day is necessary because the length of the solar cycle is closer to 365.25 days. (To be even more accurate, the years that start each new century—1700, 1800, 1900 etc.—are not leap years, but the years that start each new millennium—1000, 2000, 3000 etc.—are leap years.)

A year is divided into 12 months. The length of one month corresponds approximately, but not exactly, to the lunar cycle.

The Islamic calendar

The Islamic calendar is a lunar calendar. It has 12 months, with the length of each month set to correspond to one lunar cycle. The timing of the Moon's orbit can vary. Some Islamic groups use a system where even-numbered months have 29 days and odd numbered months have 30 days. Leap years in the Islamic calendar can occur every two or three years, where an extra day is added onto the twelfth month to keep the months aligned with the lunar cycle.

1 (a) Is the Islamic calendar shorter or longer than the Gregorian calendar? (Assume regular, not leap, years.)

(b) What is the difference in the number of days?

Year 1 of the Islamic calendar is designated as the year that the prophet Muhammad fled from Mecca. This year corresponds to the year 622 on the Gregorian calendar, so the year numbers in the Islamic and Gregorian calendars are different. An Islamic year number can be converted to a Gregorian calendar year by applying the rules below.

1 Find the Gregorian year number

Multiply the Islamic year number by 970 224, divide this by one million, then add 621.5774. The whole number part of the resulting decimal is the Gregorian year number.

2 Find the Gregorian day number

Multiply the decimal part of the number you found in step **1** by 365 and round the answer down to the nearest whole number. This will tell you the day number of the Gregorian year on which the new Islamic year begins. (You will then need to find which day of which month this is.)

2 (a) Apply the above rules to find out which Gregorian calendar year corresponds to the Islamic calendar year of 1434.

(b) On which day of the Gregorian year did the Islamic year of 1435 begin?

The Chinese traditional calendar

The Chinese traditional calendar is a 'lunisolar' calendar. This means that the length of the months corresponds to the lunar cycle, but the overall length of the year aims to follow the solar cycle. To achieve this, an extra month is added every two or three years.

Unlike most other calendars, the Chinese traditional calendar does not count the years in an infinite sequence. Instead, years go through a cycle of names. In one common traditional system, each year has two parts to its name. The first part is called the 'Celestial Stem':

1	2	3	4	5	6	7	8	9	10
jia	*yi*	*bing*	*ding*	*wu*	*ji*	*geng*	*xin*	*ren*	*gui*

The second part is an 'Earthly Branch'. Each of these Chinese words is the name of an animal in the Chinese zodiac. This is why certain years are called, for example, 'The Year of the Dragon' or 'The Year of the Pig'.

Using this system, the first year of a cycle is named by using the first word in each of these tables: *jia-zi*. The second year is then *yi-chou*, the third year is *bing-yin* and so on. When you reach the end of a table, you go back to the beginning. The tenth year is *gui-you*, the eleventh year is *jia-xu* (restarting the Celestial Stem), the twelfth year is *yi-hai*, and the thirteenth year is *bing-zi* (restarting the Earthly Branch).

3 The Gregorian calendar year 2009 had the Chinese traditional name *ji-chou*. Use this information to:

(a) find the English name of the animal for this year

(b) find the Chinese traditional name of the year 2019. What do you notice about this name?

(c) find the Chinese traditional name of the year you were born. What is the animal name for this year?

(d) find a year before and after your birth year that has the same animal name. Write the names of the years using a Celestial Stem and Earthly Branch.

(e) work out how many years it will be before a year has the name *ji-chou* again. (Hint: Use your knowledge of lowest common multiples.)

Research

Find out about some of the traditional calendars used by other cultures, such as the Indian, Jewish, Maya or Roman calendars. Are they lunar, solar or lunisolar? How many months in one year? What are the names of the months?

1	2	3	4	5	6	7	8	9	10	11	12
zi (rat)	*chou* (ox)	*yin* (tiger)	*mao* (rabbit)	*chen* (dragon)	*si* (snake)	*wu* (horse)	*wei* (sheep)	*shen* (monkey)	*you* (rooster)	*xu* (dog)	*hai* (pig)
											

Chapter review 5

Maths literacy

- 24-hour time
- area
- base
- capacity
- circumference
- composite shape
- cylinder
- diameter
- elapsed time
- hectare
- height
- kite
- parallelogram
- perimeter
- perpendicular
- prism
- radius
- rectangle
- rhombus
- square
- time zone
- trapezium
- triangle
- uniform cross-section
- volume

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 The perimeter of a circle is called its ____________.

2 The distance across a circle, through its centre, is called the ____________. Half this distance is the ____________.

3 The ____________ of a shape is the amount of surface within the boundary.

4 A ____________ is a quadrilateral with only two sides parallel. A ____________ is a quadrilateral with two pairs of opposite sides parallel.

5 The amount of space occupied by a three-dimensional object is its ____________. Its ____________ is the amount of liquid or gas it can hold.

6 There are three ____________s in Australia.

7 The volume of a ____________ is the area of the base multiplied by the height.

8 A solid with a circular base and a ____________ is called a cylinder.

9 The diagonals of a ____________ and a ____________ intersect at right angles.

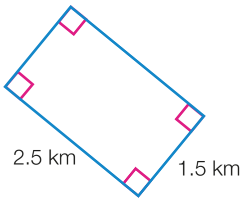
10 2:30 pm written in ____________ is 1430.

Fluency

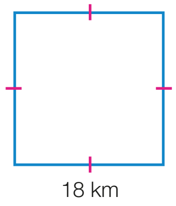
5.1

1 Find the perimeter of the following shapes.

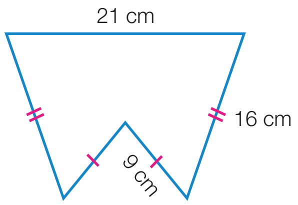
(a)



(b)



(c)



5.2

2 Estimate (do not calculate) to the nearest whole number the circumference of the following circles.

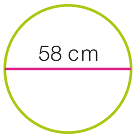
(a) diameter of 10 m

(b) radius of 4 cm

3 Calculate the circumference of each circle correct to 2 decimal places.

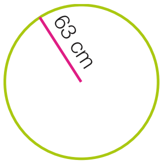
5.3

(a)



(b)

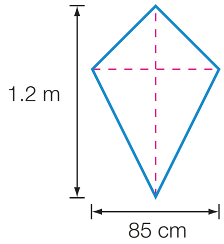
(c)



4 Find the area of each figure in m^2. State answers correct to 2 decimal places where appropriate.

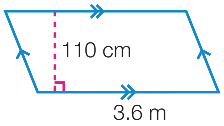
5.4, 5.5

(a)



(b)

(c)



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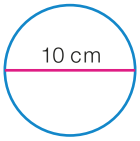

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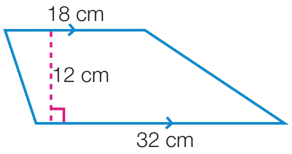
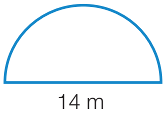


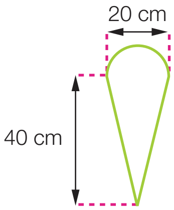
(f)



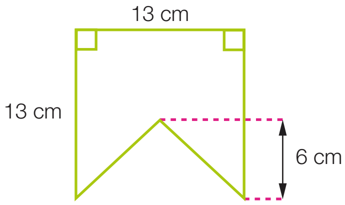
5 Calculate the area of these composite shapes to the nearest whole number.

5.6

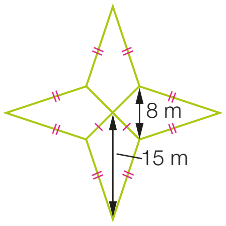
(a)



(b)



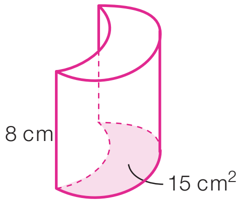
(c)



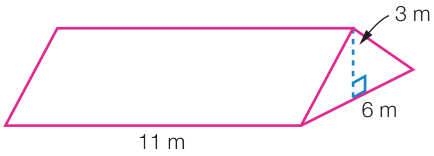
6 Calculate the volume of each solid correct to 2 decimal places.

5.7

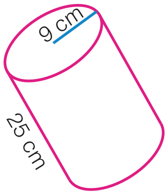
(a)



(b)



(c)



7 Copy and complete the following conversions.

5.4, 5.7

(a) 0.045 L = _____ mL

(b) 27.5 cm^2 = _____ mm^2

(c) 16 m^3 = _____ cm^3

(d) 1575 mL = _____ L

(e) 3600 m^2 = _____ ha

(f) 275 cm^3 = _____ mm^3

(g) 65 mL = _____ cm^3

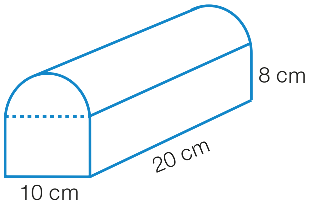
(h) 0.053 km^3 = _____ m^3

(i) 250 L = _____ m^3

(j) 0.09 km^2 = _____ m^2

8 Find the volume of this solid, correct to 2 decimal places.

5.7



9 **(a)** How much time has passed from 4:58 am to 3:41 pm? **5.8**

(b) What time is it $4\frac{1}{2}$ hours after 0914?

10 **(a)** Write 7:38 pm as a 24-hour time. **5.8**

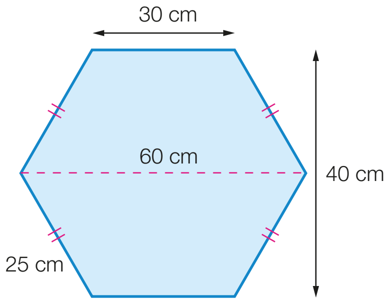
(b) Write 2321 as a 12-hour (am or pm) time.

Understanding

11 A basketball has a diameter of 23 cm. To the nearest whole number, how many times would it roll if it rolled from one end of the court to the other end 28 metres away? **5.1**

12 Which of the following has a greater capacity, and by how many mL: a rectangular prism with length = 15 cm, width = 12 cm and height = 8 cm, or a cylindrical can that is 12 cm wide and 15 cm high? **5.7**

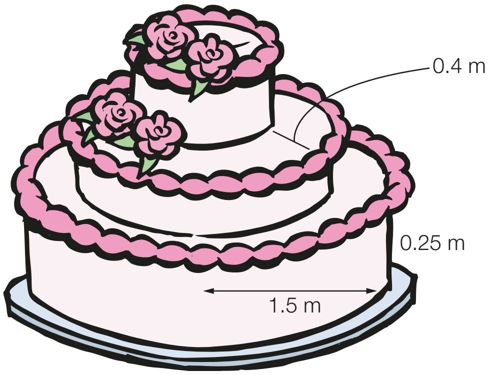
13 Find the perimeter and area of this hexagon. **5.4, 5.5**



14 Tom drove from Adelaide to Melbourne. He left Adelaide at 8:15 am. Tom spent $9\frac{1}{2}$ hours driving and also had three 20-minute breaks. What was the time in Melbourne when Tom arrived? **5.8**

Reasoning

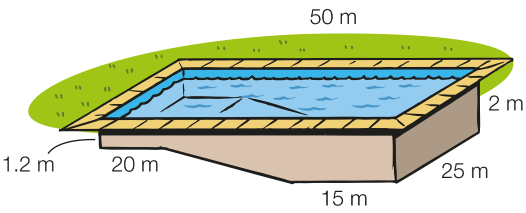
15 Katrina has to design a 'super' wedding cake for her niece's wedding. The cake is to be made of three tiers as shown. The base layer will have a radius of 1.5 m, then each additional layer's radius is reduced by 0.4 m. Each tier is 0.25 m high. **5.7**



(a) Calculate, correct to 2 decimal places, the volume of cake needed.

(b) Each tier is to have a ribbon around its circumference. How many metres of ribbon will Katrina need to buy to place around the cake? (Answer to the nearest whole number.)

16 A new swimming pool is being built by the local council. Its design is shown in the diagram, with a shallow 1.2 m section of length 20 m, and a deep 2 m section of length 15 m, joined by a section of increasing depth. **5.7**



(a) What is the volume, in cubic metres, of the new swimming pool?

After the pool is built, the council has to fill the pool with water.

(b) How many kilolitres of water will be needed to fill the pool?

(c) The only way to fill the pool is to bring the water in by tanker. The tanker has a water tank in the shape of a cylinder, with diameter 2 metres and length 10 metres. How many tanker loads will be needed to fill the pool with water?

Numeracy practice 5

Non-calculator

1 The area of a square with a perimeter of 80 cm is:

A 40 cm^2 **B** 80 cm^2 **C** 400 cm^2 **D** 1600 cm^2

2 Which metric unit would a landscape gardener use to measure the volume of soil in a pile like this?

A square metres

B cubic metres

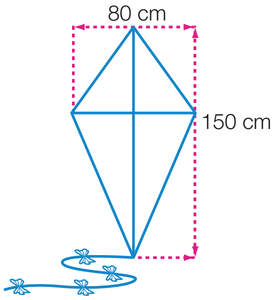
C cubic centimetres

D square centimetres

3 The number of square centimetres of fabric needed to make the kite is:

A 600 **B** 3000

C 6000 **D** 12 000



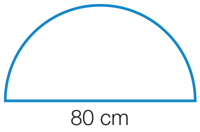
4 Sam's favourite TV show starts at 5:35 pm. If it is 12:47 pm now, how much time is there until the show starts?

A 4 h 38 min **B** 4 h 48 min **C** 5 h 12 min **D** 5 h 42 min

Calculator allowed

5 A semicircular table has a diameter of 80 cm. The perimeter of the table is closest to:

A 120 cm **B** 200 cm **C** 250 cm **D** 320 cm



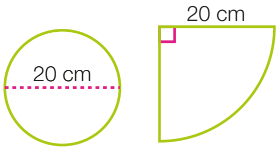
6 How many cylinders of diameter 10 cm and height 15 cm can be filled from a cubic tank of side measurement 50 cm?

A 53 **B** 106 **C** 159 **D** 212

7 What is the difference between the perimeters of these two shapes, to the nearest centimetre?

A 3 cm **B** 9 cm

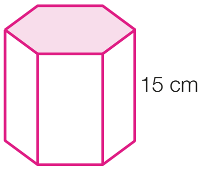
C 31 cm **D** 70 cm



8 The volume of the hexagonal prism is 500 cm^3. The area of cross-section is closest to:

A 2 cm^2 **B** 17 cm^2

C 20 cm^2 **D** 33 cm^2



6

Linear graphs

6

That's check, mate! There are more possible chess positions than there are molecules in the Earth's atmosphere, so how can players keep track of what moves have been made?

To record and communicate chess moves, one method is to imagine that the chessboard has a grid labelled 'a' to 'h' along the bottom, and numbered 1 to 8 along the side. Chess moves can then be made using these grid coordinates. For example, if a 'pawn' chess piece moves from position a2 (near the bottom left of the board) to position a4, then this move can be recorded as 'a2–a4'.

After each player has made their first move, there are 400 possible positions of pieces on the chessboard. After each player has made two moves, there are almost 200 000 possible positions. A chess master who wants to 'look ahead' five moves is faced with a trillion possibilities!

Forum

Why do you think the number of possible positions increases so quickly?

Are there other ways to record and communicate chess moves?

Can other methods be used in other games?

Why learn this?

You need to know about the Cartesian plane so that you can show relationships graphically when studying many areas of mathematics. In the real world, an atlas or a road map would be useless if there was no way of referring to the location of a country or a street. By using a systematic way of specifying position, important information can be understood. A position marked on a grid can be used to represent many quantities, such as location, temperature, height or profit.

After completing this chapter you will be able to:

- interpret line graphs
- compare graphs of different linear relationships
- plot relationships and find rules using tables of values
- plot relationships using rules
- identify different gradients
- find a rule for a set of points that lie on a straight line
- recognise linear equations that generate graphs parallel to either axis
- use linear relationships to solve problems.

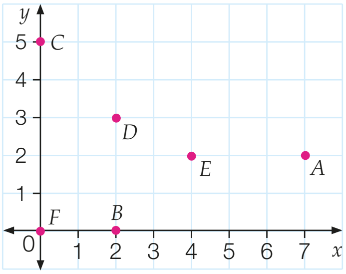
Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Draw a number plane to include x-values from -5 to 5 and y-values from -5 to 5, then plot and label each of the following points.

$P(3, -5)$, $Q(2, 0)$, $R(5, 1)$, $S(0, 4)$ and $T(-1, 5)$

2 From the diagram, list the coordinates of each of the points A to F.



3 Copy and complete the tables of values for the rules given.

(a) $y = 3x - 1$

x	1	2	3	10
y				

(b) $y = \frac{x}{2} + 4$

x	0	2	6	10
y				

4 **(a)** Substitute $T = 5$ into the equation $T = 3a - 7$, then solve for a.

(b) Substitute $x = 5$ and $y = 3$ into $y = mx + 8$ to find a value for m.

5 Solve each of the following equations for x.

(a) $5 = 2x - 3$ **(b)** $5 + 3x = 11$ **(c)** $\frac{x}{5} - 1 = 2$

6 If x represents a number, write an expression to represent:

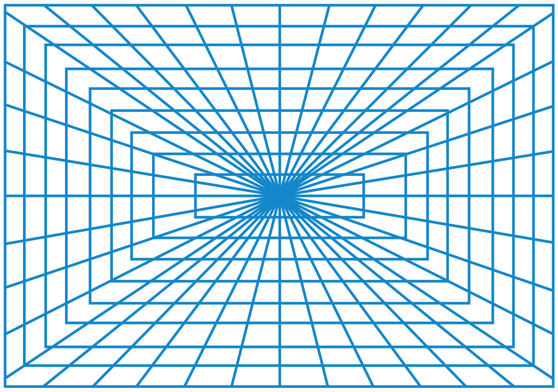
(a) three times the number **(b)** eight more than double the number.

7 Simplify:

(a) $3x + 4y + 6(x - y)$ **(b)** $3(x - 2y) - 4(x + 2y)$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.



Slopes

In this activity, you will explore the gradients of straight lines that are formed by joining sequential integers on a Cartesian plane.

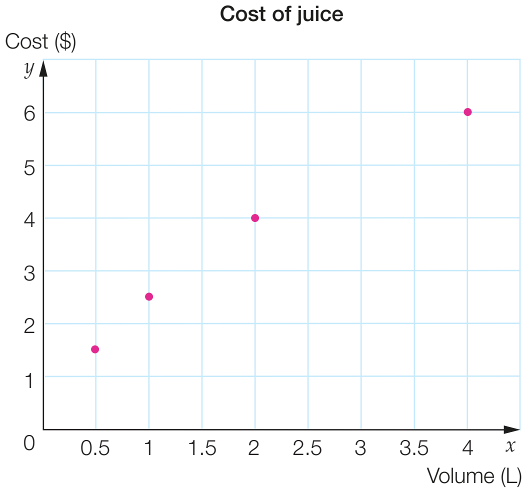
Interpreting line graphs

6.1

In Year 7, you investigated point graphs. These are graphs where the points are not joined. For example, this point graph shows the relationship between the cost of juice and the volume of juice purchased.

The points are not joined because juice can only be purchased in certain volumes (at the point values only). This graph point shows that this juice can only be purchased in volumes of 0.5 L, 1 L, 2 L and 4 L.

However, there are many situations where connecting the points on a graph with a straight line will give valuable information about the relationship and the values in between and beyond the given points.



Line graphs show the relationship between two variables and can consist of several different straight line parts.

Worked example 1

W.E. 1

Selena leaves her home to go for a walk. The following line graph shows her distance from home.

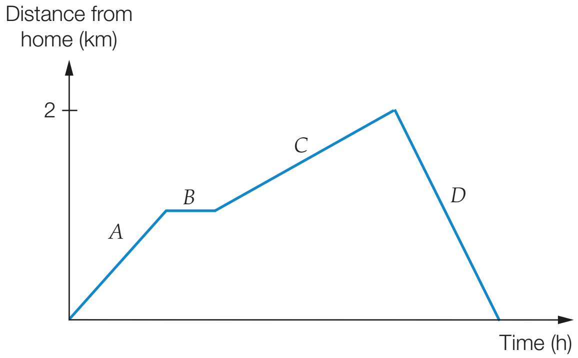
(a) Describe what information is being shown on the graph in:

- **(i)** section *A*
- **(ii)** section *B*
- **(iii)** section *D*.

(b) In which section was Selena walking the fastest?

(c) In which section was she walking the slowest?

(d) What was the total distance that she walked?



Thinking	Working
(a) **(i)** Decide where the travelling begins and then whether the distance from the starting point increases, decreases or stays the same.	(a) (i) Selena starts from her home. As the distance from home is steadily increasing as time increases, Selena is moving away from her house at a steady rate in section *A*.

(ii) Decide whether the distance from the starting point increases, decreases or stays the same.	(ii) A horizontal line means that the distance from home is not changing as time increases. Selena may have stopped somewhere to rest.
(iii) Decide whether the distance from the starting point increases, decreases or stays the same.	(iii) The distance from home is steadily decreasing as time increases, so Selena is returning to her house at a constant rate.
(b) As speed = $\frac{\text{distance}}{\text{time}}$, the steepness of the graph indicates travelling speed. Compare the steepness of each section. The steeper the graph, the faster the speed.	(b) Section *D* has the steepest line. The distance covered here is about twice the distance of section *A* or *C*, but it is covered in less time. Selena must be moving faster in section *D* than in section *A*. Perhaps she ran home.
(c) As speed = $\frac{\text{distance}}{\text{time}}$, the steepness of the graph indicates travelling speed. Compare the steepness of each section. The steeper the graph, the faster the speed.	(c) Section *A* has a steeper line than section *C*. The total distance in section *C* is about the same as the total distance in section *A* but the time taken to cover the distance is greater, so Selena is walking slower in Section *C* than in section *A*.
(d) Decide how far Selena travelled away from home, then double this distance to find the total distance she travelled.	(d) Selena walked 2 km away from home, so she walked 4 km altogether.

6.1 Interpreting line graphs

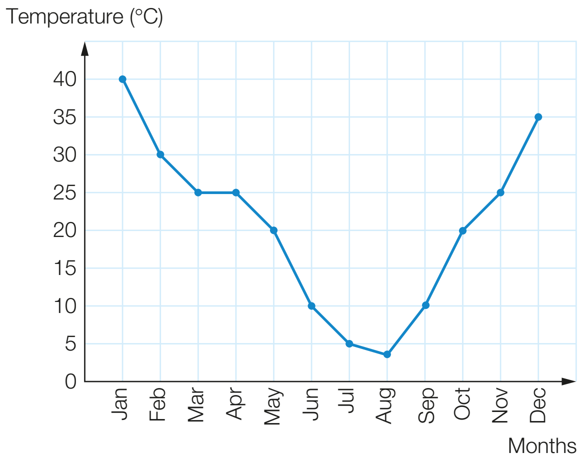
Navigator

Answers p. 648

1, 2, 3, 5, 7, 11, 14, 15	2, 3, 4, 6, 7, 8, 9, 10, 12, 14, 15, 16	2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 16

Fluency

1 The following graph shows the average temperatures during different months in a year.

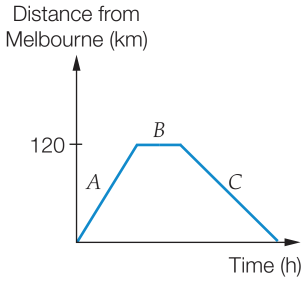


(a) Which was the coolest month of the year?

(b) Which was the hottest month of the year?

(c) Which was the second hottest month of the year?

2 A family is travelling in their car. The line graph shows their distance from Melbourne.



(a) Describe what information is being shown on the graph in:

(i) section *A* **(ii)** section *B* **(iii)** section *C*.

(b) In which section was the car travelling the fastest?

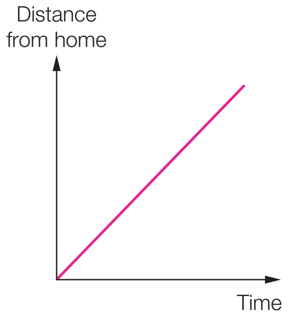
(c) In which section was the car travelling the slowest?

(d) What was the total distance travelled by the family?

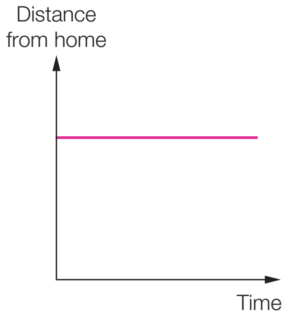
3 The following graphs show the distance Ahmid is from home while out walking during different time intervals.

Remember, a horizontal section of a distance-time graph indicates a period of rest.

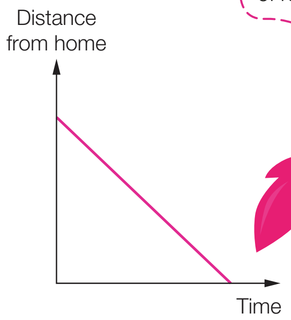
(a)



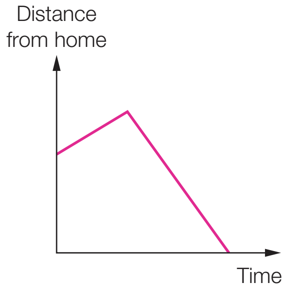
(b)



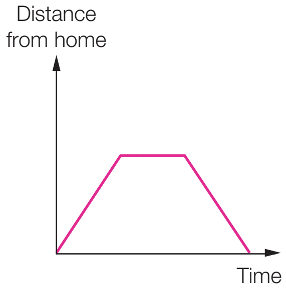
(c)



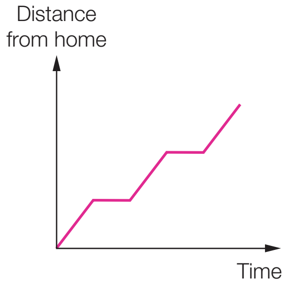
(d)



(e)



(f)



Match each graph **(a)**–**(f)** with the description of a journey **A**–**F** given below.

A Ahmid walks and stops at equal time intervals.

B Ahmid walks directly home.

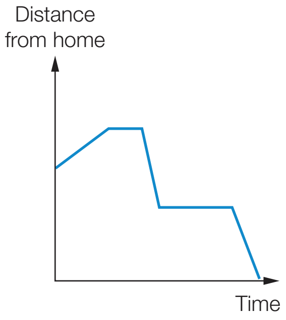
C Ahmid walks away from home at first, then stops and returns home immediately without resting.

D Ahmid is stationary (not walking).

E Ahmid walks away from home, stops for a rest, then walks home.

F Ahmid walks away from home.

4 This graph shows the distance Josie is from home while walking one afternoon.



Use the graph to answer the following questions.

(a) Did Josie start her walk from home? Explain.

(b) Josie walked to the local shop and stopped to buy milk and bread. She then started to walk back home but stopped at a friend's place. What parts of the graph show when she stopped?

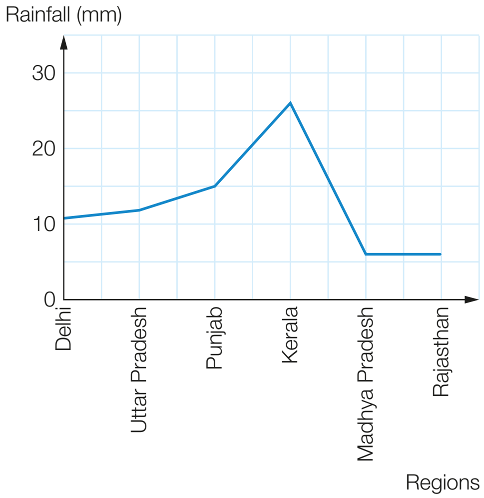
(c) Did she spend more time at the milk bar or at her friend's place?

(d) Did Josie reach her home at any time on her walk? Explain.

5 This graph shows the average rainfall recorded in different regions of India during the monsoon season.

(a) Which of these regions has the most rainfall?

(b) Which region's rainfall value is 15 mm?

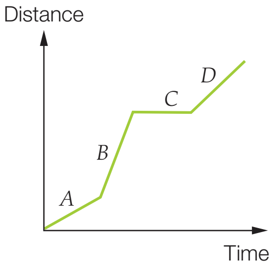


6 Rhiannan is riding her bicycle.

(a) In which section of the graph (*A*, *B*, *C* or *D*) is Rhiannan moving the fastest?

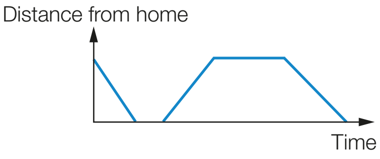
(b) In which section is she stationary?

(c) In which section is she moving the slowest?

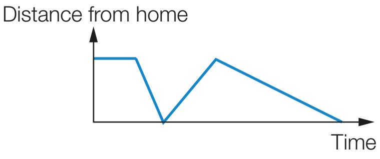


7 Josh is at work. He drives home at a constant speed, eats lunch and then drives back to work at a constant speed. After working for several hours he drives home at a constant rate. Which graph describes this?

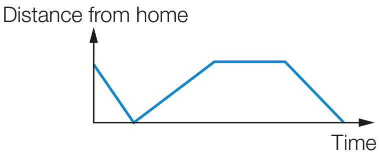
A



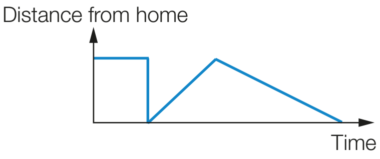
B



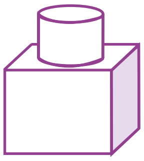
C



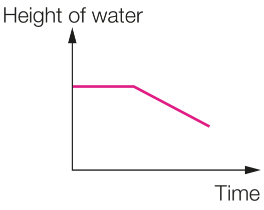
D



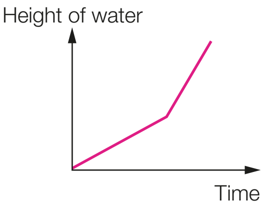
8 The container shown starts off empty. Water is then poured in at a constant rate. Which graph describes this, in terms of the height of the water in the container?



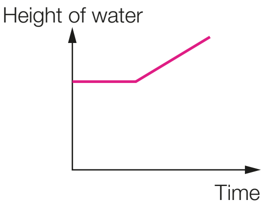
A



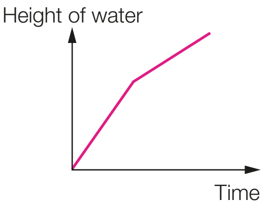
B



C



D



Understanding

9 Which journey illustrates Jason's trip as shown by the following graph?

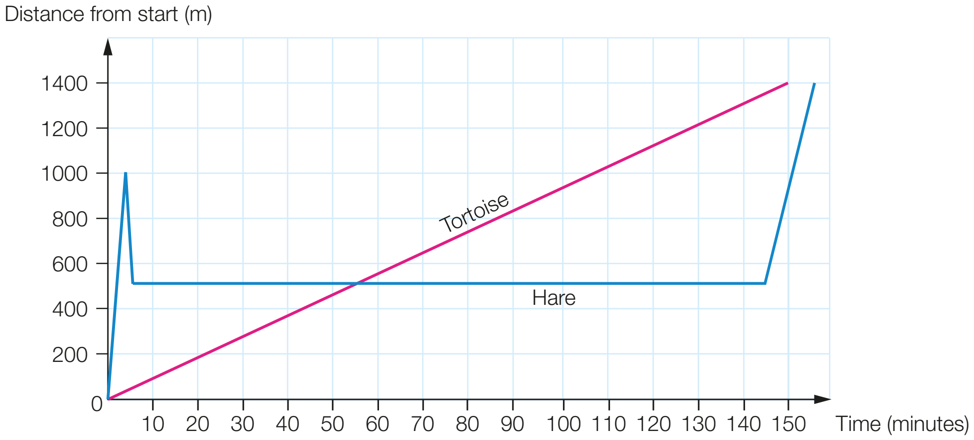
A Jason started from home, walked slowly through the park, then ran to the milk bar, bought an ice-cream and then walked back home.

B Jason started from home, ran through the park, then walked to the milk bar, bought an ice-cream and then ran back home.

C Jason started from the milk bar, ran through the park, then walked home, looked for a skateboard and then ran back to the milk bar.

D Jason started from home, ran through the park, then stopped at the milk bar, played with his skateboard and then walked back home.

10 The hare and the tortoise had a race. Below is a graph of the race.



(a) What was the length of the race?

(b) Who won the race?

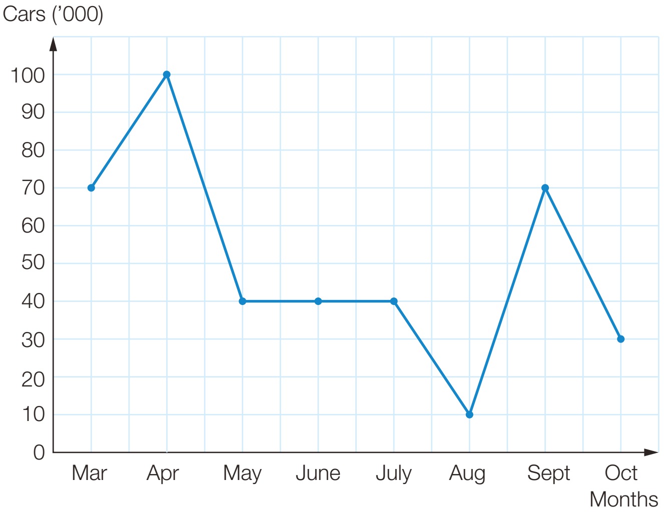
(c) What was the winning time?

(d) At what time and distance from the start were the hare and the tortoise at the same place?

(e) Describe the hare's race.

Reasoning

11 The number of cars manufactured (in thousands) by a car company during different months of a year is shown. How many more cars were manufactured in April than in August?



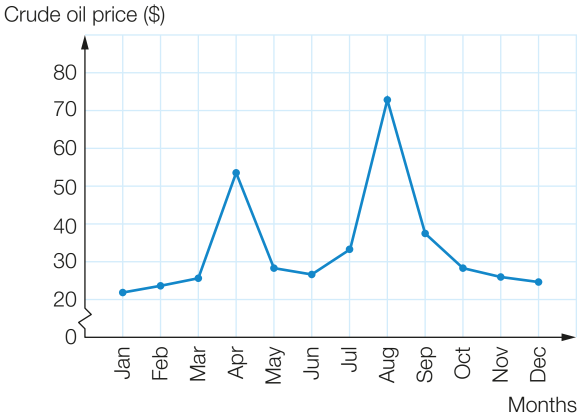
12 This graph shows changes in crude oil prices.

(a) How did the prices change from April to June?

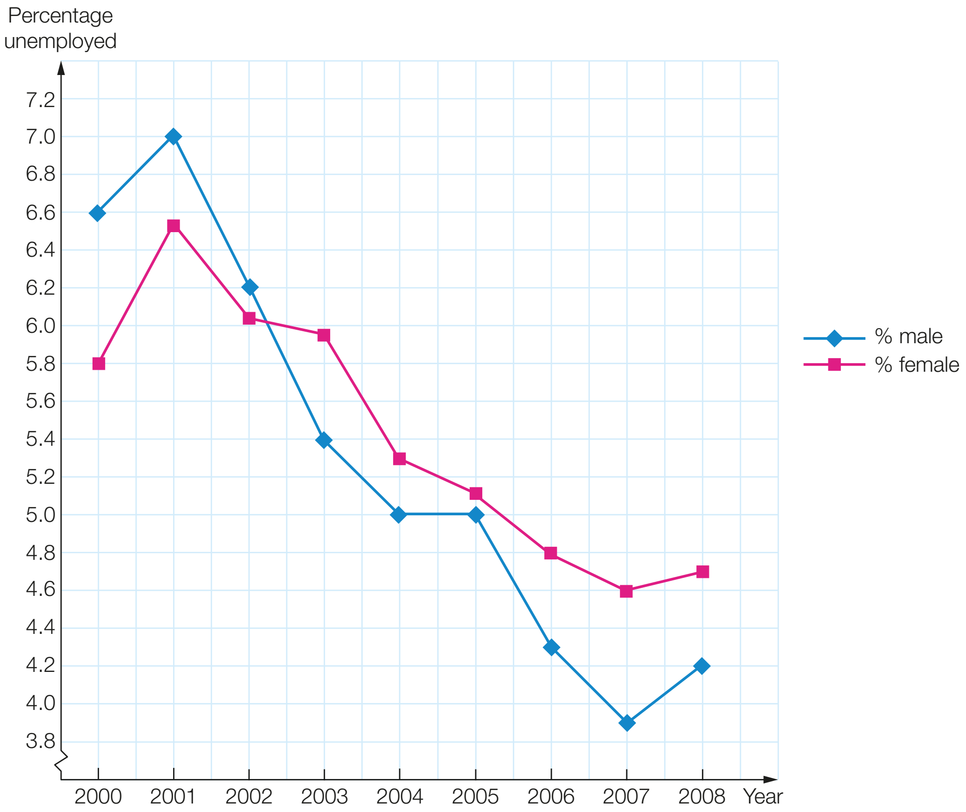
(b) Which month had the most expensive price?

(c) When was the price cheapest?

(d) How much cheaper was the price in November than in April?



13 The following graph shows information about unemployment in Australia during the first decade of the 2000s.



(a) What information is provided by the values shown on the vertical axis?

(b) In what year was the percentage of male unemployment the greatest?

(c) In what year was there the least difference between the percentage of males and the percentage of females unemployed?

(d) During what period was there a higher percentage of male unemployment than of female unemployment?

(e) What was the greatest annual percentage change in female unemployment?

(f) For how many years was the male unemployment percentage below 5%?

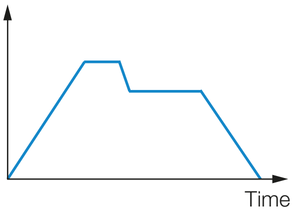
(g) State the number of years for which the female unemployment percentage was above 5.3%.

(h) The percentage of people unemployed is an indicator of a country's economy. A low unemployment percentage usually means strong economic growth. For one year in particular, the economy experienced slow growth. When do you think this occurred? Explain your answer.

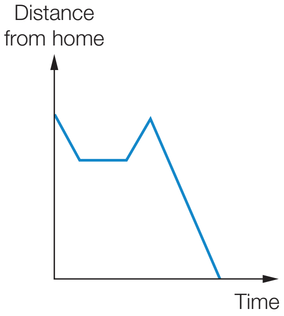
Open-ended

14 Jodie provided Darcy with some information that Darcy used to draw the line graph shown.

What might the information have been?



15 Write a story to describe the journey shown in this graph.



16 **(a)** Draw graphs that describe each of the following situations.

- **(i)** a train travelling from Rosebush station to Cedarwood station, stopping at Mapleleaf and Gumnut stations on the way
- **(ii)** a person travelling up and down in a lift that stops to let other people on and off at various floors

(b) Discuss your graphs with another student. Do they seem reasonable?

Problem solving

Where is the missing area?

Equipment required: graph paper or grid paper

There are 64 small squares in Grid A.

The four shapes (shown with different colours) are rearranged to make Grid B.

There seem to be 65 small squares in Grid B.

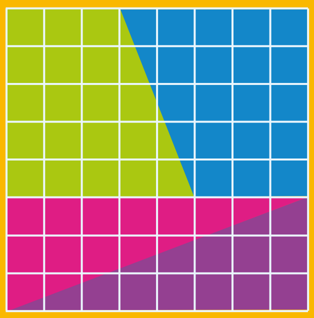
Grid A and Grid B are made from shapes of the same size, so they must have the same area.

Why does there appear to be an extra small square? Where did it come from?

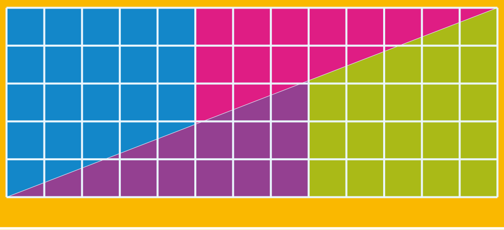
Strategy options

- Draw a diagram.
- Make a model.

Grid A



Grid B



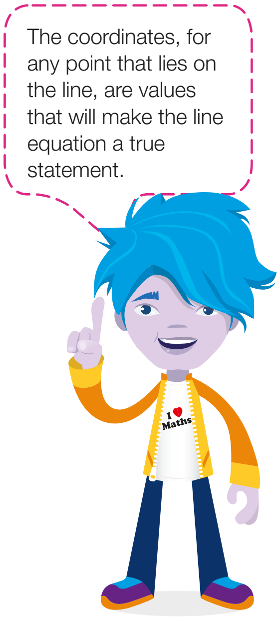
You might want to draw the shapes carefully on grid or graph paper. You can cut out the shapes to put them together yourself.

6.2

Linear relationships

Linear graphs

A **linear graph** is a straight line graph that shows the relationship between two variables. It can be drawn if you know the rule (for example, $y = x + 2$) that connects the two variables. This rule shows the **linear relationship** between the two variables. The rule or equation is used to complete a table of values that shows the x- and y-coordinates of some points on the line. These are plotted on the **Cartesian plane** and connected with a line.



Any point on the line can be represented by **coordinates**, an x-coordinate and a y-coordinate. These coordinates are written as an **ordered pair** (x, y).

The ordered pair (0, 0) is known as the **origin**.

Worked example 2

W.E. 2

(a) Copy and complete the following table of values for the rule $y = \frac{x}{2} + 4$ for values of x in the range -2 to 2.

x	-2	-1	0	1	2
y					
(x, y)					

(b) Use the table of values to draw a graph of the relationship.

Thinking	Working

(a) 1 Substitute each value of x into the rule to find the corresponding value of y.

(a) $y = \frac{x}{2} + 4$

$y = \frac{-2}{2} + 4$ where $x = -2$

$= 3$

$y = \frac{-1}{2} + 4$ where $x = -1$

$= 3.5$

$y = \frac{0}{2} + 4$ where $x = 0$

$= 4$

$y = \frac{1}{2} + 4$ where $x = 1$

$= 4.5$

$y = \frac{2}{2} + 4$ where $x = 2$

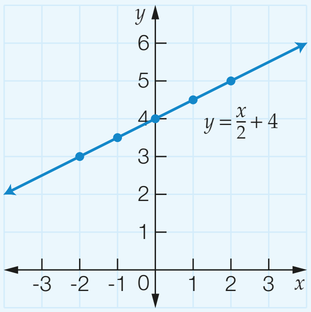
$= 5$

2 Write the y-values in the second row and enter the ordered pairs in the third row.

x	-2	-1	0	1	2
y	3	3.5	4	4.5	5
(x, y)	(-2, 3)	(-1, 3.5)	(0, 4)	(1, 4.5)	(2, 5)

(b) Draw and label the x-axis and the y-axis on the Cartesian plane, plot the points, connect them with a straight line and label the graph with the rule.

(b)



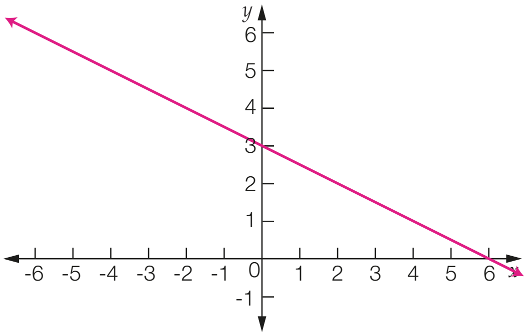
Worked example 3

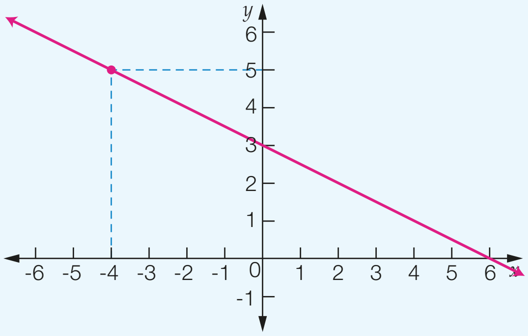
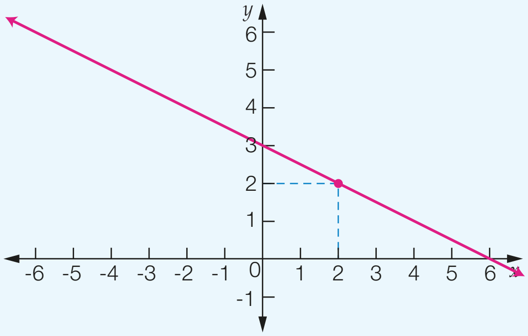
W.E. 3

For the following graph, find:

(a) the value of y where $x = -4$

(b) the value of x where $y = 2$.

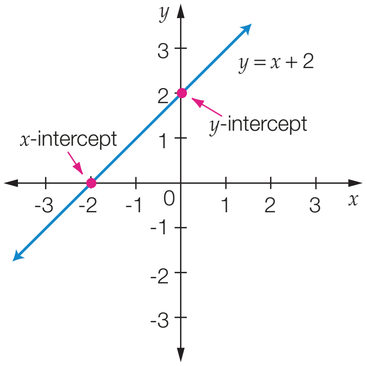


Thinking	Working
(a) Find the given point on the x-axis and draw a vertical line to meet the given line. From the point of intersection with this line, draw a horizontal line to meet the y-axis. The y-coordinate gives the required value.	(a)  Where x = -4, y = 5.
(b) Find the given point on the y-axis and draw a horizontal line to meet the given line. From the point of intersection with this line, draw a vertical line to meet the x-axis. The x-coordinate gives the required value.	(b)  Where y = 2, x = 2.

x-intercepts and *y*-intercepts

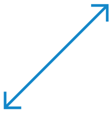
The ***x*-intercept** of a graph is the point where the line crosses the x-axis.

The ***y*-intercept** of a graph is the point where the line crosses the y-axis.

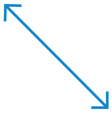


Gradient

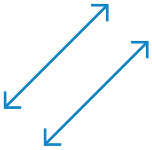
The **gradient** tells you about the **slope** of a line.



This line has a positive gradient as it slopes upwards from left to right.



This line has a negative gradient as it slopes downwards from left to right.



Lines with the same gradient are parallel.

Worked example 4

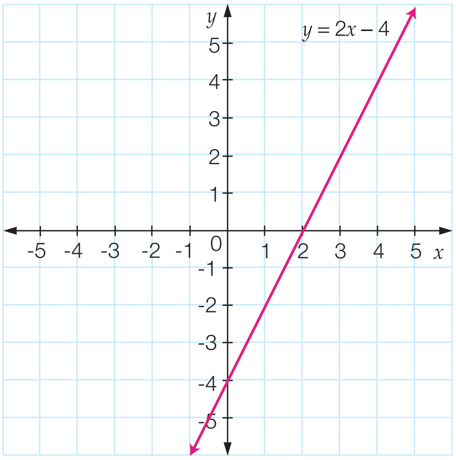
W.E. 4

For the graph with the rule $y = 2x - 4$:

(a) state the x-intercept and write the coordinates of the point

(b) state the y-intercept and write the coordinates of the point

(c) state whether the gradient of the line is positive or negative.

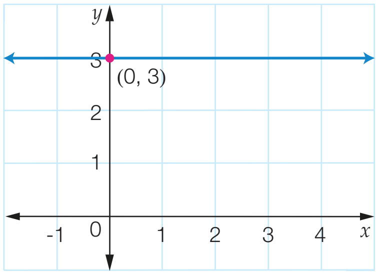


Thinking	Working
(a) The x-intercept is where the graph cuts the x-axis. Write the coordinates of the point.	(a) The x-intercept is 2; it has the coordinates (2, 0).
(b) The y-intercept is where the graph cuts the y-axis. Write the coordinates of the point.	(b) The y-intercept is -4; it has the coordinates (0, -4).
(c) The graph slopes upwards from left to right.	(c) The gradient is positive.

Horizontal and vertical lines

Below is the table of values for the equation $y = 0x + 3$ and its graph.

x	0	1	2	3	4
y	3	3	3	3	3



This equation produces a horizontal line passing through $y = 3$. The line is parallel to the x-axis. The gradient of the line is 0 (the coefficient of x) and the y-intercept is 3.

As $0x = 0$, you can write $y = 0x + 3$ as $y = 3$.

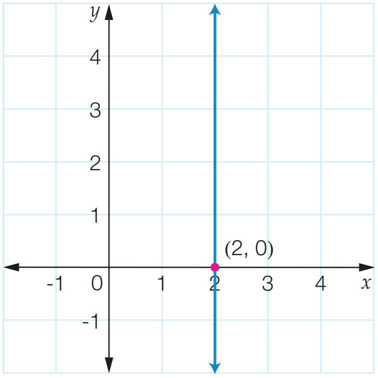
A horizontal line has a gradient of zero.

The graph of $y = c$ is a horizontal line parallel to the x-axis that passes through the point $(0, c)$. This point is the y-intercept.

Horizontal line graphs where $c \neq 0$ are the only linear graphs that do not have an x-intercept.

The graph of the values in the table below is a vertical line parallel to the y-axis and which passes through the point $x = 2$.

x	2	2	2	2	2
y	1	2	3	4	5



An undefined gradient means that the gradient cannot be evaluated.

This gradient is the steepest possible for a straight line. The gradient is described as undefined and the x-intercept is 2.

The equation of the line is $x = 2$ and there is no y-intercept for equations of this type.

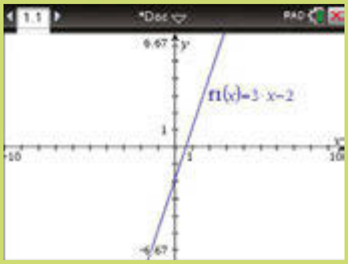
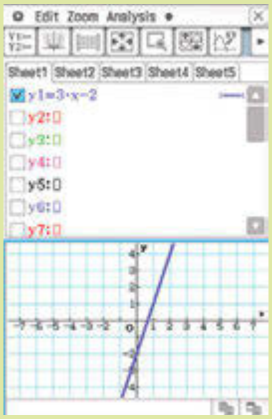
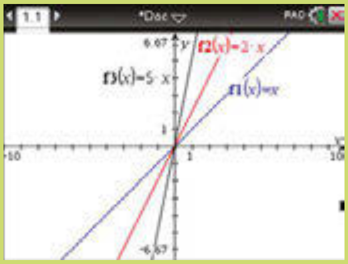
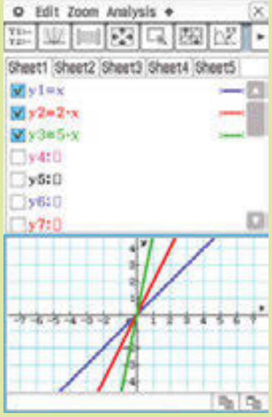
A vertical line has a gradient that cannot be evaluated (because its 'rise' is infinite and its 'run' is zero). The gradient is described as undefined.

The graph of $x = a$ is a vertical line parallel to the y-axis that passes through the point $(a, 0)$. This point is the x-intercept.

Vertical line graphs where $a \neq 0$ are the only linear graphs that do not have a y-intercept.

Using CAS to sketch linear graphs

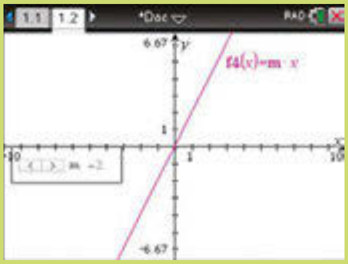
CAS technology makes graphing straight lines very easy.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Graphs** to your document and enter the equation **f1(x)=3x-2** to be graphed. The graph appears, together with its equation.  	From the menu select **Graph & Table**. Enter the equation **y1=3x-2** to be graphed and tick the box next to it. Select the graph icon to view the graph.  ☑ y1=3·x−2 ☐ y2:□ ☐ y3:□ ☐ y4:□ ☐ y5:□ ☐ y6:□ ☐ y7:□
Enter more equations to be graphed in the same way, such as $y = x$, $y = 2x$ and $y = 5x$. (To enter more equations, you can select **menu** > **Graph Entry/Edit** > **Function**.)  	Enter more equations to be graphed in the same way, such as $y = x$, $y = 2x$ and $y = 5x$.  ☑ y1=x ☑ y2=2·x ☑ y3=5·x ☐ y4:□ ☐ y5:□ ☐ y6:□ ☐ y7:□

Using TI-Nspire CAS

You can change the gradient more easily by using a slider. Add **Graphs** to a new page and enter **= m × x** for the equation to be graphed. The slider will appear automatically and you can name the variable in the slider as *m*. (Or if not, then enter **menu** > **Actions** > **Insert Slider**.)

Put the cursor (hand) over the slider and move it using the keys on the touchpad.



Using Casio ClassPad CAS

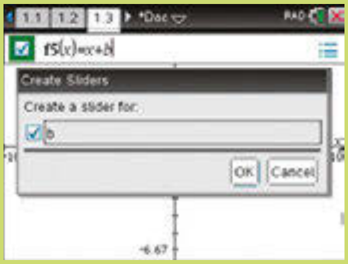
You can change the gradient more easily by using a dynamic graph. Select **Edit** > **Clear All** to clear the screen. Then enter **=mx** to be graphed, using the **Var** tab from the **Keyboard** for the *m*. Select ◆ > **Dynamic Graph** to create a slider to adjust the value of *m*. You can also select ☰ > **Settings** to adjust the slider values.

Parameter m
Value : −5
Min : −5
Max : 10
Step : 1

Watch what happens to the graph as the value of *m* changes.

Using TI-Nspire CAS

In a similar way, you can enter **=x+b** for an equation to be graphed. Create a slider for *b* to see what happens as *b* changes.



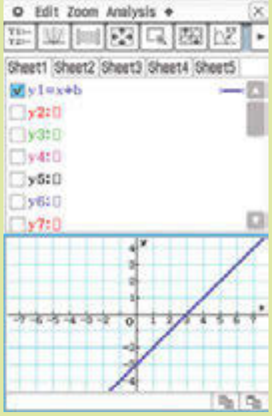
☑ f5(x)=x+b

Create a slider for:

☑ b

Using Casio ClassPad CAS

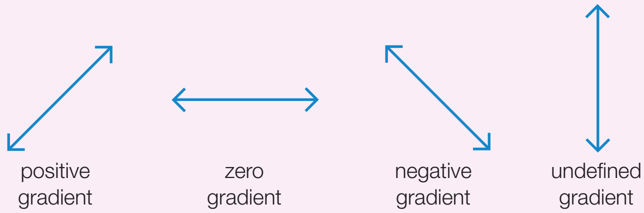
In a similar way, you can enter **=x+b** for an equation to be graphed (using the **Var** tab from the **Keyboard** for *b*). Create a slider for *b* to see what happens as *b* changes.



☑ y1=x+b

Linear graphs

- A linear relationship can be used to complete a table of values.
- A table of values gives coordinate points on a Cartesian plane that can be joined.
- The graph of a linear relationship is a straight line.
- The point where a line crosses the *x*-axis is called the *x*-intercept.
- The point where a line crosses the *y*-axis is called the *y*-intercept.
- The gradient of a line describes the slope or steepness of the line.



6.2 Linear relationships

Navigator

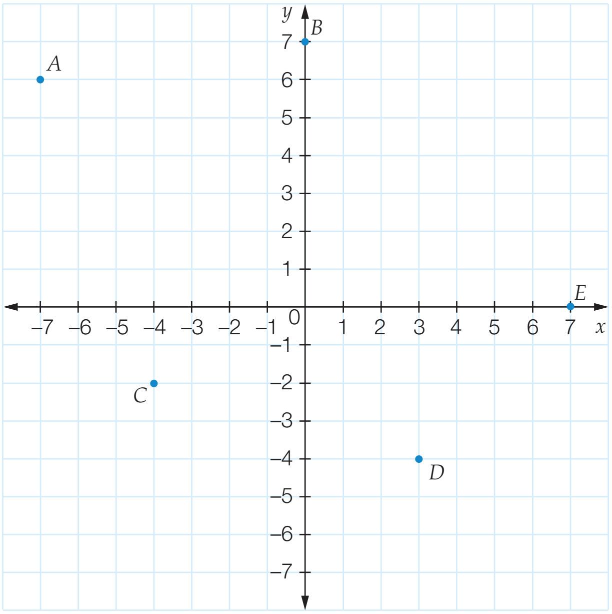
Answers p. 648

1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15 (columns 1–2), 16, 19, 23, 25	3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15 (columns 2–3), 16, 17, 19, 23, 25	3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15 (columns 2–3), 16, 17, 18, 19, 20, 21, 22, 24, 25

Equipment required: graph paper or grid paper

Fluency

1 Find the coordinates of each point shown on the graph.



2 Plot the following points on a graph.

(a) $F(-2, -3)$, $G(0, -2)$, $H(2, -1)$, $K(4, 0)$

(b) $A(0, 1)$, $B(0, 4)$, $C(0, -2)$, $D(0, -3)$. What is special about these points?

3 **(a)** Copy and complete each of the following tables of values for the rules given for values of x in the range -2 to 2.

W.E. 2

(b) Use the table of values to draw a graph of the relationship.

(i) $y = x + 1$

x	-2	-1	0	1	2
y	-1				
(x, y)	(-2, -1)				

(ii) $y = \frac{x}{2}$

x	-2	-1	0	1	2
y	-1				
(x, y)	(-2, -1)				

(iii) $y = 3x - 1$

x	-2	-1	0	1	2
$3x$	-6				
$3x - 1$	-7				
(x, y)	(-2,-7)				

(iv) $y = -x - 4$

x	-2	-1	0	1	2
$-x$	2				
$-x - 4$	-2				
(x, y)	(-2, -2)				

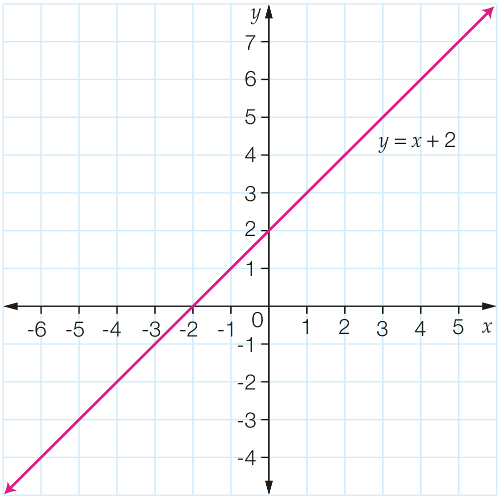
4 For each of the following graphs, find:

W.E. 3

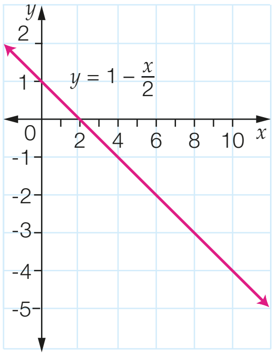
(a) the value of y where $x = 4$

(b) the value of x where $y = -3$.

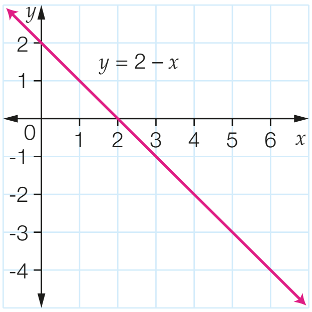
(i)



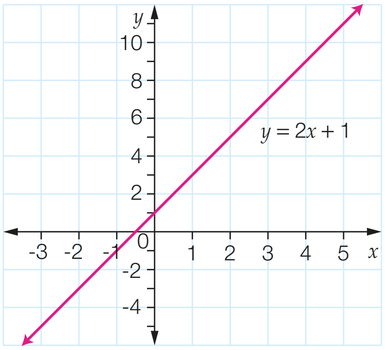
(ii)



(iii)



(iv)



W.E. 4

5 For each of the graphs in the previous question:

(a) state the x-intercept and write the coordinates of the point

(b) state the y-intercept and write the coordinates of the point

(c) state whether the gradient of the line is positive or negative.

6 The coordinates of a point that is on the graph of $y = 5x - 4$ are:

A (1, 9) B (2, -6) C (0, 4) D (-1, -9)

7 Which table of values shows coordinates from the equation $y = 2x - 3$?

A

x	-3	-2	0	2	3
y	-9	-7	-3	1	3

B

x	-3	-2	0	2	3
y	3	1	-3	-3	-9

C

x	-3	-2	0	2	3
y	-6	-5	-3	-1	0

D

x	-3	-2	0	2	3
y	3	-1	-3	7	9

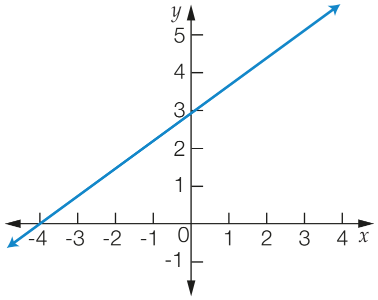
8 The coordinates of the x-intercept and the y-intercept are, respectively:

A (3, -4) and (0, 0)

B (0, 3) and (-4, 0)

C (0, -4) and (3, 0)

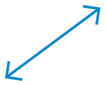
D (-4, 0) and (0, 3)



To decide which gradient is steeper, don't consider whether they are positive or negative.

9 Choose the correct answer for the following.

A


B


C


D


(a) Which line is the steepest?

(b) Which line has a gradient of zero?

(c) Which line has the smallest positive gradient?

10 (a) Copy and complete the following sentences using the words: same, negative, positive, undefined and zero.

Lines with ______ gradient slope up to the right.

Lines with ______ gradient slope down to the right.

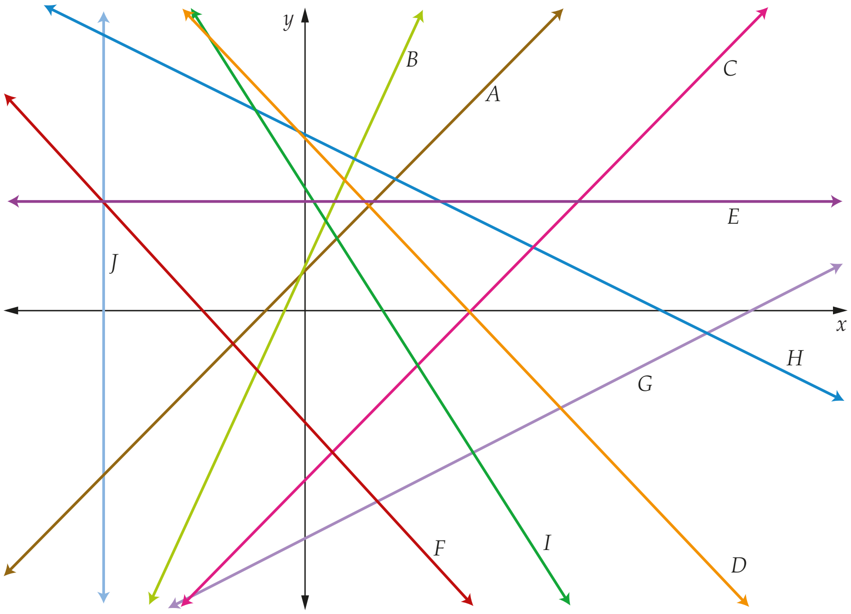
Horizontal lines have ______ gradient.

Parallel lines have the ______ gradient.

Vertical lines have ______ gradient.

(b) For the lines *A*–*J* shown below, list the letters of those:

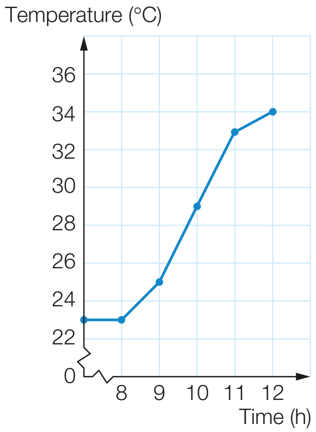
(i) with positive gradient
(ii) with negative gradient
(iii) with zero gradient
(iv) that are parallel
(v) with undefined gradient
(vi) with steepest positive gradient.



11 The following graph shows the temperature recorded every hour until noon on a hot day.

(a) What was the temperature at 11 am?

(b) When was the temperature 29 °C?



12 (a) Plot the graph of the following relationships by using the tables of values given.

(i) $y = 3$

x	0	1	2
y	3	3	3
(x, y)	(0, 3)	(1, 3)	(2, 3)

(ii) $y = -5$

x	0	1	2
y	-5	-5	-5
(x, y)	(0, -5)	(1, -5)	(2, -5)

(b) Are these relationships linear? Explain your answer.

(c) What is the gradient of each line?

(d) Are the lines parallel to the x-axis or to the y-axis?

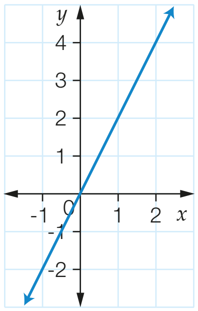
(e) State the x-intercept and the y-intercept for each graph.

Understanding

13 Which point lies on the line shown?

A (4, 2) **B** (-4, -2)

C (2, 4) **D** (-2, 4)




In mathematics, a line has zero thickness.

14 Plot the graphs for each of the following linear relationships using only three points.

(a) $y = -2x$ **(b)** $y = -3x + 9$

15 Without plotting points, draw the graphs of each of the following relationships.

(a) $y = 4$ **(b)** $y = -2$ **(c)** $x = -6$

(d) $x = 3$ **(e)** $x = 2$ **(f)** $y = 7$

16 Draw a line that passes through the points (2, 3) and (4, 1). Find the coordinates of this line's x-intercept and y-intercept.

17 **(a)** Draw the graph of each linear equation on the same Cartesian plane for values of x from -2 to 2.

(i) $y = \frac{1}{2}x$ **(ii)** $y = 2x$ **(iii)** $y = 3x$

(b) What effect does the coefficient of x have on the steepness of the line?

18 The length of the arm of a crane can be described by the equation $y = \frac{14}{5}x$, where x is the horizontal distance that the crane arm can reach.

The origin (0, 0) is 1.5 metres above the ground, at the base of the crane arm.

(a) Complete a table of values for x-values from 0 to 8.

(b) Why should you not graph any negative values for x?

(c) Draw the graph using your table of values.

(d) The crane arm's full length is 20 metres. Use your graph to estimate the horizontal distance that the crane can reach from the origin.

Reasoning

19 For the following linear relationships, state whether the slope of the graph would be positive, negative or zero.

- **(a)** water running out of a water tank (x-axis = time, y-axis = volume in tank)
- **(b)** a car accelerating at a constant rate (x-axis = time, y-axis = speed)
- **(c)** the tray on a forklift being lowered (x-axis = time, y-axis = distance from ground)
- **(d)** earning $20 per hour ($x$-axis = hours, y-axis = money earned)
- **(e)** filling a cylindrical container with lollies (x-axis = time, y-axis = number of lollies)
- **(f)** sitting on a park bench watching a football match (x-axis = time, y-axis = distance from home)

20 Cameron and Mira share 10 lollies between them.

- **(a)** Create a table of values using at least three different combinations showing different ways in which they can share these lollies.
- **(b)** Can any of the values be negative? What does this tell you about the graph?
- **(c)** Plot the values from your table of values (Cameron's lollies on the x-axis and Mira's lollies on the y-axis) and join the points together. Extend the line to show all possible combinations.
- **(d)** Is the gradient positive or negative?
- **(e)** Explain the meaning of the slope in terms of the number of lollies that Cameron and Mira share.
- **(f)**
  - **(i)** What y-coordinate corresponds to x-coordinate 4.3?
  - **(ii)** What does this mean in terms of the number of lollies Cameron and Mira share?
  - **(iii)** Considering your answers, is it sensible to join the points plotted to form a linear graph?

21 Zamira believes that the gradient of any line can be made opposite to what it is by using two of its ordered pairs and taking the negative of the x-coordinates. For example, the line joining (1, 4) to (3, 7) has a positive gradient, and changing 1 to -1 and 3 to -3 gives (-1, 4), (-3, 7), which has a negative gradient.

Is what Zamira is saying always true? If not, explain when it is not true.

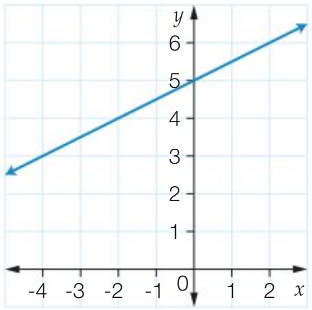
22 Liquid is being poured from a plastic container at a constant rate. The amount of liquid remaining is described by the rule $V = 6 - 2t$, where V is the volume of liquid in litres and t is the time in minutes.

At the same time, a glass container is being filled with liquid described by the rule $V = 2t$, where V is the volume of liquid in the glass container in litres and t is the time in minutes.

- **(a)** Use the rule $V = 6 - 2t$ to complete a table of values for values of t from 0 to 3.
- **(b)** What has happened at $t = 3$?
- **(c)** Use the rule $V = 2t$ to complete a table of values for values of t from 0 to 3.
- **(d)** Draw the two graphs on the same Cartesian plane.
- **(e)** Why would it not be meaningful to use values of t less than 0 or values of t greater than 3?
- **(f)** How much time did it take for both containers to have the same amount of liquid? What volume was this?

Open-ended

23 Find any three points that are on this line.



24 Rachelle completes a table for the rule $y = 2(x - 3)$. Her answers are the highlighted values.

x	3	$6\frac{1}{2}$	2
y	3	10	-2

(a) Which of Rachelle's answers are correct?

(b) Explain how she ignored the correct order of operations in each of the others.

(c) Write the correct values.

25 To answer a maths question, Julian completed a table of values and drew a linear graph that included the point (2, 1).

What might the question have been? Write three different possibilities.

Puzzle

Who's who?

Cass, Seth, Fin, Toby, Piper, Zara, Monti, Bindi, Kai and Talia are standing in a row, but not in that order. They are lined up in order from shortest to tallest.

Use the clues to name the ten people in order from shortest to tallest.

Clue 1 Counting from the left, Zara is in an even-numbered position.

Clue 2 Monti and Talia are in the middle.

Clue 3 The greatest difference in height is between Bindi and Piper.

Clue 4 Fin is standing directly next to Piper and Seth is standing next to Bindi.

Clue 5 Cass is shorter than Zara.

Clue 6 Toby is standing somewhere between Seth and Zara.

Clue 7 Monti is shorter than Talia.

Clue 8 Kai is directly next to Toby.

Clue 9 Bindi is shorter than Piper.

Clue 10 There is one person standing between Toby and Monti.

Half-time 6

6.2

1 Rain falling into a container fills it at the rate of 4 millilitres per minute.

(a) Copy and complete the following table.

x (minutes)	3	4	5	6	7	8
y (volume of water in container, mL)	22					

(b) Draw a Cartesian plane showing values of x from 0 to 8 and values of y from 0 to 45.

(c) Plot and join the points with a straight line and extend the line until it intersects with the y-axis.

(d) Using the graph, state how much water was in the container initially.

6.1

2 Melanie was 5 km from home. She waited at the bus stop for a time, then started to walk home at a constant rate. She stopped for a while to rest and then ran home at a constant rate. Draw a graph to show this information.

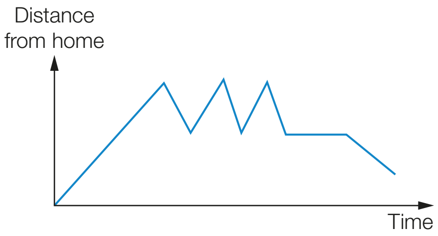
6.1

3 The graph shows Dale's early morning run starting from his house.

(a) The furthest point from home Dale reached was the oak tree in the park. How many times did he run to it?

(b) How many times did Dale rest?

(c) Did Dale return home? Explain your answer.



6.2

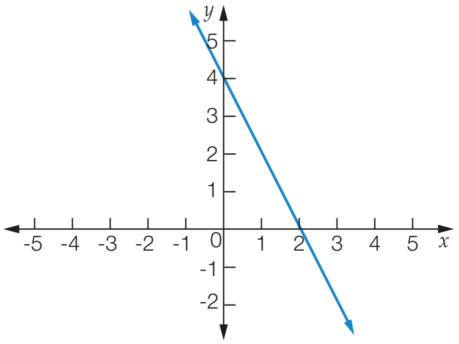
4 Which point lies on the straight line shown?

A (0, 4)

B (4, 0)

C (2, 2)

D (-2, 4)



6.1

5 The graph shows the height of two seedlings over a period of months. Both were planted at the same time and were measured at the end of each month.

(a) Which seedling was taller when planted?

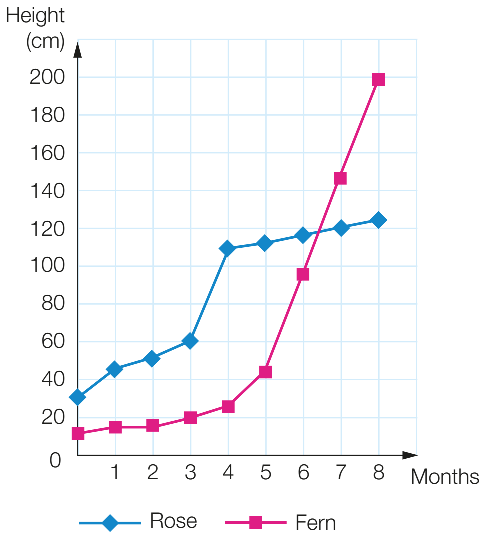
(b) When was the rose's rate of growth the fastest?

(c) During what time period was the rose taller than the fern?

(d) What was the difference between the two heights at 3 months?

(e) When did the fern reach a height of 40 cm?

(f) For how many months was the rose less than 60 cm high?

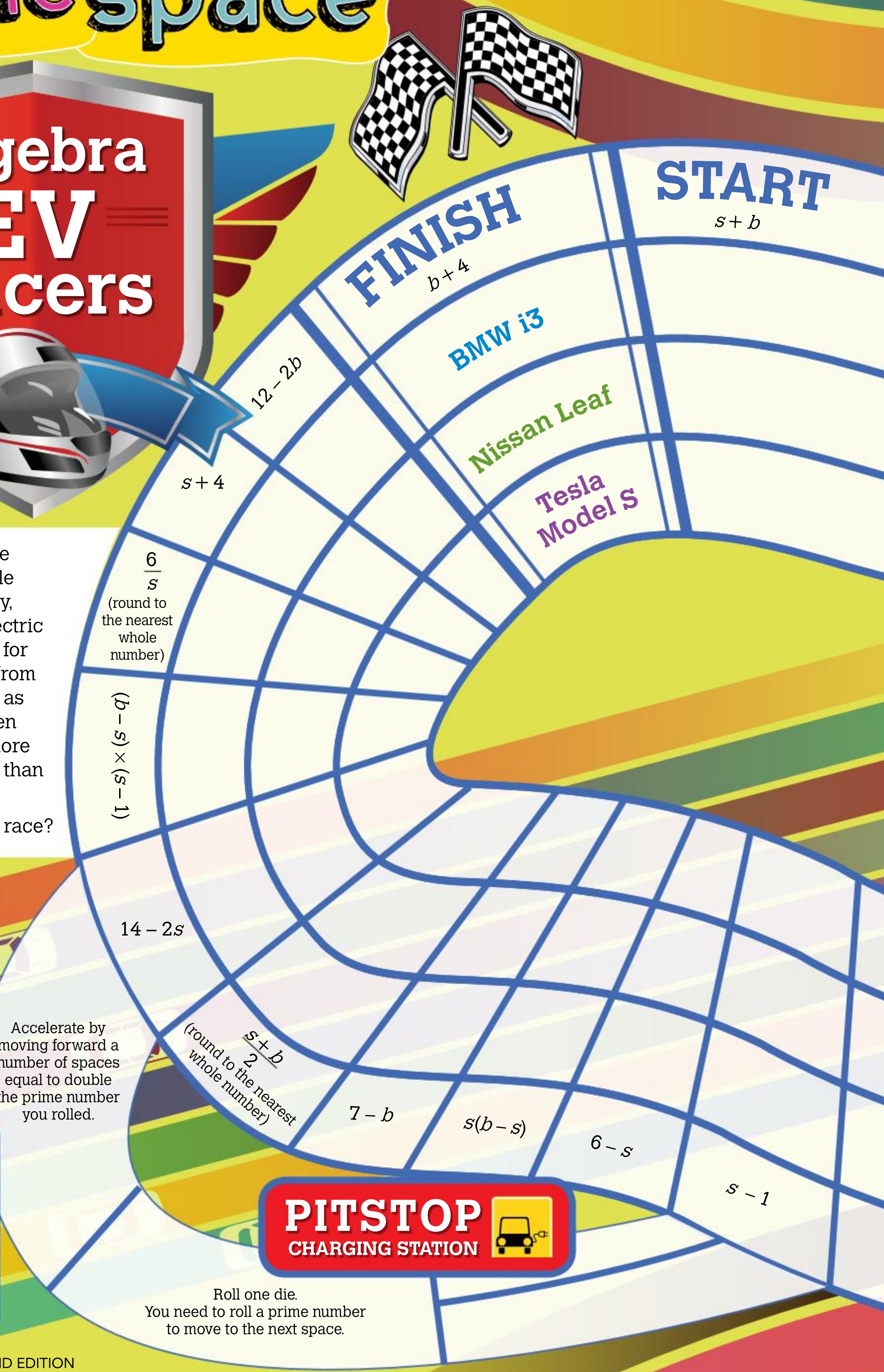


Gamespace

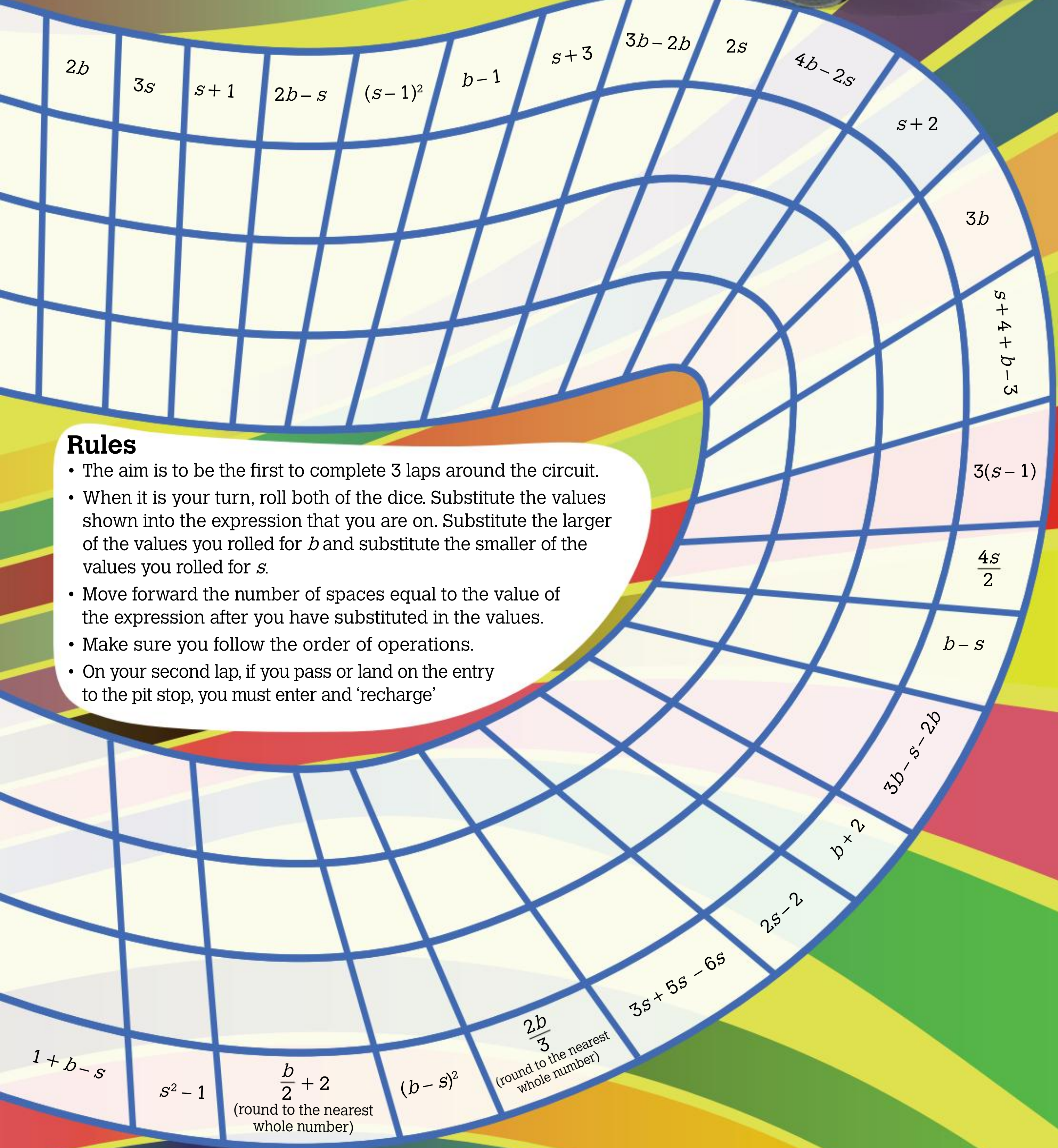
Algebra EV Racers

In Australia, more and more cars are becoming available that run on 100% electricity, sometimes called EVs ('Electric Vehicles'). If the electricity for these vehicles is sourced from renewable resources such as sunlight, wind or water, then these vehicles are much more efficient and less polluting than traditional petrol vehicles.

Which vehicle will win the race?



Equipment required:
2 dice, a counter for each player



Rules

- The aim is to be the first to complete 3 laps around the circuit.
- When it is your turn, roll both of the dice. Substitute the values shown into the expression that you are on. Substitute the larger of the values you rolled for *b* and substitute the smaller of the values you rolled for *s*.
- Move forward the number of spaces equal to the value of the expression after you have substituted in the values.
- Make sure you follow the order of operations.
- On your second lap, if you pass or land on the entry to the pit stop, you must enter and 'recharge'

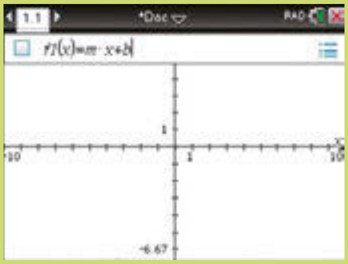
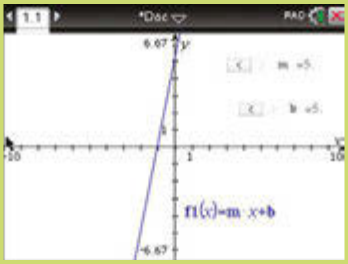
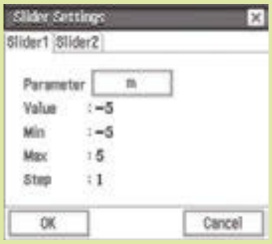
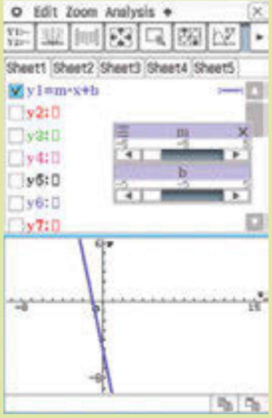
Exploration CAS

Equipment required: TI-Nspire CAS or Casio ClassPad CAS

Exploring gradients and intercepts with technology

You have seen how to plot a linear graph from a rule by making a table of values. A linear graph equation can be written in the form of $y = mx + b$, where m is the coefficient of the variable x and b is a constant. CAS technology can be used to explore the effect that m and b have on linear graphs.

Graphing a straight line

Using TI-Nspire CAS	**Using Casio ClassPad CAS**
Add **Graphs** to your document. To graph $y = mx + b$, enter **f1(*x*)=*m*×*x*+*b***, as shown.  	From the menu select **Graph & Table**. To graph $y = mx + b$, enter **y1 = *m* × *x* + *b*** as shown. Make sure to enter m and b using the **Var** tab from the **Keyboard**. 
Sliders should automatically appear so you can control the value of m and the value of b. If not, then you can select **menu** > **Actions** > **Insert Slider** to create a slider, and set the variable name to m. In the slider's **Settings…**, set the **Minimum** to -5, **Maximum** to 5 and **Step Size** to 1. Repeat this to create a slider for b, with the same settings.  	To change the values of m and b, now select ◆ **Dynamic Graph** to create sliders for m and b. You can also select ☰ > **Settings** to adjust the slider values for **Slider1** (for m) and **Slider2** (for b), as shown.  Parameter m Value :-5 Min :-5 Max :5 Step :1 
You can 'animate' your sliders to see how the graph changes as the values change. Select each slider and then select **Animate** to start and stop its values changing back and forth. See how the graph changes as the values change.	Adjust the slider values to see how the graph changes as the values change.

1 What happens when $m = 0$?

2 Now keep increasing the value of m to larger positive values. Describe what happens to the graph.

3 Write a sentence about the effect that the value of m has on the graph of the straight line $y = mx - 5$.

Now try varying the value of b. First, set the value $m = 1$, so that you will be graphing $y = x + b$. Increase the value of b from -5 and see what happens to the graph.

4 **(a)** What is the y-intercept when $b = -5$?

(b) What is the y-intercept when $b = 0$?

(c) What is the y-intercept when $b = +5$?

(d) What effect does b have on the graph of the straight line $y = x + b$?

Now see what happens with the following pairs of equations. Try to predict the answers before you check.

5 **(a)** Describe the differences between $y = 2x + 3$ and $y = -2x + 3$.

(b) Describe the differences between $y = 3x + 2$ and $y = 3x - 2$.

6 **(a)** Sketch the following graphs and check your answers by using your CAS. To draw a graph with a fractional gradient, you may need to change the step size in your settings for m.

(i) $y = x$ **(ii)** $y = -2x$

(iii) $y = \frac{1}{2}x$ **(iv)** $y = -\frac{1}{2}x$

(v) $y = -x + 3$ **(vi)** $y = 3x - 4$

(vii) $y = \frac{3}{2}x + 4$ **(viii)** $y = -\frac{3}{2}x - 3$

(b) Write a summary sentence about how you would sketch the graph of $y = mx + b$, assuming you were given the values of m and b.

7 What are the values of m and b for the equation $y = 5$? (Think about the value of m that would make mx 'disappear'.)

8 Sketch the following graphs.

(a) $y = -3$ **(b)** $y = 4$

(c) $y = 0$ **(d)** $y = \frac{2}{5}$

9 Sketch the following graphs.

(a) $x = -2$ **(b)** $x = 3$

(c) $x = 0$ **(d)** $x = -\frac{2}{5}$

10 Now use your CAS to graph $y = 2(x + 3)$ and $y = 2x + 6$.

(a) How many lines can you see displayed? Why is this?

(b) What is the gradient for $y = 2(x + 3)$?

(c) What is the y-intercept for $y = 2(x + 3)$?

(d) Check your conclusions by now graphing $y = 3(2x + 1)$ and $y = 6x + 3$ on the same axes.

Taking it further

'Families' of graphs are sets of graphs that all have something in common. For the following questions, you will create families of graphs using what you have learned. For these questions, do not use any equations that you have graphed previously.

11 Write three equations that will give parallel graphs. Graph these equations. Were they parallel? If not, try again.

12 Write three equations that will give graphs that all have the same y-intercept. Graph these equations. Did they all have the same y-intercept? If not, try again.

13 Write three equations that will give graphs that are all parallel to the x-axis. Graph these equations. Were they all parallel to the x-axis? If not, try again.

14 Write three equations that will give graphs that are all parallel to the y-axis. Graph these equations. Were they all parallel to the y-axis? If not, try again.

6.3

Finding the rule

You can identify linear relationships and find their rule using any of the following methods.

Using ordered pairs

If you have a list of ordered pairs you can look for a pattern.

(1, 8), (2, 15), (3, 22), (4, 29), (5, 36), (6, 43)

You can see that:

- the value of y increases by 7 as the x-value increases by 1, so $7x$ is a term in the rule
- 1 needs to be added to complete the rule.

Rule: $y = 7x + 1$

Using a pattern and a table of values

You should be familiar with patterns, tables of values and rules. For example:

Number of octagons (*n*)	1	2	3	4	5	6
Number of lines (*l*)	8	15	22	29	36	43

You can see that:

- each new octagon is formed by adding 7 lines to the octagon before it
- the number of lines (l) in the table increases by 7, so $7n$ is a term in the rule
- 1 needs to be added because 1 line was needed to begin the pattern.

Rule: $l = 7n + 1$ where l is the number of lines and n is the number of octagons.

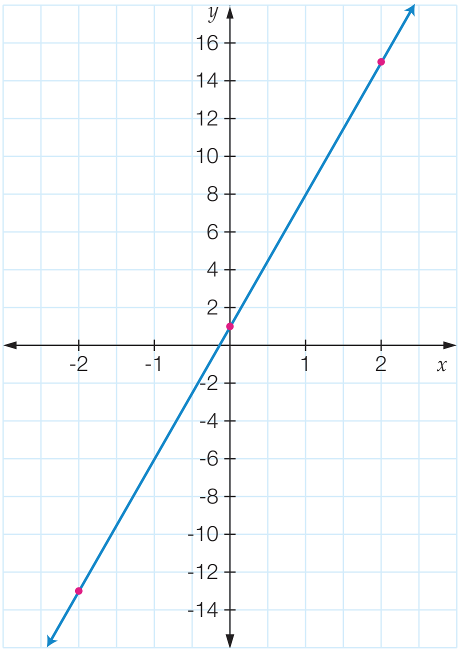
Using a graph

The ordered pairs (-2, -13), (0, 1) and (2, 15), plotted on a Cartesian plane, can be connected with a straight line. This shows a linear relationship.

You can see that:

- the y-value increases by 14 when the x-value increases by 2. That means that the y-value increases by 7 as the x-value increases by one. So, you need $7x$ in the rule.
- 1 needs to be added to find the y-value.

Rule: $y = 7x + 1$



Satisfying the rule

'Satisfying the rule' means that the ordered pairs make true statements when substituted into the equation of the rule. By substituting (-2, -13), (0, 1) and (2, 15) into $y = 7x + 1$, you can see that all points satisfy this rule:

$-13 = 7 \times -2 + 1$ $\quad 1 = 7 \times 0 + 1$ $\quad 15 = 7 \times 2 + 1$

Sometimes, when you are given a list of ordered pairs (or a table of values), only one operation has been done on x to give y, so finding the rule is not difficult. But you must always be careful to check that the rule applies to all points.

Worked example 5

W.E. 5

Find the rule for each of the following sets of points.

(a) (0, 3), (1, 4), (2, 5), (3, 6), (4, 7) **(b)** (-3, -6), (1, 2), (4, 8), (6, 12)

Thinking	Working
(a) 1 Look for a pattern. Can you do one operation on x to find y? Yes (+ 3).	(a) 3 has been added to each x-value to give y.
2 Write the rule.	$y = x + 3$
3 Check the rule by substituting one x-value from the list into the equation to find the given y-value.	Check: Using the point (4, 7) $y = x + 3$ $= 4 + 3$ where $x = 4$ $= 7$ The rule is correct.
(b) 1 Look for a pattern. Can you do one operation on x to find y? Yes (× 2).	(b) Each x-value has been multiplied by 2 to give y.
2 Write the rule.	$y = 2x$
3 Check the rule by substituting one x-value from the list into the equation to find the given y-value.	Check: Using the point (4, 8) $y = 2x$ $= 2 \times 4$ where $x = 4$ $= 8$ The rule is correct.

The rule must work for every ordered pair in the set.

When more than one operation has been done on x to give y, you may not see the relationship so easily. In these cases, you will need to look for another pattern.

Worked example 6

W.E. 6

Find the rule for the following table of values.

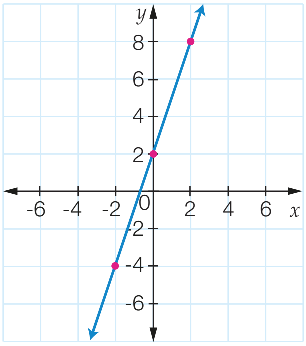
x	-3	1	5	9
y	-24	-4	16	36

Thinking	Working
1 Look for a pattern. Can you do one operation on x to find y? No. (Subtracting 21 or multiplying by 8 works for the first point, but not for the others.)	Cannot do one operation on x to give y.
2 Look for another pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This is the coefficient of x.	y increases by 20 as x increases by 4, so y increases by 5 as x increases by 1. Need $5x$ in the rule.
3 What number needs to be added or subtracted to find the y-value? This is the constant in the rule.	9 needs to be subtracted.
4 Write the rule.	$y = 5x - 9$
5 Check the rule by substituting one x-value from the table into the rule to find the corresponding y-value.	Check: use the values $x = 5$, $y = 16$ $y = 5x - 9$ $= 5 \times 5 - 9$ $= 25 - 9$ $= 16$ The rule is correct.

Worked example 7

W.E. 7

Find the rule that describes the following relationship.



Thinking	Working
1 List at least three points that lie on the line.	(-2, -4), (0, 2), (2, 8)

2 Look for a pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This number is the coefficient of x.	Cannot do one operation on x to give y. y increases by 6 as x increases by 2, so y increases by 3 as x increases by 1. Need 3x in the rule.
3 After the x-value is multiplied by the coefficient (3) found in the previous step, what number needs to be added or subtracted to find the corresponding y-value?	2 needs to be added.
4 Write the rule.	$y = 3x + 2$
5 Check the rule by substituting one x-value from the list into the equation to find the given y-value.	Check: Using the point (2, 8) $y = 3x + 2$ $= 3 \times 2 + 2$ where $x = 2$ $= 6 + 2$ $= 8$ The rule is correct.

6.3 Finding the rule

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4, 5, 6, 7, 9 (column 1), 11, 13, 19	1 (column 2), 2 (column 2), 3 (column 2), 4, 5, 6, 7, 9 (column 2), 11, 12, 13, 15, 17, 18, 19	1 (column 2), 2 (column 2), 3 (column 2), 4, 5, 6, 8, 9 (column 2), 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 651

Fluency

1 Find the rule for each of the following sets of points.

W.E. 5

(a) (0, -1), (1, 0), (2, 1), (3, 2), (4, 3)

(b) (2, 3), (3, 4), (4, 5), (6, 7), (7, 8)

(c) (-3, 3), (-1, 5), (2, 8), (3, 9), (5, 11)

(d) (-2, -5), (0, -3), (2, -1), (5, 2), (6, 3)

(e) (1, 1), (3, 3), (4, 4), (7, 7), (10, 10)

(f) (-1, -4), (0, 0), (1, 4), (2, 8), (3, 12)

(g) (-2, -10), (0, 0), (2, 10), (4, 20), (6, 30)

(h) (-2, 2), (-1, 1), (0, 0), (1, -1), (2,-2)

2 Find the rule for the following tables of values.

W.E. 6

(a)

x	-7	-5	1	4
y	-6	-4	2	5

(b)

x	-2	3	6	7
y	-5	0	3	4

(c)

x	-4	0	5	7
y	8	0	-10	-14

(d)

x	-2	-1	5	6
y	-14	-7	35	42

(e)

x	-6	-3	0	6
y	2	1	0	-2

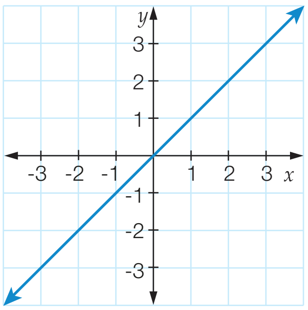
(f)

x	-5	0	10	20
y	-1	0	2	4

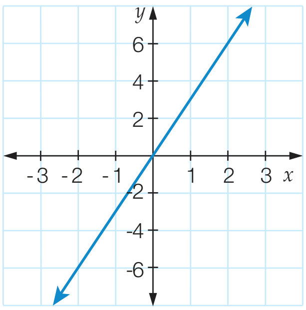
W.E. 7

3 Find the rule that describes each of the following relationships.

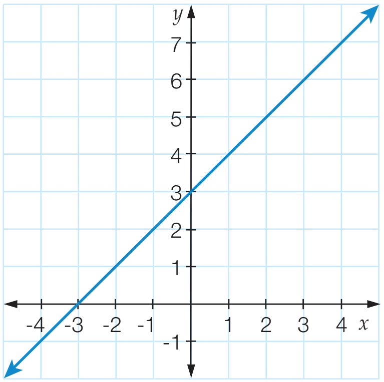
(a)



(b)



(c)



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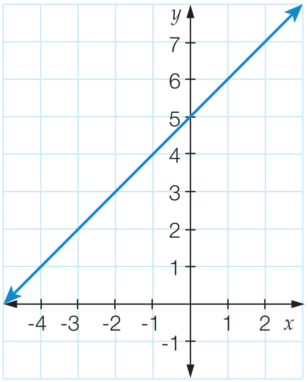

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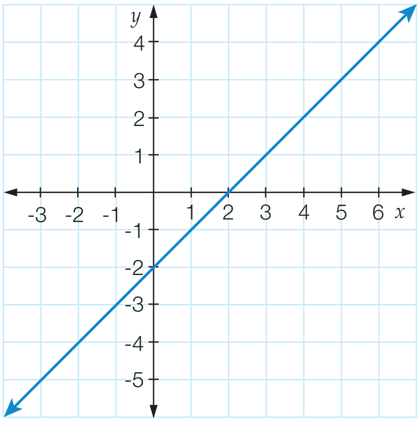
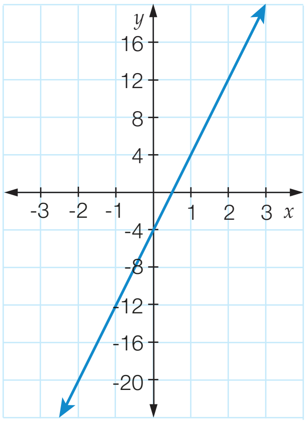
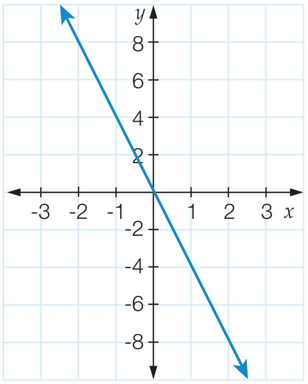


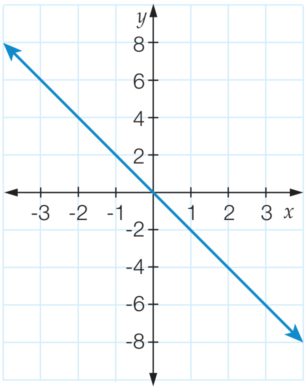
(f)



(g)



(h)



The arrows on the graph line show that the relationship goes on forever.

4 Which equation describes a horizontal line?

A $x + 3 = 2$

B $x + y = 3$

C $y - x = 0$

D $y + 3 = 8$

5 Graph the two lines $x = -3$ and $y = 6$ on the same Cartesian plane. Use this to find the coordinates of the point where the lines intersect.

6 **(a)** (1, 5), (2, 6), (3, 7), (4, 8) are coordinate pairs that lie on the line with rule:

A $y = 5x$ **B** $y = x - 4$ **C** $y = 4x$ **D** $y = x + 4$

(b) (0, 0), (1, 6), (2, 12), (3, 18) are coordinate pairs that lie on the line with the equation:

A $y = x + 5$ **B** $y = x + 6$ **C** $y = 6x$ **D** $y = 5x$

(c) (1, -4), (2, -3), (3, -2), (4, -1), (5, 0) are coordinate pairs that lie on the line with the rule:

A $y = x - 5$ **B** $y = 2x$ **C** $y = x + 6$ **D** $y = 3x$

7 The rule $y = x - 2$ is true for which ordered pairs?

A (5, 7) and (0, 2) **B** (2, -2) and (3, 1)

C (-4, 2) and (1, 3) **D** (-3, -5) and (4, 2)

8 Find the equation of the line that is parallel to the x-axis and includes the point (-2, -3).

9 Describe each of the following relationships with a rule. Hence, copy and complete each of the following tables.

(a)

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1			

(b)

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2			

(c)

x	-3	-2	-1	0	1	2	3
y				-5	-4	-3	-2

(d)

x	-3	-2	-1	0	1	2	3
y				-3	-2	-1	0

(e)

x	-3	-2	-1	0	1	2	3
y	-6		-2		2		6

(f)

x	-3	-2	-1	0	1	2	3
y		-6		0	3		9

(g)

x	-3	-2	-1	0	1	2	3
y	24		8	0		-16	

(h)

x	-3	-2	-1	0	1	2	3
y		12		0	-6		-18

Understanding

10 Match each of the worded descriptions **(a)**, **(b)**, **(c)** and **(d)** with a rule **A**, **B**, **C** or **D**.

A $y = x + 4$ **B** $y = 1 - x$ **C** $y = \frac{x}{4} + 1$ **D** $y = 1 - 4x$

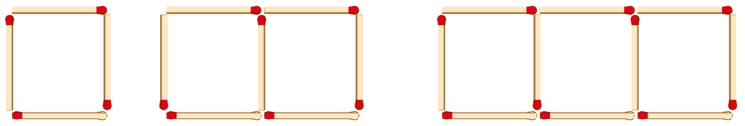
(a) The sum of the x- and y-coordinates is 1.

(b) The y-coordinate is four more than the x-coordinate.

(c) The y-coordinate is the sum of 1 added to the x-coordinate divided by four.

(d) The y-coordinate, added to four times the x-coordinate less 1, is equal to zero.

11 Consider the pattern for the number of matchsticks in each shape.



(a) Complete the table to find the relationship between the number of matchsticks and the number of squares.

Number of squares (s)	1	2	3	4
Number of matchsticks (m)	4			

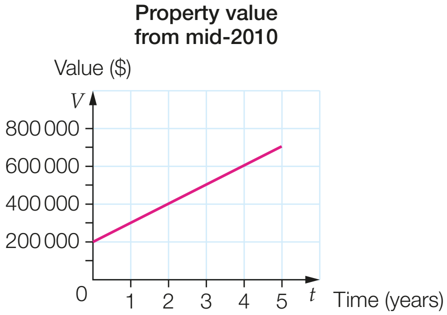
(b) How many matchsticks are needed to make 10 squares in the pattern?

Reasoning

12 The graph shows the value of a property over five years since the middle of 2010.

(a) Find the rule for the line.

(b) If the trend continues, what will the property be worth in the middle of 2022?



13 An online company sells large bottles of vitamins for \$45 and charges \$10 for delivery of any number of bottles.

(a) Find a rule to connect the total cost (C dollars) with the number of bottles delivered (n), by filling in the table of values as shown.

n	1	2	3	4
C **(\$)**	55			

(b) How many bottles will be delivered for \$325?

14 A car rental company charges a fixed fee plus a daily rate to rent a car. The cost of renting a car for two days is \$80. The cost for five days is \$140.

(a) Plot a graph of cost (in \$) against the number of days, using the above information.

(b) Find a rule connecting the cost C with the number of days d.

(c) If the cost is \$160, then for how many days was the car rented?

15 **(a)** Write a rule for the number of dots (D) in terms of the number of squares (S) in the pattern below.

(b) Explain how you found the coefficient of S in your rule.

(c) Explain how you found the amount to add on in your rule.

(d) Use your rule to find the number of dots needed to draw 8 squares with this pattern.

16 Some contractors building a fence charge a set amount for materials and an hourly rate for labour. If it takes 6 hours to build the fence, the cost will be \$880. If it takes 8 hours to build the same fence with the same materials, the cost will be \$1040.

(a) Plot the two known points and use your graph to find the rule for the total cost in dollars (C) in terms of time in hours (t).

(b) What is the hourly rate for labour?

(c) What is the cost of materials?

17 Oil is leaking from a car engine at the rate of 250 millilitres per minute. Before the leak started, the engine's oil volume was 2225 millilitres.

(a) Copy and complete the table.

x (minutes)	0	1	2	3	4	5
y (volume of oil in engine, mL)						

(b) Plot the points on the Cartesian plane and connect them with a straight line.

(c) Find the rule for y, the amount of oil remaining, after x minutes.

(d) How much time does it take for the car's engine to lose all its oil?

Open-ended

18 Write the rules of two different straight line graphs that include the point (5, 2).

19 Write three rules whose graphs show decreasing values for y where x-values increase.

Puzzle

The Liar-bird and the Tru-tru-bird

Jerry inherits his friend's three magical talking birds. He knows that two of the birds are Tru-tru-birds, which always tell the truth, and one of the birds is a Liar-bird, which always lies.

But Jerry doesn't know much about birds, so all three birds look the same to him. He needs to find out which bird is which, so he asks them.

The first bird answers but Jerry doesn't hear.

The second bird says frustratedly, 'That bird is a Liar-bird!'

The third bird says impatiently, 'Don't believe it, the second bird is lying!'

Which bird is the Liar-bird?

Exploration CAS

Equipment required: TI-Nspire CAS or Casio ClassPad CAS

Be still my beating heart

Everyone needs to pay attention to their health. One important and useful measure of health is maximum heart rate (HR_{max}). This measures heart activity after exercise, as the heart's rate of beats per minute. However, there are no set rules as to what this maximum heart rate should be. Even elite athletes in the same sport may have different healthy values of HR_{max}, varying from around 160 to 220 beats per minute.

To measure a person's HR_{max} accurately, you need expensive equipment and many tests. However, the value of HR_{max} can be estimated using a formula.

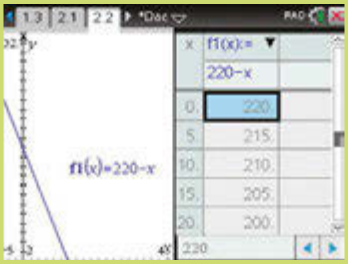
One possible formula is: $HR_{max} = 220 - \text{age}$

(Here 'age' is the person's age in years.)

You can use your CAS to explore this and other formulas, using tables of values.

Using TI-Nspire CAS

Add **Graphs** to your document. Enter the equation to be graphed as ***f*1(*x*) = 220 – *x***. Then select **menu** > **Table** > **Split-screen Table** to show the table of values for this rule. To change how the table is displayed, you can then select **menu** > **Table** > **Edit Table Settings…** and set **Table Start** to 0 and **Table Step** to 5.



x	f1(x):=
	220−x
0.	220
5.	215.
10.	210.
15.	205.
20.	200.

220

Using Casio ClassPad CAS

From the menu select **Graph & Table**. Enter the equation to be graphed as **y1=220–x** and tick the box next to it. Then select ▦ to show the table of values for this rule. To change how the table is displayed, you can select ⊠ and set **Start** to 0, **End** to 100 and **Step** to 5.

☑ y1=220−x
☐ y2:□
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

x	y1
0	220
5	215
10	210
15	205
20	200
25	195

0

This first formula is not very precise. Many medical and fitness professionals prefer the following formula: $HR_{max} = 205.8 - (0.685 \times \text{age})$.

There are several other formulas that can also be used. For example:

$HR_{max} = 206.3 - (0.711 \times \text{age})$
$HR_{max} = 217 - (0.85 \times \text{age})$
$HR_{max} = 208 - (0.7 \times \text{age})$

1 Use CAS technology to help complete the following table. Enter each new formula as a new equation.

Age in years	$HR_{max} = 220 - \text{age}$	$HR_{max} = 205.8 - (0.685 \times \text{age})$	$HR_{max} = 206.3 - (0.711 \times \text{age})$	$HR_{max} = 217 - (0.85 \times \text{age})$	$HR_{max} = 208 - (0.7 \times \text{age})$
10					
15					
20					
25					
30					
35					
40					
45					
50					
55					
60					
65					
70					
75					
80					

2 Do you think there are any problems or limitations with any of these formulas? Explain what they are.

When you are beginning a fitness program, you should also be interested in your target heart rate (THR). This is the heart rate that you want to reach during your exercise session. An effective way to calculate your THR is the Karnoven method, which is as follows:

Step 1 Find your resting heart rate (RHR): Count the number of heartbeats in a minute each morning just as you wake up. Do this for three mornings, then find the average of the three figures. This is your RHR.

Step 2 Find your HR_{max}. (You could use one of the formulas given earlier.)

Step 3 Calculate $HR_{max} - RHR$ to find your 'heart rate reserve'.

Step 4 Calculate the lower limit for your THR, which is 60% of your heart rate reserve.

Step 5 Calculate the upper limit for your THR, which is 80% of your heart rate reserve.

Step 6 Find the mean of the lower limit and upper limit values. This is your THR.

3 Can you think of a way to combine steps 3 and 4 into one calculation?

4 Imagine that you are 24 years of age and have a RHR of 80 beats per minute. Find your THR using each of the five methods for calculating the HR_{max}. How different are these values?

Taking it further

Use CAS technology to draw the graphs of these formulas. Write a few sentences to describe what these graphs show. In particular, pay attention to the heart rate values that these formulas predict for very young and very old people.

6.4 Using linear relationships

Recognising linear relationships

There are many situations where two variables are connected in a linear relationship. Situations that involve a constant rate commonly involve linear relationships: for example, a constant flow rate of a liquid, the constant speed of a vehicle, or a daily rate of pay for work done.

Defining a variable

In mathematical word problems that contain variables, you need to choose a suitable pronumeral to represent each variable. In measurement, usually h is used to represent height, V is used to represent volume, and t is used to represent time.

Worked example 8

W.E. 8

For each statement, define each of the two variables involved.

(a) The volume of water in a water tank increases during a day of heavy rain.

(b) The average speed of a car as it travels over a distance.

Thinking	Working
(a) 1 Identify the two variables.	(a) volume and time
2 Define each variable.	Let V be the volume of water in the tank. Let t be the time during which the tank is filling.
(b) 1 Identify the two variables.	(b) distance and speed
2 Define each variable.	Let d be the distance travelled. Let s be the speed the car travelled.

Worked example 9

W.E. 9

Define all the variables you are using in the following situation and then write a linear equation to represent the total cost.

The cost of a building project at a school consists of a \$1000 fixed cost to cover the materials used and a variable cost of the labour, which is calculated by a fixed rate of \$30 per hour.

Thinking	Working
1 Identify and define the variables.	Let C be the cost of the project and h be the number of hours of labour.
2 Identify the fixed costs.	Cost of materials = \$1000. Hourly rate of labour = \$30.
3 Write the equation.	$C = 30h + 1000$

Using linear graphs

A straight line can be drawn through any two points. Because of this, if you know that a relationship between two variables is linear and you are given two pieces of information that connect the two variables, then you can write the two pieces of information as two ordered pairs. You can then plot those ordered pairs as two points on a Cartesian plane, draw a linear graph using the two points, and use this to find the relationship between the variables for any other point on the line.

You only need to know the coordinates of two points to draw a straight line, but you should use at least three points to make sure you haven't made a mistake.

Worked example 10

W.E. 10

A tap is dripping at a constant rate. The number of litres of water wasted is shown in the table below.

Number of hours the tap is dripping (*d*)	0	2	6	11
Number of litres of water wasted (*w*)	0	10	30	55

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of hours tap is dripping, d' on the x-axis and 'Number of litres of water wasted, w' on the y-axis.

(b) Is this relationship linear?

(c) From your graph, decide how much water would be wasted in:

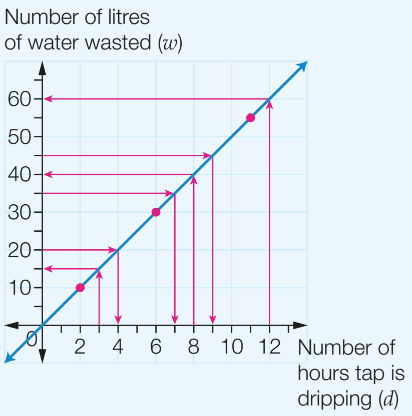
(i) 3 hours **(ii)** 8 hours **(iii)** 12 hours.

(d) From your graph, decide for how much time the tap had been dripping if the number of litres wasted was:

(i) 20 litres **(ii)** 35 litres **(iii)** 45 litres.

(e) Is it sensible to join the points in this situation?

(f) Write the rule that shows the relationship between the number of hours the tap is dripping, d, and the number of litres of water wasted, w.

Thinking	Working
(a) Draw and label your x- and y-axes and plot the points. Join them in order with straight lines. Make sure your scale will allow you to read the values required.	(a) 
(b) Decide whether all points lie on the same straight line. If so, the graph is linear.	(b) Yes, this is a linear graph.
(c) Read the y-coordinate of the point on the line that has the given x-coordinate. This is your answer.	(c) (i) 15 litres (ii) 40 litres (iii) 60 litres

(d) Read the x-coordinate of the point on the line that has the given y-coordinate. This is your answer.	(d) (i) 4 hours (ii) 7 hours (iii) 9 hours
(e) Do you measure or count the variables? Variables that can be measured are continuous, because in-between values exist, so a linear graph is sensible.	(e) Because both time and volume are continuous variables, it is sensible to use a linear graph.
(f) Look for a pattern between the number of hours the tap is left dripping (d) and the number of litres of water wasted (w).	(f) $w = 5d$

Worked example 11

W.E. 11

A Year 8 class chose to support a cancer research charity by organising a school concert as a fundraiser. The following table shows the total money raised as the number of tickets sold increased. Their teachers also donated to the amount of money raised, to start it off.

Number of tickets sold (t)	50	70	160	180
Money raised (\$$M$)	2000	2200	3100	3300

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of tickets sold (t)' on the x-axis and 'Money raised (\$$M$)' on the y-axis.

(b) Is the relationship linear?

(c) From your graph, decide how much money would be raised if the following number of tickets was sold.

(i) 90 **(ii)** 170 **(iii)** 230

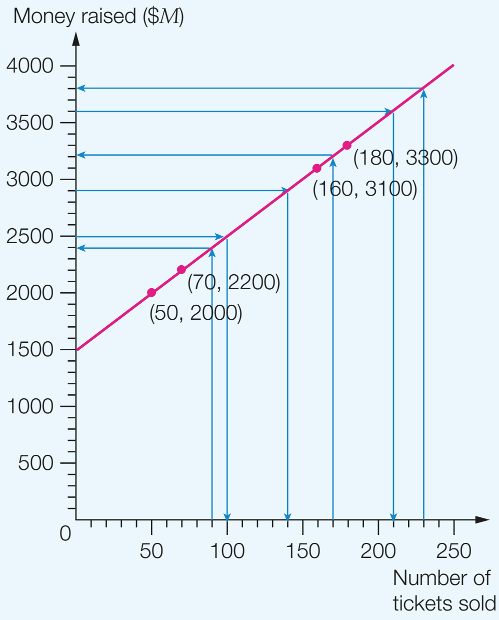
(d) From your graph, decide how many tickets would need to be sold to raise the following revenue.

(i) \$2500 **(ii)** \$2900 **(iii)** \$3600

(e) Is it sensible to join the points in this situation? If not, why use a linear graph?

(f) Write the rule that shows the relationship between the number of tickets sold (t) and the money raised (\$$M$).

(g) How much money did their teachers donate?

Thinking	Working
(a) Draw and label your x- and y-axes and plot the points. Join them in order with straight lines. Make sure your scale will allow you to read the values required.	(a) 
(b) Decide whether all points lie on the same straight line. If so, the graph is linear.	(b) Yes
(c) Read the point on the line that has an x-coordinate equal to the given value. The y-coordinate of the point is your answer.	(c) Amount of money raised (i) \$2400 (ii) \$3200 (iii) \$3800
(d) Read the point on the line that has a y-coordinate equal to the given value. The x-coordinate of the point is your answer.	(d) Number of tickets sold (i) 100 (ii) 140 (iii) 210
(e) Do you measure or count the variables? Variables that are counted are not continuous and should be shown on a point graph, but you can often model non-continuous variables with a continuous graph for convenience.	(e) These values are not continuous, but you can use a linear graph for convenience as long as you realise that in-between values are meaningless.
(f) Look for a pattern between the number of tickets sold and the money raised. By how much is t increasing? By how much is M increasing? Use this information to find the amount by which M is increasing as t increases by 1. Check that this rule works for all points.	(f) As ticket numbers increase by 20, money raised increases by \$200, so if ticket numbers increase by 1, the money raised would increase by 10. Need $10t$ in the rule. \$1500 needs to be added on. $M = 10t + 1500$
(g) The constant in the rule tells you the amount in the fund to start with. This is the teachers' donation.	(g) \$1500

6.4 Using linear relationships

Navigator

Answers p. 653

1, 2, 3, 4, 5, 7 (a–c), 8, 10, 16	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 16	1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15

Fluency

Equipment required: graph paper or grid paper

W.E. 8

1 For each statement, define each of the two variables involved.

(a) The length of a section of tram track increases as the temperature increases.

(b) The volume of water in a swimming pool decreases when the pool develops a leak.

(c) Tyre pressure increases as the volume of air increases.

(d) The radius of a balloon increases as the volume increases.

(e) A student runs a distance during a race.

W.E. 9

2 Define all the variables you are using in each of the following situations and then write a linear equation to represent the total cost of each.

(a) A solicitor charges a client a fixed fee of $50 for an initial consultation. She then charges $100 per hour for the documents she produces for her client.

(b) A taxi service charges a flagfall fee of $3.20, plus a fixed amount of $1.60 for each kilometre travelled.

(c) A call on a mobile phone costs a flagfall of 37c, plus an additional charge of 40c per minute.

3 The cost of a company's internet connection is given by the relation $C = 40 + 3d$, where d is the number of days of internet connection.

(a) Complete a table of values for d from 1 to 7.

(b) Is there a linear relation between the cost and the number of days?

(c) What would be the cost for eight days of internet connection?

4 The following table shows the perimeters P of squares with different side lengths l.

l	1	2	3	4	5
P	4	8	12	16	20

(a) Find the relationship between the square side length l and the perimeter P.

(b) Plot these points on a graph.

(c) Is this a linear relationship? Justify your answer.

(d) From the graph, what would be the perimeter of a square with side length of 1.5 m?

(e) If the perimeter of a square is 10 m, what is its side length? Use the graph to answer.

5 For a particular brand of cordial, the manufacturer recommends that the cordial syrup be diluted by adding a certain amount of water. The following table shows the corresponding amounts of cordial syrup and water needed.

W.E. 10

Amount of cordial syrup (*s*, in mL)	10	25	40	50
Amount of water needed (*w*, in mL)	30	75	120	150

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Amount of cordial syrup' on the x-axis and 'Amount of water needed' on the y-axis.

