NELSON CENGAGE Learning

8

NELSON QMATHS

for the Australian Curriculum

Stephen Swift | Ross Brodie | Stephen Corcoran | Sue Garner

8

Mangahigh.com READY

nelsonnet

This is a legal agreement between the you, (the "Customer") and Cengage Learning Australia Pty Limited (ABN 14 058 280 149) (the "Licensor") which provides the terms and conditions of this non-exclusive licence and the limited warranty for the Product. Use of the Product indicates an acknowledgement that the Customer has read and agreed to be bound by the terms and conditions of this Agreement. If you do not agree to these terms and conditions, return the Product to the place of purchase within 15 days of the date of purchase (with proof of purchase) for a full refund

1. **Licence Grant**
   You do not receive title to the Product. Copyright in the Product (which includes all images, photographs, video, animations, audio, music and text incorporated in the Product, including all of the accompanying printed material) is owned by the Licensor and/or its suppliers and is protected by Australian copyright laws. The Licensor grants you a non-exclusive licence to use the Product subject to the restrictions and terms set out in this Agreement.

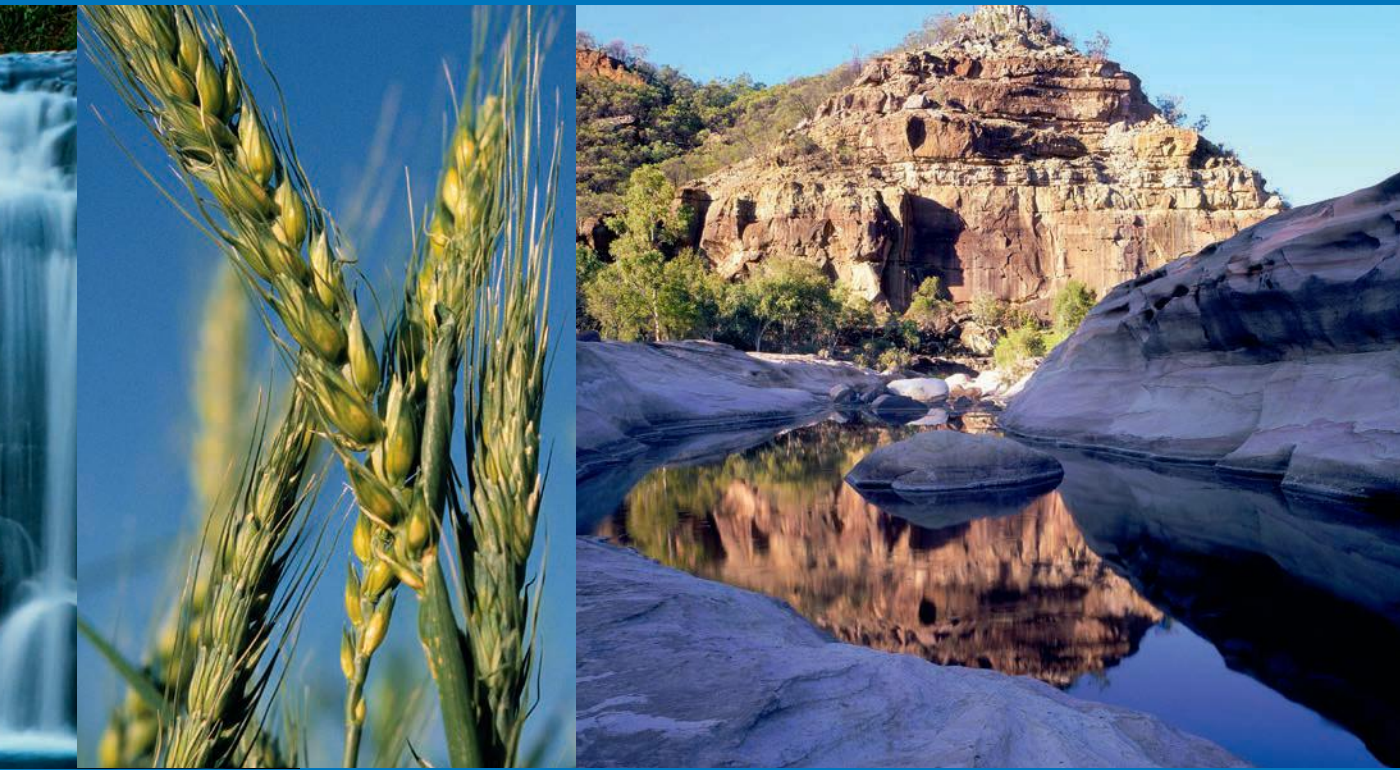
2. **A Licence allows you to:**
   Use the Product on your computer. The Customer represents that they shall in no way place the Product in the public domain or in any way compromise our copyright in the Material. You agree to take reasonable steps to protect our copyright.

3. **You may not:**
   Alter, modify, translate, reverse engineer, decompile, or adapt the software or create derivative works based on the Product. Make further copies by any means technological, electronic, digital whatsoever without the written permission of the Licensor. Rent or transfer all or any part of your rights under this Agreement. Remove or alter any copyright or other proprietary notice or label attached to the software.

4. **Termination**
   Any failure to comply with the terms and conditions of this agreement will result in the automatic termination of this licence. Upon termination of this licence for any reason, the Customer must destroy or return to the Licensor all copies of the software and accompanying documentation.

5. **Warranties**
   To the extent permitted by law, the Licensor's liability for any breach of the warranty or any term implied by law into this licence is limited to the lowest cost of replacing the goods, acquiring equivalent goods or having the goods repaired.

8



NELSON QMATHS

for the Australian Curriculum

Stephen Swift | Ross Brodie | Stephen Corcoran | Sue Garner

NELSON
CENGAGE Learning™

Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

Nelson QMaths for the Australian Curriculum 8
1st Edition
Stephen Swift
Ross Brodie
Stephen Corcoran
Sue Garner

Publishing editor: George Hook
Project editor: Alan Stewart
Editor: Anna Pang
Senior designer: Kar Heng Goh
Photo researcher: Georgina Wober
Cover images: Corbis and Shutterstock.com
Production controller: Jem Wolfenden
Typeset by: Cenveo Publisher Services

For product information and technology assistance,
in Australia call **1300 790 853**;
in New Zealand call **0800 449 725**

For permission to use material from this text or product, please email **aust.permissions@cengage.com**

National Library of Australia Cataloguing-in-Publication Data
Stephen Swift ... [et al.]
Nelson QMaths for the Australian Curriculum 8

1st ed.
9780170194778 (pbk.)
For secondary students.

Mathematics--Australia--Textbooks.
Mathematics--Study and teaching (Secondary)--Australia.

510

Cengage Learning Australia
Level 7, 80 Dorcas Street
South Melbourne, Victoria Australia 3205

Cengage Learning New Zealand
Unit 4B Rosedale Office Park
331 Rosedale Road, Albany, North Shore 0632, NZ

For learning solutions, visit **cengage.com.au**

Printed in China by China Translation & Printing Services.
1 2 3 4 5 6 7 15 14 13 12 11

Contents

9780170194778

About this book

At the beginning of each chapter

- Clear outline of chapter contents
- Strong links to the curriculum
- Prior learning provides a quick check of understanding for the individualisation of instructional sequence

Contents

Prior learning Chapter 8
MAT08SPPL00008
Parent guide Chapter 8
MAT08SPPG00008
Curriculum guide Chapter 8
MAT08SPCU00008

Australian Curriculum statements

Chance

Identify complementary events and use the sum of probabilities to solve problems.

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.

Represent such events in two-way tables and Venn diagrams and solve related problems.

9780170194778 271

Investigate: Buttered toast

A piece of buttered toast is the same on both sides, except that one side has a coating on it. You could use any square shape to represent a piece of toast, such as a square cut from a packing carton.

1 Make some pretend toast by cutting squares from a packing carton.
2 Label one side of your 'toast' as buttered.
3 Toss your 'toast' and record which side it lands.
4 Repeat the experiment many times.
5 Put together the results for the whole class.
6 Work out some conclusions about Murphy's Law.

- Parent guide provides vital information for parents
- Curriculum guide provides a rationale in words that students will understand
- Clear sectioning that is consistent with the exercises
- Many sections begin with a concrete investigation, with teacher support materials provided
- Intriguing aids for students provided

In each chapter

- Examples are clearly set out with explanations on the left and expected student answers in bold on the right
- Colour-coded examples make it easy for teachers to direct students

  Green – nearly all students will master

  Gold – most students will master

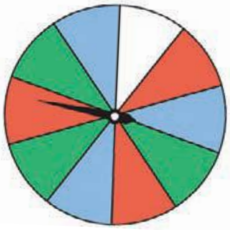
  Blue – some students should master
- Examples are sequenced in a natural learning progression that promotes chunking and effective use of working memory
- Clear examples enable parents to assist their student with confidence that they are 'doing it the way the teacher explained it'
- Important boxes highlight key terms and concepts

Example 2

The spinner shown here was spun and the colour it landed on was recorded in the table.

Colour	Red	Blue	Green	White
Frequency	28	33	31	8

a What is a trial in this experiment?
b What is the total frequency for this experiment?
c What is the relative frequency of spinning red?
d What is the relative frequency of spinning white?



Example 12

Find the complement of each of the following events.

a rolling a die and getting a 5
b it will rain tomorrow

Example 14

A coin is tossed two times. Use a tree diagram to calculate the probability that the outcome of the experiment is:

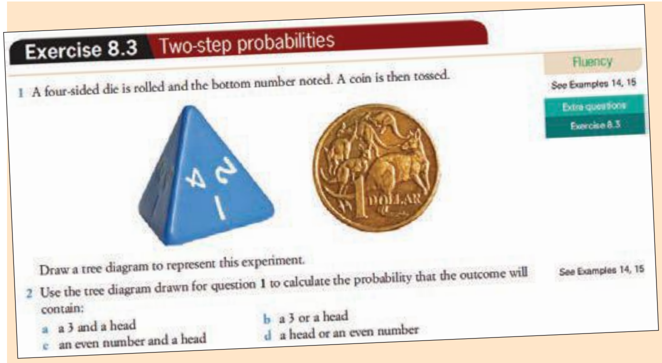
a two heads
b two tails
c a head and a tail
d a head followed by a tail

Important!

Probabilities of events

The probability of a **certain** event is 1.
The probability of an **impossible** event is 0.
The probabilities of all other events are between 0 and 1.
The sum of the probabilities of all outcomes for an experiment is 1.

9780170194778


Exercise 8.3 Two-step probabilities

Fluency

1 A four-sided die is rolled and the bottom number noted. A coin is then tossed.

See Examples 14, 15

Extra questions Exercise 8.3

Draw a tree diagram to represent this experiment.

2 Use the tree diagram drawn for question 1 to calculate the probability that the outcome will contain:

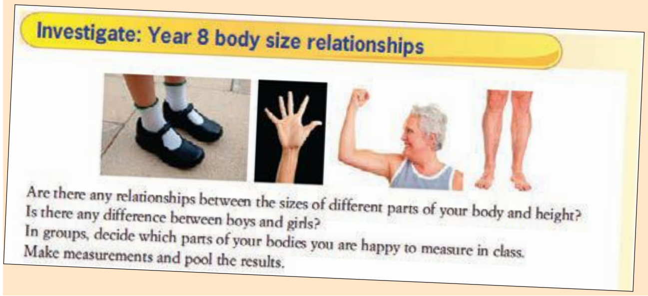
See Examples 14, 15

a a 3 and a head
b a 3 or a head
c an even number and a head
d a head or an even number

- Initial exercise questions focus on understanding or fluency and refer directly back to examples
- Questions are categorised according to the four proficiency strands - understanding, fluency, problem solving and reasoning
- Questions and parts are colour coded to be consistent with the examples

- Colour is used pedagogically to enhance learning

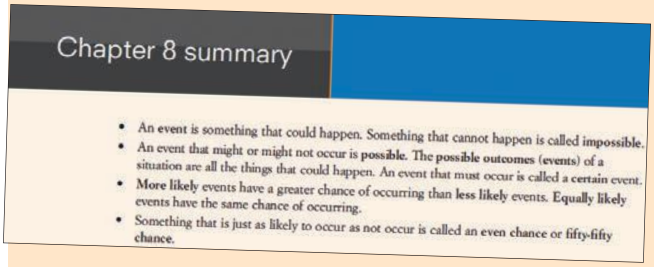
Score x	Frequency f	Score × frequency xf	Cumulative frequency
27	2	54	2
28	8	224	10
29	18	522	28
30	9	270	28 + 9 = 37


Investigate: Year 8 body size relationships

Are there any relationships between the sizes of different parts of your body and height?
Is there any difference between boys and girls?
In groups, decide which parts of your bodies you are happy to measure in class.
Make measurements and pool the results.

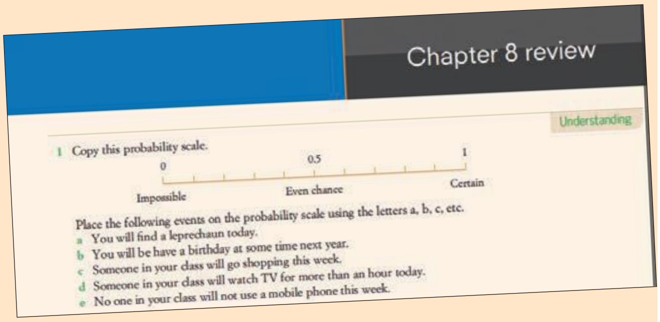
- There is an abundance of carefully designed investigations proven to aid deep understanding

At the end of each chapter


Chapter 8 summary

- An event is something that could happen. Something that cannot happen is called **impossible**.
- An event that might or might not occur is **possible**. The **possible outcomes (events)** of a situation are all the things that could happen. An event that must occur is called a **certain event**.
- **More likely** events have a greater chance of occurring than **less likely** events. **Equally likely** events have the same chance of occurring.
- Something that is just as likely to occur as not occur is called an **even chance** or **fifty-fifty chance**.

- A chapter summary that re-states all important concepts


Chapter 8 review

Understanding

1 Copy this probability scale.

0 | 0.5 | 1
Impossible | Even chance | Certain

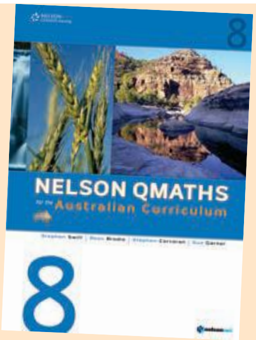
Place the following events on the probability scale using the letters a, b, c, etc.
a You will find a leprechaun today.
b You will be have a birthday at some time next year.
c Someone in your class will go shopping this week.
d Someone in your class will watch TV for more than an hour today.
e No one in your class will not use a mobile phone this week.

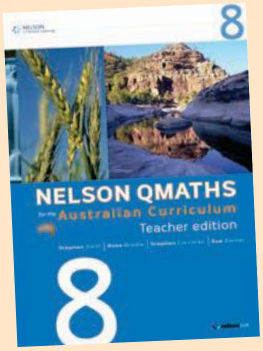
- A comprehensive review set out with example references and consistent with exercise layout
- Answers to all questions are given at the back of the book

About the series

Other books in the series

- There are five books in this Years 7 to 10 series, which has been written specifically for the Australian Mathematics Curriculum, including two books for Year 10 (see back cover).
- There is also a printed Teacher's Edition of each textbook with additional guidance and advice on implementing the Australian Mathematics Curriculum in your classroom.
- Accompanying each printed textbook is a digital textbook called the NelsonNetBook (see opposite page).





Icons in the textbook

- In the outside margins of your textbook you will find numerous icons for resources that can be accessed through the NelsonNet website or through the NelsonNetBook.

Alternative method / Moving the decimal	Explains different ways of solving a maths problem	Puzzle sheet / Find the number	Makes learning maths methods and skills fun
Animated example / Stem-and-leaf plots	Shows how to tackle difficult problems	Quiz / Expressions	Tests and marks your knowledge of a topic
Scientific calculator exercise / Fractions and decimals	Develops your scientific calculator skills	Teacher notes / Restaurant tables	Gives guidance for your teacher
Curriculum guide / Chapter 6	Explains what you need to know and be able to do	Technology / GeoGebra: Exterior angle of a triangle	Provides a spreadsheet or GeoGebra activity
Extra questions / Exercise 6.1	Allows you to do additional practice and drill	TLF learning object / Exploring order of operations (L6543)	Provides interactive ways of learning about maths*
Maths clip / Statistics	Gives the 'big picture' view of your maths topic	Video tutorial / Algebraic expressions	Fully explains maths methods and skills
Maths dictionary	Provides an illustrated explanations of all terms	Weblink / Algebra masterclass	Links to a website that makes Maths relevant
Parent guide / Chapter 6	Provides guidance for your parent to help you	Worked solutions / Exercise 6.1	Gives solutions steps for selected problems
Prior learning / Chapter 6	Finds out what you already know and can do	Worksheet / Writing a rule	Supplies tasks to be done digitally or by using a pen

* Your teacher will explain how to access The Learning Federation learning objects.

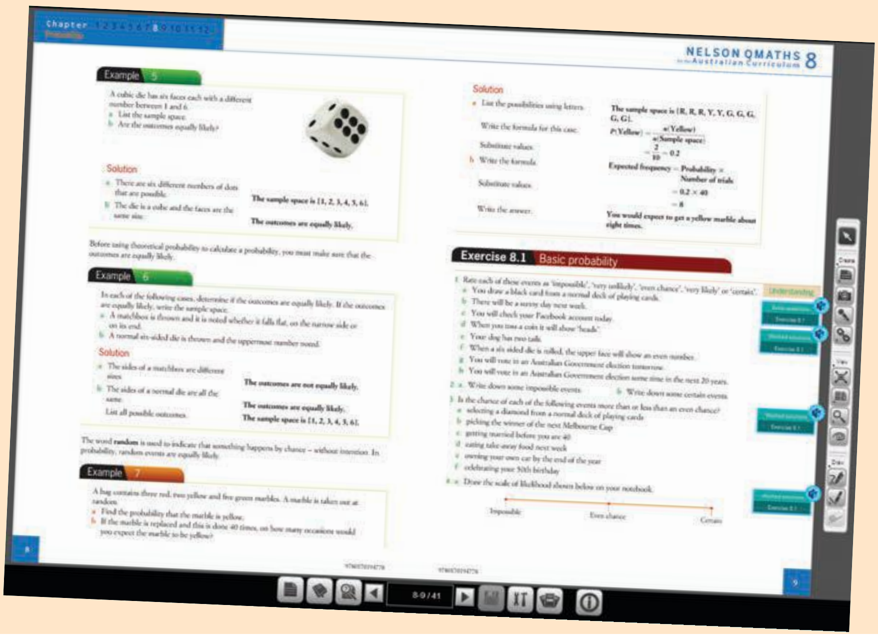
9780170194778

NelsonNet Website

- Go to www.nelsonnet.com.au to log in or find out more.
- The first screen you will see on the student website is called 'Chapter resources'.
- Click on a chapter and a list of different resources types will appear.
- Click on a resource type and a list of specific resources will pop up.
- Clicking on one of those resources will open a pdf file, start a video, or some special maths software.
- Use the blue tabs on the left of the screen to access calculator resources and the maths dictionary.

NelsonNetBook

- This is a web-based ebook that you can customise to meet your own learning needs.
- The icons with the blue NelsonNet logo are 'hotspots'. Click on an icon to open that resource.
- The tools on the vertical toolbar allow you to personalise pages in a variety of ways, including voice recordings, drawings and links to favourite websites. They also enable you to zoom in and out.
- The tools on the horizontal toolbar allow you to navigate around your ebook and change settings.
- Administration tools allow your teacher to manage classes and share annotations with you.



About the authors

Stephen Swift started teaching Mathematics, Science and Computing in 1973 and taught at all levels from Years 7 to 12 in several states, in urban and country schools until retiring in 2010 from the role of Mathematics Head of Department at Wellington Point State High School in Brisbane.

Ross Brodie has worked as a classroom teacher and Mathematics Head of Department at a number of regional and metropolitan secondary schools. He has taught Mathematics at Years 8 to 12. Ross brings a wide range of experience from education and other sectors to the writing of mathematics student books.

Stephen Corcoran is an experienced teacher who has been Head of Mathematics in four schools. He is very familiar with current trends in mathematics education. He received a National Excellence in Teaching award for inspiring students and encouraging their abilities through innovative mathematics programs.

Sue Garner is assistant Head of Mathematics at Ballarat Grammar, specialising in both Mathematical Methods (CAS) and Specialist Mathematics. She has published extensive material for maths, has lectured at The University of Ballarat and at The University of Melbourne.

Digital Resources Team Sue Garner, Alan Preston, Dawn Fernandez, Frank Moya, Greg Neal, Ian Bull, Jamos Somerville-McAlester, Roger Walter and Tobias Cooper.

Acknowledgements

Brisway © 2010 Brisway Publishing Pty Ltd. Reproduced from Brisway Edition 5 with permission.: p. 260; Corbis Australia: p. 346; Getty Images: pp. 17 (left, centre left); Bill Thomas/Imagen: p. 274; iStockphoto: pp. 19 (centre left), 25, 40, 167, 267, 349; M.C. Escher's "Symmetry Drawing E22" © 2011 The M.C. Escher Company-Holland. All rights reserved: p. 65 (left); Newspix/News Ltd: p. 24/Darren Tindale: p. 10; Photolibrary: pp. 2–3, 12, 17 (right), 19 (left), 38–39, 65 (right), 92, 93, 118–119, 144–145, 188–189, 238–239, 265, 270–271, 276, 312–313, 340–341, 351, 390, 402–403, 432, 433; Shutterstock.com: pp. 4, 5, 9, 17 (centre right), 19 (centre right, right), 20, 26, 30, 31, 75, 76, 97, 101, 112, 114, 131, 166, 177, 181, 241, 247, 252, 256, 278, 283, 284, 285, 289, 303, 308, 311, 353, 405, 408, 411, 415, 417, 425, 427, 429, 430, 439, 443, 451; Steve Lovegrove: p. 372.
The publishers would like to thank ACARA for permission to quote the Year 8 content descriptions from the Australian Mathematics Curriculum in this book. (Australian Curriculum (Sydney, 2010): Australian Curriculum, Assessment and Reporting Authority [ACARA]. ACARA does not endorse Cengage or this product.)

9780170194778

Symbols and abbreviations

$\therefore$	therefore	$\triangle ABC$	triangle ABC
$=$	is equal to	$\angle ABC$	angle ABC
$\neq$	is not equal to	$\equiv, \cong$	is congruent to
$\approx, \sim, \doteqdot$	is approximately equal to	$\vert\vert\vert, \sim$	is similar to
$>$	is greater than	$A \leftrightarrow P$	A corresponds to P
$\ngtr$	is not greater than	$\parallel$	is parallel to
$\geq$	is greater than or equal to	$\perp$	is perpendicular to
$<$	is less than	%	percentage
$\nless$	is not less than	$\frac{a}{b}$	fraction, division $a \div b$ or ratio $a : b$
$\leq$	is less than or equal to	$\pm$	plus or minus
$\propto$	is proportional to	$\sqrt{\ }$	square root
$\in$	is a member of	$\sqrt[3]{\ }$	cube root
$\notin$	is not a member of	$\sqrt[n]{\ }$	nth root
m	gradient	$45°$	45 degrees
BC	before Christ	$\bar{x}$	mean, average
BCE	before the common era	π	pi $\approx 3.1415926\ldots$
AD	Anno Domini	$\{\}, \varnothing$	null set , empty set
CE	common era	$\subseteq$	subset of
QED	*quod erat demonstrandum*	$\subset$	proper subset of
LHS	Left-hand side	1 : 100	Scale of 1 to 100
RHS	Right-hand side	$\llcorner$	right angle

$5.3\overline{7214}$ or $5.3\dot{7}21\dot{4}$ shows $5.3721472147214\ldots$

Metric system

Symbols
millimetre — mm
centimetre — cm
metre — m
kilometre — km
hectare — ha
millilitre — mL
litre — L
kilolitre — kL
megalitre — ML
milligram — mg
gram — g
kilogram — kg
tonne — t

Length
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Area
100 mm^2 = 1 cm^2
10 000 cm^2 = 1 m^2
10 000 m^2 = 1 ha
100 ha = 1 km^2

Capacity
1000 mL = 1 L
1000 L = 1 kL
1000 kL = 1 ML

Mass
1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t

Greek alphabet

A, α alpha
B, β beta
Γ, γ gamma
Δ, δ delta
E, ε epsilon
Z, ζ zeta
H, η eta
Θ, θ theta
I, ι iota
K, κ kappa
Λ, λ lambda
M, μ mu
N, ν nu
Ξ, ξ xi
O, o omicron
Π, π pi
P, ρ rho
Σ, σ sigma
T, τ tau
Y, υ upsilon
Φ, ϕ phi
X, χ chi
Ψ, ψ psi
Ω, ω omega

Statistics and probability

1 Statistics

9780170194778

Contents

Prior learning
Chapter 1
MAT08SPPL00001

Parent guide
Chapter 1
MAT08SPPG00001

Curriculum guide
Chapter 1
MAT08SPCU00001

Australian Curriculum statements

Investigate techniques for collecting data, including census, sampling and observation.

Data representation and interpretation

Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

Explore the variation of means and proportions of random samples drawn from the same population.

Investigate the effect of individual data values, including outliers, on the mean and median.

Maths clip
Statistics
MAT08SPMC00001

Weblink
Australian Bureau of Statistics
MAT08SPWB00001

Maths dictionary
MAT08ASDI00001

There are all kinds of surveys conducted in Australia. When you go into a shop, watch a TV program, buy something, check a website or even answer your phone, you may be asked to do a survey. Marketing companies are very interested in finding out why you buy particular products or services. Governments need to find out information for planning purposes. A poor survey will not be useful, so the way it is conducted is very important. In this chapter, you will look at some of the factors that influence survey design and use.

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

axis	class width	histogram	range
back-to-back plot	cumulative frequency	line graph	sample
bar chart		mean	scale
bias	data	median	score
bimodal	divided bar chart	mode	spread
census	dot plot	non-compliant	stem-and-leaf plot
central tendency	fair	outlier	survey
circle graph	frequency	picture graph	variation
class	graph	population	

1.1 Statistical measures

The main characteristics of a set of numerical data are the centre of the data and the spread of the data. The measures of central tendency are the **mode**, **median** and **mean**. The easiest measure of spread to work with is the **range**.

9780170194778

Important!

Measures of central tendency

The **mode** is the score with the highest frequency. It is the most common score, so it occurs more often than any other score. Where there are two scores with equal highest frequencies, we say that the distribution is **bimodal**.
The **median** is the *middle* score. To find the median, all scores are arranged in order from smallest to largest and the middle score is chosen. If there is an even number of scores, then the average of the middle two scores is used. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.
The **mean** is the **average** score. It is calculated by adding all the scores and dividing by the number of scores. The symbol $\bar{x}$, pronounced as 'x-bar', is often used for the mean.
The mean, median and mode are called measures of **central tendency** and are usually very similar. The mean is generally considered typical as it summarises all the scores. The relationship between the mean and the median can be changed by a few very high or very low scores. If there are a few very high or very low scores that would distort the mean, you should use the median. The mode should be used in cases where the most common score is needed.

Measure of spread

The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.

Example 1

Find the mean, median, mode and range of the following ages of grandparents of some Year 8 students.
65, 71, 71, 61, 63, 61, 63, 69, 79, 71, 63, 75, 71, 61, 65, 60, 65, 63, 79, 67, 63, 63, 73, 65, 63, 65, 59, 73, 63, 65, 61, 59, 69, 67, 59, 77, 59

Puzzle sheet
Ranges and averages
MAT08SPPS00001

Puzzle sheet
Data puzzles
MAT08SPPS00002

Worksheet
Mean, median, mode
MAT08SPWK00001

Solution

Add all the ages.

$$\textbf{Total} = \mathbf{65 + 71 + 71 + 71 + \ldots}$$
$$= \mathbf{2446}$$

Count the number of people.

Number of scores = 37

Divide to find the mean.

$$\textbf{Mean, } \bar{x} = \frac{\textbf{Total of scores}}{\textbf{Number of scores}}$$
$$= \frac{\mathbf{2446}}{\mathbf{37}}$$
$$= \mathbf{66.1081\ldots}$$

Round the answer.

The mean age is about 66.1 years old.

9780170194778

Work out which score, in order, will be halfway.	**37 ÷ 2 = 18.5, so there will be 18 scores before the 19th score and 18 scores after it.**
Write the ages in order and find the 19th score.	**59, 59, 59, 59, 60, 61, 61, 61, 61, 63, 63, 63, 63, 63, 63, 63, 63, 65, 65, 65, 65, 65, 65, 65, 67, 67, 69, 69, 71, 71, 71, 71, 73, 73, 75, 77, 79, 79**
Write the middle score.	**The median is 65 years old.**
Find the score with the highest frequency.	**There are eight 63s, more than any other score. Mode = 63 years old**
Find the range.	**Range = highest − lowest = 79 − 59 = 20**
State the answers.	**Mean ≈ 66.1, median = 65, mode = 63 and range = 20.**

On your calculator, you could enter the mean calculation as:

2446 [÷] 37 [=], which gives: 2446÷37 66.10810811

The mean can also be found using the statistics mode on your calculator, and your teacher might show you how to do this if most of the class have the same kind of calculator.

Example 2

Technology worksheet
Excel: Finding the mean, median, mode and range
MAT08SPCT00001

Find the mean, median, mode and range of the following maths test scores.
9, 22, 15, 27, 18, 23, 9, 23, 16, 28, 16, 24, 11, 23, 18, 25, 16, 21, 20, 24

Solution

Add all 20 scores.	**Total = 9 + 22 + 15 + 27 + 18 + 23 + ... = 388**
Divide by the number of scores.	**Mean($\bar{x}$) = $\frac{388}{20}$ = 19.4**
You can use your calculator.	
Enter as: 388 [÷] 20 [=].	388÷20 19.4
Arrange the scores in order.	**9, 9, 11, 15, 16, 16, 16, 18, 18, 20, 21, 22, 23, 23, 23, 24, 24, 25, 27, 28**
There's an even number, so find the middle 2 (10th and 11th).	**9, 9, 11, 15, 16, 16, 16, 18, 18, 20, 21, 22, 23, 23, 23, 24, 24, 25, 27, 28**
Find their average.	**Median = $\frac{20 + 21}{2}$ = 20.5**
There are three 16s and three 23s, more than any other score.	**It is bimodal and the modes are 16 and 23.**
Find the range.	**Range = highest − lowest = 28 − 9 = 19**
State the answers.	**Mean = 19.4, median = 20.5, modes = 16, 23 and range = 19.**

9780170194778

Example 3

Find the mean, median, mode and range for the data shown in the table below.

Score	27	28	29	30	31	32	33	34	35
Frequency	2	8	18	9	9	9	5	2	3

Solution

Set up a new table with the data arranged vertically down each column.

Put in score × frequency (xf) and cumulative frequency columns.

Complete the xf column.

Put the first frequency under the cumulative frequency heading.

Add each frequency to the previous cumulative total.

Work out the total frequency and the total of score × frequency.

Score x	Frequency f	Score × frequency xf	Cumulative frequency
27	2	54	2
28	8	224	10
29	18	522	28
30	9	270	37
31	9	279	46
32	9	288	55
33	5	165	60
34	2	68	62
35	3	105	65
Totals	65	1975	

28 + 9 = 37

Divide the totals to find the mean.

Mean $= \dfrac{1975}{65} = 30.3846\ldots$

You can use your calculator.

Enter as: 1975 ÷ 65 =.

`1975÷65    30.38461538`

Find the centre score.

$\dfrac{65}{2} = 32.5$, so the 33rd score is the median.

There are 28 scores up to 29, and the next 9 scores are all 30s.

Median = 33rd score = 30

Find the highest frequency.

Mode = 29

Find the range.

Range = highest − lowest = 35 − 27 = 8

Round and state the answers.

Mean ≈ 30.4, median = 30, mode = 29 and range = 8

Video tutorial
Analysing frequency tables
MAT08SPVT10013

Animated example
Mean, median, mode and range
MAT08SPAE00001

Technology worksheet
Excel: Organising large sets of data
MAT08SPCT00003

Worksheet
Frequency tables
MAT08SPWK00002

Worksheet
The mean and fx tables
MAT08SPWK00003

TLF learning object
Dice duels: one dice: assessment (L9778)
MAT08SPIN00001

Technology worksheet
Excel: Statistical measures calculator
MAT08SPCT00002

Technology
GeoGebra: Statistics
MAT08SPTC00001

Technology Using technology for data measures

There are many ways that computers and calculators can be used to work out statistics for large data sets. The spreadsheet on NelsonNet automatically calculates the mean, median, mode and range when you put a large set of data in. Try using the spreadsheet or any other method that your teacher suggests to work with a large set of data.

	A	B	C	D	E	F	G
1							
2	**Statistical measures calculator**						
3	Whole numbers only				Clear		Calculate
4							
5	Instructions						
6	Press the **Clear** button						
7	Type your data into the column under the heading **Data**,						
8	pressing return after each item.						
9	Press the **Calculate** button						
10	**Data**		**Mean**	**6.818182**			
11	5		**Median**	**6**			
12	6		**Mode(s)**	**6**			
13	8		**Range**	**9**			
14	9		Lowest	3			
15	6		Highest	12			
16	7						
17	4						
18	6						
19	3						
20	9						
21	12						

Worksheet
Analysing data
MAT08SPWK00006

Example 4

The scores of some Year 8 students in a spelling test are shown below. Should the mean, median or mode be used as the typical score? Explain your answer.
12, 19, 13, 16, 9, 16, 10, 17, 13, 16, 10, 19, 1, 15, 9, 16, 11, 13, 11, 20, 0, 20, 13, 16, 11, 16, 10

Solution

There are a couple of unusually low scores: 0 and 1. The others are all 9 or more.

The median should be used, because there are a few very low scores that may distort the mean.

9780170194778

Exercise 1.1 Statistical measures

Understanding

Extra questions
Exercise 1.1
MAT08SPEQ00001

See Examples 1, 2

1 Find the mean, median, mode and range for each of the following sets of data.

a 16, 20, 9, 20, 14, 21, 17, 19, 14, 23, 15, 20, 17, 19, 13, 19, 5, 20, 14, 21

b 24, 27, 22, 27, 24, 27, 23, 26, 24, 25, 25, 27, 24, 26, 22, 30, 25, 27, 22, 26, 24, 27, 25, 26

c 22, 13, 18, 15, 16, 15, 12, 15, 7, 22, 27, 17, 13, 21, 16, 24, 17, 14, 14, 15, 14, 17, 15, 20, 9, 18, 19, 17, 13

d 7, 15, 8, 17, 6, 19, 12, 13, 7, 13, 9, 15, 7, 18, 11

Fluency

See Example 3

2 Find the mean, median, mode and range for each of the following tables of data.

a

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	4	2	2	13	3	5	6	8	4	1	2

b

Score	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Frequency	2	1	2	5	3	8	5	2	2	4	5	2	1	2	2	0	1

c

Score	22	23	24	25	26	27	28	29	30	31	32
Frequency	1	4	4	3	4	6	8	7	8	4	2

d

Score	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	4	0	1	3	3	3	3	0	4	7	6	9	7	2	2	2	5	3

Problem solving

Worked solutions
Exercise 1.1
MAT08SPWS00001

3 The mean age of ten students is 13. What is the new mean when two students from Year 11, aged 16 and 17, join the group?

4 Two classes have mean ages of 14.5 and 16.5 respectively. The first class has 25 students and the second class has 22 students. What is the mean age of the combined group?

5 A group of 15 students have a mean weight of 56 kg with a median of 60 kg. Another group of 25 students have a mean weight of 65 kg with a median of 67 kg. Calculate the mean of the combined group and estimate the likely value of the median.

Worked solutions
Exercise 1.1
MAT08SPWS00001

6 A group of 16 students have a mean arm span of 160 cm, with a median of 162 cm and a mode of 157 cm. When one student leaves the group, it becomes bimodal. What is the new mean, and is there a new median?

Reasoning

See Example 4

7 For each set of data in question **1**, would you use the mean, median or mode as the typical score? Why?

8 For each table in question **2**, would you use the mean, median or mode as the typical score? Why?

9 When a new student joined a class, the median age increased from 13.5 to 14. All classes in that grade have between 24 to 28 students. How many students could now be in the class?

Worked solutions
Exercise 1.1
MAT08SPWS00001

10 A group of students has a mean height of 165 cm. When 2 more students join the group, the mode increases. Explain if it is possible for the mean to decrease. What if 3 more students joined the group and the mode increased? Is it possible for the mean to decrease in this case? Explain your reasons.

9780170194778

1.2 Organising data

When you collect data, it is important to think about how the information will be used. Otherwise you may find that using the data is more difficult than you expected. If you want to know whether big businesses pay bigger wages, then you need to collect data on the size of each business as well as the wages for each business. Finding the average wage of businesses in general would not help you work out the answer.

Important!

Statistical tables

Each measurement value is called a **score** or **datum** (data is the plural of datum). The **frequency** of a **score** is the number of times it occurs.

To make a **frequency table** (**frequency distribution** or **one-way table**) you arrange the scores in a column or row from lowest to highest. You then put the frequency of each score in the next column or row.

Data is often **grouped** into convenient intervals (**classes**) when constructing frequency tables, such as heights of 130–139 cm, 140–149 cm, etc. The **class width** is the distance from the start of one group to the start of the next.

Example 5

A survey was conducted to find whether people thought girls and boys should be in the same class. Draw up a frequency table from the results below.

Yes, Don't care, Yes, No, Yes, Yes, Don't care, No, Yes, Yes, Yes, Yes, No, Don't care, Yes, No, Yes, Yes, No, No, No, Yes, Yes, Don't care, Yes, Yes

Solution

A sensible order for the scores is Yes, Don't care, No.

Put the scores in the table, tally the marks and complete the frequency column.

Score	Tally	Frequency
Yes	𝍸 𝍸 𝍸	15
Don't care	||||	4
No	𝍸 ||	7
	Total	26

When you group scores, you should try to get about 10 groups or less, but make sure you choose sensible class widths. Widths of 3, 6, 7, 9, 11 or 13 and their multiples make it more difficult to draw graphs.

Example 6

The ages of the parents of some Year 8 students are shown below. Make a grouped frequency table of their ages.

41, 42, 41, 46, 47, 51, 41, 43, 46, 50, 44, 47, 40, 41, 44, 48, 43, 44, 47, 50, 60, 61, 37, 39, 39, 39, 39, 43, 47, 40, 47, 52, 51, 38, 40, 39, 35, 35, 36, 42, 41, 47, 48, 38, 39, 48, 51, 42, 46, 43, 46, 41, 45, 43, 48, 41, 43, 50, 42, 43, 50, 52

Solution

The ages go from 35 to 61, so class widths of 4 years would give 7 groups starting with 35–38.

Use tally marks to make sure you count accurately.

Age	Tally	Frequency
35–38	卌 I	6
39–42	卌 卌 卌 卌	20
43–46	卌 卌 卌	15
47–50	卌 卌 IIII	14
51–54	卌	5
55–59		0
60–63	II	2
	Total	62

Small data sets are best organised by hand, as shown in Examples **1–3**. However, larger sets of data would take too long to organise this way. The 1880 United States Census took eight years to count by hand. The first modern counting machine that was invented to count the 1890 census used punched cards. It was completed in one year.

Large surveys and censuses are now counted by computer using various methods to input the data. You can use a spreadsheet to organise information so that it can be sorted and counted in different ways.

9780170194778

Technology Organising large data sets

Technology worksheet
Excel: Organising census data
MAT08SPCT00004

Weblink
Census At School
MAT08SPWB00001

CensusAtSchool is a nationwide annual project of the Australian Bureau of Statistics that collects real data that is relevant to students. It provides a snapshot of the characteristics, attitudes and opinions of those students who have completed questionnaires. Students from your school can take part under the direction of teachers.

You can download information about the project and samples of the data from the CensusAtSchool website.

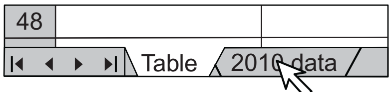
The 2010 Questionnaire and a 'clean sample' of the data are also available on the NelsonNet website. (Clean samples have had outliers or obvious errors removed.)

- Download the data sample from the website.
- The frequency table has been compiled for eye colour.

	A	B	C	D	E
1					
2	CensusAtSchool sample				
3	Column number	11		Count data	
4					
5	Score	Frequency			
6	Brown	73			
7	Blue	59			
8	Hazel	32			
9	Other	9			
10	Green	25			
11	Grey	2			
12		200			

Move the cursor down and click on the tab '2010 data' at the bottom of the screen.

Find the red column number of the column 'Q20.TrvToScl'.



Move the cursor down and click on the tab 'Table' at the bottom of the screen.

Type the red column number of the column 'Q20.TrvToScl' into the box next to the label 'Column heading'.

Go back to '2010 data' and look down through the column with the heading 'Q20.TrvToScl'.

Write down the different values of the data in the column.

Type these values into the boxes of the frequency table.

Press the button labelled 'Count data' to automatically complete the frequency table.

Try a different column.

Your teacher might want you to copy one of the frequency tables into your book or ask/show you how to save it into your school computer account.

The CensusAtSchool site can be used to find a great deal of information about Australian students. Your teacher might want you to find some particular information about students from your state or territory.

9780170194778

When you organise collected data into tables, it is important to critically examine the data. There will often be mistakes in recording data, or even results that are obviously wrong. If someone said they read 150 books every week, you would know they were exaggerating so this person's answer should be left out.

Important!

Non-compliant data

Data that is obviously wrong is said to be **non-compliant**. You should decide whether or not to use this data.

Example 7

The following are the amounts of pocket money some Year 8 students said they received every week.
The amounts are in dollars. Make a grouped frequency table of the results.
1, 20, 45, 40, 0, 10, 30, 70, 10, 20, 0, 3, 50, 400, 3, 3, 0, 40, 20, 0, 0, 0, 10, 20, 5, 20, 25, 15, 70, 10, 15

Solution

Even if pocket money includes bus fares and lunch money, $400 looks suspicious.

$400 is non-compliant so it should not be included.

The amounts go up to $70, so group in $10 increments.

Make the first group 0–9, then 10–19, . . .

Complete the table.

Pocket money ($)	Tally	Frequency
0–9	𝍸 𝍸 |	11
10–19	𝍸 |	6
20–29	𝍸 |	6
30–39	|	1
40–49	|||	3
50–59	|	1
60–69	||	0
70–79	||	2
	Total	30

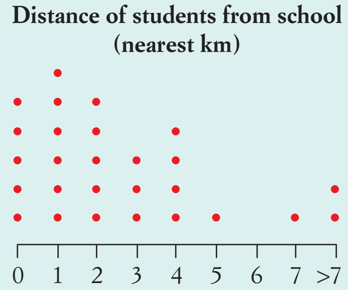
Instead of tables, you can organise data as graphs. This takes advantage of our strongest sense, sight. It can mean that the data loses accuracy, but we get a better idea of its distribution.

9780170194778

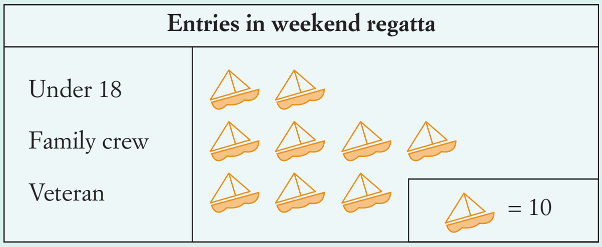
Important!

Statistical graphs

A **dot plot** is made by showing a dot for each data item above a scale. They are most useful for small sets of numeric data, particularly discrete data.

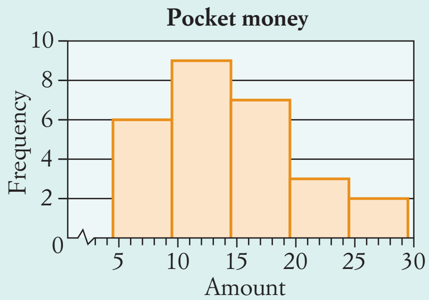


A **picture graph** (**pictogram** or **pictograph**) shows amounts with different-sized pictures or different numbers of pictures.



A chart that uses bars to show the amounts is called a **bar chart** or **bar graph**. It can be drawn so that the bars are **vertical** or **horizontal**. Bar graphs are most often used to show the relative sizes of related measurements.

A **histogram** is a column graph that shows frequencies of continuous data. It has no spaces between the columns except the columns with frequencies of zero. The edges of the columns are shown at the true class limits.

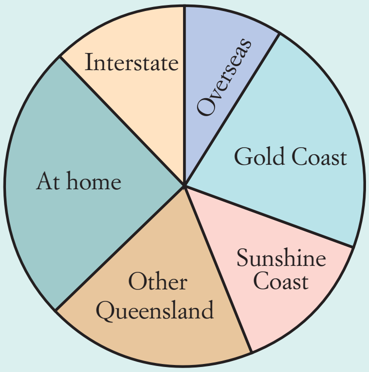


Weblink
200 years that changed the world
MAT08SPWB00001

9780170194778

A **circle graph** (**pie graph** or **sector graph**) is a circle that has been divided into sectors to show the parts of a total amount. The sector angles are proportional to the amounts.

Preferred holiday

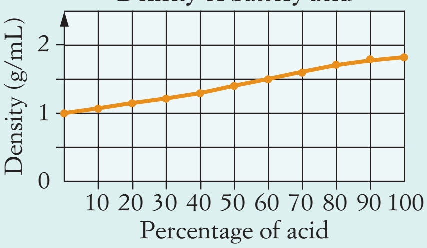


A **divided bar chart** is a long bar that has been divided into sections to show the parts of a total amount. The lengths of the sections are proportional to the amounts.

House of representatives, 1901: 75 Members

Line graphs have points connected by lines. They are most commonly used for time-series, or temperature and concentration graphs.

Density of battery acid



Words per paragraph

0	8
1	3 7 9
2	0 0 2 4 5 6
3	0 1 4 7
4	2 3 8

A **stem-and-leaf plot** is drawn to display data with a wide range. The first one or two digits are written vertically to make the **stem**. The remaining digits are written in a row across from the first digit(s) to make the **leaves**.

All charts and graphs should have a **title** saying what they are about.

An **axis** (plural **axes**) of a graph or chart is a line that shows the measurements.

The **scale** of a graph or chart (usually marked on the axes) shows the size of the measurements.

One variable is often measured as a first variable changes. The first variable is placed on the horizontal axis. The second variable is placed on the vertical axis. The first variable is often time, and something else is measured at different times.

9780170194778

Example 8




The table shows the eye colours of 200 students. Draw a circle graph to show the information.

Colour	Frequency
Brown	73
Blue	59
Hazel	32
Green	25
Grey	2
Other	9
Total	200

Solution

Redraw the table with an extra column for the angle.

Use your calculator to find the scale.

Scale $= 360° \div 200 = 1.8°$/student

Store the exact answer in the memory and multiply by each number to find the angles. For example, brown angle $= 73 \times 1.8 \approx 131°$

Sum to check that the total is 360°.

Draw a circle and mark its centre. Draw a line from the centre to the circumference.

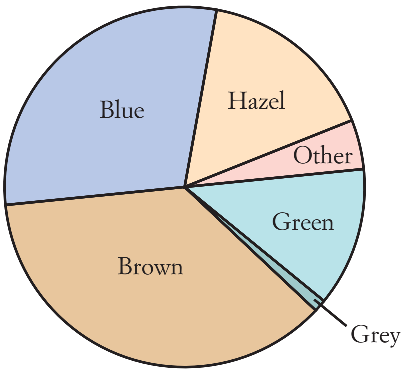
Mark each angle with your protractor, measuring the next angle from the finish of the last one.

Label and colour or shade each sector.

Write the total amount in the title.

Colour	Frequency	Angle
Brown	73	131°
Blue	59	106°
Hazel	32	58°
Green	25	45°
Grey	2	4°
Other	9	16°
Total	200	360°

200 Students' eye colours



Sometimes when you add up the angles calculated for a circle graph you will not get exactly 360° because of rounding errors.

Worksheet
Statistical match-up
MAT08SPWK00004

TLF learning object
Bar chart (L3512)
MAT08SPIN00001

Example 9

Draw a histogram to show the heights of students in a Year 8 class in widths of 5 cm.
160, 169, 156, 158, 164, 164, 170, 165, 166, 163, 165, 149, 170, 158, 1681, 63, 162, 166, 148, 162, 161, 170, 160, 150, 168, 160, 181, 163, 152, 163, 151

Solution

Heights of 1681 cm and 63 cm are impossible for Year 8 students.

Leave out 1681 cm and 63 cm.

Draw the table.

The lowest is 148 cm and the highest is 181 cm so use groups from 145–149, 150–154, . . .

Complete the table and add up the frequencies to check for errors.

Height (cm)	Tally	Frequency
145–149	ǁ	2
150–154	ǀǀǀ	3
155–159	ǀǀǀ	3
160–164	~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ǀ	11
165–169	~~ǀǀǀǀ~~ ǀ	6
170–174	ǀǀǀ	3
175–179		0
180–184	ǀ	1
	Total	**29**

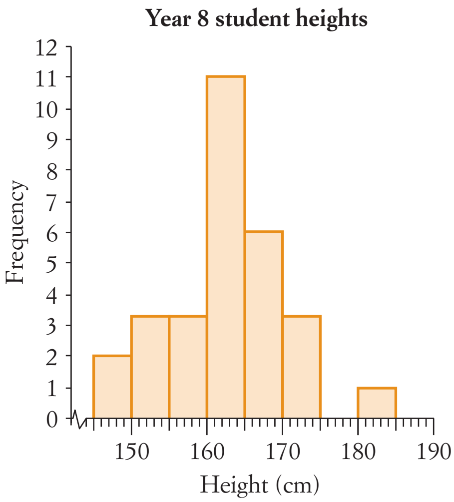
The highest frequency is 11, so go up in 1s.

Draw the horizontal scale from 140–190.

The heights are rounded to the nearest cm, so 145–149 really goes from 144.5–149.5. The column edges are actually halfway between 144 and 145, 149 and 150, and so on.

Draw a break in the horizontal scale to show that it doesn't start from 0.

Draw the histogram and shade or colour the columns.

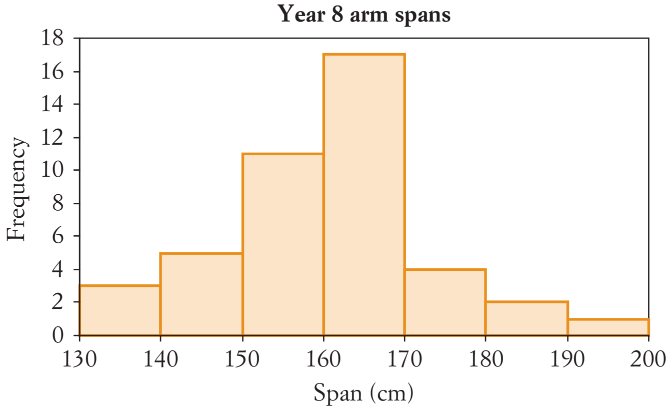


9780170194778

Investigate: Year 8 body size relationships




Are there any relationships between the sizes of different parts of your body and height?
Is there any difference between boys and girls?
In groups, decide which parts of your bodies you are happy to measure in class.
Make measurements and pool the results.



Can you make any conclusions about the students in your class?
Use the data from the CensusAtSchool site or the sample on the NelsonNet website for this book to examine the relationship for other groups of students.
Your teacher may ask you to do a report on your research.

Exercise 1.2 Organising data

Understanding

Extra questions
Exercise 1.2
MAT08SPEQ00002

See Example 5

1 Make frequency tables for the following sets of data.

a 14, 16, 16, 22, 14, 17, 12, 14, 9, 16, 11, 15, 13, 19, 18, 20, 19, 16, 18, 13, 15, 19, 14, 16, 10, 12

b 5.9, 6, 5.5, 5.6, 5.6, 6, 6.1, 5.7, 6.3, 5.8, 5.4, 5.9, 5.9, 6.1, 6.2, 6.4, 6.4, 6.2, 6.6, 5.9, 6.7, 6.1, 6.4, 5.3, 5.9, 5.2, 5.4, 5.7, 6.3, 6.2

c 125, 127, 126, 130, 130, 128, 127, 129, 129, 129, 130, 130, 129, 130, 125, 126, 132, 128, 130, 130, 124, 127, 130, 125, 129, 127, 130, 125, 127, 131, 126, 130, 129, 134, 127, 128, 130, 124, 128

Fluency

2 Make a column graph for the following numbers of DVDs that some Year 8 students said they watched in the last week.

DVDs	0	1	2	3	4	5
Frequency	2	4	8	7	4	2

3 Make a histogram for the amount of money (to the nearest dollar) that some Year 8 students had with them after lunch.

Amount	\$0	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8+
Frequency	5	6	3	2	3	2	1	2	4

See Example 6

4 Make grouped frequency tables for the following sets of data.

a 66, 63, 73, 60, 72, 81, 62, 72, 66, 74, 65, 87, 73, 68, 61, 73, 78, 63, 76, 80, 69, 68, 78, 69, 58, 77, 72, 64, 75, 73, 68, 60, 76, 84, 74 (Groups of width 4)

b 12, 19, 24, 13, 31, 28, 0, 12, 12, 13, 28, 6, 17, 17, 13, 18, 23, 8, 36, 26, 12, 19, 17, 16, 30, 18, 19, 4, 14, 22, 16, 8, 14 (Groups of width 5)

c 18.3, 27.8, 28.9, 26.1, 29.6, 29.3, 24.6, 39.1, 22.8, 24.1, 25.2, 12.6, 36.5, 19.1, 21.5, 21, 29.3, 25.5, 23.8, 17.5, 30.4, 23.7, 12.3, 24.2, 29.3, 26.5, 33.7, 26.6, 31.3, 21.9, 17.9, 27.5, 39.2, 19.8 (Groups of width 4)

See Example 8

5 Draw circle graphs for the following tables.

a

Category	A	B	C	D	E
Number	28	15	13	44	20

b

Category	A	B	C	D	E	F
Number	7	25	46	30	12	30

c

Category	A	B	C	D	E	F	G
Number	10	24	29	23	18	4	10

9780170194778

6 Draw histograms for the following frequency tables. *See* Example 9

a

Score	5–9	10–14	15–19	20–24	25–29	30–34
Number	3	7	9	6	4	1

b

Score	10–11	12–13	14–15	16–17	18–19	20–21	22–23	24–25	26–27
Number	3	4	7	8	13	12	6	3	2

c

Score	80–89	90–99	100–109	110–119	120–129	130–139
Number	7	12	22	8	6	1

7 Some Year 8 students picked the following as their favourite type of music. *See* Examples 5, 8
Pop, Pop, Pop, Heavy metal, Rap/Hip hop, Rap/Hip hop, Pop, Rap/Hip hop, Techno, Pop, Rap/Hip hop, Rap/Hip hop, Pop, Rap/Hip hop, Pop, Rock, Rap/Hip hop, Pop, Rock, Rap/Hip hop, Rock, Rock, Techno, Pop, Rock, Heavy metal, Techno, Pop, Classical, Rhythm and Blues, Pop, Jazz, Rap/Hip hop, Techno, Pop, Rap/Hip hop, Rap/Hip hop, Rock, Rock

a Make a frequency table of the results.

b Draw a circle graph of the information.

8 A group of Year 8 students measured the lengths of their feet. *See* Example 7
The results (in cm) are shown below.
28, 29, 22, 23, 20, 24, 27, 25, 25, 24, 29, 23, 25, 31, 25, 31,
24, 23, 26, 26, 27, 22, 25, 23, 28, 21, 26, 27, 33, 26, 18, 20

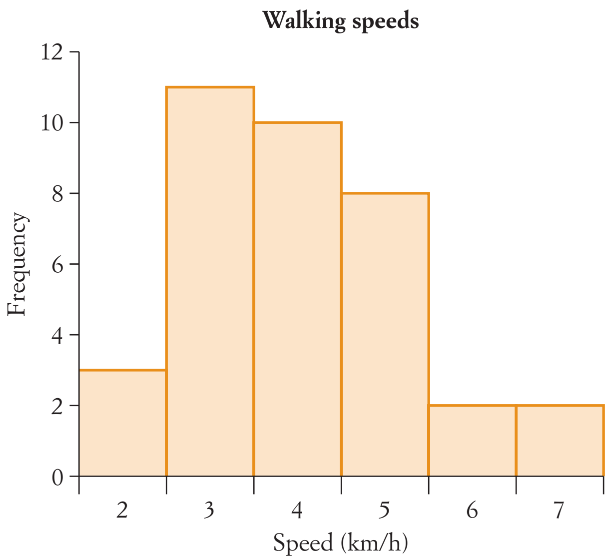
a Draw up a frequency table with class widths of 2 cm.

b Draw a histogram of the data.

Problem solving

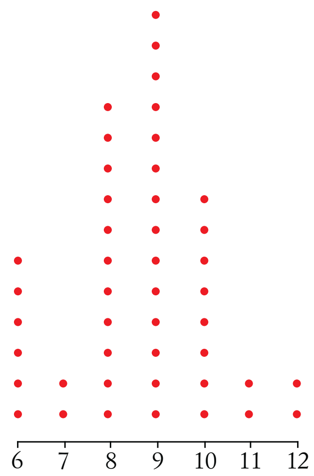
9 The histogram below shows the normal walking speeds of a group of people. Find the mean, median, mode and range of walking speeds from the graph.

Worked solutions
Exercise 1.2
MAT08SPWS00002



9780170194778

10 The dot plot on the right shows the normal hours of sleep reported by a group of students. Find the mean, median, mode and range of hours of sleep from the graph.



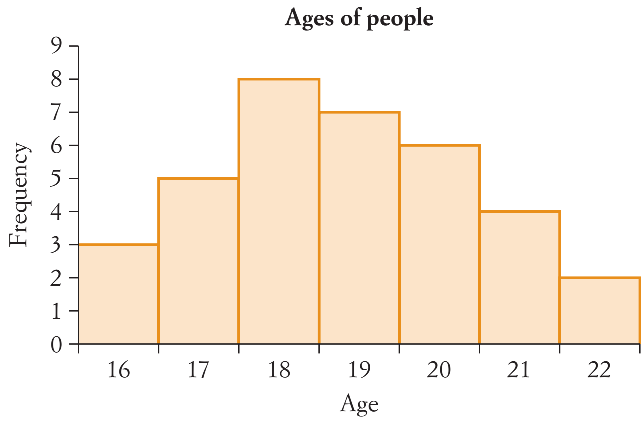
Reasoning

Worked solutions
Exercise 1.2
MAT08SPWS00002

11 The following column graph shows the ages of a group of people.

a How will the graph change during the next year?

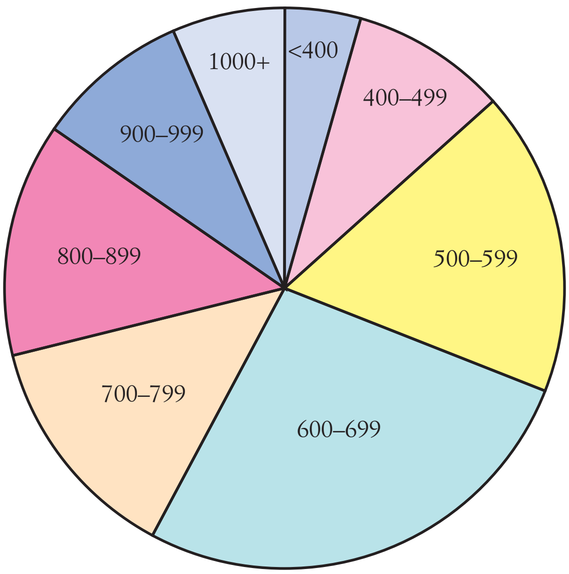
b How will the graph appear in exactly 10 years time?



Worked solutions
Exercise 1.2
MAT08SPWS00002

12 The following graph shows the weekly incomes of a group of people.

Weekly incomes of 45 people($)



a How would the graph change if everybody got an increase of $30/week?

b How would the graph change if everybody got an increase of 30%?

9780170194778

1.3 Comparing samples and populations

The results of many surveys are reported in the media. Some of these surveys may not be correct because they were not conducted fairly. The people who were surveyed may not be typical, or the questions could be unfair.

Important!

Bias and samples

When you collect information, the **population** is the complete group that the information would relate to. A **census** obtains data from the whole population.
A **sample** is part of the population. A **survey** obtains data from a sample.
A **fair sample** is like the whole population. A **biased sample** is not typical of the population.
You can also get **bias** by using poor methods or questions.
Every member of the population has an equal chance of being selected in a **random sample**.

Example 10

A teacher wanted to find out about students' access to computers at home. For each of the following, state whether the results are likely to be fair or biased.

a Students in a single Year 8 class are asked 'How many computers do you have at home?'; then asked to raise their hands for 1, 2, 3, and so on.

b Students from different classes in different places are asked to write down how many computers there are at home on pieces of paper. These were collected and counted.

c Students from different classes in different places are asked to raise their hands in answer to the questions: 'Who has at least 3 computers at home?'; 'Who has 2 computers at home?'; 'Who has only one computer at home?'; 'Does anybody not have a computer at home?'.

d Students in Year 8 on a small island are asked to write down how many computers they have at home.

Solution

a The group asked is not typical of all students. The way the question is asked is likely to make students exaggerate. **The sample and the question are both biased, so the results are likely to be biased.**

b The sample seems fair and the way the question is asked seems fair. **The results are likely to be fair.**

c The sample seems fair, but the question is likely to make students exaggerate. **The sample is fair but the question is biased, so the results are likely to be biased.**

d The group asked is not typical, but the way the question is asked seems fair. **The sample is biased but the question is fair, so the results are likely to be biased.**

You will notice that the samples that seem fair in Example **10** are those that appear to be random samples.

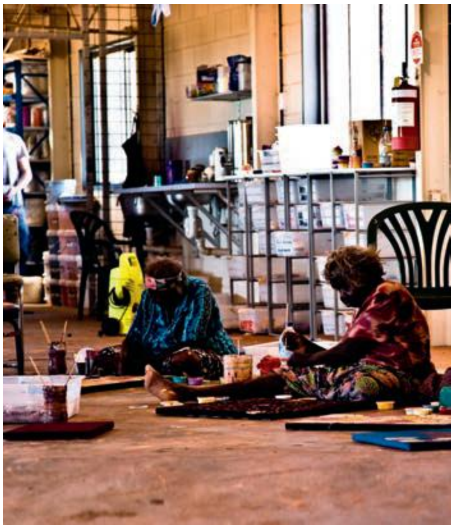
The questions you ask in a survey can be influenced by your own beliefs. This can also have an effect on the sample that you choose to survey. You may not even realise that there is a need to ask questions about some topics.

Investigate: Cultural and social bias

In this investigation, you will work in small groups to consider questions from different points of view. Each group needs to work out the kinds of answers that would be given by people from different groups about different topics.

The groups are as follows:

- People in a remote aboriginal community.
- Beef cattle farmers, including feedlots.
- Producers of free-range chicken eggs.
- People from the inner city who vote 'Green'.
- The families of Japanese whalers.
- People living in an Indian village.
- People living in a large Indian city like Mumbai.
- People working in the car industry.



There may be other groups that your teacher wants you to consider as well.

Consider the following questions:

- Is climate change caused by human activity?
- Is whaling an acceptable way to get meat?
- Should feral animals all be shot?
- Should native animals be culled if they become a problem to farmers?
- Should we build more large dams to supply water for drinking and irrigation?
- Should large companies be allowed to buy out small farmers to make larger properties?
- Should businesses in developed countries be allowed to manufacture overseas?

Your teacher may also want you to consider other questions.

Discuss the questions in your groups and then have a spokesperson communicate the results to the class.

Discuss whether different groups would use the results of surveys or scientific information in different ways.

From this activity, you should see that the results of surveys are greatly influenced by the sample that is chosen. If you want to find the opinions of a particular group of people, you should choose a random sample from that group.

9780170194778

There are many different ways of choosing fair samples. The methods chosen will depend on the size of the population, the size of the required sample, and the cost of collecting the information. When students are asked to do surveys, they often ask people they know. This might include other students in their classes. This is sometimes called **accessibility sampling**, which just means using people that are easily accessible. This method will not usually give a fair sample.
Even when fair samples are chosen, the results can be different for different samples. This is called **variation**.

Investigate: Household sizes

In this investigation, you will use samples of students from your class to investigate the average number of people in students' homes.
Give every person in the class a number, starting from the front left-hand desk and working across and back to the back right-hand desk.
You will work out the average (mean) number of people living in the home of students in the class.
Work out the average for students 1–3.
Work out the average for students 4–6.
Work out the average for students 7–9.
Continue this way until you have worked out the averages for all the students in groups of 3.
If there are extra students needed for the last group, go back to student 1 or students 1 and 2.
What is the range of the means worked out using these samples?
Now do the same thing again, but choose samples of 5, starting from 1–5, 6–10, etc.
What do you find this time about the means?
Do it again with samples of 10. What do you find?
Work out the mean for the whole class.
Record your answers to the following questions:

- Are the means the same for all samples?
- What happens to the means as the sample size is increased?
- Is this a good way to choose samples?
- Does everyone in the class have the same chance of being chosen to be in a sample?
- Can you think of a random way to choose samples from the class?

Teacher notes
Household sizes
MAT08SPTN00001

Puzzle sheet
Crowded numbers
MAT08SPPS00050

When you do a survey, it is important to identify the population for your questions. This will make it easier to choose a sample that is fair. For example, if you want to know the way people are likely to vote in an election, the population is the people who can vote. It is not much use surveying most of the people at school, because they are too young to vote. If you want to find out how people choose what soap to buy, your population is the people who buy soap.

Example 11

What is the population for each of the following? Should you use a census or would a sample be sufficient?

a Finding the clothes sizes for uniforms for a high school.
b Finding the most popular TV program in Perth.
c Working out how many trams should travel from St Kilda to the CBD in Melbourne between 7 a.m. and 9 a.m.

Solution

a Which people will wear the uniforms? — **The population is students at the high school.**

There may be a few very tall or very short students. — **You will need to do a census to make sure you cover all sizes needed.**

b Most popular means the program watched by the most people. — **The population is people in Perth who watch TV, which is nearly everybody.**

It is impractical to ask everybody. — **A sample should be used.**

c Most of the people catching a tram at that time would live in the suburb. — **The population is people who live in St Kilda who might catch a tram.**

A survey of, say, 5% of the suburb would work. — **A sample should be used.**

9780170194778

Sometimes there will be a data value that is very different from the rest. This is called an **outlier**. Of course, if it is actually a mistake, then it is not really an outlier, but non-compliant and should not be used. All outliers should be checked to see if they really are errors. They can make big changes to the values of the mean and range.

Example 12

a The boys in a class have the following heights:
167, 170, 165, 176, 166, 170, 192, 154, 170, 163, 173, 156, 174
Work out the mean and range of the boys' heights.

b Is there an outlier?

c Calculate the mean and range without the outlier.

d Comment on the change.

Solution

a Find the total.

$$\textbf{Total} = 167 + 170 + 165 + \ldots = 2196$$

Divide by the number of boys.

$$\textbf{Mean} = \frac{2196}{13} \approx 168.9 \text{ cm}$$

You can use your calculator.

Enter as: 2196 ÷ 13 =.

`2196÷13    168.9230769`

Find the range.

$$\textbf{Range} = \text{highest} - \text{lowest} = 192 - 154 = 38 \text{ cm}$$

b Write the values in order.

154, 156, 163, 165, 166, 167, 170, 170, 170, 173, 174, 176, 192

Are there any values a long way from the rest?

192 cm is an outlier.

c Find the new total.

$$\textbf{New total} = 2196 - 192 = 2004$$

$$\textbf{New mean} = \frac{2004}{12} = 167.0 \text{ cm}$$

Find the new range.

$$\textbf{Range} = \text{highest} - \text{lowest} = 176 - 154 = 22 \text{ cm}$$

d Make a comment.

Removing the outlier reduces the mean by 1.9 cm, but changes the range by much more, 16 cm.

You can use statistical measures, particularly the mean, median and range, to compare populations.

Example 13

Samples of the heights of girls and boys in Year 8 in Australian schools are given below. Use statistical measures to compare the heights of Year 8 boys and Year 8 girls.
Boys: 185, 162, 168, 162, 160, 186, 150, 1429, 157, 148, 171, 170, 178, 167, 164, 163, 153, 166, 171, 146, 167, 164, 154, 161, 163, 164
Girls: 166, 171, 170, 154, 162, 170, 170, 165, 23, 164, 170, 154, 154, 165, 171, 159, 152, 145, 170, 162, 160, 165, 163, 164, 155, 163

Solution

For the boys, ignore 1429 as it is obviously an error. Find the total for the boys.	**1429 is non-compliant. Do not use this height.** $\textbf{Total (boys)} = 185 + 162 + 168 + \ldots = 4100$
Find the mean for the boys.	$\textbf{Mean (boys)} = \frac{4100}{25} = 164 \text{ cm}$
Arrange the boys' heights in order.	**146, 148, 150, 153, 154, 157, 160, 161, 162, 162, 163, 163, 164, 164, 164, 166, 167, 167, 168, 170, 171, 171, 178, 185, 186**
Find the median (13th score).	**Median (boys) = 164 cm**
For the girls, ignore 23 as it is obviously an error. Find the total for the girls.	**23 is non-compliant. Do not use this datum.** $\textbf{Total (girls)} = 166 + 171 + 170 + \ldots = 4064$
Find the mean for the girls.	$\textbf{Mean (girls)} = \frac{4064}{25} = 162.56 \text{ cm}$
Find the range for the boys.	**Range (boys) = 186 − 146 = 40 cm**
Find the range for the girls.	**Range (girls) = 171 − 145 = 27 cm**
Arrange the girls' heights in order.	**145, 152, 154, 154, 154, 155, 159, 160, 162, 162, 163, 163, 164, 164, 165, 165, 165, 166, 170, 170, 170, 170, 170, 171, 171**
Find the median for the girls (13th score).	**Median (girls) = 164 cm**
Compare the results.	**The mean boys height (164 cm) is a little more than the mean girls height (162.6 cm), but the medians are the same, so there are as many boys as girls both above and below 164 cm. The mean boys' height is higher than the girls' because of a few very tall boys who make the boys' range much larger. The small sample sizes make the comparison a little unreliable.**

9780170194778

Sometimes it is better to compare samples using a **back-to-back** display (graph).

Important!

A **back-to-back plot** is used to compare two lots of data. One group is shown to the left of a vertical axis, and the other group is on the right. Sometimes these displays are shown above and below a horizontal axis. Histograms may be shown with side-by-side columns for comparisons.

Example 14

Video tutorial
Back-to-back stem-and-leaf plots
MAT08SPVT10014

Worksheet
Stem-and-leaf plots
MAT08SPWK00005

Show the heights from Example **13** as a back-to-back stem-and-leaf plot and comment.
Boys: 185, 162, 168, 162, 160, 186, 150, 157, 148, 171, 170, 178, 167, 164, 163, 153, 166, 171, 146, 167, 164, 154, 161, 163, 164
Girls: 166, 171, 170, 154, 162, 170, 170, 165, 164, 170, 154, 154, 165, 171, 159, 152, 145, 170, 162, 160, 165, 163, 164, 155, 163

Solution

Put the stems in the centre, from 140 to 180, leaving out the zero in each case.

Put the 'leaves' in the correct places.

Heights of Year 8 boys and girls

Female		**Male**
	14	
	15	
	16	2 8 2 0
	17	
	18	5

Complete the leaves and arrange in the correct order.

Show a key.

Heights of Year 8 boys and girls

Female (Key: 4|16 = 164)		**Male (Key: 16|4 = 164)**
5	14	6 8
9 5 4 4 4 2	15	0 3 4 7
6 5 5 5 4 4 3 3 2 2 0	16	0 1 2 2 3 3 4 4 4 6 7 7 8
1 1 0 0 0 0 0	17	0 1 1 8
	18	5 6

Comment on the patterns.

The heights of the boys are more spread out, but the centres of both the boys' and the girls' heights are about the same. There are a few very tall boys.

9780170194778

Exercise 1.3 Comparing samples and populations

Understanding

Extra questions
Exercise 1.3
MAT08SPEQ00003

See Example 11

1 Write the population for each of the following questions.
  - a The most likely person to be elected in the electorate of Brisbane in the next Federal election.
  - b The most popular 'drivetime' radio show in Townsville.
  - c The days and hours that a local library should open.
  - d The opening hours of a timber yard.
  - e The sizes of tins of dog food.

See Example 12

2 Determine if there are any possible outliers in the following sets of data.
  - a 13, 15, 13, 16, 12, 14, 12, 14, 12, 15, 10, 18, 14, 14, 14, 15, 12, 17, 13, 16, 10, 14, 11, 22, 9
  - b 16, 20, 15, 18, 15, 19, 14, 18, 15, 23, 16, 19, 16, 21, 10, 18, 15, 23, 14, 19
  - c 13, 18, 13, 17, 13, 15, 11, 17, 12, 14, 13, 15, 14, 15, 14, 16, 9, 17, 12, 14, 13, 14, 12, 20, 14
  - d 19, 25, 25, 21, 21, 21, 14, 24, 22, 25, 19, 24, 23, 16, 12, 20, 19, 15, 16, 20, 22, 22, 24, 20, 19
  - e 36, 46, 32, 48, 29, 57, 32, 43, 37, 47, 37, 42, 34, 42, 40, 40, 40, 51, 37, 42, 38, 41, 33, 59, 35

Fluency

3 The following shows the hours that some Year 8 students said that they spent watching TV over a weekend.
5, 1, 12, 25, 31, 0, 6, 5, 8, 11, 12, 15, 11, 14, 5, 2, 0, 40, 15, 20, 0, 13, 6, 7, 4, 0, 1
  - a Find the mean and median times.
  - b Remove any outliers and recalculate the mean and median.
  - c Comment on the results.

4 The following shows the price of bananas in \$/kg at different fruit shops and supermarkets.
2.15, 2.99, 2.79, 2.39, 2.75, 2.89, 2.95, 2.49, 3.39, 1.29, 2.89, 2.29, 2.79, 2.89, 2.70, 2.90, 1.98, 3.99, 3.79, 2.29, 2.35, 2.40, 3.19, 2.35, 2.95
  - a Find the mean and median prices.
  - b Remove any outliers and recalculate the mean and median.
  - c Comment on the results.

9780170194778

5 Should a sample or a census be used for each of the following? *See* Example 11

a Determining the widths of airline seats.

b Finding the variety of apple most resistant to a pest.

c Finding the tyres available in Australia with the best wet-weather performance.

d Finding the sizes that fashion shoes should be made in.

6 What is the population for each of the following questions?

a Working out the size of car parking spaces at a shopping centre.

b Working out the weight that a small road bridge should be able to support.

c The amount that people are willing to pay for a catered child's birthday party.

Problem solving

7 Use statistical measures to compare the following hourly rates (to the nearest whole dollar) of some plumbers, vets and hairdressers. Comment on the results.

Plumbers: 28, 32, 27, 19, 35, 22, 24, 27, 39, 31, 35, 28, 25, 25, 36

Vets: 25, 22, 24, 28, 24, 24, 26, 32, 23, 20, 24, 19, 22, 25, 20

Hairdressers: 12, 18, 19, 23, 16, 15, 14, 27, 12, 22, 28, 30, 25, 33, 22

Worked solutions
Exercise 1.3
MAT08SPWS00003

See Example 13

8 Use statistical measures to compare the following times taken by Year 8 boys and girls to complete a survey. The times are in minutes. What conclusions can you reach? *See* Example 14

Girls: 6.5, 3, 5, 3, 6.5, 9, 8.5, 3.5, 9, 10, 5, 4, 11, 3.5, 9.5, 7, 9.5, 13.5, 8.5, 13.5, 7.5, 13.5, 6.5, 5.5, 3.5, 9, 4.5, 4.5, 7, 11, 7.5

Boys: 4.5, 9.5, 3, 9.5, 5.5, 3.5, 7, 4.5, 4, 3.5, 6.5, 9.5, 6, 13.5, 3, 4, 4.5, 13.5, 2.5, 5.5, 9.5, 2.5, 9, 5.5, 7

9 Make a back-to-back stem-and-leaf plot of the following reaction times in seconds of Year 8 students to compare reaction times of each hand. Can you make any conclusions?

Left hand: 0.38, 0.42, 0.26, 0.39, 0.35, 0.37, 3.4, 0.43, 0.31, 0.32, 0.35, 0.31, 0.34, 0.29, 0.29, 0.48, 0.42, 0.43, 0.36, 0.48, 0.34, 0.45, 0.37, 0.34, 0.26, 0.45, 0.57, 0.32

Right hand: 0.36, 0.44, 0.23, 0.54, 0.35, 0.29, 0.4, 0.32, 0.26, 0.32, 0.29, 0.29, 0.32, 0.32, 0.39, 0.54, 0.66, 0.36, 0.37, 0.4, 0.27, 0.31, 0.26, 0.29, 0.02, 0.31, 0.35, 0.28

Worked solutions
Exercise 1.3
MAT08SPWS00003

10 Make a back-to-back stem-and-leaf plot of the times taken by some Year 8 students in different states to get to school. What did you find?
Qld: 9, 15, 5, 25, 1, 5, 45, 5, 30, 45, 20, 5, 30, 10, 15, 15, 25, 45, 4, 7, 15, 25, 6, 3, 10, 5, 25, 5, 10, 30, 20, 5, 1, 10, 15
WA: 40, 50, 15, 10, 40, 20, 25, 10, 45, 10, 20, 20, 40, 20, 5, 15, 15, 2, 15, 40, 5, 5, 20, 20, 15, 10, 30, 35, 10, 5, 1, 15, 3, 3, 5

Reasoning

See Example 10

11 A TV current affairs program about houses that needed repairs to foundations within a year of being built asked audience members if they thought the government should have more inspections of building work. Numbers were given to ring for 'Yes' and 'No' votes. Is this a fair way to find public opinion about building regulations? Explain your answer.

12 A polling organisation asked people it rang in state capitals and regional cities 'If an election were held tomorrow, which party would you vote for?'. 600 people in each capital and 400 people in the regional centres of each state were polled. Is this a fair way to gauge the likely result in each state? What about across the country? Explain your answer.

13 What is wrong with asking people in the mall on a Saturday morning about a current issue as a way of gauging the public mood on the issue?

14 A high school with a student representative council has a representative from each home class. The representatives are chosen by class members at the start of each year, but if they do not attend regularly the class teacher has to choose someone else. Will the student council under this system represent student opinion fairly?

Worked solutions
Exercise 1.3
MAT08SPWS00003

15 Three surveys were done to find people's opinions about a proposed new bridge.
Survey 1: 50 people going into a supermarket on Monday between 10 a.m. and 11 a.m. were asked.
Survey 2: 90 people in the mall on Saturday morning between 2 p.m. and 3 p.m. were asked.
Survey 3: 70 people were rung at home between 8 p.m. and 9 p.m. on a Sunday night.
Put the surveys in the order that would give the best results. Give reasons for your answers.

9780170194778

Chapter 1 summary

Quiz
Statistics
MAT08SPQZ00001

- The **mean** ($\bar{x}$) is the **average** score. It is the total of the scores divided by the number of scores.
- The **median** is the middle score. It is found by arranging all scores in order then choosing the middle score.
- **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.
- The **mode** is the score with the highest frequency. A **bimodal** distribution has two scores with equal highest frequencies.
- The mean, median and mode are called **measures of central tendency**.
- The mean is most often used as the typical score. However, if there are a few high or low scores that would distort the mean, you should use the median.
- The mode should be used in cases where the most common score is needed.
- The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.
- In statistics, each measurement value counted is called a **score** or **datum** (plural **data**). The **frequency** of a **score** is the number of times it occurs.
- A **frequency table** (**frequency distribution** or **one-way table**) has scores in a column or row from lowest to highest. The frequency of each score is tallied in the next column or row.
- Data is often **grouped** into convenient intervals (**classes**) in order to construct frequency tables, such as heights of 130–139 cm, 140–149 cm, etc. The **class width** is the distance from the start of one group to the start of the next.
- Data that is obviously wrong is called **non-compliant** and is usually not included.
- A **dot plot** is made by showing a dot for each data item above a scale. They are most useful for small sets of numeric data, particularly discrete data.
- A **picture graph** (**pictogram** or **pictograph**) shows amounts with different-sized pictures or different numbers of pictures.
- A chart with bars to show the amounts is called a **bar chart** or **bar graph**. It can be drawn so that the bars are **vertical** or **horizontal**. Bar graphs are most often used to show the relative sizes of related measurements.
- A **histogram** is a column graph that shows frequencies of continuous data. It has no spaces between the columns except the columns for frequencies of zero. The edges of the columns are shown at the true class limits.
- A **circle graph** (**pie graph** or **sector graph**) is a circle that has been divided into sectors to show the parts of a total amount. The sector angles are proportional to the amounts.
- A **divided bar chart** is a long bar that has been divided into sections to show the parts of a total amount. The lengths of the sections show the amounts.
- **Line graphs** have points connected by lines. They are most commonly used for graphs involving time, or two continuous measurements.
- A **stem-and-leaf plot** is drawn to show data with a wide range quickly and easily. The first one or two digits are written vertically to make the **stem**. The remaining digits are written in a row across from the first digit(s) to make the **leaves**.
- All charts and graphs should have a **title** saying what they are about. An **axis** (plural **axes**) of a graph or chart is a line that shows the measurements. The **scale** of a graph or chart (usually marked on the axes) shows the size of the measurements.

Chapter 1 summary

- The **population** is the complete group that statistical information would relate to. A census obtains data from the whole population.
- A **sample** is part of the population. A **survey** obtains data from a sample.
- A **fair sample** is like the whole population. A **biased sample** is not typical of the population.
- You can also get bias by using poor methods or questions.
- **Variation** refers to the differences in statistical measures resulting from the use of samples.
- A **back-to-back** display can be used to compare two lots of data. One group is shown to the left of a vertical axis, and the other group is on the right. Sometimes these displays are shown above and below a horizontal axis.
- Histograms may be shown with **side-by-side** columns for comparisons.
- An **outlier** is a score that has a very different value to the other values in a set of data.

9780170194778

Understanding

1 Find the mean, median, mode and range of the following set of data. 9, 22, 9, 14, 9, 11, 8, 16, 10, 15, 5, 18, 7, 15, 10, 16, 6, 15, 7, 12, 9, 17, 10, 12, 6, 20, 10, 17, 9, 18, 9, 10 *See* Examples 1, 2

2 Make a frequency table for each of the following sets of data. *See* Example 5
a 5, 15, 7, 14, 12, 14, 10, 13, 6, 13, 9, 13, 9, 14, 10, 13, 5, 12, 11, 13
b Yellow, Red, Red, Green, Blue, Yellow, Blue, White, Green, Black, Yellow, Yellow, White, Black, Red, Blue, Yellow, Green, Black, Yellow, Blue, Yellow, Yellow, Blue, Green

3 What is the population for each of the following questions? *See* Example 11
a The mean number of books from the school library that students read each month.
b The maximum height that Year 8 students can jump.
c The range of prices of ice-creams in Fremantle.

4 Determine if there are any possible outliers in the following sets of data. *See* Example 12
a 23, 24, 21, 23, 18, 25, 24, 25, 21, 21, 25, 20, 22, 30, 22, 20,
b 113, 99, 93, 104, 91, 99, 106, 105, 107, 103, 103, 81, 102, 107, 105, 93, 99, 104, 94, 108, 97, 93
c 8, 12, 5, 11, 5, 12, 2, 15, 7, 11, 2, 11, 8, 13, 5, 12, 4, 10, 4, 16, 8, 14, 9, 12, 9, 11

Fluency

5 Find the mean, median, mode and range of the following table of data. *See* Example 3

Score	1	2	3	4	5	6	7	8	9	10
Frequency	2	1	2	2	9	4	10	11	4	6

6 Make a grouped frequency table data with groups of width 5 for the following. *See* Example 6
53, 55, 56, 48, 39, 45, 39, 45, 41, 51, 53, 44, 43, 61, 60, 59, 51, 42, 36, 44, 60, 38, 59, 47, 38, 46, 42, 39, 35, 57, 48, 39, 37, 40, 39

7 Draw a circle graph for the following table. *See* Example 8

Category	Strongly agree	Agree	Disagree	Strongly disagree
Number	12	15	10	7

8 Draw a histogram for the following frequency table. *See* Example 9

Score	15–19	20–24	25–29	30–34	35–39	40–44
Number	5	8	7	5	3	2

9 A group of year 8 students gave their shoe sizes as follows. *See* Example 7
$7, 5, 7\frac{1}{2}, 4\frac{1}{2}, 7\frac{1}{2}, 6\frac{1}{2}, 9\frac{1}{2}, 6\frac{1}{2}, 5, 5\frac{1}{2}, 7, 6, 8\frac{1}{2}, 9, 7\frac{1}{2}, 7, 9, 4, 8\frac{1}{2}, 4, 7, 17, 7\frac{1}{2}, 4\frac{1}{2}, 11\frac{1}{2}, 5, 3\frac{1}{2}, 8\frac{1}{2}, 7, 4, 10, 5\frac{1}{2}, 1, 10\frac{1}{2}, 8, 4, 9, 4\frac{1}{2}, 10\frac{1}{2}, 7\frac{1}{2}$

a Draw up a frequency table. b Draw a column graph of the data.

10 The following shows the times in seconds that some students said they took to run 100 m.
17, 19, 17, 18, 17, 21, 17, 9, 16, 19, 18, 22, 16, 19, 14, 19, 15, 20, 16, 21, 15, 18, 18, 18, 17, 21, 17, 20
a Find the mean and median times.
b Remove any outliers and recalculate the mean and median.
c Comment on the results.

9780170194778

See Example 11

11 Should a sample or a census be used for each of the following?

a The average age of Year 8 students.

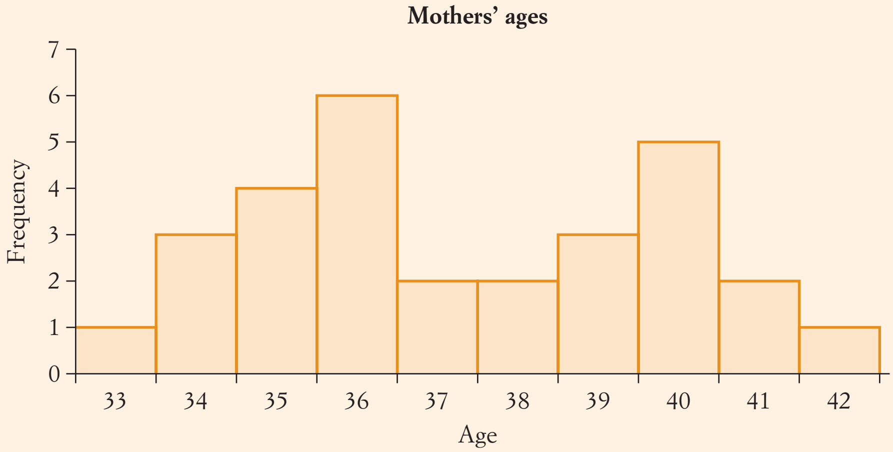
b Finding the best Year 8 triple jumper in the school.

12 What is the population for predicting the Year 7 enrolment in your high school next year?

Problem solving

13 A group of ten Year 8 students have a mean height of 168 cm. What happens to the mean when another student of height 152 cm joins the group?

14 The histogram below shows the ages of some mothers of Year 8 students. Find the mean, median, mode and range of the mothers' ages from the graph.



Reasoning

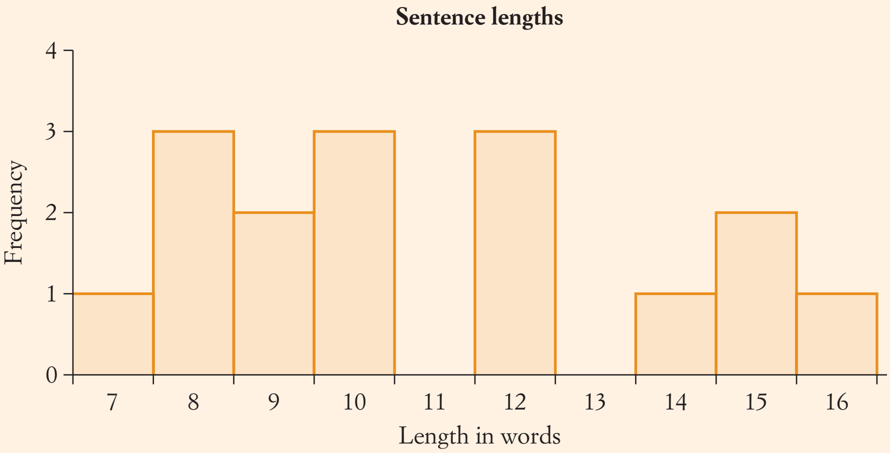
See Example 4

15 Would it be sensible to use the mean, median or mode for the typical distance that students at a high school in a large country town travel in the morning to reach school? Explain your reasoning.

See Example 10

16 After a program about the high cost of public transport from outlying areas, the host of a radio talkback show asked listeners to ring two different numbers to indicate whether they agreed or disagreed that 'No bus fares should be more than $2'. Explain what is wrong with this survey.

17 The histogram shows the sentence lengths of a book written for students in Year 4. How would you expect a graph of the sentence lengths of a book for Year 8 students to be different?



9780170194778

18 Use statistical measures to compare the following Year 8 boys' and girls' foot lengths. What conclusions can you reach? *See* Example 13

Girls: 23, 22, 23, 20, 26, 22, 28, 23, 23, 24, 20, 20, 25, 25, 27, 23, 23, 24, 22, 26, 22, 25, 23, 23, 22, 26, 27, 24, 22, 22, 34, 24, 29, 25, 22, 22

Boys: 25, 29, 25, 26, 27, 25, 33, 26, 25, 24, 22, 25, 26, 25, 29, 24, 22, 24, 27, 26, 25, 26, 26, 27, 25, 25, 29, 28, 26, 28, 27, 26, 30, 25, 25, 28, 29

19 Make a back-to-back stem-and-leaf plot of the following weekly incomes of 25 men and 25 women. Can you make any conclusions? *See* Example 14

Females: 964, 878, 1087, 1322, 1220, 1208, 1084, 1231, 1164, 945, 1355, 1108, 824, 916, 837, 992, 1112, 1149, 943, 964, 1232, 625, 992, 767, 1153

Males: 1291, 1287, 1550, 1281, 954, 1430, 1020, 961, 1353, 1113, 1526, 1469, 984, 1619, 970, 1180, 1822, 1292, 1297, 1347, 1249, 1711, 1340, 1408, 1018

20 Three surveys were conducted to find out how much people from Perth spend on holidays.

Survey 1: 200 people were phoned at home between 7 p.m. and 8 p.m. on a week night.

Survey 2: Forms were left at takeaway shops for a week with a box for them to be placed in, with a prize drawn at random from the completed forms.

Survey 3: People chosen at random from the electoral roll were asked if they were willing to be interviewed at a time of their choosing, until 50 people were surveyed.

Put the surveys in the order that would give the best results. Give reasons for your answers.

Measurement and geometry

2 Properties of shapes

9780170194778

Contents

Prior learning
Chapter 2
MAT08MGPL00002

Parent guide
Chapter 2
MAT08MGPG00002

Curriculum guide
Chapter 2
MAT08MGCU00002

Australian Curriculum statements

Geometric reasoning

Define congruence of plane shapes using transformations.

Geometric figures or flat shapes are everywhere. Look around you and see if you can see a triangular, rectangular or circular shape. It is important to be able to distinguish between different flat shapes and know their names and main features. It is also important to be able to construct geometric figures. This can be done with a ruler and pair of compasses or with computer software commonly known as computer-aided design (CAD) packages. The ability to visualise shapes is an important skill for many people including artists, architects, builders and craft workers.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

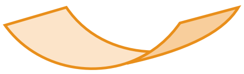
acute	degree	octagon	rotational symmetry
acute-angled	diagonal	order	scale
angle	dilation	parallelogram	scale factor
arc	dilatation	parallel	scalene
arm	dodecagon	pentagon	similar
axis of symmetry	equilateral	perpendicular	similarity
bisect	geometry	polygon	square
compasses	heptagon	protractor	symmetry
concave	hexagon	quadrilateral	tessellation
congruence	isosceles	rectangle	transformation
congruent	kite	reflection	translation
construct	line	regular	trapezium
construction	line symmetry	revolution	trapezoid
convex	magnification	right angle	triangle
corresponding	nonagon	rhombus	vertex
decagon	obtuse-angled	rotation	vertically opposite

2.1 Geometric shapes

Types of shapes

Most objects have surfaces that are **flat** or **curved** shapes. The sides of these shapes are mainly straight lines or circles (or parts of circles).

Flat surface

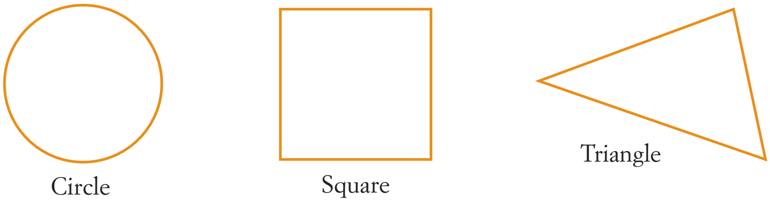


Curved surface

9780170194778

Shapes can be flat with no thickness, like squares and circles. They can also be solid, like cubes and cylinders.
To determine the size of a flat shape, we need two numbers, one each for the length and the width.
To determine the size of a solid shape, we need the length, the width and the thickness. Each measurement that we have to find is called a **dimension**.
Here are some flat and solid shapes. The correct mathematical word for a shape is a **figure**.

Flat shapes



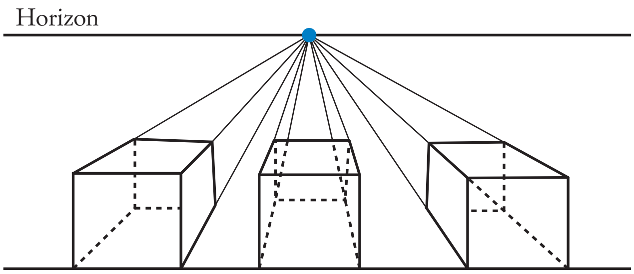
Solid shapes

Important!

Two- and three-dimensional shapes

Flat shapes can be drawn on a flat surface and are called **plane figures** or **two-dimensional figures** (**2D** for short).
Solid or hollow shapes that have to be constructed as models are called **three-dimensional figures** (**3D** for short).

We have to turn over a 3D shape if we want to look at another side. Artists use perspective to make a drawing look three-dimensional on a flat sheet of paper.

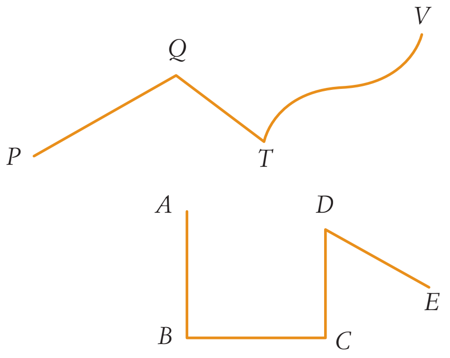


Naming plane figures

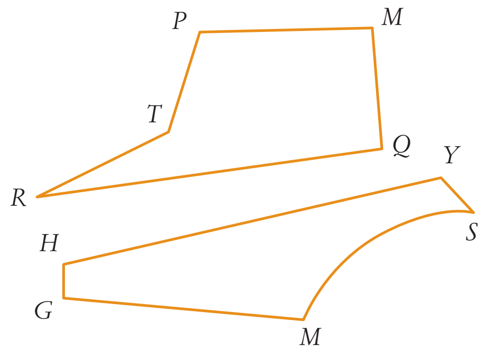
A plane figure can be **closed** or **open**. The outline of a closed figure makes a border that must be crossed to get in or out. The outlines of open figures have ends, so they can be 'straightened out' to make line segments. Some open and closed shapes are shown below.

9780170194778

Open figures



Closed figures



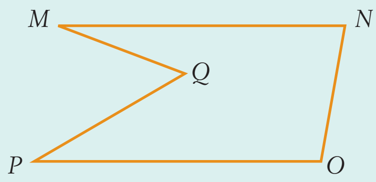
The **name** of a particular **plane figure** is determined by the names and order of its key points. For an open figure, we start at one end and finish at the other. For a closed figure, we go around the figure until we get back to the start. The shapes shown above could be called *PQTV*, *ABCDE*, *PMQRT* and *GHYSM*.

One of the most important types of closed figures is the **polygon**. Polygon means 'many angles'. Triangles and squares are polygons. A semicircle is not a polygon.

Important!

Polygons

A **polygon** is a closed figure made of line segments joined at their ends.
Each line segment is called a **side** of the polygon.
MN is a side of the polygon shown on the right.
A **vertex** (plural **vertices**) of a polygon is a point where sides meet. It is a '**corner**'. A polygon is named by listing its vertices in order. The polygon shown here could be called *MNOPQ*.



Note that points on an open figure where the line changes direction are also called vertices.

Naming polygons

Weblink
Polygon names
MAT08MGWB00002

The name of a polygon is determined by the number of sides it has. The table below shows the names of the first ten polygons.

Number of sides	Name of polygon
3	**Tri**angle
4	**Quadri**lateral
5	**Penta**gon
6	**Hexa**gon
7	**Hepta**gon

Number of sides	Name of polygon
8	**Octa**gon
9	**Nona**gon
10	**Deca**gon
11	**Undeca**gon
12	**Dodeca**gon

There are two other important properties of polygons:

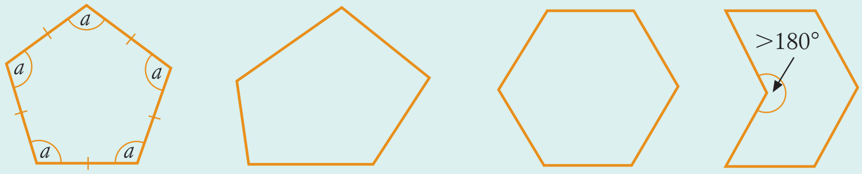
- every polygon is either **regular** or **irregular**
- every polygon is either **concave** or **convex**.

9780170194778

Important!

Types of polygons

A **regular** polygon is one with equal sides and equal angles. An **irregular** shape is one that is not regular.
The inside angles at the vertices of a **convex** polygon are *all* less than 180°. All the corners of a convex polygon point outwards.
A **concave** or **non-convex** polygon has at least one inside angle that is more than 180°. It has at least one inward-pointing corner.



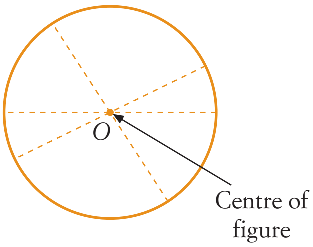
Regular pentagon | Irregular pentagon | Convex hexagon | Concave hexagon

Equilateral triangles and squares are regular polygons. Concave polygons have at least one angle that is bent backwards. It looks as if the side has been ‘pushed in’. A triangle must be convex. A **family of shapes** contains figures that have the same properties. For example, all three-sided polygons belong to the family of triangles. Polygons are assumed to be convex unless they are specifically mentioned as being concave.

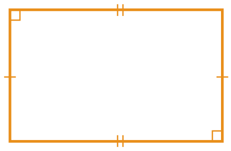
Example 1

Describe the following figures.

a



b



Solution

a This is a closed, curved figure. **The figure is not a polygon.**
Measure through the centre at different points. **The figure is a circle.**

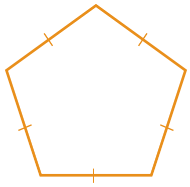
b This is a closed figure with line segments. **The figure is a polygon.**
There are four sides. **The figure is a quadrilateral.**
Opposite sides are equal and the corners are right angles. **The figure is a rectangle.**

When you identify a figure you should use enough information so that it can’t be confused with figures from the same family of shapes.

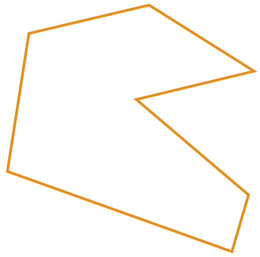
Example 2

Describe the following polygons.

a



b



Solution

a This is a closed figure with line segments. **The figure is a polygon.**
There are five sides. **The figure is a pentagon.**
All sides and angles are equal. **The figure is a regular pentagon.**

b This is a closed figure with line segments. **The figure is a polygon.**
There are seven sides. **The figure is a heptagon.**
There is an inward-pointing corner. **The figure is a concave heptagon.**

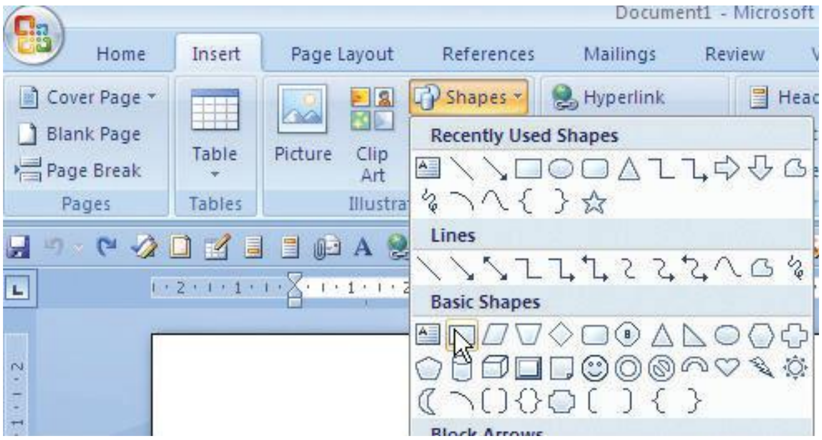
If a figure is concave, it is usual to state so. Otherwise, it is assumed that figures are convex.

Technology Drawing 2D and 3D figures

To draw a 2D shape in MS Word, do the following:

1 Click on the ‘Insert’ tab in Microsoft Word.

2 Under ‘Shapes’ , move your cursor over the ‘Basic Shapes’ and click on one that you would like to draw.



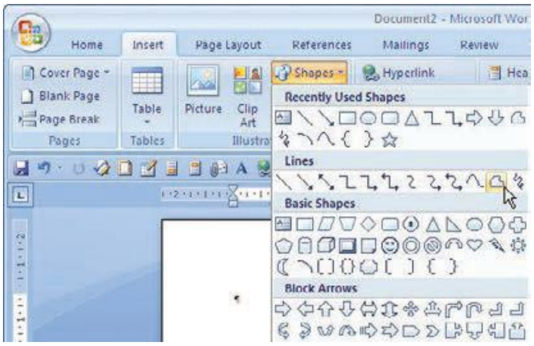
3 Draw the shape by left-clicking on the screen and dragging.

4 Move the cursor over your shape. What is the difference between the various arrow symbols such as ↔ and ↕ that appear? Try changing the appearance of the shape you have drawn.
5 You can add colour to make your shape more interesting. To do this, right-click over the object, select 'Format AutoShape' and pick a colour from the options.

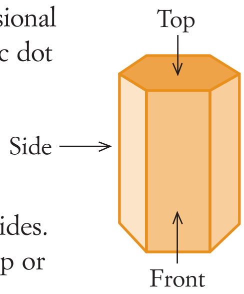
MS Word also has a '3-D Effects' button that allows you to change your plane 2D shape into a 3D one.

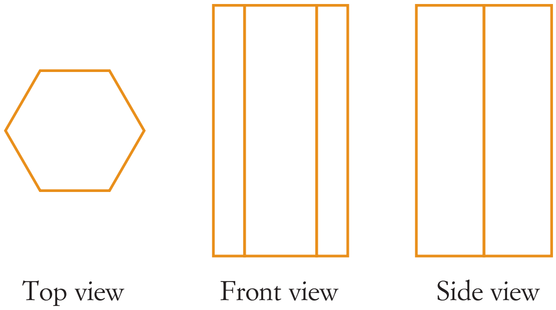
To draw a 3D shape in MS Word, do the following:
1 Click on your 2D shape. A Format tab will appear. Click on '3-D Effects' and select one of the 'Parallel' options.

2 You can rotate the shape in space and produce different lighting effects. Try this feature to see what effects you can produce.
You can also draw shapes using the 'Freeform' button as shown. Try drawing a rectangle using this function.



Three dimensional shapes or solids are not easy to represent on a two dimensional surface such as a piece of paper. You can draw them with the aid of isometric dot paper.
Solids can appear quite different when viewed from different positions.
Imagine that you could look at the hexagonal prism shown here from the top, the front or the side.
When you look at the prism from the top, you would not be able to see the sides.
When looking from the side or the front, you would not be able to see the top or bottom of the prism.
Each of these views is shown below.





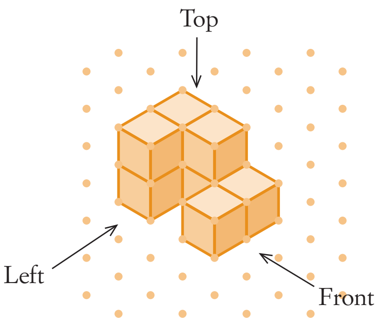
Animated example
Solid elevations and plans
MAT08MGAE00003

The views are also known as **elevations** and the top view is also called the **plan**.

Example 3

For the solid on the right, assume that there are no hidden cubes. Draw:

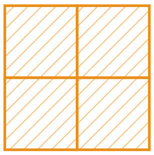
a the front elevation
b the left elevation
c the plan.



Solution

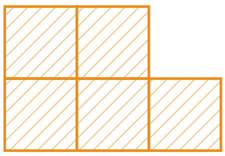
a Imagine that you are standing directly opposite the front face. You will see two vertical stacks of 2 cubes.

Draw the faces of the cubes that you would see.



b Imagine you are standing directly opposite the left face. You will see a row of 3 cubes on the bottom with a row of 2 cubes on top. The 2 cubes align with the left of the bottom row.

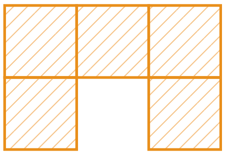
Draw the faces of the cubes that you would see.



9780170194778

c Imagine that you are hovering directly above the shape. You will see 2 cubes separated by a 1 cube space with a row of 3 cubes above them.

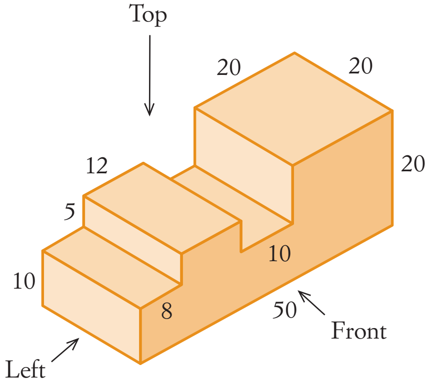
Draw the faces of the cubes that you would see.



Example 4

Look at the solid shown on the right.
All dimensions are in millimetres.
Draw:

a the front elevation
b the left elevation
c the plan.

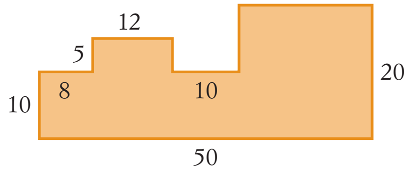


Solution

a Look at the front face.

Draw a horizontal line 50 mm long.

Use the dimensions on the diagram to draw the remaining vertical and horizontal lines.



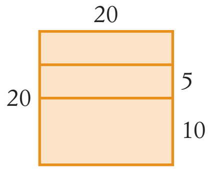
b Look at the left face of the solid.

The outside edge of the face looks like a 20 mm by 20 mm square with 2 horizontal lines across it.

One horizontal line is 10 mm above the base of the square.

The other horizontal line is 5 mm above the first.

Draw the square and horizontal lines.



c Imagine that you are hovering directly above the solid. The outside edges form a 50 mm by 20 mm rectangle.

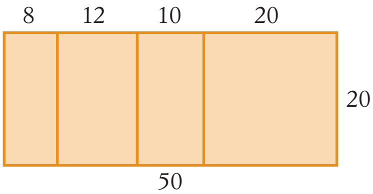
This rectangle has 3 vertical lines across it.

The leftmost vertical line is 8 mm from the left-hand side of the rectangle.

The next vertical line is 12 mm from the first.

The last vertical line is 20 mm from the right-hand side of the rectangle.

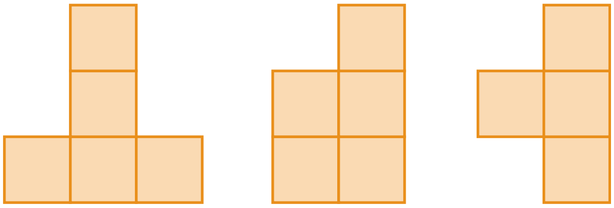
Draw the rectangle and vertical lines.



If you know what the elevations and plan of a solid look like you should be able to draw an isometric view of the solid.

Example 5

The diagrams below show the front and left elevations and plan of a solid.



Left elevation Front elevation Plan

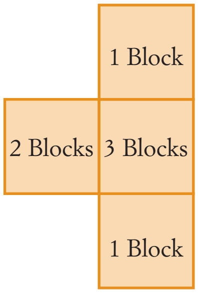
Use these diagrams to draw an isometric view of the solid.

Solution

The plan shows that there must be a row of 3 blocks with a single block attached to the middle block in that row.

The left elevation shows that there must only be 1 block at the ends of the row of 3 blocks in the plan.

The front elevation shows that the centre block in the row of 3 blocks has 2 more blocks on top. It also shows that the block attached to the middle of the row of 3 blocks has 1 more block on top.

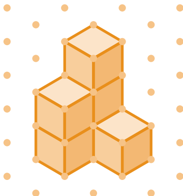


Plan

Use this information to draw the remaining blocks.

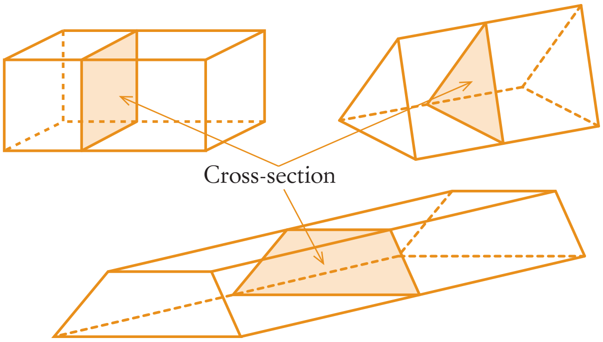
Erase unwanted lines.

Add shading to emphasise the 3D nature of the solid.

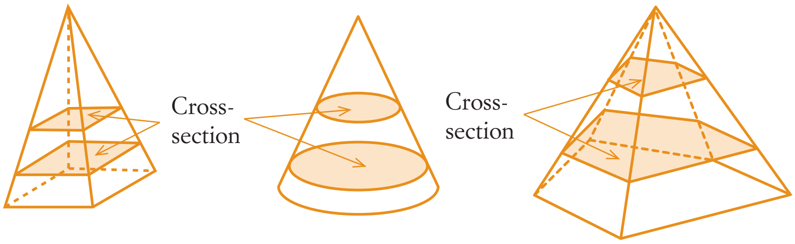


9780170194778

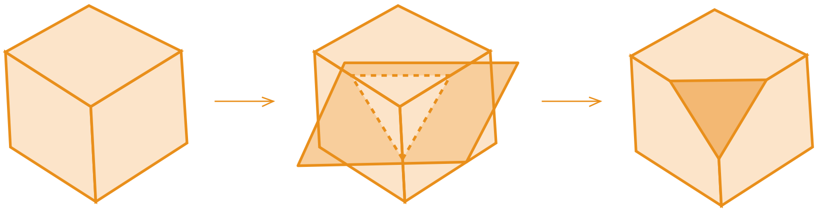
The **cross-section** of a solid is the shape obtained when a plane 'slices' across the solid parallel to the end face or base.
Prisms have cross-sections that are congruent (identical) polygons.



Pyramids and cones have cross-sections that are similar, the same shape but different sizes.



It is also possible to find other plane sections across solids. Consider the cube shown below. If we cut the cube across the corner, the shape of the section is a triangle.



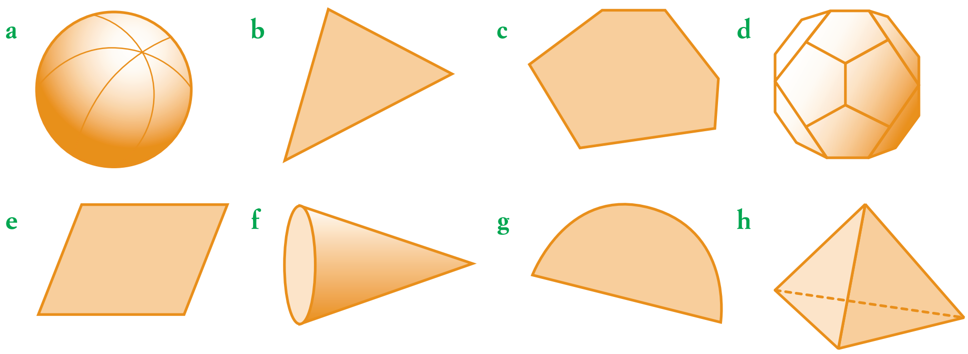
Exercise 2.1 Geometric shapes

Understanding

Extra questions
Exercise 2.1
MAT08MGEQ00004

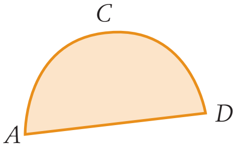
Worksheet
Geometric shapes
MAT08MGWK00009

1 Write down whether each of these drawings shows a plane figure or a three-dimensional figure.

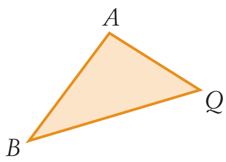


2 State whether each of the following shapes is open or closed.

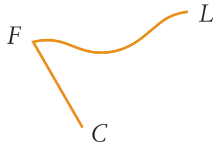
a



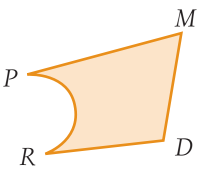
b



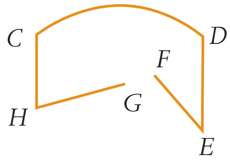
c



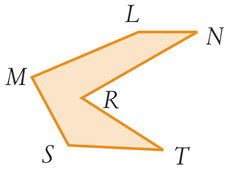
d



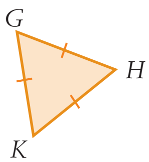
e



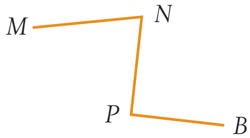
f



g



h



3 For the shapes in question **2**:

i name the vertices of each shape

ii give a possible label for each of the shapes **a**, **b**, **d**, **e** and **f**.

See Example 1

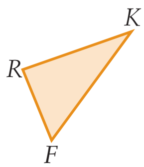
4 Describe each of the shapes **a**, **b**, **f** and **g** in question **2**.

Worked solutions
Exercise 2.1
MAT08MGWS00004

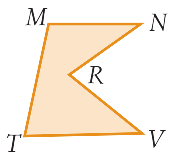
5 State which of the figures in question **2** are polygons.

6 Give a label to each of the following polygons.

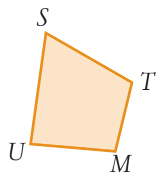
a



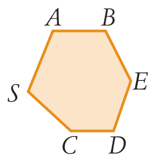
b



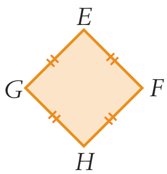
c



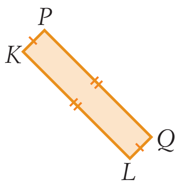
d



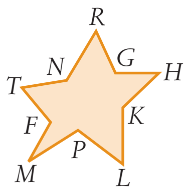
e



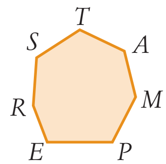
f



g



h



Worked solutions
Exercise 2.1
MAT08MGWS00004

Teacher notes
Isometric dot paper
MAT08MGTN00005

7 Draw each of the following solids on isometric dot paper.

a cube
b rectangular prism
c triangular prism
d square pyramid
e triangular pyramid
f pentagonal prism
g rectangular pyramid
h trapezoidal prism
i hexagonal prism

Fluency

8 Find and name as many different shapes as you can:

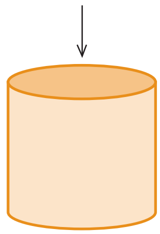
a in the objects in question **1**

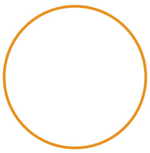
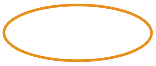
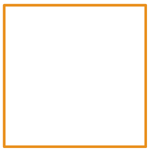
b in your pencil case

c in the classroom.

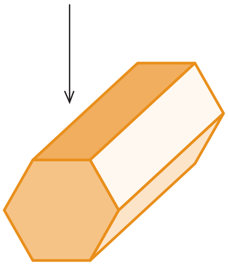
9780170194778

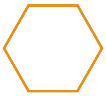
9 Select the correct view (**A**, **B**, **C** or **D**) as seen from the direction of the arrow in the diagram on the right.



A  **B**  **C**  **D** 

10 Select the correct view (**A**, **B**, **C** or **D**) as seen from the direction of the arrow in the diagram on the right.



A  **B**  **C**  **D** 

11 Describe each of the polygons in question **6**.

Worked solutions
Exercise 2.1
MAT08MGWS00004

See Example 2

12 Draw a quadrilateral.

13 Draw a convex, irregular quadrilateral.

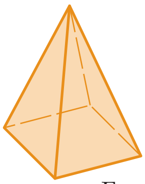
14 Draw an irregular triangle.

15 Draw a regular hexagon.

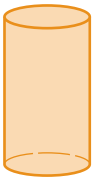
16 Draw a concave octagon.

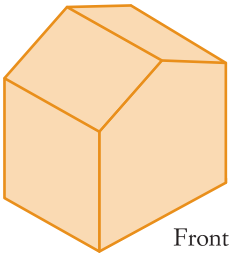
17 Draw a convex heptagon.

18 Sketch the plan, front elevation and side elevation for each of these 3D shapes.

a 

Front

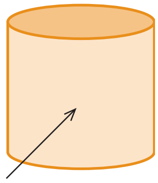
b 

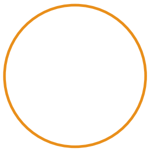
c 

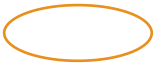
Front

9780170194778

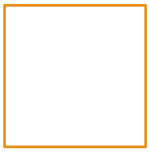
19 Select the correct view (**A**, **B**, **C** or **D**) as seen from the direction of the arrow in the diagram on the right.



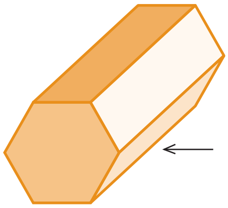
A


B


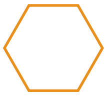
C


D


20 Select the correct view (**A**, **B**, **C** or **D**) as seen from the direction of the arrow in the diagram on the right.



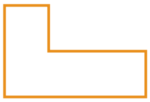
A


B


C


D


21 The left elevation, front elevation and plan of a solid are shown below. Use this information to draw a 3D sketch of the solid.


Left elevation

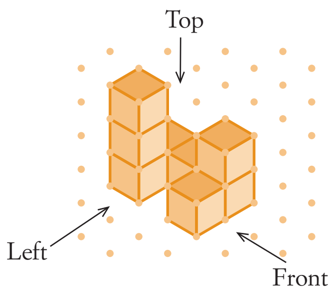

Front elevation


Plan

Worked solutions
Exercise 2.1
MAT08MGWS00004

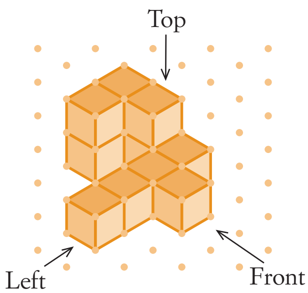
22 For the solid on the right, assume that there are no hidden cubes. Draw:

a the front elevation
b the left elevation
c the plan



23 This solid has been constructed using 11 cubes. Draw:

a the front elevation
b the left elevation
c the plan



9780170194778

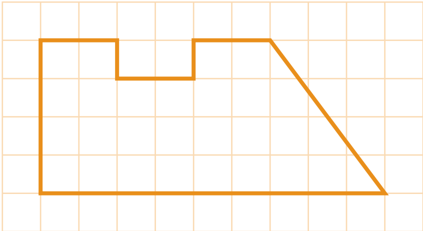
24 Here are the plan and left elevation of a prism.

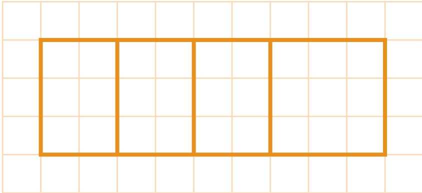
a On square dot paper, draw the front elevation of the prism.

b On isometric dot paper, draw a 3D sketch of the prism.

Teacher notes
Square dot paper
MAT08MGTN00006

Teacher notes
Isometric dot paper
MAT08MGTN00005


Left elevation

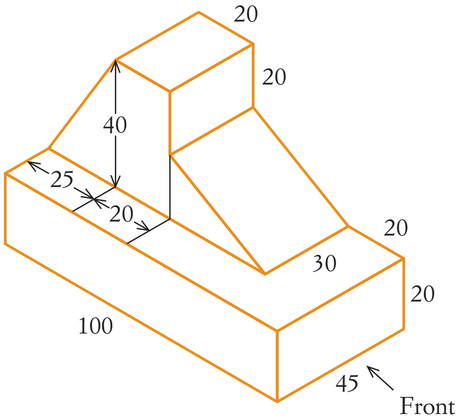

Plan

25 The diagram on the right shows a piece of metal that has been machined. All dimensions are in millimetres. Draw:

a the front elevation

b the left elevation

c the plan

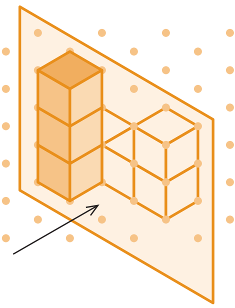


Problem solving

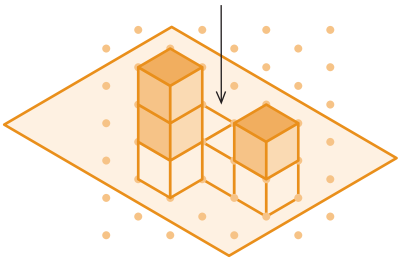
26 The solid on the right is made up of 7 blocks. It is cut by a vertical and a horizontal plane as shown in the diagrams below.

Draw the cross section that would result when looking in the direction of the arrow in each case.

a



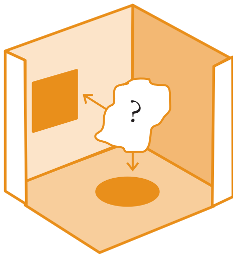
b



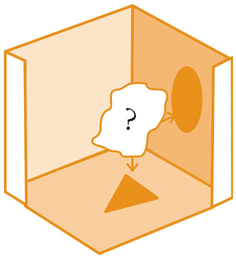
9780170194778

Worked solutions
Exercise 2.1
MAT08MGWS00004

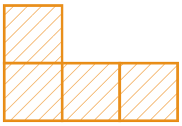
27 A certain solid object casts a circular shadow on the ground. When it is illuminated from the front, it casts a square shadow on the left-hand wall as shown in the diagram.
Name the solid.

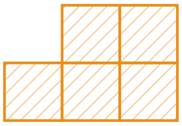


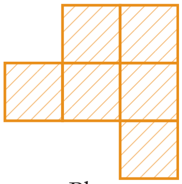
28 A different solid object casts a triangular shadow on the ground. When it is illuminated from the left, it casts a circular shadow on the right-hand wall as shown in the diagram.
Name the solid.



29 The left elevation, front elevation and plan view of a solid is shown below. Draw an isometric sketch of the solid.

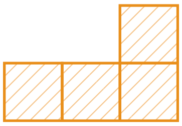

Left elevation

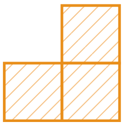

Front elevation

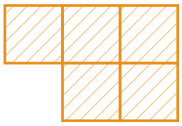

Plan

30 The front elevation, left elevation and plan view of a solid is shown in each part below. Draw an isometric sketch of the solid in each case.

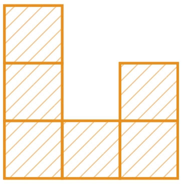
a

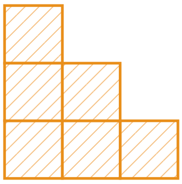

Left elevation

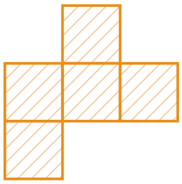

Front elevation


Plan

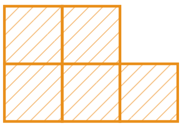
b

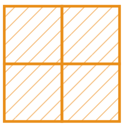

Left elevation

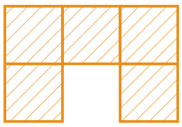

Front elevation


Plan

c


Left elevation


Front elevation


Plan

9780170194778

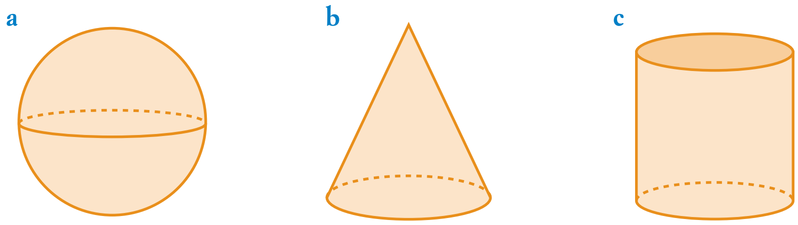
Reasoning

31 Explain why there is no regular concave polygon.

32 Explain why it is not usual to classify triangles as convex or concave.

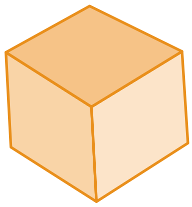
33 What shapes can you form by taking different plane sections of each of these figures?

Worked solutions
Exercise 2.1
MAT08MGWS00004



Sphere   Cone   Cylinder

34 Look at the cube shown here. Make a drawing of the cube to show how to form the following shapes by taking a plane section of the cube.



- **a** a square
- **b** an equilateral triangle
- **c** a rectangle that is not a square
- **d** a triangle that is not equilateral
- **e** a pentagon
- **f** a hexagon
- **g** a parallelogram that is not a rectangle

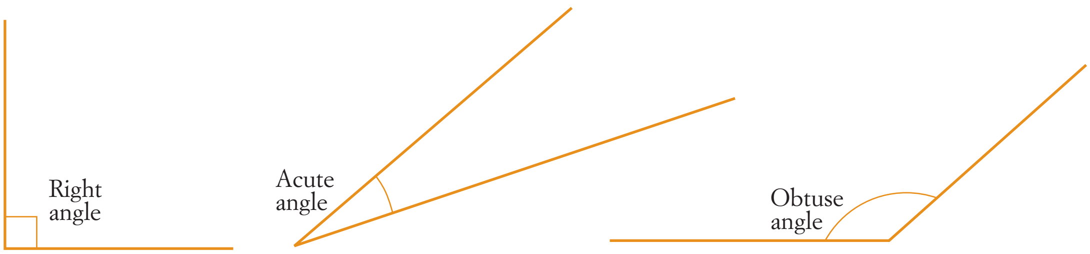
2.2 Classifying triangles and quadrilaterals

Triangles and quadrilaterals are commonly used in mathematics and other applications. Special names have been developed for members of these families of shapes.

Classifying triangles

Puzzle sheet
Classifying triangles
MAT08MGPS00003

One way of classifying or naming triangles is to use their angles. You already know that a **right angle** measures 90°. It is the angle in the corner of a square or rectangle. When we want to show a right angle, we draw a little square in the angle.
Acute angles are smaller than right angles. **Obtuse angles** are larger than right angles.



A triangle can have only one right angle. If a triangle has a right angle, it is called a **right-angled** triangle. A triangle can have only one obtuse angle. If a triangle has an obtuse angle, it is called an **obtuse-angled** triangle. A triangle must have three acute angles to be called an **acute-angled** triangle.

Important!

Classifying triangles using angles

Angles	Triangle name
All acute angles	**Acute-angled** triangle
A right angle	**Right-angled** triangle
An obtuse angle	**Obtuse-angled** triangle

Another way of classifying triangles is by using their sides. If a triangle has three identical sides (equal in length), it is called an **equilateral** triangle. A triangle with only two identical sides is called an **isosceles** triangle. A triangle with no sides the same is called a **scalene** triangle.

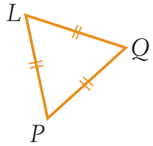
Important!

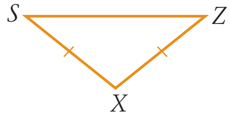
Classifying triangles using sides

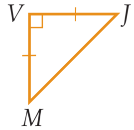
Sides	Triangle name
3 identical sides	**Equilateral** triangle
2 identical sides	**Isosceles** triangle
No identical sides	**Scalene** triangle

Example 6

Classify the following triangles.

a 

b 

c 

Solution

a All sides are identical. **$\triangle LPQ$ is equilateral.**

All angles are acute. **$\triangle LPQ$ is also acute-angled.**

b Two sides are identical. **$\triangle SZX$ is isosceles.**

There is an obtuse angle. **$\triangle SZX$ is also obtuse-angled.**

c Two sides are identical. **$\triangle MVJ$ is isosceles.**

There is a right angle. **$\triangle MVJ$ is also right-angled.**

Video tutorial
Classifying quadrilaterals
MAT08MGVT00002

Classifying quadrilaterals

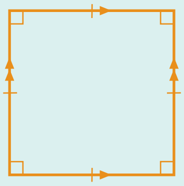
There are six special types of convex quadrilaterals that are classified using their angle and side properties. The properties stated below are the minimum needed to sort figures into one class or another. Each class of quadrilateral has many other properties that are not stated.

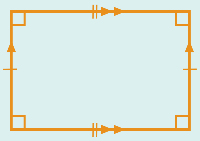
9780170194778

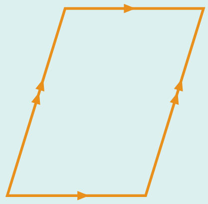
Important!

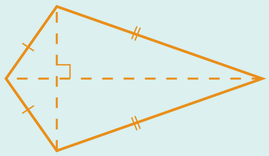
Special quadrilaterals

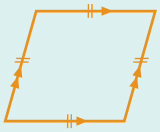
A **square** (shape **a**) has four identical sides and right angles at every vertex.
A **rectangle** (shape **b**) has right angles at every vertex.
A **parallelogram** (shape **c**) has opposite sides parallel.
A **kite** (shape **d**) has two pairs of adjacent sides that are the same length. The diagonals of a kite intersect at right angles.
A **rhombus** (shape **e**) has four identical sides. The plural of rhombus can be **rhombi** or **rhombuses**.
A **trapezium** or **trapezoid** (shape **f**) has one pair of parallel sides.

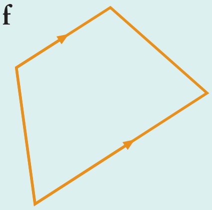
a


b


c


d


e


f


Puzzle sheet
Singing in the car
MAT08MGPS00004

Worksheet
Classifying triangles and quadrilaterals
MAT08MGWK00010

It is possible for a quadrilateral to be a member of more than one class. For example, a square is also a rectangle, a parallelogram, a kite, a rhombus and a trapezium.
Parallel lines are shown by drawing the same arrowheads on the lines that are parallel.

Technology
GeoGebra: Making quadrilaterals
MAT08MGTC00005

Investigate: Properties of quadrilaterals

Quadrilaterals have a number of properties regarding the sides, angles and diagonals. Complete the table below by answering ‘yes’ or ‘no’ to each question.

	Trapezium	Rhombus	Parallelogram	Rectangle	Square	Kite
Are opposite sides equal?						
Are opposite sides parallel?						
Are opposite angles equal?						
Are all angles equal?						
Are diagonals equal?						
Do diagonals bisect each other?						
Do diagonals intersect at right angles?						
Do diagonals bisect the angles of the quadrilateral?						

Technology
GeoGebra: Quadrilateral sides and angles
MAT08MGTC00002

Technology
GeoGebra: Quadrilateral diagonals
MAT08MGTC00003

Teacher notes
Geometric constructions
MAT08MGTN00004

The *uniformity* of a quadrilateral can be thought of as its degree of 'sameness', regularity and symmetry.

- In a paragraph, describe the features of a quadrilateral that you would measure to rate its uniformity. Also explain how you would use these measures to calculate a uniformity score.
- Compare what you have prepared with others in your class and arrive at an agreed method of calculating a uniformity score for quadrilaterals.
- Use the method you have developed to rate each type of quadrilateral.

You can construct angles and geometric shapes using instruments such as a ruler, protractor, and a pair of compasses. The extra material for Geometric constructions will show you how to accurately draw angles of various sizes and shapes such as triangles and quadrilaterals.

Exercise 2.2 Classifying triangles and quadrilaterals

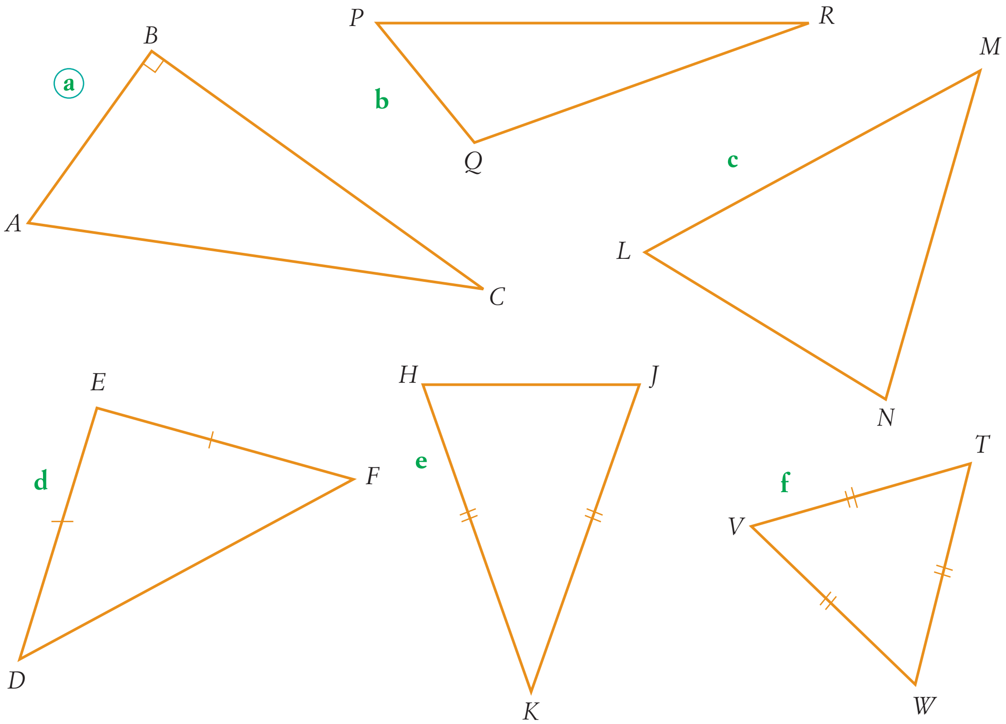
Understanding

Extra questions
Exercise 2.2
MAT08MGEQ00005

Worked solutions
Exercise 2.2
MAT08MGWS00005

See Example 6

1 Classify each of the following triangles by both its angle name and its side name.



Fluency

2 Sketch a triangle that is:

a equilateral	**b** scalene
c acute-angled and isosceles	**d** obtuse-angled and scalene
e right-angled and isosceles	**f** right-angled and scalene

9780170194778

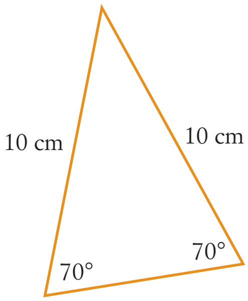
3 Which of the following best describes this triangle? Select **A**, **B**, **C** or **D**.

A isosceles and obtuse-angled

B isosceles and acute-angled

C scalene and obtuse-angled

D scalene and acute-angled

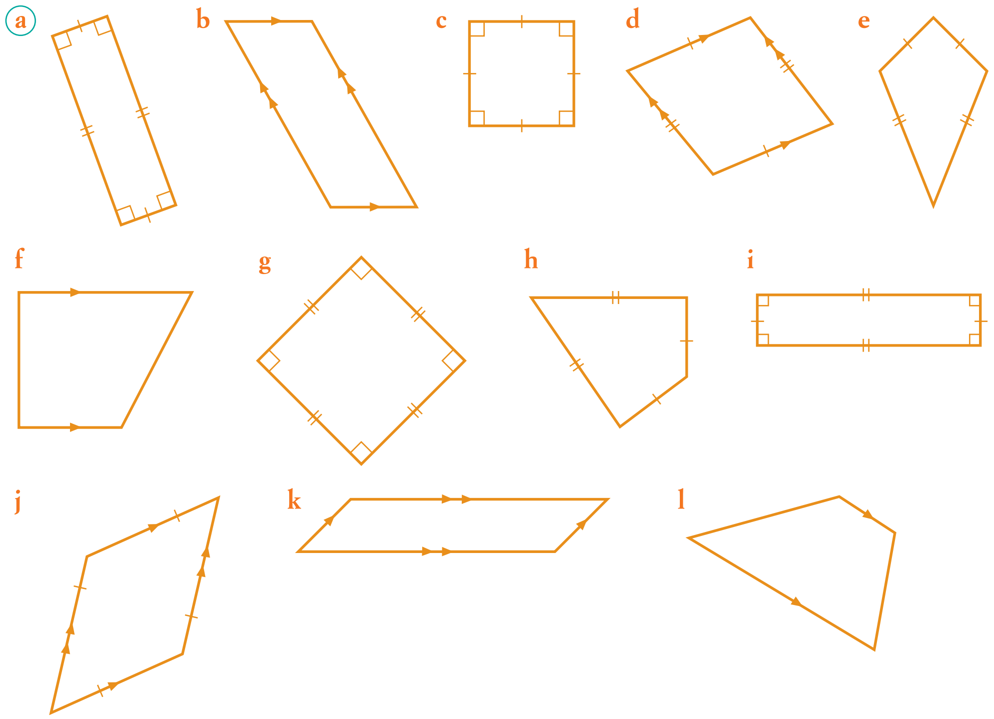


4 Classify each of the following quadrilaterals.

Worked solutions

Exercise 2.2

MAT08MGWS00005



5 What quadrilateral am I? (Note: There may be more than one answer in each case.)

a My diagonals do not bisect each other.

b My opposite sides are the same length.

c My opposite sides are not parallel.

d My adjacent sides are the same length.

e My diagonals cross at right angles.

f My opposite sides are the same length.

g My adjacent angles are equal.

Problem solving

6 Refer to $\triangle ABC$, $\triangle PQR$ and $\triangle LMN$ in question **1** to answer the following.

a Measure each side of the triangles.

b For each triangle, write down the biggest angle and longest side.

c For each triangle, write down the smallest angle and shortest side.

d For triangles, what can you say about the sizes of angles and the sides opposite them?

Worked solutions
Exercise 2.2
MAT08MGWS00005

7 Which of the following statements are true? (Explain your answers.)
a All squares are rectangles.
b All rectangles are squares.
c All rhombuses are squares.
d All squares are rhombuses.
e All rectangles are parallelograms.
f A rectangle is a quadrilateral with opposite sides parallel and equal.
g The diagonals of a trapezium meet at right angles.

8 a Draw two differently shaped parallelograms and mark their diagonals.
b Measure the position of the intersection of the diagonals in each parallelogram.
c What can you say about the intersection of diagonals in a parallelogram?

Reasoning

9 Draw rhombuses, rectangles and kites and find the properties of their diagonals.

2.3 Transformations

Important!

Transformations

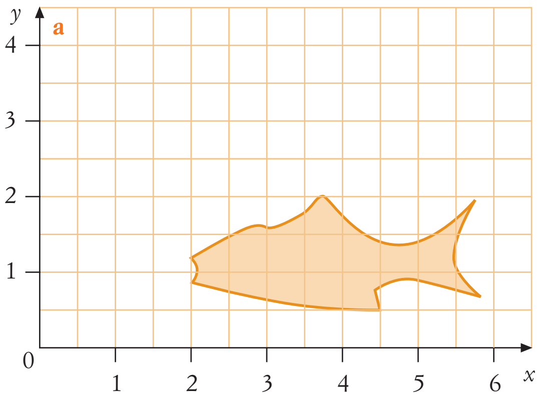
A **transformation** changes a shape or its position to form an **image**.

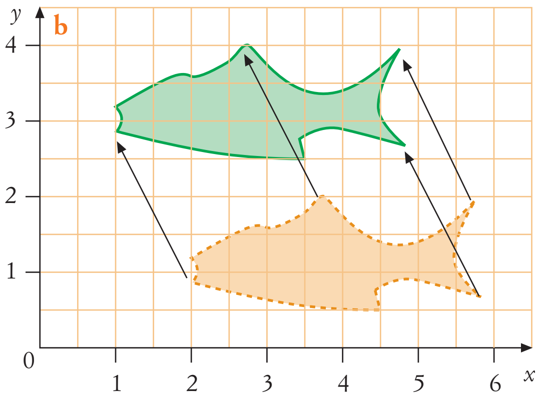
There are four common types of transformations.

Important!

Translation

A **translation** is like a sliding movement. All parts of the shape move the same distance and direction, so the vertices move the same amount. The shape and its image are exact copies (identical). In mathematics, shapes that are exact copies are called **congruent**.



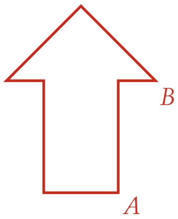


The fish in **a** has been moved left 1 unit and up 2 units in **b**. The image (green) and the shape (orange) are congruent because every point of the shape has been moved the same amount.

9780170194778

Example 7

Find the image of the shape shown after a translation of 4 to the right and 2 up.



Solution

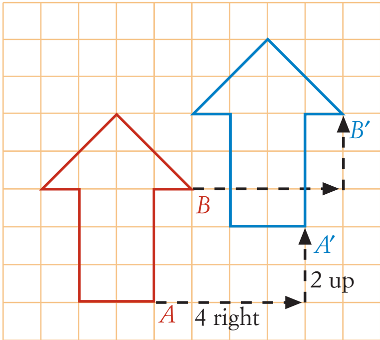
Draw the shape on some grid paper.

For each corner in the shape, mark a corresponding image corner 4 units to the right and 2 units up.

Use a prime (′) to distinguish between a point on the object and a point on the image.

$A \longrightarrow A'$, $B \longrightarrow B'$ and so on.

Join up the corners to form the image.



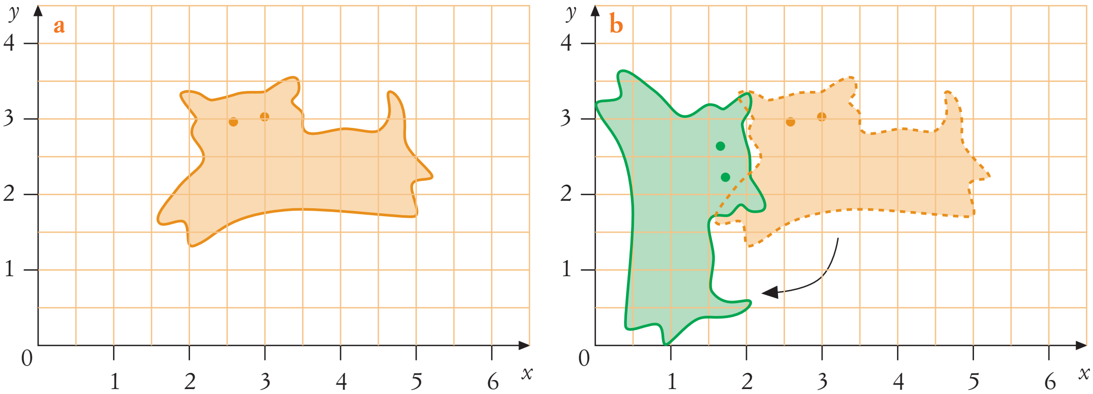
Important!

Rotation

A **rotation** is a turn. All points remain the same distance from the **centre** of the turn (the **centre of rotation**). The angle of rotation has a direction (**clockwise** or **anticlockwise**).

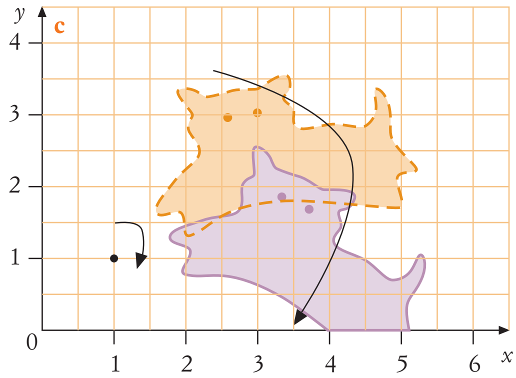
Puzzle sheet
The best butter
MAT08MGPS00005

A rotation produces an image by turning a shape around a fixed point as shown below.



The cat in **a** has been turned 90° clockwise around its ear in **b**.

9780170194778



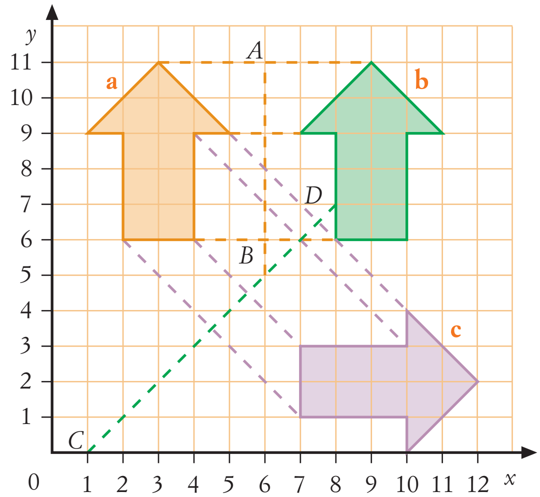
In **c** it has been turned 30° around the point (1, 1). The shape and its image are congruent.

Important!

Reflection

When a shape is **reflected** or **flipped**, all points swap to an equal distance on the other side of the **line of reflection**.

In a reflection, an image is produced as if a mirror has been placed on a line.
Arrow **b** in the diagram on the right is a reflection of arrow **a** in the line *AB*.
Arrow **c** is a reflection of arrow **a** in the sloping line *CD*. *AB* and *CD* are known as the lines of reflection.
Each point in the original shape has its reflection in the image. Each point in the original shape is the same distance from the line of reflection as its reflected image.
When a shape is reflected, it is as if it is flipped. The shape and its image are congruent.



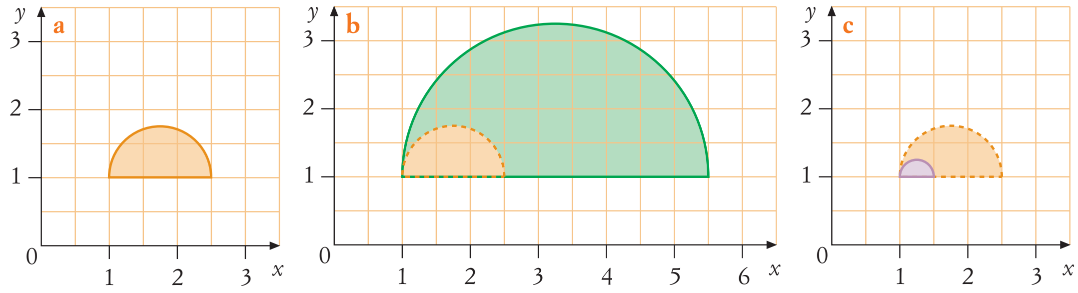
Important!

Enlargement and reduction

Enlargements and **reductions** produce an image that is a different size from the original. The amount by which the shape is enlarged or reduced is called the **magnification**.

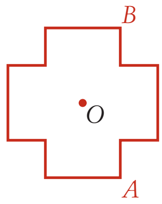
An enlargement or reduction does not produce an image that is congruent with the original shape. Enlargements and reductions are also known as **dilations**. In the following diagram, the semicircle in **a** has been enlarged to three times its size in **b**. The original in **a** has been reduced to a third of its size in **c**. The **enlargement factor** or **magnification** for **b** is 3 and the magnification for **c** is $\frac{1}{3}$.

9780170194778



Example 8

Draw the image of the shape on the right after a magnification of 2 if O is the centre of enlargement.



Worksheet
Transformations 1
MAT08MGWK00011

Worksheet
Enlargements and reductions
MAT08MGWK00017

Solution

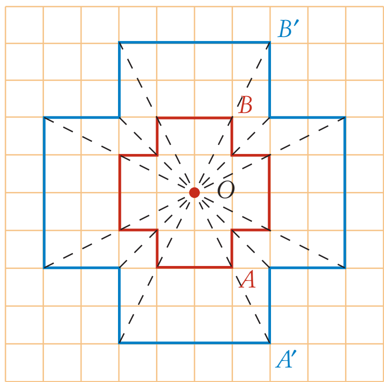
Draw the shape on some grid paper.

Draw a line from O through each corner (vertex) of the shape. Continue the lines beyond the corners (vertices).

For each corner in the shape, mark a corresponding image corner twice the distance from O.

$A \longrightarrow A'$, $B \longrightarrow B'$ and so on.

Join up the points to form the image.

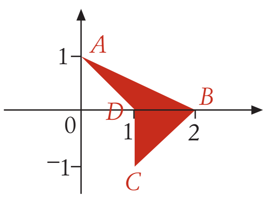


Example 9

Copy the figure on the right onto separate axes.

a Rotate it 90° clockwise around D.

b Reflect it in the line BC.



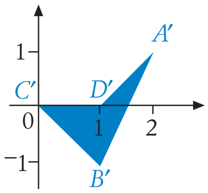
Worksheet
Transformations 2
MAT08MGWK00013

Solution

a A rotation of 360° is a full turn, so a rotation of 90° is one quarter of a full turn. For each corner in the shape, mark a corresponding image corner rotated 90° clockwise using D as the centre of rotation.

$A \longrightarrow A'$, $B \longrightarrow B'$ and so on.

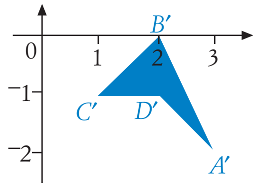
Join up the corners to form the image.



b Draw a line through BC. For each corner in the shape, mark a corresponding image corner an equal distance from and on the other side of BC.

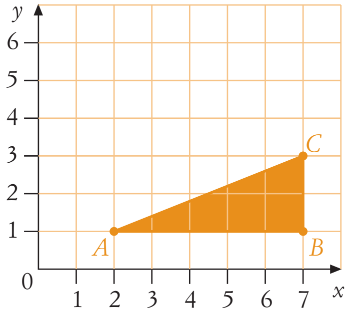
$A \longrightarrow A'$, $B \longrightarrow B'$ and so on.

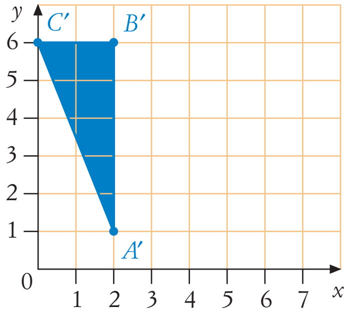
Join up the points to form the image.



Example 10

Use the coordinates of A, B, C, A', B' and C' to show the transformation of the shape below and describe the transformation in words.





Solution

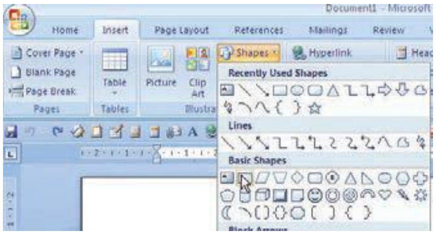
Write the coordinates of A and A'.	$A(2, 1) \longrightarrow A'(2, 1)$ **The position of A remains unchanged.**
Write the coordinates of B and B'.	$B(7, 1) \longrightarrow B'(2, 6)$
Write the coordinates of C and C'.	$C(7, 3) \longrightarrow C'(0, 6)$
Describe how the image could have been formed.	**The image and the object are congruent, so the transformation is either a translation, reflection or rotation.** **The transformation is not a translation because all points in the shape have not moved the same amount to form the image.** **The transformation is not a reflection because there is no line of reflection that will form the image.** **The transformation is a rotation. The centre of rotation is A.**
Describe the transformation.	**The shape has been rotated 90° anticlockwise around (2, 1).**

9780170194778

Technology Slide, flip and turn

1 Draw a plane shape using Microsoft Word or some other software package.
2 Experiment with the shape so that you can:
- make a copy (a congruent shape)
- magnify the shape (make a similar shape)
- translate the shape
- rotate the shape.

3 Can you reflect the shape in a vertical line?
4 Can you reflect the shape in a horizontal line?



Important!

Tessellations

A **tessellation** is a pattern that covers the plane with congruent shapes.

Tiles are fitted together on floors and walls to make eye-pleasing designs. A tessellation is a tiling pattern that uses congruent shapes to cover an area. The shapes may be rotated, translated or reflected to make the pattern. The pattern covers the area completely and no overlaps are allowed. Maurits Cornelis Escher (1898–1972) was one of the world's most famous graphic artists. His transformation prints – such as *Metamorphosis I*, *Metamorphosis II* and *Metamorphosis III*, *Sky & Water I* and *Reptiles* – are images based on tessellations. See what else you can find out about his work.

An Escher tessellation

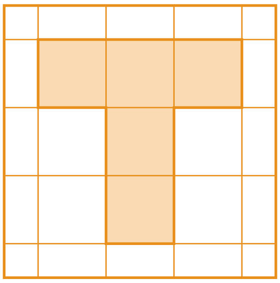
A complicated tiling tessellation

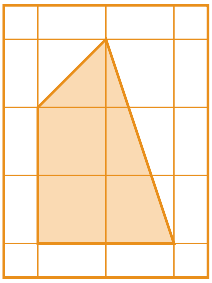
You will find isometric or grid paper helpful when making shapes for tessellations.

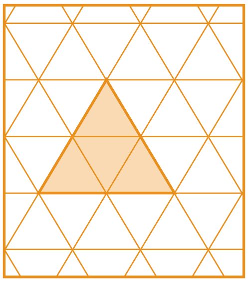
9780170194778

Example 11

Use the shapes shown below to make tessellations.

a 

b 

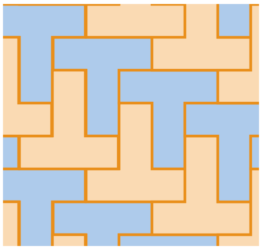
c 

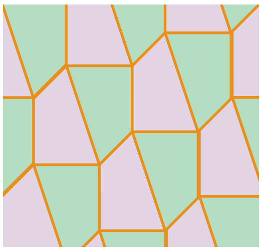
Solution

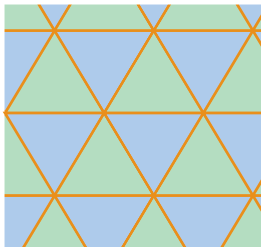
For each shape, follow these directions.

- Copy the shape onto cardboard. (You may need to draw the shape on grid or isometric paper first.)
- Cut out the cardboard shape and trace around it on a piece of paper.
- Reposition the cardboard shape to join a side with the one you have just drawn. Trace another shape.
- Repeat the procedure until the desired area is covered. Colour your outlines.

The results are as follows.

a 

b 

c 

9780170194778

Exercise 2.3 Transformations

Understanding

Extra questions
Exercise 2.3
MAT08MGEQ00006

Worked solutions
Exercise 2.3
MAT08MGWS00006

See Example 7

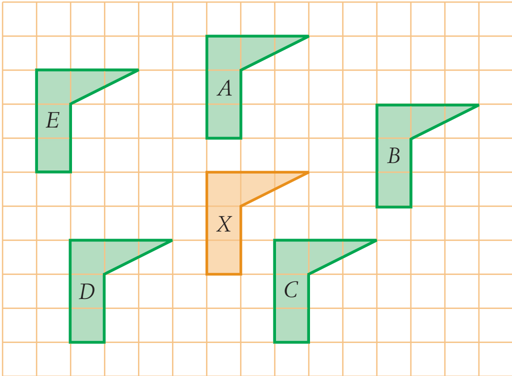
1 Shape X has been transformed to each of the images A, B, C, D and E using the same type of transformation.

a What sort of transformation has been used?

b Describe the transformation used to produce each of the images (A, B, C, D and E) from the original shape X.

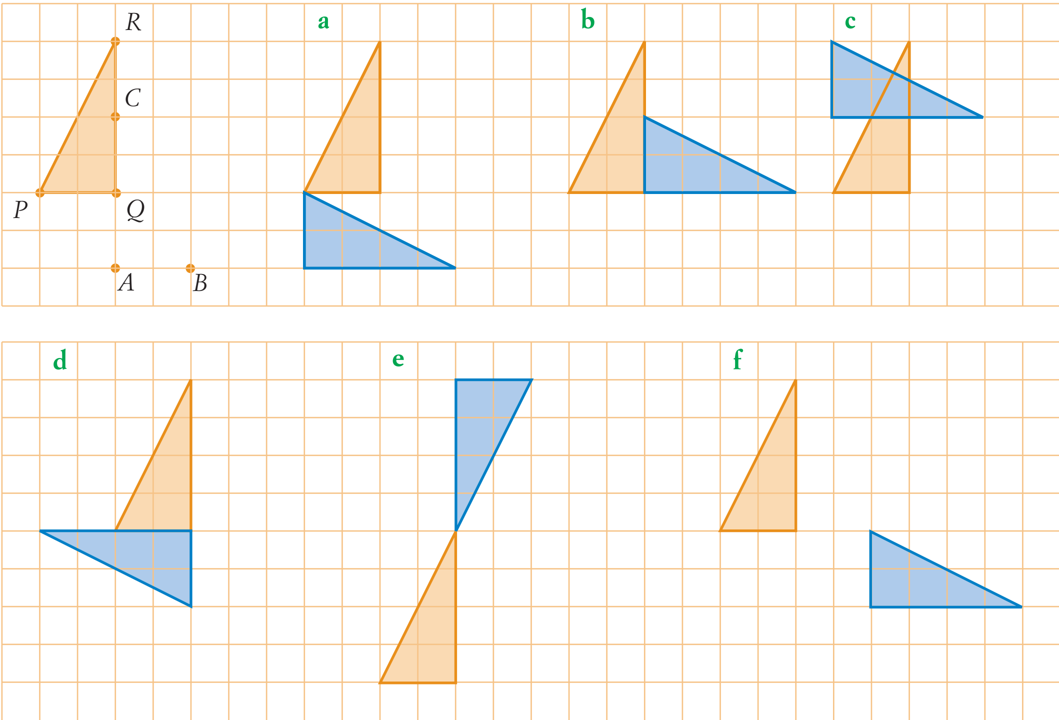
c Describe the transformation needed to slide:

- **i** shape D to shape C
- **ii** shape C to shape A
- **iii** shape E to shape B
- **iv** shape A to shape D
- **v** shape D to shape X.



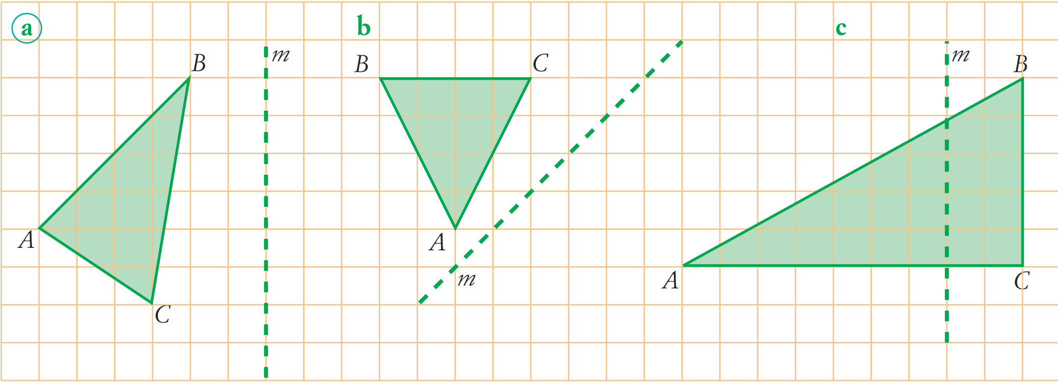
Fluency

2 $\triangle PQR$ can be rotated around any of the points P, Q, R, A, B or C. The results of some rotations are shown here. In each case, describe the rotation by identifying the centre of rotation and describing the size and direction of the rotation.



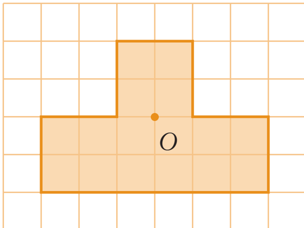
Worked solutions
Exercise 2.3
MAT08MGWS00006

3 Copy each of these diagrams onto grid paper. Draw the reflected image of $\triangle ABC$ in the line of reflection m. Each time, label the image of points A, B and C as A', B' and C' respectively.



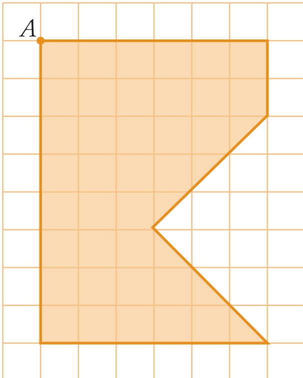
See Example 8

4 Copy this shape onto grid paper. Enlarge the shape by a factor of 3 relative to the point O.

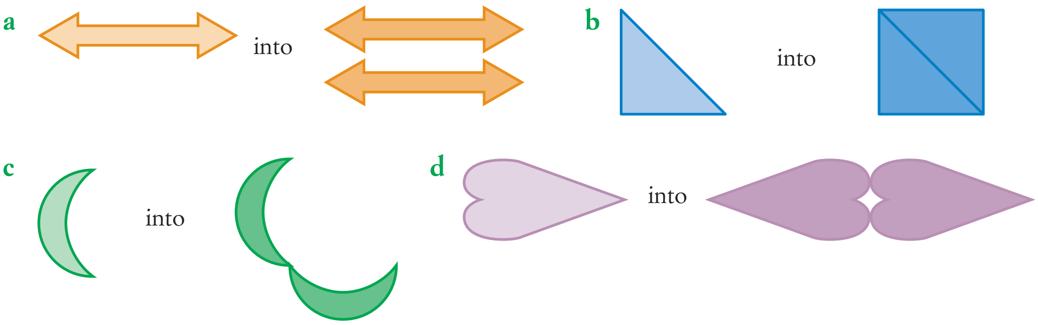


See Example 8

5 Copy this shape onto grid paper. Reduce the shape by a factor of $\frac{1}{2}$, keeping A the same.



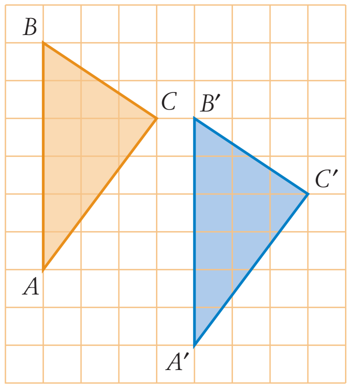
6 Draw diagrams to show where you would place a mirror to make each of the following changes.



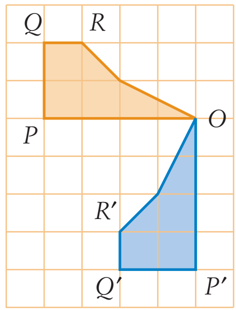
9780170194778

7 For each diagram below, state the transformation (translation, rotation or reflection) used on the original shape to form the image.

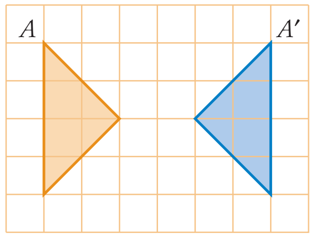
a



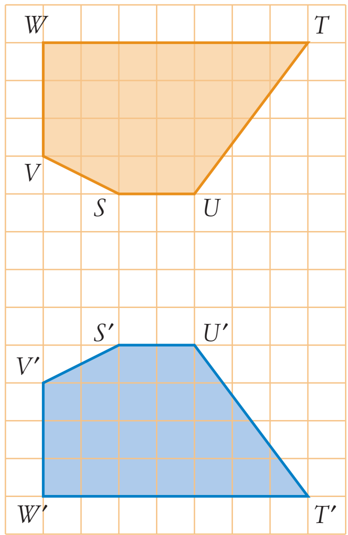
b



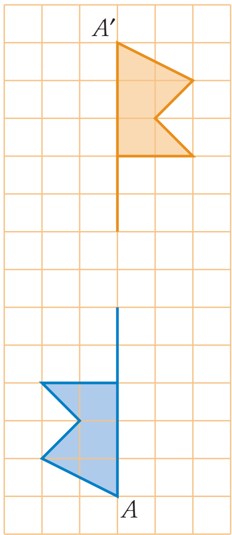
c



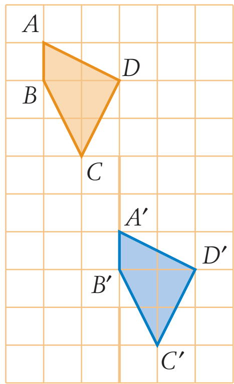
d



e

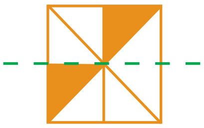


f

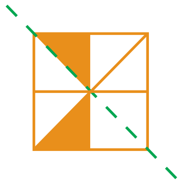


8 Copy and complete each of these patterns so that the dashed line is a line of symmetry.

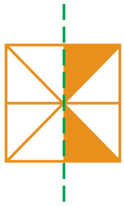
a



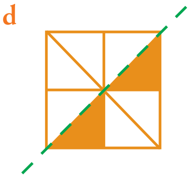
b



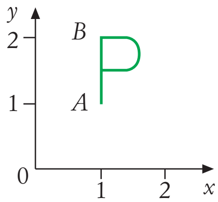
c



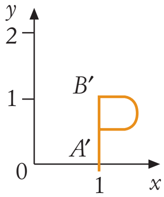
d



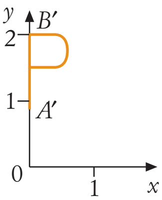
9 Use the coordinates of A, B, A' and B' to show the transformations of each shape.



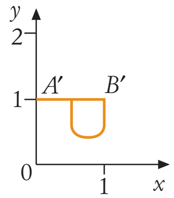
a

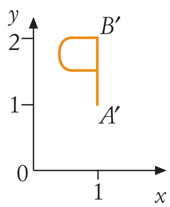


b

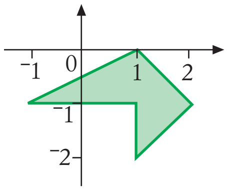


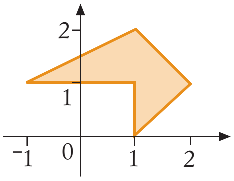
Worked solutions
Exercise 2.3
MAT08MGWS00006

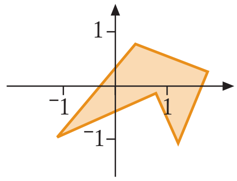
c


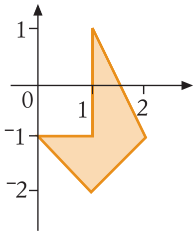
d


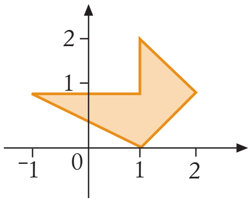
10 Identify the transformations of this shape that produce the images shown in parts **a** to **e**.

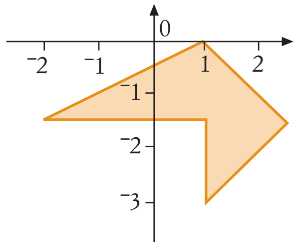


a


b


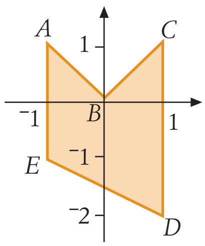
c


d


e


See Examples 9, 10

11 Copy this shape onto separate axes.

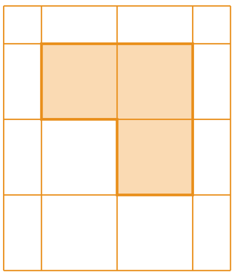


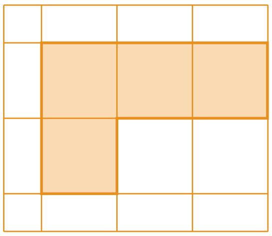
- **a** Translate it 2 right and 1 down.
- **b** Translate it 1 left and 3 up.
- **c** Rotate it 90° clockwise around A.
- **d** Rotate it 180° anticlockwise around D.
- **e** Enlarge it using a magnification of 2, keeping A the same.
- **f** Reduce it using a magnification of $\frac{1}{2}$, keeping B the same.
- **g** Reflect it in the x-axis.
- **h** Reflect it in the line BC.