(b) Is the relationship linear?

(c) From your graph, decide how much water should be added to the following quantities of cordial syrup.

(i) 15 mL **(ii)** 28 mL **(iii)** 55 mL

(d) From your graph, decide how much cordial syrup should be added to the following quantities of water.

(i) 24 mL **(ii)** 90 mL **(iii)** 135 mL

(e) Is it sensible to join the points in this situation?

(f) Write the rule that shows the relationship between the amount of cordial syrup (s) and the amount of water (w).

6 Yuni wanted to buy a number of golf clubs (g) all for the same price from an online golf store. The store adds a fixed postage and delivery cost to the cost of the clubs to find the total price (p) Yuni would be charged. The following table shows the amount Yuni could have paid for her clubs.

W.E. 11

Number of golf clubs (*g*)	2	5	7	9
Total price in dollars (*p*)	140	320	440	560

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of golf clubs (g)' on the x-axis and 'Total price in dollars (p)' on the y-axis.

(b) Is this relationship linear?

(c) From your graph, decide how much Yuni would pay for:

(i) four clubs **(ii)** six clubs **(iii)** ten clubs.

(d) From your graph, decide how many golf clubs Yuni could buy for:

(i) \$200 **(ii)** \$500 **(iii)** \$740.

(e) Is it sensible to join the points in this situation? If not, why use a linear graph?

(f) Write the rule that shows the relationship between the number of golf clubs Yuni could buy (g) and the total price she would pay (p).

(g) How much has Yuni paid for the postage and delivery of her clubs?

Understanding

7 Ribbon used to wrap Christmas gifts costs 55c per metre.

(a) Copy and complete the following table.

Length of ribbon (m)	0	1	2	3	4	5	6	7	8
Cost ($)	0	0.55	1.10						

(b) Plot a graph of this relationship. Show the length of ribbon on the horizontal axis.

(c) How much would it cost to buy 4.2 m of ribbon?

(d) What length of ribbon could be bought for $3.00?

(e) What length of ribbon could be bought for $5.00?

8 Brent is driving his car at 70 km/h.

(a) Copy and complete the following table.

Time (h)	0	1	2	3	4	5	6
Distance travelled (km)	0	70					

(b) Plot a graph of distance travelled against time. Show time on the horizontal axis.

(c) From the graph, how far does Brent travel in:

(i) 3.5 hours **(ii)** 30 minutes **(iii)** 5 hours 15 minutes?

(d) From the graph, how much time does it take Brent to travel:

(i) 175 km **(ii)** 298 km **(iii)** 400 km?

9 The table below shows the cost of buying different lengths of metal pipe.

Length of pipe (m)	2	5	8	10	15
Cost ($)	4.8	12	19.2	24	36

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes.

(b) Does the line pass through the point (0, 0)? Explain why you would expect it to do so.

(c) How much would it cost to buy a piece of pipe 12 m in length?

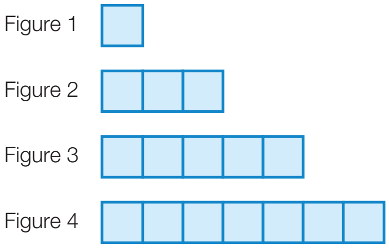
(d) How much would it cost to buy a 4 m length of pipe?

(e) Is it more economical to buy three 4 m lengths or one 12 m length?

(f) What length of pipe, to the nearest metre, can be bought for $50?

(g) What length of pipe, to the nearest metre, can be bought for $100?

10 Consider the following pattern of squares.



(a) Find a rule connecting the number of squares S in each figure with the figure number n.

(b) Complete a table of values for $n = 1$ to $n = 5$.

(c) Draw a graph of these values with n on the horizontal axis and S on the vertical axis. Is this relation linear? Justify your answer.

(d) From the graph, how many squares are in the figure with $n = 5$?

11 To convert length measurements in inches to centimetres, you can draw a graph using two points.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Length in inches' and use a scale from 0 to 15. Label the vertical axis 'Length in centimetres' and use a scale from 0 to 40.

(b) What length in centimetres is equivalent to zero inches? Write this information as an ordered pair of coordinates.

(c) 12 inches is equivalent to about 30.5 centimetres. Write this information as an ordered pair of coordinates.

(d) Plot the two points you found and carefully rule a straight line through them.

Use 1 cm on graph paper to represent 1 inch on the horizontal axis. Use 1 cm on graph paper to represent a length of 2 cm on the vertical axis.

Use your conversion graph to answer the following questions.

(e) The world's largest known spider is the Goliath birdeater tarantula. One specimen found in Venezuela in 1965 had a leg span of about 11 inches. What is this in centimetres?

(f) The smallest bird in the world is the bee hummingbird. If one of these birds has a length of 5.7 cm, how many inches long is it?

(g) The greatest reported diameter for a bubblegum bubble (blown without using hands) is 20 inches. It was blown by Chad Fell of Alabama, USA, in 2004. Convert this diameter to centimetres.

(h) The common flea (*Pulex irritans*) is a champion jumper. In 1910, scientists measured a flea's jump. They found that one made a long jump of 33 cm and a high jump of 20 cm (130 times its own height). Convert these measurements to inches.

(i) The shortest venomous snake is the Namaqua dwarf adder, which has a length of 8 inches. What is the length of this snake in cm?

(j) The world's largest cockroach measures about 3.8 inches in length and 1.7 inches across. Convert these measurements to cm.

12 Julie has savings of $S = \$650$. Every week she spends $30 from her savings to pay tutoring fees. If the relationship between her savings and number of weeks is: $S = 650 - 30w$.

(a) Construct a table of values for w from 1 to 8.

(b) How much savings will there be after 6 weeks?

Reasoning

13 Most countries in the world, including Australia, measure temperature using degrees Celsius (°C). A few other countries, including the USA, use degrees Fahrenheit (°F). A conversion graph can be used to convert temperatures from one unit to the other.

Use 1 cm on graph paper to represent 10 degrees Celsius on the horizontal axis. Use 1 cm on graph paper to represent 20 degrees Fahrenheit on the vertical axis.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Temperature in °C' and use a scale from -60 to 100. Label the vertical axis 'Temperature in °F' and use a scale from -80 to 220.

(b) The freezing point of water is 0 °C or 32 °F. Write this information as an ordered pair of coordinates.

(c) The boiling point of water is 100 °C or 212 °F. Write this information as an ordered pair of coordinates.

(d) Plot the two points you found and carefully rule a straight line through them.

Use your temperature conversion graph to answer the following questions.

(e) The highest shade temperature recorded in the world was 134 °F at Death Valley, USA, in 1913. What temperature is that in °C?

(f) The place where the coldest temperature has been recorded is the Russian Vostok Station, Antarctica, which recorded a temperature of -89 °C. Convert this temperature into °F.

(g) Which is hotter: a temperature of 30 °C or 80 °F?

(h) Human body temperature is around 98 °F. What is this in °C?

(i) An American tourist visiting Sydney reads that the maximum temperature forecast for the day is 35 °C. What is this in °F?

(j) If you were visiting New York in the USA and a weather forecaster predicted a maximum daily temperature of 50 °F, describe what you might wear for this weather.

14 The cost of renting a car from two different rental car companies is given by the following relations, where d is the number of days the car is rented.

For company A: $C_A = 4d$

For company B: $C_B = 2 + 3d$

(a) Construct a table of values for both companies, for a number of days from 1 to 5.

(b) Draw a graph for both companies' rental costs on the same coordinate plane. From which company is it cheaper to rent a car for 4 days?

(c) For what number of days do the companies cost the same?

Open-ended

15 For the linear equation $y = ax + b$:

(a) choose two different sets of values for a and b

(b) draw both graphs on the same Cartesian plane

(c) state the coordinates of the point of intersection of the two straight lines

(d) verify that this point lies on both lines by substituting the coordinates into both equations.

16 To convert centimetres to millimetres you can use the equation $m = 10c$, where m is millimetres and c is centimetres.

(a) Write two more linear equations that can be used to convert one unit of length to another unit of length.

(b) Draw each equation on a Cartesian plane, with the larger of the two units of length on the horizontal axis.

Puzzle

What number am I?

I am a three-digit number.

The sum of my digits is eighteen.

Five letters are used to spell my second digit.

The difference between my first two digits is one.

I have fewer even digits than odd digits.

My second digit is greater than my first digit.

I am a multiple of seventy-five.

If I add twenty-three, my third digit is eight.

Investigation

Equipment required: graph paper or grid paper, calculator

Captivating conversions

There are many units of measurement that are only used for specific purposes or situations. Consider the following:

- the object was travelling at a rate of 40 knots
- the whale swam 1300 nautical miles
- 5 inches of rain fell in 6 hours
- 10 acres of land for sale
- the plane flew at a height of 20 000 feet
- crude oil costs $91 per barrel
- a 3000-carat diamond
- this school bag weighs a ton
- these tyres need 24 psi of air pressure.

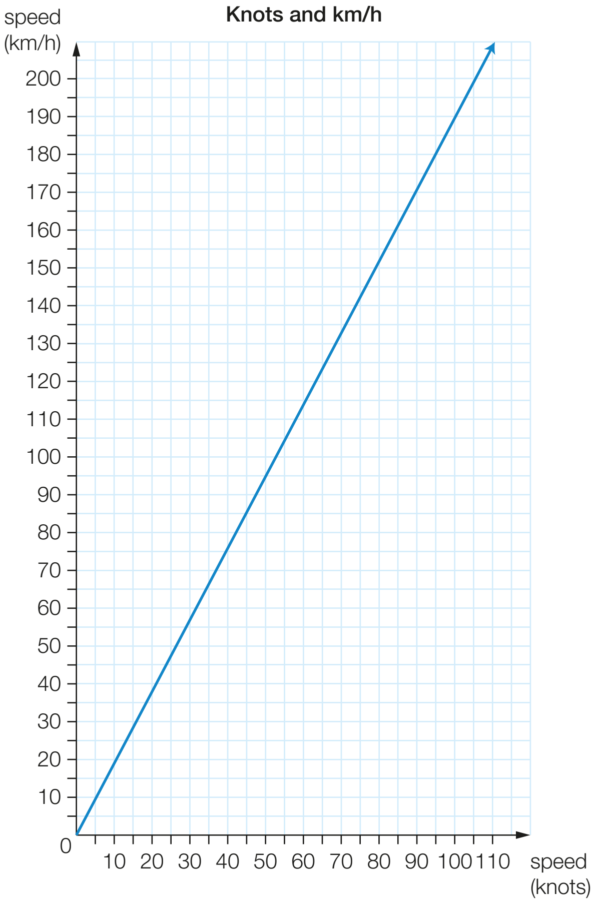
Converting measurements like these to more familiar metric measurement units can help you to understand what they mean and get an idea of their size.

The Big Question

Can conversion graphs help you to understand the meaning of different units of measurement by finding their metric equivalents?

Engage

1 Sam loves sailing and has sailed around the world in his yacht. He is often asked how fast his yacht can travel. He boasts that it can move at a speed of 40 knots. He recently took part in the Dynsey to Boarth yacht race, which is 672 nautical miles long. He finished the race in 56 hours.



(a) Use the conversion graph above to find how fast 40 knots is as a speed in km/h (to the nearest km/h).

(b) A speed of 1 knot is the same as a speed of 1 nautical mile per hour. Using the formula speed = $\frac{\text{distance}}{\text{time}}$, calculate Sam's race speed in knots.

(c) Use the same graph to find Sam's race speed in km/h (to the nearest km/h).

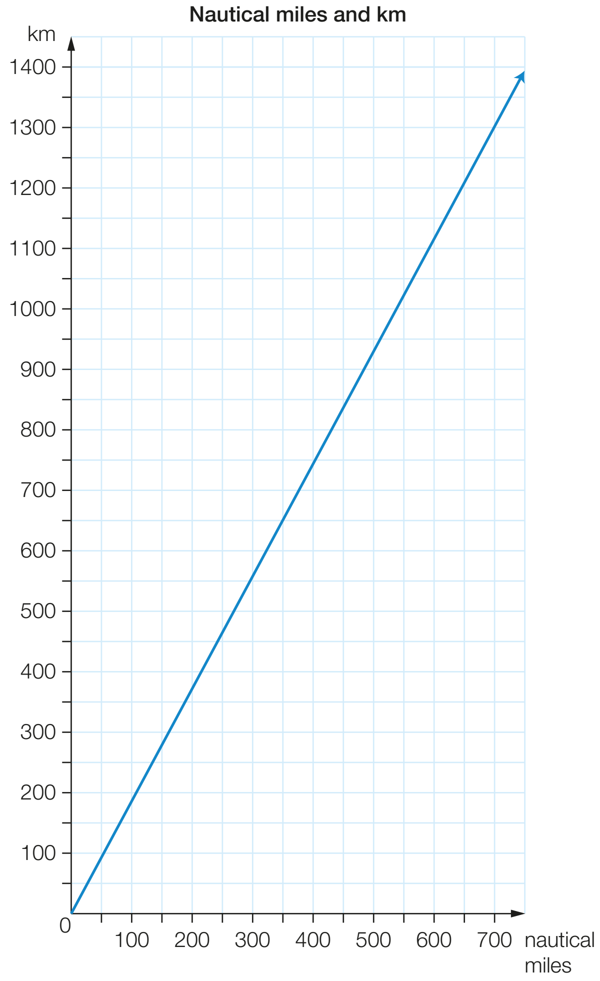
(d) Compare 40 knots with Sam's race speed, then comment on Sam's boast of travelling at 40 knots.

(e) Wind speeds of 4–27 knots are described by sailors as a 'breeze'. Find this range of wind speeds in km/h (to the nearest km/h).

(f) If the wind speed is 50 knots, what is its speed in km/h?

(g) If the wind speed is 198 km/h, what is its speed in knots?

(h) Use the conversion graph below to find the length of the race in kilometres (to the nearest km).



Explore

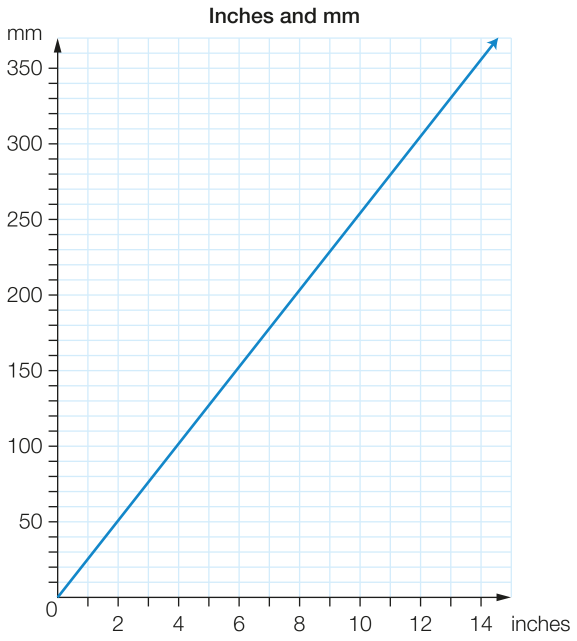
2 Jo has been growing crops for years on her farm of 100 acres. Recently it rained very heavily and Jo claimed she had 12 inches of rain in one 24-hour period.

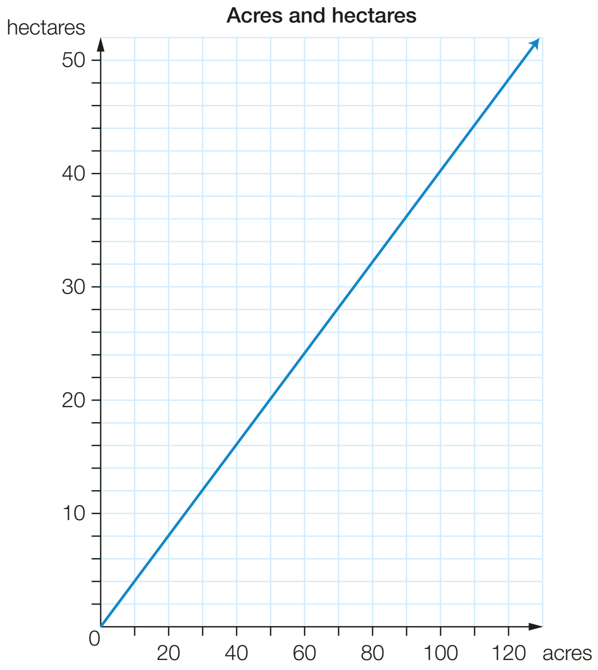
(a) Use the conversion graph 'Inches and mm' to find the number of mm of rain she measured.

(b) If 50 mm of rain was recorded in a 24-hour period, how would Jo have recorded this in inches (to the nearest inch)?

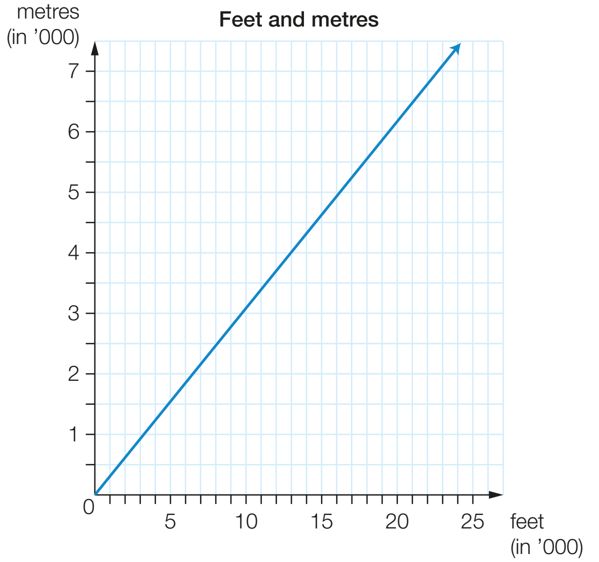
(c) Use the conversion graph 'Acres and hectares' to find how many hectares (correct to the nearest hectare) she farms.

(d) If 4 hectares was set aside for her home and garden, how many acres (correct to the nearest acre) would that be?





3 The height or altitude above sea level is measured for aircraft in terms of feet. Lauren is the pilot of a passenger aircraft with a cruising altitude between 18 000 and 24 000 feet.



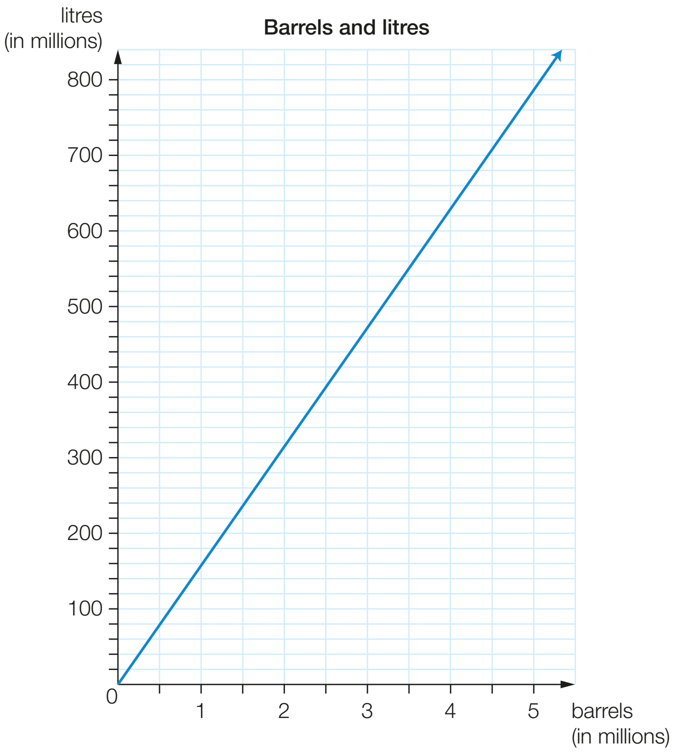
(a) Use the conversion graph 'Feet and metres' to find the altitude in metres of a plane with an altitude of:

(i) 18 000 feet **(ii)** 24 000 feet.

(b) Use the conversion graph 'Feet and metres' to find the altitude in feet of a plane with an altitude of:

(i) 6400 m **(ii)** 7500 m.

4 Oil companies measure oil by the barrel, where one barrel of oil is equivalent to approximately 159 litres. The largest accidental oil spill in history was the Deepwater Horizon disaster, which began on 20 April 2010. A drilling rig exploded while working on a well 1 mile (1.6 km) underwater in the Gulf of Mexico. It took nearly 5 months to stop the spill, with an estimated five million barrels of oil spilling into the ocean during that time.



(a) Use the conversion graph 'Barrels and litres' to find approximately how many litres of oil were spilled overall.

(b) If the spill lasted for 145 days, how many litres of oil was spilled each day?

(c) An oil well typically produces 140 000 barrels per day. How many litres is this? Compare this amount with the amount lost per day during the spill.

5 The record for the largest single crystal emerald is 7025 carats. A carat is a measure of the mass of a gemstone.

(a) If 5 carats is equivalent to 1 gram, draw a conversion graph using grid or graph paper with grams marked along the horizontal axis from 0 to 1800 and carats marked along the vertical axis from 0 to 9000. Choose an appropriate scale for each of your axes.

(b) Use your conversion graph to find the following, giving all units correct to the nearest multiple of 10 (for example 420 g, 1530 carats):

(i) the mass of the emerald described above

(ii) the mass of the 3106-carat uncut diamond called the 'Cullinan'.

(iii) the number of carats in the 'Star of Africa', a diamond of mass 106 g cut from the 'Cullinan'

(iv) the mass of a 2302-carat sapphire found in Anakie, Queensland, in 1935

(v) the number of grams cut away after a 1318-carat head of Abraham Lincoln was carved from the 2302-carat sapphire

(vi) the number of carats in a 304 g black opal called 'The Empress of Glengarry', cut from a stone found at Lightning Ridge, NSW, in 1972.

(c) (i) Which has the greatest mass: a 6465-carat ruby called 'The Eminent Star' or a 1700 g ruby called the 'Liberty Bell'?

(ii) What is the difference in mass of the two rubies?

6 Karats (usually spelt with a K instead of a C) are used to describe the purity of gold, by indicating the percentage of gold (not the mass).

(a) If '24-karat' means 100% pure gold, draw a conversion graph to convert karats of gold into a percentage of purity.

(b) Pure gold (24-karat) is too soft to use in jewellery. However, a higher number of karats means a better quality of gold. 18-karat gold is considered the best quality gold for jewellery. Use your conversion graph to find how pure this is.

(c) Which is the better quality gold, a 10-karat ring or a ring that is 45% gold?

Strategy options

- Make a table.
- Work backwards.
- Make a model.

Explain

7 **(a)** Explain why conversion graphs are a useful way of converting from one type of measurement to another.

(b) Explain the advantages and disadvantages of using a conversion graph rather than a conversion table.

(c) Explain why feet are sometimes still used to measure altitude for aircraft.

Elaborate

8 **(a)** Answer the Big Question by explaining how the conversion graphs helped you understand different types of measurement. Use your answers to the previous questions to help you.

(b) How accurate do you think conversion graphs are?

(c) When is it reasonable to use estimated conversions? Give an example.

(d) When is it important for conversions to be accurate? Would you use a conversion graph for these purposes? If not, why not?

Evaluate

9 **(a)** Did you find the conversion graphs on pages 400–402 easy to use? What difficulties, if any, did you have?

(b) How accurate do you think your answers are? How could the accuracy be improved?

(c) Did you have difficulty drawing conversion graphs for **5** and **6**? What difficulties did you encounter and what would you do differently if you had to draw them again?

Extend

10 **(a)** Investigate a famous gemstone, such as 'The Blue Hope' or 'The Excelsior', and find its mass in carats and grams.

(b) Investigate how gems are valued. Is their value just related to their size? If not, what other factors are taken into account?

(c) Investigate another unit of measurement that is used today but is not metric, such as a ton, bushel (for the weight of wheat), psi of air pressure, or troy ounces (for the weight of gold and silver). Find the comparable metric units and the conversion factor.

(d) Find an app or website for measurement unit conversions.

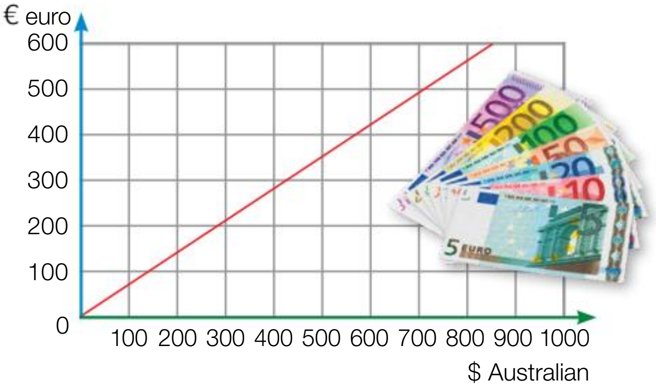
Maths 4 Real

Algebra on the move

Travelling in Europe is easier where different countries use the same currency, the euro (€). Today, most countries in Europe use the euro, but some (including the United Kingdom and Denmark) do not.

Before you travel overseas you may want to buy some of the currency that you will need…

1 The graph shows the number of euros that you could buy for a given number of Australian dollars (A$) on a particular day. The equation of this graph is $E = 0.70A$, where E is the number of euros and A is the number of dollars.



Use the graph to answer the following questions.

(a) How many euros would you get for:

(i) A$800 **(ii)** A$450 **(iii)** A$50?

(b) How many Australian dollars would you get for:

(i) €500 **(ii)** €300 **(iii)** €50?

When you travel you need to be able to quickly compare prices to decide if something is cheap or expensive. The exchange rate changes slightly every day. However, for quick mental calculations you can usually estimate the following:

€1 = A$1.50 €2 = A$3

So, the quick conversion rules are:

From A$ *to* €: $E = \frac{2}{3}A$

From € *to* A$: $A = \frac{3}{2}E$ (or $A = 1.5E$)

2 Use the quick conversion rules to find:

(a) the cost in A$ of a day tour advertised at €140

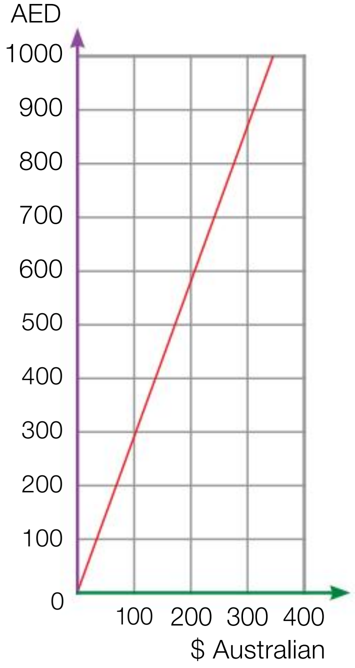
(b) the cost in A$ of a night's accommodation advertised as €110

(c) the most you can spend on a camera priced in euros if you only have A$600 left

(d) the total cost in A$ of a railway pass costing €25, lunch costing €15, a theatre ticket costing €105 and dinner costing €35.

When you fly from Sydney to Paris you decide to stop for two nights in Dubai. The currency used in Dubai is the United Arab Emirates dirham, or AED, often just called the dirham.

3 The graph shows the number of AED that you could buy for a given number of Australian dollars, on a particular day. The equation of this graph is $D = 2.95A$, where D is the number of dirham and A is the number of dollars.



Use the graph to answer the following questions.

(a) How many AED would you get for:

(i) A$300

(ii) A$100

(iii) A$250?

(b) How many Australian dollars would you get for:

(i) AED800

(ii) AED350

(iii) AED100?

A quick and easy comparison to use is that usually A$1 = AED3. The quick conversion rules for this are:

From A$ to AED: $D = 3A$

From AED to A$: $A = \frac{D}{3}$

4 Use the quick conversion rules to find:

(a) the cost in A$ of a taxi fare of AED48

(b) the cost in A$ of a night's accommodation advertised as AED375

(c) the most you can spend on a souvenir if you only have A$200 left in your account.

After your holiday in Europe you have a 4 hour stop in Dubai on the way home to Australia. You find that you have a mixture of currencies left: some euros, some United Arab Emirates dirhams and your credit card. You decide to do some duty free shopping at the airport.

5 **(a)** You have €150 left and you want to know how many dirhams this will give you. Use the first graph to change this to A$ and then use the second graph to change it to AED. How many dirhams do you get for €150?

(b) If you also had AED75 in your wallet, how many dirhams will you have in total?

(c) The cost of your shopping comes to AED1000. How much, in Australian dollars, will you have to pay?

Research

- **Find the current exchange rates and see what difference it would make to your answers. Would you need to change the quick conversion rules?**
- **Find a record of currency exchange rates changing over time. When in the past year was the best time to convert Australian dollars to pounds or dirhams? When was the best time to convert back to Australian dollars?**

Challenge 6

1 **(a)** Tom spent part of his morning visiting his friends. The graph shows his travels. He went to his friends' houses and, if they were home, stopped to see them. How many houses did he stop at?

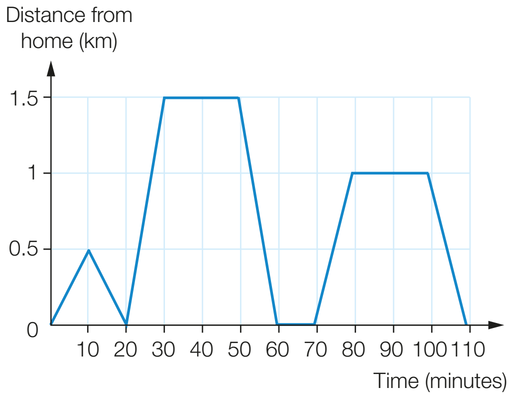
A 1 **B** 2

C 3 **D** 4

(b) How much time did he spend at home between visits?

(c) How far did he travel altogether?

(d) Between which times was he travelling the fastest?

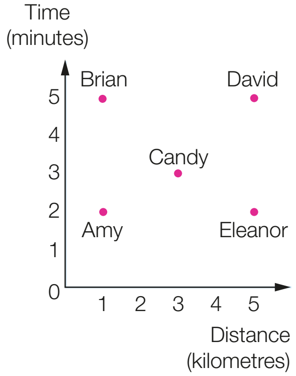


2 The graph shows the time taken by five people to travel various distances from the starting point (the origin).

(a) On average, which person travelled the fastest?

(b) On average, which person travelled the slowest?

(c) Farouk covers 4 kilometres in 4 minutes. Who else has the same average speed?



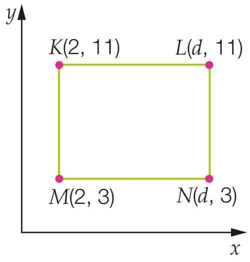
3 The coordinates of the vertices of the rectangle *KLNM* are given on the diagram. The area of the rectangle *KLNM* is 96 square units. The value of d is:

A 11

B 12

C 13

D 14



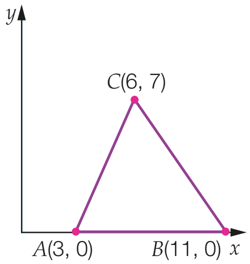
4 $A(3, 0)$, $B(11, 0)$ and $C(6, 7)$ are the vertices of a triangle as shown in the diagram. The area of ΔABC is:

A 56 square units

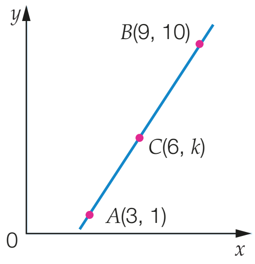
B 48 square units

C 28 square units

D 24 square units



5 The point C lies on the line AB as shown. If A and B are the points (3, 1) and (9, 10), respectively, and C is the point $(6, k)$, find the value of k.



Chapter review 6

Maths literacy

Cartesian plane
coordinate
gradient
linear graph
linear relationship
line graph
ordered pair
origin
slope
x-intercept
y-intercept

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The point (0, 0) is also known as the ____________.

2 The ____________ or ____________ of a line is positive when it goes up from left to right.

3 A number plane that is divided into four quadrants is also known as a ____________.

4 The location of a point on a Cartesian plane is given by an ____________.

5 A ____________ is a relationship that makes a straight line.

6 A ____________ is a straight-line graph.

7 The point where a line crosses the x-axis is called the ____________.

8 The point where a line crosses the y-axis is called the ____________.

Equipment required: graph paper or grid paper

Fluency

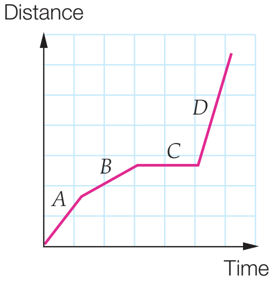
6.1

1 Liam is skating along a path.

(a) In which section of the graph (*A*, *B*, *C* or *D*) is Liam moving the fastest?

(b) In which section is he stationary?

(c) In which section is he moving the slowest, but is not stationary?



6.2

2 For each of the following:

(i) $y = 2x - 2$ (ii) $y = -x + 4$ (iii) $y = -3x - 1$

(a) Copy and complete the table of values.

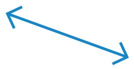
(b) Plot the points on a Cartesian plane. Join each set of points with a straight line.

(c) Write the coordinates of the x- and y-intercepts of each graph.

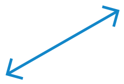
x	-2	-1	0	1	2
y					
(x, y)					

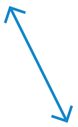
6.2

3 The line with the largest positive gradient is:

A 

B 

C 

D 

6.2

4 Match the following equations with the descriptions given below. An equation that:

(a) goes through the origin

(b) has a y-intercept of 4

(c) has a negative gradient and passes through the point (1, 3)

(d) has a zero gradient.

A $y = 8 - 5x$

B $y = -x$

C $y = 2(x + 2)$

D $y = -4$

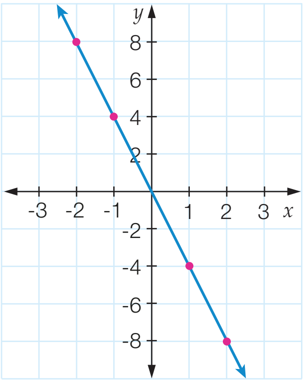
5 **(a)** Find the rule for these points:

(0, 3), (1, 4), (2, 5), (3, 6), (4, 7).

(b) Find the equation that describes the relationship shown by the graph.

(c) Find the rule for the linear relationship shown in the following table. Using the rule, copy and complete the table of values for x and y.

x	-2	-1	0	1	2
y	-4	-3	-2		



6.3

6 Write a linear equation to represent the following situation, defining all the variables you use:

The drains at Bryce's house are blocked. A plumber charges a fixed fee to go to the house. The plumber then charges an hourly rate to unblock the drains.

6.4

Understanding

7 **(a)** Sketch graphs of each of the following linear relationships.

(i) $y = x + 1$ **(ii)** $y = 5x$ **(iii)** $y = -3x + 6$

(b) State the x-intercept for each line.

(c) State the y-intercept for each line.

(d) For each, state whether the gradient is positive or negative.

6.2

8 **(a)** Sketch the graphs of each of the following linear relationships.

(i) $y = 5$ **(ii)** $x = 2$ **(iii)** $y = -4$ **(iv)** $x = -1$

(b) State the x- and y-intercept for each graph.

6.2

9 The table below shows the cost of buying different lengths of timber.

6.1, 6.2

Length of timber (m)	2	3	5	8	10
Cost ($)	10	13	19	28	34

The timber can be cut to whatever length the buyer needs.

(a) Plot the points shown in the table and join them to form a graph of this relationship. Show length of timber on the horizontal axis.

(b) Is the relationship linear? Explain your answer.

(c) How much would it cost to buy a piece of timber of length:

(i) 4 m **(ii)** 9 m **(iii)** 11 m?

(d) What length of timber could be bought for:

(i) $15 **(ii)** $20 **(iii)** $32?

10 A pool is being filled with water. The volume of water V (in litres) at any time t (in hours) is given by the equation $V = 600t$.

6.2, 6.3, 6.4

(a) Use the rule to complete a table of values of t from 0 to 40 (use increments of 10).

(b) Use the table to draw a graph for this rule.

(c) If the pool is full after 40 hours, use your graph to find how many litres it can hold.

(d) Use the graph to find:

(i) the volume after 18 hours

(ii) the time it would take for the volume to reach 18 000 L.

(e) The pool begins to leak after it is filled. The volume of water in the pool is given by the equation $V = 24\,000 - 30t$. Complete a table of volumes from 0 to 800 (use increments of 100).

(f) Use this table to draw a graph for this rule.

(g) Use the graph to find:

- **(i)** the volume of water left in the pool after $12\frac{1}{2}$ days
- **(ii)** the time it would take for the pool to lose $\frac{5}{8}$ of its volume. Give your answer in terms of days.

(h) What volume of water is lost every hour?

6.3, 6.4

11 Here is a table of values:

x	-4	-2	0	2	4
y	29	19	9	-1	-11

(a) Find the rule connecting x and y. Explain how you found it.

(b) Plot the graph.

(c) Use the graph to find the value of y where $x = 3$.

(d) Use the graph to find the value of x where $y = 14$.

Reasoning

6.4

12 One metric unit that is used for land area is the hectare. However, the unit 'acre' is also still used. A conversion graph can be useful to convert land area measurements from one unit to the other.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Area in hectares' and use a scale from 0 to 12. (Use 1 cm on the graph to represent 1 hectare.) Label the vertical axis 'Area in acres' and use a scale from 0 to 30. (Use 1 cm on the graph to represent 2 acres.)

(b) What area in acres is equivalent to zero hectares? Write this information as an ordered pair of coordinates.

(c) 10 hectares is equivalent to about 24.7 acres. Write this information as an ordered pair of coordinates.

(d) Plot the two points you found and carefully rule a straight line through them.

Use your conversion graph to answer the following questions.

(e) The playing field of the MCG has an area of about 2 hectares. How many acres is this?

(f) A property is advertised as having an area of 20 acres. Express this area in hectares.

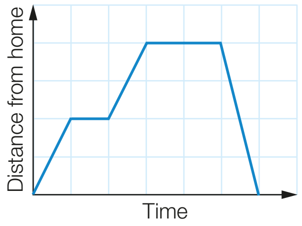
(g) Which is a larger area, 15 acres or 7 hectares?

Numeracy practice 6

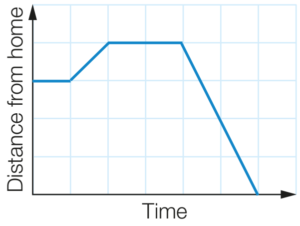
Non-calculator

1 Ivanka leaves her house to walk to the park. She stays there for a time and then goes to the shopping centre before returning home. The diagram that represents this information is:

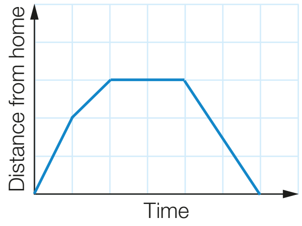
A



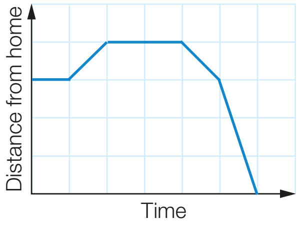
B



C

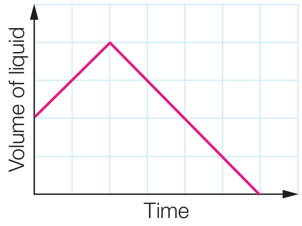


D

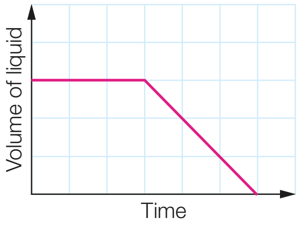


2 Jeremy noticed that water was leaking from his water bottle so that the volume of water was decreasing at a constant rate. He placed his finger over the hole to slow the leak, but the water continued to leak at a slower constant rate until the bottle was empty. The graph that describes this information is:

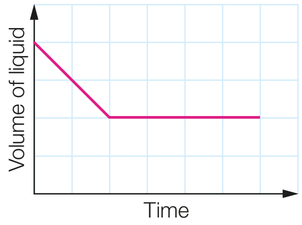
A



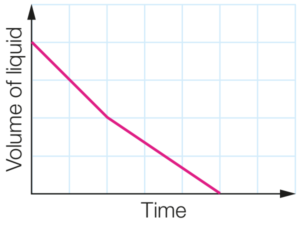
B



C



D



3 How many of the ordered pairs (7, 0), (2, -1), (-3, 5), (4, 2), (-1, 2), (0, 4), (-3, -3) lie in the second quadrant of the Cartesian plane?

A 0 **B** 1 **C** 2 **D** 3

4 Which three points (with the coordinates given) all lie on the same horizontal straight line?

A (7, -2), (-2, 7), (2,-7) **B** (-1, 4), (5, 4), (-6, 4)

C (3, 1), (3, 4), (3, -6) **D** (1, 2), (2, 1), (1, 1)

Calculator allowed

5 A rule for y in terms of x is $y = 3 - 2x$. For $x = 4\frac{1}{2}$ the value of y is:

A 12 **B** 6 **C** -6 **D** $-21\frac{1}{2}$

6 Taxi fares are calculated as a fixed 'flagfall' cost, plus the cost per kilometre travelled. The following table shows the cost of a taxi fare for several different distances.

Distance (km)	2	4	5	6
Taxi fare ($)	6.40	9.60	11.20	12.80

What is the flagfall and the cost per kilometre for this table of values?

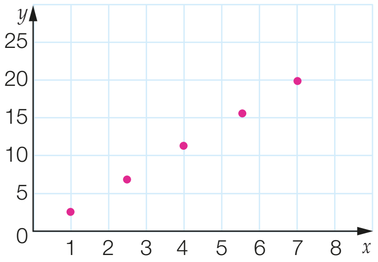
A \$1.60 flagfall + \$3.20/km **B** \$1.60 flagfall + \$1.60/km

C \$3.20 flagfall + \$3.20/km **D** \$3.20 flagfall + \$1.60/km

7 The points shown all lie on the same straight line. On this straight line, what is the y-coordinate where the x-coordinate is 6.2?

A 5 **B** 7

C 18 **D** 20



Mixed review C

Fluency

1 Simplify: **1.5**

(a) $2^4 \times 5^3 \times 2^5 \times 5^2$ **(b)** $\frac{6^3 \times 7^4}{6^2 \times 7}$

2 Find the value of the pronumeral in each of the following. **4.4**

(a) $d:10 = 3:8$ **(b)** $15:m = 12:5$ **(c)** $p:42 = 13:15$

3 Factorise: **3.7**

(a) $6ab + 9bc$ **(b)** $24x^2y^2 - 3xy$

4 Write the following recurring decimals as fractions. **2.2**

(a) $0.\dot{4}$ **(b)** $0.\dot{1}\dot{9}$

5 Expand and simplify: **3.6**

(a) $x(3y - 2x) + y(4 - 2x)$ **(b)** $3m(2m - n) - n(n - 3m)$

6 Simplify: **1.6**

(a) $(2^3 \times 3^4 \times 4)^2$ **(b)** $\left(\frac{6^2 \times 7^2}{6}\right)^3$

7 Evaluate the following expressions when $x = 3$ and $y = -4$. **3.2**

(a) $5xy - 3x^2$ **(b)** $\frac{3y^3}{8x}$

8 Share each amount given according to the ratio given in brackets. **4.6**

(a) 60 (8 : 7) **(b)** 85 (7 : 10) **(c)** 21 000 (1 : 2 : 4)

9 Find the area of the following circles, correct to 1 decimal place. **5.5**

(a) radius of 1.7 cm **(b)** diameter of 2.6 cm

10 Write the following fractions as **(i)** decimals and **(ii)** percentages. **2.5**

(a) $\frac{3}{8}$ **(b)** $\frac{13}{40}$ **(c)** $\frac{102}{125}$

11 Find the area of the following trapeziums, correct to 1 decimal place. **5.4**

(a) parallel sides 2 cm and 4 cm and perpendicular height 1.6 cm

(b) parallel sides 1.6 cm and 3.1 cm and perpendicular height 1.2 cm

12 Express the first amount as a percentage of the second amount. **2.7**

(a) 15, 60 **(b)** 41, 200 **(c)** 1, 5 **(d)** 18, 75

13 **(a)** Create a table of values for $y = 4x + 2$ from -2 to 2. **6.2**

(b) Use the table of values to plot the graph of $y = 4x + 2$.

(c) Use the graph to find the x- and y-intercepts of $y = 4x + 2$.

14 Calculate the following, writing your answers in simplest form. **2.3**

(a) $-\frac{2}{5} + \frac{3}{4}$ **(b)** $-\frac{3}{7} \times -\frac{17}{18}$ **(c)** $-2.3 + (-4.9)$ **(d)** $-5.2 \div 1.6$

Understanding

15 Find the equation for each of the following linear relationships. **6.3**

(a) (-4, -2), (-2, -1), (0, 0), (6, 3)

(b) (1, 4), (2, 7), (3, 10), (4, 13)

16 Find the gradient, x-intercept and y-intercept for the following equations. **6.2**

(a) $y = 3(2x - 5)$ **(b)** $y = \dfrac{3x + 2}{4}$

17 A store purchases a bulk order of 200 hats for $1575. **2.10**

(a) Calculate the profit made if all of the hats are sold for $11.95 each.

(b) Calculate the profit as a percentage of the cost price.

18 The Empire State Building in New York is 443 m tall. The Eiffel Tower in Paris is 300 m tall. **4.4**

(a) Express the ratio of the height of the Empire State Building to the height of the Eiffel Tower as a unit ratio. (Round decimals to 2 decimal places.)

(b) Rafal wants to make a scale model of the two buildings. If he has made the model of the Empire State Building 50 cm tall, how tall should he make the Eiffel Tower? (Answer to the nearest cm.)

Reasoning

19 A train is travelling at a speed of 120 km/h. **6.1**

(a) Write an equation showing the relationship between distance travelled, d (km), and time, t (h).

(b) Draw a graph showing up to 2 hours of travel.

(c) Use your graph to estimate the distance travelled in:

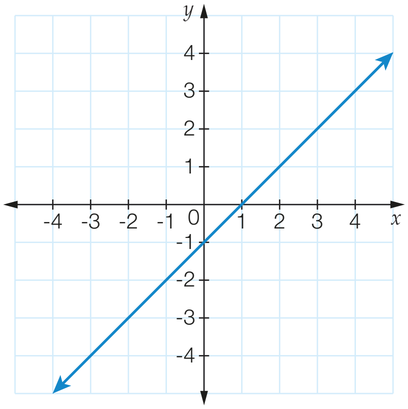
(i) 50 minutes **(ii)** 1.5 hours.

(d) Use your graph to estimate the time it takes the train to travel:

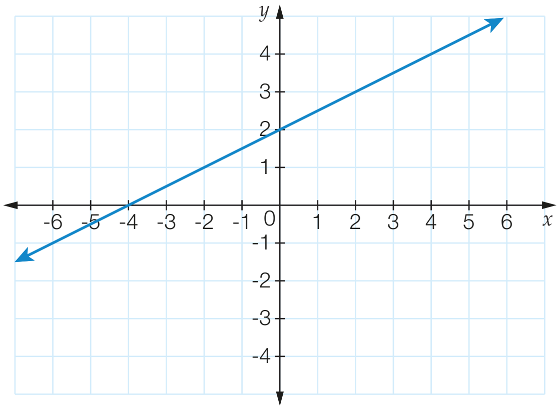
(i) 90 km **(ii)** 160 km.

20 Find the equation for each of the following graphs. **6.3**

(a)



(b)



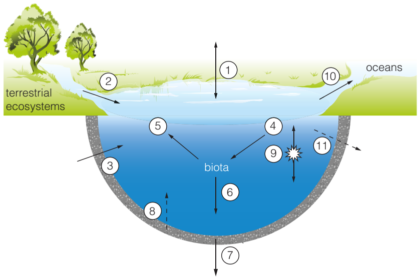
21 Show that the points (0, 4), (1, 2) and (2, 0) lie on the line with equation $y = -2x + 4$. **6.2**

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Shapes, sizes, areas and volumes

It can be difficult to understand the areas and volumes of unfamiliar shapes. What are the best ways to understand and visualise new objects? Your task is to explore this by estimating the volumes of lakes and other bodies of water.



Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Geometric patchwork ploughing

A South Australian farmer, Brian Fischer, transformed his land into a gigantic geometric pattern to fight soil erosion. The patchwork patterns have been ploughed into the ground with repeated right angles, to prevent gusting winds from blowing away the soil. Your task is to look at similar patterns and estimate what proportion of the land can still be used for farming.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

Encrypt and decrypt messages

Create a rule to encrypt your own messages using instructions that a computer can understand. Only people who know the secret rule will be able to decrypt your code to read the original message.

7

7

Linear equations

The ups and downs of equations.

You can find the speed of a roller-coaster by using an equation.

Imagine you are an engineer designing a roller-coaster that will be the most exciting ride in the show. You can use algebra to find the speed of the roller-coaster during its fall. The roller-coaster you design climbs to a great height, stops briefly, then drops quickly. As the roller-coaster falls, it accelerates due to gravity, getting faster and faster. If you ignore the effects of friction, then the speed of the roller-coaster depends on the time it takes to reach the ground. Its speed can be found using the equation:

$v = gt$

where v = speed in metres per second,
g = acceleration due to gravity and
t = the time taken for the roller-coaster to fall.

In 3 seconds, a roller-coaster could reach a speed of 29.7 metres per second, which is more than 100 km/h, and it could fall more than 44 m. What a thrill!

Forum

What other calculations might a roller-coaster engineer need to do?

What could the consequences be if a roller-coaster engineer made errors in calculations?

If you halve the height of the roller-coaster, will the speed at the bottom also be halved?

Why learn this?

Equations are a useful way of describing something mathematically. The relationship between variables such as speed and height can be expressed in an equation that describes the situation. You can then predict the effects of changing the different variables. A builder might use equations to find how much weight can hang from a ceiling. A meteorologist might use equations to predict the likelihood of rain tomorrow.

After completing this chapter you will be able to:

- write an equation in algebra
- understand equivalence
- solve equations using a variety of techniques
- use substitution to check solutions
- understand the relationship between graphs and equations
- solve equations with the unknown on both sides of the equation
- use equations to solve problems.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Which of the following are equations? (More than one option may be correct.)

A $3 = 7 - 4$ **B** $a + 5$ **C** $3m + 2 = m - 4$

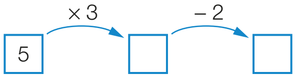
2 Solve each of the following equations. (Find the value of the pronumeral that makes the statement true.)

(a) $a + 7 = 12$ **(b)** $\frac{b}{2} = 10$ **(c)** $6c = -18$

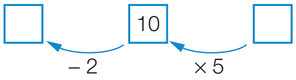
(d) $d - 4 = 13$ **(e)** $e - 3 = -8$ **(f)** $3f = 24$

3 Copy and complete these flowcharts:

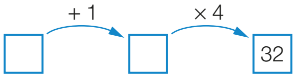
(a)



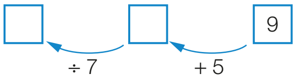
(b)



(c)



(d)



4 Find the value of each of the following expressions by substituting the value given in the brackets.

(a) $\frac{x}{2} + 9 \quad (x = 10)$ **(b)** $4(x + 5) \quad (x = 3)$ **(c)** $\frac{5x + 1}{7} \quad (x = 4)$

5 What is the inverse (or opposite) operation to each of these?

(a) multiply by 2 **(b)** add -4 **(c)** divide by 5

6 Expand each of the following to remove the brackets.

(a) $2(x + 5)$ **(b)** $3(2x - 7)$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Who is right?

In this activity, you will consider the algebraic work of two students who get different answers to the same question. How can you tell which student is right?

The language of equations

7.1

An equation is a mathematical sentence that contains two expressions connected by an equals symbol (=). The equals symbol tells you that the two expressions on either side of the = have the same value.

Equations are often used when some information is unknown. An unknown amount is called a **variable** and is represented by a **pronumeral** (which is a letter or a symbol).

For example, $x + 2 = 7$ is an equation where the pronumeral x could represent a variable such as length.

Finding the value that makes the equation a true number sentence is called **solving** the equation. You can solve the equation $x + 2 = 7$ to find the **solution** $x = 5$.

When mathematical operations are written in words, you need to write the equations using mathematical symbols before you can begin to solve them.

Worked example 1

W.E. 1

Write an equation for each of the following. Use the letter in brackets to represent the variable.

(a) Seven is added to a number (a) to give a result of ten.

(b) A number (b) is multiplied by five, then four is subtracted to give a result of sixteen.

Thinking	Working
(a) Write an expression using the variable, showing the operations that have been done on it. Equate this expression to the given result.	(a) $a + 7 = 10$
(b) **1** Write an expression using the variable, showing the operations that have been done on it. Equate this expression to the given result.	(b) $b \times 5 - 4 = 16$
2 Simplify.	$5b - 4 = 16$

Remember that $b \times 5$ is written as $5b$.

To check that the value you found for a variable is the solution, substitute the value into each side of the equation. If the left-hand side (LHS) is equal to the right-hand side (RHS), you know that you have correctly solved the equation. This is called checking by substitution. You know that $x = 5$ is the solution for $x + 2 = 7$ because $5 + 2 = 7$ is true.

Worked example 2

W.E. 2

Check by substitution whether the value given in the brackets is the solution for the equation $\frac{x+4}{6} = 2$ $(x = 14)$. (Does it make the equation true?) Answer yes or no.

Thinking	Working
1 Substitute the x-value into the left-hand side of the equation. $(x = 14)$	$\text{LHS} = \frac{x+4}{6}$ $= \frac{14+4}{6}$ where $x = 14$
2 Simplify.	$= \frac{18}{6}$ $= 3$ $\neq \text{RHS}$
3 Check whether the left-hand side of the equation equals the right-hand side.	No The value given is not the solution.

7.1 The language of equations

Navigator

Answers p. 658

1 (a–e), 2 (column 1), 3 (a–d), 4 (columns 1–2), 5 (a–d), 6, 7, 9, 11	1 (c–g), 2 (column 2), 3 (c–f), 4 (columns 2–3), 5 (a–d), 6, 7, 9, 11	1 (d–h), 2 (column 2), 3 (c–f), 4 (column 3), 5 (c–f), 6, 7, 8, 10, 11

Fluency

W.E. 1

1 Write an equation for each of the following. Use the letter in brackets to represent the variable.

(a) Eight is added to a number (a) to give a result of twelve.

(b) Four is subtracted from a number (b) to give a result of sixteen.

(c) Nine times a number (c) gives a result of sixty-three.

(d) The sum of eleven and a number (f) is zero.

(e) Seven is added to three times a number (u) to give a result of ten.

(f) A number (v) is multiplied by two, then seven is added to give a result of thirteen.

(g) Nine is added to a number (x), then the result is divided by seven to give six.

(h) The sum of six and a number (z) is multiplied by eight to give a result of zero.

W.E. 2

2 Check by substitution whether the value given in the brackets is the solution for each of the following equations. (Does it make the equation true?) Answer yes or no.

(a) $m + 2 = 9$ $(m = 7)$ **(b)** $l - 2 = -9$ $(l = -11)$

(c) $10 - p = 4$ $(p = 14)$ **(d)** $5q = 55$ $(q = 11)$

(e) $\frac{b}{4} = 8$ $(b = 2)$ **(f)** $\frac{r}{8} = 9$ $(r = 72)$

(g) $6s - 7 = 23$ $(s = 4)$ **(h)** $3a + 5 = -10$ $(a = -5)$

(i) $3(u - 8) = 6$ $(u = 6)$ **(j)** $4(5 - v) = 12$ $(v = 8)$

(k) $\frac{w + 3}{4} = 8$ $(w = 29)$ **(l)** $\frac{6 - x}{7} = 4$ $(x = 34)$

3 Find the equation that describes the following sentences.

(a) Five is added to a number to give a result of twelve.

A $5 + 12 = n$ **B** $n + 12 = 5$ **C** $n + 5 = 12$ **D** $5n = 12$

(b) A number is subtracted from nineteen to give a result of eight.

A $b - 19 = 8$ **B** $-b - 19 = 8$ **C** $b - 8 = 19$ **D** $19 - b = 8$

(c) Six is subtracted from a number to give a result of seven.

A $b - 6 = 7$ **B** $6 - b = 7$ **C** $b - 7 = 6$ **D** $7 - b = 6$

(d) Five is added to three times a number to give a result of twenty.

A $3p + 20 = 5$ **B** $5 + 20 = 3p$ **C** $3(p + 5) = 20$ **D** $3p + 5 = 20$

(e) A number is subtracted from six; this result is multiplied by four to give eight.

A $4(w - 6) = 8$ **B** $4(6 - w) = 8$ **C** $4w - 6 = 8$ **D** $8 - 4w = 6$

(f) Three is subtracted from five times a number to give a result of seven.

A $(5 - 3)k = 7$ **B** $5(k - 3) = 7$ **C** $3 - 5k = 7$ **D** $5k - 3 = 7$

4 Write each of these equations in words.

(a) $m + 2 = 9$ **(b)** $l - 2 = 9$ **(c)** $n - 6 = 3$

(d) $10 - p = 4$ **(e)** $5q = 55$ **(f)** $\frac{r}{8} = 9$

(g) $6s - 7 = 23$ **(h)** $3a - 5 = -10$ **(i)** $3(u - 8) = 6$

(j) $2 + 3p = 5$ **(k)** $\frac{w + 3}{4} = 8$ **(l)** $\frac{6 - x}{7} = 4$

Understanding

5 Write an equation for each of these rules, using the given variables for each of the quantities described.

(a) The area (A) of a rectangle is equal to its length (l) multiplied by its width (w).

(b) The area (A) of a triangle is equal to half its base (b) multiplied by its perpendicular height (h).

(c) The average speed (s) of a car is equal to the distance it travels (d) divided by the time taken (t).

(d) The area of a trapezium (A) is equal to half the sum of the lengths of the parallel sides (a and b) multiplied by the distance between them (h).

(e) The cost in dollars (C) of a cruise is 200 times the number of nights (n) plus 300.

(f) The tax-free price (F) of an item is $\frac{10}{11}$ of its retail price (R).

Reasoning

6 Mira wants to hire a cab to go to Mildura. There is a fixed flag fall charge of \$4.55 and then it costs 80 cents per km. Write an expression that shows how much she pays if she travels x km.

7 John wants to buy a guitar that costs \$1200. He has already saved \$300.

- **(a)** John saves \$$x$ per month. Write an expression to show how much money John will have saved in seven months.
- **(b)** If John is saving \$100 per month, then how many more months of saving are needed for him to be able to buy the guitar?

8 A school wants to take a class of Year 8 students on an excursion to the zoo. The bus costs \$150 and entrance to the zoo costs \$$x$ per person.

- **(a)** Write an equation to show how much it costs the school to take 25 students and 2 staff members.
- **(b)** If the zoo entrance fee is \$5 per person, how much does the excursion cost?
- **(c)** If each student pays \$$y$ to go on the excursion, write an equation to show the total amount paid by students.
- **(d)** If the students pay \$10 each for the excursion and the staff do not pay, show that this does not cover the cost of the excursion.
- **(e)** As the bus can carry up to 57 passengers, it is decided that another class of 25 students and another 2 staff should go on the excursion too. If the students are now charged \$8.50 each, use equations to show that this will cover the cost of the excursion and to know how much money will be left over.

9 A phone company charges monthly line rental of \$25 for an active phone plus 3 cents per minute for making a call.

- **(a)** How much will the monthly phone bill be, if you make x minutes of calls per month?
- **(b)** How many call minutes are made in a month if the total monthly bill is \$40?

10 Sarah wants to save enough money to buy a ticket to a concert. Tickets go on sale in 5 weeks and cost \$95.00. She has saved \$20 already.

- **(a)** Write an equation to show how much money she will have in 5 weeks if she gets \$$x$ pocket money each week for doing all her chores for the next 5 weeks.
- **(b)** If she gets \$15 per week pocket money, use your equation to show that she will have enough money to buy the ticket.
- **(c)** Sarah's friend, Rhiannon, wants to go to the same concert. She gets \$$y$ pocket money each week but owes her brother \$15. Write an equation to show how much money she will have in 5 weeks time if she repays her brother.
- **(d)** If Rhiannon gets \$20 per week pocket money, use your equation to find whether she will have enough money to buy a ticket.

Open-ended

11 Write at least three equations that have $w = 3$ as a solution, using different operations $+, -, \times$ and $\div$ in each equation.

Puzzle

Lots of letters

Given the following equations:

$A + C = A$ $\quad$ $FD = F$ $\quad$ $B - G = G$

$A + H = E$ $\quad$ $B \div H = G$ $\quad$ $E - G = F$

Find the values of A, B, C, D, E, F, G and H if the letters from A to H represent the numbers from 0 to 7.

Solving linear equations

7.2

Solving an equation means finding a value for a variable that makes the equation true.

There are many methods that can be used to solve a linear equation but they can be grouped into three categories: *numerical*, *algebraic* and *graphical*.

Solving equations numerically

You can solve equations numerically using methods such as 'inspection', or 'guess, check and improve'.

Inspection is the method you used in primary school to solve simple equations: for example, $\blacksquare + 4 = 7$; $\blacksquare = 3$. You simply look at the equation to find the solution. This method only works for very simple equations.

You can use **guess, check and improve** for more complicated equations, such as $\frac{5x-9}{2} = 8$.

Guess a value for x and substitute this value into the equation. If your guess is correct, the RHS will equal the LHS. If your guess is not correct, continue to guess until you find the solution. This can be quite a lengthy process and it may be difficult to find the exact solution, which may involve fractions and negatives. Calculators that solve equations numerically, such as graphics calculators or CAS technology, usually use some form of this method.

Worked example 3

W.E. 3

Solve the following equations numerically to find x.

(a) $5x = 35$

(b) $\frac{5x-9}{2} = 8$

Thinking	Working
(a) 1 Write the equation and identify whether this equation can be solved by inspection.	(a) $5x = 35$
2 If it can, write the answer.	$x = 7$
(b) 1 Write the equation and identify whether this equation can be solved by inspection.	(b) $\frac{5x-9}{2} = 8$
2 If it can't, use guess, check and improve. Make a guess.	Try $x = 1$ $\text{LHS} = \frac{5 \times 1 - 9}{2}$ $= \frac{5-9}{2}$ $= \frac{-4}{2}$ $= -2$ $\neq \text{RHS}$

3 If this is not the solution, guess again.

Try $x = 2$

$$\text{LHS} = \frac{5 \times 2 - 9}{2}$$
$$= \frac{10 - 9}{2}$$
$$= \frac{1}{2}$$
$$\neq \text{RHS}$$

4 Continue guessing until a solution is found. ($x = 2$ made the LHS closer to the RHS than $x = 1$, so increase the value for x.)

Try $x = 5$

$$\text{LHS} = \frac{5 \times 5 - 9}{2}$$
$$= \frac{25 - 9}{2}$$
$$= \frac{16}{2}$$
$$= 8$$
$$= \text{RHS}$$

5 Write the solution.

Because LHS = RHS, $x = 5$ is the solution.

Solving equations algebraically

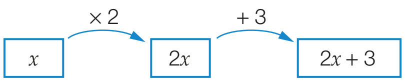
To solve equations algebraically you can use **equivalent** equations.

Equivalent equations are when a new equation can be written that does not change the value of the variable. The left-hand side of the equation is still equal to the right-hand side. To understand the process of equivalence, consider the following two strategies, 'backtracking' and 'balancing scales'.

Backtracking

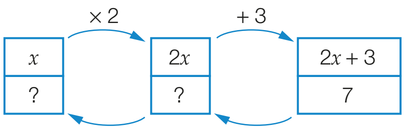
In Year 7, you may have learnt to solve equations by **backtracking** using a **flowchart**. Flowcharts can be used to show the sequence of steps required to build an expression and then to undo it by working backwards or backtracking. For example, to solve $2x + 3 = 7$, you can build the expression $2x + 3$ using a flowchart which includes necessary operations.

Remember to undo addition and subtraction *before* multiplication and division.

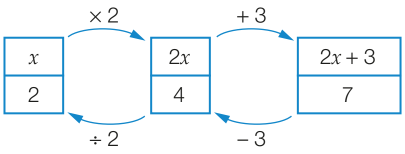


If you first added 3, then multiplied by 2, then you would get a different equation, $2(x + 3) = 7$, with a different solution.

As $2x + 3$ equals 7, you can move backwards along the flowchart to find the x-value that makes the equation true. This is called backtracking.



Remember that to 'undo' an operation you must use the **inverse** (opposite) **operation**.



$x = 2$ is the solution.

When backtracking, you can 'undo' an operation by using the inverse operation.

+ and – are inverse operations. × and ÷ are inverse operations.

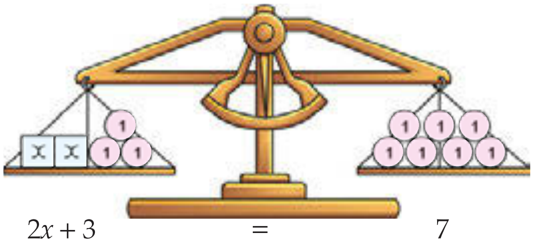
Undo + and – before × and ÷.

Balancing scales

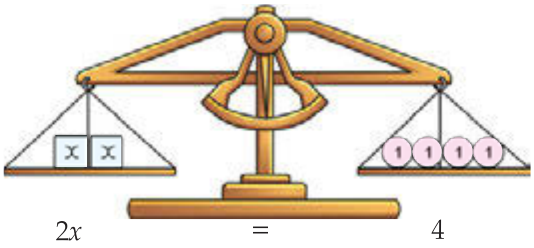
Another strategy you have used to solve equations is to imagine an equation as a set of old-fashioned scales. Like the scales, an equation is **balanced** if its left- and right-hand sides are equal to each other (equivalent). You must not do an operation (add, subtract, multiply or divide by a value) on only one side of an equation, or else it becomes unbalanced, as the two sides are no longer equivalent.

To balance the scales and the equations, you must always remember:
Whatever is done to one side of the equation must be done to the other side.

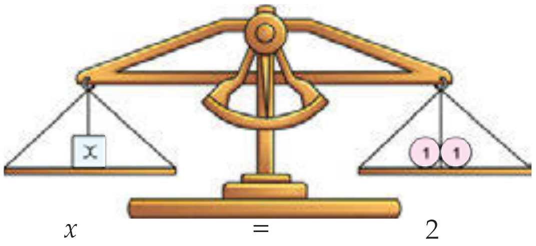
These scales show the equation $2x + 3 = 7$.



These scales show the result of removing 3 from both sides to give $2x = 4$.



These scales show the result of removing half the mass from both sides of the scales to give $x = 2$.



Solving equations using equivalence

Backtracking and balancing the scales are really the same process. Both use inverse operations to work backwards to find the value of the unknown. For example, when solving $2x + 3 = 7$, both methods use the steps 'subtract 3' and then 'divide by 2'.

Drawing flowcharts and scales are slow ways to solve equations and do not work for more complex equations. You can solve equations more quickly by using the idea of equivalence. Use inverse operations on both sides of the equals symbol, in reverse order, to find the value of the variable.

For example:

	$2x + 3 = 7$
(subtract 3 from both sides)	$2x + 3 - 3 = 7 - 3$
	$2x = 4$
(divide both sides by 2)	$\frac{2x}{2} = \frac{4}{2}$
	$x = 2$

Always check the signs in front of the constants and the coefficients. If there is a negative sign, then you need to undo the negative as well as the numerical value.

For $5 - 2x = -2x + 5$, undo by subtracting 5 and dividing by -2.

Worked example 4

W.E. 4

Solve each of the following equations using algebra.

(a) $4x - 5 = 7$ **(b)** $5 - 2x = 7$

Thinking	Working
(a) 1 Write the equation and identify the last operation to be done on the given variable (– 5). This is the first operation to be undone.	(a) $4x - 5 = 7$
2 Use the inverse operation for this operation on both sides of the equation and simplify the equation (+ 5).	$4x - 5 + 5 = 7 + 5$ $4x = 12$
3 Identify the next operation to be undone (× 4) and apply the inverse operation (÷ 4). If one side of the equation is now the variable by itself, then you have found the solution. State the solution.	$\frac{4x}{4} = \frac{12}{4}$ $x = 3$
4 Check by substitution that your answer is the solution.	Check: $\text{LHS} = 4x - 5$ $= 4 \times 3 - 5$ $= 12 - 5$ $= 7$ $= \text{RHS}$
(b) 1 Write the equation and identify the last operation to be done on the given variable (+ 5). This is the first operation to be undone.	(b) $5 - 2x = 7$
2 Use the inverse operation for this operation on both sides of the equation and simplify the equation (– 5).	$5 - 5 - 2x = 7 - 5$ $-2x = 2$

Check the sign of the coefficient of the variable. If it is negative, then you need to undo the negative as well as the number part of the coefficient.

3 Identify the next operation to be undone (× -2) and apply the inverse operation (÷ -2). If one side of the equation is now the variable by itself, then you have found the solution. State the solution.

$$\frac{-2x}{-2} = \frac{2}{-2}$$

$$x = -1$$

4 Check by substitution that your answer is the solution.

$$\begin{aligned} \text{LHS} &= 5 - 2x \\ &= 5 - 2 \times -1 \\ &= 5 + 2 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

Equations can be solved by doing inverse operations on both sides of the equals symbol.

The order in which you do the inverse operations is important. It is the opposite of the order used when the equation was made.

Solving equations graphically

Graphs can be used to find solutions to equations.

For the graph of a linear equation, all points on the line are the values that make the equation true. You can use the graph to find the *y*-coordinate for any given *x*-value and the *x*-coordinate for any given *y*-value. These pairs of *x* and *y* values must be solutions to the linear equation.

- A graph can be used to evaluate an expression. If a value for *x* is used as the *x*-coordinate of the graph, the *y*-coordinate of the point gives the value of the equation for that *x*-value.
- A graph can be used to solve an equation. If a value for *y* is used as the *y*-coordinate of the graph, the *x*-coordinate of the point gives the solution of the equation.

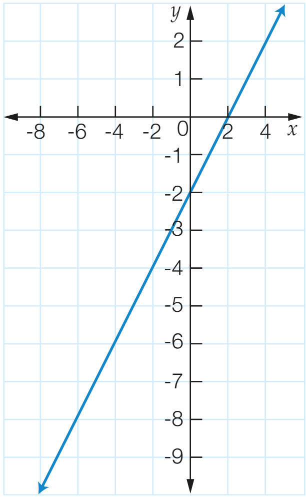
Worked example 5

W.E. 5

Use the following graph to find the value of:

(a) y for $x = -6$

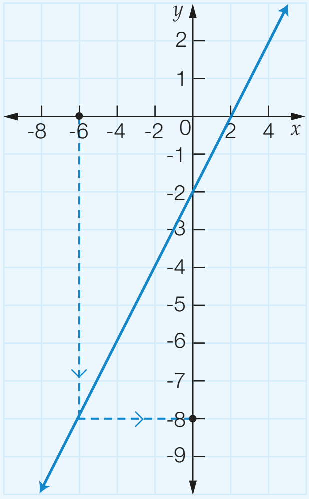
(b) x for $y = 1$.



Thinking **Working**

(a) **1** Find the x-value you need to use along the x-axis ($x = -6$). Draw a vertical line from the x-value until you reach the graph, then draw a horizontal line to the y-axis.

(a)

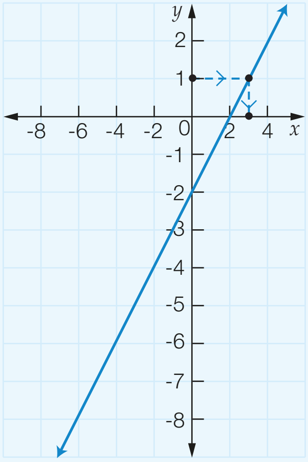


2 Read the value for y.

For $x = -6$, $y = -8$.

(b) **1** Find the y-value you need to use along the y-axis ($y = 1$). Draw a horizontal line from the y-value until you reach the graph and then draw a vertical line to the x-axis.

(b)



2 Read the value for x.

For $y = 1$, $x = 3$.

Worked example 6

W.E. 6

Use the graph of $y = 2x + 1$ to solve:

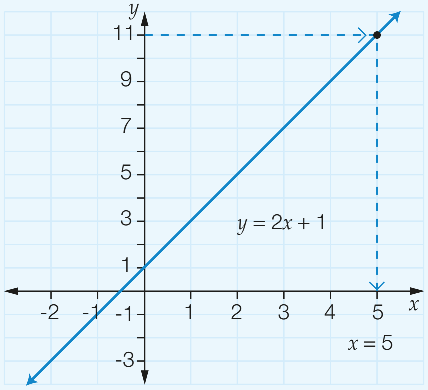
(a) $2x + 1 = 11$

(b) $2x + 1 = -3$

Thinking **Working**

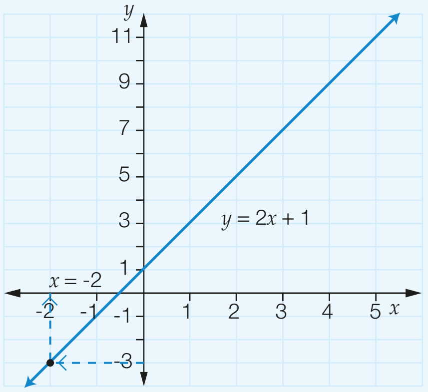
(a) **1** Use the number on the RHS of the equation as the y-value. This is the y-coordinate of a point on your graph.

(a)



2 Find the x-coordinate of that point. This is your solution. $x = 5$

(b) 1 Use the number on the RHS of the equation as the y-value. This is the y-coordinate of a point on your graph. (b)



2 Find the x-coordinate of that point. This is your solution. $x = -2$

7.2 Solving linear equations

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6 (a), 7, 8 (column 1), 9 (column 1), 11, 18, 19, 20, 22	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4, 6 (b), 7, 8 (column 2), 9 (column 2), 10, 11, 13, 14, 15, 16, 18, 19, 20, 22	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4, 6 (b), 7, 8 (column 3), 9 (column 3), 10, 12, 13, 14, 15, 16, 17, 21, 22, 23

Answers p. 658

Fluency

W.E. 3

1 Solve each of the following equations numerically to find x.

(a) $x - 5 = 11$ (b) $x - 7 = 4$ (c) $x - 6 = -2$

(d) $2x = -16$ (e) $\frac{x}{5} = 3$ (f) $\frac{x}{7} = -11$

(g) $3x - 2 = 4$ (h) $6x + 5 = 47$ (i) $3x - 7 = -16$

W.E. 4

2 Solve each of the following equations using algebra. Check your answers using substitution.

(a) $5a + 2 = 7$ (b) $3b - 8 = 4$ (c) $3c + 10 = 1$

(d) $2x - 6 = -4$ (e) $9 - 5x = 24$ (f) $24 = 12 - 2x$

(g) $7 - 3x = 4$ (h) $11 - 2x = 5$ (i) $7 - 5x = -23$

W.E. 5

3 Use the graph to find the value of:

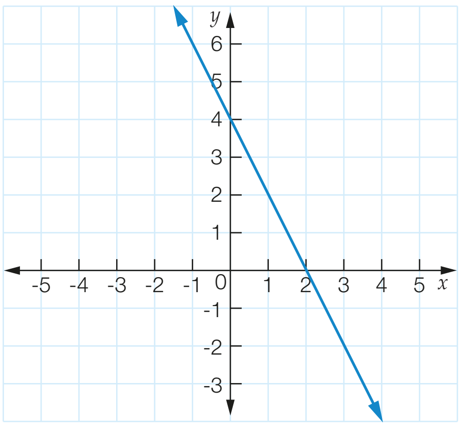
(a) y where $x = -1$ (b) y where $x = 0$

(c) y where $x = 2$ (d) x where $y = 4$

(e) x where $y = 2$ (f) x where $y = -2$

(g) y where $x = 1$ (h) y where $x = -0.5$

(i) y where $x = 2.5$



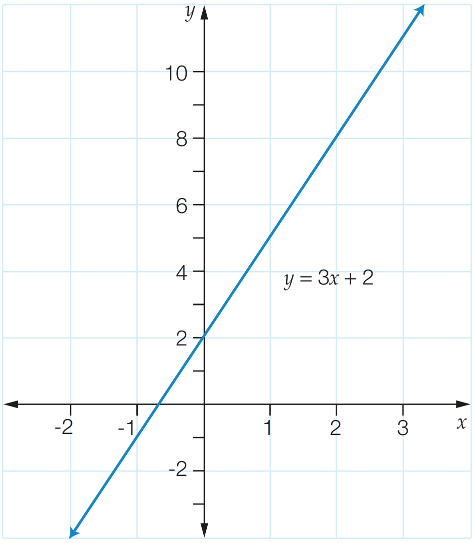
W.E. 6

4 Use the graph of $y = 3x + 2$ to solve:

(a) $3x + 2 = 8$

(b) $3x + 2 = -1$

(c) $3x + 2 = 2$



5 (a) If b is the mass of each box labelled 'b', while each silver weight represents one unit, write an equation that represents the set of scales shown.

(b) Write an equation to show the total mass of the boxes labelled 'b'.

(c) Write an equation to show the mass of each box.

(d) What is the solution to the equation you found in part **(a)**?

6 (a) Which of the following equations is equivalent to $5z - 10 = 20$?

A $5z = 10$ B $5z = 30$ C $5z = -10$ D $5z = -30$

(b) Which of the following equations is equivalent to $\frac{5p - 3}{2} = 4$?

A $5p - 3 = 20$ B $5p - 3 = 6$ C $5p - 3 = 7$ D $5p - 3 = 8$

7 (a) To find x from $4x + 1$ you need to:

A add 1, then multiply by 4

B divide by 4, then subtract 1

C subtract 1, then multiply by 4

D subtract 1, then divide by 4

(b) To find k from $-3k - 5$ you need to:

A add 5, then divide by 3

B subtract 5, then multiply by 3

C add 5, then divide by -3

D subtract 5, then divide by -3

(c) To find m from $4 - 2m$ you need to:

A subtract 2, then divide by 4

B subtract 4, then divide by -2

C add 2, then subtract 4

D add 4, then divide by -2

8 Solve each of the following using algebra.

(a) $2r + 7 = 10$
(b) $3t + 8 = 15$
(c) $7l + 11 = 23$
(d) $4a + 5 = 9$
(e) $5e + 6 = -4$
(f) $2f + 7 = -13$
(g) $9 + 4k = 1$
(h) $28 + 5h = 18$
(i) $22 + 3g = 7$
(j) $8 + 9n = -1$
(k) $12 + 7c = -16$
(l) $5 + 9k = -13$
(m) $3d + 2.4 = 8.9$
(n) $4p + 3.8 = 7.2$
(o) $12a + 5.7 = 11.3$
(p) $\frac{3x + 7}{5} = 5$
(q) $\frac{4x - 3}{3} = 11$
(r) $\frac{7x - 4}{6} = -3$
(s) $2g + 1\frac{3}{4} = 2\frac{1}{2}$
(t) $3r + \frac{5}{6} = 4\frac{1}{3}$
(u) $8p + 3\frac{1}{3} = 4\frac{2}{9}$

9 Solve each of the following equations using algebra.

(a) $-d - 1 = 7$
(b) $-e + 5 = 2$
(c) $-f + 9 = 12$
(d) $-2a + 7 = 3$
(e) $-5b + 11 = 6$
(f) $-3c - 4 = 8$
(g) $3 - 4g = -17$
(h) $8 - 3h = -10$
(i) $-9 - 2m = 7$
(j) $7 - 3r = 4$
(k) $13 - 7p = -8$
(l) $29 - 5t = -11$
(m) $2 - 3p = 4$
(n) $8 - 5l = 12$
(o) $\frac{1}{3} + x = \frac{7}{3}$
(p) $5.3 - 2a = -7.8$
(q) $6.5 - 5b = -2.1$
(r) $7.9 - 4c = -6.7$

Understanding

10 Write an equation for each of these word problems and use algebra to find the unknown number.

(a) A number is multiplied by 4, then 5 is subtracted to give a result of 23. (Let the number be *n*.)

(b) The sum of a number and seven is multiplied by three to give a result of 45. (Let the number be *m*.)

(c) The sum of five and a number is divided by nine to give a result of three. (Let the number be *p*.)

(d) A number is divided by four, then two is added to give a result of negative one. (Let the number be *q*.)

11 Match each equation in the left column with an equivalent equation in the right column.

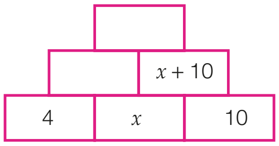
Equation	Equivalent equation
(a) $2x + 1 = 3$	**A** $x = 2$
(b) $3x - 4 = 5$	**B** $x = 9$
(c) $4x = 8$	**C** $x = -3$
(d) $\frac{x}{5} = 1$	**D** $2x = 8$
(e) $2x - 3 = 5$	**E** $2x = 2$
(f) $3x = -6$	**F** $3x = -3$
(g) $\frac{x}{3} = 3$	**G** $x = 5$
(h) $3x + 7 = 4$	**H** $x = -2$
(i) $2x - 9 = 3$	**I** $3x = 9$
(j) $-4x = 12$	**J** $2x = 12$

12 Anita's father is twice her weight, plus 16 kilograms. Her father weighs 102 kg.

(a) Write an equation using a to represent Anita's weight.

(b) Solve the equation to find Anita's weight.

13 This is an addition pyramid. Each brick is the sum of the two bricks below it.



(a) Complete the rest of the pyramid.

(b) If the top brick has a value of 20, what is the value of x?

Reasoning

14 If you multiply a number by 3 and add $\frac{5}{3}$ to it, you get $\frac{14}{3}$. What is the number?

15 Jamal has a water tank in his garden. One day, his tank had only 1000 litres left in it. During the day it rained heavily and the tank filled at a rate of 400 litres per hour. Write an equation and solve it to find how many hours it took for Jamal's tank to contain 3600 litres.

16 Jeremy wants to buy a puppy. He has been saving all his pocket money each week for the last 5 weeks, and his grandmother has given him \$25 for his birthday. He started with no money and now has \$62.50. Using p to represent the amount of pocket money he gets each week, write an equation and solve it to find how much pocket money Jeremy receives each week.

17 A light aircraft can only carry a limited weight. The pilot knows that if the passengers on board are of average weight, then the aircraft can carry 2 adults and 3 children, or 1 adult and 5 children.

(a) How many children are equivalent to one adult?

(b) If the pilot takes only children on the aircraft, use an equation to find how many could be carried.

18 Fatima drives to and from work each day from Monday to Friday. She also drives 75 kilometres on the weekend. She travels 325 kilometres every week in her car. Use an equation to find how far it is from her home to her work.

19 Paul has to take 3 litres of water with him on a boat trip. He has one 1.25 L container and two other containers of equal capacity. Together, the three containers hold 3 litres. Use an equation to find the capacity of the two other containers.

Open-ended

20 Write an equation and then solve it, using:

- **(a)** multiplication and addition
- **(b)** multiplication and subtraction.

21 The scales have 2 apples with a bunch of 20 grapes on the left and 3 apples with a bunch of 10 grapes on the right.

Claire is trying to find how many grapes have a mass equivalent to one apple. She suggests the following as possible first steps.

- Remove 20 grapes from each side.
- Take off half of everything from both sides.

- **(a)** Explain to Claire the faults in her suggested first steps.
- **(b)** Solve the problem in two steps.

22 Write at least three equations that are equivalent to $2x + 3 = 11$.

23 Jill adds a number to both sides of an equation, then divides both sides by a different number.

She gets the answer: $x = 2$.

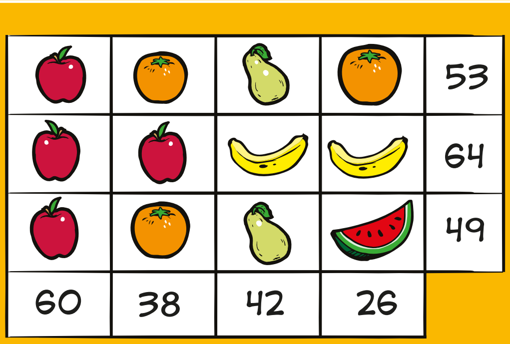
What might the original equation be?

Puzzle

Algebra fruit bowl

Every object in the puzzle at right is equal to a number. The numbers given are the sum of the objects in each row or column. Find the number value of each of the objects in the puzzle.

When only one object appears in a row or column, that makes the puzzle easier to solve. At other times, you will have to look for relationships between the objects.



7.3 Solving more complex equations

The equations in this section might look more complex, but you can still solve them in the same way, just by doing inverse operations on both sides. To find a solution to these equations, identify the order in which the equation was made around the variable, then do the correct order of inverse operations.

The equations must be undone step by step, by doing inverse operations in the reverse order to the way the equation was made. The number of operations that are done on the variable tells you how many operations need to be undone to solve the equation.

Worked example 7

W.E. 7

Solve each of the following using algebra.

(a) $\frac{y}{6} + 9 = 7$ **(b)** $\frac{t + 3}{4} = -4$

Thinking	**Working**
(a) **1** Write the equation and count the number of operations that have been done on the variable. This tells you how many inverse operations need to be done to find the solution. (Here, there are two operations, ÷ 6 and + 9.)	(a) $\frac{y}{6} + 9 = 7$ Two inverse operations need to be done.
2 Do the first inverse operation on both sides of the equation (– 9).	$\frac{y}{6} + 9 - 9 = 7 - 9$
3 Simplify the equation.	$\frac{y}{6} = -2$
4 Do the second inverse operation on both sides of the equation (× 6).	$\frac{y}{6} \times 6 = -2 \times 6$
5 Write the solution to the equation.	$y = -12$
6 Check by substitution that you have found the solution.	$\text{LHS} = \frac{y}{6} + 9$ $= \frac{-12}{6} + 9$ $= -2 + 9$ $= 7$ $= \text{RHS}$

(b) **1** Write the equation and count the number of operations that have been done on the variable. This tells you how many inverse operations need to be done to find the solution. (Here, there are two operations, + 3 and ÷ 4.)	(b) $\frac{t+3}{4} = -4$ Two operations need to be undone.
2 Do the first inverse operation on both sides of the equation (× 4). Place brackets around numerator.	$\frac{(t+3)}{4} \times 4 = -4 \times 4$
3 Simplify the equation.	$t + 3 = -16$
4 Do the second inverse operation on both sides of the equation (– 3).	$t + 3 - 3 = -16 - 3$
5 Write the solution to the equation.	$t = -19$
6 Check by substitution that you have found the solution.	$\text{LHS} = \frac{t+3}{4}$ $= \frac{-19+3}{4}$ $= \frac{-16}{4}$ $= -4$ $= \text{RHS}$

Worked example 8

W.E. 8

Solve each of the following equations using algebra.

(a) $6(3 - x) = 12$ **(b)** $5(2a - 9) = 13$

Thinking	**Working**
(a) **1** Write the equation.	(a) $6(3 - x) = 12$
2 Remove the brackets first by dividing both sides by a common factor if you can. (Here, 6.) Then use inverse operations to undo the resulting equation. (Here, there are two operations to undo, + 3 and × -1.)	$\frac{6(3-x)}{6} = \frac{12}{6}$ $3 - x = 2$
3 Continue to use inverse operations until the variable is by itself on one side of the equals symbol (– 3 first, then ÷ -1 to remove the negative from both sides).	$3 - 3 - x = 2 - 3$ $-x = -1$ $\frac{-1x}{-1} = \frac{-1}{-1}$
4 Write the solution to the equation.	$x = 1$
5 Check by substitution that you have found the solution.	$\text{LHS} = 6(3 - x)$ $= 6(3 - 1)$ $= 6 \times 2$ $= 12$ $= \text{RHS}$

(b) **1** Write the equation.	(b) $5(2a-9)=13$
2 Remove the brackets first, by dividing both sides by a common factor if you can, or by expanding using the distributive law. Then use inverse operations to undo the resulting equation. (Here, there are two operations to undo, × 10 and – 45.)	$10a-45=13$
3 Do the first inverse operation on both sides of the equation (+ 45).	$10a-45+45=13+45$
4 Simplify the equation.	$10a=58$
5 Do the second inverse operation on both sides of the equation (÷ 10).	$\frac{10a}{10}=\frac{58}{10}$
6 Write the solution to the equation.	$a=5.8$ or $5\frac{4}{5}$
7 Check by substitution that you have found the solution.	$\text{LHS}=5(2a-9)$ $=5(2\times 5.8-9)$ $=5(11.6-9)$ $=5\times 2.6$ $=13$ $=\text{RHS}$

Worked example 9

W.E. 9

Solve each of the following equations using algebra.

(a) $\frac{2a-3}{7}=-3$

(b) $\frac{6-4k}{5}=-2$

Thinking	Working
(a) **1** Write the equation and count the number of operations that have been done on the variable. This tells you how many inverse operations need to be done to find the solution. (Here, there are three operations, × 2, – 3, and ÷ 7.)	(a) $\frac{2a-3}{7}=-3$ Three operations need to be undone.
2 Multiply both sides of the equation by the number in the denominator to eliminate the fraction (× 7).	$\frac{2a-3}{7}\times 7=-3\times 7$ $2a-3=-21$
3 Continue to use inverse operations until the variable is by itself on one side of the equation (+ 3, ÷ 2).	$2a-3+3=-21+3$ $2a=-18$ $\frac{2a}{2}=\frac{-18}{2}$ $a=-9$

4 Check by substitution that you have found the solution.

$$\text{LHS} = \frac{2a-3}{7}$$
$$= \frac{2 \times -9 - 3}{7}$$
$$= \frac{-18-3}{7}$$
$$= \frac{-21}{7}$$
$$= -3$$
$$= \text{RHS}$$

(b) **1** Write the equation and count the number of operations that have been done on the variable. This tells you how many inverse operations need to be done to find the solution. (Here, there are three operations, × 4, + 6, and ÷ 5.)

(b) $$\frac{6-4k}{5} = -2$$

2 Multiply both sides of the equation by the number in the denominator to eliminate the fraction (× 5) and simplify the equation.

$$\frac{6-4k}{5} \times 5 = -2 \times 5$$
$$6 - 4k = -10$$

3 Continue to use inverse operations until the variable is by itself on one side of the equation (– 6, ÷ -4).

$$6 - 6 - 4k = -10 - 6$$
$$-4k = -16$$
$$\frac{-4k}{-4} = \frac{-16}{-4}$$
$$k = 4$$

4 Check by substitution that you have found the solution.

$$\text{LHS} = \frac{6-4k}{5}$$
$$= \frac{6 - 4 \times 4}{5}$$
$$= \frac{6-16}{5}$$
$$= \frac{-10}{5}$$
$$= -2$$
$$= \text{RHS}$$


Watch out! Here is that negative coefficient of x again.

- $-x$ is the same thing as $-1 \times x$.
- The fraction bar means division. It also acts as if there are brackets around the numerator and around the denominator. For example: $\frac{2x+5}{3} = (2x+5) \div (3)$

7.3 Solving more complex equations

Navigator

Answers p. 659

1 (column 1), 2 (column 1), 3 (column 1), 4, 5 column 1, 6, 7 (a–b), 10, 11, 14, 18, 19, 20	1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5 (column 2), 6, 7, 8, 10, 11, 12, 14, 15, 18, 19, 20, 21	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 5 (column 3), 6, 7, 8, 9, 11, 12, 13, 15, 16, 17, 20, 21

Fluency

W.E. 7

1 Solve each of the following using algebra.

(a) $\frac{c}{6}+8=10$ (b) $\frac{f}{7}+5=8$ (c) $\frac{m}{9}+3=12$

(d) $\frac{x}{2}-3=-5$ (e) $\frac{b}{3}-4=-7$ (f) $\frac{t}{4}-5=-9$

(g) $\frac{x+2}{4}=-1$ (h) $\frac{p+2}{5}=-7$ (i) $\frac{r+7}{9}=-8$

(j) $\frac{x-3}{4}=3$ (k) $\frac{y-9}{5}=7$ (l) $\frac{t-11}{12}=-5$

W.E. 8

2 Solve each of the following equations using algebra.

(a) $4(x-2)=20$ (b) $5(x+3)=-50$ (c) $8(x+2)=-24$

(d) $2(x-1)=4$ (e) $3(6x+4)=-16$ (f) $7(2x-5)=41$

(g) $3(2-x)=36$ (h) $3(4-x)=6$ (i) $7(5-x)=56$

(j) $4(2-3x)=-16$ (k) $5(3-2x)=-25$ (l) $9(5-3x)=-27$

W.E. 9

3 Solve each of the following equations using algebra.

(a) $\frac{a+3}{2}=-1$ (b) $\frac{3x-6}{4}=3$ (c) $\frac{6r+1}{5}=-7$

(d) $\frac{3r-2}{5}=8$ (e) $\frac{5n-3}{4}=8$ (f) $\frac{7p-9}{12}=8$

(g) $\frac{7-3r}{2}=2$ (h) $\frac{28-5f}{4}=2$ (i) $\frac{3-7m}{3}=8$

(j) $\frac{4-5r}{3}=-2$ (k) $\frac{5-2p}{4}=-3$ (l) $\frac{7-4k}{5}=-9$

4 (a) To find x from $\frac{x}{2}+7$ you need to:

A multiply by 2, then subtract 7 B add 7, then multiply by 2

C subtract 7, then multiply by 2 D divide by 2, then add 7

(b) To find k from $\frac{k-8}{3}$ you need to:

A add 8, then multiply by 3 B multiply by 3, then add 8

C subtract 8, then divide by 3 D multiply by 3, then subtract 8

5 Solve each of the following equations.

(a) $\frac{3a}{7} + 5 = 11$ **(b)** $\frac{5p}{3} + 8 = -7$ **(c)** $\frac{7m}{4} + 3 = -18$

(d) $5(3 - 2a) = -7$ **(e)** $4(5 - 3a) = 11$ **(f)** $3(7 - 4a) = 37$

(g) $\frac{2x + 3}{5} = 4$ **(h)** $\frac{3x + 5}{2} = 9$ **(i)** $\frac{4x - 7}{3} = 8$

(j) $\frac{1 - 2x}{3} = 3$ **(k)** $\frac{6 - 3x}{5} = 3$ **(l)** $\frac{8 - 5x}{7} = 4$

6 Match each equation (in the left column) with the appropriate inverse operations to solve it (in the right column).

Equation	Inverse operations to solve the equation
(a) $\frac{2x + 4}{3} = 5$	**A** $\times 4$, then $+ 2$, then $\div 3$
(b) $\frac{3x}{4} + 2 = 5$	**B** $- 4$, then $\times 3$, then $\div -2$
(c) $\frac{2x}{3} + 4 = 5$	**C** $\times 3$, then $- 4$, then $\div -2$
(d) $\frac{3x + 2}{4} = 5$	**D** $\times 3$, then $- 4$, then $\div 2$
(e) $\frac{2x - 4}{3} = 5$	**E** $- 2$, then $\times 4$, then $\div -3$
(f) $\frac{4 - 2x}{3} = 5$	**F** $- 2$, then $\times 4$, then $\div 3$
(g) $\frac{3x - 2}{4} = 5$	**G** $- 4$, then $\times 3$, then $\div 2$
(h) $\frac{2 - 3x}{4} = 5$	**H** $\times 3$, then $+ 4$, then $\div 2$
(i) $2 - \frac{3x}{4} = 5$	**I** $\times 4$, then $- 2$, then $\div 3$
(j) $4 - \frac{2x}{3} = 5$	**J** $\times 4$, then $- 2$, then $\div -3$

Understanding

7 Write the following statements as equations and then find the value of the unknown.

(a) A number is multiplied by three, seven is added, and the result is divided by four to give an answer of four. What is the number?

(b) A number is divided by four, one is subtracted, and the result is doubled to give an answer of six. What is the number?

(c) Two is subtracted from five times a number, then the result is divided by six to give an answer of negative twelve. What is the number?

(d) Seven is added to one and a half times a number to give a result of eight. What is the number?

8 A tennis club has 42 members. If the number of seniors is 6 more than the number of non-seniors, then how many seniors are in the club?

9 Margaret has put aside a certain amount of money to buy Christmas presents for five relatives. She spends $10 of the money on wrapping paper and cards, and intends to divide the remaining money equally between each relative. However, she then decides to add $20 to her budget to buy a special present for one relative. This present costs $48.

(a) Write an equation and solve it to find how much money Margaret initially put aside.

(b) How much money did Margaret spend altogether?

10 The base length of an isosceles triangle is 10 cm. If the perimeter of the triangle is 32 cm, then what is the length of the remaining two equal sides?

11 Julie wants to buy a car. She already has $5000 and she is saving $500 every month. How much money will she have after a year?

12 Amir has a job selling a particular product. He earns two-elevenths of the total value of the product he sells each day, plus an extra $50.

(a) Write an equation and solve it to find the total value of the products sold in one day if he earns $150.

(b) If Amir makes 22 sales of this product, what is the selling price of the product?

13 Rimesh planted some daffodil plants in his garden. The following year he found that the number of plants had doubled. He also planted another six plants. The year after that he found the number of plants had tripled to give him 108 daffodil plants.

Write an equation and solve it to find how many plants he planted at the start.

Reasoning

For each of the following, write appropriate equations and solve to answer the question.

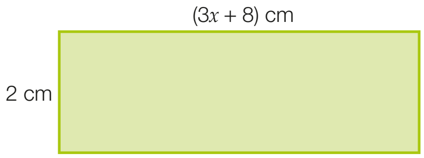
14 The sum of the digits of a two-digit number is 12. If the first digit is three times as large as the second digit, what is the number?

15 The sum of the digits of a three-digit number is 15. If the first digit is twice as large as the third digit and the second digit is one less than the third digit, what is the number?

16 Divide 120 into two different parts, so that one third of the first part is equal to one sixth of the second part. What are the parts?

17 Lyndall goes to the markets each Sunday to sell flowers. At the end of each market day she spends some of her earnings, which she keeps in a special 'market' wallet. At the beginning of July, she has a certain amount of money in her wallet. On the first weekend she doubled this amount, then spent $20. On the second weekend, she doubled the money in the wallet, then spent $25. On the third weekend, she tripled the money in the wallet and spent $15. At this point, Lyndall has $150 in her market wallet. How much did she have at the beginning of July?

18 Given that the area of this rectangle is 106 cm^2, find the value of x.



Open-ended

19 Write at least three different equations that can be solved by first adding 3, then dividing by $\frac{2}{3}$.

20 **(a)** Starting with the number 3, do three different operations on it to find an answer of 5. Then, write your sequence as an equation with x instead of 3.

(b) Make up two more three-step equations. Give the equations to your friends and see whether they can solve them.

21 Caitlyn solves the equation $2 - 5x = 8$ as follows.

$2 - 5x = 8$ step 1

$5x = 8 - 2$ step 2

$5x = 6$ step 3

$x = \frac{6}{5}$ step 4

$x = 1\frac{1}{5}$ step 5

(a) At which step did Caitlyn make a mistake? Describe how you would correct this mistake.

(b) Solve, remembering to check your answer.

Puzzle

Alphametics

Alphametics are a mathematical puzzle where digits are replaced by letters that form words and phrases. A famous example and its solution are shown below:

$$\begin{array}{cccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array} \qquad \begin{array}{cccccc} & & 9 & 5 & 6 & 7 \\ + & & 1 & 0 & 8 & 5 \\ \hline & 1 & 0 & 6 & 5 & 2 \end{array}$$

The rules of alphametics are:

1 Each letter represents a unique digit.

2 Numbers must not start with a zero.

3 The solution is unique (unless otherwise stated).

Strategy options

- Guess and check.
- Work backwards.
- Look for a pattern.

Use the example and the rules to solve the following alphametics.

1
$$\begin{array}{cccccc} & & & E & A & T \\ + & & T & H & A & T \\ \hline & A & P & P & L & E \end{array}$$

2
$$\begin{array}{cccccc} & & H & E & R & E \\ + & & & S & H & E \\ \hline & C & O & M & E & S \end{array}$$

3
$$\begin{array}{cccccc} & & M & E & M & O \\ + & & F & R & O & M \\ \hline & H & O & M & E & R \end{array}$$

4
$$\begin{array}{ccccccc} & & C & R & O & S & S \\ + & & R & O & A & D & S \\ \hline & D & A & N & G & E & R \end{array}$$

Investigation

Magic calendar squares

Equipment required: calendar

Magic squares have fascinated people for over 4000 years. A magic square is a grid of different numbers in which every row, column and diagonal add up to the same total. This total is called the 'magic sum'. The following 'Lo Shu' magic square from ancient China is a form of the simplest magic square using the numbers 1 to 9.

Lo-Shu magic square

4	9	2
3	5	7
8	1	6

According to legend, the Lo Shu magic square was marked on the shell of a turtle which came out of a flooded river during the reign of Emperor Yu.

A 'calendar' square is any 3 × 3 square drawn around a set of numbers on a calendar. Calendar squares are 'semi-magic' squares. Your exploration will show you why.

For example, consider the two different 3 × 3 squares on the calendar below:

April

Mon	Tue	Wed	Thu	Fri	Sat	Sun
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

The Big Question

Can all calendar squares be rearranged into magic squares?

Engage

1 Find the magic sum of the Lo Shu magic square by adding the numbers in each row, column and diagonal.

2 Here is another magic square:

5	10	3
4	6	8
9	2	7

(a) What is this new square's magic sum?

(b) What operation can be done on the Lo Shu magic square to make this magic square?

(c) If $x = 1$, then $x + 1 = 2$. Use this pattern to find an algebraic relationship to represent all the integers from 3 to 9.

(d) Now, replace all the numbers in the boxes in the Lo Shu magic square with algebraic expressions in terms of x, using the relationships you found in the previous step.

	x	

(e) **(i)** What is the magic sum in terms of x?

(ii) Write this in factorised form.

(f) By solving equations, find x for the following magic sums.

(i) 33 **(ii)** 54 **(iii)** -3

(g) Draw the grids for these three magic squares, calculate the numbers that will go into each square and then enter these numbers into the squares.

3 Here is another magic square:

		4
	10	
16		12

(a) Find the missing numbers.

(b) How is this square related to the Lo Shu magic square?

(c) By substituting x for 1, write all the numbers in this new square in terms of x using the relationship you found in the previous step.

Explore

4 **(a)** Select a calendar square and find the row, column and diagonal sum of it. Use these sums to explain why the squares are called ‘semi-magic’.

(b) Substitute x for the earliest date on your chosen calendar square. As you did for the Lo Shu magic square, write all the numbers in terms of x.

(c) Find the diagonal sum in terms of x.

(d) Find the sum of all the numbers in your calendar square in terms of x.

(e) What is the mean (the average value) of this sum?

(f) Find the start date for these calendar square totals by solving an appropriate equation:

(i) 180 **(ii)** 153

(g) Rearrange the numbers in your calendar square to turn your ‘semi-magic’ square into a ‘magic’ square.

Strategy options

- Guess and check.
- Look for a pattern.
- Test all possible combinations.

Explain

5 **(a)** If x is an integer, explain, in terms of x, why the magic sum of an altered Lo Shu magic square could never be 22, 38 or -2.

(b) Use the algebraic magic sum in **3** to explain why the magic sum of the squares in **3** will always be a multiple of 6.

(c) Explain why the total on a calendar square can never exceed 207 and will always be a multiple of 9.

(d) What is the smallest total? Explain how you found it.

Elaborate

6 Write your rearranged calendar square in terms of x. Substitute two other start dates into this square to generate two new calendar squares.

Evaluate

7 **(a)** Did you do this task systematically?

(b) Did you record your work carefully?

(c) Did you find any patterns? What were they?

(d) Did algebra and equation solving help you to find the rearranged magic calendar squares? If so, how did this help?

(e) Did you use any other strategies to find the rearranged magic calendar squares? If so, what were they?

Extend

Investigate some famous magic squares, such as the artist Albrecht Dürer’s 4 × 4 magic square in his woodcut *Melencolia I* or Benjamin Franklin’s 8 × 8 magic squares. See what other magic sums and patterns you can find.

Half-time 7

7.3

1 Solve the following equations using algebra.

(a) $3(5 - x) = 24$

(b) $\frac{5x}{2} + 4 = 14$

(c) $\frac{3x - 6}{8} = 9$

(d) $3(x + 3) = -6$

(e) $\frac{7 - 4x}{5} = -7$

(f) $5 - \frac{x}{3} = 25$

7.1

2 Check whether the solution given in the brackets is correct for the following equations.

(a) $x + 5 = 11 \quad (x = 7)$

(b) $2x - 3 = 7 \quad (x = 5)$

(c) $3(x - 9) = 15 \quad (x = 14)$

(d) $\frac{x - 8}{3} = 8 \quad (x = 23)$

7.3

3 Phillip, James and Sam all give money to buy a present worth $130 for Chris. James gives half the amount that Phillip gives. Sam gives $10 less than twice the amount that Phillip gives. Find the amount that each person gives towards the present.

7.2

4 Solve the following equations using algebra.

(a) $x - 9 = 15$

(b) $2x + 12 = 16$

(c) $\frac{x}{4} + 7 = 12$

(d) $8 - \frac{x}{5} = 17$

7.1

5 Write the following sentences as equations and find the value of the unknown number.

(a) Four is subtracted from a number to give a result of twelve.

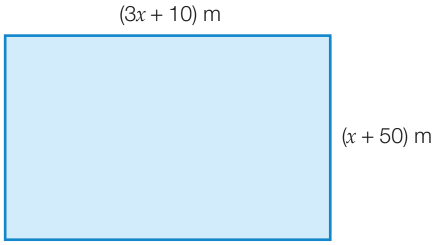
(b) A number multiplied by seven is equal to thirty-five.

(c) A number multiplied by six is equal to three times twelve.

(d) A number is multiplied by seven, then three is added to give a result of twenty-four.

7.3

6 The total distance around this rectangular field is 480 metres. Find the value of x and use this result to find the length and width of the field.



Solving equations with the unknown on both sides

7.4

So far, you have solved equations with an unknown value (the variable) on only one side of the equals symbol. However, many equations have the unknown on both sides of the equation, as in $5x + 5 = 4x + 8$.

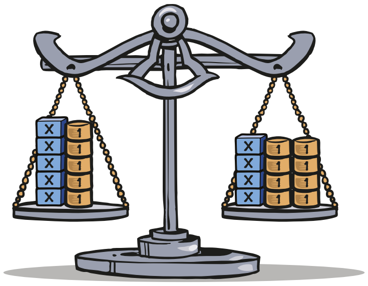
Backtracking cannot be used to solve these equations, but you can solve them using algebra. Graphs can also be used to solve these equations.

Solving with algebra

Consider the scales on the right. The left-hand side of the scales shows five blocks of mass x g and five blocks of mass 1 g. This can be written in algebraic form as $5x + 5$.

The right-hand side of the scales shows four blocks of mass x g and eight blocks of mass 1 g. This can be written in algebraic form as $4x + 8$.

How can you find the value of x? Because the scales are balanced, you can represent the situation with the equation $5x + 5 = 4x + 8$.



$5x + 5 \quad = \quad 4x + 8$

If all four blocks of mass x are removed from the right side of the balance, you will also need to remove four blocks of mass x from the left side to keep the scales balanced. This leaves one block of mass x on the left side of the equation and no blocks of mass x on the right.

$x + 5 \quad = \quad 8$

Writing in algebra:

$$5x - 4x + 5 = 4x - 4x + 8$$
$$x + 5 = 8$$

You have used the inverse operation ($- 4x$) on both sides of the equation to remove the unknown from the right side of the equation.

You can now remove the five blocks of mass 1 g from both sides of the scales. They will stay balanced. One block of mass x will remain on the left side of the scales and three blocks of mass 1 g will remain on the right side.

$x \quad = \quad 3$

Writing in algebra:

$$x + 5 - 5 = 8 - 5$$
$$x = 3$$

You have used the inverse operation (-5) on both sides of the equation to remove the number from the left side of the equation.

You have found that the unknown mass x is equal to 3 g by solving the equation $5x + 5 = 4x + 8$ and finding that $x = 3$ is the solution to the equation. You can check by substitution that the value you have found, $x = 3$, is the solution.

$$\begin{aligned}\text{LHS} &= 5x + 5 \\ &= 5 \times 3 + 5 \\ &= 15 + 5 \\ &= 20\end{aligned} \qquad \begin{aligned}\text{RHS} &= 4x + 8 \\ &= 4 \times 3 + 8 \\ &= 12 + 8 \\ &= 20\end{aligned}$$

LHS = RHS, so $x = 3$ is the solution.

To solve equations where the unknown is on both sides of the equation:

1. Do the same inverse operation on both sides of the equation to remove the variable from the RHS of the equation. This keeps the unknown on the LHS.
2. Do the same operation on both sides of the equation to remove any number on the LHS of the equation.
3. Continue to use inverse operations on both sides of the equation until a solution is found.
4. Always check by substitution that you have found the solution.

With some equations, this process will give a negative coefficient of x after the first step.

Consider $3x + 4 = 5x + 2$

Subtracting $5x$ from both sides:

$$\begin{aligned}3x - 5x + 4 &= 5x - 5x + 2 \\ -2x + 4 &= 2\end{aligned}$$

It is best to keep the coefficient of the unknown positive. You can do this by interchanging (swapping) the sides of the equation before you start.

$3x + 4 = 5x + 2$
$5x + 2 = 3x + 4$

Now, subtract $3x$ from both sides of the equation:

$$\begin{aligned}5x - 3x + 2 &= 3x - 3x + 4 \\ 2x + 2 &= 4\end{aligned}$$

This equation is much easier to solve than $-2x + 4 = 2$.

If you think about the balanced scales, this is the same as turning the scales around so the left-hand side and the right-hand side of the scales are reversed.

If the coefficient of x is greater on the RHS of the equation than on the LHS, interchange the sides before beginning the solution process.

Worked example 10

W.E. 10

Solve the following equations using algebra.

(a) $5x - 2 = 3x + 8$ **(b)** $2x + 7 = 5x - 8$

Thinking	Working
(a) 1 Write the equation.	(a) $5x - 2 = 3x + 8$
2 Use inverse operations to remove the variable from the RHS of the equals symbol. ($-\ 3x$)	$5x - 3x - 2 = 3x - 3x + 8$ $2x - 2 = 8$
3 Use inverse operations to solve for x. ($+\ 2$ first, $\div\ 2$ next)	$2x - 2 + 2 = 8 + 2$ $2x = 10$ $\frac{2x}{2} = \frac{10}{2}$ $x = 5$
4 Check by substitution that you have found the solution.	$\text{LHS} = 5 \times 5 - 2$ $\text{RHS} = 3 \times 5 + 8$ $= 23$ $= 23$ $x = 5$ is the solution
(b) 1 Write the equation.	(b) $2x + 7 = 5x - 8$
2 As the coefficient of the variable on the LHS is less than the coefficient of the variable on the RHS, interchange sides.	$5x - 8 = 2x + 7$
3 Use inverse operations to remove the variable from the RHS of the equals symbol. ($-\ 2x$)	$5x - 2x - 8 = 2x - 2x + 7$ $3x - 8 = 7$
4 Use inverse operations to solve for x. ($+\ 8$ first, $\div\ 3$ next)	$3x - 8 + 8 = 7 + 8$ $3x = 15$ $\frac{3x}{3} = \frac{15}{3}$ $x = 5$
5 Check by substitution that you have found the solution.	$\text{LHS} = 2 \times 5 + 7$ $\text{RHS} = 5 \times 5 - 8$ $= 17$ $= 17$ $x = 5$ is the solution

Worked example 11

W.E. 11

Solve the following equations using algebra.

(a) $3(x + 4) = 2(1 - x)$ **(b)** $\frac{3x + 4}{2} = \frac{2x - 1}{5}$

Thinking	Working
(a) 1 Write the equation.	(a) $3(x + 4) = 2(1 - x)$
2 Expand the brackets first, then simplify.	$3x + 12 = 2 - 2x$

3 Use inverse operations to collect the variable onto the left-hand side of the equals symbol. (+ 2*x* to both sides)

$$3x + 12 + 2x = 2 - 2x + 2x$$
$$5x + 12 = 2$$

4 Use inverse operations to solve for *x*. (− 12 first, then ÷ 5)

$$5x + 12 - 12 = 2 - 12$$
$$5x = -10$$
$$\frac{5x}{5} = \frac{-10}{5}$$
$$x = -2$$

5 Check by substitution that you have found the solution.

$$\text{LHS} = 3(-2 + 4) \qquad \text{RHS} = 2(1 - (-2))$$
$$= 6 \qquad\qquad = 6$$

$x = -2$ is the solution

(b) 1 Write the equation.

(b) $$\left(\frac{3x + 4}{2}\right) = \left(\frac{2x - 1}{5}\right)$$

2 Multiply both sides by the LCD (LCD = 10), expand and simplify.

$$^{5}\cancel{10} \times \left(\frac{3x + 4}{{}_{1}\cancel{2}}\right) = {}^{2}\cancel{10} \times \left(\frac{2x - 1}{{}_{1}\cancel{5}}\right)$$
$$5(3x + 4) = 2(2x - 1)$$
$$15x + 20 = 4x - 2$$

3 Use inverse operations to collect the variable onto the left-hand side of the equals symbol. (− 4*x* from both sides)

$$15x - 4x + 20 = 4x - 4x - 2$$
$$11x + 20 = -2$$

4 Use inverse operations to solve for *x*. (− 20 first, then ÷ 11)

$$11x + 20 - 20 = -2 - 20$$
$$11x = -22$$
$$\frac{11x}{11} = \frac{-22}{11}$$
$$x = -2$$

5 Check by substitution that you have found the solution.

$$\text{LHS} = \frac{3 \times -2 + 4}{2} \qquad \text{RHS} = \frac{2 \times -2 - 1}{5}$$
$$= -1 \qquad\qquad = -1$$

$x = -2$ is the solution

- Try to organise your equation so the variable is on the left-hand side and its coefficient is positive.
- If brackets are in the equation, then try to remove them first, by dividing both sides by a common factor or by expanding the brackets.
- If a fraction is in the equation, always multiply both sides by the LCD (lowest common denominator) and simplify the result.

Solving with graphs

You can use graphs to solve equations when the variable appears on both sides of the equals symbol by:

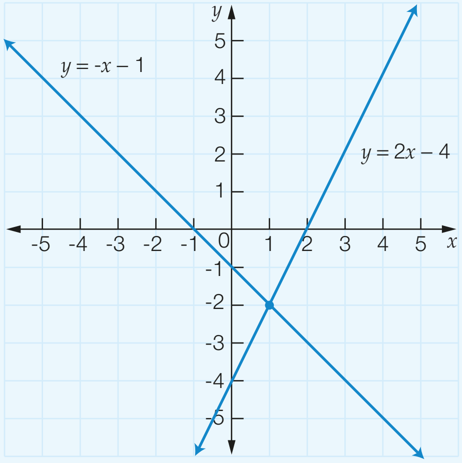
- equating the expression on each side of the equals symbol to *y*
- sketching the graph of each linear equation
- locating the point of intersection.

The *x*-coordinate of the **point of intersection** of the two graphs gives you the solution of the original equation. It is the only *x*-value that makes the original equation true.

Worked example 12

W.E. 12

Solve $-x - 1 = 2x - 4$ graphically by finding the point of intersection of relevant graphs.

Thinking	Working
1 Because the expressions on each side of the equation equal each other, you can equate both of them to y. Write each expression in the form of $y = mx + b$.	$y = -x - 1$ and $y = 2x - 4$
2 Graph each linear equation on the same Cartesian plane (you can use by-hand techniques or technology).	
3 Find the point of intersection and read the values for x and y.	Point of intersection is at (1, -2). Therefore, $x = 1$ and $y = -2$.
4 The x-value is the solution of your equation, so write this as the solution.	$x = 1$ is the solution
5 Check by substitution that you have found the solution. Substituting the x-value found will give the value of each expression and will give the y-coordinate of the point of intersection.	LHS $= -1 - 1$ $= -2$ RHS $= 2 \times 1 - 4$ $= -2$ $x = 1$ is the solution

- Finding the point of intersection of two linear graphs lets you solve equations where the variable appears on both sides of the equation, as it is the only point that satisfies both equations.
- The y-coordinate of your graph gives the value of the expression on both sides of the equals symbol.
- Some equations have no solutions. For instance, $2x + 4 = 2x - 7$ has no solution. Graphing $y = 2x + 4$ and $y = 2x - 7$ will show that they are parallel lines and so do not intersect.

Two lines that are parallel to each other have the same gradient, so they will never intersect. The gradient is related to the coefficient of the variable, so there is no solution if the coefficients of the variable are the same on both sides of the equation.

7.4 Solving equations with the unknown on both sides

Navigator

Answers p. 660

1 (columns 1–2), 2 (columns 1–2), 3 (column 1), 5, 6, 8, 12, 13, 17	1 (column 2), 2 (columns 1–2), 3 (column 2), 4, 5, 6, 7, 8, 11, 13, 14, 16, 17, 18	1 (column 3), 2 (column 3), 3 (column 3), 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 18

Equipment required: graphing calculator or software can be used for Question **3**

Fluency

W.E. 10

1 Solve the following equations using algebra.

(a) $3x - 4 = 2x$
(b) $2x - 3 = x + 4$
(c) $3a - 5 = a + 7$
(d) $5d - 4 = 2d + 17$
(e) $5e - 3 = 2e + 9$
(f) $9f + 5 = 5f - 3$
(g) $3g + 12 = 2g + 5$
(h) $2h - 7 = h - 13$
(i) $4x - 3 = 7 - x$
(j) $k + 6 = 9 - k$
(k) $10j + 1 = 25 - 2j$
(l) $3y - 22 = -4 - 3y$
(m) $2m + 5 = 4m - 3$
(n) $5 + 4n = 7n - 1$
(o) $7x - 4 = 9x - 6$
(p) $2 - y = 3 - 2y$
(q) $p + 16 = 19 - 2p$
(r) $6 + 3x = 21 - 2x$
(s) $6 - 5p = 30 - p$
(t) $14 - 5x = 5 - 2x$
(u) $17 - 15a = 39 - 4a$

W.E. 11

2 Solve the following equations using algebra.

(a) $3(x - 2) = 2x + 1$
(b) $4(x - 1) = x + 8$
(c) $5(x + 3) = 2x + 9$
(d) $5(2x + 3) = 2(3x + 1)$
(e) $7(x + 4) = 3(x - 4)$
(f) $4(3x - 2) = 3(2x + 9)$
(g) $2(3x + 7) = 3(5 - x)$
(h) $3(2x + 1) = 4(3 - x)$
(i) $4(2x + 3) = 3(4 - 5x)$
(j) $x = \frac{2}{5}(x + 1)$
(k) $\frac{4x + 2}{3} = \frac{3x - 4}{5}$
(l) $\frac{5x + 1}{3} = \frac{5x - 2}{4}$
(m) $\frac{3x + 1}{5} = \frac{2x + 7}{3}$
(n) $\frac{3x + 2}{4} = \frac{4x + 1}{5}$
(o) $\frac{2x + 5}{4} = \frac{4x + 1}{3}$

W.E. 12

3 Solve the following equations graphically by finding the point of intersection of relevant graphs.

(a) $x + 6 = -x$
(b) $5x - 2 = 2x + 7$
(c) $2x - 6 = -2x + 10$
(d) $-2x + 7 = x - 5$
(e) $5x - 1 = 3x + 7$
(f) $x - 2 = 2x + 7$
(g) $7x - 2 = 2x + 3$
(h) $5x - 1 = -x + 11$
(i) $-2x + 1 = 4x - 5$

4 **(a)** $9x + 4 = 2x - 3$ has the solution:

A $x = 1$ **B** $x = -1$ **C** $x = 2$ **D** $x = 4$

(b) $2y + 7 = -8 - 3y$ has the solution:

A $y = 2$ **B** $y = 1$ **C** $y = -2$ **D** $y = -3$

(c) $4(3k - 2) = 5(k + 4)$ has the solution:

A $k = 3$ **B** $k = -2$ **C** $k = 1$ **D** $k = 4$

Understanding

5 **(a)** Write an expression to show 11 subtracted from four times a certain number.

(b) Write an expression to show two times the number in part **(a)** subtracted from 7.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation you formed to find the unknown number.

6 **(a)** Write an expression to show 36 added to a certain number.

(b) Write an expression to show the number in part **(a)** being added to 6 and the result doubled.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation you formed to find the unknown number.

7 **(a)** Write an expression to show five less than three-quarters of a certain number.

(b) Write an expression to show twice the result of subtracting ten from the number in part **(a)**.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation you formed to find the unknown number.

8 If 10 is added to a number, the result is 2 less than 3 times the number. What is the number?

9 The denominator of a fraction is 1 more than its numerator. If you add 5 to the numerator, then the fraction evaluates to 3. What is the fraction?

10 **(a)** Show that (-3, 2) is the point of intersection of $y = 2x + 8$ and $y = -x - 1$.

(b) Write an equation without y that is equivalent to the two equations in the first part.

(c) Without using algebra or doing any calculations, write the solution to your equation.

(d) What does the y-coordinate of the point (-3, 2) tell you about your equation?

Reasoning

11 Find the value of x in the following magic square. The rule for any magic square is that the sum of each row, column and diagonal are equivalent (or add to the same value).

$x + 3$	$x + 4$	$x - 1$
$x - 2$	6	$x + 6$
9	x	$x + 1$

12 Chan's current age will be doubled after 5 years. How old is Chan now?

13 Cherry spent $50 of her savings on a dress. If she is left with two thirds of her savings, then how much savings did she have at the start?

14 Sarah has $20 more than Frank, but $40 less than Bruce. If Bruce has $10 more than twice the amount that Frank has, write an equation to find how much money each person has.

15 Solve $2(x - 5) = \frac{x + 5}{3}$ by using graphs.

16 Three times Theo's age a year ago is the same as twice his age in five years time. Write this information as an equation and then find Theo's age.

Let s = the amount of money Sarah has.

Open-ended

17 Make up three equations similar to those in Question **1**, where the solution is a positive whole number.

18 Amy solves the equation $6(x + 6) = 2(x - 4)$ as follows.

$6x + 36 = 2x - 4$ Step 1
$4x + 36 = -4$ Step 2
$4x = 32$ Step 3
$x = 8$ Step 4

(a) At which steps did Amy make a mistake? Describe how you would correct this mistake.

(b) Solve the equation. Remember to check your answer.

Puzzle

Magic algebra squares

In a magic algebra square, the sum of each row, column and diagonal must be equal.

Use these magic squares to answer the questions below.

4	9	a
$a + 1$	5	
$a + 6$	$a - 1$	6

1 **(a)** Find the value for a.

(b) Find the missing value for the empty square.

(c) Substitute the value you found for a into the magic square. Check that the sums of each row, column and diagonal are equal.

$x + 1$	$x - 10$	$x + 6$
	$x - 1$	
		$x - 3$

2 **(a)** Copy and complete the magic square by writing the missing values in terms of x.

(b) Find the value for x if the magic sum is 9.

(c) Substitute the value you found for x into the magic square. Check that the sums of each row, column and diagonal are equal.

$c + a$	$c - a - b$	
		$c + a - b$

3 **(a)** Complete the magic square in terms of c, assuming that the magic sum is $3c$.

(b) Find the values of a, b and c if the magic sum is 30, $c + a = 12$ and $c - b = 15$.

(c) Substitute the values you found for a, b and c into the magic square. Check that the sums of each row, column and diagonal are equal.

Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

How tricky was that!

You have probably seen entertainers claiming to be able to read minds. Here, you will see how you can read someone else's mind, or so it seems …

Step 1 Get a person to think of a number from 1 to 10. Then tell them to do the following steps without telling you the results.

Step 2 Double the number and then add ten.

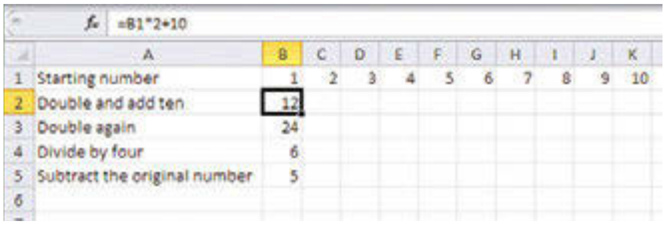
Step 3 Double the result.

Step 4 Divide the new result by four.

Step 5 Subtract the original number.

You should then wave your hands about a bit before announcing that the answer is 5. Watch the amazed look on their face.

You can set up a spreadsheet to see how these steps will always give an answer of 5.



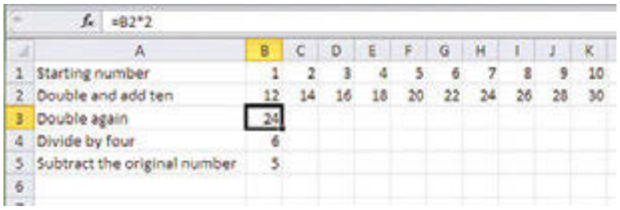
fx =B1*2+10

	A	B	C	D	E	F	G	H	I	J	K
1	Starting number	1	2	3	4	5	6	7	8	9	10
2	Double and add ten	12									
3	Double again	24									
4	Divide by four	6									
5	Subtract the original number	5									
6											

1 In row 1, enter the numbers 1 to 10 in columns **B** to **K** as shown.

2 You need to enter a formula into each of the cells **B2**, **B3**, **B4** and **B5** that will do each step, as described in cells **A2**, **A3**, **A4** and **A5**. Because the first step is 'double and add ten', the formula for cell **B2** is **=B1*2+10**, as shown.

Enter appropriate formulas for each of the other steps into the other cells in column **B**. You can then copy these formulas across to the other columns by selecting each row of cells from **B** across to **K**, then selecting the spreadsheet command **Fill Right**.



fx =B2*2

	A	B	C	D	E	F	G	H	I	J	K
1	Starting number	1	2	3	4	5	6	7	8	9	10
2	Double and add ten	12	14	16	18	20	22	24	26	28	30
3	Double again	24									
4	Divide by four	6									
5	Subtract the original number	5									
6											

If you do this correctly, you should see that the last row shows the same result 5 for every column.

How does this work? Algebra can show you how.

Let x stand for the original starting number.

Starting number $= x$
Double and add ten $= 2x + 10$
Double again $= 2(2x + 10)$
Factorise by taking out the common factor $= 2 \times 2(x + 5)$
$= 4(x + 5)$
Divide by four $= \dfrac{4(x + 5)}{4}$
$= x + 5$
Subtract the original number $= x + 5 - x$
Aha! $= 5$

This shows that for any starting number x, the final result must always be 5.

3 Here is another trick like this:

Think of a number from 1 to 10 and triple it. Then, add nine and double that answer. Then, divide by six and take away three. You should be left with the number you started with.

(a) Make a spreadsheet to show how this works.

(b) Use algebra to explain and prove how this works.

4 Do you understand how these tricks work? Here is another trick:

Again, think of a number from 1 to 10. Add two to the number and then double the result. Add five and then add the number you started with. Divide by three, then subtract three more than the number you started with. The answer should be zero.

(a) Make a spreadsheet to show how this works.

(b) Use algebra to explain and prove how this works.

5 (a) Make up your own mind-reading number trick.

(b) Make a spreadsheet to show how this works.

(c) Use algebra to explain and prove how your trick works.

Taking it further

Not much is known about Diophantus, the Greek mathematician who became known as the 'Father of Algebra'. Some scholars believe he lived some time between the years 100 and 400 CE. One of his admirers wrote the following riddle about Diophantus.

Diophantus's youth lasted one sixth of his life. He grew a beard after half more. After another one seventh of his life he married. Five years later he had a son. His son lived exactly half as long as his father and Diophantus died only four years after his son. All of this adds up to the years Diophantus lived.

(Note that the sentence '*He grew a beard after half more*' means half of his youth, not half of his life.)

6 Can you solve the riddle to find the number of years Diophantus lived?

Step 1 You can use a spreadsheet to help solve the riddle. On a new spreadsheet, in cell **A1**, enter a guess for the total number of years Diophantus lived. (For example, try 75.)

Step 2 Reading the riddle, it says that Diophantus's youth was one sixth of his life. In cell **B1**, enter the formula **=A1/6** to calculate the length of Diophantus's youth.

Step 3 Because he grew a beard after half more, enter the formula **=B1/2** into cell **C1** to calculate this time.

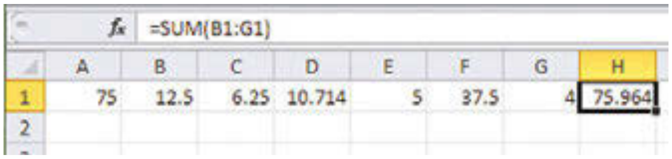
Step 4 As he married after another one seventh of his life, enter the formula **=A1/7** into cell **D1** to calculate this time.

Step 5 As he had a son 5 years later, enter 5 into cell **E1** to show this time.

Step 6 Because Diophantus's son lived exactly half as long as Diophantus, enter the formula **=A1/2** into cell **F1** to calculate this time.

Step 7 As Diophantus died 4 years after his son, enter 4 into cell **G1** to show this time.

Step 8 You can now add all the values from **B1** to **G1** using the formula **=SUM(B1:G1)** in cell **H1**, to calculate the total number of years that Diophantus lived.



fx =SUM(B1:G1)

	A	B	C	D	E	F	G	H
1	75	12.5	6.25	10.714	5	37.5	4	75.964
2								

Step 9 If the answer in **H1** is the same as the guess in **A1**, then your guess must be correct. As you can see, 75 is not the correct answer. Try another guess by changing the value in **A1**. Keep doing this until you guess correctly.

Step 10 Write how old Diophantus was when he died.

7 Write the information in the riddle as an equation. Use algebra to solve this equation, to find exactly how old Diophantus was when he died.

Solving problems using equations

7.5

In everyday life, there are some problems that can be solved with equations. To do this, you need to be able to write equations to represent the problem. If a problem is written correctly as an equation, then you will be able to solve the problem by solving the equation. This is why it is useful to understand equation-solving techniques.

To solve problems using algebraic equations:

Step 1: Identify the variable that needs to be found and represent it with a pronumeral.

Step 2: Use the information to write an equation. To make an equation out of written information, try to focus on key phrases, such as 'is more than', 'is less than' or 'triple the amount', that can be turned into mathematical operations. Remember the correct order of operations and be sure to convert all given amounts to the same units.

Step 3: Use inverse operations to solve the equation.

Step 4: Check that your answer is the solution by substituting it into the original equation and evaluating.

Worked example 13

W.E. 13

For the following situation, write an equation to show the situation, define the pronumeral clearly, and then solve the equation.

Jared has \$6.40 to spend on lollipops that cost 40 cents each. How many lollipops can he buy?

Thinking	Working
1 Identify the variable and represent it with a pronumeral.	Let n represent the number of lollipops.
2 Use the information to write an equation ensuring that you convert all given amounts to the same units. (\$6.40 = 640c)	$40n = 640$
3 Use inverse operations to solve the equation. ($\div$ 40)	$\frac{40n}{40} = \frac{640}{40}$ $n = 16$ Jared can buy 16 lollipops.
4 Check that your answer is the solution.	$\begin{aligned} \text{LHS} &= 40n \\ &= 40 \times 16 \\ &= 640 \\ &= \text{RHS} \end{aligned}$ Therefore, $n = 16$ is the solution.

Worked example 14

W.E. 14

A repair company charges a fixed travel fee of \$50 and an hourly rate of \$30 for each job. If h is the number of hours that a job takes:

(a) write an expression for the total cost of the job.

(b) Use an equation to find the number of hours the job took if the total cost of the job was \$200.

Thinking	**Working**
(a) 1 Identify the unknown and represent it with a pronumeral.	(a) Let h represent the number of hours.
2 Form an expression around the pronumeral.	$30h + 50$
(b) 1 Form an equation using the information given.	(b) $30h + 50 = 200$
2 Use inverse operations to solve the equation. ($-$ 50 first, then $\div$ 30)	$30h + 50 - 50 = 200 - 50$ $30h = 150$ $\frac{30h}{30} = \frac{150}{30}$ $h = 5$ The job takes 5 hours.
3 Check by substitution that you have found the solution.	Substitute $h = 5$ into the original equation. $\text{LHS} = 30h + 50$ $= 30 \times 5 + 50$ $= 150 + 50$ $= 200$ $= \text{RHS}$ Therefore, $h = 5$ is the solution.

7.5 Solving problems using equations

Navigator

Answers p. 661

1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 17, 19, 23, 29, 31	1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 23, 29, 31	1, 2, 3, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31

Fluency

W.E. 13

1 For each of the following situations, write an equation to show the situation, define the pronumeral clearly, and then solve the equation.

(a) Elena buys 3 apples for \$1.17. What is the cost of each apple?

(b) The mass of a shark pup is 935 kg less than that of its mother. If the pup has a mass of 9 kg, what is its mother's mass?

(c) A bus has seats for 49 passengers. If 21 passengers are sitting on the bus, how many seats are still available?

(d) After a cool change, the outside temperature fell 9 °C to reach a temperature of 27 °C. What was the outside temperature just before the cool change?

W.E. 14

2 Darren walked to the park while Ann rode her bicycle along the bike path to the park. If Darren walked x km while Ann rode 7 km, write an expression for the total distance travelled by both, and then use an equation to find the distance walked by Darren if together they travelled a total distance of 11 km.

3 Alan bought a salad roll and a container of orange juice for his lunch.

(a) If the salad roll cost \$$x$ and the orange juice cost \$2, write an expression for the total cost of Alan's lunch.

(b) Use an equation to find the cost of the salad roll if Alan spent \$4.50 on his lunch.

4 Olivia spent $\frac{3}{4}$ of her weekly pay from a part-time job on a new shirt.

(a) If Olivia's weekly pay is \$$w$, write an expression for the cost of the shirt.

(b) If the shirt cost \$24, solve an equation to find how much Olivia is paid each week.

5 Alex has four more toys than Raj. If there are 12 toys in total, how many toys do they each have?

6 The Kittens basketball team won 14 games more than it lost during the last season.

(a) If the team lost x games during the season, write an expression for the total number of games the team played.

(b) Use an equation to find the number of games won and lost by the Kittens if they played a total of 20 games in the season.

7 **(a)** Daniel is travelling at a speed of 28 km/h on his bicycle. How far does he travel in 2.5 hours if he maintains this speed?

If d represents the distance (in km) Daniel travels in 2.5 hours, the equation to solve is:

A $d + 2.5 = 28$ **B** $\frac{d}{2.5} = 28$ **C** $d = \frac{2.5}{28}$ **D** $2.5d = 28$

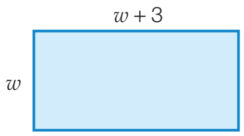
(b) Melita buys 5 cakes from the local bakery and receives \$5.50 change from \$10. Find the cost of each cake.

If c represents the cost of a cake (in cents), the equation to solve is:

A $5c + 10 = 5.5$ **B** $5c + 550 = 1000$ **C** $5c + 1000 = 550$ **D** $5c + 550 = 10$

(c) The perimeter of a rectangular swimming pool is 14 m. The pool is 3 m longer than it is wide, as shown in the diagram opposite.

If w represents the width of the pool (in m), the equation to find the width of the pool is:



A $w + 3 = 14$ **B** $4w + 3 = 14$ **C** $4w + 14 = 6$ **D** $4w + 6 = 14$

8 Six friends share a packet of trading cards equally. After dividing them up, there is one trading card left over.

(a) If each person received x trading cards, write an expression for the total number of trading cards in the bag.

(b) Use an equation to work out how many trading cards each person received if there were 25 trading cards in the packet.

9 The sum of two numbers is 48. If one number is double the other, then what are the two numbers?

10 **(a)** Write an expression for the perimeter of a rectangle, where l is the length and w is the width, and use this expression to solve each of the following problems.

(b) Cameron has a 10 m length of wire fencing to fence a rectangular vegetable garden. If the width must be 2 m, how long can the vegetable garden be?

(c) Cameron finds an extra 5 m of wire fencing, so instead of having a width of 2 m, the vegetable garden can now have a width of 3 m. Find the new length of the vegetable garden.

For each of the following questions, form an equation and solve it to find the answer.

11 A birthday card and wrapping paper cost a total of $5.65. If the card costs $2.90, how much does the wrapping paper cost? (Write 5.65 on the RHS of your equation.)

12 Samir has $70. At a sale, he buys three books (marked at the same price) and receives $11.50 change. How much did each book cost? (Write 70 on the RHS of your equation.)

13 Megan buys three ice-creams and four drinks for her family at a cost of $12. If each drink costs $1.20, find the cost of each ice-cream. (Write 12 on the RHS of your equation.)

Understanding

14 Tina has four times as many red footballs as she has yellow footballs. If she has 20 footballs altogether, then how many footballs of each colour does she have?

15 Shane travels to his friend's house in Tasmania. He travels three fifths of the distance by ferry, one third of the distance by bus and the remaining 5 km by car. What is the total distance travelled?

16 It takes Sasha two hours and twelve minutes to drive between two towns while travelling to Queensland. The computer in the car told her that her average speed was 97 km/h. Sasha wants to know the distance travelled between the two towns.

(a) Write an equation to show this situation, using d to represent the distance in km.

(b) Find the distance that Sasha travelled between the two towns.

For each of the following, write an equation and solve it to find the answer.

17 A farmer needs to fence a rectangular pig pen, but must use 4 layers of wire at different heights to keep her piglets in. She uses a 110 m roll of wire and has 2 m of wire left over. Find the perimeter of the pig pen.

18 Jack spent 7 hours at the beach. If he relaxed in the shade reading for three more hours than he surfed, then for how much time did he surf? Hence, find the time he spent reading.

19 The perimeter of a rectangular block of land is 74 m. The width is 7 m less than the length. Find the dimensions of the block of land. (First, draw a diagram.)

20 Natalie bought 5 cartons of milk and received $2 in change. If she gave $10 to the shopkeeper, find the cost of each carton of milk.

Reasoning

21 Yvonne buys a car that costs $23 000. She pays a deposit of $14 000 and then arranges to pay an amount each month for the next three years. Use an equation to find how much each equal monthly repayment should be if she wants to pay off the car in three years.

22 Thirty-six blueberries are shared among three friends. Jin gets four blueberries more than Simon. Steve gets five more than Simon. How many blueberries does Simon get?

23 The sum of three consecutive numbers is 33. What are the numbers?

24 Sandy's mother's age is now four times Sandy's age. Five years from now, Sandy's age will be one third of his mother's age. How old is Sandy now?

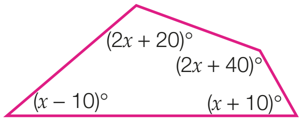
25 At the end of the term, each student in the class was given their average test score. During the term the class did three tests. For two of her tests, Leonie's results were 93% and 85%, but she has forgotten her result in the third test. Use an equation to find this unknown result for Leonie if her average test score was 91%.

26 A rectangle has an area of 27 cm^2. Its width is 4 cm and its length is longer than its width. Write an equation to show this situation, then find the length.

27 A boy is twice as tall as his little sister and 30 cm shorter than his father. The combined height of the three family members is 3.8 m. Find the height (in cm) of each person.

28 The sum of three consecutive even numbers is 102. Use an equation to find the numbers.

29 The angles inside a quadrilateral add to 360°. For the quadrilateral shown, write an equation to find the value of x and then find the value of the four angles.



Open-ended

30 Two plumbers quote for a job to renovate a bathroom. Plumber Alan quotes $1050 for material costs and $50 per hour for labour. Plumber Barry quotes $1200 for material costs and $45 per hour for labour. Choose different numbers of hours and decide which quote would be cheaper for those hours. For what number of hours will the quotes be the same?


Drawing graphs may help you to answer Question **30**.

31 The formula for the area of a triangle is:

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

Identify at least three different sets of base length (b) and height (h) to give an area of 64 m^2.

Problem solving

The perfect holiday

The cost of a cruise ship holiday from Totally Oz Tours is a booking fee of $150 plus a charge of $25 per day. The cost of food is an additional $15 per day. There is also a drinks charge of $100 for the whole cruise.

A rival company, Fun in the Sun Cruises, charges $200 as a booking fee and $45 per day, with no extra charges for food or drink.

(a) For what number of days does each company charge the same total amount?

(b) Under what circumstances will either company be a cheaper option? Explain why.

Strategy options

- Make a model.
- Guess and check.
- Test all possible combinations.

Gamespace

INTO THE MATRIX

"Wow, Alpha, check this out!" says Raydon while pointing at the history text he is reading. "Back in 2020, students used to play 3D computer games. How ancient is that?" "I know," says Alpha, "and did you know that they were outside of the game and had to watch it on a screen?" They are interrupted by Alpha's homework software "Boys, Stop reading and go play *4D Matrix Multi-Game.* You've got a test tomorrow."

Level 1: Equation labyrinth

Alpha is stuck in the Equation Labyrinth. You need to find the correct order of operations to move to the next level.

Example move

$2 = \frac{🤖}{3} - 6$

For the equation above, imagine Alpha is the variable. For Alpha to move, you need to find the operations that will 'solve' for Alpha, in the right order.

In this case, the operations are: + 6, × 3

List the operations (in the correct order) to solve each of the following:

(a) $5 = 3 \times 🤖$

(b) $3 = \frac{🤖}{4}$

(c) $1 = 🤖 + 3$

(d) $7 = 4 \times 🤖 - 7$

(e) $6 = \frac{6 \times 🤖}{-2} - 3$

Alpha needs to escape the maze with the correct code to beat the Level 1 boss.

Use your answers for (a) to (e), in the correct order, to find a path through the maze. You must start in a corner and you cannot move diagonally. The letters in each square you pass through will spell the Level 1 code.

M +7	U −3	Q ÷-2	E ÷6	A ×3
I ÷4	T −7	B +3	L ×-2	T ×6
R −3	I +7	A ÷4	S +6	U ×2
A ×4	W +3	N −3	U +7	N ×4
V ÷3	P ÷4	S +3	Q ÷6	U −5

What is the Level 1 code?

Level 2: Troopers in disguise

The Equation Troopers are hiding in disguise, ready to capture players.

Raydon must hop on the right bricks along the path to avoid being captured.

If you substitute positive whole numbers for x and y into the following statements, then they will either *always* be true, *sometimes* be true or *never* be true.

Equation Troopers are hiding behind the *sometimes* true statements and behind the *never* true statements.

Help Raydon to find them by identifying the safe bricks to complete the level. Join the safe bricks to form a path from Start to Finish.

START	$x < 2x$	$x^2 < x^3$	$x^2 + y^2 < xy$	$2(x + 4) = 2x + 8$	$x + 3 > -4$	FINISH
	$3x > x^2$	$5x + 2 > 2(2x + 1)$	$y - 2 > y - 3$	$5 - y < y - 5$	$x^y < 0$	

Level 3: Who won?

To find who has the highest ★ score, solve the following equations. (The ★ scores must be positive.)

Game scores:

Alpha's score: $5★ + 2 = 158 + 2★$

Raydon's score: $\frac{6★}{7} - 12 + 3★ = 30 + 3★$

Bonus challenge

$\frac{1}{3}\frac{(★^2 - 7)}{2} + 1 = 8$

Maths 4 Real

Virgin Galactic

'A brief moment of quiet before a wave of unimaginable, but controlled, power surges through the craft. You are instantly pinned back in your seat, overwhelmed but enthralled by the howl of the rocket motor and the eye-watering acceleration.'

Virgin Galactic wants to be the first spaceline (an airline that offers trips into space for paying passengers).

Escaping gravity

A rocket needs to reach a certain speed to escape a planet's gravity. Engineers and astronauts need to take this into account when designing and piloting spacecraft.

The formula to find the escape speed (in kilometres per second) is:

$$v = \sqrt{\frac{2b}{r}}$$

where:

v = the escape velocity (the speed, in km/s, that the rocket must reach to escape the planet)

r = the radius of the planet, in kilometres

b = a constant value that depends on the mass and gravitational force of the planet.

1 Calculate the escape velocity, in kilometres per second (km/s), needed to leave these planets.

Planet	b	r (km)
Earth	398 600	6378
Mercury	23 916	2440
Mars	42 828	3397

2 Which planet has the slowest escape velocity? Why do you think this is so?

3 If the escape velocity of Venus is 10.37 km/s, and its b value is 325 156, then what is the approximate radius of Venus, to the nearest km?

Cost of travel

4 If the cheapest tickets on Virgin Galactic cost \$200 000, and the space flight takes $2\frac{1}{2}$ hours, how many dollars per minute does the space flight cost each passenger?

5 Write a formula for the income that Virgin Galactic receives, in dollars (I), in relation to the number of paying passengers n it takes.

6 The rockets are designed to carry six passengers and Virgin Galactic only runs the flight if all the seats are booked.

(a) Using your formula for income, what income will Virgin Galactic make from one flight?

(b) If Virgin Galactic has one flight per week, then what is the income they will make in the first year?

(c) If Virgin Galactic has two flights per day, what is the income that Virgin Galactic will make in one year? (Assume this is a non-leap year.)

(d) Imagine that in one year Virgin Galactic's income is \$38 220 000. How many flights are completed in that year?

7 Building, maintaining and flying rockets costs millions of dollars. Imagine that it costs Virgin Galactic \$960 000 in maintenance, fuel, wages and other costs per flight. Assuming that each flight always carries six passengers, what would be the costs per paying passenger per flight?

8 As well as the costs mentioned in the previous question, Virgin Galactic also expects to have development costs of \$6 million each year. Write a formula for the cost per year (C) that Virgin Galactic would have, in relation to the number of paying passengers, n.

9 Profit = income − costs. Using your formulas from **5** and **8**, write a formula for Virgin Galactic's yearly profit (P).

10 What is the minimum number of passengers, and hence the minimum number of flights, that Virgin Galactic needs per year to break even (have a profit of \$0)?

Challenge 7

1 Mike is twice as old as his brother Jamie. When Mike turns 15, the sum of his and Jamie's ages will be 25. How old are Mike and Jamie now?

2 R, S, T and U are all whole, positive numbers. If $R \times S = 15$, $S \times T = 3$ and $T \times U = 7$, find the values of $R \times U$ and $S \times U$.

3 If $\frac{4}{h} = \frac{h}{36}$, find the value(s) of h.

4 Amber purchases 5 tyres for her car. If each tyre can be used for 16 000 km, how many kilometres can Amber travel using these five tyres?

5 The four numbers a, b, c and d add to give 30. The number b is double a, c is a third of b and d is 4 more than c. What are the numbers?

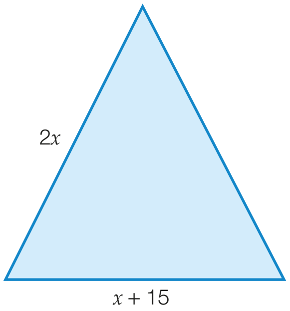
6 A square and an equilateral triangle have equal perimeters. If the side length of the triangle is 1 cm longer than the side of the square, what is the length of the side of the square?

7 If $d + e = 15$ and $e + f = 12$ and $d + e + f = 20$, find d, e and f.

8 The equilateral triangle has sides $2x$ cm and $(x + 15)$ cm as shown. The perimeter of the triangle is:

A 15 cm **B** 30 cm

C 60 cm **D** 90 cm



9 The height of a tree is measured each year. It is x metres high when planted, and grows exactly y metres each year. At its latest yearly measurement it was 23 metres high. Which of the following values are possible?

A $x = 6, y = 2$ **B** $x = 7, y = 5$ **C** $x = 4, y = 7$ **D** $x = 5, y = 3$

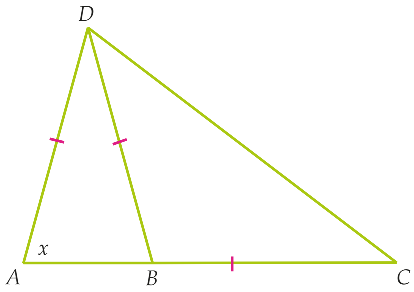
10 In a test, Sophie answered 15 out of the first 20 questions correctly. Of the remaining questions, she answered exactly one-third of them correctly. If all the questions were worth the same number of marks and Sophie scored 50% on the test, how many questions were in the test?

11 ACD is a triangle in which B lies on the side AC so that $AD = DB = BC$. $\angle DAC = x$. All angles in the diagram are a positive whole number of degrees.

(a) Find the value of x for which $\angle ADC$ has its largest value.

(b) What is the largest value of $\angle ADC$?

(c) What is the smallest value of $\angle ADC$?



Chapter review 7

Maths literacy

- backtracking
- balanced
- equivalent
- flowchart
- guess, check and improve
- inspection
- inverse operation
- point of intersection
- solution
- solving

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The ___________ of an equation is the value of the variable that makes the equation true.

2 A ___________ is a way of solving an equation by using a diagram and then ___________ to find the solution.

3 When you solve an equation, you do the same operations on each side of the equals symbol so that the equation is ___________.

4 Two equations are said to be ___________ if they have the same solution.

5 The ___________ of addition is subtraction.

Fluency

1 Write an equation for each of these sentences. **7.1**

(a) A number is multiplied by three, then four is subtracted to give a result of two.

(b) A number is added to two, then the answer is divided by 5 to give a result of six.

2 Draw a flowchart and use backtracking to solve the following equations. **7.2**

(a) $4x + 3 = 11$ (b) $3(x - 2) = 9$ (c) $\frac{x + 5}{4} = 2$ (d) $\frac{x}{7} - 6 = -3$

3 Solve each of the following by doing the same operations on both sides of the equation. **7.2**

(a) $2x + 5 = 3$ (b) $11 - 3b = 5$ (c) $19 = -4c - 1$

4 (a) The solution to the equation $3x - 2 = 7$ is: **7.2**

A $x = 1$ B $x = 2$ C $x = 3$ D $x = 4$

(b) The solution to the equation $\frac{x}{2} + 1 = -2$ is:

A $x = -2$ B $x = -3$ C $x = -4$ D $x = -6$

(c) The solution to the equation $\frac{x - 4}{4} = 2$ is:

A $x = 12$ B $x = 24$ C $x = -4$ D $x = 8$

5 Solve each of the following by doing the same operations on both sides of the equation. **7.2**

(a) $\frac{a}{3} + 7 = 10$ (b) $\frac{b - 6}{5} = -1$ (c) $\frac{3c}{2} = 12$

6 Solve each of the following equations using algebra. **7.3**

(a) $\frac{4x - 1}{3} = 5$ (b) $6 - \frac{x}{5} = 1$ (c) $\frac{5x}{4} + 3 = 13$ (d) $\frac{1 - 2x}{7} = 3$

7 Solve each of the following equations using algebra. **7.3**

(a) $\frac{3m + 4}{2} = 5$ (b) $3 - \frac{2n}{5} = 1$ (c) $4(x + 5) = 32$ (d) $\frac{3(m + 11)}{8} = -6$

8 Solve each of the following equations using algebra. **7.4**

(a) $7x + 2 = 6x - 5$ **(b)** $3x + 7 = 15 - 5x$

(c) $7(x + 1) = 2(x - 4)$ **(d)** $3(5x + 4) = 4(2 - 3x)$

9 Tony has \$25 and saves \$5 a week. Use an equation to find how many weeks it will take him to save \$80. **7.5**

10 Use equations to solve each of the following problems. **7.5**

(a) Janis gives a \$10 note to a shopkeeper to pay for 5 cans of cola. \$2.25 in change is given back. Find the cost of each can of cola.

(b) Almira has a hot dog stand at the school fete. The ingredients for the hot dogs cost \$60. If Almira sells each hot dog for \$3, how many does she need to sell to make a profit of \$24?

(c) A stick 1 m long is broken into two parts. If one piece is 9 cm longer than the other, how long are the pieces?

Understanding

11 Sam buys a new suitcase that is 70 cm wide. He discovers that two of his folders (which are identical) will fit next to each other in the suitcase with only 5 cm across to spare. **7.3**

(a) Write an equation to describe this situation, using f to represent the width of the folder.

(b) Find the width of the folder.

12 Three consecutive numbers add to 24. Use an equation to find the three numbers. **7.2**

13 Robert has 50 coins, all 5-cent coins and 10-cent coins, adding to a total of \$3.50. Use an equation to find how many of each type of coin he has. **7.3**

14 Use an equation to find x in the magic square shown. As with all magic squares the sum of each row, column and diagonal is the same. **7.3**

$4x$	$6x + 2$	28
$6x - 2$	30	$2(2x + 1)$
$4(x + 2)$	$4x - 2$	$6x$

15 If Grace is 7 years older than Harrison and Hayley is twice as old as Grace, find how old Grace is if the mean of their ages is 19. **7.5**

Reasoning

Use equations to solve each of the following.

16 **(a)** In 5 years time, Maja's age will be twice what it was 6 years ago. How old is Maja now? **7.1, 7.2, 7.3**

(b) Jake rides a motorbike. He travels to and from work three times in a week. He also travels to and from his girlfriend's place, which is one third of the distance to work, four times a week. In total, he travels 156 km each week. How far is it from his home to work?

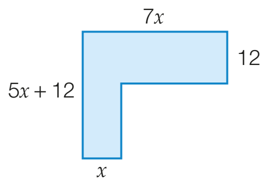
(c) Jung spent a quarter of his money on a new shirt. He then gave a donation to charity of \$3 and deposited half of the remaining money in the bank. If he is left with \$15, how much did he start with? Write an equation first using s to represent the starting money.

17 The sum of two numbers is 37. If the larger number is divided by the smaller number, the result is 3, remainder 5. Find the numbers. **7.3**

18 An isosceles triangle has a perimeter of 45 cm. Its equal sides each measure 6 cm longer than the non-equal base. How long is the base of this particular triangle? **7.4**

19 Three students take different routes home from school. Mark needs to walk twice as far as Tom, and Fred walks four times as far as Tom. If the three walk a total distance of 3.5 km (or 3500 m), how far does each person walk? **7.5**

20 The shape at right has a perimeter of 120 cm. Use the information provided in the question to generate an algebraic equation, and then use it to find the value of x cm.



7.5

21 **(a)** Using the magic square, find the magic sum in terms of a, b and c. Hence, find the expressions that belong in Box A and in Box B. Remember that the sum of each row, column and diagonal is the same.

(b) If $a = 3$, $b = 2$ and the magic sum is 27, use an equation to find c. Hence, replace the pronumerals with numbers in each of the boxes.

$a + b$	$a + 2c$	$a + c + 2b$
$a + 2b + 2c$	$a + c + b$	Box A
$a + c$	Box B	$a + 2c + b$

7.3

22 A triangular garden bed has its three sides measured at x metres, $(x + 4)$ metres and $(x + 5)$ metres. The perimeter of the garden bed is measured to be 51 metres. Find the lengths of all three sides of the garden bed.

7.5

Numeracy practice 7

Non-calculator

1 What is the value of p in this equation? $13p - 8 = 8p + 7$

2 What is the equation that best represents the following?

'I am thinking of a number—when I add 4 the number is 13'

A $x + 13 = 4$ **B** $x - 4 = 13$ **C** $x + 4 = 13$ **D** $x - 13 = 4$

3 What is the value of x in the equation $4x + 7 = 3$?

A $x = -1$ **B** $x = 1$ **C** $x = 2$ **D** $x = 11$

4 If $y = 2x - 1$ and $y = 3x + 2$, which value of x satisfies both of these equations?

A $x = -3$ **B** $x = -1$ **C** $x = 1$ **D** $x = 3$

5 The digits 4, 5 and 6 are placed in the equation $2x + y - z = 8$. If y is 4, what is the value of z?

Calculator allowed

6 Robert and Fergus shared a bag of grapes. If Fergus ate 18 more grapes than Robert did, and there were 66 grapes in the bag, how many grapes did Fergus eat?

7 The solution to the equation $6x - 1 = 4x + 5$ is:

A $x = -2$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

8 Find the value of x that makes the equation $\frac{x + 3}{4} = 5$ true.

9 The cost in dollars to make k cakes is $8 + 3k$. How many cakes are made for a cost of \$203?

10 Phil is 3 years older than his sister Bridget. The product of their ages is the same as their mother's age. If Phil's mother is 40 years old, how old is Bridget?

8

Geometry

8

What do you use to build a better stadium? Triangles, of course. Melbourne's newest stadium features the humble triangle in its award-winning design.

The Melbourne Rectangular Stadium, known as AAMI Park, is Melbourne's most innovative sports stadium. It hosts soccer and rugby matches as well as other events. The stadium seats 30 000 people and has many training facilities, which are planned to include gyms, a pool and a boxing ring.

The architects used a design made of geodesic domes, which are spherical shapes made from small equilateral triangles. These triangles give the structure strength and stability, and are very light. This design also uses 50 per cent less steel than a typical stadium roof of the same size.

The geodesic domes act as a roof, walls and supports, without the need for extra pillars or walls. This means that spectators can enjoy the events inside the stadium with a clear view from wherever they are sitting. The design also allows for the triangular panels to be changed into solar panels in the future. This would make it possible for the stadium to generate its own electricity in a sustainable way.

Forum

Why do architects use triangles in their designs?

Can you think of other buildings that have triangles in their design?

What makes a triangle such a strong shape?

If you were designing a stadium, what factors would you need to take into account?

Why learn this?

Don't underestimate the power of a simple triangle! A knowledge of geometry helps you to accurately describe the world and the natural and artificial objects around you. Understanding geometric principles allows people to design and build things that are strong, efficient and safe. This chapter will help you develop your skills in identifying and using the properties of triangles and quadrilaterals.

After completing this chapter you will be able to:

- revise and develop skills in identifying and applying angle and side properties of triangles and quadrilaterals
- use geometric symbols and notation
- use transformations to produce congruent shapes
- identify corresponding sides and angles in congruent figures of different orientations
- use the four congruency tests to show that triangles are congruent
- use congruent triangles and angle properties to prove properties of special quadrilaterals.

Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

Equipment required: protractor, ruler, grid paper

1 Draw angles of the following sizes using a protractor and a ruler.

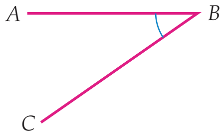
(a) 48° **(b)** 127° **(c)** 235° **(d)** 315°

2 Name the type of each angle drawn in the previous question, using words from this list: acute, right, obtuse, straight, reflex.

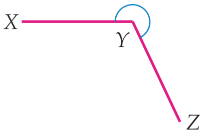
3 **(a)** Use a protractor to measure the following two angles.

(b) Name the type of each angle.

(i)



(ii)

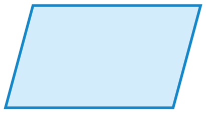


4 Solve for x.

(a) $x + 7 = 180$ **(b)** $2x + 40 + 3x = 360$ **(c)** $x + 20 + 5x - 80 = 360$

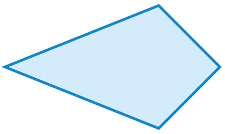
5 Copy the quadrilaterals below into your workbook. Give them their specific name and mark equal side lengths, parallel sides and equal angles.

(a)



(b)

(c)

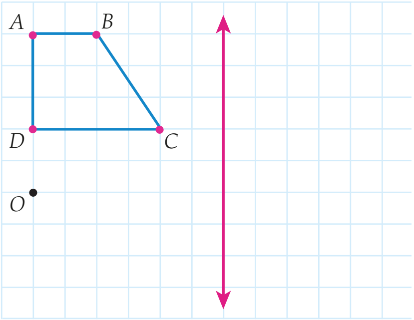


6 Use grid paper to do the following transformations on the shape *ABCD*. Draw the transformed shape and label it *A′B′C′D′*.

(a) A translation of 5 units right and 3 units down.

(b) A reflection in the vertical line shown.

(c) A rotation of 90° clockwise around the point *O*.



Exploration Task

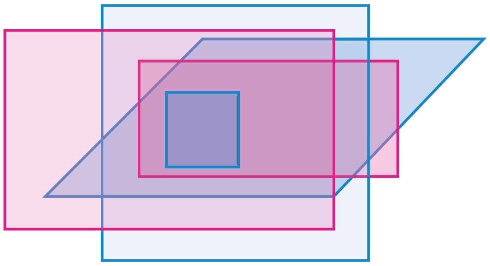
You can download this activity from the eBook or the Pearson Places website.

Squares, quadrilaterals, rectangles, parallelograms

In this activity, you will consider the following statements:

- All squares are rectangles.
- Only some rectangles are parallelograms.
- Only some squares are parallelograms.

Do you agree? Explore the properties of different shapes to understand this.

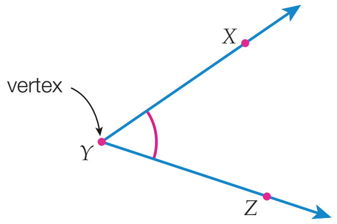


8.1

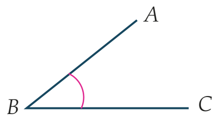
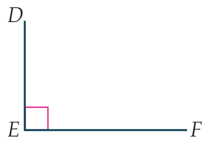
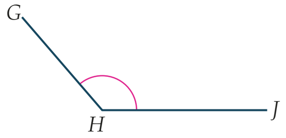
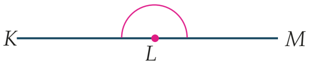
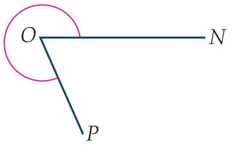
Angles review

An angle is formed when two lines (or rays, or line segments) touch or intersect at a point, which is called the **vertex**. An angle can be named using the symbol $\angle$ and the letters that mark three points that form the angle.

For example, the angle at right is called $\angle XYZ$:



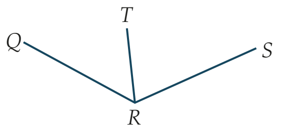
Angles at a point

Acute angle	Greater than 0° but less than 90° $0^\circ < \angle ABC < 90^\circ$	
Right angle	Exactly 90° $\angle DEF = 90^\circ$	
Obtuse angle	Greater than 90° but less than 180° $90^\circ < \angle GHJ < 180^\circ$	
Straight angle	Exactly 180° $\angle KLM = 180^\circ$	
Reflex angle	Greater than 180° but less than 360° $180^\circ < \angle NOP < 360^\circ$	
Revolution or angle of revolution	Exactly 360°	

Pairs of angles

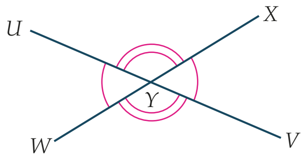
Adjacent angles share a vertex and share an arm.

Vertically opposite angles are equal.



$\angle QRT$ and $\angle TRS$ are a pair of adjacent angles.

TR is the arm shared by both angles.



The lines UV and WX intersect at Y.

$\angle UYW = \angle XYV$ as they are a pair of vertically opposite angles.

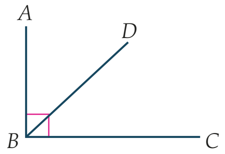
$\angle UYX = \angle WYV$ as they are a pair of vertically opposite angles.

Complementary angles are adjacent angles that add to 90°.

Each angle is called the **complement** of the other angle.

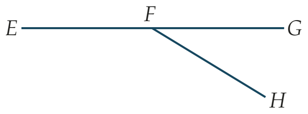
Supplementary angles are adjacent angles that add to 180°.

Each angle is called the **supplement** of the other angle.



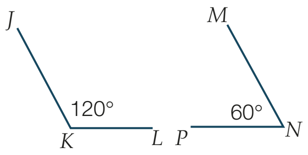
$\angle ABD + \angle DBC = 90^\circ$

$\angle ABD$ and $\angle DBC$ are complementary angles.



$\angle EFH + \angle GFH = 180^\circ$

$\angle EFH$ and $\angle GFH$ are supplementary angles.

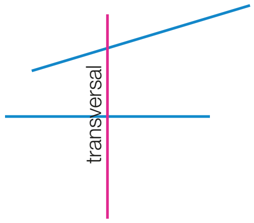


$\angle JKL$ and $\angle MNP$ are supplementary angles.

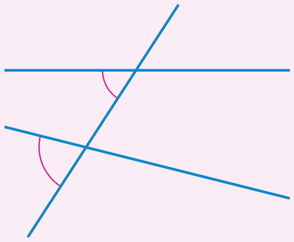
Angles and transversals

A **transversal** is a line that intersects two or more other lines, as shown.

When a transversal intersects two other lines, pairs of angles are formed. These angles are given special names.



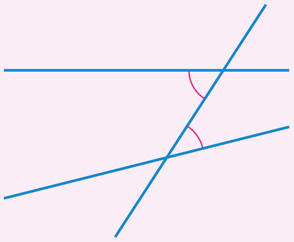
Corresponding angles



Corresponding angles are either above or below the two lines cut by the transversal, on the same side of it.

'Corresponding' means 'matching', so corresponding angles are in matching positions. Four pairs of corresponding angles are formed when a transversal cuts two lines.

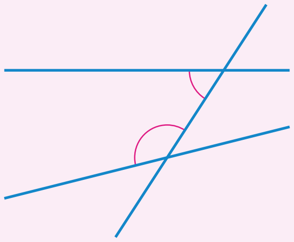
Alternate angles



Alternate angles are between the two lines cut by the transversal, but on opposite sides of it.

'Alternate' can mean 'opposite', so alternate angles swap sides of the transversal. Two pairs of alternate angles are formed when a transversal cuts two lines.

Co-interior (allied) angles

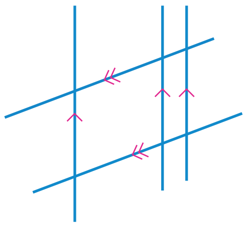


Co-interior angles (also called 'allied angles') are between the lines cut by the transversal, on the same side of it.

'Co' means 'with' and 'interior' means 'inside', so co-interior angles are inside and on the same side with the transversal.

Two pairs of co-interior angles are formed when a transversal cuts two lines.

Parallel lines and perpendicular lines



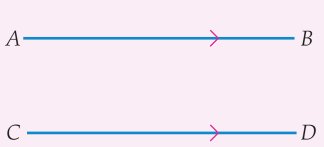
Parallel lines are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. The lines on a page of lined paper are parallel.

Lines that are parallel are marked with an arrow pointing in the same direction. If more than one set of parallel lines appear in a diagram, you can use more than one arrow.

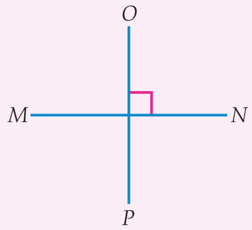
The symbol || means 'is parallel to'.

Perpendicular lines intersect at right angles.

The symbol ⊥ means 'is perpendicular to'.

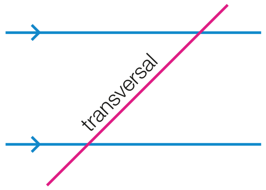


$AB \parallel CD$



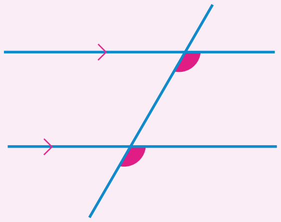
$OP \perp MN$

Angles formed when a transversal cuts parallel lines



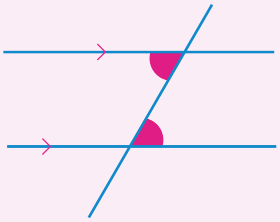
When parallel lines are crossed by a transversal, the pairs of angles made on the parallel lines have special properties.

Corresponding angles



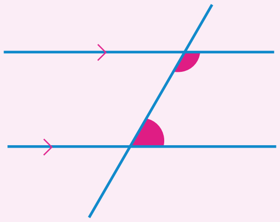
Corresponding angles on parallel lines are *equal*.

Alternate angles



Alternate angles on parallel lines are *equal*.

Co-interior (allied) angles



Co-interior angles on parallel lines are *supplementary*, so the angles add to 180°.

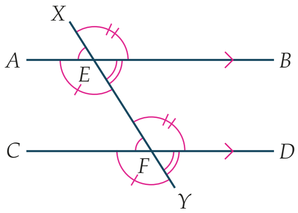
The converse of the above is also true. When two lines are cut by a transversal, if corresponding or alternate angles are equal, or if co-interior angles are supplementary, then the two lines are parallel.

A transversal is a line that intersects two or more other lines.

When a transversal intersects parallel lines, then:

- pairs of corresponding angles are equal
- pairs of alternate angles are equal
- pairs of co-interior angles are supplementary.

The lines *AB* and *CD* are parallel. You can write this as *AB* || *CD*. The transversal *XY* intersects *AD* at point *E* and *CD* at point *F*.



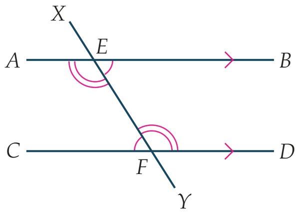
Each of the four angles formed at *E* has an equal corresponding angle at *F*, on the same side of the transversal and the same position above or below the line. Each pair of corresponding angles is marked in the same way.

$\angle AEX = \angle CFE$

$\angle FEB = \angle YFD$

$\angle AEF = \angle CFY$

$\angle DFE = \angle BEX$



Two pairs of alternate angles are formed, on opposite sides of the transversal.

$\angle AEF = \angle EFD$

$\angle BEF = \angle EFC$

Two pairs of co-interior angles are formed, one on each side of the transversal, inside the parallel lines. Co-interior angles are sometimes called 'allied' angles.

$\angle AEF + \angle EFC = 180°$

$\angle BEF + \angle EFD = 180°$

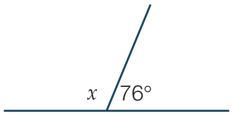
Worked example 1

W.E. 1

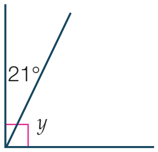
(a) What is the complement of 14°?

(b) What is the supplement of 53°?

(c) Find the value of *x*.



(d) Find the value of *y*.



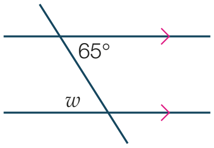
Thinking	Working
(a) Complementary angles add to 90°. Subtract the known angle from 90° to find the unknown.	(a) $90° - 14° = 76°$ The complement of 14° is 76°.
(b) Supplementary angles add to 180°. Subtract the known angle from 180° to find the unknown.	(b) $180° - 53° = 127°$ The supplement of 53° is 127°.
(c) *x* and 76° are supplementary angles. Write an equation that shows this, and solve it.	(c) $x + 76° = 180°$ (supplementary angles) $x = 104°$
(d) *y* and 21° are complementary angles. Write an equation that shows this, and solve it.	(d) $y + 21° = 90°$ (complementary angles) $y = 69°$

Worked example 2

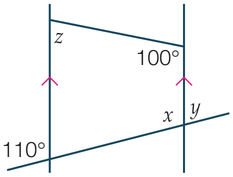
W.E. 2

Find the value of the pronumerals in each diagram. Give reasons for your answer.

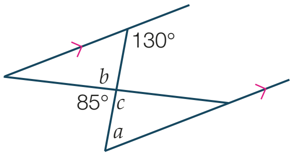
(a)



(b)



(c)



Thinking	Working
(a) Identify the relationship between the known and the unknown angle. Write this in brackets next to your answer as a reason for your answer.	(a) $w = 65^\circ$ (alternate angles on parallel lines)
(b) 1 Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	(b) $x = 110^\circ$ (corresponding angles on parallel lines) $z + 100^\circ = 180^\circ$ (co-interior angles on parallel lines) $z = 80^\circ$
2 Once you have found the value of some unknowns, use them to find others. (Here, use the value of *x* to find *y*.)	$x + y = 180^\circ$ (supplementary angles) $110^\circ + y = 180^\circ$ $y = 70^\circ$
(c) 1 Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	(c) $a + 130^\circ = 180^\circ$ (co-interior angles on parallel lines) $a = 50^\circ$
2 Once you have found the value of some unknowns, use them to find others. (Here, find the value of *c* and use it to find *b*.)	$c + 85^\circ = 180^\circ$ (supplementary angles) $c = 95^\circ$ $b = 95^\circ$ (vertically opposite angles)

8.1 Angles review

Navigator

Answers p. 664

1, 2, 3, 4 (columns 1–2), 5, 6, 7, 9, 10, 15

1, 2, 3, 4 (columns 2–3), 5, 6, 7, 8, 9, 10, 11, 12, 13, 15

2, 3, 4 (column 3), 5, 6, 7, 8, 9, 11, 12, 13, 14, 16

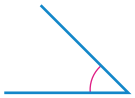
Fluency

1 Match each of the following angle types with its diagram (A–F) and its angle size in degrees (G–M).

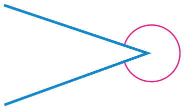
(a) acute
(b) right
(c) revolution
(d) straight
(e) obtuse
(f) reflex

A

B



C



D

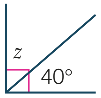
E

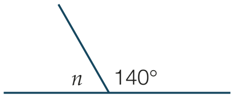
F

G 90°
H 125°
J 360°
K 45°
L 180°
M 280°

W.E. 1

2 **(a)** What is the complement of 37°?
(b) What is the supplement of 105°?
(c) Find the value of z.
(d) Find the value of n.

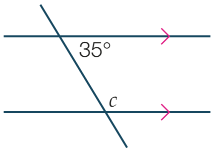




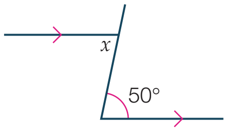
W.E. 2

3 Find the value of the pronumerals in each diagram. Give reasons for your answer.

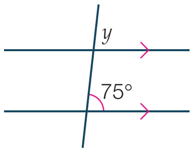
(a)



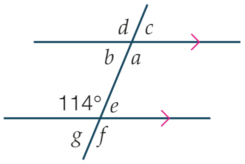
(b)



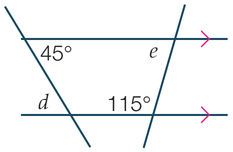
(c)



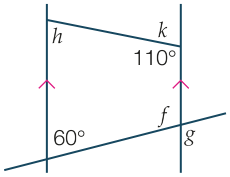
(d)



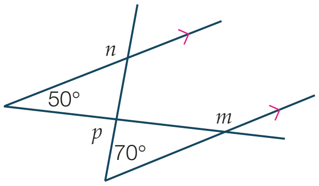
(e)



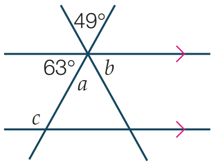
(f)



(g)

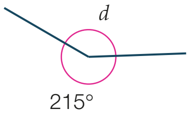


(h)

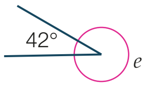


4 Find the value of the pronumerals in each diagram. Give reasons for your answer.

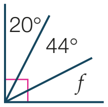
(a)



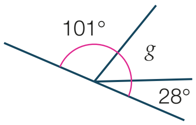
(b)



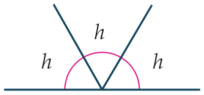
(c)



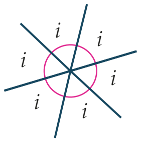
(d)



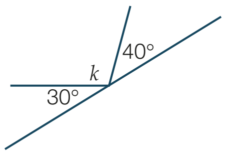
(e)



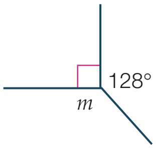
(f)



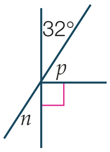
(g)



(h)

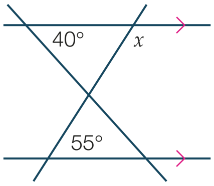


(i)



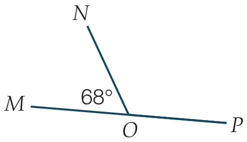
5 In the diagram, the value of x is:

A 85° **B** 95° **C** 125° **D** 140°



6 The value of $\angle NOP$ is:

A 102° **B** 112° **C** 122° **D** 132°



Understanding

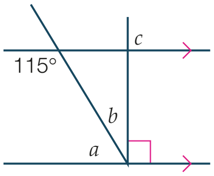
7 State whether the following sets of angles are complementary angles (C), supplementary angles (S) or neither (X).

(a) 30°, 60° **(b)** 50°, 80°, 50° **(c)** 45°, 55°

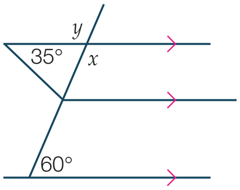
(d) 65°, 45°, 60° **(e)** 33°, 57° **(f)** 122°, 46°, 12°

8 Find the value of the pronumerals in each diagram. Give reasons for your answer.

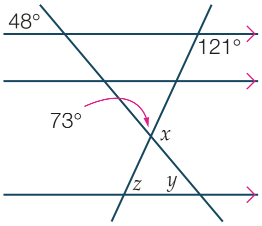
(a)



(b)



(c)



9 Write true (T) or false (F) for each of the following statements.

(a) Vertically opposite angles are always equal.

(b) Adjacent angles are always complementary.

(c) If two adjacent angles are supplementary, then one of the angles is an acute angle.

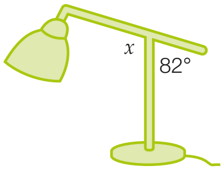
(d) If a pair of adjacent angles forms an angle of revolution, then one of the angles must be a reflex angle.

(e) Corresponding angles on parallel lines are always equal.

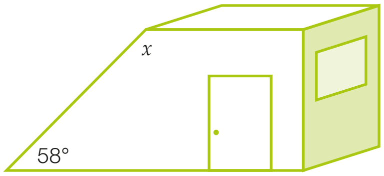
10 Find the missing angle in each of the following.

(a)

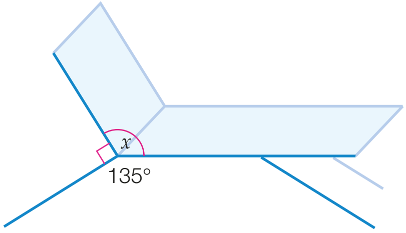
(b)



(c)



(d)

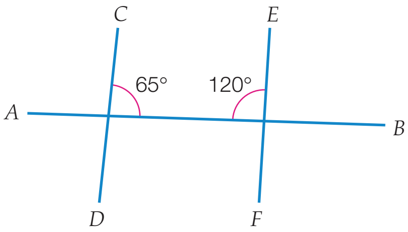


Reasoning

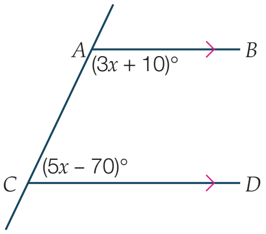
11 c and d are complementary angles. If c is 20° larger than d, what is the size of d?

12 The angles g and h are supplementary angles. g is twice as large as h. Find g and h.

13 Is CD parallel to EF? Give a reason for your answer.



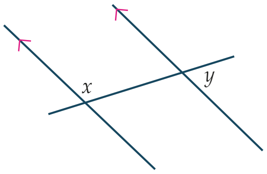
14 Find the value of x, giving reasons for your answer.



Open-ended

15 Write three possible pairs of values for x and y in the diagram shown.

Compare your answers with those of others in your class. What do you notice?

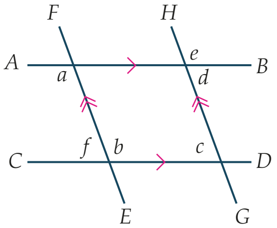


16 In the diagram, $AB \parallel CD$ (AB is parallel to CD) and $EF \parallel GH$ (EF is parallel to GH).

Some of the angles a, b, c, d, e and f are equal.

(a) Write all the pairs of equal angles you can find. Check your answer by substituting a value for a and calculating the values of the other pronumerals.

(b) Write all the pairs of supplementary angles you can find.

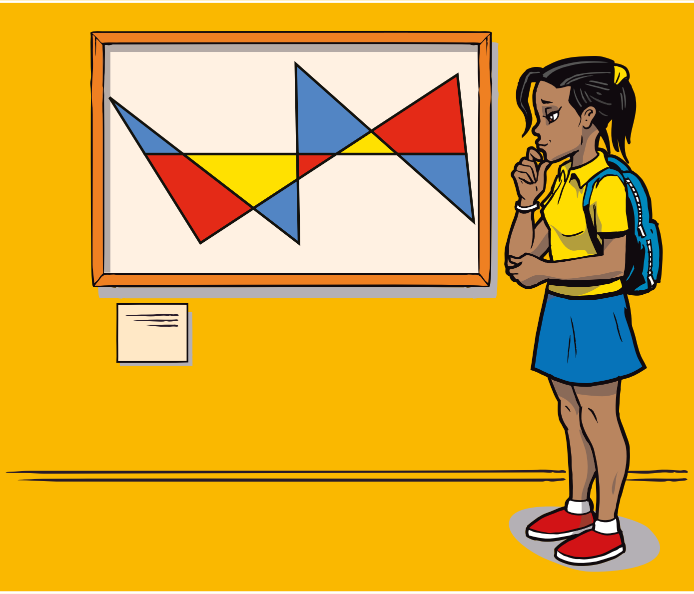


Puzzle

Angle art

Milla studies a painting at an art gallery. She notices that the painting was created using just seven straight lines, making nine closed regions. Each region was then coloured in.

Can you draw a design that has more than nine closed regions using just seven lines?



Exploration CAS

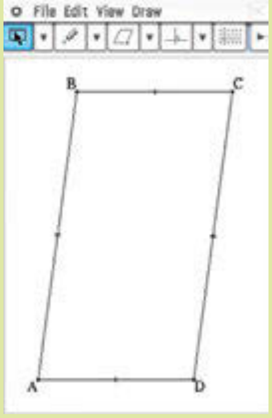
Equipment required: TI-Nspire CAS or Casio ClassPad CAS

Properties of quadrilaterals

You can use CAS technology to construct different quadrilaterals and to explore the properties of each shape. By doing this, you will become more familiar with how to use your CAS to help study and understand geometry.

Constructing a parallelogram

A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. Then select **menu** > **Points & Lines** > **Segment**. You can now select two points on the screen to draw a line segment between the points. This will be the first side of your parallelogram.  	From the menu select **Geometry**. You can draw a parallelogram as one of the built-in default shapes. Select the parallelogram icon ▱ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly. You can also select and move the vertices to change the dimensions of the shape. 

Using TI-Nspire CAS

Select **menu** > **Points & Lines** > **Point** and select a point on the screen where the parallel side of your parallelogram will be. You can now select **menu** > **Construction** > **Parallel**, then select the point and the line. This will create a parallel line through the point.

Again select **menu** > **Points & Lines** > **Point** and select the parallel line to create another point.

Select **menu** > **Points & Lines** > **Segment** as before, to draw the third side of the parallelogram.

The fourth side of the parallelogram needs to be parallel to this third side. To create this, again you can select **menu** > **Construction** > **Parallel**, then select the third side and then another point.

You can use **menu** > **Points & Lines** > **Segment** to connect adjacent vertices with line segments. Finally, you can select **Objects** and then select **ctrl** > **menu** > **Hide/Show** > **Hide Selection** to hide the extra lines and only show the sides of the parallelogram.

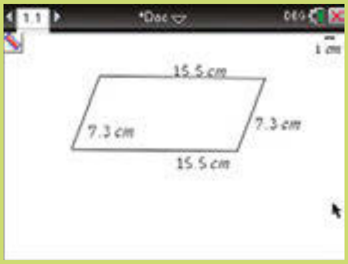
Investigating a parallelogram

To investigate the shape, first find the lengths of each side.

Using TI-Nspire CAS

To find the length of a side, select **menu** > **Measurement** > **Length** and then select one of the sides. You will then be able to place a caption showing the length.

Do this for each side to find each length. What can you say about the lengths of the opposite pairs of sides?



Using Casio ClassPad CAS

To find the length of a side, select one of the side lengths and then select **Draw** > **Measurement** > **Length**. A label for the length will appear.

Do this for each side to find each length.

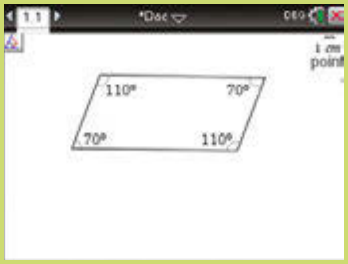
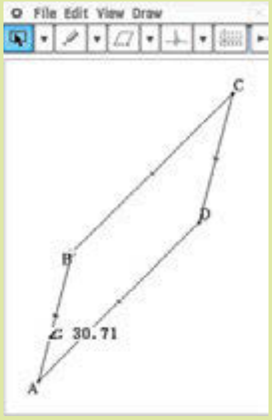
Note that you can select and move each label closer to each side.

What can you say about the lengths of the opposite pairs of sides?

You can select and drag the corner points of your parallelogram to adjust the shape. See how the length measurements change as you do this.

What can you say, in general, about the lengths of the opposite pairs of sides?

To continue investigating the shape, next measure the angles.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Select **menu** > **Measurement** > **Angle** and then select the three points that form the angle. The angle size will then be displayed. Do this for all the internal angles of the shape.  	Select the two sides that make an angle and then select **Draw** > **Measurement** > **Angle**. A label for the angle will appear. Do this for all the internal angles of the shape. 

You can select and drag the corner points of your parallelogram to adjust the shape. See how the angles change as you do this.

What can you say, in general, about the opposite angles in the parallelogram?

To further investigate the shape, it can be useful to draw the diagonals.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Draw the parallelogram's diagonals by selecting **menu** > **Points & Lines** > **Segment** to connect diagonal vertices with line segments.	Draw the parallelogram's diagonals by selecting **Draw** > **Basic Object** > **Line Segment** and then selecting the diagonal opposite corners.

Measure the lengths of the diagonal line segments.

Then see how the diagonal lengths change as you move the corner points of the parallelogram to adjust the shape.

What can you say, in general, about the diagonal lengths in the parallelogram?

Now measure the angle of intersection of the two diagonals. Is this a right angle?

Move one of the corner points and see what happens to the angle of intersection.

What can you say, in general, about the angle of intersection of the diagonals in the parallelogram?

Constructing a rectangle

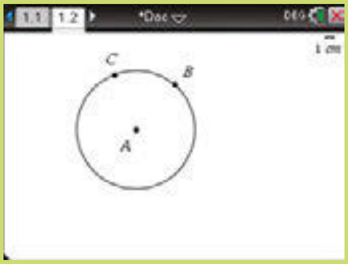
A rectangle is a parallelogram with a right angle.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. The rectangle is one of the built-in shapes. Select **menu** > **Shapes** > **Rectangle** to select the three corner points of a rectangle to be drawn.	From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen. You can now draw a rectangle as one of the built-in default shapes. Select the rectangle icon □ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly. You can also select and move the vertices to change the dimensions of the shape.

Now investigate the properties of the rectangle in the same way as you investigated the parallelogram.

Constructing a rhombus

A rhombus is a parallelogram with a pair of adjacent sides equal.

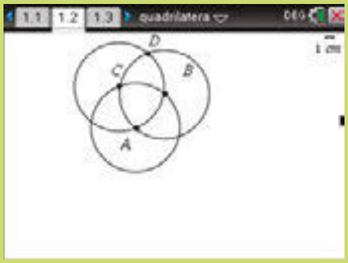
Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. To create a rhombus, you can use the properties of intersecting circles. Select **menu** > **Shapes** > **Circle** to select the centre point and radius of a circle to be drawn. You need to mark two points on this circle, so select **menu** > **Points & Lines** > **Point On** to select points on the circle to be marked. You can label the centre point *A* and the two points on the circumference *B* and *C*. 	From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen. You can now draw a rhombus as one of the built-in default shapes. Select the rhombus icon ◇ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly. You can also select and move the vertices to change the dimensions of the shape.

Using TI-Nspire CAS

You now need to make a circle with its centre at point *B* that passes through point *A*, as well as another circle with its centre at point *C* that passes through point *A*.

You can find the new intersection point of the two new circles by selecting **menu** > **Points & Lines** > **Intersection Point(s)** and selecting the two circles.

You can label this point of intersection of the two new circles as point *D*, as shown.



From these points of these three circles, you can now make a rhombus. Select **menu** > **Points & Lines** > **Segment** to create line segments joining the points *A*, *B*, *D* and *C*. You can then **Hide** the original circles, to make the rhombus easier to see.

Now investigate the properties of the rhombus in the same way as you investigated the parallelogram.

Constructing a kite

A kite is a quadrilateral with two pairs of equal adjacent sides.

Using TI-Nspire CAS

Add **Geometry** to your document.

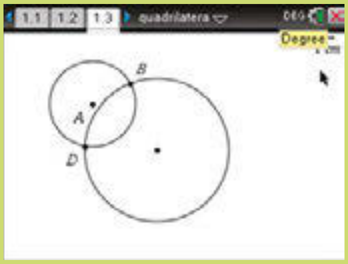
To create a kite, you can use the properties of intersecting circles.

Select **menu** > **Shapes** > **Circle** to select the centre point and radius of a circle to be drawn.

You need to mark a point on this circle, so select **menu** > **Points & Lines** > **Point On** to select a point on the circle to be marked.

Label the centre point *A* and the point on the circumference *B*.

You now need to make a circle that passes through point *B* and that has its centre outside the original circle. Select **menu** > **Shapes** > **Circle** to select a centre point outside the first circle, then select point *B*.



This new circle will have two points of intersection with the first circle, point *B* and another point. You can mark the other intersection point by selecting **menu** > **Points & Lines** > **Intersection Point(s)** and selecting the two circles.

Label this other point of intersection of the two circles as point *D*, as shown.

From these points of these two circles, you can now make a kite. Select **menu** > **Points & Lines** > **Segment** to create line segments joining the points *A*, *B*, *D* and the centre of the other circle. You can then **Hide** the original circles, to make the kite easier to see.

Using Casio ClassPad CAS

From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen.

You can now draw a kite as one of the built-in default shapes. Select the kite icon ◇ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear.

If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly.

You can also select and move the vertices to change the dimensions of the shape.

Now investigate the properties of the kite in the same way as you investigated the parallelogram.

Constructing a square

A square is a rectangle with a pair of adjacent sides equal.

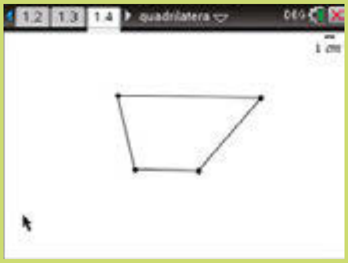
It is also a regular quadrilateral (a regular polygon with four sides).

Using TI-Nspire CAS	**Using Casio ClassPad CAS**
Add **Geometry** to your document. The square is one of the built-in shapes, as it is a regular polygon. Select **menu** > **Shapes** > **Regular Polygon** to select the centre point and then the first corner point of the square. Move the cursor around to change the number of regular polygon sides to 4, then select another corner point.	From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen. You can now draw a square as one of the built-in default shapes. Select the square icon □ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly. You can also select and move the vertices to change the dimensions of the shape.

Now investigate the properties of the square in the same way as you investigated the parallelogram.

Constructing a trapezium

A trapezium is a quadrilateral with one pair of parallel sides.

Using TI-Nspire CAS	**Using Casio ClassPad CAS**
Add **Geometry** to your document. Then select **menu** > **Points & Lines** > **Segment**. You can now select two points on the screen to draw a line segment between the points. This will be the first side of your trapezium.	From the menu select **Geometry**. Then select **Edit** > **Clear All** if necessary to clear the screen. You can now draw a trapezium as one of the built-in default shapes. Select the trapezium icon ⏢ (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. If the shape is small, select **View** > **Zoom In** or **Zoom to Fit** to see it more clearly. You can also select and move the vertices to change the dimensions of the shape.
Select **menu** > **Points & Lines** > **Point** and select a point on the screen where the parallel side of your trapezium will be. You can now select **menu** > **Construction** > **Parallel**, then select the point and the line. This will create a parallel line through the point.  You can now create another point on the parallel line, and then join the four points to make a trapezium.	

Now investigate the properties of the trapezium in the same way as you investigated the parallelogram.

Taking it further

From each group of properties, choose the best description for each of the quadrilaterals you considered (parallelogram, rectangle, rhombus, square, kite and trapezium).

Group A:

- Two pairs of opposite sides parallel
- One pair of opposite sides parallel
- No parallel sides

Group B:

- All sides equal
- Two pairs of opposite sides equal
- Two pairs of adjacent sides equal
- No sides equal

Group C:

- All angles are right angles
- Opposite angles are equal
- One pair of opposite angles equal
- No angles equal

Group D:

- Diagonals are not equal
- Diagonals bisect each other (cut each other in half)
- One diagonal is bisected
- Diagonals bisect each other at right angles
- Diagonals are equal and bisect each other
- Diagonals are equal and bisect each other at right angles

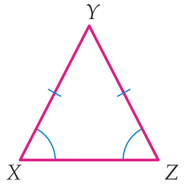
8.2

Shapes review

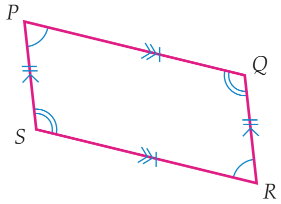
Using geometric symbols

Shapes are named by giving a capital letter to each of the vertices. You can then list the vertices in order as you move around the shape, usually in a clockwise direction.

The name of this triangle is ΔXYZ.



The name of this quadrilateral is $PQRS$.



Sides that are equal in length are marked with the same number of dashes. For example:

ΔXYZ has two equal sides, $XY = YZ$.

PQRS has two pairs of equal sides, $PQ = SR$ and $PS = QR$.

Angles that are equal in size are marked with the same number of arcs between the arms of the angle. For example:

ΔXYZ has two equal angles, $\angle YXZ = \angle YZX$.

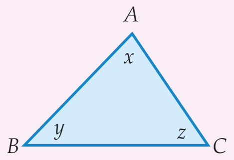
$PQRS$ has two pairs of equal angles, $\angle QPS = \angle QRS$, and $\angle PSR = \angle PQR$.

Sides that are parallel are marked with an equal number of arrows. For example:

On $PQRS$, sides PS and QR are parallel to each other, and sides PQ and SR are parallel to each other. You can write this as $PS \| QR$ and $PQ \| SR$.

The symbol '||' means 'is parallel to'. The symbol ⊥ means 'is perpendicular to'.

Angle sum of a triangle



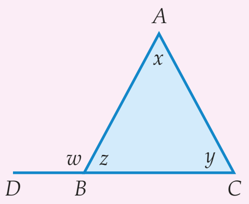
A triangle is a three-sided plane shape.

The sum of the angles of a triangle is 180°. This angle sum is written as either:

$x + y + z = 180°$ or

$\angle BAC + \angle ABC + \angle BCA = 180°$

Exterior angle of a triangle



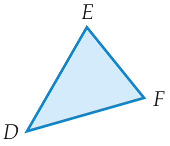
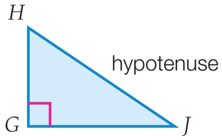
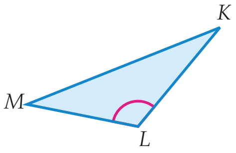
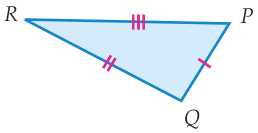
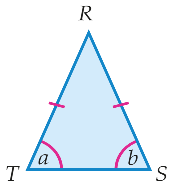
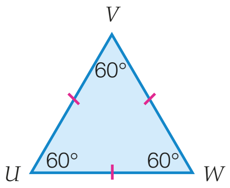
An exterior angle is produced when any side of a shape is extended. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

CB is extended to D to form the exterior angle $\angle DBA$.

$\angle DBA = \angle BAC + \angle ACB$ or

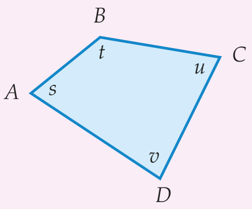
$w = x + y$

Triangle definitions and properties

Shape	Definition and properties
Acute-angled triangle  	All angles are acute. $\angle DFE < 90°$, $\angle FED < 90°$, $\angle EDF < 90°$
Right-angled triangle  	One angle equals 90°. The side opposite the right angle is called the **hypotenuse**. $\angle HGJ = 90°$ HJ is the hypotenuse.
Obtuse-angled triangle  	One angle is obtuse (the other two angles are acute). $\angle KLM > 90°$ $\angle LKM < 90°$, $\angle KML < 90°$
Scalene triangle  	All sides and angles are unequal. PQ, QR and RP are different lengths.
Isosceles triangle  	One pair of sides are equal in length. The angles opposite the equal sides are also equal in size. $RS = RT$ $\angle RST = \angle RTS$ or $b = a$
Equilateral triangle  	All sides are equal in length, as are all angles. $UV = VW = WU$ All angles are equal to 60°. $\angle UVW = \angle VWU = \angle WUV = 60°$

Quadrilaterals

Angle sum of a quadrilateral



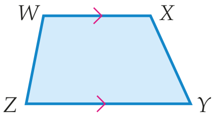
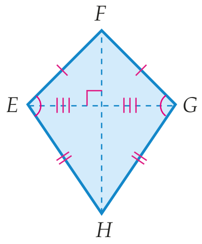
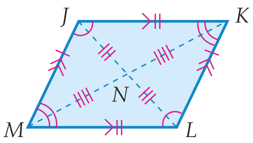
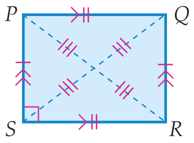
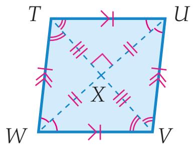
A **quadrilateral** is a four-sided plane shape.

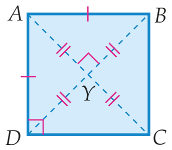
The sum of the angles of a quadrilateral is 360°. This is written as: $s + t + u + v = 360°$ or

$\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$

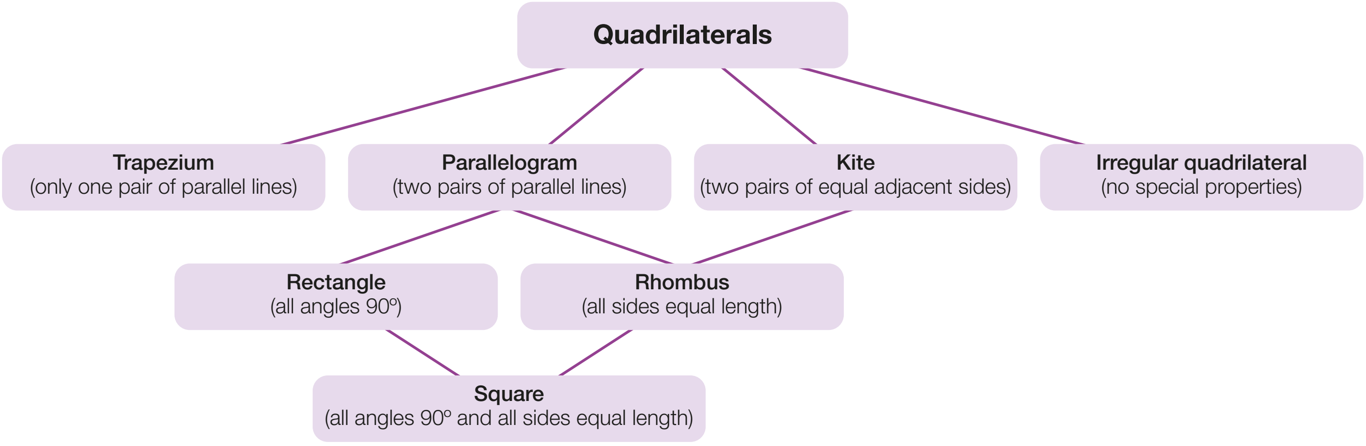
Properties of special quadrilaterals

The definition is the *minimum* information needed to define the shape.

Shape	Definition	Properties
Trapezium 	A trapezium is a quadrilateral with only one pair of opposite sides parallel. *WXYZ* is a trapezium in which $WX \parallel ZY$.	A trapezium has one pair of opposite sides that are parallel.
Kite 	A kite is a quadrilateral with two pairs of equal adjacent sides. *EFGH* is a kite in which $EF = FG$, $EH = HG$.	• A kite has one pair of opposite angles that are equal: $\angle FEH = \angle FGH$ • The diagonals are perpendicular: $FH \perp EG$ • The diagonal that passes through the equal angles is **bisected** (cut in half) by the other diagonal.
Parallelogram 	A parallelogram is a quadrilateral with both pairs of opposite sides parallel. *JKLM* is a parallelogram in which $JK \parallel ML$, $JM \parallel KL$.	A parallelogram has: • both pairs of opposite sides equal in length: $JK = ML$, $JM = KL$ • both pairs of opposite angles equal: $\angle MJK = \angle MLK$, $\angle JML = \angle JKL$ • the diagonals bisect each other: $JN = NL$, $MN = NK$
Rectangle 	A rectangle is a parallelogram with a right angle. *PQRS* is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus 	A rhombus is a parallelogram with two adjacent sides that are equal. *TUVW* is a rhombus in which $TU = TW$.	A rhombus has all the properties of a parallelogram, plus: • all the sides are equal in length: $TU = TW = WV = UV$ • the diagonals bisect the angles of the rhombus: $\angle WTV = \angle VTU$, $\angle WVT = \angle TVU$, $\angle TWU = \angle UWV$, $\angle TUW = \angle WUV$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$, $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$

Shape	Definition	Properties
Square  	A square is a rectangle with two adjacent sides equal. *ABCD* is a square in which $AB = AD$.	A square has all the properties of a rectangle, plus: • all the sides are equal: $AB = AD = DC = BC$ • the diagonals are equal in length: $AC = BD$. • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90°$
Alternative definition of a square	A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90°$.	A square has all the properties of a rhombus, plus: • all the angles are right angles: $\angle ADC = \angle DCB = \angle CBA = \angle BAD = 90°$ • the diagonals are equal: $AC = BD$

Summary of the quadrilaterals



From the above diagram, you can see that:

- rectangles, rhombuses and squares are special types of parallelograms
- a rhombus can also be classified as a special type of kite
- a square can also be classified as a special type of rectangle.

Polygons

A polygon is a plane shape with only straight sides. (A circle is curved, so it is not a polygon.)

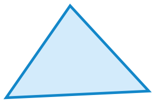
A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, a pentagon is a polygon with five sides, and so on.

A regular polygon has all its sides and angles equal. So, an equilateral triangle is a regular three-sided polygon and a square is a regular four-sided polygon. (There are no special names for other regular polygons, so for example, a regular five-sided figure is simply called a regular pentagon.)

Angle sum of a polygon

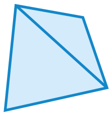
You can find the angle sum of these polygons by dividing each shape into triangles as shown.

Triangle



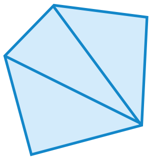
Angle sum
= 180°

Quadrilateral



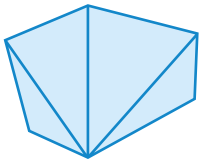
Angle sum
= 2 × 180°
= 360°

Pentagon



Angle sum
= 3 × 180°
= 540°

Hexagon



Angle sum
= 4 × 180°
= 720°

This can be written as:

Angle sum
= $(3 - 2) \times 180°$

Angle sum
= $(4 - 2) \times 180°$

Angle sum
= $(5 - 2) \times 180°$

Angle sum
= $(6 - 2) \times 180°$

The sum of the angles of a polygon with n sides = $(n - 2) \times 180°$

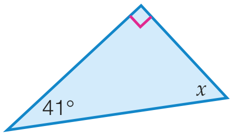
The **interior angles** (angles inside the shape) of a regular polygon are equal.

Worked example 3

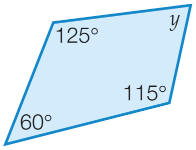
W.E. 3

Find the value of the pronumerals in each shape.

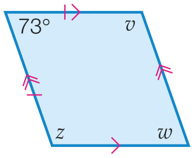
(a)



(b)



(c)



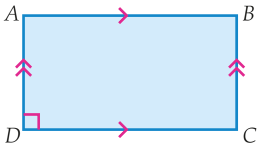
Thinking	Working
(a) 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	(a) $x + 41 + 90 = 180$ (angle sum of a triangle is 180°)
2 Solve the equation.	$x + 131 = 180$ $x = 180 - 131$ $x = 49°$
(b) 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	(b) $y + 115 + 60 + 125 = 360$ (angle sum of a quadrilateral)
2 Solve the equation.	$y + 300 = 360$ $y = 360 - 300$ $y = 60°$
(c) 1 Identify the shape.	(c) A parallelogram with one pair of adjacent sides equal is a rhombus.
2 Form an equation, with reason.	$v + 73 = 180$ (co-interior angles on parallel lines)

3	Solve the equation.	$v = 107^\circ$
4	Repeat for the other angles.	$w = 73^\circ$ (opposite angles of a rhombus are equal) $z = v$ (opposite angles of a rhombus are equal) $= 107^\circ$
5	List all answers.	$v = 107^\circ$, $w = 73^\circ$, $z = 107^\circ$

Worked example 4

W.E. 4

What kind of quadrilateral is *ABCD*?



Thinking	Working
1 Use the symbols given to list the information known about the shape. Work through the list of quadrilaterals to see which shape fits the information.	$AB \parallel DC$, $AD \parallel BC$, so *ABCD* is a parallelogram; $\angle ADC = 90^\circ$, so all other angles must be 90° (co-interior angles on parallel lines are supplementary)
2 State the answer.	*ABCD* is a rectangle.

8.2 Shapes review

Navigator

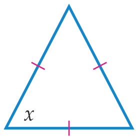
1, 2, 3, 4, 8 (a–b), 10, 11, 12, 14, 15, 16, 18, 24, 26	1, 2, 3, 4, 6, 7, 8, 10, 11, 14, 15, 16, 17, 18, 21, 23, 24, 26	2, 3, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 25, 26

Answers p. 664

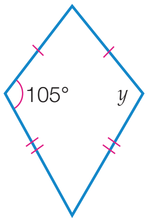
Fluency

1 Find the value of the pronumerals in each of the following. Give reasons for your answer.

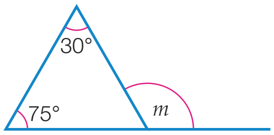
(a)



(b)



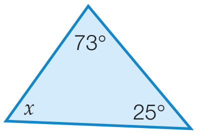
(c)



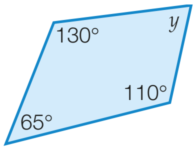
W.E. 3

2 Find the value of the pronumerals in each shape.

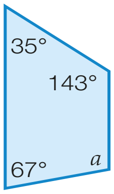
(a)



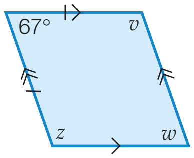
(b)



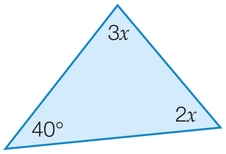
(c)



(d)



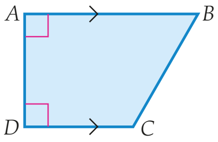
(e)



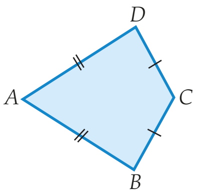
W.E. 4

3 What is the most appropriate special name for each of the following quadrilaterals *ABCD*?

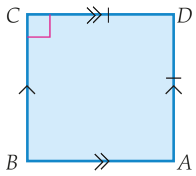
(a)



(b)

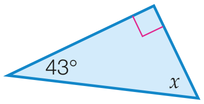


(c)



4 The value of x in this triangle is:

A 43° **B** 47° **C** 137° **D** 227°



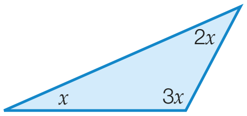
5 A hexagon (a six-sided polygon) can be divided into four triangles.

(a) What is the angle sum of a triangle?

(b) Use your answer to the previous part to find the sum of the angles in a hexagon.

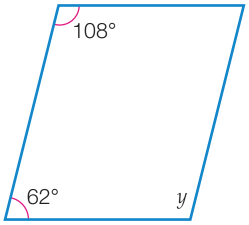
6 The three angles in a triangle are x, $2x$ and $3x$.

What is the size of each angle?



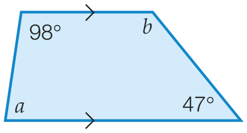
Understanding

7 What value of y will make this quadrilateral a parallelogram? Explain.

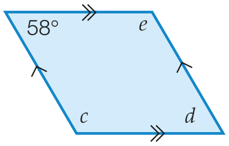


8 Find the value of the pronumerals in each shape.

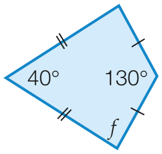
(a)



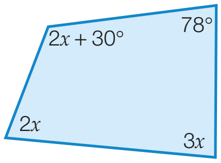
(b)



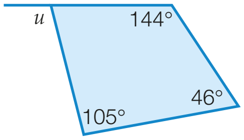
(c)



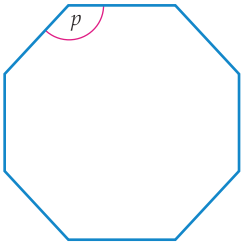
(d)



(e)



9 Find angle p in the regular octagon shown. Give reasons for your answer.



10 Write true (T) or false (F) for each of the following statements.

(a) A triangle can have two obtuse angles.

(b) A square has all the properties of a rhombus.

(c) A kite has at least one pair of parallel sides.

(d) The diagonals of a rectangle bisect each other and also bisect the angles of the rectangle.

11 If an isosceles triangle has two base angles of 48° each, what is the value of the vertex angle?

12 In which regular polygon is each angle 90°?

13 This geometric design was found in the brickwork of a wall in the ancient city of Pompeii. A regular hexagon has six squares joined to its sides. On the end of each square is an equilateral triangle. In between the squares are six rhombuses. Each rhombus is made up of two equilateral triangles.

(a) Sketch a copy of the design, then use the information given above to mark every equal side.

(b) Use the properties of a rhombus to find the angles in the rhombuses used in the design.

(c) Use the angle properties of the squares, triangles and rhombuses in the design to show that the interior angles of the hexagon are each 120°.

14 Which of the following statements is false?

A A kite has one pair of opposite angles equal.

B A trapezium cannot be a rectangle.

C A rhombus is a rectangle.

D A rhombus is a parallelogram.

15 Which statement is true?

A All quadrilaterals have at least one pair of parallel sides.

B All trapeziums have a pair of sides of equal length.

C All kites have a pair of equal angles.

D All quadrilaterals have a right angle.

16 **(a)** What is the sum of the interior angles of a regular pentagon?

(b) What is the size of each interior angle?

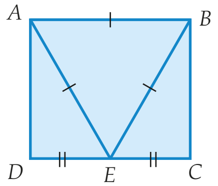
Reasoning

17 Sketch a four-sided polygon that has one pair of parallel sides that are not equal in length and one pair of opposite sides equal in length but not parallel. Name the polygon.

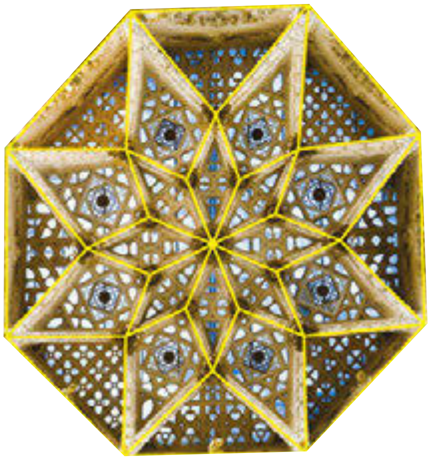
18 The rectangle *ABCD* is shown. Point *E* is half-way between *C* and *D*. *ABE* is an equilateral triangle. Find:

(a) $\angle DAE$

(b) $\angle BEC$.



19 The design of this cathedral ceiling has eight identical rhombuses with one common vertex (the point at the centre of the design). Sharing sides with the rhombuses are eight other identical quadrilaterals. Between each pair of these quadrilaterals is a right-angled isosceles triangle.



(a) Find the size of the internal angles of one rhombus.

(b) Use the side properties of rhombuses and isosceles triangles to demonstrate that the eight other quadrilaterals are kites.

(c) **(i)** What regular polygon is formed by joining the outermost vertices of the kites?

(ii) What is the sum of the angles in this polygon?

(iii) Use your answer to the previous part to find the size of one of the interior angles of this polygon.

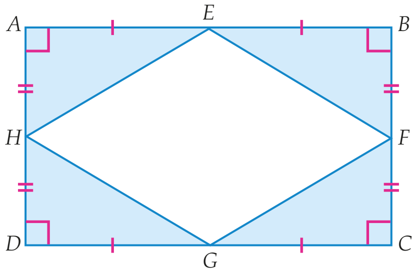
(d) Use your answers to **(a)** and **(c)** to find the interior angles of one of the kites.

20 What is the greatest number of acute angles that you can have in a quadrilateral? Explain your answer using diagrams.

21 The midpoints of each side of a rectangle *ABCD* are joined in order to form the quadrilateral *EFGH*.

(a) Measure *EF*, *FG*, *GH* and *HE* and use this information to name *EFGH*.

(b) If *ABCD* had been a square, what sort of a quadrilateral would *EFGH* be?



22 The diagonals of a quadrilateral bisect each other. Draw all the possible different quadrilaterals, naming each type.

23 Both a rhombus and a rectangle are parallelograms, but a rectangle is not necessarily a rhombus. Explain.

Open-ended

24 Draw three different right-angled triangles, stating the size of each angle in them.

25 Using as few lines as possible, draw a shape that contains:

(a) a rectangle and two squares

(b) a kite and eight triangles

(c) two rectangles, a square, a trapezium and two triangles.

26 **(a)** An isosceles triangle has one 45° angle. Suggest possible sizes for the other two angles.

(b) A triangle has one 60° angle. Must it be equilateral? Give examples of possible triangles.

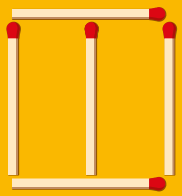
(c) Draw some examples of an obtuse-angled scalene triangle.

Puzzle

Matchstick quadrilaterals

Equipment needed: matchsticks (optional)

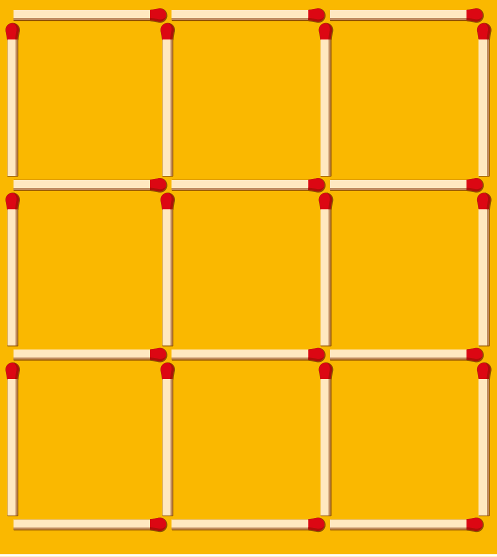
1 With five matchsticks, you can make an arrangement that contains three quadrilaterals, as shown.



(a) Using six matchsticks, make an arrangement that contains six quadrilaterals.

(b) Using six matchsticks, make an arrangement that contains nine quadrilaterals.

2 How can you remove six matchsticks from this arrangement, without moving any others, so that only three squares are left?



8.3

Congruence and transformation

You should remember the transformations of translation, reflection and rotation. These transformations all produce **congruent** figures, which are figures with an identical shape and size.

You can use the Cartesian plane to plot points, join points to form shapes, then transform the shapes.

Worked example 5

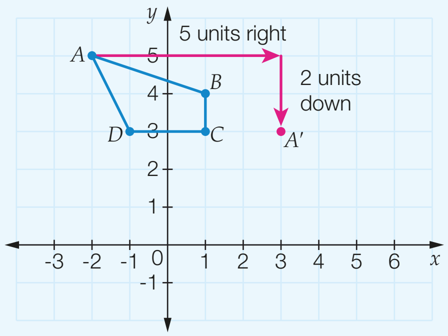
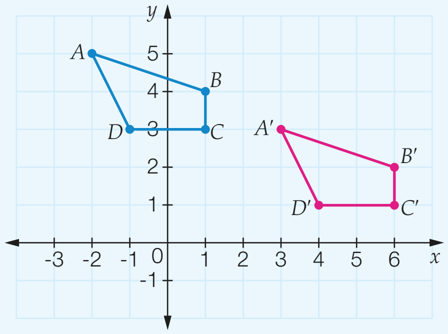
W.E. 5

Plot the following set of coordinates on a Cartesian plane: (-2, 5), (1, 4), (1, 3), (-1, 3). Join and label them *A*–*D* in the order given to form the quadrilateral *ABCD*. Do the following transformations on *ABCD*. Label the transformed figure *A′B′C′D′* and list the coordinates of figure *A′B′C′D′* for each transformation.

(a) The translation [5, -2]

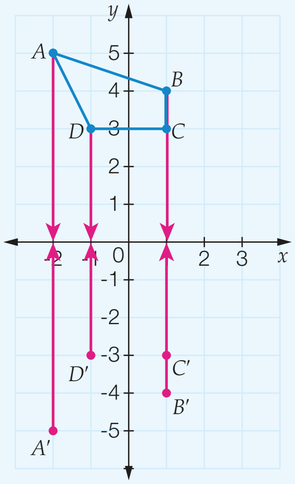
(b) A reflection in the *x*-axis

(c) A clockwise rotation of 90° about the point (-1, 1)

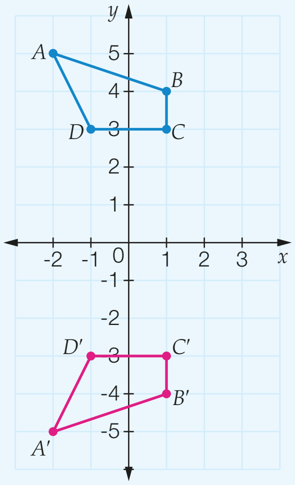
Thinking	Working
(a) **1** Plot and label the points, then join them to form the shape. Select one of the vertices and move it according to the translation given. Label it using image notation (*A′*).	(a) 
2 Repeat for the remaining vertices, label them *B′*, *C′* and *D′*, then join them to form the new image.	
3 Read the coordinates of the new points *A′*, *B′*, *C′* and *D′* and write them down.	A′(3, 3) B′(6, 2) C′(6, 1) D′(4, 1)

(b) **1** Count or measure the perpendicular distance between each vertex and the line of reflection. Use this distance to plot the image of each vertex on the opposite side of the line.

(b)



2 Label and join each of the reflected vertices to form the image of the figure.

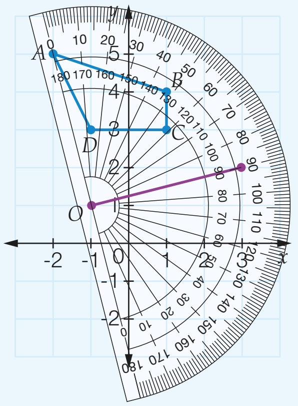


3 Read the coordinates of the new points *A′*, *B′*, *C′* and *D′* and write them down.

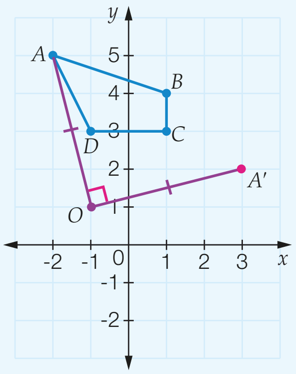
A′(-2, -5) B′(1, -4)
C′(1, -3) D′(-1, -3)

(c) **1** Plot the centre of rotation. Label it *O*. Place the centre of a protractor against the centre of rotation, with the chosen vertex (point *A* is shown here) on the base line. Measure the angle of rotation around from zero in the given direction and mark it off.

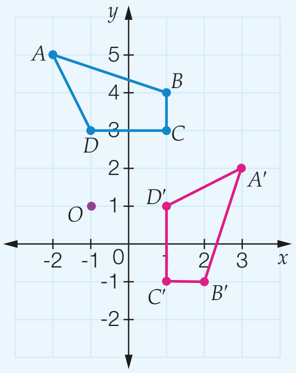
(c)



2 Plot and label the image point, using a ruler if necessary to make sure that it is the same distance from *O* as the original point.



3 Repeat Steps **1** and **2** for the other vertices.



4 Read the coordinates of the new points *A′B′C′D′* and write them down.

A′(3, 2)	B′(2, -1)
C′(1, -1)	D′(1, 1)

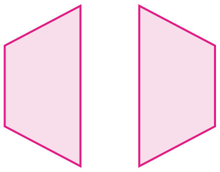
Congruent figures

If one figure can be placed on top of another, so that every side, angle and vertex matches up with the other figure, then the two figures are said to be congruent.

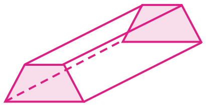
The symbol ≡ and the symbol ≅ are both used to mean ‘is congruent to’.

Congruent figures are *exactly* the same shape and the same size, but they can be in different positions and have a different orientation.

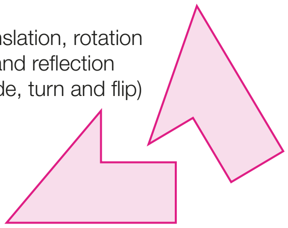
One congruent figure can be placed on top of another by one or more transformations, such as a translation (slide), reflection (flip) or rotation (turn). The pairs of figures below are congruent.



Reflection (flip)



Translation (slide)



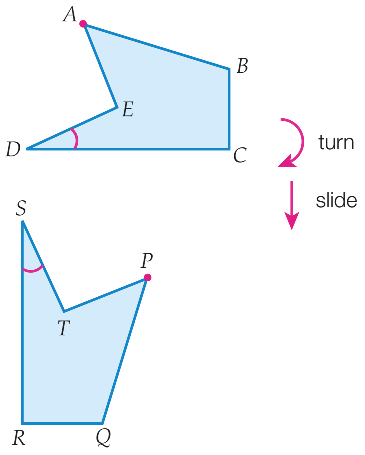
Translation, rotation and reflection (slide, turn and flip)

The sides, angles and vertices of one figure can be paired with the corresponding parts of a congruent figure. Matching sides are also called corresponding sides and the ↔ symbol can be used to show the corresponding pairs.

The two figures shown below are congruent. Figure $ABCDE$ can be rotated 90° in a clockwise direction around point A, then translated downwards to sit on top of figure $PQRST$, so that:

- vertex A corresponds to vertex P $\quad A \leftrightarrow P$
- side BC corresponds to side QR $\quad BC \leftrightarrow QR$
- angle CDE corresponds to angle RST $\quad \angle CDE \leftrightarrow \angle RST$

As the two figures shown are congruent to each other, you can write the following **congruence statement**: $ABCDE \equiv PQRST$.



When naming congruent figures, the vertices of both figures must be written in the same order, to allow corresponding vertices, angles and sides to be identified from the congruence statement.

Worked example 6

W.E. 6

For the congruent figures shown:

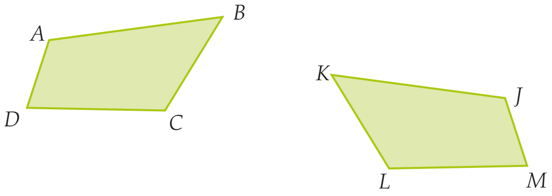
(a) identify the transformation(s) that places the first figure onto the second

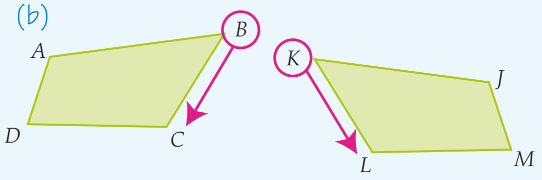
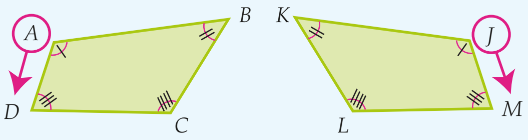
(b) pair the corresponding vertices

(c) pair the corresponding side lengths

(d) pair the corresponding angles

(e) write a congruence statement.

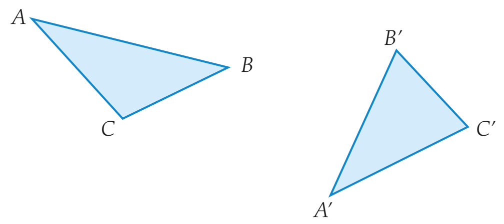


Thinking	Working
(a) Which transformation—reflection, rotation or translation—will place the first shape on top of the other?	(a) Reflection (in a vertical line), then translation
(b) Locate one part of the figure that can easily be identified in both congruent figures. (Here, locate the smallest angle in each shape.) Pair the vertices at this point, then pair the rest by moving around the figure.	(b)  $B \leftrightarrow K \quad C \leftrightarrow L \quad D \leftrightarrow M \quad A \leftrightarrow J$
(c) Pair the side lengths starting at the same easily identified point as in **(b)**.	(c) $BC \leftrightarrow KL \quad CD \leftrightarrow LM$ $DA \leftrightarrow MJ \quad AB \leftrightarrow JK$
(d) Mark pairs of corresponding angles in the same way, by marking in the angle arc from one arm to the other, then placing one, two, three or four lines across it. Three points are written to describe an angle. The vertex of the angle is at the point in the middle.	(d)  $\angle ABC \leftrightarrow \angle JKL \quad \angle BCD \leftrightarrow \angle KLM$ $\angle CDA \leftrightarrow \angle LMJ \quad \angle DAB \leftrightarrow \angle MJK$

(e) Move around the first shape, writing the vertices down in order. Start at the corresponding vertex in the second shape and move around it in the same order.

(e) $ABCD \equiv JKLM$

Corresponding vertices on congruent figures produced by transformations are shown by labelling the vertices of the second figure with a dash (also called a prime mark). You can write the vertex that corresponds to A as A' and say *A dash* or *A prime*. Refer to the second triangle shown here as $A'B'C'$.



8.3 Congruence and transformation

Navigator

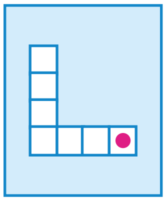
Answers p. 665

1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 17, 18	2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18	2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17

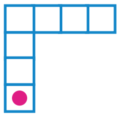
Equipment required: ruler and grid paper for Questions **3**, **6**, **8**, **15**; protractor may be used for Question **3**

Fluency

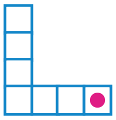
1 Label each of the following transformations of the shape on the right as a reflection, a translation or a rotation.



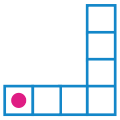
(a)



(b)



(c)



2 The vertex of a triangle is the point (5, 3). What are the coordinates of the image of the vertex, when the triangle is reflected over the x-axis?

W.E. 5

3 **(a)** Plot the following set of coordinates on a Cartesian plane: (1, 3), (0, 4), (-2, 3), (-1, 1). Join and label them A–D in the order given to form the quadrilateral $ABCD$. Do the following transformations on $ABCD$. Label the transformed figure $A'B'C'D'$ and list the coordinates of $A'B'C'D'$ for each transformation.

- **(i)** The translation [2, -3]
- **(ii)** A reflection in the x-axis
- **(iii)** A clockwise rotation of 180° about the point (0, 1)

(b) Plot the following set of coordinates on a Cartesian plane: (-3, 1), (-4, -3), (-1, -2). Join and label them D–F in the order given to form the triangle DEF. Do the following transformations on DEF. Label the transformed figure $D'E'F'$ and list the coordinates of $D'E'F'$ for each transformation.

- **(i)** The translation [-1, 4]
- **(ii)** A reflection in the y-axis
- **(iii)** An anticlockwise rotation of 90° about the point (-2, 2)

W.E. 6

4 For the congruent figures shown below:

(i) identify the transformation(s) that places the first figure onto the second

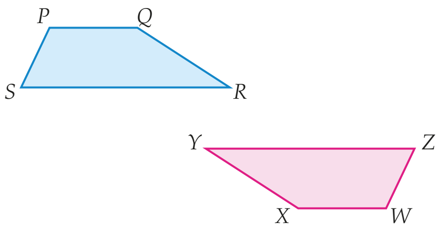
(ii) pair the corresponding vertices

(iii) pair the corresponding side lengths

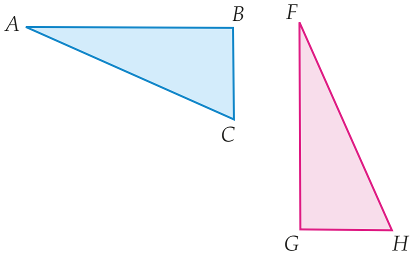
(iv) pair the corresponding angles

(v) write a congruence statement.

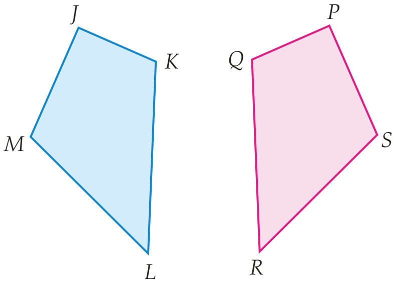
(a)



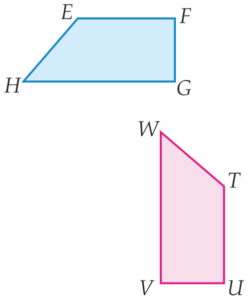
(b)



(c)

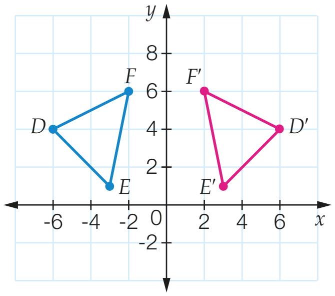


(d)

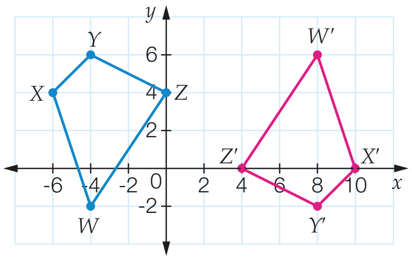


5 Describe the transformation that has been applied to produce the congruent figures shown.

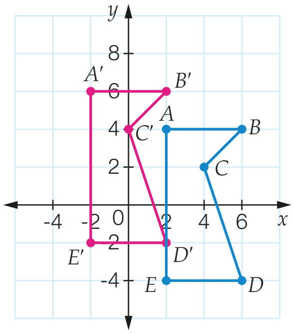
(a)



(b)



(c)



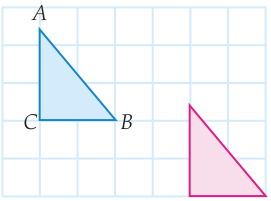
6 Copy the following congruent figures onto grid paper.

(i) Describe the transformation(s) needed to place the blue figure on top of the pink figure.

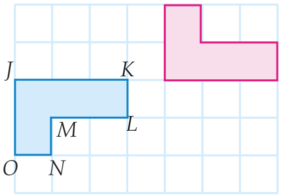
(ii) Label the corresponding vertices on the transformed figure using 'dash' notation ($A \leftrightarrow A'$, $B \leftrightarrow B'$ and so on.)

Transformations include translation, reflection and rotation. Sometimes, more than one transformation has occurred.

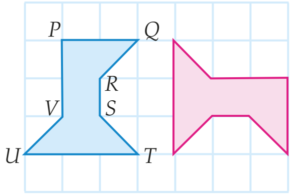
(a)



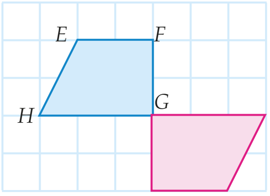
(b)



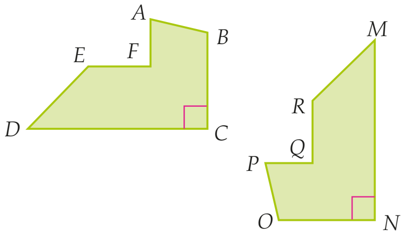
(c)



(d)



7 Refer to the congruent figures on the right.



(a) For the congruent figures shown, which pairing is *not* correct?

A $BC \leftrightarrow ON$

B $\angle NOP \leftrightarrow \angle CBA$

C $\angle DEF \leftrightarrow \angle RMN$

D $P \leftrightarrow A$

(b) Which congruence statement describes the figures shown?

A $ABCDEF \equiv MNOPQR$

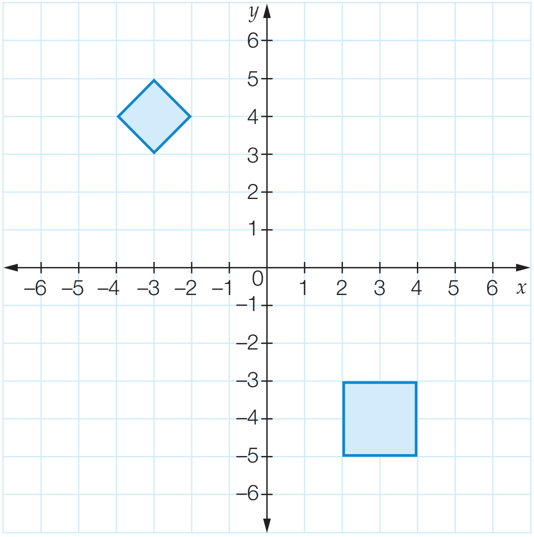
B $ABCDEF \equiv PQRMNO$

C $CDEFAB \equiv NOPQRM$

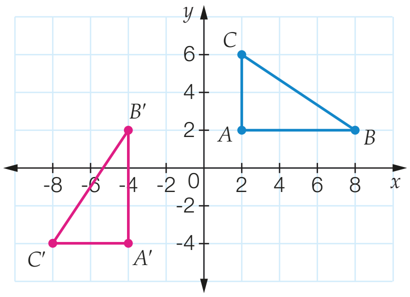
D $CDEFAB \equiv NMRQPO$

Understanding

8 Draw the following shapes on grid paper as they appear below. Apply a translation of [-5, 2] to the square and [6, -3] to the diamond.



9 Triangle *ABC* has had two transformations applied to it. Which sequence of transformations will place triangle *ABC* onto the image *A′B′C′*?



A Reflection in the y-axis, then the translation [-2, -1].

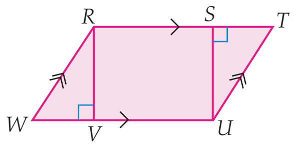
B Rotation 90° anticlockwise about the point (2, 2), then the translation [-6, -6].

C Rotation 180° clockwise about the point (0, 0), then a reflection in the x-axis.

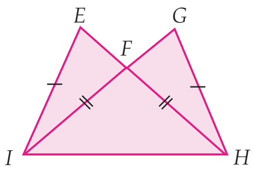
D Reflection in the x-axis, then the translation [-3, 2].

10 Write congruence statements for pairs of congruent triangles within the following figures. Some figures have more than one pair.

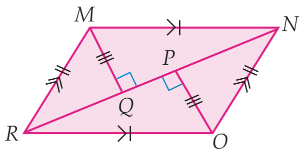
(a)



(b)



(c)



11 For the congruence statement $PQRS \equiv ABCD$, complete the following pairings.

(a) $R \leftrightarrow$

(b) $D \leftrightarrow$

(c) $PQ \leftrightarrow$

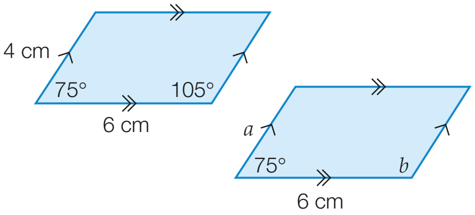
(d) $DA \leftrightarrow$

(e) $\angle PQR \leftrightarrow$

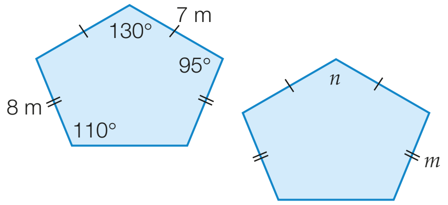
(f) $\angle CDA \leftrightarrow$

12 The pairs of figures below are congruent. Use your knowledge of corresponding angles and side lengths to find the value of the pronumerals.

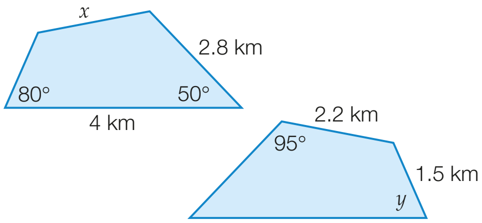
(a)



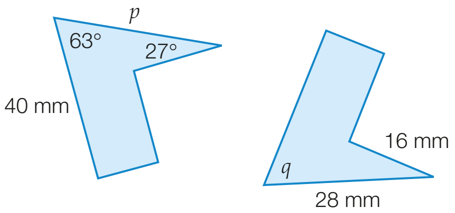
(b)



(c)

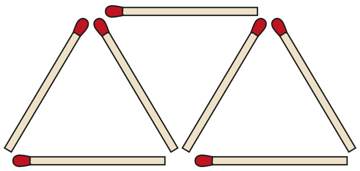


(d)



Reasoning

13 Seven matchsticks have been used to create the shape shown.

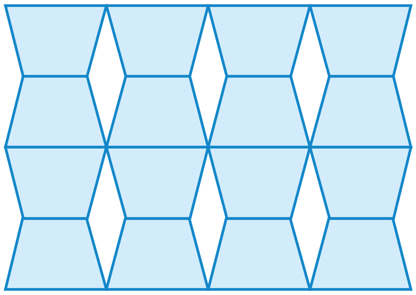


(a) How many congruent triangles are in the shape?