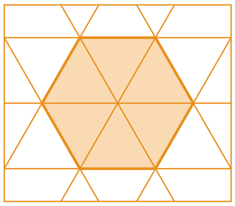
Problem solving

See Example 11

12 Make tessellations using the shapes below.

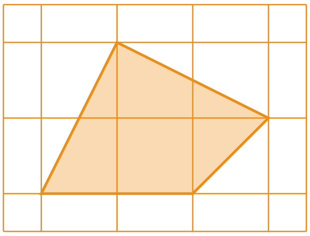
a


b


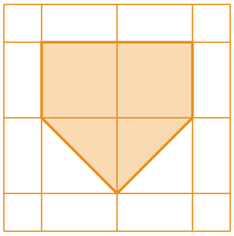
c


9780170194778

d



e



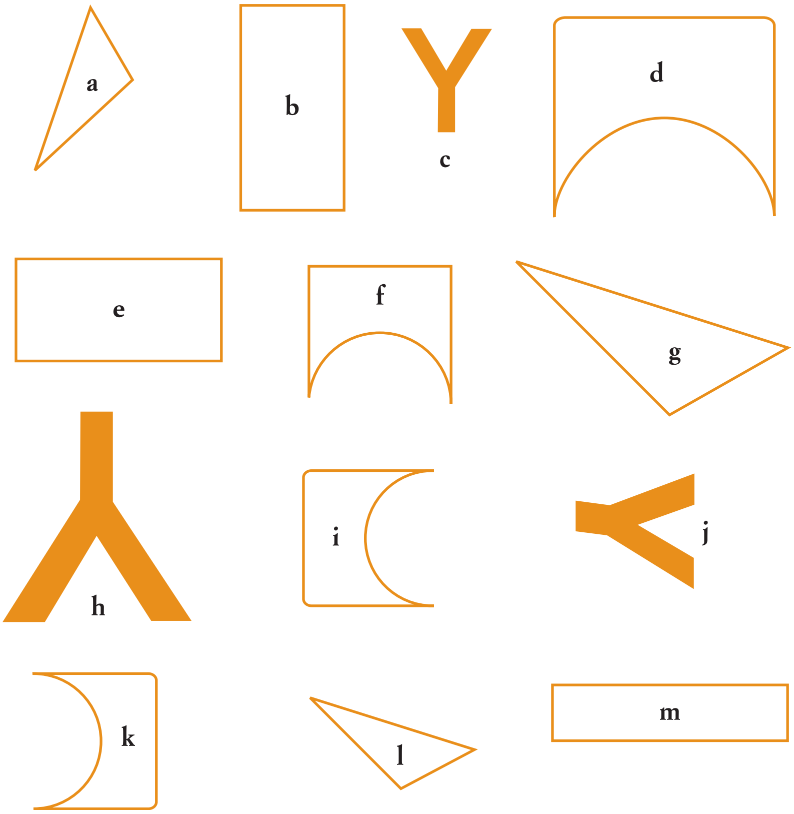
2.4 Similarity, congruence and symmetry

Some shapes can be enlarged to make new ones that are the same shape but different sizes. Some shapes can be moved to sit exactly over each other, they have the same shape and size. **Similarity** and **congruence** are the names used for these relationships between shapes. When we use an overhead projector (OHP), it enlarges what is on the OHP transparency to make a **similar** enlarged image on the screen. Telescopes, microscopes and lenses may be used to make similar enlarged or reduced images. The shapes on the same page of all the printed copies of this book are **congruent**, they are exactly the same in each copy.

You saw in the previous section of this chapter that shapes that are transformed using translations, reflections or rotations are congruent with their images.

Look at the shapes below and state which are similar and which are congruent.

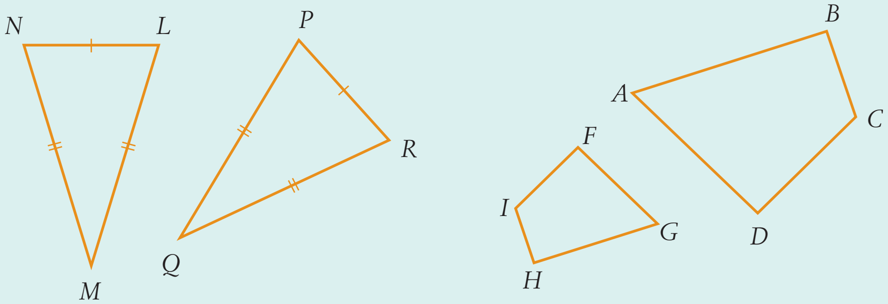
Worksheet
A page of similar shapes
MAT08MGWK00016



Important!

Congruent and similar shapes

Two shapes are **congruent** if one is an exact copy of the other. $\triangle LMN$ and $\triangle PQR$ in the diagram below are congruent. We write this as $\triangle LMN$ ||| $\triangle PQR$ or $\triangle LMN : \triangle PQR$.



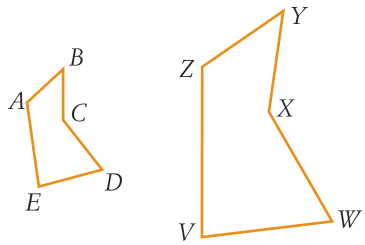
Two shapes are **similar** if one can be made by enlarging or reducing the other. All angles in one shape are the same as the angles in another shape that is similar to it. Every side in one shape is enlarged or reduced by applying the same **scale factor** to the similar shape. $ABCD$ and $GHIF$ in the diagram above are similar. We write this as $ABCD \equiv GHIF$ or $ABCD \cong GHIF$.
Two shapes that are congruent are also similar to each other because all the angles in one shape are the same as the angles in the other and the corresponding sides are the same, so the scale factor is 1.

Technology
GeoGebra: Similarity
MAT08MGTC00004

TLF learning object
Trigonometry: Similar triangles (L2327)
MAT08MGIN00002

TLF learning object
Measures: Similar shapes (L2309)
MAT08MGIN00002

Two line segments are congruent if they have the same length. When we want to show congruent line segments in a drawing, we put matching dashes through them. If there are two or more sizes, we can use several dashes. This has been done in $\triangle LMN$ and $\triangle PQR$ above.
The names of congruent or similar shapes are written so that the vertices in each name correspond.
We write $ABCDE$ ||| $ZYXWV$ to show that these figures are similar because the angle at A is the same as the angle at Z, the angle at B is the same as the angle at Y, and so on.



Technology Drawing similar shapes

There are many ways to draw shapes that are similar.

1 Draw a shape on an overhead projector transparency. Place the sheet on the overhead projector and move the projector closer and further away from the screen. Trace different-sized images of the same shape onto paper.

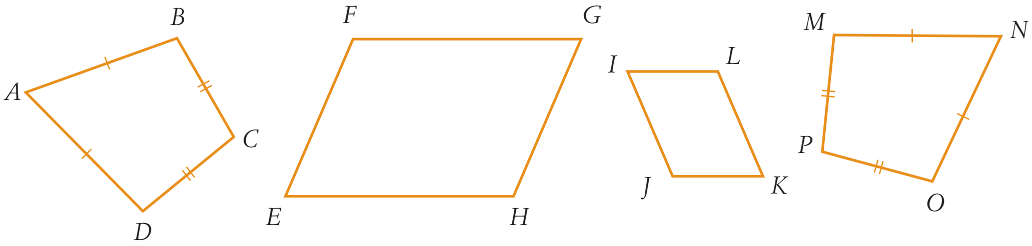
2 You can draw a variety of shapes using Microsoft Word or some other software package.
- Draw one of the basic shapes, such as a trapezium.
- Copy the shape and stretch the copy horizontally and vertically by dragging the edges of the drawing.

9780170194778

- Draw another shape and make a copy.
- Find out how to create similar shapes by enlarging or reducing the copy. Remember that the angles need to be the same in both figures for the shapes to be similar and the scale factor must be the same for all sides.
- Once you have created two shapes that are similar, find out how to rotate the shapes. Experiment with different rotations.

Example 12

Name the congruent and similar shapes in the following diagram.

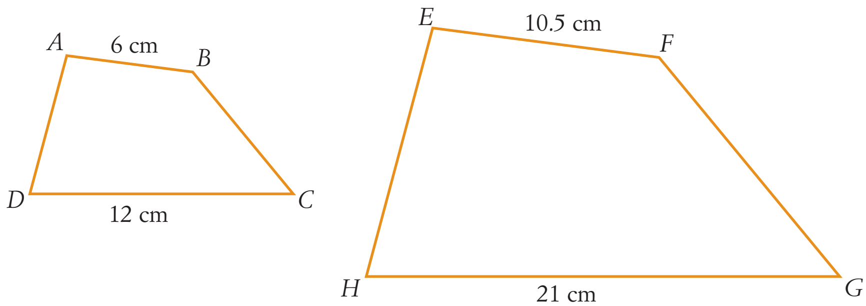


Solution

Compare the shapes by measuring sides and angles.	
Begin with *ABCD*. Identify any sides in another figure that are congruent with sides in *ABCD*.	$BC = MP$, $CD = PO$, $DA = ON$, $AB = MN$
The angles at the vertices of *ABCD* are the same as the angles at the vertices of *NMPO*.	
Identify the corresponding vertices.	$A \leftrightarrow N$, $B \leftrightarrow M$, $C \leftrightarrow P$ **and** $D \leftrightarrow O$
ABCD and *NMPO* are congruent.	$ABCD \equiv NMPO$
Compare the remaining two shapes.	
The angles at the vertices of *EFGH* are the same as the angles at the vertices of *ILKJ* and each side in *ILKJ* is half the corresponding side in *EFGH*.	
Identify the corresponding vertices.	$E \leftrightarrow I$, $F \leftrightarrow L$, $G \leftrightarrow K$ **and** $H \leftrightarrow J$
EFGH and *ILKJ* are similar.	$EFGH \parallel\!| ILKJ$

Example 13

ABCD has been enlarged to form the similar figure *EFGH*. Calculate the enlargement factor.



Solution

Identify the corresponding vertices.	$A \leftrightarrow E$, $B \leftrightarrow F$, $C \leftrightarrow G$ **and** $D \leftrightarrow H$
Identify corresponding sides.	$AB \leftrightarrow EF$, $BC \leftrightarrow FG$, $CD \leftrightarrow GH$ **and** $DA \leftrightarrow HE$
The enlargement factor is the ratio of corresponding sides.	**Enlargement factor** $= \dfrac{EF}{AB}$
Substitute known values.	$= \dfrac{10.5}{6}$
Evaluate.	$= 1.75$
State the result.	***ABCD* is magnified by an enlargement factor of 1.75 to give the figure *EFGH*.**
You can check the scale factor using the other two corresponding sides that have dimensions.	**Enlargement factor** $= \dfrac{HG}{DC}$
Substitute known values.	$= \dfrac{21}{12}$
Evaluate.	$= 1.75$

Symmetry

Some objects are more pleasing to look at than others. Look at the objects below.

9780170194778

The two shapes on the left look balanced and are said to be **symmetrical**. The two shapes on the right look out of balance and are said to be **asymmetrical**.
In mathematics there are two main types of **symmetry**: **line symmetry** and **rotational symmetry**.

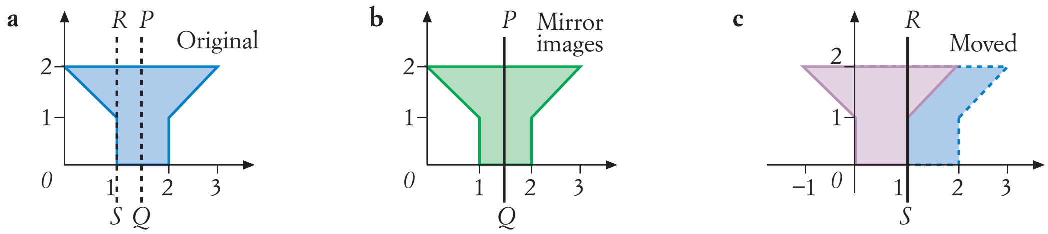
Important!

Line symmetry

A shape has **line symmetry** if it can be divided into two halves that are exact reflections of each other. The line of reflection is called a **line** (or **axis**) **of symmetry**.

Worksheet
Similarity, congruence and symmetry
MAT08MGWK00012

Look at shape **a** in the diagram below. Its reflection in the line PQ is shown in **b**. The shape stays the same, it doesn't even change position. The line PQ is a **line** (or **axis**) **of symmetry** and the shape has **line symmetry**. The shape can be folded across PQ onto itself.



If the shape above is reflected in the line RS as shown in **c**, the image is in a different position compared with the original shape. RS is therefore not a line of symmetry.

Investigate: The symmetry of faces

The left side of a human face is almost (but not quite) the mirror image of the right side. Look at the photo of a dog's face on the right. Composite pictures can be made using this photo.
If a line of symmetry is drawn vertically through the centre of the photo, each half looks like a mirror image of the other. The photos on the following page were made by merging the left (**a**) and right (**b**) halves of the first picture with their corresponding reflection. The contrast between the left and right halves is enhanced by the differences in lighting on the left and right halves in the original photo.

a 

b 

a Use a digital camera to take a photograph of each student in your group. Try to get them to look directly at the camera and not tilt their heads.
b Use a drawing or imaging program to create composite photos such as those shown above.
c Explain why the composites turn out the way they do.

Teacher notes
Symmetry of flags
MAT08MGTN00007

Many countries have national flags with geometric designs. If you want to investigate the symmetry of flags, your teacher may be able to give you some advice as to where to start. Some letters of the alphabet also have line symmetry. If you want to investigate the symmetry of the letters of the alphabet, your teacher may be able to give you a worksheet to get you started.

Puzzle sheet
Symmetry of the alphabet
MAT08MGPS00049

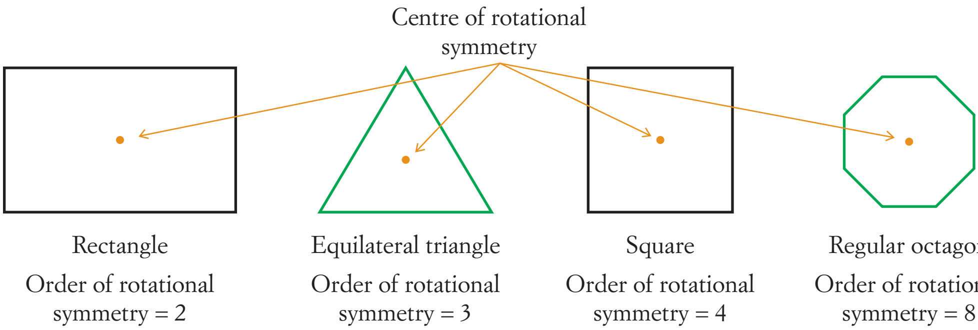
Important!

Rotational symmetry

A shape has **rotational symmetry** if when it is rotated about a central point it fits exactly on itself at least once before it completes a full revolution (360°). The number of times a shape fits exactly onto itself in one revolution (including its initial position) is called the **order of rotational symmetry** and the point about which the shape is rotated is called the **centre of rotational symmetry**.

Animated example
Rotational symmetry
MAT08MGAE00004

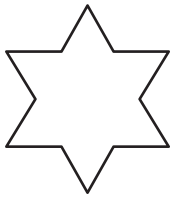
Some common shapes and their orders of rotational symmetry are shown below.



9780170194778

Example 14

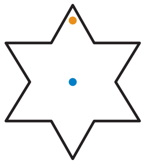
Calculate the order of rotational symmetry for this star shape.



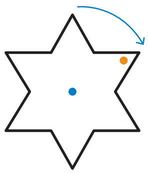
Solution

Trace the shape onto two pieces of paper. Cut out one shape.

Place a mark at the top of the cut-out shape.

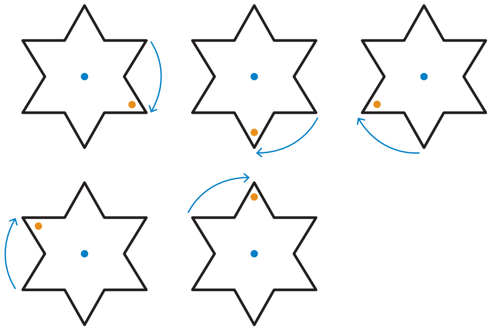


Locate the centre of rotational symmetry and pin the cut-out shape onto the shape on the piece of paper.



Rotate the cut-out shape until it exactly coincides with the original.

Repeat the process until the mark returns to its initial position, counting the number of times you do it.



The shape can be rotated 6 times in one revolution to fit exactly onto itself.

The star shape has an order of rotational symmetry of 6.

Exercise 2.4 Similarity, congruence and symmetry

Understanding

Extra questions
Exercise 2.4
MAT08MGEQ00007

Worked solutions
Exercise 2.4
MAT08MGWS00007

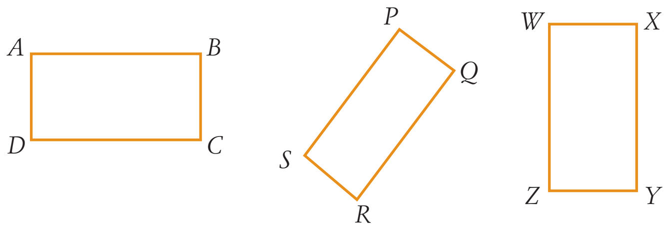
See Example 12

1 For each set of polygons shown below:

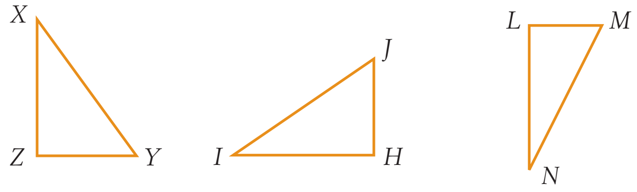
i measure the angles and sides of each polygon

ii identify corresponding points and name any congruent figures.

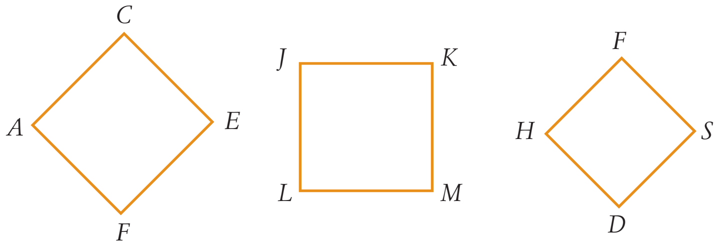
a



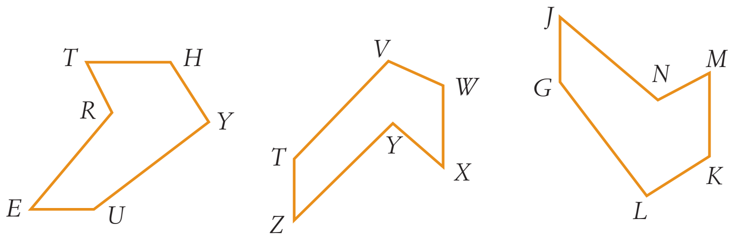
b



c



d



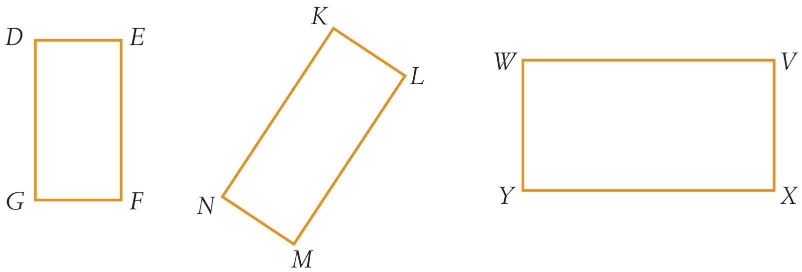
See Example 12

2 For each set of polygons shown below:

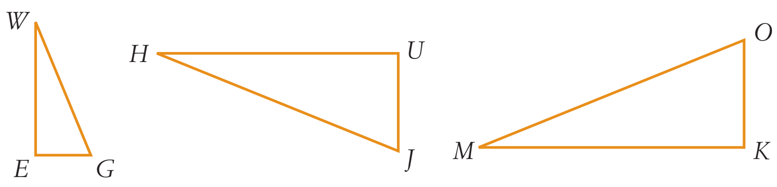
i measure the angles and sides of each polygon

ii identify corresponding points and name any similar figures.

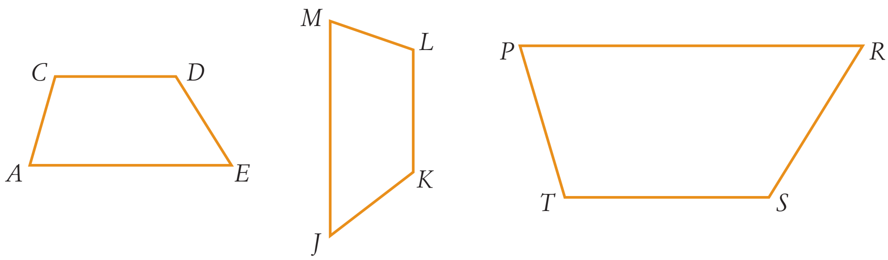
a



b



c



9780170194778

Fluency

See Example 12

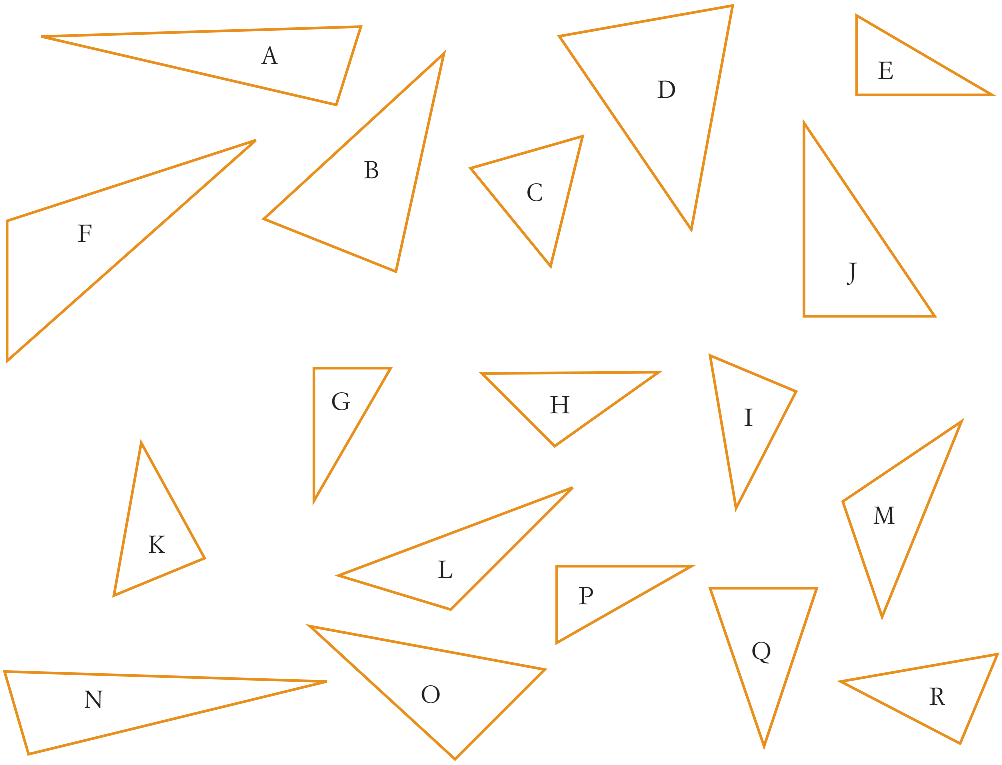
3 Write down all the congruent shapes in question **1**. (You will need to measure them.)

4 Of the triangles below:

a which is congruent to triangle **A**?

b which is congruent to triangle **E**?

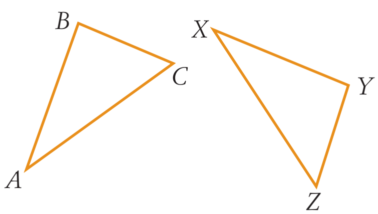
c which is congruent to triangle **I**?



5 For each pair of congruent shapes below, identify the equal sides and angles.

Worked solutions
Exercise 2.4
MAT08MGWS00007

a



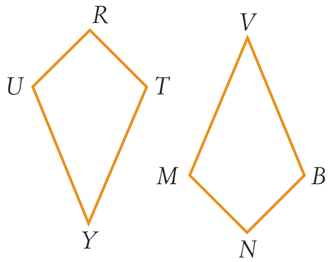
$\angle A =$

$BC =$

$\angle BCA =$

$AC =$

b



$\angle R =$

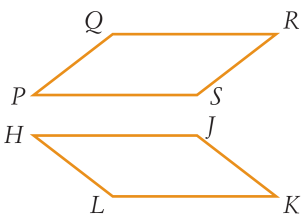
$\angle YTR =$

$RU =$

$TY =$

9780170194778

c



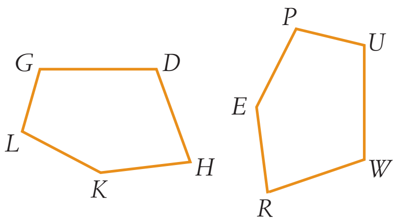
$QR =$

$RS =$

$\angle QRS =$

$\angle S =$

d



$LK =$

$\angle DHK =$

$\angle G =$

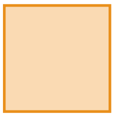
$DH =$

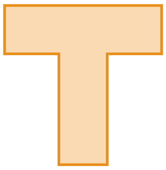
6 Look at each of the following shapes.

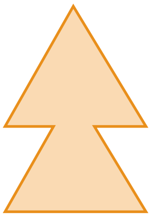
i List those shapes that have line symmetry.

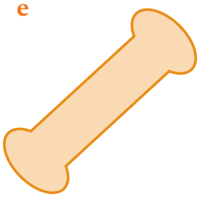
ii List those shapes that have rotational symmetry.

a

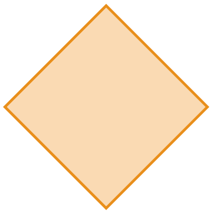

b


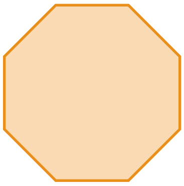
c


d


e


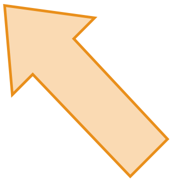
f

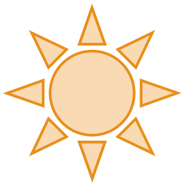

g


h


i


j


k


l


7 For each shape with line symmetry you identified in question **6**, redraw the shape and mark in the lines of symmetry.

See Example 14

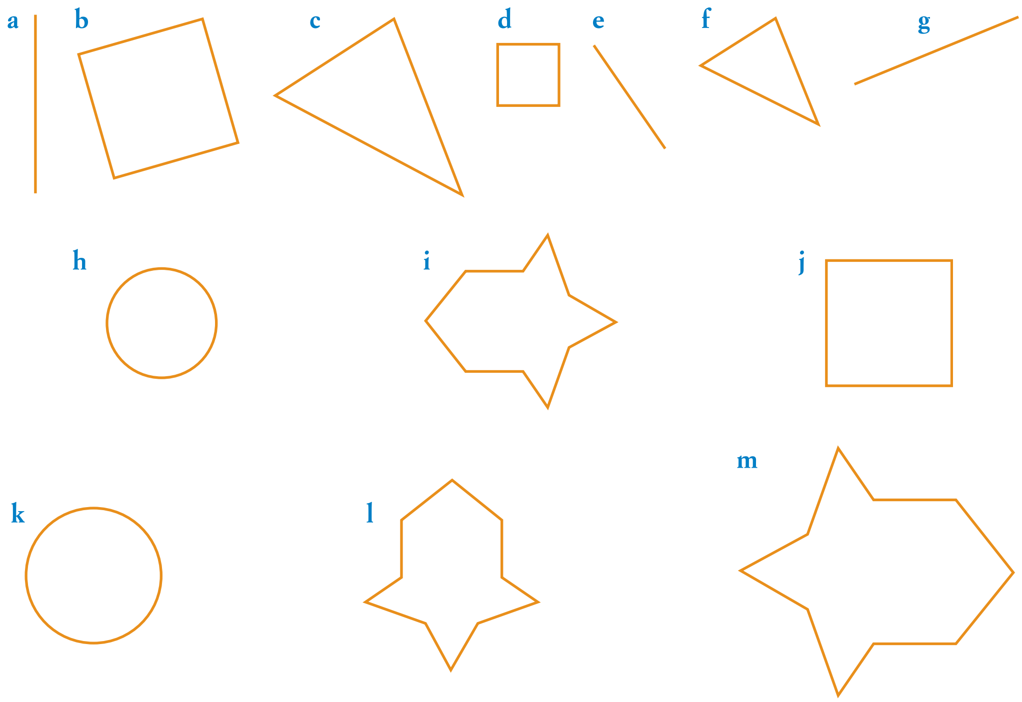
8 For each shape with rotational symmetry you identified in question **6**:

i redraw the shape and mark in the centre of rotational symmetry

ii calculate the order of rotational symmetry.

9780170194778

9 Write down all the similar shapes shown below. (You will need to measure them.) See Example 12



10 For each of the similar shapes in question **9**, measure a pair of corresponding sides and calculate the enlargement factor. See Example 13

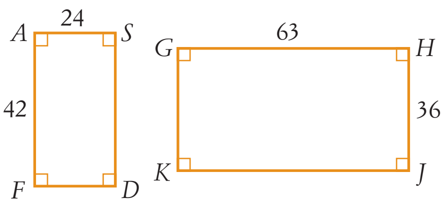
11 Decide whether each of the following pairs of figures are similar. If they are similar:

i name a pair of corresponding angles

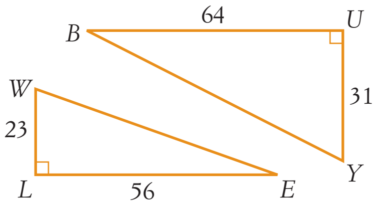
ii name a pair of corresponding sides

iii calculate the ratio of corresponding sides.

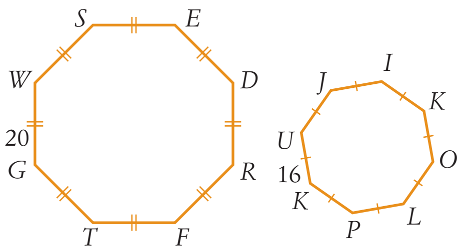
a



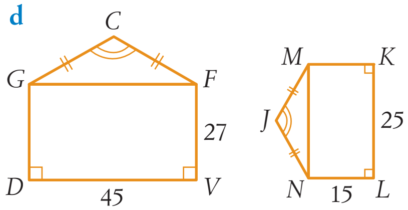
b



c



d



Problem solving

Worked solutions
Exercise 2.4
MAT08MGWS00007

12 Are all squares similar?

13 Are all rectangles similar?

14 Are all line segments similar?

15 Are all 10 cm line segments congruent?

16 Are all regular polygons similar?

17 Are all 10 cm cubes congruent?

18 Are all cylinders similar?

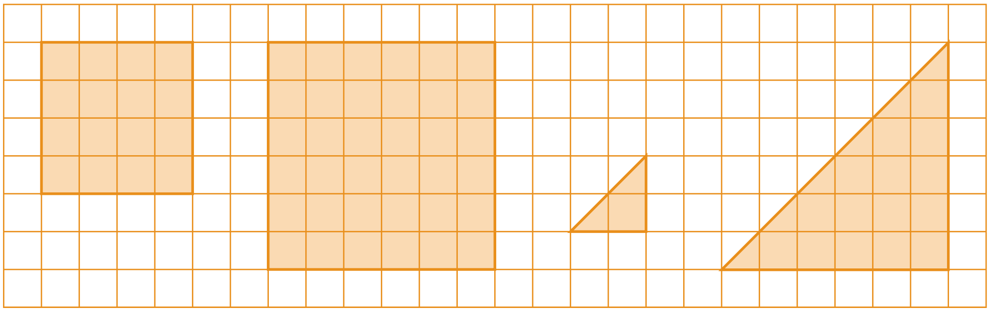
Reasoning

19 **a** Draw a shape on grid paper. Choose from a square, rectangle and right-angled triangle. Now draw some similar shapes. (Some examples are shown below.)

b Compare the lengths of the corresponding sides of the similar shapes. Also compare the areas of the similar shapes. (Count the squares to find areas.)

c Repeat the procedure using a different shape.

d Compare your results with those of others in the class.



9780170194778

Chapter 2 summary

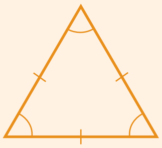
Quiz
Properties of shapes
MAT08MGQZ00002

- Most surfaces are either **flat** or **curved** shapes.
- The length, the width and the height of a solid are its **dimensions**.
- **Plane figures** or **two-dimensional** (**2D**) **figures** are flat shapes that can be drawn on a flat surface. Solid shapes are called **three-dimensional** (**3D**) **figures**.
- A plane figure can either be **closed** or **open**. Plane figures are specified by the names of their key points in order.
- A **polygon** is a closed figure made of line segments joined at their ends. Each line segment is called a **side** of the polygon. A **vertex** (plural **vertices**) of a polygon is a **corner** where sides meet. A polygon is labelled by listing its vertices in order. The number of sides a polygon has determines its name.

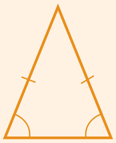
Number of sides	Name of polygon	Number of sides	Name of polygon
3	**Tri**angle	8	**Oct**agon
4	**Quadri**lateral	9	**Non**agon
5	**Pent**agon	10	**Dec**agon
6	**Hex**agon	11	**Undec**agon
7	**Hept**agon	12	**Dodec**agon

- A **regular** polygon has all its sides and angles equal. An **irregular** shape is one that is not regular.
- **Convex polygons** have vertices that all point outwards. **Concave** (non-convex) **polygons** have at least one inward pointing vertex.
- A **family of shapes** contains figures that have the same properties.
- Triangles can be classified using their side lengths or their angles as follows:

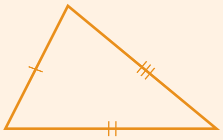
SIDES



Equilateral
Three equal sides

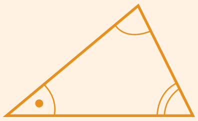


Isosceles
Two equal sides

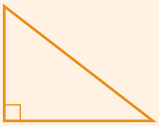


Scalene
No equal sides

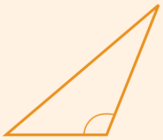
ANGLES



Acute-angled
All three angles acute



Right-angled
One right angle



Obtuse-angled
One obtuse angle

- There are six special types of convex quadrilaterals: the **trapezium** or **trapezoid**, the **parallelogram**, the **rhombus**, the **kite**, the **rectangle** and the **square**.
- A **cross-section** of a solid is the 2D shape obtained when a plane 'slices' across the solid parallel to the end face or base. Prisms have cross-sections that are congruent polygons. Pyramids and cones have cross-sections that are similar.

Chapter 2 summary

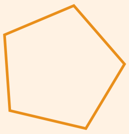
- A **transformation** changes a shape or its position to form an **image**.
- Three types of transformation produce an image that is congruent to the original shape.
  - A **translation** is a sliding movement where all parts of the shape move the same distance and direction.
  - A **rotation** is a clockwise or anticlockwise turn where all points remain the same distance from the centre of the turn (**centre of rotation**).
  - A **reflection** is a flip, with all points swapped to an equal distance on the other side of the **line of reflection**.
- An **enlargement** or a **reduction** is a transformation that produces an image that is similar to the original shape. The amount by which the shape is enlarged or reduced is called the **magnification**.
- Two shapes are **congruent** if one is an exact copy of the other. The symbol $\equiv$ means 'is congruent to'.
- Two shapes are **similar** if one can be made by enlarging or reducing the other. Similar shapes are the same shape but different sizes. The symbol ||| means 'is similar to'. The ratio of corresponding sides in similar figures is called the **scale factor**.
- A plane shape has **line symmetry** if it can be folded so that one half is a mirror image of the other. The fold is called the **axis of symmetry**. A plane shape has rotational symmetry if it can be rotated (spun) around a point called the **centre of symmetry** so that it fits onto itself before making a complete revolution.
- The number of times a shape fits exactly onto itself in one revolution is called the **order of rotational symmetry** and the point about which the shape is rotated is called the **centre of rotational symmetry**.

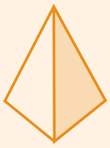
9780170194778

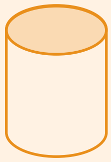
Chapter 2 review

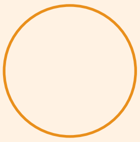
Understanding

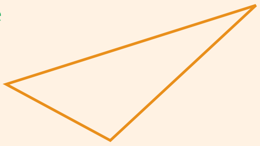
1 Write down whether each of these drawings shows a plane figure or a three-dimensional figure.

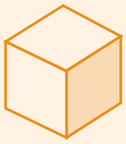
a


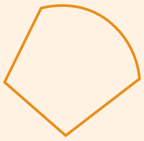
b


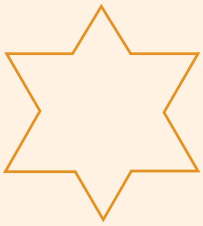
c


d


e


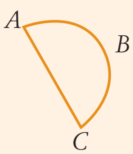
f


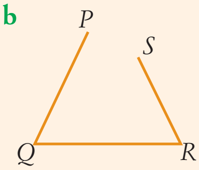
g


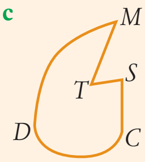
h


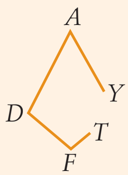
2 State whether each of the following shapes is open or closed.

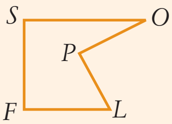
See Example 1

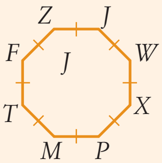
a


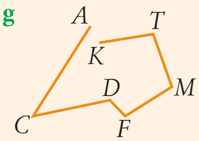
b


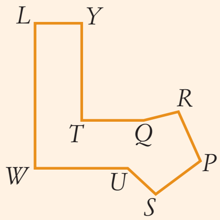
c


d


e


f


g


h


3 Name the vertices of shapes **c**, **e** and **f** in question **2**.

4 Give a possible letter name for each of the shapes **a**, **d**, **f** and **h** in question **2**.

5 State which of the figures in question **2** are polygons.

6 Draw each of the following solids on isometric dot paper.

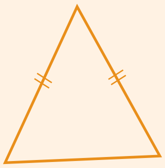
a square based pyramid **b** pentagonal prism **c** pentagonal pyramid

Weblink
Archimedian solids
MAT08MGWB00002

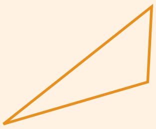
See Example 6

7 Classify each of the following triangles by both its angle name and its side name.

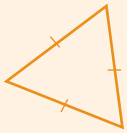
a



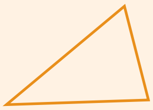
b



c



d

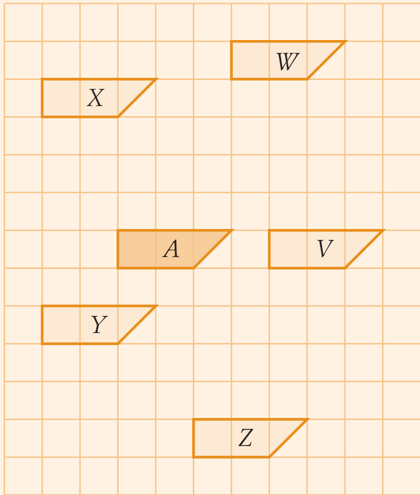


See Example 7

8 Shape A has been transformed to each of the images V, W, X, Y and Z using the same type of transformation.

a What sort of transformation has been used?

b Describe the transformation used to produce each of the images (V, W, X, Y and Z) from the original shape A.



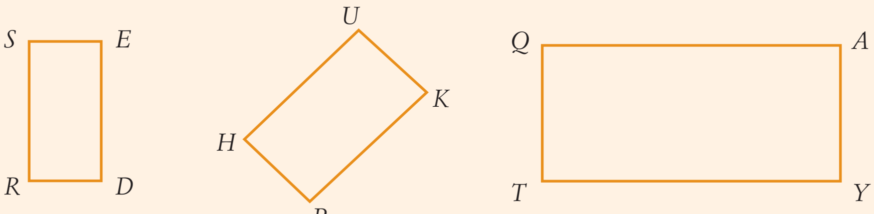
See Example 12

9 For each set of polygons shown below:

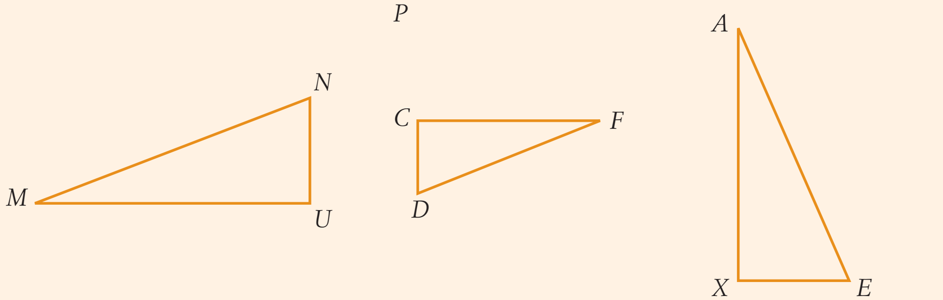
i measure the angles and sides of each polygon

ii identify corresponding points and name any congruent figures.

a

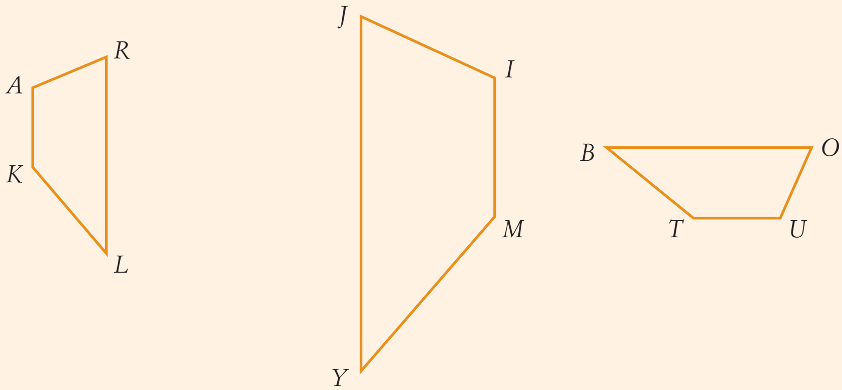


b



9780170194778

c



10 For each set of polygons shown in question **9**:

See Example 12

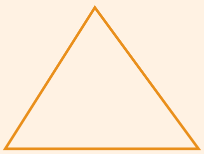
i measure the angles and sides of each polygon

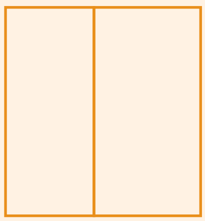
ii identify corresponding points and name any similar figures.

Fluency

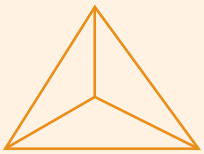
11 The plan and elevation of a solid is shown in each part below. Draw a sketch of the solid in each case.

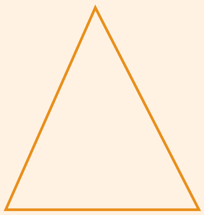
a


Plan


Elevation

b


Plan

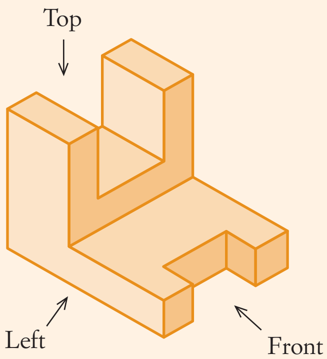

Elevation

12 For the solid shown on the right, draw:

a the front elevation

b the left elevation

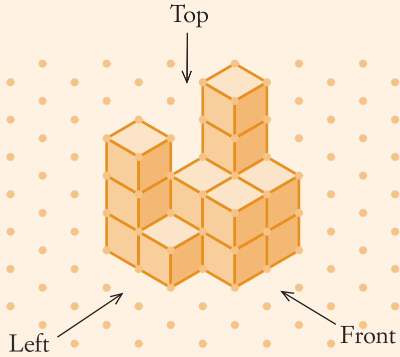
c the plan



See Example 3

13 This solid has been constructed using 14 cubes. Draw:

a the front elevation
b the left elevation
c the plan



See Example 2

14 Describe the polygons in question **2**.

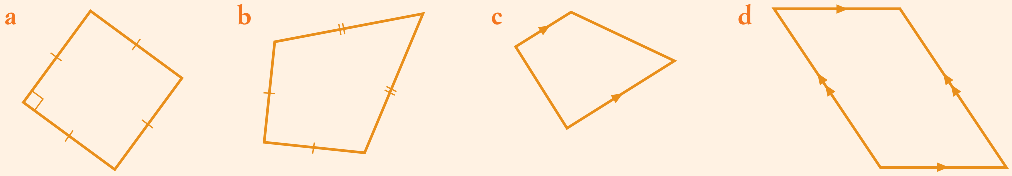
15 Draw a concave quadrilateral.

16 Draw an irregular, convex pentagon.

17 Sketch a triangle that is:

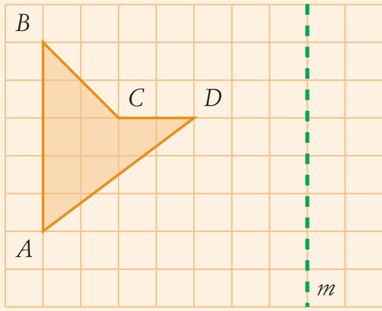
a obtuse-angled and isosceles
b acute-angled and scalene
c right-angled and isosceles

18 Classify each of the following quadrilaterals.



19 Copy this shape onto grid paper.

a Translate it 2 units right and 3 units up.
b Rotate it 90° clockwise around A.
c Reflect it in m.
d Enlarge it with a magnification of 2, keeping B the same.

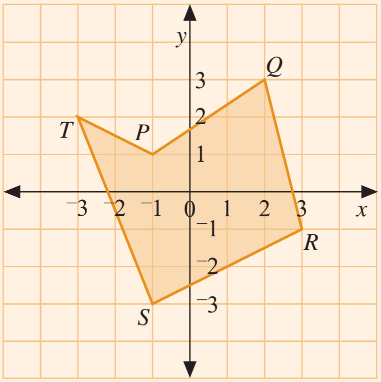


9780170194778

20 Copy this shape onto grid paper.

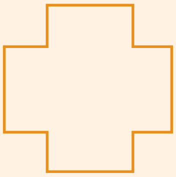
See Examples 7, 8, 9

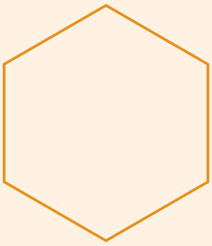
a Translate it 3 units left and 4 units down.
b Rotate it 270° clockwise around T.
c Reflect it in the x-axis.
d Enlarge it with a magnification of 3 using the origin as the centre of enlargement.
e Describe translation **a** using the coordinates of Q, R and S.
f Describe translation **c** using the coordinates of Q, R and S.



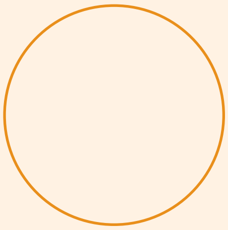
21 Which of the following shapes are similar and which are congruent?

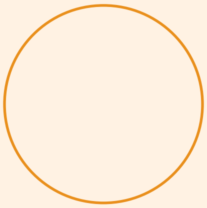
See Example 12

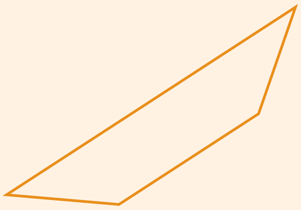
a


b


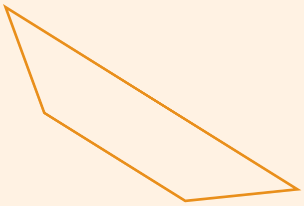
c

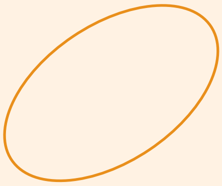

d


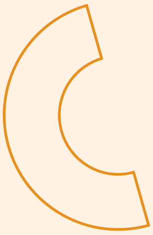
e


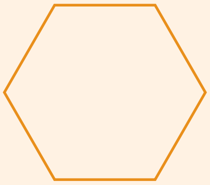
f


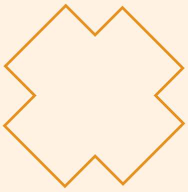
g


h


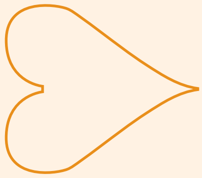
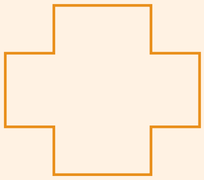
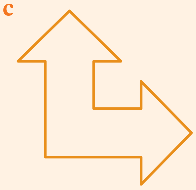
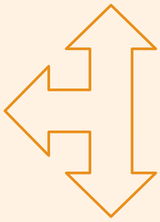
i


j


k


l


22 Which of the following shapes have line symmetry?

a

b

c

d

e

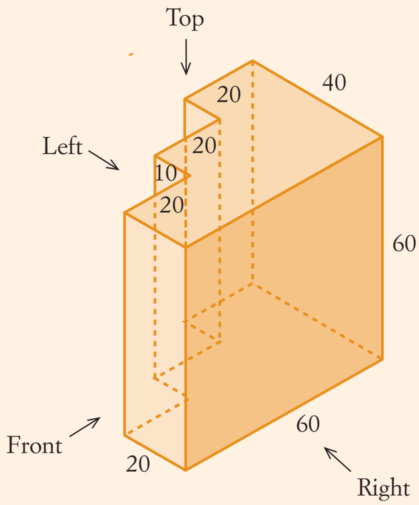
f


See Example 14
23 Which of the figures in question **22** have rotational symmetry?

24 For those figures with line symmetry identified in question **22**, draw the figure and mark in any lines of symmetry.

See Example 14
25 For those figures with rotational symmetry identified in question **22**:
- i draw the figure and mark the centre of rotational symmetry
- ii state the order of rotational symmetry

See Example 4
26 The diagram on the right shows a solid piece of metal. All dimensions are in millimetres. Draw:
- a the front elevation
- b the left elevation
- c the right elevation
- d the plan



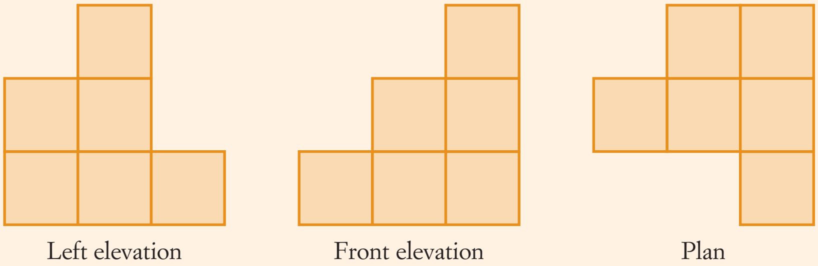
9780170194778

Problem solving

27 Why must concave quadrilaterals be irregular?

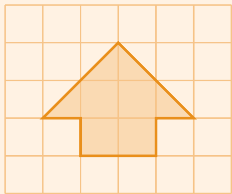
28 The front elevation, left elevation and plan view of a solid is shown in each part below. Draw an isometric sketch of the solid in each case.

See Example 5



29 Copy this shape onto grid paper and show how it tessellates.

See Example 11

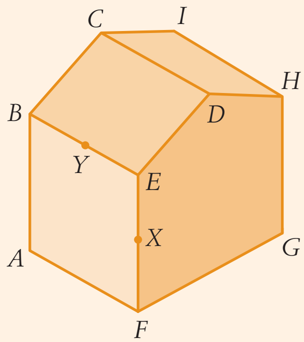


30 Are all equilateral triangles similar?

Reasoning

31 Look at the solid. Draw the shape that would result from the solid being sliced by a plane:

- **a** parallel to the base passing through X
- **b** parallel to the face $DEFGH$ passing through Y
- **c** perpendicular to the base passing through C and D
- **d** passing through C, D, F and A
- **e** perpendicular to the base passing through B and H



Number and algebra

3 Numbers and indices

9780170194778

Contents

Prior learning
Chapter 3
MAT08NAPL00003

Parent guide
Chapter 3
MAT08NAPG00003

Curriculum guide
Chapter 3
MAT08NACU00003

Australian Curriculum statements

Number and place value

Use index notation with numbers to establish the index laws with positive integral indices and the zero index.

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies.

Weblink
Aboriginal and Islander mathematics
MAT08NAWB00003

As human society became more complicated, different kinds of numbers were needed. These were invented to make it easier to think about numbers, to represent the world, and to perform accurate calculations. Very large numbers, such as the number of atoms in a teaspoonful of water or the number of stars in the Milky Way, have too many place values to be sensibly written in the normal way. Scientific notation was invented to show these numbers, and negative numbers were invented to help represent other aspects of the world.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

base	estimate	power	square
commutative	index	reciprocal	surd
cube	integer	root	
exponent	mantissa	scientific notation	

3.1 Positive and negative numbers

Positive and negative numbers can be used to show:

- movement north (positive) or south (negative)
- time interval before (negative) or after (positive) a particular event
- money paid into (positive) or taken out of (negative) a bank account.

All positive numbers are bigger than zero and all negative numbers are less than zero. You can place positive and negative numbers on the number line by extending the line to the left to include the negative numbers.

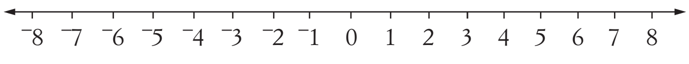
Example 1

Place the following on the number line and hence write them in order from smallest to largest.

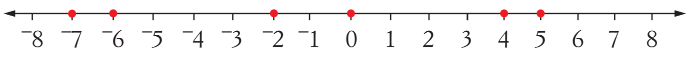
$^{-}6, 5, ^{-}2, 0, 4, ^{-}7$

Solution

Draw a number line from $^{-}8$ to 8.



Use dots to show each number.



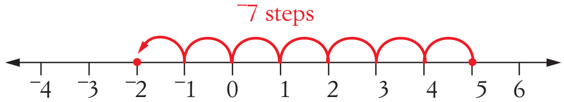
Use the line to write the numbers in order.

In order from smallest to largest, the numbers are:

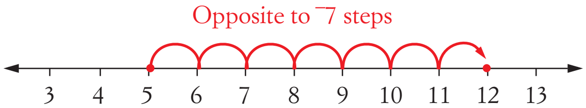
$^{-}7, ^{-}6, ^{-}2, 0, 4, 5$

9780170194778

Adding two numbers together is the mathematical equivalent of 'and', so '5 apples and 3 apples' is written as $5 + 3$.
If you move 5 km south and 3 km south, this could be written as $^{-}5 + {}^{-}3 = {}^{-}8$.
If you put \$5 into your account and take \$7 out, this could be written as $^{+}5 + {}^{-}7 = {}^{-}2$.
It is clear that $5 + {}^{-}5 = 0$, because a number and its negative are opposites. You can use this to calculate problems like $^{+}5 + {}^{-}7 = {}^{+}5 + {}^{-}5 + {}^{-}2 = {}^{-}2$. In primary school, you may have learned to add by counting on. We can use a number line to count with integers, but must count in the direction of the second number.
For $^{+}5 + {}^{-}7$, start at 5 and count 7 steps in the negative direction.



Since subtraction is the opposite of addition, we can subtract by *counting in the opposite direction to the second number*, or by adding the opposite number, so we can do $5 - 7$ as $5 + {}^{-}7$, or by counting 7 steps from 5 (but in the opposite direction to the second number $+7$).
Obviously, $5 - {}^{-}7$ must give a different answer. It is like putting \$5 into your bank account but also having the bank remove a \$7 charge it had charged earlier. You would be \$12 better off. We can do this mathematically using the opposite as $5 - {}^{-}7 = 5 + {}^{+}7 = 12$, or by stepping in the opposite direction to $^{-}7$ on a number line.
For $^{+}5 - {}^{-}7$, start at 5 and go in the opposite direction to $^{-}7$ steps.



Teacher notes
Addition and subtraction
MAT08NATN00008

Animated examples
Number lines
MAT08NAAE00002

Important!

Adding and subtracting integers

Directed numbers can be **added** by:

- starting from the first number and counting on the second number, in the direction of its sign; or
- for numbers with the same sign, adding normally and keeping the sign;
- and for numbers with opposite signs, taking away and using the sign of the part left over.

Directed numbers can be **subtracted** by:

- starting from the first number and counting on the second number, in the *opposite* direction to its sign; or
- adding the *opposite* of the number after the subtraction sign.

Worksheet
Magic squares
MAT08NAWK00020

Example 2

Work out the following.

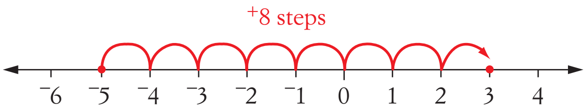
a $^{-}5 + 8$ **b** $4 + {}^{-}7$ **c** $^{-}2 + {}^{-}4 + 3$ **d** $3 - 7$
e $^{-}2 - 3$ **f** $3 - {}^{-}2$ **g** $^{-}4 - {}^{-}3$

Solution

Alternative method
Addition and subtraction
MAT08NAAM00001

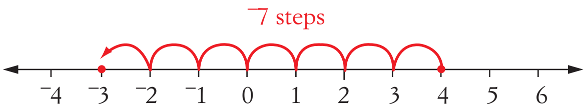
Method A: Number line

a Start from 5 and count 8 steps in the positive direction.



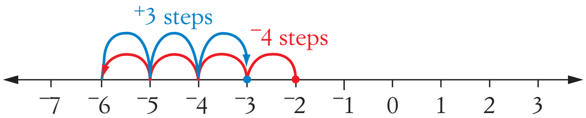
Write the answer. $\mathbf{^{-}5 + 8 = 3}$

b Start from 4 and count 7 steps in the negative direction.



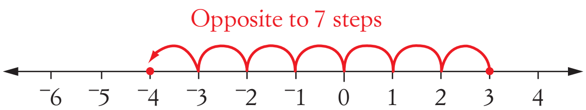
Write the answer. $\mathbf{4 + {}^{-}7 = {}^{-}3}$

c Start from $^{-}2$ and count 4 steps in the negative direction, and then 3 steps in the positive direction.



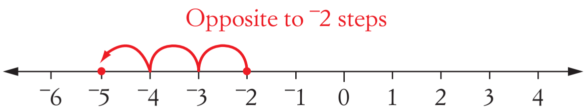
Write the answer. $\mathbf{^{-}2 + {}^{-}4 + 3 = {}^{-}3}$

d Start from 3 and count 7 steps in the opposite direction to $^{+}7$.



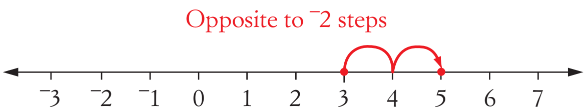
Write the answer. $\mathbf{3 - 7 = {}^{-}4}$

e Start from $^{-}2$ and count 3 steps in the opposite direction to $^{+}3$.



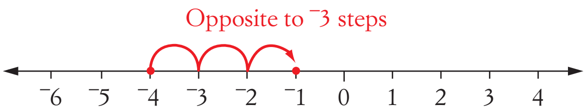
Write the answer. $\mathbf{^{-}2 - 3 = {}^{-}5}$

f Start from 3 and count 2 steps in the opposite direction to $^{-}2$.



Write the answer. $\mathbf{3 - {}^{-}2 = 5}$

g Start from $^{-}4$ and count 3 steps in the opposite direction to $^{-}3$.



Write the answer. $\mathbf{^{-}4 - {}^{-}3 = {}^{-}1}$

9780170194778

Method B: Opposites

a	Write the problem.	$^-5 + 8$
	Show 8 as 5 + 3.	$= {}^-5 + 5 + 3$
	Cancel $^-5 + 5$ and write the answer.	$= 3$
b	Write the problem.	$4 + {}^-7$
	Show $^-7$ as $^-4 + {}^-3$.	$= 4 + {}^-4 + {}^-3$
	Cancel $4 + {}^-4$ and write the answer.	$= {}^-3$
c	Write the problem.	$^-2 + {}^-4 + 3$
	Show $^-4$ as $^-1 + {}^-3$.	$= {}^-2 + {}^-1 + {}^-3 + 3$
	Cancel $^-3 + 3$ and add $^-2 + {}^-1$.	$= {}^-3$
d	Write the problem.	$3 - 7$
	Change to adding the opposite.	$= 3 + {}^-7$
	Show $^-7$ as $^-3 + {}^-4$	$= 3 + {}^-3 + {}^-4$
	Cancel and write the answer.	$= {}^-4$
e	Write the problem.	$^-2 - 3$
	Change to adding the opposite.	$= {}^-2 + {}^-3$
	Add and write the answer.	$= {}^-5$
f	Write the problem.	$3 - {}^-2$
	Change to adding the opposite.	$= 3 + 2$
	Add and write the answer.	$= 5$
g	Write the problem.	$^-4 - {}^-3$
	Change to adding the opposite.	$= {}^-4 + 3$
	Show $^-4$ as $^-1 + {}^-3$	$= {}^-1 + {}^-3 + 3$
	Cancel and write the answer.	$= {}^-1$

You can work out how to multiply and divide integers by thinking about speed, time and distance.
Suppose you are sitting on the beach when two joggers travelling south pass you.
A distance south of you would be negative and a distance north would be positive. The time before the joggers pass you would be negative and the time after they pass you would be positive.
If 4 hours before the joggers pass you they were 20 km north of you, their average speed would be $20 \div {}^-4 = {}^-5$ km/h or 5 km/h southwards.
If in the next 2 hours they go 10 km southwards, their average speed will still be $^-10 \div 2 = {}^-5$ km/h.

Their position 3 hours ago would have been $^-5 \times {}^-3 = 15$ km north of you.
Their position 6 hours from now will be $^-5 \times 6 = {}^-30$ km or 30 km south of you.

Teacher notes
Multiplication and division
MAT08NATN00009

Important!

Multiplication and division of integers

When two integers of the same sign are **multiplied** or **divided**, the answer is positive.
When two integers of different signs are **multiplied** or **divided**, the answer is negative.

Example 3

Use your calculator to work out the following.

a $^{-}8 \times 15$ **b** $^{-}23 + 18 + {}^{-}14$

Solution

a Enter as: (–) 8 × 15 =.

-8x15 -120

Write the answer. $\mathbf{^{-}8 \times 15 = {}^{-}120}$

b Enter as: (–) 23 + 18 + (–) 14 =.

-23+18+-14 -19

Write the answer. $\mathbf{^{-}23 + 18 + {}^{-}14 = {}^{-}19}$

Example 4

Technology worksheet
Excel: Integer quiz: Multiplication
MAT08NACT00005

Work out the following.

a $^{-}5 \times 3$ **b** $^{-}3 \times {}^{-}6$ **c** $^{-}12 \div {}^{-}3$ **d** $6 \div {}^{-}2$ **e** $^{+}15 \div {}^{+}3$

Solution

a Different signs, so the answer is negative. $\mathbf{^{-}5 \times 3 = {}^{-}15}$

b Same signs, so the answer is positive. $\mathbf{^{-}3 \times {}^{-}6 = 18}$

c Same signs, so the answer is positive. $\mathbf{^{-}12 \div {}^{-}3 = 4}$

d Different signs, so the answer is negative. $\mathbf{6 \div {}^{-}2 = {}^{-}3}$

e Same signs, so the answer is positive. $\mathbf{^{+}15 \div {}^{+}3 = 5}$

You should also be able to use your calculator for operations with integers. Remember that the negative key is shown as (–) on most calculators. On some calculators, to enter a negative number, you press the +/– key after the number to make it negative.
The normal order of operations also applies to positive and negative numbers.

9780170194778

Important!

Order of operations

The order of operations is as follows:
First: Work out the value within any grouping symbols, starting with the innermost grouping symbols: parentheses or round brackets ( ), square brackets [ ] and braces { }.
Second: Work out powers and roots as you come to them, going from left to right.
Third: Work out multiplication or division as you come to it, going from left to right.
Fourth: Work out addition or subtraction as you come to it, going from left to right.

Video tutorial
Order of operations
MAT08NAVT10002

TLF learning object
Exploring order of operations (L6543)
MAT08NAIN00003

Example 5

Work out the following.

a $^{-}12 - 7 + {}^{-}2 - {}^{-}8$ **b** $19 + {}^{-}8 \times 4 - {}^{-}7$ **c** $4[3 - ({}^{-}9 + 5)^2] + {}^{-}18 \div 3$

Solution

a They are all + and −, so go left to right. $\mathbf{^{-}12 - 7 + {}^{-}2 - {}^{-}8}$
Do $^{-}12 - 7$ first. $= \mathbf{^{-}19 + {}^{-}2 - {}^{-}8}$
Now do $^{-}19 + {}^{-}2$. $= \mathbf{^{-}21 - {}^{-}8}$
Do the last operation. $= \mathbf{^{-}13}$
You can use your calculator.
Enter as: (−) 12 − 7 + (−) 2 − (−) 8 =.

-12−7+-2−-8	-13

b Do the multiplication first. $\mathbf{19 + {}^{-}8 \times 4 - {}^{-}7}$
Do the addition and subtraction in order. $= \mathbf{19 + {}^{-}32 - {}^{-}7}$
$= \mathbf{^{-}13 - {}^{-}7}$
$= \mathbf{^{-}6}$
You can use your calculator.
Enter as: 19 + (−) 8 × 4 − (−) 7 =.

19+-8×4−-7	-6

c Do the innermost brackets first. $\mathbf{4[3 - ({}^{-}9 + 5)^2] + {}^{-}18 \div 3}$
Do the operations inside the brackets next, $= \mathbf{4[3 - {}^{-}4^2] + {}^{-}18 \div 3}$
doing the power before the subtraction. $= \mathbf{4[3 - 16] + {}^{-}18 \div 3}$
Do the × and ÷ before adding. $= \mathbf{4 \times {}^{-}13 + {}^{-}18 \div 3}$
Do the addition. $= \mathbf{^{-}52 + {}^{-}6}$
$= \mathbf{^{-}58}$
You can use your calculator.
Enter as:
4 × (3 − ((−) 9 + 5) x^2) + (−) 18 ÷ 3 =.

4×(3−(-9+5)²)+-18÷3	-58

Puzzle sheet
Directed numbers
MAT08NAPS00006

Puzzle sheet
Order of operations
MAT08NAPS00010

Worksheet
Integer review
MAT08NAWK00019

See Example 1

Exercise 3.1 Positive and negative numbers

Understanding

Extra questions
Exercise 3.1
MAT08NAEQ00008

1 Write the following groups of numbers in order from smallest to biggest.

a $5, {}^{-}7, {}^{-}9, 0, 6, {}^{-}4$ **b** ${}^{-}8, {}^{-}6, 4, 3, 10, {}^{-}11$ **c** $12, {}^{-}15, {}^{-}1, 10, 14, {}^{-}3$

2 Which is the largest of these numbers?

a ${}^{-}7, {}^{-}12, 5, 0, 6, {}^{-}5$ **b** $0, {}^{-}2, {}^{-}6, {}^{-}7, {}^{-}9, {}^{-}25$ **c** ${}^{-}6, 5, 3, {}^{-}11, {}^{-}19, 2$

Worked solutions
Exercise 3.1
MAT08NAWS00008

3 Which is the smallest of these numbers?

a ${}^{-}9, {}^{-}6, 5, 3, 2, {}^{-}1$ **b** ${}^{-}8, {}^{-}60, 50, 20, 65, {}^{-}50$ **c** ${}^{-}6, {}^{-}7, 3, 2, 0, {}^{-}2$

See Example 2

4 Work out the following.

a ${}^{-}6 + 4$ **b** $8 + {}^{-}5$ **c** ${}^{-}3 + {}^{-}6$ **d** ${}^{-}8 + 4$
e ${}^{-}7 + 7$ **f** $12 + {}^{-}15$ **g** ${}^{-}6 + {}^{-}14$ **h** $3 + {}^{-}8 + {}^{-}12$

See Example 2

5 Work out the following.

a $5 - {}^{-}7$ **b** ${}^{-}6 - 8$ **c** ${}^{-}5 - {}^{-}4$ **d** ${}^{-}8 - 3$
e ${}^{-}7 - {}^{-}4$ **f** $1 - 8$ **g** ${}^{-}15 - 17$ **h** ${}^{-}21 - 21$

Worked solutions
Exercise 3.1
MAT08NAWS00008

6 Work out the following.

a 3×4 **b** ${}^{-}4 \times 8$ **c** ${}^{-}5 \times {}^{-}7$ **d** $3 \times {}^{-}9$
e ${}^{-}6 \times 8$ **f** ${}^{-}7 \times {}^{-}2$ **g** ${}^{-}8 \times {}^{-}1$ **h** $4 \times {}^{-}2$

See Example 4

Fluency

See Example 5

7 Work out the following.

a ${}^{-}6 + 8 - {}^{-}4$ **b** $10 - 7 - {}^{-}5$ **c** $5 - {}^{-}4 + {}^{-}8$ **d** ${}^{-}7 - 4 - {}^{-}5$
e ${}^{-}9 + 4 - {}^{-}3$ **f** ${}^{-}4 - 3 - {}^{-}2$ **g** ${}^{-}8 + 7 - {}^{-}1$ **h** $5 + {}^{-}6 - {}^{-}4$

8 Work out the following.

a $20 \div {}^{-}4$ **b** ${}^{-}45 \div 9$ **c** ${}^{-}16 \div {}^{-}2$ **d** $\dfrac{{}^{-}18}{6}$
e ${}^{-}54 \div 9$ **f** $\dfrac{56}{{}^{-}7}$ **g** $36 \div {}^{-}6$ **h** $\dfrac{{}^{-}16}{{}^{-}4}$

9 Work out the following.

a $8 \times {}^{-}3$ **b** ${}^{-}24 \div {}^{-}6$ **c** ${}^{-}9 \times {}^{-}4$ **d** $30 \div {}^{-}5$
e $\dfrac{{}^{-}63}{7}$ **f** ${}^{-}10 \times 5$ **g** ${}^{-}7 \times {}^{-}5 \times {}^{-}2$ **h** $\dfrac{{}^{-}40}{{}^{-}5}$

10 Work out the following.

a ${}^{-}8 + 12$ **b** $6 - 11$ **c** ${}^{-}2 \times {}^{-}8$ **d** $33 \div {}^{-}11$
e ${}^{-}6 + {}^{-}11$ **f** $4 + {}^{-}19$ **g** ${}^{-}7 - {}^{-}9$ **h** ${}^{-}12 - {}^{-}3$
i $7 \times {}^{-}4$ **j** ${}^{-}35 \div 7$ **k** ${}^{-}5 \times {}^{-}9$ **l** ${}^{-}21 \div {}^{-}3$

11 Work out the following using a calculator.

a ${}^{-}30 + 26 + {}^{-}35$ **b** ${}^{-}27 - {}^{-}51 - 28$ **c** $43 - {}^{-}53 - 63$ **d** ${}^{-}29 + 51 - 36$
e ${}^{-}96 \times 14 \div {}^{-}21$ **f** $200 \div {}^{-}36 \times 27$ **g** ${}^{-}18 \times {}^{-}16 \times {}^{-}5$ **h** ${}^{-}250 \div 20 \times {}^{-}14$

12 Work out the following.

a $(10 + 2) \times 7$ **b** $29 - 4 \times 5$ **c** $11 + 5 \times 8$
d $42 \div 6 \times 5$ **e** $9 + 7 \times (6 \div 3)$ **f** $4 \times (22 - 14 + 1)$
g $7 \times 5 - 3 \times 6$ **h** $45 + 6 \times (42 - 35)$ **i** $9 + [3 \times (14 - 12) + 6]$
j $[51 - (77 \div 7)] \div [(7 + 5 \times 3) - 14]$

9780170194778

Problem solving

13 Juanita is \$53 overdrawn on her cheque account and has a debt of \$74 on her credit card. She has \$28 in her savings account and \$52 in the building society. She has only \$19 in her Christmas club account. Write her finances as directed numbers and work out her overall financial position.

14 A kingfisher dives into a river from a height 120 cm above the water. The fish it catches is 40 cm under the water. Write these as directed numbers and work out how far the bird dives.

15 Low tide is 36 cm below average water height and high tide is 36 cm above average water height. How far does the water rise up the piles of a jetty between low tide and high tide?

Reasoning

16 To a passenger looking out the window of an electric train, the power poles seem to be travelling north at 60 km/h. A truck appears to be moving north at 100 km/h and a car seems to be going south at 10 km/h. What are the true speeds of:

a the power poles? **b** the train? **c** the truck? **d** the car?

17 Juan is driving his car due east at a steady 65 km/h and Alison is going due west at 75 km/h. They pass each other at Roma at 12 noon.

a How far was each person from Roma at 10 a.m.?

b How far apart were they at 10 a.m.?

c How far will they each be from Roma at 3 p.m.?

d How far apart will they be at 3 p.m.?

18 An express lift is installed in a building. Work out the speed it is rising at (in floors/minute) if:

a 5 minutes ago it was 15 floors below

b 3 minutes ago it was 12 floors above

c 6 minutes from now it will be 24 floors below

d 8 minutes from now it will be 16 floors above.

Worked solutions
Exercise 3.1
MAT08NAWS00008

3.2 Indices

Multiplication can be extended to form **powers**.

Maths clip
Powers, indices and exponents
MAT08NAMC00002

Important!

A **power** is written with the **base** number at normal size and a small number at the top right called the **index** or **exponent**. It shows repeated multiplication. The index shows the number of times the base is in the multiplication.

A power is written in **extended form** by writing out the product in full as shown.

Base, Index

$4^5 = 4 \times 4 \times 4 \times 4 \times 4$

Power form, Extended form

The power shown is said as 'four to the fifth' or 'the fifth power of four'.

A number raised to the power 2 is called a **square**. 6^2 is said as 'six squared'.

A number raised to the power 3 is called a **cube**. 8^3 is said as 'eight cubed'.

Example 6

Work out the following powers by hand.

a 3^4 **b** Four cubed **c** 6^1

Solution

a Write the problem. $\mathbf{3^4}$

Write in extended form. $= \mathbf{3 \times 3 \times 3 \times 3}$

Multiply in steps. $= \mathbf{9 \times 3 \times 3}$

$= \mathbf{27 \times 3}$

$= \mathbf{81}$

b Write in numbers. $\mathbf{4^3}$

Write in extended form. $= \mathbf{4 \times 4 \times 4}$

Multiply in steps. $= \mathbf{16 \times 4}$

$= \mathbf{64}$

c Write the problem. $\mathbf{6^1}$

There is only one 6. $= \mathbf{6}$

The opposite of finding a power is finding a **root**.

Important!

Roots

The **surd** symbol $\sqrt{}$ is used to show roots. The nth root of a number x is shown as $\sqrt[n]{x}$ and is the number y, so $y^n = x$.
For example, $\sqrt[4]{16} = 2$ because $2^4 = 16$.
Finding the **square root** is the opposite of finding the square of a number (power 2).
The square root is usually shown without the small number ^{2}.
For example, $\sqrt{49} = 7$.
Finding a **cube root** ($\sqrt[3]{}$) is the opposite of finding a cube (power 3).
For example, $\sqrt[3]{216} = 6$.

Worksheet
Numbers and powers 1
MAT08NAWK00021

Worksheet
Numbers and powers 2
MAT08NAWK00022

Example 7

Find each of the following.

a $\sqrt{64}$ **b** $\sqrt[4]{81}$ **c** $\sqrt[3]{64}$

Solution

a Think of the square that gives 64. $\mathbf{8^2 = 64}$

Write the answer. $\mathbf{\sqrt{64} = 8}$

9780170194778

b Try some possibilities. $2^4 = 2 \times 2 \times 2 \times 2 = 16$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

Write the answer. $\sqrt[4]{81} = 3$

c Try some possibilities. $3^3 = 3 \times 3 \times 3 = 27$

$4^3 = 4 \times 4 \times 4 = 64$

Write the answer. $\sqrt[3]{64} = 4$

We can show multiplications of the same base as a pattern of powers.

$$\underset{=3}{3^1} \xrightarrow{\times 3} \underset{=9}{3^2} \xrightarrow{\times 3} \underset{=27}{3^3} \xrightarrow{\times 3} \underset{=81}{3^4} \xrightarrow{\times 3} \underset{=243}{3^5}$$

You can write this backwards to show division instead of multiplication.

$$\underset{=3}{3^1} \xleftarrow{\div 3} \underset{=9}{3^2} \xleftarrow{\div 3} \underset{=27}{3^3} \xleftarrow{\div 3} \underset{=81}{3^4} \xleftarrow{\div 3} \underset{=243}{3^5}$$

You can keep going back, but you need zero and negative exponents to continue the pattern.

$$\underset{=\frac{1}{81}=\frac{1}{3^4}}{3^{-4}} \xleftarrow{\div 3} \underset{=\frac{1}{27}=\frac{1}{3^3}}{3^{-3}} \xleftarrow{\div 3} \underset{=\frac{1}{9}=\frac{1}{3^2}}{3^{-2}} \xleftarrow{\div 3} \underset{=\frac{1}{3}}{3^{-1}} \xleftarrow{\div 3} \underset{=1}{3^0} \xleftarrow{\div 3} \underset{=3}{3^1} \xleftarrow{\div 3} \underset{=9}{3^2}$$

Working backwards with division will not work for a base of zero because we cannot divide by 0. It is clear that it will work for any other base.

Important!

Zero and negative exponents

We define $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).

Example 8

Work out:

a 8^0

b 7^{-3}

Solution

a Use the rule that $a^0 = 1$ for $a \neq 0$. $8^0 = 1$

b A power with a negative exponent is equal to the reciprocal of the positive power.

$$7^{-3} = \frac{1}{7^3} = \frac{1}{343}$$

Puzzle sheet

Index puzzle

MAT08NAPS00007

The power key on your calculator could be shown as ^, x^y or y^x. Many calculators also have a square key shown as x^2.
The root key may be shown as $\sqrt[n]{x}$, $\sqrt[x]{\ }$ or $x^{1/y}$. Many calculators also have a square root key shown just as $\sqrt{\ }$.
The negative key is shown as (−) on most calculators. It is different from the subtraction key, −. On some calculators you press the +/– key *after* the number to make it negative.
The order in which you need to enter powers may vary between calculators.

Example 9

Scientific calculator exercise
Numbers and indices
MAT08NASC00002

Use a calculator to find:

a 9^4 **b** $\sqrt{15}$ **c** 5^{-4} **d** $\sqrt[5]{200}$

Solution

a Enter as 9 x^y 4 =.

9^4 6561

Write the answer. $\mathbf{9^4 = 6561}$

b Enter as $\sqrt{\ }$ 15 =.

$\sqrt{15}$ 3.872983346

Write the answer. $\mathbf{\sqrt{15} \approx 3.873}$

c Enter as 5 x^y (−) 4 =.

5^{-4} 0.0016

Write the answer. $\mathbf{5^{-4} = 0.0016}$

d Enter as 5 $\sqrt[n]{x}$ 200 =.

$^5\sqrt{200}$ 2.885399812

Write the answer. $\mathbf{\sqrt[5]{200} \approx 2.885}$

Investigate: Powers with 5s and 9s

Your teacher may want you to work in groups for this investigation.
Work out 15^2, 25^2, 35^2 and 45^2.
Now work out 1×2, 2×3, 3×4 and 4×5.
Can you work out a rule for quickly finding the square of a number that ends in 5?
Work out 10^2 and 9^2.
Work out 20^2 and 19^2.
Work out 30^2 and 29^2.
Now work out 10×2, 20×2 and 30×2.
Can you work out a rule for quickly finding the square of a number that ends in 9?
See if you can find a rule for quickly finding the square of a number that ends in 1.

9780170194778

Example 10

Use the extended form and number laws to find the following as simple powers.

a $7^3 \times 7^5$

b $11^{10} \div 11^5$

c $(5^2)^3$

Solution

a Write the problem. $\mathbf{7^3 \times 7^5}$

Write each power in extended form. $= \mathbf{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}$

Write in index form as a single power. $= \mathbf{7^8}$

b Write the problem in fraction form. $\mathbf{\dfrac{11^{10}}{11^5}}$

Write each power in extended form. $= \dfrac{11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11}{11 \times 11 \times 11 \times 11 \times 11}$

Cancel where possible. $= \dfrac{11 \times 11 \times 11 \times 11 \times 11 \times \cancel{11 \times 11 \times 11 \times 11 \times 11}^{1}}{\cancel{11 \times 11 \times 11 \times 11 \times 11}^{1}}$

Simplify. $= 11 \times 11 \times 11 \times 11 \times 11$

Write in index form as a single power. $= \mathbf{11^5}$

c Write the problem. $\mathbf{(5^2)^3}$

Write the cube in extended form. $= \mathbf{5^2 \times 5^2 \times 5^2}$

Write each square in extended form. $= \mathbf{5 \times 5 \times 5 \times 5 \times 5 \times 5}$

Write in index form as a single power. $= \mathbf{5^6}$

Example 11

Use the extended form and number laws to find $(2 \times 7)^4$ as the product of simple powers.

Solution

Write the problem. $\mathbf{(2 \times 7)^4}$

Write in expanded form. $= \mathbf{2 \times 7 \times 2 \times 7 \times 2 \times 7 \times 2 \times 7}$

Use the commutative law. $= \mathbf{2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7 \times 7}$

Write as a product of simple powers. $= \mathbf{2^4 \times 7^4}$

Example 12

Use the extended form and number laws to find $(3^4 \times 5^2)^3$ as the product of simple powers.

Solution

Write the problem.	$(3^4 \times 5^2)^3$
Write the cube in extended form.	$= 3^4 \times 5^2 \times 3^4 \times 5^2 \times 3^4 \times 5^2$
Write each power in extended form.	$= 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
Use the commutative law.	$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
Write as a product of simple powers.	$= 3^{12} \times 5^6$

When you use operations or powers with powers, you get the same kind of results as shown in Examples **10**, **11** and **12**. You can express these using the following index laws.

Video tutorial
Index laws
MAT08NAVT10003

Important!

Index laws

1. To multiply powers of the same base, add the indices together and use the same base.
   $a^m \times a^n = a^{m+n}$
2. When one power is divided by another power of the same base, subtract the indices to give a single power to the same base.
   $a^m \div a^n = a^{m-n}$
3. When a power within brackets is raised to another power, multiply the indices together.
   $(a^m)^n \div a^{mn}$
4. A power of a product of two numbers is equal to the product of each number to that power.
   $(a \times b)^n = a^n \times b^n$

Example 13

Use the index laws to find the following as simple powers.

a $4^9 \times 4^5$ **b** $3^{12} \div 3^6$ **c** $(6^7)^3$

Solution

a	Write the problem.	$4^9 \times 4^5$
	Use the index law for multiplication of powers.	$= 4^{9+5}$
	Simplify.	$= 4^{14}$
b	Write the problem.	$3^{12} \div 3^6$
	Use the index law for division of powers.	$= 3^{12-6}$
	Simplify.	$= 3^6$

9780170194778

c Write the problem. $(6^7)^3$

Use the index law for a power of a power. $= 6^{7 \times 3}$

Simplify. $= 6^{21}$

Example 14

Use the index laws to find $(8 \times 11)^7$ as the product of simple powers.

Solution

Write the problem. $(8 \times 11)^7$

Use the index law for the power of a product. $= 8^7 \times 11^7$

Exercise 3.2 Indices

Understanding

Extra questions
Exercise 3.2
MAT08NAEQ00009

1 Work out each of the following without a calculator.

a 4^3 b 2^6 c 7^4
d 5^2 e 9^3 f 8^4
g 14^2 h 11^3 i 5^3

Worked solutions
Exercise 3.2
MAT08NAWS00009

See Example 1

2 Use a calculator to find each of the following.

a 24^3 b 12^4 c 2^{10}
d 17^3 e 9^5 f 28^3
g 122^2 h 48^3 i 7^9

See Example 9

3 Work out each of the following without a calculator.

a $\sqrt{49}$ b $\sqrt{16}$ c $\sqrt{400}$
d $\sqrt{81}$ e $\sqrt{36}$ f $\sqrt{121}$
g $\sqrt{196}$ h $\sqrt{169}$ i $\sqrt{100}$

Worked solutions
Exercise 3.2
MAT08NAWS00009

See Example 7

4 Use a calculator to find each of the following.

See Example 9

a $\sqrt{500}$ b $\sqrt{5}$ c $\sqrt{50}$
d $\sqrt{30}$ e $\sqrt{300}$ f $\sqrt{3000}$
g $\sqrt{17}$ h $\sqrt{18}$ i $\sqrt{95}$

5 Use the extended form and number laws to find the following as simple powers. See Example 10

a $4^3 \times 4^2$ b $5^3 \times 5^4$ c $7^4 \times 7^3$
d $8^3 \times 8^5$ e $9^3 \times 9^5$ f $6^3 \times 6^8$

6 Use the extended form and number laws to find the following as simple powers. See Example 10

a $4^3 \div 4^2$ b $5^7 \div 5^4$ c $7^4 \div 7^3$
d $8^7 \div 8^5$ e $9^{11} \div 9^5$ f $5^{12} \div 5^8$

7 Use the extended form and number laws to find the following as simple powers. See Example 10

a $(3^2)^3$ b $(5^3)^3$ c $(6^2)^4$
d $(5^1)^4$ e $(7^3)^4$ f $(8^0)^5$

See Example 11

8 Use the extended form and number laws to find the following as simple powers.

a $(3 \times 5)^2$ **b** $(4 \times 3)^3$ **c** $(3 \times 7)^5$
d $(3 \times 4)^6$ **e** $(5 \times 8)^4$ **f** $(1 \times 7)^8$

See Example 12

9 Use the extended form and number laws to find the following as simple powers.

a $(3^2 \times 5^3)^2$ **b** $(2^4 \times 3^2)^3$ **c** $(5^4 \times 6^2)^2$
d $(7^1 \times 5^4)^3$ **e** $(11^4 \times 9^2)^4$ **f** $(5^5 \times 4^4)^5$

Fluency

10 Use the index laws to find the following as simple powers.

See Example 13

a $6^3 \times 6^2$ **b** $7^6 \times 7^4$ **c** $5^4 \times 5^{13}$
d $8^7 \times 8^{14}$ **e** $5^0 \times 5^{15}$ **f** $6^{23} \times 6^{28}$
g $3^6 \times 3^7 \times 3^8$ **h** $5^4 \times 5^4 \times 5^4$ **i** $2^{13} \times 2^{27} \times 2^{17}$

See Example 13

11 Use the index laws to find the following as simple powers.

a $5^7 \div 5^2$ **b** $6^{17} \div 6^4$ **c** $7^4 \div 7^1$
d $3^{27} \div 3^{15}$ **e** $7^{11} \div 7^{10}$ **f** $3^{12} \div 3^{12}$

See Example 13

12 Use the index laws to find the following as simple powers.

a $(7^3)^2$ **b** $(5^4)^3$ **c** $(9^3)^5$
d $(5^{10})^4$ **e** $(3^{12})^4$ **f** $(3^1)^{99}$

See Example 13

13 Use the index laws to find the following as the product of simple powers.

a $(4 \times 5)^2$ **b** $(3 \times 7)^5$ **c** $(6 \times 7)^{15}$
d $(5 \times 4)^{16}$ **e** $(5 \times 8)^{40}$ **f** $(1 \times 10)^{11}$

See Example 14

14 Use the index laws to find the following as the product of simple powers.

a $(3^6 \times 7^2)^3$ **b** $(4^6 \times 5^2)^4$ **c** $(3^6 \times 4^2)^{10}$
d $(2^7 \times 3^2)^8$ **e** $(5^1 \times 7^{12})^9$ **f** $(3^{33} \times 7^{12})^3$

Worked solutions
Exercise 3.2
MAT08NAWS00009
See Example 8

15 Work out each of the following without a calculator.

a 6^{-2} **b** 2^{-1} **c** 8^{-2}
d 7^{-1} **e** 5^0 **f** 23^0
g 34^{-1} **h** 10^{-3} **i** 2^{-7}

See Example 9

16 Use a calculator to find each of the following.

a 16^0 **b** 31^{-2} **c** 17^{-3}
d 20^{-3} **e** 8^{-5} **f** 5^{-4}
g 50^{-3} **h** 4^{-5} **i** 25^{-2}

See Example 7

17 Work out each of the following without a calculator.

a $\sqrt[3]{8}$ **b** $\sqrt[4]{256}$ **c** $\sqrt[3]{729}$
d $\sqrt[5]{32}$ **e** $\sqrt[3]{216}$ **f** $\sqrt[3]{64}$
g $\sqrt[8]{100\ 000\ 000}$ **h** $\sqrt[4]{100\ 000\ 000}$ **i** $\sqrt[10]{1024}$

See Example 9

18 Use a calculator to find each of the following.

a $\sqrt[3]{300}$ **b** $\sqrt[4]{75}$ **c** $\sqrt[5]{190}$
d $\sqrt[3]{6}$ **e** $\sqrt[4]{6}$ **f** $\sqrt[5]{6}$
g $\sqrt[7]{27}$ **h** $\sqrt[3]{10}$ **i** $\sqrt[4]{35}$

9780170194778

Reasoning

19 a Find the values of 4^3, 2^3 and 8^3 and find a relationship between them.
b Find the values of 5^4, 3^4 and 15^4 and find a relationship.
c Find the values of 7^2, 9^2 and 63^2 and find a relationship.

20 a Find the values of $\sqrt{6}$, $\sqrt[3]{6}$, $\sqrt[4]{6}$, $\sqrt[5]{6}$ and $\sqrt[6]{6}$.
b Find the values of $\sqrt{13}$, $\sqrt[3]{13}$, $\sqrt[4]{13}$, $\sqrt[5]{13}$ and $\sqrt[6]{13}$.
c What happens to the value of $\sqrt[n]{}$ for a number as n increases?

3.3 Scientific notation

Very large and very small numbers are often used in science. A special system called **scientific notation** or **standard notation** is used to write these numbers.

Important!

Scientific notation

Numbers written in scientific notation are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order** of the number is the power. For example:

$3\,200\,000 = 3.2 \times 10^6$ has mantissa 3.2 and order 6.

mantissa order

$0.000\,56 = 5.6 \times 10^{-4}$ has mantissa 5.6 and order $^-4$.

Example 15

Alternative method
Moving the decimal point 1
MAT08NAAM00002

Write the following in scientific notation.

a 6 000 000 000
b 450
c 90 361 700 000
d 0.0079
e 0.000 000 09

Solution

a Write as a product of a number from 1 to less than 10 and a 'tens' number.

6 000 000 000
= 6 × 1 000 000 000

Write with the 'tens' number as a power.

$= 6 \times 10^9$

b Write as a product of a number from 1 to less than 10 and a 'tens' number.

450
= 4.5 × 100

Write with the 'tens' number as a power.

$= 4.5 \times 10^2$

c Write as a product of a number from 1 to less than 10 and a 'tens' number.

90 361 700 000
= 9.036 17 × 10 000 000 000

Write with the 'tens' number as a power.

$= 9.036\,17 \times 10^{10}$

d Write as a product of a number from 1 to less than 10 and a 'tens' number. 0.0079
$= 7.9 \times 0.001$

Write with the 'tens' number as a power. $= 7.9 \times 10^{-3}$

e Write as a product of a number from 1 to less than 10 and a 'tens' number. $0.000\,000\,09$
$= 9 \times 0.000\,000\,01$

Write with the 'tens' number as a power. $= 9 \times 10^{-8}$

Example 16

Write the following as ordinary numbers.

a 7×10^5 **b** 3.8064×10^3 **c** $8.156\,345 \times 10^8$ **d** 6.75×10^{-5}

Solution

a Write the problem. 7×10^5

Write the power as a 'tens' number. $= 7 \times 100\,000$

Multiply to make an ordinary number. $= 700\,000$

b Write the problem. 3.8064×10^3

Write the power as a 'tens' number. $= 3.8064 \times 1000$

Multiply to make an ordinary number. $= 3806.4$

c Write the problem. $8.156\,345 \times 10^8$

Write the power as a 'tens' number. $= 8.156\,345 \times 100\,000\,000$

Multiply to make an ordinary number. $= 815\,634\,500$

d Write the problem. 6.75×10^{-5}

Write the power as a 'tens' number. $= 6.75 \times 0.000\,01$

Multiply to make an ordinary number. $= 0.000\,067\,5$

Alternative method
Moving the decimal point 2
MAT08NAAM00003

Puzzle sheet
Strong bird
MAT08NAPS00008

Puzzle sheet
A dog at a bone
MAT08NAPS00009

Worksheet
Scientific notation 1
MAT08NAWK00023

Most scientific calculators have an EXP or 10^x key to enter numbers in scientific notation. They automatically show large or small answers in scientific notation and can be set to show all answers in scientific notation.

Example 17

Use a calculator to work out each of the following.

a $5.6 \times 10^6 \times 6.3 \times 10^8$ **b** $50\,000 \times 760\,000\,000$ **c** $6.9 \times 10^{-5} \div 7.5 \times 10^{12}$

Solution

a Enter as 5.6 EXP 6 × 6.3 EXP 8 =.

5.6x10⁶x6.3x10⁸ 3.528x10¹⁵

Write the answer. $5.6 \times 10^6 \times 6.3 \times 10^8 = 3.528 \times 10^{15}$

Scientific calculator exercise
Numbers and indices
MAT08NASC00002

Worksheet
Scientific notation 2
MAT08NAWK00024

9780170194778

b Enter as 50 000 [×] 760 000 000 [=].

50000×760000000 3.8×10^{13}

Write the answer.
$50\,000 \times 760\,000\,000 = 3.8 \times 10^{13}$

Change to the form of the question.
$= 38\,000\,000\,000\,000$

c Enter as
6.9 [EXP] [(–)] 5 [÷] 7.5 [EXP] 12 [=].

$6.9\times10^{-5}\div7.5\times10^{12}$ 9.2×10^{-18}

Write the answer.
$6.9 \times 10^{-5} \div 7.5 \times 10^{12} = 9.2 \times 10^{-18}$

If your scientific calculator has no [EXP] key, you can still enter numbers in scientific notation, but you may have to use the [10^x] or [x^y] keys.
Some calculators display scientific notation with an E or 10^x in the display, such as:

$3.8E^{13}$ or 3.8×10^{13} or

3.8E13 or $3.8_{10}{}^{13}$

It is easy to press the wrong key on a calculator, so you need to be able to mentally check answers from calculators to see if they are sensible. You can use **leading digit calculation** for this checking process.

Important!

Leading digit calculation

1. Round off the numbers in the problem to just the first (leading) digit.
2. Do the problem with the rounded numbers.
3. Round the answer to one digit, taking account of the first roundings.

Example 18

Use leading digit calculation to mentally find approximate answers for the following.

a 437 809 + 190 087 **b** 469 × 7078 **c** 5192 ÷ 28

Solution

a Think of the problem.
437 809 + 190 087

Round off to the highest place values.
≈ 400 000 + 200 000

Do the calculation in your head.
= 600 000

Write the answer.
437 809 + 190 087 is about 600 000 or 6.0×10^5.

b Think of the problem.
469 × 7078

Round off to the highest place values.
≈ 500 × 7000

Rearrange in your head.
= 5 × 7 × 100 000

Do the calculation in your head.	**= 3 500 000**
Round to one digit (down in this case).	**= 3 000 000**
Write the answer.	**469 × 7078 is about 3 000 000 or 3.0 × 10^6.**
c Think of the problem.	**5192 ÷ 28**
Round off to the highest place values.	**≈ 5000 ÷ 30**
Cancel down in your head.	**= 500 ÷ 3**
Do the calculation in your head.	**≈ 170**
Round to one digit.	**≈ 200**
Write the answer.	**5192 ÷ 28 is about 200.**

Leading digit calculation gives an **estimate** of the answer to a problem. We can use estimation in other kinds of problems.

Important!

Estimation

An **estimate** is an approximate answer to a problem, worked out using available information.

Example 19

Australia has about 22 million people. For every family with children, there are probably about 4 or 5 people without children (single adults, childless couples, elderly people). A large number of children are from single-parent families. Families generally have an average of 2 or 3 children. Estimate the number of children.

Solution

Estimate the proportion of children.	**About 2.5 out of 10 Australians are children.**
Calculate the number of children.	**Number of children = 0.25 × 22 000 000** **= 5 500 000**
Write the answer.	**There are probably about 5.5 million or 5.5 × 10^6 children in Australia.**

9780170194778

Investigate: Fermi problems

Work in groups of 4 or 5 for this investigation.
Enrico Fermi (1901–54) was an atomic physicist who liked to set problems in estimation. Several estimates were needed to solve the problems.
Work in your groups to estimate the number of piano tuners in your home state of Australia. You will first need to estimate:

- the population of the state you live in
- the proportion of people who own pianos
- the number of times a piano would be tuned every year
- the number of pianos a piano tuner could tune in a week and hence in a year.

Use the same techniques to estimate the following.

- The number of hockey teams in Australia
- The number of surfboards on Trigg Beach in Western Australia.
- The amount of money in the pockets of everyone at school today.

Your teacher may want you to discuss your findings as a class group.

Extra questions
Exercise 3.3
MAT08NAEQ00010

Exercise 3.3 Scientific notation

Understanding

Worked solutions
Exercise 3.3
MAT08NAWS00010

1 Use leading digit calculation to mentally find approximate answers for the following.

a $563 + 897$ b $9302 - 4267$ c $70\,784 + 39\,437$
d $518 + 907 - 649$ e $3142 + 4097 + 6676$ f 52×76
g 23×248 h 320×675

Fluency

See Example 18

2 Use leading digit calculation to mentally find approximate answers for the following.

a $620 \div 34$ b $849 \div 198$ c $45\,600 + 98\,312$
d $500 + 67 + 984 - 1245$ e $480\,097 - 453\,200$ f $79\,043 \div 16\,424$
g $58\,320 \div 120$ h $684 \times 47 \times 976$ i $67\,894 \div 3450$
j $5087 \times 3512 \div 2125$

Worked solutions
Exercise 3.3
MAT08NAWS00010

See Example 15

3 Write each of the following numbers in scientific notation.

a 7000   b 0.05   c 40 000 000   d 90
e 0.000 005   f 3200   g 610 000   h 5 600 000
i 0.000 576   j 0.000 000 654   k 24 000   l 0.0875

See Example 16

4 Write the following as ordinary numbers.

a 6×10^3 b 8×10^{-5} c 1×10^7 d 3×10^{-1}
e 2.8×10^3 f 6.2×10^{-6} g 3.76×10^8 h 9.089×10^{-2}
i 4.401×10^3 j 6.789×10^{-9} k 3.2×10^3 l 1.75×10^{-4}

See Example 17

5 Calculate the following and write your answers in scientific notation.

a $48\,000 \times 600\,000$ b $6 \times 10^{-5} \times 4 \times 10^{-7}$
c $3.9 \times 10^{10} \times 450$ d $7.8 \times 10^{-18} \times 3.9 \times 10^{-2}$

e $7.8 \times 10^{15} \div 3.9 \times 10^{4}$
f $3.8 \times 10^{3} \div 2 \times 10^{21}$
g $5\ 000\ 000\ 000\ 000 \times 1790$
h $4.067 \times 10^{-7} \times 614\ 000$
i $1 \times 10^{7} \div 3.6 \times 10^{-14}$

Problem solving

6 In 2008 there were 5 808 100 male and 4 751 700 female workers in Australia. Average earnings were about \$985 per week for males and about \$651 per week for females.
a What is the total weekly gross salary for all the males in Australia?
b What is the total weekly gross salary for all the females in Australia?
c How much did all of Australia earn in a week?
d How much did all of Australia earn in a year?

Reasoning

7 a Use your calculator to work out $3 \times 10^{10} + 5 \times 10^{8}$.
b Now change $3 \times 10^{10} + 5 \times 10^{8}$ to ordinary numbers before doing the calculation. Compare your answer with the calculator answer.
c Now do $5 \times 10^{12} + 6 \times 10^{2}$ on your calculator and by hand. How do the answers compare?
d Find some additions that the calculator 'gets wrong' and explain why the answers are different from those obtained by hand.

8 In 2007, 5.532×10^{6} overseas visitors came to Australia. Of these, 2.886×10^{6} came on holiday, 1.125×10^{6} came to visit friends and relatives, and 6.158×10^{5} came on business.
a How many total visitors came to Australia to visit friends and relatives, to holiday, and to conduct business?
b How many came for other reasons?

Worked solutions
Exercise 3.3
MAT08NAWS00010

9 If the population of Perth is about 1.295×10^{6}, estimate the number of children in Perth. *See* Example 19

10 Sydney has a population of about 4 272 000. Estimate how much water is used in Sydney homes each day. *See* Example 19

11 Estimate the amount of water used each day in Melbourne homes.

12 Estimate the number of cars in Adelaide.

13 Estimate the number of newspapers sold each day in Townsville, Queensland.

14 Estimate the amount paid altogether for newspapers each day in Sydney.

15 Estimate the length of sealed roads in Australia.

9780170194778

Quiz
Numbers and indices
MAT08NAQZ00003

- **Negative numbers** are shown by a raised $^{-}$ symbol and extend the number line back from zero. Together with **positive numbers**, they form the **directed numbers** (**integers**). All positive numbers are greater than 0 and all negative numbers are less than 0.
- **Addition of integers** (directed numbers) may be done by:
  - starting from the first number and counting on the second number, in the direction of its sign; or
  - for numbers with the same sign, adding normally and keeping the sign; and for numbers with different signs, taking one from the other and using the sign of the part left over.
- **Subtraction of integers** (directed numbers) may be done by:
  - starting from the first number and counting on the second number, in the opposite direction to its sign; or
  - adding the opposite of the number after the subtraction sign.
- Addition and multiplication are **commutative**, but the order of subtraction and division cannot be changed.
- When two integers of the same sign are **multiplied** or **divided**, the answer is positive.
- When two integers of different signs are **multiplied** or **divided**, the answer is negative.
- The **order of operations** is:
  1. Work out brackets first, from innermost outwards and from left to right. A vinculum (bar) works like brackets for operations on top or underneath.
  2. Work out powers and roots from left to right.
  3. Work out multiplications and divisions next, from left to right.
  4. Work out addition and subtraction last, from left to right.
- When required to add, subtract, multiply or divide numbers you should use the most suitable method: mental arithmetic, written method or calculator. Division may be written using the ÷ or / symbol with the divisor last, or by using a vinculum with the divisor written under the other number.
- A repeated multiplication may be written as a **power**. The number multiplied is written once as the **base**. The number of times it is in the multiplication is written as the small **index** (**exponent**) at the top right. The **extended form** of a power has the multiplication written out in full.

This is called the index.

This is called the base.

$$a^4 = \overbrace{a \times a \times a \times a}^{\textbf{Extended form}}$$

- You can use **index notation** to write extended products and very large or small numbers more easily.
- A **square** is a power with index 2, and a **cube** has index 3.
- Powers with **zero** and **negative exponents** are defined for non-zero bases as $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).
- $a^1 = a$ because there is only one factor a.
- The nth **root** of a number x is written using the surd symbol as $\sqrt[n]{x}$. It is the new number y that must be raised to the nth power to give x, so $y^n = x$. The **square root**, $\sqrt{x}$, is written without the index number. The third root, $\sqrt[3]{\ }$, is called the **cube root**.

Chapter 3 summary

- **Index laws**
  1. To multiply powers of the same base, add the indices together and use the same base.
     $a^m \times a^n = a^{m+n}$
  2. When one power is divided by another power of the same base, subtract the indices to give a single power to the same base.
     $a^m \div a^n = a^{m-n}$
  3. When a power within brackets is raised to another power, multiply the indices together.
     $(a^m)^n = a^{mn}$
  4. A power of a product of two numbers is equal to the product of each number to that power.
     $(a \times b)^n = a^n \times b^n$
- Numbers written in **scientific notation** are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order** of the number is the power. For example, $460 = 4.6 \times 10^2$ and $0.0067 = 6.7 \times 10^{-3}$.
- **Leading digit calculation** is usually used to check answers by an approximation. It is done by rounding numbers to the first digit and then using mental calculation.
- An **estimate** is an approximate answer to a problem, worked out using available information.

9780170194778

Understanding

1 Work out each of the following.
a 5^3 b 7^4 c $\sqrt{25}$ d $\sqrt{60}$ (with a calculator)

2 Use leading digit calculation to mentally find approximate answers to the following.
a 58 320 + 26 473 b 29×67

3 Work out the following. See Example 4
a $^{-}5 + 8$ b $^{-}6 + {}^{-}7$ c $^{-}9 - 4$ d $7 - {}^{-}3$ e $^{-}5 - {}^{-}7$
f $6 \times {}^{-}5$ g $^{-}4 \times {}^{-}7$ h $42 \div {}^{-}3$ i $^{-}60 \div {}^{-}5$ j $^{-}18 \div {}^{-}3$

4 Use the extended form and number laws to find the following as simple powers. See Example 10
a $7^3 \times 7^5$ b $11^{10} \div 11^5$ c $(5^2)^3$

Fluency

5 Use the index laws to find the following as simple powers. See Example 13
a $5^9 \times 5^4$ b $4^{17} \div 4^8$ c $(3^8)^6$

6 Use leading digit calculation to mentally find approximate answers to the following. See Example 18
a $57241 \div 320$ b $2820 \times 574 \div 345$

7 Use the index laws to find $(3^6 \times 5^2)^4$ as the product of simple powers. See Example 12

8 Work out the following using a calculator. See Example 5
a $^{-}77 + {}^{-}28 - {}^{-}51$ b $560 \times {}^{-}90 \div {}^{-}80 - {}^{-}42$

9 Work out each of the following. See Example 9
a 3^{-4} b 9^0 c 6^{-1} d $\sqrt[3]{343}$ e $\sqrt[5]{32}$

10 Write each of the following in scientific notation. See Example 15
a 3420 b 0.000 65 c 634 000 000

11 Write each of the following as ordinary numbers. See Example 16
a 5.67×10^{-6} b 4.782×10^1 c 4×10^5

12 Calculate each of the following and write your answers in scientific notation. See Example 17
a $6.52 \times 10^9 \times 4.1 \times 10^{18}$ b $7.08 \times 10^{-5} \div 1.5 \times 10^{12}$

Problem solving

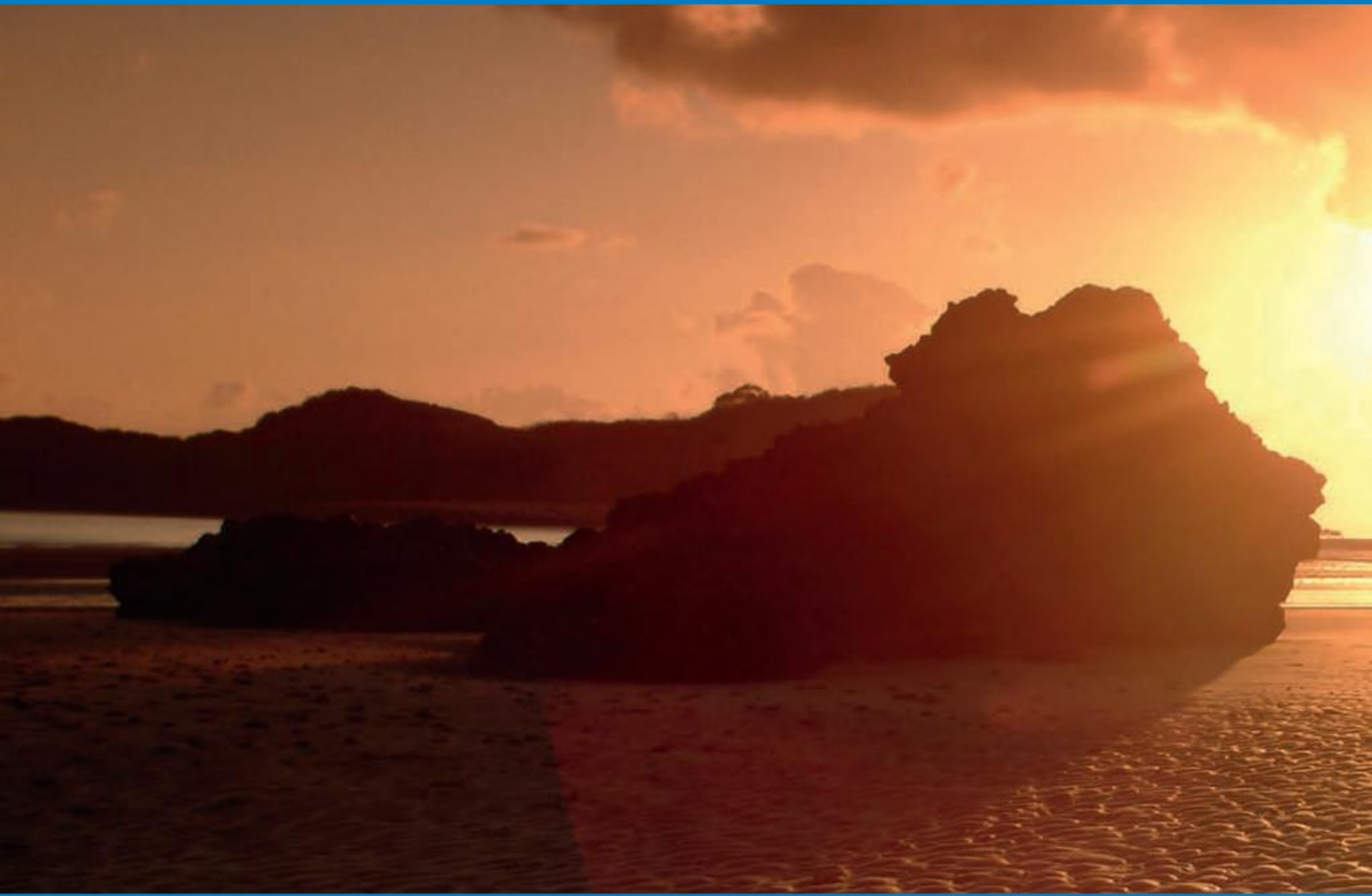
13 A submarine walkway on the Great Barrier Reef has windows 3 m below sea level. A trevally swims from the surface to a level 5 m below the windows. How far down does it swim?

14 Engines contain between 1700 and 1900 parts. Estimate the number of parts in 300 engines.

15 A crane is lifting a load of concrete to the top of a building under construction. Three seconds before it goes past a worker on the fifth floor (15 m off the ground) it is at a height of 9 m. Use directed numbers to find its speed and hence its height in another 4 s.

Reasoning

16 The charge on an electron is $^{-}1.6 \times 10^{-19}$ C (coulombs). How many electrons are needed to cancel out a charge of 0.006 C?



Number and algebra

4 Functions and graphs

9780170194778

Contents

Prior learning
Chapter 4
MAT08NAPL00004

Parent guide
Chapter 4
MAT08NAPG00004

Curriculum guide
Chapter 4
MAT08NACU00001

Australian Curriculum statements

Linear and non-linear relationships
Plot linear relationships on the Cartesian plane with and without the use of digital technologies.

The graph of a function can tell you a great deal. The way the graph changes tells you about how the function changes. The study of functions and their graphs helps scientists to accurately describe the world. This allows engineers to accurately predict how bridges, cars, trains, and other structures and machines will behave when they are built. In this chapter you will begin to learn how algebra and geometry are related through graphs of functions.

Mathematical literacy

Maths dictionary

MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

Cartesian plane	input	output	step function
coordinates	intercept	parabola	table of values
dependent	linear function	quadrant	travel graph
function	linear graph	rise	variable
gradient	midpoint	run	x-axis
independent	origin	satisfies	y-axis

4.1 Plotting points and lines

You have already learnt something about the Cartesian plane.

Puzzle sheet

Coordinates code puzzle

MAT08NAPS00013

Important!

The Cartesian plane

The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**.

The horizontal axis is called the ***x*-axis**; the vertical axis is called the ***y*-axis**. The axes cross at the **origin**.

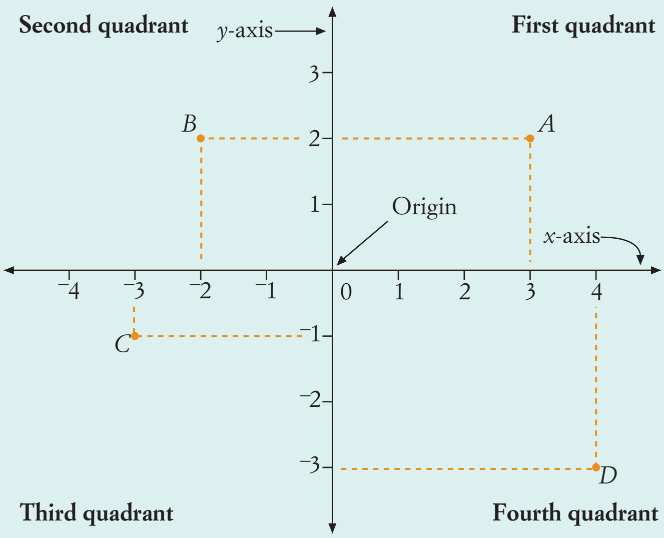
The position of a point on the Cartesian plane is determined by its **coordinates**.

The ***x*-coordinate** gives the horizontal position of the point from the origin, and the ***y*-coordinate** is the vertical position of the point from the origin. Coordinates are shown in parentheses (round brackets) with a comma between them.

The x-coordinate is always shown *first* and the y-coordinate *last*. The origin is the point (0, 0).

Each quarter of the Cartesian plane is called a **quadrant**.

In the diagram, Point A is written as (3, 2), B is $(^{-}2, 2)$, C is $(^{-}3, {}^{-}1)$ and D is $(4, {}^{-}3)$.



9780170194778

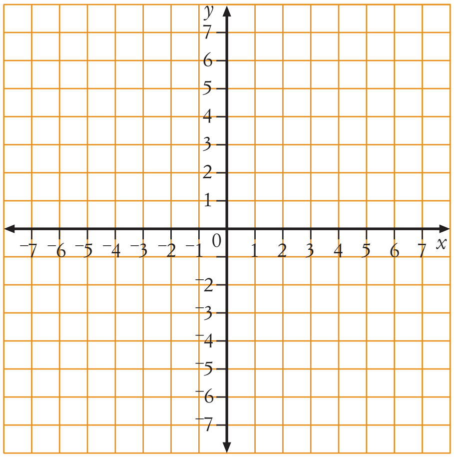
Example 1

Plot each of the following points on Cartesian axes and state its quadrant or axis.

$A(^-1, ^-3)$, $B(0, ^-8)$, $C(1, ^-2)$, $D(6, 0)$, $E(1, 3)$, $F(0, 5)$, $G(^-2, 2)$, $H(^-7, 0)$

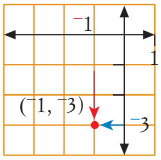
Solution

Draw and label axes to cater for the biggest number.



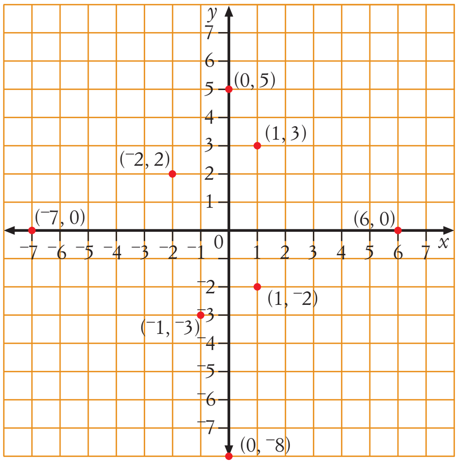
To plot $A(^-1, ^-3)$ go down from $^-1$ on the x-axis.

Then go left from $^-3$ on the y-axis.



Place a point at their intersection and label it.

Do the same for all the other points.



State the quadrant or axis of each point.

- $A(^-1, ^-3)$ is in the third quadrant
- $B(0, ^-8)$ is on the y-axis
- $C(1, ^-2)$ is in the fourth quadrant
- $D(6, 0)$ is on the x-axis
- $E(1, 3)$ is in the first quadrant
- $F(0, 5)$ is on the x-axis
- $G(^-2, 2)$ is in the second quadrant
- $H(^-7, 0)$ is on the y-axis

Worksheet
The number plane
MAT08NAWK00026

Worksheet
Plotting points and lines
MAT08NAWK00032

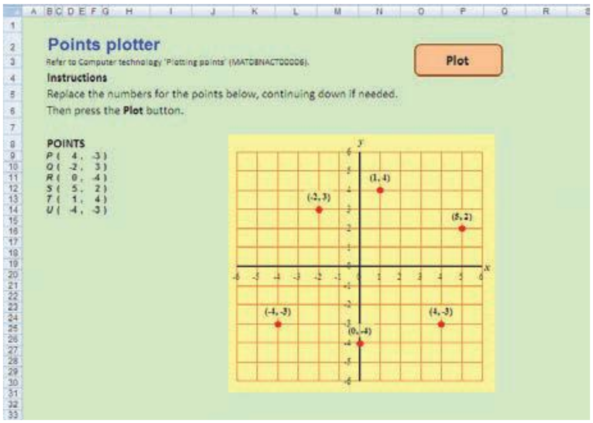
Worksheet
Number plane review
MAT08NAWK00027

Technology Plotting points

Technology worksheet
Excel: Plotting points
MAT08NACT00006

You can use digital technology to plot points. The spreadsheet 'Points plotter' on the NelsonNet website for this book automatically plots points. Other technologies may also be available at your school or demonstrated by your teacher.

Sometimes you will be required to plot points shown in a table. These could form a pattern such as a straight or curved line, or could just be a group of points.



Example 2

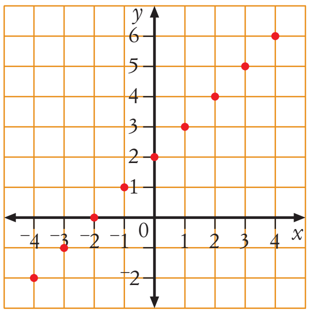
Plot the points in the table below. What shape is formed?

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4
y	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6

Solution

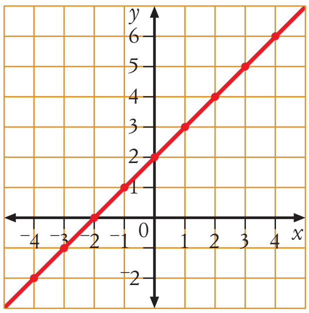
Use graph paper.

Plot the points, starting with ($^{-}4$, $^{-}2$).



The points appear to form a straight line. Check this with a ruler and use it to draw in the line.

Continue the line past the first and last points.



Write the answer.

The shape is a straight line passing through the axes at ($^{-}2$, 0) and (0, 2).

9780170194778

Example 3

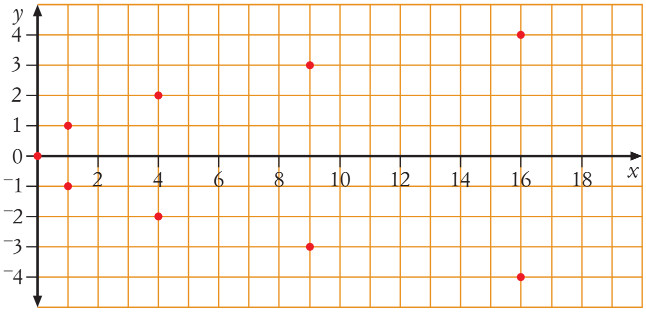
Plot the line given by the points in the table below. What shape is formed?

x	16	9	4	1	0	1	4	9	16
y	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4

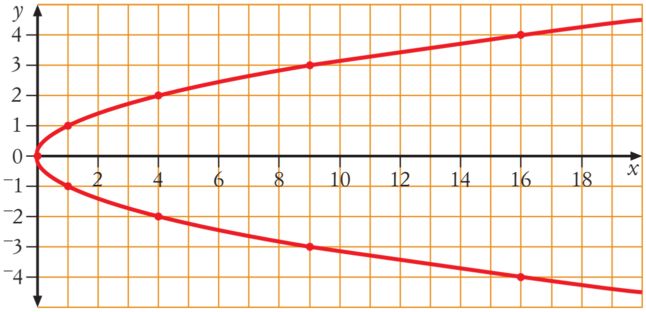
Solution

Since the x-values go from 0 to 16 and the y-values go from $^-4$ to 4, use different scales on the graph. Try to make the graph roughly square.

Plot the points, starting with (16, $^-4$).



Since the points seem to make a curve, join them in order with a smooth curve. Continue the curve past the first and last points.



Write the answer. **The shape of this line is a symmetrical curve.**

The curve in Example **3** is called a **parabola**. It is a roughly U-shaped curve symmetrical about a central line. The curve passes through the line of symmetry at a central point. Although it is roughly U-shaped, the distance between the curve and the line of symmetry increases as you get further away from the central point.

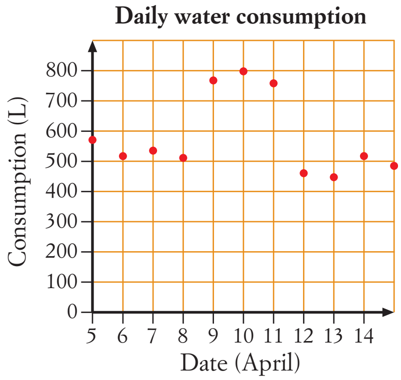
Example 4

The following data shows the water consumption of a family over ten days. Plot the graph and find out on what days the family had guests staying.

Date	5/4	6/4	7/4	8/4	9/4	10/4	11/5	12/4	13/4	14/4	15/4
Water use (L)	560	512	527	511	756	800	752	453	445	519	475

Solution

Plot the data.



Look for a pattern.

On April 5–8 and 12–15, the water consumption was around 500 L per day, but on April 9–11 it was close to 800 L per day.

Write a conclusion with reasons.

The increased water consumption from 9 April to 11 April suggests that the family had guests in that period.

Exercise 4.1 Plotting points and lines

Understanding

Extra questions
Exercise 4.1
MAT08NAEQ00011
See Example 1

1 Plot each of the following points on Cartesian axes.
$A(2, 3)$, $B(^-4, 5)$, $C(4, 7)$, $D(^-8, 6)$, $E(8, 5)$, $F(^-6, {}^-5)$, $G(^-4, {}^-1)$, $H(^-5, {}^-5)$, $I(^-5, {}^-6)$, $J(7, 3)$, $K(8, 7)$, $L(6, {}^-3)$, $M(2, {}^-5)$, $N(6, 2)$, $P(5, {}^-1)$, $Q(2, {}^-6)$, $R(^-5, {}^-2)$, $S(^-6, 4)$, $T(2, 6)$, $U(^-6, 3)$, $V(^-3, 0)$, $W(^-4, 4)$, $X(7, {}^-2)$, $Y(8, {}^-1)$, $Z(1, {}^-5)$

Fluency

2 Plot each of the following points on Cartesian axes and state its quadrant or axis.
$A(^-5, 1)$, $B(^-6, {}^-3)$, $C(6, 6)$, $D(2, {}^-3)$, $E(^-6, {}^-7)$, $F(3, {}^-5)$, $G(2, 2)$, $H(7, 6)$, $I(1, 5)$, $J(^-1, {}^-5)$, $K(^-4, 2)$, $L(0, 4)$, $M(^-3, {}^-5)$, $N(7, 3)$, $P(^-3, {}^-8)$, $Q(^-4, 0)$, $R(7, {}^-7)$, $S(^-3, 1)$, $T(^-2, 5)$, $U(6, {}^-3)$

See Example 2

3 Plot the points in the table below. What shape is formed?

x	$^-4$	$^-2$	0	2	4	6
y	$^-7$	$^-4$	$^-1$	2	5	8

9780170194778

4 Plot the points in the table below. What shape is formed?

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4
y	12	10	8	6	4	2	0	$^{-}2$	$^{-}4$

5 Plot the points in the table below. What shape is formed?

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6
y	$^{-}7$	$^{-}6$	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3

6 Plot the points in the table below. What shape is formed?

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6
y	9	8	7	6	5	4	3	2	1	0	$^{-}1$

7 Plot the points in the table below. What shape is formed?

x	$^{-}12$	$^{-}10$	$^{-}8$	$^{-}6$	$^{-}4$	$^{-}2$	0	2	4
y	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6

8 Plot the points in the table below. What shape is formed?

x	$^{-}10$	$^{-}8$	$^{-}6$	$^{-}4$	$^{-}2$	0	2
y	9	6	3	0	$^{-}3$	$^{-}6$	$^{-}9$

9 Without plotting, state the quadrant or axis of each of the following points.
$A(7, {^{-}3})$, $B(5, 6)$, $C(4, 8)$, $D({^{-}3}, {^{-}1})$, $E(1, 2)$, $F(6, 3)$, $G({^{-}5}, 2)$, $H(8, {^{-}6})$, $I({^{-}5}, 7)$, $J(7, {^{-}8})$, $K(6, {^{-}7})$, $L(2, {^{-}7})$, $M(3, {^{-}7})$, $N(3, 0)$, $P({^{-}5}, {^{-}7})$, $Q(1, 5)$, $R({^{-}2}, 7)$, $S(5, {^{-}7})$, $T({^{-}2}, {^{-}7})$, $U(0, {^{-}5})$, $V({^{-}5}, 3)$, $W({^{-}1}, 3)$, $X({^{-}1}, 6)$, $Y(5, {^{-}5})$, $Z({^{-}1}, 2)$

Problem solving

10 Plot the points in the table below. What shape is formed?

See Example 3

x	$^{-}6$	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6	7	8
y	46	33	22	13	6	1	$^{-}2$	$^{-}3$	$^{-}2$	1	6	13	22	33	46

11 Plot the points in the table below. What shape is formed?

Worked solutions
Exercise 4.1
MAT08NAWS00011

x	$^{-}5$	$^{-}4$	$^{-}3$	0	3	4	5	4	3	0	$^{-}3$	$^{-}4$	$^{-}5$
y	0	3	4	5	4	3	0	$^{-}3$	$^{-}4$	$^{-}5$	$^{-}4$	$^{-}3$	0

12 Plot the points in the table below. What shape is formed?

x	$^{-}2$	$^{-}1$	0	1	2	3	4	3	2	1	0	$^{-}1$	$^{-}2$
y	1	0	$^{-}1$	$^{-}2$	$^{-}1$	0	1	2	3	4	3	2	1

13 The table below shows the rain recorded on the Gold Coast in the first 10 days of February. Plot the points and comment on the results.

See Example 4

Date in February, d	1	2	3	4	5	6	7	8	9	10
Recorded rain, r (mm)	0.6	4	0.2	0	0	0	2.6	0.8	7.6	0.6

14 The table below shows the average height of 10 seedlings for a fortnight from when they first appeared. Plot the points and comment on the results.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Height (mm)	1	5	15	50	90	120	140	155	165	175	180	185	188	190

Worked solutions
Example 4.1
MAT08NAWS00011

15 The table below shows the costs of tyres in dollars and the average distances the tyres lasted. Plot the points and comment on the results.

Tyre cost in dollars	200	225	193	147	122	190	228	308	333	362
Distance, '000 km	54	27	37	46	29	48	22	49	59	36

Reasoning

Worked solutions
Example 4.1
MAT08NAWS00011

16 The table below shows the average weight in kg of fruit produced by tomato plants when different amounts of fertiliser (in mL) are added after flowering. Plot the results and decide on how much fertiliser should be used if the cost of the fertiliser is not important. Is this the way that tomato growers would make the decision?

Fertiliser (mL)	0	50	100	150	200	250	300	350	400	450	500
Tomatoes (kg)	4	9	11	12.5	13.5	14.5	14.3	14	13.4	12.8	12

17 A flood gauge by the side of a creek shows its water level. The creek is considered to be in flood if the level is over 2.5 m. The readings at hourly intervals on a particularly wet day were as shown below. Plot the points and state the flood time. What kind of flood was it?

Time	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900
Flood level (m)	0.8	0.8	0.9	2	6	7	4	1.5	1	0.9	0.85	0.8

18 After the air conditioner is switched on in a hot room (41°C), the room temperature starts to drop as shown by the information below. Plot the information to estimate how long it will be before the temperature drops below 30°C. When will the temperature fall below 0°C?

Time (minutes)	0	1	2	3	4	5	6	7
Temperature (°C)	41	40.2	39.4	38.6	37.8	37	36.2	35.4

4.2 Linear graphs

You have already done some work on graphs with straight lines in previous years.
A **travel graph** shows the distance travelled by something over time. When the speed is constant, you get a straight line, as shown in the following example.

Example 5

Animated example
Travel graphs
MAT08NAAE00005

A snail moves at 8 cm/minute. It moves forward for 3 minutes, then stays still for 2 minutes while it eats a lettuce leaf. Make a table of the distance it has moved from its starting point and draw a travel graph of its movement.

Solution

Show the time t for $t = 0$ to 5 minutes and the distance as d cm.

Time, t (minutes)	0	1	2	3	4	5
Distance, d (cm)						

It moves 8 cm every minute for the first 3 minutes.

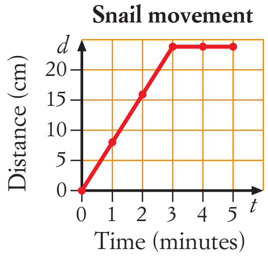
Time, t (minutes)	0	1	2	3	4	5
Distance, d (cm)	0	8	16	24		

It doesn't change its position in the next two minutes.

Time, t (minutes)	0	1	2	3	4	5
Distance, d (cm)	0	8	16	24	24	24

9780170194778

Draw the graph with time on the bottom and distance up the side.



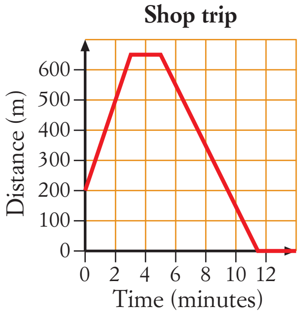
Notice that when the snail is moving forward, the graph slopes upwards. When the snail is stationary, the graph is flat. What would the graph look like if it moved back for the next minute? A travel graph can also tell you about what happened during a trip.

Example 6

Worksheet
The hare and the tortoise
MAT08NAWK00048

This graph shows John and Henry going to the shop from John's house and then going to Fred's house.

a How long did it take them to get to the shop?
b What does the flat part show?
c How long did they spend at the shop?
d How long did it take them to get back from the shop to Fred's house?
e How far was it from John's house to the shop?
f Find their speeds going to and from the shop. When did they travel faster?
g Who lived further from the shop?



Solution

a They left at 0 and got there at 3 minutes. **They took 3 minutes to get to the shop.**

b They are not moving when it's flat. **It shows them not moving; they are at the shop.**

c They got there at 3 and left at 5 minutes. **They spent 2 minutes at the shop.**

d They left at 5 and got there at $11\frac{1}{2}$ minutes. **It took them $6\frac{1}{2}$ minutes to get to Fred's.**

e They were at John's house at 200 m and at the shop at 650 m. **John's house is 450 m from the shop.**

f They took 3 minutes to travel 450 m to the shop. Work out the speed to the shop.

$$\textbf{Speed to shop} = \frac{\textbf{Distance}}{\textbf{Time}} = \frac{\textbf{450 m}}{\textbf{3 minutes}} = \textbf{150 m/minute}$$

Work out the speed from the shop.

$$\textbf{Speed to shop} = \frac{\textbf{Distance}}{\textbf{Time}} = \frac{\textbf{650 m}}{\textbf{6}\frac{1}{2}\textbf{ minutes}} = \textbf{100 m/minute}$$

Which was faster? — **They were faster on the way to the shop.**

g Compare the distances. — **John lives 450 m and Fred lives 650 m from the shop, so Fred is further from the shop.**

In the example above, you should be able to see that the graph is steeper when they were travelling faster. This was on the way to the shop.

Video tutorial
Graphing linear equations
MAT08NAVT10023

Important!

Linear graphs

A number rule is called a **function** in algebra. One number is changed into another one by the rule. The number put into the rule (usually on the right-hand side) is called the **input** or **independent variable**. The number that is produced by the rule is called the **output** or **dependent variable**.

A function can be **plotted** by putting different values into the function and calculating the **outputs**. This is usually put into a table called a **table of values**.

A rule that produces a straight line is called a **linear function** and the graph is called a **linear graph**. Linear functions are of the form $y = ?x \pm ??$, where ? and ?? are constants.

Puzzle sheet
Matching linear equations
MAT08NAPS00011

Puzzle sheet
Graphing functions
MAT08NAPS00014

Worksheet
Tables of values
MAT08NAWK00025

Worksheet
Graphing linear equations
MAT08NAWK00029

Example 7

a Make a table of values for the function $g = 3n - 2$ from $n = {}^-2$ to 4.
b Plot the function.
c Describe the function.

Solution

a Draw up the table with spaces for the n values from $^-2$ to 4.

n	$^-2$	$^-1$	0	1	2	3	4
g							

Work out the value of g by substituting $n = {}^-2$.

$$g = 3n - 2$$
$$= 3 \times {}^-2 - 2$$

Simplify.
$$= {}^-6 - 2$$

Calculate the answer.
$$= {}^-8$$

You can use your calculator.

Enter as: 3 [×] [(–)] 2 [–] 2 [=].

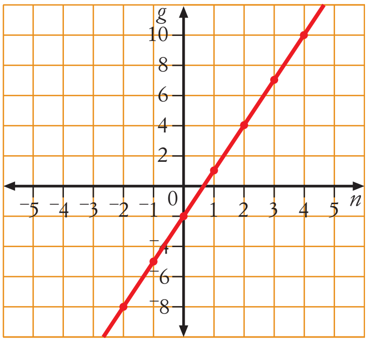
3x-2-2 -8

Put the answer into the table and work out the rest.

n	$^-2$	$^-1$	0	1	2	3	4
g	$^-8$	$^-5$	$^-2$	1	4	7	10

9780170194778

b Plot the points onto a Cartesian grid and join them up by drawing a straight line with a ruler.

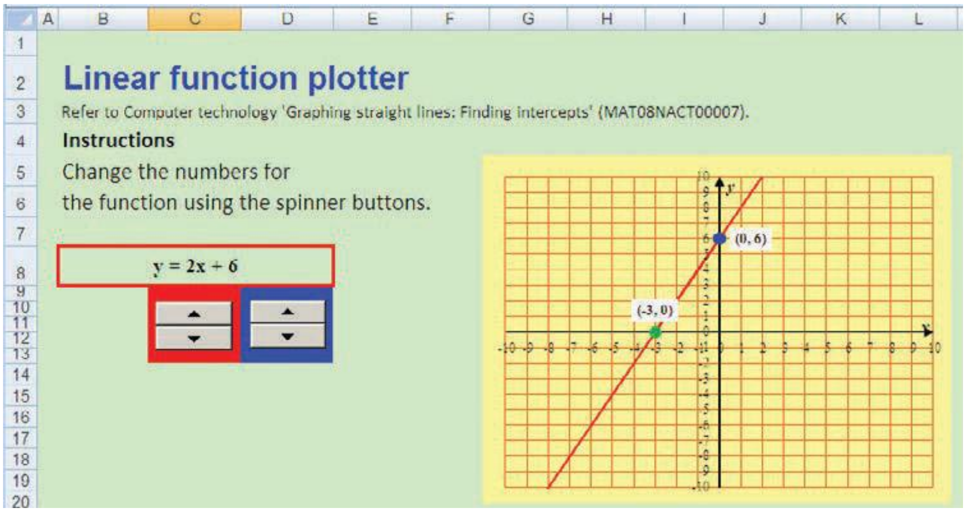


c Describe the graph of the function. **It is a linear function passing through the axes at (0, $^-$2) and ($\frac{2}{3}$, 0).**

Technology Linear functions

You can use digital technology to plot linear functions.
The spreadsheet 'Linear function plotter' on the NelsonNet website automatically plots points.
Other technologies may also be available on your school intranet (LAN) or demonstrated by your teacher.

Technology worksheet
Excel: Graphing straight lines: Finding intercepts
MAT08NACT00007



You can use linear functions and graphs to model many real-life situations.

Example 8

TLF learning object
Exploring linear equations (L6553)
MAT08NAIN00004

Weblink
Barbie Bungee
MAT08NAWB00004

A 5-bedroom holiday house that sleeps up to 12 people is available for rent for \$320 per night for two people plus \$20 for each additional person. Construct an algebraic model of the rent per night and plot the function. Then find the rent for 3 days for 5 people.

Solution

Write the rent for 2 people. **Rent for 2 people is \$320.**

If 5 people stay, there are 3 *extra*. **For n people, there are $(n - 2)$ extra people.**

Write the additional rent for n people. **Extra rent for n people $= 20 \times (n - 2)$**

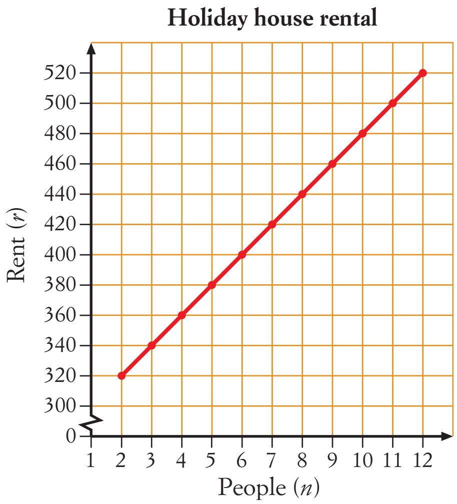
Write the total rent r for n people. $r = 320 + 20(n - 2)$

Complete a table of values for n from 2 to 12.

n	2	3	4	5	6	7	8	9	10	11	12
r	320	340	360	380	400	420	440	460	480	500	520

Plot the graph.

Since n is only from 2 to 12, do not draw the line past these limits.



Use the graph to find the rent for 5 people for three days. **The rent for 5 people is \$380/night. For three days, the rent $= 3 \times \$380 = \$1140$**

Write the answer. **The rent for 3 days for 5 people is \$1140.**

A **step function** has only flat parts on its graph, but the values usually get bigger at particular values of x. Like number lines, values at the end of the 'steps' that are not included are shown by an open, uncoloured circle. Values at the end that are included are shown by a coloured-in circle.

9780170194778

Example 9

A fruit shop gives a discount to customers who buy larger amounts of fruit. Apples are \$6.50/kg for up to 1 kg, \$5.50/kg for amounts up to another 3 kg, \$4.50/kg for amounts up to another 4 kg and \$4.00/kg for amounts over 8 kg. Draw a graph of the unit price function and describe it.

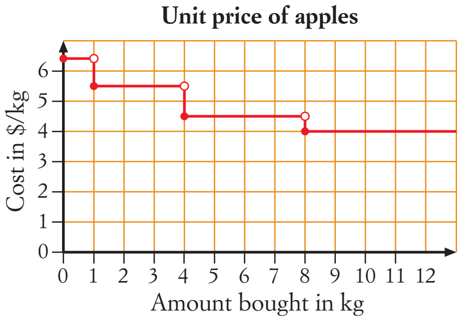
Solution

Draw the graph.

The unit price is \$6.50 from 0 kg up to, but not including, \$1. The line is flat for this part and is open at the \$1 end.

From 1 kg up to 4 kg it is at \$5.50 and is open at the 4 kg end. It is closed at the 1 kg end.

Keep going in a similar way.



Describe the graph.

The graph looks like uneven steps going down.

In Example **9**, it is cheaper to buy just over 1 kg than to buy 1 kg.

Exercise 4.2 Linear graphs

Understanding

Extra questions
Exercise 4.2
MAT08NAEQ00012

See Example 5

1 A car moves away from the lights in the left-hand lane at about 3 m/sec. After about 5 seconds the lanes merge and the car increases its speed to 5 m/s for 2 seconds before speeding up to 10 m/s. Draw a graph of its progress for 10 seconds and find how long it takes to travel 40 m.

2 A blue-tongued skink moves across a wide concrete path at 20 cm/s to a spot in the sun. It has to move a distance of 1.3 m across the path. Only 3 seconds after it settles, it hears someone coming along the path and moves back off the path, taking only 2 seconds to get off the path. Draw a graph of its movement and find how long it took to find the right spot.

Fluency

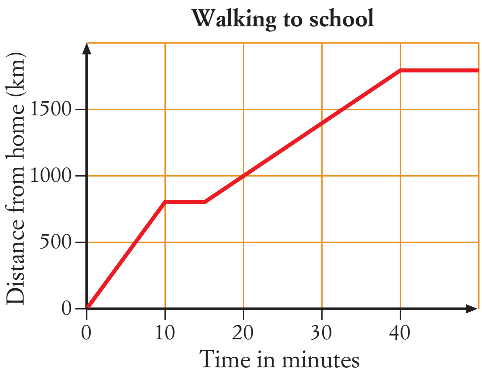
See Example 6

3 The travel graph on the right shows Jemima walking to school.

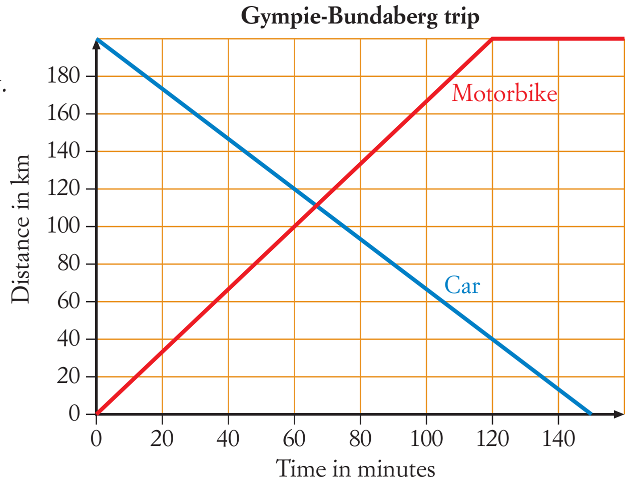
a How far is it from her house to the school?

b How long does she wait at her friend's house
before continuing to school?

c Give an explanation of the shape of the graph.



4 The travel graph on the right shows the journeys of a motorbike and a car between two locations on the highway.
a How long was the trip for the motorbike?
b How long was the trip for the car?
c What time did they pass each other?
d How far apart are the places that they started from?



See Example 7

5 a Make a table of the function $y = 2x + 3$ from to $^{-}5$ to 3.
b Plot the function. c Describe the function.

6 a Make a table of the function $y = 4 - 2x$ from to $^{-}4$ to 4.
b Plot the function. c Describe the function.

7 a Make a table of the function $y = 3x - 4$ from to $^{-}2$ to 3.
b Plot the function. c Describe the function.

8 a Make a table of the function $y = -x - {}^{-}2$ from to $^{-}5$ to 5.
b Plot the function. c Describe the function.

Problem solving

See Example 8

9 It costs \$60 to hire a sailboard for up to 2 hours and an extra \$20 an hour after that. Construct an algebraic model of the cost of hire up to 10 hours and plot the function. Find how long it can be hired if you have \$120.

Worked solutions
Exercise 4.2
MAT08NAWS00012

10 Pine bark costs \$35/m^3, with a delivery charge of \$45 to suburbs within the delivery area of the landscape garden supplier. Construct an algebraic model of the delivered cost of pine bark up to 6 m^3 and plot the functions. Find the cost for 4.5 m^3 of pine bark with delivery.

See Example 9

11 A courier of legal documents and small parcels based in the CBD charges for delivery based on the distance from the GPO. For distances up to 2 km, the charge is \$2/km. The charge increases for further distances by \$0.50/km for every extra 2 km or part thereof, to a maximum charge of \$5/km for the part over 12 km. The courier does not deliver to addresses further than 20 km from the city. Draw a graph of the charge and describe it. Find the cost of delivering an item to a location 9 km from the city.

Worked solutions
Exercise 4.2
MAT08NAWS00012

12 A produce merchant sells oats as feed for cattle at a cost of \$400/tonne for up to 3 tonnes. For extra quantities up to another 4 tonnes, the price is \$350/tonne, for extra quantities over 7 tonnes the price is \$300/tonne. Draw a graph of the price/tonne and describe it. Find the cost of 9 tonnes of oats.

Reasoning

Worked solutions
Exercise 4.2
MAT08NAWS00012

13 Explain how you would draw a graph of the total price per item for items delivered by the courier in question **11** to different distances from the city centre. Draw the graph and describe it.

14 Explain how you would draw a graph of the total price of different amounts of oats bought from the grain merchant in question **12**. Draw the graph and describe it.

9780170194778

4.3 Connecting algebra and geometry

Investigate: Linear rules and graphs: Part I

Use graph paper for this investigation.
On one sheet of graph paper, plot the graphs of the following linear functions (rules) from $^-5$ to 5.

$$y = x + 3$$
$$y = x + 5$$
$$y = x - 1$$
$$y = x - 4$$
$$y = x$$

Think of the last three functions as $y = x + {}^-1$, $y = x + {}^-4$, and $y = x + 0$. Write the rules for all five functions along their lines.
Use a different sheet of graph paper to plot the following linear functions from $^-5$ to 5.

$$y = 2x + 1$$
$$y = 2x + 4$$
$$y = 2x - 3$$
$$y = 2x - 1$$
$$y = 2x$$

Think of the last three functions as $y = 2x + {}^-3$, $y = 2x + {}^-1$, and $y = 2x + 0$. Write the rules for all five functions along their lines.
Use another sheet of graph paper to plot the following linear functions from $^-5$ to 5.

$$y = 5 - x$$
$$y = 3 - x$$
$$y = 1 - x$$
$$y = {}^-2 - x$$
$$y = {}^-x$$

Think of these functions as $y = -x + 5$, $y = -x + 3$, $y = -x + 1$, $y = -x + {}^-2$, and $y = -x + 0$. Write the rules for all five functions along their lines.
Make a prediction about each of the lines for the functions $y = {}^-2x + 3$, $y = 3x - 4$ and $y = 1.5x + 2$. Check your prediction for each line.

Video tutorial
Linear functions
MAT08NAVT00004

Teacher notes
Computer technology: Linear functions
MAT08NATN00022

Important!

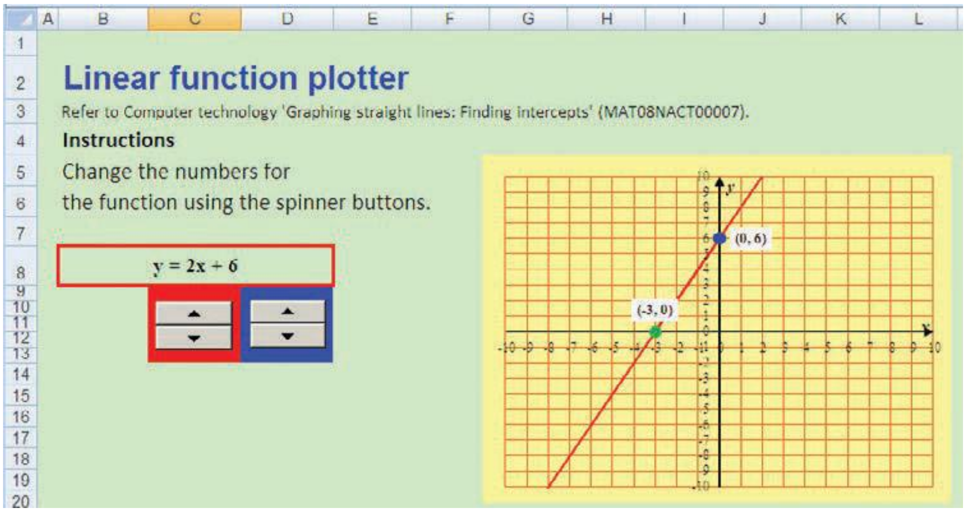
Intercepts

The ***x*-intercept** of a straight line is the part of the *x*-axis from the origin to the point where the line crosses the axis. The point itself or the *x*-coordinate of the point are also taken as being the intercept.
The ***y*-intercept** is taken as the *y*-coordinate of the intersection (or the point) of the line and the *y*-axis.

Technology Linear functions

Technology worksheet
Excel: Graphing straight lines: Changing m- and c- values
MAT08NACT00008

You can use digital technology to find the intercepts of linear functions.
Use the spreadsheet 'Linear function plotter' on the NelsonNet website or some other digital technology to find the intercepts of the functions given by your teacher.



Since linear functions are always straight lines, you only need to plot two points to find the line. However, it is a good idea to plot three as a check for calculation errors.

Example 10

Draw the line $y = 2x - 3$ and find the intercepts.

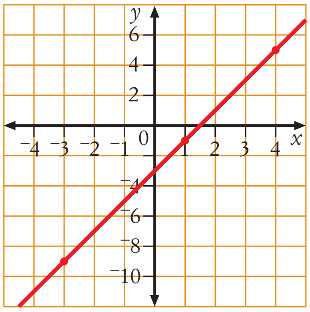
Solution

It is a linear function, so draw up a short table of values.

x	$^{-}3$	1	4
y	$^{-}9$	$^{-}1$	5

9780170194778

Plot the points and draw the line.



Use the graph to find the intercepts.

The x-intercept is $1\frac{1}{2}$ and the y-intercept is $^-3$.

The intercepts in Example **10** could also be written as $\left(1\frac{1}{2}, 0\right)$ and $(0, {}^-3)$.

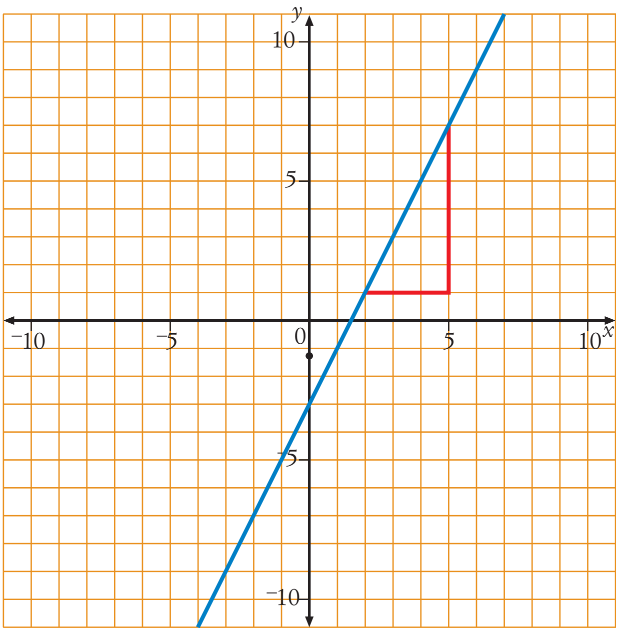
Investigate: Linear rules and graphs: Part II

Use graph paper for this investigation.

Graph A

On a sheet of graph paper, plot the graph of the linear function $y = 2x - 3$ from $^-10$ to 10.

Draw a right-angled triangle in red on the line from $x = 2$ to $x = 5$. It should look like this diagram.



Teacher notes

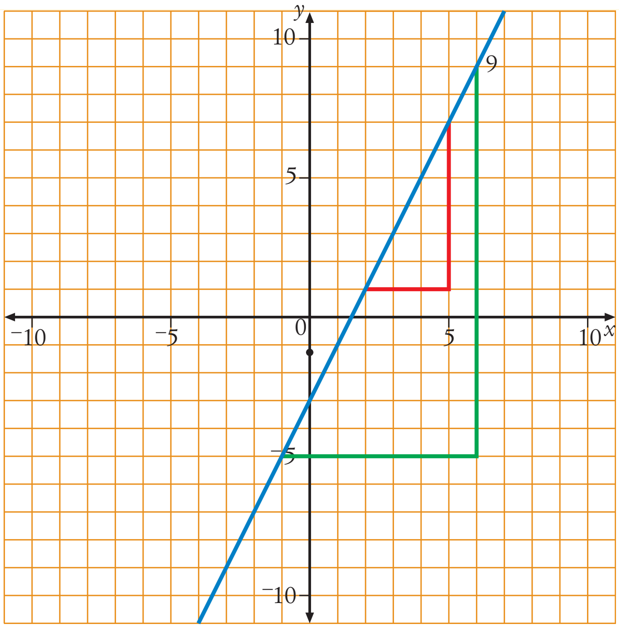
Computer technology: Functions and graphs

MAT08NATN00023

TLF learning object

EagleCat: Linear graph (L10090)

MAT08NAIN00004



The lengths of the sides of this triangle have special names. We call the horizontal side the **run**. The vertical side is called the **rise**.
Measure the rise and the run.
Divide the rise by the run. What do you get?
Now draw a new right-angled triangle in another colour on the line from $x = {}^{-}1$ to $x = 6$. It should now look like this diagram.
Measure the rise and the run on the new triangle. Divide the rise by the run. What do you get?
Now draw a new right-angled triangle in a third colour on the line from $x = {}^{-}3$ to $x = 5$.
Measure the rise and the run on the third triangle. Divide the rise by the run. What do you get?
Draw a fourth triangle on the line anywhere you want, find the rise and the run, and divide again.

Graph B

Using a new sheet of graph paper, plot the graph of the linear function $y = {}^{-}2x + 4$ from $^{-}10$ to 10.
Draw a right-angled triangle in red on the line from $x = {}^{+}1$ to $x = 5$. Measure the rise and the run. Check your answers for the rise and the run with your teacher before dividing.
Draw right-angled triangles from $^{-}4$ to 6, $^{-}2$ to 3 and one other of your choice and find the result from rise divided by run. What do you find for these?

Graph C

With a third piece of graph paper, plot the graph of the linear function $y = x + 3$ from $^{-}10$ to 10.
Draw right-angled triangles from $^{-}3$ to 7, $^{-}2$ to 4, 1 to 8 and one other of your choice and find the result for rise divided by run. What do you find for these?
Your teacher might want you to draw some more lines.
Look at your pieces of graph paper and compare the results of rise divided by run with the rules for the functions. What do you find?

Important!

Gradient of a Line

For two points on a straight line in a Cartesian plane, the **rise** is the vertical distance between the points and the **run** is the horizontal distance between the points. The **gradient** of a straight line is calculated as gradient $= \frac{\text{rise}}{\text{run}}$ for any two points on the line. The usual symbol for gradient is $\boldsymbol{m}$.

9780170194778

Example 11

Animated example
Linear graphs
MAT08NAAE00006

Technology
GeoGebra: Equation of a line
MAT08NATC00006

Plot the line $y = 5 - 2x$ and find the gradient.

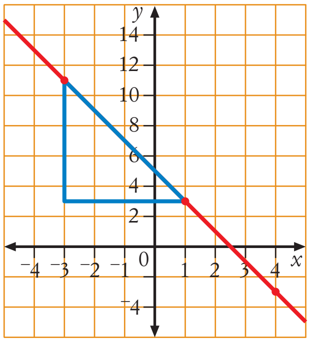
Solution

It is a linear function, so draw up a short table of values.

x	$^-3$	1	4
y	11	3	$^-3$

Plot the points and draw the graph.

Draw a right-angled triangle on some convenient points on the graph.



Read off the rise and the run from the triangle. **Rise = $^-8$ and run = 4**

Work out the gradient.

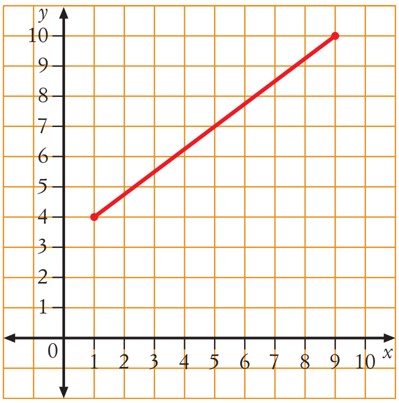
$$\textbf{Gradient} = \frac{\textbf{rise}}{\textbf{run}} = \frac{^-8}{4} = {}^-2$$

Write the answer. **The gradient of $y = 5 - 2x$ is $^-2$.**

In geometry, you can find the **midpoint** of a line by bisecting the line segment. You can also use algebra to find a midpoint.

Investigate: Midpoint of a line segment

Use graph paper for this investigation.
Plot the points (1, 4) and (9, 10) on your graph paper.
Draw a line segment between the points and find the point halfway along it.
Write down this point. It is the midpoint of the line segment between the points (1, 4) and (9, 10).
Use the same method to find the midpoints of the line segments between the points:
($^-4$, 5) and (1, $^-8$)
($^-4$, $^-6$) and (4, $^-2$)
(2, $^-5$) and (10, 10)
Now look at each pair of points and the midpoint of the line segment.
What pattern do you find?



Important!

Midpoint of a Line Segment

The values of the coordinates of the midpoint of a line segment between two points are the averages of the coordinates of the points.

Example 12

Find the midpoint of the line segment between ($^-5$, 1) and (3, 8).

Solution

Find the average of the x-coordinates.

$$x\text{-coordinate} = \frac{^-5 + 3}{2} = \frac{^-2}{2}$$

Find the average of the y-coordinates.

$$y\text{-coordinate} = \frac{1 + 8}{2}$$
$$\frac{9}{2} = 4\tfrac{1}{2}$$

Write the answer. **The midpoint of ($^-5$, 1) and (3, 8) is $\left(^-1, 4\frac{1}{2}\right)$.**

Notice that in the answer to Example **12**, the midpoint of the line segment is just shortened to the midpoint of the points.

Example 13

(3, 5) is the midpoint of (6, $^-3$) and another point. Find the other point.

Solution

Choose variables for the unknowns. **Let the other point have coordinates (a, b).**

Use the rule for the midpoint to write an equation for the x-coordinate.

$$\frac{6 + a}{2} = 3$$

Solve to find the x-coordinate. $a = 0$

Use the rule for the midpoint to write an equation for the y-coordinate.

$$\frac{^-3 + b}{2} = 5$$

Solve to find the y-coordinate. $b = 13$

Write the answer. **The other point is (0, 13).**

You may already have noticed that the rules of functions that are not linear do not make straight lines. Some of the curved lines they make are quite interesting shapes. In some cases, you may need to find values of x from values of y instead; or you may need to find values of y from values of x as well.

9780170194778

Example 14

Use values of x from $^-4$ to 4 to plot the function $y = \frac{1}{2}x^2$.

Scientific calculator exercise
Functions and graphs
MAT08NASC00003

Solution

Make a table of values.

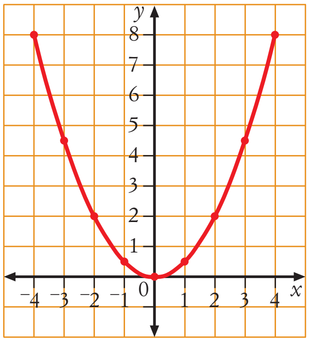
x	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	8	4.5	2	0.5	0	0.5	2	4.5	8

Use brackets on your calculator to find $(^-4)^2$.

Enter as 0.5 [×] [(] [(–)] 4 [)] [x^2] [=].

0.5×(-4)² 8

Plot the points and join them with a smooth line.



You saw earlier in this chapter that the shape in Example **14** is called a parabola.

Example 15

Use values of x between $^-4$ and 4 to plot the line made by $3x^2 + 4y^2 = 48$.

Solution

Rearrange the equation by using inverse operations or backtracking to find the value of y.

$$\begin{aligned} 3x^2 + 4y^2 &= 48 \\ 4y^2 &= 48 - 3x^2 \\ y^2 &= \frac{48 - 3x^2}{4} \\ y &= \sqrt{\frac{48 - 3x^2}{4}} \end{aligned}$$

Make a table of values, using your calculator to find approximate square roots.

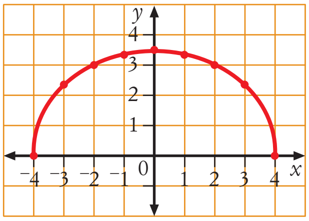
x	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4
y	0	2.3	3	3.4	3.5	3.4	3	2.3	0

Use brackets in your calculator.

Enter as: [√] [(] [(] 48 [–] 3 [×] [(] [(–)] 3 [)] [x^2] [)] [÷] 4 [)] [=].

√((48–3×(-3)²)÷4) 2.291287847

Plot the points and join them with a smooth line.



In Example **15**, there are actually two answers for y for each value of x. What shape do you get if you use both possible values of y?

Exercise 4.3 Connecting algebra and geometry

Understanding

Extra questions
Exercise 4.3
MAT08NAEQ00013

1 Plot each of the following lines and find the intercepts.

a $y = 2x + 4$ **b** $y = 3x - 6$ **c** $y = -x + 5$ **d** $y = 2x - 6$
e $y = x - 3$ **f** $y = {}^-2x + 6$ **g** $y = {}^-3x - 9$ **h** $y = 4x - 4$

Fluency

See Example 11

2 Plot each of the following functions and find the gradients.

a $y = x - 1$ **b** $y = {}^-2x + 8$ **c** $y = 3x - 2$ **d** $y = -x - 3$
e $y = 4 - 3x$ **f** $y = \frac{1}{2}x - 6$ **g** $y = \frac{1}{4}x + 2$ **h** $y = {}^-3x + 1$

3 Plot each of the following pairs of points and find their midpoints.

a $({}^-3, 1), (8, 7)$ **b** $({}^-4, {}^-1), ({}^-6, 0)$ **c** $({}^-9, {}^-4), ({}^-4, {}^-9)$ **d** $(7, {}^-1), (5, 7)$
e $(7, {}^-5), (1, {}^-8)$ **f** $(2, {}^-7), ({}^-5, {}^-4)$ **g** $({}^-9, {}^-3), (7, 6)$ **h** $(4, {}^-6), ({}^-4, 1)$

See Example 12

4 Find the midpoints of the following pairs of points without plotting.

a $({}^-9, {}^-1), ({}^-9, {}^-7)$ **b** $(7, {}^-6), ({}^-6, 3)$ **c** $(2, 8), ({}^-3, 9)$ **d** $({}^-5, {}^-1), ({}^-9, {}^-3)$
e $(5, {}^-4), ({}^-1, {}^-9)$ **f** $({}^-6, 0), (3, {}^-1)$ **g** $({}^-4, 5), (1, 2)$ **h** $(4, {}^-8), (2, 8)$

See Example 14

5 Use values of x from $^-4$ to 4 to plot the line $y = x^2$.

6 Use values of x from $^-3$ to 3 to plot the line $y = 2x^2$

See Example 15

7 Use values of y from $^-4$ to 4 to plot the line $x = y^2$.

8 Use values of y from $^-4$ to 4 to plot the line $x = \frac{1}{2}y^2$.

Worked solutions
Exercise 4.3

9 Use values of x from $^-3$ to 3 to plot the line $x^2 + y^2 = 9$.

MAT08NAWS00013

10 Use values of x from $^-4$ to 4 to plot the line $x^2 + 2y^2 = 16$.

Problem solving

11 $\left({}^-3, {}^-\frac{1}{2}\right)$ is the midpoint of $({}^-9, 6)$ and another point. Find the other point.

See Example 13

12 $(1, {}^-2)$ is the midpoint of $(2, {}^-8)$ and another point. Find the other point.

Worked solutions
Exercise 4.3

13 $(1, {}^-1)$ is the midpoint of $(a, 2)$ and $({}^-5, b)$. Find a and b.

MAT08NAWS00013

14 $\left(1\frac{1}{2}, {}^-4\frac{1}{2}\right)$ is the midpoint of $(a, {}^-2)$ and $({}^-5, b)$. Find a and b.

Reasoning

15 Explain how you could find the gradient of a line between two points without drawing the line segment.

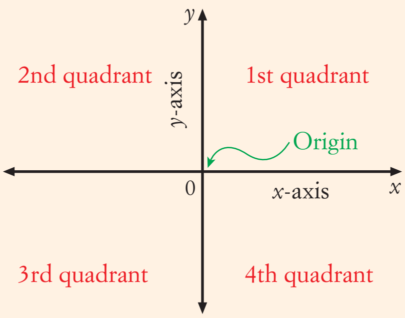
Worked solutions
Exercise 4.3
MAT08NAWS00013

16 Explain how you can tell if a rule is going to give a vertical parabola or a horizontal half-parabola.

9780170194778

Chapter 4 summary

- The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**.
- The horizontal axis is called the ***x*-axis**. The vertical axis is called the ***y*-axis**. The axes cross at the **origin**.
- Every point on the Cartesian plane is opposite a number on each axis. The points are named using numbers called **coordinates**.
- Coordinates are shown in parentheses (round brackets) with a comma between them. The origin is the point (0, 0). The ***x*-coordinate** is always shown *first* and the ***y*-coordinate** *last*.
- Each quarter of the Cartesian plane is called a **quadrant**.



- A **parabola** is a roughly U-shaped curve that is symmetrical about a central line.
- A **travel graph** shows the distance travelled by something over time. Something moving at a constant speed produces a straight line and a stationary object produces a horizontal line.
- A mathematical rule that changes numbers into new ones is called a **function**. The starting number is called the **input** or **independent variable**. The number produced is called the **output** or **dependent variable**.
- The rule for a function is usually written with the output variable on the left-hand side of the equals sign and the rule on the right in terms of the input variable.
- A function can be **plotted** by putting different values into the function and calculating the outputs. This is usually put into a table called a **table of values**.
- A rule that produces a straight line is called a **linear function** and the graph is called a **linear graph**. Linear functions are of the form $y = ?x \pm ??$, where ? and ?? are constants.
- The ***x*-intercept** of a straight line is the part of the x-axis where the line crosses the axis.
- The ***y*-intercept** is the y-coordinate (or the point) of the intersection between the line and the y-axis.
- For two points on a straight line in the Cartesian plane, the **rise** is the vertical distance between the points and the **run** is the horizontal distance between the points. The gradient of a straight line is calculated as Gradient $= \frac{\text{rise}}{\text{run}}$ for any two points on the line. The usual symbol for the gradient is $\boldsymbol{m}$.
- The values of the coordinates of the **midpoint** of a line segment between two points are the averages of the coordinates of the points. The midpoint of a line segment is often stated as being the midpoint of the points at its ends.

Quiz
Functions and graphs
MAT08NAQZ00004

Puzzle sheet
Linear equations crossword
MAT08NAPS00012

9780170194778

Chapter 4 review

Understanding

See Example 1

1 Plot each of the following points on Cartesian axes and state its quadrant or axis.
$A(^-3, 1)$, $B(2, ^-4)$, $C(^-5, ^-2)$, $D(4, 0)$, $E(^-5, 6)$, $F(5, 2)$, $G(0, ^-2)$

See Example 5

2 Saeed is late for the bus and starts running down the street to the bus stop at 3 m/s. He turns the corner and after a minute he sees that the bus has not come and there are lots of people waiting. So he slows to a walk, travelling at 1 m/s for half a minute. Then he sees the bus coming and resumes running at 2 m/s for another minute to catch the bus. Draw a travel graph of his movement and find how far he ran altogether.

See Example 10

3 Plot each of the following lines and find the intercepts.
a $y = x - 6$ **b** $y = 2x + 7$ **c** $y = {}^-2x + 4$ **d** $y = {}^-3x - 6$

Fluency

See Example 2

4 Plot the points in the following table. What shape is formed?

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	8	6	4	2	0	$^-2$	$^-4$

5 Without plotting, state the quadrant or axis of each of the following points.
$A(^-5, ^-1)$, $B(2, 1)$, $C(^-3, 0)$, $D(^-4, 1)$, $E(5, ^-3)$, $F(0, 6)$, $G(^-3, ^-5)$, $H(7, ^-1)$

See Example 7

6 **a** Make a table of the function $y = {}^-2x + 5$ from $^-3$ to 5.
b Plot the function. **c** Describe the function.

See Example 11

7 Calculate the gradient of each of these functions.
a $y = 2x - 1$ **b** $y = 4 - 3x$ **c** $y = \frac{1}{2}x + 2$ **d** $y = -x - 5$

8 Plot each of the following pairs of points and find their midpoints.
a $(6, 1)$, $(2, 9)$ **b** $(^-3, 4)$, $(^-7, ^-3)$

See Example 12

9 Find the midpoints of the following pairs of points without plotting.
a $(^-5, ^-3)$, $(^-1, 5)$ **b** $(2, ^-6)$, $(^-4, 9)$

See Example 14

10 Use values of x from $^-4$ to 4 to plot the line $y = x^2 - 2$.

11 Use values of y from $^-3$ to 3 to plot the line $x = y^2 + 1$.

See Example 15

12 Use values of x from $^-4$ to 4 to plot the line $3x^2 + y^2 = 48$.

Problem solving

13 Plot the points in the following table. What shape is formed?

x	**$^-3$**	**$^-2$**	**$^-1$**	**0**	**1**	**2**	**3**
y	**$^-6$**	**$^-1$**	**2**	**3**	**2**	**$^-1$**	**$^-6$**

See Example 8

14 It costs \$50 to hire a cement mixer for a day and an extra \$30 for each day after that. Construct an algebraic model of the cost of hire for up to 6 days and plot the function. Find how long it can be hired if you have \$160.