(b) If the shape is extended to 11 matchsticks, how many congruent triangles are there now?

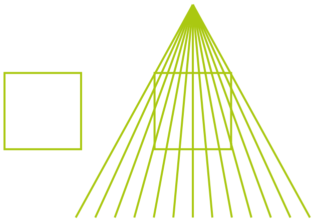
(c) In the shape shown, there are two congruent rhombuses. Draw each of these separately.

14 Copy a part of the tessellation below and describe the transformations used.

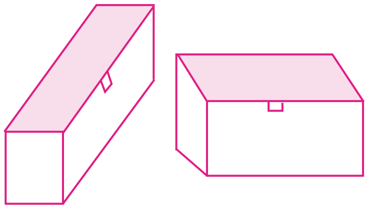


15 Look carefully at the figures below.

(a) Are the two squares congruent?



(b) Are the shaded shapes congruent?



Use a ruler to check your answers. Did the results surprise you? Explain the results.

Open-ended

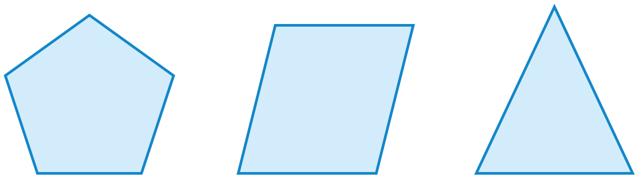
16 Keira was answering some homework questions relating to two congruent figures with different orientations. Her answers are shown.

(a) $GH = 4$ cm (b) $\angle XYZ = 90°$ (c) $EFGH \equiv WXYZ$

What might the two figures look like?

17 Two figures are congruent if they are exactly the same shape and size. State two real-life examples of congruent figures.

18 Design your own tiles for a bathroom by creating a tessellation using one of the shapes below. Use colours of your choice.



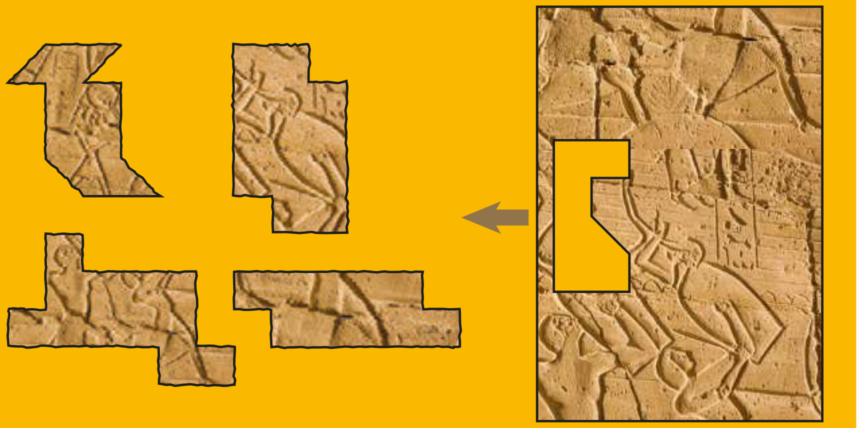
Puzzle

The ancient tomb

A clever archaeologist uncovered the tomb of a ruler of an ancient civilisation. The stone walls of the tomb had a keyhole and an arrow pointing to four stone shapes on the opposite wall.

According to legend, pushing one of the four stone shapes will open the tomb's secrets, but pushing the wrong stone will trigger an ancient trap and possible death.

The archaeologist realised that each of the stone shapes could be divided into two congruent halves. One of the shapes, divided in this way, matched the shape of the keyhole.

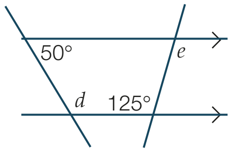


The archaeologist escaped death and unearthed historical treasures from the tomb. Which stone did the archaeologist push to open the door?

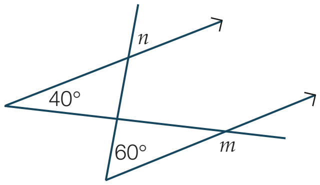
Half-time 8

1 Find the value of the pronumerals in each figure. **8.1**

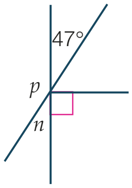
(a)



(b)

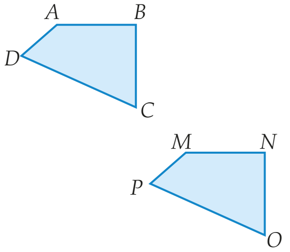


(c)

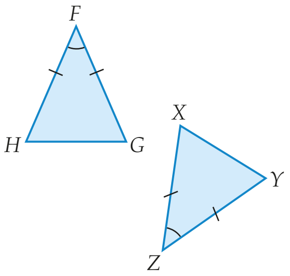


2 Write a congruence statement to describe the congruent figures shown. **8.3**

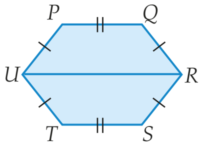
(a)



(b)

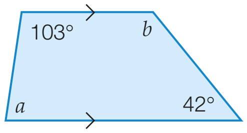


(c)

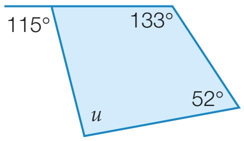


3 Find the value of the pronumerals in each shape. **8.2**

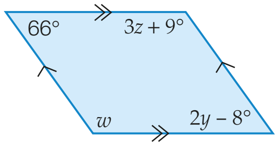
(a)



(b)

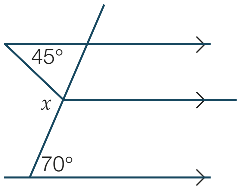


(c)



4 The value of x in the diagram is: **8.1, 8.2**

A 45° **B** 65° **C** 70° **D** 115°



5 Plot the points (1, 1), (3, 4), (5, 4) and (5, 2) on a Cartesian plane. Join them and label them *A*–*D* in the order given to form the quadrilateral *ABCD*. Do the following transformations on *ABCD*. Label the transformed figure *A′B′C′D′* and list the coordinates of *A′B′C′D′* for each transformation. **8.3**

(a) A reflection in the y-axis.

(b) A rotation 90° clockwise about the point (0, 0).

(c) The translation [-5, -2].

6 Which statements are true? **8.2**

A All squares are rectangles.

B All parallelograms are quadrilaterals.

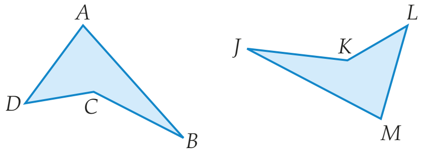
C All kites have equal diagonals.

D The diagonals of a parallelogram bisect each other at right angles.

7 Complete the following pairings for the congruent figures shown. **8.3**

(a) $B \leftrightarrow$ **(b)** $AD \leftrightarrow$

(c) $JM \leftrightarrow$ **(d)** $\angle JKL \leftrightarrow$



Maths 4 Real

The art of M.C. Escher

Art and tessellation

Shapes are said to *tessellate* if they fit together without any gaps between them. A pattern made up of tessellating shapes is called a tessellation.

Tessellating geometric shapes are used in many areas of art and design. Examples can be found on the wall of Spain's Alhambra Palace, and in Indian Rangoli patterns. Maurits Cornelis Escher was a famous Dutch artist who often used transforming and tessellating shapes to dramatic effect.

1 Describe the transformation (flip, slide or turn) that Escher has used to create the congruent copies of the fish in this tessellation.

2 The 'repeating unit' of a tessellation is the smallest figure (or group of figures) that can be used to create the entire tessellation by using transformations (flips, slides or turns).

For each of the Escher designs shown below:

(a) identify the repeating unit

(b) identify the transformation that is applied to the repeating unit to create the tessellation.

(i)

(ii)

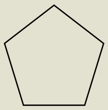
Investigating tessellations

Equipment required: paper, scissors

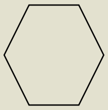
A *perfect* or *regular* tessellation uses shapes of only one type, which are all congruent.

To test and see whether a shape will tessellate, fold a piece of paper in half, then in half again, and again, so that there are eight layers of paper. Draw the shape in the middle of the top layer and cut it out. If the eight shapes fit together with no gaps, then the shape forms a regular tessellation.

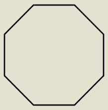
3 Which of the following shapes tessellate? Test them using the method described above.



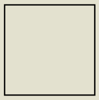
Pentagon (Regular)



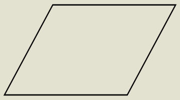
Hexagon (Regular)



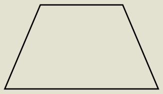
Octagon (Regular)



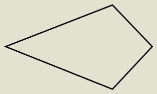
Square



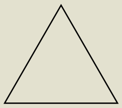
Parallelogram



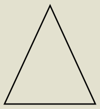
Trapezium



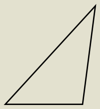
Kite



Equilateral Triangle



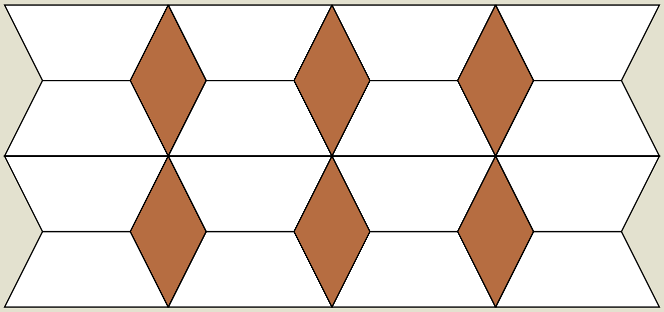
Isosceles Triangle



Scalene Triangle

4 What property do the tessellating shapes have in common?

A tessellation can also be made with two different shapes. This is called a *partial* or *semi-regular* tessellation.



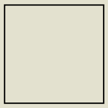
5 Investigate shapes that will tessellate when paired with one other shape.

(a) Draw a semi-regular tessellation.

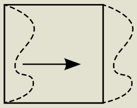
(b) Explain why the two shapes that you selected in (a) form a semi-regular tessellation.

6 Make an Escher-style drawing of your own.

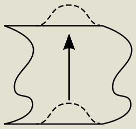
(a) Start with a square.



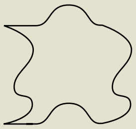
(b) Move a shape from the left to the right.



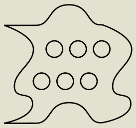
(c) Move a piece from the bottom to the top.



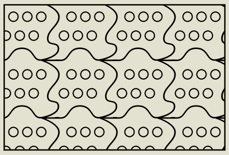
(d) You now have an unusual shape that will tessellate.



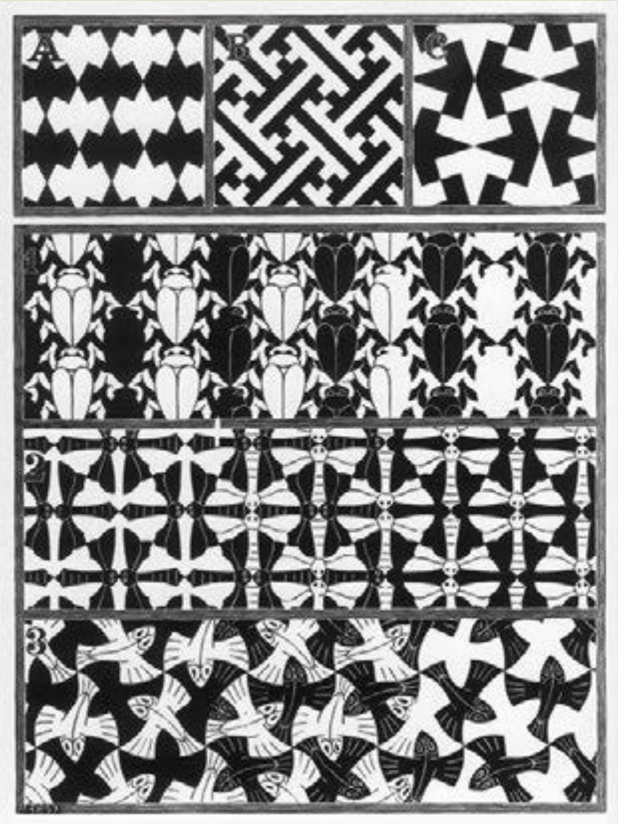
(e) Add detail to the inside of your shape.



(f) Copy your shape and place the shapes together to fill an area. The area could be any shape: square, rectangle, triangle, circle and so on.



7 Look at the print *Regular Division of the Plane II*, below. Discuss how Escher has used the basic shapes and patterns in the top three squares in the designs below.



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8 *Gravity* by M.C. Escher is a lithograph.

(a) What is a lithograph?

(b) What geometrical solid is shown here?

9 Find out more about M.C. Escher and his art.

10 Find out about other artists who use tessellations or other mathematical properties in their art.

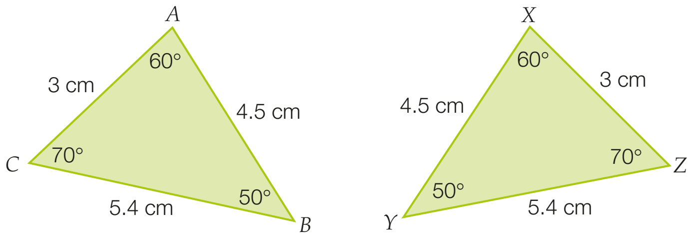
11 Find examples of tessellation being used in other areas, such as architecture or fashion designs.

8.4

Congruent triangles

Triangles that have corresponding sides of the same length *and* corresponding angles of the same size are known as **congruent triangles**.

The triangles shown below are congruent. The congruence statement to describe the two triangles is written as $\Delta ABC \equiv \Delta XYZ$. This is said as 'triangle *ABC* is congruent to triangle *XYZ*'.



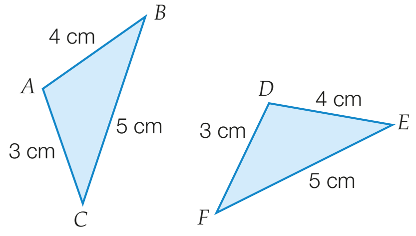
ΔXYZ is a reflection of ΔABC. All matching sides are the same length and all matching angles are the same size. It is not necessary, however, to know all side lengths and all angles to find whether two triangles are congruent.

Tests for congruency

There are four tests you can apply to find whether two triangles are congruent. Each of the tests needs three pieces of information.

- Side, Side, Side (SSS)

  Triangles are congruent if all corresponding sides are the same length.

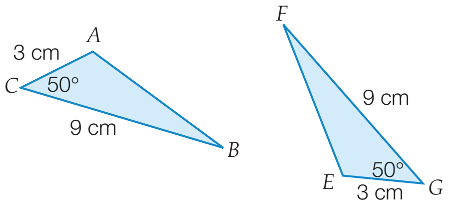


$AB = DE$
$BC = EF$
$AC = DF$

Therefore, $\Delta ABC \equiv \Delta DEF$ (SSS)

- Side, Angle, Side (SAS)

  Triangles are congruent if they have two sides of equal lengths and the angle formed by the two sides is equal (this is the **included angle**).

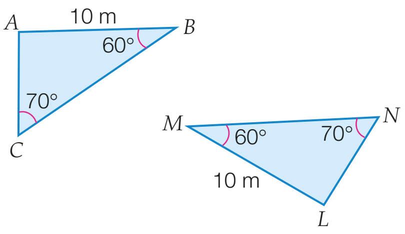


$AC = EG$
$\angle ACB = \angle EGF$
$BC = FG$

Therefore, $\Delta ABC \equiv \Delta EFG$ (SAS)

- Angle, Side, Angle (ASA)

  Triangles are congruent if two corresponding angles are the same size and one pair of corresponding sides are the same length.

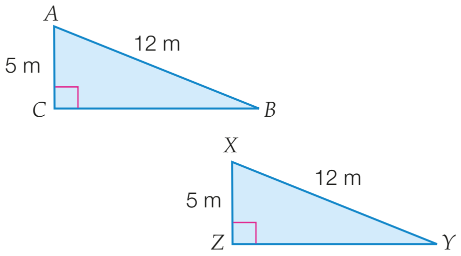


$$\angle ABC = \angle LMN$$
$$AB = LM$$
$$\angle ACB = \angle LNM$$

Therefore, $\Delta ABC \equiv \Delta LMN$ (ASA)

- Right angle, Hypotenuse, Side (RHS)

  Right-angled triangles are congruent if the hypotenuses (the sides opposite the right angles) and a pair of corresponding second sides have equal lengths.



$$\angle ACB = \angle XZY = 90°$$
$$AB = XY$$
$$AC = XZ$$

Therefore, $\Delta ABC \equiv \Delta XYZ$ (RHS)

The four congruency tests are:

- Side, Side, Side (SSS)
- Side, Angle, Side (SAS)
- Angle, Side, Angle (ASA)
- Right angle, Hypotenuse, Side (RHS).

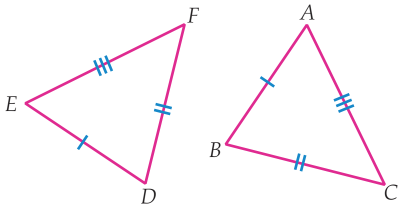
After a congruence statement, write in brackets the initials of the test used, e.g. (SSS).

Worked example 7

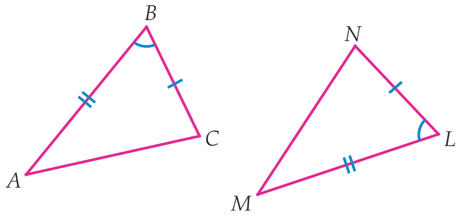
W.E. 7

Show that each of the following pairs of triangles are congruent, stating the congruence test used.

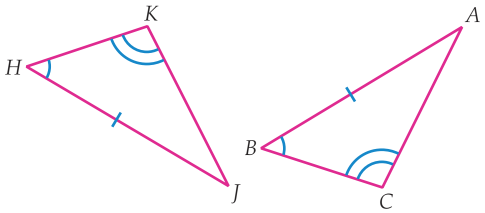
(a)



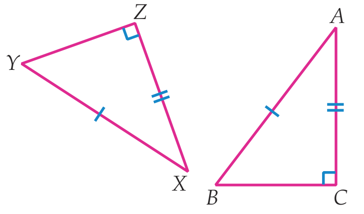
(b)



(c)



(d)



Thinking	Working
(a) **1** State the corresponding sides that are equal.	(a) $AB = ED$ (Side) $BC = DF$ (Side) $CA = FE$ (Side)

	2	State that the triangles are congruent, taking care to state corresponding sides in the same order. In brackets, write the initials of the congruence test used.	Therefore $\Delta ABC \equiv \Delta EDF$ (SSS)
(b)	1	State the corresponding sides and angles that are equal.	(b) $AB = ML$ (Side) $\angle ABC = \angle MLN$ (Angle) $BC = LN$ (Side)
	2	State that the triangles are congruent, taking care to state corresponding vertices in the same order. In brackets, write the initials of the congruence test used.	Therefore $\Delta ABC \equiv \Delta MLN$ (SAS)
(c)	1	State the corresponding sides and angles that are equal.	(c) $\angle ABC = \angle JHK$ (Angle) $\angle ACB = \angle JKH$ (Angle) $AB = JH$ (Side)
	2	State that the triangles are congruent, taking care to state corresponding sides and angles in the same order. In brackets, write the initials of the congruence test used.	Therefore $\Delta ABC \equiv \Delta JHK$ (ASA)
(d)	1	State the corresponding sides and angles that are equal.	(d) $\angle ACB = \angle XZY = 90°$ (Right angle) $AB = XY$ (Hypotenuse) $CA = ZX$ (Side)
	2	State that the triangles are congruent, taking care to state corresponding sides and angles in the same order. In brackets, write the initials of the congruence test used.	Therefore $\Delta ABC \equiv \Delta XYZ$ (RHS)

Constructing congruent triangles

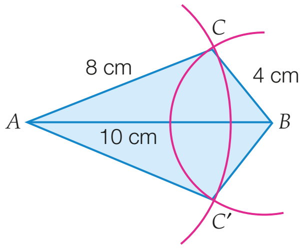
Side, Side, Side (SSS)

You can construct congruent triangles with three given side lengths (SSS) by carrying out the following steps. The example below is for a triangle with sides of 10 cm, 8 cm and 4 cm.

Step 1 Rule a side of length 10 cm and label it *AB*.

Step 2 Set your compass to a radius of 8 cm. Place the point on *A*. Draw an arc, extending it above and below the line *AB*.

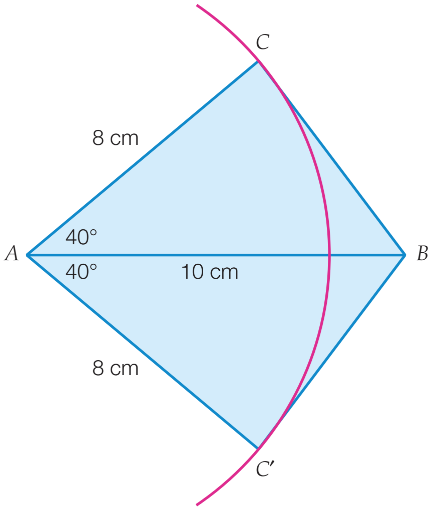
Step 3 Set your compass to a radius of 4 cm. Place the point on *B*. Draw another arc, extending it again above and below *AB*. The two arcs should intersect at two points. Label these points *C* and *C*′.



Step 4 Use your ruler to join the sides *AC*, *BC*, *AC*′ and *BC*′.

Notice that you have drawn two congruent triangles: Δ*ABC*′ is a reflection of Δ*ABC* in the line *AB*.

Side, Angle, Side (SAS)



You can construct congruent triangles with two given side lengths and a given included angle (SAS) in a similar way. The following steps produce triangles with sides of 10 cm, 8 cm and an included angle of 40°.

Step 1 Rule a side of length 10 cm and label it *AB*.

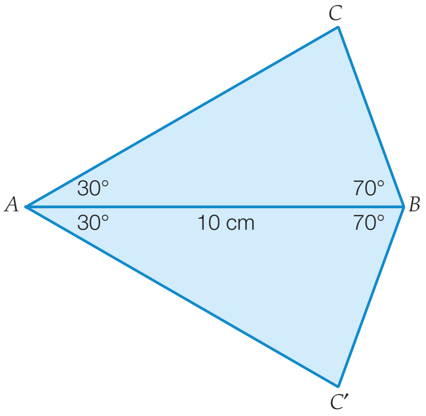
Step 2 Use a protractor to construct a 40° angle above and below the line at point *A*.

Step 3 Set your compass to a radius of 8 cm. Place the point on *A* and draw an arc above and below *AB* so that it intersects the angle arms drawn in Step 2. Label the points of intersection *C* and *C*′ respectively.

Step 4 Use your ruler to join the sides *BC* and *BC*′.

You have now drawn two congruent triangles with Δ*ABC*′ being a reflection of Δ*ABC* in the line *AB*.

Angle, Side, Angle (ASA)



You can construct congruent triangles with a given side length and two given angles (ASA). The following steps produce triangles with sides of 10 cm, and angles of 30° and 70°.

Step 1 Rule a side of length 10 cm and label it *AB*.

Step 2 Use a protractor to construct a 30° angle above and below the line at point *A*.

Step 3 Use a protractor to construct a 70° angle above and below the line at point *B*.

Step 4 Extend the arms of the angles until they intersect. Label the points of intersection *C* and *C*′.

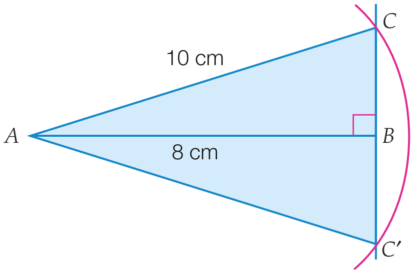
You have now drawn two congruent triangles with Δ*ABC*′ being a reflection of Δ*ABC* in the line *AB*.

Right angle, Hypotenuse, Side (RHS)

You can construct congruent right-angled triangles with a given hypotenuse and side (RHS). The following steps produce a triangle with a hypotenuse of 10 cm and a shorter side of 8 cm.

Step 1 Rule a side of length 8 cm and label it *AB*.

Step 2 Using a compass or a protractor, construct a line perpendicular to *AB* to pass through point *B,* extending above and below point *B*.



Step 3 Set your compass to a radius of 10 cm. Place the point on *A*. Draw an arc, extending it above and below the line *AB*. The arc should intersect the perpendicular line at two points. Label these points *C* and *C*′.

Step 4 Use your ruler to join the sides *AC* and *AC*′.

Again, you have drawn two congruent triangles, with Δ*ABC*′ being a reflection of Δ*ABC* in the line *AB*.

8.4 Congruent triangles

Navigator

Answers p. 668

1, 2, 3, 4, 5, 7 (a–c), 8, 13, 15, 16	1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 15, 16	1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

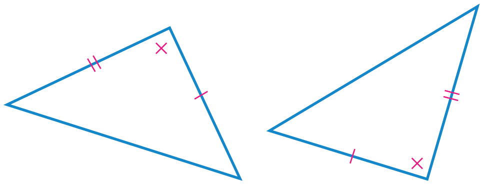
Equipment required: ruler, protractor and compass for Questions **8**, **11–16**

Fluency

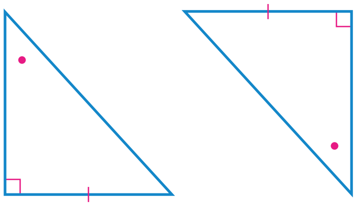
W.E. 7

1 Are the following pairs of triangles congruent? If yes, state the congruence test used. *Note:* Angles marked with the same symbol have the same value.

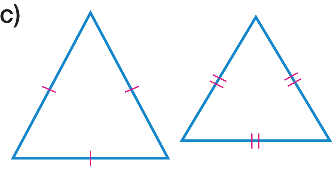
(a)



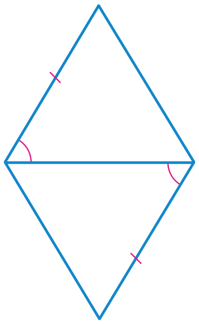
(b)



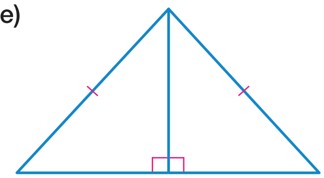
(c)



(d)



(e)



2 If $\Delta ABC \equiv \Delta QRS$, answer the following questions.

(Hint: Draw the triangles to help you visualise the situation.)

(a) Which angle is included by the sides AB and BC?

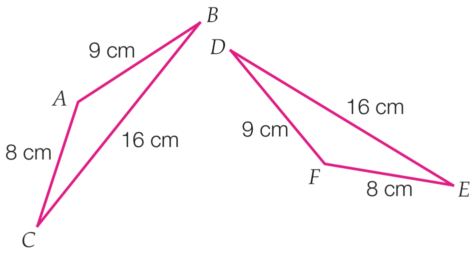
(b) Which side is included by $\angle SQR$ and $\angle RSQ$ in ΔQRS?

(c) Which angle in ΔQRS must be congruent to $\angle BCA$ of ΔABC?

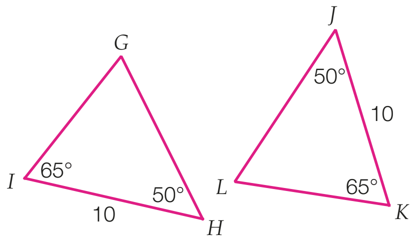
(d) Which side in ΔABC is congruent to QR in ΔQRS?

3 Show that each of the following pairs of triangles are congruent, stating the congruence test used.

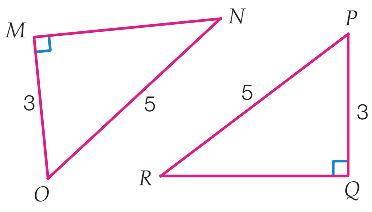
(a)



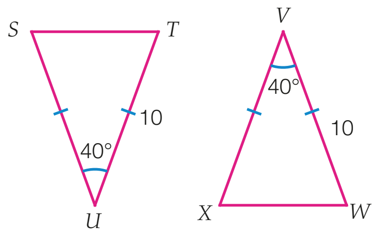
(b)



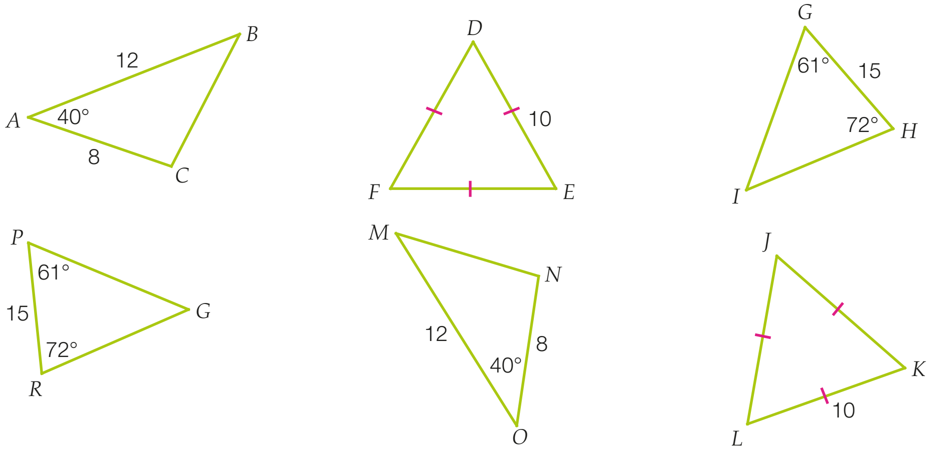
(c)



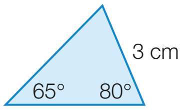
(d)



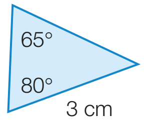
4 Find three pairs of congruent triangles among the following diagrams. Write a congruence statement for each pair and state the congruence test used.



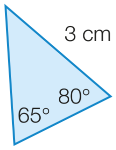
5 Which of the following triangles is not congruent to the triangle shown?



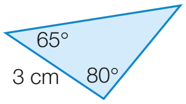
A



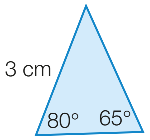
B



C



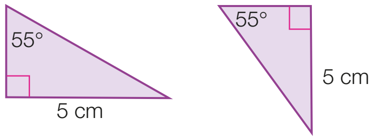
D



6 Which test could be used to show that these two triangles are congruent?

A SSS **B** SAS

C ASA **D** RHS



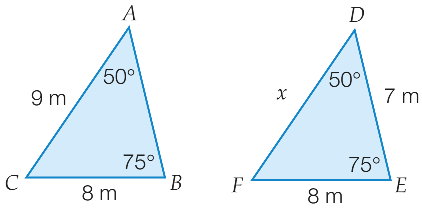
Understanding

7 For each pair of triangles below:

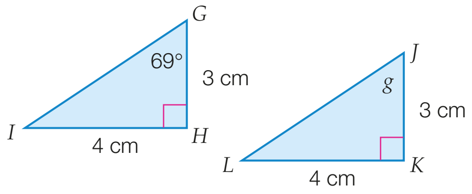
(i) show that they are congruent, stating the relevant congruency test

(ii) state the value of the pronumeral.

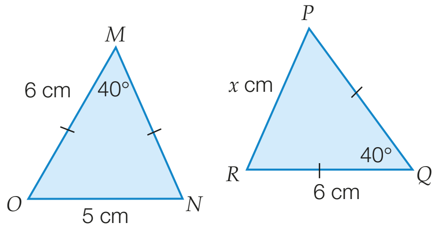
(a)



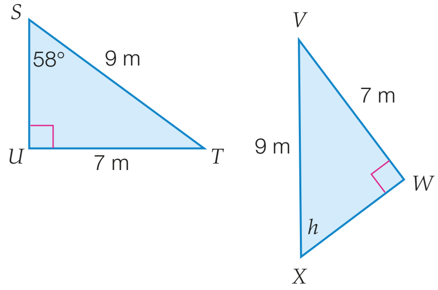
(b)



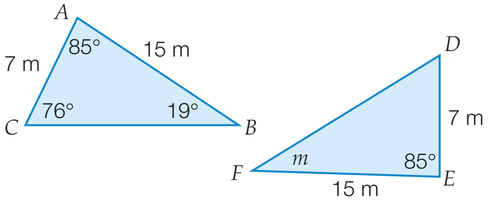
(c)



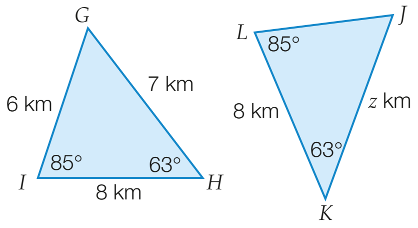
(d)



(e)



(f)



8 Use a protractor, compass and ruler to construct the following unique triangles.

(a) sides 7 cm and 4 cm, with included angle 30°

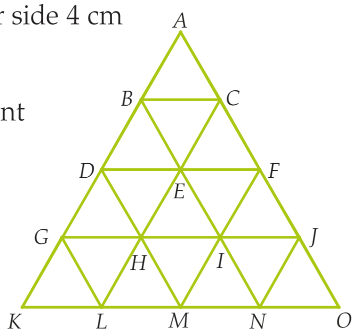
(b) angles of 20° and 100° with the side between the angles 8 cm in length

(c) right-angled triangle with hypotenuse 6 cm and another side 4 cm

(d) sides 3 cm, 5 cm and 8 cm

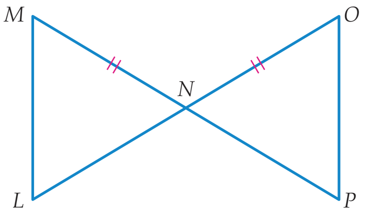
9 Find three pairs of congruent triangles, each pair of a different size, in this diagram. (The small triangles are all equilateral.)

State the congruence test that you used.



Reasoning

10 Ayda's teacher tells her that the two triangles in this diagram were drawn using only 4 straight lines, and are congruent by the test of SAS (Side, Angle, Side). What sides or angles must be equal for this to be true?



11 Triangle *ABC* has the following measurements: $AB = 6$ cm, $\angle ABC = 30°$ and $AC = 4$ cm.

(a) Use a protractor and ruler to draw two different triangles with these measurements.

(b) Explain why two triangles with only these measurements stated do not have sufficient information to show that they are congruent.

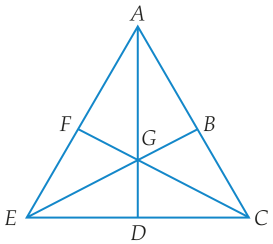
(c) Write three further measurements for triangle *ABC* to make the triangle unique.

12 Explain why having two pairs of corresponding equal sides does not necessarily mean that a pair of triangles is congruent. Use a protractor, compass and ruler to draw a diagram to support your answer.

13 Explain why two triangles with three equal angles may not necessarily be congruent. Use a protractor, compass and ruler to draw a diagram to support your answer.

14 The word 'bisect' means 'cut in half'. Each angle of the equilateral triangle *ABC* shown has been bisected. Three different sizes of congruent triangle are formed. The side length of the triangle *ABC* is 6 cm.

(a) Using a ruler and a protractor, draw the three different-sized triangles formed inside the equilateral triangle on the right.



(b) For each set of congruent triangles formed in the equilateral triangle above:

- **(i)** write a congruency statement
- **(ii)** state a congruency test to prove that they are congruent.

(c) Draw an example of the following types of triangles and bisect all angles using a protractor.

- **(i)** isosceles triangle
- **(ii)** scalene triangle

(d) How do the congruent triangles formed in **(c)** differ from the congruent triangles in an equilateral triangle? Why is this?

Open-ended

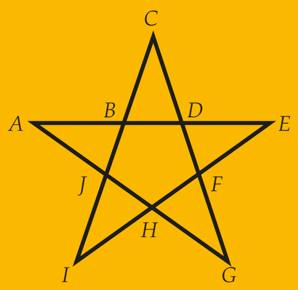
15 Construct a pair of congruent triangles of different orientation, using only a pencil, ruler and protractor. State the minimum information necessary for each triangle to show that they are congruent.

16 Construct a pair of congruent triangles of different orientation, using only a pencil, ruler and compass. State the minimum information necessary for each triangle to show that they are congruent.

Problem solving

The magical pentagram

When the non-adjacent vertices of a regular pentagon are joined with five straight lines, a pentagram is formed. Within a pentagram there are different congruent figures.



(a) Find **(i)** two congruent kites
- **(ii)** two large congruent isosceles triangles
- **(iii)** two small congruent isosceles triangles.

If the adjacent vertices of the pentagram are now joined to form a regular pentagon, more congruent shapes are formed.

(b) Find **(i)** two congruent rhombuses
- **(ii)** two congruent acute-angled isosceles triangles, not congruent with those in part **(a)**
- **(iii)** two congruent obtuse-angled isosceles triangles, not congruent with those in part **(a)**
- **(iv)** two congruent trapeziums
- **(v)** two more different-sized congruent trapeziums.

(c) How many different-sized types of isosceles triangles are there?

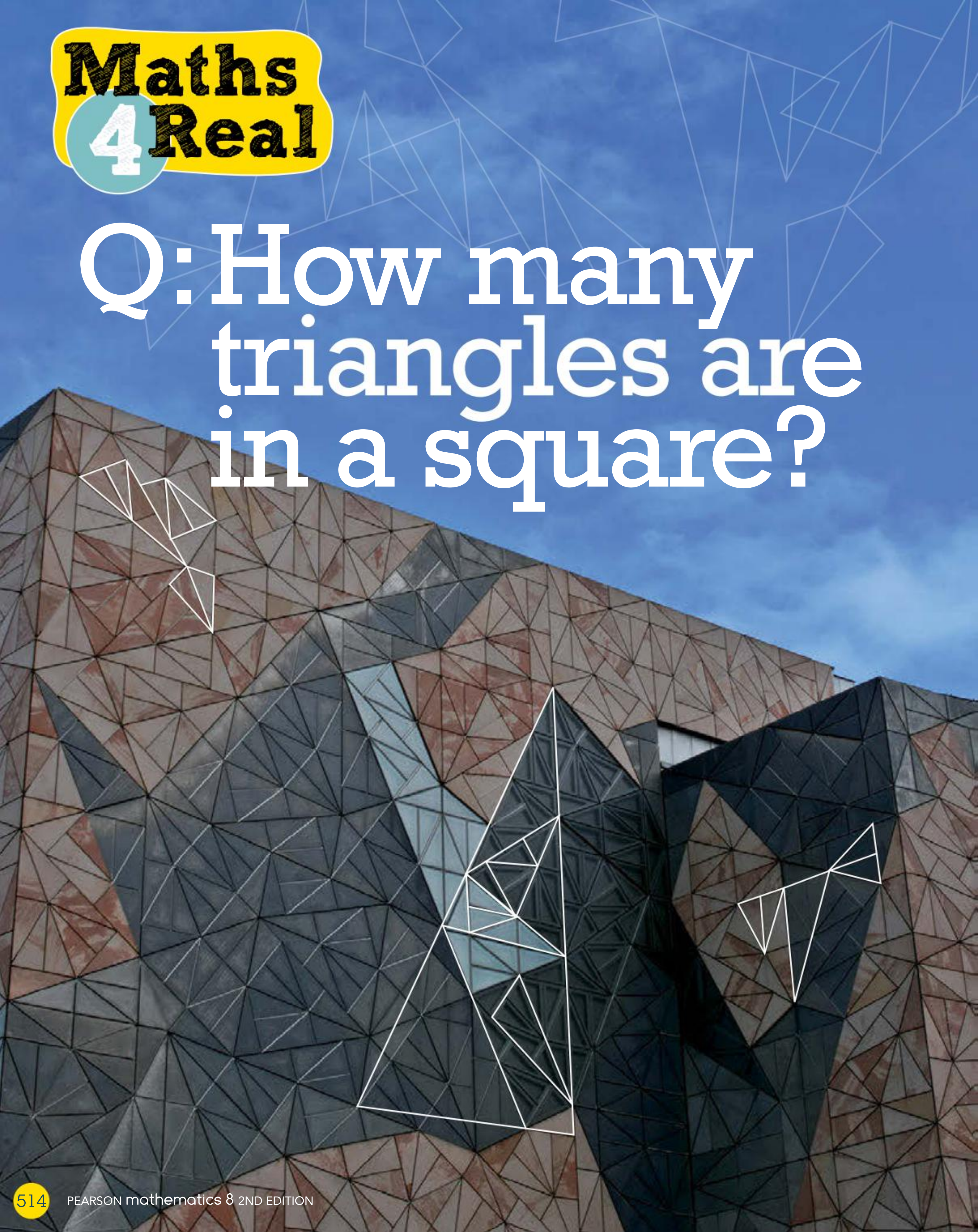
(d) How many congruent triangles are there of each size of isosceles triangle?

(e) How many triangles are there altogether in the diagram?

Strategy options
- Draw a diagram
- Look for a pattern.

Maths 4 Real

Q: How many triangles are in a square?



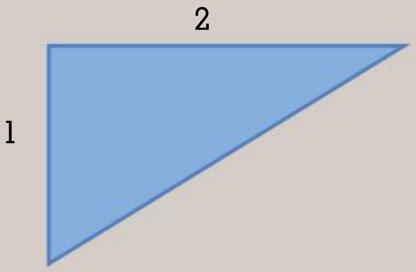
A: Far too many to count, if the square is Federation Square in Melbourne, Victoria. The buildings are covered with small right-angled triangles as seen in the photo. Many people love the design for its creative way of using a simple triangle to create patterns that have such a visual impact. Others believe that such modern buildings have no place among the old and familiar architecture of the past.

If you look closely, you can see that the triangles join together to make many different shapes.

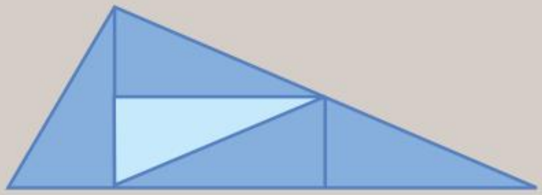
1 Find an isosceles triangle, an irregular quadrilateral, a parallelogram, a rectangle, a square, a kite, a trapezium and a pentagon in the design.

Draw diagrams to show what you have found.

How do all the triangular tiles fit together? Start with a right-angled triangle in which the middle-length side is twice the length of the shortest side.

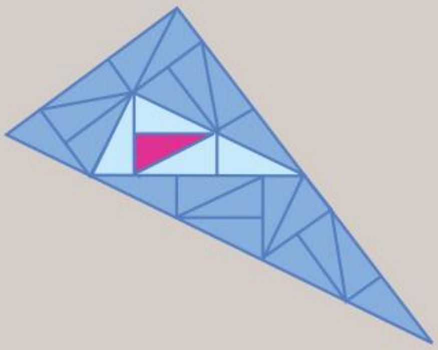


2 Draw four congruent triangles around this triangle to make a larger triangle as shown below.



3 How many triangles can you find in this shape? There are more than the five original triangles.

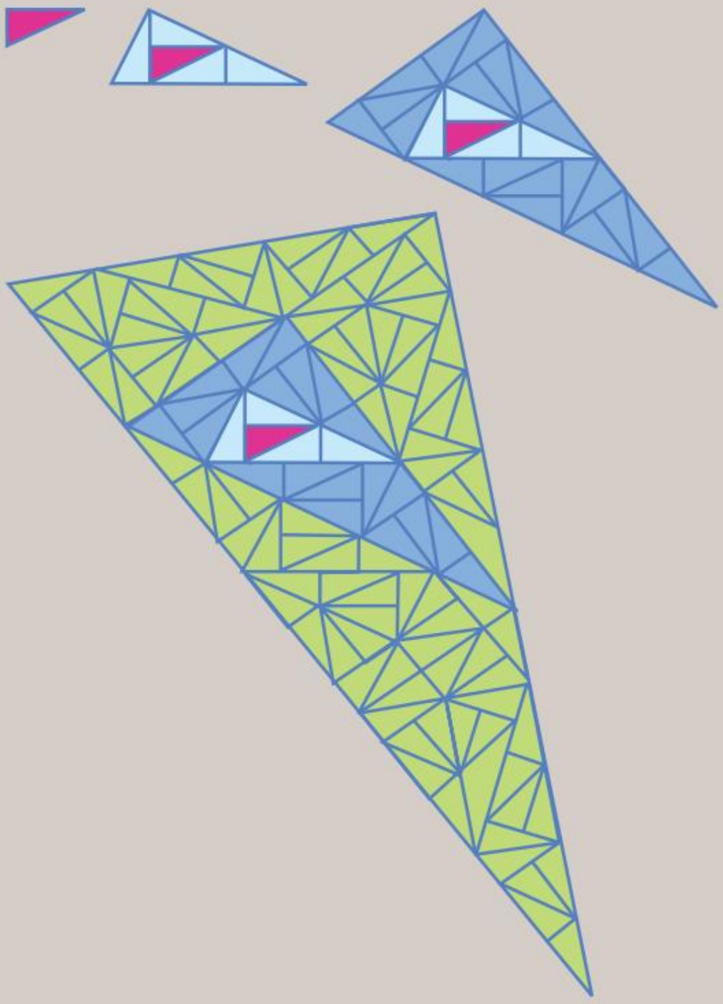
4 Using five of these new triangles, construct an even larger triangle as shown below.



5 How many triangles can you find now?

6 Find the number of triangles in the next size as shown below.

Use your answer to Question **5** to find a pattern that will help you.



7 Look closely at the sequence of diagrams above. What is happening to the orientation of the triangles as the largest triangle increases in size?

(Hint: This kind of shape tiling is called pinwheel tiling, after the pinwheel toy that rotates in the wind. This effect was named and analysed by the mathematicians John Conway and Charles Radin.)

8 Does pinwheel tiling make a repeating (tessellating) pattern? Refer to the picture of Federation Square on the opposite page.

Research

Find another example of pinwheel tiling in your local area, your capital city or anywhere else in the world.

Investigation

The Möbius strip

Equipment required: paper, scissors, sticky tape or glue

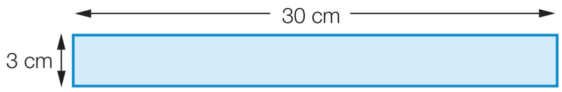
The Möbius strip is a loop with amazing properties discovered by the German mathematician and astronomer August Ferdinand Möbius (1790–1868).

The Big Question

Can a piece of paper have just one surface?

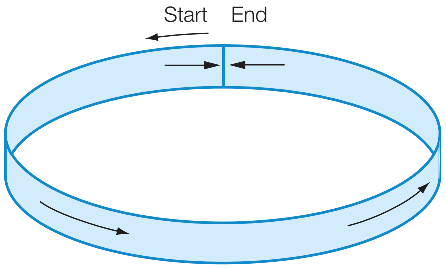
Engage

Cut several strips of paper about 3 cm wide and 30 cm long.

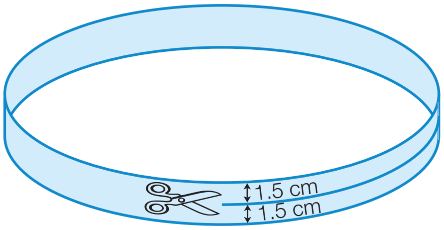


Take one of the strips and stick the ends together to form a loop as shown.

Draw a line all around the outside of the loop, starting at the join in the loop and not lifting your pencil until you get back to the join.

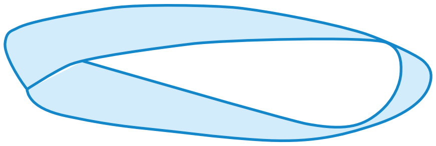


1. How many distinct (separate) surfaces does this loop have?
2. Cut the loop down the middle, all the way around. After doing this cut, how many separate pieces of paper are there? What shapes are they?



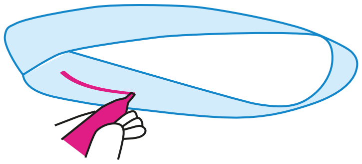
Explore

To investigate the Big Question, you need to make a Möbius strip. Take a new strip of paper, but before you stick the ends together, give one end a half-twist. This will form a Möbius strip, as shown.



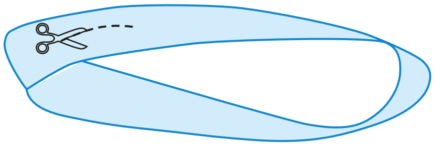
3. How many surfaces do you think the Möbius strip has?

To check your prediction, start at a point and begin drawing a line along the loop. Do not take the pencil off the loop until you reach the starting point again.



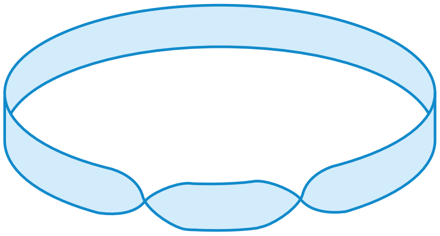
4. How many surfaces does a Möbius strip have?

Now, cut the Möbius strip down the middle, all the way around the loop.



5. After doing this cut, how many separate pieces of paper are there? What shape(s)?

Take a new strip of paper. Give one end two half-twists (that is, one complete 360° twist) and then stick the ends together.



Again, draw a line along the loop without lifting the pencil.

6. How many surfaces does this loop have?

Strategy options

- Make a model.
- Look for a pattern.

7 Predict what will result from cutting this loop along the middle, lengthways. Now, cut the loop.

Explain

8 Copy and complete the table below.

Number of half-twists	Number of surfaces	Figure result from cutting along middle
0		
1		
2		

9 Does a Möbius strip have an inside and an outside surface? Explain.

10 Describe the effect of cutting a Möbius strip along the middle. Why do you think this happens?

Elaborate

11 **(a)** Predict how many surfaces a loop with three half-twists will have.

(b) Check your prediction.

12 **(a)** If a strip of paper is given three half-twists to form a loop, what figure do you predict will result from cutting it along the middle all the way around? How confident are you that your prediction is correct?

(b) Check your prediction.

13 **(a)** Can you make a conclusion that connects the number of twists with the number of surfaces? Do you have enough evidence for this conclusion?

(b) Answer the Big Question.

Evaluate

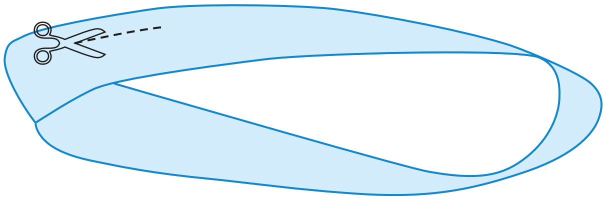
14 Were you surprised by some of your observations in this investigation? Explain why or why not.

15 How is the mathematical thinking that you need for a task like this different from the thinking needed for other maths tasks, such as measurement calculations or algebra?

Extend

Investigate the properties of the Möbius strip further.

Start by making another Möbius strip, but instead of cutting it lengthways along the middle, cut the strip starting one-third from the edge.

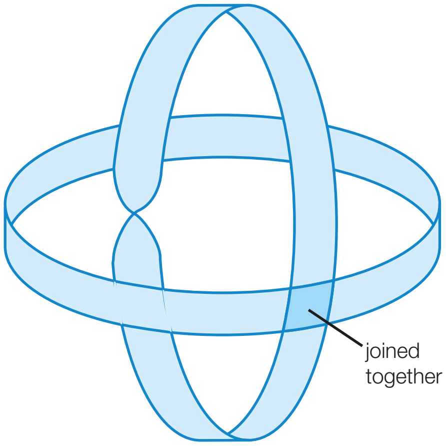


16 What figure is created by cutting a Möbius strip one-third of the way in from the edge?

17 **(a)** Predict the figure that will result from cutting a loop in this way for loops that have different numbers of half-twists.

(b) Check your prediction.

Stick together an ordinary loop and a Möbius strip as shown.



18 What happens if both of these loops are now cut lengthways?

8.5

Congruence and quadrilaterals

Definitions and properties of geometric figures

It is important to know the difference between a **definition** and a **property** of a geometric figure.

A definition is the *minimum* amount of information necessary to describe a figure *exactly*.

A property is any characteristic that a figure has after it has been defined.

Because the definition of a figure only needs the minimum information, the definition will usually not include all of the properties of the figure, only some of the properties.

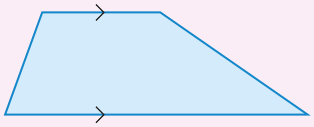
The other properties can be found by considering the consequences of the definition.

Definitions of special quadrilaterals

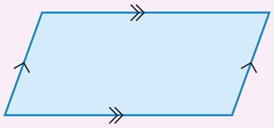
A quadrilateral is a polygon with four straight sides.

The definitions of six special quadrilaterals (trapezium, parallelogram, rectangle, kite, rhombus and square) are given below. You can use these definitions to prove angle and side properties for these quadrilaterals.

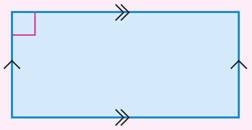
A trapezium is a quadrilateral with only one pair of parallel sides.



A parallelogram is a quadrilateral with both pairs of opposite sides parallel.



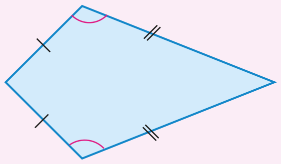
A rectangle is a quadrilateral with both pairs of opposite sides parallel and a right angle.



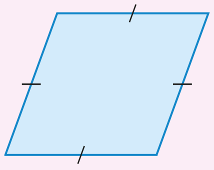
It can also be defined as:

- a parallelogram with a right angle.

A kite is a quadrilateral with two pairs of adjacent sides equal in length.



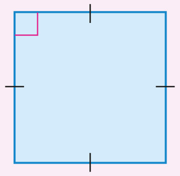
A rhombus is a quadrilateral with all sides equal in length.



It can also be defined as:

- a parallelogram with adjacent sides equal
- a kite with all sides equal in length.

A square is a quadrilateral with all sides equal and a right angle.

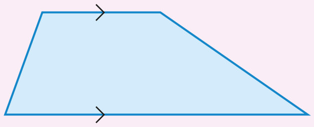


It can also be defined as:

- a parallelogram with all sides equal and a right angle
- a rectangle with all sides equal
- a rhombus with a right angle.

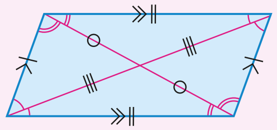
Properties of special quadrilaterals

Trapezium



- One pair of opposite sides are parallel.

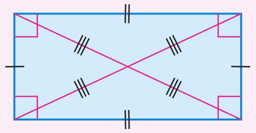
Parallelogram



- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.

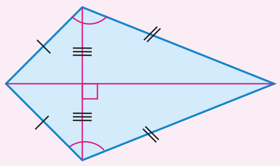
Rectangle

A special type of parallelogram



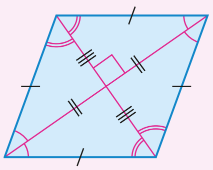
- Both pairs of opposite sides are equal in length.
- All angles are right angles.
- The diagonals are equal in length and bisect each other.

Kite



- One pair of opposite angles is equal.
- The diagonals intersect at right angles.
- The diagonal that passes through the two equal angles is bisected by the other diagonal.

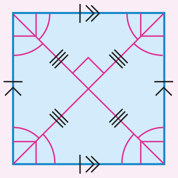
Rhombus



A special type of parallelogram and kite

- Both pairs of opposite sides are parallel.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other at right angles.
- Each diagonal bisects the two interior angles through which it passes.

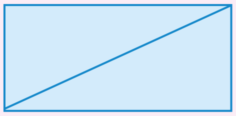
Square



A special type of parallelogram, rhombus, rectangle and kite

- All angles are right angles.
- The diagonals are equal in length and bisect each other at right angles.
- Each diagonal bisects the two interior angles through which it passes and each angle formed equals 45°.

Proving special properties

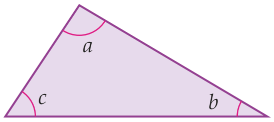


Any quadrilateral can be divided into two triangles with a diagonal line connecting two vertices. If you can prove that the two triangles are congruent using the definition of the special quadrilaterals, then you can prove the properties of the special quadrilateral.

Remember that to *prove* something is to demonstrate that it is true for all cases.

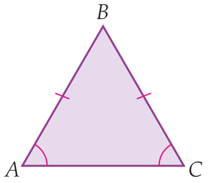
You can use your knowledge of angle sums and angles on parallel lines to help you investigate the properties of special quadrilaterals.

- Angles in a triangle add to 180°.



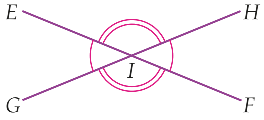
$a + b + c = 180°$

- Base angles of isosceles triangles are equal.



$\angle BAC = \angle BCA$

- Vertically opposite angles are equal.

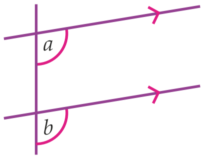


$\angle EIG = \angle FIH$

$\angle EIH = \angle GIF$

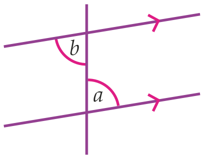
When two parallel lines are crossed by a transversal (a third line), then:

- Corresponding angles are equal.



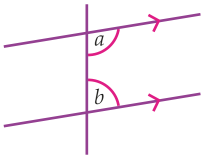
$a = b$

- Alternate angles are equal.



$a = b$

- Co-interior (allied) angles add to 180°.



$a + b = 180°$

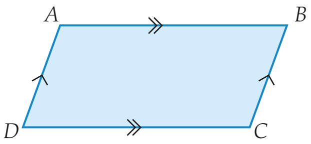
Worked example 8

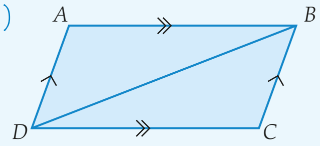
W.E. 8

Use the definition of a parallelogram and congruent triangles to prove that:

(a) both pairs of opposite sides are equal in length

(b) both pairs of opposite angles are equal.



Thinking	Working
(a) **1** Divide the parallelogram into two triangles by joining opposite vertices. (Join *B* to *D*.)	(a) 
2 Find whether the two triangles are congruent by writing down known information about sides and angles. One side is common and two pairs of angles are equal because they are alternate angles on parallel lines ($\angle ABD$ and $\angle CDB$, $\angle ADB$ and $\angle CBD$). Therefore, the two triangles are congruent (ASA).	$\angle ABD = \angle CDB$ (alternate angles) $\angle ADB = \angle CBD$ (alternate angles) $BD = DB$ (common side)
3 Write a congruence statement with the relevant congruence test in brackets.	Therefore, $\Delta ABD \equiv \Delta CDB$ (ASA)
4 Corresponding sides of congruent triangles are equal.	Therefore, $AB = CD$ and $AD = CB$
5 State the property you have just proved.	Therefore, both pairs of opposite sides are equal in length.

(b)	**1**	State the congruent triangles found in part **(a)**.	(b) $\Delta ABD \equiv \Delta CDB$
	2	Corresponding angles of congruent triangles are equal.	Therefore, $\angle DAB = \angle BCD$
	3	Use co-interior angles to show that the other pair of angles are equal.	$\angle ADC = 180° - \angle DAB$ (co-interior) and $\angle CBA = 180° - \angle DAB$ (co-interior) Therefore, $\angle ADC = \angle CBA$
	4	State the property you have just proved.	Therefore, both pairs of opposite angles are equal.

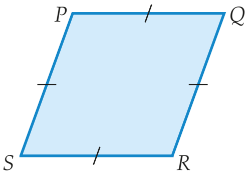
Worked example 9

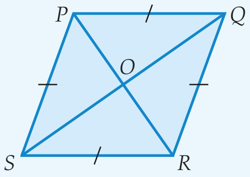
W.E. 9

Use the definition of a rhombus to prove that:

(a) the diagonals bisect each other at right angles

(b) each diagonal bisects the two interior angles it passes through.



		Thinking	Working
(a)	**1**	Divide the rhombus into four triangles by drawing both diagonals. (Join P to R and Q to S.) Label the point of intersection O.	(a) 
	2	Prove that two large triangles are congruent by using the definition of a rhombus (four equal sides).	$PS = QR$ $SR = PQ$ PR is common
	3	Write a congruence statement.	Therefore, $\Delta PSR \equiv \Delta RQP$ (SSS)
	4	Prove two small triangles opposite each other are congruent, using properties of the larger congruent triangle if necessary.	$\angle POS = \angle QOR$ (vertically opposite) $PS = QR$ (given) $\angle SPR = \angle QRP$ ($\Delta PSR \equiv \Delta RQP$)
	5	Write a congruence statement.	$\Delta POS \equiv \Delta ROQ$ (ASA)
	6	Corresponding sides of the congruent triangles are equal.	Therefore, $PO = RO$ and $SO = QO$
	7	Prove two small triangles adjacent to each other are congruent.	$PS = PQ$ (given) PO is common $SO = QO$ (proven)
	8	Write a congruence statement.	Therefore, $\Delta POS \equiv \Delta POQ$ (SSS)
	9	Because there are two congruent triangles with a common side, you can make a statement about the adjacent angles.	Therefore, $\angle POS = \angle POQ = 90°$ (angles on a straight line) Similarly, $\angle ROS = \angle ROQ = 90°$

	10	State the property you have just proved.	The diagonals bisect each other at right angles.
(b)	1	Write any information needed that was proved in part **(a)**.	(b) $\angle SPR = \angle QRP$
	2	Use the base angles of congruent isosceles triangles to find more equal angles. (*PSR* and *PQR* are isosceles.)	$\angle SPR = \angle SRP$ (base angles of an isosceles triangle) and $\angle QRP = \angle QPR$ (base angles of isosceles triangle) Therefore, $\angle SPR = \angle QPR = \angle SRP = \angle QRP$
	3	Show all angles that are equal to each other.	So, $\angle SPR = \angle QPR$ and $\angle SRP = \angle QRP$ Similarly, $\angle PSQ = \angle RSQ = \angle RQS = \angle PQS$
	4	State the property you have just proved.	Each diagonal bisects the two interior angles it passes through.

8.5 Congruence and quadrilaterals

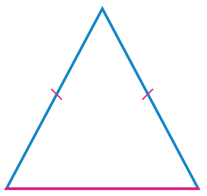
Navigator

Answers p. 669

1, 2, 3, 4, 6, 7, 8, 10, 13, 14	2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (a), 13, 14	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Fluency

1 What kind of quadrilateral is formed when two identical isosceles triangles are joined at their base?

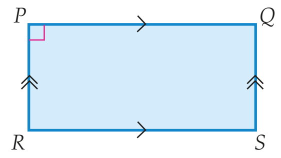


W.E. 8

2 Use the definition of a rectangle and congruent triangles to prove that:

(a) both pairs of opposite sides are equal in length

(b) both pairs of opposite angles are equal.

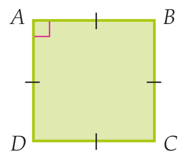


W.E. 9

3 Use the definition of a square to prove that:

(a) the diagonals bisect each other at right angles

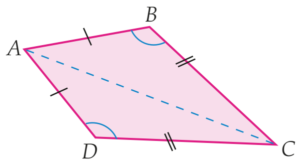
(b) each diagonal bisects the two interior angles it passes through.



4 A kite *ABCD* has been divided into two congruent triangles. Which of the following statements is true?

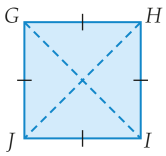
A $AD \| BC$ **B** $\angle ABC = \angle CDA$

C $\Delta BCD \equiv \Delta DAB$ **D** $CD = AB$

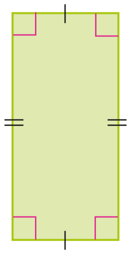


5 Prove that the diagonals of a kite intersect at right angles by first dividing it into two congruent triangles.

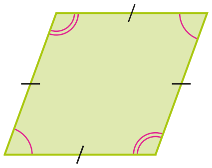
6 Square *GHIJ* is shown. Diagonals *GI* and *HJ* divide the square into four congruent triangles. Which of the following shapes will also divide into four congruent triangles when both diagonals are drawn?



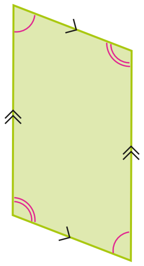
A



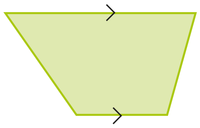
B



C



D



Understanding

7 Which statement best describes a trapezium?

A The diagonals are congruent.

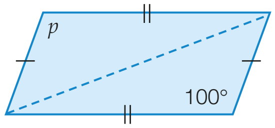
B There is a right angle.

C There is only one pair of parallel sides.

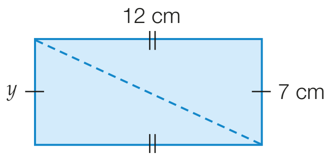
D All sides are equal.

8 Use congruent triangles and known angle facts to find the value of the pronumerals in the following quadrilaterals. State the reason for each of your answers.

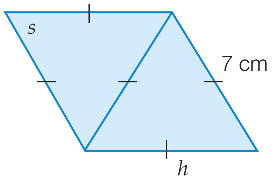
(a)



(b)



(c)



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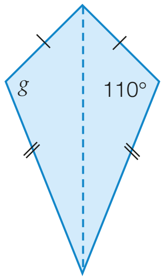
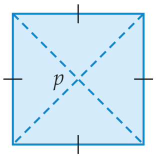


(e)



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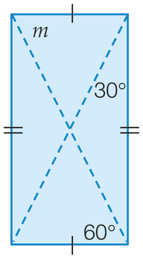
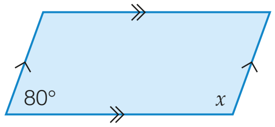
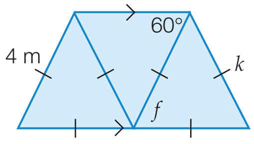


(g)



(h)

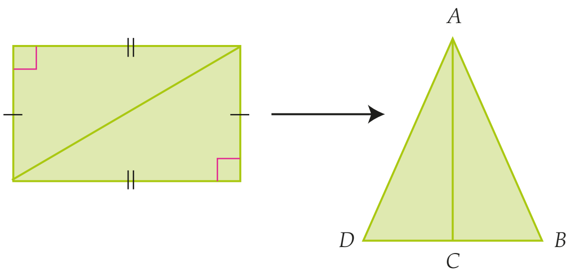


9 A rectangle has been cut along one diagonal to divide it into two triangles. The triangles are then rearranged to form another triangle.

(a) Mark equal side lengths and known angles on the triangle.

(b) Prove $\angle ADC = \angle ABC$.

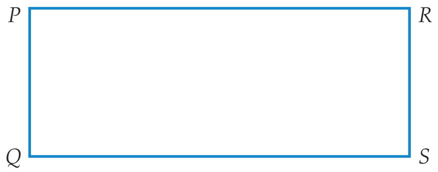
(c) What type of triangle is this?



Reasoning

10 You are given the following information about the quadrilateral below (which has not been drawn to scale):

- *PQ* is congruent to *RS*
- *PR* is congruent to *QS*.



(a) Has enough information been given for you to know whether this is a parallelogram? Explain your answer.

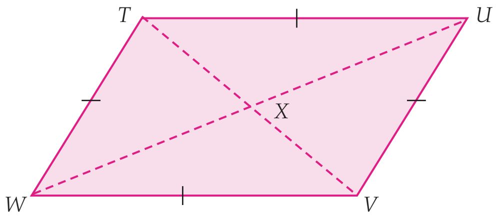
(b) Give one other property that would need to be stated to prove that the quadrilateral is a rectangle.

11 Draw a rectangle with length 6 cm and width 4 cm. Rule a diagonal line, dividing the rectangle into two triangles.

(a) Are the two triangles congruent? State a congruency test that proves the triangles will be congruent in all rectangles.

(b) The two congruent triangles in the rectangle you drew can be rearranged to form two other special types of quadrilateral. State the names of the two special quadrilaterals. Write a statement to show the congruent triangles formed within the quadrilaterals.

12 Rhombus *TUVW* is shown. Diagonals *UW* and *VT* divide it into four triangles, intersecting at point *X*.



(a) Prove $\angle TWV = \angle TUV$.

(b) What type of triangle is *TUW*?

(c) Prove $TX = XV$.

(d) Do any other quadrilaterals share these properties?

Open-ended

13 Draw a diagram to support each of the following statements.

(a) A parallelogram can be divided into two congruent parallelograms.

(b) A parallelogram can be divided into two parallelograms which are not congruent.

(c) A parallelogram can be divided into a rectangle and two congruent triangles.

14 Draw a quadrilateral in which one diagonal will divide the shape into congruent halves and the other diagonal will not.

Puzzle

The tablet of Geo Met Tree

An archaeologist digs up an ancient stone tablet. The tablet is found wrapped inside a scroll that explains the tablet's legendary mystical powers. The scroll reads:

'This tablet may seem incomplete. But you will have a happy life if, with two straight cuts, you can arrange all pieces into a square.'

Can you cut the tablet with two straight cuts and rearrange the pieces into the shape of a square?

Strategy options

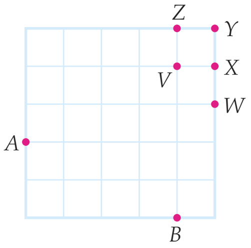
- Guess and check.
- Make a model.

Challenge 8

1 The points A, B, V, W, X, Y, Z are marked on a square grid as shown.

When joined to the points A and B, some of the points W, X, Y and Z will form an isosceles triangle. These points are:

A V and X **B** X and Y **C** Y and Z **D** V and Z



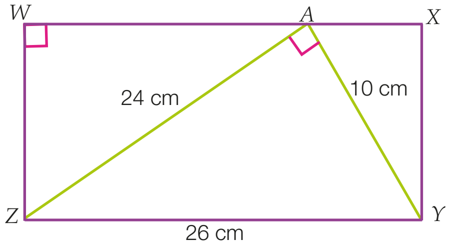
2 What is the area of the rectangle $WXYZ$?

A 120 cm^2

B 240 cm^2

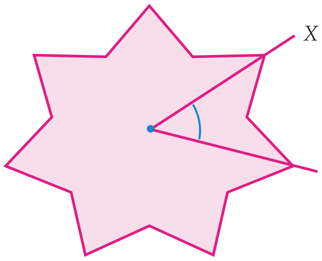
C 260 cm^2

D 624 cm^2



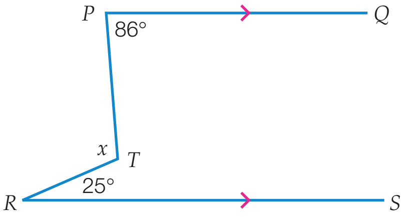
3 The large star on the Australian flag has 7 points, one for each state and one for the territories. The angle between X and Y as seen from the centre of the star is:

A 50° **B** $50\frac{3}{7}°$ **C** $51\frac{3}{7}°$ **D** $60\frac{3}{7}°$



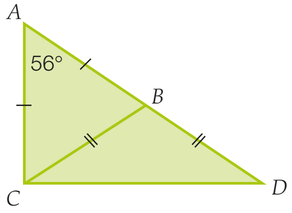
4 In the diagram, PQ is parallel to RS. The value of x is:

A 121° **B** 111° **C** 101° **D** 61°



5 In the diagram, $AB = AC$ and $BC = BD$. If $\angle BAC = 56°$, then $\angle CDB$ equals:

A 28° **B** 31° **C** 59° **D** 62°



6 Twenty-seven small cubes have been stacked to form this shape.

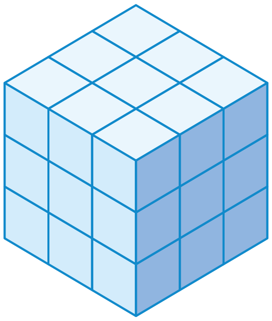
How many cubes have:

(a) no faces showing

(b) one face showing

(c) two faces showing

(d) three faces showing?



Chapter review 8

Maths literacy

- acute-angled triangle
- adjacent angles
- alternate angles
- bisect
- co-interior angles
- complementary angles
- congruence statement
- congruent
- congruent triangle
- corresponding angles
- equilateral triangle
- exterior angle
- hypotenuse
- included angle
- interior angle
- isosceles triangle
- obtuse-angled triangle
- quadrilateral
- right-angled triangle
- scalene triangle
- supplementary angles
- transversal
- vertex
- vertically opposite angles

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The point at which two arms of an angle meet is called the __________.

2 Shapes that can be placed on top of each other so that every side and vertex match up are said to be __________.

3 $EFGH \equiv PQRS$ is an example of a __________.

4 Triangles with one equal side length and two equal angles marked can be proven to be __________.

5 __________ add to 180°.

6 A __________ is a line that intersects two or more parallel lines.

7 A __________ has three sides, none of which are equal in length.

8 __________ are equal and lie on either side of a transversal.

9 __________ have a common arm and a common vertex.

10 An __________ has three equal sides and three equal angles.

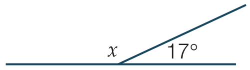
Fluency

Equipment required: ruler, protractor and compass for Questions **13**, **18**

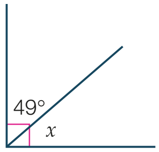
8.1

1 Find the value of x in the following diagrams.

(a)



(b)



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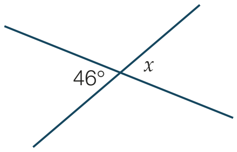

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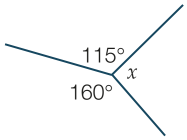

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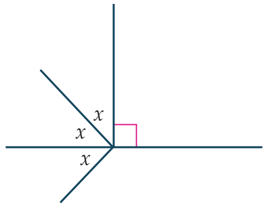

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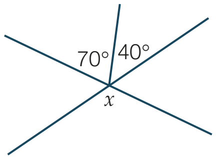
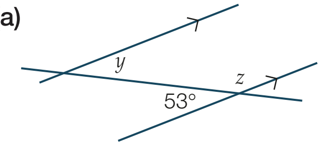


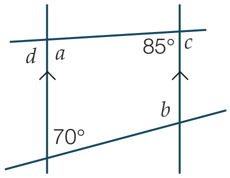
8.1

2 Find the value of the pronumerals in the following diagrams.

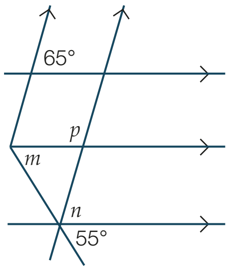
(a)



(b)

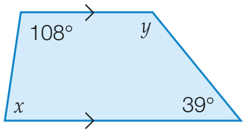


(c)

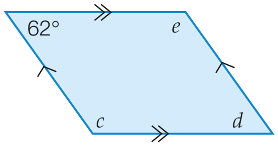


3 Find the value of the pronumerals in the following diagrams. 8.2

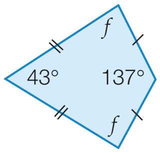
(a)



(b)



(c)



4 Match the name of the shape with its description. 8.2

(a) rectangle

(b) kite

(c) isosceles triangle

(d) trapezium

A adjacent sides of equal length, no parallel sides

B one pair of opposite sides parallel

C opposite sides equal in length, all angles 90°

D two sides equal in length

5 Plot the following points on the Cartesian plane: (-2, 1), (-3, 4), (1, 5), (1, 2). Join and label them *W*–*Z* in the order given to form the quadrilateral *WXYZ*. Do the following transformations on *WXYZ*. Label the transformed shape *W′X′Y′Z′*. 8.3

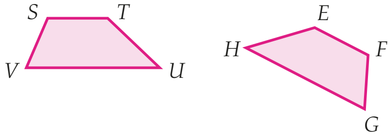
(a) A reflection in the x-axis.

(b) The translation [3, -4].

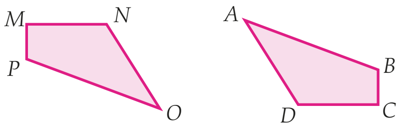
(c) A rotation 180° anticlockwise about the point (-3, 0).

6 For the two pairs of congruent figures shown below, complete the following pairings. 8.3

(i)



(ii)



(a) $H \leftrightarrow$

(b) $N \leftrightarrow$

(c) $TU \leftrightarrow$

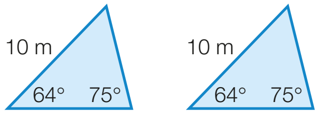
(d) $AD \leftrightarrow$

(e) $\angle CDA \leftrightarrow$

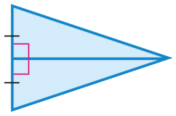
(f) $\angle EFG \leftrightarrow$

7 State the congruency test used to show that each of the following pairs of triangles are congruent. 8.4

(a)

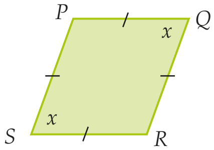


(b)

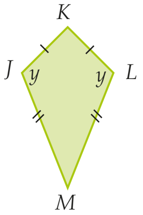


8 Use congruent triangles to prove that the opposite angles shown in the shapes below are equal: 8.5

(a)



(b)



Understanding

9 A heptagon can be divided into five triangles. 8.2

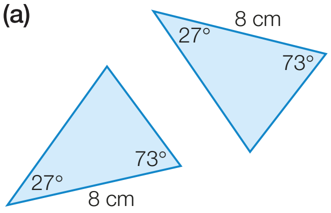
(a) Find the sum of the angles in a heptagon.

(b) Use your answer to the previous part to find the size of the interior angle in a regular heptagon.

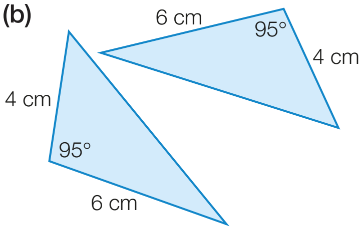
10 Explain why each of the following pairs of triangles are congruent.

8.4

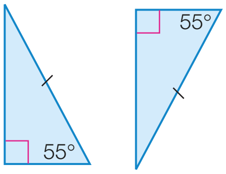
(a)



(b)

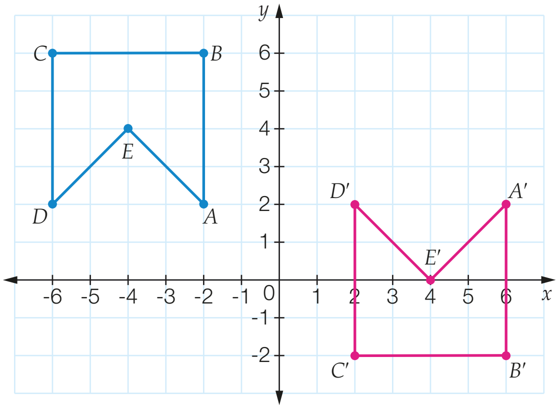


(c)



11 The shape *ABCDE* has had two transformations applied to it. Which sequence of transformations will place *ABCDE* onto the image *A′B′C′D′E′*?

8.3



A Reflection in the y-axis, then the translation [0,-2].

B Rotation 90° clockwise about the point (-2, 2), then the translation [2, 0].

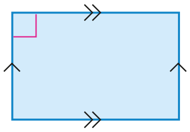
C Rotation 180° anticlockwise about the point (0, 0), then the translation [0, 2].

D Reflection in the x-axis, then the translation [8, 4].

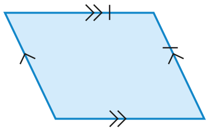
12 Identify each of the following quadrilaterals using the name that corresponds to all the properties shown.

8.2

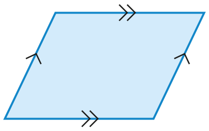
(a)



(b)



(c)



13 **(a)** Use a pencil, ruler and protractor to construct the following.

8.4

(i) a triangle with angles of 35° and 65°, and a side in between them of 7 cm

(ii) a triangle with sides of 9 cm and 6.5 cm, and an angle in between them of 48°

(b) Use a pencil, ruler and compass to construct the following.

(i) a triangle with sides of 7 cm, 9 cm and 11 cm

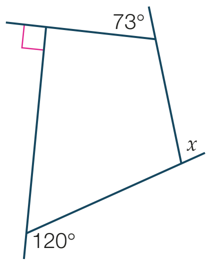
(ii) a right-angled triangle with a hypotenuse of 13 cm and a shorter side of 12 cm

Reasoning

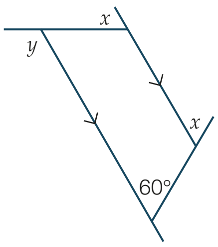
14 Find the value of the pronumerals in each of the following quadrilaterals.

8.1

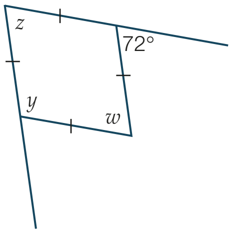
(a)



(b)



(c)

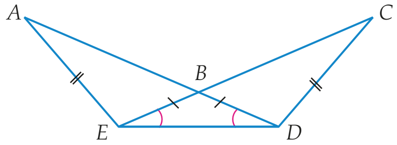


15 Which of the following special quadrilaterals will always divide into two congruent triangles along a diagonal? (More than one option may be correct.)

8.5

A kite **B** trapezium **C** rhombus **D** rectangle

16 Two triangles are placed so that an isosceles triangle forms where the two triangles overlap. Is there enough information to prove the triangles are congruent? Why or why not? **8.4**



17 Triangle ABC has the following measurements: $\angle ABC = 55°$, $AC = 10$ cm. Triangle DEF also has the same measurements (for $\angle DEF$ and side DF). **8.4**

(a) Use a protractor and ruler to draw two different triangles with these measurements.

(b) Explain why there is not enough information to show that ΔABC and ΔDEF are congruent.

(c) Give extra information that would ensure that the two triangles are congruent.

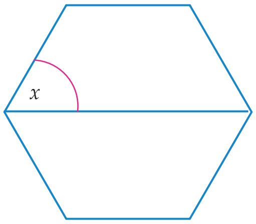
18 Explain how a square can be both a special type of rhombus and a special type of rectangle. **8.2**

Numeracy practice 8

Non-calculator

1 Two identical trapeziums fit together to make the regular hexagon shown.

What is the value of x?



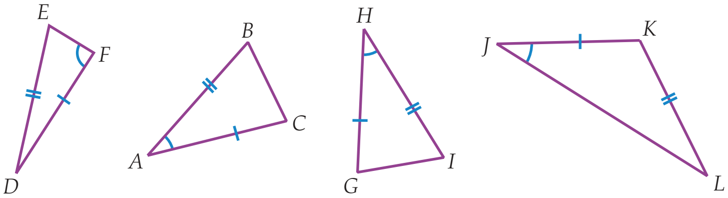
2 Which pair of these triangles is congruent?

A ΔABC and ΔDEF

B ΔGIH and ΔKLJ

C ΔGHI and ΔCAB

D ΔFDE and ΔJKL



3 Which of these are not always parallel?

A the opposite sides of a square

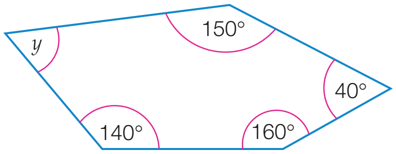
B the opposite sides of a parallelogram

C the opposite sides of a rhombus

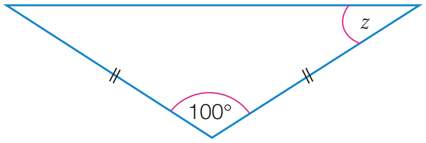
D the opposite sides of a kite

Calculator allowed

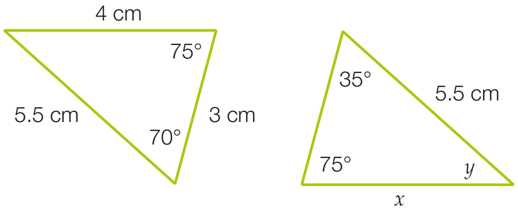
4 What is the value of y?



5 What is the value of z?



6 The following two triangles are congruent. Find the value of the pronumerals.



9

9

Statistics and probability

Counting the catch. The harvesting of seafood, such as abalone, is a profitable business in which statistics plays a vital role.

Ecologists and fishing authorities are concerned that overfishing could permanently reduce the numbers of fish and other sea creatures. To keep fishing sustainable, there must be limits on how much fishing operators can catch. A sustainable catch depends on knowing how big the populations are, and this is where statistics can be useful.

For example, abalone is a shellfish that lives in the waters off the south and central coast of Australia. It is harvested by divers. It would be impossible to count all the abalone, so how can population sizes be found?

Abalone numbers are sampled, and the population size is estimated using statistics. Then fishing authorities decide catch limits that should keep abalone safe from over-harvesting.

Forum

How do you think catch limits are enforced when a species is in danger of over-harvesting? Does this apply only to shellfish? How do fishing authorities make sure the sampling gives a good measure of the actual population?

Why learn this?

Making sense of data is an important part of life. People make decisions every day based on statistics and probability, sometimes without even knowing that they are doing so. For example, you might check tables to see which supermarket is the cheapest in the area, or to decide which car insurance to buy. Statistics affect these situations.

After completing this chapter you will be able to:
- use sample statistics to predict population statistics
- understand why random sampling is necessary to avoid bias
- calculate measures of centre and spread from grouped and ungrouped data
- construct frequency tables and graphs with and without technology
- understand that probabilities range between 0 and 1
- calculate theoretical probabilities by considering the sample space
- calculate complementary probabilities
- use Venn diagrams and tables to calculate probabilities involving criteria of 'and', 'or', 'at least' and 'not'.

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

Equipment required: calculator for Questions **2**, **4**

1 Students in a Year 8 class were surveyed as to how many pets they had, and the results are shown below.

1, 4, 0, 1, 3, 6, 2, 1, 1, 1, 2, 3, 2, 1, 1, 2, 3, 5, 1, 1, 2, 2, 2, 1, 2

(a) Tally these results and construct a frequency table.

(b) Construct a frequency column graph showing this information.

2 Find the mean, median and mode for the following sets of data. Use a calculator where necessary and round your answers to 1 decimal place.

(a) 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1

(b) 40, 20, 60, 30, 50, 10

3 State which of the two probabilities represents a 'more likely' chance of an event.

(a) $\frac{1}{4}$ or $\frac{3}{4}$

(b) 0.6 or 0.8

4 Find the mean, median and mode for the following data set that represents the number of cars in each of the households for the students in a class. Use a calculator where necessary and round your answers to 1 decimal place.

Number of cars	Frequency
0	1
1	5
2	9
3	6
4	3
5	1

5 Find the median for the data set shown in this stem-and-leaf plot.

Stem	Leaf
2_L	2 2 4
2_H	6 7 7 9
3_L	0 0 0 1 1 3
3_H	5 5 6
4_L	0 0 1 1 2 3

Key: 2 | 2 = 22

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Mean versus median

In this activity, you will explore how the possible number of texts sent by students may vary with different mean and median numbers. How many texts could each student have sent?

Population sampling

9.1

Statistics can be calculated using data collected from a **population** or from a **sample**.

- 'Population' means the entire category. This often refers to people, but in statistics it can be used to describe any defined category, such as birds, insects, vases, cars and so on.
- 'Sample' means a small proportion of the population.

When dealing with statistics, it is important to know whether you are using **population statistics** or **sample statistics**. You also need to understand how you can use data from samples instead of an entire population to gain information about the population.

Population statistics

- A **census** is the process of collecting data from every member of the population.
- Population statistics are statistics calculated from the data collected by a census.

Sample statistics

A census can be expensive and time-consuming, because a population may be very large. However, statistics techniques can be used to find reliable information by using data from a sample instead of from the entire population.

Because of this, samples are frequently used. For example, market research companies often use a sample of selected people to find out whether a new product is likely to be successful. If this is done well, then the views of the selected people (the sample) can be taken to represent the views of the population as a whole.

- Sample statistics only use data from a sample.

Polls

- A **poll** is a survey given to a sample of the population. A poll is used to collect data that can be used to calculate sample statistics. These statistics are then used to predict the population statistics.

You will often see the results of political polls in the news, especially when there is an election about to be held. These polls, which usually consist of about 1500 people, are used to predict the result of the election, when millions of votes may be cast. Some predictions are extremely accurate, while others get it wrong altogether.

Political polls are a good example of sample statistics being used to predict population statistics. The differences in predicted results are usually due to differences in the way the samples have been taken. Sample statistics from well-designed surveys should match the population statistics. This would mean that the poll accurately predicts the result of an election.

Uses of sampling

If you know the proportion of all the different categories in a sample and you know the population size, then you can estimate the number of each category in the population.

Worked example 1

W.E. 1

A large box is filled with 500 building blocks that are all of the same size. The blocks are yellow, red and blue. A sample of 20 blocks is taken and it is found that 6 are yellow, 9 are red and the rest are blue. Make an estimate for the number of each colour of block in the box.

Thinking	Working
1 Find the proportion of each colour in the sample.	$\text{Proportion of yellow} = \frac{6}{20} = \frac{3}{10}$ $\text{Proportion of red} = \frac{9}{20}$ $\text{Proportion of blue} = \frac{5}{20}$
2 Multiply the total number by the separate proportions to estimate the total population, rounding to the nearest whole number if necessary.	$\text{Estimated number of yellow} = \frac{3}{10} \times 500 = 150$ $\text{Estimated number of red} = \frac{9}{20} \times 500 = 225$ $\text{Estimated number of blue} = \frac{5}{20} \times 500 = 125$

Another use of population sampling is to estimate wildlife populations. For example, an environmental scientist will capture animals of interest, tag them and release them back into the wild. After the tagged wildlife has rejoined the wild population, a second sample is taken. To estimate the population, the following proportion can be used:

$$\frac{\text{tagged wildlife in 2nd sample}}{\text{total wildlife in 2nd sample}} = \frac{\text{tagged wildlife in population}}{\text{total wildlife in population}}$$

This can be rearranged to give:

$$\text{total wildlife in population} = \frac{\text{total wildlife in 2nd sample}}{\text{tagged wildlife in 2nd sample}} \times \text{tagged wildlife in population}$$

Worked example 2 W.E. 2

A sample of 200 fish is taken from a lake, tagged and released. Some time later, a second sample of 150 fish is taken and 15 of these have tags. What is the estimated fish population for the lake?

Thinking	Working
1 Write the three known values.	Tagged fish in population: 200 Tagged fish in 2nd sample: 15 Total fish in 2nd sample: 150
2 Use the rule to estimate the population of fish in the lake, rounding to the nearest whole number if necessary.	Total fish in lake $= \frac{\text{total fish in 2nd sample}}{\text{tagged fish in 2nd sample}} \times \text{tagged fish in population}$ $= \frac{150}{15} \times 200$ $= 2000$
3 State the answer.	There are about 2000 fish in the lake.

Sampling bias

The word **bias** refers to a distortion or problem with information, or a preference or belief that is not supported by evidence. In statistics, a sampling bias is when the sample does not truly reflect the population, because the sample was selected in a way that was unfair or distorted.

Worked example 3 W.E. 3

State whether or not each of the following survey methods will result in a biased sample. For those that you think are biased, give a reason why.

(a) You want to know the favourite sport of a city, so you ask people at an AFL football match.

(b) You are trying to find out how much time each day a person spends using a mobile phone, so you question people who are entering a mobile phone shop.

(c) You want to do a survey on the use of parks in your city, so a survey form is sent to all home and business owners in the city.

(d) You want to know about changes to speed zones in your local area, so you randomly select people from the local area phone book and conduct a phone poll.

(e) A very large business wants to know if its employees would use gym facilities if they were provided, so every third name on a list of employees is surveyed.

Thinking	Working
(a) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(a) Biased—People who are not at the match may prefer other sports. People who go to an AFL match are more likely to name AFL football as their favourite sport.

(b) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(b) Biased—There are a lot of people who own a mobile phone who do not need to go near a mobile phone store. Those who do go to the store may be higher mobile phone users than those who don't.
(c) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(c) Biased—There are people who live in the area that do not own a home (people who rent) or a business, so they are excluded. It also relies on people returning the survey they have been sent.
(d) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(d) Biased—Although people have been randomly selected, people with silent numbers or who do not have a landline phone are excluded.
(e) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(e) Fair—By selecting every third name, a random sample has been generated and, as the list contained all employees' names, everyone has an equal chance of being surveyed.

Sampling bias is often due to inappropriate methods of sampling, such as **judgemental sampling** or **convenience sampling**.

Judgemental sampling

News reporters often talk to people who have seen something important (such as an accident), to interview them for their opinion. This is judgemental sampling, where the sample of people is selected because of their special knowledge. This can be unreliable and biased, because opinions given by a specific group of people are not likely to reflect the opinions of the whole population of the general public.

However, 'judgemental' samples can be useful for finding opinions about a special topic. For example, a survey of roof repair specialists would give more useful opinions about the best kind of roof materials than a random sample of people who are not roof repair specialists.

Convenience sampling

Another common method of sampling is to ask people who are easy to find. For example, an interviewer might survey people who are walking through a particular shopping centre on a particular day. This is convenience sampling, where the sample of people is selected because they are convenient to select. This is also likely to be unreliable and biased. It is unlikely that the opinions given by people who happen to be convenient to interview will reflect the opinions of the whole population.

Another common way to find information quickly is to ask people to 'phone in' or to respond to a survey online. This is likely to be unreliable and biased, because only people who are motivated to respond will be included.

Note that a sample may be both judgemental sampling and also convenience sampling. For example, this would happen if you sampled a group of people who have special knowledge, but only spoke to those who were conveniently nearby.

Worked example 4

W.E. 4

For each of the following situations, state whether judgemental sampling or convenience sampling has been used, and list some factors that could produce bias.

(a) People coming out of a cinema on Wednesday afternoon are asked their opinion of the movie they have just seen.

(b) You ask everyone who sits with you at lunch whether they prefer cats or dogs.

(c) All scientists who have qualifications in climate studies are asked for their forecasts regarding climate change.

Thinking	Working
(a) Is the sample chosen because the people have special knowledge, or because they are convenient?	(a) This is judgemental sampling, because people who have seen the movie are sampled because they have special knowledge (they need to have seen the movie to give a meaningful opinion). It is also convenience sampling, because the sample is limited to people who happened to be there one Wednesday afternoon. There may be a bias because people who go to the cinema at this time may not be a good representation of the whole population.
(b) Is the sample chosen because the people have special knowledge, or because they are convenient?	(b) This is convenience sampling. There is likely to be bias, because people who sit together are more likely to be similar to each other, and may not be a good representation of the whole population.
(c) Is the sample chosen because the people have special knowledge, or because they are convenient?	(c) This is judgemental sampling, because people with expertise in climate studies are needed for their special knowledge (as they need to understand climate studies to give a meaningful opinion). However, this does not mean that their opinions will reflect the opinions of the whole population.

Random sampling

For a large or widespread population, it can be difficult to collect unbiased data. The best way to avoid bias is to use **simple random sampling**. This is where each member of the population has an equally likely chance of being chosen in the sample. This method helps to make sure that the sample will accurately reflect the entire population.

Picking names out of a hat is a common method of random sampling for small groups. For larger populations, it can be possible to give each member of the population a number, and then use a random number generator to select the sample.

For a sample to be reliable and unbiased in representing the population, it helps if the size of the sample is as large as possible. Repeating a process and then averaging the results also helps to reduce the effect of unusual or unrealistic data in a sample.

Random samples can still lead to bias if different segments of the population are not represented equally or appropriately in the sample. To avoid this, other sampling techniques are needed.

For example, you might use random sampling to create separate random samples taken from different age groups. This way, the samples together will include representation from all the different age groups.

Bias in experiments

Coins or dice are often used in experiments to collect data. This data is then used to calculate sample statistics. To use this data to predict population statistics, you must be sure that the coins or dice used are unbiased, or 'fair'.

For a coin, 'unbiased' (or 'fair') means that heads or tails are equally likely outcomes. For dice, 'unbiased' means that any side of the die is equally likely to be rolled.

To see whether a coin or die (or some other source of data) is biased, you can run a large number of trials to see what the data shows. The larger the number of trials, the more sure you can be about what the data shows. A small number of trials may seem to indicate bias (for example, tossing a coin that shows 'heads' five times in a row), but a larger number of trials can show that it is actually not biased (for example, tossing a coin that shows 'heads' five times in a row, followed by 'tails' five times in a row, followed by an even mix of heads and tails).

It is important to remember that even unbiased surveys will sometimes produce unlikely results.

Worked example 5

W.E. 5

A die is rolled 60 times and the result for each outcome is recorded. This experiment is repeated so that five sets of results are collected. These results are shown in the table opposite.

(a) Use each set of results separately to decide whether you think the die is fair.

(b) Using the separate decisions you made in the first part, make an overall decision about the fairness of the die.

(c) Combine the results and comment again about the fairness of the die.

Outcome	Set 1	Set 2	Set 3	Set 4	Set 5
1	10	10	11	11	7
2	14	7	8	11	9
3	13	12	7	10	11
4	6	11	17	9	11
5	12	10	7	11	9
6	5	10	10	8	13

Thinking	Working
(a) 1 If the die is fair, you expect each outcome to occur the same number of times, so divide the total number of rolls by the number of possible outcomes (60 ÷ 6).	(a) Number of expected results $= \frac{60}{6}$ $= 10$

2 Compare this number to the results in each column in the table.

Set 1—Biased. There are too many 2s and 3s and too few 4s and 6s.

Set 2—Biased. There are too many 3s and too few 2s.

Set 3—Biased. There are too many 4s and too few 3s and 5s.

Set 4—Fair. All results are about the same value.

Set 5—Biased. There are too many 6s and too few 1s.

(b) Make a statement about the fairness of the die by looking at the decision made for each set.

(b) Based on the results of each set, it seems the die is not fair. However, the results are not either consistently low or consistently high for any particular outcome.

(c) 1 Total the numbers for all five sets for each outcome.

(c)

Number	Total
1	49
2	49
3	53
4	54
5	49
6	46

2 Find the result you expect by dividing the total number of rolls by the number of possible outcomes (300 ÷ 6).

$$\text{Number of expected results} = \frac{300}{6}$$
$$= 50$$

3 Compare the number to the total results and make a statement about the fairness of the die.

The die is fair. All results are about the same value and close to the 50 expected.

9.1 Population sampling

Navigator

1, 2, 3, 4, 5, 8, 14, 15	1, 2, 3, 4, 5, 6, 8, 9, 14, 15, 16	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16

Answers p. 671

Equipment required: coin for Question **14**, die for Question **16**

Fluency

W.E. 1

1 **(a)** A large jar is filled with 2500 jelly beans. The jelly beans are red, green and black. A sample of 50 jelly beans is taken and it is found that 20 are green, 18 are red and the rest are black. Make an estimate for the number of each colour of jelly bean in the jar.

(b) Another jar is filled with 1800 jelly beans. The jelly beans are red, green and black. A sample of 60 jelly beans is taken and it is found that 24 are green, 15 are red and the rest are black. Make an estimate for the number of each colour of jelly bean in the jar.

W.E. 2

2 (a) A sample of 300 fish is taken from a lake, tagged and released. Some time later, a second sample of 150 fish is taken and 20 of these have tags. What is the estimated fish population for the lake?

(b) A sample of 100 koalas is taken from a national park, tagged and released. Months later, a second sample of 50 koalas is taken and 10 of these have tags. What is the estimated koala population for the national park?

W.E. 3

3 For each of the following survey methods, state whether or not it will result in a biased sample. For those that you think are biased, give a reason why.

(a) You want to find the average height of adults in your local area, so you measure the height of 100 adults chosen at random at a local shopping centre.

(b) You are trying to find the average hand span of teenagers, so you measure the hand span of all the Year 8 students in your school.

(c) You want to find the average weekly income of families in your area, so you question people who come into the local petrol station.

(d) You want to find out the health issues in your community, so you question people who are attending the local medical clinic.

(e) You want to find the average age of cars on the road, so you get the local mechanic to record the age of each car she works on over a 1-month period.

(f) You are trying to find the preferred music style for your community, so you question people chosen at random at a shopping centre.

W.E. 4

4 For each of the following situations, decide whether judgemental sampling or convenience sampling techniques have been used. List any factors that could produce bias.

(a) Students arriving at school through a particular gate between 8 am and 8:15 am are asked if they like reading.

(b) All of the members of a quilting group are asked what brand of sewing machine they prefer.

(c) A random sample of mechanics in Australia are asked which brand of car they would recommend.

W.E. 5

5 A die is rolled 60 times and the result for each outcome is recorded. This experiment is repeated so that five sets of results are collected. These results are shown in the table below.

Outcome	Set 1	Set 2	Set 3	Set 4	Set 5
1	13	9	10	7	6
2	8	12	9	11	12
3	10	4	9	12	6
4	12	15	12	8	14
5	9	11	8	15	8
6	8	9	12	7	14

Remember, there are six possible outcomes for rolling a die: 1, 2, 3, 4, 5, 6.

(a) Use each set of results separately to decide whether you think the die is fair.

(b) Using the separate decisions you made in the first part, make an overall decision about the fairness of the die.

(c) Combine the results and comment again about the fairness of the die.

Understanding

6 Scientists capture and tag 300 penguins and then release them. After several months, the scientists begin a monthly sampling program, with the following results.

	Month 1	Month 2	Month 3	Month 4
Sample size	150	175	200	125
Number of tagged penguins	55	65	72	51

(a) Estimate the size of the penguin population for each of the months.

(b) Combine the four samples into one big sample and use it to estimate the penguin population.

(c) Which answer for the total population do you think is more reliable? Give some reasons for your answer.

7 The following table shows the results from groups of 60 rolls of a die labelled 1–6 that you have been told is biased.

	Set 1	Set 2	Set 3	Set 4	Set 5
1	10	9	8	15	10
2	15	12	6	10	8
3	7	11	10	9	15
4	10	11	9	7	3
5	10	8	10	3	10
6	8	9	17	16	14

In what way do you think this die is biased? Does each sample show the same apparent bias?

8 Manufacturers should always be interested in quality control. However, they can usually test only a sample of their product.

(a) Explain why a baker cannot taste-test every loaf of bread.

(b) Matches are sold in boxes labelled ‘Average contents 50 safety matches’. If you counted the contents of 50 such boxes of matches, what range of values do you think you might find?

Reasoning

9 Public transport systems are sometimes criticised for late and cancelled services. Explain how you could choose an unbiased sample to survey the general public's opinions of public transport. Make sure you clearly state what population you are considering.

10 The following articles were published in *The Australian* newspaper in December 2015.

Walking slowly may be early sign of dementia

WALKING more slowly may not simply be a harmless indication that you are getting doddery in old age—it could be the first sign of dementia.

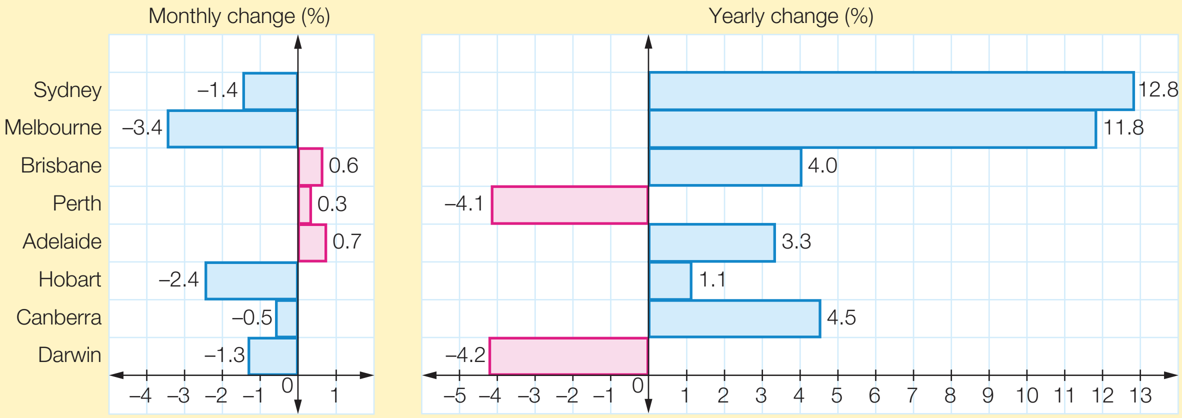
A study has found that subtle changes in the gait of old people are associated with Alzheimer's disease, even when there are none of the classic cognitive impairments normally used to diagnose the condition.

For the research, published in the journal *Neurology*, the walking speed of 128 elderly people was measured. Their brains were then scanned to look for any build-up of amyloid plaques.

Tom Whipple, *The Times London*, 3/12/2015

Signs point to sales cooling over summer

Heat comes off home values



Source: CoreLogic RP Data

THE brakes have come on in the country's key housing markets, with Melbourne prices falling 3.4% last month and Sydney's once white-hot market also easing.

Turi Condon, *The Australian*, 2/12/2015

(a) Which article uses data from judgemental sampling techniques? Say why this may or may not give accurate values for the whole population on this occasion.

(b) Which of the articles uses data from a random sample? What further steps could be taken to ensure more confidence in these results?

11 The following article appeared on the website of *The Courier*.

Support to cut speed limits on country roads

NEW road safety research shows strong support for lower speed limits on country roads. The Monash University study of 4100 drivers showed the majority thought a speed limit of 100 km/h for country roads was too high.

Seventy-five per cent believed there should be a reduction in speed limits on country roads from 100 km/h to 90 km/h and 92 per cent considered a limit of 80 km/h appropriate for gravel roads.

Only 14 per cent of drivers said the 50 km/h speed limit for local residential roads was too high.

However, there was some discrepancy between the views of city and country drivers to lower speed limits.

More than 80 per cent of city drivers backed a 90 km/h limit for country roads compared to 61.8 per cent of country drivers. While 93.2 per cent of city drivers and 88.5 per cent of country drivers supported the 80 km/h limit for gravel roads.

Senior research fellow Dr Bruce Corben of Monash University Accident Resource Centre said the difference in opinion could reflect a desire by country drivers to save travel time and greater familiarity with the roads.

He said lowering the speed limit was a way of reducing road trauma at a low cost.

"I think governments have been reluctant to move in that direction because there is a perception that the community is opposed to it."

"With these results is an opportunity to re-think what's the appropriate limit for rural roads."

Cathy Morris, *The Courier*, 1/4/2010

(a) The article does not state the numbers of country and city drivers who were surveyed. Does this make a difference to your interpretation of the surveys?

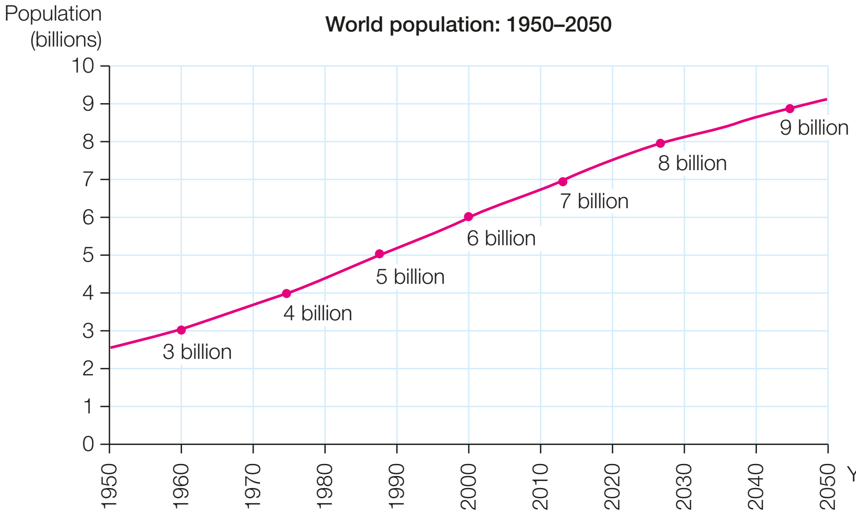
(b) An article in Victoria's *Herald Sun* about the same study stated that 1217 of the drivers surveyed were Victorians. Do you think this is an unbiased or a biased survey? Give reasons for your answer.

(c) If this survey was to be conducted Australia-wide, how could you ensure that each state and territory had fair representation?

(d) The Monash University Accident Research Centre, authors of the report, gave the following additional information: Victoria contributed 1217 (29.7%) of the respondents, South Australia 1175 (28.6%), Western Australia 1135 (27.7%) and Tasmania 573 (14%) and the survey was web-based with individuals on existing databases sent an invitation to participate.

Does this additional information change your opinion about the survey? Why or why not? Give a detailed answer.

12 Below is a line graph of the world population from 1950 to 2015. The line graph has been projected to predict the world population from 2015 to 2050.



(a) Estimate the world population in 1959.

(b) What happened to the world population in the 40 years from 1960 to 2000?

(c) To draw the graph into the future, assumptions have been made by statisticians. Name one assumption that has been made.

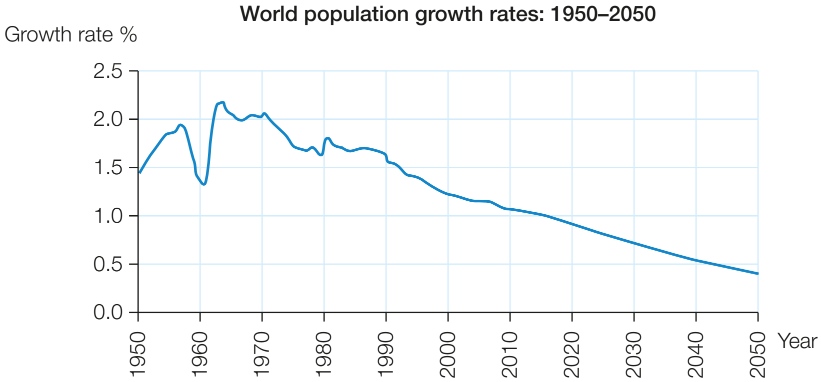
(d) Are these assumptions the same as a bias? Explain.

(e) How many years after 2000 will it take for the population to increase by the same amount that it did from 1960 to 2000?

(f) What are some of the issues the world will face if the world population increases as this graph predicts?

(g) How would this data have been collected?

13 The graph below shows the rate at which the world population grew from 1950 to 2010 and was predicted to grow from 2010 to 2050.



(a) During what 10-year period was the world growth rate the greatest?

(b) Babies born in the years following World War II, which ended in 1945, were called the 'Baby Boomers'. What connection does this have to your answer to the previous part?

(c) What was the world growth rate in 2010?

(d) What has happened to the world growth rate from 1980 to 2010?

(e) If the prediction is correct, will the world population be increasing or decreasing in 2050? Explain your answer.

(f) A combination of natural disasters and a decrease in agricultural production in China affected the world population rate for a couple of years. Looking at the graph, when do you think this occurred and what effect did it have on the shape of the graph?

Open-ended

14 Flip a coin 20 times, recording the number of heads and tails results. Flip the coin another 20 times and compare the results. Do you think your coin is biased?

15 For each of the following situations, describe a group that could be sampled for a judgemental sample.

(a) Finding the best grass stain remover for football uniforms.

(b) Finding the best fish-and-chip shop on your side of town.

16 **(a)** If you rolled a standard six-sided die 30 times, how many times would you expect each number to appear?

(b) If you rolled a die 30 times and 3 showed eight times, would you think that the die was biased?

(c) How many of a particular number appearing would make you feel confident that the die was biased?

(d) What can you do to check a die you think is biased?

(e) Roll a die 30 times and note the frequency for each value. Do you think your die is biased?

Problem solving

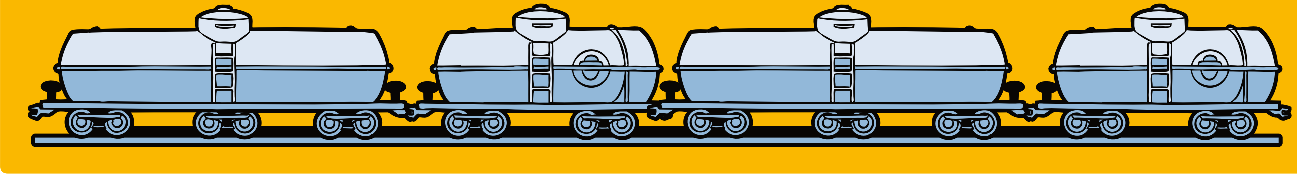
How many trains?

A train company transports freight around the country. They use freight cars of two different types: Roomy and Extra Roomy. An Extra Roomy car is twice as long as a Roomy car.

How many different combinations of cars are there, if the total length of the train must be as long as ten Roomy cars? (If the order of the cars is different, then you should count it as a different combination, even if it has the same number of each type of car as another combination.)

Strategy options

- Make a table.
- Solve a simpler problem.
- Test all possible combinations.



9.2 Using sample measures of centre and spread

When a population is sampled, the **mean**, **median** and **mode** are used to represent the **measures of centre** for the population (also called **measures of location**). Sometimes, it is difficult to select a sample that accurately represents a population. Statisticians must avoid bias when sampling, and they must also avoid any underlying bias when they analyse data. Consider the following.

Every five years in Australia, the Australian Bureau of Statistics conducts a census of the Australian population. According to the census in 2011, the median age of the Australian population at June 2011 was 37.3 years, which is higher than the median age of 36.6 years at June 2006 (the previous census). Would a random sample of Australians in 2011 have found the same median?

To select a sample that best represents the ages of the Australian population, where would you go? Sports events, festivals or shopping centres with large crowds are unlikely to include the very old or very young who live in Australia. These places may also be biased in other ways. A hospital may include the very old and very young, but it would not include the healthy population.

Measures of centre

To understand the meaning of statistical information, you can use three important statistical measures of centre: the mean, median and mode.

Mean

The mean is the arithmetic average of a data set:

$$\text{Mean} = \frac{\text{sum of all the data values}}{\text{number of data values}}$$

Median

The median is the middle value of a data set, when all the values are written in order from smallest to largest. If there are two middle values, then the median is the average of these two values.

Mode

The mode is the value that occurs most often in a data set.

There may be no mode, if no value occurs more than once.

If there are two modes, the data set is called bi-modal.

If there are more than two modes, the data set is called multimodal, but the mode is usually not useful in these cases. A data set of this type is often considered to have no mode.

Worked example 6

W.E. 6

Sarah collects rewards points from her supermarket. The points she has earned in the last 12 months are listed below.

72, 39, 57, 82, 36, 42, 64, 54, 62, 48, 74, 42

Find the **(a)** mean, **(b)** median and **(c)** mode of this data set.

Thinking	Working
(a) To find the mean, add all the data values and divide by the number of data values.	(a) $\text{Mean} = \frac{72+39+57+82+36+42+64+54+62+48+74+42}{12}$ $= 56$
(b) To find the median, arrange in ascending (or descending) order and identify the middle value. If there are two middle values, average them.	(b) Ascending order: 36, 39, 42, 42 48, 54, 57, 62, 64, 72, 74, 82 Middle values: 54 and 57 $\text{Median} = \frac{54+57}{2}$ $= 55.5$
(c) To find the mode, identify the value that occurs the most frequently.	(c) Mode = 42

Measures of spread

To help understand the data, you can also measure the spread of the data using the **range**.

Range

Range = (highest value in the data set) − (lowest value in the data set)

The effect of outliers

You have probably seen that sometimes some data is quite different from the rest of the data set. This can be because of unusual circumstances, natural variations, or sometimes because of errors in collecting or recording the data. These unusual or extreme data values may be **outliers**.

You need to know what effect an outlier or outliers have on the statistics you can calculate. The most reliable statistics will be those in which outliers do not have a great effect on the statistics. Look at the following example to see how possible outliers can distort statistics.

Worked example 7

W.E. 7

(a) For each of the following data sets calculate the: **(i)** mean **(ii)** median and **(iii)** range.

Set 1: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 30

Set 2: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 16

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the sample statistics you have calculated?

Thinking	Working
(a) **(i)** To find the mean, add the values and divide by the number of values.	(a) (i) Set 1: Mean = $\frac{136}{17} = 8$ Set 2: Mean = $\frac{122}{17} = 7.18$
(ii) The median is the middle value.	(ii) Set 1: 1 1 2 3 4 4 4 5 ⑥ 7 8 9 10 12 14 16 30 Median = 6 Set 2: 1 1 2 3 4 4 4 5 ⑥ 7 8 9 10 12 14 16 16 Median = 6
(iii) The range is the highest value – the lowest value.	(iii) Set 1: Range = 30 − 1 = 29 Set 2: Range = 16 − 1 = 15
(b) Compare the means, medians and ranges of Set 1 and Set 2 and comment on similarities and differences.	(b) The ranges are quite different. The means are a little different. There is no difference between the medians.

In the above example, the 30 in the data set in part **(a)** could be an outlier, as it does not seem to fit the rest of the data. It is an extreme value. It had no effect on the median. It had a slight effect on the mean and a significant effect on the range. This is why, in the presence of outliers, the median is often used to describe data sets in preference to the mean and the range.

9.2 Using sample measures of centre and spread

Navigator

1, 2, 3, 4, 6 (a–c), 7 8 (a–b), 11, 12, 13	1, 2, 3, 4, 6, 7, 8, 11, 12, 13	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13

Answers p. 672

Fluency

W.E. 6

1 **(a)** Rohan recorded his points from the last 15 rounds of a boardgame:

23, 37, 42, 36, 45, 52, 25, 65, 47, 37, 54, 36, 50, 48, 33

Find the **(i)** mean, **(ii)** median and **(iii)** mode of this data set.

(b) Stevie recorded the points her netball team scored in the 18 games they played this season.

43, 45, 56, 26, 36, 44, 64, 54, 67, 34, 42, 63, 51, 29, 41, 30, 55, 39

Find the **(i)** mean, **(ii)** median and **(iii)** mode of this data set.

W.E. 7

2 **(a)** For each of the following data sets calculate the: **(i)** mean **(ii)** median and **(iii)** range.