See Example 9

15 A fruit and vegetable shop sells bananas at \$4.90/kg loose. It has 3 kg bags of bananas for \$12 and sells 10 kg boxes of bananas for \$33 a box. Assume that the unit price for 3 kg applies to any quantity over 3 kg and the unit price for 10 kg applies to any quantity over 10 kg. Draw a graph of the unit cost (cost/kg) of bananas from 0–15 kg and describe it. What is the cost of 9 kg of bananas?

9780170194778

16 $(^-3, ^-\frac{1}{2})$ is the midpoint of $(^-9, 6)$ and another point. Find the other point. *See* Example 13

Reasoning

17 The table below shows the amount of weekly practice that different swimming squads have and the average distance three members of the squad can swim in 10 minutes. Plot the points and comment on the effectiveness of training.

Training (hrs)	5	9	12	15	20
Distance (m)	247	310	336	376	396

18 Explain how you would draw a graph of the total cost of different quantities of bananas in question **15** if you had to buy a mixture of loose, bags and boxes to make up in-between amounts and pay the corresponding prices. Draw the graph, describe it and comment.

19 Show that all functions with x^2 in their rule do not have the same shape.

Number and algebra

5 Fractions and decimals

9780170194778

Contents

Prior learning
Chapter 5
MAT08NAPL00005

Parent guide
Chapter 5
MAT08NAPG00005

Curriculum guide
Chapter 5
MAT08NACU00005

Australian Curriculum statements

Number and place value
Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies.

Real numbers
Investigate terminating and recurring decimals
Investigate the concept of irrational numbers, including π.

Maths clip
Working out fractions
MAT08NAMC00003

Weblink
Maths is fun!
MAT08NAWB00005

If you measure the distance from your home to the school, it is unlikely to be an exact number of kilometres. The mass or height of a chair is unlikely to be an exact number. Fractions and decimals were invented so that parts of a whole number could be used. The ancient Egyptians and Babylonians used fractions thousands of years ago, but decimal fractions were only invented about 400 years ago by John Napier and other mathematicians of the time. Fractions and decimals are mathematically considered to be just different ways of writing numbers called **rational numbers**.

Mathematical literacy

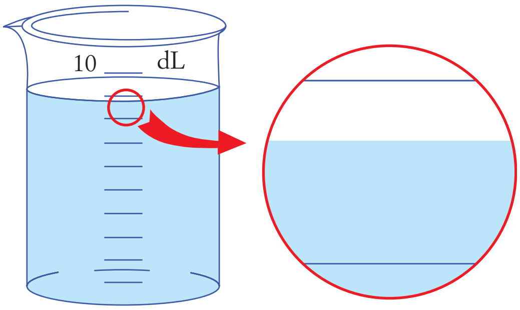
Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

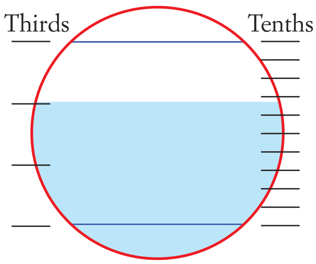
common denominator	improper fraction	numerator	simplest form
cross-product	irrational number	proper fraction	terminating decimal
denominator	lowest common denominator	rational number	vinculum
equivalent fraction	mixed fraction	recurring decimal	vulgar fraction
		significant figure	

5.1 Comparing rational and irrational numbers

The diagram on the right shows the water level in a measuring cylinder. The diagram next to it shows a close-up. It is obvious that there is between 8 and 9 dL in the cylinder.



To work out the amount more precisely, we could divide the gap between 8 and 9 into parts as shown in the diagram below. Then we could write the amount as being close to $8\frac{2}{3}$ or 8.7 dL. In the case of the $\frac{2}{3}$, two out of three parts of the gap are filled. In the case of the 0.7, about seven out of ten parts of the gap are filled.



9780170194778

Important!

Fractions and decimal fractions

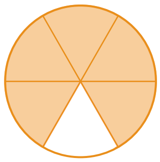
A **fraction** shows part of a whole.
A **common fraction** (**vulgar fraction**) has a **numerator** on top separated by a **vinculum** (bar) from the **denominator** underneath. The denominator shows the number of equal shares in the whole and the numerator shows the number of these shares in the fraction.
A common fraction may be shown horizontally instead of vertically, using a **slash** to separate the numerator and denominator, for example 2/3.
Common fractions are said or written in words with the numerator first, followed by the denominator with the 'th' or 'ths' ending (except halves, thirds and quarters). A hyphen is usually placed between the numerator and denominator of small fractions, such as five-eighths ($\frac{5}{8}$ or 5/8).
A **decimal fraction** (or just **decimal**) extends the normal place value system to the right of the decimal point, so that each place value is 10 times smaller than the place value to its left.
The first **decimal place** has a value of a tenth ($\frac{1}{10}$), the second decimal place has a value of a hundredth ($\frac{1}{100}$), and so on.
The decimal fraction part of a number is read as a series of digits after the word 'point', so 13.763 is said as 'thirteen point seven six three'.
A decimal may be written in extended form by writing the fraction for each decimal place in common fraction form.

Weblink
The TIMES project
MAT08NAWB00005

Weblink
Mathematics enhancement program: UK
MAT08NAWB00005

Example 1

a Write the fraction represented by the shaded part of the diagram on the right in words and symbols.



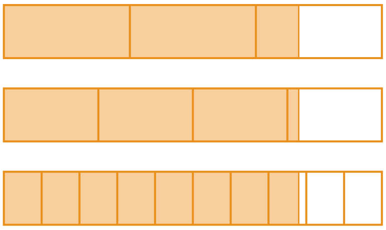
b Estimate the fraction represented by the shaded part of the diagram below. Give your answer as both a common fraction and a decimal fraction and write it using both words and symbols.
Choose from $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ and 0.4, 0.5, 0.6, 0.7, 0.8.

Solution

a The circle is divided into six equal parts, so the denominator is 6. Five of the parts are shaded, so the numerator is 5. Write the answer.

Five-sixths $\left(\frac{5}{6}\right)$ is shaded.

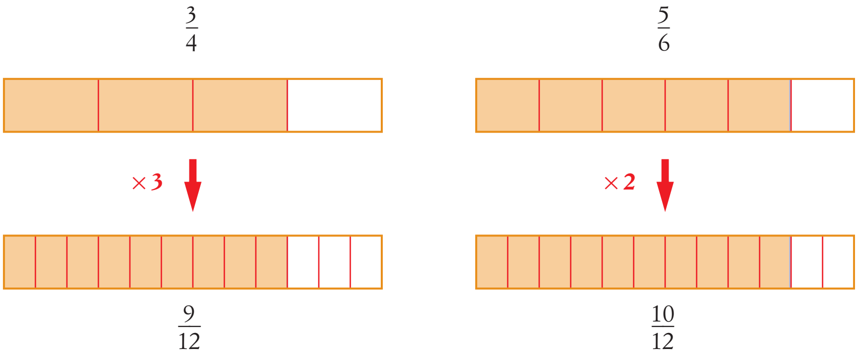
b Imagine the whole bar divided into parts as shown here.



Write the answers that appear closest.

The shaded part is about three-quarters $\left(\frac{3}{4}\right)$ or about zero point eight (0.8).

We can change a whole number to fraction format so that a number with a whole and a fraction part, such as $8\frac{2}{3}$ can be written entirely as a fraction. In this case, 8 would be $\frac{24}{3}$, so $8\frac{2}{3}$ would be $\frac{26}{3}$. It is easier to compare the sizes of fractions such as $\frac{3}{4}$ and $\frac{5}{6}$ if we write them with the same denominator. We can do this diagrammatically, as shown below.



Using the common denominator 12, it is easy to see that $\frac{3}{4} < \frac{5}{6}$.

When comparing decimals and fractions, we should change them both to the same format. When writing answers to fraction problems, it is normal to change the fractions so that they have the smallest possible numerators and denominators.

Important!

Formats of rational numbers

The numerator and denominator of a fraction can be multiplied or divided by the same number to obtain an **equivalent fraction** that represents the same amount. Equivalent fractions have equal **cross-products**.

Different fractions can always be written with the same denominator. It is usual to use the smallest possible number, called the **lowest common denominator**.

A **proper fraction** such as $\frac{2}{3}$ is less than 1. Its numerator is smaller than its denominator.

An **improper fraction** such as $\frac{26}{3}$ is more than 1. Its numerator is bigger than its denominator.

A **mixed fraction** (**mixed number**) such as $8\frac{2}{3}$ has a whole-number part and a proper fraction.

The **simplest form** of a fraction is the equivalent fraction (or mixed number) with the lowest possible denominator. It is obtained by changing an improper fraction to a mixed fraction and dividing the numerator and denominator by common factors. This is called **cancelling** (or **cancelling down**).

9780170194778

A fraction can be changed to a decimal by treating the fraction as a division problem. A non-recurring decimal can be changed to a fraction by writing the decimal in extended form and cancelling down.
Common fractions and decimals are known as **rational numbers**. Integers are also included if we think of them as equivalent fractions, such as $5 = \frac{5}{1}$ or $\frac{10}{2}$.

Example 2

Animated example
Rational numbers
MAT08NAAE00007

Scientific calculator exercise
Fractions and decimals
MAT08NASC00004

Worksheet
Decimal fractions
MAT08NAWK00034

a Write $3\frac{4}{5}$ as an improper fraction.
b Change $\frac{20}{3}$ to a mixed fraction.
c Write $\frac{18}{24}$ in simplest form.
d Change $\frac{5}{8}$ to a decimal.
e Change 0.36 to a fraction.
f Change $\frac{7}{12}$ to a decimal.
g Change $2\frac{4}{7}$ to a decimal.
h Show that $\frac{8}{12}$ and $\frac{10}{15}$ are equivalent.

Solution

a Change the 3 to fifths and add the 4.

$$3\frac{4}{5} = \frac{3 \times 5 + 4}{5} = \frac{19}{5}$$

b Divide by 3 to get the whole number.

$$\frac{20}{3} = 6 \text{ R } 2$$

Write the answer as a mixed fraction.

$$= 6\frac{2}{3}$$

c Divide by the common factor, 6.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

d Perform the division as a decimal.

$$\begin{array}{r} 0.\ 6\ 2\ 5 \\ 8\,\overline{)5.{}^{5}0{}^{2}0{}^{4}0} \end{array}$$

Write the answer.

$$\frac{5}{8} = 0.625$$

e Write in extended form.

$$0.36 = \frac{36}{100}$$

Cancel down and write the answer.

$$= \frac{9}{25}$$

f Perform the division, adding as many zeros as needed.

$$\begin{array}{r} 0.\ 5\ \ 8\ 3\ 3\ 3\ldots \\ 12\,\overline{)7.{}^{7}0{}^{10}0{}^{4}0{}^{4}0{}^{4}0\ldots} \end{array}$$

Use a bar to show that the 3 is repeating.

$$\frac{7}{12} = 0.58\overline{3}$$

g Perform the division for the fraction part.

$$\begin{array}{r} 0.\ 5\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ 1\ldots \\ 7\,\overline{)4.{}^{4}0{}^{5}0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0\ldots} \end{array}$$

Use a bar to show the repeating digits.

$$2\frac{4}{7} = 2.\overline{571428}$$

h Work out cross-products.

$$\frac{8}{12} \quad \frac{10}{15}$$

$$8 \times 15 = 120 \text{ and } 12 \times 10 = 120$$

Write the answer.

The cross-products are the same, so $\frac{8}{12}$ and $\frac{10}{15}$ are equivalent fractions.

9780170194778

Example 3

Insert <, = or > in the squares to make true statements.

a $\frac{4}{5} \square \frac{11}{15}$ **b** $7.52 \square 7.452$ **c** $\frac{3}{7} \square \frac{7}{15}$

d $^{-}1.46 \square {}^{-}1.475$ **e** $\frac{^{-}5}{6} \square \frac{^{-}5}{8}$ **f** $1.87 \square \frac{20}{11}$

g $0.063 \square 7 \times 10^{-2}$

Solution

a Change $\frac{4}{5}$ to 15ths.

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

Compare as 15ths.

$$\frac{12}{15} > \frac{11}{15}$$

Write in the original form.

$$\frac{4}{5} > \frac{11}{15}$$

b Add an extra zero to make the decimal parts the same length.

$$7.52 \square 7.452$$

$$7.520 \square 7.452$$

The units are the same, but $\frac{520}{1000}$ is more than $\frac{452}{1000}$.

$$7.520 > 7.452$$

Write in the original form.

$$7.52 > 7.452$$

c **Method 1**

Change to a common denominator.

$$\frac{3 \times 15}{7 \times 15} \square \frac{7 \times 7}{15 \times 7}$$

Compare as 105ths.

$$\frac{45}{105} < \frac{49}{105}$$

Write in the original form.

$$\frac{3}{7} < \frac{7}{15}$$

Method 2

Use your calculator to change both fractions to decimals.

Enter $\frac{3}{7}$ as 3 [÷] 7 [=].

3÷7 0.4285714286

Enter $\frac{7}{15}$ as 7 [÷] 15 [=].

7÷15 0.4666666667

Compare as decimals.

$$0.428\ldots < 0.466\ldots$$

Write in the original form.

$$\frac{3}{7} < \frac{7}{15}$$

d Add an extra zero to make the decimal parts the same length.

$$^{-}1.46 \square {}^{-}1.475$$

$$^{-}1.460 \square {}^{-}1.475$$

The units are the same, but $^{-}0.460$ is more than $^{-}0.475$.

$$^{-}1.460 > {}^{-}1.475$$

Write in the original form.

$$^{-}1.46 > {}^{-}1.475$$

9780170194778

e The lowest common denominator is 24, so change both to 24ths. $\frac{^-5}{6} \square \frac{^-5}{8}$

$\frac{^-20}{24} \square \frac{^-15}{24}$

$^-20$ is less than $^-15$. $\frac{^-20}{24} < \frac{^-15}{24}$

Write in the original form. $\frac{^-5}{6} < \frac{^-5}{8}$

f Change the fraction to a decimal. $\frac{20}{11} = 1.\overline{81}$

Compare as decimals. $1.87 > 1.\overline{81}$

Write in the original form. $1.87 > \frac{20}{11}$

g Change both to the same form. $7 \times 10^{-2} = 0.07$

Compare as decimals. $0.063 < 0.07$

Write in the original form. $0.063 < 7 \times 10^{-2}$

Important!

Number lines for rational numbers

The letter x is used to represent any rational number, including integers, fractions and decimals.

On a number line, an **open** circle shows that the boundary of a group of numbers is *not included* ($<$ or $>$). A **closed** circle shows that the boundary of a group of numbers *is included* ($\leq$ or $\geq$).

Example 4

Show the following on separate number lines.

a $2\frac{1}{4} < x < 4\frac{3}{5}$ **b** $x > 5.8$ **c** $^-4\frac{2}{3} < x \leq 2\frac{6}{11}$

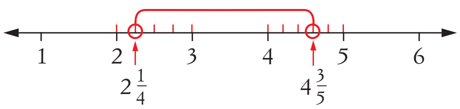
d $^-17.35 \leq x < {^-6.74}$ **e** $^-14.63 \leq x \leq 11\frac{7}{9}$

Solution

a Draw the number line from, say, 1 to 6 with arrows.

Divide the space between 2 and 3 into four equal parts and the space between 4 and 5 into five equal parts.

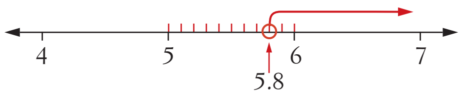
Mark the endpoints with open circles to show that they are not included. Show all the numbers between, using an arc.



9780170194778

b Draw the number line from, say, 4 to 7 with arrows.

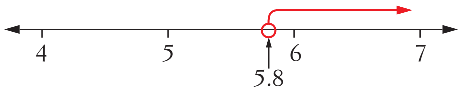
Divide the space between 5 and 6 into ten equal parts.



Mark the endpoint with an open circle to show that it is not included.

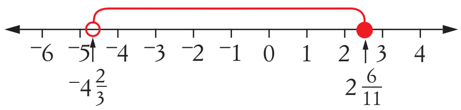
Show the numbers going up, using an arc with an arrow.

Some teachers will prefer you to *estimate* the position of 5.8, as shown here.



c Draw the number line from $^{-}6$ to 4 with arrows.

Mark the approximate positions of the endpoints.

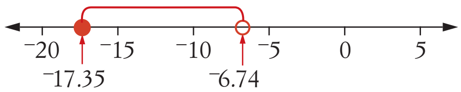


Use an open circle at $^{-}4\frac{2}{3}$ to show that it is not included.

Use a closed circle at $2\frac{6}{11}$ to show that it is included.

Show all the numbers between, using an arc.

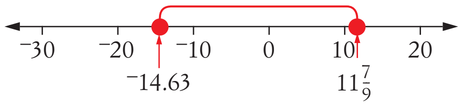
d Draw the number line from before $^{-}17$ to after $^{-}6$ in intervals of 5.



Mark $^{-}17.35$ and $^{-}6.74$, and put a closed circle at $^{-}17.35$ and an open circle at $^{-}6.74$. Show all the numbers between, using an arc.

e Draw the number line from before $^{-}14$ to after 11 in intervals of 10.

Mark $^{-}14.63$ and $11\frac{7}{9}$ and put closed circles at the endpoints to show that they are included. Show all the numbers between, using an arc.



Important!

Decimals such as 0.375 are called **terminating decimals**. ‘Terminate’ means ‘to stop’. Sometimes when a common fraction is converted to a decimal, we get a **repeating** or **recurring decimal**. One or more of the digits in the decimal repeats forever, for example, $\frac{1}{9} = 0.111...$

The repeating or recurring pattern occurs in the numbers following the decimal point. To show that the pattern goes on forever, we use dots or a line to identify the repeating section, for example, $0.285714\ 285714\ 285714... = 0.\dot{2}8571\dot{4}$ or $0.\overline{285714}$.

9780170194778

Example 5

Change each of the following to a decimal.

a $\frac{5}{9}$ **b** $\frac{3}{11}$

Solution

a $\frac{5}{9}$ means $5 \div 9$

With a calculator: 5 ÷ 9 = 0.5555 . . .

$$9\overline{)5.0000\ldots} = 0.555\ldots$$

$$\frac{5}{6} = 0.555\ldots = 0.\dot{5} \text{ or } 0.\overline{5}$$

b $\frac{3}{11}$ means $3 \div 11$

With a calculator: 3 ÷ 11 = 0.272727. . .

$$11\overline{)3.00000\ldots} = 0.27272\ldots$$

$$\frac{3}{11} = 0.272727\ldots = 0.\dot{2}\dot{7} \text{ or } 0.\overline{27}$$

Important!

Recurring decimals and fractions

To change a recurring decimal to a fraction:

- Multiply the decimal by 10^n, where n is the number of recurring digits.
- Subtract the number.
- Divide by $10^n - 1$ and simplify if possible.

Example 6

Animated example
Changing recurring decimals into fractions
MAT08NAAE00008

Change $0.\overline{234}$ to a fraction.

Solution

Note how many figures recur.	**It has 3 digits recurring.**
Write the number out a bit.	**Number = 0.234 234 234 234 . . .**
Multiply by 1000 for the 3 digits.	**1000 × Number = 234.234 234 . . .**
Subtract the number.	**999 × Number = 234**
Divide by 999.	**Number** $= \frac{234}{999}$
Cancel if possible.	$= \frac{26}{111}$
Write the answer.	$0.\overline{234} = \frac{26}{111}$

Sometimes the number does not recur from immediately after the decimal point. In this case, first multiply by a power of 10 so that it does.

Example 7

Change $0.08\overline{3}$ to a fraction.

Solution

Multiply by 100 to make it recur just after the decimal point.	$100 \times \textbf{number} = 8.\overline{3}$
Note how many digits recur.	**It has 1 recurring digit.**
Write the number out a bit.	$100 \times \textbf{Number} = 8.333\,333\ldots$
Multiply by 10 for the 1 digit.	$10 \times 100 \times \textbf{Number} = 83.333\,333\ldots$
Subtract.	$9 \times 100 \times \textbf{Number} = 75$
Divide by 9×100.	$\textbf{Number} = \frac{75}{900}$
Cancel if possible.	$= \frac{15}{180} = \frac{3}{36} = \frac{1}{12}$
Write the answer.	$0.08\overline{3} = \frac{1}{12}$

All fractions can be converted to terminating or recurring decimals and vice versa, so these numbers are called rational (as they can be written as a ratio or fraction).

Important!

An **irrational number** is not rational. It cannot be written as a terminating or recurring decimal. An **irrational number** cannot be expressed as a fraction.

Investigate: Irrational numbers

If you are asked to identify whether a number is rational or irrational, first write the number in decimal form. If the number terminates, then it is rational. If it goes on forever, then look for a repeated pattern of digits. If there is no repeated pattern, then the number is irrational.

1 Investigate the following set of numbers to determine if they are rational or irrational.

$4.07 \quad \sqrt{2} \quad \sqrt{729} \quad \frac{1}{11} \quad 0$

$\sqrt{4} \quad \sqrt{3} \quad \sqrt{\frac{1}{1024}} \quad \frac{1}{\sqrt{5}} \quad \pi$

Hint: Use your calculator to convert each to a decimal.

The mathematical constant π (Pi) is a famous irrational number that has been around for over 4000 years. Pi is a non-terminating decimal with no repeated pattern. By definition, Pi is the ratio of the circumference of a circle to its diameter.

$\pi \approx 3.\,14159\,26535\,89793\,23846\,26433\,83279\,50288\,41971\,69399\,37510\ldots$

9780170194778

Over the years mathematicians have been able to calculate Pi to a greater number of decimal places.

2 Investigate the history and evolution of Pi. Who has been responsible for increasing its accuracy and how has technology influenced this evolution?

Investigate: Fraction estimations

You will need six strips of paper 10 cm long, six circles about 5 cm in diameter with marked centres, a ruler, a protractor and a pencil.

For this investigation, work in groups of three. There are six sessions in a round, and in each session you change roles as follows. Your teacher will probably put about 20 fractions on the board to choose from.

Session	Setter	Trier	Judge
1	Student 1	Student 2	Student 3
2	Student 2	Student 3	Student 1
3	Student 3	Student 1	Student 2
4	Student 1	Student 3	Student 2
5	Student 2	Student 1	Student 3
6	Student 3	Student 2	Student 1

The person who is the *setter* picks a fraction and draws it, using a ruler or protractor to measure the distances and angles. In each session, you do one on a strip and one on a circle. The *trier* has to do the same fraction by hand, on a strip and on a circle. The *judge* then measures the fractions and allocates points to the *trier* as follows.

Match between drawings of setter and trier		Points
Within 2 cm (20 mm)	or 40°	1
Within 1 cm (10 mm)	or 20°	2
Within 5 mm	or 10°	3
Within 2.5 mm	or 5°	4

At the end of the round, everyone will have a score out of 16.

Exercise 5.1 Comparing rational and irrational numbers

Understanding

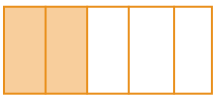
Extra questions

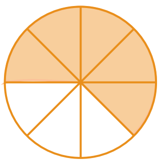
Exercise 5.1

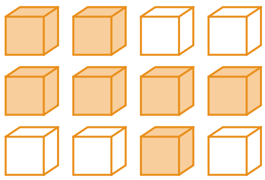
MAT08NAEQ00014

See Example 1

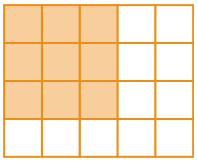
1 Write the fractions represented by the shaded parts of these diagrams in words and symbols.

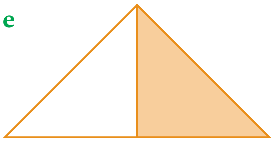
a 

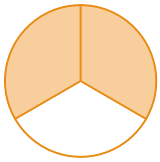
b 

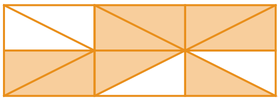
c 

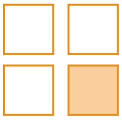
9780170194778

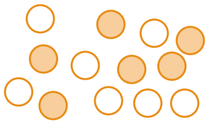
d 

e 

f 

g 

h 

i 

Worked solutions
Exercise 5.1
MAT08NAWS00014

See Example 2

2 Write each of these mixed fractions as an improper fraction.

a $3\frac{1}{2}$ b $1\frac{2}{3}$ c $4\frac{1}{3}$ d $6\frac{1}{4}$
e $5\frac{3}{4}$ f $1\frac{2}{5}$ g $4\frac{3}{5}$ h $5\frac{1}{8}$

See Example 2

3 Write each of these improper fractions as a mixed fraction.

a $\frac{17}{5}$ b $\frac{5}{2}$ c $\frac{19}{4}$ d $\frac{8}{3}$
e $\frac{11}{2}$ f $\frac{13}{4}$ g $\frac{28}{5}$ h $\frac{20}{7}$

See Example 2

4 Write each of these fractions in simplest form.

a $\frac{5}{10}$ b $\frac{5}{20}$ c $\frac{8}{20}$ d $\frac{3}{18}$
e $\frac{4}{12}$ f $\frac{6}{8}$ g $\frac{8}{12}$ h $\frac{8}{40}$
i $\frac{12}{32}$ j $\frac{4}{48}$ k $\frac{80}{100}$ l $\frac{15}{36}$

See Example 2

5 Change each of these fractions to a decimal.

a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{2}{5}$ d $\frac{3}{8}$
e $\frac{3}{4}$ f $\frac{4}{5}$ g $\frac{18}{100}$ h $\frac{9}{50}$
i $\frac{17}{50}$ j $\frac{7}{25}$ k $\frac{19}{25}$ l $\frac{7}{20}$

Fluency

6 Complete each of the following as equivalent fractions.

a $\frac{1}{2}=\frac{?}{4}=\frac{?}{6}=\frac{4}{?}=\frac{?}{10}=\frac{6}{?}$
b $\frac{?}{3}=\frac{4}{6}=\frac{?}{12}=\frac{10}{?}=\frac{?}{24}=\frac{24}{?}$
c $\frac{2}{5}=\frac{?}{10}=\frac{?}{15}=\frac{8}{?}=\frac{?}{40}=\frac{40}{?}$
d $\frac{?}{3}=\frac{3}{9}=\frac{5}{?}=\frac{2}{?}=\frac{4}{?}=\frac{?}{24}$
e $\frac{4}{5}=\frac{?}{15}=\frac{16}{?}=\frac{?}{100}=\frac{32}{?}=\frac{?}{10}$
f $\frac{1}{12}=\frac{?}{24}=\frac{5}{?}=\frac{?}{36}=\frac{6}{?}=\frac{?}{108}$

7 Change each of these decimals to a fraction and cancel if possible.

a 0.6 b 0.8 c 0.7 d 0.25
e 0.75 f 0.24 g 0.58 h 0.36
i 0.98 j 0.72 k 0.128 l 0.125

Worked solutions
Exercise 5.1
MAT08NAWS00014

See Example 3

8 Insert <, = or > where the boxes are to make true statements.

a $\frac{3}{8}\ \square\ \frac{1}{2}$ b $\frac{5}{8}\ \square\ \frac{8}{5}$ c $\frac{5}{9}\ \square\ \frac{5}{10}$
d $\frac{3}{5}\ \square\ \frac{4}{5}$ e $0.325\ \square\ 0.4$ f $2.5\ \square\ 1.789$
g $6.32\ \square\ 7.16$ h $5.85\ \square\ 5.495$ i $\frac{2}{3}\ \square\ \frac{4}{5}$
j $\frac{5}{8}\ \square\ \frac{6}{7}$ k $\frac{1}{4}\ \square\ \frac{1}{7}$ l $\frac{3}{5}\ \square\ \frac{7}{10}$

See Example 4

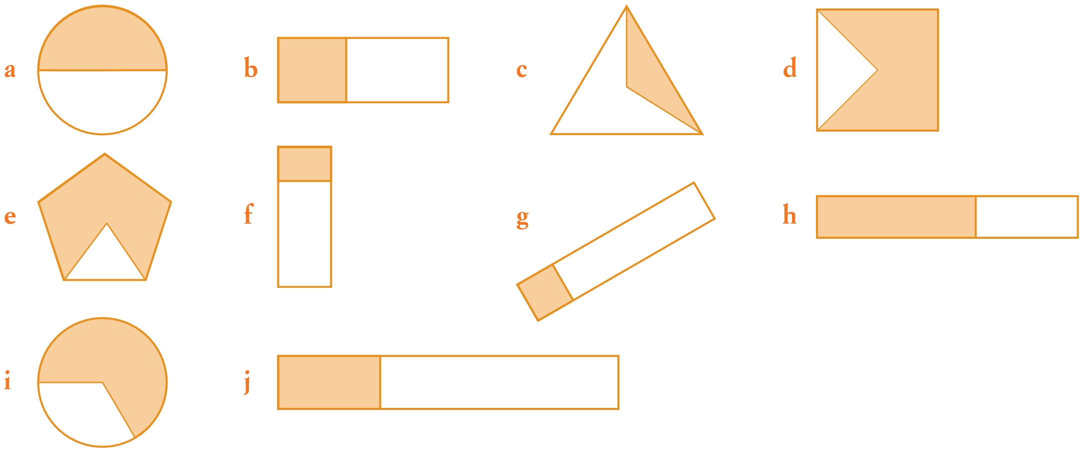
9 Show the following on separate number lines.

a $x < 5\frac{1}{2}$ b $x > 3\frac{4}{5}$ c $3\frac{1}{3} < x < 6\frac{2}{3}$
d $x > 4.7$ e $x > 3.2$ f $x < 4.5$

9780170194778

g $3.1 < x < 5.8$ **h** $6\frac{1}{5} < x < 8\frac{1}{4}$ **i** $x > 4\frac{2}{5}$
j $x < 5\frac{7}{8}$ **k** $x < 7.25$ **l** $3.5 < x < 6.2$

10 Estimate the fractions represented by the shaded parts of the diagrams below in words and symbols as both vulgar and decimal fractions. Choose from: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{3}, \frac{2}{5}, \frac{3}{4}, \frac{3}{5}, \frac{4}{5}$ and 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9.



11 Complete each of the following as equivalent fractions.

a $\frac{?}{2} = \frac{6}{?} = \frac{36}{24} = \frac{15}{?} = \frac{?}{12} = \frac{?}{6}$ **b** $\frac{4}{?} = \frac{?}{60} = \frac{?}{30} = \frac{20}{75} = \frac{12}{?} = \frac{?}{120}$

c $\frac{?}{24} = \frac{?}{16} = \frac{28}{?} = \frac{63}{72} = \frac{35}{?} = \frac{?}{8}$ **d** $\frac{?}{144} = \frac{?}{12} = \frac{22}{?} = \frac{33}{?} = \frac{55}{60} = \frac{44}{?}$

e $\frac{32}{?} = \frac{40}{?} = \frac{8}{?} = \frac{?}{144} = \frac{64}{72} = \frac{?}{18}$ **f** $\frac{?}{28} = \frac{15}{?} = \frac{6}{?} = \frac{?}{21} = \frac{24}{56} = \frac{?}{7}$

12 Write each of these fractions in simplest form.

a $\frac{30}{36}$ **b** $\frac{36}{24}$ **c** $\frac{60}{55}$ **d** $\frac{36}{16}$ **e** $\frac{64}{72}$ **f** $\frac{63}{96}$

g $\frac{60}{16}$ **h** $\frac{35}{75}$ **i** $\frac{24}{21}$ **j** $\frac{30}{18}$ **k** $\frac{18}{60}$ **l** $\frac{28}{12}$

13 State whether the fractions in each of the following pairs are equivalent.

a $\frac{10}{16}, \frac{15}{25}$ **b** $\frac{24}{28}, \frac{28}{35}$ **c** $\frac{10}{15}, \frac{20}{25}$ **d** $\frac{16}{24}, \frac{40}{60}$

e $\frac{18}{55}, \frac{24}{66}$ **f** $\frac{24}{42}, \frac{60}{105}$ **g** $\frac{14}{24}, \frac{34}{60}$ **h** $\frac{16}{36}, \frac{20}{45}$

14 Change each of these fractions to a decimal.

a $\frac{5}{6}$ **b** $\frac{1}{3}$ **c** $\frac{4}{9}$ **d** $\frac{3}{11}$ **e** $\frac{5}{7}$ **f** $\frac{3}{7}$

g $\frac{1}{18}$ **h** $\frac{11}{14}$ **i** $\frac{7}{12}$ **j** $\frac{1}{6}$ **k** $\frac{9}{13}$ **l** $\frac{4}{13}$

15 Change each of these decimals to a fraction and cancel if possible.

a 0.348 **b** 0.448 **c** 0.375 **d** 0.327
e 0.211 **f** 0.986 **g** 0.3125 **h** 0.5625

16 Insert <, = or > to make true statements.

a $\frac{3}{7} \square \frac{4}{9}$ **b** $^{-}4.6 \square {}^{-}6.4$ **c** $\frac{4}{15} \square \frac{7}{20}$ **d** $\frac{5}{8} \square \frac{11}{19}$

e $^{-}0.3 \square {}^{-}0.24$ **f** $^{-}3.45 \square 2.14$ **g** $\frac{9}{5} \square \frac{11}{6}$ **h** $\frac{6}{11} \square \frac{7}{15}$

i $\frac{5}{2} \square \frac{7}{4}$ **j** $\frac{7}{9} \square \frac{10}{12}$ **k** $\frac{7}{2} \square \frac{13}{4}$ **l** $\frac{2}{7} \square \frac{4}{13}$

17 Write in order from smallest to largest.

a $2.3, 3.4, {}^-2.1, {}^-3.1$ **b** ${}^-6.31, {}^-6.1, {}^-6.3$ **c** ${}^-7.6, 6.7, 7.67$
d ${}^-8.2, {}^-8.22, 8.02$ **e** $\frac{1}{3}, \frac{1}{2}, \frac{1}{5}, \frac{1}{4}$ **f** $2\frac{1}{2}, 3\frac{1}{2}, 2\frac{1}{3}$
g $1\frac{1}{4}, 1\frac{3}{4}, 1\frac{2}{3}$ **h** $\frac{5}{8}, \frac{4}{5}, \frac{7}{9}$

18 Show the following on separate number lines.

a $x \geq 4.1$ **b** $x < {}^-2\frac{1}{4}$ **c** ${}^-4.3 \leq x < 1.7$
d ${}^-5\frac{2}{7} < x \leq 4\frac{5}{11}$ **e** $x \leq 3\frac{5}{9}$ **f** $x > {}^-7.57$
g ${}^-5\frac{2}{3} \leq x \leq {}^-2\frac{5}{6}$ **h** ${}^-17.95 < x < 28.7$ **i** ${}^-5\frac{1}{6} \leq x < 5\frac{14}{15}$
j $2\frac{3}{4} < x < 11\frac{1}{6}$ **k** ${}^-35.78 \leq x \leq {}^-11.56$ **l** ${}^-14\frac{3}{5} < x \leq {}^-5\frac{7}{15}$

See Example 5

19 Change each of the following to a decimal.

a $\frac{1}{3}$ **b** $\frac{2}{9}$ **c** $\frac{5}{11}$
d $\frac{7}{9}$ **e** $\frac{2}{3}$ **f** $\frac{1}{7}$

See Example 6

20 Change each of the following to a fraction.

a $0.\overline{57}$ **b** $0.\overline{24}$ **c** $0.\overline{314}$
d $0.\overline{572}$ **e** $0.\overline{324}$ **f** $0.\overline{1122}$

See Example 3

21 Insert $<$, $=$ or $>$ to make true statements.

a $\frac{5}{8} \square 0.6$ **b** $\frac{1}{4} \square 0.2$ **c** $\frac{1}{2} \square 0.5$
d ${}^-\frac{1}{3} \square {}^-\frac{1}{4}$ **e** $\frac{{}^-3}{5} \square {}^-0.6$ **f** $\frac{{}^-9}{10} \square \frac{{}^-10}{11}$
g ${}^-3.3 \square {}^-3\frac{1}{3}$ **h** $2\frac{1}{4} \square 2.\overline{25}$ **i** $5.6 \times 10^{-1} \square \frac{1}{5}$
j $0.6 \square 8.9 \times 10^{-2}$ **k** $\frac{3}{7} \square 7.3 \times 10^{-2}$ **l** $\frac{1}{45} \square 1.5 \times 10^{-2}$

22 Write in order from smallest to largest.

a $0.6, \frac{6}{7}, \frac{7}{8}, 0.9$ **b** $5.2, 5\frac{1}{2}, 5.4, 5\frac{1}{4}$ **c** $0.7, \frac{3}{4}, \frac{2}{3}, 0.8$
d $\frac{1}{5}, 0.1, \frac{1}{4}, \frac{2}{15}$ **e** $0.63, \frac{7}{10}, \frac{3}{5}, 0.61$ **f** $0.3, \frac{3}{11}, \frac{1}{4}, 0.31$
g $\frac{5}{9}, \frac{6}{11}, 0.6, 0.5$ **h** ${}^-1\frac{3}{4}, {}^-1.7, {}^-1\frac{2}{3}, {}^-1.8$ **i** ${}^-0.5, {}^-\frac{3}{4}, 0, \frac{1}{4}$
j $5.5 \times 10^{-1}, \frac{5}{11}, 0.5, \frac{5}{9}$ **k** $6.3 \times 10^{-2}, \frac{1}{20}, 0.02, \frac{3}{50}$ **l** $\frac{{}^-2}{3}, \frac{4}{5}, 0.7, {}^-0.8$

23 Show the following on separate number lines.

a ${}^-17\frac{3}{5} \leq x \leq {}^-2.89$ **b** ${}^-5.78 \leq x < 20\frac{3}{8}$
c ${}^-28.3 < x < 12\frac{7}{9}$ **d** ${}^-51\frac{3}{8} < x \leq 14\frac{6}{11}$
e ${}^-31.83 \leq x < {}^-13\frac{3}{4}$ **f** ${}^-7\frac{5}{12} \leq x \leq {}^-2.52$
g $53.45 < x < 91\frac{1}{14}$ **h** $2.1 \times 10^{1} < x < 28.7$
i ${}^-4.3 \times 10^{-1} \leq x < 6.4 \times 10^{-1}$ **j** ${}^-2.35 \times 10^{1} < x < 2.35 \times 10^{-1}$
k ${}^-3.7 \times 10^{-2} \leq x \leq {}^-\frac{1}{56}$ **l** ${}^-12\frac{5}{8} < x \leq 1.25 \times 10^{1}$

See Example 7

24 Change each of the following to a fraction.

a $0.06\overline{7}$ **b** $0.06\overline{1}$ **c** $0.00\overline{2}$

9780170194778

Problem solving

25 The old imperial measure of 12 inches is 1 foot and 3 feet is 1 yard.

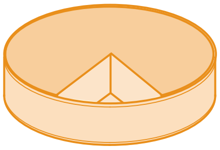
a What fraction of a foot is 5 inches?

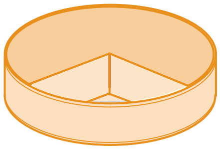
b What fraction of a yard is 5 inches?

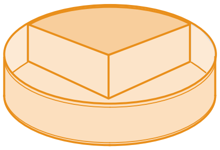
c What fraction of a foot is 8 inches?

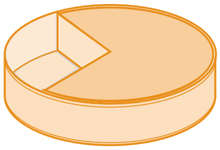
d What fraction of a yard is 10 inches?

26 For each cheesecake, estimate the fraction that has been eaten and the fraction left.

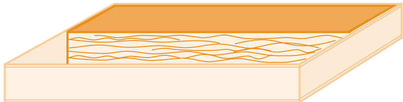
a


b


c


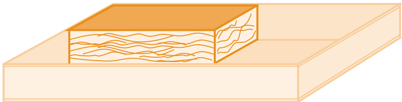
d


27 For each tray of lasagne, estimate the fraction that has been eaten and the fraction left.

a


b


c


d


Reasoning

28 Which would be preferable?

a Half a dollar or 55c?

b 0.6 or $\frac{2}{3}$ of a pie?

c 0.27 or $\frac{1}{4}$ of a chocolate?

29 A working shift at a cannery is 8 hours. Peter worked for half the shift, Brenda worked for 5 hours and Phillipa worked for 0.6 of the shift. Arrange their working times in order.

5.2 Adding and subtracting rational numbers

You can do mental addition and subtraction of some decimals in the same way as for whole numbers. For numbers with the same number of decimal places, you can treat the problem as if it were a whole-number problem.

9780170194778

TLF learning object
Circle3 (L3507)
MAT08NAIN00005

Example 8

Work out the following in your head.

a 6.3 + 5.7 **b** 0.052 − 0.037 **c** 0.642 + 0.853
d 50.6 − 21.8 **e** 25.5 + 1.797 **f** 5.8 − 2.957

Solution

a Treat as 63 + 57 tenths. **6.3 + 5.7 = 12.0**

b Treat as 52 − 37 thousandths. **0.052 − 0.037 = 0.015**

c Treat as 642 + 853 thousandths. **0.642 + 0.853 = 1.495**

d Treat as 506 − 218 (= 500 − 212) tenths. **50.6 − 21.8 = 28.8**

e Add the overlap (0.5 + 0.7) and leave the rest.

$$\begin{aligned}\mathbf{25.5 + 1.797} &= \mathbf{25 + 1 + (0.5 + 0.7) + 0.097}\\ &= \mathbf{26 + 1.2 + 0.097}\\ &= \mathbf{27.297}\end{aligned}$$

f Treat as 5.8 − 3.0 plus 0.1 − 0.057, so as 2.8 + 0.043.

$$\begin{aligned}\mathbf{5.8 - 2.957} &= \mathbf{5.8 - 3.0 + (0.1 - 0.057)}\\ &= \mathbf{2.8 + 0.043}\\ &= \mathbf{2.843}\end{aligned}$$

For decimal addition and subtraction using pen and paper, you need to set out the problem with the same place values under each other.

Important!

Written addition and subtraction of decimals

For positive numbers:

1. Put a decimal point at the right end of whole numbers.
2. Set out vertically with the decimal points under each other.
3. Put zeros on the right if needed, to make all decimal parts the same length.
4. Add or subtract and put the decimal point of the answer under the decimal point in the problem.

For negative numbers:
Change to the opposite operation with positive numbers.

Example 9

Use written methods to solve the following.

a 20.56 + 4.975 + 0.387 + 45 **b** 321.6 − 47.867
c 15.6 + 8.95 − 5.97 + 6.1 − 12.784 **d** 14.75 + $^-$5.96 − $^-$2.6
e $^-$7.9 − 15.92 + $^-$5.83

9780170194778

Solution

a Set out vertically with the decimal points under each other.

Add extra zeros so that all the decimals are the same length.

Add in the usual way, and put the decimal point in place.

$$\begin{array}{r} {}^{1\,1\,2\,1} \\ 20.560 \\ 4.975 \\ 0.387 \\ +\ \underline{45.000} \\ 70.992 \end{array}$$

b Set out vertically with the decimal points under each other.

Add extra zeros so that all the decimals are the same length.

Subtract in the usual way, and put the decimal point in place.

$$\begin{array}{r} 2\ {}^{1}1\ {}^{1}0\ {}^{1}5\ 9 \\ \cancel{3}\ \cancel{2}\ \cancel{1}.\cancel{6}\,{}^{1}\cancel{0}\,{}^{1}0 \\ -\ \underline{4\ 7.8\ 6\ 7} \\ 2\ 7\ 3.7\ 3\ 3 \end{array}$$

c Rearrange for efficiency.

$$15.6 + 8.95 - 5.97 + 6.1 - 12.784$$
$$= 15.6 + 8.95 + 6.1 - 5.97 - 12.784$$

Add extra zeros and add each part.

$$\begin{array}{r} {}^{2\,1} \\ 15.60 \\ 8.95 \\ +\ \underline{6.10} \\ 30.65 \end{array} \qquad \begin{array}{r} {}^{1\,1} \\ 5.970 \\ +\ \underline{12.784} \\ 18.754 \end{array}$$

Now subtract the totals.

$$\begin{array}{r} 2\ 9^{1}\ 5^{1}4 \\ \cancel{3}\,{}^{1}\cancel{0}.\cancel{6}\,\cancel{5}\,{}^{1}0 \\ -\ \underline{1\ 8.7\ 5\ 4} \\ 1\ 1.8\ 9\ 6 \end{array}$$

d Change the negative numbers and operations.

$$14.75 + {}^{-}5.96 - {}^{-}2.6$$

Rearrange.

$$= 14.75 - 5.96 + 2.6$$
$$= 14.75 + 2.6 - 5.96$$

Complete the operations.

$$\begin{array}{r} 14^{1}.75 \\ +\ \underline{2.60} \\ 17.35 \end{array} \qquad \begin{array}{r} 6^{1}2 \\ 1\cancel{7}.\cancel{3}\,{}^{1}5 \\ -\ \underline{5.9\ 6} \\ 11.3\ 9 \end{array}$$

e Change the negative numbers and operations.

$${}^{-}7.9 - 15.92 + {}^{-}5.83$$

Rearrange.

$$= {}^{-}7.9 - 15.92 - 5.83$$
$$= {}^{-}(7.9 + 15.92 + 5.93)$$

Complete the addition.

$$\begin{array}{r} {}^{1}7^{2}.90 \\ 15.92 \\ +\ \underline{5.83} \\ 29.65 \end{array}$$

Make the total negative.

$${}^{-}7.9 - 15.92 + {}^{-}5.83 = {}^{-}29.65$$

9780170194778

You should also be able to use your calculator to perform addition and subtraction of decimals. For example, the calculation $14.75 + {}^{-}5.96 - {}^{-}2.6$ would be entered as

14.75 [+] [(–)] 5.96 [–] [(–)] 2.6 [=].

`14.75+-5.96--2.6    11.39`

Mental addition and subtraction of fractions can be done when they have the same denominator or can easily be changed to the same denominator.

Example 10

Work out the following in your head.

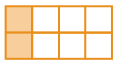
a $\frac{7}{8} - \frac{5}{8}$ **b** $\frac{2}{3} + \frac{5}{6}$ **c** $\frac{4}{5} - \frac{^{-}3}{5}$ **d** $\frac{1}{2} - \frac{3}{4}$

Solution

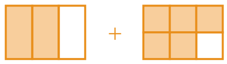
a Think of the parts as pictures.

$\frac{7}{8} - \frac{5}{8}$

Simplify the answer in your head.

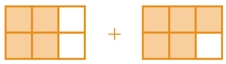
$=$  $= \frac{2}{8} = \frac{1}{4}$

b Think of the parts as pictures.



$\frac{2}{3} + \frac{5}{6}$

Make them all sixths.

$=$  $= \frac{9}{6}$

Simplify the answer in your head.

$= 1\frac{1}{2}$

c Change the negative to adding the opposite.

$\frac{4}{5} - \frac{^{-}3}{5}$

$= \frac{4}{5} + \frac{3}{5}$

Complete the operation and simplify in your head.

$= \frac{7}{5}$

$= 1\frac{2}{5}$

d Think of the half in quarters.

$\frac{1}{2} - \frac{3}{4}$

$= \frac{2}{4} - \frac{3}{4}$

Complete in your head.

$= \frac{^{-}1}{4}$

Standard written methods for adding and subtracting fractions rely on changing the fractions so that they have the same denominator.

9780170194778

Important!

Written methods for adding and subtracting fractions

1 Use multiples to find the **lowest common denominator**: this will be the smallest number that is a multiple of all the denominators.
2 Change the fractions to equivalent fractions with the lowest common denominator.
3 For mixed fractions, calculate the whole-number part separately.
4 Add or subtract the numerators of the fractions and simplify the answer.

Example 11

Use written methods to solve the following.

a $5\frac{1}{3}+4\frac{5}{6}$ **b** $8\frac{1}{4}-3\frac{7}{8}$ **c** $3\frac{1}{2}+5\frac{2}{3}+2\frac{5}{12}$ **d** $7\frac{7}{10}-2\frac{4}{15}$

Solution

a Write the problem. $5\frac{1}{3}+4\frac{5}{6}$

Add the whole numbers and change the fractions to the lowest common denominator, 6. $=9+\frac{2}{6}+\frac{5}{6}$

Add the numerators. $=9+\frac{7}{6}$

$=9+1\frac{1}{6}$

Simplify. $=10\frac{1}{6}$

b Write the problem. $8\frac{1}{4}-3\frac{7}{8}$

Subtract the whole numbers and change the fractions to the lowest common denominator, 8. $=5+\frac{2}{8}-\frac{7}{8}$

Change $5\frac{2}{8}$ to $4\frac{10}{8}$ so that you can subtract. $=4\frac{10}{8}-\frac{7}{8}$

Subtract the numerators. $=4\frac{3}{8}$

c Write the problem. $3\frac{1}{2}+5\frac{2}{3}+2\frac{5}{12}$

Add the whole numbers and change the fractions to the lowest common denominator, 12. $=10+\frac{6}{12}+\frac{8}{12}+\frac{5}{12}$

Add the numerators. $=10\frac{19}{12}$

Simplify. $=11\frac{7}{12}$

d Write the problem. $7\frac{7}{10}-2\frac{4}{15}$

Subtract the whole numbers and change the fractions to the lowest common denominator, 30. $=5+\frac{21}{30}-\frac{8}{30}$

Subtract the numerators. $=5\frac{13}{30}$

Alternative methods
Subtracting in steps
MAT08NAAM00006

Puzzle sheet
Adding and subtracting fractions
MAT08NAPS00018

Worksheet
Fraction arithmagons
MAT08NAWK00035

You can use your calculator to add and subtract fractions.

Example 12

Scientific calculator exercise
Fractions and decimals
MAT08NASC00004

Use your calculator to solve the following.

a $5\frac{2}{3}+4\frac{1}{8}-6\frac{5}{12}$ **b** $^{-}4\frac{1}{2}+2\frac{3}{4}-{}^{-}1\frac{3}{5}$

Solution

a Enter as a 5 [a b/c] 2 [a b/c] 3 [+] 4 [a b/c] 1 [a b/c] 8 [−] 6 [a b/c] 5 [a b/c] 12 [=].

3⌋3⌋8 **or** 3∪3/8 **or** 3r3r8

b Enter as [(−)] 4 [a b/c] 1 [a b/c] 2 [+] 2 [a b/c] 3 [a b/c] 4 [−] [(−)] 1 [a b/c] 3 [a b/c] 5 [=].

−3⌋20 **or** −3/20 **or** −3r20

Different models of calculators will display fractions in different ways.

When solving problems involving fractions or decimals, you should choose the most efficient and appropriate method: mental, written or calculator. However, you need to be able to solve a range of problems using each method.

Exercise 5.2 Adding and subtracting rational numbers

Understanding

Extra questions
Exercise 5.2
MAT08NAEQ00015

Worked solutions
Exercise 5.2
MAT08NAWS00015

1 Calculate the following in your head.

a 0.36 + 0.49 **b** 0.086 − 0.045 **c** 7.2 − 5.8
d 0.0056 + 0.0078 **e** 0.074 + 0.088 **f** 0.74 − 0.37
g 6.7 + 3.5 **h** 0.0045 − 0.0029 **i** 0.77 − 0.68
j 0.048 + 0.084 **k** 0.66 + 0.75 **l** 8.3 − 6.7

See Example 8

2 Calculate the following in your head.

a 19.6 + 23.9 **b** 0.576 − 0.435 **c** 0.0453 + 0.0378
d 5.21 − 3.52 **e** 0.0409 − 0.0215 **f** 28.6 − 13.8
g 0.409 + 0.864 **h** 1.85 + 7.23 **i** 0.00576 − 0.00278
j 47.9 + 69.7 **k** 0.607 + 0.866 **l** 32.6 − 17.9

See Example 10

3 Calculate the following in your head.

a $\frac{4}{5}+\frac{2}{5}$ **b** $\frac{5}{7}+\frac{6}{7}$ **c** $\frac{3}{4}+\frac{3}{8}$ **d** $\frac{5}{6}-\frac{1}{6}$
e $\frac{1}{2}-\frac{1}{8}$ **f** $\frac{3}{10}+\frac{4}{5}$ **g** $\frac{9}{10}-\frac{1}{2}$ **h** $\frac{5}{6}-\frac{1}{3}$
i $\frac{5}{8}+\frac{3}{4}$ **j** $\frac{1}{6}+\frac{1}{3}$ **k** $\frac{3}{4}-\frac{1}{8}$ **l** $\frac{2}{3}+\frac{2}{9}$

Fluency

See Example 9

4 Use written methods to solve the following.

a 17.56 + 23.21 + 418.46 + 89.076 **b** 14.69 − 3.411
c 228.461 − 3.4621 **d** 19 + 2.974 + 8.456 + 0.083 04 + 416.9
e 18.11 + 42.052 + 0.053 + 416.9 **f** 19.4 − 0.034 103
g 304 − 5.1047 **h** 13.03 + 900 + 0.098 + 1.62 + 21.214

i $50 - 49.306$ j $41.76 - 5.864$
k $122.9 + 40 + 0.0679 + 408.905 + 0.19$ l $2182.46 + 18.6 + 5.097 + 0.06 + 4.395 + 0.388$

5 Work out the following on your calculator.
a $34.8 + 5.896 + 19.605 + 5.7098$ b $21.87 + 19.295 + 480 + 21.9871 + 15.89$
c $45.2 - 9.70875$ d $172.967 - 48.73$
e $985 + 21.64 + 0.0786 + 3.9764 + 821.75$ f $525.1 - 221.907$
g $4.917 - 0.0489$ h $17.76 + 38 + 19.8045 + 4560.9 + 91.236$

6 Use written methods to solve the following.
a $28.6 + 5.67 - 0.769 + 234.8 - 263.97$
b $2.097 + 19.6 - 14.709 + 15.3342 - 9.067$
c $18.904 - 7.0907 + 2.9606 - 12.98 + 8.6$
d $907.85 + 21.65 - 432.708 - 21.544 + 9.8$
e $14.78 + 45.765 - 32.45 - 21.674 + 9.0897$
f $0.645 + 0.097 - 0.0865 + 1.6 - 0.8707$

7 Work out the following on your calculator.
a $405.6 + 21.304 - 28.88 - 102.03 + 19.5$ b $2.097 + 4.08 - 3.609 - 1.7906 + 2.9065$
c $42.76 + 51.306 - 32.9809 - 5.403 - 10.8$ d $19.76 + 8.235 - 9.805 - 0.0654 - 4.302$
e $6.7 - 8.93 + 9.64 - 1.26 + 6.098 + 4.5$ f $34.7 - 4.895 - 32.6309 + 12.37 + 19.8$

8 Use written methods to solve the following. *See* Example 11
a $\frac{3}{4} - \frac{5}{12}$ b $\frac{2}{3} - \frac{1}{12}$ c $2\frac{11}{12} + 5\frac{1}{2}$ d $4\frac{1}{4} - 1\frac{3}{8}$
e $\frac{2}{3} + \frac{4}{15}$ f $8\frac{1}{2} - 2\frac{7}{10}$ g $7\frac{3}{8} - 2\frac{3}{4}$ h $\frac{4}{5} + \frac{7}{20}$
i $9\frac{2}{5} + 3\frac{7}{10}$ j $7\frac{5}{6} - 4\frac{1}{3}$ k $4\frac{1}{2} + 3\frac{7}{8}$ l $7\frac{1}{15} - 1\frac{3}{5}$

9 Work out the following on your calculator. *See* Example 12
a $\frac{4}{5} + \frac{3}{8} - \frac{3}{10}$ b $1\frac{2}{3} + 3\frac{3}{4} - \frac{5}{12}$ c $12\frac{1}{4} - \frac{5}{8} - 2\frac{3}{10}$ d $6\frac{7}{8} + 3\frac{3}{4} - 4\frac{2}{3}$
e $4\frac{5}{6} - \frac{7}{12} - \frac{2}{3}$ f $4\frac{5}{6} - \frac{5}{12} - 1\frac{2}{3}$ g $6\frac{1}{2} + 4\frac{3}{8} - 5\frac{3}{4}$ h $9\frac{1}{4} + 3\frac{4}{5} - 5\frac{7}{10}$

10 Work out the following in your head.
a $21.34 + 0.765$ b $19.8 - 3.564$ c $15.672 + 12.8$ d $134.7 + 1.9764$
e $7.99 - 3.4765$ f $21.789 - 1.99$ g $18.25 + 7.7563$ h $45.98 - 6.857$

11 Use written methods to solve the following.
a $^{-}4.67 + 8.975$ b $9.234 - {}^{-}3.4$ c $^{-}5.783 + {}^{-}21.77$ d $9.63 - 12.876$
e $^{-}19.03 - {}^{-}3.407$ f $^{-}1.2306 + 8.904$ g $5.78 - {}^{-}7.0905$ h $^{-}4.098 + 52.8$
i $3.0987 - 5.776$ j $^{-}7.804 - 6.0302$ k $89.07 + {}^{-}32.705$ l $^{-}5.8 + 19.3027$

12 Work out the following on your calculator.
a $^{-}6.83 + 5.743 + {}^{-}3.75 - 7.5$ b $18.54 - 21.3404 + {}^{-}0.896 - {}^{-}4.7 + 0.0706$
c $42.08 - 13.76 + {}^{-}0.9089 - {}^{-}6.0876$ d $32.709 + {}^{-}5.409 - 15.08 + {}^{-}45.456 - 12.8$
e $^{-}0.308 + 4.07 - {}^{-}9.0907 - 42.32 + 12.76$ f $18.76 + {}^{-}102.3 - 5.78 - {}^{-}54.342 + 8$

13 Work out the following in your head.
a $\frac{1}{2} - \frac{3}{4}$ b $\frac{^{-}3}{5} + \frac{4}{5}$ c $\frac{^{-}5}{6} - \frac{1}{2}$ d $\frac{3}{5} - \frac{^{-}7}{10}$
e $\frac{^{-}3}{4} - \frac{^{-}7}{8}$ f $\frac{1}{6} - \frac{2}{3}$ g $\frac{5}{8} + \frac{^{-}3}{4}$ h $\frac{^{-}1}{4} - \frac{7}{8}$
i $\frac{1}{6} - \frac{^{-}1}{2}$ j $\frac{^{-}3}{8} + \frac{^{-}7}{8}$ k $\frac{1}{5} + \frac{^{-}3}{10}$ l $\frac{2}{3} + \frac{^{-}1}{2}$

14 Use written methods to solve the following.

a $2\frac{7}{8}+7\frac{2}{3}$ **b** $8\frac{1}{3}-3\frac{3}{8}$ **c** $6\frac{1}{2}-4\frac{2}{5}$ **d** $\frac{1}{6}+\frac{2}{3}+\frac{1}{2}$

e $\frac{1}{2}-\frac{1}{3}+\frac{3}{4}$ **f** $\frac{5}{8}+\frac{7}{12}-\frac{1}{6}$ **g** $\frac{3}{5}+\frac{1}{4}+\frac{2}{3}$ **h** $4\frac{5}{6}-1\frac{1}{2}-1\frac{3}{4}$

i $1\frac{1}{6}+3\frac{7}{10}-\frac{11}{15}$ **j** $3\frac{2}{7}+1\frac{1}{2}+5\frac{2}{3}$ **k** $2\frac{1}{2}-1\frac{7}{9}+2\frac{5}{36}$ **l** $6\frac{1}{4}+4\frac{11}{12}-9\frac{3}{8}$

15 Work out the following on your calculator.

a $^{-}2\frac{1}{2}+1\frac{1}{2}$ **b** $^{-}4\frac{2}{3}+1\frac{1}{2}$ **c** $2\frac{2}{5}-3\frac{5}{8}$ **d** $^{-}5\frac{1}{4}-{}^{-}3\frac{2}{3}$

e $^{-}2\frac{4}{5}+3\frac{1}{2}$ **f** $6\frac{2}{7}-6\frac{2}{3}$ **g** $1\frac{2}{5}-3\frac{7}{10}+\frac{1}{2}$ **h** $\frac{3}{8}-2\frac{1}{2}$

i $^{-}5\frac{2}{3}+4\frac{5}{8}-2\frac{5}{6}$ **j** $3\frac{2}{3}-5\frac{4}{5}+{}^{-}1\frac{2}{3}$ **k** $^{-}4\frac{5}{8}-{}^{-}6\frac{1}{3}+1\frac{3}{4}$ **l** $6\frac{1}{4}-8\frac{7}{8}+{}^{-}1\frac{4}{5}$

Problem solving

16 Lucy had $21.54 and her big brother James had $20 in their bank accounts. Their little sister Tania had only 53 cents. How much did they have altogether?

17 A 'special' coolant for formula 1 car engines is made by mixing together 5.3 L of water, 0.685 L of glycol and 1.25 L of methylated spirits. The engine cooling system has a capacity of 8 L.

a How much coolant is there?

b The radiator is topped up with water. How much water is actually in the cooling system?

18 It is found that a new home has accidentally been built across the boundary between two blocks of land. The home builder's block is 0.0675 ha in area. The neighbour agrees to sell the home builder a strip of land of area 0.0201 ha. The neighbour's block was originally 0.0928 ha. How big is each block now?

19 As well as restricting the mass of passengers' baggage, airlines sometimes limit baggage by the total linear dimensions. Giulio's expanding suitcase is 15.2 cm thick, 75.34 cm long and 50 cm wide. The limit for total linear dimensions is 145 cm.

a What are the total linear dimensions of his suitcase?

b How much thicker can he make it and still stay within the limit?

20 Cora has some concrete blocks left over from building a barbecue. There are 2 full blocks, 5 halves, 4 three-quarters and 2 quarters (all cut vertically).

a What is the longest single row of blocks she can make with the leftovers?

b Draw an arrangement of two equal rows with staggered joints.

9780170194778

21 If you buy $2\frac{1}{4}$ m of material and use $\frac{3}{4}$ m for a cushion cover and $\frac{5}{8}$ m for table napkins, how much is left?

Worked solutions
Exercise 5.2
MAT08NAWS00015

22 Samantha can only buy paint in 1 L or 4 L tins. She needs 21 L for the eaves of her house and $\frac{3}{4}$ L for the gutters.
a How much paint should she buy?
b How much will she have left over?

Reasoning

23 Scientists working for a chemical company are extracting a new drug. In the laboratory they find that 57.5 g of deadly nightshade berries contains 39.34 g of water and 15.214 g of cellulose. Of the remaining part of the berries, 2.856 g is not useful.
If 0.0362 g of the drug is lost in the extraction, how much is actually obtained?

24 Sylvia is a sales representative. She invites all her clients to a Christmas party. The Christmas cake is a whopper! The baker used 28.5 kg of sugar, 42.37 kg of flour, 22.5 kg of fats, 0.31 kg of spices, 5 kg of eggs, 20 kg of water and 41.07 kg of fruit. How much does the cake weigh if 1.37 kg of water and 0.086 kg of carbon dioxide were lost in cooking?

Worked solutions
Exercise 5.2
MAT08NAWS00015

25 A recipe includes $\frac{1}{4}$ tsp of nutmeg, $\frac{1}{2}$ tsp of ground cloves, $\frac{3}{4}$ tsp of ground ginger, $\frac{1}{3}$ tsp of oregano and $\frac{1}{8}$ tsp of dried basil. What is the total quantity of herbs and spices?

26 Farmer Giles has $\frac{1}{2}$ a haystack in one paddock, $2\frac{1}{4}$ in another, and $\frac{5}{6}$ in a third. How much hay has he altogether?

27 Peta's garden can only be watered between 4 p.m. and 10 p.m. in winter. There are three flower beds that need $\frac{3}{4}$ hour, $1\frac{1}{2}$ hours and 2 hours of watering respectively. The lawn needs $1\frac{2}{3}$ hours. How much time is left for the vegetable garden?

5.3 Multiplying and dividing rational numbers

You should be able to multiply or divide by 0.01, 0.1, 10, 100, 1000 and so on in your head by working out the effect on place values. Multiplication or division by numbers such as 20 and 200 can be done by thinking of them as 2×10 or 2×100. You can also adapt methods from mental calculation with whole numbers.

Example 13

Work out the following in your head.

a 28.3×1000 **b** 5×16.8 **c** $\frac{28.6}{1000}$ **d** $7.64 \div 0.001$
e 25×7.63 **f** $29.7 \div 300$ **g** 2.9×5.8 **h** 0.025×5.84

Solution

a Increase the values by three places, so 8 changes to 8000.
$$28.3 \times 1000 = 28\,300$$

b Treat 5 as $10 \div 2$, so increase the values by one place and halve.
$$\begin{aligned} 5 \times 16.8 &= 168 \div 2 \\ &= 84 \end{aligned}$$

c Decrease the values by three places, so 8 changes to 0.008.
$$\frac{28.6}{1000} = 0.0286$$

d Increase the values by three places, so 7 changes to 7000.
$$7.64 \div 0.001 = 7640$$

e Treat 25 as $100 \div 4$, so increase the values by two places and divide by 4.
$$\begin{aligned} 25 \times 7.63 &= 763 \div 4 \\ &= 190.75 \end{aligned}$$

f Treat 300 as 100×3, so decrease the values by two places and divide by 3.
$$\begin{aligned} 29.7 \div 300 &= 0.297 \div 3 \\ &= 0.099 \end{aligned}$$

g Treat as $3 \times 5.8 - 0.1 \times 5.8$, so calculate $17.4 - 0.58$.
$$\begin{aligned} 2.9 \times 5.8 &= 3 \times 5.8 - 0.1 \times 5.8 \\ &= 16.82 \end{aligned}$$

h Treat 0.025 as $\frac{1}{40}$ so decrease the values by one place and divide by 4.
$$\begin{aligned} 0.025 \times 5.84 &= 0.584 \div 4 \\ &= 0.146 \end{aligned}$$

The written methods for multiplication and division of decimals are set out below. These methods take account of the place values of the decimals by counting or changing decimal places.

Important!

Written multiplication and division of decimals

For multiplication:

1. Ignore the decimal point and multiply as whole numbers.
2. Count the total number of decimal places in the numbers.
3. Insert the decimal point to give this total number of decimal places in the answer.

For division:

1. Change both numbers by the same number of decimal places to make the divisor a whole number.
2. Perform the new division, adding zeros as needed to complete the calculation.
3. If the answer does not terminate, indicate the repeating digits with a dot over the top of the first and last numbers of a sequence of repeating digits.

9780170194778

Example 14

Use written methods to solve the following.

a 5.72×6.4 **b** $7.25 \div 0.4$ **c** $56.5 \times 0.000\,324$

d $\frac{9.858}{0.11}$ **e** $5.6 \times {}^{-}4.8$ **f** ${}^{-}9.8 \div {}^{-}0.08$

Solution

a Ignore the decimal points and set out as 572 × 64. So put the 2 over the 4.

Multiply out.

Count the total number of decimal places and put in the decimal point.

$$\begin{array}{rrl} & 5.7\,2 & \text{2 places} \\ \times & 6.4 & \text{1 place} \\ \hline & 2\,2\,8\,8 & \text{3 places} \\ & 3\,4\,3\,2\,0 & \\ \hline & 3\,6.6\,0\,8 & \end{array}$$

Write the answer.

5.72 × 6.4 = 36.608

b Change both numbers by the same number of places to divide by a whole number.

$$7.25 \div 0.4$$
$$= 72.5 \div 4$$

Now set out as usual, putting the decimal point for the answer above the decimal point in the question.

$$\begin{array}{r} 1\;8\,.1\;2\;5 \\ 4\,\overline{)\,7^{3}2\,.5^{1}0^{2}0} \end{array}$$

Write the answer.

7.25 ÷ 0.4 = 18.125

c Ignore the decimal points and set out as 565 × 324. So put the 5 over the 4.

Multiply out.

Count the total number of decimal places and put in the decimal point.

Write the answer.

$$\begin{array}{rrl} & 5\,6.5 & \text{1 place} \\ \times & 0.0\,0\,0\,3\,2\,4 & \text{6 places} \\ \hline & 2\,2\,6\,0 & \text{7 places} \\ & 1\,1\,3\,0\,0 & \\ & 1\,6\,9\,5\,0\,0 & \\ \hline & 0.0\,1\,8\,3\,0\,6\,0 & \end{array}$$

d Change both numbers by the same number of places to divide by a whole number.

$$\frac{9.858}{0.11}$$
$$= 985.8 \div 11$$

Now set out as usual, putting the decimal point for the answer above the decimal point in the question.

$$\begin{array}{r} 8\;\;9.\;6\;1\;8\;1\;8\ldots \\ 11\,\overline{)\,98^{10}5.^{6}8^{2}0^{9}0^{2}0^{9}0\ldots} \end{array}$$

Write the answer, using a bar for repeating digits.

$$\frac{9.858}{0.11} = 89.6\overline{18}$$

e Ignore signs and set out as 56 × 48.

Multiply out and put in the decimal point.

$$\begin{array}{rrl} & 5.6 & \text{1 place} \\ \times & 4.8 & \text{1 place} \\ \hline & 4\,4\,8 & \text{2 places} \\ & 2\,2\,4\,0 & \\ \hline & 2\,6.8\,8 & \end{array}$$

Take account of signs and write the answer.

$5.6 \times {}^{-}4.8 = {}^{-}26.88$

Puzzle sheet
Operations with decimals
MAT08NAPS00017

Worksheet
Decimal number grids
MAT08NAWK00033

f Ignore signs and calculate as $9.8 \div 0.08$. $\quad 9.8 \div 0.08$

Change the decimal places. $\quad = 980 \div 8$

Perform the division.

$$\begin{array}{r} 1\ 2\ 2.\ 5 \\ 8\,\overline{)\,9^{1}8^{2}0.^{4}0} \end{array}$$

Take account of signs and write the answer. $\quad {}^{-}9.8 \div {}^{-}0.08 = 122.5$

For problems involving many digits, it may be more efficient to use a calculator than mental or written methods for decimal multiplication or division.

Some multiplications by unit fractions can be done in your head by treating the problem as a division, perhaps rewriting the other fraction to obtain a suitable numerator. Some divisions can be done by partitioning.

Example 15

Work out the following in your head.

a $\frac{1}{3} \times 18$ **b** $\frac{1}{3} \times \frac{5}{8}$ **c** $1\frac{2}{3} \div \frac{1}{6}$ **d** $1\frac{2}{5} \div \frac{3}{10}$

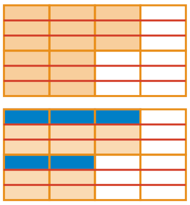
Solution

a Treat multiplication by $\frac{1}{3}$ as division by 3.

$$\frac{1}{3} \times 18 = \frac{18}{3} = 6$$

b Think of the 8ths as 24ths so that the numerator can be divided by 3.

Now perform the division by taking a third of the shaded parts.

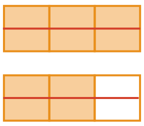


$$\frac{5}{8} = \frac{15}{24}$$

$$\frac{1}{3} \times \frac{5}{8} = \frac{15}{24} \div 3 = \frac{5}{24}$$

c Think of $1\frac{2}{3}$ in 6ths.

Think of the division as 'how many sixths in 10 sixths?'

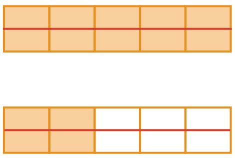


$$1\frac{2}{3} = \frac{10}{6}$$

$$1\frac{2}{3} \div \frac{1}{6} = 10$$

d Think of $1\frac{2}{5}$ in 10ths.

Think of the division as 'how many lots of 3 (tenths) in 14 (tenths)?'



$$1\frac{2}{5} = \frac{14}{10}$$

$$14 \div 3 = 4\frac{2}{3}$$

$$1\frac{2}{5} \div \frac{3}{10} = 4\frac{2}{3}$$

9780170194778

A fraction can be written as the numerator multiplied by the unitary fraction of the denominator, so $\frac{3}{5}$ can be written as $3 \times \frac{1}{5}$. From this you can also see that multiplication by a unitary fraction is the same as division by the denominator.
Therefore, multiplication by a fraction can be done as multiplication by the numerator and division by the denominator. For example, $20 \times \frac{3}{5} = 60 \div 5 = 12$.
This gives the following method for the written multiplication of fractions.

Teacher notes
Multiplication of fractions
MAT08NATN00011

Important!

Written multiplication of fractions

1. Change mixed fractions to improper fractions.
2. Multiply the numerators and multiply the denominators.
3. Simplify the answer, including cancellation by common factors. If desired, cancellation by common factors can be done before the multiplications.

Example 16

Work out the following using written methods.

a $\frac{1}{2} \times \frac{4}{5}$ **b** $\frac{3}{5} \times \frac{4}{7}$ **c** $\frac{5}{8} \times \frac{^-4}{15}$ **d** $1\frac{1}{4} \times 3\frac{1}{2}$

Alternative method
Cancelling at the end
MAT08NAAM00004

Solution

a Write the problem. $\frac{1}{2} \times \frac{4}{5}$

It is quickest to cancel first if possible. $= \frac{1}{{}_1\cancel{2}} \times \frac{\cancel{4}^2}{5}$

Multiply the top and multiply the bottom. $= \frac{1 \times 2}{1 \times 5}$

Write the answer. $= \frac{2}{5}$

b Write the problem. $\frac{3}{5} \times \frac{4}{7}$

There is no cancellation, so multiply. $= \frac{3 \times 4}{5 \times 7}$

Write the answer. $= \frac{12}{35}$

c Write the problem without signs. $\frac{5}{8} \times \frac{4}{15}$

Cancel where possible. $= \frac{{}^1\cancel{5}}{{}_2\cancel{8}} \times \frac{\cancel{4}^1}{\cancel{15}_3}$

Multiply the top and multiply the bottom. $= \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

Write the answer, taking signs into account. $\frac{5}{8} \times \frac{^-4}{15} = \frac{^-1}{6}$

d Write the problem. $1\frac{1}{4} \times 3\frac{1}{2}$

Change to improper fractions. There is no cancellation, so multiply. $= \frac{5}{4} \times \frac{7}{2}$

$= \frac{35}{8}$

Simplify and write the answer. $= 4\frac{3}{8}$

By thinking of division by a fraction as partitioning, we obtain the following rules for dividing fractions.

Important!

Written division of fractions

1. Change mixed fractions to improper fractions.
2. Multiply by the reciprocal of the divisor; that is, turn the second fraction upside-down and change the operation to multiply.

Teacher notes
Division of fractions
MAT08NATN00010

Example 17

Work out the following using written methods.

a $\frac{5}{8} \div \frac{3}{4}$ **b** $\frac{1}{2} \div \frac{1}{9}$ **c** $\frac{^-3}{5} \div 3$ **d** $7\frac{1}{3} \div 5\frac{1}{2}$ **e** $^-2\frac{2}{3} \div \frac{^-4}{5}$

Alternative methods
Cancelling the multiplication at the end
MAT08NAAM00005

Worksheet
Multiplying and dividing fractions number grids
MAT08NAWK00036

Solution

a Write the problem. $\frac{5}{8} \div \frac{3}{4}$

Invert $\frac{3}{4}$ and change to multiplication. $= \frac{5}{8} \times \frac{4}{3}$

Cancel where possible. $= \frac{5}{_2\cancel{8}} \times \frac{\cancel{4}^1}{3}$

Multiply and write the answer. $= \frac{5 \times 1}{2 \times 3}$

$= \frac{5}{6}$

b Write the problem. $\frac{1}{2} \div \frac{1}{9}$

Invert $\frac{1}{9}$ and change to multiplication. $= \frac{1}{2} \times \frac{9}{1}$

There is no cancellation, so multiply. $= \frac{9}{2}$

Simplify and write the answer. $= 4\frac{1}{2}$

9780170194778

c Write the problem without signs, putting the 3 in fraction format. $= \frac{3}{5} \div \frac{3}{1}$

Invert $\frac{3}{1}$ and change to multiplication. $= \frac{3}{5} \times \frac{1}{3}$

Cancel where possible. $= \frac{{}^{1}\cancel{3}}{5} \times \frac{1}{\cancel{3}_{1}}$

Multiply the tops and multiply the bottoms. $= \frac{1}{5}$

Write the answer, taking signs into account. $\frac{^{-}3}{5} \div 3 = \frac{^{-}1}{5}$

d Write the problem. $7\frac{1}{3} \div 5\frac{1}{2}$

Change to improper fractions. $= \frac{22}{3} \div \frac{11}{2}$

Invert $\frac{11}{2}$ and change to multiplication. $= \frac{22}{3} \times \frac{2}{11}$

Cancel where possible. $= \frac{{}^{2}\cancel{22}}{3} \times \frac{2}{\cancel{11}_{1}}$

Multiply. $= \frac{2 \times 2}{3 \times 1} = \frac{4}{3}$

Simplify and write the answer. $= 1\frac{1}{3}$

e Write the problem without signs. $2\frac{2}{3} \div \frac{4}{5}$

Change 2 to an improper fraction. $= \frac{8}{3} \div \frac{4}{5}$

Invert $\frac{4}{5}$ and change to multiplication. $= \frac{8}{3} \times \frac{5}{4}$

Cancel where possible. $= \frac{{}^{2}\cancel{8}}{3} \times \frac{5}{\cancel{4}_{1}}$

Multiply. $= \frac{2 \times 5}{3 \times 1} = \frac{10}{3}$

Simplify. $= 3\frac{1}{3}$

Write the answer, taking signs into account. $^{-}2\frac{2}{3} \div \frac{^{-}4}{5} = 3\frac{1}{3}$

It is important that you are able to use your calculator to perform fraction operations so that you can choose whether a mental, written or calculator method will be the most efficient for any particular problem.

Scientific calculator exercise
Fractions and decimals
MAT08NASC00004

Example 18

Use your calculator to solve the following.

a $\frac{3}{5} \times \frac{4}{9}$ **b** $2\frac{3}{4} \div 5\frac{1}{8}$ **c** $^{-}3\frac{2}{3} \times 5\frac{1}{2}$ **d** $^{-}11\frac{7}{8} \div {}^{-}4\frac{1}{6}$

Solution

a Enter as 3 [a b/c] 5 [×] 4 [a b/c] 9 [=].

4⌋15	or	4/15	or	4r15

b Enter as 2 [a b/c] 3 [a b/c] 4 [÷] 5 [a b/c] 1 [a b/c] 8 [=].

22⌋41	or	22/41	or	22r41

c Enter as [(−)] 3 [a b/c] 2 [a b/c] 3 [×] 5 [a b/c] 1 [a b/c] 2 [=].

−20⌋1⌋6	or	−20∪1/6	or	−20r1r6

d Enter as [(−)] 11 [a b/c] 7 [a b/c] 8 [÷] [(−)] 4 [a b/c] 1 [a b/c] 6 [=].

2⌋17⌋20	or	2∪17/20	or	2r17r20

Example 19

Work out the following.

a $^{-}2.\dot{3} + \frac{5}{8} - \frac{^{-}7}{12}$ **b** $2\frac{5}{8} + {}^{-}7.964 - {}^{-}1\frac{7}{12} + 2.31$

Solution

a This is an exceptional case. Change the decimal to a fraction.

$$^{-}2.\dot{3} + \frac{5}{8} - \frac{^{-}7}{12}$$
$$= {}^{-}2\frac{1}{3} + \frac{5}{8} - \frac{^{-}7}{12}$$

Now solve by changing to a common denominator.

$$= \frac{^{-}56}{24} + \frac{15}{24} + \frac{14}{24}$$

You could also do this on your calculator using the fraction key like Example **15**.

$$= \frac{^{-}27}{24} = \frac{^{-}9}{8} \text{ or } {}^{-}1\frac{1}{8}$$

b Change to decimals.

$$2\frac{5}{8} + {}^{-}7.964 - {}^{-}1\frac{7}{12} + 2.31$$

Now do the calculation.

$$\approx 2.625 + {}^{-}7.964 - {}^{-}1.583 + 2.31$$
$$= {}^{-}1.446$$

You might be able to do Example **16**, part **b** on your calculator using the fraction key as well as putting in the decimals as follows:

2 [a b/c] 5 [a b/c] 8 [+] [(−)] 7.964 [−] [(−)] 1 [a b/c] 7 [a b/c] 12 [+] 2.31 [=]

giving the answer $^{-}1.445666667$ which rounds to $^{-}1.446$.

9780170194778

Exercise 5.3 Multiplying and dividing rational numbers

Understanding

Extra questions
Exercise 5.3
MAT08NAEQ00016

Worked solutions
Exercise 5.3
MAT08NAWS00016

1 Work out the following in your head.

a 0.067×100 b $5.678 \div 1000$ c 15.8×0.1
d $\dfrac{0.5695}{0.01}$ e $0.06 \div 100$ f 9860×1000
g $0.0698 \div 0.001$ h 98.3×0.001 i 5×0.28
j $\dfrac{21.97}{2}$ k $0.0078/5$ l 2×0.053

2 Use written methods to solve the following.

See Example 13
See Example 14

a 3.4×2.9 b $0.069 \div 3$ c 9.25×3.4 d $\dfrac{5.83}{0.2}$
e 21.2×0.88 f $\dfrac{5.8}{0.005}$ g 1.25×0.96 h $6.59 \div 0.4$
i 4.53×6.4 j 0.835×0.26 k $5.44/0.5$ l $2.7 \div 0.08$

3 Use a calculator to solve the following.

a 28.56×5.785 b $9.637 \div 0.054$ c $28.076 \div 5.65$ d $0.9268 \times 0.005\,44$
e $182.9 \div 5.6$ f 64.707×22.8 g $145.8 \times 0.060\,98$ h $16.82 \div 0.0757$

4 Work out these multiplications in your head.

See Example 15

a $\frac{1}{3} \times 45$ b $68 \times \frac{1}{2}$ c $725 \times \frac{1}{5}$ d $\frac{1}{4} \times 836$
e $42 \times \frac{1}{4}$ f $26 \times \frac{1}{3}$ g $14 \times \frac{1}{5}$ h $29 \times \frac{1}{8}$

Fluency

5 Work out the following in your head.

a 0.057×25 b $28.6 \div 200$ c 0.038×0.002 d $\dfrac{0.008\,56}{8}$
e 0.6×0.8 f $18.6 \div 0.03$ g 5.92×5 h $0.0042 \div 600$
i 33.5×0.04 j $\dfrac{0.0063}{90}$ k 0.089×0.002 l $4.8/0.06$

6 Use written methods to solve the following.

a 3.47×2.98 b 34.5×2.48 c $\dfrac{19.2}{0.09}$ d $23.08 \div 1.1$
e $2.904 \div 0.07$ f 98.45×3.67 g $17.8 \times 0.006\,04$ h $\dfrac{15.22}{0.6}$
i 2.02×0.0505 j $0.00867/0.12$ k $452.1 \div 1.3$ l 29.6×4.9074

7 Work out the following in your head.

See Example 15

a $\frac{5}{8} \times \frac{1}{2}$ b $\frac{1}{4} \times \frac{8}{15}$ c $\frac{3}{4} \div \frac{1}{8}$ d $2\frac{1}{2} \div \frac{1}{4}$
e $\frac{3}{5} \times \frac{1}{3}$ f $2\frac{1}{4} \div \frac{1}{8}$ g $\frac{2}{3} \div \frac{1}{6}$ h $\frac{3}{5} \times \frac{1}{4}$
i $3\frac{2}{3} \div \frac{1}{9}$ j $\frac{4}{5} \times \frac{1}{3}$ k $4\frac{1}{3} \div \frac{1}{6}$ l $\frac{9}{10} \times \frac{1}{3}$

8 Use written methods to solve the following.

See Example 16

a $\frac{3}{5} \times \frac{5}{8}$ b $\frac{5}{7} \times \frac{3}{8}$ c $\frac{5}{8} \div \frac{3}{4}$ d $\frac{2}{3} \div \frac{5}{6}$
e $\frac{3}{4} \div \frac{1}{5}$ f $\frac{4}{9} \times \frac{3}{10}$ g $\frac{7}{8} \times \frac{9}{14}$ h $\frac{4}{5} \div \frac{7}{8}$
i $\frac{2}{5} \times \frac{15}{16}$ j $\frac{1}{9} \div \frac{2}{3}$ k $\frac{7}{10} \div \frac{3}{5}$ l $\frac{5}{18} \times \frac{3}{10}$

See Example 17

9 Use a calculator to solve the following.

a $1\frac{1}{2} \times \frac{2}{9}$ **b** $2\frac{4}{5} \times 2\frac{1}{2}$ **c** $1\frac{1}{4} \div 1\frac{1}{2}$ **d** $3\frac{1}{3} \div 1\frac{2}{3}$

e $3\frac{1}{2} \times 4\frac{1}{2}$ **f** $2\frac{1}{8} \div \frac{1}{4}$ **g** $6\frac{3}{5} \times 8\frac{1}{3}$ **h** $3\frac{3}{4} \div 9\frac{3}{8}$

i $1\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{3}$ **j** $3\frac{1}{5} \times 2\frac{1}{4} \div 2\frac{2}{15}$ **k** $6\frac{3}{4} \div 9 \times \frac{4}{5}$ **l** $2\frac{2}{3} \times 1\frac{5}{16} \div 3\frac{1}{2}$

10 Work out the following in your head.

a $0.76 \div 0.4$ **b** 59×0.035 **c** 0.05×1.64 **d** $\frac{5.78}{0.025}$

e 9.9×0.074 **f** $0.0068 \div 0.005$ **g** 0.0079×2.5 **h** 0.039×0.45

11 Use written methods to solve the following.

a $^{-}5.3 \times 4.22$ **b** $4.85 \div {}^{-}0.4$ **c** $3.6 \times {}^{-}0.047$ **d** $\frac{^{-}0.086}{^{-}0.6}$

e $^{-}5.824 \div 0.07$ **f** $^{-}0.0457 \times {}^{-}1.4$ **g** $0.975/{}^{-}0.008$ **h** $^{-}0.058 \times 0.75$

i $^{-}7.8 \times {}^{-}1.694$ **j** $0.886 \times {}^{-}0.79$ **k** $^{-}7.7 \div {}^{-}0.009$ **l** $\frac{^{-}0.008\,07}{1.5}$

12 Use a calculator to solve the following.

a $^{-}5.785 \times {}^{-}0.0974$ **b** $14.66 \div {}^{-}5.82$ **c** $\frac{^{-}0.0185}{0.95}$ **d** $0.0732 \times {}^{-}0.058\,05$

e $^{-}164.74 \div {}^{-}14.84$ **f** $^{-}25.82 \times 16.074$ **g** $^{-}63.24/178.92$ **h** $^{-}5.97 \times {}^{-}7.8082$

See Example 17

13 Work out the following in your head.

a $2\frac{5}{8} \div \frac{3}{8}$ **b** $2\frac{1}{2} \times \frac{4}{5}$ **c** $2\frac{2}{5} \div \frac{4}{5}$ **d** $4\frac{1}{2} \div \frac{3}{4}$

e $3\frac{1}{2} \times \frac{5}{7}$ **f** $3\frac{1}{3} \div \frac{5}{6}$ **g** $3\frac{3}{4} \div 1\frac{1}{4}$ **h** $5\frac{1}{2} \div 1\frac{3}{8}$

i $1\frac{1}{2} \times \frac{3}{4}$ **j** $5\frac{1}{4} \div \frac{3}{4}$ **k** $7\frac{1}{2} \div \frac{5}{8}$ **l** $1\frac{1}{5} \times \frac{1}{12}$

See Example 17

14 Use written methods to solve the following.

a $\frac{1}{2} \times \frac{^{-}3}{5}$ **b** $4\frac{1}{2} \times \frac{2}{3}$ **c** $7\frac{1}{3} \div {}^{-}5\frac{1}{2}$ **d** $\frac{^{-}1}{2} \times \frac{2}{3} \times \frac{^{-}2}{5}$

e $^{-}6\frac{1}{2} \div {}^{-}2\frac{3}{5}$ **f** $1\frac{4}{5} \div 4\frac{1}{2}$ **g** $5\frac{1}{3} \times 2\frac{1}{4}$ **h** $4\frac{3}{8} \times 1\frac{3}{5}$

i $^{-}2\frac{3}{4} \div 5\frac{1}{2}$ **j** $^{-}3\frac{3}{4} \div {}^{-}2\frac{1}{2}$ **k** $1\frac{2}{3} \times {}^{-}7\frac{1}{2}$ **l** $2\frac{2}{3} \div {}^{-}1\frac{7}{9}$

See Example 18

15 Use a calculator to solve the following.

a $2\frac{3}{4} \times 4\frac{1}{2} \div 3\frac{1}{4} \div 1\frac{4}{5}$ **b** $7\frac{1}{2} \div 3\frac{3}{4} \times 2\frac{1}{2}$ **c** $1 \div \frac{3}{4} \times 7 \div 3\frac{1}{2}$

d $^{-}2 \times 1\frac{3}{4} \div 4\frac{2}{3}$ **e** $^{-}3\frac{1}{2} \times 1\frac{5}{7} \times \frac{^{-}7}{8}$ **f** $\frac{5}{16} \div 1\frac{1}{2} \div \frac{^{-}5}{8}$

g $^{-}2\frac{1}{4} \div \frac{7}{15} \times \frac{3}{10}$ **h** $^{-}1\frac{2}{3} \div 3\frac{3}{4} \times {}^{-}5\frac{1}{3}$ **i** $\left(5\frac{1}{2}\right)^2$

j $\left(2\frac{3}{4}\right)^3$ **k** $\left(\frac{^{-}1}{2}\right)^3$ **l** $\left({}^{-}2\frac{1}{3}\right)^5$

See Example 19

16 Work out the following.

a $^{-}1.\overline{3} - \frac{^{-}7}{9}$ **b** $2\frac{1}{4} - 1.875 + \frac{1}{12}$

c $^{-}4.35 + \frac{3}{8} + \frac{^{-}4}{5}$ **d** $2.\overline{6} + 5\frac{1}{6} - 1.\overline{3} + 2\frac{5}{18}$

e $1\frac{5}{16} + {}^{-}8.172 - {}^{-}1\frac{7}{8} + 3.61$ **f** $^{-}1\frac{5}{9} + {}^{-}7.\overline{1} - 1\frac{7}{18} + 2.31 - {}^{-}11.004$

Problem solving

17 A ready-mixed concrete company sells standard concrete for \$87.53 per cubic metre. If a customer buys 3.85 cubic metres, how much does it cost?

18 A dog breeder uses 57.8 kg of dog food in a week. She has 4 dogs. What is the average daily consumption of this brand of dog food for each dog?

9780170194778

19 Timber can only be bought in lengths that are multiples of 0.3 m. The longest dressed timber usually available is about 5 m long (say 5.1 m). What lengths of timber should be bought so that the skirting boards in each of the passageways of a new hotel have the fewest joins? The passages have lengths of:

a 34 m   b 26 m   c 47 m

20 Five-eighths of a garden bed is planted with gerberas. A third of the gerberas are white and a quarter are orange. Half of the remainder of the bed is covered by Condamine couch.

a What fraction of the bed has white gerberas?
b What fraction has orange gerberas?
c What fraction has Condamine couch?
d Which is there more of: orange gerberas or Condamine couch?

Worked solutions
Exercise 5.3
MAT08NAWS00016

21 Three-quarters of the 8100 bricks needed for a house are delivered in January. The rest are supplied in June. Unfortunately, the long delay in starting work has caused two-ninths of the bricks delivered in January to become unusable because of staining.

a What fraction of all the bricks have become unusable?
b How many bricks were delivered in January?
c How many replacement bricks must be bought in June?

22 A blouse is made with 1.24 m of material. A clothing company has rolls containing 28.6 m and 57.4 m of material suitable for the blouse.

a How many blouses could be made using the small roll?
b How much of the small roll would be wasted?

If the large roll is used, how much would be wasted?

23 A loaf of bread contains 0.48 kg of flour. How many loaves of bread can be made from a 25 kg bag of flour?

Reasoning

Worked solutions
Exercise 5.3
MAT08NAWS00016

24 Two-thirds of the people at a reunion have blue eyes. Of these, two-ninths have blond hair. There are eight blue-eyed blonds.

a What fraction of the people are blue-eyed blonds?
b How many people are at the reunion?

25 An old book gives instructions for making a counting board in inches. The counting board consists of a wooden strip with holes drilled into it as peg holes. Each hole is to be $\frac{3}{16}$ of an inch in diameter and is separated from the next one by half an inch. The book recommends a strip 12 inches long. How many holes are drilled along the length?

5.4 Applications of fractions and decimals

When decimals and fractions are mixed together, it is almost always easier to change the fractions to decimals before doing any calculations. If the calculations are complex, it may be more efficient to use a calculator. In this case, you can use the fraction key or use division to enter fractions.

For mixed fractions, you will need to use brackets if you enter them as divisions. You may already know the following fraction conversions. They can save time in mixed calculations.

Important!

Common fraction conversions

$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{2}{3} = 0.\overline{6}$	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	$\frac{1}{5} = 0.2$
$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$	$\frac{4}{5} = 0.8$	$\frac{1}{6} = 0.1\overline{6}$	$\frac{5}{6} = 0.8\overline{3}$	$\frac{1}{8} = 0.125$
$\frac{3}{8} = 0.375$	$\frac{5}{8} = 0.625$	$\frac{7}{8} = 0.875$	$\frac{1}{9} = 0.\overline{1}$	$\frac{2}{9} = 0.\overline{2}$	$\frac{4}{9} = 0.\overline{4}$
$\frac{5}{9} = 0.\overline{5}$	$\frac{7}{9} = 0.\overline{7}$	$\frac{8}{9} = 0.\overline{8}$	$\frac{1}{10} = 0.1$	$\frac{1}{20} = 0.05$	$\frac{1}{25} = 0.04$

Example 20

Work out the following.

a $2\frac{3}{4} + 2.56 - 3$ **b** $7.8 + 3.2 \times 8.7 - 11.4$ **c** $3\frac{1}{4} \times \left(3.4 + 2\frac{4}{5}\right) \div 0.4$

Solution

a Write the problem.

$$2\frac{3}{4} + 2.56 - 3$$

Change the fraction to a decimal.

$$= 2.75 + 2.56 - 3$$

Perform addition and subtraction from left to right.

$$= 5.31 - 3 = 2.31$$

b Write the problem.

$$7.8 + 3.2 \times 8.7 - 11.4$$

Do the multiplication first.

$$= 7.8 + 27.84 - 11.4$$

Perform addition and subtraction from left to right.

$$= 35.64 - 11.4 = 24.24$$

c Write the problem.

$$3\frac{1}{4} \times \left(3.4 + 2\frac{4}{5}\right) \div 0.4$$

Change the fractions to decimals.

$$= 3.25 \times (3.4 + 2.8) \div 0.4$$

Evaluate the brackets first.

$$= 3.25 \times 6.2 \div 0.4$$

Perform multiplication and division from left to right.

$$= 20.15 \div 0.4 = 50.375$$

In the example above, $3\frac{1}{4} \times \left(3.4 + 2\frac{4}{5}\right) \div 0.4$ could be worked out on a calculator by entering:

3 [$\frac{\square}{\square}$] 1 [$\frac{\square}{\square}$] 4 [×] [(] 3.4 [+] 2 [$\frac{\square}{\square}$] 4 [$\frac{\square}{\square}$] 5 [)] [÷] 0.4 [=]

or

3 [$a^b/_c$] 1 [$a^b/_c$] 4 [×] [(] 3.4 [+] 2 [$a^b/_c$] 4 [$a^b/_c$] 5 [)] [÷] 0.4 [=]

or

[(] 3 [+] 1 [÷] 4 [)] [×] [(] 3.4 [+] [(] 2 [+] 4 [÷] 5 [)] [)] [÷] 0.4 [=]

9780170194778

Answers to mixed fraction and decimal problems can be estimated using leading digit calculation. Fractions can be changed to approximate decimal equivalents or can be made into approximate unit fractions and dealt with as divisions.

Example 21

Estimate answers to the following.

a 0.0573×907.85 **b** $1.97 \times \frac{5}{11} \div 0.0438 \div 4\frac{5}{6}$

Solution

a Write the problem.	$\mathbf{0.0573 \times 907.85}$
Round to leading digits.	$\approx \mathbf{0.06 \times 900}$
The hundredths and hundreds cancel out.	$= \mathbf{54}$
Round to the leading digit.	$\approx \mathbf{50}$
b Write the problem.	$\mathbf{1.97 \times \frac{5}{11} \div 0.0438 \div 4\frac{5}{6}}$
Round the decimals to leading digits.	$\approx \mathbf{2 \times \frac{5}{11} \div 0.04 \div 4\frac{5}{6}}$
$\frac{5}{11}$ is just under $\frac{1}{2}$ and $4\frac{5}{6} \approx 5$.	$\approx \mathbf{2 \times \frac{1}{2} \div 0.04 \div 5}$
Work through.	$= \mathbf{1 \div 0.04 \div 5}$
	$= \mathbf{100 \div 4 \div 5}$
	$= \mathbf{5}$

Answers to decimal problems can be estimated using scientific notation instead of leading digit calculation. The accuracy of numbers can be stated using the number of figures needed to show the mantissa. This is called the **number of significant figures**, so 43 200 has 3 **significant figures**. Leading digit calculation really uses just 1 significant figure.
When you multiply decimals, you add the decimal places. This means that when you multiply numbers in scientific notation you add the orders. It follows that you subtract the orders for division.

Example 22

Estimate an approximate answer to $0.000\,057\,8 \times 6432 \div 1\,973\,000$.

Solution

Write each number in scientific notation, while rounding to 1 significant figure.	$\mathbf{0.000\,057\,8 \times 6432 \div 1\,973\,000}$ $\approx \mathbf{6 \times 10^{-5} \times 6 \times 10^{3} \div (2 \times 10^{6})}$
Rearrange.	$= \mathbf{6 \times 6 \div 2 \times 10^{-5} \times 10^{3} \div 10^{6}}$
Complete the calculations.	$= \mathbf{18 \times 10^{-8}}$
Round off and write correctly.	$\approx \mathbf{2 \times 10^{-9}}$
Write the answer in the form of the question.	$= \mathbf{0.000\,000\,002}$

Example 23

Work out the following.

a $^-2.\overline{6} \times \frac{5}{8} \div \frac{1}{6}$ **b** $1\frac{3}{4} \times {}^-3.96 \div 2\frac{7}{8} \times 2.51$

Solution

a This is an exceptional case. Change the decimal to a fraction.

$$^-2.\overline{6} \times \frac{5}{8} \div \frac{1}{6} = {}^-2\frac{2}{3} \times \frac{5}{8} \div \frac{1}{6}$$

Change to an improper fraction and invert to change the division sign to multiplication.

$$= \frac{^-8}{3} \times \frac{5}{8} \times \frac{6}{1}$$

Cancel where possible and multiply across the numerator and denominator.

$$= -\frac{{}^1\cancel{8}}{\cancel{3}_1} \times \frac{5}{\cancel{8}_1} \times \frac{{}^1\cancel{6}}{1}$$

You could also do this on your calculator using the fraction key like Example **15**.

$$= {}^-10$$

b Change to decimals. $\mathbf{1.75 \times 3.96 \div 2.875 \times 2.51}$

Now do the calculation. $\approx {}^-\mathbf{6.05}$

Extra questions
Exercise 5.4
MAT08NAEQ00017

Exercise 5.4 Applications of fractions and decimals

Understanding

Worked solutions
Exercise 5.4
MAT08NAWS00017

1 Work out the following.

a $3.48 + 2\frac{1}{4}$ **b** $4\frac{1}{2} + 7.36$ **c** $8.2 - 3\frac{4}{5}$ **d** $9.65 - 5\frac{3}{20}$
e $3\frac{3}{4} \times 5.6$ **f** $\frac{3}{8} \times 0.14$ **g** $6.58 \div \frac{1}{5}$ **h** $7.97 \div \frac{1}{2}$

2 State the number of significant figures in each of the following numbers.

a 520 000 **b** 0.007 56 **c** 28.6 **d** 300
e 0.000 42 **f** 578 400 **g** 1.6 **h** 0.003

Fluency

3 Estimate answers to the following.

a $0.0786 + 0.067$ **b** 45.72×0.0683 **c** $0.4897 \div 0.0765$ **d** 42.78×302.77
e $45.62 - 28.909$ **f** $\frac{36.785}{0.479}$ **g** $0.006\,78 \div 2.953$ **h** $3.907 \times 0.006\,32$

See Example 20

4 Work out the following.

a $\frac{5}{8} \times 2.75$ **b** $3\frac{4}{5} \div 1.6 \times 2\frac{7}{20}$ **c** $3.4 \times 2.8 - 4.9$
d $7.9 + 2.15 \div 0.5$ **e** $(3.2 - 1.84) \times 0.45$ **f** $91.3 - 7.8 \times 6.09$
g $15\frac{3}{4} + 5.6 \div 0.02$ **h** $4.14 \div 0.03 - 0.021$ **i** $\left(4\frac{4}{5} + 2.3\right) \div 0.009$
j $8.4 - 21.5 \times 0.04$ **k** $6.8 + 3.04 \times 1.69$ **l** $5.67 \times 2.46 - 9.9076$

See Example 21

5 Estimate answers to the following.

a $26.54 \times \frac{4}{15} \div 1.246$ **b** $\frac{5}{32} \div 0.078 \times 53.89 \times \frac{3}{13}$
c $15.6 \div \frac{2}{9} \times 0.0074 \times \frac{5}{16}$ **d** $\frac{9}{13} \times 0.007\,42 \div 423.8 \div \frac{3}{50}$
e $11.34 \div \frac{2}{3} \div 0.021 \times \frac{7}{20}$ **f** $\frac{2}{15} \div 6.804 \times \frac{5}{8} \times 0.0045$

9780170194778

6 Work out the following.

a $\left(4\frac{1}{2} - 3\frac{4}{5}\right) \times 2\frac{3}{4}$

b $4\frac{1}{4} \div 5\frac{3}{5} - 2\frac{3}{8} \times \frac{2}{9}$

c $8\frac{1}{5} - 6\frac{3}{4} + 5\frac{3}{8} \times 2\frac{4}{9} \div \frac{4}{11}$

d $\left(7\frac{1}{2} - 2\frac{1}{3}\right) \times \left(\frac{5}{12} + 5\frac{3}{4} \times 7\frac{5}{6}\right)$

e $5.8 + \frac{2}{3} \times 7.95$

f $2\frac{3}{5} \times 4.9 - 3\frac{9}{16} \times 1.15$

g $78 \div 1\frac{5}{8} + 4.76 \times 2\frac{5}{7}$

h $8.65 - \frac{9}{10} \div 0.06 + 8.9 \times 7\frac{5}{8}$

7 Estimate answers to the following. *See* Example 22

a 45 600 × 0.0654 ÷ 15 000

b 28.9 ÷ 0.000 042 × 3495

c 6230 ÷ 1.89 × 0.000 67 ÷ 4230

d 0.0052 × 678 000 ÷ 2350 ÷ 0.000 062

e 5.75 × 0.0068 ÷ 2305 × 56 000

f 809 600 × 0.0178 × 340.96 ÷ 0.0008

8 Work out the following. *See* Example 23

a $^{-}1.\overline{6} \times \frac{^{-}7}{9}$

b $2\frac{1}{5} \div 1.875 \times \frac{1}{12}$

c $^{-}4.15 \times \frac{3}{10} \times \frac{^{-}4}{5}$

d $4.\overline{6} \times 5\frac{1}{3} \div 1.\overline{3} \times 2\frac{5}{12}$

e $1\frac{5}{32} \times {}^{-}5.172 \div {}^{-}1\frac{7}{16} \times 3.61$

f $^{-}1\frac{2}{9} \div {}^{-}7.\overline{1} \div 1\frac{7}{18} \times 2.31 \div {}^{-}21.004$

Problem solving

Worked solutions

Exercise 5.4

MAT08NAWS00017

9 Australia has 7.682 million square kilometres of land. Six twenty-fifths of Australia is desert. Three-fifths of the remainder is classified as arid and suitable only for low-intensity grazing. Two-thirds of the total land area is used for some kind of agriculture. How much is used for non-arid agriculture?

Reasoning

Worked solutions

Exercise 5.4

MAT08NAWS00017

10 The recipe for the Lebanese dish babaganoush uses 2 cloves of garlic and half a teaspoon of chilli powder for a small eggplant. It is estimated that a large eggplant is two and a third times as big as a small one. If 4 large eggplants are used to make babaganoush for a banquet, how much chilli powder is needed?

11 A wedding cake was sealed after the wedding and small amounts were eaten on successive anniversaries. Three-quarters of the cake was eaten at the wedding and two-thirds of what remained was eaten at each successive anniversary. After the fifth wedding anniversary, only one slice was left. Into how many slices was the original cake cut?

12 A pool is $\frac{3}{5}$ as wide as it is long. It is 4.8 m long and 2.1 m deep. Andrew and Tania have a swimming race, but as Andrew is younger he swims across the pool and Tania swims the length. After 20 minutes, Andrew has completed eight widths and Tania has swum half that number of lengths. How far has each child swum?

13 A rectangular house is 8.2 m wide and 15.6 m long. The eaves overhang the walls by 0.6 m. The eaves are 2.4 m from the ground. Downpipes are needed for at least every 13 m of guttering and must go 0.2 m into the ground. What length of downpipe has to be purchased if the guttering goes all the way around the house?

Chapter 5 summary

Quiz
Fractions and decimals
MAT08NAQZ00005

- A **common fraction** (**vulgar fraction**) has a **numerator** on top separated by a **vinculum** (**bar**) from the **denominator** underneath. The denominator shows the number of equal shares in the whole and the numerator shows the number of these shares in the fraction. A common fraction may be written horizontally instead of vertically, using a **slash** to separate the numerator and denominator.
- Common fractions are read with the numerator first, followed by the denominator with the ending '**ths**', as in three-fifths.
- A **decimal fraction** (or just **decimal**) extends the normal place value system to the right of the **decimal point**, so that each place value is 10 times smaller than the place value to its left. The first **decimal place** has a value of a tenth ($\frac{1}{10}$), the second decimal place has a value of a hundredth ($\frac{1}{100}$) and so on.
- **Equivalent fractions** represent the same amount using different denominators. They can be obtained by multiplying or dividing both the numerator and the denominator by the same number. Equivalent fractions have equal **cross-products**.
- A **proper fraction** such as $\frac{2}{3}$ is less than 1, while an **improper fraction** such as $\frac{26}{3}$ is more than 1. Its numerator is bigger than its denominator. A **mixed fraction** (mixed number) such as $8\frac{2}{3}$ has a whole number part and a proper fraction.
- The **simplest form** of a fraction is the equivalent fraction (or mixed number) with the lowest possible denominator. It is obtained by **cancelling down** to the equivalent fraction with the lowest possible denominator and changing an improper fraction to a mixed fraction.
- A fraction can be changed to a decimal by treating the fraction as a division problem.
- A non-recurring decimal is changed to a fraction by writing the decimal in extended form and cancelling down.
- You should know the decimal equivalents of common fractions such as $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \ldots$
- Common fractions, decimals and integers are known as **rational numbers**.
- Different fractions can always be changed to equivalent fractions with the same denominator (called a **common denominator**). It is usual to use the smallest possible, called the **lowest common denominator**.
- When rational numbers are shown on a number line:
  - The letter x is used to represent any rational number.
  - An **open** circle shows that the boundary of a group of numbers *is not included* ($<$ or $>$).
  - A **closed** circle shows that the boundary of a group of numbers *is included* ($\leq$ or $\geq$).
- A **recurring decimal** is a decimal that has one or more digits that repeat forever. We show that digits are repeating by putting a bar (or dots) over the repeating digits. A **terminating decimal** is a decimal that is not recurring, but comes to an end.
- An **irrational number** is not rational. It cannot be written as a terminating or recurring decimal. An **irrational number** cannot be expressed as a fraction.
- When decimals are added or subtracted on paper:
  1. Put a decimal point at the right end of whole numbers.
  2. Set out vertically with the decimal points under each other.
  3. Put zeros on the right if needed, to make all decimal parts the same length.
  4. Add or subtract and put the decimal point under the decimal point in the problem.

9780170194778

Chapter 5 summary

- When fractions are added or subtracted on paper, they must have the same denominator.
  1. Use multiples to find the lowest common denominator: the smallest number that is a multiple of all the denominators.
  2. Change the fractions to equivalent fractions with the lowest common denominator.
  3. For mixed fractions, calculate the whole-number part separately.
  4. Add or subtract the numerators of the fractions and simplify the answer.
- When decimals are multiplied on paper:
  1. Ignore the decimal point and multiply as whole numbers.
  2. Count the total number of decimal places in the numbers.
  3. Insert the decimal point to give this total number of decimal places in the answer.
- When decimals are divided on paper:
  1. Change *both* numbers by the same number of decimal places to make the divisor a whole number.
  2. Perform the new division, adding zeros as needed to complete the calculation.
  3. If the answer does not terminate, show the repeating digits with a dot over the top of the first and last numbers in the sequence of recurring numbers.
- When fractions are multiplied on paper:
  1. Change mixed fractions to improper fractions.
  2. Multiply the numerators and multiply the denominators.
  3. Simplify the answer, including cancellation by common factors. If desired, cancellation by common factors can be done before the multiplications.
- A fraction is **partitioned** by breaking it into equal-sized parts.
- When fractions are divided on paper:
  1. Change mixed fractions to improper fractions.
  2. Multiply by the reciprocal of the divisor; that is, turn the second fraction upside-down and change the operation to multiply.
- When decimals and fractions are mixed together in a calculation, it is almost always easier to change the fractions to decimals before doing any calculations.
- Answers to decimal problems can be estimated using scientific notation instead of leading digit calculation. The number of digits needed to show the mantissa of a number is called the number of significant figures and is a measure of accuracy.

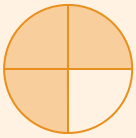
Chapter 5 review

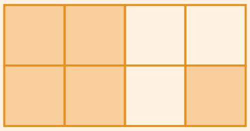
Understanding

Worksheet
Fractions and decimals review
MAT08NAWK00037

See Example 1

1 Write the fractions represented by the shaded parts, in words and symbols.

a 

b 

See Example 2

2 Write $2\frac{4}{5}$ as an improper fraction.

See Example 2

3 Write $\frac{23}{4}$ as a mixed fraction.

See Example 2

4 Complete as equivalent fractions : $\frac{3}{5} = \frac{6}{?} = \frac{?}{30} = \frac{21}{?}$

See Example 2

5 Write $\frac{12}{18}$ in simplest form.

See Example 2

6 Change $\frac{3}{5}$ to a decimal and 0.45 to a fraction.

See Example 8

7 Calculate the following in your head.

a $0.79 + 0.65$ **b** $0.0076 - 0.0038$

Fluency

8 Use written methods to solve the following.

a $42.67 + 4.027 + 19.076 + 53 + 4.92$ **b** $38.74 - 9.657$

See Example 9

9 Work out the following on your calculator.

a $9645.78 + 40.067 + 28.9754 + 2.9876$ **b** $152.52 - 35.8079$

10 Work out the following in your head.

a 0.0056×1000 **b** $67.24 \div 100$ **c** 0.072×0.1

d $0.0765 \div 0.001$ **e** 5×0.34 **f** $\dfrac{14.6}{2}$

g $28 \times \frac{1}{4}$ **h** $\frac{1}{5} \times 32$

11 Use written methods to solve the following.

a 15.24×5.6 **b** $0.058 \div 4$ **c** $\dfrac{13.6}{0.02}$ **d** $21.72/0.6$

12 Use a calculator to solve the following.

a 23.452×8.625 **b** $23.805 \div 0.164$

13 Work out $6.65 + 5\frac{1}{4}$.

14 Work out an approximate answer to $0.098\,04 \times 665.78$.

15 Estimate the fraction represented by the shaded part of this diagram in words and symbols as both vulgar and decimal fractions. Choose from $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{8}, \frac{7}{8}$ and 0.4, 0.5, 0.6, 0.8.

16 Write $\frac{90}{42}$ in simplest form.

See Example 2

17 State whether the fractions in each of the following pairs are equivalent.

a $\frac{10}{12}, \frac{25}{65}$ **b** $\frac{21}{28}, \frac{18}{24}$

9780170194778

18 Change $\frac{6}{7}$ and $\frac{5}{12}$ to decimals.

19 Write in ascending order.

a 5.3, 4.8, $^-4.9$, $^-5.2$ **b** $3\frac{3}{4}, 4\frac{1}{3}, 3\frac{2}{9}, 4\frac{1}{10}$

20 Show the following on separate number lines.

a $x \geq {}^-3.71$ **b** $x \leq 1\frac{5}{16}$ **c** $^-1.63 \leq x < 2.27$ **d** $^-4\frac{2}{3} < x \leq 2\frac{3}{13}$

21 Insert <, = or > to make true statements. See Example 3

a $\frac{3}{5}\ \square\ \frac{5}{12}$ **b** $0.7\ \square\ 0.689$

22 Show the following on separate number lines. See Example 4

a $x < 4\frac{2}{3}$ **b** $x > 5.3$ **c** $1\frac{4}{5} < x < 3\frac{3}{4}$ **d** $3.1 < x < 6.8$

23 Change each of the following to a decimal. See Example 5

a $\frac{1}{6}$ **b** $\frac{4}{9}$ **c** $\frac{7}{11}$

24 Change each of the following to a fraction. See Example 6

a $0.\overline{17}$ **b** $0.\overline{93}$ **c** $0.\overline{144}$

25 Calculate the following in your head.

a $0.0764 + 0.0975$ **b** $6.27 - 3.59$ **c** $\frac{2}{3} - \frac{1}{6}$ **d** $\frac{5}{8} + \frac{3}{4}$

26 Use written methods to solve the following.

a $426.8 - 209.89 + 41.097 - 17.48 - 63.893$ **b** $\frac{5}{6} - \frac{1}{4}$ **c** $\frac{1}{8} + \frac{5}{6}$

d $\frac{4}{5} + \frac{9}{10}$ **e** $4\frac{2}{3} + 8\frac{1}{2}$ **f** $5\frac{1}{4} - 2\frac{2}{3}$ **g** $7\frac{4}{5} - 3\frac{2}{3}$

27 Work out the following on your calculator.

a $23.87 + 823.509 - 56.098 - 203.46 + 17.6$ **b** $5\frac{3}{4} - 2\frac{5}{8} + 4\frac{2}{3} - 6\frac{7}{9}$

28 Work out the following in your head.

a 0.0018×25 **b** $47.2 \div 4000$ **c** 26.4×5 **d** $\frac{12.9}{0.03}$

e $\frac{9}{10} \times \frac{1}{3}$ **f** $2\frac{3}{5} \div \frac{1}{10}$ **g** $1\frac{2}{3} \div \frac{1}{6}$ **h** $\frac{2}{3} \times \frac{1}{5}$

29 Use written methods to solve the following.

a 16.24×0.523 **b** $18.26 \div 0.7$ **c** $29.1 \div 0.12$ **d** $0.0462 \times 0.005\,06$

e $\frac{4}{15} \times \frac{5}{8}$ **f** $\frac{3}{5} \div \frac{4}{9}$ **g** $\frac{6}{7} \times \frac{4}{5}$ **h** $\frac{2}{5} \div \frac{7}{10}$

30 Use a calculator to solve the following. See Example 18

a $3\frac{2}{5} \times 1\frac{1}{6}$ **b** $6\frac{1}{4} \div 2\frac{1}{2}$ **c** $4\frac{6}{11} \div 3\frac{1}{3}$ **d** $2\frac{3}{4} \div 4\frac{2}{3} \times 3\frac{1}{2}$

31 Work out the following. See Example 20

a $16.38 - 4.85 \times 2.04$ **b** $\left(4\frac{1}{2} + 6.2\right) \times 3.6 \div 1\frac{1}{5}$

32 Find an approximate solution to $\frac{1}{13} \times 308.96 \div \frac{7}{106}$. See Example 21

9780170194778

Chapter 5 review

See Example 3

33 Insert <, = or > to make true statements.

a $\frac{2}{3}\ \square\ 0.7$ **b** $\frac{^-3}{4}\ \square\ ^-0.77$ **c** $0.08\ \square\ 7.9 \times 10^{-1}$

34 Write in ascending order.

a $0.1,\ \frac{1}{7},\ 0.67,\ 0.15,\ \frac{1}{8}$ **b** $\frac{^-4}{5},\ ^-0.85,\ 0.8,\ \frac{^-3}{4},\ \frac{1}{2},\ 6.3 \times 10^{-2}$

35 Show the following on separate number lines.

a $^-5\frac{2}{11} \le x \le {}^-3.42$ **b** $^-28.72 \le x < 14\frac{3}{5}$ **c** $^-8.3 \times 10^{-1} < x \le \frac{1}{40}$

36 Calculate the following in your head.

a $146.3 + 9.4766$ **b** $39.6 - 14.583$ **c** $\frac{^-5}{8} + \frac{1}{2}$ **d** $\frac{2}{3} - \frac{^-5}{6}$

e $\frac{^-4}{5} + \frac{^-7}{10}$ **f** $\frac{1}{4} - \frac{2}{3}$ **g** $\frac{^-3}{8} + \frac{3}{4}$ **h** $\frac{^-7}{10} - \frac{^-2}{5}$

37 Use written methods to solve the following.

a $^-5.96 + 3.784$ **b** $6.072 - {}^-1.84$ **c** $^-21.3 + {}^-421.77$ **d** $19.75 - 42.621$

e $\frac{2}{5} - \frac{1}{4} + \frac{5}{6}$ **f** $\frac{5}{9} + \frac{7}{18} - \frac{5}{12}$ **g** $2\frac{2}{5} + 5\frac{1}{4} - \frac{7}{10}$ **h** $5\frac{7}{10} - 2\frac{3}{4} - 1\frac{1}{5}$

38 Work out the following on your calculator.

a $^-4.96 - 5.874 + 2.14 - {}^-8.24 - 2.3$ **b** $^-4\frac{1}{3} + 6\frac{3}{8} - {}^-3\frac{1}{6} + 7\frac{7}{12} + {}^-4\frac{3}{4}$

39 Work out the following in your head.

a 4.9×0.043 **b** $1.86 \div 0.4$ **c** 0.51×0.025 **d** $0.034 \div 0.0005$

e $2\frac{2}{5} \div \frac{3}{5}$ **f** $4\frac{1}{2} \div \frac{3}{4}$ **g** $2\frac{2}{5} \times 1\frac{1}{6}$ **h** $7\frac{1}{2} \div 1\frac{1}{4}$

40 Use written methods to solve the following.

a $^-2.4 \times {}^-5.32$ **b** $6.75 \div {}^-0.6$ **c** $^-0.45 \times 0.0364$ **d** $\frac{^-0.477}{^-0.04}$

e $^-4\frac{2}{3} \div 3\frac{1}{4}$ **f** $4\frac{1}{2} \div {}^-5\frac{1}{4}$ **g** $^-3\frac{4}{5} \times {}^-1\frac{1}{9}$ **h** $^-5\frac{2}{3} \times 3\frac{1}{3}$

41 Use a calculator to solve the following.

a $^-42.985 \times {}^-0.073 \div 0.000\,67$ **b** $4\frac{1}{5} \times 3\frac{3}{8} \div 4\frac{9}{10}$

c $5\frac{1}{3} \div {}^-2\frac{6}{7} \times {}^-1\frac{1}{4}$ **d** $\left({}^-2\frac{1}{3}\right)^4$

See Example 19

42 Work out the following.

a $^-5.\overline{3} + \frac{3}{4} - \frac{^-7}{12}$ **b** $1\frac{5}{8} + {}^-7.314 - {}^-3\frac{7}{12} + 2.41$

43 Work out the following.

a $\left(6\frac{5}{8} - 4\frac{2}{3}\right) \times \frac{2}{9}$ **b** $5.8 \times 4.64 - 2\frac{1}{4} \times 3\frac{2}{5}$

See Example 22

44 Estimate an answer to $310\,503 \div 0.0256 \times 0.000\,074 \div 189.7$.

See Example 23

45 Work out the following.

a $^-5.75 \times \frac{3}{7} \times \frac{^-4}{5}$ **b** $5.\overline{6} \times 3\frac{1}{3} \div 4.\overline{3} \times 2\frac{5}{9}$

Problem solving

46 What fraction of the months in a year are long months?

9780170194778

47 A length of timber is sold as being 5.4 m long, but is actually 0.15 m longer than the specified length. Pieces of length 1.85 m and 2.08 m are cut from it, with each saw cut taking 0.003 m from the length. How much is left?

48 Mixed nuts from a health food shop cost \$8.55/kg. How much will 3.2 kg cost?

49 Would you prefer to have $\frac{5}{8}$ or 0.65 of a bag of delicious lollies?

50 Peter ran 4.52 km, walked 600 m and jogged 2.18 km away from home. How far did he have to come back?

51 From $2\frac{1}{2}$ kg of sugar, Alex used $\frac{3}{4}$ kg for jam and $\frac{1}{5}$ kg for a syrup. How much did he have left:

a as a fraction of a kilogram? b in grams?

52 Donna went to a party plan meeting and spent a fifth of her money on some face cream, a third of what was left on make-up remover and three-quarters of what was now left on lip gloss. She went home with \$4.16 left on her card. How much did she spend?

53 An industrial process uses 45.723 kg of soda, 324.7 kg of silica and 4.906 kg of fluxes. During heating, 50.762 kg of carbon dioxide and other gases are lost in the chemical reaction. What is the mass of product?

Reasoning

54 Andrew divided his vegetable garden into 12 plots and planted 2 of them with beans and $2\frac{1}{2}$ with sweet corn. He divided one plot into three parts and planted two of the parts with onion seed. What fraction of his vegetable garden was left unplanted?

55 How many 1.2 kg bags of jubes can be made up from a bulk container with 25.8 kg, and how much will be left over?

56 A wholesaler adds $\frac{1}{3}$ to the cost price of furniture as his mark-up. The retailer adds $\frac{1}{2}$ to the price he pays, but at a sale reduces the marked price by $\frac{1}{5}$. What multiple of the wholesaler's cost price is paid by a customer at the sale? Give your answer as a fraction.

Measurement and geometry

6 Shapes and angles

9780170194778

Contents

Prior learning
Chapter 6
MAT08MGPL00006

Parent guide
Chapter 6
MAT08MGPG00006

Curriculum guide
Chapter 6
MAT08MGCU00006

Australian Curriculum statements

Geometric reasoning

Develop the conditions for congruence of triangles.
Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

You can see geometrical figures everywhere. Angles are formed when two lines meet. If you look carefully you will see triangles, quadrilaterals and other geometric shapes in your home, in the classroom, in buildings, in art and in the natural environment. Studying and understanding geometrical figures can help you better understand the world around you.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

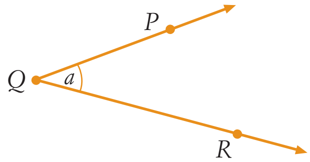
allied angles	equilateral	polygon	shape
alternate	exterior angle	protractor	side
angle sum	interior angle	quadrilateral	square
co-interior	interval	ratio	superimpose
complementary	isosceles	rectangle	supplementary
congruent	kite	reduction	trapezium
construct	parallel	rhombus	triangle
corresponding	parallelogram	scalene	vertex/vertices
diagonal	perpendicular	set square	vertically opposite

6.1 Using angle relationships

Weblink
History of Mathematics
MAT08MGWB00006

Weblink
Why math: What's math got to do with anything?
MAT08MGWB00006

Weblink
Numb3rs scene: Everything is numbers, math is everywhere
MAT08MGWB00006



An **angle** is formed by two lines with the same starting point, known as the **vertex** of the angle. The lines that form the angle are known as the **arms** or **sides** of the angle.

Angles are usually marked with a curved line called an **arc**. This shows the size of the turn when one arm is rotated to meet the other.

The arms QP and QR form the angle shown on the right. The vertex of the angle is Q. The arrowheads on arms QP and QR show that the arms extend indefinitely. It is common not to show these arrowheads when drawing angles.

There are different ways of naming angles. You can use the angle symbol $\angle$ and the points on the angle. The vertex is written as the middle letter of the angle name. The angle above could be written as $\angle PQR$ or $\angle RQP$.

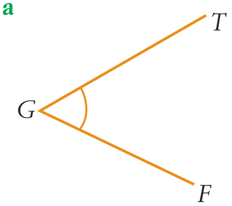
This angle could also be referred to as a.

When there is only one angle and it is clearly indicated with an arc from one arm to the other as shown above, it can be named using just the vertex. So in this case $\angle PQR$ could be referred to as $\angle Q$.

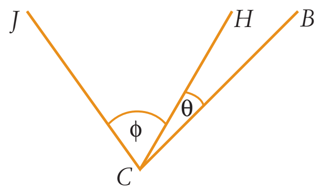
Example 1

Name the angles below.

a



b



9780170194778

Solution

a Use the points on the angle. **This angle is either ∠*TGF* or ∠*FGT*.**

The angle could also be called ∠*G*.

b There are many angles here. $\angle JCB = \angle BCJ$

Write the angle names. $\angle JCH = \angle HCJ = \phi$

$\angle HCB = \angle BCH = \theta$

Ancient Greeks, such as Euclid, were the first people to study and write about angles and geometry. Because of this, Greek letters are often used as angle names. Greek letters that are commonly used to name angles are alpha (α), beta (β), gamma (γ), theta (θ) and phi (ϕ).

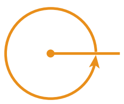
Classifying angles

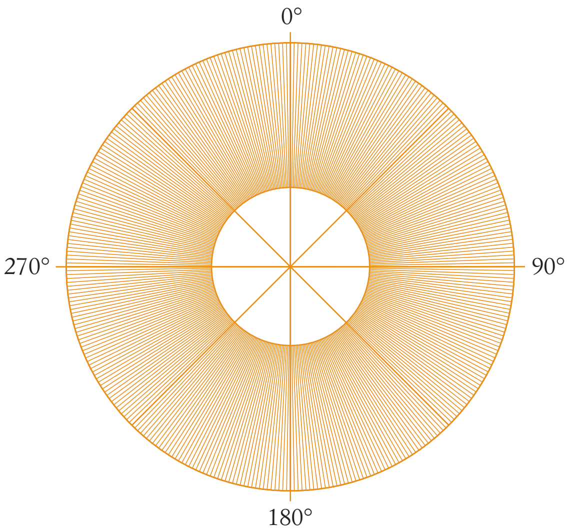
TLF learning object

Biscuit factory: Gear direction (L2283)

MAT08MGIN00006

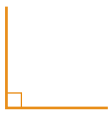
There are 360° in a complete **turn** or a **full rotation**.



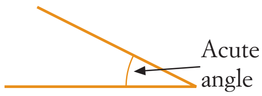


This means that a **half-turn** is 180°.
It is also known as a **straight angle**.

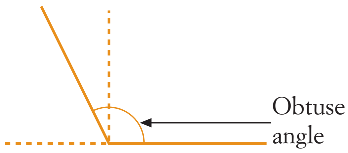
A quarter turn is 90°. It is also known as a **right angle.**



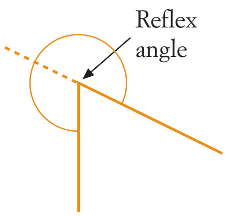
Angles can be classified according to their size.
An **acute angle** is smaller than a right angle (90°).



An **obtuse angle** is larger than a right angle (90°) but less than a straight angle (180°).

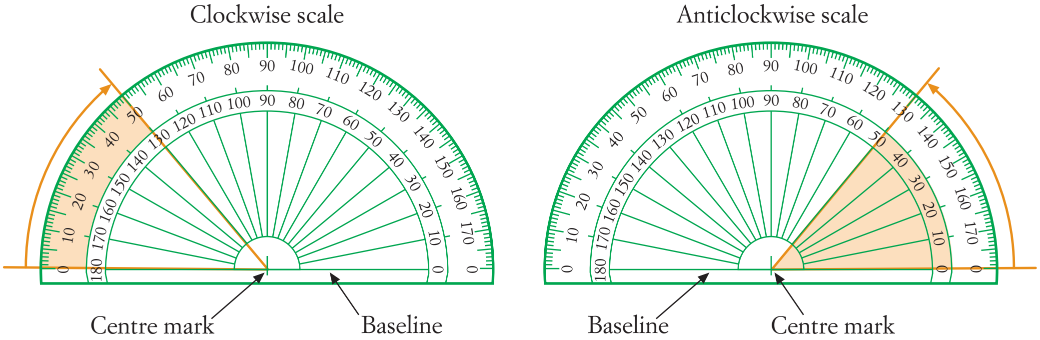


A **reflex angle** is larger than a straight angle (180°) but less than a full rotation (360°).

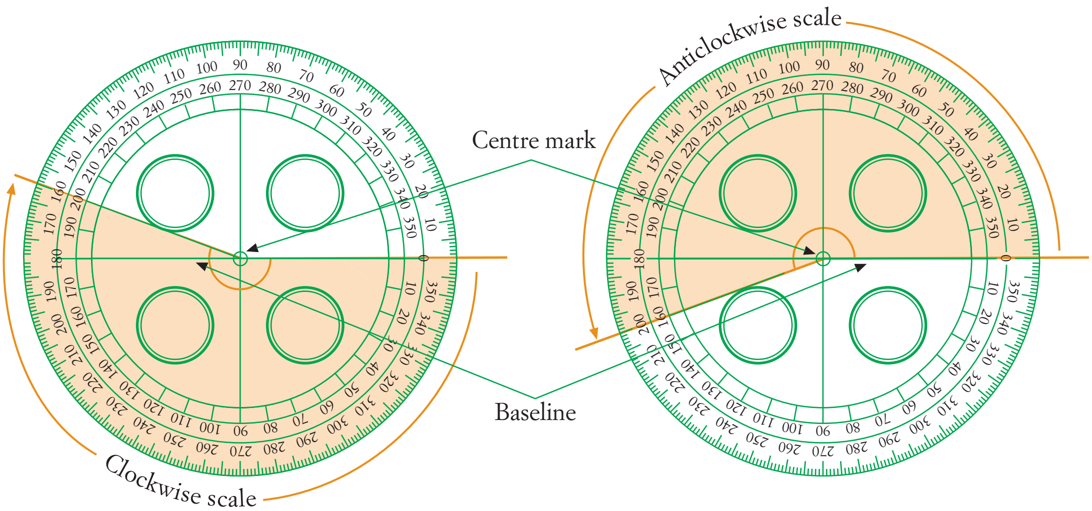


Measuring angles

To measure an angle, we use a **protractor** marked in degrees. A 180° protractor has two scales. The following diagram shows how an angle of 50° can be measured using either scale.



You can also use a 360° protractor to measure angles. A 360° protractor has two scales. The following diagram shows how an angle of 200° can be measured using either the clockwise (inside) scale or anticlockwise (outside) scale.



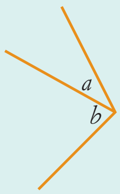
Teacher notes
Reflex angles
MAT08MGTN00012

If you don't have a 360° protractor, you can still measure a reflex angle using a 180° protractor. This method is described in the resource on the NelsonNet website.
A number of useful angle relationships are listed below.

Important!

Angle relationships

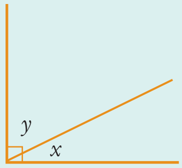
Adjacent angles are next to each other. They share a common arm and a common vertex.
In the diagram on the right:
a and b are adjacent.



Complementary angles have a sum of 90°.
For example, 60° and 30° are **complementary**.

In the diagram on the right:

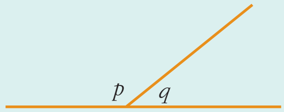
$$x + y = 90°$$



Supplementary angles have a sum of 180°.
For example, 120° and 60° are **supplementary**.

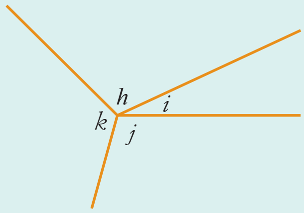
In the diagram on the right:

$$p + q = 180°$$



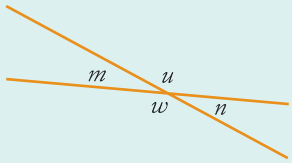
Angles at a point (in a revolution) add to 360°.
In the diagram on the right:

$$h + i + j + k = 360°$$



Vertically opposite angles are equal.
In the diagram on the right:

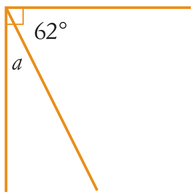
$$m = n \text{ and } u = w$$



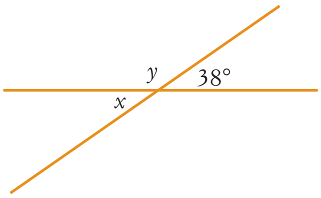
Example 2

Find the value of each unknown angle.

a



b



Puzzle sheet
Angles: A dog day
MAT08MGPS00020

Worksheet
Straight angles, right angles and revolutions
MAT08MGWK00038

Solution

a 62° and a are complementary angles. $\quad$ $\mathbf{62° + a = 90°}$

Subtract 62° from both sides to calculate a. $\quad$ $\mathbf{a = 28°}$

b 38° and x are vertically opposite angles. $\quad$ $\mathbf{x = 38°}$

38° and y are on a straight line so they must be supplementary. $\quad$ $\mathbf{38° + y = 180°}$

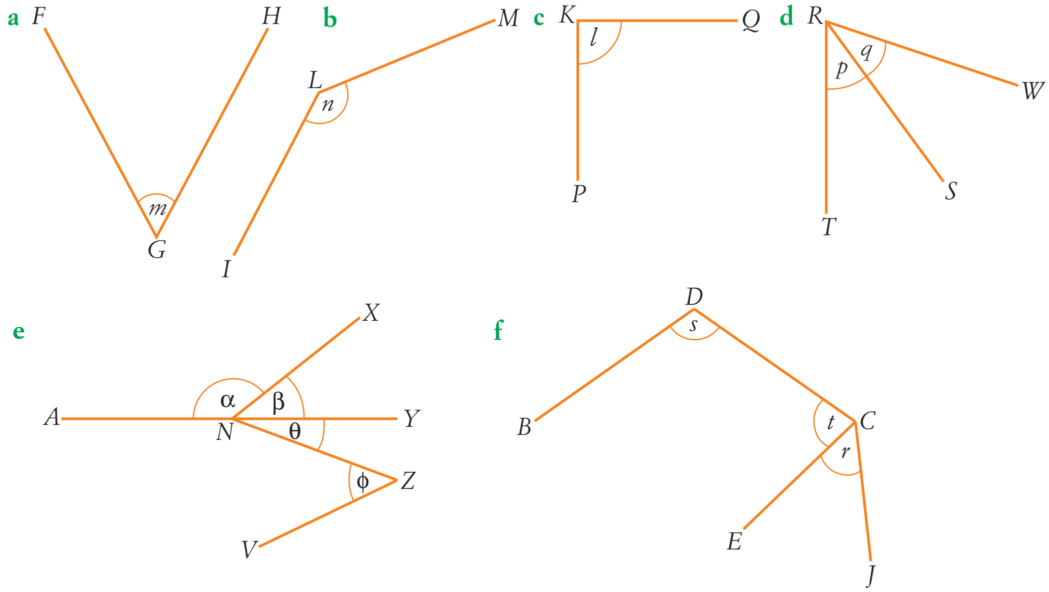
Subtract 38° from both sides of the equation. $\quad$ $\mathbf{y = 142°}$

Exercise 6.1 Using angle relationships

Understanding

Extra questions
Exercise 6.1
MAT08MGEQ00018
See Example 1

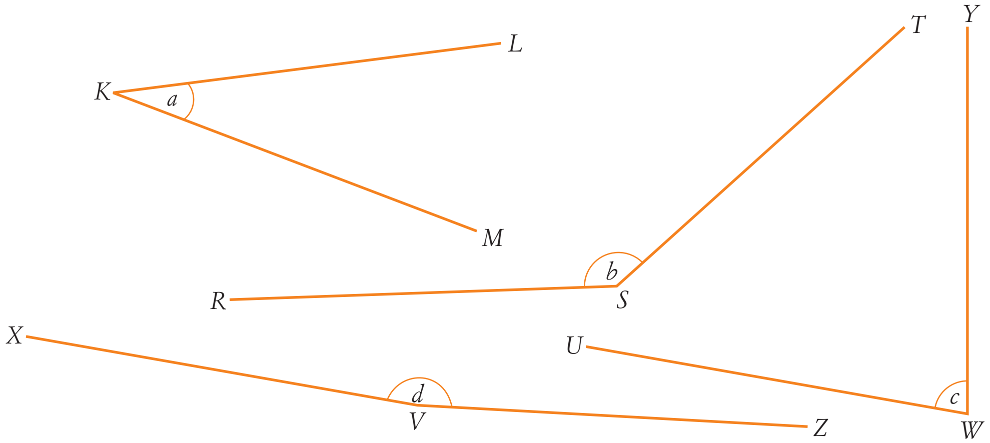
1 Name each of the following angles using the letter inside the angle and both three-letter names, as in $a = \angle ABC = \angle CBA$.



Worked solutions
Exercise 6.1
MAT08MGWS00018

2 Which of the angles in question **1** can be named by the capital letter at the vertex without causing confusion?

3 Measure the angles shown below and write your answers in the form $\angle ABC = 80°$.

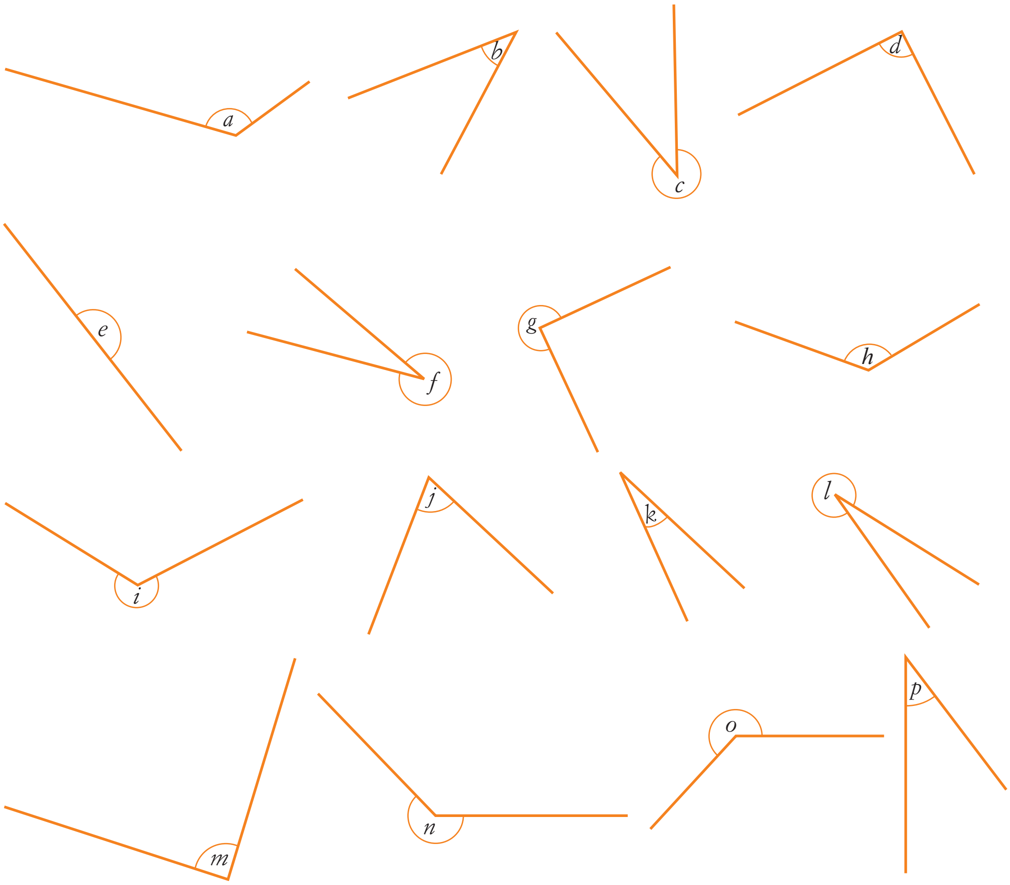


4 What type of angle is each of the following?

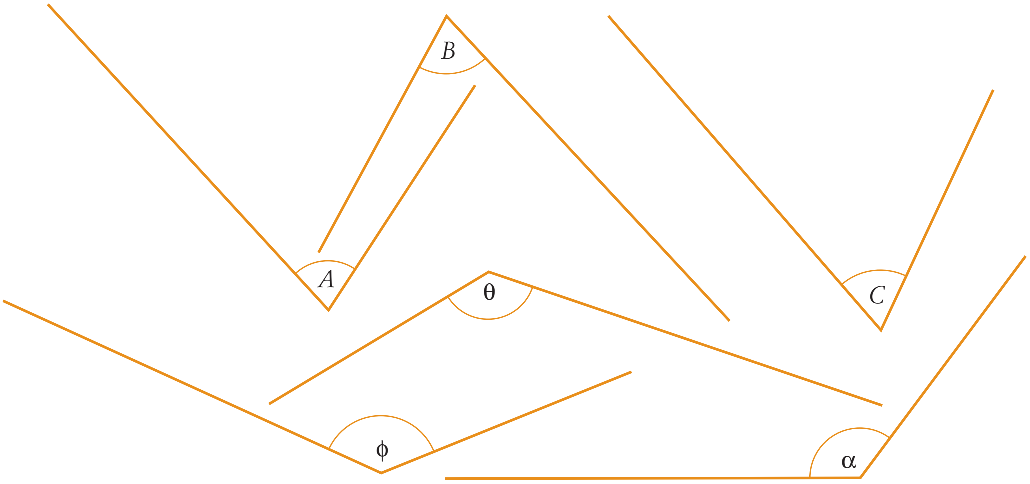
a 23°	**b** 123°	**c** 360°	**d** 180°
e 285°	**f** 90°	**g** 320°	**h** 4°

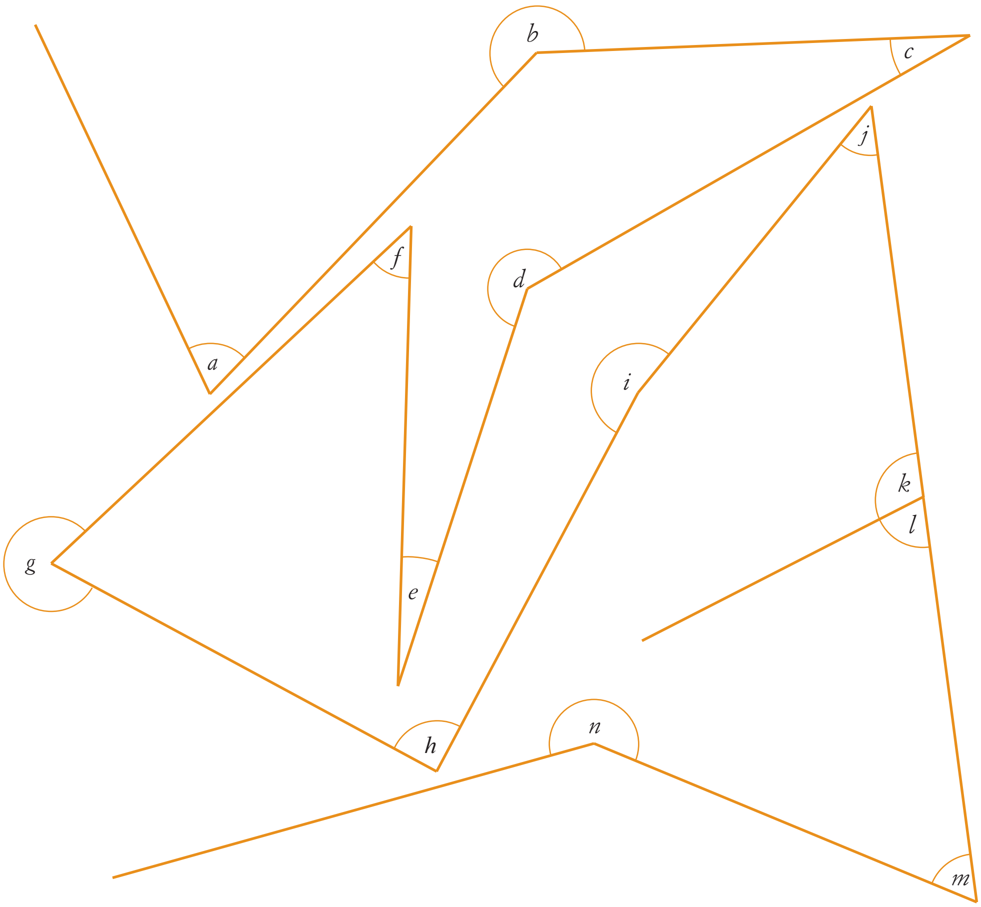
9780170194778

5 Classify each of the following angles as acute, right, obtuse or reflex.



6 Measure the angles shown below.





Fluency

7 What is the complement of:

See Example 2

a 19° b 35° c 76° d 6°?

8 What is the supplement of:

Worked solutions

Exercise 6.1

MAT08MGWS00018

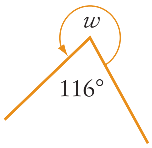
See Example 2

a 46° b 134° c 105° d 62°?

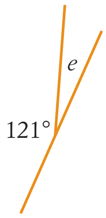
9 Find the value of each unknown angle, giving reasons in each case.

See Example 2

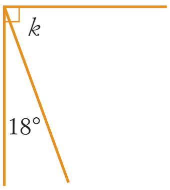
a



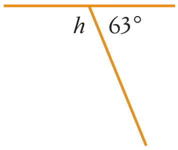
b



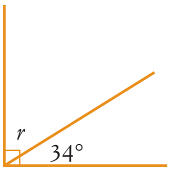
c



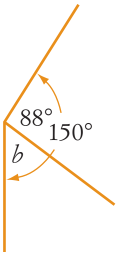
d



e

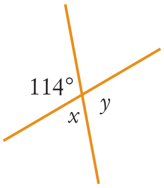


f

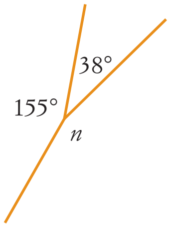


9780170194778

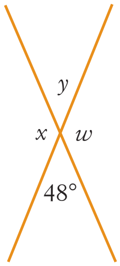
g



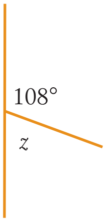
h



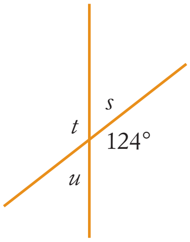
i



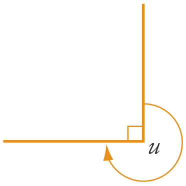
j



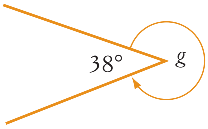
k



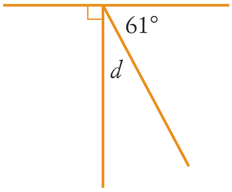
l



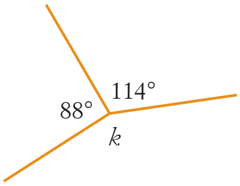
m



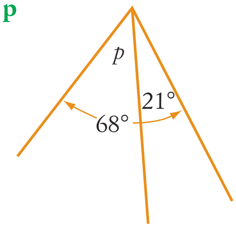
n



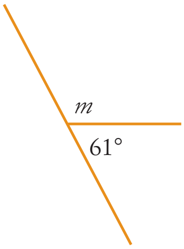
o



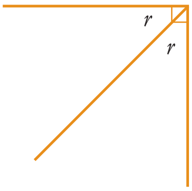
p



q



r



10 Find the value of each unknown angle, giving reasons in each case.

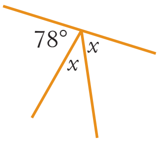
Worked solutions

Exercise 6.1

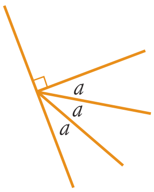
MAT08MGWS00018

See Example 2

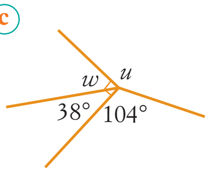
a



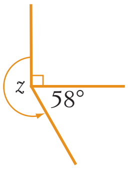
b



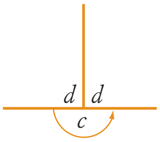
c



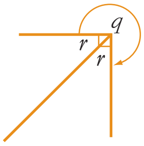
d



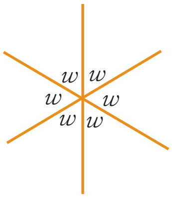
e



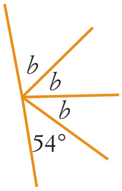
f



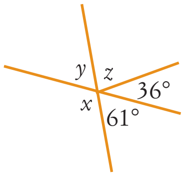
g



h



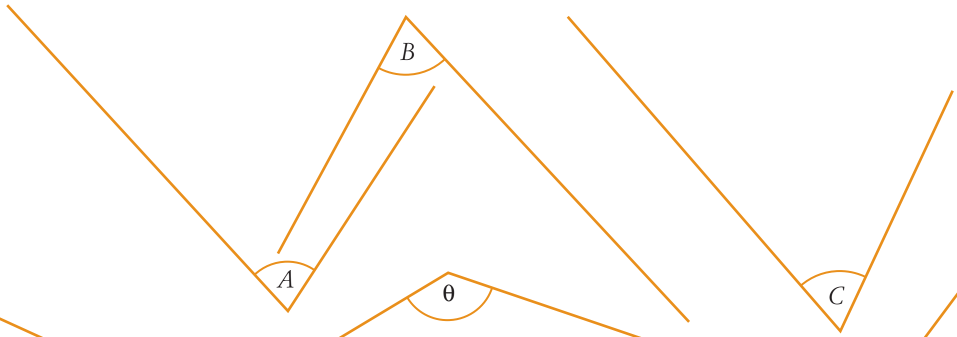
i



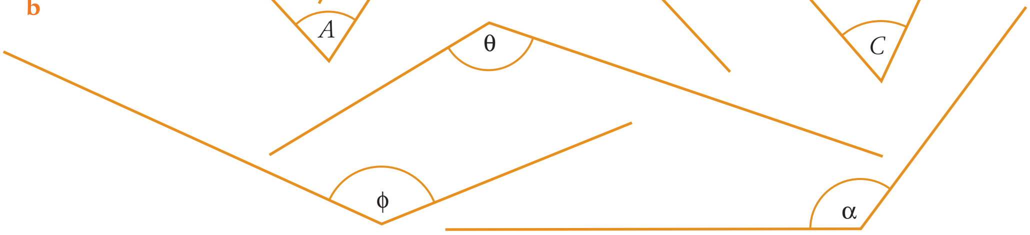
Problem solving

11 Guess which are the biggest and smallest angles in parts **a** and **b** below. Use a protractor to check your answers.

a



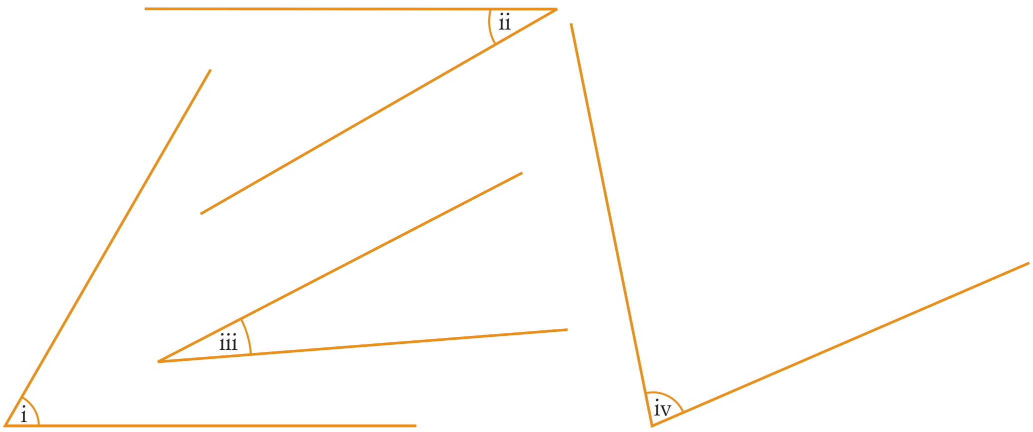
b



12 **a** Draw up the following table to record your results for this question.

Angle	Estimate	Measurement	Error (°)	% Error
i				
ii				
iii				
iv				

b Estimate the size of each of the angles at the top of the page opposite in degrees and record each estimate in the table you have constructed.



9780170194778

c Check your estimate by measuring each angle with a protractor. Record the measurements in the table.

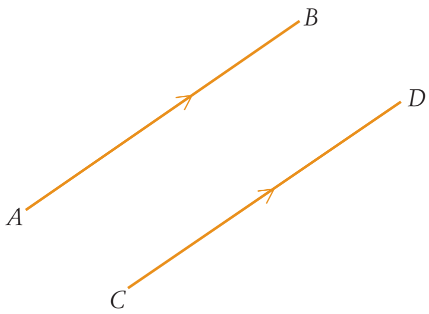
d Calculate the difference between your estimate and the actual measurement. Record this in the table as the error in degrees.

e Calculate the percentage error for each estimate. Record these in the table.

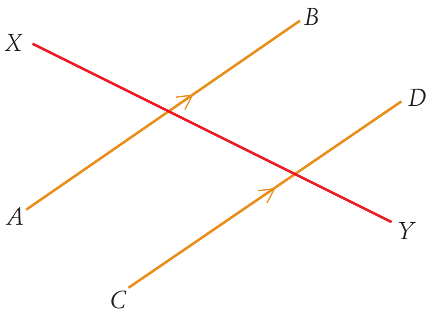
f Which estimate was the most accurate and which was the least accurate? Compare your results with others in the class.

6.2 Calculating angles on parallel lines

Lines that do not intersect (meet) are said to be **parallel** and this is indicated by matching arrows on the lines.



AB is parallel to *CD* and we write this as $AB \parallel CD$.
A line that crosses two or more other lines is called a **transversal**.



XY is a transversal for *AB* and *CD*.

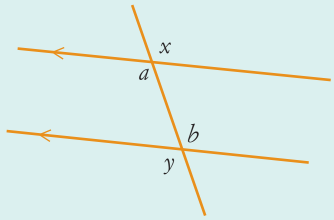
The angles that are formed when a transversal crosses parallel lines have special names and properties.

Important!

Angles on parallel lines

Alternate angles on parallel lines are equal. Alternate angles lie on opposite sides of the transversal.

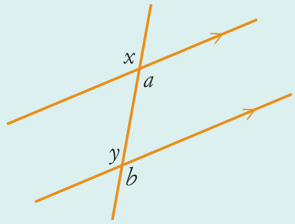
Alternate angles may be interior (a and b) or exterior (x and y).



In the diagram above:

$$a = b \text{ and } x = y$$

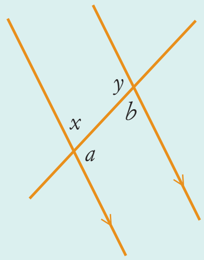
Corresponding angles on parallel lines are equal. Corresponding angles lie on the same side of the transversal.



In the diagram on the right:

$$a = b \text{ and } x = y$$

Co-interior or **allied angles** on parallel lines are supplementary.
Co-interior angles lie on the same side of the transversal and between the parallel lines.



In the diagram on the right:

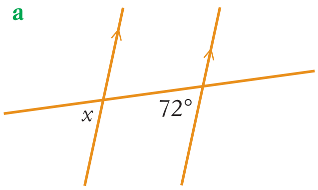
$$a + b = 180° \text{ and } x + y = 180°$$

Example 3

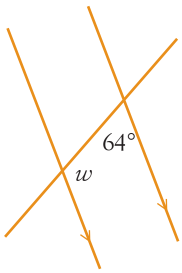
Puzzle sheet
Angles in parallel lines
MAT08MGPS00021

Find the value of each unknown angle.

a



b

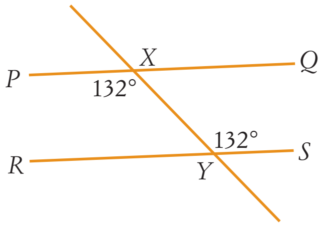


a 72° and x are corresponding angles on parallel lines.	$\boldsymbol{x = 72°}$
b 64° and w are co-interior angles on parallel lines.	$\boldsymbol{64° + w = 180°}$
Subtract 64° from both sides of the equation.	$\boldsymbol{w = 116°}$

9780170194778

Example 4

Show that PQ and RS are parallel.



Solution

Describe the position of $\angle PXY$ and $\angle XYS$ on PQ and RS.	**$\angle PXY$ and $\angle XYS$ are alternate angles.**
Write the values of $\angle PXY$ and $\angle XYS$.	$\boldsymbol{\angle PXY = \angle XYS = 132°}$
As the angles are equal the lines must be parallel.	$\boldsymbol{PQ \parallel RS}$**.**

Worksheet
Angles in parallel lines
MAT08MGWK00039

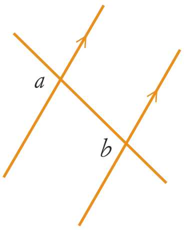
Exercise 6.2 Calculating angles on parallel lines

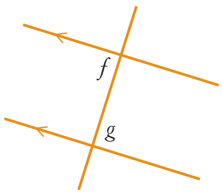
Understanding

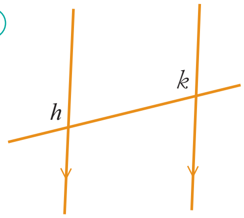
Extra questions
Exercise 6.2
MAT08MGEQ00019

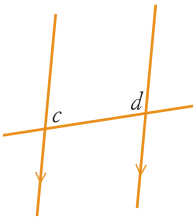
Worked solutions
Exercise 6.2
MAT08MGWS00019

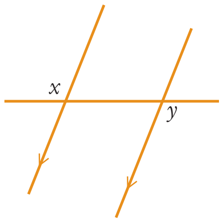
1 Identify each named pair of angles as either alternate, corresponding or co-interior.

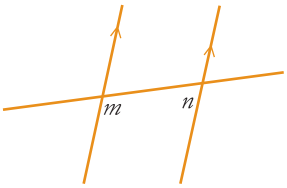
a


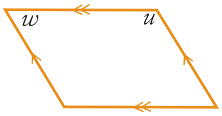
b


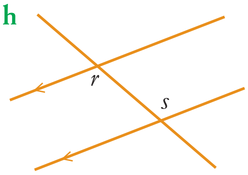
c


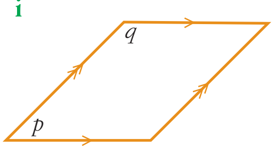
d


e


f


g


h


i


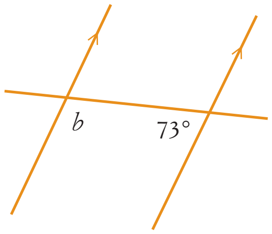
9780170194778

Fluency

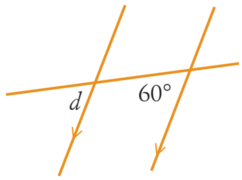
See Example 3

2 Find the value of the unknown angle in each of these diagrams. (Give reasons for your answers.)

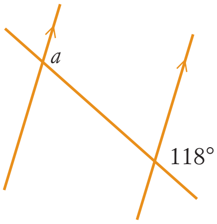
a



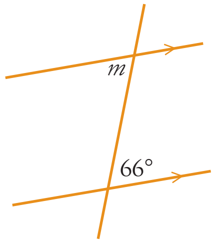
b



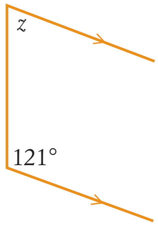
c



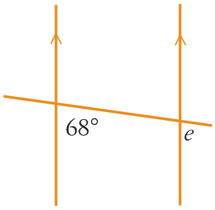
d



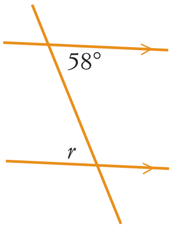
e



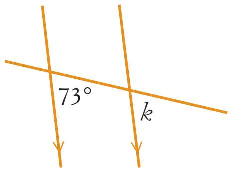
f



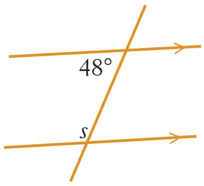
g



h



i



Worked solutions

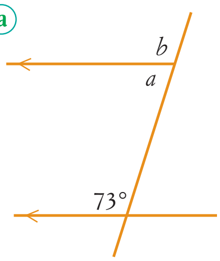
Exercise 6.2

MAT08MGWS00019

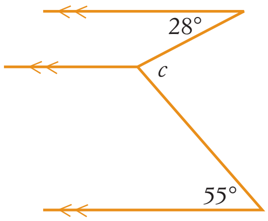
See Example 3

3 Find the value of the unknown angle in each of these diagrams. (Give reasons for your answers.)

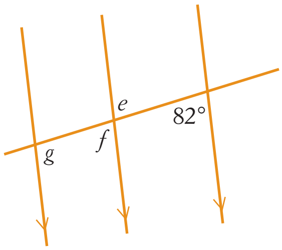
a



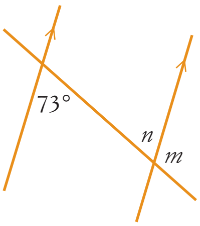
b



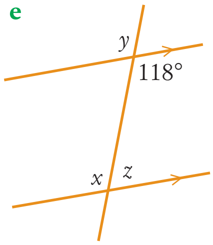
c



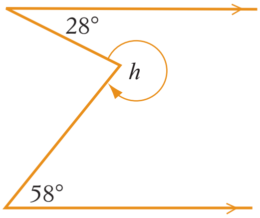
d



e

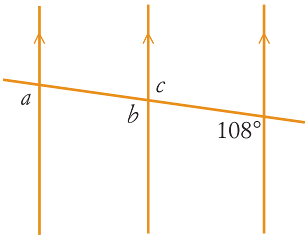


f

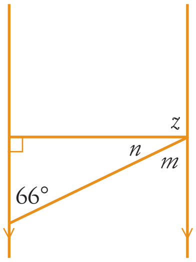


9780170194778

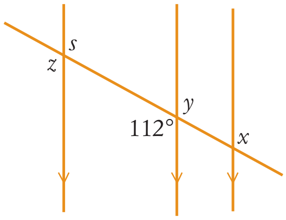
g



h



i



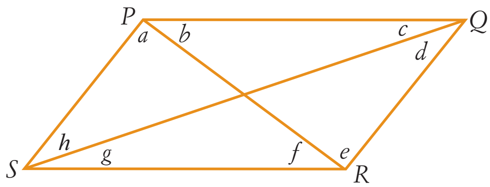
4 In the diagram on the right, *PQRS* is a parallelogram. Use this diagram to complete each of the following. Give a reason for your answer in each case.

a $a = \ldots$

b $h = \ldots$

c $a + b + \ldots + \ldots = 180°$

d $f + e + \ldots + \ldots = 180°$



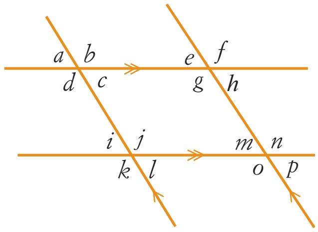
5 Use the diagram on the right to answer the following questions.

a Name all angles congruent to *f*.

b Name all angles congruent to *a*.

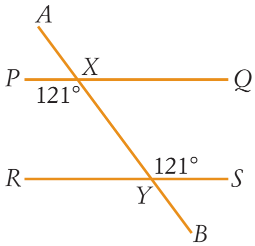
c Name all angles supplementary to *c*.

d If $b = 126°$ what is the value of *j* and *e*?

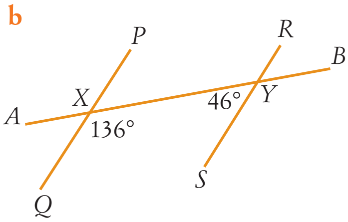


6 For each diagram, determine if $PQ \parallel RS$. If the lines are parallel, then prove it. *See* Example 4

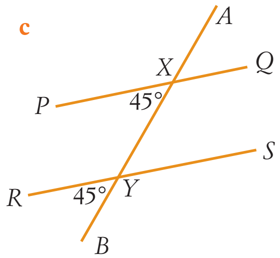
a



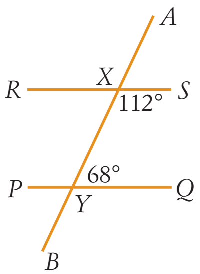
b



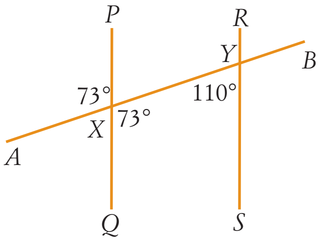
c



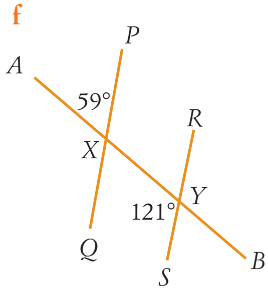
d



e



f



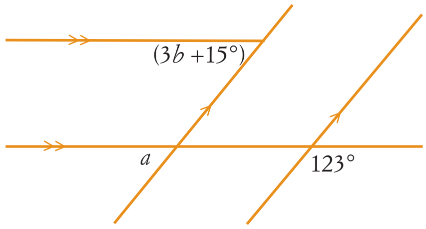
Problem solving

Worked solutions

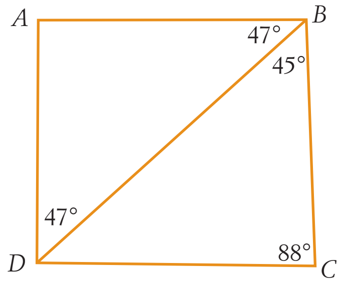
Exercise 6.2

MAT08MGWS00019

7 Calculate the values of a and b in the diagram below, giving reasons for your answers.

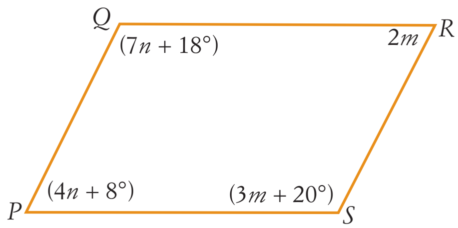


8 Identify any parallel lines in the diagram below, giving reasons for your answers.

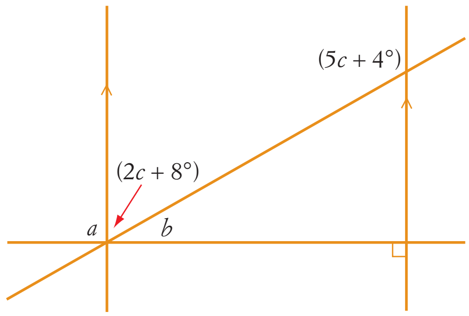


Reasoning

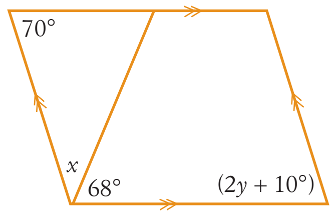
9 $PQRS$ is a parallelogram in the diagram below. Calculate the values of m and n, giving reasons for your answers.



10 Calculate the values of a, b and c in the diagram below, giving reasons for your answers.



11 Calculate the values of x and y in the diagram below, giving reasons for your answers.



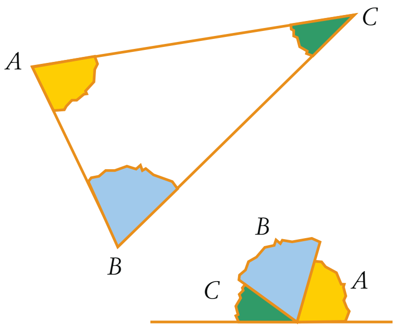
9780170194778

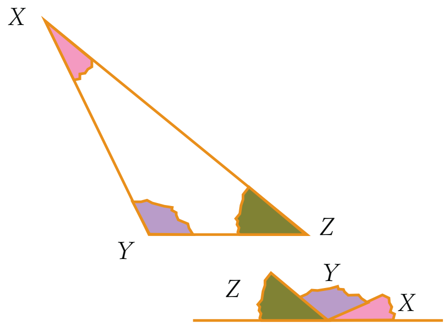
6.3 Calculating angles in triangles

Video tutorial
Angles in polygons
MAT08MGVT00006

Investigate: Sum of the interior angles of a triangle

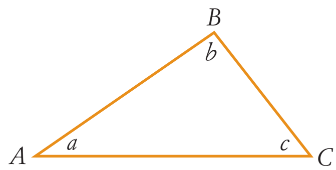
1 Draw a **triangle** on a piece of paper and then cut it out.
2 The angles formed by the sides of the triangle are known as the **interior angles**. Tear off the interior angles (corners) of the triangle.
3 Place the torn-off corners on the desk and rearrange them to form a straight line.





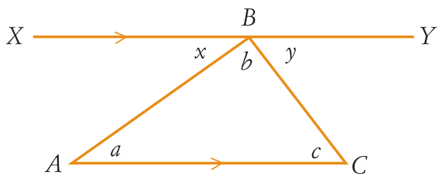
4 Repeat this procedure with a variety of differently shaped triangles.
5 Copy and complete the following statement.
The three angles of a triangle can be arranged to form a straight angle. This means that the three interior angles of a triangle add to a total of _____°.
6 For a regular (**equilateral**) triangle, what is the size of each internal angle?

The previous investigation shows that the sum of the interior angles of certain triangles is 180°. It is possible to prove that this property exists for all triangles.
Draw any triangle ABC, and name its interior angles a, b and c.



Draw a line XY through B that is also parallel to AC.

Name the angles formed as x and y.

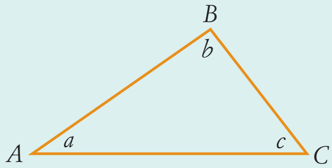


The angles x, b and y are supplementary. $\quad$ $\boldsymbol{x + b + y = 180°}$
a and x are alternate angles on parallel lines. $\quad$ $\boldsymbol{a = x}$
c and y are alternate angles on parallel lines. $\quad$ $\boldsymbol{c = y}$
Substitute for x and y in the first equation. $\quad$ $\boldsymbol{a + b + c = 180°}$
State the result. $\quad$ **The sum of the interior angles of a triangle is 180°.**

Important!

Angle sum of a triangle

The sum of the interior angles of a triangle is 180°.

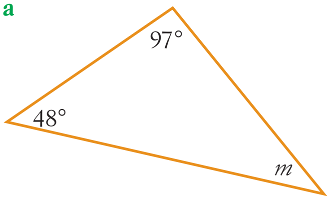


In $\triangle ABC$, $a + b + c = 180°$.

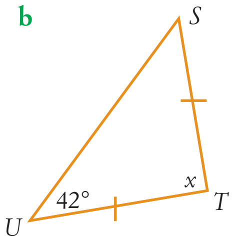
Example 5

Calculate the value of the unknown angle in each triangle shown.

a



b



Solution

a Use the angle sum of a triangle rule. $\quad \mathbf{48° + 97° + \mathit{m} = 180°}$

Evaluate. $\quad \mathbf{145° + \mathit{m} = 180°}$

Subtract 145° from both sides. $\quad \mathbf{\mathit{m} = 35°}$

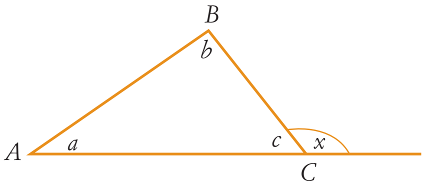
b $\triangle UST$ is **isosceles**. $\quad \mathbf{\angle \mathit{SUT} = \angle \mathit{TSU} = 42°}$

Use the **angle sum** of a triangle rule. $\quad \mathbf{42° + 42° + \mathit{x} = 180°}$

Evaluate. $\quad \mathbf{84° + \mathit{x} = 180°}$

Subtract 84° from both sides. $\quad \mathbf{\mathit{x} = 96°}$

An **exterior angle** to a triangle is formed by extending any side of a triangle.

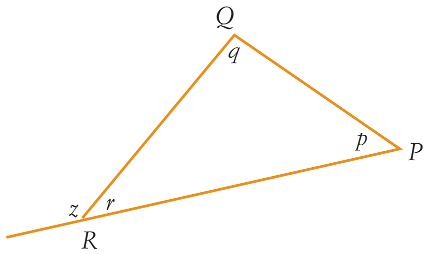


As shown here, x is an exterior angle to $\triangle ABC$.

9780170194778

Investigate: Exterior angle of a triangle

1 Draw a triangle, say $\triangle PQR$ on a piece of paper.



2 Extend RP to form an exterior angle, z, as shown here.
3 Measure the exterior angle and the three interior angles p, q and r.
4 It is possible to arrange the interior angles in 3 different pairs: p and q, p and r, and q and r.
5 Complete the following.

$$p + q = \ldots$$
$$p + r = \ldots$$
$$q + r = \ldots$$

6 Compare the sums of the pairs of interior angles with the size of z.
7 Complete the following.

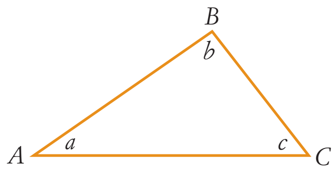
$$z = \ldots + \ldots$$

8 Describe the position of the interior angles compared to the exterior angle.
9 Repeat steps 1 to 8 for other triangles.

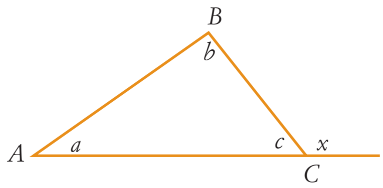
Video tutorial
Angle sums of triangles and quadrilaterals
MAT08MGVT10007

Technology
GeoGebra: Exterior angles of a triangle
MAT08MGTC00007

The previous investigation showed that the exterior angle of a triangle is equal to the sum of the two interior opposite angles. It is possible to prove that this property exists for all triangles. Draw any triangle ABC, and name the interior angles a, b and c.



Extend AC and name the exterior angle x.

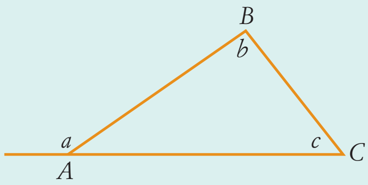


c and x are supplementary.	$\boldsymbol{x + c = 180°}$
The interior angles of $\triangle ABC$ are supplementary.	$\boldsymbol{a + b + c = 180°}$
The angle sums are equal.	$\boldsymbol{x + c = 180° = a + b + c}$
Subtract c from each side of the equation.	$\boldsymbol{x = a + b}$
State the result.	**The exterior angle of a triangle is equal to the sum of the two interior opposite angles.**

Important!

Exterior angle of a triangle

The exterior angle of a triangle is equal to the sum of the interior opposite angles.

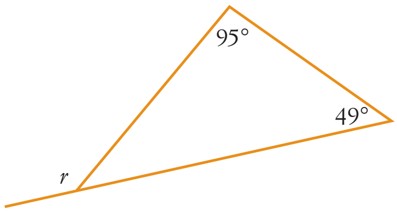


In $\triangle ABC$, $a = b + c$.

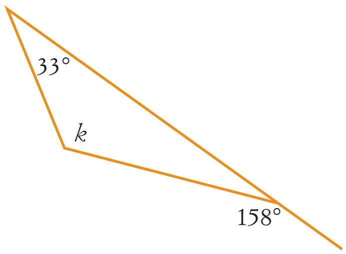
Example 6

Calculate the value of the unknown angle in each of the following triangles below.

a



b



Solution

a Use the exterior angle of a triangle rule. $\boldsymbol{r = 95° + 49°}$

Evaluate. $\boldsymbol{r = 144°}$

b Use the exterior angle of a triangle rule. $\boldsymbol{158° = k + 33°}$

Reverse the equation and subtract 33° from both sides. $\boldsymbol{k = 158° - 33°}$

Evaluate. $\boldsymbol{k = 125°}$

Puzzle sheet
Angles in triangles
MAT08MGPS00022

Puzzle sheet
Mixed angles
MAT08MGPS00023

Worksheet
Angles in triangles
MAT08MGWK00040

9780170194778

Exercise 6.3 Calculating angles in triangles

Understanding

Do not use a protractor to calculate any of the angles for the questions in this exercise.

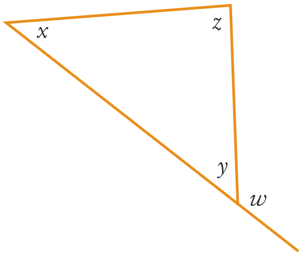
Extra questions
Exercise 6.3
MAT08MGEQ00020

1 In each triangle below, name:

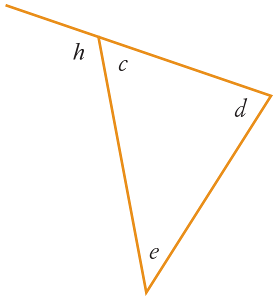
i the exterior angle

ii the interior opposite angles for the exterior angle.

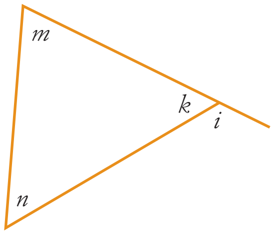
a



b



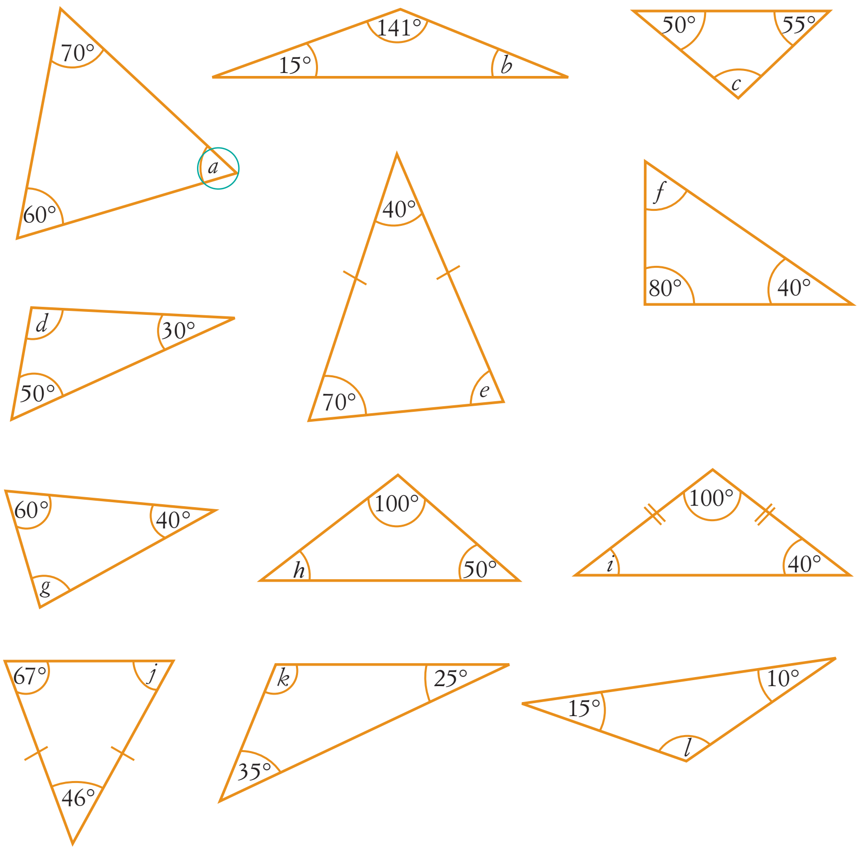
c



Fluency

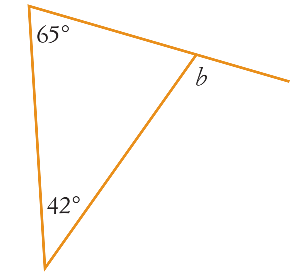
Worked solutions
Exercise 6.3
MAT08MGWS00020
See Example 5

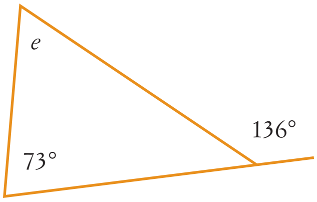
2 Find the unknown angles in each of the following triangles.

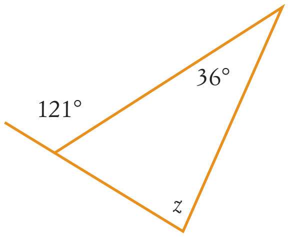


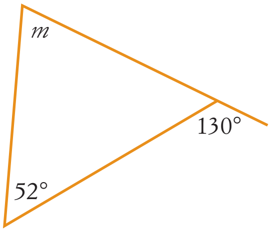
See Example 6

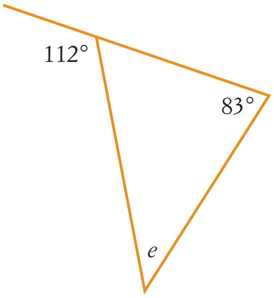
3 Find the unknown angles in each of the following triangles.

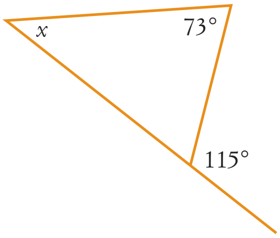
a


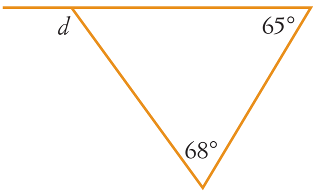
b


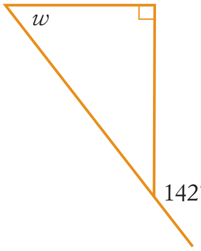
c


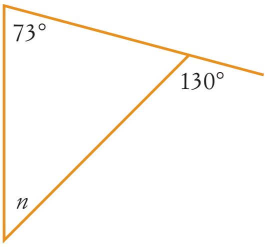
d


e


f


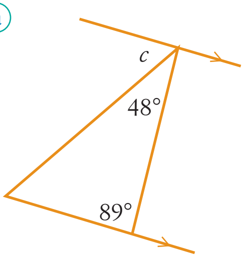
g


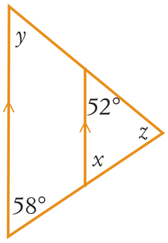
h


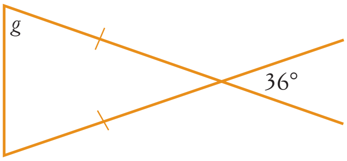
i


Worked solutions
Exercise 6.3
MAT08MGWS00020

4 Find the unknown angles in each of the following diagrams.

a


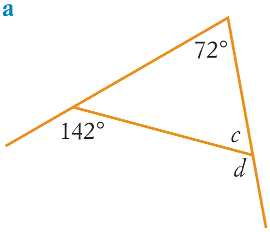
b


c


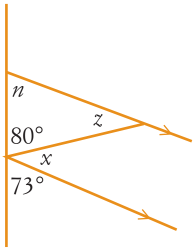
9780170194778

5 Find the unknown angles in each of the following diagrams.

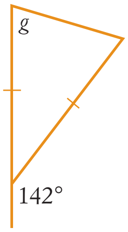
a



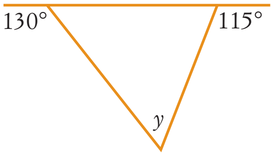
b



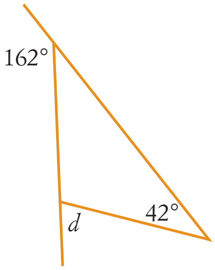
c



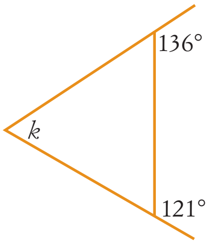
d



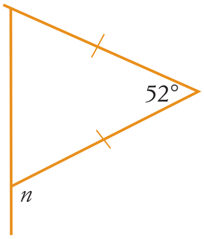
e



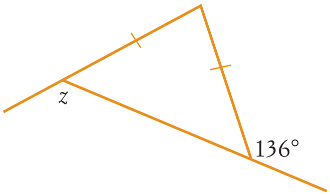
f



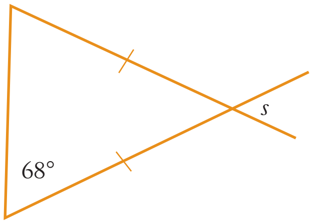
g



h

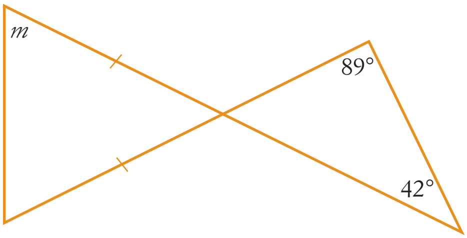


i

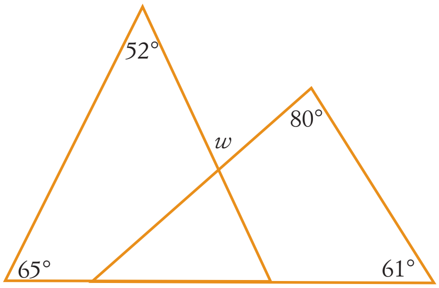


Problem solving

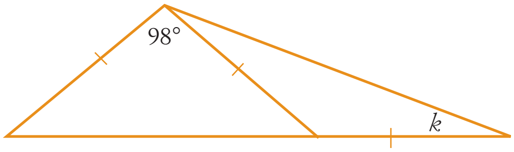
6 Find the unknown angle in the diagram.



7 Find the unknown angle in the diagram.



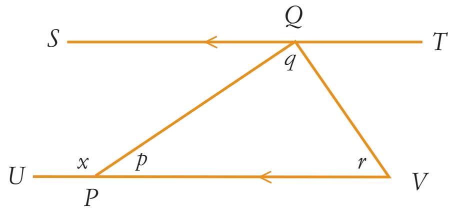
8 Find the unknown angle in the diagram.



Reasoning

Worked solutions
Exercise 6.3
MAT08MGWS00020

9 In the diagram below, $ST \parallel UV$.

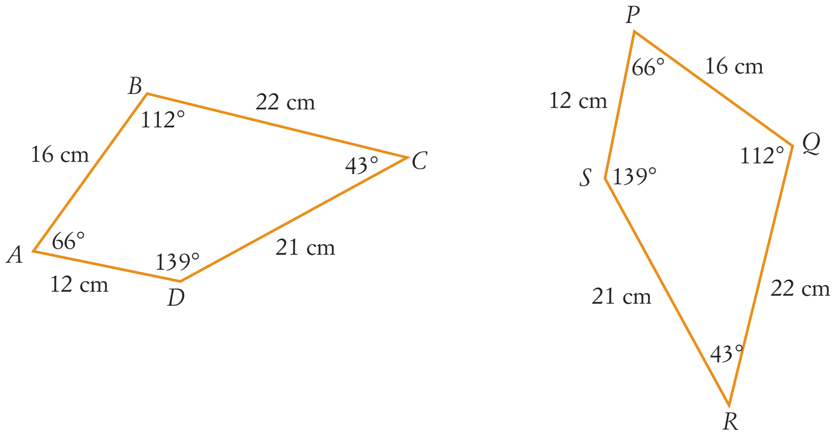


Use the properties of angles on parallel lines to prove that the exterior angle of $\triangle PQV$ is equal to the sum of the two interior opposite angles.
In other words, use angles on parallel lines to prove $x = q + r$.

10 If the angles of a triangle are in the ratio of 1 : 2 : 3, prove that the triangle is right-angled.

6.4 Congruent triangles

You have already seen that **congruent** figures have the same size and shape.



Comparing the quadrilaterals above, you can see that $\angle A = \angle P$, $\angle B = \angle Q$, $AB = PQ$, $BC = QR$ and so on. In fact, the quadrilateral on the right can be exactly **superimposed** on the other one so they are congruent. The points in the figures that would coincide if the figures were superimposed are said to be **corresponding**. We can show correspondence between these points by writing $A \leftrightarrow P$ (A corresponds to P), $B \leftrightarrow Q$, $C \leftrightarrow R$ and $D \leftrightarrow S$.
When we write a statement of congruence, corresponding points must be written in the same order for each figure:

$$ABCD \equiv PQRS \text{ (or } ABCD \cong PQRS\text{)}$$

9780170194778

We can check if figures are congruent by measuring all angles and sides but this isn't always convenient. In the case of triangles, special tests have been developed to test for congruency.

Investigate: Tests for congruent triangles

Activity 1

You will need coloured straws, sticky tape, scissors and a ruler.

Step 1: Cut three red straws to lengths of 18 cm, 15 cm and 10 cm. Use sticky tape to join them together to make a triangle.

Step 2: Now make a green triangle the same way.

Step 3: Make a third triangle using blue straws and the same measurements.

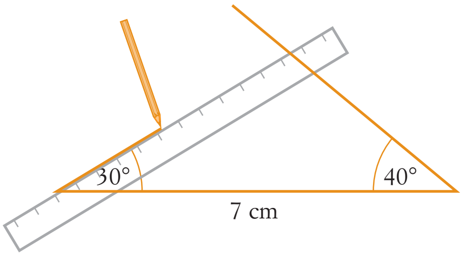
Compare the three triangles. What do you find?

Choose three different side lengths and repeat the process. What do you find?

You will need a protractor, pencil, paper, a compass and a ruler for the remaining activities.

Activity 2

Let's see how many different triangles can be drawn with a side of 7 cm between angles of 30° and 40°. The 7 cm side is called the **contained** side because it is contained between the two angles.



Step 1: Draw a side of length 7 cm.

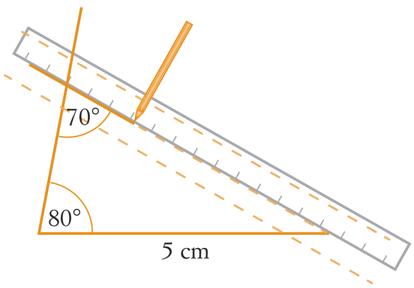
Step 2: From one end draw a side that makes an angle of 30° with the first side.

Step 3: From the other end draw a side that makes an angle of 40° with the other side and complete the triangle.

Is it possible to draw a different triangle with angles of 30° and 40° containing a side of length 7 cm?

Activity 3

Let's see how many different triangles can be drawn with angles of 70° and 80° and a side of length 5 cm that is *not* contained between the angles.

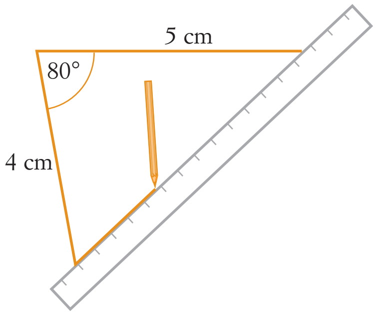


Step 1: Draw a side of length 5 cm.
Step 2: From one end draw a second side that makes an angle of 80° with the first side.
Step 3: Now draw a light pencil line that makes an angle of 70° with the second side.
Step 4: Slide your ruler parallel with the light pencil line until it meets the 5 cm side and complete the triangle.

Is it possible to draw a different triangle with angles of 80° and 70° and a non-contained side of length 5 cm?

Activity 4

Let's see how many different triangles can be drawn with sides of 4 cm and 5 cm and an angle of 80° between the two sides. The 80° angle is called an **included** angle.

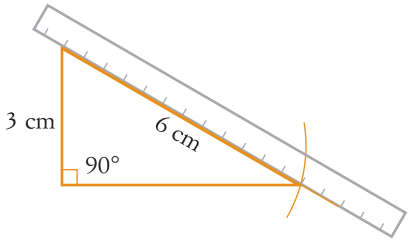


Step 1: Draw a side of length 4 cm.
Step 2: From one end draw a second side that makes an angle of 80° with the first side.
Step 3: Measure 5 cm down this side and complete the triangle.

Is it possible to draw a different triangle with sides of 4 cm and 5 cm and an included angle of 80°?

Activity 5

Let's see how many different triangles can be drawn with a right angle, a hypotenuse of 6 cm and another side of 3 cm.



Step 1: Draw a side of length 3 cm.
Step 2: From one end draw a second side that makes an angle of 90° with the first side.
Step 3: Set the compass to a distance of 6 cm.
Step 4: Place the point of the compass at the other end of the 3 cm side and mark off a point on the second side of the triangle. Now draw in the hypotenuse of 6 cm.

Is it possible to draw a different right-angled triangle with a hypotenuse of 6 cm and another side of 3 cm?

9780170194778

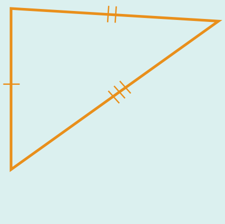
From the investigation, you can see that there are a number of ways in which congruent triangles can be identified using angle and side properties.

Important!

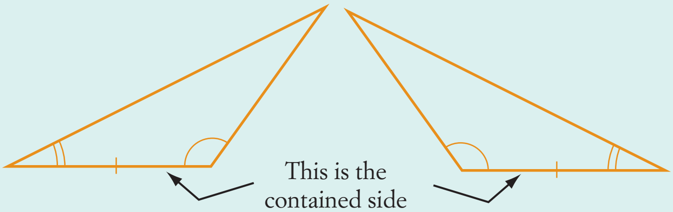
Tests for congruent triangles

The four tests for congruent triangles are as follows.

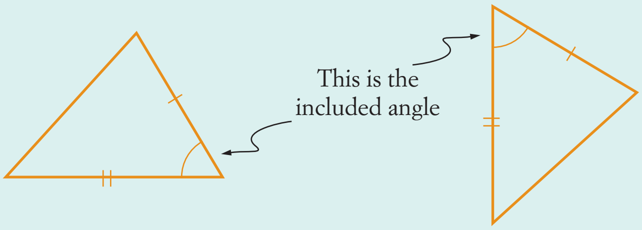
- **Side, side, side** or **SSS**
  Triangles are congruent if three corresponding sides are equal.



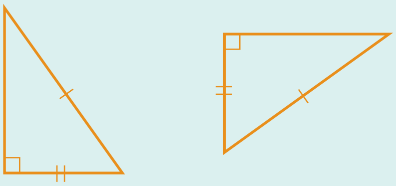
- **Angle, side, angle** or **ASA**
  Triangles are congruent if two angles and the side they contain are equal.



- **Side, angle, side** or **SAS**
  Triangles are congruent if two sides and the included angle are equal.



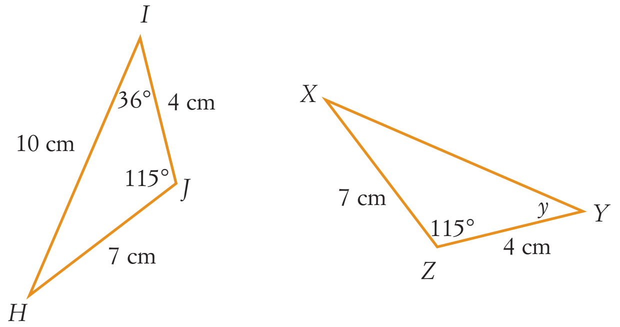
- **Right angle, hypotenuse, side** or **RHS**
  Triangles are congruent if a right angle, the hypotenuse and a corresponding side are equal.



Animated example
Congruent triangles
MAT08MGAE00009

Example 7

Look at the triangles below.



a Show that the triangles are congruent by using a test for congruency.
b Find the value of y.

Solution

a Identify congruent sides and angles in the triangles.	$\angle \boldsymbol{HIJ} = \angle \boldsymbol{XYZ}$ $\boldsymbol{IJ} = \boldsymbol{YZ}$ $\boldsymbol{JH} = \boldsymbol{ZX}$
$\angle HIJ$ and $\angle XYZ$ are included angles between corresponding sides that are equal.	$\triangle \boldsymbol{HIJ} \equiv \triangle \boldsymbol{XYZ}$ **(SAS)**
b Corresponding angles are equal.	$\angle \boldsymbol{HIJ} \leftrightarrow \angle \boldsymbol{XYZ}$
$\angle HIJ = 30°$.	$\boldsymbol{y} = \mathbf{30°}$

A statement of congruency identifies corresponding points in figures.

Example 8

If $\triangle PQR \equiv \triangle STU$ complete the following.

a $PR = \ldots$ **b** $\angle QRP = \angle \ldots$

Solution

a $\triangle PQR \equiv \triangle STU$.	$\boldsymbol{P} \leftrightarrow \boldsymbol{S}$, $\boldsymbol{Q} \leftrightarrow \boldsymbol{T}$ **and** $\boldsymbol{R} \leftrightarrow \boldsymbol{U}$.
Identify the side that corresponds to PR.	$\boldsymbol{PR} = \boldsymbol{SU}$
b $\angle QRP \leftrightarrow \angle TUS$.	$\angle \boldsymbol{QRP} = \angle \boldsymbol{TUS}$

9780170194778

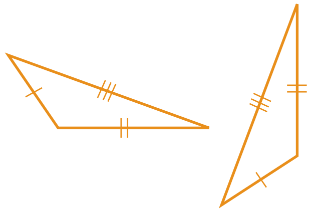
Exercise 6.4 Congruent triangles

Understanding

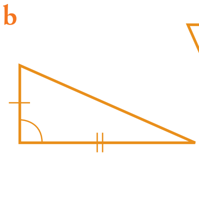
Extra questions
Exercise 6.4
MAT08MGEQ00021

1 For each of the following, which test could be used to show that the pair of triangles is congruent? If no test is possible, state 'None'.

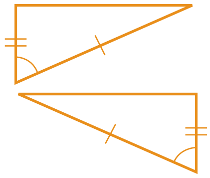
a



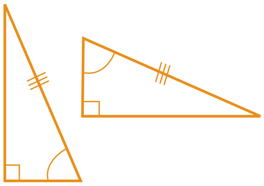
b



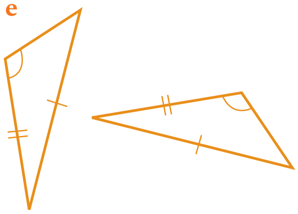
c



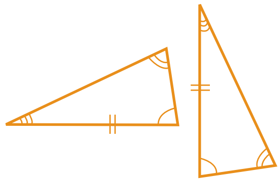
d



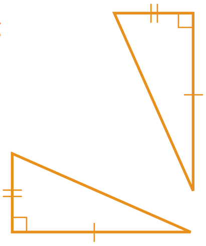
e



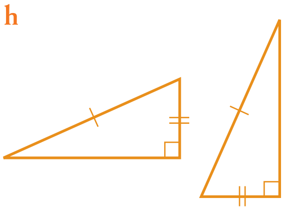
f



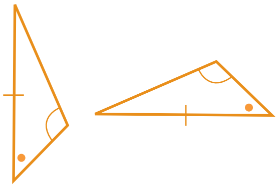
g



h



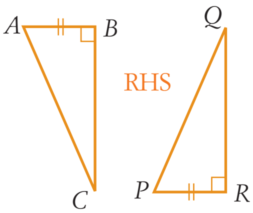
i



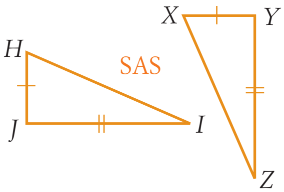
2 In each case below and at the top of the next page, a pair of triangles and a congruency test is shown. Name the additional equal corresponding parts that would be needed to prove that the triangles are congruent using the stated test in each case.

Worked solutions
Exercise 6.4
MAT08MGWS00021

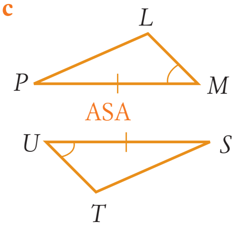
a



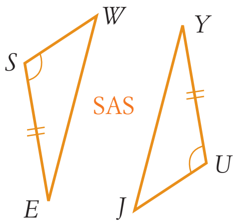
b



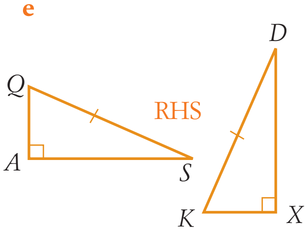
c



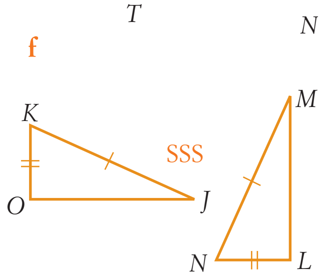
d



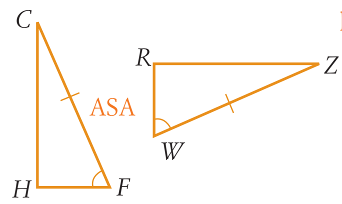
e



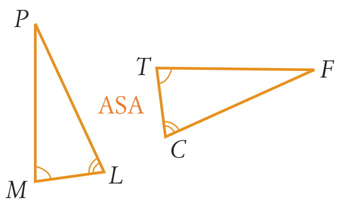
f



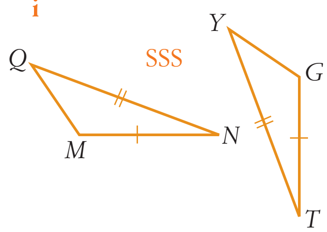
g



h



i



Fluency

See Example 8

3 If $\triangle NQM \equiv \triangle WIZ$ select from **A**, **B**, **C** and **D** below to complete the statement $MQ = \ldots$

A WI **B** IW **C** IZ **D** ZI

See Example 8

4 If $\triangle LMN \equiv \triangle ACE$ select from **A**, **B**, **C** and **D** below to complete the statement $\angle N = \ldots$

A $\angle A$ **B** $\angle E$ **C** $\angle C$ **D** $\angle M$

See Example 8

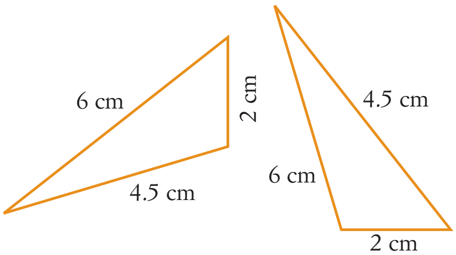
5 If $\triangle DSE \equiv \triangle PLK$ select from **A**, **B**, **C** and **D** below to complete the statement $\triangle LKP = \ldots$

A $\triangle ESD$ **B** $\triangle SED$ **C** $\triangle DSE$ **D** $\triangle SDE$

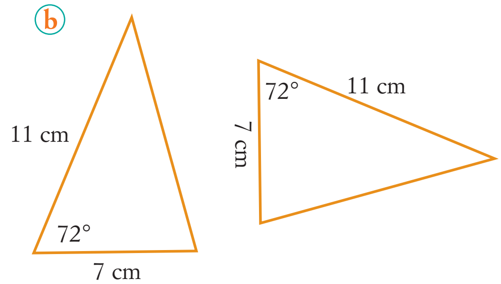
Worked solutions
Exercise 6.4
MAT08MGWS00021
See Example 7

6 Determine whether each of the following pairs of triangles is congruent. If the triangles are congruent, state the test used.

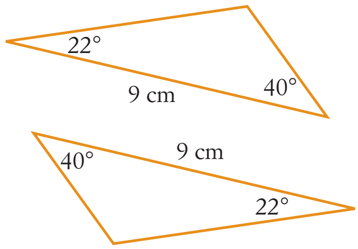
a



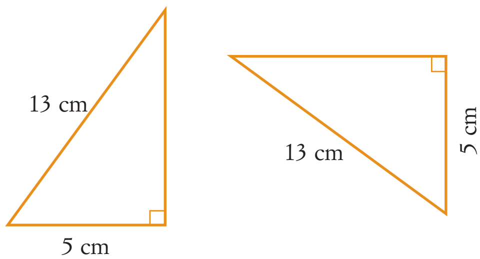
b



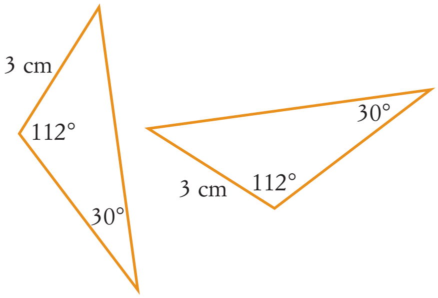
c



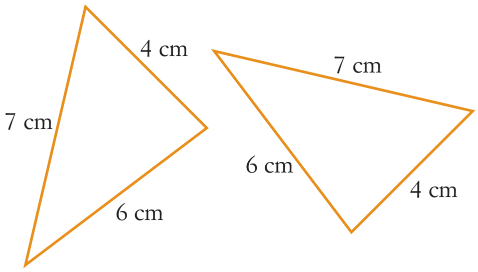
d



e



f

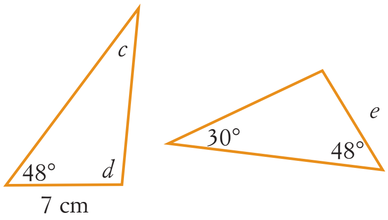


9780170194778

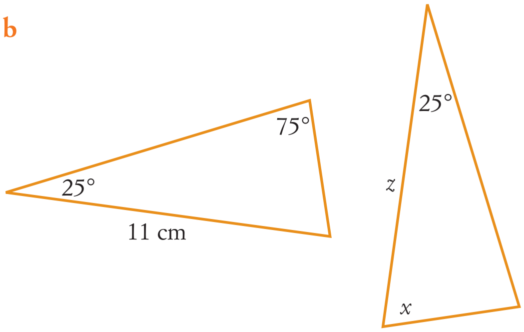
7 In each case below the pair of triangles is congruent. Find the value of the unknown sides and angles indicated.

See Example 7

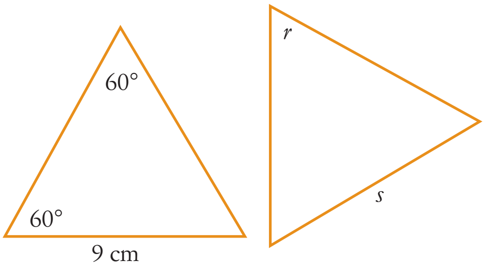
a



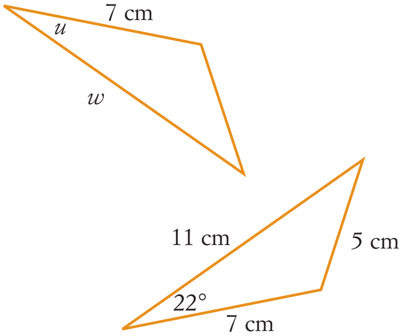
b



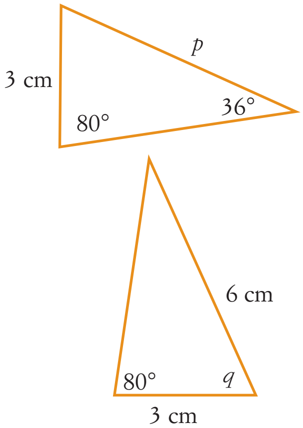
c



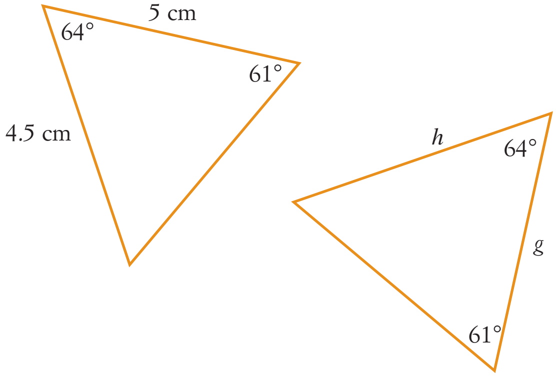
d



e



f



Problem solving

8 Salima wants to prove that $\triangle PQR \equiv \triangle XYZ$. She knows that $PQ = XY$ and $PR = XZ$. What other information must she have to prove that the triangles are congruent? Select the best answer from **A** to **H** below.

See Example 8

A $\angle R = \angle Z$

B $\angle P = \angle X$

C $\angle Q = \angle Y$

D $QR = YZ$

E **A** or **D** (either will work)

F **B** or **D** (either will work)

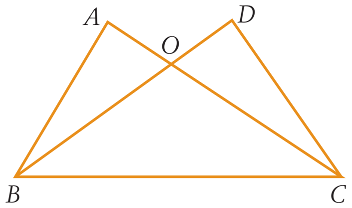
G **C** or **D** (either will work)

H None of the above

Reasoning

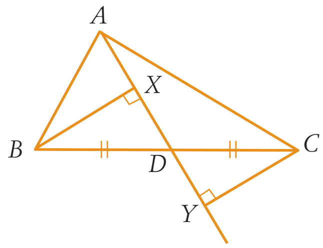
Worked solutions
Exercise 6.4
MAT08MGWS00021

9 In the diagram below, $OA = OD$ and $OB = OC$.

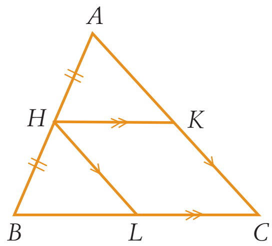


Prove that $\angle BAC = \angle BDC$.

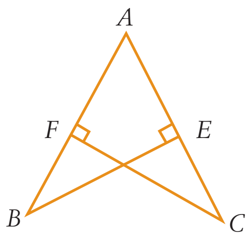
10 In the diagram below, prove that $BX = CY$.



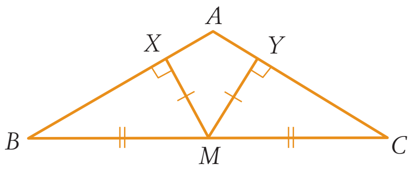
11 In the diagram below, prove that $HK = BL$.



12 In the diagram below, prove that $BE = CF$.



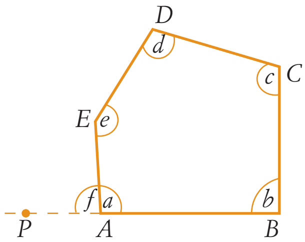
13 In the diagram below, prove that $\triangle BXM \equiv \triangle CYM$ and use this result to show that $AB = AC$.



9780170194778

6.5 Calculating angles in polygons

We have previously looked at the interior and exterior angles of a triangle. A triangle is a convex polygon. We will now extend that examination to other convex polygons.
Look at the pentagon *ABCDE* below.



Angles a, b, c, d and e all lie within the boundary of the pentagon and are called interior angles. If the side AB is extended through to P, the angle formed ($\angle EAP = f$) is known as an exterior angle of the pentagon.

Important!

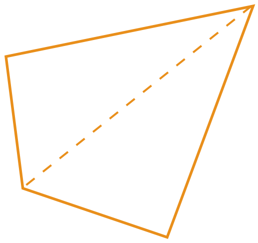
Interior angles of a polygon

A **polygon** has only one inside region, called its **interior**. The angles that lie in this region are all called **interior angles**.

Investigate: Interior angles of polygons

Video tutorial
Angles in polygons
MAT08MGVT00006

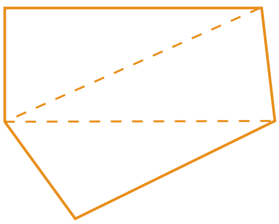
Quadrilaterals



1 Draw a quadrilateral.
2 Draw in one diagonal.
3 What are the two shapes that are formed?
4 What is the sum of the interior angles of each shape formed?
5 What is the sum of the interior angles of the quadrilateral?
6 Copy and complete the following statement:
A quadrilateral can be divided to form two triangles. Therefore the interior angles of a quadrilateral add to a total of _____°.
7 For a regular quadrilateral (**square**), what is the size of each internal angle?

Pentagon

1 Draw a pentagon.
2 From any vertex, draw in diagonals to each of the other opposite vertices.



3 What are the three shapes that are formed?
4 What is the sum of the interior angles of each shape formed?
5 What is the sum of the interior angles of the pentagon?
6 Copy and complete the following statement:
A pentagon can be divided to form three triangles. Therefore the interior angles of a pentagon add to a total of _____°.
7 For a regular pentagon, what is the size of each internal angle?

Other convex polygons

Use the results of this investigation to complete the following table.

Polygon	Number of sides	Number of triangles in polygon	Total degrees in polygon
Triangle	3	1	180°
Quadrilateral	4	2	
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		

A hexagon has 6 sides.

$$\begin{aligned}\text{The sum of the angles in a hexagon} &= (\text{angle sum of a triangle}) \times 4\\ &= 180° \times 4\\ &= 720°\end{aligned}$$

A heptagon has 7 sides.

$$\begin{aligned}\text{The total of the angles in a heptagon} &= 180° \times 5\\ &= 900°\end{aligned}$$

Use the information above to complete the following sentence.
The angle sum of a convex polygon with n sides is:
$A = 180 \times (\textit{number of triangles} - \_\_\_\_\_\_)° = 180 \times (n - \_\_\_)°$

It can also be shown that the same result holds for concave (non-convex) polygons.

Important!

Sum of interior angles in a polygon

The sum of the interior angles of a polygon with n sides is given by:

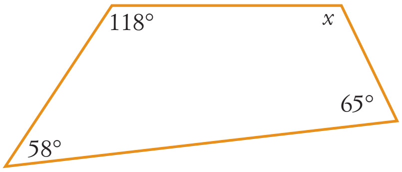
$$A = 180(n - 2)°$$

where n is the number of sides in the polygon.
In particular, the sum of the interior angles of a **quadrilateral** is $180(4 - 2)° = 360°$.

9780170194778

Example 9

Find the size of x in this quadrilateral.



Solution

Write the rule for the angle sum of a polygon. $A = 180(n - 2)°$

The quadrilateral has 4 sides so $n = 4$. $= 180(4 - 2)°$

You can use your calculator.

Enter as: 180 (4 − 2) = .

180(4−2) 360

$= 360°$

The sum of the angles is 360°. $x + 65° + 58° + 118° = 360°$

$x + 241° = 360°$

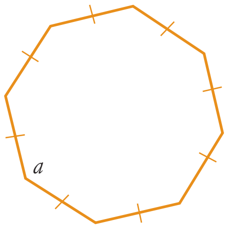
Take 241° away from both sides of the equation. $x = 360° - 241°$

Evaluate. $= 119°$

State the result. $x = 119°$

Example 10

Find the size of each internal angle in a regular octagon.



Solution

Write the rule for the angle sum of a polygon. $A = 180(n - 2)°$

The octagon has 8 sides so $n = 8$. $= 180(8 - 2)°$

You can use your calculator.

Enter as: 180 (8 − 2) = .

180(8−2) 1080

$= 1080°$

In a regular octagon, the 8 interior angles are equal. $8a = 1080°$

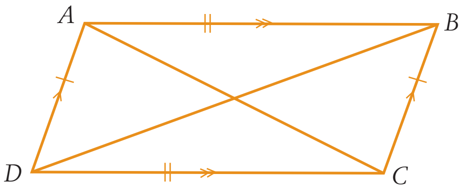
Divide both sides of the equation by 8. $a = \frac{1080°}{8}$

Evaluate. $a = 135°$

Worksheet
Angles in regular polygons
MAT08MGWK00041

Scientific calculator exercise
Shapes and angles
MAT08MGSC00001

You have seen that the sum of the interior angles of a quadrilateral is 360°. You can use congruent triangles to establish a special angle relationship for parallelograms.
Look at the parallelogram *ABCD* below.



We know that opposite sides of a **parallelogram** are equal.
Now look at $\triangle ABD$ and $\triangle CDB$.
$AB = CD$
$AD = CB$
DB is common to both triangles.
So $\triangle ABD \equiv \triangle CDB$ (SSS)
$\therefore \angle DAB = \angle BCD$
$\angle DAB$ and $\angle BCD$ are called the opposite angles of the parallelogram.
So the opposite angles in a parallelogram are equal.
Use $\triangle ADC$ and $\triangle CBA$ to show that $\angle ADC = \angle CBA$.

Important!

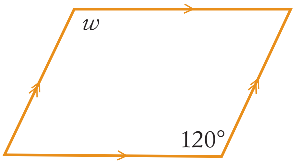
Opposite angles in a parallelogram

The opposite angles in a parallelogram are equal.

$$a = b$$
$$x = y$$

Example 11

Find the value of w in the diagram below.



Solution

Identify the quadrilateral.	**The quadrilateral is a parallelogram.**
The 120° angle and w are opposite angles in the parallelogram.	$\boldsymbol{w} = \mathbf{120°}$

9780170194778

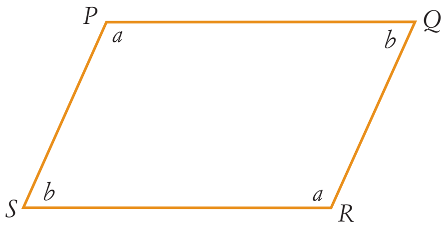
Example 12

The quadrilateral shown below has equal opposite angles.

Prove that the quadrilateral is a parallelogram.

Solution

Mark in the equal angles.



The sum of the interior angles of the quadrilateral is 360°.	$\boldsymbol{a + a + b + b = 360°}$
Simplify by collecting like terms.	$\boldsymbol{2a + 2b = 360°}$
Divide both sides by 2.	$\boldsymbol{a + b = 180°}$
Use the fact that a and b are co-interior angles.	$\boldsymbol{PQ \parallel SR}$ **and** $\boldsymbol{PS \parallel QR}$
Opposite sides of $PQRS$ are parallel.	***PQRS*** **is a parallelogram.**

Important!

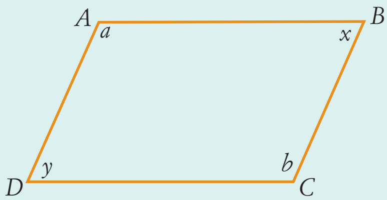
Opposite angles in a parallelogram

A quadrilateral in which opposite angles are equal must be a parallelogram.

If $\quad a = b$

and $\quad x = y$

then $\quad ABCD$ is a parallelogram.



Exercise 6.5 Calculating angles in polygons

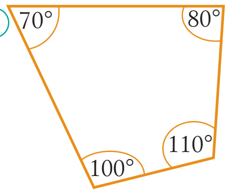
Understanding

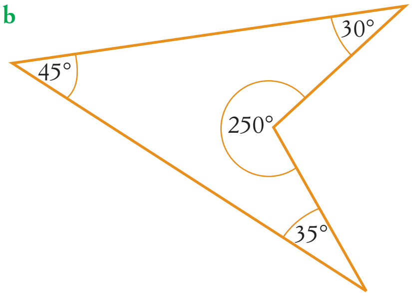
Extra questions
Exercise 6.5
MAT08MGEQ00022

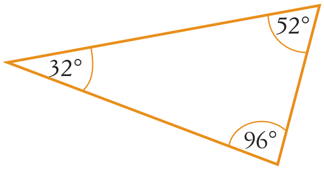
Worked solutions
Exercise 6.5
MAT08MGWS00022

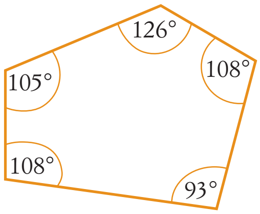
See Examples 9, 10

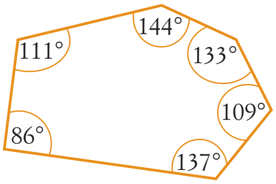
1 Find the sum of the interior angles of the following polygons.

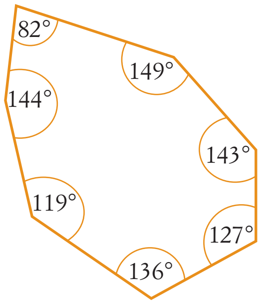
a


b


c


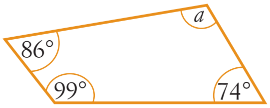
d


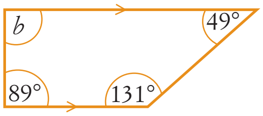
e


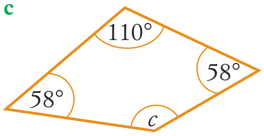
f


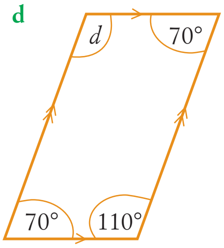
See Example 9

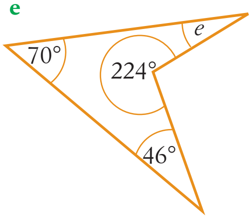
2 Find the unknown angles in each of the following quadrilaterals without using a protractor.

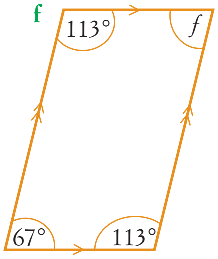
a


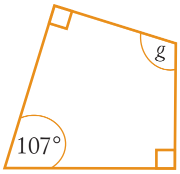
b


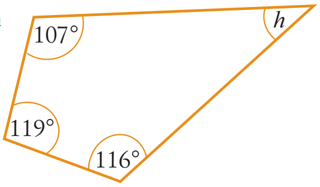
c


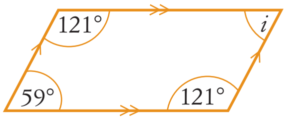
d


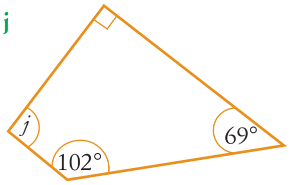
e


f


g


h


i


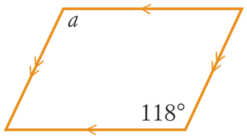
j


9780170194778

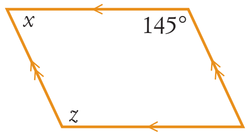
3 Find the value of each variable:

See Example 11

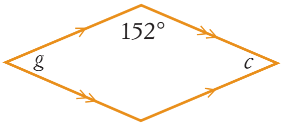
a



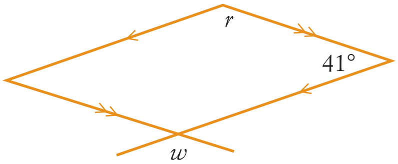
b



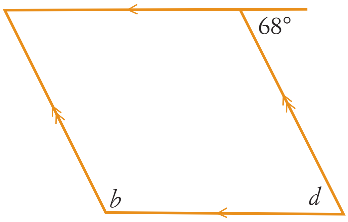
c



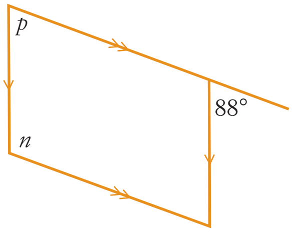
d



e



f



Fluency

4 Copy and complete the following table.

	Polygon	**Number of sides**	**Sum of interior angles**
a	Pentagon		
b	Nonagon		
c	Hexagon		
d	Decagon		
e	Dodecagon		

5 Calculate the sum of the interior angles for a convex polygon with:

a 14 sides **b** 20 sides **c** 25 sides **d** 100 sides

6 Find the number of sides in a polygon if the sum of its interior angles is:

a 2340° **b** 3600° **c** 2700° **d** 5760°

Worked solutions
Exercise 6.5
MAT08MGWS00022

7 For each of the following regular polygons, calculate the size of one interior angle.

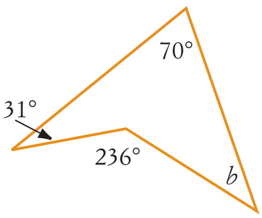
See Example 10

a square **b** hexagon **c** nonagon **d** dodecagon

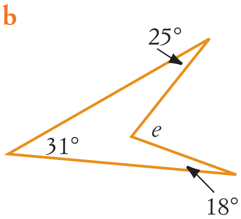
8 Find the unknown angles in each of the following quadrilaterals without using a protractor.

See Example 9

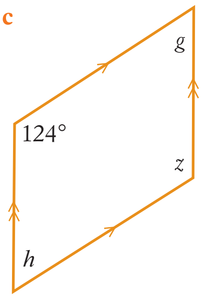
a



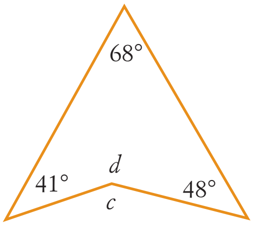
b



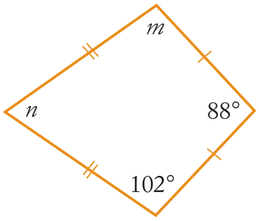
c



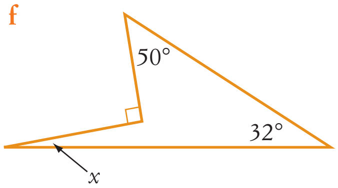
d



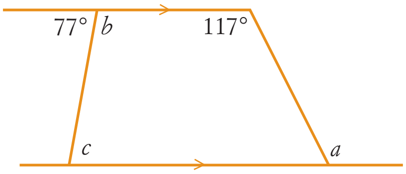
e



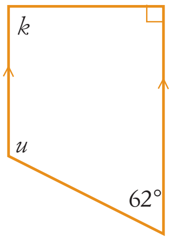
f



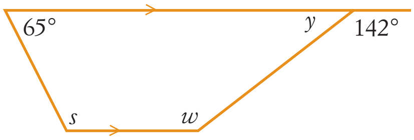
g



h



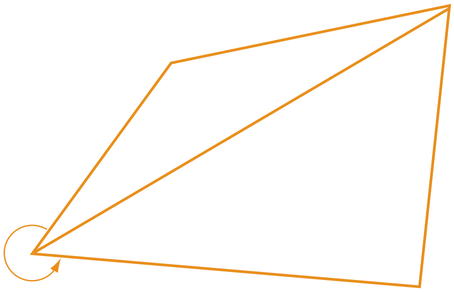
i



Problem solving

Worked solutions
Exercise 6.5
MAT08MGWS00022

9 Draw a convex quadrilateral and rule in a diagonal to divide the quadrilateral into two triangles as shown in the diagram below.

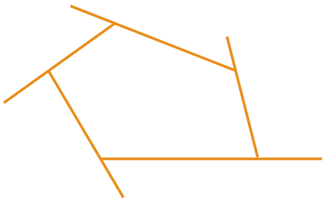


Now prove that the sum of the three interior opposite angles of the quadrilateral is equal to the reflex angle at the fourth vertex.

10 The interior angles of a regular convex quadrilateral each measure 156°. How many sides does the polygon have?

Reasoning

11 Draw a convex pentagon. Extend each of the sides in order as shown below.

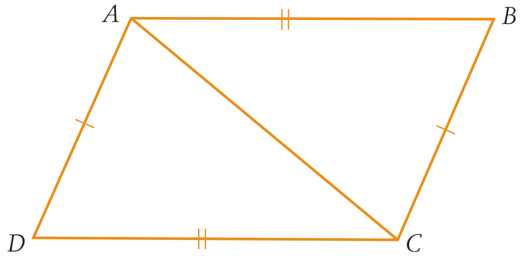


Prove that the sum of the exterior angles is equal to 360°.

9780170194778

12 The quadrilateral $ABCD$ shown below has $AB = CD$ and $AD = CB$.

See Example 12



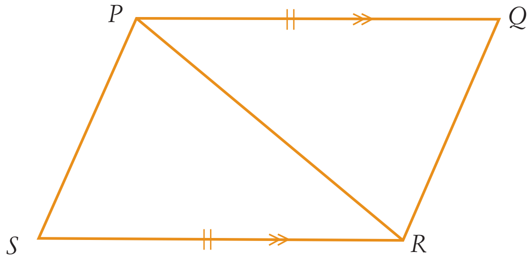
a Show that $\triangle ABD \equiv \triangle CDB$.

b Use this result to show that $AB \parallel CD$.

c Show that $ABCD$ is a parallelogram.

13 The quadrilateral $PQRS$ shown below has $PQ = RS$ and $PQ \parallel RS$.

See Example 12



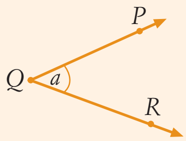
a Show that $\triangle PQR \equiv \triangle RSP$.

b Show that $PQRS$ is a parallelogram.

Chapter 6 summary

Quiz
Shapes and angles
MAT08MGQZ00006

- An angle is formed by two lines called the arms or sides of the angle with a common starting point or vertex. When naming an angle, we use the symbol $\angle$. The vertex is written as the middle letter of the angle name, so $\angle RQP$ has a vertex at Q and arms QR and QP. $\angle RQP$ can also be referred to as $\angle Q$ or a.



- There are 360° in a complete **turn** or a **full rotation**. A **half-turn**, also known as a **straight angle**, measures 180°. A quarter turn, known as a **right angle**, measures 90°.
- An **acute angle** is smaller than 90°. An **obtuse angle** is larger than 90° but less than 180°. A **reflex angle** is larger than 180° but less than 360°.
- We measure angles using a **protractor** marked in degrees.
- **Adjacent angles** are next to each other. They share a common arm and a common vertex. **Complementary angles** have a sum of 90°. **Supplementary angles** have a sum of 180°. **Angles at a point** (in a revolution) add to 360°. **Vertically opposite** angles are equal.
- **Parallel** lines are ones that do not intersect. They are indicated by matching arrows on the lines. If AB is parallel to CD we write this as $AB \parallel CD$. A **transversal** is a line that crosses parallel lines.
- **Alternate angles** on parallel lines are equal. **Corresponding angles** on parallel lines are equal. **Co-interior angles** on parallel lines are supplementary.

  The sum of the interior angles of a triangle is 180°.
- An **exterior angle** to a triangle is formed by extending any side of a triangle. The exterior angle of a triangle is equal to the sum of the interior opposite angles.
- Figures that can be exactly superimposed are said to be **congruent** and the points in the figures that coincide are called **corresponding**. The symbol $\equiv$ means congruent. When we write a statement of congruence, corresponding points must be written in the same order for each figure.
- The four tests for congruent triangles are:
  - **Side, Side, Side (SSS)** – Triangles are congruent if three corresponding sides are equal.
  - **Angle, Side, Angle (ASA)** – Triangles are congruent if two angles and the side they contain are equal.
  - **Side, Angle, Side (SAS)** – Triangles are congruent if two sides and the included angle are equal.
  - **Right angle, Hypotenuse, Side (RHS)** – Triangles are congruent if a right angle, the hypotenuse and a corresponding side are equal.
- The angles that lie inside a polygon are called **interior angles**.
- The sum of the interior angles of a polygon with n sides is given by:

$$A = 180(n - 2)°$$

  where n is the number of sides in the polygon.
- The opposite angles in a parallelogram are equal.
- A quadrilateral in which opposite angles are equal must be a parallelogram.

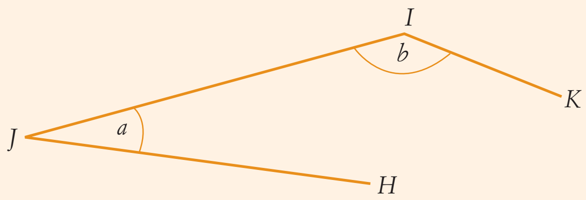
9780170194778

Understanding

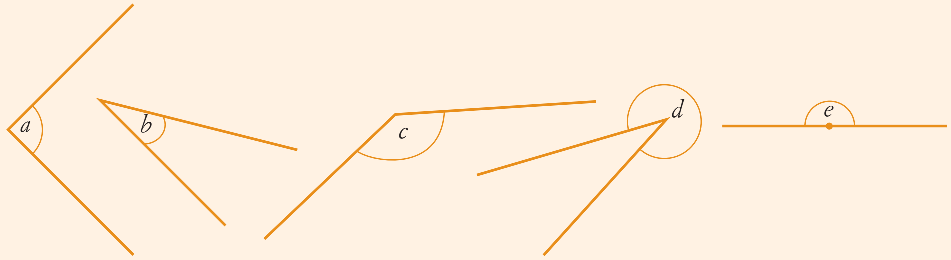
Worksheet
Angles review
MAT08MGWK00043
See Example 1

Weblink
Ask Dr Math
MAT08MGWB00006

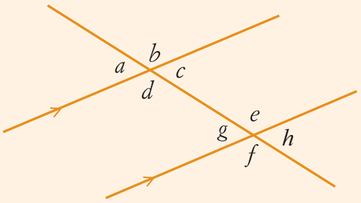
1 Name and measure the angles in this diagram.



2 Classify each of the following angles as acute, obtuse, straight, reflex or right.



3 Describe the following pairs of angles in the diagram on the right as either alternate, corresponding or co-interior.

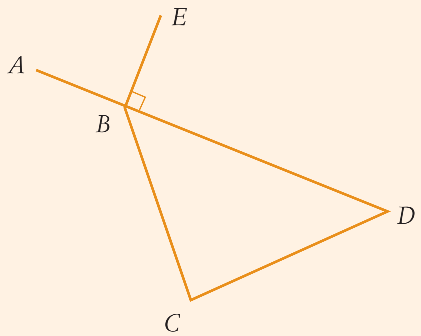


a b and e	**b** d and e	**c** d and f
d c and e	**e** a and h	**f** d and g

4 For the triangle below, name:

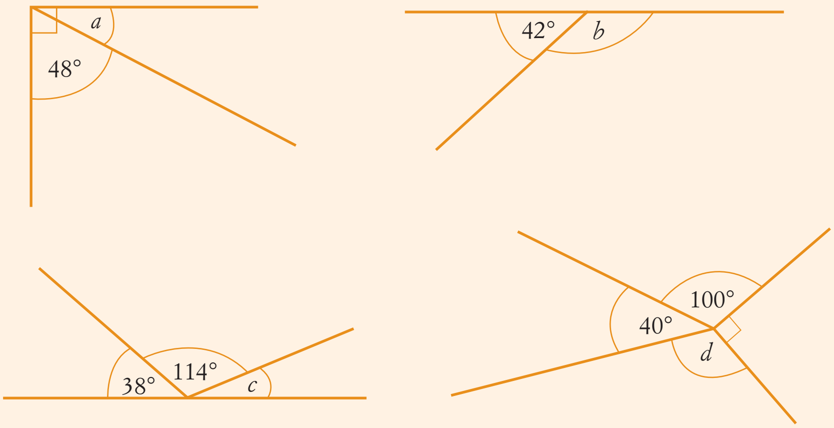
a the exterior angle

b the interior opposite angles to the exterior angle



See Example 2

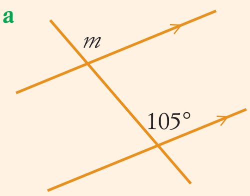
5 Find the unknown angles in each of the following without using a protractor.



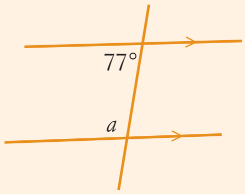
See Example 3

6 Find the unknown angle in each case below.

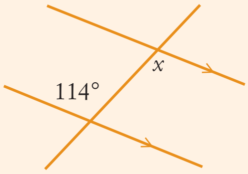
a



b



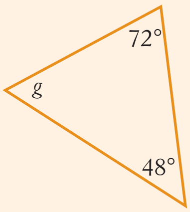
c



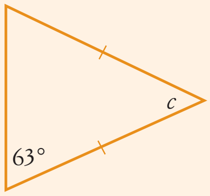
See Example 5

7 Find the unknown angle in each case below.

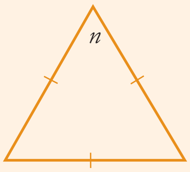
a



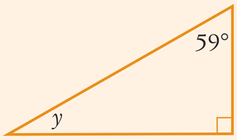
b



c



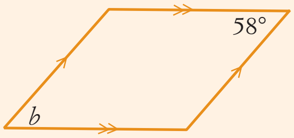
d



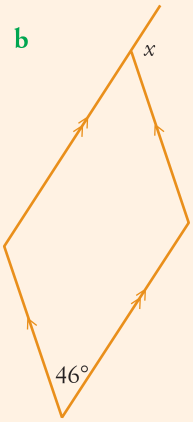
See Example 11

8 Calculate the value of the variable in each of the following.

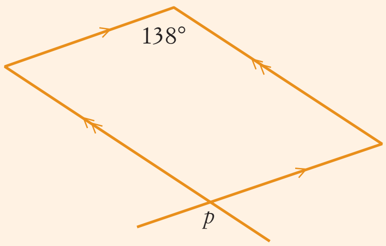
a



b



c



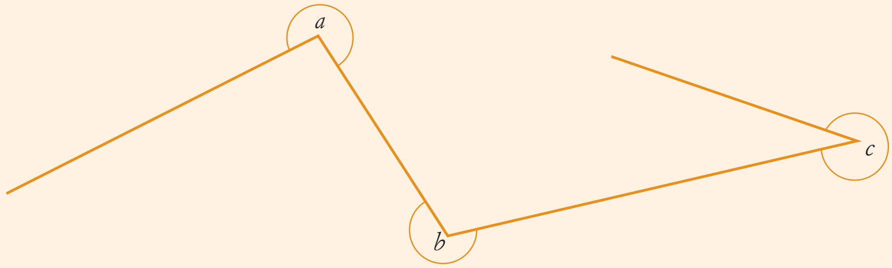
9780170194778

Fluency

9 Classify each of the following angles using their size.

a 41° **b** 143° **c** 180° **d** 280°

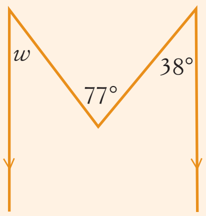
10 Use a protractor to measure the angles shown here.



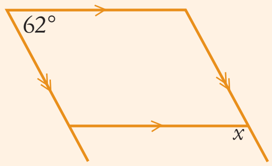
11 Find the unknown angle in each case below.

See Example 3

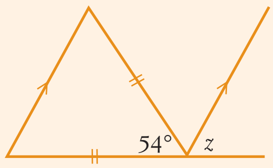
a



b



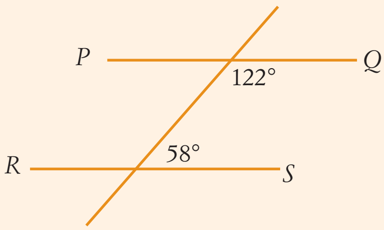
c



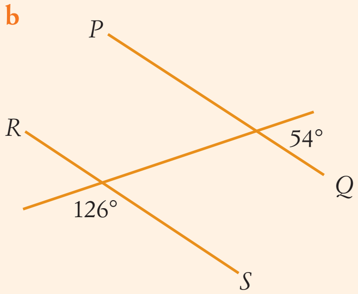
12 In each case below determine if $PQ \parallel RS$.

See Example 4

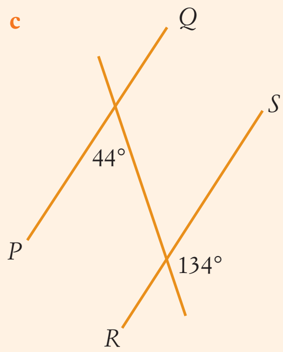
a



b

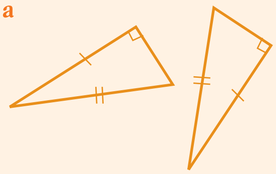


c

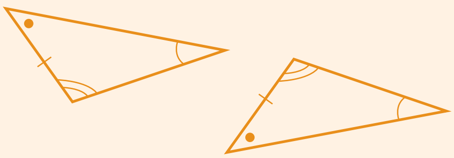


13 For each of the following, which test could be used to show that the pair of triangles is congruent? If no test is possible, state 'None'.

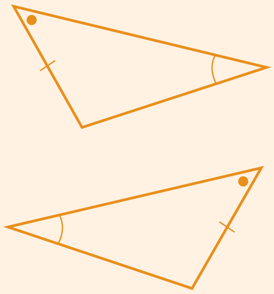
a



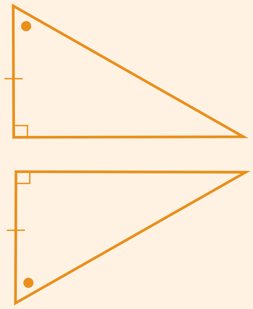
b



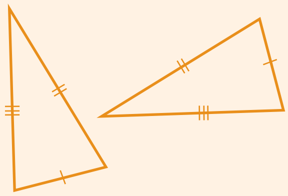
c



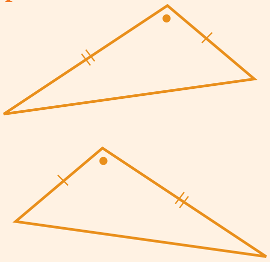
d



e

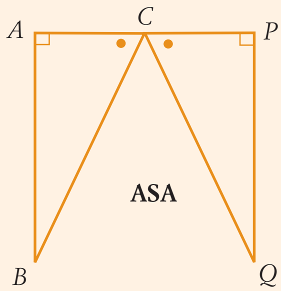


f

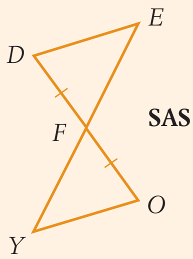


14 Name the additional corresponding side(s) or angle(s) needed to prove that the pairs of triangles shown below are congruent by the congruency test indicated.

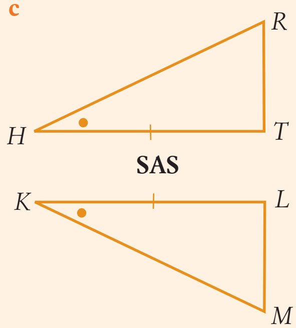
a



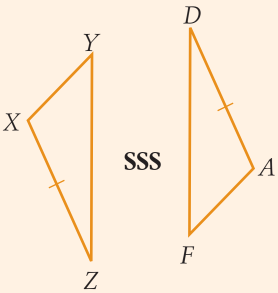
b



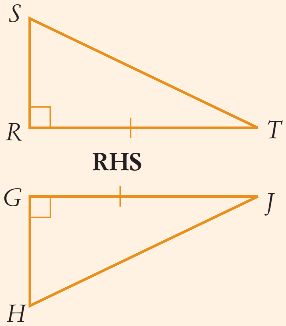
c



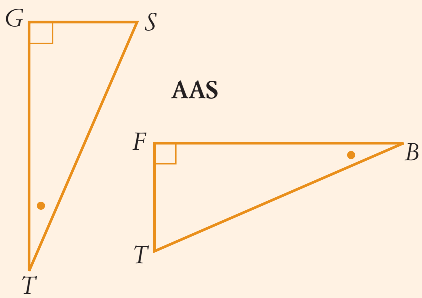
d



e



f

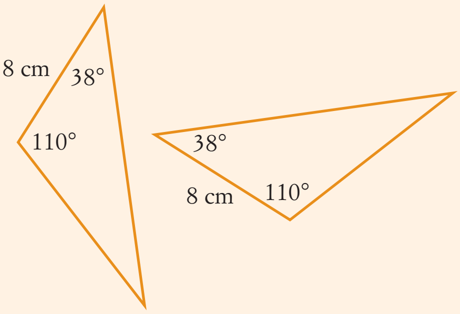


See Example 7

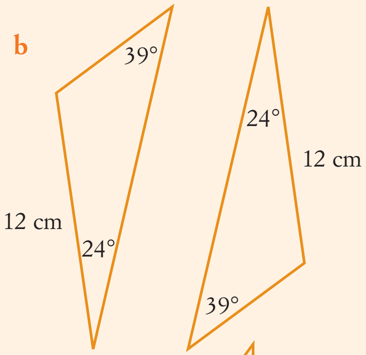
15 Determine whether each of the pairs of triangles at the top of the next page is congruent. If the triangles are congruent, state the test used.

9780170194778

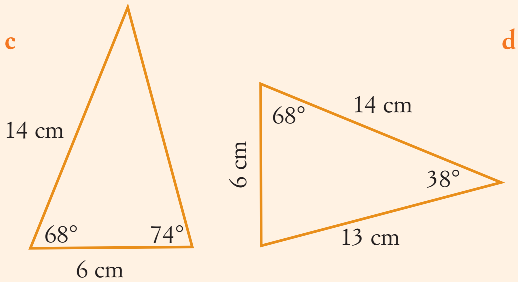
a



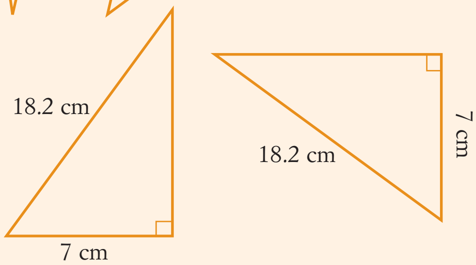
b



c



d



16 If $\triangle YVW \equiv \triangle DIG$ select from **A**, **B**, **C** and **D** below to complete the statement $VW = \ldots$ See Example 8

A DI **B** ID **C** DG **D** IG

17 If $\triangle FAD \equiv \triangle TSR$ select from **A**, **B**, **C** and **D** below to complete the statement $\angle R = \ldots$ See Example 8

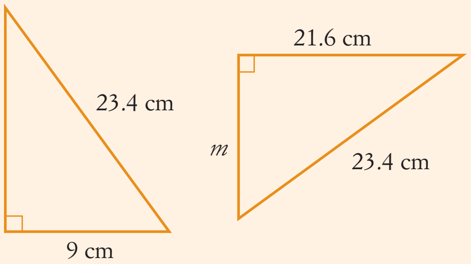
A $\angle A$ **B** $\angle D$ **C** $\angle F$ **D** $\angle S$

18 If $\triangle QJS \equiv \triangle UPH$ select from **A**, **B**, **C** and **D** below to complete the statement $\triangle JSQ = \ldots$ See Example 8

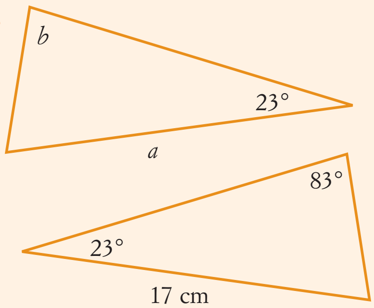
A $\triangle PHU$ **B** $\triangle HPU$ **C** $\triangle PUH$ **D** $\triangle HUP$

19 In each case below the pair of triangles is congruent. Find the value of the unknown sides and angles indicated. See Example 7

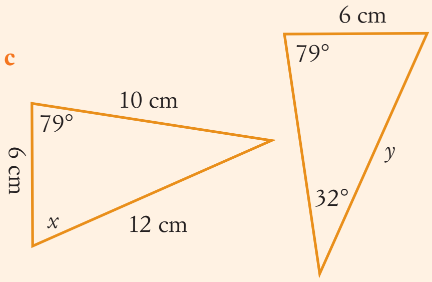
a



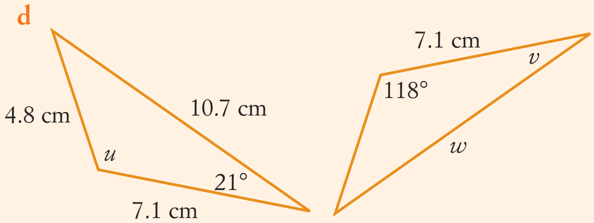
b



c

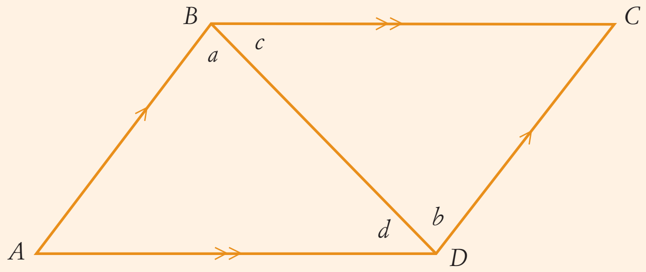


d



9780170194778

20 Use the information below to show that $\triangle ABD \equiv \triangle CDB$.

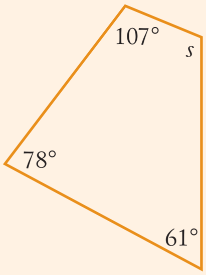


See Example 9

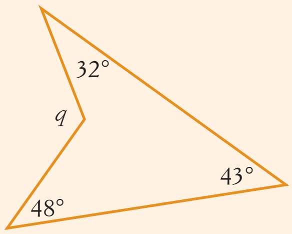
21 Find the unknown angle in each case below.

a

b



c



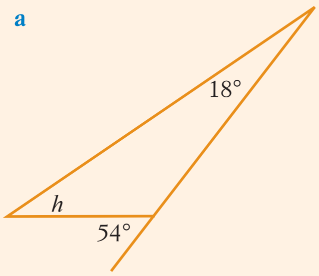
See Example 10

22 Calculate the sum of the internal angles of an octagon.

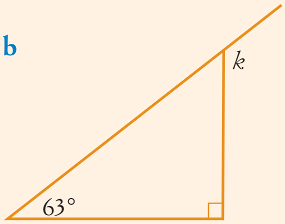
See Example 6

23 Find the unknown angle in each case below.

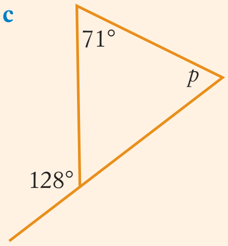
a



b

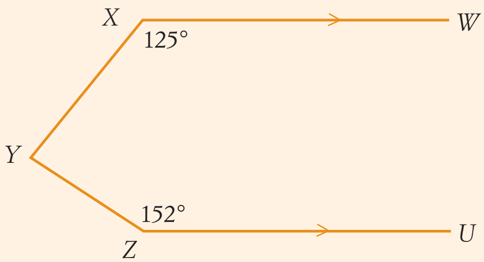


c



Problem solving

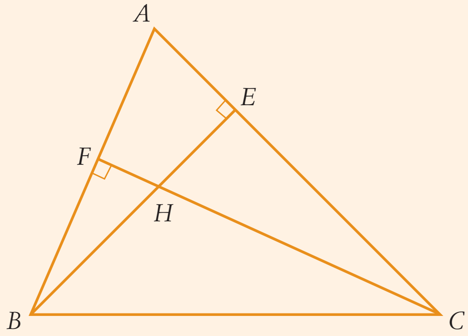
24 Calculate the size of $\angle XYZ$ in the diagram below without using a protractor.



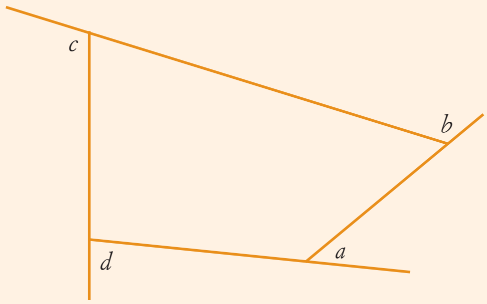
9780170194778

Reasoning

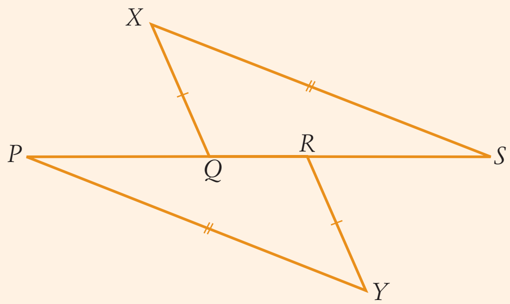
25 In the diagram below, prove that $\angle ECH = \angle FBH$.



26 The sides of a quadrilateral are extended as shown in the diagram below.
If $a + b = 180°$ show that $c + d = 180°$.

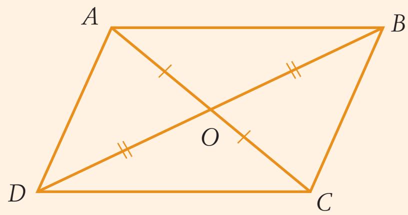


27 In the diagram below, if $PQ = RS$, prove that XQ is parallel to RY.

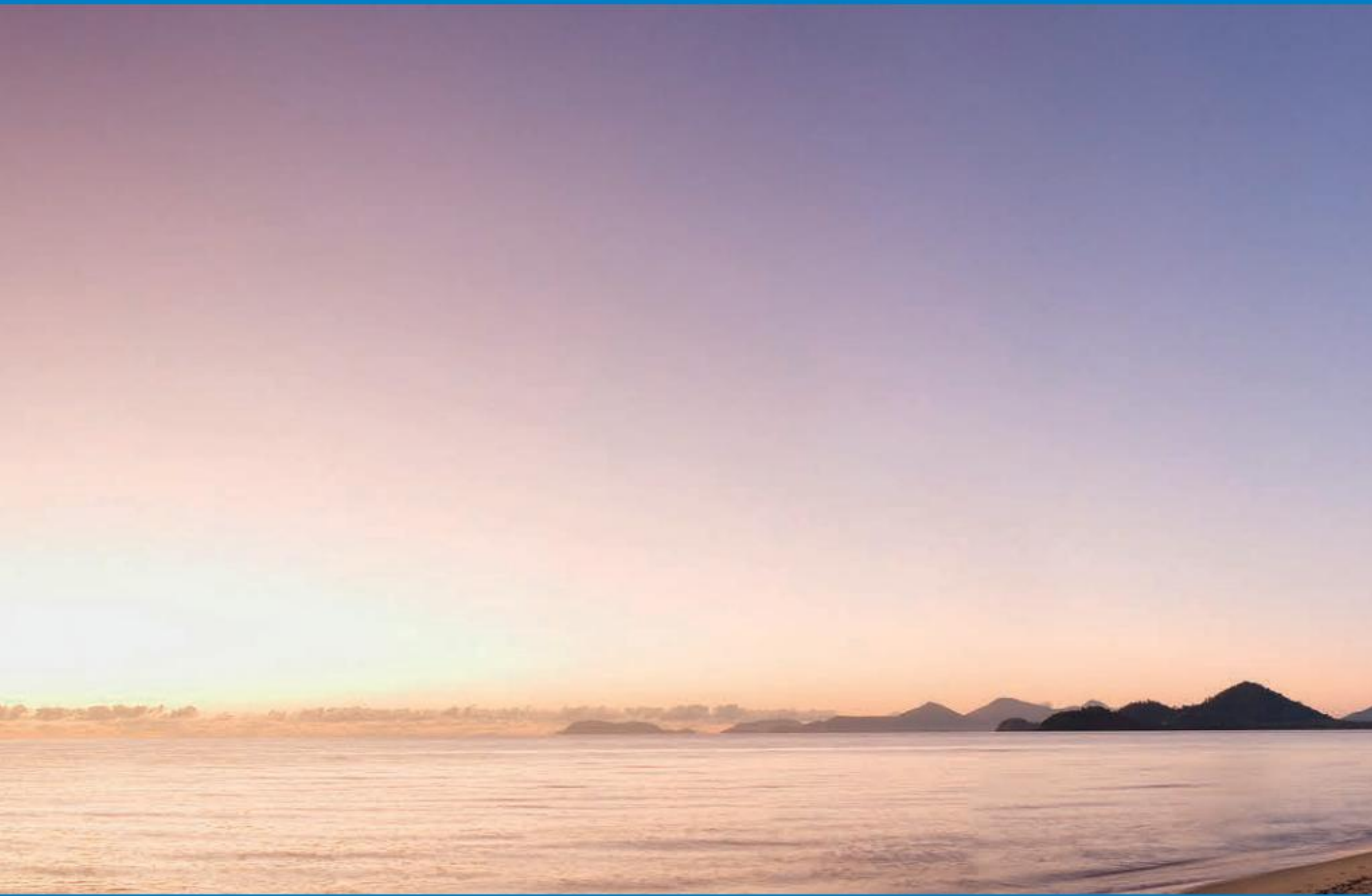


28 The quadrilateral $ABCD$ shown below has diagonals that intersect at O. If $AO = CO$ and $BO = DO$:

See Example 12



a show that $\triangle AOB \equiv \triangle COD$
b show that $ABCD$ is a parallelogram



Number and algebra

7 Applying ratios and rates

9780170194778

Contents

Prior learning
Chapter 7
MAT08NAPL00007

Parent guide
Chapter 7
MAT08NAPG00007

Curriculum guide
Chapter 7
MAT08NACU00007

Australian Curriculum statements

Real Numbers

Solve a range of problems involving rates and ratios, with and without digital technologies.

Maths clip
Why do we have ratios?
MAT08NAMC00004

At a fruit and vegetable shop, the prices are usually displayed as rates. The quantities of tomatoes that different customers buy may be different, but the price per kilogram is the same. Most products where the amount that we can buy varies are priced in this way. There are many other situations where a quantity may vary, but the rate or ratio relative to something else remains the same, such as scale drawings, electric currents, force and acceleration. We use rates, ratios and proportions to perform calculations in circumstances like these.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

cross-product	kilowatt	reduction	speed
enlargement	rate	scale	
equivalent	ratio	scale factor	

7.1 Rates

Video tutorial
Rates problems
MAT08NAVT10021

A **rate** compares related measurements. The measurements that are compared within a rate are usually referred to as **quantities**.

Important!

Rates

A **rate** is obtained by dividing one quantity by a different, related quantity. A rate can be written as a fraction.

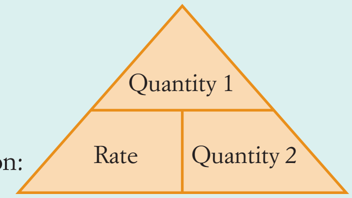
The measurement units for the quantities in a rate are separated by the word ‘per’ or the symbol /, with the second unit being the denominator.

Prices, pay rates and **speeds** are the most common rates.

For example, $\text{Average speed} = \dfrac{\text{distance}}{\text{time}}$

Rates can be used to calculate one quantity from the other.

A triangle of the rate can be used to remember the operation:



$$\text{Rate} = \frac{\text{quantity 1}}{\text{quantity 2}} \quad \textit{or} \quad \text{Quantity 1} = \text{rate} \times \text{quantity 2} \quad \textit{or} \quad \text{Quantity 2} = \frac{\text{quantity 1}}{\text{rate}}$$

Example 1

Teacher notes
Triangle method
MAT08NATN00013

Puzzle sheet
Rates problems
MAT08NAPS00027

Elena earned \$66.80 working for 5 hours.

a What was her rate of pay?

b What would she earn by working for $3\frac{1}{2}$ hours?

Solution

a Write the rate equation. $\textbf{Rate of pay} = \dfrac{\textbf{pay}}{\textbf{time}}$

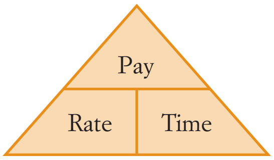
9780170194778

Substitute values. $= \dfrac{\$66.80}{5\text{ h}}$

Perform the division and write as a rate. $= \$13.36\text{/h}$

Write the answer. **Her rate of pay was $13.36/h.**

b Draw the triangle.



Write the equation. $\textbf{Pay} = \textbf{rate} \times \textbf{time}$

Substitute values. $= \$13.36 \times 3.5\text{ h}$

Perform the calculation. $= \$46.76$

Worksheet
Rates
MAT08NAWK00044

Example 2

A cyclist travels 46 km in 2 hours.

a What is the average speed?

b Change the speed to m/s.

Scientific calculator exercise
Applying ratios and rates
MAT08NASC00005

Puzzle sheet
Speed
MAT08NAPS00024

Solution

a Write the formula. $\text{Average speed} = \dfrac{\text{distance}}{\text{time}}$

Substitute values. $= \dfrac{46\text{ km}}{2\text{ h}}$

Perform the division and write the speed. $= 23\text{ km/h}$

b Write the speed as a division. $23\text{ km/h} = \dfrac{23\text{ km}}{1\text{ h}}$

Change to metres and seconds. $= \dfrac{23 \times 1000\text{ m}}{1 \times 60 \times 60\text{ s}}$

The 60s on the bottom are *both divided*, so you need to use brackets or divide both on your calculator.

Enter as: 23 [×] 1000 [÷] 60 [÷] 60 [=].

or 23 [×] 1000 [÷] [(] 60 [×] 60 [)] [=].

Use your calculator.

23×1000÷60÷60
6.388888889

$\approx 6.4\text{ m/s}$

Example 3

Change 15 m/s to km/h.

Solution

Write the speed as a division.	$\mathbf{15\ m/s} = \dfrac{\mathbf{15\ m}}{\mathbf{1\ s}}$
Change to kilometres and hours.	$= \dfrac{\mathbf{15 \div 1000\ km}}{\mathbf{1 \div 60 \div 60\ h}}$
Invert division to get multiplication.	$= \dfrac{\mathbf{15 \times 60 \times 60}}{\mathbf{1 \times 1000}}\ \mathbf{km/h}$
Use your calculator.	$= \mathbf{54\ km/h}$

Animated example
Rates: triangle method
MAT08NAAE00010

Worksheet
Speed
MAT08NAWK00045

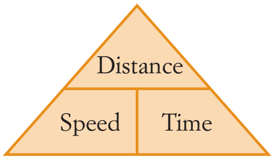
Example 4

A bus travels at a speed of 85 km/h.

a How far will the bus travel in 7 hours?
b How long will it take the bus to travel 380 km?

Solution

a Draw the triangle.



Write the equation.	$\mathbf{Distance} = \mathbf{speed} \times \mathbf{time}$
Substitute values.	$= \mathbf{85\ km/h \times 7\ h}$
Perform the calculation.	$= \mathbf{595\ km}$
b Use the triangle to write the equation.	$\mathbf{Time} = \dfrac{\mathbf{distance}}{\mathbf{speed}}$
Substitute values.	$= \dfrac{\mathbf{380\ km}}{\mathbf{85\ km/h}}$
Perform the calculation.	$\approx \mathbf{4.47\ h}$
Change to hours and minutes.	$\approx \mathbf{4\ h\ 28\ min}$

If you live in Western Australia, the residential **A1 Tariff** is used for electricity costs. If a business has an attached residence or if you run a business from home and there is only one meter, then the **K1 Tariff** is used. It includes an allowance of 20 units per day for the residential use. The **K1 Tariff** applies to many small factories, farms, churches, doctors and so on.

9780170194778

2011 Synergy A1 and K1 Tariff Charges (All prices are inclusive of GST)

K1 Charges	Cost	A1 Charges	Cost
Supply charge (cents) per day	40.14	Supply charge (cents) per day	40.14
Electricity charge (cents) per unit for:		Electricity charge (cents) per unit	21.87
First 20 units per day	21.87		
Between 21–1650 units per day	27.41		
More than 1650 units per day	24.75		

Source: http://www.synergy.net.au

Important!

Electrical Power Units

1 **kilowatt** (kW) = 1000 watts (W)
1 kilowatt hour (kWh) = 1 kilowatt for 1 hour

The amount of electricity used is measured in **kilowatt hours**. This is the amount of electrical energy used in 1 hour by an object rated at a power of 1000 W (watts). A toaster uses about 1000 W, but an electric drill only uses about half as much.

Example 5

How much electricity is used in 4 hours by an air-conditioner rated at 1800 W?

Solution

Change power rating to kilowatts.	**1800 W = 1.8 kW**
Multiply by the time.	**Amount used = 1.8 kW × 4 h**
Calculate the answer.	**= 7.2 kWh**
Write the answer.	**The air-conditioner uses 7.2 kWh of electricity.**

The K1 Tariff for electricity supply is an example of a stepped charge, where the rate changes with the amount. You need to divide this type of charge into sections.

Example 6

During a week in July a dentist who operates an office out of his home uses 1000 units/day for 5 days.
What is the electricity bill for 5 days?

Solution

Calculate the supply charge.	**0.4014 × 5 days = $ 2.01**
Calculate the first 20 units of each day.	**(20 × 0.2187) × 5 days = $21.87**
Calculate the remaining 980 (1000 – 20) units of each day.	**(980 × 0.2741) × 5 days = $1343.09**
You can use your calculator.	

Enter as:

5 (0.4014 + 20 × 0.2187 + (1000 − 20) × 0.2741) = .

1366.967

Total owed for five days. **\$2.01 + \$21.87 + \$1343.09 = \$1366.97**

Example 7

A dishwasher cycle takes 2 h 20 min and is rated at 2200 W. What is the average cost of the electricity to run the dishwasher?

Solution

Change the power rating to kilowatts. **2200 W = 2.2 kW**

Multiply by the time. **Amount used** $= 2.2\,\text{kW} \times 2\frac{1}{3}\,\text{h}$

≈ 5.1333 **kWh**

Use the home rate of 21.87 c/kWh. **Cost = \$0.2187 × 5.1333**

You can use your calculator.

Enter as: 2.2 × 2 a b/c 1 a b/c 3 × 0.2187 = .

1.12266

≈ **\$1.12**

Write the answer, ignoring the supply charge. **The electricity costs about \$1.12 per cycle.**

Investigate: Population densities

Australia has the third lowest population density of any country. The only other recognised countries with lower population densities are Namibia and Mongolia. What do Australia, Namibia and Mongolia have in common? The country with the highest population density in the world is the old city-state of Monaco in Europe. The second, third, fifth, sixth, eighth and ninth most populous countries by density are all island nations.

The table below shows the areas and populations of some south-east Asian countries.

Country	Population	Area (km^2)	Population density
Australia	22 662 184	7 682 300	
Bangladesh	162 221 000	143 998	
Cambodia	14 805 000	181 035	
China	1 343 070 000	9 640 821	
India	1 195 740 000	3 287 240	
Indonesia	237 556 363	1 904 569	
Japan	127 387 000	377 873	
Malaysia	28 306 700	329 847	
Mongolia	2 671 000	1 564 116	

9780170194778

Country	Population	Area (km^2)	Population density
Myanmar (Burma)	50 020 000	676 578	
New Zealand	4 315 800	270 534	
North Korea	24 051 706	120 538	
Pakistan	175 540 000	803 940	
Papua New Guinea	6 732 000	462 840	
Philippines	92 226 600	300 076	
Republic of China (Taiwan)	23 069 345	35 980	
Singapore	5 076 700	710.2	
South Korea	48 456 369	99 538	
Thailand	64 232 760	513 115	
Vietnam	85 789 573	331 689	

Which of the countries listed has the largest area? Which has the smallest area?
Which has the largest population? Which has the smallest population?
Calculate the population density for each country.
What do the high population density countries (apart from Singapore) have in common?
How are they different from Australia and Mongolia?

Exercise 7.1 Rates

Understanding

Extra questions
Exercise 7.1
MAT08NAEQ00023
See Example 1

1 Express the following as rates in simplest form.
  a Aldo earned \$264 by working for 10 hours.
  b Beatrice was paid \$174 for 6 hours of work.
  c A 3 kg bag of tomatoes cost \$4.65.
  d It takes 3 L of water to dissolve 200 g of solid pool chlorine.
  e A CD spun 480 revolutions in 5 seconds.

See Example 2

2 Find the average speed in each of the following cases.
  a A car travels 100 km in 2 hours.
  b A skateboard rider travels 120 m in 16 seconds.
  c A cyclist covers 480 km in 15 hours.
  d A greyhound runs 780 m in 40 seconds.
  e A cheetah runs 414 m in 12 seconds.
  f A jet flies 3150 km in 6 hours.

Fluency

Worked solutions
Exercise 7.1
MAT08NAWS00023
See Example 5

3 How much electricity is used by:
  a a small fridge rated at 150 W running for a day?
  b a stove rated at 11 000 W running for 3 hours?
  c an automatic washing machine rated at 900 W running for 4 hours?
  d a large air-conditioner rated at 2400 W running for 3 days?
  e a TV, stereo, computer and game console running on standby (each at a rating of only 10 W on standby) for a month?

4 Change the following speeds to m/s.
a 72 km/h b 96 km/h c 100 km/h
d 75 km/h e 450 km/h f 40 km/h

See Example 3
5 Change the following speeds to km/h.
a 48 m/s b 36 m/s c 100 m/s
d 65 m/s e 200 m/s f 8 m/s

See Example 6
6 Use the 2011 rates for WA from page 243 to find the total electricity bill for five days at Tariff K1.
a 18 units/day b 150 units/day c 1200 units/day
d 1550 units/day e 1850 units/day f 2100 units/day

See Example 7
7 Find the average cost of electricity to run:
a a 1000 W toaster for 1 hour a day for one week.
b a 1800 W kettle for $\frac{1}{2}$ an hour each day for a quarter.
c a 40 W light bulb for a whole quarter (24 hours a day).
d a vacuum cleaner rated at 600 W for 4 hours.
e a 300 W hair dryer for an hour a day for one week.

Problem solving

See Example 4
8 A motorcycle is travelling at 120 km/h. How far does it travel in:
a 1 hour? b 5 hours? c $\frac{1}{2}$ an hour?
d $\frac{1}{4}$ of an hour? e $3\frac{1}{2}$ hours? f $6\frac{1}{3}$ hours?

9 The price of potatoes is $1.15/kg. How much will each of the following masses of potatoes cost?
a 7 kg b 12 kg c 25 kg
d 2.5 kg e 3.72 kg f 4.16 kg

Worked solutions
Exercise 7.1
MAT08NAWS00023

10 The speed of sound is approximately 331 m/s. How far does sound travel in:
a 1 second? b 5 seconds? c 32 seconds?

11 What mass of the potatoes in question **9** could you buy for:
a $10.35 b $69? c $8.50?

12 Using the information from question **10**, calculate the time taken for sound to travel:
a 600 m b 3000 m c 8 km

13 Peter earns $6.72/h. How long would it take him to earn each of the following amounts, with no overtime loading?
a $47.04 b $30 c $200

14 The exchange rate for the Australian dollar (AUD) and the Euro (€) was given as €0.6235. This means that AUD1 was equal to €0.6235.
a Find the Euro value of:
i AUD240 ii AUD63.75 iii AUD800
b Find the Australian dollar value of:
i €60 ii €45.80 iii €700

Reasoning

15 A customer is deciding between two different 250 L hot water systems. One is rated at 2400 W and the other is rated at 3000 W. Using Tariff A1 from the table on page 243, calculate: (Do not include supply charge)
a how much it would cost to run each hot water system for 1 hour a day for 30 days

9780170194778

b how much money you would save per month using the 2400 W hot water system.

c Which one would you choose? Why?

16 A motorist averaged 60 km/h for the first half of a 120 km trip, and 40 km/h through roadworks for the last 60 km. What was his average speed? *Hint*: It is not 50 km/h.

Worked solutions
Exercise 7.1
MAT08NAWS00023

17 Freda the farmer can travel between her farm and the nearest town in 4 hours if her average speed is 80 km/h.

a How far is her farm from the town?

b Find the average speed she would need to complete the trip in 5 hours.

c If she averaged only 50 km/h, how long would the trip take?

d What is the shortest possible time in which Freda can legally complete the journey if the speed limit is 100 km/h?

Weblink
All about ratios
MAT08NAWB00007

18 In a 1000 m race, Jacqui finishes 100 m ahead of Lisa and 190 m in front of Fran. If Lisa and Fran continue to run at the same speed, how far in front of Fran will Lisa finish?

7.2 Ratios

If you compare quantities of the same kind, you obtain a **ratio**. The ratio order is important.

Important!

Ratios

A ratio compares quantities of the *same kind* in a definite order, such as 5 : 8 lollies.
To simplify a ratio, you must write the quantities in the *same units*, such as changing hours and minutes both to minutes.
A ratio is stated without units, using:

- the word 'to', as in 5 to 8 *or*
- the symbol :, as in 5 : 8 *or*
- fraction notation, with the second quantity as the denominator, as in $\frac{5}{8}$.

Ratios are normally expressed with whole numbers, but for some purposes they are expressed as decimals, with one part equal to 1, such as 1 : 1.6.

We normally write ratios in simplest form using whole numbers, in the same way as fractions are written in simplest form.

Example 8

Australia was visited by 82 596 tourists in August and 121 907 in November. There were 1 142 561 visitors in the year. Write the ratios of visitors for:

a August compared with November
b Year : November
c Change the ratios to the form . . . : 1.

Solution

a Write the ratio with August first.	**Aug : Nov = 82 596 : 121 907**
b Write the ratio with the year first.	**Year : Nov = 1 142 561 : 121 907**
c Divide each ratio by the second number.	**82 596 ÷ 121 907 : 1**
	≈ 0.68 : 1
	1 142 561 ÷ 121 907 : 1
	≈ 9.37 : 1

Example 9

Scientific calculator exercise
Applying ratios and rates
MAT08NASC00005

Puzzle sheet
Simplifying ratios
MAT08NAPS00025

Worksheet
Simplifying ratios
MAT08NAWK00046

Write each of the following as ratios in simplest form.

a 8 to 10 **b** 1.5 L : 500 mL **c** 0.09 : 1.5 **d** $4\frac{1}{2} : \frac{5}{6}$

Solution

a Write the ratio.	**8 to 10**
Write in fraction form.	$= \dfrac{\mathbf{8}}{\mathbf{10}}$
Cancel by 2.	$= \dfrac{\mathbf{8 \div 2}}{\mathbf{10 \div 2}} = \dfrac{\mathbf{4}}{\mathbf{5}}$
Write in ratio form.	**= 4 : 5**
b Write the ratio.	**1.5 L : 500 mL**
Write in the same units.	**= 1500 mL : 500 mL**
Cancel by 500.	**= 1500 ÷ 500 : 500 ÷ 500**
	= 3 : 1
c Write the ratio.	**0.09 : 1.5**
Multiply both by 100 to make whole numbers.	**= 9 : 150**
Cancel by 3.	**= 9 ÷ 3 : 150 ÷ 3**
	= 3 : 50

9780170194778

d Write the ratio.	$4\frac{1}{2} : \frac{5}{6}$
Write both as improper fractions.	$= \frac{9}{2} : \frac{5}{6}$
Multiply both by 6 to make whole numbers.	$= \frac{9}{2} \times 6 : \frac{5}{6} \times 6 = 27 : 5$
Alternatively, you could just multiply both parts of the ratio by the LCD (6) on your calculator.	
Enter as: 4 [a b/c] 1 [a b/c] 2 [×] 6 [=].	4 1/2×6 27
and 5 [÷] 6 [×] 6 [=].	5÷6×6 5

In cases that require a quantity to be divided into shares in a particular ratio, it is simplest to use a **unitary method**, finding the value of one part of the total parts of a ratio.

Important!

Unitary method

A unitary method involves two main steps:

1. Calculate the value of a single part of the ratio.
2. Multiply this value by the number of parts to calculate the desired quantity.

Example 10

Share $42 in the ratio 3 : 4.

Solution

Work out the total number of parts.	**Total parts** $= 3 + 4$ $= 7$
Calculate the amount for one part.	**Value of one part** $= \$42 \div 7$ $= \$6$
Calculate the first share.	**First share** $= 3$ **parts** $= 3 \times \$6$ $= \$18$
Calculate the second share.	**Second share** $= 4$ **parts** $= 4 \times \$6$ $= \$24$
Check the total.	**Total amount** $= \$18 + \$24$ $= \$42$ ✓ **OK**
Write the answer.	**The amount is shared as $18 : $24.**

Animated example
Shares in ratios
MAT08NAAE00011

Scientific calculator exercise
Applying ratios and rates
MAT08NASC00005

Technology worksheet
Excel worksheet: Applying ratios
MAT08NACT00009

Video tutorial
Dividing a quantity in a given ratio
MAT08NAVT10020

The unitary method can also be used for ratios with more than two shares.

Worksheet
Ratio applications
MAT08NAWK00047

TLF learning object
In proportion: ratios (L8098)
MAT08NAIN00007

TLF learning object
In proportion: variables in ratios (L8101)
MAT08NAIN00007

Example 11

A jar containing 450 g of curry powder has the spices chilli, coriander and turmeric in the ratio 2 : 4 : 3. How much of each spice is in the mixture?

Solution

Work out the total number of parts.	**Total parts** $= 2 + 4 + 3$ $= 9$
Calculate the amount for one part.	**Value of one part** $= 450 \div 9$ $= 50$ g
Calculate the chilli.	**Chilli** $= 2$ parts $= 2 \times 50$ g $= 100$ g
Calculate the coriander.	**Coriander** $= 4$ parts $= 4 \times 50$ g $= 200$ g
Calculate the turmeric.	**Turmeric** $= 3$ parts $= 3 \times 50$ g $= 150$ g
Check the total.	**Total amount** $= 100 + 200 + 150$ g $= 450$ g ✓ OK
Write the answer.	**The curry powder consists of 100 g of chilli, 200 g of coriander and 150 g of turmeric.**

Two jellybean makers both claimed to have 'more black beans' than any other brand. Over the years, the makers argued about which one had more black beans. To settle the row, they decided to each open one of their packets and do a count.

Beanie Beans — 48 black and 64 others	Bongo Beans — 39 black and 52 others
Black : total	Black : total
$= 48 : (48 + 64)$	$= 39 : (39 + 52)$
$= 48 : 112$	$= 39 : 91$
$= 48 \div 16 : 112 \div 16$	$= 39 \div 13 : 91 \div 13$
$= 3 : 7$	$= 3 : 7$

The ratios are the same! So *neither* could rightly claim to have more black beans.

Important!

Equivalent ratios

Ratios that are the same are called **equivalent ratios**. Ratios can be shown to be **equivalent** by:

- multiplying or dividing each part of one ratio by the same number to get the other *or*
- cancelling both to the simplest form *or*
- **cross-multiplying** to get the same answer.

9780170194778

Example 12

Determine whether the following pairs of ratios are equivalent.

a 15 : 45 and 12 : 36 **b** $\frac{1}{3} : \frac{1}{2}$ and $\frac{5}{3} : \frac{7}{10}$

Solution

a Write the first ratio.	**15 : 45**
Cancel 15 : 45 to simplest form.	$= \mathbf{15 \div 15 : 45 \div 15 = 1 : 3}$
Write the second ratio.	**12 : 36**
Cancel 12 : 36 to simplest form.	$= \mathbf{12 \div 12 : 36 \div 12 = 1 : 3}$
Compare the simplest forms and write the answer.	**The ratios are equivalent because they are both equivalent to 1 : 3.**
b Write the first ratio.	$\mathbf{\frac{1}{3} : \frac{1}{2}}$
Multiply both by 6 to make whole numbers.	$= \mathbf{\frac{1}{3} \times 6 : \frac{1}{2} \times 6}$
This ratio is in simplest form.	$= \mathbf{2 : 3}$
Write the second ratio.	$\mathbf{\frac{5}{8} : \frac{7}{10}}$
Multiply both by 40 and cancel to make whole numbers.	$= \mathbf{\frac{5}{\cancel{8}_1} \times \cancel{40}^5 : \frac{7}{\cancel{10}_1} \times \cancel{40}^4}$
This ratio is in simplest form.	$= \mathbf{25 : 28}$
Compare the simplest forms and write the answer.	**The ratios are not equivalent because their simplest forms are different.**

Example 13

Use cross-multiplication to determine whether the following pairs of ratios are equivalent.

a 6 : 8 and 9 : 15 **b** 91 : 52 and 49 : 28

Solution

a Write the ratios in fraction form.	$\mathbf{\frac{6}{8}}$ **and** $\mathbf{\frac{9}{15}}$
Cross-multiply.	$\mathbf{\frac{6}{8} \quad \frac{9}{15}}$ $\mathbf{6 \times 15 = 90}$ **and** $\mathbf{8 \times 9 = 72}$
Compare the **cross-products** and write the answer.	**The cross-products are different, so the ratios are not equivalent.**
b Write the ratios in fraction form.	$\mathbf{\frac{91}{52}}$ **and** $\mathbf{\frac{49}{28}}$
Cross-multiply.	$\mathbf{\frac{91}{52} \quad \frac{49}{28}}$ $\mathbf{91 \times 28 = 2548}$ **and** $\mathbf{52 \times 49 = 2548}$
Compare the cross-products and write the answer.	**The cross-products are equal, so the ratios are equivalent.**

9780170194778

Example 14

Which is the larger ratio, 3 : 7 or 5 : 12?

Solution

Change 3 : 7 to the form . . . : 1.	**3 : 7**
	$= 3 \div 7 : 7 \div 7 \approx 0.429 : 1$
Change 5 : 12 to the form . . . : 1.	**5 : 12**
	$= 5 \div 12 : 12 \div 12 \approx 0.417 : 1$
Compare the ratio values.	**$0.429 > 0.417$**
Write the answer.	**3 : 7 is larger than 5 : 12.**

Investigate: Newspaper ratios

Obtain copies of two different daily newspapers. Make sure that they are for the same day of the week.

1. Compare the newspapers by doing the following things.
   - Find the number of pages in each paper.
   - Measure the length and width of the pages of each paper and calculate the area of one page for each.
   - Measure the length and width of the columns of each paper and calculate the area of a column in each paper.
2. Now use areas or columns to determine the space each newspaper devotes to different features as follows. Compare the spaces by finding the ratio of the feature to the total for each newspaper.
   - Which newspaper uses more space for news headlines?
   - Which newspaper uses more space for news?
   - Which newspaper uses more space for cartoons?
   - Which newspaper uses more space for advertisements?
   - Which newspaper uses more space for sport?
3. Use your results to decide which is the better newspaper for a family.

Sometimes different quantities are related by the same ratios. For example, if you put more apples in a bag, then the price goes up in the same ratio as the weight (number) of apples. Increasing the speed of a conveyer belt decreases the time taken to transfer the same amount of material. Problems of this nature can be done using equivalent ratios. Take care in putting them the right way round!

9780170194778

Example 15

A pile of sand takes 2 men 36 minutes to barrow from the front to the back of a house. How long would it have taken 3 men?

Solution

The time to move the sand is the opposite ratio to the number of men. Write this as equivalent ratios.	$\dfrac{\text{1st number of men}}{\text{2nd number of men}} = \dfrac{\text{2nd time}}{\text{1st time}}$
Put in the numbers.	$\dfrac{2}{3} = \dfrac{\text{2nd time}}{36}$
Cross-multiply the ratios.	**2nd time × 3 = 2 × 36 = 72 minutes**
Divide by 3.	**2nd time = 72 ÷ 3 = 24 minutes**
Write the answer.	**3 men would have taken 24 minutes.**

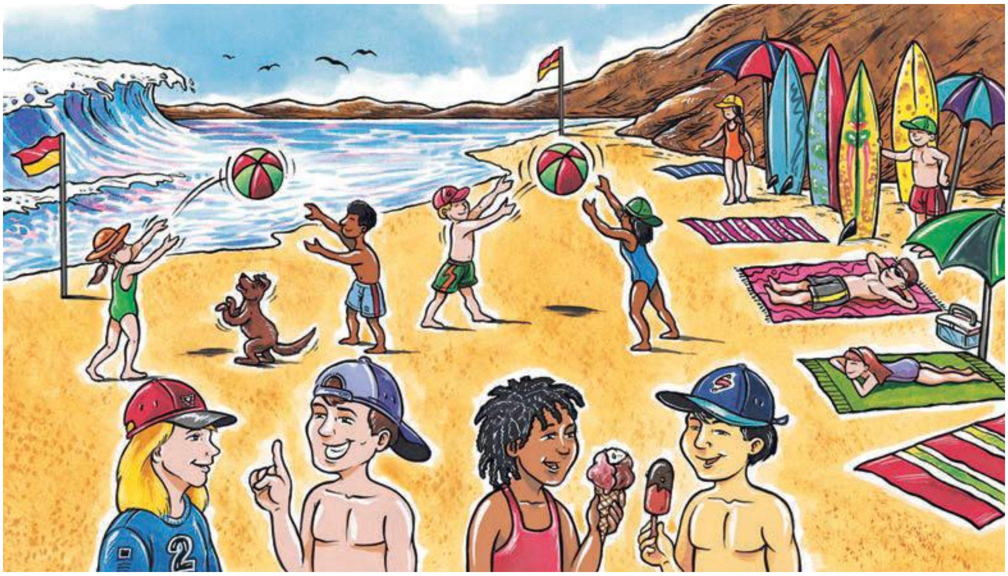
Exercise 7.2 Ratios

Understanding

Extra questions
Exercise 7.2
MAT08NAEQ00024

See Example 8

1 The drawing below shows a beach scene. Find the ratios below, simplifying if necessary.



a Beach balls to surfboards
b Boys : surfboards
c Beach balls to girls
d Boys with hats : girls
e Girls to boys with hats
f Boys with hats to girls with hats
g Girls with hats to boys with hats
h Beach towels to umbrellas

Worked solutions
Exercise 7.2
MAT08NAWS00024

See Example 9

2 Write the following as ratios, simplifying if necessary.
- a A class of 10 boys and 15 girls
- b A box of chocolates with 25 hard centres and 15 soft
- c The angle sum of a rectangle to that of a triangle
- d An angle of 30° compared with a right angle
- e The days in a week to the days in February (non-leap year)
- f A chessboard with 12 white and 8 black pieces

See Example 9

3 Write the following ratios in simplest form.

a 8 : 16	b 12 : 6	c 6 : 30
d 12 : 14	e 15 : 24	f 44 : 33
g 24 : 32	h 80 : 25	i 35 : 42

Fluency

See Example 9

4 Write the following ratios in simplest form.

a 0.5 : 2	b 1.2 : 0.3	c 2.4 : 0.4
d 3.5 : 7.5	e 0.001 : 0.1	f 1.2 : 6
g 1.6 : 0.04	h 2.5 : 4.5	i 10 : 2.25

See Example 9

5 Write the following ratios in simplest form, changing units as necessary.

a 50c : \$2	b 4 m : 150 cm	c 750 mL : 2.5 L
d 2 days : 1 week	e 2.4 km : 900 m	f \$1.20 : 80c
g 3.6 L : 27 mL	h 4.5 min : 180 s	i 350 kg : 4.2 t

See Example 9

6 Write the following ratios in simplest form.

a $\frac{1}{2} : 2$	b $1 : \frac{1}{4}$	c $\frac{3}{4} : \frac{1}{2}$
d $\frac{7}{9} : \frac{2}{3}$	e $1\frac{1}{4} : \frac{5}{8}$	f $\frac{2}{3} : 2\frac{1}{6}$
g $1\frac{1}{2} : \frac{9}{10}$	h $\frac{4}{5} : 2\frac{3}{10}$	i $4\frac{1}{2} : 2\frac{1}{2}$

See Example 10

7 Write the following ratios in the form . . . : 1, rounding to 1 decimal place if necessary.

a 150 : 50	b 240 : 48	c 112 : 560
d 1245 : 231	e 24 980 : 4790	f 3291 : 15 289

See Example 10

8 Write the following ratios in the form 1 : . . ., rounding to 1 decimal place if necessary.

a 24 : 36	b 45 : 81	c 124 : 93
d 80 : 188	e 345 : 287	f 4380 : 23 670

See Example 10

9 For each of the following, find the total number of parts and the value of 1 part.
- a 20 pets are cats and dogs in the ratio 2 : 3.
- b 100 g of cat food has meat and fish in the ratio 4 : 6.
- c 2800 sports fans support the Reds and the Blues in the ratio 11 : 9.
- d 56 000 viewers watch commercial channels and the ABC in the ratio 6 : 1.
- e An 800 g jumper contains wool and acrylic in the ratio 4 : 1.
- f 108 minutes of a television show consist of programs and commercials in the ratio 7 : 2

See Example 10

10 Share the following amounts as stated.

a \$24 in the ratio 1 : 2	b 81 kg in the ratio 4 : 5
c 124 mL in the ratio 1 : 3	d 84 cm in the ratio 5 : 2

9780170194778

11 Determine whether each of the following pairs of ratios are equivalent. See Example 12

a 5 : 6 and 15 : 18 b 24 : 16 and 7 : 5
c 20 : 32 and 12 : 18 d \$2 to 450c and 20 : 45
e 0.2 kg to 45 g and 2 L to 450 mL

12 Use cross-multiplication to determine whether these pairs of ratios are equivalent. See Example 13

a 24 : 7 and 36 : 10.5 b 43 : 26 and 24 : 15
c 4.8 : 12 and 14.4 : 36 d 27 : 24 and 135 : 120
e 15 : 27 and 25 : 45

13 For the following map measurements, find the real measurements.

a 5 cm at 1 : 100 b 4.6 cm at 1 : 1000 c 3.5 cm at 1 : 200
d 3 cm at 1 : 100 000 e 53 mm at 1 : 20 000

14 If scaled drawings are made for the following real measurements, what size should they be?

a 200 cm at 1 : 200 b 5 km at 1 : 100 000 c 18 km at 1 : 200 000
d 2400 m at 1 : 10 000 e 5.2 m at 1 : 100

15 Share the following amounts as stated. See Example 11

a 245 g in the ratio 1 : 2 : 4 b 4180 in the ratio 3 : 2 : 6

16 Use cross-multiplication to find the unknowns in the following ratios.

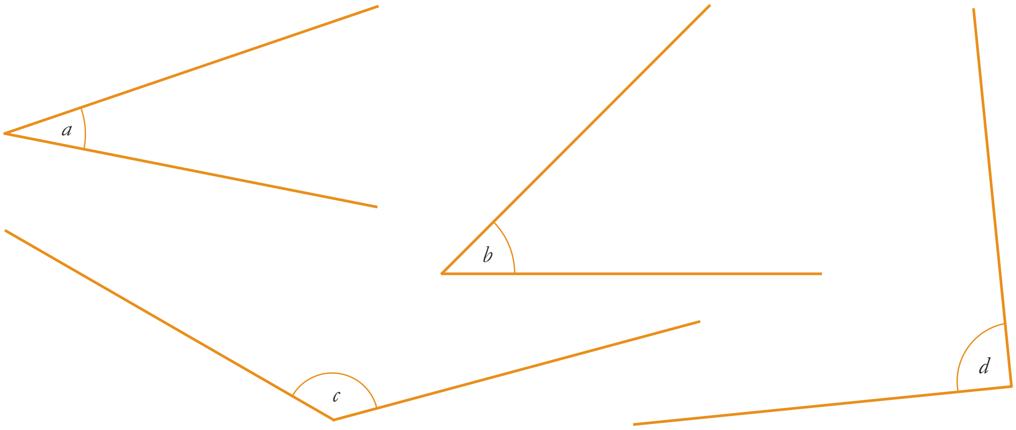
a $a : 5 = 12 : 15$ b $9 : 12 = 27 : m$ c $24 : 33 = x : 11$
d $\frac{p}{10} = \frac{1.2}{4}$ e $\frac{3.6}{d} = \frac{9}{2}$ f $\frac{f}{6.5} = \frac{1.5}{2.5}$

17 Insert >, < or = to make true statements. See Example 14

a $2 : 3 \boxed{} 12 : 18$ b $20 : 14 \boxed{} 11 : 7$ c $42 : 16 \boxed{} 22 : 8$
d $15 : 9 \boxed{} 60 : 36$ e $27 : 55 \boxed{} 36 : 73$ f $\frac{1}{2} : 5 \boxed{} \frac{2}{5} : 11$

Problem solving

18 Measure the angles below, then use your measurements to complete the table.



Pair	a, b	a, c	a, d	c, b	c, d	b, d
Ratio of angles						

9780170194778

19 Andrea and Frank bought a $20 lottery ticket and won the major prize of $1 000 000. Andrea put in $11 for the ticket. How should they share the prize?

Worked solutions
Exercise 7.2
MAT08NAWS00024

20 There are 42 students in the cast of the school musical. If the ratio of boys to girls is 3 : 4, how many girls are in the cast?

See Example 11

21 The flying distance from Brisbane to Hobart is 1800 km. Sydney divides the distance in the ratio 5 : 7. How far is Sydney from:

a Brisbane? b Hobart?

See Example 11

22 In a certain novel, the ratio of vowels to consonants on a page is 5 : 11. If there are 3024 letters on the page, how many of them are:

a vowels? b consonants?

See Example 11

23 Concrete is made using sand, gravel and cement in the ratio 2 : 4 : 1. If 112 shovels of ingredients are mixed for the slab of a garden shed, how much of each is used?

See Example 11

24 The Dragons and the Scorpions played their annual 'grudge' match of softball last week. After three innings, the Dragons scored runs in the ratio 3 : 1 : 2 for a total of 18 runs.

a How many runs did they score in the second innings?

b If the Scorpions scored runs in the ratio 1 : 1 : 4, with 4 runs in the second innings, which team is winning and by how much?

Reasoning

See Example 15

25 A super-flip is made with the following ingredients: 1 large egg, 3 teaspoons of sugar, 4 drops of vanilla and 1 cup of milk.

a How much vanilla is needed for 6 super-flips?

b If 12 teaspoons of sugar are needed, how many flips are being made?

See Example 15

26 When programmed in a certain way, a computer spends 30% of its time accepting (and processing) input data, 40% of its time doing calculations and the remainder printing output data. The program is rewritten to do the calculations twice as fast to reduce the overall computer time used. What would be the new percentages for input, calculations and output? (*Hint*: Think about what happens to the total time.)

Worked solutions
Exercise 7.2
MAT08NAWS00024
See Example 15

27 When 30 cattle are put in a paddock, it takes 5 days for the feed to be consumed. How long would the feed in the same paddock last if only 25 cattle were in it?

See Example 15

28 A lifeboat has enough provisions to last a group of 10 people for 4 days on full rations. How long would the provisions last a group of 12 people on half rations?

9780170194778

7.3 Practical rates and ratios

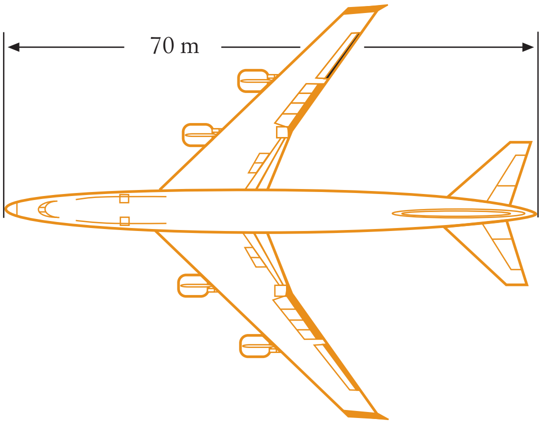
Puzzle sheet
Scale enlargement
MAT08NAPS00026

The word *scale* has many meanings. Earlier in this chapter, scale was used to refer to the markings on a measuring device. A **scale drawing** of an object is a copy that has the same shape and proportions as the real object. A scale drawing is *similar* to the outline of the object, so it is an *enlargement* or *reduction*. The length on the drawing is divided by the corresponding real length to obtain the **scale**. A scale of $\frac{1}{1000}$ is usually written as 1 : 1000.

Scale drawings are used by builders, designers, artists, scientists and engineers.
Here is a photo of an ant.
The head and body of the real ant is 4 mm long.
In the photo, the ant is 40 mm long.

$$\begin{aligned}\text{Scale} &= \frac{\text{photo length}}{\text{real length}}\\ &= 40\text{ mm} \div 4\text{ mm}\\ &= 10 : 1\end{aligned}$$

This means that the **scale factor** is 10. Therefore 10 mm on the drawing represents 1 mm on the actual object.
This photo is an **enlargement** of the actual size.
Here is a scale drawing of a plane.
The real plane is 70 m long.
In the drawing the plane is 70 mm long.



$$\begin{aligned}\text{Scale} &= \frac{\text{drawing length}}{\text{real length}}\\ &= \frac{70\text{ mm}}{70\text{ m}}\\ &= \frac{70\text{ mm}}{70\,000\text{ m}}\\ &= \frac{1}{1000}\\ &= 1 : 1000\end{aligned}$$

This means that the scale factor is $\frac{1}{1000}$.
Therefore 1 mm on the drawing represents 1000 mm (1 m) on the actual object.
This scale drawing is a **reduction** of the actual size.

Important!

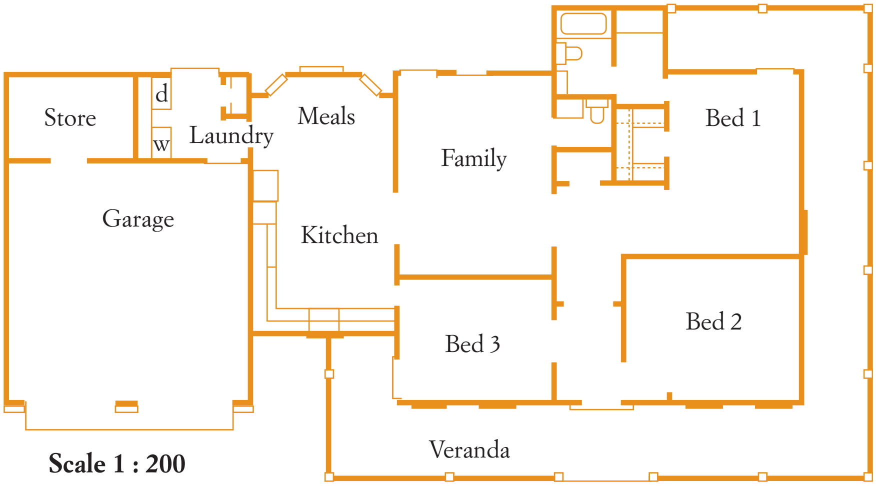
Scales

The standard way of writing a **scale** is:
Length in drawing : Length in real object

Example 16

Look at the floor plan below of a house.

a How long is the house in the scale drawing?
b How long is the real house (including the garage and veranda)?
c How many bedrooms are shown in the scale drawing?
d How many bedrooms are in the real house?
e Which is the smallest bedroom shown in the scale drawing?
f Which is the smallest bedroom in the real house?



Solution

a Measure the length. — **The house is 11.5 cm long in the drawing.**

b The drawing scale is 1 : 200. This means that 1 cm in the drawing = 200 cm in the house.

Multiply the drawing length by 200. — **Length of house = 11.5 cm × 200**
= 2300 cm

Convert to metres. — **= 23 m**

c Look at the drawing. — **There are 3 bedrooms in the drawing.**

d The drawing truly represents reality. — **There are 3 bedrooms in the real house.**

e Measure the bedrooms in the drawing. — **Bedroom 3 is the smallest in the drawing.**

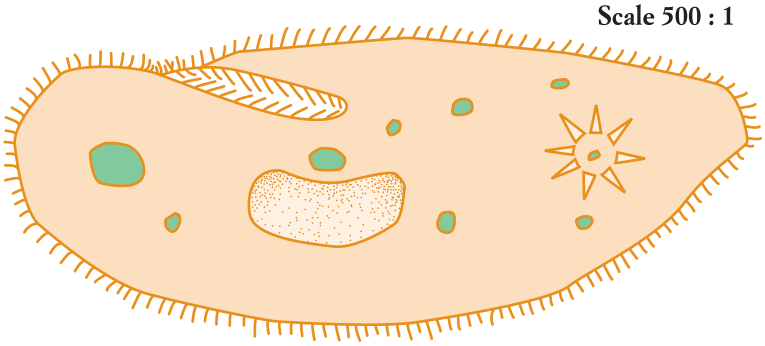
f The drawing is a true representation. — **Bedroom 3 is the smallest in the real house.**

9780170194778

Example 17

This is a scale drawing of a freshwater microorganism called a *Paramecium*.

Scale 500 : 1



a How long is the *Paramecium* in the scale drawing?
b How long is the real *Paramecium*?

The tip of a ballpoint pen is about 0.6 mm wide.
c How wide would such a pen tip be at the same scale?
d Would a marble, a cricket ball or a basketball be the best object to show the size of a pen tip using this scale?

Solution

a Measure the length. — **The *Paramecium* is 10 cm long in the drawing.**

b The scale is 500 : 1. — **500 mm in the drawing = 1 mm in the *Paramecium***

Divide length by 500. — **Length of *Paramecium* = 10 cm ÷ 500**

Convert to mm. — **= 100 mm ÷ 500**

Calculate. — **= 0.2 mm**

c Write the scale. — **Drawing width : Real width = 500 : 1**

This means that the width is 500 times greater in the drawing.

Pen tip measurement × 500. — **Pen tip in drawing = 0.6 mm × 500**

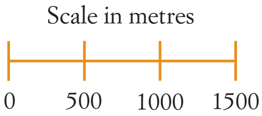
= 300 mm

Convert to cm. — **= 30 cm**

d Compare the diameters. — **The basketball would be the best object to show the size.**

9780170194778

Map scales can be written in many ways. For example, look at this scale.

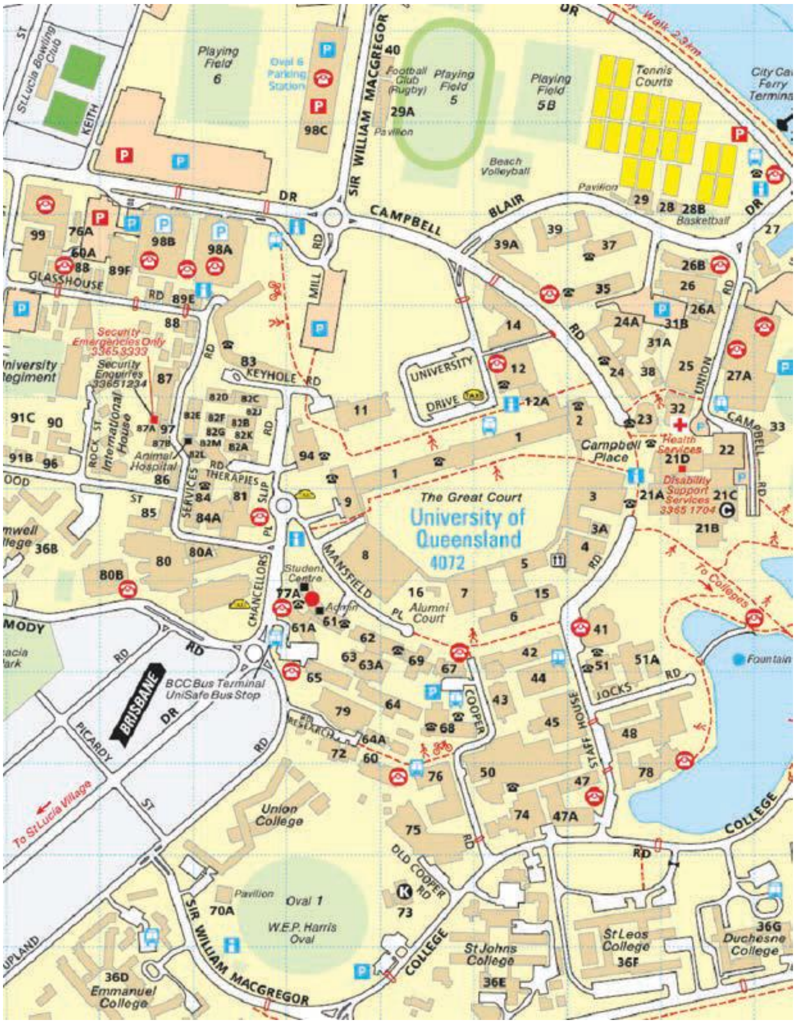


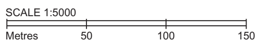
This scale shows that 3 cm on the map represents 1500 m in real life. This could be written as:

	3 cm : 1500 m
or	3 cm : 150 000 cm
or	1 cm : 50 000 cm
or simply	1 : 50 000.

Example 18

The following map shows a part of the St Lucia campus of the University of Queensland.





a How many metres does 5 cm on the map represent?
b How far is it between the centres of WEP Harris Oval and Field No. 5B?

9780170194778

Solution

a Write the scale. **1 : 5000**

Write the scale for 5 cm on the map. **5 cm : 25 000 cm**

Convert the real measurement to metres. **25 000 cm = 250 m**

b Measure the map distance. **Map distance = 10.8 cm**

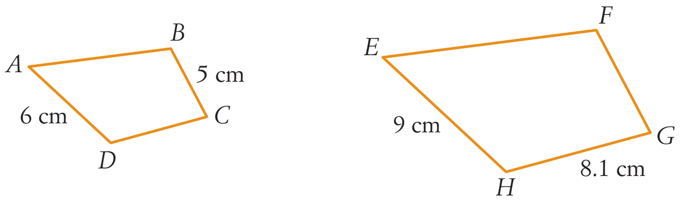
Multiply by 5000. **Real distance = 54 000 cm**

Convert to metres. **= 540 m**

You can use the scale of similar drawings to find unknown sides, just as you use the scale of a scale drawing or map.

Example 19

The quadrilaterals shown below are similar.



a Find the scale that relates *ABCD* to *EFGH*.
b Find the length of *FG*.
c Find the length of *CD*.

Solution

a Identify known corresponding sides. ***EH* corresponds to *AD*.**

Use them to find the scale factor. $\text{Scale factor} = \frac{EH}{AD}$

$$= \frac{9}{6} = 1.5 \text{ or } \frac{3}{2}$$

Write the answer. **The scale factor is 1.5. *EFGH* is an enlargement of *ABCD* by 1.5 times.**

b Identify the side corresponding to *FG*. ***FG* corresponds to *BC*.**

Relate using the scale factor.

$$FG = 1.5 \times BC$$
$$= 1.5 \times 5 \text{ cm}$$
$$= 7.5 \text{ cm}$$

c Identify the side corresponding to *CD*. ***CD* corresponds to *GH*.**

Relate using the scale factor. $GH = 1.5 \times CD$

Divide by 1.5 to find *CD*. $CD = \frac{GH}{1.5}$

Substitute the value of *GH*. $= \frac{8.1}{1.5}$

Calculate the answer. $= 5.4 \text{ cm}$

Exercise 7.3 Practical rates and ratios

Understanding

Extra questions
Exercise 7.3
MAT08NAEQ00025
See Example 16

1 The plans of a house are drawn to a scale of 1 : 100. Find the length in metres for the following measurements on the plan.

- **a** width of the back door, 1 cm
- **b** length of the family room, 5 cm
- **c** length of the hall cupboard, 2 cm
- **d** width of the hallway, 2 cm
- **e** height of the window, 1.5 cm
- **f** length of the front wall, 22.3 cm
- **g** length of the kitchen bench, 3.84 cm
- **h** width of the kitchen, 3.65 cm
- **i** width of the garage, 4.75 cm
- **j** length of the second bedroom, 3.35 cm

Fluency

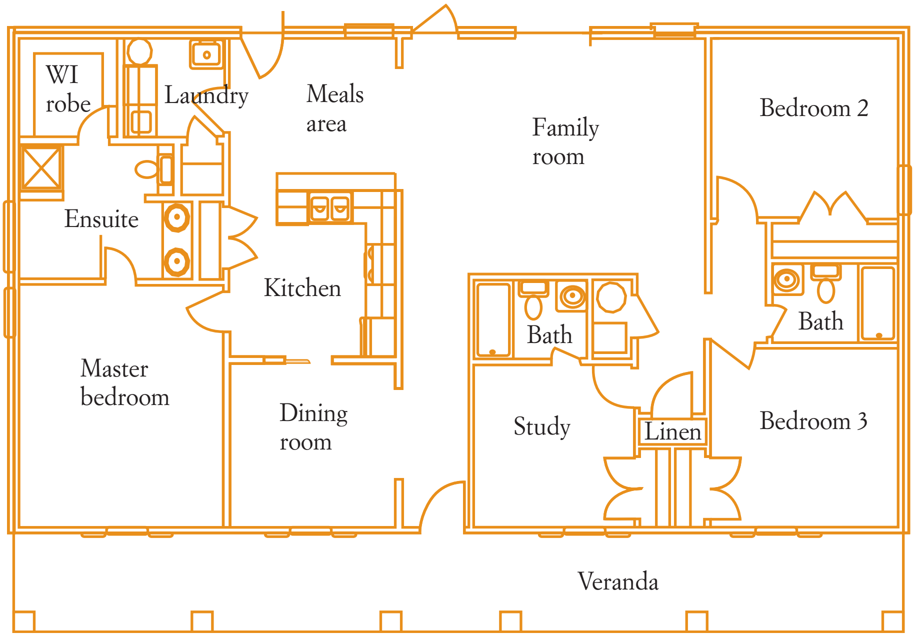
See Example 16

2 For the house plan shown in Example **16**, find the real-life dimensions (length × width) of the following rooms and write your answers in metres.

- **a** bedroom 2
- **b** the garage
- **c** the family room
- **d** the kitchen

Worked solutions
Exercise 7.3
MAT08NAWS00025
See Example 16

3 The house plan below is drawn to a scale of 1 : 150.



- **a** Find the dimensions of the veranda.
- **b** Find the dimensions of the master bedroom.
- **c** What is the length of the house (without the veranda)?
- **d** What is the overall width of the meals area and family room?
- **e** What are the dimensions of the walk-in wardrobe (WI robe) in the master bedroom?

9780170194778

4 A tourist map uses a scale of 1 : 250 000. What are the real lengths in kilometres for the following distances measured on the map?

a 4 cm **b** 7 cm **c** 15 cm
d 18 cm **e** 9.5 cm **f** 10.4 cm

5 An orienteering map uses a scale of 1 : 25 000. What are the real lengths in metres for the following distances measured on the map?

Worked solutions
Exercise 7.3
MAT08NAWS00025

a 4 cm **b** 2 cm **c** 7 cm
d 9 cm **e** 10.5 cm **f** 6.4 cm

6 Using an orienteering map with a scale of 1 : 25 000, find the length on the map (in cm) for the following real distances.

a 2.5 km **b** 5 km **c** 7 km
d 4.5 km **e** 8 km **f** 6.4 km

7 Find the real lengths for the following distances as measured on a map with the scale shown.

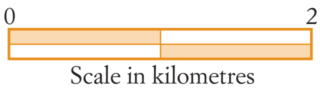
a 5 cm (1 : 100) **b** 8 cm (1 : 1000) **c** 9 cm (1 : 250)
d 14 cm (1 : 150) **e** 7.2 cm (1 : 12 500) **f** 12.8 cm (1 : 25 000)

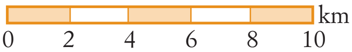
8 For each map scale shown, find the real-life distance equal to a map distance of 12 cm.

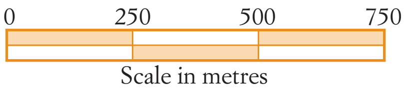
a


b

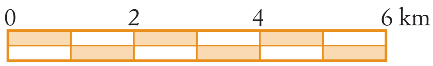

c


d


e


f


g


h


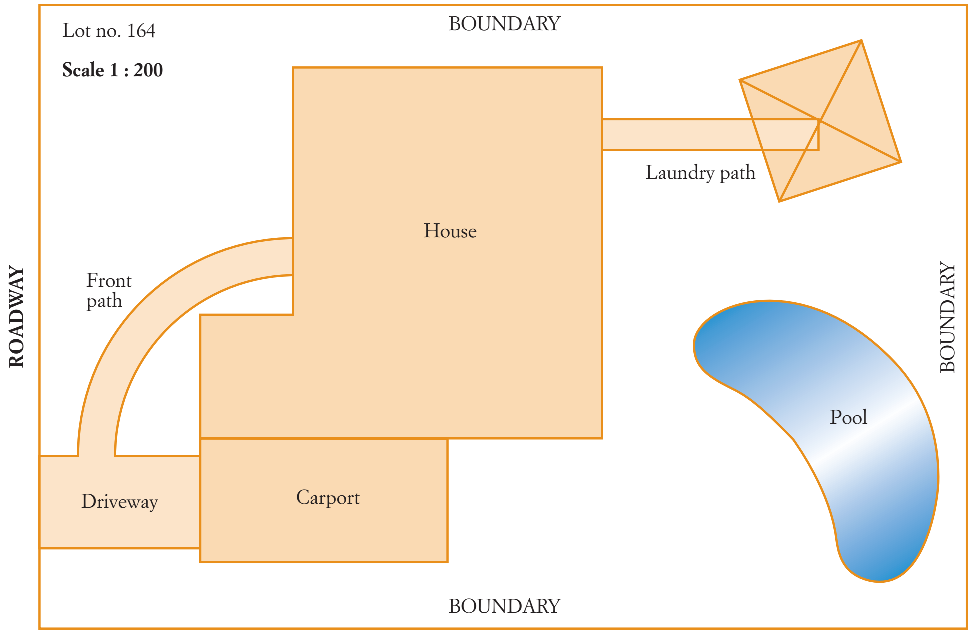
9 Refer to the map of the University of Queensland shown in Example **18** to answer the following questions.

See Example 18

a What are the actual dimensions of Playing Field 5B?
b How far is it from the pavilion next to the tennis courts to the WEP Harris pavilion?
c How wide is the Great Court?
d How far is it from St Johns College to Union College?
e Approximately how long is Staff House Road?

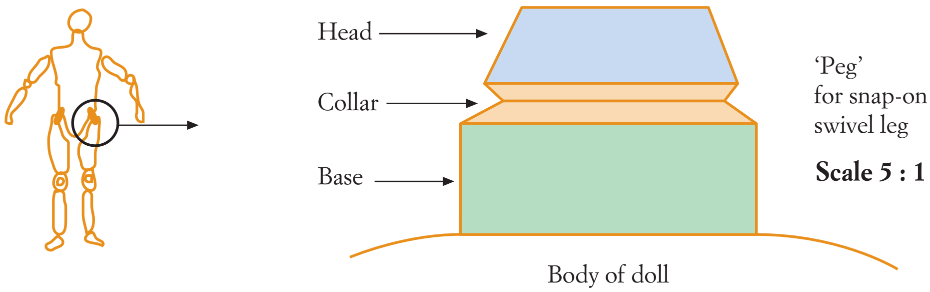
Problem solving

10 Here is a site plan of a house on a suburban block of ground.



a What are the frontage and the depth of the block on the drawing?
b What are the real frontage and depth of the block?
c What are the width and the length of the carport on the drawing?
d What are the real width and length of the carport?
e What is the real width of the driveway?
f What are the real length and width of the laundry path?
g What are the real length and width of the house (excluding the carport)?
h What is the real width of the front path?

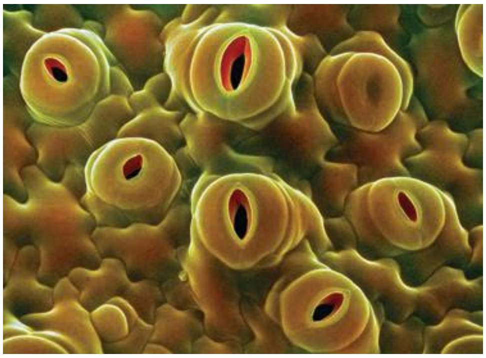
11 This is a detailed drawing of the peg on a doll's body that the leg fits onto.



a What is the length (mm) of the drawing of the peg base?
b What is the real length of the peg base?
c What is the real length of the collar?
d What is the real length of the top of the peg head?
e How much does the peg stick out from the doll's body?
f How much narrower is the collar than the base?

9780170194778

12 This is an image of the surface of a leaf taken using an electron microscope. The scale is 350 : 1.



a What does 1 cm on the photograph represent in reality?

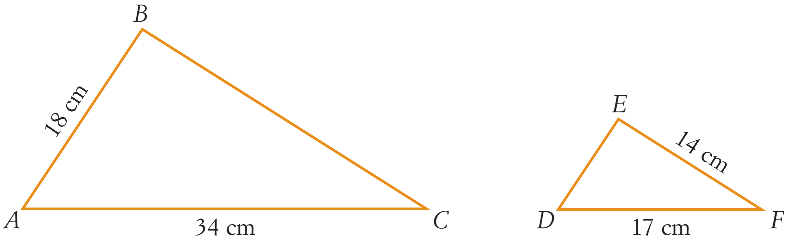
b How far apart are the centres of the two circular structures (called stomata) at the bottom of the photograph?

c What is the real diameter of the topmost circular structure?

Reasoning

See Example 19

13 The triangles shown below are similar.



a Find the scale that relates *ABC* to *DEF*.

b Find the length of *DE*.

c Find the length of *BC*.

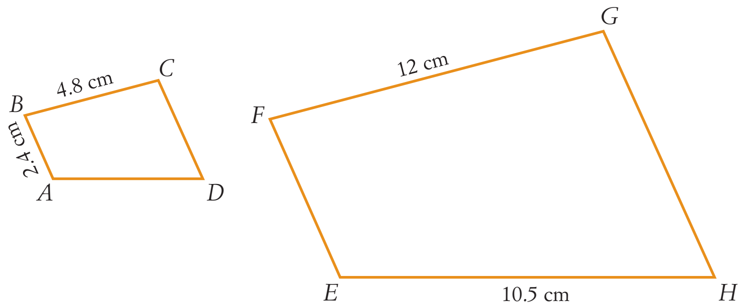
Worked solutions

Exercise 7.3

MAT08NAWS00025

See Example 19

14 The quadrilaterals shown below are similar.



a Find the scale that relates *ABCD* to *EFGH*.

b Find the length of *EF*.

c Find the length of *AD*.

15 Use measuring tapes and rulers to make a scale drawing of your classroom. A scale of 1 : 10 or 1 : 20 would probably be best.

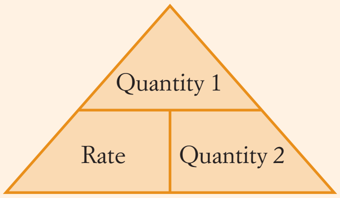
Chapter 7 summary

Quiz
Applying ratios and rates
MAT08NAQZ00007

- A **rate** is obtained by dividing one quantity by another. The units of measurement of a rate are separated by the word 'per' or the symbol /. The second quantity is the denominator. Average speed is a rate.

$$\text{Average Speed} = \frac{\text{distance}}{\text{time}}$$

- A triangle of a rate can be used to remember how to obtain one quantity from the other.



- A **ratio** compares two quantities of the same kind in a definite order. It is stated without units using the word 'to' or the symbol : between the parts.
- A ratio may be shown in fraction notation, which sometimes helps to simplify the ratio. Ratios are usually shown with whole numbers, but can be written as 1 : . . . or . . . : 1.
- A **unitary method** can be used to help divide a quantity into a particular ratio. The value of one part of the ratio is found first before finding the value of each share.
- Equivalent ratios are equal. They can be shown to be equal by **cross-multiplying**.
- A scale is a ratio given in the order **size of drawing : size of real thing**. The **scale factor** n is obtained by writing the scale in the form 1 : n.

For scale drawings: Real size = scale factor × drawing size

- The amount of electricity you use is measured in **kilowatt hours** (**kWh**). It is the amount used in 1 hour by something rated at a power of 1000 W.
- A **scale drawing** of an object is a copy that has the same shape and proportions as the real object. The standard way of writing a scale is:

Length in drawing : Length in real object

9780170194778

Understanding

1 Express 30 L in 4 hours as a rate in simplest form.

See Example 1

2 Find the average speed when a car travels 420 km in 6 hours.

Fluency

See Example 5

3 How much electricity is used by a 3600 W welder running for 5 hours?

See Example 2

4 Change 48 km/h to m/s.

See Example 9

5 Write each of the following in simplest form.

a 14 to 35 b 1 kg : 250 g c 28 cm : 42 mm

d 0.4 : 2 e $\frac{3}{5} : \frac{9}{10}$ f $3\frac{3}{4} : 1\frac{7}{8}$

Worksheet
Ratios and rates review
MAT08NAWK00049

See Example 8

6 Write 28 : 2050 in the given form.

a 1 : . . . b . . . : 1

See Example 12

7 Determine whether the following pairs of ratios are equivalent.

a 6 : 15 and 9 : 24 b 8 : 22 and 20 : 55

See Example 10

8 Share $60 in the ratio 7 : 8.

See Example 18

9 A map is drawn at a scale of 1 : 10 000. What is the real distance for a measurement of 2.4 cm on the map?

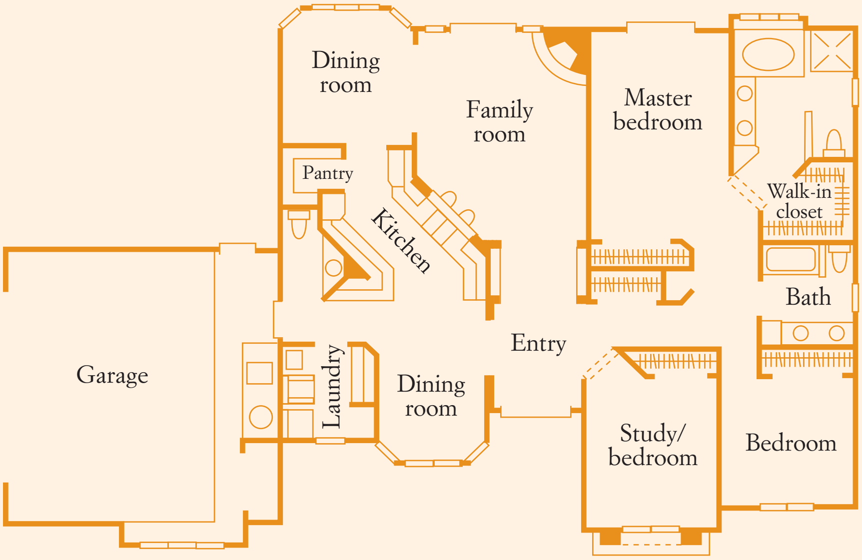
See Example 18

10 The plans of a house are drawn to a scale of 1 : 100. Find the length in metres for the following measurements on the plan.

a length of the lounge room, 5 cm
b length of the front wall, 23.4 cm
c width of the front door, 1.8 cm
d width of the garage door, 2.4 cm

See Example 18

11 The house plan below is drawn to a scale of 1 : 200.



a Find the dimensions of the garage.
b Find the dimensions of the study.
c What is the length of the house (including the garage)?
d What is the overall width of the house along the wall furthest from the garage?
e What are the dimensions of the entry?

12 An orienteering map uses a scale of 1 : 25 000. What are the real lengths in metres for the following distances measured on the map?

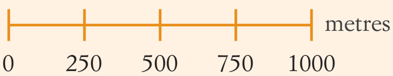
a 4 cm   **b** 7 cm   **c** 11.4 cm

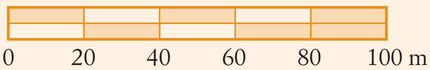
13 Using an orienteering map with a scale of 1 : 25 000, find the length on the map (in cm) for the following real distances.

a 3 km   **b** 5 km   **c** 4.5 km

14 For each map scale shown, find the real-life distance equal to a map distance of 8 cm.

a


b


c


d


15 Change 40 m/s to km/h.

9780170194778

16 Use the 2011 charges shown to find the total electricity bill for a WA customer on the K1 Tariff who used 1450 units/day for 7 days. *See* Example 6
The price of electricity in Western Australia is as follows.

2011 Synergy K1 Tariff Charges (All prices are inclusive of GST)

K1 Charges	**Cost**
Supply charge (cents) per day	40.14
Electricity charge (cents) per unit for:	
First 20 units per day	21.87
Between 21–1650 units per day	27.41
More than 1650 units per day	24.75

Source: http://www.synergy.net.au

17 Insert >, < or = to make true statements.

a 5 : 8 $\square$ 7 : 13 **b** 50 : 3 $\square$ 106 : 7 **c** 42 : 16 $\square$ 22 : 8

18 Share $180 in the ratio 2 : 3 : 4.

Problem solving

19 A snail moves at 3.5 cm/s. How far does it travel in 14 seconds?

20 Bananas cost $1.75/kg. What mass of bananas could you buy for $10? *See* Example 4

21 David and Phan share deliveries of advertising literature in their suburb. Last week they were paid $15. David covered 120 houses and Phan covered 180 houses. How much should Phan get? *See* Example 4

Reasoning

22 This is a photo of an ant. The ant is actually 6.1 mm long. *See* Example 17
a What is the scale?
b What does 1 cm on the photograph represent in reality?
c What is the real distance across the ant's abdomen?

23 The average mass of a watermelon is 3.4 kg. At the Rocklea markets, watermelon is sold in lots of 40 kg or more for $0.58/kg. A fruit shop bought watermelons and applied a mark-up of 80%. How many watermelons could you buy from the shop for $20?

24 An industrial dishwasher is rated at 4500 W and runs for 12 hours a day in a local restaurant. Use the electrical charges shown in question **12** to calculate the amount of electricity the dishwasher uses in a 30-day period. How much would this cost?

25 A recipe uses 300 g of self-raising flour, 150 mL of milk, 125 g of sugar, 4 eggs and 30 g of butter to make a sponge cake that weighs 500 g after cooking. What amounts of each ingredient should be used to make an 800 g cake?

Statistics and probability

8 Probability

9780170194778

Contents

Prior learning
Chapter 8
MAT08SPPL00008

Parent guide
Chapter 8
MAT08SPPG00008

Curriculum guide
Chapter 8
MAT08SPCU00008

Australian Curriculum statements

Chance

Identify complementary events and use the sum of probabilities to solve problems.

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.

Represent such events in two-way tables and Venn diagrams and solve related problems.

Maths clip
Using probability
MAT08SPMC00005

There are endless situations that involve making decisions. Your ability to make an informed decision can be assisted by accurately estimating the probability or likelihood of future events taking place.
Decisions involving probability are made by everyone – from the business person examining a new business opportunity, to the scientist testing a new treatment for a disease, to the office worker deciding whether to take an umbrella to work on a particular day.
Politicians are another group of people who have a keen interest in the likelihood of events – especially at election times or when certain issues become 'hot' topics for debate. They often look at the results of opinion polls to help with their decision making.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

and	event	likelihood	theoretical probability
certain	expected frequency	likely outcome	tree diagram
chance	experiment	or	trial
complement	experimental probability	outcome	two-step probability
complementary event	favourable outcome	possible	two-way table
die/dice	fifty-fifty chance	probability	unlikely
element	impossible	random	Venn diagram
equally likely		sample point	
even chance		sample space	
		simulation	

8.1 Basic probability

Weblink
Real life probability
MAT08SPWB00008

When you consider what could happen in the future, you make judgements about the **chance** or **likelihood** of particular things occurring.
People often have a 'feeling' that something will happen. Some things appear to be certain while others appear to have little or no chance of occurring. In mathematics, there are terms that can be used to describe the chance of something happening.

Important!

Likelihood

Something that cannot happen is called **impossible**.
Something that might or might not occur is **possible**. The **possible outcomes** (**events**) of a situation are all the things that could happen.
Something that must occur is called a **certain** event.
Something that is just as likely to occur as not occur is called an **even chance** or **fifty-fifty chance**.
When we compare possible events, we say that the event with a greater chance of occurring is the **more likely** event and the other is the **less likely** event.
Possible events that have the same chance of occurring are called **equally likely** events.

9780170194778

Example 1

A buttered piece of bread falls on the floor. Compare the chances of the possible outcomes.

Solution

List what can happen.	**The possible outcomes are that the toast lands buttered-side down or buttered-side up.**
Compare the chances.	**The chances of landing buttered-side up or buttered side down are really the same.**
Write the answer.	**When a piece of toast falls on the floor, it is equally likely to land buttered-side up or buttered-side down.**

Investigate: Certain and impossible events

Work in groups of 4 or 5 for this investigation.
There are a number of sayings about the certainty or impossibility of events.
Benjamin Franklin (1706–1790), one of the Founding Fathers of the United States, claimed that he wrote the following in a letter to a friend: 'Our Constitution is in actual operation. Everything appears to promise that it will last; but in this world nothing is certain but death and taxes.' This has given rise to the now famous saying 'Nothing is certain but death and taxes'.

- Discuss the certainty of different events with your group.
- Try to identify some events that are certain.
- Make a list of any events you agree are certain.

Another saying that relates to the likelihood of events comes from Charles Dickens's *Pickwick Papers* (1837). Dickens wrote: 'Nothing is impossible, anything can happen, as in Mary said Tom would never call her again, but I told her, "Never say never." '

- What does the adage 'Never say never' suggest about impossibility of events?
- Try to identify some events that are impossible.
- Make a list of any events you agree are impossible.
- Discuss the findings of your group with other groups in the class.
- What do you conclude about the wisdom of these sayings?

You may have heard of Murphy's Law, which says that 'Anything that can go wrong, will go wrong'. According to Murphy's Law, buttered toast will always land buttered-side down. This is not really true, but you are more likely to notice this outcome because you then have to clean up the mess. When you notice something more often, you tend to think it occurs more often. If you wanted to check the likelihood of different events, you could perform an experiment to work out which event occurred more often. In some cases, you could use existing information.

Investigate: Buttered toast

A piece of buttered toast is the same on both sides, except that one side has a coating on it. You could use any square shape to represent a piece of toast, such as a square cut from a packing carton.

1. Make some pretend toast by cutting squares from a packing carton.
2. Label one side of your 'toast' as buttered.
3. Toss your 'toast' and record which side it lands.
4. Repeat the experiment many times.
5. Put together the results for the whole class.
6. Work out some conclusions about Murphy's Law.

Video tutorial
Experimental probability
MAT08SPVT10017

In mathematics, **probability** is the measure of how likely an event is. **Experimental probability** uses real data to work out chances or likelihoods.

Important!

Probability

Each time we try an experiment it is called a **trial**.
The result of the trial is called an **outcome**.
An **event** is one or more outcomes of an experiment.
The result that we are interested in is called the **favourable outcome** of a trial.
More likely outcomes have higher frequencies than less likely outcomes.

$$\text{Relative frequency} = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

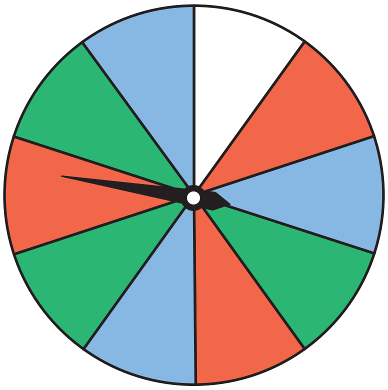
The relative frequency of an event is its **probability**. Probabilities can be expressed as decimals between 0 and 1, as fractions, or as percentages.
You can use the probability of an event to predict the number of times an event is likely to occur in the future. This is called the **expected frequency**.

$$\text{Expected frequency} = \text{Probability} \times \text{Number of trials}$$

9780170194778

Example 2

The spinner shown here was spun and the colour it landed on was recorded in the table.



Colour	Red	Blue	Green	White
Frequency	28	33	31	8

a What is a trial in this experiment?
b What is the total frequency for this experiment?
c What is the relative frequency of spinning red?
d What is the relative frequency of spinning white?
e If the same spinner was spun 25 times how many times would you expect it to finish on green?

Solution

a Each time you do an experiment is a trial. — **A trial is one spin of the spinner.**

b Add up the frequencies. — **Total frequency** $= 28 + 33 + 31 + 8 = 100$

c The frequency of red is 28. — **Relative frequency of red** $= \frac{28}{100}$

Simplify. — $= 0.28$ **or** 28%

d The frequency of white is 8. — **Relative frequency of white** $= \frac{8}{100}$

Simplify. — $= 0.08$ **or** 8%

e Calculate the relative frequency of green. — **Relative frequency of green** $= \frac{31}{100}$

Simplify. — $= 0.31$

Write as a probability. — **Probability of green** $= 0.31$

Write the formula. — **Expected frequency** $=$ **Probability** $\times$ **Number of trials**

Substitute values. — $= 0.31 \times 25$

$= 7.75$

Write the answer. — **You would expect the spinner to land on green about 8 times in 25 spins.**

Technology worksheet
Excel: Simulating a spinner
MAT08SPCT00010

TLF learning object
Snakes and spinners: Assessment (L8868)
MAT08SPIN00008

9780170194778

Example 3

Thirty people in McLeod Street, Cairns, were asked where they were born. The results were:

Place of birth	Cairns	Elsewhere in Queensland	Interstate	Overseas
Frequency	9	9	4	8

a Which is more likely, being born in Cairns or interstate?
b What is the relative frequency of being born overseas?
c If another 50 people in Shields Street were asked, how many would you expect to have been born in Cairns?
d If you asked the same question in the Flinders Street Mall in Townsville, would you expect similar results? Why or why not?

Solution

a Being born in Cairns has the higher frequency. **Being born in Cairns is more likely.**

b Write the formula. **Relative frequency of overseas birth**

$$= \frac{\textbf{Frequency of event}}{\textbf{Total frequency}}$$

Substitute values. $= \frac{8}{30}$

Simplify. $= \frac{4}{15} \approx 0.27 \text{ or } 27\%$

On your calculator. **4** ÷ **15** =. `4÷15 0.2666666667`

c Find the relative frequency. **Relative frequency of Cairns birth**

$$= \frac{\textbf{Frequency of event}}{\textbf{Total frequency}}$$

Substitute values and simplify. $= \frac{9}{30} = \frac{3}{10} = 0.3$

9780170194778

Write the formula.	**Expected frequency = Probability × Number of trials**
Substitute values.	**= 0.3 × 50**
Evaluate.	**= 15**
Write the answer in words.	**You would expect 15 people to be born in Cairns.**
d The conditions are not the same.	**You would not expect similar results because the conditions are different. For example, you would expect fewer born in Cairns.**

You do not need to perform experiments in order to calculate probabilities. You already know from experience that the possible outcomes of certain experiments are equally likely. For example, you know from experience that, when you toss a coin, there is no difference between the outcomes of heads and tails that would make one more likely than the other.

Important!

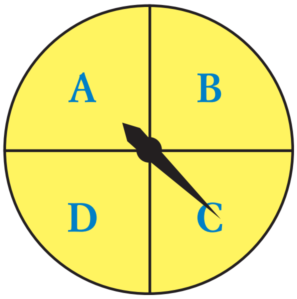
Theoretical probability

Collected data is not used to calculate **theoretical probability**.
It is based on a list, called the **sample space**, that contains all the possible outcomes. Each outcome in the sample space is listed exactly as many times as it is possible for it to occur.
Each element of the sample space is called an **element** or **sample point**.
The number of ways that an event can occur is written as n(Event).
The **probability** of an event is written as P(Event) and is a measure of its likelihood.
Probability is calculated from the sample space using the formula:

$$P(\text{Event}) = \frac{n(\text{Event})}{n(\text{Sample space})}$$

Example 4

The spinner shown on the right has four letters.
a What is the sample space for this spinner?
b Are the outcomes equally likely?



Solution

a There are four possible letters that the spinner can land on. **The sample space is {A, B, C, D}.**

b Each region of the pointer is the same size—a quarter of the circle. **The outcomes are equally likely.**

9780170194778

Example 5

A cubic die has six faces each with a different number between 1 and 6.

a List the sample space.
b Are the outcomes equally likely?

Solution

a There are six different numbers of dots that are possible. **The sample space is {1, 2, 3, 4, 5, 6}.**

b The die is a cube and the faces are the same size. **The outcomes are equally likely.**

Before using theoretical probability to calculate a probability, you must make sure that the outcomes are equally likely.

Example 6

Puzzle sheet
Basic probability
MAT08SPPS00028

Worksheet
Basic probability
MAT08SPWK00050

In each of the following cases, determine if the outcomes are equally likely. If the outcomes are equally likely, write the sample space.

a A matchbox is thrown and it is noted whether it falls flat, on the narrow side or on its end.
b A normal six-sided die is thrown and the uppermost number noted.

Solution

a The sides of a matchbox are different sizes. **The outcomes are not equally likely.**

b The sides of a normal die are all the same. **The outcomes are equally likely.**
List all possible outcomes. **The sample space is {1, 2, 3, 4, 5, 6}.**

The word **random** is used to indicate that something happens by chance – without intention. In probability, random events are equally likely.

Example 7

Animated example
Theoretical probability
MAT08SPAE00013

A bag contains three red, two yellow and five green marbles. A marble is taken out at random.

a Find the probability that the marble is yellow.
b If the marble is replaced and this is done 40 times, on how many occasions would you expect the marble to be yellow?

9780170194778

Solution

a List the possibilities using letters. **The sample space is {R, R, R, Y, Y, G, G, G, G, G}.**

Write the formula for this case.
$$\boldsymbol{P(\text{Yellow}) = \frac{n(\text{Yellow})}{n(\text{Sample space})}}$$

Substitute values.
$$\boldsymbol{= \frac{2}{10} = 0.2}$$

b Write the formula. **Expected frequency = Probability × Number of trials**

Substitute values.
$$\boldsymbol{= 0.2 \times 40}$$
$$\boldsymbol{= 8}$$

Write the answer. **You would expect to get a yellow marble about eight times.**

Exercise 8.1 Basic probability

Understanding

Extra questions
Exercise 8.1
MAT08SPEQ00026

Worked solutions
Exercise 8.1
MAT08SPWS00026

1 Rate each of these events as 'impossible', 'very unlikely', 'even chance', 'very likely' or 'certain'.
   a You draw a black card from a normal deck of playing cards.
   b There will be a sunny day next week.
   c You will check your Facebook account today.
   d When you toss a coin it will show 'heads'.
   e Your dog has two tails.
   f When a six sided die is rolled, the upper face will show an even number.
   g You will vote in an Australian Government election tomorrow.
   h You will vote in an Australian Government election some time in the next 20 years.

2 **a** Write down some impossible events. **b** Write down some certain events.

Worked solutions
Exercise 8.1
MAT08SPWS00026

3 Is the chance of each of the following events more than or less than an even chance?
   a selecting a diamond from a normal deck of playing cards
   b picking the winner of the next Melbourne Cup
   c getting married before you are 40
   d eating take-away food next week
   e owning your own car by the end of the year
   f celebrating your 50th birthday

Worked solutions
Exercise 8.1
MAT08SPWS00026

4 **a** Draw the scale of likelihood shown below in your notebook.

9780170194778

b Mark the position of each of these types of chances on the scale you have drawn.

highly likely	rarely	often
never	perhaps	fifty-fifty
very uncertain	maybe	sure thing
doubtful	highly unlikely	against the odds
no chance	likely	some chance
improbable	probable	not likely

See Example 1

5 Compare the chances of the outcomes for each of the following possibilities.

a Getting a head or getting a tail when tossing a coin.

b Getting a 6 or getting a number other than 6 when rolling a die.

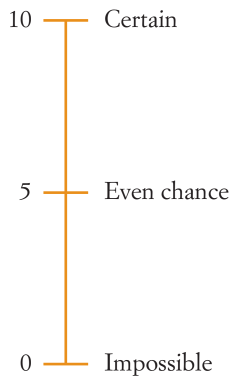
c Winning or losing a car race against the world champion driver.

d Winning or losing a game of scrabble against a 5-year-old.

e Being dealt a pair or being dealt a flush in five cards.

6 Consider the Scale of likelihood on the right where a certain event has a likelihood score of '10', an even chance has a likelihood score of '5' and an impossible event has a likelihood score of '0'.
Use the scale to give a likelihood score between 0 and 10 to each of the following events.