Set 1: 8 9 9 10 10 10 10 10 14 15 16 18 19 20 22

Set 2: 1 9 9 10 10 10 10 10 14 15 16 18 19 20 22

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the sample statistics you have calculated?

3 In the 2015 NRL season, the Manly-Warringah Sea Eagles scored the following points in their games.

12	24	12	4	16	12
16	12	30	10	14	10
30	8	28	38	12	32
44	28	26	16	10	14

(a) Calculate the mean and median number of points scored for the season. Round the mean to 1 decimal place.

(b) Which of these statistics, the mean or the median, better represents their scores for the season?

4 **(a)** For the data set 1 2 2 2 3 4 5 5 6 6 6 8 8 8 8 9, the median is:

A 5 **B** 5.5 **C** 6 **D** 8

(b) For the data set 2 3 3 4 5 6 6 6 7 7 7 9 9 9 9, the mode is:

A 6 **B** 7 **C** 8 **D** 9

(c) For the data set 1 1 1 2 2 3 3 3 3 3 5 6 6 6 6 6 6 7 7 7 7 8 8 9 9, the mean is:

A 3 **B** 5 **C** 6 **D** 7

Understanding

5 In November 2015 the Australian cricket team played New Zealand in three Test matches. The following is a list of the scores (number of runs) made by each individual Australian cricketer during the three Test matches.

Cricketer	1st Test		2nd Test		3rd Test	
	1st innings	2nd innings	1st innings	2nd innings	1st innings	2nd innings
Warner	163	116	253	24	1	35
Burns	71	129	40	0	14	11
Smith	48	1	27	138	53	14
Voges	83	1	41	119	13	28
M Marsh		2	34	1	4	28
Nevill			19	35	66	10
Starc			0	28	24	0
Hazlewood			8	2	4	0
Lyon			4		34	0
Khawaja	174	9	121			
Johnson			2	29		
S Marsh					2	49
Siddle					0	9

(a) Calculate the overall mean of the cricketer's scores. Round the answer to 1 decimal place.

(b) Calculate the overall median of the cricketer's scores.

(c) Which cricketer's score was the closest to the overall mean score in each innings?

(d) Over time, would you expect the mean score for each cricketer to be similar to the mean score for the team? Explain your answer.

6 An orange farmer expects to harvest all oranges at once. To help predict the total mass of oranges, the farmer picks the oranges from one tree and weighs each one. The masses, in grams, are recorded to the nearest 5 g:

175	180	190	195	175	175
180	165	185	175	175	175
175	175	175	180	175	175
175	185	165	180	175	175
195	190	185	190	175	165
175	170	165	195	190	185
180	190	175	170	165	155
185	180	185	175	175	190
195	175	175	185	170	185

(a) Calculate the mean, median and mode for these oranges, to the nearest whole number.

(b) Which of the statistics do you think should be used to estimate the total mass of oranges on the farm? State the assumptions you make to answer this question.

(c) If there are 250 trees on the farm, make an estimate for the total mass of oranges harvested. State the assumptions you make to get this value.

(d) Four random samples of 5 oranges were taken and the weights recorded:

Set 1: 175 155 175 185 180 Set 2: 180 175 165 190 170
Set 3: 170 175 190 195 175 Set 4: 170 195 165 175 195

(i) Find the mean, median and mode for each set. In each case state your answer correct to the nearest whole number.

(ii) Which of these statistics stayed closer to the value of the statistics found in part **(a)** for the population of oranges on this tree?

7 The following data set appears to have two outliers, one at the start of the data and one at the end.

3 10 12 12 14 14 15 16 17 18 18 19 28

You have already seen that one outlier affects the mean and the range, but what happens here where there are two outliers?

(a) Calculate the mean, median and range for the data set.

(b) Recalculate these values after changing the 3 to 10 and changing the 28 to 19.

(c) What conclusions can you draw from your calculations?

Reasoning

8 The following table shows the minimum and maximum temperatures recorded for Melbourne each day in March 2015.

Date	1	2	3	4	5	6	7	8	9	10	11
Min °C	15.2	11.6	15.5	15.4	12.6	14.1	15.4	16.2	14.4	16.2	11.2
Max °C	20.8	23.0	22.7	23.5	18.6	21.1	22.6	24.4	23.8	21.7	23.8

Date	12	13	14	15	16	17	18	19	20	21	22
Min °C	13.0	14.4	9.4	13.1	10.5	13.0	17.8	16.5	14.2	11.9	11.3
Max °C	19.9	20.4	30.1	18.3	25.5	26.2	24.4	33.5	19.6	22.7	28.8

Date	23	24	25	26	27	28	29	30	31
Min °C	13.6	12.6	13.0	9.8	11.6	12.7	8.5	10.6	11.2
Max °C	27.0	15.3	20.4	18.0	18.1	19.1	22.7	23.1	25.3

(a) Calculate the mean minimum and mean maximum temperatures for March 2015 in Melbourne correct to 1 decimal place.

Based on the records kept since 1855, the mean minimum temperature for Melbourne in March is 13.2 °C and the mean maximum temperature is 23.9 °C.

(b) How did the mean minimum and maximum temperatures in March 2015 in Melbourne compare to these long-term statistics?

If the mean March temperatures are calculated using just the figures from 1981 to the present day, then the mean minimum temperature is 14.5 °C and the mean maximum temperature is 24.4 °C.

(c) How did the mean minimum and maximum temperatures in March 2015 in Melbourne compare to these restricted statistics?

(d) Do you think it is better to use the historical means (since 1855) or the more recent means (since 1981)? Explain why.

9 A growth chart for boys aged 13 years indicates that 95% of them will be less than 169 cm tall, 90% less than 166.5 cm, 75% less than 163 cm, 50% less than 157 cm, 25% less than 151 cm, 10% less than 147 cm and 5% less than 143 cm.

(a) You are given the following set of heights and told that they are from boys of approximately the same age as each other.

149	147	159	167	171	177	155	180	160	170
173	166	162	150	155	169	155	179	160	161
172	155	149	154	150	168	171	151	152	141
171	156	168	155	156	158	161	160	159	168

Do you think these boys are 13-year-olds? Give a reason for your answer.

(b) A random sample of eight values was chosen from this data.

160, 155, 171, 168, 177, 159, 160, 161

(i) Find the mean and median for this sample. Round the value of the mean to 1 decimal place.

(ii) By first calculating the population mean and median, say which statistic you think is the better measure of centre. Give a reason for your choice.

10 The following tables list how much caffeine is in various coffee drinks and how much coffee is consumed in various countries.

(a) How does the average annual coffee consumption in Australia compare with that in Slovakia? Calculate the Australia value as a percentage of the Slovakia value, to the nearest whole number.

(b) The amount of caffeine in a 250 mL cup of espresso is usually an amount from 30 mg to 50 mg. How many mg of caffeine is left after 25 mL of the 250 mL cup has been consumed? State your answer as a range.

(c) If data from all eight countries listed in Table 2 was combined, do you think the coffee consumption per person per year would be closest to 2.8, 4.7 or 9.2 kg? Justify your answer.

(d) Do you think the figures in Table 2 were from sampling or from a census? Explain your answer.

(e) Do the range of values in Table 1 give you information about the average (mean) amount of caffeine in each type of coffee? Explain your answer.

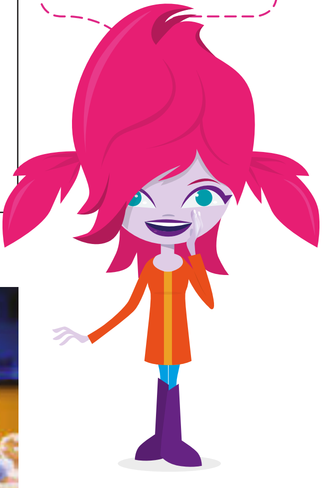
Coffee type (225 mL cup)	Caffeine range (mg)
Brewed/plunger	80–350
Instant	60–100
Mocchaccino	35–55
Cappuccino	30–50
Decaf, brewed	2–4
Decaf, instant	1–4

Table 1

Country	Coffee consumption per person (kg) per year
Slovakia	10.8
Iceland	9.2
France	5.5
Italy	5.3
Canada	4.7
USA	4.6
Australia	2.8
UK	2.5

Table 2

Remember, there are 1000 g in a kg and 1000 mg in a g.



Open-ended

11 Make up 10 pieces of data so that the mean is 12.3, the median is 13 and the mode is 11.

12 Make up 10 pieces of data so that the mean is 12.3, the median is 13 and the mode is 11 and there is an outlier of 32. (32 is one of your 10 pieces of data.)

13 Consider the following data set:

0, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10, 11, 11, 11, 11, 12, 12, 12, 14, 14, 14, 14, 14, 20

Find the mean, median and range of this data set. Now randomly select five values, remove them from the data set, and recalculate the new mean, median and range. How does the new (sample) mean, median and range compare to the original (population) mean, median and range?

9.3 Frequency tables and graphs

If data is discrete (so that the possible number values can only take particular separate values), such as the counted number of cars in a car park, or if data is categorical (in categories), such as the colours of different cars in a car park, then this data can be graphed as a column graph (bar chart).

If data is continuous (so that the possible number values can take any values within a range), such as the measured lengths of different cars in a car park, then this data can also be displayed in a **frequency column graph** where each column represents a segment of the continuous data values. The height of each column represents the frequency of data within that segment. This kind of frequency column graph (with continuous data) is called a **histogram**.

Worked example 8

W.E. 8

A packet of licorice strips contains 50 strips, but the strips are different lengths. Draw a histogram to represent the number of licorice strips of different lengths in a packet.

Licorice strip length (cm)	Frequency
10–<11	5
11–<12	6
12–<13	14
13–<14	8
14–<15	7
15–<16	6
16–<17	3

Thinking

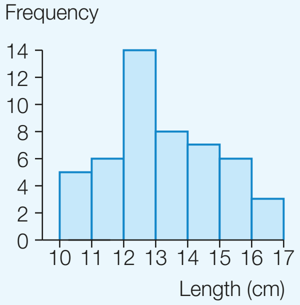
1 Find a suitable vertical scale. The highest frequency is 14, so go up in 2s. Label the axis.

2 On the horizontal axis, leave a half-column space next to the vertical axis, then equally space the rectangular columns. Label the axis.

3 Choose a suitable title for the graph.

Working

Licorice strip length



Grouping data

Class intervals are useful for grouping if there is a large amount of data and the range is large (greater than 10).

The statistical measures of centre (mean, mode and median) cannot be read from the graph for **grouped data**, but the graph does give a good idea of the spread of the data. The shape of the graph also gives important information.

Finding class intervals

A **class interval** is found by dividing the range into a number of equally sized intervals, so you have between five and 10 columns in your frequency column graph. For example, if a discrete data set has values ranging from 10 to 75, the range is 65. If a class interval of 10 was used, this would give seven columns with intervals 10–19, 20–29, … 70–79.

Discrete grouped data can be presented as a frequency column graph. The class intervals for discrete data are presented as discrete groups, as shown in the following example.

Worked example 9

W.E. 9

The following data shows the number of customers that have eaten at a restaurant each day for 50 days.

11, 45, 29, 43, 37, 30, 58, 80, 70, 82, 23, 31, 27, 74, 41, 48, 65, 11, 65, 77, 70, 36, 50, 50, 65, 50, 45, 60, 37, 67, 2, 41, 62, 43, 27, 81, 56, 35, 31, 50, 2, 48, 98, 51, 72, 43, 67, 43, 43, 26

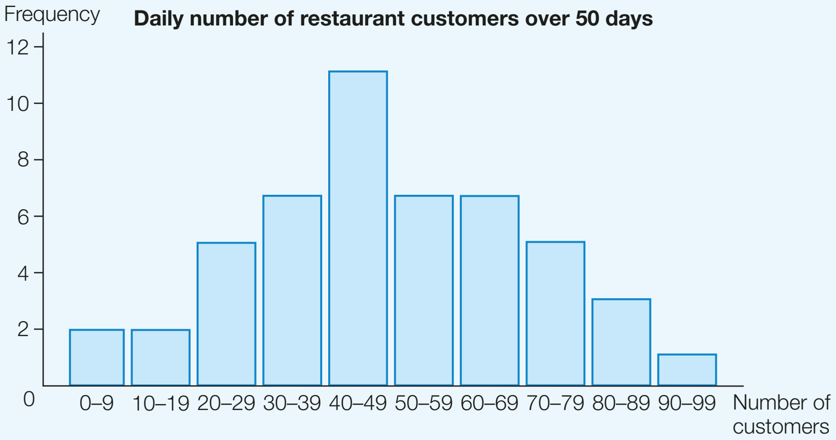
Select a suitable class interval, construct a frequency table with the grouped data and draw a frequency column graph of the data.

Thinking	Working
1 Find the range and divide it by 5 and 10 to find a suitable class interval.	Range $= 98 - 2$ $= 96$ $\frac{96}{5} = 19.2$ and $\frac{96}{10} = 9.6$ Use a class interval of 10.
2 Decide where the class interval should start.	Use 0–9, 10–19, …

3 Construct a frequency table for the grouped data. Use a tally column to help you count the frequency in each class interval.

Number of customers	Tally	Frequency
0–9	||	2
10–19	||	2
20–29	𝍸	5
30–39	𝍸 ||	7
40–49	𝍸 𝍸 |	11
50–59	𝍸 ||	7
60–69	𝍸 ||	7
70–79	𝍸	5
80–89	|||	3
90–99	|	1

4 Construct a frequency column graph for the grouped data. Label and number each axis.



Continuous grouped data is commonly presented as a histogram. The class intervals are presented as continuous groups, as shown in the following example.

Worked example 10

W.E. 10

The following data shows the winning times for the 50 m freestyle swimming final, recorded for the last 50 years at a school's annual swimming carnival. Times are in seconds.

32.12, 31.23, 31.56, 33.28, 33.73, 32.17, 34.49, 32.55, 34.28, 33.74, 31.58, 35.47, 30.44, 34.65, 33.89, 31.14, 35.36, 29.99, 30.69, 32.16, 33.75, 31.17, 30.45, 34.46, 30.78, 35.95, 33.35, 34.74, 30.12, 33.88, 32.19, 33.42, 31.57, 35.67, 32.96, 31.45, 32.64, 32.64, 33.68, 34.54, 32.14, 33.76, 30.45, 34.23, 34.28, 32.54, 31.99, 32.43, 30.76, 32.24

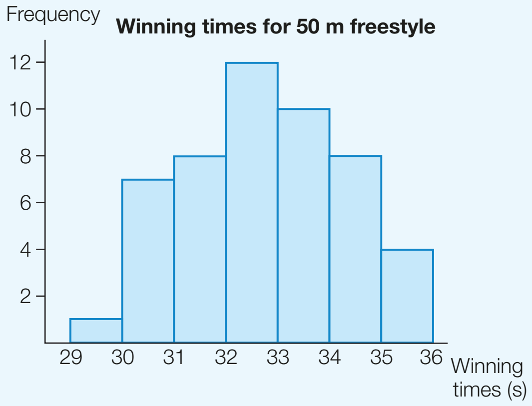
Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

Thinking	Working
1 Find the range and divide it by 5 and 10 to find a suitable class interval.	Range $= 35.95 - 29.99$ $= 5.96$ $\frac{5.96}{5} = 1.19$ and $\frac{5.96}{10} = 0.596$ Use a class interval of 1.
2 Decide where the class interval should start.	Use 29–<30, 30–<31, …

3 Construct a frequency table for the grouped data. Use a tally column to help you count the frequency in each class interval.

Winning times (s)	Tally	Frequency
29–<30	I	1
30–<31	𝍸 II	7
31–<32	𝍸 III	8
32–<33	𝍸 𝍸 II	12
33–<34	𝍸 𝍸	10
34–<35	𝍸 III	8
35–<36	IIII	4

4 Construct a histogram. Place the endpoints of the intervals at either end of the base of each rectangle. Label the axes and add a title.



When drawing any statistical graph, you should ensure your graph has:

- labelled axes
- a consistent scale on both axes
- a title.

9.3 Frequency tables and graphs

Navigator

1, 2, 3, 4, 5, 7 (a–b), 8, 10, 12, 14	1, 2, 3, 4, 5, 6, 7 (c–e), 8, 9, 10, 11, 12, 13, 14	1, 2, 3, 4, 5, 6, 7 (c–e), 8, 9, 10, 11, 12, 13, 14

Answers p. 673

Fluency

1 For each of the following frequency tables, draw a histogram to represent the results.

W.E. 8

(a)

Height of pancake stack (cm)	Frequency
10–<11	4
11–<12	3
12–<13	6
13–<14	3
14–<15	2
15–<16	0
16–<17	4

(b)

Distance travelled to school (km)	Frequency
0–<1	14
1–<2	3
2–<3	3
3–<4	5
4–<5	10
5–<6	11
6–<7	4
7–<8	3

W.E. 9

2 (a) The following data shows the number of plants a nursery sells over a 60-day period.

52, 38, 65, 29, 109, 123, 63, 72, 118, 35, 76, 82, 44, 54, 85, 132, 153, 98, 42, 58, 77, 94, 105, 113, 104, 88, 74, 53, 67, 93, 73, 64, 23, 52, 118, 109, 73, 43, 95, 27, 83, 122, 143, 79, 81, 84, 35, 41, 107, 97, 115, 123, 145, 78, 42, 97, 133, 125, 102, 94

Using an interval of 20–39 for the first class, construct a frequency table with the grouped data and draw a frequency column graph of the plant sales data.

(b) Plant sales for the same nursery for the same 60-day period the previous year are given below.

40, 43, 57, 64, 36, 74, 54, 21, 86, 64, 53, 86, 46, 75, 88, 75, 97, 47, 53, 22, 89, 96, 103, 110, 112, 52, 77, 94, 69, 99, 113, 87, 64, 78, 85, 102, 95, 89, 93, 108, 113, 114, 127, 88, 76, 55, 45, 37, 67, 89, 95, 75, 36, 59, 63, 79, 91, 72, 84, 92

Construct a frequency table and draw a frequency column graph of the plant sales data using the same class intervals as before.

W.E. 10

3 (a) The following data shows the heights of 50 Year 8 students. Heights are in cm.

127.5, 135.5, 130, 124.5, 129.5, 140, 132, 144, 141, 142.5, 130.5, 132, 135, 133, 144.5, 128.5, 141.5, 133, 132.5, 147, 138.5, 130, 140.5, 138, 133, 157, 143, 132, 129, 142, 152, 142.5, 144, 135, 155, 149.5, 146, 126.5, 132, 127, 153, 142.5, 139, 127.5, 145, 155, 149.5, 167, 156, 152.5

Using an interval of 120-<125 for the first class, construct a frequency table with the grouped data. Draw a histogram of the data with the endpoints of the class intervals forming the scale for the data axis.

(b) The following data shows the heights of the same 50 students measured 1 year later. Heights are in cm.

131.5, 140.5, 136, 129.5, 133, 145, 135.5, 149, 144, 147.5, 135, 136, 139.5, 138, 146.5, 138.5, 151.5, 141, 139.5, 155, 146.5, 137, 146.5, 146, 140, 161, 153, 140, 136, 149, 156, 144.5, 149, 145, 159, 153.5, 157, 138, 139.5, 137, 164, 150.5, 147, 135.5, 153, 161, 155.5, 172, 167, 161.5

Construct a frequency table and draw a histogram of the data using similar class intervals to before.

4 A class of 26 Year 8 students was asked how many hours they spent browsing social media on a 'typical' weekend. The data given below is the time given, rounded up to the next whole hour, for each student.

4, 6, 10, 5, 4, 8, 12, 14, 18, 6, 4, 6, 13, 2, 16, 8, 6, 4, 10, 14, 6, 4, 7, 8, 9, 12

Using 1–2 hours as the first class interval, summarise the results in a frequency table. Display the data using the endpoints of the class intervals to scale the data axis.

5 A lolly shop owner weighed a number of bags of sweets. The data below gives the mass to the nearest gram for each bag of sweets.

383, 433, 280, 314, 251, 386, 450, 257, 296, 305, 442, 355, 450, 363, 411, 375, 262, 262, 287, 312, 330, 495, 405, 499

Using 251–275 grams as the first class interval, summarise the results in a frequency table. Display the data using the values 250, 275 and so on for the scale on the data axis.

6 **(a)** If you had 70 data values spread between 15 and 105, a suitable first interval would be:

A 0–15 **B** 7–15 **C** 15–19 **D** 15–24

(b) If you had 200 data values spread between 0 and 50, a suitable first interval would be:

A 0–2 **B** 0–5 **C** 0–15 **D** 0–20

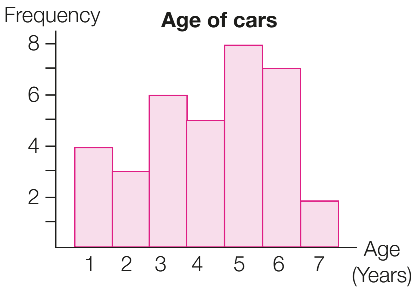
(c) If you had 500 data values spread between 70 and 270, a suitable first interval would be:

A 70–75 **B** 70–80 **C** 70–89 **D** 70–99

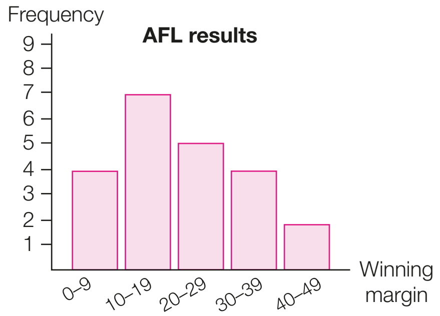
Understanding

7 Construct a frequency table for each of the following graphs.

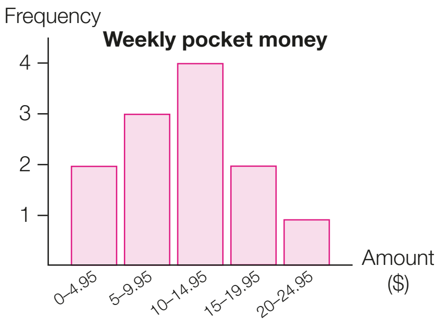
(a)



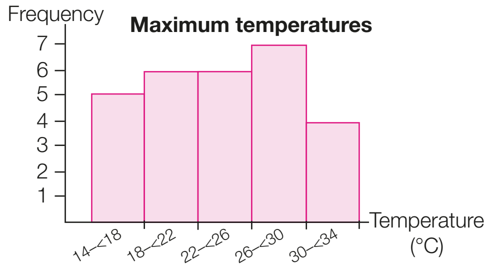
(b)



(c)



(d)



(e)

8 When people give their age, they usually round down to the whole number of years at their last birthday. For each of the following frequency tables, draw a histogram with a suitable scale on the data axis.

(a) Ages of players in a basketball team

Age	Frequency
11	2
12	0
13	6
14	8
15	1

(b) Ages of people in a movie theatre for a Friday 8 pm session.

Age	Frequency
10–13	23
14–17	48
18–21	15
22–25	8
26–29	8

9 Why might it be difficult to complete the following frequency table?

Number of customers	Frequency
0–10	
10–20	
20–30	
30–40	
40–50	

Reasoning

10 Refer to the two frequency column graphs constructed in Question **2**. Compare your histograms and comment on any similarities and differences in plant sales over the two years.

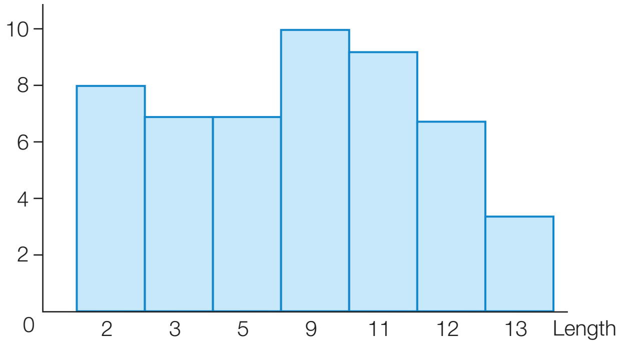
11 Refer to the two histograms constructed in Question **3**. Compare your histograms and comment on what has happened to the heights of students.

Open-ended

12 Make up 30 pieces of discrete data that could be grouped into 6 classes with a class interval of 10.

13 Make up 40 pieces of continuous data that could be grouped into 8 classes with a class interval of 5.

14 What is wrong with the histogram shown?



Problem solving

Missing frequencies

1 Complete the frequency table by finding what A and B represent so that the mean is $3\frac{11}{19}$, the median is 4 and there are 19 scores in total.

Score	Frequency
1	4
2	A
3	2
4	2
5	B
6	4

2 Complete the frequency table by finding what C and D represent so that the mean is $6\frac{12}{29}$, the median is 6 and there are 29 scores in total.

Score	Frequency
4	3
5	C
6	4
7	5
8	2
9	D

Strategy options

- Work backwards.
- Test all possible combinations.

Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Can technology improve sporting achievement?

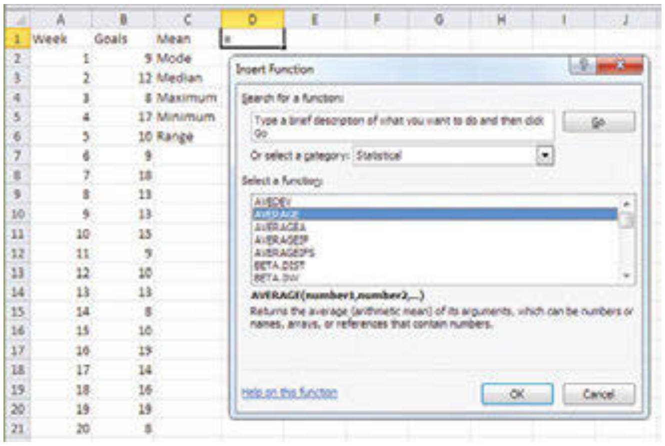
Lacey, an enthusiastic netballer, is very keen to improve her goal-shooting skills in netball. Her coach suggests that she should keep a record of her weekly performance. The following data represents the number of goals that Lacey scores each week during the first 20 weeks of the netball season:

9, 12, 8, 17, 10, 9, 18, 13, 13, 15,
9, 10, 13, 8, 10, 19, 14, 16, 19, 8

Lacey's coach reminds Lacey that statistics could help her understand and use this data. Lacey thinks that the mean, mode, median and range could be useful. Her coach suggests that she use a spreadsheet to analyse these statistics.

1 Use a spreadsheet to enter Lacey's results, as shown below. To calculate the mean, mode, median, maximum, minimum and range, you can use the spreadsheet's built-in formulas. In Excel, under the **Formulas** tab, select **Insert Function**, then find the appropriately described function to use from the list. For example, the function to find the mean is called **Average**. Then you can select the cells **B2:B21** as the values for the function.

Note that to find the range, you must write a formula to find the difference between the maximum and minimum values.



	A	B	C	D
1	Week	Goals	Mean	=
2	1	9	Mode	
3	2	12	Median	
4	3	8	Maximum	
5	4	17	Minimum	
6	5	10	Range	
7	6	9		
8	7	18		
9	8	13		
10	9	13		
11	10	15		
12	11	9		
13	12	10		
14	13	13		
15	14	8		
16	15	10		
17	16	19		
18	17	14		
19	18	16		
20	19	19		
21	20	8		

Insert Function

Search for a function:

Type a brief description of what you want to do and then click Go

Or select a category: Statistical

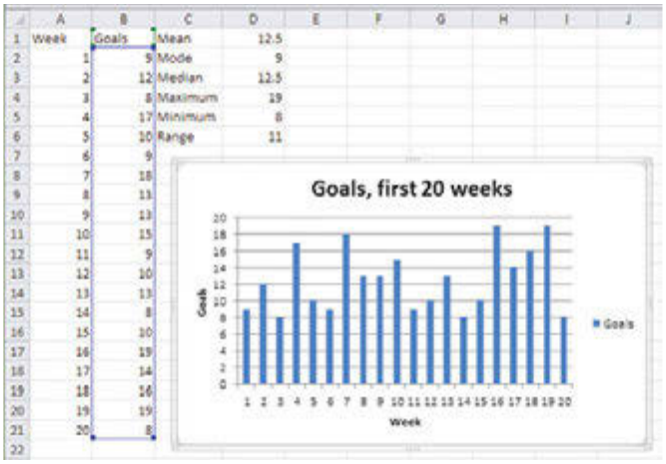
Select a function:

AVEDEV
AVERAGE
AVERAGEA
AVERAGEIF
AVERAGEIFS
BETA.DIST
BETA.INV

AVERAGE(number1,number2,...)

Returns the average (arithmetic mean) of its arguments, which can be numbers or names, arrays, or references that contain numbers.

2 Lacey would like to see how her season looks on a graph. The spreadsheet can be used to plot goals against weeks. Select the 'Goals' column of values, cells **B1:B21**. To create a column graph in Excel, under the **Insert** tab, select **Column** > **2-D Column** > **Clustered Column** and a column graph will appear. You can then use the options under the **Chart Tools** > **Layout** tab to set this graph's chart title, axis titles and other features appropriately.



Week	Goals	Mean	12.5
1	9	Mode	9
2	12	Median	12.5
3	8	Maximum	19
4	17	Minimum	8
5	10	Range	11
6	9		
7	18		
8	13		
9	13		
10	15		
11	9		
12	10		
13	13		
14	8		
15	10		
16	19		
17	14		
18	16		
19	19		
20	8		

3 In the last four weeks of the season, she scores 17, 15, 11 and 18 goals. Add this data to your spreadsheet and recalculate the statistics for the complete 24-week season, using your graph and the summary statistics. How does this new data affect the summary statistics? Did Lacey improve in the last four weeks of the season?

4 From week 10 onwards, Lacey tried some new training techniques. Recalculate the summary statistics and the graph for the last 10 weeks of the season. Using your graph and the summary statistics, comment on whether you think the new training techniques are effective.

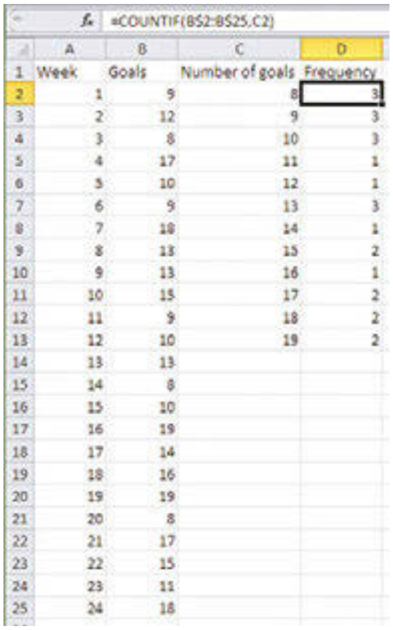
Taking it further

5 Lacey decided that she would like to produce a frequency graph using the season statistics.

To do this, the frequency data needs to be listed in the spreadsheet. Label a new column 'Number of goals' and use this column to list the different numbers of goals scored, in order from 8 to 19, as shown. Label the next column 'Frequency' and use this column to list the frequency of the 'number of goals' values (that is, the number of times that each number of goals appears).

This frequency can be found using the formula **=COUNTIF(B$2:B$25,C2)**, which will count the number of cells from **B2** to **B25** that are the same as cell **C2**. Because cell **C2** is the number 8, this will count the number of times that 8 goals appears in column **B** (the frequency).

Use **Fill Down** to copy this formula down the 'Frequency' column. The **$** symbol makes sure that the range **B2:B25** is the same every time, while the cell **C2** changes to match the different cells **C3**, **C4** and so on in each row.



=COUNTIF(B$2:B$25,C2)

Week	Goals	Number of goals	Frequency
1	9	8	3
2	12	9	3
3	8	10	3
4	17	11	1
5	10	12	1
6	9	13	3
7	18	14	1
8	13	15	2
9	13	16	1
10	15	17	2
11	9	18	2
12	10	19	2
13	13		
14	8		
15	10		
16	19		
17	14		
18	16		
19	19		
20	8		
21	17		
22	15		
23	11		
24	18		

You can now produce a frequency graph in the same way as before, this time selecting the Frequency column of data to be graphed. You will also need to select **Select Data** to make the horizontal axis numbers match the numbers of goals. Remember to give your graph a title and label your axes.

6 What does this graph tell you about Lacey's performance throughout the season?

7 Which graph do you think gave you the most information?

8 Which statistics gave you the most information?

9 Do you think the spreadsheet could help Lacey to improve her game? Give your reasons using graphs and/or statistics.

Statistics from grouped data

9.4

You should already know how to find the mean, median and mode from an ordinary set of numerical data. You should also recall that the mean, median and mode can be found from a frequency table of ungrouped data, as in the following example.

Worked example 11

W.E. 11

A school has 165 Year 8 students. Each student is asked how many pet animals they have at home. The results are shown on the right.

Create a frequency table of the data and use it to:

(a) calculate the mean, correct to 1 decimal place

(b) calculate the median

(c) calculate the mode

(d) draw a column graph of the data

(e) identify any possible outliers.

3 2 4 0 2 0 0 3 3 2 1 2 1 2 1
1 1 2 1 1 3 2 1 1 2 1 3 3 1 0
2 1 1 1 4 1 4 1 2 1 2 1 1 2 2
1 4 2 1 1 2 1 0 0 3 1 1 2 2 0
2 1 1 7 1 0 3 2 2 2 2 3 1 2 1
0 2 0 1 3 2 1 0 2 0 4 0 2 1 2
2 1 1 4 3 1 0 2 1 9 2 2 0 1 2
0 0 2 1 0 2 3 0 3 2 2 1 2 1 3
1 1 1 1 3 1 4 2 2 0 2 3 1 1 0
3 1 1 2 0 0 1 2 2 4 1 2 1 1 2
1 1 4 1 2 2 1 3 0 1 2 1 2 0 4

Thinking

Working

(a) **1** Complete the frequency table by counting or tallying the number of each data value.

2 Multiply each data value by its frequency to find the subtotal for that value.

3 Find the total of all data values, Σxf, and the number of data values, Σf.

(a)

Number of pets x	Frequency f	Totals $x \times f$
0	25	0
1	60	60
2	50	100
3	18	54
4	10	40
7	1	7
9	1	9
Total	165	270

4 Calculate the mean:

$$\text{mean} = \frac{\text{sum of the data}}{\text{number of data}}$$

$$\text{Mean} = \frac{\text{sum of the data}}{\text{number of data}}$$

$$= \frac{270}{165}$$

$$= 1.636\ldots$$

1.6 pets per home

(b) Find the median by counting down the frequencies to locate the middle data value.

(b) Position of the middle data value:

$$\frac{165+1}{2}=\frac{166}{2}=83$$

Adding frequencies from the top:

$25 + 60 = 85$

i.e. the 26th to the 85th data are all 1.

Median is 1 pet per home.

(c) The mode is the data value with the highest frequency.

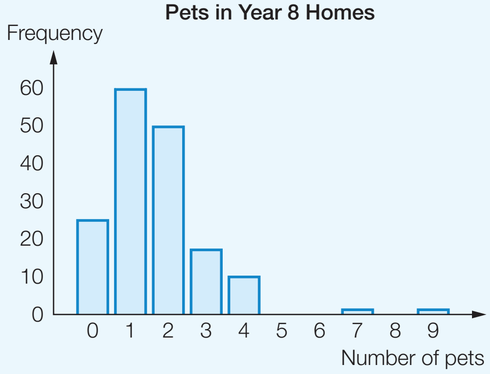
$f = 60 \quad x = 1$

Mode is 1 cat or dog per home.

(d) **1** Write the data values on the horizontal axis, including the missing data values that had a frequency of zero.

2 Scale the vertical axis appropriately, going up in tens to 60.

3 Label the axes and add a title.



(e) Identify extreme data values that are very different from the others.

(e) 7 or 9 pets in a home could both be outliers.

Measures of centre from grouped data

With grouped data, you can estimate the mean even if you do not know every individual data value.

Instead of finding the mean of every data value, you can first find the **class centre** or **midpoint** of each grouped class interval, then use the frequencies to find the mean of these values.

Class centre for discrete data

Add the endpoints of the class and divide by 2.

If the class interval is 0–9, the midpoint is 4.5. If the class interval is 0–4, the midpoint is 2.

Class centre for continuous data

Add the endpoints of the class and divide by 2.

If the class interval is 0–<10, the midpoint is 5. If the class interval is 0–<5, the midpoint is 2.5.

The class centre values are listed in their own table column as x. The mean of the data is estimated by finding the mean of these x values. Using the frequency value f for each class, you can calculate this mean as usual:

$$\text{mean} = \frac{\text{sum of } xf}{\text{sum of } f} = \frac{\Sigma xf}{\Sigma f}$$

This is shown in the examples that follow.

For grouped data, you can also find the **modal class interval**, which is the class interval that has the highest frequency. The **median class interval** is the class interval that contains the median value.

Worked example 12

W.E. 12

A manufacturer has orders from 50 different retailers for a particular product each week. One week the number of orders is:

207, 312, 333, 215, 145, 332, 177, 149, 263, 147, 238, 319, 336, 479, 284, 167, 352, 123, 198, 352, 346, 163, 133, 197, 227, 482, 265, 187, 316, 214, 287, 324, 469, 375, 471, 328, 183, 318, 236, 255, 353, 227, 165, 276, 175, 389, 299, 273, 365, 328

Use grouped data to:

(a) calculate the mean number of orders, giving your answer correct to 2 decimal places

(b) calculate the median class interval for the number of orders

(c) calculate the modal class interval for the number of orders

(d) draw a frequency column graph of the orders

(e) identify any possible outliers.

Thinking | **Working**

(a) **1** To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Number of orders	x	f	xf
100–149	124.5	5	622.5
150–199	174.5	9	1570.5
200–249	224.5	7	1571.5
250–299	274.5	8	2196.0
300–349	324.5	11	3569.5
350–399	374.5	6	2247.0
400–449	424.5	0	0.0
450–499	474.5	4	1898.0
		$\Sigma f = 50$	$\Sigma xf = 13\,675$

2 The mean is then found by calculating $\frac{\Sigma xf}{\Sigma f}$. It does not need to be a whole number.

$$\text{Mean} = \frac{\Sigma xf}{\Sigma f}$$

$$= \frac{13\,675}{50}$$

$$= 273.50$$

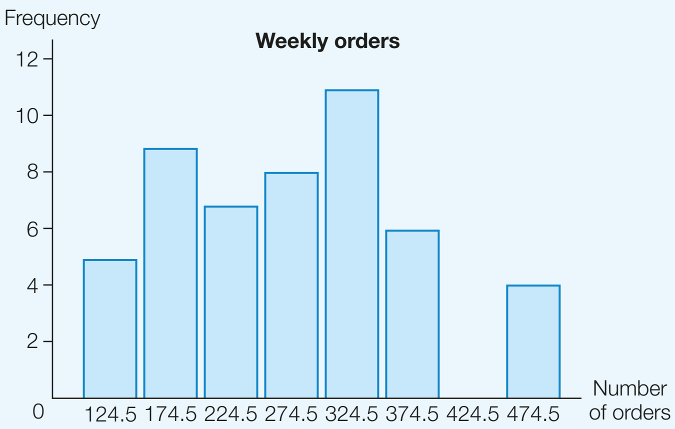
(b) Find the median in the usual way by counting down the frequency column until the middle is found. (Here, there are 50 values, so the median is between the 25th and 26th values.)

(b) Both the 25th and 26th values are in the 250–299 class, so this is the median class interval.

(c) The modal class interval is the class interval with the highest frequency.

(c) Modal class interval is 300–349.

(d) Draw the frequency column graph, remembering to leave half a column width before starting the first column. The midpoint can be used on the horizontal axis.	*(d)*
(e) Identify whether any data values appear to be quite different from the other data.	*(e) There don't appear to be any outliers.*



Worked example 13

W.E. 13

A pumpkin farmer wants to grow the biggest, heaviest pumpkins to win a prize at the show. The breeder decides to use statistics to help plan a growing program. The data in the frequency table gives the weight, in kg, of the pumpkins currently growing in the patch.

Weight (kg)	Frequency
25–<30	2
30–<35	8
35–<40	14
40–<45	18
45–<50	15
50–<55	11
55–<60	6

(a) Calculate an estimate for the mean weight of the pumpkins. Give your answer correct to the nearest whole number.

(b) Calculate the median class interval for the weights of the pumpkins.

(c) Calculate the modal class interval for the weights of the pumpkins.

(d) Draw a histogram of the data.

(e) Identify any possible outliers.

Thinking | **Working**

(a) **1** To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Weight (kg)	x	Frequency	xf
25–<30	27.5	2	55
30–<35	32.5	8	260
35–<40	37.5	14	525
40–<45	42.5	18	765
45–<50	47.5	15	712.5
50–<55	52.5	11	577.5
55–<60	57.5	6	345
		$\Sigma f = 74$	$\Sigma xf = 3240$

2 The mean is then found by calculating $\frac{\Sigma xf}{\Sigma f}$.

$$\text{Mean} = \frac{\Sigma xf}{\Sigma f}$$

$$= \frac{3240}{74}$$

$$= 43.783\ldots$$

$= 44$ kg (correct to nearest kg)

(b) Find the median by counting down the frequency column until the middle is found. (Here, there are 74 values, so the median is between the 37th and 38th values.)

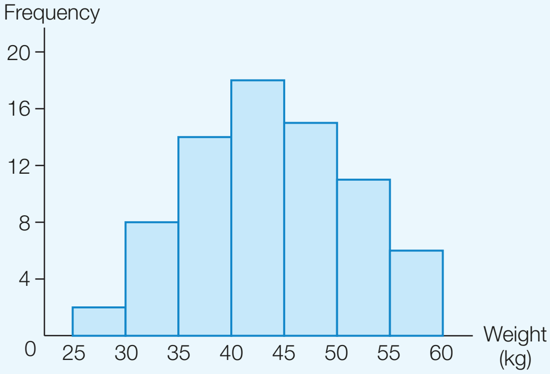
(b) Both the 37th and 38th values are in the 40–<45 class, so this is the median class interval.

(c) The modal class interval is the class interval with the highest frequency.

(c) Modal class interval is 40–<45 kg.

(d) Draw the histogram remembering to leave half a column width before starting the first column.

(d)



(e) Identify whether any data values appear to be quite different from the other data values.

(e) There don't appear to be any outliers.

Using technology

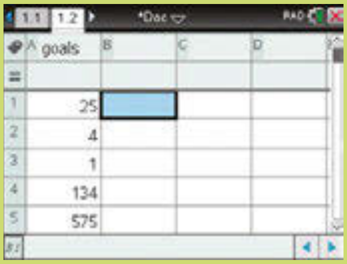
If you have a large list of raw data spread out over a large range, then it can take some time to sort it into class intervals and then draw a frequency graph. However, you can use a spreadsheet or CAS technology to make this much easier and faster.

To do this using CAS technology, consider the following data set, the number of goals scored by each player in an AFL team:

25	4	1	134	575	198	27	2	106	14	73	0	0
84	49	423	66	0	55	153	17	1	3	19	0	0
11	1	6	15	17	57	0	40	19	0	0	26	0
2	0	0	0	0	0	0						

Using TI-Nspire CAS

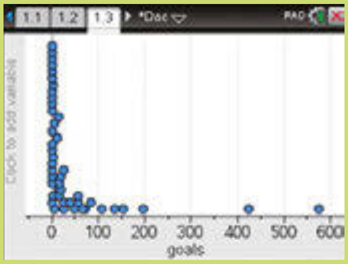
1 Add **Lists & Spreadsheet** to your document. Enter the name of column **A** as **goals**. Enter all the data from above into this column, starting in the first row.



	A goals	B	C	D
1	25			
2	4			
3	1			
4	134			
5	575			

2 Add **Data & Statistics** to your document. This will show a scattering of data points.

Use the cursor to select the bottom of the screen, where the text shows **Click to add variable**, and select the variable **goals**. This will then display a dot plot.



Using Casio ClassPad CAS

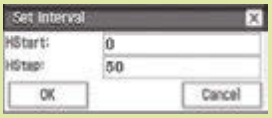
1 From the menu select **Statistics**. Enter all the data from above into **list1**, starting in the first row.

	list 1	list 2	list 3
1	25		
2	4		
3	1		
4	134		
5	575		
6	198		
7	27		
8	2		
9	106		
10	14		
11	73		
12	0		
13	0		
14	84		
15	49		
16	423		
17	66		
18	0		

2 To create a frequency graph (histogram), select **SetGraph** > **Setting…** and set the **Type** to **Histogram** and the **XList** to **list1** as shown.

Draw: ● On ○ Off
Type: Histogram
XList: list1
Freq: 1

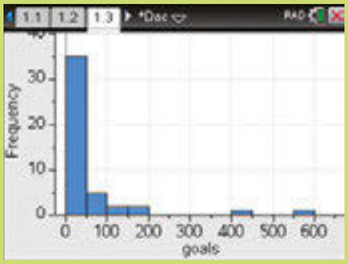
Then select the draw graph symbol 📊 and set the histogram start point **HStart** at 0 and the step value **HStep** at 50 as shown.



HStart: 0
HStep: 50

Using TI-Nspire CAS

3 To create the graph, select **menu** > **Plot Type** > **Histogram**.



By default, the class interval has been set as 50. To change the class interval, you can select **menu** > **Plot Properties** > **Histogram Properties** > **Bin Settings** > **Equal Bin Width**.

Using Casio ClassPad CAS

3 The frequency graph will appear in the lower screen.

	list 1	list 2	list 3
1	25		
2	4		
3	1		
4	134		
5	575		
6	198		
Cal▸			

```
[ 47]=
```

To change the class interval, you can go back to the data list and then do this again with a different **HStep** value.

The frequency graph shows that the data are positively skewed and that there are probably two outliers, 423 and 575. Before you started, you may have suspected these would be outliers!

Using TI-Nspire CAS

4 To find the key statistical values, go back to the list of data by selecting the tab **1.1** at the top of the window. Then select **menu** > **Statistics** > **Stat Calculations** > **One-Variable Statistics…**. Set **Num of Lists** to 1 and then select **goals** as the **X1 List**. You may have to scroll to see all the results.

	A goals	B	C	D
=				=OneVar(
1	25		Title	One-Va…
2	4		$\bar{x}$	48.3261
3	1		Σx	2223.
4	134		Σx²	632319.
5	575		sx := sn…	108.001

```
D1 ="One-Variable Statistics"
```

The statistics values should be given as follows:

Mean ($\bar{x}$) = 48.3261
Median (**MedianX**) = 8.5
IQR (**Q_3X** – **Q_1X**) = 49
Range (**MaxX** – **MinX**) = 575

Note that the range and IQR are not displayed, so they must be calculated from the given five-figure summary.

Using Casio ClassPad CAS

4 To find the key statistical values, select **Calc** > **One-Variable** and set the **XList** to **list1**. You may have to scroll to see all the results.

```
One-Variable

x̄      =48.326087
Σx     =2223
Σx²    =632319
σx     =106.82087
sx     =108.00104
n      =46
minX   =0
Q1     =0
Med    =8.5
```

The statistics values should be given as follows:

Mean ($\bar{x}$) = 48.3261
Median (**Med**) = 8.5
Mode = 0
IQR (**Q_3** – **Q_1**) = 49
Range (**maxX** – **minX**) = 575

Note that the range and IQR are not displayed, so they must be calculated from the given five-figure summary.

If you are working with grouped data, then you need to use the midpoint of each class interval instead of the individual (ungrouped) data values. You can then enter the frequencies into the second column and use the CAS similarly to find the statistics.

9.4 Statistics from grouped data

Navigator

Answers p. 676

1, 2, 3, 5, 6, 7, 8 (a–b), 12, 14	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (a–c), 12, 14	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14

Equipment required: spreadsheet or CAS technology may be used for Questions **1**, **3**

Fluency

W.E. 11

1 The Wembley High soccer coach recorded the number of goals scored in every soccer game played by the team. The results are as follows:

3 2 1 2 5 0 2 2 0 1 3 2 1 2 2 1 1 0 0 0 5 2 2 2 0 1 0 0 5 3
0 8 0 2 1 1 4 3 9 0 1 2 2 0 0 2 0 0 1 3

Use a frequency table of the data to:

(a) calculate the mean, giving your answer correct to 1 decimal place

(b) calculate the median

(c) calculate the mode

(d) draw a frequency column graph of the data

(e) identify any possible outliers.

W.E. 12

2 A bakery records the number of sales of a new type of speciality bread over a 60-day period. The number of sales of the bread are recorded below.

78, 62, 68, 93, 123, 87, 92, 77, 113, 105, 85, 121, 113, 127, 105, 99, 108, 110, 116, 128, 123, 111, 109, 93, 132, 103, 94, 117, 124, 132, 117, 118, 122, 107, 110, 125, 106, 88, 138, 109, 128, 103, 138, 104, 128, 134, 122, 118, 99, 111, 119, 128, 121, 116, 105, 108, 135, 137, 127, 124

Use grouped data with a first interval of 60–69 to:

(a) calculate an estimate for the mean number of orders, giving your answer correct to the nearest whole number

(b) calculate the median class interval for the number of orders

(c) calculate the modal class interval for the number of orders

(d) draw a frequency column graph of the orders

(e) identify any possible outliers.

W.E. 13

3 A horse trainer has a large stable of horses and needs to keep records of each horse's weight. The weights are entered into a frequency table so that the trainer can gather some statistics on the horses. The data in the frequency table gives the weight, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	3
300–<350	13
350–<400	17
400–<450	20
450–<500	16
500–<550	14
550–<600	4

(a) Calculate an estimate for the mean weight of the horses. Give your answer correct to the nearest whole number.

(b) Calculate the median class interval for the weights of the horses.

(c) Calculate the modal class interval for the weights of the horses.

(d) Draw a histogram of the data.

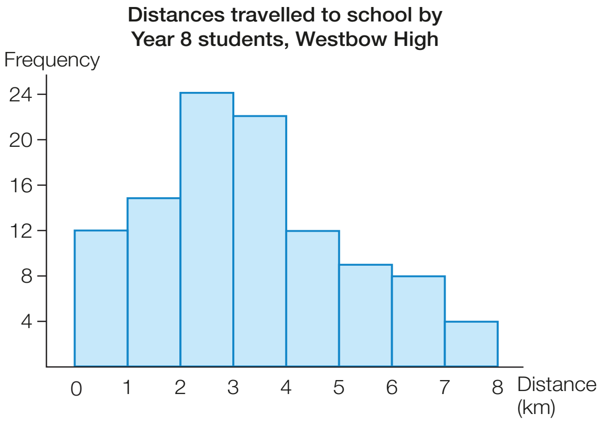
(e) Identify any possible outliers.

4 The following histogram shows the distance travelled to school, measured in km, by each of the students in Year 8 at Westbow High School.

(a) State the modal class.

(b) Find the median class.

(c) Find an estimate for the mean distance travelled to school (correct to 1 decimal place).



5 **(a)** The class centre for a class interval of 0–19 is:

A 9 **B** 9.5 **C** 10 **D** 10.5

(b) The class centre for a class interval of 10–20 is:

A 14.5 **B** 15 **C** 15.25 **D** 15.5

To find the class centre, add the endpoints and divide by 2.

Understanding

6 Consider the following stem-and-leaf plot.

(a) Find the mean and median from the raw scores. Calculate the mean correct to 1 decimal place.

(b) By organising the data into a frequency table, with 40–49 as the first class interval, calculate an estimate for the mean. Round your answer to 1 decimal place.

(c) State the modal class.

Stem	Leaf
4	8
5	2 3 4 5 6 7 7
6	4 5 5 6 7
7	3 4 5
8	2
9	1
10	9

Key: 4 | 8 = 48

7 The numbers that follow are the points scored by the 26 players on the list of the Brisbane Broncos who played at least one game of the 2015 season.

4	0	12	4	8	0	0	12	16	12	56	80	60
20	8	61	0	4	56	0	0	144	44	8	12	0

(a) Using 0–19 as the first class, group the data. Display the results as a frequency column graph.

(b) Do you think any of the values might be possible outliers? If so, which ones?

(c) Remove all of the values that are over 20 and construct a frequency column graph of the remaining data. Let the first interval be 0–4.

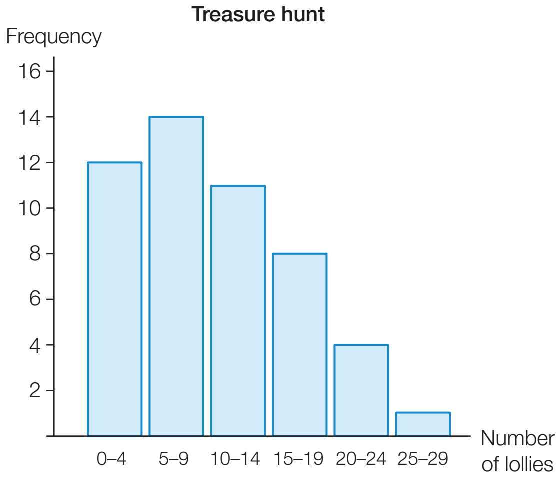
(d) Comment on the benefits of the different histograms.

8 The graph opposite shows the number of lollies found by a group of children at a birthday party treasure hunt.

(a) How many children attended the party?

(b) Calculate an estimate for the mean number of lollies found.

(c) Can you calculate the exact number of lollies found? Explain your answer.



9 Find the class intervals for each of the following lists of class centres for discrete data.

(a) 5.5, 15.5, 25.5, 35.5, 45.5

(b) 2, 7, 12, 17, 22

(c) 24.5, 74.5, 124.5, 174.5, 224.5

(d) 25.5, 75.5, 125.5, 175.5, 225.5

(e) 11, 14, 17, 20, 23

(f) 27, 32, 37, 42, 47

Reasoning

10 The following table shows the number of AFL games played by each of the 45 players on the list of the Sydney Swans at the beginning of the 2016 season.

Number of games	Frequency
0–39	27
40–79	6
80–119	2
120–159	5
160–199	2
200–239	1
240–279	2

(a) Calculate an estimate for the mean number of games played. Round your answer to the nearest whole number.

(b) State the interval that represents the modal class.

(c) State the interval that represents the median class.

(d) In fact, 14 of the players have yet to play a game. Recalculate the three measures of centre, leaving out these players.

(e) Does this give a better representation of the measures of centre for the number of games? Explain your reasoning.

11 The following frequency table shows the time taken, in minutes, by the students at a particular school to complete a cross-country race.

(a) Calculate an estimate for the mean time taken by the students to complete the cross-country race. Round your answer to the nearest minute.

(b) Five students did not complete the race within the time limit. They all walked to the finish together, 45 minutes after the start. What is the mean time with these students included? Round your answer to the nearest minute.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

Open-ended

12 **(a)** Write two different lists of class intervals that have class centres 5 apart.

(b) Write two different lists of class intervals that have class centres 10 apart.

13 Alphaville is a small town at an elevation of 221 m; Betaborough is a country town at an elevation of 107 m; and Deltatown is a coastal town at an elevation of 3 m. The following table gives some statistics related to the annual mean temperatures of these towns.

	Max (°C)	Max range (°C)	Min (°C)	Min range (°C)
Betaborough	28.2	18.7	13.4	18.1
Alphaville	20.6	17.6	6.6	9.2
Deltatown	25.1	7.5	17.2	9.8

Choose one of the towns and construct a table of the monthly maximum and minimum temperatures that fit the given statistics.

14 **(a)** Construct a stem-and-leaf plot with at least 20 values such that the median is bigger than the mean.

(b) Construct a stem-and-leaf plot with at least 20 values such that the median is smaller than the mean.

Puzzle

Complete the patterns

In each case, find the next number in the pattern.

1

4	8	10	11	11.5	

2

3	4	9	24	81	144	

3

4	2	3	1	2	0	

4

4	5	9	14	23	

Now, see if you can make up another four patterns of your own and get one of your classmates to try to solve them.

Maths 4 Real

Which statistic is best in sport?

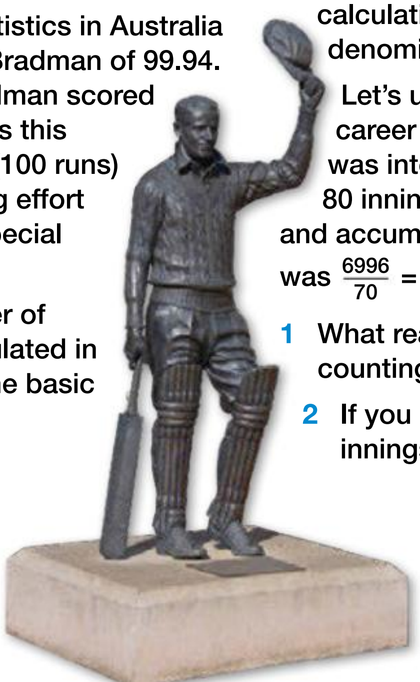
Cricket

Cricket uses lots of statistics, such as batting averages, bowling averages, run rates, and the greatest number of innings without scoring a 'duck' (zero).

One of the best known cricket statistics in Australia is the run average for Sir Donald Bradman of 99.94. This means that, on average, Bradman scored 99.94 runs every time he batted. (Is this actually possible?) As a 'century' (100 runs) is considered to be an outstanding effort in any innings, this average is a special achievement. But is it accurate?

The average (mean) for the number of runs scored by a cricketer is calculated in a slightly strange way. It follows the basic formula for an average:

$$\text{Run average} = \frac{\text{total runs scored}}{\text{number of innings}}$$



However, only completed innings (where the cricketer goes 'out') are counted in the denominator. So, if the cricketer is not out when the team's innings is completed, then their runs are counted in this calculation, but the innings is not added to the denominator value.

Let's use Bradman as an example. His Test career spanned the years 1928–1948, but was interrupted by World War II. He batted 80 innings in Test cricket, with 10 'not out' innings, and accumulated 6996 runs. So, his average (mean) was $\frac{6996}{70} = 99.94$.

1 What reasons do you think there would be for not counting the 'not outs'?

2 If you did count the 'not outs' as completed innings, what would be Bradman's run average?

You also know how to calculate the median and the mode. The following is a list of Bradman's scores, innings by innings. The symbol * means that score was a 'not out'.

18	1	79	112	40	58	123	37*	8	131
254	1	334	14	232	4	25	223	152	43
0	226	112	2	167	299*	0	103*	8	66
76	24	48	71	29	25	36	13	30	304
244	77	38	0	0	82	13	270	26	212
169	51	144*	18	102*	103	16	187	234	79
49	0	56*	12	63	185	13	132	127*	201
57*	138	0	38	89	7	30*	33	173*	0

3 Calculate the median and mode for Bradman's results.

4 Of the four numbers calculated (the average, the average including 'not outs', median and mode), which do you think is the fairest number that best describes Bradman's career?

Now consider the statistics for two cricketers at different stages in their careers.

The following are the results for Marcus North's first 10 innings:

117	5	38	0	125*	0	6	12	96	110

The following are the results for Michael Hussey's first 70 innings:

1	29	137	31*	133*	30*	23	58	122	31
45	6	14*	75	73	89	23	37	182	86
91	61*	74*	103	6	37	133	132	34*	2
36	41	145*	0	46	22	56	1	10	40
12	18	146	31	54	1	53	90	19	35
0	70	0	8	0	2	30	45*	4	0
50	19	20	39	3	51	27	0	64	10

5 Calculate the four different results for each of these players.

6 Now, make a few comments about which result you now think is the fairest one to use to analyse a cricketer's performance.

AFL

The Coleman Medal is awarded to the player in the Australian Football League who kicks the greatest number of goals in the regular home-and-away season. In 2010, Jack Riewoldt of the Richmond football club kicked 78 goals to win the medal. A year earlier, his cousin, Nick Riewoldt of the St Kilda football club, kicked 68 goals to finish third in the medal tally.

Jack Riewoldt played 22 games in the 2010 home and away season and kicked the following number of goals:

2	0	3	3	2	3	2	4	6	4	6	10	3	5	5	3	2	2	1	2	7	3

Nick Riewoldt played 21 games in the 2009 home and away season and kicked the following number of goals:

0	2	1	5	4	4	5	3	4	4	3	5	2	3	1	4	5	2	2	3	6

7 For each player, calculate the mean, median and modal number of goals.

8 If you were to comment on the number of goals kicked by Jack or Nick in an 'average' game, what answer would you give?

9 Nick Riewoldt also played 3 games in the 2009 finals, scoring 5, 4 and 1 goals. How does this affect his mean, median and mode?

Another way of comparing the players is to look at their accuracy when kicking for goal.

Over the course of four seasons these results were scored by the two players. (In Australian Rules football a goal is worth 6 points and a behind is worth 1 point.)

	Jack Riewoldt		Nick Riewoldt	
	Goals	Behinds	Goals	Behinds
2007	7	3	42	26
2008	18	8	65	39
2009	32	27	78	47
2010	78	39	34	27

10 (a) Calculate the number of goals as a percentage of their scoring shots.

(b) Which player would you say is a 'better' kick for goal?

Half-time 9

9.3 **1** For the following data list find a suitable class interval, record the results in a frequency table, and then display the data using a frequency column graph.

25, 45, 67, 99, 18, 24, 67, 87, 56, 65, 43, 21, 19, 91, 37, 67, 54, 23, 19, 22, 54, 88, 93, 86, 54 67, 43, 22, 27, 28, 38, 46, 98, 25, 33, 62, 90, 30, 21, 28, 78, 18, 77, 71, 50, 43, 40, 20, 77, 91

9.2 **2** The following data values represent the number of games played by players on the list of the South Sydney Rabbitohs.

74	33	0	23	61	10	9	4	0	8
20	110	34	68	63	4	126	37	0	46
30	178	122	69	9	0	9	4	12	

(a) Using the raw data, calculate the:

- **(i)** mean number of games played correct to 2 decimal places
- **(ii)** median number of games played
- **(iii)** mode of the number of games played.

(b) Do you think any of the results represents an outlier? If so, which one(s)? If not, how many games would need to be played for you to consider it an outlier?

(c) Four players have yet to play a game. Remove these results from your data and recalculate the mean, median and mode. Did this make much of a difference to your statistics?

(d) Which of the above statistics do you think does the best job of measuring the centre of this data?

3 Each student in a class was asked how much pocket money they received each week. The following table summarises the results. 9.4

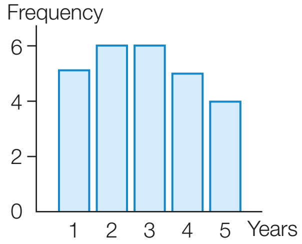
(a) Find the mean amount of pocket money received by the students.

(b) Find the class interval that represents the median amount of pocket money received.

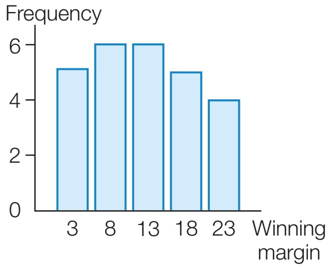
Amount of pocket money	Frequency
0–<5	6
5–<10	13
10–<15	3
15–<20	2
20–<25	1

4 Construct a frequency table from each of the following frequency column graphs. 9.3

(a)



(b)



5 Each student in a class was asked how many times they had travelled to a different Australian state or territory. The following table summarises the results. 9.2

(a) Find the mean, correct to 2 decimal places.

(b) Find the median number of interstate visits for these students.

Number of interstate visits	Frequency
0	8
1	7
2	3
3	4
4	1
5	1
6	0
7	0
8	2

6 Draw a histogram for the data displayed in the following table. 9.3

Time waiting for train (min)	Frequency
8	12
9	8
10	13
11	11
12	9
13	16
14	10

7 Researchers are trying to find out the number of freshwater crocodiles in a particular lagoon. To do this, they have captured and tagged 30 crocodiles and then released them back into the wild. Some time later, they capture another 20 crocodiles and find that two of them have tags. What should be their estimate for the total crocodile population in the lagoon? 9.1

Investigation

Who's the best? Here's the test

Every year in November, a group of young footballers are brought together and taken through a series of physical tests.

The Big Question

Was the group of young footballers better in 2015 or in 2016?

Engage

One test that young footballers do is the 'beep' test. They must run 20 m in a time interval set by two electronically timed beeps. The time interval starts at 9 seconds for level 1, but the time gets shorter and shorter and the distance run gets longer and longer at each level as players progress through the levels of the test. Each level takes about 1 minute. The score is the highest level a player reaches, keeping in time with the beeps. The higher the level, the fitter the player.

The best ten results for beep test levels in 2016 were:
14.8, 14.5, 14.5, 14.4, 14.3, 14.3, 14.2, 14.2, 14.2, 14.1

In 2015, the best ten results were:
15.2, 15.1, 14.9, 14.9, 14.8, 14.8, 14.7, 14.5, 14.4, 14.3

1. Find the mean, mode and median for each group.
2. Find the range for each group.
3. Draw a histogram for each set of data.

Explore

Using only one set of results does not really give a balanced comparison of the two groups. The best ten results for the 3 km time trial (given in minutes and seconds) for the two years are given below.

2016: 10:17, 10:32, 10:41, 10:42, 10:42, 10:43, 10:44, 10:53, 10:57, 10:57

2015: 10:23, 10:25, 10:32, 10:42, 10:48, 10:49, 10:49, 10:49, 10:51, 10:54

4 Find the mean, mode and median for each group.

5 Find the range for each group.

6 Draw a histogram for each set of data.

Another set of results for the agility test (seconds) is given below.

2016: 7.77, 8.03, 8.18, 8.25, 8.27, 8.31, 8.32, 8.32, 8.34, 8.35

2015: 7.91, 8.05, 8.07, 8.22, 8.26, 8.29, 8.32, 8.36, 8.38, 8.39

7 Find the mean, mode and median for each group.

8 Find the range for each group.

9 Draw a histogram for each set of data.

Strategy options

- Have I seen a similar problem?
- Look for a pattern.

Explain

10 Using the 'beep' test statistics, which group do you think is better at this test?

11 Using the statistics from the 3 km time trial, which group do you think is better at this test? Give reasons for your decision.

12 Using the statistics from the agility trial, which group do you think is better at this test? Give reasons for your decision.

Elaborate

13 Using all three results, which group do you think is better overall? Give reasons for your decision.

14 What do you think 'better' means in this context?

Evaluate

15 Each year, 75 young footballers attend the camp. Do you think that using the best ten results is the fairest way to make a comparison between the groups? Can you suggest a better method for comparison?

16 Do you think you can make a fair comparison on the basis of three tests?

17 Which statistics were the most useful in helping you to make a decision about the best group?

18 Were the histograms helpful in making a decision about the better group?

19 What other graphs could you have used?

Extend

Here are some more results from the two groups.

Repeated 30 m sprint (seconds)
2016: 24.82, 24.97, 24.97, 25.10, 25.10, 25.14, 25.44, 25.44, 25.48, 25.49
2015: 23.59, 23.96, 24.01, 24.08, 24.17, 24.22, 24.22, 24.25, 24.37, 24.48

Vertical jump (cm)
2016: 78, 74, 72, 72, 70, 70, 69, 69, 69, 67
2015: 81, 74, 73, 72, 71, 70, 70, 70, 68, 68

20 m sprint (seconds)
2016: 2.80, 2.82, 2.83, 2.86, 2.89, 2.90, 2.91, 2.92, 2.92, 2.92
2015: 2.83, 2.88, 2.88, 2.89, 2.91, 2.91, 2.92, 2.92, 2.93, 2.94

20 Use this additional data to make a final decision about whether the 2015 or the 2016 group was better.

21 Do you think that these tests would be good predictors of future performance in football? What other factors need to be considered when selecting a top footballer?

Maths 4 Real

Hold the front page

The deadline is looming for tomorrow's edition of *The Morning News* but Katrina, the editor, is not happy about some of the stories her reporters have submitted. You will have to help her meet her deadline by writing some new stories!

Retention rates

This story is supposed to be based on the following information.

For a school, the retention rate compares the number of students still enrolled at that school at the end of a time period with the number at the beginning and expresses this as a percentage. As an example, if there were 100 students in Year 7 in 2014 and 94 students in Year 9 in 2016 then the retention rate is 94%.

The table shows the retention rate for Australian public schools.

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Year 7/8 to Year 9 retention rate	99.7	101.1	101.1	100.6	102.0	99.9	98.2	98.3
Year 10 to Year 12 retention rate	67.6	74.2	70.7	66.0	67.6	61.8	74.6	96.6
Year 7/8 to Year 12 retention rate	64.6	71.9	70.3	64.7	68.2	61.3	67.6	95.9

ABS, 4221.0 Schools, Australia 2008

The story, of about 300 words, should include the following, as a minimum:

- a graph showing a comparison on a state by state basis
- an explanation of how the rate could be more than 100%
- a discussion of why the rates are so different for the different time periods
- a discussion of which state or territory does best at retaining its students in public schools
- an explanation of where the students who are not retained go.

How good is Australia at sport?

The following table shows some information about medal results at the 2014 Commonwealth Games in Glasgow, Scotland, and the 2010 Commonwealth Games in New Delhi, India.

	Gold		Silver		Bronze		**Total**	
	2014	2010	2014	2010	2014	2010	2014	2010
Australia	49	74	42	55	46	48	137	177
Canada	32	26	16	17	34	32	82	75
England	58	37	59	59	57	46	174	142
India	15	38	30	27	19	36	64	101
Jamaica	10	2	4	4	8	1	22	7
Kenya	10	12	10	11	5	9	25	32
Malaysia	6	12	7	10	6	13	19	35
New Zealand	14	6	14	22	17	8	45	36
Scotland	19	9	15	10	19	7	53	26
South Africa	13	12	10	11	17	10	40	33

You need to present this information in a more informative manner. At the moment, it is in alphabetical order only, but this makes it difficult to see which countries did best. You might like to draw some sort of graph as well. When you have done that you need to write a brief report, using as many statistical terms and values as you can. Come up with a headline for your report.

9.5 Understanding probability

An event in probability is simply something that happens. Examples of events in probability are a coin toss that shows heads, rolling a 4 on a die, or having a day when it rains.

The **probability** of an event happening is the likelihood, or chance, of that event happening, expressed as a number between 0 and 1. This probability is labelled as Pr(event) and may be written as a decimal, fraction or percentage.

A probability of 0 means that the chance of an event is *impossible*. A probability of 1 means that the chance of the event is *certain*. A probability of 0.5 is an 'even chance', equally likely to happen or to not happen.

For example, the probability of rolling a 7 on a standard six-sided die would be 0, while the probability of rolling a number less than 7 would be 1 (because the only possibilities are 1, 2, 3, 4, 5, 6).

The result of an event is called an **outcome**. For example, when tossing a coin, there are two possible outcomes: heads or tails. The 'successful' outcome is the result found by the probability of the event, Pr(event).

Probabilities of actual events can be found through experiments and by collecting long-term data. You can also often determine the theoretical probability by investigating the situation mathematically.

Sample space

To calculate a theoretical probability, you need to know the **sample space**. A sample space is a list of all possible outcomes. For simple situations, the outcomes are all equally likely.

The successful outcomes are some or all of the outcomes listed in the sample space. The sample space from tossing a coin is heads (H) and tails (T). If the successful outcome is 'heads', then there is only one successful outcome out of two outcomes in the sample space, so:

$$\Pr(H) = \frac{1}{2}$$

The sample space from rolling a standard six-sided die is 1, 2, 3, 4, 5, 6. If the successful outcome is rolling a 2, then there is only one successful outcome possible out of six outcomes in the sample space, so:

$$\Pr(2) = \frac{1}{6}$$

When more than one outcome is considered a success, you can calculate the probability by counting the number of successes overall, or by calculating each probability separately and adding them.

For example, the probability of rolling a 1 or a 2 on a die can be calculated in two ways.

There are two successful outcomes possible out of six outcomes in the sample space, so:

$$\Pr(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$$

Alternatively, by considering the probability of each event separately and adding them:

$$\Pr(1) = \frac{1}{6} \text{ and } \Pr(2) = \frac{1}{6} \text{ so } \Pr(1 \text{ or } 2) = \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Calculating probability

If all outcomes are equally likely, then:

$$\Pr(\text{event}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

Worked example 14

W.E. 14

In Kayla's class there are 24 students. Five were born in Cambodia, three in Vietnam, six in Australia, two in Croatia, five in Serbia, one in England, one in New Zealand and one in South Africa. A member of the class is selected at random. What is the probability that a student chosen at random was born in:

(a) Vietnam **(b)** Croatia or Serbia?

Thinking	Working
(a) 1 Write the number of successful outcomes.	(a) Number born in Vietnam = 3
2 Write the total number of outcomes.	Total = 24
3 Use the formula to express the probability as a fraction and then cancel down to simplest terms (if possible).	$\Pr(\text{born in Vietnam}) = \frac{3}{24}$ $= \frac{1}{8}$
(b) 1 Calculate the number of successful outcomes.	(b) Number born in Croatia or Serbia $= 2 + 5$ $= 7$
2 Write the total number of outcomes.	Total = 24
3 Use the formula to express the probability as a fraction and then cancel down to simplest terms (if possible).	$\Pr(\text{born in Croatia or Serbia}) = \frac{7}{24}$

Note: If you added together each of the individual probabilities for country of birth they would have to equal 1, because it is certain that each student was born in one of the listed countries.

Complements

All situations can be written so that there are only two possible outcomes: an event may happen or it may not happen. These two probabilities add up to 1, because it is certain that any event must either happen or not happen. When two outcomes are the only two possibilities, then each outcome is called the **complement** of the other. For example, the probabilities of 'winning' or 'not winning' are called complementary probabilities. (However, you should note that in many games 'winning' and 'losing' are not complementary, because there may be another possibility, such as a draw or tie.)

Pr(event occurring) + Pr(event not occurring) = 1

Sometimes, the probability you want to calculate is easier to find if you calculate the probability of its complement, then subtract this from 1.

Pr(event not occurring) = 1 – Pr(event occurring)

Consider the probability of getting a 2, 3, 4, 5 or 6 when rolling a die. This probability is the same as the probability of 'not getting a 1'. So, you can calculate the probability of 'getting a 1' and subtract this from 1 to find the required probability.

Worked example 15

W.E. 15

If you roll a standard six-sided die, what is the probability of:

(a) not rolling a 6 **(b)** not rolling a 5 or a 6?

Thinking	Working
(a) **1** Write the known probability. (In this case, the probability of rolling a 6.)	(a) Pr(rolling a 6) $= \frac{1}{6}$
2 Use the formula Pr(event not occurring) = 1 – Pr(event occurring) and calculate the answer.	Pr(not rolling a 6) = 1 – Pr(rolling a 6) $= 1 - \frac{1}{6}$ $= \frac{5}{6}$
(b) **1** Write the known probabilities.	(b) Pr(rolling a 5) $= \frac{1}{6}$ and Pr(rolling a 6) $= \frac{1}{6}$
2 Find the total known probability.	Pr(rolling a 5 or a 6) $= \frac{1}{6} + \frac{1}{6}$ $= \frac{2}{6}$ $= \frac{1}{3}$
3 Use the formula Pr(event not occurring) = 1 – Pr(event occurring) and calculate the answer.	Pr(not rolling a 5 or a 6) $= 1 - \frac{1}{3}$ $= \frac{2}{3}$

9.5 Understanding probability

Navigator

1, 2, 3, 4, 5, 6, 8, 11, 12, 15 (a), 16, 17	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17

Answers p. 678

Fluency

W.E. 14

1 A class consists of 26 students. Ten study Japanese and the rest study Italian. What is the probability that a student chosen at random studies:

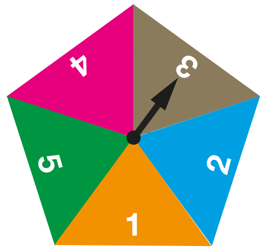
(a) Japanese (b) Italian?

W.E. 15

2 If you spin a five-sided spinner with numbers 1, 2, 3, 4, 5, then what is the probability of:

(a) not getting a 2

(b) not getting a 1 or a 2?



3 Illona is running in a race at the local athletics track. If her probability of winning is $\frac{1}{8}$ then what is the probability that she will not win?

4 There are 13 teachers in the staff room: eight junior teachers and five senior teachers, one of whom is Mr Antonac. A student knocks at the door wanting to see Mr Antonac. What is the probability that the teacher who answers the door is:

(a) senior (b) junior (c) Mr Antonac?

5 The hockey training squad consists of eight forwards, five half-backs, four full-backs and Jarrod, who plays in goal. One player is chosen at random from the training squad to represent the team at a fundraising function.

What is the probability that the person is:

(a) a forward (b) a half-back (c) Jarrod, the goalkeeper?

6 There are 10 000 tickets sold in the school raffle. If Alf bought 10 tickets, find:

(a) Pr(Alf wins the raffle) (b) Pr(Alf does not win the raffle).

7 My fruit bowl contains five bananas, four apples, one orange and five peaches. If I choose one piece of fruit from the bowl at random, find:

(a) Pr(a banana) (b) Pr(a peach or an apple)

(c) Pr(not an apple) (d) Pr(not an orange or a banana).

8 A bag of potatoes contains five green potatoes and fourteen good potatoes (which are not green). If a potato is chosen at random, the probability of it being green is:

A $\frac{5}{19}$ B $\frac{5}{14}$ C $\frac{1}{2}$ D $\frac{14}{19}$

Understanding

9 What is the probability that three digits on a car number plate will consist of 2, 3 or 4 in any order? (Assume that each digit can be any of the ten digits from 0 to 9.)

10 For seven of the last twelve years, Dennis has received a birthday present of socks and handkerchiefs from his grandparents. Assuming his grandparents don't change their present-giving behaviour, what is the probability that he doesn't receive socks and handkerchiefs from his grandparents for his next birthday?

11 Dominic rolls a standard six-sided die. Find:

(a) Pr(a 2 or a 4)
(b) Pr(a number larger than 2)
(c) Pr(a number less than 2)
(d) Pr(a multiple of 3)
(e) Pr(not a 5)
(f) Pr(not a number larger than 3).

12 A bag contains 26 small tiles, each marked with a different letter of the alphabet. Celeste selects one tile from the bag. Choose the answer with the correct probability that the tile Celeste draws is:

(a) a vowel (a, e, i, o, u)

A $\frac{2}{13}$ **B** $\frac{5}{26}$ **C** $\frac{1}{5}$ **D** $\frac{1}{4}$

(b) b, d, g or z

A $\frac{2}{13}$ **B** $\frac{4}{25}$ **C** $\frac{3}{13}$ **D** $\frac{1}{4}$

(c) one of the letters in the word 'chance'.

A $\frac{2}{13}$ **B** $\frac{5}{26}$ **C** $\frac{3}{13}$ **D** $\frac{6}{25}$

13 The numbers from one to twenty are written on separate counters and placed in a hat. If one counter is drawn at random, find:

(a) Pr(a number between 5 and 11)
(b) Pr(a number with a 1 in it)
(c) Pr(a number less than 30)
(d) Pr(a multiple of 5)
(e) Pr(not a multiple of 5)
(f) Pr(not odd).

Reasoning

14 The 2015 motor vehicle census showed there were more than 18 million vehicles registered in Australia. Express your answers to the following questions as percentages to the nearest whole number.

Vehicle type	Number registered in 2015
Passenger vehicles	13 549 449
Camper vans	58 288
Light commercial vehicles	2 907 006
Light rigid trucks	140 625
Heavy rigid trucks	331 699
Articulated trucks	94 975
Non-freight carrying vehicles	23 361
Buses	95 149
Motorcycles	807 215
Total motor vehicles	18 007 767

(a) Find the probability that a vehicle selected at random from the register is:

- **(i)** a passenger vehicle
- **(ii)** a rigid truck
- **(iii)** not a motorcycle
- **(iv)** neither a bus nor a camper van.

(b) Is it more likely that a vehicle chosen at random is a motorcycle, or any type of truck?

(c) The previous year's data is shown in this table. Find the 2015 and 2014 probabilities for both passenger vehicles and rigid trucks. Is there any significant change?

Vehicle type	Number registered in 2014
Passenger vehicles	13 297 260
Camper vans	55 757
Light commercial vehicles	2 824 052
Light rigid trucks	135 658
Heavy rigid trucks	329 464
Articulated trucks	93 853
Non-freight carrying vehicles	23 144
Buses	94 131
Motorcycles	780 174
Total motor vehicles	17 633 493

15 Draw spinners that will satisfy each of the following conditions.

(a) Pr(green) = Pr(blue) = Pr(orange) = $\frac{1}{3}$.

(b) Three colours only, and Pr(red) = 3 × Pr(blue) and Pr(green) = $\frac{1}{5}$.

(c) Four colours only, and Pr(brown) = 5 × Pr(green), Pr(red) = 3 × Pr(green) and Pr(yellow) = Pr(green).

Open-ended

16 **(a)** Put the following words and expressions in order of probability, from impossible to certain. Then, give a percentage probability to each of them. How does your ordering compare with other students' answers?

50-50
impossible
most likely
probably
unusual
certain
little chance
most often
sure thing
usually
even chance
more often than not
most unlikely
unlikely
very unlikely

(b) Can you add some other words or expressions for probabilities to the list?

17 Decide what the probability is for each of the following situations in your life.

(a) getting to school on time every day

(b) having an apple for lunch

(c) going to the movies at least once a month

(d) getting a phone call from a friend today

(e) running a marathon in your lifetime

(f) taking a space ride to the Moon

(g) playing sport on the weekend

(h) sleeping after 9 am on a Saturday

Puzzle

Track the bracket

Insert brackets to make each of the following equations true.

1 $3 \times 4 - 5 + 6 \times 2 + 5 = 39$

2 $5 - 2 \times 3 - 4 \div 6 - 3 = 1$

3 $18 + 36 \div 3 - 3 \times 5 \times 2 - 3 \times 1 = 9$

9.6

Theoretical probability

There are many situations where the probabilities of experiments can be calculated theoretically. This means you can calculate the probabilities without actually doing an experiment. It can be useful to do this for situations that involve choices and randomness: tossing a coin or rolling a die, using a spinner, picking a card from a pack of cards, or picking a coloured marble out of a bag are all such examples.

You can list the sample space that gives the total number of outcomes, then count the number of successful outcomes. When all outcomes are equally likely, then you can calculate the required probability using the formula:

$$\text{Pr(event)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

Worked example 16

W.E. 16

A standard pack of 52 playing cards has four 'suits' of 13 cards each, which can be listed as follows.

♥: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
♦: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
♣: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
♠: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

(♥ = hearts, ♦ = diamonds,
♣ = clubs, ♠ = spades, A = Ace,
J = Jack, Q = Queen, K = King)

Alvin selects one card from the pack. Find the probability of that card being:

(a) a diamond

(b) a King or a Queen.

(c) a 5 or a spade (assume that it could be both)

(d) a Queen or a heart, but not both.

Thinking	Working
(a) **1** Count the number of successful outcomes in the sample space. Count the total number of outcomes.	(a) Number of successful outcomes = 13 Total number of outcomes = 52
2 Calculate the probability by using $\text{Pr(event)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	Pr(diamond) $= \frac{13}{52}$ $= \frac{1}{4}$
(b) **1** Count the number of successful outcomes.	(b) Number of ways of selecting a King or a Queen = 8
2 Calculate the probability by using $\text{Pr(event)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	Pr(King or Queen) $= \frac{8}{52}$ $= \frac{2}{13}$

(c) **1** Count the number of successful outcomes, being careful not to count the same card twice.	(c) Number of 5s: 4 Number of spades: 13 Overlap: 1 (the 5 of spades) Number of '5 or spades': $4 + 13 - 1 = 16$
2 Calculate the probability.	$\Pr(5 \text{ or spades}) = \frac{16}{52}$ $= \frac{4}{13}$
(d) **1** Count the number of successful outcomes, being careful not to count cards that have been excluded.	Number of Queens that are not hearts: $4 - 1 = 3$ Number of hearts that are not Queens: $13 - 1 = 12$ Number of 'Queen or hearts but not both': $3 + 12 = 15$
2 Calculate the probability.	$\Pr(\text{Queen or hearts but not both}) = \frac{15}{52}$

9.6 Theoretical probability

Navigator

1, 2, 3 (a–f), 4, 7 (a–b), 9 (a), 13	1, 2, 3 (c–i), 4, 5, 6, 7, 8, 9, 11, 13, 14	1, 2, 3 (c–i), 4, 5, 6, 7, 8, 9, 10, 11, 12, 14

Answers p. 678

Fluency

W.E. 16

1 Phoebe has a standard pack of 52 playing cards. She selects one card from the pack.

Find the probability of that card being:

(a) a club

(b) an Ace or a 10

(c) a 7 or a diamond

(d) a Jack or a spade but not both.

2 For each of the following questions **(i)** list the sample space and **(ii)** find the probability of the event occurring.

(a) tossing a coin and getting heads

(b) rolling a standard six-sided die and getting a 4

(c) spinning the spinner shown at right and getting blue

(d) picking an Ace of hearts out of a standard pack of 52 playing cards

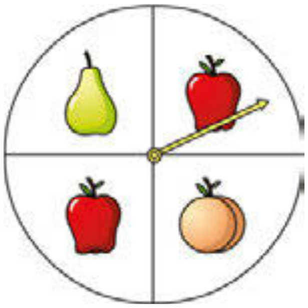
(e) picking a red marble out of a bag containing 1 red, 1 blue, 1 green and 1 orange marble

3 Find the probability of the following events occurring.

(a) rolling a standard six-sided die and getting a 3 or a 6

(b) rolling a standard six-sided die and getting an odd number

(c) spinning the spinner shown below and the needle pointing to apple



(d) spinning the following spinner and the needle pointing to tomatoes

(e) picking a black card out of a standard pack of 52 playing cards

(f) picking a spade out of a standard pack of 52 playing cards

(g) picking a heart or a diamond out of a standard pack of 52 playing cards

(h) picking a red marble out of a bag containing 2 red, 2 blue and 2 black marbles

(i) picking a red marble out of a bag containing 2 red, 2 blue, 2 black and 2 green marbles

4 The spinner shown at right has 6 equal sections, numbered from 1 to 6. Find the probability of spinning the spinner and getting each of these events:

(a) even number or red (3)

(b) odd number or yellow (5)

(c) number greater than 3, or green (6), but not both

(d) not orange (1) and not an even number

5 Find the probability of the following events occurring.

(a) rolling a standard six-sided die and not getting a 3 or a 6

(b) picking a card that is not a spade out of a standard pack of 52 playing cards

(c) picking a marble that is not red out of a bag containing 2 red, 2 blue, 2 black and 2 green marbles

6 For a packet of coloured jelly beans, it is known that:

Pr(green) = 0.2 Pr(red) = 0.3 Pr(white) = 0.2

Pr(purple) = 0.2 Pr(yellow) = 0.1

Find:

(a) Pr(red or purple) **(b)** Pr(yellow or white)

(c) Pr(not yellow or white) **(d)** Pr(not green or red).

7 You have a twelve-sided die, with faces numbered from 1 to 12.

(a) The probability of rolling a multiple of 3 is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(b) The probability of rolling a multiple of 4 is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(c) The probability of rolling a number that is a multiple of 3 and 4 is:

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{12}$

(d) The probability of rolling a number that is a multiple of 3 or 4 is:

A $\frac{1}{2}$　　B $\frac{1}{3}$　　C $\frac{1}{4}$　　D $\frac{1}{12}$

(e) The probability of rolling a number that is a multiple of 3 or 4 but not both is:

A $\frac{11}{12}$　　B $\frac{7}{12}$　　C $\frac{5}{12}$　　D $\frac{1}{12}$

Understanding

8 For a six-sided die, show how you would colour the faces so that:

(a) Pr(white) = $\frac{1}{2}$, Pr(blue) = $\frac{1}{3}$, Pr(green) = $\frac{1}{6}$

(b) Pr(purple) = Pr(silver) = $\frac{1}{6}$, Pr(grey) = Pr(orange) = $\frac{1}{3}$

9 You are to make up a bag of at least twenty coloured counters. Show how you could do this so that:

(a) Pr(yellow) = 0.2, Pr(blue) = 0.4, Pr(red) = 0.3, Pr(black) = 0.1

(b) Pr(yellow) = Pr(pink) = 0.25, Pr(purple) = 0.15, Pr(green) = 0.05,
Pr(black) = Pr(blue) = Pr(white) = 0.1

10 One hundred counters are each blue or yellow and also have the numbers 1, 2 or 3 on them, as shown in the table. The counters are all placed in a bag and a single counter is taken at random.

	Blue	Yellow
1	10	15
2	12	20
3	21	22

(a) Find the probability that the counter is the following:

(i) 2 or blue　　**(ii)** yellow or an odd number

(iii) blue or 1, but not both　　**(iv)** blue and 3

(b) A yellow counter falls out of the bag. What is the probability that it is a 2?

(c) If all of the counters marked '1' are placed in a new bag and one counter is taken at random, then what is the probability it is yellow?

Reasoning

11 Two standard dice are rolled. Copy and complete the table below showing the sample space to find the probability that the difference between the two numbers rolled is 2.

		Second die					
		1	**2**	**3**	**4**	**5**	**6**
First die	**1**	(1, 1)	(1, 2)				
	2	(2, 1)					
	3						
	4						
	5						
	6						

12 A Scrabble set uses 100 pieces: 98 pieces with a letter of the alphabet written on them, and two pieces that are blank.

The distribution of letters on the pieces (that is, how many pieces there are of each letter) is as follows.

A 9	B 2	C 2	D 4	E 12	F 2	G 3	H 2	I 9	J 1	K 1	L 4	M 2	N 6
O 8	P 2	Q 1	R 6	S 4	T 6	U 4	V 2	W 2	X 1	Y 2	Z 1	Blank 2	

(a) Why do the numbers of letters differ so much?

(b) Find the probability that the first piece selected is:

(i) a blank tile **(ii)** the letter O **(iii)** the letter R **(iv)** a vowel.

(c) You are selecting the seven pieces for your first turn, one at a time.

- **(i)** Find Pr(E chosen first).
- **(ii)** With E already chosen, find Pr(second letter is A).
- **(iii)** With E and A already chosen, find Pr(third letter is another E).
- **(iv)** With E, A and E already chosen, find Pr(fourth letter is I).

Open-ended

13 Draw and label a spinner that has at least four colours where the probability of one colour is at least twice the probability of one of the other colours.

14 Make up your own bag of 40 counters as follows.

- Each counter is red, blue or orange.
- There are at least three counters of each colour.
- Each counter has a number from 1 to 5.
- At least two counters have each number.

Using your counters, find the probabilities that a particular counter is:

(a) Pr(red and 5)

(b) Pr(blue or an even number)

(c) Pr(orange or 3 but not both).

Problem solving

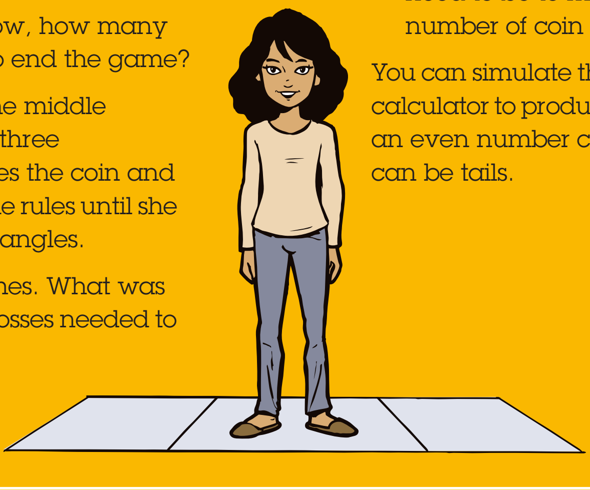
Random walks

Emily is playing a game. She stands in the middle of a strip of rectangles (like a footpath) and tosses a coin for each move. If the coin shows heads, she moves one rectangle right. If the coin shows tails, she moves one rectangle left. This is called a random walk.

In the problems that follow, how many coin tosses are needed to end the game?

1 Emily is standing in the middle rectangle of a strip of three rectangles. Emily tosses the coin and moves according to the rules until she leaves the strip of rectangles.

Play this game ten times. What was the mean number of tosses needed to end the game?



2 Emily is now standing in the middle rectangle of a strip of five rectangles. The rules are the same as before. What is the potential problem this time?

How many rectangles do you think there would need to be to make it too difficult to predict the number of coin tosses required?

You can simulate the tossing of the coin by using your calculator to produce random numbers. For example, an even number can be heads and an odd number can be tails.

Strategy options

- Make a table.
- Act it out.

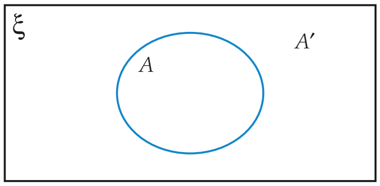
Venn diagrams and sample space

9.7

Sometimes, you want to know the probabilities involved when two different events, such as tossing a coin and rolling a die, occur within the same experiment. You might want to know the probability of both events occurring, of one or the other event occurring, of one event occurring given that the other event has occurred or the probability of neither event occurring. In these situations it is useful to use tables or **Venn diagrams** to help understand the combined sample space and to help calculate the probabilities.

Venn diagrams

- A Venn diagram is usually represented by a rectangle that contains all the possible outcomes, the **universal set**. The universal set is usually labelled with the symbol ξ, ε, $\mathcal{E}$ or U.
- The outcomes of an event are contained in a circled region with a letter name, such as A.



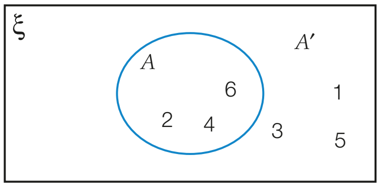
- A', the complement of A, lies outside this region. This is the same as saying '*not A*'.

Consider rolling a die.

Then, the universal set $\xi = \{1, 2, 3, 4, 5, 6\}$ and all the outcomes are written inside the rectangle.

If A represents the outcomes of rolling an even number, then $A = \{2, 4, 6\}$ and these outcomes are in the region labelled A.

A', the complement of A, would be rolling an odd number. (This is the same as saying not rolling an even number.) $A' = \{1, 3, 5\}$ and these outcomes would be outside A.



Finding probabilities with Venn diagrams

In probability, you are interested in the number of outcomes for an event. You can show this in a Venn diagram and write it as n(event). For example, $n(A)$ is the number of outcomes for event A.

$$\Pr(\text{event}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$\Pr(A) = \frac{\text{number of outcomes in set } A}{\text{number of outcomes in the universal set}}$$

$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

If you are counting the even numbers rolled when rolling a die, $n(A) = 3$.

$$\begin{aligned}\Pr(A) &= \frac{n(A)}{n(\xi)}\\ &= \frac{3}{6}\\ &= \frac{1}{2}\end{aligned}$$

Worked example 17

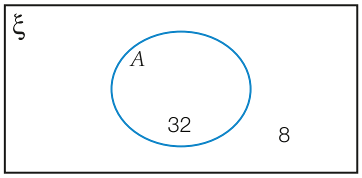
W.E. 17

Using this Venn diagram that shows the number of people in a group, find:

(a) $n(A)$ **(b)** $n(\text{not } A)$ **(c)** $n(\xi)$

If a person is chosen at random, find:

(d) $\Pr(A)$ **(e)** $\Pr(\text{not } A)$



Thinking	Working
(a) Write the number inside the region as the number in the required set (A).	(a) $n(A) = 32$
(b) Write the number outside the circle as the number in the complement of the required set (not A).	(b) $n(\text{not } A) = 8$
(c) Write the sum of these numbers as the number in the universal set (ξ).	(c) $n(\xi) = n(A) + n(\text{not } A)$ $= 32 + 8$ $= 40$
(d) Write the probability using: $\Pr(\text{event}) = \dfrac{n(\text{event})}{n(\xi)}$	(d) $\Pr(A) = \dfrac{n(A)}{n(\xi)}$ $= \dfrac{32}{40}$ $= \dfrac{4}{5}$
(e) Write the probability using: Pr(complement of the event) $= \dfrac{n(\text{complement})}{n(\xi)}$ or you can use $\Pr(\text{not } A) = 1 - \Pr(A)$	(e) $\Pr(\text{not } A) = \dfrac{n(\text{not } A)}{n(\xi)}$ or $\Pr(\text{not } A) = 1 - \Pr(A)$ $= \dfrac{8}{40}$ $= 1 - \dfrac{4}{5}$ $= \dfrac{1}{5}$ $= \dfrac{1}{5}$

Unions and intersections

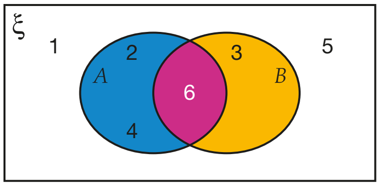
It is often useful to represent two different events using a Venn diagram. For example, the outcomes of a second event can be shown in a region labelled *B*.

Again consider rolling a die, where ξ = {1, 2, 3, 4, 5, 6}, the event *A* = {2, 4, 6}, and the event *B* = {multiples of 3}.

A and *B* will overlap, because they both contain the number 6.

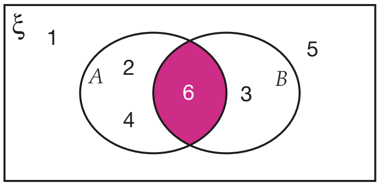
The **union** of *A* and *B* includes all the outcomes in *A or B*.

A or *B* = {2, 3, 4, 6}

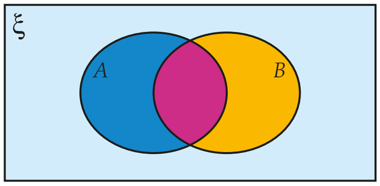


The **intersection** of *A* and *B* includes the outcomes that are in both *A and B*.

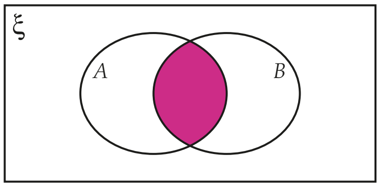
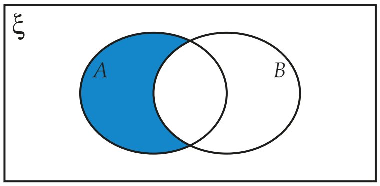
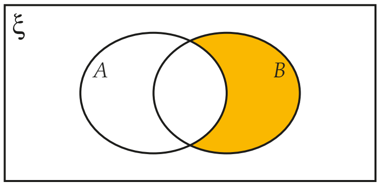
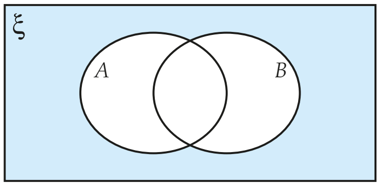
A and *B* = {6}



Consider the following diagram.



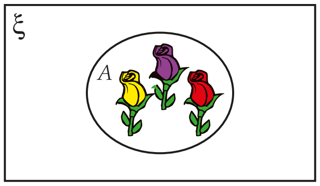
There are four distinct areas in this diagram.

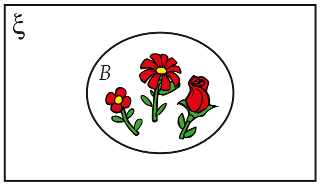
1 The area enclosed by the overlap of *A* and *B*: • is known as the intersection of *A and B* • represents the outcomes in which *A and B* both occur.	
2 The area enclosed by the remainder of *A*: • contains the outcomes in *A* not in the overlap with *B* • represents the outcomes in which *A* occurs but *B* does not.	
3 The area enclosed by the remainder of *B*: • contains the outcomes in *B* not in the overlap with *A* • represents the outcomes in which *B* occurs but *A* does not.	
4 The area outside both circles: • contains the outcomes in the universal set not in either *A* or *B* • represents the outcomes in which neither *A* nor *B* occurs.	

Summary of probability notation

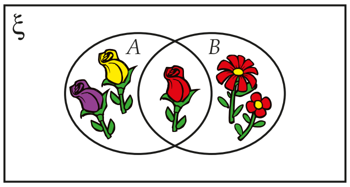
- ξ: universal set, all possible outcomes.
- Intersection of sets A and B: contains outcomes that are common to A and B.
- Union of sets A and B: contains outcomes that are in A or B (some outcomes belong to both A and B).
- A': complement of A, also written as 'not A'
- $n(\xi)$: number of elements in the universal set
- $\Pr(A) = \dfrac{n(A)}{n(\xi)}$

For example, if ξ = {flowers}, A = {roses}, and B = {red flowers}, then

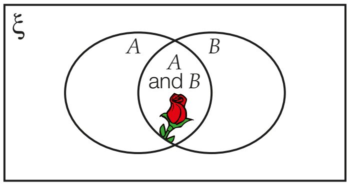




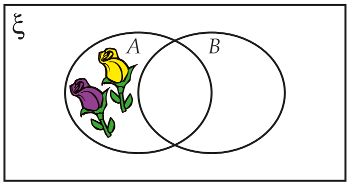
A or B = {roses of any colour *or* any red flowers}



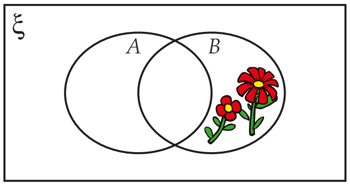
A and B = {flowers that are roses *and* that are red}
= {red roses}



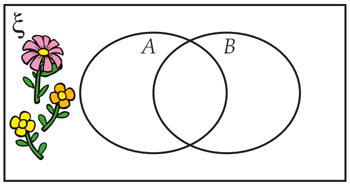
A and not B = {flowers that are roses *and* are not red}
= {roses of any colour except red}



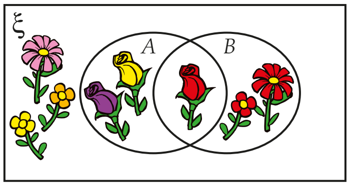
not A and B = {flowers that are not roses *and* that are red}
= {red flowers that are not roses}



not A and not B = {flowers that are not roses *and* are not red}
= {any flower that is not red or a rose}



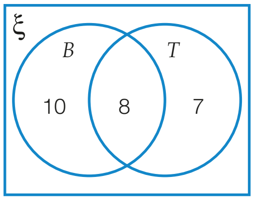
ξ = the universal set
= (A and B) + (A and not B) + (B and not A) + (not A and not B)



Worked example 18

W.E. 18

In a Year 8 class of 25 students, all of whom enjoy watching either basketball or tennis or both basketball and tennis, 18 enjoy watching basketball and 15 enjoy watching tennis. This information is shown in the Venn diagram at right. If a student is chosen at random, find the probability that they:



(a) enjoy watching both basketball and tennis

(b) only enjoy watching tennis.

Thinking	Working
1 Define the sets you will be using.	Let B = {students who enjoy watching basketball} T = {students who enjoy watching tennis} ξ = {Year 8 class}
2 Write the number in each of the sets defined, including the universal set and the intersection.	$n(B) = 18$, $n(T) = 15$, $n(\xi) = 25$, $n(B \text{ and } T) = 8$
(a) Find the probability of the intersection by dividing the number in the intersection by the total number.	(a) Pr(student enjoys basketball and tennis) $= \Pr(B \text{ and } T)$ $= \dfrac{n(B \text{ and } T)}{n(\xi)}$ $= \dfrac{8}{25}$
(b) Find the probability of the set required by dividing the number in the set by the number in the universal set.	(b) Pr(student enjoys tennis only) $= \Pr(T \text{ only})$ $= \dfrac{n(T \text{ only})}{n(\xi)}$ $= \dfrac{7}{25}$

Tables and sample space

You can use a table to help work out the combined sample space and the probability of an outcome from combined events. For example, all the outcomes from rolling two dice can be listed as shown.

Die 1

Die 2

	1	2	3	4	5	6
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)

There are a total of 36 different possible outcomes, so $n(\xi) = 36$.

You can find probabilities by looking at the table and counting the successful outcomes.

To find Pr(rolling at least one 6), count the number of outcomes that contain at least one 6 and divide by the total $n(\xi)$. Therefore, Pr(rolling at least one 6) = $\frac{11}{36}$.

Worked example 19

W.E. 19

A coin is tossed and a die is rolled. Use a table to list all the possible outcomes.

From the table, find the probability of the following:

(a) heads

(b) heads and a multiple of 2

(c) tails and a number greater than 4.

Thinking	Working
(a) 1 Construct a table.	(a)

coin

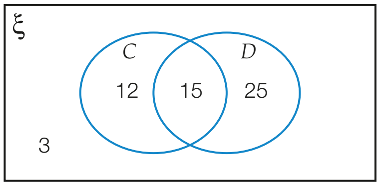
die		H	T
	6	(H, 6)	(T, 6)
	5	(H, 5)	(T, 5)
die	4	(H, 4)	(T, 4)
	3	(H, 3)	(T, 3)
	2	(H, 2)	(T, 2)
	1	(H, 1)	(T, 1)

Thinking	Working
2 Count the total number of outcomes.	$n(\xi) = 12$
Count the number of successful outcomes.	$n(H) = 6$
Write the probability.	$\Pr(H) = \frac{6}{12}$ $= \frac{1}{2}$
(b) Count the number of successful outcomes using the table. (In this case the successful outcomes are: (H, 2), (H, 4) and (H, 6).)	(b) $n(\text{H and multiple of 2}) = 3$
Write the probability.	$\Pr(\text{H and multiple of 2}) = \frac{3}{12}$ $= \frac{1}{4}$
(c) Count the number of successful outcomes using the table. (In this case the successful outcomes are (T, 5) and (T, 6).)	(c) $n(\text{T and number greater than 4}) = 2$
Write the probability.	$\Pr(\text{T and number greater than 4}) = \frac{2}{12}$ $= \frac{1}{6}$

Worked example 20

W.E. 20

Some students were asked their preferences regarding cats (*C*) and dogs (*D*). The results are shown in the Venn diagram.



If one person is chosen at random from those surveyed, find:

(a) Pr(they like cats)

(b) Pr(they like cats but not dogs)

(c) Pr(they don't like cats or dogs)

Thinking	Working
(a) Count the total number in the universal set ξ (all the students surveyed).	(a) $n(\xi) = 12 + 15 + 25 + 3$ $= 55$
Count the number in *C* (all the students who like cats).	$n(C) = 12 + 15$ $= 27$
Find the probability using: $\text{Pr(event)} = \dfrac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	$\Pr(C) = \dfrac{27}{55}$
(b) State the total number in ξ.	(b) $n(\xi) = 55$
State the number in *C* but not *D* (all students who like cats but don't like dogs).	$n(C \text{ and not } D) = 12$
Find the probability.	$\Pr(C \text{ and not } D) = \dfrac{12}{55}$
(c) State the total number in ξ.	(c) $n(\xi) = 55$
State the number outside both *C* and *D* (students who don't like cats and who also don't like dogs).	$n(\text{not } C \text{ and not } D) = 3$
Find the probability.	$\Pr(\text{not } C \text{ and not } D) = \dfrac{3}{55}$

Two-way tables

Two-way tables can help to organise a set of data that is categorised in two different ways. Like Venn diagrams, these tables can help you to calculate the probabilities.

Worked example 21

W.E. 21

A school has 280 junior students and 320 senior students. Of the 400 students who play sport, 240 are senior students.

(a) Construct a two-way table showing this information.

(b) Use the table to answer the following questions.

What is the probability that:

- **(i)** a student selected at random is a junior student
- **(ii)** a student selected at random plays sport
- **(iii)** a junior student who plays sport is selected
- **(iv)** a junior student is selected given that they play sport
- **(v)** a student who does not play sport is selected, given that the student is a senior student?

Thinking

(a) **1** Set up a two-way table with one variable and its complement as the row headings ('Plays sport' and 'Does not play sport'), and the other variable and its complement as the column headings ('Senior students' and 'Junior students').

2 Use the information given to fill in the table. (240 in 'Plays sport' row and 'Senior students' column). Subtract this number from 400 to find the number of junior students who play sport. Subtract 240 from 320 to find the number of senior students who don't play sport. Add the number of senior students and junior students to find the total number of students. Subtract 400 from this total to find the number of students who don't play sport and finally fill in the last cell.

Working

(a)

	Senior students	Junior students	Total
Plays sport	240	160	400
Does not play sport	80	120	200
Total	320	280	600

(b) **(i)** Use the number of successful outcomes and the total number in ξ to find the probability (the number of junior students and the total number of students).

(b) (i) Pr(junior student) $= \frac{280}{600}$

$= \frac{7}{15}$

(ii) Use the number of successful outcomes and the total number in ξ to find the probability (the number of students who play sport and the total number of students).

(ii) Pr(student who plays school sport)

$= \frac{400}{600}$

$= \frac{2}{3}$

(iii) Use the number of successful outcomes and the total number in ξ to find the probability (the number of junior students who play sport and the total number of students).	(iii) Pr(junior student who plays sport) $= \frac{160}{600}$ $= \frac{4}{15}$
(iv) Use the number of successful outcomes and the total number in the restricted universe to find the probability (the number of junior students who play sport and the total number of students who play sport).	(iv) Pr(junior student given they play sport) $= \frac{160}{400}$ $= \frac{2}{5}$
(v) Use the number of successful outcomes and the total number in the restricted universe to find the probability (the number of senior students who don't play sport and the total number of senior students).	(v) Pr(does not play sport, given that the student is a senior student) $= \frac{80}{320}$ $= \frac{1}{4}$

9.7 Venn diagrams and sample space

Navigator

1, 2, 3, 4, 5, 7, 11 (a–c), 12, 17	1, 2, 3, 4, 5, 6, 7, 8, 9, 11 (a–c), 12, 13, 14, 16, 17, 18	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18

Answers p. 679

Fluency

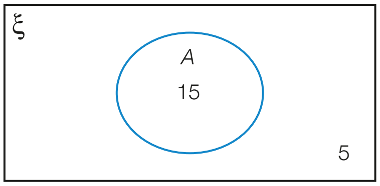
W.E. 17

1 For this Venn diagram that shows the number of people in a group, write:

(a) $n(A)$ **(b)** $n(\text{not } A)$ **(c)** $n(\xi)$

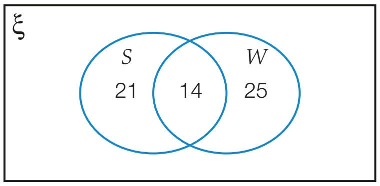
If a person is chosen at random, find:

(d) $\Pr(A)$ **(e)** $\Pr(\text{not } A)$



W.E. 18

2 In a survey of 60 music students at a school, all of them play either a string instrument, a wind instrument or both. Of these, 35 play a string instrument and 39 play a wind instrument. This information is shown in the Venn diagram at right. If a student is chosen at random, find the probability that they:



(a) play both a string instrument and a wind instrument

(b) only play a wind instrument.

W.E. 19

3 A coin is tossed and a die is rolled. Use a table to list all the possible outcomes. From the table, find the probability of:

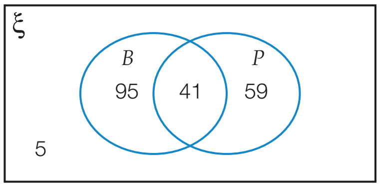
(a) tails

(b) tails and a multiple of 3

(c) heads and a number less than 4.

W.E. 20

4 A sample of 200 people was surveyed as to their access to broadband internet (B) and pay TV (P). The results are shown in the Venn diagram.



If one person is chosen at random from those surveyed, find:

(a) Pr(they have access to broadband internet)

(b) Pr(they have access to pay TV but not broadband internet)

(c) Pr(they have access to neither of these technologies).

W.E. 21

5 A mouse breeder has 18 white mice and 12 brown mice. Of the 14 male mice, 10 are white.

(a) Construct a two-way table showing this information.

(b) Use your table to calculate the probability that:

- **(i)** a mouse selected at random is white
- **(ii)** a mouse selected at random is male
- **(iii)** a mouse selected at random is brown and female.

6 A die is rolled. $\xi = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$ and $B = \{1, 2\}$

(a) Draw a Venn diagram to represent this information.

(b) Hence, find:

(i) A and B **(ii)** A or B **(iii)** not A

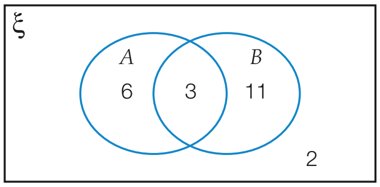
(iv) $n(A)$ **(v)** $n(B)$ **(vi)** $n(\xi)$

(vii) $n(A$ and $B)$ **(viii)** $n(A$ or $B)$ **(ix)** $n($not $B)$

(c) If a number is chosen at random from ξ, find the probability that it came from:

(i) A **(ii)** A and B **(iii)** A or B

7 This Venn diagram shows the number of elements in each of two sets A and B.



(a) $n(A)$ is:

A 3 **B** 6

C 9 **D** 22

(b) $n(B$ only) is:

A 3 **B** 9 **C** 11 **D** 14

(c) $n(A$ or $B)$ is:

A 3 **B** 17 **C** 20 **D** 22

(d) $n(A$ and $B)$ is:

A 2 **B** 3 **C** 9 **D** 20

(e) $n(\xi)$ is:

A 22 **B** 20 **C** 17 **D** 9

(f) $n($not $B)$ is:

A 2 **B** 6 **C** 8 **D** 9

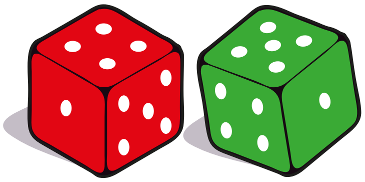
8 Two standard six-sided dice, one green and one red, are rolled. Construct a table and use it to write the outcomes that match the following descriptions.

(a) a 6 on either of the dice

(b) a 6 on both dice

(c) a 6 on the green die

(d) a 6 on the green die but not the red die



9 Two dice, one green and one red, are rolled. Use your table and answers from the previous question to find the probability of getting:

(a) a 6 on either of the dice

(b) a 6 on both dice

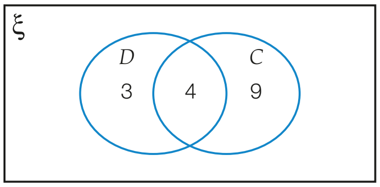
(c) a 6 on the green die

(d) a 6 on the green die but not on the red die

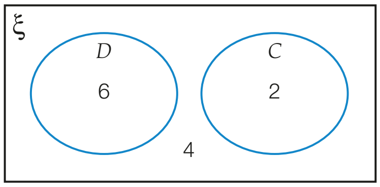
Understanding

10 Sally decided to survey some Year 8 students about whether they owned a dog, a cat, both or neither. She conducted two surveys of two separate groups of students and recorded her results on the following Venn diagrams where D = {Year 8 students who own a dog} and C = {Year 8 students who own a cat}.

(a)



(b)



For each diagram:

(i) set up a table showing the information as probabilities.

Hence, for each diagram, find:

(ii) Pr(D)
(iii) Pr(D and C)
(iv) Pr(D or C)
(v) Pr(C only)
(vi) Pr(not D)
(vii) Pr(not C)

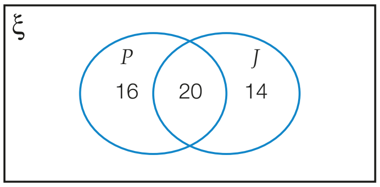
11 Two standard six-sided dice, one red and one green, are rolled. Find the probability of getting:

(a) a sum of 7

(b) a 3 on the green die

(c) even numbers on both dice.

12 In a survey of 50 people, it was found that 36 people liked peanut butter and 34 people liked jam. Of all those surveyed, 20 said they liked both peanut butter and jam. This information is shown on a Venn diagram where
P = {people who liked peanut butter} and
J = {people who liked jam}.



If one person is chosen at random, find:

(a) Pr(P)
(b) Pr(P and J)
(c) Pr(J)
(d) Pr(J only)

Reasoning

13 A shopkeeper found that, out of the shop's first 20 customers, 13 bought milk and 15 bought a newspaper. Four customers bought neither milk nor a newspaper.

- **(a)** Represent this information in a table with headings 'Bought milk', 'Didn't buy milk', etc.
- **(b)** How many customers bought both milk and a newspaper?
- **(c)** Find the probability that a customer, chosen at random from the first 20:
  - **(i)** bought milk only
  - **(ii)** bought milk and a newspaper
  - **(iii)** bought milk or a newspaper
  - **(iv)** did not buy a newspaper.

14 A group of 40 Year 8 students were asked if they preferred reading text on screen or on paper. Ten students said they liked reading both, seven said they liked reading on screen only, while two said they did not like reading either.

Find the probability that a student chosen at random from this group:

- **(a)** liked reading on paper only
- **(b)** liked reading on screen or on paper
- **(c)** did not like reading on paper
- **(d)** did not like reading on screen or on paper.

15 At a school camp, 50 Year 8 students were surveyed about whether they preferred pizza or fish and chips.

Ten students said they liked both and four students said they liked neither. The number of students who liked pizza was the same as the number who liked fish and chips. Find the probability that a student, selected at random:

- **(a)** likes both pizza and fish and chips
- **(b)** likes pizza only
- **(c)** does not like fish and chips.

16 In a Year 8 class of 25 students, 15 said they liked athletics, 17 said they liked swimming and 4 said they didn't like either. Find the probability that a student chosen at random:

- **(a)** likes swimming
- **(b)** likes athletics only
- **(c)** likes swimming or athletics.

Open-ended

17 Survey a number of students about whether they travel to school by bus, train, both or neither. Construct a Venn diagram with this information and use it to find the probability that a student chosen at random travels by bus only.

18 Robbie was working out the probability of getting a 3 on at least one die when rolling two dice. He decided that there were 12 successful outcomes out of a possible 36: the first die could be 3 while the second die could be anything (6 outcomes), and also the second die could be 3 while the first die could be anything (another 6 outcomes).
He wrote:

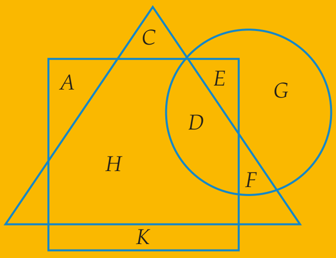
$$\begin{aligned}\text{Pr(3 on one die)} &= \frac{12}{36}\\ &= \frac{1}{3}\end{aligned}$$

Show Robbie why he is wrong and find the correct answer.

Problem solving

Inventive Venn

All questions relate to the following diagram.



1 Which letter or letters appear in the intersection of all three shapes?

2 Which letter or letters appear in the triangle and the square but not in the circle?

3 Which letter or letters appear in the triangle only?

4 Which letter or letters appear in either the square or the triangle?

5 Which letter or letters appear in neither the circle nor the triangle?

6 How would you describe the location of *F*?

7 Make up another three similar questions based on the diagram.

Strategy options

- Act it out.
- Break problem into manageable parts.