Scale of likelihood



a You will represent Australia in basketball in the next Olympic games.

b It will rain next week.

c You will catch a fish without bait on the hook.

d When you toss a coin it will show 'heads'.

e You will draw a spade from a normal deck of playing cards.

f You will roll a number less than 7 when you roll a six-sided die.

g There will be a full moon tonight.

h It will snow in Darwin tomorrow.

i You will pass your next maths test.

7 Arrange these events in order, from the least likely to the most likely.

a You have a brother or a sister.

b The next person you speak to is female.

c Your favourite sports team will win their next game.

d There will be no car accidents in Australia tomorrow.

e No one at your school has a birthday today.

f It will not rain tomorrow.

g You will eat take away food this week.

h A cyclone will cross the Australian coastline this week.

i The Sun will set tomorrow.

j It will be cold tomorrow.

9780170194778

8 **a** Copy this table into your workbook. *See* Example 2

Outcome	Tally	Frequency
Tail		
Head		

b Toss a coin 50 times and record the outcome of each toss in the table.

c What is a trial in this experiment?

d What is the total frequency?

e What is the relative frequency of a head?

f What is the relative frequency of a tail?

9 **a** Copy this table into your workbook. *See* Example 2

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

b Roll a die 40 times and record the outcome of each roll in the table.

c What is a trial in this experiment?

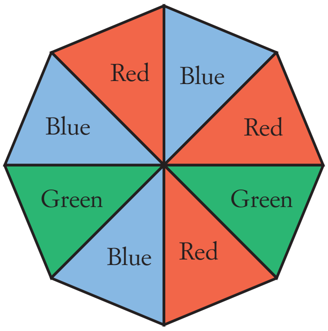
d What is the total frequency?

e What is the relative frequency of a 6?

f What is the relative frequency of a 2?

g What is the relative frequency of an even number?

10 Make a copy of the spinner shown on the right. Cut it out and stick it onto some light cardboard. Place a toothpick or something similar through the centre so that it can be spun so that it lands on one of the flat edges.



Worked solutions
Exercise 8.1
MAT08SPWS00026

a Copy this table into your workbook.

Outcome	Tally	Frequency
Blue		
Green		
Red		

b Spin the spinner 40 times and record the outcome of each spin in the table.

c What is a trial in this experiment?

d What is the total frequency?

e What is the relative frequency of blue?

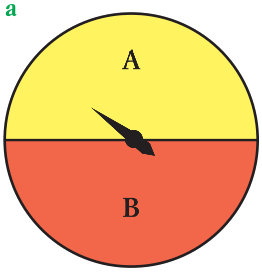
f What is the relative frequency of green?

g What is the relative frequency of red?

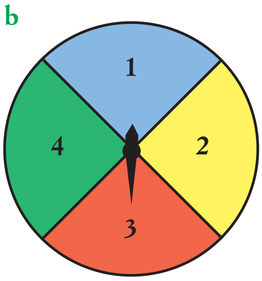
See Example 4

11 Use the spinners below to answer the following questions.

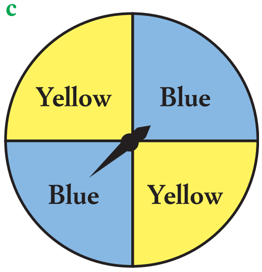
a



b



c



i List the sample space for the each of the spinners.

ii Are each of the events equally likely?

See Example 5

12 A bag contains five green marbles, three blue marbles, two red marbles and two yellow marbles. One marble is drawn at random from the bag.

a List the sample space.

b Are the four different colour outcomes equally likely?

13 For the spinners in question **11**, calculate the probability of the pointer landing on yellow.

14 For the situation in question **12**, calculate the probability of drawing:

a a red marble **b** a green marble

Fluency

15 Use the results of question **8** to answer the following questions.

a According to your results, which outcome (head or tail) is more likely?

b If a coin was tossed 100 times, what would you expect to be the frequency of heads?

c If a coin was tossed 500 times, what would you expect to be the frequency of tails?

16 Use the results of question **9** to answer the following questions.

a According to your results, which outcome (1, 2, 3, 4, 5 or 6) is most likely?

b Compare the probabilities of the outcomes: an even number and an odd number.

c If a die was rolled 1000 times, how many times would you expect it to result in a 5?

d If a die was rolled 200 times, how many times would you expect it to result in an odd number?

17 Use the results of question **10** to answer the following questions.

a According to your results, which outcome (blue, green or red) is least likely?

b Compare the probabilities of the outcomes: red and green.

c If the spinner was spun 100 times, how many times would you expect it to finish on red?

d If the spinner was spun 400 times, how many times would you expect it to finish on green?

Worked solutions

Exercise 8.1

MAT08SPWS00026

See Example 3

18 Two coins were tossed and the following results obtained.

TH	HT	TT	TH	TT	TT	HT	TH	TH	HH
HT	HT	TH	HH	TT	TH	HT	TH	HT	TT

9780170194778

a Construct a frequency table.
b Calculate the relative frequencies of each outcome as decimals.
c According to these results, are you more likely to get 'two tails' or 'heads and tails'?

19 A pack of cards was cut and the suit was noted. This was done 25 times and the results were:

See Example 3

Hearts (♥)	Diamonds (♦)	Clubs (♣)	Spades (♠)
7 times	5 times	5 times	8 times

a Calculate the relative frequencies of each outcome as percentages.
b Does it appear that a black card or a red card is more likely?
c Does it appear that a spade or a diamond is more likely?

20 The numbers of oranges in 2 kg bags of oranges were counted. The numbers in each bag were:

See Example 3

14	15	16	14	15	14	15	17	14	15
16	17	14	20	16	17	17	13	15	15
17	17	15	15	14	15	18	17	16	17

a Draw up a frequency table.
b Calculate the relative frequencies of each outcome as fractions.
c Based on these results, are you more likely to get a bag with 15 or 20 oranges?

21 In each of the following cases, state whether the events are equally likely, and if so write the sample space accordingly.

See Example 6

a There are five red, three green and four blue marbles in a bag. One marble is chosen from the bag.
b A coin is tossed twice and the number of tails is noted.
c A drawing pin is thrown and it is noted whether it lands point up or point down.
d The score from throwing a dart at a dartboard is noted.
e Two multiple-choice questions have answers A, B and C, with one correct answer for each. Answers are chosen at random and the number of correct answers is noted.

22 There are four red, three green and eight blue marbles in a bag. One marble is taken out.

Worked solutions
Exercise 8.1
MAT08SPWS00026
See Example 7

a List the sample space.
b What is n(Red marble)?
c Work out P(Red marble).
d Find P(Green marble).
e Work out P(Blue marble).
f What do all the probabilities add up to?

9780170194778

Worked solutions
Exercise 8.1
MAT08SPWS00026

23 A flea-market stall has unlabelled cans of sliced beetroot, pineapple pieces and potato salad for sale. The stallholder says that eight of the cans are pineapple, six are potato salad and two are beetroot. You buy one can.

a Use SB, PP and PS to list the sample space.
b Work out $P(\text{SB})$.
c Work out $P(\text{PP})$.
d Work out $P(\text{PS})$.

24 In a loose leaf folder there are three blank sheets, two graph sheets and five lined sheets of paper. A sheet of paper is taken out at random.

a Write down the sample space.
b Work out $n(\text{blank})$ and $P(\text{blank})$.
c Work out $n(\text{graph})$ and $P(\text{graph})$.
d Work out $n(\text{lined})$ and $P(\text{lined})$.
e What is the most likely kind of paper to be picked out?

See Example 7

25 A normal die is rolled.

a What is the probability of throwing a 6?
b What is $P(2)$?
c What is $P(\text{even number})$?
d What is the probability of more than 4?
e What is $P(\text{prime number})$?
f If the die is rolled 150 times, on how many occasions would you expect the result to be a 2?

Problem solving

Worked solutions
Exercise 8.1
MAT08SPWS00026

26 'Incy wincy spider climbed up the water spout. Down came the rain and washed the spider out!' Some children counted the legs on spiders washed down the gutter after a storm. The numbers of legs they found were as follows:

7	6	7	7	8	7	5	7	7	7
7	6	5	6	8	7	7	8	7	6
8	8	8	7	6					

a Draw up a frequency table.
b Are you more likely to find a spider with six legs or seven legs?

9780170194778

27 Lettuce plants are supposed to grow better if they are planted out small. Unfortunately, the snails eat more of the plants when they are small. The lettuces are usually big enough after 8 days to survive. The table shows the number of lettuces eaten each day when 50 lettuces were planted.

Day	1	2	3	4	5	6	7	8	9	10
Number eaten	6	4	6	3	5	6	5	0	0	0

a How many lettuces were eaten in the first seven days?

b Are lettuces more likely to be eaten by snails in the first three days or the next three days?

28 Refer to question **20**.

a From 140 bags of oranges, how many would you expect to have 20 oranges?

b From 500 bags of oranges, how many would you expect to have 15 oranges?

Reasoning

29 Refer to question **27**.

a What is the relative frequency of lettuces being eaten by snails?

b What is the relative frequency of lettuces surviving snail attacks?

c How many lettuces would probably be lost if 80 were planted?

d How many lettuces should be planted to end up with 50?

30 Would it be wise to use the results of question **19** to predict the number of diamonds you should expect from cutting a pack of cards 300 times?

31 A lucky dip has 4 prizes worth \$2, 6 prizes worth \$1, 10 prizes worth 20c and 180 prizes worth 10c. It costs 20c per dip to play. Assume that you are the first person to play.

a What is the probability of getting a prize worth more than 20c?

b What is the probability of getting only 10c?

c If you played this game 600 times, how much would you expect to win or lose?

32 A fair coin has been tossed five times and there have been five heads in a row. What is the probability that the next time it lands on tails? Explain your answer.

8.2 Two-way tables and complementary events

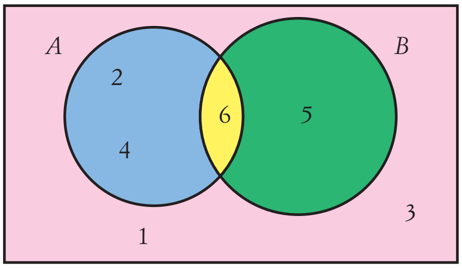
Video tutorial
Venn diagrams
MAT08SPVT10016

TLF learning object
Red hot dealer (R12169)
MAT08SPIN00008

You can use **Venn diagrams** to help understand situations. These show outcomes in a box with events shown as circles within the box.
Consider the experiment where a die is rolled and the uppermost number is noted. The sample space for this experiment is {1, 2, 3, 4, 5, 6}. Choose:

$$\text{Event } A = \text{an even number} = \{2, 4, 6\}$$
$$\text{Event } B = \text{a number greater than } 4 = \{5, 6\}$$

The sample space for this experiment is shown in the Venn diagram below.



By looking at the Venn diagram above you can see the following.

- Event A is shown as the circle shaded blue and yellow and event B is shown as the circle shaded green and yellow.
- The yellow region is the intersection of events A and B and it contains the element that is common to both events: 6.
- The blue region contains elements that are included in event A but not in event B: 2 and 4.
- The green region contains an element that is included in event B but not in event A: 5.
- The pink region contains elements that are not included in event A or event B: 1 and 3.

Important!

The meanings of 'and', 'or'

In mathematics, '. . . and . . .' means *both*, but '. . . or . . .' means *at least one*.

Peter has blue eyes and fair hair means that he has *both* blue eyes and fair hair.
Daria has brown eyes or dark hair means she could have brown eyes and fair hair, brown eyes and dark hair, blue eyes and dark hair, green eyes and dark hair, or other combinations as long as she has *at least one* of brown eyes and dark hair.

9780170194778

Example 8

Animated example
Venn diagrams 1
MAT08SPAE00014

A six-sided die is rolled and the uppermost number noted. What is the probability that the number is:

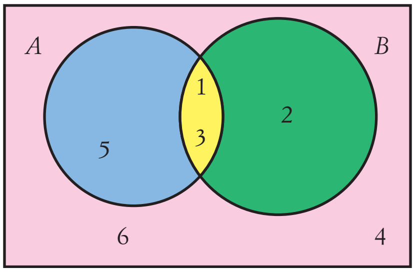
a odd? **b** less than 4? **c** odd or less than 4? **d** odd and less than 4?

Solution

Let event A be 'odd number' and event B be 'number less than 4'. List the sample space and the outcomes of each event.

Sample space $= \{1, 2, 3, 4, 5, 6\}$
Event $A =$ **odd number** $= \{1, 3, 5\}$
Event $B =$ **number less than 4** $= \{1, 2, 3\}$

Draw a Venn diagram for this experiment showing events A and B.



a Use the rule for probability.

$$P(\text{Odd}) = \frac{n(\text{Odd})}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{3}{6}$$

Evaluate.

$$= \frac{1}{2}$$

b Use the rule for probability.

$$P(<4) = \frac{n(<4)}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{3}{6}$$

Evaluate.

$$= \frac{1}{2}$$

c Use the Venn diagram to list the outcomes that are either odd or less than 4.

Odd or $< 4 = \{1, 2, 3, 5\}$

Use the rule for probability.

$$P(\text{Odd or} < 4) = \frac{n(\text{Odd or} < 4)}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{4}{6}$$

Evaluate.

$$= \frac{2}{3}$$

d Use the Venn diagram to list the outcomes that are both odd and less than 4.

Odd and $< 4 = \{1, 3\}$

Use the rule for probability.

$$P(\text{Odd and} < 4) = \frac{n(\text{Odd and} < 4)}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{2}{6}$$

Evaluate.

$$= \frac{1}{3}$$

When using Venn diagrams, you don't have to list individual elements of the events making up the sample space. Sometimes you only need to list the number of elements in the various events.

Example 9

Animated example
Venn diagrams 2
MAT08SPAE00015

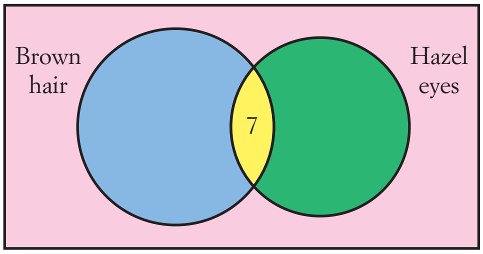
Out of a group of 34 students, 22 have brown hair and 19 have hazel coloured eyes. If 7 of the students have brown hair and hazel eyes, find the probability that a student selected at random:

a does not have hazel eyes

b does not have brown hair.

Solution

a Draw a Venn diagram for this experiment showing events 'Brown hair' and 'Hazel eyes'. There are 7 students who have brown hair and hazel eyes. Enter 7 in the intersection of the circles representing the events.



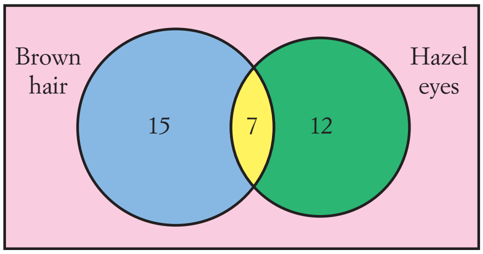
22 students have brown hair. Of these, 7 have hazel eyes.

Number students with brown hair and eyes that are not hazel $= 22 - 7$
$= 15$

19 students have hazel eyes. Of these, 7 have brown hair.

Number students with hazel eyes who do not have brown hair $= 19 - 7$
$= 12$

Complete the Venn diagram.



Use the rule for probability.

P(**Student does not have hazel eyes**)

$= \dfrac{n(\textbf{Eyes not hazel})}{n(\textbf{Sample space})}$

Substitute required values.

$= \dfrac{15}{34}$

Evaluate.

$\approx 44.1\%$

9780170194778

b Use the rule for probability. $P(\textbf{Student does not have brown hair})$

$= \dfrac{n(\textbf{Hair not brown})}{n(\textbf{Sample space})}$

Substitute required values. $= \dfrac{\mathbf{19}}{\mathbf{34}}$

Evaluate. $\approx \mathbf{55.9\%}$

Tables can also be useful aids to understanding the outcomes of experiments. Consider the situation where a coin is tossed and a die is rolled.

This experiment consists of two parts and so you can represent the outcomes of this experiment using a **two-way table**. The two-way table is constructed by listing the possible outcomes of one part of the experiment along the top row and the outcomes of the other part down the left-hand column as shown here.

	1	**2**	**3**	**4**	**5**	**6**
H						
T						

The table is completed by filling in the cells as shown.

	1	**2**	**3**	**4**	**5**	**6**
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Puzzle sheet
Two-way probability tables
MAT08SPPS00029

Worksheet
Two-way probability tables
MAT08SPWK00051

Example 10

Use the two-way table for tossing a coin and rolling a six-sided die to find the probability that the outcome is:

a a head and an even number
b a head or an even number
c a tail or a number less than 3

Solution

a Draw up the table.

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Use the table to find the number of elements in the sample space. $n(\text{Sample space}) = 12$

List the elements of the outcome. $\text{H and even} = \{\text{H2, H4, H6}\}$

Find the number of elements in the desired outcome. $n(\text{H and even}) = 3$

Use the rule for probability. $P(\text{H and even}) = \dfrac{n(\text{H and even})}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{3}{12}$

Evaluate. $= \dfrac{1}{4}$

b List the elements of the outcome. $\text{H or even} = \{\text{H1, H2, H3, H4, H5, H6, T2, T4, T6}\}$

Use the table to find the number of elements in the desired outcome. $n(\text{H or even}) = 9$

Use the rule for probability. $P(\text{H or even}) = \dfrac{n(\text{H or even})}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{9}{12}$

Evaluate. $= \dfrac{3}{4}$

c List the elements of the outcome. $\text{T or} < 3 = \{\text{T1, T2, T3, T4, T5, T6, H1, H2}\}$

Use the table to find the number of elements in the desired outcome. $n(\text{T or} < 3) = 8$

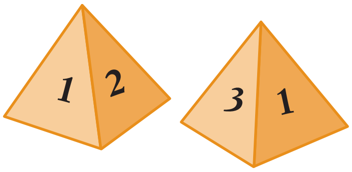
Use the rule for probability. $P(\text{H or even}) = \dfrac{n(\text{T or} < 3)}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{8}{12}$

Evaluate. $= \dfrac{2}{3}$

9780170194778

The faces of two four-sided dice are numbered 1 through 4. The dice are rolled and the numbers on the bottom faces are noted. The two-way table for this experiment is shown here.



	1	2	3	4
1	1, 1	1, 2	1, 3	1, 4
2	2, 1	2, 2	2, 3	2, 4
3	3, 1	3, 2	3, 3	3, 4
4	4, 1	4, 2	4, 3	4, 4

The probability that the total of the bottom faces is less than 10 is:

$$P(\text{Total} < 10) = \frac{n(\text{Total} < 10)}{n(\text{Sample space})} = \frac{8}{8} = 1$$

The probability that the total of the bottom faces is greater than 10 is:

$$P(\text{Total} > 10) = \frac{n(\text{Total} > 10)}{n(\text{Sample space})} = \frac{0}{8} = 0$$

For this experiment, the event 'total $<$ 10' is a certain event. The event will occur every time. Certain events have a probability of 1.
For this experiment, the event 'total $>$ 10' is an impossible event. The event will never occur. Impossible events have a probability of 0.
When a six-sided die is rolled, the sample space is {1, 2, 3, 4, 5, 6}.
The probability of each outcome is equally likely and is $\frac{1}{6}$.
Adding the probabilities of all possible outcomes for this experiment gives:

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

These results are true for all experiments.

Important!

Probabilities of events

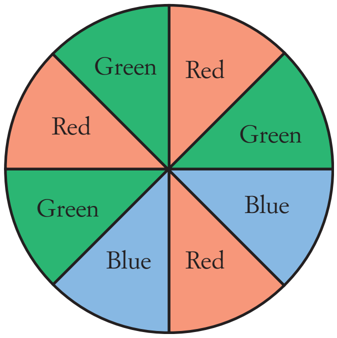
The probability of a **certain** event is 1.
The probability of an **impossible** event is 0.
The probabilities of all other events are between 0 and 1.
The sum of the probabilities of all outcomes for an experiment is 1.

9780170194778

Example 11

For this spinner, what is the probability of spinning:

a green
b a colour other than green
c green or a colour other than green?



Solution

a List the sample space.

The sample space is {G, G, G, R, R, R, B, B}.

Use the probability formula for this case.

$$P(\mathbf{G}) = \frac{n(\mathbf{G})}{n(\textbf{Sample space})}$$

Substitute values.

$$= \frac{3}{8} = 0.375$$

b 'A colour other than green' can be written as 'not green'. List the favourable outcomes.

Not green = {R, R, R, B, B}

Use the probability formula.

$$P(\textbf{Not G}) = \frac{n(\textbf{Not G})}{n(\textbf{Sample space})}$$

$$= \frac{5}{8} = 0.625$$

c List the favourable outcomes.

Green or not green = {G, G, G, R, R, R, B, B}

The event 'green or not green' includes all possible outcomes. The event 'green or not green' is certain.

$P(\textbf{Green or not green}) = 1$

Video tutorial
Complementary events
MAT08SPVT10015

In the previous example you saw that the events 'green' and 'a colour other than green' together made up the entire sample space for the experiment. Events such as these are called **complementary events**.
Because an event and its complement together make up the entire sample space for an experiment, the sum of the probabilities of an event and its complement must equal 1.

Important!

Complementary events

For any event A, the **complement** of the event is shown as A' or $\overline{A}$.
The sum of the probabilities of an event and its complement is 1.
The probability of the complement of event A is the same as the probability that event A will not occur.

$$P(A) + P(A') = 1$$

So $P(A') = 1 - P(A)$

or $P(A) = 1 - P(A')$

9780170194778

Example 12

Find the complement of each of the following events.

a rolling a die and getting a 5

b it will rain tomorrow

Solution

a List the sample space. — **The sample space is {1, 2, 3, 4, 5, 6}.**

The complement of getting a 5 is not getting a 5. — **The complement of 'getting a 5' is 'getting a 1, 2, 3, 4 or 6'.**

b The event is that it will rain tomorrow. — **The complement of 'raining tomorrow' is 'not raining tomorrow'.**

Example 13

Puzzle sheet
Complementary events
MAT08SPPS00030

A spinner only has whole number outcomes. The probability of getting an odd number is 0.38. Calculate the probability of getting an even number.

Solution

Getting an odd number and getting an even number are complementary events.

Use the rule for the probabilities of an event and its complement.	$P(A) + P(A') = 1$
Substitute in the event and its complement.	$P(\text{Odd}) + P(\text{Even}) = 1$
Substitute for P(odd).	$0.38 + P(\text{Even}) = 1$
Subtract 0.38 from each side of the equation.	$P(\text{Even}) = 1 - 0.38 = 0.62$
State the result.	**The probability of getting an even number is 0.62.**

Exercise 8.2 Two-way tables and complementary events

Fluency

Extra questions
Exercise 8.2
MAT08SPEQ00027

See Example 8

1 A six-sided die is rolled and the uppermost number noted. If A = {an even number} and B = {a number > 2}:

a represent the outcomes of the experiment and events A and B using a Venn diagram.

What is the probability that the number is:

b even?

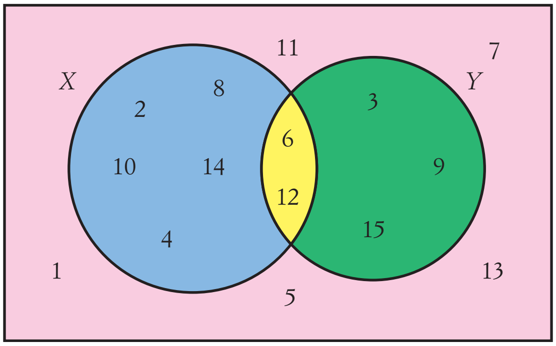
c greater than 2?

d even or greater than 2?

e even and greater than 2?

See Example 8

2 Numbered discs are randomly drawn from a container. $X = \{\text{an even number}\}$ and $Y = \{\text{a multiple of 3}\}$. The outcomes are shown in the Venn diagram below.



What is the probability that a selected number is:

a even?
b a multiple of 3?
c even or a multiple of 3?
d even and a multiple of 3?
e not even?
f not a multiple of 3?

See Example 9

3 Out of a group of 40 students, 29 listen to an MP3 player on the way to school and 28 have a Facebook account. 17 of the students listen to an MP3 player on the way to school and have a Facebook account.

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a student selected at random:

b does not listen to an MP3 player on the way to school
c does not have a Facebook account

See Example 9

4 A second-hand car yard has 233 vehicles. Of these, 63 are red, 44 are four wheel drives and 16 are red four wheel drives.

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a vehicle selected at random:

b is red
c is a four wheel drive
d is not red
e is not a four wheel drive
f is neither red nor a four wheel drive.

Worked solutions
Exercise 8.2
MAT08SPWS00027
See Example 10

5 Two six-sided dice are rolled and the total of the uppermost faces is noted.

a Construct a two-way table to represent this experiment.

Use the two-way table to find the probability that the total is:

b even
c prime
d greater than 6
e less than 5
f even or greater than 6
g even and less than 6

See Example 10

6 A pizza restaurant sells pizzas in four different sizes: family, large, medium and small. The fillings available are meat lovers, Hawaiian, supreme, four cheeses and vegetarian. Over the years, the chef knows that all fillings and sizes are equally likely to be ordered.

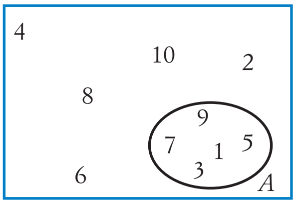
a Construct a two-way table to represent the possible outcomes when a pizza is ordered.

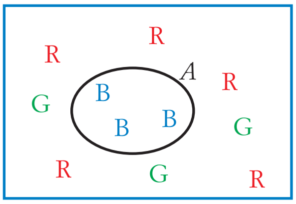
Use the two-way table to find the probability that the next pizza ordered is:

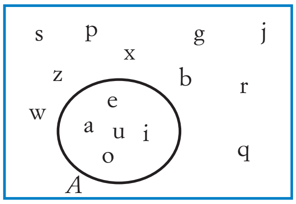
b a large size with any filling
c a supreme of any size
d a family size with any filling
e a vegetarian of any size
f a medium meat lovers
g a medium or a meat lovers

9780170194778

7 Each Venn diagram below shows the sample space for an experiment and an event A. In each case, list elements of the complement of A.

a


b


c


8 Find the complement of each of the following events.

a Getting a head when you toss a coin.

b Selecting a letter of the alphabet and getting a vowel.

c Selecting a month of the year and getting a month that ends in 'er'.

d Winning a game of tennis.

e Getting a number greater than 7 on a spinner with segments numbered 1 through 10.

f Getting an even number when a six-sided die is rolled.

9 A six-sided die is rolled. List the complement of each of the following events.

a getting a 6

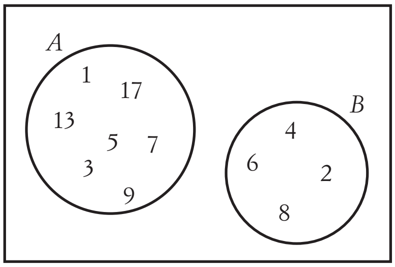
b getting an odd number

c getting a prime number

d getting a number less than 3

Worked solutions
Exercise 8.2
MAT08SPWS00027

10 A bag contains 11 numbered cards. The Venn diagram on the right shows two events A and B that result when a card is drawn and the number on it noted.



Calculate:

a $P(A)$

b $P(B)$

c $1 - P(A)$

d $1 - P(B)$

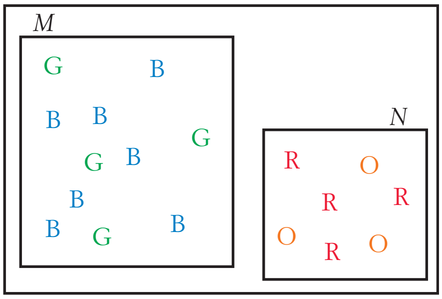
e $P(A) + P(B)$

Complete the following:

f $P(A') = \ldots$

g $P(B') = \ldots$

11 A jar contains 18 marbles: each one coloured green, blue, red or orange. The Venn diagram on the right shows two events M and N that result when a marble is drawn and its colour noted.



Calculate:

a $P(M)$

b $P(N)$

c $1 - P(M)$

d $1 - P(N)$

e $P(M) + P(N)$

Complete the following:

f $P(M') = \ldots$

g $P(N') = \ldots$

12 A six-sided die is rolled and the number on the uppermost face is noted. What is the complement of the event: a number greater than 4?
Select from **A**, **B**, **C** or **D** below.

A a number less than 4

B 1 and 2

C 5 and 6

D a number less than or equal to 4

Worked solutions
Exercise 8.2
MAT08SPWS00027

13 A bag contains 2 green, 4 red, 6 blue and 8 yellow marbles.
A marble is selected at random. What is the probability of:

a getting a red
b getting a marble that isn't red
c getting a yellow marble
d getting a marble that isn't yellow

Worked solutions
Exercise 8.2
MAT08SPWS00027

14 Two coins are tossed and the uppermost faces are noted.

a Draw a two-way table to show all the possible outcomes.

Use the table to find the probability that the outcome is:

b two heads
c not two heads
d a head and a tail
e other than a head and a tail

15 Two six-sided dice are rolled and the numbers on the uppermost faces are noted.

a Draw a two-way table to show all the possible outcomes.

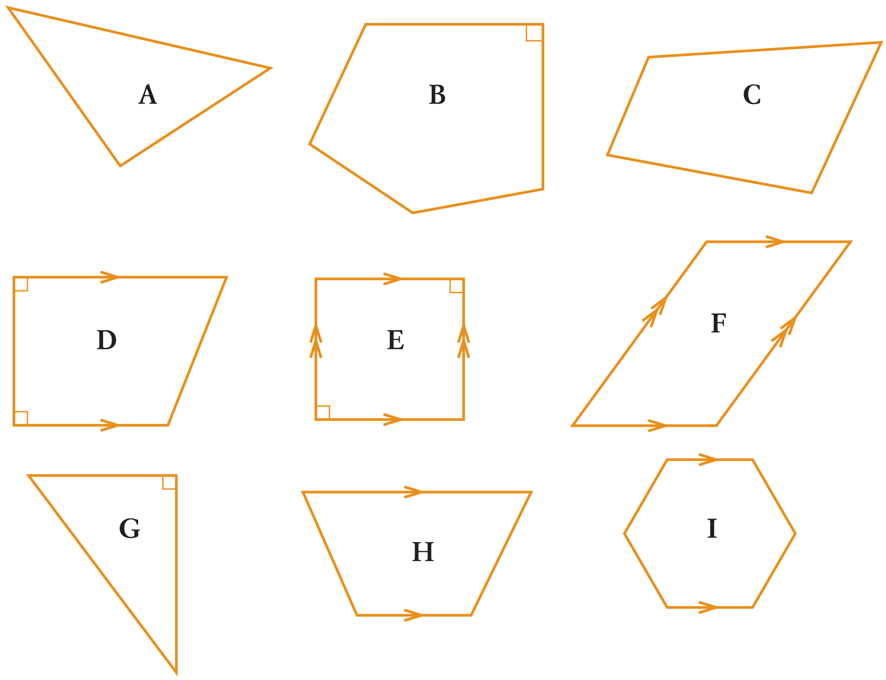
Use the table to find the probability that the total of the faces is:

b 12
c not 12
d an even number
e an odd number
f a prime number
g not a prime number
h greater than 8
i less than or equal to 8

Problem solving

Worked solutions
Exercise 8.2
MAT08SPWS00027

16 Some plane shapes are shown below. In an experiment, a shape is selected at ramdom.



If:

$P = \{\text{a shape with four sides}\}$
$Q = \{\text{a shape with a right angle}\}$
$R = \{\text{a shape with a pair of parallel sides}\}$

a Represent the experiment using a Venn diagram.

9780170194778

Use the Venn diagram to calculate the probability that a selected shape:

b has four sides and a right angle
c has four sides or a right angle
d has four sides or parallel sides
e has four sides, no right angle and no parallel sides
f does not have parallel sides
g does not have four sides but does have a right angle
h has four sides, a right angle and parallel sides

17 The letters of the alphabet are written on separate cards and placed in a bag. A card is selected from the bag at random.

a How many possible outcomes are there?
b Are the outcomes equally likely?

What is the probability that the selected card is:

c a, c, d, e, f, g, h, or j
d not a, c, d, e, f, g, h, or j
e a vowel
f not a vowel

18 Marbles numbered 1 through 20 are placed in a bag and one is selected from the bag at random.

Worked solutions
Exercise 8.2
MAT08SPWS00027

a How many possible outcomes are there?
b Are the outcomes equally likely?

What is the probability that the selected marble is:

c an even number
d not even
e a prime number
f not prime
g a number less than 30
h a number greater than 30

19 Two hundred tennis players were asked which of these strokes they considered their weakest stroke(s): the serve, the drop shot, the lob.
Of the 200 players questioned:

- 30 players said none of these were their weakest stroke.
- 20 players said all three of these were their weakest stroke.
- 30 players said their serve and drop shot were their weakest strokes.
- 20 players said only their serve and lob were their weakest strokes.
- 25 players said their drop shot but not their lob was their weakest stroke.
- 42 players said only their lob was their weakest stroke.
- 85 players said their serve was their weakest stroke.

a Represent the experiment using a Venn diagram.

One of the 200 players is randomly selected. Use the Venn diagram to calculate the probability that a selected player thinks:

b only their serve and lob are their weakest strokes
c their serve is not their weakest stroke
d their weakest stroke is their serve
e only their lob is their weakest stroke

Reasoning

20 Twenty-five men and 25 women were asked whether they would prefer to listen to music, play sport or watch TV in their spare time.
Of those questioned:

- 4 men said they preferred to listen to music
- 13 men said they preferred to play sport
- a total of 15 people said they preferred to listen to music
- a total of 19 people said they preferred to play sport.

a Represent the experiment using a two-way table.

If a randomly selected group of 100 men and 100 women were asked the same question:

b how many people would you expect to say they preferred to listen to music?

c how many women would you expect to say they preferred to watch TV?

d how many men would you expect to say they preferred to play sport?

21 Marbles numbered 1 through 100 are placed in a bag. A marble is taken from the bag at random. What is the probability that the selected marble is:

a a multiple of 3 **b** not a multiple of 3?

8.3 Two-step probabilities

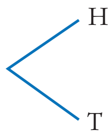
Puzzle sheet
Donkey
MAT08SPPS00031

A probability experiment that involves two steps is known as a **two-step chance experiment**. An example of a simple two-step experiment is tossing a coin twice. One way of representing the outcomes of a two-step experiment is by using a two-way table. You saw how a two-way table is constructed in the previous section.

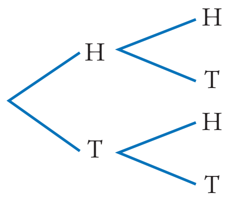
Another method of representing the outcomes of a two-step experiment is called a **tree diagram**.

We will construct a tree diagram for tossing a coin twice.

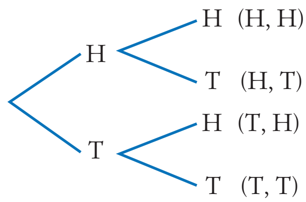
Begin by marking a point on the page. The outcomes of the first step (H and T) are then listed under each other to the right of the point and a 'branch' is drawn from the point to each outcome.



List the outcomes of the second step to the right of each of the outcomes of the first step and draw branches between each of the outcomes.



Finally, at the end of each branch write the two outcomes in order.



9780170194778

Example 14

A coin is tossed two times. Use a tree diagram to calculate the probability that the outcome of the experiment is:

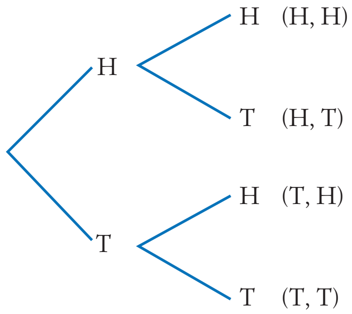
a two heads **b** two tails
c a head and a tail **d** a head followed by a tail

Technology worksheet
Excel worksheet: Random number generator
MAT08SPCT00011

Weblink
Birth month paradox
MAT08SPWB00008

Solution

a Draw the tree diagram to show the sample space for this experiment.



Use the probability formula. $P(2\text{H}) = \dfrac{n(2\text{H})}{n(\text{Sample space})}$

There is only one outcome that is two heads. $= \dfrac{1}{4}$

b Use the probability formula. $P(2\text{T}) = \dfrac{n(2\text{T})}{n(\text{Sample space})}$

There is only one outcome that is two tails. $= \dfrac{1}{4}$

c Use the probability formula. $P(\text{H and T}) = \dfrac{n(\text{H and T})}{n(\text{Sample space})}$

There are two favourable outcomes: H, T and T, H. $= \dfrac{2}{4}$

Simplify. $= \dfrac{1}{2}$

d Use the probability formula. $P(\text{H, T}) = \dfrac{n(\text{H, T})}{n(\text{Sample space})}$

There is one favourable outcome: H, T. $= \dfrac{1}{4}$

Example 15

A drawer contains three pairs of socks: one blue, one red and one green. A pair of socks is taken out of the drawer, the colour noted and returned to the drawer. This step is then repeated. Use a tree diagram to calculate the probability that the selected pairs of socks are:

a the same colour **b** different colours
c red and green **d** red followed by green

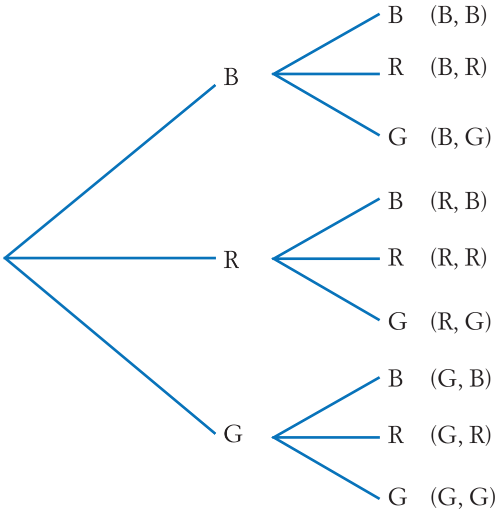
Worksheet
Two-step probabilities
MAT08SPWK00053

Worksheet
Tree diagrams
MAT08SPWK00052

9780170194778

Solution

a Draw the tree diagram to show the sample space for this experiment.



Use the probability formula.

$$\boldsymbol{P(\text{Same colour}) = \frac{n(\text{Same colour})}{n(\text{Sample space})}}$$

There are three favourable outcomes: B, B; R, R and G, G.

$$\boldsymbol{= \frac{3}{9}}$$

Simplify.

$$\boldsymbol{= \frac{1}{3}}$$

b The events 'same colour' and 'different colours' are complementary.

$$\boldsymbol{P(\text{Same colour}) + P(\text{Different colours}) = 1}$$

Substitute the known value.

$$\boldsymbol{\frac{1}{3} + P(\text{Different colours}) = 1}$$

Subtract $\frac{1}{3}$ from both sides of the equation.

$$\boldsymbol{P(\text{Different colours}) = 1 - \frac{1}{3}}$$

Simplify.

$$\boldsymbol{= \frac{2}{3}}$$

c Use the probability formula.

$$\boldsymbol{P(\text{R and G}) = \frac{n(\text{R and G})}{n(\text{Sample space})}}$$

There are two favourable outcomes: R, G and G, R.

$$\boldsymbol{= \frac{2}{9}}$$

d Use the probability formula.

$$\boldsymbol{P(\text{R, G}) = \frac{n(\text{R, G})}{n(\text{Sample space})}}$$

There is only one favourable outcome.

$$\boldsymbol{= \frac{1}{9}}$$

In the previous example, the outcomes of each step were the same because the first pair of socks was replaced before the second pair was withdrawn. The situation where the outcomes of each step of an experiment are the same is said to involve **replacement**. In some experiments there is no replacement.

9780170194778

Example 16

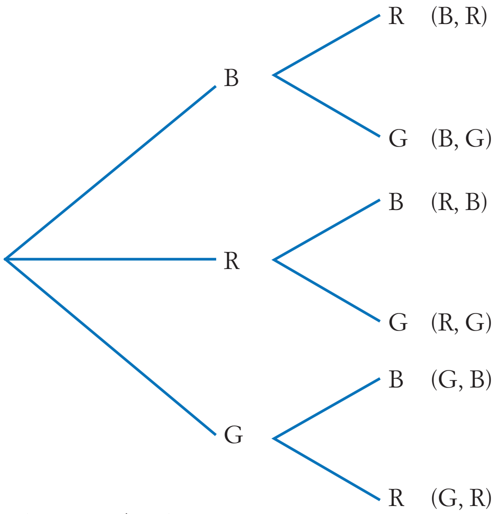
A jar contains a red, blue and green disc. A disc is taken out of the jar, the colour noted and not returned. Another disc is then removed and the colour noted. Use a tree diagram to calculate the probability that the selected discs are:

a the same colour **b** different colours

c red and green **d** red followed by green

Solution

a Draw the tree diagram to show the sample space for this experiment.



There are no favourable outcomes. $P(\text{Same colour}) = 0$

b Every outcome consists of different colours. $P(\text{Different colours}) = 1$

c Use the probability formula. $P(\text{R and G}) = \dfrac{n(\text{R and G})}{n(\text{Sample space})}$

There are two favourable outcomes: R, G and G, R. $= \dfrac{2}{6}$

Simplify. $= \dfrac{1}{3}$

d Use the probability formula. $P(\text{R, G}) = \dfrac{n(\text{R, G})}{n(\text{Sample space})}$

There is only one favourable outcome. $= \dfrac{1}{6}$

It is also possible to show the outcomes of a two-step experiment without replacement using a two-way table. Consider the situation in the previous example. The two-way table is set up as shown on the following page.

	B	R	G
B			
R			
G			

There is no replacement for this experiment, so if a blue disc is selected in the first step, a blue disc can't be selected in the second step. Using this reasoning, the table can be completed as follows:

	B	R	G
B		B, R	B, G
R	R, B		R, G
G	G, B	G, R	

Example 17

A bag contains one red, two blue and two green marbles. A marble is taken out of the bag, the colour noted and not returned. Another marble is then removed and the colour noted. Use a two-way table to calculate the probability that the selected marbles are:

a the same colour
b different colours
c blue and green
d blue followed by green

Solution

a Draw the table and complete the cells. Remember to block out the cells that are not possible due to no replacement.

	R	B	B	G	G
R		R, B	R, B	R, G	R, G
B	B, R		B, B	B, G	B, G
B	B, R	B, B		B, G	B, G
G	G, R	G, B	G, B		G, G
G	G, R	G, B	G, B	G, G	

Use the probability formula.

$$P(\text{Same colour}) = \frac{n(\text{Same colour})}{n(\text{Sample space})}$$

There are 4 favourable outcomes.

$$= \frac{4}{20}$$

Simplify.

$$= \frac{1}{5}$$

b The events 'same colour' and 'different colours' are complementary.

$$P(\text{Same colour}) + P(\text{Different colours}) = 1$$

Substitute the known value.

$$\frac{1}{5} + P(\text{Different colours}) = 1$$

Subtract $\frac{1}{5}$ from both sides of the equation.

$$P(\text{Different colours}) = 1 - \frac{1}{5}$$

Simplify.

$$= \frac{4}{5}$$

9780170194778

c Use the probability formula.

$$P(\text{B and G}) = \frac{n(\text{B and G})}{n(\text{Sample space})}$$

There are 8 favourable outcomes: 4 lots of B, G and 4 lots of G, B.

$$= \frac{8}{20}$$

Simplify.

$$= \frac{2}{5}$$

d Use the probability formula.

$$P(\text{B, G}) = \frac{n(\text{B, G})}{n(\text{Sample space})}$$

There are 4 favourable outcomes.

$$= \frac{4}{20}$$

Simplify.

$$= \frac{1}{5}$$

Exercise 8.3 Two-step probabilities

Fluency

1 A four-sided die is rolled and the bottom number noted. A coin is then tossed.

See Examples 14, 15

Extra questions
Exercise 8.3
MAT08SPEQ00028

Draw a tree diagram to represent this experiment.

2 Use the tree diagram drawn for question **1** to calculate the probability that the outcome will contain:

See Examples 14, 15

a a 3 and a head **b** a 3 or a head
c an even number and a head **d** a head or an even number

3 A bag contains four cards numbered 1 through 4. A two digit number is formed by drawing a card, writing down its number, returning it to the bag and then repeating the first step. Draw a tree diagram to represent this experiment.

See Examples 14, 15

4 Use the tree diagram drawn for question **3** to calculate the probability of obtaining:

Worked solutions
Exercise 8.3
MAT08SPWS00028

See Examples 14, 15

a a number ending in 4 **b** an even number
c a number greater than 20 **d** a number less than or equal to 20
e a number with the digits 1 and 4 **f** the number 14

5 Astrid wants to buy a new car. She doesn't care whether it's a 4-door sedan, a 2-door sedan or a hatchback. The only colours she likes are black, yellow or green and she likes these colours equally. Draw a tree diagram to represent the various cars that Astrid is likely to buy.

See Examples 14, 15

See Examples 14, 15

6 Use the tree diagram drawn for question **5** to calculate the probability that Astrid bought:

a a 4-door sedan of any colour
b a green car
c a yellow 2-door sedan
d a car that is black or yellow

7 A drawer contains five pairs of socks: one red, two grey and two blue. A pair of socks is taken out of the drawer, the colour noted and returned to the drawer. This step is then repeated. Draw a two-way table to represent this experiment.

Worked solutions
Exercise 8.3
MAT08SPWS00028

8 Use the two-way table drawn for question **7** to calculate the probability that the selected pairs of socks are:

a the same colour
b different colours
c blue and grey
d blue and then grey
e both red
f both grey

9 A card is drawn from a normal deck of 52 playing cards, the suit noted and then returned to the deck. This step is then repeated. Draw a two-way table to represent this experiment.

10 Use the two-way table drawn for question **9** to calculate the probability that the selected cards are:

a both ♥
b a ♥ and a ♦
c both black cards
d the same suit
e a ♠ followed by a ♦
f the same colour
g a red card and a black card
h a red card followed by a black card

See Example 16

11 A bag contains four cards numbered 1 through 4. A two digit number is formed by first drawing a card and writing down its number. This card is not returned to the bag. Another card is then drawn from the bag and its number written down. Draw a tree diagram to represent this experiment.

See Example 16

12 Use the tree diagram drawn for question **11** to calculate the probability of obtaining:

a a number ending in 4
b an even number
c a number greater than 20
d a number less than or equal to 20
e a number with the digits 1 and 4
f the number 14

Problem solving

13 A card is drawn from a normal deck of 52 playing cards, the suit noted and then returned to the deck. Another card is then drawn from the deck and its rank (A, 2, 3, 4, 5, . . ., J, Q, K) is noted. Draw a two-way table to represent this experiment and calculate the probability that the selected cards are:

a a ♥ and then a K
b a red card and a 4
c a ♠ or a ♦ and a 10
d a black card and a J or Q

See Example 16

14 A drawer contains three pairs of socks: two red and one blue. A pair of socks is taken out of the drawer, the colour noted and then put aside. Another pair of socks is then taken out of the drawer and its colour noted. Draw a tree diagram to represent this experiment and calculate the probability that the selected pairs of socks are:

a the same colour
b different colours
c red followed by blue
d blue followed by red
e both red
f both blue

9780170194778

Worked solutions
Exercise 8.3
MAT08SPWS00028

15 A drawer contains five pairs of socks: one red, two grey and two blue. A pair of socks is taken out of the drawer, the colour noted and then put aside. Another pair of socks is then taken out of the drawer and its colour noted. Draw a two-way table to represent this experiment and calculate the probability that the selected pairs of socks are:

See Example 17

a the same colour **b** different colours **c** blue and grey
d blue and then grey **e** both red **f** both grey

16 A bag contains six cards numbered 1 through 6. A 2-digit number is formed by first drawing a card and writing down its number. This card is not returned to the bag. Another card is then drawn from the bag and its number written down. Draw a two-way table to represent this experiment and calculate the probability of obtaining:

See Example 17

a an odd number **b** a number ending in 3
c a number greater than 30 **d** a number less than or equal to 30
e a number with the digits 2 and 5 **f** the number 25

17 A family has two children. What is the probability of:

a no boys **b** two girls
c the younger being a girl **d** a boy and a girl

Reasoning

18 In the game rock-paper-scissors, two players simultaneously display the gesture for a rock, scissors or paper using their hands. The objective is to select a gesture which defeats that of the opponent. The outcome of each round is settled as follows:

- rock beats scissors
- scissors beats paper
- paper beats rock

If both players choose the same gesture, the game is tied.

If you are playing an opponent and each of you randomly chooses the rock, scissors or paper gesture each time you play, find the probability that:

a you win the round **b** you lose the round **c** the round is tied

19 How many different 2-digit numbers can be formed from the digits 1, 3, 5, 7 and 9 if:

a no digit can be repeated **b** repetitions are allowed?

20 A coin is tossed three times. Find the probability of getting at least two tails.

21 A coin is tossed four times. Find the probability that at least one head occurs.

Chapter 8 summary

Quiz
Probability
MAT08SPQZ00008

Puzzle sheet
Probability: Little minds
MAT08SPPS00032

Worksheet
Probability review
MAT08SPWK00054

- An **event** is something that could happen. Something that cannot happen is called **impossible**.
- An event that might or might not occur is **possible**. The **possible outcomes** (**events**) of a situation are all the things that could happen. An event that must occur is called a **certain** event.
- **More likely** events have a greater chance of occurring than **less likely** events. **Equally likely** events have the same chance of occurring.
- Something that is just as likely to occur as not occur is called an **even chance** or **fifty-fifty chance**.
- **Experimental probability** uses real data to work out chances. Each time we try an experiment it is called a **trial**. The result that we are interested in is called the **favourable outcome** of a trial. More likely outcomes have higher frequencies than less likely outcomes.
- The **probability** of an outcome can be measured as a decimal between 0 and 1, a fraction or a percentage using the **relative frequency**.

$$\text{Relative frequency} = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

- **Expected frequency** = Probability × Number of trials
- **Theoretical probability** is based on the **equally likely outcomes**.
- The **sample space** lists all possible outcomes, each known as an **element** or **sample point**.
- The number of ways that an event can occur is written as n(**event**).
- The **probability** of an event is written as P(**Event**) and is calculated from the sample space using the formula:

$$P(\text{Event}) = \frac{n(\text{Event})}{n(\text{Sample space})}$$

- The probability of a certain event is 1 and the probability of an impossible event is 0. The probabilities of all other events lie between 0 and 1.
- A **random** event happens by chance.
- A **tree diagram** or a **two-way table** can be used to represent the outcomes of an experiment. Events can also be represented using **Venn diagrams**.
- The sum of the probabilities of all outcomes for an experiment is 1.
- In mathematics, '**and**' means *both* but '**or**' means *at least one*.
- An event (E) and its **complement** (E' or $\overline{E}$) together make up the entire sample space for the experiment. The probability of the complement of event E is the same as the probability that event E will not occur.

$$P(E) + P(E') = 1$$

$$\text{So} \qquad P(E') = 1 - P(E)$$

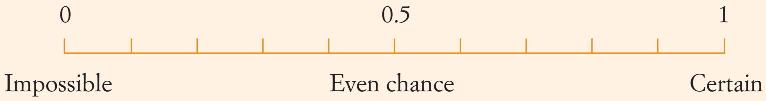
$$\text{or} \qquad P(E) = 1 - P(E')$$

- A probability experiment that involves two stages or steps is known as a **two-step chance experiment**.
- The situation where the outcomes of each step of a two-step chance experiment are the same is said to involve **replacement**.

9780170194778

Understanding

1 Copy this probability scale.



Place the following events on the probability scale using the letters a, b, c, etc.

a You will find a leprechaun today.
b You will be have a birthday at some time next year.
c Someone in your class will go shopping this week.
d Someone in your class will watch TV for more than an hour today.
e No one in your class will not use a mobile phone this week.

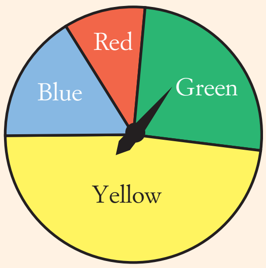
2 Is the chance of each of the following events more than or less than an even chance?

a selecting a club from a normal deck of playing cards
b a student in your class will drive a car home from school today
c a student at your school will complete year 12
d getting a number greater than 2 when a six-sided die is rolled
e students in your class will attend classes at school on Christmas Eve

3 Compare the chance of tossing a head with the chance of drawing a king from a standard pack of playing cards. *See* Example 1

4 Use the spinner shown on the right to answer these questions. *See* Example 2

a Are the outcomes equally likely?
b Which colour is most likely to be spun?
c Which colour is least likely to be spun?
d List the outcomes in order, from most likely to least likely.



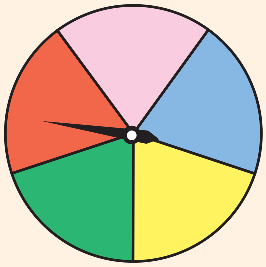
5 The table below shows the numbers of different breeds of cattle on a particular property. *See* Example 3

Breed	Angus	Hereford	Brahman	Simmental	Charolais
Number	120	350	200	180	130

One of the cattle was stuck in a waterhole. What breed is the steer most likely to be?

6 This spinner was spun and the colour where the pointer landed was noted in the table below. *See* Example 3

Outcome	**Tally**	**Frequency**
Blue	卌 卌 卌 卌 ||||	
Green	卌 卌 卌 卌	
Red	卌 卌 卌 |||	
Yellow	卌 卌 卌 ||	
Pink	卌 卌 卌 卌 |	



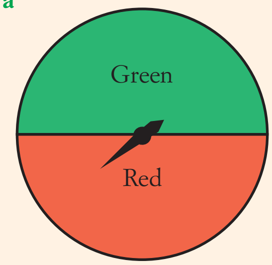
9780170194778

a Complete the frequency column of the table.
b What is the total frequency?
c What is the relative frequency of blue?
d What is the relative frequency of red?
e What is the relative frequency of pink?

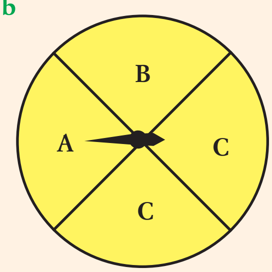
See Examples 4-6

7 Use the spinners below to answer the following questions.

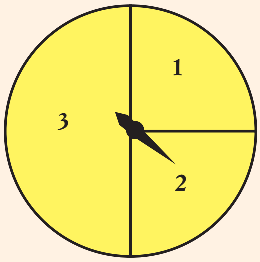
a



b

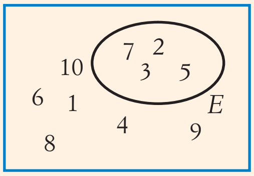


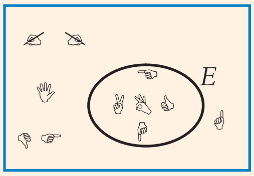
c

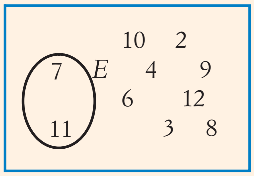


i List the sample space for the each of the spinners.
ii Are each of the events equally likely?

8 Each Venn diagram below shows the sample space for an experiment and an event E. In each case list elements of the complement of E.







Fluency

9 Write down some equally likely events.

10 Calculate the relative frequencies of each breed in question **5**.

See Example 3

11 A six-sided die was rolled and the number of dots on the upper face noted.
The results were:

1	2	3	4	5	6
8 times	6 times	5 times	10 times	6 times	5 times

a Calculate the total frequency for this experiment.
b Calculate the relative frequencies of each outcome as percentages.
c Which number appears most likely?
d Which number appears least likely?
e Does it appear that an odd or even number is more likely?

See Examples 4-6

12 A bag contains six red and nine green marbles.
a List the sample space.
b Find the probability of drawing a red marble.

9780170194778

13 There are 9 green, four red and one pink and 6 blue marbles in a bag. One marble is taken out. *See* Examples 4-6

a List the sample space.
b What is n(blue marble)?
c Work out P(blue marble).
d Find P(green marble).
e Work out P(red marble).
f Work out P(pink marble).
g What do all the probabilities add up to?

14 For the spinner shown on the right: *See* Examples 4-6

a list all the outcomes
b find the probabilities of each of the outcomes
c what is P(even number)?
d what is P(odd number)?
e find $P(\text{number} < 10)$.

15 A six-sided die is rolled. *See* Example 7

a What is the probability of rolling a 5?
b What is the probability of rolling an even number?

16 Discs numbered 1 through 20 are randomly drawn from a container. *See* Example 8

$$A = \{\text{an odd number}\} \text{ and } B = \{\text{multiple of 3}\}$$

a Represent the outcomes of this experiment using a Venn diagram.

Use the Venn diagram to calculate the probability that a randomly selected number is:

b odd
c a multiple of 3
d an odd multiple of 3
e odd or a multiple of 3
f not odd
g not a multiple of 3

17 A group of 63 students goes on a picnic. Of those, 27 went swimming, 19 played volleyball and 33 went kayaking. Nine students went swimming and played volleyball, 6 played volleyball and went kayaking, 12 went swimming and went kayaking and 4 did all three activities. *See* Example 9

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a student selected at random:

b participated in none of the three events
c participated in exactly one event
d only went kayaking

18 Two six-sided dice are rolled and the total of the uppermost faces is noted. *See* Example 10

a Construct a two-way table to represent this experiment.

Use the two-way table to find the probability that the total is:

b odd
c not prime
d less than 7
e greater than 8
f even or greater than 8
g even and greater than 8

19 A six-sided die is rolled. What is the probability of: *See* Examples 11, 12

a rolling a 3?
b not rolling a 3?

See Examples 11, 12 **20** A bag contains 8 red marbles, 2 black marbles and 5 white marbles. One marble is randomly taken out of the bag.

a How many different outcomes are possible?
b What is the probability that a black marble is selected?
c What is the probability that the selected marble is not black?

See Examples 14, 15 **21** A bag contains four cards numbered 2, 5, 6 and 9. A two digit number is formed by drawing a card, writing down its number, returning it to the bag and then repeating the first step.

a Draw a tree diagram to represent this experiment.

Use this tree diagram to calculate the probability of obtaining:

b a number ending in 6 **c** an even number
d a number greater than 55 **e** a number less than or equal to 55
f a multiple of 5 **g** not a multiple of 5

See Example 17 **22** A box contains five chocolates – one milk chocolate, two dark chocolates and two white chocolates. A chocolate is taken out of the box and eaten. Another chocolate is then taken and eaten.

a Draw a two-way table to represent this experiment.

Use this two-way table to calculate the probability that the selected chocolates were:

b the same type **c** different types
d a dark and a white one **e** a dark one followed by a white one
f both white **g** both dark

Problem solving

23 The amount of petrol (in litres) sold by a service station is shown below.

63	25	50	21	18	26	39	29	45	31
35	53	19	50	44	63	26	20	53	40
58	48	49	21	50					

Use this information to decide if the next petrol sale is likely to be more or less than 30 L.

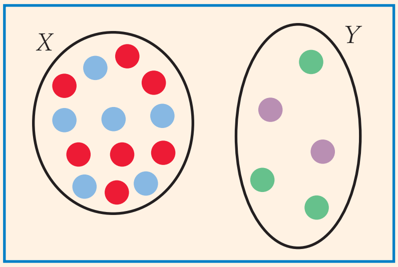
24 A normal coin was tossed 150 times and landed tails 72 times.

a What is the relative frequency of obtaining:

i a head **ii** a tail?

b Compare the results of part **a** with the theoretical probability of tossing a head or a tail.

See Example 12 **25** A jar contains 18 discs: each one coloured either red, blue, green or purple. The Venn diagram on the right shows two events X and Y that result when a disc is drawn and its colour noted.



Calculate:

a $P(X)$ **d** $1 - P(Y)$
b $P(Y)$ **e** $P(X) + P(Y)$
c $1 - P(X)$

Complete the following:

f $P(X') = \ldots$ **g** $P(Y') = \ldots$

26 If 100 head of cattle from the property in question **5** are sold, how many would you expect to be Brahmans?

9780170194778

27 It is estimated that 4% of the Australian population have red hair. If there are 260 people in a cinema, how many would you expect to be redheads?

28 Of a total of 100 people at a business conference 26 pay their bills using BPAY, 33 use internet banking and 31 use telephone banking. Eight said they use BPAY and internet banking, 12 said they use internet banking and telephone banking and 11 said they use telephone banking and BPAY. Five said they use all three methods to pay their bills. What is the probability that a randomly selected conference goer: *See* Example 9

a only uses internet banking to pay bills

b does not use any of the three methods (BPAY, internet banking or telephone banking) to pay bills?

29 A family has three children. What is the probability of:

a no girls

b exactly two boys

c the youngest being a boy

d a boy and a two girls?

Reasoning

30 A six-sided die is rolled 1000 times. Do you think that you would be able to accurately predict the number of times a 4 will be rolled based on the results of question **11**?

31 The faces of a twelve-sided die are numbered 1 through 12 and the faces on a six-sided die are numbered 1 through 6. The two dice are rolled and the numbers on the upper faces are noted. This is repeated 500 times.

a How many times would you expect to get two sixes?

b How many times would you expect to get a total of 10?

c How many times would you expect to get a total greater than 10?

32 A coin is tossed three times and the outcomes noted. *See* Example 15

a Draw a tree diagram to represent this experiment.

b List the sample space.

c Find P(exactly two tails).

d What is P(at least two tails)?

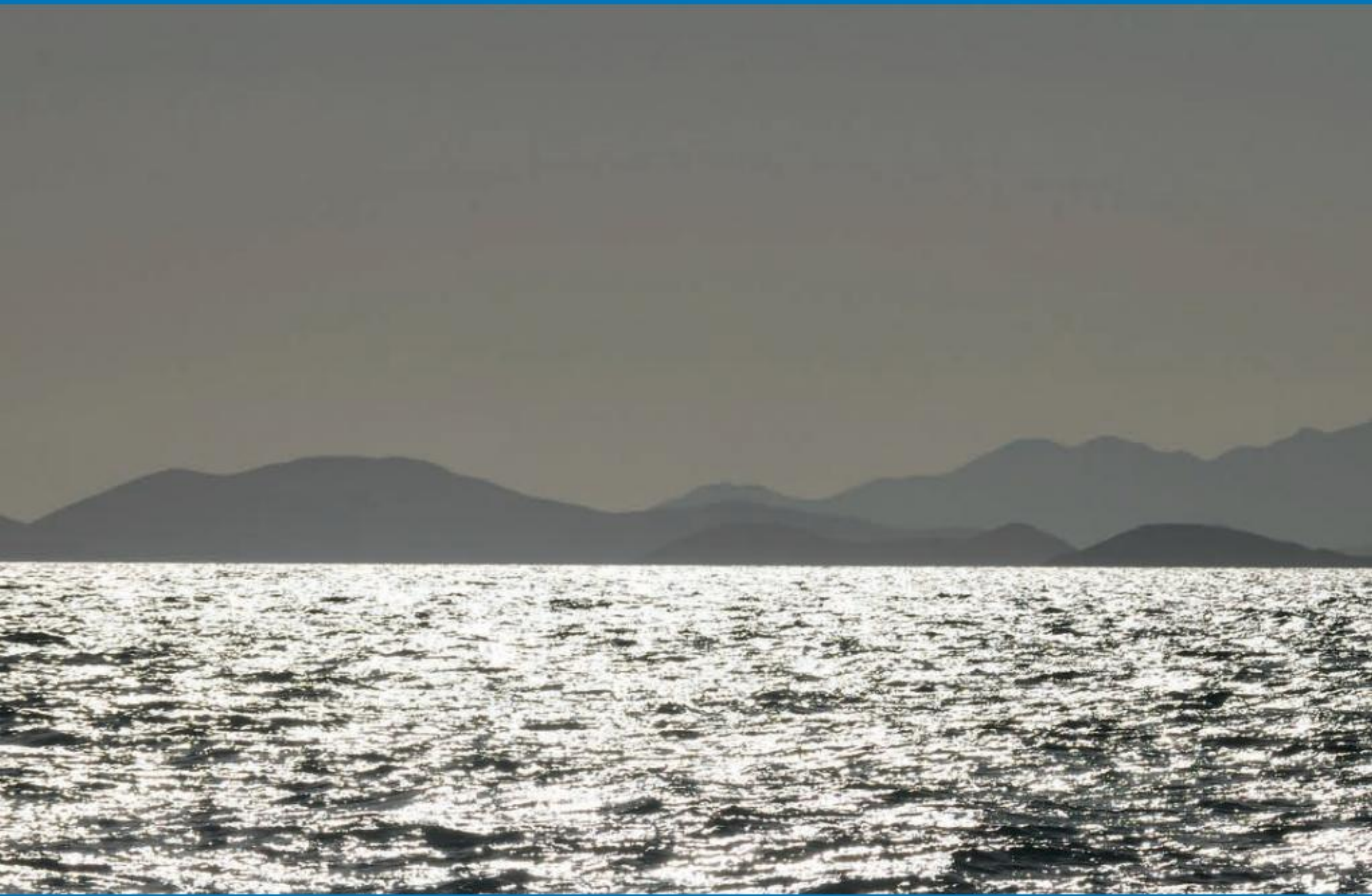
e Find P(a head and two tails).

The experiment is repeated 100 times.

f How many times would you expect to get at least one tail?

g How many times would you expect to get a tail and two heads?

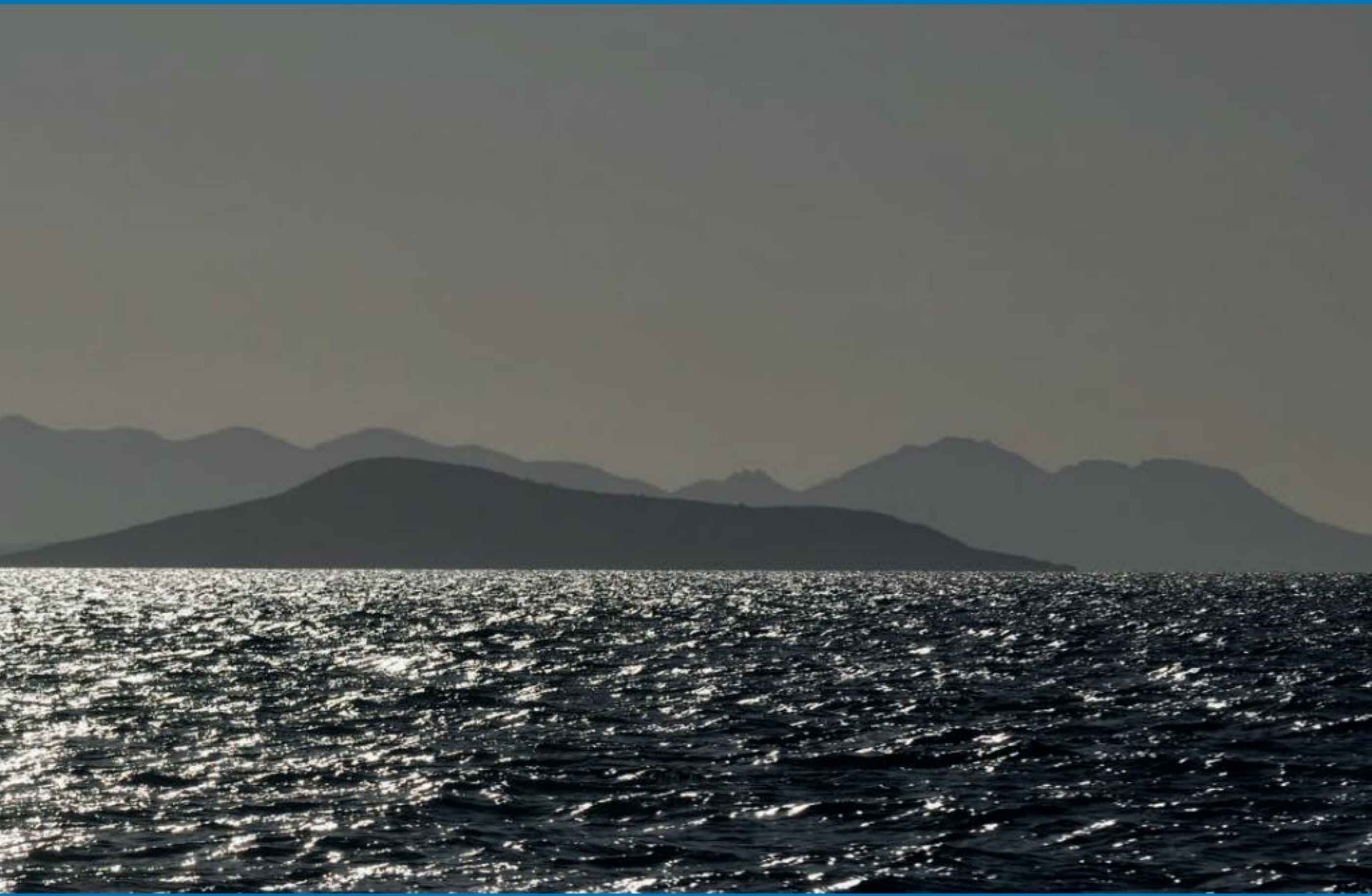
33 A coin is tossed four times. Find the probability that the outcome has exactly two tails.



Number and algebra

9 Algebraic expressions

9780170194778



Contents

Prior learning
Chapter 9
MAT08NAPL00009

Parent guide
Chapter 9
MAT08NAPG00009

Curriculum guide
Chapter 9
MAT08NACU00009

Australian Curriculum statements

Patterns and algebra

Extend and apply the distributive law to the expansion of algebraic expressions.

Factorise algebraic expressions by identifying numerical factors.

Simplify algebraic expressions involving the four operations.

When the council is going to build new roads, bridges, buildings and so on they often make models for people to look at. When an architect designs a new art gallery, he will often make a model. When we are children, we play with model cars, model people, model bricks, etc. Scientists make models to explain how the world works.
In mathematics, models are very useful tools for solving problems. Many problems are solved with algebraic models. Expressions and equations are actually models. In this chapter you will learn more about expressions.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

algebraic expression	common factor	expression	simplify
arithmetic expression	constant	factorise	subject
binomial brackets	distributive law	formula	substitute
coefficient	evaluate	highest common factor	term
collecting like terms	expand	like terms	variable

9.1 Algebraic modelling

You use algebra to write down what calculations you should perform to work something out. For example, if you wanted to write down how to change the length of a film in hours and minutes just to minutes, you could write $60h + m$. This would mean: 'multiply the hours by 60 and add the minutes on.'

Weblink
Algebraic dancing
MAT08NAWB00009

Important!

Variables and expressions

A **variable** is a letter or symbol that stands for a number. A **constant** is a number.
An **expression** has variables and/or numbers connected by arithmetic operations like +, ÷ and powers. An expression that just has numbers is an **arithmetic expression**, while one with variables is an **algebraic expression**. The numbers that are multiplied by the variables are called **coefficients**, the parts separated from the rest by + or − are called **terms**, and a number on its own is called a **constant term**.

Coefficients Constant term
Terms

$$5x + 2p - 6$$

In algebra, you usually leave out the multiply symbol × between a constant and a variable, and between variables. So in the expression shown above, $5x$ means $5 \times x$ and $2p$ means $2 \times p$.
To **evaluate** an algebraic expression you use **substitution**. You put in values for the variables (**substitute** values) and work out the answer.

9780170194778

Example 1

Write an expression to change an amount in d dollars and c cents to the same amount in only cents.

Solution

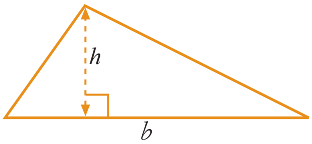
Write what the variables mean.	**d is the amount of dollars and c is the number of cents.**
Every dollar is 100 cents, so multiply the dollars by 100.	**100 d**
Add on the cents.	**The amount is $100d + c$.**

Example 2

Worksheet
Working with formulas
MAT08NAWK00077

The expression $\frac{1}{2}bh$ is used to work out the area of a triangle with base length b and height h. Evaluate the area of a triangle with base 8 cm and height 7 cm.

Solution



Write the expression.	$A = \frac{1}{2}bh$
Substitute $b = 8$ and $h = 7$.	$= \frac{1}{2} \times 8 \times 7$
Work out the answer.	$= 28$
Write the area, remembering to use square units.	**The area is 28 cm^2.**

Important!

Formulas

A **formula** is an equation with a variable by itself on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula. The value of the subject variable is calculated by evaluating the expression on the right using given values for the variables in the expression.

9780170194778

TLF learning object
Bridge builder: Complex squares (L1925)
MAT08NAIN00009

Example 3

The voltage across a resistor is given by the formula $V = IR$ where I is the current in amps and R is the resistance in ohms. Find the voltage when a current of 4 amps passes through a resistor of 60 ohms.

Solution

Write the formula.	$V = IR$
Substitute values.	$= 4 \times 60$
Write the units.	$= 240$ volts
Write the answer.	**The voltage is 240 volts.**

Complicated algebraic expressions can often be made easier. This is called **simplification** because it makes the expression simpler.

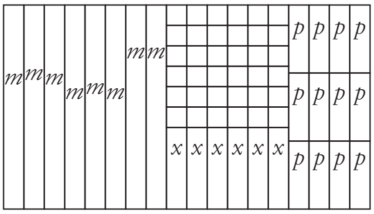
Investigate: Simplifying expressions

Teacher notes
Simplifying expressions
MAT08NATN00016

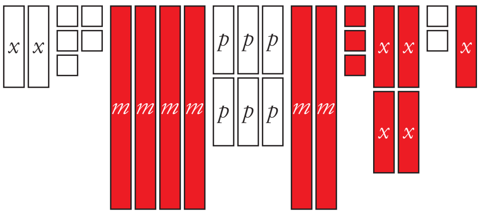
TLF learning object
The divider: With or without remainders (L2006)
MAT08NAIN00009

You can use a model to show simplification of expressions by combining terms.
You will need some small squares and strips of paper coloured on one side to model numbers and variables.
You can make the squares and strips as follows:

- Draw lines 1 cm apart on a blank piece of paper in the pattern shown on the right.
- It doesn't matter how long the strips are, as long as they are not an exact centimetre in length. Label the strips of equal length m, p and x.
- Use a highlighter or coloured pencil to shade the other side of the paper.
- Cut out the squares and strips and label the reverse sides of the strips.

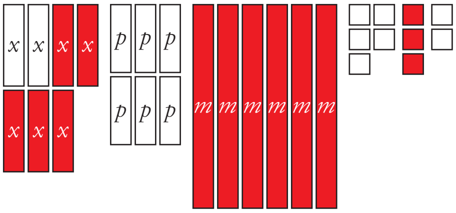


The white side represents positive and the coloured side represents negative.
Use your strips and squares to model $2x + 5 - 4m + 6p - 2m - 3 - 4x + 2 - x$ using your strips and squares. Your model should look like this:

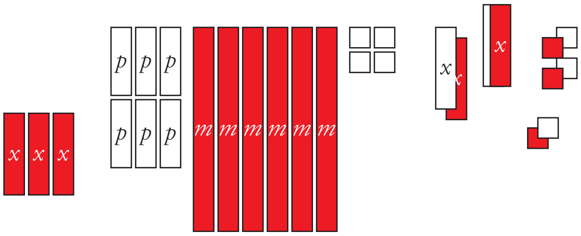


9780170194778

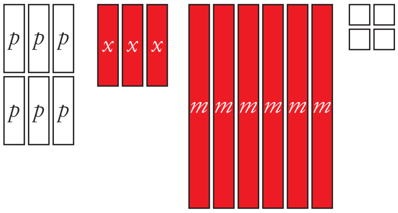
Now move your model around so the same parts are together. Put the numbers at the right, like this:



A positive x and a negative x will cancel out, so rearrange to pair up whatever positives and negatives you can and put the pairs at the end like this:



Remove the cancelling pairs and put a group of positives at the front if you can.



Write down the result.

$2x + 5 - 4m + 6p - 2m - 3 - 4x + 2 - x$

$\qquad = 6p - 3x - 6m + 4$

Use your modelling method to simplify the following:

$7m - 2m + 4m$	$2p + 6 + 3p - 8$	$x - 4 + 3x + 1$
$2p - 5 - 7p + 2 + p$	$x + 5m - 4x + 2m$	$3p - 5x + 7p - 2x + 4$
$8p + 6x - 7 - 5p + x - 4p + 3$	$m + x + 5 - 3x + 5m + 8 - 6x + m - 3$	

Your teacher might want you to try some others as well.

Worksheet
Posts and rails
MAT08NAWK00055

Worksheet
House patterns
MAT08NAWK00056

The investigation above shows that some algebraic expressions can be simplified. The parts with the same variables or constants can be put together to make a simpler expression.

Important!

Like terms

If an algebraic expression contains terms with exactly the same variables, we call these **like terms**. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**.

Example 4

Identify the like terms in each of the following expressions.

a $3p, 5p, 7, 6x, 4p^2, p, 4px$

b $2x, 3xy, 4p, 5py, 7xy, 5x^2, 4yx$

c $4m, 6m^2, {}^-4m, 7, m, mn, 2mn, 7m^2n$

Solution

a $p^2 = p \times p$, 7 and $4p \times x$ are not like p. **The like terms are $3p, 5p, p$**

b $yx = y \times x = x \times y = xy$. **The like terms are $3xy, 7xy, 4yx$**

c m and mn are different. **The like terms are $4m, {}^-4m, m$ and $mn, 2mn$**

When you collect like terms to simplify an expression think of the sign in front of the term as being part of the term.

Example 5

Video tutorial
Adding and subtracting terms
MAT08NAVT10004

Animated example
Collecting like terms
MAT08NAAE00012

Puzzle sheet
Adding and subtracting like terms
MAT08NAPS00033

Worksheet
Simplifying expressions with like terms
MAT08NAWK00058

Simplify each of the following expressions by collecting like terms.

a $5b - 7b + b$

b ${}^-3y + 5z + 8y - 7z$

c $4c + 8 - 5c + 9 + 6c - 5$

d $3x + 5xy^2 - 3 + xy - 5x + 2xy^2 - 4 + 8x^2y + x - 6xy$

e $x^3 + 4x^2 - 5x + 2 - 9x^2 - 3x - 7$

Solution

a Think of b as $1b$. — $\mathbf{5b - 7b + 1b}$

Think $5 - 7 + 1 = {}^-1$ — $\mathbf{= {}^-1b}$

Write $1b$ as just b. — $\mathbf{= -b}$

b Identify the like terms. — $\mathbf{{}^-3y + 5z + 8y - 7z}$

For the y terms, think ${}^-3 + 8 = {}^-5$. — $\mathbf{{}^-5y - 2z}$ **[or $-5y - 2z$]**

For the z terms, think $5 - 7 = {}^-2$.

c The constants are different to the variables. — $\mathbf{4c + 8 - 5c + 9 + 6c - 5}$

Rearrange the longer ones if you like. — $\mathbf{= 4c - 5c + 6c + 8 + 9 - 5}$

For the c terms, think $4 - 5 + 6 = 5$. — $\mathbf{= 5c + 12}$

For the constant terms, think $8 + 9 - 5 = 12$.

d Identify the like terms. — $\mathbf{3x + 5xy^2 - 3 + xy - 5x + 2xy^2 - 4 + 8x^2y + x - 6xy}$

Rearrange to group the like terms. — $\mathbf{3x - 5x + x + 5xy^2 + 2xy^2 + xy - 6xy - 3 - 4 + 8x^2y}$

For the x terms, think $3 - 5 + 1 = {}^-1$. — $\mathbf{= -x + 7xy^2 - 5xy - 7 + 8x^2y}$

For the xy^2 terms, think $5 + 2 = 7$.

9780170194778

For the xy terms, think $1 - 6 = {}^-5$.

For the constant terms, think ${}^-3 - 4 = {}^-7$.

Put a positive term first if possible. $= \mathbf{7xy^2 - x - 5xy - 7 + 8x^2y}$

e Identify the like terms. $\mathbf{x^3 + 4x^2 - 5x + 2 - 9x^2 - 3x - 7}$

Arrange with higher powers first. $= \mathbf{x^3 + 4x^2 - 9x^2 - 5x - 3x + 2 - 7}$

For the x^2 terms, think $4 - 9 = {}^-5$. $= \mathbf{x^3 - 5x^2 - 8x - 5}$

For the x terms, think ${}^-5 - 3 = {}^-8$.

For the constant terms, think $2 - 7 = {}^-5$.

When you multiply simple algebraic terms with a coefficient and variable(s), you can use the commutative and associative laws to change the order of multiplication.
For example:

$$\begin{aligned} 5k^2 \times 3km &= 5 \times k \times k \times 3 \times k \times m \\ &= 5 \times 3 \times k \times k \times k \times \mathrm{m} \\ &= 15 \times k^3 \times m \\ &= 15k^3m \end{aligned}$$

You can divide in a similar way but you do need to remember to divide by all the parts of the divisor. When dividing, most people find it simpler to treat simple algebraic terms like fractions: cancelling between the top and bottom. When the bottom won't cancel completely it is definitely easier this way.
For example:

$$\begin{aligned} 18m^3n \div 9mn &= (18 \div 9) \times (m^3 \div m) \times (n \div n) \\ &= 2 \times m^2 \times 1 \\ &= 2m^2 \end{aligned}$$

or

$$\begin{aligned} 18m^3n \div 9mn &= \frac{{}^{2}\cancel{18}\,\cancel{m^3}^{2}\,\cancel{n}^{1}}{{}_{1}\cancel{9}\,{}_{1}\cancel{m}\,{}_{1}\cancel{n}} \\ &= \frac{2 \times m^2 \times 1}{1 \times 1 \times 1} \\ &= 2m^2 \end{aligned}$$

Important!

Multiplying and dividing terms

When you multiply or divide algebraic terms, the coefficients and each different variable are multiplied or divided separately.

Example 6

Simplify the following:

a $6h \times 5g$

b $7p^2q \times {}^-2pq^3 \times 4pqr$

c $12ab \div 3a$

d $20p^2q \div {}^-24pq^3$

9780170194778

Solution

a Write the problem. $6h \times 5g$

Multiply the coefficients separately. $= 6 \times 5 \times h \times g$

Complete the answer $= 30hg$

b Write the problem. $7p^2q \times {}^{-}2pq^3 \times 4pqr$

Separate the coefficients and variables. $= 7 \times {}^{-}2 \times 4 \times p^2 \times p \times p \times q \times q^3 \times q \times r$

Perform the multiplications. $= {}^{-}56p^4q^5r$

c Write the problem as a fraction. $\dfrac{12ab}{3a}$

Cancel where possible. $= \dfrac{{}^{4}\cancel{12}\cancel{a}b}{{}_{1}\cancel{3}\cancel{a}}$

Write the answer. $= 4b$

d Write the problem as a fraction. $\dfrac{20p^2q}{{}^{-}24pq^3}$

Cancel where possible. Positive divided by negative is negative. $= -\dfrac{{}^{5}\cancel{20}\cancel{p^2}\cancel{q}}{{}_{6}\cancel{24}\cancel{p}\cancel{q^3}{}_{q^2}}$

Write the answer. $= -\dfrac{5p}{6q^2}$

Example 7

Simplify $\dfrac{7x - x}{2y + 6y} \times \dfrac{7y - 2y}{4a + 6a}$.

Solution

Write the problem with the implied brackets on the top and bottom. $\dfrac{(7x - x)}{(2y + 6y)} \times \dfrac{(7y - 2y)}{(4a + 6a)}$

Now simplify the terms in each of the brackets. $= \dfrac{6x}{8y} \times \dfrac{5y}{10a}$

Cancel where possible. $\dfrac{{}^{3}\cancel{6}x}{{}_{4}\cancel{8}\cancel{y}} \times \dfrac{{}^{1}\cancel{5}\cancel{y}}{{}_{2}\cancel{10}a}$

Write the answer. $= \dfrac{3x}{8a}$

Exercise 9.1 Algebraic modelling

Understanding

Extra questions

Exercise 9.1

MAT08NAEQ00029

See Example 4

1 Identify the like terms in each of the following sets of expressions.

a $3p \quad 4p \quad 3pq \quad 4q \quad {}^{-}3p$

b $6x \quad 2x^2 \quad {}^{-}7x \quad 4x^3 \quad x$

c ${}^{-}2mn \quad 7mn^2 \quad 4m^2n \quad 5m^2n^2 \quad {}^{-}7mn^2$

d $-a \quad bc \quad 4b \quad {}^{-}3bc \quad {}^{-}b \quad abc \quad 6b$

e $5ef \quad {}^{-}f \quad 2ef \quad 4f^2 \quad 7e^2 \quad {}^{-}ef \quad 6f^2$

f $6 \quad {}^{-}m^2 \quad {}^{-}7 \quad 8m \quad 2m^3 \quad m$

9780170194778

g $2k - k^2 - \frac{1}{2}k - 8k^3 - 5 - {}^{-}k$　**h** $4x - 2x^2 - 3xy - \frac{x^2}{2} - \frac{xy^2}{3} - \frac{1}{4}xy$

i $3y - 4(x + y) - 4xy - \frac{x + y}{5} - 4x$

Weblink
The RITEMATHS project
MAT08NAWB00009

Fluency

2 Write an expression to work out the number of days in w weeks.

See Example 1

3 Write an expression to work out the unit cost of pineapples if there are 30 pineapples in a container and each container costs $\$C$.

4 Write an expression to work out the length in mm of something p m, q cm and r mm long.

5 Write an expression to work out the temperature in degrees Kelvin from its temperature T in degrees Celsius if absolute zero is ${}^{-}273.15$ °C.

6 Simplify each of the following expressions by collecting like terms.

See Example 5

a $3a + 3a$　**b** $7b + 2b$　**c** $4m + 3m$　**d** $6n + 2n$
e $3q + q$　**f** $2a + 4a$　**g** $2w + 5w$　**h** $9p + 7p$
i $8a - 3a$　**j** $7a - 7a$　**k** $9a - 3a$　**l** $6y - y$
m $7k - 6k$　**n** $10q - q$　**o** $4n - n$　**p** $15b - b$

7 Simplify each of the following if possible.

a $7a - 3a$　**b** $4m + m$　**c** $7t + 15t$　**d** $3a - 2b$
e $rt + 3rt$　**f** $6 + 12d$　**g** $a + a$　**h** $a - 1$

8 Simplify each of the following.

a $8a + 3a - 2a$　**b** $3n + 2n + 7n$　**c** $4c + 8a + 9c$　**d** $18t - 15t + 15t$
e $10y + 3y + 4y$　**f** $5g + 8g - 10g$　**g** $6w - w + 2w$　**h** $3a + 2a - a$

9 Simplify each of the following.

a ${}^{-}5a + q + 4 + 7a - 4q$　**b** ${}^{-}6t - 6m - 5 - 3t - 4m + 2$
c $2t + 4c - 3 - 6t - 6c - 5$　**d** ${}^{-}6g + 7h + 8 + 7g + 6h - 2$
e $-k + 7g + 1 + 7k + 5g - 9$　**f** ${}^{-}7b + 2b + 7b - 4b + 8$
g ${}^{-}6x + 2 + 3w + 2w - 8$　**h** $4g - 6 + m + 3g - m + 8$

10 Simplify each of the following.

a ${}^{-}5v + 2h + 2b + 6h + 5b + 2v$　**b** $3z + 4d - 6a + d - 6z + 3a$
c ${}^{-}5m - 5d + 4h + 7d - 4h - 4m$　**d** $7y - 4n + m - 3y - m + 4n$
e ${}^{-}3p + 2y + 4y + 4 - 7x - 5x - 2p + 3$　**f** $w - x - 6k - 5 + 5x - 6k - 3w + 1$
g $2c - 7w + 4 - 3c + 6u - 2u + 4w - 9$　**h** ${}^{-}3k - m - 7 + 7z + m - 5z - 2 - 5k$

11 Simplify each of the following.

a $3x^2 + 2x + 5 - x^2 - 3x + 7$　**b** $m^2 - 7m + 6 + 4m^2 - 8m - 2$
c $4t^2 + 3t - 5 + t^2 + 7t + 6$　**d** $2w^2 - 5w - 8 - 3w^2 + 2w - 2$

12 Simplify each of the following.

a $-ab + 3a + 5ab - 6b - 7b + 3ab$
b $2x^2 + 4x - 2 + 5xy - y^2 - 5y + 7 + x^2 + y^2 - 8 - xy$
c $5xyz + 2xy + 5y - xyz + 3xy + 5z + 2xy$
d $6ab + 2ab^3 + 2a^2b - 4ab^2 - 42a^2b + ab$

13 Simplify each of the following.

See Example 6

a $3 \times 4r$　**b** $8 \times s$　**c** $2d \times 3g$　**d** $2 \times 5a$
e $4m \times 3n \times p$　**f** $2x \times 6y$　**g** $ab \times 8$　**h** $8uv \times 3w$

14 Simplify each of the following.

a $8h \div 2$ b $16cd \div 4$ c $\frac{25gh}{5}$ d $40xy \div 4x$
e $30pq \div 6q$ f $\frac{18hk}{6kh}$ g $56m \div 8m$ h $\frac{32fg}{16g}$

15 Simplify each of the following expressions.

a $2tg^2n \times -tg$ b $^-5yu^2 \times 3y^3u$ c $^-7vn^3 \times 3v^2dn$
d $4h^2k \times -h^2dk$ e $7cd^2h \times 3ch^2$ f $5b^2q^3 \times 7a^2q^2b$
g $^-3aq^2 \times {}^-2am^3q^2$ h $n^2p \times {}^-8hp^2n$

16 Simplify each of the following expressions.

a $2p^3k \div {}^-8pw$ b $10td^2 \div {}^-14h^3d$ c $^-21t^2k \div {}^-6tk^3$
d $^-21c^4d \div 24cd^2$ e $12u^2d \div 16u^3d$ f $x^3p^3c \div 4x^5cp$
g $^-8w^3n \div 20w^2c$ h $27q^2x^3 \div 6m^3x^2$

See Example 7

17 Simplify each of the following expressions.

a $\frac{3y + 9y}{5z} \times \frac{10z}{4a}$ b $\frac{8a + 2a}{4b^2 - b^2} \times \frac{3b + 2b}{3c + c}$ c $\frac{2p + 3p}{5pq - pq} \times \frac{10qr - 2qr}{12t + 3t}$

d $\frac{15mn - 8mn}{3n^3 + n^3} \times \frac{5mp - mp}{4p^2 + 3p^2}$ e $\frac{3xy^3 + 17xy^3}{9x^3y + 6x^3y} \times \frac{7x + 2x}{4z + z}$

Problem solving

See Example 2

18 The expression πD is used to calculate the circumference of a circle. Calculate the circumference of a circle using the ancient Egyptian value of $\pi \approx 3\frac{1}{6}$ for a circle of diameter 12 m.

Worked solutions
Exercise 9.1
MAT08NAWS00029

19 The expression $\frac{5(f-32)}{9}$ is used to change degrees Fahrenheit (used in the US) to degrees Celsius. The temperature in New York during a heatwave was said to be 96 degrees. If the same temperature was recorded in Melbourne, what would it be given as?

20 Write an expression for the cost of m kg of brussel sprouts priced at $\$c$/kg. Use your expression to find the price for 2.4 kg priced at \$6.99/kg and state the cash price as well.

See Example 3

21 The formula for the distance d metres travelled by an object moving at speed v m/s for t seconds is $d = vt$. How far does an object moving at 5 m/s travel in 8 seconds?

22 The formula for the force F Newtons on an object of mass m kilograms accelerating at a m/s^2 is $F = ma$. What is the force exerted on an object of mass 10 kg accelerating at 5 m/s^2?

23 The formula for the average acceleration of an object a starting from rest is given by $a = \frac{v}{t}$, where v is the speed reached after time t. A stone dropped from a tower has accelerated to a speed of 29.4 m/s after 3 seconds. What is the acceleration of gravity?

Worked solutions
Exercise 9.1
MAT08NAWS00029

24 Write an expression for the amount of sugar required to make n individual hot strawberry soufflés if you need 5 egg whites, 1 cup of sugar and a punnet of strawberries to make 4 soufflés. Use your expression to find the amount of sugar needed for 6 individual soufflés.

Reasoning

25 Write a formula for the total mass m of a truck with an unloaded mass of u kg carrying V m^3 of sand with a density of d kg/m^3. Explain your formula and modify it to give the mass in tonnes.

Worked solutions
Exercise 9.1
MAT08NAWS00029

26 Write a formula to work out the cost d of painting a wall w m wide and h m high with paint that has a coverage of a m^2/L and costs $\$c$ per litre. Explain your formula and state why it may underestimate the cost.

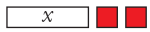
9780170194778

9.2 Expanding and simplifying

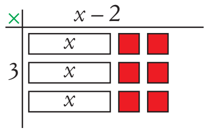
Investigate: Expanding brackets

You can use a simple model to work out the results of multiplying out algebraic brackets. Use the same pieces of paper that you used in the investigation 'Simplifying expressions'. Remember that the white side represents positive and the coloured side represents negative.

First make a model for $x - 2$. It should look like this:

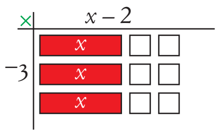


Now make another two models so that you have $3(x - 2)$. Place all three models on a multiplication cross like this:



Write the result of $3(x - 2)$ by counting what is inside the cross.

To change the multiplication cross to $^{-}3(x - 2)$, you need to flip the pieces inside the cross over to change the sign to give:



Write down the result of $^{-}3(x - 2)$.

Your teacher may want you to do some models of other expressions.

Teacher notes
Expanding brackets
MAT08NATN00014

Video tutorial
Expanding brackets
MAT08SPVT00008

One of the important laws of arithmetic is the **distributive law**. It is also used in algebra to **expand** brackets.

Important!

The distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers.

This can be written in symbols as:

$$a \times (b - c) = a \times b - a \times c$$

and

$$a \times (b + c) = a \times b + a \times c$$

The distributive law can be summarised as: $a(b \pm c) = ab \pm ac$.

Example 8

Expand $4(y + 8)$.

Solution

Write the problem.	$\mathbf{4(y + 8)}$
Use the distributive law.	$\mathbf{= 4 \times y + 4 \times 8}$
Simplify the answer.	$\mathbf{= 4y + 32}$

If the number outside is negative, you need to make sure that you write it properly when multiplying.

Puzzle sheet
Expandominoes
MAT08NAPS00035

Example 9

Expand $^{-}5(a + 3)$.

Solution

Write the problem.	$\mathbf{^{-}5(a + 3)}$
Use the distributive law.	$= \mathbf{^{-}5 \times a + {}^{-}5 \times 3}$
Simplify the answer.	$= \mathbf{^{-}5a + {}^{-}15}$
Write in the usual form.	$= \mathbf{^{-}5a - 15}$

If there is a subtraction in the brackets, you may find it simpler to write it as adding a negative number rather than as a subtraction.

Example 10

Expand each of:

a $6(k - 4)$ **b** $^{-}4(x - 5)$

Solution

a	Write the problem.	$\mathbf{6(k - 4)}$
	Write the subtraction as adding a negative.	$= \mathbf{6(k + -4)}$
	Use the distributive law.	$= \mathbf{6 \times k + 6 \times {}^{-}4}$
	Simplify the answer.	$= \mathbf{6k + {}^{-}24}$
	Write in the usual form.	$= \mathbf{6k - 24}$
b	Write the problem.	$\mathbf{^{-}4(x - 5)}$
	Write the subtraction as adding a negative.	$= \mathbf{^{-}4(x + {}^{-}5)}$
	Use the distributive law.	$= \mathbf{^{-}4 \times x + {}^{-}4 \times {}^{-}5}$
	Simplify the answer.	$= \mathbf{^{-}4x + 20}$
	Write with a positive term first.	$= \mathbf{20 - 4x}$

When there is more than one variable or powers involved, you need to be a little more careful but the same methods can be used.

9780170194778

Example 11

Expand each of the following.

a $3a(a + 2b)$ **b** $^{-}5x(4x^2 - 2y)$ **c** $3pq^2(2p^2 - 5q)$ **d** $^{-}4c(2c^3 - 2c^2 + 5c - 8)$

Solution

a Write the problem. $\mathbf{3a(a + 2b)}$

Use the distributive law. $\mathbf{= 3a \times a + 3a \times 2b}$

Simplify. $\mathbf{= 3a^2 + 6ab}$

b Write the problem with the subtraction written as addition of a negative. $\mathbf{^{-}5x(4x^2 + {}^{-}2y)}$

Use the distributive law. $\mathbf{= {}^{-}5x \times 4x^2 + {}^{-}5x \times {}^{-}2y}$

Simplify. $\mathbf{= {}^{-}20x^3 + 10xy}$

Write a positive term first. $\mathbf{= 10xy - 20x^3}$

c Write the problem with the subtraction written as addition of a negative. $\mathbf{3pq^2(2p^2 + {}^{-}5q)}$

Use the distributive law. $\mathbf{= 3pq^2 \times 2p^2 + 3pq^2 \times {}^{-}5q}$

Simplify and write in the usual way. $\mathbf{= 6p^3q^2 - 15pq^3}$

d Write the problem with the subtraction written as addition of a negative. $\mathbf{^{-}4c(2c^3 + {}^{-}2c^2 + 5c + {}^{-}8)}$

Use the distributive law. $\mathbf{= {}^{-}4c \times 2c^3 + {}^{-}4c \times {}^{-}2c^2 + {}^{-}4c \times 5c + {}^{-}4c \times {}^{-}8}$

Simplify. $\mathbf{= {}^{-}8c^4 + 8c^3 + {}^{-}20c^2 + 32c}$

Write with a positive term first. $\mathbf{= 8c^3 - 8c^4 - 20c^2 + 32c}$

You might need to do some simplification by collecting like terms after an expansion.

Example 12

Expand and simplify each of the following.

a $7m - 2n - 3(4m - n)$ **b** $4v(3v + 7n - m) - 3v(5v - 3n + 7m)$

Solution

a Write the problem with subtractions written as addition of negatives. $\mathbf{7m + {}^{-}2n + {}^{-}3(4m + {}^{-}1n)}$

Use the distributive law. $\mathbf{= 7m + {}^{-}2n + {}^{-}12m + 3n}$

Simplify. $\mathbf{= {}^{-}5m + n}$

Write with a positive term first. $\mathbf{= n - 5m}$

Puzzle sheet
Expanding
MAT08NAPS00034

Worksheet
Expanding brackets
MAT08NAWK00057

b Write the problem with subtractions written as addition of negatives. $4v(3v + 7n + {}^{-}m) + {}^{-}3v(5v + {}^{-}3n + 7m)$

Use the distributive law. $= 12v^2 + 28vn + {}^{-}4vm + {}^{-}15v^2 + 9vn + {}^{-}21vm$

Simplify and write in the usual way. $= 37vn - 3v^2 - 25vm$

The word **binomial** actually means two numbers. In algebra, **binomial products** are actually double brackets like $(x + 2)(x - 5)$.

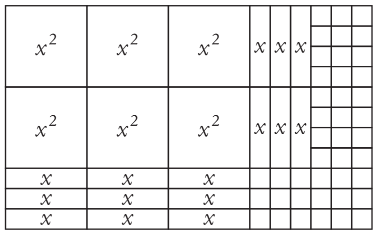
Investigate: Expansion of binomial brackets

For this investigation you will only look at binomial brackets with the variable x.

You will need the pieces of paper you used in the investigation 'Simplifying expressions' with x and the small squares. You will also need some larger squares with sides the same length as your x-strips.

Alternatively, you could make up a new set using a piece of paper ruled up in the way shown below.

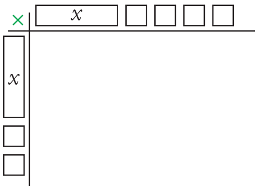
The small squares are 1 cm square and the strips are 1 cm wide.



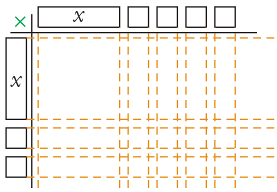
When you have ruled it up, use a highlighter or coloured pencil to shade the other side of the small squares and strips. Cut out all the shapes and strips and label the reverse sides.

The coloured sides of the strips and small squares represent negatives.

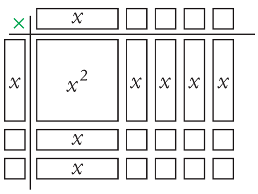
Draw a multiplication cross and show $(x + 2)(x + 4)$ along its sides like this:



Now picture lines running across and down from the strips and squares to make shapes in the multiplication cross.



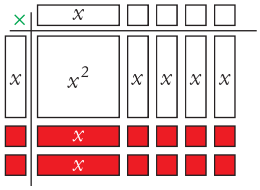
Put shapes into the spots that you have pictured to complete the model.



The model shows that $(x + 2)(x + 4) = x^2 + 6x + 8$.

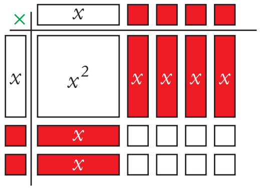
9780170194778

The model can easily be changed to show $(x - 2)(x + 4)$ by flipping shapes to give:



What is the result for $(x - 2)(x + 4)$?

You can flip shapes again to show $(x - 2)(x - 4)$.



What do you get for $(x - 2)(x - 4)$?

Now make models to find the expansions for the following.

$(x + 3)(x - 4)$ $(x + 2)(x + 3)$ $(x - 4)(x - 3)$ $(2x + 3)(x + 3)$ $(2x - 1)(2x - 3)$

Check your results with your teacher.

Binomial brackets can also be expanded using the distributive law, but you need to apply it several times. You need to be careful when multiplying negatives.

Example 13

Expand and simplify each of the following.

a $(x + 3)(x + 4)$ **b** $(g + 6)(g - 3)$ **c** $(h + 2)(h - 5)$

d $(p - 2)(p + 3)$ **e** $(m - 4)(m - 3)$

Solution

a Write the expression. $\quad (x + 3)(x + 4)$

Apply the distributive law to the first bracket. $\quad = x(x + 4) + 3(x + 4)$

Expand both brackets. $\quad = x^2 + 4x + 3x + 12$

Simplify. $\quad = x^2 + 7x + 12$

b Write the expression. $\quad (g + 6)(g - 3)$

Apply the distributive law to the first bracket. $\quad = g(g - 3) + 6(g - 3)$

Expand both brackets. $\quad = g^2 - 3g + 6g - 18$

Simplify. $\quad = g^2 + 3g - 18$

c Write the expression. $\quad (h + 2)(h - 5)$

Apply the distributive law to the first bracket. $\quad = h(h - 5) + 2(h - 5)$

Expand both brackets. $\quad = h^2 - 5h + 2h - 10$

Simplify. $\quad = h^2 - 3h - 10$

d Write the expression. $(p - 2)(p + 3)$
Apply the distributive law to the first bracket. $= p(p + 3) - 2(p + 3)$
Expand both brackets. $= p^2 + 3p - 2p - 6$
Simplify. $= p^2 + p - 6$

e Write the expression. $(m - 4)(m - 3)$
Apply the distributive law to the first bracket. $= m(m - 3) - 4(m - 3)$
Expand both brackets. $= m^2 - 3m - 4m + 12$
Simplify. $= m^2 - 7m + 12$

The method is still the same even when the variable does not have a coefficient of 1, or if there are several variables.

Example 14

Expand and simplify each of the following.

a $(3p + 2)(2p + 1)$ **b** $(4x - 3)(2x - 1)$ **c** $(2a - b)(3a + 2b)$ **d** $(g + 3h)(3g - h)$

Solution

a Write the expression. $(3p + 2)(2p + 1)$
Apply the distributive law to the first bracket. $= 3p(2p + 1) + 2(2p + 1)$
Expand both brackets. $= 6p^2 + 3p + 4p + 2$
Simplify. $= 6p^2 + 7p + 2$

b Write the expression. $(4x - 3)(2x - 1)$
Apply the distributive law to the first bracket. $= 4x(2x - 1) - 3(2x - 1)$
Expand both brackets. $= 8x^2 - 4x - 6x + 3$
Simplify. $= 8x^2 - 10x + 3$

c Write the expression. $(2a - b)(3a + 2b)$
Apply the distributive law to the first bracket. $= 2a(3a + 2b) - b(3a + 2b)$
Expand both brackets. $= 6a^2 + 4ab - 3ab - 2b^2$
Simplify. $= 6a^2 + ab - 2b^2$

d Write the expression. $(g + 3h)(3g - h)^2$
Apply the distributive law to the first bracket. $= g(3g - h) + 3h(3g - h)$
Expand both brackets. $= 3g^2 - gh + 9gh - 3h^2$
Simplify. $= 3g^2 + 8gh - 3h^2$

9780170194778

Exercise 9.2 Expanding and simplifying

Understanding

Extra questions
Exercise 9.2
MAT08NAEQ00030

See Example 8

1 Expand each of the following.

a $6(a+3)$ b $5(r+5)$ c $3(w+4)$ d $5(g+8)$ e $8(d+8)$
f $6(a+7)$ g $2(h+7)$ h $3(t+7)$ i $6(t+7)$ j $3(r+5)$

See Example 9

2 Expand each of the following.

a $^{-}8(m+3)$ b $^{-}3(n+7)$ c $^{-}8(x+7)$ d $^{-}4(u+4)$ e $^{-}2(z+3)$
f $^{-}3(y+3)$ g $^{-}3(r+3)$ h $^{-}6(d+3)$ i $^{-}3(p+4)$ j $^{-}5(v+7)$

See Example 10

3 Expand each of the following.

a $2(v-4)$ b $2(t-3)$ c $6(r-3)$ d $2(p-2)$ e $2(h-7)$
f $3(p-8)$ g $2(g-8)$ h $3(h-6)$ i $5(r-4)$ j $2(q-5)$

4 Expand each of the following.

a $^{-}3(r-4)$ b $^{-}8(a-2)$ c $^{-}6(r-5)$ d $^{-}3(x-8)$ e $^{-}8(w-4)$
f $^{-}8(q-4)$ g $^{-}3(v-7)$ h $^{-}5(x-4)$ i $^{-}6(a-3)$ j $^{-}3(q-4)$

Fluency

5 Expand each of the following.

a $2(5u-2)$ b $^{-}6(3a-2)$ c $3(4g-3)$ d $7(3w+2)$ e $^{-}6(8u-3)$
f $^{-}5(2t-7)$ g $^{-}5(3q-4)$ h $3(4g+5)$ i $2(3a-5)$ j $4(7h+4)$

See Example 11

6 Expand each of the following.

a $2y(9y-2m)$ b $^{-}9h(8h-8x)$ c $^{-}3c(c+9r)$
d $8n(2n^2-8c)$ e $6x^2(4x-9u)$ f $4vh(8v+4h)$
g $^{-}4bx(5b+2x)$ h $^{-}5x^2m(9x+3m-n)$ i $^{-}5am^2(2a-5m+4p)$
j $^{-}2w(2w^2+3wz-5z^2w-2)$

7 Expand each of the following.

a $4b(8b-5a+4d)$ b $^{-}4m(6m-5n+8z)$ c $3y(2y+5n+2h)$
d $8a(6a+7b-4p)$ e $5c(7c-7n-8y)$ f $^{-}5k(4k+8q+t)$
g $7x(9x-5p-4g)$ h $7d(7d+9g+2m)$ i $^{-}5m(3m-3z-8y)$
j $4r(4r+7c+6z)$

See Example 12

8 Expand and simplify each of the following.

a $4t+30+5(t+4)$ b $3t-15+4(t-7)$ c $8w+28+5(w-2)$
d $n-24+2(n+4)$ e $d-15+2(d-3)$ f $6p+1-2(p+4)$
g $8n+24-8(n-8)$ h $2w-14-6(w-5)$ i $3r+15-8(r+5)$
j $8h-16-6(h+5)$

9 Expand and simplify each of the following.

a $7(n-2)+6(n-8)$ b $4(w+2)-8(w+5)$ c $4(x+2)+5(x+8)$
d $5(a+3)-8(a-8)$ e $3(b-5)+6(b+2)$ f $3(w-4)+6(w+2)$
g $3(p-8)-3(p+5)$ h $2(b-3)-8(b+4)$ i $6(b-7)-2(b+7)$
j $6(q+7)-6(q-8)$

10 Expand and simplify each of the following.

a $6q(3q - 4z - 7h) - 5q(5q - 5z + 7h)$ **b** $9g(7g + 7q + 6m) - 2g(g - 9q - m)$
c $6u(6u + 7t + t) - 2u(2u + 5t - 6t)$ **d** $9h(7h + 5r + 6n) - 4h(4h + 7r + 3n)$
e $^{-}6r(5r + 2d - 9k) - 2r(5r - 4d - 9k)$ **f** $^{-}9g(5g + 9b + 7t) - 3g(g + 9b - t)$
g $8a(a - p - 6g) + a(6a + 6p - 5g)$ **h** $2c(7c - 8u - 7e) + 8c(c - u - 7e)$
i $7w(9w + 6u - 5y) - 2w(4w + u + 5y)$ **j** $^{-}9d(6d - 7b + v) - 7d(9d - 8b - v)$

See Example 13

11 Expand each of the following.

a $(w + 6)(w + 4)$ **b** $(r + 6)(r + 5)$ **c** $(z - 3)(z - 5)$ **d** $(t + 9)(t + 5)$
e $(p - 6)(p + 3)$ **f** $(u - 9)(u - 3)$ **g** $(x - 4)(x + 8)$ **h** $(t + 9)(t + 3)$
i $(r - 4)(r - 3)$ **j** $(k - 9)(k + 4)$ **k** $(t - 4)(t + 7)$ **l** $(q - 6)(q + 3)$
m $(q - 4)(q - 8)$ **n** $(w + 9)(w + 3)$ **o** $(t + 4)(t - 9)$ **p** $(x - 4)(x + 2)$
q $(m - 6)(m - 3)$ **r** $(c - 7)(c + 5)$ **s** $(d - 4)(d - 4)$ **t** $(u + 8)(u + 2)$

See Example 14

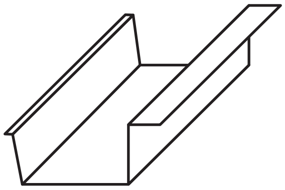
12 Expand each of the following.

a $(4q - 3d)(7q + 8d)$ **b** $(5x - 2t)(4x + 3t)$ **c** $(5d - 4y)(4d + 9y)$
d $(8u + 3t)(2u + 9t)$ **e** $(5g - 8p)(2g - 3p)$ **f** $(4w - 5p)(8w - 9p)$
g $(2w + 5q)(9w + 4q)$ **h** $(8z - 7n)(4z - 5n)$ **i** $(8g - 3w)(7g + 5w)$
j $(4v + 3n)(3v - 2n)$ **k** $(5x - 7g)(7x + 3g)$ **l** $(2d - 5y)(7d + 6y)$

Problem solving

Worked solutions
Exercise 9.2
MAT08NAWS00030

13 Rainwater gutters are made by cutting lengths of steel or aluminium into strips. These are then folded up on opposite sides as shown in the diagram on the right.



Different sized gutters are made by making the strips different widths.
Altogether, the sides and back-folds at the top take 22 cm from the width of the strip, with the height being 8 cm.
The strips are cut from sheet metal of length 6 m. If a strip is w cm wide, write an expression for the volume of water that can be held by a length of guttering made from the strip. Expand the expression. Find the volume for a strip of width 34 cm.

Worked solutions
Exercise 9.2
MAT08NAWS00030

14 Metal trays of different sizes are made from rectangular sheets 2.5 times as long as they are wide. The corners are cut out and the edges folded up a distance of 2.5 cm to make the sides. Write an expression for the volume of such a tray made from a sheet w cm wide. Expand and simplify the expression. Find the volume held by a tray 20 cm wide.

Reasoning

15 Explain how you work out the width needed for a strip to make guttering with a capacity of 72 litres per length for the guttering in question **13**.

Worked solutions
Exercise 9.2
MAT08NAWS00030

16 Explain how to find expressions for the expansions of $(x + y)^2$ and $(x + y)^3$.

9780170194778

9.3 Factorising

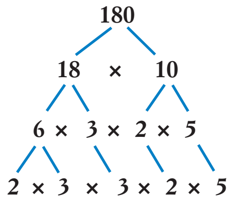
You have already seen what is meant by a **common factor** and the **highest common factor** of two numbers.

Example 15

Find the highest common factor of 180 and 168.

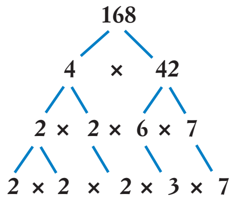
Solution

Draw a factor tree to find the prime decomposition of 180.



Write the prime decomposition of 180. $180 = 2^2 \times 3^2 \times 5$

Draw a factor tree to find the prime decomposition of 168.



Write the prime decomposition of 168. $168 = 2^3 \times 3 \times 7$

Write out the powers and find the primes that are in both 180 and 168.

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$168 = 2 \times 2 \times 2 \times 3 \times 7$

Use these for the highest common factor. $\text{HCF} = 2 \times 2 \times 3 = 2^2 \times 3\ (= 12)$

You can use the same technique to find the highest common factor of algebraic terms.

Example 16

Find the highest common factor of $20xy^2z$ and $24x^2ym$.

Solution

Write the decomposition of $20xy^2z$. $20xy^2z = 2^2 \times 5 \times x \times y^2 \times z$

Write the decomposition of $24x^2ym$. $24x^2ym = 2^3 \times 3 \times x^2 \times y \times m$

Write out the powers of each to find the common parts.

$20xy^2z = 2 \times 2 \times 5 \times x \times y \times y \times z$

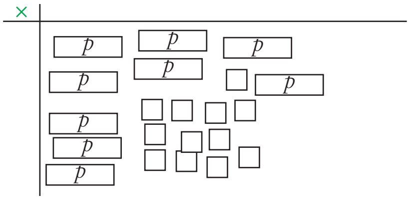
$24x^2ym = 2 \times 2 \times 2 \times 3 \times x \times x \times y \times m$

Use these for the highest common factor. $\text{HCF} = 2 \times 2 \times x \times y = 4xy$

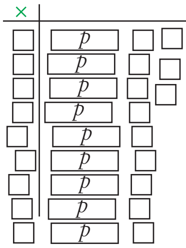
Investigate: Modelling factorisation

Teacher notes
Modelling factorisation
MAT08NATN00015

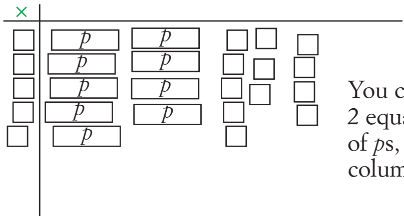
For this investigation you will need the pieces of paper that you used in the investigation 'Simplifying expressions'. You will also use a multiplication cross like you did in the 'Expanding brackets' investigation.
Place 9 p strips and 12 unit squares on your multiplication cross like this:



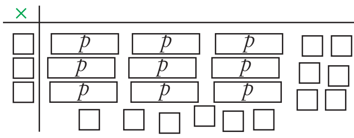
This shows $\mathbf{9p + 12}$. There are more ps than squares so start with a column of all the ps. Put spare squares at the side of the column. Now try to arrange the squares inside the cross to make rows that are all the same.



You can't fit the squares evenly, so try 2 columns of ps.

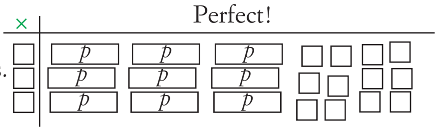


You can't make 2 equal columns of ps, so try 3 columns of ps.

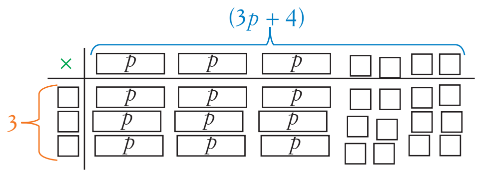


Try to fit the squares.

Perfect!

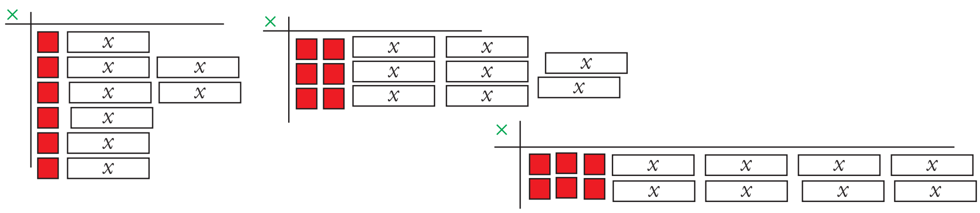


Finish the multiplication cross by completing the top with spare strips and squares.



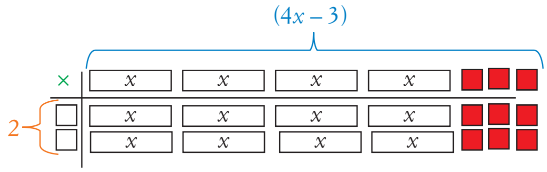
Now write the result: $9p + 12 = 3(3p + 4)$. [3 rows of $(3p + 4)$]
Notice that 3 is the highest common factor of $3p$ and 12.

Now try arranging $8x - 6$ on a multiplication grid. You need to use 6 coloured squares to represent negative 6. Since there are more xs than unit squares you must start with a column of the negative unit squares. The arrangement steps are shown below:

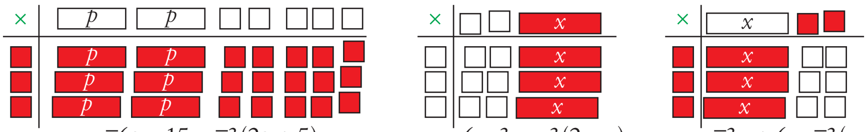


9780170194778

If possible put the positives first. If there is a choice, make it the variable. Put spare xs and squares at the top and side to complete the multiplication cross.



Now write the result: $8x - 6 = 2(4x - 3)$ [2 rows of $(4x - 3)$]
Notice that 2 is the highest common factor of $8x$ and $^-6$.
Some more completed multiplication crosses are shown below:



$^-6p - 15 = {}^-3(2p + 5)$ $\quad 6 - 3x = 3(2 - x)$ $\quad {}^-3x + 6 = {}^-3(x - 2)$

Notice that the last two completed crosses are actually the same thing, so $3(2 - x) = {}^-3(x - 2)$.

In the previous investigation your multiplication squares looked the same as the ones you got in the investigation ‘Expanding brackets’.

Important!

Factorisation

Factorisation is the opposite process of multiplying out brackets. In the **common factor** method the highest common factor of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.

Example 17

Video tutorial
Factorising expressions
MAT08NAVT10006

Puzzle sheet
Factorising
MAT08NAPS00036

Factorise each of the following.

a $12x + 30$ **b** $25a + 10b - 20c$

Solution

a Write the expression.	$\mathbf{12x + 30}$
Write each term using the highest common factor.	$= \mathbf{6 \times 2x + 6 \times 5}$
Take out the HCF and leave the rest in brackets.	$= \mathbf{6(2x + 5)}$
b Write the expression.	$\mathbf{25a + 10b - 20c}$
Write each term using the HCF of all three terms.	$= \mathbf{5 \times 5a + 5 \times 2b + 5 \times {}^-4c}$
Take out the HCF and leave the rest in brackets.	$= \mathbf{5(5a + 2b + {}^-4c)}$
Write the answer in the usual way.	$= \mathbf{5(5a + 2b - 4c)}$

When all the terms are negative, use a negative number as the HCF.

Example 18

Factorise $^-14t - 21$.

Solution

Write the expression.	$\mathbf{^-14\boldsymbol{t} - 21}$
Since all terms are negative include the negative sign in the HCF.	$\mathbf{^-7 \times 2\boldsymbol{t} + {^-7} \times 3}$
Take out the HCF and leave the rest in brackets.	$\mathbf{= {^-7}(2\boldsymbol{t} + 3)}$

If the expression starts with a negative but there is a positive term, there are several ways of writing the answer.

Example 19

Puzzle sheet
Factorising with negative terms
MAT08NAPS00037

Factorise each of the following.

a $^-15m + 20n$ **b** $^-8p - 4q + 12$

Solution

a Method 1

Write the expression.	$\mathbf{^-15\boldsymbol{m} + 20\boldsymbol{n}}$
Write with $^-5$ as the HCF.	$\mathbf{= {^-5} \times 3\boldsymbol{m} + {^-5} \times {^-4}\boldsymbol{n}}$
Take out the HCF and write the brackets as usual.	$\mathbf{= {^-5}(3\boldsymbol{m} - 4\boldsymbol{n})}$

Method 2

Write the expression with a positive term first.	$\mathbf{20\boldsymbol{n} + {^-15}\boldsymbol{m}}$
Write with 5 as the HCF.	$\mathbf{= 5(4\boldsymbol{n} - 3\boldsymbol{m})}$
Take out the HCF and write the brackets.	

b Method 1

Write the expression.	$\mathbf{^-8\boldsymbol{p} - 4\boldsymbol{q} + 12}$
Write with $^-4$ as the HCF.	$\mathbf{= {^-4} \times 2\boldsymbol{p} + {^-4} \times \boldsymbol{q} + {^-4} \times {^-3}}$
Take out the HCF and write the brackets as usual.	$\mathbf{= {^-4}(2\boldsymbol{p} + \boldsymbol{q} - 3)}$

Method 2

Write the expression with a positive term first.	$\mathbf{12 + {^-8}\boldsymbol{p} - 4\boldsymbol{q}}$
Write with 4 as the HCF.	$\mathbf{= 4 \times 3 + 4 \times {^-2}\boldsymbol{p} + 4 \times {^-\boldsymbol{q}}}$
Take out the HCF and write the brackets.	$\mathbf{= 4(3 - 2\boldsymbol{p} - \boldsymbol{q})}$

You can see from Example **19** that $^-5(3m - 4n) = 5(4n - 3m)$ and $^-4(2p + q - 3) = 4(3 - 2p - q)$. In general, if you change the sign of the common factor outside the brackets you must also change the sign of *every* term inside the brackets.

You may have to use one or more of the variables as part of the highest common factor.

9780170194778

Example 20

Factorise each of the following.

a $4x^2 + 6xy$ **b** $24tu - 20u^2 + 32uv$

Solution

a Write the expression. $\mathbf{4x^2 + 6xy}$

Write using the HCF of $2x$. $= \mathbf{2x \times 2x + 2x \times 3y}$

Take out the HCF and write the brackets. $= \mathbf{2x(2x + 3y)}$

b Write the expression. $\mathbf{24tu - 20u^2 + 32uv}$

Write using the HCF of $4u$. $= \mathbf{4u \times 6t + 4u \times {}^{-}5u + 4u \times 8v}$

Take out the HCF and write the brackets. $= \mathbf{4u(6t - 5u + 8v)}$

Exercise 9.3 Factorising

Understanding

Extra questions

Exercise 9.3

MAT08NAEQ00031

1 Find the highest common factor of each of the following pairs.

a 385 – 35 **b** 1089 – 66 **c** 385 – 10 **d** 126 – 231
e 1078 – 539 **f** 175 – 21 **g** 495 – 75 **h** 48 – 36
i 231 – 33 **j** 231 – 121

See Example 15

2 Find the highest common factor of each of the following pairs.

See Example 16

a $8g^2y - 28gy^2$ **b** $9hv - 33hr$ **c** $6p^2b - 9pb$ **d** $5m^2qb - 2mq^2b$
e $5w^2br - 7wb^2$ **f** $4g^2u^2 - 10g^2u$ **g** $5zd^2 - 11z$ **h** $15g^2a - 9gv$
i $15pm - 9m$ **j** $8g^2q^2 - 44gqz$

3 Factorise each of the following.

See Example 17

a $5w + 45$ **b** $4t + 8$ **c** $3r + 6$ **d** $5z + 10$ **e** $4k + 32$
f $8d + 8$ **g** $5t + 45$ **h** $7x + 63$ **i** $5w + 45$ **j** $4w + 20$

4 Factorise each of the following.

a $40m + 30$ **b** $12m + 18$ **c** $4d + 32$ **d** $54v + 36$ **e** $3c + 15$
f $48z + 56$ **g** $14r + 2$ **h** $36k + 24$ **i** $18n + 36$ **j** $45x + 40$

5 Factorise each of the following.

a $24t + 12q$ **b** $8w + 72$ **c** $10u + 18$ **d** $32c + 24$ **e** $16r + 24$
f $24m + 36$ **g** $12h + 6$ **h** $24h + 42$ **i** $4k + 12$ **j** $48h + 24$

6 Factorise each of the following.

a $30u - 18w - 42g$ **b** $45p - 9u + 63a$ **c** $8r + 10x + 14n$ **d** $20h - 12b + 4d$
e $24n + 9u - 21v$ **f** $24u - 4p - 12a$ **g** $72g + 72b - 63b$ **h** $45n - 45r + 27w$
i $56d - 64d - 72v$ **j** ${}^{+}72u - 24m + 40v$

7 Factorise each of the following.

See Example 18

a ${}^{-}5z - 15x$ **b** ${}^{-}12h - 6t$ **c** ${}^{-}12y - 18b$ **d** ${}^{-}40k - 16w$
e ${}^{-}9u - 36a$ **f** ${}^{-}20b - 20k$ **g** ${}^{-}49y - 56w$ **h** ${}^{-}24v - 28w$
i ${}^{-}18n - 27z$ **j** ${}^{-}9d - 24v$

9780170194778

Fluency

See Example 19

8 Factorise each of the following.

a $^{-}8d + 12q$ b $^{-}16g + 8m$ c $^{-}32g + 8h$ d $12h + 20u$
e $^{-}21k + 49a$ f $42c - 12k$ g $36z - 45t$ h $^{-}27r + 63h$
i $^{-}45y - 30g$ j $40b - 20k$ k $48b - 8r$ l $^{-}14u - 6r$
m $21c - 24d$ n $^{-}18x + 12t$ o $24t - 12m$ p $72c - 9w$
q $^{-}42v - 49r$ r $^{-}6b - 10x$ s $28y + 4n$ t $^{-}20a + 15y$

9 Factorise each of the following.

a $^{-}18t + 54g + 27w$ b $^{-}16z + 10v - 14q$ c $36z - 18d + 12w$
d $10k - 30r + 35y$ e $^{-}8w + 40y + 56c$ f $^{-}42t - 36r + 18d$
g $^{-}72x + 32u - 16g$ h $^{-}18u + 3t + 12v$ i $^{-}4b + 28x + 20y$
j $^{-}35y - 7k - 42v$ k $3a - 24h - 18b$ l $^{-}16t + 48y + 32q$
m $16q + 8v + 64a$ n $^{-}6b + 18h - 3t$ o $7y + 7n + 14q$
p $^{-}12c + 18x + 42n$ q $6b + 24y + 18d$ r $21z - 3w + 3c$
s $32h + 8m + 20v$ t $8w - 12z + 14c$

See Example 20

10 Factorise each of the following.

a $^{-}9q^2 - 63qu$ b $28vz - 16v^2$ c $4w^2 - 18wz$ d $81v^2 + 36vu$
e $^{-}2r^2 - 2rv$ f $6x^2 + 36xh$ g $7g^2 + 21gh$ h $^{-}18d^2 - 9dw$
i $6tm - 9t^2$ j $35t^2 - 40tu$ k $54w^2 + 36wb$ l $2bw - 14b^2$
m $48wp - 6w^2$ n $^{-}35k^2 - 28kn$ o $6a^2 - 18an$ p $^{-}64c^2 - 56cw$
q $35rt - 14t^2$ r $42r^2 + 21rk$ s $24uz - 36u^2$ t $28t^2 - 32td$

11 Factorise each of the following.

a $42y^2 - 36yg + 24yt$ b $^{-}49v^2 - 21vm - 14vg$ c $42u^2 - 14uq - 35uw$
d $^{-}64k^2 + 72kr - 48kn$ e $8q^2 - 24qt + 12qa$ f $12q^2 - 12qc + 36qz$
g $28m^2 - 36mu - 12mb$ h $12q^2 - 16qd + 8qa$ i $^{-}18b^2 - 27bq + 3bm$
j $^{-}16z^2 - 12zb + 10zk$ k $^{-}72uv^2 + 18ur + 81uq$ l $21r^2 - 42rw - 49rd$
m $18y^2 - 54ya - 36yd$ n $5n^2 + np - nr$ o $^{-}9x^2 - 6xy - xk$
p $27x^2 - 36xw + 18xm$ q $18n^2 - 21nq + 18nu$ r $40n^2 - 72nc + 40nk$
s $^{-}30p^2 + 36py + 6pa$ t $8d^2 + 40dc + 16da$

Problem solving

Worked solutions
Exercise 9.3
MAT08NAWS00031

12 Factorise $3x(4b + 7) + 5(4b + 7)$

13 Factorise $2m^2(m^2 + 1) - 3(m^2 + 1)$

Reasoning

Worked solutions
Exercise 9.3
MAT08NAWS00031

14 The following expression can be factorised. Perform the factorisation and explain the steps in detail.
$(3ac + 6bc) - 7ad - 14bd$

Worked solutions
Exercise 9.3
MAT08NAWS00031

15 The following expression can be factorised. Perform the factorisation and explain the steps in detail.
$9vx - 15vy + 12ux - 20uy$

9780170194778

Chapter 9 summary

Quiz
Algebraic expressions
MAT08NAQZ00009

- A **variable** is a letter or symbol that stands for a number. A **constant** is a number by itself.
- An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $-$, $\times$, $\div$ and powers. An expression that has only numbers is an **arithmetic expression**, while one with variables is an **algebraic expression**.
- The numbers in an expression that are multiplied by variables are called **coefficients**. The parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**.
- To **evaluate** an algebraic expression you **substitute** values for the variables and work out (evaluate) the answer.
- A **formula** is an equation with a variable by itself on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula.
- Algebraic terms with exactly the same variables are called **like terms**. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**.
- When you multiply or divide algebraic terms, the coefficients and each different variable are multiplied or divided separately.
- The **distributive law** states that the product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c \quad \text{and} \quad a \times (b - c) = a \times b - a \times c.$$

- **Expansion of brackets** means the removal of brackets using the distributive law.
- **Binomial products** are double brackets like $(x + 2)(x - 5)$. They can be expanded using the distributive law several times.
- The **highest common factor** of two algebraic terms is the largest expression that will divide exactly into each of the given terms.
- **Factorisation** is the opposite process of multiplying out brackets. In the **common factor** method, the highest common factor of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.

Chapter 9 review

Understanding

See Example 15

1 Find the highest common factor of each of the following pairs of numbers.

a 66 – 165 **b** 252 – 168

See Example 4

Worksheet

Algebraic expressions review

MAT08NAWK00059

2 Identify the like terms in each of the following sets of expressions.

a $3d - 4de - 3d^2 - {}^{-}4ed - \frac{1}{4}de - d^3$

b $-kmg - 4kg - 5gmk - 5k^2m - 2kg - (k + g) - 2kmg$

See Examples 8–10

3 Expand each of the following.

a $5(x + 8)$ **b** ${}^{-}4(c + 2)$ **c** $7(k - 4)$ **d** ${}^{-}6(t - 5)$

See Example 16

4 Find the highest common factor of each of the following pairs of terms.

a $12k - 28m$ **b** $15x^2yz - 35xy^2u$

See Examples 17–18

5 Factorise each of the following.

a $7h + 21$ **b** $45u + 75$ **c** $16p - 28q + 20r$ **d** ${}^{-}9b - 12c$

Fluency

6 Write an expression to work out the volume in mL of k litres and m mL.

See Example 1

7 Write an expression to work out the number of days in n years if it includes m leap years.

See Example 5

8 Simplify each of the following expressions.

a $7x + 5x$ **b** $8m - m$

c $2p + q - 5p + 7q$ **d** ${}^{-}3m^2 + mn - 6n^2 + 6 - 8m^2 + 9mn + 2n^2$

See Example 6

9 Simplify each of the following expressions.

a $7 \times 8f$ **b** $28c \div 4$ **c** $4mn \times {}^{-}5mp$ **d** ${}^{-}7xy^2 \times {}^{-}2x^3z$ **e** ${}^{-}30g^3hk \div 18g^2h^4p^2$

10 Expand each of the following.

a ${}^{-}3(2u + 5)$ **b** $7(5d - 6)$ **c** ${}^{-}3(4p - 3)$ **d** $4(3m - 8)$

See Example 11

11 Expand each of the following.

a $2x(3x - 4y)$ **b** ${}^{-}5p(2q - 3p)$ **c** $4h(6h - 2k + 3)$ **d** ${}^{-}3u(4u + 3v - 5w)$

See Example 12

12 Expand and simplify each of the following.

a $2m + 12 - 5(m - 3)$ **b** $6(p + 2) + 4(p - 5)$ **c** $7m(2m - 4n + 3p) - 5m(4m - 5n + 2p)$

13 Simplify $\dfrac{3kp + 5kp}{20p^2 - 6p^2} \times \dfrac{15mn + 6mn}{3m^2 + m^2}$

See Examples 13–14

14 Expand each of the following.

a $(z + 2)(z + 5)$ **b** $(c + 2)(c - 7)$ **c** $(4k - 3d)(3k - 5d)$ **d** $(5x - 2y)(3x + 2y)$

See Examples 17–19

15 Factorise each of the following.

a ${}^{-}18x + 24y - 30z$ **b** ${}^{-}42m + 28n$ **c** $15n - 6p + 18q$ **d** $52a - 39b$

See Example 20

16 Factorise each of the following.

a $24y^2 - 9yz$ **b** ${}^{-}75e^2 + 35ae$ **c** ${}^{-}72b^2 - 24bc - 28bd$ **d** $12r^2 - 15rt + 30ru$

Problem solving

17 The expression $2(l + w)$, where l is the length and w is the width, is used to calculate the perimeter of a rectangle. Find the perimeter of a rectangle of length 3.8 m and width 2.1 m.

See Example 2

18 The radius of a circle with a circumference of C is calculated using the expression $\dfrac{C}{2\pi}$, where $\pi \approx 3.14$. Find the radius of a circle with a circumference of 40 cm.

9780170194778

19 The formula for the area of a rhombus is given by $A = \frac{1}{2}xy$, where x and y are the lengths of its diagonals. Find the area of a rhombus with diagonals of lengths 12 cm and 8 cm. *See* Example 3

20 The formula $T \approx 2\pi\sqrt{\frac{L}{g}}$ seconds, where L is in metres and g is the acceleration of gravity, is used to calculate the time a pendulum takes to swing from one side to the other and back again (the period). Using $g = 9.8$ m/s^2 and $\pi \approx 3.14$, find the period of a pendulum of length 1.5 m.

21 Factorise each of the following.

a $g^2(4m + 3) - 2h(4m + 3)$

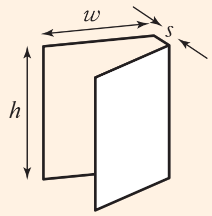
b $(2mn - 12n^2) + 5pm - 30pn$

Reasoning

22 Write a formula for the cost H of hiring earth-moving equipment for n days if the fixed cost (insurance, transport, contract fees and so on) is \$$f$ and the daily charge is \$$D$ per day. Explain your formula and find the cost of hire of a bulldozer for 3 days if its fixed cost is \$800 and the daily charge is \$1200 per day.

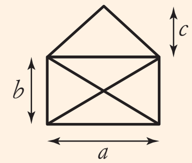
23 Write a formula for the cost C of bricks for a wall w metres long and h metres high if you need 56 bricks/m^2 and they cost \$$b$ per hundred. Explain your formula and calculate the cost of a wall 12 m long and 1.2 m high if the bricks used cost \$780/100. Explain why this may underestimate the cost of the bricks.

24 Inserts for plastic DVD and CD cases are made by printing, cutting, trimming and folding light cardboard to make the shape shown at the right. It is normal to leave an extra 0.5 cm all the way around for trimming. The inserts are printed on very large sheets measuring 0.841 m by 1.189 m before they are cut. Explain and write a formula for the area A used to make an insert. Expand the brackets in your formula.



Find the number of inserts that can be cut from one sheet of cardboard if the length and height are both 12 cm and the 'spine' is 0.7 cm wide.

25 A classic child's challenge is to draw the 'house' shape shown here without lifting their pen from the paper or retracing a line. Explain how an expression for the total area is found and factorise your expression for the total area. Find the area of such a shape where the width is 6 cm, the height is 4 cm and the height of the roof is 3 cm.



Measurement and geometry

10 Measuring shapes and time

9780170194778

Contents

Prior learning
Chapter 10
MAT08MGPL00010

Parent guide
Chapter 10
MAT08MGPG00010

Curriculum guide
Chapter 10
MAT08MGCU00010

Australian Curriculum statements

Using units of measurement

Choose appropriate units of measurement for area and volume and convert from one unit to another.
Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.
Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Solve problems involving duration, including using 12- and 24-hour time within a single time zone.

Weblink
The Mac Tutor history of Mathematics
MAT08MGWB00010

Weblink
Disposable cup snake
MAT08MGWB00010

The ability to calculate area and volume is necessary in many situations at home and work. If you want to paint the walls of a room, you will need to calculate the area to be painted so that you can buy the right amount of paint for the job. If you want to turf an area of ground, you need to be able to calculate the area before you order the turf.

Houses have foundations made from slabs of concrete. A builder needs to order the correct amount of concrete for the slab. The amount required has to be calculated using the dimensions of the slab. The concrete truck that delivers the concrete has a bowl that is designed to carry a particular amount of concrete. The designers of the truck need to be able to work out the space inside the bowl from its shape and size.

You need to know the capacity of a swimming pool to work out how much chlorine or other chemicals need to be added to treat the water.

These are all examples of situations where the calculation of area or volume is important.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

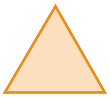
The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

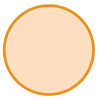
area	hectare	perpendicular height	surface area
base	hypotenuse	prism	trapezium
composite shape	length	rectangle	triangle
cross-section	net	right-angled	volume
cylinder	parallelogram	square	
formula	perimeter		

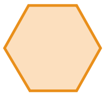
10.1 Areas of triangles, rectangles and circles

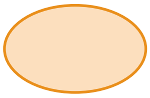
Units of **area** are used to measure **surfaces** such as walls, desks and floors. To measure area, we need to use a unit that will completely cover a surface without leaving gaps or overlapping. **Square units** such as cm^2 and mm^2 are ideal for this. All we need do is count the number of squares needed.

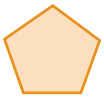
Why would it be difficult to use the following non-square units to measure area?


Triangular units


Circular units


Hexagonal units

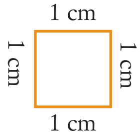

Elliptical units


Pentagonal units

9780170194778

Area units

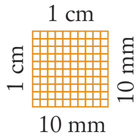
This is 1 square millimetre: $1\ \text{mm}^2 = 1\ \text{mm} \times 1\ \text{mm}$

This is 1 square centimetre: 

$1\ \text{cm}^2 = 1\ \text{cm} \times 1\ \text{cm}$

The cm^2 can be drawn to show mm^2 as well, as shown here.
From this diagram you can see that:

$$\begin{aligned} 1\ \text{cm}^2 &= 1\ \text{cm} \times 1\text{cm} \\ &= 10\ \text{mm} \times 10\ \text{mm} \\ \text{or } 1\text{cm}^2 &= 100\ \text{mm}^2 \end{aligned}$$



Investigate: Body areas

Use 1 cm^2 grid paper for this activity or draw some yourself.

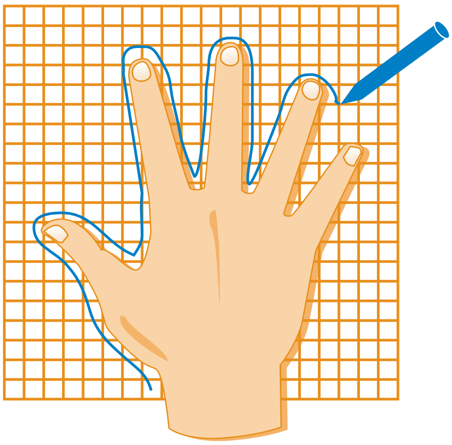
1 Place your hand palm-down on the grid paper and trace around it.

2 Estimate the area of your hand in cm^2 by counting the squares. You will need a method for counting parts of cm^2. Discuss this with your friends to find a good way of doing it.

3 Record the area of your hand.

4 On another sheet of cm^2 paper, trace around your foot and estimate its area as you did for your hand. Record the area.

5 Calculate the value of the fraction $\dfrac{\text{Area of hand}}{\text{Area of foot}}$ correct to 1 decimal place.



6 Compare your result with the results of others in the class. Comment on your findings.

7 Collect information from other class members and draw a graph showing hand area versus height. Comment on the graph.

Square centimetres and square millimetres are used to measure small areas. Larger areas are measured by using one of the following metric units.

Important!

Metric units of area

Large Areas	
Square metre (m^2)	$1\ m^2 = 1\ m \times 1\ m$
Square kilometre (km^2)	$1\ km^2 = 1\ km \times 1\ km$ $= 1000\ m \times 1000\ m = 1\ 000\ 000\ m^2$
Hectare (ha)	$1\ ha = 100\ m \times 100\ m = 10\ 000\ m^2$
Small Areas	
Square millimetre (mm^2)	$1\ mm^2 = 1\ mm \times 1\ mm$
Square centimetre (cm^2)	$1\ cm^2 = 1\ cm \times 1\ cm$ $= 10\ mm \times 10\ mm = 100\ mm^2$

We have seen how areas can be measured by counting square units on a grid. This method is not very useful for measuring the areas of objects in real life. Rules for calculating area have been developed for many shapes. Your teacher may wish to show you how these rules have been developed.

Teacher notes
Development of area rules
MAT08MGTN00017

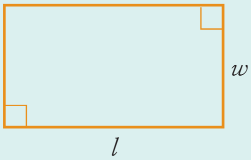
Worksheet
Approximating the area of a circle
MAT08MGWK00067

Video tutorial
Area of a circle
MAT08MGVT10008

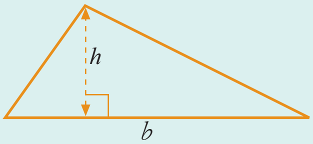
Important!

Area rules

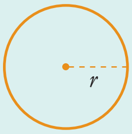
Rectangle: $A = l \times w$



Triangle: $A = \frac{1}{2} b \times h$



Circle: $A = \pi r^2$
where $\pi \approx 3.14$



Squares are just a special type of rectangle, so you can use the rule for the area of a rectangle to find the area of a square.
When you calculate the area of a circle, use the π key on your calculator.

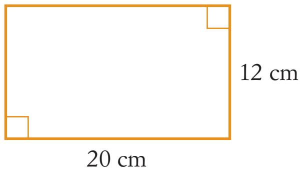
Puzzle sheet
Circle areas
MAT08MGPS00045

Worksheet
Rectangle and triangle areas
MAT08MGWK00065

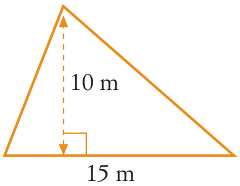
Example 1

Use a rule to find the area of each of the following.

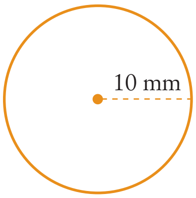
a



b



c



9780170194778

Solution

a Write the rule for the area of a rectangle. $A = l \times w$

Substitute the values for l and w. $= 20 \text{ cm} \times 12 \text{ cm}$

Evaluate. $= 240 \text{ cm}^2$

b Write the rule for the area of a triangle. $A = \frac{1}{2} b \times h$

Substitute the values for b and h. $= \frac{1}{2} \times 15 \text{ m} \times 10 \text{ m}$

Evaluate. $= 75 \text{ m}^2$

c Write the rule for the area of a circle. $A = \pi r^2$

Substitute the value for r. $= \pi \times 10 \text{ mm} \times 10 \text{ mm}$

You can use your calculator.

Enter as: π × 10 × 10 = . `π×10×10    314.1592654`

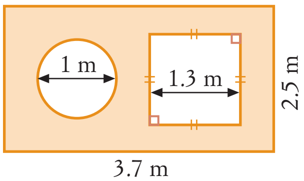
Round off to the nearest mm^2. $\approx 314 \text{ mm}^2$

Combined shapes

Combined shapes are made up of other shapes. The area of a combined shape can be found by dividing the shape into squares, rectangles, triangles and circles (or parts of circles) and finding the areas of these.

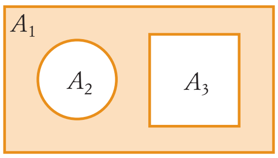
Example 2

Find the shaded area.



Solution

Identify and name the areas that make up the shape.



A_1 is a rectangle. $A_1 = l \times w$

Substitute the values for l and w. $= 3.7 \text{ m} \times 2.5 \text{ m}$

$= 9.25 \text{ m}^2$

A_2 is a circle. $A_2 = \pi r^2$

Substitute the value for r. $= \pi \times 0.5 \text{ m} \times 0.5 \text{ m}$

Evaluate and round off to 2 decimal places. $\approx 0.79 \text{ m}^2$

Animated example
Combined shapes
MAT08MGAE00017

Scientific calculator exercise
Measurement and geometry
MAT08MGSC00007

A_3 is a square. $A_3 = l \times w$

Substitute the values for l and w. $= 1.3\text{ m} \times 1.3\text{ m}$

$= 1.69\text{ m}^2$

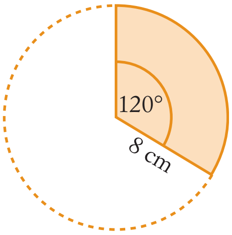
Calculate the shaded area. **Shaded area** $= A_1 - A_2 - A_3$

Substitute the areas. $\approx 9.25\text{ m}^2 - 0.79\text{ m}^2 - 1.69\text{ m}^2$

$= 6.77\text{ m}^2$

Example 3

Find the shaded area of this circle.



Solution

Calculate the shaded fraction of the circle. **Shaded fraction** $= \frac{120°}{360°} = \frac{1}{3}$

Shaded area is $\frac{1}{3}$ of the full area. **Shaded area** $= \frac{1}{3} \times \pi r^2$

Substitute for r. $= \pi r^2 \div 3$

$= \pi \times 8\text{ cm} \times 8\text{ cm} \div 3$

Evaluate and round off to the nearest cm^2. $\approx 67\text{ cm}^2$

Investigate: Estimating large areas

Work in groups of three or four for this investigation. You will need:

- blackboard and chalk
- tent pegs or witches' hats
- a large tape measure or a trundle wheel
- a metre rule.

9780170194778

1 On the blackboard, draw a square measuring 1 metre by 1 metre.
   a What area does this represent?
   b Using this as a reference, estimate the area of your classroom. Discuss your estimate with others in the class. Measure the room and calculate its area. Comment on the results.

2 Now go to different student work areas in the school, such as home economics, manual arts and science. Select a number of student work areas and complete the following table in your pad.

Description of area	Estimated area	Actual area	Number of students normally in this area	Area per student

Comment on your group's findings.

3 Go to the school oval or any other large area of land in the school. Use the trundle wheel (or a tape measure) and the pegs to mark out an area 10 m by 10 m. Walk around this area so that you have a good idea of how much ground it covers.
   a Use the facts that 1 ha = 10 000 m^2 and 10 m × 10 m = 100 m^2 to write the area you have marked out as a fraction of a hectare.
   b How many such areas would it take to make up 1 ha?
   c Estimate the area of the school oval using your marked-out area as a reference. Next calculate the actual area of the oval. How close was your estimate? Comment on your results.

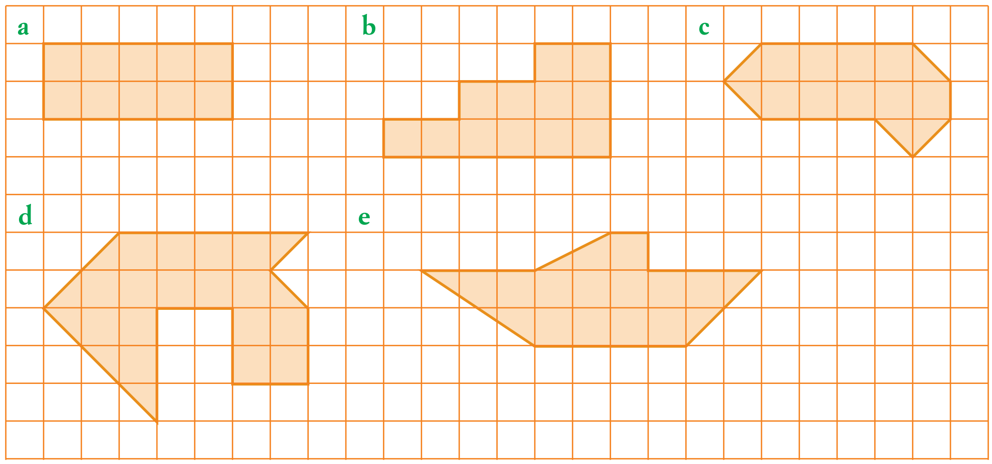
Exercise 10.1 Areas of triangles, rectangles and circles

Understanding

Extra questions
Exercise 10.1
MAT08MGEQ00032

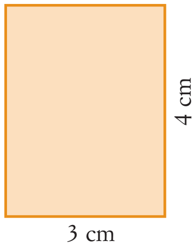
Puzzle sheet
Areas 1
MAT08MGPS00043

1 Calculate the area of figures **a** to **e** below in square units.

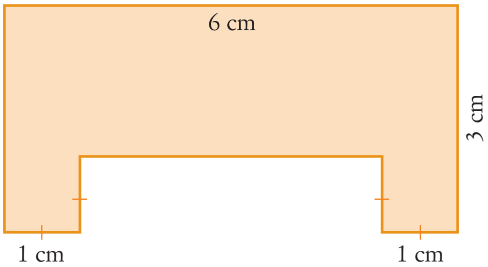


2 Draw the following on grid paper and calculate the area of each figure in cm^2 and mm^2.

a



b

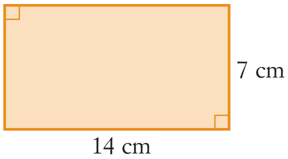


Fluency

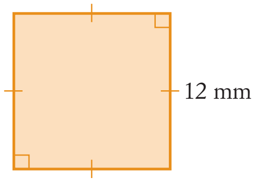
Worked solutions
Exercise 10.1
MAT08MGWS00032
See Example 1

3 Find the area of each of the following figures.

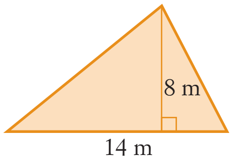
a



b



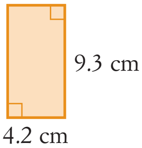
c



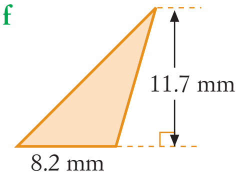
d

(Correct to 1 decimal place)

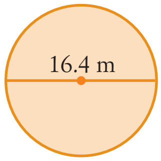
e



f



g



4 Calculate the area of each of the following.

a rectangle with $l = 17$ cm and $w = 8$ cm

b triangle with base = 7.2 m and height = 1.8 m

c circle with radius = 15 mm (answer correct to nearest mm^2)

d square with side = 1.7 km

5 Use the metric units of area table (on page 344) to convert the following areas to the units shown in brackets.

a 3 cm^2 (mm^2) **b** 0.08 m^2 (cm^2) **c** 1.04 ha (m^2)

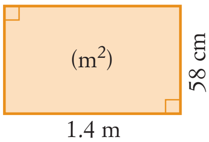
d 9.2 km^2 (ha) **e** 370 mm^2 (cm^2) **f** 58 000 m^2 (ha)

g 9500 cm^2 (m^2) **h** 0.002 ha (m^2) **i** 0.013 m^2 (cm^2)

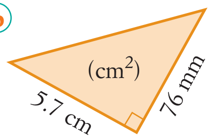
Worked solutions
Exercise 10.1
MAT08MGWS00032

6 Find the area of each of the following, giving your answer in the units indicated.

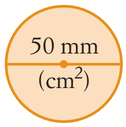
a



b

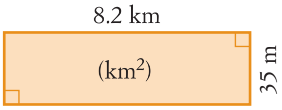


c

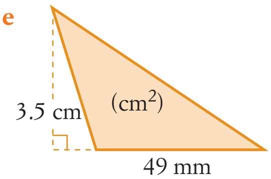


(Correct to nearest cm^2)

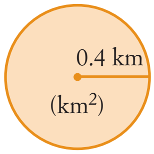
d



e



f



(Correct to 1 decimal place)

9780170194778

Problem solving

7 Select the most likely area measurement for each of these objects.

a	The cover of this book	400 mm^2	400 cm^2	0.4 m^2
b	A suburban block of land	900 cm^2	900 m^2	0.9 km^2
c	Your desk top	50 mm^2	50 cm^2	0.5 m^2
d	A three-bedroom low-set home	8500 cm^2	85 m^2	850 m^2
e	A 5-cent piece	25 mm^2	25 cm^2	0.25 m^2
f	A school's grounds	2000 m^2	2 ha	2 km^2
g	Australia	8 000 000 m^2	8 000 000 ha	8 000 000 km^2
h	A pinhead	1.2 mm^2	0.12 cm^2	21.2 cm^2

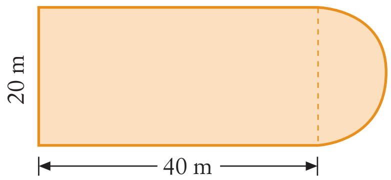
8 A driveway measuring 23 m by 7 m is paved using half bricks. The cost of the pavers is \$42/m^2 and the cost of labour is \$33/m^2. Find the total cost of paving the driveway.

9 Use a geoboard or some grid paper to make different shapes with an area of 12 square units. Draw them in your book. How many different shapes can you draw?

10 Find the areas of the shaded regions.

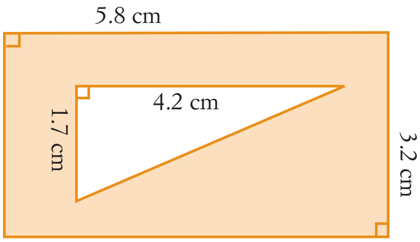
See Examples 2, 3

a



(Answer correct to 1 decimal place)

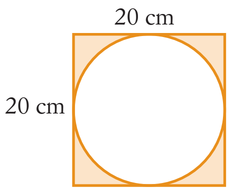
b



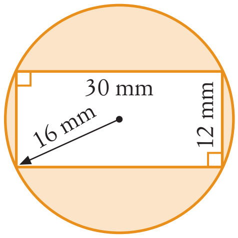
See Example 2

11 Find the area of the shaded region in each of the following. (Give all answers correct to 1 decimal place.)

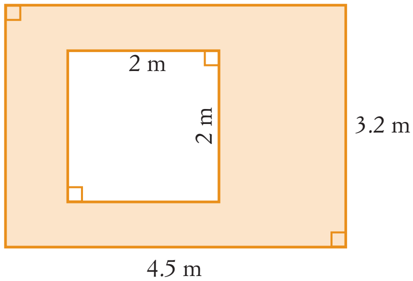
a



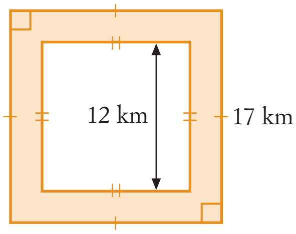
b



c



d

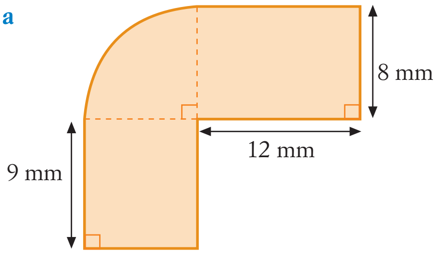


Worked solutions
Exercise 10.1
MAT08MGWS00032

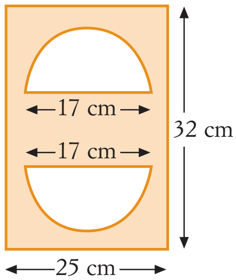
See Examples 2, 3

12 Find the areas of the shaded regions. (Give all answers correct to 1 decimal place.)

a



b



Reasoning

13 A large room measuring 9.5 m by 7.3 m needs new carpet. The carpet comes on a 4 m wide roll.

a Draw a diagram of the room to show that there are two ways in which the carpet could be laid.

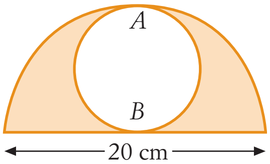
b Calculate the length of carpet needed for each method of laying.

c If the carpet sells for \$512/m on the roll, find the saving that would be made by laying the carpet the more efficient way.

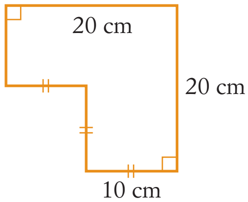
14 A multistorey office block has rectangular floors measuring 38 m by 15 m. The total floor space of the building is 7980 m^2. How many floors does the office building have?

9780170194778

15 The small circle touches the semicircle at A and B. Find the ratio of the area of the semicircle to the area of the small circle. This ratio always works out to have the same value, no matter what the diameter of the semicircle happens to be. Why?



16 Draw this figure on cm^2 grid paper. Show how it can be cut into four pieces that are the same shape and size. Try your solution to see if it works.

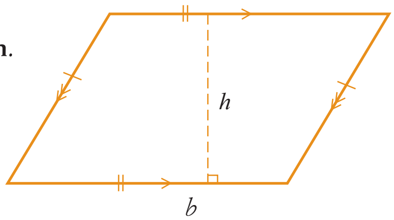


10.2 Areas of other shapes

Video tutorial
Area formulas for triangles and quadrilaterals
MAT08MGVT00010

You know the **formula** for calculating the area of a rectangle. There are also formulas for calculating the areas of other quadrilaterals including the parallelogram, kite, rhombus and trapezium.

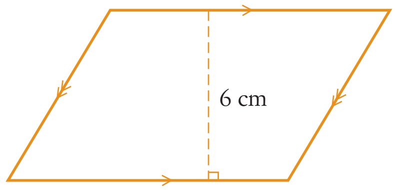
A **parallelogram** is a quadrilateral with opposite sides parallel. Because opposite sides are parallel, they are also equal in **length**.



We can call one side the **base** (b). The distance between the base and the opposite side is called the **perpendicular height** or just the height (h).

Investigate: Area of a parallelogram

1 Draw a parallelogram with a base length of 10 cm and a height of 6 cm on a sheet of paper.



2 Draw in a dashed line to indicate the height anywhere inside the parallelogram.
3 Cut out the parallelogram of paper.
4 Cut along the dashed line and arrange the two pieces you have made to form a rectangle.
5 Measure the length and breadth of the rectangle.
6 Complete the following.

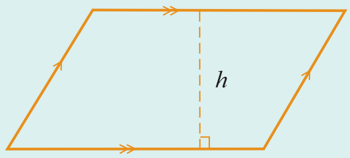
$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \cdots \\ &= 10 \text{ cm} \times \cdots\end{aligned}$$

7 The rectangle was formed from the parallelogram you drew. Complete the following.

$$\begin{aligned}\text{Area of parallelogram} &= \text{area of rectangle} \\ &= 10 \text{ cm} \times \cdots \\ &= b \times \cdots\end{aligned}$$

Important!

Area of a parallelogram



$$\begin{aligned}\text{Area of a parallelogram} &= \text{base} \times \text{perpendicular height} \\ &= bh\end{aligned}$$

9780170194778

Example 4

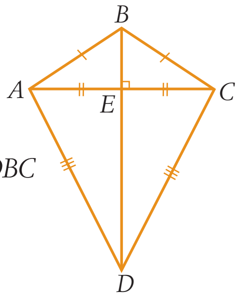
Find the area of this parallelogram.

Solution

Write the rule for the area of a parallelogram.	$\boldsymbol{A = bh}$
The base is 52 cm and the height is 34 cm.	$= \mathbf{52 \times 34}$
Evaluate.	$= \mathbf{1768\ cm^2}$

Area of a kite

A kite has two pairs of adjacent sides equal.
The diagonals of a kite intersect at right angles.
The area of the kite is equal to the sum of the areas of its triangles.
Kite $ABDC$ is made up of $\triangle DAB$ and $\triangle DBC$.

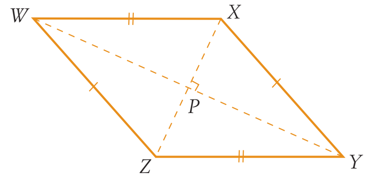


$$\text{Area of kite } ABDC = \text{area of } \triangle DAB + \text{area of } \triangle DBC$$
$$\text{Use the rule for triangle areas} = \frac{1}{2}BD \times AE + \frac{1}{2}BD \times EC$$
$$\text{Apply the distributive law} = \frac{1}{2}BD(AE + EC)$$
$$AE + EC = AC = \frac{1}{2}BD \times AC$$

So the area of a kite is half the product of its diagonals.

Investigate: Area of a rhombus

For a rhombus, all its sides are equal and its diagonals bisect each other at right angles.



So, for rhombus $WXYZ$:

$$WX = XY = YZ = ZW$$
$$WP = PY$$
and
$$ZP = PX$$

Complete the following.

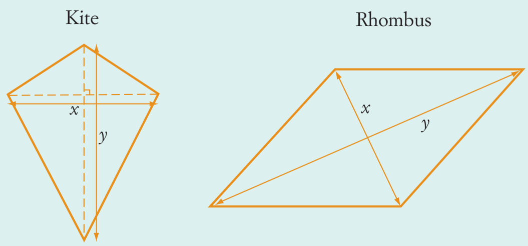
$$\begin{aligned}\text{Area of rhombus } WXYZ &= \text{area of } \triangle\cdots\\ &= \frac{1}{2}WY \times \cdots + \frac{1}{2}WY \times \cdots\\ &= \frac{1}{2}WY(\cdots + \cdots)\cdots\\ &= \frac{1}{2}WY \times \cdots\end{aligned}$$

The area of a rhombus is ___ the product of its ________.

From the previous work you can see that the same rule is used to calculate the area of a kite and a rhombus.

Important!

Area of a kite or a rhombus

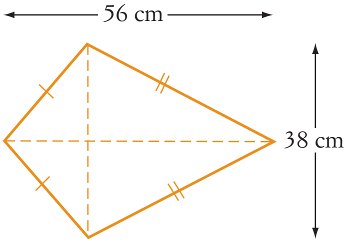


$$\begin{aligned}A &= \tfrac{1}{2} \times \text{diagonal 1} \times \text{diagonal 2}\\ &= \tfrac{1}{2}xy\end{aligned}$$

9780170194778

Example 5

Find the area of this kite.



Solution

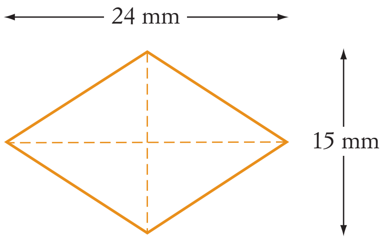
Write the rule for the area of a kite. $A = \frac{1}{2}xy$

The diagonals measure 56 cm and 38 cm. $= \frac{1}{2} \times 56 \times 38$

Evaluate. $= 1064 \text{ cm}^2$

Example 6

Find the area of this rhombus.



Solution

Write the rule for the area of a rhombus. $A = \frac{1}{2}xy$

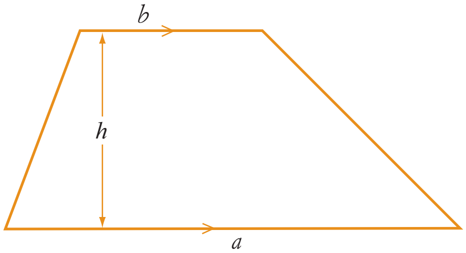
The diagonals measure 24 mm and 15 mm. $= \frac{1}{2} \times 24 \times 15$

Evaluate. $= 180 \text{ mm}^2$

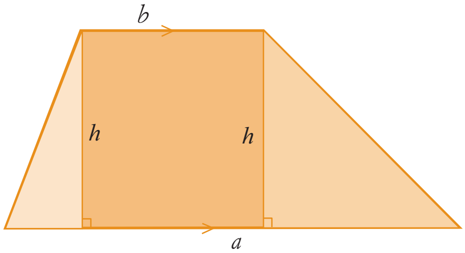
Technology
GeoGebra: Area of a trapezium
MAT08MGTC00009

Investigate: Area of a trapezium

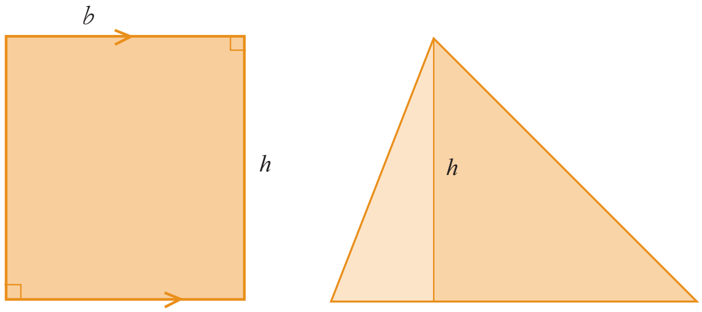
Draw a **trapezium** similar to the one on the right on a piece of paper. Make sure that it is large enough for you to cut out.



Label the parallel sides a (longer side) and b (shorter side) and the perpendicular height, h. Now draw in perpendicular lines from each end of b to meet a as shown in the diagram on the right.
This divides the trapezium into two triangles and a rectangle.



Cut off the two triangles and slide them together to form another triangle as shown here. The area of the trapezium must be the same as the combined areas of the rectangle and the large triangle you have formed.



The length of the base of the triangle must be the difference between the two parallel sides of the trapezium, or $a - b$.

9780170194778

$$\text{Area of trapezium} = \text{area of rectangle} + \text{area of triangle}$$

$$\text{Use the area formulas} = bh + \tfrac{1}{2}(a - b) \times h$$

$$\text{Expand using the distributive law} = bh + \tfrac{1}{2}ah - \tfrac{1}{2}bh$$

$$= bh - \tfrac{1}{2}bh + \tfrac{1}{2}ah$$

$$= \tfrac{1}{2}bh + \tfrac{1}{2}ah$$

$$\text{Factorise using the distributive law} = \tfrac{1}{2}h(a + b)$$

$$\text{Rearrange} = \tfrac{1}{2}(a + b) \times h$$

Use the result above to complete the statement below:

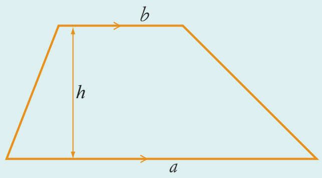
The area of a trapezium is the product of ______ the sum of the parallel ______ and its ______.

Important!

Area of a trapezium

$$A = \tfrac{1}{2} \times \text{sum of parallel sides} \times \text{height}$$

$$= \tfrac{1}{2}(a + b) \times h$$



Example 7

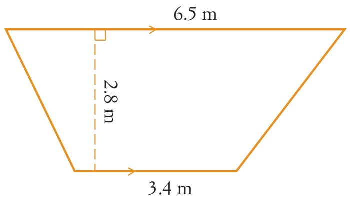
Scientific calculator exercise

Measurement and geometry

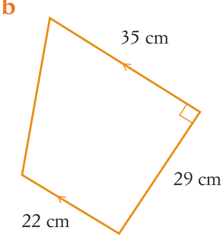
MAT08MGSC00007

Find the area of these trapeziums.

a



b



Solution

a Write the rule for the area of a trapezium. $\quad A = \frac{1}{2}(a + b) \times h$

The parallel sides measure 6.5 m and 3.4 m and the height is 2.8 m. $\quad = \frac{1}{2}(6.5 + 3.4) \times 2.8$

You can use your calculator. Enter as:
0.5 (6.5 + 3.4) × 2.8 =. $\quad$ 0.5(6.5+3.4)×2.8 13.86

Evaluate. $\quad = \mathbf{13.86\ m^2}$

b Write the rule for the area of a trapezium. $A = \frac{1}{2}(a + b) \times h$

The parallel sides measure 35 cm and 22 cm and the height is 29 cm. $= \frac{1}{2}(35 + 22) \times 29$

You can use your calculator.

Enter as: 0.5 (35 + 22) × 29 = . 0.5(35+22)x29 826.5

Evaluate. $= 826.5 \text{ cm}^2$

Exercise 10.2 Areas of other shapes

Understanding

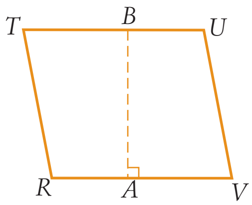
Extra questions
Exercise 10.2
MAT08MGEQ00033

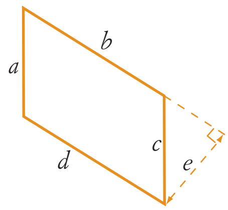
Technology worksheet
Excel: Area calculator
MAT08MGCT00012

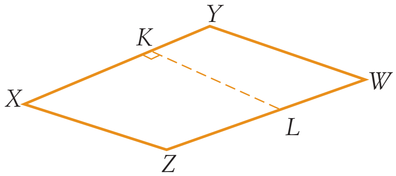
Puzzle sheet
Areas of quadrilaterals and triangles
MAT08MGPS00044

Worksheet
Quadrilateral and triangle areas
MAT08MGWK00066

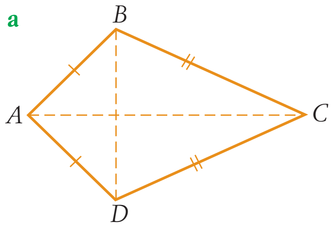
1 Write the rule for finding the area of each of the parallelograms below.