Challenge 9

1 Georgina has green socks, red socks and blue socks that she keeps loose at the top of her wardrobe. She can't see the socks when she grabs them.

(a) How many socks does she need to select before she has a pair?

(b) How many more socks must she select to be sure she has a second pair?

2 Two children and one adult have to cross a river in a boat that can only hold one adult or two children at a time. How can they do this?

3 Three playing cards are placed in a row. The club is to the right of the heart and the diamond. The 5 is to the left of the heart. The eight is to the right of the 4. From left to right, the cards are:

A 5 of diamonds, 4 of hearts, 8 of clubs

B 5 of hearts, 4 of diamonds, 8 of clubs

C 8 of diamonds, 5 of clubs, 4 of hearts

D 4 of hearts, 8 of clubs, 5 of diamonds

4 **(a)** At a local fundraiser, a ring toss game is played. Three rings are tossed over any of three pegs. A ring over peg X is worth *one* point, over peg Y *three* points and over peg Z *five* points. If all three rings land on pegs, how many different point totals are possible?

A 6 **B** 7 **C** 8 **D** 9

(b) Which total(s) were the most common?

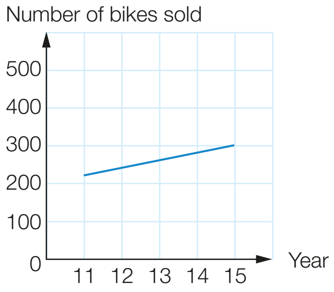
(c) How is it possible to get an even total?

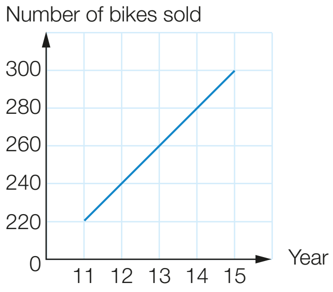
5 For the set of numbers {3, 4, 5, 8, x}, the mean, median and the mode are all the same number.

(a) Considering the mode is known, what are the only possible values of x?

(b) What is the value of x?

6 Margaret claims the two graphs below represent the same information. Do you agree? Give reasons for your answer.





Chapter review 9

Maths literacy

- bias
- census
- class centre
- class intervals
- complement
- frequency column graphs
- grouped data
- histogram
- intersection
- mean
- measures of centre
- median
- median class interval
- midpoint
- modal class interval
- mode
- outcome
- outlier
- poll
- population
- population statistics
- random sampling
- range
- sample
- sample space
- sample statistics
- simple random sampling
- two-way table
- union
- universal set
- Venn diagram

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 When dealing with a large data set you can group the data into __________.

2 To find the __________ of a class interval you add together the two end points and divide by two.

3 The difference between the largest value and the smallest value is called the __________.

4 The __________ is the list of outcomes possible in a mathematical experiment.

5 When you find the elements in common between two sets you are finding the __________.

6 The set that contains all the elements in the universal set not included in *A* is called the __________ of *A*.

7 A data value that is quite different from other values is called an __________.

8 When data is collected from all members of a population it is called a __________.

Fluency

1 A sample of 50 parrots are captured, tagged and released. Some time later, a second sample of 75 parrots are captured, of which 10 are found to have tags. Use these values to estimate the parrot population in the area. **9.1**

2 For each of the following data sets, calculate the: **9.2**

- **(i)** mean (correct to 2 decimal places)
- **(ii)** median
- **(iii)** range

(a) 12 23 23 25 26 27 28 29 29 31 32 33 35 35 36 38

(b) 1 23 23 25 26 27 28 29 29 31 32 33 35 35 36 38

(c) Comment on any similarities and differences you found.

3 A class of Year 8 students spend time each week collecting insects from their backyards. The following data shows the number of insects of different sizes collected over a year.

9.3

Insect size (length in cm)	Insects collected
1–<2	1044
2–<3	986
3–<4	569
4–<5	1012
5–<6	758
6–<7	999

Select a suitable scale and draw a histogram of the results.

4 For the following data, select a suitable size for the class interval, record the results in a frequency table, and draw a frequency column graph.

9.3

4, 6, 10, 5, 4, 8, 12, 14, 18, 6, 4, 6, 13, 2, 16, 8, 6, 4, 10, 14, 6, 4, 7, 8, 9, 12

5 The data in the following frequency table gives the weight, in grams, of fish of a particular species caught one day by a commercial fisher.

9.4

Weight (g)	Frequency
325–<350	5
350–<375	7
375–<400	11
400–<425	24
425–<450	31
450–<475	29

(a) Calculate the mean weight of the fish correct to 2 decimal places.

(b) Calculate the median class interval for the weights of the fish.

(c) Calculate the modal class interval for the weights of the fish.

(d) Draw a histogram of the data.

6 A 10-sided die with the numbers 1 to 10 is rolled once. Find the probability of rolling:

9.5

(a) a 5

(b) a 6 or a 7 or an 8

(c) an 11

(d) a prime number

(e) a number greater than 3

(f) a number that is not a multiple of 3.

7 Find, for one spin of the spinner shown:

9.6

(a) Pr(blue)

(b) Pr(red)

(c) Pr(blue or red)

(d) Pr(not red).

8 For rolling a standard six-sided die, find the following probabilities. 9.6

(a) Pr(rolling a 5)

A $\frac{1}{6}$ **B** $\frac{1}{5}$ **C** $\frac{5}{6}$ **D** 1

(b) Pr(rolling a 4 or 6)

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{4}{5}$ **D** 1

(c) Pr(rolling a number that is not 4 or 6)

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

9 Draw and label a spinner that would produce the following probabilities. 9.6

Pr(red) = $\frac{1}{3}$ Pr(blue) = $\frac{1}{2}$ Pr(yellow) = $\frac{1}{6}$

10 $\xi = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$ and $B = \{1, 2\}$

(a) Draw a Venn diagram to represent this information.

(b) Hence, find:

(i) *A* or *B* **(ii)** *A* and *B* **(iii)** not *A*

(c) Find:

(i) Pr(*A* or *B*) **(ii)** Pr(*A* and *B*) **(iii)** Pr(not *A*)

Understanding

11 The Australian cricket team toured New Zealand in March 2010 and played two Test matches. The following is a list of the scores made by each individual New Zealand cricketer during the first innings of the first Test match. 9.2

4 46 11 138 4 15 5 7 22 7 0

(a) Calculate the mean score correct to 2 decimal places.

(b) Calculate the median score.

(c) Calculate the range.

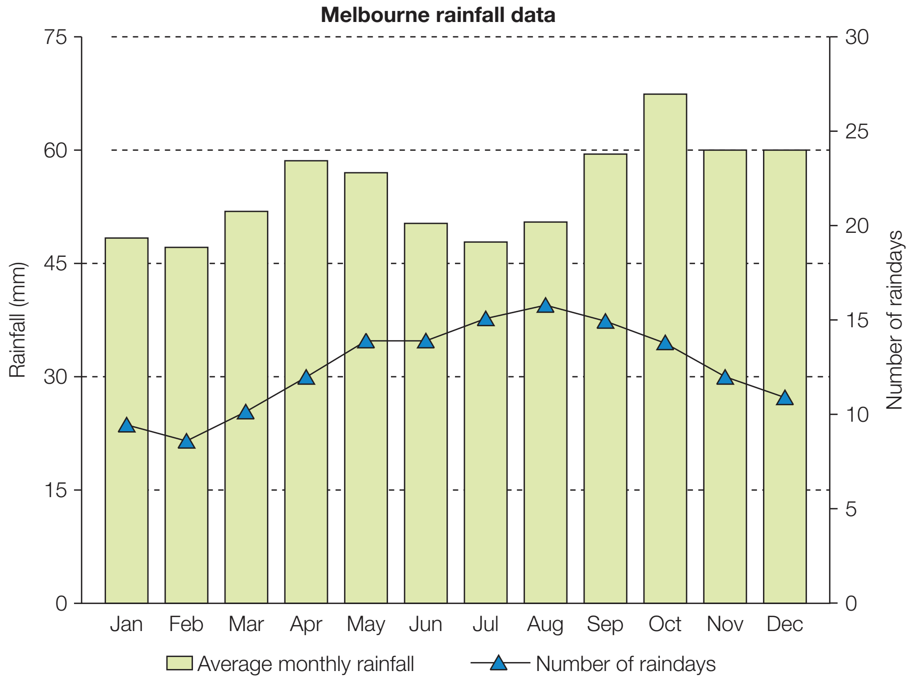
(d) Is there a possible outlier?

In the second innings of the same Test match the New Zealand cricketers scored:

24 19 29 22 58 22 51 3 45 0 5

(e) Describe these scores using the mean, median and range as you did for the first innings.

12 The following graph shows rainfall data for the Melbourne weather station. **9.3**



Bureau of Meteorology

(a) Why does the graph have two different vertical scales marked on it?

(b) Which month records the highest mean rainfall?

(c) Which month has the most raindays?

(d) **(i)** Which month would you say produces, on average, the most rain on the days when it actually rains?

(ii) Which month would you say produces, on average, the least rain on the days when it actually rains?

13 A motor company launches a new car, the Merlin. The car is available as either a sedan, a station wagon, a coupe or a ute. It is available in white, black, red, yellow or silver. **9.5**

(a) How many different variations of the Merlin are available?

(b) If Anissa buys a Merlin at random, find:

(i) Pr(it is a yellow coupe)

(ii) Pr(it is not black)

(iii) Pr(it is red or a ute or both).

14 In a survey of 60 people, it was found that 40 people liked lemonade and 30 people liked cola. Two people said they disliked both. **9.7**

(a) Present this information as a Venn diagram.

(b) If one person is chosen at random, find the probability that this person:

(i) likes lemonade

(ii) likes lemonade only

(iii) likes cola

(iv) likes both lemonade and cola

(v) does not like lemonade

(vi) likes lemonade or cola.

15 For a 10-sided die, describe how you would colour the faces so that:

9.5, 9.6

(a) Pr(blue) = Pr(green) = 2 × Pr(red) where red, green and blue are the only available colours

(b) Pr(blue) = 5 × Pr(black) where blue, black and pink are the only available colours.

16 A string orchestra of 20 musicians has 12 members who play the violin. There are 16 members who are right-handed and 2 left-handers play the violin. If a member of the orchestra is chosen at random, find the probability that he or she:

9.5, 9.6

(a) is right-handed and plays the violin

(b) is left-handed and doesn't play the violin.

Reasoning

17 The following data values are the height (in cm) of the 46 players on the list of the Fremantle Dockers at the beginning of an AFL season.

9.2, 9.3, 9.4

174	191	176	182	188	185	189	180	190
182	186	181	186	189	190	192	194	193
186	193	194	188	191	191	199	184	193
192	203	211	184	188	181	180	194	177
187	182	184	191	191	192	189	194	184
186								

(a) Using the raw data calculate the:

(i) mean height, correct to 2 decimal places

(ii median height

(iii) modal height.

(b) Which of these do you think is the best measure of centre for height?

(c) Do you think any of the values represents a possible outlier? If so, which one(s). If not, what would the height need to be for you to consider it a possible outlier?

(d) Group the data in intervals beginning 171–175, 176–180 and so on. Use the table to find an estimate for the mean height to the nearest cm.

(e) It should have been easy to do the calculation from the table, but was there a loss of accuracy? Explain.

Numeracy practice 9

Non-calculator

Consider the following data set: 2, 3, 4, 2, 4, 6, 1, 5, 2, 9, 7

1 The median is:

A 2 **B** 4 **C** 4.5 **D** 6

2 The range is:

A 2 **B** 5 **C** 8 **D** 11

3 The mode is:

A 2 **B** 4 **C** 5 **D** 8

4 At an athletics meet, Julie recorded the heights jumped by 30 students in the high jump event. They are recorded below.

Stem	Leaf
15_L	1 2 2 2 4 4
15_U	5 8 9 9
16_L	0 0 1 2 2 4 4
16_U	5 5 5 6 7 7 8 8 9 9
17_L	0 1 2

Key: 17 | 2 = 172

What is the median height jumped?

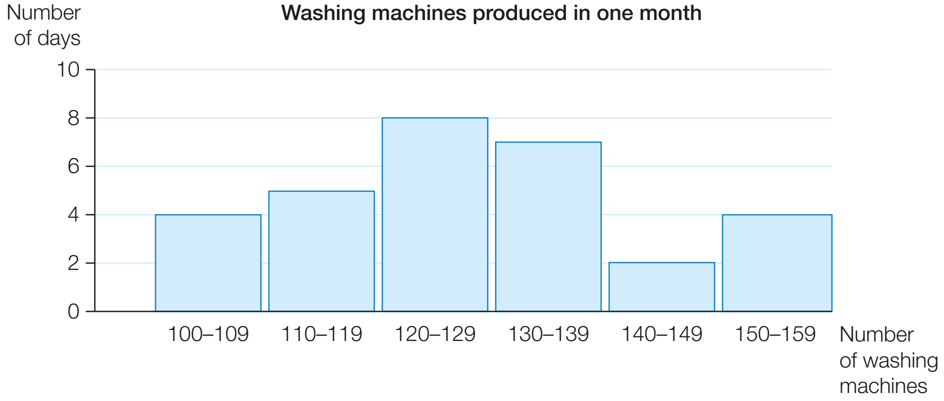
A 163 cm **B** 164 cm **C** 165 cm **D** 166 cm

Calculator allowed

5 Find an estimate for the mean of the following data set. Give your answer correct to 2 decimal places.

Score	Frequency
1–<2	4
2–<3	7
3–<4	11
4–<5	6
5–<6	9
6–<7	2

6 This graph shows data on how many washing machines are produced each day by a factory in one month.

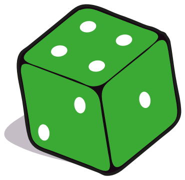


On how many days did the factory produce more than 129 washing machines?

A 8 **B** 9 **C** 13 **D** 21

7 A standard die has 6 faces that are numbered from 1 to 6.
Nicky rolls the die 1000 times to test whether it is fair.

Number on top face	1	2	3	4	5	6
Frequency	150	166	194	148	160	182



What is the probability of rolling a 5 with this die, based on Nicky's results?

A $\frac{1}{160}$ **B** $\frac{4}{25}$ **C** $\frac{1}{6}$ **D** $\frac{16}{25}$

Mixed review D

Fluency

1 Change the following percentages to fractions in simplest form. **2.6**

(a) $16\frac{3}{8}\%$ **(b)** 31.55%

2 A card is taken from a standard pack of cards. What is the probability of: **9.5**

(a) not getting the 2 of hearts **(b)** not getting a black card

(c) not getting a Queen **(d)** not getting a diamond?

3 Evaluate each of the following expressions when $a = -2$ and $b = 5$. **3.2**

(a) $ab + a$ **(b)** $6b - \frac{80}{ab}$ **(c)** $5(a - b) + 7b$

4 Change the following fractions to percentages. **(a)** $\frac{31}{60}$ **(b)** $1\frac{5}{6}$ **2.5**

5 Increase 850 by the following percentages and give your answers to 2 decimal places. **2.9**

(a) 13% **(b)** 8% **(c)** 56.5%

6 Solve each of the following equations. **7.2**

(a) $5g - 6 = 4$ **(b)** $3(m + 7) = 27$ **(c)** $\frac{s}{9} + 2 = 7$

7 On a particular day, 1 Australian dollar (A\$) was worth 0.55 United States dollars (US\$). **6.4**

(a) Construct a set of axes showing A\$ on the horizontal axis and US\$ on the vertical axis. Extend the horizontal axis to \$100 and the vertical axis to \$60.

(b) Zero A\$ must always be worth zero US\$. What is this as an ordered pair of coordinates?

(c) How much would A\$10 be worth? Write this as an ordered pair of coordinates.

(d) Plot the two coordinate pairs on the graph and draw a straight line to connect them, extending the line to fit the axes.

(e) Use the graph to find how much A\$45 is worth in US\$.

(f) Use the graph to find how much US\$50 is worth in A\$.

8 Find the probability of each of the following events occurring. **9.5**

(a) rolling a standard six-sided die and getting an odd number

(b) picking a spade out of a standard pack of 52 playing cards

(c) picking a red or white marble out of a bag containing 3 red, 5 white, 2 green and 6 orange marbles

9 Joanna and Stephen went bushwalking and travelled 7.5 km in the first 3 hours. **4.7**

(a) What was their average speed in the first 3 hours?

(b) If they continued at this speed for 4.5 more hours, what distance would they have travelled in the day?

10 Simplify the following: **(a)** $\frac{2^4 \times 3^5}{(2 \times 3)^3}$ **(b)** $\left(\frac{2^2}{3}\right)^5$ **1.5, 1.6**

11 Simplify the following: **(a)** $(4^2)^3 \times 5^0$ **(b)** $\frac{(6 \times 8)^3}{6^2 \times 8}$ **1.5, 1.6**

12 Simplify the following expressions by expanding the brackets and collecting like terms. **3.6**

(a) $5(2d + e) - 7d$ **(b)** $4j(3k + 2) + 5(jk - 7)$

13 Draw a frequency column graph or histogram for each of the following data sets. **9.3**

(a)

Score	Frequency
1	5
2	7
3	8
4	6
5	2

(b)

Values	Frequency
15–<20	3
20–<25	5
25–<30	8
30–<35	2
35–<40	1

14 Solve for x: **(a)** $\frac{3x + 2}{5} = -11$ **(b)** $8(3x - 2) = 32$ **7.3**

15 A standard six-sided die is rolled. **9.6**

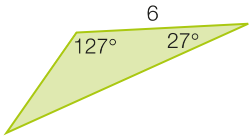
(a) List all possible outcomes.

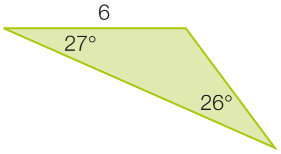
(b) Find: **(i)** Pr(4) **(ii)** Pr(12) **(iii)** Pr(2 or 5) **(iv)** Pr(0)

16 If $A = \frac{1}{2}(a + b)h$, find: **3.2**

(a) A when $a = 2$, $b = 6$ and $h = 10$ **(b)** h when $A = 40$, $a = 2$ and $b = 8$.

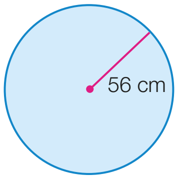
17 Show that these shapes are congruent and state the congruence test used. **8.4**



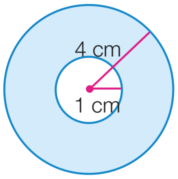


18 Find the shaded area of the following shapes, correct to 2 decimal places. **5.5, 5.6**

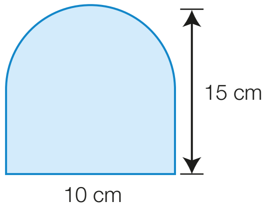
(a)



(b)



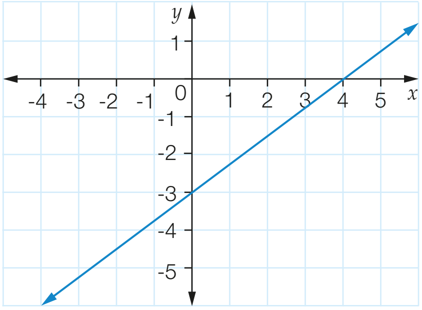
(c)



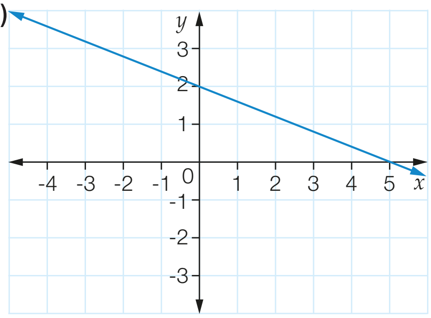
19 Solve for x: **(a)** $8x + 5 = 2x - 13$ **(b)** $\frac{x}{4} + 1 = \frac{x}{2} - 1$ **7.4**

20 Find the rule that describes each of the following relationships. **6.3**

(a)



(b)



Understanding

21 You have a bag of 20 coloured counters. Given that Pr(yellow) = 0.2, Pr(green) = 0.1, Pr(red) = 0.4 and Pr(brown) = 0.3, how many of each coloured counter do you have? **9.5**

22 A car is travelling at an average speed of 90 km/h. **6.4**

(a) Write an equation showing the relationship between distance travelled, d km, and time, t hours.

(b) Draw a graph showing 3 hours of travel for the car.

(c) Use your graph to estimate the distance travelled in:

(i) 45 minutes **(ii)** 2 hours 20 minutes.

(d) Use your graph to estimate the time it takes to travel:

(i) 70 km **(ii)** 250 km.

23 Write equations to help you solve the following. **7.2, 7.5**

(a) Six less than a third of a number is 27. What is the number?

(b) A number is doubled, then divided by six and finally has ten added to it. The result is the number first thought of. Find the number.

24 Find the area of an annulus that has an inner diameter of 6 cm and an outer diameter of 13 cm. Give your answer correct to 1 decimal place. **5.5, 5.6**

25 In a sample of the staff at a particular firm, 78% said they were happy with the treatment they receive from their manager. **9.1**

(a) If the company has 500 staff, approximately how many would you expect to be happy with the treatment they receive?

(b) What would you think if a census of the population revealed that 405 staff were happy with the treatment they received?

26 Michelle's pay is increased by 40%. If her fortnightly pay was $1480 before, what is her income now? **2.9**

27 On one day Billy recorded that he slept for 8.4 hours, watched TV for 2.6 hours, was at school for 8.75 hours, played sport for 2.3 hours and spent the rest with his family. Write each of these as a percentage of a day, to the nearest whole number. **2.7**

28 Twenty counters labelled 11 to 30 are placed in a container. If a counter is drawn at random, find: **9.6**

(a) Pr(not 15) **(b)** Pr(not divisible by 5) **(c)** Pr(not less than 17).

Reasoning

29 The cost of hiring a bicycle is a $10 payment, plus $5 per half-hour. **6.1, 6.2**

(a) Copy and complete the table of costs for up to 8 hours of hire.

Time (h)	0	2	4	6	8
Cost ($)					

(b) Draw a graph of this relationship.

(c) What does the y-intercept represent?

(d) To buy a bike equivalent to the type hired costs $350. For how many hours could you hire the bike for the same cost as this purchase price?

Answers

Worked solutions and answers to *all* activities appear in the Teacher Companion.

Chapter 1

Recall 1

1 **(a)** **(i)** 9 °C **(ii)** 15 °C **(iii)** 6 °C
(b) **(i)** 3rd floor **(ii)** 1st floor

2 **(a)** 8 **(b)** 81 **(c)** 25 **(d)** 1 000 000
(e) 6 **(f)** 7 **(g)** 4 **(h)** 2

3 **(a)** +3 **(b)** -9

4 **(a)** 16 **(b)** 63 **(c)** 228 **(d)** 285
(e) 6 **(f)** 6 **(g)** 108 **(h)** 154

5 **(a)** 9 **(b)** 54 **(c)** 43

6 **(a)** $2^3 \times 3$ **(b)** $2^2 \times 3^2$ **(c)** $2^3 \times 3 \times 5$

7 **(a)** 25 **(b)** 29 **(c)** 28 **(d)** 72
(e) 400 **(f)** 945 **(g)** 9 **(h)** 14

Exercise 1.1

1 **(a)** +13 **(b)** -2 **(c)** +8 **(d)** -4
(e) -3 **(f)** +5 **(g)** +18 **(h)** -10
(i) -7 **(j)** -1 **(k)** -21 **(l)** -8
(m) +7 **(n)** -7 **(o)** -26 **(p)** +6

2 **(a)** -8 **(b)** 10 **(c)** -16 **(d)** 96 **(e)** 45
(f) -34 **(g)** -56 **(h)** 15 **(i)** 29

3 **(a)** +1 **(b)** -2 **(c)** +16 **(d)** -1
(e) +5 **(f)** -11 **(g)** -17 **(h)** +20
(i) +6 **(j)** +3 **(k)** +8 **(l)** -7
(m) -4 **(n)** -22 **(o)** -6 **(p)** -11

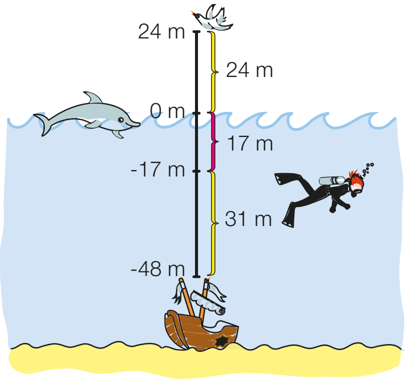
4 **(a)** +350 **(b)** -4800 **(c)** -6 **(d)** +73
(e) -2 **(f)** -19 **(g)** +10 750 **(h)** +34
(i) -200 **(j)** +8100

5 **(a)** $+2 < +6$ **(b)** $-3 < +1$ **(c)** $+5 > -4$ **(d)** $-1 > -3$
(e) $+5 > -10$ **(f)** $-6 > -8$ **(g)** $+16 > -16$ **(h)** $-33 < -12$

6 **(a)** +11, +4, 0, -2, -7 **(b)** +7, 1, 0, -9, -23
(c) 11, 4, 1, 0, -3, -15 **(d)** 19, 8, 6, -2, -5, -43
(e) 26, 14, 5, -1, -38, -72 **(f)** 32, 17, 4, 0, -19, -56

7 **(a)** 4 **(b)** 2 **(c)** -10 **(d)** 2
(e) 2 **(f)** -6 **(g)** 14 **(h)** -24

8 **(a)**



(b) **(i)** 41 m **(ii)** 48 m

9 **(a)** 200 – 110

(b)

Month	Money owing
Initially	$90
After 1 month	$75
After 2 months	$60
After 3 months	$45
After 4 months	$30
After 5 months	$15
After 6 months	$0

10 **(a)** Wednesday **(b)** Sunday
(c) **(i)** 11 °C **(ii)** 13 °C
(d) Wednesday

11 **(a)** -120 – 87 + 243 + 50 – 109
(b) **(i)** less **(ii)** $23 less

12 2411 m

13 **(a)** +5 **(b)** +8 **(c)** +17 **(d)** +15
(e) -1 **(f)** -11 **(g)** +14

14 **(a)** magic total = -9

-6	-1	-2
1	-3	-7
-4	-5	0

(b) magic total = -12

6	-4	4	-18
-16	2	-6	8
-14	0	-8	10
12	-10	-2	-12

15 **(a)** -13 **(b)** -23 **(c)** -25
(d) -60 **(e)** -111 **(f)** -133
(g) -109 **(h)** -45 **(i)** -127

16 **(a)** **(i)** 1 **(ii)** 6 **(iii)** 9 **(iv)** 13
(b) **(i)** $+3 - 2 = 1$ **(ii)** $+11 - 5 = 6$
(iii) $+17 - 8 = 9$ **(iv)** $+34 - 21 = 13$
(c) The answers to **(a)** and **(b)** are the same.
(d) commutative law
(e) **(i)** 11 **(ii)** 16 **(iii)** 27 **(iv)** 15

Open-ended – Sample answers

17 **(a)** 1 °C **(b)** 6 °C
18 +1, +5, -10
19 -18, -19, -20

Exercise 1.2

1 **(a)** 30 **(b)** 21 **(c)** 77 **(d)** 54
(e) -40 **(f)** -24 **(g)** -27 **(h)** -35
(i) -8 **(j)** -16 **(k)** -25 **(l)** -16
(m) 30 **(n)** 8 **(o)** 6 **(p)** 15
(q) -60 **(r)** -140 **(s)** -90 **(t)** 132

2 **(a)** 24 **(b)** 12 **(c)** 15 **(d)** -40
(e) -18 **(f)** -24 **(g)** -50 **(h)** -20
(i) -32 **(j)** -27 **(k)** -8 **(l)** -1000

3 **(a)** -3, -6, -9 **(b)** 0, 2, 4 **(c)** -2, -4, -6

4 **(a)** D **(b)** D

5 **(a)** +3 **(b)** -4 **(c)** -5 **(d)** -3
(e) +6 **(f)** +6 **(g)** +8 **(h)** -9

6 **(a)** 36 **(b)** -36 **(c)** 100 **(d)** -100
(e) 50 **(f)** 75 **(g)** -400 **(h)** -700
(i) 16 **(j)** -100 **(k)** 81 **(l)** -256

7 **(a)** -6 **(b)** 0 **(c)** 15 **(d)** -2 **(e)** -6 **(f)** -2

8 **(a)** **(i)** -\$7 **(ii)** -\$6
(b) -\$161 **(c)** -\$114 **(d)** -\$275

9 **(a)** -9 **(b)** -35 **(c)** -13 **(d)** 18
(e) 2 **(f)** -8 **(g)** -20 **(h)** -10

10 **(a)** **(i)** 1 **(ii)** -1 **(iii)** 1 **(iv)** -1 **(v)** 1 **(vi)** -1
(b) even, odd
(c) **(i)** 1 **(ii)** -1 **(iii)** -1 **(iv)** 1

11 **(a)** $-4 \times -4 \times -4 = -64$ **(b)** $-3 \times -3 \times -3 \times -3 = 81$
(c) $-2 \times -2 \times -2 \times -2 \times -2 = -32$ **(d)** $-(-6 \times -6) = -36$

12 **(a)** **(i)** 3 × -\$25 = -\$75 for gas
(ii) 3 × -\$35 = -\$105 for electricity
(b) **(i)** gas: \$93 – \$75 = \$18
electricity: \$102 – \$105 = -\$3

(ii) Hong’s direct debit of \$25 per month for gas has resulted in her account being underpaid by \$18. She should consider increasing the direct debit amount to \$30 per month, which would give a 3-monthly total very close to her current bill.

Hong’s direct debit of \$35 for electricity has resulted in her account being slightly overpaid by \$3. She doesn’t need to adjust it now, but should monitor it closely.

Open-ended – Sample answers

13 $3 \times 4 \times -2 = -24$; $8 \times 3 \times -1 = -24$; $-6 \times -2 \times -2 = -24$

14 3 ♥, 3 ♦, 1 ♣ $(3 \times -5 + 3 \times -3 + 2 = -22)$
4 ♥, 2 ♦, 1 ♠ $(4 \times -5 + 2 \times -3 + 4 = -22)$

Exercise 1.3

1 **(a)** -3 **(b)** -3 **(c)** -4 **(d)** -3
(e) -2 **(f)** -6 **(g)** -9 **(h)** -8
(i) 4 **(j)** 9 **(k)** 7 **(l)** 6
(m) 7 **(n)** -9 **(o)** -4 **(p)** -10
(q) -30 **(r)** -100 **(s)** -20 **(t)** 230

2 **(a)** -16 **(b)** -7 **(c)** -13 **(d)** -15
(e) -26 **(f)** 31 **(g)** 21 **(h)** -24
(i) 15 **(j)** -20 **(k)** -9 **(l)** 3

3 **(a)** D **(b)** B

4 **(a)** -5 **(b)** 7 **(c)** -11 **(d)** 2
(e) 4 **(f)** -3

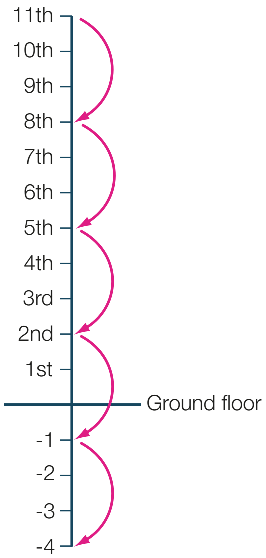
5 **(a)** negative **(b)** negative **(c)** negative

6 **(a)** -4 **(b)** -8 **(c)** -5 **(d)** -1
(e) -5 **(f)** 29

7 **(a)** -4 **(b)** 4 **(c)** 32

8 -3

9 **(a)**



(b) 5 stops from the top to the bottom level
(c) floors: 8, 5, 2, -1, -4

10 -$40

11 (a) 16 minutes (b) 48 minutes

12 -4 °C

13 (a) 43 months

(b) 3 years and 7 months

(c) -$6200

14 -72, -30, -24, -18, -15, -12, -9, -8, -5, -4, -3, -2, 1, 2, 3, 6, 8, 10, 12, 16, 18, 20, 36, 48, 60

Open-ended – Sample answers

15 Must be a negative multiple of 8; e.g. -8, -24, -80.

16 6 ÷ 5 > -1 5 ÷ 6 > -1
5 ÷ 3 > -1 3 ÷ 5 > -1
-8 ÷ -7 > -1 -7 ÷ -8 > -1
-4 ÷ 5 > -1 3 ÷ -4 > -1

Half-time 1

1 (a) 9 – 6 = 3 (b) 14 – 21 = -7
(c) -9 – 8 = -17 (d) -31 – 27 = -58
(e) -8 – 8 = -16 (f) -14 – 20 = -34
(g) 7 + 10 = 17 (h) -35 + 21 = -14

2 (a) -24 (b) 56 (c) -140 (d) 24
(e) 9 (f) -25 (g) -8 (h) -64

3 (a) +14 °C (b) -8 °C

4 (a) 5 (b) -9 (c) -5 (d) 7
(e) -8 (f) -8 (g) 3 (h) 4

5 (a) -37, -30, -3, 0, 3, 7 (b) -40, -20, -5, -1, 54

6 (a) 1 (b) -2 (c) -1 (d) -14
(e) -3 (f) -5 (g) -11 (h) 4

7 (a) $80 (b) -30

8 (a) -120 (b) +7

9 (a) $20 000 (b) -10 000

10 (a) -22 (b) -26 (c) -63 (d) 12

Exercise 1.4

1 (a) 8, -10, -9, -11 (b) -8, +9, 5, -8, +45, 31

2 (a) 10 (b) 20 (c) 4 (d) -16
(e) 6 (f) 55 (g) -3 (h) -4
(i) -26 (j) 36 (k) 9 (l) 48

3 (a) -10 (b) -25 (c) 0 (d) 44
(e) -30 (f) -24 (g) 60 (h) 300
(i) 10 (j) 22

4 (a) -18 (b) 39 (c) 41 (d) 15
(e) 243 (f) 252 (g) 45 (h) -144
(i) -44 (j) -2

5 (a) C (b) A

6 (a) $100 (b) $40

(c) After the second $80 withdrawal in Week 2, which made the account balance -$60.

(d) $60

7 $24 million loss

8 (a) 12 °C (b) 0 °C (c) -4 °C (d) -16 °C

9 (a) 72 + (1 × -2) + (4 × -1) + (5 × 0) + (7 × +1) + (1 × +10)

(b) (i) 83 (ii) 11 over par

(c) (i) 69 (ii) 3 under par

10 (a) (i) 11 (ii) 41

(b) Without brackets, the calculation, starting with 19 and moving from left to right is: subtract 7 (to give 12), then subtract 15 (to give -3), then add 14 (to give 11).

Calculating 7 – 15 in brackets first gives 19 – (-8), which gives 27. Adding 14 to this gives 41.

11 (a) 8 + 16 – 23 – 9 = -8

(c) 11 × -8 + 24 ÷ -2 = -100

Open-ended – Sample answers

12 (a) 24 (b) -53

(c) Without the brackets, the calculation becomes:
$-15 + 11 \times -2 + -4^2$.

Evaluating the index number and the multiplication first gives:
-15 + -22 + -16
= -37 + -16
= -53

13 -3 – (48 ÷ (-6)) = 5
The brackets must give -8.

Exercise 1.5

1 (a) 2^4 (b) 4^7 (c) 8^8
(d) 11^6 (e) 13^6 (f) 9^9
(g) $5^4 \times 6^3$ (h) $2^3 \times 3^2$ (i) $4^4 \times 10^4$
(j) $7^5 \times 8^6$ (k) $2^3 \times 3^4 \times 5^2$ (l) $11^3 \times 13^6$
(m) $6^4 \times 7^3 \times 9^2$ (n) 12×14^3

2 (a) 5 × 5 × 5 × 5 × 5 × 5 × 5 (b) 4 × 4 × 4 × 4 × 4 × 4
(c) 7 × 7 × 7 (d) 9 × 9 × 9 × 9 × 9 × 9
(e) 10 × 10 × 10 × 10
(f) 3 × 3 × 3 × 3 × 3 × 3 × 3
(g) 2 × 2 × 2 × 5 × 5 × 5 × 5
(h) 3 × 3 × 3 × 3 × 3 × 3 × 3 × 11 × 11 × 11 × 11 × 11
(i) 9 × 9 × 9 × 9 × 9 × 7 × 7 × 7 × 7 × 7
(j) 2 × 4 × 4 × 4 × 4 × 4 × 4 × 6 × 6 × 6 × 6
(k) 3 × 3 × 3 × 3 × 7 × 7 × 12
(l) 9 × 9 × 9 × 11 × 17 × 17

3 (a) 5^5 (b) 3^8 (c) 6^6 (d) 4^7
(e) 9^9 (f) 17^{11} (g) 63^{16} (h) $4^9 \times 5^5$
(i) $2^{10} \times 6^7$ (j) $7^4 \times 8^{12}$ (k) $9^7 \times 11^7$ (l) $3^{15} \times 10^{11}$

4 **(a)** $(-2)^7 = -128$ **(b)** $(-6)^9 = -10\,077\,696$

(c) $(-3)^9 = -19\,683$ **(d)** $(-7)^4 = 2401$

(e) -675 **(f)** -64

(g) -147 456 **(h)** 3969

5 **(a)** 7^2 **(b)** 3^5 **(c)** 2^3 **(d)** 13^2

(e) 17^3 **(f)** 25^3 **(g)** $(-2)^2$ **(h)** -4

(i) -6 **(j)** $3^2 \times 2^3$ **(k)** $6^2 \times 4$ **(l)** $9^2 \times 13$

(m) $5^{12} \times 7$ **(n)** 11×8^2 **(o)** $2^2 \times 3^5 \times 4$

6 **(a)** C **(b)** D **(c)** C **(d)** A

7

Words	Index form	Expanded form	Value
Seven squared	7^2	7×7	49
Six to the power of five	6^5	$6 \times 6 \times 6 \times 6 \times 6$	7776
Eight to the power of three	8^3	$8 \times 8 \times 8$	512
Eleven to the power of six	11^6	$11 \times 11 \times 11 \times 11 \times 11 \times 11$	1 771 561

8 **(a)** -16 **(b)** 64 **(c)** 256 **(d)** -1024

(e) 24 **(f)** -24

9 **(a)** **(i)** 50×2^3 **(ii)** 50×2^5 **(iii)** 50×2^8

(b) **(i)** 400 **(ii)** 1600 **(iii)** 12 800

10 **(a)**

$(-5)^1$	-5
$(-5)^2$	25
$(-5)^3$	-125
$(-5)^4$	625
$(-5)^5$	-3 125
$(-5)^6$	15 625
$(-5)^7$	-78 125
$(-5)^8$	390 625
$(-5)^9$	-1 953 125
$(-5)^{10}$	9 765 625
$(-5)^{11}$	-48 828 125
$(-5)^{12}$	244 140 625

(b) The answers alternate between negative and positive.

(c) Odd powers give a negative answer, even powers give a positive answer.

11 **(a)** $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ **(b)** 128

12 **(a)** 8 **(b)** 125 **(c)** 1

(d) -64 **(e)** 81 **(f)** -64

13 **(a)** **(i)** $4^5 \times 4^3$
$= 1024 \times 64$
$= 65\,536$

(ii) $7^5 \div 7^2$
$= 16\,807 \div 49$
$= 343$

(iii) $(9^2 \times 9^3) \div 9^4$
$= (81 \times 729) \div 6561$
$= 59\,049 \div 656$
$= 9$

(iv) $(5^3 \times 5^4) \div (5^3 \times 5^4)$
$= (125 \times 625) \div (125 \times 625)$
$= 78\,125 \div 78\,125$
$= 1$

(b) **(i)** $4^5 \times 4^3$
$= 4^8$
$= 65\,536$

(ii) $7^5 \div 7^2$
$= 7^3$
$= 343$

(iii) $(9^2 \times 9^3) \div 9^4$
$= 9^5 \div 9^4$
$= 9$

(iv) $(5^3 \times 5^4) \div (5^4 \times 5^3)$
$= 5^7 \div 5^7$
$= 1$

(c) The process used in part **(b)** is more efficient.

14 **(a)** **(i)** 1 **(ii)** 1 **(iii)** 1

(iv) 0 **(v)** 0 **(vi)** 0

(b) **(i)** 1 raised to any positive whole number power is equal to 1.

(ii) 0 raised to any positive whole number power is equal to 0.

15 **(a)** **(i)** negative **(ii)** negative

(iii) positive **(iv)** negative

(b) A negative number raised to an odd power will give a negative answer regardless of brackets, as seen in **(i)** and **(ii)**. A negative number raised to an even power will give a positive answer when enclosed in brackets, as seen in **(iii)**, but if not enclosed in brackets will remain negative, as seen in **(iv)**.

(c) **(i)** -432 **(ii)** 144 **(iii)** 800

(iv) -1600 **(v)** -128 **(vi)** -128

16 **(a)**

Time (min)	Number of Bacteria A	Number of Bacteria B
0	1	1
20	2	1
40	4	2
60	8	2
80	16	4
100	32	4
120	64	8
140	128	8

(b) Bacteria A: 512; Bacteria B: 16

(c) 496

(d) Bacteria A, because after a period of time there would be many more of Bacteria A than Bacteria B.

Open-ended – Sample answers

17 **(a)** 32 (2^5), 64 (2^6) **(b)** 27 (3^3), 81 (3^4)

(c) 16 (2^4 and 4^2) or 64 (2^6 and 4^3)

18 (a) Joel has multiplied the base and the index together.

(b) He got 9^1 correct, as he would have calculated 9×1 to achieve an answer of 9.

(c) An index number is a short way of writing repeated multiplication of the same number. Writing each number in expanded form might help him see this.

19 Hayden was correct because he added like terms. Tao was incorrect because he multiplied the bases and then added like terms.

20 (a)–(b) $4^2 \div 2^3$; $10^2 \div (2 \times 5^2)$; $4^3 \div 2^5$; $6^2 \div (2 \times 3^2)$

Exercise 1.6

1 (a) 9^{12} (b) 14^{21} (c) 6^{64} (d) 10^{25}
(e) 4^6 (f) $1\ (5^0)$ (g) 2^{20} (h) $1\ (7^0)$

2 (a) $4^2 \times 3^2$ (b) $2^3 \times 5^3$ (c) $3^5 \times 2^5$ (d) $4^4 \times 7^4$
(e) $7^3 \times 10^3$ (f) 8×9 (g) $2^6 \times 3^6$ (h) $4^2 \times 5^2 \times 6^2$
(i) $\frac{3^3}{5^3}$ (j) $\frac{2^2}{9^2}$ (k) $\frac{10^3}{13^3}$ (l) $\frac{12^2}{13^2}$
(m) $\frac{1^5}{2^5}$ (n) $\frac{1^3}{10^3}$ (o) $\frac{1^6}{11^6}$ (p) $\frac{1^2}{12^2}$

3 (a) 1 (b) 1 (c) 1 (d) 1 (e) 1
(f) 3 (g) 4 (h) -1 (i) -6 (j) 3

4 (a) D (b) A

5 C

6 (a) F, $(3 + 4)^2 = 7^2$ (b) T
(c) F, $(-3)^2 + (6)^3 = 15^2$ (d) T
(e) F, $(5)^2 \times (2)^3 = 2 \times 10^2$ (f) T

7 (a) $8^4 \times 7^6$ (b) $\frac{6^6}{7^6}$ (c) $2^4 \times 4^5$ (d) 5^4
(e) $\frac{4^7}{9^2}$ (f) $\frac{3^2}{4^5}$ (g) $7^2 \times 8^4$ (h) 10^3
(i) 4^{10} (j) $2^4 \times 3^2$ (k) $\frac{3}{2}$ (l) $\frac{2^2}{3^{12}}$

8 (a) $(5 \times 2)^6$ (b) $(3 \times 7)^3$
(c) $(7 \times 5)^2$ (d) $(3 \times 3 \times 5)^4$
(e) $(2 \times 5 \times 7)^5$ (f) $(7 \times 11)^7$

9 (a) $2^2 \times 3^2$ (b) $2^4 \times 3$ (c) $3^2 \times 7$
(d) $2^3 \times 3^2$ (e) 3×5^2 (f) $2^2 \times 3 \times 7$

10 (a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{45}{22}$

11 (a) 2^{33} (8 589 934 592) is the largest power of 2 for which all digits can fit on a 10-digit calculator display.

(b) 9^{10} (3 486 784 401) is the largest power of 9 for which all digits can fit on a 10-digit calculator display.

12 (a) a^6 (b) $\frac{c^4}{d^4}$
(c) x^5y^5 (d) 1
(e) $2^2x^2 = 4x^2$ (f) 5
(g) $\frac{b^{10}}{c^5}$ (h) $\frac{3^3a^3}{b^3} = \frac{27a^3}{b^3}$

13 (a) 7 (b) 18 (c) 10 (d) 7 (e) 3 (f) 1

14 (a) Terry used the index number rules to simplify the fraction inside the brackets before raising it to the power of 2. Kelvin raised the numerator and the denominator to the power of 2, worked out their values and then simplified the answer.

(b) Terry answered the question more efficiently. By simplifying the fraction inside the brackets first, he didn't have to deal with any large numbers on a calculator.

15 (a) $(2^2)^2$ (b) $(3^2)^2$ (c) $(5^2)^2$
(d) $(3^2)^3$ or $(3^3)^2$ (e) $(4^2)^2$ or $(2^4)^2$ (f) $(10^3)^2$ or $(10^2)^3$

Open-ended – Sample answers

16 (a) $(4 \times 6)^3$, $(8 \times 3)^3$ or $(2 \times 12)^3$
(b) $(4 \times 9)^5$, $(3 \times 12)^5$ or $(2 \times 18)^5$
(c) $(6 \times 8)^2$, $(4 \times 12)^2$ or $(24 \times 2)^2$
(d) $(4 \times 20)^4$, $(16 \times 5)^4$ or $(40 \times 2)^4$

17 $625\,000 = 5^4 \times 10^3$ or $25^2 \times 10^3$

18 Values for m and n must multiply to give 24.

19 (a) Powers were multiplied instead of added; 3^6
(b) Powers were divided instead of subtracted; 7^6
(c) Powers were raised to a power instead of multiplied; 5^8
(d) Any number raised to the power of 0 is equal to 1. Therefore, $13 \times 1 = 13$

Challenge 1

1 B

2 D

total Reece spent = \$100 + \$130 = \$230
total Reece got back = \$110 + \$150 = \$260
profit = \$260 – \$230 = \$30

3 Let the consecutive numbers be $x - 1$, x, $x + 1$.
$2(x + 1) + 3(x - 1) = -31$
$5x - 1 = -31$, $x = -6$
The numbers are -7, -6, -5.

4 I am x years old. Father is $x + 30$, mother is $x + 24$.
$x + 30 = 2(x + 24)$, $x = -18$. My father's age was double my mother's age 18 years before I was born.

5 $64 = 2^6 = 4^3 = 8^2$; hence, $x = 2, y = 6$; $x = 4, y = 3$; $x = 8, y = 2$; $x = 64, y = 1$.

6 (a) $2^{15} = (2^3)^5 = 8^5$
$3^{10} = (3^2)^5 = 9^5$
So, $2^{15} < 3^{10}$ and statement is false.

(b) $2^{27} = (2^3)^9 = 8^9$

$3^{18} = (3^2)^9 = 9^9$

So, $3^{18} > 2^{27}$

7 B $a = (2^4)^{20} = 16^{20}$; $b = (3^3)^{20} = 27^{20}$; $c = (5^2)^{20} = 25^{20}$

8 D **9** A **10** B **11** A

12 $256 = 256^1 = 16^2 = 4^4 = 2^8$; hence $y = 256, x = 1$; $y = 16, x = 2$; $y = 4, x = 4$; $y = 2, x = 8$.

13 B

14 $4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256, 4^5 = 1024$. If the power is odd, then the last digit in the expansion is 4, so 4^{3827} ends in 4.

15 $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729, 3^7 = 2187$, $3^8 = 6561$.

There is a pattern in the last digit that is repeated every fourth number: 3, 9, 7, 1, 3, 9, 7, 1, …

$2004 \div 4 = 501$. As there is no remainder, 2004 is the 4th digit in the pattern, so 3^{2004} ends in 1.

Chapter review 1

1 **(a)** -23 **(b)** +840 000 **(c)** +350 **(d)** +4

2 **(a)** -23, -6, 0, 9, 14 **(b)** -15, -7, -2, 5, 8 **(c)** -11, 0, 6, 12, 34

3 **(a)** 7 **(b)** -2 **(c)** -11 **(d)** -7

(e) 3 **(f)** -24 **(g)** 20 **(h)** -8

4 **(a)** 2 **(b)** 2 **(c)** 0 **(d)** -7

(e) 4 **(f)** -1

5 **(a)** -28 **(b)** -20 **(c)** 36 **(d)** 84

(e) -135 **(f)** -176 **(g)** -1800 **(h)** 5600

(i) 36 **(j)** −49 **(k)** -36 **(l)** 16

6 **(a)** -12 **(b)** -5 **(c)** 7 **(d)** -9

(e) -6 **(f)** 13 **(g)** -8 **(h)** 9

(i) -20 **(j)** -44 **(k)** -40 **(l)** -13

7 **(a)** -14 **(b)** 0 **(c)** 35 **(d)** 14

(e) 0 **(f)** 21

8 **(a)** 9^5, 59 049 **(b)** 6^7, 279 936

(c) $3^4 \times 4^5$, 82 944 **(d)** $8^2 \times 10^3$, 64 000

9 **(a)** $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

(b) $13 \times 13 \times 13 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

(c) $4 \times 4 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9 \times 9$

(d) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 10 \times 10 \times 17$

10 **(a)** 7^5 **(b)** 3^8 **(c)** 5^4 **(d)** 2^8

(e) $3^9 \times 2^{10}$ **(f)** $4^5 \times 7^3$ **(g)** $5^6 \times 2^6$ **(h)** $7^7 \times 11^6$

11 **(a)** 5^4 **(b)** 7^3 **(c)** 11^2 **(d)** 2^3

(e) 3×4 **(f)** $7^2 \times 10^2$ **(g)** 8×5^3 **(h)** $2 \times 9 \times 13^2$

12 **(a)** 81 **(b)** -81 **(c)** -125

(d) -1 **(e)** -144 **(f)** 1296

13 **(a)** $3^{10} = 59\,049$ **(b)** $7^6 = 117\,649$

(c) $3^{15} = 14\,348\,907$ **(d)** $2^{16} = 65\,536$

(e) 1 **(f)** 6 **(g)** 1 **(h)** 1

14 **(a)** $4^3 \times 11^3$ **(b)** $8^5 \times 9^5$ **(c)** $3^7 \times 5^7$ **(d)** $7^4 \times 10^4$

(e) $\frac{1}{2^3}$ **(f)** $\frac{3^2}{4^2}$ **(g)** $\frac{5^4}{6^4}$ **(h)** $\frac{8^7}{9^7}$

15 -1 °C **16** Ava 0; Georgia 1; Rose -4; Wei -7

17 **(a)**

Time (number of days)	0	1	2	3	4
Number of cells	1	2	4	8	16

(b)

Time (number of days)	0	1	2	3	4	5	6	7
Number of cells	1	2	4	8	16	32	64	128

(c) $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$

(d) $N = 2^{14} = 16\,384$

18 **(a)** \$70 loss **(b)** \$10 profit **(c)** \$9410 profit

19 -7 °C per hour

20 **(a)** A **(b)** C

21 **(a)** 1 **(b)** 1 **(c)** 1 **(d)** 1

(e) -3 **(f)** 25 **(g)** -32 **(h)** -27

(i) 4 **(j)** -10 000 **(k)** 27 **(l)** $\frac{9}{128}$

22 **(a)** $\frac{10}{9}$ **(b)** $\frac{20}{49}$ **(c)** $\frac{3}{10\,000}$

23 **(a)** 3 **(b)** 8 **(c)** 10

24 **(a)** $(6^2)^2$ **(b)** $(2^{14})^2$ **(c)** $(10^9)^2$ **(d)** $(70^2)^7$

25 **(a)** -20

(b)

4	-6	2	-20
-18	0	-8	6
-16	-2	-10	8
10	-12	-4	-14

(c)

-2	3	-1	10
9	0	4	-3
8	1	5	-4
-5	6	2	7

Magic sum is 10.

(d)

-8	12	-4	40
36	0	16	-12
32	4	20	-16
−20	24	8	28

Magic sum is 40.

26 32 players

27 **(a)** $(-3 - 4) \times -2 = 14$ **(b)** $(2 + -3) \times 4 + -2 = -6$

(c) $16 \div -4 + 3 \times (-2 + 1) = -7$

(d) $4 + -3 \times -2 \div -6 \times (-1 + 5) = 0$

(e) $2 - 3 + 6 \times (3 + 2) \times -1 - 6 = -37$

(f) $24 \div -2 \div -3 + 4 \times (-2 - 5) = -24$

28 **(a)** F **(b)** F **(c)** F **(d)** T **(e)** T **(f)** F

Numeracy practice 1

1 -8, 3 2 $9 3 C

4 (a)

Shape	1	2	3	4	5
Number of flowers	1	4	9	16	25

(b) the perfect squares

(c) 100

5 B 6 7 minutes

7 $A = 4, B = 8, C = 2$ 8 C

Chapter 2

Recall 2

1 (a) $1\frac{4}{15}$ (b) $\frac{7}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

2 (a) 13.041 (b) 0.459 (c) 63.5
(d) 13.02 (e) 0.465 (f) 316

3 (a) 2.03 (b) 3.19 (c) 2.910 (d) 4.086

4 (a) $\frac{3}{5}$ (b) $\frac{4}{23}$ (c) $2\frac{2}{3}$

5 (a) $\frac{5}{2}$ (b) $\frac{38}{9}$ (c) $\frac{67}{20}$

6 (a) $2\frac{3}{7}$ (b) $4\frac{4}{9}$ (c) $9\frac{1}{2}$

7 (a) 70.8 (b) 490 (c) 80 (d) 0.0521
(e) 3.465 (f) 0.004 037

8 (a) $75 (b) 105 kg (c) 225 m

9 (a) 50% (b) 75% (c) 40% (d) 70%

10 (a) 35% (b) 90% (c) 4% (d) 127%

Exercise 2.1

1 (a) $\frac{4}{5}$ (b) $\frac{1}{20}$ (c) $\frac{1}{500}$ (d) $\frac{9}{10\,000}$
(e) $\frac{7}{50}$ (f) $\frac{31}{50}$ (g) $\frac{31}{100}$ (h) $\frac{17}{20}$
(i) $\frac{711}{1000}$ (j) $\frac{171}{250}$ (k) $\frac{5}{8}$ (l) $\frac{16}{125}$
(m) $\frac{203}{1000}$ (n) $\frac{47}{500}$ (o) $\frac{39}{250}$ (p) $\frac{7009}{10\,000}$

2 (a) $0.382, 0.399, \frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ (b) $0.88, 0.89, 0.899, \frac{9}{10}, \frac{9}{8}$
(c) $0.07, 0.099, 0.112, \frac{1}{8}, \frac{1}{4}$ (d) $0.2, 0.3, \frac{1}{3}, \frac{3}{8}, \frac{2}{5}$
(e) $\frac{1}{2}, 0.55, 0.555, 0.58, \frac{3}{5}$ (f) $\frac{2}{9}, 0.291, \frac{3}{10}, 0.302, \frac{2}{3}$
(g) $2.278, 2\frac{3}{4}, 2\frac{4}{5}, 2\frac{9}{10}, 2.932$ (h) $1.029, 1\frac{1}{5}, 1.243, 1\frac{3}{8}, 1\frac{2}{3}$
(i) $4.199, 4\frac{1}{5}, 4.201, 4\frac{1}{4}, 4.295$
(j) $3\frac{1}{5}, 3.439, 3.45, 3.482, 3\frac{2}{3}$

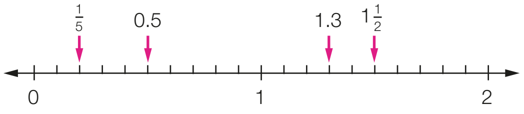
3 (a) 6 pieces; 20 cm remaining
(b) 6 cups; 30 g remaining
(c) 16 cups; 50 mL remaining
(d) 13 buses; 26 people on the 13th bus

4 (a) C (b) C

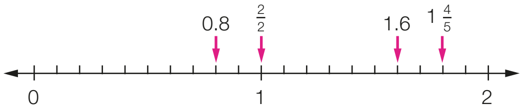
5 (a) $7\frac{1}{2}$ (b) $3\frac{2}{5}$ (c) $1\frac{1}{4}$ (d) $5\frac{16}{25}$
(e) $13\frac{1}{50}$ (f) $27\frac{24}{25}$ (g) $9\frac{9}{200}$ (h) $124\frac{353}{500}$

6 (a) 51.7 (b) 28.12 (c) 7.125 (d) 67.28
(e) 39.0625 (f) 5.15 (g) 14.1625 (h) 24.0775

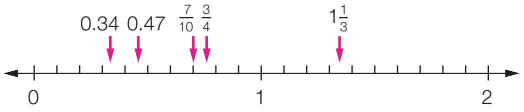
7 (a)



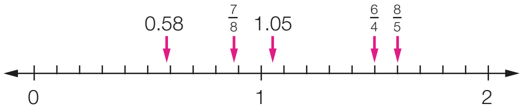
(b)



8 (a)



(b)



9 (a) < (b) < (c) = (d) = (e) >
(f) < (g) > (h) < (i) <

10 B

11 (a) (i) $\frac{5}{8}$ (ii) $\frac{80}{129}$ (iii) $\frac{403}{1000}$ (iv) $1\frac{81}{125}$
(v) $1\frac{11}{20}$ (vi) $9\frac{3}{5}$
(b) (i) 0.625 (ii) 0.620 (iii) 0.403 (iv) 1.648
(v) 1.55 (vi) 9.6

12 (a) $53.95 (b) $105.30 (c) $5.30

13 (a) US$7.20 (b) US$36 (c) US$72
(d) US$192.24 (e) US$1328.40

14 $24.75 15 $2653.50 16 7 17 14

18 (a) 0.9 m, 0.45 m, 0.27 m (b) 1.08 m (c) $\frac{2}{5}$

19 (a) Sample answer: Angela can calculate how many lollies are in each packet to find out how many she has in total, and then divide that amount among the 50 gift bags.
(b) 10
(c) Sample answer: 36; this answer may not be exact as we can't be sure how many lollies are in each packet.

20 (a) 13 kg (b) 1.9 kg

Open-ended – Sample answers

21 $\frac{1}{4}, \frac{3}{10}, \frac{7}{20}$ **22** 0.95, 0.99999, 1.102 **23** $\frac{8}{10}, \frac{12}{15}$

24 **(a)** Leo is treating the decimals as whole numbers and not comparing place value. He needs to compare the place value of each digit in the number. 1.7 has 7 in the tenths place value, while 1.25 only has 2 in that same position. Leo's answer to the first question is only correct because the numbers have a digit in the same place value position (units and tenths).

(b) Compare the whole number parts of the two decimal numbers first. If one is larger than the other, then that decimal number is larger. If the whole number parts are the same, compare the digits in the tenths place value column. If one is larger than the other, then that decimal number is larger. If they are the same, compare the digits in the hundredths place value column. Keep moving down the place value columns comparing digits until you find one digit larger than the other corresponding digit. This digit belongs to the larger number.

Exercise 2.2

1 **(a)** $0.1\dot{6}$ **(b)** $0.8\dot{3}$ **(c)** $0.\dot{1}$ **(d)** $0.\dot{5}$
(e) $0.\dot{0}\dot{9}$ **(f)** $0.\dot{1}\dot{8}$ **(g)** $0.\dot{2}\dot{7}$ **(h)** $0.\dot{1}5384\dot{6}$
(i) $0.0\dot{6}$ **(j)** $0.1\dot{3}$ **(k)** $0.2\dot{6}$ **(l)** $0.4\dot{6}$
(m) $0.0\dot{5}$ **(n)** $0.2\dot{7}$ **(o)** $0.3\dot{8}$ **(p)** $0.6\dot{1}$

2 **(a)** $\frac{4}{9}$ **(b)** $\frac{1}{9}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{7}{9}$
(e) $\frac{14}{33}$ **(f)** $\frac{32}{99}$ **(g)** $\frac{17}{99}$ **(h)** $\frac{8}{33}$
(i) $\frac{4}{333}$ **(j)** $\frac{71}{333}$ **(k)** $\frac{332}{999}$ **(l)** $\frac{224}{999}$
(m) $\frac{113}{900}$ **(n)** $\frac{25}{198}$ **(o)** $\frac{3211}{9990}$ **(p)** $\frac{8197}{1998}$

3 **(a)** 1.125 terminating **(b)** 6.5 terminating
(c) 21.189 620 1… irrational **(d)** 9.165 151 39… irrational
(e) 0.555 5… recurring **(f)** 7.641 989 27… irrational
(g) 0.181 818… recurring **(h)** 7.333 33… recurring
(i) 14.696 938 46… irrational **(j)** 2.5 terminating
(k) 23.1 terminating **(l)** 0.916 66… recurring
(m) 18.5 terminating **(n)** 18.973 665 96… irrational
(o) 3.888 8… recurring **(p)** 9.818 350 16… irrational

4 **(a)** D **(b)** C

5 D

6 C

7 **(a)** 1.7 and 1.8; closer to 1.7 **(b)** 2.2 and 2.3; closer to 2.2
(c) 3.4 and 3.5; closer to 3.5 **(d)** 4.4 and 4.5; closer to 4.5
(e) 2.8 and 2.9; closer to 2.8 **(f)** 2.6 and 2.7; closer to 2.6
(g) 6.8 and 6.9; closer to 6.9 **(h)** 8.2 and 8.3; closer to 8.2

8 7.920 m

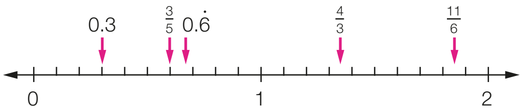
9 0.125

10 **(a)** $0.\dot{6}\dot{3}$; 64 cents
(b) 65 cents if you buy only one

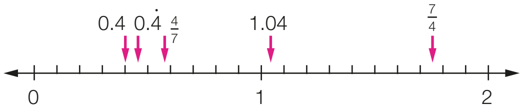
11 $\frac{9}{9} = 1$

12 **(a)** $\frac{1}{3}$, 0.9, 1.3, $\sqrt{3}$, $\sqrt{9}$ **(b)** 0.5, $\frac{5}{9}$, $\frac{9}{5}$, $\sqrt{5}$, $\sqrt{59}$
(c) 0.36, $\frac{4}{11}$, 0.4, $\sqrt{4}$, $\sqrt{11}$ **(d)** $\frac{1}{16}$, $\frac{1}{6}$, 1.6, $\sqrt{6}$, $\sqrt{16}$

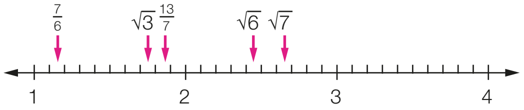
13 **(a)**



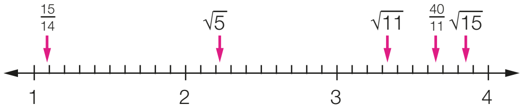
(b)



14 **(a)**



(b)



15 **(a)** B **(b)** B

16 **(a)** $\frac{1}{60}$ **(b)** $0.1\dot{6}$

17 **(a)** $\frac{1}{3}$ **(b)** $0.\dot{3}$ **(c)** yes

18 **(a)** 0.111 111 11 and 0.090 909 09
(b) 0.202 020 2 **(c)** $\frac{20}{99} = 0.\dot{2}\dot{0}$ **(d)** yes
(e) 0.666 666 66 and 0.555 555 55 which add to give 1.222 222 21; the fractional sum is $1\frac{2}{9} = 1.\dot{2}$.
(f) The final digit in the decimal sum is incorrect as there was no number to carry into this column. When the fractions were written as decimals with only 8 digits after the decimal point, the error was introduced.

Open-ended – Sample answers

19 $0.\dot{3}$ is bigger, it is more than $\frac{33}{100}$ and 0.3 is $\frac{30}{100}$

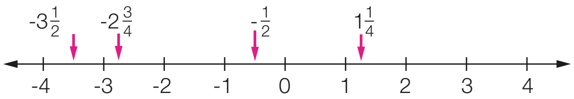
20 $\frac{9}{99} = 0.\dot{0}\dot{9}$, $\frac{7}{9} = 0.\dot{7}$, $\frac{13}{999} = 0.\dot{0}1\dot{3}$, $\frac{58}{9999} = 0.\dot{0}05\dot{8}$,

$\frac{78\,465}{99\,999} = 0.\dot{7}84\,6\dot{5}$

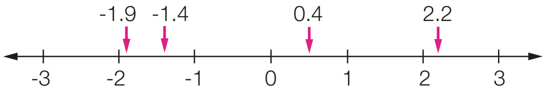
The recurring cycle is just the numerator of the fraction.

Exercise 2.3

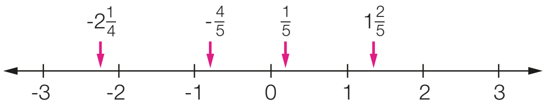
1 **(a)**



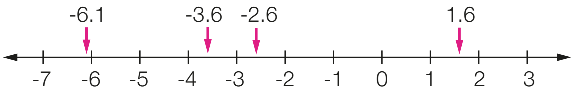
(b)



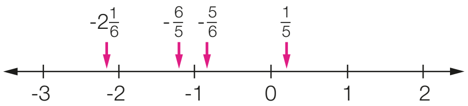
(c)



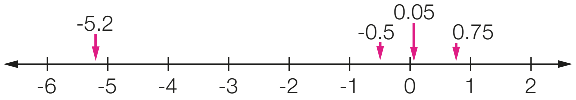
(d)



(e)



(f)



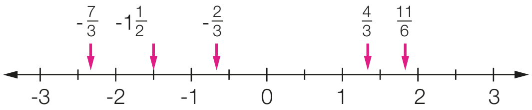
2 **(a)** -0.1 **(b)** -1 **(c)** -3.1 **(d)** -1.1
(e) -0.4 **(f)** -14.2 **(g)** -8.5 **(h)** 9.5
(i) 1.69 **(j)** 14.45 **(k)** -1.54 **(l)** 1.43
(m) -1.97 **(n)** 7.8 **(o)** -10.07 **(p)** 31.56

3 **(a)** $-\frac{3}{2}$ or $-1\frac{1}{2}$ **(b)** $-\frac{4}{5}$ **(c)** $-\frac{3}{7}$
(d) $-\frac{7}{9}$ **(e)** $-\frac{17}{20}$ **(f)** $-\frac{1}{40}$
(g) $\frac{1}{30}$ **(h)** $\frac{1}{6}$ **(i)** $\frac{3}{5}$
(j) $\frac{5}{2}$ or $2\frac{1}{2}$ **(k)** $\frac{2}{3}$ **(l)** $-\frac{1}{4}$
(m) $\frac{3}{4}$ **(n)** $-\frac{35}{6}$ or $-5\frac{5}{6}$ **(o)** $\frac{49}{10}$ or $4\frac{9}{10}$
(p) $\frac{11}{12}$

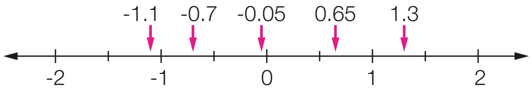
4 **(a)** $-\frac{3}{14}$ **(b)** -2 **(c)** $-\frac{5}{6}$
(d) $-\frac{9}{5}$ or $-1\frac{4}{5}$ **(e)** -32 **(f)** $\frac{1}{8}$
(g) -9 **(h)** $\frac{1}{2}$ **(i)** 4
(j) $-7\frac{1}{3}$ **(k)** $-6\frac{2}{3}$ **(l)** $-7\frac{1}{2}$

5 **(a)** -0.32 **(b)** -0.3 **(c)** -4.5 **(d)** -0.9
(e) 2.4 **(f)** 90 **(g)** -2.4 **(h)** -0.37
(i) 89.78 **(j)** -7.06 **(k)** -331.29 **(l)** -5.56

6



7



8 B

9 **(a)** Friday **(b)** Monday
(c) Wednesday (difference of 7.8 °C)
(d) **(i)** average maximum: 2.6 °C
(ii) average minimum: -3.1 °C

10 6.33 **11** 21.6 m **12** -21.7 m

13 **(a)** +1.2%, 0%, $+\frac{3}{4}$%, -0.8%, $-1\frac{1}{4}$%, 0%, $-\frac{1}{2}$%, -0.6%
(b) **(i)** $-1\frac{1}{5}$ **(ii)** -1.2

14 -3.4, -3, -0.9, $-\frac{3}{4}$, 0, $\frac{3}{10}$, 1, 1.7

15 $-2\frac{1}{5}$, -2, -1.8, $-\frac{5}{4}$, 0, $\frac{3}{4}$, $\frac{4}{5}$, 1.1

16 **(a)** -157.536 **(b)** -335.5 **(c)** 35.155
(d) -80.64 **(e)** -1.536 **(f)** -4.682
(g) -7119.009 **(h)** -12 150.073 **(i)** 0.782
(j) -2839.396

17 **(a)** Chloe's account is overdrawn. She owes the bank $12.91.
(b) Her account was still overdrawn. There was no money available.
(c) $473.85

18 **(a)** $\frac{7}{12}$ hours **(b)** 35 minutes

19 The further a number lies to the right on the number line, the larger it is.

$\frac{1}{2}$ lies to the right of $\frac{1}{4}$, so $\frac{1}{2} > \frac{1}{4}$.

$-\frac{1}{4}$ lies to the right of $-\frac{1}{2}$, so $-\frac{1}{2} < -\frac{1}{4}$.

Open-ended – Sample answers

20 -1.9, -1.75, -1.24 **21** $-\frac{5}{6}$, $-\frac{1}{2}$, $-\frac{1}{10}$

22 **(a)** Dan has ignored the signs of the numbers he is adding and subtracting, and not applied the rules for adding and subtracting directed numbers. In the first problem, he has added 4.5, not -4.5. In the second problem, he has subtracted 2.9, not -2.9.

(b) When the two symbols are opposites (+ – or – +), replace them with one – symbol and subtract.

When the two symbols are the same (+ + or – –), replace them with one + symbol and add.

Exercise 2.4

1 **(a)** soda water **(b)** lemon squash
(c) creamy soda **(d)** lemonade

2 **(a)** gauge 3 **(b)** gauge 5 **(c)** gauge 1 **(d)** gauge 6

3 **(a)** D **(b)** B **(c)** A

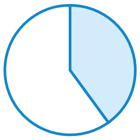
4 **(a)** F **(b)** D **(c)** A **(d)** C

5 **(a)** 90% **(b)** 70% **(c)** 40% **(d)** 10%

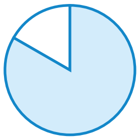
6 35% (approx.) 7 55% (approx.)

8 40%

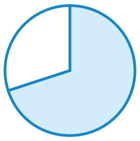
9 **(a)** **(i)**



(ii)



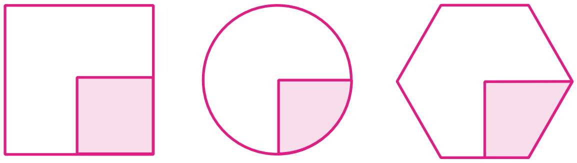
(iii)



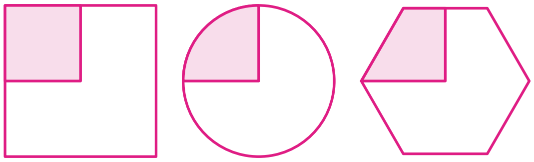
(b) Students' own answers. However, students will probably find it easier to estimate the percentage amount in **(a)**.

Open-ended – Sample answers

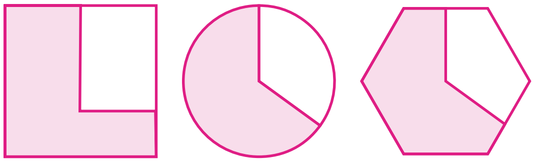
10 **(a)**



(b)



(c)



11 **(a)** Helga mistakenly believes that the large square contains 100 individual squares and so has shaded in 2 columns and 4 squares in the belief that each column is 10 units long and that she has shaded in 24 individual squares out of 100, or 24%. However, the large square contains only 64 individual squares, so by shading in 2 columns and 4 squares, Helga has actually shaded in 20 squares out of 64, or 31.25%.

(b) Helga mistakenly believes that the total circle contains 10 segments and so has shaded in 3 of the segments in the belief that she has shaded in three tenths or 30%. However, the total circle contains only 8 segments, not 10, so shading in 3 out of 8 segments equals 37.5%, not 30%.

Exercise 2.5

1 **(a)** 9% **(b)** 10% **(c)** 14% **(d)** 95%
(e) 60% **(f)** 25% **(g)** 150% **(h)** 200%
(i) $33.\dot{3}\%$ or $33\frac{1}{3}\%$ **(j)** $66.\dot{6}\%$ or $66\frac{2}{3}\%$
(k) $16.\dot{6}\%$ or $16\frac{2}{3}\%$ **(l)** $22.\dot{2}\%$ or $22\frac{2}{9}\%$
(m) 80% **(n)** 56.25% or $56\frac{1}{4}\%$
(o) 70% **(p)** 43.75% or $43\frac{3}{4}\%$
(q) 91.25% or $91\frac{1}{4}\%$ **(r)** $21.\dot{6}\%$ or $21\frac{2}{3}\%$
(s) $83.\dot{3}\%$ or $83\frac{1}{3}\%$ **(t)** $25.\dot{5}\%$ or $25\frac{5}{9}\%$

2 **(a)** 125% **(b)** 140% **(c)** 320% **(d)** 550%
(e) 237.5% **(f)** 275% **(g)** 570% **(h)** 462.5%
(i) $466.\dot{6}\%$ **(j)** $533.\dot{3}\%$ **(k)** $177.\dot{7}\%$ **(l)** $783.\dot{3}\%$

3 **(a)** 90% **(b)** 40% **(c)** 80% **(d)** 60%
(e) 17% **(f)** 47% **(g)** 82% **(h)** 53%
(i) 5.1% **(j)** 43.8% **(k)** 0.7% **(l)** 34.2%
(m) 920% **(n)** 510% **(o)** 202% **(p)** 901%

4 **(a)** D **(b)** D

5 **(a)** B **(b)** C

6

Percentage	Fraction	Decimal
5%	$\frac{1}{20}$	0.05
10%	$\frac{1}{10}$	0.1
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\dot{3}$
40%	$\frac{2}{5}$	0.4
50%	$\frac{1}{2}$	0.5
60%	$\frac{3}{5}$	0.6
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\dot{6}$
75%	$\frac{3}{4}$	0.75
80%	$\frac{4}{5}$	0.8
100%	1	1

7 10% **8** 70% **9** 375%

10 Ron 75% and Andrew 66.67%

11 **(a)** Measurement 83%, Algebra 87%, Geometry 80%

(b) Algebra

12 **(a)** 70.83% **(b)** 29.17%

13 **(a)** 67.5% **(b)** 95% **(c)** 90% **(d)** 55%

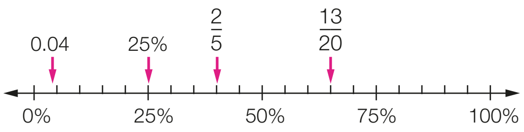
(e) 42.5% **(f)** 82.5%

14 32

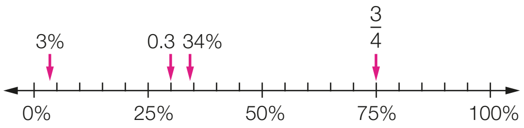
15 24.6% calcium, 3.4% magnesium, 0.12% potassium

16 **(a)** 2 cm **(b)** 1.23%

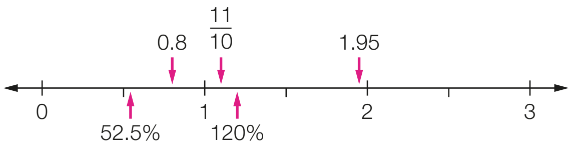
17 **(a)**



(b)



18 **(a)**



(b)

19 **(a)** $\frac{3}{14}$ **(b)** 21.43%

(c) Yes, the amount of extra fibre is more than 20% of the original amount.

20 **(a)** first store: 61%, 65%, $83\frac{1}{3}\%$;

second store: 48%, $52\frac{1}{2}\%$, 70%

(b) first store by 13%, $12\frac{1}{2}\%$, $13\frac{1}{3}\%$

(c) first store by 14%

21 **(a)** 13.3%, 3%, 0.73%, 59.4%, 17.2%

(b) 8.8% **(c)** 58.9%

(d) not accepted: 1.25% sodium

22 **(a)** 1.33% **(b)** 2.67%

23 **(a)** 27.5% **(b)** 22.5% **(c)** 15%

Open-ended – Sample answers

24 1.32 = 132%, 2.39 = 239%, 5.30 = 530%

25 $\frac{73}{100}, \frac{37}{50}, \frac{18}{25}$

Half-time 2

1 65–70% full

2 **(a)** 2.5, terminating **(b)** $0.\dot{6}\dot{3}$, recurring

(c) 7.348 469 228…, irrational

(d) 13.2, terminating **(e)** $6.91\dot{6}$, recurring

(f) 8.660 254 038…, irrational

3 **(a)** 11 boxes **(b)** 15 cm left over

4 **(a)** 456% **(b)** 75% **(c)** $88\frac{3}{4}\%$ or 88.75%

(d) 43.2% **(e)** 140%

5 **(a)** $1\frac{1}{3}$ **(b)** $3\frac{112}{495}$ or $\frac{1597}{495}$ **(c)** $5\frac{1901}{9990}$ or $\frac{51\,851}{9990}$

6 **(a)** $-0.7, -\frac{5}{9}, \frac{3}{8}, \frac{1}{2}, 1.099, \sqrt{5}$

(b) $-1.23, -\frac{2}{3}, 0.874, \frac{17}{8}, \sqrt{13}$

7 **(a)** $0.\dot{6}$ **(b)** $0.\dot{7}14\ 28\dot{5}$ **(c)** $0.\dot{1}$ **(d)** $0.\dot{4}\dot{5}$

8 **(a)** $\frac{219}{500}$ **(b)** $1\frac{9}{125}$ **(c)** $2\frac{253}{5000}$ **(d)** $13\frac{1401}{2000}$

9 **(a)** $\frac{3}{10}$ **(b)** $\frac{3}{14}$ **(c)** $\frac{17}{60}$ **(d)** $-1\frac{7}{8}$

(e) -13.29 **(f)** 11.76 **(g)** -7.93 **(h)** -15

10 55% to 65%

Exercise 2.6

1 **(a)** $\frac{17}{100}$ **(b)** $\frac{12}{25}$ **(c)** $\frac{9}{100}$ **(d)** $\frac{13}{20}$

(e) $\frac{29}{50}$ **(f)** $\frac{19}{25}$ **(g)** $1\frac{17}{100}$ **(h)** $1\frac{29}{100}$

(i) $2\frac{2}{5}$ **(j)** $3\frac{3}{20}$ **(k)** $1\frac{19}{50}$ **(l)** $3\frac{3}{5}$

2 **(a)** $\frac{1}{200}$ **(b)** $\frac{1}{500}$ **(c)** $\frac{3}{700}$ **(d)** $\frac{3}{800}$

(e) $\frac{1}{1000}$ **(f)** $\frac{1}{150}$ **(g)** $\frac{1}{120}$ **(h)** $\frac{1}{450}$

(i) $\frac{9}{400}$ **(j)** $\frac{13}{200}$ **(k)** $\frac{7}{125}$ **(l)** $\frac{7}{80}$

3 **(a)** $\frac{83}{1000}$ **(b)** $\frac{1229}{10\,000}$ **(c)** $\frac{101}{200}$ **(d)** $\frac{5}{8}$

(e) $\frac{21}{1000}$ **(f)** $\frac{17}{500}$ **(g)** $\frac{11}{2000}$ **(h)** $\frac{1}{250}$

(i) $\frac{41}{400}$ **(j)** $\frac{7}{8}$ **(k)** $1\frac{251}{500}$ **(l)** $4\frac{41}{200}$

4 **(a)** 0.8 **(b)** 0.27 **(c)** 0.4 **(d)** 0.15

(e) 0.66 **(f)** 0.92 **(g)** 0.03 **(h)** 0.05

(i) 1.1 **(j)** 2.65 **(k)** 7 **(l)** 3.04

(m) 0.005 **(n)** 0.072 **(o)** 0.405 **(p)** 0.0625

5 **(a)** 0.0025 **(b)** 0.001 25 **(c)** 0.007 **(d)** 0.004
(e) 0.025 **(f)** 0.1075 **(g)** 0.372 **(h)** 0.188
(i) $0.25\dot{3}$ **(j)** $0.05\dot{6}$ **(k)** $0.02\dot{5}$ **(l)** $0.238\dot{3}$

6 C **7** A **8** B **9** B

10 $\frac{43}{50}$ **11** $\frac{19}{200}$ **12** 0.45 **13** 0.925

14 red roses $\frac{2}{5}$, pink roses $\frac{8}{25}$, yellow roses $\frac{7}{25}$

15 $\frac{3}{40}$ **16** 0.008

17 **(a)** $30\% = \frac{3}{10}$; $16\% = \frac{4}{25}$; $9\% = \frac{9}{100}$
(b) $55\% = \frac{11}{20}$ **(c)** $45\% = \frac{9}{20}$

18 **(a)** 0.02, 20%, $\frac{1}{4}$, $\frac{2}{5}$ **(b)** 4.5%, 0.45, $\frac{4}{5}$, $\frac{5}{4}$
(c) 0.03, $\frac{3}{10}$, 33%, $\frac{1}{3}$ **(d)** 7.2%, $7\frac{1}{2}$%, 0.72, $\frac{7}{2}$

19 $\frac{7}{25}$ **20** $\frac{9}{20}$ (45%)

21 **(a)** $\frac{17}{20}$ **(b)** $\frac{12}{25}$ **(c)** $\frac{37}{100}$ **(d)** $\frac{8}{25}$

Open-ended – Sample answers

22 **(a)** $\frac{13}{25}$ (52%), $\frac{27}{50}$ (54%), $\frac{11}{20}$ (55%)
(b) 0.255 (25.5%), 0.2504 (25.04%), 0.250 03 (25.003%)

23 **(a)–(b)** $3\frac{1}{4}$ is greater than 1 whole, so it is greater than 100%.

$3 = 300\%$, so $3\frac{1}{4} = 300 + 25\% = 325\%$.

3.25% is one hundredth the size of 325% ($325 \div 100 = 3.25$). Its value lies between 3% and 4%.

Exercise 2.7

1 **(a)** **(i)** 26% **(ii)** 26%
(b) **(i)** 80% **(ii)** 80%
(c) **(i)** 60% **(ii)** 60%
(d) **(i)** $87\frac{1}{2}$% **(ii)** 87.5%
(e) **(i)** $43\frac{3}{4}$% **(ii)** 43.75%
(f) **(i)** $83\frac{1}{3}$% **(ii)** $83.\dot{3}$ %
(g) **(i)** $56\frac{1}{4}$% **(ii)** 56.25%
(h) **(i)** $55\frac{5}{9}$% **(ii)** $55.\dot{5}$ %
(i) **(i)** $159\frac{7}{27}$% **(ii)** $159.\dot{2}5\dot{9}$ %
(j) **(i)** $128\frac{4}{7}$% **(ii)** $128.\dot{5}71\,42\dot{8}$ %
(k) **(i)** $109\frac{1}{11}$% **(ii)** $109.\dot{0}\dot{9}$ %
(l) **(i)** $118\frac{2}{11}$% **(ii)** $118.\dot{1}\dot{8}$ %

2 **(a)** 6.25% **(b)** 2% **(c)** 35% **(d)** 12.5% **(e)** 30%
(f) 17.86% **(g)** 8% **(h)** 4.25% **(i)** 8.57% **(j)** 2.08%

3 **(a)** A **(b)** C

4 95% **5** 60% **6** $63\frac{1}{3}$% **7** 8%

8 15% **9** 15% **10** 37.76% **11** 7.14%

12 $66\frac{2}{3}$% **13** $31\frac{1}{4}$% **14** $22\frac{1}{2}$% **15** 62.5%

16 $151.\dot{1}$ % **17** $8\frac{1}{3}$ % **18** 33%

19 **(a)** Nathan = 75%, Dayo = 77.8%.
Dayo is the more accurate scorer.
(b) Nathan = 73.5%, Dayo = 78.1%
(c) Dayo has improved his percentage from the first game to the second.

20 **(a)** 8.3% **(b)** 12.5% **(c)** 22.2%
(d) The value of the percentage increases because the number of hours representing the 'whole' is decreasing.

21 **(a)** 40% **(b)** 25.6% **(c)** 34.4%

Open-ended – Sample answers

22 \$1.30, \$1.35, \$1.45

23 Bella 160 cm and dad 180 cm; Bella 156 cm and dad 175 cm; Bella 171 cm and dad 192 cm.

Exercise 2.8

1 **(a)** \$12 **(b)** \$30 **(c)** \$32 **(d)** \$20.40
(e) \$209.96 **(f)** \$42.89 **(g)** 45 kg **(h)** 40 kg
(i) 3.6 kg **(j)** 9.28 min **(k)** 29.75 min **(l)** 9.42 min
(m) 1.7 L **(n)** 46.5 L **(o)** 18 L

2 **(a)** B **(b)** B

3 **(a)** C **(b)** C

4 68 questions

5 150 grams

6 42 kg

7 9 minutes

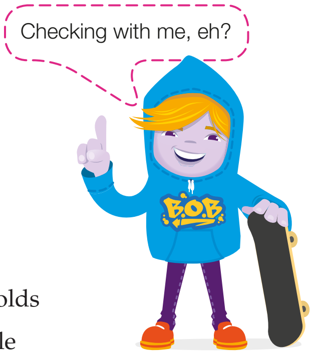
8 392 pages

9 **(a)** 80% **(b)** 4400 households

10 **(a)** 28.6% **(b)** 117 714 people

11 **(a)** 116.4 grams
(b) \$5238

12 **(a)** 10 350 people **(b)** 12 650 people



13 (a) (i) $720 (ii) $1080
(b) (i) $150 (ii) $300

14 104 people 15 4385 tonnes

16 (a) 15 people (b) 97.9% (c) 685

17 (a) 62.5 mL (b) 187.5 mL

18 81.2%

19 (a) $1125 (b) $225 (c) $2610

20 (a) (i) 2240 litres (ii) 13 440 litres
(b) 67.2 litres (c) 201.6 litres (d) 1.25%
(e) No, the water production is less than 1.5% of total volume.

Open-ended – Sample answers

21 (a) That would be 12.6 books and the answer needs to be a whole number.
(b) 20%, 25%, 30%

22 (a) The second class has 15 more basketball players.
(b) 20% is the percentage amount and so means the same as one fifth of the 25 students in the class, or 5 students. 20 students is the number of students out of the class, or 80% of the class. It is easier to compare things when they are in the same form.

Exercise 2.9

1 (a) (i) 1725 (ii) 1875 (iii) 2175 (iv) 3300
(b) (i) 1980 (ii) 1320 (iii) 748 (iv) 110
(c) (i) $103.01 (ii) $368.50 (iii) $86.68 (iv) $86.82
(d) (i) 4.89 km (ii) 1.35 km (iii) 5.05 km (iv) 5.11 km

2 (a) 3.9% (b) 4.2% (c) 14.4% (d) 6.8%

3 C 4 $862.50 5 9.65% 6 6.2%

7 (a) $39 600 (b) $29 250 (c) $5250

8 (a) 68.23 s (b) 3.41 s (c) 4%

9 (a) $419 650
(b) $451 123.75
(c) 22%

10 (a) $2.80
(b) $2.60
(c) 4% increase

11 (a) 16 320
(b) (i) 15 667
(ii) 15 040
(c) The three 4% decreases don’t give the same result as one 12% decrease, because you are decreasing by 4% of a smaller number each time, rather than by 4% of the original number.

12 (a) 32 (b) three more tests

13 (a) 10% increase (b) 9% decrease
(c) In (a), 150 cm represented 100%.
In (b), 165 cm represented 100%.

14 (a) 5865 tonnes (b) 3 years
(c) Theoretically, no, because the decrease gets smaller every time without ever reaching zero. However, it would eventually become unprofitable to continue.

Open-ended – Sample answers

15 Sample answer for a class of 13 students with brown eyes and 11 students with non-brown eyes:
(a) 54.17% (b) 1.83% (c) 45.83% (d) 2.17%
(e) No, the percentage increase for each group is not the same. The group that represents the lower fraction of the class has a greater gain by adding one extra student.

16 Sample answer for a 160 cm student: 12.8 cm or 13 cm taller

Exercise 2.10

1 (a) $416 (b) $16.20 (c) $30.19 (d) $259.97

2 (a) $22.95 (b) $43.50 (c) $20 (d) $54

3 (a) $109.89 (b) $58 (c) $39.74 (d) $241.43

4 (a) (i) $26 (ii) 40%
(b) (i) $450 (ii) 23%

5 (a) 167% profit (b) 56% loss
(c) 63% profit (d) 79% profit

6 D 7 D 8 C

9 (a) (i) $37.90 (ii) $341.10
(b) (i) $79.60 (ii) $119.40
(c) (i) $6 (ii) $14
(d) (i) $24.99 (ii) $74.96

10 (a) $8000 (b) 26.7%

11 (a) (i) $56 (ii) $616
(b) (i) $6.40 (ii) $70.40
(c) (i) $1.90 (ii) $20.90
(d) (i) $264 (ii) $2904

12 (a) $720 (b) $72 (c) $792

13 (a) $245 (b) $2695

14 (a) $125 (b) 64.1%

15 (a) $162.50 (b) 46.43%

16 (a) $550 (b) 22.45%

17 $450

18 (a) $180 (b) $55.56%

19 (a) $29.95 (b) $253.64 (c) $27 271.82 (d) $36.32

20 $5.14

21 They are easy for people to estimate or calculate mentally, as they correspond to common fractions.

22 **(a)** \$2.30 **(b)** \$4.25 **(c)** 315%

23 **(a)** **(i)** 20% **(ii)** 40% **(iii)** 3.57% **(iv)** 25%

(b) **(i)** 16.67% **(ii)** 28.57% **(iii)** 3.45% **(iv)** 20%

(c) Calculating the profit using the cost price gives a higher percentage. This is because when you have made a profit the cost price will be lower than the selling price, so calculating the same amount as a percentage of the cost price will always give a higher figure.

24 **(a)** \$57.38 **(b)** 27.5% **(c)** 22.5% less profit

25 \$28

26 The second store that is offering 20% off. Here, the T-shirts will cost \$47.88, \$2 less than the other store's 'buy two, get the third half price' offer.

27 **(a)** \$47 250 **(b)** \$10 500

(c) 4.5 years, but if profit is paid only once a year, then 5 years.

28 **(a)** **(i)** $25\% = \frac{1}{4}$ **(ii)** $10\% = \frac{1}{10}$

(iii) $20\% = \frac{2}{10}$ or $\frac{1}{5}$ **(iv)** $33\frac{1}{3}\% = \frac{1}{3}$

(b) To find 25% off, Chantal can find a quarter of the price by dividing by 4 (or halving twice), then subtracting this amount. To find 10% off, Chantal should divide by 10, which is as simple as moving the decimal point 1 place to the left, then subtracting this amount. To find 20% off, Chantal could find 10%, then multiply this amount by 2, or divide by 5 and then subtract this amount. To find $33\frac{1}{3}\%$, she needs to find a third of the price by dividing by 3, and subtracting this amount.

Open-ended – Sample answers

29 50% off, sale price = 0.5 × \$299 = \$149.50
25% off, sale price = 0.75 × \$299 = \$224.25
10% off, sale price = 0.9 × \$299 = \$269.10

30 Sometimes it will look more impressive for a store to give a saving as a percentage, particularly when the price is high and a percentage is easily identifiable. In other cases, a dollar discount may look more attractive, particularly when the equivalent percentage discount is relatively small.

31 The second discount of 20% applies to the reduced price, not to the original price. The shop assistant discounted \$150 by 30% to give a sale price of \$105, which is then discounted by a further 20%; i.e. 0.8 × \$105 = \$84.

Challenge 2

1 A

$$\frac{2}{3} + 0.7 + 55\% = \frac{2}{3} + \frac{7}{10} + \frac{55}{100} = \frac{40}{60} + \frac{42}{60} + \frac{33}{60} = \frac{115}{60}$$

$$\frac{115}{60} \div 3 = \frac{115}{180} = \frac{23}{36}$$

2 Ate $\frac{1}{4}$, then ate $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$, then ate $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Amount left $= 1 - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3}\right) = \frac{1}{6}$.

$\frac{1}{6}$ of the tub of yoghurt remained.

3 $\left(\frac{1}{4} + \frac{1}{2}\right) \div 2 = \frac{3}{4} \div 2 = \frac{3}{8}$

4 Area of new piece is 4 times original piece.
cost = \$25 × 4 = \$100

5 A

save \$1.70 gives a price of 5.04 − 1.70 = \$3.34

cost price of \$3.20 + 5% = 3.2 × 1.05 = \$3.36

discount of $33\frac{1}{3}\% = 5.04 \times 66\frac{2}{3}\% = 5.04 \times \frac{2}{3} = \$3.36$

buy 2, get 1 free $= \frac{5.04 \times 2}{3} = \$3.36$

6 8 × 8 + 8 ÷ 8 = 65

7 $\frac{1}{2} + \frac{0.1}{2} + \frac{1}{0.2} = 0.5 + 0.05 + 5 = 5.55$

8 C largest $= 5\frac{3}{4}$, smallest $= 1\frac{2}{5}$

difference $= 5\frac{3}{4} - 1\frac{2}{5} = 4\frac{7}{20}$

9 2, B and D are true

10 B 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28,
2 + 3 + 4 + 5 + 6 + 7 + 8 = 28 + (-1 + 8) = 35.

Each time you need to add 7 to the first of the seven consecutive numbers to get the last number in the next set.

11 $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

$\frac{7}{8} + \frac{1}{16} = \frac{14}{16} + \frac{1}{16} = \frac{15}{16}$

This pattern continues until

$\frac{4095}{4096} + \frac{1}{8192} = \frac{8190}{8192} + \frac{1}{8192} = \frac{8191}{8192}$

12 Glass contains $\frac{5}{12}$ of the first liquid and $\frac{7}{12}$ of the second liquid:

percentage water $= \frac{5}{12} \times 22.5\% + \frac{7}{12} \times 27\% = 25.125\%$

Chapter review 2

1 **(a)** $\frac{29}{50}$ **(b)** $\frac{31}{2500}$ **(c)** $2\frac{121}{200}$

2 **(a)** $-\frac{3}{10}$, -0.283, $\frac{2}{9}$, 0.26, $\frac{3}{4}$, $\sqrt{7}$

(b) $-\frac{1}{3}$, -0.26, 0.22, 0.253, $\frac{7}{10}$, $\sqrt{2}$

3 **(a)** 28.625 **(b)** 15.075 **(c)** 53.815 625

4 Four 60 cm lengths, with 10 cm left over.

5 **(a)** 13.25 terminating **(b)** 9.230 384 6… irrational
(c) 1.222 222… recurring **(d)** 11.180 339… irrational
(e) 5.5 terminating

6 B

7 **(a)** $\frac{2}{9}$ **(b)** $\frac{29}{90}$ **(c)** $\frac{32}{99}$

8 **(a)** $\frac{1}{4}$ **(b)** $-\frac{5}{18}$ **(c)** $\frac{1}{4}$ **(d)** -15

9 **(a)** -0.6 **(b)** -1.6 **(c)** -25.62 **(d)** -4.08

10 B

11 **(a)** 46% **(b)** 120% **(c)** 280% **(d)** $62\frac{1}{2}\%$

12 **(a)** 60% **(b)** 35% **(c)** 112% **(d)** 3%

13 **(a)** $\frac{13}{20}$ **(b)** $\frac{77}{50}$ or $1\frac{27}{50}$ **(c)** $\frac{7}{120}$ **(d)** $\frac{47}{1000}$

14 **(a)** 0.8 **(b)** 0.07 **(c)** 0.0209 **(d)** 0.0625

15 **(a)** 90% **(b)** 218.75% **(c)** 2% **(d)** 6.67%

16 **(a)** 29.4 m **(b)** 88 kg **(c)** 330 L **(d)** $6.72
(e) $23.60 **(f)** $8.55

17 **(a)** **(i)** $607.50 **(ii)** $752.40
(b) **(i)** $15.96 **(ii)** $18.45

18 **(a)** $76.50 **(b)** $1710 **(c)** $89 **(d)** $70.27

19 17.24% loss **20** C **21** -$61.90

22 **(a)** 13 bags **(b)** 125 g left

23 $\frac{7}{40}$ **24** 22 500 tonnes

25 70.92% **26** 3 830 040 km^2

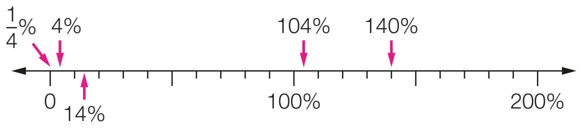
27 **(a)** $3200 **(b)** 34%

28 20% **29** $614

30 **(a)** 28 cents per pen **(b)** 35 cents per pen

31 56.5% **32** -1.3 °C

33



34 45 g **35** **(a)** B **(b)** B

36 **(a)** 602 896.5 ML **(b)** 110 440.5 ML **(c)** 30 778.5 ML

37 **(a)** $65.85 **(b)** $5.99 **(c)** $26.61
(d) $49.40 **(e)** $4.49 **(f)** $11.66

38 **(a)** $39.58\dot{3}$ cents **(b)** 40 cents **(c)** 5 cents
(d) This extra amount comes from the difference between

$40 \times 12 - 39.58\dot{3} \times 12 = 5$

Numeracy practice 2

1 3 **2** D **3** $449 400 **4** B

5 B **6** B **7** 25 cents **8** 70%

Mixed review A

1 **(a)** 16 **(b)** -13 **(c)** -22 **(d)** -12
(e) 36 **(f)** -21 **(g)** -4 **(h)** -6

2 **(a)** $\frac{9}{25}$ **(b)** $\frac{1}{20}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{1}{500}$

3 **(a)** $436.80 **(b)** $996.80

4 **(a)** 200 **(b)** 81 000 **(c)** -80 **(d)** -48 **(e)** 45
(f) 3136 **(g)** 12 **(h)** 32 **(i)** 108

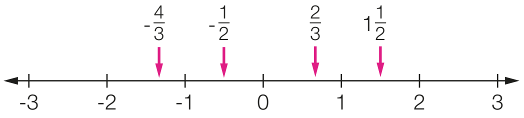
5 **(a)** 85% **(b)** 60% **(c)** 62.5% **(d)** 57.14%

6 **(a)** $0.\dot{3}$ **(b)** $0.\dot{6}$ **(c)** $0.1\dot{6}$ **(d)** $0.\dot{5}$

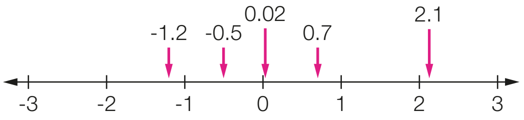
7 **(a)** **(i)** 0.7 **(ii)** $\frac{7}{10}$
(b) **(i)** 0.45 **(ii)** $\frac{9}{20}$
(c) **(i)** 1.2 **(ii)** $\frac{6}{5}$
(d) **(i)** 0.08 **(ii)** $\frac{2}{25}$

8 **(a)** $48 **(b)** 455 kg **(c)** 3.57 m
(d) 0.48 s **(e)** $15.13 **(f)** 440 m

9 **(a)**



(b)



10 C

11 **(a)** -12 **(b)** 7

12 **(a)** $29.60 **(b)** $44.40

13 **(a)** $6^5 \times 5^9 = 15\,187\,500\,000$
(b) $7^6 \times 10^4 \div (5^3 \times 2^2) = 2\,352\,980$
(c) $10^2 = 100$ **(d)** $4 \times 5^2 = 100$

14 **(a)** **(i)** 30×2^6 **(ii)** 30×2^{12}
(b) **(i)** 1920 **(ii)** 122 880

15 $-6\frac{2}{3}$ °C

16 $(2 \times 3)^2 = 6^2$
$= 36$
$2^2 \times 3^2 = 4 \times 9$
$= 36$

17 **(a)** $52 **(b)** $71.20
(c) $19.20 **(d)** $6.80

18 Match 1: 71.4% Match 2: 75%

Match 3: 72.2% Match 4: 75%

Matches 2 and 4; Match 3; Match 1

Casey is not steadily improving—her accuracy fluctuates. She is fairly consistent, however.

19 (a) B (b) B

20 Ali has interpreted 2^4 as 2×4, 3^3 as 3×3 and 5^2 as 5×2. He needs to remember that the index number tells you how many of the base number are multiplied together; e.g. $2^4 = 2 \times 2 \times 2 \times 2$.

Chapter 3

Recall 3

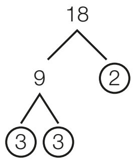
1 (a) $x + 10$ (b) $\frac{m}{3}$ (c) $2l$ (d) $20 - y$

2 (a) 18 (b) -36 (c) 2 (d) 4

3 (a) $l = 9$ (b) $l = -3$

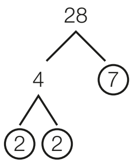
4 (a) F (b) T (c) F (d) T

5 (a)



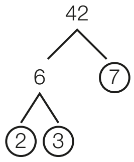
$18 = 2 \times 3^2$

(b)



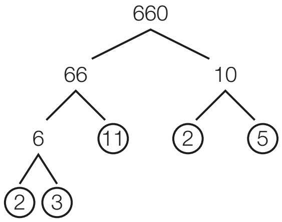
$28 = 2^2 \times 7$

(c)



$42 = 2 \times 3 \times 7$

(d)



$660 = 2^2 \times 3 \times 5 \times 11$

6 (a) 3^4 (b) 6^3 (c) 14^2 (d) 5^7

7 (a) $9 \times 9 = 81$

(b) $(-3) \times (-3) \times (-3) \times (-3) = 81$

(c) $10 \times 10 \times 10 \times 10 - 2 \times 2 \times 2 \times 2 \times 2 = 9968$

8 (a) 2 (b) 5 (c) 2

Exercise 3.1

1 (a) (i) expression (ii) 3 (iii) x, $3b$, -4
(iv) x, b (v) -4

(b) (i) equation (ii) 4 (iii) $-2a$, $4b$, $-c$, 5

(iv) a, b, c (v) 5

(c) (i) equation (ii) -1 (iii) $7a$, $-b$, $3c$, a

(iv) a, b, c (v) none

(d) (i) expression (ii) 7 (iii) $3d$, $4f$, $-3bd$, -3, $7b$

(iv) d, f, b (v) -3

(e) (i) equation (ii) -3 (iii) $5g$, $-3r$, $5rf$, 7, $-3b$

(iv) g, r, f, b (v) 7

(f) (i) expression (ii) 1 (iii) b, f, $-3bf$, 8

(iv) b, f (v) 8

2 (a) $4cd$ (b) $\frac{2t}{7}$ (c) $4gk^2$ (d) $\frac{6}{fg}$

3 (a) $w + r$ (b) $w + r = 8$ (c) $2w$

(d) $r - 2$ (e) $2w + r - 2 = 14$ (f) $w + r - 2$

4 (a) A (b) C (c) A (d) B

5 (a) A (b) D

6 (a) $\frac{x}{6}$ (b) $\frac{h}{9}$ (c) $\frac{6a}{11}$ (d) $\frac{15}{3r}$

(e) $\frac{21}{12v}$ (f) $\frac{4s}{19}$ (g) $\frac{8}{x} - \frac{u}{6}$ (h) $\frac{h}{5} + \frac{4}{i}$

(i) $\frac{cu}{5} + 9y$ (j) $\frac{q}{7c} - \frac{gh}{4}$ (k) $\frac{vz}{6} - \frac{8}{fs}$ (l) $\frac{3}{rt} + \frac{6w}{yz}$

(m) $\frac{4bh}{2r}$ (n) $\frac{6ac}{5eu}$

7 D

8 (a) $4x$ (b) $5 + x$ (c) $2x - 10$ (d) $4(x + 2)$

9 (a) Kayla has seven more pencils than Andrew.

(b) Simon has twice as many pencils as Andrew.

(c) Emily has two fewer pencils than Simon.

(d) $y - 3$ (e) $y + 10$ (f) $2y - 3$ (g) $2y + 1$

10 (a) $m + n$ (b) $n - 2$ (c) $m + n - 2$

(d) $m + 8$ (e) $m + n + 6$

11 (a) $2x$

(b) There are 3 times as many people as at 3 pm.

(c) $3x - 5$ (d) $3x + 2$

12 (a) $2g$ (b) $g - 5$ (c) $g + 4$

13 (a) $2m + 1$ (b) $2m + 4$ (c) $\frac{m}{3} + 2$

14 D

15 (a) 10:10 pm (b) 7 points (c) 10:40 pm

(d) 10 points (e) 10:25 pm, 10:35 pm

16 At 6:00 pm there were 20 fewer people than expected. Two people arrived at 10 past, another three at quarter past, and four more at 20 past. Five minutes later one left, but another six people arrived five minutes after that. At 7:00 pm three more people arrived; this was the greatest number for the night. Ten left at 8:00 pm, another five at 9:30 pm, and the last two at 11:00 pm.

Open-ended – Sample answers

17 (a) The number of kangaroos originally in the park.

(b) Twice the number of kangaroos originally in the park.

(c) Four more than the number of kangaroos originally in the park.

(d) One less than half the number of kangaroos originally in the park.

18 Refer to Exercise 3.1 Question **3**.

Exercise 3.2

1 **(a)** 15 **(b)** 21 **(c)** 36 **(d)** 18 **(e)** 54
(f) 177 **(g)** 0 **(h)** 174 **(i)** 18 **(j)** 108
(k) 432 **(l)** 108 **(m)** -207 **(n)** 9 **(o)** 54
(p) 3 **(q)** $\frac{3}{4}$ **(r)** 2 **(s)** 2 **(t)** 96
(u) $224\frac{1}{2}$

2 **(a)** -7 **(b)** -64 **(c)** -45 **(d)** 3 **(e)** -4
(f) 11 **(g)** 20 **(h)** 34 **(i)** -31 **(j)** 21
(k) -133 **(l)** -125

3 **(a)**

x	-2	0	-4	-8
y	-4	-2	-6	-10

(b)

x	-5	-3	-2	-7
y	-9	-7	-6	-11

(c)

x	-7	-1	-5	-9
y	-16	-10	-14	-18

(d)

x	1	2	3	4
y	3	7	11	15

(e)

x	-2	0	1	2
y	-8	-4	-2	0

(f)

x	-4	-7	-20	11
y	2	-7	-46	47

(g)

x	-3	-2	-1	-5
y	18	12	6	30

(h)

x	-1	0	3	5
y	2	-3	-18	-28

(i)

x	-3	-1	2	4
y	15	7	-5	-13

(j)

x	8	6	12	20
y	4	3	6	10

(k)

x	15	9	12	30
y	3	1	2	8

(l)

x	12	-4	8	-24
y	2	-2	1	-7

(m)

x	0	1	2	3
y	0	1	4	9

(n)

x	-2	0	2	4
y	-17	-1	15	127

(o)

x	0	2	4	8
y	1	11	29	89

4 **(a)** 24 **(b)** 150 **(c)** 95 **(d)** 2
(e) -20 **(f)** $-\frac{5}{2}$ **(g)** −2 **(h)** -12
(i) 12 **(j)** 113 **(k)** 13 **(l)** 238

5 **(a)** 22 **(b)** 0 **(c)** 0
(d) 180 **(e)** -36 **(f)** -60
(g) $\frac{1}{3}$ **(h)** $\frac{4}{5}$ **(i)** 5
(j) -2 **(k)** $-\frac{1}{3}$ **(l)** $-\frac{33}{8}$ or $-4\frac{1}{8}$

6 D **7** A **8** C

9 **(a)** dy **(b)** km **(c)** $(km - dy)$
(d) $(5 \times 21) - (12 \times 7) = 105 - 84 = \$21$ profit
(e) No, because 21 – 21 = -\$0. Boon makes no profit.

10 **(a)** $3c - a$ **(b)** $a + 2b - 3c$ **(c)** \$25

11 **(a)** 2, otherwise Ravi would have scored a negative number of runs.
(b) $4 = 2 \times 4 - 4$
(c) Ravi scored more runs, as if $r > 4$, $2r - 4 > r$.

12 **(a)** 45 minutes
(b) $\frac{6600}{50} = 132$ minutes, which is more than 2 hours.
(c) $\frac{6600}{55} = 120$ minutes, so it is now possible to package the 2200 biscuits in 2 hours.

Open-ended – Sample answers

13 $M = 1, N = 1; M = 3, N = 1$

14 **(a)** $a = -6, b = -2, c = 8$
(b) $a = -10, b = 10, c = 1; a = 2, b = 10, c = -5; a = 4, b = 5, c = -5$

15 $a = 2, b = 1; a = 3, b = 2; a = 4, b = 4$

Exercise 3.3

1 **(a)** 40 m **(b)** 22 m **(c)** $33\frac{1}{3}$ m **(d)** 8.2 m

2 **(a)** 122 **(b)** 201 **(c)** 871

3 **(a)** 50 °F **(b)** 41 °F **(c)** 95 °F

4 **(a)** 0 °C **(b)** 5 °C **(c)** 10 °C

5 **(a)** 32 **(b)** 6.75

6 D **7** B

8 **(a)** **(i)** \$4 **(ii)** \$8 **(iii)** \$17.60
(b) not correct

9 **(a)** \$250 **(b)** \$750 **(c)** 8

10 **(a)** 19.6 m **(b)** 78.4 m **(c)** 490 m

11 **(a)** 50 °F **(b)** The answers are the same. **(c)** 40 °F
(d) The answer to part **(c)** is 1 °F less than the answer to Question **3(b)**.

(e) 100 °F

(f) The answer to part **(e)** is 5 °F greater than the answer to Question **3(c)**.

(g) Sample answer: Around 10 °C there is little difference. Lower than 10 °C it gives slightly lower temperatures, but at higher temperatures it gives a much greater difference in temperature. At 0 °C it is 2 °F less, at 40 °C it is 6 °F more.

12 **(a)** 48 000 cm^3 **(b)** 36 000 cm^3 **(c)** 180 000 cm^3

13 **(a)** **(i)** 163 **(ii)** 171

(b) No: the number of fish calculated is quite close

(c) 5

14 **(a)** \$7 **(b)** \$600

(c) **(i)** Aussie Bank **(ii)** \$10 **(iii)** Matilda Bank; \$40

15 **(a)** 15 000 m^3 **(b)** 3000

Open-ended – Sample answers

16 10 cm × 2 cm; 5 cm × 4 cm

17 100, 20 and 21; 100, 20 and 24

Exercise 3.4

1 **(a)** like **(b)** not like **(c)** not like **(d)** like

(e) not like **(f)** like **(g)** like **(h)** not like

(i) like **(j)** like **(k)** like **(l)** not like

2 **(a)** $17a$ **(b)** $21f$ **(c)** $5v$

(d) $-3v$ **(e)** $10w$ **(f)** $3d$

(g) $12d + 9$ **(h)** $12f + 5$ **(i)** $7r - 3u$

(j) $-7m - 11n$ **(k)** $3f + 2fg$ **(l)** $6ij + 20j$

3 **(a)** $3dhw + 2dh + 5d + 8$ **(b)** $10x + 4xyz + 8$

(c) $-8x^2 - 10y + 2$ **(d)** $-5x^3 - 6y + xy$

(e) $18ab + 24a + 15b + 40$ **(f)** $4s - 60st + 20$

(g) $-b - 5b^3 - 5ab$ **(h)** $2a^3 - 8a^2 - 9a - 61$

4 B **5** D **6** D

7 **(a)** $15t + 7d$, 52 **(b)** $90w + 134v$, 358 **(c)** $10a + 2b$, 22

(d) $13f + 3g$, 25 **(e)** $-4b$, -20 **(f)** $5b$, 20

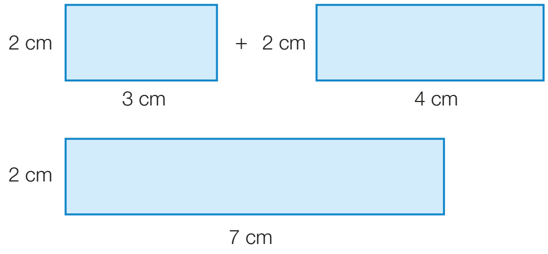
8 **(a)** T **(b)** F **(c)** T

9 **(a)** $8c + 5d$ **(b)** $7c + 6d$

(c) $8c + 5d + 7c + 6d = 15c + 11d$

10 **(a)** $27d$ **(b)** $40d$ **(c)** \$200

11 **(a)**



(b) $3y + 4y = 7y$

12 **(a)** A, x and x^2 are not like; D, x^2y and xy^2 are not like

(b) B: $5x^2$; x^2 and x^2 are like

C: $8xy$; xy and yx are like

E: $6xy$; xy and xy are like

F: $3xy$; xy and xy are like

13 **(a)** $11x$

(b) Ryan has 4 times as many coins as Jess.

(c) $510x$ cents or \$$5.1x$

(d) No. You don't know how many coins he has of each value.

14 **(a)** $32x$ **(b)** $63x$ **(c)** \$$63vx$

(d) Sample answer: $x = 3$, $v = 2$

Open-ended – Sample answers

15 $a = 10$, $b = 10$; $a = 3$, $b = 4$

16 $x = 1$, $y = 1$; $x = 2$, $y = 2$

17 **(a)** Lindy forgot that xy means $x \times y$ and that $xy = 1xy$. Only coefficients can be added and subtracted.

(b) If Lindy wrote a 1 in front of the xy, it might remind her to add and subtract only the coefficients of like terms.

Half-time 3

1 \$$(2n + m)$

2 **(a)** $17x$ **(b)** $5y$ **(c)** $2x + 6y$

(d) $6x^2 + 3xy$ **(e)** $10b - 16a$ **(f)** $x + 2a + 2a^2$

3 **(a)** 1 **(b)** 22 **(c)** 8 **(d)** 58 **(e)** 485 **(f)** 255

4 **(a)**

x	-2	0	1	2
y	-10	-4	-1	2

(b)

x	-3	-1	2	4
y	-3	1	7	11

(c)

x	-1	0	3	5
y	5	3	21	53

5 **(a)** 36 **(b)** 38 **(c)** -1 **(d)** $11\frac{1}{9}$ **(e)** 520

6 **(a)** $t + 1$ **(b)** $t - 1$

7 **(a)** $9xy + 3x - 2y$ **(b)** $7x^2 - 4x - 3xy$

(c) $9m^2 - 4m + 5$ **(d)** $8pq + 4p^2q - 3p^2q^2 + 4pq^2$

8 **(a)** **(i)** 2 **(ii)** 2 **(iii)** 3

(b) **(i)** 3 **(ii)** -5 **(iii)** -4

(c) **(i)** 3 **(ii)** 1 **(iii)** 7

(d) **(i)** 2 **(ii)** -2 **(iii)** 0

(e) **(i)** 3 **(ii)** $-\frac{1}{2}$ **(iii)** -5

(f) **(i)** 3 **(ii)** 3 **(iii)** 6

(g) **(i)** 3 **(ii)** 1 **(iii)** -2

(h) **(i)** 2 **(ii)** -12 **(iii)** -10

(i) **(i)** 1 **(ii)** 7 **(iii)** 0

Exercise 3.5

1 (a) $15a$ (b) $12a$ (c) $18g$ (d) $56z$
(e) $36z$ (f) $54p$ (g) $66ef$ (h) $72mn$
(i) $132jk$ (j) $18ghk$ (k) $48ghk$ (l) $90beh$
(m) $-20y$ (n) $21y$ (o) $16a$ (p) $-44ab$
(q) $18qru$ (r) $-56qru$ (s) $80jk$ (t) $-60jk$
(u) $-48uv$

2 (a) $2a$ (b) $4b$ (c) $7e$ (d) 3
(e) $\frac{5}{6}$ (f) $\frac{1}{15}$ (g) $3ab$ (h) $3f$
(i) $2b$ (j) $\frac{f}{8}$ (k) $\frac{n}{4}$ (l) $\frac{e}{7}$
(m) -2 (n) -2 (o) 4 (p) $\frac{1}{5g}$
(q) $\frac{1}{11h}$ (r) $\frac{1}{4j}$

3 (a) $4c$ (b) $10d$ (c) $2g$ (d) 6
(e) $\frac{2}{3}$ (f) $\frac{10}{11}$ (g) $2cd$ (h) $7cd$
(i) $5gh$ (j) $\frac{h}{3}$ (k) $\frac{5h}{2}$ (l) $\frac{25h}{8}$
(m) $-2d$ (n) $-\frac{5b}{3}$ (o) $-5c$ (p) $-\frac{3}{2}$
(q) $\frac{1}{6}$ (r) 11

4 D 5 D 6 A 7 B

8 $60abcd$

9 (a) $18hlw$ (b) $3hjl$
(c) $72hl$ and $36hl$ (d) 2

10 (a) $x, 1.2x, 1.44x$ (b) $3.64x$ mm
(c) $\$3.64dx$ (d) $2y$
(e) $\$6cy$ (f) $\$(3.64dx + 6cy + f)$
(g) $1540

Open-ended – Sample answers

11 Joshua missed the × sign. The correct answer is $-6k^2$.

12 $-3p^2r$ and $7pr^2$ 13 $3mn$, $2mn^2$ and mn^2

Exercise 3.6

1 (a) 2121 (b) 1734 (c) 1616 (d) 2254 (e) 4059
(f) 3589 (g) 594 (h) 38 380 (i) 70 400

2 (a) $5x + 60$ (b) $2s - 10$ (c) $12a + 60$
(d) $4s + st$ (e) $6l - lr$ (f) $2a + ah$
(g) $12x + 15$ (h) $54x - 9$ (i) $6h + 8$
(j) $2mn + 2mr$ (k) $3pr - 3pz$ (l) $6pr + 6pt$
(m) $5t^2 + 10t$ (n) $10p^2 - 14p$ (o) $16m + 8m^2$
(p) $15m^2 + 21mn$ (q) $4kp - 20k^2$ (r) $20x^2 - 50xy$

3 (a) $-2a - 8$ (b) $-3b + 27$ (c) $-c - d$ (d) $-d + x$
(e) $-ex - 7e$ (f) $-f^2 + fy$ (g) $-15g - 50$ (h) $-24 + 56h$
(i) $-22i - 44l$ (j) $-16jx - 24jy$ (k) $-12klx + 45kly$
(l) $-36mnp + 60mn^2p$

4 (a) $35 + 6d$ (b) $24 - 20e$ (c) $-6mn - 24n$
(d) $-3xy - 8x$ (e) $35t - 60kt$ (f) $-5v - 90vw$
(g) $74mn - 40m$ (h) $-14ab - 4a$ (i) $75x - 21xy$
(j) $18tz - 108t$

5 (a) $-x - 8$ (b) $3x - 2$ (c) $2 - 3x$
(d) $21 - 3x$ (e) $6 - 10x$ (f) $4x + 6$
(g) $7 - x$ (h) -68 (i) $-26x + 27$
(j) -14 (k) $x - 27y$ (l) $2x - 24y$
(m) $cd - 6ce$ (n) $de + 2df - 16bd$ (o) $2mn - 5mt + nt$
(p) $2ap + 2aq - 2pq$

6 (a) C (b) C

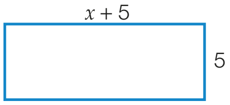
7 D

8 $2a(3a - 2b) = 6a^2 - 4ab$

9 (a) (i) $33x + 165$ (ii) 3465
(b) (i) $105y + 315$ (ii) 3465
(c) Both are ways of evaluating 105 × 33 using the distributive law.

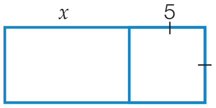
10 (a) $v - u$ (b) $6v - 6u$ (c) $9v - 6u$

11 (a)



(b) $5(x + 5)$

(c)



(d) area of left part = $5x$
area of right part = 25

(e) The area of the first rectangle is $5 \times (x + 5)$ and the second rectangle is $5x + 25$. Both rectangles are the same size, therefore $5 \times (x + 5) = 5x + 25$.

12 She multiplied $x\left(x + \frac{1}{x}\right) = x^2 + 1$.

Open-ended – Sample answers

13 Simplify $x(18 - y) + 2x$.

14 (a) The −2 at the end of the first line should be +2.
(b) $3(2x - 1) - (5x - 2) = x - 1$
(c) If there is a minus in front of a bracket, write a 1 between the minus and the bracket; i.e. $3(2x - 1) - 1(5x - 2)$.

15 $6e(2f + 2)$, $2ef(6 + 2g)$, $f(12e + 6f + 4)$

Exercise 3.7

1 **(a)** 24 **(b)** 150 **(c)** 126 **(d)** 16 **(e)** 5 **(f)** 9

2 **(a)** 3 **(b)** 7 **(c)** 11 **(d)** 4 **(e)** 15
(f) 6 **(g)** $2p^2$ **(h)** $3t$ **(i)** $5d^3$ **(j)** fm^2
(k) a^2b **(l)** x^2y^2 **(m)** $5mn^2$ **(n)** $3st$ **(o)** $3gh$

3 **(a)** $3(t+5)$ **(b)** $2(h+7)$ **(c)** $2(a-4)$
(d) $3(3p+1)$ **(e)** $5(4p-1)$ **(f)** $3(5z+1)$
(g) $4(3-5b)$ **(h)** $2(12+5y)$ **(i)** $2(9-7q)$
(j) $3(px+4)$ **(k)** $2(kt-10)$ **(l)** $2(2hs+7)$
(m) $-8(ab+2)$ **(n)** $-5(mn+3)$ **(o)** $-12(kr+11)$

4 **(a)** $3x(l+5)$ **(b)** $10m(p-2)$ **(c)** $14q(p+2)$
(d) $3x(2v-1)$ **(e)** $10a(3b+2)$ **(f)** $2y(9x-20)$
(g) $2p(8+7q)$ **(h)** $5l(3-4m)$ **(i)** $11j(2-5l)$
(j) $10m(3z-2)$ **(k)** $4k(8h+9)$ **(l)** $4h(10+7i)$
(m) $-4b(3a-4)$ **(n)** $-7x(2y-3)$ **(o)** $-8m(9+7n)$

5 **(a)** $a(a+1)$ **(b)** $b(b+4)$ **(c)** $c(c-1)$ **(d)** $d(1-d)$

6 **(a)** $3a(a+1)$ **(b)** $2b(b+2)$ **(c)** $5c^2(1+c)$ **(d)** $6d(d-1)$

7 D **8** B **9** $\$5(ab+5c)$

10 **(a)** $\frac{mv^2}{2}+mgh$ **(b)** $m\left(\frac{v^2}{2}+gh\right)$

11 **(a)** $(x+1)(3-y)$ **(b)** $(3x-4)(7+y)$ **(c)** $(x+7)(5+2y)$

12 **(a)** $a(x+y)$ **(b)** $b(x+y)$ **(c)** $a(x+y)+b(x+y)$
(d) $(x+y)(a+b)$ **(e)** $(3+y)(x+b)$

13 **(a)** $(a+1)$ **(b)** $(a+1)(4+b)$

Open-ended – Sample answers

14 $8a^2b$ and $16c$; $16b^3$ and $24c^2$; $48pr$ and $56s$

15 **(a)** area $= x(10-x) \Rightarrow L = 10 - x$
(b) $x = 1, 2, 3$ or 4

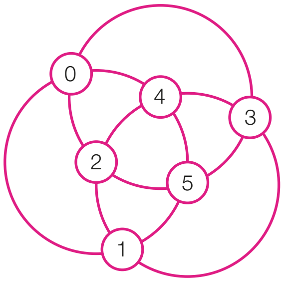
16 Mohammed is correct. Stephen has made the mistake of not taking the factor out of the constant term, 3. He can avoid this mistake by expanding the brackets to check that his factorised expression equals his original expression.

Challenge 3

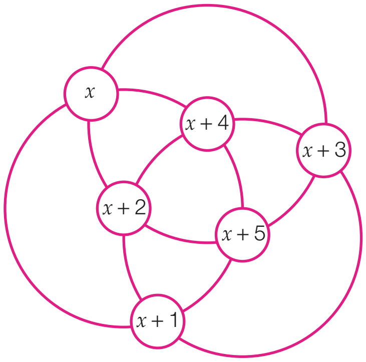
1 $x + (x+2) + (x+4) + (x+6) = 140$, $4x + 12 = 140$, $4x = 128$, $x = 32$. Numbers are 32, 34, 36, 38.

2 $1+3+5+7$, $1+5+5+5$, $1+3+3+9$, $1+1+3+11$, $1+1+5+9$, $1+1+7+7$, $1+1+1+13$, $3+3+5+5$, $3+3+3+7$. Nine ways.

3 **(a)** The circles contain the numbers: 0, 2, 3, 5; 0, 1, 4, 5; 1, 2, 3, 4.



(b) Match the position of the expressions with the positions of 0, 1, 2, 3, 4, 5 in that order from **(a)**.



(c) 3, 4, 5, 6, 7, 8 or any $x+1$, $x+2$, $x+3$, $x+4$, $x+5$, $x+6$

4 **(a)** $3^6 = (3^2)^3 = 9$; $4^6 = (4^2)^3 = 16^3$
(b) $(3^2)^3 + (4^2)^3 = 9^3 + 16^3 = 25 \times 193$
Hence, 25 is a factor.
(c) $(3^2)^3 - 5^3 = 9^3 - 5^3 = 4 \times 151$
Hence, 4 is a factor.

5

$x+8$	$x-6$	$x-5$	$x+5$
$x-3$	$x+3$	$x+2$	x
$x+1$	$x-1$	$x-2$	$x+4$
$x-4$	$x+6$	$x+7$	$x-7$

6 A

$$\frac{a+\frac{1}{b}}{b+\frac{1}{a}} = \frac{\frac{ab}{b}+\frac{1}{b}}{\frac{ab}{a}+\frac{1}{a}} = \frac{\frac{ab+1}{b}}{\frac{ab+1}{a}} = \frac{ab+1}{b} \times \frac{a}{ab+1} = \frac{a}{b}$$

Chapter review 3

1 **(a)** They have three more power points in their house than you do.
(b) They have two fewer power points in their house than you do.
(c) They have six times as many power points as you do.

2 **(a)** 22 **(b)** 28 **(c)** 20 **(d)** 4

3 **(a)** 7 **(b)** 120 **(c)** -48

4 **(a)** -4 **(b)** 11

5 **(a)** 7 **(b)** 8 **(c)** -10

6 **(a)** 630 **(b)** ±9

7 **(a)** $-10j$ **(b)** $3ty$ **(c)** $3ghi$ **(d)** $19ab+7$
(e) $-a+23b$
(f) cannot be simplified by collecting like terms

8 **(a)** $30ab$ **(b)** $66abc$ **(c)** $14ab$

9 **(a)** $\frac{6}{5}$ **(b)** $5b$ **(c)** $\frac{b}{11}$

10 (a) $14 + 7a$ (b) $2ab + 6a$ (c) $12a^2 + 20a$

11 (a) $8 + 9a$ (b) $74a + 13ab$ (c) $21a^6 + 16a^3$

12 (a) $3(a - 5)$ (b) $4(5ab + 4)$ (c) $-5(3p - 4q)$

13 Melissa has one packet, and for the next four minutes she eats 2 jelly beans per minute, and 6 during the fifth minute. Roderigo buys two packets in the first minute. He then eats 10 in the second minute, finishes the first packet in the third minute, eats 6 more in the fourth minute, and finishes the second packet in the fifth minute. Huong buys three packets in the second minute. He then eats 5 in the third minute, finishes the first pack in the fourth minute, and eats 8 in the fifth minute.

14 (a) $t + g + b$ (b) $t + g + \frac{b}{2} - 14$ (c) $t + g + \frac{b}{2} - 7$

15 Mei 158 cm; Sienna 159 cm, Liam 157 cm

16 Yumi 180 cm **17** -24

18 (a) $u + r + d$ (b) $10(u + r + d)$ (c) 140 cm

19 1.40 metres per second

20 (a) $2x + 4$ (b) $6x - 2$

(c) $3(2x + 4) + 5(6x - 2)$ (d) $36x + 2$

(e) 434 apples

21 (a) $7LW$ (b) \$14.00

22 (a) $2x(3x + 2)$ (b) $6x^2 + 4x$ (c) 112 cm^2

23 (a) (i) $\frac{l}{4}(w - 6)$ (ii) $\frac{lw}{4} - \frac{3l}{2}$

(b)
$$\begin{aligned} A_{\text{left}} &= lw - \left(\frac{lw}{4} - \frac{3l}{2}\right) \\ &= lw - \frac{lw}{4} + \frac{3l}{2} \\ &= \frac{3lw}{4} + \frac{3l}{2} \end{aligned}$$

(c) $\frac{3l}{2}\left(\frac{w}{2} + 1\right)$

(d) (i) $A = 320 - 50 = 270\text{ cm}^2$

(ii) $A = \frac{3 \times 20}{2}\left(\frac{16}{2} + 1\right) = 270\text{ cm}^2$

24 (a) $b + 1$ (b) $a(b + 1)$ (c) $c(b + 1)$

(d) $a(b + 1) + c(b + 1)$ (e) $(a + c)(b + 1)$

(f) (i) $\frac{(a + c)(b + 1)}{a + c}$ (ii) $b + 1$ (iii) b

(iv) It is the same as the second number, b.

(g) Result should be 7, which is equal to b.

Numeracy practice 3

1 D **2** B **3** B **4** C

5 D **6** B **7** A **8** C

Chapter 4

Recall 4

1 (a) 200 cm (b) 3600 m (c) 4000 kg (d) 0.35 L

2 (a) $\frac{8}{12}$ (b) $\frac{7}{2}$

3 (a) $\frac{3}{4}$ (b) $1\frac{2}{3}$ (c) $\frac{5}{7}$ (d) $1\frac{6}{19}$

4 (a) $\frac{15}{7}$ (b) $\frac{19}{8}$

5 (a) (i) $2\frac{2}{5}$ (ii) $4\frac{1}{2}$ (iii) $3\frac{1}{8}$ (iv) $5\frac{1}{5}$

(b) (i) 2.4 (ii) 4.5 (iii) 3.125 (iv) 5.2

6 (a) 0.53 (b) 3.90

7 (a) 45 (b) 3.2 (c) 77.52

8 (a) 25% (b) 56.25%

9 (a) $x = 21$ (b) $d = 6$

Exercise. 4.1

1 (a) 5 : 3 (b) 3 : 5 (c) 1 : 5 (d) 1 : 3

(e) 5 : 9 (f) 1 : 9 (g) 1 : 3 (h) 5 : 4

2 (a) 4 : 9 (b) 1 : 5 (c) 20% (d) $\frac{7}{9}$

3 (a) 1 : 4 (b) 1 : 5 (c) 1 : 4 (d) 2 : 5

(e) 6 : 17 (f) 6 : 5 (g) 1 : 15 (h) 1 : 8

(i) 9 : 13 (j) 5 : 2 : 1 (k) 6 : 1 : 2 (l) 11 : 5 : 9

(m) 1 : 4 : 2 (n) 9 : 1 : 4 (o) 134 : 100 : 43

4 (a) D (b) A (c) C

5 (a) 3 : 8 : 2 (b) 8 : 13 (c) 3 : 13 (d) $\frac{2}{13}$

6 (a) 2 : 3 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$

7 (a) 2 : 1 (b) 1 : 3 (c) $\frac{1}{2}$ (d) $66.\dot{6}\%$

8 (a) 24 : 1 (b) 4800 mL (4.8 L)

9 (a) 39 : 23 (b) 170%

10 (a) 4 : 1 (b) $\frac{1}{5}$

11 (a) 1 : 49 (b) $\frac{1}{50}$

12 (a) (i) 1 : 1 (ii) 2 : 1

(b) Milan's mixture should produce a paler shade of pink because he has equal parts of red and white, while Ali has twice as much red as white.

13 (a) 20% (b) $\frac{3}{11}$

Open-ended – Sample answers

14 **(a)** 10:4; 15:6; 20:8 **(b)** 10:15; 2:3; 100:150

15 The ratio of people who like kangaroos to people who like platypuses is 3:7. The ratio of people who like quokkas to people who like koalas is 8:5. The fraction of the total group who liked platypuses is $\frac{7}{23}$. The number of people who like koalas is $\frac{5}{7}$ of people who like platypuses.

Exercise 4.2

1 **(a)** 1:2 **(b)** 2:1 **(c)** 2:3 **(d)** 5:4
(e) 2:5 **(f)** 10:1 **(g)** 7:8 **(h)** 12:13
(i) 25:26 **(j)** 5:3 **(k)** 1:2:3 **(l)** 3:5:2
(m) 7:5:4

2 **(a)** 50:3 **(b)** 1:2 **(c)** 50:6:3 **(d)** 16:10:9
(e) 3:10 **(f)** 70:9 **(g)** 13:70 **(h)** 12:1
(i) 3:8 **(j)** 4:25 **(k)** 2:3 **(l)** 5:4
(m) 8:3 **(n)** 30:1 **(o)** 9:10:2 **(p)** 2:14:7
(q) 8:25:2 **(r)** 95:100:3

3 **(a)** 7:2 **(b)** 12:11 **(c)** 12:7 **(d)** 41:16
(e) 5:3 **(f)** 5:4 **(g)** 20:9 **(h)** 31:24
(i) 12:7 **(j)** 23:54 **(k)** 49:72 **(l)** 9:10

4 **(a)** 47:31 **(b)** 17:52 **(c)** 29:34 **(d)** 5:8
(e) 15:31 **(f)** 17:21 **(g)** 94:103 **(h)** 44:29
(i) 35:22 **(j)** 162:55 **(k)** 55:166 **(l)** 71:170

5 B

6 **(a)** 7:6 **(b)** 1:2 **(c)** 11:5 **(d)** 4:5
(e) 16:9 **(f)** 9:5

7 **(a)** 9:26 **(b)** $\frac{9}{26}$

8 **(a)** 3:5:7 **(b)** 250:600:1000 (or 5:12:20)

9 **(a)** 684:11 **(b)** $\frac{11}{684}$ **(c)** 170:89 **(d)** 191%

10 **(a)** 5:3 **(b)** $\frac{3}{5}$ **(c)** 40%

11 **(a)** 40:25:9 **(b)** 37:20 **(c)** 34%

12 **(a)** 4:3 **(b)** 7:4

13 3:2

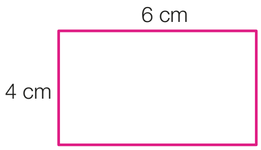
14 **(a)** 3:2 **(b)** 6:4:5 **(c)** $\frac{2}{5}$

15 **(a)** 10:5:3 **(b)** $\frac{1}{6}$ **(c)** 250 mL

16 **(a)** 3:80:20 **(b)** 2:7:4 **(c)** ride, 54%
(d) You can cycle faster than you can run, and you can run faster than you can swim.

Open-ended – Sample answers

17 Your classmate's rectangle may be a different size, but the ratio of side lengths will be the same.



18 Troy could spend equal time training for each discipline, or he could use one of the ratios found in Question **16** to structure his training. Alternatively, if he feels that he is weaker in one particular discipline, then he may want to focus his training on that.

Exercise 4.3

1 **(a)** 1.06:1 **(b)** 1.40:1 **(c)** 2.33:1 **(d)** 0.78:1
(e) 0.21:1 **(f)** 1.47:1 **(g)** 4.89:1 **(h)** 5.31:1
(i) 0.71:1 **(j)** 0.75:1 **(k)** 1.12:1 **(l)** 2.17:1

2 **(a)** 170.5 cm **(b)** \$595
(c) 88.1 days **(d)** 57.14 g

3 **(a)** **(i)** 2.7 L **(ii)** 22.2 kg
(b) 29 610 000 sheep
(c) \$50 **(d)** 45.53 cm

4 **(a)** 21.25 **(b)** 10.83 **(c)** 1.80 **(d)** 5.29

5 6667 km

6 **(a)** 267 067 km **(b)** 133.11 cm or 1.33 m

7 0.32 L or 320 mL

8 **(a)** 0.25:1 **(b)** 3.97

9 105 kilometres per hour

10 **(a)** 1.78:1 **(b)** 42.19 cm

11

Quantities	Ratio	Unit ratio	Scale factor	Explanation
$a:b$	4:5	0.8:1	0.8	a is 0.8 times the size of b
$x:y$	7:1	7:1	7	x is seven times as large as y
$m:p$	12:4	3:1	3	m is three times as large as p
$g:h$	240:12	20:1	20	h is one-twentieth the size of g
$r:j$	3.8:15.2	0.25:1	0.25	r is one-quarter the size of j
$d:e$	4000:20	200:1	200	d is 200 times as large as e

12 **(a)** 36 **(b)** 30 **(c)** 72 **(d)** 5280
(e) 2 miles **(f)** multiply by 36 **(g)** multiply by 5280

13 **(a)** 1.035:1 **(b)** \$348.99

14 1.89 m tall son or 1.62 m tall son. The 1.62 m is the more likely as the 12-year-old son is more likely to be shorter and not taller than his father.

15 **(a)**

Length (inches)	Width (inches)	*L* : *W*	Unit ratio
6	4	6 : 4	1.5 : 1
7	5	7 : 5	1.4 : 1
8	6	8 : 6	1.33 : 1
12	8	12 : 8	1.5 : 1
16	12	16 : 12	1.33 : 1
18	12	18 : 12	1.5 : 1

(b) 6 by 4, 12 by 8 or 18 by 12. These paper sizes have the same unit aspect ratio as the image, 1.5 : 1.

(c) **(i)** 7.5 inches

(ii) This image length is too long for the paper.

(d) The extra 0.5 inches of length could be ‘cropped’ from the image. Alternatively, recommend that the image be printed on different-sized paper.

Open-ended – Sample answers

16 **(a)** 2-year-old dog = 11.4 human years;
10-year-old cat = 63 human years;
6-month-old mouse = 12.5 human years

(b)

Animal	Life expectancy (years)
Blue-tongue lizard	21
Horse	20
Dog	13
Cat	12
Budgie	10
Rabbit	6
Guinea pig	4
Mouse	3

(c) A high value means a short-lived animal.

17 **(a)** Possible dimensions in the ratio 4 : 3 are: 100 cm long and 75 cm wide, 80 cm long and 60 cm wide, 120 cm long and 90 cm wide.

(b) Possible dimensions in the ratio 16 : 9 are: 112 cm long and 63 cm wide, 160 cm long and 90 cm wide, 168 cm long and 94.5 cm wide.

(c) The aspect ratio of the wall space dimensions is 1.4 : 1—closer to the aspect ratio of 4 : 3 ($1.\dot{3} : 1$).

(d) Multiplying the width of 1.2 m by the aspect ratio 4 : 3 gives a length of 1.6 m, only 10 cm short of the maximum amount of wall space.

Exercise 4.4

1 **(a)** 9 **(b)** 56 **(c)** 45 **(d)** 6
(e) 10 **(f)** 30 **(g)** 117 **(h)** 500
(i) 400 **(j)** 108 **(k)** 50 **(l)** 180

2 **(a)** 750 mL **(b)** 1050 mL

3 162 g

4 **(a)** 50 mL **(b)** 200 mL cola cordial with 800 mL water

5 **(a)** 20 mL **(b)** 9 lessons
(c) 16 teachers **(d)** 160 mL

6 **(a)** 16.5 **(b)** 12.8 **(c)** 4.44 **(d)** 27.5
(e) 13.5 **(f)** 52.5 **(g)** 0.6 **(h)** 4.4
(i) 10.5 **(j)** 3.25 **(k)** 7.5 **(l)** 7.2

7 **(a)** 449 cm **(b)** 42 cm **(c)** 42
(d) **(i)** 1.8 hours **(ii)** 1 hour 48 minutes

8 **(a)** D **(b)** C

9 **(a)** A **(b)** C

10 5.4 m

11 **(a)** 135 g **(b)** 210 g

12 **(a)** $835 714 **(b)** $1 285 714

13 2.5 kg of cement, 5 kg of sand

14 $2.05

15 **(a)** 5100 votes **(b)** 4250 votes **(c)** $\frac{1}{14}$ **(d)** 11 900

16 **(a)** 125 : 132 : 138 **(b)** $1.37, $1.44 **(c)** $1.18, $1.31

Open-ended – Sample answers

17 7–9: 1 group, 10–13: only one group (some miss out), 14–18: 2 groups, 19 and 20: take only 2 groups (some miss out), all other numbers up to 90 can be arranged so that the group sizes are a minimum of 7 without anyone missing out.

18 171 cm, 114 cm; 162 cm, 108 cm; 168 cm, 112 cm

19 **(a)** 10, 12, 8; 25, 30, 20

(b) As 15 does not divide evenly into 100, the test cannot be made to add to exactly 100; 90 and 105 are the closest.

Half-time 4

1 **(a)** 7 : 3 **(b)** 17 : 5 **(c)** 1 : 1

2 $125\ \text{m}^2$

3 **(a)** 7 : 8 **(b)** 3 : 2 **(c)** 200 : 7 **(d)** 15 : 2
(e) 50 : 9 **(f)** 13 : 3

4 **(a)** 5 : 2 : 3 **(b)** $\frac{1}{5}$ **(c)** 50%

5 **(a)** 24 **(b)** 6 **(c)** 60 **(d)** 286

6 84 years **7** 452 m

8 **(a)** 11 : 14 **(b)** 127% **(c)** 27%

Exercise 4.5

1 **(a)** 20 cm **(b)** 9 m **(c)** 4.5 km **(d)** 0.5 m

2 **(a)** 200 000 cm **(b)** 2 km
(c) 8 km **(d)** 15 cm

3 **(a)** 500 **(b)** 297 m **(c)** 1.2 cm

4 **(a)** $1 : \underline{600}$ **(b)** $1 : \underline{400}$ **(c)** $1 : \underline{250}$
(d) $1 : \underline{60}$ **(e)** $1 : \underline{800}$ **(f)** $1 : \underline{500}$
(g) $1 : \underline{25}$ **(h)** $1 : \underline{125}$

5 **(a)** 2 m **(b)** 3 m **(c)** 5 m
(d) 0.8 m **(e)** 2.3 m **(f)** 1.9 m
(g) 40.3 m **(h)** 92.5 m **(i)** 120 m

6 **(a)** 5 mm **(b)** 10 mm **(c)** 20 mm **(d)** 1 mm
(e) 100 mm **(f)** 200 mm **(g)** 8 mm **(h)** 3 mm
(i) 13 mm **(j)** 12 mm **(k)** 2 mm **(l)** 0.6 mm

7 **(a)** 60 m **(b)** 100 m **(c)** 200 m
(d) 52 m **(e)** 768 m **(f)** 86 m
(g) 18 m **(h)** 35.8 m **(i)** 102 m

8 **(a)** no **(b)** no **(c)** yes **(d)** yes **(e)** no **(f)** no

9 D

10 **(a)** 7 m **(b)** 5 m **(c)** 80 m **(d)** 40 m
(e) 0.3 m **(f)** 120 m **(g)** 630 m **(h)** 5200 m
(i) 176 m

11 **(a)** 20 mm **(b)** 8 mm **(c)** 3500 mm
(d) 4000 mm **(e)** 240 mm **(f)** 3 mm
(g) 25 mm **(h)** 8 mm **(i)** 90 mm

12 **(a)** 19 m **(b)** 35 m **(c)** 10 m **(d)** 2 m

13 **(a)** 23 m **(b)** 11.5 m

14 **(a)** 850 km **(b)** 1200 km **(c)** 3000 km **(d)** 3700 km

15 **(a)** **(i)** 40 mm × 100 mm (or 4 cm × 10 cm)
(ii) 80 mm × 200 mm (or 8 cm × 20 cm)
(iii) 100 mm × 250 mm (or 10 cm × 25 cm)
(iv) 50 mm × 125 mm (or 5 cm × 12.5 cm)
(b) 1 : 200

16 **(a)** 29 mm **(b)** 55 mm **(c)** 17.5 mm **(d)** 10 mm

17 $a = 25$ mm, $b = 19$ mm, $c = 6.5$ mm, $d = 13.5$ mm

18 0.6 mm

19 **(a)** 7.2 cm^2 **(b)** 2700 cm by 2160 cm
(c) 5 832 000 cm^2
(d) 6480 cm^2. No. To find the actual area, both the length and the width have been multiplied by 900, so the actual area has been multiplied by 900^2.
(e) 27 m by 21.6 m
(f) 583.2 m^2
(g) Area in $\text{m}^2 = 10\,000 \times$ area in cm^2

Open-ended – Sample answers

20 (minimum) 1 : 700; 1 : 500; (maximum) 1 : 280

21 1 : 300 000, 1 : 380 000
Even though the second scale will fill the page more completely, the first scale is easier to work with.

22 drawing plans of small pieces of machinery, drawing insects, drawing microscopic objects

Exercise 4.6

1 **(a)** 20 **(b)** 27 **(c)** 48

2 **(a)** 12 : 18 **(b)** 15 : 5 **(c)** 35 : 7 **(d)** 6 : 33
(e) 64 : 20 **(f)** 24 : 14 **(g)** 28 : 21 : 7 **(h)** 12 : 24 : 6
(i) 6 : 21 : 27 **(j)** 6 : 15 : 12 **(k)** 10 : 15 : 25 **(l)** 8 : 56 : 32

3 **(a)** Kayla 27 jelly beans; little brother 18 jelly beans
(b) 14 lamingtons in one container, 49 in another and 21 in the last

4 won 15, lost 6

5 400 mL of concentrate and 1600 mL of water

6 200 mL oil, 4800 mL petrol (or 0.2 L oil, 4.8 L petrol)

7 140 boys and 160 girls

8 Karine $23 750; André $14 250; Vivian $9500; Julian $9500

9 **(a)** N 900 kg, P 600 kg, K 1200 kg
(b) 3 : 3 : 4

10 **(a)** first group 16 squares; second group 8 squares
(b) 4 **(c)** 2

11 256 cm or 384 cm from one end

Open-ended – Sample answers

12 The bill could be divided equally and each pays approximately $38.33. Or 3 drinks, 3 main courses and 1 dessert means there is a total of 7 parts: Kelly and Petra could pay $\frac{2}{7}$ or $32.86 each, while Nadine could pay $\frac{3}{7}$ or $49.29. Or a fairer method might be to let a main course be 2 parts, and drinks and dessert 1 part each, making a total of 10 parts: Nadine pays $\frac{4}{10}$ or $46, while the others pay $\frac{3}{10}$ or $34.50 each.

13 **(a)** Ms Footsie $15 million, Ms Nikkei $27 million
(b) That way, they each get back their investment and then share the profit equally.

Exercise 4.7

1 **(a)** The speed limit for this road is 100 km per hour, or 100 km/h.
(b) Jackie covers 100 km for every 1 hour travelled.
(c) 200 km

2 **(a)** In 1 minute Robert reads 294 words.
(b) In 1 hour Jack earns $21.
(c) Every 100 g of cereal costs $0.55.
(d) For every 100 km travelled the sportscar uses 11.8 L of fuel.

3 **(a)** 40 km/h **(b)** 50 km/h **(c)** 66.67 km/h
(d) 38.46 km/h **(e)** 93.33 km/h **(f)** 8.20 m/s
(g) 2.03 m/s **(h)** 10.5 km/h

4 **(a)** 521 g **(b)** $49.38 **(c)** $54.17 **(d)** 8.25 stalks

5 **(a)** **(i)** Sydney 1520 people/week,
Melbourne 1755 people/week,
Brisbane 694 people/week,
Australia 6253 people/week
(ii) Sydney 2.18%, Melbourne 1.81%, Brisbane 1.69%, Australia 1.40%
(b) Sydney 4 548 891, Melbourne 4 346 409, Brisbane 2 213 587, Australia 23 941 459

6 **(a)** D **(b)** B

7 **(a)** 123.3 c/L **(b)** 125.6 c/L

(c) 145.5 c/L **(d)** 142.1 c/L

8 **(a)** 86 400 beats per day **(b)** 115 200 beats per day

9 **(a)** 5.5 hours **(b)** 240 km

10 $13\frac{1}{3}$ cups self-raising flour, $6\frac{2}{3}$ tablespoons icing sugar, $2\frac{1}{2}$ cups milk, $2\frac{1}{2}$ cups thickened cream, 3 or 4 eggs (since it is difficult to have $3\frac{1}{3}$ eggs)

11 **(a)** 4.3 runs per over **(b)** 4.325 runs per over

(c) 1.25 runs per ball **(d)** 0.764 runs per ball

12 **(a)** **(i)** \$6.50 **(ii)** \$3.25

(b) **(i)** \$0.90 **(ii)** \$2.70

(c) **(i)** \$28 **(ii)** \$70

(d) **(i)** \$7.50 **(ii)** \$10.50

13 **(a)** 29.2345 kL **(b)** 9.2334 kL

(c) 12.4972 kL **(d)** 10.5445 kL

14 **(a)** **(i)** US\$494.57 **(ii)** 826.23 Brazilian reals

(iii) 309.20 British pounds

(b) A\$3057.35 **(c)** A\$574.56

15 **(a)**

World	**(i)**	7516.7 million	**(ii)**	7601.6 million
Australia	**(i)**	24.6 million	**(ii)**	25.0 million
China	**(i)**	1388.7 million	**(ii)**	1395.0 million
India	**(i)**	1342.7 million	**(ii)**	1358.8 million
Indonesia	**(i)**	263.6 million	**(ii)**	266.7 million

(b) **(i)** Australia **(ii)** China

(c) The population is not changing.

(d) The population is decreasing.

16 **(a)** 87 m/min **(b)** \$3.50/kg **(c)** \$9/m

(d) \$1.25/pen **(e)** 970 L/h **(f)** 0.3 shirts/min

17

	Per serving: 17 g (one biscuit)	Per 100 g
Protein	0.884 g	5.2 g
Fat – total	4.539 g	26.7 g
Fat – saturated	2.227 g	13.1 g
Carbohydrate – total	11.237 g	66.1 g
Carbohydrate – sugars	4.692 g	27.6 g

18 **(a)** 2.17 km/h **(b)** 18.42 km

19 **(a)** 90 m/min **(b)** 5.4 km/h

20 **(a)** 10.4 m/s **(b)** 1 hour 7 minutes

(c) 9.53 m/s, 5.87 m/s **(d)** marathon, 0.91

(e) 4.90

21 **(a)** **(i)** \$230.11 **(ii)** \$273.22

(iii) \$199.47 **(iv)** \$350.87

(b) **(i)** 905.51 kWh **(ii)** 1047.92 kWh

(iii) 1066.38 kWh **(iv)** 1155.82 kWh

22 **(a)** no change in population

(b) The population actually decreased (when the rate dropped below zero).

(c) A decade around 2%, briefly reached up over 3%, then dropped to be around 1%–1.5% for several decades, until recently climbing back up over 2%.

(d) A sharp drop down to nearly -1%, then a sharp climb back up to nearly 3% for a decade. Slowly decreased to around 1–1.5%, before decreasing further in recent years to around 0.5%.

(e) Severe famine and natural disasters drove the population of China in the 1960s. Later on, the 'one-child' policy brought the rate down slowly. Australia has never had such a policy. Australia also had a lot of migration in the 1960s and 1970s. Australia also has an ageing population, meaning that people are living longer.

(f) The actual populations of the two countries in 2009.

23 **(a)** Babies being born, people moving to Australia (immigration)

(b) Even though the growth rate is getting smaller, it is still positive. The population is still growing, it is just growing more slowly.

Open-ended – Sample answers

24 A high birth rate, people moving to or from the city (or country), people living longer (and therefore a lower death rate), natural disaster.

25 If your hair grows 2 cm and you go to the hairdresser every 8 weeks, your hair grows at a rate of 2.5 mm/week.

Challenge 4

1 To be half an hour apart they must each have to travel for 30 minutes. At 80 km/h, 30 min is 40 km; at 90 km/h, 30 min is 45 km. They are 85 km apart.

2 Kevin takes 6 hours to do one-third of the work, Sanna takes 6 hours to do two-thirds of the work. Kevin takes 18 hours on his own.

3 Two cats eat three tins of cat food in three days. Two cats eat six tins of cat food in six days.

4 Glass contains $\frac{5}{12}$ of the first liquid and $\frac{7}{12}$ of the second liquid.

Percentage water = $\frac{5}{12} \times 22.5\% + \frac{7}{12} \times 27\% = 25.125$

5 Let the distance from A to B be x km.

Time from A to B = $\frac{x}{40}$ hours. Time from B to A = $\frac{x}{60}$ hours.

Total time to cover $2x$ km = $\frac{x}{40} + \frac{x}{60} = \frac{3x + 2x}{120} = \frac{5x}{120}$

$= \frac{x}{24}$ hours

Average speed = $2x \div \left(\frac{x}{24}\right) = \frac{48x}{x} = 48$ km/h

6 In one hour, the first pipe would fill $\frac{1}{3}$ of the tank, the second pipe would fill $\frac{1}{5}$ of the tank and the third pipe would fill $\frac{1}{10}$ of the tank.

Hence, in 1 hour you fill $\frac{1}{3}+\frac{1}{5}+\frac{1}{10}=\frac{10+6+3}{30}=\frac{19}{30}$ of the tank.

Hence, it takes $\frac{30}{19}=1\frac{11}{19}$ hours or 1 hour 34 minutes 44 seconds to fill the tank using all three pipes.

7 Each revolution of the first wheel results in $\frac{32}{36}=\frac{8}{9}$ of a revolution of the second wheel.

Thus, the first wheel rotates 9 times for the second wheel to rotate 8 times, and the cogs would again come together if they had started together.

First wheel: 128 rev/s = 128×60 rpm
= $128 \times 60 \times 60$ rev/hour
= $128 \times 60 \times 60 \times 24$ revolutions in a 24-hour period.

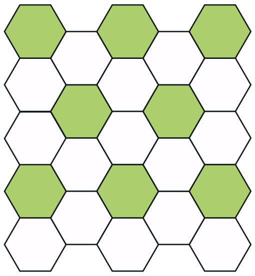
Number of times together if they start apart

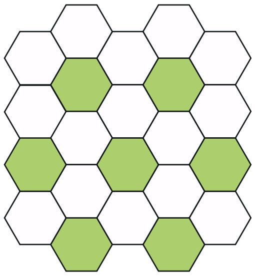
$$=\frac{128 \times 60 \times 60 \times 24}{9}=1\,228\,800 \text{ times.}$$

If they started together, they would come together 1 228 801 times.

8 **(a)** Each green tile is surrounded by 6 white tiles. Each white tile is surrounded by 3 white and 3 green tiles. Hence, each white tile is used with 3 different green tiles, so the ratio of green tiles to white tiles is 1 : 2.

(b) Each diagram shows all the tiles needed to cover an approximately square area.





First diagram has 8 green tiles and 15 white tiles, ratio green : white = 8 : 15 ≈ 1 : 2.

The second diagram has 7 green tiles and 15 white tiles, ratio green : white = 7 : 15 ≈ 1 : 2.

9 **(a)** **(i)** 50 days **(ii)** 83.3 days **(iii)** 8.3 days
(iv) 2.5 days **(v)** 0.83 days

(b) The more people you have, the more likely they will get in each other's way. Eventually, adding more people will not help.

Chapter review 4

1 **(a)** **(i)** 4 : 5 **(ii)** 5 : 4 : 3 **(iii)** 3 : 4 **(iv)** 5 : 12
(b) 42% **(c)** $\frac{3}{4}$
2 **(a)** 7 : 12 **(b)** 5 : 1 : 2 **(c)** 5 : 2
(d) 9 : 2 **(e)** 4 : 7 **(f)** 2 : 1
3 D
4 **(a)** 2.93 : 1 **(b)** 0.52 : 1 **(c)** 0.60 : 1
5 2.7 **6** 1.42 L
7 **(a)** 77 **(b)** 140 **(c)** 11
(d) 18 **(e)** 56 **(f)** 13.5
8 **(a)** 8 **(b)** 5
9 **(a)** 1 : 25 **(b)** 1.2 L (1200 mL)
10 **(a)** $1:\underline{200}$ **(b)** $1:\underline{2\,400\,000}$ **(c)** $1:\underline{5000}$
11 **(a)** 4 m **(b)** 280 m **(c)** 260 m
12 **(a)** 11.3 mm **(b)** 190 mm **(c)** 490 mm
13 **(a)** 20 : 30 **(b)** 42 : 30 **(c)** 12 : 20
14 **(a)** 4 km/h **(b)** 75 km/h
(c) $24 000 per game **(d)** $1.67 per litre
(e) **(i)** $9 **(ii)** $7.50
(f) $10\frac{1}{2}$ potatoes, 700 g pumpkin
15 10:30 am
16 **(a)** 2 : 5 **(b)** 40%
17 9.6 cups of fruit
18 125 kg **19** 45 L
20 971.5 g **21** 1.6 goals
22 120 coins, 160 coins and 240 coins
23 **(a)** 3.375 m × 3.25 m **(b)** 7.02 cm^2
(c) 109 687.5 cm^2 **(d)** 15 625
24 **(a)** 1.09% **(b)** 14 979
25 315 mL
26 **(a)** at least 9.3 runs per over
(b) 10.25 runs per over
(c) 33 runs

Numeracy practice 4

1 A **2** B **3** D **4** D
5 8.75 L **6** D **7** 300 g
8 B **9** A **10** B

Mixed review B

1 (a) $\frac{23}{100}$ (b) $\frac{1}{250}$ (c) $\frac{1}{8}$ (d) $\frac{7}{1000}$

2 (a) 14% (b) 3% (c) 28.6% (d) 190%

3 (a) 7.2 (b) 3.12 (c) 283.8

4 (a) $68 (b) $552.50

5 (a) $2^6 = 64$ (b) $5^2 = 25$ (c) $15^0 = 1$

6 (a) $2a^2$ (b) $7t^2 + 6t$ (c) $8q^2 - 3p^2 + 12q$

7 (a) 2 (b) 19 (c) 318

8 (a) $35jk$ (b) $7p$ (c) $6a^3b$

9 (a) $5(h - 9)$ (b) $y(y + 7)$ (c) $4m(3n + 2)$

10 (a) 0.17 (b) 0.065 (c) 4 (d) 0.000 06

11 (a) -15 (b) 9 (c) -30 (d) -4

(e) $-\frac{43}{40}$ or $-1\frac{3}{40}$ (f) $\frac{-7}{24}$ (g) -5.33 (h) -1.44

12 (a) 12 (b) 11 (c) 7 (d) 15 (e) 7.2 (f) 10.8

13 (a) 30 : 18 (b) 140 : 490 (c) 16 : 24 : 48

14 84%

15 (a) 9% (b) 85% (c) $18\frac{3}{4}$% (d) 225%

16 (a) 654 (b) 58.32 (c) 270

17 (a) 5 : 11 (b) 7 : 3 (c) 5 : 12 (d) 7 : 10

(e) 15 : 11 (f) 5 : 158

18 84 km/h

19 (a) 0.7, 76%, $\frac{7}{9}$ (b) 0.105, $\frac{3}{20}$, 15.2% (c) 500%, 5.06, $5\frac{1}{5}$

20 (a) $5610 (b) $5329.50

21 187 cm

22 (a) $104 (b) 19.66%

23 31.25 24 $1343.75

25 (a) 32 °F (b) 68 °F (c) 95 °F (d) 302 °F

26 Gerard $5000, Hao $7500

27 (a) $133.10 (b) $110 (c) 21%

Chapter 5

Recall 5

1 (a) 7.82 (b) 15.00 (c) 0.99 (d) 106.50

2 (a) 24.78 (b) 27.56 (c) 37.68 (d) 67.31

(e) 59.85 (f) 845.68

3 (a) 4500 m (b) 0.69 m (c) 48 mm (d) 4.6 cm

(e) 6.78 m (f) 0.09 km

4 (a) 4.65 m (b) 5.064 m

5 (a) (i) $P = 20$ cm (ii) $A = 24$ cm^2

(b) (i) $P = 12$ cm (ii) $A = 6$ cm^2

6 (a) 420 m^3 (b) 91.125 cm^3

Exercise 5.1

1 (a) 27 km (b) 46 m (c) 203 cm

(d) 15 m (e) 24 cm (f) 15.2 cm

(g) 160 mm (h) 104 mm (i) 106 m

(j) 15 m (k) 200 cm (l) 180 cm

2 (a) 28.5 cm (b) 96 mm (c) 7.2 cm

(d) 14.8 cm (e) 146 m (f) 82 mm

(g) 124 m (h) 21 km (i) 184 mm

3 (a) 228 mm (b) 33 m (c) 196 cm (d) 17 mm

4 73.2 m

5 (a) 74 mm or 7.4 cm (b) 88 mm or 8.8 cm

(c) 340 cm or 3.4 m (d) 270 cm or 2.7 m

(e) 10 600 m or 10.6 km (f) 6100 m or 6.1 km

6 74 m

7 (a) D (b) D

8 (a) 8000 m (b) $46 000 (c) 400 h

9 (a) 15 cm (b) 12 cm

10 (a) 800 cm (8 m)

(b) 1200 cm (12 m), 2160 cm (21.6 m), 4080 cm (40.8 m)

Open-ended – Sample answers

11 Triangle: 3 different lengths that add to 120 cm where the total of the 2 smallest must be more than the longest; e.g. 35 cm, 40 cm, 45 cm.

Quadrilateral: 4 different lengths that add to 120 cm where the total of the 3 smallest must be more than the longest; e.g. 20 cm, 25 cm, 30 cm, 45 cm. Pentagon: 5 different lengths that add to 120 cm where the total of the 4 smallest must be more than the longest; e.g. 10 cm, 20 cm, 25 cm, 30 cm, 35 cm.

12 (a) 20 m, 20 m, 16 m; 16 m, 16 m, 24 m; 22 m, 22 m, 12 m

(b) 15, 30, 30 or 19, 38, 38

Exercise 5.2

1 Approximate answers:

Circle	Radius *r* (cm)	Diameter *d* (cm)	Circumference *C* (cm)
A	1	2	6.3
B	2	4	12.6
C	3	6	18.8
D	4	8	25.1
E	5	10	31.4
F	6	12	37.7

2 (a) The diameter is twice the radius. The circumference is approximately three times the diameter, or six times the radius.

(b) Students' own answers.

3 (a) 3 (b) 6 (c) double

4 (a) 2 m (b) 12 m

5 **(a)** doubled **(b)** doubled

6 **(a)** hard to follow curve of circle with string, measuring instruments not very accurate; human error

(b) **(i)** They would both be smaller.

(ii) They would both be bigger.

7 **(a)** In a non-terminating, non-recurring number, the digits do not form a repeating pattern and there is never a final digit.

(b) **(i)** 3.14 **(ii)** 3.142 **(iii)** 3.1416

Open-ended – Sample answers

8 **(a)** $d = 10$ cm, $C = 31.4$ cm **(b)** $d = 4$ cm, $C = 12.6$ cm

$d = 16$ cm, $C = 50.2$ cm $d = 6$ cm, $C = 18.8$ cm

$d = 13$ cm, $C = 40.8$ cm $d = 5$ cm, $C = 15.7$ cm

9 diameter = 75 m, lap of honour = 235 m

Exercise 5.3

1 **(a)** 12.57 m **(b)** 6.28 cm **(c)** 15.71 mm

(d) 94.25 cm **(e)** 113.10 m **(f)** 144.51 m

(g) 141.37 m **(h)** 94.25 mm **(i)** 182.21 mm

(j) 62.83 cm **(k)** 502.65 km **(l)** 219.91 cm

2 **(a)** 23.14 m **(b)** 41.13 m **(c)** 7.14 cm

(d) 10.71 cm **(e)** 16.85 m **(f)** 39.13 m

3 **(a)** C **(b)** B

4 7.54 m **5** 12.57 m **6** 520 mm

7 **(a)** Square: $P = 56.57$ cm, $\frac{P}{D} = 2.83$

Hexagon: $P = 60$ cm, $\frac{P}{D} = 3$

Octagon: $P = 61.2$ cm, $\frac{P}{D} = 3.06$

Circle: $P \approx 62.83$ cm, $\frac{P}{D} \approx 3.14$

(b) hexagon **(c)** octagon

(d) Regular nonagon or decagon, or any other regular shape with more than 8 sides.

8 **(a)** 56.56 cm **(b)** 80 cm

(c) 62.83 cm; the value seems suitable, as the circumference should be somewhere between the perimeter of the inside square and the perimeter of the outside square.

Open-ended – Sample answers

9 Any value between approximately 9.72 cm and 11.67 cm is suitable.

10 $C = \pi d$ where $\pi \approx 3.14$, so $\pi \times 100 \approx 314$, $\pi \times 1000 \approx 3140$ etc.; therefore $d = 1000, 10\,000, \ldots$

11 **(a)** $8 \times 4 = 32$ cm gives perimeter of a square that is bigger than the circle, so 50.27 cm is too big. $11 \times 3 = 33$ cm gives perimeter of an equilateral triangle smaller than the quarter circle, so 17.28 cm is too small.

(b) First: $2\pi r$ has been used where d was given, not r. Second: Straight parts were not included.

(c) 25.13 cm, 39.28 cm

Exercise 5.4

1 **(a)** **(i)** 1000 m **(ii)** 1 000 000 m^2

(b) **(i)** 0.01 m **(ii)** 0.0001 m^2

(c) **(i)** 2800 cm **(ii)** 280 000 cm^2

(d) **(i)** 5.4 km **(ii)** 0.0054 km^2

2 **(a)** 50 000 cm^2 **(b)** 65 cm^2 **(c)** 9400 mm^2

(d) 976 mm^2 **(e)** 120 000 m^2 **(f)** 10 030 m^2

(g) 5 ha **(h)** 98 cm^2 **(i)** 6.7 m^2

(j) 0.095 m^2 **(k)** 236 000 m^2 **(l)** 0.34 ha

(m) 0.046 m^2 **(n)** 34 500 mm^2 **(o)** 9 km^2

(p) 55.7 km^2

3 **(a)** **(i)** square **(ii)** $A = l^2$

(b) **(i)** kite **(ii)** $A = \frac{1}{2}xy$

(c) **(i)** trapezium **(ii)** $A = \frac{1}{2}(a + b)$

(d) **(i)** parallelogram **(ii)** $A = bh$

4 **(a)** 26.66 m^2 **(b)** 30.25 km^2 **(c)** 65 cm^2

(d) 60 m^2 **(e)** 144 m^2 **(f)** 80 cm^2

(g) 2.34 cm^2 **(h)** 105 m^2 **(i)** 120 mm^2

(j) 0.88 km^2 **(k)** 60 cm^2 **(l)** 224 mm^2

5 **(a)** 33 cm^2 **(b)** 40 mm^2 **(c)** 40 m^2

(d) 52 mm^2 **(e)** 104 m^2 **(f)** 150 cm^2

6 **(a)** 117 cm^2 **(b)** 7.5 m^2 **(c)** 130 mm^2

(d) 147 cm^2 **(e)** 12 km^2 **(f)** 360 mm^2

7 **(a)** **(i)** 1.68 m^2 **(ii)** 16 800 cm^2

(b) **(i)** 1.54 m^2 **(ii)** 15 400 cm^2

(c) **(i)** 0.7 m^2 **(ii)** 7000 cm^2

(d) **(i)** 0.39 m^2 **(ii)** 3900 cm^2

8 **(a)** D **(b)** A

9 3.45 m^2 **10** C

11 **(a)** 32 m^2 **(b)** \$1024

12 D

13 **(a)** 750 000 m^2 **(b)** 75 ha **(c)** 0.75 km^2

14 1487.5 m^2

15 **(a)** 10 368 cm^2 **(b)** 640 cm **(c)** 2304

16 50 cm^2

17 **(a)** 6.4 cm **(b)** 16 cm

18 **(a)** 24 cm

(b) 16 cm, 32 cm, parallel sides add to 32 cm

19 **(a)** **(i)** 30 cm **(ii)** 16 cm

(b) **(i)** 13.5 cm **(ii)** 3 cm

20 1.3 m and 3.2 m

21 **(a)** 69.36 m^2 **(b)** 32.04 m

Open-ended – Sample answers

22 6 m × 8 m, 12 m × 4 m, 8 m × 6 m

23 **(a)** 9 m and 7 m, 10 m and 6 m, 11 m and 5 m, 12 m and 4 m, 13 m and 3 m, 14 m and 2 m, 8 m and 8 m

(b) 16 m

24 **(a)** Missed the brackets around (10 + 4).

(b) 42 cm^2

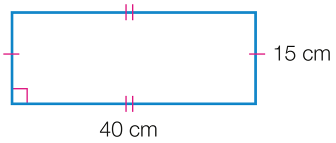
Half-time 5

1 **(a)** 94.25 cm **(b)** 10.05 m

2 312 cm

3 840 cm^2, 120 cm

4

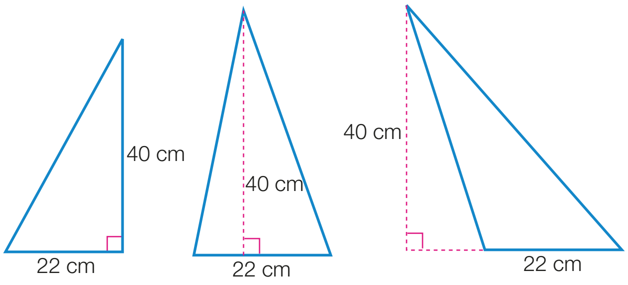


5 128 mm

6 **(a)** 205.66 cm **(b)** 60.70 mm

7 **(a)** 440 cm^2

(b) possible triangles:



8 31 m

9 17.28 cm

10 an estimate between 75 cm and 85 cm

Exercise 5.5

1 **(a)** 78.54 m^2 **(b)** 153.94 cm^2 **(c)** 12.57 cm^2

(d) 50.27 m^2 **(e)** 113.10 km^2 **(f)** 28.27 cm^2

(g) 254.47 mm^2 **(h)** 615.75 m^2 **(i)** 1385.44 cm^2

2 **(a)** 39.27 cm^2 **(b)** 6.28 cm^2 **(c)** 14.14 m^2

(d) 3.14 m^2 **(e)** 28.27 mm^2 **(f)** 38.48 mm^2

3 **(a)** 3848 cm^2 **(b)** 3632 cm^2

4 **(a)** C **(b)** C

5 110.84 cm^2

6 **(a)** 8825 m^2 **(b)** \$66 187.50

7 31.42 cm^2

8 **(a)** 201.06 m^2 **(b)** 28.27 m^2 **(c)** 172.79 m^2

9 B

10 7.07 m^2

11 **(a)** 13 $units^2$ **(b)** 13 $units^2$ **(c)** 25 $units^2$ **(d)** 29 $units^2$

12 D

13 **(a)** 78.54 cm^2 **(b)** 1.18 m^2 **(c)** 0.72 m^2

14 **(a)**

Radius r (cm)	Circumference C (cm)	C^2 (cm^2)	Area A (cm^2)	$\frac{C}{A}$	$\frac{C^2}{A}$
1	6.28	39.48	3.14	2	12.57
2	12.57	157.91	12.57	1	12.57
5	31.42	986.96	78.54	0.4	12.57
10	62.83	3947.84	314.16	0.2	12.57

(b) $\frac{C}{A}$ is not constant; $\frac{C^2}{A} = 12.57$ is constant

(c) $\frac{12.57}{\pi} = 4$

(d) $\frac{C^2}{A} = 4\pi$ or $C^2 = 4\pi A$

Open-ended – Sample answers

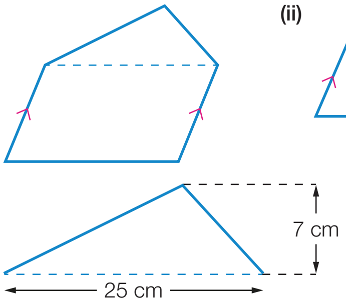
15 $r = 16$ cm, $l = 21$ cm, $w = 17$ cm gives $A \approx 447$ cm^2

16 **(a)** No, the correct answer is 33.18 m^2.

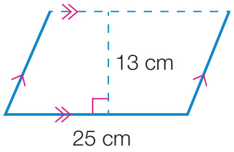
(b) Simon has used the diameter instead of the radius in his working. He must be careful to check what value he is substituting into his formula. Ajay has doubled the radius instead of squaring it. He must be sure he knows the difference between multiplying a number by 2 and multiplying a number by itself.

Exercise 5.6

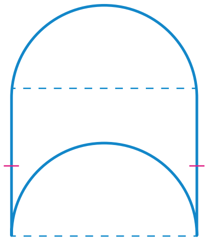
1 **(a)** **(i)**



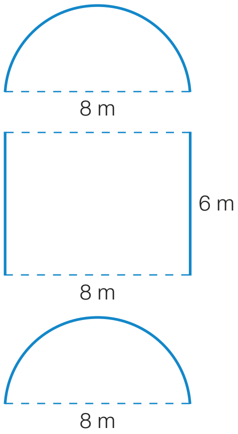
(ii)



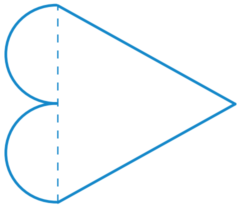
(b) **(i)**



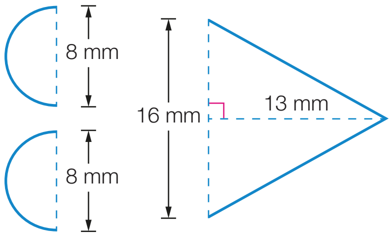
(ii)



(c) (i)



(ii)



2 (a) 2160 cm^2 (b) 4000 cm^2 (c) 12.28 m^2
(d) 25.13 cm^2 (e) 9.42 cm^2 (f) 12 cm^2

3 (a) 1264 cm^2 (b) 15.31 m^2 (c) 52.73 cm^2
(d) 1884.25 cm^2 (e) 3.14 m^2 (f) 16.57 cm^2

4 C 5 4.21 m^2 6 12.57 m^2

7 (a) 194.25 cm, 893.14 cm^2 (b) 12.28 m, 10.28 m^2

8 (a) 53 m^2 (b) $344.50

9 (a) 465.13 m^2 (b) 310.08 m^2 (c) 389.70 m^2

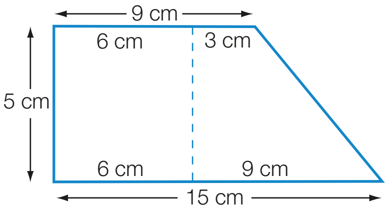
10 B

11 (a) (i) 399.34 m (ii) 462.17 m
(b) (i) 10 390.39 m^2 (ii) 14 697.91 m^2 (iii) 4307.52 m^2

12 (a) 0.43 m^2
(b) (i) 2.60 m^2 (ii) 3.14 m^2 (iii) 0.54 m^2

Open-ended – Sample answers

13



14 (a) Perimeter: for the semicircle, diameter is not part of the boundary; the curve is only half of a circumference; for the total, 5 should be added twice. Area: for the rectangle, the perimeter has been used instead of that for area; for the semicircle, the formula for diameter has been used instead of radius; the final subtraction has been done in the wrong order.
(b) $P = 66$ cm, $A = 161$ cm^2

Exercise 5.7

1 (a) 477 cm^3 (b) 196 cm^3 (c) 198 cm^3
(d) 80 cm^3 (e) 1188 cm^3 (f) 1125 m^3
(g) 452.39 cm^3 (h) 21.99 mm^3 (i) 942.48 cm^3

2 (a) 20 000 mm^3 (b) 195 000 mm^3 (c) 8 300 000 cm^3
(d) 25 460 000 cm^3 (e) 750 000 000 m^3
(f) 57 000 000 000 m^3 (g) 0.15 cm^3
(h) 3.83 cm^3 (i) 0.047 9 m^3 (j) 0.006 07 m^3
(k) 480 000 cm^3 (l) 0.000 097 5 km^3 (m) 0.000 36 km^3
(n) 2 500 000 m^3 (o) 40 mm^3 (p) 35 600 cm^3

3 (a) 2 L (b) 55 L (c) 0.8 L
(d) 0.04 L (e) 5000 mL (f) 95 000 mL
(g) 30 mL (h) 500 mL (i) 25 mL
(j) 48 mL (k) 33 cm^3 (l) 140 cm^3
(m) 5.8 kL (n) 1.7 kL (o) 0.13 m^3
(p) 0.05 m^3

4 (a) 630.0 mL (b) 15 878.9 mL (c) 83.2 mL

5 (a) (i) 980 000 cm^3 (ii) 0.98 m^3
(b) (i) 360 000 cm^3 (ii) 0.36 m^3
(c) (i) 729 000 cm^3 (ii) 0.729 m^3
(d) (i) 753 982.24 cm^3 (ii) 0.75 m^3
(e) (i) 3 716 504.11 cm^3 (ii) 3.72 m^3
(f) (i) 3 216 990.88 cm^3 (ii) 3.22 m^3

6 (a) A (b) C

7 440 cm^3

8 (a) 1.696 m^3 (b) 1696 L

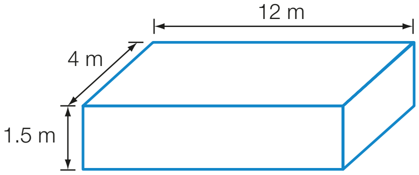
9 (a) $972 \div 2 = 486$ m^3 (b) $40.5 \times 12 = 486$ m^3

10 1.70 m^3 11 4.5 L 12 C

13 (a) 795 mL (b) 0.795 L

14 368 mL

15 (a)



(b) $120.96

16 1000 pinheads

17 3 with about 190 mL left over

18 27 chocolates

19 (a) 1000 L (b) 400 m^2

Open-ended – Sample answers

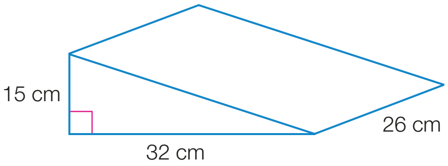
20

d (cm)	3	4	5	6	7
h (cm)	70.7	39.8	25.5	17.7	13.0

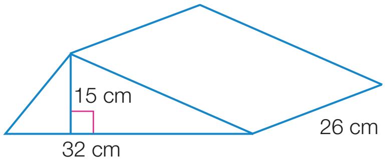
A ‘handy’ size might be best.

21 **(a)** The bases of the triangular prisms have the same dimensions as the base of the rectangular prism (15 cm and 26 cm). The base of the rectangular prism can be cut to produce the base of both triangular prisms. Extending this cut along the height of the rectangular prism (32 cm) gives the triangular prisms. The volume of each triangular prism is half of the volume of the rectangular prism.

(b)



or



Exercise 5.8

1 **(a)** **(i)** 0230 **(ii)** 1430
(b) **(i)** 0615 **(ii)** 1815
(c) **(i)** 1143 **(ii)** 2343
(d) **(i)** 1200 **(ii)** 0000

2 **(a)** 1:54 pm **(b)** 8:33 am **(c)** 5:39 am
(d) 4:34 pm **(e)** 6:30 pm **(f)** 7:02 pm
(g) 1:47 am **(h)** 3:20 am

3 **(a)** 2 h 46 min **(b)** 4 h 15 min **(c)** 2 h 46 min
(d) 4 h 51 min **(e)** 2 h 17 min **(f)** 3 h 3 min
(g) 10 h 9 min **(h)** 22 h 41 min **(i)** 14 h 33 min
(j) 17 h 12 min

4 **(a)** 7:15 am **(b)** 0623 **(c)** 1402 **(d)** 7:19 pm
(e) 4:18 pm **(f)** 0730 **(g)** 1251 **(h)** 8:38 pm
(i) 5:04 pm **(j)** 2113

5 **(a)** 1:00 pm **(b)** 3:45 pm **(c)** 12:15 am **(d)** 8:40 am

6 **(a)** 2130 **(b)** 2100

7 **(a)** A **(b)** B **(c)** C

8 **(a)** T **(b)** T **(c)** T **(d)** F **(e)** F **(f)** T

9 7:37 am

10 **(a)** 8:32 am **(b)** 27 min

11 A **12** 0655

13 **(a)** 4:50 am **(b)** 6:50 am

14 **(a)** 10:35 am and 10:30 pm **(b)** 4:01 am and 3:17 pm
(c) 39 min **(d)** 5 h 57 min
(e) 8 June and 9 June **(f)** 2 h 39 min

15 8 am Thursday

16 **(a)**

State	Standard time	Daylight saving time
WA	10 am	10 am
NT	11:30 am	11:30 am
SA	11:30 am	12:30 pm
Qld	12 noon	12 noon
NSW	12 noon	1 pm
Vic	12 noon	1 pm
Tas	12 noon	1 pm

(b) 2:00 pm **(c)** 11:30 am

(d)

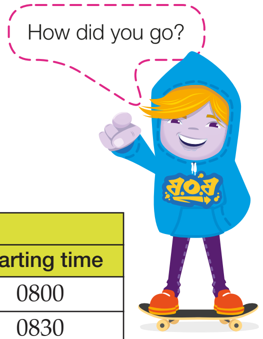
Perth	Darwin
 09:45 am	 11:15 am

Adelaide	Sydney
 12:15 pm	 12:45 pm

Melbourne	Brisbane
 12:45 pm	 11:45 am

17 Yes he will, as he arrives in Milan at 1805.

18 25 years, 10 months, 11 days (or 12 days depending on whether the current day is counted)



19

Recruit training day	
Activity	**Starting time**
Parade ground drill	0800
Navigation lesson	0830
Military history lesson	0930
Fitness session	1030
Lunch/shower	1200
Weapons handling lesson	1235
Weapons handling practice	1320
Bush camp	1535
End	1635

20 **(a)** 4 h **(b)** 4 h 53 min

21 arrive Hong Kong at 1:50 pm, arrive Tokyo at 4:30 pm, arrive Auckland at 12:45 pm, arrive Bangkok at 2:30 pm

Open-ended – Sample answers

22 Elizabeth has written two of the times incorrectly as decimal numbers. 1 h 30 min is $1\frac{1}{2}$, or 1.5 h, not 1.3 h. 45 min is $\frac{3}{4}$ of an hour, or 0.75, not 0.45 h. There are 60 minutes in 1 hour, not 100, so times cannot be written as decimals—the 'point' between hours and minutes is a separator, not a decimal point.

A correct way to add up the times would be to add the hours: $1 + 2 + 1 + 1 = 5$, and then add the minutes: $30 + 45 = 75$ min $= 1\frac{1}{4}$ hours, so the total time is $6\frac{1}{4}$ hours.

23 Five breaks of 4 min 24 s each

24 Advantage: Get up soon after daybreak, giving more daylight time in the evening.
Disadvantage: Difficult to fall asleep when the Sun is up late and it's still hot.

Challenge 5

1 C Half full 1 minute before 10 am, one-quarter full 1 minute before that.

2 C $2\pi r = 14 \times 4$. $r = \frac{28}{\pi}$. $A = \pi \times \left(\frac{28}{\pi}\right)^2 = 249.55$

3 B

4 Area $GDEH = \frac{1}{2}(30 + 90) \times 60 = 3600$. Let $CG = x$.

$90x + 1800 = 3600$; $CG = 20$ cm

5 A (journey takes 45 minutes)

6 29 August is a Thursday, so 31 August is a Saturday. 7 September is the first Saturday, so 14 September is the second Saturday in September.

7 $20 \div 2 = 10$, so she will take the tablets on 10 occasions, the last one being $9 \times 8 = 72$ hours after she takes the first ones, or 3 days.

8 10 hours after 9 am the time is 7 pm; the clock will have lost $10 \times 3 = 30$ minutes, so it will show 6:30 pm. Correct time is 7 pm.

9 To get 5 pieces you need 4 cuts, so each cut takes $40 \div 4 = 10$ seconds. To get 8 pieces you need 7 cuts, so it takes $7 \times 10 = 70$ seconds, or 1 min 10 seconds.

10 Turn them both before cooking the rice. When the 7-minute timer runs out, start cooking, as there are 4 minutes left in the 11-minute timer. After the 4 minutes is over (the 11-minute timer has run out) turn the 11-minute timer upside-down to cook for another 11 minutes.

11 From when the train enters the tunnel to when the end just leaves the tunnel, the front of the train will have travelled $2 + 1 = 3$ km. At 20 km/h it takes $3 \div 20$ hours; that is, 9 minutes.

12 A normal year has 52 weeks and 1 day, so a given date falls a day later each year. A leap year has 52 weeks and 2 days, so a given date falls 2 days later from March of a leap year.

2009, Friday; 2010 Saturday; 2011 Sunday; 2012 leap year, Tuesday; 2013 Wednesday; 2014 Thursday; 2015 Friday. The year is 2015.

13 8th to 14th of the month

Chapter review 5

1 **(a)** 8 km **(b)** 72 km **(c)** 71 cm

2 **(a)** 31 m

(b) 24 cm, 25 cm or 26 cm (all acceptable)

3 **(a)** 182.21 cm **(b)** 232.48 mm **(c)** 395.84 cm

4 **(a)** 0.51 m^2 **(b)** 5 309 291.59 m^2 **(c)** 3.96 m^2

(d) 0.01 m^2 **(e)** 0.03 m^2 **(f)** 76.97 m^2

5 **(a)** 557 cm^2 **(b)** 130 cm^2 **(c)** 240 m^2

6 **(a)** 120 cm^3 **(b)** 99 m^3 **(c)** 6361.73 cm^3

7 **(a)** 45 mL **(b)** 2750 mm^2 **(c)** 16 000 000 cm^3

(d) 1.575 L **(e)** 0.36 ha **(f)** 275 000 mm^3

(g) 65 cm^3 **(h)** 53 000 000 m^3 **(i)** 0.25 m^3

(j) 90 000 m^2

8 2385.40 cm^3

9 **(a)** 10 h 43 min **(b)** 1344

10 **(a)** 1938 hours **(b)** 11:21 pm

11 39 times

12 The cylindrical can has the greater capacity by 256 mL.

13 160 cm, 1800 cm^2 **14** 7:15 pm

15 **(a)** 3.10 m^3 **(b)** 21 m

16 **(a)** 1950 m^3 **(b)** 1950 kL **(c)** 63 tankers

Numeracy practice 5

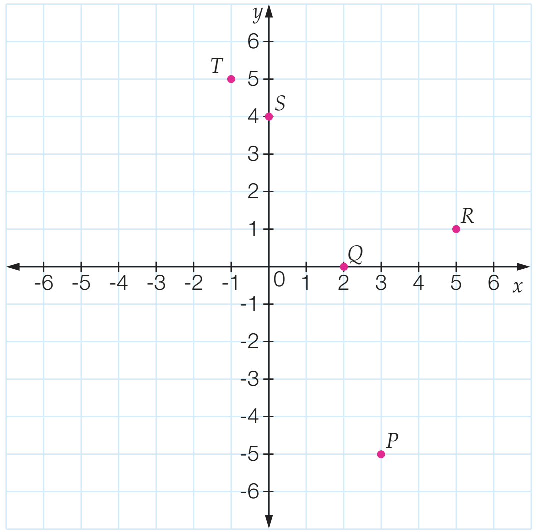
1 C **2** B **3** C **4** B

5 B **6** B **7** B **8** D

Chapter 6

Recall 6

1



2 $A(7, 2)$, $B(2, 0)$, $C(0, 5)$, $D(2, 3)$, $E(4, 2)$, $F(0, 0)$

3 (a)

x	1	2	3	10
y	2	5	8	29

(b)

x	0	2	6	10
y	4	5	7	9

4 (a) $a = 4$ (b) $m = -1$

5 (a) $x = 4$ (b) $x = 2$

(c) $x = 15$

6 (a) $3x$ (b) $2x + 8$

7 (a) $9x - 2y$ (b) $-x - 14y$

Exercise 6.1

1 (a) August (b) January (c) December

2 (a) (i) The family start their journey in Melbourne. They travel away from Melbourne (constant speed).

(ii) The car is stationary.

(iii) The family travel back to Melbourne (constant speed).

(b) section A (c) section C (d) 240 km

3 (a) F (b) D (c) B (d) C (e) E (f) A

4 (a) No, as the line does not start at zero on the 'Distance from home' axis.

(b) The horizontal parts of the graph show when Josie stopped.

(c) friend's place

(d) Yes, as the line finishes at zero distance from home.

5 (a) Kerala (b) Punjab

6 (a) section B (b) section C (c) section A

7 A

8 B

9 B

10 (a) 1400 m (b) tortoise (c) 150 min

(d) 55 min into the race, 500 m from the start

(e) The hare ran very quickly for 1000 m, then went back 500 m to see where the tortoise was. The hare decided to wait for the tortoise and probably fell asleep for about 140 min. The hare then tried to catch the tortoise, but was too late. The tortoise won the race.

11 90 000 cars

12 (a) decreased (b) August (c) January (d) $28

13 (a) unemployment rate, expressed as a percentage

(b) 2001 (c) 2005 (d) 2000 to 2002

(e) 0.7% (f) 3 years (g) 4 years

(h) 2001, the year of greatest unemployment.

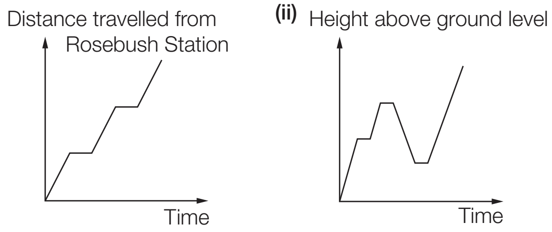
Open-ended – Sample answers

14 Students' own answers. For example:

Let the vertical axis represent Jodie's distance from school. Jodie walks at a constant rate away from school. She reaches the shop where she stops for a time. She then walks at a constant rate back towards school. She stops to rest at the park and then walks at a constant rate all the way back to school.

15 At a particular distance from home, a person travels towards home, then stops at a position for a period of time, then returns to the place they started from. From there, the person travels directly home.

16 (a) (i) (ii)

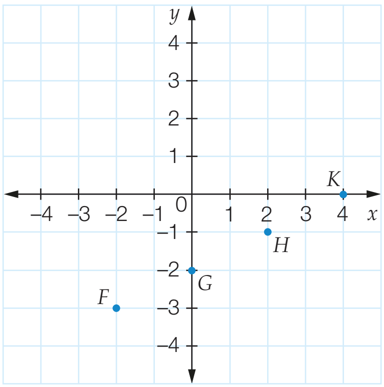


(b) Students' own answers.

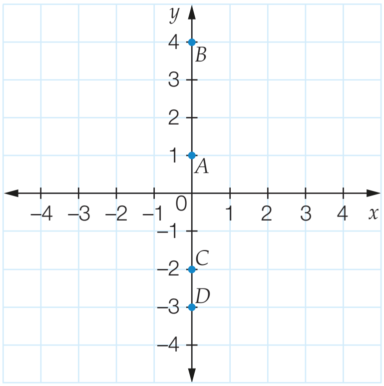
Exercise 6.2

1 $A(-7, 6)$, $B(0, 7)$, $C(-4, -2)$, $D(3, -4)$, $E(7, 0)$

2 (a)



(b)



All points are on the y-axis.

3 (a) (i)

x	-2	-1	0	1	2
y	-1	0	1	2	3
x, y	(-2, -1)	(-1, 0)	(0, 1)	(1, 2)	(2, 3)

(ii)

x	-2	-1	0	1	2
y	-1	-0.5	0	0.5	1
x, y	(-2, -1)	(-1, -0.5)	(0, 0)	(1, 0.5)	(2, 1)

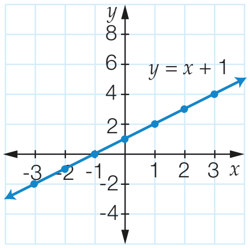
(iii)

x	-2	-1	0	1	2
$3x$	-6	-3	0	3	6
$3x - 1$	-7	-4	-1	2	5
(x, y)	(-2, -7)	(-1, -4)	(0, -1)	(1, 2)	(2, 5)

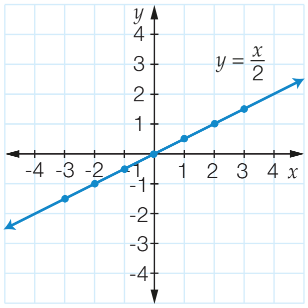
(iv)

x	-2	-1	0	1	2
$-x$	2	1	0	-1	-2
$-x - 4$	-2	-3	-4	-5	-6
(x, y)	(-2, -2)	(-1, -3)	(0, -4)	(1, -5)	(2, -6)

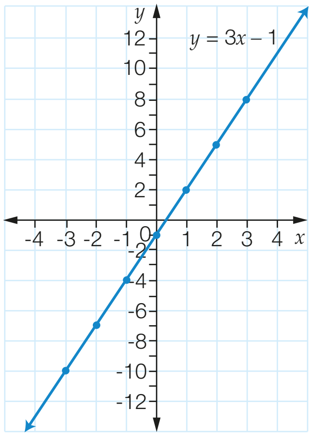
(b) (i)



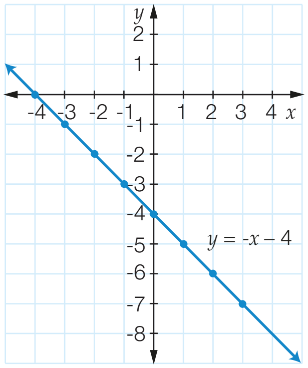
(ii)



(iii)



(iv)



4 (a) **(i)** 6 **(ii)** -1 **(iii)** -2 **(iv)** 9

(b) **(i)** -5 **(ii)** 8 **(iii)** 5 **(iv)** -2

5 (a) **(i)** -2, (-2, 0) **(ii)** 2, (2, 0) **(iii)** 2, (2, 0)
(iv) -0.5, (-0.5, 0)

(b) **(i)** 2, (0, 2) **(ii)** 1, (0, 1) **(iii)** 2, (0, 2) **(iv)** 1, (0, 1)

(c) **(i)** positive **(ii)** negative **(iii)** negative **(iv)** positive

6 D **7** A **8** D

9 (a) A **(b)** C **(c)** D

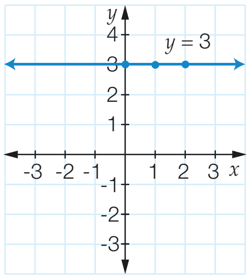
10 (a) Lines with *positive* gradient slope up to the right.
Lines with *negative* gradient slope down to the right.
Horizontal lines have *zero* gradient.
Parallel lines have the *same* gradient.
Vertical lines have *undefined* gradient.

(b) **(i)** *A*, *B*, *C*, *G* **(ii)** *D*, *F*, *H*, *I* **(iii)** *E*

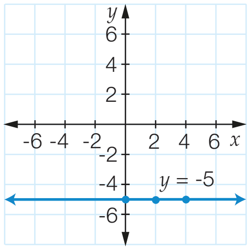
(iv) *A* and *C*; *F* and *D* **(v)** *J* **(vi)** *B*

11 (a) 33 °C **(b)** 10 am

12 (a) **(i)**



(ii)



(b) Each relationship is linear, as the graph is a straight line.

(c) zero

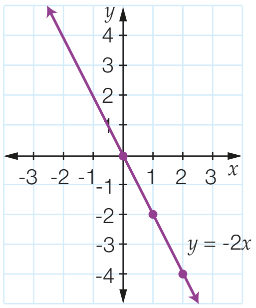
(d) parallel to the x-axis

(e) **(i)** no x-intercept; y-intercept is 3

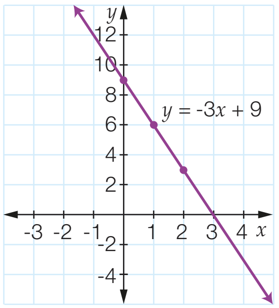
(ii) no x-intercept; y-intercept is -5

13 C

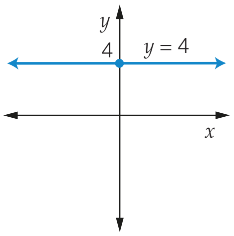
14 **(a)**



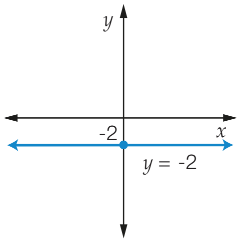
(b)



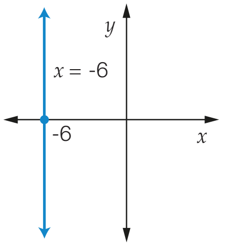
15 **(a)**



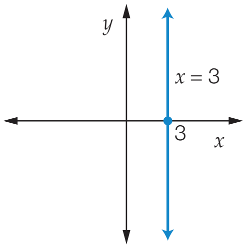
(b)



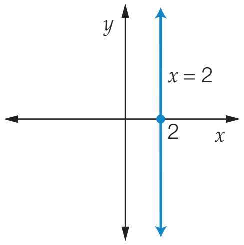
(c)



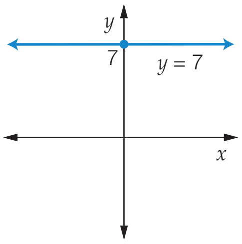
(d)



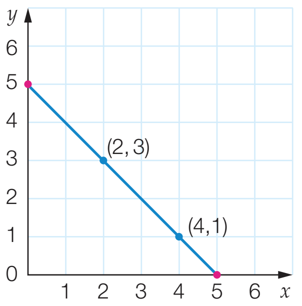
(e)



(f)

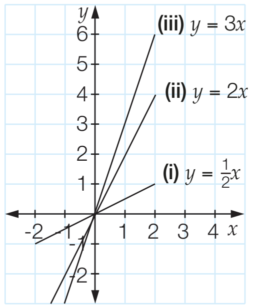


16



x-intercept is (5, 0) and y-intercept is (0, 5)

17 **(a)**



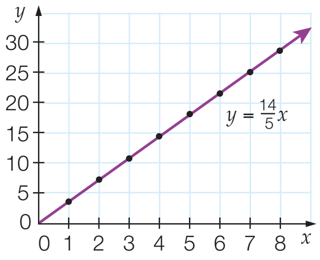
(b) The larger the coefficient, the steeper the line for positive coefficients, but for negative coefficients, the smaller the coefficient the steeper the line.

18 **(a)**

x	y	(x, y)
0	0	(0, 0)
1	2.8	(1, 2.8)
2	5.6	(2, 5.6)
3	8.4	(3, 8.4)
4	11.2	(4, 11.2)
5	14	(5, 14)
6	16.8	(6, 16.8)
7	19.6	(7, 19.6)
8	22.4	(8, 22.4)

(b) The horizontal distance cannot be less than zero.

(c)



(d) 7.1 metres

19 **(a)** negative **(b)** positive **(c)** negative **(d)** positive

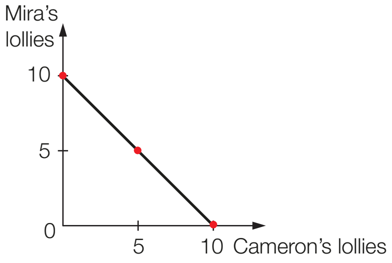
(e) positive **(f)** zero

20 **(a)** Sample answer:

Cameron	10	5	0
Mira	0	5	10

(b) Neither value can be negative, so the graph can't cross the x- or y-axis.

(c)



(d) negative

(e) As Mira gets fewer lollies, Cameron gets more.

(f) **(i)** 5.7

(ii) Cameron would have 4.3 lollies, Mira would have 5.7 lollies.

(iii) No, because each person can only have a whole number of lollies.

21 It is true when the line is not vertical or horizontal.

22 **(a)**

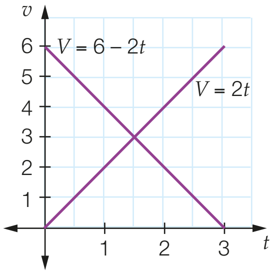
t	0	1	2	3
V	6	4	2	0
(t, V)	(0, 6)	(1, 4)	(2, 2)	(3, 0)

(b) The container is empty.

(c)

t	0	1	2	3
V	0	2	4	6
(t, V)	(0, 0)	(1, 2)	(2, 4)	(3, 6)

(d)



(e) Time and volume must be positive quantities. Therefore, it does not make sense to use values of t less than 0 or greater than 3.

(f) 1.5 minutes, 3 litres

Open-ended – Sample answers

23 (-2, 4), $(1, 5\frac{1}{2})$, (-4, 3), (2, 6)

24 **(a)** $x = 2, y = -2$

(b) $x = 3, y = 2 \times 3 - 3 = 3, x = 6\frac{1}{2}, y = 2 \times 6\frac{1}{2} - 3 = 10$

Rachelle ignored the brackets.

(c)

x	3	8	2
y	0	10	-2

25 Students' own answers.

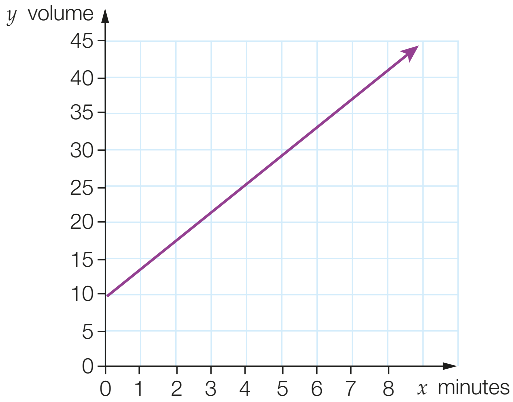
For example: Complete a table of values and draw the graph using the rule $y = x - 1$.

Half-time 6

1 **(a)**

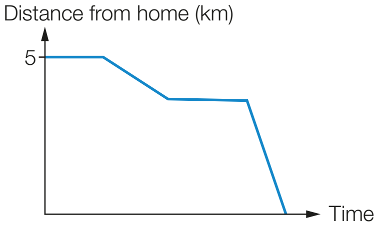
x (minutes)	3	4	5	6	7	8
y (volume of water in container, mL)	22	26	30	34	38	42

(b)–(c)



(d) 10 millilitres

2



3 **(a)** three times

(b) Dale stopped to rest once.

(c) No. The end of the graph does not finish on the horizontal axis.

4 A

5 **(a)** Rose **(b)** Between month 3 and month 4.

(c) Between months 1 and 7 (the fern overtook the rose during month 7)

(d) 40 cm **(e)** month 5 **(f)** 3 months

Exercise 6.3

1 **(a)** $y = x - 1$ **(b)** $y = x + 1$ **(c)** $y = x + 6$

(d) $y = x - 3$ **(e)** $y = x$ **(f)** $y = 4x$

(g) $y = 5x$ **(h)** $y = -x$

2 **(a)** $y = x + 1$ **(b)** $y = x - 3$ **(c)** $y = -2x$

(d) $y = 7x$ **(e)** $y = -\frac{x}{3}$ **(f)** $y = \frac{x}{5}$

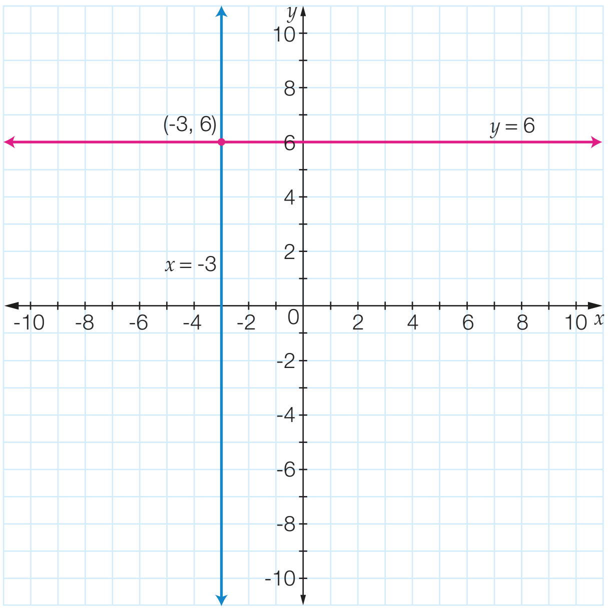
3 **(a)** $y = x$ **(b)** $y = 3x$ **(c)** $y = x + 3$

(d) $y = x + 5$ **(e)** $y = x - 2$ **(f)** $y = 8x - 4$

(g) $y = -4x$ **(h)** $y = -2x$

4 D

5



Point of intersection: (-3, 6)

6 **(a)** D **(b)** C **(c)** A

7 D

8 $y = -3$

9 **(a)** $y = x + 1$

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

(b) $y = x + 2$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

(c) $y = x - 5$

x	-3	-2	-1	0	1	2	3
y	-8	-7	-6	-5	-4	-3	-2

(d) $y = x - 3$

x	-3	-2	-1	0	1	2	3
y	-6	-5	-4	-3	-2	-1	0

(e) $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

(f) $y = 3x$

x	-3	-2	-1	0	1	2	3
y	-9	-6	-3	0	3	6	9

(g) $y = -8x$

x	-3	-2	-1	0	1	2	3
y	24	16	8	0	-8	-16	-24

(h) $y = -6x$

x	-3	-2	-1	0	1	2	3
y	18	12	6	0	-6	-12	-18

10 **(a)** B **(b)** A **(c)** C **(d)** D

11 **(a)**

s	1	2	3	4
m	4	7	10	13

$m = 4 + 3(s - 1)$

(b) 31 matches

12 **(a)** $V = 100\,000t + 200\,000$ **(b)** \$1 400 000

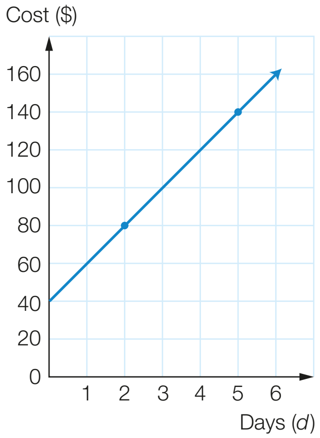
13 **(a)**

n	1	2	3	4
C (\$)	55	100	145	190

$C = 45n + 10$

(b) 7

14 **(a)**



(b) $C = 20d + 40$ **(c)** 6 days

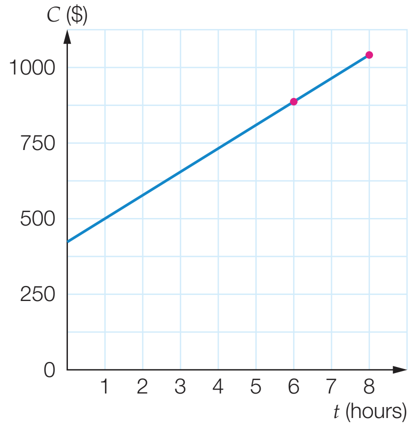
15 **(a)** $D = 2S + 4$

(b) 2 dots are added to make each new square.

(c) 4 dots needed: 3 at one end and 1 at the other end.

(d) 20 dots

16 **(a)** $C = 80t + 400$

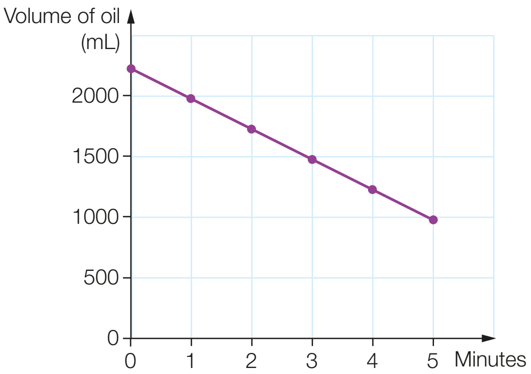


(b) \$80 per hour **(c)** \$400

17 **(a)**

x (minutes)	0	1	2	3	4	5
y (volume of oil in engine, mL)	2225	1975	1725	1475	1225	975

(b)



(c) $y = 2225 - 250x$

(d) 8.9 minutes (8 minutes, 54 seconds)

Open-ended – Sample answers

18 $y = x - 3; y = \frac{x}{5} + 1; y = \frac{x+1}{3}$

19 Students' own answers.
For example: $y = -x; y = 1 - x; y = 2 - x$

Exercise 6.4

1 **(a)** l = length of track, T = temperature

(b) V = volume of water, t = time

(c) p = tyre pressure, V = volume of air

(d) r = radius of balloon, V = volume

(e) d = distance run, t = time taken

2 **(a)** s = solicitor's fee in \$
h = hours spent producing documents
$s = 50 + 100h$

(b) f = taxi fee in \$
k = number of km travelled
$f = 3.2 + 1.6k$

(c) c = cost of a phone call in \$
m = number of minutes spent on phone
$c = 0.37 + 0.4m$

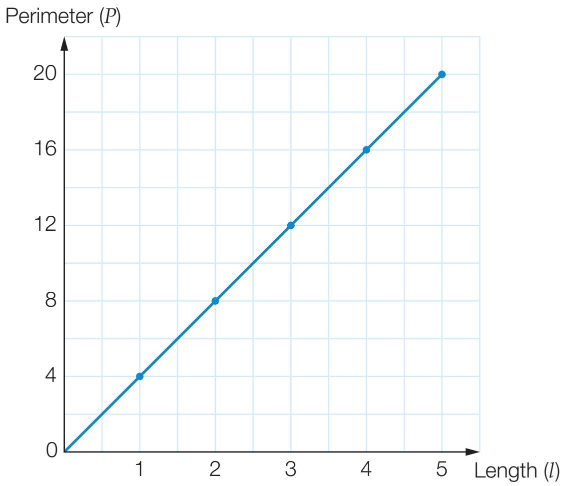
3 **(a)**

d	1	2	3	4	5	6	7
C	43	46	49	52	55	58	61

(b) yes **(c)** \$64

4 **(a)** $P = 4l$

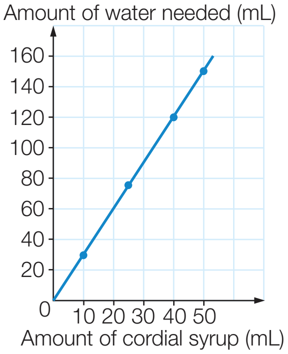
(b)



(c) Yes. The graph is a straight line, so the relationship is linear.

(d) 6 m **(e)** 2.5 m

5 **(a)**



(b) yes

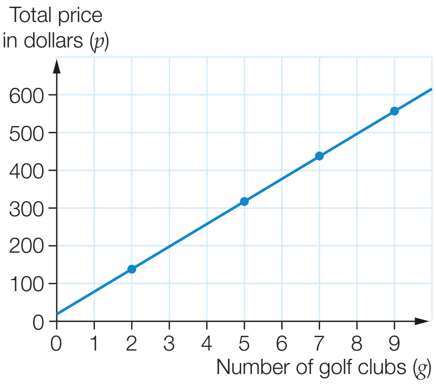
(c) **(i)** 45 mL **(ii)** 84 mL **(iii)** 165 mL

(d) **(i)** 8 mL **(ii)** 30 mL **(iii)** 45 mL

(e) Yes, because both variables are continuous.

(f) $w = 3s$

6 **(a)**



(b) yes

(c) **(i)** \$260 **(ii)** \$380 **(iii)** \$620

(d) **(i)** 3 **(ii)** 8 **(iii)** 12

(e) No, because the number of golf clubs isn't a continuous variable. Use a linear graph for convenience.

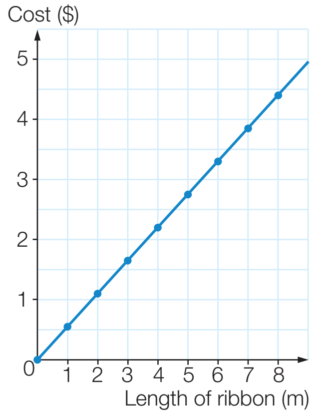
(f) $p = 60g + 20$

(g) \$60 per club and \$20 for postage and delivery

7 **(a)**

Length of ribbon (m)	0	1	2	3	4	5	6	7	8
Cost ($)	0	0.55	1.10	1.65	2.20	2.75	3.30	3.85	4.40

(b)

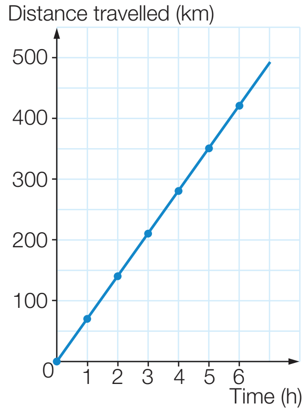


(c) $2.30 **(d)** 5.5 m **(e)** 9 m (9.1 m)

8 **(a)**

Time (h)	0	1	2	3	4	5	6
Distance travelled (km)	0	70	140	210	280	350	420

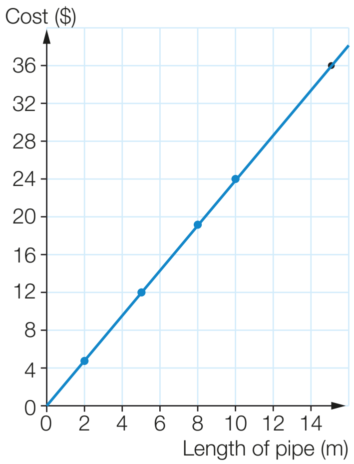
(b)



(c) **(i)** 245 km **(ii)** 35 km **(iii)** 370 km

(d) **(i)** 2.5 h **(ii)** 4.3 h **(iii)** 5.7 h

9 **(a)**



(b) Yes, it doesn't cost anything to buy 0 m of pipe.

(c) $28.80 **(d)** $9.60

(e) It costs the same amount.

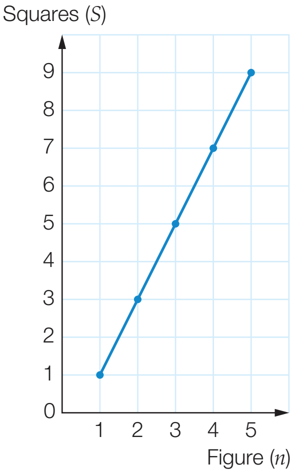
(f) 21 m **(g)** 42 m

10 **(a)** $S = 2n - 1$

(b)

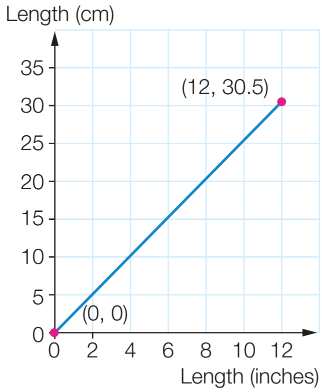
n	1	2	3	4	5
S	1	3	5	7	9

(c) From the graph it is a straight line. Therefore, it is a linear relation.



(d) 9

11 **(a), (d)**



(b) (0, 0) **(c)** (12, 30.5) **(e)** 28 cm

(f) 2.2 inches **(g)** 51 cm **(h)** 13 inches, 8 inches

(i) 20 cm **(j)** 9.7 cm, 4.3 cm

12 **(a)**

w	1	2	3	4	5	6	7	8
S	620	590	560	530	500	470	440	410

(b) $470

13 **(b)** (0, 32) **(c)** (100, 212) **(e)** 56.7 °C **(f)** -128 °F

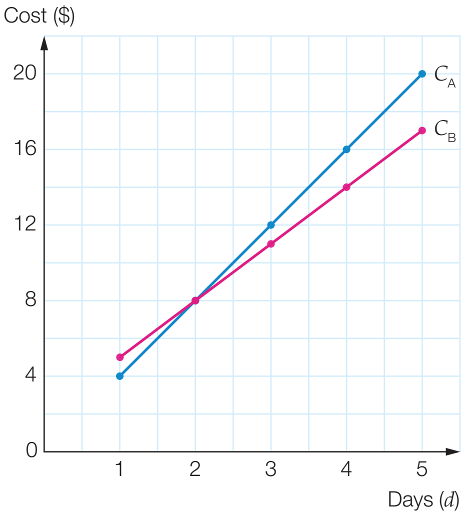
(g) 30 °C (86 °F) is hotter than 80 °F **(h)** 37 °C

(i) 95 °F **(j)** 10 °C, wear something warm

14 **(a)**

d	1	2	3	4	5
C_A	4	8	12	16	20
C_B	5	8	11	14	17

(b)



Company B

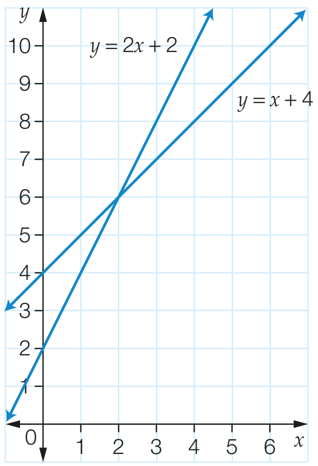
(c) 2 days

Open-ended – Sample answers

15 Students' own answers. For example:

(a) $a = 2$, $b = 2$ and $a = 1$, $b = 4$

(b)



(c) (2, 6)

(d) Where $x = 2$:

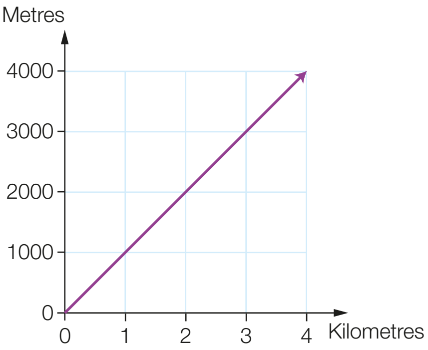
$y = 2x + 2 = 6$

$y = x + 4 = 6$

16 Students' own answers. For example:

(a) $m = 1000k$, where m is metres and k is kilometres

(b)



Challenge 6

1 **(a)** B **(b)** 10 minutes **(c)** 6 km

(d) Between 20 and 30 minutes, and between 50 and 60 minutes.

2 **(a)** Eleanor **(b)** Brian **(c)** Candy and David

3 D **4** C

5 $k = 5.5$. Using the link between gradients, $\frac{k-1}{6-3} = \frac{10-1}{9-3}$.

Or, find the equation of AB and substitute the coordinates of C.

Chapter review 6

1 **(a)** D **(b)** C **(c)** B

2 **(a)** **(i)**

x	-2	-1	0	1	2
y	-6	-4	-2	0	2
(x, y)	(-2, -6)	(-1, -4)	(0, -2)	(1, 0)	(2, 2)

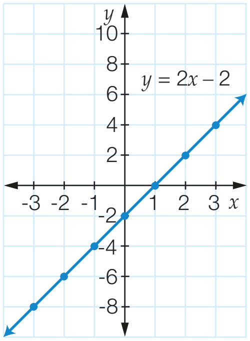
(ii)

x	-2	-1	0	1	2
y	6	5	4	3	2
(x, y)	(-2, 6)	(-1, 5)	(0, 4)	(1, 3)	(2, 2)

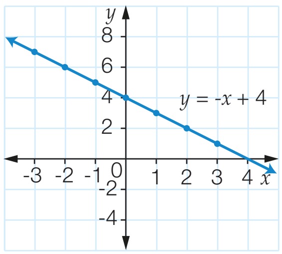
(iii)

x	-2	-1	0	1	2
y	5	2	-1	-4	-7
(x, y)	(-2, 5)	(-1, 2)	(0, -1)	(1, -4)	(2, -7)

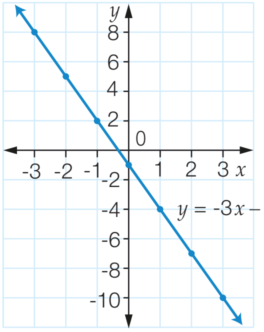
(b) **(i)**



(ii)



(iii)



(c) **(i)** x-intercept (1, 0); y-intercept (0, -2)

(ii) x-intercept (4, 0); y-intercept (0, 4)

(iii) x-intercept $(-\frac{1}{3}, 0)$; y-intercept (0, -1)

3 C

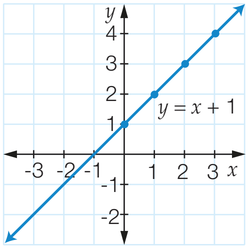
4 **(a)** B **(b)** C **(c)** A **(d)** D

5 **(a)** $y = x + 3$ **(b)** $y = -4x$ **(c)** $y = x - 2$

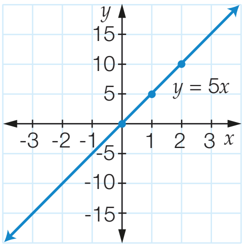
x	-2	-1	0	1	2
y	-4	-3	-2	-1	0

6 Sample answer: $C = a + bt$, where C is the total cost in dollars, a = fixed fee, b = hourly charge, t = time in hours.

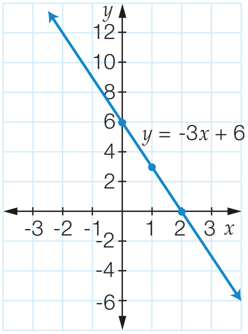
7 **(a)** **(i)**



(ii)



(iii)

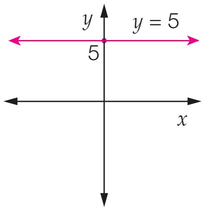


(b) **(i)** -1 **(ii)** 0 **(iii)** 2

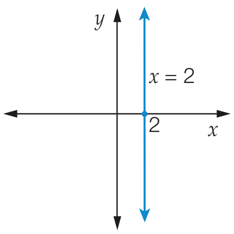
(c) **(i)** 1 **(ii)** 0 **(iii)** 6

(d) **(i)** positive **(ii)** positive **(iii)** negative

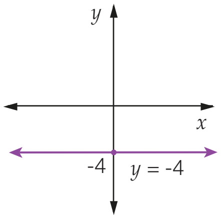
8 **(a)** **(i)**



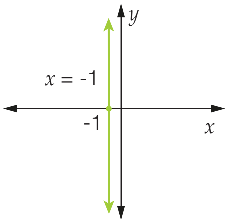
(ii)



(iii)



(iv)



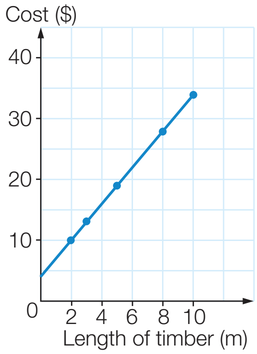
(b) **(i)** no x-intercept, y-intercept is 5

(ii) x-intercept is 2, no y-intercept

(iii) no x-intercept, y-intercept is -4

(iv) x-intercept is -1, no y-intercept

9 **(a)**



(b) The relationship is linear because the graph is a straight line.

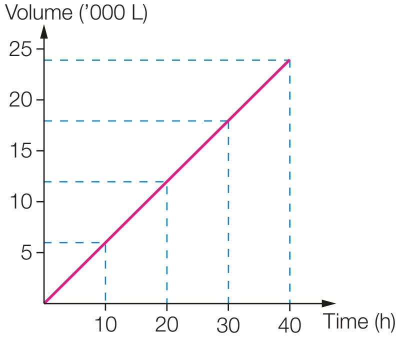
(c) **(i)** \$16 **(ii)** \$31 **(iii)** \$37

(d) **(i)** 3.7 m **(ii)** 5.3 m **(iii)** 9.3 m

10 **(a)**

t	0	10	20	30	40
V	0	6000	12 000	18 000	24 000

(b)



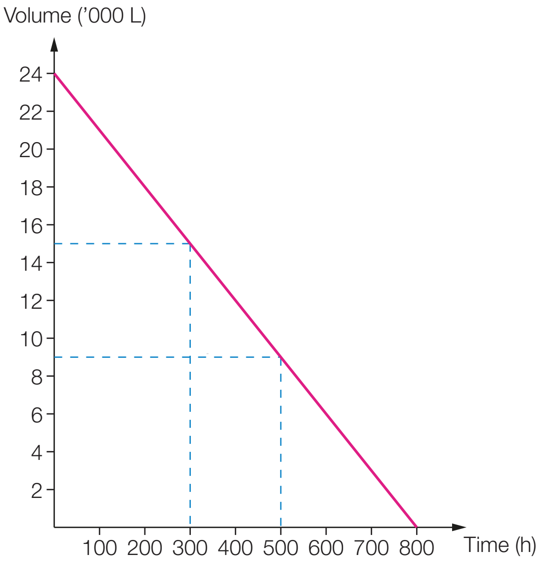
(c) 24 000 L

(d) **(i)** 10 800 L **(ii)** 30 hours

(e)

t	0	100	200	300	400	500	600	700	800
V	24 000	21 000	18 000	15 000	12 000	9000	6000	3000	0

(f)

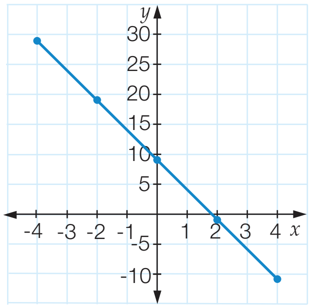


(g) **(i)** 15 000 L **(ii)** 500 hours = $20\frac{5}{6}$ days

(h) 30 L

11 **(a)** $y = 9 - 5x$
y-intercept is 9, so the constant is 9.
If x goes up 2, y goes down 10, so the coefficient of x = -5.

(b)



(c) -6 **(d)** -1

12 **(b)** (0, 0) **(c)** (10, 24.7)

(e) approximately 5 acres **(f)** approximately 8 hectares

(g) 7 hectares is larger than 15 acres (≈ 6 hectares).

Numeracy practice 6

1 A **2** D **3** C **4** B

5 C **6** D **7** C

Mixed review C

1 **(a)** $2^9 \times 5^5$ **(b)** 6×7^3

2 **(a)** $d = 3.75$ **(b)** $m = 6.25$ **(c)** $p = 36.4$

3 **(a)** $3b(2a + 3c)$ **(b)** $3xy(8xy - 1)$

4 **(a)** $\frac{4}{9}$ **(b)** $\frac{19}{99}$

5 **(a)** $xy - 2x^2 + 4y$ **(b)** $6m^2 - n^2$

6 **(a)** $2^6 \times 3^8 \times 4^2$ **(b)** $6^3 \times 7^6$

7 **(a)** -87 **(b)** -8

8 **(a)** 32 : 28 **(b)** 35 : 50 **(c)** 3000 : 6000 : 12 000

9 **(a)** 9.1 cm^2 **(b)** 5.3 cm^2

10 **(a)** **(i)** 0.375 **(ii)** 37.5%

(b) **(i)** 0.325 **(ii)** 32.5%

(c) **(i)** 0.816 **(ii)** 81.6%

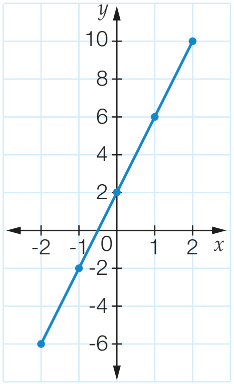
11 **(a)** 4.8 cm^2 **(b)** 2.8 cm^2

12 **(a)** 25% **(b)** 20.5% **(c)** 20% **(d)** 24%

13 **(a)**

x	-2	-1	0	1	2
y	-6	-2	2	6	10

(b)



(c) x-intercept = $-\frac{1}{2}$, y-intercept = 2

14 **(a)** $\frac{7}{20}$ **(b)** $\frac{17}{42}$ **(c)** -7.2 **(d)** -3.25

15 **(a)** $y = \frac{1}{2}x$ **(b)** $y = 3x + 1$

16 **(a)** $m = 6$, x-intercept = $\frac{5}{2}$, y-intercept = -15

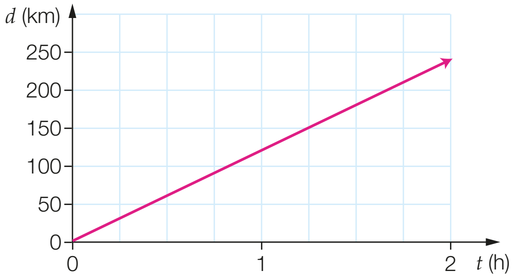
(b) $m = \frac{3}{4}$, x-intercept = $-\frac{2}{3}$, y-intercept = $\frac{1}{2}$

17 **(a)** \$815 **(b)** 52%

18 **(a)** 1.48 : 1 **(b)** 34 cm

19 **(a)** $d = 120t$

(b)



(c) **(i)** 100 km

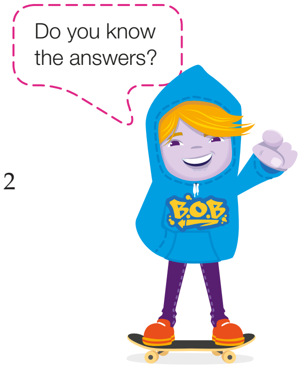
(ii) 180 km

(d) **(i)** 45 minutes

(ii) 1 hour 20 minutes

20 **(a)** $y = x - 1$ **(b)** $y = \frac{x}{2} + 2$

21 (0, 4), (1, 2) and (2, 0)
$x = 0, y = -2 \times 0 + 4 = 4$
$x = 1, y = -2 \times 1 + 4 = 2$
$x = 2, y = -2 \times 2 + 4 = 0$



Chapter 7

Recall 7

1 A, C

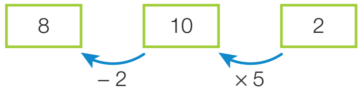
2 (a) $a = 5$ (b) $b = 20$

(c) $c = -3$ (d) $d = 17$

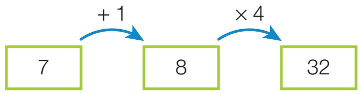
(e) $e = -5$ (f) $f = 8$

3 (a)

(b)



(c)



(d)

4 (a) 14 (b) 32 (c) 3

5 (a) divide by 2 (b) subtract -4 (c) multiply by 5

6 (a) $2x + 10$ (b) $6x - 21$

Exercise 7.1

1 (a) $a + 8 = 12$ (b) $b - 4 = 16$ (c) $9c = 63$

(d) $11 + f = 0$ (e) $7 + 3u = 10$ (f) $2v + 7 = 13$

(g) $\frac{9 + x}{7} = 6$ (h) $8(6 + z) = 0$

2 (a) yes (b) no (c) no (d) yes

(e) no (f) yes (g) no (h) yes

(i) no (j) no (k) yes (l) no

3 (a) C (b) D (c) A (d) D (e) B (f) D

4 (a) A number is added to two to give the result nine.

(b) Two less than a number is equal to nine.

(c) Six is taken away from a number to give the result three.

(d) A number is taken away from ten and the result is four.

(e) A number is multiplied by five and the result is fifty-five.

(f) A number is divided by eight and the result is nine.

(g) Seven is taken away from six times a number and the result is twenty-three.

(h) Five less than three times a number equals negative ten.

(i) Eight is taken away from a number, then the result is multiplied by three to give six.

(j) Two is added to three times a number to give a result of five.

(k) Three is added to a number, then this answer is divided by four and the result is eight.

(l) A number is taken away from six, then this answer is divided by seven to give the result of four.

5 (a) $A = l \times w$ (b) $A = \frac{1}{2}bh$ (c) $s = \frac{d}{t}$

(d) $A = \frac{1}{2}h(a + b)$ (e) $C = 200n + 300$ (f) $F = \frac{10}{11}R$

6 $\$(4.55 + 0.80x)$

7 (a) $300 + 7x$ (b) 2 months

8 (a) $C = 150 + 27x$ (b) \$285 (c) $A = 25y$

(d) $25 \times \$10 = \$250$, which will not cover the cost.

(e) cost = \$420, charge = \$425, left over = \$5

9 (a) $\$(25 + 0.03x)$ (b) 500 calls

10 (a) $S = \$(20 + 5x)$ (b) $20 + 5 \times 15 = \$95$

(c) $R = \$(5y - 15)$ (d) No, $5 \times 20 - 15 = \$85$

Open-ended – Sample answers

11 $w + 5 = 8$; $5w = 15$; $\frac{w}{3} = 1$

Exercise 7.2

1 (a) $x = 16$ (b) $x = 11$ (c) $x = 4$

(d) $x = -8$ (e) $x = 15$ (f) $x = -77$

(g) $x = 2$ (h) $x = 7$ (i) $x = -3$

2 (a) $a = 1$ (b) $b = 4$ (c) $c = -3$

(d) $x = 1$ (e) $x = -3$ (f) $x = -6$

(g) $x = 1$ (h) $x = 3$ (i) $x = 6$

3 (a) $y = 6$ (b) $y = 4$ (c) $y = 0$

(d) $x = 0$ (e) $x = 1$ (f) $x = 3$

(g) $y = 2$ (h) $y = 5$ (i) $y = -1$

4 (a) $x = 2$ (b) $x = -1$ (c) $x = 0$

5 (a) $2b + 3 = 7$ (b) $2b = 4$ (c) $b = \frac{4}{2}$ (d) $b = 2$

6 (a) B (b) D

7 (a) D (b) C (c) B

8 (a) $r = 1\frac{1}{2}$ (b) $t = 2\frac{1}{3}$ (c) $l = 1\frac{5}{7}$

(d) $a = 1$ (e) $e = -2$ (f) $f = -10$

(g) $k = -2$ (h) $h = -2$ (i) $g = -5$

(j) $n = -1$ (k) $c = -4$ (l) $k = -2$

(m) $d = 2\frac{1}{6}$ (n) $p = 0.85$ (o) $a = \frac{7}{15}$

(p) $x = 6$ (q) $x = 9$ (r) $x = -2$

(s) $g = \frac{3}{8}$ (t) $r = 1\frac{1}{6}$ (u) $p = \frac{1}{9}$

9 (a) $d = -8$ (b) $e = 3$ (c) $f = -3$

(d) $a = 2$ (e) $b = 1$ (f) $c = -4$

(g) $g = 5$ (h) $h = 6$ (i) $m = -8$

(j) $r = 1$ (k) $p = 3$ (l) $t = 8$

(m) $p = -\frac{2}{3}$ (n) $l = -\frac{4}{5}$ (o) $x = 2$

(p) $a = 6.55$ (q) $b = 1.72$ (r) $c = 3.65$

10 (a) $4n - 5 = 23; n = 7$ (b) $3(m + 7) = 45; m = 8$

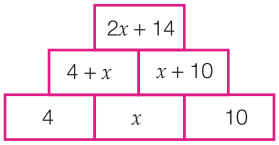
(c) $\frac{p + 5}{9} = 3; p = 22$ (d) $\frac{q}{4} + 2 = -1; q = -12$

11 (a) E (b) I (c) A (d) G (e) D

(f) H (g) B (h) F (i) J (j) C

12 (a) $2a + 16 = 102$ (b) $a = 43$ kg

13 (a)



(b) $x = 3$

14 1 **15** $6\frac{1}{2}$ hours

16 $5p + 25 = 62.5; p = 7.5$. He receives $7.50 each week.

17 (a) 2

(b) 7 children. The aircraft can carry 1 adult and 5 children, or $a + 5c$; 1 adult is equivalent to 2 children, so:

$$\begin{aligned} a + 5c &= 2c + 5c \\ &= 7c \end{aligned}$$

18 25 km. Total distance for 10 weekday trips, where w is the distance from home to work, plus the weekend distance:

$$\begin{aligned} 10w + 75 &= 325 \\ w &= 25 \end{aligned}$$

19 $1.25 + 2m = 3; m = 0.875$ L = 875 mL each

Open-ended – Sample answers

20 (a) $2x + 5 = 13, x = 4$ (b) $3x - 7 = 5, x = 4$

21 (a) The steps will result in negative 10 grapes and one-and-a-half apples.

(b) Remove 10 grapes on each side, then remove 2 apples on each side. The answer is 1 apple = 10 grapes.