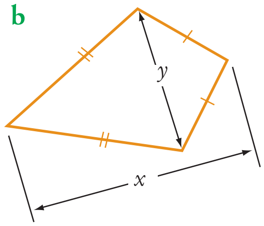
a


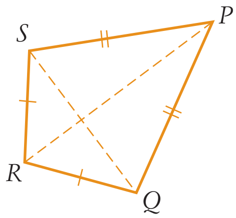
b


c


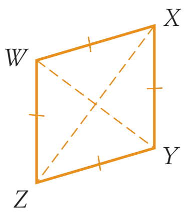
2 Write the rule for finding the area of each of the kites below.

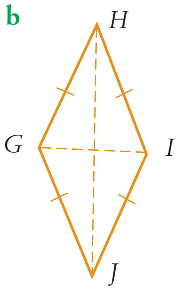
a


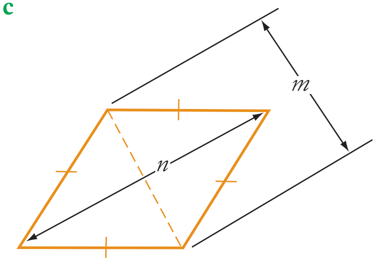
b


c


3 Write the rule for finding the area of each of the following rhombuses.

a


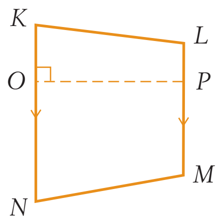
b


c


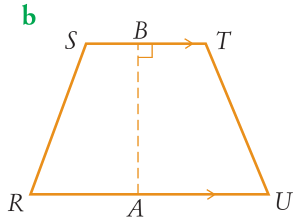
9780170194778

4 Write the rule for finding the area of each of the trapeziums below.

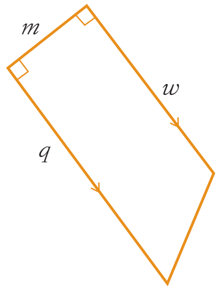
a



b



c

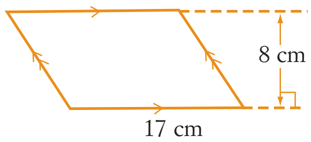


Fluency

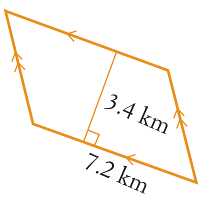
See Example 4

5 Find the area of each of these parallelograms.

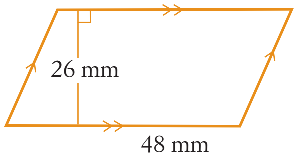
a



b



c

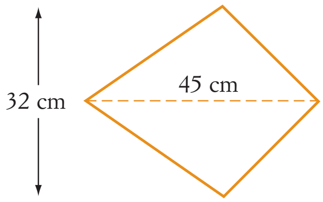


Worked solutions
Exercise 10.2
MAT08MGWS00033

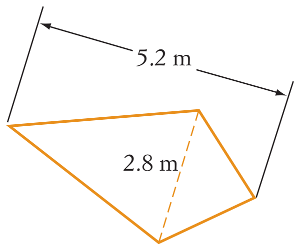
See Example 5

6 Find the area of each of these kites.

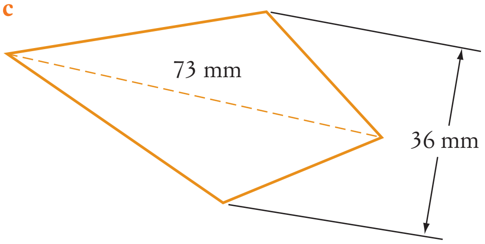
a



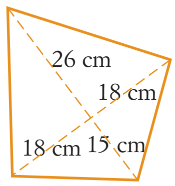
b



c



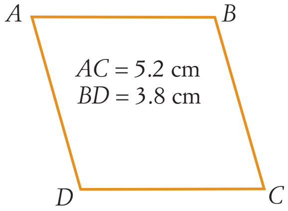
d



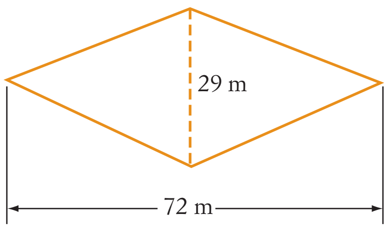
See Example 6

7 Find the area of each of these rhombuses.

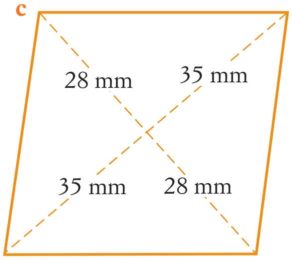
a



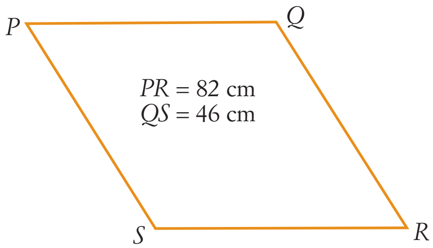
b



c



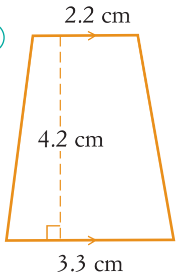
d



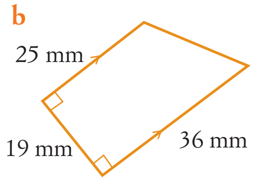
Worked solutions
Exercise 10.2
MAT08MGWS00033
See Example 7

8 Find the area of each of these trapeziums.

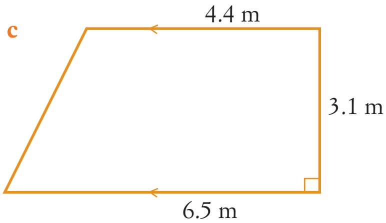
a



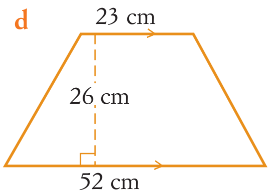
b



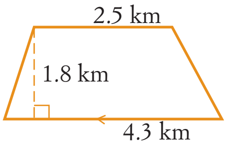
c



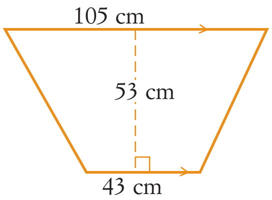
d



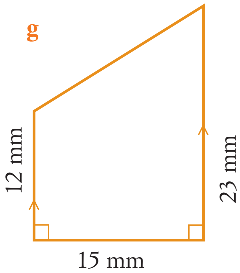
e



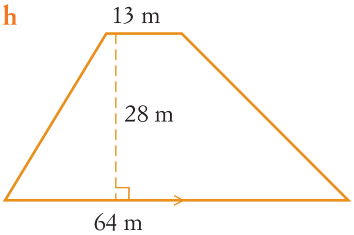
f



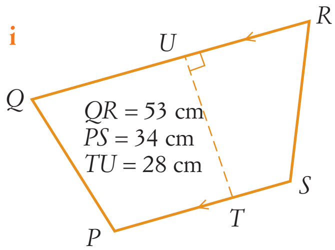
g



h



i

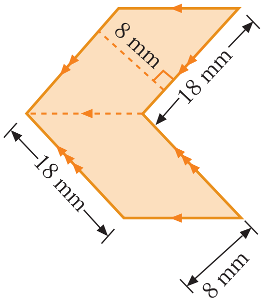


Problem solving

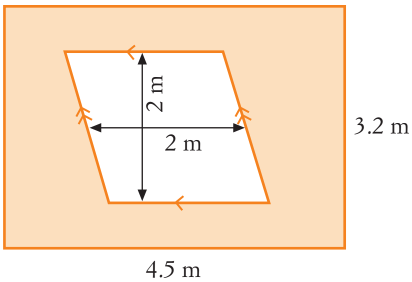
Worked solutions
Exercise 10.2
MAT08MGWS00033

9 Find the area of each shaded region below.

a



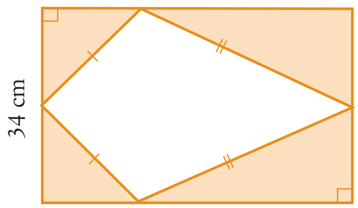
b



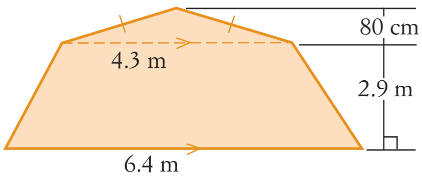
9780170194778

10 Find the area of each shaded region below.

a



b



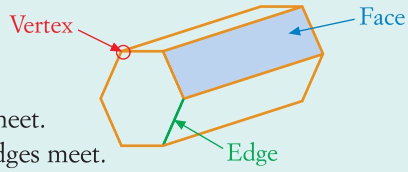
10.3 Surface area

Three-dimensional shapes are described using particular terms for the sides, points and lines that form the shape.

Important!

3D shape terms

The **faces** of a 3D shape are the sides that separate the inside from the outside.
The **edges** of a 3D shape are the lines where the faces meet.
The **vertices** of a 3D shape are the corners where the edges meet.
These terms apply to 3D shapes with flat faces, but they are also used for some parts of shapes with curved faces.

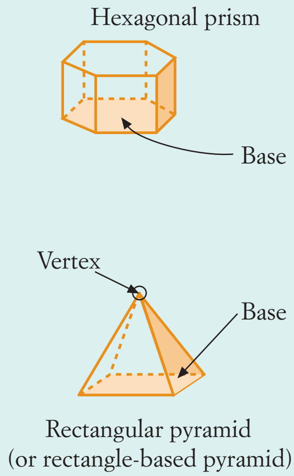


Three-dimensional shapes are classified and named according to their faces and the way they are joined together. The shapes based on circles have special names.

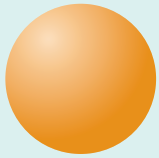
Important!

Common 3D shapes

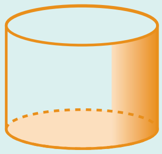
A **prism** has two parallel faces that are polygons of the same shape and size.
One of these faces is called the **base** of the prism.
A prism takes its name from the polygon that makes the base, so the figure on the right is a hexagonal prism.
The side faces of a prism are formed by the lines joining the vertices of the base and its opposite face.
A **pyramid** has a *base* that is a polygon.
The other faces are triangles that all meet at one vertex, the 'point' of the pyramid.
A pyramid's name is derived from the shape of its base, hence the figure on the right is a rectangular pyramid.



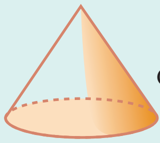
A **sphere** is a ball shape that is completely symmetrical.


Sphere

A **cylinder** is like a prism, except it has a circular base.


Cylinder

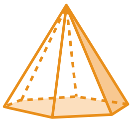
A **cone** is like a pyramid, except it has a circular base.

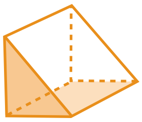

Cone

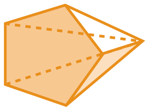
Example 8

a State whether each of the following shapes is a prism, a pyramid or some other shape.
b Give the full name of each shape.

i


ii


iii


iv


Solution

a **i** The side faces are rectangles. — **The shape is a prism.**

ii The side faces are triangles meeting at a point. — **The shape is a pyramid.**

iii The side faces are rectangles. The triangle faces do not meet at a point; they make the base and top. — **The shape is a prism.**

iv The side faces are triangles meeting at a point. — **The shape is a pyramid.**

b **i** The base is a rectangle. — **Rectangular prism**

ii The base is a hexagon. — **Hexagonal pyramid**

iii The base is a triangle. — **Triangular prism**

iv The base is a pentagon. — **Pentagonal pyramid**

There are other shapes that are not prisms or pyramids. Some of these shapes have special names.

9780170194778

Important!

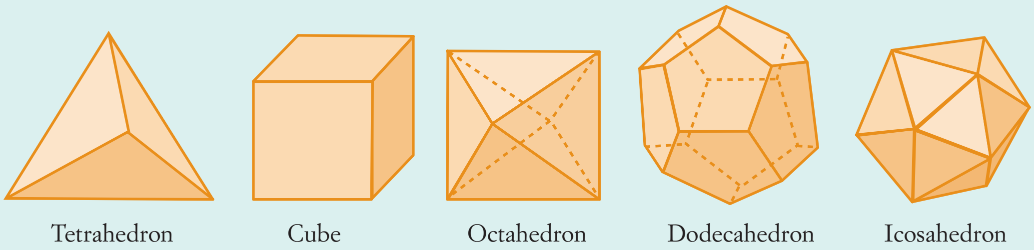
Hedrons

A 3D shape with flat sides that has only one or two different-sized polygons generally gets its name from the Greek for the number of faces and the suffix **-hedron**. For example, a tetrahedron has 4 faces (from the Greek *tetra*, meaning 'four').

The five **platonic** solids are the hedrons for which every face is the same regular polygon and every vertex joins the same number of faces.

The **tetrahedron**, **octahedron** and **icosahedrons** have faces that are equilateral triangles. The **cube** has square faces while the faces of the **dodecahedron** are regular pentagons.

A tetrahedron has 4 faces, a cube has 6 faces, an octahedron has 8 faces, a dodecahedron has 12 faces and an icosahedron has 20 faces.

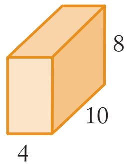


You saw in Chapter **2** that the view of a 3D shape will differ depending on the viewpoint.
A **plan** of a 3D object shows how it appears from directly above.
An **elevation** of a 3D object shows what it looks like from a particular side.
A **side elevation** shows it from one side while a **front elevation** shows it from the front.

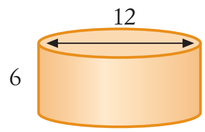
Example 9

Sketch views of the following from the front, right side and top, where the front is the side facing out of the page.

a



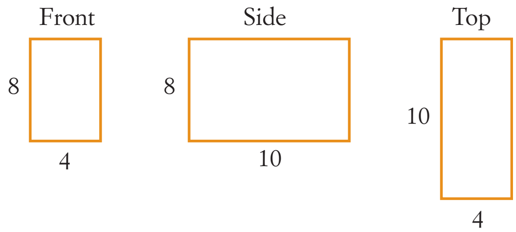
b



Solution

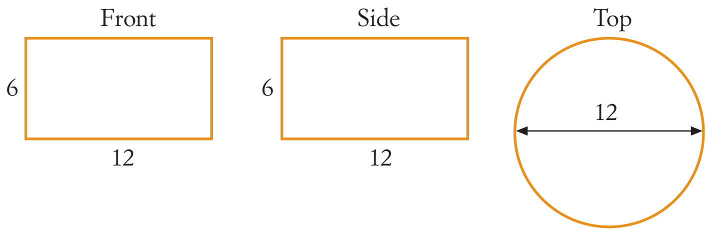
a Each view is a rectangle.

Write the sizes on each view.

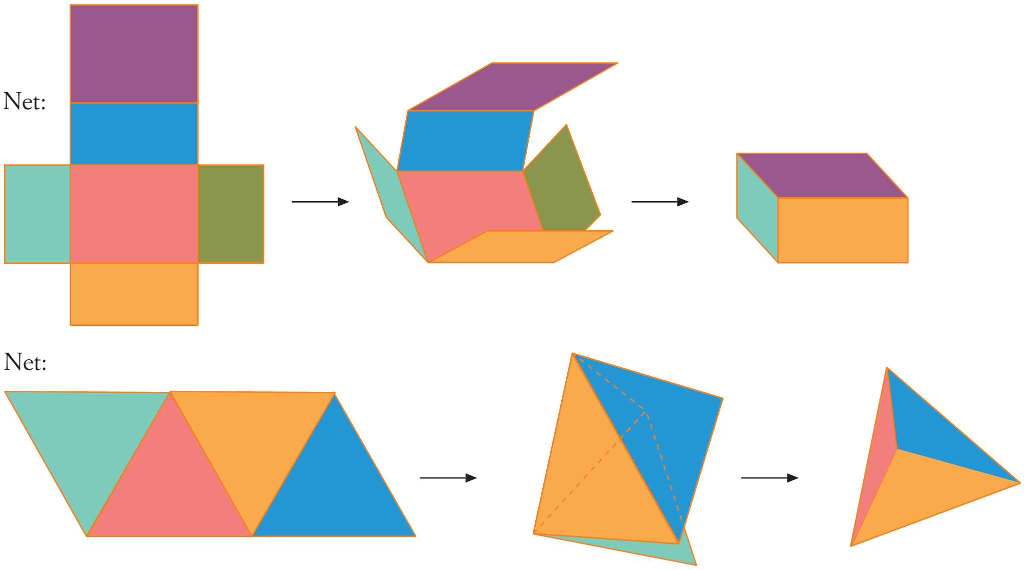


9780170194778

b The front and side look like the same rectangle, and the top is a circle.



A **net** of a 3D shape is a 2D shape that can be folded up to make the 3D shape. The diagrams below show how the nets of a box and tetrahedron could be folded up to make the 3D shape.



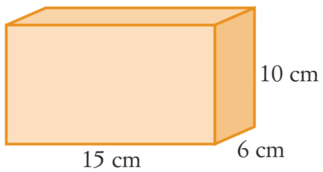
You should be able to draw nets for simple 3D shapes and tell whether a given 2D layout is really a net. There is more than one possible net for most shapes.

Example 10

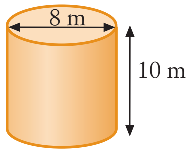
Animated example
Nets
MAT08MGAE00018

Draw nets for the following shapes, including measurements.

a



b



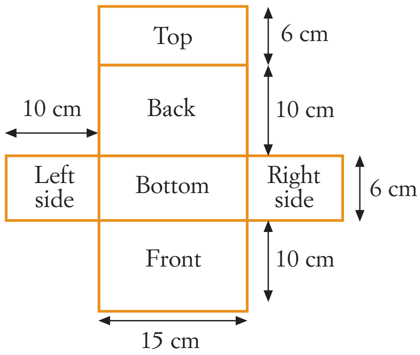
Solution

a The box shape has six faces, with front and back the same (15 cm by 10 cm), top and bottom the same (6 cm by 15 cm), and left and right sides the same (6 cm by 10 cm).

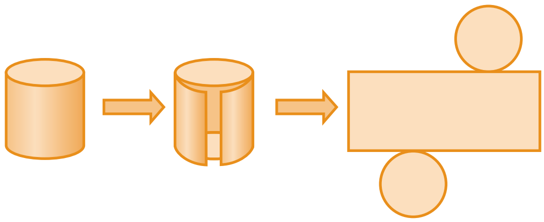
The top and back are connected along the 15 cm edge, the bottom is connected to the back along the 15 cm edge, and the front is connected to the bottom along the 15 cm edge.

9780170194778

The left and right are connected to the bottom along the 6 cm edge. Draw each face and connect along the edges mentioned to make a net. Write the measurements on the net.



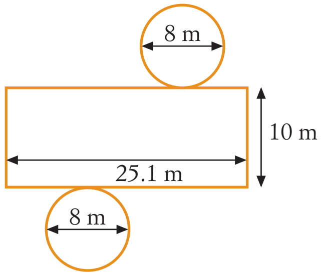
b Imagine cutting the cylinder from top to bottom, then unrolling the curved part.



Find the circumference of the circle.

$$\begin{aligned} C &= \pi D \\ &= \pi \times \mathbf{8} \\ &\approx \mathbf{25.1\ m} \end{aligned}$$

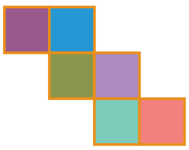
Write the sizes on the net.



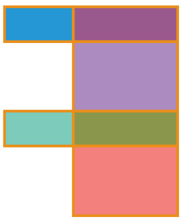
Example 11

Determine if the layouts below form nets. Sketch the 3D shape of each of the nets.

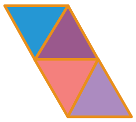
a



b

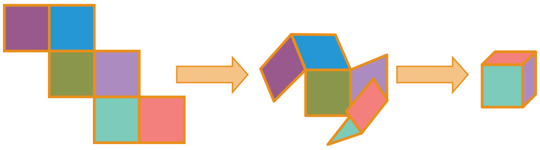


c



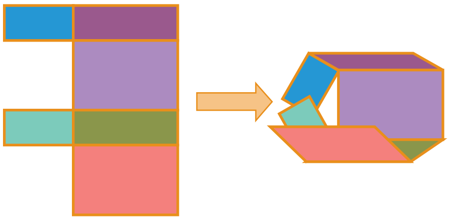
Solution

a Imagine folding the shape.



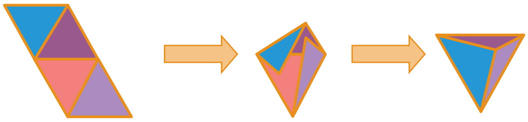
Write the answer. **The layout is the net of a cube.**

b Imagine folding the shape. Two flaps would be at the same end.



Write the answer. **The layout is not a net.**

c Imagine folding the shape.



Write the answer. **The layout is the net of a tetrahedron.**

Investigate: Making shapes from nets

Teacher notes
Nets
MAT08MGTN00018

Your teacher may have photocopies of large nets for you to work with.
Cut out and colour the nets before folding them to make 3D shapes.
Try to colour the net before folding so that you can turn the shape to see just one colour, then turn it again to see another colour.

When you make a container or paint a 3D object, you may need to know the total area of its surface.

Important!

Surface area

The **surface area** of a 3D shape is the total area of all its faces.

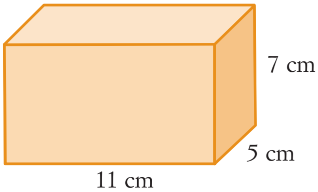
It is often useful to draw the net of a shape before calculating the surface area.

9780170194778

Example 12

Scientific calculator exercise
Measurement and geometry
MAT08MGSC00007

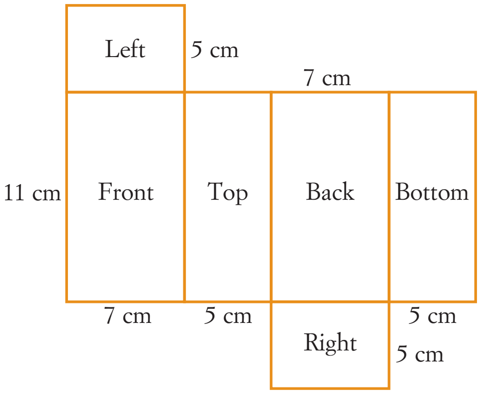
Find the surface area of this shape.



Solution

Draw a net of the shape.

Write the measurements on the net.



Work out the area of each face.

Left $= l \times w = 5 \times 7 = 35\ \text{cm}^2$

Front $= l \times w = 11 \times 7 = 77\ \text{cm}^2$

Top $= l \times w = 11 \times 5 = 55\ \text{cm}^2$

Back $= l \times w = 11 \times 7 = 77\ \text{cm}^2$

Bottom $= l \times w = 11 \times 5 = 55\ \text{cm}^2$

Right $= l \times w = 5 \times 7 = 35\ \text{cm}^2$

Find the total.

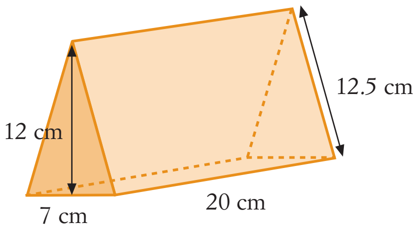
Surface area $= 35 + 77 + 55 + 77 + 55 + 35\ \text{cm}^2$

$= 724\ \text{cm}^2$

Example 13

Animated example
Surface area
MAT08MGAE00021

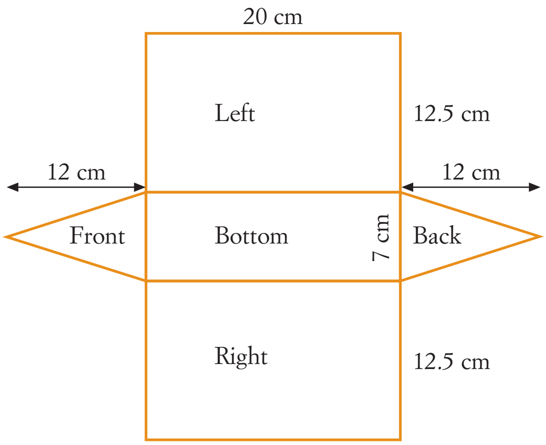
Find the surface area of the shape.



Solution

Draw a net of the shape.

Write the measurements on the net.



Work out the area of each face.

Left $= l \times w = 20 \times 12.5 = 250$ **cm**^{2}

Front $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 7 \times 12 = 42$ **cm**^{2}

Bottom $= l \times w = 20 \times 7 = 140$ **cm**^{2}

Back $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 7 \times 12 = 42$ **cm**^{2}

Right $= l \times w = 20 \times 12.5 = 250$ **cm**^{2}

Find the total.

Surface area $= 250 + 42 + 140 + 42 + 250$ **cm**^{2}

$= 724$ **cm**^{2}

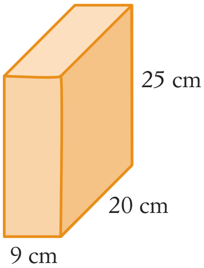
Example 14

A cereal packet is 9 cm by 20 cm by 25 cm. Every outside surface is printed with pictures and writing. What is the printed area?

Solution

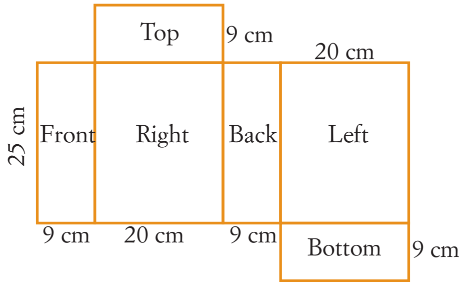
The outside surface is the surface area.

Sketch the packet, with measurements.



Draw a net of the shape.

Write the measurements on the net.



9780170194778

Work out the area of each face.	**Top** $= l \times w = 20 \times 9 = 180 \text{ cm}^2$
	Front $= l \times w = 25 \times 9 = 225 \text{ cm}^2$
	Right $= l \times w = 25 \times 20 = 500 \text{ cm}^2$
	Back $= l \times w = 25 \times 9 = 225 \text{ cm}^2$
	Left $= l \times w = 25 \times 20 = 500 \text{ cm}^2$
	Bottom $= l \times w = 20 \times 9 = 180 \text{ cm}^2$
Find the total.	**Surface area** $= 180 + 225 + 500 + 225 + 500 + 180 \text{ cm}^2$
	$= 1810 \text{ cm}^2$
Write the answer.	**The printed area is 1810 cm^2.**

Exercise 10.3 Surface area

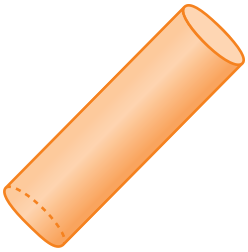
Understanding

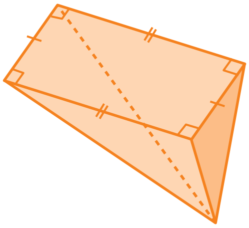
Extra questions
Exercise 10.3
MAT08MGEQ00034
See Example 8

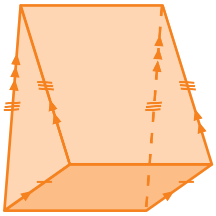
Worksheet
Surface areas 1
MAT08MGWK00068

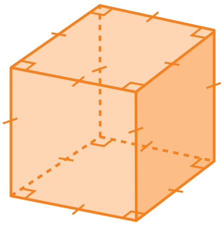
Worksheet
Surface areas 2
MAT08MGWK00069

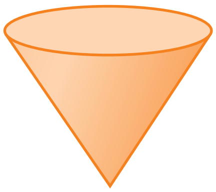
1 State whether each of the following shapes is a prism, a pyramid or some other shape.

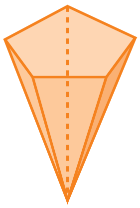
a


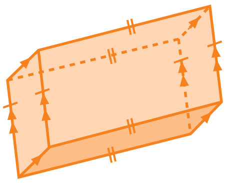
b


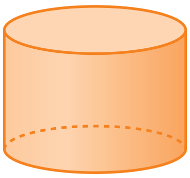
c


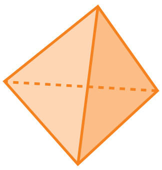
d


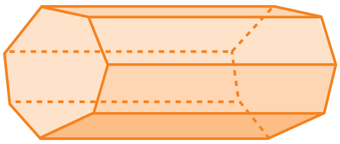
e


f


g


h


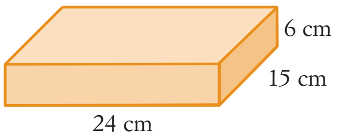
i


j


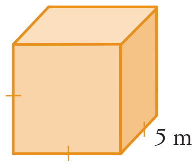
See Example 9

2 Sketch views of the following from the front, right side and top.

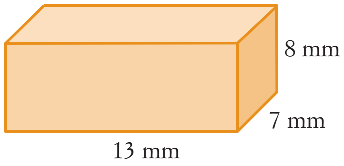
a



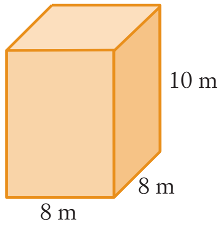
b



c



d



See Example 10

3 Draw nets for the shapes from question **2**, including measurements.

See Example 8

4 Give the full name of each shape shown in question **1**.

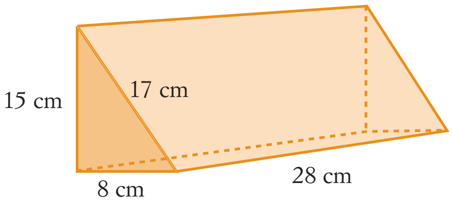
Fluency

See Example 9

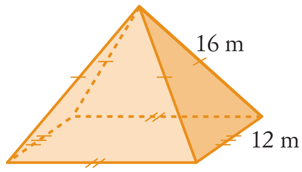
Worked solutions
Exercise 10.3
MAT08MGWS00034

5 Sketch views of the following from the front, right side and top.

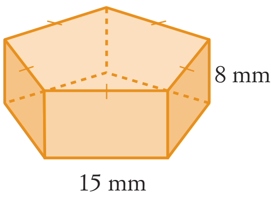
a



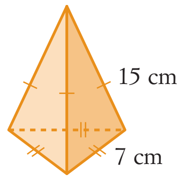
b



c



d

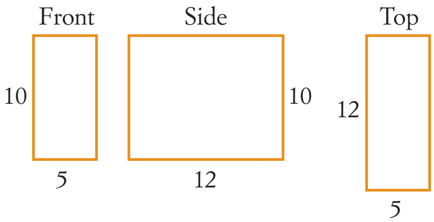


See Example 10

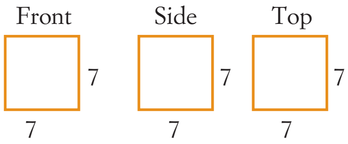
6 Draw nets for the shapes in question **5**, including measurements.

7 Sketch the 3D shape for each of the following.

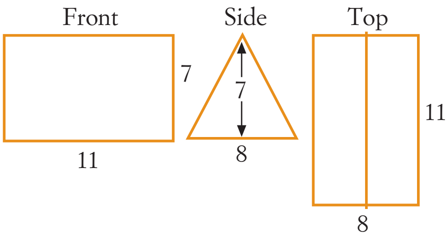
a



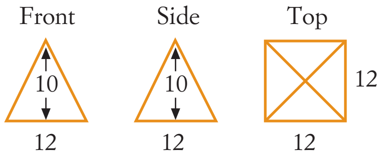
b



c



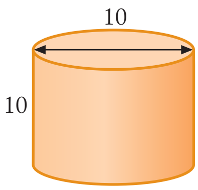
d



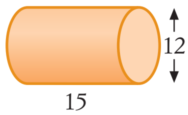
9780170194778

8 Sketch front and side elevations and plans for each of the following cylinders. *See* Example 9

a



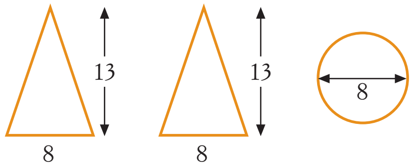
b



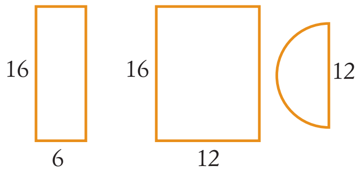
9 Draw nets for the cylinders in question **8**, including measurements where appropriate. *See* Example 10

10 Sketch the 3D shape for each of the following.

a



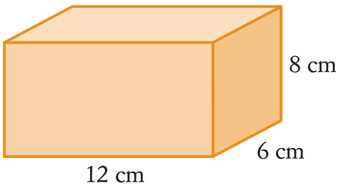
b



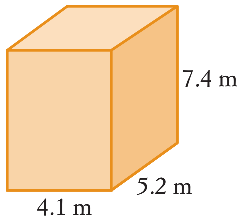
11 Find the surface area of these shapes.

Worked solutions
Exercise 10.3
MAT08MGWS00034
See Example 12

a

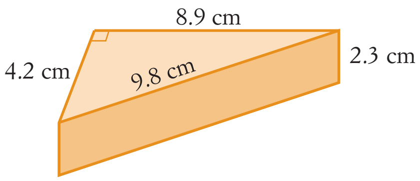


b

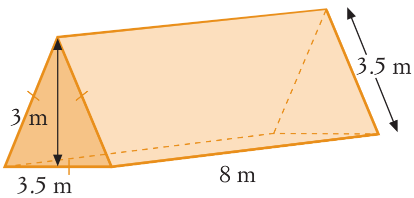


12 Find the surface areas of these shapes. *See* Example 13

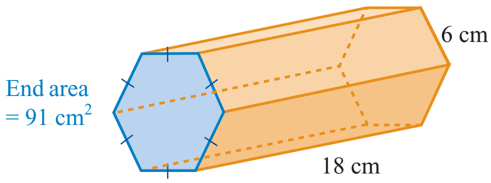
a



b



13 Find the surface area of this shape.



14 Calculate the surface area of a rectangular box measuring 12 cm by 15 cm by 28 cm. *See* Example 14

Problem solving

15 A manufacturer wants to produce breakfast cereal containers. The packet is made of cardboard.

a What is the area of cardboard needed to make 1 packet, without seam allowance?

b If cardboard costs $\$4/\text{m}^2$, how much would the cardboard for 1000 packets cost?

Note: $1\ \text{m}^2 = 10\ 000\ \text{cm}^2$.

Worked solutions
Exercise 10.3
MAT08MGWS00034
See Example 14

16 This flat-roofed garden shed is made from zincalume. It is 3.7 m long, 4.6 m deep and 2.2 m high. The window has an area of 1.5 m^2, and the shed is built on a concrete slab.

a Find the surface area of the shed, including the roof.

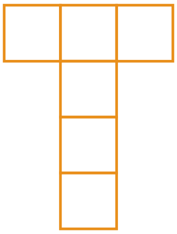
b What is the area of zincalume, including the doors?

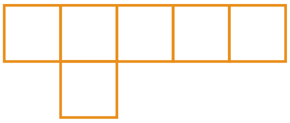
c Calculate the cost of painting the zincalume on the outside with two coats if 1 L of paint covers 15 m^2 and a 4 L tin costs $88.50.

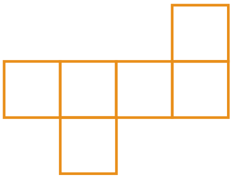
Reasoning

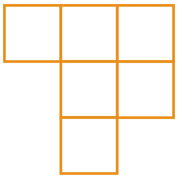
See Example 11

17 Determine if each of the following forms a net. For each one that does, sketch the 3D shape.

a


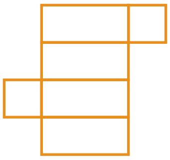
b


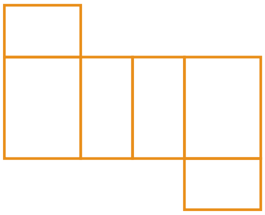
c


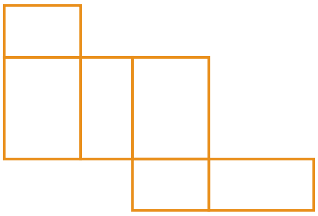
d


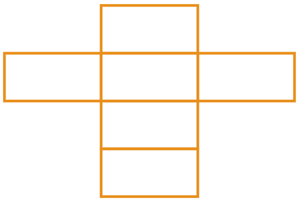
See Example 11

18 Determine if each of the following forms a net. For each one that does, sketch the 3D shape.

a


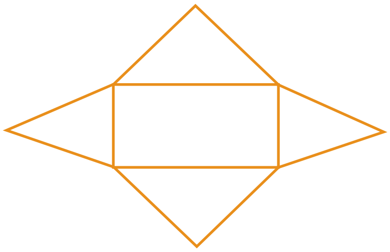
b


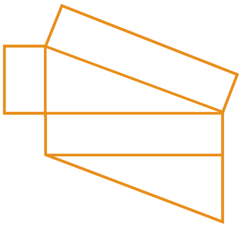
c


d


See Example 11

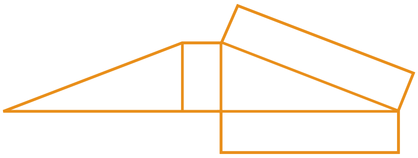
19 Determine if each of the following forms a net. For each one that does, sketch the 3D shape.

a


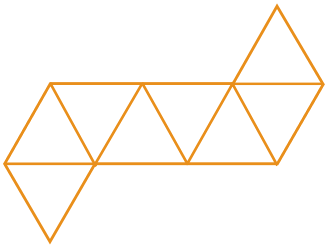
b


9780170194778

c



d



20 How many *different* nets of a cube are there, not counting those that are just rotations or reflections of another net?

10.4 Volumes of prisms and cylinders

You have seen that the area of a flat shape can be measured by working out how many squares would be needed to cover the space. A similar technique is used to work out the space taken by a 3D object.

Video tutorial
Volume of a prism
MAT08MGVT10009

Important!

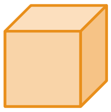
Volume

The **volume** of a 3D object is a measure of the amount of space it occupies. It is calculated by working out how many cubes of unit size would be needed to fill the same space. Volume is measured in **cubic** units such as **cubic centimetres** (**cm^3** or **cc**) or **cubic metres** (**m^3**).

TLF learning object
The Metrix: New planet (L8146)
MAT08MGIN00010

TLF learning object
The Metrix: New planet: Decimals (L8149)
MAT08MGIN00010

The drawing on the right shows a cubic centimetre.
Your teacher may have some centicubes that are each 1 cm by 1 cm by 1 cm, and therefore have a volume of 1 cm^3.



Investigate: Classroom cubes

You will need 12 one-metre rulers (or sticks such as garden stakes cut to be exactly 1 metre long) and some sticky tape. If you have more rulers available, you will be able to make more cubes.

- Join the 12 one-metre rulers together with sticky tape to make a cube in one corner of the classroom.
- If you have more metre rulers, join them onto the first cube to make extra metre cubes.
- Estimate how many of these cubes would be needed to fill the classroom.
- Now measure the classroom and work out how many would be needed.
- How many students will fit into a metre cube?
- Compare the size of a centicube with the size of your cubic metre.
- Estimate how many centicubes are needed to fill the cubic metre (you won't have enough to fill it up).

Example 15

What is the most suitable unit for measuring each of the following volumes?

a a cup

b a shipping container

Solution

a You would use centimetres to measure the dimensions of a cup, so the corresponding cubic measure should be used for volume.

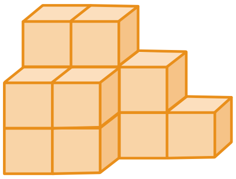
The volume of a cup is measured in cm^3.

b You would use metres to measure a container.

A shipping container is measured in m^3.

Example 16

The drawing shows a shape composed of cubes that are 1 cm in size.
What is the volume of the shape?



Solution

Count the cubes on the top. **There are 2 cubes on the top layer.**

Count the cubes in the middle. **There are 5 cubes in the middle layer.**

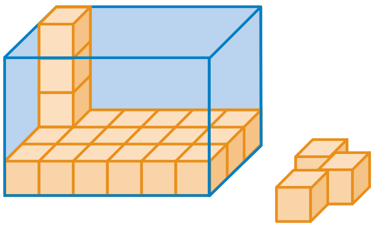
Count the cubes on the bottom. **There are 6 cubes in the bottom layer.**

Add and state the volume. **The volume is 13 cm^3.**

Investigate: Volumes of rectangular prisms

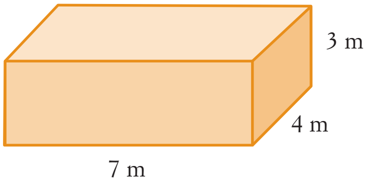
For this investigation, you will need some small containers such as lunch boxes or food containers, together with some centicubes.

- Put a layer of centicubes on the bottom of the container.
- Build up the centicubes in one corner to find out how many layers would be needed to fill the container.
- Work out the volume of the container.
- If the container has a volume written on the bottom, compare your answer with this figure.
- Repeat the procedure for several containers.



Example 17

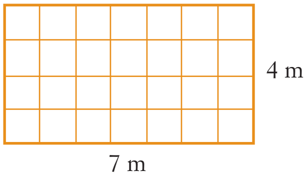
Use a strategy of 'filling' with cubes to determine the volume of the shape shown.



Solution

Draw the base of the shape.

Draw lines showing where cubes should be placed to cover the bottom.



Write the number of cubes needed to cover the bottom. **Number to cover base = $7 \times 4 = 28$**

Write the number of layers. **Three layers are needed for a height of 3 m.**

Write the total volume. **Volume = $28 \times 3 = 84$ m^3**

From Example **17** and the investigation with centicubes, you should be able to confirm the following rule for the calculation of volume.

Important!

Volume of a prism

The volume of a prism with a base area of A and height of h is given by $V = A \times h$.

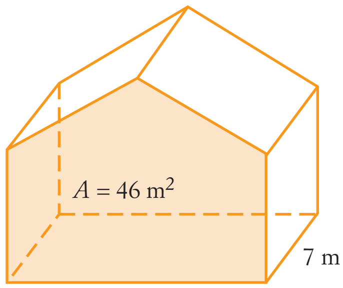
TLF learning object

Turn up the volume - unit of work (R11505)

MAT08MGIN00010

Example 18

Find the volume of the prism.



Solution

Write the rule for the area of a prism. $\boldsymbol{V = A \times h}$

The area of the base is 46 m^2 and the height is 7 m. $\boldsymbol{= 46 \times 7}$

Evaluate. $\boldsymbol{= 322}$ **m^3**

Write the answer. **The volume is 322 m^3.**

Example 19

Calculate the volume of this prism.

Solution

Calculate the area of the rectangular base.

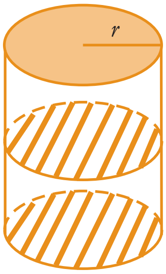
$$\begin{aligned} \boldsymbol{A} &= \boldsymbol{l \times w} \\ &= \mathbf{11\ cm \times 7\ cm} \\ &= \mathbf{77\ cm^2} \end{aligned}$$

Calculate the volume of the prism.

$$\begin{aligned} \boldsymbol{V} &= \boldsymbol{A \times h} \\ &= \mathbf{77\ cm^2 \times 3.5\ cm} \\ &= \mathbf{269.5\ cm^3} \end{aligned}$$

Write the answer. **The volume is 269.5 cm^3.**

Look at the cylinder.



The **cross-sections** taken parallel to the circular base are uniform.
This means that a cylinder can be treated as a prism when we calculate its volume. So the formula, $V = Ah$, can be used to calculate the volume of a cylinder.
For a circle, $A = \pi r^2$, so the volume formula $V = Ah$ becomes:

$$\begin{aligned} V &= \pi r^2 \times h \\ &= \pi r^2 h \end{aligned}$$

Important!

Volume of a cylinder

The volume of a cylinder with a base of radius r and height of h is given by:

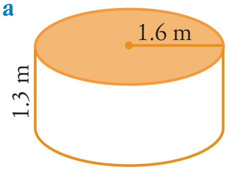
$$V = \pi r^2 h$$

9780170194778

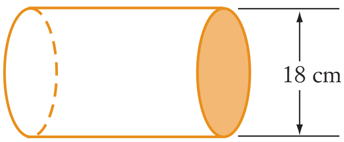
Example 20

Find the volume of these cylinders correct to one decimal place.

a



b



Solution

a Write the rule for the volume of a cylinder. $V = \pi r^2 h$

Substitute for the height and the radius of the cylinder. $= \pi \times (1.6)^2 \times 1.3$

You can use your calculator.

Enter as: π × 1.6 x^2 × 1.3 =. `π×1.6²×1.3   10.45522035`

Evaluate. $= 10.455\ 220\ldots$

Round off and write the answer. **The volume is approximately 10.5 m^3.**

b Write the rule for the volume of a cylinder. $V = \pi r^2 h$

The diameter is 18 cm, so the radius is 9 cm. Substitute for the height and the radius. $= \pi \times 9^2 \times 32$

You can use your calculator.

Enter as: π × 9 x^2 × 32 =. `π×9²×32   8143.008158`

Evaluate. $= 8143.008\ 158\ldots$

Round off and write the answer. **The volume is approximately 8143 cm^3.**

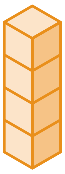
Scientific calculator exercise
Measurement and geometry
MAT08MGSC00007

Exercise 10.4 Volumes of prisms and cylinders

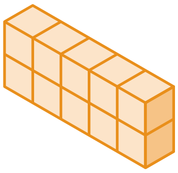
Understanding

1 Find the volume of each of the following shapes, if each cube is 1 cubic centimetre in size.

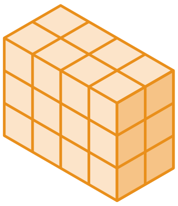
a



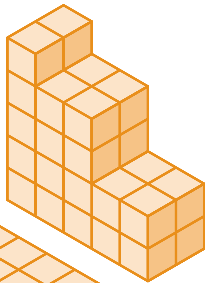
b



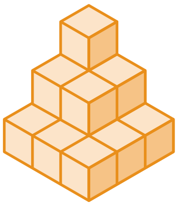
c



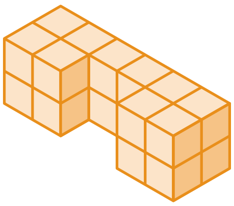
d



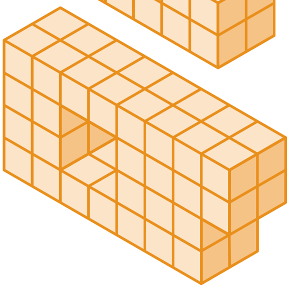
e



f



g



Extra questions
Exercise 10.4
MAT08MGEQ00035
See Example 16

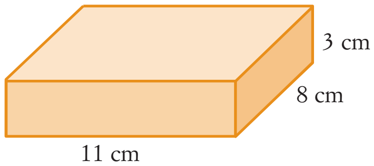
Worksheet
Volume
MAT08MGWK00070

Worksheet
Measuring shapes review
MAT08MGWK00071

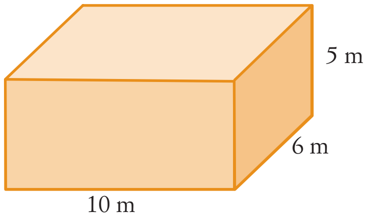
See Example 17

2 Use a 'filling' strategy to find the volume of each of the following.

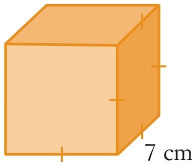
a



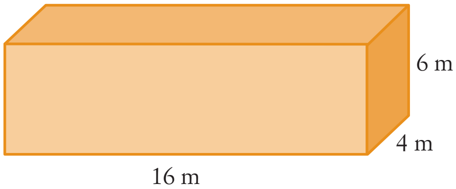
b



c



d



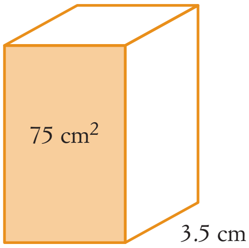
Fluency

Worked solutions
Exercise 10.4
MAT08MGWS00035

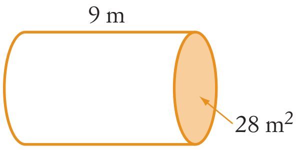
See Example 18

3 Calculate the volume of each of the following, correct to one decimal place if necessary.

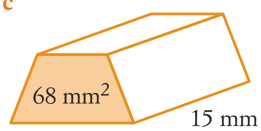
a



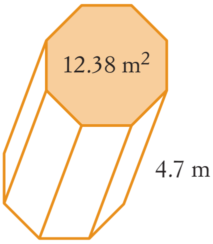
b



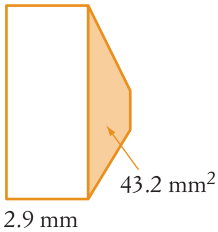
c



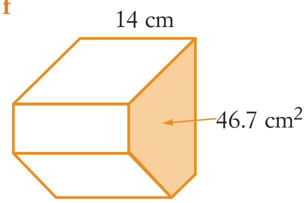
d



e



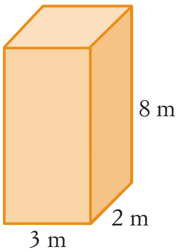
f



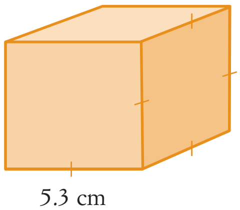
See Example 19

4 Calculate the volume of each of these prisms.

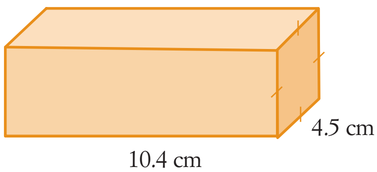
a



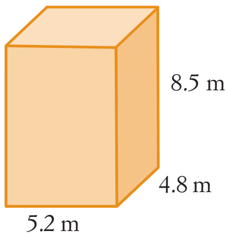
b



c



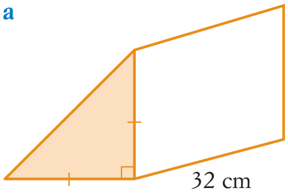
d



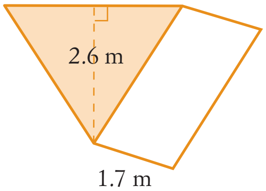
9780170194778

5 Calculate the volume of each of these prisms.

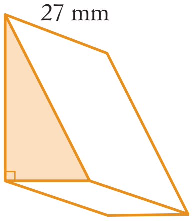
a



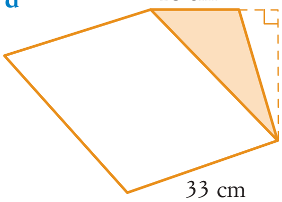
b



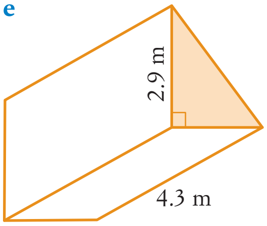
c



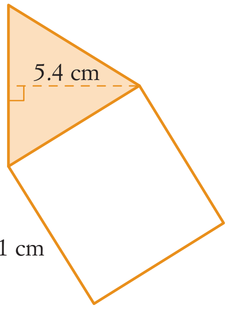
d



e



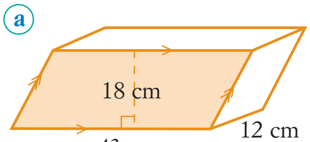
f



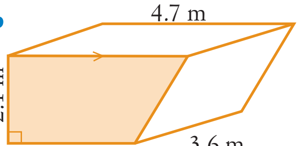
6 Calculate the volume of each of these prisms.

Worked solutions
Exercise 10.4
MAT08MGWS00035

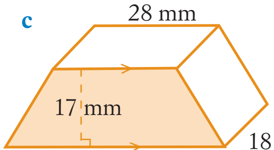
a



b



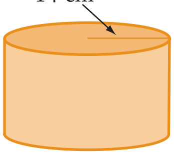
c



7 Calculate the volume of each of these cylinders correct to one decimal place.

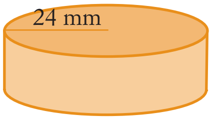
Worked solutions
Exercise 10.4
MAT08MGWS00035
See Example 20

a

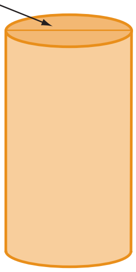


b

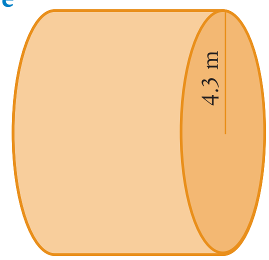
c



d



e



f

Problem solving

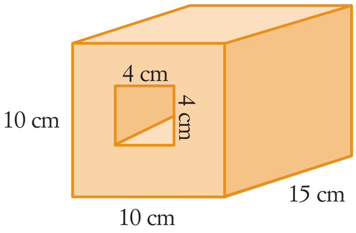
8 What is the most suitable unit for measuring the volume of each of the following?

See Example 15

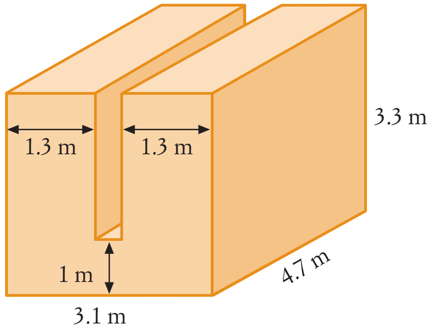
a swimming pool
b cake tin
c builder's wheelbarrow
d amount of concrete in a house foundation
e bottle of medicine
f your mouth

9 Find the volume of each of the following shapes.

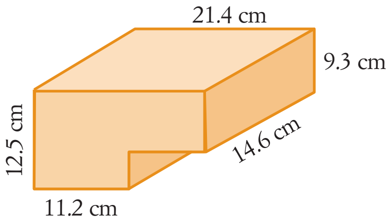
a



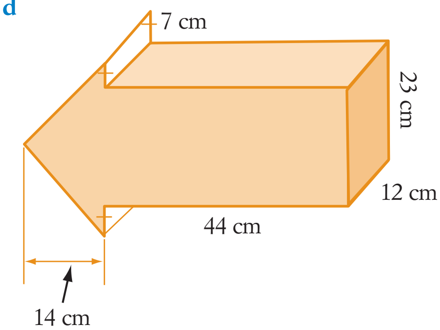
b



c

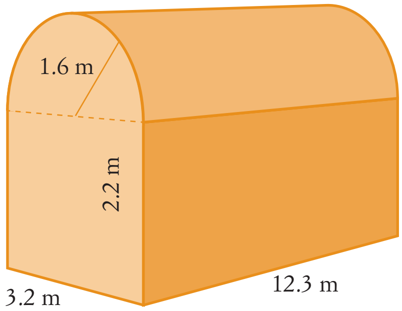


d

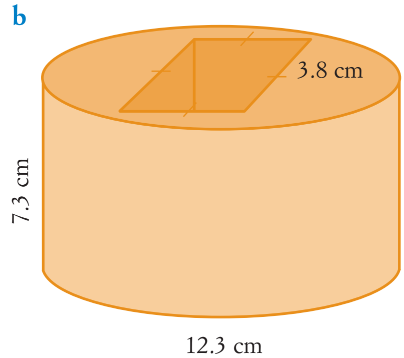


10 Find the volume of each of the following shapes correct to two decimal places.

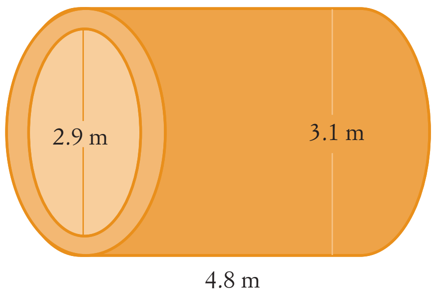
a



b



11 This concrete culvert has an inner diameter of 2.9 m and an outer diameter of 3.1 m. If the culvert is 4.8 m long, calculate the volume of concrete (correct to two decimal places) needed to make it.



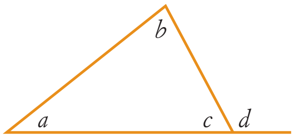
Reasoning

12 A cylindrical can has a volume of 40 000 cm^3. If the can has a diameter of 30 cm, calculate the height of the can to the nearest 0.1 cm.

9780170194778

10.5 Measuring triangles

You have previously looked at angle relationships for triangles.
For example, in the triangle on the right:



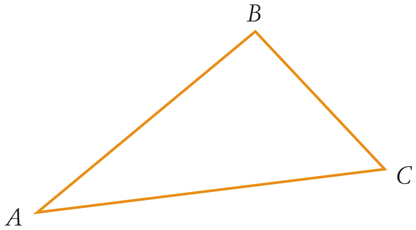
$$a + b + c = 180°$$

and $$d = a + c$$

Other triangle relationships are also important.

Investigate: Angles and sides of triangles

1 Look at $\triangle ABC$ on the right.



- Measure the interior angles and the sides of the triangle and record your results in a table below.

Angles		Sides	
$\angle A$		BC	
$\angle B$		AC	
$\angle C$		AB	

2 Now draw a triangle of your own and label the vertices.
- Measure the interior angles and the sides of the triangle and record your results in a table similar to the one above.

3 Repeat the procedure for another triangle.

4 Use the results in the table to complete the following statements.

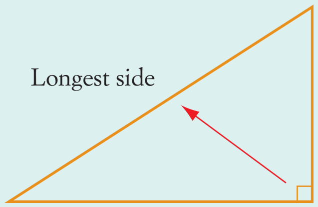
The shortest side of a triangle is opposite the __________ *interior angle.*
The longest side of a triangle is opposite the __________ *interior angle.*

The result established in the previous investigation is always true.

Important!

Sides and angles of triangles

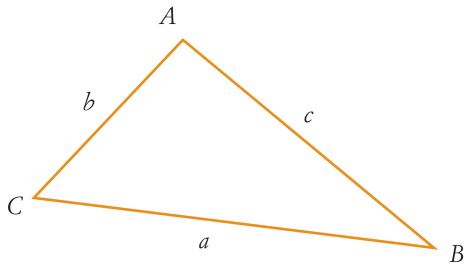
The shortest side of a triangle is always opposite the smallest interior angle.
The longest side of a triangle is always opposite the largest interior angle.



In a **right-angled** triangle, the right angle must be the largest angle. Why is this always true? This means that, in a right-angled triangle the longest side is always opposite the right angle.

Investigate: The triangle inequality

1 Draw a triangle similar to the one shown here and label the sides and vertices.



2 Measure the sides and record the measurements.
3 Select any side and compare its length with the sum of the lengths of the other two sides.
4 Select a different side and compare its length with the sum of the lengths of the other two sides.
5 Compare the length of the remaining side with the sum of the other two sides.
6 Use the results to complete the following statement.
Any side of a triangle is ________ than the sum of the other two sides.
7 Now draw another triangle with side lengths that are different from the one you drew in the first part of this investigation.
8 Repeat steps 2–6 using this new triangle.

The relationship that you established in the previous investigation is true for all triangles and is known as the triangle inequality theorem.

Important!

Triangle inequality theorem

Any side of a triangle is always shorter than the sum of the other two sides.
In any $\triangle ABC$ with sides a, b and c:

$$a < b + c$$

The **converse** of a theorem is like the *reverse* of it.

The reverse of	a is less than $b + c$
is	a is not greater than or equal to $b + c$

Important!

Converse of the triangle inequality theorem

Any side of a triangle is not equal to or longer than the sum of the other two sides.
In any $\triangle ABC$ with sides a, b and c:

$$a \ngeq b + c$$

The converse of the triangle inequality theorem means that a triangle cannot be constructed from three line segments if any of them is longer than the sum of the other two.

Example 21

Is it possible to construct a triangle with sides measuring 18 cm, 7 cm and 9 cm?

Solution

Calculate the sums of various pairs of sides of the triangle.	$\mathbf{18 + 7 = 25}$ $\mathbf{18 + 9 = 27}$ $\mathbf{9 + 7 = 16}$
Compare the sum of each pair of sides with the remaining side of the triangle.	$\mathbf{18 + 7 = 25 > 9}$ $\mathbf{18 + 9 = 27 > 7}$ $\mathbf{9 + 7 = 16 < 18}$
Compare the results with the converse of the triangle inequality theorem.	**The 18 cm side is longer than the sum of the other two sides of the triangle.**
State the result.	**It is not possible to construct a triangle with sides measuring 18 cm, 7 cm and 9 cm.**

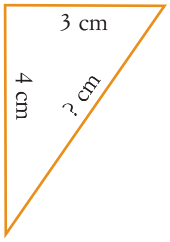
There is a special relationship between the lengths of the sides of right-angled triangles. The following investigation will help you understand this relationship.

Investigate: Sides of right-angled triangles

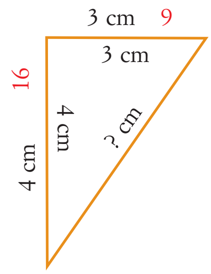
Teacher notes
Sides of right-angled triangles
MAT08MGTN00019

In this investigation, you will measure the side lengths of some right-angled triangles.

1 Draw a right-angled triangle as follows:
- Draw a line 4 cm long straight down your page.
- Now draw the second side from the top of the first line.
- Draw it 3 cm long straight across the page. It should make a right angle with the first line.
- Join the ends of the lines to make the third side.
- Measure the length of the third side (nearest 0.1 cm).



2 Draw more right-angled triangles with sides measuring:
- 12 cm down and 9 cm across
- 12 cm down and 5 cm across
- 6 cm down and 4.5 cm across
- 7.5 cm down and 4 cm across

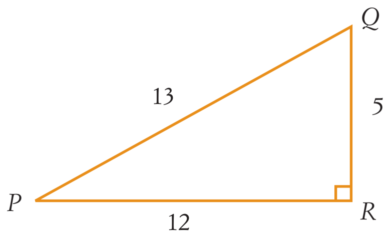


3 Measure the third sides of your new triangles, and write the lengths on the inside of each side.
4 Which side is the longest in a right-angled triangle?
5 Draw each of the five triangles on the board.
6 Under direction from your teacher, write your measurements for the third side of each triangle on the board.
7 Use the measurements of the whole class to decide what the measurements should be.
8 Write the 'correct' measurements on the outside of each triangle.
9 In a different colour (red), write the squares of each side on the outsides of the triangles.
10 Look for a pattern in the red numbers for each triangle.

Example 22

$\triangle PQR$ shown here is right-angled.
The dimensions are shown in centimetres.

a Calculate the square of each side of the triangle.
b Complete the following.
$PQ^2 = \_\_\_\_ + \_\_\_\_$



Solution

a Square each of the sides.

$PQ^2 = 169$
$QR^2 = 25$
$PR^2 = 144$

b Compare the squared side lengths.

$169 = 25 + 144$

Use this result to complete the statement.

$PQ^2 = QR^2 + PR^2$

9780170194778

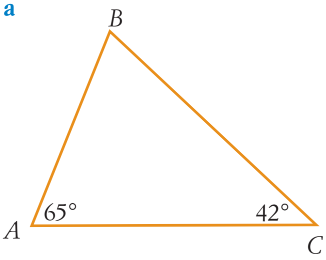
Exercise 10.5 Measuring triangles

Understanding

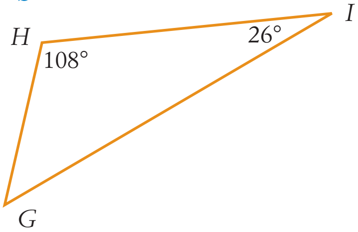
Extra questions
Exercise 10.5
MAT08MGEQ00036

1 Without using a ruler, determine the longest side in each of the following triangles.

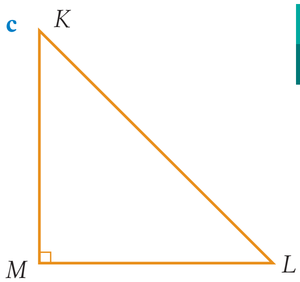
a



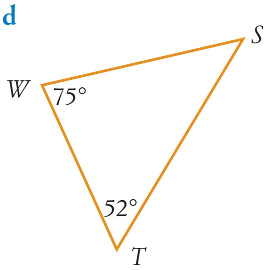
b



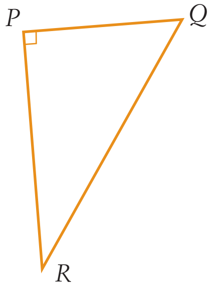
c



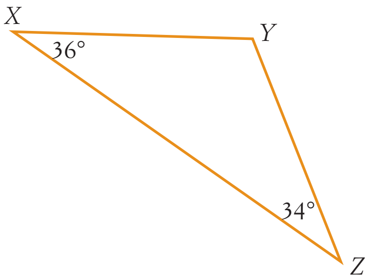
d



e

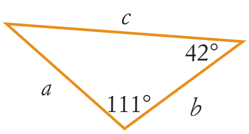


f

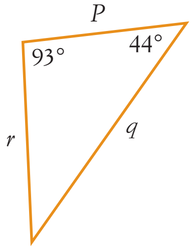


2 Without using a ruler, determine the shortest side in each of the following triangles.

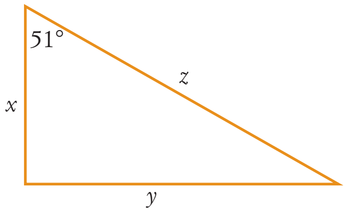
a



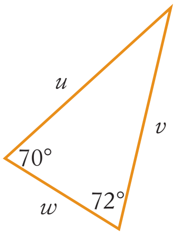
b



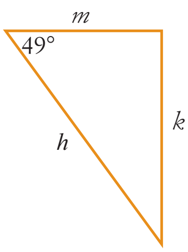
c



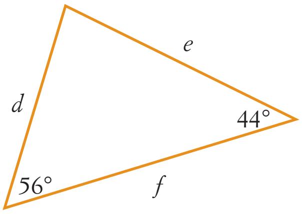
d



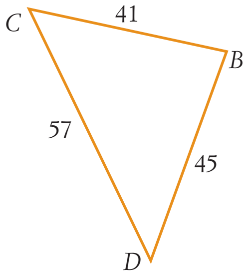
e

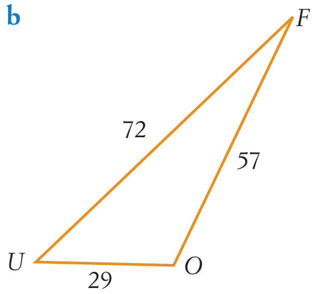


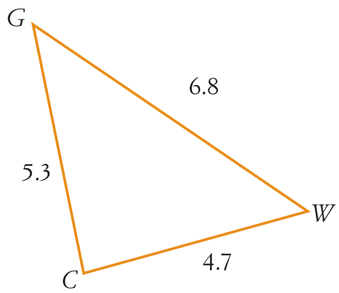
f

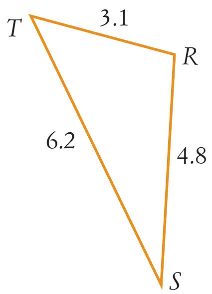


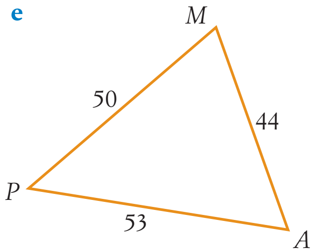
3 Without using a protractor, determine the smallest interior angle in each of the following triangles.

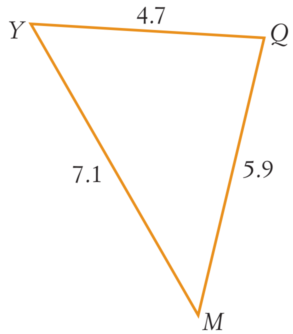
a


b


c


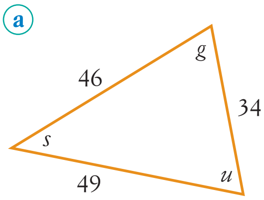
d


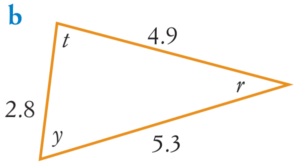
e


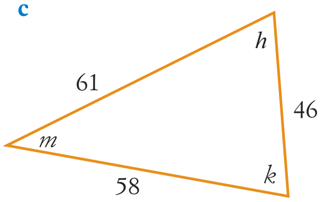
f


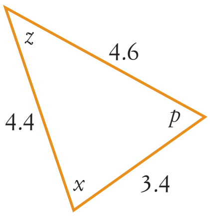
Worked solutions
Exercise 10.5
MAT08MGWS00036

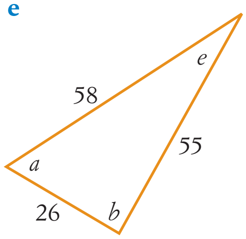
4 Without using a protractor, determine the largest interior angle in each of the following triangles.

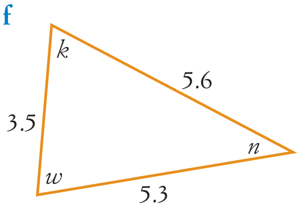
a


b


c


d


e


f


Worked solutions
Exercise 10.5
MAT08MGWS00036
See Example 21

5 Is it possible to construct a triangle with sides measuring:

a 28 cm, 15 cm and 18 cm
b 12 mm, 9 mm and 6 mm
c 19 cm, 26 cm and 11 cm
d 9 m, 19 m and 8 m
e 4.7 cm, 1.6 cm and 3.9 cm
f 27 mm, 32 mm and 63 mm?

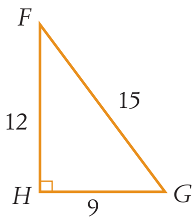
9780170194778

6 A triangle has sides a, b and c with $a = 17$ cm and $b = 9$ cm. What is the greatest length that c can be? *See* Example 21

7 For each of the following situations:
  i Calculate the square of each side of the triangle.
  ii Complete the statement regarding the sides of the triangle.

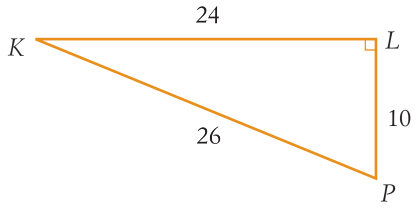
Worked solutions
Exercise 10.5
MAT08MGWS00036
See Example 22

a



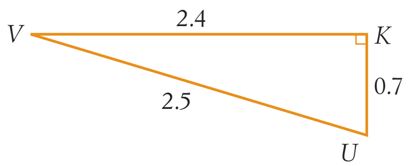
$FG^2 =$ _____ + _____

b



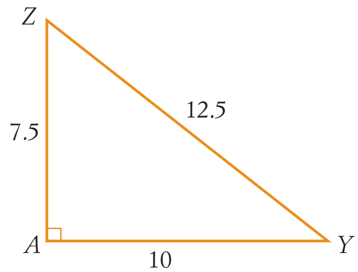
$KP^2 =$ _____ + _____

c



$VK^2 =$ _____ + _____

d



$ZY^2 =$ _____ + _____

10.6 Measuring time intervals

Calculating time intervals is an important life skill. There are a number of conventions that are important when writing and speaking about time. The middle of the day is called **noon** (or **midday**) and the middle of the night is called **midnight**. Other times are written or said as 12-hour or 24-hour times.

Weblink
The French republican calendar
MAT08MGWB00010

Important!

12-hour time

Times from midnight to noon are written with 'a.m.' at the end while times from noon to midnight are written with 'p.m.' at the end.
The hours, minutes and seconds are written in order and separated by a colon (:).

24-hour time

24-hour times are given as four digits. The first two show the number of hours after midnight and the last two show the minutes after the hour. The time is stated as 'hours' (h).

To convert from 24-hour time to 12-hour time, look at the first two digits.

- If they are 00, then it is just after 12:00 midnight.
- If they are less than 12, then it is 'a.m.' (morning) time. Write 'a.m.'.
- If they are exactly 12, it is 'p.m.' (evening) time. Write 'p.m.'.
- If they are 13 or more, it is 'p.m.' (evening) time. Subtract 12 and write 'p.m.'.

Then insert a colon before the last two digits.

Example 23

Convert:

a 8:37 a.m. to 24-hour time **b** 1408 h to 12-hour time

Solution

a It is morning, so remove the colon and add 0 to the front and an h to the end. **8:37 a.m. = 0837 h**

b The first two digits are greater than 12, so it is p.m. Subtract 12 from the first two digits and insert a colon. **1408 h = 2:08 p.m.**

Video tutorial
Time differences
MAT08MGVT10022

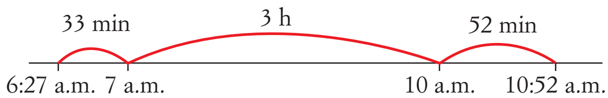
When calculating time intervals, you need to treat the hours and minutes separately. Sometimes, diagrams can help in calculating time intervals.

Example 24

Calculate the time interval between 6:27 a.m. and 10:52 a.m. on the same day.

Solution

Draw a line and mark in 6:27 a.m. at the start.



First move up to the next hour.

From 6:27 a.m. to 7:00 a.m. = 33 min.

From 7:00 a.m. to 10:00 a.m. = 3 h.

From 10:00 a.m. to 10:52 a.m. = 52 min.

Use the diagram to work out the time interval. **Time interval = 33 min + 3 h + 52 min**
= 3 h 85 min

Convert 85 minutes to hours and minutes. **= 3 h + 1 h 25 min**

Evaluate. **= 4 h 25 min**

9780170194778

You don't need to use a diagram to find time intervals.

Example 25

Find the length of time between 9:48 a.m. and 3:17 p.m. on the same day.

Solution

Calculate the time interval from 9:48 a.m. to midday.	**12 h 00 min − 9 h 48 min**
	= 11 h 60 min − 9 h 48 min
Subtract the hours and minutes separately.	**= (11 − 9) h + (60 − 48) min**
	= 2 h + 12 min
Calculate the time interval from midday to 3:17 p.m.	**3:17 p.m. is 3 h 17 min after midday.**
Find the total time interval.	**Time interval = 2 h 12 min + 3 h 17 min**
	= 5 h 29 min

There is another way to do time calculations. First convert times to 24-hour time. Then use the calculator's [° ' "] or [DMS] key to enter hours and minutes, and subtract the times.

Example 26

Use a calculator to find the time difference between 7:46 a.m. and 8:39 p.m.

Solution

Convert the times to 24-hour times.	**7:46 a.m. = 0746 h**
	8:39 p.m. = 2039 h

Use the calculator's [° ' "] or [DMS] key to enter hours and minutes, and subtract the times.

20 [° ' "] 39 [° ' "] [−] 7 [° ' "] 46 [° ' "] [=].	20°30°–7°46° 12°53°0°
State the result.	**8:39 p.m. − 7:46 a.m. = 12 h 53 min**

Investigate: Local time

The time in your region is called **local time**. You may have friends or relatives who live in other states of Australia or other countries. If you do, you probably know that their local time may not be the same as yours

- Obtain a map of South-East Asia and make a list of cities in various locations across the region. Try to find at least 15 cities and include your current location in the list. Make sure that you include cities in New Zealand, Fiji and other places east of your location.

- Make a note of whether each location is east or west of yours.
- The internet has a large number of sites that show the local time in places all over the world. Use the internet or some other means to find the local time in each of the cities in your list.
- You will notice that some times are earlier than (before) your local time while others are later (after). Make a note of whether each location is before or after the local time of your region.
- What generalisations can you make regarding whether cities are before or after local time and whether they are east or west of your location?
- Why do different locations on the Earth's surface have different local times?

Exercise 10.6 Measuring time intervals

Understanding

Extra questions
Exercise 10.6
MAT08MGEQ00037

1 Change each of the following to 24-hour time.

a 3 p.m.	**b** 3 a.m.	**c** 12 noon	**d** 10:40 p.m.
e 8:19 p.m.	**f** 9:06 p.m.	**g** 11:20 a.m.	**h** 11:40 p.m.
i 3:07 a.m.	**j** 8:45 a.m.	**k** 10:02 p.m.	**l** 12:05 a.m.
m 6:42 a.m.	**n** 2:15 a.m.	**o** 11:47 p.m.	**p** 10:23 a.m.

See Example 23

2 Change each of the following to 12-hour time.

a 0900 hours	**b** 1500 hours	**c** 0530 hours	**d** 1740 hours
e 0842 hours	**f** 2230 hours	**g** 1550 hours	**h** 2230 hours
i 0010 hours	**j** 1350 hours	**k** 0410 hours	**l** 1835 hours
m 2133 hours	**n** 0231 hours	**o** 1052 hours	**p** 2101 hours

See Example 24

3 Use a diagram to help you calculate the time interval between:

a 7:15 p.m. and 8:20 p.m.	**b** 1016 h and 1206 h
c 4:09 a.m. and 9:53 a.m.	**d** 11:15 p.m. and 3:08 a.m.
e 0727 h and 1312 h	**f** 9:36 p.m. and 9:14 a.m.
g 7:45 p.m. and 10:10 p.m.	**h** 2:24 a.m. and 3:07 a.m.
i 4:15 p.m. and 6:02 p.m.	**j** 10:25 a.m. and 1433 h
k 8:40 a.m. and 4:19 p.m.	**l** 0645 h and 2010 h

9780170194778

Fluency

4 Find:

a 3 h 26 min + 5 h 18 min
b 2 h 22 min + 6 h 28 min
c 9 h 21 min + 6 h 46 min
d 5 h 44 min + 7 h 31 min
e 8 h 41 min + 3 h 37 min
f 6 h 47 min + 3 h 39 min

5 Find:

a 3 h 54 min – 1 h 17 min
b 10 h 43 min – 2 h 8 min
c 10 h 32 min – 3 h 51 min
d 7 h 17 min – 2 h 38 min
e 9 h 26 min – 5 h 42 min
f 8 h 17 min – 4 h 33 min

6 Calculate the time interval between:

Worked solutions
Exercise 10.6
MAT08MGWS00037
See Example 25

a 6:23 p.m. and 9:54 p.m.
b 1122 h and 1348 h
c 9:11 a.m. and 10:58 a.m.
d 10:25 p.m. and 4:09 a.m.
e 0638 h and 1517 h
f 8:34 p.m. and 7:26 a.m.
g 6:55 p.m. and 11:18 p.m.
h 4:47 a.m. and 6:04 a.m.
i 3:22 p.m. and 10:09 p.m.
j 11:17 a.m. and 1627 h
k 10:27 a.m. and 6:22 p.m.
l 0736 h and 2108 h

7 Use a calculator to find the time interval between:

Worked solutions
Exercise 10.6
MAT08MGWS00037
See Example 26

a 9:18 p.m. and 10:34 p.m.
b 1034 h and 1327 h
c 7:12 a.m. and 10:46 a.m.
d 10:21 p.m. and 5:09 a.m.
e 0826 h and 1421 h
f 6:38 p.m. and 7:17 a.m.
g 8:46 p.m. and 11:20 p.m.
h 3:43 a.m. and 6:09 a.m.
i 5:22 p.m. and 9:05 p.m.
j 11:28 a.m. and 1324 h
k 5:46 a.m. and 7:22 p.m.
l 0542 h and 2109 h

Problem solving

8 The time shown on a Blu-ray recorder is 1954 h. Twenty-five minutes later Yasmine started watching a movie that lasted 138 minutes. She then made a phone call to her friend that lasted 26 minutes. When she finished talking to her friend, Yasmine checked the time on the clock of the Blu-ray player. What time did it show?

9 The table below shows a train service timetable.

Worked solutions
Exercise 10.6
MAT08MGWS00037

City to Shorncliffe Outbound — Monday to Friday Service

Comes from		IPS	CVN	CVN	IPS	IPS	CQD	IPS	IPS	IPS	CQD	CQD
Station	am	am	am	am	am	am	am	am	am	am	am	am
Toowong	.	5:53	1	2	7:09	exp	7:51	8:13	8:43	9:12	9:38	10:08
Auchenflower	.	5:55	Via Sth Bne	Via Sth Bne	7:11	exp	7:53	8:15	8:45	9:15	9:41	10:11
Milton	----	5:57			7:13	7:37	7:55	8:17	8:47	9:17	9:43	10:13
Roma Street pform	#9	#9	#6	#6	#9	#9	#9	#9	#9	#9	#9	#9
Roma Street	5:24	6:00	6:18	6:49	7:17	7:42	7:58	8:21	8:50	9:20	9:46	10:16
Central arrive	**5:27**	**6:03**	**6:21**	**6:52**	**7:21**	**7:46**	**8:01**	**8:24**	**8:53**	**9:23**	**9:49**	**10:19**
Central pform	**#6**	**#6**	**#3**	**#3**	**#6**	**#6**	**#6**	**#6**	**#6**	**#6**	**#6**	**#6**
Central depart	**5:29**	**6:05**	**6:23**	**6:54**	**7:22**	**7:47**	**8:03**	**8:26**	**8:55**	**9:26**	**9:52**	**10:22**
Boondall	**5:54**	**6:29**	**6:51**	**7:22**	**7:49**	**8:12**	**8:28**	**8:51**	**9:21**	**9:51**	**10:17**	**10:47**
North Boondal	5:56	6:31	6:53	7:24	7:51	8:14	8:30	8:53	9:23	9:53	10:19	10:49
Deagon	5:58	6:33	6:55	7:26	7:53	8:16	8:32	8:55	9:25	9:55	10:21	10:51
Sandgate	6:00	6:35	6:57	7:28	7:55	8:18	8:34	8:57	9:27	9:57	10:23	10:53
Shorncliffe	**6:02**	**6:37**	**6:59**	**7:30**	**7:57**	**8:20**	**8:36**	**8:59**	**9:29**	**9:59**	**10:25**	**10:55**

1 This service runs as per the following times - Park Road: 6:08am, South Bank: 6:11am, South Brisbane: 6:13am
2 This service runs as per the following times - Park Road: 6:39am, South Bank: 6:42am, South Brisbane: 6:44am

a What is the time taken to travel from Toowong to Shorncliffe on the 9:38 a.m. train?

b Jamal has to be in Shorncliffe at 8:30 a.m. for a meeting. It will take him 18 minutes to walk from the station to his meeting. At what time must he catch the train from Roma Street to ensure that he is at the meeting on time?

c Nima travels from Auchenflower, arriving at North Boondall at 8:53 a.m. How long did the trip take?

d Sheng arrived at Shorncliffe station at 6:37 a.m. His friend Yuan was running very late and caught the train at Milton at 9:17 a.m. How long did Sheng have to wait at Shorncliffe before Yuan arrived?

10 The table below shows a bus service timetable from Melbourne to Brisbane.

MELBOURNE - BRISBANE

TOWN	CODE		GX340 WED-FRI ONLY DST	PICK-UP AND SET-DOWN POINT
MELBOURNE	**MEL**	**Dep**	**7:15P**	**Greyhound Terminal, Spencer St, Southern Cross Terminal**
Seymour *ONR*	SYR		8:30P	Interstate Bus Stop, Emily St. Outside Trade Centre
Wahring	**WAH**	**Arr**	**8:55P**	**Caltex 24hr**
		Dep	**9:30P**	
Shepparton	SHT		10:00P	Railway Station, Purcell St
Katamatite *ONR*	KTE		10:30P	Opposite Bakery
NEW SOUTH WALES				
West Wyalong	WWL		3:00A	Country Link Bus stop, Church St
Forbes	FBS		4:10A	Caltex Service Station
Parkes	**PKE**	**Arr**	**4:40A**	**BP Roadhouse, Newell Highway**
		Dep	**5:20A**	
Peak Hill *ONR*	PKH		5:55A	Rural Supplies, Caswel St
Narrabri Meal Break	**NAM**	**Arr**	**10:15A**	Shell 24r Roadhouse
		Dep	**10:45A**	
Narrabri	NAA		10:55A	Tourist Information Centre, Newell Highway
Bellata	BEL		11:35A	Independent Service Station
Moree	MRZ		12:05P	Tourism Information Centre, Newell Highway
Boggabilla *ONR*	BGG		1:25P	Shell Roadhouse (24hr)
QUEENSLAND				
Goondiwindi	**GOO**	**Arr**	**12:35P**	**Caltex Roadhouse, Boundry Rd**
		Dep	**1:10P**	
Millmerran	MIL		2:45P	Opp Bus Stop, Commens St, Near QATB
Pittsworth *ONR*	PIT		3:15P	Bus Shelter Short St
Toowoomba	**TWB**	**Arr**	**3:50P**	**Greyhound Terminal, 28-30 Neil St**
		Dep	**4:00P**	
Withcott	WCT		-	Bus stop at the Hotel
Gatton	GAT		-	Opposite Civic Centre
Ipswich	IPS		-	Transit Centre, Mansfield Place
BRISBANE	**BNE**		**5:40P**	**Greyhound Terminal, Level 3, Roma St Transit Centre**

'ONR' – 'On request stop'

a How long does the trip from Melbourne to Brisbane take?

b Kamal joins the bus at Parkes and gets off at Toowoomba. How long was his trip?

c Nailah joins the bus at Shepparton and gets off at Moree. How long was her trip?

d Fatima caught the bus at Forbes and arranged for her father to pick her up at the Brisbane terminal. He couldn't get there until 7:30 p.m. on the night she arrived. How long after she left Forbes did she meet her father?

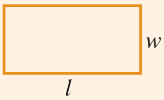
9780170194778

- Units of **area** are used to measure flat **surfaces**. We use **square units** to measure area. Large areas are measured in square metres (m^2), square kilometres (km^2) or hectares (ha) (1 ha = 10 000 m^2). Small areas are measured using square millimetres (mm^2) or square centimetres (cm^2).
- There are rules for finding the areas of common plane figures.

Quiz
Measuring shapes and time
MAT08MGQZ00010

Rectangle

$A = l \times w$

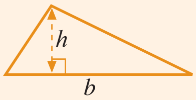


Circle

$A = \pi r^2$

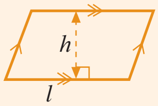
Triangle

$A = \frac{1}{2} b \times h$



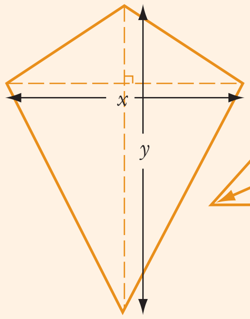
Parallelogram

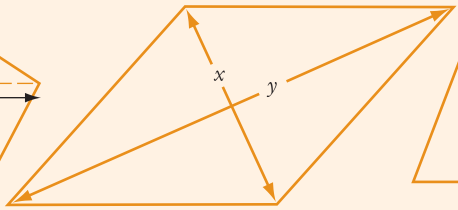
$A = l \times h$



Kite or rhombus

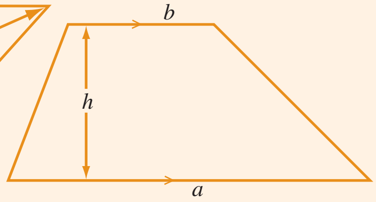
$A = \frac{1}{2}xy$



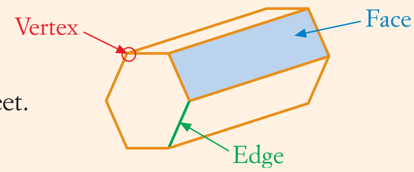


Trapezium

$A = \frac{1}{2}(a + b) \times h$



- Combined shapes are made up of other shapes. The area of a combined shape can be found by dividing the shape into other shapes whose areas may be found.
- The **faces** of a 3D shape are the sides that separate the inside from the outside.
- The **edges** of a 3D shape are the lines where the faces meet.
- The **vertices** of a 3D shape are the corners where the edges meet.



- A **prism** has two parallel faces that are identical polygons. One of these faces is called the base of the prism. The other faces of a prism are made by the lines joining the vertices of the base and the parallel face.
- A prism's name is determined by the polygon that makes the base.
- A **pyramid** has a base that is a polygon. The other faces are triangles that all meet at one vertex, the 'point' of the pyramid.
- A **sphere** is a ball shape that is completely symmetrical.
- A **cylinder** is like a prism except that it has a circular base.
- A **cone** is like a pyramid except that it has a circular base.

Chapter 10 summary

- A **hedron** is a 3D shape that has only one or two different-sized polygons as its faces and is generally named using the Greek for the number of faces and the suffix **-hedron**.
- The **platonic** solids are the hedrons for which every face is the same regular polygon and every vertex joins the same number of faces. The **tetrahedron** (4 sides), **octahedron** (8 sides) and **icosahedron** (20 sides) have faces that are equilateral triangles. The **cube** has 6 square faces while the **dodecahedron** has 12 faces that are regular pentagons.
- A **plan** of a 3D object shows how it appears from directly above. An **elevation** of a 3D object shows what it looks like from a particular side. A **side elevation** shows it from one side while a **front elevation** shows it from the front.
- A **net** of a 3D shape is a 2D shape that can be folded up to make the 3D shape.
- The **surface area** of a 3D shape is the total area of all its faces.
- The **volume** of a 3D object is a measure of the amount of space it occupies. It is calculated by working out how many cubes of unit size would be needed to fill the same space. Volume is measured in **cubic** units such as **cubic centimetres** (**cm^3** or **cc**) or **cubic metres** (**m^3**).
- The volume of a prism with a base area of A and height of h is given by $V = A \times h$.
- The shortest side of a triangle is always opposite the smallest interior angle. The longest side of a triangle is always opposite the largest interior angle.
- The side of a triangle is always shorter than the sum of the other two sides. In any $\triangle ABC$ with sides a, b and c:

$$a < b + c$$

- No side of a triangle is equal to or longer than the sum of the other two sides.
  In any $\triangle ABC$ with sides a, b and c:

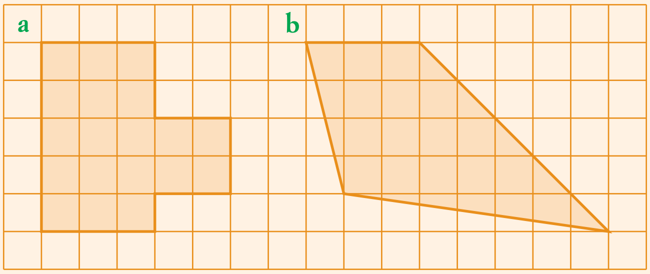
$$a \ngeq b + c$$

- In **12-hour time**, times from midnight to noon are written with ‘a.m.’ at the end while times from noon to midnight are written with ‘p.m.’ at the end. The hours, minutes and seconds are written in order and separated by a colon (:).
- **24-hour time**s are given as four digits. The first two show the number of hours after midnight and the last two show the minutes after the hour. The time is stated as ‘hours’ (h).

9780170194778

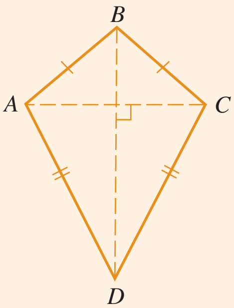
Understanding

1 Calculate the area of the figures below in square units.

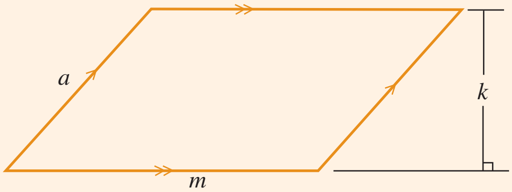


2 Write the rule for finding the area of these figures.

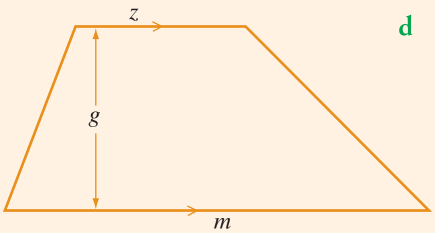
a



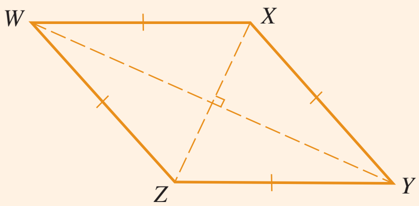
b



c

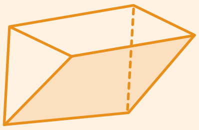


d

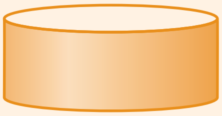


3 State whether each of the following is a prism, a pyramid or some other shape. *See* Example 8

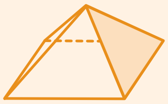
a



b

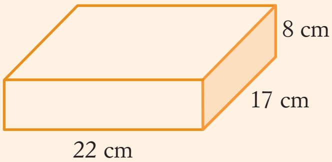


c

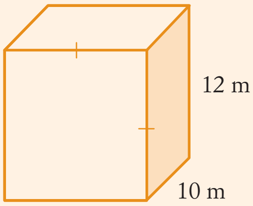


4 Sketch views of each of the following from the front, right side and top. *See* Example 9

a



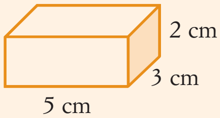
b



See Example 10
5 Draw nets for each of the shapes in question **4**, including measurements.

See Example 16
6 What is the volume of this shape, if each cube is one metre long?

See Example 17
7 Use a 'filling' strategy to work out the volume of this shape.



See Example 23
8 Change each of the following to 24-hour time.
a 7 a.m. **b** 10:16 a.m. **c** 6:24 p.m. **d** 11:18 p.m.

See Example 23
9 Change each of the following to 12-hour time.
a 0300 hours **b** 1720 hours **c** 0032 hours **d** 1926 hours

See Example 24
10 Use a diagram to help you calculate the time interval between:
a 3:21 a.m. and 3:45 p.m. **b** 0642 h and 1451 h

Fluency

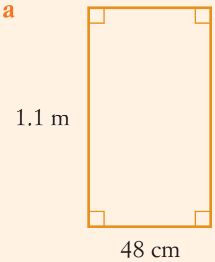
11 Select the most likely area measurement for each object.

a	table top	1500 mm^2	150 cm^2	1.5 m^2
b	page from a book	500 mm^2	500cm^2	0.5 m^2
c	eraser	800 mm^2	80 cm^2	0.8 m^2

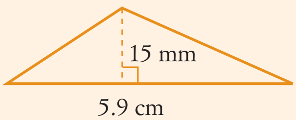
12 Use the summary provided here to convert the following areas to the units shown in brackets.
a 7 cm^2 (mm^2) **b** 0.055m^2 (cm^2)
c 6.52 ha (m^2) **d** 1.07 km^2 (ha)

See Example 1
13 Calculate the areas of the following (give answers correct to 1 decimal place).

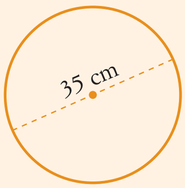
a



b



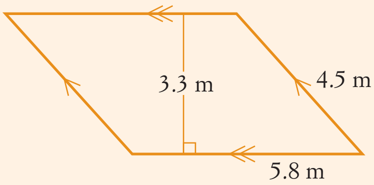
c



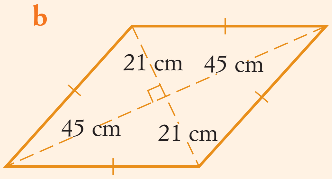
9780170194778

14 Calculate the areas of the following. *See* Example 4–7

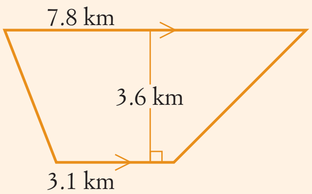
a



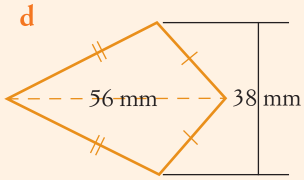
b



c

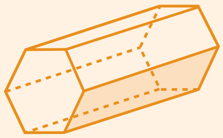


d

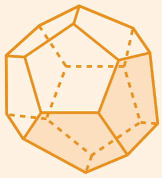


15 Give the full name of each shape shown below. *See* Example 8

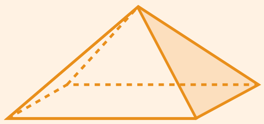
a



b

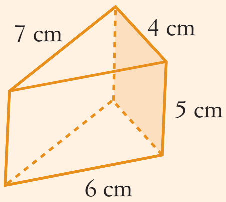


c

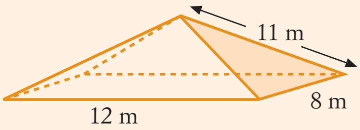


16 Sketch views of each of the following from the front, right side and top. *See* Example 9

a

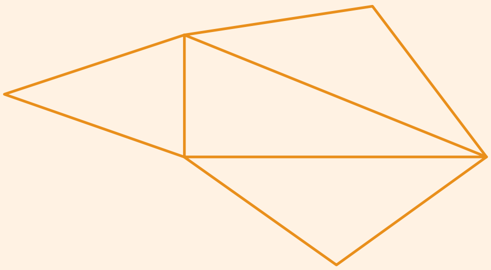


b



17 Draw nets for each of the shapes in question **5**, including measurements. *See* Example 10

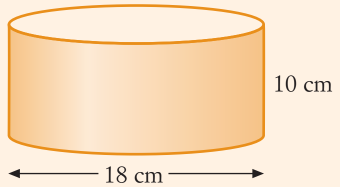
18 Sketch the 3D shape for the following net.



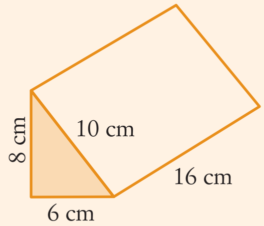
See Example 9

19 Sketch views of each of the following from the front, right side and top.

a



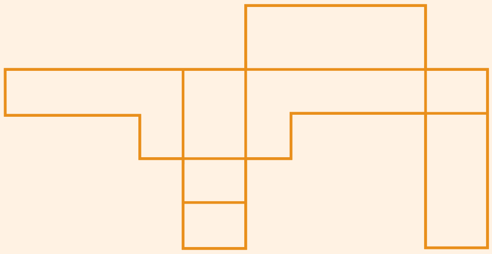
b



See Example 10

20 Draw nets for each of the shapes in question **19**.

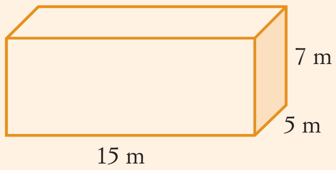
21 Sketch the 3D shape for the following net.



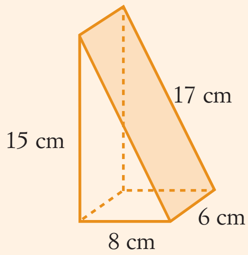
See Example 12, 13

22 Find the surface area of each of the following.

a

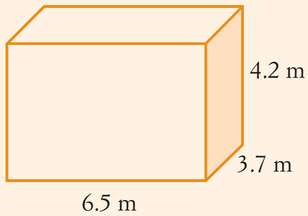


b



See Example 19

23 Find the volume of this shape.



24 Find:

a 4 h 43 min + 2 h 37 min **b** 6 h 44 min + 2 h 45 min

25 Find:

a 7 h 36 min – 2 h 27 min **b** 7 h 15 min – 5 h 38 min

See Example 25

26 Calculate the time interval between:

a 2:37 p.m. and 8:14 p.m. **b** 1021 h and 1415 h

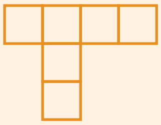
9780170194778

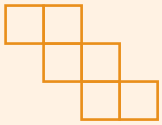
27 Use a calculator to find the time interval between: See Example 26

a 7:14 a.m. and 11:44 a.m. b 1054 h and 1402 h

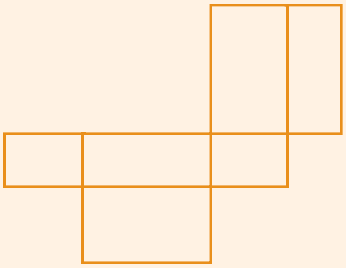
Problem solving

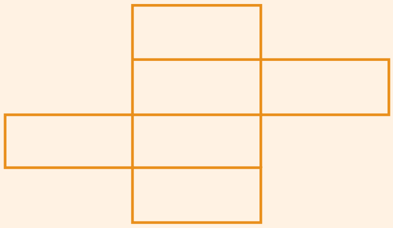
28 Determine if each of the following shapes forms a net. For each one that does, sketch the 3D shape. See Example 11

a


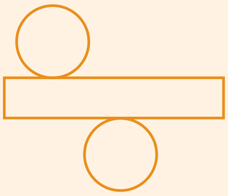
b


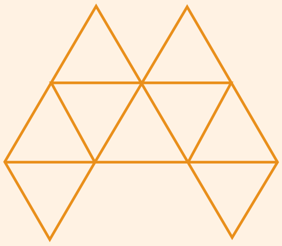
29 Determine if each of the following shapes forms a net. For each one that does, sketch the 3D shape. See Example 11

a


b


30 Determine if each of the following shapes forms a net. For each one that does, sketch the 3D shape. See Example 11

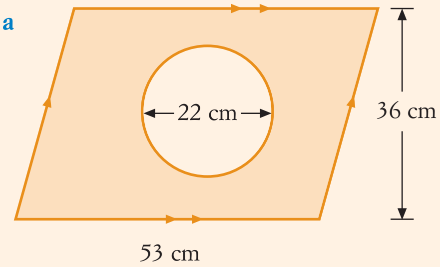
a


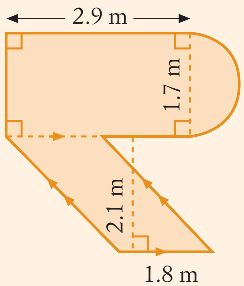
b


31 A tissue box is 11 cm by 11 cm by 22 cm. How much cardboard is needed to make the box?

32 What is the most suitable unit for measuring the volume of a house brick?

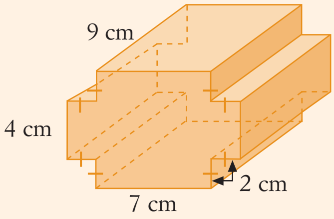
33 Calculate the areas of the shaded regions (give answers correct to 1 decimal place). See Examples 2, 3

a


b


See Example 13

34 Find the volume of the following shape.



See Example 20

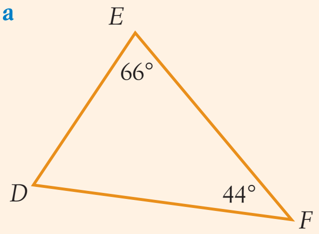
35 Find the volume of the triangular prism shown in question **19**.

See Example 20

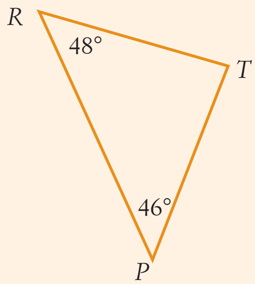
36 Find the volume of the cylinder shown in question **19**.

37 Without using a ruler, determine the longest side in each of the following triangles.

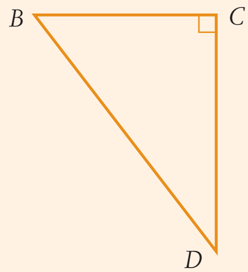
a



b

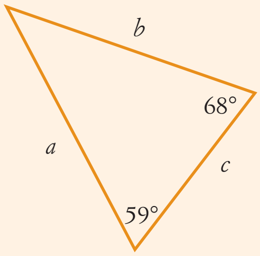


c

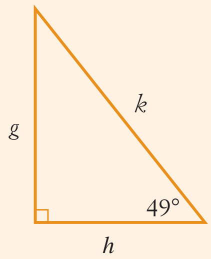


38 Without using a ruler, determine the shortest side in each of the following triangles.

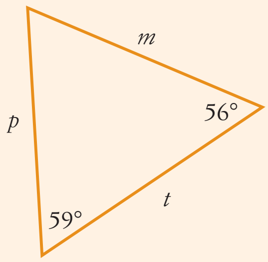
a



b

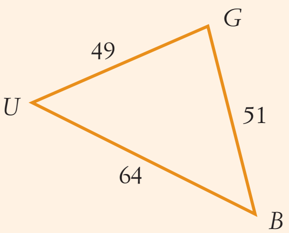


c

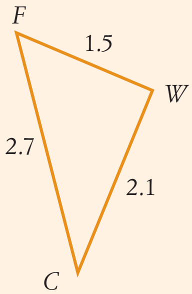


39 Without using a protractor, determine the smallest interior angle in each of the following triangles.

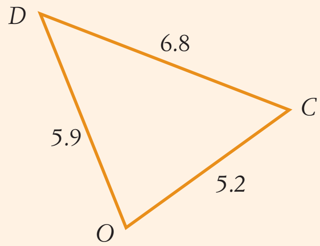
a



b



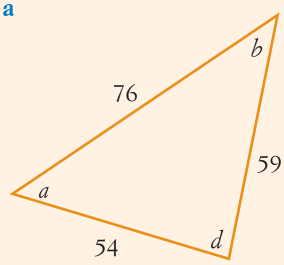
c



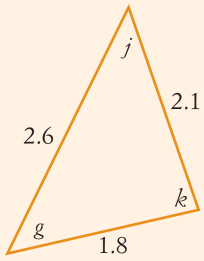
9780170194778

40 Without using a protractor, determine the largest interior angle in each of the following triangles.

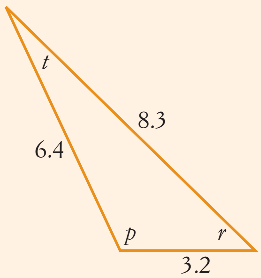
a



b



c



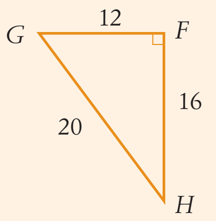
41 Is it possible to construct a triangle with sides measuring: *See* Example 21

a 36 cm, 18 cm and 28 cm **b** 42 mm, 19 mm and 16 mm
c 35 cm, 21 cm and 19 cm **d** 2.3 m, 5.9 m and 1.8 m?

42 For each of the following situations: *See* Example 22

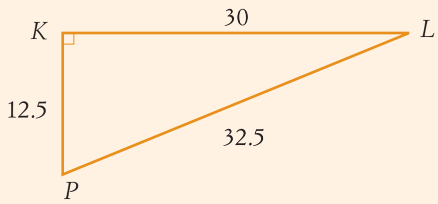
i calculate the square of each side of the triangle
ii complete the statement regarding the sides of the triangle.

a



$GH^2 = \_\_\_ + \_\_\_$

b



$PL^2 = \_\_\_ + \_\_\_$

Reasoning

43 A cylindrical can is designed to hold 5000 cm^3 of petrol. If the can has an internal height of 22 cm, what is its diameter?

9780170194778

Number and algebra

11 Personal business calculations

9780170194778

Contents

Prior learning
Chapter 11
MAT08NAPL00011

Parent guide
Chapter 11
MAT08NAPG00011

Curriculum guide
Chapter 11
MAT08NACU00011

Australian Curriculum statements

Real numbers
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies.

Money and financial mathematics
Solve problems involving profit and loss, with and without digital technologies.

Maths clip
Business maths
MAT08NAMC00006

How do you know whether you will have enough money to pay your bills? What about saving for a holiday or a deposit on a house? Can you afford to buy a DVD burner on an instalment-purchase scheme? What is the cheapest loan?

Many people buy mobile phones on a $0 cost basis, but have to join a 'plan' that involves paying monthly amounts over a number of years. What is the best plan for you? Should you have a pre-paid phone?

These are all questions about money management and budgeting. This chapter will help you to work out how to better manage your own money.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

balance	Goods and Services Tax (GST)	loss	retainer
budget	income	marked price	selling price
commission	instalment	percentage	simple interest
cost price	interest	principal	unitary method
discount	interest rate	profit	
expenditure		repayment	

11.1 Accounts and budgets

You can use simple ratios or rates to compare the value of different brands or package sizes of the same things. You usually work out the cost of the same amount of the items you want to compare. This is called a **unitary method**.

Important!

Unitary method

To use the unitary method, a ratio or rate is used to work out the value of one quantity for a fixed amount (often 1) of another. This is particularly useful in price comparisons.

Example 1

Technology worksheet
Excel: Price comparison
MAT08NACT00013

TLF learning object
School canteen: Estimate and check (L1934)
MAT08NAIN00011

You can buy toothpaste in 100 g or 150 g tubes. The 100 g tube costs $3.90. The 150 g tube cost $5.80. Which is the best buy?

Solution

In this case, you can see immediately that both tubes have a simple multiple of 50 g in them.

Work out the cost of 50 g for the 100g tube.

Cost of 50 g for 1st tube = \$3.90 ÷ 2
= \$1.95/50 g

9780170194778

Work out the cost of 50 g for the 150 g tube.	**Cost of 50 g for 150 g tube** $= \$5.80 \div 3$ $\approx \$1.93/50$ **g**
Compare the cost.	**The 150 g tube is cheaper for 50 g.**
Write the answer.	**The 150 g tube is the best buy.**

When you pay for goods, there may be a minimum amount if you want to use a credit card or get a **cash-out**. Most banks and some stores charge a fee for credit card transactions. When you pay cash, the amount is rounded to the nearest 5 cents. When comparing costs, you should take any fees into account.

Example 2

Sandra is competing in the Goju-Kai Karate World Championships. She needs to buy a mouth guard for $58.70, two arm guards at $11.49 each and four bottles of sports drink at $1.99 each. Paying by credit card would cost an extra 50 cents.

a What is the exact bill?
b What is the actual cash price?
c How much extra would using a credit card actually cost?

Solution

a Set out the costs and totals in a table to make it easier to follow. You can use your calculator to multiply the cost per unit by the number of units to get each amount.

Item	Cost/unit	Units	Amount
Mouth guard	$58.70	1	$58.70
Arm guard	$11.49	2	$22.98
Sports drink	$1.99	4	$7.96
		Total	$89.64

Add to get the total.
Write the answer. **The exact bill is $89.64.**

b Round to the nearest 5 cents. **The cash price is $89.65.**

c Add the credit card charge. **Credit card price** $= \$89.64 + \$0.50$
$= \$90.14$

Find the difference from cash. **Extra cost** $= \$90.14 - \$89.65$
$= \$0.49$

Write the answer. **Using a credit card would cost 49 cents extra.**

The charges for mobile phones are often difficult to calculate. Mobile phone 'plans' are stated with a monthly cost and include the cost of the phone, but they also use different charging rates. The actual cost of mobile phones varies from about \$100 for basic phones up to about a thousand dollars for phones with built-in camera and Internet capabilities. Many people send SMS messages, as they are charged only the connection fee for a message of up to 160 characters. Thus a 330-character message would be charged as 3 messages.

Example 3

Rianna signed up for a \$35 monthly mobile phone plan that included a mobile phone. In the first month, she made an average of 5 calls a day, lasting an average of 3 minutes each. Her contract was for 2 years and included \$30 of 'free' calls each month. The call rate was 90 c/min with a call connection fee of 25c.

a What was the cost of a 180-character SMS message?
b What was her average cost of a call?
c What was her bill for the first month?
d What would the total cost of the phone be over 2 years if she continued this way?

Solution

a 180-character message is charged as two messages.

SMS message cost = \$0.25 × 2
= \$0.50

Write the answer.

The cost of the message was 50c.

b Multiply the call rate by the average time.

Timed cost for call = \$0.90 c/min × 3 min
= \$2.70

Add the connection fee.

Total for call = \$2.70 + \$0.25
= \$2.95

Write the answer.

Her average call cost was \$2.95.

c Work out the cost of 5 calls in a day.

Daily call cost = \$2.95 × 5
= \$14.75

Assume there are 30 days in a month.

Call cost for month = \$14.75 × 30
= \$442.50

Find the total cost for the month.

Total for month = call cost + monthly charge − 'free calls'
= \$442.50 + \$35 − \$30
= \$447.50

Write the answer.

Her bill for the first month was \$447.50.

d Work out the cost for 24 months.

Cost for 2 years = \$447.50 × 24
= \$10 740

Write the answer.

The cost for 2 years would be \$10 740.

To manage money properly, you should spend only what you can afford. You can draw up a **budget** to make sure that this is the case.

9780170194778

Important!

Budgets

A **budget** is a financial plan. It is often presented as a table with two columns, called a **balance sheet**. The first column shows **income** and the second shows **expenditure**. Amounts can be **carried forward** from one period to the next. This is done in business to produce a **balanced budget**, where the income and expenses are equal.

Example 4

Joshua had \$18.70 at the start of the week. He received \$10 pocket money, but had to pay \$1.20 each weekday for bus fares. During the week he sold 5 old CDs to a friend for \$4.50 each. He also bought a Top 20 CD for \$27.50. He spent \$5 to go to the blue-light disco on Friday night.

a Draw up a budget showing his income and expenses for the week.

b How much did he have left?

Solution

a Show the income on the left and the expenses on the right.

Income		Expenses	
Carried forward	\$18.70	Bus fares	\$6.00
Pocket money	\$10.00	CD	\$27.50
Sale of CDs	\$22.50	Disco	\$5.00
Total	\$51.20	Total	\$38.50

b Find the difference.

Amount left = \$51.20 − \$38.50
= \$12.70

Technology worksheet
Excel: Income and expenses
MAT08NACT00014

TLF learning object
Financial maths - Purchasing and pricing (R10713)
MAT08NAIN00011

Example 5

A school excursion to an art exhibition is planned, and 280 students are expected to go. The buses cost \$190 each to hire and can carry 65 pupils each. It costs \$3 each for the students to get into the exhibition. The Parents and Friends Association at the school is willing to subsidise the trip for up to \$500. Work out a budget showing income and expenses, and decide how much the students should be asked to pay.

Solution

Show the income on the left and the expenses on the right.

Income		Expenses	
P&F subsidy	\$500	5 Buses @ \$190	\$950
		Entry fees, 280 @ \$3	\$840
Total	\$500	Total	\$1790

Find the difference.

Amount left to pay = \$1790 − \$500
= \$1290

Technology worksheet
Excel: Hamburger stall profit
MAT08NACT00016

Puzzle sheet
Money and change
MAT08NAPS00038

9780170194778

Divide by the number of students.	**Amount each** $= \$1290 \div 280 = \$4.61$
Round up to a realistic number.	$\approx \$5.00$
Multiply by the number of students.	**Student payment** $= \$5 \times 280$ $= \$1400$
Find the new subsidy.	**Subsidy** $= \$1790 - \$1400$ $= \$390$

Write a balanced budget.

Income		Expenses	
P&F subsidy	$390	5 Buses @ $190	$950
280 students @ $5	$1400	Entry fees, 280 @ $3	$840
Total	$1790	Total	$1790

Write the answer. **Students should be asked to pay $5, to make a round amount.**

Investigate: Mobile phone costs

The cost of mobile phone 'plans' depends on the type of phone included, or whether you have to buy the phone separately. The costs of calls vary according to the plan and depending on the phone company.
Different plans offer different conditions for SMS, data and games, as well as the times at which you can make cheaper calls. Mobile phone companies have websites that have details of the phone plans.
Use the Internet to check mobile phone plans and compare the total costs of different plans over two years for different uses of the mobile phone. For example, you could compare costs for people who:

- make 1 phone call and send 10 SMSs each day
- make 6 phone calls each day
- make 3 phone calls and send 5 SMSs each day
- download 3 GB data from the internet.

9780170194778

Exercise 11.1 Accounts and budgets

Understanding

Extra questions
Exercise 11.1
MAT08NAEQ00038

1 Find the cost for each of the following mobile phone calls.
 a A 150-character SMS message with a 25c connection fee
 b A 400-character SMS message with a connection fee of 20c
 c A 5-minute call at 80 c/min with a 25c connection fee
 d A 3-minute call at 95 c/min with a 20c connection fee
 e A 4-minute 35-second call at 88 c/min, charged in 30-second blocks, with a 25c connection fee

Fluency

Worked solutions
Exercise 11.1
MAT08NAWS00038

See Example 1

2 Which is the best buy?
 a 300 g of ham for \$4.49 or 450 g of ham at a cost of \$5.90
 b 650 g of Vegemite for \$6.99 or 750 g of Vegemite priced at \$7.85
 c A 1 gigabyte memory stick for \$4.99 or a 6 gigabyte memory stick for \$29.90
 d 645 g of rump steak for \$9.89 or 950 g of rump steak costing \$13.60
 e A twin pack of two 300 g tins of hairspray for \$7.39 or one 750 g tin of hairspray priced at \$9.55

Problem solving

See Example 2

3 At a party and novelty shop, you purchase the following:
2 packets of balloons @ \$3.49/packet
7 party crackers @ \$4.99 each
1 table centrepiece @ \$15.90
4 boxes of silver tree balls @ \$8.98/box.
It costs \$2 to use a credit card.
 a What is the total bill?
 b What is the cash price?
 c How much would you pay by credit card?

See Example 1

4 At a motor accessories shop, you make the following purchases:
1 steering-wheel cover @ \$18.95
2 containers of oil @ \$13.45/container
4 wheel flaps @ \$7.98 each
2 bumper stickers @ \$3.96 each.
It costs \$1.50 to use a credit card.
 a What is the total bill?
 b What is the cash price?
 c How much would you pay by credit card?

See Example 2

5 At an office supplies shop, you buy the following:
2 boxes of high-quality computer paper (1000 sheets) @ \$24.95/box
3 reams of copy paper @ \$8.99/ream
2 boxes of HB pencils @ \$3.98/box
4 erasers @ 87c each.
It costs \$2.50 to use a credit card.
 a What is the total bill?
 b What is the cash price?
 c How much would you pay by credit card?

See Example 3

6 Thomas has a \$20, 2-year mobile phone plan that includes a mobile phone and \$15 worth of free calls a month. The call rate is 95 c/min with a call connection fee of 25c. In the first month, he made an average of 5 calls a day, lasting an average of 2 minutes each.

a What was his bill for the first month?
In the second month he started using SMS more, so he reduced his calls to 2 a day, but sent 5 SMS messages each day (all under 160 characters).

b What was his bill for the second month?

Worked solutions
Exercise 11.1
MAT08NAWS00038

7 Ciara bought a pre-paid mobile phone for \$155, which included \$25 worth of calls. She used this credit up in a few days. The cost of the calls was 80 c/min with a 25c connection fee. How many 3-minute calls would she be able to make if she limited herself to \$50 credit each month?

See Example 3

See Example 3

8 Jack signed for a \$50, 2-year mobile phone plan that included a stylish new mobile phone and \$40 worth of free calls a month. He made an average of 6 calls a day, lasting an average of 4 minutes each, and also sent about 10 SMS messages each day. The call rate was 65 c/min with a call connection fee of 25c. What was his bill for a month?

9 Use the information from question **6** to find the total cost over 2 years for Thomas' mobile phone if he:

a continues the way he did in the first month

b continues the way he did in the second month.

Reasoning

Worked solutions
Exercise 11.1
MAT08NAWS00038

10 Find the cost of Ciara's mobile phone from question **7** over 2 years if she kept going at \$50 credit each month.

11 Find the cost of Jack's mobile phone over 2 years in question **8** if he kept going the same way.

See Example 4

12 Daniel gets \$12 pocket money each week and earns \$5 for washing the family car. He started the week with a present of \$10 from his Nan. During the week he spent \$3 each day at the canteen. He bought 2 orchids at \$4 each for his Mum's birthday.

a Draw up a budget showing his income and expenditure for the week.

b How much did he have left?

See Example 4

Worksheet
Money and change
MAT08NAWK00060

13 Lucy works part-time delivering groceries after school. She earns \$32 each day for working on Monday, Wednesday and Friday. She also works on Saturday for twice as long at the same hourly rate. One week she spent \$12.80 on a new tube for her bike, and bought some cosmetics that cost \$25.60. On Thursday she took \$90 out of the bank to get a new pair of jeans that cost \$105.50.

a Draw up a budget showing her income and expenditure for the week.

b How much cash did she have left at the end of the week?

See Example 4

14 Julie has just moved away from home to share a flat with Debbie. She has \$940 in the bank at the beginning of the week. The rent for the flat is \$260 a week, and Julie earns \$420 a week after tax. She spends \$170 during the week on food and other house-keeping, and also buys a new kettle for \$56. Bus fares to work cost her \$6 a day and on Friday she spends \$20 on chocolates. On Saturday morning she buys a new blouse for \$75, but has to get some money out of the bank to pay for it. She needs to have \$40 in cash to start the next week.

a Work out a budget showing her income and expenses for the week, including the bank withdrawal.

b Work out how much she has left in the bank.

c Work out how long before she has to move back home if she keeps spending like this.

9780170194778

See Example 5

15 A primary school excursion for 420 pupils is planned to attend a play. The cost of hiring 65-seat buses is \$195 per bus, and the cost of admission is \$3.50 per student.

a Draw up a budget and work out how much each student should be charged.

b How much will be left over if every student is present?

c After the buses have been ordered they must be paid for. On the day, only 310 pupils are present because of a flu epidemic. How much money does the school lose?

11.2 Percentages

When you want to compare quantities that are 'out of' different amounts, it is usual to use **percentages**. Percentages are all 'out of' 100.

Weblink
Cake cutting algorithm
MAT08NAWB00011

Important!

Percentages

A **percentage** is a fraction with a denominator of 100, but the /100 is replaced by the % sign. For example, $\frac{9}{100} = 9\%$.
A common fraction or decimal can be changed to a percentage by multiplying by 100%. To find a percentage of a quantity, change the percentage to a fraction or decimal and multiply by the quantity.

Example 6

Puzzle sheet
Percentominoes
MAT08NAPS00016

Express each of the following as a percentage.

a $\frac{1}{5}$ **b** 3 out of 10 **c** 0.28

d $1\frac{2}{5}$ **e** 0.784 **f** 8 out of 13

Solution

a Write the fraction. $\mathbf{\frac{1}{5}}$

Multiply by 100%. $= \mathbf{\frac{1}{5} \times 100\% = 20\%}$

b Write 3 out of 10 as a fraction. $\mathbf{\frac{3}{10}}$

Change the denominator to 100. $= \mathbf{\frac{30}{100}}$

Write as a percentage. $= \mathbf{30\%}$

c Write the decimal. 0.28

Write in extended form. $= \frac{28}{100}$

Write as a percentage. $= 28\%$

d Write the fraction. $1\frac{2}{5}$

Change to an improper fraction. $= \frac{7}{5}$

Multiply by 100%. $= \frac{7}{5} \times 100\% = 140\%$

e Write the decimal 0.784

Multiply by 100%. $= 0.784 \times 100\% = 78.4\%$

f Write 8 out of 13 as a fraction. $\frac{8}{13}$

Multiply by 100%. $\frac{8}{13} \times 100\% = 61\frac{7}{13}\% \approx 61.54\%$

Example 7

Use percentages to find the better score: 11 out of 15 for English, or 16 out of 21 for Art.

Solution

Write the English score as a fraction. **English score** $= \frac{11}{15}$

Change to a percentage. $= \frac{11}{15} \times 100\% \approx 73.33\%$

Write the Art score as a fraction. **Art score** $= \frac{16}{21}$

Change to a percentage. $= \frac{16}{21} \times 100\% \approx 76.19\%$

Choose the larger percentage. **16 out of 21 for Art is the better score.**

Example 8

Alternative method
Finding the percentage left
MAT08NAAM00009

A 5 L juice container has 3.6 L of juice left. What percentage has been used?

Solution

Find the amount used. **Amount used** $= 5 - 3.6$

$= 1.4$ L

Calculate the percentage. **Percentage used** $= \frac{1.4}{5} \times 100\%$

$= 28\%$

Write the answer. **28% of the juice has been used.**

9780170194778

Example 9

a Change 55% to a fraction.
b Change 38% to a decimal.
c Change 42.5% to a decimal.
d Change 135% to a fraction.
e Change $16\frac{2}{3}$ to a fraction.

Puzzle sheet
Percentages to decimals
MAT08NAPS00015

Solution

a Write the percentage. $\mathbf{55\%}$

Write % as hundredths. $= \mathbf{\frac{55}{100}}$

Cancel down. $= \mathbf{\frac{11}{20}}$

b Write the percentage. $\mathbf{38\%}$

Write % as hundredths. $= \mathbf{\frac{38}{100}}$

Write as a decimal. $= \mathbf{0.38}$

c Write the percentage. $\mathbf{42.5\%}$

Write % as hundredths. $= \mathbf{\frac{42.5}{100}}$

Treat as division by 100. $= \mathbf{0.425}$

d Write the percentage. $\mathbf{135\%}$

Write % as hundredths. $= \mathbf{\frac{135}{100}}$

Cancel down. $= \mathbf{1\frac{7}{20}}$

e Write the percentage. $\mathbf{16\frac{2}{3}\%}$

Write as an improper fraction. $= \mathbf{\frac{50}{3}\%}$

Write % as hundredths. $= \mathbf{\frac{50}{3 \times 100}}$

Cancel down. $= \mathbf{\frac{1}{6}}$

Investigate: Population comparisons

From 2006 to 2010 the resident population of Pilbara in WA increased from 44 089 to 48 610. In the same period, Broken Hill in NSW decreased from 20 131 to 19 818. Work out the percentage changes and explain the difference.
The resident populations of Australia, Japan, Malaysia, Pakistan and Singapore increased from 20.7, 127.7, 24.4, 155.8 and 3.53 to 22.3, 128.1, 28.3, 169.7 and 3.77 million respectively. The *total* population of Singapore went from 4.40 to 5.08 million. Calculate the percentage increases and explain the differences. Why is there such a difference between the resident and total population in Singapore?

Alternative method
Percentage calculations using fractions
MAT08NAAM00013

Scientific calculator exercise
Personal business calculations
MAT08NASC00008

Puzzle sheet
Percentage cross number
MAT08NAPS00019

Puzzle sheet
Percentages
MAT08NAPS00039

Worksheet
Percentages
MAT08NAWK00061

Example 10

Find each of the following.

a 25% of \$600 **b** 37% of \$427 **c** 12% of \$143.70
d 8.4% of 23.4 kg **e** $7\frac{1}{3}$% of 240 L

Solution

a Write the problem. **25% of \$600**

Think of 25% as $\frac{1}{4}$. $= \frac{1}{4}$ **of \$600**

Divide by 4 in your head. $= \$600 \div 4$

$= \$150$

b Write the problem. **37% of \$427**

Enter as 37 [÷] 100 [×] 427 [=]. `37÷100×427 157.99`

Write the answer. **37% of \$427 is \$157.99.**

c Write the problem. **12% of \$143.70**

Enter as 12 [÷] 100 [×] 143.70 [=]. `12÷100×143.70 17.244`

Round and write the answer. **12% of \$143.70 is \$17.24.**

d Write the problem. **8.4% of 23.4 kg**

Enter as 8.4 [÷] 100 [×] 23.4 [=]. `8.4÷100×23.4 1.9656`

Round and write the answer. **8.4% of 23.4 kg is 1.966 kg**

e Write the problem. $7\frac{1}{3}$**% of 240 L**

Write as an improper fraction. $= \frac{22}{3}$**% of 240 L**

Enter as 22 [÷] 3 [÷] 100 [×] 240 [=]. `22÷3×100 1.76`

Write the answer. $7\frac{1}{3}$**% of 240 L is 17.6 L.**

Example 11

A cash advance until payday from a loan shop costs 30% a week. Sue borrowed \$150 till payday the same week.

a How much was she charged? **b** How much did she have to pay back?

Solution

a Write the problem. **Charge = 30% of \$150**

Write the percentage as a decimal. $= 0.30 \times \$150$

Work out the answer. $= \$45$

b Find the total. **Amount to pay back = \$150 + \$45**

$= \$195$

9780170194778

When solving percentage problems, it is often useful to consider the percentages and the amounts as being two different scales on the same 'ruler'. This method allows you to calculate answers using ratios or proportions and to work backwards to find original amounts.

Example 12

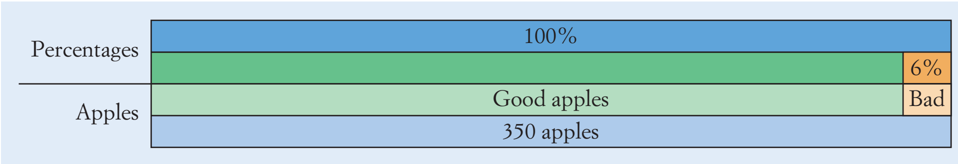
Alternative method
Finding the opposite percentage
MAT08NAAM00008

Animated examples
Percentage problems
MAT08NAAE00016

A bulk container normally has about 6% bad apples. When packed, a container holds 350 apples. How many will arrive in good condition?

Solution

Show the percentages on the top scale of a 'ruler' and the apples on the bottom scale.



Find the percentage of good apples. $\textbf{Percentage good} = 100\% - 6\%$

$= 94\%$

Write as a ratio problem. $\dfrac{\textbf{Good apples}}{350} = \dfrac{94\%}{100\%}$ **or** $\dfrac{\textbf{Good apples}}{94\%} = \dfrac{350}{100}\%$

Solve for the number of good apples. $\textbf{Good apples} = \dfrac{94}{100} \times 350$

Do the calculation. $= 329$

Write the answer. **About 329 apples will arrive in good condition.**

Notice that when you write the ratios it doesn't matter whether you match the percentages and the amounts, or the good apples and the totals. You still get the same answer.

Example 13

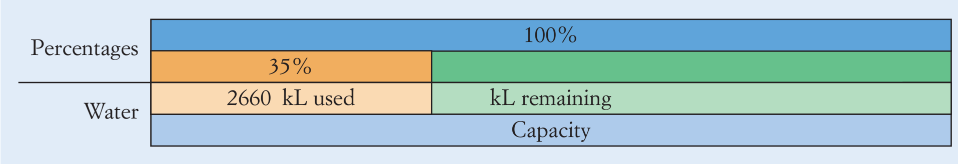
Alternative method
Finding capacity
MAT08NAAM00007

Video tutorial
The unitary method
MAT08NAVT10011

When townspeople have used 2660 kL from a full water reservoir, the amount of water has been reduced by 35%. What is the capacity of the water reservoir?

Solution

Show the percentages on the top scale of a 'ruler' and the water on the bottom scale.



Write as a ratio problem. $\dfrac{\textbf{Capacity}}{\textbf{2660}} = \dfrac{\textbf{100\%}}{\textbf{35\%}}$

Solve for the capacity. $\textbf{Capacity} = \dfrac{\textbf{100}}{\textbf{35}} \times \textbf{2660} = \textbf{7600 kL}$

Write the answer. **The capacity is 7600 kL.**

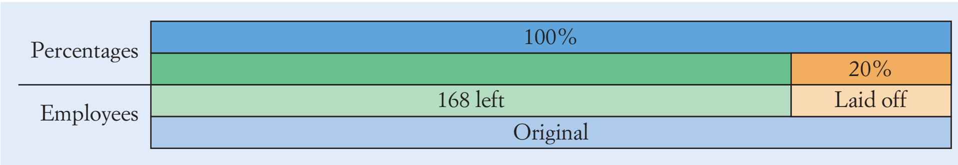
Example 14

Alternative method
Matching the information 1
MAT08NAAM00010

After a 20% reduction in employment, a factory has 168 employees left. How many were laid off?

Solution

Show the percentages on the top scale of a 'ruler' and the employees on the bottom scale.



Find the percentage left. $\textbf{Percentage left} = \textbf{100\%} - \textbf{20\%}$

$= \textbf{80\%}$

Write as a ratio problem. $\dfrac{\textbf{Laid off}}{\textbf{168}} = \dfrac{\textbf{20\%}}{\textbf{80\%}}$

Solve for the number laid off. $\textbf{Laid off} = \dfrac{\textbf{20}}{\textbf{80}} \times \textbf{168} = \textbf{42}$

Write the answer. **42 employees were laid off.**

Exercise 11.2 Percentages

Understanding

Extra questions
Exercise 11.2
MAT08NAEQ00039
See Example 6

1 Express each of the following as a percentage.

a $\frac{2}{5}$ **b** $\frac{3}{10}$ **c** 0.65 **d** 5 out of 20

e 12 out of 40 **f** 0.8 **g** $\frac{3}{4}$ **h** 45 out of 60

i 17 out of 20 **j** 17 out of 25 **k** $\frac{4}{50}$ **l** 0.08

9780170194778

2 Change each of the following to a fraction. *See* Example 9

a 35% b 25% c 20% d 80%
e 70% f 22% g 15% h 96%

3 Change each of the following to a decimal. *See* Example 9

a 45% b 75% c 60% d 6%
e 8% f 16% g 21% h 43%

4 Work out the following in your head. *See* Example 10

a 50% of \$300 b 25% of 800 kg c 20% of 70 g d 10% of 490 L
e 1% of \$3000 f 5% of 500 mg g 20% of 1 hour h 50% of a day

Fluency

5 Work out the following using your calculator. *See* Example 10

a 43% of \$500 b 40% of \$350 c 67% of 15 kg d 21% of 46 m
e 7% of 28 L f 19% of \$56 g 10% of 48 t h 75% of 60 min

6 Express each of the following as a percentage.

a $1\frac{3}{4}$ b 29 out of 40 c 0.365 d 28 out of 77
e 105 out of 147 f $\frac{5}{7}$ g 8 h 17 out of 22
i $\frac{62}{43}$ j 5 out of 9 k 1.24 l $2\frac{3}{11}$

7 Change each of the following to a fraction. *See* Example 9

a 125% b 250% c 120% d 145%
e 110% f 37.5% g 62.5% h 153%

8 Change each of the following to a decimal. *See* Example 9

a 42.5% b 15.6% c 0.7% d 124%
e 207% f 140% g 23.6% h 5.6%

9 Work out the following. *See* Example 9

a 12.5% of \$300 b 9.4% of 22 kg c 5.7% of 270 g d 16.8% of \$28
e 14.2% of \$21.46 f 36.4% of 2.89 L g 125% of 7.6 s h 122% of 240 g

10 Change each of the following to a fraction. *See* Example 9

a $33\frac{1}{3}\%$ b $12\frac{1}{2}\%$ c $82\frac{1}{2}\%$ d $66\frac{2}{3}\%$
e $83\frac{1}{3}\%$ f $41\frac{2}{3}\%$ g $122\frac{1}{2}\%$ h $116\frac{2}{3}\%$

11 Work out the following. *See* Example 10

a $15\frac{1}{2}\%$ of 520 g b $5\frac{1}{2}\%$ of \$7000 c $16\frac{2}{3}\%$ of \$1200 d $37\frac{1}{2}\%$ of 4 t
e $6\frac{2}{3}\%$ of 900 kg f $8\frac{3}{4}\%$ of 4600 L g $17\frac{1}{2}\%$ of \$5000 h $4\frac{2}{5}\%$ of 80 kg

12 If 23% of a number is 161, what is the number?

Problem solving

Worked solutions
Exercise 11.2
MAT08NAWS00039

13 Matthew spent 20 minutes watering and 60 minutes weeding in his garden. What percentage of his time in the garden did he spend weeding?

14 There were 8 black jellybeans in a packet of 40 jellybeans. What percentage were black?

See Example 8

15 Jayne spends 40% of the money she earns from her part-time job on her mobile phone. She earned \$180 one month. How much did she spend on her mobile that month?

Worked solutions
Exercise 11.2
MAT08NAWS00039

16 Which is the better mark: 19 out of 25 for Maths, or 25 out of 35 for Science?

17 Margaret and Kate are in different classes for Music. Margaret got 15 out of 18 for her test, and Kate got 20 out of 25 for hers. Who did better in Music?

See Example 7
See Example 7

18 Stefan bought a desk for \$60 and sold it for \$75 at the local flea market. He also sold a terrarium for \$46. The terrarium had cost him \$36. He can buy and sell as many desks or terrariums as he likes at these prices.

a How much profit did he make on each deal?

b Would it be better to deal in desks or terrariums?

See Example 8

19 Copper makes up 30% of a particular alloy. How much copper is there in a 50 kg ingot of the alloy?

See Example 11

20 Gold is very soft, so it is combined with other metals to make most jewellery. In 18-carat gold there is 75% gold. How much gold is in 20 g of 18-carat gold?

See Example 11

21 Seawater is about 3% salt. How much salt could be extracted from 2 t of seawater?

See Example 11

22 Cast iron is about 5% carbon. If an ingot of cast iron contains 3 kg of carbon, how much does the ingot weigh?

Reasoning

Worked solutions
Exercise 11.2
MAT08NAWS00039

23 Caimin lost 70% of his money. He had only \$15 left. How much did he have to start with?

24 When a piece of pottery is made, it loses 35% of the mass it had as clay. What mass of clay is used to make a piece that weighs:

a 325 g? b 260 g? c 650 g? d 2 kg?

See Example 12
See Example 13
See Example 14

25 A spin dryer removes 92% of the water from a pair of jeans. Before being hung out to dry, they weigh 1.47 kg. When they are thoroughly dry, they weigh 1.29 kg. How much water did they contain before they were spun?

9780170194778

11.3 Personal business

Percentages are often used in business as a convenient way to work out and compare prices and returns on different items. Many government charges are also expressed as percentages.

Important!

Business percentages

The **cost price** (**buying price**) of an item is the price that was originally paid by the person who is now selling the item.
The **selling price** of an item is the price it is actually sold for. The normal selling price (before any discount) is also called the **marked price**.
A **profit** is made when goods are sold for more than what they cost.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

A **loss** is made when goods are sold for less than what they cost.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

A **markup** is added to the cost price to get the **marked price** (normal selling price).

$$\text{Marked price} = \text{cost price} + \text{markup}$$

A **discount** is an amount taken off a normal price (the marked price).

$$\text{Discount} = \text{normal marked price} - \text{price actually paid}$$

A **commission** is money paid for a service that a person has carried out. It is usually expressed as a percentage of the value of the service, such as the value of goods sold by a sales representative. A fixed **retainer** may be paid even if no sales are made. When a discount, markup, profit, loss or commission is calculated as a percentage, it is usually considered to be 'out of' the original price. You can think of this as

$$\text{Percentage amount} = \frac{\text{amount}}{\text{original price}} \times 100\%$$

In Australia, the **Goods and Services Tax** (**GST**) is paid to the federal government whenever items are sold or a service is provided. It is currently set at 10%.

Video tutorial
Percentage increase and decrease
MAT08NAVT10010

Video tutorial
GST
MAT08NAVT10012

Example 15

A calculator marked at $60 is actually sold for $44.

a What is the discount?

b What is the percentage discount?

Solution

a Find the amount off the marked price.

Discount = marked price − price paid
= $60 − $44 = $16

b The marked price is the original price.

$$\textbf{Percentage discount} = \frac{\textbf{discount}}{\textbf{marked price}} \times \mathbf{100\%}$$
$$= \frac{\$16}{\$60} \times 100\% \approx 26.7\%$$

Write the answer.

The discount is about 26.7%.

Scientific calculator exercise
Personal business calculations
MAT08NASC00008

Puzzle sheet
Discounts
MAT08NAPS00040

Worksheet
Discounts
MAT08NAWK00062

Puzzle sheet
Profit and loss
MAT08NAPS00041

Example 16

A second-hand furniture shop bought a desk for \$240 and sold it for \$310.

a What is the profit?
b What is the percentage profit?

Solution

a Work out the increase.

$$\begin{aligned}\textbf{Profit} &= \textbf{selling price} - \textbf{cost price}\\ &= \$310 - \$240\\ &= \$70\end{aligned}$$

b **Method 1**

The cost price is the original price.

$$\begin{aligned}\textbf{Percentage profit} &= \frac{\textbf{profit}}{\textbf{cost price}} \times 100\%\\ &= \frac{\$70}{\$240} \times 100\%\\ &\approx 29.2\%\end{aligned}$$

Write the answer. **The percentage profit is about 29.2%.**

Method 2

Write as a ratio problem.

$$\frac{\textbf{profit }\%}{100\%} = \frac{70}{240}$$

Find the percentage profit.

$$\begin{aligned}\textbf{Profit }\% &= \frac{70}{240} \times 100\%\\ &= 29.2\%\end{aligned}$$

Write the answer. **The percentage profit is about 29.2%.**

Example 17

A sales representative works on a commission of 15%. What does he earn for selling 50 pairs of bolt cutters at \$60 each?

Solution

Calculate the total sales.

$$\begin{aligned}\textbf{Sales} &= 50 \times \$60\\ &= \$3000\end{aligned}$$

Calculate the commission.
Write 15% as a decimal.

$$\begin{aligned}\textbf{Commission} &= 15\% \textbf{ of sales}\\ &= 0.15 \times \$3000\\ &= \$450\end{aligned}$$

Write the answer. **He earns \$450.**

9780170194778

In problems where you are not given the original amount, you will find the diagram and ratio method particularly useful.

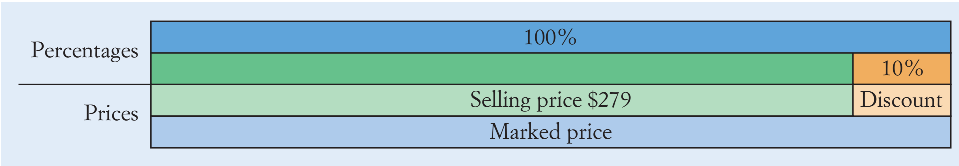
Example 18

Alternative method
Matching the information 2
MAT08NAAM00011

Bettina paid \$279 for a jacket after getting a discount of 10%. What was the marked price?

Solution

Show the percentages on the top scale of a 'ruler' and the prices on the bottom scale.



Find the selling price percentage.

$$\textbf{Selling price percentage} = 100\% - 10\% \textbf{ of marked price}$$
$$= 90\%$$

Write as a ratio problem.

$$\frac{\textbf{Marked price}}{\$279} = \frac{100\%}{90\%}$$

Solve for the marked price.

$$\textbf{Marked price} = \frac{100}{90} \times \$279$$
$$= \$310$$

Write the answer.

The marked price was \$310.

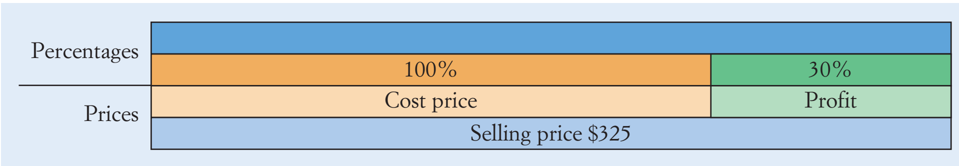
Example 19

Alternative method
Matching the information 3
MAT08NAAM00012

A 'cash liquidator' shop sold an electric guitar for \$325, making a profit of 30%. What did the shop pay for the guitar?

Solution

Show the percentages on the top scale of a 'ruler' and the prices on the bottom scale.



Find the selling price percentage.

$$\textbf{Selling price percentage} = 100\% + 30\% \textbf{ of cost price}$$
$$= 130\%$$

Write as a ratio problem.

$$\frac{\textbf{Cost price}}{\$325} = \frac{100\%}{130\%}$$

Solve for the cost price.

$$\textbf{Cost price} = \frac{100}{130} \times \$325$$
$$= \$250$$

Write the answer.

The shop bought the guitar for \$250.

Exercise 11.3 Personal business

Understanding

Extra questions
Exercise 11.1
MAT08NAEQ00040
See Example 15

1 Work out the discount for each of the following items.

	Item	Marked price	Selling price
a	Joggers	$80	$65
b	iPod	$55	$45
c	Hairspray	$8.40	$7
d	Skateboard	$125	$115
e	DVD player	$295	$245

See Example 16

2 For each of the following, work out the profit or loss.

	Item	Cost price	Selling price
a	Computer	$1540	$2070
b	Car stereo system	$780	$640
c	Body board	$78	$92
d	Bikini	$45	$35
e	Terracotta pot	$145	$220

Fluency

See Example 15
See Example 16

3 Calculate the percentage discount for each item in question **1**.

4 Calculate the percentage profit or loss for each item in question **2**.

5 Calculate the selling price for each of the following.

	Item	Cost price	Markup
a	Handbag	$60	80%
b	Toolbox	$35	45%
c	TV	$220	60%
d	Wireless mouse	$25	120%
e	Hamburger	$2.40	150%

6 Calculate the selling price for each of the following.

	Item	Marked price	Discount
a	Mexican-style poncho	$140	20%
b	Riding boots	$95	15%
c	Light fitting	$116	10%
d	Gas BBQ and trolley	$255	30%
e	BMX bike	$640	25%

See Example 17

7 Calculate the commission earned in each case below.

	Sales	Commission
a	$560	20%
b	$435	35%
c	$5600	15%
d	$30 000	3%
e	$240 000	2%

9780170194778

8 Calculate the marked price for each of the following.

See Example 18

	Item	Discount	Selling price
a	Carpet	20%	$1120
b	Oil	15%	$7.14
c	Model plane	10%	$243
d	Dining table	12%	$528
e	Encyclopaedia set	30%	$875

Problem solving

9 A market gardener had 40 watermelon vines growing, and each vine had 7 watermelons on it. Unfortunately, 30 watermelons had sooty mould and could not be sold. He sent the rest to market, but was told that 53 had fallen off the truck and were no good. The melons reached the market late on Friday, and 24 unsold ones were carried over to the next sale day. It was a long weekend, and by the time the next sale came only 10 melons were in good enough condition to sell. The watermelons cost $1.20 each to grow and they sold for $3.00 each.

Worked solutions
Exercise 11.3
MAT08NAWS00040

a How many watermelons had the farmer grown?

b How many were sold altogether?

c How much did they cost the farmer to grow?

d What profit or loss did the farmer make?

10 What percentage profit or loss did the farmer in question **9** make?

Worked solutions
Exercise 11.3
MAT08NAWS00040

11 An electrical store buys light bulbs for $1.74 and sells them for $2.40. It buys 50 m reels of speaker wire for $31 and sells the wire for 90 c/m.

a What is the percentage profit for each item?

b Which is the more profitable item?

12 At a storewide sale, a shop offers 10% off the ticketed price of all items except food and clothes. It allows 5% off the price of clothes. Calculate the amount taken off, and the new price, for each of the following ticketed prices.

a Man's suit $187

b Special sharpener $5.40

c Electric iron $88.50

d Bike helmet $56.29

e Bicycle $195

f Barbecue $249.90

13 Peter is a salesman. He gets a commission of 35% on sales. What does he get for selling:

a a vacuum cleaner for $470?

b a food processor for $87?

c 12 vacuum cleaner bags for $3.70 each?

d 15 desk lamps for $38.40 each?

e 11 colour TVs for $645.80 each?

f a stereo system for $976?

14 Candice gets a retainer of \$200 a week and a commission of 15% of sales. In the second week of October she sold \$3600 worth of goods. How much did she earn?

15 What is the GST payable on a bill of \$80 for lawn mowing?

16 A disposals store buys goods from fire sales, liquidation sales and so on. It bought some toasters at cost for \$15 each and found that they were marked at \$31.50. It put a sticker on them saying 'Reduced by 30%, only \$25'. They didn't sell well, so the store reduced the price by 20% from \$25. The toasters then sold out. Work out:

a the percentage profit the toasters would have originally made
b the price for a 30% reduction from \$31.50
c the actual percentage reduction from \$31.50
d the last price the toasters sold for
e the percentage profit the store made at each of the two prices for which it sold toasters.

Reasoning

17 A frost-free freezer costs a store \$600. The store applies a markup of 40%, but later discounts the price by 20%. Work out the actual selling price and the profit as a percentage. (Hint: Do in stages.)

Worked solutions
Exercise 11.3
MAT08NAWS00040

18 What is the actual price paid for a surfboard that costs a dealer \$580 if a markup of 50% is applied, tax of 10% is due and a discount of 15% is given?

19 A wholesaler has a markup of 20%, a distributor has 15%, and a retailer adds 30%.

a What is the final selling price of a digital video camera that cost the wholesaler \$850?
b What is the total percentage markup?

20 A shop marks up items by 25%. What is the biggest percentage discount the shopkeeper can give without losing money?

21 At a sale, Vance paid \$35.70 for an iron after 15% discount. What was the marked price?

22 Janine is a sales representative working on 35% commission. If she made \$241.50 in one week, how much were her sales worth?

23 Simon is a silent partner in a bakery. He takes an 18% share of the profits. His share in one year was \$27 900. What was the total profit?

24 Diana receives a 2% commission for sales of houses. What prices were paid for the houses for which she received the following commissions?

a \$8160	b \$13 800	c \$6960
d \$10 100	e \$15 400	f \$19 780

25 A new coat was discounted by 20%. The customer paid \$96. What was the marked price?

26 When Vasily bought a used car for \$4200, the dealer claimed to be making a loss of 30%. What had the dealer paid for the car?

27 In the third week of October, Candice from question **14** earned \$905. What was the value of the goods she sold?

28 A home theatre system priced at \$1980 includes GST. How much is the GST?

9780170194778

11.4 Borrowing and investing money

When you borrow money, you normally pay back more than you borrow. The extra amount is called **interest**. When you put money into a savings account, the bank normally pays you extra to use the money in the account. This is also interest.

Weblink
Money stuff!
MAT08NAWB00011

Important!

Interest

Interest is the charge made for a loan. It is called the **interest rate** and is normally worked out as a percentage for each year, often written as, say, 5% p.a. (per annum).
The amount lent or borrowed is called the **principal**.
The amount that has to be paid back each week or month for a loan is called the **instalment** or **repayment**.
Simple interest is calculated using the whole loan for the whole time it is borrowed. It is also called **flat-rate interest**.

Example 20

Teacher notes
Interest
MAT08NATN00020

Theo bought a surfboard that normally costs \$600 over 2 years for \$40 a month.

a What is the total amount paid?
b What is the total interest?
c What is the interest rate?

Solution

a Calculate the total for the 24 months. **Total paid** $= 24 \times \$40 = \$960$

b Find the extra amount paid. **Total interest** $= \$960 - \$600 = \$360$

c Find the interest for 1 year. **Annual interest** $= \$360 \div 2 = \$180$

Calculate as a percentage of the principal.

$$\textbf{Interest rate} = \frac{\textbf{annual interest}}{\textbf{principal}} \times 100\% = \frac{\$180}{\$600} \times 100\% = 30\%$$

Puzzle sheet
Simple interest
MAT08NAPS00042

Technology worksheet
Excel: Borrowing money and repayments
MAT08NACT00015

Worksheet
Simple interest
MAT08NAWK00063

Technology
GeoGebra: Simple interest
MAT08NATC00008

Example 21

Gina borrowed \$3000 over 3 years at 16% flat-rate interest. Work out her monthly instalments to repay the loan.

Solution

Find the interest for 1 year.	**Annual interest** $= 16\%$ **of** \$3000 $= 0.16 \times \$3000$ $= \$480$
Find the interest for 3 years.	**Total interest** $= 3 \times \$480$ $= \$1440$
Find the amount to pay back.	**Total to repay** = **principal** + **interest** $= \$3000 + \$1440$ $= \$4440$
Find the monthly repayment.	**Instalment** $= \dfrac{\text{total to repay}}{\text{number of payments}}$ $= \$4440 \div 36$
You can use your calculator, but you should press equals after each separate calculation to avoid getting the result for $0.16 \times 3000 + 3000 \div 36$, which is about 563.	
Enter as: 3 [×] 0.16 [×] 3000 [=].	3x0.16x3000 1440
Then do [+] 3000 [=].	Ans+3000 4440
Then do [÷] 36 [=].	Ans÷36 123.3333333
	$\approx$ **\$123.33**
Write the answer.	**The monthly instalment is \$123.33.**

In Example **21**, you will get the wrong answer if you don't press the equals key as shown above. A scientific calculator uses the correct order of operations. If you have a scientific calculator, it will do $0.16 \times 3000 + 3000 \div 36$ as $(0.16 \times 3000) + (3000 \div 36)$.

Exercise 11.4 Borrowing and investing money

Understanding

Extra questions
Exercise 11.4
MAT08NAEQ00041

1 Find the simple interest on each of the following loans.

a \$400 at 12% for 2 years
b \$2000 at 15% for 3 years
c \$5000 at 18% for 4 years
d \$1500 at 16% for 3 years
e \$100 at 15% for 5 years
f \$500 at 15% for 5 years
g \$3200 at 14% for 2 years
h \$1600 at 21% for 4 years

9780170194778

Fluency

2 Find the final account balance for each of the following investments.
a $3000 at 12% simple interest for 3 years.
b $5000 at 16% simple interest for 3 years.
c $2500 at 15% simple interest for 2 years.
d $1800 at 12% simple interest for 18 years.
e $6000 at 17% simple interest for 25 years.

Worked solutions
Exercise 11.4
MAT08NAWS00041

Problem solving

See Example 20

3 Jack has a Harley. He buys the latest leather gear on no-deposit flat-rate terms. The total price is $840. He pays it over 6 months at $40 a week.
a What is the total he pays?
b What is the interest?
c What is the interest rate?

See Example 20

4 Shelley took a loan to buy an old MG sports car for $16 000. She paid a deposit of $4000 and is repaying the loan at $400 a month over 6 years.
a How much did she actually borrow?
b How much will she pay in instalments altogether?
c What is the actual amount she will pay for the car?
d How much interest will she pay?
e What is the interest rate?

Worked solutions
Exercise 11.4
MAT08NAWS00041

See Example 20

5 Hank is a fishing fanatic. He bought a new aluminium boat for $3000 and paid it off at $45 a week for 2 years. What was the interest rate?

See Example 21

6 Maria pays a deposit of $500 on a new windsurfer priced at $1200. She agrees to pay it off at 15% simple interest over the next year.
a How much does she borrow?
b What is her monthly repayment?

See Example 21

7 Andrea took a loan of $12 000 to buy a car over 4 years. The interest rate was 23% flat-rate. What was her monthly repayment?

Reasoning

Worked solutions
Exercise 11.4
MAT08NAWS00041

8 Juliet bought a second-hand car for $3500. She gave the dealer a $700 deposit and signed a 4-year instalment-purchase contract at 24% flat-rate interest. After 2 years she falls behind in the payments and has to make an extra payment as a penalty.
a How much did she actually borrow?
b What is her monthly repayment?
c What is the total amount she will pay for the car (including the penalty)?
d How much extra will she pay for the car (compared with the original price)?

Chapter 11 summary

Quiz
Personal business calculations
MAT08NAQZ00011

- Cash payments are rounded to the nearest 5 cents. Credit card payment may incur an extra charge.
- A **cash-out** is extra money withdrawn from your account in cash at a checkout.
- The amount of electricity you use is measured in **kilowatt hours** (**kWh**). It is the amount used in 1 hour by something rated at a power of 1000 W.
- A **budget** is a financial plan. It is often presented as a **balance sheet** with two columns. The first column shows **income** and the second shows **expenditure**. A **balanced budget** has equal income and expenses.
- A **percentage** is a number written as a fraction with a denominator of 100, but the /100 is replaced by the % sign.
- A fraction or decimal can be changed to a percentage by multiplying by 100%.
- To find a percentage of an amount, the percentage is changed to a fraction or decimal and multiplied by the amount.
- The **cost price** (**buying price**) of an item is the price paid by the person selling the item.
- The **selling price** of an item is the price it is actually sold for.
- A **discount** is an amount taken off a normal price.

$$\text{Discount} = \text{marked price} - \text{price paid}$$

- Shops add a **markup** to the cost price to get the **marked price** of goods they sell.

$$\text{Marked price} = \text{cost price} + \text{markup}$$

- A **profit** is made when goods are sold for more than what they cost.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

- A **loss** is made when goods are sold for less than what they cost.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

- A **commission** is money paid for a service actually carried out, particularly when the amount paid is calculated as a percentage. A **retainer** may be paid in addition to a commission.
- When a discount, profit, loss, markup or commission is calculated as a percentage, it is usually considered to be 'out of' the original price.

$$\text{Percentage amount} = \frac{\text{amount}}{\text{original price}} \times 100\%$$

- In Australia the **Goods and Services Tax** (**GST**) is currently set at 10%. It is collected by the seller or service provider and paid to the federal government.
- **Interest** is the charge made for a loan. It is normally worked out as a percentage for each year called the interest rate. The amount lent or borrowed is called the **principal**.
- The amount that has to be paid back each week or month for a loan is called the **instalment** or **repayment**.
- **Simple interest** is calculated using the whole loan for the whole time it is borrowed. It is also called **flat-rate interest**.
- The **unitary method** is particularly useful in price comparisons, a ratio or rate is used to work out the value of one quantity for a fixed amount (often 1) of another.

9780170194778

Chapter 11 review

Understanding

1 Find the cost of a 5-minute phone call at 65 c/min with a 33c connection fee.

2 Express each of the following as a percentage. See Example 6

a $\frac{3}{5}$ b $\frac{7}{20}$ c 0.15

3 Change 65% to both a fraction and a decimal. See Example 9

4 Work out 48% of \$350. See Example 10

Fluency

5 A game console marked at \$210 is sold for \$195. What is the discount? See Example 15

6 Constantine bought trays of mangoes for \$3.50 each and sold them for \$4.80 each. What profit did he make? See Example 16

Worksheet

Personal business calculations review

MAT08NAWK00064

7 Which is the best buy? 700g of pasta for \$3.19 or 450 g of pasta at a cost of \$1.90. See Example 1

8 Express each of the following as a percentage. See Example 5

a $2\frac{5}{8}$ b 27 out of 38 c 0.236

9 Change 245% to both a fraction and a decimal.

10 Work out 6.7% of \$247.40. See Example 10

11 Calculate the percentage discount and percentage profit in questions **5** and **6**.

12 A hat cost a shop \$35 and it used a markup of 40%. What was the marked price?

13 What commission is earned by selling \$1200 worth of goods on 15% commission? See Example 17

14 How much simple interest is charged for a loan of \$4500 for 3 years at 8.2%?

15 Find the final amount and simple interest when \$9000 is invested at 8% for 2 years.

16 Find the cost of these mobile phone calls at 85 c/min and a connection fee of 25c. See Example 3

a A 200-character SMS message b A 4-minute call

17 Change $26\frac{2}{3}$% to a fraction.

9780170194778

See Example 10
18 Work out $38\frac{8}{9}\%$ of \$135.

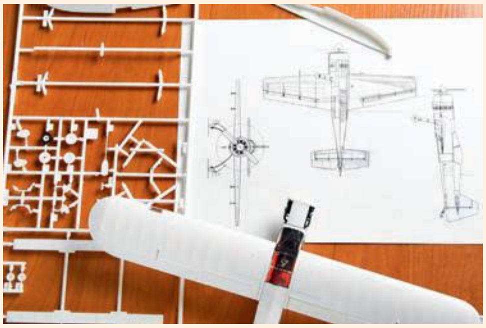
See Example 13
19 28% of a number comes to 36.4. What is the number?

See Example 18
20 After getting a discount of 12%, a customer paid only \$136.40 for a briefcase. What was the marked price of the briefcase?

Problem solving

See Example 2
21 Nino buys the following at a hobby shop. It costs \$2.50 to use a credit card.

2 tubes of craft glue @ \$3.98 each
3 stickers @ \$1.24 each
4 model kits @ \$7.96 each



a What is the total bill?
b What is the cash price?
c What would he pay by credit card?

22 At the weekend, Cameron spent 4 hours on schoolwork, including 1 hour and 12 minutes on Maths. What percentage of the time did he spend on Maths?

23 A shopkeeper bought 8 boxes of candles for \$12.50 each and sold the individual candles for 80c each. Each box had 25 candles in it. He still had five candles left at the end and used them himself. What profit or loss did he make?

See Example 7
24 Which is the better mark: 13 out of 20 for Science, or 17 out of 25 for English?

See Example 8
25 65% of a computer's CPU time is spent idle. If the computer is on for 5 hours, how much time does the CPU spend working?

26 What percentage profit or loss did the shopkeeper in question **23** make?

27 A takeaway shop has estimated that it costs \$1.80 to prepare a hamburger and keep it hot. On Tuesday they prepare 500 hamburgers during the day and price them to sell at an 80% profit. At 7:45 pm they have 40 left and sell them at a 50% discount.
a What is their overall percentage profit?
b What percentage profit or loss do they make on the last 40 hamburgers?

28 A coat is priced at \$170, ex-GST. What is the price including GST?

See Example 3
29 Carla has a \$35, 2-year mobile plan that includes \$15 of free calls a month. The call rate is 90 c/min with a 25c connection fee. In the first month, she averaged 2 calls (each about 3 minutes) and 6 SMS messages each day. What was her bill for the month?

30 Find the cost of Carla's mobile phone (question **29**) over 2 years if she continues the same pattern of use.

9780170194778

31 Gavin is planning a trip for a group of four. Each person will have to pay $7.40 for the bus fare. They will rent a small fishing dinghy for $12.00 an hour. They plan to fish for 5 hours, then go to an amusement park for the rest of the day. The entry fee for the park is $16 per person. Gavin estimates that it will cost $28 altogether to buy bread and fillings to make their lunch. *See* Example 4

a Draw up a budget for the trip.

b How much will the trip cost each of them?

Reasoning

32 An alloy contains 32% chromium and 7% nickel. The rest is iron. There is 6.3 kg of nickel in one order of the alloy. How much iron is present?

33 Alicia works on a 12% commission with a retainer of $220 a week. One week she earned $640. What was the value of the goods she sold?

34 A bedroom suite is priced at $3289 including GST. How much is the GST?

35 Tanya bought a new flat-screen TV priced at $2500 on terms of $142 a month over 2 years. What interest rate was she charged? *See* Example 20

36 Tom saw a boat for $6500. He had $800 deposit and could get finance at 13.6% flat-rate interest over 4 years for the rest of the money. What monthly payment would be needed? *See* Example 21

Number and algebra

12 Solving equations

9780170194778

Contents

Prior learning
Chapter 12
MAT08NAPL00012

Parent guide
Chapter 12
MAT08NAPG00012

Curriculum guide
Chapter 12
MAT08NACU00012

Australian Curriculum statements

Linear and non-linear relationships
Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

'Peter is two years older than Ishmael.' This is an English sentence that contains numerical information. Equations are one of the most important mathematical models. They give you a way of writing down numerical sentences. When one of the numbers is not known, you may be able to find the unknown number using an equation as a model. Linear equations are the simplest kind of equations, and in this chapter you will learn more about modelling and solving problems using linear equations.

Mathematical literacy

Maths dictionary
MAT08ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

backtracking	flow chart	modelling	solution
balance	formal solution	predecessor	solving
consecutive	graphing	product	successor
difference	guess and check	quotient	sum
distributive law	inspection	right-hand side	unknown
equation	inverse operation	root	variable
expression	left-hand side	satisfies	

12.1 Creating equations

You have already done some work on equations in Year 7. In many cases, the first step is to model a problem as an equation.

Important!

Equation

An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The expressions can be arithmetic or algebraic.

An equation can be true, false, or true for some circumstances, such as a particular value (or values) of a variable. You can decide whether or not arithmetic equations are true by working out each side. If the left-hand side (LHS) has the same value as the right-hand side (RHS), then it must be true.

9780170194778

Example 1

Write each of the following as an equation and consider whether it is true or false.

a Three times eight is the same as six times four.

b The sum of five and nine is the same as the product of five and three.

Solution

a 'is the same as' means equals.	$\mathbf{3 \times 8 = 6 \times 4}$
Evaluate the left-hand side.	**LHS** $= \mathbf{3 \times 8 = 24}$
Evaluate the right-hand side.	**RHS** $= \mathbf{6 \times 4 = 24}$
Compare the sides.	**LHS = RHS**
Write your conclusion.	**The equation is true.**
b Write as an equation.	$\mathbf{5 + 9 = 5 \times 3}$
Evaluate the left-hand side.	**LHS** $= \mathbf{5 + 9 = 14}$
Evaluate the right-hand side.	**RHS** $= \mathbf{5 \times 3 = 15}$
Evaluate the right-hand side.	**LHS** $\neq$ **RHS**
Compare the sides.	**The equation is false.**

You cannot always decide whether an algebraic equation is true, false, or true for some values just by looking at it. You may need to try out some values first.

Example 2

Write each of the following as an equation or expression and consider whether it is true or false, explaining your steps.

a Two numbers add up to twenty-one.

b The square of a natural number is seven more than the number

c For three consecutive whole numbers the square of the middle number is one more than the product of the others.

d The successor to double a particular number is even.

Solution

a Choose variables for the numbers.	**Let the numbers be** x **and** y**.**
Write the equation.	$\mathbf{x + y = 21}$
Try $x = 5$ and $y = 15$.	**With 5 and 15, LHS** $= \mathbf{5 + 15 = 20 \neq 21}$
Try $x = 7$ and $y = 14$.	**With 7 and 14, LHS** $= \mathbf{7 + 14 = 21}$
Try $x = 24$ and $y = {}^{-}3$.	**With 24 and** $\mathbf{{}^{-}3}$**, LHS** $= \mathbf{24 + {}^{-}3 = 21}$
Try $x = {}^{-}3$ and $y = 12$.	**With** $\mathbf{{}^{-}3}$ **and 12, LHS** $= \mathbf{{}^{-}3 + 12 = 9 \neq 21}$
Write your conclusion.	**The equation is true for some cases and false for others.**

b Write the equation of the natural number n.	$n^2 = n + 7$
Consider $n = 1$.	**For $n = 1$, LHS $= 1^2 = 1$, RHS $= 1 + 7 = 8$, LHS $\neq$ RHS**
Consider $n = 2$.	**For $n = 2$, LHS $= 2^2 = 4$, RHS $= 2 + 7 = 9$, LHS $\neq$ RHS**
Consider $n = 3$.	**For $n = 3$, LHS $= 3^2 = 9$, RHS $= 3 + 7 = 10$, LHS $\neq$ RHS**
Consider $n = 4$.	**For $n = 4$, LHS $= 4^2 = 16$, RHS $= 4 + 7 = 11$, LHS $\neq$ RHS**
Consider $n = 5$.	**For $n = 5$, LHS $= 5^2 = 25$, RHS $= 5 + 7 = 12$, LHS $\neq$ RHS**
Write your conclusion.	**From the pattern, it is obvious that the equation is false for all natural numbers.**
c Choose a variable for the first number.	**Let the first number be n.**
Write the next number.	**Next number $= n + 1$**
Write the last number.	**Last number $= n + 2$**
Write the equation.	$(n + 1)^2 = n(n + 2) + 1$
Simplify the LHS.	**LHS** $= (n + 1)^2$
Write the square in extended form.	$= (n + 1)(n + 1)$
Use the distributive law.	$= n(n + 1) + 1(n + 1)$
Use the distributive law again.	$= n^2 + n + n + 1$
Simplify.	$= n^2 + 2n + 1$
Simplify the RHS.	**RHS** $= n(n + 2) + 1$
Use the distributive law.	$= n^2 + 2n + 1$
Compare the sides.	**LHS $=$ RHS, for any value of n.**
Write your conclusion.	**The equation is always true.**
d Choose a variable.	**Let the number be n.**
Write the first part of the expression.	**Double the number $= 2n$**
Complete the expression, E.	**The successor to double the number, $E = 2n + 1$**
Try $n = 1$.	$E = 2 + 1 + 1 = 3$
Try $n = 2$.	$E = 2 + 2 + 1 = 5$
Try $n = 3$.	$E = 2 + 3 + 1 = 7$
Write your conclusion.	**From the pattern 3, 5, 7, . . . the statement is never true.**

9780170194778

There are many English words and phrases that you need to be able to translate into mathematical operations. Some of these are shown in the table below.

Word/phrase	Meaning	Example	Symbolic form
Sum	Add	The sum of a number and 6.	$n + 6$
Quotient	Divide	The quotient of 20 and a number.	$20 \div n$ or $\frac{20}{n}$
Product	Multiply	The product of a number and negative 8.	$^{-}8n$ or $-8n$
Difference	Subtract	The difference between 30 and a number.	$30 - n$
(A number) more than	Add to	A number is increased by 5.	$n + 5$
(A number) less than	Subtract from	8 less than a number.	$n - 8$
Decrease by (a number)	Subtract from	A number is decreased by 12.	$n - 12$
Decrease by (a factor of)	Divide by	A number is decreased by a factor of 5.	$n \div 5$ or $\frac{n}{5}$
Increase by (a number)	Add to	x is increased by 7.	$x + 7$
Increase by (a factor)	Multiply by	A number is increased by a factor of 8.	$8n$
Double	Multiply by 2	A number doubled.	$2n$
Triple	Multiply by 3	Triple a number.	$3n$
Twice	Multiply by 2	Twice a number.	$2n$
Thrice	Multiply by 3	Thrice a number.	$3n$
Quadruple	Multiply by 4	A number is quadrupled.	$4n$
Half of	Divide by 2	A number is halved.	$n \div 2$ or $\frac{n}{2}$
A third of	Divide by 3	A third of a number.	$n \div 3$ or $\frac{n}{3}$
A quarter of	Divide by 4	A quarter of a number.	$n \div 4$ or $\frac{n}{4}$
Comes to	Equals	. . . comes to eleven	$\ldots = 11$
Amounts to	Equals	. . . amounts to \$200.	$\ldots = \$200$
The same as	Equals	. . . is the same as the sum of n and 3.	$\ldots = n + 3$
Successor	Next	The successor to a natural number.	$n + 1$
Predecessor	Previous	The predecessor to an integer.	$n - 1$
Treble	Multiply by 3	Treble a number.	$3n$
Reduce by (a number)	Subtract from	A number is reduced by 2.	$n - 2$
Reduce by (a factor of)	Divide by	A number is reduced by a factor of 8.	$n \div 8$ or $\frac{n}{8}$

In many cases, several operations will be combined in an equation. In which case you need to follow the normal order of operations.