22 $4x + 6 = 22; 2x = 8; 2x + 12 = 20$

23 $3x - 4 = 2$

Exercise 7.3

1 (a) $c = 12$ (b) $f = 21$ (c) $m = 81$ (d) $x = -4$

(e) $b = -9$ (f) $t = -16$ (g) $x = -6$ (h) $p = -37$

(i) $r = -79$ (j) $x = 15$ (k) $y = 44$ (l) $t = -49$

2 (a) $x = 7$ (b) $x = -13$ (c) $x = -5$ (d) $x = 3$

(e) $x = -1\frac{5}{9}$ (f) $x = 5\frac{3}{7}$ (g) $x = -10$ (h) $x = 2$

(i) $x = -3$ (j) $x = 2$ (k) $x = 4$ (l) $x = 2\frac{2}{3}$

3 (a) $a = -1$ (b) $x = 6$ (c) $r = -6$ (d) $r = 14$

(e) $n = 7$ (f) $p = 15$ (g) $r = 1$ (h) $f = 4$

(i) $m = -3$ (j) $r = 2$ (k) $p = 8.5$ (l) $k = 13$

4 (a) C (b) B

5 (a) $a = 14$ (b) $p = -9$ (c) $m = -12$ (d) $a = 2\frac{1}{5}$

(e) $a = \frac{3}{4}$ (f) $a = -1\frac{1}{3}$ (g) $x = 8.5$ (h) $x = 4\frac{1}{3}$

(i) $x = 7\frac{3}{4}$ (j) $x = -4$ (k) $x = -3$ (l) $x = -4$

6 (a) D (b) F (c) G (d) I (e) H

(f) C (g) A (h) J (i) E (j) B

7 Writing the unknown as x:

(a) $\frac{3x + 7}{4} = 4, x = 3$ (b) $2\left(\frac{x}{4} - 1\right) = 6, x = 16$

(c) $\frac{5x - 2}{6} = -12, x = -14$ (d) $\frac{3x}{2} + 7 = 8, x = \frac{2}{3}$

8 24

9 (a) $\frac{x - 10}{5} + 20 = 48, x = \$150$ (b) $170

10 11 cm

11 $11 000

12 (a) $\frac{2x}{11} + 50 = 150, x = \$550$ (b) $25

13 $3(2x + 6) = 108, x = 15$

14 $3d + d = 12; d = 93$

15 $2d + (d - 1) + d = 15; d = 834$

16 $\frac{a}{3} = \frac{b}{6}; a + b = 120; a = 40, b = 80$

17 Week 1: $w_1 = 2m - 20$

Week 2: $w_2 = 2 \times (2m - 20) - 25$
$= 4m - 65$

Week 3: $w_3 = 3 \times (4m - 65) - 15$
$150 = 12m - 210$
$m = \$30$

18 $2 \times (3x + 8) = 106$
$x = 15$

Open-ended – Sample answers

19 $\frac{2x}{3} - 3 = 0; \frac{2y}{3} - 3 = 21; \frac{2m}{3} - 3 = -1$

20 (a) $\frac{7x + 4}{5} = 5$

(b) Students' own answers.

21 (a) Step 2. The $5x$ must remain negative, then divide both sides by -5.

(b) $x = -1\frac{1}{5}$

Half-time 7

1 (a) $x = -3$ (b) $x = 4$ (c) $x = 26$

(d) $x = -5$ (e) $x = 10.5$ (f) $x = -60$

2 (a) incorrect (b) correct (c) correct (d) incorrect

3 Phillip = \$40, James = \$20, Sam = \$70

4 (a) $x = 24$ (b) $x = 2$ (c) $x = 20$ (d) $x = -45$

5 (a) $x - 4 = 12, x = 16$ (b) $7x = 35, x = 5$

(c) $6x = 3 \times 12, x = 6$ (d) $7x + 3 = 24, x = 3$

6 $x = 45$, length = 145 m, width = 95 m

Exercise 7.4

1 (a) $x = 4$ (b) $x = 7$ (c) $a = 6$ (d) $d = 7$

(e) $e = 4$ (f) $f = -2$ (g) $g = -7$ (h) $h = -6$

(i) $x = 2$ (j) $k = 1\frac{1}{2}$ (k) $j = 2$ (l) $y = 3$

(m) $m = 4$ (n) $n = 2$ (o) $x = 1$ (p) $y = 1$

(q) $p = 1$ (r) $x = 3$ (s) $p = -6$ (t) $x = 3$

(u) $a = -2$

2 (a) $x = 7$ (b) $x = 4$ (c) $x = -2$

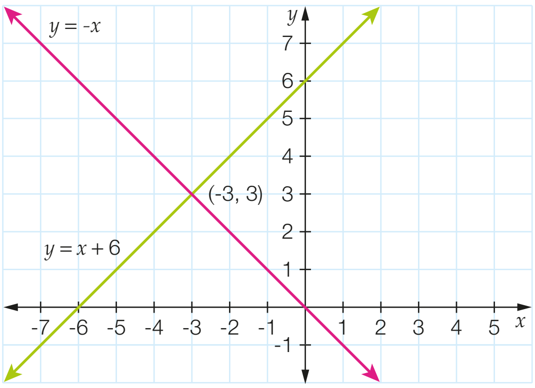
(d) $x = -\frac{13}{4} = -3\frac{1}{4}$ (e) $x = -10$ (f) $x = \frac{35}{6} = 5\frac{5}{6}$

(g) $x = \frac{1}{9}$ (h) $x = \frac{9}{10}$ (i) $x = 0$

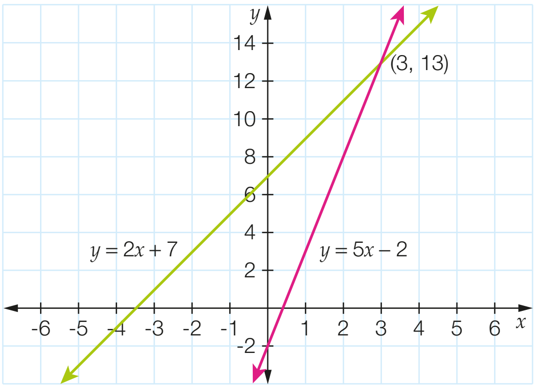
(j) $x = \frac{2}{3}$ (k) $x = -2$ (l) $x = -2$

(m) $x = -32$ (n) $x = 6$ (o) $x = \frac{11}{10}$

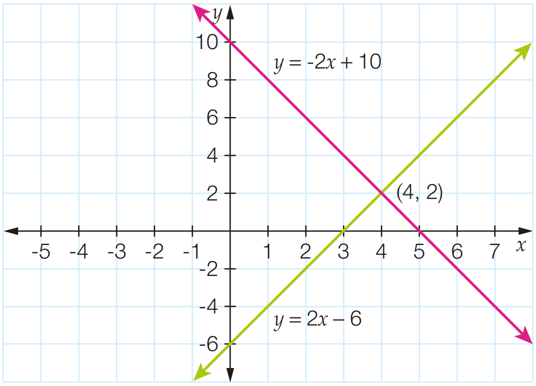
3 (a)



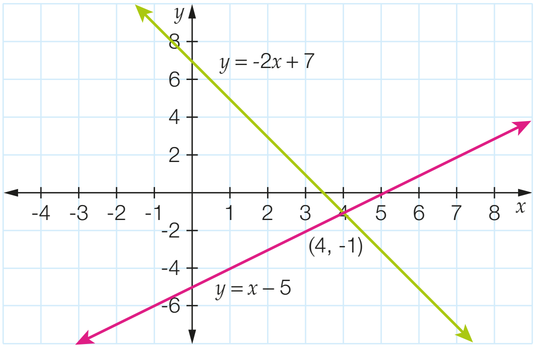
(b)



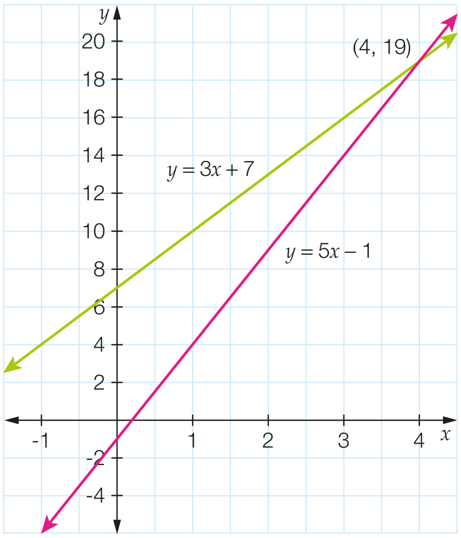
(c)



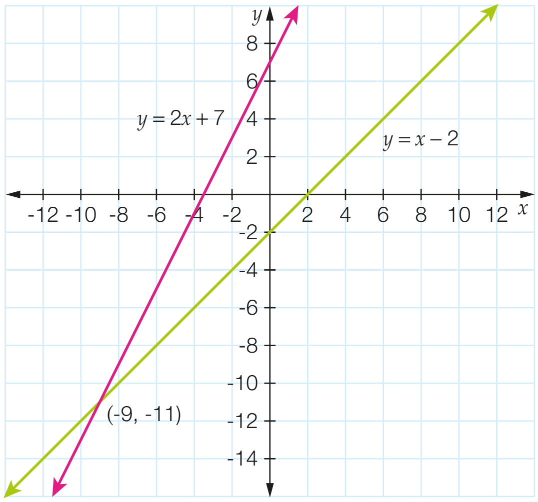
(d)



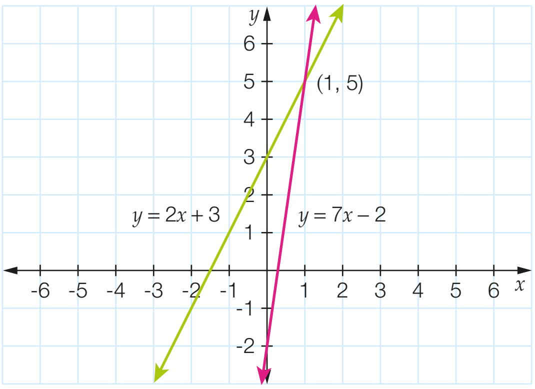
(e)



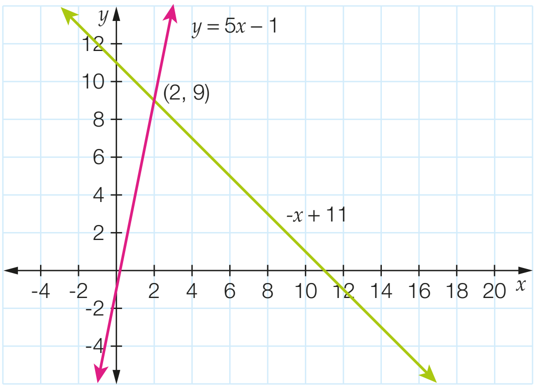
(f)



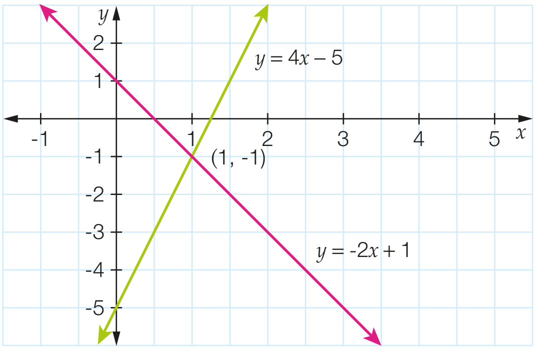
(g)



(h)



(i)



4 **(a)** B **(b)** D **(c)** D

5 **(a)** $4x - 11$ **(b)** $7 - 2x$

(c) $4x - 11 = 7 - 2x$ **(d)** $x = 3$

6 **(a)** $x + 36$ **(b)** $2(x + 6)$

(c) $x + 36 = 2(x + 6)$ **(d)** $x = 24$

7 **(a)** $\frac{3x}{4} - 5$ **(b)** $2(x - 10)$

(c) $\frac{3x}{4} - 5 = 2(x - 10)$ **(d)** $x = 12$

8 6

9 $\frac{1}{2}$

10 **(a)** $y = 2x + 8$
$= 2 \times -3 + 8$ where $x = -3$
$= 2$

(-3, 2) lies on $y = 2x + 4$

$y = -x - 1$
$= - -3 - 1$ where $x = -3$
$= 2$

(-3, 2) lies on $y = -x - 1$

Therefore, (-3, 2) must be the point of intersection.

(b) $2x + 8 = -x - 1$

(c) $x = -3$

(d) 2 is the value of $2x + 8$ and $-x - 1$ where $x = -3$

11 $x = 4$ **12** 5 years **13** $150

14 Equations:

$s = f + 20 = b - 40$

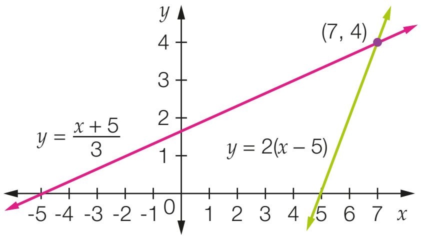
$b = 2f + 10$

$f + 20 = 2f + 10 - 40$

Solve for f, then calculate s and b.

Sarah = $70, Frank = $50, Bruce = $110

15



$x = 7$

16 $3(x - 1) = 2(x + 5)$, $x = 13$

Open-ended – Sample answers

17 $2x + 3 = 8 - 3x$ ($x = 1$); $4x - 5 = 2x + 3$ ($x = 4$); $3x - 2 = 2x + 7$ ($x = 9$)

18 **(a)** Step 1—error in expanding the right bracket. Correction: expand both terms by 2 to get the result $2x - 8$.

Step 3—error in inverse operation of +36. Correction: subtract 36 from both sides.

(b) $x = -11$

Exercise 7.5

1 **(a)** $3c = 117$; $c = 39$ cents **(b)** $m - 935 = 9$; $m = 944$ kg

(c) $n + 21 = 49$; $n = 28$ seats **(d)** $t - 9 = 27$; $t = 36$ °C

2 $x + 7 = 11$; Darren walked 4 km.

3 **(a)** $$(x + 2)$

(b) $$(x + 2) = 4.5; cost of salad roll is $2.50.

4 **(a)** $\$\frac{3w}{4}$

(b) $\$\frac{3w}{4} = \$24$; Olivia is paid $32.

5 Raj has 4 toys, Alex has 8 toys

6 **(a)** $x + x + 14$

(b) $2x + 14 = 20$; the Kittens lost 3 games and won 17 games.

7 **(a)** B **(b)** B **(c)** D

8 **(a)** $6x + 1$

(b) $6x + 1 = 25$; each person receives 4 trading cards.

9 16, 32

10 **(a)** $2l + 2w$

(b) $2l + 4 = 10$; the vegetable garden can be 3 m long.

(c) $2l + 6 = 15$; now, the vegetable garden can be 4.5 m long.

11 $w + 2.90 = 5.65$; the wrapping costs $2.75.

12 $3b + 11.50 = 70$; each book costs $19.50.

13 $3i + 4 \times 1.20 = 12$; each ice-cream costs $2.40.

14 16 red, 4 yellow 15 75 km

16 **(a)** $s = \frac{d}{t}$; $97 = \frac{d}{2.2}$ **(b)** $d = 213.4$ km

17 $4p + 2 = 110$; the perimeter of the pen is 27 m.

18 $2s + 3 = 7$; Jack surfed for 2 hours and read for 5 hours.

19 $2(2l - 7) = 74$; the block of land has length 22 m and width 15 m.

20 $5m + 2 = 10$; the carton of milk costs $1.60.

21 $250 each month for 3 years

22 9 23 10, 11, 12 24 10 years 25 95%

26 $4l = 27$

$l = \frac{27}{4}$

$l = 6.75$ cm

27 sister = 70 cm, brother = 140 cm and father = 170 cm

28 32, 34 and 36

29 $x = 50$; angles in an anticlockwise direction = 40°, 60°, 120°, 140°

Open-ended – Sample answers

30 $h = 20$, plumber Alan is cheaper
$h = 50$, plumber Barry is cheaper
$h = 30$, both quotes will be the same.

31 $b = 16$ cm and $h = 8$ cm; $b = 32$ cm and $h = 4$ cm; $b = 64$ m and $h = 2$ cm; $b = 128$ cm and $h = 1$ cm; $b = 8$ cm and $h = 16$ cm

Challenge 7

1 10 and 5

2 $R = 5, S = 3, T = 1, U = 7$, so $R \times U = 35$ and $S \times U = 21$

3 ±12 4 20 000 km 5 $a = 6, b = 12, c = 4, d = 8$

6 3 cm 7 $d = 8, e = 7, f = 5$

8 D 9 D 10 50

11 **(a)** $\angle DBA = \angle DAB = x$, $\angle ADB = (180° - 2x)$,

$\angle DBC = 180° - x$, $\angle BDC = \angle BCD = \frac{x}{2}$,

$\angle ADC = 180° - 2x + \frac{x}{2} = \left(\frac{360° - 3x}{2}\right)$

For $\angle ADC$ to be a whole number of degrees, x must be even. The smallest even value for x is 2. This gives the largest value for $\angle ADC$.

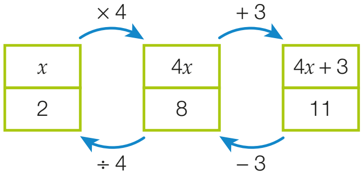
(b) 177°

(c) The largest even value for x is 88. The smallest value of $\angle ADC = 48°$.

Chapter review 7

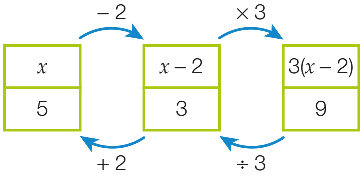
1 **(a)** $3x - 4 = 2$ **(b)** $\frac{2 + x}{5} = 6$

2 **(a)**



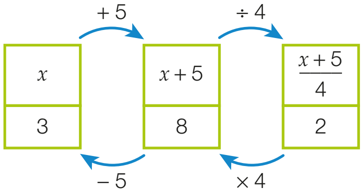
Solution is $x = 2$

(b)



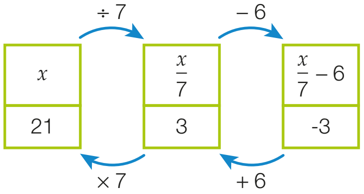
Solution is $x = 5$

(c)



Solution is $x = 3$

(d)



Solution is $x = 21$

3 **(a)** $x = -1$ **(b)** $b = 2$ **(c)** $c = -5$

4 **(a)** C **(b)** D **(c)** A

5 **(a)** $a = 9$ **(b)** $b = 1$ **(c)** $c = 8$

6 **(a)** $x = 4$ **(b)** $x = 25$ **(c)** $x = 8$ **(d)** $x = -10$

7 **(a)** $m = 2$ **(b)** $n = 5$ **(c)** $x = 3$ **(d)** $m = -27$

8 **(a)** $x = -7$ **(b)** $x = 1$ **(c)** $x = -3$ **(d)** $x = -\frac{4}{27}$

9 11 weeks

10 **(a)** Each can of cola costs \$1.55 ($5c + 2.25 = 10$).

(b) Almira needs to sell 28 hot dogs to make a profit of \$24. ($3h - 60 = 24$)

(c) One piece is 45.5 cm long and the other is 54.5 cm long. ($x + x + 9 = 100$)

11 **(a)** $2f + 5 = 70$ **(b)** $f = 32.5$ cm

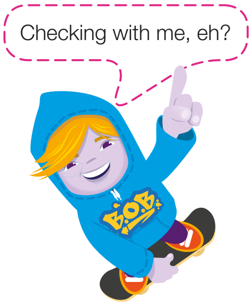
12 7, 8 and 9 **13** 30 and 20 **14** $x = 6$

15 16 years old

16 **(a)** 17 years old

(b) 18 km

(c) $s = \$44$



17 8, 29

18 base = 11 cm

19 Tom = 500 m, Mark = 1000 m and Fred = 2000 m

20
$$\begin{aligned} 7x + (5x + 12) + 12 + x + (7x - x) + (5x + 12 - 12) &= 120 \\ 24x + 24 &= 120 \\ 24x &= 96 \\ x &= 4 \text{ cm} \end{aligned}$$

21 **(a)** Box A: a; Box B: $a + 2b$ **(b)**

5	11	11
15	9	3
7	7	13

22 $x = 14$; side 1 = 14 m, side 2 = 18 m and side 3 = 19 m

Numeracy practice 7

1 $p = 3$ **2** C **3** A **4** A

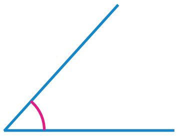
5 $z = 6$ **6** 42 **7** C **8** $x = 17$

9 65 cakes **10** 5 years old

Chapter 8

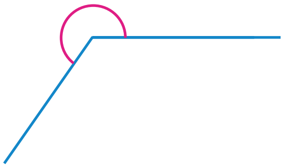
Recall 8

1 **(a)**

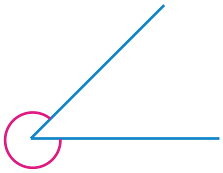


(b)

(c)



(d)



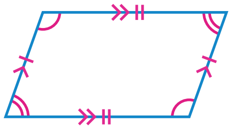
2 **(a)** acute **(b)** obtuse **(c)** reflex **(d)** reflex

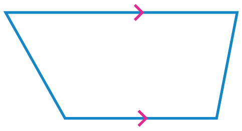
3 **(a)** **(i)** 35° **(ii)** 245°

(b) **(i)** acute **(ii)** reflex

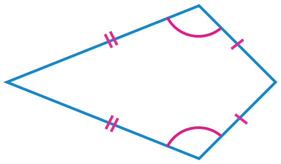
4 **(a)** $x = 173$ **(b)** $x = 64$ **(c)** $x = 70$

5 **(a)** parallelogram **(b)** trapezium

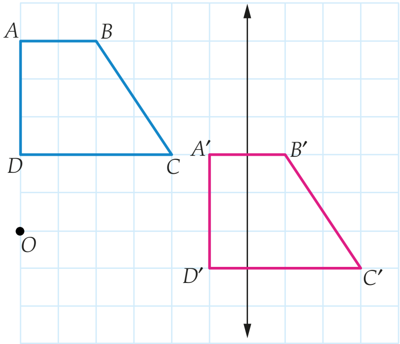




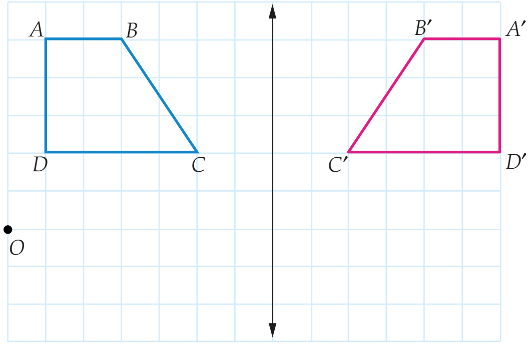
(c) kite



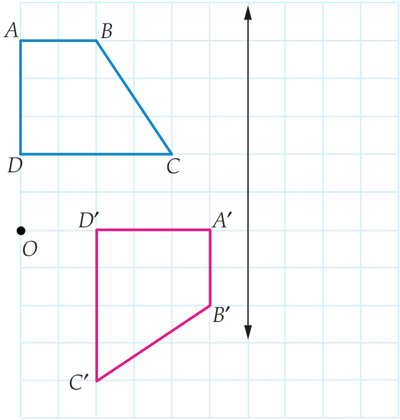
6 **(a)**



(b)



(c)



Exercise 8.1

1 (a) B, K (b) D, G (c) E, J (d) A, L
(e) F, H (f) C, M

2 (a) 53° (b) 75° (c) 50° (d) 40°

3 (a) $c = 145^\circ$, co-interior angles supplementary on parallel lines

(b) $x = 50^\circ$, alternate angles on parallel lines

(c) $y = 75^\circ$, corresponding angles

(d) $a = 114^\circ$, alternate angles on parallel lines; $b = 66^\circ$, co-interior angles supplementary on parallel lines; $c = 66^\circ$, vertically opposite angles equal; $d = 114^\circ$, straight angle; $e = 66^\circ$, straight angle; $f = 114^\circ$, vertically opposite angles equal; $g = 66^\circ$, straight angle

(e) $d = 45^\circ$, alternate angles on parallel lines; $e = 65^\circ$, co-interior angles supplementary on parallel lines

(f) $f = 120^\circ$, co-interior angles supplementary on parallel lines; $g = 120^\circ$, vertically opposite angles equal; $h = 70^\circ$, co-interior angles supplementary on parallel lines; $k = 70^\circ$, straight angle

(g) $m = 130^\circ$, co-interior angles supplementary on parallel lines; $n = 110^\circ$, alternate angles equal and supplementary adjacent angles; $p = 50^\circ + 70^\circ = 120^\circ$, exterior angle of triangle

(h) $a = 49^\circ$, vertically opposite angles equal; $b = 68^\circ$, straight angle; $c = 117^\circ$, co-interior angles supplementary on parallel lines

4 (a) $d = 145^\circ$, angle of revolution

(b) $e = 318^\circ$, angle of revolution

(c) $f = 26^\circ$, right angle

(d) $g = 51^\circ$, straight angle

(e) $h = 60^\circ$, straight angle

(f) $i = 60^\circ$, angle of revolution

(g) $k = 110^\circ$, straight angle

(h) $m = 142^\circ$, angle of revolution

(i) $p = 58^\circ$, complementary angles; $n = 32^\circ$, vertically opposite angles

5 C 6 B

7 (a) C (b) S (c) X (d) X (e) C (f) S

8 (a) $a = 65^\circ$, co-interior angles supplementary on parallel lines; $b = 25^\circ$, complementary angles; $c = 90^\circ$, corresponding angles on parallel lines

(b) $x = 120^\circ$, co-interior angles supplementary on parallel lines; $y = 120^\circ$, vertically opposite angles equal

(c) $x = 107^\circ$, straight angle; $y = 48^\circ$, corresponding angles on parallel lines; $z = 59^\circ$, co-interior angles supplementary on parallel lines

9 (a) T (b) F

(c) F (they could both be right angles)

(d) F (they could both be straight angles)

(e) T

10 (a) 65° (b) 98° (c) 122° (d) 135°

11 $d = 35^\circ$

12 $g = 120^\circ$, $h = 60^\circ$

13 No; as the angles marked do not add to 180°, they are not co-interior angles on parallel lines, so CD and EF are not parallel.

14 $(3x + 10)^\circ + (5x - 70)^\circ = 180^\circ$, co-interior angles supplementary on parallel lines; $8x - 60^\circ = 180^\circ$, $x = 30^\circ$

Open-ended – Sample answers

15 $x = 100^\circ$, $y = 80^\circ$; $x = 105^\circ$, $y = 75^\circ$; $x = 110^\circ$, $y = 70^\circ$
The pairs of values are always supplementary.

16 (a) $a = b = e$, $c = d = f$

(b) a and c; a and d; a and f; b and c; b and d; b and f; e and c; e and d; e and f

Exercise 8.2

1 (a) $x = 60^\circ$, equilateral triangle

(b) $y = 105^\circ$, opposite angles in a kite are equal

(c) $m = 105^\circ$, exterior angle of a triangle is equal to the sum of two interior opposite angles

2 (a) $x = 82^\circ$ (b) $y = 55^\circ$

(c) $a = 115^\circ$ (d) $w = 67^\circ$, $v = z = 113^\circ$

(e) $x = 28^\circ$

3 (a) trapezium (b) kite (c) square

4 B

5 (a) 180° (b) $4 \times 180^\circ = 720^\circ$

6 30°, 60°, 90°

7 $y = 108^\circ$, opposite angles in a parallelogram must be equal for parallel sides

8 (a) $a = 82^\circ$, $b = 133^\circ$

(b) $d = 58^\circ$, $c = e = 122^\circ$

(c) $f = 95^\circ$

(d) $2x + 3x + 78^\circ + (2x + 30^\circ) = 360^\circ$, $7x + 108^\circ = 360^\circ$, $7x = 252^\circ$, $x = 36^\circ$

(e) $u = 180^\circ - 65^\circ = 115^\circ$

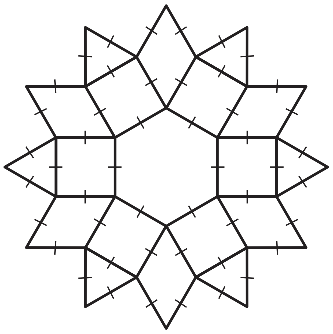
9 135°, angles in an octagon

10 (a) F (b) T (c) F (d) F

11 84°

12 square

13 **(a)**



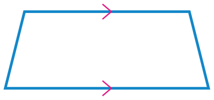
(b) two angles 60°, two angles 120°

(c) 2 × 90° (squares) + 60° (rhombus) = 240°
360° – 240° = 120°

14 C **15** C

16 **(a)** 540° **(b)** 108°

17 trapezium



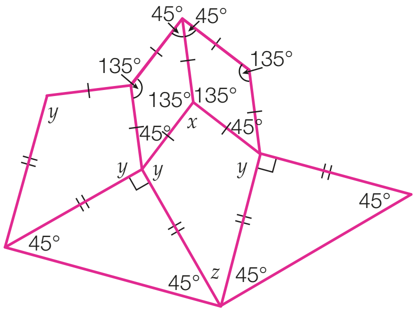
18 **(a)** 30° **(b)** 60°

19 **(a)** 45°, 45°, 135°, 135°

(b) Two adjacent sides of the kite are the sides of a rhombus, which must be equal. The other pair of adjacent sides are the sides of the isosceles triangle, which also must be equal. Two pairs of equal adjacent sides means that the quadrilateral is a kite.

(c) **(i)** octagon **(ii)** 1080° **(iii)** 135°

(d)



$x = 360° – (2 × 135°)$
$= 90°$

$2y = 360° – (45° + 90°)$
$= 225°$
$y = 112.5°$

$z = 360° – 315°$
$= 45°$

20 A rectangle has four 90° angles; three of them can be made acute, making the fourth obtuse. An example would be a quadrilateral with angles of 75°, 80°, 85°, 120°.

21 **(a)** *EFGH* is a rhombus.

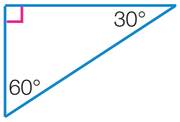
(b) square (regular quadrilateral)

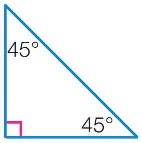
22 Types are: parallelogram, rectangle, rhombus, square (and, for the thinkers, an isosceles trapezium)

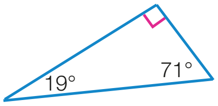
23 A rectangle has angles of 90°, but a rhombus does not have 90° angles.

Open-ended – Sample answers

24

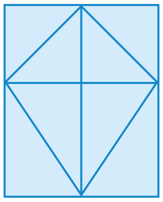




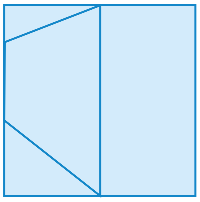


25 **(a)**

(b)



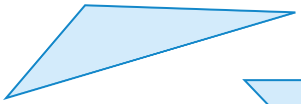
(c)



26 **(a)** 90°, 45°; 67.5°, 67.5°

(b) no; angles could be 60°, 30°, 90°; or 60°, 40°, 80°

(c)

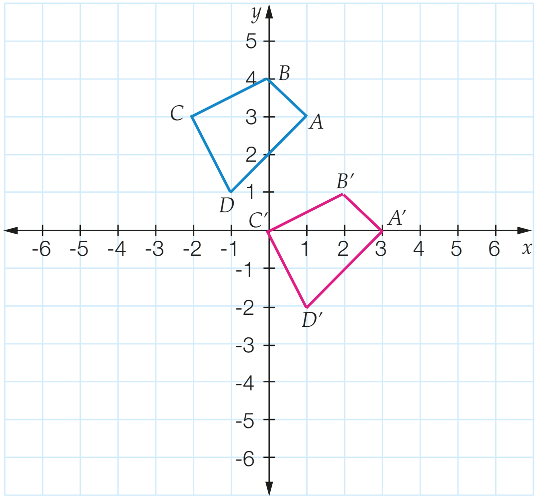


Exercise 8.3

1 **(a)** rotation **(b)** translation **(c)** reflection

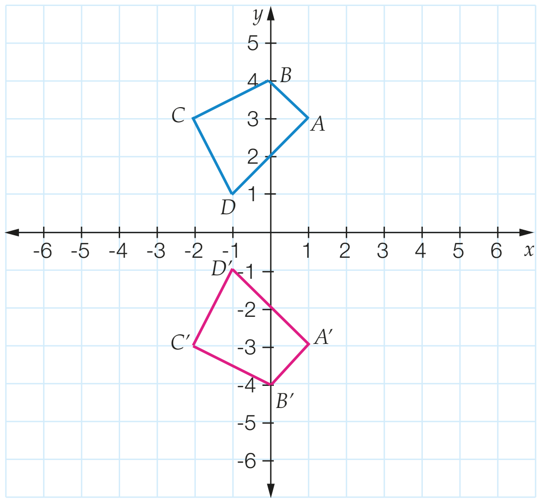
2 (5, -3)

3 **(a)** **(i)**



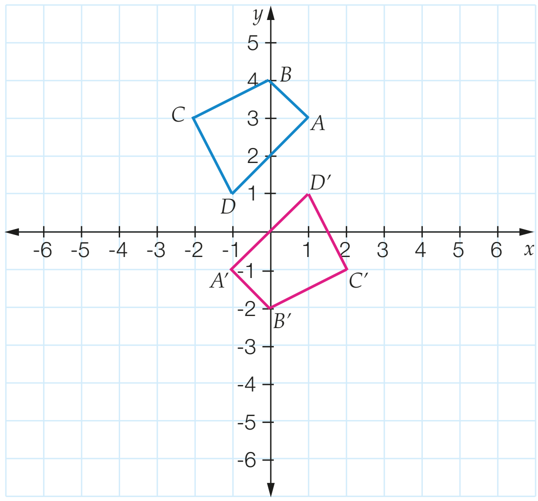
A′(3, 0), *B*′(2, 1), *C*′(0, 0), *D*′(1, -2)

(ii)



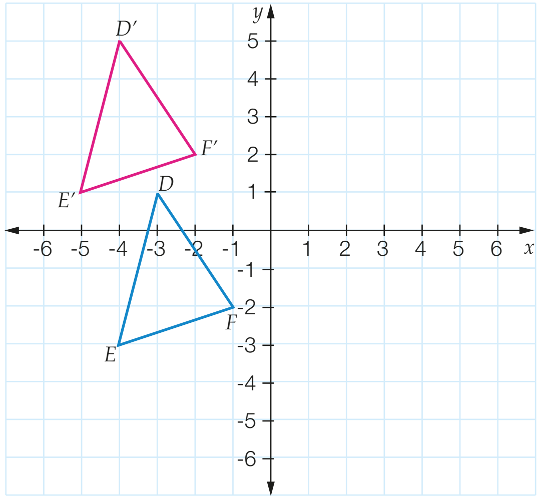
$A'(1, -3)$, $B'(0, -4)$, $C'(-2, -3)$, $D'(-1, -1)$

(iii)



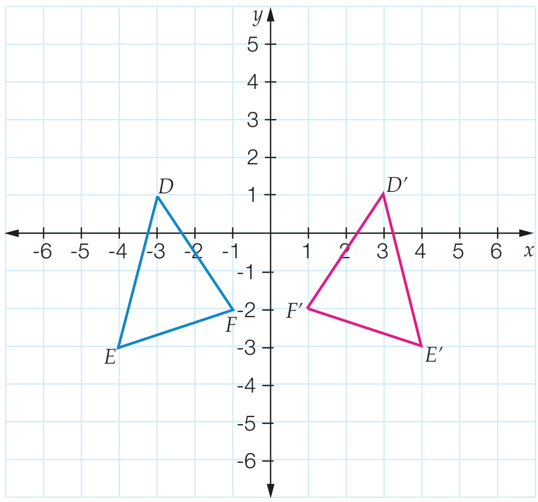
$A'(-1, -1)$, $B'(0, -2)$, $C'(2, -1)$, $D'(1, 1)$

(b) (i)



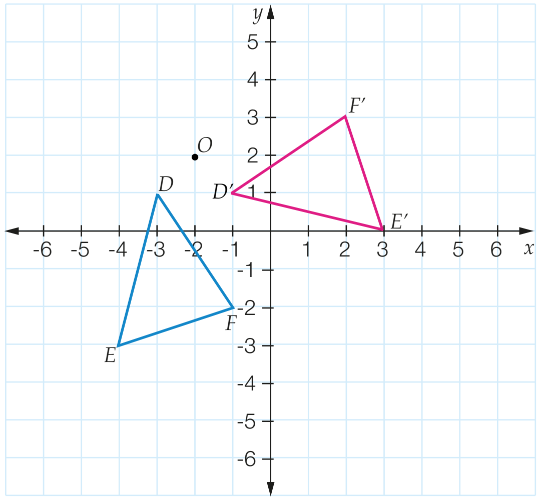
$D'(-4, 5)$, $E'(-5, 1)$, $F'(-2, 2)$

(ii)



$D'(3, 1)$, $E'(4, -3)$, $F'(1, -2)$

(iii)



$D'(-1, 1)$, $E'(3, 0)$, $F'(2, 3)$

4 **(a)** **(i)** rotation (180°) and translation

(ii) $P \leftrightarrow W$, $Q \leftrightarrow X$, $R \leftrightarrow Y$, $S \leftrightarrow Z$

(iii) $PQ \leftrightarrow WX$, $QR \leftrightarrow XY$, $RS \leftrightarrow YZ$, $SP \leftrightarrow ZW$

(iv) $\angle PQR \leftrightarrow \angle WXY$, $\angle QRS \leftrightarrow \angle XYZ$, $\angle RSP \leftrightarrow \angle YZW$, $\angle SPQ \leftrightarrow \angle ZWX$

(v) $PQRS \equiv WXYZ$

(b) **(i)** rotation (90° clockwise), reflection (in a vertical line) and translation

(ii) $A \leftrightarrow F$, $B \leftrightarrow G$, $C \leftrightarrow H$

(iii) $AB \leftrightarrow FG$, $BC \leftrightarrow GH$, $AC \leftrightarrow FH$

(iv) $\angle ABC \leftrightarrow \angle FGH$, $\angle BCA \leftrightarrow \angle GHF$, $\angle BAC \leftrightarrow \angle GFH$

(v) $\Delta ABC \equiv \Delta FGH$

(c) **(i)** reflection (in a vertical line)

(ii) $J \leftrightarrow P$, $K \leftrightarrow Q$, $L \leftrightarrow R$, $M \leftrightarrow S$

(iii) $JK \leftrightarrow PQ$, $KL \leftrightarrow QR$, $LM \leftrightarrow RS$, $MJ \leftrightarrow SP$

(iv) $\angle MJK \leftrightarrow \angle SPQ$, $\angle JKL \leftrightarrow \angle PQR$, $\angle KLM \leftrightarrow \angle QRS$, $\angle LMJ \leftrightarrow \angle RSP$

(v) $JKLM \equiv PQRS$

(d) **(i)** rotation (90° clockwise) and translation

(ii) $E \leftrightarrow T, F \leftrightarrow U, G \leftrightarrow V, H \leftrightarrow W$

(iii) $EF \leftrightarrow TU, FG \leftrightarrow UV, GH \leftrightarrow VW, HE \leftrightarrow WT$

(iv) $\angle HEF \leftrightarrow \angle WTU, \angle EFG \leftrightarrow \angle TUV, \angle FGH \leftrightarrow \angle UVW, \angle GHE \leftrightarrow \angle VWT$

(v) $EFGH \equiv TUVW$

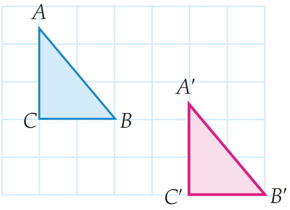
5 **(a)** reflection in the y-axis

(b) rotation 180° clockwise about the point (2, 2)

(c) translation of [-4, 2]

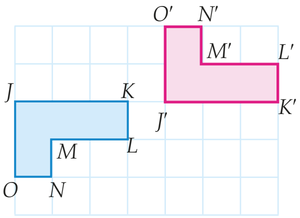
6 **(a)** **(i)** translation 4 units right, 2 units down

(ii)



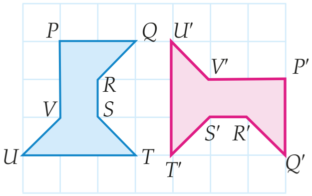
(b) **(i)** reflection along the line JK, then translation 4 units right

(ii)



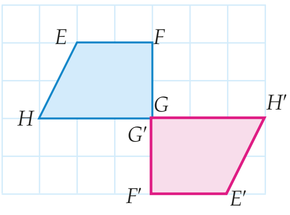
(c) **(i)** rotation 90° clockwise around T, then translation 1 unit right

(ii)



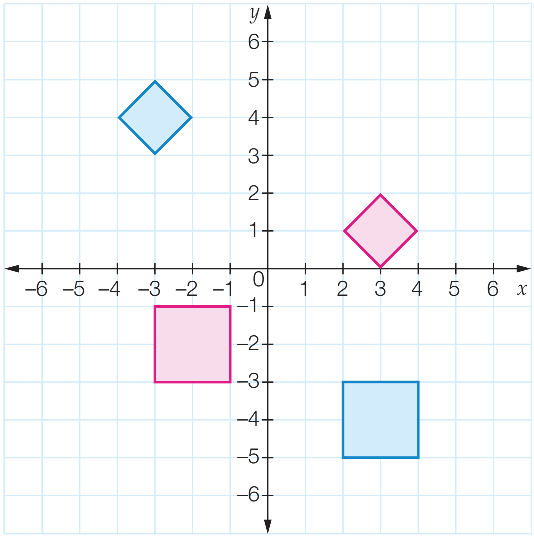
(d) **(i)** rotation 180° about G

(ii)



7 **(a)** C **(b)** D

8



9 B

10 **(a)** $RVW \equiv UST$

(b) $IFE \equiv HFG, IEH \equiv HGI$

(c) $RMQ \equiv NOP, ROP \equiv NMQ, RNO \equiv NRM$

11 **(a)** C **(b)** S **(c)** AB

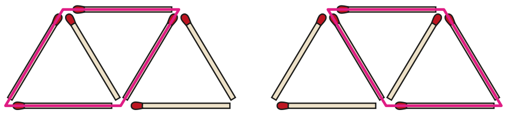
(d) SP **(e)** $\angle ABC$ **(f)** $\angle RSP$

12 **(a)** $a = 4$ cm, $b = 105°$ **(b)** $m = 8$ m, $n = 130°$

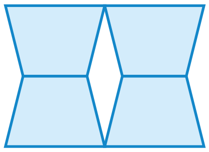
(c) $x = 2.2$ km, $y = 80°$ **(d)** $p = 28$ mm, $q = 63°$

13 **(a)** 3 **(b)** 5

(c)



14



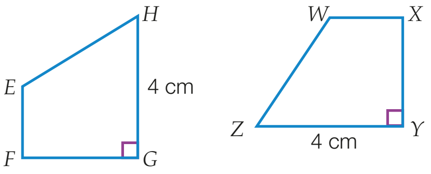
The trapeziums are reflected and then translated each time to create the tessellation.

15 **(a)** yes **(b)** yes

In **(a)** the lines make the second square appear as a trapezium and in **(b)** the perspective makes the first shape appear longer and thinner than the second.

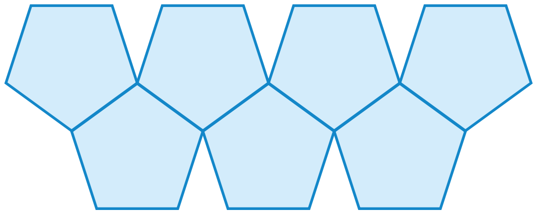
Open-ended – Sample answers

16



17 squares on a chessboard, opposite faces of boxes and cans

18 Sample answer:



Half-time 8

1 (a) $d = 130^\circ$, $e = 125^\circ$ (b) $m = 140^\circ$, $n = 60^\circ$

(c) $n = 47^\circ$, $p = 133^\circ$

2 (a) $ABCD \equiv MNOP$ (b) $FGH \equiv ZXY$

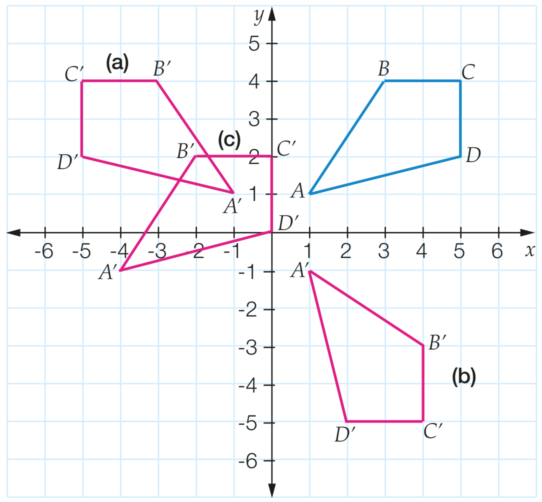
(c) $UPQR \equiv RSTU$

3 (a) $a = 77^\circ$, $b = 138^\circ$ (b) $u = 110^\circ$

(c) $w = 114^\circ$, $y = 37^\circ$, $z = 35^\circ$

4 D

5



(a) $A'(-1, 1)$, $B'(-3, 4)$, $C'(-5, 4)$, $D'(-5, 2)$

(b) $A'(1, -1)$, $B'(4, -3)$, $C'(4, -5)$, $D'(2, -5)$

(c) $A'(-4, -1)$, $B'(-2, 2)$, $C'(0, 2)$, $D'(0, 0)$

6 A, B

7 (a) $B \leftrightarrow J$ (b) $AD \leftrightarrow ML$

(c) $JM \leftrightarrow BA$ (d) $\angle JKL \leftrightarrow \angle BCD$

Exercise 8.4

1 (a) yes, SAS (b) yes, ASA (c) no

(d) yes, SAS (e) yes, RHS

2 (a) $\angle ABC$ (b) QS (c) $\angle RSQ$ (d) AB

3 (a) SSS (b) ASA (c) RHS (d) SAS

4 $\Delta BAC \equiv \Delta MON$ (SAS)

$\Delta DEF \equiv \Delta KLJ$ (SSS)

$\Delta HGI \equiv \Delta RPG$ (ASA)

5 C **6** C

7 (a) (i) ASA (ii) $x = 9$ m

(b) (i) SAS (ii) $g^\circ = 69^\circ$

(c) (i) SAS (ii) $x = 5$ cm

(d) (i) RHS (ii) $h^\circ = 58^\circ$

(e) (i) SAS (ii) $m^\circ = 19^\circ$

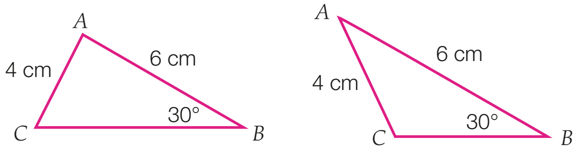
(f) (i) ASA (ii) $z = 7$ km

8 Students' own answers; many possible solutions.

9 Students' own answers.

10 Ayda knows that side LN = side PN, for the test to be correct.

11 (a) Possible answer:

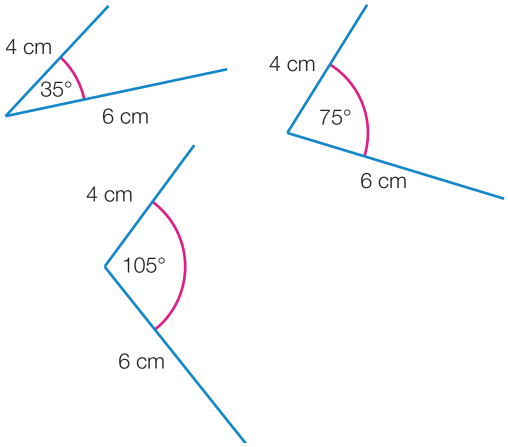


(b) If the lengths of two sides are given, the included angle must be stated to create a unique triangle.

(c) Any other valid angle or side measurement will make the triangle unique.

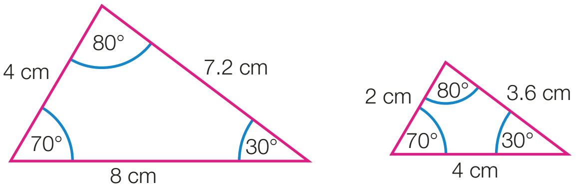
12 The corresponding sides can meet at a variety of angles—like opening or closing a pair of scissors. Specifying the angle in between these two sides 'fixes', or 'anchors' them into position (the SAS test).

e.g.

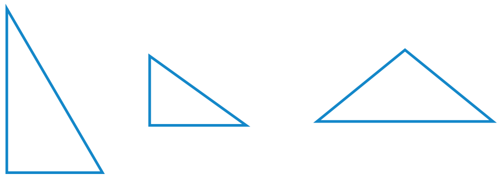


13 Three equal angles means that the triangles will be the same shape, but not necessarily the same size.

e.g.



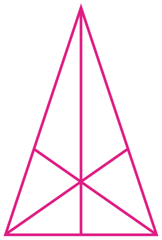
14 (a)



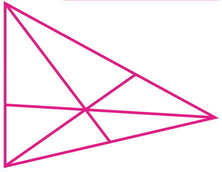
(b) (i) Answers may vary: $\Delta ADC \equiv \Delta ADE$; $\Delta AGF \equiv \Delta AGB$; $\Delta EGC \equiv \Delta CGA$.

(ii) All congruency tests can be applied to prove that the different-sized triangles are congruent.

(c) (i) An isosceles triangle has six pairs of congruent triangles of different sizes.



(ii) A scalene triangle has three pairs of congruent triangles of different sizes.



(d) Bisecting the sides of an equilateral triangle produces six congruent triangles because the three sides of the original triangle are all the same length. Bisecting the sides of an isosceles triangle produces four congruent triangles of one size, and two of another size, as two of the original triangle's sides are equal. Bisecting the sides of a scalene triangle produces three pairs of congruent triangles, as none of the original triangle's sides are equal.

Open-ended – Sample answers

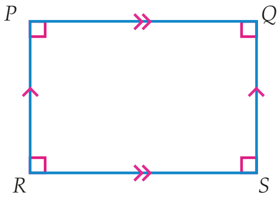
15 Students' own answers. Triangles constructed in this way are SAS or ASA.

16 Students' own answers. Triangles constructed in this way are SSS or RHS.

Exercise 8.5

1 rhombus

2



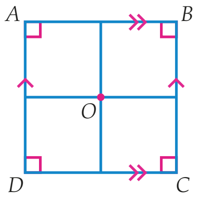
(a) $\angle PQR = \angle SRQ$ (alternate angles)
$\angle PRQ = \angle SQR$ (alternate angles)
$RQ = QR$ (common side)

Therefore, $\Delta PQR \equiv \Delta SRQ$ (ASA)
Therefore, $PR = QS$ and $PQ = RS$
Therefore, both pairs of opposite sides are equal in length.

(b) $\Delta PQR = \Delta SRQ$
Therefore, $\angle RPQ = \angle QSR$
$\angle PRS = 180° - \angle RPQ$ (co-interior angles)
but $\angle SQP = 180° - \angle RPQ$ (co-interior angles)

Therefore, $\angle PRS = \angle SQP$
Therefore, both pairs of opposite angles are equal.

3

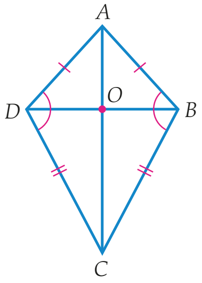


(a) $AD = BC$, $AB = DC$
Therefore, $\Delta ABC \equiv \Delta ADC$ (SSS)
$\angle DAC = \angle BCA$
$\angle AOD = \angle BOC$ (vertically opposite)
$AD = BC$ (given)
Therefore, $\Delta AOD = \Delta BOC$ (ASA)
Therefore, $AO = CO$ and $DO = BO$
$AD = AB$ (given)
Therefore, $\Delta AOD = \Delta AOB$ (SSS)
Therefore, $\angle AOD = \angle AOB = 90°$(angles on a straight line).
Similarly, $\angle COD = \angle COB = 90°$
Therefore, the diagonals bisect each other at right angles.

(b) $\angle DAC = \angle BCA$
$\angle DAC = \angle DCA$ (base angles of isosceles triangles)
$\angle BCA = \angle BAC$
Therefore, $\angle DAC = \angle BCA = \angle BAC = \angle BCA$
So, $\angle DAC = \angle BAC$ and $\angle DCA = \angle BCA$
Similarly, $\angle ADB = \angle CDB = \angle CBD = \angle ABD$
Therefore, each diagonal bisects the two interior angles it passes through.

4 B

5



$AB = AD$
$BC = CD$
Therefore, $\Delta ACD \equiv \Delta ACB$ (SAS)
Therefore, $\angle DAC = \angle BAC$
Therefore, $\Delta AOB \equiv \Delta AOD$ (SAS)
Therefore, $\angle DOA = \angle BOA$
$\angle DOA + \angle BOA = 180°$ (angles on a straight line)
Therefore, the diagonals intersect at right angles.

6 B **7** C

8 (a) $p = 100°$, opposite angles are equal

(b) $y = 7$ cm, opposite side lengths are equal

(c) $s = 60°$, $h = 7$ cm, all angles and side lengths are equal in an equilateral triangle

(d) $g = 110°$ (SSS)

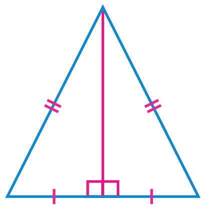
(e) $p = 90°$ (SSS), diagonals cross at right angles in a square

(f) $m = 60°$, alternate angles between parallel sides are equal

(g) $x = 100°$, co-interior angles add to 180°

(h) $f = 60°$, $k = 4$ m, equilateral triangles

9 **(a)**



(b) $\Delta ABC \equiv \Delta ADC$ (SSS), $\angle ADC \leftrightarrow \angle ABC$, therefore, angles are equal.

(c) isosceles

10 **(a)** Yes: if both pairs of opposite sides are equal in length then the quadrilateral is a parallelogram.

(b) All angles are right angles.

11 **(a)** yes (SSS)

(b) parallelogram (opposite sides and angles are equal) and kite (one pair of equal angles)

12 **(a)** $\Delta TWV \equiv \Delta TUV$ (ASA), $\angle TWV \leftrightarrow \angle TUV$, therefore, opposite angles are equal.

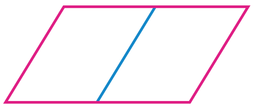
(b) isosceles

(c) $\angle TUX = \angle VUX$ (diagonals bisect), therefore, $\Delta TUX \equiv \Delta VUX$ (SAS), $TX \leftrightarrow VX$ (corresponding sides are equal).

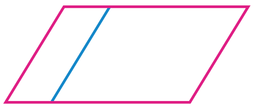
(d) The diagonals of a rhombus intersect at right angles. A square has this property.

Open-ended – Sample answers

13 **(a)**

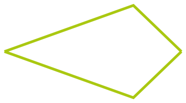


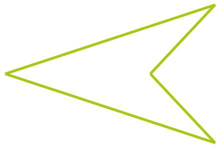
(b)



(c)

14





Challenge 8

1 D

$AB = \sqrt{20}$, $BW = \sqrt{10}$, $BX = \sqrt{17}$, $BY = \sqrt{26}$, $BV = 4$, $BZ = 5$, $AW = \sqrt{26}$, $AX = \sqrt{29}$, $AY = \sqrt{34}$, $AV = \sqrt{20}$, $AZ = 5$.

$AB = AV = \sqrt{20}$, hence ΔABV is isosceles. $AZ = BZ = 5$, hence ΔABZ is isosceles.

2 B **3** C **4** B **5** B

6 **(a)** 1 **(b)** 6 **(c)** 12 **(d)** 8

Chapter review 8

1 **(a)** $x = 163°$ **(b)** $x = 41°$ **(c)** $x = 46°$

(d) $x = 85°$ **(e)** $x = 45°$ **(f)** $x = 110°$

2 **(a)** $y = 53°, z = 127°$ **(b)** $a = 95°, b = 110°, c = 95°, d = 85°$

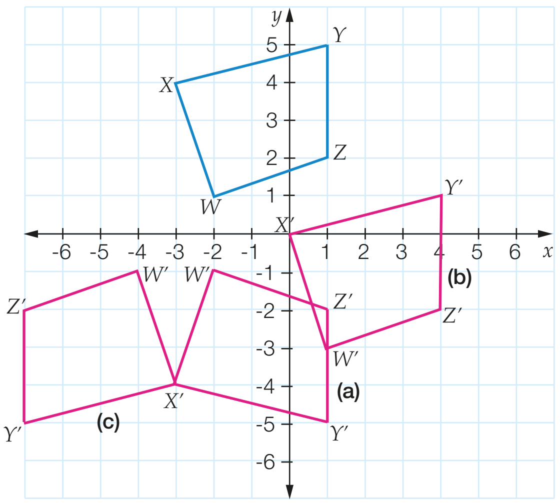
(c) $m = 55°, n = 65°, p = 115°$

3 **(a)** $x = 72°, y = 141°$ **(b)** $c = 118°, d = 62°, e = 118°$

(c) $f = 90°$

4 **(a)** C **(b)** A **(c)** D **(d)** B

5



6 **(a)** $H \leftrightarrow U$ **(b)** $N \leftrightarrow D$ **(c)** $TU \leftrightarrow EH$

(d) $AD \leftrightarrow ON$ **(e)** $\angle CDA \leftrightarrow \angle MNO$

(f) $\angle EFG \leftrightarrow \angle TSV$

7 **(a)** ASA **(b)** SAS

8 **(a)** Join PR, then SSS. **(b)** Join KM, then SSS.

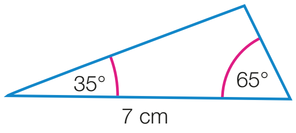
9 **(a)** $5 \times 180° = 900°$ **(b)** $900 \div 7 = 128.6°$

10 **(a)** ASA **(b)** SAS **(c)** ASA

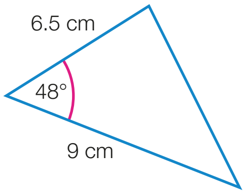
11 D

12 **(a)** rectangle **(b)** rhombus **(c)** parallelogram

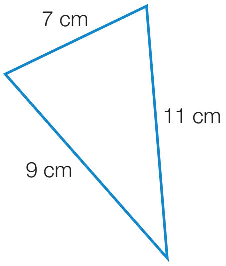
13 **(a)** **(i)**



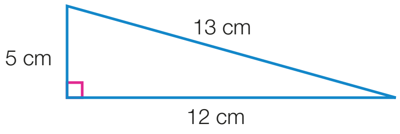
(ii)



(b) **(i)**



(ii)



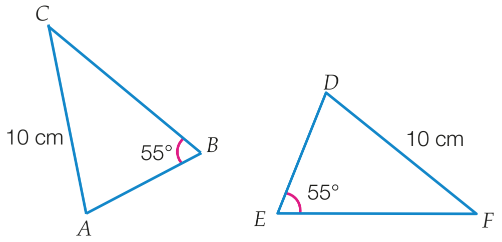
14 **(a)** $x = 77°$ **(b)** $x = 60°, y = 120°$

(c) $w = 72°, y = 108°, z = 72°$

15 C, D

16 Triangles are congruent: two sides are equal and one angle, but as it is not the included angle, more information is needed. Triangles overlapping to make an isosceles triangle show that *EC* and *AD* are equal in length and that ∠*A* = ∠*C*.

17 **(a)**



(b) For the triangles to be congruent, one side and one angle is not enough information. At least one more angle is needed for ASA, or one more side for SAS.

(c) Answers may vary. One other side or angle on each triangle should be specified.

18 A square is a rhombus with 90° angles. A square is a rectangle with all four sides equal in length.

Numeracy practice 8

1 60° **2** C **3** D **4** $y = 50°$

5 $z = 40°$ **6** $x = 4$ cm, $y = 70°$

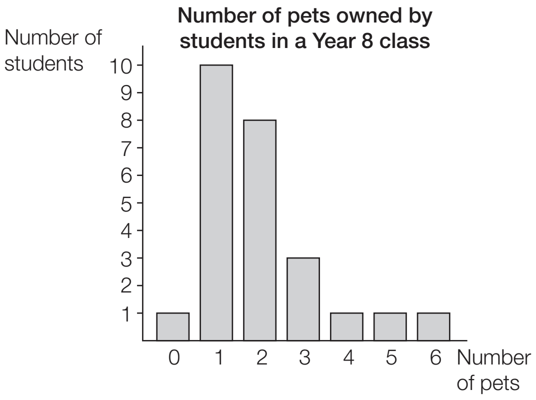
Chapter 9

Recall 9

1 **(a)** Number of pets owned by students in a Year 8 class

Number of pets	Tally	Frequency
0	|	1
1	𝍸 𝍸	10
2	𝍸 |||	8
3	|||	3
4	|	1
5	|	1
6	|	1

(b)



2 **(a)** mean = 3.5, median = 3, mode = 2

(b) mean = 35, median = 35, no mode

3 **(a)** $\frac{3}{4}$ **(b)** 0.8

4 mean = 2.4, median = 2, mode = 2

5 31

Exercise 9.1

1 **(a)** estimated number of green = 1000
estimated number of red = 900
estimated number of black = 600

(b) estimated number of green = 720
estimated number of red = 450
estimated number of black = 630

2 **(a)** 2250 fish **(b)** 500 koalas

3 **(a)** not biased

(b) Biased: not all teenagers are in Year 8 and, as you get older, your hand span increases.

(c) Biased: poorer families may not have a car.

(d) Biased: healthy people are less likely to be at the medical centre, so the rate of illness may be overstated.

(e) Biased: new cars under warranty are less likely to be serviced by the local mechanic.

(f) not biased

4 **(a)** Convenience sampling. Bias: students arriving at school early or late may be more or less likely to enjoy reading.

(b) Convenience and judgemental sampling. Bias: group members may have been influenced by advice on a particular brand being used within the group.

(c) Judgemental sampling. Bias: mechanics may recommend cars that they like to work on.

5 **(a)** Set 1: biased – too many 1s
Set 2: biased – too many 4s
too few 3s
Set 3: fair
Set 4: biased – too many 5s
too few 1s + 6s
Set 5: biased – too many 4s + 6s
too few 1s + 3s

(b) Based on the individual results it seems that the die is not fair. The number 4 seems to appear too often.

(c) number of expected results $= \frac{300}{6}$

$= 50$

Number	Total
1	45
2	52
3	41
4	61
5	51
6	50

The number 3 appears 9 times fewer than it should and the number 4 appears 11 times more than it should. This makes it seem that the die could be biased, and a higher number of rolls would support or reject this.

6 **(a)** Month 1: 818, Month 2: 808, Month 3: 833, Month 4: 735

(b) 802

(c) The estimate of all the samples put together is likely to be the more reliable, as you are using a larger sample size to calculate the population.

7 More 6s occur than expected from a fair die. However, the individual samples do not show this very clearly. The total of all 300 rolls shows 64 6s, when the theoretical expected value is 50. The next highest total is 52. There are also two very low totals—4s with 40 and 5s with 41. In summary, the die does appear to be biased.

8 **(a)** If every loaf was tested there would be none left to sell.

(b) Students' own answers.

9 Students should provide their own answers, but this should include comments about sample size and how people would be selected.

10 **(a)** 'Signs point to sales cooling over summer' uses judgemental sampling techniques. This is a good way to compare values over time but it may not give an accurate picture of all house values if, for example, more expensive homes were generally kept for longer periods than less expensive homes, or if houses with pools attracted higher prices in the summer months.

(b) 'Walking slowly may be early sign of dementia' uses data from a random sample. You could be more confident if other studies gave similar results, or if the sample size was increased.

11 **(a)** If it is expected that views will be different, then the difference in the numbers surveyed should be proportional to the population.

(b) This shows that the people surveyed were from more than just one state, so the survey could be unbiased.

(c) To ensure that each state had a fair representation, you could take a proportional number of people to be surveyed in each state according to the population of that state.

(d) The additional information probably makes you more wary about accepting the conclusions. As a starting point, it does not include any people from the most populous state (NSW), and people voluntarily opted into the survey, which is not usually a sound method.

12 **(a)** 3 billion

(b) The world population doubled.

(c) There will be no unusual events that will reduce the world population dramatically, for instance Earth being hit by a meteorite.

(d) No, the statisticians' assumptions are (presumably) based on evidence of what has happened in the past; a bias is a belief that has no evidence to support it.

(e) 45 years

(f) Food and housing shortages, environment damage – loss of species, global warming, running out of resources.

(g) Many countries submit census data to be collated. This would be very accurate. The population of other countries is estimated through sampling and other statistical techniques.

13 **(a)** 1962–1971

(b) The large number of babies born after World War II would themselves be having children.

(c) 1.1%

(d) It has declined steadily.

(e) The population would still be increasing, but more slowly than at any time in this period.

(f) The growth rate would have decreased. This would have happened from 1957 to 1960.

Open-ended – Sample answers

14 If values anywhere near 10 come up in each sample you should not claim bias.

15 **(a)** All of the people who wash football uniforms.

(b) All of the people on your side of town who buy fish and chips.

16 **(a)** 5 times

(b) The sample is not large enough to come to a conclusion about bias but the early indication does point to bias.

(c) Answers will vary but should indicate a large proportion.

(d) Answers will vary. Continue the experiment to see if the trend continues.

(e) Answers based on experimental results.

Exercise 9.2

1 **(a)** **(i)** 42 **(ii)** 42 **(iii)** bi-modal (36 and 37)

(b) **(i)** 45.5 **(ii)** 43.5 **(iii)** no mode

2 **(a)** **(i)** Set 1: mean = 13.33 Set 2: mean = 12.87

(ii) Set 1: median = 10 Set 2: median = 10

(iii) Set 1: range = 14 Set 2: range = 21

(b) Changing the first piece of data has an effect on both the mean and range of the data, but has no effect on the median.

3 **(a)** The mean score is 19.1 and the median is 15 points.

(b) The median score represents the games generally as the two highest scores push the mean value up to above almost 60% of the scores.

4 **(a)** B **(b)** D **(c)** B

5 **(a)** 40.0 **(b)** 24

(c) 1st Test, 1st innings: Smith
1st Test, 2nd innings: Khawaja
2nd Test, 1st innings: Burns
2nd Test, 2nd innings: Nevill
3rd Test, 1st innings: Lyon
3rd Test, 2nd innings: Warner

(d) No—some cricketers are specialist batsmen and would expect to average a higher score than the other cricketers who may be specialist bowlers.

6 (a) mean: 179 g median: 175 g mode: 175 g

(b) Use the mean because the formula is:

$$\text{mean} = \frac{\text{total mass}}{\text{number of oranges}}$$

Assumptions: this tree is typical in its mass of oranges and that you know the total number of oranges.

(c) $250 \times 52 \times 178.7 = 2\,323\,100$ kg. Assumption: the trees average the same number of oranges as this tree.

(d) (i)

	Mean	Median	Mode
Set 1	174	175	175
Set 2	176	175	no mode
Set 3	181	175	175
Set 4	180	175	195

(ii) The median stayed closer.

7 (a) mean: 15.08 median: 15 range: 25

(b) mean: 14.92 median: 15 range: 9

(c) Having an outlier at each end of the data set can balance out the mean, but there is a major impact on the range. The median is not affected.

8 (a) mean minimum = 13.1 °C, mean maximum = 22.7 °C

(b) The average minimum is nearly the same as the historical mean but the average maximum is more than a degree less, so you would have to say that March in 2015 was slightly cooler than usual in Melbourne.

(c) Compared to the restricted figures, which are both lower than the historical means, Melbourne was cooler than usual in March 2015.

(d) A good argument could be mounted for either point of view. The historical mean has the advantage of taking into account every year for which records have been kept, but the restricted means might better reflect the current trend in temperatures. If it is currently a warm spell in general, as far as temperatures are concerned, you might be able to better compare a particular year to its era.

9 (a) The mean and median values are larger than expected for 13-year-old boys, so these boys are probably older than 13 years. The median is 160 cm, the mean is 161 cm and the mode is 155 cm.

(b) (i) mean 163.9 cm, median 160.5 cm

(ii) Population: mean 161 cm, median 160 cm. The sample median is better. It is close to the median for the data set given. The mean is quite different. The inclusion of a possible outlier of 177 cm made no difference to the median.

10 (a) 26% (b) 27–45 mg

(c) The combined coffee consumption would be closest to 4.7 kg per capita per year, because this is closest to the quantity in the countries with the largest populations (USA, France, Italy and UK), and the countries with a high consumption have small populations.

(d) The figures were from a sample, as a census would have been prohibitively expensive and time-consuming.

(e) As you don't know about the spread of the data in the provided range, you can't accurately estimate any of the averages. However, when the range is smaller, you can provide more accurate estimates than with a larger range.

Open-ended – Sample answers

11 2, 4, 10, 11, 11, 15, 16, 17, 18, 19

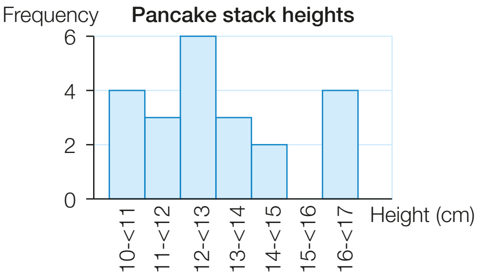
12 0, 1, 2, 11, 11, 15, 16, 17, 18, 32

13 population mean = 10, population median = 9.5, population range = 20

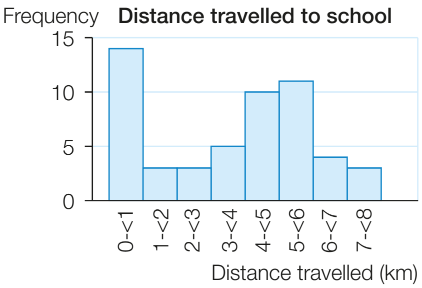
Results will vary, but some sample means will be higher and some will be lower than (and some will be the same as) the population mean. Some sample medians will be higher and some will be lower than the population median (but cannot be the same since they will all be whole numbers). If just one outlier was included, a bigger difference from the population mean occurs, but the median is unaffected. If both outliers were in the same sample, the sample mean would be close to the population mean and again the median is unaffected. The only time a sample range is the same as the population range is when both outliers are included. The range is quite large with just one outlier, but most samples would have no outliers and hence a small range.

Exercise 9.3

1 (a)

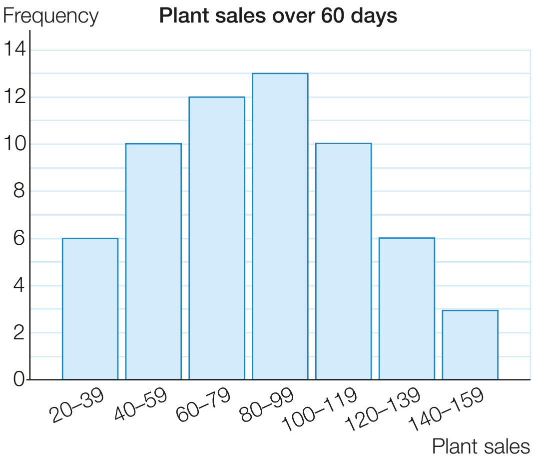


(b)



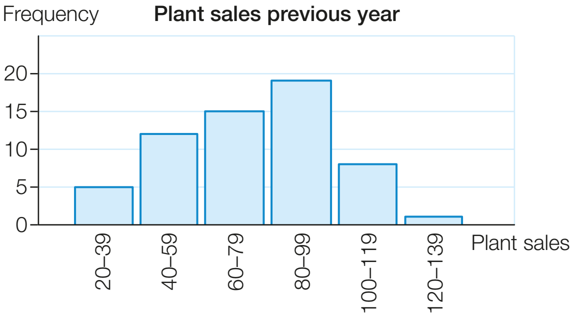
2 (a)

Plant sales	Frequency
20–39	6
40–59	10
60–79	12
80–99	13
100–119	10
120–139	6
140–159	3



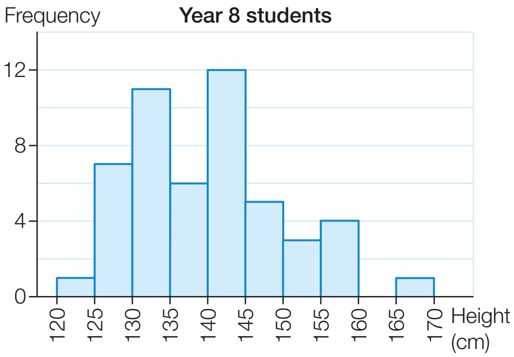
(b)

Plant sales	Frequency
20–39	5
40–59	12
60–79	15
80–99	19
100–119	8
120–139	1



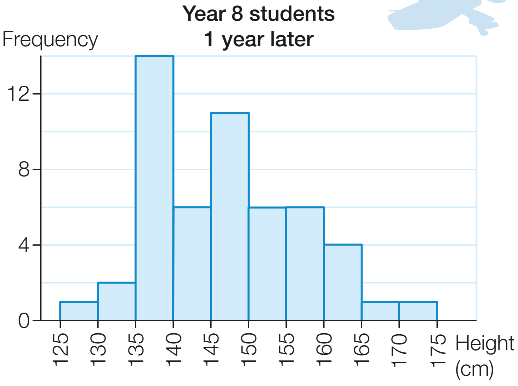
3 (a)

Height in cm	Frequency
120–<125	1
125–<130	7
130–<135	11
135–<140	6
140–<145	12
145–<150	5
150–<155	3
155–<160	4
160–<165	0
165–<170	1



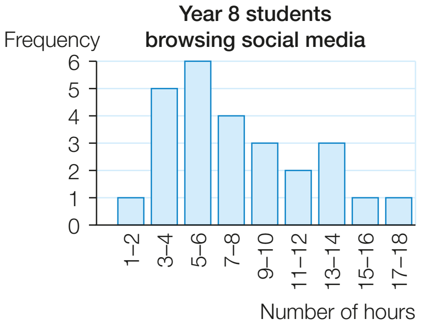
(b)

Height in cm	Frequency
125–<130	1
130–<135	2
135–<140	14
140–<145	6
145–<150	11
150–<155	5
155–<160	5
160–<165	4
165–<170	1
170–<175	1



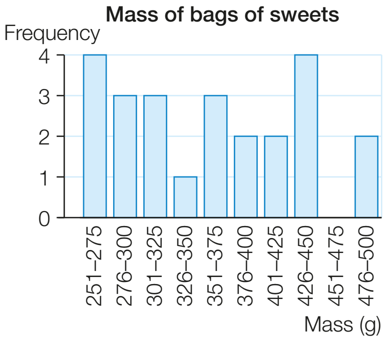
4

Hours browsing social media	Frequency
1–2	1
3–4	5
5–6	6
7–8	4
9–10	3
11–12	2
13–14	3
15–16	1
17–18	1



5

Mass of bags of sweets (g)	Frequency
251–275	4
276–300	3
301–325	3
326–350	1
351–375	3
376–400	2
401–425	2
426–450	4
451–475	0
476–500	2



6 (a) D (b) B (c) C

7 (a)

Age (years)	Frequency
1	4
2	3
3	6
4	5
5	8
6	7
7	2

(b)

Winning margin	Frequency
0–9	4
10–19	7
20–29	5
30–39	4
40–49	2

(c)

Amount ($)	Frequency
0–4.95	2
5–9.95	3
10–14.95	4
15–19.95	2
20–24.95	1

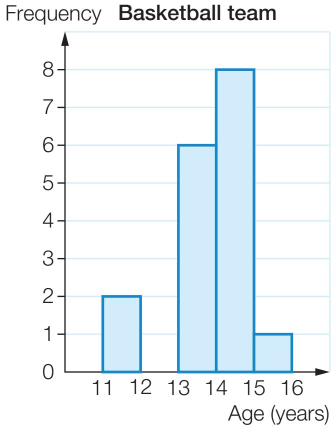
(d)

Temperature (°C)	Frequency
14–<18	5
18–<22	6
22–<26	6
26–<30	7
30–<34	4

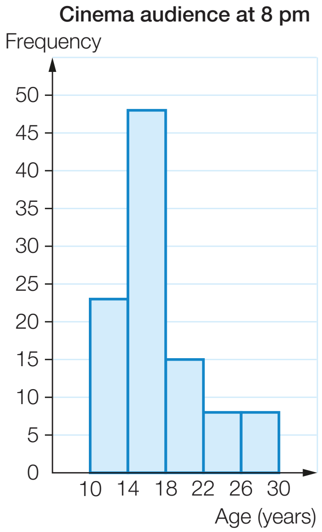
(e)

Number of books	Frequency
1–5	4
6–10	5
11–15	0
16–20	3
21–25	6
26–30	6
31–35	1

8 (a)



(b)



9 There is overlap of the class intervals. For example, the number 10 should be part of the first interval but not the second interval, or vice versa.

10 Sales of plants are slightly higher in the 60-day period of the current year (graph (a)), with three of the days in the 140–159 sales range. Both years have the most common number of sales per day from 80 to 99.

11 As you would expect, the overall height has increased. In the second graph there is no one in the (shorter) 120–<125 cm class and the 125–<130 cm class has fewer students. The one student in the 165–<170 class a year ago has been joined by another student and one of them is at least 170 cm tall. The most common height range is now 135–<140 cm with the 130–<135 cm class having fewer students, since some have grown.

Open-ended – Sample answers

12 25, 18, 33, 27, 12, 30, 45, 41, 21, 57, 5, 33, 47, 21, 15, 51, 36, 48, 9, 20, 19, 31, 35, 22, 46, 30, 15, 27, 10, 41

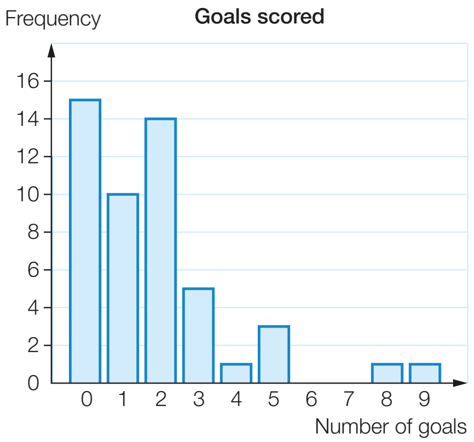
13 41.5, 47.3, 36.8, 53.3, 31.2, 40.5, 51.2, 39.9, 41.4, 44.9, 27.1, 58.3, 38.7, 48.0, 60.0, 44.4, 39.6, 54.7, 52.3, 47.1, 44.5, 34.6, 62.2, 49.1, 37.6, 42.2, 46.5, 57.8, 33.7, 31.4, 61.3, 40.0, 25.9, 50.6, 47.7, 30.5, 45.3, 52.1, 62.4, 30.8

14 The numbers along the horizontal axis are not consistently scaled. Length requires units. The vertical axis needs the label 'Frequency'. A title is needed to give some idea of what is being shown.

Exercise 9.4

1 **(a)** 1.8 goals **(b)** 1.5 goals **(c)** 0 goals

(d)



(e) Possible outliers 8, 9.

2

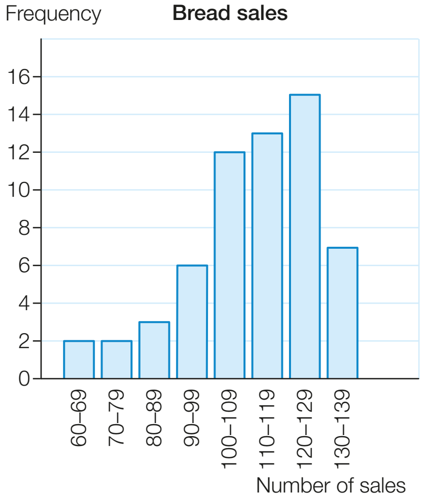
Class interval	Midpoint	Frequency	*fx*
60–69	64.5	2	129
70–79	74.5	2	149
80–89	84.5	3	253.5
90–99	94.5	6	567
100–109	104.5	12	1254
110–119	114.5	13	1488.5
120–129	124.5	15	1867.5
130–139	134.5	7	941.5
		60	6650

(a) mean = 111

(b) median class interval = 110–119 sales

(c) modal class interval = 120–129 sales

(d)



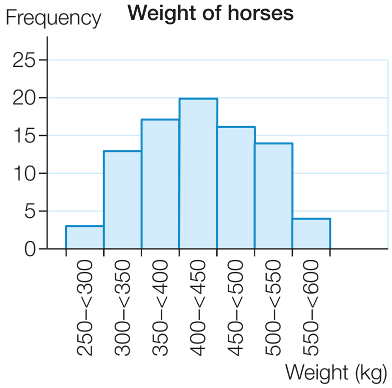
(e) no outliers

3 **(a)** mean = 427 kg

(b) median class interval = 400–<450 kg

(c) modal class interval = 400–<450 kg

(d)



(e) no outliers

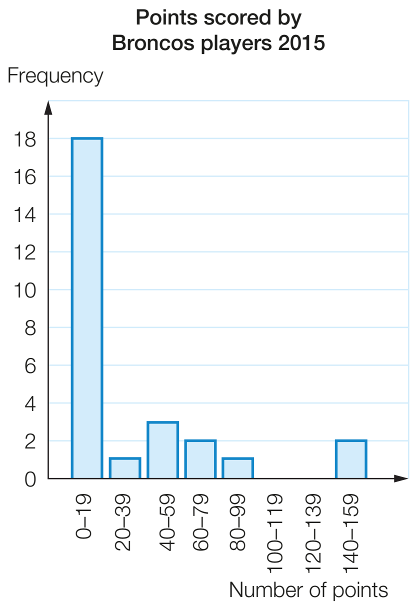
4 **(a)** (2–<3) km **(b)** (3–<4) km **(c)** 3.3 km

5 **(a)** B **(b)** B

6 **(a)** mean = 66.5, median = 65

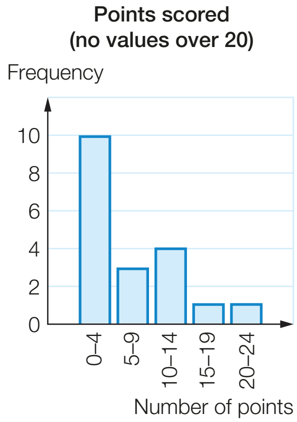
(b) 66.1 **(c)** 50–59

7 **(a)**



(b) 56, 80, 60, 61, 56, 144, 44 could all be outliers.

(c)



(d) The first column graph allows you to see the contrast in points between the very high scoring players and the rest. The second column graph gives much more detail of scoring by the majority of players.

8 **(a)** 50 children

(b) 10.1

(c) No; from the data, you don't know the exact number each child found.

9 **(a)** 1–10, 11–20, 21–30, 31–40, 41–50

(b) 0–4, 5–9, 10–14, 15–19, 20–24

(c) 0–49, 50–99, 100–149, 150–199, 200–249

(d) 1–50, 51–100, 101–150, 151–200, 201–250

(e) 10–12, 13–15, 16–18, 19–21, 22–24

(f) 25–29, 30–34, 35–39, 40–44, 45–49

10 **(a)** 64 games **(b)** 0–39 games **(c)** 0–39 games

(d) mean = 84 games, mode class = 0–39 games, median class = 40–79 games

(e) Yes, 14 out of 45 is nearly one third of the team that hasn't played a game yet. Saying the players have played 84 games each 'on average' makes more sense than saying 64 games including non-starters.

11 **(a)** 26 minutes **(b)** 27 minutes

Open-ended – Sample answers

12 **(a)** 20–24, 25–29, 30–34; 11–15, 16–20, 21–25

(b) 0–9, 10–19, 20–29; 51–60, 61–70, 71–80

13 Students will produce their own set of temperatures for one of these places.

14 **(a)**

Stem	Leaf
0	3 6 8
1	0 1 2
2	0 5 8 9
3	3 3 5 6 7 9
4	1 3 3 4 6

Key: 2 | 5 = 25

(b)

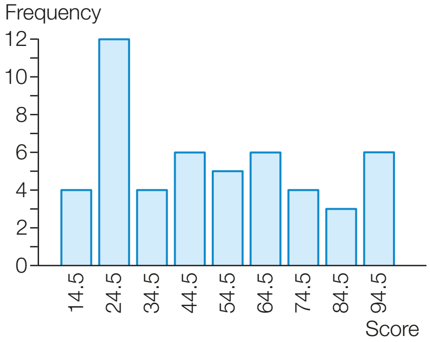
Stem	Leaf
2	1 3 4 7 9
3	0 5 5 5 8 9
4	1 4 7
5	3 6 8 8
6	6 8 9

Key: 2 | 3 = 23

Half-time 9

1

Score	Frequency
10–19	4
20–29	12
30–39	4
40–49	6
50–59	5
60–69	6
70–79	4
80–89	3
90–99	6



2 **(a)** **(i)** 40.10 **(ii)** 23 **(iii)** 0

(b) 178, as this value is quite a bit higher than all the other values. It is unusual for any player to play so many games.

(c) mean: 46.52, median: 33, mode: 4 and 9. Yes, all of the statistics changed significantly.

(d) The median didn't change much when the outlier was removed, so is probably the best measure of centre.

3 **(a)** \$8.30 **(b)** \$(5–<10)

4 **(a)**

Age	Frequency
1	5
2	6
3	6
4	5
5	4

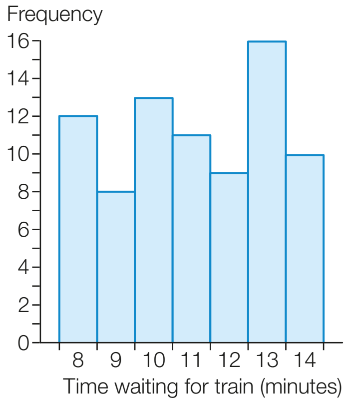
(b)

Margin	Frequency
1–5	5
6–10	6
11–15	6
16–20	5
21–25	4

5 **(a)** mean = 1.92

(b) median = 1

6



7 300 crocodiles

Exercise 9.5

1 **(a)** $\frac{5}{13}$ **(b)** $\frac{8}{13}$

2 **(a)** $\frac{4}{5}$ **(b)** $\frac{3}{5}$

3 $\frac{7}{8}$

4 **(a)** $\frac{5}{13}$ **(b)** $\frac{8}{13}$ **(c)** $\frac{1}{13}$

5 **(a)** $\frac{4}{9}$ **(b)** $\frac{5}{18}$ **(c)** $\frac{1}{18}$

6 **(a)** $\frac{1}{1000}$ **(b)** $\frac{999}{1000}$

7 **(a)** $\frac{1}{3}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{11}{15}$ **(d)** $\frac{3}{5}$

8 A **9** $\frac{3}{500}$ or 0.006 **10** $\frac{5}{12}$

11 **(a)** $\frac{1}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{5}{6}$ **(f)** $\frac{1}{2}$

12 **(a)** B **(b)** A **(c)** B

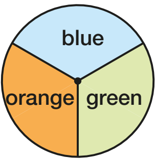
13 **(a)** $\frac{1}{4}$ **(b)** $\frac{11}{20}$ **(c)** 1 **(d)** $\frac{1}{5}$ **(e)** $\frac{4}{5}$ **(f)** $\frac{1}{2}$

14 **(a)** **(i)** 75% **(ii)** 3% **(iii)** 96% **(iv)** 99%

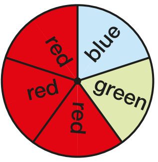
(b) motorcycle

(c) no change for either probability

15 **(a)**



(b)



(c)

Open-ended – Sample answers

16 **(a)** impossible – 0%, unusual – 30%, usually – 70%, very unlikely – 5%, even chance – 50%, most likely – 75%, most unlikely – 10%, 50-50 – 50%, most often – 80%, little chance – 20%, more often than not – 60%, sure thing – 99%, unlikely – 25%, probably – 65%, certain – 100%

(b) rare, occasionally, frequently, possibly, good chance

17 **(a)** 95% **(b)** 70% **(c)** 90% **(d)** 65%

(e) 50% **(f)** 1% **(g)** 60% **(h)** 70%

Exercise 9.6

1 **(a)** $\frac{1}{4}$ **(b)** $\frac{2}{13}$ **(c)** $\frac{4}{13}$ **(d)** $\frac{15}{52}$

2 **(a)** **(i)** T, H **(ii)** $\frac{1}{2}$

(b) **(i)** 1, 2, 3, 4, 5, 6 **(ii)** $\frac{1}{6}$

(c) **(i)** red, blue, green, yellow **(ii)** $\frac{1}{4}$

(d) **(i)** A♥, A♦, A♠, A♣;
2♥, 2♦, 2♠, 2♣;
3♥, 3♦, 3♠, 3♣;
4♥, 4♦, 4♠, 4♣;
5♥, 5♦, 5♠, 5♣;
6♥, 6♦, 6♠, 6♣;
7♥, 7♦, 7♠, 7♣;
8♥, 8♦, 8♠, 8♣;
9♥, 9♦, 9♠, 9♣;
10♥, 10♦, 10♠, 10♣;
J♥, J♦, J♠, J♣;
Q♥, Q♦, Q♠, Q♣;
K♥, K♦, K♠, K♣

(ii) $\frac{1}{52}$

(e) **(i)** red, blue, green, orange **(ii)** $\frac{1}{4}$

3 **(a)** $\frac{1}{3}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{2}$

(f) $\frac{1}{4}$ **(g)** $\frac{1}{2}$ **(h)** $\frac{1}{3}$ **(i)** $\frac{1}{4}$

4 **(a)** $\frac{2}{3}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{3}$

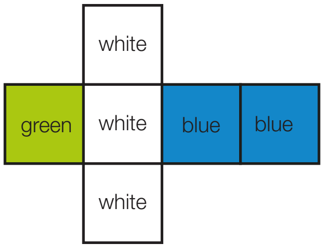
5 **(a)** $\frac{2}{3}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{4}$

6 **(a)** 0.5 **(b)** 0.3 **(c)** 0.7 **(d)** 0.5

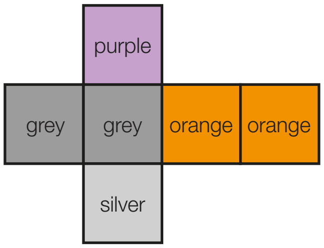
7 **(a)** B **(b)** A **(c)** D **(d)** A **(e)** C

8 Positions of die faces may vary.

(a)



(b)



9 **(a)** for 20 counters: 4 yellow, 8 blue, 6 red, 2 black

(b) for 20 counters: 5 yellow, 5 pink, 3 purple, 1 green, 2 black, 2 blue, 2 white

10 **(a)** **(i)** $\frac{63}{100} = 0.63$ **(ii)** $\frac{22}{25} = 0.88$

(iii) $\frac{12}{25} = 0.48$ **(iv)** $\frac{21}{100} = 0.21$

(b) $\frac{20}{57}$ **(c)** $\frac{3}{5} = 0.6$

11

		Second die					
		1	2	3	4	5	6
First die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$\frac{2}{9}$

12 **(a)** The letters that are used most frequently in words have more pieces so that words can be formed right through the game.

(b) **(i)** $\frac{1}{50}$ **(ii)** $\frac{2}{25}$ **(iii)** $\frac{3}{50}$ **(iv)** $\frac{21}{50}$

(c) **(i)** $\frac{3}{25}$ **(ii)** $\frac{1}{11}$ **(iii)** $\frac{11}{98}$ **(iv)** $\frac{9}{97}$

Open-ended – Sample answers

13

14 **(a)**

	Red	Blue	Orange
1	3	0	0
2	4	4	2
3	5	2	7
4	1	6	2
5	1	2	1

Pr(red and 5) = $\frac{1}{40}$

(b) For the possible distribution given in (a), Pr(blue or even) = $\frac{23}{40}$

(c) For the possible distribution given in (a), Pr(orange or 3 but not both) = $\frac{3}{10}$ or 0.3.

Exercise 9.7

1 **(a)** 15 people **(b)** 5 people **(c)** 20 people

(d) $\frac{3}{4}$ **(e)** $\frac{1}{4}$

2 **(a)** $\frac{7}{30}$ **(b)** $\frac{5}{12}$

3

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

(a) $\frac{1}{2}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$

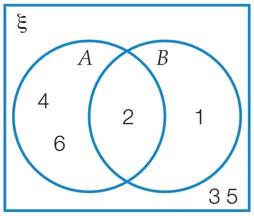
4 **(a)** $\frac{136}{200} = \frac{17}{25}$ **(b)** $\frac{59}{200}$ **(c)** $\frac{5}{200} = \frac{1}{40}$

5 **(a)**

	White	Brown
Male	10	4
Female	8	8

(b) **(i)** $\frac{3}{5}$ **(ii)** $\frac{7}{15}$ **(iii)** $\frac{4}{15}$

6 **(a)**



(b) (i) {2} (ii) {1, 2, 4, 6} (iii) {1, 3, 5}

(iv) 3 (v) 2 (vi) 6

(vii) 1 (viii) 4 (ix) 4

(c) (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$ (iii) $\frac{2}{3}$

7 (a) C (b) C (c) C (d) B (e) A (f) C

8

		Green die					
		1	2	3	4	5	6
Red die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(a) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) (1, 6) (2, 6) (3, 6) (4, 6) (5, 6)

(b) (6, 6)

(c) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(d) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5)

9 (a) $\frac{11}{36}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{5}{36}$

10 (a) (i)

	D	not D
C	$\frac{1}{4}$	$\frac{9}{16}$
not C	$\frac{3}{16}$	0

(ii) $\frac{7}{16}$ (iii) $\frac{1}{4}$ (iv) 1

(v) $\frac{9}{16}$ (vi) $\frac{9}{16}$ (vii) $\frac{3}{16}$

(b) (i)

	D	not D
C	0	$\frac{1}{6}$
not C	$\frac{1}{2}$	$\frac{1}{3}$

(ii) $\frac{1}{2}$ (iii) 0 (iv) $\frac{2}{3}$

(v) $\frac{1}{6}$ (vi) $\frac{1}{2}$ (vii) $\frac{5}{6}$

11 (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

12 (a) $\frac{18}{25}$ (b) $\frac{2}{5}$ (c) $\frac{17}{25}$ (d) $\frac{7}{25}$

13 (a)

	Bought milk	Didn't buy milk
Bought a newspaper	12	3
Didn't buy a newspaper	1	4

(b) 12

(c) (i) $\frac{1}{20}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{5}$ (iv) $\frac{1}{4}$

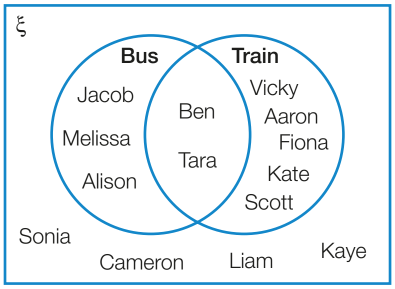
14 (a) $\frac{21}{40}$ (b) $\frac{19}{20}$ (c) $\frac{9}{40}$ (d) $\frac{1}{20}$

15 (a) $\frac{1}{5}$ (b) $\frac{9}{25}$ (c) $\frac{11}{25}$

16 (a) $\frac{17}{25}$ (b) $\frac{4}{25}$ (c) $\frac{21}{25}$

Open-ended – Sample answers

17



$\frac{3}{14}$

18 Robbie double-counted the (3, 3) outcome.

Pr(3 on one die) = $\frac{11}{36}$

Challenge 9

1 (a) 4

(b) 2. If she only selected one sock, it might be the same as the pair she already had, so she would not have a second pair.

2 The two children cross first, one comes back. The adult crosses, the child comes back. The two children now cross together.

3 A

4 (a) B. 3, 5, 7, 9, 11, 13, 15

(b) 7, 9 or 11. Each occurred twice.

(c) The only way to get an even total is if one of the rings misses a peg.

5 (a) 3, 4, 5, 8 (b) $x = 5$

6 The only difference between the graphs is the vertical scale. The second graph appears to be growing faster. They represent the same information.

Chapter review 9

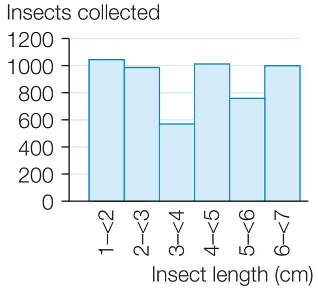
1 375 parrots

2 **(a)** **(i)** 28.88 **(ii)** 29 **(iii)** 26

(b) **(i)** 28.19 **(ii)** 29 **(iii)** 37

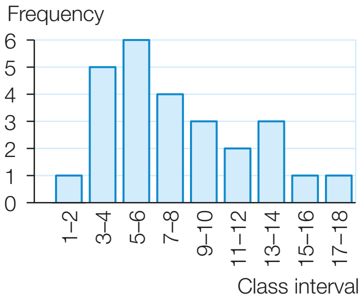
(c) The mean has been reduced slightly and the range has increased significantly. The median is unaffected. The outlier (1) has caused the changes, again showing that the median is unaffected by outliers.

3



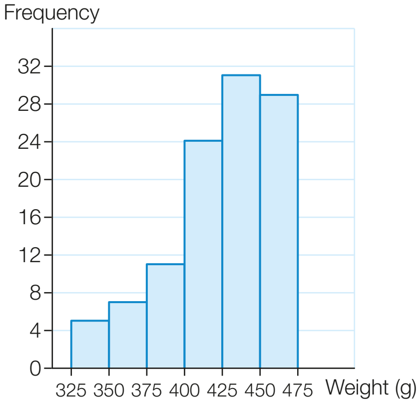
4

Class interval	Frequency
1–2	1
3–4	5
5–6	6
7–8	4
9–10	3
11–12	2
13–14	3
15–16	1
17–18	1



5 **(a)** 423.95 g **(b)** 425–<450 g **(c)** 425–<450 g

(d)

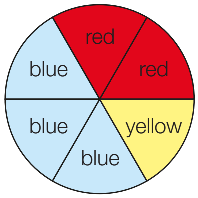


6 **(a)** $\frac{1}{10}$ **(b)** $\frac{3}{10}$ **(c)** 0 **(d)** $\frac{2}{5}$ **(e)** $\frac{7}{10}$ **(f)** $\frac{7}{10}$

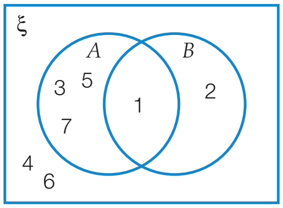
7 **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{5}{6}$

8 **(a)** A **(b)** A **(c)** D

9



10 **(a)**



(b) **(i)** {1, 2, 3, 5, 7} **(ii)** {1} **(iii)** {2, 4, 6}

(c) **(i)** $\frac{5}{7}$ **(ii)** $\frac{1}{7}$ **(iii)** $\frac{3}{7}$

11 **(a)** 23.55 **(b)** 7 **(c)** 138

(d) 138 is clearly an outlier.

(e) Mean is 25.27, median is 22 and range is 58. There doesn't appear to be an outlier.

12 **(a)** Two pieces of information on the graph—each has its own scale.

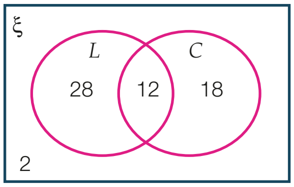
(b) October **(c)** August

(d) **(i)** December **(ii)** August

13 **(a)** 20 cars

(b) **(i)** $\frac{1}{20}$ **(ii)** $\frac{4}{5}$ **(iii)** $\frac{2}{5}$

14 **(a)**



(b) **(i)** $\frac{2}{3}$ **(ii)** $\frac{7}{15}$ **(iii)** $\frac{1}{2}$

(iv) $\frac{1}{5}$ **(v)** $\frac{1}{3}$ **(vi)** $\frac{29}{30}$

15 **(a)** four blue, four green and two red

(b) five blue, one black and four pink

16 **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{10}$

17 **(a)** **(i)** 188.20 cm **(ii)** 188.5 cm **(iii)** 191 cm

(b) Either the mean or the median. The two values are very similar.

(c) 211 could be an outlier.

(d)

Interval class (cm)	Frequency
171–175	1
176–180	4
181–185	10
186–190	13
191–195	15
196–200	1
201–205	1
206–210	0
211–215	1

The estimate for the mean is 188.43 cm.

(e) In this case, there was almost no loss of accuracy. In part, this was because of the tight bunching of the data.

Numeracy practice 9

1 B **2** C **3** A **4** A

5 3.88 **6** C **7** B

Mixed review D

1 (a) $\frac{131}{800}$ (b) $\frac{631}{2000}$

2 (a) $\frac{51}{52}$ (b) $\frac{1}{2}$ (c) $\frac{12}{13}$ (d) $\frac{3}{4}$

3 (a) -12 (b) 38 (c) 0

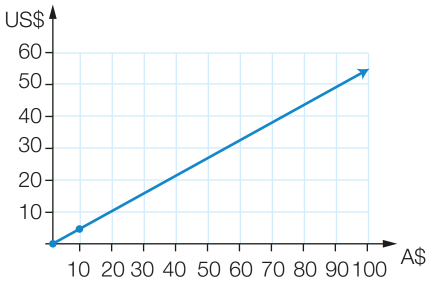
4 (a) $51\frac{2}{3}\%$ (b) $183\frac{1}{3}\%$

5 (a) 960.5 (b) 918 (c) 1330.25

6 (a) $g = 2$ (b) $m = 2$ (c) $s = 45$

7 (b) (0, 0) (c) US\$5.50, (10, 5.5)

(a), (d)



(e) A\$45 is worth approximately US\$25

(f) US\$50 is worth approximately A\$90

8 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$

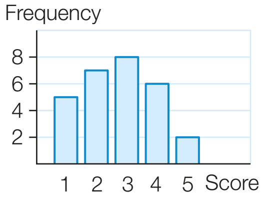
9 (a) 2.5 km/h (b) 18.75 km

10 (a) 2×3^2 (b) $\frac{2^{10}}{3^5}$

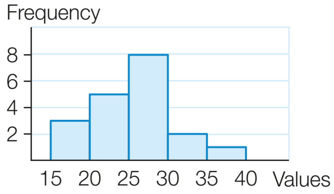
11 (a) 4^6 (b) 6×8^2

12 (a) $3d + 5e$ (b) $17jk + 8j - 35$

13 (a)



(b)



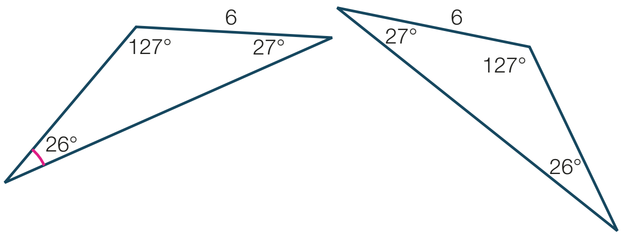
14 (a) $x = -19$ (b) $x = 2$

15 (a) 1, 2, 3, 4, 5, 6

(b) (i) $\frac{1}{6}$ (ii) 0 (iii) $\frac{1}{3}$ (iv) 0

16 (a) 40 (b) 8

17



Find the missing angle to show ASA congruency.

18 (a) 86.59 cm^2 (b) 47.12 cm^2 (c) 178.54 cm^2

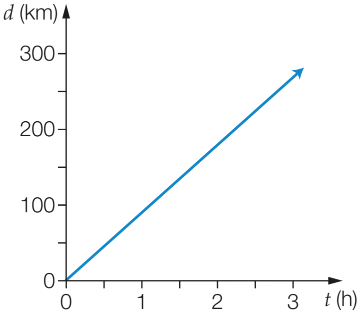
19 (a) $x = -3$ (b) $x = 8$

20 (a) $y = \frac{3x}{4} - 3$ (b) $y = -\frac{2x}{5} + 2$

21 4 yellow, 2 green, 8 red, 6 brown

22 (a) $d = 90t$

(b)



(c) (i) 67.5 km (ii) 210 km

(d) (i) 48 min (ii) 2 h 48 min

23 (a) $\frac{x}{3} - 6 = 27$, the number is 99

(b) $\frac{2x}{6} + 10 = x$, the number is 15

24 104.5 cm^2

25 **(a)** 390

(b) The survey had done a reasonable job of representing the population.

26 $2072 per fortnight

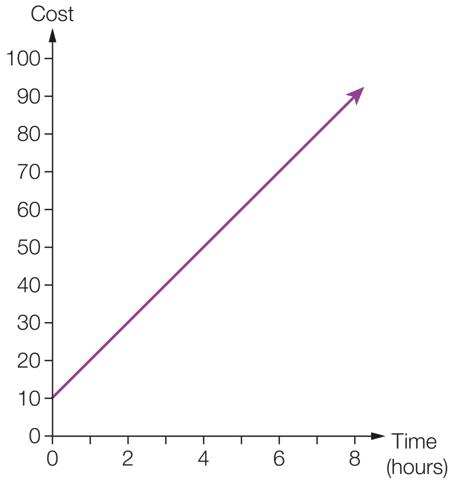
27 sleep: 35%; tv: 11%; school: 36%; sport: 10%; family: 8%

28 **(a)** $\frac{19}{20}$ **(b)** $\frac{4}{5}$ **(c)** $\frac{7}{10}$

29 **(a)**

Time (h)	0	2	4	6	8
Cost ($)	10	30	50	70	90

(b)



(c) cost for 0 hours hire

(d) 34 hours

Glossary and index

24-hour time a system of writing time that numbers the hours from 0 to 23, with no use of 'am' or 'pm'. (p. 334)

acute-angled triangle a triangle in which all angles are acute (less than 90°) (p. 485)

adjacent angles two angles that share a common arm, and a common vertex (p. 469)

alternate angles angles found between two lines but on opposite sides of a transversal; alternate angles between parallel lines are equal (pp. 470, 471)

area the amount of surface within a boundary (p. 291)

associative law the order in which numbers are added or multiplied can be swapped without changing the result; in algebra, $a + (b + c) = (a + b) + c$ and also $a \times (b \times c) = (a \times b) \times c$ (p. 180)

backtracking working backwards along a flowchart (p. 422)

balanced even, the same on both sides of scales or an equation (p. 423)

base

1. the number that is raised to an index or power (p. 34)
2. the side of a triangle, parallelogram or trapezium that is perpendicular to the height
3. the face of a prism that is perpendicular to the height (p. 293)

bias preference or belief not supported by evidence (p. 535)

bisect to cut in half (p. 486)

capacity volume of produce (often liquid) that a container can hold (unit is litre) (p. 328)

Cartesian plane a grid marked with a horizontal x-axis number line and a vertical y-axis number line, so that any point in the plane can be described by its coordinates (x, y) from the origin $(0, 0)$ (p. 362)

census when an entire population is surveyed, not just a sample from the population (p. 533)

circumference the perimeter of a circle (p. 278)

class centre the middle of a class interval (p. 564)

class interval the range of data values used to group data into sets for statistics (p. 555)

coefficient the number factor in a term (p. 151)

co-interior angles angles found between two lines and on the same side of a transversal; co-interior angles on parallel lines are supplementary (pp. 470, 471)

complement

1. an angle that adds to another angle to equal 90° (p. 470)
2. (of a set A) the set of elements that are not in A; written as A' or 'not A'' (p. 584)

complementary angles a pair of angles that add to 90° (p. 470)

composite shape a shape made up of more than one basic shape (p. 314)

congruence statement if two shapes $ABCD$ and $EFGH$ are congruent, this can be written as the congruence statement $ABCD \equiv EFGH$ where the symbol '$\equiv$' means 'is congruent to'; vertices of shapes must be written in the same order so that corresponding vertices can be identified (p. 497)

congruent exactly the same in size and shape (p. 494)

congruent triangles triangles that are exactly the same shape and size, identified using the four tests AAA (Angle, Angle, Angle), SAS (Side, Angle, Side), ASA (Angle, Side, Angle) and RHS (Right angle, Hypotenuse, Side) (p. 506)

constant a term with no pronumeral part (p. 151)

convenience sampling selecting a sample to survey because it is convenient to select (p. 536)

coordinate an x-value or a y-value used to locate a point (p. 362)

corresponding angles

1. matching angles in congruent or similar figures
2. angles found either both above or both below two lines on the same side of a transversal; corresponding angles on parallel lines are equal (pp. 470, 471)

cost price (CP) the amount that a retailer buys an item for (p. 126)

cube to multiply a number by itself, then multiply by itself again (p. 34)

cylinder a solid with a circular base and uniform cross-section (p. 325)

definition the minimum amount of information necessary to describe a figure exactly (p. 518)

diameter the distance from one side of a circle to the other side through the centre (p. 278)

discount a reduction of the selling price (p. 126)

distributive law the way to 'multiply out the brackets', $a(b + c) = ab + ac$ (p. 184)

elapsed time a period of time that has passed (p. 335)

equation two expressions either side of an equals symbol (p. 151)

equilateral triangle a triangle in which all sides are equal in length, as are all angles (p. 485)

equivalent the same as (but not necessarily written the same way) (p. 422)

equivalent ratios a pair of ratios in which the numbers in one ratio are multiples of the numbers in the other (p. 208)

evaluate to calculate the value of a formula or expression (p. 35)

exact decimal form using a dash or dot(s) to indicate the repeating digits of a recurring decimal (p. 67)

expanded form a power expression written as a string of multiplications (p. 35)

expanding removing brackets using the distributive law (p. 184)

exponent *see* index (p. 34)

expression a collection of terms added or subtracted together (p. 151)

exterior angle an angle on the outside of a shape (p. 484)

factorised form an expression written as a product of factors (p. 192)

factorising writing an expression as the product of factors (p. 192)

flowchart a diagram that shows the order in which tasks are to be done (p. 422)

formula (*plural* **formulas** *or* **formulae**) a rule that is expressed using pronumerals and an equals symbol (p. 163)

frequency column graph a column graph that shows the frequency of different values in a data set (p. 554)

Goods and Services Tax (GST) an extra 10% of the selling price that businesses must add onto the price of most products and services sold in Australia (p. 126)

gradient the steepness of a line (p. 364)

grouped data data that has been combined into class intervals to simplify the data and the calculation of statistics from the data (p. 555)

guess, check and improve solving equations by choosing a number, substituting it into the equation and repeating this process until the sides are equal (p. 421)

hectare a unit of area equal to 10 000 square metres (p. 291)

height the distance from the base to the top of a shape, perpendicular to the base (p. 293)

highest common factor (HCF) the largest factor that occurs in each of the sets of factors of a group of terms or expressions (p. 180)

histogram frequency column graph for continuous numerical data (p. 554)

hypotenuse the longest side on a right-angled triangle; it is always opposite the right angle (p. 485)

included angle the angle between the two lines (p. 506)

index (*plural* **indices**) a number, usually written smaller and above another number (the base), that tells how many times the base is multiplied by itself (p. 34)

index form a number multiplied by itself, written as a base raised to an index (p. 34)

inspection solving an equation by looking at the equation and seeing the solution (p. 421)

integers all of the positive whole numbers 1,2, 3, …, the negative whole numbers -1, -2, -3, … and zero (p. 5)

interior angle an angle on the inside of a shape (p. 488)

intersection

1. *see* point of intersection
2. the elements that belong to all sets in a grouping of two or more sets (p. 595)

inverse operation an 'opposite' operation; for example, + and – are inverse operations of each other (p. 422)

irrational number a number that cannot be expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$; when written as a decimal number, the digits are non-terminating and non-recurring (p. 70)

isosceles triangle a triangle with two equal sides; the angles opposite the equal sides are equal angles (p. 485)

judgemental sampling selecting a sample of people to survey because they special knowledge (p. 536)

kite a quadrilateral with two pairs of adjacent sides equal in length (p. 295)

like terms terms that have the same pronumeral part, but may have different coefficients; e.g. $3ab$ and $5ba$ are like terms, but $2a$ and $3a^2$ are not (p. 172)

line graph a graph that shows the relationship between two variables and can consist of several straight line sections (p. 355)

linear graph a straight-line graph (p. 362)

linear relationship a relationship between two quantities that gives a straight line when graphed (p. 362)

loss when an item's selling price is less than its cost price, the difference between these prices (p. 126)

mark-up the amount added to the cost price of an item to make its selling price (p. 126)

mean the average value of a data set (that is, the sum of the data values divided by the number of data values) (p. 546)

measure of centre or **measure of location** a single value that represents key information about a data set (p. 546)

median the middle value when a set of data values are put in order (p. 546)

median class interval a class interval in grouped data that contains the median (middle) value (p. 564)

midpoint

1. half-way between two points on an interval
2. the middle of a class interval (p. 564)

modal class interval the class interval that contains the mode (p. 564)

mode the most frequently occurring data value in a data set (p. 546)

negative used to describe numbers less than zero on a number line; associated with losses or decreases (p. 5)

non-terminating decimal a decimal number that has an infinite number of digits, going on forever (p. 67)

not like terms or **non-like terms** terms that have different pronumeral parts (p. 172)

obtuse-angled triangle a triangle in which one angle is obtuse (the other two angles are acute) (p. 485)

ordered pair a pair of numbers (coordinates) that uniquely determine the location of a point (usually given as an x-value followed by a y-value) (p. 362)

origin the point of intersection of the vertical and horizontal axes on a number plane; it has the coordinates (0, 0) (p. 362)

outcome the result of a probability experiment or event (p. 582)

outlier an unusual extreme data value, compared to other values in the data set (p. 548)

parallel lines lines in the same plane that are always the same distance apart and will never intersect (p. 471)

parallelogram quadrilateral with both pairs of opposite sides parallel (p. 293)

part : part ratio a ratio that compares two or more parts of a whole (p. 207)

part : whole ratio a ratio that compares a part (or parts) to the whole (p. 207)

per cent literally 'for each hundred' (p. 84)

percentage a value expressed as an amount out of 100 (p. 84)

percentage error the difference between an exact value and an approximate value, expressed as a percentage of the exact value (p. 92)

perimeter the distance around the outside of a shape (p. 271)

perpendicular at right angles to (p. 293)

plane a flat two-dimensional surface (p. 471)

point of intersection the point where two or more lines, rays or line segments meet (p. 446)

poll a survey given to a sample of a population (p. 533)

population an entire category or group (p. 533)

population statistics statistics calculated from the data collected by a census (p. 533)

positive used to describe numbers greater than zero on a number line; associated with gains or increases (p. 5)

power *see* index (p. 34)

prism a three-dimensional shape with a uniform cross-section, where the cross-section is a polygon and the edges are straight (p. 324)

probability the likelihood of a specific event, expressed as a number from 0 to 1 (p. 582)

profit when a product's selling price is more than its cost price, the difference between these prices (p. 126)

pronumeral a letter or symbol that represents a number (pp. 151, 417)

property any characteristic of a particular geometrical figure (p. 518)

quadrilateral a four-sided plane shape (p. 485)

quotient the result of a division calculation

radius the distance from the centre of a circle to its edge (p. 278)

random sampling sampling for which every member of a group has the same chance of being selected as any other member of that group (p. 537)

range the difference between the largest value and smallest value in a data set (p. 547)

rate a way of comparing two amounts of different quantities, in terms of how much of one quantity corresponds to how much of the other quantity (p. 252)

ratio a comparison of two or more different amounts in relation to each other, usually written using a : symbol (p. 207)

rectangle a parallelogram with right angles (p. 292)

recurring decimal a decimal number with a pattern of digits that repeats (p. 60)

rhombus a parallelogram with adjacent sides equal in length (p. 295)

right-angled triangle a triangle with one right (90°) angle (p. 485)

sale price the new price when a selling price is reduced by a discount (p. 126)

sample a small selection of a population (p. 533)

sample space a listing of all the possible outcomes for a particular situation (p. 582)

sample statistics statistics calculated from sample data (p. 533)

scale factor the number that relates the size of an enlarged or reduced figure to its original; the number that the original side lengths are multiplied by, to produce the new figure (p. 220)

scale ratio a ratio on a map or diagram that indicates the actual lengths represented by the lengths shown; for example, a scale ratio of 1:100 indicates that 1 cm on a map corresponds to a distance of 100 cm in real life (p. 241)

scalene triangle a triangle with no equal sides or angles (p. 485)

selling price (SP) the amount that an item is sold for (that is, the cost price plus the mark-up) (p. 126)

simple random sampling sampling for which every member of the population has the same chance of being selected (p. 537)

simplest form when the numbers in a ratio or fraction have no common factors (p. 208)

slope *see* gradient

solid a three-dimensional object that has length, width and depth (p. 324)

solution the value of a pronumeral that makes the equation true; the answer (p. 417)

solving finding a value that makes an equation a true number sentence (p. 417)

speed the ratio of distance travelled to time taken (p. 252)

square

1. to multiply a number by itself (p. 34)
2. a rectangle with a pair of adjacent sides equal (p. 293)

substitution replacing a pronumeral with a given number (p. 158)

supplement an angle that adds to another angle to equal 180° (p. 470)

supplementary angles a pair of angles that add to 180° (p. 470)

surd an irrational number that can only be expressed exactly using the radical ($\sqrt{\ }$) symbol (p. 70)

term a single item that is added or subtracted in an expression; for example, a number or a pronumeral by itself ($7, x$), a number multiplied by pronumerals: ($12c, 5k, 3xy, 4a^2, 5b^2c$), pronumerals multiplied together (mn, abc, a^3, b^2) (p. 151)

terminating decimal a decimal number that has a limited (not infinite) number of digits (p. 67)

time zone a geographical region of the world where clocks are all set to the same time (p. 336)

transversal a line that crosses (transverses) other lines (p. 470)

trapezium a quadrilateral with only one pair of opposite sides parallel (p. 294)

triangle a three-sided plane shape (p. 293)

two-way table a table that categorises information in rows and columns according to two different criteria (p. 599)

uniform cross-section when an object's cross-section is identical to its base, no matter where the cross-section is cut parallel to the base (p. 324)

union the elements that belong to at least one set in a grouping of two or more sets (p. 595)

unit ratio a ratio expressed so that at least one number in the ratio is 1 (p. 220)

unitary method when a rate is expressed as an amount of one quantity corresponding to 1 unit of the other quantity (p. 253)

universal set the set of all elements (all possible outcomes) in a situation; it is often labelled by a symbol ξ, ε, ε or U (p. 593)

variable in algebra, an unknown amount that may vary, represented by a pronumeral (pp. 151, 417)

Venn diagram a diagram used to show the relationships between sets, using areas to show the elements (or number of elements) in different sets (p. 593)

vertex (*plural* **vertices**) a point where lines or edges meet (p. 469)

vertically opposite angles pairs of angles that are opposite each other formed where two lines intersect (p. 469)

volume the amount of space occupied by a three-dimensional object (p. 324)

***x*-intercept** a point where a graph intersects the x-axis (p. 364)

***y*-intercept** a point where a graph intersects the y-axis (p. 364)

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