Example 3

Animated example
Writing equations
MAT08NAAE00019

Write as equations:

a When 7 is added to four times a number, the answer is 31.
b Double the sum of a number and six comes to 28.
c The product of a number and its predecessor is 56.
d The quotient of eight less than a number and 4 is the sum of three times the number and 5.

Solution

a	Decide which operation is first.	**Multiplication is done before addition.**
	Use brackets to show which operation is done first.	**When 7 is added to (four times a number) . . .**
	Write the first operation in symbols.	$4n$
	Now include the second operation.	$7 + 4n$
	Complete the equation.	$7 + 4n = 31$ **or** $4n + 7 = 31$
b	Decide which operation is first.	**'Double the sum' means that the addition is done first.**
	Use brackets to show which operation is done first.	**Double (the sum of a number and six)**
	Write the first operation in symbols.	$(n + 6)$
	Now include the second operation.	$2 \times (n + 6)$
	Write the multiplication in the usual way.	$2(n + 6)$
	Complete the equation.	$2(n + 6) = 28$
c	Use brackets to show what is done first.	**The product of a number and (its predecessor)**
	The predecessor is one less.	$n - 1$
	'Product of' means multiply.	$n \times (n - 1)$
	'is' means equals, so write the equation.	$n(n - 1) = 56$
d	Separate the sides of the equation.	**LHS = 'The quotient of the eight less than a number and 4'**
	Use brackets to show what is done first.	**The quotient of the (eight less than a number) and 4**
	Write the first operation in symbols.	$(n - 8)$
	Now do the second operation.	$\frac{(n - 8)}{4}$
	Now do the right-hand side.	**The sum of (three times the number) and 5.**
	Write the first operation in symbols.	$3n$
	Now include the second operation.	$3n + 5$
	Now write the complete equation.	$\frac{(n - 8)}{4} = 3n + 5$
	You don't need brackets as well as the vinculum in the final equation.	$\frac{n - 8}{4} = 3n + 5$

9780170194778

In most cases, you won't be told to use 'a number' but will have to decide what you are going to use for the variable(s). If you have a choice, it is often easier to choose the smaller number as the variable.

Example 4

Skis, poles and boots can be hired for \$50 plus a daily charge of \$20 at a ski resort. Jose paid \$150 for the hire of his ski gear for the days he was at the resort. Write an equation for this situation.

Solution

Choose a variable.	**Let the number of days be *d*.**
Show the first operation with brackets.	**\$50 plus (a daily charge of \$20)**
Write the first operation in symbols.	$\mathbf{20}\boldsymbol{d}$
Include the second operation.	$\mathbf{20}\boldsymbol{d} + \mathbf{50}$
Write the full equation.	$\mathbf{20}\boldsymbol{d} + \mathbf{50} = \mathbf{150}$

Example 5

Akbah's younger brother is called Mohammed. His age is 5 less than half of Akbah's age, which is 24. Write an equation for this situation.

Solution

Choose a variable.	**Let Mohammed's age be *m*.**
Reverse the information so that it uses Mohammed's age, the smallest number.	**When you add five to Mohammed's age and multiply by 2, you get 24.**
Show the first operation with brackets.	**When you (add five to Mohammed's age) and multiply by 2, you get 24.**
Write the first operation in symbols.	$(\boldsymbol{m} + \mathbf{5})$
Include the second operation.	$\mathbf{2} \times (\boldsymbol{m} + \mathbf{5})$
Write the full equation with the brackets in the usual form.	$\mathbf{2}(\boldsymbol{m} + \mathbf{5}) = \mathbf{24}$

Worksheet
Equation problems
MAT08NAWK00075

A common type of puzzle that appears in newspapers uses the ages of relatives at different times. In this case you should still choose the smaller number as the variable, but you may need to write some steps to show the different times. Do this in order of size.

Example 6

Ying is 7 times older than her daughter Tara. In two years time she will only be 5 times as old as Tara. Write an equation for this situation.

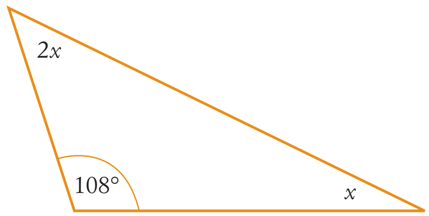
Solution

Choose a variable.	**Let Tara's present age be t.**
Write her age in 2 years time.	**Tara's age in 2 years $= (t + 2)$**
Write Ying's present age.	**Ying's present age $= 7 \times t = 7t$**
Write Ying's age in 2 year's time.	**Ying's age in 2 years time $= (7t + 2)$**
Write the other information in the question.	**Ying's age in 2 years time is 5 times Tara's age in 2 years time.**
Write the equation.	$(7t + 2) = 5 \times (t + 2)$
You don't need the brackets on the LHS in the answer.	$7t + 2 = 5(t + 2)$

Some geometry problems can be solved using equations.

Example 7

Write the equation you would use to find the unknown angles in this diagram.



Solution

State the relevant geometric property.	**Angles in a triangle add up to 180°.**
Write the angle sum.	**Angle sum $= 108° + x + 2x$**
Write the equation.	$108° + x + 2x = 180°$
Simplify the equation.	$3x + 108 = 180$

Equations arise quite frequently in financial calculations.

9780170194778

Example 8

A discount of 15% reduces the price of a pair of designer jeans by \$48. Write the equation you would use to find the original price of the jeans.

Solution

Choose the variable.	**Let original price = \$*p***
Write the discount in terms of the price.	**Discount = 15% of *p***
Show how to work it out.	$= \frac{15}{100} \times p$
Write the equation.	$\frac{15}{100} \times p = 48$
Simplify and write the answer.	$\frac{3p}{20} = 48$

You can also get equations from statistics and probability situations.

Example 9

The probability that an orange in a bulk bin is rotten is 0.06. There are actually 21 rotten oranges in the bin. Write the equation you would use to find the number of oranges in the bin.

Solution

Choose the variable.	**Let the number of oranges in the bin = *n***
Write the probability as a decimal.	**Probability = 0.06**
Write the probability rule.	$\text{Probability} = \frac{\text{Number of rotten oranges}}{\text{Number in bin}}$
Substitute to get the equation.	$0.006 = \frac{21}{n}$
Write the answer with the variable on the left.	$\frac{21}{n} = 0.06$

Example 10

Three people have an average height of 160 cm. One has a height of 152 cm and another has a height of 176 cm. Write the equation you would use to find the height of the third person.

Solution

Choose the variable.	**Let the height of the third person be *h*.**
Write the equation for the average.	$\text{Average} = \frac{\text{Total of scores}}{\text{Number of scores}}$
Substitute the values.	$160 = \frac{152 + 176 + h}{3}$
Simplify and write the answer with the variable on the left.	$\frac{228 + h}{3} = 160$

Exercise 12.1 Creating equations

Understanding

Extra questions
Exercise 12.1
MAT08NAEQ00042

1 What operation is meant by each of the following?
a Quotient
b Decrease (by 5)
c Sum
d Difference
e Increase (by a factor of 6)

2 Change each of the following into a mathematical expression:
a The sum of a number and 4.
b The quotient of 8 and n.
c The product of x and 7.
d The successor to p.
e Decrease a number by 9.
f The difference between y and 11.
g Thrice a number.
h A quarter of a number.
i 12 more than m.
j 16 is decreased by a factor of g.

Fluency

See Example 1, 3

3 Change each of the following into a mathematical equation:
a The sum of a number and 11 is 4.
b The difference between 5 and a number is 1.
c The product of a number and 12 is the same as the difference between 130 and the number.
d When a number is increased by 11, the result is the same as when it is trebled.
e The difference between a number and 16 is the same as the difference between 28 and the number.
f Three successive integers have a sum of 42.
g A number and its predecessor have a sum of 53.
h The average of 4 successive numbers is 27.
i The product of a number and its successor is 54.
j The quotient of a number and its predecessor is a quarter of the number.

4 Change each of the following into a mathematical equation.
a Three less than four times a number is the same as the difference between 50 and the number.
b The quotient of eleven more than a number and 15 amounts to three.
c 28 less than the product of a number and 21 works out to 119.
d The sum of thrice a number and its successor is the same as 50 more than the number.
e The difference between double a number and 18 is the same as three times the number.
f The product of a number and five more than the number is the difference between 40 and the number.
g The quotient of a number and the difference between the number and two comes to the same thing as the quotient of nine and the number.
h The product two more than a number and two less than a number is the same as five times the number.
i The quotient of a number and its successor is the same as the sum of three-nineteenths and the quotient of the number and five more than the number.
j The product of three more than a number and seven less than a number comes to the product of 15 more than the number and the difference between the number and 11.

9780170194778

Problem solving

See Example 4

5 The cost of materials for a hamburger stall at a school fete was \$300. The hamburgers are sold for \$2 each. The stall made a profit of \$500 after selling all the hamburgers. Write an equation for this situation.

6 A white elephant stall at the fete sold donated goods. A generous parent paid \$200 for one item, but all the other items at the stall were sold for \$5 each. The stall made a total of \$600. Write an equation involving the number of items donated.

See Example 5

7 Adam and his father Stephen share a birthday. On Stephen's birthday in 2011, he turned 60. He was then 15 years less than three times Adam's age. Write an equation for this situation.

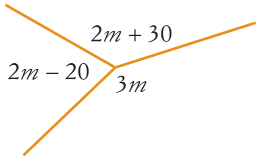
Worked solutions
Exercise 12.1
MAT08NAWS00042

8 Aldis and Inara were talking about their ages to Bobbie. Bobbie said that he was 57. Inara said, "Although I'm younger than Aldis, your age is three times bigger than 40 years less than my age". Write an equation for Inara's age.

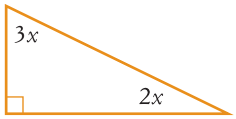
See Example 7

9 Write the equation you would use to find the unknown angles in each of these diagrams:

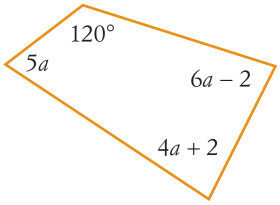
a



b



c



See Example 8

10 A sofa with a mark-up of 60% is sold at profit of \$210. Write the equation you would use to find what the shop paid for the sofa.

11 Germaine got 5% more for her Maths test than she did on English, for which she got 59%. Germaine actually got 48 marks on her maths test. Write the equation you would use to find out what the maths test was out of.

See Example 9

12 Some incomplete packs of cards are used for building card houses. The probability of getting a king from the cards is 0.025. If there are actually 120 cards altogether, write an equation to find the total number of kings.

13 A supermarket buys fluoro light bulbs in large boxes. The probability of getting a faulty one is 0.08. If there were actually 69 good bulbs in the box, write the equation you would use to find the number of lights in a box.

See Example 10

14 48 scores have a median of 27.5. The 24th score is 25. Write the equation you would use to find the 25th score.

15 Two couples have an average mass of 74 kg. The average mass of the group goes up by 4 kg when the group is joined by a fifth person. Write the equation you would use to find the mass of the fifth person.

See Example 6

16 Jan is 4 times as old as her daughter Aleysha. In another 4 years she will only be 3 times older than Aleysha. Write the equation you would use to find Aleysha's age.

Worked solutions
Exercise 12.1
MAT08NAWS00042

17 Col is a third of Peter's age. In ten years time, he will be 5 years less than half Peter's age. Write the equation you would use to find Col's present age.

Reasoning

Worked solutions
Exercise 12.1
MAT08NAWS00042
See Example 2

18 Write each of the following as an equation or expression and consider its truth, explaining your answer.

- **a** The product of two numbers is 48.
- **b** The difference between successive numbers is double their sum.
- **c** The product of two consecutive integers is negative 6.
- **d** The sum of a number and its successor is odd.
- **e** The product of two integers is an even number.
- **f** The product of a number and its successor is even.
- **g** The product of the successor and predecessor of a number is odd.
- **h** The product of a number and its predecessor is the same as the difference between the square of the number and the number itself.
- **i** The product of a number and its successor exceeds the product of the number and its predecessor by twice the number.
- **j** The square of a number's predecessor is 3 more than the product of the number and the difference between the number and two.

12.2 Equations solution methods

The purpose of writing an equation is to model a situation in order to work something out. You need to find the value(s) of the variable that make the equation true.

Important!

Solution of an equation

A **solution** of an algebraic equation is a value of the variable that makes the equation true. The process of finding the solution to an equation is called **solving** an equation.

The simplest equations are just rearrangements of basic number problems. The equation $5n = 20$ says 'What number do you multiply by 5 to get 20?' It is just another way of saying $20 \div 5$. Similarly, $p - 7 = 4$ is a rearrangement of $4 + 7$. However, $7 - p = 4$ is a rearrangement of $7 - 4$. You don't need any working for equations like these. You just think about the number fact that is involved. This is called solving **by inspection**.

9780170194778

Example 11

Puzzle sheet
One-step equations
MAT08NAPS00046

Worksheet
One-step equations
MAT08NAWK00073

Solve each of the following equations.

a $3k = 21$ **b** $m \times 5 = 15$
c $a + 4 = 11$ **d** $7 + d = 12$
e $g - 5 = 9$ **f** $12 - c = 4$
g $\frac{v}{6} = 7$ **h** $\frac{27}{t} = 3$

Solution

a Write the equation. $\mathbf{3k = 21}$
This is a rearrangement of $21 \div 3$. $\mathbf{k = 7}$

b Write the equation. $\mathbf{m \times 5 = 15}$
This is a rearrangement of $15 \div 5$. $\mathbf{m = 3}$

c Write the equation. $\mathbf{a + 4 = 11}$
This is a rearrangement of $11 - 4$. $\mathbf{a = 7}$

d Write the equation. $\mathbf{7 + d = 12}$
This is a rearrangement of $12 - 7$. $\mathbf{d = 5}$

e Write the equation. $\mathbf{g - 5 = 9}$
This is a rearrangement of $9 + 5$. $\mathbf{g = 14}$

f Write the equation. $\mathbf{12 - c = 4}$
This is a rearrangement of $12 - 4$. $\mathbf{c = 8}$

g Write the equation. $\mathbf{\frac{v}{6} = 7}$
This is a rearrangement of 7×6. $\mathbf{v = 42}$

h Write the equation. $\mathbf{\frac{27}{t} = 3}$
This is a rearrangement of $27 \div 3$. $\mathbf{t = 9}$

In the example above, **f** and **h** have the variable second. They are not rearrangements of the opposite operation to the one in the equation. **f** has the variable subtracted and **h** has the variable divided. All the equations with addition and multiplication work in the same way, whether the variable is first or last. What property of multiplication and addition makes this true?

The simplest way to solve an equation is to try different values until you find the one that makes the equation true. This is called **guess and check** or **trial and error**.

Example 12

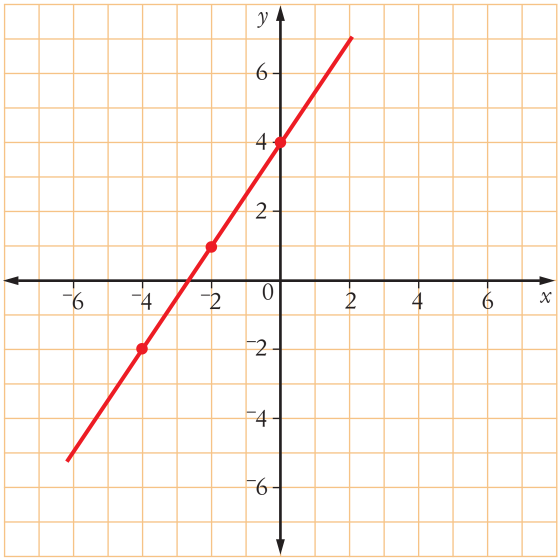
Use guess-and-check to find solutions to these equations.

a $9m + 7 = 61$ **b** $5x - 3 = 4x + 7$

Solution

a Write the equation.	$\mathbf{9m + 7 = 61}$
Try $m = 4$.	$\mathbf{9 \times 4 + 7 = 43}$
Too small, so try $m = 8$.	$\mathbf{9 \times 8 + 7 = 79}$
Too big, so try $m = 5$.	$\mathbf{9 \times 5 + 7 = 52}$
Closer. Try $m = 6$.	$\mathbf{9 \times 6 + 7 = 61}$
Correct! Write the solution.	$\mathbf{m = 6}$
b Write the equation.	$\mathbf{5x - 3 = 4x + 7}$
Try $x = 5$. Work out each side.	$\mathbf{LHS = 5 \times 5 - 3 = 22}$ $\mathbf{RHS = 4 \times 5 + 7 = 27}$
LHS < RHS, so try $x = 8$.	$\mathbf{LHS = 5 \times 8 - 3 = 37}$ $\mathbf{RHS = 4 \times 8 + 7 = 39}$
LHS < RHS, so try $x = 11$.	$\mathbf{LHS = 5 \times 11 - 3 = 52}$ $\mathbf{RHS = 4 \times 11 + 7 = 51}$
Almost there, but LHS > RHS. Try $x = 10$.	$\mathbf{LHS = 5 \times 10 - 3 = 47}$ $\mathbf{RHS = 4 \times 10 + 7 = 47}$
Correct! Write the solution.	$\mathbf{x = 10}$

You can also solve an equation using a graph. You saw in Chapter **4** how to use a table of values to plot a graph. You only need two points for a straight line, but it is a good idea to do a third one as a check.



9780170194778

Example 13

a Draw a graph of the equation $y = 3x + 4$.
b Use the graph to solve the equation $3x + 4 = 10$.

Solution

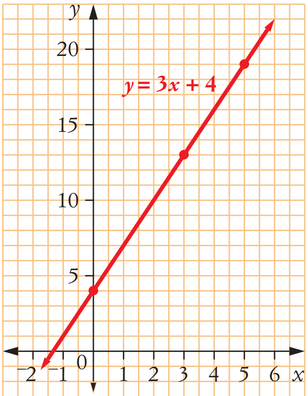
a Draw up a table for x and y values. Choose three values for x.

x	0	3	5
y			

Complete the table.

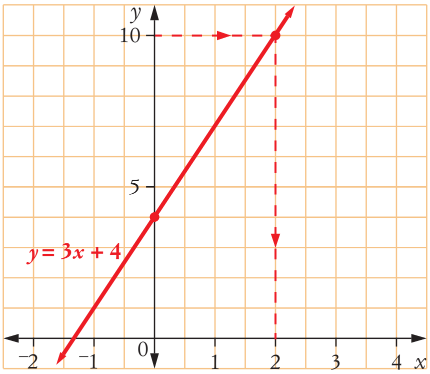
x	0	3	5
y	4	13	19

Plot the points and join them. Extend the graph past the points you have plotted.



b To solve $3x + 4 = 10$, use the graph to find when y $(= 3x + 4) = 10$.

Draw a line across from $y = 10$ until it hits the graph line. Then draw a line down until it hits the x-axis.



State the result.

The solution to $10 = 3x + 4$ is $x = 2$.

Technology
GeoGebra: Solving equations
MAT08NATC00010

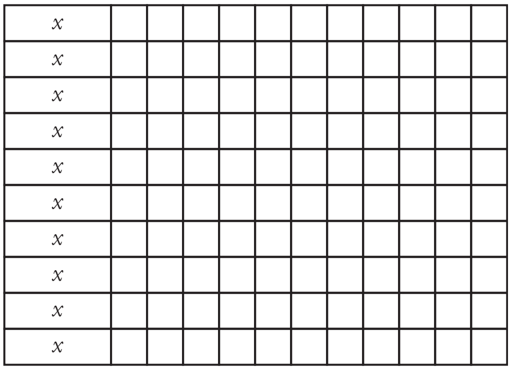
TLF learning object
Mobile phone plans: Find the best deal (L1107)
MAT08NAIN00012

TLF learning object
Mobile phone plans: Peak and off-peak (L1108)
MAT08NAIN00012

Investigate: Modelling equations

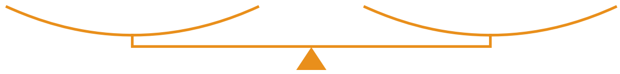
Teacher notes
Modelling equations
MAT08NATN00021

Work in pairs for this investigation.
You can use a simple model for equations similar to the methods used to model expressions in Chapter **9**. You will again need some Algebra Investigation Materials.
Make the squares and strips like this:
Draw lines 1 cm apart on a blank piece of paper in the pattern shown on the right.

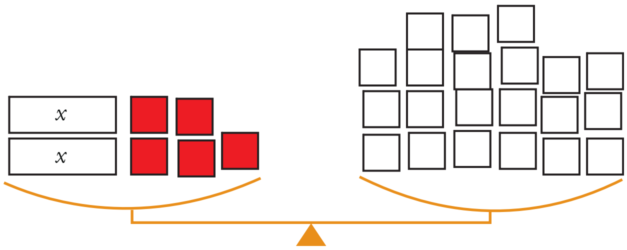


- It doesn't matter how long the strips are, as long as they are not an exact centimetre length. Label the strips of equal length x.
- Use a highlighter or coloured pencil to shade the other side of the paper.
- Cut out the squares and strips and label the reverse sides of the strips.

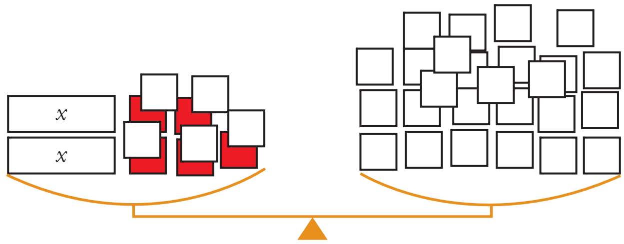
The white side represents positive and the coloured side represents negative.
Draw a 'balance' in your maths pad or on another piece of paper like this:



Now use your Algebra Investigation Materials and balance to model the equation $2x - 5 = 21$ using your strips and squares. Your model should look a bit like this:

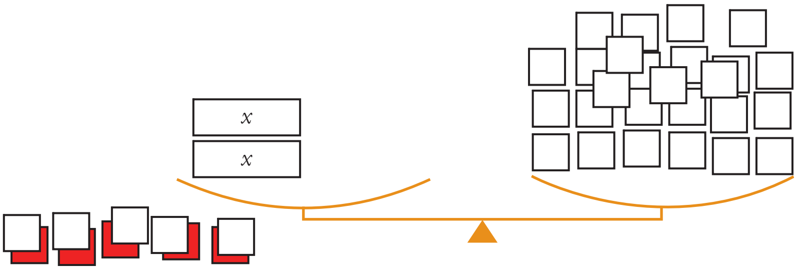


You don't need to carefully separate the strips and squares as shown above. This has been done just to show that the correct numbers of pieces are actually there.
Remember from your previous work that squares of opposite colours cancel out. x-strips of opposite colours also cancel out.
RULE: Whatever you put on one side, you must put on the other side.
Now put 5 extra squares on both sides. It should look like this:

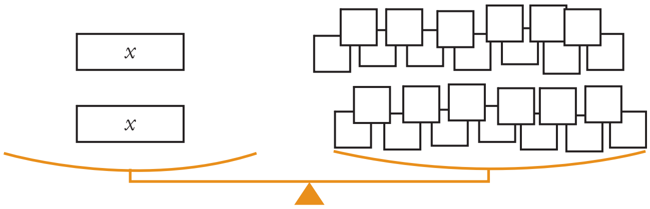


9780170194778

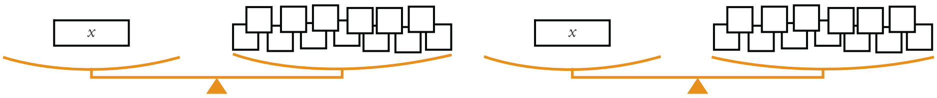
Remove the pairs of opposite coloured squares form the left-hand side. Each one of the pairs really means $1 - 1 = 0$, so it is not making any change to the balance.



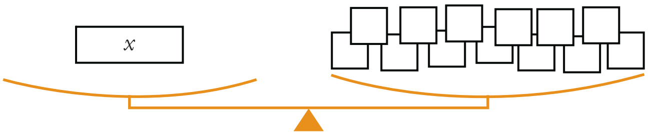
The second last step in modelling the solution is to partition the strips and squares. Rearrange the squares and strips into equal rows like this:



If you had a second balance, you could move the partitions onto different balances like this:



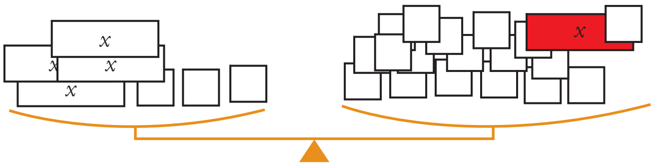
The result is as shown below:



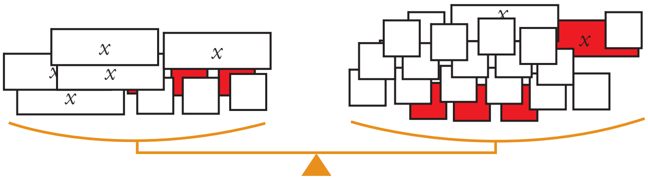
This gives you the answer:

The solution to $2x - 5 = 21$ is $x = 13$.

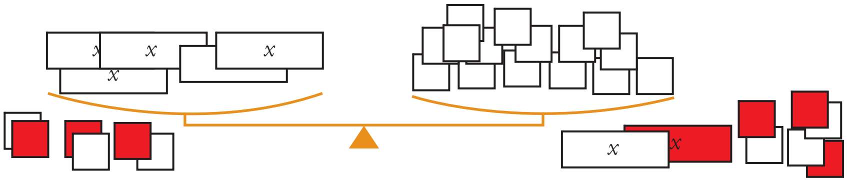
Now do the model for $4x + 3 = 18 - x$. Your model should look like this:



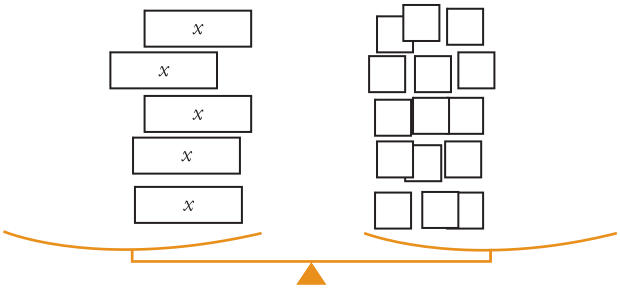
Put three negative squares and a positive x-strip on each side.



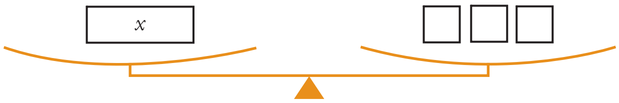
Remove the opposite coloured pairs:



Now partition into equal rows:



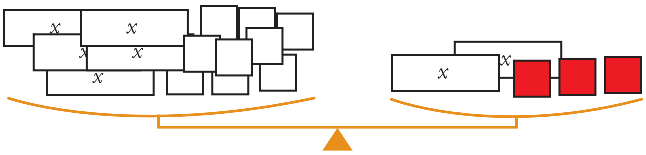
Get rid of all except one partition:



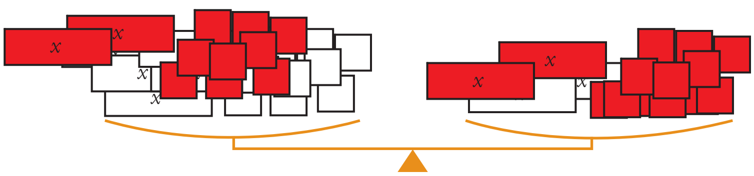
Write down the answer:

The solution to $4x + 3 = 18 - x$ is $x = 3$.

Now do the model for $5x + 9 = 2x - 3$. Your model should look like this:

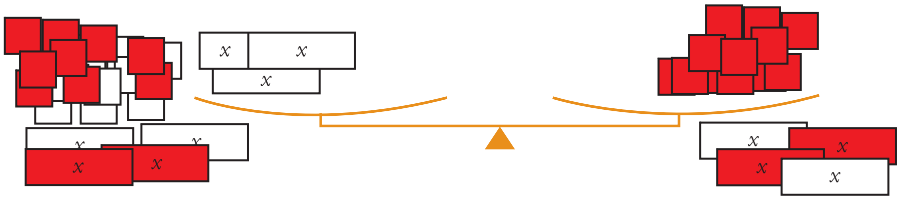


Do the first step:

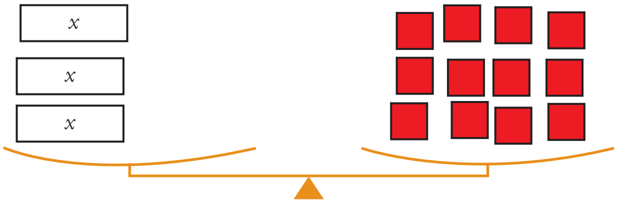


9780170194778

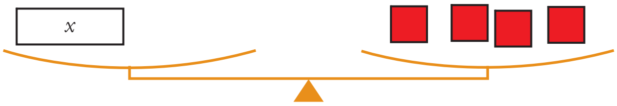
Remove the matched pairs:



Partition:



Get rid of all except one partition:



Write down the answer:

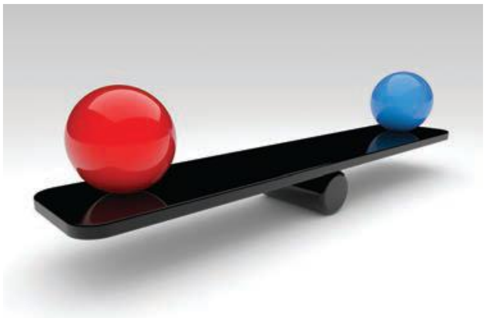
The solution to $5x + 9 = 2x - 3$ is $x = {}^{-}4$.

Now try to do these ones:

$4x + 7 = 15$
$2x - 13 = 3$
$5x + 12 = 2$
$2x - 3 = 15 - x$
$5x + 8 = x - 12$
$3 - 4x = 5x - 6$

Hint: you might want to turn the last one around.
Your teacher probably has more for you to try on your own.

Weblink
The RITEMATHS program
MAT08NAWB00012



Last year you may have drawn **flow charts** to model equations. A flow chart shows the steps involved on the left-hand side of an equation using arrows between a line of boxes. The variable is shown in the first box. The answer from the right-hand side of the equation is shown in the last box. The boxes in between are for the numbers that result from each operation. The operations are shown above the arrows between the boxes.
You do **backtracking** to solve the equation by moving backwards along the boxes from the right-hand side of the flow chart to the beginning. At each step, the operation is 'undone' by doing the opposite operation.

Example 14

Worksheet
Backtracking
MAT08NAWK00072

a Write the equation $4m + 5 = 37$ as a flow chart.
b Use backtracking to solve the equation.

Solution

a Write the equation.

$$\mathbf{4}\boldsymbol{m} + \mathbf{5} = \mathbf{37}$$

Make all the operations on the LHS obvious.

$$\mathbf{4} \times \boldsymbol{m} + \mathbf{5} = \mathbf{37}$$

Put the variable in the first box.

$$\boxed{m}$$

Choose the first operation to show on the flow chart.

According to the order of operations, $\times$ comes before $+$.

Show the first operation on an arrow to a second box.

$$\boxed{m} \xrightarrow{\times 4} \boxed{}$$

Show the next operation.

$$\boxed{m} \xrightarrow{\times 4} \boxed{} \xrightarrow{+5} \boxed{}$$

There are no more operations on the LHS, so show the result (RHS) in the last box.

$$\boxed{m} \xrightarrow{\times 4} \boxed{} \xrightarrow{+5} \boxed{37}$$

b Show the opposite operations underneath each one shown on the flow chart.

$$\boxed{} \xrightarrow[\div 4]{\times 4} \boxed{} \xrightarrow[-5]{+5} \boxed{37}$$

Do the first backward operation.

$$\boxed{} \xrightarrow[\div 4]{\times 4} \boxed{32} \xrightarrow[-5]{+5} \boxed{37}$$

Do the last backward operation.

$$\boxed{8} \xrightarrow[\div 4]{\times 4} \boxed{32} \xrightarrow[-5]{+5} \boxed{37}$$

You can use your calculator, but you must press equals after each arrow calculation.

Enter as: 37 $\boxed{-}$ 5 $\boxed{=}$.

37–5	32

Then do $\boxed{\div}$ 4 $\boxed{=}$.

Ans÷4	8

State the result.

The solution of $4m + 5 = 37$ is $m = 8$.

If you don't press equals after the first part of the calculation your scientific calculator will follow the correct order of operations, so $37 - 5 \div 4$ will come to 35.75.

9780170194778

Example 15

Use backtracking to solve the following equations.

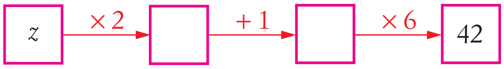
a $42 = 6(2z + 1)$ **b** $\dfrac{4a - 26}{3} = {}^{-}18$

Solution

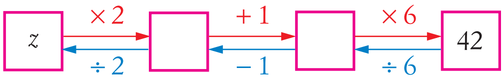
a Swap the equation around so the variable is on the LHS. $\mathbf{6(2z + 1) = 42}$

Make all the operations obvious. $\mathbf{6 \times (2 \times z + 1) = 42}$

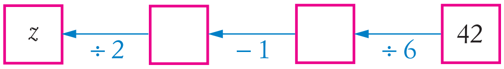
Do the flow chart.



Put in the backward operations.



Do the backward operations.

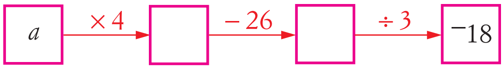


State the result. **The solution of $42 - 6(2z + 1)$ is $z = 3$.**

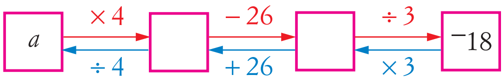
b Write the equation. $\mathbf{\dfrac{4a - 26}{3} = {}^{-}18}$

Make all the operations obvious. $\mathbf{(4 \times a - 26) \div 3 = {}^{-}18}$

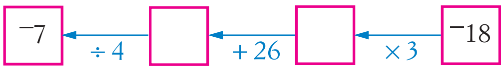
Do the flow chart.



Put in the backward operations.



Do the backward operations.



State the result. **The solution of $\dfrac{4a - 26}{3} = {}^{-}18$ is $a = {}^{-}7$**

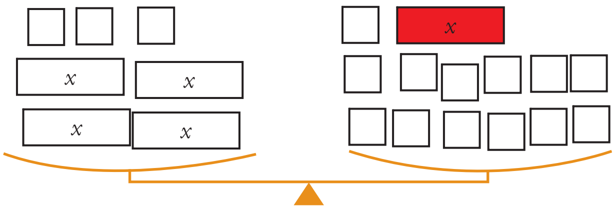
Scientific calculator exercise
Solving equations
MAT08NASC00009

Puzzle sheet
Two-step and three-step equations
MAT08NAPS00047

Worksheet
Two-step and three-step equations
MAT08NAWK00074

Before you look at formally solving equations it is helpful to go through an example like the investigation in your head.

Example 16



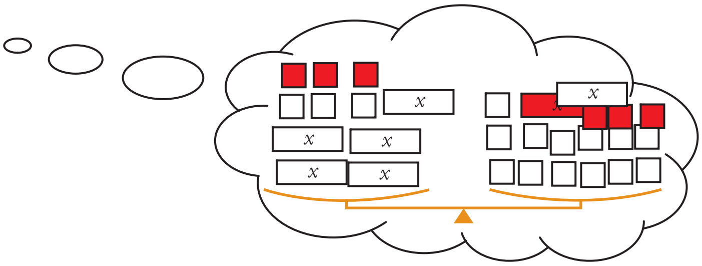
Write the equation shown by this balance diagram and solve the equation mentally by putting in extra variable and unit shapes.

Solution

Write the equation. $4x + 3 = 13 - x$

Think what you would do with a model.

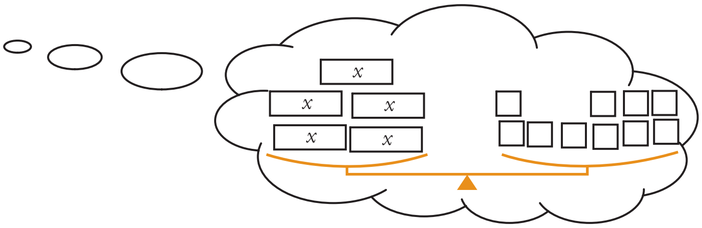
You would add x and $^{-}3$ to both sides.



Do this with the equation. $4x + 3 + {^{-}3} + x = 13 - x + {^{-}3} + x$

Think what you would do with a model.

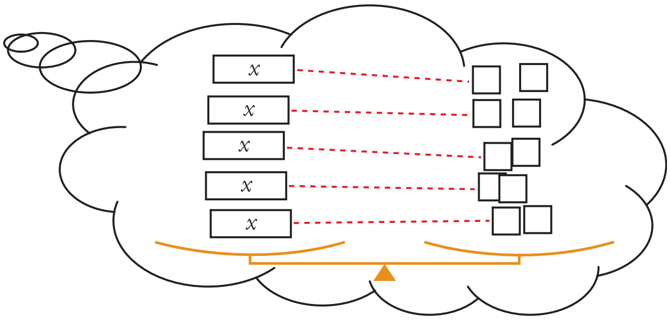
You would remove the matching pairs.



Do this with the equation, using $4x + x = 5x$ and $13 + {^{-}3} = 10$. $5x = 10$

You would then partition into 5 parts.

This is really division.



Divide both sides by 5. $\frac{5x}{5} = \frac{10}{5}$

Simplify and write the answer. **The solution of $4x + 3 = 13 - x$ is $x = 2$.**

Both the model and the backtracking method use backward operations to solve equations. These are called **inverse operations**. The inverse operations for the operations you have learnt so far are shown in the table in the box below.

9780170194778

Important!

Formal solution of equations

Operation	Symbol	Inverse operation	Symbol
Addition	$+$	Subtract	$-$
Subtraction	$-$	Add	$+$
Multiplication	$\times$	Divide	$\div$
Division	$\div$	Multiply	$\times$
Square	$\square^2$	Square root	$\sqrt{}$
Square root	$\sqrt{}$	Square	$\square^2$
Cube	$\square^3$	Cube root	$\sqrt[3]{}$
Cube root	$\sqrt[3]{}$	Cube	$\square^3$

To formally solve equations, use inverse operations in the reverse order to get the variable on its own on the LHS with only an arithmetic expression on the RHS.
Whatever you do to one side must be done to the other.
You must show your steps mathematically.

Example 17

Video tutorial
Excel: Solving equations
MAT08NAVT00012

Formally solve the following equations:

a $7b + 11 = 67$

b $\frac{m}{4} - 2 = 3$

Solution

a Write the equation. $7b + 11 = 67$

Make all the operations obvious. $7 \times b + 11 = 67$

In calculations, × comes before +, so undo the + 11 first to solve the equation. $7 \times b + 11 - 11 = 67 - 11$

Simplify both sides. $7 \times b = 56$

Undo the × 7 by dividing by 7. $\frac{7 \times b}{7} = \frac{56}{7}$

Simplify to get the answer. $b = 8$

b Write the equation. $\frac{m}{4} - 2 = 3$

In calculations, ÷ comes before –, so undo the – 2 first to solve the equation. $\frac{m}{4} - 2 + 2 = 3 + 2$

Simplify both sides. $\frac{m}{4} = 5$

Undo the ÷ 4 by multiplying by 4. $4 = \frac{m}{4} = 4 \times 5$

Simplify to get the answer. $m = 20$

9780170194778

If one side of an equation does not have the variable term first, it is often easier to change it so that it does. If a variable term is subtracted, make it addition of a negative.

Example 18

Technology worksheet
Excel: Solving linear equations
MAT08NACT00017

Formally solve the following equations.

a $4 - 2x = 28$
b $5p - 3 = {}^{-}38$

Solution

a Write the equation.	$\mathbf{4 - 2x = 28}$
Write the variable term first as ${}^{-}2x$.	$\mathbf{{}^{-}2x + 4 = 28}$
Make all the operations obvious.	$\mathbf{{}^{-}2 \times x + 4 = 28}$
In calculations, $\times$ comes before $+$, so undo the $+ 4$ first to solve the equation.	$\mathbf{{}^{-}2 \times x + 4 - 4 = 28 - 4}$
Simplify.	$\mathbf{{}^{-}2 \times x = 24}$
Undo the $\times\ {}^{-}2$ by dividing by ${}^{-}2$.	$\mathbf{\dfrac{{}^{-}2 \times x}{{}^{-}2} = \dfrac{24}{{}^{-}2}}$
Simplify to get the answer.	$\mathbf{x = {}^{-}12}$
b Write the equation.	$\mathbf{5p - 3 = {}^{-}38}$
Make all the operations obvious.	$\mathbf{5 \times p - 3 = {}^{-}38}$
In calculations, $\times$ comes before $-$, so undo the $- 3$ first to solve the equation.	$\mathbf{5 \times p - 3 + 3 = {}^{-}38 + 3}$
Simplify both sides.	$\mathbf{5 \times p = {}^{-}35}$
Undo the $\times 5$ by dividing by 5.	$\mathbf{\dfrac{5 \times p}{5} = \dfrac{{}^{-}35}{5}}$
Simplify to get the answer.	$\mathbf{p = {}^{-}7}$

When you have to solve an equation with variables on both sides, you need to move all the variable terms to the left-hand side. This changes the equation into ones like the previous examples.

9780170194778

Example 19

Formally solve the following equations.

a $3a + 4 = 7 + 2a$ **b** $2d - 8 = {}^{-}71 - 5d$

Solution

a Write the equation. $\mathbf{3a + 4 = 7 + 2a}$

Get the + $2a$ to the LHS by subtracting $2a$. $\mathbf{3a + 4 - 2a = 7 + 2a - 2a}$

Simplify both sides. $\mathbf{a + 4 = 7}$

Undo the + 4 by subtracting 4. $\mathbf{a + 4 - 4 = 7 - 4}$

Simplify to get the answer. $\mathbf{a = 3}$

b Write the equation. $\mathbf{2d - 8 - {}^{-}71 = 5d}$

Get the − $5d$ to the LHS by adding. You can do the − 8 at the same time. $\mathbf{2d - 8 + 5d + 8 = {}^{-}71 - 5d + 5d + 8}$

Simplify both sides. $\mathbf{7d = {}^{-}63}$

Undo the × 7 by dividing by 7. $\mathbf{\frac{7d}{7} = \frac{{}^{-}63}{7}}$

Simplify to get the answer. $\mathbf{d = {}^{-}9}$

Video tutorial
Equations with variables on both sides
MAT08NAVT10018

Animated example
Formally solving equations
MAT08NAAE00020

Puzzle sheet
Equations with the unknown on both sides
MAT08NAPS00048

Worksheet
Equations with unknowns on both sides
MAT08NAWK00076

In an equation with variables on both sides, make sure the equation is written with the larger variable term (whether positive or negative) on the left-hand side.

Example 20

Formally solve the equation $5 - 7x = 13 - 3x$.

Solution

Write the equation. $\mathbf{5 - 7x = 13 - 3x}$

${}^{-}3x$ is bigger than ${}^{-}7x$, so turn the equation around so that it is on the LHS. $\mathbf{13 - 3x = 5 - 7x}$

Put the variable terms at the front. $\mathbf{{}^{-}3x + 13 = {}^{-}7x + 5}$

Get the variables on the LHS and the constants on the RHS. $\mathbf{{}^{-}3x + 13 + 7x - 13 = {}^{-}7x + 5 + 7x - 13}$

Simplify each side. $\mathbf{4x = {}^{-}8}$

Divide by 4. $\mathbf{\frac{4x}{4} = \frac{{}^{-}8}{4}}$

Simplify to get the answer. $\mathbf{x = {}^{-}2}$

To solve an equation with brackets, is usually easier to multiply out the brackets before doing anything else. This makes the equation just like the ones you have already seen.

Example 21

Video tutorial
Equations with brackets
MAT08NAVT10019

Formally solve the following equations.

a $5y - 2 = 2(y + 7) - 1$ **b** $20 - 5(2a - 6) = 5(a + 1)$

Solution

a Write the equation.	$\mathbf{5y - 2 = 2(y + 7) - 1}$
Multiply out the brackets.	$\mathbf{5y - 2 = 2y + 14 - 1}$
Simplify the RHS.	$\mathbf{5y - 2 = 2y + 13}$
Get the variables on the LHS and the constants on the RHS.	$\mathbf{5y - 2 - 2y + 2 = 2y + 13 - 2y + 2}$
Simplify each side.	$\mathbf{3y = 15}$
Divide by 3.	$\mathbf{\frac{3y}{3} = \frac{15}{3}}$
Simplify to get the answer.	$\mathbf{y = 5}$
b Write the equation.	$\mathbf{20 - 5(2a - 6) = 5(a + 1)}$
Remove the brackets, being careful with signs.	$\mathbf{20 - 10a + 30 = 5a + 5}$
Simplify the LHS and put variables at the front.	$\mathbf{{}^{-}10a + 50 = 5a + 5}$
$5a$ is bigger than $^{-}10a$, so turn the equation around so it is on the LHS.	$\mathbf{5a + 5 = {}^{-}10a + 50}$
Get the variables on the LHS and the constants on the RHS.	$\mathbf{5a + 5 + 10a - 5 = {}^{-}10a + 50 + 10a - 5}$
Simplify each side.	$\mathbf{15a = 45}$
Divide by 15.	$\mathbf{\frac{15a}{15} = \frac{45}{15}}$
Simplify to get the answer.	$\mathbf{a = 3}$

Examples **17–21** show a general principle used in mathematics: try to make more difficult problems into easier ones that you already know how to do.

9780170194778

Exercise 12.2 Equation solution methods

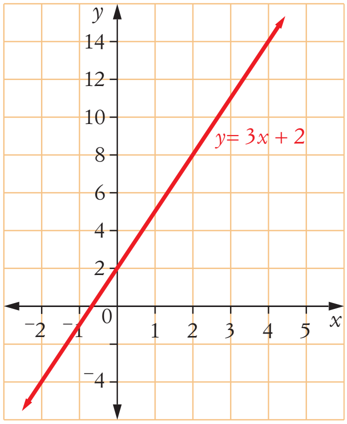
Understanding

Extra questions
Exercise 12.2
MAT08NAEQ00043

See Example 13

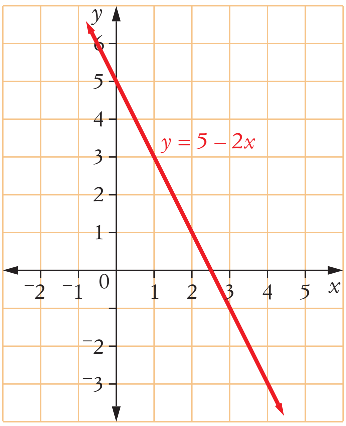
1 This is the graph of the equation $y = 3x + 2$.
Use the graph to answer the following.

a If $x = 1$, find y.
b If $x = 3$, find y.
c If $x = 0$, find y.
d If $y = 8$, find x.
e If $y = 14$, find x.
f Solve the equation $3x + 2 = 2$.
g Solve the equation $3x + 2 = 11$.
h Solve the equation $3x + 2 = 8$.
i Solve the equation $3x + 2 = 14$.



2 Use the graph of the equation $y = 5 - 2x$ shown to answer the following.

a If $x = 0$, find y.
b If $x = 4$, find y.
c If $x = 2$, find y.
d If $y = 3$, find x.
e If $y = {}^{-}1$, find x.
f Solve the equation $5 - 2x = 5$.
g Solve the equation $5 - 2x = {}^{-}3$.
h Solve the equation $5 - 2x = 3$.
i Solve the equation $5 - 2x = 0$.



3 Copy the following flow charts and insert inverse (opposite) operations.

See Example 14

a $\boxed{g} \xrightarrow{\times 7} \boxed{} \xrightarrow{-5} \boxed{37}$

b $\boxed{k} \xrightarrow{\times 4} \boxed{} \xrightarrow{+9} \boxed{41}$

c $\boxed{a} \xrightarrow{\div 3} \boxed{} \xrightarrow{-11} \boxed{5}$

d $\boxed{x} \xrightarrow{\div 5} \boxed{} \xrightarrow{+13} \boxed{17}$

e $\boxed{y} \xrightarrow{\times 6} \boxed{} \xrightarrow{-12} \boxed{} \xrightarrow{\div 2} \boxed{3}$

f $\boxed{h} \xrightarrow{\div 4} \boxed{} \xrightarrow{+9} \boxed{} \xrightarrow{\times 3} \boxed{45}$

4 Solve each of the equations represented by question **3**.

5 Draw flow charts for each of the following.

a $5g + 7 = 22$
b $6a - 11 = 25$
c $44 = 5h + 9$
d $58 = 8u - 14$
e $\dfrac{7 + 2r}{3} = 5$
f $\dfrac{3w - 1}{5} = 7$

6 Solve each of the equations represented in question **5**.

See Example 15

7 Use flow charts and backtracking to solve the following equations.

a $3g + 7 = 19$ **b** $5a - 12 = 23$ **c** $\frac{h}{4} + 9 = 15$

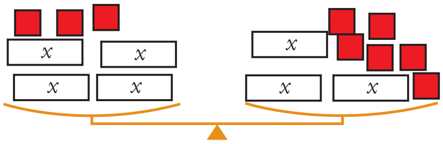
d $3(q + 5) = 36$ **e** $4(a - 11) = 16$ **f** $6m + 7 = 25$

g $33 = 7w - 23$ **h** $\frac{3d - 2}{5} = 2$ **i** $\frac{3r + 6}{9} = 3$

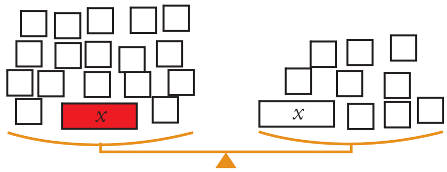
See Example 16

8 Write and solve the equations represented by each of the following.

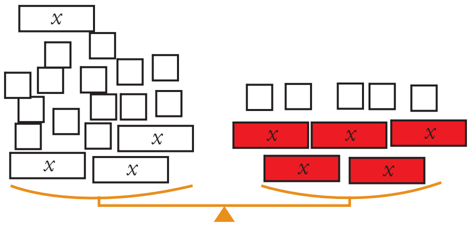
a



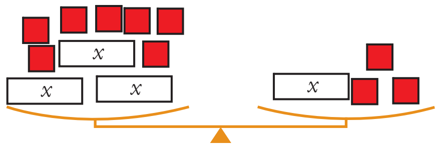
b



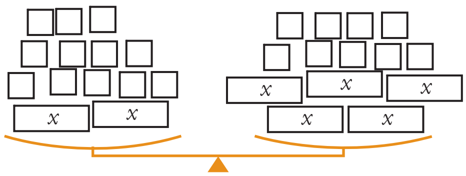
c



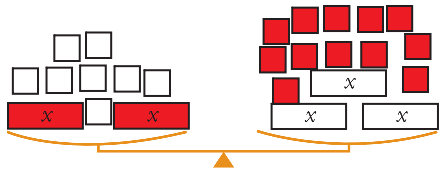
d



e



f



See Example 16

9 Use inverse operations to complete the steps shown by . . . to solve each equation.

a
$$\begin{aligned} 4d + 7 &= 31 \\ 4d + 7 \ldots &= 31 \ldots \\ 4d &= \ldots \\ \frac{4d}{\ldots} &= \frac{\ldots}{\ldots} \\ d &= \ldots \end{aligned}$$

b
$$\begin{aligned} 7a + 6 &= 41 \\ 7a + 6 \ldots &= 41 \ldots \\ 7a &= \ldots \\ \frac{7a}{\ldots} &= \frac{\ldots}{\ldots} \\ a &= \ldots \end{aligned}$$

c
$$\begin{aligned} 3x - 5 &= 22 \\ 3x - 5 \ldots &= 22 \ldots \\ 3x &= \ldots \\ \frac{3x}{\ldots} &= \frac{\ldots}{\ldots} \\ x &= \ldots \end{aligned}$$

d
$$\begin{aligned} \frac{w + 9}{8} &= 2 \\ \frac{w + 9}{8} \ldots &= 2 \ldots \\ w + 9 &= \ldots \\ w + 9 \ldots &= \ldots \\ w &= \ldots \end{aligned}$$

e
$$\begin{aligned} \frac{b + 13}{3} &= 7 \\ \frac{b + 13}{3} \ldots &= 7 \ldots \\ b + 13 &= \ldots \\ b + 13 \ldots &= \ldots \\ b &= \ldots \end{aligned}$$

f
$$\begin{aligned} \frac{m - 6}{5} &= 4 \\ \frac{m - 6}{5} \ldots &= 4 \ldots \\ m - 6 &= \ldots \\ m - 6 \ldots &= \ldots \\ m &= \ldots \end{aligned}$$

9780170194778

Fluency

10 Solve each of the following equations by inspection. *See* Example 11

a $8 + m = 14$ **b** $h - 12 = 7$ **c** $7y = 35$ **d** $24 \div j = 4$

e $k + 7 = 12$ **f** $8x = 56$ **g** $\frac{m}{7} = 4$ **h** $24 \div p = 2$

11 Solve each of the following equations by inspection.

a $m + 8 = 12$ **b** $a + 12 = 20$ **c** $17 + q = 22$

d $h - 6 = 14$ **e** $z + 13 = 22$ **f** $15 - y = 2$

g $4m = 24$ **h** $6a = 48$ **i** $h \div 4 = 6$

j $56 = 8e$ **k** $\frac{d}{7} = 12$ **l** $36 = 9u$

12 Use trial and error to solve each of the following equations. *See* Example 12

a $2f + 3 = 15$ **b** $3b - 5 = 4$ **c** $2d + 7 = d + 9$

d $4(3 + 2w) = 28$ **e** $3g \div 5 = 6$ **f** $\frac{4x}{3} - 8 = 4$

g $25 - 2f = 9$ **h** $16 = 3r - 11$ **i** $2(w - 5) = 24$

j $\frac{d + 5}{6} = 3$ **k** $4a - 4 = a + 8$ **l** $\frac{1}{2}y + 7 + 12$

13 Use trial and error to solve each of the following equations.

a $6d - 14 = 22$ **b** $7m + 18 = 81$ **c** $12k - 31 = 77$

d $8z + 15 = 103$ **e** $\frac{2x}{3} + 5 = 11$ **f** $85 - 7n = 29$

g $6y + 14 = 4y + 48$ **h** $\frac{3x}{5} + 7 = 13$ **i** $8q + 11 = 3q + 41$

14 Use the method of guess and check to solve the following equations.

a $11 - 4q = {}^{-}13$ **b** $5h + 32 = 7$ **c** $7y - 9 = {}^{-}51$

d $15 - 8v = 47$ **e** $\frac{3r}{4} - 7 = {}^{-}13$ **f** $26 - 6z = 50$

g $\frac{7 - d}{4} + 5 = 9$ **h** $6 - 5x = 7x + 14$ **i** $11 - 3u = {}^{-}8u + 46$

15 Draw a graph of the equation $y = 3x - 5$ using values of x between ${}^{-}2$ and 6, then use the graph to solve the following equations.

a $3x - 5 = 1$ **b** $3x - 5 = 10$ **c** $3x - 5 = 4$ **d** $3x - 5 = {}^{-}8$

16 Draw a graph of the equation $y = 7 - 4x$ using values of x between ${}^{-}2$ and 6, then use the graph to solve the following equations.

a $7 - 4x = 3$ **b** $7 - 4x = 11$ **c** $7 - 4x = {}^{-}5$ **d** $7 - 4x = {}^{-}13$

17 Formally solve the following equations. *See* Examples 17, 18

a $5m + 3 = 28$ **b** $3a + 13 = 25$ **c** $6q + 9 = 45$

d $7c - 11 = 10$ **e** $12 + 4n = 44$ **f** $11 - 3w = 2$

g $8x - 9 = 15$ **h** $22 - 4y = 2$ **i** $27 - 3d = 9$

j $26 - 2m = 8$ **k** $42 - 6z = 12$ **l** $7 + 8b = 63$

m $19 - 3f = 7$ **n** $29 - 4q = 5$ **o** $31 - 6k = 7$

18 Formally solve the following equations.

a $\frac{v}{8} + 3 = 6$ **b** $4 - \frac{y}{2} = {}^-4$ **c** $\frac{t}{8} - 8 = 4$

d $\frac{w}{4} + 3 = 5$ **e** $9 - \frac{k}{4} = 14$ **f** $7 - \frac{q}{2} = {}^-7$

g $\frac{m}{2} + 8 = 9$ **h** $\frac{v}{6} - 5 = 2$ **i** $6 - \frac{g}{5} = {}^-4$

j $\frac{d}{5} + 5 = 7$ **k** $\frac{d}{5} - 9 = 6$ **l** $3 - \frac{q}{9} = {}^-7$

m $\frac{x}{4} + 4 = 9$ **n** $3 - \frac{v}{9} = {}^-2$ **o** $\frac{p}{8} - 6 = 8$

19 Formally solve the following equations.

a ${}^-4g + 7 = 19$ **b** $39 = 15 - 6y$ **c** ${}^-5a - 8 = 22$

d $10 = {}^-6q - 8$ **e** $13 - 4t = 41$ **f** ${}^-11 + 7p = 10$

g ${}^-2(3w + 4) = 10$ **h** $42 = 6(3 - 2m)$ **i** $5({}^-3d + 7) = {}^-25$

Problem solving

20 The graph below shows the amount of profit (P) in dollars from selling n tickets to a school disco. The graph is based on the equation $P = 4n - 300$.

a What is the price of one ticket?

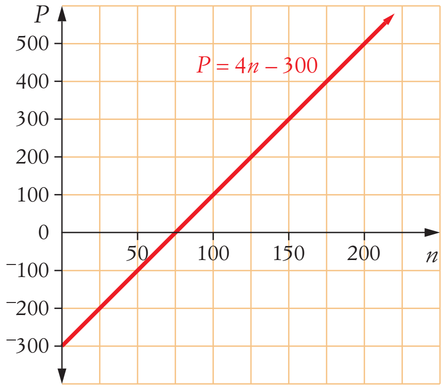
b What will the profit be if no tickets are sold?

c How many tickets must be sold for the disco organisers to cover costs?

d What profit will be made if 100 tickets are sold?

e What profit will be made if 200 tickets are sold?

f How many tickets need to be sold to make a profit of $200?



21 A group of students are planning to sell hamburgers to raise funds for an end-of-year trip. They buy enough rolls, mince and salad to make a maximum of 50 hamburgers.
The equation $P = 1.5n - 30$ shows their profit in dollars (P) for selling n hamburgers.

a Draw a graph of their profit, then use it to answer the following questions.

b How many burgers do they need to sell before they start to make a profit?

c How much will they lose if only 10 burgers are sold?

d What profit will be made if 40 burgers are sold?

e What is the maximum profit they can make?

See Examples 19, 20

22 Formally solve the following equations.

a $6a = 4a + 12$ **b** $5m + 7 = 3m + 15$

c $8x - 10 = 3x$ **d** $4y + 6 = 2y + 18$

e $9f + 7 = 2f + 28$ **f** $4 + 3r = 28 - r$

g $9 + 7k = 2k + 24$ **h** $4w - 10 = 2w + 14$

i $29 - 6g = 17 - 3g$ **j** $4 - 8p = 8p - 28$

k $2r - 15 + 3r = 8r - 30$ **l** $4e + 12 = 28 - 2e + 5e$

m $10x - 15 + 2x = 8x + 13$ **n** $15 - 3y + 6 + y = 7y - 15$

9780170194778

23 Formally solve the following equations.

See Example 21

a $2d + 3(d - 2) = 4$
b $2(y + 5) = y + 25$
c $3(w + 2) + 2(w + 5) = 41$
d $2(a + 3) + a + 1 = 25$
e $3(p + 1) + 2(p - 1) = 51$
f $5m + 3 = 2(m + 9)$
g $2(1 - g) = 3(2 - g)$
h $x - 7 = 2(x - 4)$
i $4(2d - 5) = 2(3d + 1)$
j $5(k - 2) = 2(k + 7)$
k $2(2r - 5) = r + 11$
l $5(2e - 4) = 3(e + 5)$
m $4(3b + 4) - 6b = 46$
n $4(9 - 2x) + 3(x - 4) = 3x$

24 Formally solve the following equations.

Worked solutions
Exercise 12.2
MAT08NAWS00043

a $5 - k = 3 - 2k$
b $f + 9 = 3f + 13$
c $b - 4 = 2b - 11$
d $10m + 12 = 6 + 8m$
e $7 - 5a = 3a - 17$
f $10h - 9 = 7h - 30$
g $2m + 2 = 5m + 8$
h $11q + 6 = 12q - 4$

25 Formally solve the following equations.

Worked solutions
Exercise 12.2
MAT08NAWS00043

a $a + 8 - 3(a - 2) = 2$
b $7f + 11 = -(7 - f)$
c $3(x - 4) - (x + 2) = 10$
d $2y - 8 = {}^{-}4(y + 2)$
e $4m - 3(m - 5) = 11$
f $d - (d + 7) = 9 - 2d$
g $r - (2r - 8) = 13$
h $5(n - 2) - 3(7 - 2n) = 2$
i $1 - (y - 8) = 2(2y - 3)$
j $k - 3(2k - 2) = 31$
k $2p - 3(1 - p) = 12$
l $4x - 3(2x + 6) = 2(x - 3)$
m $2(y - 5) + 3(y + 3) = 24$
n $a - 3(a + 2) = 14 + 2a$

Reasoning

26 The solutions to the equations below are not necessarily correct. Check each one and if it is incorrect, give the letter of the line where the first error occurs.

Worked solutions
Exercise 12.2
MAT08NAWS00043

a
$$\begin{aligned} 2(m + 6) &= 20 & \\ 2m + 12 &= 20 & \text{(A)} \\ 2m &= 8 & \text{(B)} \\ m &= 4 & \text{(C)} \end{aligned}$$

b
$$\begin{aligned} 4(3 + a) &= 8 & \\ 12 + 4a &= 8 & \text{(A)} \\ 4a &= 20 & \text{(B)} \\ a &= 5 & \text{(C)} \end{aligned}$$

c
$$\begin{aligned} 3(b + 4) &= 18 & \\ 3b + 4 &= 18 & \text{(A)} \\ 3b &= 14 & \text{(B)} \\ b &= 4\tfrac{2}{3} & \text{(C)} \end{aligned}$$

d
$$\begin{aligned} 3(2y - 7) &= 27 & \\ 6y - 21 &= 27 & \text{(A)} \\ 6y &= 48 & \text{(B)} \\ y &= 8 & \text{(C)} \end{aligned}$$

e
$$\begin{aligned} 2(a + 4) + (a - 5) &= 15 & \\ 2a + 8 + a - 5 &= 15 & \text{(A)} \\ 3a + 3 &= 15 & \text{(B)} \\ 3a &= 12 & \text{(C)} \\ a &= 12 & \text{(D)} \end{aligned}$$

f
$$\begin{aligned} 5(x + 3) + 2(x - 2) &= 26 & \\ 5x + 15 + 2x + 4 &= 26 & \text{(A)} \\ 7x + 19 &= 26 & \text{(B)} \\ 7x &= 7 & \text{(C)} \\ x &= 1 & \text{(D)} \end{aligned}$$

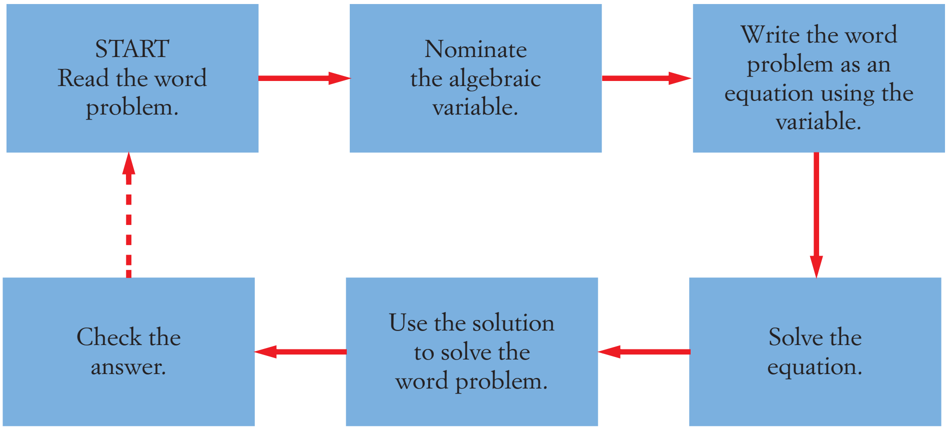
g $$\begin{aligned} b+3(1+b) &= b+12 \\ b+3+3b &= b+12 \quad \text{(A)} \\ 3b+3 &= b+12 \quad \text{(B)} \\ 2b+3 &= 12 \quad \text{(C)} \\ 2b &= 9 \quad \text{(D)} \\ b &= 4\tfrac{1}{2} \quad \text{(E)} \end{aligned}$$

h $$\begin{aligned} 3(m-4) &= 4(2-m)+8 \\ 3m-12 &= 8-m+8 \quad \text{(A)} \\ 4m-12 &= 8+7 \quad \text{(B)} \\ 4m &= 16+12 \quad \text{(C)} \\ 4m &= 28 \quad \text{(D)} \\ m &= 7 \quad \text{(E)} \end{aligned}$$

i $$\begin{aligned} x+2(2x-4) &= 7 \\ x+4x-8 &= 7 \quad \text{(A)} \\ 5x-8 &= 7 \quad \text{(B)} \\ 5x &= 15 \quad \text{(C)} \\ x &= 3 \quad \text{(D)} \end{aligned}$$

12.3 Applying linear equations

You are now ready to solve word problems by writing an equation and then solving it. When you do this, you will follow the steps shown below.



9780170194778

Example 22

Write and formally solve an equation for each of the following.

a The difference between 40 and triple a number is 16.

b The product of three more than a number and five comes to 65.

Solution

a Decide which operation is first.	**Multiplication is done before subtraction.**
Use brackets to show which operation is done first.	**The difference between 40 and (triple a number) . . .**
Write the first operation in symbols, using a variable for the unknown number.	$3n$
Now include the second operation.	$40 - 3n$
Complete the equation.	$40 - 3n = 16$
Rewrite with the variable at the front.	$^{-}3n + 40 = 16$
Do the + 40 first.	$^{-}3n + 40 - 40 = 16 - 40$
Simplify both sides.	$^{-}3n = {}^{-}24$
Divide by $^{-}3$.	$\frac{^{-}3n}{^{-}3} = \frac{^{-}24}{^{-}3}$
Simplify to get the answer.	$n = 8$
b Decide which operation is first.	**Multiplication is done before addition.**
Use brackets to show which operation is done first.	**The product of (three more than a number) . . .**
Write the first operation in symbols, using a variable for the unknown number.	$x + 3$
Now include the second operation.	$5(x + 3)$
Complete the equation.	$5(x + 3) = 65$
Multiply out the brackets.	$5x + 15 = 65$
Do the + 15 first.	$5x + 15 - 15 = 65 - 15$
Simplify both sides.	$5x = 50$
Divide by 5.	$\frac{5x}{5} = \frac{50}{5}$
Simplify to get the answer.	$x = 10$

Example 23

Gemma bought 3 tins of tennis balls and emptied them into her sports bag. She already had 7 balls in the bag. When she emptied her bag out there were 19 balls altogether. How many balls were in each new tin?

Solution

Choose a variable.	**Let the number of balls in a tin be b.**
Write the equation.	$3b + 7 = 19$
Take 7 from both sides.	$3b + 7 - 7 = 19 - 7$
Simplify both sides.	$3b = 12$
Divide by 3 to get the answer.	$b = 4$
Check the answer. Write the LHS first.	$3b + 7$
Substitute the answer.	$= 3 \times 4 + 7$
Evaluate the expression.	$= 19$ ✓**OK**
Write the answer in a sentence.	**There were 4 tennis balls in each tin.**

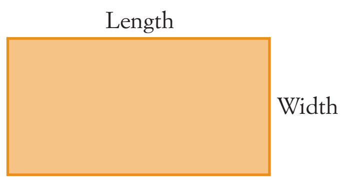
For most real problems, you need to know how to work something out. In the following example, you need to know how to work out the perimeter of a rectangle. You should always draw a diagram for problems involving measurements, areas, volumes and so on.

Example 24

A rectangular swimming pool has a perimeter of 58 m. It is 5 m longer than it is wide. Find its dimensions.

Solution

Draw a diagram.



Choose a variable.	**Let the width be w.**
Write an expression for the length.	**Length** $= w + 5$
What is meant by perimeter?	**Perimeter = Distance around the outside**
Write an expression for the perimeter.	**Perimeter** $= w + (w + 5) + w + (w + 5)$
Simplify the expression.	$= 4w + 10$
Write the equation.	$4w + 10 = 58$
Take 10 from both sides.	$4w + 10 - 10 = 58 - 10$
Simplify.	$4w = 48$

9780170194778

Divide by 4 to get the answer.	$w = 12$
Write the dimensions.	**Width = 12 m, length = 17 m**
Check the answer.	**Perimeter** $= 12 + 17 + 12 + 17$
Work out the perimeter.	$= 58$ ✓OK
Write the answer in a sentence.	**The rectangle is 12 m wide and 17 m long.**

Example 25

Because she paid cash, Tammy got a discount of 8% on a silk blouse. She actually got \$3.20 off the price. What was the original price?

Solution

Choose a variable.	**Let the original price be p.**
Write the discount in terms of the price.	**Discount = 8% of p**
Write the equation.	$\frac{8}{100} \times p = 3.2$
Simplify.	$\frac{2p}{25} = 3.2$
Multiply by 25.	$\frac{2p}{25} \times 25 = 3.2 \times 25$
Simplify.	$2p = 80$
Divide by 2.	$p = 40$
Check the answer.	**8% of \$40**
Write the percentage as a decimal instead of a fraction to check.	$= 0.08 \times \$40$
Work out the answer.	$= \$3.20$ ✓OK
Write the answer in a sentence.	**The blouse originally cost \$40.**

Example 26

Marcus is three times as old as Pieter. 9 years ago he was six times as old as Pieter. What are their ages now?

Solution

Choose a variable.	**Let Pieter's present age = a.**
Write Marcus's age now.	**Marcus's present age = $3a$**
Write their ages 9 years ago.	**Pieter's age 9 years ago = $a - 9$**
	Marcus's age 9 years ago = $3a - 9$
Write the equation.	$3a - 9 = 6(a - 9)$
Multiply out the brackets.	$3a - 9 = 6a - 54$

Swap the equation so the greater variable term is on the left-hand side.	$6a - 54 = 3a - 9$
Take away $3a$ and add 54 to both sides.	$6a - 54 - 3a + 54 = 3a - 9 - 3a + 54$
Simplify.	$3a = 45$
Divide by 3.	$a = 15$
State the answers.	**Pieter is 15 and Marcus** $= 3 \times 15 = 45$
Check the ages 9 years ago.	**9 years ago, Pieter was 6 and Marcus was 36, which is 6 times as much. ✓OK**
Write the answers in a sentence.	**Marcus is now 45 and Pieter is 15 years old.**

Exercise 12.3 Applying linear equations

Fluency

Extra questions
Exercise 12.3
MAT08NAEQ00044

See Example 22

1 Write an equation and find the number(s) in each of the following.
   a If 4 is added to three times a number, the result is 19.
   b If twice a number n is subtracted from 30, the result is 14.
   c If 11 is added to twice a number n, the result is 5.
   d Three successive integers have a sum of 36.
   e Three more than quadruple a number and the product of six and the number comes to 143.
   f Five more than the product of six and a number is 11 less than eight times the number.
   g Seven less than the product of a number and 4 is 73 more than nine times the number.
   h Triple a number's predecessor is 57.
   i The product of ten less than a number and four is two more than seven times the number.
   j Six times the difference between 12 and a number is the predecessor to the product of five and the difference between the number and three.

Problem solving

For questions **2** to **17**, write an algebraic equation and then solve it to answer the question.

See Example 23

2 Kylie needed 47 balls of wool to knit a jumper. She bought 5 packets and 2 separate balls of wool. How many balls were in each packet?

3 Hao bought 1000 bricks to build 6 supports for a fence. After he had finished, there were 76 bricks left over. How many bricks went into each support?

See Example 24

4 A rectangle has an area of 87 m^2. If one side is 3 m long, what is the length of the other?

5 The perimeter of a parallelogram is 62 cm. One of its sides is 19 cm. What are the lengths of the other sides?

6 Kelly is 5 years older than Ben. The sum of their ages is 33. How old is Ben?

7 Adam and Britney have \$480 between them. Adam has seven times as much as Britney. How much does each have?

See Example 25

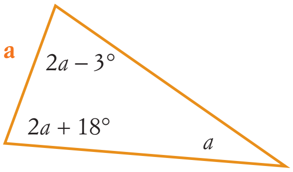
8 A used car salesman worked on a commission of 30% of the selling price. He made \$2400 when he sold a second-hand SV6 Commodore. What was the selling price and what price did the caryard pay?

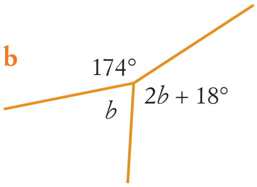
9780170194778

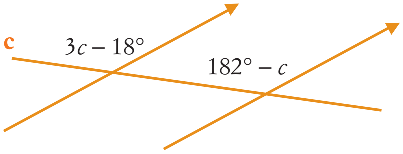
9 In a Year 9 class there were 28 students. If there were 4 more girls than boys, how many boys were in the class?

10 A rectangular garden bed is 5 m longer than it is wide. Its perimeter is 62 m. Find the length of the bed.

11 Write equations and solve them to calculate the unknowns in these geometric diagrams.

a


b


c


12 A large tank is half full of petrol. After another 300 L of fuel is drained out, the tank is now found to be one-third full. What fraction is drained out and how much does the tank hold when it is full?

13 Bonita has 26 more CDs than Aidan. Her number of CDs is also three times as many as his. How many CDs does each have?

14 Silvio is 10 years older than Maria. The sum of their ages is equal to four times Maria's age. How old are Silvio and Maria?

15 Juan bought 6 punnets of lettuce plants for his vegetable garden, but 16 plants were dead. Angela bought 4 punnets of lettuce plants from the same nursery. How many plants were in each punnet, if Juan and Angela finished up with the same number of plants?

Worked solutions
Exercise 12.3
MAT08NAWS00044

16 Toby's mother was 26 years old when he was born. Now she is three times older than Toby. What are the ages of Toby and his mother now?

See Example 26

17 Don's mother had another baby after he had gone overseas. She called the baby Ellen. Some years later, he went home and was there on Ellen's birthday. He was then seven times as old as Ellen. 8 years later he was only 3 times Ellen's age. How old was he when Ellen was born?

Worked solutions
Exercise 12.3
MAT08NAWS00044

Reasoning

18 The average height of 4 people is 165 cm. When Hayden joined the group, the average went up to 167 cm. Jane said that Hayden must have been 169 cm. Work out his actual height and what Jane did wrong in working out her answer.

Worked solutions
Exercise 12.3
MAT08NAWS00044

19 Donna got a new HD tuner at a discount of 20%. When she said that this came to $150 off the price, her friend Aimee said she must have paid $750 for it, but Donna said that was wrong. Explain what Aimee did wrong and work out the original price and what Donna paid.

20 On his Maths test, Billy got an answer of 18.4 for the question "When double the difference between a number and 4 is taken from 100, an answer 7 times the original number was obtained. Find the number." He was sure he was right but actually got the question wrong. Work out the correct answer and what Billy did wrong.

Chapter 12 summary

Quiz
Solving equations
MAT08NAQZ00012

- An **equation** is a mathematical sentence. It has an equals sign between two expressions. It can be true or false, or true for some particular value(s).
- A **solution** or **root** of an equation is a value of the variable that makes the equation true. The solution satisfies the equation and finding the solution is known as **solving** the equation.
- Informal techniques used for solving equations include **inspection**, **guess and check**, **graphing**, **backtracking** using **flow charts**, and **modelling** as a **balance**.
- The **formal solution** of equations is done using **inverse operations**. Inverse operations are done in the reverse order to the normal order of operations. Addition is the inverse of subtraction, and multiplication is the inverse of division.
- The rules for solving equations are:
  - What is done to one side must be done to the other.
  - To 'undo' or cancel an operation, apply its inverse.
  - Keep equals signs under each other.
- In cases where the same variable is on both sides of an equation, put the side with the larger variable term on the left, move all variable terms to the left and all constants to the right-hand side.
- When solving equations involving brackets, it is usually easier to expand the brackets first using the distributive law.
- The following steps should be used to solve word problems:
  1. Read the word problem.
  2. Choose a variable.
  3. Write the word problem as an equation using the variable.
  4. Solve the equation.
  5. Use the solution to solve the word problem.
  6. Check the answer in the original equation (or real situation).

9780170194778

Understanding

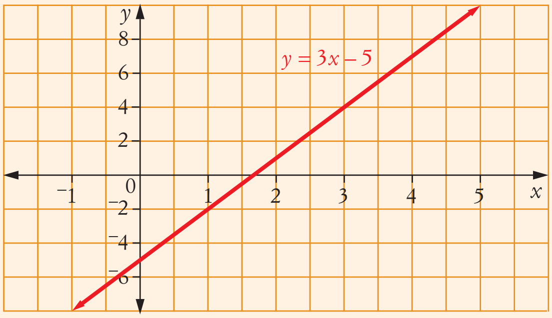
Worksheet
Solving equations review
MAT08NAWK00078

1 Change each of the following into a mathematical expression.

- **a** The sum of a number and 5
- **b** The successor to a number
- **c** The quotient of a number and 12
- **d** The quotient of 12 and a number
- **e** The difference between a number and 8
- **f** Thrice a number
- **g** Increase a number by 6
- **h** Decrease a number by a factor of 3
- **i** The product of a number and 5
- **j** Increase a number by a factor of 6.

See Example 13

2 This is the graph of the equation $y = 3x - 5$. Use the graph to answer the following.



- **a** If $x = 3$, find y.
- **b** If $x = 0$, find y.
- **c** If $y = 7$, find x.
- **d** If $y = {}^-2$, find x.
- **e** Solve the equation $3x - 5 = 1$.
- **f** Solve the equation $3x - 5 = {}^-8$.

See Example 14

3 Copy the following flow charts and insert inverse operations on each.

a $\boxed{y} \xrightarrow{\times 3} \boxed{} \xrightarrow{-7} \boxed{17}$

b $\boxed{a} \xrightarrow{\times 5} \boxed{} \xrightarrow{-10} \boxed{} \xrightarrow{\div 6} \boxed{5}$

4 Write the following equations as flow charts.

- **a** $3k + 5 = 17$
- **b** $\dfrac{3h - 18}{2} = 3$

See Example 15

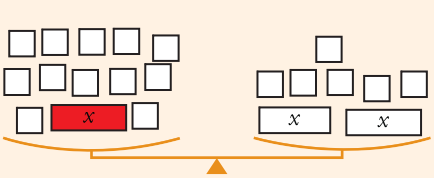
5 Use flow charts and backtracking to solve the following equations.

- **a** $4e + 5 = 17$
- **b** $3(m - 5) = 18$
- **c** $\dfrac{4m + 1}{3} = 7$

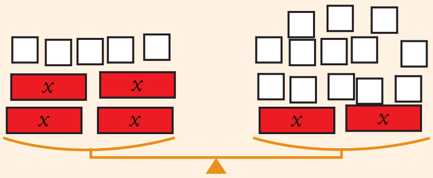
See Example 16

6 Write and solve the equations represented by each of the following.

a



b



Fluency

See Example 1, 3

7 Change each of the following into a mathematical equation.

- **a** When a number is increased by 5 the answer is 20.
- **b** The product of a number and 8 is 56.
- **c** 5 less than 8 times a number is 35.
- **d** Thrice the difference between 9 and a number is 51.
- **e** Nine more than the product of number's predecessor and successor is 65.
- **f** The quotient of three less than five times a number and 2 is the same as double the number's successor.

8 Solve each of the following by inspection.

a $p + 8 = 20$ **b** $\frac{n}{4} = 8$ **c** $3h = 12$ **d** $11 - q = 4$ **e** $z - 10 = 5$

9 Use trial and error to solve the following equations.

a $3g - 4 = 11$ **b** $2(5 + 3a) = 46$ **c** $\frac{3d}{2} - 2 = 7$ **d** $9b + 11 = 137$

e $\frac{3a}{8} - 11 = {}^-2$ **f** $23 - 8d = {}^-81$ **g** $\frac{9 - a}{7} + 6 = 10$

10 Draw a graph of the equation $y = 4x - 7$ using values of x between ${}^-1$ and 5. Use the graph to solve the following equations.

a $4x - 7 = 9$ **b** $4x - 7 = {}^-11$

See Example 17, 18

11 Formally solve the following equations.

a $4g + 7 = 39$ **b** $5a - 16 = 14$ **c** $11 = 9r - 16$ **d** $\frac{k}{3} + 5 = 11$

e $\frac{3h}{5} = 6$ **f** $\frac{7w + 4}{5} = 12$ **g** $3k + 20 = 5$ **h** $4 - 5z = 39$

i $4p + 3 = {}^-29$ **j** ${}^-7e - 4 = {}^-39$

See Example 22

12 Write an equation and find the number in each of the following.

a The difference between 40 and the product of y and 2 is 16.

b Four times a number's successor is the same as 13 less than five times its predecessor.

Problem solving

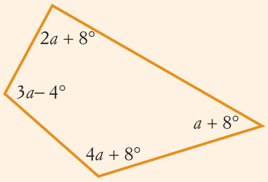
See Example 4, 25

13 A fruit shop bought a tray of punnets of strawberries for \$30. They did not sell all the strawberries, but made a profit of \$24 on the tray. Given that the punnets were sold for \$3 each, write an equation for the situation and find the number of punnets sold.

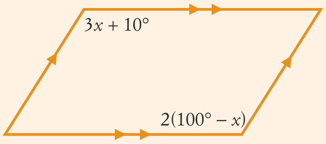
See Example 7

14 Write equations for each of the diagrams below and solve the equations to find the values of the pronumerals.

a



b



See Examples 18, 25

15 David marked up the furniture in his shop by 120%. He made a profit of \$498 on a sofa bed. Write an equation and find what he paid for it.

See Example 9

16 The probability of drawing a king from a pack of cards with 2 missing is 0.06. Write an equation and find the number of kings in the pack.

See Example 10

17 Janine is the goal attack in her netball team. Over the first four games of the season she has scored an average of 10 goals. What would she have to score in the next game to lift her average to 14? Comment on your answer.

18 A group of students hire an inflatable 'fun castle' to operate at their school's open day. They know from experience that no more than 200 children will pay to use the fun castle on the open day. Their profit in dollars (P) is calculated using the equation $P = 4n - 300$, where n is the number of children who pay to use the castle.

9780170194778

a Draw a graph of their profit, then use it to answer the following questions.
b How many children need to use the castle before the students start to make a profit?
c What profit will they make if 100 children pay to use the castle?
d What is their maximum profit for the day?

19 Hayley had four packets of Smarties. Three of the packets were full and one of them had only 15 left. She counted all the Smarties and found that there were 141 Smarties altogether. How many were in each full packet? *See* Example 23

20 Formally solve the following equations. *See* Examples 19, 20

a $7y = 16 + 3y$ b $6u - 11 = 10 + 3u$

21 Formally solve the following equations. *See* Example 21

a $3(b + 4) = 21$ b $4(2w - 5) = 36$ c $5j + 2(j - 4) = 20$
d $3(4p - 5) = 7(p + 5)$ e $4(^{-}3d + 20) = {}^{-}28$ f $x - 12 - 4(3x + 2) = 2$
g $5(y - 4) - 3(3 + 4y) = 6$

22 Adam is 35 years younger than his father, but his father is ten years more than twice Adam's age. Write an equation for this situation and find their ages. *See* Example 5

23 At a hospital there are 100 patients. Some are served three meals a day, while others are served only two. A total of 260 meals are prepared every day. How many patients have three meals a day?

24 Juanita is thirty years older than her son and in six years time will be four times her son's age. Write an equation and find their ages *See* Examples 6, 26

Reasoning

25 Write each of the following as an equation or expression and consider its truth, explaining your answer. *See* Example 2

a A number and its successor add up to 1 000 000 000.
b Double a number is the same as the sum of half the number and an even constant.
c The square of 2 more than a number is the same as the square of a number and twice its successor.
d One less than the cube of a number is the same as the product of its predecessor and the sum of its successor and square.

26 The solutions to the equations below are not necessarily correct. Check each one and if it is incorrect, give the letter of the line where the first error occurs.

a
$$\begin{aligned} 3(a-8)+6 &= 8-2(a-1) & \\ 3a-24+6 &= 8-2a+2 & \text{(A)}\\ 3a-30 &= 10-2a & \text{(B)}\\ 3a-30-2a+30 &= 10+2a-2a+30 & \text{(C)}\\ a &= 40 & \text{(D)} \end{aligned}$$

b
$$\begin{aligned} 6x-5(2x+4) &= 4-2(x+2) & \\ 6x-10x-20 &= 4-2x-4 & \text{(A)}\\ -4x-20 &= -2x & \text{(B)}\\ 2x &= 20 & \text{(C)}\\ c &= 10 & \text{(D)} \end{aligned}$$

27 A puzzle in a newspaper said that a father was 12 times as old as his son but that in 6 years he would be triple the difference between 23 and his son's age. Frank said the problem was impossible because the son's age came to a negative number but David said the answer came to nearly 65. When they looked in the paper the next day it said the Dad was 36. Who was right? Give reasons for your answer.

Answers

Chapter 1

Exercise 1.1

1 **a** Mean = 16.8, median = 18, mode = 20, range = 18
b Mean ≈ 25.2, median = 25, mode = 27, range = 8
c Mean ≈ 16.4, median = 16, mode = 15, range = 20
d Mean ≈ 11.8, median = 12, mode = 7, range = 13

2 **a** Mean = 4.7, median = 5, mode = 3, range = 10
b Mean ≈ 19.9, median = 19, mode = 18, range = 16
c Mean ≈ 27.6, median = 28, mode = 28, 30, range = 10
d Mean ≈ 12.5, median = 13, mode = 14, range = 17

3 About 13.6

4 About 15.44

5 Combined mean = 61.625, median ≈ 64 or 65

6 New mean = 160.2. The median could stay the same or it might increase.

7 **a** median **b** mean **c** mean **d** mean

8 **a** mean **b** median **c** mean **d** median

9 It must have been an odd number, so it was 25 or 27. Now it must be 26 or 28.

10 If the mode has changed, then both students must have been the same height, which must be larger than the old mode. If this is less than the mean, the mean could decrease. The same reasoning applies for 3 students, except that one need not be the same.

Exercise 1.2

1 **a**

Score	Frequency
9	1
10	1
11	1
12	2
13	2
14	4
15	2
16	5
17	1
18	2
19	3
20	1
21	0
22	1
Total	26

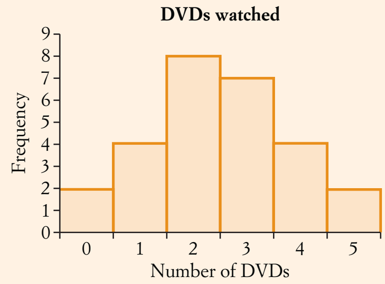
b

Score	Frequency
5.2	1
5.3	1
5.4	2
5.5	1
5.6	2
5.7	2
5.8	1
5.9	5
6.0	2
6.1	3
6.2	3
6.3	2
6.4	3
6.5	0
6.6	1
6.7	1
Total	30

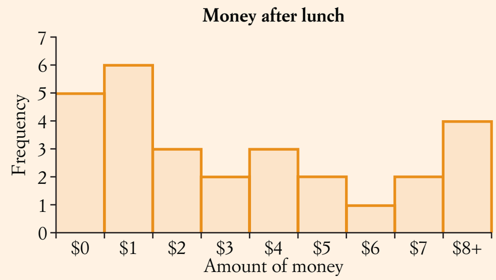
c

Score	Frequency
124	2
125	4
126	3
127	6
128	4
129	6
130	11
131	1
132	1
133	0
134	1
Total	39

2



3



4 **a**

Score	Frequency
58–61	4
62–65	5
66–69	7
70–73	7
74–77	6
78–81	4
82–85	1
86–89	1
Total	35

b

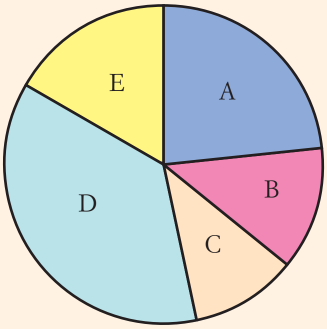
Score	Frequency
0–4	2
5–9	3
10–14	9
15–19	10
20–24	3
25–29	3
30–34	2
35–39	1
Total	33

9780170194778

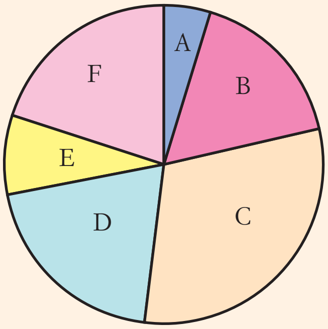
c

Score	Frequency
12.0–15.9	2
16.0–19.9	5
20.0–23.9	6
24.0–27.9	10
28.0–31.9	7
32.0–35.9	1
36.0–39.9	3
Total	34

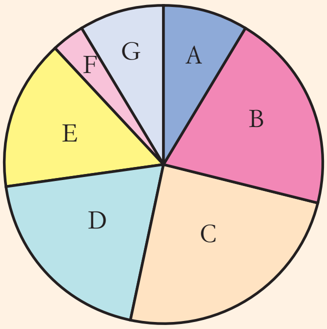
5 a



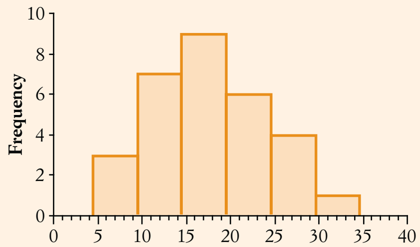
b



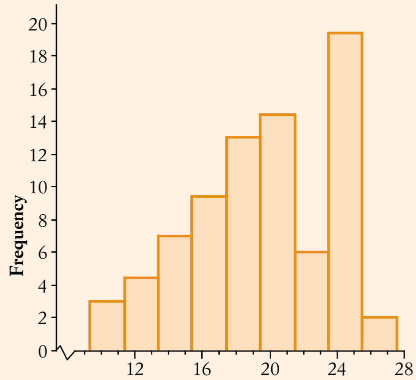
c



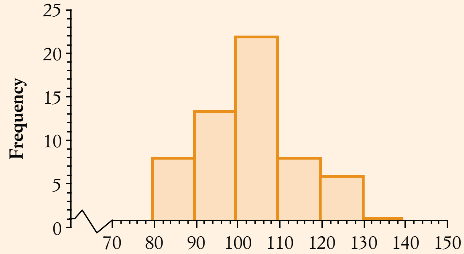
6 a



b



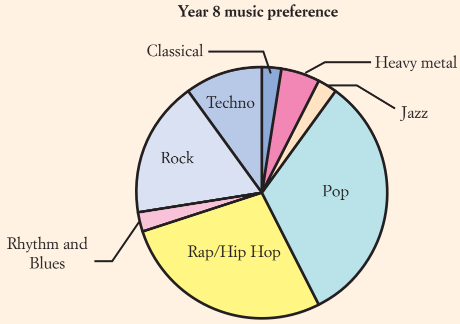
c



7 a

Music	Frequency
Classical	1
Heavy metal	2
Jazz	1
Pop	12
Rap/Hip hop	11
Rhythm and blues	1
Rock	7
Techno	4
Total	39

b

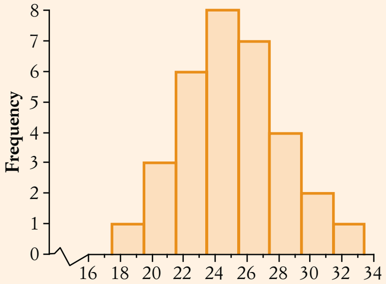


8 a The lengths 253 cm and 1 cm have been omitted.

Length (cm)	Frequency
18–19	1
20–21	3
22–23	6
24–25	8
26–27	7
28–29	4
30–31	2
32–33	1

9780170194778

b



9 Mean ≈ 4.03, median = 4, mode = 3, range = 5 km/h

10 Mean ≈ 8.66, median = 9, mode = 9, range = 6 h

11 a As people have their birthdays, the graph will gradually shift right until it is the same shape again, but one year to the right at the same time next year.
b It will be the same shape, but the age scale will shift across to 26–32.

12 a Since \$30 is 30% of a category, about 30% of each group will shift up to the next category. The 700–799, 900–999 and 1000+ sectors will get bigger, the 800–899 will stay about the same, and the others will get smaller.
b Since 30% of \$300 is \$90, the <400 sector will probably disappear. Some of the 600–699 and all of the 700–799, 800–899 and 900–999 will move into the 1000+ sector, and the other sectors will become much smaller, so the 1000+ sector will occupy nearly half the circle.

Exercise 1.3

1 a People on the electoral roll for Brisbane.
b People who listen to the radio at that time in Townsville.
c People in the local area who may use the library.
d Customers of the timber yard, probably mostly tradespeople.
e Dog owners who feed their dogs tinned dog-food.

2 a Yes, 22 **b** No **c** Yes, 9, 20
d Yes, 12 **e** Yes, 57, 59

3 a Mean ≈ 9.96, median = 7 h
b Without 31, 40, mean ≈ 7.92, median = 6 h
c The few very high scores raise the mean substantially. The median is raised a little. If 25 is also removed, the mean drops to 7.2 and the median stays at 6.

4 a Mean ≈ \$2.71, median = \$2.79
b Without \$1.29, mean ≈ \$2.77, median = \$2.79
c The very low price (probably for mushy ones) lowers the mean.

5 a A large sample (with some passengers possibly being charged for 2 seats)
b Population
c Population (all tyres available)
d Sample

6 a Registered cars (one of each kind)
b Likely traffic (with a weight limit sign)
c Parents with sufficient income to do a catered party

7 Plumbers: Mean ≈ \$28.87, median = \$28; Vets: Mean ≈ \$23.87, median = \$24; Hairdressers: Mean ≈ \$21.07, median = \$22. The plumbers earn the most and the hairdressers the least, probably because of the number of each of these professionals.

8 Girls: Mean ≈ 7.39, median = 7, range = 10.5 minutes; Boys: Mean ≈ 6.26, median = 5.5, range = 11.5 minutes. While the fastest girls and boys took about the same time, and similarly with the slowest, girls generally took more time than boys. However, the small samples mean we should be cautious about these conclusions.

9 Omitting 3.4 (left) and 0.02 (right) as non-compliant, you should get:

Reaction times (Key: 6|2 = 0.26 s = 2|6)		
Left hand		**Right hand**
9 9 6 6	2	3 6 6 7 8 9 9 9 9
9 8 7 7 6 5 5 4 4 4 2 2 1 1	3	1 1 2 2 2 2 5 5 6 6 7 9
8 8 5 5 3 3 2 2	4	0 0 4
7	5	4 4
	6	6

The right hand is generally faster, but there are some right hands that are quite slow. This may indicate that there are more right-handed than left-handed or ambidextrous students.

10

Times to get to school (Key: 5|1 = 15 minutes = 1|5)		
Qld		**WA**
9 7 6 5 5 5 5 5 5 5 4 3 1 1	0	1 2 3 3 5 5 5 5 5
5 5 5 5 5 0 0 0 0	1	0 0 0 0 0 5 5 5 5 5
5 5 5 5 0 0	2	0 0 0 0 0 0 5
0 0	3	0 5
5 5 5	4	0 0 0 0 5
	5	0

9780170194778

WA students generally take a little more time to get to school than Queensland students, perhaps because of longer distances. Although the sample is small, the pattern is very clear.

11 No, because the people who continued to watch a program like that would be biased towards a 'yes' answer.

12 It seems reasonably fair overall for each state, except that it leaves out people who don't live in a capital or other major city. It is not fair for the country because different numbers of people live in each state.

13 It would not reflect the general population, as those in a city mall on a Saturday morning would not include people who worked on Saturday or people who don't live close to the city centre.

14 It will tend to have a bias towards compliant students, but this is unavoidable. At least everybody would know who to speak to about issues.

15 Survey 3 is probably the best because most people are home on Sunday night. The other 2 surveys are biased towards shoppers. 1 and 2 both select on availability at a particular time and place, so 2 is better than 1 because it has more people.

Chapter review

1 Mean ≈ 11.91, median = 10, mode = 9, range = 17

2 **a**

Score	5	6	7	8	9	10	11	12	13	14	15
Frequency	2	1	1	0	2	2	1	2	5	3	1

b

Color	Red	Green	Yellow	White	Black	Blue
Frequency	3	4	8	2	3	5

3 **a** Students at the school **b** Year 8 students
c Shops that sell ice-creams in Fremantle

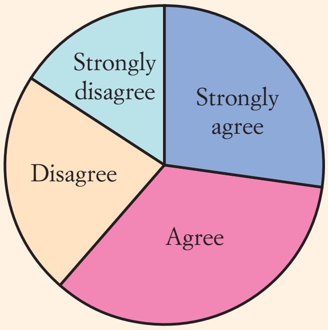
4 **a** Yes, 30 **b** Yes, 81 **c** No

5 Mean ≈ 6.69, median = 7, mode = 8, range = 9

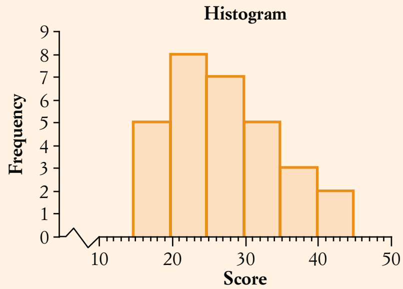
6

Score	Frequency
35–39	5
40–44	3
45–49	12
50–54	1
55–59	5
60–64	3

7

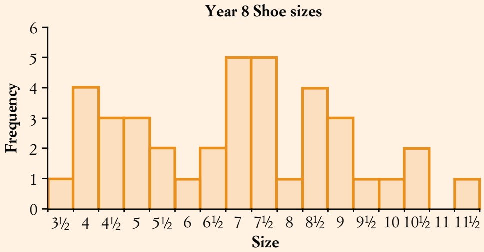


8



9 **a** Omitting sizes 1 and 17 as non-compliant, we get:

Size	Frequency
$3\frac{1}{2}$	1
4	4
$4\frac{1}{2}$	3
5	3
$5\frac{1}{2}$	2
6	1
$6\frac{1}{2}$	2
7	5
$7\frac{1}{2}$	5
8	1
$8\frac{1}{2}$	4
9	3
$9\frac{1}{2}$	1
10	1
$10\frac{1}{2}$	2
11	0
$11\frac{1}{2}$	1



10 **a** Mean ≈ 17.64, median = 18 sec
b Mean ≈ 17.96, median = 18 sec
c Removing the 9 seconds raises the mean.

11 **a** Sample **b** Census

12 The Year 6 students in nearby primary schools and Year 6 students in the school itself if it also has lower grades.

13 It goes down to about 166.5 cm.

14 Mean ≈ 37.31, median = 37, mode = 36, range = 9

15 The median, because there might be a few students who travel a very long way.

16 Only some people listen to talk-back radio and the nature of the program would bias responses towards agreement.

17 There would be more sentences that were longer, and some would be longer than 16 words, and the frequencies would be higher because there would be more text on one page in a Year 8 book.

18 Girls: Mean ≈ 23.92 cm, median = 23, range = 14 cm; Boys: Mean ≈ 26.22 cm, median = 26, range = 11 cm. The sizes of the girls' feet vary more, but the boys generally have bigger feet.

19

Weekly incomes
(Key: 45|12 = $1245/week = 12|45)

Females		Males
25	6	
67	7	
78 37 24	8	
92 92 64 64 45 43 16	9	54 61 70 84
87 84	10	18 20
64 53 49 12 08	11	13 80
32 31 20 08	12	49 81 87 91 92 97
55 22	13	40 47 53
	14	08 30 69
	15	26 50
	16	19
	17	11
	18	22

While some women earn more than some men, generally men earn more than women.

20 Survey 2 is the worst because anybody, including children, could fill in the forms. Although survey 3 probably has more accurate information, the number involved in survey 1 means that it is probably the best.

Chapter 2

Exercise 2.1

1 Planes are **b**, **c**, **e** and **g**; 3D shapes are **a**, **d**, **f** and **h**

2 **a** Closed **b** Closed **c** Open **d** Closed
e Open **f** Closed **g** Closed **h** Open

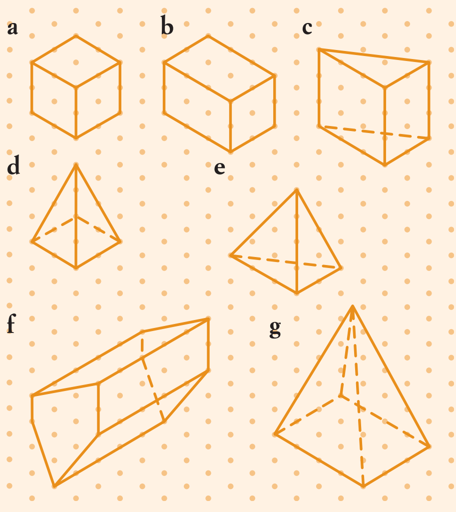
3 **i** **a** *A*, *D* **b** *A*, *B*, *Q*
c *C*, *F*, *L* **d** *M*, *D*, *R*, *P*
e *G*, *H*, *C*, *D*, *E*, *F* **f** *M*, *L*, *N*, *R*, *T*, *S*
g *G*, *H*, *K* **h** *M*, *N*, *P*, *B*

ii **a** *ACD*, *CDA*, *DAC*, *CAD*, *ADC* or *DCA*
b *BAQ*, *QBA*, etc. **d** *PMDR*, *DMPR*, etc.
e *GHCDEF* or *FEDCHG*
f *STRNLM*, *LNRTSM*, etc.

4 **a** Semi-circle **b** Scalene triangle
f Concave hexagon **g** Equilateral triangle

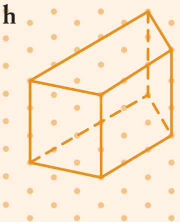
5 **b**, **f**, and **g** are polygons.

6 **a** *RKF* **b** *MNRVT* **c** *STMU*
d *ABEDCS* **e** *EFHG* **f** *PQLK*
g *TNRGHKLPMF* **h** *STAMPER*

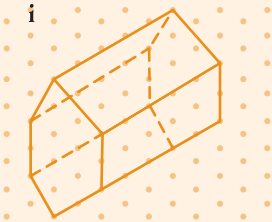
7


9780170194778

h



i



8 a **a** Sphere **b** Triangle
c Hexagon **d** Dodecahedron
e Parallelogram **f** Cone
g Semicircle **h** Triangular pyramid

b You should find circles, cylinders, rectangles, triangles and so on. Some shapes may not be perfect, for example, a rubber that was originally in the shape of a rectangular prism but now has rounded comers.

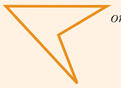
c There are many possibilities here. Check your list with those of others in the class to see who has the most.

9 A

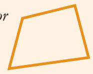
10 C

11 a Triangle
b Concave pentagon
c Convex quadrilateral
d Convex hexagon
e Regular quadrilateral (rhombus)
f Convex quadrilateral (rectangle)
g Concave decagon
h Convex heptagon

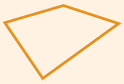
12



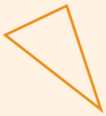
or



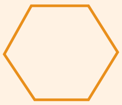
13



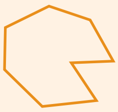
14



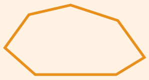
15



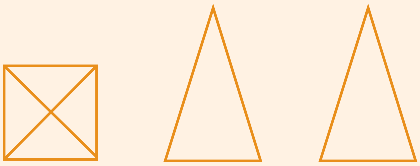
16



17

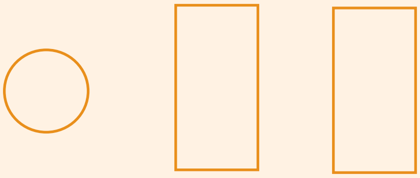


18 a



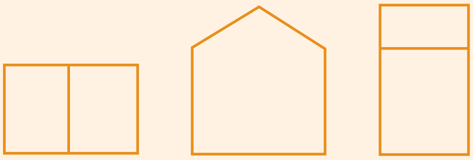
Plan Front elevation Side elevation

b



Plan Front elevation Side elevation

c

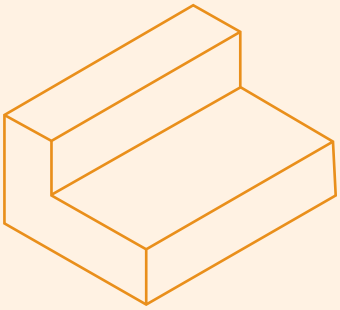


Plan Front elevation Side elevation

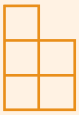
19 D

20 D

21

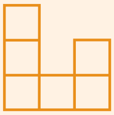


22 a



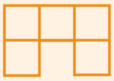
Front elevation

b



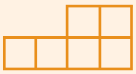
Left elevation

c



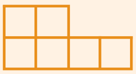
Plan

23 a



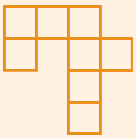
Front elevation

b



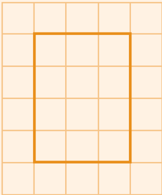
Left elevation

c



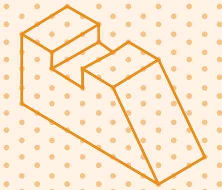
Plan

24 a

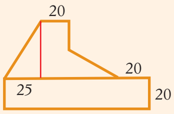


Front elevation

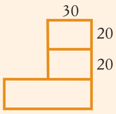
b



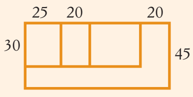
25



Left elevation

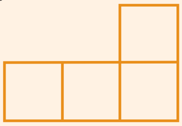


Front elevation

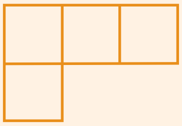


Plan

26 a



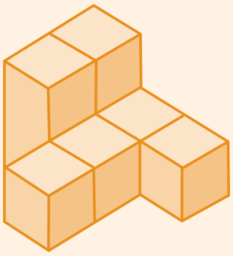
b



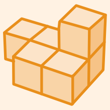
27 A cylinder whose height equals the diameter of its base.

28 A cone

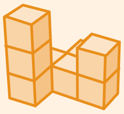
29



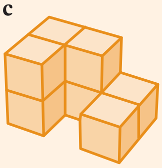
30 a



b



c



31 A polygon cannot have equal angles if one internal angle is greater than 180 degrees, so a concave polygon cannot be regular.

32 Triangles can only be convex.

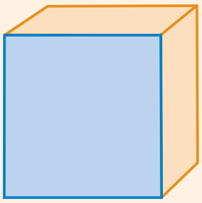
33 a Any plane section of a sphere will be a circle.

b Possible plane sections are circles (when a plane cuts parallel to the circular base), ellipses (when a plane cuts at an angle, not parallel to the circular base and not intersecting the base of the cone), parabolas (when a plane cuts parallel to the edge of the cone, not intersecting the vertex but intersecting the base), and hyperbolas (when a plane cuts perpendicular to the base, but not intersecting the vertex).

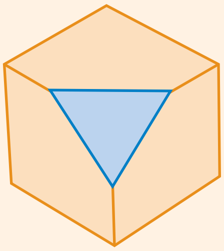
c Possible plane sections are circles (when a plane cuts parallel to the base), rectangles (when a plane cuts perpendicular to the base), and ellipses (when a plane cuts at an angle, not parallel to the circular base and not intersecting the base of the cylinder).

34 Other answers are possible.

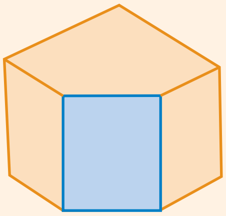
a Slice the cube parallel to one of its sides.



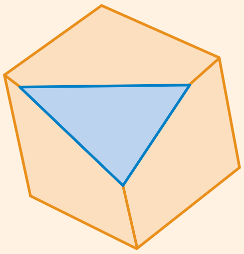
b Slice the cube with a plane that cuts through the midpoints of the three edges coming from any vertex.



c Slice the cube with a plane perpendicular to one of its faces and parallel to the four vertical edges.

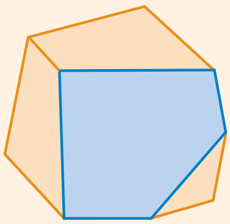


d Slice the cube so that the plane intersects three edges at different distances from the vertices.

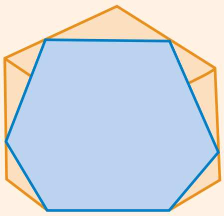


9780170194778

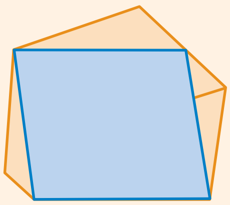
e Slice the cube with a plane going through five of the six faces of the cube.



f Slice the cube with a plane going through all six faces of the cube.



g Slice the cube with a plane from the top face to the bottom. The slice cannot be parallel to any side of the top face, and the slice must not be vertical.



Exercise 2.2

1 **a** Right-angled, scalene
b Obtuse-angled, scalene
c Acute-angled, scalene
d Obtuse-angled, isosceles
e Acute-angled, isosceles
f Acute-angled, equilateral

2 Teacher to check

3 B

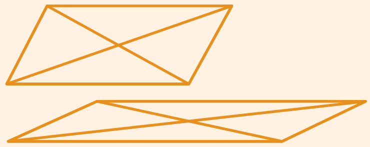
4 **a** Rectangle **b** Parallelogram
c Square **d** Parallelogram
e Kite **f** Trapezium
g Square **h** Kite
i Rectangle **j** Rhombus
k Parallelogram **l** Trapezium (trapezoid)

5 **a** Trapezium
b Parallelogram, rhombus, rectangle, square
c Trapezium, kite
d Kite, rhombus, square
e Kite, rhombus, square
f Parallelogram, rhombus, rectangle, square
g Rectangle, square

6 **a** $\triangle ABC$: $AB = 28$ mm, $BC = 54$ mm, $CA = 61$ mm; $\triangle PQR$: $PQ = 20$ mm, $QR = 47$ mm, $RP = 57$ mm; $\triangle LMN$: $LM = 50$ mm, $MN = 45$ mm, $NL = 37$ mm
b $\triangle ABC$: B, AC; $\triangle PQR$: Q, PR; $\triangle LMN$: N, LM
c $\triangle ABC$: C, AB; $\triangle PQR$: R, PQ; $\triangle LMN$: M, NL
d The sizes of the angles are in the same order as the lengths of the opposite sides. The largest angle is opposite the largest side, and so on.

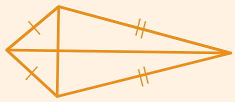
7 **a**, **d**, **e** and **f**

8 **a** and **b**

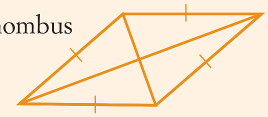


c The diagonals of a parallelogram cut each other in half.

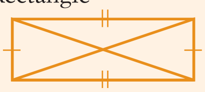
9 Kite



Rhombus



Rectangle



The diagonals of rhombuses and rectangles also cut each other in half.
The diagonals of rhombuses and kites are perpendicular.
The diagonals of a rectangle are equal.

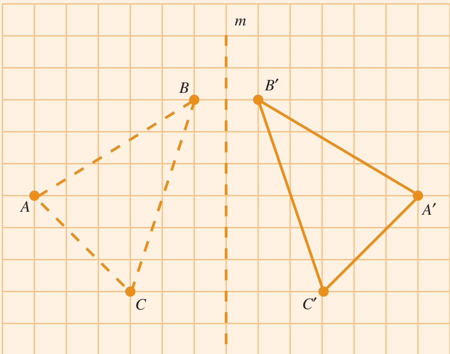
Exercise 2.3

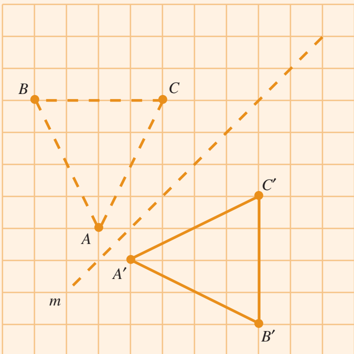
1 **a** Translation
b $X \to A$: 4 up; $X \to B$: 5 right, 2 up; $X \to C$: 2 right, 2 down; $X \to D$: 4 left, 2 down; $X \to E$: 5 left, 3 up

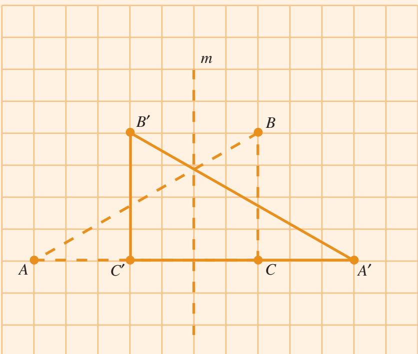
c i 6 right ii 2 left, 6 up
iii 10 right, 1 down iv 4 left, 6 down
v 4 right, 2 up

2 a 90° clockwise about P
b 90° clockwise about Q
c 90° clockwise about C
d 270° clockwise about Q
e 180° clockwise about R
f 90° clockwise about A
(Note: Could also be written as anticlockwise rotations where 90° clockwise = 270° anticlockwise, 180° clockwise = 180° anticlockwise, 270° clockwise = 90° anticlockwise.)

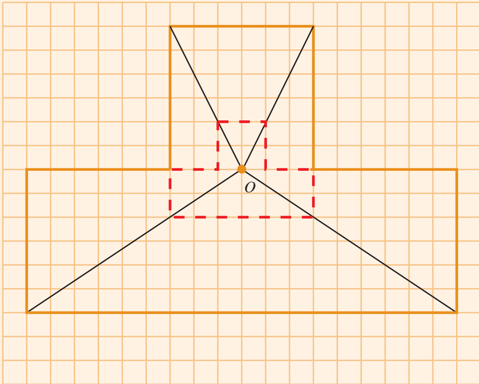
3



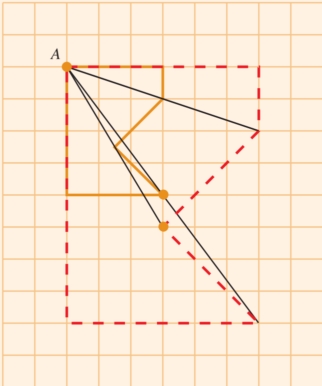




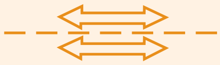
4



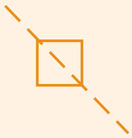
5



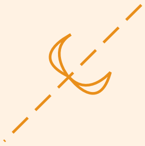
6 a



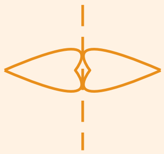
b



c

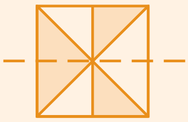


d

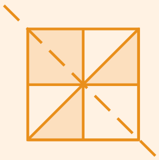


7 a Translation b Rotation c Reflection
d Reflection e Rotation f Translation

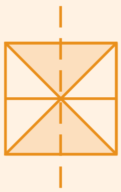
8 a



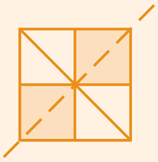
b



c



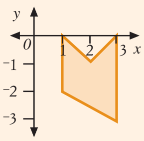
d

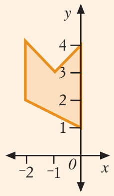


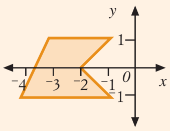
9780170194778

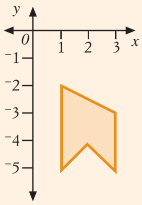
9 **a** $A(1, 1) \rightarrow A'(1, 0)$; $B(1, 2) \rightarrow B'(1, 1)$
b $A(1, 1) \rightarrow A'(0, 1)$; $B(1, 2) \rightarrow B'(0, 2)$
c $A(1, 1) \rightarrow A'(0, 1)$; $B(1, 2) \rightarrow B'(1, 1)$
d $A(1, 1) \rightarrow A'(1, 1)$; $B(1, 2) \rightarrow B'(1, 2)$

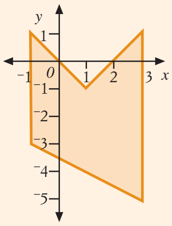
10 **a** Translation **b** Rotation **c** Rotation
d Reflection **e** Enlargement

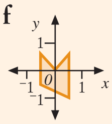
11 **a**


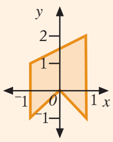
b


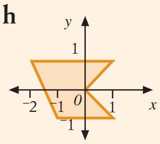
c


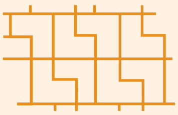
d


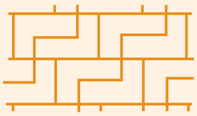
e


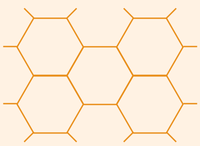
f


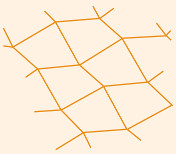
g


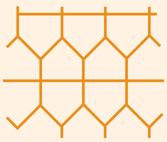
h


12 **a**


b


c


d


e


Exercise 2.4

1 **a** $ABCD \equiv XYZW$ **b** $\triangle XYZ \equiv \triangle IJH$
c $CEFA \equiv JKML$ **d** $THYUER \equiv MKLGJN$

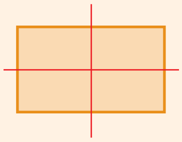
2 **a** $DEFG \; ||| \; WVXY$ **b** $\triangle WGE \; ||| \; \triangle HJU$
c $CDEA \; ||| \; STPR$

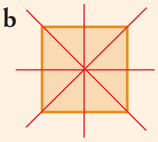
3 **a** ≡ **g**; **b** ≡ **j**; **i** ≡ **l**

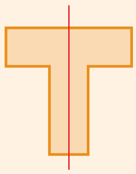
4 **a** **N** **b** **G** and **P** **c** **K** and **R**

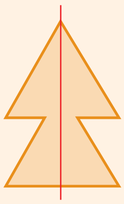
5 **a** $\angle A = \angle X$, $BC = YZ$, $\angle BCA = \angle YZX$, $AC = XZ$
b $\angle R = \angle N$, $\angle YTR = \angle VMN$, $RU = NB$, $TY = MV$
c $QR = LK$, $RS = KJ$, $\angle QRS = \angle LKJ$, $\angle S = \angle J$
d $LK = PE$, $\angle DHK = \angle WRE$, $\angle G = \angle U$, $DH = WR$

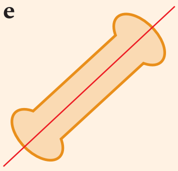
6 **i** All shapes have line symmetry
ii **a**, **b**, **e**, **f**, **g**, **h**, **j** and **l** have rotational symmetry

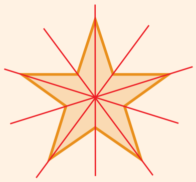
7 **a**


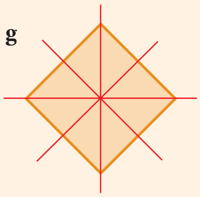
b


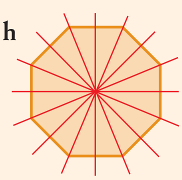
c


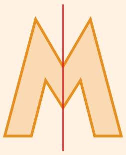
d


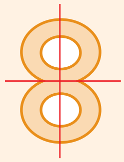
e


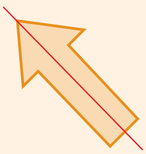
f


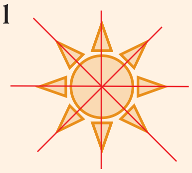
g


h


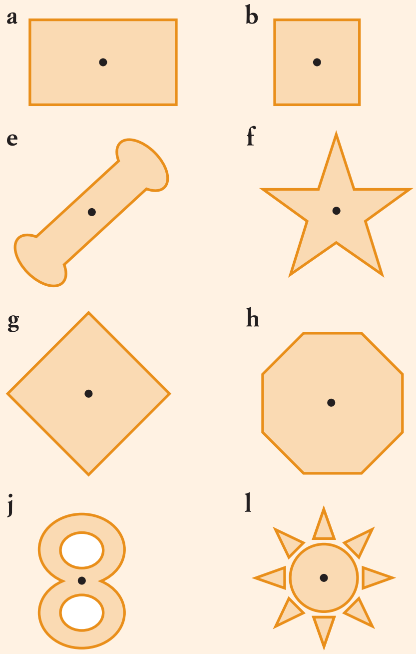
i


j


k


l


8 **i**



ii **a** 2, **b** 4, **f** 5, **g** 4, **h** 8, **j** 2 and **l** 8

9 **a** $\mathrel{|||}$ **e** $\mathrel{|||}$ **g**; **b** $\mathrel{|||}$ **d** $\mathrel{|||}$ **j**; **c** $\mathrel{|||}$ **f**; **h** $\mathrel{|||}$ **k**; **i** $\mathrel{|||}$ **l** $\mathrel{|||}$ **m**

10 **a** 1.5 **b** 2 **c** 1.7

11 **a** **i** $\angle A \leftrightarrow \angle K$, $\angle S \leftrightarrow \angle G$, $\angle D \leftrightarrow \angle H$, $\angle F \leftrightarrow \angle J$ or $\angle A \leftrightarrow \angle H$, $\angle S \leftrightarrow \angle J$, $\angle D \leftrightarrow \angle K$, $\angle F \leftrightarrow \angle G$, etc.
ii $AS \leftrightarrow KG$, $SD \leftrightarrow GH$, etc. or $AS \leftrightarrow HJ$, $SD \leftrightarrow JK$, etc.
iii 3 : 2 or 1.5
b Not similar
c **i** Any angle from the first figure corresponds to any angle from the second figure
ii Any side from the first figure corresponds to any side from the second figure
iii 5: 4 or 1.25
d **i** $\angle G \leftrightarrow \angle N$, $\angle C \leftrightarrow \angle J$, $\angle F \leftrightarrow \angle M$, $\angle V \leftrightarrow \angle K$, $\angle D \leftrightarrow \angle L$
ii $GC \leftrightarrow NJ$, $CF \leftrightarrow JM$, $FV \leftrightarrow MK$, $VD \leftrightarrow KL$, $DG \leftrightarrow LN$
iii 5 : 9 or $0.\overline{5}$

12 Yes

13 No

14 Yes

15 Yes

16 No, only those with the same number of sides.

17 Yes

18 No

19 **a** There are many possible shapes.
b The ratio of sides is constant for any two similar shapes. The ratio of areas is the square of the ratio of sides.

Chapter review

1 plane figures: **a**, **d**, **e**, **g** and **h**; 3D figures: **b**, **c** and **f**

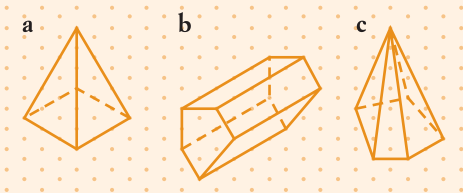
2 Open: **b**, **d** and **g**; Closed: **a**, **c**, **e**, **f** and **h**.

3 **c** M, T, S, C and D **e** S, O, P, L and F
f Z, J, W, X, P, M, T and F

4 **a** ABC **d** $TFDAY$
f $ZJWXPMTF$ **h** $YTQRPSUWL$

5 **e**, **f** and **h**

6



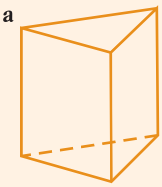
7 **a** Isosceles triangle, acute-angled triangle
b Scalene triangle, obtuse-angled triangle
c Equilateral triangle, acute-angled triangle
d Scalene triangle, acute-angled triangle

8 **a** Translation
b 4 units right (V), 3 units right and 5 units up (W), 2 units left and 4 units up (X), 2 units left and 2 units down (Y), 2 units right and 5 units down (Z)

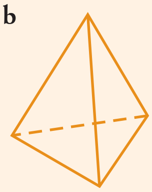
9 **a** No congruent figures **b** $\triangle MNU \equiv \triangle AEX$
c $KARL \equiv TUOB$

10 **a** $SEDR \mathrel{|||} TQAY$ **b** $\triangle MNU \mathrel{|||} \triangle FDC$
c $KARL \mathrel{|||} MIJY$

11 **a**

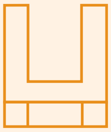


b



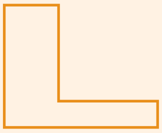
9780170194778

12 **a**



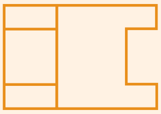
Front elevation

b



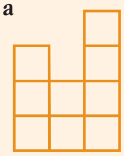
Left elevation

c



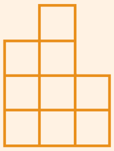
Plan

13 **a**



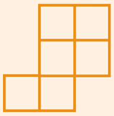
Front elevation

b



Left elevation

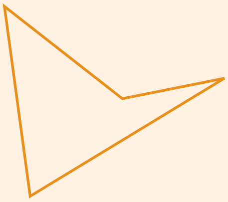
c



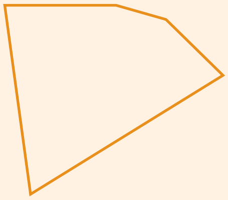
Plan

14 **e** Concave pentagon **f** Regular octagon
h Concave nonagon

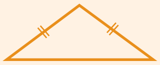
15



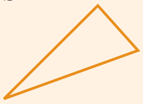
16



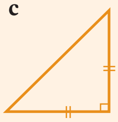
17 **a**



b

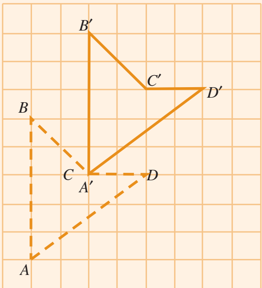


c

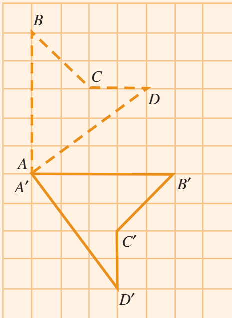


18 **a** Square **b** Kite
c Trapezium **d** Parallelogram

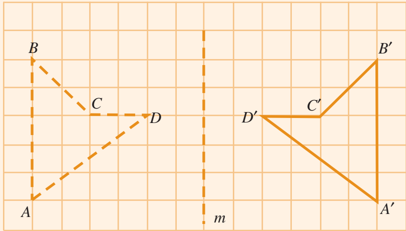
19 **a**



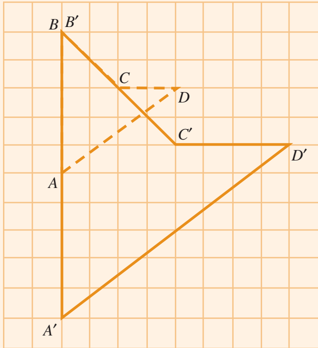
b



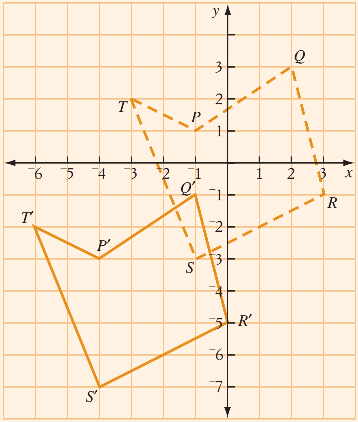
c



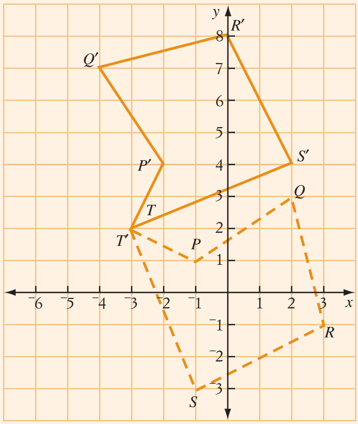
d



20 **a**

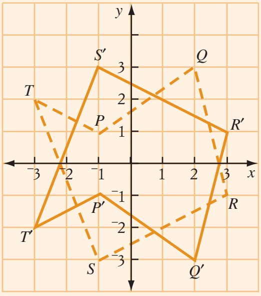


b

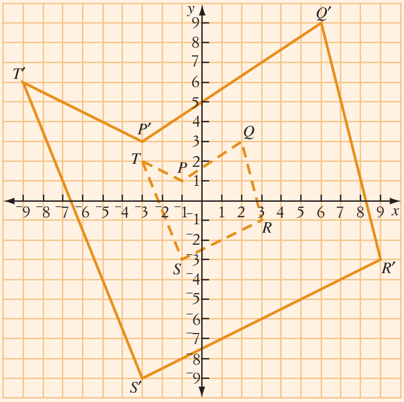


9780170194778

c



d



e $Q(2, 3) \to Q'(^-1, ^-1)$, $R(3, ^-1) \to R'(0, ^-5)$ and $S(^-1, ^-3) \to S'(^-4, ^-7)$

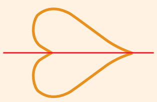
f $Q(2, 3) \to Q'(2, ^-3)$, $R(3, ^-1) \to R'(3, 1)$ and $S(^-1, ^-3) \to S'(^-1, 3)$

21 Similar figures: **k** ||| **b**; **d** ||| **e**
Congruent figures: **a** ≡ **l**; **c** ≡ **j**; **f** ≡ **h**; **g** ≡ **i**

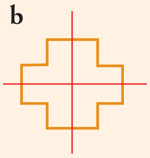
22 **a**, **b**, **c**, **d** and **e** have line symmetry.

23 **b** and **e** have rotational symmetry.

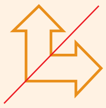
24 **a**



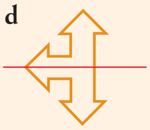
b



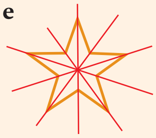
c



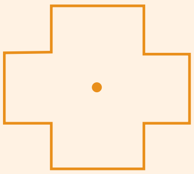
d



e

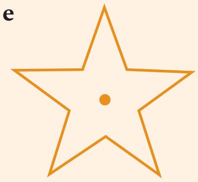


25 **i** **b**



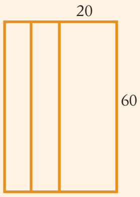
ii Order of rotational symmetry = 2

i **e**



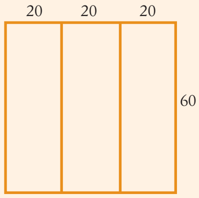
ii Order of rotational symmetry = 5

26 **a**



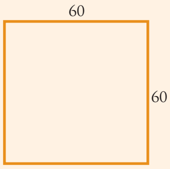
Front elevation

b



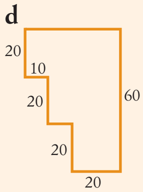
Left elevation

c



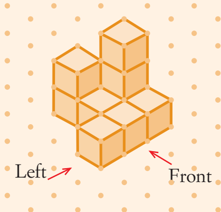
Right elevation

d



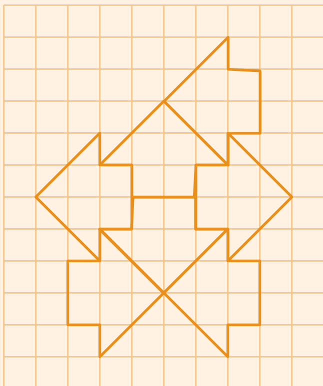
Plan

27



28 You can't draw a four-sided figure with an inward pointing corner if all sides are equal.

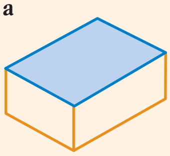
29



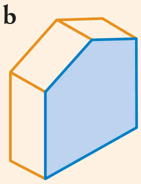
9780170194778

30 Yes

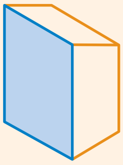
31 **a**



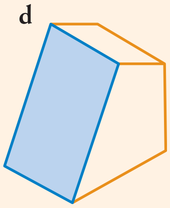
b



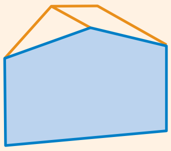
c



d



e



Chapter 3

Exercise 3.1

1 **a** $^{-}9, ^{-}7, ^{-}4, 0, 5, 6$ **b** $^{-}11, ^{-}8, ^{-}6, 3, 4, 10$
c $^{-}15, ^{-}3, ^{-}1, 10, 12, 14$

2 **a** 6 **b** 0 **c** 5

3 **a** $^{-}9$ **b** $^{-}60$ **c** $^{-}7$

4 **a** $^{-}2$ **b** 3 **c** $^{-}9$ **d** $^{-}4$
e 0 **f** $^{-}3$ **g** $^{-}20$ **h** $^{-}17$

5 **a** 12 **b** $^{-}14$ **c** $^{-}1$ **d** $^{-}11$
e $^{-}3$ **f** $^{-}7$ **g** $^{-}32$ **h** $^{-}42$

6 **a** 12 **b** $^{-}32$ **c** 35 **d** $^{-}27$
e $^{-}48$ **f** 14 **g** 8 **h** $^{-}8$

7 **a** 6 **b** 8 **c** 1 **d** $^{-}6$
e $^{-}2$ **f** $^{-}5$ **g** 0 **h** 3

8 **a** $^{-}5$ **b** $^{-}5$ **c** 8 **d** $^{-}3$
e $^{-}6$ **f** $^{-}8$ **g** $^{-}6$ **h** 4

9 **a** $^{-}24$ **b** 4 **c** 36 **d** $^{-}6$
e $^{-}9$ **f** $^{-}50$ **g** $^{-}70$ **h** 8

10 **a** 4 **b** $^{-}5$ **c** 16 **d** $^{-}3$
e $^{-}17$ **f** $^{-}15$ **g** 2 **h** $^{-}9$
i $^{-}28$ **j** $^{-}5$ **k** 45 **l** 7

11 **a** $^{-}39$ **b** $^{-}4$ **c** 33 **d** $^{-}14$
e 64 **f** $^{-}150$ **g** $^{-}1440$ **h** 175

12 **a** 84 **b** 9 **c** 51 **d** 35
e 23 **f** 36 **g** 17 **h** 87
i 21 **j** 5

13 $\$(^{-}53 + {}^{-}74 + 28 + 52 + 19) = {}^{-}\$28$.
She owes \$28.

14 $120 - {}^{-}40 = 160$. It dives 160 cm.

15 72 cm

16 **a** 0 km/h **b** 60 km/h south
c 40 km/h north **d** 70 km/h south

17 **a** Juan 130 km west, Alison 150 km east
b 280 km
c Juan 195 km east, Alison 225 km west
d 420 km

18 **a** 3 floors/min **b** $^{-}4$ floors/min
c $^{-}4$ floors/min **d** 2 floors/min

Exercise 3.2

1 **a** 64 **b** 64 **c** 2401
d 25 **e** 729 **f** 4096
g 196 **h** 1331 **i** 125

2 **a** 13 824 **b** 20 736 **c** 1024
d 4913 **e** 59 049 **f** 21 952
g 14 884 **h** 110 592 **i** 40 353 607

3 **a** 7 **b** 4 **c** 20 **d** 9 **e** 6
f 11 **g** 14 **h** 13 **i** 10

4 **a** About 22.36 **b** About 2.236
c About 7.071 **d** About 5.477
e About 17.32 **f** About 54.77
g About 4.123 **h** About 4.243
i About 9.747

5 **a** 4^5 **b** 5^7 **c** 7^7 **d** 8^8
e 9^8 **f** 6^{11}

6 **a** 4^1 **b** 5^3 **c** 7^1 **d** 8^2
e 9^6 **f** 5^4

7 **a** 3^6 **b** 5^9 **c** 6^8 **d** 5^4
e 7^{12} **f** $8^0 = 1$

8 **a** $3^2 \times 5^2$ **b** $4^3 \times 3^3$ **c** $3^5 \times 7^5$
d $3^6 \times 4^6$ **e** $5^4 \times 8^4$ **f** $1^8 \times 7^8$

9 **a** $3^4 \times 5^6$ **b** $2^{12} \times 3^6$ **c** $5^8 \times 6^4$
d $7^3 \times 5^{12}$ **e** $11^{16} \times 9^8$ **f** $5^{25} \times 4^{20}$

10 **a** 6^5 **b** 7^{10} **c** 5^{17} **d** 8^{21} **e** 5^{15}
f 6^{51} **g** 3^{21} **h** 5^{12} **i** 2^{57}

11 **a** 5^5 **b** 6^{13} **c** 7^3 **d** 3^{12}
e 7^1 **f** $3^0 = 1$

12 **a** 7^6 **b** 5^{12} **c** 9^{15}
d 5^{40} **e** 3^{48} **f** 3^{99}

13 **a** $4^2 \times 5^2$ **b** $3^5 \times 7^5$ **c** $6^{15} \times 7^{15}$
d $5^{16} \times 4^{16}$ **e** $5^{40} \times 8^{40}$ **f** 10^{11} or $1^{11} \times 10^{11}$

14 **a** $3^{18} \times 7^6$ **b** $4^{24} \times 5^8$ **c** $3^{60} \times 4^{20}$
d $2^{56} \times 3^{16}$ **e** $5^9 \times 7^{108}$ **f** $3^{99} \times 7^{36}$

15 **a** $\frac{1}{36}$ **b** $\frac{1}{2}$ **c** $\frac{1}{64}$ **d** $\frac{1}{7}$ **e** 1
f 1 **g** $\frac{1}{34}$ **h** $\frac{1}{1000}$ **i** $\frac{1}{128}$

16 **a** 1 **b** About 0.001 041
c About 0.000 204 **d** 0.000 125
e About 0.000 031 **f** 0.0016
g 0.000 008 **h** About 0.000 977
i 0.0016

17 **a** 2 **b** 4 **c** 9 **d** 2 **e** 6
f 4 **g** 10 **h** 100 **i** 2

18 **a** About 6.694 **b** About 2.943
c About 2.856 **d** About 1.817
e About 1.565 **f** About 1.431
g About 1.601 **h** About 2.154
i About 2.432

19 **a** $8^3 = 512 = 4^3 \times 2^3$
b $15^4 = 50\,625 = 5^4 \times 3^4$
c $63^2 = 3969 = 7^2 \times 9^2$

20 **a** 2.449, 1.817, 1.565, 1.431, 1.348
b 3.606, 2.351, 1.899, 1.670, 1.533
c It gets smaller, but remains more than 1 (for whole numbers).

Exercise 3.3

1 **a** 1000 **b** 5000 **c** 100 000 or 1.0×10^5
d 800 **e** 10 000 **f** 4000
g 4000 **h** 200 000 or 2.0×10^5

2 **a** 20 **b** 4 **c** 100 000
d 400 **e** 0 **f** 4
g 600 **h** 30 000 000 **i** 20
j 10 000

3 **a** 7×10^3 **b** 5×10^{-2} **c** 4×10^7
d 9×10^1 **e** 5×10^{-6} **f** 3.2×10^3
g 6.1×10^5 **h** 5.6×10^6 **i** 5.76×10^{-4}
j 6.54×10^{-7} **k** 2.4×10^4 **l** 8.75×10^{-2}

4 **a** 6000 **b** 0.000 08
c 10 000 000 **d** 0.3
e 2800 **f** 0.000 006 2
g 376 000 000 **h** 0.090 89
i 4401 **j** 0.000 000 006 789
k 3200 **l** 0.000 175

5 **a** 2.88×10^{10} **b** 2.4×10^{-11}
c 1.755×10^{13} **d** 3.042×10^{-19}
e 2×10^{11} **f** 1.9×10^{-18}
g 8.95×10^{15} **h** $2.497\,138 \times 10^{-1}$
i About 2.778×10^{20}

6 **a** About \$5.7 billion or \$5.7 $\times 10^9$
b About \$3.1 billion or \$3.1 $\times 10^9$
c About \$8.8 billion or \$8.8 $\times 10^9$
d About \$458 billion or \$4.6 $\times 10^{11}$

7 **a** 3.05×10^{10}
b The answer is the same.
c The answers are not the same because of rounding on the calculator.
d Any additions where the place values are different by more than the display length will be done incorrectly on the calculator because of rounding.

8 **a** About 4.6268×10^6 or 4.6 million visitors
b About 9.052×10^6 or 904 thousand visitors

For questions 9–15, answers should be within 20 times the answers given.

9 About 324 000 or 3.24×10^5 children

10 About 1800 million or 1.8×10^9 litres

11 About 1400 million or 1.4×10^9 litres

12 About 400 000 or 4.0×10^5 cars

13 About 25 000 newspapers

14 About \$900 000 or \$9.0 $\times 10^5$

15 About 40 000 000 or 4.0×10^7 km

Chapter review

1 **a** 125 **b** 2401 **c** 5
d About 7.746

2 **a** 90 000 **b** 2100

3 **a** 3 **b** $^{-}13$ **c** $^{-}13$ **d** 10 **e** 2
f $^{-}30$ **g** 28 **h** $^{-}14$ **i** 12 **j** 6

4 **a** 7^8 **b** 11^5 **c** 5^6

9780170194778

5 **a** 5^{13} **b** 4^{9} **c** 3^{48}

6 **a** 200 **b** 6000

7 $3^{24} \times 5^{8}$

8 **a** $^{-}54$ **b** 672

9 **a** $\frac{1}{81}$ **b** 1 **c** $\frac{1}{6}$ **d** 7

e 2

10 **a** 3.24×10^{3} **b** 6.5×10^{-4} **c** 6.34×10^{8}

11 **a** 0.000 005 67 **b** 47.82 **c** 400 000

12 **a** 2.6732×10^{28} **b** 4.72×10^{-17}

13 8 m down

14 540 000 parts

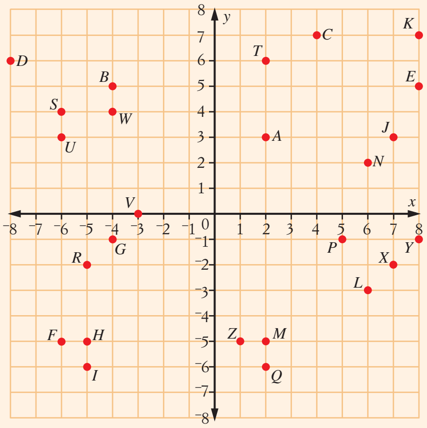
15 2 m/s, 23 m off the ground

16 3.75×10^{16}

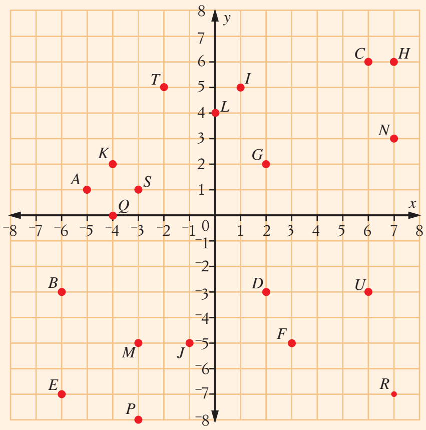
Chapter 4

Exercise 4.1

1

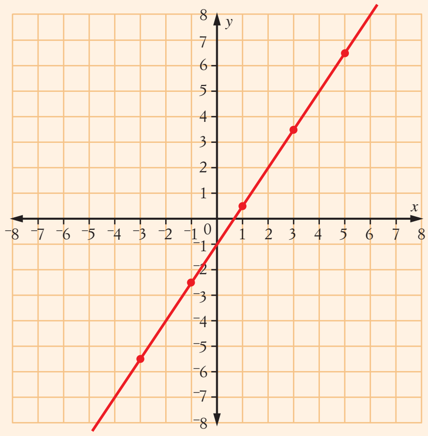


2

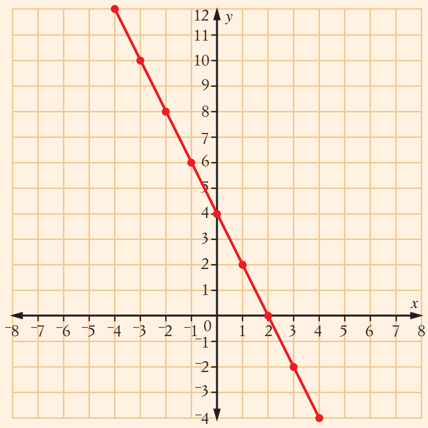


$A(^{-}5, 1)$, 2nd quadrant, $B(^{-}6, ^{-}3)$, 3rd quadrant, $C(6, 6)$, 1st quadrant, $D(2, ^{-}3)$, 4th quadrant, $E(^{-}6, ^{-}7)$, 3rd quadrant, $F(3, ^{-}5)$, 4th quadrant, $G(2, 2)$, 1st quadrant, $H(7, 6)$, 1st quadrant, $I(1, 5)$, 1st quadrant, $J(^{-}1, ^{-}5)$, 3rd quadrant, $K(^{-}4, 2)$, 2nd quadrant, $L(0, 4)$, y-axis, $M(^{-}3, ^{-}5)$, 3rd quadrant, $N(7, 3)$, 1st quadrant, $P(^{-}3, ^{-}8)$, 3rd quadrant, $Q(^{-}4, 0)$, x-axis, $R(7, ^{-}7)$, 4th quadrant, $S(^{-}3, 1)$, 2nd quadrant, $T(^{-}2, 5)$, 2nd quadrant, $U(6, ^{-}3)$, 4th quadrant.

3 The shape is a straight line passing through the axes at $(^{-}1, 0)$ and $(0, \frac{1}{2}, 0)$.

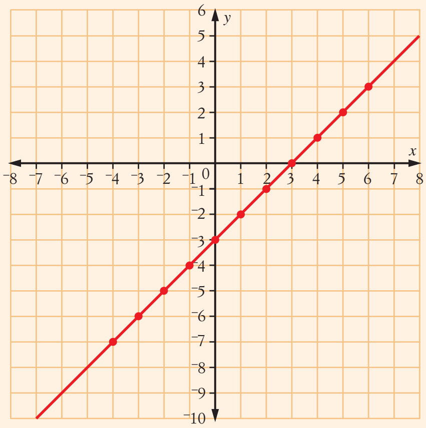


4 The shape is a straight line passing through the axes at (0, 4) and (2, 0).

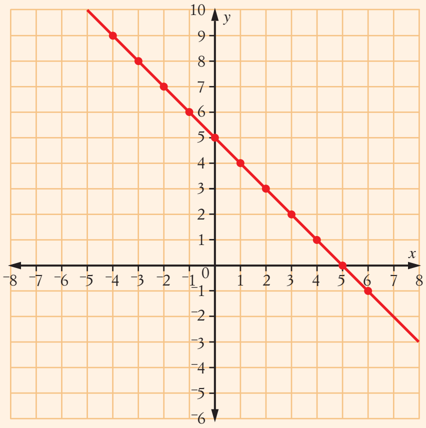


9780170194778

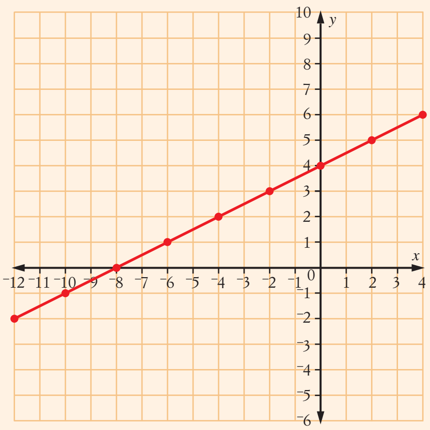
5 The shape is a straight line passing through the axes at (0, $^{-}3$) and (3, 0).



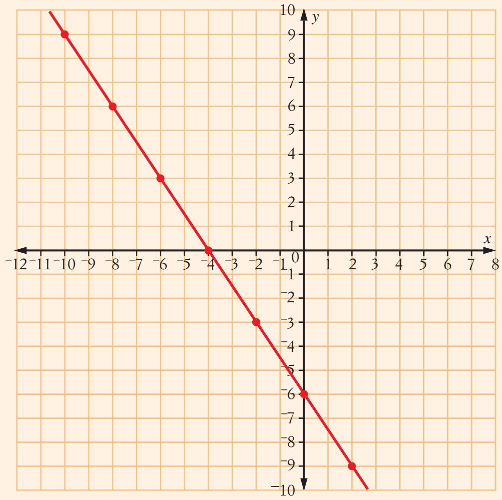
6 The shape is a straight line passing through the axes at (5, 0) and (0, 5).



7 The shape is a straight line passing through the axes at ($^{-}8$, 0) and (0, 4).

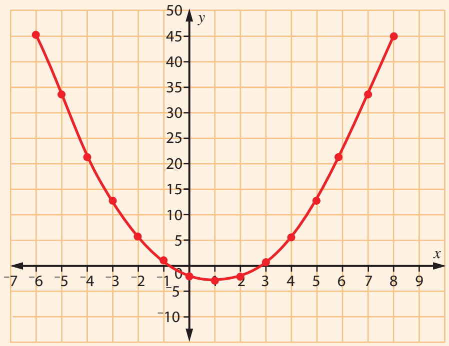


8 The shape is a straight line passing through the axes at (0, $^{-}4$) and (4, $^{-}6$).

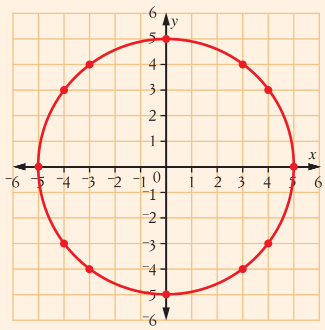


9 A(7, $^{-}3$), 4th quadrant, B(5, 6), 1st quadrant, C(4, 8), 1st quadrant, D($^{-}3$, $^{-}1$), 3rd quadrant, E(1, 2), 1st quadrant, F(6, 3), 1st quadrant, G($^{-}5$, 2), 2nd quadrant, H(8, $^{-}6$), 4th quadrant, I($^{-}5$, 7), 2nd quadrant, J(7, $^{-}8$), 4th quadrant, K(6, $^{-}7$), 4th quadrant, L(2, $^{-}7$), 4th quadrant, M(3, $^{-}7$), 4th quadrant, N(3, 0), x-axis, P($^{-}5$, $^{-}7$), 3rd quadrant, Q(1, 5), 1st quadrant, R($^{-}2$, 7), 2nd quadrant, S(5, $^{-}7$), 4th quadrant, T($^{-}2$, $^{-}7$), 3rd quadrant, U(0, $^{-}5$), y-axis, V($^{-}5$, 3), 2nd quadrant, W($^{-}1$, 3), 2nd quadrant, X($^{-}1$, 6), 2nd quadrant, Y(5, $^{-}5$), 4th quadrant, Z($^{-}1$, 2), 2nd quadrant

10 The shape of the line is a parabola.

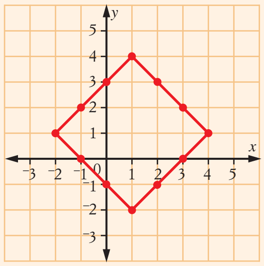


11 The shape of the line is a circle.

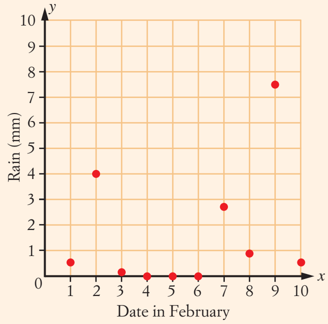


9780170194778

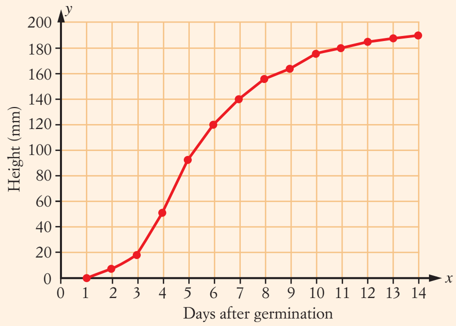
12 The shape is a diamond (square turned 45°).



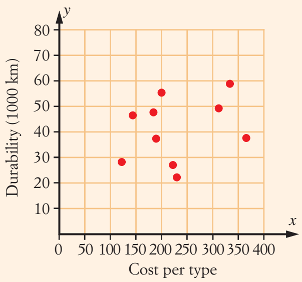
13 Apart from a couple of days, the beginning of February was dry (less than 3 mm of rain) on the Gold Coast.



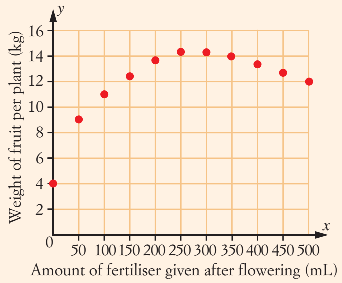
14 The seedlings grow very quickly to start with, and then the growth slows down.



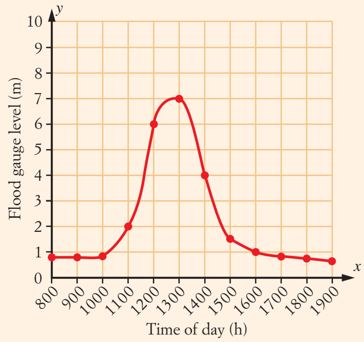
15 There does not appear to be any pattern in the cost of tyres and how long they last.



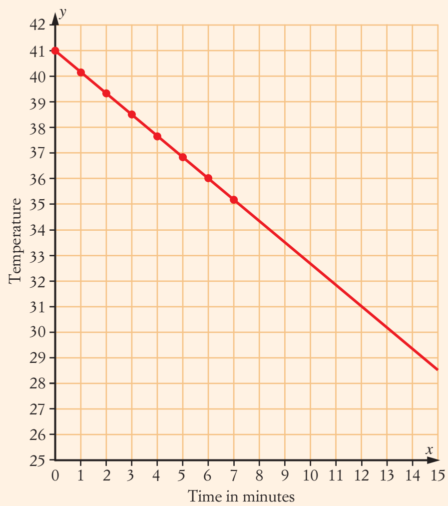
16 Fertiliser increases the quantity of tomatoes, but too much fertiliser makes the amount go down again. The ideal amount seems to be about 250 mL of fertiliser, but tomato growers would have to compare the cost of fertiliser to the quantity of extra tomatoes obtained.



17 The creek was in flood between about 1115 h and 1430 (11:15 a.m. and 2:30 p.m.). It was a flash flood, which rose and fell quickly.

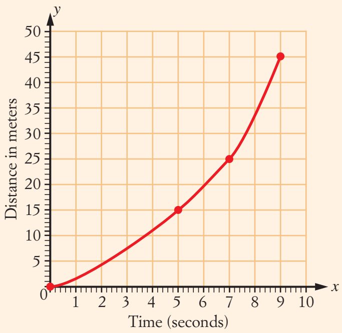


18 If the temperature continues to fall at the same rate, then it will drop below 30°C about $13\frac{1}{2}$ minutes after it has been switched on. It will not fall to 0°C because the temperature eventually will start to fall more slowly, and after a while it will stop falling altogether.

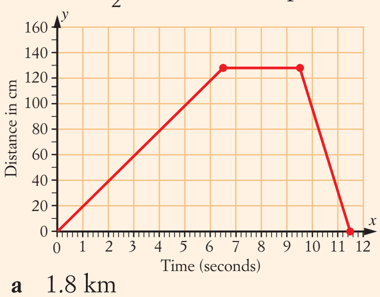


Exercise 4.2

1 It takes about $8\frac{1}{2}$ seconds to travel 40 m.



2 It takes $6\frac{1}{2}$ seconds to creep to the right spot.



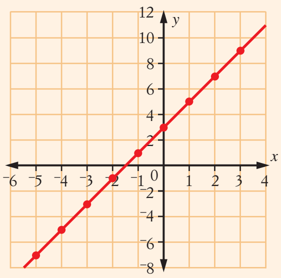
3 **a** 1.8 km
b 5 minutes
c She walked quickly to her friend's house 800 m away, waited 5 minutes for her, and then they walked the rest of the way to school.

4 **a** 2 hours
b $2\frac{1}{2}$ hours
c About 67 minutes after they started.
d 200 km

5 **a**

x	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}7$	$^{-}5$	$^{-}3$	$^{-}1$	1	3	5	7	9

b

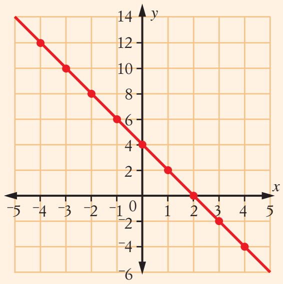


c It is a linear function passing through the axes at $(^{-}1\frac{1}{2}, 0)$ and (0, 3).

6 **a**

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4
y	12	10	8	6	4	2	0	$^{-}2$	$^{-}4$

b

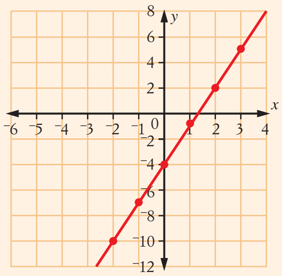


c It is a linear function passing through the axes at (0, 4) and (2, 0).

7 **a**

x	$^{-}2$	$^{-}1$	0	1	2	3
y	10	$^{-}7$	$^{-}4$	$^{-}1$	2	5

b

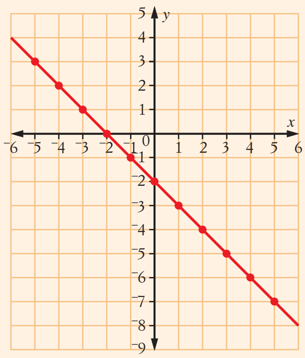


c It is a linear function passing through the axes at $(^{-}4, 0)$ and $(1\frac{1}{3}, 0)$.

8 **a**

x	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5
y	3	2	1	0	$^{-}1$	$^{-}2$	$^{-}3$	$^{-}4$	$^{-}5$	$^{-}6$	$^{-}7$

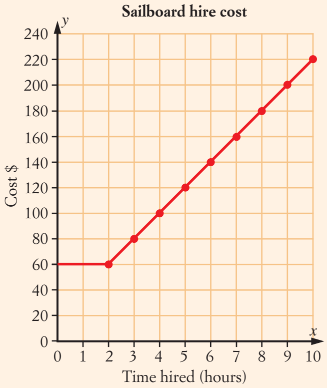
b



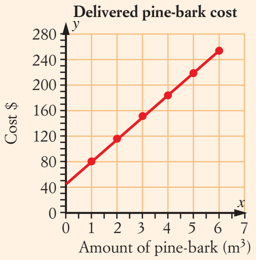
c It is a linear function passing through the axes at $(^{-}2, 0)$ and $(0, ^{-}2)$.

9780170194778

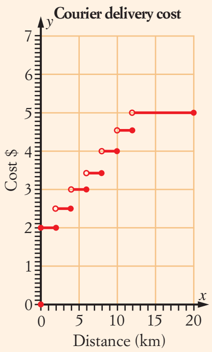
9 $h = 60 + 20(h - 2)$, 5 hours



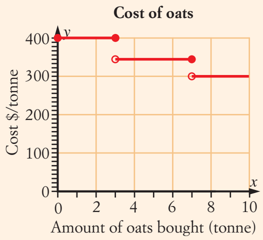
10 $c = 35p + 45$, $202.50



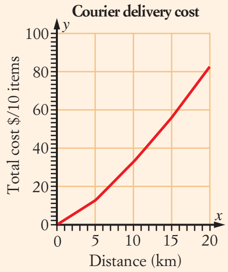
11 It looks like steps going up, $26



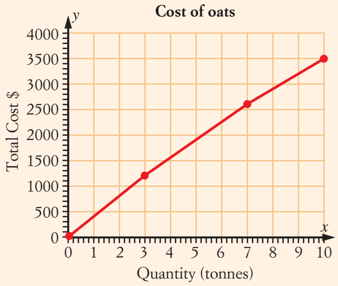
12 It looks like steps going down, $3200



13 For different distances, you need to calculate the cumulative cost of delivery to that distance to make a new graph. The graph looks like a sloping line that bends upwards to slope more steeply at each change of pricing until the price increases stop. It then goes up with the same slope.

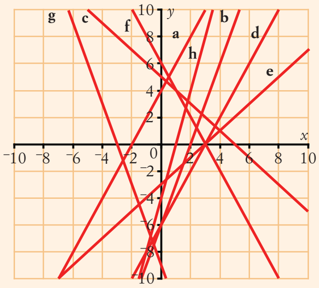


14 For different amounts of oats, you need to calculate the cumulative cost of that amount of oats to make a new graph. The graph looks like a sloping line that becomes less steep at each change of pricing.



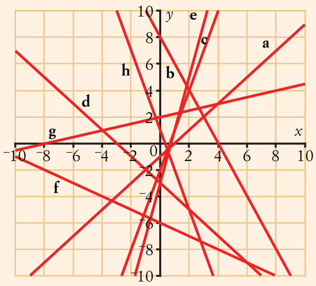
Exercise 4.3

1



a x-intercept $= {}^{-}2$, y-intercept $= 4$
b x-intercept $= 2$, y-intercept $= {}^{-}6$
c x-intercept $= 5$, y-intercept $= 5$
d x-intercept $= {}^{-}3$, y-intercept $= {}^{-}6$
e x-intercept $= 3$, y-intercept $= {}^{-}3$
f x-intercept $= 3$, y-intercept $= 6$
g x-intercept $= 3$, y-intercept $= {}^{-}9$
h x-intercept $= 1$, y-intercept $= {}^{-}4$

2



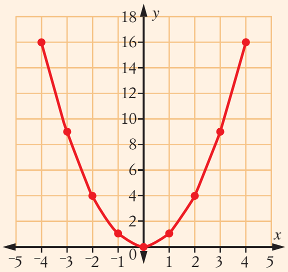
9780170194778

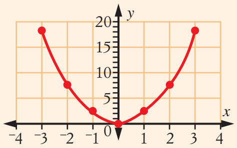
Answers

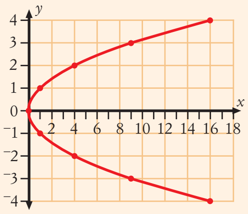
a 1 **b** $^-2$ **c** 3 **d** $^-1$
e $^-3$ **f** $\frac{1}{2}$ **g** $\frac{1}{4}$ **h** $^-3$

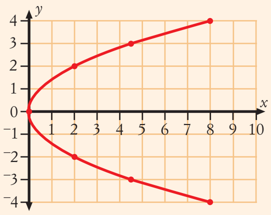
3 **a** $(2\frac{1}{2}, 4)$ **b** $(^-5, -\frac{1}{2})$ **c** $(^-6\frac{1}{2}, ^-6\frac{1}{2})$
d $(6, 3)$ **e** $(4, ^-6\frac{1}{2})$ **f** $(^-1\frac{1}{2}, ^-5\frac{1}{2})$
g $(^-1, 1\frac{1}{2})$ **h** $(0, -2\frac{1}{2})$

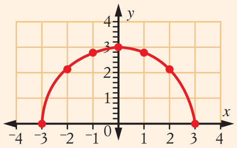
4 **a** $(^-9, ^-4)$ **b** $(\frac{1}{2}, -1\frac{1}{2})$ **c** $(^-\frac{1}{2}, 8\frac{1}{2})$
d $(^-7, ^-2)$ **e** $(2, ^-6\frac{1}{2})$ **f** $(^-1\frac{1}{2}, ^-\frac{1}{2})$
g $(^-1\frac{1}{2}, 3\frac{1}{2})$ **h** $(3, 0)$

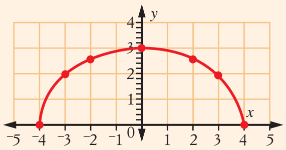
5


6


7


8


9


10


11 $(3, ^-7)$

12 $(0, 4)$

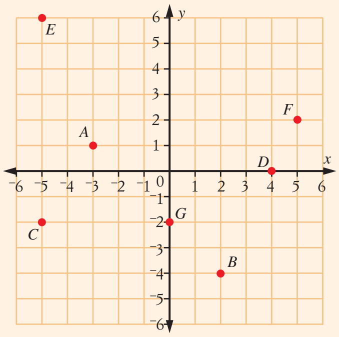
13 $a = 7, b = ^-4$

14 $a = 8, b = ^-7$

15 Find the rise as the difference between the y-coordinates and the run as the difference between the x-coordinates. Then find the quotient.

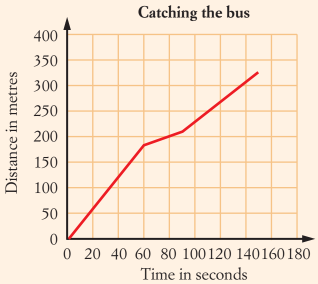
16 If it has $y = \boxed{?}\, x^2$ it will be vertical but if it has $x = \boxed{?}\, y^2$ it will be horizontal.

Chapter review

1


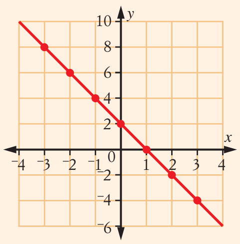
$A(^-3, 1)$ 2nd quadrant, $B(2, ^-4)$ 4th quadrant, $C(^-5, ^-2)$ 3rd quadrant, $D(4, 0)$ x-axis, $E(^-5, 6)$ 2nd quadrant, $F(5, 2)$ 1st quadrant, $G(0, ^-2)$ y-axis

2 He ran 300 m and walked 30 m.



3 **a** x-intercept 6, y-intercept 6
b x-intercept $^-3\frac{1}{2}$, y-intercept 7
c x-intercept 2, y-intercept 4
d x-intercept $^-2$, y-intercept $^-6$

4 The shape is a straight line passing through $(0, 2)$ and $(1, 0)$.



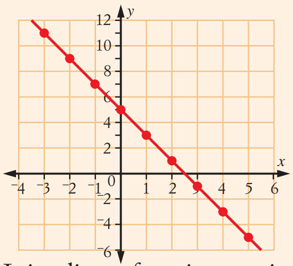
9780170194778

5 $A(^{-}5, {}^{-}1)$ 3rd quadrant, $B(2, 1)$ 1st quadrant, $C(^{-}3, 0)$ x-axis, $D(^{-}4, 1)$ 2nd quadrant, $E(5, {}^{-}3)$ 4th quadrant, $F(0, 6)$ y-axis, $G(^{-}3, {}^{-}5)$ 3rd quadrant, $H(7, {}^{-}1)$ 4th quadrant

6 a

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5
y	11	9	7	5	3	1	$^{-}1$	$^{-}3$	$^{-}5$

b



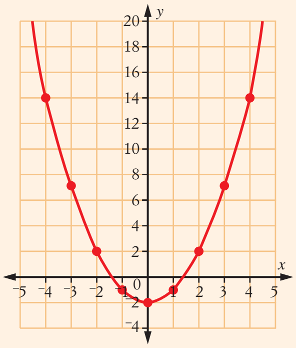
c It is a linear function passing through the axes at (0, 5) and (3, 0), with a negative gradient.

7 a 2 **b** $^{-}3$ **c** $\frac{1}{2}$ **d** $^{-}1$

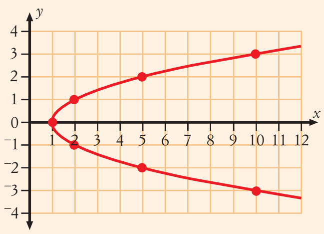
8 a (4, 5) **b** $(^{-}5, \frac{1}{2})$

9 a $(^{-}3, 1)$ **b** $(^{-}1, 1\frac{1}{2})$

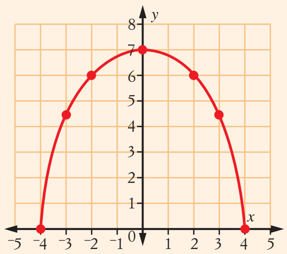
10



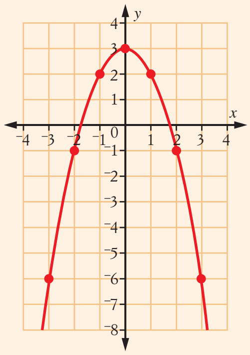
11



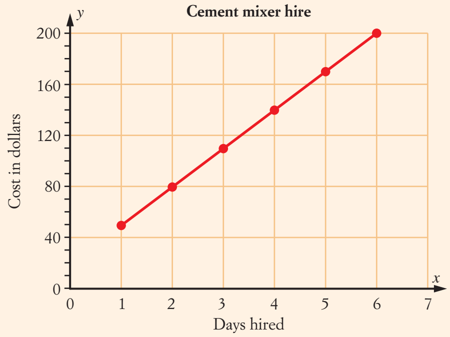
12



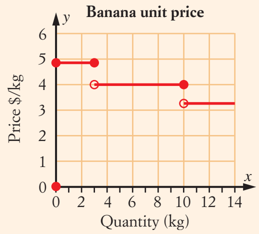
13 The shape is an upside-down parabola.



14 $c = 50 + 30(d - 1)$. $4\frac{2}{3}$ days or 4 days with \$20 left over if they don't have part-day hire.

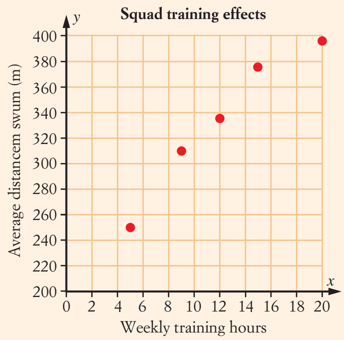


15 It looks like steps going down. \$36.



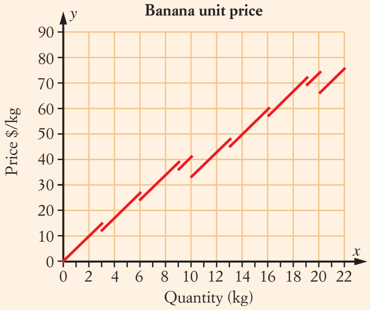
16 $(3, {}^{-}7)$

17 More training seems to increase swimming performance, but it looks as if it might start to level off after a while. More data is needed to be sure.



9780170194778

18 Below 3 kg, the quantity would be multiplied by \$4.90. At 3 kg the price would drop to \$12. Between 3kg and 6 kg, the price would be \$12 plus \$4.90 times the amount over 3 kg. A similar thing happens between 6 kg and 9 kg, between 9 kg and 10 kg, between 10 kg and 12 kg, and between 12 kg and 15 kg, etc. It is actually cheaper to buy 3 kg, 6 kg, 10 kg, etc than to buy over 2.45 kg, over 5.45 kg, over 7.84 kg, etc and pay the loose price (or the price for 3 bags).



19 Use (say) $x^2 + y^2 = 16$ and plot from $x = {}^-4$ to 4. Then use $y = x^2$ (say) and compare.

Chapter 5

Exercise 5.1

1 **a** Two-fifths, $\frac{2}{5}$ **b** Five-eighths, $\frac{5}{8}$
c Seven-twelfths, $\frac{7}{12}$ **d** Nine twentieths, $\frac{9}{20}$
e A half, $\frac{1}{2}$ **f** Two-thirds, $\frac{2}{3}$
g Eight-twelfths, $\frac{8}{12}$ **h** A quarter, $\frac{1}{4}$
i Six-thirteenths, $\frac{6}{13}$

2 **a** $\frac{7}{2}$ **b** $\frac{5}{3}$ **c** $\frac{13}{3}$ **d** $\frac{25}{4}$
e $\frac{23}{4}$ **f** $\frac{7}{5}$ **g** $\frac{23}{5}$ **h** $\frac{41}{8}$

3 **a** $3\frac{2}{5}$ **b** $2\frac{1}{2}$ **c** $4\frac{3}{4}$ **d** $2\frac{2}{3}$
e $5\frac{1}{2}$ **f** $3\frac{1}{4}$ **g** $5\frac{3}{5}$ **h** $2\frac{6}{7}$

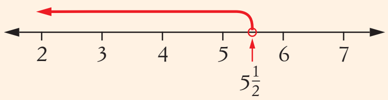
4 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{2}{5}$ **d** $\frac{1}{6}$
e $\frac{1}{3}$ **f** $\frac{3}{4}$ **g** $\frac{2}{3}$ **h** $\frac{1}{5}$
i $\frac{3}{8}$ **j** $\frac{1}{12}$ **k** $\frac{4}{5}$ **l** $\frac{5}{12}$

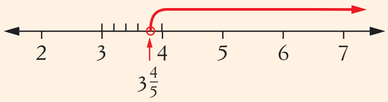
5 **a** 0.5 **b** 0.25 **c** 0.4 **d** 0.375
e 0.75 **f** 0.8 **g** 0.18 **h** 0.18
i 0.34 **j** 0.28 **k** 0.76 **l** 0.35

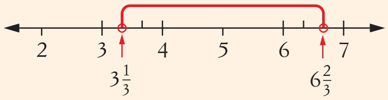
6 **a** $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$
b $\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{10}{15} = \frac{16}{24} = \frac{24}{36}$
c $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{16}{40} = \frac{40}{100}$
d $\frac{1}{3} = \frac{3}{9} = \frac{5}{15} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$
e $\frac{4}{5} = \frac{12}{15} = \frac{16}{20} = \frac{80}{100} = \frac{32}{40} = \frac{8}{10}$
f $\frac{1}{12} = \frac{2}{24} = \frac{5}{60} = \frac{3}{36} = \frac{6}{72} = \frac{9}{108}$

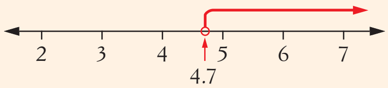
7 **a** $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{7}{10}$ **d** $\frac{1}{4}$
e $\frac{3}{4}$ **f** $\frac{6}{25}$ **g** $\frac{29}{50}$ **h** $\frac{9}{25}$
i $\frac{49}{50}$ **j** $\frac{18}{25}$ **k** $\frac{16}{125}$ **l** $\frac{1}{8}$

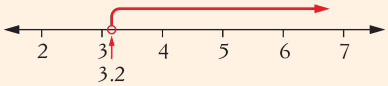
8 **a** $\frac{3}{8} < \frac{1}{2}$ **b** $\frac{5}{8} < \frac{8}{5}$ **c** $\frac{5}{9} > \frac{5}{10}$
d $\frac{3}{5} < \frac{4}{5}$ **e** $0.325 < 0.4$ **f** $2.5 > 1.789$
g $6.32 < 7.16$ **h** $5.85 > 5.495$ **i** $\frac{2}{3} < \frac{4}{5}$
j $\frac{5}{8} < \frac{6}{7}$ **k** $\frac{1}{4} > \frac{1}{7}$ **l** $\frac{3}{5} < \frac{7}{10}$

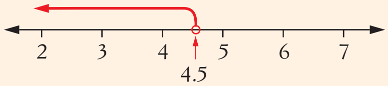
9 **a**


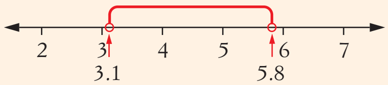
b


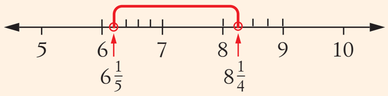
c


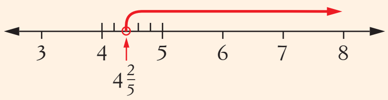
d


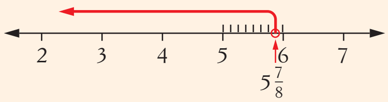
e


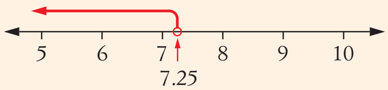
f


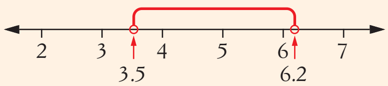
g


h


i


j


k


l


10 **a** $\frac{1}{2}$, 0.5 **b** $\frac{2}{5}$, 0.4 **c** $\frac{1}{3}$, 0.3
d $\frac{3}{4}$, 0.7 or 0.8 **e** $\frac{4}{5}$, 0.8 **f** $\frac{1}{5}$, 0.2
g $\frac{1}{5}$, 0.2 **h** $\frac{3}{5}$, 0.6 **i** $\frac{2}{3}$, 0.7
j $\frac{1}{4}$ or $\frac{2}{5}$, 0.3

11 **a** $\frac{3}{2} = \frac{6}{4} = \frac{36}{24} = \frac{15}{10} = \frac{18}{12} = \frac{9}{6}$

9780170194778

b $\frac{4}{15} = \frac{16}{60} = \frac{8}{30} = \frac{20}{75} = \frac{12}{45} = \frac{32}{120}$

c $\frac{21}{24} = \frac{14}{16} = \frac{28}{32} = \frac{63}{72} = \frac{35}{40} = \frac{7}{8}$

d $\frac{132}{144} = \frac{11}{12} = \frac{22}{24} = \frac{33}{36} = \frac{55}{60} = \frac{44}{48}$

e $\frac{32}{36} = \frac{40}{45} = \frac{8}{9} = \frac{128}{144} = \frac{64}{72} = \frac{16}{18}$

f $\frac{12}{28} = \frac{15}{35} = \frac{6}{14} = \frac{9}{21} = \frac{24}{56} = \frac{3}{7}$

12 **a** $\frac{5}{6}$ **b** $1\frac{1}{2}$ **c** $1\frac{1}{11}$ **d** $2\frac{1}{4}$
e $\frac{8}{9}$ **f** $\frac{21}{32}$ **g** $3\frac{3}{4}$ **h** $\frac{7}{15}$
i $1\frac{1}{7}$ **j** $1\frac{2}{3}$ **k** $\frac{3}{10}$ **l** $2\frac{1}{3}$

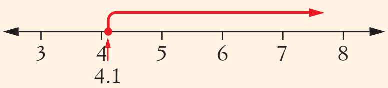
13 **a** No **b** No **c** No **d** Yes
e No **f** Yes **g** No **h** Yes

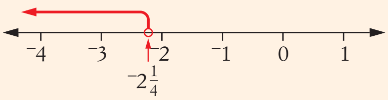
14 **a** $0.8\overline{3}$ **b** $0.\overline{3}$ **c** $0.\overline{4}$
d $0.\overline{27}$ **e** $0.\overline{714285}$ **f** $0.\overline{428\,571}$
g $0.0\overline{5}$ **h** $0.7\overline{85\,7142}$ **i** $0.58\overline{3}$
j $0.1\overline{6}$ **k** $0.\overline{692\,307}$ **l** $0.\overline{307\,692}$

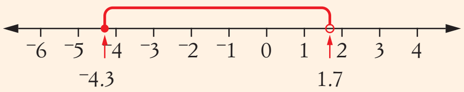
15 **a** $\frac{87}{250}$ **b** $\frac{56}{125}$ **c** $\frac{3}{8}$ **d** $\frac{327}{1000}$
e $\frac{211}{1000}$ **f** $\frac{493}{500}$ **g** $\frac{5}{16}$ **h** $\frac{9}{16}$

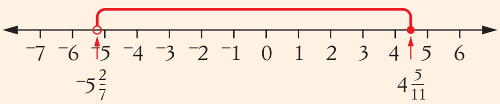
16 **a** $\frac{3}{7} < \frac{4}{9}$ **b** $^{-}4.6 > {^{-}6.4}$ **c** $\frac{4}{15} < \frac{7}{20}$
d $\frac{5}{8} > \frac{11}{19}$ **e** $^{-}0.3 < {^{-}0.24}$ **f** $^{-}3.45 < 2.14$
g $\frac{9}{5} < \frac{11}{6}$ **h** $\frac{6}{11} > \frac{7}{15}$ **i** $\frac{5}{2} > \frac{7}{4}$
j $\frac{7}{9} < \frac{10}{12}$ **k** $\frac{7}{2} > \frac{13}{4}$ **l** $\frac{2}{7} < \frac{4}{13}$

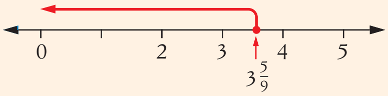
17 **a** $^{-}3.1, {^{-}2.1}, 2.3, 3.4$ **b** $^{-}6.31, {^{-}6.3}, {^{-}6.1}$
c $^{-}7.6, 6.7, 7.67$ **d** $^{-}8.22, {^{-}8.2}, 8.02$
e $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ **f** $2\frac{1}{3}, 2\frac{1}{2}, 3\frac{1}{2}$
g $1\frac{1}{4}, 1\frac{2}{3}, 1\frac{3}{4}$ **h** $\frac{5}{8}, \frac{7}{9}, \frac{4}{5}$

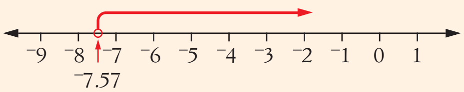
18 **a**


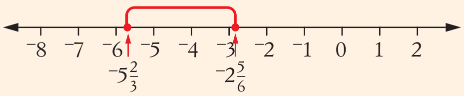
b


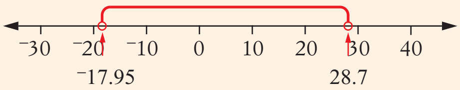
c


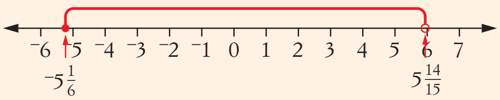
d


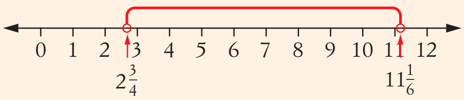
e


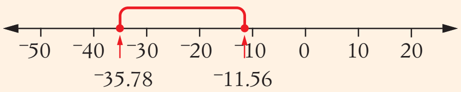
f


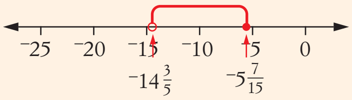
g


h


i


j


k


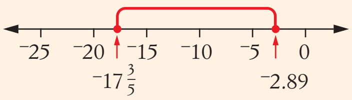
l


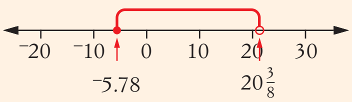
19 **a** $0.\dot{3}$ **b** $0.\dot{2}$ **c** $0.\overline{45}$
d $0.\dot{7}$ **e** $0.\dot{6}$ **f** $0.\overline{142857}$

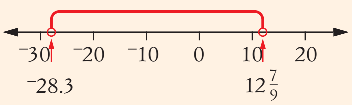
20 **a** $\frac{57}{99}$ **b** $\frac{8}{33}$ **c** $\frac{314}{999}$
d $\frac{572}{999}$ **e** $\frac{36}{111}$ **f** $\frac{374}{3333}$

21 **a** $\frac{5}{8} > 0.6$ **b** $\frac{1}{4} > 0.2$
c $\frac{1}{2} = 0.5$ **d** $^{-}\frac{1}{3} < {^{-}\frac{1}{4}}$
e $^{-}\frac{3}{5} = {^{-}0.6}$ **f** $^{-}\frac{9}{10} > {^{-}\frac{10}{11}}$
g $^{-}3.3 > {^{-}3\frac{1}{3}}$ **h** $2\frac{1}{4} < 2.\overline{25}$
i $5.6 \times 10^{-1} > \frac{1}{5}$ **j** $0.6 > 8.9 \times 10^{-2}$
k $\frac{3}{7} > 7.3 \times 10^{-2}$ **l** $\frac{1}{45} > 1.5 \times 10^{-2}$

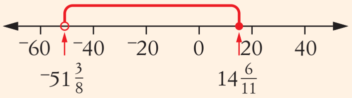
22 **a** $0.6, \frac{6}{7}, \frac{7}{8}, 0.9$ **b** $5.2, 5\frac{1}{4}, 5.4, 5\frac{1}{2}$
c $\frac{2}{3}, 0.7, \frac{3}{4}, 0.8$ **d** $0.1, \frac{2}{15}, \frac{1}{5}, \frac{1}{4}$
e $\frac{3}{5}, 0.61, 0.63, \frac{7}{10}$ **f** $\frac{1}{4}, \frac{3}{11}, 0.3, 0.31$
g $0.5, \frac{6}{11}, \frac{5}{9}, 0.6$ **h** $^{-}1.8, {^{-}1\frac{3}{4}}, {^{-}1.7}, {^{-}1\frac{2}{3}}$
i $^{-}\frac{3}{4}, {^{-}0.5}, 0, \frac{1}{4}$ **j** $\frac{5}{11}, 0.5, 5.5 \times 10^{-1}, \frac{5}{9}$
k $0.02, \frac{1}{20}, \frac{3}{50}, 6.3 \times 10^{-2}$ **l** $^{-}0.8, \frac{2}{3}, 0.7, \frac{4}{5}$

23 **a**


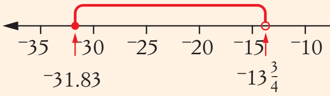
b


c


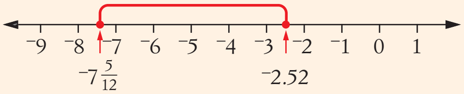
d



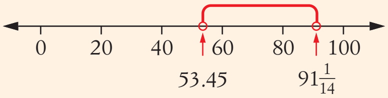
e



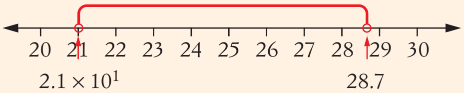
f



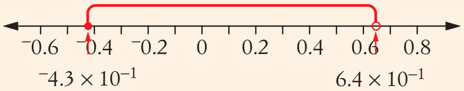
g



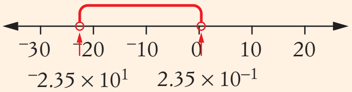
h



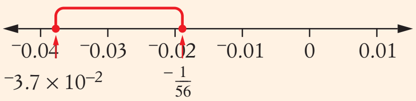
i



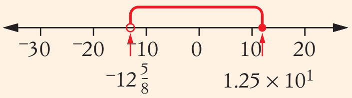
j



k



l



24 **a** $\frac{61}{900}$ **b** $\frac{11}{180}$ **c** $\frac{1}{450}$

25 **a** $\frac{5}{12}$ **b** $\frac{5}{36}$ **c** $\frac{2}{3}$ **d** $\frac{5}{18}$

26 **a** $\frac{1}{6}, \frac{5}{6}$ **b** $\frac{1}{3}, \frac{2}{3}$ **c** $\frac{2}{3}, \frac{1}{3}$ **d** $\frac{1}{4}, \frac{3}{4}$

27 **a** $\frac{2}{5}, \frac{3}{5}$ **b** $\frac{2}{3}, \frac{1}{3}$ **c** $\frac{1}{4}, \frac{3}{4}$ **d** $\frac{3}{4}, \frac{1}{4}$

28 **a** 55c is more **b** $\frac{2}{3}$ is more **c** 0.27 is more

29 Peter worked least, Phillipa the middle amount and Brenda the most.

Exercise 5.2

1 **a** 0.85 **b** 0.041 **c** 1.4 **d** 0.0134
e 0.162 **f** 0.37 **g** 10.2 **h** 0.0016
i 0.09 **j** 0.132 **k** 1.41 **l** 1.6

2 **a** 43.5 **b** 0.141 **c** 0.0831
d 1.69 **e** 0.0194 **f** 14.8
g 1.273 **h** 9.08 **i** 0.002 98
j 117.6 **k** 1.473 **l** 14.7

3 **a** $1\frac{1}{5}$ **b** $1\frac{4}{7}$ **c** $1\frac{1}{8}$ **d** $\frac{2}{3}$
e $\frac{3}{8}$ **f** $1\frac{1}{10}$ **g** $\frac{2}{5}$ **h** $\frac{1}{2}$
i $1\frac{3}{8}$ **j** $\frac{1}{2}$ **k** $\frac{5}{8}$ **l** $\frac{8}{9}$

4 **a** 548.306 **b** 11.279 **c** 224.9989
d 30.513 04 **e** 477.115 **f** 19.385 897
g 298.8953 **h** 935.962 **i** 0.694
j 35.896 **k** 572.0629 **l** 2211

5 **a** 66.0108 **b** 559.0421 **c** 35.49125
d 124.237 **e** 1832.445 **f** 303.193
g 4.8681 **h** 4727.7005

6 **a** 4.331 **b** 13.2552 **c** 10.3939
d 485.048 **e** 15.5107 **f** 1.3848

7 **a** 315.494 **b** 3.6839 **c** 44.8821
d 13.8226 **e** 16.748 **f** 29.3441

8 **a** $\frac{1}{3}$ **b** $\frac{7}{12}$ **c** $8\frac{5}{12}$ **d** $2\frac{7}{8}$
e $\frac{14}{15}$ **f** $5\frac{4}{5}$ **g** $4\frac{5}{8}$ **h** $1\frac{3}{20}$
i $13\frac{1}{10}$ **j** $3\frac{1}{2}$ **k** $8\frac{3}{8}$ **l** $5\frac{7}{15}$

9 **a** $\frac{7}{8}$ **b** 5 **c** $9\frac{13}{40}$ **d** $5\frac{23}{24}$
e $3\frac{7}{12}$ **f** $2\frac{3}{4}$ **g** $5\frac{1}{8}$ **h** $7\frac{7}{20}$

10 **a** 22.105 **b** 16.236 **c** 28.472 **d** 136.6764
e 4.5135 **f** 19.799 **g** 26.0063 **h** 39.123

11 **a** 4.305 **b** 12.634 **c** -27.553
d -3.246 **e** -15.623 **f** 7.6734
g 12.8705 **h** 48.702 **i** -2.6773
j -13.8342 **k** 56.365 **l** 13.5027

12 **a** -12.337 **b** 1.0742 **c** 33.4987
d -46.036 **e** -16.7073 **f** -26.978

13 **a** $\frac{-1}{4}$ **b** $\frac{1}{5}$ **c** $-1\frac{1}{3}$ **d** $1\frac{3}{10}$
e $\frac{1}{8}$ **f** $\frac{-1}{2}$ **g** $\frac{-1}{8}$ **h** $-1\frac{1}{8}$
i $\frac{2}{3}$ **j** $-1\frac{1}{4}$ **k** $\frac{-1}{10}$ **l** $\frac{1}{6}$

14 **a** $10\frac{13}{24}$ **b** $4\frac{23}{24}$ **c** $2\frac{1}{10}$ **d** $1\frac{1}{3}$
e $\frac{11}{12}$ **f** $1\frac{1}{24}$ **g** $1\frac{31}{60}$ **h** $1\frac{7}{12}$
i $4\frac{2}{15}$ **j** $10\frac{19}{42}$ **k** $2\frac{31}{36}$ **l** $1\frac{19}{24}$

15 **a** -1 **b** $-3\frac{1}{6}$ **c** $-1\frac{9}{40}$ **d** $-1\frac{7}{12}$
e $\frac{7}{10}$ **f** $\frac{-8}{21}$ **g** $-1\frac{4}{5}$ **h** $-2\frac{1}{8}$
i $-3\frac{7}{8}$ **j** $-3\frac{4}{5}$ **k** $3\frac{11}{24}$ **l** $-4\frac{17}{40}$

16 $42.07

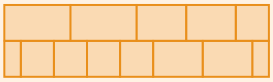
17 **a** 7.235 L **b** 0.765 L

18 Home builder 0.0876 ha, neighbour 0.0727 ha

9780170194778

19 **a** 140.54 cm **b** 4.46 cm

20 **a** 8 blocks
b Many are possible:



21 $\frac{7}{8}$ m

22 **a** 5 × 4 L tins and 2 × 1 L tins
b $\frac{1}{4}$ L

23 0.0538 g

24 158.294 kg

25 $1\frac{23}{24}$ tsp

26 $3\frac{7}{12}$ haystacks

27 $\frac{1}{12}$ hour (5 minutes)

Exercise 5.3

1 **a** 6.7 **b** 0.005 678 **c** 1.58
d 56.95 **e** 0.0006 **f** 9 860 000
g 69.8 **h** 0.0983 **i** 1.4
j 10.985 **k** 0.001 56 **l** 0.106

2 **a** 9.86 **b** 0.023 **c** 31.45
d 29.15 **e** 18.656 **f** 1160
g 1.2 **h** 16.475 **i** 28.992
j 0.2171 **k** 10.88 **l** 33.75

3 **a** 165.2196 **b** About 178.463
c About 4.969 **d** 0.005 041 792
e About 32.66 **f** 1475.3196
g 8.890 884 **h** About 222.2

4 **a** 15 **b** 34 **c** 145 **d** 209
e $10\frac{1}{2}$ **f** $8\frac{2}{3}$ **g** $2\frac{4}{5}$ **h** $3\frac{5}{8}$

5 **a** 1.425 **b** 0.143 **c** 0.000 076
d 0.001 07 **e** 0.48 **f** 620
g 29.6 **h** 0.000 007 **i** 1.34
j 0.000 07 **k** 0.000 178 **l** 80

6 **a** 10.3406 **b** 85.56 **c** $213.\overline{3}$
d $20.9\overline{81}$ **e** $41.4\overline{857142}$ **f** 361.3115
g 0.107 512 **h** $25.3\overline{6}$ **i** 0.102 01
j 0.7225 **k** $347.\overline{769230}$ **l** 145.259 04

7 **a** $\frac{5}{16}$ **b** $\frac{2}{15}$ **c** 6 **d** 10
e $\frac{1}{5}$ **f** 18 **g** 4 **h** $\frac{3}{20}$
i 33 **j** $\frac{4}{15}$ **k** 26 **l** $\frac{3}{10}$

8 **a** $\frac{3}{8}$ **b** $\frac{15}{56}$ **c** $\frac{5}{6}$ **d** $\frac{4}{5}$
e $3\frac{3}{4}$ **f** $\frac{2}{15}$ **g** $\frac{9}{16}$ **h** $\frac{32}{35}$
i $\frac{3}{8}$ **j** $\frac{1}{6}$ **k** $1\frac{1}{6}$ **l** $\frac{1}{12}$

9 **a** $\frac{1}{3}$ **b** 7 **c** $\frac{5}{6}$ **d** 2
e $15\frac{3}{4}$ **f** $8\frac{1}{2}$ **g** 55 **h** $\frac{2}{5}$
i 5 **j** $3\frac{3}{8}$ **k** $\frac{3}{5}$ **l** 1

10 **a** 1.9 **b** 2.065 **c** 0.082
d 231.2 **e** 0.7326 **f** 1.36
g 0.019 75 **h** 0.017 55

11 **a** $^-22.366$ **b** $^-12.125$ **c** $^-0.1692$
d $0.14\overline{3}$ **e** $^-83.2$ **f** 0.063 98
g $^-121.875$ **h** $^-0.0435$ **i** 13.2132
j $^-0.699\ 94$ **k** $855.\overline{5}$ **l** $^-0.005\ 38$

12 **a** 0.563 459 **b** About $^-2.519$
c About $^-0.019\ 47$ **d** $^-0.004\ 249\ 26$
e About 11.10 **f** $^-415.030\ 68$
g About $^-0.3535$ **h** 46.614 954

13 **a** 7 **b** 2 **c** 3 **d** 6
e $2\frac{1}{2}$ **f** 4 **g** 3 **h** 4
i $1\frac{1}{8}$ **j** 7 **k** 12 **l** $\frac{1}{10}$

14 **a** $\frac{^-3}{10}$ **b** 3 **c** $^-1\frac{1}{3}$ **d** $\frac{2}{15}$
e $2\frac{1}{2}$ **f** $\frac{2}{5}$ **g** 12 **h** 7
i $\frac{^-1}{2}$ **j** $1\frac{1}{2}$ **k** $^-12\frac{1}{2}$ **l** $^-1\frac{1}{2}$

15 **a** $2\frac{3}{26}$ **b** 5 **c** $2\frac{2}{3}$ **d** $\frac{^-3}{4}$
e $5\frac{1}{4}$ **f** $\frac{^-1}{3}$ **g** $^-1\frac{25}{56}$ **h** $2\frac{10}{27}$
i $30\frac{1}{4}$ **j** $20\frac{51}{64}$ **k** $\frac{^-1}{8}$ **l** $^-69\frac{40}{243}$

16 **a** $\frac{^-5}{9}$ **b** $\frac{11}{24}$ **c** $^-4.775$
d $8\frac{7}{9}$ **e** $^-1.3745$ **f** $3.258\overline{4}$

17 \$337 (\$336.99)

18 2.064 kg

19 **a** 6 × 5.1 m and 3.6 m
b 5 × 5.1 m and 0.6 m
c 9 × 5.1 m and 1.2 m

20 **a** $\frac{5}{24}$ **b** $\frac{5}{32}$ **c** $\frac{3}{16}$
d Condamine couch

21 **a** $\frac{1}{6}$ **b** 6075 bricks **c** 1350 bricks

22 **a** 23 blouses **b** 0.08 m **c** 0.36 m

9780170194778

Answers

23 52 loaves

24 **a** $\frac{4}{27}$ **b** 54 people

25 17 holes, allowing for space at the ends

Exercise 5.4

1 **a** 5.73 **b** 11.86 **c** 4.4 **d** 4.5
e 21 **f** 0.0525 **g** 32.9 **h** 15.94

2 **a** 2 s.f. **b** 3 s.f. **c** 3 s.f. **d** 1 s.f.
e 2 s.f. **f** 4 s.f. **g** 2 s.f. **h** 1 s.f.

3 **a** 0.1 **b** 3 **c** 0.06 **d** 10 000
e 20 **f** 80 **g** 0.002 **h** 0.02

4 **a** 1.718 75 **b** 5.581 25 **c** 4.62
d 12.2 **e** 0.612 **f** 43.798
g 295.75 **h** 137.979 **i** $788.\overline{8}$
j 7.54 **k** 11.9376 **l** 4.0406

5 **a** 7 **b** 20 **c** 0.1
d 0.0002 **e** 300 **f** 0.000 05

6 **a** $1\frac{37}{40}$ **b** $\frac{233}{1008}$ **c** $37\frac{419}{720}$ **d** $234\frac{125}{144}$
e 11.1 **f** 8.643 125 **g** 60.92 **h** 61.5125

7 **a** 0.2 **b** 2×10^9 **c** 0.0005
d 30 000 **e** 1 **f** 6×10^9

8 **a** $1\frac{8}{27}$ **b** $\frac{22}{225}$ **c** $\frac{249}{250}$ or 0.996
d $45\frac{1}{9}$ or $45.\overline{1}$ **e** ≈ 15.018 **f** $\approx {}^{-}0.0136$

9 1.618 million square km

10 $4\frac{2}{3}$ teaspoons

11 972 slices

12 Andrew 23.04 m, Tania 19.2 m

13 16 m for 5 downpipes, allowing for the bend back to the eaves

Chapter review

1 **a** Three-quarters, $\frac{3}{4}$
b Five-eigh ths, $\frac{5}{8}$

2 $\frac{14}{5}$

3 $5\frac{3}{4}$

4 $\frac{3}{5} = \frac{6}{10} = \frac{18}{30} = \frac{21}{35}$

5 $\frac{2}{3}$

6 $\frac{3}{5} = 0.6, 0.45 = \frac{9}{20}$

7 **a** 1.44 **b** 0.0038

8 **a** 123.693 **b** 29.083

9 **a** 9717.81 **b** 116.7121

10 **a** 5.6 **b** 0.6724 **c** 0.0072 **d** 76.5
e 1.7 **f** 7.3 **g** 7 **h** $6\frac{2}{5}$

11 **a** 85.344 **b** 0.0145 **c** 680 **d** 36.2

12 **a** 202.2735 **b** About 145.15

13 11.9

14 About 70

15 $\frac{2}{3}$, 0.6

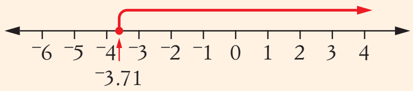
16 $2\frac{1}{7}$

17 **a** No **b** Yes

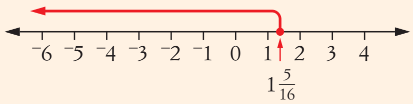
18 $0.\overline{857\,142}$, $0.41\overline{6}$

19 **a** ${}^{-}5.2$, ${}^{-}4.9$, 4.8, 5.3 **b** $3\frac{2}{9}$, $3\frac{3}{4}$, $4\frac{1}{10}$, $4\frac{1}{3}$

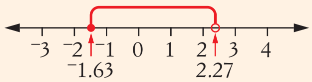
20 **a**



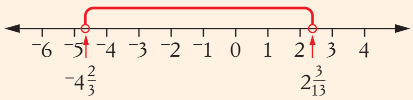
b



c

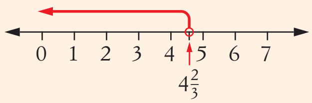


d

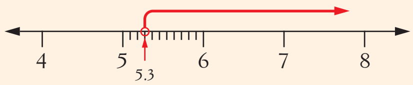


21 **a** $\frac{3}{5} > \frac{5}{12}$ **b** $0.7 > 0.689$

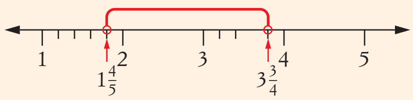
22 **a**



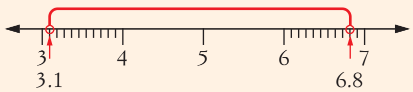
b



c



d



23 **a** $0.1\overline{6}$ **b** $0.\overline{4}$ **c** $0.\overline{63}$

9780170194778

24 **a** $\frac{17}{99}$ **b** $\frac{31}{33}$ **c** $\frac{16}{111}$

25 **a** 0.1739 **b** 2.68 **c** $\frac{1}{2}$ **d** $1\frac{3}{8}$

26 **a** 176.634 **b** $\frac{7}{12}$ **c** $\frac{23}{24}$ **d** $1\frac{7}{10}$
e $13\frac{1}{6}$ **f** $2\frac{7}{12}$ **g** $4\frac{2}{15}$

27 **a** 605.421 **b** $1\frac{1}{72}$

28 **a** 0.045 **b** 0.0118 **c** 132 **d** 430
e $\frac{3}{10}$ **f** 26 **g** 10 **h** $\frac{2}{15}$

29 **a** 8.493 52 **b** $26.0\overline{857\,142}$ **c** 242.5
d 0.000 233 772 **e** $\frac{1}{6}$ **f** $1\frac{7}{20}$
g $\frac{24}{35}$ **h** $\frac{4}{7}$

30 **a** $3\frac{29}{30}$ **b** $2\frac{1}{2}$ **c** $1\frac{4}{11}$ **d** $2\frac{1}{16}$

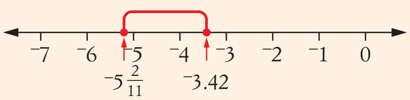
31 **a** 6.486 **b** 32.1

32 300

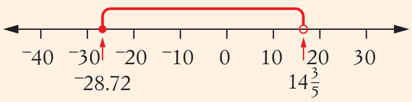
33 **a** $\frac{2}{3} < 0.7$ **b** $\frac{^-3}{4} > {}^-0.77$ **c** $0.08 < 7.9 \times 10^{-1}$

34 **a** 0.1, $\frac{1}{8}$, $\frac{1}{7}$, 0.15, 0.67
b $^-0.85$, $\frac{^-4}{5}$, $\frac{^-3}{4}$, 6.3×10^{-2}, $\frac{1}{2}$, 0.8

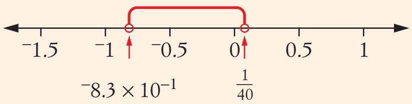
35 **a**



b



c



36 **a** 155.7766 **b** 25.017 **c** $\frac{^-1}{8}$ **d** $1\frac{1}{2}$
e $^-1\frac{1}{2}$ **f** $\frac{^-5}{12}$ **g** $\frac{3}{8}$ **h** $\frac{^-3}{10}$

37 **a** $^-2.176$ **b** 7.912 **c** $^-443.07$
d $^-22.871$ **e** $\frac{59}{60}$ **f** $\frac{19}{36}$
g $6\frac{19}{20}$ **h** $1\frac{3}{4}$

38 **a** $^-2.754$ **b** $8\frac{1}{24}$

39 **a** 0.2107 **b** 4.65 **c** 0.01275 **d** 68
e 4 **f** 6 **g** $2\frac{4}{5}$ **h** 6

40 **a** 12.768 **b** $^-11.25$ **c** $^-0.016\,38$
d 11.925 **e** $^-1\frac{17}{39}$ **f** $\frac{^-6}{7}$
g $4\frac{2}{9}$ **h** 18

41 **a** About 4683 **b** $2\frac{25}{28}$ **c** $2\frac{1}{3}$ **d** $29\frac{52}{81}$

42 **a** $^-4$ **b** $0.304\overline{3}$

43 **a** $\frac{47}{108}$ **b** 19.262

44 4

45 **a** $1\frac{34}{35}$ **b** ≈ 11.1396

46 $\frac{7}{12}$

47 1.614 m

48 \$27.36

49 0.65 is more than $\frac{5}{8}$.

50 7.3 km

51 **a** $1\frac{11}{20}$ kg **b** 1550 g

52 \$31.20

53 324.567 kg

54 $\frac{41}{72}$

55 21 bags with 0.6 kg left over

56 $1\frac{3}{5}$ times the wholesaler's price

Chapter 6

Exercise 6.1

1 **a** $m = \angle FGH = \angle HGF$
b $n = \angle ILM = \angle MLI$
c $l = \angle PKQ = \angle QKP$
d $p = \angle TRS = \angle SRT$; $q = \angle SRW = \angle WRS$
e $\alpha = \angle ANX = \angle XNA$; $\beta = \angle XNY = \angle YNX$; $\theta = \angle YNZ = \angle ZNY$; $\phi = \angle VZN = \angle NZV$
f $s = \angle BDC = \angle CDB$; $t = \angle DCE = \angle ECD$; $r = \angle ECJ = \angle JCE$

2 $m = \angle G$, $n = \angle L$, $l = \angle K$, $\phi = \angle Z$ and $s = \angle D$

3 **a** $\angle KLM = 28°$ **b** $\angle RST = 140°$
c $\angle UWY = 80°$ **d** $\angle XVZ = 173°$

4 **a** acute **b** obtuse **c** full rotation
d straight **e** reflex **f** right
g reflex **h** acute

5 **a** obtuse **b** acute **c** reflex
d right **e** straight **f** reflex
g reflex **h** obtuse **i** reflex
j acute **k** acute **l** reflex
m acute (nearly 90°) **n** reflex **o** reflex
p acute

9780170194778

6	**A** 76°	**B** 72°	**C** 66°	ϕ 134°
	θ 130°	α 128°	**a** 69°	**b** 224°
	c 27°	**d** 223°	**e** 16°	**f** 45°
	g 288°	**h** 90°	**i** 191°	**j** 47°
	k 110°	**l** 70°	**m** 60°	**n** 218°

7 **a** 71° **b** 55° **c** 14° **d** 84°

8 **a** 134° **b** 46° **c** 75° **d** 118°?

For questions **9** and **10** other reasons are possible.

9 **a** $w = 244°$ angles at a point
b $e = 59°$ supplementary angles
c $k = 72°$ complementary angles
d $h = 117°$ supplementary angles
e $r = 56°$ complementary angles
f $b = 62°$ adjacent angles
g $x = 66°$ supplementary angles, $y = 114°$ vertically opposite angles
h $n = 167°$ angles at a point
i $y = 48°$ vertically opposite angles, $w = 132°$ supplementary angles, $x = 132°$ vertically opposite angles
j $z = 72°$ supplementary angles
k $t = 124°$ vertically opposite angles, $u = 56°$ supplementary angles, $s = 56°$ vertically opposite angles
l $u = 270°$ angles at a point
m $g = 322°$ angles at a point
n $d = 29°$ complementary angles
o $k = 158°$ angles at a point
p $p = 47°$ adjacent angles
q $m = 119°$ supplementary angles
r $r = 45°$ complementary angles

10 **a** $x = 51°$ supplementary angles
b $a = 30°$ supplementary angles
c $w = 52°$ complementary angles, $u = 166°$ angles at a point
d $z = 212°$ angles at a point
e $c = 180°$ straight angle, $d = 90°$ supplementary angles
f $r = 45°$ complementary angles, $q = 270°$ angles at a point
g $w = 60°$ angles at a point
h $b = 42°$ supplementary angles
i $y = 61°$ vertically opposite angles, $x = 119°$ supplementary angles, $z = 83°$ supplementary angles

11 **a** A is biggest, C is smallest
b ϕ is biggest, α is smallest

12 **c** **i** 60° **ii** 30° **iii** 23° **iv** 78°

Exercise 6.2

1 **a** Corresponding angles **b** Alternate angles
c Corresponding angles **d** Co-interior angles
e Alternate angles **f** Co-interior angles
g Co-interior angles **h** Alternate angles
i Co-interior angles

2 **a** $b = 107°$, co-interior angles
b $d = 60°$, corresponding angles
c $a = 118°$, corresponding angles
d $m = 66°$, alternate angles
e $z = 59°$, co-interior angles
f $e = 68°$, corresponding angles
g $r = 58°$, alternate angles
h $k = 73°$, corresponding angles
i $s = 132°$, co-interior angles

Other reasons are possible for Question 3.

3 **a** $b = 73°$ corresponding angles, $a = 107°$ supplementary angles
b $c = 83°$ co-interior angles
c $f = 82°$ corresponding angles, $e = 82°$ vertically opposite angles, $g = 98°$ co-interior angles
d $n = 73°$ corresponding angles, $m = 107°$ supplementary angles
e $x = 118°$ alternate angles, $z = 62°$ supplementary angles, $y = 118°$ corresponding angles
f $h = 274°$ co-interior angles,
g $b = 108°$ corresponding angles, $a = 108°$ corresponding angles, $c = 108°$ alternate angles
h $z = 90°$ alternate angles, $m = 66°$ alternate angles, $n = 24°$ complementary angles
i $z = 112°$ corresponding angles, $s = 112°$ alternate angles, $y = 112°$ corresponding angles, $x = 112°$ corresponding angles,

4 **a** $a = e$ alternate angles
b $h = d$ alternate angles
c $a + b + h + g = 180°$ co-interior angles or $a + b + c + d = 180°$ co-interior angles
d $f + e + c + d = 180°$ co-interior angles or $f + e + h + g = 180°$ co-interior angles

5 **a** n, o, g, b, d, j and k **b** i, l, c, e, h, m and p
c g, f, d, b, k, j, n and o **d** $f = 126°$. $e = 54°$

6 **a** $PQ \parallel RS$, $\angle PXY = \angle XYS$ (alternate angles)
b $PQ \nparallel RS$
c $PQ \parallel RS$, $\angle PXY = \angle RYB$ (corresponding angles)

9780170194778

d $PQ \parallel RS$, $\angle SXY + \angle XYQ = 180°$ (co-interior angles)
e $PQ \nparallel RS$
f $PQ \parallel RS$, $\angle QXY + \angle SYX = 180°$ (co-interior angles)

7 $a = 123°$ corresponding angles and supplementary angles
$b = 14°$ corresponding angles

8 $AB \parallel DC$ co-interior angles

9 $m = 32°$ co-interior angles, $n = 14°$ co-interior angles

10 $c = 24°$ co-interior angles, $a = 90°$ corresponding angles, $b = 34°$ angles at a point

11 $x = 42°$ co-interior angles, $y = 30°$ co-interior angles

Exercise 6.3

1 a i w **ii** x and z
b i h **ii** d and e
c i i **ii** n and m

2 $a = 50°$ $b = 24°$ $c = 75°$ $d = 100°$
$e = 70°$ $f = 60°$ $g = 80°$ $h = 30°$
$i = 40°$ $j = 67°$ $k = 120°$ $l = 155°$

3 a $b = 107°$ **b** $e = 63°$ **c** $z = 85°$
d $m = 78°$ **e** $e = 29°$ **f** $x = 42°$
g $d = 133°$ **h** $w = 52°$ **i** $n = 57°$

4 a $c = 43°$
b $x = 58°$, $y = 52°$, $z = 70°$
c $g = 72°$

5 a $c = 70°$, $d = 110°$ **b** $x = 27°$, $n = 73°$, $z = 27°$
c $g = 71°$ **d** $y = 65°$
e $d = 60°$ **f** $k = 77°$
g $n = 116°$ **h** $z = 136°$
i $s = 44°$

6 $m = 65.5°$

7 $w = 78°$

8 $k = 20.5°$

9–10 Proofs

Exercise 6.4

1 a SSS **b** SAS **c** SAS
d None **e** None **f** ASA
g SAS **h** RHS **i** None

2 a $AC = PQ$ **b** $\angle J = \angle Y$
c $\angle P = \angle S$ **d** $WS = JU$
e $QA = KX$ or $AS = XD$ **f** $OJ = LM$
g $\angle C = \angle Z$ **h** $ML = TC$
i $QM = YG$

3 D

4 B

5 B

6 a SSS **b** SAS **c** AAS
d RHS **e** Not necessarily congruent **f** SSS

7 a $e = 7$ cm, $d = 102°$ **b** $z = 11$ cm, $x = 80°$
c $s = 9$ cm, $r = 60°$ **d** $w = 11$ cm, $u = 22°$
e $p = 6$ cm, $q = 64°$ **f** $g = 5$ cm, $h = 4.5$ cm

8 F

9–13 Proofs

Exercise 6.5

1 a 360° **b** 360° **c** 180°
d 540° **e** 720° **f** 900°

2 a 101° **b** 91° **c** 134°
d 110° **e** 20° **f** 67°
g 73° **h** 18° **i** 59°
j 99°

3 a $a = 118°$ **b** $z = 145°$, $x = 35°$
c $g = 28°$, $c = 28°$ **d** $r = 139°$, $w = 139°$
e $d = 68°$, $b = 112°$ **f** $n = 92°$, $p = 88°$

4 Copy and complete the following table.

	Polygon	Number of sides	Sum of interior angles
a	Pentagon	5	540°
b	Nonagon	9	1260°
c	Hexagon	6	720°
d	Decagon	10	1440°
e	Dodecagon	12	1800°

5 a $2160°$ b $3240°$ c $4140°$ d $17\,640°$
6 a 15 b 22 c 17 d 34
7 a $90°$ b $120°$ c $140°$ d $150°$
8 a $b = 135°$
b $e = 74°$
c $g = 56°, h = 56°, z = 124°$
d $c = 157°, d = 203°$
e $m = 102°, n = 68°$
f $x = 8°$
g $a = 117°, b = 103°, c = 77°$
h $k = 90°, u = 118°$
i $y = 38°, w = 142°, s = 115°$

9 Proof

10 15

11–13 Proofs

Chapter review

1 $a = \angle HJI = \angle IJH = 23°$, $b = \angle JIK = \angle KIJ = 142°$

2 a Right b Acute c Obtuse
d Reflex e Straight

3 a Corresponding b Alternate c Corresponding
d Co-interior e Alternate f Co-interior

4 a $\angle ABC$ b $\angle BCD$ and $\angle BDC$ or $\angle C$ and $\angle D$

5 $a = 42°$ $b = 138°$ $c = 28°$ $d = 130°$

6 a $m = 105°$ b $a = 103°$ c $x = 114°$

7 a $g = 60°$ b $c = 54°$ c $n = 60°$ d $y = 31°$

8 a $b = 58°$ b $x = 134°$ c $p = 138°$

9 a Acute b Obtuse c Straight d Reflex

10 $a = 264°$ $b = 250°$ $c = 328°$

11 a $w = 39°$ b $x = 118°$ c $z = 63°$

12 a $PQ \parallel RS$ b $PQ \parallel RS$ c $PQ \nparallel RS$

13 a RHS b ASA c None
d ASA e SSS f SAS

14 a $AC = PC$ b $FE = FY$
c $RH = MK$ d $XY = AF$ and $YZ = FD$
e $ST = HJ$ f $GT = FB$

15 a ASA b Not congruent c SAS d RHS

16 D

17 B

18 A

19 a $m = 9$ cm b $a = 17$ cm, $b = 83°$
c $y = 12$ cm, $x = 69°$ d $w = 10.7$ cm, $v = 21°$, $u = 118°$

20 Proof

21 a $r = 90°$ b $s = 114°$ c $q = 123°$

22 $1080°$

23 a $h = 36°$ b $k = 153°$ c $p = 57°$

24 $83°$

25–28 Proofs

Chapter 7

Exercise 7.1

1 a \$26.40/h b \$29/h c \$1.55/kg
d 66.7 g/L e 96 rev/s

2 a 50 km/h b 7.5 m/s c 32 km/h
d 19.5 m/s e 34.5 m/s f 525 km/h

3 a 3.6 kWh b 33 kWh c 3.6 kWh
d 172.8 kWh e 28.8 kWh

4 a 20 m/s b 26.7 m/s c 27.8 m/s
d 20.8 m/s e 125 m/s f 11.1 m/s

5 a 172.8 km/h b 129.6 km/h c 360 km/h
d 234 km/h e 720 km/h f 28.8 km/h

6 a \$21.69 b \$202.04 c \$1641.07
d \$2120.74 e \$2505.29 f \$2814.67

7 a \$1.53 b \$17.71 c \$18.90
d \$0.52 e \$0.46

8 a 120 km b 600 km c 60 km
d 30 km e 420 km f 760 km

9 a \$8.05 b \$13.80 c \$28.75
d \$2.88 e \$4.28 f \$4.78

10 a 331 m b 1.655 km c 10.592 km

11 a 9 kg b 60 kg c 7.39 kg

12 a 1.8 s b 9.1 s c 24.2 s

13 a 7 hours b 4 h 28 min c 29 h 46 min

9780170194778

14 a i €149.64 ii €39.75 iii €498.80
b i AUD96.23 ii AUD73.46 iii AUD1122.69

15 a 2400 W \$15.75/month, 3000 W \$19.68/month
c \$3.93
d Answers will vary

16 48 km/h

17 a 320 km **b** 64 km/h **c** 6.4 h **d** 3.2 h

18 100 m

Exercise 7.2

1 a 1 : 2 **b** 3 : 2 **c** 1 : 3 **d** 2 : 3
e 3 : 2 **f** 1 : 1 **g** 1 : 1 **h** 5 : 3

2 a 2 : 3 **b** 5 : 3 **c** 2 : 1
d 1 : 3 **e** 1 : 4 **f** 3 : 2

3 a 1 : 2 **b** 2 : 1 **c** 1 : 5
d 6 : 7 **e** 5 : 8 **f** 4 : 3
g 3 : 4 **h** 16 : 5 **i** 5 : 6

4 a 1 : 4 **b** 4 : 1 **c** 6 : 1
d 7 : 15 **e** 1 : 100 **f** 1 : 5
g 40 : 1 **h** 5 : 9 **i** 40 : 9

5 a 1 : 4 **b** 8 : 3 **c** 3 : 10
d 2 : 7 **e** 8 : 3 **f** 3 : 2
g 400 : 3 **h** 3 : 2 **i** 1 : 12

6 a 1 : 4 **b** 4 : 1 **c** 3 : 2
d 7 : 6 **e** 2 : 1 **f** 4 : 13
g 5 : 3 **h** 8 : 23 **i** 9 : 5

7 a 3 : 1 **b** 5 : 1 **c** 0.2 : 1
d 5.4 : 1 **e** 5.2 : 1 **f** 0.2 : 1

8 a 1 : 1.5 **b** 1 : 1.8 **c** 1 : 0.8
d 1 : 2.4 **e** 1 : 0.8 **f** 1 : 5.4

9 a 5 parts, 4 pets **b** 10 parts, 10 g
c 20 parts, 140 fans **d** 7 parts, 8000 viewers
e 5 parts, 160 g **f** 9 parts, 12 minutes

10 a \$8 : \$16 **b** 36 kg : 45 kg
c 31 mL : 93 mL **d** 60 cm : 24 cm

11 a Yes **b** No **c** No
d Yes **e** Yes

12 a Yes **b** No **c** Yes
d Yes **e** Yes

13 a 5 m **b** 46 m **c** 7 m
d 3 km **e** 1.06 km

14 a 1 cm **b** 5 cm **c** 9 cm
d 24 cm **e** 5.2 cm

15 a 35 g : 70 g : 140 g **b** 1140 : 760 : 2280

16 a $a = 4$ **b** $m = 36$ **c** $x = 8$
d $p = 3$ **e** $d = 0.8$ **f** $f = 3.9$

17 a 2 : 3 = 12 : 18 **b** 20 : 14 < 11:7
c 42 : 16 < 22 : 8 **d** 15 : 9 = 60 : 36
e 27 : 55 < 36 : 73 **f** $\frac{1}{2} : 5 > \frac{2}{5} : 11$

18

Pair	a, b	a, c	a, d	c, b	c, d	b, d
Ratio of angles	2 : 3	2 : 9	1 : 3	3 : 1	3 : 2	1 : 2

19 Andrea \$550 000, Frank \$450 000

20 24 girls

21 a 750 km **b** 1050 km

22 a 945 vowels **b** 2079 consonants

23 32 shovels of sand, 64 shovels of gravel, 16 shovels of cement

24 a 3 runs
b Scorpions lead by 6 runs.

25 a 24 drops **b** 4 flips

26 Input 37.5%, calculations 25%, output 37.5%

27 6 days

28 $6\frac{2}{3}$ days

Exercise 7.3

1 a 1 m **b** 5 m **c** 2 m
d 2 m **e** 1.5 m **f** 22.3 m
g 3.84 m **h** 3.65 m **i** 4.75 m
j 3.35 m

2 Answers are based on the external dimensions of rooms.
a Length = 4.8 m, width = 4 m
b Length = 6.6 m, width = 6.6 m
c Length = 5.4 m, width = 4.2 m
d Length = 3.6 m, width = 6.8 m

3 Answers are based on the external dimensions of rooms.
   a Length = 17.7 m, width = 1.95 m
   b Length = 4.05 m, width = 4.8 m
   c 18 m
   d 9.6 m
   e Length = 2.25 m, width = 2.25 m

4 **a** 10 km **b** 17.5 km **c** 37.5 km
   d 45 km **e** 23.75 km **f** 26 km

5 **a** 1000 m **b** 500 m **c** 1750 m
   d 2250 m **e** 2625 m **f** 1600 m

6 **a** 10 cm **b** 20 cm **c** 28 cm
   d 18 cm **e** 32 cm **f** 25.6 cm

7 **a** 5 m **b** 80 m **c** 22.5 m
   d 21 m **e** 900 m **f** 3.2 km

8 **a** 6000 m or 6 km **b** 3000 m or 3 km
   c 120 km **d** 6 km
   e 30 km **f** 1800 m or 1.8 km
   g 1200 m **h** 14.4 km

9 **a** 60 m × 42.5 m **b** 500 m **c** 144 m
   d 165 m **e** 225 m

10 **a** Frontage = 8.2 cm, depth = 12.3 cm
   b Frontage = 16.4 m, depth = 24.6 m
   c Width = 1.6 cm, length = 3.3 cm
   d Width = 3.2 m, length = 6.6 m
   e 2.4 metres
   f Length = 5.8 m, width = 0.8 m
   g Length = 10 m, width = 10.6 m
   h 1 m

11 **a** 39 mm **b** 7.8 mm **c** 6 mm
   d 5 mm **e** 6 mm **f** 1.8 mm

12 **a** 0.029 mm **b** 0.047 mm **c** 0.04 mm

13 **a** Triangle *ABC* is an enlargement of scale factor 2 of triangle *DEF*.
   b 9 cm
   c 28 cm

14 **a** Quadrilateral *EFGH* is an enlargement of scale factor 2.5 of quadrilateral *ABCD*.
   b 6 cm
   c 4.2 cm

15 Compare your answer with those of other class members. Make displays of the drawings.

Chapter review

1 7.5 L/h

2 70 km/h

3 18 kWh

4 $13\frac{1}{3}$ m/s

5 **a** 2 : 5 **b** 4 : 1 **c** 20 : 3 **d** 1 : 5
   e 2 : 3 **f** 2 : 1

6 **a** 1 : 73.2 **b** 0.014 : 1

7 **a** No **b** Yes

8 \$28 : \$32

9 240 m

10 **a** 5 m **b** 23.4 m **c** 1.8 m **d** 2.4 m

11 (Answers are based on the external dimensions of rooms.)
   a Length = 7.2 m, width = 5.6 m
   b Length = 5.0 m, width = 3.6 m
   c 22.6 m
   d 12.8 m
   e Length = 4.6 m, width = 2.6 m

12 **a** 1000 m **b** 1750 m **c** 2850 m

13 **a** 12 cm **b** 20 cm **c** 18 cm

14 **a** 800 m **b** 2000 m **c** 160 m **d** 16 km

15 144 km/h

16 \$2777.17

17 **a** 5 : 8 > 7 : 13 **b** 50 : 3 > 106 : 7
   c 42 : 16 < 22 : 8

18 \$40 : \$60 : \$80

19 49 cm

20 5.71 kg

21 \$9

22 **a** 10 : 1 **b** 1 mm **c** 0.8 mm

23 5 whole watermelons (5.6)

24 \$442.93 and \$400.95 if the shop uses more than 1650 units/day, not counting the dishwasher. Note: When considering individual appliances, the supply charge and the first 20 units would not be considered. You are considering the extra cost incurred by the machine.

9780170194778

25 480 g of self-raising flour, 240 mL of milk, 200 g of sugar, 6 large eggs or 7 small eggs, 48 g of butter

Chapter 8

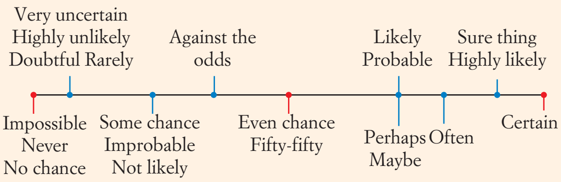
Exercise 8.1

1 a Even chance **b** Very likely
c Answers will vary **d** Even chance
e Very unlikely **f** Even chance
g Impossible **h** Very likely

2 a Answers can be anything that cannot happen.
b Answers can be anything that must happen.

3 a less **b** less **c** more
d more **e** less **f** more

4 Positions are not intended to be exact.



5 a They are equally likely.
b Getting a 6 is less likely than not getting a 6.
c You are more likely to lose than to win.
d You are more likely to win than to lose.
e You are more likely to get a pair than a flush.

6 a 0 **b** Answers will vary
c 1 or 2 **d** 5
e 2 or 3 **f** 10
g 1 or 2 **h** 0
i Answers will vary

7 Answers will vary depending on time of year etc. Your teacher will check these answers.

8–10 Answers will vary. Your teacher will check.

11 a i {A, B} **ii** Each event is equally likely.
b i {1, 2, 3, 4} **ii** Each event is equally likely.
c i {Yellow, blue} **ii** Each event is equally likely.

12 a {G, G, G, G, G, B, B, B, R, R, Y, Y}
b The different colour outcomes are not equally likely.

13 a $P(\text{Y}) = \frac{1}{2}$ **b** $P(\text{Y}) = \frac{1}{4}$ **c** $P(\text{Y}) = \frac{1}{2}$

14 a $P(\text{R}) = \frac{1}{6}$ **b** $P(\text{G}) = \frac{5}{12}$

15–17 Answers will vary. Your teacher will check.

18 a, b

Outcome	Frequency	Relative frequency
HH	2	0.1
HT	6	0.3
TH	7	0.35
TT	5	0.25
Total	20	

c You are more likely to get ‘heads and tails’ than ‘two tails’.

19 a

Suit	Frequency	Relative Frequency
Hearts (♥)	7	0.28
Diamonds (♦)	5	0.2
Clubs (♣)	5	0.2
Spades (♠)	8	0.32
Total	25	1.00

b Black or red are equally likely.
c A spade appears more likely, but only a small number of trials was done.

20 a, b

Number	Frequency	Relative frequency
13	1	$\frac{1}{30}$
14	6	$\frac{1}{5}$
15	9	$\frac{3}{10}$
16	4	$\frac{2}{15}$
17	8	$\frac{4}{15}$
18	1	$\frac{1}{30}$
19	0	0
20	1	$\frac{1}{30}$
Total	30	

c A bag with 15 oranges is more likely.

21 a No
b Yes, a H or a T are equally likely; {HH, HT, TH, TT}
c No
d No
e Yes, random choice; {AA, AB, AC, BA, BB, BC, CA, CB, CC}

22 a {R, R, R, R, G, G, G, B, B, B, B, B, B, B, B}
b 4
c $\frac{4}{15}$ or about 0.27 or 27%
d $\frac{1}{5}$ or 0.2 or 20%

e $\frac{8}{15}$ or about 0.53 or 53%

f 1

23 a {SB, SB, PP, PP, PP, PP, PP, PP, PP, PP, PS, PS, PS, PS, PS, PS}

b $\frac{1}{8}$ or 0.125 or 12.5%

c $\frac{1}{2}$ or 0.5 or 50%

d $\frac{3}{8}$ or 0.375 or 37.5%

24 a {B, B, B, G, G, L, L, L, L, L}

b 3 and $\frac{3}{10}$ or 0.3 or 30%

c 2 and $\frac{1}{5}$ or 0.2 or 20%

d 5 and $\frac{1}{2}$ or 0.5 or 50%

e Lined paper

25 a $\frac{1}{6}$ or about 0.17 or 17%

b $\frac{1}{6}$ or about 0.17 or 17%

c $\frac{1}{2}$ or 0.5 or 50%

d $\frac{1}{3}$ or about 0.33 or 33%

e $\frac{1}{2}$ or 0.5 or 50%

f 25 times

26 a

Legs	Frequency
5	2
6	5
7	12
8	6
Total	**25**

b Seven legs

27 a 35 b The first three days.

28 a 4 or 5 b 150

29 a 0.7 b 0.3
c 56 d 167 (166.7)

30 No, as there are not enough trials

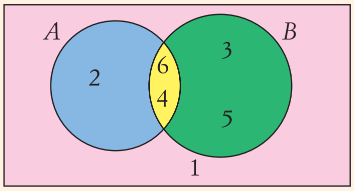
31 a $\frac{1}{20}$ or 0.05 or 5% b $\frac{9}{10}$ or 0.9 or 90%

c Lose about $18

32 The probability is $\frac{1}{2}$ or 0.5 or 50%. The past history does not change the probability.

Exercise 8.2

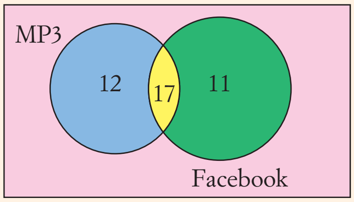
1 a



b $\frac{1}{2}$ c $\frac{2}{3}$ d $\frac{5}{6}$ e $\frac{1}{3}$

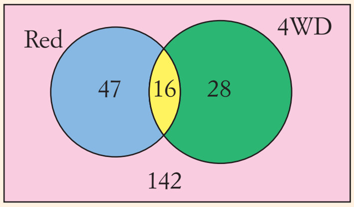
2 a $\frac{7}{15}$ b $\frac{1}{3}$ c $\frac{2}{3}$
d $\frac{2}{15}$ e $\frac{8}{15}$ f $\frac{2}{3}$

3 a



b $\frac{11}{40}$ c $\frac{3}{10}$

4 a



b $\frac{63}{233} \approx 27.0\%$ c $\frac{44}{233} \approx 18.9\%$ d $\frac{170}{233} \approx 73.0\%$
e $\frac{189}{233} \approx 81.1\%$ f $\frac{142}{233} \approx 60.9\%$

5 a

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{2}$ c $\frac{5}{12}$ d $\frac{7}{12}$
e $\frac{1}{6}$ f $\frac{5}{6}$ g $\frac{1}{9}$

6 a

Type	F	L	M	S
Meat lovers (ML)	F, ML	L, ML	M, ML	S, ML
Hawaiian (H)	F, H	L, H	M, H	S, H
Supreme (Sup)	F, Sup	L, Sup	M, Sup	S, Sup
4Cheeses (4C)	F, 4C	L, 4C	M, 4C	S, 4C
Vegetarian (V)	F, V	L, V	M, V	S, V

9780170194778

b $\frac{1}{4}$ c $\frac{1}{5}$ d $\frac{1}{4}$
e $\frac{1}{5}$ f $\frac{1}{20}$ g $\frac{2}{5}$

7 **a** {2, 4, 6, 8, 10}
b {R, R, R, R, R, G, G, G}
c {b, g, j, p, q, r, s, w, x, z}

8 **a** a tail
b a consonant
c {January, February, March, April, May, June, July, August}
d losing
e {1, 2, 3, 4, 5, 6, 7}
f an odd number

9 **a** {1, 2, 3, 4, 5} **b** {2, 4, 6}
c {1, 4, 6} **d** {3, 4, 5, 6}

10 **a** $\frac{7}{11}$ **b** $\frac{4}{11}$ **c** $\frac{4}{11}$
d $\frac{7}{11}$ **e** 1 **f** $P(A') = P(B)$
g $P(B') = P(A)$

11 **a** $\frac{11}{18}$ **b** $\frac{7}{18}$ **c** $\frac{7}{18}$
d $\frac{11}{18}$ **e** 1 **f** $P(M') = P(N)$
g $P(N') = P(M)$

12 D

13 **a** 0.2 **b** 0.8 **c** 0.4 **d** 0.6

14 **a**

	H	T
H	HH	HT
T	TH	TT

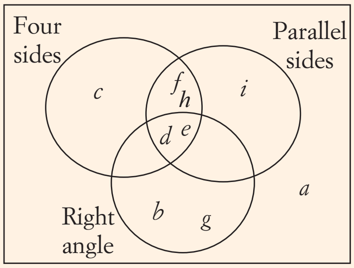
b $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$

15 **a**

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b $\frac{1}{36}$ **c** $\frac{35}{36}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$
f $\frac{5}{12}$ **g** $\frac{7}{12}$ **h** $\frac{5}{18}$ **i** $\frac{13}{18}$

16 **a**

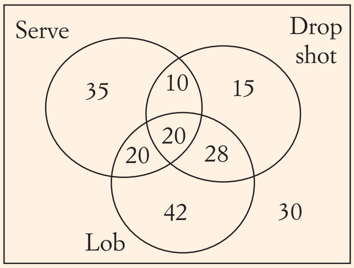


b $\frac{2}{9}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$ **e** $\frac{1}{9}$
f $\frac{4}{9}$ **g** $\frac{2}{9}$ **h** $\frac{2}{9}$

17 **a** 26 **b** Outcomes are equally likely.
c $\frac{4}{13}$ **d** $\frac{9}{13}$ **e** $\frac{5}{26}$ **f** $\frac{21}{26}$

18 **a** 20 **b** Outcomes are equally likely.
c $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{2}{5}$ **f** $\frac{3}{5}$
g 1 **h** 0

19 **a**



b $\frac{1}{10}$ **c** $\frac{23}{40}$ **d** $\frac{17}{40}$ **e** $\frac{21}{100}$

20 **a**

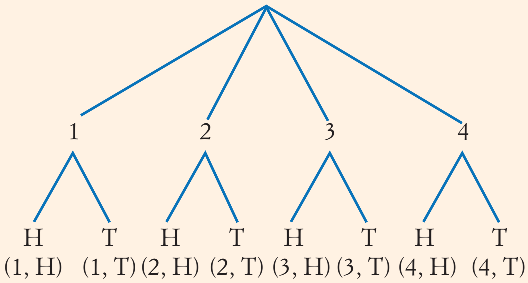
	Music	Sport	TV	Totals
Men	4	13	8	25
Women	11	6	8	25
Totals	15	19	16	50

b 60 **c** 32 **d** 52

21 **a** $\frac{33}{100}$ **b** $\frac{67}{100}$

Exercise 8.3

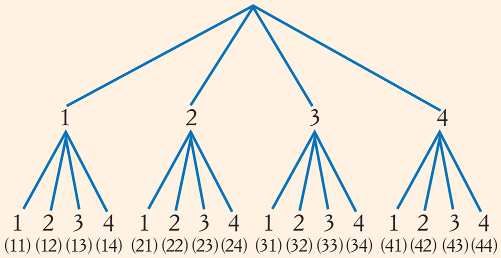
1



2 **a** $\frac{1}{8}$ **b** $\frac{5}{8}$ **c** $\frac{1}{4}$ **d** $\frac{3}{4}$

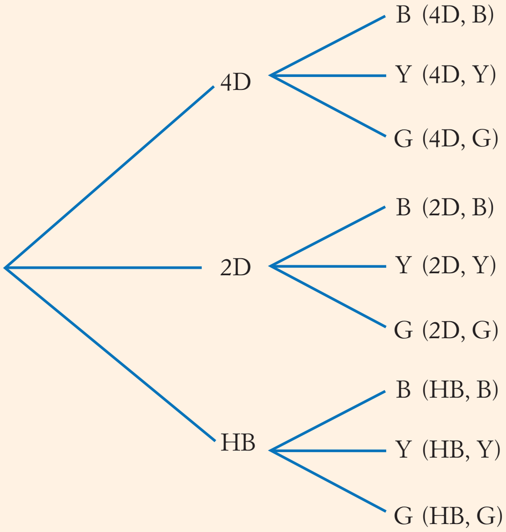
9780170194778

3



4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
d $\frac{1}{4}$ **e** $\frac{1}{8}$ **f** $\frac{1}{16}$

5



6 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{1}{9}$ **d** $\frac{2}{3}$

7

	R	**G**	**G**	**B**	**B**
R	RR	RG	RG	RB	RB
G	GR	GG	GG	GB	GB
G	GR	GG	GG	GB	GB
B	BR	BG	BG	BB	BB
B	BR	BG	BG	BB	BB

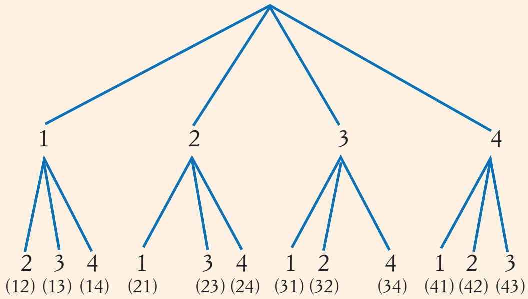
8 **a** $\frac{9}{25}$ **b** $\frac{16}{25}$ **c** $\frac{8}{25}$
d $\frac{4}{25}$ **e** $\frac{1}{25}$ **f** $\frac{4}{25}$

9

	♥	♦	♣	♠
♥	♥♥	♥♦	♥♣	♥♠
♦	♦♥	♦♦	♦♣	♦♠
♣	♣♥	♣♦	♣♣	♣♠
♠	♠♥	♠♦	♠♣	♠♠

10 **a** $\frac{1}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{4}$ **d** $\frac{1}{4}$
e $\frac{1}{16}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{4}$

11



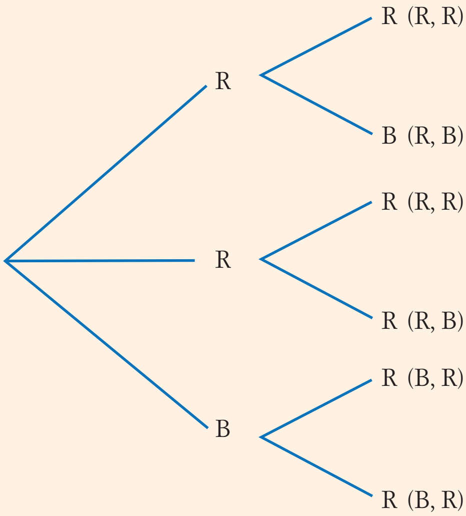
12 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
d $\frac{1}{4}$ **e** $\frac{1}{6}$ **f** $\frac{1}{12}$

13

	♥	♦	♣	♠
A	A♥	A♦	A♣	A♠
2	2♥	2♦	2♣	2♠
3	3♥	3♦	3♣	3♠
4	4♥	4♦	4♣	4♠
5	5♥	5♦	5♣	5♠
6	6♥	6♦	6♣	6♠
7	7♥	7♦	7♣	7♠
8	8♥	8♦	8♣	8♠
9	9♥	9♦	9♣	9♠
10	10♥	10♦	10♣	10♠
J	J♥	J♦	J♣	J♠
Q	Q♥	Q♦	Q♣	Q♠
K	K♥	K♦	K♣	K♠

a $\frac{1}{52}$ **b** $\frac{1}{26}$ **c** $\frac{1}{26}$ **d** $\frac{1}{13}$

14



a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$
d $\frac{1}{3}$ **e** $\frac{1}{3}$ **f** 0

9780170194778

15

	R	G	G	B	B
R		RG	RG	RB	RB
G	GR		GG	GB	GB
G	GR	GG		GB	GB
B	BR	BG	BG		BB
B	BR	BG	BG	BB	

a $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{2}{5}$
d $\frac{1}{5}$ **e** 0 **f** $\frac{1}{10}$

16

	1	2	3	4	5	6
1		12	13	14	15	16
2	21		23	24	25	26
3	31	32		34	35	36
4	41	42	43		45	46
5	51	52	53	54		56
6	61	62	63	64	65	

a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$
d $\frac{1}{3}$ **e** $\frac{1}{15}$ **f** $\frac{1}{30}$

17 a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

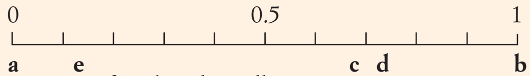
18 a $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{1}{3}$

19 a 20 **b** 25

20 $\frac{1}{2}$

21 $\frac{15}{16}$

Chapter review

1


The positions of c, d and e will vary.

2 a less than an even chance
b less than an even chance
c more than an even chance
d more than an even chance
e less than an even chance

3 There is a greater chance of tossing heads than drawing a king from a normal pack.

4 a No **b** Yellow
c Red **d** Yellow, green, blue, red.

5 Hereford

6 a

Outcome	Tally	Frequency
Blue	卌 卌 卌 卌 IIII	24
Green	卌 卌 卌 卌	20
Red	卌 卌 卌 III	18
Yellow	卌 卌 卌 II	17
Pink	卌 卌 卌 卌 I	21

b 100 **c** 0.24 **d** 0.18 **e** 0.21

7 a i {Green, pink}
ii Each event is equally likely.
b i {A, B, C, D}
ii Each event is equally likely.
c i {1, 2, 3}
ii The events are not equally likely.

8 a {1 4 6 8 9 10}
b {🖎, ✍, 🖓, ☞, ☝, 🖐}
c {2, 3, 4, 6, 8, 9, 10, 12}

9 A coin falling heads or tails; cutting a black or red card; etc.

10

Breed	Relative frequency
Angus	0.122, 12.2%
Hereford	0.357, 35.7%
Brahman	0.204, 20.4%
Simmental	0.184, 18.4%
Charolais	0.133, 13.3%

11 a 40
b 1: 20%, 2: 15%, 3: 12.5%, 4: 25%, 5: 15%, 6: 12.5%
c 4
d 3 or 6
e Maybe an even number (21 to 19 not significant in such a small sample)

12 a {R R R R R R G G G G G G G G G}
b 0.4

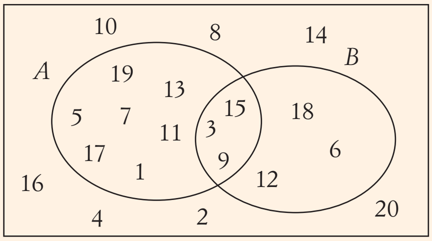
13 a {G G G G G G G G G R R R R P **B B B B B B**}
b 6 **c** 0.3 **d** 0.45
e 0.2 **f** 0.05 **g** 1

14 a {2 3 5 6 7 9 11 13}
b probabilities of each outcome = 0.125
c 0.25
d 0.75
e 0.75

9780170194778

15 a $\frac{1}{6}$ **b** $\frac{1}{2}$

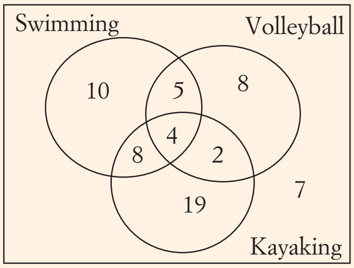
16 a



b $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** $\frac{3}{20}$

e $\frac{13}{20}$ **f** $\frac{1}{2}$ **g** $\frac{7}{10}$

17 a



b $\frac{7}{63} = \frac{1}{9}$ **c** $\frac{37}{63}$ **d** $\frac{19}{63}$

18 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

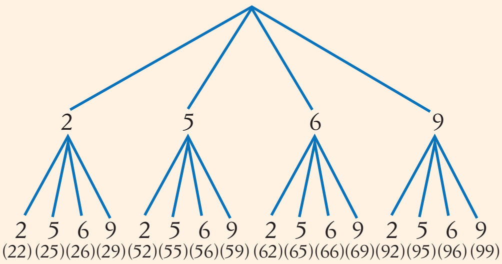
b $\frac{1}{2}$ **c** $\frac{7}{12}$ **d** $\frac{5}{12}$

e $\frac{5}{18}$ **f** $\frac{2}{3}$ **g** $\frac{1}{9}$

19 a $\frac{1}{6}$ **b** $\frac{5}{6}$

20 a 3 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$

21 a



b $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{5}{8}$

e $\frac{3}{8}$ **f** $\frac{1}{4}$ **g** $\frac{3}{4}$

22 a

	M	D	D	W	W
M		M, D	M, D	M, W	M, W
D	D, M		D, D	D, W	D, W
D	D, M	D, D		D, W	D, W
W	W, M	W, D	W, D		W, W
W	W, M	W, D	W, D	W, W	

b $\frac{1}{5}$ **c** $\frac{4}{5}$ **d** $\frac{2}{5}$

e $\frac{1}{5}$ **f** $\frac{1}{10}$ **g** $\frac{1}{10}$

23 Over 30 litres is likely because the frequency is higher.

24 a i 0.52 **ii** 0.48

b $P(H) = P(T) = 0.5$

25 a $\frac{13}{18}$ **b** $\frac{5}{18}$ **c** $\frac{5}{18}$

d $\frac{13}{18}$ **e** 1 **f** $P(X') = P(Y)$

g $P(Y') = P(X)$

26 About 20

27 10 or 11

28 a $\frac{9}{50}$ **b** $\frac{9}{25}$

29 a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{3}{8}$

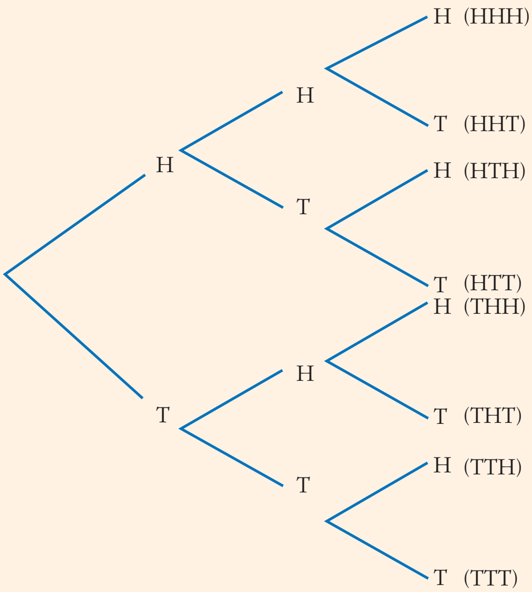
30 No. There were too few trials on which to base a prediction.

31 a About 7 times

b 41 or 42 times

c About 229 times

32 a



9780170194778

b {HHH HHT HTH HTT THH THT TTH TTT}

c $\frac{3}{8}$ d $\frac{1}{2}$ e $\frac{3}{8}$

f 87 or 88 times g 37 or 38 times

33 $\frac{3}{8}$

Chapter 9

Exercise 9.1

1 a $3p, 4p, {}^{-}3p$
b $6x, {}^{-}7x, x$
c $7mn^2, {}^{-}7mn^2$
d $bc, {}^{-}3bc$ and $4b, -b, 6b$
e $5ef, 2ef, -ef$ and $4f^2, 6f^2$
f $6, {}^{-}7$ and $8m, m$
g $2k, \frac{1}{2}k, -k$
h $2x^2, x^2/2, 3xy, \frac{1}{4}xy$
i $4(x+y), \dfrac{x+y}{5}$

2 $7w$

3 $\$\dfrac{C}{30}$

4 $1000p + 10q + r$

5 $T + 273.15$ °K

6 a $6a$ b $9b$ c $7m$ d $8n$
e 4θ f 6α g $7w$ h $16p$
i $5a$ j 0 k $6a$ l $5y$
m k n 9θ o $3n$ p $14b$

7 a $4a$ b $5m$ c $22t$ d $3a - 2b$
e $4rt$ f $6 + 12d$ g $2a$ h $a - 1$

8 a $9a$ b $12n$ c $8a + 13c$ d $18t$
e $17y$ f $3g$ g $7w$ h $4a$

9 a $2a - 3q + 4$ b ${}^{-}9t - 10m - 3$
c ${}^{-}4t - 2c - 8$ d $g + 13h + 6$
e $6k + 12g - 8$ f $8 - 2b$
g $5w - 6x - 6$ h $7g + 2$

10 a $8h - 3v + 7b$ b $5d - 3z - 3a$
c $2d - 9m$ d $4y$
e $6y - 5p - 12x + 7$ f $4x - 2w - 12k - 4$
g $4u - c - 3w - 5$ h $2z - 8k - 9$

11 a $2x^2 - x + 12$ b $5m^2 - 15m + 4$
c $5t^2 + 10t + 1$ d $-w^2 - 3w - 10$

12 a $7ab + 3a - 13b$
b $3x^2 + 4x - 3 + 4xy - 5y$
c $4xyz + 7xy + 5y + 5z$
d $7ab + 2ab^3 - 4ab^2 - 40a^2b + ab$

13 a $12r$ b $8s$ c $6dg$ d $10a$
e $12mnp$ f $12xy$ g $8ab$ h $24uvw$

14 a $4h$ b $4cd$ c $5gh$ d $10y$
e $5p$ f 3 g 7 h $2f$

15 a ${}^{-}2t^2g^3n$ b ${}^{-}15y^4u^3$ c ${}^{-}21v^3n^4d$
d ${}^{-}4h^4k^2d$ e $21c^2d^2h^3$ f $35a^2b^3q^5$
g $6a^2q^4m^3$ h ${}^{-}8n^3p^3h$

16 a $\dfrac{{}^{-}p^2k}{4w}$ b $\dfrac{{}^{-}5td^2}{7h^3}$ c $\dfrac{7t}{2k^2}$ d $\dfrac{{}^{-}7c^3}{8d}$
e $\dfrac{3}{4u}$ f $\dfrac{p^2}{4x^2}$ g $\dfrac{{}^{-}2wn}{5c}$ h $\dfrac{9q^2x}{2m^3}$

17 a $\dfrac{6y}{a}$ b $\dfrac{25a}{6bc}$ c $\dfrac{2r}{3t}$ d $\dfrac{m^2}{n^2p}$
e $\dfrac{12y^2}{5xz}$

18 About 38 **19** About 35.6 °C

20 mc, \$16.78, \$16.80

21 40 m **22** 50 N **23** 9.8 m/s^2

24 $\dfrac{n}{4}$ cups, $1\frac{1}{2}$ cups

25 $u + Vd$ kg, the mass of the truck and the mass of the load, $\dfrac{u+Vd}{1000}$t.

26 $\dfrac{dwc}{a}$, the area of the wall divided by the coverage and multiplied by the cost per litre. It may underestimate the cost because you cannot buy paint in any quantity; usually only 500 mL, 1 L, 4 L and 10 L tins.

Exercise 9.2

1 a $6a + 18$ b $5r + 25$ c $3w + 12$
d $5g + 40$ e $8d + 64$ f $6a + 42$
g $2h + 14$ h $3t + 21$ i $6t + 42$
j $3r + 15$

9780170194778

2 **a** $^{-}8m - 24$ **b** $^{-}3n - 21$ **c** $^{-}8x - 56$
d $^{-}4u - 16$ **e** $^{-}2z - 6$ **f** $^{-}3y - 9$
g $^{-}3r - 9$ **h** $^{-}6d - 18$ **i** $^{-}3p - 12$
j $^{-}5v - 35$

3 **a** $2v - 8$ **b** $2t - 6$ **c** $6r - 18$
d $2p - 4$ **e** $2h - 14$ **f** $3p - 24$
g $2g - 16$ **h** $3h - 18$ **i** $5r - 20$
j $2q - 10$

4 **a** $12 - 3r$ **b** $16 - 8a$ **c** $30 - 6r$
d $24 - 3x$ **e** $32 - 8w$ **f** $32 - 8q$
g $21 - 3v$ **h** $20 - 5x$ **i** $18 - 6a$
j $12 - 3q$

5 **a** $10u - 4$ **b** $^{-}18a + 12$ **c** $12g - 9$
d $21w + 14$ **e** $^{-}48u + 18$ **f** $^{-}10t + 35$
g $^{-}15q + 20$ **h** $12g + 15$ **i** $6a - 10$
j $28h + 16$

6 **a** $18y^2 - 4ym$ **b** $72hx - 72h^2$
c $^{-}3c^2 - 27cr$ **d** $16n^3 - 64nc$
e $24x^3 - 54x^2u$ **f** $32v^2h + 16vh^2$
g $^{-}20b^2x - 8bx^2$
h $5x^2mn - 45x^3m - 15x^2m^2$
i $25am^3 - 10a^2m^2 - 20am^2p$
j $10w^2z^2 - 4w^3 - 6w^2z + 4w$

7 **a** $32b^2 - 20ba + 16bd$
b $^{-}24m^2 + 20mn - 32mz$
c $6y^2 + 15yn + 6yh$ **d** $48a^2 + 56ab - 32ap$
e $35c^2 - 35cn - 40cy$ **f** $^{-}20k^2 - 40kq - 5kt$
g $63x^2 - 35xp - 28xg$
h $49d^2 + 63dg + 14dm$
i $^{-}15m^2 + 15mz + 40my$
j $16r^2 + 28rc + 24rz$

8 **a** $9t + 50$ **b** $7t - 43$ **c** $13w + 18$
d $3n - 16$ **e** $3d - 21$ **f** $4p - 7$
g 88 **h** $16 - 4w$ **i** $^{-}5r - 25$
j $2h - 46$

9 **a** $13n - 62$ **b** $^{-}4w - 32$ **c** $9x + 48$
d $79 - 3a$ **e** $9b - 3$ **f** $9w$
g $^{-}39$ **h** $^{-}6b - 38$ **i** $4b - 56$
j 6

10 **a** $qz - 7q^2 - 77qh$ **b** $61g^2 + 81gq + 56gm$
c $32u^2 + 50ut$ **d** $47h^2 + 17hr + 42hn$
e $72rk - 40r^2 - 4rd$
f $^{-}48g^2 - 108gb - 60gt$
g $14a^2 - 2ap - 53ag$ **h** $22c^2 - 24cu - 70ce$
i $55w^2 + 40wu - 45wy$
j $119db - 117d^2 - 2dv$

11 **a** $w^2 + 10w + 24$ **b** $r^2 + 11r + 30$
c $z^2 - 8z + 15$ **d** $t^2 + 14t + 45$
e $p^2 - 3p - 18$ **f** $u^2 - 12u + 27$
g $x^2 + 4x - 32$ **h** $t^2 + 12t + 27$
i $r^2 - 7r + 12$ **j** $k^2 - 5k - 36$
k $t^2 + 3t - 28$ **l** $q^2 - 3q - 18$
m $q^2 - 12q + 32$ **n** $w^2 + 12w + 27$
o $t^2 - 5t - 36$ **p** $x^2 - 2x - 8$
q $m^2 - 9m + 18$ **r** $c^2 - 2c - 35$
s $d^2 - 8d + 16$ **t** $u^2 + 10u + 16$

12 **a** $28q^2 + 11qd - 24d^2$ **b** $20x^2 + 7xt - 6t^2$
c $20d^2 + 29dy - 36y^2$ **d** $16u^2 + 78ut + 27t^2$
e $10g^2 - 31gp + 24p^2$ **f** $32w^2 - 76wp + 45p^2$
g $18w^2 + 53wq + 20q^2$ **h** $32z^2 - 68zn + 35n^2$
i $56g^2 + 19gw - 15w^2$ **j** $12v^2 + 1vn - 6n^2$
k $35x^2 - 34xg - 21g^2$ **l** $14d^2 - 23dy - 30y^2$

13 $600 \times 8(w - 22) = 4800w - 13\,200\ \text{cm}^3$ or $\dfrac{48w - 132}{10\,000}\ \text{m}^3$, $0.0576\ \text{m}^3$

14 $2.5(w - 5)(2.5w - 5) = 6.25w^2 - 8.75w + 62.5\ \text{cm}^3$, $1687.5\ \text{cm}^3 = 1.6875$ L

15 Change the 72 L to 72 000 cm^3 and solve the equation $4800w - 13\,200 = 72\,000$.

16 Write $(x + y)^2 = (x + y)(x + y)$ and expand the brackets, then write $(x + y)^3 = (x + y)(x + y)^2$, put in the expression for $(x + y)^2$ and expand the brackets again.

Exercise 9.3

1 **a** 35 **b** 33 **c** 5 **d** 21 **e** 77
f 7 **g** 15 **h** 12 **i** 33 **j** 11

2 **a** $4gy$ **b** $3h$ **c** $3pb$ **d** mqb **e** wb
f $2g^2u$ **g** z **h** $3g$ **i** $3m$ **j** $4gq$

3 **a** $5(w + 9)$ **b** $4(t + 2)$ **c** $3(r + 2)$
d $5(z + 2)$ **e** $4(k + 8)$ **f** $8(d + 1)$
g $5(t + 9)$ **h** $7(x + 9)$ **i** $5(w + 9)$
j $4(w + 5)$

4 **a** $10(4m + 3)$ **b** $3(4m + 6)$ **c** $4(d + 8)$
d $18(3v + 2)$ **e** $3(c + 5)$ **f** $8(6z + 7)$

9780170194778

g $2(7r+1)$ **h** $12(3k+2)$ **i** $18(n+2)$
j $5(9x+8)$

5 **a** $12(2t+q)$ **b** $8(w+9)$ **c** $2(5u+9)$
d $8(4c+3)$ **e** $8(2r+3)$ **f** $12(2m+3)$
g $6(2h+1)$ **h** $6(4h+7)$ **i** $4(k+3)$
j $24(2h+1)$

6 **a** $6(5u-3w-7g)$ **b** $9(5p-1u+7a)$
c $2(4r+5x+7n)$ **d** $4(5h-3b+1d)$
e $3(8n+3u-7v)$ **f** $4(6u-1p-3a)$
g $9(8g+8b-7b)$ **h** $9(5n-5r+3w)$
i $8(7d-8g-9v)$ **j** $8(9u-3m+5v)$

7 **a** $^-5(z+3x)$ **b** $^-6(2h+t)$
c $^-6(2y+3b)$ **d** $^-8(5k+2w)$
e $^-9(u+4a)$ **f** $^-20(b+k)$
g $^-7(7y+8w)$ **h** $^-4(6v+7w)$
i $^-9(2n+3z)$ **j** $^-3(3d+8v)$

8 **a** $4(3q-2d)$ **b** $8(m-2g)$ **c** $8(h-4g)$
d $4(3h+5u)$ **e** $7(7a-3k)$ **f** $6(7c-2k)$
g $9(4z-5t)$ **h** $9(7h-3r)$ **i** $^-15(3y+2g)$
j $20(2b-k)$ **k** $8(6b-r)$ **l** $^-2(7u+3r)$
m $3(7c-8d)$ **n** $6(2t-3x)$ **o** $12(2t-m)$
p $9(8c-w)$ **q** $^-7(6v+7r)$ **r** $^-2(3b+5x)$
s $4(7y+n)$ **t** $5(3y-4a)$

9 **a** $9(6g-2t+3w)$ **b** $2(5v-8z-7q)$
c $6(6z-3d+2w)$ **d** $5(2k-6r+7y)$
e $8(5y-w+7c)$ **f** $6(3d-7t-6r)$
g $8(4u-9x-2g)$ **h** $3(t-6u+4v)$
i $4(7x-b+5y)$ **j** $^-7(5y+k+6v)$
k $3(a-8h-6b)$ **l** $16(3y-t+2q)$
m $8(2q+v+8a)$ **n** $3(6h-2b-t)$
o $7(y+n+2q)$ **p** $6(3x-2c+7n)$
q $6(b+4y+3d)$ **r** $3(7z-w+c)$
s $4(8h+2m+5v)$ **t** $2(4w-6z+7c)$

10 **a** $^-9q(q+7u)$ **b** $4v(7z-4v)$
c $2w(2w-9z)$ **d** $9v(9v+4u)$
e $^-2r(r+v)$ **f** $6x(x+6h)$
g $7g(g+3g)$ **h** $^-9d(2d+w)$
i $3t(2m-3t)$ **j** $5t(7t-8u)$
k $18w(3w+2b)$ **l** $2b(w-7b)$
m $6w(8p-w)$ **n** $^-7k(5k+4n)$
o $6a(a-3n)$ **p** $^-8c(8c+7w)$
q $7t(5r-2t)$ **r** $21r(2r+k)$
s $12u(2z-3u)$ **t** $4t(7t-8d)$

11 **a** $6y(7y-6g+4t)$ **b** $^-7v(7v+3m+2g)$
c $7u(6u-2q-5w)$ **d** $8k(9r-8k-6n)$
e $4q(2q-6t+3a)$ **f** $12q(q-c+3z)$
g $4m(7m-9u-3b)$ **h** $4q(3q-4d+2a)$
i $3b(m-6b-9q)$ **j** $2z(5k-8z-6b)$
k $9u(2r-8v^2+9q)$ **l** $7r(3r-6w-7d)$
m $18y(y-3a-2d)$ **n** $n(5n+p-r)$
o $-x(9x+6y+k)$ **p** $9x(3x-4w+2m)$
q $3n(6n-7q+6u)$ **r** $8n(5n-9c+5k)$
s $6p(6y-5p+a)$ **t** $8d(d+5c+2a)$

12 $(3x+5)(4b+7)$, using $(4b+7)$ as a common factor

13 $(2m^2-3)(m^2+1)$, using (m^2+1) as a common factor

14 Create a common factor by factorising the first two terms and the last 2 terms separately to give $3c(a+2b)-7d(a+2b)=(3c-7d)(a+2b)$.

15 Create a common factor by factorising the first two terms and the last 2 terms separately to give $3v(3x-5y)+4u(3x-5y)=(3v+4u)(3x-5y)$.

Chapter review

1 **a** 33 **b** 84

2 **a** $4de$, ^-4ed, $\frac{1}{4}de$
b $-kmg$, $2kmg$ and $4kg$, $2kg$

3 **a** $5x+40$ **b** $^-4c-8$ **c** $7k-28$ **d** $30-6t$

4 **a** 4 **b** $5xy$

5 **a** $7(h+3)$ **b** $15(3u+5)$
c $4(4p-7q+5r)$ **d** $^-3(3b+4c)$

6 $1000k+m$

7 $365n+m$

8 **a** $12x$ **b** $7m$ **c** $8q-3p$
d $10mn-11m^2-4n^2+6$

9 **a** $56f$ **b** $7c$ **c** $^-20m^2np$
d $14x^4y^2z$ **e** $\dfrac{^-5gk}{3h^3p^2}$

10 **a** $^-6u-15$ **b** $35d-42$
c $9-12p$ **d** $12m-32$

11 **a** $6x^2-8xy$ **b** $15p^2-10pq$
c $24h^2-8hk+12h$ **d** $15uw-12u^2-9uv$

12 **a** $27-3m$
b $10p-8$
c $53mn-8m^2-31mp$

9780170194778

13 $\dfrac{3kn}{pm}$

14 a $z^2 + 7z + 10$ b $c^2 - 5c - 14$
c $12k^2 - 29kd + 15d^2$ d $15x^2 + 4xy - 4y^2$

15 a $6(4y - 3x - 5z)$ b $14(2n - 3m)$
c $3(5n - 2p + 6q)$ d $13(4a - 3b)$

16 a $3y(8y - 3z)$ b $5e(7a - 15e)$
c $^{-}4b(18b + 6c + 7d)$ d $3r(4r - 5t + 10u)$

17 11.8 m

18 About 6.4 cm

19 48 cn^2

20 About 2.5 s

21 a $(g^2 - 2h)(4m + 3)$ b $(2n + 5p)(m - 6n)$

22 $H = f + nD$, fixed cost plus daily charge. $4400

23 $C = \dfrac{56whb}{100} = \dfrac{14whb}{25}$, area of bricks multiplied by 56, multiplied by the cost per 100 divided by 100. $6289.92, but if the bricks can only be bought in lots of 100 or pallets (which might have more bricks in them) it may be more.

24 The width of each cover will be $2w + s + 1$ cm and the height will be $h + 1$ cm, so $A = (h + 1)(2w + s + 1)$ cm^2 $= 2wh + hs + h + 2w + s + 1$ cm^2. The area of the insert is 334.1 cm^2 and the sheet is 9999.49 cm^2, so $9999.49 \div 334.1 \approx 29$ can be made if the inserts fit almost exactly on the sheet. Otherwise it will be less.

25 Add the area of the rectangle and triangle, so the area is $ab + \frac{1}{2}ac = a(b + \frac{1}{2}c)$ or $\dfrac{a(2b + c)}{2}$. 33 cm^2

Chapter 10

Exercise 10.1

1 a 10 b 12 c $11\frac{1}{2}$
d $19\frac{1}{2}$ e 15

2 a 12 cm^2, 1200 mm^2 b 14 cm^2, 1400 mm^2

3 a 98 cm^2 b 144 mm^2 c 56 m^2
d 78.5 km^2 e 39.06 cm^2 f 47.97 mm^2
g 211.1 m^2

4 a 136 cm^2 b 6.48 m^2 c 707 mm^2
d 2.89 km^2

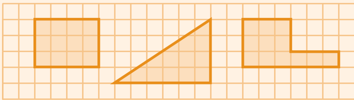
5 a 300 mm^2 b 800 cm^2 c 10 400 m^2
d 920 ha e 3.7 cm^2 f 5.8 ha
g 0.95 m^2 h 20 m^2 i 130 cm^2

6 a 0.812 m^2 b 21.66 cm^2 c 20 cm^2
d 0.287 km^2 e 8.575 cm^2 f 0.5 km^2

7 a 400 cm^2 b 900 m^2 c 0.5 m^2
d 85 m^2 e 25 mm^2 f 2 ha
g 8 000 000 km^2 h 1.2 mm^2

8 $12 075

9 Examples:

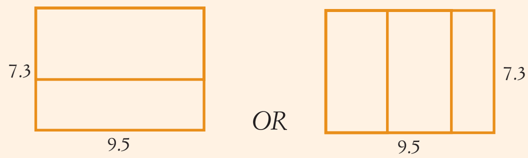


10 a 957.1 m^2 b 14.99 cm^2

11 a 85.8 cm^2 b 444.2 mm^2 c 10.4 m^2
d 145 km^2

12 a 218.3 mm^2 b 573.0 cm^2

13 a

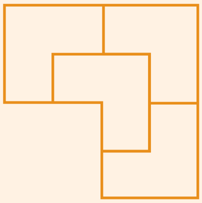


b 19 m; 21.9 m c Saving = $1484.80

14 14

15 2 : 1 because the radii are always in the same ratio.

16



Exercise 10.2

1 a $A = RV \times AB$ b $A = b \times e$
c $A = XY \times KL$

2 a $A = \frac{1}{2}BD \times AC$ b $A = \frac{1}{2}xy$
c $A = \frac{1}{2}RP \times SQ$

3 a $A = \frac{1}{2}WY \times XZ$ b $A = \frac{1}{2}GI \times HJ$
c $A = \frac{1}{2}nm$

9780170194778

4 a $A = \frac{1}{2}(KN + LM) \times OP$
  b $A = \frac{1}{2}(RU + ST) \times BA$
  c $A = \frac{1}{2}(q + w) \times m$

5 a 136 cm^2 b 24.48 km^2 c 1248 mm^2

6 a 720 cm^2 b 7.28 m^2 c 1314 mm^2
  d 738 mm^2

7 a 9.88 cm^2 b 1044 m^2 c 1960 mm^2
  d 1886 cm^2

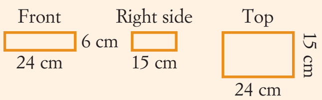
8 a 11.55 cm^2 b 579.5 mm^2 c 16.895 m^2
  d 975 cm^2 e 6.12 km^2 f 3922 cm^2
  g 262.5 mm^2 h 1078 m^2 i 1218 cm^2

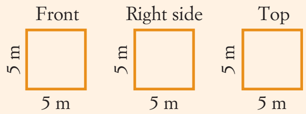
9 a 288 mm^2 b 10.4 m^2

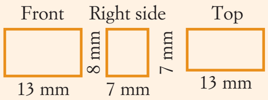
10 a 935 cm^2 b 17.235 m^2

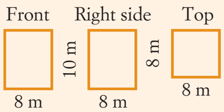
Exercise 10.3

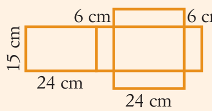
1 a Cylinder b Pyramid c Prism
  d Prism e Cone f Pyramid
  g Prism h Cylinder i Pyramid
  j Prism

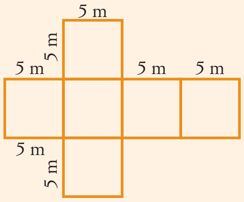
2 a


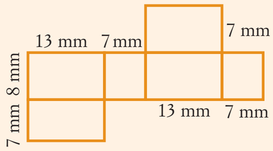
  b


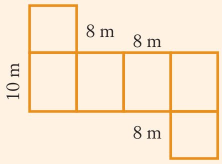
  c


  d


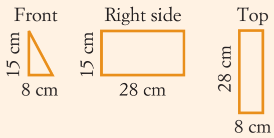
3 a


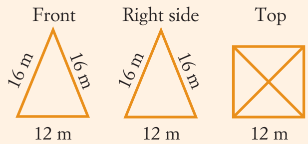
  b


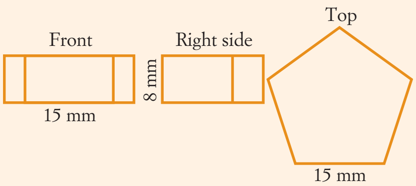
  c


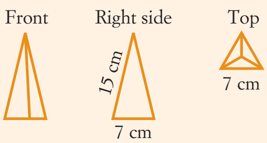
  d


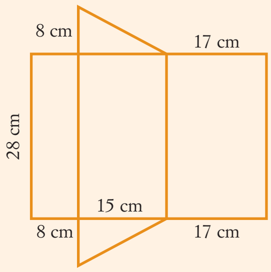
4 a Cylinder b Rectangular pyramid
  c Triangular prism d Square prism (or cube)
  e Cone f Pentagonal pyramid
  g Rectangular prism h Cylinder
  i Triangular pyramid j Heptagonal prism

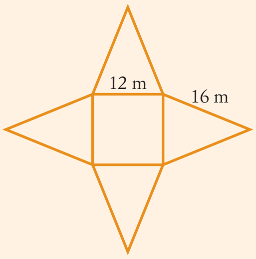
5 a


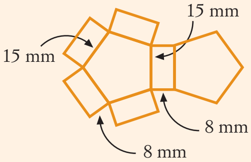
  b


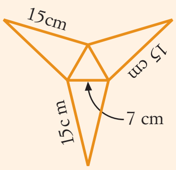
  c


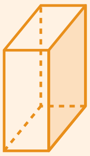
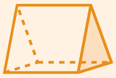
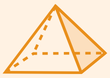
  d


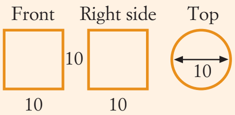
6 a


b 

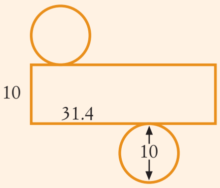
c 

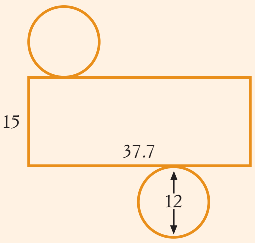
d 

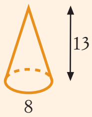
7 a 
b 
c 
d 

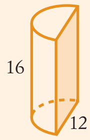
8 a 

b 

9 a 

b 

10 a 

b 

11 a 432 cm^2 b 180.28 m^2

12 a 90.05 cm^2 b 94.5 m^2

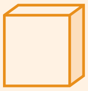
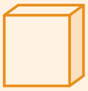
13 830 cm^2

14 1872 cm^2

15 a 1946 cm^2 b \$778.40

16 a 53.54 m^2 (excluding the floor)
b 52.04 m^2 c \$177

17 a 
b Not a net
c 
d Not a net

18 a 
b 
c 
d Not a net

19 a 
b Not a net
c 
d 

20 Four, but many more by rotation or reflection

Exercise 10.4

1 a 4 cm^3 b 10 cm^3 c 24 cm^3 d 34 cm^3
e 14 cm^3 f 20 cm^3 g 52 cm^3

2 a 264 cm^3 b 300 m^3 c 343 cm^3 d 384 m^3

3 a 262.5 cm^3 b 252 m^3 c 1020 mm^3
d 58.2 m^3 e 125.3 mm^3 f 653.8 cm^3

4 a 48 m^3 b 148.877 cm^3 c 210.6 cm^3
d 212.16 m^3

5 a 10 816 cm^3 b 8.398 m^3 c 8262 mm^3
d 7128 cm^3 e 13.717 m^3 f 126.522 cm^3

6 a 9288 cm^3 b 32.13 m^3 c 12 240 mm^3

9780170194778

7 **a** 12 315.0 cm^3 **b** 52.9 m^3 **c** 14 476.5 mm^3
d 114 991.7 cm^3 **e** 273.0 m^3 **f** 192.1 cm^3

8 **a** m^3 **b** cm^3 **c** m^3
d m^3 **e** cm^3 **f** cm^3

9 **a** 1260 cm^3 **b** 42.7 m^3 **c** 3429 cm^3
d 15 252 cm^3

10 **a** 136.05 m^3 **b** 761 cm^3

11 4.52 m^3

12 1256.6 cm

Exercise 10.5

1 **a** AC **b** GI **c** KL **d** ST
e QR **f** XZ

2 **a** b **b** p **c** x **d** w
e m **f** d

3 **a** $\angle D$ **b** $\angle F$ **c** $\angle G$ **d** $\angle S$
e $\angle P$ **f** $\angle M$

4 **a** g **b** t **c** k **d** x
e b **f** w

5 **a** Yes **b** Yes **c** Yes **d** No
e Yes **f** No

6 $c < 26$ cm

7 **a** **i** 144, 81, 225 **ii** $FG^2 = FH^2 + HG^2$
b **i** 576, 100, 676 **ii** $KP^2 = KL^2 + LP^2$
c **i** 5.76, 0.49, 6.25 **ii** $VU^2 = VK^2 + KU^2$
d **i** 56.25, 100, 156.25 **ii** $ZY^2 = ZA^2 + AY^2$

Exercise 10.6

1 **a** 1500 h **b** 0300 h **c** 1200 h
d 2240 h **e** 2019 h **f** 2106 h
g 1120 h **h** 2340 h **i** 0307 h
j 0845 h **k** 2202 h **l** 0005 h
m 0642 h **n** 0215 h **o** 2347 h
p 1023 h

2 **a** 9:00 a.m. **b** 3:00 p.m. **c** 5:30 a.m.
d 5:40 p.m. **e** 8:42 a.m. **f** 10:30 p.m.
g 3:50 p.m. **h** 10:30 p.m. **i** 12:10 a.m.
j 1:50 p.m. **k** 4:10 a.m. **l** 6:35 p.m.
m 9:33 p.m. **n** 2:31 a.m. **o** 10:52 a.m.
p 9:01 p.m.

3 **a** 1 h 5 min **b** 1 h 50 min **c** 5 h 44 min
d 3 h 53 min **e** 5 h 45 min **f** 11 h 38 min
g 2 h 25 min **h** 43 min **i** 1 h 47 min
j 4 h 8 min **k** 7 h 39 min **l** 13 h 25 min

4 **a** 8 h 44 min **b** 8 h 50 min **c** 16 h 7 min
d 13 h 15 min **e** 12 h 18 min **f** 10 h 26 min

5 **a** 2 h 37 min **b** 8 h 35 min **c** 6 h 41 min
d 4 h 39 min **e** 3 h 44 min **f** 3 h 44 min

6 **a** 3 h 31 min **b** 2 h 26 min **c** 1 h 47 min
d 5 h 44 min **e** 8 h 39 min **f** 10 h 52 min
g 4 h 23 min **h** 1 h 17 min **i** 6 h 47 min
j 5 h 10 min **k** 7 h 55 min **l** 13 h 32 min

7 **a** 1 h 16 min **b** 2 h 53 min **c** 3 h 34 min
d 6 h 48 min **e** 5 h 55 min **f** 12 h 39 min
g 2 h 34 min **h** 2 h 26 min **i** 3 h 43 min
j 1 h 56 min **k** 13 h 36 min **l** 15 h 27 min

8 23:03 h.

9 **a** 47 min **b** 7:17 a.m. **c** 38 min
d 3 h 22 min

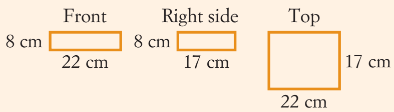
10 **a** 22 h 25 min **b** 11 h 10 min **c** 14 h 5 min
d 15 h 20 min

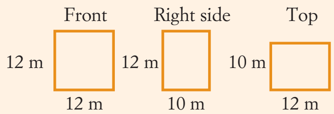
Chapter review

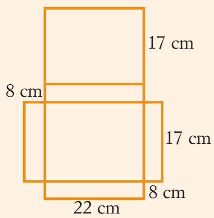
1 **a** 19 square units **b** 21 square units

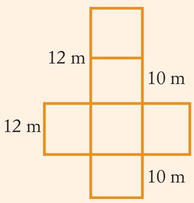
2 **a** $A = \frac{1}{2} AC \times BD$ **b** $A = m \times k$
c $A = \frac{1}{2}(m + z) \times g$ **d** $A = \frac{1}{2} WY \times ZX$

3 **a** Prism **b** Cylinder **c** Pyramid

4 **a**


b


5 **a**


b


6 27 m^3

Answers

7 30 cm^3

8 **a** 0700 h **b** 1016 h **c** 1824 h **d** 2318 h

9 **a** 3:00 a.m. **b** 5:20 p.m. **c** 12:32 a.m.
d 7:26 p.m.

10 **a** 12 h 24 min **b** 8 h 9 min

11 **a** 1.5 m^2 **b** 500 cm^2 **c** 800 mm^2

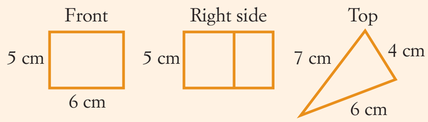
12 **a** 700 mm^2 **b** 550 cm^2 **c** 65 200 m^2
d 107 ha

13 **a** 5280 cm^2 **b** 442.5 mm^2 **c** 962.1 cm^2

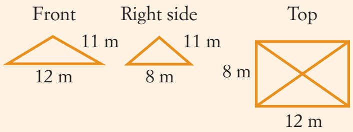
14 **a** 19.15 m^2 **b** 1890 cm^2 **c** 19.62 km^2
d 1064 mm^2

15 **a** Hexagonal prism **b** Dodecahedron
c Rectangular pyramid

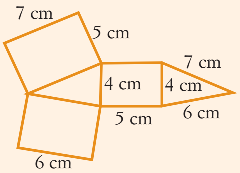
16 **a**



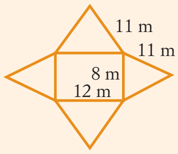
b



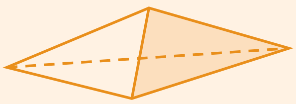
17 **a**



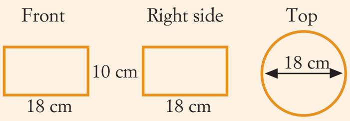
b



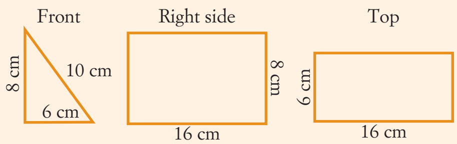
18



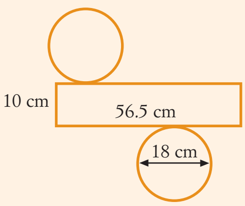
19 **a**



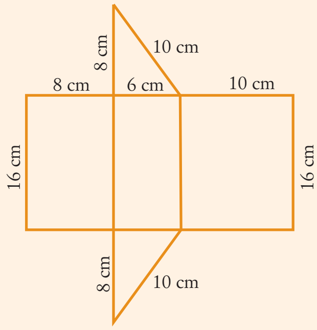
b



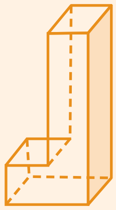
20 **a**



b



21



22 **a** 430 m^2 **b** 360 cm^2

23 101.01 m^3

24 **a** 7 h 20 min **b** 9 h 29 min

25 **a** 5 h 9 min **b** 1 h 37 min

26 **a** 5 h 37 min **b** 3 h 54 min

27 **a** 4 h 30 min **b** 3 h 8 min

28 **a** Not a net. **b** Net of a cube: 

29 **a** Net of a rectangular prism: 
b Not a net.

30 **a** Net of a cylinder: 
b Not a net (too many triangles).

31 1210 cm^2

32 Cubic centimetres

33 **a** 1527.9 cm^2 **b** 9.8 m^2

34 648 cm^3

35 384 cm^3

36 2544.7 cm^3

37 **a** *FE* **b** *RP* **c** *BD*

38 **a** *c* **b** *h* **c** *p*

39 **a** $\angle GBU$ **b** $\angle FCW$ **c** $\angle ODC$

40 **a** *d* **b** *k* **c** *p*

41 **a** Yes **b** No **c** Yes **d** No

9780170194778

42 **a** **i** $GH^2 = 400$, $FH^2 = 256$, $FG^2 = 144$
ii $GH^2 = FH^2 + FG^2$
b **i** $PL^2 = 1056.25$, $KL^2 = 900$, $KP^2 = 156.25$
ii $PL^2 = KL^2 + KP^2$

43 17 cm

Chapter 11

Exercise 11.1

1 **a** \$0.25 **b** \$0.60 **c** \$4.25 **d** \$3.05
e \$4.65

2 **a** 450 g **b** 750 g **c** 6 gig
d 950 g **e** Twin pack

3 **a** \$93.73 **b** \$93.75 **c** \$95.73

4 **a** \$85.69 **b** \$85.70 **c** \$87.19

5 **a** \$88.31 **b** \$88.30 **c** \$90.81

6 **a** \$327.50 **b** \$171.50

7 18 calls

8 \$598

9 **a** \$7860 **b** \$4116

10 \$1355 (including the cost of the phone)

11 \$14 352

12 **a**

Income	
Pocket money	\$12.00
Car washing	\$ 5.00
Present	\$10.00
Total	\$27.00

Expenses	
Tuckshop	\$15.00
Orchids	\$ 8.00
Total	\$23.00

b He had \$4 left.

13 **a**

Income	
Deliveries	\$160.00
Bank withdrawal	\$ 90.00
Total	\$250.00

Expenses	
Bike tube	\$ 12.80
Cosmetics	\$ 25.60
Jeans	\$105.50
Total	\$143.90

b She had \$106.10 left.

14 **a**

Income	
Earnings	\$420
Bank withdrawal	\$101
Total	\$521

Expenses	
Share of flat rent	\$130
Food and housekeeping	\$170
Kettle	\$ 56
Bus fares	\$ 30
Chocolates	\$ 20
Blouse	\$ 75
Cash for next week	\$ 40
Total	\$521

b \$839 **c** 8 weeks

15 **a**

Income	
Student fees (\$7 each)	\$2940
Total	\$2940

Expenses	
Seven buses	\$1365
Admission	\$1470
Total	\$2835

b \$105 **c** \$280

Exercise 11.2

1 **a** 40% **b** 30% **c** 65% **d** 25%
e 30% **f** 80% **g** 75% **h** 75%
i 85% **j** 68% **k** 8% **l** 8%

2 **a** $\frac{7}{20}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{4}{5}$
e $\frac{7}{10}$ **f** $\frac{11}{50}$ **g** $\frac{3}{20}$ **h** $\frac{24}{25}$

3 **a** 0.45 **b** 0.75 **c** 0.6 **d** 0.06
e 0.08 **f** 0.16 **g** 0.21 **h** 0.43

4 **a** \$150 **b** 200 kg **c** 14 g
d 49 L **e** \$30 **f** 25 mg
g 12 minutes **h** 12 hours

9780170194778

5 a $215 b $140 c 10.05 kg
d 9.66 m e 1.96 L f $10.64
g 4.8 t h 45 minutes

6 a 175 % b 72.5% c 36.5%
d 36.4% e 71.4% f 71.4%
g 800% h 77.3% i 144.2%
j 55.6% k 124% l 227.3%

7 a $1\frac{1}{4}$ b $2\frac{1}{2}$ c $1\frac{1}{5}$ d $1\frac{9}{20}$
e $1\frac{1}{10}$ f $\frac{3}{8}$ g $\frac{5}{8}$ h $1\frac{53}{100}$

8 a 0.425 b 0.156 c 0.007
d 1.24 e 2.07 f 1.4
g 0.236 h 0.056

9 a $37.50 b 2.068 kg c 15.39 g
d $4.70 e $3.05 f 1.05 L
g 9.5 s h 292.8 g

10 a $\frac{1}{3}$ b $\frac{1}{8}$ c $\frac{33}{40}$ d $\frac{2}{3}$
e $\frac{5}{6}$ f $\frac{5}{12}$ g $1\frac{9}{40}$ h $1\frac{1}{6}$

11 a 80.6 or $80\frac{3}{5}$ g b $385 c $200
d 1.5 t or $1\frac{1}{2}$ t e 60 kg f 402.5 or $402\frac{1}{2}$ L
g $875 h 3.52 kg

12 700

13 75%

14 20%

15 $72

16 Maths (76%) is better than Science (71.4%).

17 Margaret (83.3%) did better than Katie (80%).

18 a Desk $15, terrarium $10 b Terrariums

19 15 kg

20 15 g

21 60 kg

22 60 kg

23 $50

24 a 500 g b 400 g c 1000 g d 3.077 kg

25 2.25 kg

Exercise 11.3

1 a $15 b $10 c $1.40 d $10
e $50

2 a $530 profit b $140 loss c $14 profit
d $10 loss e $75 profit

3 a 18.75% b 18.2% c 16.7% d 8%
e 16.9%

4 a 34.4% profit b 17.9% loss c 17.9% profit
d 22.2% loss e 51.7% profit

5 a $108 b $50.75 c $352 d $30
e $3.60

6 a $112 b $80.75 c $104.40 d $178.50
e $480

7 a $112 b $152.25 c $840 d $900
e $4800

8 a $1400 b $8.40 c $270 d $600
e $1250

9 a 280 melons b 183 melons
c $336 d $213 profit

10 63.4% profit

11 a Light bulbs 37.9%, speaker wire 45.2%
b Speaker wire

12 a $9.35, $177.65 b $0.54, $4.86
c $8.85, $79.65 d $5.63, $50.66
e $19.50, $175.50 f $24.99, $224.91

13 a $164.50 b $30.45 c $15.54
d $201.60 e $2486.33 f $341.60

14 $740

15 $8

16 a 110% b $22.05 c 20.6%
d $20 e 66.7%, 33.3%

17 $672, 12%

18 $813.45

19 a $1524.90 b 79.4%

20 20%

21 $42

22 $690

23 $155 000

24 a $408 000 b $690 000 c $348 000
d $505 000 e $770 000 f $989 000

25 $120

26 $6000

27 $4700

28 $180

9780170194778

Exercise 11.4

1 a $96 b $900 c $3600 d $720
e $75 f $375 g $896 h $1344

2 a $4080 b $7400 c $3250
d $5688 e $31 500

3 a $1040 b $200 c 47.6%

4 a $12 000 b $28 800 c $32 800
d $16 800 e 23.3%

5 28%

6 a $700 b $67.08

7 $480

8 a $2800 b $114.33 c $6302.17
d $2802.17

Chapter review

1 $3.58

2 a 60% b 35% c 15%

3 $\frac{13}{20}$, 0.65

4 168

5 $15

6 $1.30 per mango

7 450 g

8 a $265\frac{1}{2}$%
b 71.05% (rounded to 2 decimal places)
c 23.6%

9 $2\frac{9}{20}$, 2.45

10 $16.58

11 7.1%, 37.1% (rounded to 1 decimal place)

12 $49

13 $180

14 $1107

15 $10 440, $1440

16 a $0.50 b $3.65

17 $\frac{4}{15}$

18 $52.50

19 130

20 $155

21 a $43.52 b $43.50 c $46.02

22 30%

23 $56 profit

24 English

25 1 hour 45 min

26 56%

27 a 73.2%
b 11.1% loss.
Note: For 80% profit on $1.80, selling price is $3.24. They would sell hamburgers for $3.25 each. A 50% discount means selling at $1.625, since $3.25 is slightly higher than 80% profit, assume they sell at $1.60 each, or $64, a loss of $8 = 8.9%.

28 $187

29 $242

30 $5808

31 a

Expenses	
Bus fares	$7.40 × 4 = $29.60
Dinghy	$12.00 × 5 = $60.00
Amusement park	$16 × 4 = $64.00
Lunch	$28.00
Total	$181.60

b Each pays $181.60 ÷ 4 = $45.40

32 54.9 kg

33 $3500

34 $299

35 18.16%

36 $183.35

Chapter 12

Exercise 12.1

1 a Divide b Subtract (5) c Add
d Subtract e Multiply (by 6)

2 a $n + 4$ b $8 \div n$ or $\frac{8}{n}$ c $7x$
d $p + 1$ e $n - 9$ f $y - 11$

9780170194778

g $3n$ **h** $n \div 4$ or $\frac{n}{4}$ **i** $m + 12$
j $16g$

3 **a** $n + 11 = 4$ **b** $5 - n = 1$
c $12n = 130 - n$ **d** $n + 11 = 3n$
e $n - 16 = 28 - n$ **f** $n + n + 1 + n + 2 = 42$
g $n + n - 1 = 53$ **h** $\frac{4n + 6}{4} = 27$
i $n(n + 1) = 54$ **j** $\frac{n}{n - 1} = \frac{n}{4}$

4 **a** $4n - 3 = 50 - n$ **b** $\frac{n+11}{15} = 3$
c $21n - 28 = 119$ **d** $3n + n + 1 = n + 50$
e $2n - 18 = 3n$ **f** $n(n + 5) = 40 - n$
g $\frac{n}{n - 2} = \frac{9}{n}$ **h** $(n + 2)(n - 2) = 5n$
i $\frac{n}{n + 1} = \frac{3}{19} + \frac{n}{n + 5}$
j $(n + 3)(n - 7) = (n + 15)(n - 11)$

5 $2n - 300 = 500$

6 $5(n - 1) + 200 = 600$

7 $3a - 15 = 60$

8 $3(i - 40) = 57$

9 **a** $7m + 10 = 360$ **b** $5x + 90 = 180$
c $15a + 120 = 360$

10 $\frac{60p}{100} = 210$

11 $\frac{48}{m} = \frac{64}{100}$

12 $\frac{k}{120} = 0.025$

13 $\frac{69}{b} = 1 - 0.08$

14 $\frac{25 + s}{2} = 27.5$

15 $\frac{4 \times 74 + f}{5} = 78$

16 $3(a + 4) = 4a + 4$

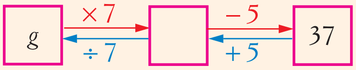
17 $c + 10 = \frac{3c + 10}{2} - 5$

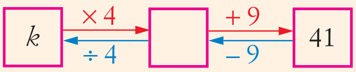
18 **a** $nm = 48$, true for some numbers and not others
b $n + 1 - n = 2(n + n + 1)$, not true for any whole numbers
c $n(n + 1) = {}^{-}6$, not true because $n(n + 1)$ is always positive or zero
d $n + n + 1$, from the pattern of $1 + 2$, $2 + 3$, $3 + 4$, . . . is always odd
e nm is sometimes odd and sometimes even
f $n(n + 1)$, from the pattern of 1×2, 2×3, 3×4, . . . is always even
g $(n + 1)(n - 1)$, from the pattern of 1×3, 2×4, 3×5, . . . is sometimes odd and sometimes even.
h $n(n - 1) = n^2 - n$, expanding brackets on the LHS shows it is always true
i $n(n + 1) - n(n - 1) = 2n$, expanding and simplifying the LHS shows it is always true
j $(n - 1)^2 = n(n - 2) + 3$, expansion and simplification shows it is never true

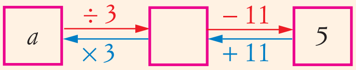
Exercise 12.2

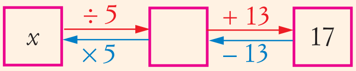
1 **a** $y = 5$ **b** $y = 11$ **c** $y = 2$
d $x = 2$ **e** $x = 4$ **f** $x = 0$
g $x = 3$ **h** $x = 2$ **i** $x = 4$

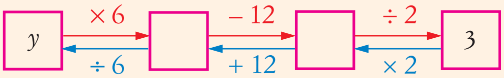
2 **a** $y = 5$ **b** $y = {}^{-}3$ **c** $y = 1$
d $x = 1$ **e** $x = 3$ **f** $x = 0$
g $x = 4$ **h** $x = 1$ **i** $2\frac{1}{2}$

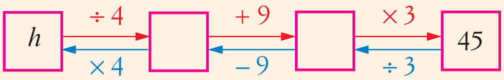
3 **a**


b


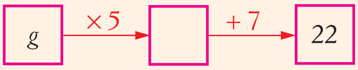
c


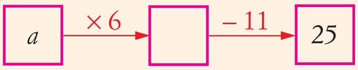
d


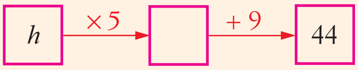
e


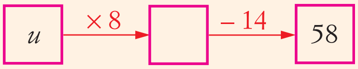
f


4 **a** $g = 6$ **b** $k = 8$ **c** $a = 48$
d $x = 20$ **e** $y = 3$ **f** $h = 24$

5 **a**


b


c


d


9780170194778

e $\boxed{r} \xrightarrow{\times 2} \boxed{} \xrightarrow{+7} \boxed{} \xrightarrow{\div 3} \boxed{5}$

f $\boxed{w} \xrightarrow{\times 3} \boxed{} \xrightarrow{-1} \boxed{} \xrightarrow{\div 5} \boxed{7}$

6 **a** $g = 3$ **b** $a = 6$ **c** $h = 7$
d $u = 9$ **e** $r = 4$ **f** $w = 12$

7 **a** $g = 4$ **b** $a = 7$ **c** $h = 24$
d $q = 7$ **e** $a = 15$ **f** $m = 3$
g $w = 8$ **h** $d = 4$ **i** $r = 7$

8 **a** $4x - 3 = 3x - 6$; $x = 3$
b $17 - x = x + 9$; $x = {}^-4$
c $4x + 14 = 5 - 5x$; $x = 1$
d $3x - 7 = x - 3$; $x = 2$
e $2x + 12 = 5x + 9$; $x = 1$
f $8 - 2x = 3x - 12$; $x = 4$

9 **a**
$$\begin{aligned} 4d + 7 &= 31 \\ 4d + 7 - 7 &= 31 - 7 \\ 4d &= 24 \\ \frac{4d}{4} &= \frac{24}{4} \\ d &= 6 \end{aligned}$$

b
$$\begin{aligned} 7a + 6 &= 41 \\ 7a + 6 - 6 &= 41 - 6 \\ 7a &= 35 \\ \frac{7a}{7} &= \frac{35}{7} \\ a &= 5 \end{aligned}$$

c
$$\begin{aligned} 3x - 5 &= 22 \\ 3x - 5 + 5 &= 22 + 5 \\ 3x &= 27 \\ \frac{3x}{3} &= \frac{27}{3} \\ x &= 9 \end{aligned}$$

d
$$\begin{aligned} \frac{w + 9}{8} &= 2 \\ \frac{w + 9}{8} \times 8 &= 2 \times 8 \\ w + 9 &= 16 \\ w + 9 - 9 &= 16 - 9 \\ w &= 7 \end{aligned}$$

e
$$\begin{aligned} \frac{b + 13}{3} &= 7 \\ \frac{b + 13}{3} \times 3 &= 7 \times 3 \\ b + 13 &= 21 \\ b + 13 - 13 &= 21 - 13 \\ b &= 8 \end{aligned}$$

f
$$\begin{aligned} \frac{m - 6}{5} &= 4 \\ \frac{m - 6}{5} \times 5 &= 4 \times 5 \\ m - 6 &= 20 \\ m - 6 + 6 &= 20 + 6 \\ m &= 26 \end{aligned}$$

10 **a** $m = 6$ **b** $h = 19$ **c** $y = 5$ **d** $j = 6$
e $k = 5$ **f** $x = 7$ **g** $m = 28$ **h** $p = 12$

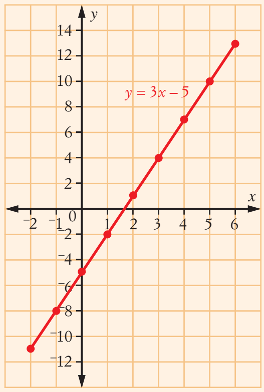
11 **a** $m = 4$ **b** $a = 8$ **c** $q = 5$
d $h = 20$ **e** $z = 9$ **f** $y = 13$
g $m = 6$ **h** $a = 8$ **i** $h = 24$
j $e = 7$ **k** $d = 84$ **l** $u = 4$

12 **a** $f = 6$ **b** $b = 3$ **c** $d = 2$
d $w = 2$ **e** $g = 10$ **f** $x = 9$
g $f = 8$ **h** $r = 9$ **i** $w = 17$
j $d = 13$ **k** $a = 4$ **l** $y = 10$

13 **a** $d = 6$ **b** $m = 9$ **c** $k = 9$
d $z = 11$ **e** $x = 9$ **f** $n = 8$
g $y = 17$ **h** $x = 10$ **i** $q = 6$

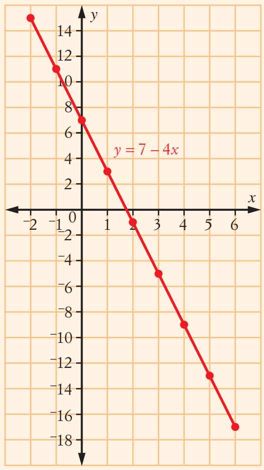
14 **a** $q = 6$ **b** $h = {}^-5$ **c** $y = {}^-6$
d $v = {}^-4$ **e** $r = {}^-8$ **f** $z = {}^-4$
g $d = {}^-9$ **h** $x = -\frac{2}{3}$ **i** $u = 7$

15



a $x = 2$ **b** $x = 5$ **c** $x = 3$ **d** $x = {}^-1$

16



a $x = 1$ **b** $x = {}^-1$ **c** $x = 3$ **d** $x = 5$

17 **a** $m = 5$ **b** $a = 4$ **c** $q = 6$
d $c = 3$ **e** $n = 8$ **f** $w = 3$
g $x = 3$ **h** $y = 5$ **i** $d = 6$
j $m = 9$ **k** $z = 5$ **l** $b = 7$
m $f = 4$ **n** $q = 6$ **o** $k = 4$

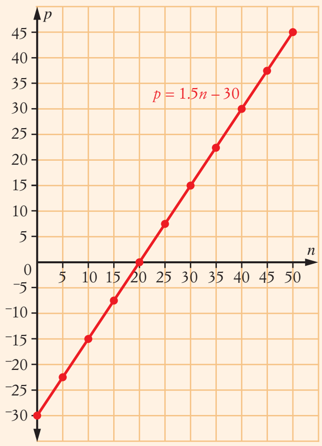
18 **a** $v = 24$ **b** $y = 16$ **c** $t = 96$
d $w = 8$ **e** $k = {}^-20$ **f** $q = 28$
g $m = 2$ **h** $v = 42$ **i** $g = 50$

j $d = 10$ **k** $d = 75$ **l** $q = 90$
m $x = 20$ **n** $v = 45$ **o** $p = 112$

19 **a** $g = {}^{-}3$ **b** $y = {}^{-}4$ **c** $a = {}^{-}6$
d $q = {}^{-}3$ **e** $t = {}^{-}7$ **f** $p = 3$
g $w = {}^{-}3$ **h** $m = {}^{-}2$ **i** $d = 4$
j $p = 8$ **k** $r = {}^{-}5$ **l** $u = {}^{-}2$
m $f = 3$ **n** $d = {}^{-}2$ **o** $w = 3$

20 **a** \$4 **b** ${}^{-}$\$300 **c** 75 tickets
d \$100 **e** \$500 **f** 125 tickets

21 **a**



b 20 burgers **c** \$15 **d** \$30 **e** \$45

22 **a** $a = 6$ **b** $m = 4$ **c** $x = 2$
d $y = 6$ **e** $f = 3$ **f** $r = 6$
g $k = 3$ **h** $w = 12$ **i** $g = 4$
j $p = 2$ **k** $r = 5$ **l** $e = 16$
m $x = 7$ **n** $y = 4$

23 **a** $d = 2$ **b** $y = 15$ **c** $w = 5$
d $a = 6$ **e** $p = 10$ **f** $m = 5$
g $g = 4$ **h** $x = 1$ **i** $d = 11$
j $k = 8$ **k** $r = 7$ **l** $e = 5$
m $b = 5$ **n** $x = 3$

24 **a** $k = {}^{-}2$ **b** $f = {}^{-}2$ **c** $b = 7$ **d** $m = {}^{-}3$
e $a = 3$ **f** $h = {}^{-}7$ **g** $m = {}^{-}2$ **h** $q = 10$

25 **a** $a = 6$ **b** $f = {}^{-}3$ **c** $x = 12$
d $y = 0$ **e** $m = {}^{-}4$ **f** $d = 8$
g $r = {}^{-}5$ **h** $n = 3$ **i** $y = 3$
j $k = {}^{-}5$ **k** $p = 3$ **l** $x = {}^{-}3$
m $y = 5$ **n** $a = {}^{-}5$

26 **a** True **b** B **c** A
d True **e** D **f** A
g B **h** A **i** True

Exercise 12.3

1 **a** $4 + 3n = 19$, $n = 5$
b $30 - 2n = 14$, $n = 8$
c $11 + 2n = 5$, $n = {}^{-}3$
d $3n + 3 = 36$, $n = 11$, so 11, 12, 13
e $4n + 3 + 6n = 143$, $n = 14$
f $6n + 5 = 8n - 11$, $n = 8$
g $4n - 7 = 9n + 73$, $n = {}^{-}16$
h $3(n - 1) = 57$, $n = 20$
i $4(n - 10) = 7n + 2$, $n = {}^{-}14$
j $6(12 - n) = 5(n - 3) - 1$, $n = 8$

2 $5p + 2 = 47$; number of balls in a packet $(p) = 9$

3 $6s + 76 = 1000$; number of bricks in a support $(s) = 154$

4 $3s = 87$; length of side $(s) = 29$ m

5 $2(19 + s) = 62$; length of side $(s) = 12$ cm

6 $b + (5 + b) = 33$; Ben's age $(b) = 14$

7 $b + 7b = 480$; Britney's money $(b) = \$60$, so Adam has \$420

8 $\frac{3p}{10} = 2400$, selling price = \$8000, buying price = \$5400

9 $b + (b + 4) = 28$; number of boys $(b) = 12$

10 $2(w + w + 5) = 62$; width $(w) = 13$ m, so length = 18 m

11 **a** $5a + 15 = 180°$, $a = 33°$
b $3b + 192° = 360°$, $b = 56°$
c $3c - 18° = 182° - c$, $c = 50°$

12 $\frac{1}{6}; \frac{t}{6} = 300$; capacity of tank $(t) = 1800$ L

13 $26 + a = 3a$; Aidan's CDs $(a) = 13$, so Bonita has 39 CDs

14 $m + m + 10 = 4m$; Maria's age $(m) = 5$, so Silvio is 15

15 $6p - 16 = 4p$; number of plants in a punnet $(p) = 8$

16 $t + 26 = 3t$; Toby's age $(t) = 13$, so his mother is 39

17 $7e + 8 = 3(e + 8)$, $e = 4$ and he was 24 when she was born.

9780170194778

18 Using $\frac{4 \times 165 + h}{5} = 167$, his height is 175 cm.
She did it as $\frac{165 + h}{5} = 167$.

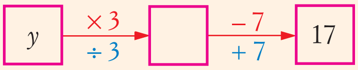
19 Using $\frac{20p}{100} = 150$, the *original* price was \$750.
Amee thought this was the price Donna paid, which was actually \$600.

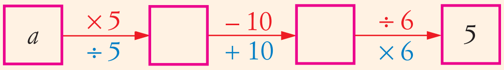
20 Using $100 - 2(n - 4) = 7n$, the number is 12. Billy made two errors. He multiplied out as $100 - 2n - 8 = 7n$ and then he obtained $5n = 92$ instead of $9n$.

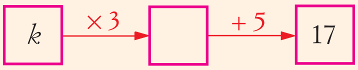
Chapter review

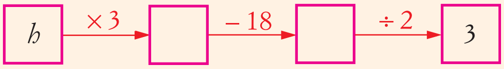
1 **a** $n + 5$ **b** $n + 1$ **c** $\frac{n}{12}$ **d** $\frac{12}{n}$
e $n - 8$ **f** $3n$ **g** $n + 6$ **h** $\frac{n}{3}$
i $5n$ **j** $6n$

2 **a** $y = 4$ **b** $y = {}^{-}5$ **c** $x = 4$
d $x = 1$ **e** $x = 2$ **f** $x = {}^{-}1$

3 **a**


b


4 **a**


b


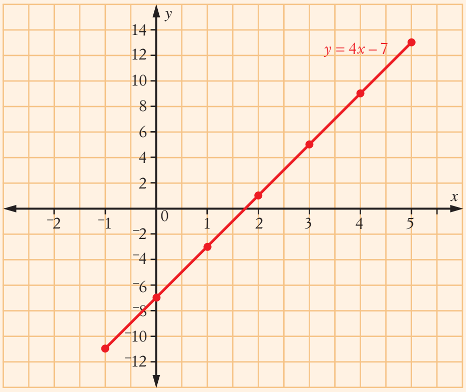
5 **a** $e = 3$ **b** $m = 11$ **c** $m = 5$

6 **a** $12 - x = 2x + 6$; $x = 2$
b $5 - 4x = 13 - 2x$; $x = {}^{-}4$

7 **a** $n + 5 = 20$ **b** $8n = 56$
c $8n - 5 = 35$ **d** $3(n - 9) = 51$
e $(n - 1)(n + 1) + 9 = 65$ **f** $\frac{5n - 3}{2} = 2(n + 1)$

8 **a** $p = 12$ **b** $n = 32$ **c** $h = 4$
d $q = 7$ **e** $z = 15$

9 **a** $g = 5$ **b** $a = 6$ **c** $d = 6$ **d** $b = 14$
e $a = 24$ **f** $d = 13$ **g** $a = {}^{-}19$

10


a $x = 4$ **b** $x = {}^{-}1$

11 **a** $g = 8$ **b** $a = 6$ **c** $r = 3$
d $k = 18$ **e** $h = 10$ **f** $w = 8$
g $k = {}^{-}5$ **h** $z = {}^{-}7$ **i** $p = {}^{-}8$
j $e = 5$

12 **a** $40 - 2y = 16$, $y = 12$
b $4(n + 1) = 5(n - 1) - 13$, $n = 22$

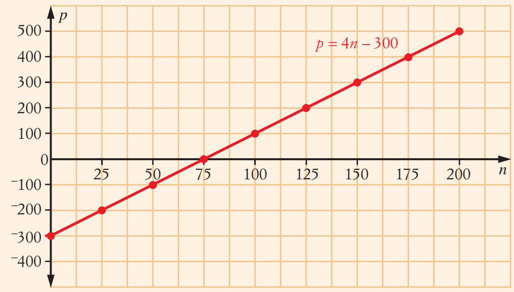
13 $3n - 30 = 24$, $n = 18$

14 **a** $10a + 20° = 360°$, $a = 34°$
b $3x + 10° = 2(100° - x)$, $x = 38°$

15 $\frac{6p}{5} = 498$, David paid \$415

16 $\frac{k}{50} = 0.06$, there are 3 kings.

17 $\frac{40 + g}{5} = 14$, she would need to score 30, so would probably have to swap to goal shooter.

18 **a**


b 75 children **c** \$100 **d** \$500

19 $3p + 15 = 141$; There are 42 smarties in a full packet.

20 **a** $y = 4$ **b** $u = 7$

21 **a** $b = 3$ **b** $w = 7$ **c** $j = 4$ **d** $p = 10$
e $d = 9$ **f** $x = {}^{-}2$ **g** $y = {}^{-}5$

22 $a + 35 = 2a + 10$, Adam is 25 and his father is 60.

23 $3x + 2(100 - x) = 260$, 60 patients have 3 meals a day

24 $s + 30 + 6 = 4(s + 6)$, Juanita is 34 and her son is 4

25 **a** $n + n + 1 = 1\ 000\ 000$ is not true because $2 \times 499\ 999 + 1 = 999\ 999$ and $2 \times 500\ 000 = 1\ 000\ 001$, so there cannot be such a sum of a number and its successor.

b $2n = \frac{n}{2} + m$, where m is even. For $n = 3$, $6 = 3 + m$, so $m = 3$; for $n = 4$, $m = 6$; for $n = 5$, $m = 7.5$, for $n = 12$, $m = 18$, so it is true for some numbers and not true for others.

c $(n + 2)^2 = n^2 + 2(n + 1)$. Expanding and simplifying brackets, LHS $= n^2 + 4n + 4$ and RHS $= n^2 + 2n + 2$, so it is never true for any natural number (solution to equation is $n = {}^{-}1$).

d $n^3 - 1 = (n - 1)(n + 1 + n^2)$; expanding the LHS gives $n^3 - 1$, so it is always true.

26 **a** B **b** C

27 The newspaper was right. Frank had calculated 'the difference between the son's age and 23' to get $12a + 6 = 3(a + 6 - 23)$ instead of $12a + 6 = 3[23 - (a + 6)]$. David had missed writing the $(a + 6)$ in brackets, using $12a + 6 = 3[23 - a + 6]$.

9780170194778

Basic scientific calculator skills

Keys

Key	Name	Use
+ − × ÷	Plus, minus, multiply, and divide	Carry out basic operations
=	Equals sign	Gives the answer
.	Decimal point	Inserts a decimal point
DEL	Delete	Deletes previous entry
ANS	Answer	Retrieves the previous answer
REPLAY	Arrow pad	Moves around the screen
SHIFT MODE 2ndF	Shift, mode, second function	Accesses other operations
(−) or +/−	Negative number	Makes a number negative
a b/c or $\frac{\square}{\square}$ or a/b	Fraction	Allows fractions to be entered
( ) or ( )	Bracket	Inserts brackets
x^2	Square	Squares a number
$\sqrt{\ }$	Square root	Finds the square root of a number
x^3	Cube	Cubes a number
$\sqrt[3]{\ }$	Cube root	Finds the cube root of a number
y^x a^b ^	*y* to the power of *x* *a* to the power of *b* Power	Working with powers
$\sqrt[n]{x}$ or $\sqrt[x]{\ }$ or $x^{1/y}$	*n*th root of *x*	Working with roots other than 2 and 3

9780170194778

Basic scientific calculator skills

Key	Name	Use
EXP or $\times 10^x$	Exponent key	To enter numbers in scientific notation
° ' " or DMS	Degrees-minutes-seconds	Working with degrees, minutes and seconds or hours, minutes and seconds
STO	Store	Stores an answer
A	Memory A	Used with STO and RCL keys to store and retrieve a number from memory
RCL	Recall	Recalls an answer
π	Pi	Inserts the value of Pi

Basic operations

Performing a calculation

Question	Calculator steps	Answer
$34 + 6 - 16$	34 + 6 − 16 =	24
$5 + 6 \times 7$	5 + 6 × 7 =	47
7.4×9.1	7.4 × 9.1 =	67.34
$234 \div 1.5$	234 ÷ 1.5 =	156
$45 \div (0.2 + 4.3)$	45 ÷ (0.2 + 4.3) =	10
$45 \div 0.2 + 4.3$	45 ÷ 0.2 + 4.3 =	229.3

9780170194778

Basic scientific calculator skills

Correcting a wrong entry

Question	Calculator steps	Answer
$34 + 6 - 16$	Wrong entry: 34 + 8 − 16 = To correct this entry, press the left side of the arrow pad REPLAY to go back to the 8, then press DEL 6 =.	26 ✗ 24 ✓

Performing calculations involving integers

Question	Calculator steps	Answer
$^-4 + 9$	(−) 4 + 9 =	5
$17 \times {}^-2$	17 × (−) 2 =	$^-34$
$18 - {}^-3 \times 4$	18 − (−) 3 × 4 =	30
$25 - (7 \times (5 - 3) + 4)$	25 − (7 × (5 − 3) + 4) =	7
$^-12 - 7 + {}^-2 - {}^-8$	(−) 12 − 7 + (−) 2 − (−) 8 =	$^-13$
$^-5 \times (5 - 7)$	(−) 5 × (5 − 7) =	10

Performing calculations involving powers or roots

Question	Calculator steps	Answer
16^2	16 x^2 =	256
9^3	9 x^3 =	729
5^4	5 a^b 4 = or 5 y^x 4 =	625
$\sqrt{196}$	$\sqrt{\ }$ 196 =	14

Question	Calculator steps	Answer
$\sqrt[3]{4913}$	$\sqrt[3]{\ }$ 4913 =	17
$\sqrt[5]{200}$	5 $\sqrt[n]{x}$ 200 =	$2.885399812 \approx 2.885$

Converting fractions and decimals

Question	Calculator steps	Answer
Simplify $\frac{16}{20}$	16 $a^b/_c$ 20 =	$\frac{4}{5}$
Change $1\frac{3}{4}$	1 $a^b/_c$ 3 $a^b/_c$ 4 = SHIFT $a^b/_c$	$\frac{7}{4}$
Change $\frac{19}{4}$ to a proper fraction	19 $a^b/_c$ 4 =	$4\frac{3}{4}$
Change 1.7 to a fraction	1.7 = $a^b/_c$	$1\frac{7}{10}$
Change $\frac{5}{6}$ to a decimal	5 ÷ 6 =	$0.8333333333 = 0.8\dot{3}$

Performing operations involving fractions

Question	Calculator steps	Answer
$\frac{1}{2}+\frac{1}{3}$	1 $a^b/_c$ 2 + 1 $a^b/_c$ 3 =	$\frac{5}{6}$
$\frac{2}{3}-\frac{1}{2}$	2 $a^b/_c$ 3 − 1 $a^b/_c$ 2 =	$\frac{1}{6}$
$\frac{2}{3}\times\frac{2}{5}$	2 $a^b/_c$ 3 × 2 $a^b/_c$ 5 =	$\frac{4}{15}$
$\frac{1}{2}\div\frac{1}{3}$	1 $a^b/_c$ 2 ÷ 1 $a^b/_c$ 3 =	$1\frac{1}{2}$
$\frac{1}{2}\div\frac{1}{3}-\frac{2}{3}\times\frac{2}{5}$	1 $a^b/_c$ 2 ÷ 1 $a^b/_c$ 3 − 2 $a^b/_c$ 3 × 2 $a^b/_c$ 5 =	$1\frac{7}{30}$

9780170194778

Basic scientific calculator skills

Question	Calculator steps	Answer
$\frac{1}{2} \div (\frac{1}{3} - \frac{2}{3}) \times \frac{2}{5}$	1 a b/c 2 ÷ (1 a b/c 3 − 2 a b/c 3) × 2 a b/c 5 =	$\frac{-3}{5}$
$9\frac{2}{3} - 3\frac{3}{4}$	9 a b/c 2 a b/c 3 − 3 a b/c 3 a b/c 4 =	$5\frac{11}{12}$
$3\frac{2}{5} \times 1\frac{2}{3}$	3 a b/c 2 a b/c 5 × 1 a b/c 2 a b/c 3 =	$5\frac{2}{3}$

Storing and recalling answers

Question	Calculator steps	Answer
Find the angle required to show 210 out of 1200 Year 7 students on a circle graph	Find degrees per student: 360 ÷ 1200 =	0.3
	Then store the answer: ANS	
	Multiply by the stored answer to find the angle: 210 × ANS =	63 (angle is 63°)

Performing calculations involving time

Question	Calculator steps	Answer
Find 7 hours 5 mins − 3 hours 24 mins	7 ° ' " 5 ° ' " − 3 ° ' " 24 ° ' " =	3° 41′ = 3 h 41 mins

Converting units

Question	Calculator steps	Answer
Convert 23 km/h to m/s	23 × 1000 ÷ 60 ÷ 60 =	6.388888889 ≈ 6.4 m/s
Convert 23° 16′ 30″ to decimal degrees	23 ° ' " 16 ° ' " 30 ° ' " = SHIFT ° ' "	23.275°

9780170194778

Basic scientific calculator skills

Performing calculations involving π

Question	Calculator steps	Answer
Find the area of a circle with $r = 20$ ($A = \pi r^2$)	$\boxed{\pi}$ $\boxed{\times}$ 20 $\boxed{x^2}$ $\boxed{=}$	$1256.637061 \approx 1257$

Performing calculations involving percentages

Question	Calculator steps	Answer
Change $\frac{15}{37}$ to a percentage	15 $\boxed{\div}$ 37 $\boxed{\times}$ 100 $\boxed{=}$	40.54054054 $\approx 40.54\%$
Find 36% of $380	36 $\boxed{\div}$ 100 $\boxed{\times}$ 380 $\boxed{=}$	136.8

Performing calculations involving numbers in scientific notation

Question	Calculator steps	Answer
$5.6 \times 10^6 \times 6.3 \times 10^8$	5.6 $\boxed{\text{EXP}}$ 6 $\boxed{\times}$ 6.3 $\boxed{\text{EXP}}$ 8 $\boxed{=}$ OR 5.6 $\boxed{\times 10^x}$ 6 $\boxed{\times}$ 6.3 $\boxed{\times 10^x}$ 8 $\boxed{=}$	3.528^{15} or 3.528E15 $= 3.528 \times 10^{15}$

9780170194778

Word	Definition
acute (angle)	Angle between 0° and 90°.
allied (angles)	Inside angles on the same side of a transversal over two lines. *See also* co-interior angles.
alternate (angles)	Inside opposite (or outside opposite) angles on a transversal over two lines.
area	The amount of a surface enclosed by the boundary of a shape.
axis of symmetry	A line that divides a shape into halves like reflections in a mirror.
axis	A vertical or horizontal line with markings showing values; plural axes.
back-to-back	Display for the data of two groups, one on the left and the other on the right.
backtrack	Undo operations of an equation to find the solution.
base (power)	The bottom number of a power, such as the 5 in 5^4.
base (shape)	The 'bottom' of a prism or pyramid.
bias	Data or method that is likely to give results generally different to the population. *See also* fair.
bimodal	Having two modes; two scores with equal highest frequency.
binomial products, binomial brackets	Product of two brackets, such as $(2x + 3)(4x - 1)$.
Cartesian plane	A flat surface with horizontal and vertical axes intersecting at 0.
centre of rotation	The point around which a rotation occurs.
centre of symmetry	The centre of a shape with point symmetry; it rotates about this point.
certain	Will always happen.
circumference	The boundary of a circle; the length of this boundary; the perimeter of a circle.
class	A group of data such as 130–139 cm. The class width is the distance between the starts of successive classes.
closed (2D shape)	Flat shape with a continuous boundary that must be crossed to get from the inside to the outside.
co-interior (angles)	Inside angles on the same side of a transversal over two lines. *See also* allied angles.
coefficient	Number in front of a variable; number that is multiplied by a variable.
collect (like terms)	Add or subtract like terms to simplify an expression.
commission	Money paid for a service such as selling goods.
common factor	Number or algebraic term that is a factor of each of some given numbers or terms.

9780170194778

Word	Definition
common fraction	Fraction shown with a denominator and numerator; such as $\frac{5}{8}$. *See also* vulgar fraction.
commutative (law)	Addition (or multiplication) of two numbers is the same for either number order.
complementary (angles)	Adjacent angles that together form a right angle (90°).
complementary (events)	Two separate events that together make the entire sample space.
concave (shape)	Shape with at least one internal angle greater than 180°; an inward-pointing vertex.
cone	A shape equivalent to a pyramid with a circular base.
congruent (shapes), congruence	Shapes that are the same. They may be exactly superimposed, possibly by turning one over.
consecutive	Immediately after each other : 5, 6 and 7 are consecutive numbers.
constant	A number, particularly a term of an expression that is just a number.
convex (shape)	Shape with internal angles all less than 180°; vertices all point outwards.
coordinates	The numbers showing position in a Cartesian plane; e.g. (3, 5).
corresponding (angles)	Angles on the same side of a transversal and the same sides of the lines it crosses.
cost price	The amount paid by someone who intends to sell an item.
cross-multiplication, cross-product	Multiply the first part of each ratio by the second part of the other to determine equivalence.
cross-section	The 2D shape made by cutting through a solid (3D) shape.
cube (number)	The third power of a number, such as 5^3.
cube root	The number whose cube is the original number, such as $\sqrt[3]{216} = 6$.
cumulative frequency	The progressive total of frequencies of ordered numeric data.
data	Information collected or used for analysis or reference; singular datum.
degree	Angle measure such that a complete revolution is 360 degrees, written as 360°.
denominator	The number at the bottom of a fraction; the number of equal parts into which the whole is divided.
dependent (variable)	The variable of the output of a rule or function; its value is calculated from the input(s).
diameter	The distance from one side of a circle to the other through the centre; double the radius.
difference	The result of subtraction; the difference of 3 and 7 is $^{-}4$.
directed number	Any of zero, positive or negative whole numbers. *See also* integer.

9780170194778

Word	Definition
discount	An amount taken off the usual price of an article.
distributive (law)	The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers; $a(b \pm c) = ab \pm ac$.
dodecahedron	Regular polyhedron with twelve regular pentagon faces.
element	A basic probability outcome; '5' on a die. *See also* simple outcome, sample point.
equilateral (triangle)	A triangle with all sides (and angles) the same.
equivalent fractions	Fractions with different denominators that have the same value, for example, $\frac{4}{6}$ and $\frac{10}{15}$.
evaluate	Calculate the answer to an expression.
event (probability)	An individual outcome or collection of outcomes.
event space	List of all the simple outcomes of a situation. *See also* sample space.
expand	Multiply out brackets using the distributive law.
expected frequency	The number of times an event is expected from a particular number of trials.
experimental probability	The frequency of an outcome divided by the total frequency. *See also* relative frequency.
exponent	The raised number of a power, such as the 4 in 5^4. *See also* index.
expression	Variables and/or numbers connected by operations.
exterior (angle of a triangle)	Angle on the outside of a triangle made by extending one side.
face	Flat or curved surface that forms part of the boundary of a solid (3D) shape.
factor (of a number)	Whole number that divides evenly into a given number; 5 is a factor of 20. *See also* multiple.
factorise	Use the distributive law to change an expression into factors, include a bracketed factor.
fair	A sample/question/method that does not favour particular results and will be truly representative of the population. *See also* bias.
favourable outcome	The particular outcome that you are looking for.
formal solution	Show the steps for the solution of an equation using inverse operations.
formula	An equation that gives one quantity in terms of others. The quantity on its own is called the subject.
frequency distribution	The frequencies for all the scores of some data.
frequency	The number of times a particular value/outcome occurs.

Word	Definition
function	A rule that changes a number (input) to a new one (output).
Goods and Services Tax (GST)	Tax paid on the selling price of goods or service; currently 10% in Australia.
gradient	The slope of a line obtained by dividing the vertical distance between two points (rise) by the horizontal (run) distance.
hectare	The area of a square 100 m by 100 m; 10 000 m^2.
highest common factor (HCF)	The biggest number that is a factor of all given numbers.
histogram	A column graph of frequencies shown without gaps between columns.
icosahedron	Regular polyhedron with twenty equilateral triangle faces.
impossible	Will never happen.
improper fraction	A fraction more than 1; the numerator is bigger than the denominator.
independent (variable)	Variable used to find the value of a rule or function.
index	The raised number of a power, such as the 4 in 5^4; plural *indices*. *See also* exponent.
inspection (solving by)	Solving an equation by (mentally) substituting values for the variable.
instalment	Equal payments used to repay a loan at fixed intervals, such as monthly instalments.
integer	Any of zero, positive or negative whole numbers. *See also* directed number.
intercept	Position where a line intersects an axis.
interest rate	Interest stated as a yearly percentage of the amount borrowed.
interest	Charge made for the use of money (obtained as a loan). The amount of the loan is called the principal.
inverse operations	Operations that reverse each other; × and ÷, etc.
irrational (number)	A number that is not rational. It cannot be expressed as a fraction, terminating or recurring decimal.
isosceles (triangle)	Triangle with two equal sides.
kite	Quadrilateral with two pairs of equal sides, each pair joined at one vertex.
like terms	Terms of an expression containing exactly the same variable parts.
line graph	A graph where points joined by line(s) show data.
line symmetry	Property of a shape with one or more lines of symmetry.
linear (rule, function)	A rule or function whose graph is a straight line.

9780170194778

Word	Definition
loss	Difference between the cost price and selling price of an article when the selling price is lower than the cost price.
lowest common denominator (LCD)	The smallest number that is a denominator of all given fractions.
lowest common multiple (LCM)	The smallest number that is a multiple of all given numbers.
mantissa	The value part of a number in scientific notation: The mantissa of 3.52×10^4 is '3.52'.
mean	The arithmetic average, $\bar{x}$ = total ÷ number; for 3, 6, 12, $\bar{x} = 21 \div 3 = 7$.
median	The central value (by count) of ordered statistical data.
metric system	The system of measurement based on powers of 10 that was first devised in France around 1790.
midpoint	The halfway point between two given points.
mixed number, mixed fraction	A number with whole and fraction parts; such as $3\frac{1}{4}$.
mode	The value that occurs most often in statistical data; the score with the highest frequency.
multiple (of a number)	A whole number that divides evenly by a given number; 24 is a multiple of 8. *See also* factor.
negative (number)	Number less than 0; for example, $^-6$ or $^-23.75$.
net	A flat (2D) shape that can be folded or rolled to form a 3D shape.
non-compliant (data)	Statistical data that is obviously wrong.
numerator	The number at the top of a fraction; the number of equal parts in the fraction.
obtuse (angle)	Angle between 90° and 180°.
octahedron	Regular polyhedron with eight equilateral triangle faces.
order (of symmetry)	For point symmetry, the number of different rotations that make the identical shape.
outcomes	The possible results in a trial.
outlier	A data value well outside the range of a majority of numerical data.
parallel	Lines or surfaces in the same direction, which do not intersect.
parallelogram	A quadrilateral whose opposite sides are parallel.
percentage	A number written as a fraction out of 100, with the vinculum and 100 replaced by %; such as $\frac{2}{5} = \frac{40}{100} = 40\%$.
perimeter	The length of the boundary of a (flat) shape.
perpendicular	A line through another line or surface at a angle of 90°.

Word	Definition
point symmetry	The property that a shape can be rotated less than 360° to make the same shape. *See also* rotational symmetry.
polygon	Flat (2D) closed shape with straight sides.
polyhedron	Solid shape with faces that are polygons; plural *polyhedra* or *polyhedrons*.
population	The entire group that could be relevant to a survey.
positive (number)	Number more than 0, sometimes shown with a + sign; $^{+}7$, $^{+}42$, 5.63.
power	A number multiplied by itself, written as 5^4, say, for $5 \times 5 \times 5 \times 5$.
predecessor	The number immediately before another: the predecessor of 5 is 4.
principal	The initial amount of a loan.
prism	Polyhedron where every cross-section parallel to one face is identical to that face. *See also* base.
probability	The chance of an event; Experimental or theoretical probability.
product	The result of multiplying two or more numbers.
profit	The difference between selling price and buying price when the selling price is greater.
proper fraction	A fraction less than 1; the denominator is bigger than the numerator.
pyramid	Polyhedron where cross-sections parallel to one face are similar and reduce evenly in size to a vertex called the apex opposite this face.
quadrant	One of the four sections of a Cartesian plane bounded by the axes.
quadrilateral	Flat (2D) closed shape with four straight sides.
quotient	The result of dividing one number by another: the quotient of 2 and 8 is 0.25.
radius	A line from the centre to the circumference of a circle; the length of this line.
range	The difference between the highest and lowest scores in a set of numeric data.
rate	The result of dividing one quantity by a different related quantity; $2.40/kg.
ratio	Comparison of quantities of the same kind; 7 : 2.
rational number	Number that can be expressed as a fraction, terminating or recurring decimal.
reciprocal	The result of turning a fraction upside down; its product with the fraction is 1.
recurring decimal	A decimal with a pattern of digits that repeats endlessly. *See also* repeating decimal.
reflex (angle)	Angle between 180° and 360°.

9780170194778

Word	Definition
regular (polygon, polyhedron)	Polygon (or polyhedron) with identical sides and angles (faces and edges).
repeating decimal	A decimal with a pattern of digits that repeats endlessly. *See also* recurring decimal.
retainer	When working on commission, an amount paid without consideration of sales.
revolution	The angle made by rotating a line through a complete circle; 360°.
rhombus	A quadrilateral with all sides identical.
right (angle)	Angle of 90°.
root (1)	A square root, cube root, fourth root, etc.
root (2)	The solution of an equation.
rotational symmetry	The property that a shape can be rotated less than 360° to make the same shape. *See also* point symmetry.
round	Find an approximation with only some place values, such as $5.87 \approx 5.9$ or $7245 \approx 7000$.
sample point	A basic probability outcome; '5' on a die. *See also* element, simple outcome.
sample space	List of all the simple outcomes of a situation. *See also* event space.
sample	Part of a population (usually used for a survey).
satisfy	Make true; (3, 7) satisfies $y = 2x + 1$.
scale factor	The ratio of the sizes of an enlargement or reduction of a shape.
scalene (triangle)	A triangle with no sides the same.
scientific notation	Notation used to write a value as a number between 1 and 10, multiplied by a power of 10.
significant figures	The (number) of digits in the mantissa when a number is written in scientific notation.
similar	Shapes that are enlargements or reductions of each other.
similarity	Property of shapes that are enlargements or reductions of each other.
simple interest	Interest charged as a percentage of the initial loan, multiplied by the period of the loan.
simple outcome	A basic probability outcome; '5' on a die. *See also* element, sample point.
simplest form (fraction)	The equivalent form with the lowest possible denominator; $\frac{10}{15} = \frac{2}{3}$.
simplify	Multiply out brackets and/or collect like terms.
solution	The answer to a problem, particularly an equation.
solve	Find the answer to a problem, particularly an equation.
spread	Measure of the differences between the values of a set of data.

Word	Definition
square (number)	The product of a number with itself.
square root	The number whose square is the original number, such as $\sqrt{25} = 5$.
statistical (data)	Data that is counted to obtain information.
stem-and-leaf plot	Display with high place values on a vertical 'stem' and the other place values on horizontal 'leaves'.
straight angle	An angle of 180°.
subject	The variable on its own (on the left) that is calculated from others in a formula.
substitution	Calculation of the value of an expression for a particular value(s) of the variable(s).
successor	The number immediately after another: the successor of 5 is 6.
sum	The result of adding two or more numbers.
superimpose	Place a shape exactly over another.
supplementary (angles)	Adjacent angles that form a straight line; their sum is 180°.
surd (symbol)	The symbol $\sqrt{\ }$ used to show a root (1).
surface area	The area of a surface, particularly the total area of the faces of a solid shape.
term (number patterns)	One of the values in a number pattern; 6 is the second term of 3, 6, 9, 12, 15, . . .
term (equations and expressions)	Part of an expression separated from other parts by addition or subtraction.
terminating decimal	Decimal with a finite number of digits after the decimal point; it ends.
tessellation	A tiling pattern that uses congruent shapes to cover an area.
tetrahedron	Regular polyhedron with four identical faces that are equilateral triangles.
theoretical probability	The number of simple outcomes in a probability event divided by the total number of simple outcomes.
time line	A line with times and events placed along it.
time zone	A region of the Earth where all the clocks are set to the same time.
transformation	Change in a shape: a rotation, reflection, enlargement, reduction or translation.
translation	Sliding movement of a shape; all points slide the same amount in the same direction.
transversal	A line that crosses two or more others.
trapezium	A quadrilateral with one pair of opposite sides parallel.

9780170194778

Word	Definition
travel graph	A graph showing distance travelled during a trip. The time is shown horizontally.
two-way table	A table that shows one set of outcomes horizontally and another set vertically.
unitary method	Method using the value of a unit quantity to calculate a result.
unknown	A symbol, particularly a letter, which stands for a number. *See also* pronumeral, variable.
variable	A symbol, particularly a letter, which stands for a number. *See also* pronumeral, unknown.
variation	Differences in outcomes as a result of using different samples in probability or statistics.
Venn diagram	A diagram that shows the elements of events (or sets) in overlapping circles within a rectangle.
vertex	The intersection of two sides of a polygon or three or more faces of a solid shape; plural vertices.
vertically opposite (angles)	Opposite angles at the intersection of two lines; they are not adjacent.
vinculum	The line between the top and bottom of a fraction.
vulgar fraction	Fraction shown with a denominator and numerator; such as $\frac{5}{8}$. *See also* common fraction.
***x*-axis**	The horizontal axis of a Cartesian plane.
***x*-coordinate**	The first coordinate of a point in the Cartesian plane; the 4 in (4, 3).
***y*-axis**	The vertical axis of a Cartesian plane.
***y*-coordinate**	The second coordinate of a point in the Cartesian plane; the 3 in (4, 3).

Index

9780170194778

9780170194778

Index

9780170194778

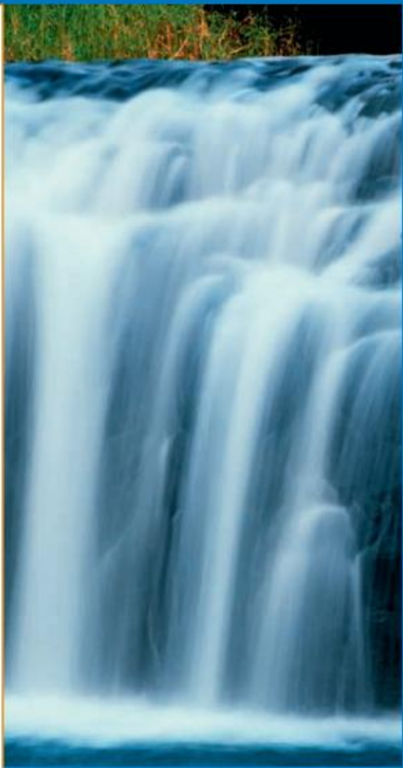
9780170194778

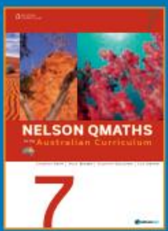
NELSON QMATHS 8
for the **Australian Curriculum**

Nelson QMaths is an innovative **series** written by an experienced team of authors to address the Australian Mathematics Curriculum Years 7–10. It includes two Year 10 pathways for students.

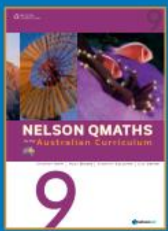
For each year level, the **suite of resources** includes:

- A student book that focuses explicitly on developing the four **proficiencies** (understanding, fluency, problem solving and reasoning), and provides numerous **graduated examples** and **exercises** that cater for the range of abilities in most classrooms.
- An exciting web-based, interactive **NelsonNetBook** that students and teachers can **personalise** and **annotate**, with additional tools for teachers to share their annotations with particular groups of students.
- A **NelsonNet student website** that provides an extensive collection of resources, including worksheets, puzzle sheets, quizzes, video tutorials, spreadsheet tasks, calculator exercises, animated examples and Geogebra activities.
- A **NelsonNet teacher website** that includes secure topic tests, prior learning checks, curriculum grids and guides, teaching plans, as well as access to the free ExamView question bank software.
- A printed, full colour **Teacher's Edition** of the student book, with page-by-page wraparound commentary providing useful advice and practical ideas on implementing the new curriculum, nurturing the **general capabilities** and integrating the **cross-curriculum priorities.**

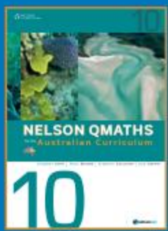




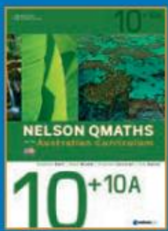
Nelson QMaths 7
Student Book
9780170194730



Nelson QMaths 9
Student Book
9780170194815



Nelson QMaths 10
Student Book
9780170194853



Nelson QMaths 10+10A
Student Book
9780170194860

ISBN: 978-0170194778
9 780170 194778

NELSON
CENGAGE Learning
For learning solutions, visit **cengage.com.au**